Checkpoints Chapter 6 CIRCULAR MOTION

Question 1

In uniform circular motion, there is a net force acting radially inwards. This net force causes the velocity to change (in direction). Since the speed is constant, the KE (½mv^2) is also constant.

∴ B, D (ANS)

The answers in the book do not give D.

Question 2

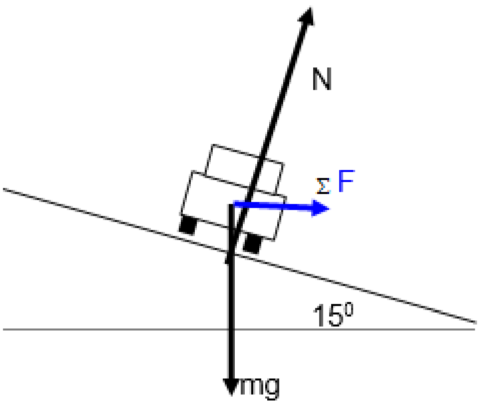
$$\Sigma F = \frac{mv^2}{r}$$
$$= R - mg$$
$$\therefore R = mg + \frac{mv^2}{r}$$

The reaction force, $R = mg + \frac{mv^2}{r}$.

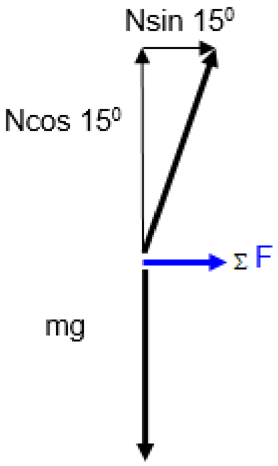
∴ R > mg, so Jim 'feels' heavier.

∴ C (ANS)

Question 3



Since the car is moving in a horizontal circle, the net force must be acting radially inwards (horizontally).
For this you can't resolve the weight force, because it is perpendicular to the net force.
You have to resolve the normal reaction.



$$\Sigma F = N\sin 15^0$$

∴ D (ANS)

Question 4

$$N\cos 15 + mg = 0$$
$$|N\cos 15| = |mg|$$

∴ C (ANS)

Question 5 (2013 Q5a, 1m, 68%)

Since the mass is travelling at a constant speed, the net force is radially inwards.

∴ D (ANS)

Question 6

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$
$$= \frac{2\pi r}{T}$$
$$= \frac{2 \times \pi \times 10^8}{1 \times 10^5}$$

= 6,283 m s^{-1} (ANS)

∴ C (ANS)

Question 7

The net force is calculated from

$$F = m\frac{v^2}{r}$$

Be careful with your substitutions; the mass is 1 × 10^5, radius is 1 × 10^8 m

$$F = 1 \times 10^5 \times \frac{6283^2}{1 \times 10^8}$$

∴

∴ The magnitude of the net force is
F = 39,476 N

∴ B (ANS)

Question 8

There are two forces acting, the weight (down) and the normal reaction. At the top of the hill, the net force is down, as the rollercoaster car is undergoing circular motion, therefore mg > N. When the carriage is moving at the bottom, (in the valleys), undergoing circular motion, the net force is up, therefore N > mg.

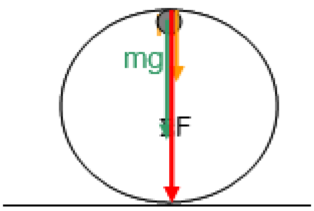
∴ **D (ANS)**

Question 9

Centripetal forces act radially inwards.

∴ **D (ANS)**

Question 10



At the point D,

$$\Sigma F = \frac{mv^2}{r} = mg + N$$

$$\therefore N = \frac{mv^2}{r} - mg$$

∴ **B (ANS)**

Question 11a

For all circular motion the net force is always radially inwards. ∴The force is always towards Griselda.

Question 11b

$$\Sigma F = \frac{mv^2}{r}$$

$$\therefore v = \sqrt{\frac{720 \times 1.2}{4}}$$

∴ v = **14.7 m s^{-1}** **(ANS)**

Question 11c

As soon as the force ceases to be applied, the hammer will continue in the direction it was travelling. The hammer will leave tangentially to the circle it was moving in.

