**Diffraction patterns, wavelike properties of matter**

- **investigate and describe theoretically and practically the effects of varying the width of a gap or diameter of an obstacle on the diffraction pattern produced by light and apply this to limitations of imaging using light**
- **interpret electron diffraction patterns as evidence for the wave-like nature of matter**
- **distinguish between the diffraction patterns produced by photons and electrons**
- **calculate the de Broglie wavelength of matter: $\lambda = \frac{h}{p}$.**
- **interpret the single photon/electron double slit experiment as evidence for the dual nature of light/matter**
- **compare the momentum of photons and of matter of the same wavelength including calculations using: $p = \frac{h}{\lambda}$**

**Heisenberg's uncertainty principle**

- **explain how diffraction from a single slit experiment can be used to illustrate Heisenberg's uncertainty principle**
- **explain why classical laws of physics are not appropriate to model motion at very small scales.**

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| Paper | Multiple choice | Short Answer | Idea | Marks | % | Type |
|---|---|---|---|---|---|---|
| 2022 | 14 | | Wave nature of light | 1 | 68% | Concept |
| | | 16a | Diffraction α $\frac{\lambda}{w}$ | 1 | 63% | Concept |
| | | 16b | Diffraction α $\frac{\lambda}{w}$ | 2 | 40% | Explanation |
| | | 17a | $\lambda = \frac{h}{p}$, with $p = \sqrt{2mKE}$ | 4 | 42% | Calculation |
| | | 17b | $\lambda = \frac{h}{mv}$ | 3 | 62% | Calculation |
| 2022 NHT | 19 | | Diffraction pattern concept | 1 | NA | Concept |
| | 20 | | Definition | 1 | NA | Concept |
| | | 16a | X-ray wavelength | 2 | NA | Show |
| | | 16b | Speed of electron | 3 | NA | Calculation |
| | | 18 | Classical/quantum model | 3 | NA | Explanation |
| 2021 | 17 | | $\lambda = \frac{h}{mv}$ | 1 | 68% | Calculation |
| | | 17a | Photon momentum | 1 | 55% | Concept |
| | | 17b | $F\Delta t = \Delta p$, photons | 3 | 11% | Calculation |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | 18a | De Broglie wavelength | 1 | 55% | Concept |
| | | 18b | Diffraction pattern | 4 | 21% | Calculation |
| **2021 NHT** | 17 | | De Broglie wavelength | 1 | NA | Calculation |
| | | 16a | $E = \frac{hc}{\lambda}$ | 3 | NA | Calculation |
| | | 16b | Diffraction α $\frac{\lambda}{w}$ | 3 | NA | Explanation |
| **2020** | | 16a | Electron KE | 2 | 64% | Calculation |
| | | 16b | Photon energy | 3 | 41% | Calculation |
| **2019** | 14 | | de Broglie wavelength | 1 | 84% | Calculation |
| | 15 | | Diffraction pattern | 1 | 40% | Concept |
| | | 17a | Diffraction pattern concept | 3 | 49% | Explanation |
| | | 17b | Photon frequency | 4 | 22% | Calculation |
| **2019 NHT** | 15 | | Single slit diffraction | 1 | NA | Concept |
| | | 11a | Diffraction pattern concepts | 2 | NA | Explanation |
| | | 11b | Photon energy | 4 | NA | Calculation |
| | | 18 | Single slit diffraction | 3 | NA | Explanation |
| **2018** | 15 | | Single slit diffraction | 1 | 49% | Concept |
| | | 18a | $E = \frac{hc}{\lambda}$ | 2 | 55% | Calculation |
| | | 18b | $E = \frac{h^2}{2m\lambda^2}$ | 3 | 33% | Calculation |
| **2018 NHT** | | 13a | $\lambda = \frac{h}{\sqrt{2mE}}$ | 3 | NA | Calculation |
| | | 13b | Diffraction pattern concept | 2 | NA | Calculation |
| | | 13c | Diffraction pattern concept | 2 | NA | Explanation |
| **2017** | 16 | | Single slit | 1 | 66% | Concept |
| | | 19 | Diffraction pattern concept | 4 | 39% | Explanation |

