Energy conservation.

- **investigate and apply theoretically and practically the concept of work done by a constant force using:**
  – work done = constant force × distance moved in direction of net force
  – work done = area under force-distance graph
- **analyse transformations of energy between kinetic energy, strain potential energy, gravitational potential energy and energy dissipated to the environment (considered as a combination of heat, sound and deformation of material):**

  –kinetic energy at low speeds: $E_k = \frac{1}{2} mv^2$; elastic and inelastic collisions with reference to conservation of kinetic energy
  –strain potential energy: area under force-distance graph including ideal springs obeying Hooke's Law: $E_s = \frac{1}{2} k\Delta x^2$
  –gravitational potential energy: $E_g = mg\Delta h$ or from area under a force-distance graph and area under a field - distance graph multiplied by mass.

Paper	Multiple choice	Short Answer	Idea	Marks	%	Type
2022	8		Energy stored in spring, graph	1	72%	Calculation
2022 NHT	8		Spring constant, from graph	1	NA	Calculation
	9		mg = kΔx	1	NA	Calculation
		7a	$\Delta mgh = \frac{1}{2}mv^2$	2	NA	Show
		7b	$\Delta mgh = \frac{1}{2}mv^2$	2	NA	Explanation
		7c	$T - mg = \frac{mv^2}{r}$	3	NA	Calculation
2021	11		Spring constant, from graph	1	63%	Calculation
	12		Energy stored in spring, graph	1	68%	Calculation
		8b	Kinetic energy calculation	2	45%	Calculation
		8c	ΔGPE, g vs d graph	3	22%	Calculation
		8d	ΔKE and ΔGPE, work done	3	31%	Explanation
		9a	Roller coaster, Δmgh = ΔKE	2	63%	Calculation
2021 NHT	12		Vertical spring	1	NA	Concept
		8c	GPE = mgΔh	3	NA	Calculation
		9a	Net force	4	NA	Concept
		9b	$\Delta mgh = \frac{1}{2}k\Delta x^2$	3	NA	Calculation
		18b	ΔKE in collision	2	NA	Explanation
2020	10		WD = f × d	1	80%	Calculation
		9a	Horizontal spring	2	69%	Calculation
		9b	ΔGPE + ΔKE = constant	3	51%	Calculation
2019	20		Energy concept	1	36%	Concept
		8a	ΔGPE + ΔKE = constant	3	57%	Calculation
2019 NHT		5a	Vertical spring	2	NA	Calculation
		5b i	Vertical spring	2	NA	Calculation
		5b ii	Vertical spring	4	NA	Calculation
2018		6a	Vertical spring	3	50%	Calculation
		6b	Vertical spring	2	15%	Calc/Explain
		6c	Vertical spring	2	15%	Calculation
2018 NHT		9a	Vertical spring compression	2	NA	Calculation
		9b	Vertical spring compression	3	NA	Calculation
		9c	Vertical spring compression	4	NA	Calculation
2017	12		Horizontal spring	1	89%	Calculation
	13		Horizontal spring	1	58%	Calculation
		8b	ΔPE + ΔKE = constant	2	38%	Calculation
		13a	Vertical spring	3	32%	Calculation
		13b	Vertical spring	4	45%	Explanation

Energy

questions can be grouped into the following ideas.

Kinetic Energy

The Kinetic Energy is the energy of a body due to its motion.

KE = $\frac{1}{2}mv^2$

WD = ΔKE = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Gravitational Potential Energy.

When changes in height 'Δh' are small compared to the radius of the Earth, the potential energy U_g of a body near the earth's surface is given by $U_g = mg\Delta h$.

Work Done

A fundamental principle of nature is that energy cannot be created or destroyed, only transformed or transferred to another body. A body that has energy may transfer some, or all, of its energy to another body. The total amount of energy remains constant (conserved), even if it has been transformed to another type. The amount of energy transformed is called **work**. The body losing energy does work, the body gaining energy has work done on it.

Work done = Force × displacement, it is a scalar quantity, with the units of Joule.

The area under a force-displacement graph shows the work done. If the force is constant then the area under the graph is given by **W = F × d**, where F is the force, d is the distance over which the force acts. This assumes that the force and the displacement are in the same direction. If they aren't then the work is the product of the resolved part of the force (in the direction of motion) × the displacement.

The work done is often equal to the change in the Kinetic Energy. If work is done on an object travelling in a horizontal plane, then the work done equals the loss of kinetic energy.

$\therefore$ WD = ΔKE

$\therefore$ ΔKE = F × d

If the force isn't constant, then the work done is equal to the area under the F - d graph.

Power

Power is the rate of doing work.

$$\therefore P = \frac{W}{\Delta t} = \frac{Fd}{\Delta t} = Fv.$$

Power (scalar) with units of joule sec^{-1} or **Watt**.

Strain potential energy

Strain energy (elastic potential energy) is the energy stored in any material that has been stretched or compressed from its normal shape. Springs and elastic bands are good examples. For a spring not stretched beyond its elastic limit, the force, F, applied is proportional to the extension, Δx, produced.

$F = k\Delta x$ where k is called the spring constant. (the gradient of the F - Δx graph)

This is known as Hooke's Law. The spring constant (force constant), k, gives a measure of the **stiffness** of the material.

As F varies with Δx, the force will not be constant, therefore the acceleration is not constant. This means that the SUVAT equations (used only for constant acceleration), cannot be used.