Question 12a

Use $\Sigma F = \frac{mv^2}{r} = mg + N$

Where N = 0, for the minimum speed possible.

$$\therefore N = \frac{mv^2}{r} - mg$$

$$\therefore 0 = \frac{mv^2}{r} - mg$$

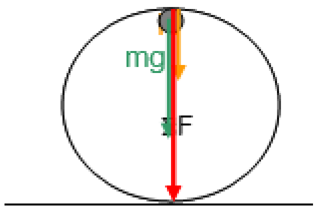
$$\therefore \frac{v^2}{r} = g$$

$$\therefore \frac{v^2}{0.25} = 9.8$$

$$\therefore v^2 = 9.8 \times 0.25$$

∴ **v = 1.57 m s^{-1}** **(ANS)**

Question 12b



$$\Sigma F = \frac{mv^2}{r} = mg + N$$

$$\therefore N = \frac{mv^2}{r} - mg$$

$$= \frac{0.05 \times 2^2}{0.25} - 0.05 \times 10$$

= 0.3 N **(ANS)**

Question 13

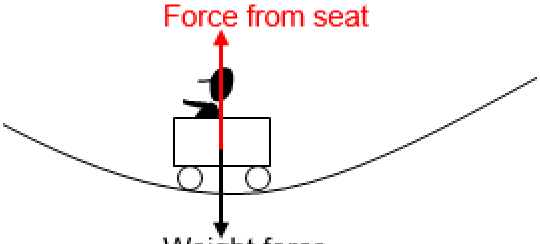
$$\Sigma F = \frac{mv^2}{r}$$

$$\therefore 8000 = \frac{400 \times v^2}{5}$$

$$\therefore v^2 = \frac{5 \times 8000}{400} = 100$$

∴ **v = 10 m s^{-1}** **(ANS)**

Question 14a



Because the carriage is moving, it is undergoing circular motion at this point, so the force from the seat must be greater than the weight force,

$\therefore$ the net force is acting up.

Question 14b

$$\Sigma a = \frac{v^2}{r}$$

$$= \frac{10^2}{20}$$

$= \mathbf{5\ m\ s^{-2}}$ **(ANS)**

Question 14c

The reaction force, $R = mg + \frac{mv^2}{r}$.

$\therefore R = 65 \times 9.8 + \frac{65 \times 10^2}{20}$

$= 637 + 325$

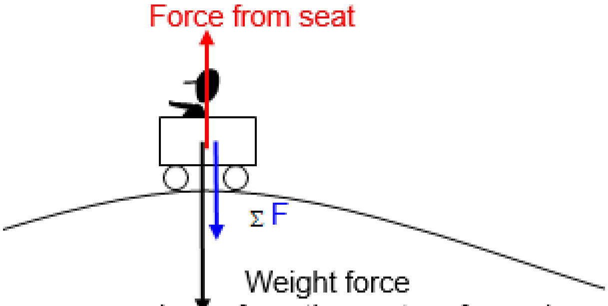
$= 962\text{N}$

$\therefore$ Jim will feel $\frac{962}{637} = 1.5$ times as heavy

$\therefore$ Jim will feel 50% heavier

Question 14d

At the top of a hill the force diagram looks like this:



$$\Sigma F = \frac{mv^2}{r} = mg - R$$

$\therefore R = mg - \frac{mv^2}{r}$

$\therefore$ R is less than mg, so Jim 'feels' lighter.

Question 15a

Use $a = \frac{v^2}{r}$

$= \frac{10^2}{10}$

$= \mathbf{10\ m\ s^{-2}}$ **down** **(ANS)**

Question 15b

36 km hr^{-1} = 10 m s^{-1}

$$\Sigma F = \frac{mv^2}{r}$$

$= mg - R$

$\therefore R = mg - \frac{mv^2}{r}$

$\therefore R = 10m - \frac{m \times 10^2}{10}$

$\therefore R = 10m - 10m$

$\therefore R = 0$

This means that since the road is not pushing up on the truck, then the truck is not pushing down on the road. Therefore there isn't a contact force between the truck and the road, so the truck is not in contact with the road.

Question 16a

The loss in PE must equal the gain in KE.

