Chapter 1 – Review of percentages and ratios

Solutions to 1A Now Try This Questions

1 $\frac{14}{70} : \frac{14}{70} \times 100\ \%$

$= \frac{140}{7} = 20\ \%$

2 $0.25 : 0.25 \times 100\ \%$

$= 25\ \%$

3 $78\% = \frac{78}{100}$

$= \frac{78 \div 2}{100 \div 2}$

$= \frac{39}{50}$

4 $45\% = \frac{45}{100} = 0.45$

5 $30\% \text{ of } \$90 = \frac{30}{100} \times 90$

$= \$27$

6 $\frac{10}{25} \times 100\%$

$= 40\%$

7 $60 \text{ cm} = 60 \times 10 \text{ mm}$

$= 600\text{mm}$

$\frac{18}{600} \times 100\% = 3\%$

Solutions to Exercise 1A

1 **a** $17\% = \frac{17}{100}$

b $94\% = \frac{94}{100}$

c $71\% = \frac{71}{100}$

2 **a** $7\% = \frac{7}{100}$

b $13\% = \frac{13}{100}$

c $50\% = \frac{50}{100}$

$= \frac{1}{2}$

d $10\% = \frac{10}{100}$

$= \frac{1}{10}$

e $20\% = \frac{20}{100}$

$= \frac{1}{5}$

3 **a** $\frac{11}{100} = 11\ \%$

b $\frac{23}{100} = 23\ \%$

c $\frac{79}{100} = 79\ \%$

4 **a** $\frac{1}{2} : \frac{1}{2} \times 100\ \%$

$= 50\%$

b $\frac{2}{5} : \frac{2}{5} \times 100\ \%$

$= 40\%$

c $\frac{1}{4} : \frac{1}{4} \times 100\ \%$

$= 25\%$

5 **a** $0.78 : 0.78 \times 100\ \%$

$= 78\%$

b $0.37 : 0.37 \times 100\ \%$

$= 37\%$

c $0.561 : 0.561 \times 100\ \%$

$= 56.1\%$

6 **a** $\frac{1}{4} = \frac{1}{4} \times 100$

$= 25\ \%$

b $\frac{4}{5} = \frac{4}{5} \times 100$

$= 80\ \%$

c $\frac{3}{20} = \frac{3}{20} \times 100$

$= 15\ \%$

d $\frac{7}{10} = \frac{7}{10} \times 100$

$= 70\ \%$

e 0.19
$= 0.19 \times 100$
$= 19\ \%$

f 0.79
$= 0.79 \times 100$
$= 79\ \%$

g 2.15
$= 2.15 \times 100$
$= 215\ \%$

h 39.57
$= 39.57 \times 100$
$= 3957\ \%$

i 0.073
$= 0.073 \times 100$
$= 7.3\ \%$

j 1
$= 1 \times 100$
$= 100\ \%$

7 **a** 25%

i $\frac{25}{100} = \frac{1}{4}$

ii $25\% = \frac{25}{100} = 0.25$

b 50%

i $\frac{50}{100} = \frac{1}{2}$

ii $50\% = \frac{50}{100} = 0.5$

c 75%

i $\frac{75}{100} = \frac{3}{4}$

ii $75\% = \frac{75}{100} = 0.75$

d 68%

i $\frac{68}{100} = \frac{34}{50} = \frac{17}{25}$

ii $68\% = \frac{68}{100} = 0.68$

e 5.75%

i $\frac{5.75}{100} = \frac{575}{10000} = \frac{23}{400}$

ii $5.75\% = \frac{5.75}{100} = 0.0575$

f 27.2%

i $\frac{27.2}{100} = \frac{272}{1000} = \frac{34}{125}$

ii $27.2\% = \frac{27.2}{100} = 0.272$

g 0.45%

i $\frac{0.45}{100} = \frac{45}{10000} = \frac{9}{2000}$

’i

ii $0.45\% = \frac{0.45}{100} = 0.0045$

h 0.03%

i $\frac{0.03}{100} = \frac{3}{10000}$

ii $0.03\% = \frac{0.03}{100} = 0.0003$

i 0.0065%

i $\frac{0.0065}{100} = \frac{65}{1\,000\,000} =$

$\frac{65}{1\,000\,000} = \frac{13}{200000}$

ii $0.0065\% = \frac{0.0065}{100}$

$= 0.000\,065$

j 100%

i $\frac{100}{100} = 1$

ii $100\% = \frac{100}{100} = 1$

8 **a** 15% of \$760

$= \frac{15}{100} \times 760$

= \$114

b 22% of \$500

$= \frac{22}{100} \times 500$

= \$110

c 17% of 150 m

$= \frac{17}{100} \times 150$

= 25.5 m

d $13\frac{1}{2}$% of \$10 000

$= \frac{13.5}{100} \times 10\,000$

= \$1350

e 2% of 79.34 cm

$= \frac{2}{100} \times 79.34$

= 1.5868

= 1.59 cm, correct to 3 sig. figs.

f 19.6% of 13.46

$= \frac{19.6}{100} \times 13.46$

= 2.63816

= 2.64 correct to 3 sig figs.

g 0.46% of 35 €

$= \frac{0.46}{100} \times 35$

= 0.161 €

h 15.9% of \$28 740

$= \frac{15.9}{100} \times 28\,740$

= 4569.66

= \$4 570, correct to 3 sig. figs.

i 22.4% of \$346 900

$= \frac{22.4}{100} \times 346\,900$

= 77 705.6

= \$77 700, correct to 3 sig figs.

j 1.98% of \$1 000 000

$= \frac{1.98}{100} \times 1\,000\,000$

= \$19 800

9 $\frac{28}{35} \times 100 = 80\%$

10 $\frac{450}{1200} \times 100 = 37.5\%$

11 16 were defective, so there were (360 – 16) = 344 that were satisfactory.

$$\frac{344}{360} \times 100 = 95.55\ldots\%$$
$$= 95.6\%$$

12 $\frac{300}{360} \times 100 = 83.33\%$

13 $\frac{125}{624} \times 100 = 20.03\ldots$
$$= 20\%$$

14 2 m = 2 × 100 cm
$\frac{75}{200} \times 100 = 37.5\%$

15 $\frac{2\,115\,000}{3\,250\,000} \times 100 = 65.076\ldots\%$
$$= 65.08\%$$

16 Profit made is \$1.50 – 0.60 = \$ 0.90
$\frac{0.90}{0.60} \times 100 = 150\ \%$

17 5% of 160 = $\frac{5}{100} \times 160 = 8$
Tim had 160 + 8 = 168 letters.

18 20% of 70 = $\frac{20}{100} \times 70 = 14$
Martin had 70 − 14 = 56 marbles.

19 Let x be the number of balloons needed.
30% of $x = 15$
$\frac{30}{100} \times x = 15$
$\frac{3x}{10} = 15$
$3x = 150$
$x = 50$

Solutions to 1B Now Try This Questions

8 Method 1 10% of 950

$= \frac{10}{100} \times 950$

$= 95$

$950 + 95$

$= 1045$

Weekly wage $= \$1045$

Method 2 An increase of 10%

$= (100 + 10)\% = 110\%$

110 % of \$950

$= \$1045$

9 Method 1 8 % of 12 km

$= \frac{8}{100} \times 12$

$= 0.96$

$12 - 0.96 = 11.04$ km

Method 2 A decrease of 8%

$= (100 - 8)\% = 92\%$

92% of 12

$= \frac{92}{100} \times 12$

$= 11.04$

The new distance is 11.04 km

10 Method 1 20 % of \$150

$= \$30$

$\$150 - \$30 = \$120$

Method 2 A discount of 20%

$= (100 - 20)\% = 80\%$

80% of \$150

$= \frac{80}{100} \times 150$

$= \$120$

11 Percentage increase $= \frac{\text{increase}}{\text{original}} \times 100$

$= \frac{70}{365} \times 100$

$= 19.17808...$

$= 19.18\%$ to two decimal places

12

Percentage decrease $= \frac{\text{decrease}}{\text{original}} \times 100$

(decrease $= \$599 - \$529 = \$70$)

$= \frac{70}{599} \times 100$

$= 11.68614.....$

$= 12$ % to the nearest whole number

Solutions to Exercise 1B

1 **a** 20% of \$190

$= \frac{20}{100} \times 190$

$= \$38$

b \$190-\$38 = \$152

2 **a** 10% of \$30

$= \frac{10}{100} \times 30$

$= \$3$

b \$30+\$3 = \$33

3 **a** 5% of \$25

$= \frac{5}{100} \times 25$

=$1.25

b $25-$1.25 = $23.75

4 a 20% of $185

$= \frac{20}{100} \times 185$

= $37

b Sale price = $185 – $37

= $148

5 a 5% of $89.99

$= \frac{5}{100} \times 89.99$

= 4.4995

= $4.50

Sale price = $89.99 – $4.50

= $85.49

b 10% of $189

$= \frac{10}{100} \times 189$

= $18.90

Sale price = $189 – $18.90

= $170.10

c 15% of $499.00

$= \frac{15}{100} \times 499$

= $74.85

Sale price = $499 – $74.85

= $424.15

d 20% of $249.00

$= \frac{20}{100} \times 249$

= $49.80

Sale price = $249 – $49.80

= $199.20

e 22.5% of $79.95

$= \frac{10}{100} \times 79.95$

= 17.988..

= $17.99

Sale price = $79.95 – $17.99

= $61.96

f 25% of $22.95

$= \frac{25}{100} \times 22.95$

= 5.7373

= $5.74

Sale price = $22.95 – $5.7

= $17.21

g 27.5% of $600

$= \frac{27.5}{100} \times 600$

= 165.0

= $165

Sale price = $600 – $165

= $435

h 30% of $63.50

$= \frac{30}{100} \times 63.50$

= $19.05

Sale price = $63.50 – $19.05

= $44.45

i 33% of $1000

$= \frac{33}{100} \times 1000$

= $330

Sale price = $1000 – $330

= $670

6 a 12.5% of $1200

$= \frac{12.5}{100} \times 1200$

= $150

New price = $1200 – $150

= $1050

Alternative method:

100 – 12.5 = 87.5

87.5% of $1200

$= \frac{87.5}{100} \times 1200$

= \$1050

b 12.5% discount on \$1400

100 – 12.5 = 87.5

87.5% of \$1400

$= \frac{87.5}{100} \times 1400$

= 1225.0

= \$1225

Alternative method:

12.5% of \$1400

$= \frac{12.5}{100} \times 1400$

= 175

New price = 1400 – 175

= 1225

= \$1225

c 12.5% discount on \$246

100 – 12.5 = 87.5

87.5% of \$246

$= \frac{87.5}{100} \times 246$

= \$215.25

Alternative method:

12.5% of 246

$= \frac{12.5}{100} \times 246$

= \$30.75

New price = \$246 – \$30.75

= \$215.25

d 12.5% discount on \$750

100 – 12.5 = 87.5

87.5% of \$750

$= \frac{87.5}{100} \times 750$

= \$656.25

Alternative method:

12.5% of 750

$= \frac{12.5}{100} \times 750$

= \$93.75

New price = \$750 – \$93.75

= \$656.25

7 \$95.95 + \$29.95 + 2 × \$45

= \$215.90

a Total saving = 6% of \$215.90

$= \frac{6}{100} \times 215.90$

= \$12.95

b Amount paid = \$215.90 – \$12.95

= \$202.95

8 6% increase means the new amount will be original amount (100%) plus 6%.

106% of 14 000

$= \frac{106}{100} \times 14\,000$

= 14 840

9 \$23 960 – \$18 700 = \$5260

$$\text{Percentage change} = \frac{\textit{change}}{\textit{original}} \times 100$$

$$= \frac{5260}{23960} \times 100$$

= 21.95%

10 12% more means 112%.

112% of 24 000

$= \frac{112}{100} \times 24\,000$

= 26 880

The new tyre should average 26 880 km.

11 **a** \$60 – \$52 = \$8

Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{8}{60} \times 100$

= 13.33

= 13%

b $250 – $185 = $65

Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{65}{250} \times 100$

= 26%

c $5000 – $4700 = $300

Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{300}{5000} \times 100$

= 6%

d $3.80 – $2.90 = $0.90

Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{.9}{3.8} \times 100$

= 23.68

= 24%

e $29.75 – $24.50 = $5.25

Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{5.25}{29.75} \times 100$

= 17.64…

= 18%

f $12.95 – $10 = $2.95

Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{2.95}{12.95} \times 100$

= 22.77…

= 23%

12 $13 990 – $13 000 = $990

Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{990}{13990} \times 100$

= 7.0764…

= 7.08%, correct to 2 dec. pl.

13 **a** $79.99 – $65 = $14.99

Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{14.99}{79.99} \times 100$

= 18.73…

= 19%

b $29.99 – $19.99 = $10

Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{10}{29.99} \times 100$

= 33.34…

= 33%

c $1099 – $599 = $500

Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{500}{1099} \times 100$

= 45.49 …

= 45%

d \$49.99 – \$39.99 = \$10
Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{10}{49.99} \times 100$

= 20.004…
= 20%

e \$14.95 – \$10 = \$4.95
Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{4.95}{14.95} \times 100$

= 33.11…
= 33%

f \$299 – \$250 = \$49
Percentage discount

$= \frac{discount}{original} \times 100$

$= \frac{49}{299} \times 100$

= 16.38…
= 16%

14 **a** \$20 increased to \$25 = \$5 increase
Percentage increase

$= \frac{increase}{original} \times 100$

$= \frac{5}{20} \times 100$

= 25%

b \$300 increased to \$420 = \$120 increase
Percentage increase

$= \frac{increase}{original} \times 100$

$= \frac{120}{300} \times 100$

= 40%

c \$540 increased to \$580.50 = \$40.50 increase
Percentage increase

$= \frac{increase}{original} \times 100$

$= \frac{40.50}{540} \times 100$

= 7.5%

15 For an increase of 10% :
110% of \$50

$= \frac{110}{100} \times 50$

= \$55
For a decrease of 8% :
92% of \$60

$= \frac{92}{100} \times 60$

= \$55.20
Decreasing \$60 by 8% results in larger sum of money.

18 1.92-1.86=0.06
Percentage increase $= \frac{0.06}{1.86} \times 100$
=3.225...
=3.23%

Solutions to 1C Now Try This Questions

13 $$\text{Amount of GST} = \frac{\text{cost without GST}}{10}$$
$$= \frac{850}{10}$$
$$= \$85$$

14 Method 1 10% of 1400
$$= \frac{10}{100} \times 1400$$
$$= 140$$
$$1400 + 140 = 1540$$
Price with GST is $1540

Method 2 $100 + 10 = 110\%$

110%of 1400
$$= \frac{110}{100} \times 1400$$
$$= 1540$$
Price with GST is $1540

15 $$\text{Original cost} = \frac{\text{cost with GST}}{1.1}$$
$$= \frac{737}{1.1}$$
$$= 670$$
The price without GST is $670.

16 $$\text{Amount of GST} = \frac{\text{cost with GST}}{11}$$
$$= \frac{1155}{11} = \$105$$

Solutions to Exercise 1C

1 **a** 10% of $900
$$= \frac{10}{100} \times 900$$
$$=\$90$$

b 10% of $760
$$= \frac{10}{100} \times 760$$
$$=\$76$$

c 10% of $599
$$= \frac{10}{100} \times 599$$
$$=\$59.90$$

d 10% of $65
$$= \frac{10}{100} \times 65$$
$$=\$6.50$$

e 10% of $2572
$$= \frac{10}{100} \times 2572$$
$$=\$257.20$$

f 10% of $48755
$$= \frac{10}{100} \times 48755$$
$$=\$4875.50$$

2 **a** 10% of $121.30
$$= \frac{10}{100} \times 121.30$$
$$=\$12.13$$

b 10% of \$367.50

$= \frac{10}{100} \times 367.50$

=\$36.75

c 10% of \$1085.50

$= \frac{10}{100} \times 1085.50$

=\$108.55

d 10% of \$395

$= \frac{10}{100} \times 395$

=\$39.50

3 a 10% of \$139

$= \frac{10}{100} \times 139$

=\$13.90

\$139 + \$13.90 = \$152.90

Alternatively: 110% of \$139

$= \frac{110}{100} \times 139$

=\$152.90

b 10% of \$139

$= \frac{10}{100} \times 2678$

=\$267.80

\$2678 + \$267.80 = \$2945.80

Alternatively: 110% of \$2678

$= \frac{110}{100} \times 2678$

=\$2945.80

c 10% of \$9850

$= \frac{10}{100} \times 9850$

=\$985

\$9850 + \$985 = \$10835

Alternatively: 110% of \$9850

$= \frac{110}{100} \times 9850$

=\$10 835

d 10% of \$1395

$= \frac{10}{100} \times 1395$

=\$139.50

\$1395 + \$139.50 = \$1534.50

Alternatively: 110% of \$1395

$= \frac{110}{100} \times 1395$

=\$1534.50

4 $\frac{2399}{1.1}$ = \$2180.91

5 $\frac{39990}{11}$ = \$3635.45

6 a $\frac{109.78}{1.1}$ = \$99.80

b \$109.78 – \$99.80 = \$9.98

7 a $\frac{57300}{11}$ = \$5209.09

b \$57300 – \$5209.09 = \$52090.91

8 $\frac{1599}{11}$ = \$145.36

9 10% of \$695 = \$69.50

\$695 + \$69.50 = \$764.50
The second mower costs \$764.50 which is 50 cents cheaper than the first mower advertised for \$765. He should buy the second mower and save 50 cents.

Solutions to 1D Now Try This Questions

18 8 prefer English

11 prefer Mathematics

1 prefers Computer Science

The ratio is 8 : 11 : 1

19 $14 : 49$

$= \frac{14}{7} : \frac{49}{7}$

$= 2 : 7$

20 52 mm : 8 cm

52 mm : 8 × 10 mm

= 52 mm : 80 mm

$= \frac{52}{4} : \frac{80}{4}$

$= 13 : 20$

21 $4 : 11 = x : 88$

$$\frac{4}{11} = \frac{x}{88}$$

$$\frac{4}{11} \times 88 = \frac{x}{88} \times 88$$

$$x = 32$$

Solutions to Exercise 1D

1 **a** wholemeal rolls : sourdough rolls
17 : 9

b sourdough rolls : wholemeal rolls
9 : 17

c wholemeal rolls : total number of rolls
17 : 26

2 **a** $25 : 75 = 25 \div 25 : 75 \div 25$
$= 1 : 3$

b $0.2 : 0.5 = 0.2 \times 10 : 0.5 \times 10$
$= 2 : 5$

c 32 mm : 5 cm
32 mm : 5 × 10mm
$= 32 : 50$
$= \frac{32}{2} : \frac{50}{2}$
$= 16 : 25$

3 35 : 15

4 **a** Whale to horse is 80 : 40

b Elephant to kangaroo is 70 : 9

c Whale to tortoise is 80 : 120

d Chimpanzee to mouse is 40 : 4

e Horse to mouse to whale is 40 : 4 : 80

5 **a** 12 : 15 (divide by 3)
= 4 : 5

b 10 : 45 (divide by 5)
= 2 : 9

c 22 : 55 : 33 (divide by 11)
= 2 : 5 : 3

d 1.3 : 3.9 (multiply by 10)
= 13 : 39 (divide by 13)

= 1 : 3

e 2.7 : 0.9 (multiply by 10)
= 27 : 9 (divide by 9)
= 3 : 1

f $\frac{5}{3} : \frac{1}{4}$ (multiply by 3)
= 5 : $\frac{3}{4}$ (multiply by 4)
= 20 : 3
Alternatively:
$\frac{5}{3} : \frac{1}{4}$ (multiply by 12)
= 20 : 3

g 18 : 8 (divide by 2)
= 9 : 4

6 a 60L : 25L (divide by 5)
12 : 5

b $2.50 : $50 (multiply by)
= 25 : 500 (divide by 25)
= 1 : 20

c 75 cm : 2 m
= 75 cm : 200 cm (divide by 25)
= 3 : 8

d 5 kg : 600 g
= 5 000 : 600 (divide by 100)
= 50 : 6
= 25 : 3 (divide by 2)

e 15 mm : 50 cm : 3 m
= 15 mm : 500 mm : 3000 mm
(divide by 5)
= 3 : 100 : 600

f 1 km : 1 m : 1 cm
= 100 000 cm : 100 cm : 1 cm
= 100 000 : 100 : 1

g 5.6 g : 91 g (multiply by 10)
= 56 : 910 (divide by 2)
= 28 : 455 (divide by 7)
= 4 : 65

h $30 : $6 : $1.20 : 0.60 (multiply by 10)
= 300 : 60 : 12 : 6 (divide by 6)
= 50 : 10 : 2 : 1

7 a $\frac{1}{4} = \frac{x}{20}$ (multiply both sides by 20)
(or use Solve on CAS calculator)
$x = 5$

b $\frac{15}{8} = \frac{135}{x}$
(use Solve on CAS calculator)
$x = 72$
Alternatively, invert both fractions
$\frac{8}{15} = \frac{x}{135}$
(multiply both sides by 135)
$x = 72$

c $\frac{600}{5} = \frac{x}{1}$
$x = 120$

d $\frac{2}{5} = \frac{2000}{x}$ (use Solve on CAS calculator)

$x = 5000$
Alternatively, invert both fractions
$\frac{5}{2} = \frac{x}{2000}$ (multiply by 2000)
$x = 5000$

e $\frac{3}{7} = \frac{x}{56}$ (multiply both sides by 56)
(or use Solve on CAS calculator)
$x = 24$

8 a The ratio 4 : 3 is the same as 3 : 4
FALSE
4 : 3 ≠ 3 : 4
Order is important in ratios.

b The ratio 3 : 4 is the same as 20 : 15
FALSE
3 : 4 ≠ 20 : 15
Order is important in ratios.
Correct statement is 3 : 4 = 15 : 20

c 9 : 45 = 1 : 5
TRUE

d Statement is FALSE
60 : 12 (divide by 4)
= 15 : 3 (divide by 3)
= 5 : 1
Correct statement is
60 : 12 = 15 : 3 = 5 : 1

e father's age : girl's age
= 7 : 1 (multiply by 8)
= 56 : 8
FALSE
Girl would be 8 years old.

f my allowance : friend's allowance
20 : 32 (divide by 32)

$= \frac{20}{32} : 1$

$= \frac{5}{8} : 1$

TRUE

9 **a** 100 : 60 : 175 : 125 : 125

b Divide by 5 which gives
20 : 12 : 35 : 25 : 25

c Since recipe makes 25 biscuits, you will need to multiply recipe by 3 to make 75 biscuits.
300 g rolled oats, 180 g coconut, 525 g plain flour, 375 g brown sugar, 375 g butter, 9 tbsp. boiling water, 6 tbsp. golden syrup, 3 tsp bicarb soda.

Solutions to 1E Now Try This Questions

22 $1 + 2 + 3 = 6$

$36 \div 6 = 6$

One part is $1 \times \$6 = \$6$

Two parts is $2 \times \$6 = \$12$

Three parts is $3 \times \$6 = \$18$

$\$6, \ \$12$ and $\$18$.

Solutions to Exercise 1E

1 **a** $1 : 1$
$1 + 1 = 2$ parts
$96 \div 2 = 48$
One part is 48
48, 48

b $2 : 1$
$2 + 1 = 3$ parts
$96 \div 3 = 32$
One part is 32.
Two parts is $2 \times 32 = 64$
64 and 32

c $5 : 3$
$5 + 3 = 8$ parts
$96 \div 8 = 12$
One part is 12
5 parts is $5 \times 12 = 60$
3 parts is $3 \times 12 = 36$
60 and 36

d $19 : 5$
$19 + 5 = 24$ parts
$96 \div 24 = 4$
One part is 4
19 parts is $19 \times 4 = 76$
5 parts is $5 \times 4 = 20$
76 and 20

2 **a** $4 : 1$
$4 + 1 = 5$ parts
$40 \div 5 = 8$
One part is 8 m
$4 \times 8 = 32$
Rope lengths will be 32 m and 8 m.

b $1 : 7$ $1 + 7 = 8$ parts
$40 \div 8 = 5$
One part is 5 m
$7 \times 5 = 35$
Rope lengths will be 5 m and 35 m.

c $6 : 2$
$6 + 2 = 8$ parts
$40 \div 8 = 5$
One part is 5 m
$6 \times 5 = 30$
$2 \times 5 = 10$
Rope lengths will be 30 m and 10 m.

d $4 : 4$
This simplifies to $1 : 1$
$1 + 1 = 2$ parts
$40 \div 2 = 20$
One part is 20 m
Rope lengths will be 20 m and 20 m.

3 **a** $6 : 4$ $6 + 4 = 10$ parts

$500 ÷ 10 = $50
One part is $50
6 × 50 = 300
4 × 50 = 200
One person would receive $300 and the other would receive $200.

b 1 : 4 : 5   1 + 4 + 5 = 10 parts
$500 ÷ 10 = $50
One part is $50
4 × 50 = 200
5 × 50 = 250
One person would receive $50, one would receive $200 and the other would receive $250.

c 1 : 8 : 1
1 + 8 + 1 = 10 parts
$500 ÷ 10 = $50
One part is $50
8 × 50 = 400
One person would receive $50, one would receive $400 and the other would receive $50.

d 8 : 9 : 8
8 + 9 + 8 = 25 parts
$500 ÷ 25 = $20
One part is $20
8 × 20 = 160
9 × 20 = 180
One person would receive $160, one would receive $180 and the other would receive $160.

e 10 : 5 : 4 : 1   10 + 5 + 4 + 1 = 20 parts
$500 ÷ 20 = $25
One part is $25
10 × 25 = 250
5 × 25 = 125
4 × 25 = 100
One person would receive $250, one would receive $125, one would receive $100 and the other would receive $25.

4 bananas : mangos : pineapples
= 10 : 1 : 4
Since there are 20 pineapples, multiply each number in the ratio by 5 (giving 20 pineapples).
= 50 : 5 : 20

a There will be 50 bananas.

b There will be 5 mangos.

c There will be 50 + 5 + 20 = 75 pieces of fruit.

5 1 : 4   1 + 4 = 5 parts
7.5 L ÷ 5 = 1.5 L
One part is 1.5
4 × 1.5 = 6
Cordial : water
= 1 : 4
= 1.5 : 6

a 1.5 litres of cordial is required.

b 6 litres of water is required.

6 1 : 20 000 (multiply by 15)
=15 : 300 000
Convert 300 000 cm to kilometres.
= 300 000 ÷ 100 m
= 3 000 m
= 3 000 ÷ 1000 km
= 3 km.

7 4:3:3
4+3+3=10 parts
3 600 000 ÷ 10 = 360 000
Daughter receives 4× $ 360 000
=$1 440 000
The sons receive 3× $360 000
=$1 080 000 each.

Solutions to 1F Now Try This Questions

23 $\$32 \div 8 = \$4$

$15 \times \$4 = \$60$

Solutions to Exercise 1F

1 a i $\$36 \div 12 = \$3$

ii $11 \times \$3 = \$33$

iii $\$18 \div \$3 = 6$
6 mangos

b i 14 oranges cost $12
Half of $12 = $6, so you can buy half as many oranges.
Half of 14 = 7 oranges.

ii $\$12 \times 3 = \$36$
$14 \times 3 = 42$ oranges.

iii $\$12 \times 4 = \$48$
$14 \times 4 = 56$ oranges.

2 a 12 cakes cost $14.40

1 cake costs $\dfrac{\$14.40}{12} = \$1.20$

$\therefore$ 13 cakes cost $\$1.20 \times 13 = \$15.60$

b Clock gains 20 seconds over 5 days

$\therefore$ Clock gains $\dfrac{20}{5} = 4$ seconds per day

$\therefore$ Clock gains $4 \times 21 = 84$ seconds in 3 weeks

c 17 books cost $501.50

$\therefore$ 1 book costs $\dfrac{\$501.50}{17} = \$29.50$

$\therefore$ 30 books cost $\$29.50 \times 30 = \$885$

d 4.5 km per 18 minutes

So, speed is $\dfrac{4.5\,\text{km}}{18\,\text{min}} = 0.25\dfrac{\text{km}}{\text{min}}$

Over 40 minutes the athlete travels:
$0.25 \times 40 = 10$ km

3 5 tins of orange paint requires:
1 tin of red paint
4 tins of yellow paint

Thus, 35 tins (= 5 × 7) of orange paint requires:
7 (= 1 × 7) tins of red paint
28 (= 4 × 7) tins of yellow paint

4 1 hour 50 minutes $= 1 + \dfrac{50}{60}$ hours

$= \dfrac{11}{6}$ hours

$$\frac{165\,\text{km}}{\frac{11}{6}\,\text{hours}} = 90\,\text{km/h}$$

a $90 \times 3 = 270$ km

b $90 \times 2\frac{1}{2} = 225$ km

c 20 minutes $= \dfrac{20}{60}$ hours $= \dfrac{1}{3}$ hours

$90 \times \dfrac{1}{3} = 30$ km

d 70 minutes $= \dfrac{70}{60}$ hours $= \dfrac{7}{6}$ hours

$90 \times \dfrac{7}{6} = 105$ km

e 3 hours 40 minutes $= 3 + \frac{40}{60}$ hours

$$= \frac{11}{3} \text{ hours}$$

$$90 \times \frac{11}{3} = 330 \text{ km}$$

f $90 \times \frac{3}{4} = 67.5$ km

5 For 35 g cone:

$$\frac{35\text{ g}}{\$1.25} = 28\text{ g per }\$1$$

For 73 g cone:

$$\frac{73\text{ g}}{\$2} = 36.5\text{ g per }\$1$$

The 73 g cone for $2 offers better value based on its weight to price ratio

6 Brand A : 2L costs $2.99
Brand B : 1L costs $1.95 so 2L cost
$2 \times \$1.95 = \$3.90$
Brand C : 600mL costs $1.42 so 1mL
costs $\$\frac{1.42}{600}$
Thus 2L of Brand C costs
$2000 \times \frac{1.42}{600} = \$4.73$
Thus Brand A is the best buy.

7 6 eggs for 2 chocolate cakes
Therefore 3 eggs for 1 chocolate cake.
For 17 chocolate cakes, you need
$17 \times 3 = 51$ eggs.

8 **a** 45 litres for 495 kilometres.
$\frac{45}{45}$ for $\frac{495}{45}$ kilometres which is
1 litre for 11 kilometres.
So 1×50 litres for11×50 kms gives
50 litres for 550 kilometres.

b 45 litres for 495 kilometres.
$\frac{45}{495}$ for $\frac{495}{495}$ kilometres which is
$\frac{45}{495}$ litres for 1 kilometre.
Multiply both sides by 187.
$\frac{45}{495} \times 187$ litres for 187 kms.
So 17 litres for 187 kms.

9 Let x be the amount.
110% of $x = 330$

$$\frac{110}{100} \times x = 330$$

$$\frac{110}{100} \times \frac{100}{110} \times x = 330 \times \frac{100}{110}$$

$x = \$300$

10 A$1 = US$0.72

$$\frac{\$1}{0.72} = US\frac{\$0.72}{0.72}$$

$$A\frac{\$1}{0.72} = US\$1$$

So US$180 is $A\frac{\$1}{0.72} \times 180$

US$180 is A$250

Solutions to Skills Checklist Questions

1 $\frac{3}{4} : \frac{3}{4} \times 100\%$

$= 75\%$

2 $0.67 : 0.67 \times 100\%$

$= 67\%$

3 $40\% = \frac{40}{100} = \frac{4}{10} = \frac{2}{5}$

4 $85\% = \frac{85}{100} = 0.85$

5 30% of $\$50 = \frac{30}{100} \times 50 = \$15$

6 $100\% - 20\% = 80\%$

80% of $\$150 = \frac{80}{100} \times 150$

$= \$120$

7 increase $= \$7 - \$4.50 = \$2.50$

$\%$ increase $= \frac{2.50}{4.50} \times 100$

$= 55.555... = 56\%$

8 Amount of GST $= \frac{\text{cost without GST}}{10}$

$= \frac{60000}{10}$

$= \$6000$

9 orignal cost $= \frac{\text{cost with GST}}{1.1}$

$= \frac{\$1098}{1.1}$

$= \$998.18$

10 Amount of GST $= \frac{\text{cost with GST}}{11}$

$= \frac{\$1595}{11}$

$= \$145$

11 Ten 16 years old

Twenty 17 years old

$10 : 20 = 1 : 2$

12 $64 : 96 = \frac{64}{2} : \frac{96}{2}$

$= \frac{32}{8} : \frac{48}{8}$

$= 4 : 6$

$= 2 : 3$

13 $1 + 3 + 12 = 16$ parts

$1024 \div 16 = 64$

One part is 64

Three parts is $3 \times 64 = 192$

Twelve parts is $12 \times 64 = 768$

So 64, 192 and 768

14 $\$2500 \div 50 = \$50$

The cost of one item is $50

The cost of 10 items is

$\$50 \times 10 = \$500$

Solutions to Chapter Review Multiple-Choice Questions

1 $4 + 7 \times 3$
$= 4 + 21$
$= 25$ **E**

2 $3 + (6 \div 3) - 2$
$= 3 + 2 - 2$
$= 5 - 2$
$= 3$ **A**

3 $(8.7 - 4.9) \times (5.4 + 2.8)$
$= 3.8 \times 8.2$
$= 31.16$ **B**

4 $(-3) \times 4 \times 5$
$= (-12) \times 5$
$= -60$ **C**

5 $(-2) + 8$
$= 6$ **B**

6 $(-2) - (-3)$
$= (-2) + 3$
$= 1$ **C**

7 $5 - (-9)$
$= 5 + 9$
$= 14$ **E**

8 3.895
= 3.895
Count 2 decimal places.
Next number is 5 so round up.
= 3.90 **D**

9 4679
Number after 6 is greater than 5 so round up.
= 4700 **C**

10 5.21×10^5
= 521 000
Decimal point moves 5 places to the right. **B**

11 0.0048
$= 4.8 \times 10^{-3}$ **D**

12 28 037.2
Count 2 significant figures from the left.
28 037.2
The next digit (0) is less than 5, so the 8 remains unchanged.
= 28 000 **A**

13 0.03069
Count 2 significant figures from the left.
0.03069
The next digit is 6 which is greater than 5 so round up.
= 0.031 **D**

14 5.1 m^2
$= 5.1 \times 100^2 \text{ cm}^2$
$= 51\,000 \text{ cm}^2$ **C**

15 56%
$= \frac{56}{100}$
$= \frac{28}{50}$
$= \frac{14}{25}$ **E**

16 15% of \$1600
$= \frac{15}{100} \times 1600$
= \$240 **C**

17 30% markup
130% of \$450
$= \frac{130}{100} \times 450$
= \$585 **A**

18 5 green, 7 blue, 3 yellow
Total = 5 + 7 + 3 = 15
blue : total

= 7 : 15 **C**

19 1 : 3 : 2

1 + 3 + 2 = 6 parts

\$750 ÷ 6 = \$125

1 part is \$125

3 parts is 3 × 125 = \$375

2 parts is 2 × 125 = \$250

\$125 is smallest share. **B**

20 450g : 3 kg

= 450g : 3000 g

= 45 : 300

= 9 : 60

= 3 : 20 **A**

Solutions to Chapter Review Short-Answer Questions

1 **a** $3 + 2 \times 4$

$= 3 + 8$

$= 11$

b $25 \div (10 - 5) + 5$

$= 25 \div 5 + 5$

$= 5 + 5$

$= 10$

c $14 - 21 \div 3$

$= 14 - 7$

$= 7$

d $(12 + 12) \div 12 + 12$

$= 24 \div 12 + 12$

$= 2 + 12$

$= 14$

e $27 \div 3 \times 5 + 4$

$= 9 \times 5 + 4$

$= 45 + 4$

$= 49$

f $4 \times (-2) + 3$

$= -8 + 3$

$= -5$

g $\frac{10 - 8}{2}$

$= \frac{2}{2}$

$= 1$

h $\frac{4(3 + 12)}{2}$

$= \frac{4(15)}{2}$

$= \frac{60}{2}$

$= 30$

i $\frac{-5 + 9}{2}$

$= \frac{4}{2}$

$= 2$

2 **a** $5^3 = 125$

b $\sqrt{64} - 5$

$= 8 - 5$

$= 3$

c $9^{\frac{1}{2}} + 9^{\frac{1}{2}}$

$= 3 + 3$

$= 6$

d $\sqrt{8} = 2.8284\ldots$

$= 2.83$

e $\sqrt{25 - 9}$

$= \sqrt{16}$

$= 4$

f $\sqrt{25} - 9$

$= 5 - 9$

$= -4$

g $\dfrac{6^3}{(10 \div 2)^2}$

$= \dfrac{216}{5^2}$

$= 8.64$

h $\sqrt{(6^2 + 10^2)}$

$= \sqrt{36 + 100}$

$= \sqrt{136}$

$= 11.6619..$

$= 11.66$

3 **a** $2945 = 2.945 \times 10^3$

b $0.057 = 5.7 \times 10^{-2}$

c $369\,000 = 3.69 \times 10^5$

d $850.9 = 8.509 \times 10^2$

4 **a** $7.5 \times 10^3 = 7500$

(Decimal place moves 3 places to the right.)

b $1.07 \times 10^{-3} = 0.001\,07$

(Decimal place moves 3 places to the left)

c $4.56 \times 10^{-1} = 0.456$

(Decimal place moves 1 place to the left)

5 **a** 8.916 (2)

Count 2 significant figures from the left.

8.916

The next number (1) is less than 5 so 9 remains unchanged.

Answer is 8.9

b 0.0589 (2)

Count 2 sig. figs. from the left.

0.0**58**9

The next number (9) is greater than 5 so round up.

Answer is 0.059

c **8**09 (1)

Count 1 sig. fig from the left.

809

The next number (0) is less than 5 so 8 remains unchanged.

Answer is 800.

6 **a** 7.145 (2)

7.**14**5

Since next number is 5, round up.

Answer is 7.15

b 598.241 (1)

598.**2**41

The next number (4) is less than 5, so 2 remains unchanged.

Answer is 598.2

c 4.0789 (3)

4.**078**9

The next number (9) is greater than 5, so round up.

Answer is 4.079

7 **a** 7.07 cm (mm)

$= 7.07 \times 10$

$= 70.7$ mm

b 2170 m (km)

$= 2170 \div 1000$

$= 2.170$ km

c 0.1 m^2 (cm^2)

$= 0.1 \times 100^2$

$= 1000$ cm^2

d 2.5 km^2 (m^2)

$= 2.5 \times 1000^2$
$= 2\,500\,000 \text{ m}^2$

e $0.000\,5 \text{ m}^2(\text{cm}^2)$
$= 0.000\,5 \times 100^2$
$= 5 \text{ cm}^2$

f $0.000\,53 \text{ cm}^3\ (\text{mm}^3)$
$= 0.000\,53 \times 10^3$
$= 0.53 \text{ mm}^3$

g 5.8 kg (mg)
$= 5.8 \times 1000 \times 1000$
= 5 800 000 mg

h 0.07 L (mL)
$= 0.07 \times 1000$
= 70 mL

8 **a** 75%
$= \frac{75}{100}$
= 0.75

b 40%
$= \frac{40}{100}$
= 0.4

c 27.5%
$= \frac{27.5}{100}$
= 0.275

9 **a** 10%
$= \frac{10}{100}$
$= \frac{1}{10}$

b 20%
$= \frac{20}{100}$
$= \frac{1}{5}$

c 22%
$= \frac{22}{100}$
$= \frac{11}{50}$

10 **a** 30% of 80
$= \frac{30}{100} \times 80$
= 24

b 15% of \$70
$= \frac{15}{100} \times 70$
= \$10.50

c $12\frac{1}{2}$ % of \$106
$= \frac{12.5}{100} \times 106$
= \$13.25

11 **a** 5% of \$1038
$= \frac{5}{100} \times 1038$
= \$51.90

b Sale price = \$1038 – \$51.90
= \$986.10

12 Increase of 15% means (100 + 15) = 115% of original wage.
115% of \$750
$= \frac{115}{100} \times 750$
= \$862.50

13 percentage increase $= \frac{\textit{change}}{\textit{original}} \times 100$
Change = \$15 – \$12.50 = \$2.50
% increase $= \frac{2.50}{12.50} \quad \times 100$
= 20%

14 \$516 – \$278 = \$238

$\%\text{ discount} = \dfrac{\textit{change}}{\textit{original}} \times 100$

$= \dfrac{238}{516} \times 100$

= 46.12. . .

≈ 46%

15 Melissa

$\%\text{ loss} = \dfrac{4}{78} \times 100$

= 5.128. . .

≈ 5.13%

Jody

$\%\text{ loss} = \dfrac{3}{68} \times 100$

= 4.411. . .

≈ 4.41%

16 **a** False. 3 : 2 is NOT the same as 2 : 3

Order in ratios is important,

b False. 1 : 5 ≠ 3 : 12

c False. To compare ratios, they must have the same units.

d True. 3 : 4 = 9 : 12

17 **a** 4 : 6

There are 4 + 6 = 10 parts

\$800 ÷ 10 = \$80

One part is \$80

4 parts is 4 × \$80 = \$320

6 parts is 6 × \$80 = \$480

One person receives \$320 and the other receives \$480.

b 1 : 4

There are 1 + 4 = 5 parts

\$800 ÷ 5 = \$160

One part is \$160

4 parts is 4 × \$160 = \$640

One person receives \$160 and the other receives \$640.

c 2 : 3 : 5

There are 2 + 3 + 5 = 10 parts

\$800 ÷ 10 = \$80

One part is \$80

2 parts is 2 × \$80 = \$160

3 parts is 3 × \$80 = \$240

5 parts is 5 × \$80 = \$400

One person receives \$160, another receives \$240 and the other receives \$400.

d 2 : 2 : 4

There are 2 + 2 + 4 = 8 parts

\$800 ÷ 8 = \$100

One part is \$100

2 parts is 2 × \$100 = \$200

4 parts is 4 × \$100 = \$400

Two people receive \$200 and the other person receives \$400.

18 wholemeal : plain

3 : 4

3 × **6** : 4 × **6**

18 : 24

24 cups of plain flour requires 18 cups of wholemeal flour.

19 **a** 1 : 1000

1 × **2.7** : 1000 × **2.7**

1 : 2700

2.7 cm on the map represents 2700 cm in actual distance.

2700 cm = 2700 ÷ 100 m = 27 m

b 140 mm = 140 ÷ 10 cm = 14 cm

1 : 1000

1 × **14** : 1000 × **14**

14 : 14 000

14 cm on the map represents an actual distance of 14 000 cm.

14 000 cm = 14 000 ÷ 100 m

$= 140$ m

20 5 kg cost \$50
1 kg cost \$10
2 kg cost \$20.

21 12 litres for 86 km
$\frac{12}{\mathbf{12}}$ for $\frac{86}{\mathbf{12}}$

1 litre for $\frac{86}{12}$ km
$1 \times \mathbf{42}$ for $\frac{86}{12} \times \mathbf{42}$ km
42 litres for 301 km.

22 $6 - 3 = 3$
$10^3 = 1000$
An earthquake of magnitude 6 is 1000 times stronger than an earthquake of magnitude 3.

Chapter 2 – Investigating and comparing data distributions

Now try this 2A

5 There are four possible values for *travel mode*. These are: walk, car, bus, other. Count the number of times each occurs, and enter the frequencies into the table. Check that the sum of the frequencies is 25, then then calculate the percentage frequencies as shown.

	Frequency	
Travel	Number	%
walk	4	$\frac{4}{25} \times 100 = 16.0$
car	12	$\frac{12}{25} \times 100 = 48.0$
bus	8	$\frac{8}{25} \times 100 = 32.0$
other	1	$\frac{1}{25} \times 100 = 4.0$
Total	25	100.0

6 To construct the bar chart, label the horizontal axis with the values walk, car, bus, other. These can be in any order, and the width of the bars can be any width, but the width must all be the same. The vertical scale should start at 0, and extend to slightly more than the maximum frequency in your table, which is 12. The heights of each of the four bars are equal to the frequency for that value.

To construct the percentage bar chart, again label the horizontal axis with the values walk, car, bus, other. These can be in any order, and the width of the bars can be any width, but the width must all be the same. The vertical scale should start at 0, and extend to slightly more than the maximum percentage frequency in your table, which is 48. The heights of each of the four bars are equal to the percentage frequency for that value.

Solutions to Exercise 2A

1 **a** Definition based. Categorical.

b Definition based. Numerical.

c Definition based. Categorical.

d Definition based. Numerical.

e Definition based. Categorical.

2 **a** Definition based. Nominal.

b Definition based. Ordinal.

c Definition based. Ordinal.

d Definition based. Nominal.

3 **a** *number of pages* can take only whole number values. Discrete.

b *price* can take only values to the nearest whole cent. Discrete.

c *volume* can take any value within a range. Continuous.

d *time* can take any value within a range. Continuous.

e *number of people* can take only whole number values. Discrete.

4 **a** Definition based. Nominal.

b Definition based. Ordinal.

c Definition based. Numerical - discrete.

d Definition based. Ordinal.

5 **a** Definition based. Nominal.

b There are two possible values for *gender* given in the table. Count the number of times each occurs, and enter the frequencies into the table. Check that the total of the frequencies is equal to the number of values, then then calculate the percentage frequencies using the rule:

$$percentage\ frequency = \frac{frequency}{total} \times 100$$

Gender	Frequency	
	Count	Percentage
Female	5	33.3
Male	10	66.7
Total	15	100.0

6 **a** Definition based. Ordinal.

b There are six possible values for *shoe size* given in the table. Count the number of times each occurs, and enter the frequencies into the table. Check that the total of the frequencies is equal to the number of values, then then calculate the percentage frequencies using the rule:

$$percentage\ frequency = \frac{frequency}{total} \times 100$$

Shoe size	Frequency	
	Count	Percentage
7	3	15
8	7	35
9	4	20
10	3	15
11	2	10
12	1	5
Total	20	100

7 **a**

Food type	Frequency	
	Count	Percentage
Hamburger	23	33.3
Chicken	7	10.1
Fish and chips	6	8.7
Chinese	7	10.1
Pizza	18	26.1
Other	8	11.6
Total	69	99.9

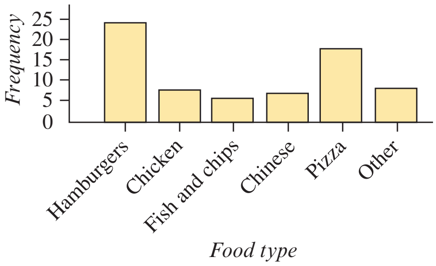
b Definition based. Nominal.

c Read from table: 7 students prefer Chinese.

d Read from table: 10.1% of students prefer Chicken.

e Read from table: hamburger (most frequent response – 23 students)

f



8 **a**

Attitude to capital punishment	Frequency	
	Count	Percentage
Strongly agree	21	8.2
Agree	11	4.3
Don't know	42	16.4
Disagree	53	20.7
Strongly disagree	129	50.4
Total	256	100.0

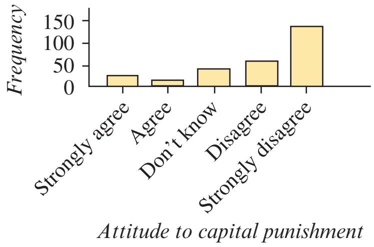
b Definition based. Ordinal.

c Read from table: 21 people strongly agree.

d Read from table: 50.4% of people strongly disagree.

e Read from table: strongly disagree (was most frequent response – 129 people)

f



9 **a**

Type of book	Frequency	
	Count	Percentage
Children	53	22.8
Fiction	89	38.4
Cooking	42	18.1
Travel	15	6.5
Other	33	14.2
Total	232	100.0

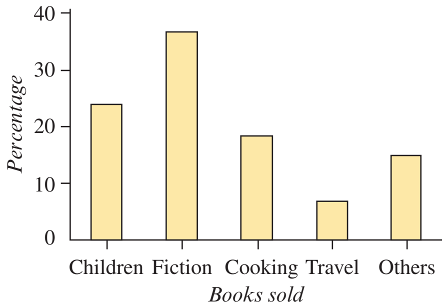
b Definition based. Nominal.

c Read from table: 89 books purchased were classified as fiction.

d Read from table: 22.8% of books were classified as children's.

e Read from table: 232 books were purchased in total.

f



10

Leisure activity	Frequency	
	Count	Percentage
Watch TV	84	42
Read	26	13
Listen to music	46	23
Watch a video	24	12
Phone friends	8	4
Other	12	6
Total	200	100

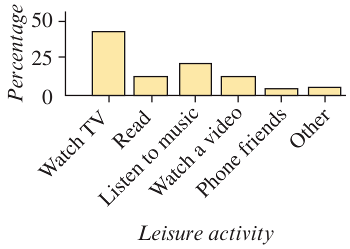
a Read from table: 200 students were surveyed.

b Definition based. Nominal.

c Read from table: 4% of students prefer to phone friends.

d Read from table: watching TV (was most frequent response – 84 students)

e



11 **a** Definition based. Ordinal

b To determine the number of people surveyed add the frequencies for each value of the variable which need to be read off the bar chart.
Slight =7, Moderate = 55, A lot = 18, so number surveyed= 7 + 55 + 18 = 80.

c 18

d modal category is the one with the highest frequency = moderate, percentage choosing that response = $\frac{55}{80} \times 1100 = 68.75\%$

12 **a** Definition based. Ordinal

b The modal income category is \$60,000-\$79,999 which has a percentage frequency of 40% (read from the bar chart).

Since there are 200 people in total, the number is category is $200 \times \frac{40}{100} = 80$.

Now try this 2B

7 There are many ways of writing this report, the one given is only an example of what you might write.

8 There are many ways of writing this report, the one given is only an example of what you might write.

Solutions to Exercise 2B

1 A group of 69 students were asked their favourite type of fast food. The most popular response was hamburgers (33.3%), followed by pizza (26.1%). The rest of the group were almost evenly split between chicken, fish and chips, Chinese and other, all around 10%.

2 A group of 256 people were asked whether they agreed that there should be a return to capital punishment in their state. The majority of these people strongly disagreed (50.4%), followed by 20.7% who disagreed. Levels of support for return to capital punishment were quite low, with only 4.3% agreeing and 8.2% strongly agreeing. The remaining 16.4% said that they didn't know.

3 A group of 200 students were asked how they prefer to spend their leisure time. The most popular response was using the internet and digital games (42%), followed by listening to music (23%), reading (13%), watching TV or going to a movie (12%) and phoning friends (4%). The remaining 6% said 'other'. Watching TV for this group of students was clearly the most popular leisure time activity.

4 A group of 600 employees from a large company were asked about the importance to them of the salary that they earned in the job. The majority of employees said that it was important (55%), or very important (30%). Only a small number of employees said that it was somewhat important (10%) with even fewer saying that it was not at all important (5%). Salary was clearly important to almost all of the employees in this company.

