Relativity

Einstein's postulates

- **describe Einstein's two postulates for his theory of special relativity that:**
  - **– the laws of physics are the same in all inertial (non-accelerated) frames of reference**
  - **– the speed of light has a constant value for all observers regardless of their motion or the motion of the source**
- **compare Einstein's theory of special relativity with the principles of classical physics**

Time dilation, length contraction

- **describe proper time (t_0) as the time interval between two events in a reference frame where the two events occur at the same point in space**
- **describe proper length (L_0) as the length that is measured in the frame of reference in which objects are at rest**
- **model mathematically time dilation and length contraction at speeds approaching *c* using the equations: $t = t_0\gamma$ and $L = \frac{L_0}{\gamma}$ where $\gamma = \left(\sqrt{1 - \frac{v^2}{c^2}}\right)^{-1}$**

Particles

- **explain why muons can reach Earth even though their half-lives would suggest that they should decay in the outer atmosphere.**

Energy

- **interpret Einstein's prediction by showing that the total 'mass-energy' of an object is given by: $E_{tot} = E_k + E_0 = \gamma mc^2$ where $E_0 = mc^2$, and where kinetic energy can be calculated by: $E_k = (\gamma - 1)mc^2$**
- **describe how matter is converted to energy by nuclear fusion in the Sun, which leads to its mass decreasing and the emission of electromagnetic radiation.**

Paper	Multiple choice	Short Answer	Idea	Marks	%	Type
2022	18		Inertial frame definition	1	82%	Concept
	19		Lorentz factor	1	85%	Calculation
		9	Fusion, Δm	2	47%	Explanation
		11	Muons	2	41%	Explanation
2022 NHT	10		Length contraction	1	NA	Concept
	11		$E_{rest} = mc^2$	1	NA	Calculation
		11a	Time dilation	2	NA	Calculation
		11b	Length contraction	2	NA	Concept
2021	20		Inertial frame definition	1	49%	Concept
		10a	Length contraction	2	57%	Explanation
		10b	Length contraction	2	63%	Calculation
2021 NHT	13		Time dilation	1	NA	Concept
	20		$E_{rest} = mc^2$, Δm	1	NA	Calculation
		10a	Lorentz factor, sig figs	2	NA	Calculation
		10b	$t = \frac{d}{v}$, sig figs	2	NA	Calculation
2020	12		Gamma calculation	1	82%	Calculation
	13		Rate of matter to energy	1	56%	Calculation
		11a	Length contraction	2	29%	Calculation
		11b	Proper time	2	44%	Explanation
2019	13		Length contraction	1	75%	Calculation
		11	Einstein's postulates	3	40%	Explanation
2019 NHT	16		Lorentz factor	1	NA	Calculation
	17		Muons, time dilation, γ	1	NA	Calculation
	18		Kinetic energy	1	NA	Calculation
		17	Time dilation	3	NA	Calculation
		18	Length contraction	3	NA	Explanation
		19	$E_{rest} = mc^2$	2	NA	Calculation
2018	13		Lorentz factor	1	58%	Concept
	14		Kinetic energy	1	60%	Concept
		14	Frames of reference	2	13%	Explanation
		15	$KE = (\gamma - 1)mc^2$	3	43%	Calculation
		16	Time dilation	2	55%	Calculation
2018 NHT	10		Lorentz factor	1	NA	Calculation
	11		$t = \frac{s}{v}$	1	NA	Calculation
		14	Time dilation	3	NA	Calculation
		15	$E_{rest} = mc^2$	2	NA	Calculation
2017	10		Frames of reference	2	75%	Concept
	11		$E_{rest} = mc^2$	1	66%	Calculation
		10	Length contraction	1	58%	Concept
		11a	Particles	2	44%	Calculation
		11b	Particles	2	39%	Calculation
		11c	Particles	3	27%	Explanation

Relativity basics questions can be grouped into the following ideas.

Frames of reference

A reference frame is the physical object to which we attach our co-ordinate system. In everyday life, that object is the ground.