$\therefore \Delta mgh = \Delta KE$

$\frac{1}{2} mv^2 = 250 \times 9.8 \times 22$

$\therefore v^2 = 2 \times 250 \times 9.8 \times 22 \div 250$

$\therefore v^2 = 431.2$

$\therefore v = 20.76$

$\therefore \mathbf{v = 21\ m\ s^{-1}}$ **(ANS)**

Question 16b

$KE_D = KE_C - mgh$

$= 250 \times 9.8 \times 22 - 250 \times 9.8 \times 16$

$= 53\,900 - 39\,200$

$= 14\,700$ J

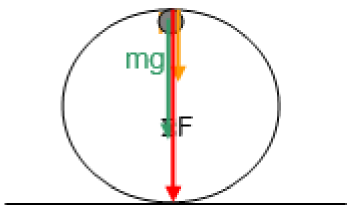
$\therefore \frac{1}{2} mv^2 = 14\,700$

$\therefore v^2 = 2 \times 14\,700 \div 250$

$\therefore v^2 = 117.6$

$\therefore \mathbf{v = 10.8\ m\ s^{-1}}$ **(ANS)**

Question 16c



$$\Sigma F = \frac{mv^2}{r} = mg + N$$

$$\therefore N = \frac{mv^2}{r} - mg$$

$$= \frac{250 \times 10.8^2}{8} - 250 \times 9.8$$

= 1 225 N (ANS)

Question 16d

With no other forces, the combined weight of the car and passengers is W = mg = 2450. The normal reaction at the top of the loop is 50% of this value, hence the passengers will feel a **50% reduction** in their weight.

Question 17a

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$= \frac{2\pi r}{T}$$

$$= \frac{2 \times \pi \times 10}{20}$$

= 3.14 m s^{-1} (ANS)

Question 17b

$$\Sigma F = \frac{mv^2}{r}$$

$$= \frac{60 \times 3.14^2}{10}$$

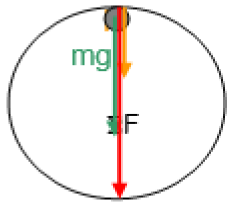
∴ 59 N (ANS)

Question 17c

At the top of the wheel

$$\Sigma F = \frac{mv^2}{r} = mg + N$$

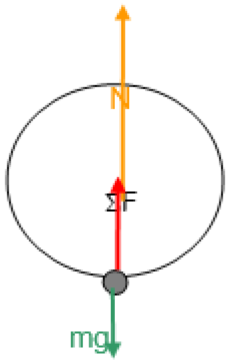
$$\therefore N = \frac{mv^2}{r} - mg$$



At the bottom of the wheel

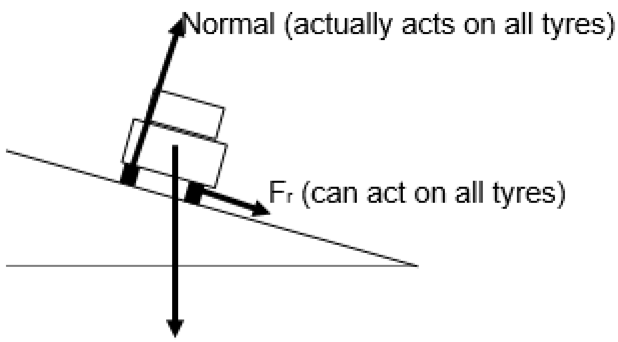
$$\Sigma F = \frac{mv^2}{r} = N - mg$$

$$\therefore N = \frac{mv^2}{r} + mg$$



So at the top, because $N = \frac{mv^2}{r} - mg$, you actually feel lighter and at the bottom $N = \frac{mv^2}{r} + mg$, makes you feel heavier.

Question 18a



Question 18b

$$\Sigma F = N\sin 15^0$$

$$\Sigma F = \frac{mv^2}{r} = N \sin 15^0$$

$$N\cos 15 + mg = 0$$

$$\therefore N = \frac{mg}{\cos 15^0}$$

$$\frac{mv^2}{r} = \frac{mg\sin15^0}{\cos15^0}$$

$$\therefore \frac{v^2}{r} = g\tan15^0$$

$$\therefore v = \sqrt{gr\tan15^0}$$

$$\therefore v = \sqrt{10 \times 167 \times \tan15^0}$$

∴ v = 21 m s^{-1} (ANS)

Question 19a

Line C is acting vertically downwards from the centre of mass,
∴ it is the gravity force on the weight.
Line A is acting perpendicular to the road surface, if it was shown acting from the road, not the centre of mass, then it would be the normal reaction force of road on car.
As the car is travelling faster than the recommended speed it is going to need some extra assistance (from the road) in getting a large enough force acting radially inwards. The road is going to need to supply this extra force. Line B represents the force from the road. The component of this force acting horizontal and radially inwards will assist the car to complete the corner.

