Circular motion

- **investigate and analyse theoretically and practically the uniform circular motion of an object moving in a horizontal plane: ($F_{net} = \frac{mv^2}{r}$), including:**
  – a vehicle moving around a circular road
  – a vehicle moving around a banked track
  – an object on the end of a string
- **model natural and artificial satellite motion as uniform circular motion**
- **investigate and apply theoretically Newton's second law to circular motion in a vertical plane (forces at the highest and lowest positions only)**

Paper	Multiple choice	Short Answer	Idea	Marks	%	Type
2022		8a	Horizontal, $F = \frac{mv^2}{r}$	2	80%	Calculation
		8b	Horizontal, force direction	1	84%	Concept
		8c	Horizontal, force acting	2	42%	Explanation
2022 NHT		9a	KE calculation	1	NA	Show
		9b	Vertical, normal force at top	4	NA	Calculation
2021		9b	Vertical, speed at top	2	50%	Concept
		9c	Vertical, normal force	3	15%	Calculation
		9d	Vertical, effect of energy loss	3	51%	Explanation
2021 NHT						
2020	8		Horizontal, speed	1	56%	Concept
		8a	Conical pendulum, force	2	58%	Concept
		8b	Conical pendulum, speed	4	50%	Calculation
2019		8a	$mg\Delta h = \Delta KE$	3	57%	Calculation
		8b	Vertical, normal force	3	53%	Calculation
		8c	Vertical, normal concept	3	23%	Explanation
2019 NHT	8		Vertical, speed	1	NA	Calculation
	9		Vertical, net force	1	NA	Concept
2018		10a	Vertical, N = 0	2	64%	Calculation
		10b	Vertical, N = 0 explanation	2	59%	Explanation
2018 NHT		8a	Vertical, speed at top	2	NA	Calculation
		8b	Vertical, energy	3	NA	Calculation
2017		2b	$F = \frac{mv^2}{r}$	2	67%	Calculation
		7a	Banked curve, forces	2	67%	Concept
		7b	Banked curve, angle	2	68%	Calculation
		8a	Vertical, normal = 0	2	66%	Calculation

Circular motion questions can be grouped into the following ideas.

Application $F = \frac{mv^2}{r}$	Worked example 1
Horizontal circles	
Forces acting, resultant force.	Worked example 2
Speed (velocity) at any point.	Worked example 3
Banked curves	
Forces acting.	Worked example 4
Angle θ calculation	Worked example 5
Conical pendulum	
Forces acting.	Worked example 6
Speed	Worked example 7
Vertical circles	
Speed at top/bottom.	Worked example 8
Normal force at top/bottom.	Worked example 9
N = 0	1991 Question 11, 1 mark
Energy/ΔEnergy	Worked example 10
Roller coaster	
Speed at top/bottom.	
Energy at top/bottom.	Worked example 13
Forces acting, including N = 0	Worked example 14
Air resistance	Worked example 15

Horizontal circles

Uniform circular motion results when a resultant force of constant magnitude acts normal to the motion of the body.

When a body travels in a circle, the force is always towards the centre, while the velocity is tangential. The force is always at right angles to the velocity, this is the definition for circular motion.

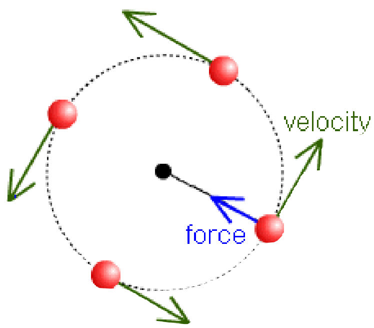
The formula for the magnitude of a net force necessary to keep an object in circular motion is:

$$F = \frac{mv^2}{r}$$

Acceleration is given by $a = \frac{v^2}{r}$

A mass moving with a uniform speed in a circular path of radius ‘r’ and with a period ‘T’ has an average speed given by