Now try this 2C

9 The minimum value is 0 and the maximum is 5.
Make sure that your frequencies add to 30, then calculate percentage frequencies as shown:

Number of widgets	Frequency	
	Number	%
0	8	$\frac{8}{30} \times 100 = 33.3$
1	8	$\frac{8}{30} \times 100 = 33.3$
2	4	$\frac{4}{30} \times 100 = 16.7$
3	2	$\frac{2}{30} \times 100 = 8.3$
4	1	$\frac{1}{30} \times 100 = 4.2$
6	1	$\frac{1}{30} \times 100 = 4.2$
Total	30	100.0

10 The minimum value is 0 and the maximum is 63. You intervals should be 0-9 (10 values), 10-19 (10 values) and so on.

Tally the number of diners in each interval, and then make sure that your frequencies add to 30. Calculate percentage frequencies as shown:

	Frequency	
Dined in restaurant	Number	%
0−9	17	$\frac{17}{30} \times 100 = 56.7$
10−19	5	$\frac{5}{30} \times 100 = 16.7$
20−29	3	$\frac{3}{30} \times 100 = 10.0$
30−39	0	$\frac{0}{30} \times 100 = 0$
40−49	1	$\frac{1}{30} \times 100 = 3.3$
50−59	3	$\frac{3}{30} \times 100 = 10$
60−69	1	$\frac{1}{30} \times 100 = 3.3$
Total	30	100

12 Follow the steps in Example 12 to create a histogram of the data in NTT 9.

13 Follow the steps in Example 13 to create the histogram of the data.

Solutions to Exercise 2C

1 **a** Count the number of times each value occurs and enter into the Frequency column.

b Even though our data is only taking whole number values, it is usual to create the histogram with no gaps between the columns. The second column will run from 1.5-2.5, the third from 2.5-3.5 etc. The height of each columns is equal to the frequency of the value.

2 **a** Count the number data values occurring in each interval and enter into the Frequency column.

b Complete the histogram as shown. The height of each columns is equal to the frequency of the values within the interval.

3 Since the data are whole numbers from 0 to 5 there is no need to group the data. Count the number of each data value and enter into a frequency table. Check the total is 15 and then calculate percentage frequencies using

$$\textit{percentage frequency} = \frac{\textit{number}}{15} \times 100.$$

Number of	Frequency	
magazines	Count	Percentage
0	4	26.7
1	4	26.7
2	3	20.0
3	2	13.3
4	1	6.7
5	1	6.7
Total	15	100.1

The total is greater than 100% due to rounding.

4 Since the data can take many different value we need to group the data. Intervals which start at 0 and have a width of 5 \$0-\$4.99, \$5.00-\$9.99, \$10.00-\$14.99 and so on. Count the number of data values within each of these intervals and enter into a frequency table. Check the total frequency is 20, and then calculate percentage frequencies using

$$\textit{percentage frequency} = \frac{\textit{number}}{20} \times 100.$$

Amount of money	Frequency	
	Count	Percentage
0.00–4.99	13	65
5.00–9.99	3	15
10.00–14.99	2	10
15.00–19.99	1	5
20.00–24.99	1	5
Total	20	100

5 **a** **i** Read from table: 2 students.

ii Read from table: 3 students.

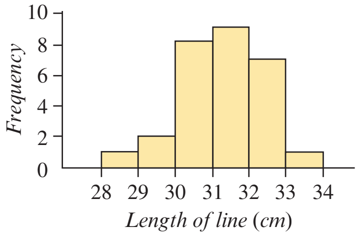
iii Read from table: 8 students.

b **i** Read from table: 32.1% of students.

ii Read from table: 39.3% of students.

iii Read from table: 89.3% of students.

c



6 **a** Read from graph: 4 students.

b Read from graph: 2 children.

c Read from graph: 5 students.

d Read from graph: 28 students (total each of the columns).

7 **a** Read from graph: 0 students.

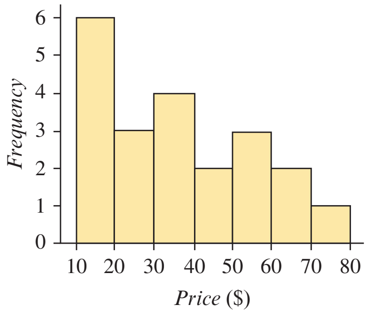
b Read from graph: 48 students.

c Read from graph: definition based. 60–69 is the modal class.

d Read from graph: 33 students.

8 **a** Enter the data into your CAS calculator and follow the instructions on page 58

(TI) and 61 (CASIO) to construct the histogram. The solution should look like this:

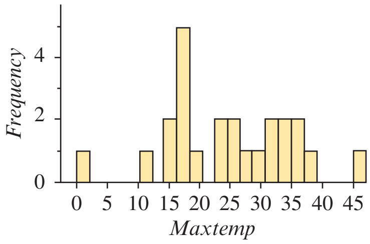


b **i** Read from graph: \$30 to < \$40.

ii Read from graph: 4 books.

iii Read from graph: \$10 to < \$20.

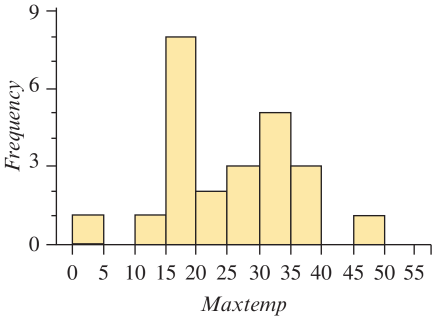
9 **a** Enter the data into your CAS calculator and follow the instructions on page 58 (TI) and 61 (CASIO) to construct the histogram. The solution should look like this:



b **i** Read from calculator: 11°C.

ii Read from calculator: 1 city.

c The new graph should look like this:



d **i** Read from calculator: 2 cities.

ii Definition based. Read from graph: modal class 15°C to < 20°C

10 You can use your calculator for this. Enter the data into your CAS calculator and follow the instructions on page 58 (TI) and 61 (CASIO) to construct the histogram.

a Move the cursor across the histogram to find the frequencies needed to construct the frequency table.

b See calculator.

c There is no unique mode, as several values have the same (highest) frequency of 5.

d Looking at the histogram we can see that 3 students made 5 errors, 3 students made 6 errors, 3 made 8 error, and 5 made 9 errors, giving a total of 14 students, or $\frac{14}{30} \times 100 = 46.7\%$ of students who will need to resit the test.

11 You can use your calculator for this. Enter the data into your CAS calculator and follow the instructions on page 58 (TI) and 61 (CASIO) to construct the histogram.

a Move the cursor across the histogram to find the frequencies needed to construct the frequency table.

b See calculator.

c The mode is the interval 10-19 minutes, with a frequency of 10 or $\frac{10}{3} \times 100 =$ 33.3%.

Now try this 2D

14 Follow the steps in Example 14 to construct the dot plot.

15 Follow the steps in Example 15 to construct the stem plot.

Solutions to Exercise 2D

1 **a** Definition based. Positively skewed.

b Definition based. Negatively skewed.

c Definition based. Symmetric.

2 **a** Definition based. They differ in location. The two distributions have an approximately equal spread but different locations.

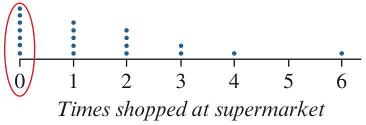
b Definition based. They differ in neither. The two distributions have approximately equal spread and centres.

c Definition based. They differ in both. The two distributions differ in both spread and centres.

3 **a** Construct the dot plot following the steps in Example 14.

b Read from dot plot: 2 children in the family is the most common result (mode).

4 **a** Construct the dot plot following the steps in Example 14,



b Read from dot plot: 7 people did not shop at the supermarket in the last week.

5 **a** Definition based. Negatively skewed.

b Definition based. Approximately symmetric

c Definition based. Positively skewed.

6 **a** Construct the dot plot following the steps in Example 14, using a horizontal axis with a scale from 19-30, marked off in intervals of 1.

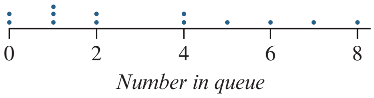
b Read from dot plot: there is no unique mode, with equal highest frequencies at 24 and 25.

c The distribution is approximately symmetric.

d Count the dots from left to right, there are 13 players younger than 25 years $= \frac{13}{22} \times 100 = 59\%$.

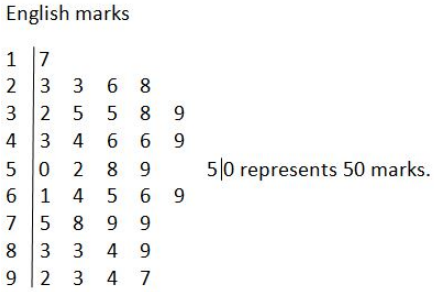
7 Construct the dot plot following the steps in Example 14, using a horizontal axis with a scale from 0-8, marked off in intervals of 1.

a



b Read from table: 12.25 pm is the time of peak demand at the cafe (8 people).

8 **a** Follow the steps in Example 15 to construct the stem plot.



English marks

1 | 7
2 | 3 3 6 8
3 | 2 5 5 8 9
4 | 3 4 6 6 9
5 | 0 2 8 9
6 | 1 4 5 6 9
7 | 5 8 9 9
8 | 3 3 4 9
9 | 2 3 4 7

5|0 represents 50 marks.

b Count the values on the from stem and leaf plot from 50 to 99, there are 21, so 21 students obtained 50 or more marks.

c Read from stem and leaf plot: 17 was the lowest mark.

9 It might help to turn the page on its side.

a Approximately symmetric.

b Since tail of the distribution is towards smaller values this distribution is negatively skewed.

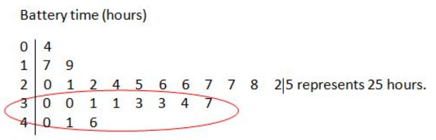
c Since tail of the distribution is towards higher values this distribution is positively skewed.

10 **a** Count the number of values in the stem plot = 40 people.

b The distribution is approximately symmetric.

c Count from the minimum value (2)to the the largest value that is less than 43 (42). There are 21 people.

11 **a**



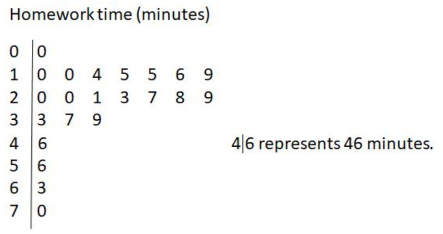
Battery time (hours)

0	4
1	7 9
2	0 1 2 4 5 6 6 7 7 8
3	0 0 1 1 3 3 4 7
4	0 1 6

2|5 represents 25 hours.

b Read from stem and leaf plot: 9 batteries lasted for more than 30 hours. Do not include the two 30 values, as the question asks ‘more than’.

12 **a**



Homework time (minutes)

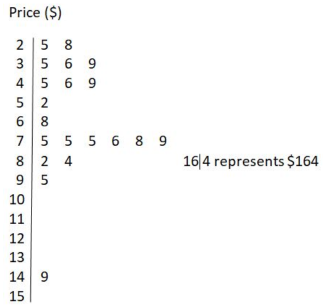
0	0
1	0 0 4 5 5 6 9
2	0 0 1 3 7 8 9
3	3 7 9
4	6
5	6
6	3
7	0

4|6 represents 46 minutes.

b Read from stem and leaf plot:
2 students spend more than
60 minutes on homework.

c Positively skewed. More values towards the positive end of the distribution.

13 **a**



Price ($)

2	5 8
3	5 6 9
4	5 6 9
5	2
6	8
7	5 5 5 6 8 9
8	2 4
9	5
10	
11	
12	
13	
14	9
15	

16|4 represents $164

b Definition based. Approximately symmetric. Values distributed relatively evenly around centre. One outlier (149)

14 Begin by constructing a single horizontal axis with scale encompassing both data sets, from 0 to 18.

a Construct the dot plot of the minimum temperatures using one colour.

b Construct the dot plot of the maximum temperatures on the same axis using a different colour.

c The centre of the minimum temperature is much lower than the centre for the maximum temperature. The minimum temperatures also look to be more spread out.

15 **a** Follow the steps in Example 15 to construct the stem plot.

b Subtracting 2 from every data points would just be a location change for the distribution, reducing the centre by 2, and leaving the speed unchanged.

16 **a** Symmetric. We would expect the number of both very short and very tall people to reduce similarly as we get further away from the centre of the distribution.

b Positively skewed. There would be a long positive tail as there are many expensive houses.

c Negatively skewed, as many babies are born before full term, and not many after.

Now try this 2E

18 **a** $n = 19 \Rightarrow$ the median is the 10^{th} value. Counting from left to right we see the 10^{th} value=5.

b $n = 20 \Rightarrow$ the median is $\dfrac{10^{th} + 11^{th}}{2}$.

Counting from left to right we see the 10^{th} value=3 and the 11^{th} value=4.

Hence the median $= \dfrac{3+4}{2} = 3.5$

Solutions to Exercise 2E

1 **a** $\Sigma x = 26$

b $\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{26}{11} = 2.36$

2 **a** 11 12 17 19 21 24 32 34 35 53 62 63 95

b $n = 13 \Rightarrow$ the median is the 7^{th} value. Counting from left to right we see the 7^{th} value=32.

3 **a**
$$\begin{aligned}\bar{x} &= \frac{\sum x}{n}\\ &= \frac{2+2+5+7+9}{5}\\ &= \frac{25}{5} = 5\end{aligned}$$
Mean = 5

b
$$\begin{aligned}\bar{x} &= \frac{\sum x}{n}\\ &= \frac{1+3+4+5+6+11}{6}\\ &= \frac{30}{6} = 5\end{aligned}$$
Mean = 5

c
$$\begin{aligned}\bar{x} &= \frac{\sum x}{n}\\ &= \frac{5+10+15+20+25}{5}\\ &= \frac{75}{5} = 15\end{aligned}$$
Mean = 15

d
$$\begin{aligned}\bar{x} &= \frac{\sum x}{n}\\ &= \frac{96+97+98+101+105+109}{6}\\ &= \frac{606}{6} = 101\end{aligned}$$
Mean = 101

e
$$\begin{aligned}\bar{x} &= \frac{\sum x}{n}\\ &= \frac{0.2+1.2+1.9+2.3+3.4+7.8}{6}\\ &= \frac{16.8}{6} = 2.8\end{aligned}$$

Mean = 2.8

4 **a** $n = 9 \Rightarrow$ the median is the 5^{th} value. Counting from left to right we see the 5^{th} value=9

b $n = 10 \Rightarrow$ the median is $\dfrac{5^{th} + 6^{th}}{2}$

Counting from left to right we see the 5^{th} value=6 and the 6^{th} value=7

Hence the median $= \dfrac{6 + 7}{2} = 6.5$

c $n = 10 \Rightarrow$ the median is $\dfrac{5^{th} + 6^{th}}{2}$

Counting from left to right we see the 5^{th} value=27 and the 6^{th} value=27

Hence the median $= \dfrac{27 + 27}{2} = 27$

d $n = 8 \Rightarrow$ the median is $\dfrac{4^{th} + 5^{th}}{2}$

Counting from left to right we see the 4^{th} value=106 and the 5^{th} value=107

Hence the median $= \dfrac{106 + 107}{2} = 106.5$

e $n = 11 \Rightarrow$ the median is the 6^{th} value. Counting from left to right we see the 6^{th} value=1.2

5 **a** $n = 12 \Rightarrow$ the median is $\dfrac{6^{th} + 7^{th}}{2}$

Counting down from the top we see the 6^{th} value=57 and the 7^{th} value=57

Hence the median $= \dfrac{57 + 57}{2} = 57$

b $n = 24 \Rightarrow$ the median is $\dfrac{12^{th} + 13^{th}}{2}$

Counting down from the top we see the 12^{th} value=27 and the 13^{th} value=28

Hence the median $= \dfrac{27 + 28}{2} = 27.5$

6 **a** $\bar{x} = \dfrac{\sum x}{n}$

$$= \frac{2.1 + 2.3 + 3.2 + 3.4 + 3.6 + 4.2 + \enclose{circle}{7.4} + 9.0 + 11.3 + 19.4 + 27.6 + 28.4 + 40.3}{13}$$

$$= \frac{162.2}{13} = 12.5$$

Mean = 12.5 ha

Median = 7.4 ha

b The median is a better measure of centre for the data set, as it is typical of more suburbs.

7 **a** $\bar{x} = \dfrac{\Sigma x}{12} = 55.42$

b $\bar{x} = \dfrac{\Sigma x}{24} = 55.42$

8 **a** $\bar{x} = \dfrac{\sum x}{n}$

$= \dfrac{293\,400 + 318\,000 + \ldots + 600\,000 + 750\,000}{14}$

$= \dfrac{5\,507\,400}{14}$

$= 393\,386$

Mean = \$393 386

Median = \$340 000

b The median is a better measure of centre for the data set, as it is typical of more house prices.

9 mean= $\dfrac{12 + 13 + (2 \times 15) + (3 \times 16) + (5 \times 17) + (4 \times 18) + (2 \times 19) + 20 + 21}{19}$ =16.95

$n = 20 \Rightarrow$ the median is $\dfrac{10^{th} + 11^{th}}{2}$

Counting left to right we see the 10^{th} value=17 and the 11^{th} value=17

Hence the median = 17

10 First mean: $\bar{x} = \dfrac{\Sigma x}{8} = 23.6 \Rightarrow \Sigma x = 23.6 \times 8 = 188.8$

Second mean: $\bar{x} = \dfrac{\Sigma x}{0} = 24.9 \Rightarrow \Sigma x = 24.9 \times 9 = 224.1$

The value added= $224.1 - 188.8 = 35.3$

11 The sum of the scores for both classes = $(76 \times 23) + (72 \times 20) = 2981$

There are a total of 23 + 20 + 43 students, so the mean across both classes

$= \dfrac{2981}{43} = 69.3$

12 **a** Mean of data with error: $\dfrac{\Sigma x}{20} = 15.6 \Rightarrow \Sigma x = 15.6 \times 20 = 312$

Since she added 1.6 instead of 10.6 we need to subtract 1.6 and add 10.6 to this sum ie.

Correct value of $\Sigma x = 312 - 1.6 + 10.6 = 321$

Correct of $\bar{x} = \dfrac{321}{21} = 16.05$

b Since she made a mistake in the smallest value, the order of the data is unchanged, and the median is still15.

13 From the histogram the total number of children is 28.

a $n = 28 \Rightarrow$ the median is $\dfrac{14^{th} + 15^{th}}{2}$
Adding the frequencies from left to right we see the 10^{th} value=2 and the 15^{th} value=3
Hence the median = 2.5

b $\bar{x} = \dfrac{(1 \times 4) + (2 \times 10) + (3 \times 5) + (4 \times 4) + (6 \times 2) + 7 + (9 \times 2)}{28} = \dfrac{92}{28} = 3.3$

14 From the histogram the total number of students is 48.

a $n = 48 \Rightarrow$ the median is $\dfrac{24^{th} + 55^{th}}{2}$
The 24th value is in the interval 50 - 59, and the 25th value is in the interval 60 - 69. Thus we could infer that the median is about 60, although we cannot give it an exact value.

b We cannot determine the exact value of the mean from a histogram where the data have been grouped. However, as the distribution is approximately symmetric we could infer that it would be similar in value to the median.

15 **a** The median would be in the interval where the cumulative percentage frequency is 50%. Thus it is in the interval 180.0 - 184.9, although we cannot give it an exact value.

b We cannot determine the exact value of the mean from a histogram where the data have been grouped. However, as the distribution is approximately symmetric we could infer that it would be similar in value to the median.

Now try this 2F

19 range = 6.4 − 0.5 = 5.9 kg

20 $n = 19 \Rightarrow$ the median is the $10^{th} = 3.9$
When n is odd, the middle value is omitted when working out the quartiles, meaning here are 9 values in each half.
$Q_1 = 5^{th}$ value when counting from the top down = 2.5
$Q_3 = 5^{th}$ value when counting from the bottom up = 4.8
$IQR = 4.8 - 2.5 = 2.3$kg

21 Follow the steps in Example 21 to complete the table.

	x	$(x - \bar{x})$	$(x - \bar{x})^2$
	0	-2.25	5.0625
	1	-1.25	1.5625
	3	0.75	0.5625
	5	2.75	7.5625
Sum	9	0	14.75

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}} = \sqrt{\frac{14.75}{4 - 1}} = 2.217$$

Solutions to Exercise 2F

1 **a** 0 1 1 1 2 2 2 3 6 7

b $R = 7 - 0 = 7$

c $n = 10 \Rightarrow$ there are 5 values in each half.
Q_1= middle of the bottom half= 3^{rd} value when counting from left to right = 1

d Q_3= middle of the top half= 3^{rd} value when from right to left = 3

e $IQR = 3 - 1 = 2$

2 Set up a table and follow the steps in Example 21 to complete.

3 **a** 2, 2, | 5, 7, $\textcircled{9}$, 11, 12, | 16, 23
Range = 23 – 2
= 21
IQR = 14 – 3.5
= 10.5

b 1, $\textcircled{3}$, 3, 5, 6, | 7, 9, $\textcircled{11}$, 12, 12
Range = 12 – 1
= 11
IQR = 11 – 3
= 8

c 21, 23, $\textcircled{24}$, 25, 27, | 27, 29, $\textcircled{31}$, 32, 33
Range = 33 – 21
= 12
IQR = 31 – 24
= 7

d 101, 101, | 105, 106, | 107, 107,
| 108, 109
Range = 109 – 101
= 8
IQR = 107.5 – 103
= 4.5

e 0.2, 0.9, $\textcircled{1.0}$, 1.1, 1.2, $\textcircled{1.2}$, 1.3, 1.9, $\textcircled{2.1}$, 2.2, 2.9
Range = 2.9 – 0.2
= 2.7
IQR = 2.1 – 1.0
= 1.1

4 **a** $n = 12 \Rightarrow$ there are 6 values in each half.

Q_1= middle of the bottom half = $\dfrac{49 + 50}{2} = 49.5$

Q_3= middle of the top half= $\dfrac{59 + 59}{2} = 59$

$IQR = 59 - 49.5 = 9.5$ mm

b $n = 23 \Rightarrow$ there are 11 values in each half.

Q_1= middle of the bottom half (6^{th} value down from the top) = 22

Q_3= middle of the top half (6^{th} value up from the bottom)= 33

$IQR = 33 - 22 = 11$ hours

5 $n = 20 \Rightarrow$ there are 10 values in each half.

$M = 3$

$Q_1 = \dfrac{1 + 1}{2} = 1$

$Q_3 = \dfrac{5 + 6}{2} = 5.5$

6 Follow calculator instructions on Page 90 (TI) or page 91 (CASIO).

7 Follow calculator instructions on Page 90 (TI) or page 91 (CASIO).

8 Enter the data into your calculator and follow calculator instructions on Page 90 (TI) or page 91 (CASIO).

9 Enter the data into your calculator and follow calculator instructions on Page 90 (TI) or page 91 (CASIO).

10 **a** Enter the data into your calculator and follow calculator instructions on Page 90 (TI) or page 91 (CASIO).

b Enter the data into your calculator and follow calculator instructions on Page 90 (TI) or page 91 (CASIO).

c The error does not affect the median or the interquartile range very much. It doubles the mean and increases the standard deviation by a factor of 20.

Solutions to Exercise 2G

1 **a** $15.8 \pm 2.3 = (13.5, 18.1)$

b $15.8 \pm 2 \times 2.3 = (11.2, 20.4)$

c $15.8 \pm 3 \times 2.3 = (8.9, 22.7)$

2 **a** $435.6 \pm 53.3 = 382.3, 488.9)$

b $435.6 \pm 2 \times 53.3 = (329.0, 542.2)$

c $435.6 \pm 3 \times 53.3 = (595.5)$

3 **a** $163 \pm 2 \times 8 = (147, 179) \Rightarrow 95\%$

b $163 \pm 3 \times 8 = (139, 187) \Rightarrow 99.7\%$

4 **a** $100 \pm 3 \times 15 = (55.145) \Rightarrow 99.7\%$

b $100 \pm 2 \times 15 = (70, 130) \Rightarrow 95\%$

5 **a** $60 \pm 1 \times 3 = (57, 63) \Rightarrow 68\%$

b $68 \pm 3 \times 3 = (51, 69) \Rightarrow 99.7\%$

6 **a** $24 \pm 1 \times 4 = (24, 28) \Rightarrow 68\%$

b $68 \pm 2 \times 3 = (16, 32) \Rightarrow 95\%$

7 $a = 1.0 - 2 \times 0.002 = 0.996\text{L or } 960\text{mL}$
$b = 1.0 + 2 \times 0.002 = 1.004\text{L or } 1004\text{mL}$

8 **a** $a = 91\,000 - 2 \times 26\,500 = 38\,000$
$b = 91\,000 + 2 \times 26\,500 = 144\,000$

b $c = 91\,000 - 3 \times 26\,500 = 11\,500$
$d = 91\,000 + 3 \times 26\,500 = 170\,500$

9 **a** $a = 6.2 - 2 \times 1.6 = 3$ hours
$b = 6.2 + 2 \times 1.6 = 9.4$ hours

b $c = 6.2 - 3 \times 1.6 = 1.4$ hours
$d = 6.2 + 3 \times 1.6 = 11.0$ hours

c 50% of the distribution is less than the mean $\Rightarrow e = 6.2$ hours.

d Use the diagrams on page 96 to help with this.
$6.2 + 1 \times 1.6 = 7.8 \Rightarrow$ 16% of people exercise more than 7.8 hours.
$6.2 + 2 \times 1.6 = 9.4 \Rightarrow$ 2.5% of people exercise more than 9.4 hours.
So the percentage of people exercising between 7.8 and 9.4 hours is
16% − 2.5% = 13.5%

Solutions to NTT 2H

24 Follow the steps in Example 24 to construct the boxplot.

25 $n = 22 \Rightarrow \quad min = 16 \quad Q_1 = 35 \quad M = 40 \quad Q_3 = 48 \quad max = 75$
$IQR = 48 - 35 = 13$
Lower fence=$Q_1 - 1.5 \times IQR = 35 - 1.5 \times 13 = 15.5$ $\Rightarrow$ there are no values less than 15.5 so no outliers at the lower end.
Upper fence=$Q_3 + 1.5 \times IQR = 48 + 1.5 \times 13 = 67.5$ $\Rightarrow$ there are two values more than 67.5 (72, 75).These are outliers at the upper end. Now follow the steps in Example 24 to construct the boxplot, remembering that the upper whisker will stop and the largest value which is NOT an outlier, which is 60.

26 $min = 5 \quad Q_1 = 11 \quad M = 17 \quad Q_3 = 26 \quad max = 61$
Follow the steps in Example 26 to determine the percentages.

Solutions to Exercise 2H

1 Follow the steps in Example 24 to complete a simple boxplot.

2 **a** Enter the data into you calculate and follow the instructions on page 90 (TI) or page 91 (CASIO) to find the 5-number summary for these data.

b Follow the steps in Example 24 to complete a simple boxplot.

3 $IQR = 19 - 13 = 6$

Lower fence$=Q_1 - 1.5 \times IQR = 13 - 1.5 \times 6 = 4$

Upper fence$=Q_3 + 1.5 \times IQR = 19 + 1.5 \times 6 = 28$

4 **a** $IQR = 65 - 45 = 20$

Lower fence$=Q_1 - 1.5 \times IQR = 45 - 1.5 \times 20 = 15$

Upper fence$=Q_3 + 1.5 \times IQR = 659 + 1.5 \times 20 = 95$

b 14 is an outlier at the lower end, 99 is an outlier at the upper end.

5 **a** $Q_1 - 1.5 \times \text{IQR} = 7.75 - 1.5 \times 7.25$

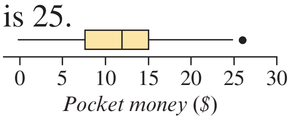
$= -3.125$

$Q_3 + 1.5 \times \text{IQR} = 15.00 + 1.5 \times 7.25$

$= 25.875$

b Hence, 26.00 is the only outlier.

c Now follow the steps in Example 24 to construct the boxplot, remembering that the upper whisker will stop and the largest value which is NOT an outlier, which is 25.



6 $min = 1 \quad Q_1 = 12 \quad M = 18 \quad Q_3 = 24 \quad max = 50$

Follow the steps in Example 26 to determine the percentages.

7 $min = 20 \quad Q_1 = 54 \quad M = 55 \quad Q_3 = 69 \quad max = 75$

Follow the steps in Example 26 to determine the percentages.

8 Enter the data into your calculator and follow the instructions on page 105 (TI) and page 106 (CASIO) to construct a boxplot.

9 **a** Enter the data into your calculator and follow the instructions on page 105 (TI) and page 106 (CASIO) to construct a boxplot.

b $k = Q_1 = 6$

10 **a** Enter the data into your calculator and follow the instructions on page 105 (TI) and page 106 (CASIO) to construct a boxplot.

b Scroll the cursor across the boxplot to find the values of the outliers.

c $Q_3 = 13$

11 **a** Enter the data into your calculator and follow the instructions on page 105 (TI) and page 106 (CASIO) to construct a boxplot.

b Scroll the cursor across the boxplot to find the values of the outliers.

c $M = \$579\,000$ $\quad$ $max = \$1\,625\,000$

12 **a** Enter the data into your calculator and follow the instructions on page 105 (TI) and page 106 (CASIO) to construct a boxplot.

b Scroll the cursor across the boxplot to find the values of the outliers.

c $Q_3 = 11$

13 **a** Read from the boxplot $M = \$800$

b Read from the boxplot $Q_1 = 400$ $\quad$ $Q_3 = 1600 \Rightarrow IQR = \$1200$

c $Q_3 = \$1600$

d upper fence= $1600 + 1.5 \times 1200 = \$3400$, which is more than \$3200 so this value is not an outlier.

14 **a** Enter the data into your calculator and follow the instructions on page 105 (TI) and page 106 (CASIO) to construct a boxplot.

b Scroll the cursor across the boxplot to find the values of the outliers.

c $Q_1 = 59.9$

Solutions to NTT 2I

27 Follow the steps in Example 27. Exact wording of answers will vary.

The median number of hours spent relaxing by this group of males was higher (M = 30 hours) and the median for this group of females (M = 24.5 hours). The spread of number of hours spent relaxing was lower for males (IQR = 14.15 hours) than for the females (IQR = 19 hours). In conclusion, the males spent more time relaxing that the females, and this amount of time was less variable for the males than the females.

28 Follow the steps in Example 28. Exact wording of answers will vary.

The median amount of time spent in an exam was slightly higher for the females in the group ($M \approx 122$ mins) than for the males ($M \approx 116$ mins). The spread for the two groups was almost the same, (females: $IQR \approx 45$ mins, males: $IQR \approx 44$ mins.) In conclusion, females tended to spend longer in the exam than males, with similar variability. There was one females who spent an unusually short time in the exam, only 40 minutes.

Solutions to Exercise 2I

1 a Test A: $n = 18$ $M = 57$ Test B: $n = 18$ $M = 73$

b lower

c Test A: $IQR = 61 - 45 = 16$ Test B:$IQR = 79 - 76 = 12$

d more

2 Read the 5 number summary off each boxplot.

Males: $min = 20$ $Q_1 = 25$ $M = 26$ $Q_3 = 28.5$ $max = 40$

Females: $min = 20$ $Q_1 = 22$ $M = 24$ $Q_3 = 25.5$ $max = 40$

a The foot length for males (median = 26 cm) is longer than the foot length for females (median = 24 cm).

b The foot length for males (IQR = 3.5cm) is the similar in variability as the foot length for females (IQR = 3.5 cm).

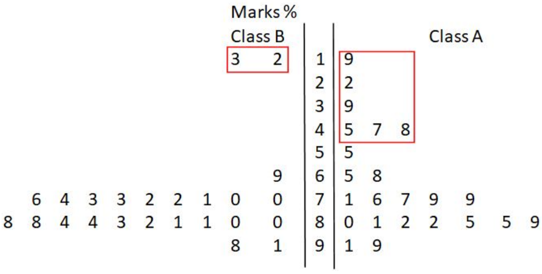
3 a

	Females		Males				
	9	0					4 | 0 = 40 years
⑤	0	1	3	6			
	7	2	①	4	5|6	7	
	7|1	3	④				
③	0	4	0		7		
	0	5					
		6					
	9	7					

Males:
$M = 25.5$ years
IQR = 34 – 21 = 13 years
Females:
$M = 34$ years
IQR = 43 – 15 = 28 years

b Report: The median age of the females ($M = 34$ years) was higher than the median age of males ($M = 25.5$ years). The spread of ages of the females (IQR = 28 years) was greater than the spread of ages of the males (IQR = 13 years). In conclusion, the median age of the females admitted to the hospital on that day was higher than the males. Their ages were also more variable.

4 a

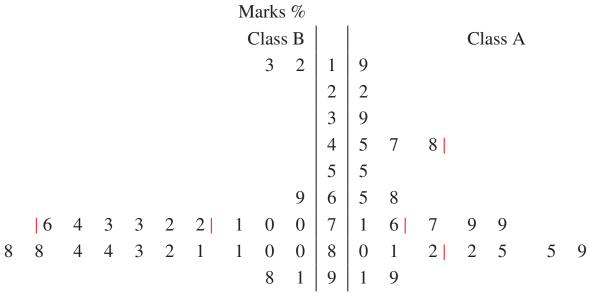


Marks %

Class B		Class A
3 2	1	9
	2	2
	3	9
	4	5 7 8
	5	5
9	6	5 8
6 4 3 3 2 2 1 0 0	7	1 6 7 9 9
8 8 4 4 3 2 1 1 0 0	8	0 1 2 2 5 5 9
8 1	9	1 9

From the stem plot above, we can see that there are 6 students in Class A, and 2 students in Class B, scored less than 50%.

b



Marks %

Class B		Class A
3 2	1	9
	2	2
	3	9
	4	5 7 8
	5	5
9	6	5 8
6 4 3 3 2 2 1 0 0	7	1 6 7 9 9
8 8 4 4 3 2 1 1 0 0	8	0 1 2 2 5 5 9
8 1	9	1 9

For Class A:
M = 76.5%
IQR = 82 – 51.5 = 30.5%
For Class B:
M = 78%
IQR = 83.5 – 71.5 = 12%

c Report: The median mark for Class A (M = 76.5) was lower than the median mark for Class B (M = 78). The spread of marks for Class A (IQR = 30.5) was greater than the spread of marks of Class B (IQR = 12). In conclusion, Class B had a higher median mark than Class A and their marks were less variable.

5 a Japanese A: M = 17 nights,
IQR = 16.5 nights;
Australians : M = 7 nights,
IQR = 10.5 nights

b Report: The median time spent away from home by the Japanese (M = 17 nights) was much higher than the median time spent away from home by the Australians (M = 7 nights). The spread in the time spent away from home by the Japanese (IQR = 16.5 nights) was also greater than the time spent away from home by the Australians (IQR = 10.5). In conclusion, the median time spent away from home by the Japanese was longer than the Australians and the time they spent away from home more variable.

6 a From the boxplots:
Year 12: *M* = 5.5 hours, IQR = 4.5 hours
Year 8: *M* = 3 hours, IQR = 2.5 hours (these values can vary slightly)

b Report: The median homework time for the year 12 students (*M* = 5.5 hours/week) was higher than the median homework time for year 8 students (*M* = 3 hours/week). The spread in the homework time for the year 12 students (IQR = 4.5 hours/week) was also greater than the year 8 students (IQR = 2.5 hours/week). In conclusion, the median homework time for the year 12 students was higher than the year 8 students and the homework time was more variable.

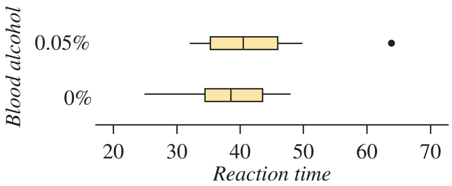
7 a From the boxplots:
Males: *M* = 22%, IQR = 15%
Females: *M* = 19%, IQR = 14%
(these values can vary slightly)

b Report: The median smoking rate for males (*M* = 22%) was higher than for females (*M* = 19%). The spread in smoking rates for males (IQR = 15%) was similar to females (IQR = 14%). In conclusion, median smoking rates were higher for males than females but the variability in smoking rates was similar.

8 a Before: *M* = 26, IQR = 4, OL = 45;
After: *M* = 30, IQR = 6, OL = 48 and 52
(these values can vary slightly)

b Report: The median number of sit ups before the fitness class (*M* = 26) was lower than after the fitness class (*M* = 30). The spread in number of sit ups before the fitness class (IQR = 6) was less than after the fitness class (IQR = 9). There was one outlier before the fitness class, the person who did 45 sit ups. There were two outliers after the fitness class, the person who did 48 sit ups and the person who did 52 sit ups. In conclusion, the median number of sit ups increased after taking the fitness class and there was more variability in the number of sit ups that could be done.

9 a



b Report: The median time is slightly higher for the 0.05% blood alcohol group (*M* = 40.5) than for the 0% blood alcohol group (*M* = 38.5). The spread in time is also slightly higher for the 0.05% blood alcohol group (IQR = 9.5) than for 0%

blood alcohol (IQR = 9.0). There was one outlier, the person with 0.05% blood alcohol who had a very long time of 64 seconds.

In conclusion, the median time was longer for the 0.05% blood alcohol group than for the 0% blood alcohol group but the variability in times was similar.

10 a People who were not thinking of changing to a different kind of work tended to work in occupations with higher median occupational prestige score (M = 48), than those who sometimes think of changing (M = 44) , who in turn tended to work in occupations with higher median occupational prestige score than those who were thinking of changing (M = 39). The variability in occupational prestige scores was highest for those thinking of changing work (IQR = 17), and similar for those not thinking of changing (IQR = 20) and sometimes thinking of changing (IQR = 22).

b Yes, there appears to be an association between the variables, with people who are occupations with higher occupational prestiges score tending to be less likely to want to change to a different kind of work.

Solutions to Skills Checklist chapter 2

1 **a** Defintion, continuous

b Defintion, discrete

c Defintion, nominal

d Definition, ordinal

2 Count teh number of times each answer occurs and enter into a frequency table. Since there are 20 values:
Percentage frequency= $\frac{number}{20} \times 100$

	Frequency	
Canteen rating	Number	%
bad	6	30
ok	8	40
good	6	30
Total	25	100

3 mode is the answer with the highest frequency = ok

4 Follow the instructions in Example 6 to construct a bar chart from the frequency table.

5 A group of 20 children were asked rate their canteen as bad, ok or good. The most popular response was ok, chosen by 40% of the students. The responses bad and good received the same percentage of responses, 30% each.

6 Enter the data into your calculator and follow the instructions on page 58 (TI) or 61 (CASIO) to construct a histogram.

7 Compare the histogram to the diagrams on page 67-68 ⇒positively skewed

8 Follow the steps in Example 14 to construct a dot plot.

9 Follow the steps in Example 15 to construct a stem plot.

10 Enter the data into your calculator and follow the instruction on page 90 (TI) and 91 (CASIO) to find $M = 24.5, \bar{x} = 29.15$

The median is preferable as the distribution is positively skewed (see page 81).

11 Enter the data into your calculator and follow the instruction on page 90 (TI) and 91 (CASIO) to find $Q_1 = 20$, $Q_3 = 34$, IQR = 14

12 Enter the data into your calculator and follow the instruction on page 90 (TI) and 91 (CASIO) to find s = 13.18

13 $3.5 \pm 3 \times 0.5 = (2, 5) \Rightarrow 68\%$. See page 96.

14 Use the stem plot from Question 9 to find the five number summary
$min = 17, Q_1 = 20, M = 24.5, Q_3 = 34, max = 66$
Follow the steps in example 24 to construct a simple boxplot.

15 lower fence = -1, upper fence = 55, thus 56 and 66 are outliers.

16 See Example 27 and Example 38.

Solutions to Chapter Review Multiple-Choice Questions

1 Definition based. The data is not numerical. **D**

2 A bar chart is the most appropriate display for a categorical variable. The only categorical variable in this list is *Hair Colour*. **D**

3 A histogram is the most appropriate display for a numerical variable. The only numerical variable in this list is *Distance*. **C**

4 *level of sunlight* is ordinal, *growth* is numerical. **C**

5 Read from histogram: 2% **B**

6 Read from histogram: 6% **D**

7 Modal interval is the most frequent response, and therefore ‘40 to less than 50’ with a frequency of 39% is the interval with the most frequent number of hours worked. **D**

8 The median is the middle number, which falls in the ‘40 to less than 50 hours’ interval. **D**

9 The range is 44 – 1 = 43. **D**

10 The median is the middle number. The data is already arranged in ascending order: 1, 1, 1, 3, 4, 5, 5, 6, 7, 9, 10, 12, 14, 14, 16, 22, 23, 44. The median is 8. **D**

11 $Q_1 = 4$
$Q_3 = 14$
$IQR = 10$ **D**

12 To calculate the mean and standard deviation enter the data into your calculator and follow the instructions on page 90 (TI) and 91 (CASIO) to find $\bar{x} = 184.4$, $s = 8.0$ **A**

13 Read from dot plot: 3 students scored 56. **C**

14 The number of students who scored between 40 and 80 = 16. To calculate percentage, divide by the number of students (20) and multiply by 100.
$16 \div 20 \times 100 = 80\%$ **C**

15 Range is given by the highest value – lowest value which is approximately 50 from the graph. **E**

16 Read from boxplot: The median is closest to 9. **B**

17 Read from boxplot: $Q_3 - Q_1 = IQR$
$15 - 5 = 10$ **B**

18 $Q_3 = 16$, so the percentage of people who have lived at the address for more than 16 years is 25%. **B**

19 $65 \pm 2 \times 8 = (49, 81) \Rightarrow 95\%$ **C**

20 $a = 65 - 3 \times 8 = 41$
$b = 65 + 3 \times 8 = 89$
D

21 Read from boxplots: company 3 has a lower median and IQR than the other two companies. **C**

22 Read from boxplots: company 1 has the largest IQR. **A**

21 Read from boxplots: company 3’s median is approximately \$85 000, but 75% of company 2’s employees earned less than approximately $110 000, so statement C is not true.

C

Solutions to Chapter Review Short-Answer Questions

1 **a** The *number of phone calls* a hotel receptionist receives each day is discrete data – it is numerical, and the number of phone calls is countable.

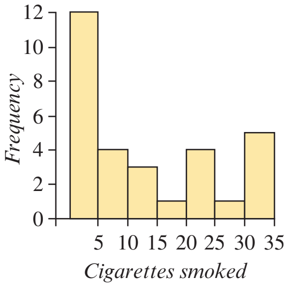
b *Interest in politics on a scale from 1 to 5, where 1 = very interested, 2 = quite interested, 3 = somewhat interested, 4 = not very interested, and 5 = uninterested* is ordinal data – it is categorical and the data can be placed in order.

2 **a** The level of measurement of *Type of company worked for* is categorical.

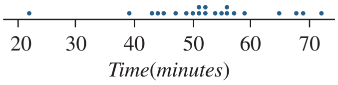
b From the bar chart, approximately 7.5% of people are self-employed.

3 First arrange the data in a frequency table.

Number of cigarettes smoked	*Frequency (count)*
0 – 4	12
5 – 9	4
10 – 14	3
15 – 19	1
20 – 24	4
25 – 29	1
30 – 34	5
Total	30



4 **a**



b Time (minutes)

2	2
3	9
4	3 4 5 7 9
5	0 1 1 2
2	4 5 6 6 7 9
6	5 8 9
7	2

4\|7 represents 47 minutes

c From the stem-and-leaf plot, M = 52 mins, Q_1 = 47 mins, Q_3 = 57 mins

5 Enter the data into your calculator and follows the instructions on page 90 (TI) or 91 (CASIO) to find:

$\bar{x}$ = \$283.57

s = \$122.72, M = \$267.50, IQR = \$90,

R = \$495

6 Follow the steps in Example 24 to construct a simple boxplot.

7 **a** Enter the data into your calculator and follows the instructions on page 105 (TI) or 106 (CASIO) to construct a boxplot

b Use you cursor to read the median value from the boxplot. M = 14.5

c Mark off and count the data values that are greater than 20, there are 10. Since there are a total of 36 data values $\frac{10}{36} \times 100 = 27.8\%$.

Solutions to Chapter Review Extended-Response Questions

1 **a** *General health* is an ordinal variable.

b Complete the table as shown:

General health	Frequency	
	Count	Percentage
Very healthy	255	51.0
Pretty healthy	$500 \times \frac{35.6}{100} = 178$	35.6
A little unhealthy	500-(255+178+29)=38	$\frac{38}{500} \times 100 = 7.6$
Very unhealthy	29	$\frac{29}{500} \times 100 = 5.8$
Total	500	100.0

c Follow the steps in Example 6 to construct a bar chart.

d Five hundred people were asked about their level of general health. The vast majority of the respondents indicated they were healthy, with 50.1% choosing very healthy and 35.6% choosing pretty healthy. Only 7.6% responded that they were a little unhealthy, and even fewer that they were very unhealthy (5.8%).

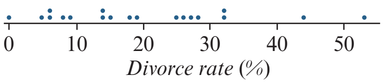
2 **a** *Divorce rate* is a numerical variable.

b **Divorce rate (%)**

0	0 5 6 6 8 9
1	4 4 5 8 9
2	5 6 7 8
3	2 2
4	4
5	3

3|2 represents 32%

c



d The distribution of divorce rates is positively skewed.

e Percentage of countries with divorce rates greater than 30

$$= \frac{\text{numbers of countries with divorce rate greater than 30}}{19} \times 100$$

$$= \frac{4}{19} \times 100$$

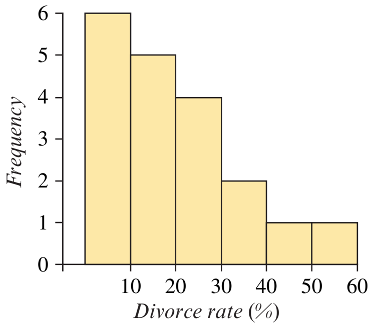
= 21.05%, correct to 2 decimal places

f Enter the data into your calculator and follow the instructions on page 90 (TI) or 91 (CASIO) to summary statistics.

$\bar{x} = 20.05 \qquad M = 18$

Therefore the mean and median of the distribution of divorce rates are 20.05% and 18% respectively.

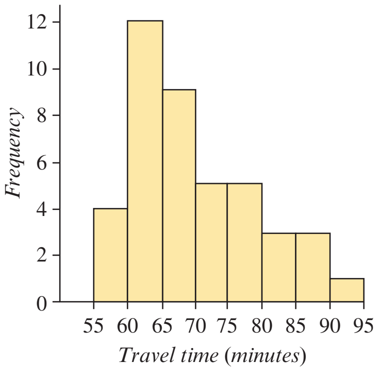
g



i The histogram is positively skewed, consistent with our answer to **d**.

ii Five countries had divorce rates from 10% to less than 20%.

3 a



i The trip took from 65-69 minutes on 9 days.

ii The histogram is positively skewed.

iii Percentage of trains that took less than 65 minutes to reach Flinders St

$$= \frac{\text{numbers of trains that took less then 65 minutes to reach Flinders Street}}{\text{total number of trains timed}} \times 100$$

$$= \frac{4 + 12}{42} \times 100$$

$= 38.1\%$, correct to 1 decimal place

b

$\bar{x} = 69.60$ minutes

$s = 9.26$ minutes

Min $= 57$ minutes

$Q_1 = 62$ minutes

$M = 68$ minutes

Q_3 = 76 minutes
Max = 90 minutes

c **i** The mean time taken from Lilydale to Flinders Street was 69.60 minutes

ii 50% of the trains took more than 68 minutes to travel from Lilydale to Flinders Street

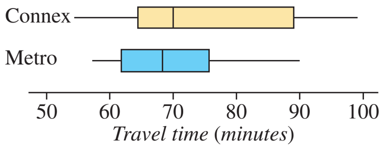
iii Range: R = largest data value – smallest data value
= Max – Min
= 90 – 57
= 33
IQR = $Q_3 - Q_1$
= 76- 62
= 14

The range of travelling times was 33 minutes while the interquartile range was 14 minutes.

iv 25% of trains took more than 76 minutes to travel to Flinders Street (Q_3 = 76).

v The standard deviation of travelling times was 9.26 minutes (s = 9.26).

d



e The distributions of travel times are both positively skewed. The travel times for Connex (M = 70 minutes) tend to be longer than the travel times for Metro (M = 68 minutes). The spread of times is also longer for Connex (IQR = 24 minutes) compared to Metro (IQR = 14). Overall, Metro has improved the service on the Lilydale line.

Chapter 3 – Sequences and finance

Solutions to Exercise 3A

1 **a** Difference between 2 and 5 is 3 so try adding 3 to each term:
$2 + 3 = 5$
$5 + 3 = 8$
$8 + 3 = 11$
$11 + 3 = 14$
Thus, the numbers in the box are 8 and 14.

b Difference between 1 and 3 is 2 but difference between 3 and 9 is 6. Thus, rule does not involve adding. Instead, each term is multiplied by 3:
$1 \times 3 = 3$
$3 \times 3 = 9$
$9 \times 3 = 27$
$27 \times 3 = 81$
Thus, the number in the box is 27.

2 **a** Starting value: 10.
Rule: subtract 2 from the previous value

b Starting value: *a*
Rule: add three letters along in the alphabet

3 **a** Starting value is 5

b Rule: Add 6 to the previous value

c The next three terms are determined as follows: $29 + 6 = 35$
$35 + 6 = 41$
$41 + 6 = 47$

4 **a** Constant

b Increasing

c Oscillating

d Oscillating and limiting to zero

e Decreasing

f Increasing

5 **a** Add 4 to previous term:
$15 + 4 = 19$

b Subtract 1 from previous term:
$7 - 1 = 6$

c Pattern alternates between 1 and 2: next term is 1.

d Subtract 7 from previous term:
$10 - 7 = 3$

e Add 8 to the previous term:
$40 + 8 = 48$

f Oscilating between −2, 2 and 0.
2

6 **a** To find the next term, multiply the previous term by 2:
$32 \times 2 = 64$

b To find the next term, divide the previous term by 2:
$6 \div 2 = 3$

c To find the next term, multiply the

previous term by 3:
$27 \times 3 = 81$

d To find the next term, divide the previous term by 3:
$9 \div 3 = 3$

e To find the next term, divide the previous term by 2:
$125 \div 2 = 62.5$

f To find the next term, multiply the previous term by 3:
$54 \times 3 = 162$

7 **a** This is a list of months so the next month after February is March.

b This is a list of every second letter in the alphabet so the next letter would be i after you skip h.

c This is a list that alternates between spades, diamonds, hearts and clubs so the next symbol would be ◊.

d This is a list that alternattes between left and right facing arrows so the next symbol would be ⇒.

e This is a list of the days of the week so the next day after Tuesday is Wednesday.

f This is a list of arrows, each rotated by 90° from the previous one in a clockwise direction. Thus, the next symbol is ↑.