The energy stored in the spring is = $\frac{1}{2}$ base × height

= $\frac{1}{2}\Delta x.k\Delta x$

= $\frac{1}{2}k(\Delta x)^2$ (the strain energy).

The energy stored in the spring is also the work done, which is the area under the graph.

Springs

Energy is conserved in spring questions (but not in real life).
Therefore the Total Energy is constant.
Typically the total energy is the sum of the kinetic energy, the gravitational potential energy, and the elastic potential energy.

$\therefore$ TE = KE + GPE + EPE.

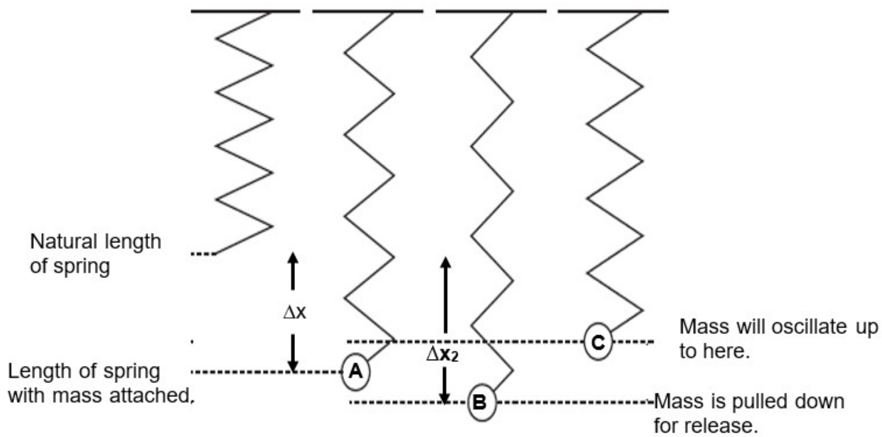
Horizontal springs solution process

Typically use $F = k\Delta x$, and $\frac{1}{2}k(\Delta x)^2 = \Delta KE$.
The greater the compression of the spring, the greater the force exerted by it (or acting on it). At maximum compression, the velocity =0, but the force (acceleration) is at its maximum value.

As the spring is horizontal the Gravitational Potential energy is constant, so the loss in KE is equal to the gain in spring energy. The energy stored in the spring, given by $\frac{1}{2}k(\Delta x)^2$ will be converted into KE of a mass when the spring releases its stored energy. Therefore equate the two expressions to solve for the unknown variable.

If an object is slowed to a stop by compressing a spring, then the initial KE is stored in the spring as EPE given by $\frac{1}{2}k(\Delta x)^2$. Equate the two to find the unknown variable.

Vertical springs solution process



Typically the spring is loaded with masses, and the extension is recorded.

If the spring is vertical, then gravitational potential energy needs to be considered. The total energy of the system (if we ignore friction and air resistance) will be constant.

The total energy is given by the sum of:
kinetic energy + gravitational potential energy + elastic potential energy

$\therefore$ TE = $\frac{1}{2}mv^2 + mgh + \frac{1}{2}k\Delta x^2$.

Spring constant "k"

To find the spring constant, consider the point **A**. Here there are two forces acting on the mass, the weight down and an upward force from the spring.
As the mass is in equilibrium, the net force on it is zero.

$\therefore mg + k\Delta x = 0$ Use this to find the unknown quantity.

Once you know the spring constant, the question turns into an Energy question.

The mass will oscillate up and down.
Typically, the mass is pulled further down to "**B**" and then released. The mass will oscillate about a mean position "**A**", between "**B**" and "**C**". The motion is symmetrical about "**A**".

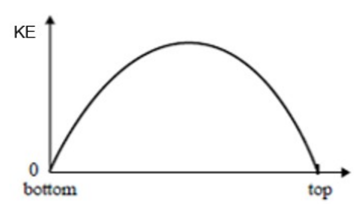
To solve problems, find the TE at one point and then find the individual components at other points.

You will need to find the total energy of the system. This can be found at either **B** or **C**. At both **B** and **C,** the kinetic energy is zero, as the mass is momentarily stationary. It is often simplest to assume that the gravitational potential energy is zero at **B** and so the only energy of the system is the elastic potential energy stored in the spring. This is given by EPE = $\frac{1}{2}k\Delta x_2^2$. (be careful with Δx_2).

Once you know the TE at one point, you can use this to find individual components at other points in the oscillation.
At the midpoint, the KE is at its maximum value , the GPE is half its maximum value and the EPE = $\frac{1}{2}k\Delta x^2$.

The graphs of the three component energies and the graph of the total energy need to be well understood.

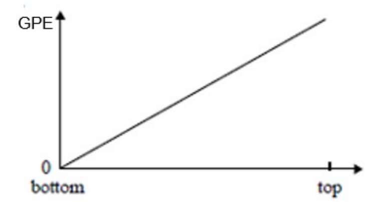
Kinetic Energy



The kinetic energy is zero at the top, increases to a maximum in the middle and then decreases to zero at the bottom.

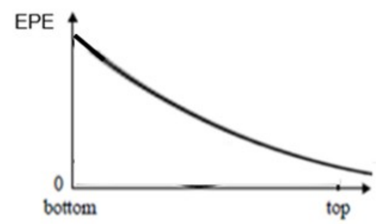
Gravitational potential energy

The gravitational potential energy is greatest at the top, and least (often 0) at the bottom.



