

Checkpoints Chapter 7 Relativity**Multiple Choice Questions****Question 1**Use $\Delta t = \gamma \Delta t'$

$\Delta t' = 510$ ps and $\Delta t = 5.1$ ps. (Note $\Delta t'$ is time is measured in a frame of reference in which the observer is stationary with respect to the particles)

$$\begin{aligned} \therefore 510 \times 10^{-12} &= \frac{5.1 \times 10^{-12}}{\sqrt{1 - \frac{v^2}{c^2}}} \\ \therefore \sqrt{1 - \frac{v^2}{c^2}} &= \frac{5.1 \times 10^{-12}}{510 \times 10^{-12}} \\ \therefore 1 - \frac{v^2}{c^2} &= \frac{1}{100^2} \\ \therefore \frac{v^2}{c^2} &= 1 - \frac{1}{10000} \\ \therefore \frac{v^2}{c^2} &= 0.9999 \\ \therefore \frac{v}{c} &= 0.999949 \\ \therefore v &= 99.995\% c \\ \therefore \mathbf{B} \quad (\text{ANS}) \end{aligned}$$

Question 2

$$L = \frac{L_0}{\gamma}$$

Use

Where $L = \frac{1}{4} L_0$

$$\begin{aligned} \therefore \gamma &= \frac{L_0}{\frac{1}{4} L_0} \\ \therefore \gamma &= 4 \\ \therefore \sqrt{1 - \frac{v^2}{c^2}} &= \frac{1}{4} \\ \therefore 1 - \frac{v^2}{c^2} &= \frac{1}{16} \\ \therefore 1 - \frac{v^2}{c^2} &= \frac{1}{16} \\ \therefore \frac{v^2}{c^2} &= 1 - \frac{1}{16} \\ \therefore \frac{v^2}{c^2} &= 0.9375 \end{aligned}$$

$$\begin{aligned} \therefore \frac{v}{c} &= 0.9682 \\ \therefore v &= 2.9 \times 10^8 \text{ m s}^{-1} \\ \therefore \mathbf{C} \quad (\text{ANS}) \end{aligned}$$

Question 3Use $E_k = (\gamma - 1)m_0c^2$.

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1 - \frac{(0.3c)^2}{c^2}}} \\ \therefore \gamma &= 1.0483. \\ \therefore E_k &= (1.0483 - 1) m_0c^2 \\ &= 0.0483 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \\ &= 3.95 \times 10^{-15} \\ \therefore \mathbf{D} \quad (\text{ANS}) \end{aligned}$$

Question 4The KE is given by $E_k = (\gamma - 1)m_0c^2$.

$$\therefore \mathbf{D} \quad (\text{ANS})$$

Question 5**B (ANS)****Question 6**Use $E = mc^2$.

The energy is given to you in eV. To convert from eV to Joule you need to multiply by 1.6×10^{-19} . (This will be covered in detail in Light and Matter).

$$\begin{aligned} \therefore 1.773 \times 10^9 \times 1.6 \times 10^{-19} &= m \times (3 \times 10^8)^2 \\ \therefore m &= 3.152 \times 10^{-27} \\ \therefore \mathbf{A} \quad (\text{ANS}) \end{aligned}$$

Question 7 (2012 Q4, 2m, 73%)

The best answer is B.

D is also true, but it is only one example of an inertial frame, therefore it is not adequate to describe the properties.

$$\therefore \mathbf{B} \quad (\text{ANS})$$

Question 8 (2013 Q1, 2m, 78%)

$$\mathbf{D} \quad (\text{ANS})$$

Question 9 (2104 Q1, 2m, 52%)

From the definition.

$$\mathbf{C} \quad (\text{ANS})$$

Question 10

A proper measurement is taken in the same inertial frame.

$$\therefore \mathbf{D} \quad (\text{ANS})$$

Question 11

$$\therefore \gamma = 11.5$$

$$\therefore \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 11.5$$

$$\therefore \sqrt{1 - \frac{v^2}{c^2}} = 0.0869565$$

$$\therefore 1 - \frac{v^2}{c^2} = 0.00756$$

$$\therefore \frac{v^2}{c^2} = 0.99244$$

$$\therefore \frac{v}{c} = 0.9962$$

$$\therefore v = 2.989 \times 10^8 \text{ m s}^{-1}$$

\therefore **A (ANS)**

Question 12 (2015 Q4, 2m 71%)

To measure the proper time, the observer needs to be at rest relative to the clock .