8 **a** Add 3 to the previous term:
$14 + 3 = 17$
$17 + 3 = 20$

b Add 9 to the previous term:
$46 + 9 = 55$
$55 + 9 = 64$

c Subtract 4 from the previous term:
$26 - 4 = 22$
$22 - 4 = 18$

d Subtract 8 from the previous term:
$42 - 8 = 34$
$34 - 8 = 26$

e Multiply the previous term by 2:
$24 \times 2 = 48$
$48 \times 2 = 96$

f Multiply the previous term by 3:
$108 \times 3 = 324$
$324 \times 3 = 972$

g Divide the previous term by 2:
$16 \div 2 = 8$
$8 \div 2 = 4$

h Multiply the previous term by −2:
$-24 \times -2 = 48$
$48 \times -2 = -96$

i Add the previous two terms together:
$3 + 5 = 8$
$5 + 8 = 13$

9 The sequence starts with the value 3 and then 2 is added to each term to find the next term.
$3 + 2 = 5$
$5 + 2 = 7$
$7 + 2 = 9$
$9 + 2 = 11$
Thus, the first five terms of the sequence

are 3, 5, 7, 9, 11

10 The sequence starts with the value 90 and then 6 is subtracted from each term to find the next term.
$90 - 6 = 84$
$84 - 6 = 78$
$78 - 6 = 72$
$72 - 6 = 66$
Thus, the first five terms of the sequence are 90, 84, 78, 72, 66

11 The sequence starts with the value 5 and then each term is multiplied by 2 before 1 is added.
$5 \times 2 + 1 = 11$
$11 \times 2 + 1 = 23$
$23 \times 2 + 1 = 47$
$47 \times 2 + 1 = 95$
Thus, the first five terms of the sequence are 5, 11, 23, 47, 95

12 **a** In this sequence, the sequence 1, 2, 3, . . . are cubed:
$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
Thus, the next term in the sequence is 125.

b In this sequence, the numbers are multiplied by 2, 3, 4, . . .:
$1 \times 2 = 2$
$2 \times 3 = 6$
$6 \times 4 = 24$
$24 \times 5 = 120$
Thus, the next term in the sequence is 120.

c In this sequence, the numbers are multiplied by 2, 3, 4, . . .:
$-1 \times 2 = -2$
$-2 \times 3 = -6$
$-6 \times 4 = -24$
$-24 \times 5 = -120$
$-120 \times 6 = 720$
Thus, the next term in the sequence is −720.

13 **a** The numbers in the sequence are multiplied by 2, 3, 4, ... The next terms are found by multiplying by 5 and then 6:
$48 \times 5 = 240$
$240 \times 6 = 1440$

b The numbers in the sequence are squares of 1, 2, 3, ... The next terms are found by finding the squares of 6 and 7:
$6^2 = 36$
$7^2 = 49$

c The numbers in the sequence are squares of odd numbers 1, 3, 5, ... The next terms are found by finding the squares of 9 and 11:
$9^2 = 81$
$11^2 = 121$

14 The sequence is found by multiplying each term by −2. Thus:
$16 \times -2 = -32$
$-32 \times -2 = 64$
$64 \times -2 = -128$
$-128 \times -2 = 256$
$256 \times -2 = -512$
Thus, the next five terms are:
−32, 64, −128, 256, −512.

Solutions to Exercise 3B

1 The starting value t_0 is the first term in the sequence, 20.

2 **a** $8, 4, 3, 11, 14, \ldots.$

b Remember that the first term is t_0 so t_0, t_1, t_2, t_3, t_4 is written below your answer to part **a**.

3 **a** The starting value of the sequence is 9

b To obtain the next term, add 2 to the current term, and repeat the process.

4 Consider the subscript of the required term where the first term is t_0 and read off the values from the sequence.

a $t_0 = 6$

b $t_3 = 21$

c $t_2 = 16$

d $t_4 = 26$

e $t_5 = 31$

f $t_6 = 36$

5 Consider the subscript of the required term where the first term is t_0 and read off the values from the sequence.

a **i** $t_0 = 6$

ii $t_3 = 18$

iii $t_1 = 10$

b **i** $t_0 = 2$

ii $t_3 = 128$

iii $t_1 = 8$

c **i** $t_0 = 29$

ii $t_3 = 8$

iii $t_1 = 22$

d **i** $t_0 = 96$

ii $t_3 = 12$

iii $t_1 = 48$

6 Consider the subscript of the required term where the first term is t_0 and read off the values from the sequence where possible. If the term isn't listed, then the sequence is extended by adding four to generate the next term.

a $t_3 = 20$

b $t_2 = 16$

c $t_0 = 8$

d $t_4 = t_3 + 4 = 20 + 4 = 24$

e $t_5 = t_4 + 4 = 24 + 4 = 28$

f Extending the sequence gives $8, 12, 16, 20, 24, 28, 32, 36, 40, \ldots$ so $t_8 = 40$

7 Consider the subscript of the required term where the first term is t_0 and read off the values from the sequence. To find t_6, establish the rule so that the next terms of the sequence can be found first.

a **i** $t_0 = 14$

ii $t_3 = 32$

iii Adding 6 to subsequent terms, giving the sequence $14, 20, 26, 32, 38, 44, 50, \ldots$ so $t_6 = 50$

b **i** $t_0 = 2$

ii $t_3 = 54$

iii Multiply by 3 to find subsequent terms, giving the sequence $2, 6, 18, 54, 162, 486, 1458, \ldots$ so $t_6 = 1458$

c **i** $t_0 = 40$

ii $t_3 = 16$

iii Subtract 8 to find subsequent terms, giving the sequence $40, 32, 24, 16, 8, 0, -8, \ldots$ so $t_6 = -8$

d **i** $t_0 = 8000$

ii $t_3 = 1000$

iii Divide by 2 to find subsequent terms, giving the sequence $8000, 4000, 2000, 1000, 500, 250, 125, \ldots$ so $t_6 = 125$

8 Write down the first term t_0 and then apply the rule to generate five terms.

a Starting value is 1 and the rule is to add 2.
1, 3, 5, 7, 9

b Starting value is 100 and the rule is to subtract 10.
100, 90, 80, 70, 60

c Starting value is 52 and the rule is to add 12.
52, 64, 76, 88, 100

9 To start, identify the starting value and write it as V_0 = starting value. The rule should be written as $V_{n+1} = V_n +$ difference or $V_{n+1} = V_n -$ difference.

a $V_0 = 3, \qquad V_{n+1} = V_n + 7$

b $V_0 = 9, \qquad V_{n+1} = V_n + 4$

c $V_0 = 16, \qquad V_{n+1} = V_n - 3$

10 To generate the sequence, begin with the starting value and then apply the rule to generate the next term. Repeating this process four times will give the first five terms (including the starting value).

a Starting value is 3 and then 7 is added each time. $V_0 = 3$
$V_1 = V_0 + 7 = 3 + 7 = 10$
$V_2 = V_1 + 7 = 10 + 7 = 17$
$V_3 = V_2 + 7 = 17 + 7 = 24$
$V_4 = V_3 + 7 = 24 + 7 = 31$
Thus, the first five terms are:
$3, 10, 17, 24, 31$

b Starting value is 9 and then 4 is added

each time. $V_0 = 9$
$V_1 = V_0 + 4 = 9 + 4 = 13$
$V_2 = V_1 + 4 = 13 + 4 = 17$
$V_3 = V_2 + 4 = 17 + 4 = 21$
$V_4 = V_3 + 4 = 21 + 4 = 25$
Thus, the first five terms are:
$9, 13, 17, 21, 25$

c Starting value is 16 and then 3 is subtracted each time. $V_0 = 16$
$V_1 = V_0 - 3 = 16 - 3 = 13$
$V_2 = V_1 - 3 = 13 - 3 = 10$
$V_3 = V_2 - 3 = 10 - 3 = 7$
$V_4 = V_3 - 3 = 7 - 3 = 4$
Thus, the first five terms are:
$16, 13, 10, 7, 4$

11 Let V_n be the value after n iterations. Then V_0 is the starting value and so is the first listed value in the sequence. To find the rule, establish how much is being added or subtracted to generate each term.

a 11 is the value of the first term and then 4 is added to generate the next term.
$V_0 = 11, \qquad V_{n+1} = V_n + 4$

b 43 is the value of the first term and then 4 is subtracted to generate the next term.
$V_0 = 43, \qquad V_{n+1} = V_n - 4$

c 3 is the value of the first term and then 4 is subtracted to generate the next term.
$V_0 = 3, \qquad V_{n+1} = V_n - 4$

12 To find the value for the required terms, it is easiest to write out the sequence using the recurrence relation. The starting value is 20 and the rule is to add 3 to each term. This gives the sequence:
$20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, \ldots$

a Reading of the list and recalling that the first term is T_0, the term name that has a value of 23 is T_1.

b Similarly, $T_4 = 32$.

c Similarly, $T_8 = 44$.

13 **a** The starting value can be read off: 4

b The value added to get the second term is 5, but the value added to get the third term is 10. Thus, the sequence isn't generated by adding a term.

c The first term would need to be multiplied by $\frac{9}{4}$ to get the second term whereas the second term would need to be multiplied by $\frac{19}{9}$ to get the third term.

d The starting value is 4. The rule is to add 5 times the number of iterations that have been applied.

e Let V_n be the value after n iterations. Since the staring value is 4, $V_0 = 4$. Each term is generated by adding 5 times the number of iterations. That is, by adding $5 \times n$. Thus, to find the V_{n+1}, you need to add $5 \times (n + 1)$ to V_n.
The recurrence relation is:
$V_0 = 4, \qquad V_{n+1} = V_n + 5(n + 1)$

Solutions to Exercise 3C

1 Recall that the first term is t_0 so t_3 is the fourth term in the sequence.

a 8

b 16

c 42

2 To find the difference between t_1 and t_0 and then between t_2 and t_1, calculate $t_1 - t_0$ and $t_2 - t_1$.

a $t_1 - t_0 = 15 - 11 = 4,$
$t_2 - t_1 = 19 - 15 = 4$

b $t_1 - t_0 = 3 - 1 = 2,$
$t_2 - t_1 = 1 - (-1) = 2$

3 **a** $t_0 = 4, t_1 = 11, t_2 = 18, t_3 = 25, t_4 = 32$

b Reading off, $t_3 = 25$.

c $t_1 - t_0 = 11 - 4 = 7,$
$t_2 - t_1 = 18 - 11 = 7,$
$t_3 - t_2 = 25 - 18 = 7,$
$t_4 - t_3 = 32 - 25 = 7$
Since the difference between each pair of terms is 7, the sequence is arithmetic.

d To find t_5, add 7 to t_4.
$t_5 = t_4 + 7 = 39.$
Similarly for t_6: $t_6 = t_5 + 7 = 46.$

4 **a** $D = t_2 - t_1 = 11 - 5 = 6$
$t_3 = 23$

b $D = t_2 - t_1 = 13 - 16 = -4$
$t_3 = 5$

c $D = t_2 - t_1 = 15 - 11 = 4$
$t_3 = 23$

d $D = t_2 - t_1 = 4 - 8 = -4$
$t_3 = -4$

e $D = t_2 - t_1 = 30 - 35 = -5$
$t_3 = 20$

f $D = t_2 - t_1 = 2 - 1.5 = 0.5$
$t_3 = 3$

5 **a** $d = t_2 - t_1 = 23 - 17 = 6$
$t_5 = t_4 + 6 = 35 + 6 = 41$
$t_6 = t_5 + 6 = 41 + 6 = 47$

b $d = t_2 - t_1 = 11 - 13 = -3$
$t_5 = t_4 + -3 = 8 - 3 = 2$
$t_6 = t_5 + -3 = 2 - 3 = -1$

c $d = t_2 - t_1 = 1\frac{1}{2} - 2 = -\frac{1}{2}$
$t_5 = t_4 + -\frac{1}{2} = \frac{1}{2} - \frac{1}{2} = 0$
$t_6 = t_5 + -\frac{1}{2} = 0 - \frac{1}{2} = -\frac{1}{2}$

d $d = t_2 - t_1 = 35 - 27 = 8$
$t_5 = t_4 + 8 = 51 + 8 = 59$
$t_6 = t_5 + 8 = 59 + 8 = 67$

e $d = t_2 - t_1 = 21 - 33 = -12$
$t_5 = t_4 + -12 = -3 - 12 = -15$
$t_6 = t_5 + -12 = -15 - 12 = -27$

f $d = t_2 - t_1 = 1.1 - 0.8 = 0.3$
$t_5 = t_4 + 0.3 = 1.7 + 0.3 = 2.0$
$t_6 = t_5 + 0.3 = 2.0 + 0.3 = 2.3$

6 **a** $t_2 - t_1 = 11 - 8 = 3$
$t_3 - t_2 = 14 - 11 = 3$
$t_4 - t_3 = 17 - 14 = 3$
Sequence is arithmetic, $d = 3$.

b $t_2 - t_1 = 15 - 7 = 8$
$t_3 - t_2 = 22 - 15 = 7$
Sequence is not arithmetic.

c $t_2 - t_1 = 7 - 11 = -4$
$t_3 - t_2 = 3 - 7 = -4$
$t_4 - t_3 = -1 - 3 = -4$
Sequence is arithmetic, $d = -4$.

d $t_2 - t_1 = 9 - 12 = -3$
$t_3 - t_2 = 6 - 9 = -3$
$t_4 - t_3 = 3 - 6 = -3$
Sequence is arithmetic, $d = -3$.

e $t_2 - t_1 = 8 - 16 = -8$
$t_3 - t_2 = 4 - 8 = -4$
Sequence is not arithmetic.

f $t_2 - t_1 = 1 - 1 = 0$
$t_3 - t_2 = 1 - 1 = 0$
$t_4 - t_3 = 1 - 1 = 0$
Sequence is arithmetic, $d = 0$.

7 To use your CAS, type in the starting value and press ENTER. Then type in + difference (or - difference) and press enter. Doing this repeatedly will generate the sequence.

a The starting value is 3 and the difference is 5 (so press "+5"). This gives $3, 8, 13, 18, 23$.

b The starting value is 16 and the difference is -7 (so press "-7"). This gives $16, 9, 2, -5, -12$.

c The starting value is 1.6 and the difference is 2.3 (so press "+2.3"). This gives $1.6, 3.9, 6.2, 8.5, 10.8$.

d The starting value is 8.7 and the difference is -3.1 (so press "-3.1"). This gives $8.7, 5.6, 2.5, -0.6, -3.7$.

e The starting value is 293 and the difference is -67 (so press "-67"). This gives $293, 226, 159, 92, 25$.

8 To use your CAS, type in the starting value and press ENTER. Then type in + difference (or - difference) and press enter. Adding (or subtracting) the required value the correct number of times will give the required term.

a The starting value is 1 and the difference is 5. Since we are looking for t_6, press "+5" 6 times to give $t_6 = 31$. The first seven terms of the sequence are: $1, 6, 11, 16, 21, 26, 31$.

b The starting value is 45 and the difference is -2. Since we are looking for t_{12}, press "-2" 13 times to give $t_{12} = 21$. The first thirteen terms of the sequence are: $45, 43, 41, 39, 37, 35, 33, 31, 29, 27, 25, 23, 21$.

c The starting value is 15 and the difference is -1. Since we are looking for t_{10}, press "-1" 10 times to give $t_{10} = 5$. The first eleven terms of the sequence are: $15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5$.

d The starting value is 0 and the difference is 3. Since we are looking for t_{15}, press "+3" 15 times to give $t_{15} = 45$. The first sixteen terms of the sequence are: $0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45$

9 The starting value is 320 as this was the initial number of wine glasses. 15 glasses break each week, the common difference is 15. Using your CAS, enter 320 and press enter. Then press "−15" and enter until you get to 200. Count the number of times you press "−15" to get the answer of 8 weeks.

10 The starting value is 100 and the common difference is 7.

a Enter 100 on your calculator and press enter. Then press "+7" and enter 4 times to get the answer of 128.

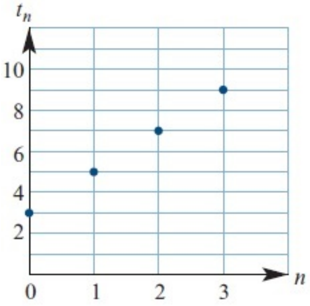
b Similarly, press "+7" and enter a total of 12 times get the answer of 184 songs.

11 **a** **i** The rule is to add 2 so the next value is 9.

ii The first line of the table has the numbers 0, 1, 2 and 3, indicating the number of iterations that have been applied. The second line has the value of the sequence.

n	0	1	2	3
t_n	3	5	7	9

iii The first row of the table is plotted along the x-axis and the second row of the table is plotted along the y-axis.



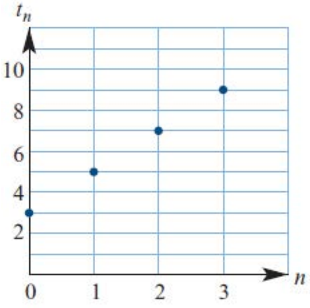
iv The points are in a line that is sloping upwards (as you move from left to right) so we say that it has a positive slope.

b **i** The rule is to subtract 3 so the next value is 2.

ii The first line of the table has the numbers 0, 1, 2 and 3, indicating the number of iterations that have been applied. The second line has the value of the sequence.

n	0	1	2	3
t_n	11	8	5	2

iii The first row of the table is plotted along the x-axis and the second row of the table is plotted along the y-axis.



iv The points are in a line that is sloping downwards (as you move from left to right) so we say that it has a negative slope.

12 **a** In this sequence, 5 is added to each term to generate the next term. The missing terms are 28 and 33.

b In this sequence, 6 is subtracted from each term to generate the next term. The missing terms are −10 and −16.

c In this sequence, 9 is added to each term to generate the next term. The missing terms are 24 and 33.

d In this sequence, 5 is subtracted from each term to generate the next term. The missing terms are 13 and 8.

e In this sequence, 8 is added to each term to generate the next term. The missing terms are 11 and 19.

f In this sequence, 8 is added to each term to generate the next term. The missing terms are 13 and 21.

g In the sequence, 11 is subtracted from each term to generate the next term. The missing terms are 29 and 18.

h In the sequence, 7 is subtracted from each term to generate the next term. The missing terms are 29 and 15.

i In the sequence, 8 is added to each term to generate the next term. The missing terms are 23, 39 and 55.

13 **a** To find the commond difference, calculate $t_1 - t_0 = -5 - (-2) = -3$. Thus, $D = -3$ so 3 is subtracted from each term to generate the next term.

b t_{100} is the term after 100 applications of the rule. That is, 3 is subtracted 100 times. This means that you would need to subtract 300 from the starting value.

c To find t_{100}, subtract 300 from the starting value of −2. This gives $t_{100} = t_0 - 300 = -2 - 300 = -302$.

d To find t_{200}, subtract $3 \times 200 = 600$ from the starting value of −2. This gives $t_{100} = t_0 - 600 = -2 - 600 = -602$.

Solutions to Exercise 3D

1 Recall that a is used to represent the starting term and D represents the difference in an arithmetic sequence.

a $a = 7$
$D = t_1 - t_0 = 11 - 7 = 4$

b $a = 8$
$D = t_1 - t_0 = 5 - 8 = -3$

c $a = 14$
$D = t_1 - t_0 = 23 - 14 = 9$

d $a = 62$
$D = t_1 - t_0 = 35 - 62 = -27$

e $a = -9$
$D = t_1 - t_0 = -4 - (-9) = 5$

f $a = -13$
$D = t_1 - t_0 = -(-19) - (-13) = -6$

2 Substitute the value of n, a and D into $t_n = a + n \times D$ where $n = 5$.

a $t_5 = 7 + 5 \times 4 = 62$

b $t_5 = 8 + 5 \times -3 = -7$

c $t_5 = 14 + 5 \times 9 = 59$

d $t_5 = 62 + 5 \times -27 = -73$

e $t_5 = -9 + 5 \times 5 = 16$

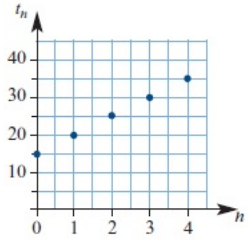
f $t_5 = -13 + 5 \times -6 = -43$

3 The starting value of the sequence is the first number so $a + 12$. The common difference is $t_1 - t_0 = 10$ so $D = 10$. Since we are finding t_{20}, $n = 20$.

4 To complete the blanks, note the value of n for each line, that $a = 7$ and that $D = 3$.

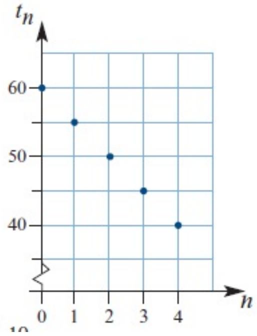
$$t_1 = t_0 + 1 \times 3 = 7 + 1 \times 3 = 10$$
$$t_2 = t_0 + 2 \times 3 = 7 + \ldots \times 3 = 13$$
$$t_3 = t_0 + 3 \times 3 = 7 + \ldots \times 3 = 16$$
$$t_{20} = t_0 + 20 \times 3 = 7 + \ldots \times 3 = 67$$

5 **a** The starting term is 15 and then 5 is added to generate the next term. Thus, the first five terms of the sequence are 15, 20, 25, 30, 35.



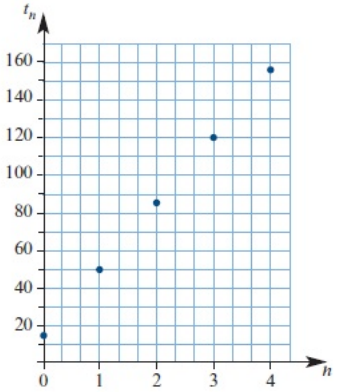
b To find t_{44}, calculate
$t_{44} = 15 + 44 \times 5 = 235$

6 **a** The starting term is 60 and then 5 is subtracted to generate the next term. Thus, the first five terms of the sequence are 60, 55, 50, 45, 40.



b To find t_{10}, calculate
$t_{10} = 60 - 10 \times 5 = 10$

7 **a** The starting term is 15 and then 35 is added to generate the next term. Thus, the first five terms of the sequence are 15, 50, 85, 120, 155.



b To find t_{15}, calculate $t_{15} = 15 + 15 \times 35 = 540$

8 **a** The starting value is 18 ($a = 18$), the common difference is 3 ($D = 3$) and $n = 35$. Thus, $t_{35} = 18 + 35 \times 3 = 123$.

b The starting value is −14 ($a = -14$), the common difference is 8 ($D = 8$) and $n = 41$. Thus, $t_{41} = -14 + 41 \times 8 = 314$.

c The starting value is 27 ($a = 27$), the common difference is −7 ($D = -7$) and $n = 37$. Thus, $t_{37} = 27 + 37 \times (-7) = -454$.

d The starting value is 16 ($a = 16$), the common difference is 15 ($D = 15$) and $n = 29$. Thus, $t_{29} = 16 + 29 \times 15 = 451$.

e The starting value is −19 ($a = -19$), the common difference is −4 ($D = -4$) and $n = 26$. Thus, $t_{26} = -19 + 26 \times (-4) = -123$.

f The starting value is 0.8 ($a = 0.8$), the common difference is 0.7 ($D = 0.7$) and $n = 36$. Thus, $t_{36} = 0.8 + 36 \times 0.7 = 26$.

g The starting value is 82 ($a = 82$), the common difference is 14 ($D = 14$) and $n = 21$. Thus, $t_{21} = 82 + 21 \times 14 = -212$.

h The starting value is 9.4 ($a = 9.4$), the common difference is 0.6 ($D = 0.6$) and $n = 29$. Thus, $t_{29} = 9.4 + 29 \times 0.6 = -8$.

9 Note that $a = 11$, $D = 8$ and $n = 40$ so $t_{40} = 11 + 8 \times 40 = 331$.

10 Note that $a = 27$, $D = 19$ and $n = 100$ so $t_{100} = 27 + 19 \times 100 = 1927$.

11 Note that $a = 100$, $D = -7$ and $n = 20$ so $t_{20} = 100 - 7 \times 20 = -40$.

12 Since the initial height was 1.5 m, $a = 1.5$. The tree grew by 0.75m per year so $D = 0.75$. Since we want to know the height of the tree after 18 years, $t_{18} = 1.5 + 0.75 \times 12 = 15$m.

13 **a** Substituting in $t_4 = 10$ and $n = 4$ gives $10 = a + 4D$.

b Substituting in $t_8 = 18$ and $n = 8$ gives $18 = a + 8D$.

c A CAS calculator or algebra can be used to solve the two equations simultaneously. For example, Subtracting the equation in **a** from the equation in **b** gives $8 = 4D$ so $D = 2$. Substituting $D = 2$ back into either equation gives $a = 2$.

d Using these values,
$t_5 = 2 + 2 \times 5 = 12$ and
$t_6 = 9 + 2 \times 9 = 20$.

e The first term is the starting value, 2. The next two values are found by adding 2. Thus, the first three terms are 2, 4, 6.

14 The terms in this sequence are found by adding 2. The sequence continues as follows: 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25. Thus, the number of terms that are **less** than 25 is 10.

Solutions to Exercise 3E

1 The amount of interest per year is found by calculating 4% of $5000.

$$\frac{4}{100} \times 5000 = \$200$$

2 The amount of depreciation per year is found by calculating 12% of $3000.

$$\frac{12}{100} \times 3000 = \$360$$

3 The recurrence relation can be written by noting that after n years, D is added n times to the initial value of V_0. Thus, $V_n = V_0 + n \times D$.

4 To calculate the amount of interest paid each year given an interest rate of r%, find $r\% \times$ investment.

a $\frac{7.3}{100} \times 10\,000 = \$730$

b $\frac{3.8}{100} \times 16\,500 = \$627$

b $\frac{5.4}{100} \times 214\,600 = \$11\,588.40$

5 **a** $V_1 = V_0 + 450 = 10\,000 + 450 = 10\,450$

$V_2 = V_1 + 450 = 10\,450 + 450 = 10\,900$

$V_3 = V_2 + 450 = 10\,900 + 450 = 11\,350$

b To find when the value will first exceed $14 500, continue adding $450 until you first exceed $14 500. Start at 10 000 and use repeated addition of 450 on your calculator until the value $14 500 is reached.

10 000, 10 450, 10 900, 11 350,
11 800, 12 250, 12 700, 13 150,
13 600, 14 050, 14 500

Without counting the starting value of 10 000 with see that 10 additions (years) were required to reach a value of 14 500.The investment will have a value of $14 500 after 10 years.

6 **a** $V_1 = V_0 + 400 = 8000 + 400 = 8400$

$V_2 = V_1 + 400 = 8400 + 400 = 8800$

$V_3 = V_2 + 400 = 8800 + 400 = 9200$

b To find when the value will first exceed $12 000, continue adding $400 until you first exceed $12 000. Start at 8000 and use repeated addition of 400 on your calculator until the value $12 000 is reached.

8000, 8400, 8800, 9200,
9600, 10 000, 10 400, 10 800,
11 200, 11 600, 12 000

Without counting the starting value of 8000 with see that 10 additions (years) were required to reach a value of 12 000.

The investment will have a value of $12 000 after 10 years.

c The initial investment was the value of V_0 which was $8000.

d The amount of interest earned was $400. To find the interest rate, calculate:

$$\frac{400}{8000} \times 100 = 5\%$$

7 **a** $V_0 = 90\,000$

$V_1 = V_0 - 9000 = 90\,000 - 9000 =$ \$81 000

$V_2 = V_1 - 9000 = 81\,000 - 9000 =$ \$72 000

$V_3 = V_2 - 9000 = 90\,000 - 9000 =$ \$63 000

b Start at 90 000 and use repeating subtraction of 9000 on your calculator until 5 subtractions have been applied.
90 000, 81 000, 72 000, 63 000
54 000, 45 000
Without counting the starting value of 90 000 with see that 5 subtractions (years) resulted in a value of 45 000.
The tractor will have a value of \$45 000 after 5 years.

c Start at 90 000 and use repeated subtraction of 9000 on your calculator until a value of zero has been reached.
90 000, 81 000, 72 000, 63 000
54 000, 45 000, 36 000, 27 000
18 000, 9 000, 0
Without counting the starting value of 90 000 with see that 10 subtractions (years) resulted in a value of zero.
The tractor will have a zero value after 10 years.

d The original value is \$95 000,
$V_0 = 95\,000$
The value depreciates at a flat rate of 12% per year.
12% of \$95 000 = $0.12 \times 95\,000 =$ 11 400
The value decreases by \$11 400 after each year so:
$V_{n+1} = V_n - 11\,400$
The recurrence relation is:
$V_0 = 95\,000, \quad V_{n+1} = V_n - 11\,400$

8 **a** The recurrence relation is:
$V_0 = 2400, V_{n+1} = V_n - 300$
$V_0 = 2400$, so the computer had a value of \$2400 when new.

b $V_0 = 2400, V_{n+1} = V_n - 300$, so the value in the next year would by \$300 less than the value in the current year.
The computer depreciated by \$300 after each year.

c Solving $\frac{r}{100} \times 2400 = 300$ for r gives $r = 12.5$
The percentage flat rate of depreciation was 12.5%.

d Start at 2400 and use repeated subtraction of 300 on your calculator until a value of 600 has been reached:
2400, 2100, 1800, 1500, 1200, 900, 600
Without counting the starting value of 2400 with see that 6 subtractions (years) resulted in a value of 600.
The computer will have a value of \$600 after 6 years.

e Half of its new price = $0.5 \times 2400 =$ 1200
From part **d** we see that it was at 1200 after 4 years.

9 **a** The value after making 5000 pairs of jeans is found when $n = 5$.
$V_0 = \$18\,000$
$V_1 = V_0 - 200 = 18\,000 - 200 =$ \$17 800

$V_2 = V_0 - 200 = 17\ 800 - 200 =$ \$17 600
$V_3 = V_0 - 200 = 17\ 600 - 200 =$ \$17 400
$V_4 = V_0 - 200 = 17\ 400 - 200 =$ \$17 200
$V_5 = V_0 - 200 = 17\ 200 - 200 =$ \$17 000
Thus the value of the sewing machine after making 5000 pairs of jeans is \$17 000.

b The value of the machine after making 20 000 pairs of machines is found when $n = 20$. By subtracting \$200 from \$18 000 a total of 20 times, the vaue is found to be \$14 000.

c Since \$200 is subtracted for every 1000 pairs of jeans, we can calculate \$18 000 ÷ \$200 to find the number of times we can subtract \$200 before reaching 0. Thus, \$18 000 ÷ \$200 = 90.
Thus, 90 000 pairs of jeans can be made before the value of the machine reaches \$0.

d Since the starting value is \$20 000, $V_0 = 20\ 000$. The machine depreciates by \$250 for every 1000 pairs of jeans. Thus, we subtract 250 from the current value to find the next value. That is, $V_{n+1} = V_n - 250$.
Thus, the recurrence relation is:
$V_0 = 20\ 000 \qquad V_{n+1} = V_n - 250$

10 **a** The starting value of the scissor-lift is given by V_0 which was \$26 500.

b Considering the recurrence relation, the value of the next term decreases by 70. Thus, the scissor-lift depreciates by \$70 every 1000 uses.

c The value of the scissor-lift when it is half of its original value is 26 500 ÷ 2 = 13 250. Since the value declines by 70 for every 1000 uses, we want to solve $26\ 500 - 70n = 13\ 250$ for n. This gives 190 which means that the scissor-lift devalues to less than half of its original value after 190 000 uses.

11 **a** Looking at the recurrence relation, $V_{n+1} = V_n + 800$ we can see that the value of V_{n+1} is 800 more than V_n. Thus, the value of the motorbike increases by \$800 each year.

b Since \$800 is added each year, the amount added to the initial investment (of \$32 000) after 15 years is 15 × \$800. That is, $V_{15} = 32\ 000 + 15 \times 800 = \$44\ 000$

c The rule for the value of V_n is $V_n = 32\ 000 + n \times 800$. To find when the value of the investment reaches \$40 000, solve $40\ 000 = 32\ 000 + n \times 800$ for n to give $n = 10$. Thus, it takes 10 years for the investment to reach \$40 000.

12 **a** Looking at the recurrence relation, $V_{n+1} = V_n - 500$ we can see that the value of V_{n+1} is 500 less than V_n. Thus, the value of the motorbike depreciates by \$500 each year.

b The motorbike depreciates by \$500 each year for four years, thus, the value is $V_4 = 4000 - 4 \times 500 = \$2000$.

c The rule for the value of the motorbike after n years is $V_n = 4000 - n \times 500$. To find the number of years that it takes for the bike to be worth \$0, solve $0 = 4000 - n \times 500$ for n. This gives $n = 8$ so it takes 8 years for the bike to be worth \$0.

13 Bruce is receiving \$459 in simple interest each year so the common difference, D, is 459. The value after 5 years is $V_5 = 10\,795$, thus, we want to solve $V_5 = a + n \times D$ or equivalently, $10\,795 = a + 5 \times 459$ for a, where a is the initial invesment. Thus, the initial investment is \$8500.

Solutions to Exercise 3F

1 **a** The numerator and denominator both have a common factor of 3 so $\frac{3}{6} = \frac{1}{2}$

b The numerator and denominator both have a common factor of 3 so $\frac{81}{3} = \frac{27}{1} = 27$

c The numerator and denominator both have a common factor of 9 so $\frac{27}{9} = \frac{3}{1} = 3$

d The numerator and denominator both have a common factor of 4 so $\frac{4}{64} = \frac{1}{16}$

2 **a** $\frac{8}{4} = 2$ so the common ratio is 2.

b $\frac{3}{1} = 3$ so the common ratio is 3.

c $\frac{16}{8} = 2$ so the common ratio is $\frac{1}{2}$.

d $\frac{24}{48} = \frac{1}{2}$ so the common ratio is $\frac{1}{2}$.

3 **a** Arithmetic: Add 2 to generate each term.

b Geometric: Mutliply each term by 4.

c Geometric: Multiply each term by $\frac{1}{3}$.

d Arithmetic: Subtract 5 to generate each term.

4 **a** $\frac{6}{3} = 2$ so the common ratio is 2.

b $\frac{16}{64} = \frac{1}{4}$ so the common ratio is $\frac{1}{4}$.

c $\frac{30}{6} = 5$ so the common ratio is 5.

d $\frac{8}{2} = 4$ so the common ratio is 4.

e $\frac{16}{32} = \frac{1}{2}$ so the common ratio is $\frac{1}{2}$.

f $\frac{12}{2} = 6$ so the common ratio is 6.

g $\frac{100}{10} = 10$ so the common ratio is 10.

h $\frac{21}{3} = 7$ so the common ratio is 7.

5 **a** Geometric: $\frac{8}{4} = \frac{16}{8} = \frac{32}{16} = 2$. There is a common ratio of 2.

b Geometric: $\frac{3}{1} = \frac{9}{3} = \frac{27}{9} = 3$. There is a common ratio of 3.

c Not Geometric: Sequence is arithmetic as there is a common difference of 5 since 5 is added to generate subsequent terms.

d Geometric: $\frac{15}{5} = \frac{45}{15} = \frac{135}{45} = 3$. There is a common ratio of 3.

e Geometric: $\frac{24}{12} = \frac{12}{6} = \frac{6}{3} = \frac{1}{2}$. There is a common ratio of $\frac{1}{2}$.

f Not Geometric: To obtain the second term, multiply by 2 but to obtain the fourth term, multiply by 1.5.

g Not Geometric: Sequence is arithmetic as there is a common difference of 4 since 4 is added to generate subsequent terms.

h Geometric: $\frac{9}{27} = \frac{3}{9} = \frac{1}{3} = \frac{1}{3}$. There is a common ratio of $\frac{1}{3}$.

i Geometric: $\frac{4}{2} = \frac{8}{4} = \frac{16}{8} = 2$. There is a common ratio of 2.

6 a The common ratio is 2 since $\frac{14}{7} = 2$. Thus, the missing tems are $28 \times 2 = 56$ and $56 \times 2 = 112$.

b The common ratio is 5 since $\frac{15}{3} = 5$. Thus, the missing tems are $75 \times 5 = 375$ and $375 \times 5 = 1875$.

c The common ratio is 3 since $\frac{12}{4} = 3$. Thus, the missing tems are $12 \times 3 = 36$ and $36 \times 3 = 108$.

d The common ratio is 2 since $\frac{40}{20} = 2$. Thus, the missing tems are $x \times 2 = 20$ so $x = 10$ and $y \times 2 = 10$ so $y = 5$. Thus, the two missing numbers are 5 and 10.

e The common ratio is 4 since $\frac{128}{32} = 4$. Thus, the missing tems are $2 \times 4 = 8$ and $128 \times 4 = 512$.

f The common ratio is 5 since $\frac{729}{243} = 3$. Thus, the missing tems are $3 \times 3 = 9$ and $27 \times 3 = 81$.

7 a $a = 7, R = 5$

Type 7. Press ENTER (or EXE).

Type × 5. Press ENTER (or EXE) six more times to find t_6.

5, 35, 175, 875, 4375, 21 875, 109 375

$t_6 = 109\ 375$

b $a = 3, r = 6$

Type 3. Press ENTER (or EXE).

Type × 6. Press ENTER (or EXE) six more times to find t_6.

6, 18, 108, 648, 3888, 23 328, 139 968

$t_6 = 139\ 968$

c $a = 96, r = \frac{1}{2}$

Type 96. Press ENTER (or EXE).

Type $\times\frac{1}{2}$. Press ENTER (or EXE) six more times to find t_6.

96, 48, 24, 12, 6, 3, 1.5

$t_6 = 1.5$

d $a = 4, r = 7$

Type 4. Press ENTER (or EXE).

Type × 7. Press ENTER (or EXE) six more times to find t_6.

4, 28, 196, 1372, 9604, 67 228, 470 596

$t_6 = 470\ 596$

e $a = 160, r = \frac{1}{2}$

Type 160. Press ENTER (or EXE).

Type $\times\frac{1}{2}$. Press ENTER (or EXE) six more times to find t_6.

160, 80, 40, 20, 10, 5, 2.5

$t_6 = 2.5$

f $a = 11, r = 9$

Type 11. Press ENTER (or EXE).

Type × 9. Press ENTER (or EXE) six more times to find t_6.

11, 99, 891, 8019, 72 171, 649 539, 5 845 851

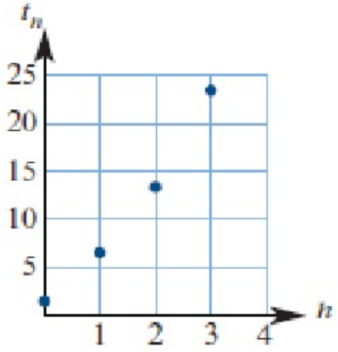
$t_6 = 5\ 845\ 851$

8 **a** **i** $t_3 = t_2 \times 2 = 24$

ii

n	0	1	2	3
t_n	3	6	12	24

iii



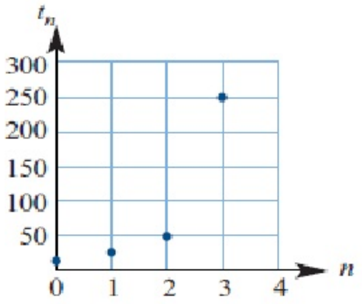
iv The graph is a curve with values that are increasing at an increasing rate.

i $t_3 = t_2 \times 5 = 250$

ii

n	0	1	2	3
t_n	2	10	50	250

iii



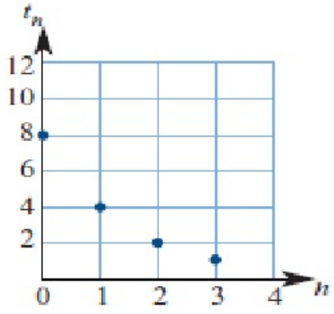
iv The graph is a curve with values that are increasing at an increasing rate.

9 **a** **i** $t_3 = t_2 \times \frac{1}{2} = 1$

ii

n	0	1	2	3
t_n	8	4	2	1

iii



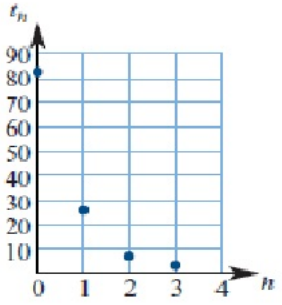
iv The graph is a curve with values decreasing and approaching zero.

i $t_3 = t_2 \times \frac{1}{3} = 3$

ii

n	0	1	2	3
t_n	81	27	9	3

iii



iv The graph is a curve with values decreasing and approaching zero.

10 **a** When a number increases by 10%, we need to multiply it by 110%. Since % per "per 100", this is the same as multiplying it by $\frac{110}{100} = 1.1$.

b Multipying 20 by 1.1 gives the next term of 22. Repeatedly multiplying by 1.1 gives the first four terms of the sequence: 20, 22, 24.2, 26.62

11 **a** When a number decreases by 10%, we need to multiply it by 100% − 10% = 90%. Since % per "per 100", this is the same as multiplying it by $\frac{90}{100} = 0.9$.

b Multipying 100 by 0.9 gives the next term of 90. Repeatedly multiplying by 0.9 gives the first four terms of the sequence: 100, 90, 81, 72.9

12 **a** $\frac{12\,000}{10\,000} = 1.2$

b $\frac{14\,400}{12\,000} = 1.2$ and $\frac{17\,280}{14\,400} = 1.2$

c Since the ratio between successive terms is 1.2, there is a 20% increase from t_0 to t_1.

d To find the next term, calculate:
17 280 × 1.2 = 20 736.

13 **a** On the CAS, type 500 and press ENTER.
Type ×4 − 5 and then press ENTER 5 four times so that you have a total of 5 terms:
500, 195, 73, 24.2, 4.68

b The sequence is not arithmetic as each term in the sequence cannot be generated by simply adding or subtracting a common difference. The sequence is not geometric as each term in the sequence cannot be generated by simply multiplying by the same value.

c As seen in the sequence in part **a**, the terms are decreasing from 500 and so it is impossible to get to 800.

d The easiest way to do this is to use finance solver on your CAS. Trial and error can also be used but it would take a long time as there are 20 736 iterations!

Solutions to Exercise 3G

1 Recall that *a* represents the starting value and *R* represents the common ratio.

a The starting value is 2 so $a = 2$.
$\frac{6}{2} = 3$ so $R = 3$.

b The starting value is 5 so $a = 5$.
$\frac{20}{5} = 4$ so $R = 4$.

c The starting value is 5 so $a = 5$.
$\frac{10}{5} = 2$ so $R = 2$.

d The starting value is 3 so $a = 3$.
$\frac{12}{3} = 4$ so $R = 4$.

2 **a** From **1a**, $a = 2$ and $R = 3$ so the rule is $t_n = 3^n \times 2$.
Thus, $t_5 = 3^5 \times 2 = 486$.

a From **1b**, $a = 5$ and $R = 5$ so the rule is $t_n = 5^n \times 5$.
Thus, $t_5 = 5^5 \times 5 = 5120$.

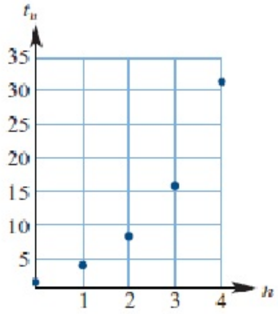
a From **1a**, $a = 5$ and $R = 2$ so the rule is $t_n = 5^n \times 2$.
Thus, $t_5 = 5^5 \times 2 = 160$

a From **1a**, $a = 3$ and $R = 4$ so the rule is $t_n = 3^n \times 4$.
Thus, $t_5 = 3^5 \times 4 = 3072$

3

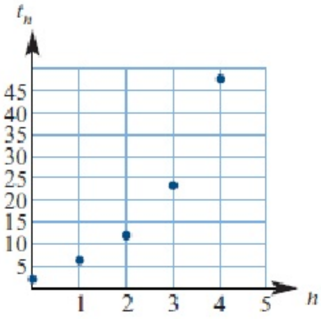
$$t_1 = R^1 \times t_0 = 2^1 \times 6 = 2 \times 6 = 12$$
$$t_2 = R^2 \times t_0 = 2^2 \times 6 = 4 \times 6 = 24$$
$$t_3 = R^3 \times t_0 = 2^3 \times 6 = 8 \times 6 = 48$$
$$t_{20} = R^{20} \times t_0 = 2^{20} \times 6 = 1\,048\,576 \times 6$$
$$= 6\,291\,456$$

4 **a** Using the starting value of 2 and the rule of multiplying by 2, the first five terms are 2, 4, 8, 16, 32.
To graph, place the values of *n* along the *x*-axis and the terms of the sequence along the *y*-axis.



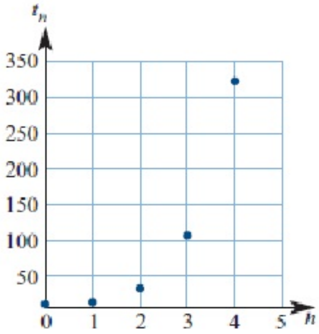
b The value of t_{10} can be obtained by typing 2 into the CAS and pressing ENTER. Then press ×2 and press ENTER 10 times. This gives $t_{10} = 2048$.

5 **a** Using the starting value of 3 and the rule of multiplying by 2, the first five terms are 3, 6, 12, 24, 48.
To graph, place the values of *n* along the *x*-axis and the terms of the sequence along the *y*-axis.



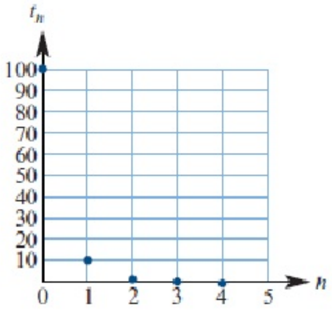
b The value of t_{12} can be obtained by typing 3 into the CAS and pressing ENTER. Then press ×2 and press ENTER 12 times. This gives $t_{10} = 12\ 288$.

6 **a** Using the starting value of 4 and the rule of multiplying by 3, the first five terms are 4, 12, 36, 108, 324.
To graph, place the values of n along the x-axis and the terms of the sequence along the y-axis.



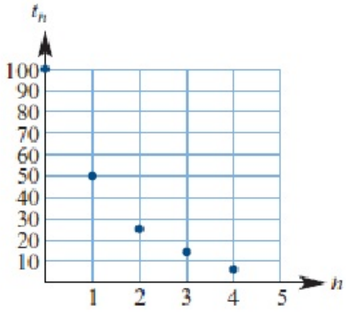
b The value of t_{10} can be obtained by typing 4 into the CAS and pressing ENTER. Then press ×3 and press ENTER 10 times. This gives $t_{10} = 236\ 196$.

7 **a** Using the starting value of 100 and the rule of multiplying by $\frac{1}{10}$, the first five terms are $100, 10, 1, \frac{1}{10}, \frac{1}{100}$.
To graph, place the values of n along the x-axis and the terms of the sequence along the y-axis.



b The value of t_{10} can be obtained by typing 100 into the CAS and pressing ENTER. Then press $\times\frac{1}{10}$ and press ENTER 10 times. This gives $t_{10} = \frac{1}{100\ 000\ 000}$.

8 **a** Using the starting value of 100 and the rule of multiplying by $\frac{1}{2}$, the first five terms are 100, 50, 25, 12.5, 6.25.
To graph, place the values of n along the x-axis and the terms of the sequence along the y-axis.



b The value of t_{15} can be obtained by typing 100 into the CAS and pressing ENTER. Then press $\times\frac{1}{2}$ and press ENTER 15 times. This gives $t_{10} = 0.00305$.

9 Type 10 on your CAS and ENTER then ×2 the required number of times.

a Press ENTER a total of 3 times: $t_3 = 80$.

b Press ENTER a total of 5 times: $t_5 = 320$.

c Press ENTER a total of 12 times: $t_{12} = 40\,960$.

d Press ENTER a total of 15 times: $t_{15} = 327\,680$.

e Press ENTER a total of 20 times: $t_{20} = 10\,485\,760$.

f Press ENTER a total of 25 times: $t_{25} = 335\,544\,320$.

10 **a** Type in the starting value of 6 and ENTER then ×2. Press ENTER a total of 30 times: $t_{30} = 6\,442\,450\,944$.

b Type in the starting value of 4 and ENTER then ×3. Press ENTER a total of 30 times: $t_{30} = 823\,564\,528\,378\,596$.

c Type in the starting value of 2000 and ENTER then ×0.5. Press ENTER a total of 30 times: $t_{30} = 0.00000186$.

d Type in the starting value of 10 000 and ENTER then ×0.5. Press ENTER a total of 30 times: $t_{30} = 0.00000931$.

11 Type in the starting value of 4 and ENTER then ×2 since the common ratio is 2. Press ENTER a total of 20 times to get $t_{20} = 4\,194\,304$.

12 Type in the starting value of 5 and ENTER then ×2 since the common ratio is 2. Press ENTER a total of 10 times to get $t_{10} = 2048$.

13 Type in the starting value of 5000 and ENTER then $\times\frac{1}{2}$ since the common ratio is $\frac{1}{2}$. Press ENTER a total of 20 times to get $t_{20} = 0.004768$.

14 Since the paper starts as a 1 m^2 square, the starting value is 1. Being cut in half each day means that the common ratio is $\frac{1}{2}$. On your CAS, type in the starting value of 1 and ENTER then $\times\frac{1}{2}$ a total of 7 times to get $t_7 = 0.0078125$. Thus, the area of the paper left at the end of the 7th day is 0.0078125m^2.

15 **a** Since $n = 2$ and $t_2 = 12$, we can substitute in these values into the rule $t_n = R^n \times t_0$ to get $12 = R^2 \times t_0$.

b Since $n = 5$ and $t_5 = 96$, we can substitute in these values into the rule $t_n = R^n \times t_0$ to get $96 = R^5 \times t_0$.

c On your CAS, enter the two equations: $12 = R^2 \times t_0$ and $96 = R^5 \times t_0$

and solve for the two unknown variables R and t_0. This gives $R = 2$ and $t_0 = 3$.

d Using the values found in **15c**, the recurrence relation is $t_0 = 3, \qquad t_{n+1} = 2 \times t_n$.

e Substituting in $n = 3$, $t_3 = 24$ and then $n = 4$ gives $t_4 = 48$. Checking by performing the iterations:

$t_0 = 3$
$t_1 = 6$
$t_2 = 12$
$t_3 = 24$
$t_4 = 48$

f Continuing in the same manner by multiplying the previous term by 2 or by using the rule to calculate t_6, the first fix terms are: 3, 6, 12, 24, 48, 96

Solutions to Exercise 3H

1 a Since the common ratio is $4 > 1$ and nothing is added or subtracted, the recurrence relation models geometric growth.

b Since the common ratio is $\frac{1}{5} < 1$ and nothing is added or subtracted, the recurrence relation models geometric decay.

c Since 2 is subtracted, the recurrence relation models neither geometric growth nor decay.

d Since 10 is added, the recurrence relation models neither geometric growth nor decay.

e Since the common ratio is $0.2 < 1$ and nothing is added or subtracted, the recurrence relation models geometric decay.

f Since the common ratio is $1.1 > 1$ and nothing is added or subtracted, the recurrence relation models geometric growth.