$$\frac{\text{distance travelled}}{\text{time taken}} = \frac{2\pi r}{T}$$



$$F_{net} = ma$$

$$= \frac{mv^2}{r}$$

$$= m\frac{4\pi^2 r}{T^2}$$

$$\therefore a = \frac{v^2}{r}$$

$$= \frac{4\pi^2 r}{T^2}$$

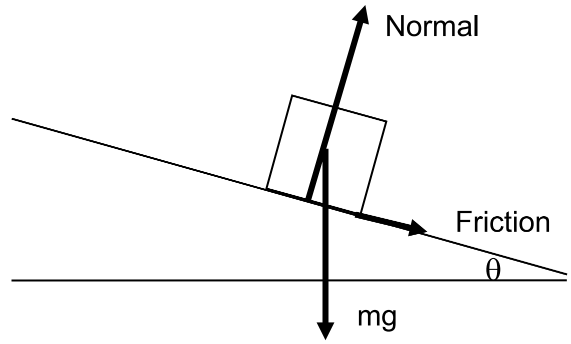
In circular motion the movement is always at right angles to the force, since work done = force × distance moved in the direction of the force. The work done in circular motion is zero.

Definition of circular motion

For an object travelling at a constant speed, if a constant force is always perpendicular to the motion, then the resultant path is circular.

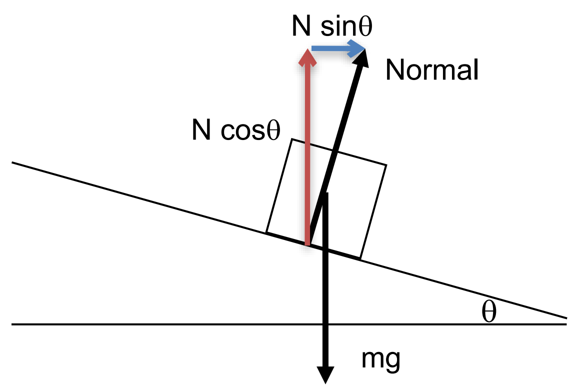
Banked surfaces

Surfaces (eg. roads, velodromes) are banked to reduce the need for frictional forces to assist in circular motion. The forces acting are shown here.



For the object to travel in circular motion the net force acting needs to be radially inwards.

If we want the frictional force to be zero, then the normal force is resolved into perpendicular and horizontal components.



Here N cosθ = mg (because the object is not accelerating in the vertical direction) and the unbalanced force is N sinθ

$\therefore N \sin\theta = \frac{mv^2}{R}$ dividing this by N cosθ = mg gives

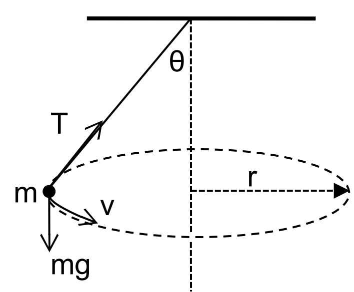
$$\frac{N\sin\theta}{N\cos\theta} = \frac{mv^2}{R} \times \frac{1}{mg}$$

$\tan\theta = \frac{v^2}{Rg}$ this formula is very convenient and should be on your formula sheet.

Conical Pendulum

A conical pendulum is a mass suspended by a string, that moves in a horizontal circle at constant speed.

There are two forces acting, the tension in the string, and the weight.

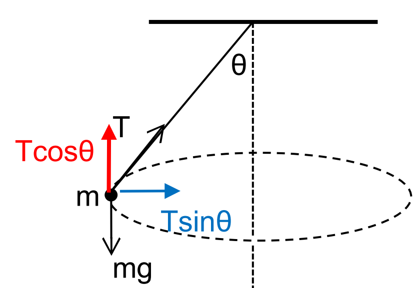


Since the mass will move in a horizontal circle, $F_{net\ (vertical)} = 0$

$\therefore T \cos\theta = mg$

In the horizontal direction, the unbalanced force is T sinθ. and the mass is moving in a circle,

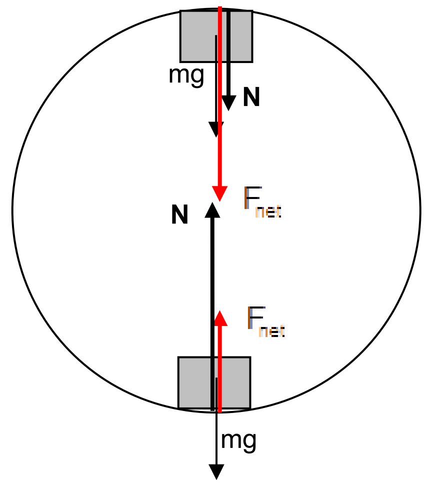
$\therefore T\sin\theta = \frac{mv^2}{r}$



These two equations can be arranged to give $\tan\theta = \frac{v^2}{rg}$

Vertical circles

Consider an object travelling around a vertical loop.