**Diffraction pattern questions can be grouped into the following ideas.**

Matter Diffraction patterns

| | |
|---|---|
| Concepts | Worked example 2 |
| Using $mv = \frac{h}{\lambda}$ | Worked example 1 |
| Using $p = \frac{h}{\lambda} = \frac{hf}{c} = \frac{E}{c}$ | Worked example 3 |
| Using $f\Delta t = \Delta p$ | Worked example 4 |
| Using $p = \sqrt{2mKE}$, $E_K = \frac{p^2}{2m}$ | Worked example 5 |
| Using $E = hf = \frac{hc}{\lambda}$ | Worked example 6 |
| Using $E = \frac{1}{2}mv^2 = \frac{h^2}{2m\lambda^2}$ | Worked example 7 |
| $\frac{\lambda}{w}$ ratio, equal λ, equal p | Worked example 8 |

Heisenberg's uncertainty principle

| | |
|---|---|
| Basic concepts/single slit | Worked example 9 |
| Classical/quantum model | Worked example 10 |

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**Diffraction**

When light passes through a narrow aperture, a hole, or a slit, it spreads out, i.e. it diffracts, $\propto \frac{\lambda}{d}$.

**The wave nature of electrons**

If electrons can behave as waves inside an atom, they might be able to exhibit other wave properties like interference. This was shown in electron scattering experiments - definite patterns of reinforcement (antinodes) and reduction (nodes) were found in scattering. The gaps that the electrons were passing through in scattering were very small, in the order of the radius of an atom; hence the electrons wavelengths must be much smaller than that of light.

**The wave-like nature of matter**

deBröglie speculated that if light waves could behave like particles, then particles of matter should behave like waves. He argued that the equation $p = \frac{h}{\lambda}$ should apply to particles as well as waves. Experiments with electrons clearly show that they can diffract and interfere with each other. Protons and neutrons also have been shown to exhibit wave-like behaviour.

The wavelength of a particle of matter could be found by using: $\lambda = \frac{h}{p} = \frac{h}{mv}$

The wavelength is inversely proportional to the momentum. With any mass that is not sub-atomic, the product of 'mv' is so large that $\lambda$ is always of the order $10^{-33}$ m, too small for us to see. When sound and light waves pass through narrow slits, they show diffraction effects only when the slit is about 1 – 50 $\lambda$. Thus, a particle could be expected to show diffraction only if it is passing through a gap 1 - 50 times its de Bröglie wavelength.

For most large particles this is impractical as the physical size of the particle is far too large. Only in the case of small particles such as electrons, is the de Bröglie wavelength enough for diffraction and interference effects to occur.

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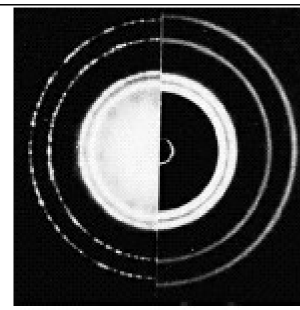
**Electron Diffraction Patterns**




One experiment that is used to show the wave nature of electrons is the electron diffraction pattern. Electrons are sent through a piece of metal as they pass through, they leave a pattern on a screen. Only half of the pattern is projected onto the screen, then x-rays are sent through the metal and the pattern from the x-rays is projected onto the other half of the screen. If the two patterns align, then the x-rays must have the same wavelength as the electrons, since the amount of diffraction is governed by the ratio of $\frac{\lambda}{w}$, and since w is the same, then λ must be the same. If the both have the same λ, they must both have the same momentum.

The pattern is formed from diffraction as the wave fronts travel between the atoms. Then the wavefronts interfere with each other to form bright and dark rings.

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**The wave-like nature of individual photons**

Water waves and sound waves demonstrate interference by interacting with each other. With photons it is not quite so straight forward. When the double slit experiment is performed using light, it is possible to lower the intensity so that only one photon of light passed through the slits at a time. (this was done by Taylor) i.e., there was no chance of the photons interacting, but interference was still observed.

