# How fast can things go?

## Newton's laws of motion

 investigate and apply theoretically and practically Newton's three laws of motion in situations where two or more coplanar forces act along a straight line and in two dimensions

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
1, 2	1, 6, 7	1, 2		3	1, 2, 3, 7, 8	3, 4a, b, 5a, b, c, d.	1a, b 2a, b	1a, b	2a, b	1a, b

The following relationships hold for straight line motion.

$$v_{av} = \frac{x_2 - x_1}{\Delta t}$$
$$a = \frac{v - u}{t} = \frac{\Delta v}{\Delta t}$$
$$v = u \pm at$$
$$v^2 = u^2 \pm 2ax$$
$$x = ut \pm \frac{1}{2}at^2$$
$$x = \frac{(u + v)}{2}t$$

When given a graph in the exam, look at three things on the graph before even reading the question:

type of graph (x - t, v - t,)

the units on the axis

the limit on each axis.

Graph type	x - t	v - t	a-t
Direct reading	'x' at any 't'	'v' at any 't'	'a' at any 't'
	't' at any 'x'	't' at any 'v'	't' at any 'a'
Gradient	Instantaneous velocity at any point. v <sub>av</sub> between any two points	Instantaneous 'a' Average 'a'	
Area under graph		Change in position	Change in velocity

To convert from km/h to m/s you need to divide by 3.6, this should be on your cheat sheet.

A car of mass 2000 kg is driven along a straight flat road by an engine that provides a constant driving force. It accelerates from rest to a speed of 30 m s<sup>-1</sup> in 10 seconds. Assume that friction and air resistance forces remain constant at a total of 500 N.

#### Example 1.1: 1984 Question 1 (1 mark)

What is the acceleration of the car?

#### Example 1.2: 1984 Question 2 (1 mark)

What is the magnitude of the total driving force acting on the car?

Example 1.3: 1984 Question 3 (1 mark)

How far does the car travel in the 10 seconds?

Example 1.4: 1984 Question 4 (1 mark)

How much work has the car engine done in the 10 seconds?

Example 1.5: 1984 Question 5 (1 mark)

What is the power developed by the engine when the car is travelling at 20 m s<sup>-1</sup>?

#### Newton's Laws of motion

#### 1<sup>st</sup> Law

Every object remains at rest, or in uniform motion in a straight line (the velocity may be a non-zero constant), unless acted on by some external force.



A cyclist pedals up a 15° slope at a constant speed as shown. The total mass of rider and bicycle is 100 kg.

### Example 1.6: 2001 Question 1 (2 marks)

What is the magnitude of the **net** force on the bicycle and rider when travelling up the slope?

## Example 1.7: 2001 Question 3 (3 marks)

Sometime later the rider turns around and rolls at constant speed down the slope. Calculate the magnitude of the total frictional forces that are acting on the rider and bicycle while travelling down the slope.

### Newton's 2<sup>nd</sup> Law

The rate of change of momentum is equal to the resultant force causing the change.

$$\mathbf{F} = \frac{m\mathbf{v} - m\mathbf{u}}{t}$$
 since  $\mathbf{a} = \frac{\mathbf{v} - \mathbf{u}}{t}$   $\therefore$   $\mathbf{F} \cdot \mathbf{t} = m\mathbf{v} - m\mathbf{u}$ 

Impulse (change in momentum) i.e.  $I = p_2 - p_1$ . is always the area under F-t graph. When the force is constant, it also equals F $\Delta$ t.

Area under "**F** - t" graph = Impulse =  $\triangle$  momentum = **F**  $\triangle$  t

# TOTAL MOMENTUM BEFORE IMPACT EQUALS THE TOTAL MOMENTUM AFTER IMPACT.

: **P**<sub>(total)</sub> is constant before, during and after the collision.

### Example 1.8: 2002 Question 7 (2 marks)

A car travelling at 11.0 m s<sup>-1</sup> collides head-on with a concrete pillar. The car comes to rest in a time of 0.10 s. The car comes to rest against the pillar. The mass of the car and occupants is 1.30 tonne. Determine the average force on the car during the impact with the pillar.

## Example 1.9: 2002 Question 8 (3 marks)

Explain how the crumple zone of the car can minimise the extent of injuries experienced by the occupants of the car. (Assume that the occupants are wearing seatbelts.)

#### Newton's 3<sup>rd</sup> Law

Action and reaction are equal and opposite. The force on A by B is equal and opposite to the force on B by A.

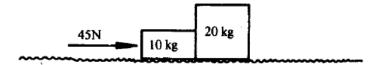
This is summarised as  $F_{on A by B} = -F_{on B by A}$  (the minus sign is to give the direction of the force vector).

For the forces to be an action reaction pair, the subjects of the two statements need to be interchanged.

This concept is often tested using a book on a table. The action reaction pair of forces are: the weight force of the **on the book by Earth**, and the force of the **on the Earth by the book**. Many students get this incorrect, because they use the normal reaction as the other part of the pair.

Two masses, 10 kg and 20 kg, are placed in contact on a rough surface as shown. A person exerts a force of 45 N on the 10 kg mass.

The magnitude of the frictional force acting on the 10 kg mass is 10 N and the magnitude of the frictional force acting on the 20 kg mass is 20 N.



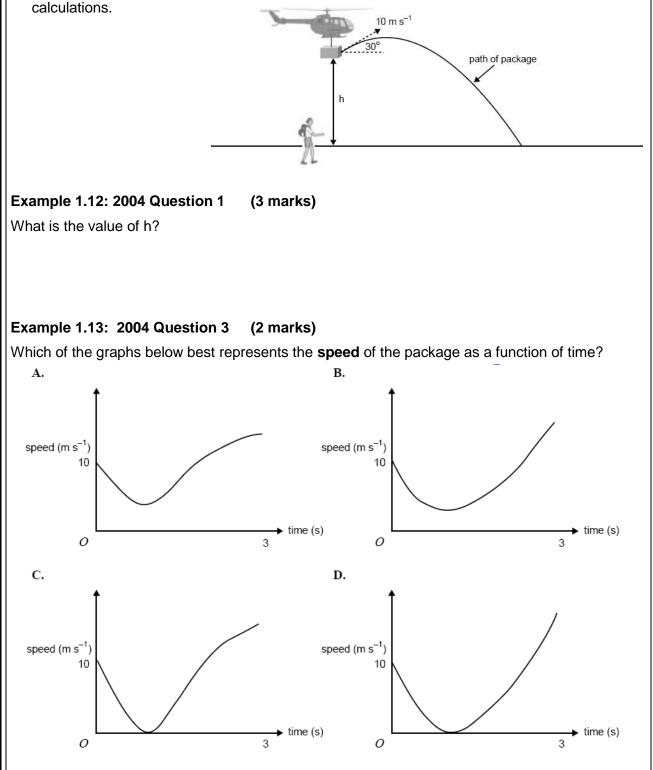
### Example 1.10: 1984 Question 28 (1 mark)

What is the acceleration of the system of two masses?

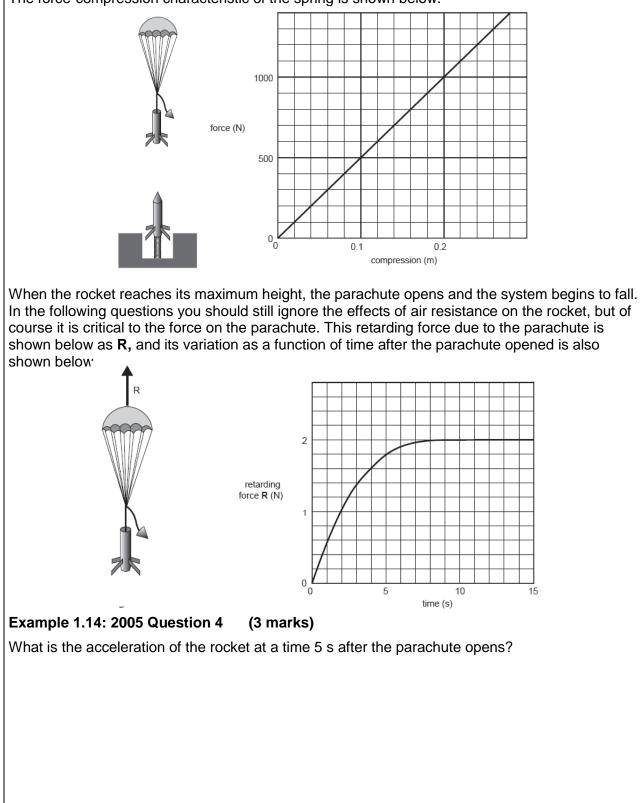
## Example 1.11: 1984 Question 29 (1 mark)

What is the force exerted by the 20 kg mass on the 10 kg mass while they are in motion?

A bushwalker is stranded while walking. Search and rescue officers drop an emergency package from a helicopter to the bushwalker. They release the package when the helicopter is a height (h) above the ground, and directly above the bushwalker. The helicopter is moving with a velocity of 10 m s<sup>-1</sup> at an angle of 30° to the horizontal, as shown below. The package lands on the ground 3.0 s after its release. Ignore air resistance in your calculations.



A model rocket of mass 0.20 kg is launched by means of a spring, as shown below. The spring is initially compressed by 20 cm, and the rocket leaves the spring as it reaches its natural length. The force-compression characteristic of the spring is shown below.



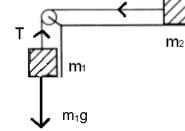
### **Connected Bodies**

Problems involving the motion of two bodies connected by strings are solved on the following assumptions; the string is assumed light and inextensible so its weight can be neglected and there is no change in length as the tension varies. In these cases, tension is the condition of a body subjected to equal but opposite forces which attempt to increase its length, and tension forces are pulls exerted by a string on the bodies to which it is attached.

To solve these problems, consider the vertical direction first.  $m_1g-T=m_1a \label{eq:mass}$ 

The direction of this acceleration must be downwards. This leads to:  $T=m_1g-m_1a$ 

The tension in the string is the same in both directions, therefore  $T = m_2 a$ .



Т

Since both bodies are connected by an inextensible string,

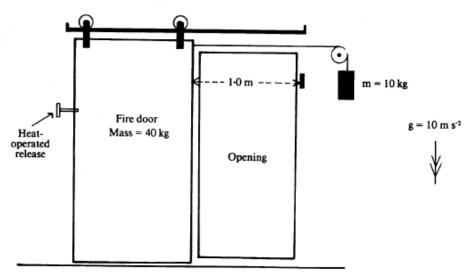
both bodies must have the same acceleration.

The vertical forces acting on m<sub>2</sub>, (not shown) cancel each other out, and do not impact on its motion.

Combing these two equations gives  $a = \frac{m_1}{m_1 + m_2}g$ 

$$T = \frac{m_1 m_2}{m_1 + m_2} g$$

A fireproof door of mass 40 kg closes across an opening 1.0 m wide, in an emergency. Heat from the fire causes the door to be released and it's then moved by a falling mass of 10 kg as shown in the diagram. Ignore friction.

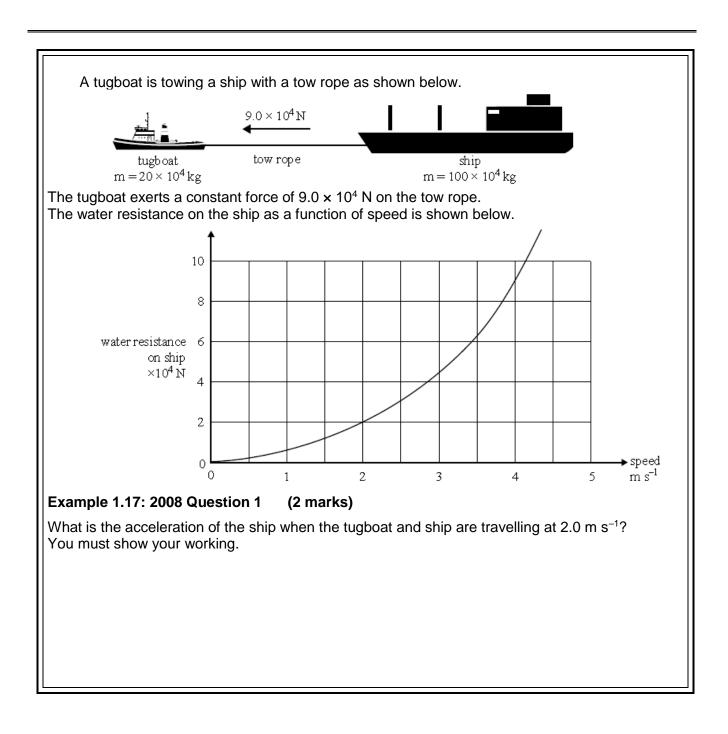


## Example 1.15: 1987 Question 13 (1 mark)

What is the acceleration of the door when it is released?

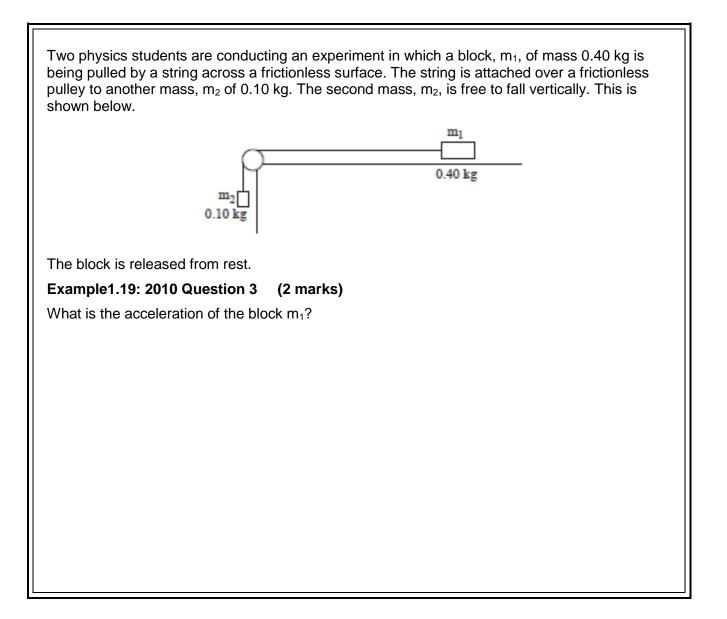
# Example 1.16: 1987 Question 14 (1 mark)

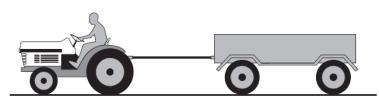
Calculate the time (in seconds) it takes the door to fully close after it is released.