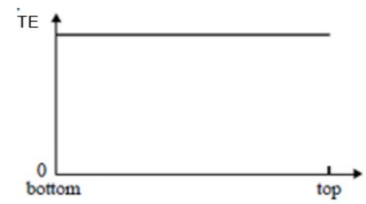
Elastic potential energy

The elastic potential energy is going to be a minimum at the top of the oscillation and increase to a maximum at the bottom. It is given by EPE = $\frac{1}{2}k\Delta x_2^2$, so it will be parabolic. It may not end at zero if the top point of the oscillation is below the natural length of the spring.



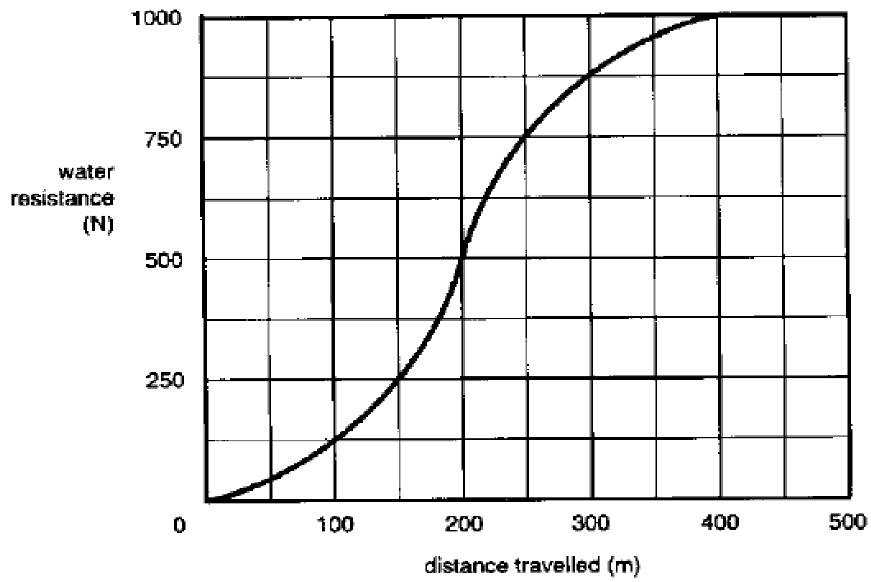
Total energy

The total energy of the system remains constant throughout the oscillation.



Worked example 1: Application of F = ma.

A motorboat of mass 800 kg is accelerated from rest, until it reaches a constant speed. The boat is always driven by a constant force. It is opposed by the resistance of the water. This opposing force is not constant. The graph below shows the **force of the water opposing the boat's motion,** as a function of the distance it has travelled. The boat reaches its final, constant speed after a distance of 400 m.



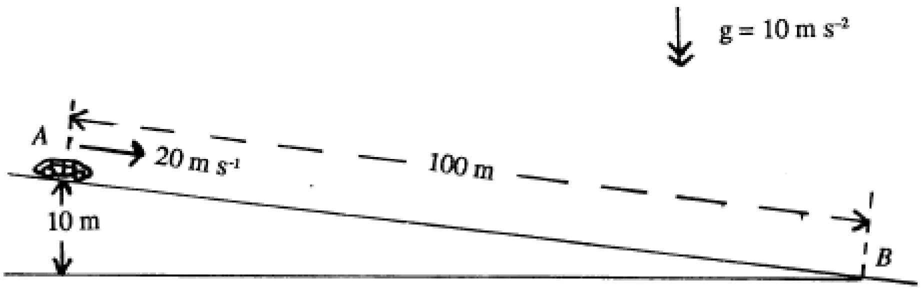
1995 Question 11, 1 mark

After the boat has travelled a distance of 400 m, what is the resultant force acting on the boat?

Solution	**Current study design:**
Since the boat reaches a final constant speed after a distance of 400 m, the resultant force acting on the boat must be zero, since there is no acceleration. ∴ 0 N (ANS), (27%)	**2021 NHT Question 9a** **2019 Question 20 (36%)**

Worked example 2: KE = $\frac{1}{2}$ mv^2

In a braking test a car of mass 1000 kg was travelling down a hill on a straight road which has a constant slope of 1 in 10 as shown in the diagram.
The car was travelling at 20 m s^{-1} at *A* where a constant braking force was applied so that the car came to a stop at *B*, 100 m from *A*.

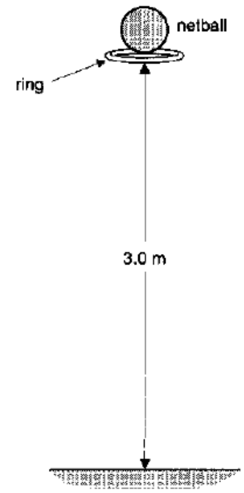


1988 Question 6, 1 mark

What is the kinetic energy of the car at *A*?

Solution	**Current study design:**
The KE of the car = $\frac{1}{2}$ mv^2 ∴ KE = $\frac{1}{2}$ × 1000 × 20^2 = 2.0×10^5 J (ANS)	**2021 Question 8b (45%)**

Worked example 3: TE = KE + GPE



A netball of mass 0.25 kg falls 3.0 m to the floor, from the ring where it was **briefly at rest.** The acceleration due to gravity can be taken as 10 m s^{-2}. Air resistance can be neglected.