Inertial frames

Inertial frames of reference are where the frame is not accelerating, therefore they are either at ‘rest’ or moving with constant velocity with respect to other *inertial frames of reference.* **Newton’s first law is obeyed in all Inertial frames of reference**.

Non-inertial frames of reference

An accelerated frame of reference is called a non-inertial frame of reference. **This is beyond the limits of this course.**

Relative velocity

$\text{velocity}_{\text{A relative to B}} = \boldsymbol{v}_{ab} = \boldsymbol{v}_a - \boldsymbol{v}_b$.

Einstein’s postulates

The Principle of Relativity

All the laws of physics are the same in all inertial frames.

The Constancy of the Speed of Light

The speed of light in vacuum is the same (3×10^8 m s^{-1}) in all inertial frames [i.e. there is no ether, and the speed of light is the same regardless of the motion and the source of light].

Proper Time t_0

The time measured in the frame of reference where the clock is stationary. Proper time is always measured with one clock. If it requires more than one clock to measure the time it must be the dilated time that is being measured. (Less clocks, less time).

Proper Length L_0

Measured length in the frame where the object (or distance between two events) is stationary.

<u>Note</u>
Proper length and proper time are measured in different frames of reference. Proper time is measured in the moving frame and proper length in the stationary frame.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Gamma, γ (Lorentz factor) **∴ γ ≥ 1**

γ	v
1.1	0.140 c
1.5	0.745 c
2	0.866 c
5	0.980 c
10	0.995 c

v	c	γ
0.1 c	10%	1.005
0.5 c	50%	1.155
0.8 c	80%	1.667
0.95 c	95%	3.203
0.99 c	99%	7.089

For calculations it is sometimes more convenient to consider $\beta = \frac{v}{c}$.

This gives $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ which leads to $\beta = \sqrt{1-\frac{1}{\gamma^2}}$

Many students find it useful to have $c^2 = 9.0 \times 10^{16}$ on their cheat sheet, as this aids in remembering to square the values in the calculations.

Length Contraction

The length of a moving object will be measured to be less than the length of the same object measured in the frame when the object is stationary. This is ***length contraction***.

In order to measure the **proper** length of an object, it must be measured from a frame of reference in which it is at rest. If it is being measured as it is moving, we can use the following formula to calculate its **relativistic length contraction**.

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$ or $$L = \frac{L_0}{\gamma}$$

Where:
L = length in moving frame
L_0 = length in stationary frame
v = relative velocity of the moving frame
c = speed of light

Time dilation

Proper time (t_0) as the time interval between two events in a reference frame where the two events occur at the same point in space, in other words the time interval can be measured by one clock. Time in a frame of reference that is moving will be observed to run slower than time in your own frame. This is ***time dilation***.

We need to use a correction factor, called a Lorentz Conversion.

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$ or $$t = t_0\gamma$$

Where:
t = elapsed time in moving frame
t_0 = time in a stationary frame
v = relative velocity of the moving frame
c = speed of light

Light year (ly)

A light year is a distance. It is the distance that light travels in a "Julian" year (365.25 days).

Strictly speaking it is 299 792 458 × 365.25 × 24 × 60 × 60 = 9.4607×10^{15} m

Correct to two sig figs (since $c = 3.0 \times 10^8$, on the formula sheet) **1 ly = 9.5×10^{15} m** (sometimes it is quoted as 9.46×10^{15} m)

Problem solving process.

Identify the two events.

Identify the rest frame and the moving frame.

The measured length is always longest in the rest frame i.e. the length of a moving object is always measured to be shorter.

The measured time is always shortest in the rest frame.

If the question requires measurements from another frame either multiply or divide by gamma.

Cosmic Muons

Muons are elementary particles created in the upper atmosphere by cosmic rays. They are unstable, and decay with an average half life of 2.2 μs (2.2×10^{-6} s) when measured at rest. This means that in the reference frame of the muons, half of them decay in each time interval of 2.2 μs,

Laboratory Muons

Muons are very short-lived particles that are created when energetic protons collide with each other. A beam of muons can be produced by very-high-energy particle accelerators.