## Example 1.18: 2008 Question 2 (2 marks)

After a time, the tugboat and ship are travelling at a constant speed. What is this constant speed?



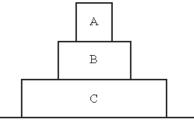


A tractor, including the driver, has a mass of 500 kg and is towing a trailer of mass 2000 kg as shown above. The tractor and the trailer are accelerating at 0.50 m s<sup>-2</sup>. Ignore any retarding friction forces. Ignore the mass of the towing rope. The tractor and the trailer start from rest.

Example 1.20: 2011 Question 2 (2 marks)

What is the tension in the rope connecting the tractor and trailer?

Three children's toy blocks A (0.050 kg), B (0.10 kg) and C (0.20 kg), are sitting on a table as shown in the diagram below.



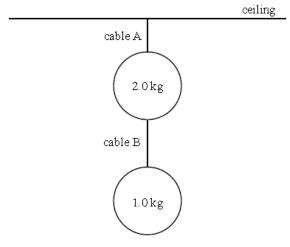
## Example 1.21: 2011 Question 7 (2 marks)

What is the force by block C on Block B? Explain your answer in terms of Newton's laws.

## Example 1.22: 2011 Question 8 (2 marks)

The table is now removed and the blocks free fall. What is now the force by block B on Block C? Ignore air resistance. Explain your answer.

Two metal spheres hang from the ceiling as shown below. Cable A runs between the ceiling and the upper sphere of mass 2.0 kg. Cable B runs between the 2.0 kg sphere and the 1.0 kg sphere. Assume that the cables have no mass.

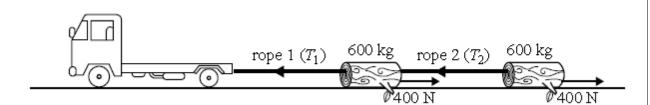


#### Example 1.23: 2012 Question 4b (2 marks)

Newton's third law is sometimes stated as 'To every action there is an equal and opposite reaction'. If the weight (the gravitational force by Earth) of the 2.0 kg sphere is taken as the 'action' force, identify the corresponding 'reaction' force and give its direction.

A truck is dragging two logs along level ground in a straight line. The mass of each log is 600 kg and each log experiences a constant retarding friction force of 400 N with the ground.

The connections between the truck and the logs are made with ropes that have a breaking force of 2 400 N.  $T_1$  and  $T_2$  are the tensions in the ropes as shown below. The truck and the logs are moving towards the **left**.



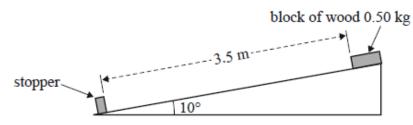
## Example 1.24: 2012 Question 5d (3 marks)

The ropes have a breaking force of 2 400 N.

Rope 1 connects the truck to the front log and rope 2 connects the two logs.

The truck, still on level ground, increases its acceleration until one of the ropes is about to break. Identify which rope is about to break, and calculate the magnitude of the acceleration of the truck and the logs at this instant.

Students set up an inclined plane surface, as shown below. It is angled at 10° to the horizontal. They release the block of wood, of mass 0.50 kg at a distance of 3.5 m from the stopper. When the block is sliding down the incline, there is a constant frictional force between it and the surface. They find that it takes 6.0 s to reach the stopper at the bottom.



### Example 1.25: 2013 Question 1b (3 marks)

Calculate the magnitude of the frictional force of the plane surface acting on the block.

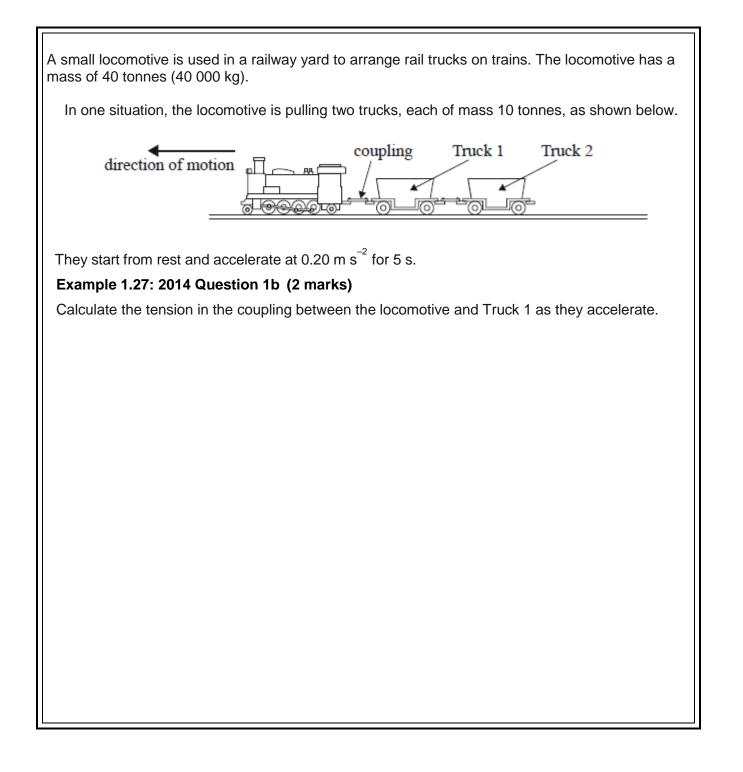
Students set up an experiment that consists of two masses,  $m_1$ , of 2.0 kg, and  $m_2$ , of 6.0 kg, connected by a string, as shown below. The mass of the string can be ignored. The surface is frictionless. The pulley is frictionless.



At the start of the experiment, the bottom of mass  $m_1$  is 1.2 m above the floor and both masses are stationary.

#### Example 1.26: 2013 Question 2b (2 marks)

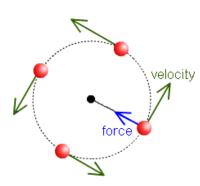
Calculate the tension in the string as  $m_1$  is falling.



# Newton's laws of motion

- investigate and analyse theoretically and practically the uniform circular motion of an object moving in a horizontal plane: ( $F_{net} = \frac{mv^2}{r}$ ), including:
  - a vehicle moving around a circular road
  - a vehicle moving around a banked track
  - an object on the end of a string
- model natural and artificial satellite motion as uniform circular motion
- investigate and apply theoretically Newton's second law to circular motion in a vertical plane (forces at the highest and lowest positions only)

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
4, 5		3, 4	3, 4 12	1, 2, 5, 6, 9	4, 5, 6	7a, b, c	4a, b 5a, b	4c	3a, b 4a, b	2a, b



Uniform circular motion results when a resultant force of constant magnitude acts normal to the motion of the body.

When a body travels in a circle, the force is always towards the centre, while the velocity is tangential. The force is always at right angles to the velocity. The force is often called a tension (particularly when using a string)

A mass moving with a uniform speed in a circular path of radius 'r' and with a period 'T' has an average speed given by

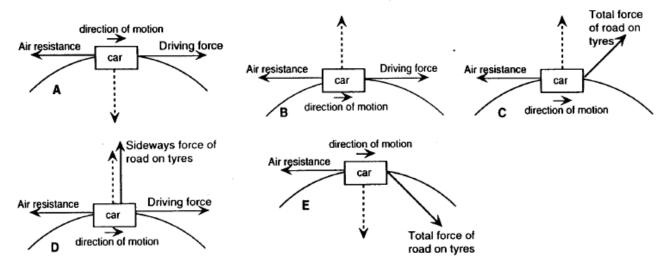
$$\frac{\text{distance travelled}}{\text{time taken}} = \frac{2\pi r}{T}$$
$$\Sigma F = ma$$
$$= \frac{mv^2}{r}$$
$$= m\frac{4\pi^2 r}{T^2}$$
$$\therefore a = \frac{v^2}{r}$$
$$= \frac{4\pi^2 r}{T^2}$$

In circular motion the movement is always at right angles to the force, since work done = force  $\times$  distance moved in the direction of the force. The work done in circular motion is zero.

A car is travelling in a horizontal path around a circular curve. The car's speed is constant and it is travelling from left to right in the diagrams below.

## Example 1.28: 1992 Trial (2 marks)

Which of the following diagrams best shows the horizontal forces (shown as solid lines) and their resultant force (shown as a dashed line)? Give reasons for your choice.

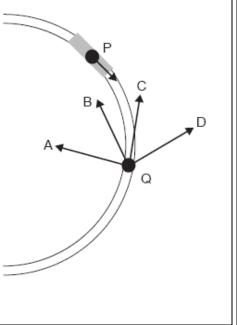


The safe speed for a train taking a curve on level ground is determined by the force that the rails can take before they move sideways relative to the ground. From time to time trains derail because they take curves at speeds greater than that recommended for safe travel.

The figure below shows a train at position P taking a curve on horizontal ground, at a constant speed, in the direction shown by the arrow.

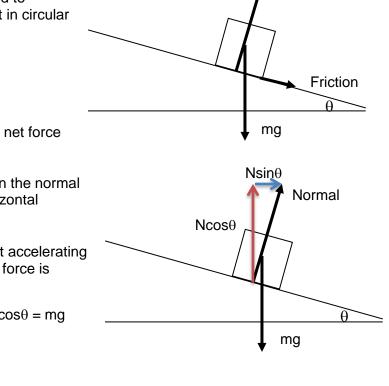
#### Example 1.29: 2005 Question 7 (2 marks)

At point Q the driver applies the brakes to slow down the train on the curve. Which of the arrows (A - D) indicates the direction of the **net** force exerted **on the wheels** by the rails?



## Banked surfaces

Surfaces (eg. roads, velodromes) are banked to reduce the need for frictional forces to assist in circular motion. The forces acting are shown below.



For the object to travel in circular motion the net force acting needs to be radially inwards.

If we want the frictional force to be zero, then the normal force is resolved into perpendicular and horizontal components.

Here N  $cos\theta$  = mg (because the object is not accelerating in the vertical direction) and the unbalanced force is N  $sin\theta$ 

 $\therefore N \sin\theta = \frac{mv^2}{R}$  dividing this equation by N  $\cos\theta = mg$  gives

$$\frac{\text{Nsin}\theta}{\text{Ncos}\theta} = \frac{\text{mv}^2}{\text{R}} \times \frac{1}{\text{mg}}$$
$$\tan\theta = \frac{\text{v}^2}{\text{Rg}} \text{ this formula}$$

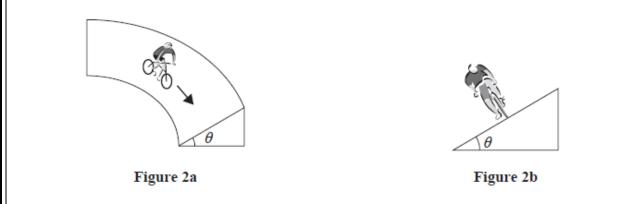
 $n\theta = \frac{v}{Rg}$  this formula is very convenient and should be on your own formula sheet.

In designing a bicycle track at a racing track, the designer wants to bank the track on a particular corner so that the bicycles will go around the corner with no sideways frictional force required between the tyres and the track at 10 m s<sup>-1</sup>.

Figures 2a and 2b below show the banked track and the bicycle.

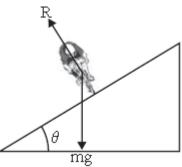
## Example 1.30: 2010 Question 5 (2 marks)

On **Figure 2b** draw two arrows to show the two forces acting on the bicycle and rider (treated as a single object).



Normal

A demonstration at a show involved a bike being ridden around a circular banked track. The horizontal path that the bike takes is a circle of radius 20 m, and the bike travels at a constant speed of 15 m s<sup>-1</sup>. The bike and the rider have a total mass of 300 kg. Ignore retarding friction.



Example 1.31: 2011 Question 5 (2 marks)

The diagram above shows the only two forces on the bike and the rider. On this diagram draw an arrow to show the direction of the net force on the bike and rider. Explain how the two forces shown cause this net effect.

#### Vertical circles

You feel your 'weight' as the normal reaction of the surface on you, because you can only feel things that act on you. So if the normal reaction increases you 'feel' heavier, and if the normal reaction decreases you 'feel' lighter.

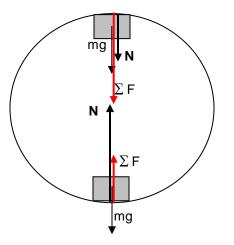
Consider an object travelling around a vertical loop

#### At the top

$$\Sigma F = N + mg = \frac{mv^2}{r}$$

At the bottom

$$\Sigma F = N - mg = \frac{mv^2}{r}$$



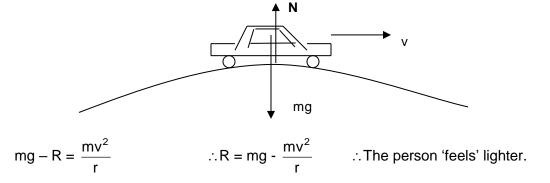
Person on a swing



At the bottom of the swing, the forces on the person are the reaction from the seat and the weight force.

R - mg =  $\frac{mv^2}{r}$ . In this case R > mg, so the person 'feels' heavier.

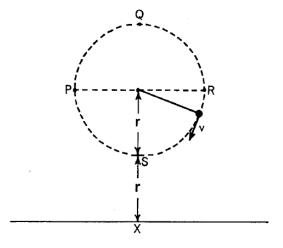
Person in a car going over a hump.



An object is rotated in a vertical circle of radius r at constant speed of v. The centre of the circle is at height 2r above a horizontal surface. The acceleration due to gravity is 'g'.

#### Example 1.32: 1977 Question 15 (1 mark)

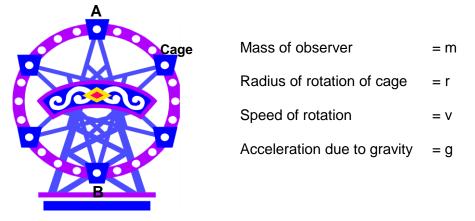
If the object is released at point P, how far will it land from point X?



## Example 1.33: 1977 Question 16 (1 mark)

If instead, the object were released from point Q, how far would it land from point X?

A fairground amusement machine consists of cages turned at a constant speed in a vertical circle. The cage just clears the ground at the bottom of its path. The cage pivots during the motion, so that an observer in the cage always remains in a vertical standing position.



Use either A or B to answer the following question. If neither position is appropriate, write N.

