Projectile motion

- **Investigate and analyse theoretically and practically the motion of projectiles near Earth's surface, including a qualitative description of the effects of air resistance.**

Paper	Multiple choice	Short Answer	Idea	Marks	%	Type
2022						
2022 NHT		10a	Oblique, time of flight	2	NA	Show
		10b	Oblique, landing height	4	NA	Calculation
2021	9		Horizontal, range	1	77%	Calculation
	10		Horizontal, time of flight	1	69%	Calculation
2021 NHT	10		Oblique, range	1	NA	Calculation
	11		Oblique, force, air resistance	1	NA	Concept
2020						
2019	12		Horizontal, concept	1	61%	Concept
		10a	Oblique, time of flight	2	77%	Calculation
		10b	Oblique, range	2	62%	Calculation
2019 NHT		6a	Oblique, max height	2	NA	Calculation
		6b	Oblique, landing height	4	NA	Calculation
2018		7a	Horizontal, range	1	79%	Calculation
		7b	Horizontal, launch height	2	67%	Calculation
		7c	Horizontal, landing speed	3	50%	Calculation
2018 NHT		6a	Horizontal, time of flight	3	NA	Calculation
		6b	Horizontal, range	2	NA	Calculation
		6c	Horizontal, Energy	2	NA	Calculation
2017		9a	Oblique, landing height	3	45%	Calculation

Projectiles can be grouped into the following ideas.

Launched horizontally

Range	Worked example 1
Landing speed	Worked example 2
Launch height	Worked example 3
Time of flight	Worked example 4

Launched obliquely, symmetrical

Maximum height	Worked example 6
Range	Worked example 7

Launched obliquely

Maximum height	Worked example 8
Range	Worked example 10
Time of flight	Worked example 11
Landing/launch height, $h = ut - \frac{1}{2}gt^2$	Worked example 9

Energy

KE on landing, work done	Worked example 5

Forces acting

Resultant force/acc during flight	Worked example 12
Force/acc. at max h, air resistance	Worked example 13

Projectile motion

Projectile motion is motion under a constant unbalanced force. A projectile is a body that has been thrown or projected. Air resistance is to be considered as negligible in the quantitative questions involving calculations but may need to be included in qualitative questions.

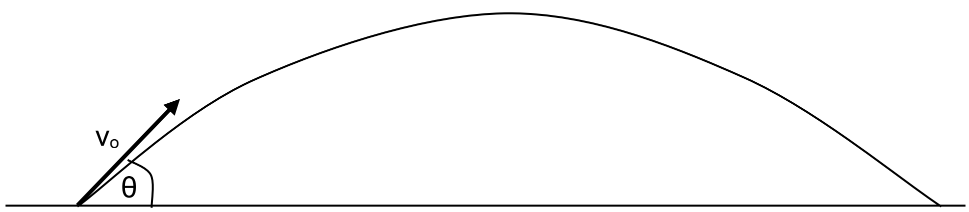
Projectiles can be categorised as one of four types.

1. Launched vertically.
2. Launched horizontally.
3. Launched obliquely (at an angle) and lands at the same height.
4. Launched obliquely (at an angle) and lands at a different height.

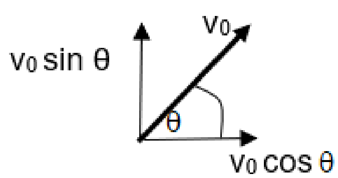
For both the vertical and inclined projectiles:

- the only force acting is the weight, ie. the bodies are in free fall, acceleration = 9.8 m s^{-2}, including at the top of the motion.
- instantaneous velocity is tangential to the path.
- the total energy (KE & PE) is constant, between any two points $\Delta KE = -\Delta PE$.
- paths are symmetrical for time, if air resistance can be ignored.

Inclined Projectiles, resolving the initial velocity



The vector representing the initial velocity can be resolved into two components



Inclined or oblique projections

- the only force acting is vertically down, so the acceleration and change in velocity are vertical
- horizontally, there is no component of force, so constant horizontal velocity.
- maximum range is when angle of projection is 45^0.

Horizontal projection

For projectiles thrown horizontally and dropped from rest, the vertical motions are the same.

