**Resonance, standing waves and diffraction**

- **explain resonance as the superposition of a travelling wave and its reflection, and with reference to a forced oscillation matching the natural frequency of vibration.**
- **analyse the formation of standing waves in strings fixed at one or both ends.**
- **investigate and explain theoretically and practically diffraction as the directional spread of various frequencies with reference to different gap width or obstacle size, including the qualitative effect of changing the $\frac{\lambda}{w}$ ratio.**

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| Paper | Multiple choice | Short Answer | Idea | Marks | % | Type |
|---|---|---|---|---|---|---|
| 2022 | 13 | | Rope, fixed both ends | 1 | 63 | Calculation |
| 2022 NHT | | | | | | |
| 2021 | | | | | | |
| 2021 NHT | 14 | | Standing wave, nodes | 1 | NA | Concept |
| | | 14 | $\frac{\lambda}{w}$ ratio, sound | 2 | NA | Explanation |
| 2020 | 14 | | Diffraction ratio, water | 1 | 72% | Concept |
| | | 13a | v = fλ | 2 | 67% | Calculation |
| | | 13b | String, fixed both ends | 2 | 77% | Concept |
| 2019 | | 13a | v = fλ | 1 | 81% | Calculation |
| | | 13b | String, fixed both ends | 2 | 38% | Explanation |
| 2019 NHT | | 14a | String, fixed both ends | 2 | NA | Concept |
| | | 14b | String, fixed both ends | 2 | NA | Calculation |
| | | 14c | v = fλ | 2 | NA | Calculation |
| 2018 | | 11b | Standing wave, 2 speakers | 3 | 20% | Calculation |
| 2018 NHT | | 11c | Single slit | 2 | NA | Explanation |
| 2017 | 14 | | Diffraction effects | 1 | 71% | Concept |
| | | 16a | String, fixed both ends | 2 | 63% | Calculation |
| | | 16b | String, fixed both ends | 2 | 58% | Calculation |
| | | 16c | Concept | 2 | 43% | Explanation |

**Resonance questions can be grouped into the following ideas.**

(For some reason there have not been any questions on this section of the study design over the last 7 years)

| | |
|---|---|
| Basic properties and definitions | |
| Pipe open at both ends. | Worked example 1 |
| Pipe open at one end. | Worked example 2 |

**Standing wave questions can be grouped into the following ideas.**

| | |
|---|---|
| Concept | Worked example 3 |
| Application of $v = f\lambda$ | Worked example 4 |
| Between two sources | Worked example 5 |
| Strings. | Worked example 6 |

**Diffraction questions can be grouped into the following ideas.**

| | |
|---|---|
| $\frac{\lambda}{w}$ ratio | Worked example 7 |
| Effects of diffraction | Worked example 8 |

## Superposition

The displacement of two waves combining with each other is calculated by the vector addition of the two components. The two pulses pass through each other without being altered.

Constructive interference is when the two pulses pass through each other and superimpose and reinforce each other to give a maximum disturbance; this results in a louder sound. Destructive interference is when the two pulses pass through each other and superimpose and cancel each other out to give minimum or zero disturbance; this results in a quieter sound.

## Resonance

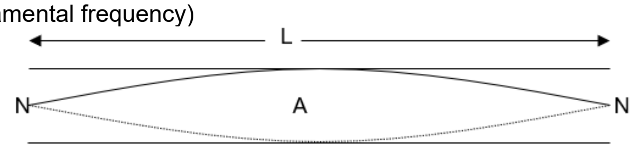
Resonance is the condition when a natural vibrating system responds to an external driving frequency, it occurs when a forcing frequency, the same as the natural frequency, is applied. Each object has its own natural or resonant frequency.