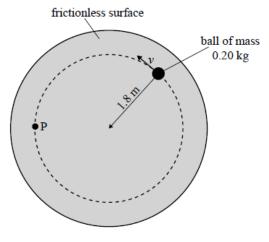
## Example 1.34: 1978 Question 18 (1 mark)

At which position(s) will the observer exert a force on the cage floor *greater than* his/her normal weight? (One or more answers)

## Example 1.35: 1978 Question 19 (1 mark)

By how much will the force on the cage floor by the observer differ between position A and B?

Kim has attached a ball of mass 0.20 kg to a piece of string of length 1.8 m and is making it move in a **horizontal** circle on a frictionless surface. Kim gradually increases the speed, *v*, of the ball. The situation is shown from above in the diagram below. The string has a breaking force of 4.0 N.



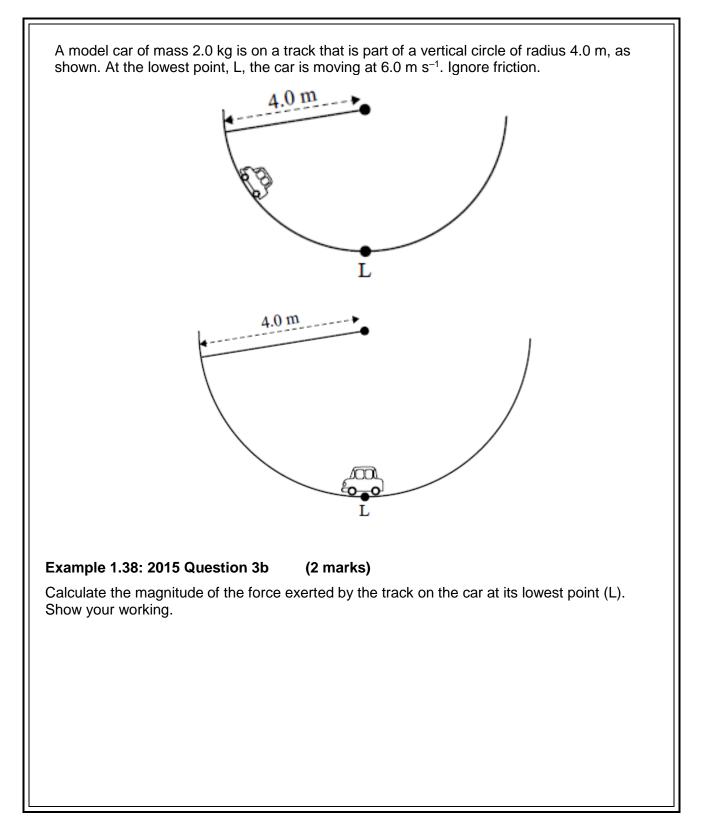
### Example 1.36: 2012 Question 7c (3 marks)

On another occasion, Kim swings the same ball of mass of 0.20 kg in a **vertical** circle at constant speed. She uses a much stronger string, which does not break.

She notices that the tension in the string is greater at the bottom of the circle than it is at the top of the circle.

Explain why the tension at the bottom is greater than the tension at the top. You may include a diagram as part of your explanation.

A mass of 2.0 kg is being swung by a light rod in a vertical circle of radius 1.0 m at a constant speed of 7.0 m s<sup>-1</sup>, as shown below. light rod Q  $v = 7.0 \text{ m s}^{-1}$ (011 Ρ R  $\mathbf{S}$ Example 1.37: 2013 Question 5b (3 marks) Calculate the magnitude of the tension in the light rod at point S. Show the steps of your working.



A satellite moves in uniform circular motion around Earth.

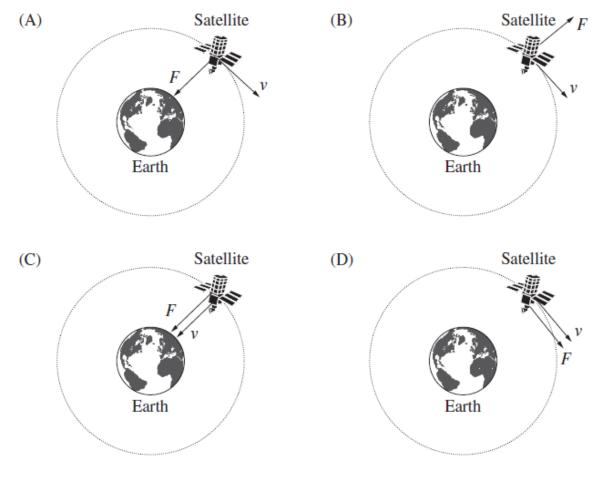
The following table shows the symbols used in the diagrams below. These diagrams are NOT drawn to scale.

## Key

F	net force on satellite
v	velocity of satellite

## Example 1.39: NSW 2000 Question 2 (1 mark)

Which diagram shows the direction of F and v at the position indicated?



## Newton's laws of motion

## Investigate and analyse theoretically and practically the motion of projectiles near Earth's surface, including a qualitative description of the effects of air resistance

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
6, 7, 8	14,15, 16, 17	5, 6, 7	10, 11	10, 11	12, 13	6a, b	8a, b	3a, b	5a, b	5a, b

Projectile motion is motion under a constant unbalanced force. A projectile is a body that has been thrown or projected. Air resistance is to be considered as negligible in the quantitative questions involving calculations, but may need to be included in qualitative questions.

### For both the vertical and inclined projectiles:

- the only force acting is the weight, ie. the bodies are in free fall, acceleration is always 10 m s<sup>-2</sup>, including at the top
- instantaneous velocity is tangential to the path
- the total energy (KE & PE) is constant, between any two points  $\Delta KE = -\Delta PE$
- paths are symmetrical for time, if air resistance can be ignored

## Inclined or oblique projections

- the only force acting is vertically down, so the acceleration and change in velocity are vertical
- · horizontally there is no component of force, so constant horizontal velocity
- maximum range is when angle of projection is 45°

## Horizontal projection

For projectiles thrown horizontally and dropped from rest, the vertical motions are the same.

Horizontal:	velocity always = Vhorizontal	Vertical:	Velocity changing v = u - gt
	acceleration $= 0$		acceleration = -g
	displacement = $x = v_{horizontal} \times t$		displacement $y = ut - \frac{1}{2} gt^2$

To find the 'total' velocity, add  $v_{vertical}$  and  $v_{horizontal}$  using vectors.

## Symmetrical flights

If there is no air resistance, and the projectile starts and ends at the same height, then the range is

given by:  $R = \frac{v^2 \sin 2\theta}{g}$  R is the range, v is the initial speed and  $\theta$  the angle of projection.

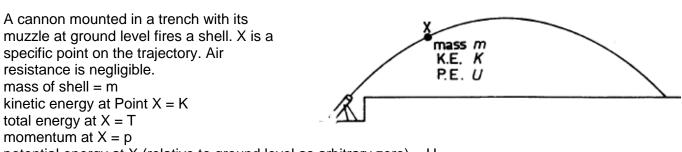
# Total Energy (TE)

If air resistance is negligible, then the total energy is constant. TE = KE + PE. At ground level PE = 0, so TE = KE. As the projectile rises it gains PE, so it must lose KE. At the top of its flight, the PE is maximum and the KE is minimum. (KE is not zero, because the projectile still has some KE due to its horizontal motion).

At any point on the way up or the way down, the TE is constant. If you know the horizontal component of the velocity, then you can use this to find the maximum height, by using the TE at ground level and working out what he PE must be at the top when  $v_{vertical} = 0$ , but  $v_{horizontal} = constant$ .

## Projectile problem methodology

- 1. Draw a fully labelled diagram
- 2. Treat the motion as two separate motions, horizontal and vertical. List the data under vertical and horizontal
- 3. For vertical motion use,  $v^2 = u^2 + 2ax$ , and v = u + at. (use a = -q)
- 4. To go from one direction to another, the common link is the time of flight, t.
- 5. Remember that it will take the same time to go up as to come down (if air resistance can be ignored).
- 6. Label the direction (+ or -) for all variables except time.
- 7. Sometimes the question is phrased 'with a strong tail wind.' This means that the tail wind cancels out any effects of air resistance.



potential energy at X (relative to ground level as arbitrary zero) = U

#### Example 1.40: 1971 Question 4 (1 mark)

Write an expression for K in terms of p and m only.

#### Example 1.41: 1971 Question 5 (1 mark)

The initial kinetic energy of the shell is

Α. Τ	В.	T+U	C.	T - U
<b>D.</b> T + K	E.	Т – К	F.	not determinable from the

#### Example 1.42: 1971 Question 6 (1 mark)

The velocity of the shell at its maximum height is

- A. Zero В. C. T – K Ε. F.
- information given.

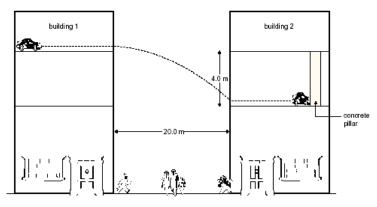
p m

dependent on the angle of elevation of the cannon.

In the movie, *Car Escape*, Taylor and Jones drove their sports car across a horizontal car park in building 1 and landed it in the car park of building 2, landing one floor lower.

Building 2 is 20 metres from building 1, as shown.

The floor where the car lands in building 2 is 4.0 m below the floor from which it started in building 1. In the questions below, treat the car as a point particle and assume air resistance is negligible.



### Example 1.43: 2002 Question 5 (3 marks)

Calculate the minimum speed at which the car should leave building 1 in order to land in the car park of building 2.

## Example 1.44: 2002 Question 6 (2 marks)

In order to be sure of landing in the car park of building 2, Taylor and Jones in fact left building 1 at a speed of 25 m s<sup>-1</sup>. Calculate the magnitude of the **velocity** of the car just prior to landing in the car park of building 2.

A rocket of mass 0.50 kg is set on the ground, pointing vertically up, as shown below. When ignited, the gunpowder burns for a period of 1.5 s, and provides a constant force of 22 N. The mass of the gunpowder is very small compared to the mass of the rocket, and can be ignored. The effects of air resistance can also be ignored.

#### Example 1.45: 2006 Question 6 (2 marks)

What is the magnitude of the resultant force on the rocket?

# Example 1.46: 2006 Question 7 (2 marks)

After 1.5 s, what is the height of the rocket above the ground?

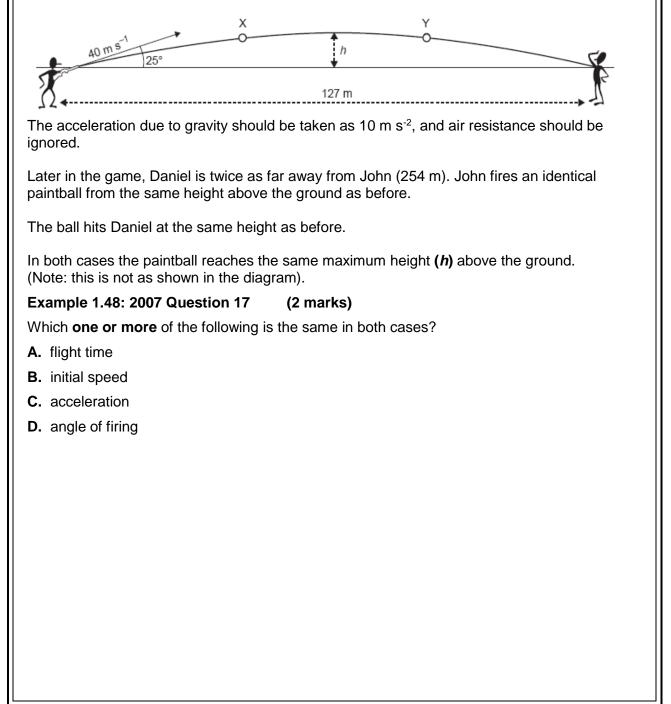
A second identical rocket, that again provides a constant force of 22 N for 1.5 s, is now launched horizontally from the top of a 50 m tall building. Assume that in its subsequent motion the rocket always points horizontally.

#### Example 1.47: 2006 Question 8 (4 marks)

After 1.5 s, what is the speed of the rocket, and at what angle is the rocket moving relative to the ground?

Daniel and John are playing paintball. Daniel fires a 'paintball' at an angle of  $25^{\circ}$  to the horizontal and a speed of  $40.0 \text{ m s}^{-1}$ . The paintball hits John, who is 127 m away.

The height at which the ball hits John and the height from which the ball was fired are the same. The situation is shown below.

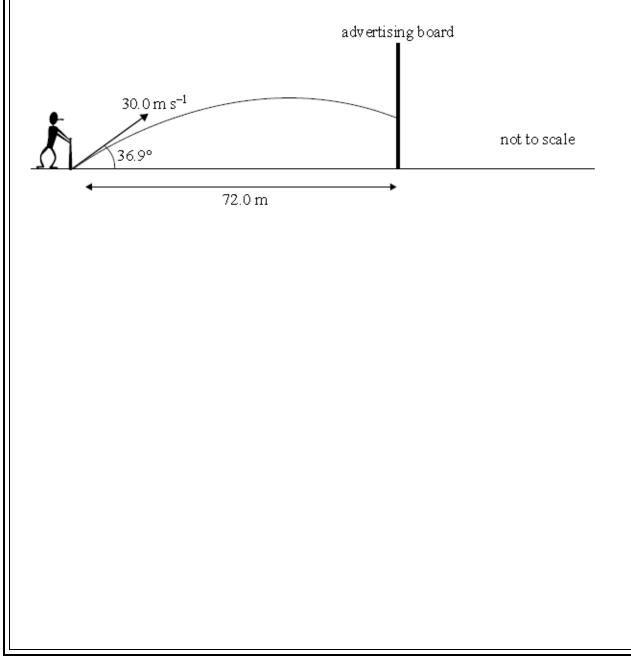


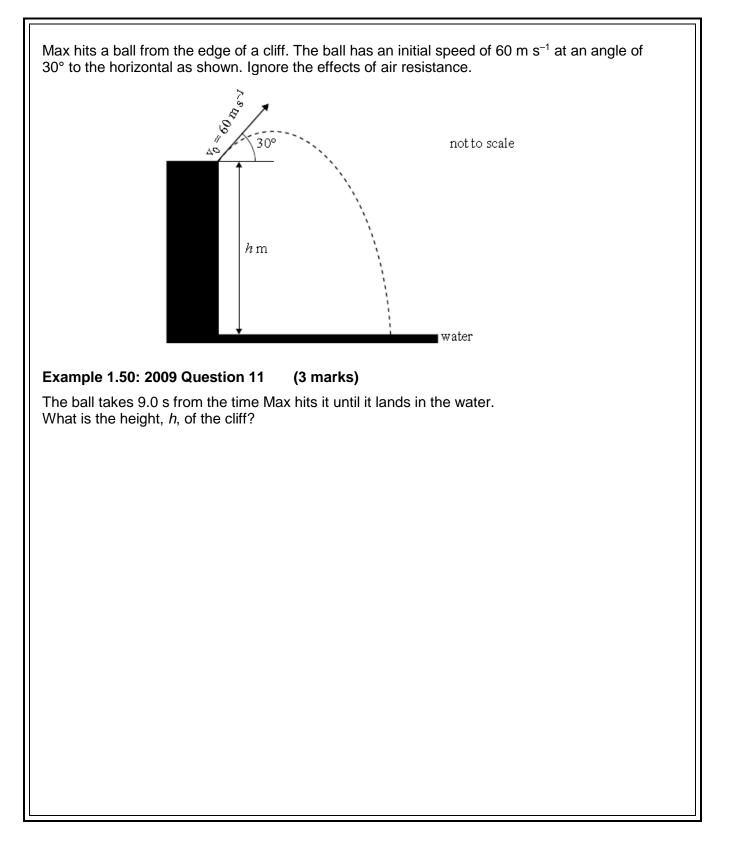
A batsman hits a cricket ball (from ground level) at a speed of 30.0 m s<sup>-1</sup> and at an angle of 36.9° to the horizontal.