1995 Question 7, 1 mark

What is the kinetic energy of the ball as it reaches the floor?

Solution

The loss in gravitational PE is equal to the gain in KE. Since it starts from rest, this means that it will also be its KE.

$\therefore \Delta mgh = \Delta KE$

$\therefore 0.25 \times 10 \times 3.0$

= 7.5 J (ANS), (65%)

Current study design:

2022 NHT Question 7a

2022 NHT Question 7b

2021 Question 9a (63%)

2020 Question 9b (51%)

2019 Question 8a (57%)

2017 Question 8b (38%)

Worked example 4: Springs: EPE = $\frac{1}{2}k\Delta x^2$

Shortly after the ball first contacts the floor it comes to rest momentarily. At this point the ball has been compressed by 2.0 cm (0.020 m). In the question below, you should model the compression of the ball as if it were a spring with a force constant k = 3.40× 10^4 N m^{-1}.

1995 Question 8, 1 mark

How much elastic potential energy has been stored in the ball when the compression is complete?

Solution

Elastic potential energy stored in the ball is given by:

$EPE = \frac{1}{2} k\Delta x^2$

$= \frac{1}{2} \times 3.40 \times 10^4 \times 0.02^2$

= 6.8 J (ANS), (68%)

Current study design:

2022 Question 8, (72%)

Worked example 5: ΔKE, vertical collision with floor.

It should be clear from your answers to Questions 7 and 8 that not all of the kinetic energy of the ball as it reached the floor, has been stored as elastic potential energy. However from the *Principle of Conservation of Energy* we know that energy cannot be created or destroyed.

1995 Question 9, 1 mark

Describe where the 'missing' energy might have gone.

Solution

Sound and heat are generated during the impact of the ball with the floor. Deformation of the ball and the floor surface also occurs. These energy losses account for the difference between 7.5 J and 6.8 J. (68%)

Current study design:

2021 NHT Question 18b

Worked example 6: WD = F × d, constant force.

In a test, an unpowered toy car of mass 4.0 kg is held against a spring, compressing the spring by 0.50 m, and then released, as shown below.
There is negligible friction while the car is in contact with the spring.
The figure below also shows the force–extension graph for the spring.

2016 Question 4d, 3 marks

After the car leaves the spring at 2.0 m s^{-1}, the car has a constant frictional resistance of 2.0 N. Calculate how far the car travels before it stops. Show your working.

Solution

Use the work done is the change in KE.

$\therefore F \times d = \Delta KE$

$KE_i = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 4 \times 2^2$

= 8 J

$\therefore F \times d = 8$

$\therefore 2.0 \times d = 8$

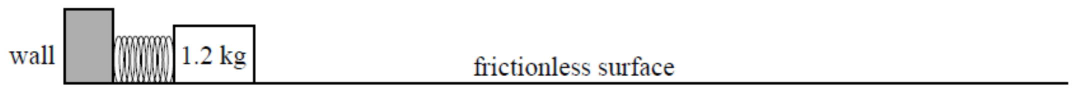
$\therefore d = 4$ m (ANS), (55%)

Current study design:

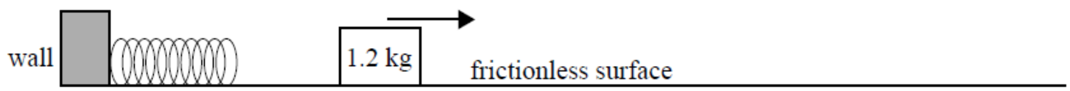
2020 Question 10 (80%)

Worked example 7: WD = ΔKE.

A spring is resting against a wall. The spring is compressed by a distance of 8.0 cm from its uncompressed length. Jemima holds a block of mass 1.2 kg stationary against the compressed spring as shown below.



Jemima releases the block, and it slides to the right on a frictionless surface. It leaves the spring with a kinetic energy of 5.4 J and slides at constant speed as shown below.



2012 Question 1b, 1 mark

Calculate the work done by the spring on the block.

Solution

Work done by the spring on the block is the energy stored in the spring, which is given to the block as KE.

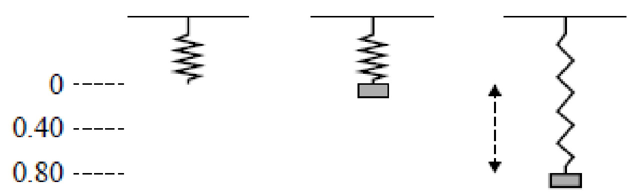
∴ 5.4 J (ANS), (63%)

Current study design:

2021 Question 8d (31%)

Worked example 8: GPE = mgΔh.

A mass of 2.0 kg is suspended from a spring, with spring constant k = 50 N m^{-1}, as shown below.
It is released from the unstretched position of the spring and falls a distance of 0.80 m. Take the zero of gravitational potential energy at its lowest point.



2015 Question 6a, 1 mark

Calculate the change in gravitational potential energy as the mass moves from the top position to the lowest position.

