Momentum and Impulse.

- **investigate and apply theoretically and practically the laws of energy and momentum conservation in isolated systems in one dimension**

  – kinetic energy at low speeds: $E_k = \frac{1}{2}mv^2$; elastic and inelastic collisions with reference to conservation of kinetic energy

- **investigate and analyse theoretically and practically impulse in an isolated system for collisions between objects moving in a straight line: *FΔt = mΔv***

Paper	Multiple choice	Short Answer	Idea	Marks	%	Type
2022	6		Momentum conservation	1	78%	Calculation
		7b i	Momentum conservation	3	67%	Calculation
		7b ii	Elastic/Inelastic	3	69%	Calculation
2022 NHT						
2021						
2021 NHT	3		FΔt = mΔv	1	NA	Calculation
		9c	p = mv	2	NA	Calculation
		9d	Momentum conservation	1	NA	Concept
		18a	FΔt = mΔv	4	NA	Calculation
		18c	Momentum conservation	1	NA	Concept
2020		10a	Momentum conservation	4	83%	Calculation
		10b	Elastic/Inelastic	3	64%	Explanation
		10c i	Impulse	3	58%	Calculation
		10c ii	Impulse	2	69%	Concept
2019		9	Collision, direction change	3	59%	Calculation
2019 NHT		7a	Impulse	3	NA	Calculation
		7b	FΔt = mΔv	2	NA	Calculation
		7c	Elastic/Inelastic	2	NA	Calculation
2018	8		Collision	1	86%	Calculation
	9		Elastic/Inelastic	1	71%	Concept
2018 NHT	12		FΔt = mΔv	1	NA	Calculation
		7	Elastic/Inelastic	3	NA	Calculation
2017	8		FΔt = mΔv	1	87%	Calculation
		12	Elastic/Inelastic	3	63%	Calculation

Momentum and Impulse questions can be grouped into the following ideas.

Momentum	
Applications of p = mv	Worked example 1
Impulse	
Straight substitution $F\Delta t = m\Delta v$	Worked example 2
Horizontal collision, $F\Delta t = m\Delta v$, direction	Worked example 7
Change of direction, 'Δv' calculations	Worked example 3
Change of direction, $F\Delta t = m\Delta v$	Worked example 4
$I = F\Delta t$	Worked example 5
Newton's third law	Worked example 9
Vertical collision, $F\Delta t = m\Delta v$, direction	Worked example 10
Momentum collisions/explosions	
Conservation of p during collision	Worked example 11
No change of direction, 'p' calculations	Worked example 6
Change of direction, 'p' calculations	Worked example 12
Elastic/Inelastic collision	Worked example 8
Conservation of 'p' involving Earth	Worked example 13

Momentum

The momentum (**p**) of a body is the product of its mass and velocity.

p = m **v.**

The unit is kilogram metre per second (kg m s^{-1})

Momentum is a vector. It has a magnitude and a direction.

Momentum is conserved in all collisions/explosions.

Momentum is conserved when no external force acts. It is transferred to the Earth whenever a body hits the ground or slides to a halt.

Momentum transfer involving the Earth

Vertical

1. Body rises under gravity - slows down and loses momentum to the Earth.
2. Body falling under gravity - speeds up giving the Earth equal and opposite momentum change.
3. Falling body hits the ground - its **p** is transferred to the Earth.

Horizontal

4. Body slowed due to friction - gives the Earth and equal and opposite **p**.
5. Body accelerated due to friction - gives the Earth an equal and opposite **p**.

Collisions

When A and B collide, the action on A by B is equal and opposite to that on B by A. (Newtons 3rd) Hence the rate of change of momentum of A is equal and opposite to the rate of change of momentum of B. Since the time of contact is the same for both, then the change in momentum of A is equal and opposite to the change in momentum of B.

Collisions problem solving

Remember that a sign convention is essential.
If the bodies collide and stay together, then the momentum after the collision

$$\mathbf{p}_{\text{final}} = \Sigma\, \mathbf{p}_{\textbf{initial}}$$
$$\Sigma\, m\, \mathbf{v}_{\text{final}} = \Sigma\, m\, \mathbf{v}_{\text{initial}}$$

Mathematically, problems on 'collision' or 'explosion' are similar, except that for an explosion, the momentum of the system before the blast can be zero.

$$\mathbf{p}_{\text{(total before the collision)}} = \mathbf{p}_{\text{(total after the collision)}}$$

Always draw a diagram

Any unit may be used for mass or velocity, as long as such units are consistent within the equation.

