SPECIALIST MATHEMATICS

Unit 4 SAC 2

Problem solving task

The berry farm

Student Name……..

Student Number……..

This task is divided into three parts to be completed over a period of 1 week in 2–3 hours. The SAC assesses the 3 outcomes, with the marking allocation as outlined below.

Marking allocation

Outcomes	**Marks allocated**
Outcome 1	7
Outcome 2	10
Outcome 3	8
TOTAL MARKS	**25**

Berry farm

Contents covered: linear combination of random variables, and hypothesis testing.

Throughout this SAC, leave your answer correct to four decimal places where appropriate.

William bought a berry farm and started a new family business. As a new owner, William tries to familiarise himself with the fruit production and sale process. He applies Mathematics knowledge to estimate future production levels from historical statistics. Analysing the fruit samples also helps him to better control the quality of each variety of berries.

Problem One

The workers at the farm pack blueberries in '250 g' boxes for sale. Suppose it is known that the standard deviation of the weight for each box is 3 g. For quality control, William samples some boxed blueberries and weights them.

a. Find the probability that a randomly selected box of blueberry has a weight of more than 253 g.

b. State the mean and standard deviation of the distribution of the combined weight of six randomly selected boxes of blueberries.

c. Hence, find the probability the combined weight of these six boxes is greater than 1518 g.

d. State the mean and standard deviation of the distribution of the average weight of six randomly selected boxes of blueberries.

e. Hence or otherwise, find the probability that the average weight of these six boxes is greater than 253 g.

f. Compare and comment on the answers you found in part **a** and part **e**.

Different varieties of blueberry have different levels of sweetness, measured by the amount of sugar per serving of fruits. William's farm produces two kinds of blueberries. The rabbit eye blueberries have a mean sugar level of 12.8 g, with a standard deviation of 1.2 g. The southern highbush blueberries have a mean sugar level of 13.5 g, with a standard deviation of 1.5 g.

g. Find the probability that a randomly selected serving of rabbit eye blueberries has a higher sugar level than a randomly selected serving of southern highbush blueberries.

h. A sample of six servings of rabbit eye and southern highbush blueberries are randomly selected. Based on their mean sugar levels, find the probability that the southern highbush variety is sweeter than the rabbit eye blueberries.

William would like to introduce a new variety of blueberries, the northern highbush, into his farm. He researches this variety and finds that the time it takes to become fully ripe depends on the number of days, t, elapsed since 1st January each year. The relationship can be approximately modelled by the probability density function

$$f(t) = \begin{cases} \frac{-3}{4000}(t^2 - 8t - 84), & -6 \leq t \leq 14 \\ 0, & \text{otherwise} \end{cases}$$

i. Find the average number of days elapsed since 1st January that the northern highbush blueberries take to become fully ripe.

j. Find the probability that 25 randomly selected northern highbush blueberries samples all become fully ripe by 5th January.

k. State any assumptions you made in part **j**, and explain the reasoning for this assumption.

Problem Two

William renovates the berry farm with a more efficient watering system. He expects this would help to increase the size of blueberries produced. Historically, the diameter of blueberries was normally distributed with a mean of 25 mm and variance 1 mm. William takes a random sample of 16 blueberries produced using the new water system. Their diameters are measured and have a mean of 25.7 mm.

Let the random variable $\bar{X}$ denote the average diameter of blueberries sampled.

a. Write down suitable null and alternative hypotheses to test whether this sample provides evidence that the size of blueberries increased.

b. Find the p-value for this test.

c. State the conclusion of this statistical test at a level of significance of 0.05, explain your reasoning.

d. If William takes another random sample of 16 blueberries, could his conclusion be different? Explain what William needs to do as your reasoning.

Instead of calculating the p-value for each sample of blueberries, William wants to use a more sufficient method to test the same hypotheses. The following calculation would help with his tests.

e. Find the value of k such that $\Pr(\bar{X} > k) = 0.05$.

f. State the criteria to reject the null hypothesis at a level of significance of 0.05. Explain your reasoning.

g. What is the probability of a Type I error for this test?

h. Use the value of k found in part **e** to calculate $\Pr(\bar{X} < k | \mu = 25.5)$. Assume the variance remains the same.

i. Interpret the probability found above in part **h**.

William takes another random sample of 16 blueberries and finds their mean diameter is 24.6 mm. He now suspects the new watering system might help increase the size of some blueberries but decrease the size of others due to lack of water.

j. Using the new sample mean, conduct a statistical test at a level of significance of 0.05 to determine whether William's suspicion can be supported. Make sure you clearly state:

- The null and alternative hypotheses
- Distribution of the test statistics
- The p–value of the test
- Your conclusion and reasoning

k. Find an alternative approach that would lead to the same conclusion you found in part **j**. Explain your reasoning.