Question 19b

Line C is acting vertically downwards from the centre of mass,
∴ it is the gravity force on the weight.
Line A is acting perpendicular to the road surface, if it was shown acting from the road, not the centre of mass, then it would be the normal reaction force of road on car.
As the car is travelling slower than the recommended speed it is going to exert a force on the road that is outward as in line D

Question 20a

The net force is calculated from

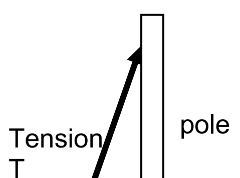
$$F = m\frac{v^2}{r}$$

Be careful with your substitutions; the mass of 100 g is 0.100 kg and the radius of 80 cm is 0.80 m

$$\therefore F = 0.100 \times \frac{5^2}{0.80}$$

∴ F = 3.1 N (ANS)

Question 20b



Question 20c

The net force acting radially inwards is 3.125N.
∴ the horizontal component of the tension Tsinθ gives:

$\therefore T\sin\theta = 3.125$

and the vertical component of the tension Tcosθ gives:

$T\cos\theta = mg$

$T\cos\theta = 0.100 \times 10$

$= 1$

Square both equations

$\therefore T^2\sin^2\theta + T^2\cos^2\theta = 3.125^2 + 1^2$

$\therefore T^2(\sin^2\theta + \cos^2\theta) = 10.7656$

$\therefore T^2 = 10.7656$

∴ T = 3.3 N (ANS)

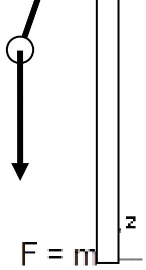
Question 20d

Use $T\cos\theta = 1$

$\therefore 3.28(11)\cos\theta = 1$

$$\therefore \cos\theta = \frac{1}{3.2811}$$

∴ θ = 72^0 (ANS)

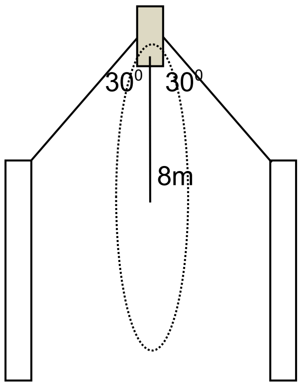


Question 21a

At the top of the ride, $\Sigma F = m\frac{v^2}{r}$

$\therefore \Sigma F = 250 \times 10^2 \div 8$

$\mathbf{= 3.1 \times 10^3\ N}$ **(ANS)**



Question 21b

The actual forces applied to the cage are the tensions from each cable and the weight.

$\therefore$ 3 125 = 2Tcos30⁰ + mg

$\therefore$ 3 125 = 1.732T + 2 450

$\therefore$ T = (3 125 – 2 450) ÷ 1.732

$\therefore$ T = 389.7

$\therefore$ T is close to 390 N

Question 21c

The total energy of the system will remain constant.

At the top Total Energy

= $KE_{top} + GPE_{top}$

= $½mu^2 + mgh_{top}$

where u is the speed at the top

At the bottom, Total Energy

= $KE_{bottom} + GPE_{bottom}$

= $½mv^2 + mgh_{bottom}$

where v is the speed at the bottom

$\therefore ½mu^2 + mgh_{top} = ½mv^2 + mgh_{bottom}$

$\therefore mgh_{top} - mgh_{bottom} = ½mv^2 - ½mu^2$

$\therefore mg(h_{top} - h_{bottom}) = ½ \times 250 \times v^2 – ½ \times 250 \times 10^2$

$\therefore 250 \times 9.8\ (16) = 125v^2 – 12\,500$

(where the distance between the top and the bottom is the diameter of 16 m)

$\therefore 39\,200 + 12\,500 = 125v^2$

$\therefore v^2 = 413.6$

$\therefore v = 20.34$

$\therefore$ **v = 20.3 m s⁻¹** **(ANS)**

Question 21d

At the bottom of the ride, $\Sigma F = m\frac{v^2}{r}$

$\therefore \Sigma F = 250 \times 20.3^2 \div 8$

= 12 878

The actual forces applied to the cage are the tensions from each cable and the force of gravity.