An advertising board is now placed on the boundary of the cricket ground at a distance 72.0 m from the batsman as shown below. Air resistance can be ignored.

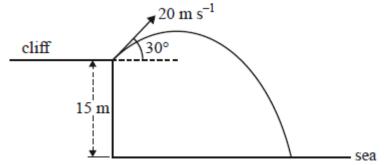
#### Example 1.49: 2008 Question 7 (3 marks)

Assuming the ball is hit exactly the same way, at what height above the ground will the ball strike the advertising board? You must show your working.





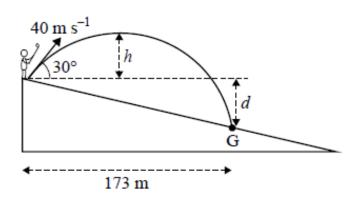
A stone is thrown from the top of a 15 m high cliff above the sea at an angle of  $30^{\circ}$  to the horizontal. It has an initial speed of 20 m s<sup>-1</sup>. The situation is shown below. Ignore air resistance.



## Example 1.51: 2013 Question 8b (3 marks)

Calculate the magnitude and direction of the velocity of the stone immediately before it reaches the sea. Give the direction as the magnitude of the angle between the velocity and the horizontal.

A golfer hits a ball on a part of a golf course that is sloping downwards away from him, as shown.



The golfer hits the ball at a speed of 40 m s<sup>-1</sup> and at an angle of 30° to the horizontal. Ignore air resistance.

## Example 1.52: 2015 Question 5b (3 marks)

The ball lands at a point at a horizontal distance of 173 m from the hitting-off point, as shown. Calculate the vertical drop, *d*, from the hitting-off point to the landing point, G.

## Newton's laws of motion

 investigate and apply theoretically and practically the laws of energy and momentum conservation in isolated systems in one dimension.

		2011	2012	2013	2014	2015	2016
3, 10,         3, 4, 5         8, 9           11         10, 7	12	14, 15	2	3a, b, c	1c	1a, c	4c

Momentum is conserved in all collisions. If a collision is elastic, kinetic energy is also conserved. If KE is lost, the collision is inelastic. The energy that has gone 'missing' is usually converted into heat, sound or energy of deformation.

Momentum equation  $\mathbf{p} = \mathbf{m} \times \mathbf{v}$ .

#### Momentum transfer involving the Earth

#### Vertical

- 1. Body rises under gravity slows down and loses momentum to the earth.
- 2. Body falling under gravity speeds up giving the earth equal and opposite momentum change.
- 3. Falling body hits the ground its **p** is transferred to the earth.

#### Horizontal

- 4. Body slowed due to friction gives the earth and equal and opposite **p**.
- 5. Body accelerated due to friction gives the earth an equal and opposite **p**.

A small truck of mass 3.0 tonne collides with a stationary car of mass 1.0 tonne. They are locked together as they move off. The speed immediately after the collision was known to be 7.0 m s<sup>-1</sup> from the jammed reading on the car speedometer. Robin, one of the police investigating the crash, uses 'conservation of momentum' to estimate the speed of the truck before the collision.

Example 1.53: 2005 Question 8 (3 marks)

What value did Robin obtain?

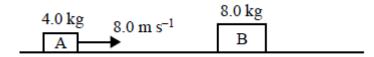
# Example 1.54: 2005 Question 9 (2 marks)

The calculated value is questioned by the other investigator, Chris, who believes that 'conservation of momentum' only applies in elastic collisions.

Explain why Chris's comment is wrong.

	A 1.2 kg block moves to the right along the frictionless surface and collides elastically with a stationary block of mass 2.4 kg as shown below.							
	,	•	stationary					
Before	1.2 kg		2.4 kg	frictionless surface				
After the elas	tic collision, the 1.2	kg block moves	to the left as sho	wn.				
After	1.2 kg		2.4 kg	<ul> <li>frictionless surface</li> </ul>				
After the colli the 1.2 kg blc Explain why t	ock had before the	m of the 2.4 kg blo collision.	ock is <b>greater</b> th	an the momentum that				

Block A, of mass 4.0 kg, is moving to the right at a speed of 8.0 m s<sup>-1</sup>, as shown below. It collides with a stationary block, B, of mass 8.0 kg, and rebounds to the left. Its speed after the collision is 2.0 m s<sup>-1</sup>.



Example 1.56: 2015 Question 1a (2 marks)

Calculate the speed of block B after the collision.

# Relationships between force, energy and mass

# • investigate and analyse theoretically and practically impulse in an isolated system for collisions between objects moving in a straight line: $F\Delta t = m\Delta v$

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
10, 11	3, 4, 5	8, 9, 10, 11	1	12	14, 15	2	3a, b, c	1c	1a, c	1c, e

Consider a body of mass 'm' changing its velocity from 'u' to 'v' in time 't' under the action of a constant force 'F'.

 $\mathbf{F} = \mathbf{ma} \qquad \text{since} \quad \mathbf{a} = \frac{(\mathbf{v} - \mathbf{u})}{\mathbf{t}}$  $\mathbf{F} = \frac{\mathbf{m}\mathbf{v} - \mathbf{m}\mathbf{u}}{\mathbf{t}}$  $\mathbf{F} = \frac{\mathbf{m}\Delta\mathbf{v}}{\mathbf{t}}$  $\therefore \mathbf{F}\Delta\mathbf{t} = \mathbf{m}\Delta\mathbf{v}$ 

Since a net force gives a change in momentum over a change in time, it can be written that a change in momentum is given by:  $\Delta p = F\Delta t$ 

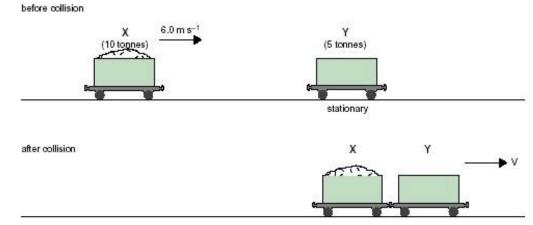
The product of a constant force and the time for which it acts is called the **IMPULSE (I)** of the force.  $I = \Delta p = F \Delta t$ . The unit is Newton second, (but this is identical to kg m s<sup>-1</sup>).

 $\therefore \mathbf{I} = \mathbf{p}_2 - \mathbf{p}_1.$ 

The impulse of a force can be measured by the change in momentum. **Impulse** and **momentum** are vector quantities. Both impulse and momentum have the same units.

Note: NO collision can result in an INCREASE in the TOTAL KINETIC ENERGY of a system.

A moving railway truck (X) of mass 10 tonnes, moving at 6.0 m s<sup>-1</sup>, collides with a stationary railway truck (Y) of mass 5.0 tonnes. After the collision they are joined together and move off as one. This situation is shown below.



# Example 1.57: 2002 Question 9 (2 marks)

Calculate the final speed of the joined railway trucks after the collision.

## Example 1.58: 2002 Question 10 (2 marks)

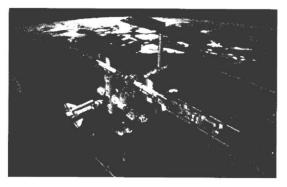
Calculate the magnitude of the total impulse that truck Y exerts on truck X during the collision.

#### Example 1.59: 2002 Question 11 (3 marks)

Explain why this is an example of an inelastic collision. Calculate numerical values to justify your answer.

The figure below shows a space shuttle docking with the International Space Station. Imagine that you are an astronaut floating in space at rest relative to the International Space Station. You watch the space shuttle, of mass 6000 kg, dock. You observe the shuttle approaching the space station with a speed of  $5.00 \text{ m s}^{-1}$ .

After docking, the space station's speed has increased by 0.098 m s<sup>-1</sup>.

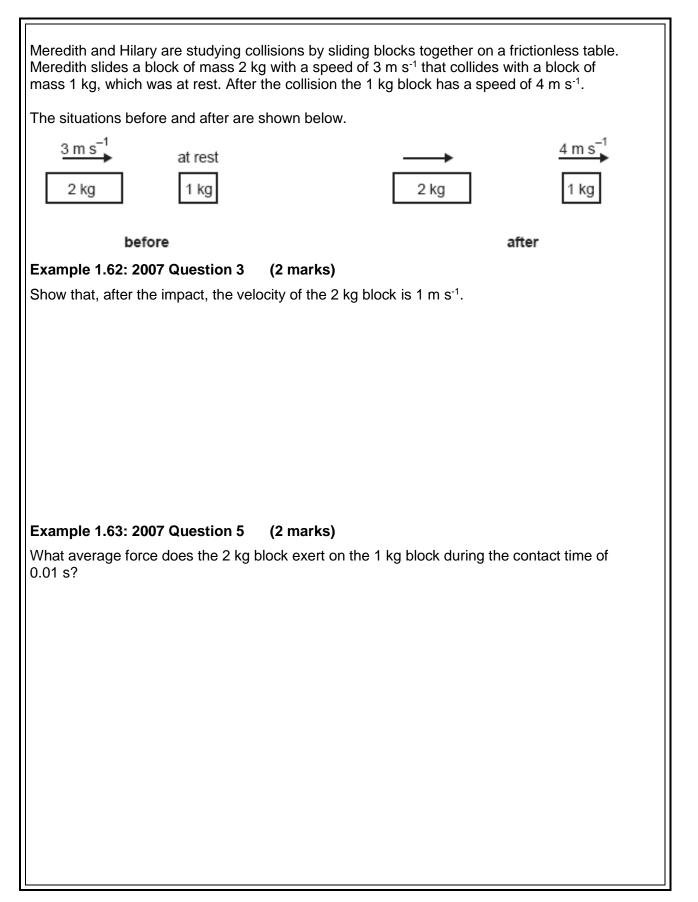


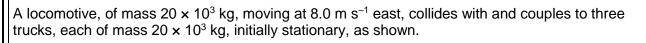
## Example 1.60: 2006 Question 10 (3 marks)

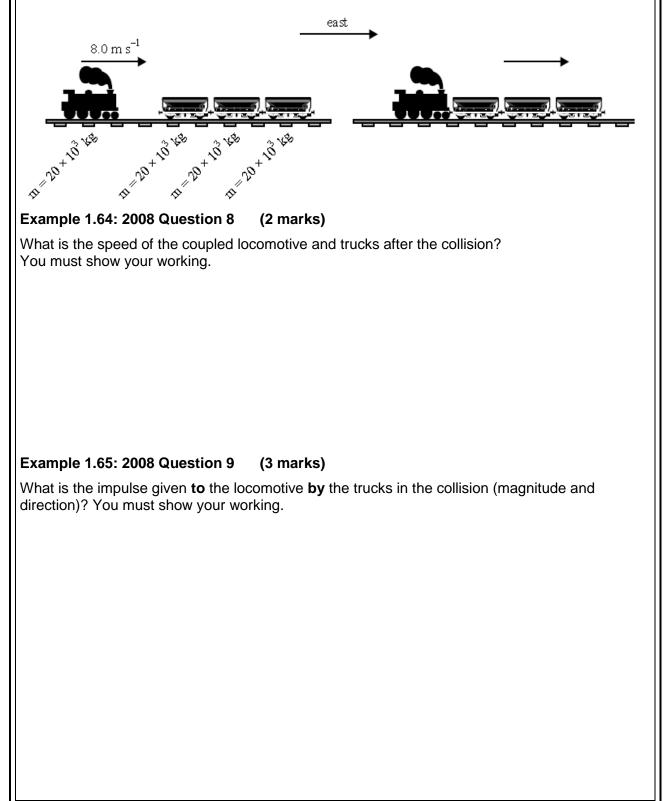
Show that the mass of the space station is  $3 \times 10^5$  kg.

# Example 1.61: 2006 Question 11 (3 marks)

After first making contact, it takes 20 s for the shuttle to come to rest with the space station. Calculate the average force exerted on the shuttle by the space station.







# Relationships between force, energy and mass

- investigate and apply theoretically and practically the concept of work done by a constant force using:
  - work done = constant force × distance moved in direction of net force
  - work done = area under force-distance graph

3 8	4d

A fundamental principle of nature is that energy cannot be created or destroyed, only transformed or transferred to another body. A body that has energy may transfer some, or all, of its energy to another body. The total amount of energy remains constant (conserved), even if it has been transformed to another type. The amount of energy transformed is called **work**. The body losing energy does work, the body gaining energy has work done on it.

Work = Force  $\times$  displacement, it is a scalar quantity, with the units of Joule.

The area under a force-displacement graph shows the work done. If the force is constant then the area under the graph is given by  $\mathbf{W} = \mathbf{F} \times \mathbf{d}$ , where F is the force, d is the distance over which the force acts. This assumes that the force and the displacement are in the same direction. If they aren't then the work is the product of the resolved part of the force (in the direction of motion) × the displacement.

If the force isn't constant, then the work done is equal to the area under the F - d graph.

**Power** is the rate of doing work.  $P = \frac{W}{\Delta t} = \frac{Fd}{\Delta t} = Fv$ . Power (scalar) with units of joule sec<sup>-1</sup> or Watt.