At the top

There are two forces acting downwards, the normal reaction and the weight.

$$F_{net} = \frac{mv^2}{r}$$

$$\therefore N + mg = \frac{mv^2}{r}$$

At the bottom

The normal reaction and the weight are in opposite directions.
The normal reaction is greater than the weight because the net force is always radially inwards.

$$F_{net} = \frac{mv^2}{r}$$

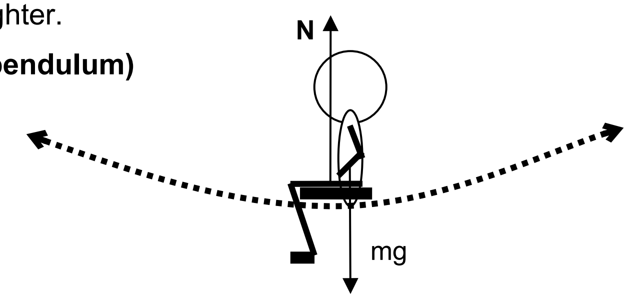
$$N - mg = \frac{mv^2}{r}$$

Apparent weight

The weight of a mass is the downward force exerted on it by the earth's gravity. The **apparent weight** is the upward normal or reaction force.

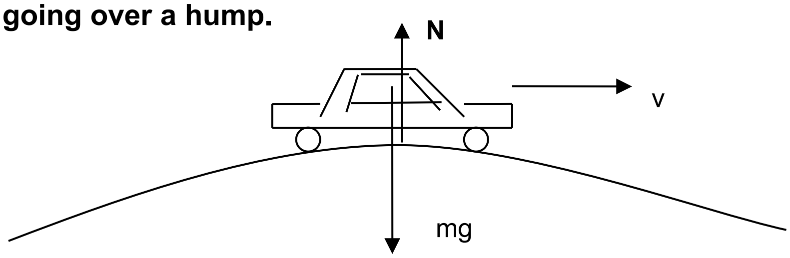
You feel your 'weight' as the normal reaction of the surface on you, because you can only feel things that act on you. If the normal reaction increases you 'feel' heavier, and if the normal reaction decreases you 'feel' lighter.

Person on a swing (pendulum)



At the bottom of the swing, the forces on the person are the reaction from the seat and the weight force. $N - mg = \frac{mv^2}{r}$. In this case N > mg, so the person 'feels' heavier.

Person in a car going over a hump.

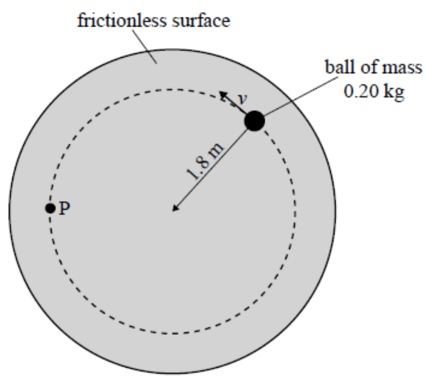


$mg - N = \frac{mv^2}{r}$ $\therefore N = mg - \frac{mv^2}{r}$ $N < mg$, $\therefore$ the person 'feels' lighter.

$$F = \frac{mv^2}{r}$$

Worked example 1: Application

Kim has attached a ball of mass 0.20 kg to a piece of string of length 1.8 m and is making it move in a **horizontal** circle on a frictionless surface. Kim gradually increases the speed, *v*, of the ball. The situation is shown from above in the diagram below. The string has a breaking force of 4.0 N.



2012 Question 7a, 2 marks

Calculate the greatest speed the ball can reach before the string breaks.

Solution

Use $F_{net} = ma$

$$\therefore T = \frac{mv^2}{r}$$

$$\therefore 4.0 = \frac{0.20 \times v^2}{1.8}$$

$$\therefore v^2 = \frac{4.0 \times 1.8}{0.2}$$

$\therefore v = 6 \text{ m s}^{-1}$ (ANS). (79%)

Current study design:

2017 Question 2b (67%)

Worked example 2: Horizontal: forces acting/resultant force.