\therefore **A (ANS)**

Question 13 (2015 Q5, 2m 52%)

Spacecraft S66 is moving, therefore the time will be dilated as it is non-proper.

\therefore **D (ANS)**

Question 14 (2015 Q8, 2m 82%)

B, C and D are all moving relative to the landing area, therefore they will not measure the proper length.

\therefore **A (ANS)**

Question 15 (2014 Q6, 2m 37%)

To measure proper time the two events need to occur at the same place relative to the observer, and thus can be timed with a single clock located at that place. This is not the case.

\therefore **D (ANS)**

Question 16 (2013 Q9, 2m 76%)

To measure a proper length, the observer needs to be stationary with respect to the length being measured.

\therefore **A (ANS)**

Question 17 (2012 Q8, 2m, 46%)

Since $t = \gamma t_0$,

Where t = elapsed time in moving frame

t_0 = time in a stationary frame (proper time)

$$\text{therefore } t_0 = \frac{t}{\gamma}$$

γ is always greater than 1.

Therefore t_0 is always less than t .

\therefore **A (ANS)**

Question 18

As the speed tends to c , the KE increases at an increasing rate due to γ .

\therefore **B (ANS)**

Question 19 (2014 Q7, 2m, 40%)

Proper time is measured when the observer is at rest relative to the event being measured.

This occurs in both cases, so the observer measures proper time each time.

\therefore **B (ANS)**

Question 20 (2014 Q4, 2m, 25%)

The classical interpretation would have the speed of the radio wave signal travelling at a speed of c irrespective of the motion of the transmitter. Using classical relativity

$$\text{velocity}_{\text{A relative to B}} = v_a - v_b.$$

$$\therefore \text{velocity}_{\text{signal relative to Hector}} = c - v_{\text{hector}}.$$

$$\therefore c - 0.4c = 0.6c$$

\therefore **B (ANS)**

Question 21 (2014 Q5, 2m, 62%)

He navigator will measure proper values, so $d = c \times t$

$$\therefore d = 3 \times 10^8 \times 0.0100$$

$$\therefore d = 3 \times 10^6 \text{ m}$$

\therefore **A (ANS)**

Question 22 (2013 Q4, 2m, 81%)

The speed of light is constant, c .

D (ANS)

Question 23 (2015 Q9, 2m, 74%)

Use $E = mc^2$

The nucleus loses $1.8 \times 10^{-13} \text{ J}$ of energy.

$$\therefore 1.8 \times 10^{-13} = m \times (3.0 \times 10^8)^2$$

$$\therefore m = 2.0 \times 10^{-30} \text{ kg}$$

Therefore final mass = $M_i - 2 \times 10^{-30}$

\therefore **C (ANS)**

Question 24 (2016 Q1, 2m, 48%)

Both clocks are at rest relative to the relevant observer, so the observed period of vibration will be the same, i.e. undilated.

\therefore **C (ANS)**

Question 25 (2016 Q2, 2m, 45%)

In Anna's reference frame, Barry is moving away from her (the distance between them is increasing, and she sees herself as being stationary) and the space lab is moving towards her. This means that from Anna's reference frame the signal has a shorter distance to travel to the lab. The signal travels at the speed of light in all directions, so the space lab will receive the signal first.

∴ **B (ANS)**

Question 26 (2016 Q3, 2m, 79%)

As Carla is moving towards the sound source, (as compared with being stationary) she will encounter wavefronts more frequently. This will give the speed of sound relative to Carla as $v_s + v_c$

The speed of light depends only on the properties of the medium that it is travelling in. In this case it will be 'c'.

∴ **C (ANS)**

Question 27 (2016 Q5, 2m, 69%)

The proper time (undilated) is the time the clock measures in its own reference frame

∴ **B (ANS)**

Question 28 (2016 Q8, 2m, 47%)

The particle is accelerating, so Special Relativity does not apply in this case.