Be extra careful when one of the masses reverses its direction after colliding. This requires taking into consideration the vector nature of momentum.

Impulse

Consider a body of mass 'm' changing its velocity from 'u' to 'v' in time 't' under the action of a constant force 'F'.

$\mathbf{F} = m\mathbf{a}$ since $\mathbf{a} = \frac{(v - u)}{t}$

$\mathbf{F} = \frac{mv - mu}{t}$

$\mathbf{F} = \frac{m\Delta v}{t}$

$\therefore \mathbf{F}\Delta t = m\Delta v$

Since a net force gives a change in momentum over a change in time, it can be written that a change in momentum is given by: $\Delta p = F\Delta t$

The product of a constant force and the time for which it acts is called the **IMPULSE (I)** of the force.

$\mathbf{I} = \Delta p = \mathbf{F}\Delta t$. The unit is Newton second, (N s) (This is identical to kg m s^{-1}).

$\therefore \mathbf{I} = \mathbf{p_2} - \mathbf{p_1}$.

The impulse of a force can be measured by the change in momentum. **Impulse** and **momentum** are vector quantities. Both impulse and momentum have the same units.

Collisions

From an Energy perspective we are concerned with two types of collisions, elastic or inelastic.

Elastic collisions

If the collision is elastic, then both momentum and kinetic energy is conserved in the collision.
$\therefore E_{final} = E_{initial}$ and $p_{final} = p_{initial}$

Inelastic collisions

If the collision is inelastic, momentum is conserved, but some energy is 'lost' to the environment. The energy that is 'lost' to the environment is usually transformed into heat energy, sound energy and energy of deformation. $\therefore p_{final} = p_{initial}$ but $E_{initial} > E_{final}$.

If KE is lost, the collision is inelastic. The energy that has gone 'missing' is usually converted into heat, sound or energy of deformation.

Note: NO collision can result in an INCREASE in the TOTAL KINETIC ENERGY of a system.

Typical exam questions will allocate:

one mark for identifying and using conservation of momentum,

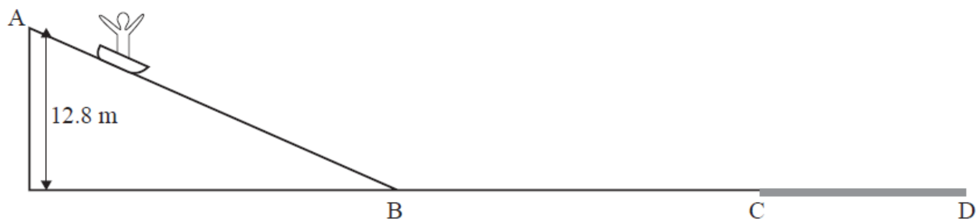
one mark for calculating the initial KE and

one for calculating the final KE **and** stating if the collision is elastic or inelastic.

Worked example 1: Momentum p = mv

Fred is riding his sled on snow. Fred and the sled have a total mass of 60 kg. He travels downhill from A to B. The sled starts from rest.

A is a vertical height of 12.8 m above B. At B he travels along a horizontal snowfield to point C. From A to C (on snow) there is no friction force.



2011 Question 14, 2 marks

What is the momentum of Fred and his sled at point C?

Solution

Use p = mv
To find v, use $\Delta mgh = \Delta PE$
$\therefore 60 \times 10 \times 12.8 = \frac{1}{2} \times 60 \times v^2$
$\therefore v^2 = 256$
$\therefore v = 16 \text{ m s}^{-1}$
Substitute into p = mv
$\therefore p = 60 \times 16$
$\therefore p = 960 \text{ kg m s}^{-1}$ (ANS), (69%)

Current study design:

2021 NHT Question 9c

Worked example 2: Impulse, $F\Delta t = m\Delta v$

At point C, he runs of snow onto grass where there is now a (constant) friction force and he slows to a stop at D after a time of 6.0 s.

2011 Question 15, 2 marks

What is the magnitude of the friction force as he travels from point C to point D?