Horizontal: velocity always = $v_{horizontal}$
acceleration = 0
displacement = $x = v_{horizontal} \times t$

Vertical: Velocity changing $v = u - gt$
acceleration $= -g$
displacement $y = ut - \frac{1}{2}gt^2$

To find the 'total' velocity, add $v_{vertical}$ and $v_{horizontal}$ using vectors.

Range formula for symmetrical flights

If there is no air resistance, and the projectile starts and ends at the same height, then the range is given by: R = $\frac{v^2 \sin 2\theta}{g}$ R = range, v = initial speed, and θ = angle of projection.

Total Energy (TE)

If air resistance is negligible, then the total energy is constant. TE = KE + PE. At ground level PE = 0, so TE = KE. As the projectile rises it gains PE, so it must lose KE. At the top of its flight, the PE is maximum and the KE is minimum. (KE is not zero, because the projectile still has some KE due to its horizontal motion).

At any point on the way up or the way down, the TE is constant. If you know the horizontal component of the velocity, then you can use this to find the maximum height, by using the TE at ground level and working out what he PE must be at the top when $v_{vertical} = 0$, but $v_{horizontal}$ = constant.

General Projectile problem methodology

1. Draw a fully labelled diagram
2. Treat the motion as two separate motions, horizontal and vertical. List the data under vertical and horizontal
3. For vertical motion use, $v^2 = u^2 + 2ax$, and $v = u + at$. (use a = -g)
4. To go from one direction to another, the common link is the time of flight, t.
5. Remember that it will take the same time to go up as to come down (if air resistance can be ignored).
6. Label the direction (+ or -) for all variables except time.
7. Sometimes the question is phrased 'with a strong tail wind.' This means that the tail wind cancels out any effects of air resistance.

Projectile problem methodology

1. Vertical launch

Acceleration is -9.8 m s^{-2}. At maximum height velocity = 0, acceleration = -9.8 m s^{-2}.

Use $v = u – gt$ to find time to highest position or use $v^2 – u^2 = 2gh$, to find height.

If launched from one height and lands at a different height, use $x = ut + \frac{1}{2}gt^2$ to find unknown.

This is just accelerated motion in a vertical direction.

2. Horizontal launch

Divide the problem into two parts, horizontal and vertical.

In the **horizontal** the acceleration is 0, therefore the speed is constant.

Use $d = v_{hor} \times t$, to find the horizontal distance travelled.

In the **vertical** the acceleration is -9.8 m s^{-2}. The initial speed = 0, use $x = ut + \frac{1}{2}gt^2$ to find the time to fall. Use ‘t’ to find the horizontal distance travelled.

3. Angled launch, landing at same height

Divide the problem into two parts, horizontal and vertical.

In the **horizontal** the acceleration is 0, therefore the speed is constant. Use $d = v_{hor} \times t$, to find the horizontal distance travelled.

In the **vertical** the acceleration is -9.8 m s^{-2}. Divide into two sections up and down.

Up Use $v^2 – u^2 = 2gh$, to find the maximum height above the launch height or use $v = u – gt$ to find time to highest position

Down Motion is symmetrical, times will be the same.

Projectile problem methodology

4. Angled launch, landing at same height

Divide the problem into two parts, horizontal and vertical.

In the **horizontal** the acceleration is 0, therefore the speed is constant. Use $d = v_{hor} \times t$, to find the horizontal distance travelled.

In the **Vertical** the acceleration is -9.8 m s^{-2}. Divide into two sections up and down.