A series of bright and dark bands were eventually formed on the photographic plate that was being used. The pattern of interference was an interference pattern as predicted by the wave model. The photons behave like particles in that they go through either one slit or the other, but they don't form a pattern consisting of two narrow lines that you would expect from particles. The photons don't interact with each other, yet after passing through the slit each photon has a high probability of heading towards one of the bright bands.

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**Wave-particle duality of light**

The argument about whether light was a wave or a particle was settled in the 1920's. The wave model explained refraction, diffraction and interference of light. The particle model explained the photoelectric effect. Light is neither a wave nor a particle. Photons exhibit both wave and particle properties. This is called WAVE-PARTICLE DUALITY.

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**Photons**

The modern theory of light is a merging of the wave and the particle models. Light is modelled as a stream of packets or *quanta* of energy. The energy carried by each quantum is proportional to the frequency of light and can be found from Planck's equation: $E = hf = \frac{hc}{\lambda}$. Greater intensity of light has more quanta - each quantum still has the same energy. The quanta of energy are called photons. Photons are neither particles nor waves.

Low frequency photons such as radio-waves and microwaves exhibit distinctly wave-like behaviours such as diffraction and interference, but have no particle-like properties. Around the middle of the spectrum in the visible light region, photons have both wave and particle properties. They interfere and diffract like waves, and also interact with electrons in the photoelectric effect as particles do. At the high frequency end of the spectrum, X-ray and gamma ray photons behave much more like particles than waves.

**The momentum of photons**

In 1923, Compton showed that X-ray photons could collide with electrons and scatter, leaving with a longer wavelength (less energy) than before. This is only possible if the photons were able to transfer momentum and hence energy to the electrons.

Maxwell suggested that photons do have momentum given by

$p = \frac{E}{c}$ *where c is the speed of light and E is the energy of the photon.*

As the energy of the photon is related to its frequency by Planck's equation, and, since $v = f\lambda$ for waves, the momentum equation can be written as $p = \frac{hf}{c} = \frac{h}{\lambda}$

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**Heisenberg's uncertainty principle**

The wave particle duality of Quantum physics (not classical physics), assumes that a matter wave, like a light wave, is a probability wave. Heisenberg proposed that measured values cannot be given to position (x) and momentum (p) of a particle simultaneously with unlimited precision.

This uncertainty is an outcome of both wave-particle duality and the interactions between the object being observed and the effect of the observation on that object. For the normal 'nonquantum' world $\Delta x$ and $\Delta p_x$ are so small they are considered insignificant, but at the atomic scale, this level of uncertainty is significant.

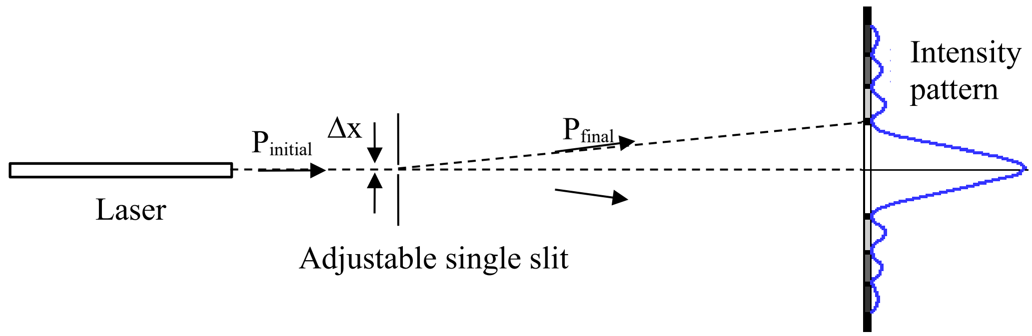
To measure the precise location of a free particle (e.g. electron), it needs to be hit with another particle (e.g. photon). This will cause the electron to move (or move differently) as energy is transferred from the photon. Therefore the act of measuring causes a change in the value of what is being measured.

If $\Delta x$ is position uncertainty, and $\Delta p$ momentum uncertainty then use $\Delta x \times \Delta p_x \geq \frac{h}{4\pi}$ to find the minimum uncertainty allowed. As $\Delta x$ decreases $\Delta p_x$ has to increase. Note $\Delta$ means "uncertainty of'. Simply put, the more we know the position of a particle, the less we know the momentum of the particle.

