GRAVITATIONAL FIELDS
Multiple-choice questions

Question 1

Rearrange a formula involving G such as

$mg = \frac{GMm}{R^2}$ then make G the subject

$\therefore G = \frac{gR^2}{M}$

$= \frac{m}{s^2} \times m^2 \times \frac{1}{kg}$

$= m^3 \times s^{-2} \times kg^{-1}$

$\therefore$ **B (ANS)**

Question 2

$g = \frac{GM}{R^2}$, therefore for g to be half its value at Earth's surface, $\frac{1}{R_{new}^2} = \frac{1}{2(R_{original}^2)}$

R is the distance between the centres of mass of the two objects.

$\therefore R_{new}^2 = 2(R_{original}^2)$

$\therefore R_{new} = \sqrt{2}\, R_{original}$

The altitude is the height above the Earth's surface.

$\therefore R_{new} = \sqrt{2}\, R - R$

$\therefore$ **B (ANS)**

(This answer is different to the back of the book)

Question 3

The potential energy is a function of the height above the surface of the Earth.

$\therefore$ **A (ANS)**

Question 4

Period is independent of mass, merely dependent on radius. The answer is 1

$\therefore$ **C (ANS)**

Question 5

The moon is much smaller than the Earth, as a result of this the gravitational attraction on the moon is approximately one sixth of that on Earth.

Using W = mg, the weight is ~ 100 000 N

$\therefore$ **D (ANS)**

Question 6

Use W = mg, on Earth 76 = m × 9.8

$\therefore$ m = 7.755

On Mars, W = mg

$\therefore$ W = 7.755 × 3.8

$\therefore$ W = 29. 5 N

$\therefore$ **B (ANS)**

Question 7

All satellites orbiting the Earth are in circular motion, therefore there is a net force acting on them. This force is the gravitational attraction between the two masses.

$\therefore$ **A (ANS)**

Question 8

All satellites orbiting the Earth are in circular motion, therefore there is a net force acting on them. This force is the gravitational attraction between the two masses.

$\therefore$ mg = ma

$\therefore$ **B (ANS)**

Question 9

A geostationary satellite has a period that is the length of one day. This way it takes the same time to orbit the Earth as the Earth takes to rotate. Hence it will always be above the same place on Earth, hence its name, geostationary.

1 day = 24 × 60 × 60 s

= 86 400 s

$\therefore$ **B (ANS)**

Question 10

The potential energy is given by mgh, if they are at the same height, then g and h are identical for both satellites, but the PE depends on mass.

The orbital period is given by $R^3 = kT^2$, so both satellites will have the same period/

$\therefore$ **B (ANS)**

Extended questions

Question 11

$F = \frac{GMm}{R^2}$

$\therefore F = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(0.4)^2}$

$\therefore$ **F = 4.2 × 10^{-10}N (ANS)**

Question 12

Gravitational Potential energy is given by

$$U = -\frac{GMm}{R}$$

$$U = -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times2.0\times10^{5}}{6.6\times10^{6}}$$

$$\therefore U = -12.13\times10^{12}\,J$$

Note that the radius is the radius of the Earth plus 200 km.

The kinetic energy is half the <u>size</u> of the Potential energy when in stable circular orbit so

E_k = 6.1 × 10^{12} J

The other way of doing this is:

$$\frac{mv^2}{r} = \frac{GMm}{R^2}$$

$$\therefore mv^2 = \frac{GMm}{R}$$

$$\therefore \tfrac{1}{2}mv^2 = \tfrac{1}{2}\times\frac{GMm}{R}$$

$$\therefore KE = \tfrac{1}{2}\times 12.13\times10^{12}$$

E_k = 6.1 × 10^{12} J (ANS)

Question 13

Use $g = \dfrac{4\pi^2R}{T^2}$

Where $R = \dfrac{C}{2\pi}$

$= \dfrac{2.4\times10^{9}}{2\pi}$

(don't forget to convert from km to m)

= 3.82 × 10^8

and T = 27.3 × 24 × 3600

= 2.36 × 10^6 sec.

$$\therefore g = \frac{4\pi^2\times3.82\times10^{8}}{(2.36\times10^{6})^2}$$

= 0.0027 m s^{-2} (ANS)

Question 14

A satellite orbits about the <u>centre</u> of the Earth. If it is not in the plane of the equator it will be alternately above a point in the Northern then Southern hemispheres. To remain above a fixed point on the Earth it must be above the equator.

Question 15

Therefore the shuttle is 10 R_E from the centre of the Earth.

g at the Earth's surface is 9.8 m s^{-2}

$$g = \frac{GM}{R^2}$$

We will use g_1 as the gravitational field at the surface of the Earth.