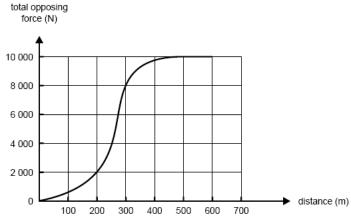
A seaplane of mass 2 200 kg takes off from a smooth lake. It starts from rest, and is driven by a **constant** force generated by the propeller. After travelling a distance of 500 m, the seaplane is travelling at a constant speed, and then it lifts off after travelling a further 100 m.

The total force opposing the motion of the seaplane is not constant. The figure below shows the total force opposing the motion of the seaplane as

a function of the distance travelled.

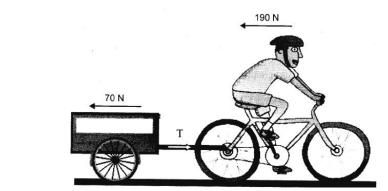
# Example 1.66: 1999 Question 5 (4 marks)

Estimate the work done by the seaplane against the opposing forces in travelling for a distance of 500 m from the start.



A cyclist is towing a small trailer along a level bike track show. The cyclist and bike have a mass of 90 kg, and the trailer has a mass of 40 kg. There are opposing constant forces of 190 N on the rider and bike and 70 N on the trailer. These opposing forces do not depend on the speed of the bike.

The bike and trailer are initially travelling at a constant speed of 6.0 m s<sup>-1</sup>. The cyclist stops pedalling, and the bike and trailer slow down



Example 1.67: 2006 Question 3 (3 marks)

How far will the bike and trailer travel before they come to rest?

# Relationships between force, energy and mass

- analyse transformations of energy between kinetic energy, strain potential energy, gravitational potential energy and energy dissipated to the environment (considered as a combination of heat, sound and deformation of material):
  - kinetic energy at low speeds:  $E_k = \frac{1}{2}mv^2$ ; elastic and inelastic collisions with

reference to conservation of kinetic energy

- strain potential energy: area under force-distance graph including ideal springs obeying Hooke's Law:  $E_s = \frac{1}{2} k\Delta x^2$
- gravitational potential energy:  $E_g = mg\Delta h$  or from area under a force-distance graph and area under a field- distance graph multiplied by mass.

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
	9, 10, 11	12, 13, 14	2, 5, 6	4, 7, 13, 14, 15, 16, 17	16, 17, 18,19, 20	1a, b, c, d	6a, b, c	1d, 2a, b, c, d	1b, 6a, b, c, d	1d, 3a, b, 4a, b

Kinetic Energy (KE), the energy of a body due to its motion.

 $KE = \frac{1}{2} mv^2$  $WD = \Delta KE = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$ 

**Gravitational Potential Energy.** When changes in height 'h' are small compared to the radius of the Earth, the potential energy  $U_g$  of a body near the earth's surface is given by  $U_g = mgh$ .

**Elastic potential energy** (strain energy) is the energy stored in any material that has been stretched or compressed from its normal shape. Springs and elastic bands are good examples. For a spring not stretched beyond its elastic limit, the force, F, applied is proportional to the extension, Δx, produced.

 $F = k\Delta x$  where k is called the spring constant. (the gradient of the F -  $\Delta x$  graph)

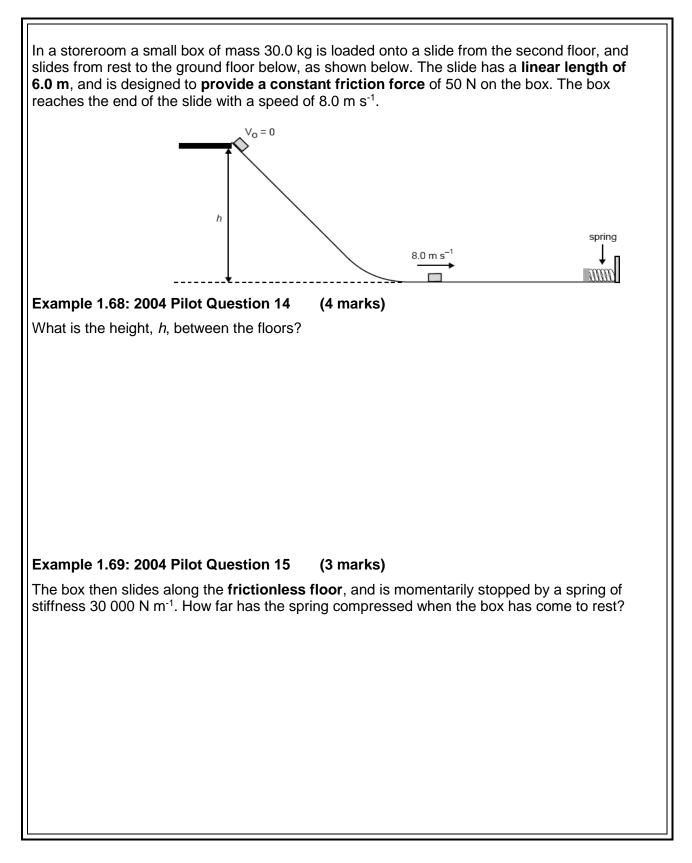
This is known as Hooke's Law. The spring constant (force constant), k, gives a measure of the **stiffness** of the material.

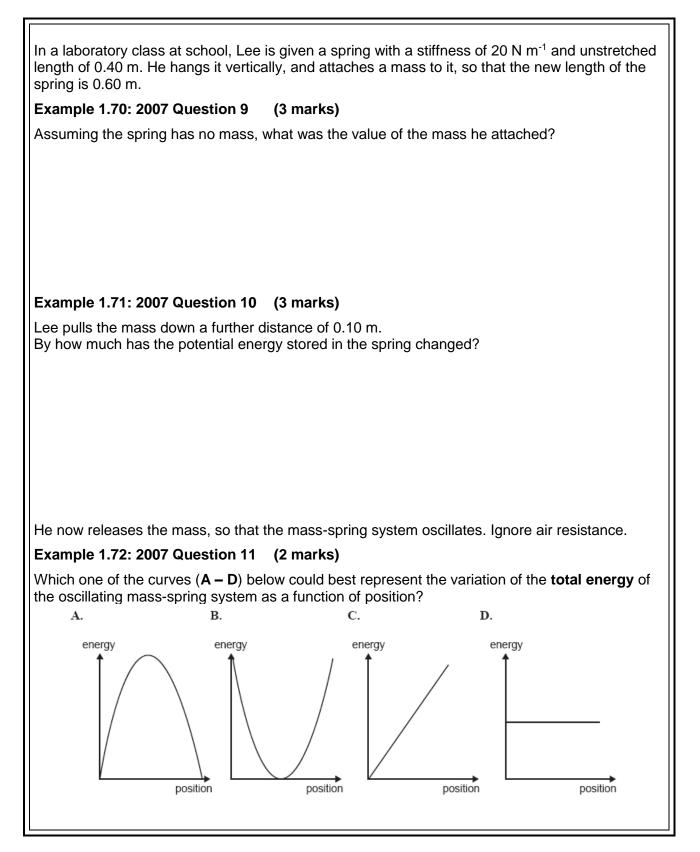
The energy stored in the spring is =  $\frac{1}{2}$  base × height

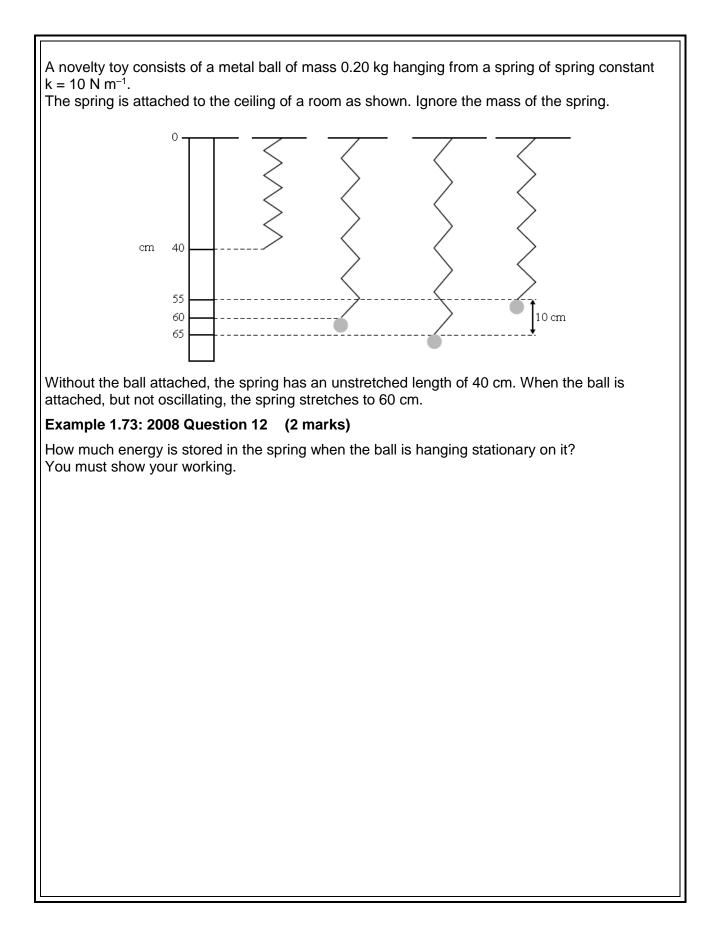
$$=\frac{1}{2}\Delta x.k\Delta x$$

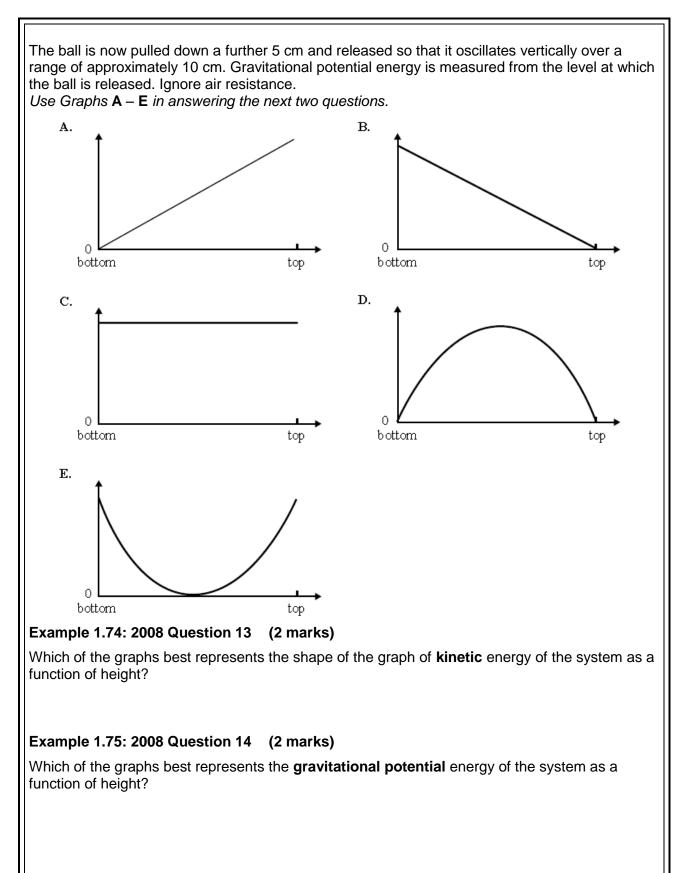
=  $\frac{1}{2} k(\Delta x)^2$  (the strain energy).

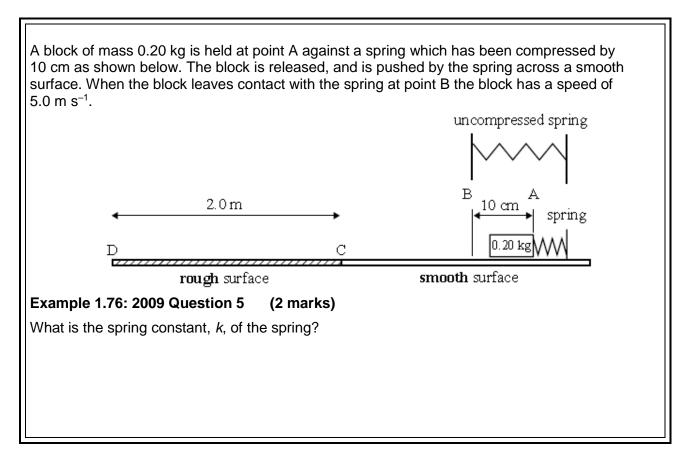
The energy stored in the spring is also the work done, which is the area under the graph.

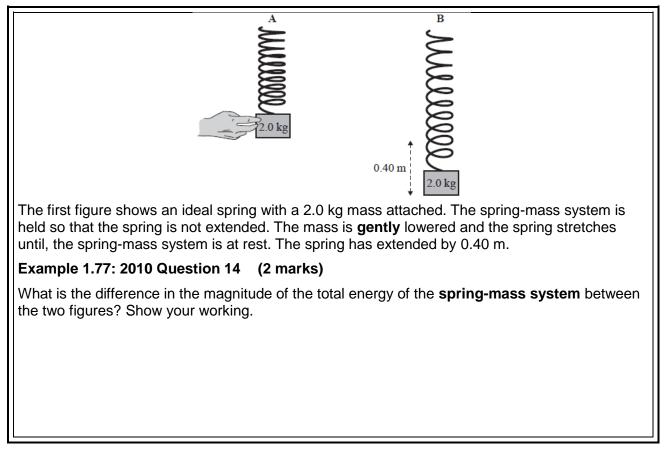








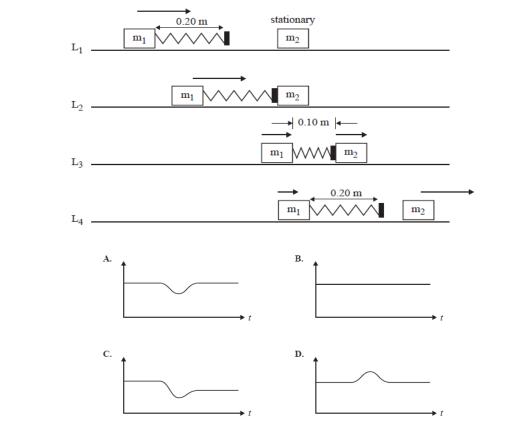




Physics students are conducting a collision experiment using two trolleys,  $m_1$  of mass 0.40 kg and  $m_2$  of mass 0.20 kg.

• Trolley m<sub>1</sub> has a light spring attached to it.