2 a The amount of money that is initially invested is the starting value which is \$5000.

b To generate a new term, the current value is multiplied by 1.05, or $\frac{105}{100}\%$. Thus, the values are growing by 5% so the annual interest rate is 5%.

c The value of the investment after one year is given by
$V_1 = 1.05 \times 5000 = \$5250$.

3 a The initial value of the asset is given by its starting value which is \$40 000.

b To generate a new term, the current value is multiplied by 0.90, or $\frac{90}{100}\%$. Thus, the values are decreasing by 10% so the annual percentage depreciation of the asset is 10%.

c The value of the investment after one year is given by
$V_1 = 0.9 \times 40\,000 = \$36\,000$.

4 a Quarterly means that payments are made 4 times a year so the annual rate is divided by 4, giving an interest rate per quarter of $\frac{4.5}{4}\% = 1.25\%$.

b Monthly means that payments are made 12 times a year so the annual rate is divided by 12, giving an interest rate per month of $\frac{4.5}{12}\% = 0.375\%$.

5 a Using the recurrence relation:
$V_0 = 10\,000$
$V_1 = 1.045 \times 10\,000 = 10\,450$
$V_2 = 1.045 \times 10\,450 = 10\,920.25$
$V_3 = 1.045 \times 10\,920.25 = 11\,411.66$

b Type the initial value of 10 000 into the CAS and press ENTER. Type ×1.045 and press ENTER until the value exceeds \$12 000. Count the number of times that you multiplied by 1.045. Since you pressed ENTER 5 times, it takes 5 years for the value of the investment to first exceed \$12 000, with $V_5 = 12\,161.82$.

6 **a** Using the recurrence relation:
$V_0 = 200\,000$
$V_1 = 1.052 \times 200\,000 = 210\,400$
$V_2 = 1.052 \times 210\,400 = 221\,340.80$
$V_3 = 1.052 \times 221\,340.80 =$
$232\,850.52$

b Type the initial value of 200 000 into the CAS and press ENTER. Type ×1.052 and press ENTER until the value exceeds \$265 000. Count the number of times that you multiplied by 1.052. Since you pressed ENTER 6 times, it takes 6 years for the value of the investment to first exceed \$265 000, with $V_6 = 271\,096.83$.

7 **a** The starting value of \$8000 gives the initial amount that was invested.

b On a CAS, type 8000 and press ENTER. Then type ×1.075 and press ENTER. Continue pressing ENTER until the value first exceeds \$10 000. Since the value was multiplied by 1.075 on four occassions to give $V_4 = 10\,683.75$, it takes 4 years for the value to first exceed \$10 000.

c The common ratio is 1.075 meaning that the value is increasing by 7.5% per year so the interest rate is 7.5%.

8 **a** Using the recurrence relation:
$V_0 = 100\,000$
$V_1 = 1.063 \times 100\,000 = 106\,300$
$V_2 = 1.063 \times 106\,300 = 112\,996.90$
$V_3 = 1.063 \times 112\,996.90 =$
$120\,115.70$

b Double the initial investment is $100\,000 \times 2 = 200\,000$. Type the initial value of 100 000 into the CAS and press ENTER. Type ×1.063 and press ENTER until the value exceeds \$200 000. Count the number of times that you multiplied by 1.063. Since you pressed ENTER 12 times, it takes 12 years for the value of the investment to first exceed \$200 000, with $V_{12} = 208\,160.91$.

9 **a** Since the annual interest rate is 6% and interest is paid monthly, we need to divide the annual rate by 12. Thus, the monthly interest rate is $\frac{6}{12} = 0.5$.

b The initial investment is \$80 000 so the starting value is $V_0 = 80\,000$. Since the value of the investment is increasing by 0.5% each month, the common ratio is 1.005, giving the second part of the recurrence relation as $V_{n+1} = 1.005V_n$.

c Using the recurrence relation:
$V_0 = 80\,000$
$V_1 = 1.005 \times 80\,000 = 80\,400$
$V_2 = 1.005 \times 80\,400 = 80\,802$
$V_3 = 1.005 \times 80\,802 = 81\,206.01$

10 **a** Using the recurrence relation:
$V_0 = 90\,000$
$V_1 = 0.85 \times 90\,000 = 76\,500$
$V_2 = 0.85 \times 76\,500 = 65\,025$

$V_3 = 0.85 \times 65\,025 = 55\,271.25$
$V_4 = 0.85 \times 55\,271.25 = 46\,980.56$
$V_5 = 0.85 \times 46\,980.56 = 39\,933.48$

b As the tractor will be sold after 8 years, we need to find the value of V_8 which can be done by continuing to use the recurrence relation as in **10a**. Thus, $V_8 = 24\,524.15$

11 **a** The value of the refrigerator when it was new is the starting value given as $1200.

b To find when the refrigerator will first be less than $200, enter the starting value on the CAS of 1200. Then type ×0.56 and press ENTER until the value is first less than $200. Since you need to press ENTER 15 times, it takes 15 years for the value to first be below $200.

c Half of the original value of the refrigerator is 1200 ÷ 2 = $600. To find when the value of the refrigerator will first be less than $600, enter the starting value on the CAS of 1200. Then type ×0.56 and press ENTER until the value is first less than $600. Since you need to press ENTER 6 times, it takes 6 years for the value to first be below $200.

d The percentage rate of depreciation is 100 – 56 = 44 so there is a 44% rate of depreciation each year.

12 **a** The value of the car when it was due is given by the starting value of $84 000.

b To find the value of the car after 1 year, note that the car has depreciated by 3.5% so we need to multiply the value by 100 – 3.5 = 96.5%. Thus, $V_1 = 0.965 \times 84\,000 = 81\,060$.

c Thus, the recurrence relation can be written as $V_0 = 84\,000$ for the starting value and $V_{n+1} = 0.965 \times V_n$ since the value declines to 96.% of the previous value.

d As shown above.

e To generate the sequence, start with the starting value and then multiply the current term by 0.965 to generate the next term. This gives the first five calues as follows: $84 000, $81 060, $78 222.90, $75.485.10, $72 843.12

f To find how much the value of the car has declined in five years, you need to subtract the value of the car after five years from the initial value: 84 000 – 72 843.12 = $13 706.39.

13 To determine which option gives Jackson the largest value after five years, the value must be calculated for each option:
A: $20\,000 + \dfrac{5 \times 7}{100} \times 20\,000 = \$27\,000$
B: $20\,000 \times 1.065^5 = \$27\,401.73$
C: $20\,000 \times 1.005^{60} = \$26\,977.00$
Since **B** has the highest value, this is the best option for Jackson. The amount of interest that he earns is 27 401.73 – 20 000 = $7401.73

Solutions to Exercise 3I

1 We can use the rule $t_n = R^n \times t_0$ where $R = 3$ and $t_0 = 5$.

a To find t_1, we let $n = 1$ in the rule: $t_1 = 3^1 \times 5 = 15$.

b To find t_3, we let $n = 3$ in the rule: $t_3 = 3^3 \times 5 = 135$.

c To find t_5, we let $n = 5$ in the rule: $t_1 = 3^5 \times 5 = 1215$.

d To find t_7, we let $n = 7$ in the rule: $t_3 = 3^7 \times 5 = 10\,935$.

2 **a** Here, $n = 4$, $R = 3$ and $V_0 = 5$ so $V_4 = 3^4 \times 5 = 405$.

b Here, $n = 4$, $R = 2$ and $V_0 = 10$ so $V_4 = 2^4 \times 10 = 160$.

c Here, $n = 4$, $R = 0.5$ and $V_0 = 1$ so $V_4 = 0.5^4 \times 1 = 0.0625$.

d Here, $n = 4$, $R = 0.25$ and $V_0 = 200$ so $V_4 = 0.25^4 \times 200 = 0.78125$.

3 Here, $R = 1.05$, $V_0 = 5000$

a $n = 6$ so: $V_6 = 1.05^6 \times 5000 = 6700.48$

b $n = 10$ so: $V_{10} = 1.05^{10} \times 5000 = 8144.47$

c $n = 100$ so: $V_{100} = 1.05^{100} \times 5000 = 657\,506.29$

4 **a** Since the starting value is 10 000, the initial amount of money invested was \$10 000.

b Since $R = 1.1$, the value of the investment increases by 0.1 of the value each year which corresponds to an interest rate of 10% per year.

c Using the starting value and $R = 1.1$, the rule for the value of the investment after n years is $V_n = 1.1^n \times 10\,000$.

d Using the rule with $n = 5$, $V_5 = 1.1^5 \times 10\,000 = \$16\,105$.

e The answer from **4d** tells us the value of the investment after 5 years.

5 **a** Since the starting value is 12 000, the initial amount of money invested was \$12 000.

b Since $R = 1.08$, the value of the investment increases by 0.08 of the value each year which corresponds to an interest rate of 8% per year.

c Using the starting value and $R = 1.08$, the rule for the value of the investment after n years is $V_n = 1.08^n \times 12\,000$.

d Using the rule with $n = 4$, $V_4 = 1.08^4 \times 12\,000 = \$16\,326$.

6 **a** Since the starting value is 18 500, the initial value of the car was \$18 500.

b Since $R = 0.9$, the value of the investment decreases by 0.1 of the value each year which corresponds to a depreciation rate of 10% per year.

c Using the starting value and $R = 0.9$, the rule for the value of the investment after n years is $V_n = 0.9^n \times 18\,500$.

d Using the rule with $n = 4$, $V_4 = 0.9^4 \times 18\,500 = \$10\,924$.

e The answer from **6d** tells us the value of the car after 5 years.

7 **a** Since the starting value is 9 500, the initial value of the boat was \$9500.

b Since $R = 0.95$, the value of the investment decreases by 0.05 of the value each year which corresponds to a depreciation rate of 10% per year.

c Using the starting value and $R = 0.95$, the rule for the value of the investment after n years is $V_n = 0.95^n \times 9500$.

d Using the rule with $n = 10$, $V_{10} = 0.95^{10} \times 9\,500 = \$5688$.

e The answer from **7d** tells us the value of the boat after 10 years.

8 **a** Since the starting value is 520 000, the initial value of the loan was \$520 000.

b Since $R = 0.9965$, the value of the loan decreases by $1 - 0.9965 = 0.0035$ of the value each month which corresponds to a depreciation rate of 0.35% per month.

c Using the starting value and $R = 0.9965$, the rule for the value of the investment after n years is $V_n = 0.9965^n \times 520\,000$.

d Using the rule with $n = 6$, $V_6 = 0.9965^6 \times 520\,000 = \$509\,175$.

e The answer from **8d** tells us the value of the boat after 6 years.

9 **a** Using the rule with $n = 5$, $V_5 = 1.045^5 \times 10\,000 = \$12\,461.82$.

b The amount of interest is the total increase in the value of the investment: $12\,461.82 - 10\,000 = \$2461.82$.

c To find the amount of interest earned in any one year, we need to find how much the investment has increased in that year.
$V_5 - V_4 = 1.045^5 \times 10\,000 - 1.045^4 \times 10\,000 = 12\,461.82 - 11\,925.19 = 536.63$

d With an annual interest rate of 4.5%, the monthly interest rate is $\frac{4.5}{12} = 0.375$. Thus, $R = 1.00375$.
A rule for the value of the investment after n months is:
$I_n = 1.00375^n \times 10\,000$.

e The value of the investment after 5 years (60 months) is $I_{60} = 1.00375^{60} \times 10\,000 = \$12\,517.96$.

10 **a** Using the rule with $n = 10$, $V_{10} =$

$1.09^{10} \times 300\,000 = \$710\,209.10$.

b The amount of interest is the total increase in the value of the investment: $710\,209.10 - 300\,000 = \$410\,209.10$.

c To find the amount of interest earned in any one year, we need to find how much the investment has increased in that year. $V_{10} - V_9 = 1.09^{10} \times 300\,000 - 1.09^{10} \times 300\,000 = 710\,209.10 - 651\,567.98 = 58\,641.12$

d With an annual interest rate of 9%, the monthly interest rate is $\frac{9}{12} = 0.75$. Thus, $R = 1.0075$. A rule for the value of the investment after n months is: $I_n = 1.0075^n \times 300\,000$.

e The value of the investment after 10 years (120 months) is $I_{120} = 1.0075^{120} \times 10\,000 = \$735\,407.12$.

11 **a** Since the starting value is \$24 000, $V_0 = 24\,000$. Depreciation at a rate of 9.5% per year means that the common ratio is $1 - 0.095 = 0.905$, thus the rule is $V_n = 0.905^n \times 24\,000$.

b To find the value of the printer after 5 years, $n = 5$ so $V_5 = 0.905^5 \times 24\,000 = \$14\,569.82$.

c The total depreciation over 5 years is the difference in value: $V_5 - V_0 = 14\,569.82 - 24\,000 = \$9430.18$.

12 **a** The interest rate is 6% paid annually on an investment of \$5000, meaning that $\frac{6}{100} \times 5000 = \$300$ is paid each year. Thus, at the end of three years, the investment is valued based on the initial investment plus three years of interest: $5000 + 3 \times 300 = \$5900$.

b For a compound interest loan with an interest rate of 5.5%, the common ratio, R, is 1.055. Thus, the rule for the value of the investment after n years is $V_n = 1.055^n \times 5000$. If $n = 4$, then $V_4 = 1.055^4 \times 5000 = \$5871.21$.

c If a credit card payment is made on day 18 when the statement length is 30 days plus 12 days, then the number of interest free days is 24. This means that if a payment is made 800 days later then interest is paid on $800 - 24 = 776$. The daily interest rate when interest is 20% per annum is $\frac{20}{365 \times 100}$. Thus, the amount that must be paid is $\left(1 + \frac{20}{365 \times 100}\right)^{776} \times 5000 = \$7648.63$.

d In this case, the initial purchase, plus the initial fee plus a monthly fee of \$8 each month for three years must be paid: $5000 + 50 + 8 \times 3 \times 12 = \$5338$.

e Comparing the above answers, the cheapest option is the Buy-now, pay-later option.

13 **a** Using the formula:

$$\left(1 + \frac{18.9}{365 \times 100}\right)^{52} \times 2000 = \$2054.57$$

Thus, the amount of interest to be paid is $2054.57 - 2000 = \$54.57$

b Using the formula:

$$\left(1 + \frac{24}{365 \times 100}\right)^{200} \times 785 = \$895.29$$

Thus, the amount of interest to be paid is 895.29 – 785 = $110.29

c Using the formula:

$$\left(1 + \frac{22.5}{365 \times 100}\right)^{60} \times 12\,000 = \$12\,452.00$$

Thus, the amount of interest to be paid is 12 452 – 12 000 = $452

d Using the formula:

$$\left(1 + \frac{21.7}{365 \times 100}\right)^{90} \times 837 = \$882.99$$

Thus, the amount of interest to be paid is 882.99 – 837 = $45.99

14 To answer this question, we need to calculate the payment from each card so we can ascertain which one works out to be cheapest. We use the formula for each credit card.

a **A**: Only pay the cost of the item. $2000

B: Only pay the cost of the item. $2000

Both credit cards cost the same.

b **A**: Only pay the cost of the item. $2000

B: Pay interest on 20 days:

$$\left(1 + \frac{19}{365 \times 100}\right)^{20} \times 2000 = \$2020.93$$

Use Credit Card A.

c **A**: Pay interest on 30 days:

$$\left(1 + \frac{22}{365 \times 100}\right)^{30} \times 2000 = \$2036.48$$

B: Pay interest on 50 days:

$$\left(1 + \frac{19}{365 \times 100}\right)^{50} \times 2000 = \$2052.72$$

Use Credit Card A.

d **A**: Pay interest on 180 days:

$$\left(1 + \frac{22}{365 \times 100}\right)^{180} \times 2000 = \$2256.15$$

B: Pay interest on 200 days:

$$\left(1 + \frac{19}{365 \times 100}\right)^{200} \times 2000 = \$2196.40$$

Use Credit Card B.

15 **a** Inflation is applied for one year of 2.7% on the price of $3.50:

$$\frac{1.027}{100} \times 3.50 = \$3.59$$

b Inflation is applied for one year of 2.6% on the price of $3.50 then for one year at the rate of 3.5% on the new price. Alternatively, you can apply the rate of 3.5% to the answer generated in **15a**:

$$\frac{1.035}{100} \times 3.59\times = \$3.72$$

16 **a** Using the formula with an annual inflation rate of 1.9% and a price of $1.80: $\left(1 + \frac{1.9}{100}\right)^{20} \times 1.80 = \$2.62$

b Using the formula with an annual inflation rate of 7.1% and a price of $1.80: $\left(1 + \frac{7.1}{100}\right)^{20} \times 1.80 = \$7.10$

17 **a** Using the formula with an annual inflation rate of 3% and a final of $200 000: Solve $200\,000 = \left(1 + \frac{3}{100}\right)^{10} \times P$ for P gives $148 818.78

b Using the formula with an annual inflation rate of 13% and a final of $200 000: Solve $200\,000 = \left(1 + \frac{3}{100}\right)^{10} \times P$ for P

gives \$58 917.67

18 In this case, the initial value is \$7500 and the final value is \$10 155.61. If the interest rate is 6.25% we can set up an equation and solve for t as follows: $10\,155.61 = \left(1 + \frac{6.25}{100}\right)^t \times 7500$ to give

5 years

18 In this case, the initial value is \$5000 and the final value is \$20 000. If the interest rate is 4.75% we can set up an equation and solve for t as follows: $20\,000 = \left(1 + \frac{4.75}{100}\right)^t \times 5000$ to give 30 years

Solutions to Chapter Review Multiple-Choice Questions

1 The sequence is arithmetic with a common difference of 3. That is 3 is added. To find the next term, add 3 to 13 to get $13 + 3 = 16$ **E**

2 An arithmetic sequence has a common difference. Only C has a common difference,$D = 2$. **C**

3 $D = t_2 - t_1 = 19 - 27 = -8$ **B**

4 For this recurrence relation, the rule is $t_n = -7n + 63$. Since $n = 15$, $t_{15} = -42$ **A**

5 Note that the loan repayment will include 3 years of interest repayments plus the original amount borrowed:

$$110\,000 = \frac{4.6}{100} \times 110\,000 \times 3 = \$125\,180$$

E

6 A geometric has a common ratio. Only A has a common ratio, $R = 2$. **A**

7 Calculating the common ratio: $\frac{9}{27} = \frac{1}{3}$. **D**

8 For this recurrence relation, the rule is $t_n = 0.2^n \times 10\,000$. Since $n = 8$, $t_8 = 0.0256$ **A**

9 For this recurrence relation, the rule is $V_n = 1.06^n \times 12\,000$. Since $n = 3$, $V_3 = \$14\,292.19$ **D**

10 For this recurrence relation, the rule is $V_n = 1.0125^n \times 135\,000$. Since $n = 3$, $V_3 = \$192\,216.80$. The amount of interest earned is the difference between V_3 and V_0, giving interest of \$57 216.80 **D**

Solutions to Chapter Review Short-Answer Questions

1 **a** This sequence is an arithmetic sequence where 3 is added to generate the next term since $2 + 3 = 5, 5 + 3 = 8, 8 + 3 = 11$. The next two terms in the sequence can be generated: $11 + 3 = 14, 14 + 3 = 17$. Thus, the next two terms are 14 and 17.

b This sequence is an arithmetic sequence where 9 is added to generate the next term since $47 + 9 = 56, 56 + 9 = 65.65 + 9 = 74$. The next two terms in the sequence can be generated: $74 + 9 = 83, 83 + 9 = 92$. Thus, the next two terms are 83 and 92.

c This sequence oscillates between 16 and 60. The next two terms are 16 and 60.

d This sequence is a geometric sequence where each term is multiplied by 3 to generate the next term since $2 \times 3 = 6, 6 \times 3 = 18, 18 \times 3 = 54$. The next two terms in the sequence can be generated: $54 \times 3 = 162, 162 \times 3 = 486$. Thus, the next two terms are 162 and 486.

e This sequence is a geometric sequence where each term is multiplied by $\frac{1}{2}$ to generate the next term since $1000 \times \frac{1}{2} = 500, 500 \times \frac{1}{2} = 250, 250 \times \frac{1}{2} = 125.$ The next two terms in the sequence can be generated: $125 \times \frac{1}{2} = 62.5, 62.5 \times \frac{1}{2} = 31.25.$ Thus, the next two terms are 62.5 and 31.25.

2 **a** This sequence has a starting value of 12 and the rule is to add 6 to generate the next term.

i Since the first term is 12, $t_0 = 12$.

ii t_3 corresponds to the term after 3 iterations of the rule have been applied, or equivalently, the fourth term in the sequence. Thus, $t_3 = 30$.

ii t_5 corresponds to the term after 5 iterations of the rule have been applied, or equivalently, the sixth term in the sequence. Thus, $t_5 = 42$.

b This sequence has a starting value of 20 and the rule is to subtract 2 to generate the next term.

i Since the first term is 20, $t_0 = 20$.

ii t_3 corresponds to the term after 3 iterations of the rule have been applied, or equivalently, the fourth term in the sequence. Thus, $t_3 = 14$.

ii t_5 corresponds to the term after 5 iterations of the rule have been applied, or equivalently, the sixth term in the sequence. Thus, $t_5 = 10$.

c This sequence has a starting value of 2 and the rule is to multiply by 5 to generate the next term.

i Since the first term is 12, $t_0 = 2$.

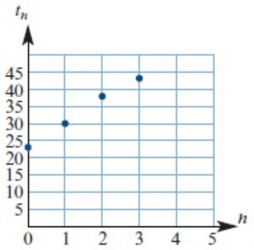
ii t_3 corresponds to the term after 3 iterations of the rule have been applied, or equivalently, the fourth term in the sequence. Thus, $t_3 = 250$.

ii t_5 corresponds to the term after 5 iterations of the rule have been applied, or equivalently, the sixth term in the sequence. Thus, $t_5 = 6250$.

3 The sequence 7, 11, 15, 19, . . . has a starting value of 7 and the rule is: add 4 to generate the next term. This means the rule is $t_n = 4n + 7$. Since we are looking for t_{20}, $n = 20$. Thus, $t_{20} = 87$.

4 With a starting value of 32 and a common difference of 11, the rule is $t_n = 11n + 32$. Thus, $t_{100} = 11 \times 100 + 32 = 1132$.

5 Using the recurrence relation:
$V_0 = 23$
$V_1 = V_0 + 7 = 23 + 7 = 30$
$V_2 = V_1 + 7 = 30 + 7 = 37$
$V_3 = V_2 + 7 = 37 + 7 = 44$



6 **a** $V_0 = 2000$
$V_1 = V_0 + 800 = 20\,000 + 800 =$ \$20 800
$V_2 = V_1 + 800 = 20\,800 + 800 =$ \$21 600
$V_3 = V_2 + 800 = 21\,600 + 800 =$ \$22 400

b Solve $30\,000 = 800n + 20\,000$ for n on the CAS. Thus, it takes 13 years.

7 **a** The value of the printer when it was new is given as the starting value (V_0) of \$2000.

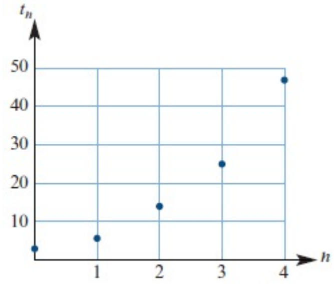
b The amount of depreciation is the amount that the value goes down by each year which is \$250.

c The percentage flat rate of depreciation is found by finding $\frac{250}{2000} \times 100 = 12.5\%$.

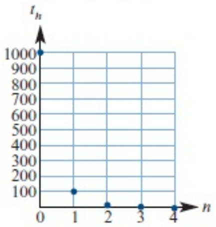
d To find when the printer will have a value of \$600, solve the rule associated with the recurrence relation: $V_n = -250n + 2000$. That is, $600 = -250n + 2000$ for n using your CAS. This gives $n = 6$ so it takes 6 years.

e Half of the new price is \$1000 so solve $1000 = -250n + 2000$ for n on the CAS. Since $n = 4$, it takes 4 years.

8 Using the recurrence relation:
$V_0 = 3$
$V_1 = 2V_0 = 2 \times 3 = 6$
$V_2 = 2V_1 = 2 \times 6 = 12$
$V_3 = 2V_2 = 2 \times 12 = 24$
$V_4 = 2V_3 = 2 \times 24 = 48$



9 **a** Using the recurrence relation:
$V_0 = 1000$
$V_1 = 0.1V_0 = 0.1 \times 1000 = 100$
$V_2 = 0.1V_1 = 0.1 \times 100 = 10$
$V_3 = 0.1V_2 = 0.1 \times 10 = 1$
$V_4 = 0.1V_3 = 0.1 \times 1 = 0.1$



b

c Using the rule and $n = 10$,
$V_{10} = 0.1^{10} \times 1000 = 0.0000001$.

10 With a first term of 8 and a common ratio of 2, the rule can be rewritten as $t_n = 2^n \times 8$. Thus, $t_{10} = 2^{10} \times 8 = 8192$.

11 **a** Using the recurrence relation:
$V_0 = 20\,000$
$V_1 = 1.04V_0 = 1.04 \times 20\,000 =$
\$20 800
$V_2 = 1.04V_1 = 1.04 \times 100 =$ \$21 632
$V_3 = 1.04V_2 = 1.04 \times 10 =$
\$22 497.28

b To find when the investment will first reach more than \$30 000, solve $30\,000 = 1.04^n \times 20\,000$ for n on the CAS. Thus, $n = 11$ so it takes 11 years (note $V_{11} = 30\,789.08$).

12 **a** The value of the work station is the starting value of \$2000.

b The value at the end of the first year is $V_1 = 0.9 \times 2000 =$ \$1800. Thus, the amount of depreciation is \$200.

c The percentage flat rate depreciation is 10%. This can be seen by noting that $1 - 0.9 = 0.1 = 10\%$ or by calculating $\frac{200}{2000} \times 100\% = 10\%$.

d To find when the value will first be worth less than \$600, solve $600 = 0.9^n \times 2000$ for n on your CAS. Thus, $n = 12$ so it takes 12 years.

e Half of the original price is \$1000, so solve $1000 = 0.9^n \times 2000$ for n on your CAS. Thus, $n = 7$ so it takes 7 years.

13 Given that $V_5 = 2048$ and $R = 4$, then we can write down the rule when $n = 5$ to solve for the starting value, a. Solve $V_5 = 4^5 \times a$ for a on the CAS to give $a = 2$.
The recurrence relation can be written as: $V_0 = 2, \quad V_{n+1} = 4 \times V_n$.

Solutions to Chapter Review Written-Response Questions

1 **a** The amount of depreciation each year is calculated as 8% of \$4100. $\frac{8}{100} \times 4100 = \$328$.

b If the motor scooter depreciates by \$328 each year, the value after one year is $4100 - 328 = \$3772$.

c If the motor scooter depreciates by a further \$328, the value after two years is $3772 - 328 = \$3444$.

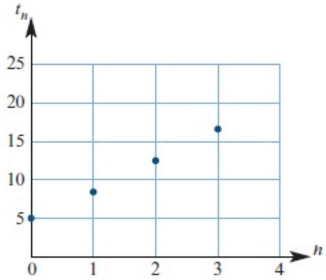
d Since \$328 is subtracted to find the next value, $V_1 = V_0 - 328$ and $V_2 = V_1 - 328$.

e Given a starting value of \$4100 and that the motor scooter depreciates by \$328 each year, the recurrence relation can be written as:
$V_0 = 4100, \quad V_{n+1} = V_n - 328$.

f To find when the value of the motor scooter will first be less than \$1000, solve $1000 = 4100 - 328n$ for n on the CAS. Thus, the motor scooter will first be worth less than \$1000 after 10 full years have passed.

2 **a** With a starting value of 5 and a common difference of 4:
$V_0 = 5$
$V_1 = V_0 + 4 = 5 + 4 = 9$
$V_2 = V_0 + 4 = 9 + 4 = 13$
$V_3 = V_0 + 4 = 13 + 4 = 17$
In a table:

n	0	1	2	3
t_n	5	9	13	17

b 

c The points lie on a straight line with a positive slope. Thus, the sequence is arithmetic.

3 **a** The value of the van when it was new is given by V_0 as \$64 000.

b To find the next term, we subtract \$3200. This means that the van depreciates by \$3200 each year.

c To find the percentage flat rate depreciation that was applied we calculate $\frac{3200}{64\,000} \times 100\% = 5\%$.

d To find when the van will first be worth less than \$40 000, we solve 40 000 = 64 000 – 3200*n* for *n* on the CAS. This tells us that it takes 8 full years for the van to first be worth less than \$40 000.

e Half of the original value of the van is \$32 000 so to find when the van reaches this value, solve 32 000 = 64 000 – 3200*n* for *n* on the CAS. This tells us that it will take 10 years.

4 Given the recurrence relation, the rule for V_n can be formed as $V_n = 30\,000 - 0.15n$ given the starting value of \$30 000 and that the depreciation is \$0.15 per unit produced.

a Substituting 1000, 2000 and 3000 for *n* into the rule gives the value of the machine:
$V_{1000} = 30\,000 - 0.15 \times 1000 = \$29\,850$
$V_{2000} = 30\,000 - 0.15 \times 2000 = \$29\,700$
$V_{3000} = 30\,000 - 0.15 \times 3000 = \$29\,550$

b A machine producing 20 000 units per year for three years will produce 60 000 units in three years. This means that it's value at the end of three years is:
$V_{30\,000} = 30\,000 - 0.15 \times 30\,000 = \$21\,000$

c The machine is worth half of its original value when it is worth \$15 000. To find how many units it must produce to get to this value, solve 30 000 – 0.15 × *n* for *n* on your CAS. This tells us that 100 000 units are produced.

5 **a** The amount that was initially invested was \$6200 given by V_0.

b The interest rate for the investment is found from the common ratio 1.08. This causes the value to increase by 8% so the interest rate is 8%.

c Using the recurrence relation:
$V_0 = 6200$
$V_1 = 1.08V_0 = 1.08 \times 6200 = \$6696$
$V_2 = 1.08V_0 = 1.08 \times 6200 = \$7231.68$
$V_3 = 1.08V_0 = 1.08 \times 6200 = \$7810.21$

d To find the interest earned, we find the difference between each subsequent year:
First year: $V_1 - V_0 = \$496$
Second year: $V_2 - V_1 = \$535.68$
Third year: $V_3 - V_2 = \$578.53$

6 **a** The bike depreciates by 15% so to find how much it depreciates each year, we calculate 15% of the starting value of \$9200. $\frac{15}{100} \times 9200 = \$1380$.

b Since the starting value is \$9200, we write $V_0 = 9200$. The bike's value reduces by 15% each year so we multiply by $1 - 0.15 = 0.85$ to get $V_{n+1} = 0.85V_n$.

c To find the value of the bike after 5 years, we can use the rule $V_n = 0.85^n \times 9200$ where $n = 5$. Thus, $V_5 = 0.85^5 \times 9200 = 4082$.

d To find when the motorbike will first have a value of less than \$5000, solve $5000 = 0.85^n \times 9200$ for n on the CAS. Thus, it will take 4 years.

7 **a** To compare the three options, we calculate the interest after one year from each.
A: $\frac{5.5}{100} \times 20\,000 = \$1100$
B: $\frac{5}{100} \times 20\,000 = \$1000$
C: $\left(\frac{4.5}{100 \times 12}\right)^{12} \times 20\,000 = \$919$

b To compare the value of each option at the end of five years, we need to compute the value in turn.
A: $20\,000 + 5 \times \frac{5.5}{100} \times 20\,000 = \$25\,500$
B: $\left(1 + \frac{5}{100}\right)^5 \times 20\,000 = \$25\,525.63$
C: $\left(1 + \frac{4.5}{100 \times 12}\right)^{60} \times 20\,000 = \$25\,035, 92$
The option with the highest value is **B**.

c The amount of interest earned over the five years from option **B** is \$25 525.63-\$20 000=\$525.63.

8 **a** The initial value of the car is given by V_0 as \$22 500.

b The annual depreciation of the car is found from the common ratio as $1 - 0.85 = 0.15$. Thus, the depreciation rate is 15%.

c The value of the car after n years, V_n, can be found from these two values as $V_n = 0.85^n \times V_0$.

d Using the rule and $n = 5$, $V_5 = 0.85^5 \times 22\,500 = \$9983$.

e Half of the starting value is \$11 250 so we want to solve $11\,250 = 0.85^n \times 22\,500$ for n. Thus it takes 5 years.

Chapter 4 – Matrices

Solutions to Exercise 4A

1 **a** Count the number of rows: 2

b Count the number of coloums: 4

c The order lists the number of rows and the number of columns with a multiplication symbol inbetwee: 2×4

d The number of elements can be found by multiplying the number of rows and columns together: $2 \times 4 = 8$

2 **a** The number of girls that play sport is read from the table (first row, second column) as 53.

b The number of boys who play sport is read from the table (second row, second column) as 36.

c The number of students who play a musical instruments can be found by adding up the first column: $38 + 45 = 83$.

3 **a** $\text{Order } (C) = \text{rows} \times \text{columns}$

$$= 3 \times 4$$

b **i** $c_{13} = \text{row 1, column 3}$

$$= 16$$

ii $c_{24} = \text{row 2, column 4}$

$$= 3$$

iii $c_{31} = \text{row 3, column 1}$

$$= 5$$

c $5 + 6 + 10 + 1 = 22$

d $4 + 8 + 6 = 18$

4 **a** **i** Remember that rows are listed before columns: Order $(A) = 2 \times 3$

ii Find the right row and then the right column: $a_{12} = 6$ and $a_{22} = 7$

b **i** Remember that rows are listed before columns: Order $(B) = 1 \times 3$

ii Find the right row and then the right column: $b_{13} = 2$ and $b_{11} = 6$

c **i** Remember that rows are listed before columns: Order $(C) = 3 \times 2$

ii Find the right row and then the right column: $c_{32} = -4$ and $c_{12} = 5$

d **i** Remember that rows are listed before columns: Order $(D) = 3 \times 1$

ii Find the right row and then the right column: $d_{31} = 9$ and $d_{11} = 8$

e **i** Remember that rows are listed before columns: Order $(E) = 2 \times 2$

ii Find the right row and then the right column: $e_{21} = 15$ and $e_{12} = 12$

f **i** Remember that rows are listed before columns: Order $(F) = 3 \times 4$

ii Find the right row and then the right column: $f_{34} = 20$ and $f_{23} = 5$

5 **a** A row matrix has only one row: B

b A column matrix has only one column: D

c A square matrix has the same number of rows and columns: E

6 A symmetric matrix is one where $a_{ij} = a_{ji}$: B, C and D.

7 **a** Remember to locate the row and then the column: $d_{23} = 9$

b Remember to locate the row and then the column: $d_{45} = 2$

c Remember to locate the row and then the column: $d_{11} = 3$

d Remember to locate the row and then the column: $d_{24} = 10$

e Remember to locate the row and then the column: $d_{42} = 8$

8 **a** To find the order, first count the number of rows and the number of columns. Order $(A) = 4 \times 3$, Order $(B) = 2 \times 1$, Order $(C) = 1 \times 2$, Order $(D) = 2 \times 5$,

b To locate a particular element in a matrix, first look up the row then the column. $a_{32} = 4$ $b_{21} = -5$, $c_{11} = 8$, $d_{24} = 7$

9 **a** To find the number of Year 11 students who prefer basketball, we consider the second row (Year 11) and the second column (Basketball). Since $S_{22} = 32$, 32 Year 11 students prefer basketball.

b To find the order a matrix, count the number of rows and then the number of columns. Here, there are three rows and four columns so Order $(S) = 3 \times 4$.

c The second row corresponds to Year 11 and the third column to Football so s_{23} tells us that 22 Year 11 students prefer Football.

10 **a** **i** Farm X is row 1 and sheep are listed in column 3 so we find f_{13} = 75 ha

ii Farm X is row 1 and cattle are listed in column 2 so we find f_{12} = 300 ha

iii Farm Y is row 1 and wheat is listed in column 1 so we find f_{21} = 200 ha

b Total hectares used by both farms for wheat $= f_{1\,1} + f_{2\,1} = 150 + 200 = 350$

c **i** Farm Y uses 0 ha for cattle.

ii Farm X uses 75 ha for sheep.

iii Farm X uses 150 ha for wheat.

d **i** Farm Y is listed in Row 2 and Sheep are listed in Column 3: $f_{2\ 3}$

ii Farm X is listed in Row 2 and Cattle are listed in Column 2: $f_{1\ 2}$

iii Farm Y is listed in Row 2 and Wheat is listed in Column 1: $f_{2\ 1}$

e Order $F = 2 \times 3$

11 Use the first two dot points to set up the structure of the matrix. Since Bakery 2 sold 165 pies and 181 sausage rolls, put 165 in Row 2, column 1 and 181 in Row 2, column 2.
As Bakery 1 sold 30 more pies than Bakery 2, they sold 165 + 30 = 195 pies. Put this in Row 1, Column 1.
As Bakery 1 sold 40 fewer sausage rolls than Bakery 2, they sold 181 − 40 = 141 sausage rolls. Put this in Row 1, Column 2.

$$B = \begin{array}{c} \begin{array}{cc} Pies & SausageRolls \end{array} \\ \left[\begin{array}{cc} 195 & 141 \\ 165 & 181 \end{array}\right] \end{array} \begin{array}{l} \\ Bakery\ 1 \\ Bakery\ 2 \end{array}$$

12 Since 137 students want to go mountain bike riding and 83 girls are already listed, $S_{12} = 137 - 83 = 54$.
Since $s_{23} = 0.5 \times s_{1\ 1} = 0.5 \times 28 = 14$
Since the total number of boys is 146,
$s_{11} + s_{12} + s_{13} + s_{14} = 28 + 54 + 29 + s_{14}$.
Thus, $s_{14} = 35$.
Since the number of girls who wanted to go skiing was 5 more than the number of girls who wanted to go kayaking,
$s_{21} = s_{24} + 5 = 31 + 4 = 35$.
Thus:

$$S = \begin{array}{c} \\ Boys \\ Girls \end{array} \begin{array}{c} \begin{array}{cccc} Snow & Mountain\ bike & Hike & Kayak \end{array} \\ \left[\begin{array}{cccc} 28 & 54 & 29 & 35 \\ 36 & 83 & 14 & 31 \end{array}\right] \end{array}$$

Solutions to Exercise 4B

1 Matrices can be added together if they have the same order. A, C and G can be added together since they are 2×2 matrices. D and F can be added together since they are 2×3 matrices and E and H can be added together since they are 3×2 matrices.

2 False since the entry in the first row and second column should be 4: $-1 + 5 = 4$, not 6.

3 To perform this calculation, we perform the calculation on the corresponding elements of the matrices:

$$\begin{aligned}&\begin{bmatrix}-3 & 3\\ 9 & 12\end{bmatrix}-\begin{bmatrix}-3 & 3\\ 9 & 12\end{bmatrix}\\ &=\begin{bmatrix}-3--3 & 3-3\\ 9-9 & 12-12\end{bmatrix}\\ &=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\end{aligned}$$

Note that since the two matrices were the same, our result is a zero matrix with zeros in all spaces.

4 a
$$\begin{aligned}\begin{bmatrix}4 & 3\\ 0 & 2\end{bmatrix}+\begin{bmatrix}5 & 7\\ 6 & 1\end{bmatrix} &= \begin{bmatrix}4+5 & 3+7\\ 0+6 & 2+1\end{bmatrix}\\ &= \begin{bmatrix}9 & 10\\ 6 & 3\end{bmatrix}\end{aligned}$$

b
$$\begin{aligned}\begin{bmatrix}8 & 6\\ 9 & 3\end{bmatrix}+\begin{bmatrix}-1 & 2\\ 4 & 0\end{bmatrix} &= \begin{bmatrix}8-1 & 6+2\\ 9+4 & 3+0\end{bmatrix}\\ &= \begin{bmatrix}7 & 8\\ 13 & 3\end{bmatrix}\end{aligned}$$

c
$$\begin{aligned}\begin{bmatrix}3 & 5\\ 7 & 0\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} &= \begin{bmatrix}3+0 & 5+0\\ 7+0 & 0+0\end{bmatrix}\\ &= \begin{bmatrix}3 & 5\\ 7 & 0\end{bmatrix}\end{aligned}$$

d
$$\begin{aligned}\begin{bmatrix}9\\ 8\end{bmatrix}-\begin{bmatrix}0\\ 0\end{bmatrix} &= \begin{bmatrix}9-0\\ 8-0\end{bmatrix}\\ &= \begin{bmatrix}9\\ 8\end{bmatrix}\end{aligned}$$

e
$$\begin{aligned}\begin{bmatrix}8 & 6\\ 2 & 9\end{bmatrix}+\begin{bmatrix}5 & 2\\ 1 & 3\end{bmatrix} &= \begin{bmatrix}8-5 & 6-2\\ 2-1 & 9-3\end{bmatrix}\\ &= \begin{bmatrix}3 & 4\\ 1 & 6\end{bmatrix}\end{aligned}$$

f
$$\begin{aligned}\begin{bmatrix}7 & 4\\ 5 & 1\end{bmatrix}+\begin{bmatrix}3 & 6\\ 2 & -8\end{bmatrix} &= \begin{bmatrix}7-3 & 4-6\\ 5-2 & 1+8\end{bmatrix}\\ &= \begin{bmatrix}4 & -2\\ 3 & 9\end{bmatrix}\end{aligned}$$

g
$$\begin{aligned}\begin{bmatrix}4 & 2\end{bmatrix}+\begin{bmatrix}8 & 5\end{bmatrix} &= \begin{bmatrix}4+8 & 2+5\end{bmatrix}\\ &= \begin{bmatrix}12 & 7\end{bmatrix}\end{aligned}$$

h
$$\begin{aligned}\begin{bmatrix}7 & -5\end{bmatrix}-\begin{bmatrix}7 & -5\end{bmatrix} &= \begin{bmatrix}7-7 & -5+5\end{bmatrix}\\ &= \begin{bmatrix}0 & 0\end{bmatrix}\end{aligned}$$

i
$$\begin{aligned}\begin{bmatrix}4 & -3\end{bmatrix}+\begin{bmatrix}-4 & 3\end{bmatrix} &= \begin{bmatrix}4-4 & -3+3\end{bmatrix}\\ &= \begin{bmatrix}0 & 0\end{bmatrix}\end{aligned}$$

5 a
$$\begin{aligned}\begin{bmatrix}3 & -2\\ 2 & 4\end{bmatrix}+\begin{bmatrix}5 & 7\\ 1 & 3\end{bmatrix} &= \begin{bmatrix}3+5 & -2+7\\ 2+1 & 4+3\end{bmatrix}\\ &= \begin{bmatrix}8 & 5\\ 3 & 7\end{bmatrix}\end{aligned}$$

b $B + A = A + B = \begin{bmatrix}8 & 5\\ 3 & 7\end{bmatrix}$

c $$\begin{bmatrix}3 & -2\\ 2 & 4\end{bmatrix} - \begin{bmatrix}5 & 7\\ 1 & 3\end{bmatrix} = \begin{bmatrix}3-5 & -2-7\\ 2-1 & 4-3\end{bmatrix}$$
$$= \begin{bmatrix}-2 & -9\\ 1 & 1\end{bmatrix}$$

d $$\begin{bmatrix}5 & 7\\ 1 & 3\end{bmatrix} - \begin{bmatrix}3 & -2\\ 2 & 4\end{bmatrix} = \begin{bmatrix}5-3 & 7+2\\ 1-2 & 3-4\end{bmatrix}$$
$$= \begin{bmatrix}2 & 9\\ -1 & -1\end{bmatrix}$$

e The matrices do not have the same order, so this is not possible.

f $$\begin{bmatrix}6 & 2\\ 1 & 0\\ 3 & -8\end{bmatrix} + \begin{bmatrix}-3 & 5\\ 4 & -2\\ 1 & 7\end{bmatrix} = \begin{bmatrix}6-3 & 2+5\\ 1+4 & 0-2\\ 3+1 & -8+7\end{bmatrix}$$
$$= \begin{bmatrix}3 & 7\\ 5 & -2\\ 4 & -1\end{bmatrix}$$

g The matrices do not have the same order, so this is not possible.

h $$\begin{bmatrix}-3 & 5\\ 4 & -2\\ 1 & 7\end{bmatrix} - \begin{bmatrix}6 & 2\\ 1 & 0\\ 3 & -8\end{bmatrix}$$
$$= \begin{bmatrix}-3-6 & 5-2\\ 4-1 & -2-0\\ 1-3 & 7+8\end{bmatrix}$$
$$= \begin{bmatrix}-9 & 3\\ 3 & -2\\ -2 & 15\end{bmatrix}$$

6 Add the two matrices together:
$$\begin{bmatrix}19 & 21 & 7 & 3\\ 18 & 7 & 11 & 4\end{bmatrix} + \begin{bmatrix}24 & 21 & 3 & 2\\ 19 & 20 & 6 & 5\end{bmatrix}$$
$$= \begin{bmatrix}19+24 & 21+21 & 7+3 & 3+2\\ 18+19 & 17+20 & 11+6 & 4+5\end{bmatrix}$$
This gives:

$$\begin{array}{ccccc} & \mathit{Li} & \mathit{La} & \mathit{In} & \mathit{Gr}\\ \mathit{Men} & 43 & 42 & 10 & 5\\ \mathit{Women} & 37 & 37 & 17 & 9\end{array}$$

7 To find the change, we subtract the first matrix from the second matrix.
$$\begin{bmatrix}26 & 10 & 25 & 26\\ 12 & 12 & 21 & 31\\ 22 & 5 & 30 & 18\end{bmatrix} - \begin{bmatrix}32 & 10 & 82 & 41\\ 29 & 17 & 75 & 44\\ 22 & 12 & 103 & 61\end{bmatrix}$$
$$= \begin{bmatrix}26-32 & 10-10 & 25-82 & 26-41\\ 12-29 & 12-17 & 21-75 & 31-44\\ 22-22 & 5-12 & 30-103 & 18-61\end{bmatrix}$$
$$= \begin{bmatrix}-6 & 0 & -57 & -15\\ -17 & -5 & -54 & -13\\ 0 & -7 & -73 & -43\end{bmatrix}$$
A negative entry tells us that the stock of that item has declined.

8 a $$\begin{bmatrix}38 & 52 & 57 & 63\\ 150 & 163 & 167 & 170\end{bmatrix}$$
$$- \begin{bmatrix}32 & 44 & 59 & 56\\ 145 & 155 & 160 & 164\end{bmatrix}$$
$$= \begin{bmatrix}38-32 & 52-44 & 57-59 & 63-56\\ 150-145 & 163-155 & 167-160 & 170-164\end{bmatrix}$$

$$\begin{array}{lcccc} & \mathit{Ar} & \mathit{Be} & \mathit{Ca} & \mathit{Da}\\ \mathit{Weight}\ (\mathit{kg}) & 6 & 8 & -2 & 7\\ \mathit{Height}\ (\mathit{cm}) & 5 & 8 & 7 & 6\end{array}$$

b Beni gained the most weight (8 kg).

c Beni grew the most (8 cm).

9 Since $3 + a = 7$, $a = 4$. Since $11 + b = 11$, $b = 0$. Since $5 + c = 2$, $c = -3$. Since $-1 + d = 8$, $d = 9$.

10 Since $8 - a = 7$, $a = 1$. Since $4 - b = 10$, $b = -6$. Since $-2 - c = -9$, $c = 7$. Since $12 - d = 0$, $d = 12$.

11 The first dot point tells us that Jack kicked a total of 9 goals. Since he kicked 4 in round 1 and 3 in round 3, he must have kicked 2 goals in round 2.
The second dot point tells us that Mykola kicked 6 more goals in round 1 than round 3. This tells us that Mykola kicked 12 kicks in round 3. Mykola's total number of kicks was 39.
The third dot point tells us that Nick had a total of 43 kicks. Since he had 13 kicks in round 2 and 20 kicks in round 3, he must have had 10 kicks in round 1 and had 43 kicks in total.
The fourth dot point tells us that Jack had twice as many handballs than in the two rounds combined. Since he had $2 + 8 = 10$ handballs in the first two rounds, he must have had 20 handballs in round 3. This means that Jack had 30 handballs in total.