The high-speed muons produced for an experiment by the Fermilab accelerator are measured to have a lifetime of 5.0 microseconds. When the lifetime is measured in muons frame of reference, the lifetime is measured to be 2.2 microseconds.

Relativistic Energy, Mass-energy $E_{tot} = E_k + E_0 = \gamma mc^2$, rest mass energy $E_0 = mc^2$

Einstein linked not only space and time but also mass and energy. A piece of matter, even at rest and not interacting with anything else, has an "energy of being". This is called its *rest energy.* Einstein concluded that it takes energy to make mass and that energy is released if mass disappears. This linkage has resulted in the term mass-energy.

The **mass-energy** of any object is given by $E = \gamma mc^2$.

With mass-energy, a moving body has kinetic energy and rest energy.

$$\therefore\ E_{tot} = E_k + E_{rest} \text{ (or } E_0)$$

$$\therefore\ \gamma mc^2 = E_k + mc^2$$

$$\therefore\ E_k = \gamma mc^2 - mc^2$$

$$\therefore\ \mathbf{E_k = (\gamma - 1)mc^2}.$$

Note 1: It can be shown that for all values of '$v > 0$', the relativistic KE will **ALWAYS** be greater that the Newtonian KE.

Note 2: The study design uses $E_0 = mc^2$, whereas the formula sheet (up to 2022) uses $E_{rest} = mc^2$.

Our Sun

In 1 second, 4.5 million tonnes of mass is converted into radiant energy in the sun. The sun is so massive, however, that in 1 million years only one ten millionth of the sun's mass will have been converted to radiant energy.

Combining Atoms – Nuclear Fusion

Nuclear fusion is the process of fusing two atoms together and creating a larger atom.
The energy from the Sun (heat and light) originates from this **nuclear fusion process** that is occurring inside the core of the Sun. The specific type of fusion that occurs inside of the Sun is known as **proton-proton fusion**.

Inside the Sun, this process begins with protons (a lone hydrogen nucleus) and through a series of steps, these protons fuse together and are turned into helium.

Worked example 1: Einstein's postulates: Inertial frame of reference.

2013 Question 1, 2 marks

James is stationary ($v = 0$) on a footpath while Amanda drives past at a constant speed of 60 km h^{-1}.

Which one of the following statements is correct?

A. Amanda is in a non-inertial reference frame because she is moving relative to James.

B. James must be in a non-inertial reference frame because he is stationary at the moment.

C. James is not stationary in his reference frame because he is moving in Amanda's reference frame.

D. Amanda is stationary in her reference frame even though she is moving in James's reference frame.

Solution

An inertial frame of reference is non-accelerating. Amanda, travelling at a constant speed is in an inertial frame. James travelling at a constant speed, (v = 0), is also in an inertial frame. James is stationary in his reference frame.

∴D (ANS), (78%)

Current study design:

2022 Question 18 (82%)

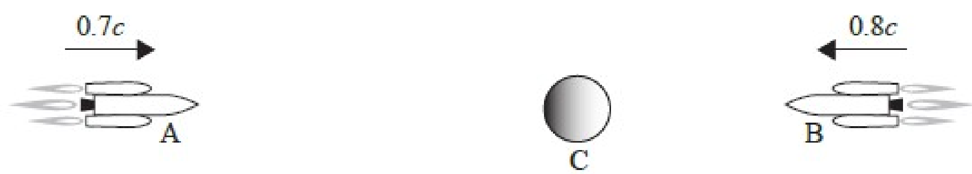
2021 Question 20 (49%)

2018 Question 14 (13%)

2017 Question 10 (75%)

Worked example 2: Einstein's postulates: Speed of light.