**Single slit diffraction.**

Taylor carried out the single slit experiment using light so feeble that only one photon passed randomly through the slit at a time. Interference fringes built up on the screen (over 3 months), even though the photons could not have been interacting with each other.

As the photon passes through the slit, $\Delta x$ is the slit width, its position is known with some uncertainty.



Davisson and Germer (1928) demonstrated the wave nature of electrons, so single slit diffraction pattern can also be observed with particles, e.g. electrons, protons, neutrons etc.

Heisenberg explained this by saying that the single slit introduces some uncertainty in $\Delta x$ and hence $\Delta p_x$, so the beam spreads out and the interference pattern becomes wider. If $\Delta x$ becomes smaller, $\Delta p_x$ must be greater so beam spreads out more.

For more information

Veritasium video on Heisenberg https://www.youtube.com/watch?v=a8FTr2qMutA

This video requires some explanation. For me, the diagram showing the momentum arrows can be misleading. Just need to point out that we are only interested in $\Delta p_x$.

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**Momentum and Energy connections**

Start with $p = mv$

Square both sides $p^2 = m^2v^2$

Divide both sides by 2m $\frac{p^2}{2m} = \frac{m^2v^2}{2m}$

Leads to $\frac{p^2}{2m} = \frac{mv^2}{2}$

$$\therefore \frac{p^2}{2m} = KE$$

Rearranging gives $p^2 = 2mKE$

$$\therefore p = \sqrt{2mKE}$$

Both of these are very useful, but only work if $m > 0$.

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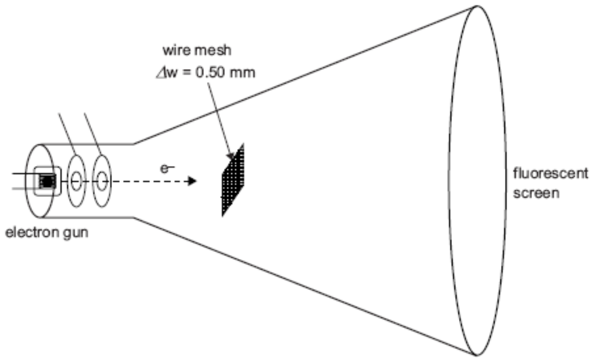
*Worked example 1: Matter Diffraction patterns: Using $mv = \frac{h}{\lambda}$.*

A sketch of a cathode ray tube (CRT) is shown below.

In this device, electrons of mass $9.10 \times 10^{-31}$ kg are accelerated to a velocity of $2.0 \times 10^{7}$ m s$^{-1}$.

A fine wire mesh in which the gap between the wires is w = 0.50 mm has been placed in the path of the electrons, and the pattern produced is observed on the fluorescent screen.

Planck's constant: $h = 6.63 \times 10^{-34}$ J s




**2005 Question 6, 3 marks**

Calculate the de Broglie wavelength of the electrons. You must show your working.

**Solution**

The momentum of the electron

**$m \times v = 9.1 \times 10^{-31} \times 2.0 \times 10^{7}$**

**$= 1.82 \times 10^{-23}$**

**Using** $p = \frac{h}{\lambda}$

$\therefore \lambda = \frac{6.63 \times 10^{-34}}{1.82 \times 10^{-23}}$

**$= 3.64 \times 10^{-11}$ m (ANS), (71%)**

**Current study design:**

2022 Question 17b (62%)

2022 NHT Question 16b

2021 Question 17 (68%)

2021 Question 18a (55%)

2021 NHT Question 17

2019 Question 14 (84%)

*Worked example 2: Matter Diffraction patterns: Basic concepts.*

**2005 Question 7, 2 marks**

Explain, with reasons, whether or not the students would observe an electron diffraction pattern on the fluorescent screen due to the presence of the mesh.

**Solution**

No.

For diffraction, the gap width must be the same order of magnitude as the wavelength.

**From the diagram the mesh spacing is $5 \times 10^{-4}$ m. This is much greater than the wavelength of the electron. (71%)**

**Current study design:**

2022 Question 14 (68%)

2022 Question 16a (63%)

2022 Question 16b (40%)

2022 NHT Question 19

2017 Question 19 (39%)

*Worked example 3: Matter Diffraction patterns: Using* $p = \frac{h}{\lambda} = \frac{hf}{c} = \frac{E}{c}$.