$\therefore$ 12 878 = 2Tcos30⁰ – mg

(Assuming the positive direction is up)

$\therefore$ 12 878 = 1.732T – 2 450

$\therefore$ T = (12 878 + 2 500) ÷ 1.732

$\therefore$ T = 8 850 N

Therefore if the speed at the bottom of the ride is 20.3 m s⁻¹, then the tension in the cable at the bottom of the ride is greater than the tension in the cable at the top of the ride (390 N from before).

Question 21e

At the bottom of the ride, $\Sigma F = m\frac{v^2}{r}$

$\therefore \Sigma F = 250 \times 8^2 \div 8$

= 2 000

The actual forces applied to the cage are the tensions from each cable and the force of gravity.

$\therefore$ 2 000 = 2Tcos30⁰ – mg

(Assuming the positive direction is up)

$\therefore$ 2 000 = 1.732T – 2 450

$\therefore$ T = (2 000 + 2 450) ÷ 1.732

$\therefore$ **T = 2 569 N (ANS)**

Question 22a (2012 Q7a, 2m, 80%)

Use ΣF = ma

$\therefore T = \frac{mv^2}{r}$

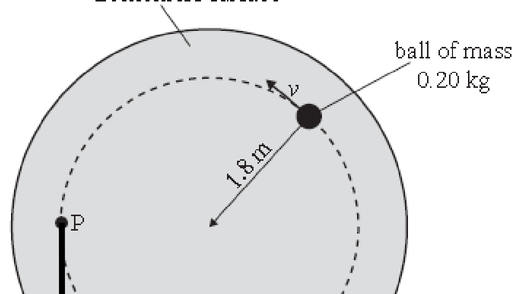
$\therefore 4.0 = \frac{0.20 \times v^2}{1.8}$

$\therefore v^2 = \frac{4.0 \times 1.8}{0.2}$

$\therefore$ **v = 6 m s⁻¹** **(ANS)**

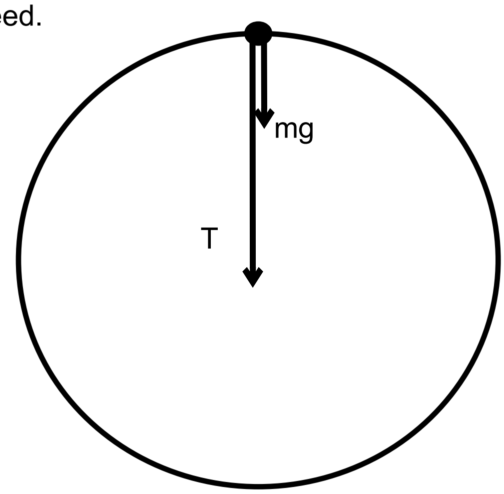
Question 22b (2012 Q7b, 1m, 75%)

The sphere will fly off tangentially.



Question 22c (2012 Q7c, 3m, 33%)

Consider the motion to be vertical, at **constant** speed.



At the top of the path, there are two forces acting on the sphere, its weight, mg, and the tension from the string T.

$\therefore \Sigma F = ma$

Becomes $T + mg = \frac{mv^2}{r}$

$\therefore T = \frac{mv^2}{r} - mg.$

At the bottom of the circle, the situation looks like.

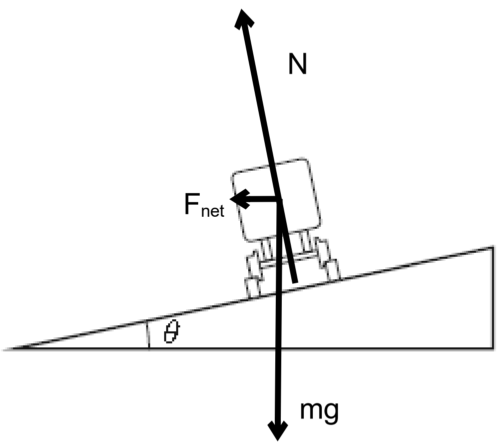
$\therefore \Sigma F = ma$

Becomes $T - mg = \frac{mv^2}{r}$

$\therefore T = \frac{mv^2}{r} + mg.$

Since $\frac{mv^2}{r}$ is constant, the tension at the bottom is 2mg larger than the force at the top. $\therefore$ the tension in the string is greater at the bottom of the circular path.