The spaceship *Andromeda* (A) is travelling at 0.7*c* towards the asteroid Ceres (C). It sends a light pulse to the nearby ship *Bradbury* (B), which is approaching the asteroid from the far side at 0.8*c*, as shown below.



2013 Question 4, 2 marks

The speed of the light pulse as measured from each body is

A. greatest for A and least for B.

B. greatest for B and least for A.

C. greatest for C and least for B.

D. the same for each body.

Solution

The speed of light is constant, *c*.

∴D (ANS), (81%)

Current study design:

2019 Question 11 (40%)

Worked example 3: Einstein's postulates: $s = v \times t$, $t = \frac{s}{v}$

2010 Question 1, 2 marks

On a planet a long way away, a racing car is moving at high speed (0.9*c*) along a straight track. It is heading straight for a post. Jim is standing next to the post. The situation is shown below.

When the racing car is 1.00 km from the post (as measured by Jim), the driver sends a flash of light from the car.

Which of the following is closest to the time that the flash of light takes to reach the post (as measured by Jim)?

A. 1.5 microseconds

B. 1.8 microseconds

C. 3.3 microseconds

D. 3.7 microseconds

Solution

Use $t = \frac{d}{v}$

where v = c

$\therefore t = \frac{1000}{3 \times 10^8}$

$\therefore t = 3.3 \times 10^{-6}$ s

∴ t = 3.3 microseconds

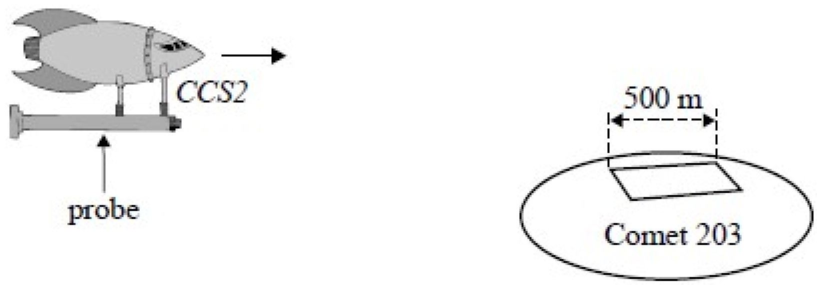
∴C (ANS), (61%)

Current study design:

2018 NHT Question 11

Worked example 4: Time dilation, length contraction: Proper length, length contraction.

Comet-chasing spacecraft *CCS2* travels at a speed for which γ = 1.5 relative to the nearest stars. It approaches Comet 203, which is effectively stationary relative to the nearest stars, as shown below. There is a landing probe attached to *CCS2*.



2015 Question 7, 2 marks

The designated landing area on the comet has length 500 m in the comet's frame and is parallel to spacecraft *CCS2*'s velocity.

What is the length of this landing area, as measured by instruments on *CCS2*?

A. 750 m

B. 500 m

C. 408 m

D. 333 m

Solution

The spacecraft is measuring the length of a relatively moving object, so it measures a contracted length.
The landing area is not moving in the frame of the comet, therefore it is the proper length.

Use $L = \frac{L_0}{\gamma}$ with γ = 1.5

$\therefore L = \frac{500}{1.5}$

$\therefore L = 333$

∴D (ANS), (74%)

Current study design:

2022 NHT Question 10

2022 NHT Question 11b

2021 Question 10a, (57%)

2021 Question 10b (63%)

2019 Question 13 (75%)

2019 NHT Question 18

2017 Question 10, (58%)

Worked example 5: Time dilation, length contraction: Proper length, length contraction.

2015 Question 8, 2 marks

Spacecraft *CCS2* releases a probe that will land on the comet. Near touchdown, the probe is at the same velocity as the comet.

Which one of the following is able to measure the proper length for the landing area?