∴ **D (ANS)**

Question 29 (2017 Q10, 1m, 75%)

The laws of physics are the same in all inertial frames, so the results from experiments will be identical across all inertial frames. An inertial frame is when the frame is not accelerating. (Therefore moving at a constant speed and stationary, are identical frames). The results will be different in an accelerating frame.

∴ **B (ANS)**

Question 30 (2017 Q11, 1m, 66%)

Use $E = mc^2$

$$\therefore 3.8 \times 10^{26} = m \times (3.0 \times 10^8)^2$$

$$\therefore m = \frac{3.8 \times 10^{26}}{(3.0 \times 10^8)^2}$$

$$\therefore m = \frac{3.8 \times 10^{26}}{9.0 \times 10^{16}}$$

$$\therefore m = 4.2 \times 10^9 \text{ kg}$$

∴ **B (ANS)**

Extended questions**Question 31 (2016 Q6, 2m, 71%)**

The proper time is in the reference frame of the pion. In the Earth's reference frame time will be dilated by a factor of γ .

$$\text{Where } \gamma = \frac{1}{\sqrt{1 - \frac{(0.98c)^2}{c^2}}}$$

$$\therefore \gamma = 5.025$$

Therefore the measured half life in the Earth's frame of reference is $26 \times 10^{-9} \times 5.025$

$$= \mathbf{131 \text{ ns (ANS)}}$$

Question 32 (2016 Q7, 2m, 58%)

In the particle's frame of reference, the lab is moving towards it at $0.91c$, $\gamma = 2.4$, so lengths will be contracted.

$$\text{Use } L = \frac{L_0}{\gamma}$$

$$\therefore L = \frac{2}{2.4}$$

$$\therefore \mathbf{L = 0.83 \text{ m (ANS)}}$$

Question 33

An inertial frame is any frame that is not accelerating.

Question 34

A spaceship travelling away from the Earth emits a beam of light. The speed of the light beam measured on Earth and by the spaceship are the same, c .

Question 35a

$$\gamma = 2$$

$$\therefore \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2$$

$$\therefore \sqrt{1 - \frac{v^2}{c^2}} = 0.5$$

$$\therefore 1 - \frac{v^2}{c^2} = 0.25$$

$$\therefore \frac{v^2}{c^2} = 0.75$$

$$\therefore \frac{v}{c} = 0.866$$

$$\therefore \mathbf{v = 2.6 \times 10^8 \text{ m s}^{-1} \text{ (ANS)}}$$

Question 35b

Sally will observe Sam's clock to be 'ticking' at half the rate hers ticks at.

Question 35c

$$\gamma = 2$$

$$\text{Use } L = \frac{L_0}{\gamma}$$

$$\begin{aligned} \therefore 500 &= \frac{L_0}{2} \\ \therefore L_0 &= \mathbf{1000 \text{ m}} \quad (\text{ANS}) \end{aligned}$$

Question 36

Due to the speed of the train, the observer will see the train as contracted, but the tunnel will still be 600 m long

To find the contracted length of the train, use

$$L = \frac{L_0}{\gamma}$$

$$\text{Where } \gamma = \frac{1}{\sqrt{1 - \frac{(0.85c)^2}{c^2}}}$$

$$\therefore \gamma = 1.898$$

This gives the contracted length of the train to be

$$\begin{aligned} L &= \frac{700}{1.8983} \\ \therefore L &= 368 \text{ m} \end{aligned}$$

The train will "fit" in the tunnel. So the observer will see the train enter the tunnel and some time later emerge from the other side of the tunnel. The train will not be visible to the observer for some time whilst it is in the tunnel.

This is similar to the back of the book.