We will use g_2 as the gravitational field at 10 R_E from the centre of the Earth.

$$\frac{g_2}{g_1} = \frac{GM}{R_2^2}\times\frac{R_1^2}{GM}$$

$$\frac{g_2}{g_1} = \frac{R_1^2}{R_2^2}$$

Sub values in for R_1 and R_2

$$\frac{g_2}{g_1} = \left(\frac{1R_E}{10R_E}\right)^2 \quad \frac{g_2}{g_1} = \left(\frac{1}{10}\right)^2 \quad \frac{g_2}{g_1} = \frac{1}{100}$$

get g_2 by itself

$$g_2 = \frac{g_1}{100}$$

$\therefore$ g_2 = 0.098 ms^{-2}

Using the equation of motion:

v = u + at

$\therefore$ v = 0 + 0.098 × 200

$\therefore$ v = 20 m s^{-1} (ANS)

Question 16

Equating gravitational field strengths,

$$\frac{GM_S}{(R-x)^2} = \frac{GM_E}{x^2}$$

We can simplify such that $\dfrac{M_S}{(R-x)^2} = \dfrac{M_E}{x^2}$

Question 17

Use

$$W = mg$$
$$= \frac{mv^2}{r}$$
$$= \frac{2 \times \frac{1}{2}mv^2}{r}$$
$$= \frac{2E_k}{r}$$
$$= \frac{2 \times 3.0 \times 10^{10}}{8.0 \times 10^7}$$

= 750 N (ANS)

This is another example of combining the Kinetic Energy formula with the Centripetal Force expression from circular motion

Question 18

A balance will work in any place where there is a gravitational field by comparing the gravitational attraction on two masses, so it will work on the Moon. In a satellite in orbit, the balance and masses would all be in free fall so the gravitational attraction [apparent weight] will be zero.

Question 19

The original kinetic energy will be converted to Gravitational Potential Energy as it moves through the Gravitational field. If the graph supplied was a Force vs height graph, the energy thus transformed would be given by the area under that graph. To convert the g vs height graph, the 'g' values are multiplied by the mass involved – 50 kg.

The area is approximately a Trapezium.
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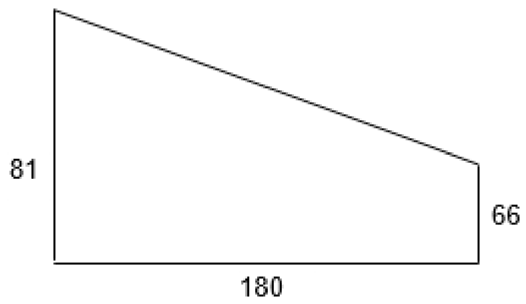

Area $= (81 + 66)/2 \times 180\,000$
$= 13\,230\,000$ J
$\therefore \frac{1}{2} \times 50 \times v^2 = 13\,230\,000$
$\therefore$ v = 730 ms^{-1} (ANS)

Question 20

See the notes for the derivation of Kepler's Law.

$$\frac{GM}{R^2} = \frac{4\pi^2 R}{T^2}$$
$$\therefore R^3 = \frac{GM}{4\pi^2}T^2$$

This is the standard manipulation of the formula obtained when equating the Gravitational field strength to the centripetal acceleration.

Question 21a

To do this question you must get an equation that has both R and T, where R is the orbital radius and T is the period.
You can use Kepler's Law, which is;

$$\frac{R^3}{T^2} = \text{constant}$$

Therefore any increase in R must also have an increase in T.
Kepler's Law shows there is no dependence on mass.

Question 21b

$$\frac{R^3}{T^2} = \text{constant}$$
$$\frac{(1.1 \times 10^9)^3}{7.16^2} = \frac{(1.87 \times 10^9)^3}{T^2}$$

(you could use any other set of data)
Solution of the equation yields
T = 17 days (ANS)

Question 22a

Use $g = \frac{GM_{Phobos}}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 1.07 \times 10^{16}}{(11.3 \times 10^3)^2}$$

= 0.0056 N kg^{-1} (ANS)

Question 22b

At the centre of any spherical body with a uniform density, the gravitational force is zero. This is because the centre of the planet is evenly surrounded by mass in all three dimensions, therefore the attraction will balance out, therefore the net force acting will be zero.