A. the probe when it is travelling at the same velocity as the comet

B. *CCS2* because it has far more accurate radar instruments than the probe

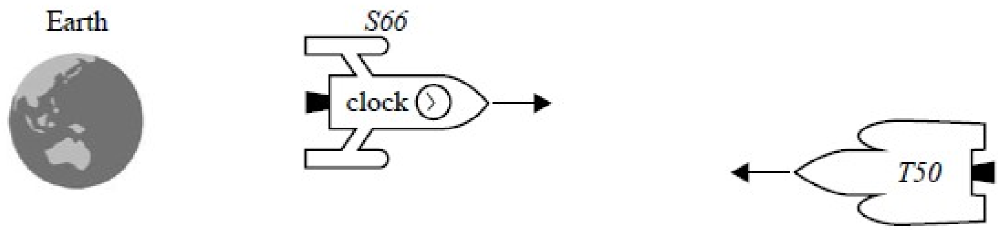
C. *CCS2* at the instant it passes by the landing area on the comet

D. a radar pulse from *CCS2* because the pulse will momentarily be stationary when it bounces off the landing area

Solution	**Current study design:**
The proper length is measured by an observer at rest relative to the object being measured.	**2022 NHT Question 10**
∴ A (ANS), (82%)	**2022 NHT Question 11b**
	2021 Question 10a, (57%)
	2021 Question 10b, (63%)
	2019 Question 13 (75%)
	2019 NHT Question 18
	2017 Question 10 (58%)

Worked example 6: Time dilation, length contraction: Proper time, time dilation.

Spacecraft *S66* is travelling at high speed away from Earth carrying a highly accurate atomic clock. Another spacecraft, *T50*, is travelling in the opposite direction to *S66*, as shown below.



2015 Question 4, 2 marks

Which one of the following observers will be able to measure proper time using this clock?

A. an astronaut seated on spacecraft *S66* five metres behind the clock's position.

B. a scientist on Earth at the clock's original position.

C. no observer can measure proper time since light within the clock moves at the speed of light.

D. the navigator of the other spacecraft, *T50*, travelling at the moment when that navigator is opposite the clock.

Solution

To measure proper time using this clock the two events need to occur at the same location as the clock. This can only occur in the spaceship.

∴A (ANS), (71%)

Current study design:

2022 NHT Question 11a

2021 NHT Question 13

2020 Question 11b (44%)

2018 Question 16 (55%)

2018 NHT Question 14

Worked example 7: Time dilation, length contraction: Proper time, time dilation.

2015 Question 5, 2 marks

An observer, *E*, on Earth emits a short radio pulse to spacecraft *S66*, which reflects it directly back towards the observer. The time elapsed for *E* between sending and receiving the pulse is 20.0 ms.

Which one of the following is true?

A. According to *E*, spacecraft *S66* was more than 3000 km away when the pulse reached it.

B. According to *E*, the pulse took longer to reach spacecraft *S66* than it did to return from spacecraft *S66* to *E*.

C. The 20.0 ms interval measured by *E* is not a proper time because the radio pulse travelled away and back.

D. According to spacecraft *S66*, the time interval between the signal being sent and being received back by *E* is greater than 20.0 ms.

Solution	**Current study design:**
In the observer's, E, frame of reference the time interval 20 ms, is the proper time, t_0. The spacecraft, S66, measure the time interval of something in a relatively moving frame, so it is measuring the dilated time, *t*.	**2022 NHT Question 11a** **2021 NHT Question 13** **2020 Question 11b (44%)** **2018 Question 16 (55%)** **2018 NHT Question 14**
Therefore, S66 will measure a longer time. ∴ D (ANS), (52%)	

Worked example 8: Time dilation, length contraction: Lorentz factor.

An electron with a Lorentz factor of 4 travels in a straight line a distance of 600 m as measured in the laboratory frame of reference.

2008 Question 11, 2 marks

Which one of the following best gives the speed of the electron?