A motorcyclist is riding around a circle of radius of 100 m. The surface is flat and horizontal. The motorcyclist is travelling at a constant speed of 32.0 m s^{-1}.

The motorcycle with rider has a mass of 250 kg.

2009 Question 3, 2 marks

What is the magnitude of the net force on the motorcycle with rider?

Solution

Since the motorcyclist is riding in a circle the net force acting on the rider can be calculated by the following:

$$F_{net} = m\frac{v^2}{r}$$

$$F_{net} = 250 \times \frac{32^2}{100}$$

$\therefore$ **F_{net} = 2560 N (ANS). (90%)**

Current study design:

2022 Question 8a (80%)

2022 Question 8b (84%)

2022 Question 8c (42%)

Worked example 3: Horizontal: speed (velocity) at any point.

A racing car of mass 700 kg (including the driver) is travelling around a corner at a constant speed. The car's path forms part of a circle of radius 50 m, and the track is horizontal.

The magnitude of the central force provided by friction between the tyres and the ground is 11 200 N.

2010 Question 1, 2 marks

What is the speed of the car?

Solution

Use $F_{net} = m\frac{v^2}{r}$

$\therefore$ 11 200 = 700 × $\frac{v^2}{50}$

$\therefore$ v = 28.3 m s^{-1} (ANS), (85%)

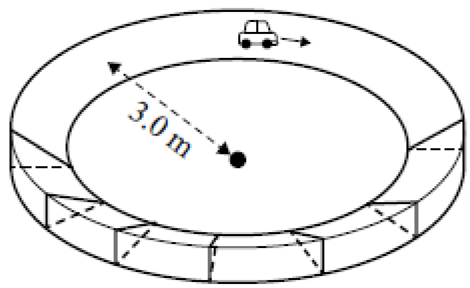
Current study design:

2020 Question 8 (56%)

Worked example 4: Banked curves: forces acting.

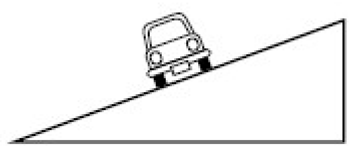
A model car of mass 2.0 kg is placed on a banked circular track. The car follows a path of radius 3.0 m. The motor maintains it at a constant speed of 2.0 m s^{-1}, as shown below.

The angle of bank is such that there are no sideways frictional forces between the wheels and the track.

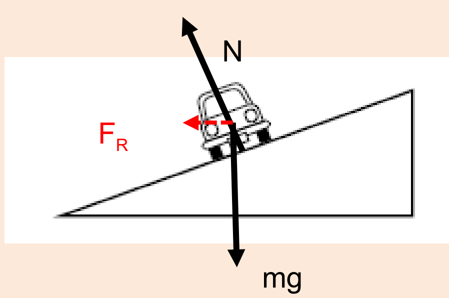


2015 Question 4a, 3 marks

On the diagram below, draw the forces acting on the car using solid lines and label each force. Show the resultant force as a dotted line, labelled F_R.



Solution



(87%)

Current study design:

2017 Question 7a (67%)

Worked example 5: Banked curves: angle θ calculation.

2015 Question 4b, 3 marks

Calculate the required angle of bank of the track, in degrees, to maintain the 2.0 kg car at 3.0 m s^{-1} on the 3.0 m circular path with no sideways friction between the wheels and the track. Show your working.

Solution

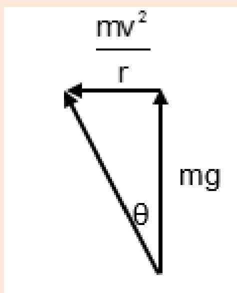
If we resolve N into two components, one vertical and the other horizontal. In the horizontal direction we get

$$N\sin\theta = \frac{mv^2}{r}$$

In the vertical direction we get

$$N\cos\theta = mg$$

Drawing that gives



$$\therefore \tan\theta = \frac{\frac{mv^2}{r}}{mg}$$

$$\therefore \tan\theta = \frac{v^2}{rg}$$

$$\therefore \tan\theta = \frac{3.0^2}{3.0 \times 10}$$

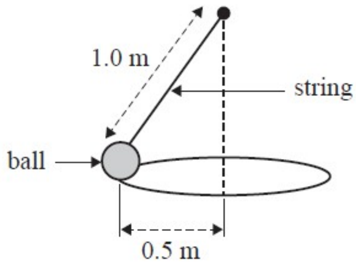
$\therefore \theta = 16.7^0$ (ANS), (67%)

Current study design:

2017 Question 7b (68%)

Worked example 6: Conical pendulum: forces acting.