$\therefore$ 0 N kg^{-1} (ANS)

Question 23

$$\frac{GM}{R^2} = \frac{4\pi^2 R}{T^2}$$

$$\therefore M = \frac{4\pi^2 R^3}{GT^2}$$

$$\frac{4 \times \pi^2 \times (3.8 \times 10^8)^3}{6.67 \times 10^{-11} \times (28 \times 24 \times 3600)^2}$$

$\therefore$ **M = 5.5 × 10^{24} kg (ANS)**

Question 24

$$\frac{GM}{R^2} = \frac{4\pi^2 R}{T^2} \Rightarrow T^2 = \frac{4\pi R^3}{GM}$$

From this equation the period is independent of the mass of the satellite. If they have the same radius of orbit and the same period, then they must both have the same speed.

Question 25a (2010 Q18, 2m, 60%)

$$\text{Weight} = \frac{GM_e m_{iss}}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 3.04 \times 10^5}{(6.72 \times 10^6)^2}$$

= 2.69 × 10^6 N (ANS)

Question 25b (2010 Q19, 2m, 50%)

The period can be calculated from Kepler's Law:

$$\frac{GM}{4\pi^2} = \frac{R^3}{T^2}$$

$$\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4\pi^2} = \frac{(6.72 \times 10^6)^3}{T^2}$$

$$\therefore T^2 = \frac{(6.72 \times 10^6)^3 \times 4\pi^2}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}$$

$\therefore$ **T = 5 480 secs (ANS)**

This question can also be done using

$$F = \frac{mv^2}{r}$$

$$= \frac{m}{r}\left(\frac{2\Pi r}{T}\right)^2$$

$$= \frac{m4\Pi^2 r}{T^2}$$

$$\therefore F = \frac{m4\Pi^2 r}{T^2}$$

$$\therefore T^2 = \frac{m4\Pi^2 r}{F}$$

$$\therefore T^2 = \frac{4\pi^2 \times (6.72 \times 10^6)}{2.69 \times 10^6}$$

$\therefore$ **T = 5 480 secs (ANS)**

Question 25c (2010 Q20, 1m, 60%)

Same (ANS)

From Kepler's Law, the period is independent of the mass of the satellite.

Question 26a (2011 Q21, 1m, 80%)

W = mg, if the visitor weighs the same then g must equal 10 on both the planet and the Earth.

$\therefore$ **10 N kg^{-1} (ANS)**

Question 26b (2011 Q22, 2m, 50%)

$$\text{Use } F = mg = \frac{GMm}{r^2}$$

$$\therefore g_{Visitor} = \frac{GM_V}{r_v^2}$$

$$\therefore 10 = \frac{GM_V}{r_v^2}$$

$$\therefore 10 = \frac{6.67 \times 10^{-11} \times M_V}{(30 \times 10^3)^2}$$

$$\therefore M_V = \frac{10 \times (30 \times 10^3)^2}{6.67 \times 10^{-11}}$$

$\therefore$ **M$_V$ = 1.35 × 10^{20} kg (ANS)**

Question 26c (2011 Q23, 2m, 50%)

$$\text{Use } \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\therefore \frac{GM}{r} = v^2$$

$$\text{Use } v = \frac{2\pi r}{T}$$

$$\therefore \frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$\therefore T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\therefore T^2 = \frac{4\pi^2 \times (1 \times 10^9)^3}{6.67 \times 10^{-11} \times 5.7 \times 10^{25}}$$

$\therefore T^2 = 1.04 \times 10^{13}$

$\therefore$ **T = 3.2 × 10^6 secs (ANS)**

Question 27 (2012 Q8a, 4m, 58%)

Use $F = \frac{GMm}{R^2}$ and $F = \frac{mv^2}{R}$, combined with $v = \frac{2\Pi R}{T}$ to get $\frac{R^3}{T^2} = \frac{GM}{4\Pi^2}$.
This is known as Kepler's Law. It is not on the course, but it is extremely useful.

$$R^3 = \frac{GMT^2}{4\Pi^2}$$

$$\therefore R^3 = \frac{6.67\times10^{-11} \times 7.36\times10^{22}\times(2\times60\times60)^2}{4\Pi^2}$$

$\therefore$ $R^3 = 6.45 \times 10^{18}$
$\therefore$ $R = 1.86 \times 10^6$
This is the radius of orbit, the question asks for the height above the moon's surface.
$\therefore$ $h = 1.86 \times 10^6 - 1.74 \times 10^6$
$\therefore$ h = 1.22 × 10⁵ m (ANS)

Question 28a (2013 Q7a, 1m, 40%)

Period = 24 hours
= 24 × 60 × 60
= 8.64×10^4 s (ANS)

Question 28b (2013 Q7b, 2m, 40%)

Use $\frac{r^3}{t^2} = \frac{GM}{4\pi^2}$ which is Kepler's Law.