Question 23a (2013 Q4a, 2m, 60%)



The two forces acting are the weight force (from the centre of mass) acting vertically downwards, and the Normal reaction, which is a force perpendicular to the banked surface.

The net force is a vector acting radially inwards (it had to be horizontal).
The question didn't ask you to label mg, and N. Good physics means that you do. Also it's better to be safe than sorry.

Question 23b (2013 Q4b, 2m, 54%)

This question reinforces the need to have the formula on the cheat sheet, even though it is not on the course.

Use $\tan\theta = \dfrac{v^2}{rg}$

$\therefore \tan\theta = \dfrac{50^2}{2000 \times 10}$

$\therefore$ **θ = 7.1⁰ (ANS)**

Question 24 (2013 Q5b, 3m, 43%)

At the point S there are two forces acting on the mass. The weight is acting down, and the tension force is up.
Since the net force is up,
ΣF = ma
$\therefore$ ma = T – mg

$a = \dfrac{v^2}{r}$

$= \dfrac{7^2}{1}$

$= 49\ m/s^2$

$\therefore$ 2.0 × 49 = T – 2.0 × 10

$\therefore$ **T = 118 N (ANS)**

Question 25a (2014 Q4a, 2m, 70%)

For Mary and Bob to feel weightless, the Normal reaction must be zero.

$\therefore mg = mv^2/r$

$\therefore g = v^2/r$

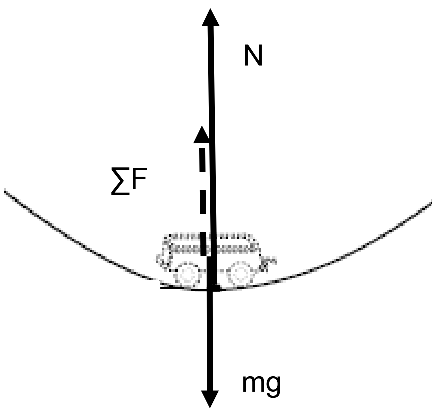
$\therefore 10 = v^2/20$

$\therefore v^2 = 200$

$\therefore v = 14.14$

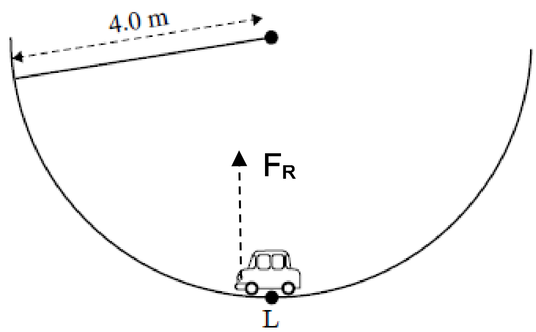
$\therefore$ **v = 14 m s⁻¹ (ANS)**

Question 25b (2014 Q4c, 3m, 67%)

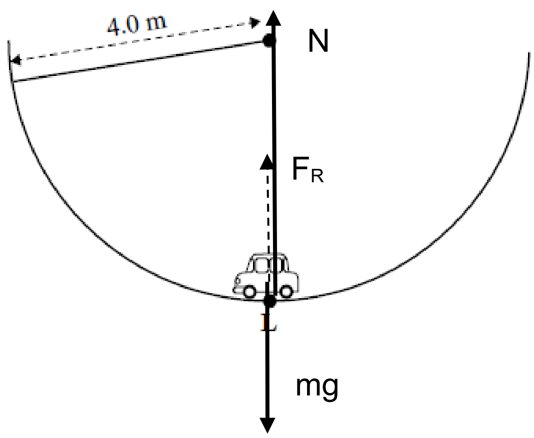


The weight is acting down from the centre of mass.
The normal is vertically upwards, from the surface. The normal vector is larger than the weight vector.
The resultant force is acting upwards

Question 26a (2015 Q3a, 1m, 80%)



Question 26b (2015 Q3b, 3m, 50%)