Solution

Use $F \Delta t = m \times \Delta v$,
$\therefore F \times 6 = 960$
$\therefore F = 160$ N (ANS), (63%)

Current study design:

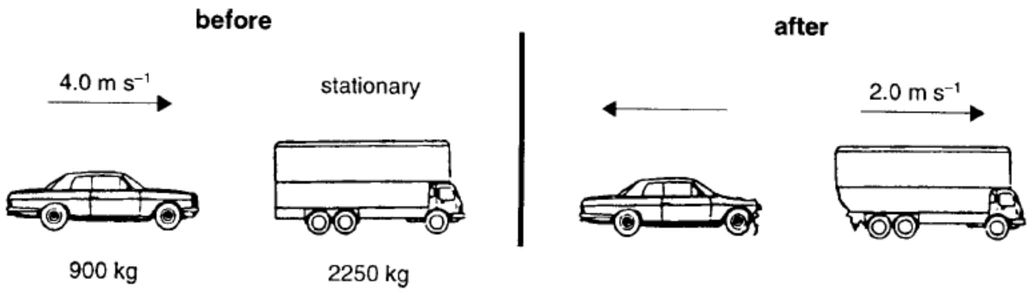
2021 NHT Question 3

2018 NHT Question 12

Worked example 3: Impulse, change of direction, Δv calculations.

A car of mass 900 kg, travelling on a horizontal road with a speed of 4.0 m s^{-1}, runs into the rear of a stationary truck of mass 2250 kg as shown below. Immediately after the collision the truck moves forward with a speed of 2.0 m s^{-1} and the car rebounds in the opposite direction.

In modelling this collision you should assume that
- there is no driving force from either engine during the collision
- no braking takes place during the collision
- the car and truck remain in a straight line.



1997 Question 6, 3 marks

What is the speed of the car immediately after the collision?

Solution

Momentum must be conserved, so $p_i = p_f$

$\therefore p_i = 900 \times 4.0 + 2250 \times 0 = 3600$ to the right.

$\therefore p_f = 3600$ (to the right) $= 2250 \times 2.0 - 900 \times v$

$$\therefore v = \frac{4500 - 3600}{900}$$

$\therefore v = 1.0$ m s^{-1} (ANS), (75%)

You need to be careful to consider the direction of the momenta.

Current study design:

2019 NHT Question 7a

Worked example 4: Impulse, change of direction, FΔt = mΔv.

1997 Question 8, 3 marks

The car and the truck were in contact for 22 ms (2.2×10^{-2} s). What was the magnitude of the average force on the car during the collision?

Solution

Use $F\Delta t = m\Delta v$

$$\therefore F = \frac{m\Delta v}{\Delta t}$$

$$= \frac{900 \times 5}{22 \times 10^{-3}}$$

$= 204545$

$= 2.0 \times 10^{5}$ N (ANS), (80%)

Current study design:

2019 NHT Question 7b

Worked example 5: Impulse, $I = F\Delta t$.

In a game of tennis, the ball is thrown vertically upward. At the top of its motion the ball is momentarily stationary. At this point it is hit forward horizontally by the racquet.

In a particular serve, the ball of mass 57 g is given a horizontal impulse of 1.70 kg m s^{-1}.

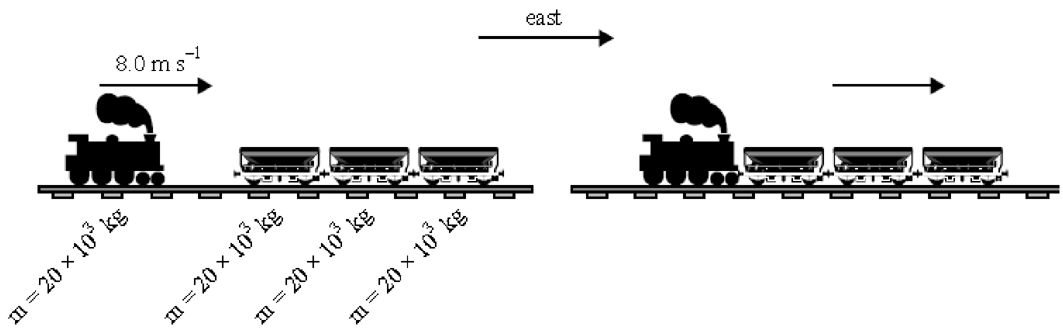
2010 Question 12, 2 marks

Assuming that the racquet and ball were in contact for a period of 0.0080 s, what was the average force exerted on the ball by the racquet?