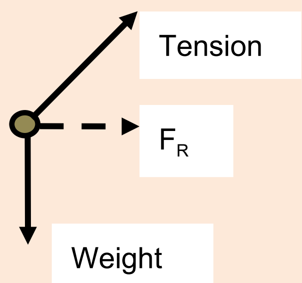
A steel ball of mass 2.0 kg is swinging in a circle of radius 0.50 m at a constant speed of 1.7 m s^{-1} at the end of a string of length 1.0 m, as shown below.



2016 Question 2a, 2 marks

On the second figure above, draw all the forces **on the ball**. Label all forces. Draw the resultant force as a dotted line labelled F_R.

Solution



(65%)

Current study design:

2022 NHT Question 7c

2020 Question 8a (58%)

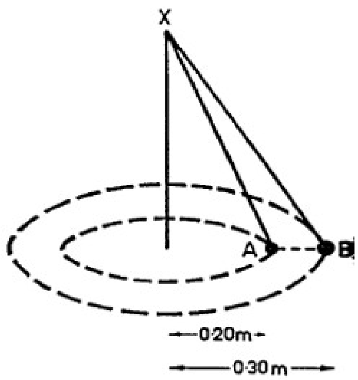
Worked example 7: Conical pendulum: speed.

Two spheres *A* and *B* are attached to the ends of light strings *XA* and *XB.*

They are swung in horizontal circles as indicated in the diagram.

The time for one complete revolution is the same for both *A* and *B.*

The radius of A's path is 0·20 m, and the radius of B's path is 0·30 m.



1967 Question 28, 1 mark

What is the value of the ratio $\dfrac{\text{Speed of sphere B}}{\text{Speed of sphere A}}$?

Solution

Since the time for both is the same, let us say T, then the speed is given by:

$$v = \frac{2\pi r}{T}$$

$$\therefore v_B = \frac{2\pi \times 0.30}{T}$$

$$\therefore v_A = \frac{2\pi \times 0.20}{T}$$

$$\therefore \frac{\text{Speed of sphere B}}{\text{Speed of sphere A}} = \frac{0.30}{0.20}$$

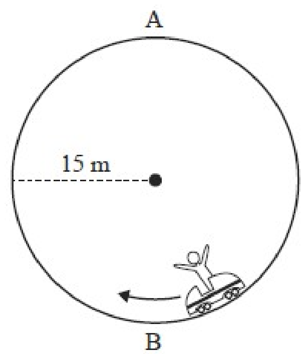
∴ 1.5 (ANS), (94%)

Current study design:

2020 Question 8b (50%)

Worked example 8: Vertical circles: speed at top/bottom.

A ride at a fun fair involves a car travelling around a vertical circle of radius 15 m. The arrow indicates the direction of travel as shown below.



2011 Question 9, 2 marks

What is the minimum speed that the car can have at position A if it is not to leave the rails and start to fall?

Solution

When the car is just about to leave the rails, the normal reaction is zero.

At A, $F_{net} = N + mg$

$\therefore \frac{mv^2}{r} = N + mg$, but $N = 0$

$\therefore \frac{mv^2}{r} = mg$

$\therefore \frac{v^2}{r} = g$

$\therefore v = \sqrt{gr}$

$\therefore v = \sqrt{9.8 \times 15}$

$\therefore v = 12.1 \text{ m s}^{-1}$ (ANS), (74%)

Current study design:

2019 NHT Question 8

Worked example 9: Vertical circles: Normal force at top/bottom.

2011 Question 10, 2 marks

What is the minimum speed that the car can have at position A if it is not to leave the rails and start to fall?

Solution

Since the normal is zero, Melanie will be apparently weightless. This is because she is in free fall accelerating down at 9.8 m s^{-2}.