### Pressure variation in pipes (Open at both ends)

First harmonic (fundamental frequency)

$\lambda_1 = 2L$

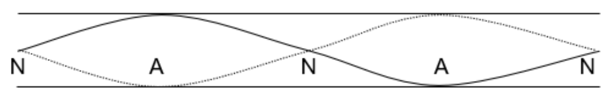
$f_1 = \frac{v}{2L}$




Second harmonic (first overtone)

$\lambda_2 = \frac{2L}{2} = L$

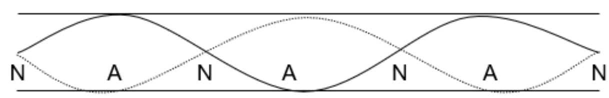
$f_2 = \frac{2v}{2L} = 2f_1$




third harmonic (second overtone)

$\lambda_3 = \frac{2L}{3}$

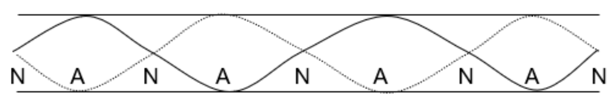
$f_3 = \frac{3v}{2L} = 3f_1$




fourth harmonic (third overtone)

$\lambda_4 = \frac{2L}{4} = \frac{L}{2}$

$f_4 = \frac{4v}{2L} = 4f_1$

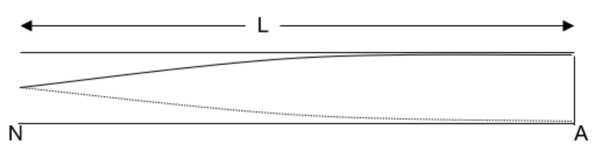



### Pressure variation in pipes (Closed at one end)

First harmonic (fundamental frequency)

$\lambda_1 = 4L$

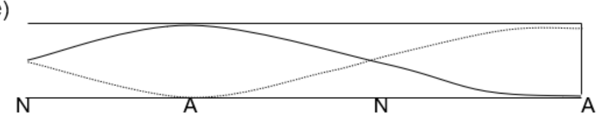
$f_1 = \frac{v}{4L}$




third harmonic (first overtone)

$\lambda_2 = \frac{4L}{3}$

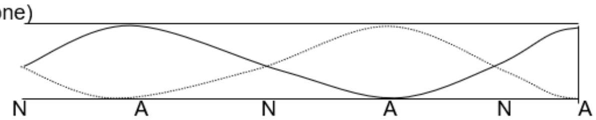
$f_2 = \frac{3v}{4L} = 3f_1$




fifth harmonic (second overtone)