Solution	**Current study design:**
Using Impulse = Average force × time, $\therefore F = \frac{1.7}{0.0080}$ = 212.5 = 210 N (ANS), (80%)	**2017 Question 8 (87%)**

Worked example 6: Impulse, no change in direction, ‘p’ calculations.

A locomotive, of mass 20 × 10^3 kg, moving at 8.0 m s^{-1} east, collides with and couples to three trucks, each of mass 20 × 10^3 kg, initially stationary, as shown.



2008 Question 8, 2 marks

What is the speed of the coupled locomotive and trucks after the collision?

You must show your working.

Solution	**Current study design:**
Momentum is conserved in this collision. ∴ $p_i = p_f$ ∴ 20 × 10^3 × 8 = 80 × 10^5 × v ∴ v = 2 m s^{-1} (ANS), (79%)	**2018 Question 8 (86%)**

Worked example 7: Impulse, Horizontal collision, $F\Delta t = m\Delta v$

2008 Question 9, 3 marks

What is the impulse given **to** the locomotive **by** the trucks in the collision (magnitude and direction)? You must show your working.

Solution	**Current study design:**
The impulse given to the locomotive by the trucks is equal and opposite to the impulse given to the trucks by the locomotive. Impulse on the trucks is the change in momentum of the trucks. = $m\Delta v$ = 60 × 10^3 × 2 = 1.2 × 10^5 kg m s^{-1} West (ANS), (53%)	**2020 Question 10c i (58%)**

Worked example 8: Horizontal collision, Inelastic/elastic.

2008 Question 10, 3 marks

Was this collision elastic or inelastic?

Support your conclusion by appropriate calculation.

Solution

If the collision is elastic, then $KE_{final} = KE_{initial}$.

$KE_{initial} = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 20 \times 10^3 \times 8^2$

$= 64 \times 10^4$

$= 6.4 \times 10^5$ J

$KE_{final} = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 80 \times 10^3 \times 2^2$

$= 1.6 \times 10^5$

$KE_{final} < KE_{initial}$

∴ **KE is lost**

∴ **Collision is inelastic (ANS), (73%)**

Current study design:

2022 Question 7b ii (69%)

2020 Question 10b (64%)

2019 NHT Question 7c

2018 Question 9 (71%)

2018 NHT Question 7

2017 Question 12 (63%)

Worked example 9: Horizontal collision, Newton 3

During the collision, the magnitude of the average force exerted by the locomotive on the trucks is F_L and the magnitude of the average force exerted by the trucks on the locomotive is F_T.

2008 Question 11, 3 marks

Will F_L be greater, equal to, or less than F_T? Explain your answer.

Solution

The magnitude of the impulse on both the locomotive and the trucks is the same.

Since $I = F \Delta t$

and the time of the collision is the same for both.

∴ $F_L = F_T$ (ANS), (51%)

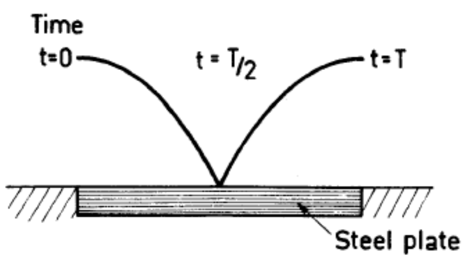
(This is an example of Newton's Third Law, where $F_{A\ on\ B} = -F_{B\ on\ A}$)

Current study design:

2020 Question 10c ii (69%)

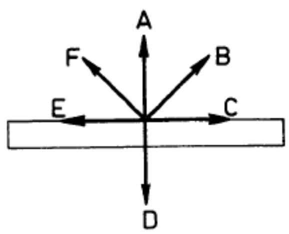
Worked example 10: Vertical collision, $F\Delta t = m\Delta v$.

A steel ball is projected horizontally and makes a perfectly elastic collision with a steel plate embedded in the ground.



1971 Question 38, 1 mark

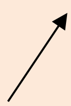
Which arrow (A - F) best describes the direction of the impulse of the plate on the ball?



Solution

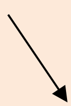
Just after the collision the momentum is tangential to the path.

$\therefore p_f =$



Just before the collision the momentum is tangential to the path.