A source is designed to produce X-rays with a wavelength of $1.4 \times 10^{-10}$ m.

**2009 Question 9, 2 marks**

What is the momentum of one of these X-ray photons?

**Solution**

**Current study design:**

2021 Question 17a (55%)

**Use** $p = \frac{h}{\lambda}$

$= \frac{6.63 \times 10^{-34}}{1.4 \times 10^{-10}}$

**$\therefore 4.7 \times 10^{-24}$ N s (ANS), (57%)**

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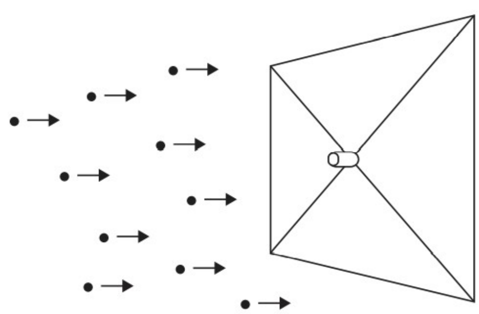
*Worked example 4: Matter Diffraction patterns: Using* $f\Delta t = \Delta p$.

A 'space sail' mounted on a tiny interstellar cylindrical probe relies on the momentum of photons from a nearby star to exert a propulsive force, as shown below.

The photons strike the sail at 90° to its surface and reflect elastically. Scientists need to calculate the force exerted by the photons, which have a frequency of $7.0 \times 10^{15}$ Hz and a momentum equal to $1.55 \times 10^{-26}$ kg m s$^{-1}$.



**2021 Question 17b, 3 marks**

$2.0 \times 10^{18}$ photons of this frequency strike the space sail every second. Calculate the force that the reflecting photons exert on the space sail. Show your working. Give your answer correct to two significant figures.

**Solution**

**Current study design:**

2021 Question 17b (11%)

Use $f\Delta t = \Delta p$, This is a variation of the impulse equation, $f\Delta t = m\Delta v$. Where f is the force exerted by the photons, $\Delta t = 1$ sec, and $\Delta p$ = the change in momentum of the electrons on reflection from the 'space sail'.

**Initial momentum is $1.55 \times 10^{-26}$ kg m s$^{-1}$ the final momentum is the same.**

The change in momentum = final – initial,

**$\therefore \Delta p = 1.55 \times 10^{-26} - - 1.55 \times 10^{-26}$.**

**$\therefore \Delta p = 3.10 \times 10^{-26}$.**

**$\therefore f \times 1 = 3.10 \times 10^{-26}$**

**$\therefore f_{(per\ photon)} = 3.10 \times 10^{-26}$ N**

**As there are $2.0 \times 10^{18}$ photons striking the sail each second, the force applied to the sail = $2.0 \times 10^{18} \times 3.10 \times 10^{-26}$**

**$\therefore f = 6.2 \times 10^{-8}$ N (ANS), (11%)**

**(correct to 2 sig figs)**

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*Worked example 5: Matter Diffraction patterns: Using p =* $\sqrt{2mKE}$, $E_k = \frac{p^2}{2m}$.

**1999 Question 5, 2 marks**

Electrons of kinetic energy 54 eV are used to investigate the spacing of atoms in a nickel crystal. These electrons have a de Broglie wavelength of $1.67 \times 10^{-10}$ m and the atomic spacing in the nickel crystal is $2.15 \times 10^{-10}$ m.

Calculate the momentum of a 54 eV electron. ($h = 6.63 \times 10^{-34}$ J s)

| Solution | Current study design: |
|---|---|
| If KE = $\frac{1}{2}mv^2$ and p = mv, then KE = $\frac{p^2}{2m}$<br>p = $\sqrt{2mKE}$<br>= $\sqrt{2 \times 9.1 \times 10^{-31} \times 54 \times 1.6 \times 10^{-19}}$<br>**= $3.97 \times 10^{-24}$ Ns**<br>**$\therefore 4.0 \times 10^{-24}$ N s (ANS), (70%)** | 2022 Question 17a (42%) |

*Worked example 6: Matter Diffraction patterns: Using E = hf =* $\frac{hc}{\lambda}$.