Use $F_R = ma$

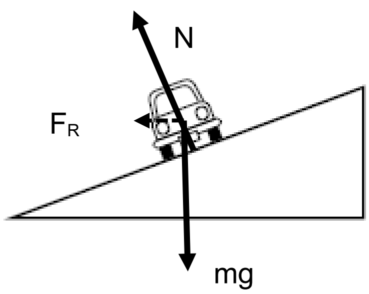
$\therefore \dfrac{mv^2}{r} = N - mg$

$\therefore \dfrac{2 \times 6^2}{4} = N - 2 \times 10$

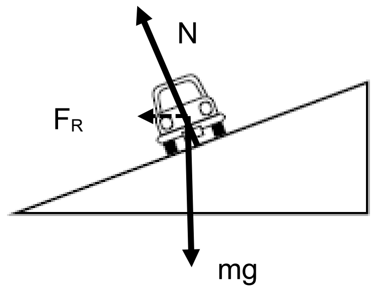
$\therefore$ N = 18 + 20

$\therefore$ **N = 38 N (ANS)**

Question 26c (2015 Q4a, 2m, 77%)



Question 26d **(2015 Q4b, 3m, 67%)**



If we resolve N into two components, one vertical and the other horizontal.

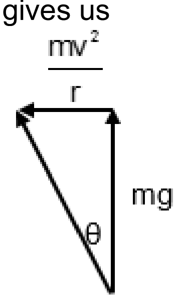
In the horizontal direction we get

$N\sin\theta = \frac{mv^2}{r}$

In the vertical direction we get

$N\cos\theta = mg$

Drawing that gives us



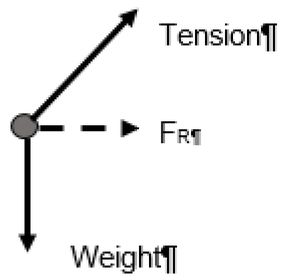
$\therefore \tan\theta = \frac{\frac{mv^2}{r}}{mg}$

$\therefore \tan\theta = \frac{v^2}{rg}$

$\therefore \tan\theta = \frac{3.0^2}{3.0 \times 10}$

$\therefore \theta = 16.7^0$ (ANS)

Question 27a **(2016 Q2a, 2m, 65%)**



Question 27b (2016 Q2b, 3m, 53%)

Consider the horizontal direction

$T\sin\theta = \frac{mv^2}{r}$

$\therefore T\sin\theta = \frac{2.0 \times 1.7^2}{0.5}$

From the diagram, $\sin\theta = \frac{O}{H}$

$\therefore \sin\theta = \frac{0.5}{1.0}$

$= 0.5$

$\therefore T = 2 \times \frac{2.0 \times 1.7^2}{0.5}$

$\therefore$ T = 23.1 N (ANS)

This can also be solved using the vertical direction

$\Sigma F = T\cos\theta - mg$

$\therefore T\cos\theta = 2.0 \times 10$

(Trigonometry gives $\theta = 30^0$)

$\therefore T \times 0.866 = 20$

$\therefore$ T = 23.1 N (ANS)

Question 28 (2017 Q2b, 3m)

Use $f = \frac{mv^2}{r}$

$\therefore 8.2 \times 10^{-8} = \frac{9.1 \times 10^{-31} \times v^2}{53 \times 10^{-12}}$

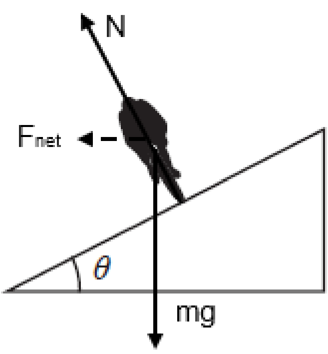
$\therefore v^2 = \frac{53 \times 10^{-12} \times 8.2 \times 10^{-8}}{9.1 \times 10^{-31}}$

$\therefore v^2 = 4.7758 \times 10^{12}$

$\therefore v = 2.185 \times 10^6$

$\therefore$ v = 2.2 × 10^6 m s^{-1} (ANS)

Question 29a (2017 Q7a, 2m)



Even though the question doesn't ask for it, don't forget to label your arrows.

Question 29b (2017 Q7b, 2m)

You need to use $\tan\theta = \frac{v^2}{Rg}$

$\therefore \tan\theta = \frac{10^2}{20 \times 9.8}$

$\therefore \tan\theta = 0.5102$

$\therefore \theta = 27.03$

$\therefore \theta = 27^0$ (ANS)

Question 30 (2017 Q8a, 2m)

If the normal reaction is zero, then $g = \frac{v^2}{r}$

$\therefore 9.8 = \frac{v^2}{6.4}$

$\therefore v^2 = 62.72$

$\therefore$ v = 7.9 m s^{-1} (ANS)