$\lambda_3 = \frac{4L}{5}$

$f_3 = \frac{5v}{4L} = 5f_1$



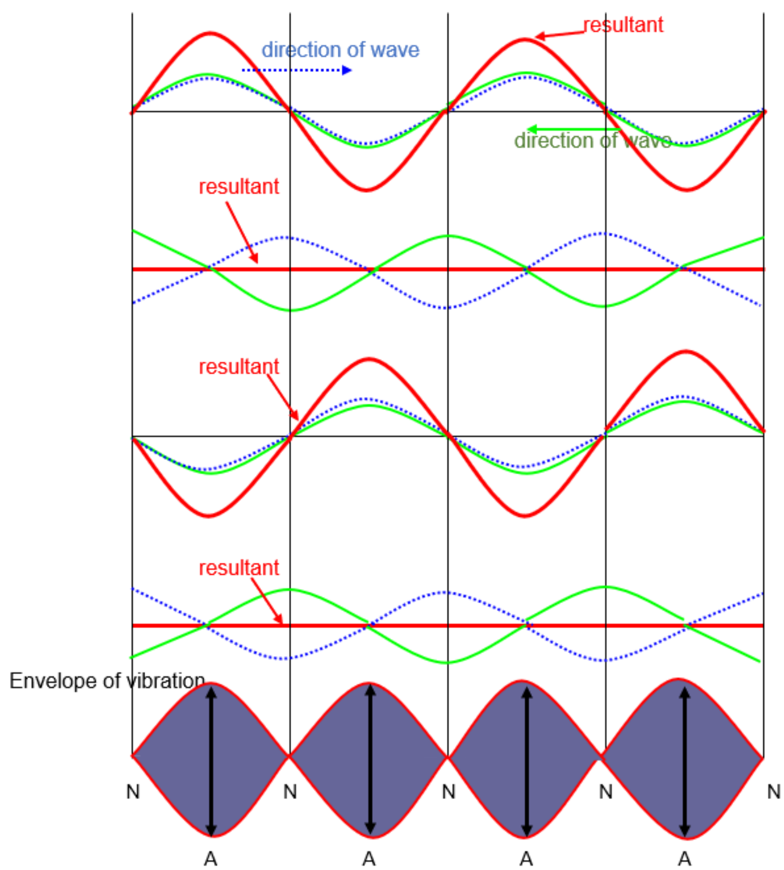

The term *overtone* is applied to harmonics other than the fundamental frequency.

**Standing waves**

If we have two identical waves travelling in the opposite directions in one medium we get a standing or stationary wave. The superposition principle is used to obtain the waveform.

- Certain points marked N = node = point of zero displacement, a node or nodal point.
- Loops or antinodes, marked A, are points of maximum displacement, midway between the nodes.
- The wave does not progress through the medium.
- Wavelength is the same as that of the components.
- Maximum amplitude of the resultant wave is twice that of the components.
- The distance between adjacent nodes or antinodes is $\frac{\lambda}{2}$.
- Any particles between any two successive nodes are in phase. Their motions correspond at any instant. They have zero displacement and maximum displacement at the same instant.
- They can only be produced by the superposition of two waves of equal amplitude and frequency travelling in the opposite direction.
- They are the result of resonance and occur only at the natural frequencies of the vibration.

Nodes are a result of destructive interference, there is little or no variation in air pressure, so it is perceived by the listener as a region of soft sound. Antinodes are a result of constructive interference, so there will be maximum variation of air pressure, which will be perceived as a region of loud sound.



**Reflection in strings**

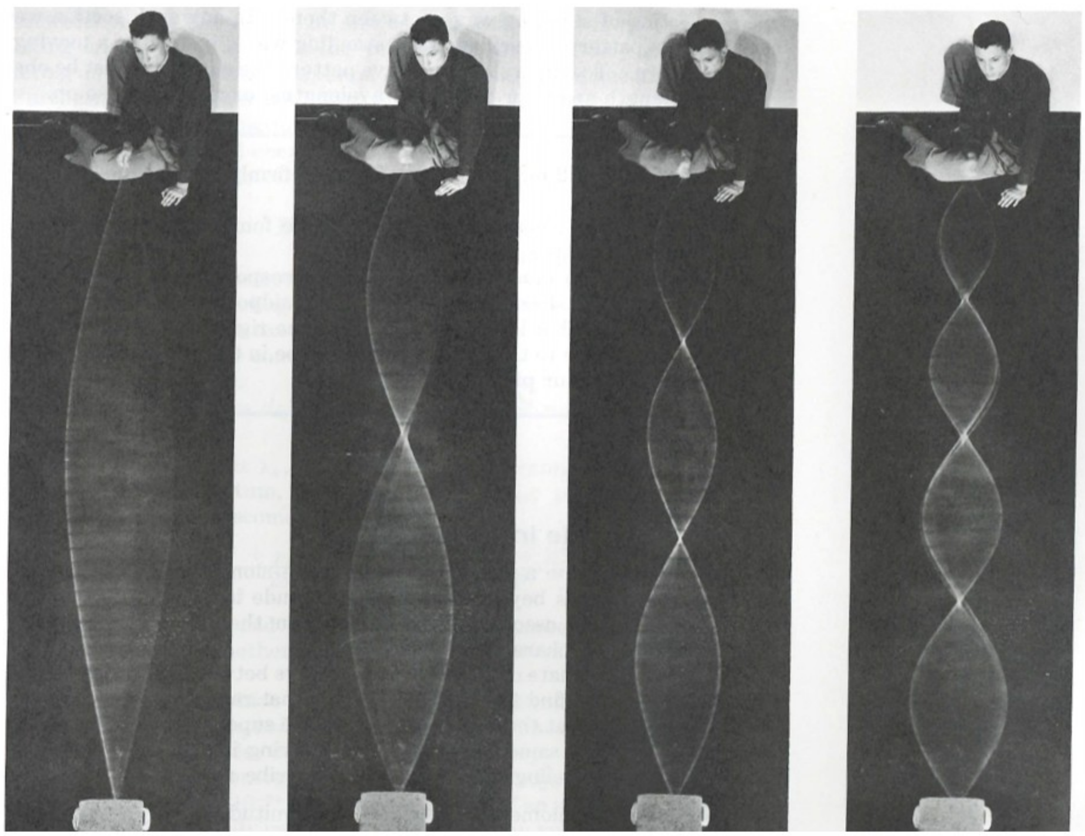
When a wave reaches a free end, or yielding boundary, it will reflect with crests as crests and troughs as troughs. If the wave reflects from a fixed end, a crest will reflect as a trough and a trough will reflect as a crest.