Round 1:

$$\begin{array}{r|ccc|} & \textit{Kicks} & \textit{Goals} & \textit{Handballs} \\ \textit{Jack} & 10 & 4 & 2 \\ \textit{Nick} & 10 & 0 & 26 \\ \textit{Gary} & 18 & 1 & 12 \end{array}$$

Round 2:

$$\begin{array}{r|ccc|} & \textit{Kicks} & \textit{Goals} & \textit{Handballs} \\ \textit{Jack} & 7 & 2 & 8 \\ \textit{Nick} & 13 & 2 & 12 \\ \textit{Gary} & 9 & 2 & 11 \end{array}$$

Round 3:

$$\begin{array}{r|ccc|} & \textit{Kicks} & \textit{Goals} & \textit{Handballs} \\ \textit{Jack} & 6 & 3 & 20 \\ \textit{Nick} & 20 & 1 & 19 \\ \textit{Gary} & 12 & 4 & 11 \end{array}$$

Total:

$$\begin{array}{r|ccc|} & \textit{Kicks} & \textit{Goals} & \textit{Handballs} \\ \textit{Jack} & 23 & 9 & 30 \\ \textit{Nick} & 43 & 3 & 57 \\ \textit{Gary} & 39 & 7 & 34 \end{array}$$

Solutions to Exercise 4C

1 Calculating each element of the matrix independently gives $3 \times 2 = 6$, $3 \times -1 = -3$, $3 \times 8 = 24$ and $3 \times 7 = 21$.

$$\begin{bmatrix} 6 & -3 \\ 24 & 21 \end{bmatrix}$$

2 Each element in the matrix is multiplied by 5 so:

$$\begin{bmatrix} 5 \times -6 & 5 \times 7 \\ 5 \times 3 & 5 \times -2 \end{bmatrix}$$

3 Each element of the first matrix is multiplied by 2 and each element in the second matrix is multiplied by 3. The corresponding elements are then added together to give:

$$\begin{bmatrix} 2 \times -2 + 3 \times 4 & 2 \times 5 + 3 \times -1 \\ 2 \times 1 + 3 \times 0 & 2 \times 0 + 3 \times 3 \end{bmatrix}$$

4 In each of the following, we multiply each element of the matrix by the number in front of the matrix.

a
$$2\begin{bmatrix} 7 & -1 \\ 4 & 9 \end{bmatrix} = \begin{bmatrix} 2 \times 7 & 2 \times -1 \\ 2 \times 4 & 2 \times 9 \end{bmatrix} = \begin{bmatrix} 14 & -2 \\ 8 & 18 \end{bmatrix}$$

b
$$5\begin{bmatrix} 0 & -2 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 5 \times 0 & 5 \times -2 \\ 5 \times 5 & 5 \times 7 \end{bmatrix} = \begin{bmatrix} 0 & -10 \\ 25 & 35 \end{bmatrix}$$

c
$$\begin{aligned} &-4\begin{bmatrix} 16 & -3 \\ 1.5 & 3.5 \end{bmatrix} \\ &= \begin{bmatrix} -4 \times 16 & -4 \times -3 \\ -4 \times 1.5 & -4 \times 3.5 \end{bmatrix} \\ &= \begin{bmatrix} -64 & 12 \\ -6 & -14 \end{bmatrix} \end{aligned}$$

d
$$1.5\begin{bmatrix} 1.5 & 0 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1.5 \times 1.5 & 1.5 \times 0 \\ 1.5 \times -2 & 1.5 \times 5 \end{bmatrix} = \begin{bmatrix} 2.25 & 0 \\ -3 & 7.5 \end{bmatrix}$$

e
$$3\begin{bmatrix} 6 & 7 \end{bmatrix} = \begin{bmatrix} 3 \times 6 & 3 \times 7 \end{bmatrix} = \begin{bmatrix} 18 & 21 \end{bmatrix}$$

f
$$6\begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 6 \times -2 \\ 6 \times 5 \end{bmatrix} = \begin{bmatrix} -12 \\ 30 \end{bmatrix}$$

g
$$\begin{aligned} &\frac{1}{2}\begin{bmatrix} 4 & 6 & 0 \\ 0 & 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{2} \times 4 & \frac{1}{2} \times 6 & \frac{1}{2} \times 0 \\ \frac{1}{2} \times 0 & \frac{1}{2} \times 3 & \frac{1}{2} \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 3 & 0 \\ 0 & 1\frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{aligned}$$

h
$$\begin{aligned} &-1\begin{bmatrix} 3 & 6 & -8 \end{bmatrix} \\ &= \begin{bmatrix} -1 \times 3 & -1 \times 6 & -1 \times 8 \end{bmatrix} \\ &= \begin{bmatrix} -3 & -6 & 8 \end{bmatrix} \end{aligned}$$

5 **a**
$$3\begin{bmatrix} 3 & -4 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 \times 3 & 3 \times -4 \\ 3 \times 2 & 3 \times 5 \end{bmatrix} = \begin{bmatrix} 9 & -12 \\ 6 & 15 \end{bmatrix}$$

b

$$\begin{aligned}
&2\begin{bmatrix}7 & 6\\1 & -4\end{bmatrix}+4\begin{bmatrix}-3 & 4\\-2 & -5\end{bmatrix}\\
&=\begin{bmatrix}2\times 7 & 2\times 6\\2\times 1 & 2\times -4\end{bmatrix}+\begin{bmatrix}4\times -3 & 4\times 4\\4\times -2 & 4\times -5\end{bmatrix}\\
&=\begin{bmatrix}14 & 12\\2 & -8\end{bmatrix}+\begin{bmatrix}-12 & 16\\-8 & -20\end{bmatrix}\\
&=\begin{bmatrix}14-12 & 12+16\\2-8 & -8-20\end{bmatrix}\\
&=\begin{bmatrix}2 & 28\\-6 & -28\end{bmatrix}
\end{aligned}$$

c

$$\begin{aligned}
&5\begin{bmatrix}3 & -4\\2 & 5\end{bmatrix}-2\begin{bmatrix}7 & 6\\1 & -4\end{bmatrix}\\
&=\begin{bmatrix}5\times 3 & 5\times -4\\5\times 2 & 5\times 5\end{bmatrix}-\begin{bmatrix}2\times 7 & 2\times 6\\2\times 1 & 2\times -4\end{bmatrix}\\
&=\begin{bmatrix}15 & -20\\10 & 25\end{bmatrix}-\begin{bmatrix}14 & 12\\2 & -8\end{bmatrix}\\
&=\begin{bmatrix}15-14 & -20-12\\10-2 & 25+8\end{bmatrix}\\
&=\begin{bmatrix}1 & -32\\8 & 33\end{bmatrix}
\end{aligned}$$

d

$$\begin{aligned}
2\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} &= \begin{bmatrix}2\times 0 & 2\times 0\\2\times 0 & 2\times 0\end{bmatrix}\\
&=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}
\end{aligned}$$

e $3B + O = 3B$

$$\begin{aligned}
3\begin{bmatrix}7 & 6\\1 & -4\end{bmatrix} &= \begin{bmatrix}3\times 7 & 3\times 6\\3\times 1 & 3\times -4\end{bmatrix}\\
&=\begin{bmatrix}21 & 18\\3 & -12\end{bmatrix}
\end{aligned}$$

6 Methods will vary depending on the calculator being used.

a Read result from calculator:

$$\begin{bmatrix}79 & -31\\68 & -36\end{bmatrix}$$

b Read result from calculator:

$$\begin{bmatrix}-121 & 50\\-84 & 103\end{bmatrix}$$

c Read result from calculator:

$$\begin{bmatrix}13 & -2\\36 & 53\end{bmatrix}$$

d Read result from calculator:

$$\begin{bmatrix}69 & -27\\60 & -30\end{bmatrix}$$

7 **a** $3A + 4B$

$$\begin{aligned}
&=3\begin{bmatrix}1\\1\\0\end{bmatrix}+4\begin{bmatrix}0\\1\\1\end{bmatrix}\\
&=\begin{bmatrix}3\\3\\0\end{bmatrix}+\begin{bmatrix}0\\4\\4\end{bmatrix}\\
&=\begin{bmatrix}3\\7\\4\end{bmatrix}
\end{aligned}$$

b $5C - 2D$

$$\begin{aligned}
&=5\begin{bmatrix}0 & 1 & 1 & 1\end{bmatrix}-2\begin{bmatrix}0 & 0 & 1 & 1\end{bmatrix}\\
&=\begin{bmatrix}0 & 5 & 5 & 5\end{bmatrix}-\begin{bmatrix}0 & 0 & 2 & 2\end{bmatrix}\\
&=\begin{bmatrix}0 & 5 & 3 & 3\end{bmatrix}
\end{aligned}$$

c $2(3A + 4B)$

$$\begin{aligned}
&=2\begin{bmatrix}3\\7\\4\end{bmatrix} \quad \text{from } \mathbf{a}\\
&=\begin{bmatrix}6\\14\\8\end{bmatrix}
\end{aligned}$$

d $3(5C - 2D)$

$$\begin{aligned}
&=3\begin{bmatrix}0 & 5 & 3 & 3\end{bmatrix} \quad \text{from } \mathbf{b}\\
&=\begin{bmatrix}0 & 15 & 9 & 9\end{bmatrix}
\end{aligned}$$

8 a In this case we start by multiplying the matrix by 3:

$$3\begin{bmatrix} 300 & 150 & 125 & 425 \\ 250 & 170 & 260 & 320 \end{bmatrix}$$

$$= \begin{bmatrix} 900 & 450 & 375 & 1075 \\ 750 & 510 & 780 & 960 \end{bmatrix}$$

Then add each column to find the amount of each chemical required.

$$\begin{array}{cccc} A & B & C & D \\ \hline 1650 & 960 & 1155 & 2235 \end{array}$$

b The amount of chemical B required to make 3 litres of each product can be read off the new matrix in the second column. You will need 960 mL.

9 a Subtract Costs from Sales matrix:

$$\begin{bmatrix} 18 & 12 & 24 \\ 16 & 9 & 26 \\ 19 & 13 & 12 \end{bmatrix} - \begin{bmatrix} 12 & 10 & 15 \\ 11 & 8 & 17 \\ 15 & 14 & 7 \end{bmatrix} =$$

$$\begin{bmatrix} 18-12 & 12-10 & 24-15 \\ 16-11 & 9-8 & 26-17 \\ 19-15 & 13-14 & 12-7 \end{bmatrix}$$

$$\begin{array}{c|ccc} & Cl & Fu & El \\ \hline Store\ A & 6 & 2 & 9 \\ Store\ B & 5 & 1 & 9 \\ Store\ C & 4 & -1 & 5 \end{array}$$

b Since losses are not taxed, replace any negative numbers in the above matrix with zero, then multiply the matrix by 0.3. $0.3\begin{bmatrix} 6 & 2 & 9 \\ 5 & 1 & 9 \\ 4 & 0 & 5 \end{bmatrix} =$

$$\begin{bmatrix} 0.3\times 6 & 0.3\times 2 & 0.3\times 9 \\ 0.3\times 5 & 0.3\times 1 & 0.3\times 9 \\ 0.3\times 4 & 0.3\times 0 & 0.3\times 5 \end{bmatrix}$$

$$\begin{array}{c|ccc} & Cl & Fu & El \\ \hline Store\ A & 1.8 & 0.6 & 2.7 \\ Store\ B & 1.5 & 0.3 & 2.7 \\ Store\ C & 1.2 & 0 & 1.5 \end{array}$$

10 a

$$\begin{array}{cccccc} & Wins & & Wins & & Wins \\ Rings & 1 & + & 1 & + & 1 \\ Bars & 0 & & 1 & & 1 \end{array}$$

$$= \begin{array}{c|c} & Wins \\ \hline Rings & 1+1+1 \\ Bars & 0+1+1 \end{array}$$

$$= \begin{array}{c|c} & Wins \\ \hline Rings & 3 \\ Bars & 2 \end{array}$$

b

$$\begin{array}{c|c} & \$ \\ \hline Rings & 50\begin{bmatrix} 3 \\ 2 \end{bmatrix} \\ Bars & \end{array}$$

$$= \begin{array}{c|c} & \$ \\ \hline Rings & 150 \\ Bars & 100 \end{array}$$

11 a $5A - 2B$ is found by computing:

$$5\begin{bmatrix} 6 & 3 \\ -2 & 14 \end{bmatrix} - 2\begin{bmatrix} 15 & -7 \\ 8 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 15 \\ -10 & 70 \end{bmatrix} - \begin{bmatrix} 30 & -14 \\ 16 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 29 \\ -26 & 70 \end{bmatrix}$$

b Since C and D have different dimensions, they cannot be added or subtracted. Thus, $C - 2D$ is undefined.

c Since B and D have different dimentions, they cannot be added or subtracted. Thus, $2B - D$ is undefined.

12 We can find the value of each pronumeral in turn.

a $3a + 2 \times 3 = 3$ so $3a + 6 = 3$ so $a = -1$
$3 \times 5 + 2b = 19$ so $15 + 2b = 19$ so $b = 2$
$3 \times -2 + 2 \times -1 = c$ so $-6 - 2 = c$ so $c = -8$
$3 \times 6 + 2 \times 7 = d$ so $18 + 14 = d$ so $d = 32$

b $5a - 3 \times 6 = 2$ so $5a - 18 = 2$ so $a = 4$
$5 \times -7 - 3b = -47$ so $-35 - 3b = -47$ so $-3b = -12$ so $b = 4$
$5 \times 10 - 3 \times -4 = 2d$ so $50 + 12 = 2d$ so $62 = 2d$ so $d = 31$
$5 \times 9 - 3d = c$ since $d = 31$,
$45 - 3 \times 31 = c$ sp $45 - 93 = c$ so $c = -48$

Solutions to Exercise 4D

1 Recall that the first entry is found by combining the first row and first column following the matrix multiplication algorithm.

$$\begin{bmatrix} 2 \times 6 + (-5) \times 7 \\ 3 \times 6 + 1 \times 7 \end{bmatrix}$$

2 Matrix A has 2 rows and 3 columns so the order is 2×3.
Matrix B has 3 rows and 2 columns so the order is 3×2. Since A has the same number of columns as rows that B has, the two matrices can be multiplied: AB.

3 Matrix A has 4 rows and 3 colums so the order is 4×4.
Matrix B has 3 rows and 4 colums so the order is 3×4.
When A and B are multiplied together as AB, the resulting matrix will have 4 rows and 4 columns so the order is 4×4.

4 The identity matrix has 1's on the main diagonal and zeros in all other positions.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5 **a** $\begin{array}{cc} A & B \\ 2 \times 2 & 2 \times 1 \end{array}$

As the middle two numbers are the same, the matrix can be defined.
Order $(AB) = 2 \times 1$

$$\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \times 5 + 2 \times 4 \\ 3 \times 5 + 1 \times 4 \end{bmatrix}$$
$$= \begin{bmatrix} 38 \\ 19 \end{bmatrix}$$

b $\begin{array}{cc} B & A \\ 2 \times 1 & 2 \times 2 \end{array}$

The two middle numbers are not the same, so the matrix cannot be defined.

c $\begin{array}{cc} C & B \\ 3 \times 2 & 2 \times 1 \end{array}$

As the middle two numbers are the same, the matrix can be defined.
Order $(CB) = 3 \times 1$

$$\begin{bmatrix} 1 & 3 \\ 0 & 8 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \times 5 + 3 \times 4 \\ 0 \times 5 + 8 \times 4 \\ 2 \times 5 - 5 \times 4 \end{bmatrix}$$
$$= \begin{bmatrix} 17 \\ 32 \\ -10 \end{bmatrix}$$

d $\begin{array}{cc} B & C \\ 2 \times 1 & 3 \times 2 \end{array}$

The two middle numbers are not the same, so the matrix cannot be defined.

e $\begin{array}{cc} A & A \\ 2 \times 2 & 2 \times 2 \end{array}$

As the middle two numbers are the same, the matrix can be defined.
Order $(AA) = 2 \times 2$

$$\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 6 \times 6 + 2 \times 3 & 6 \times 2 + 2 \times 1 \\ 3 \times 6 + 1 \times 3 & 3 \times 2 + 1 \times 1 \end{bmatrix}$$
$$= \begin{bmatrix} 42 & 14 \\ 21 & 7 \end{bmatrix}$$

f $B \quad B$

$2 \times 2 \quad 1 \times 1$

The two middle numbers are not the same, so the matrix cannot be defined.

g $A \quad C$

$2 \times 2 \quad 3 \times 2$

The two middle numbers are not the same, so the matrix cannot be defined.

h $C \quad A$

$3 \times 2 \quad 2 \times 2$

As the middle two numbers are the same, the matrix can be defined.

Order $(CA) = 3 \times 2$

$$\begin{bmatrix} 1 & 3 \\ 0 & 8 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 6 + 3 \times 3 & 1 \times 2 + 3 \times 1 \\ 0 \times 6 + 8 \times 3 & 0 \times 2 + 8 \times 1 \\ 2 \times 6 - 5 \times 3 & 2 \times 2 - 5 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 5 \\ 24 & 8 \\ -3 & -1 \end{bmatrix}$$

6 **a** $1\times2 \quad 2\times1$

As the middle terms are the same, matrix multiplication is defined.

$$\begin{bmatrix} 3 & 4 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$= [3 \times 6 + 4 \times 5]$

$= [18 + 20]$

$= [38]$

b $1\times2 \quad 3\times1$

As the middle terms are not the same, matrix multiplication is not defined.

c $1\times3 \quad 3\times1$

As the middle terms are the same, matrix multiplication is defined.

$$\begin{bmatrix} 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$= [0 \times 0 + 1 \times 1 + 1 \times 0]$

$= [0 + 1 + 0]$

$= [1]$

d $1\times3 \quad 2\times1$

As the middle terms are not the same, matrix multiplication is not defined.

e $1\times4 \quad 4\times1$

As the middle terms are the same, matrix multiplication is defined.

$$\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$= [1 \times 1 + 1 \times 0 + 0 \times 1 + 1 \times 1]$

$= [1 + 0 + 0 + 1]$

$= [2]$

f $1\times4 \quad 3\times1$

As the middle terms are not the same, matrix multiplication is not defined.

7 **a** $$\begin{bmatrix} 4 & 2 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \times 3 + 2 \times 5 \\ 1 \times 3 + 6 \times 5 \end{bmatrix}$$

$$= \begin{bmatrix} 22 \\ 33 \end{bmatrix}$$

b $$\begin{bmatrix} 8 & 4 \\ 5 & 9 \end{bmatrix}\begin{bmatrix} 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \times 7 + 4 \times 2 \\ 5 \times 7 + 9 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 64 \\ 53 \end{bmatrix}$$

c $$\begin{aligned}&\begin{bmatrix}3&5\\1&8\end{bmatrix}\begin{bmatrix}2&4\\-2&-3\end{bmatrix}\\&=\begin{bmatrix}3\times2+5\times-2&3\times4-5\times-3\\1\times2+8\times-2&1\times4+8\times-3\end{bmatrix}\\&=\begin{bmatrix}-4&-3\\-14&-20\end{bmatrix}\end{aligned}$$

d $$\begin{aligned}&\begin{bmatrix}8&3\\5&2\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix}\\&=\begin{bmatrix}8\times1+3\times0&8\times0+3\times1\\5\times1+2\times0&5\times0+2\times1\end{bmatrix}\\&=\begin{bmatrix}8&3\\5&2\end{bmatrix}\end{aligned}$$

e $$\begin{aligned}\begin{bmatrix}7&4\\1&0\\6&1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}&=\begin{bmatrix}7\times1+4\times1\\1\times1+0\times1\\6\times1+1\times1\end{bmatrix}\\&=\begin{bmatrix}11\\1\\7\end{bmatrix}\end{aligned}$$

f $$\begin{aligned}\begin{bmatrix}3&2\end{bmatrix}\begin{bmatrix}1\\9\end{bmatrix}&=\begin{bmatrix}3\times1+2\times9\end{bmatrix}\\&=\begin{bmatrix}21\end{bmatrix}\end{aligned}$$

g $$\begin{aligned}\begin{bmatrix}4&1&2\end{bmatrix}\begin{bmatrix}0\\6\\1\end{bmatrix}&=\begin{bmatrix}4\times0+1\times6+2\times1\end{bmatrix}\\&=\begin{bmatrix}8\end{bmatrix}\end{aligned}$$

h $$\begin{aligned}\begin{bmatrix}6&2\end{bmatrix}\begin{bmatrix}4\\3\end{bmatrix}&=\begin{bmatrix}6\times4+2\times3\end{bmatrix}\\&=\begin{bmatrix}30\end{bmatrix}\end{aligned}$$

i $$\begin{aligned}\begin{bmatrix}3&2&1\end{bmatrix}\begin{bmatrix}6\\5\\8\end{bmatrix}&=\begin{bmatrix}3\times6+2\times5+1\times8\end{bmatrix}\\&=\begin{bmatrix}36\end{bmatrix}\end{aligned}$$

8 **a** $$\begin{aligned}AB&=\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}5&6\\7&8\end{bmatrix}\\&=\begin{bmatrix}1\times5+2\times7&1\times6+2\times8\\3\times5+4\times7&3\times6+4\times8\end{bmatrix}\\&=\begin{bmatrix}19&22\\43&50\end{bmatrix}\end{aligned}$$

b $$\begin{aligned}BA&=\begin{bmatrix}5&6\\7&8\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}\\&=\begin{bmatrix}5\times1+6\times3&5\times2+6\times4\\7\times1+8\times3&7\times2+8\times4\end{bmatrix}\\&=\begin{bmatrix}23&34\\31&46\end{bmatrix}\end{aligned}$$

c AB does not equal BA.

9 Type the matrices into your CAS using the instructions given on page 246 and 247.

a

$$\begin{bmatrix}0&-8\\4&2\end{bmatrix}$$

b

$$\begin{bmatrix}16&14\\16&14\end{bmatrix}$$

c

$$\begin{bmatrix}83\end{bmatrix}$$

d

$$\begin{bmatrix}4\end{bmatrix}$$

e

$$\begin{bmatrix}3&3\end{bmatrix}$$

f

$$\begin{bmatrix}31\\35\\21\end{bmatrix}$$

10 These questions are best done using a CAS calculator.

a **i** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

ii $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

iii $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

b **i** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

ii $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

iii $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c **i** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

ii $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

iii $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

11 Matrices can only be multiplied together if the first matrix has the same number of columns as rows in the second column. Thus:
AC: Since A is 3×1 and C is 1×3 giving a 3×3 matrix.
CA: Since C is 1×3 and A is 3×1 giving a 1×1 matrix.
BC: Since B is 2×1 and C is 1×3 giving a 2×3 matrix.
DB: Since D is 2×2 and B is 12×1 giving a 2×1 matrix.

12 This is best performed with a CAS calculator.

$$\begin{bmatrix} 27 & 5 & 13 \\ 18 & 15 & -3 \end{bmatrix}$$

13 You could use simultaneous equations to solve this by noting that:
$-6 \times a + 3 \times c = -27$
$-2 \times a + 5 \times c = -37$
$-6 \times b + 3 \times d = -30$
$-2 \times b + 5 \times d = -18$
Using your CAS to solve these gives $a = 1$, $b = 4$, $c = -7$ and $d = -2$. So the second matrix is

$$\begin{bmatrix} 1 & 4 \\ -7 & -2 \end{bmatrix}$$

Solutions to Exercise 4E

1 True - when the product of two matrices gives the identity matrix, the two matrices are inverses of each other.

2 Use your CAS. The result of the multiplication is the identity matrix which means that D is the inverse of C (and vice versa).

3 **a** $3x + 7y = 27$ is equivalent to saying $3 \times x + 7 \times y = 27$ Similarly, $5x + 6y = 28$ is equivalent to saying $5 \times x + 6 \times y = 28$. Thus, we can represent the two equations in matrix form as follows:

$$\begin{bmatrix} 3 & 7 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 27 \\ 28 \end{bmatrix}$$

b $2x + 8y = -2$ is equivalent to saying $2 \times x + 8 \times y = -2$ Similarly, $3x + 20y = 5$ is equivalent to saying $2 \times x + 8 \times y = 5$. Thus, we can represent the two equations in matrix form as follows:

$$\begin{bmatrix} 2 & 8 \\ 3 & 20 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

4 These questions can be answered using your CAS by following the instructions on page 250 and 251 of your textbook.

a $\begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$

b $\begin{bmatrix} 1 & -2 \\ -2 & 4.5 \end{bmatrix}$

c $\begin{bmatrix} -4 & 9 \\ 1 & -2 \end{bmatrix}$

d $\begin{bmatrix} -1.5 & 3.5 \\ 1 & -2 \end{bmatrix}$

e $\begin{bmatrix} 0.5 & -0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 \end{bmatrix}$

f $\begin{bmatrix} 0.1 & -0.2 & 0.35 \\ 0.4 & 0.2 & -0.6 \\ -0.1 & 0.2 & 0.15 \end{bmatrix}$

g $\begin{bmatrix} 16 & 10 & -15 \\ -8 & -5 & 8 \\ 3 & 2 & -3 \end{bmatrix}$

h $\begin{bmatrix} -0.25 & 0.125 & 0.5 \\ 0 & 0.5 & 0 \\ 0.5 & -0.25 & 0 \end{bmatrix}$

5 Use your CAS calculator using pages 250 and 251 of the textbook as a guide.

a $\begin{bmatrix} -\frac{-6}{31} & \frac{1}{31} \\ \frac{7}{31} & \frac{4}{31} \end{bmatrix}$

b $\begin{bmatrix} -\frac{25}{566} & \frac{19}{566} \\ \frac{7}{283} & \frac{6}{283} \end{bmatrix}$

c $\begin{bmatrix} \frac{5}{344} & \frac{41}{2752} \\ -\frac{1}{86} & \frac{9}{688} \end{bmatrix}$

d $\begin{bmatrix} -\frac{767}{32} & \frac{21}{4} \\ -\frac{103}{8} & 13 \end{bmatrix}$

6 To find the determinant, find $ad - bc$ of the matrix.

a $\det(A) = 3 \times 5 - (-1) \times 4 = 15 - (-4) = 19$

b $\det(B) = -7 \times 1 - 2 \times 3 = -7 - 6 = -13$

c $\det(C) = -4 \times 8 - 9 \times 6 = -32 - 54 = -86$

d $\det(D) = 0 \times -1 - 8 \times 4 = 0 - 32 = -32$

7 Use your CAS calculator.

a $\begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{7} & \frac{1}{28} \end{bmatrix}$

b $\begin{bmatrix} \frac{1}{14} & -\frac{3}{14} \\ \frac{3}{28} & \frac{5}{28} \end{bmatrix}$

c $\begin{bmatrix} -\frac{3}{98} & \frac{1}{98} \\ \frac{5}{196} & \frac{13}{392} \end{bmatrix}$

d $\begin{bmatrix} -\frac{3}{98} & \frac{1}{98} \\ \frac{5}{196} & \frac{13}{392} \end{bmatrix}$

8 **a** Note that $5x + y = 13$ can be expressed as $5 \times x + 1 \times y = 13$ and that $3x + 2y = 12$ can be expressed as $3 \times x + 2 \times y = 12$. This means we can express the equations in matrices as:

$$\begin{bmatrix} 5 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ 12 \end{bmatrix}$$

b Note that $x + 2y = 10$ can be expressed as $1 \times x + 2 \times y = 12$ and that $4x + y = 5$ can be expressed as $4 \times x + 1 \times y = 5$. This means we can express the equations in matrices as:

$$\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

c Note that $7x - 2y = -31$ can be expressed as $7 \times x - 2 \times y = -31$ and that $-3x + 2y = -1$ can be expressed as $-3 \times x + 2 \times y = -1$. This means we can express the equations in matrices as:

$$\begin{bmatrix} 7 & -2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -31 \\ -1 \end{bmatrix}$$

d Note that $6x + 5y = 38$ can be expressed as $6 \times x + 5 \times y = 38$ and that $9x + 3y = 66$ can be expressed as $9 \times x + 3 \times y = 66$. This means we can express the equations in matrices as:

$$\begin{bmatrix} 6 & 5 \\ 9 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 38 \\ 66 \end{bmatrix}$$

9 Convert each of the following to matrix form and then use your CAS calculator to solve.

a
$$\begin{aligned}\begin{bmatrix}3x+2y\\5x+y\end{bmatrix} &= \begin{bmatrix}12\\13\end{bmatrix}\\ \begin{bmatrix}3&2\\5&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} &= \begin{bmatrix}12\\13\end{bmatrix}(A\times B=C)\\ \begin{bmatrix}x\\y\end{bmatrix} &= A^{-1}\times C\\ &= \begin{bmatrix}2\\3\end{bmatrix}\end{aligned}$$
$x=2, y=3$

b
$$\begin{aligned}\begin{bmatrix}4x+3y\\x+2y\end{bmatrix} &= \begin{bmatrix}10\\5\end{bmatrix}\\ \begin{bmatrix}4&3\\1&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} &= \begin{bmatrix}10\\5\end{bmatrix}(A\times B=C)\\ \begin{bmatrix}x\\y\end{bmatrix} &= A^{-1}\times C\\ &= \begin{bmatrix}1\\2\end{bmatrix}\end{aligned}$$
$x=1, y=2$

c
$$\begin{aligned}\begin{bmatrix}4x-3y\\3x+y\end{bmatrix} &= \begin{bmatrix}10\\1\end{bmatrix}\\ \begin{bmatrix}4&-3\\3&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} &= \begin{bmatrix}10\\1\end{bmatrix}(A\times B=C)\\ \begin{bmatrix}x\\y\end{bmatrix} &= A^{-1}\times C\\ &= \begin{bmatrix}1\\-2\end{bmatrix}\end{aligned}$$
$x=1, y=-2$

d
$$\begin{aligned}\begin{bmatrix}8x+3y\\5x+2y\end{bmatrix} &= \begin{bmatrix}50\\32\end{bmatrix}\\ \begin{bmatrix}8&3\\5&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} &= \begin{bmatrix}50\\32\end{bmatrix}(A\times B=C)\\ \begin{bmatrix}x\\y\end{bmatrix} &= A^{-1}\times C\\ &= \begin{bmatrix}4\\6\end{bmatrix}\end{aligned}$$
$x=4, y=6$

e
$$\begin{aligned}\begin{bmatrix}6x+7y\\4x+5y\end{bmatrix} &= \begin{bmatrix}68\\46\end{bmatrix}\\ \begin{bmatrix}6&7\\4&5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} &= \begin{bmatrix}68\\46\end{bmatrix}(A\times B=C)\\ \begin{bmatrix}x\\y\end{bmatrix} &= A^{-1}\times C\\ &= \begin{bmatrix}9\\2\end{bmatrix}\end{aligned}$$
$x=9, y=2$

f
$$\begin{aligned}\begin{bmatrix}6x-5y\\7x+4y\end{bmatrix} &= \begin{bmatrix}-27\\-2\end{bmatrix}\\ \begin{bmatrix}6&-5\\7&4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} &= \begin{bmatrix}-27\\-2\end{bmatrix}(A\times B=C)\\ \begin{bmatrix}x\\y\end{bmatrix} &= A^{-1}\times C\\ &= \begin{bmatrix}-2\\3\end{bmatrix}\end{aligned}$$
$x=-2, y=3$

10 Since $AA^{-1}=I$ and $A^{-1}A=I$, then
$A\times(AA^{-1})\times(A^{-1}A)\times A^{-1}=AA^{-1}=I$
or equivalently, $\begin{bmatrix}1&0\\0&1\end{bmatrix}$

11 Multiplying the first two matrices gives:
$3\times x+(-2)\times y=23$
$7\times x+(-5)\times y=55$
Rewriting, $3x-2y=23$ and
$7x-5y=55$

12 **a** Performing the matrix multiplication gives:

$-a + 7b = 1, -b + 7d = 0$

$3a + 8c = 0, 3b + 8d = 1$

Solving each pair of equations simultaneously using the CAS gives:

$a = -\frac{8}{29}, b = \frac{7}{29}, c = \frac{3}{29}, d = \frac{1}{29}$

b Performing the matrix multiplication gives:

$5a + 11b = 1, -6a + 4b = 0$

$5c + 11d = 0, -6c + 4d = 1$

Solving each pair of equations simultaneously using the CAS gives:

$a = \frac{2}{43}, b = \frac{3}{43}, c = -\frac{11}{86}, d = \frac{5}{86}$

Solutions to Exercise 4F

1 There is a line between *B* and *A* and *B* and *E* so town B is directly connected to both *A* and *E*.

2 Since there are two lines between *A* and *C* then these two towns are directly connected by two rides.

3 If two towns are connected by one road then a 1 should be placed in the corresponding position. If the two towns are connected by 2 roads then place a 2 in the corresponding position. For example, *A* and *C* are connected by 2 roads and so we place a 2 in the 3rd row and 1st column. Since *B* and *D* are not connected, we place a 0 in the 4th row and 2nd column.

Continuing in this way gives:

$$\begin{array}{cc} \begin{array}{ccccc} A & B & C & D & E \end{array} & \\ \begin{bmatrix} 0 & 1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} \end{array}$$

4 **a** **i** There are two towns *A* and *B*, so use a 2 × 2 matrix to show direct connections. Label the columns *A*, *B* and the rows *A*, *B*.

There are no roads connecting *A* to itself so enter 0 where column *A* crosses row *A*.

There are 3 roads from *B* to *A* so enter 3 where column *B* crosses row *A*.

There are 3 roads from *A* to *B* so enter 3 where column *A* crosses row *B*.

There are no roads connecting *B* to itself so enter 0 where column *B* crosses row *B*.

ii – **vi** There are three towns *A B* and *C*, so use a 3 × 3 matrix, with columns and rows labelled *A*, *B* and *C*, to show direct connections using the method above.

b The second column gives the number of roads directly connecting *B* to *A*, *B* and *C*. The sum of the second column gives the total number of roads directly connected to *B*.

5 **a** **i** Draw three points to represent towns *A*, *B* and *C*. Read down column *A* to row *A* to find that there are 0 roads directly connecting *A* to *A*. So no road from *A* looping back to itself is needed.

Read down column *A* to row *B* to find that there is 1 road directly connecting *A* to *B*. So draw 1 road from *A* to *B*. Read down column *A* to row *C* to find that there is 1 road directly connecting *A* to *C*. So draw 1 road from *A* to *C*. Repeat this procedure working down columns *B* and *C*.

ii – **iv** Use the method described in **i**.

b The first column gives the number of roads directly connecting *A* to *A*, *A* to *B* and *A* to *C*. The sum of the first column gives the total number of roads directly connected to *A*.

6 **a**

$$\begin{array}{c|cccc|} & A & B & C & D \\ A & 0 & 1 & 0 & 1 \\ B & 1 & 0 & 1 & 1 \\ C & 0 & 1 & 0 & 0 \\ D & 1 & 1 & 0 & 0 \end{array}$$

b To determine who has met the most people, add up each row (or column). The row (or column) with the largest value has met the most people.

c Read from matrix: *B* has met 3 people.

d Read from matrix: *C* has met 1 person.

7 Reading down column *A* to row *D*, the 0 means *A* does not communicate with *D*. However, reading down column *D* to row *A* the 1 means *D* does communicate with *A*. A contradiction, because all communications were assumed to be two-way.

8 **a** Label the columns and rows of a 3×3 matrix as *A*, *B* and *C*.
Where column *A* meets row *A* write 0, as communication must be between two people.
Where column *A* meets row *B* write 1, as an arrow joins *A* to *B*.
Where column *A* meets row *C* write 0, as no arrow joins *A* to *C*.
Repeat for column *B*, then column *C*, entering 1 when an arrow joins the people, otherwise 0.

$$\begin{array}{|ccc|c} A & B & C & \\ 0 & 1 & 0 & A \\ 1 & 0 & 1 & B \\ 0 & 1 & 0 & C \end{array}$$

b Label the columns and rows of a 4×4 matrix *D*, *E*, *F* and *G*.
Use the method described in part **a** to obtain:

$$\begin{array}{|cccc|c} D & E & F & G & \\ 0 & 1 & 1 & 1 & D \\ 1 & 0 & 0 & 0 & E \\ 1 & 0 & 0 & 1 & F \\ 1 & 0 & 1 & 0 & G \end{array}$$

c Label the columns and rows of a 4×4 matrix *J*, *K*, *L* and *M*.
Use the method described in part **a** to obtain:

$$\begin{array}{|cccc|c} J & K & L & M & \\ 0 & 1 & 1 & 0 & J \\ 1 & 0 & 1 & 1 & K \\ 1 & 1 & 0 & 0 & L \\ 0 & 1 & 0 & 0 & M \end{array}$$

9 **a** Draw three dots to represent the towns *A*, *B* and *C*.
Read the value where each column crosses a row. Finding a 1 there means that a road should join the towns. However, 0 indicates that a road should not join the towns.
For example, where column *A* meets row *A* the 0 means there is no road from *A* looping back to *A*.
Where column *A* meets row *B* the 0 means there is no road from *A* to *B*.
Where column *A* meets row *C* the 1

means there is a road from A to C. Read the values at the intersections of the columns and rows and join the dots representing the towns when the value is 1 to obtain the diagram shown in the answers. A 0 means no road joins the towns. Other variations of the diagrams may be possible.

b Draw four dots to represent the towns P, Q, R and S. Use the method described in part **a**.

c Draw four dots to represent the towns T, U, V and W. Use the method described in part **a**.

10 a Label the columns and rows of a 4×4 matrix as C, E, K and R.
Where column C meets row C write 0, as communication must be between two people.
Where column C meets row E write 1, as an arrow joins C to E.
Where column C meets row K write 1, as an arrow joins C to K.
Where column C meets row R write 1, as an arrow joins C to R.
Repeat for column E, then column K and R, entering 1 when an arrow joins the people, otherwise 0.

$$Q = \begin{array}{cccc|c} C & E & K & R & \\ 0 & 1 & 1 & 1 & C \\ 1 & 0 & 0 & 1 & E \\ 1 & 0 & 0 & 1 & K \\ 1 & 1 & 1 & 0 & R \end{array}$$

b The sum of a column (or row) gives the number of people that person communicates with. So the sum of column R tells us that Remy communicates with 3 people.

c i Using a calculator to find Q^2 :

$$Q^2 = \begin{array}{cccc|c} C & E & K & R & \\ 3 & 1 & 1 & 2 & C \\ 1 & 2 & 2 & 1 & E \\ 1 & 2 & 2 & 1 & K \\ 2 & 1 & 1 & 3 & R \end{array}$$

ii The sum of column E is 6, so Eli communicates with 6 people via another person (two step communications).

iii From the diagram Eli has the following two-step communications:

$E \rightarrow R \rightarrow C$
$E \rightarrow R \rightarrow E$
$E \rightarrow C \rightarrow E$
$E \rightarrow R \rightarrow K$
$E \rightarrow C \rightarrow K$
$E \rightarrow C \rightarrow R$

11 a Label the columns and rows of a 4×4 matrix as E, F, G and H.
Write 1 where the column for a town meets the row of a town if there is a road directly connecting the two towns. Otherwise, write 0.
For example, write 0 where column E intersects row E as there is no road directly connecting town E to itself.
Write 1 where column E intersects row F as one road directly connects town E to town F. And, so on, to give the matrix:

$$R = \begin{array}{cccc|c} E & F & G & H & \\ 0 & 1 & 1 & 0 & E \\ 1 & 0 & 1 & 1 & F \\ 1 & 1 & 0 & 1 & G \\ 0 & 1 & 1 & 0 & H \end{array}$$

b The sum of column F reveals that 3 towns are directly connected to town F.

c **i** Using a calculator:

$$R^2 = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{cccc|c} E & F & G & H & \\ 2 & 1 & 1 & 2 & E \\ 1 & 3 & 2 & 1 & F \\ 1 & 2 & 3 & 1 & G \\ 2 & 1 & 1 & 2 & H \end{array}$$

ii The sum of column F in matrix R^2 is 7.
So there are 7 ways to travel from Fields to a town via another town. This includes travelling from Fields to another town then travelling back to Fields.

iii The list of possible ways is:
$F \to G \to E$.
$F \to E \to F$.
$F \to G \to F$.
$F \to H \to F$.
$F \to E \to G$.
$F \to H \to G$.
$F \to G \to H$.

12 **a** Label the columns and rows of a 5×5 matrix as A, B, C, D and E.
Write 1 where the column for a persom meets the row of a person when there is communication between them. Otherwise, write 0.
For example, write 0 where column E intersects row E as there is no communication from E to itself.
Write 1 where column E intersects row B as Evangeline and Beth communicate. And, so on, to give the matrix:

$$F = \begin{array}{ccccc|c} A & B & C & D & E & \\ 0 & 1 & 1 & 1 & 0 & A \\ 1 & 0 & 1 & 0 & 1 & B \\ 1 & 1 & 0 & 0 & 1 & C \\ 1 & 0 & 0 & 0 & 0 & D \\ 0 & 1 & 1 & 0 & 0 & E \end{array}$$

b Using your CAS, calculate F^2 then F^3, then F^4 and so on until the only 0s are on the main diagonal. Thus, the number of steps is 3 since F^3 satisfies this.

c From **b**, we know that there are 2 possible paths. These are:
$E \to C \to A \to D$
$E \to B \to A \to D$

13 **a** Construct a 10 × 10 matrix and label each column and row with each team. Write a 1 when the column for a team meets the row of a team that have played each other. Otherwise, write a 0.
For example, write a 1 where column N intersects with row S as they play each other in round 1.

b Add up each row and column separately. Each should add up to 3 as there have been three rounds and all teams played one game in each round.

c After 8 rounds, each row and column should add up to 8 as each team has played 8 games.

d After 9 rounds, each team should have played each of the other 9 teams once so there should be 1s in all positions except along the main diagonal, which contains 0s.

e In round 10, all teams have already played each other once and so each match will be a rematch. Thus, there will be a 2 in each row and column to indicate that the teams have now played each other twice. Otherwise, there are 0s on the main diagonal and 1s in all other positions.

Solutions to Exercise 4G

1 Recall that the columns in a transition matrix add up to 1. In the first column, 1 − 0.8 = 0.2 must be in the second row and in the second column, 1 − 0.3 = 0.7 in the second row.

$$B = \begin{array}{cc} H & C \\ \left[\begin{array}{cc} 0.8 & 0.4 \\ 0.2 & 0.6 \end{array}\right] & \begin{array}{c} H \\ C \end{array} \end{array}$$

2 **a** Since 60% of children on the Ferris wheel go on the Ferris wheel again, 1 − 60% = 40% go on the Merry-go-round next time.

b Since 40% of children on the Merry-go-round go on the Merry-go-round again, 1 − 50% = 50% go on the Ferris wheel next time.

3 **a** There are 50 people at location *A* and 90% of these people remain at *A*. That is, 50 × 90% = 45 people remain at *A*.

b There are 50 people at location *B* and 30% of these people change to *A*. That is, 50 × 30% = 15 people change to *A*.

c The predicted number at *A* is the sum of these two: 45 + 15 = 60 people expected at *A* next period.

4 **a** Since 60% of people who buy a pie this week will buy a pie next week, 1 − 60% = 40% will buy dim sims next week.

b Since 80% of people who buy dim sims this week will buy dim sims next week, 1 − 80% = 20% will buy a pie next week.

c A transition matrix is formed by putting labels on the rows and columns. The entries are given in decimal form.

$$T = \begin{array}{cc} \begin{array}{cc} P & D \end{array} & \\ \left[\begin{array}{cc} 0.6 & 0.2 \\ 0.4 & 0.8 \end{array}\right] & \begin{array}{c} P \\ D \end{array} \end{array}$$

5 **a** **i** If it rains today, there is a 70% chance it will rain tomorrow so there is a 30% chance it won't rain tomorrow.

ii If it does not rain today, there is a 60% chance it will rain tomorrow so there is a 40% chance it won't rain tomorrow.

b Putting these numbers into a matrix with row and column labels with *R* for rain and *N* for not rain gives:

$$T = \begin{array}{cc} \begin{array}{cc} R & N \end{array} & \\ \left[\begin{array}{cc} 0.7 & 0.6 \\ 0.4 & 0.4 \end{array}\right] & \begin{array}{c} R \\ N \end{array} \end{array}$$

6 To construct a transition matrix, we need to interpret the diagram.
Note that of those currently in state *A*, 85% stay in state *A* and 15% transition to state *B*. This means that the first column is:

$$\begin{bmatrix} 0.85 \\ 0.15 \end{bmatrix}$$

Of those currently in state B, 55% stay in state B and 45% transition to state A. This means that the second column is:
$$\begin{bmatrix} 0.45 \\ 0.55 \end{bmatrix}$$
Putting this together gives the matrix with lables A and B for state A and B respectively.

$$T = \begin{array}{cc} \begin{array}{cc} A & B \end{array} & \\ \begin{bmatrix} 0.85 & 0.45 \\ 0.15 & 0.55 \end{bmatrix} & \begin{array}{c} A \\ B \end{array} \end{array}$$

7 **a** **i** From the matrix, the entry in the first colum and first row is 0.75 which corresponds to the percentage of people who buy a latte on Monday who then buy a latte on Tuesday. Thus, 75% of whose who buy a latte on Monday are predicted to buy a latte on Tuesday.

ii From **ai**, this means that $1 - 0.75 = 0.25$ or 25% are predicted to buy a flat white.

b If 160 lattes are sold on Monday, then $160 \times 0.75 = 120$ lattes are sold to those people on Tuesday.

c Since 120 of the 160 people who bought lattes on Monday bought lattes on Tuesday, then $160 - 120 = 40$ people bought flat whites.

d Considering both those who bought lattes and flat whites on Monday, the number of lattes sold on Tuesday is: $160 \times 0.75 + 100 \times 0.4 = 120 + 40 = 160$.

e This can be calculated in two ways. Either $160 + 100 = 260$ people in total and 160 bought lattes on Tuesday so $260 - 160 = 100$ bought flat whites on Tuesday, or $160 \times 0.25 + 100 \times 0.6 = 40 + 60 = 100$.

8 **a** On Monday, 170 students were on time and 90% of them were predicted to be on time on Tuesday so $170 \times 90\% = 153$.

b On Monday, 30 students were late and 70% of them were predicted to be on time on Tuesday so $30 \times 70\% = 21$.

c In total, this means that we expect $153 + 21 = 174$ students to be on time on Tuesday.

d In total there are $170 + 30 = 200$ students. This means we expect $200 - 174 = 26$ students to be late on Tuesday.

9 Looking at the diagram we can see that 40% transition from X to Y so we can put 0.4 on the entry on the first row and second column.
Considering Y, we see that 30% transition from Y to X and 10% from Y to Z so the second column is
$$\begin{bmatrix} 0.3 \\ 0.6 \\ 0.1 \end{bmatrix}$$
Considering Z, we see that 50% transition from Z to X, 20% from Z to Y and 30% remain at Z so the third column is

$$\begin{bmatrix} 0.5 \\ 0.2 \\ 0.3 \end{bmatrix}$$

This gives:

$$\begin{array}{cc} \begin{array}{ccc} X & Y & Z \end{array} & \\ \begin{bmatrix} 0.4 & 0.3 & 0.5 \\ 0.4 & 0.6 & 0.2 \\ 0.2 & 0.1 & 0.3 \end{bmatrix} & \begin{array}{c} X \\ Y \\ Z \end{array} \end{array}$$

Solutions to Exercise 4H

1 To construct, make sure that you label each row of the matrix and put the number of machines in the right row. Since there are 100 machines in operating, 100 should go in the O row while 25 should go in the B row for broken.

$$\begin{bmatrix} 100 \\ 25 \end{bmatrix} \begin{array}{c} O \\ B \end{array}$$

2 There are two ways of answering this. One is to use your CAS to multiply the two matrices and then consider the number in the required row (first row). Alternatively you can interpret the matrices as follows:
Above a C: $0.85 \times 60 + 0.1 \times 140 = 65$
Below a C: $0.15 \times 60 + 0.9 \times 140 = 135$
Thus, 65 students get above a C and 135 get below a C.

3 **a** Use your CAS.

b Use you CAS.

c Use your CAS.

4 **a** Use your CAS to calculate $T \times S_0$. To find how many are predicted to go to Bill's, read off the first line.

b To find the numbers after 4 weeks, use your CAS to calculate $T^4 \times S_0$. The first number (195) gives the predicted number at Bill's Barnacles while the second number (155) predicts the number of people at Sally's Seafood.

c To find the numbers after 12 weeks, use your CAS to calculate $T^{12} \times S_0$. The first number (194) gives the predicted number at Bill's Barnacles while the second number (156) predicts the number of people at Sally's Seafood.

d To calculate what is expected in the long run, use your CAS to calculate, say $T^{20} \times S_0$ and $T^{30} \times S_0$. If these two numbers are very close, then we've reached the long run. The first number (194) gives the predicted number at Bill's Barnacles while the second number (156) predicts the number of people at Sally's Seafood.

5 **a** Note that if 90% of players who are available in one week are available in the next week, then 10% are injured the next week. If 40% of injured players are available next week, the 60% remain injured. putting this information into a matrix in decimal form gives:

$$T = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} A & I \\ \end{array}$$

$$T = \begin{array}{ccc} A & I & \\ \left[\begin{matrix} 0.9 \\ 0.1 \end{matrix}\right. & \left.\begin{matrix} 0.4 \\ 0.6 \end{matrix}\right] & \begin{matrix} A \\ I \end{matrix} \end{array}$$

b If all 50 players are available, then 0 are injured. These can go into a column matrix as follows:

$$S_0 = \begin{bmatrix} 50 \\ 0 \end{bmatrix}$$

c Perform the multiplication using your CAS to find $T \times S_0$. Read off the first line to find that 45 players will be available.

d To calculate what is expected in the long run, use your CAS to calculate, say $T^{20} \times S_0$ and $T^{30} \times S_0$. If these two numbers are very close, then we've reached the long run. The number of people expected on the injury list is the second number (10).

6 **a** Construct a 2 × 2 matrix with the labels R and I for read and ignore along the top and side.

$$T = \begin{array}{cc} R & I \\ \left[\begin{array}{cc} 0.8 & 0.25 \\ 0.2 & 0.75 \end{array}\right] & \begin{array}{c} R \\ I \end{array} \end{array}$$

b The initial numbers of 4000 and 2000 can be placed in a column matrix with the number for R listed first so that the matrix is consistent with the matrix T in **a**.

$$S_0 = \begin{bmatrix} 4000 \\ 2000 \end{bmatrix}$$

c There are 4000 people who currently read the paper. Of these, 80% of them will read the paper next week, then $4000 \times 80\% = 3200$ will read it next week. This means that $4000 - 3200 = 800$ will ignore it next week.

d There are 2000 people who currently ignore the paper. Of these, 75% of them will read the paper next week, then $2000 \times 25\% = 500$ will read it next week. This means that $2000 - 500 = 1500$ will ignore it next week.

e To calculate the number of people who read and ignore the paper after 6 weeks, we perform the matrix multiplication $T^6 \times S_0$ on the CAS. We can then read the numbers off the resulting column matrix where the number who read is listed first and those who ignore is listed second.

f To calculate what is expected in the long run, use your CAS to calculate, say $T^{20} \times S_0$ and $T^{30} \times S_0$. If these two numbers are very close, then we've reached the long run. The first number (3333) listed is the number of people that we expect to read the paper in the long run while the second number (2667) listed is the number of people we expect will ignore the paper in the long run.

7 **a** From the diagram, we construct a 2 × 2 matrix with rows and columns labeled as D and P.
Since 90% of those driving continue to drive and 10% of those driving catch public transport the next day, the first column will be:

$$\begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$$

Since 80% of those using public transport continue to get public transport and 20% of those who catch public transport drive the next day, the second column will be:

$$\begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$$

Putting this together gives the required matrix.

$$T = \begin{array}{cc} D & P & \\ \left[\begin{array}{cc} 0.9 & 0.2 \\ 0.1 & 0.8 \end{array}\right] & \begin{array}{c} D \\ P \end{array} \end{array}$$

b The initial state matrix contains the number of those who drive in the first row and the public who initially catch public transport in the second row.

$$S_0 = \begin{bmatrix} 40 \\ 160 \end{bmatrix}$$

c Since 40 residents initially drive to work and 90% of them continue to do so, $40 \times 90\% = 36$ of these people are expected to drive tomorrow. This means that $40 - 36 = 4$ of those who drive are expected to get public transport tomorrow.

d Since 160 residents initially catch public transport to work and 80% of them continue to do so, $160 \times 80\% = 128$ of these people are expected to drive tomorrow. This means that $160 - 128 = 32$ of those who drive are expected to get public transport tomorrow.

e Use your CAS to calculate $T^5 \times S_0$ and read off the second row (82).

f To calculate what is expected in the long run, use your CAS to calculate, say $T^{20} \times S_0$ and $T^{30} \times S_0$. If these two numbers are very close, then we've reached the long run. The number listed in the second row is how many are expected to get public transport and the number listed in the top row is the number expected to drive to work.