Question 37

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1 - \frac{(0.7711c)^2}{c^2}}} \\ \therefore \gamma &= \frac{1}{\sqrt{1 - 0.594595}} \\ \therefore \gamma &= \frac{1}{\sqrt{0.405405}} \\ \therefore \gamma &= 0.636714 \\ \therefore \gamma &= \mathbf{1.571} \quad (\text{ANS}) \end{aligned}$$

Question 38a

$$\begin{aligned} \gamma &= 2.35 \\ \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} &= 2.35 \\ \sqrt{1 - \frac{v^2}{c^2}} &= 0.42553 \\ 1 - \frac{v^2}{c^2} &= 0.18108 \\ \frac{v^2}{c^2} &= 0.81892 \\ \frac{v}{c} &= 0.9049 \\ \therefore v &= \mathbf{0.90 c} \quad (\text{ANS}) \end{aligned}$$

Question 38b

$$\begin{aligned} \text{To find the length of the beamline, use } L &= \frac{L_0}{\gamma} \\ \therefore L &= \frac{4.2 \times 10^3}{2.35} \\ \therefore L &= \mathbf{1.79 \text{ km}} \quad (\text{ANS}) \end{aligned}$$

Question 38c

The time will be given by $t = \frac{d}{c}$, using the scientists frame of reference.

$$\begin{aligned} \therefore t &= \frac{4.2 \times 10^3}{3 \times 10^8} \\ \therefore t &= \mathbf{1.4 \times 10^{-5} \text{ s}} \quad (\text{ANS}) \end{aligned}$$

(This is close to the back of the book)

Question 38d

$$\begin{aligned} \text{Use } E_K &= (\gamma - 1)mc^2 \\ \therefore E_K &= (2.35 - 1) \times 1.67 \times 10^{-27} \times (3 \times 10^8)^2 \\ \therefore E_K &= 2.02 \times 10^{-10} \text{ J} \end{aligned}$$

To convert J to eV divide by 1.6×10^{-19}

$$\begin{aligned} \therefore E_K &= \frac{2.02 \times 10^{-10}}{1.6 \times 10^{-19}} \\ \therefore E_K &= \mathbf{1 \text{ 263 MeV}} \quad (\text{ANS}) \end{aligned}$$

The question specifies the answer to be in MeV.

Question 39a

The time will be given by $d = c \times t$, using the Earth's frame of reference.

The time it will take to reach the spacecraft is half the total time.

$$\therefore d = 3.0 \times 10^8 \times 10.0 \times 10^{-3}$$

$$\therefore d = 3.0 \times 10^6 \text{ m}$$

$$\therefore \mathbf{d = 300 \text{ km (ANS)}}$$

Question 39b

$$\text{Use } \gamma = \frac{0.115}{0.100}$$

$$\therefore \gamma = 1.15$$

$$\therefore \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = 1.15$$

$$\therefore \sqrt{1 - \frac{v^2}{c^2}} = 0.869565$$

$$\therefore 1 - \frac{v^2}{c^2} = 0.75614$$

$$\therefore \frac{v^2}{c^2} = 0.243856$$

$$\therefore \frac{v}{c} = 0.49382$$

$$\therefore v = 0.49 c$$

$$\therefore \mathbf{v = 1.48 \times 10^8 \text{ m s}^{-1} \quad \text{(ANS)}}$$

Question 40

To find the length of the landing strip, use

$$L = \frac{L_0}{\gamma}$$

$$\therefore L = \frac{500}{1.5}$$

$$\therefore \mathbf{L = 333 \text{ m (ANS)}}$$

Question 41

Use $L = \frac{L_0}{\gamma}$, where L is the contracted length of the ruler.

$$\therefore L = 1.90 \text{ m.}$$

$$\frac{200}{\gamma}$$

$$\therefore \gamma = \frac{190}{200}$$

$$\therefore \gamma = 1.05263$$

$$\therefore \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = 1.05263$$

$$\therefore \sqrt{1 - \frac{v^2}{c^2}} = 0.95$$

$$\therefore 1 - \frac{v^2}{c^2} = 0.9025$$

$$\therefore \frac{v^2}{c^2} = 0.0975$$

$$\therefore \frac{v}{c} = 0.31225$$

$$\therefore v = 0.31225 c$$

$$\therefore \mathbf{v = 9.37 \times 10^7 \text{ m s}^{-1} \quad \text{(ANS)}}$$

Question 42a

To find the distance, use $L = \frac{L_0}{\gamma}$

$$\frac{5.00}{\gamma}$$

$$\therefore L = \frac{5.00}{1.25}$$

$$\therefore \mathbf{L = 4.00 \text{ m (ANS)}}$$

Question 42b

The time will be given by $t = \frac{d}{c}$,

$$\frac{5.00 \times 2}{3 \times 10^8}$$

$$\therefore t = \frac{5.00 \times 2}{3 \times 10^8}$$

$$\therefore t = 3.3 \times 10^{-8} \text{ s}$$

Time in galaxy frame

$$= 1.25 \times 3.3 \times 10^{-8}$$

$$= 4.2 \times 10^{-8}$$

$$= \mathbf{42 \text{ ns (ANS)}}$$

Question 43

The limousine, is twice as long as the garage, so $\gamma = 2$, so that the contracted length is half the rest length.