- When uncompressed, this spring has a length 0.20 m.
- Trolley  $m_1$  is initially moving to the right. Trolley  $m_2$  is stationary.
- The trolleys collide, compressing the spring to a length of 0.10 m.
- The trolleys then move apart again, and the spring reverts to its original length (0.20 m), and both trolleys move off to the right.
- The collision is elastic.
- The trolleys do not experience any frictional forces.



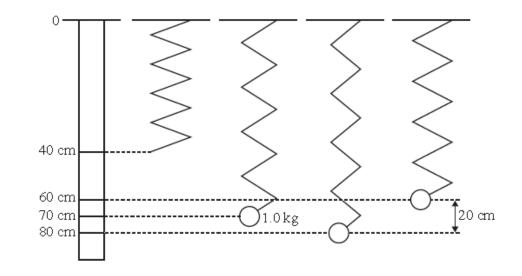
Example 1.78: 2010 Question 15 (2 marks)

Which graph best shows how the total kinetic energy of the system varies with time before, during and after the collision? Explain your answer.

# Example 1.79: 2010 Question 17 (2 marks)

If the collision had been inelastic, which graph would best show how the magnitude of the total momentum of the system varies with time before, during and after the collision? Explain your answer.

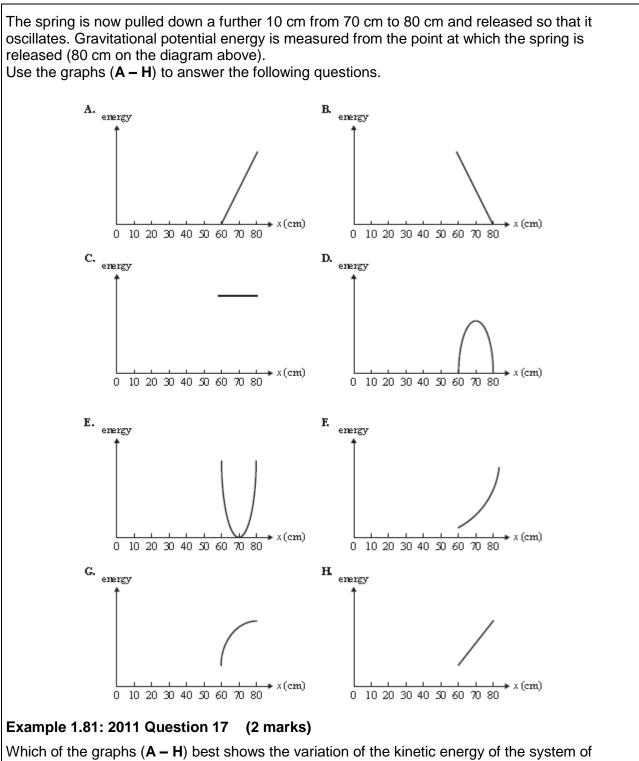
Physics students are conducting an experiment on a spring that is suspended from the ceiling. Ignore the mass of the spring.



Without the mass attached, the spring has an unstretched length of 40 cm. A mass of 1.0 kg is then attached. When the 1.0 kg mass is attached, with spring and mass stationary, the spring has a length of 70 cm.

#### Example 1.80: 2011 Question 16 (2 marks)

What is the spring constant, k, of the spring?



Which of the graphs (A - H) best shows the variation of the kinetic energy of the system o spring and mass plotted against the length of the stretched spring?

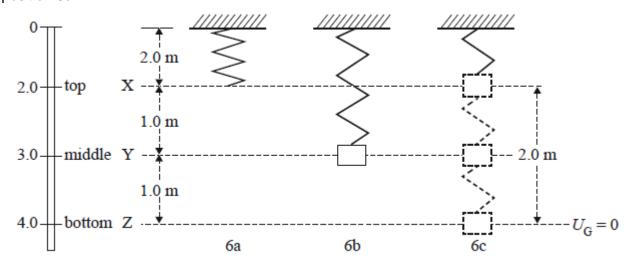
## Example 1.82: 2011 Question 19 (2 marks)

Which of the graphs (A - H) best shows the variation of the gravitational potential energy of the system of spring and mass (measured from the lowest point as zero energy) plotted against the length of the stretched spring?

#### Example 1.83: 2011 Question 20 (3 marks)

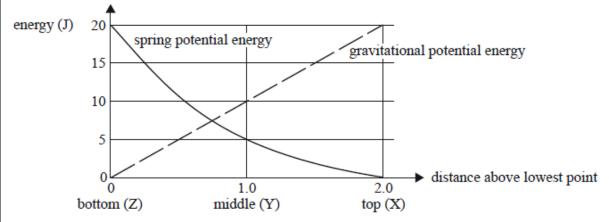
Which of the graphs (A - H) best shows the variation of the spring (strain) potential energy plotted against the length of the stretched spring? Give reasons for choosing this answer for the spring (strain) potential energy.

A spring is resting against a wall. The spring is compressed by a distance of 8.0 cm from its uncompressed length. Jemima holds a block of mass 1.2 kg stationary against the compressed spring as shown below. wall 1.2 kg frictionless surface Jemima releases the block and it slides to the right on a frictionless surface. It leaves the spring with a kinetic energy of 5.4 J and slides at constant speed as shown below. wall 1.2 kg frictionless surface Leaves the spring frictionless surface Kall 1.2 kg frictionless surface Example 1.84: 2012 Question 1c (2 marks) Calculate the spring constant, *k*, of the spring. Assume that the spring obeys Hooke's law. Students hang a mass of 1.0 kg from a spring that obeys Hooke's law with  $k = 10 \text{ N m}^{-1}$ . The spring has an unstretched length of 2.0 m. The mass then hangs stationary at a distance of 1.0 m below the unstretched position (X) of the spring, at Y, as shown at position 6b. The mass is then pulled a further 1.0 m below this position and released so that it oscillates, as shown in position 6c.



The zero of gravitational potential energy is taken to be the bottom point (Z).

The spring potential energy and gravitational potential energy are plotted on a graph.

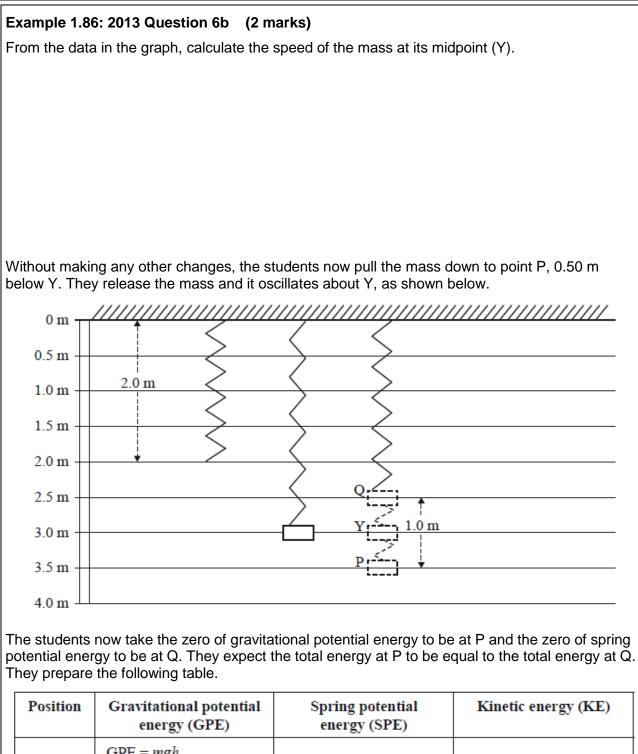




Calculate the total energy of the system when the mass is at its lowest point (Z).

Q

Ρ



energy (GPE)	energy (SPE)	
$GPE = mgh$ $= 1.0 \times 10 \times 1.0 = 10 J$	SPE = 0	KE = 0
GPE = 0	SPE = $\frac{1}{2}k(\Delta x)^2$ = $\frac{1}{2} \times 10 \times 1.0^2 = 5.0 \text{ J}$	KE = 0

However, their calculation of the total energy (GPE + SPE + KE) at Q (10 J) is different from their calculation of the total energy at P (5.0 J).

## Example 1.87: 2013 Question 6c (3 marks)

Explain the mistake that the students have made.

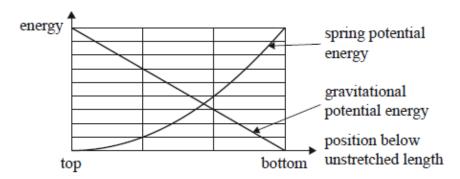
Jo and Sam are conducting an experiment using a mass attached to a spring. The spring has an unstretched length of 40 cm. The situation is shown below. 40 cm 40 cm Figure 3a Figure 3b Figure 3c They begin their experiment by measuring the spring constant of the spring by progressively adding 50 g masses to it, as shown in Figure 3b. They measure the resultant length of the spring with the mass stationary and record the following data. Number of 50 g masses 0 1 2 3 Length of spring 40 cm 50 cm 60 cm 70 cm Example 1.88: 2014 Question 2a (2 marks) Show that the spring constant is equal to 5.0 N m<sup>-1</sup>.

Jo and Sam now attach four 50 g masses to the spring and release it from its unstretched position, which is a length of 40 cm. They allow the masses to oscillate freely, as shown in Figure 3c.

# Example 1.89: 2014 Question 2b (3 marks)

Find the **extension** of the spring at the lowest point of its oscillation (when it is momentarily stationary). Ignore frictional losses. Show your reasoning.

Jo and Sam measure the position of the four masses as they oscillate freely up and down, as described previously. From this data, they plot graphs of the gravitational potential energy and spring potential energy. Their results are shown below



Jo says their calculation must be wrong because the graphs should add to a constant amount, the total energy of the system. However, Sam says that the graphs are correct.

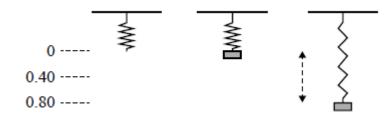
## Example 1.90: 2014 Question 2c (2 marks)

Explain why Jo is incorrect. Your explanation should include the reason that the spring potential energy and the gravitational potential energy do not add to a constant amount at each point.

# Example 1.91: 2014 Question 2d (4 marks)

Calculate the maximum speed of the masses during the oscillation. Show your working.

A mass of 2.0 kg is suspended from a spring, with spring constant k = 50 N m<sup>-1</sup>, as shown. It is released from the unstretched position of the spring and falls a distance of 0.80 m. Take the zero of gravitational potential energy at its lowest point.

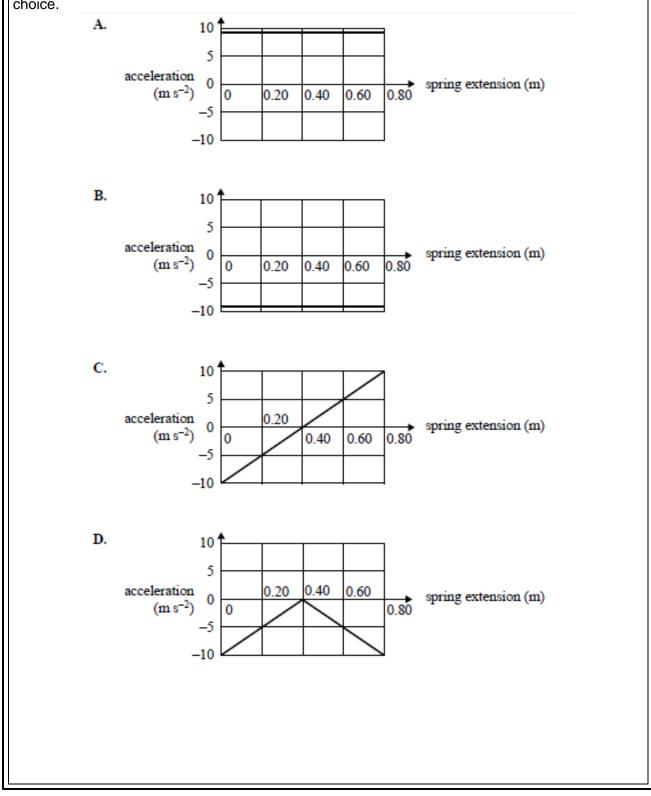


## Example 1.92: 2015 Question 6c (3 marks)

Calculate the speed of the mass at its midpoint (maximum speed).

# Example 1.93: 2015 Question 6d (2 marks)

Which one of the following graphs (A - D) best shows the acceleration of the mass as it goes from the highest point to the lowest point? Take upwards as positive. Give a reason for your choice.



Einstein's theory of special relativity

- describe Einstein's two postulates for his theory of special relativity that:
  - the laws of physics are the same in all inertial (non-accelerated) frames of reference
  - the speed of light has a constant value for all observers regardless of their motion or the motion of the source
- compare Einstein's theory of special relativity with the principles of classical physics

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
2, 5, 6	2, 3	7, 8	1, 13	1, 3, 5, 12	5, 6, 8	1, 3, 4	1, 4, 7	1, 4	1, 2, 3	1, 2, 3, 5,

A reference frame is the physical object to which we attach our co-ordinate system. In everyday life, that object is the ground.

#### **Inertial frames**

Inertial frames of reference are where the frame is not accelerating, therefore they are either at 'rest' or moving with constant velocity with respect to other *inertial frames of reference*. **Newton's first law is obeyed in all Inertial frames of reference**.

#### Non-inertial frames of reference

An accelerated frame of reference is called a non-inertial frame of reference. This is beyond the limits of this course.

#### **Relative velocity**

velocity<sub>A relative to B</sub> =  $v_{ab} = v_a - v_b$ .

Water in a river 1 km wide flows with a speed of 4 km/hr. A ferry, which points towards the opposite bank, crosses the river. By the time the ferry reaches the opposite bank it has been carried 2 km down-stream.

#### Example 1.94: 1969 Question 1 (1 mark)

If there had been no current the ferry would have had a speed of

- **A.** 4 km/hr
- **B.** 1 km/hr
- C 2 km/hr
- **D.**  $\sqrt{5}$  km/hr

**E.** 
$$\frac{\sqrt{5}}{2}$$
 km/h

# Example 1.95: 1969 Q2, (1 mark)

The time taken to reach the other side is

- **A.** 2 hr **B.**  $\frac{\sqrt{5}}{2}$  hr
- $\mathbf{C} = \frac{1}{2}$  hr
- **D.**  $\frac{1}{4}$  hr
- **E.**  $2\sqrt{5}$  hr

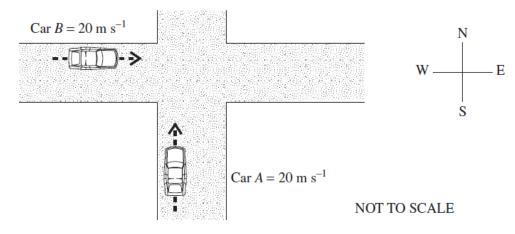
#### Example 1.96: 1969 Q3, (1 mark)

A ferry captain wishes to take his boat directly across the river with a speed of 2 km/hr with respect to the ground. He finds he has to point his boat up-stream, making an angle  $\theta$  with the direct path.