Strings in musical instruments are always fixed at both ends. It can be shown that the wavelength of the standing waves corresponding to the natural harmonics is

$\lambda_n = \frac{2L}{n}$ or $f = \frac{nv}{2L}$.

All harmonics ($n$ = 1, 2, 3, …) may be present, the ratio of frequencies $f_1 : f_2 : f_3 = 1 : 2 : 3$.

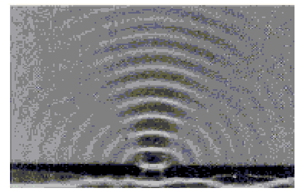
The fundamental frequency of a stretched wire depends on: length, tension and mass per unit length.



**Diffraction**

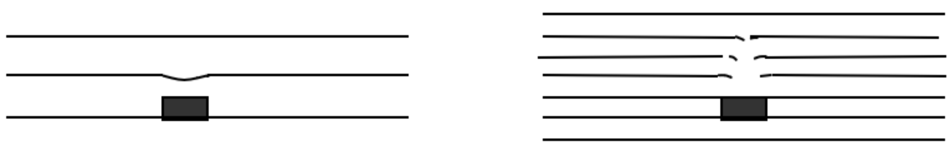
When light (waves) pass through a narrow aperture, a hole, a slit or an obstacle, it spreads out, this is known as diffraction. Diffraction is the bending of the path of the wave from its initial direction. The amount of diffraction $\propto \frac{\lambda}{d}$. To observe diffraction effects with the small wavelength of visible light, a very narrow slit must be used.

**Diffraction through gaps.**



When sound travels through a narrow opening, such as a doorway, the waves bend around both sides of the opening and are diffracted on both sides of the doorway. The amount of diffraction is given by the value of the ratio $\frac{\lambda}{w}$ where w is the width of the opening. As this ratio increases, so does the amount of diffraction (bending). If $\frac{\lambda}{w}$ << 1, very little diffraction occurs, if the ratio $\frac{\lambda}{w} \geq 1$, then it is complete diffraction, i.e. defined as bending through $180^0$.

**Diffraction around obstacles.**

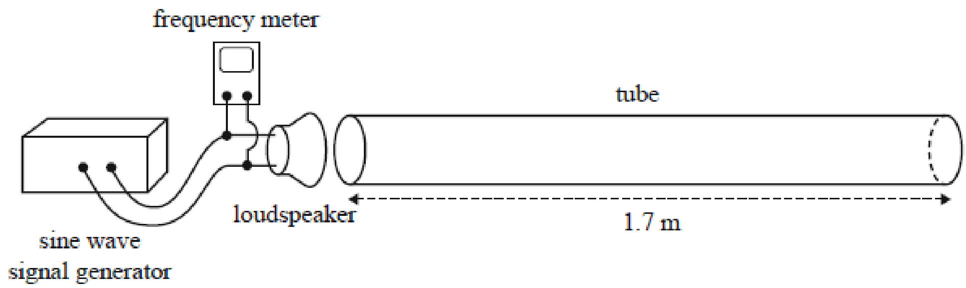


long wavelengths, low frequencies — short wavelengths, high frequencies

When the obstacle is small compared with the wavelength, there is very little disturbance. Larger 'shadows' occur when the obstacle is much larger than the wavelength of the incident wave.