$$\therefore \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = 2$$

$$\therefore \sqrt{1 - \frac{v^2}{c^2}} = 0.5$$

$$\therefore 1 - \frac{v^2}{c^2} = 0.25$$

$$\therefore \frac{v^2}{c^2} = 0.75$$

$$\therefore \frac{v}{c} = 0.866025$$

$$\therefore v = 0.8660 c$$

$$\therefore v = 2.60 \times 10^8 \text{ m s}^{-1} \quad \text{(ANS)}$$

Question 44

$$\therefore \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma$$

It is simplest to calculate the two values and then find the ratio.

$$\therefore \gamma_1 = \frac{1}{\sqrt{1 - \frac{(1.4 \times 10^8)^2}{(3.0 \times 10^8)^2}}}$$

$$\therefore \gamma_1 = \frac{1}{\sqrt{1 - 0.217778}}$$

$$\therefore \gamma_1 = \frac{1}{\sqrt{0.78222}}$$

$$\therefore \gamma_1 = \frac{1}{0.88443}$$

$$\therefore \gamma_1 = 1.13066$$

$$\therefore \gamma_2 = \frac{1}{\sqrt{1 - \frac{(2.8 \times 10^8)^2}{(3.0 \times 10^8)^2}}}$$

$$\therefore \gamma_2 = \frac{1}{\sqrt{1 - 0.87111}}$$

$$\therefore \gamma_2 = \frac{1}{\sqrt{0.12889}}$$

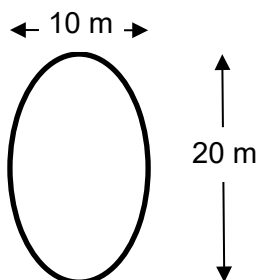
$$\therefore \gamma_2 = \frac{1}{0.3590}$$

$$\therefore \gamma_2 = 2.7854$$

$$\therefore \frac{\gamma_2}{\gamma_1} = \frac{2.7854}{1.13066} = 2.46 \quad \text{(ANS)}$$

Question 45a

Briony will observe the length of the window to be contracted in the horizontal plane. The vertical dimension will remain at 20 m. $\gamma = 2$, therefore the length will be halved.

**Question 45b**

Use $t = \frac{d}{v}$, $d = 10$ m, as we are in Briony's frame.

$$\therefore t = \frac{10}{0.866 \times 3 \times 10^8} \\ \therefore t = 3.85 \times 10^{-8} \text{ s} \quad \text{(ANS)}$$

Question 46a

Use $t = \gamma t_0$,

$$\therefore t = (1 + [5 \times 10^{-11}]) \times 1 \\ \therefore t = 1 + [5 \times 10^{-11}] \quad \text{(ANS)}$$

Question 46b

The satellite is moving horizontally, so there will not be any length contraction in the vertical direction. So the satellite will be 20 000 km above ground.

$$\therefore 20\,000 \text{ km} \quad \text{(ANS)}$$

Question 47

The speed of light is measured to be $3 \times 10^8 \text{ m s}^{-1}$ in all frames of reference, regardless of the speed of the source or the observer.

Therefore Student A is incorrect, and Student B is correct.