Up Use $v^2 – u^2 = 2gh$, to find the maximum height above the launch height.

or use $v = u – gt$ to find time to highest position

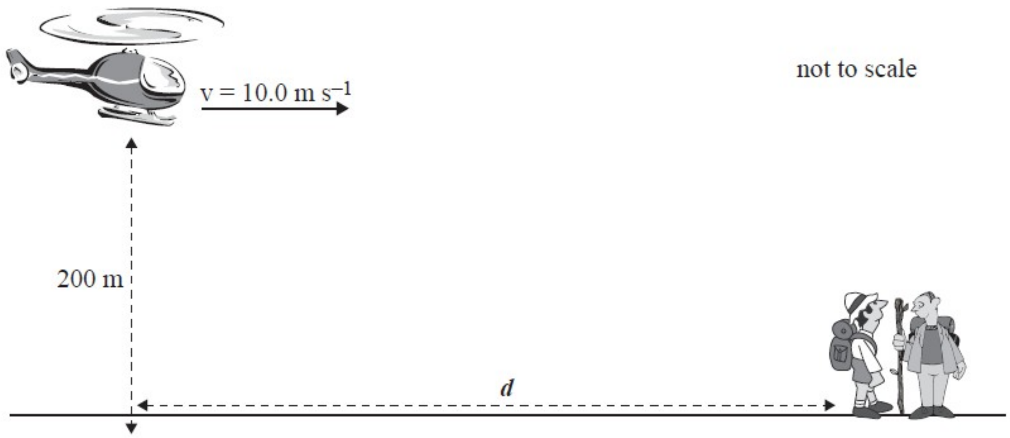
Down Once you know the maximum height, treat as a horizontal launch problem or if you know the time of flight, use $y = ut - \frac{1}{2}gt^2$. If y is negative then it is the distance below the launch point.

Worked example 1: Horizontal projection, Range.

Fred is riding his sled on snow. Fred and the sled have a total mass of 60 kg. He travels downhill from A to B. The sled starts from rest.

A is a vertical height of 12.8 m above B. At B he travels along a horizontal snowfield to point C. From A to C (on snow) there is no friction force.

A helicopter is to drop a rescue package to a group of hikers. The helicopter is travelling with a speed of 10.0 m s^{-1} at a constant height of 200 m over level ground. The situation is shown below. You should ignore air resistance.

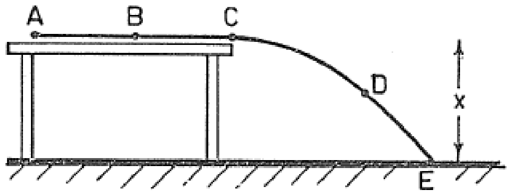


2010 Question 10, 2 marks

The pilot wants the package to land beside the hikers. At what horizontal distance, ***d***, from the hikers must the package be released from the helicopter?

Solution	**Current study design:**
The time it will take for the package to land is given by $x = ut + \frac{1}{2}at^2$	**2021 Question 9 (77%)**
$\therefore 200 = 0 + \frac{1}{2} \times 9.8 \times t^2$	**2018 Question 7a (79%)**
$\therefore t^2 = 40.8$	**2018 NHT Question 6b**
$\therefore t = 6.39$ s	
The horizontal velocity will be unchanged, as we are ignoring air resistance, $\therefore d = u \times t$	
$\therefore d = 63.9$	
$\therefore d = 64$ m (ANS), (64%)	

Worked example 2: Horizontal projection, Landing speed.



A mass slides with uniform speed on a horizontal frictionless table. At point C it leaves the table and moves under the influence of gravity along the path CDE, striking the floor at the point E.

The point E is a vertical distance *x* below the table top. The mass reaches the point E with a horizontal velocity component *U* and a vertical velocity component *V*.

Answers to the next *two* questions should be given in terms of one or more of the quantities *U, V* and *g*.

1968 Question 49, 1 mark

Write and expression for the speed at E.

Solution	**Current study design:**
The speed at E will be the vector addition of U (horizontal) and V (vertical) $\therefore \sqrt{u^2 + v^2}$ (ANS), (61%)	**2018 Question 7c (50%)**

Worked example 3: Horizontal projection, Launch height.