$$\therefore r^3 = \frac{6.7\times10^{-11}\times6.0\times10^{24}(8.64\times10^4)^2}{4\pi^2}$$

$\therefore$ $r^3 = 7.60 \times 10^{22}$
$\therefore$ r = 4.24×10^7 m (ANS)

Question 29a (2014 Q5a, 4m, 60%)

Period = 1200 hours
= 1200 × 60 × 60
= 4.32×10^6 s

Use $\frac{r^3}{t^2} = \frac{GM}{4\pi^2}$ which is Kepler's Law.

$$\therefore \frac{(7.0\times10^{10})^3}{(4.32\times10^6)^2} = \frac{6.67\times10^{-11}\times M}{4\pi^2}$$

$$\therefore M = \frac{1.354\times10^{34}}{1244.78208}$$

$\therefore$ $M = 1.09 \times 10^{31}$
$\therefore$ M = 1.1×10^{31} (ANS)

Question 29b (2014 Q5b, 2m, 40%)

No. From $\frac{r^3}{t^2} = \frac{GM}{4\pi^2}$, if we only know 'r', 't', 'G' and 'M', we can't find 'm', the mass of the planet.

Question 30a (2015 Q7a, 1m, 90%)

T = 10.25 × 60 × 60
$\therefore$ T = 36900
$\therefore$ T = 3.69×10^4 s (ANS)

Question 30b (2015 Q7b, 3m, 67%)

Use $\frac{r^3}{t^2} = \frac{GM}{4\pi^2}$ which is Kepler's Law.

$$\therefore \frac{r^3}{(3.69\times10^4)^2} = \frac{6.67\times10^{-11}\times5.68\times10^{26}}{4\pi^2}$$

$$\therefore r = \sqrt[3]{\frac{6.67\times10^{-11}\times5.68\times10^{26}\times(3.69\times10^4)^2}{4\pi^2}}$$

$\therefore$ R = 1.09×10^8 m (ANS)

Question 31

At the centre of any spherical body with a uniform density, the gravitational field is zero. This is because the centre of the planet is evenly surrounded by mass in all three dimensions, therefore the attraction will balance out, therefore the net field acting will be zero, BUT, this doesn't mean that there aren't any forces acting, and any object at the centre of Earth would feel forces pulling on it in all directions, so it would feel stretched in all directions.

Question 32a (2016 Q6a, 3m, 27%)
A geostationary orbit needs to have the same period as that of the Earth, approximately 24 hours.
It needs to travel in the same direction as the Earth is rotating.
It needs to be above the equator, so that the net force acting radial inwards is the gravitational force of attraction between the satellite and the Earth.

For a satellite in a stable circular orbit, the only force acting on a satellite is the gravitational attraction between it and the central body. This force acting on it is always perpendicular to its motion. Therefore the energy of the satellite is unchanged as it orbits. The kinetic energy

and gravitational potential energy both stay
the same.

Question 32b (2016 Q6b, 3m, 63%)
Emily is incorrect.
Any person or object will experience apparent weightlessness when they have an acceleration equal to that of gravity; i.e. when they are in free fall.
The person or object will experience zero normal reaction force at this time.

Question 33a (2017 Q4a,3m, 80%)

Use g = $G\frac{M}{r^2}$

$\therefore$ g = 6.67 × 10^{-11} × $\frac{1.3 \times 10^{22}}{(1.2 \times 10^{6})^2}$

$\therefore$ **g = 0.60 N kg^{-1} (m s^{-2}) (ANS)**

Question 33b (2017 Q4b, 3m, 43%)

Use $G\frac{M}{r^2} = \frac{v^2}{r}$ and v = $\frac{2\pi r}{T}$

to get $\frac{4\pi^2 r}{T^2} = G\frac{M}{r^2}$

$\therefore T^2 = \frac{4\pi^2 r^3}{GM}$

$\therefore T = \sqrt{\frac{4\pi^2 r^3}{GM}}$

$\therefore T = \sqrt{\frac{4 \times \pi^2 \times (1.8 \times 10^7)^3}{6.67 \times 10^{-11} \times 1.30 \times 10^{22}}}$

$\therefore$ T = 515292.75 s

$\therefore$ **T = 5.2 × 10^5 s (ANS)**

Question 33c (2017, Q4c, 3m, 37%)

Melissa is correct, Rick and Nam are incorrect.

Using $T = \sqrt{\frac{4\pi^2 r^3}{GM}}$, the period depends on the radius. If the radius is the same, then the period is the same and the speed is given by v = $\frac{2\pi r}{T}$, so the speed will be the same. The speed is independent of the mass.