*Worked example 1: Resonance: Pipe open at both ends.*

Students are conducting an experiment to observe sound standing waves in an air column in a hollow tube, as shown below. The length of the tube is 1.7 m and it is open at both ends. A sine wave signal generator drives a loudspeaker mounted at the end of the tube. A frequency meter measures the frequency. Speed of sound in air = 340 m $s^{-1}$.




The students increase the frequency from zero until they detect the first resonance (first harmonic).

**2014 Question 6, 2 marks**

Which one of the following best describes the frequency at which the students will observe this first resonance?

**A.** 17 Hz

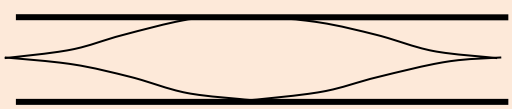
**B.** 50 Hz

**C.** 100 Hz

**D.** 170 Hz

**Solution**

This will occur when there are nodes at both ends and an antinode in the middle.



In this case the half a wavelength fits in the tube. Therefore λ = 2 × L

∴ λ = 3.4 m

Using v = fλ, we get

340 = f × 3.4

∴ f = 100 Hz

**∴ C (ANS), (83%)**

**Current study design:**

**There have not been any questions on this dot point from 2017.**

*Worked example 2: Resonance: Pipe open at one end.*

**2014 Question 7, 2 marks**

The students place a cap on the end of the tube furthest from the loudspeaker so that it is now closed at one end and open at the other. They increase the frequency until they observe the first resonance, then they increase it again until they observe the next resonance.

Which one of the following best describes the frequency at which the students will observe the second resonance?

**A.** 170 Hz

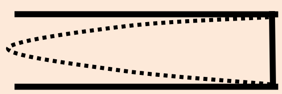
**B.** 150 Hz

**C.** 100 Hz

**D.** 50 Hz

**Solution**

With the pipe closed at one end, the first resonance will look like



Therefore one quarter λ = L
From v = fλ,
We get 340 = $f_0$ × (4 × 1.7)
∴ $f_0$ = 50 Hz
The next time this occurs will be at $3f_0$. (only odd harmonics exist with a pipe closed at one end).
∴ $3f_0$ = 3 × 50
= 150 Hz
**∴ B (ANS), (67%)**