Solution

GPE = mgΔh

∴ 2.0 × 9.8 × 0.80 = 15.68

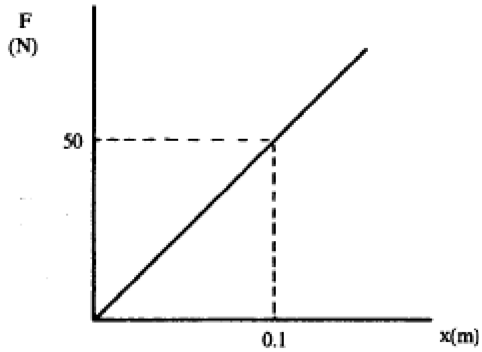
∴ 16 J (ANS), (87%)

Current study design:

2021 NHT Question 8c

Worked example 9: Springs: Spring constant = gradient f vs Δx graph.

The spring constant (*k* of a spring can be defined as the force per unit distance required to extend the spring. Thus a strong spring will have a large value of *k*, while a weak spring will have a small value. The force-extension curve of a spring is shown.



1981 Question 21, 1 mark

What is the value of *k*, the spring constant? (Give both magnitude and unit).

Solution

Using F = kΔx, the spring constant is the gradient of the graph as the graph is F vs x.

$$\therefore k = \frac{50}{0.1}$$

$\therefore$ k = 500 N m^{-1} (ANS) (81%)

You must remember to include the units as they were specifically asked for in the question.

Current study design:

2022 NHT Question 8

2021 Question 11 (63%)

2018 NHT Question 9a

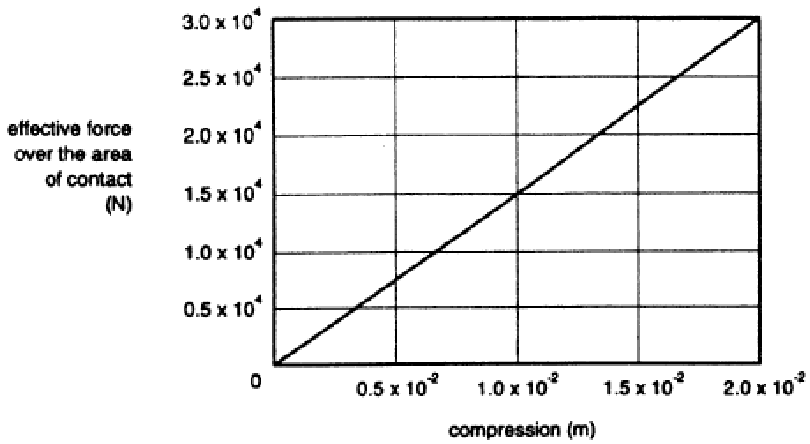
2017 Question 12 (89%)

Worked example 10: Springs: ΔE = area under f vs Δx graph.

Most sports safety helmets are designed to protect the head during impact with the ground following a fall. A helmet usually consists of a liner of polystyrene foam about 2.5 cm thick which is moulded to fit the shape of a human head. Some helmets are covered with a hard outer shell of plastic.

When helmets are designed and tested, it is assumed that the head of the sportsperson behaves like a freely falling mass. This is an example of modelling: replacing a complex, real situation with a simpler one to which the principles of physics can more easily be applied.

As a head wearing a helmet hits the ground, the effective force over the area of contact increases as the foam liner becomes more and more compressed.



For one particular foam liner, the amount by which the foam is compressed, as the effective force over the area of contact increases, is shown in the graph above.

1993 Question 10, 1 mark

How much work is done to compress the foam liner by 2.0 cm over the area of contact?

Solution

The work done to compress the foam liner is the area under the force/compression graph between 0 and 2.0 cm.

$\therefore A = \frac{1}{2} bh$

$\therefore A = \frac{1}{2} \times 2.0 \times 10^{-2} \times 3.0 \times 10^{4}$

$\therefore A = 3.0 \times 10^{2}$ J (ANS)

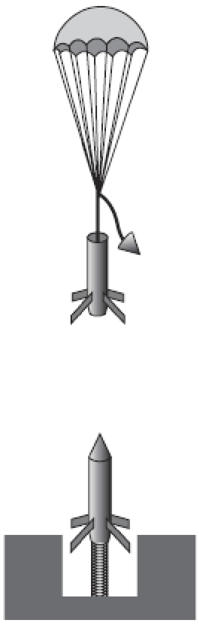
Current study design:

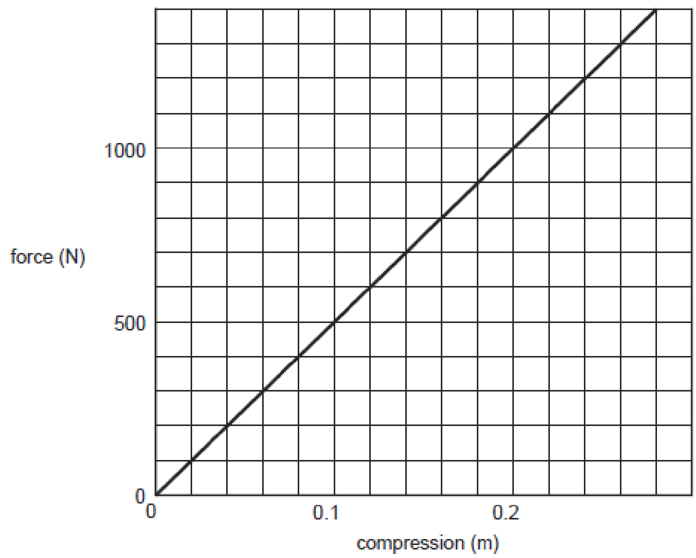
2021 Question 12 (68%)

2018 NHT Question 9b

Worked example 11: Springs: ∆KE = area under f vs ∆x graph.