Question 48

$$\text{Use velocity}_{\text{Alarm sound relative to A}} = \mathbf{v}_{\text{Alarm sound}} - \mathbf{v}_A \\ = 335 - 5 \text{ (in the same direction)} \\ = 330 \text{ m s}^{-1} \quad \text{(A)} \quad \text{(ANS)}$$

$$\text{Use velocity}_{\text{Alarm sound relative to B}} = \mathbf{v}_{\text{Alarm sound}} - \mathbf{v}_B \\ = 335 - (-5) \text{ (in the opposite direction)} \\ = 340 \text{ m s}^{-1} \quad \text{(B)} \quad \text{(ANS)}$$

Question 49

Since Lee is travelling away from the sound source the speed Lee will measure, will be the speed of sound $- 30 \text{ m s}^{-1}$

$$\therefore 340 - 30 = 310 \text{ m s}^{-1}$$

Since Sung is travelling towards the sound source the speed Sung will measure will be the speed of sound $+ 30 \text{ m s}^{-1}$

$$\therefore 340 + 30 = 370 \text{ m s}^{-1}$$

$$\therefore \text{LEE} \quad 310 \text{ m s}^{-1} \\ \text{SUNG} \quad 370 \text{ m s}^{-1} \quad \text{(ANS)}$$

Question 50aUse $E = mc^2$

$$\therefore 4.5 \times 10^{-11} = m \times (3 \times 10^8)^2$$

$$\therefore m = 5 \times 10^{-28} \text{ kg}$$

∴ rest mass of the pion is half of this

$$\therefore m = 2.5 \times 10^{-28} \text{ kg (ANS)}$$

Question 50bWork done = $(\gamma - 1)m_0c^2$

$$\therefore E = (3 - 1) \times 2.5 \times 10^{-28} \times (3 \times 10^8)^2$$

$$\therefore E = 2 \times 2.5 \times 3^2 \times 10^{-28} \times 10^{16}$$

$$\therefore E = 45 \times 10^{-12}$$

$$\therefore E = 4.5 \times 10^{-11} \text{ J (ANS)}$$

Question 51Work done = $(\gamma - 1)m_0c^2$

$$\therefore E = (4.5 - 1) \times 1.67 \times 10^{-27} \times (3 \times 10^8)^2$$

$$\therefore E = 3.5 \times 1.67 \times 3^2 \times 10^{-27} \times 10^{16}$$

$$\therefore E = 52.605 \times 10^{-11}$$

$$\therefore E = 5.3 \times 10^{-10} \text{ J (ANS)}$$

Question 52Use $E = E_k + E_{\text{rest}}$

$$\therefore E = 9.00 \times 10^{-11} + 1.67 \times 10^{-27} \times (3 \times 10^8)^2$$

$$\therefore E = 9 \times 10^{-11} + 15.03 \times 10^{-11}$$

$$\therefore E = 24 \times 10^{-11}$$

$$\therefore E = 2.4 \times 10^{-10} \text{ J (ANS)}$$

Question 53Find both values for γ .

$$\therefore \gamma_1 = \frac{1}{\sqrt{1 - \frac{(9.0 \times 10^7)^2}{(3.0 \times 10^8)^2}}}$$

$$\therefore \gamma_1 = \frac{1}{\sqrt{1 - 0.09}}$$

$$\therefore \gamma_1 = \frac{1}{\sqrt{0.91}}$$

$$\therefore \gamma_1 = 0.9539$$

$$\therefore \gamma_1 = 1.0483$$

$$\therefore \gamma_2 = \frac{1}{\sqrt{1 - \frac{(1.5 \times 10^8)^2}{(3.0 \times 10^8)^2}}}$$

$$\therefore \gamma_2 = \frac{1}{\sqrt{1 - 0.25}}$$

$$\therefore \gamma_2 = \frac{1}{\sqrt{0.75}}$$

$$\therefore \gamma_2 = \frac{1}{0.8660}$$

$$\therefore \gamma_2 = 1.1547$$

Then use $\Delta E_k = (\gamma_2 - 1)m_0c^2 - (\gamma_1 - 1)m_0c^2$

$$\therefore \Delta E_k = \Delta\gamma \times mc^2$$

$$\therefore \Delta E_k = (1.1547 - 1.0483) \times 6.64 \times 10^{-27} \times (3 \times 10^8)^2$$

$$\therefore \Delta E_k = 0.1064 \times 6.64 \times 10^{-27} \times (3 \times 10^8)^2$$

$$\therefore \Delta E_k = 6.36 \times 10^{-11} \text{ J (ANS)}$$

This is close to the answer in the back of the book.