A beam of electrons is produced in an electron gun.

The de Broglie wavelength of each electron is 0.36 nm.

An experiment is undertaken to compare the diffraction of these electrons and X-rays. With a similar gap spacing, the diffraction patterns are found to be nearly identical.

**2016 Question 20b, 3 marks**

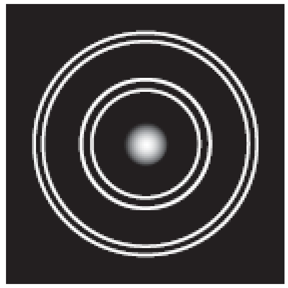
Calculate the energy of the X-rays. Show each step of your working.

| Solution | Current study design: |
|---|---|
| Since the two patterns are nearly identical, they must both have the same wavelength.<br>**For an X-ray, E =** $\frac{hc}{\lambda}$<br>**$\therefore$ E =** $\frac{4.14 \times 10^{-15} \times 3 \times 10^{8}}{0.36 \times 10^{-9}}$<br>**$\therefore$ E = $34.5 \times 10^{2}$**<br>**$\therefore$ E = $3.5 \times 10^{3}$ eV (ANS), (47%)** | 2022 NHT Question 16a<br>2021 NHT Question 16a<br>2020 Question 16b (41%)<br>2019 Question 17b (22%)<br>2018 Question 18a (55%)<br>2018 NHT Question 13a<br>2018 NHT Question 13b |

*Worked example 7: Matter Diffraction patterns: Using E = $\frac{1}{2}$ mv² = $\frac{h^2}{2m\lambda^2}$ .*

A beam of electrons is travelling at a constant speed of $1.5 \times 10^5$ m s$^{-1}$.

The beam shines on a crystal and produces a diffraction pattern. The pattern is shown below. Take the mass of one electron to be $9.1 \times 10^{-31}$ kg.



**2012 Question 3a, 2 marks**

Calculate the kinetic energy (in eV) of one of the electrons.

**Solution**

**KE = $\frac{1}{2}mv^2$**

**= $\frac{1}{2} \times 9.1 \times 10^{-31} \times (1.5 \times 10^5)^2$**

**= $1.024 \times 10^{-20}$ J**

**Convert into eV by dividing by $1.6 \times 10^{-19}$**

**∴ KE = 0.064 eV (ANS), (60%)**

**Current study design:**

**2020 Question 16a (64%)**

**2019 NHT Question 11b**

**2018 Question 18b (33%)**

## *Worked example 8: Matter Diffraction patterns: $\frac{\lambda}{w}$ ratio, equal λ, equal p*

Students study diffraction of electrons by a crystal lattice. The apparatus is shown in Figure A. In this apparatus electrons of mass $9.1 \times 10^{-31}$ kg are accelerated to a speed of $1.5 \times 10^{7}$ m s$^{-1}$.

The electrons pass through the crystal, and the diffraction pattern is observed on a fluorescent screen. The pattern the students observe is shown in Figure B.

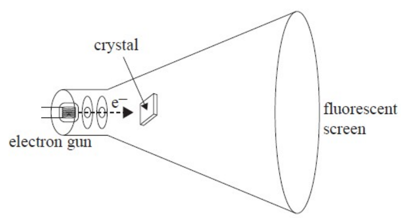



**Figure A**

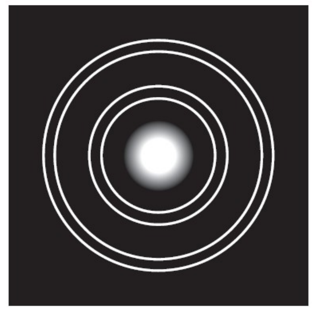
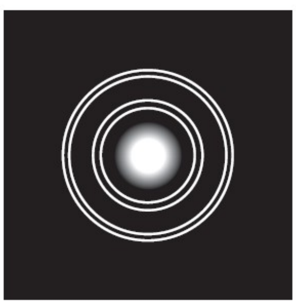


**Figure B**

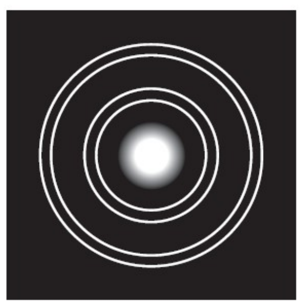
**2010 Question 8, 2 marks**

The students now increase the accelerating voltage and hence the speed of the electrons.