A model rocket of mass 0.20 kg is launched by means of a spring, as shown below. The spring is initially compressed by 20 cm, and the rocket leaves the spring as it reaches its natural length. The force-compression characteristic of the spring is also shown below.

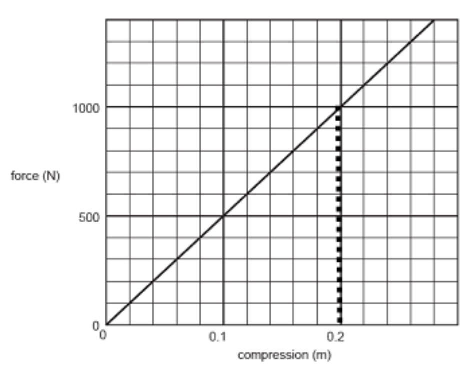




2005 Question 1, 2 marks

How much energy is stored in the spring when it is compressed?

Solution



The energy stored is given by the area under the graph.

Area = 0.5 × 0.2 × 1000

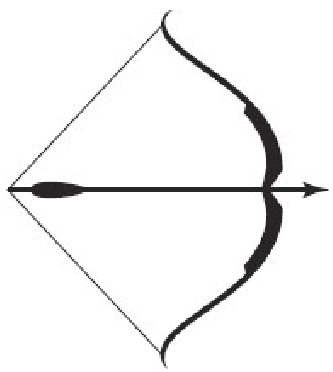
= 100 J (ANS) (57%)

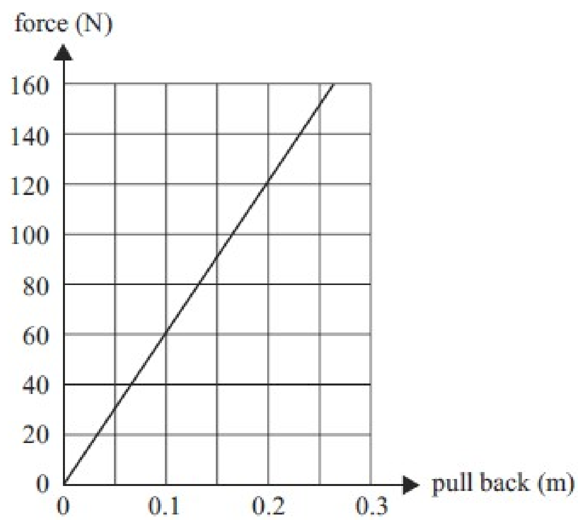
Current study design:

2017 Question 13 (58%)

Worked example 12: Horizontal springs: $\Delta KE = \frac{1}{2} k\Delta x^2$.

A bow can be treated as a simple spring, as shown below. The force–distance graph of the bow is shown below. The *horizontal*-axis shows the distance the archer pulls back the bow string. An arrow has a mass of 0.03 kg.





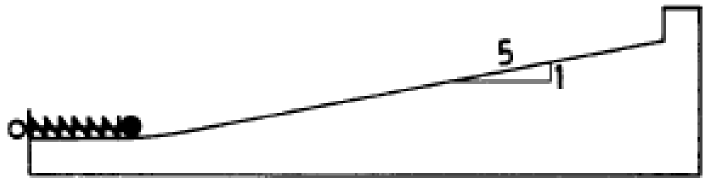
2017 NHT Question 5c, 2 marks

The bow is pulled back 0.20 m and released.

Calculate the speed at which the arrow leaves the bow.

Solution	**Current study design:**
The energy stored in the bow (area under graph) will be transferred to the KE of the arrow. $\therefore \frac{1}{2} F\Delta x = \frac{1}{2} mv^2$ $\therefore \frac{1}{2} \times 120 \times 0.2 = \frac{1}{2} \times 0.03 \times v^2$ $\therefore v^2 = 800$ $\therefore v = 28.3$ $\therefore v = 3 \times 10^1$ m s^{-1} (ANS)	**2020 Question 9a (69%)**

Worked example 13: Horizontal springs: $\Delta mgh = \frac{1}{2} k\Delta x^2$.



1973 Question 39, 1 mark (modified)

In a pinball machine the plunger is pulled to compress the spring. When it is released, the spring projects the steel ball.

The above arrangement is used on the pinball table as shown. A ball of mass 0.250 kg is placed on the plunger. The spring is fully compressed (storing 0.25 J of energy) and then released. Assuming that the plunger and the spring have negligible mass, and that the table has a slope of 1 in 5.

How far along the surface of the table could the ball travel?

Solution

The initial energy of the ball is 0.25 J.
The energy stored in the spring is 0.25 J.
The gain in gravitational PE must equal the loss in EPE.

$\therefore mg\Delta h = \Delta EPE$

$\therefore g\Delta h = \frac{0.25}{0.250}$

$\therefore g\Delta h = 1$

$\therefore \Delta h = \frac{1.0}{9.8}$

$\therefore \Delta h = 0.102$

The distance the ball travels along the surface is given by 5 × Δh.

$\therefore d = 5 \times 0.102$
$= 0.51$ m (ANS)

Current study design:

2021 NHT Question 9b

Worked example 14: Vertical springs: force equivalence, $mg = k\Delta x$.

A spring has a spring constant, *k*, of 20 N m^{-1}. Point P shows the unstretched length of the spring.