Question 54aSince the Sun is giving off $3.85 \times 10^{26} \text{ W}$, this is $3.85 \times 10^{26} \text{ joules every second}$.

$$\Delta E = \Delta mc^2$$

$$\therefore 3.85 \times 10^{26} = \Delta m \times (3 \times 10^8)^2$$

$$\therefore \Delta m = 4.278 \times 10^{10}$$

$$= 4.3 \times 10^9 \text{ kg (ANS)}$$

Question 54bIf the Sun is losing $4.3 \times 10^9 \text{ kg}$ every second, it will lose $4.3 \times 10^9 \times (365.25 \times 24 \times 60 \times 60) \times 10^9$ in one billion years.

$$\therefore \Delta m = 4.3 \times 10^9 \times (365.25 \times 24 \times 3600) \times 10^9$$

$$\therefore \Delta m = 1.4 \times 10^{26} \text{ kg}$$

$$\% \text{ loss} = \frac{1.4 \times 10^{26}}{2 \times 10^{30}} \times 100\%$$

$$\therefore \% \text{ loss} = 0.007\% \text{ (ANS)}$$

Question 55

(2016 Q9, 2m, 68%)

Use $E = (\gamma - 1)m_0c^2$

$$1.2 \times 10^{-10} = (\gamma - 1) \times 1.67 \times 10^{-27} \times (3.0 \times 10^8)^2$$

$$\therefore (\gamma - 1) = \frac{1.2 \times 10^{-10}}{1.67 \times 10^{-27} \times (3.0 \times 10^8)^2}$$

$$\therefore (\gamma - 1) = \frac{1.2 \times 10^{-10}}{1.503 \times 10^{-10}}$$

$$\therefore (\gamma - 1) = 0.798$$

$$\therefore \gamma = 1.8 \text{ (ANS)}$$

Question 56

(2016 Q10, 2m, 38%)

Use conservation of energy.

The initial energy of the proton is

$$(\gamma - 1)m_0c^2$$

Final energy of the system is $m_0c^2 + KE_{\text{nucleus}}$

$$\therefore (3 - 1)m_0c^2 = m_0c^2 + KE_{\text{nucleus}}$$

$$\therefore 2m_0c^2 = m_0c^2 + KE_{\text{nucleus}}$$

$$\therefore KE_{\text{nucleus}} = m_0c^2 \text{ (ANS)}$$

Question 57 (2017 Q10, 2m, 60%)

Use $L = \frac{L_0}{\gamma}$ where $L_0 = 1$, and $L = 0.333$

$$\therefore \gamma = \frac{1}{0.333}$$

$$\therefore \gamma = 3$$

$$\therefore \sqrt{1 - \frac{v^2}{c^2}} = 3$$

$$\therefore \sqrt{1 - \frac{v^2}{c^2}} = 0.333$$

$$\therefore 1 - \frac{v^2}{c^2} = 0.111$$

$$\therefore \frac{v^2}{c^2} = 0.888$$

$$\therefore \frac{v}{c} = 0.94$$

$$\therefore v = 0.94 c \quad (\text{ANS})$$

Question 58a (2017 Q11a, 2m, 45%)

Use $t = \frac{d}{v}$, $d = 9.14 \times 10^{-5}$ m, in the scientist's frame.

$$\therefore t = \frac{9.14 \times 10^{-5}}{0.99875 \times 3 \times 10^8}$$

$$\therefore t = 3.05 \times 10^{-13} \text{ s} \quad (\text{ANS})$$

Question 58b (2017 Q11b, 2m, 40%)

The laboratory is not the proper frame, so the distance travelled must be less than 9.14×10^{-5} . Common sense says use the gamma factor to reduce this length.

To find the length as measured in the particles

reference frame, use $L = \frac{L_0}{\gamma}$

$$\therefore L = \frac{9.14 \times 10^{-5}}{20}$$

$$\therefore L = 4.6 \times 10^{-6} \text{ m} \quad (\text{ANS})$$

Question 58c (2017 Q11c, 2m, 27%)

More particles reach the detector in the laboratory frame than predicted from classical physics due to time dilation. The particle is moving fast, so its half-life, as measured in the scientist's frame of reference, will be longer. This means that fewer particles will decay before reaching the detector, hence more will arrive at the detector.