$\therefore$ 0 N (ANS), (48%)

Current study design:

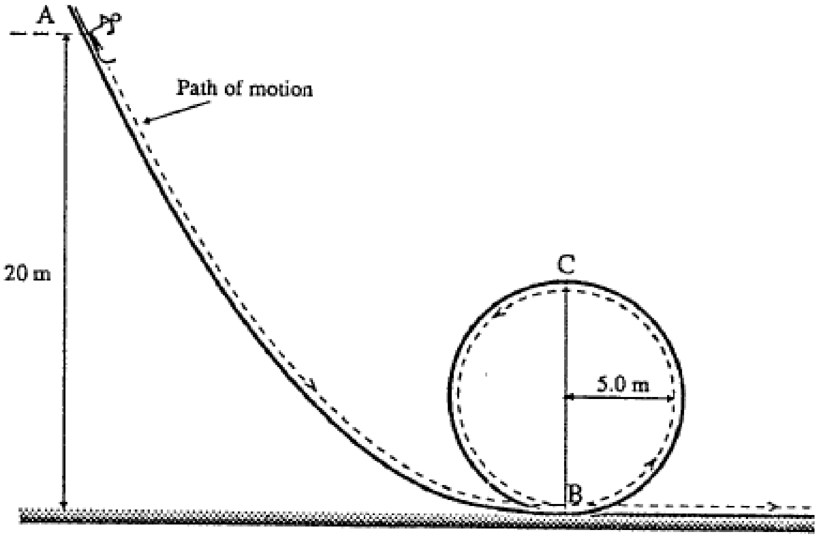
2019 NHT Question 9

2018 NHT Question 8a

Worked example 10: Vertical circles: Energy/ΔEnergy.

At a fun fair in Canada, Tim rides a sled down an icy slope and around the inside of an iced loop as shown below. In this situation friction is small and can be ignored in the question below. At no stage does the sled leave the track. The total mass of the sled and Tim is 150 kg.

The sled is initially at point A, 20 m higher than point B.



1991 Question 11, 1 mark

Find the loss of potential energy associated with the sled and Tim as they move from point A to point B.

Solution	**Current study design:**
Use $\Delta GPE = mg\Delta h$ $\therefore \Delta GPE = 150 \times 9.8 \times 20$ $\therefore \Delta GPE = 29\,400$ $\therefore \Delta GPE = 2.9 \times 10^4$ J (ANS)	**2022 NHT Question 9a** **2018 NHT Question 8b**

Worked example 11: Vertical circles: N = 0.

1991 Question 13, 4 marks

Tim believes that he will be 'weightless' at point C.

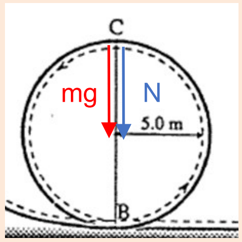
- **i.** Explain what the term 'weightless' means.

You may wish to include a diagram showing all the forces acting on Tim at point C, and you should use any calculations that you need.

- **ii.** What is the minimum speed that the sled could have at point C if it is not to lose contact with the track?
- **iii.** Is Tim correct?

Solution

Weightless is defined as when the normal reaction is zero.



At the point C there are two forces acting on Tim and the sled, the weight, which is acting vertically downwards and the normal reaction from the track. If the normal reaction (N) = 0, then Tim will feel weightless. The net force acting at Point C is:

$$F_{net} = \frac{mv^2}{r}$$

$$\therefore \frac{mv^2}{r} = mg + N$$

The point C is 10 m above B.
TE at B = 29 400 J

$$\therefore 29\ 400 = mg\Delta h + \tfrac{1}{2}mv^2$$

$$\therefore 29\ 400 = 14\ 700 + \tfrac{1}{2}mv^2$$

$$\therefore \tfrac{1}{2}mv^2 = 14\ 700$$

$$\therefore mv^2 = 29\ 400.$$

(gives Tim's speed as 14.0 m s^{-1})

Use $\frac{mv^2}{r} = mg + N$ to find N.

$$\therefore \frac{29400}{5} = 150 \times 9.8 + N$$

$$\therefore 5880 = 1470 + N$$

$$\therefore N = 4410.$$

The minimum speed that Tim could have at point C (and not lose contact) is given by

$$\frac{mv^2}{r} = mg + N \text{ when } N = 0$$

$$\therefore \frac{mv^2}{r} = mg$$

$$\therefore \frac{v^2}{r} = g$$

$$\therefore v^2 = 49$$

$$\therefore v = 7.0 \text{ m s}^{-1}$$

This means that Tim is incorrect as he will not feel weightless, as the normal reaction acting on him is 4.4×10^3 N.