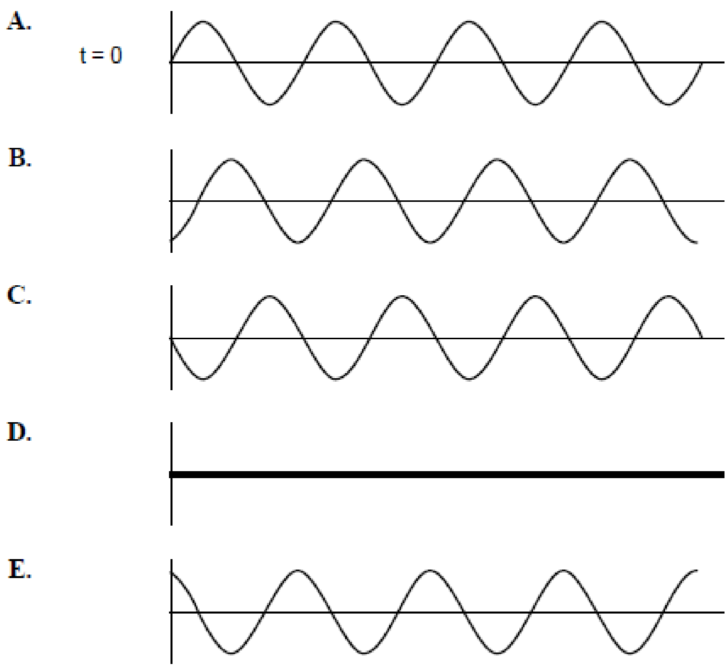
**Current study design:**

**There have not been any questions on this dot point from 2017.**

*Worked example 3: Standing waves: concept.*

The following diagrams represent the variation of air pressure of a sound wave of period T, as a function of distance, at a particular time. Diagram A was recorded at time t = 0.

The other diagrams (**B – E**) show the pressure variation at later times.




**2002 Question 4, 2 marks**

Write the letters (**A–E**) of the diagrams that correspond to a standing wave at times t = $\frac{T}{4}$, and $\frac{T}{2}$, later than diagram A.

**Solution**

If the diagram corresponds to a standing wave, then it is showing nodes and antinodes. The nodes remain stationary, and the antinodes vary from positive maximum through to negative maximum and back.

**∴ $\frac{1}{4}$ of a cycle later the maximum crests have moved to the zero line. ∴ D**

**After $\frac{1}{2}$ of a cycle, the positive maximum has moved to show a negative maximum, etc. ∴ C**

**$\frac{T}{4}$ = D $\frac{T}{2}$ = C (ANS), (42%)**

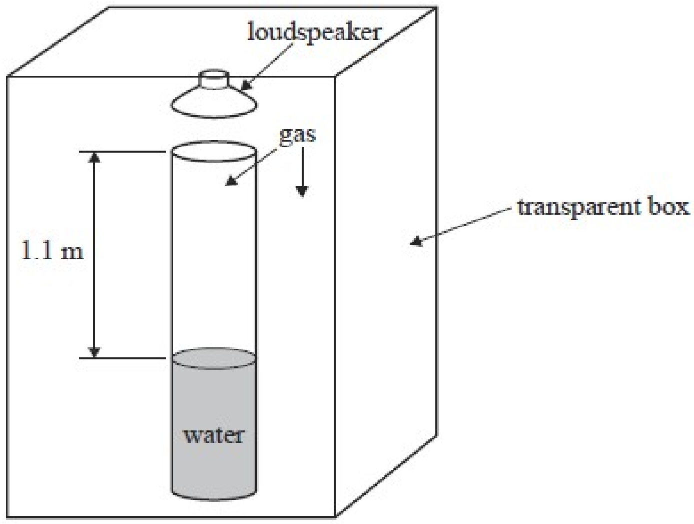
**Current study design:**

**2021 NHT Question 14**

**2019 Question 13b (38%)**

**2017 Question 16c (43%)**

*Worked example 4: Standing waves: Application of v = fλ.*




Lee has been given the task of determining the speed of sound in a particular gas. The equipment he uses is shown below. A large transparent box is filled with the gas and Lee sets up a tube, containing water, with a small loudspeaker above it. The water level is set as shown above to give a length of gas column of 1.1 m. The frequency of the speaker is increased from zero until Lee hears the first (lowest) resonance at 90 Hz.

**2011 Question 7, 2 marks**

Which of the following options is the best estimate of the speed of sound in the gas?

**A.** 300 m $s^{-1}$

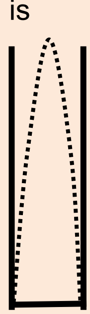
**B.** 320 m $s^{-1}$

**C.** 360 m $s^{-1}$

**D.** 400 m $s^{-1}$

**Solution**

With a pipe open at one end, you get a pressure node at the open end, (because it mixes with the air) and a pressure anti-node at the closed end. (You must get opposite effects because one end is open and the other end is closed).



From the diagram $\frac{1}{4}$ of a wavelength is 1.1 m.