A. 0.25 c

B. 0.94 c

C. 0.97 c

D. 0.99 c

Solution	Current study design:
Use $\beta = \sqrt{1 - \frac{1}{\gamma^2}}$ $\therefore \beta = \sqrt{1 - \frac{1}{4^2}}$ $\therefore \beta = 0.9682$ $\therefore v = 0.968\ c$ ∴ C (ANS), (67%)	**2022 Question 19 (85%)** **2021 NHT Question 10a** **2020 Question 12 (82%)** **2019 NHT Question 16** **2018 Question 13 (58%)** **2018 NHT Question 10**

Worked example 9: Time dilation, length contraction: Light years.

2019 NHT Question 17, 3 marks

A spaceship is travelling from Earth to the star system Epsilon Eridani, which is located 10.5 light-years from Earth as measured by Earth-based instruments.

If the spaceship travels at 0.85*c* (γ = 1.90), determine the duration of the flight as measured by the astronauts on the spaceship travelling to Epsilon Eridani. Take one light-year to be 9.46×10^{15} m. Show your working. (Give your answer in years.)

Solution	Current study design:
From the astronaut's frame of reference, the distance travelled is given by $\frac{10.5 \times 9.46 \times 10^{15}}{1.9}$ $\therefore d = 5.305 \times 10^{16}$ m. In the astronaut's frame, the time taken to travel this distance is given by $t = \frac{d}{v}$ $\therefore t = \frac{5.305 \times 10^{16}}{0.85 \times 3 \times 10^8}$ $\therefore t = 2.050 \times 10^8$ s. $\therefore$ t = 6.5 years (ANS)	**2021 NHT Question 10b** **2020 Question 11a (29%)** **2019 NHT Question 17**

Worked example 10: Particles: Lifetime in rest frame.

Pions are particles that are present in cosmic rays striking Earth. Pions decay, with a half-life of 26 ns.

The half-life is the time taken for half of a large number of pions to decay.

2016 Question 5, 2 marks

In which frame of reference will the undilated value of the half-life be correctly observed?

A. in the frame of the high-energy source of each pion

B. in each pion's own frame

C. in any inertial frame

D. in Earth's frame

Solution	**Current study design:**
The proper time (undilated) is the time the clock measures in its own reference frame. The proper time is measured in the frame of reference where the pion is at rest. ∴ B (ANS), (69%)	**2019 NHT Question 17** **2017 Question 11a (44%)**

Worked example 11: Particles: Lifetime in moving frame.

2016 Question 6, 2 marks

Consider one pion approaching Earth at a speed of 0.98*c*. It decays 26 ns in its own frame of reference after it is formed.

How long did the pion exist as observed in Earth's frame of reference?

A. 5.2 ns

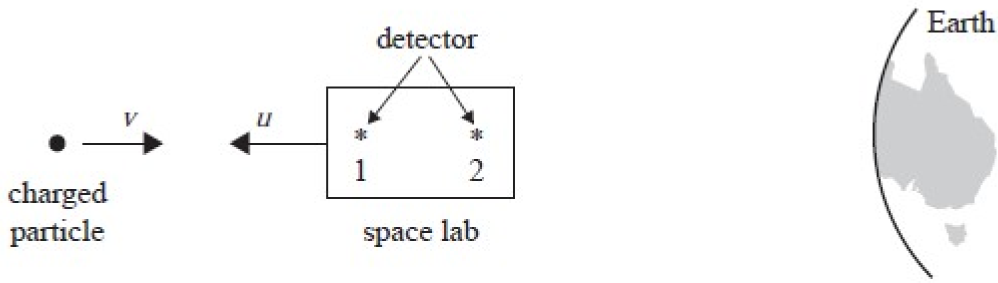
B. 26 ns

C. 130 ns

D. 650 ns

Solution	**Current study design:**
The proper time is in the reference frame of the pion. In the Earth's reference frame measured time will be dilated by γ. Where $\gamma = \frac{1}{\sqrt{1 - \frac{(0.98c)^2}{c^2}}}$ ∴ γ = 5.025 Therefore the measured half life in the Earth's frame of reference is $26 \times 10^{-9} \times 5.025$ = 131 ns ∴ C (ANS), (71%)	**2017 Question 11c (44%)**

Worked example 12: Particles: Distance measured in moving frame.