Current study design:

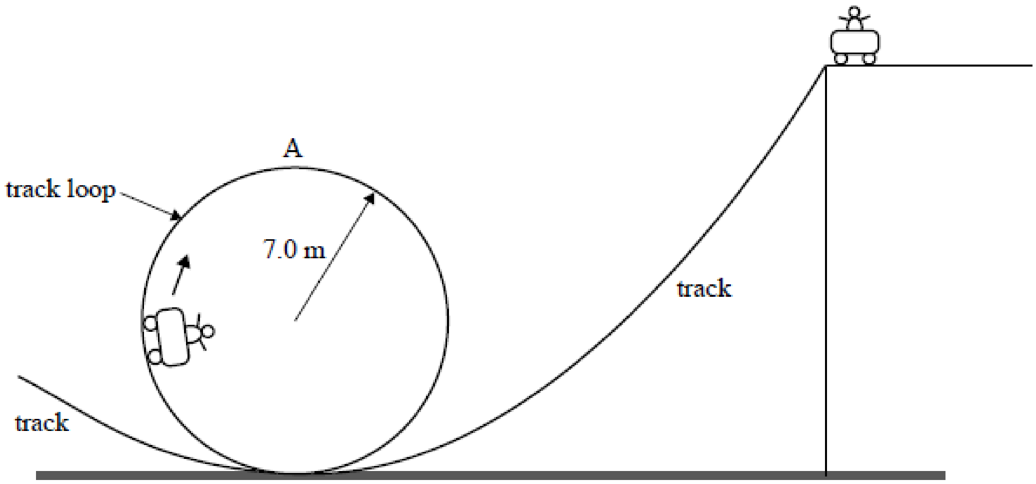
2018 Question 10a (64%)

2018 Question 10b (59%)

Worked example 12: Roller coasters: Speed at top/bottom.

On a loop-the-loop roller coaster, a loop in the track has a radius of 7.0 m as shown below.

On a particular occasion, the mass of the trolley and riders is 600 kg.



2009 Question 12, 3 marks

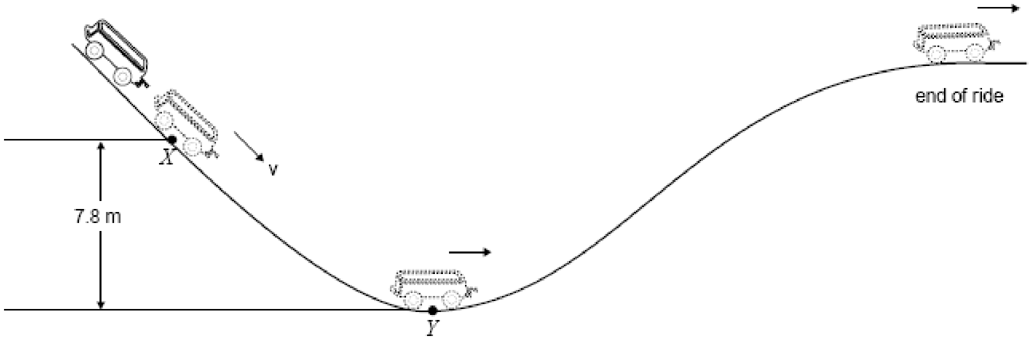
To go safely around the loop, the trolley wheels must not leave the rails at point A.

What is the minimum speed that the trolley must have at point A so that it does not leave the rails?

Solution	**Current study design:**
To not leave the rails the trolley must have a net force of: $F_{net} = m\frac{v^2}{r}$ At the top of the loop the net force is provided by the weight force of the trolley. So: $mg = m\frac{v^2}{r}$ ∴ $600 \times 9.8 = 600 \times \frac{v^2}{7}$ ∴ $v^2 = 68.6$ ∴ $v = 8.3$ m s^{-1} (ANS), (64%)	**2021 Question 9b (50%)**

Worked example 13: Roller coasters: Energy at top/bottom.

A cart on a roller coaster rolls down a track as shown below. The upper section of the track is straight and the lower section forms part of a circle. The effect of friction can be ignored in the following questions.



The mass of the cart is 500 kg, and at point *X* it is travelling at a speed of 10 m s^{-1}.

1998 Question 15, 2 marks

Calculate the speed of the cart at the point *Y*.

Solution

The total energy must be constant.