8 **a** Using the diagram and the order B, M and R:
70% of those in state B stay in state B, 20% transition to M and 10% transition to R. This means that the first column is

$$\begin{bmatrix} 0.7 \\ 0.2 \\ 0.1 \end{bmatrix}$$

15% of those in state M transition to state B, 55% stay in state M and 30% transition to R. This means that the second column is

$$\begin{bmatrix} 0.15 \\ 0.55 \\ 0.3 \end{bmatrix}$$

20% of those in state R transition to state B, 20% transition to M and 60% remain in state R. This means that the third column is

$$\begin{bmatrix} 0.7 \\ 0.2 \\ 0.1 \end{bmatrix}$$

b The initial state matrix contains the number of those who buy a Banana Bonanza, a Mango Magic and a Radical Raspberry in the first, second and third row respectively.

$$S_0 = \begin{bmatrix} 300 \\ 200 \\ 50 \end{bmatrix}$$

c Perform matrix multiplication on your CAS.

d Use your CAS to calculate $T \times S_0$ for Tuesday, $T^2 \times S_0$ for Wednesday, $T^3 \times S_0$ for Thursday, $T^4 \times S_0$ for

Friday and similarly for Saturday and Sunday. Add together the first row for each day (including Monday) to find how many of each type of drink is sold in the first week.
B: 1609, *M*: 1332, *R*: 1009

e To calculate what is expected in the long run, use your CAS to calculate, say $T^{20} \times S_0$ and $T^{30} \times S_0$. If these two numbers are very close, then we've reached the long run. Read each number off the resulting column matrix.
B: 203, *M*: 169, *R*: 178

Solutions to Exercise 4I

1 **a** Matrix P is a column matrix as it has only one column.

b Remember that matrix multiplication follows the algorithm where we go add the products of rows and columns.

$$\begin{bmatrix} 214 \times 3 + 103 \times 2 \\ 162 \times 3 + 189 \times 2 \end{bmatrix}$$

c The top line of the resulting matrix tells us how much Joe earns from selling cans (214 × 3) plus how much he earns from selling bottles (103 × 2).

2 $$\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 1400 \\ 1000 \end{bmatrix} = [2 \times 1400 + 3 \times 1000]$$
$$= [5800]$$

Helen had 5800 kJ for lunch.

3 $$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \times \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \times 4 + 3 \times 2 & 2 \times 5 + 3 \times 1 \\ 1 \times 4 + 4 \times 2 & 1 \times 5 + 4 \times 1 \end{bmatrix}$$
$$= \begin{array}{cc} & \begin{array}{cc} W & S \end{array} \\ \begin{array}{c} Smith \\ Jones \end{array} & \begin{bmatrix} 14 & 13 \\ 12 & 9 \end{bmatrix} \end{array}$$

4

$$H \times P$$
$$= \begin{bmatrix} 2 & 13 & 5 \end{bmatrix} \begin{bmatrix} 20 \\ 5 \\ 1 \end{bmatrix}$$
$$= [40 + 65 + 5]$$
$$= [110]$$

5 **a**
$$\begin{array}{cc} & \begin{array}{ccc} Qu & So & Co \end{array} \\ Sold & \begin{bmatrix} 18 & 12 & 64 \end{bmatrix} \end{array}$$

b
$$\begin{array}{cc} & \$ \\ \begin{array}{c} Quiche \\ Soup \\ Coffee \end{array} & \begin{bmatrix} 5 \\ 8 \\ 3 \end{bmatrix} \end{array}$$

c

$$\begin{bmatrix} 18 & 12 & 64 \end{bmatrix} \begin{bmatrix} 5 \\ 8 \\ 3 \end{bmatrix}$$
$$= [18 \times 5 + 12 \times 8 + 64 \times 3]$$
$$= [378]$$

6 **a**
$$\begin{array}{cc} & \begin{array}{cccc} Ch & Pa & Pi & S.R \end{array} \\ Sold & \begin{bmatrix} 90 & 84 & 112 & 73 \end{bmatrix} \end{array}$$

b
$$\begin{array}{cc} & \$ \\ \begin{array}{c} Ch \\ Pa \\ Pi \\ S.R \end{array} & \begin{bmatrix} 4 \\ 5 \\ 5 \\ 3 \end{bmatrix} \end{array}$$

c
$$\begin{bmatrix} 90 & 84 & 112 & 73 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 5 \\ 3 \end{bmatrix}$$
$$= [1559]$$

Thus, $1559

7 **a**
$$\begin{bmatrix} 2 & 3 & 2 & 3 \\ 1 & 4 & 0 & 2 \\ 3 & 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 + 3 + 2 + 3 \\ 1 + 4 + 0 + 2 \\ 3 + 4 + 3 + 2 \end{bmatrix}$$
$$= \begin{array}{cc} I & \\ J & \\ K & \end{array} \!\!\! \begin{bmatrix} 10 \\ 7 \\ 12 \end{bmatrix}$$

b $$\frac{1}{4}\begin{bmatrix}10\\7\\12\end{bmatrix}$$

$$=\begin{array}{cc}\begin{matrix}I\\J\\K\end{matrix} & \begin{bmatrix}2.5\\1.75\\3\end{bmatrix}\end{array}$$

c $$\begin{bmatrix}1 & 1 & 1\end{bmatrix}\times H$$

$$=\begin{bmatrix}1 & 1 & 1\end{bmatrix}\times\begin{bmatrix}2 & 3 & 2 & 3\\1 & 4 & 0 & 2\\3 & 4 & 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2+1+3 & 3+4+4 & 2+0+3 & 3+2+2\end{bmatrix}$$

$$=\begin{array}{cccc} M & Tu & W & Th\\ \hline 6 & 11 & 5 & 7\end{array}$$

d $$=\frac{1}{3}\begin{array}{cccc} M & Tu & W & Th\\ \hline 6 & 11 & 5 & 7\end{array}$$

$$=\begin{array}{cccc} M & Tu & W & Th\\ \hline 2 & 3.7 & 1.7 & 2.3\end{array}$$

8 a $$R\times\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$$

$$=\begin{array}{cc}\begin{matrix}E\\F\\G\\H\end{matrix} & \begin{bmatrix}87 & 91 & 94 & 86 & 88\\93 & 76 & 89 & 62 & 95\\73 & 61 & 58 & 54 & 83\\66 & 79 & 83 & 90 & 91\end{bmatrix}\end{array}\times\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}$$

$$=\begin{array}{cc}\begin{matrix}E\\F\\G\\H\end{matrix} & \begin{bmatrix}446\\415\\329\\409\end{bmatrix}\end{array}$$

b $$\begin{array}{cc}\begin{matrix}E\\F\\G\\H\end{matrix} & \begin{bmatrix}446\\415\\329\\409\end{bmatrix}\end{array}\times\frac{1}{5}$$

$$=\begin{array}{cc}\begin{matrix}E\\F\\G\\H\end{matrix} & \begin{bmatrix}89.2\\83\\65.8\\81.8\end{bmatrix}\end{array}$$

c $$\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}\times R$$

$$=\begin{bmatrix}1 & 1 & 1 & 1\end{bmatrix}\times\begin{bmatrix}87 & 91 & 94 & 86 & 88\\93 & 76 & 89 & 62 & 95\\73 & 61 & 58 & 54 & 83\\66 & 79 & 83 & 90 & 91\end{bmatrix}$$

$$=\begin{array}{cccccc} & T1 & T2 & T3 & T4 & T5\\ Sum & 319 & 307 & 324 & 292 & 357\end{array}$$

d $$\frac{1}{4}\begin{array}{ccccc} T1 & T2 & T3 & T4 & T5\\ \hline 319 & 307 & 324 & 292 & 357\end{array}$$

$$=\begin{array}{cccccc} & T1 & T2 & T3 & T4 & T5\\ Avg. & 79.75 & 76.75 & 81 & 73 & 89.25\end{array}$$

9 If we take the 2 × 2 identity matrix

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

and multiply it by the scalar k, we get

$$\begin{bmatrix}k & 0\\0 & k\end{bmatrix}$$

Hence, given the definition, this is a scalar matrix.

10 a $P \times E$ tells us the mobile phone bill of each person in the third quarter because E has a 1 in row 3 and a 0 in all other rows. Performing the multiplication gives:

$$\begin{bmatrix}52\\64\\44\end{bmatrix}$$

b To extract the second quarter, we need a 1 in the second row and a zero in the other rows. The matrix will be a 4 × 1 matrix as there are four columns in matrix *P*.

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

c To list Charlie's quarterly costs, we only want the the last row of matrix *P*. This will be a matrix with 1 row and 4 columns so it is a 1 × 4 matrix.

d To get a 1 × 4 matrix, we need to multiply *P* by a 1 × 3 matrix by pre-multiplying. This is because we need the first matrix to have 1 row and the second matrix to have 4 columns. Further, we need the number of columns in the first matrix to have the number of rows of the second matrix.

e Since Charlie is the third row, we want the third row so the required matrix is

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Solutions to Chapter Review Multiple-Choice Questions

1 Read from matrix: order(F) = 2×3 **B**

2 Read from matrix: $F_{2\,1} = 5$ **E**

3 Read from matrix: Emir's family has 3 PCs. **C**

4 To find the number of towns directly connecting to town E add the numbers in column (or row) E. So $2 + 0 + 3 = 5$ towns are directly connected to town E. **D**

5 For two matrices to be equal elements in the same position must be equal. $3x = 6$ so $x = 2$. **A**

6

$$\begin{aligned} M + N &= \begin{bmatrix} 7 & 6 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 7+5 & 6-2 \\ 4+1 & 3+0 \end{bmatrix} \\ &= \begin{bmatrix} 12 & 4 \\ 5 & 3 \end{bmatrix} \end{aligned}$$

D

7

$$\begin{aligned} M - N &= \begin{bmatrix} 7 & 6 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 7-5 & 6+2 \\ 4-1 & 3-0 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 8 \\ 3 & 3 \end{bmatrix} \end{aligned}$$

D

8

$$\begin{aligned} N - N &= \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 5-5 & -2+2 \\ 1-1 & 0-0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

C

9

$$\begin{aligned} 2N &= 2\begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 2\times 5 & 2\times -2 \\ 2\times 1 & 2\times 0 \end{bmatrix} \\ &= \begin{bmatrix} 10 & -4 \\ 2 & 0 \end{bmatrix} \end{aligned}$$

A

10

$$\begin{aligned} 2M + N &= 2\begin{bmatrix} 7 & 6 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 14 & 12 \\ 8 & 6 \end{bmatrix} + \begin{bmatrix} 5 & -2 \\ 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 14+5 & 12-2 \\ 8+1 & 6+0 \end{bmatrix} \\ &= \begin{bmatrix} 19 & 10 \\ 9 & 6 \end{bmatrix} \end{aligned}$$

E

11 $n = p$ when order(A) = $m \times n$ and order(B) = $p \times q$. Consider the options:

Order(P) = 2×3
Order(Q) = 3×1
Order(R) = 1×2
Order(S) = 2×2

Matrix multiplication is not defined for PS. **D**

12 Definition based: $m \times q$.
Order(QR) = 3×2 **B**

13 Definition based: $m \times q$
Order(RP) = 1×3 **E**

14

$$PQ = \begin{bmatrix} 5 & 4 & 1 \\ 7 & 6 & 8 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \times 5 + 4 \times 6 + 1 \times 0 \\ 7 \times 2 + 5 \times 6 + 8 \times 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 + 24 + 0 \\ 14 + 36 + 0 \end{bmatrix}$$

$$= \begin{bmatrix} 34 \\ 50 \end{bmatrix}$$

A

15 Definition based: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **D**

16 Define matrix A as ‘*a*’ on calculator:

$$\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix} \rightarrow a$$

To find the inverse enter a^{-1}:

$$a^{-1} = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} \frac{3}{2} & -\frac{5}{2} \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$$

E

Solutions to Chapter Review Short-Answer Questions

Questions 1 – 4 *refer to matrix* A:

$$A = \begin{bmatrix} 4 & 2 & 1 & 0 \\ 3 & 4 & 7 & 9 \end{bmatrix}$$

1 The order of matrix A is 2×4, i.e. there are 2 rows and 4 columns.

2 The element a_{13} is 1, i.e. the value in the first row and third column is 1.

3 If $C = \begin{bmatrix} 5 & 6 \end{bmatrix}$,

$$\begin{aligned} CA &= \begin{bmatrix} 5 & 6 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 & 0 \\ 3 & 4 & 7 & 9 \end{bmatrix} \\ &= \begin{bmatrix} 20 + 18 & 10 + 24 & 5 + 42 & 0 + 54 \end{bmatrix} \\ &= \begin{bmatrix} 38 & 34 & 47 & 54 \end{bmatrix} \end{aligned}$$

4 If the order of B was 4×1, thenAB would be a 2×4 matrix multiplied by a 4×1 matrix, resulting in a matrix of order 2×1.

5 Label the columns and rows of a 3×3 matrix as P, Q and R.
Where the column for a town meets the row of a town enter the number of roads directly connecting the two towns.
If no road connects the two towns enter 0.
For example, write 2 where column P intersects row Q as two roads directly connect town$P$ to town Q.
Enter 0 where column P intersects row R as there is no road directly connecting town$P$ to$R$.
And so on, to give the matrix:

$$\begin{array}{cl} \begin{matrix} P & Q & R \end{matrix} & \\ \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 3 \\ 0 & 3 & 0 \end{bmatrix} & \begin{matrix} P \\ Q \\ R \end{matrix} \end{array}$$

6 **a**

$$\begin{aligned} 3A &= 3 \times \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 3 \\ 12 & 6 \end{bmatrix} \end{aligned}$$

b

$$\begin{aligned} A + B &= \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 7 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 6 \\ 11 & 8 \end{bmatrix} \end{aligned}$$

c

$$\begin{aligned} B - A &= \begin{bmatrix} 0 & 5 \\ 7 & 6 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 4 \\ 3 & 4 \end{bmatrix} \end{aligned}$$

d

$$\begin{aligned} 2A + B &= 2 \times \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 7 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 2 \\ 8 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 7 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 7 \\ 15 & 10 \end{bmatrix} \end{aligned}$$

e

$$\begin{aligned} A - A &= \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

f

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 5 \\ 7 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 3 \times 0 + 1 \times 7 & 3 \times 5 + 1 \times 6 \\ 4 \times 0 + 2 \times 7 & 4 \times 5 + 2 \times 6 \end{bmatrix} \\ &= \begin{bmatrix} 7 & 21 \\ 14 & 32 \end{bmatrix} \end{aligned}$$

g

$$BA = \begin{bmatrix}0 & 5\\7 & 6\end{bmatrix} \times \begin{bmatrix}3 & 1\\4 & 2\end{bmatrix} = \begin{bmatrix}0\times 3+5\times 4 & 0\times 1+5\times 2\\7\times 3+6\times 4 & 7\times 1+6\times 2\end{bmatrix} = \begin{bmatrix}20 & 10\\45 & 19\end{bmatrix}$$

h Follow the steps given in **How to find the inverse of a matrix using a CAS calculator** (p. 250-251) to obtain:

$$A^{-1} = \begin{bmatrix}3 & 1\\4 & 2\end{bmatrix}^{-1} = \begin{bmatrix}1 & -0.5\\-2 & 1.5\end{bmatrix}$$

i

$$A^2 = \begin{bmatrix}3 & 1\\4 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\4 & 2\end{bmatrix} = \begin{bmatrix}3\times 3+1\times 4 & 3\times 1+1\times 2\\4\times 3+2\times 4 & 4\times 1+2\times 2\end{bmatrix} = \begin{bmatrix}13 & 5\\20 & 8\end{bmatrix}$$

j

$$AI = \begin{bmatrix}3 & 1\\4 & 2\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}3\times 1+1\times 0 & 3\times 0+1\times 1\\4\times 1+2\times 0 & 4\times 0+2\times 1\end{bmatrix} = \begin{bmatrix}3 & 1\\4 & 2\end{bmatrix} = A$$

k

$$AA^{-1}$$

$$= \begin{bmatrix}3 & 1\\4 & 2\end{bmatrix}\begin{bmatrix}1 & -0.5\\-2 & 1.5\end{bmatrix} \quad \text{from } \mathbf{h}$$

$$= \begin{bmatrix}3\times 1+1\times(-2) & 3\times(-0.5)+1\times 1.5\\4\times 1+2\times(-2) & 4\times(-0.5)+2\times 1.5\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$$

Solutions to Extended-Response Questions

1 **a**

$$\begin{array}{l|ccc} & \text{Cattle} & \text{Pigs} & \text{Sheep} \\ \hline \text{Farm } A & 420 & 50 & 100 \\ \text{Farm } B & 300 & \mathbf{40} & 220 \end{array}$$

There are 40 pigs on Farm B.

b

$$\begin{array}{l|ccc} & \text{Cattle} & \text{Pigs} & \text{Sheep} \\ \hline \text{Farm } A & 420 & 50 & \mathbf{100} \\ \text{Farm } B & 300 & 40 & \mathbf{220} \end{array}$$

$100 + 220 = 320$

There is a total of 320 sheep on both farms.

c

$$\begin{array}{l|ccc} & \text{Cattle} & \text{Pigs} & \text{Sheep} \\ \hline \text{Farm } A & 420 & 50 & 100 \\ \text{Farm } B & 300 & 40 & 220 \end{array}$$

For Farm A: $420 + 50 + 100 = 570$

For Farm B: $300 + 40 + 220 = 560$

Farm A has the largest total number of livestock.

2 **a**

$$\begin{array}{l|ccc} & \text{Cakes} & \text{Pies} & \text{Rolls} \\ \hline A & 12 & 25 & 18 \\ B & 15 & \mathbf{21} & 16 \end{array}$$

Shop$B$ sold 21 pies.

b

$$\begin{array}{l|c} & \$ \\ \hline \text{Cakes} & 3 \\ \text{Pies} & \mathbf{2} \\ \text{Rolls} & 1 \end{array}$$

The selling price of pies is \$2.

c

$$\begin{aligned} SP &= \begin{bmatrix} 12 & 25 & 18 \\ 15 & 21 & 16 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 12 \times 3 + 25 \times 2 + 18 \times 1 \\ 15 \times 3 + 21 \times 2 + 16 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 104 \\ 103 \end{bmatrix} \end{aligned}$$

d Matrix SP gives the value of sales for each shop.

e

$$\begin{array}{cc} & \$ \\ Shop\,A & \begin{bmatrix}\mathbf{104}\\103\end{bmatrix} \\ Shop\,B & \end{array}$$

Shop *A* had the largest income from its sales. The takings for Shop *A* were $104.

3 **a** $A = \begin{bmatrix}4 & 1\\3 & 2\end{bmatrix}$

b $B = \begin{bmatrix}2 & 1500\\3 & 2500\end{bmatrix}$

c

$$\begin{aligned} AB &= \begin{bmatrix}4 & 1\\3 & 2\end{bmatrix}\begin{bmatrix}2 & 1500\\3 & 2500\end{bmatrix} \\ &= \begin{bmatrix}4\times 2+1\times 3 & 4\times 1500+1\times 2500\\3\times 2+2\times 3 & 3\times 1500+2\times 2500\end{bmatrix} \\ &= \begin{bmatrix}11 & 8500\\12 & 9500\end{bmatrix}\end{aligned}$$

$$AB = \begin{array}{c} \\ Patsy \\ Geoff \end{array}\begin{array}{c}\begin{array}{cc}\$ & kJ\end{array}\\ \begin{bmatrix}11 & 8500\\12 & 9500\end{bmatrix}\end{array}$$

4 **a** Since 60% of those who have an accident are predicted to have an accident next year, then 1 – 60% = 40% are predicted to not have an accident.

b From part **a**, the first column is given as

$$\begin{bmatrix}0.6\\0.4\end{bmatrix}$$

Since 5% of those who did not have an accident are predicted to have an accident next year, then 1 – 5% = 95% are predicted to not have an accident. Using this we can form the second column as

$$\begin{bmatrix}0.6\\0.4\end{bmatrix}$$

Labelling the rows and columns and putting the two columns together gives the transition matrix:

$$T = \begin{array}{c|cc} & A & N \\ \hline A & 0.6 & 0.05 \\ N & 0.4 & 0.95 \end{array}$$

c Since 4000 people had an accident, this number goes in the first row of the column matrix. Since 26 000 people did not have an accident, we put 26 000 in the second row of the column matrix to get:

$$S_0 = \begin{bmatrix}4000\\26000\end{bmatrix}$$

d Performing the matrix multiplication $T \times S_0$ on your CAS and then reading off the the first row will give the number of people expected to have an accident to be 3700. Note that the number on the second row is the predicted number of people who do not have an accident.

e To find what happens in five years time, we compute $T^5 \times S_0$ using a CAS. Reading the top line gives the number of people expected to have an accident in five years time to be 3367.

f To calculate what is expected in the long run, use your CAS to calculate, say $T^{20} \times S_0$ and $T^{30} \times S_0$. If these two numbers are very close, then we've reached the long run. The number listed in the first row tells us how many people we expect will have an accident (3333) while the second row tells us how many people we expect will not have an accident (26 667).

5 a To find the total number of items sold we need to consider how many items are in each packet *A* and then the number of packets sold (either *C* or *D*). Since *A* is a 1×3 matrix, we can either compute *AC* or *DA* to find the answer. This can be done on the CAS or by hand as $12 \times 100 + 8 \times 50 + 4 \times 30 = 1720$.

b To find the total value of sales we need to consider the price per item *B* and then the number of packets sold (either *C* or *D*). Since *B* is a 3×1 matrix, we can either compute *BD* or *DB* on the CAS or by hand as $100 \times 8 + 50 \times 6 + 30 \times 5 = \$1250$.

6 a $6x + 5y = 14$

b Apple \$1.50, banana \$1

Chapter 5 – Linear relations and equations

Solutions to 5A Now Try This Questions

1 $C = 330 + 80t$

$= 330 + 80(10)$

$= 330 + 800$

$= 1130$

The cost is \$1130.

2 $P = 2L + 2W$

$= 2(26.5) + 2(14.8)$

$= 53 + 29.6$

$= 82.6$

The perimeter is 82.6 cm.

3 $P = 4L$

If $L = 0$, $P = 4(0) = 0$

If $L = 10$, $P = 4(10) = 40$

If $L = 20$, $P = 4(20) = 80$

If $L = 30$, $P = 4(30) = 120$

If $L = 40$, $P = 4(40) = 160$

If $L = 50$, $P = 4(50) = 200$

If $L = 60$, $P = 4(60) = 240$

If $L = 70$, $P = 4(70) = 280$

If $L = 80$, $P = 4(80) = 320$

If $L = 90$, $P = 4(90) = 360$

If $L = 100$, $P = 4(100) = 400$

x	0	10	20	30	40	50	60	70	80	90	100
y	0	40	80	120	160	200	240	280	320	360	400

Solutions to Exercise 5A

1 $C = 300 + 120t$

a $t = 1$ hour

$C = 300 + 120(1)$

$= 300 + 120$

$= \$420$

b $t = 2$ hours

$C = 300 + 120(2)$

$= 300 + 240$

$= \$540$

2 $P = 2L + 2W$

a $L = 4,\ W = 2$

$P = 2(4) + 2(2)$

$= 8 + 4$

$= 12$ cm

b $L = 5.8,\ W = 3.5$

$P = 2(5.8) + 2(3.5)$

$= 11.6 + 7$

$= 18.6$ cm

c $L = 7.9,\ W = 2.7$

3 **a** $43 \times \$3 = \$129$

b $44 \times \$3 = \$132$

c

x	40	41	42	43	44	45
C ($)	120	123	126	129	132	135

4 $C = 1200 + 50t$

a $t = 4$ hours

$C = 1200 + 50(4)$

$= 1200 + 200$

$= \$1400$

b $t = 6$ hours

$C = 1200 + 50(6)$

$= 1200 + 300$

$= \$1500$

c $t = 4.5$ hours

$C = 1200 + 50(4.5)$

$= 1200 + 225$

$= \$1425$

5 $d = v \times t$

$v = 95$ km/h; $t = 4$ hours

$d = (95) \times (4)$

$= 380$ km

6 $F = 1.3K + 4$

a $K = 5$ km

$F = 1.3(5) + 4$

$= 6.50 + 4$

$= \$10.50$

b $K = 8$ km

$F = 1.3(8) + 4$

$= 10.40 + 4$

$= \$14.40$

c $K = 20$ km

$F = 1.3(20) + 4$

$= 26 + 4$

$= \$30$

7 $C = 2\pi r$

a $r = 3$ mm

$C = 2\pi(3)$

$= 6\pi$

$= 18.85$ mm

b $r = 7.2$ m

$C = 2\pi(7.2)$

$= 14.4\pi$

$= 45.24$ m

8 $A = L \times W$

a $L = 3;\ W = 4$

$A = 3 \times 4$

$= 12 \text{ units}^2$

b $L = 15;\ W = 8$

$A = 15 \times 8$

$= 120 \text{ units}^2$

c $L = 2.5;\ W = 9$

$A = 2.5 \times 9$

$= 22.5\text{units}^2$

9 $A = \frac{1}{2}h(a + b)$

a $h = 1;\ a = 3;\ b = 5$

$$\begin{aligned} A &= \frac{1}{2}(1)(3 + 5) \\ &= \frac{1}{2}(8) \\ &= 4 \text{ units}^2 \end{aligned}$$

b $h = 5;\ a = 2.5;\ b = 3.2$

$$\begin{aligned} A &= \frac{1}{2}(5)(2.5 + 3.2) \\ &= 14.25 \text{ units}^2 \end{aligned}$$

10 $C = \frac{5}{9}(F - 32)$

a $F = 50°\text{F}$

$$\begin{aligned} C &= \frac{5}{9}(50 - 32) \\ &= \frac{5}{9}(18) \\ &= 10°\text{C} \end{aligned}$$

b $F = 0°\text{F}$

$$\begin{aligned} C &= \frac{5}{9}(0 - 32) \\ &= \frac{5}{9}(-32) \\ &= -17.8°\text{C} \end{aligned}$$

c $F = 212°\text{F}$

$$\begin{aligned} C &= \frac{5}{9}(212 - 32) \\ &= \frac{5}{9}(180) \\ &= 100°\text{C} \end{aligned}$$

d $F = 92°\text{F}$

$$\begin{aligned} C &= \frac{5}{9}(92 - 32) \\ &= \frac{5}{9}(60) \\ &= 33.3°\text{C} \end{aligned}$$

11 $I = \frac{PRT}{100}$

a $P = \$5000;\ R = 12\%;\ T = 4$

$$\begin{aligned} I &= \frac{(5000)(12)(4)}{100} \\ &= \frac{240\,000}{100} \\ &= \$2400 \end{aligned}$$

b $P = \$8500;\ R = 7.9\%;\ T = 3$

$$\begin{aligned} I &= \frac{(8500)(7.9)(3)}{100} \\ &= \frac{201\,450}{100} \\ &= \$2014.50 \end{aligned}$$

12 $P = 6G + B$

a **i** $G = 2;\ B = 3$

$$\begin{aligned} P &= 6(2) + 3 \\ &= 12 + 3 \\ &= 15 \text{ points} \end{aligned}$$

ii $G = 8;\ B = 20$

$$\begin{aligned} P &= 6(8) + 20 \\ &= 48 + 20 \\ &= 68 \text{ points} \end{aligned}$$

b Redteam points = 6(4) + 2

= 26 points

Greenteam points = 6(3) + 10

= 28 points

Greenteam wins the match.

13 $C = 2\pi r$

$r = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$

$C = 2\pi(0.4)$ (0.5, 0.6, . . . , 1.0)

$= 2.513$ cm (3.142, 3.77, . . . , 6.283)

r (cm)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
C (cm)	0	0.628	1.257	1.885	2.513	3.142	3.770	4.398	5.027	5.655	6.283

14 $C = 40 + 0.18n$

$n = 80, 90, 100, \dots, 130$

$C = 40 + 0.18(80)$ (90, 100, . . . , 130)

$= 40 + 14.4$

$= 54.4$ cm (56.2, 58, . . . , 63.4)

n	50	60	70	80	90	100	110	120	130
C ($)	49	50.8	52.6	54.4	56.2	58	59.8	61.6	63.4

15 $E = 110 + 9M$

$M = 70, 75, 80, \dots, 120$

$E = 110 + 9(70)$ (75, 80, . . . , 120)

$= 110 + 630$

$= 740$ kJ (785, 830, . . . , 1190)

M (kg)	60	65	70	75	80	85	90	95	100	105	110	115	120
E (kJ)	650	695	740	785	830	875	920	965	1010	1055	1100	1145	1190

16 $I = \dfrac{PRT}{100}$

$P = \$10000;\ R = 4.5\%;$

$T = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

$I = \dfrac{10000 \times 4.5 \times (1)}{100}$ (2, 3, . . . , 10)

$= \dfrac{45000}{100}$

$= \$450$ (900, 1350, . . . , 4500)

T (years)	1	2	3	4	5	6	7	8	9	10
I ($)	450	900	1350	1800	2250	2700	3150	3600	4050	4500

17 $S = 90(2n - 4)$

$n = 5, 6, 7, 8, 9, 10$

$S = 90(2(5) - 4)$ (6, 7, 8, 9, 10)

$= 90(10 - 4)$

$= 90(6)$

$= 540\,^\circ$ (720, 900, 1080, 1260, 1440)

n	3	4	5	6	7	8	9	10
S (°)	180	360	540	720	900	1080	1260	1440

18 $m = a + (n - 1)d$

$a = 3;\ d = 2$

a $n = 6$

$m = 3 + (6 - 1)(2)$

$= 3 + 5 \times 2$

$= 3 + 10$

$= 13$

b $n = 11$

$m = 3 + (11 - 1)(2)$

$= 3 + 10 \times 2$

$= 3 + 20$

$= 23$

c $n = 50$

$m = 3 + (50 - 1)(2)$

$= 3 + 49 \times 2$

$= 101$

19 **a** **i** a 2 kg chicken

Time $(t) = 45(2) + 20$

$= 90 + 20$

$= 110$ min

$= 1$ hr 50 min

ii 2.25 kg beef (well done)

$t = 65(2.25) + 30$

$= 176.25$ min

$= 2$ hr 56 min

iii 2.4 kg lamb (well done)
$t = 65(2.4) + 30$

$= 186$ min

$= 3$ hr 6 min

iv 2.5 kg beef (medium)

$t = 55(2.5) + 20$

$= 157.5$ min

$= 2$ hr 38 min

b 2 kg lamb (med)
$t = 55(2) + 25$

$= 110 + 25$

$= 135$ min

$= 2$ hr 15 min.
So to be ready at 7.30pm, the lamb will need to be put in oven 2 hours 15 min before 7.30, at 5.15 pm.

Solutions to 5B Now Try This Questions

4
$$2x - 6 = 10$$
$$2x - 6 + 6 = 10 + 6$$
$$2x = 16$$
$$2x \div 2 = 16 \div 2$$
$$x = 8$$

5 Use the **solve(** function on a CAS calculator.

solve($-5 - 2x = 15, x$) $x = -10$

$x = -10$

6 Let P be the perimeter.

$$P = 10 + 10 + x + x$$
$$= 20 + 2x$$

7 Let n be the number.

$$n + 7 = 49$$
$$n + 7 - 7 = 49 - 7$$
$$n = 42$$

8 Let d be the number of days that the van is hired for.

$$56 + 80d = 500$$
$$56 + 80d - 56 = 500 - 56$$
$$80d = 444$$
$$80d \div 80 = 444 \div 80$$
$$d = 5.55$$

Car hire works on a daily rate so 5.55 days is not a option. The van can be hired for 5 days.

Solutions to Exercise 5B

1 a
$$x + 6 = 15$$
$$x + 6 - 6 = 15 - 6$$
$$x = 9$$

b
$$y + 11 = 26$$
$$y + 11 - 11 = 26 - 11$$
$$y = 15$$

c
$$m - 5 = 1$$
$$m - 5 + 5 = 1 + 5$$
$$m = 6$$

d
$$m - 5 = -7$$
$$m - 5 + 5 = -7 + 5$$
$$m = -2$$

e
$$6 + e = 9$$
$$6 - 6 + e = 9 - 6$$
$$e = 3$$

f
$$-n + 5 = 1$$
$$-n + 5 - 5 = 1 - 5$$
$$-n = -4$$
$$n = 4$$

2 a
$$5x = 15$$
$$5x \div 5 = 15 \div 5$$
$$x = 3$$

b
$$3g = 27$$
$$3g \div 3 = 27 \div 3$$
$$g = 9$$

c $6j = -24$

$6j \div 6 = -24 \div 6$

$j = -4$

d $\frac{r}{3} = 4$

$\frac{r}{3} \times 3 = 4 \times 3$

$r = 12$

e $\frac{t}{-2} = 6$

$\frac{t}{-2} \times -2 = 6 \times -2$

$t = -12$

f $\frac{h}{-8} = -5$

$\frac{h}{-8} \times -8 = -5 \times -8$

$h = 40$

3 **a** $2a + 15 = 27$

$2a + 15 - 15 = 27 - 15$

Subtract 15 from both sides of equation.

b $2a = 12$

$2a \div 2 = 12 \div 2$

Divide both sides by 2.

c $a = 6$

4 **a** $\frac{y}{4} - 10 = 0$

$\frac{y}{4} - 10 + 10 = 0 + 10$

Add 10 to both sides of equation.

b $\frac{y}{4} = 10$

$\frac{y}{4} \times 4 = 10 \times 4$

Multiply both sides by 4.

c $y = 40$

5 **a** $v + 7 = 2$

$v + 7 - 7 = 2 - 7$

$v = -5$

b $9 - k = 2$

$9 - 9 - k = 2 - 9$

$-k = -7$

$k = 7$

c $3 - a = -5$

$3 - 3 - a = -5 - 3$

$-a = -8$

$a = 8$

d $-5b = -25$

$-5b \div -5 = -25 \div -5$

$b = 5$

e $13 = 3r - 11$

$13 + 11 = 3r - 11 + 11$

$24 = 3r$

$r = 8$

f $\frac{x + 1}{3} = 2$

$\frac{x + 1}{3} \times 3 = 2 \times 3$

$x + 1 = 6$

$x + 1 - 1 = 6 - 1$

$x = 5$

6 **a**

$$\begin{aligned}
2(y-1) &= 6\\
\frac{2(y-1)}{2} &= \frac{6}{2}\\
y-1 &= 3\\
y-1+1 &= 3+1\\
y &= 4
\end{aligned}$$

b

$$\begin{aligned}
8(x-4) &= 56\\
\frac{8(x-4)}{8} &= \frac{56}{8}\\
x-4 &= 7\\
x-4+4 &= 7+4\\
x &= 11
\end{aligned}$$

c

$$\begin{aligned}
3(g+2) &= 12\\
\frac{3(g+2)}{3} &= \frac{12}{3}\\
g+2 &= 4\\
g+2-2 &= 4-2\\
g &= 2
\end{aligned}$$

d

$$\begin{aligned}
3(4x-5) &= 21\\
\frac{3(4x-5)}{3} &= \frac{21}{3}\\
4x-5 &= 7\\
4x-5+5 &= 7+5\\
4x &= 12\\
\frac{4x}{4} &= \frac{12}{4}\\
x &= 3
\end{aligned}$$

e

$$\begin{aligned}
8(2x+1) &= 16\\
\frac{8(2x+1)}{8} &= \frac{16}{8}\\
2x+1 &= 2\\
2x+1-1 &= 2-1\\
2x &= 1\\
\frac{2x}{2} &= \frac{1}{2}\\
x &= 0.5
\end{aligned}$$

f

$$\begin{aligned}
3(5m-2) &= 12\\
\frac{3(2m-2)}{3} &= \frac{12}{3}\\
5m-2 &= 4\\
5m-2+2 &= 4+2\\
5m &= 6\\
\frac{5m}{5} &= \frac{6}{5}\\
m &= 1.2
\end{aligned}$$

g

$$\begin{aligned}
\frac{2(a-3)}{5} &= 6\\
\frac{2(a-3)}{5}\times 5 &= 6\times 5\\
2(a-3) &= 30\\
\frac{2(a-3)}{2} &= \frac{30}{2}\\
a-3 &= 15\\
a-3+3 &= 15+3\\
a &= 18
\end{aligned}$$

h
$$\begin{aligned}\frac{4(r+2)}{6} &= 10\\ \frac{4(r+2)}{6}\times 6 &= 10\times 6\\ 4(r+2) &= 60\\ \frac{4(r+2)}{4} &= \frac{60}{4}\\ r+2 &= 15\\ r+2-2 &= 15-2\\ r &= 13\end{aligned}$$

7 a
$$\begin{aligned}2x &= x+5\\ 2x-x &= x-x+5\\ x &= 5\end{aligned}$$

b
$$\begin{aligned}2a+1 &= a+4\\ 2a-a+1 &= a-a+4\\ a+1 &= 4\\ a+1-1 &= 4-1\\ a &= 3\end{aligned}$$

c
$$\begin{aligned}4b-10 &= 2b+8\\ 4b-2b-10 &= 2b-2b+8\\ 2b-10 &= 8\\ 2b-10+10 &= 8+10\\ 2b &= 18\\ b &= 9\end{aligned}$$

d
$$\begin{aligned}7-5y &= 3y-17\\ 7-5y+5y &= 3y+5y-17\\ 7 &= 8y-17\\ 7+17 &= 8y-17+17\\ 24 &= 8y\\ \frac{24}{8} &= \frac{8y}{8}\\ y &= 3\end{aligned}$$

e
$$\begin{aligned}3(x+5)-4 &= x+11\\ 3x+15-4 &= x+11\\ 3x+11 &= x+11\\ 3x+11-11 &= x+11-11\\ 3x &= x\\ 3x-x &= x-x\\ 2x &= 0\\ x &= 0\end{aligned}$$

f
$$\begin{aligned}6(c+2) &= 2(c-2)\\ 6c+12 &= 2c-4\\ 6c-2c+12 &= 2c-2c-4\\ 4c+12 &= -4\\ 4c+12-12 &= -4-12\\ 4c &= -16\\ \frac{4c}{4} &= \frac{-16}{4}\\ c &= -4\end{aligned}$$

g
$$\begin{aligned}2f+3 &= 2-3(f+3)\\ 2f+3 &= 2-3f-9\\ 2f+3 &= -7-3f\\ 2f+3f+3 &= -7-3f+3f\\ 5f+3 &= -7\\ 5f+3-3 &= -7-3\\ 5f &= -10\\ \frac{5f}{5} &= \frac{-10}{5}\\ f &= -2\end{aligned}$$

h
$$\begin{aligned} 5(1-3y)-2(10-y) &= -10y \\ 5-15y-20+2y &= -10y \\ -15-13y &= -10y \\ -15-13y+13y &= -10y+13y \\ -15 &= 3y \\ \frac{-15}{3} &= \frac{3y}{3} \\ -5 &= y \\ y &= -5 \end{aligned}$$

8 a
$$\begin{aligned} 3a+5 &= 11 \\ 3a+5-5 &= 11-5 \\ 3a &= 6 \\ 3a \div 3 &= 6 \div 3 \\ a &= 2 \end{aligned}$$

b
$$\begin{aligned} 4b+3 &= 27 \\ 4b+3-3 &= 27-3 \\ 4b &= 24 \\ 4b \div 4 &= 24 \div 4 \\ b &= 6 \end{aligned}$$

c
$$\begin{aligned} 2w+5 &= 9 \\ 2w+5-5 &= 9-5 \\ 2w &= 4 \\ 2w \div 2 &= 4 \div 2 \\ w &= 2 \end{aligned}$$

d
$$\begin{aligned} 7c-2 &= 12 \\ 7c-2+2 &= 12+2 \\ 7c &= 14 \\ 7c \div 7 &= 14 \div 7 \\ c &= 2 \end{aligned}$$

e
$$\begin{aligned} 3y-5 &= 16 \\ 3y-5+5 &= 16+5 \\ 3y &= 21 \\ 3y \div 3 &= 21 \div 3 \\ y &= 7 \end{aligned}$$

f
$$\begin{aligned} 4f-1 &= 7 \\ 4f-1+1 &= 7+1 \\ 4f &= 8 \\ 4f \div 4 &= 8 \div 4 \\ f &= 2 \end{aligned}$$

g
$$\begin{aligned} 3+2h &= 13 \\ 3-3+2h &= 13-3 \\ 2h &= 10 \\ 2h \div 2 &= 10 \div 2 \\ h &= 5 \end{aligned}$$

h
$$\begin{aligned} 2+3k &= 6 \\ 2-2+3k &= 6-2 \\ 3k &= 4 \\ 3k \div 3 &= 4 \div 3 \\ k &= \frac{4}{3} = 1.3\dot{3} \end{aligned}$$

i
$$\begin{aligned} -4(g-4) &= -18 \\ -4(g-4) \div -4 &= -18 \div -4 \\ g-4 &= \frac{9}{2} \\ g-4+4 &= \frac{9}{2}+4 \\ g &= \frac{17}{2} = 8.5 \end{aligned}$$

j $\dfrac{2(s-6)}{7} = 4$

$\dfrac{2(s-6)}{7} \times 7 = 4 \times 7$

$2(s-6) = 28$

$2(s-6) \div 2 = 28 \div 2$

$s - 6 = 14$

$s - 6 + 6 = 14 + 6$

$s = 20$

k $\dfrac{5(t+1)}{2} = 8$

$\dfrac{5(t+1)}{2} \times 2 = 8 \times 2$

$5(t+1) = 16$

$5(t+1) \div 5 = 16 \div 5$

$t + 1 = \dfrac{16}{5}$

$t + 1 - 1 = \dfrac{16}{5} - 1$

$t = \dfrac{11}{5} = 2.2$

l $\dfrac{-4(y-5)}{5} = 2.4$

$\dfrac{-4(y-5)}{5} \times 5 = 2.4 \times 5$

$-4(y-5) = 12$

$-4(y-5) \div -4 = 12 \div -4$

$y - 5 = -3$

$y - 5 + 5 = -3 + 5$

$y = 2$

m $2(x-3) + 4(x+7) = 10$

$2x - 6 + 4x + 28 = 10$

$6x + 22 = 10$

$6x + 22 - 22 = 10 - 22$

$6x = -12$

$6x \div 6 = -12 \div 6$

$x = -2$

n $5(g+4) - 6(g-7) = 25$

$5g + 20 - 6g + 42 = 25$

$-g + 62 = 25$

$-g + 62 - 62 = 25 - 62$

$-g = -37$

$-g \div -1 = -37 \div -1$

$g = 37$

o $5(p+4) = 25 + (7-p)$

$5p + 20 = 25 + 7 - p$

$5p + 20 = 32 - p$

$5p + 20 - 20 = 32 - 20 - p$

$5p = 12 - p$

$5p + p = 12 + p$

$6p = 12$

$6p \div 6 = 12 \div 6$

$p = 2$

9 **a** $P = x + 15 + 12$

$= 27 + x$

b $P = x + x + x + x$

$= 4x$

c $P = a + b + a + b$

$= 2a + 2b$

d $P = 8 + y + 6 + 5$

$= 19 + y$

10 **a** $P = m + 12 + 10$

$= 22 + m$

b $P = 30$ cm

$= m + 22$

$(30) = m + 22$

$30 - 22 = m + 22 - 22$

$m = 8$ cm

11 **a** $P = y + y + y + y$

$= 4y$

b $P = 52$ cm

$= 4y$

$(52) = 4y$

$52 \div 4 = 4y \div 4$

$y = 13$ cm

12 **a** Let n = the unknown number.

$n + 7 = 15$

b $n + 7 = 15$

$n + 7 - 7 = 15 - 7$

$n = 8$

13 Let n = the unknown number.

$2n + 5 = 17$

$2n + 5 - 5 = 17 - 5$

$2n = 12$

$2n \div 2 = 12 \div 2$

$n = 6$

14 Let n = the unknown number.

$2n - 15 = 103$

$2n - 15 + 15 = 103 + 15$

$2n = 118$

$2n \div 2 = 118 \div 2$

$n = 59$

15 **a** Let t = time

b $C = 20 + 15t$

c **i** For $t = 2$:

$C = 20 + 15(2)$

$= 20 + 30$

$= \$50$

ii For $t = 5$:

$C = 20 + 15(5)$

$= 20 + 75$

$= \$95$

16 **a** $P = x + (x + 6) + x + (x + 6)$

$= 4x + 12$

b $P = 84$ cm

$= 4x + 12$

$(84) = 4x + 12$

$84 - 12 = 4x + 12 - 12$

$72 = 4x$

$72 \div 4 = 4x \div 4$

$x = 18$ cm

c Length $(L) = x + 6$

$= 18 + 6$

$= 24$ cm

Width $(W) = x$

$= 18$ cm

17
$$\begin{aligned} P &= \$350 \\ &= 15n - 820 \\ (350) &= 15n - 820 \\ 350 + 820 &= 15n - 820 + 820 \\ 1170 &= 15n \\ 1170 \div 15 &= 15n \div 8 \\ n &= 78 \text{ tickets} \end{aligned}$$

18 $C = 60 + 2.5n$

where $n =$ number of cards
$$\begin{aligned} 122.50 &= 60 + 2.5n \\ 122.50 - 60 &= 60 - 60 + 2.5n \\ 62.50 &= 2.5n \\ 62.50 \div 2.5 &= 2.5n \div 2.5 \\ n &= 25 \text{ invitations} \end{aligned}$$

19 Let A = Anne and B = Barry.
$$\begin{aligned} A &= 3B \\ A + B &= 1000 \\ (3B) + B &= 1000 \\ 4B &= 1000 \\ 4B \div 4 &= 1000 \div 4 \\ B &= \$250 \\ A &= 3B \\ &= 3(250) \\ &= \$750 \end{aligned}$$

Anne gets $750 and Barry gets $250.

20 dist. walked + dist. cycled = 45 km.

If x is distance walked, then $\frac{x}{2}$ is the distance cycled.
$$\begin{aligned} x + \frac{x}{2} &= 45 \\ \frac{3x}{2} &= 45 \\ \frac{3x}{2} \times 2 &= 45 \times 2 \\ 3x &= 90 \\ 3x \div 3 &= 90 \div 3 \\ x &= 30 \text{ km.} \end{aligned}$$

21 Let x = number of hours until Ben and Amy meet.

a Ben travels $12x$ km.

Amy travels $10x$ km.
$$\begin{aligned} 12x + 10x &= 17.2 \\ 22x &= 17.2 \\ 22x \div 22 &= 17.2 \div 22 \\ x &= 0.78 \text{ hours} \\ &= 47 \text{ mins} \end{aligned}$$

b Ben travels $12 \times 0.78 = 9.4$ km.

Amy travels $10 \times 0.78 = 7.8$ km.

Solutions to 5C Now Try This Questions

9 **a** x lemon tarts cost
$x \times 3.50 = 3.5x$
y apple crumble tarts cost
$y \times 4.75 = 4.75y$ $C = 3.5x + 4.75y$

b For 10 lemon tarts and 15 apple crumble tarts

$$\begin{aligned} C &= 3.5x + 4.75y \\ &= 3.5(10) + 4.75(15) \\ &= 35 + 71.25 \\ &= 106.25 \end{aligned}$$

Cost is \$106.25.

Solutions to Exercise 5C

1 **a** \$4.50x

b \$14.30y

c $C = 4.5x + 14.3y$

2 **a** Read from formula,
$C = 4x + 2.5y = \$4$

b Read from formula,
$C = x + 2.5y = \$2.50$

c $8 \times \$4 + 4 \times \$2.50 = \$32 + \$10$
$= \$42$

3 Let x = balloons and y = streamers.

a $C = 0.5x + 0.2y$

b $x = 25;\ y = 20$

$$\begin{aligned} C &= 0.5x + 0.2y \\ &= 0.5(25) + 0.2(20) \\ &= 12.5 + 4 \\ &= \$16.50 \end{aligned}$$

4 Let x = adults and y = children.

a $C = 40x + 25y$

b $x = 150;\ y = 315$

$$\begin{aligned} C &= 40x + 25y \\ &= 40(150) + 25(315) \\ &= 6000 + 7875 \\ &= \$13875 \end{aligned}$$

5 Let x = chocolate bars and y = muesli bars.

a $C = 1.6x + 1.4y$

b $x = 55;\ y = 38$

$$\begin{aligned} C &= 1.6x + 1.4y \\ &= 1.6(55) + 1.4(38) \\ &= 88 + 53.2 \\ &= \$141.20 \end{aligned}$$

6 Let x = custard tarts and y = iced doughnuts.

a $C = 1.75x + 0.7y$

b $x = 25;\ y = 12$

$$\begin{aligned} C &= 1.75x + 0.7y \\ &= 1.75(25) + 0.7(12) \\ &= 43.75 + 8.4 \\ &= \$52.15 \end{aligned}$$

7 Let x = coffee and y = milkshake.

a $C = 3.5x + 5y$

b $x = 52;\ y = 26$

$$\begin{aligned} C &= 3.5x + 5y \\ &= 3.5(52) + 5(26) \\ &= 182 + 130 \\ &= \$312 \end{aligned}$$

8 Let x = budgerigars and y = parrots.

a $C = 30x + 60y$

b $x = 60; y = 28$

$$\begin{aligned} C &= 30x + 60y \\ &= 30(60) + 60(28) \\ &= 1800 + 1690 \\ &= \$3480 \end{aligned}$$

9 Let x = 50c and y = 20c.

a $N = x + y$

b $V = 0.5x + 0.2y$

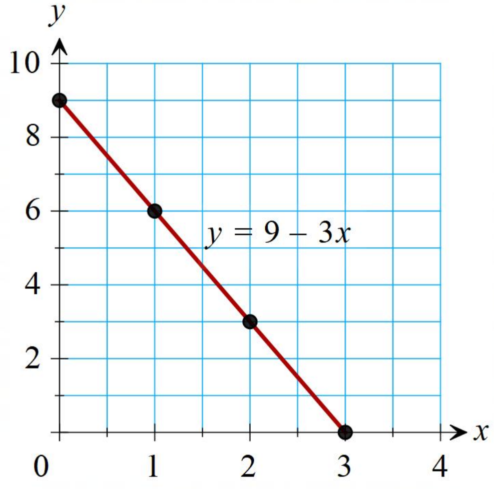
c $x = 45; y = 77$

$$\begin{aligned} V &= 0.5x + 0.2y \\ &= 0.5(25) + 0.2(77) \\ &= 22.5 + 15.4 \\ &= \$37.90 \end{aligned}$$

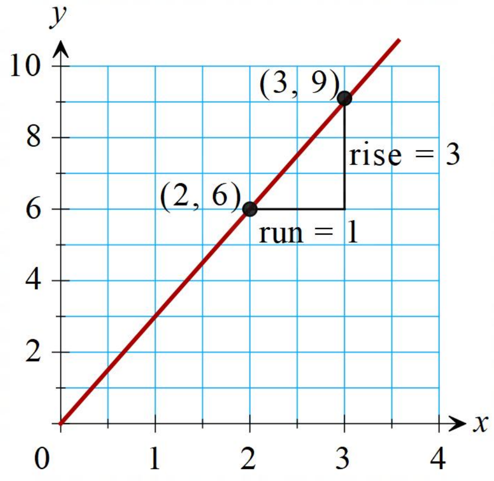
Solutions to 5D Now Try This Questions

10 If $x = 0, y = 9 - 3(0) = 9 - 0 = 9$
If $x = 1, y = 9 - 3(1) = 9 - 3 = 6$
If $x = 2, y = 9 - 3(2) = 9 - 6 = 3$
If $x = 3, y = 9 - 3(3) = 9 - 9 = 0$

x	0	1	2	3
y	9	6	3	0



11 Sketch a graph showing the two points:



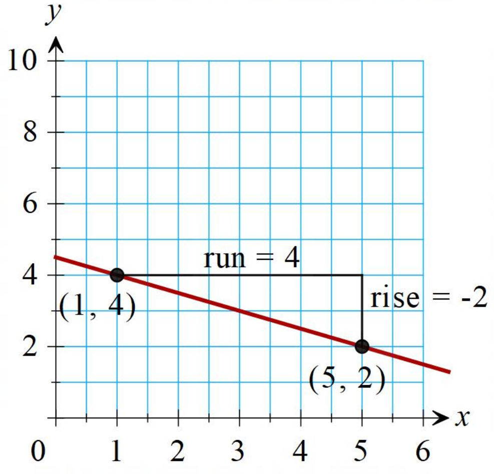
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

rise = $9 - 6 = 3$

run = $3 - 2 = 1$

$$\text{slope} = \frac{3}{1} = 3$$

12 Sketch a graph showing the two points.



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

rise = $2 - 4 = -2$

run = $5 - 1 = 4$

$$\text{slope} = \frac{-2}{4} = -0.5$$

Solutions to Exercise 5D

1 The slope of *A* falls from left to right so is negative.
The slope of *B* rises from left to right so is positive.
C is not defined and *D* has no slope so is zero.