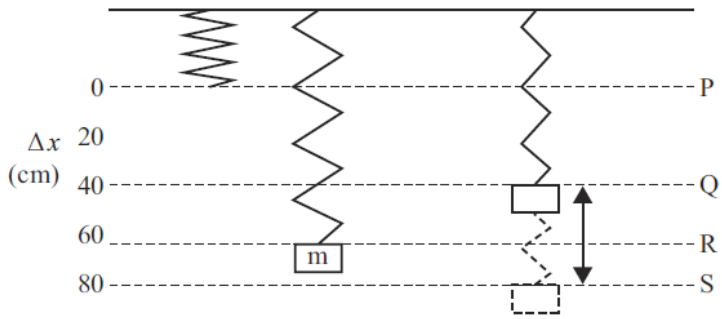


Figure 5a **Figure 5b**

not to scale

2017 NHT Question 4a, 2 marks

A mass, m, is hung from the spring. It extends the spring 0.60 m to point R, as shown in Figure 5a. Calculate the mass of m.

Solution	**Current study design:**
Use $F = k\Delta x$, where $F = mg$ $\therefore m \times 9.8 = 20 \times 0.60$ $\therefore m = 1.2$ kg (ANS)	<u>**2022 NHT Question 9**</u> **2019 NHT Question 5a** **2018 Question 6a (50%)**

Worked example 15: Vertical springs: $TE = \frac{1}{2}mv^2 + mgh + \frac{1}{2}k\Delta x^2$

2017 NHT Question 4b, 2 marks

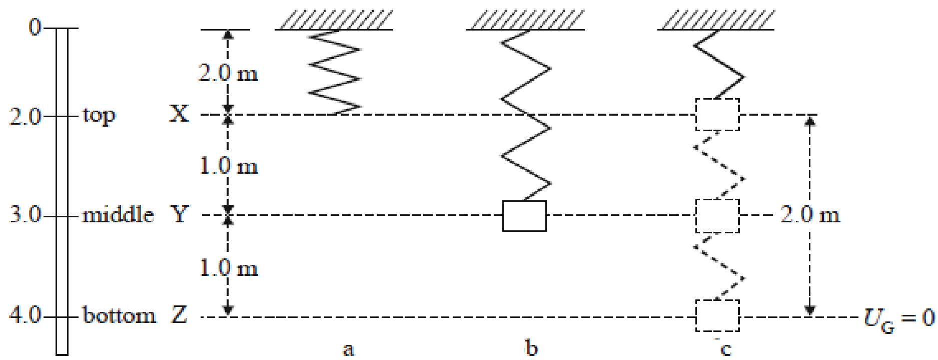
The mass is now raised to point Q and released, so that it oscillates between points Q and S, as shown in Figure 5b.

Calculate the change in spring potential energy in moving from point Q to point S. Show your working.

Solution	**Current study design:**
Use $EPE = \frac{1}{2}k(\Delta x)^2$, where Q is 0.40 m and S is 0.80 m $\therefore \Delta EPE = \frac{1}{2} \times 20 \times (0.80^2 - 0.40^2)$ $\therefore \Delta EPE = \frac{1}{2} \times 20 \times (0.64 - 0.16)$ $\therefore \Delta EPE = 4.8$ J (ANS)	**2019 NHT Question 5b i** **2018 NHT Question 9c** **2017 Question 13a (58%)**

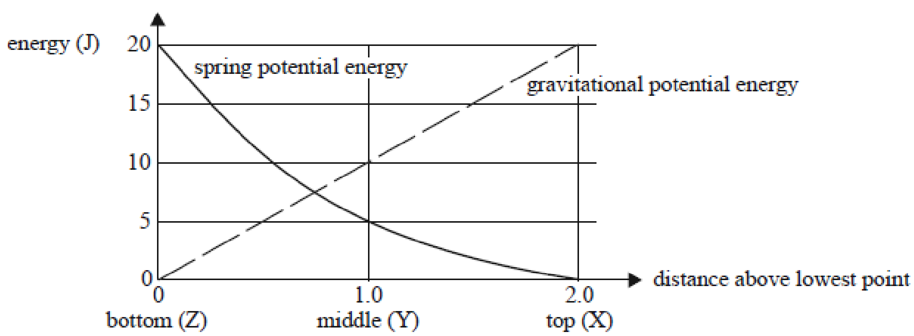
Worked example 16: Vertical springs: finding TE_{middle}, hence v_{max}, ∴ a = 0.

Students hang a mass of 1.0 kg from a spring that obeys Hooke's law with k = 10 N m^{-1}. The spring has an unstretched length of 2.0 m. The mass then hangs stationary at a distance of 1.0 m below the unstretched position (X) of the spring, at Y, as shown at position 'b' in the figure below. The mass is then pulled a further 1.0 m below this position and released so that it oscillates, as shown in position 'c'.



The zero of gravitational potential energy is taken to be the bottom point (Z).

The spring potential energy and gravitational potential energy are plotted on a graph, as shown below.



2013 Question 6b, 2 marks

From the data in the graph, calculate the speed of the mass at its midpoint (Y).

Solution

From the graph, at Z, the GPE is zero, the KE = 0, as the mass is momentarily stationary, so the total energy is 20 J.

From the graph, at the point Y, the SPE = 5 J, the GPE = 10 J. Since the TE = 20 J, and remains constant, the KE must be 5 J.