A space lab travelling at u = 0.8c (γ = 1.67) away from Earth can record high-energy charged particles passing through its detectors. One particle is travelling towards Earth at v = 0.91c (γ = 2.4) relative to the space lab.

Two detectors, numbered 1 and 2 in the figure above, are 2.0 m apart in the space lab's frame.

2016 Question 7, 2 marks

How far apart are the two detectors in this particular particle's frame?

A. 0.83 m

B. 1.2 m

C. 3.3 m

D. 4.8 m

Solution

The proper length of the distance between the two detectors is given as L_0 = 2.0 m. In the particles' frame the detector is in relative motion, so they're measuring the length of a relatively moving object, so they're measuring the contracted length, L. The Lorentz factor has been given, γ = 2.4.

Use L = $\frac{L_0}{\gamma}$

∴ γ = 2.4

∴ L = $\frac{2.0}{2.4}$

∴ L = 0.833

∴ A (ANS), (58%)

Current study design:

2022 Question 11 (41%)

2017 Question 11b (39%)

Worked example 13: Energy: $E = \Delta mc^2$.

A pion and its antiparticle, each at rest, annihilate to produce two photons whose total energy is 4.5×10^{-11} J. Apart from the two photons, nothing else is produced in this process. The masses of a pion and its antiparticle are the same.

2014 Question 10, 2 marks

The rest mass of the pion is

A. 1.3×10^{-28} kg

B. 2.5×10^{-28} kg

C. 5.0×10^{-28} kg

D. 7.5×10^{-20} kg

Solution

$E_{rest} = mc^2$, so the mass, m, converted to other forms of energy (the two photons) is given by

$4.5 \times 10^{-13} = m \times (3.0 \times 10^8)^2$

$\therefore m = 5.0 \times 10^{-28}$

$\therefore$ mass of pion = 2.5×10^{-28} kg

$\therefore$ B (ANS), (52%)

Current study design:

2022 Question 9 (47%)

2022 NHT Question 11

2021 NHT Question 20

2019 NHT Question 19

2018 NHT Question 15

2017 Question 11 (66%)

Worked example 14: Energy: $E_k = (\gamma - 1)mc^2$.

2016 Question 9, 2 marks

When a proton is accelerated from rest, it gains a kinetic energy of 1.20×10^{-10} J.

What value of γ is reached? (The rest mass of a proton is 1.67×10^{-27} kg.)

A. 2.2

B. 1.8

C. 1.5

D. 1.3

Solution

Use $E = (\gamma - 1)mc^2$

$1.2 \times 10^{-10} = (\gamma - 1) \times 1.67 \times 10^{-27} \times (3.0 \times 10^8)^2$

$$\therefore (\gamma - 1) = \frac{1.2 \times 10^{-10}}{1.67 \times 10^{-27} \times (3.0 \times 10^8)^2}$$

$$\therefore (\gamma - 1) = \frac{1.2 \times 10^{-10}}{1.503 \times 10^{-10}}$$

$\therefore (\gamma - 1) = 0.798$

$\therefore \gamma = 1.8$

$\therefore$ B (ANS), (61%)

Current study design:

2019 NHT Question 18

2018 Question 14 (60%)

2018 Question 15 (43%)

Worked example 15: Energy: Power.

2007 Question 10, 2 marks

The Sun produces energy by fusing light elements such as helium into heavier elements. In this fusion process mass is lost. The Sun radiates energy at a rate of 4.0×10^{26} W.

How much mass does the Sun lose each second?

Solution

Use power = $\frac{\Delta E}{s}$

Use E (J) = Δmc^2

$\therefore 4.0 \times 10^{26} = \Delta m \times (3.0 \times 10^8)^2$

$\therefore \Delta m = 4.4 \times 10^9$ kg per sec.

$\therefore \Delta m = 4.4 \times 10^9$ kg (ANS), (43%)

Current study design:

2020 Question 13 (56%)