2 $y = 3 + 2x$

Substitute vales for x into equation to find y.

e.g. If $x = -2,\ y = 3 + 2 \times (-2)$

$$= 3 - 4$$

$$= -1$$

x	-2	-1	0	1	2	3
y	-1	1	3	5	7	9

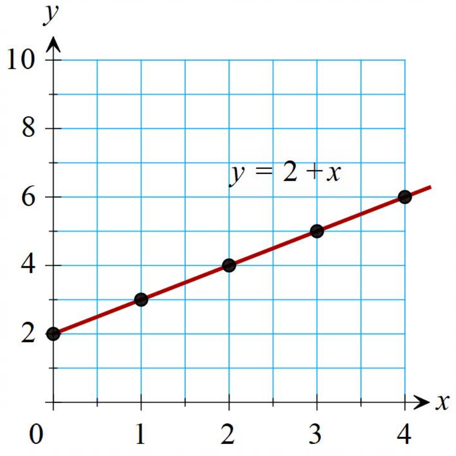
3 **a** From graph, count up vertically from B to vertical height of A.
rise = 3

b From graph, count across horizontally from B to horizontal distance of A.
run = 1

c $\dfrac{\text{rise}}{\text{run}} = \dfrac{3}{1} = 3$

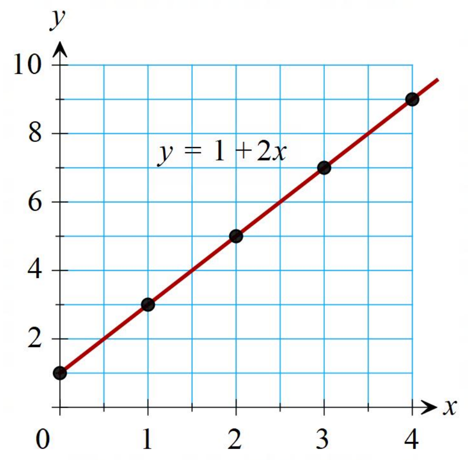
4 **a** $y = 2 + x$

x	0	1	2	3	4
y	2	3	4	5	6



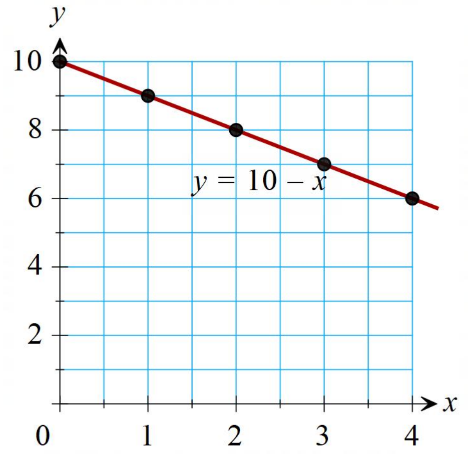
b $y = 1 + 2x$

x	0	1	2	3	4
y	1	3	5	7	9



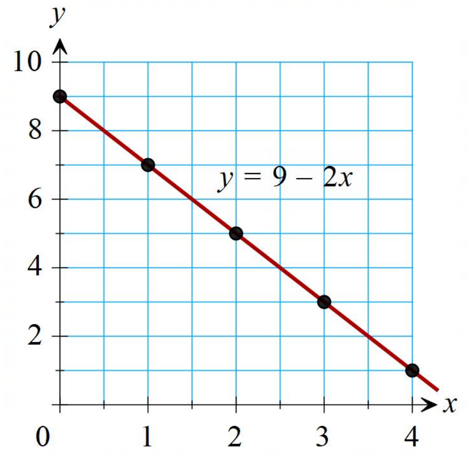
c $y = 10 - x$

x	0	1	2	3	4
y	10	9	8	7	6



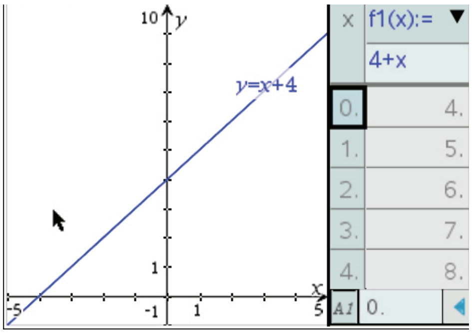
d $y = 9 - 2x$

x	0	1	2	3	4
y	9	7	5	3	1



5 **a** $y = 4 + x$

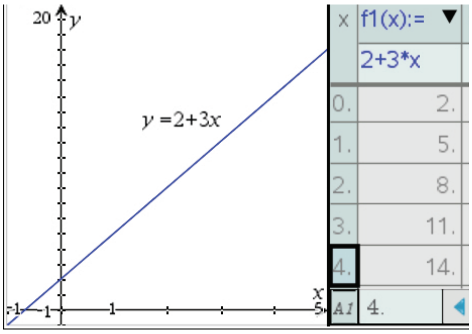
x	0	1	2	3	4
y	4	5	6	7	8



x	f1(x):=
	4+x
0.	4.
1.	5.
2.	6.
3.	7.
4.	8.
A1	0.

b $y = 2 + 3x$

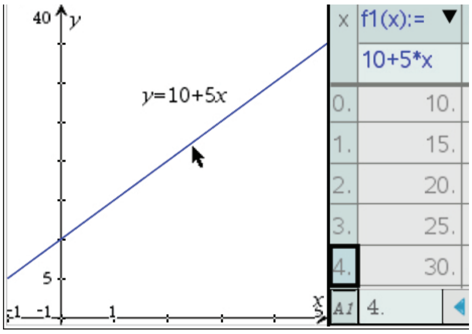
x	0	1	2	3	4
y	2	5	8	11	14



x	f1(x):=
	2+3*x
0.	2.
1.	5.
2.	8.
3.	11.
4.	14.
A1	4.

c $y = 10 + 5x$

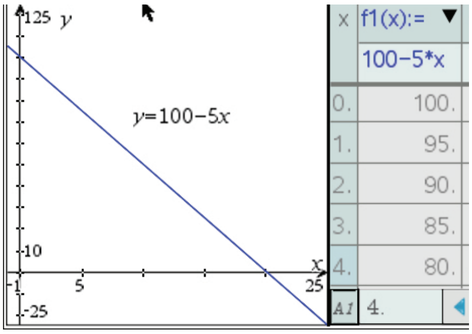
x	0	1	2	3	4
y	10	15	20	25	30



x	f1(x):=
	10+5*x
0.	10.
1.	15.
2.	20.
3.	25.
4.	30.
A1	4.

d $y = 100 - 5x$

x	0	1	2	3	4
y	100	95	90	85	80



x	f1(x):=
	100−5*x
0.	100.
1.	95.
2.	90.
3.	85.
4.	80.
A1	4.

6 **a** Read from graph:
(0, 4), (2, 6), (3, 7), (5, 9)

b Read from graph:
(0, 8), (1, 6), (2, 4), (3, 2)

7 *A*: (1, 8), (4, 1)

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{1-8}{4-1}$$
$$= \frac{-7}{3} = -2.33$$

B: (0, 2), (4, 9)

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{9-2}{4-0}$$
$$= \frac{7}{4} = 1.75$$

C: (1, 1), (5, 5)

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{5-1}{5-1}$$
$$= \frac{4}{4} = 1.00$$

8 *A*: (0, 4), (3, 10)

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{10-4}{3-0}$$
$$= \frac{6}{3} = 2$$

B: (1, 10), (3, 4)

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{4 - 10}{3 - 1}$$
$$= \frac{-6}{2} = -3$$

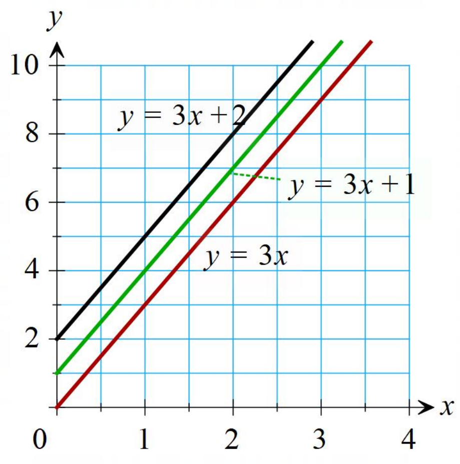
C: (1, 2), (4, 2)

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{2 - 2}{4 - 1} \quad = \frac{0}{3} = 0$$

9 Use the 2 points (-1, 1) and (0, $\frac{1}{2}$)

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{\frac{1}{2} - 1}{0 - -1}$$
$$= \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$$

10 **a**



b Select any two points on each line and use

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

The slope of $y = 3x$ is 3.

The slope of $y = 3x + 1$ is 3.

The slope of $y = 3x + 2$ is 3.

c The slopes are all 3. The lines are parallel.

Solutions to 5E Now Try This Questions

13 $$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-8}{6-2} = \frac{-5}{4} = -1.25$$

14 **a** $y = 4 - 2x$

In the intercept-slope form,

$y = a + bx$, a is the y-intercept and b is the slope.

y-intercept $= 4$, slope $= -2$

b $y = 3x$

In the intercept-slope form,

$y = a + bx$, a is the y-intercept and b is the slope.

y-intercept $= 0$, slope $= 3$

c $y - 3x = 6$

$y = 6 + 3x$

In the intercept-slope form,

$y = a + bx$, a is the y-intercept and b is the slope.

y-intercept $= 6$, slope $= 3$

15 **a** y-intercept $= 5$, slope $= 3$

$y = 5 + 3x$

b y-intercept $= 6$, slope $= -2$

$y = 6 - 2x$

c y-intercept $= -4$, slope $= 5$

$y = -4 + 5x$

16 $y = 5 + 3x$

The y-intercept is 5.

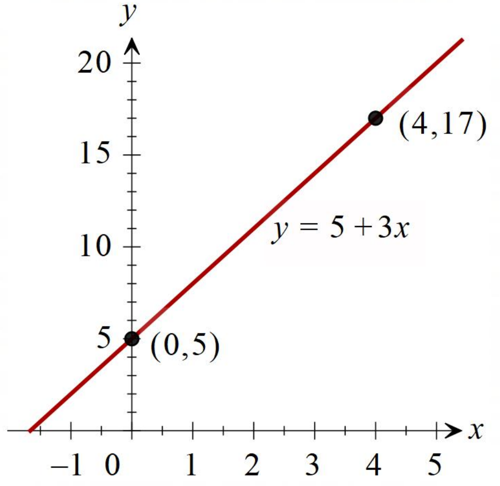
Choose another point on the graph.

e.g. $x = 4$

Find y:

$y = 5 + 3x = 5 + 3(4) = 17$

Plot the y-intercept and the point (4,17) on the line.



Solutions to Exercise 5E

1 **a** $$\text{slope} = \frac{9-3}{5-2} = \frac{6}{3} = 2$$

b $$\text{slope} = \frac{30-0}{15-0} = \frac{30}{15} = 2$$

c slope $= \frac{17 - 3}{5 - (-2)}$

$= \frac{14}{7}$

$= 2$

d slope $= \frac{7 - 4}{2 - 5}$

$= \frac{3}{-3}$

$= -1$

e slope $= \frac{5 - 0}{-1 - 4}$

$= \frac{5}{-5}$

$= -1$

f slope $= \frac{6 - 1}{-1 - 5}$

$= \frac{5}{-6}$

$= -\frac{5}{6}$

2 a $y = 5 + 2x$

When the equation is in the form $y = a + bx$, the slope and the y-intercept can be read from the equation. a is the y-intercept and b is the slope.

y-intercept = 5

slope = 2 (coefficient of x)

b $y = 6 - 3x$

This is of the form $y = a + bx$.

y-intercept = 6

slope = −3 (coefficient of x)

c $y = 15 - 5x$

This is of the form $y = a + bx$.

y-intercept = 15

slope = −5 (coefficient of x)

d $y = 3x$

This if of the form $y = a + bx$.

y-intercept = 0

slope = 3 (coefficient of x)

3 a $y + 3x = 10$

Subtract $3x$ from both sides.

$y + 3x - 3x = 10 - 3x$

$y = 10 - 3x$

This is of the form $y = a + bx$.

y-intercept = 10

slope = −3 (coefficient of x)

b $4y + 8x = -20$

Subtract $8x$ from both sides.

$4y + 8x - 8x = -20 - 8x$

$4y = -20 - 8x$

Divide each term by 4.

$\frac{4y}{4} = \frac{-20}{4} - \frac{8x}{4}$

$y = -5 - 2x$

This is of the form $y = a + bx$.

y-intercept = −5

slope = −2

c $x = y - 4$

Subtract 4 from both sides.

$x + 4 = y - 4 + 4$

$x + 4 = y$

Rearrange in the form $y = a + bx$.

$y = x + 4$

$y = 4 + x$

This is of the form $y = a + bx$.

y-intercept = 4

slope = 1

d $x = 2y - 6$

Subtract 6 from both sides.

$x + 6 = 2y - 6 + 6$

$x + 6 = 2y$

$2y = x + 6$

Divide each term by 2.

$$\frac{2y}{2} = \frac{x}{2} + \frac{6}{2}$$

$$y = 3 + \frac{x}{2}$$

This is of the form $y = a + bx$.

y-intercept $= 3$

slope $= \frac{1}{2} = 0.5$

e $2x - y = 5$

Subtract $2x$ from both sides.

$2x - y - 2x = 5 - 2x$

$$-y = 5 - 2x$$

Divide each term by -1.

$y = -5 + 2x$

This is of the form $y = a + bx$.

y-intercept $= -5$

slope $= 2$

f $y - 5x = 10$

Add $5x$ to both sides.

$y - 5x + 5x = 10 + 5x$

$$y = 10 + 5x$$

This is of the form $y = a + bx$.

y-intercept $= 10$

slope $= 5$

g $2.5x + 2.5y = 25$

Subtract $2.5x$ from both sides.

$2.5x - 2.5x + 2.5y = 25 - 2.5x$

$$2.5y = 25 - 2.5x$$

Divide each term by 2.5.

$$\frac{2.5y}{2.5} = \frac{25}{2.5} - \frac{2.5x}{2.5}$$

$$y = 10 - x$$

This is of the form $y = a + bx$.

y-intercept $= 10$

slope $= -1$

h $y - 2x = 0$

Add $2x$ to both sides.

$y - 2x + 2x = 0 + 2x$

$$y = 2x$$

This is of the form $y = a + bx$.

y-intercept $= 0$

slope $= 2$

i $y + 3x - 6 = 0$

Add 6 to both sides.

$y + 3x - 6 + 6 = 0 + 6$

$$y + 3x = 6$$

Subtract $3x$ from both sides.

$y + 3x - 3x = 6 - 3x$

$$y = 6 - 3x$$

This is of the form $y = a + bx$.

y-intercept $= 6$

slope $= -3$

j $y = 3$

This is of the form $y = a + bx$.

y-intercept $= 3$

slope $= 0$

k $4x - 5y - 8 = 7$

Add 8 to both sides.

$4x - 5y - 8 + 8 = 7 + 8$

$$4x - 5y = 15$$

Subtract $4x$ from both sides.

$4x - 5y - 4x = 15 - 4x$

$$-5y = 15 - 4x$$

Divide each term by -5.

$$\frac{-5y}{-5} = \frac{15}{-5} - \frac{4x}{-5}$$

$$y = -3 + 0.8x$$

This is of the form $y = a + bx$.

y-intercept $= -3$

slope $= 0.8$

l $2y - 8 = 2(3x - 6)$

Expand brackets:

$2y - 8 = 6x - 12$

Add 8 to both sides.

$2y - 8 + 8 = 6x - 12 + 8$

$$2y = 6x - 4$$

Divide each term by 2.

$$\frac{2y}{2} = \frac{6x}{2} - \frac{4}{2}$$

$$y = 3x - 2$$

$$y = -2 + 3x$$

This is of the form $y = a + bx$.

y-intercept $= -2$

slope = 3

4 **a** y-intercept = 2, slope = 5
Use $y = a + bx$ where
a = y-intercept and b = slope.
$y = 2 + 5x$

b y-intercept = 5, slope = 10
Use $y = a + bx$ where
a = y-intercept and b = slope.
$y = 5 + 10x$

c y-intercept = −2, slope = 4
Use $y = a + bx$ where
a = y-intercept and b = slope.
$y = -2 + 4x$

d y-intercept = 0, slope = −3
Use $y = a + bx$ where
a = y-intercept and b = slope.
$y = -3x$

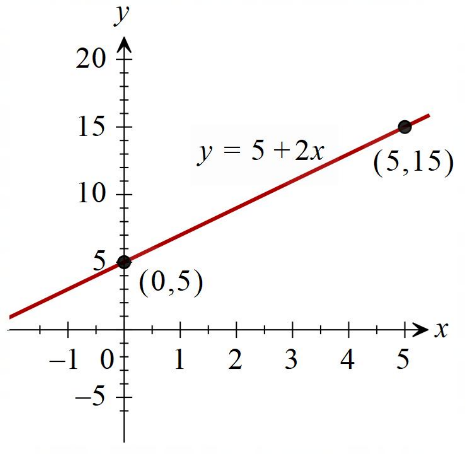
e y-intercept = −2, slope = 0
Use $y = a + bx$ where
a = y-intercept and b = slope.
$y = -2$

f y-intercept = 1.8, slope = −0.4
Use $y = a + bx$ where
a = y-intercept and b = slope.
$y = 1.8 - 0.4x$

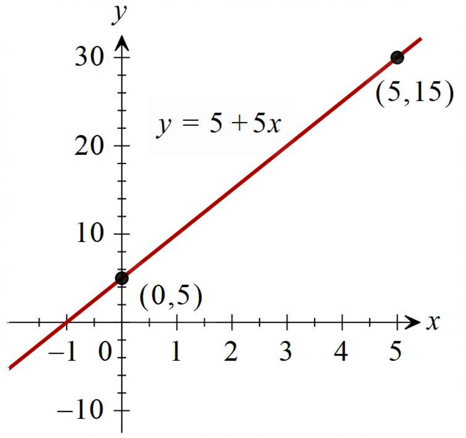
g y-intercept = 2.9, slope = −2
Use $y = a + bx$ where
a = y-intercept and b = slope.
$y = 2.9 - 2x$

h y-intercept = −1.5, slope = −0.5
Use $y = a + bx$ where
a = y-intercept and b = slope.
$y = -1.5 - 0.5x$

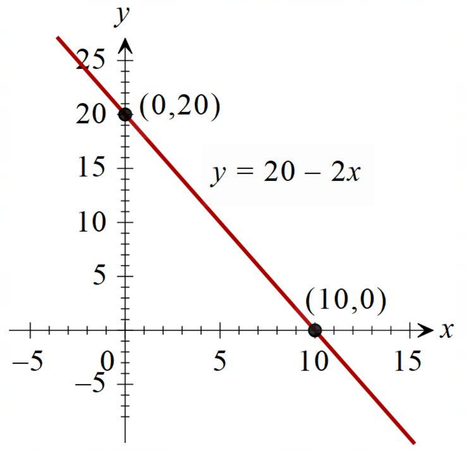
5 **a** $y = 5 + 2x$
y-intercept = 5
When $x = 5$:
$y = 5 + 2(5) = 5 + 10 = 15$.
So (5, 15) is a point on the line.



b $y = 5 + 5x$
y-intercept = 5
When $x = 5$:
$y = 5 + 5(5) = 5 + 25 = 30$.
So (5, 30) is a point on the line.



c $y = 20 - 2x$
y-intercept = 20
When $x = 10$:
$y = 20 - 2(10) = 20 - 20 = 0$.
So (10, 0) is a point on the line.



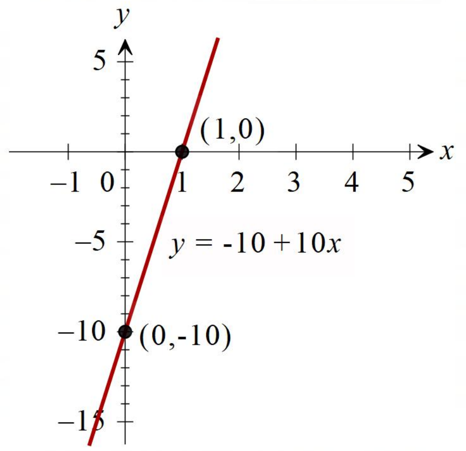
d $y = -10 + 10x$

y-intercept $= -10$

When $x = 1$:

$y = -10 + 10(1) = -10 + 10 = 0.$

So $(1, 0)$ is a point on the line.



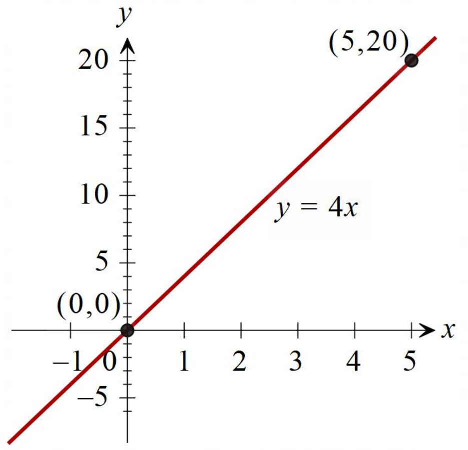
e $y = 4x$

y-intercept $= 0$

When $x = 5$:

$y = 4(5) = 20.$

So $(5, 20)$ is a point on the line.



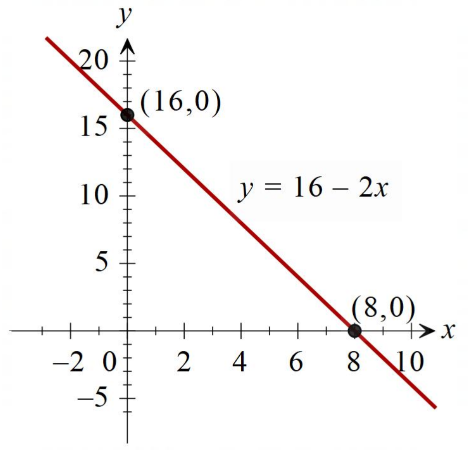
f $y = 16 - 2x$

y-intercept $= 16$

When $x = 8$:

$y = 16 - 2(8) = 16 - 16 = 0.$

So $(8, 0)$ is a point on the line.



6 **a** $y - x = 3$

Rearrange equation so that it is of the form $y = a + bx$.

Add x to both sides.

$y - x + x = 3 + x$

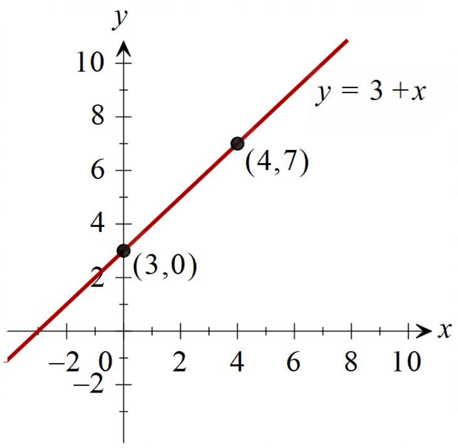
$y = 3 + x$

y-intercept $= 3$

When $x = 4$:

$y = 3 + (4) = 7.$

So $(4, 7)$ is a point on the line.



b $y + 2x = 1$

Rearrange equation so that it is of the form $y = a + bx$.

Subtract $2x$ from both sides.

$y + 2x - 2x = 1 - 2x$

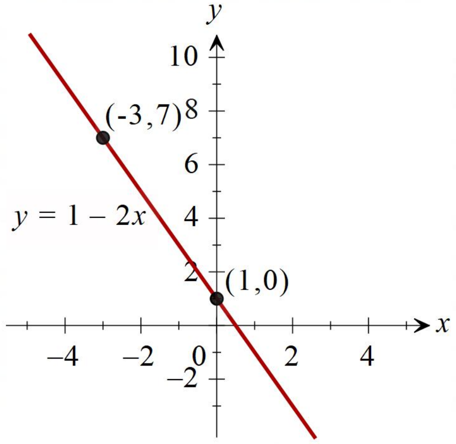
$$y = 1 - 2x$$

y-intercept = 1

When $x = -3$:

$y = 1 - 2(-3) = 1 + 6 = 7$.

So $(-3, 7)$ is a point on the line.



c $y - 3x - 4 = 0$

Rearrange equation so that it is of the form $y = a + bx$.

Add $3x$ and 4 to both sides.

$y - 3x + 3x - 4 + 4 = 0 + 3x + 4$

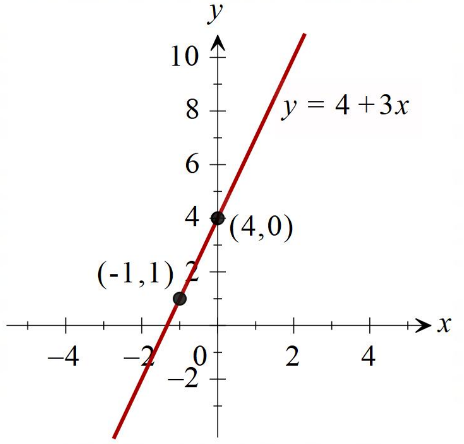
$$y = 4 + 3x$$

y-intercept = 4

When $x = -1$:

$y = 4 + 3(-1) = 4 - 3 = 1$.

So $(-1, 1)$ is a point on the line.



Solutions to 5F Now Try This Questions

17 The equation of a straight line is $y = a + bx$.

From the graph it can be seen that the y-intercept is 3. Thus $a = 3$.

To find the slope, b, use the two points (0, 3) and (1, 5).

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{5-3}{1-0} = \frac{2}{1} = 2$$

So $a = 3,\ b = 2$

Thus $y = 3 + 2x$

18 Equation of straight line is $y = a + bx$

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{10-4}{3-2} = \frac{6}{1} = 6$$

So $y = a + 6x$

To find a, use the point (2, 4) (or you can use the point (3, 10)) and substitute into equation to find a.

$y = 4,\ x = 2$

$$4 = a + 6(2)$$

$$4 = a + 12$$

$$4 - 12 = a + 12 - 12$$

$$-8 = a$$

$a = -8,\ b = 6$

$y = -8 + 6x$

Solutions to Exercise 5F

1 **a** Read from graph on y-axis.

y-intercept = 1

b $\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Use the points (0, 1) and (4, 7)

$$\text{slope} = \frac{7-1}{4-0} = \frac{6}{4} = 1.5$$

c $y = a + bx$

a is the y-intercept = 1

b is the slope = 1.5

$y = 1 + 1.5x$

2 **a** $\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Use the points (1, −4) and (3, 0)

$$\text{slope} = \frac{0-(-4)}{3-1} = \frac{4}{2} = 2$$

b $y = a + bx$

b is the slope = 2

$y = a + 2x$

Substitute the point (3, 0) into equation. (i.e. $x = 3, y = 0$)

$0 = a + 2 \times 3$

$0 = a + 6$

$a = -6$

y-intercept = −6

3 **a** $\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Use the points (−1, 5) and (1, −1)

$$\text{slope} = \frac{-1-5}{1-(-1)} = \frac{-6}{2} = -3$$

b $y = a + bx$

b is the slope = −3

$y = a - 3x$

Substitute the point (−1, 5) into equation. (i.e. $x = -1,\ y = 5$)

$5 = a + (-3) \times (-1)$

$5 = a + 3$

$a = 2$

y-intercept = 2

4 A: (0, 10), (4, 1)
y-intercept = 10

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 10}{4 - 0} = \frac{-9}{4} = -2.25$$

$y = 10 - 2.25x$

B: (0, 2), (4, 9)
y-intercept = 2

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 2}{4 - 0} = \frac{7}{4} = 1.75$$

$y = 2 + 1.75x$

C: (0, 0), (5, 5)
y-intercept = 0

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{5 - 0} = \frac{5}{5} = 1$$

$y = x$

5 A: (0, 4), (3, 10)
y-intercept = 4

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{3 - 0} = \frac{6}{3} = 2$$

$y = 4 + 2x$

B: (0, 8), (2, 5)

y-intercept = 8

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 8}{2 - 0} = \frac{-3}{2} = -1.5$$

$y = 8 - 1.5x$

C: (0, 2), (5, 5)
y-intercept = 2

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{5 - 0} = \frac{3}{5} = 0.6$$

$y = 2 + 0.6x$

6 A: (1, 10), (3, 1)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 10}{3 - 1} = \frac{-9}{2} = -4.5$$

$y = a + bx$
Substitute b = slope = −4.5
$y = a - 4.5x$
To find a, substitute values for x and y from either point (1, 10) or (3, 1).
Choose (1, 10).
$10 = a - 4.5 \times 1$
$10 = a - 4.5$
Add 4.5 to both sides.
$a = 14.5$
Equation is $y = 14.5 - 4.5x$

Alternative method

$$y - y_1 = \text{slope} \times (x - x_1)$$
$$y - 10 = -4.5(x - 1)$$
$$y - 10 + 10 = -4.5x + 4.5 + 10$$

$$y = 14.5 - 4.5x$$

B: (1, 0), (3, 10)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{10 - 0}{3 - 1}$$
$$= \frac{10}{2} = 5$$

$y = a + bx$
Substitute b = slope = 5
$y = a + 5x$
To find a, substitute values for x and y from either point (3, 10) or (1, 0).
Choose (3, 10).
$10 = a + 5 \times 3$
$10 = a + 15$
Subtract 15 from both sides.
$a = -5$
Equation is $y = -5 + 5x$

Alternative method

$$y - y_1 = \text{slope} \times (x - x_1)$$
$$y - 0 = 5(x - 1)$$
$$y = -5 + 5x$$

C: (2, 1), (5, 10)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{10 - 1}{5 - 2}$$
$$= \frac{9}{3} = 3$$

$y = a + bx$
Substitute b = slope = 3
$y = a + 3x$
To find a, substitute values for x and y from either point (5, 10) or (2, 1).
Choose (5, 10).
$10 = a + 3 \times 5$
$10 = a + 15$
Subtract 15 from both sides.
$a = -5$

Equation is $y = -5 + 3x$

Alternative method

$$y - y_1 = \text{slope} \times (x - x_1)$$
$$y - 1 = 3(x - 2)$$
$$y - 1 + 1 = 3x - 6 + 1$$
$$y = -5 + 3x$$

7 A CAS calculator is to be used for this question. An alternative method by hand is shown below.
A: (1, 10), (5, 4)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4 - 10}{5 - 1}$$
$$= \frac{-6}{4} = -1.5$$
$$y - y_1 = \text{slope} \times (x - x_1)$$
$$y - 10 = -1.5(x - 1)$$
$$y - 10 + 10 = -1.5x + 1.5 + 10$$
$$y = 11.5 - 1.5x$$

B: (1, 0), (2, 10)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{10 - 0}{2 - 1}$$
$$= \frac{10}{1} = 10$$
$$y - y_1 = \text{slope} \times (x - x_1)$$
$$y - 0 = 10(x - 1)$$
$$y = -10 + 10x$$

C: (0, 2), (5, 8)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8 - 2}{5 - 0}$$

$$= \frac{6}{5} = 1.2$$

$$y - y_1 = \text{slope} \times (x - x_1)$$

$$y - 2 = 1.2(x - 0)$$

$$y - 2 + 2 = 1.2x + 2$$

$$y = 2 + 1.2x$$

8 (–1, 1.5), (4, –1)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1 - 1.5}{4 - (-1)}$$

$$= \frac{-2.5}{5} = -0.5$$

$y = a + bx$

Substitute b = slope = −0.5

$y = a - 0.5x$

To find a, substitute values for x and y from either point (–1, 1.5) or (4, –1).

Choose (4, –1).

$-1 = a - 0.5 \times 4$

$-1 = a - 2$

Add 2 to both sides.

$a = 1$

Equation is $y = 1 - 0.5x$

9 (0, 2), (5, 9)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{9 - 2}{5 - 0}$$

$$= \frac{7}{5} = 1.4$$

$y = a + bx$

Substitute b = slope = 1.4

$y = a + 1.4x$

To find a, substitute values for x and y from either point (0, 2) or (5, 9).

Choose (0, 2).

$2 = a + 1.4 \times 0$

$2 = a$

Equation is $y = 2 + 1.4x$

10 (2, −4), (−4, 8)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8 - (-4)}{-4 - 2}$$

$$= \frac{12}{-6} = -2$$

$y = a + bx$

Substitute b = slope = −2

$y = a - 2x$

To find a, substitute values for x and y from either point (2, −4) or (−4, 8).

Choose (−4, 8).

$8 = a - 2 \times -4$

$8 = a + 8$

Subtract 8 from both sides.

$a = 0$

Equation is $y = -2x$

Solutions to 5G Now Try This Questions

19 **a** Read from the graph.
When $t = 0$, $V = 4000$ L

b Read from the graph.
When $t = 100$, $V = 7000$ L

c Read from the graph.
When $V = 10\,000$ L,
$t = 200$ minutes
200 minutes = 3 hours, 20 minutes

d $V = a + bt$

$$b = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Use 2 points from graph
e.g. (0, 4 000), (200, 10 000)

$$b = \frac{10\,000 - 4\,000}{200 - 0}$$
$$= \frac{6\,000}{200} = 30$$

y-intercept = a = 4 000
So $V = 4\,000 + 30t$

e Substitute $t = 150$ into equation.

$$V = 400 + 30(150)$$
$$= 8\,500 \text{ L}$$

f The rate is given by the slope of the graph which is 30 litres per min.

20 **a** Read from the graph.
When $t = 0$, $V = \$40\,000$

b Read from the graph.
When $t = 3$, $V = \$25\,000$

c $V = a + bt$

$$b = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

Use 2 points from graph
e.g. (0, 40 000), (3, 25 000)

$$b = \frac{25\,000 - 40\,000}{3 - 0}$$
$$= \frac{-15\,000}{3} = -5\,000$$

y-intercept = a = 40 000
So $V = 40\,000 - 5\,000t$

d The slope of the line is −5 000 so the car depreciates in value by $5 000 per year.

e Let $V = 0$

$$0 = 40\,000 - 5\,000t$$
$$5\,000t = 40\,000$$
$$t = \frac{40\,000}{5\,000}$$
$$t = 8$$

The car will have zero value at 8 years.

Solutions to Exercise 5G

1 Reading from graph.

a 45 litres

b 40 litres

c 25 litres

2 **a** 3 cm (read from graph)

b 6 cm (read from graph)

c 9 cm (read from graph)

d Use any two points from graph

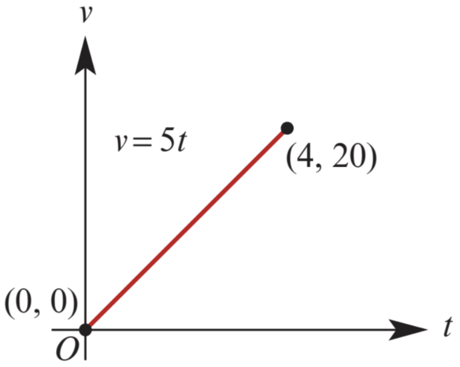
e.g. (0, 3) and (4, 6)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 3}{4 - 0} = \frac{3}{4} = 0.75$$

e $h = a + bt$
$a = 3, b = 0.75$
$h = 3 + 0.75t$

3 a $V = 5t + 0$
$V = 5t$

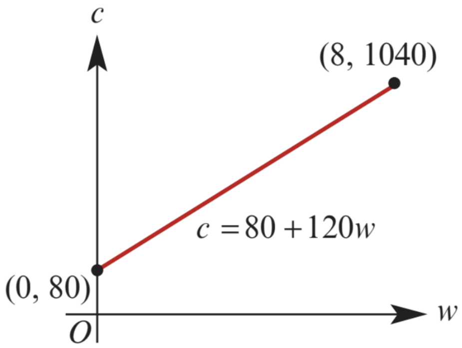
b



c Let $t = 3.2$
$V = 5 \times 3.2$
$= 16$ L

4 a Initial charge is $80
So $a = 80$
$120 for each cubic metre, so
$b = 120$
$C = 80 + 120w$

b



c Let $w = 5$
$C = 80 + 120 \times 5$
$= \$680$

5 a Read from graph: monthly service fee = $10.

b Read from graph: they charge you $17.50 per month to make 100 calls.

c y-intercept = (0, 10)
Use the y-intercept and another point on the graph, in this case (100, 17.50).

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{17.50 - 10}{100 - 0} = \frac{7.50}{100} = 0.075$$

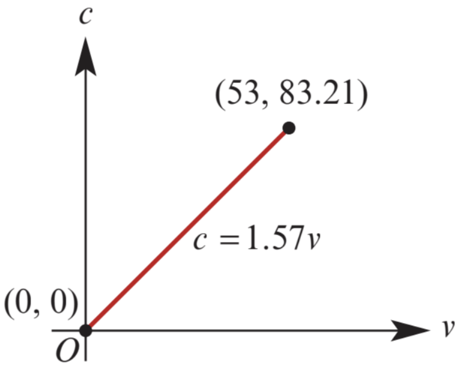
$C = 10 + 0.075n$

d $n = 300$
$C = 10 + 0.075n$
$= 10 + 0.075(300)$
$= 10 + 22.50$
$= \$32.50$

e Cost per call is $0.075 (slope), or 7.5 cents. This rounds off to 8 cents.

6 a Motorist needs to purchase v litres of petrol at $1.57
$C = 1.57v$

b



c Let $v = 60 - 7 = 53$
$C = 1.57 \times 53$
$= \$83.21$

7 a Read from graph: there was 500 mL of saline in the reservoir at the start.

b Read from graph: after 40 minutes, there is 400 mL of saline left in the drip.

c Read from graph: it takes 200 minutes for the drip to empty.

d y-intercept = (0, 500)
Use the y-intercept and another point on the graph, in this case (200, 0)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 500}{200 - 0} = \frac{-500}{200} = -2.5$$

$V = 500 - 2.5t$

e $t = 115$ minutes
$V = 500 - 2.5t$
$= 500 - 2.5(115)$
$= 500 - 287.5$
$= 212.5$ mL left in the drip.

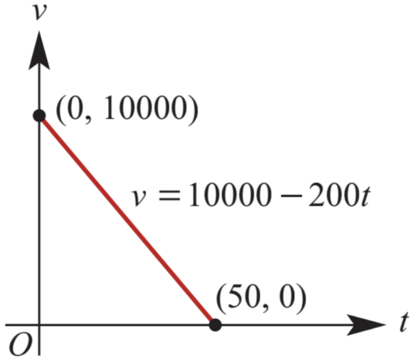
f The saline is flowing out of the drip at a rate of 2.5 mL per minute.

8 a $V = 10\,000 - 200t$

b $0 = 10\,000 - 200t$
Solve for t

$$t = \frac{10\,000}{200} = 50 \text{ days}$$

c



d Let $t = 30$
$V = 10,000 - 200 \times 30$
$= 4\,000$ L

9 a (0, 32), (40, 104)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{104 - 32}{40 - 0} = \frac{72}{40} = 1.8$$

y-intercept is 32. So $a = 32$.

$y = 32 + 1.8x$
$F = 32 + 1.8C$

b i $C = 50°$
$F = 32 + 1.8C$
$= 32 + 1.8(50)$
$= 32 + 90$
$= 122°$

ii $C = 150°$
$F = 32 + 1.8C$
$= 32 + 1.8(150)$
$= 32 + 270$
$= 302°$

iii $C = -40°$

$$\begin{aligned} F &= 32 + 1.8C \\ &= 32 + 1.8(-40) \\ &= 32 - 72 \\ &= -40° \end{aligned}$$

c When the temperature in Celsius increases by 1 degree, the temperature in Fahrenheit increases by 1.8 degrees.

Solutions to 5H Now Try This Questions

21 From the graph, it can be seen that the point of intersection is (−0.5, − 2).

Solutions to Exercise 5H

1 **a** Read from graph: $x = -1$

b Read from graph: $x = 3$

2 **a** Read from graph: (1, 1)

b Read from graph: (1, −1)

3 The method used to find the solution will depend on the calculator being used.

a Read from calculator: (2, −4)

b Read from calculator: (−3, 2)

c Read from calculator: (1.5, 2.5)

d Read from calculator: (7, 2)

e Read from calculator: (0, 3)

f Read from calculator: (1, 5)

g Read from calculator: (0.4, −2.6)

h Read from calculator: (7, 25)

4 **a** Read from calculator: (4, −1)

b Read from calculator: (0.5, 2)

c Read from calculator: (−1, −2)

d Read from calculator: (2, 3)

e Read from calculator: (1.3, 3.5)

f Read from calculator: (1.5, −0.6)

Solutions to 5I Now Try This Questions

22 Let P represent plain croissant and C represent chocolate croissant.

$6P + 6C = 46.20$

$5P + 3C = 30.10$

Use CAS calculator to solve the 2 simultaneous equations.

$P = 3.5,\ C = 4.2$

Plain croissants cost \$3.50 and chocolate croissants cost \$4.20 each.

23 Let L = length and W = width.

$L = 4w$

$P = 64 = 2L + 2W$

Use CAS calculator to solve the two simultaneous equations.

$W = 6.4,\ L = 25.6$

The width is 6.4 cm and the length is 25.6 cm.

24 Let A be the number of adult tickets and let C be the number of children's tickets.

$A + C = 498$

$26.95A + 6C = 9\,692$

Use CAS calculator to solve the two simultaneous equations.

$A = 320,\ C = 178$

320 adult tickets were sold.

Solutions to Exercise 5I

1 c is cost of a crayon and p is cost of a pencil.

$5c + 6p = 12.75$

$7c + 3p = 13.80$

2 **a** $50p + 5m = 109$

$75p + 5m = 146$

b Solve the simultaneous equations on CAS calculator.

A litre of petrol, p costs \$1.48

c A litre of motor oil, m costs \$7.

3 **a** Let a = cost of an orange and

b = cost of a banana.

$6a + 10b = 7.10$

$3a + 8b = 4.60$

b Solve the simultaneous equations on CAS calculator.

An orange costs 60 cents.

c A banana costs 35 cents.

4 Let x = weight of a box of nails and

y = weight of a box of screws.

$x + y = 2.5$

$4x + y = 7$

Solve the simultaneous equations on a CAS calculator.

A box of nails weighs 1.5 kg.

A box of screws weighs 1 kg.

5 Let x = number of wombats and

y = number of emus.

$x + y = 28$

$4x + 2y = 88$

Solve the simultaneous equations on CAS calculator.

There are 12 emus and 16 wombats.

6 Let x = length and y = width.

$2x + 2y = 36$ ①
$x = 2y$ ②

Solve the simultaneous equations on CAS calculator.
Length = 12 cm, width = 6 cm.
Alternative method by hand:
Substitute second equation, ②, into the first equation ① :

$$2(2y) + 2y = 36$$
$$4y + 2y = 36$$
$$6y = 36$$
$$6y \div 6 = 36 \div 6$$
$$y = 6 \text{ cm}$$

Substitute $y = 6$ into ②:

$$x = 2(6)$$
$$= 12 \text{ cm}$$

7 Let x = unknown number 1 and
y = unknown number 2.

$x + y = 52$
$y - x = 8$

Solve the simultaneous equations on CAS calculator.
The numbers are 22 and 30.

8 Let the two numbers be x and y.

$x + y = 35$
$y - x = 19$

Solve the simultaneous equations on CAS calculator.
The numbers are 8 and 27.

9 Let x = Bruce's age and y = Michelle's age.

$x = y + 4$
$x + y = 70$

Solve the simultaneous equations on CAS calculator.
Bruce is 37 and Michelle is 33.

10 Let x = boy's present age.
His sister is $x - 6$.

$$x + 3 = 2(x - 6 + 3)$$
$$x + 3 = 2(x - 3)$$
$$x + 3 = 2x - 6$$
$$x + 3 - x = 2x - 6 - x$$
$$3 = x - 6$$
$$3 + 6 = x - 6 + 6$$
$$x = 9$$

Boy is 9 and his sister is 3.

11 **a** Let x = cost of a thickshake
and y = cost of a smoothie.

$x = y + 2$
$3x + 4y = 27$

Solve the simultaneous equations on a CAS calculator.
The cost of a thickshake is \$5.

b The cost of a fruit smoothie is \$3.

12 Let x = mother's age and y = son's age.

$x + 4 = 3(y + 4)$ (in 4 years time)
$x + 4 - 4 = 3y + 12 - 4$
$x = 3y + 8$ ①
$x - 4 = 5(y - 4)$ (4 years ago)
$x - 4 + 4 = 5y - 20 + 4$
$x = 5y - 16$ ②

Solve the simultaneous equations, ① and ②.
Mother is 44 and son is 12.

13 Let:
x = no. of students aged 8-12

y = no. of students aged 13 and over

$$1.20x + 2y = 188.40$$
$$x + y = 125$$

Solve the simultaneous equations on CAS Calculator.
$x = 77$ which is the number of students aged between 8 and 12 entering the competition.

14 Let:
x = no. of standard models
y = no. of deluxe models

Manufacture : $3x + 5.5y = 250$ ①
Assembly : $2x + 1.5y = 80$ ②

Solve the simultaneous equations on CAS calculator.
10 standard models
40 deluxe models

15 Let: x = volume of 40% solution
y = volume of 15% solution
700 litres: $x + y = 700$ ①

24% solution:

$$0.4x + 0.15y = 0.24 \times 700$$
$$0.4x + 0.15y = 168$$ ②

Solve the simultaneous equations ① and ② on CAS calculator.
252 litres of 40% solution
448 litres of 15% solution

16 Let x = number of boys
y = number of girls

5% more boys: $x = 1.05y$ ①
246 members: $x + y = 246$ ②

Solve the simultaneous equations using CAS calculator.
126 boys and 120 girls.

17 Let:
x = volume of unleaded petrol
y = volume of diesel

Total of 10 000 L:
$x + y = 10\ 000$ ①
\$14 495 Sales:
$1.42x + 1.54y = 14\ 495$ ②

Solve the simultaneous equations ① and ② using CAS calculator.
7 542 Litres of unleaded fuel
2 458 Litres of diesel

18 Let: x = amount invested at 5%
y = amount invested at 8%

\$30 000 to invest:
$x + y = 30\ 000$ ①
\$2 100 interest
$0.05x + 0.08y = 2\ 100$ ②

Solve the simultaneous equations ① and ② using CAS calculator.
\$10 000 invested at 5%
\$20 000 invested at 8%

19 Perimeter: $2w + 2l = 120$ ①
$l = 1.5w$ ②

Use CAS calculator to solve the simultaneous equations ① and ②.
Width is 24 m, length is 36 m.

Solutions to 5J Now Try This Questions

25 **a** $C = 20 + 3x \quad (0 \le x < 4)$
$C = 60 + 20x \quad (4 \le x \le 10)$

i When $x = 3.5$
$C = 20 + 30(3.5) = 125$
The cost for 3.5 m^3 is \$125.

ii When $x = 4$
$C = 60 + 20(4) = 140$
The cost for 4 m^3 is \$140.

iii When $x = 8$
$C = 60 + 20(8) = 220$
The cost for 8 m^3 is \$220.

b The graph has two line segments. Find the co-ordinates of the end points of both lines.
$x = 0: \; C = 20 + 30(0) = 20$
$x = 4: \; C = 20 + 30(4) = 140$
$x = 4: \; C = 60 + 20(4) = 140$
$x = 10: \; C = 60 + 20(10) = 260$
Draw a set of labelled axes and mark in the points with their co-ordinates. Join up the end point of each line segment with a straight line. Label each line segment with its equation.

Solutions to Exercise 5J

1 **a** Since 2 is within $0 \le x \le 4$, you would use $C = 90 + 10x$

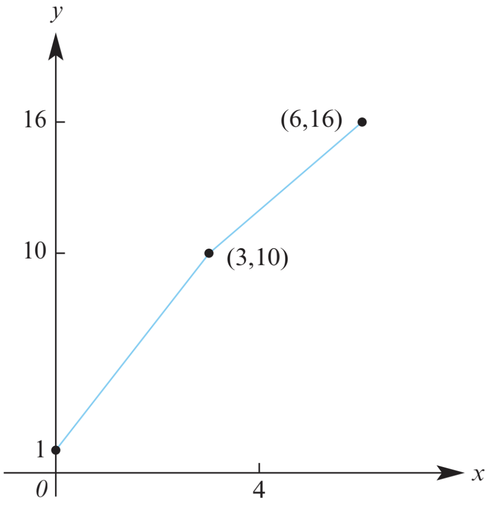
b Since 6 is within $4 \le x \le 8$, you would use $C = 50 + 20x$

2 **a** Since 2 is within $0 \le t < 3$, you would use $D = 45 - 5t$

b Since 3 is within $3 \le t < 8$, you would use $D = 90 - 20t$

c Since 7 is within $3 \le t < 8$, you would use $D = 90 - 20t$

3



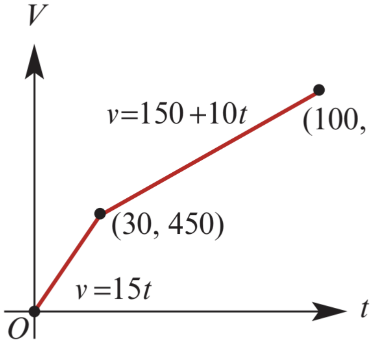
4 **a** **i** $V = 15 \times 20 = 300$ L

ii $V = 15 \times 30 = 450$ L

iii $V = 150 + 10 \times 60 = 750$ L

iv $V = 150 + 10 \times 100 = 1150$ L

b



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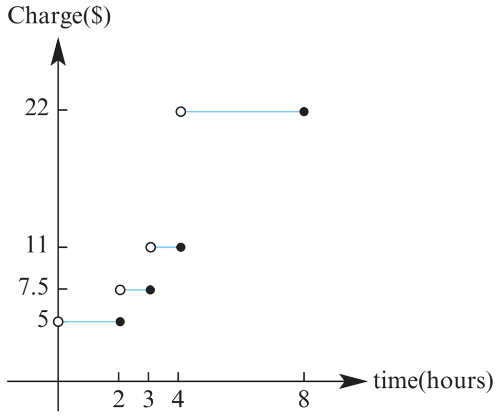


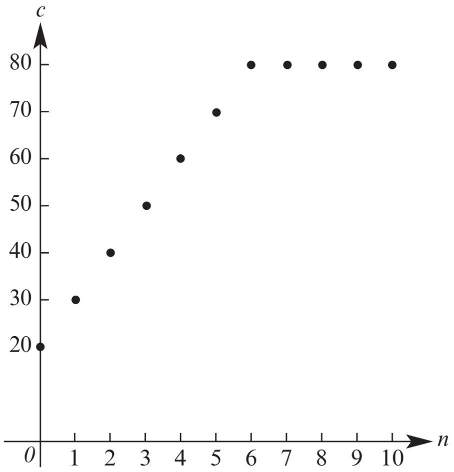
6 **a** Read value from graph.
$0

b Read value from graph.
$3

c Read value from graph.
$5

7 **a** $C = 20 + 10n \quad 1 \leq n \leq 6$
$C = 80 \quad 7 \leq n \leq 10$

b



c **i** $C = 20 + 10n$
$C = 20 + 10 \times 4$
$C = \$60$

ii Since there are 10 people, the cost remains constant at $80.
$C = \$80$

Solutions to Skills Checklist Questions

1 $C = 10t + 50$

Substitute $t = 6$

$C = 10(6) + 50 = 110$

The cost is $110.