The angle  $\theta$  is given by

A.  $\sin \theta = \frac{1}{2}$ B.  $\tan \theta = \frac{1}{2}$ C.  $\cos \theta = \frac{2}{\sqrt{5}}$ D.  $\tan \theta = 2$ E.  $\sin \theta = \frac{1}{\sqrt{5}}$ 

Car A and Car B approach an intersection as shown.



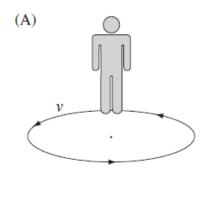
## Example 1.97: NSW 2000 Question 2 (1 mark)

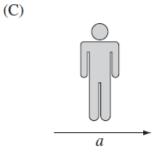
What is the approximate velocity of Car *B* relative to Car *A*?

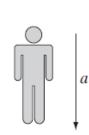
- **A.** 28 m s<sup>-1</sup> NE
- **B.** 28 m s<sup>-1</sup> NW
- **C.** 28 m s<sup>-1</sup> SE
- **D.** 28 m s<sup>-1</sup> SW

# Example 1.98: NSW 2013 Question 8 (1 mark)

Which of the diagrams best represents an example of an inertial frame of reference?

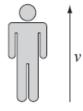






(D)

**(B)** 



#### Einstein's postulates

#### The Principle of Relativity

All the laws of physics are the same in all inertial frames. (This compares with Newton's assumptions that the laws of <u>mechanics</u> are the same in all inertial frames)

#### The Constancy of the Speed of Light

The speed of light in vacuum is the same  $(3 \times 10^8 \text{ m s}^{-1})$  in all inertial frames [ie there is no ether and the speed of light is the same regardless of the motion and the source of light].

# Example 1.99: 2010 Question 5 (2 marks)

Two physics students are conducting accurate experiments to test Newton's second law of motion ( $\Sigma F = ma$ ). Each student is in a windowless railway carriage. One carriage (carriage A) is moving at a constant velocity of 0.9*c*. The other carriage (carriage B) is moving at 10 m s<sup>-1</sup> and decelerating.

Which one of the following best describes the likely results of their experiments?

- A. Only the experiment in carriage A confirms Newton's second law of motion.
- B. Only the experiment in carriage B confirms Newton's second law of motion.
- **C.** Neither experiment confirms Newton's second law of motion.
- **D.** Both experiments confirm Newton's second law of motion.

## Example 1.100: 2012 Question 1 (2 marks)

Which of the following factors affects the speed of light?

- **A.** the electrical properties of the medium through which light is travelling
- **B.** the speed of the observer of the light
- C. the speed of the light-emitting source
- **D.** none of the above; the speed of light never changes

# Example 1.101: 2015 Question 2 (2 marks)

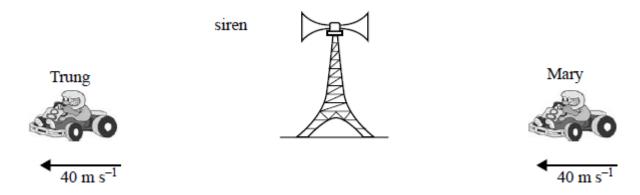
The following statements reflect views held before Einstein proposed the special theory of relativity.

Which one of these views is now still considered to be true?

- **A.** The observed speed of light in a medium depends on the speed of an observer relative to that medium.
- **B.** The speed of light in a region depends only on the values of electric and magnetic properties of that region.
- **C.** Light is a wave comprising oscillating electric and magnetic fields that cannot travel through empty space.
- **D.** The inertial frame of a medium in which light is travelling is a special frame and, in that frame, the speed of light is *c*.

#### Use the following information to answer Questions 7 and 8.

Trung and Mary are driving along a road at 40 m s<sup>-1</sup> in the same direction. A stationary siren is situated between them. The speed of sound in air is 340 m s<sup>-1</sup>. The situation is shown below.

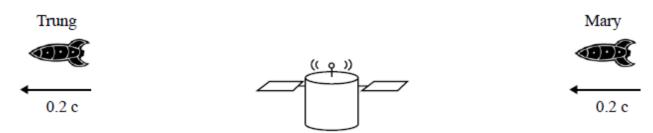


# Example 1.102: 2008 Question 7 (2 marks)

Which one of the following gives the speed of sound from the siren, in m s<sup>-1</sup>, as measured by Trung and Mary?

	Trung	Mary
Α.	340	340
В.	300	380
C.	380	300
D.	320	320

A similar situation now occurs in space, except that Trung and Mary are travelling in two rocket ships in the same direction at 0.2 c. Instead of the siren, a stationary space station between them is emitting light of speed  $3.0 \times 10^8$  m s<sup>-1</sup> in all directions, as shown



## Example 1.103: 2008 Question 8 (2 marks)

Which one of the following gives the speed of light from the space station as measured by Trung and Mary?

	Trung	Mary
Α.	1.2 c	0.8 c
В.	С	С
C.	0.8 c	1.2 c
D.	1.1 c	1.1 c

On a planet a long way away, a racing car is moving at high speed (0.9*c*) along a straight track. It is heading straight for a post. Jim is standing next to the post. The situation is shown below.



#### Example 1.104: 2010 Question 1 (2 marks)

When the racing car is 1.00 km from the post (as measured by Jim), the driver sends a flash of light from the car.

Which of the following is closest to the time that the flash of light takes to reach the post (as measured by Jim)?

- A. 1.5 microseconds
- B. 1.8 microseconds
- C. 3.3 microseconds
- D. 3.7 microseconds

#### Example 1.105: 2010 Question 2 (2 marks)

The driver of the racing car, Susanna, measures the distance between herself and the post at exactly the same time that she sends the flash of light.

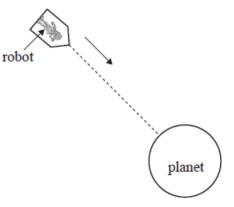
Which one of the following is closest to the distance that she measures?

- **A.** 0.44 km
- **B.** 0.90 km
- **C.** 1.00 km
- **D.** 2.29 km

#### The following information relates to Questions 6 - 8.

A robot is heading radially towards the surface of a planet in the *Hoth* system at a constant speed of 0.85*c*. Observers on the surface of the planet observe it at a time when it is a distance *x* above the surface in their reference frame. The observers calculate the time that the robot will take to reach the surface of the planet as 784 microseconds.

The situation is shown below.



## Example 1.106: 2010 Question 6 (2 marks)

Which one of the following is closest to the distance x?

- **A.** 105 km
- **B.** 200 km
- **C.** 235 km
- **D.** 380 km

#### Example 1.107: 2010 Question 7 (2 marks)

Which one of the following is the best estimate of the time, as measured by the **robot**, for it to reach the surface of the planet?

- A. 413 microseconds
- B. 666 microseconds
- C. 784 microseconds
- D. 1488 microseconds

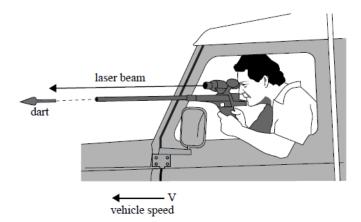
# Example 1.108: 2010 Question 8 (2 marks)

Which one of the following best describes the time of the robot's descent to the planet surface as measured by the robot, and the time as measured by the observers on the surface of the planet?

- A. They are both measurements of proper time in their own reference frames.
- B. Neither are measures of proper time.
- **C.** Only the observers measure the proper time.
- **D.** Only the robot measures the proper time.

A conservation scientist is using a stun dart fired from a rifle to tranquilize a kangaroo to tag it for conservation research. He is travelling in a specially designed vehicle at a speed V in a straight line. He fires a dart from his rifle straight ahead.

The dart has a speed U, measured relative to his rifle. At the same time, a flash of light is emitted from the laser sight mechanism on his rifle. This is shown in the figure below.



# Example 1.109: 2010 Question 12 (2 marks)

Which one of the following choices is the best estimate of the speed of the dart and the light flash, as measured by a stationary observer on the ground?

	Speed of dart relative to stationary observer	Speed of light flash relative to stationary observer
А.	U + V	С
B.	U + V	$V + \sqrt{\frac{V^2}{c^2}}$
C.	U - V	<i>V</i> + <i>c</i>
D.	U	с

# Einstein's theory of special relativity

- describe proper time ( $t_0$ ) as the time interval between two events in a reference frame where the two events occur at the same point in space
- describe proper length ( $L_0$ ) as the length that is measured in the frame of reference in which objects are at rest
- model mathematically time dilation and length contraction at speeds approaching c

using the equations:  $t = t_0 \gamma$  and  $L = \frac{L_0}{\gamma}$  where  $\gamma = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)^2$ 

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
3, 4, 9	4, 5, 6, 8, 9	1, 2, 4, 9, 10, 11, 12	2, 3, 4	2, 4, 6, 7, 8	4, 9, 11, 12	2, 7, 8, 9, 10	5, 8, 9, 10, 11			5, 6, 7, 8, 9, 11

# Proper Time to

The time measured in the frame of reference where the clock is stationary

# Proper Length L<sub>0</sub>

Length in the frame where the object (or distance) is stationary.

#### <u>Note</u>

Proper length and proper time are measured in different frames of reference. Proper time is measured in the moving frame and proper length in the stationary frame.

## Gamma y

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

γ	v
1.1	0.140 c
1.5	0.745 c
2	0.866 c
3	0.943 c
4	0.968 c
5	0.980 c
10	0.995 c

v	C	γ
0.1 c	10%	1.005
0.25 c	25%	1.033
0.5 c	50%	1.155
0.8 c	80%	1.667
0.9 c	90%	2.294
0.95 c	95%	3.203
0.99 c	99%	7.089

### Length Contraction

The length of a moving object will be measured to be less than the length of the same object measured in the frame when the object is stationary. This is *length contraction*.

In order to measure the **proper** length of an object, it must be measured from a frame of reference in which it is at rest. If it is being measured as it is moving, we can use the following formula to calculate its **relativistic length contraction**.

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} \qquad \text{or } L = \frac{L_0}{v}$$

# **Time dilation**

Time in a frame of reference that is moving will be observed to run slower than time in your own frame. This is *time dilation*. Note that  $t_0$  is the proper time between the two events. We need to use a correction factor, called a Lorentz Conversion.

Here it is:

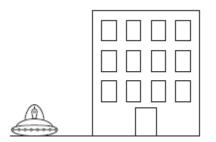
$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ or } t$	$t = t_0 \gamma$	Where: t = elapsed time in moving frame $t_0 =$ time in a stationary frame v = relative velocity of the moving frame c = speed of light

# Example 1.110: NSW 2007 Question 2 (1 mark)

A spaceship sitting on its launch pad is measured to have a length L. This spaceship passes an outer planet at a speed of 0.95 *c*. Which observations of the length of the spaceship are correct?

	Observer on the spaceship	Observer on the planet	
Α.	No change	Shorter than L	
В.	No change	Greater than L	
C.	Shorter than L	No change	
D.	Greater than L	No change	

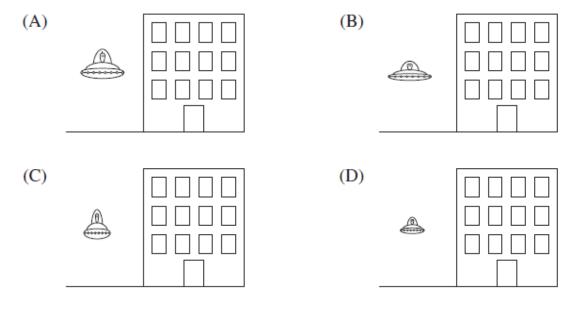
The diagram shows a stationary spacecraft next to a building, as seen by an observer across the street.



A short time later the spacecraft is observed to be travelling vertically upwards at 0.8c, relative to the building.

### Example 1.111: NSW 2011 Question 9 (1 mark)

Which diagram best represents the appearance of the moving spacecraft, as seen by the observer?



The rest length of a train is 200 m and the rest length of a railway platform is 160 m. The train rushes past the platform so fast that, when observed in the platform's frame of reference, the train and the platform are the same length.

#### Example 1.112: NSW 2014 Question 19 (1 mark)

How fast is the train moving?

- **A.** 0.60 *c*
- **B.** 0.75 *c*
- **C.** 0.80 *c*
- **D.** 1.25 *c*

#### Use the following information to answer Questions 4 - 6.

Two spacecraft travel in opposite directions, with spacecraft *Ajax* travelling at a speed of 0.5*c* and spacecraft *Hector* travelling at a speed of 0.4*c*. Both are travelling relative to the inertial frame of the galaxy. The situation is shown below.



A radio signal is emitted by *Ajax* towards *Hector*. The navigator of *Hector* uses the classical physics understanding of radio waves travelling at a speed relative to a medium fixed with respect to the galaxy.

# Example 1.113: 2014 Question 4 (2 marks)

Using this classical understanding, the speed of the radio signal relative to *Hector* is expected to be

**A.** c

- **B.** 0.6*c*
- **C.** 0.5*c*
- **D.** 0.1*c*

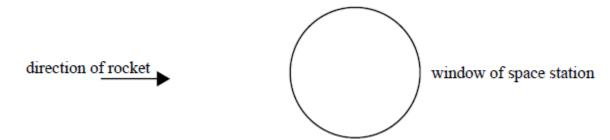
# Example 1.114: 2014 Question 6 (2 marks)

How can proper time be measured for the interval between the radio signal being emitted on *Ajax* and the signal reaching *Hector*?

- A. Use measurements made by the crew on Ajax.
- **B.** Use measurements made by the crew on *Hector*.
- **C.** Use measurements made by an observer stationary at the point where the signal was emitted.
- **D.** No single observer can measure proper time for this case.

Use the following information to answer Questions 1 and 2.