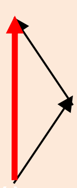
$\therefore p_i =$



The change in momentum = Impulse

$\therefore I = p_f - p_i$

$\therefore I =$



$\therefore$ A (ANS), (70%)

Current study design:

2021 NHT Question 18a

Worked example 11: Horizontal collision/explosions, ‘p’ conservation.

A small truck of mass 3.0 tonne collides with a stationary car of mass 1.0 tonne. They remain locked together as they move off. The speed immediately after the collision was known to be 7.0 m s^{-1} from the jammed reading on the car speedometer. Robin, one of the police investigating the crash, uses ‘conservation of momentum’ to estimate the speed of the truck before the collision.

The calculated value is questioned by the other investigator, Chris, who believes that ‘conservation of momentum’ only applies in elastic collisions.

2005 Question 9, 2 marks

Explain why Chris’s comment is wrong.

Solution

Chris’s comment is wrong because momentum is conserved in all collisions in an isolated system. In elastic collisions kinetic energy is also conserved. In inelastic collisions kinetic energy is lost as heat and sound and energy used in deformation. (65%)

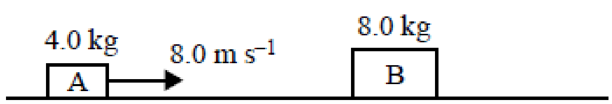
Current study design:

2022 Question 6 (78%)

2022 Question 7b i (67%)

Worked example 12: Horizontal collision/explosions, change of direction.

Block A, of mass 4.0 kg, is moving to the right at a speed of 8.0 m s^{-1}, as shown below. It collides with a stationary block, B, of mass 8.0 kg, and rebounds to the left. Its speed after the collision is 2.0 m s^{-1}.



2015 Question 1a, 2 marks

Calculate the speed of block B after the collision.

Solution

Momentum is conserved, so the final momentum is equal to the initial momentum.
$p_i = 4.0 \times 8.0 + 8.0 \times 0$
$\therefore p_i = 32.0$
Assume to the right is positive, therefore the final momentum of Block A is negative.
$\therefore 32.0 = 8.0 \times v - 4.0 \times 2$
$\therefore p_f = 32.0$
$\therefore 32 = 8v - 8$
$\therefore v = 5$ m s^{-1} (ANS), (43%)

Current study design:

2020 Question 10a (83%)

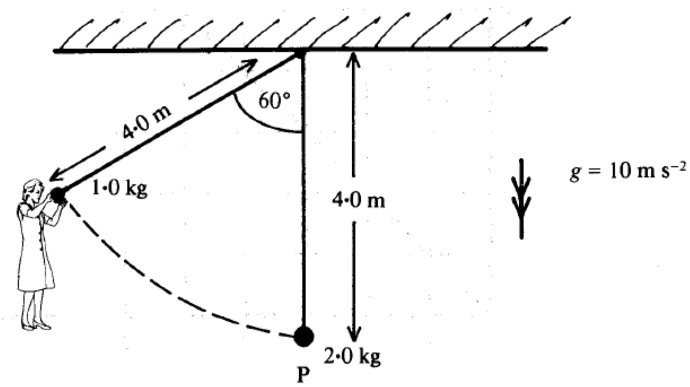
2019 Question 9 (59%)

Worked example 13: Conservation of 'p' involving Earth.

A student investigates the collision between two balls of plasticine, whose masses are 1.0 *kg and* 2.0 kg respectively, using the following procedure.

The balls are attached to light strings of length 4.0 m, the upper ends of the strings being attached to the same point on the ceiling of the laboratory. The 1.0 kg ball is then drawn aside so that its string remains taut and makes an angle of 60^0 with the vertical. This ball is released so that it swings down and makes a head-on collision with the other ball.

The situation is pictured below.



1985 Question 10, 1 mark

When the masses have reached the top of their swing their momentum has dropped to zero. Which of the following statements (**A - D**) below best explains where this momentum has gone?

A. It has been transferred to the earth via the string.

B. It has been stored in the string.

C. It has been stored as potential energy in the plasticine.

D. It has been dissipated as heat and sound.

Solution	**Current study design:**
Momentum is always conserved. The momentum that the ball had just before the collision has been transferred to the earth (through the connection to the ceiling). ∴ **A (ANS), (68%)**	**2021 NHT Question 9d** **2021 NHT Question 18c**