∴ $\frac{1}{2}mv^2 = 5$ J

∴ $0.5 \times 1 \times v^2 = 5$

∴ $v^2 = 10$

∴ $v = 3.16$ m s^{-1} (ANS), (44%)

Current study design:

2021 NHT Question 12

2019 NHT Question 5b ii

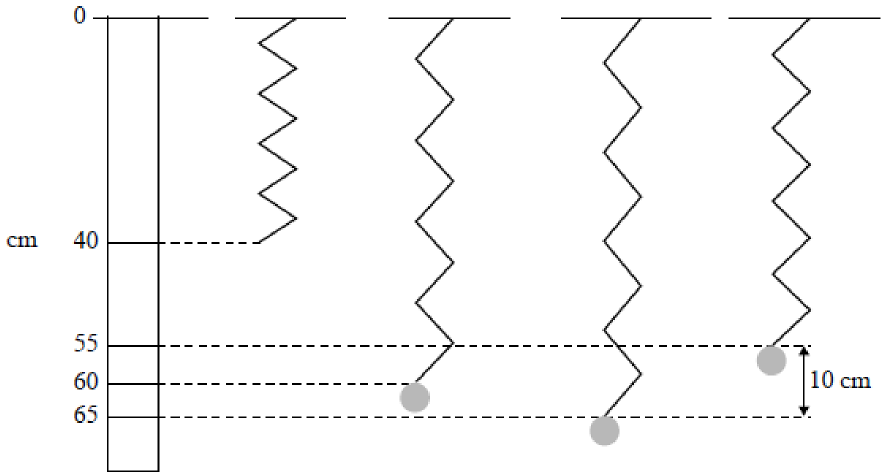
2018 Question 6b (13%)

2018 Question 6c (14%)

Worked example 17: Vertical springs: Energy transformations

A novelty toy consists of a metal ball of mass 0.20 kg hanging from a spring of spring constant k = 10 N m^{-1}.

The spring is attached to the ceiling of a room as shown below. Ignore the mass of the spring.

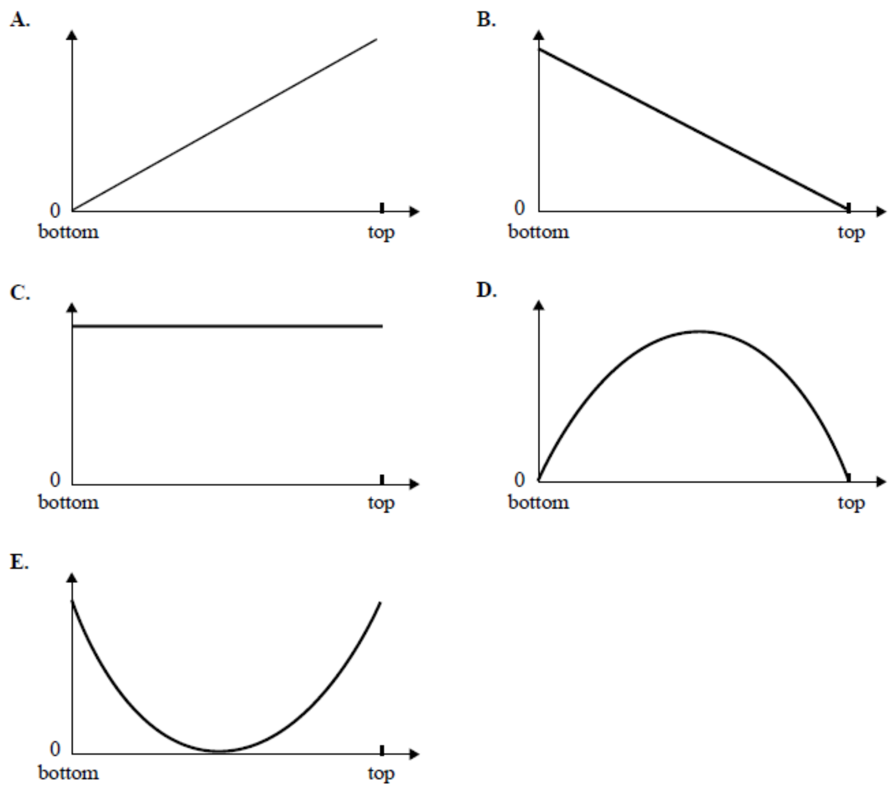


Without the ball attached, the spring has an unstretched length of 40 cm. When the ball is attached, but not oscillating, the spring stretches to 60 cm.

The ball is now pulled down a further 5 cm and released so that it oscillates vertically over a range of approximately 10 cm.

Gravitational potential energy is measured from the level at which the ball is released. Ignore air resistance.

Use Graphs A–E in answering Questions 13 and 14.



2008 Question 13, 2 marks

Which of the graphs best represents the shape of the graph of **kinetic** energy of the system as a function of height?

2008 Question 14, 2 marks

Which of the graphs best represents the **gravitational potential** energy of the system as a function of height?

Solution	**Current study design:**
Kinetic Energy The ball will be stationary (momentarily) at the top and the bottom of the oscillation. Therefore the KE will be zero at these points. The KE will be a maximum at the midpoint. **∴ D (ANS), (38%)** **Gravitational Potential Energy** The gravitational potential energy is measured from the point of release. **∴ at the bottom, PE = 0.** **∴ A (ANS), (50%)**	**2017 Question 13b (45%)**