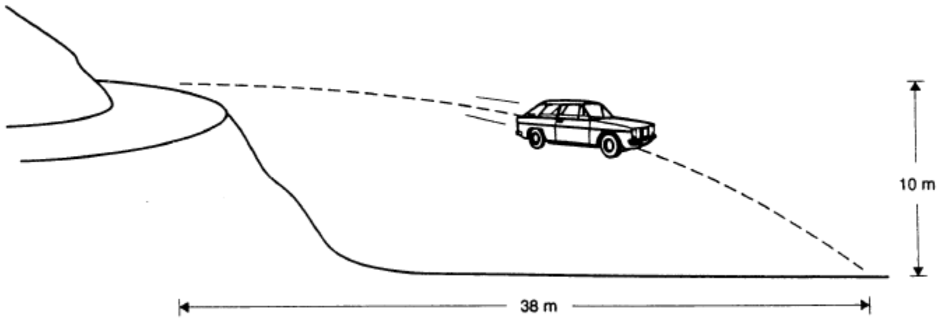
1968 Question 50, 1 mark

Write and expression for *x*.

Solution	**Current study design:**
)The increase in KE in the vertical direction is the same as the loss of GPE $\therefore mgx = \frac{1}{2} mv^2$ $\therefore gx = \frac{1}{2} v^2$ $\therefore x = \frac{\frac{1}{2}v^2}{g}$ (ANS), (51%)	**2018 Question 7b (67%)**

Worked example 4: Horizontal projection, Time of flight.

A car, mass 1.5×10^3 kg, is travelling along a horizontal mountain road. It fails to take a sharp bend and plunges over the side of the road. The car lands at a vertical distance of 10 m below the road, and a horizontal distance of 38 m from the road (see below).
Use g = 9.8 m s^{-2}.



Ignore any effects of air resistance when answering the questions below.

1993 Question 1, 1 mark

Calculate the time for which the car is in the air, between leaving the road and landing.

Solution

The time required is the time it takes for the car to fall from a height of 10 m.

Use $h = ut + \frac{1}{2}gt^2$

$\therefore 10 = 0 + \frac{1}{2} \times 9.8 \times t^2$

$\therefore t^2 = \frac{20}{9.8}$

$\therefore t^2 = 2.0408$

$\therefore t = 1.428$

$\therefore t = 1.4$ s (ANS)

Current study design:

2021 Question 10 (69%)

2018 NHT Question 6a

Worked example 5: Energy, KE on landing, work done.

1993 Question 3, 1 mark

How much work is done by gravity on the car, between the point where it left the road and the point where it lands again?

Solution

The work done by gravity is given by $mg\Delta h$

$WD = 1.5 \times 10^3 \times 9.8 \times 10$

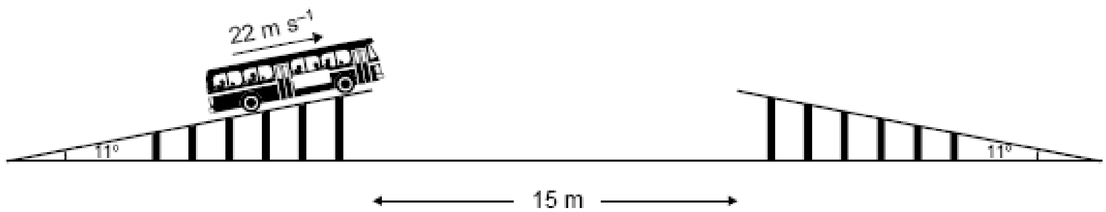
$\therefore WD = 1.5 \times 10^5$ J (ANS)

Current study design:

2018 NHT Question 6c

Worked example 6: Oblique projection, symmetrical, Maximum height.

In the film *Speed* a bus travelling at 22 m s^{-1} is driven over a 15 m gap in an incomplete freeway as shown. The take-off angle is 11° above the horizontal. In calculating the answers to questions 11 and 12 assume that air resistance is negligible.



1997 Question 11, 3 marks

After take-off, what time does it take for the bus to reach the **highest** point in its flight?

Solution	**Current study design:**
Consider the vertical component of the velocity. **v = u - gt** **where v = 0 and u = 22 sin11°** **∴ 0 = 22sin11° – 9.8 × t** **∴ t = 4.198 ÷ 9.8** **∴ t = 0.428** **∴ t = 0.43 sec (ANS), (40%)**	**2019 Question 10a (77%)**

Worked example 7: Oblique projection, symmetrical, Range.