∴ the wavelength is 4.4 m

Using v = fλ

v = 90 × 4.4

= 400 m $s^{-1}$

**∴D (ANS), (73%)**

**Current study design:**

**2022 Question 13 (63%)**

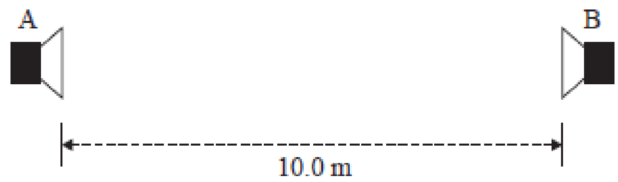
**2020 Question 13a (67%)**

**2019 Question 13a (81%)**

**2019 NHT Question 14c**

*Worked example 5: Standing waves: Between two sources.*

Yasmin and Paul set up the following experiment in a large open area. They connect two speakers that are facing each other, as shown below. Both speakers are connected 10 m apart to the same signal generator and amplifier, which is producing a sound with a wavelength of 1.0 m.




**2016 Question 7, 2 marks**

Yasmin stands in the centre, equidistant to speakers A and B. She then moves towards Speaker B and experiences a sequence of loud and quiet regions. She stops at the second region of quietness.

How far is she from Speaker B?

**A.** 0.75 m

**B.** 1.25 m

**C.** 2.75 m

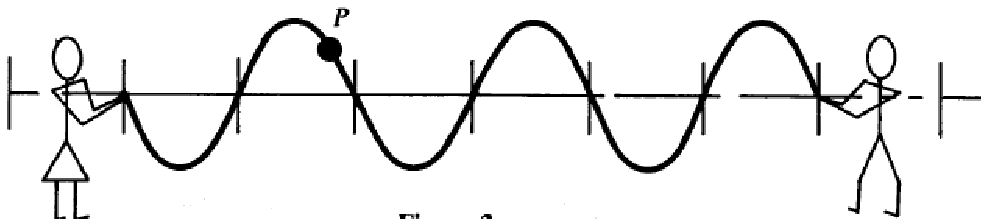
**D.** 4.25 m

| **Solution** | **Current study design:** |
|---|---|
| The middle between the two speakers will be a local maximum, as the waves from both speakers will interfere constructively. The speakers will create a standing wave which will have a wavelength of half of the wavelength of the original sound. Half a wavelength either side of the centre will be a local minimum. This will be 4.75 m from B, as the standing wave wavelength is 0.5 m. Yasmin will move another 0.5 metre towards B for the next local minimum. She will be 4.25 m from B. ∴**D (ANS), (38%)** | **2018 Question 11b (20%)** |

## *Worked example 6: Standing waves: Strings.*

Two physics students, Penny and John, use a length of light spring to demonstrate waves to their class. They set up a standing wave pattern on the spring as shown below.

At the instant shown below the pattern is at its maximum displacement.




**Key** **A.** It is moving to the right.

**B.** It is moving to the left.

**C.** It is moving upwards.

**D.** It is moving downwards.

**E.** It is momentarily stationary.

**F.** It is always stationary.

**1988 Question 36, 1 mark**

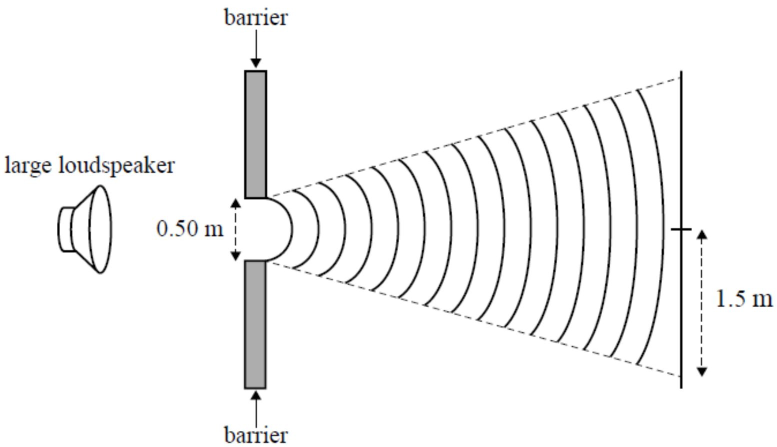
Using the key above, which of the statements (**A - F**) best describes the motion of the point P, at the instant shown.