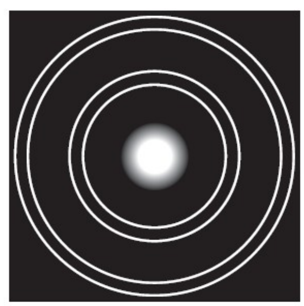
Which one of the options below now best shows the pattern they will observe on the screen? Explain your answer.



option A



option B (identical to original pattern)



option C

**Solution**

Increasing the accelerating voltage will increase the speed of the electrons. This will increase their momentum.

**From $\lambda = \frac{h}{mv}$ increasing the momentum will decrease the deBroglie wavelength of the electrons.**
**The amount of diffraction (bending) is given by $\frac{\lambda}{w}$, so the smaller wavelength means smaller spacing between diffraction lines.**

**∴ A (ANS), (52%)**

**Current study design:**

**2021 Question 18b (21%)**

**2021 NHT Question 16b**

**2019 Question 15 (40%)**

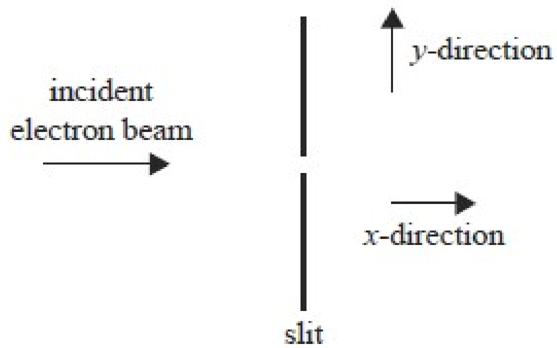
**2019 Question 17a (49%)**

**2019 NHT Question 11a**

**2018 NHT Question 13c**

*Worked example 9: Heisenberg's uncertainty principle: Basic concepts/single slit.*

A diffraction pattern is produced by a stream of electrons passing through a narrow slit, as shown in the diagram below.




**2017 Question 16, 1 mark**

This electron diffraction pattern can be used to illustrate Heisenberg's uncertainty principle. This is because knowing the uncertainty in the

**A.** electron's speed is large leads to the uncertainty in its kinetic energy being small.

**B.** slit width is small leads to a large uncertainty in the electron's momentum in the *y*-direction.

**C.** electron's momentum in the *y*-direction is small leads to a large uncertainty in the slit's width.

**D.** electron's angle of approach to the slit leads to a large uncertainty in the electron's momentum in the *y*-direction

**Solution**

Heisenberg's uncertainty principle states the more we know about the position of the electron, then the more uncertainty we have with knowing its momentum.
For an electron going through a very narrow slit, as it passes through the slit, we know its position (in the y-direction) quite precisely. This means that its momentum in the y-direction has a large uncertainty. This results in the spread of the electrons which is evidenced by a spread in the diffraction pattern.

**∴ B (ANS), (66%)**

**Current study design:**

2022 NHT Question 20
2019 NHT Question 15
2018 Question 15 (49%)
2018 NHT Question 18
2017 Question 16 (66%)

*Worked example 10: Heisenberg's uncertainty principle: Classical/quantum model.*

**2022 NHT Question 18, 3 marks**

Provide an example of an instance in which classical laws of physics cannot describe motion at very small scales and explain why they cannot.

| **Solution** | **Current study design:** |
|---|---|
| The interference pattern established by electrons when the pass through two narrow slits cannot be explained by classical laws of physics. Classical laws predict that the electrons would travel in straight lines through the slits, and produce two lines on the screen. To explain the resulting diffraction pattern requires an understanding that moving electrons exhibit wave like properties. | **2022 NHT Question 18** |