∴ the sum of PE and KE = constant.

At X TE (total energy)

$= mgh + \frac{1}{2}mv^2$

$= 500 \times 9.8 \times 7.8 + \frac{1}{2} \times 500 \times 10^2$

$= 63\,220$ J

At Y TE

$63\,220 = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 500 \times v^2$

$v^2 = \frac{63220}{250}$

∴ v = 15.90

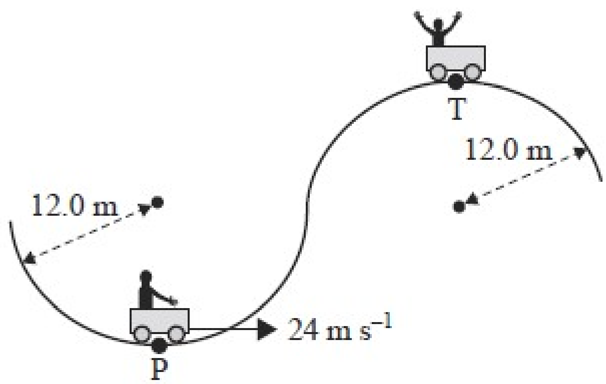
∴ v = 16 m s^{-1} (ANS), (25%)

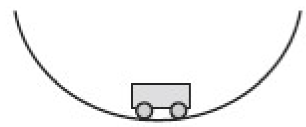
Current study design:

2019 Question 8a (57%)

Worked example 14: Roller coasters: Forces acting including N = 0.

An amusement park has a car ride consisting of vertical partial circular tracks, as shown below. The track is arranged so that the car remains upright at both the top and bottom positions. The track has a radius of 12.0 m and its lowest point is point P.

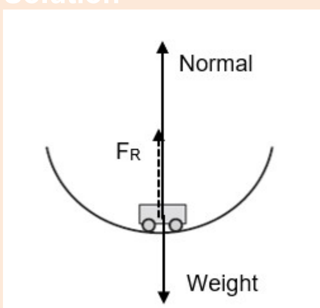




2017 NHT Question 3a, 3 marks

On the second diagram, draw labelled arrows showing all of the forces on the car at point P and draw the resultant force with a dotted arrow labelled F_R.

Solution



Current study design:

2022 NHT Question 9b

2021 Question 9c (15%)

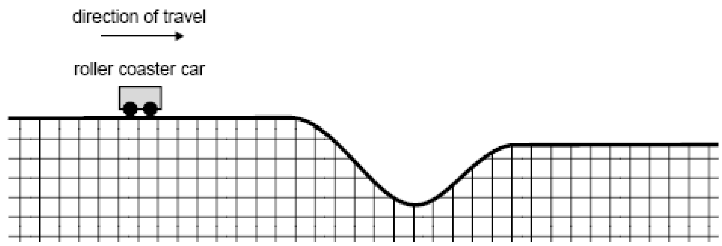
2019 Question 8b (53%)

2019 Question 8c (23%)

2017 Question 8a (66%)

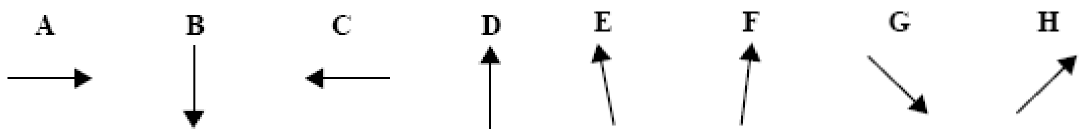
Worked example 15: Roller coasters: Forces acting including N = 0.

The diagram below shows a section of a roller coaster track, with a roller coaster car travelling at constant speed from the left.



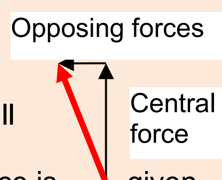
2001 Question 10, 2 marks

Which of the arrows (**A–H**) best indicates the direction of the net force on the roller coaster car at the lowest point, assuming that friction and air resistance **cannot** be neglected?



Solution

At the lowest point, the roller coaster is in the bottom part of a section of circular motion. This means that the force causing this acceleration is directly up. If we now take friction and air resistance into consideration, (these will both be opposing the motion) then the net force is given by the sum of the two vectors.



∴ E (ANS), (35%)

Current study design:

2021 Question 9d (51%)