| **Solution** | **Current study design:** |
|---|---|
| Since it is a standing wave at its maximum displacement, everywhere on the spring is momentarily stationary. | **2020 Question 13b (77%)** |
| ∴E (ANS) | **2019 NHT Question 14a** |
| | **2019 NHT Question 14b** |
| | **2017 Question 16a (63%)** |
| | **2017 Question 16b (58%)** |

## Worked example 7: Diffraction: $\frac{\lambda}{w}$ ratio.

A group of students is conducting experiments to study the diffraction of sound.

The first experiment is conducted on the school oval. The arrangement is shown below.




The frequency is 1200 Hz. The width of the gap between the two barriers is 0.5 m.

At some distance from the gap, the students note that the edge of the diffraction pattern is 1.5 m off the centre line.

**2015 Question 10, 2 marks**

The students increase the frequency to 3000 Hz.

Which one of the following is most likely to be observed?

**A.** The edge of the pattern will still be approximately 1.5 m off the centre line.

**B.** The edge of the pattern will be closer to the centre line.

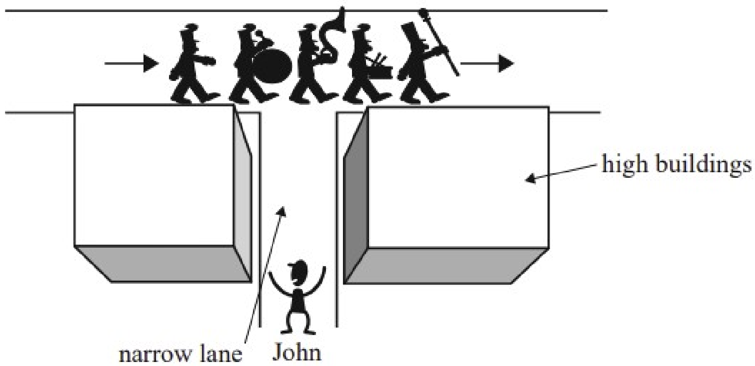
**C.** The edge of the pattern will be further out than 1.5 m.

**D.** There will now be no edge of the pattern.

| **Solution** | **Current study design:** |
|---|---|
| **Increasing the frequency will decrease the wavelength. The diffraction varies as $\frac{\lambda}{d}$, where d is the width of the gap, and λ is the wavelength. Therefore the decrease in λ will lead to less spread in the observed pattern.** <br> **∴B (ANS), (71%)** | **2021 NHT Question 14** <br> **2020 Question 14 (72%)** |

*Worked example 8: Diffraction: Effects.*

A band is marching along a street. The musicians are spread out, with the lower-frequency instruments leading at the front and the higher-frequency instruments towards the rear. All the instruments are played with the same loudness. The band passes the entrance to a narrow lane, where John is standing. He can hear each separate instrument. The situation is shown below.




**2014 Question 11, 2 marks**

Which one of the following best describes what John hears as the instruments pass?

**A.** He will hear each instrument for the same length of time.

**B.** He will hear the lower-frequency instruments for a longer time than the higher-frequency instruments.

**C.** He will hear the lower-frequency instruments for a shorter time than the higher-frequency instruments.

**D.** The length of time that he hears each different instrument depends only on the speed of sound on the day.

**Solution**

John will hear the sound via two methods. Firstly, he will hear the sound as the musicians move past the end of the lane. The sound will travel directly to him. Once the band has moved on, the lower frequencies will diffract more and will persist longer down the laneway.

∴B (ANS), (79%)

**Current study design:**

**2018 NHT Question 11c**

**2017 Question 14 (71%)**