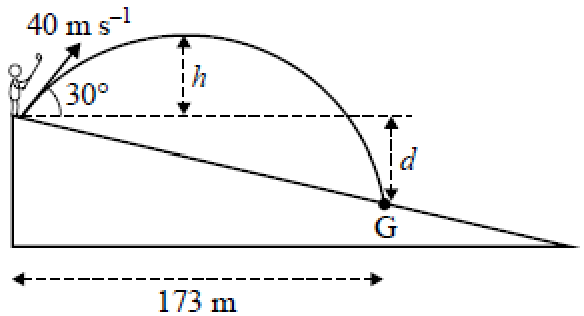
1997 Question 12, 3 marks

Does the bus land on the other section of the freeway? Show your calculations and reasoning.

Solution	**Current study design:**
Yes You need to assume that the other part of the bridge is the same height as the first side. If it takes 0.428 sec to get to the top, then it will take 0.857 sec to be back to the original height. The horizontal distance travelled in this time: = vcos11 × 0.857 = 22cos 11 × 0.857 = 18.5 m **Therefore the bus reaches the other side. (40%)**	**2019 Question 10b (62%)**

Worked example 8: Oblique projection, maximum height.

A golfer hits a ball on a part of a golf course that is sloping downwards away from him, as shown below.



The golfer hits the ball at a speed of 40 m s^{-1} and at an angle of 30° to the horizontal. Ignore air resistance.

2015 Question 5a, 2 marks

Calculate the maximum height, *h*, that the ball rises above its initial position.

Solution	**Current study design:**
In the vertical direction the initial speed is $v_v = 40 \times \sin 30^0$ $\therefore v_v = = 20$ m s^{-1}. At the top of its flight the vertical component of the ball's velocity is zero. Use $v^2 - u^2 = 2gh$. $\therefore 0^2 - 20^2 = 2 \times (-9.8) \times h$ $\therefore 400 = 19.6 \times h$ $\therefore h = 20.4$ $\therefore h = 20$ m (ANS), (80%)	**2019 NHT Question 6a**

Worked example 9: Oblique projection, landing/launching height $h = ut - \frac{1}{2}gt^2$

2015 Question 5b, 3 marks

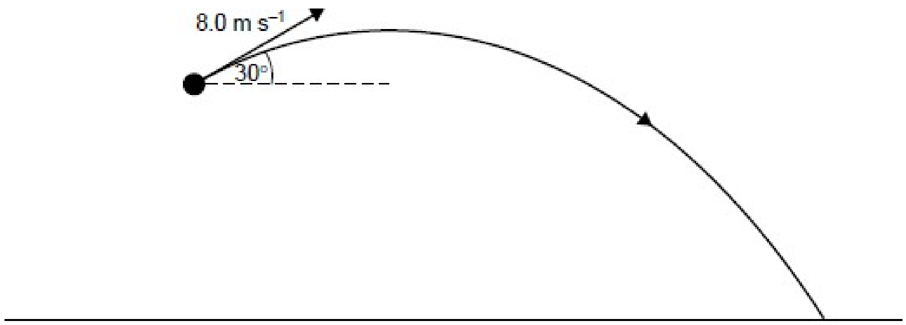
The ball lands at a point at a horizontal distance of 173 m from the hitting-off point, as shown above.

Calculate the vertical drop, *d*, from the hitting-off point to the landing point, G.

Solution	**Current study design:**
Find the time taken to get to the point G, by using the initial horizontal speed. In the horizontal direction $d = v \times t$ **$\therefore 173 = 40 \cos 30^0 \times t$** **$\therefore t = 4.99$** **$\therefore t = 5.$** **Use $h = ut - \frac{1}{2}gt^2$ to get the vertical position of the ball at 5 seconds.** **$\therefore h = 20 \times 5 - \frac{1}{2} \times 9.8 \times 5^2$** **$\therefore h = -22.5$ m** **$\therefore d = 23$ m (ANS), (50%)**	**2022 NHT Question 10b** **2019 NHT Question 6b**

Worked example 10: Oblique projection, range.

At the school track and field events, Anthony released a shotput from shoulder height above the ground, at 8.0 m s^{-1} at an angle of 30° above the horizontal. For one particular throw, the time of flight was 1.15 s. (Neglect air resistance in your calculations.) The figure shows the flight of the shotput.