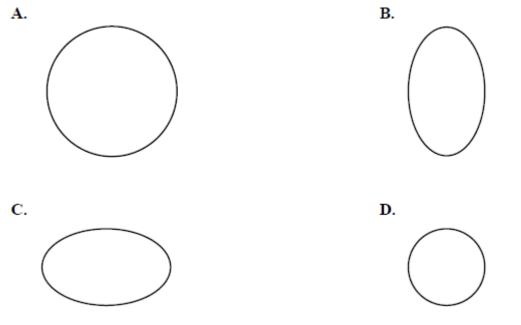
A rocket passes a space station at a speed 0.80 c parallel to the side of the space station. There is a circular window on the space station, as shown below.



A person inside the passing rocket observes the window.

# Example 115: 2008 Question 1 (2 marks)

Which of the following figures best shows how the window would look to the person on the passing rocket?



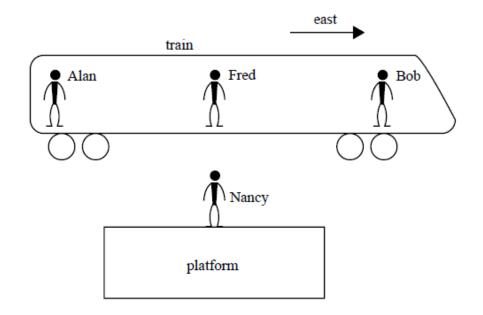
# Example 1.116: 2008 Question 2 (2 marks)

For which of the following values of the Lorentz factor,  $\gamma$ , would relativistic changes in time, length and mass **not** be observed?

- A. close to 0
- B. significantly less than 1
- C. approximately equal to 1
- D. significantly greater than 1

The figure below shows Fred in a futuristic train travelling at constant relativistic speed in an easterly direction in a straight line. Fred is halfway between two people, Alan and Bob, who are at opposite ends of the carriage.

The train passes a platform. Nancy is standing on the platform.



# Example 1.117: 2008 Question 4 (2 marks)

Fred measures the carriage he is travelling in to be 20 m long. Nancy has measured the platform she is standing on to be 10 m long. The train rushes past at such a speed that Nancy sees the carriage and the platform to be the same length.

How fast was the train moving?

- **A.** 0.50 c
- **B.** 0.75 c
- **C.** 0.87 c
- **D.** 0.97 c

#### Example 1.118: 2008 Question 9 (2 marks)

Roger is in a spacecraft travelling at constant speed, passing Bridget who is stationary. Roger is holding a ruler. Roger measures its length to be 1.0 m. The ruler can be seen by Bridget through a window as the spacecraft passes.

Bridget measures the length of the ruler to be 0.8 m.

Which one of the following gives the proper length of the ruler?

- **A.** 0.20 m
- **B.** 0.80 m
- **C.** 1.0 m
- **D.** 1.2 m

### Example 1.119: 2011 Question 11 (2 marks)

Which of the following statements about proper length is the most accurate?

- **A.** The proper length of an object is always greater than or equal to another measure of the length of the object.
- **B.** The proper length of an object is always less than another measure of the length of the object.
- **C.** The proper length of an object is sometimes less than another measure of the length of the object, and sometimes greater than or equal to another measure of the length of the object.
- **D.** The proper length of an object can only be measured by an observer who is moving relative to the object.

#### Example 1.120: 2011 Question 9 (2 marks)

Scientists observe the path of a short-lived elementary particle in a detector. It is created in the detector and exists only for a short time, leaving a path of length 5.4 mm long. The scientists measure its speed as  $2.5 \times 10^8$  m s<sup>-1</sup>, giving  $\gamma = 1.81$ . What is the proper lifetime of the particle?

- **A.** 5.3 × 10<sup>-11</sup> s
- **B.** 3.3 × 10<sup>-11</sup> s
- **C.**  $1.8 \times 10^{-11}$  s
- **D.**  $1.2 \times 10^{-11}$  s

#### Example 1.121: 2008 Question 10 (2 marks)

An electron with an initial speed of 0.99 c enters an accelerator, which increases the electron's speed and energy.

Which one of the following best describes the effect of the accelerator on the electron's speed and energy?

- A. The speed increases substantially, the energy increases substantially.
- B. The speed increases slightly, the energy increases substantially.
- C. The speed increases slightly, the energy increases slightly.
- D. The speed increases substantially, the energy increases slightly.

#### Use the following information to answer Questions 11 and 12.

An electron with a Lorentz factor of 4 travels in a straight line a distance of 600 m as measured in the laboratory frame of reference.

#### Example 1.122: 2008 Question 11 (2 marks)

Which one of the following best gives the speed of the electron?

- **A.** 0.25 c
- **B.** 0.94 c
- **C.** 0.97 c
- **D.** 0.99 c

#### Example 1.123: 2008 Question 12 (2 marks)

As measured in the electron's frame of reference, what would be the approximate length of the linear section?

- **A.** 2 400 m
- **B.** 600 m
- **C.** 300 m
- **D.** 150 m

#### Example 1.124: 2012 Question 8 (2 marks)

Which of the following statements about the **proper time** between two events is the most accurate?

- **A.** It is always shorter than or equal to another measurement of the time interval between the two events.
- **B.** It is always longer than or equal to another measurement of the time interval between the two events.
- **C.** It may be greater than, equal to or less than another measurement of the time interval between the two events.
- **D.** It can never be measured by an observer who is located at the same position as the two events.

# Example 1.125: 2010 Question 4 (2 marks)

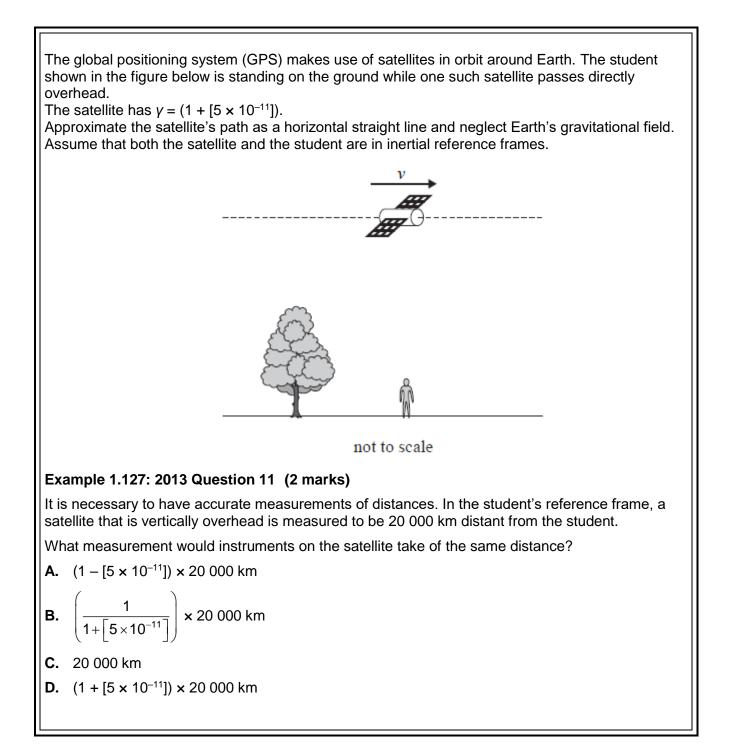
Which one of the following is the best description of the **proper length** of an object travelling with constant velocity?

- **A.** The length when measured by any observer at the same location.
- **B.** The length when measured by an observer at rest relative to the object.
- **C.** The length when both ends of the object are measured at the same time.
- **D.** The length when measured with a proper standard measuring stick.

### Example 1.126: 2013 Question 8 (2 marks)

Which one of the following statements is correct?

- **A.** Proper time cannot be measured on a moving clock.
- **B.** Proper time is the time interval between two events that is measured by a stationary clock.
- **C.** Proper time is the shortest possible time interval between two events that any observer can measure.
- **D.** An observer who measures a proper time is the only observer performing a correct measurement of the time between two events.



# Einstein's theory of special relativity

• explain why muons can reach Earth even though their half-lives would suggest that they should decay in the outer atmosphere.

2	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
	7, 8			9, 10	10						

Muons are elementary particles created in the upper atmosphere by cosmic rays. They are unstable, and decay with a half-life of  $2.2 \ \mu s$  ( $2.2 \ \times 10^{-6} \ s$ ) when measured at rest. This means that in the reference frame of the muons, half of them decay in each time interval of  $2.2 \ \mu s$ ,

Muons are very short-lived particles that are created when energetic protons collide with each other. A beam of muons can be produced by very-high-energy particle accelerators.

The high-speed muons produced for an experiment by the Fermilab accelerator are measured to have a lifetime of 5.0 microseconds. When these muons are brought to rest, their lifetime is measured to be 2.2 microseconds.

# Example 1.128: NSW 2001 Question 16a (1 mark)

Name the effect demonstrated by these observations of the lifetimes of the muons.

# Example 1.129: NSW 2001 Question 16b (3 marks)

Calculate the velocity of the muons as they leave the accelerator.

Muons are subatomic particles which at rest have a lifetime of 2.2 microseconds ( $\mu$ s).

When they are produced in Earth's upper atmosphere, they travel at 0.9999 c.

Using classical physics, the distance travelled by a muon in its lifetime can be calculated as follows:

# $x = v \times t$ = 660 m

# Example 1.130: NSW 2016 Question 19 (1 mark)

Which row of the table correctly summarises the behaviour of these muons?

	Muon's referen	nce frame	Earth's reference frame			
	Distance travelled (m)	Lifetime (µs)	Distance travelled (m)	Lifetime (µs)		
(A)	660	2.2	> 660	> 2.2		
(B)	> 660	> 2.2	660	2.2		
(C)	660	2.2	< 660	< 2.2		
(D)	< 660	< 2.2	660	2.2		

# Relationships between force, energy and mass

- interpret Einstein's prediction by showing that the total 'mass-energy' of an object is given by:  $E_{tot} = E_k + E_0 = \gamma mc^2$  where  $E_0 = mc^2$ , and where kinetic energy can be calculated by:  $E_k = (\gamma 1)mc^2$
- describe how matter is converted to energy by nuclear fusion in the Sun, which leads to its mass decreasing and the emission of electromagnetic radiation.

2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
10, 11	10	11, 13	7, 8, 11,12	11	1, 2, 3, 10	6, 11, 12	3, 6	10, 11	9, 10, 11	10

# Example 1.131: 2008 Question 13 (2 marks)

In the fusion process, a proton of rest mass  $1.673 \times 10^{-27}$  kg and a neutron of rest mass  $1.675 \times 10^{-27}$  kg combine to form a deuterium nucleus of rest mass  $3.344 \times 10^{-27}$  kg, with a release of energy.

According to Einstein's postulate of the equivalence of mass and energy, which one of the following is the best estimate of the energy released in this interaction?

- **A.**  $1.2 \times 10^{-21} \text{ J}$
- **B.** 3.6 × 10<sup>-13</sup> J
- **C.**  $4.0 \times 10^{-3}$  J
- **D.**  $3.6 \times 10^{14} \text{ J}$

# Example 1.132: 2010 Question 11 (2 marks)

In a particle accelerator, an alpha particle of mass  $6.64424 \times 10^{-27}$  kg is accelerated from rest to high speed. The total work done on the alpha particle is equal to  $7.714 \times 10^{-10}$  J. Which one of the following is closest to its final speed?

- **A.** 0.90*c*
- **B.** 0.95*c*
- **C.** 0.85*c*
- **D.** 0.80*c*

# Example 1.133: 2011 Question 1 (2 marks)

An experiment is done where two protons with very high kinetic energy collide in order to try to create a single stationary 'Higgs' particle.

**Each** proton in the reaction has a kinetic energy of  $1.1 \times 10^{-6}$  J. No other particles are produced in the reaction and the protons will not exist after the production of the Higgs particle. The proton rest mass is equal to  $1.6726 \times 10^{-27}$  kg.

Which of the following options is the best estimate of the mass of the Higgs particle?

**A.**  $1.2 \times 10^{-23} \text{ kg}$ 

**B.**  $2.4 \times 10^{-23}$  kg

- **C.**  $3.3 \times 10^{-27}$  kg
- **D.** 1.7 × 10<sup>-27</sup> kg

### Example 1.134: 2011 Question 3 (2 marks)

When stationary, a proton has a rest mass-energy of  $1.50 \times 10^{-10}$  J. A proton is accelerated from a speed with  $\gamma = 1.05$  to a speed with  $\gamma = 1.10$ . Which of the following is closest to the work done on the proton during its acceleration from the first speed to the second speed?

- **A.** 2.9 × 10<sup>6</sup> J
- **B.** 3.2 × 10<sup>-10</sup> J
- **C.**  $7.5 \times 10^{-12} \text{ J}$
- **D.** 8.3 × 10<sup>-29</sup> J

#### Example 1.135: 2011 Question 10 (2 marks)

Which of the following statements best explains why it is impossible to accelerate particles (such as electrons) so that they are travelling at the speed of light?

- A. It is directly forbidden by one of Einstein's postulates.
- **B.** As particles increase in speed, the rest mass (m<sub>0</sub>) tends towards an infinite value.
- **C.** The kinetic energy of particles, given by  $E_{\rm K} = (\gamma 1)m_0c^2$  tends towards an infinite value.
- **D.** The speed of particles is given by L/t; this is equal  $L_0/t_0\gamma^2$  to and this value tends towards an infinity.

A pion and its antiparticle, each at rest, annihilate to produce two photons whose total energy is  $4.5 \times 10^{-11}$  J. Apart from the two photons, nothing else is produced in this process. The masses of a pion and its antiparticle are the same.

# Example 1.136: 2014 Question 11 (2 marks)

The pion is now accelerated from rest before colliding with its antiparticle. What work must be done on one pion so that it has  $\gamma$  equal to 3.00?

- **A.** 4.5 × 10<sup>-11</sup> J
- **B.** 2.2 × 10<sup>-11</sup> J
- **C.** 1.1 × 10<sup>-11</sup> J
- **D.**  $9.0 \times 10^{-11} \text{ J}$

# Example 1.137: 2015 Question 10 (2 marks)

A particle of rest mass  $m_0$  is accelerated from rest to 0.6*c* relative to Earth's frame. Which one of the following statements is true?

- **A.** In its own frame, the mass of the particle is now  $1.25m_0$ .
- **B.** The work done to accelerate the particle is equal to the kinetic energy of the particle.
- **C.** The kinetic energy of the particle in Earth's frame is  $\frac{1}{2}m_0(0.6c)^2$ .
- **D.** The increase in total energy of the particle (measured in Earth's frame) is due to an increase in both the kinetic energy of the particle and the rest energy of the particle.