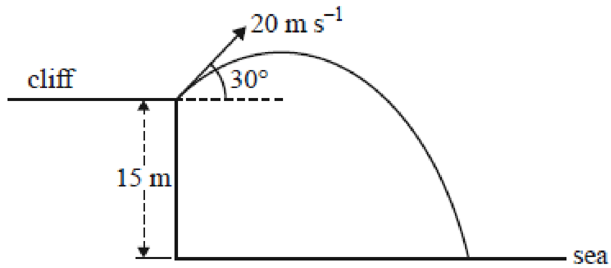
1997 Question 11, 3 marks

What was the horizontal distance that the shotput landed from the thrower? (Neglect air resistance in your calculations.)

Solution	**Current study design:**
Use the horizontal component of the initial velocity. $\therefore d = v_{hor} \times t$ $\therefore d = 8.0 \times \cos30 \times 1.15$ $\therefore d = 7.97$ $\therefore d = 8.0$ m (ANS)	**2021 NHT Question 10**

Worked example 11: Oblique projection, time of flight.

A stone is thrown from the top of a 15 m high cliff above the sea at an angle of 30° to the horizontal. It has an initial speed of 20 m s^{-1}. The situation is shown below. Ignore air resistance.



2013 Question 8a, 3 marks

Calculate the time taken for the stone to reach the sea.

Solution

The methodical way to complete this is to divide the problem into two parts, up and down.

Consider 'up'

Initial velocity is 10 m s^{-1}

Final velocity = 0

$\therefore$ v = u – gt

$\therefore$ 0 = 10 – 9.8 t

$\therefore$ t = 1.02 sec.

Height at top

x = ut – $\frac{1}{2}$ gt^2

gives x = 10 × 1 – $\frac{1}{2}$ × 9.8 × 1.02^2

$\therefore$ x = 4.9 m

$\therefore$ height = 19.9 m.

Consider 'down'

x = ut + $\frac{1}{2}$ gt^2

$\therefore$ 19.9 = 0 + $4.9t^2$

$\therefore$ t^2 = 4.06

$\therefore$ t = 2.02

$\therefore$ Total time = 1.02 + 2.02

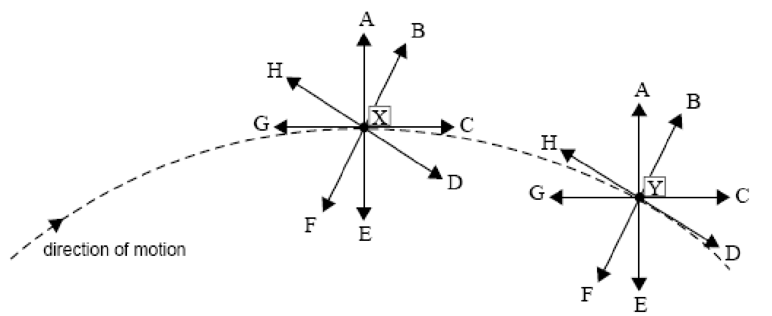
= 3.0 sec (ANS), (50%)

Current study design:

2022 NHT Question 10a

Worked example 12: Forces acting: Resultant force/acc during flight.

A car takes off from a ramp and the path of its centre of mass through the air is shown below.



First, model the motion of the car assuming that air resistance is small enough to neglect.

1999 Question 11, 1 mark

Which **one** of the directions (**A–H**) best shows the direction of the **acceleration** of the car at point Y?

Solution	**Current study design:**
The only force acting is the weight force, so the acceleration is down. ∴ E (ANS), (63%)	**2019 Question 12 (61%)**

Worked example 13: Forces acting: Force/acc. at max h, air resistance.

Now, suppose that **air resistance cannot be neglected**.

1999 Question 11, 1 mark

Which **one** of the directions (**A–H**) best shows the direction of the **acceleration** of the car at point X?

Solution	**Current study design:**
Now there are two forces acting, the weight force down (vertical) and the air resistance that is opposing the motion (horizontal). Therefore, the acceleration must be somewhere between the two. ∴ F (ANS), (63%)	**2021 NHT Question 11**