2 $W = 790 + 40n$

When $n = 5$, $W = 790 + 40(5) = 990$

When $n = 6$, $W = 790 + 40(6) = 1030$

When $n = 7$, $W = 790 + 40(7) = 1070$

When $n = 8$, $W = 790 + 40(8) = 1110$

When $n = 9$, $W = 790 + 40(9) = 1150$

When $n = 10$, $W = 790 + 40(10) =$ 1190

n	5	6	7	8	9	10
W	990	1030	1070	1110	1150	1190

3 $2x - 5 = 25$

$2x - 5 + 5 = 25 + 5$

$2x = 30$

$x = 15$

4 $C = 100 + 60h$

When $t = 5$, $C = 100 + 60(5) = 400$

He earns $400.

5 $C = 2.9x + 2.5y$

6 Draw up a table of values. Choose suitable x values.

x	0	1	2	3
y	5	3	1	−1

Draw up axes and plot points on graph. Draw a straight line going through all points.

7 slope $= \dfrac{y_2 = y_1}{x_2 - x_1}$

Let (1, 8) be x_1, y_1 and (4, 2) be x_2, y_2

$$\text{slope} = \frac{2-8}{4-1} = \frac{-6}{3} = -2$$

8 $y = -7 + 2x$

This is in the form $y = a + bx$ where a is the y-intercept and b is the slope.

y-intercept $= -7$ and the slope is 2

9 $y = a + bx$

$b = 3$

Substitute the point (0, 5) in for x and y to find a. (or you may notice that the y-intercept is 5).

$5 = a + 3(0)$

$a = 5$

So the equation of line is $y = 5 + 3x$.

10 slope $= \dfrac{y_2 = y_1}{x_2 - x_1}$

Let $(3, \; -1)$ be $(x_1, \; y_1)$ and $(-2, \; 7)$ be $(x_2, \; y_2)$

$$\text{slope} = \frac{7-(-1)}{-2-3} = \frac{8}{-5} = -1.6$$

Substitute $(3, \; -1)$ and slope of -1.6, into equation $y = a + bx$ to find a, the y-intercept.

$-1 = a - 1.6(3)$

$-1 = a - 4.8$

Add 4.8 to both sides of the equation.

$a = 3.8$

The equation of the line is $y = 3.8 - 1.6x$.

11 Let t be the number of months. Plant is initially 70 cm tall and it grows 3 cm every month.

$h = 70 + 3t$

12 Use a CAS calculator to solve the two simultaneous equations $5x - 2y = 17$ and $3x + 4y = 20$.
$x = 4.1538...,\ y = 1.8846...$
$x = 4.15,\ y = 1.88$ correct to 2 decimal places.

13 Let C represent croissants and E represent chocolate eclair.
$6C + 4E = 45.20$
$5C + 8E = 62.40$
Use a CAS calculator to solve the two simultaneous equations.
$C = 4, E = 5.3$
A croissant costs \$4.

14 $C = 12t \quad (0 \le t < 4)$
$C = 8 + 4t \quad (4 \le t \le 10)$
When $t = 8$, $C = 8 + 10(8) = 88$
It costs \$88 to park for 8 hours.

15 For each time interval, draw the appropriate line segment.

Solutions to Chapter Review Multiple-Choice Questions

1 $a = 4$

$$\begin{aligned} 3a + 5 &= 3(4) + 5 \\ &= 12 + 5 \\ &= 17 \end{aligned}$$

B

2 $b = 1$

$$\begin{aligned} 2b - 9 &= 2(1) - 9 \\ &= 2 - 9 \\ &= -7 \end{aligned}$$

B

3 $C = 50t + 14,\ t = 8$

$$\begin{aligned} C &= 50(8) + 14 \\ &= 400 + 14 \\ &= 414 \end{aligned}$$

B

4 $P = 2L + 2W;\ L = 6,\ W = 2$

$$\begin{aligned} P &= 2(6) + 2(2) \\ &= 12 + 4 \\ &= 16 \end{aligned}$$

C

5 $x = -2,\ y = 3$ and $z = 7$

$$\begin{aligned} \frac{z - z}{y} &= \frac{7 - (-2)}{3} \\ &= \frac{7 + 2}{3} \\ &= \frac{9}{3} \\ &= 3 \end{aligned}$$

D

6 $a = 2,\ b = 5,\ c = 6,\ d = 10$

$$\begin{aligned} bd - ac &= (5) \times (10) - (2) \times (6) \\ &= 50 - 12 \\ &= 38 \end{aligned}$$

C

7

$$\begin{aligned} 4x &= 24 \\ 4x \div 4 &= 24 \div 4 \\ x &= 6 \end{aligned}$$

B

8

$$\begin{aligned} \frac{x}{3} &= 8 \\ 3 \times \frac{x}{3} &= 3 \times 8 \\ x &= 24 \end{aligned}$$

B

9

$$2v + 5 = 11$$
$$2v + 5 - 5 = 11 - 5$$
$$2v = 6$$
$$2v \div 2 = 6 \div 2$$
$$v = 3$$

A

10

$$3k - 5 = -14$$
$$3k - 5 + 5 = -14 + 5$$
$$3k = -9$$
$$3k \div 3 = -9 \div 3$$
$$k = -3$$

B

11 Cost = \$60 + 0.25 × 750

= \$247.50 **C**

12 $v = u + at$; $v = 11.6$, $u = 6.5$, $a = 3.7$

$$11.6 = 6.5 + 3.7t$$
$$11.6 - 6.5 = 6.5 - 6.5 + 3.7t$$
$$5.1 = 3.7t$$
$$3.7t \div 3.7 = 5.1 \div 3.7$$
$$t = 1.38$$

C

13 $y = 5x$ ①

$y = 2x + 6$ ②

Solve the simultaneous equations using CAS calculator.

$x = 2,\ y = 10$

Alternative method by hand:

Let ① = ②:

$$5x = 2x + 6$$
$$5x - 2x = 2x - 2x + 6$$
$$3x = 6$$
$$3x \div 3 = 6 \div 3$$
$$x = 2$$

Substitute $x = 2$ into ①:

$$y = 5x$$
$$= 5(2)$$
$$= 10$$

Solution is (2, 10). **D**

14 Read from graph: (2, 5) **D**

15 $2x + 3y = -6$ ①

$x + 3y = 0$ ②

Solve the simultaneous equations using CAS calculator.

$x = -6,\ y = 2$

Solution is (−6, 2). **E**

16 $y = 4 + 3x$

When $x = 2$, $y = 4 + 3(2)$

$= 4 + 6$

$= 10$ **E**

17 $y = 5 + 4x$

$y = a + bx$, where $a = y$-intercept

Therefore the y-intercept is (0, 5). **D**

18 $y = 10 - 3x$

$y = a + bx$, where b = slope (gradient)

Therefore the gradient = –3. **A**

19 $y - 2x = 3$

$y = 3 + 2x$

$y = a + bx$, where b = slope (gradient)

Therefore the gradient = 2. **D**

20 (5, 8), (9, 5)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 8}{9 - 5}$$
$$= \frac{-3}{4}$$
$$= -0.75$$

C

21 $y = 4 + 5x$

The y-intercept is (0, 4).

Gradient = 5, so slope is positive.

The only graph that fits is C. **C**

22 $y = 15 - 3x$

The y-intercept is (0, 15).

Gradient = −3, so slope is negative.

Both D and E fit these parameters, but the point (3, 0) in D does not fit the equation:

$0 \neq 15 - 3(3)$

The only graph that fits is E. **E**

23 Read from graph: (0, 2) **C**

24 Choose 2 points: (0, 2), (5, 8).

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8 - 2}{5 - 0}$$

$$= \frac{6}{5}$$

$$= 1.2$$ **B**

25 Definition based: read from graph.

Not defined. **E**

26 y-intercept is (0, 8)

(0, 8), (4, 0)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - 8}{4 - 0}$$

$$= \frac{-8}{4} = -2$$

$y = a + bx$, where a = y-intercept (8)

and b = gradient (–2).

$y = (8) + (-2)x$

$= 8 - 2x$ **C**

27 $y = -5 + 10x$

$(1, -5)$: $(-5) \neq -5 + 10(1)$

$(1, 5)$: $(5) = -5 + 10(1)$ ✓

$(1, 15)$: $(15) \neq -5 + 10(1)$

$(2, 20)$: $(20) \neq -5 + 10(2)$

$(2, 25)$: $(25) \neq -5 + 10(2)$

Therefore point B lies on the line. **B**

28 Growth rate refers to the gradient.

Take two points: (0, 15), (1, 20)

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{20 - 15}{1 - 0}$$

$$= \frac{5}{1}$$

$= 5$ cm/week **C**

Solutions to Chapter Review Short-Answer Questions

1 a $x + 5 = 15$

$x = 15 - 5$

$= 10$

b $x - 7 = 4$

$x = 7 + 4$

$= 11$

c $16 + x = 24$

$x = 24 - 16$

$= 8$

d $9 - x = 3$

$9 - 3 = x$

$x = 6$

e $2x + 8 = 10$

$2x = 10 - 8$

$2x = 2$

$x = 1$

f $3x - 4 = 17$

$3x = 17 + 4$

$3x = 21$

$x = 7$

g $x + 4 = -2$

$x = -2 - 4$

$= -6$

h $3 - x = -8$

$3 + 8 = x$

$x = 11$

i $6x + 8 = 26$

$6x = 26 - 8$

$6x = 18$

$x = 3$

j $3x - 4 = 5$

$3x = 5 + 4$

$3x = 9$

$x = 3$

k $\frac{x}{5} = 3$

$x = 3 \times 5$

$= 15$

l $\frac{x}{-2} = 12$

$x = 12 \times (-2)$

$= -24$

2 **a** When $l = 12$ and $b = 8$:

$P = 2l + 2b$

$P = 2 \times l2 + 2 \times 8$

$= 40$

b When $l = 40$ and $b = 25$:

$P = 2l + 2b$

$P = 2 \times 40 + 2 \times 25$

$= 130$

3 **a** When $b = 6$ and $h = 10$:

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \times 6 \times 10$

$= 30$

b When $b = 12$ and $h = 9$:

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \times 12 \times 9$

$= 54$

4 When $r = 15$:

$C = 2\pi r$

$C = 2 \times \pi \times 15$

$= 94.25,$ correct to 2 decimal places

The circumference of the circle is 94.25 cm, correct to 2 decimal places.

5

x	y
–20	$33 \times (-20) - 56 = -716$
–15	$33 \times (-15) - 56 = -551$
–10	$33 \times (-10) - 56 = -386$
–5	$33 \times (-5) - 56 = -221$
	$33 \times 0 - 56 = -56$
5	$33 \times 5 - 56 = 109$
10	$33 \times 10 - 56 = 274$
15	$33 \times 15 - 56 = 439$
20	$33 \times 20 - 56 = 604$
25	$33 \times 25 - 56 = 769$

a $y = 274$ when $x = 10$

b When $y = -221$, $x = -5$

6 Let x be the unknown number.

Double the number: $2x$

Add 4: $2x + 4$

The result is 6: $2x + 4 = 6$

$\therefore 2x = 6 - 4$

$2x = 2$

$x = 1$

The original number is 1.

7 Let x be the unknown number.

Multiply the number by 3: $3x$

Subtract 4: $3x - 4$

The result is 11: $3x - 4 = 11$

$\therefore 3x = 11 + 4$

$3x = 15$

$x = 5$

The number is 5.

8 a $y = x + 2$

$y = 6 - 3x$

Solve the simultaneous equations using CAS calculator.

$x = 1, \ y = 3$

The point of intersection of the lines with rule $y = x + 2$ and $y = 6 - 3x$ is (1, 3).

b $y = x{-}3$

$2x - y = 7$

Solve the simultaneous equations using CAS calculator.

The point of intersection of the lines with rule $y = x - 3$ and $2x - y = 7$ is (4, 1).

c $x + y = 6$

$2x - y = 9$

Solve the simultaneous equations using CAS calculator.

$x = 5, \ y = 1$

The point of intersection of the lines with rule $x + y = 6$ and $2x - y = 9$ is (5, 1).

9 a $y = 5x - 2$

$2x + y = 12$

Use CAS calculator to solve the simultaneous equations.

$x = 2, \ y = 8$

b $x + 2y = 8$

$3x - 2y = 4$

Use CAS calculator to solve the simultaneous equations.

$x = 3, \ y = 2.5$

c $2p - q = 12$

$p + q = 3$

Use CAS calculator to solve the simultaneous equations.

$p = 5, \ q = -2$

d $3p + 5q = 25$

$2p - q = 8$

Use CAS calculator to solve the simultaneous equations.

$p = 5, \ q = 2$

e $3p + 2q = 8$

$p - 2q = 0$

Use CAS calculator to solve the simultaneous equations.

$p = 2, \ q = 1$

10 a

x	y
0	$2 + 5 \times 0 = 2$
1	$2 + 5 \times 1 = 7$
2	$2 + 5 \times 2 = 12$
3	$2 + 5 \times 3 = 17$
4	$2 + 5 \times 4 = 22$

b

x	y
0	$12 - 0 = 12$
1	$12 - 1 = 11$
2	$12 - 2 = 10$
3	$12 - 3 = 9$
4	$12 - 4 = 8$

c

x	y
0	$-2 + 4 \times 0 = -2$
1	$-2 + 4 \times 1 = 2$
2	$-2 + 4 \times 2 = 6$
3	$-2 + 4 \times 3 = 10$
4	$-2 + 4 \times 4 = 14$

11 **a** Let $w = 6$

$c = 95 + 110 \times 6$

$= \$755$

b From the equation, \$110 (slope).

12 Line A falls from left to right, so the slope is negative. Choose any two points on line A, say (0, 9) and (5, 3).

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Rise} = 3 - 9 = -6$$

$$\text{Run} = 5 - 0 = 5$$

$$\therefore \text{Slope} = \frac{-6}{5} = -1.2$$

The slope of line A is –1.2.

Line B rises from left to right, so the slope is positive. Choose any two points on line B, say (0, 2) and (5, 5).

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Rise} = 5 - 3 = 3$$

$$\text{Run} = 5 - 0 = 5$$

$$\therefore \text{Slope} = \frac{3}{5} = -0.6$$

The slope of line B is 0.6.

13 Line A rises from left to right, so the slope is positive. Choose any two points on line A, say (1, 0) and (5, 9).

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Rise} = 9 - 0 = -9$$

$$\text{Run} = 5 - 1 = 4$$

$$\therefore \text{Slope} = \frac{9}{4} = 2.25$$

The slope of line A is 2.25.

Line B falls from left to right, so the slope is negative.
Choose any two points on line B, say (0, 8) and (3, 0).

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Rise} = 0 - 8 = -8$$

$$\text{Run} = 3 - 0 = 3$$

$$\therefore \text{Slope} = \frac{-8}{3} = -2.67,$$

correct to 2 decimal places

The slope of line B is –2.67, correct to 2 decimal places.

Solutions to Chapter Review Written-Response Questions

1 $C = 25 + 8h$

a When the boat is hired for 4 hours,
$h = 4$, and $C = 25 + 8 \times 4 = 57$
If the boat is hired for 4 hours, the cost is \$57.

b If the cost was \$81, $C = 81$, and

$81 = 25 + 8h$ $8h = 56$ $h = 7$

2 $C = 25 + 0.50n$

a

n	C
60	$25 + 0.5 \times 60 = 55$
70	$25 + 0.5 \times 70 = 60$
80	$25 + 0.5 \times 80 = 65$
90	$25 + 0.5 \times 90 = 70$
100	$25 + 0.5 \times 100 = 75$
110	$25 + 0.5 \times 110 = 80$
120	$25 + 0.5 \times 120 = 85$
130	$25 + 0.5 \times 130 = 90$
140	$25 + 0.5 \times 140 = 95$
150	$25 + 0.5 \times 150 = 100$
160	$25 + 0.5 \times 160 = 105$

b The cost of making 160 phone calls is \$105.

3 a The linear equation for the total charge, C, of any job is $C = 80 + 45h$.

b For a 3-hour job, $h = 3$, and
$C = 80 + 45 \times 3 = 215$
A 3-hour job would cost \$215.

4 a Let a and c be the price of an adult's and child's ticket respectively.
$3a + 5c = 73.5$ ①
$2a + 3c = 46.5$ ②

b Use a CAS calculator to solve the simultaneous equations.
The cost of an adult's ticket is \$12.00.

c The cost of a child's ticket is \$7.50.

5 Let:
x = width
Perimeter = $10x$

$$\begin{aligned}(x + 9) + x + (x + 9) + x &= 10x\\ 4x + 18 &= 10x\\ 6x &= 18\\ x &= 3 \text{ m}\\ \text{width} &= 3 \text{ m}\end{aligned}$$

6 Let
f = French
i = Indonesian

j = Japanese

Total no. of students = 105

$f + i + j = 105$ ①

The Indonesian class has 2/3 the number of students of the French Class:

$i = \frac{2}{3}f$ ②

The Japanese class has 5/6 the number of students of the French Class:

$j = \frac{5}{6}f$ ③

Sub ② and ③ into ①:

$$f + \frac{2}{3}f + \frac{5}{6}f = 105$$
$$\frac{6}{6}f + \frac{4}{6}f + \frac{5}{6}f = 105$$
$$\frac{15}{6}f = 105$$
$$15f = 630$$
$$f = 42$$

Sub into ②: Sub into ③:

$i = \frac{2}{3} \times 42$ $j = \frac{5}{6} \times 42$

$i = 28$ $j = 35$

French = 42 students
Indonesian = 28 students
Japanese = 35 students

7 **a** From the graph, after 20 months, the value of the machine is \$200 000.

b The line predicts that the machine will have no (zero) value after 60 months (or 5 years).

c Choose any two points on the line, say $(t_1, V_1) = (0, 300)$, $(t_2, V_2) = (60, 0)$.

Then, the equation of the straight line is given by:

$$\text{slope} = \frac{300 - 0}{0 - 60} = -5$$

$$V = a - 5t$$

Substitute $(0, 300)$

$$300 = a - 5 \times 0$$

$$\therefore \quad a = 300$$

$$\therefore \quad V = 300 - 5t$$

d After 3 years, $t = 36$. So $V = 300 - 5 \times 36 = 120$.
The value of the machine after 3 years is \$120 000.

e The slope of the equation is –5, which indicates the machine depreciates by \$5 000 per month.

8 **a** From the graph, when there were 500 000 machines the amount of money transacted through ATMs was \$80 billion.

b Choose any two points on the line, say $(N_1, A_1) = (0, 0)$ and $(N_2, A_2) = (1000, 160)$.

Then, the equation of the straight line is given by:

$$\text{slope} = \frac{160 - 0}{1000 - 60} = 0.16$$

$$A = a + 0.16N$$

Substitute $(0, 0)$

$$0 = a + 0.16 \times 0$$

$$\therefore \quad a = 0$$

$$\therefore \quad A = 0.16N$$

c When there were 600 000 machines, $N = 600$.
When $N = 600$, $A = 0.16 \times 600 = 96$.
The amount transacted when there were 600 000 machines is predicted to be \$96 billion.

d When there are 1 500 000 machines, $N = 1500$.
When $N = 1500$, $A = 0.16 \times 1500 = 240$.
The amount transacted when there are 1 500 000 machines is predicted to be \$240 billion.

e The amount transacted through ATM machines is increasing by \$0.16 billion (slope) with each 1 000 extra ATMs.

9 a Choose any two points on the line, say $(A_1, H_1) = (0, 80)$, $(A_2, H_2) = (3, 100)$.
Then, the equation of the straight line is given by:

$$\text{slope} = \frac{100 - 80}{3 - 0} = 6.67$$

$$H = a + 6.67A$$

Substitute (0, 80)

$$80 = a + 6.67 \times 0$$

$$\therefore \quad a = 80$$

$$\therefore \quad H = 80 + 6.67A$$

b When $A = 3$, $H = 6.67 \times 3 + 80 = 100$
The height of a child aged 3 is predicted to be 100 cm.

c The slope is 6.67, so the equation of the line of best fit tells us that, each year, children's heights increase by 6.67 cm.

10 a i When $x = 20$, $C = 5 + 0.40 \times 20 = 13$.
The charge for using 20 kL of water is $13.

ii When $x = 30$, $C = -31 + 1.6 \times 30 = 17$.
The charge for using 30 kL of water is $17.

iii When $x = 50$, $C = -31 + 1.6 \times 50 = 49$
The charge for using 50 kL of water is $49.

b i When you use less than 30 kL of water, the linear equation $C = 5 + 0.40x$ is used.
The slope is 0.40, which means that a kilolitre of water costs $0.40, or 40 cents.

ii When you use more than 30 kL of water, the linear equation $C = -31 + 1.6x$ is used.
The slope is 1.6, which means that a kilolitre of water costs $1.60.

c The graph below is the segmented graph for $0 \le x \le 50$ for the equations

$C = 5 + 0.40\,x \; (0 \le x < 30)$

$C = -31 + 1.6\,x \; (x \ge 30)$

Chapter 6 – Revision of Unit 1

Solutions to Multiple-choice questions

Chapter 2: Investigating and comparing data distributions

1 *level of water* and *size* are both ordinal variables. **A**

2 A bar chart is suitable for a categorical variable. The only variable on this list which is categorical is *coffee size*. **E**

3 Histograms, dots plots and stem plots are all suitable for numerical variables. **D**

4 Read off from the vertical axis of the histogram, the frequency for 5 to less than 10 years is 26. Since there are a total of 100 people, this is also the percentage. **B**

5 $n = 100$, so the median is the average of the 50^{th} and 51^{st} values. Looking the frequencies from left to right we see values 30 to 55 are in the 5 to less than 10 years column, so this is where the median will lie. **B**

6 The modal interval has the highest frequency, so it is from 0 to less than 5 years. **A**

7 Since the tail is the the right this is positively skewed. **A**

8 Enter the data into your calculator and follow the instructions on page 90 (TI) or 91 (CASIO) to find the value of the mean and standard deviation. **B**

9 $IQR = 35 - 5 = 30$

lower fence $= 5 - 1.5 \times 30 = -40$

upper fence $= 35 + 1.5 \times 30 = 80$

D

10 $80 \pm 2 \times 10 = (60, 100) \Rightarrow 95\%$

C

11 From the boxplot, the lowest pulse rate for males is about 46. **D**

12 The median pulse rate for females in higher than the median pulse rate for males. The IQR for females is also higher than the IQR for males (no need to work these out, just compared the widths of the two boxes in the boxplots).

C

13 $75.5 \pm 2 \times 1.5 = (72.5, 78.5)$

Since 95% of the lengths are between 72.5 and 78.5, then by symmetry 2.5% are less than 72.5, and 2.5% are greater than 78.5.

A

Chapter 3: Sequences and finance

14 The starting value is 6 and then 3 is added to generate the next term.

B

15 The starting value is 6 and then 4 is subtracted to generate the next term. **C**

16 An arithmetic sequence increases or decreases by a constant amount.

The first option oscillates, the second sequence is geometric (common ratio of 10), the fourth sequence is a list of the square numbers and the fifth sequence has repeated numbers. The third sequence increases by 100 to generate each term. **C**

17 The first sequence decreases by 9, 10 then 11 so is not arithmetic. **A**

18 Since Brian initially has 2 Camelias in his garden, so the initial value is 2. Since Brian plants 3 each week, the sequence is generated by adding 3 to each value. **A**

19 Since Lee invests $40 000, then $V_0 = 40\ 000$. Simple interest of 5.1% on $40 000 means that Lee earns $2040 in interest each year. Thus, to generate the next term, 2040 is added to each term. **D**

20 The recurrence relation tells us that we have an initial value of 50 and subtract 20 to generate each term. **B**

21 The computer was purchased for $3200 so the initial value was $3200. Depreciation is 15% which is equivalent to $480 per year. After four years, the value of the computer is $3200 - 480 \times 4 = \$1280$. **A**

22 The sequence has an initial value of 39 and a common difference is −3, that is 3 is subtracted to generate each term. The rule is $V_n = 39 - 3n$ so $V_8 = 39 - 3 \times 8 = 15$. **B**

23 Since $t_1 = 26$ then $t_1 = t_0 + d \times 1 = 26$ and $t_5 = t_0 + d \times 5 = 10$. Solving these two equations simultaneously gives $d = -4$ and $t_0 = 30$. Using these values, $t_3 = 30 - 4 \times 3 = 30 - 12 = 18$. **B**

24 Rearranging $t_{n+1} - t_n = -7$ gives $t_{n+1} = t_n - 7$. Thus, 7 is subtracted from a term to generate the next term. The initial value is 3 so $t_n = 3 - 7n$. **E**

25 A geometric sequence is one with a common ratio. Sequence **C** is not a geometric sequence since $\frac{21}{3} \neq \frac{55}{21}$ meaning that there is no common ratio. **C**

26 As the original purchase price was $25 000, $V_0 = 25\ 000$. Depreciation of 20% per year means that the remaining value is 80%. We are interested in the value after 6 years so calculating $0.8^6 \times 25\ 000 = 6553.60$ which is closest to $7000. **A**

27 The Common Ratio is found using any two consecutive terms. $\frac{500}{2000} = 0.25$ **B**

28 Solve on CAS $72\ 000 = 1.03^4 \times P$ for P since the value is $72 000 after 4 years. **B**

Chapter 4: Matrices

29 Since matrix A has 3 rows and 2 columns, the order is 3×2. **C**

30 The element a_{12} corresponds to the element in the first row and the second column from matrix A which is given as -5. **B**

31 Matrices are added by adding their corresponding elements.

$$\begin{aligned} A + B &= \begin{bmatrix} 3 & -5 \\ 1 & 8 \\ 2 & -4 \end{bmatrix} + \begin{bmatrix} 5 & -2 \\ -1 & 0 \\ 3 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 3+5 & -5+(-2) \\ 1+(-1) & 8+0 \\ 2+3 & (-4)+7 \end{bmatrix} \\ &= \begin{bmatrix} 8 & -7 \\ 0 & 8 \\ 5 & 3 \end{bmatrix} \end{aligned}$$

Thus: **D**

32 First multiply each element in matrix A by 2 then subtract the corresponding elements from matrix B.

$$\begin{aligned} 2A - B &= \begin{bmatrix} 6 & -10 \\ 2 & 16 \\ 4 & -8 \end{bmatrix} - \begin{bmatrix} 5 & -2 \\ -1 & 0 \\ 3 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 6-5 & -10-(-2) \\ 2-(-1) & 16-0 \\ 4-3 & (-8)-7 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -8 \\ 3 & 16 \\ 1 & -15 \end{bmatrix} \end{aligned}$$

Thus: **D**

33 Matrix multiplication is defined when the number of columns of the first matrix is the same as the number of rows in the second matrix. Thus, EH is not defined as E has 3 columns and H has 2 rows. **E**

34 The order of matrix FG is found by finding the number of rows in F (3) and the number and the number of columns in G (2) to give 3×2. **C**

35 The inverse of a matrix can be found using a CAS. **B**

36 To find the number of pets owned by Freddy's family we add up the elements in row 3: $1 + 3 + 0 = 4$. **D**

37 The number of ways that Manesh connects with others can be found by adding up the elements in row 1 (or in column 1): $0 + 3 + 2 = 5$. **E**

38 There are two ways this can be done. First, you could construct a diagram from the matrix and then trace through. Second, you could trace through the matrix. We see that D is connected to both C and E so we need to look at the second step. C is connected to A. **A**

39 Multiply out the matrix with the column vector to form the equations. You can use your CAS or do this by hand. **A**

Chapter 5: Linear relations and modelling

40 $a = 2, b = 3, c = 5$

$$\begin{aligned} ab - c &= 2(3) - 5 \\ &= 1 \end{aligned}$$

B

41

$$\begin{aligned} 2x + 3 &= 12 \\ 2x + 3 - 3 &= 12 - 3 \\ 2x &= 9 \\ \therefore x &= \frac{9}{2} \\ x &= 4.5 \end{aligned}$$

B

42

$$\begin{aligned} y &= 3x - 7 \\ y &= 3(5) - 7 \\ &= 15 - 7 \\ &= 8 \end{aligned}$$

C

43 $C = 400$

\$150 per hour = $150t$

$\therefore C = 400 + 150t$ **D**

44 $y = 3 - 2x$

The intercept-slope form of a straight line is $y = a + bx$, where a is the y-intercept and b is the slope. Thus the y-intercept is 3 and the slope is -2. **C**

45 $2x - 3y = 5$

$7x + 8y = 3$

Use a CAS calculator to solve the simultaneous equations.

$x = 1.32,\ y = -0.78$ **D**

46 Let n be the number.

$$\begin{aligned} 5 \times n + 71 &= 101 \\ 5n + 71 &= 101 \\ 5n + 71 - 71 &= 101 - 71 \\ 5n &= 30 \\ \therefore n &= 6 \end{aligned}$$

B

47 $(3, -1)$ and $(4, 1)$

$$\begin{aligned} \text{slope} &= \frac{1 - (-1)}{4 - 3} \\ &= \frac{2}{1} = 2 \end{aligned}$$

D

48 $y = a + bx$

$a = y\text{-intercept} = -3$

For b = slope, select two points on the graph. Choose $(0, -3)$ and $(2, 0)$.

$$\text{slope} = \frac{0 - (-3)}{2 - 0} = \frac{3}{2} = 1.5$$

Thus $y = -3 + 1.5x$.

This is the same as $y = 1.5x - 3$.

D

49 The y-intercept (a) is 7 and the slope (b) is -2.

The only graphs that have a y-intercept of 7 and a negative gradient are **A** and **B**. Use two points on each to find the slope.

For graph **A**, use $(0, 7)$ and $(3.5, 0)$.

$$\text{slope} = \frac{0 - 7}{3.5 - 0} = \frac{-7}{3.5} = -2$$

For graph **B**, use $(0, 7)$ and $(2, 0)$.

$$\text{slope} = \frac{0 - 7}{2 - 0} = \frac{-7}{2} = -3.5$$

Graph **A** has a slope of -2 and a y-intercept of 7. **A**

50 $3x + 6y = 60$

$x + 9y = 69$

Use a CAS calculator to solve the simultaneous equations.

$x = 6, y = 7$ **B**

51 $(4, 0)$ and $(0, 7)$

$$\text{slope} = \frac{7 - 0}{0 - 4} = \frac{7}{-4} = -\frac{7}{4}$$

B

52 Let s = number of soccerballs and b = number of basketballs.

$$s + b = 8$$
$$15s + 30b = 165$$

Use CAS calculator to solve the simultaneous equations.

$s = 5,\ b = 3$

So the number of basketballs is 3.

C

53 Each postcard cost 70 cents to make and sells for \$3.50.

Thus the profit made on each is \$3.50 − 0.70 = \$2.80.

Sebastian sells 62 cards giving a profit of 62 × \$3.50 = \$173.60.

However, Sebastian's overall profit is \$153.60. So the fixed cost for making a batch of postcards is \$173.60 − \$153.60 = \$20. **A**

54 $(6, 4)$ and $(-2, -8)$

$$\text{slope} = \frac{-8-4}{-2-6} = \frac{-12}{-8} = \frac{3}{2}$$

By observation, the point $(4, 0)$ cannot be on the line nor can the points $(5, 3)$ or $(15, 15)$. This leaves options **C** and **D** to check which one gives a slope of $\frac{3}{2}$ with the point $(6, 4)$ (or you can use the point $(-2, -8)$).

Try the point $(7, 5)$ with $(6, 4)$.

$$\text{slope} = \frac{4-5}{6-7} = \frac{-1}{-1} = 1$$

Try the point $(12, 13)$ with $(6, 4)$.

$$\text{slope} = \frac{4-13}{6-12} = \frac{-9}{-6} = \frac{3}{2}$$

Thus the point $(12, 13)$ is on the line. **D**

55 The slope of the line is $\frac{3}{2}$.

Use $(6, 4)$ and $(p, 0)$

$$\frac{3}{2} = \frac{0-4}{p-6}$$
$$\frac{3}{2} = \frac{-4}{p-6}$$
$$3 = \frac{-4 \times 2}{p-6}$$
$$3(p-6) = -8$$
$$3p - 18 = -8$$
$$3p = 10$$
$$\therefore p = \frac{10}{3}$$

E

Solutions to Written-response questions

Chapter 2: Investigating and comparing data distributions

1 a *General Happiness* is a categorical variable.

b

General Happiness	Frequency	
	Number	%
Very happy	53	26.5
Pretty happy	200 − (53 + 29) = 118	59.0
Not too happy	29	$\frac{29}{100} \times 100 = 14.5$
Total	200	100.0

c Follow the steps in Example 6 to construct a percentage bar chart.

d A group of 200 people was asked to describe their general happiness by choosing one of the responses very happy, pretty happy, not too happy. The highest level of response was pretty happy, chosen by 59.0% of the people. Another 26.5% of people chose very happy. Only 14.5% of people said they were not too happy.

2 a Follow the steps in Example 14 to construct a dot plot of the data.

b Enter the data into you calculator and follow the instructions on page 90 (TI) or 91 (CASIO) to find the summary statistics for the data set.

i range = 10

ii median = 1

iii $IQR = 3$

c Number of 0 rain days = 10 ⇒ percentage $= \dfrac{10}{31} \times 100 = 32.2\%$

d Follow the instructions on page 58 (TI) or 61 (CASIO) to construct a histogram using your calculator.

3 a i Follow the steps in Example 24 to construct a simple boxplot.

ii $IQR = 92\,000 - 49\,000 = 43\,000$
lower fence $= 49\,000 - 1.5 \times 43\,000 = -\$15\,500$
upper fence $= 92\,000 + 1.5 \times 43\,000 = \$156\,500$

iii $Q_1 = \$49\,000 \Rightarrow 75\%$ earned more than this.

b i Read from the boxplots: 2016 median = $74 000, 2020 median = $82 000.

ii The median salary in 2020 ($M = \$82\,000$) was higher than the median salary in 2016 ($M = \$74\,000$). The spread of salaries in 2020 ($IQR = \$25\,000$) was slightly higher than the spread in 2016 ($IQR = \$23\,000$). There was one outlier in 2016, who had a salary of $121 000 which was high compared to the rest of the Sales team. There were two outliers in 2020, who had high salaries of $131 000 and $155 000 respectively. In conclusion, the median salary has certainly increased over the four years from 2016 to 2020, while at the same time salaries have become more a little more variable.

4 a *Team* is a categorical variable.

b *Score*

Team A		Team B
4 2 0 0	0	0 2 2 2 2 4
9 8 8 7 6	0	6 6
4 1 0	1	0 2 3 3
9 6	1	5
0	2	
	2	
	3	0 1

c Team A: Min = 1, $Q_1 = 4$, $M = 8$, $Q_3 = 14$, Max = 20.
Team B: Min = 1, $Q_1 = 2$, $M = 6$, $Q_3 = 13$, Max = 31.

d Team A: lower fence = −6, upper fence = 29, no outliers.
Team B: lower fence = −14.5, upper fence = 22.5. Outliers 30 and 31.

e The mean is higher because of the influence of the outliers.

Chapter 3: Sequences and finance

5 a Calculate:
$V_0 = 2000$
$V_1 = V_0 + 76 = 2000 + 76 = 2076$
$V_2 = V_1 + 76 = 2076 + 76 = 2152$
$V_3 = V_2 + 76 = 2152 + 76 = 2228$
Thus the values at the end of 1, 2 and 3 years are \$2076, \$2152 and \$2228.

b Using the CAS, repeatedly add 76 until the value first exceeds \$3000. Since 76 must be added 14 times, the value of the investment will first exceed \$3000 after 14 years and have a value of \$3064.

c If \$1500 is initially invested then $V_0 = 1500$. If interest accures at a rate of 15% then 1500 × 15% = \$90 is added each year so $V_{n+1} = V_n + 90$.

6 a i The total amount repaid is the original amount borrowed by \$750 per month for 10 years. 50 000 + 750 × 12 × 10 = \$140 000.

ii The equivalent interest rate per annum can be found by first finding the amount paid annually (750 × 12 = \$9000), then calculating:

$$\frac{900}{50\,000} \times 100 = 18\%$$

b i First calculate the monthly interest rate:

$$\frac{6.75}{12} = 0.5625\%$$

Then use the formula for 12 × 10 = 120 months:

$$1.005625^{120} \times 50\,000 = 98\,016.09$$

ii Solve $A^{120} \times 50\,000 = 140000$ for A gives $A = 1.0086170769$.
Subtracting 1 then multipying by 12 and 100 gives:
$0.0086170769 \times 12 \times 100 = 10.3\%$

7 a Calculate:
$V_0 = 2500$
$V_1 = V_0 - 400 = 2500 - 400 = 2100$
$V_2 = V_1 - 400 = 2100 - 400 = 1700$
$V_3 = V_2 - 400 = 1700 - 400 = 1300$
Thus the values at the end of 1, 2 and 3 years are \$2100, \$1700 and \$1300.

b Using your CAS, solve $1000 = 2500 - 400n$ for n to find how long it will take for the computer to reach a value of \$1000. This gives $n = 3.75$ so it will take 4 years.

c Since the computer was purchased for \$1800, $V_0 = 1800$. The computer depreciates by \$350 each year which means that the value declines by 350 each year so $V_{n+1} = V_n - 350$.

8 a i Since emissions reduce by 130 kilograms each day, we subtract 130 to find the required numbers to give the missing numbers as:
$1240 - 130 = 1110$, $1110 - 130 = 980$, $980 - 130 = 850$,
$850 - 130 = 720$, $720 - 130 = 590$, $590 - 130 = 460$

ii Since the starting value is 1500, $E_0 = 1500$. 130 is subtracted each day, $E_{i+1} = E_i - 130$.

iii At any point in time, we can subtract 130 for each day from the starting value of 1500. This means we can write the rule as $E_n = 1500 - 130n$.

b i Since the emissions need to decrease from 1500 to 200 in 5 days, this means that there needs to be a reduction of $1500 - 200 = 1300$ in total and so 260 each day.

ii A graph can be drawn by plotting the points 1500, 1240, 980, 720, 460, 200.

iii Since the starting value is 1500, $K_0 = 1500$. 260 is subtracted each day so $K_{n+1} = K_n - 260$.

iv 260 is subtracted for each of the n days from the original value of 1500. This means that $K_n = 1500 - 260n$.

9 i Since there were initially 360 000 rabbits and the population increases by

12 000 per week, the population will be 360 000 + 12 000 = 372 000 after one week.

ii The population increases by 12 000 each week for n weeks so we add 12 000n to the initial value of 360 000 to give $V_n = 360\,000 + 12\,000n$.

iii Use CAS to solve 600 000 = 360 000 + 12 000n for n to give $n = 20$ so it will take 20 weeks.

iv Here we want to start with 360 000 and after 40 weeks have 1 000 000 rabbits so we need to solve 1 000 000 = 360 000 + $R \times 40$ for R on the CAS. We find $R = 16\,000$ which means that the rabbit population must increase by 16 000 each week over the 40 weeks.

b **i** To show that the common ratio is 1.5, find the ratio of subsequent terms:
$\frac{768}{512} = 1.5, \quad \frac{1152}{768} = 1.5$

ii Since the common ratio is 1.5, we can work out the number of rabbits that die each week:
Week 1: 512
Week 2: 768
Week 3: 1152
Week 4: 1152 × 1.5 = 1728
Week 5: 1728 × 1.5 = 2592

iii To find the total number that die during the first six weeks we calculate the number that die each week and then add them together:
Week 6: 2592 × 1.5 = 3888
Total: 512 + 768 + 1152 + 1728 + 2592 + 3888 = 10 640

Chapter 4: Matrices

10 **a** To find how many kilometres Mary travelled last week, we add up the first row: 38 + 85 + 4 = 127

b To find the total distance that Mary and Derek ran last week we add up the first column (run): 38 + 12 = 50

c Mary cycles to and from work each day which means that she cycles 7 × 2 = 14 km each day or 14 × 5 = 70 from Monday to Friday. This leaves 85 − 70 = 15 km on the weekend.

11 **a** Read off Row 2, Column 1 to find that Shop B sold 45 Handrolls.

b The selling price of the Sashimi packs is found in matrix P in the second row:

$15.

c Use your CAS to calculate SP or perform the matrix by hand as follows.

$$SP = \begin{bmatrix} 60 \times 4 + 15 \times 15 + 8 \times 10 \\ 45 \times 4 + 21 \times 15 + 10 \times 10 \end{bmatrix} = \begin{bmatrix} 545 \\ 595 \end{bmatrix}$$

d Since we are multiplying how many packs each store sold by the price of the packs, matrix SP tells us the total revenue for each store. The top line tells us the revenue of store A and the second line tells us the revenue of store B.

e Based on **c** we can see that store B had the highest sales as they sold $595 worth of sushi.

Chapter 5: Linear relations and modelling

12 Let s be the number of standard boxes and d be the number of deluxe boxes.
Peaches: 12 kg of peaches were sold and each standard box had 1kg peaches and each deluxe box had 2 kg peaches. This gives the equation:
$s + 2d = 12$

Apples: 14 kg of apples were sold and each standard box had 2 kg of apples and each deluxe box had 1.5 kg apples. This gives the equation:
$2s + 1.5d = 14$

Use a CAS calculator to solve the simultaneous equations.
$s = 4,\ d = 4$
4 standard boxes and 4 deluxe boxes were sold.

13 **a** Fixed fee of $50 and $15 for each hour ($t$):
$C = 50 + 15t$

b y-intercept is 50.
Find another point on the graph. For example, let $t = 8$.
$C = 50 + 15(8) = 50 + 120 = 170$
Plot graph, using the points (0, 50) and (8, 120).

c $76.25 = 50 + 15t$

$26.25 = 15t$

$$t = \frac{26.25}{15} = 1.75$$

The gardener worked for 1.75 hours.

14 **a** Female TBW $= -2.097 + 0.1069 \times 175 + 0.2466 \times 62$

$= 31.8997$

$= 31.90$ litres correct to 2 dec. places

b Male TBW $= 2.447 - 0.09516 \times 45 + 0.1074 \times 184 + 0.3362 \times 87$

$= 47.1758$

$= 47.18$ litres correct to 2 dec. places

c $32 = -2.097 + 0.1069 \times h + 0.2466 \times 62$
Use Solve on CAS calculator to solve for h.
$h = 175.938$
$\therefore h = 176$ cm (to the nearest cm)

d Male TBW $= 2.447 - 0.09516 \times 78 + 0.1074 \times 174 + 0.3362 \times 80$

$= 40.608$

$= 40.61$ litres correct to 2 dec. places

e Male TBW $= 2.447 - 0.09516 \times 78 + 0.1074 \times 174 + 0.3362 \times 95$

$= 44.6512$

$= 45.65$ litres correct to 2 dec. places

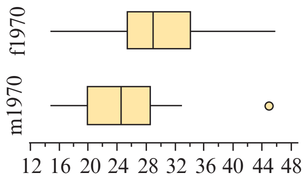
f Use the formula for Male TBW with age = 22 and height = 185.

W(kg)	60	65	70	75	80	85	90	95	100	105	110	115	120
TBW(litres)	40.39	42.08	43.76	45.44	47.12	48.80	50.48	52.16	53.84	55.52	57.20	58.89	60.57

Solutions to Investigations

Statistics investigation

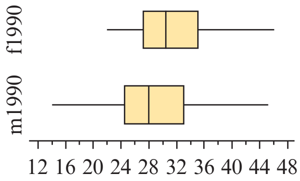
1 a i



ii mothers: M= 23.5, fathers: M = 29

iii In 1970, the median age for mothers ($M = 23.5$) was lower than that for fathers ($M = 29$). The spread of ages for the mothers ($IQR = 8.5$) was the same for fathers ($IQR = 9.0$). There was one outlier – a mother of age 45 years. This mother was much older than the remainder of the mothers.

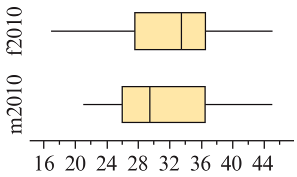
b i



ii mothers: M= 28, fathers: M=31

iii In 1990, the median age for mothers ($M = 28$) was still lower than the median age for fathers ($M = 31$). The spread of ages for the mothers ($IQR = 9.0$) was the same as the 1970 spread and was slightly higher than the spread for fathers ($IQR = 9.0$).

c **i**



ii mothers: M = 31, fathers: M=33.5

iii In 2010, the median age for mothers ($M = 31$) was again lower than the median age for fathers ($M = 33.5$). The spread of ages for the mothers ($IQR = 10.5$) was higher than the spread for fathers ($IQR = 9.5$).

d Report

The median age for mothers has increased steadily over the years, from 23.5 in 1970, to 28 in 1990 and 31 in 2010. The spread in ages for mothers was the same in 1970 ($IQR = 8.5$) and 1990 ($IQR = 9$), but increased in 2010 ($IQR = 10.5$). In 1970, a mother of age 45 was considered an outlier, but in 1990 and 2010, the age of 45 was not unusual enough to be an outlier.

The median age for fathers has increased steadily over the years, from 29 in 1970, to 31 in 1990 and to 33.5 in 2010. The spread in ages for fathers has remained reasonably steady during this time (1970: $IQR = 9$; 1990: $IQR = 9$; 2010: $IQR = 9.5$)

The difference in age between mothers and fathers has changed little over time. Fathers have typically been older than mothers by the same amount (1970: 5.5 y; 1990: 3 y; 2010: 4 y)

Matrices investigation - Encoding and decoding

3 **a** **i**

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} F & B & I & \\ K & N & O & W \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 6 & 2 & 9 & 27 \\ 11 & 14 & 15 & 23 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & 18 & 33 & 77 \\ 17 & 16 & 24 & 50 \end{bmatrix}$$

ii

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} M & A & P & \\ L & O & S & T \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 13 & 1 & 16 & 27 \\ 12 & 15 & 19 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 38 & 17 & 51 & 74 \\ 25 & 16 & 35 & 47 \end{bmatrix}$$

iii

$$\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}A & P & E & \\F & A & C & E\end{bmatrix}$$

$$= \begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 16 & 5 & 27\\6 & 1 & 3 & 5\end{bmatrix}$$

$$= \begin{bmatrix}8 & 33 & 13 & 59\\7 & 17 & 8 & 32\end{bmatrix}$$

iv

$$\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}F & I & N & D\\ & T & O & M\end{bmatrix}$$

$$= \begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}6 & 9 & 14 & 4\\27 & 20 & 15 & 13\end{bmatrix}$$

$$= \begin{bmatrix}39 & 38 & 43 & 21\\33 & 29 & 29 & 17\end{bmatrix}$$

v

$$\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}N & O & & G & U & A & R & D\\ & T & O & N & I & G & H & T\end{bmatrix}$$

$$= \begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}14 & 15 & 27 & 7 & 21 & 1 & 18 & 4\\27 & 20 & 15 & 14 & 9 & 7 & 8 & 20\end{bmatrix}$$

$$= \begin{bmatrix}55 & 50 & 69 & 28 & 51 & 9 & 44 & 28\\41 & 35 & 42 & 21 & 30 & 8 & 26 & 24\end{bmatrix}$$

vi

$$\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}M & E & E & T & & A & N & N\\A & T & & J & O & H & N & S\end{bmatrix}$$

$$= \begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}13 & 5 & 5 & 20 & 27 & 1 & 14 & 14\\1 & 20 & 27 & 10 & 15 & 8 & 14 & 19\end{bmatrix}$$

$$= \begin{bmatrix}27 & 30 & 37 & 50 & 69 & 10 & 42 & 47\\14 & 25 & 32 & 30 & 42 & 9 & 28 & 33\end{bmatrix}$$

b $a = 2, b = 1, c = 3, d = 2$

$\det(S) = ad - bc$

$= 2 \times 2 - 1 \times 3$

$= 4 - 3 = 1$

Inverse matrix is $\begin{bmatrix}2 & -1\\-3 & 2\end{bmatrix}$

i

$$\begin{bmatrix}2 & -1\\-3 & 2\end{bmatrix}\begin{bmatrix}10 & 27 & 33 & 62\\19 & 45 & 53 & 97\end{bmatrix}$$

$$= \begin{bmatrix}1 & 9 & 13 & 27\\8 & 9 & 7 & 8\end{bmatrix}$$

$$= \begin{bmatrix}A & I & M & \\H & I & G & H\end{bmatrix}$$

ii $$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 28 & 41 & 52 & 59 \\ 47 & 62 & 85 & 91 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & 20 & 19 & 27 \\ 10 & 1 & 14 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} I & T & S & \\ J & A & N & E \end{bmatrix}$$

iii $$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 20 & 39 & 63 & 59 \\ 34 & 66 & 101 & 91 \end{bmatrix}$$
$$= \begin{bmatrix} 6 & 12 & 25 & 27 \\ 8 & 15 & 13 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} F & L & Y & \\ H & O & M & E \end{bmatrix}$$

iv $$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 37 & 65 & 29 & 79 \\ 56 & 109 & 44 & 131 \end{bmatrix}$$
$$= \begin{bmatrix} 18 & 21 & 14 & 27 \\ 1 & 23 & 1 & 25 \end{bmatrix}$$
$$= \begin{bmatrix} R & U & N & \\ A & W & A & Y \end{bmatrix}$$

v $$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 22 & 45 & 48 & 67 & 73 & 15 & 36 & 59 \\ 40 & 75 & 82 & 114 & 119 & 23 & 57 & 91 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & 15 & 14 & 20 & 27 & 7 & 15 & 27 \\ 14 & 15 & 20 & 27 & 19 & 1 & 6 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} D & O & N & T & & G & O & \\ N & O & T & & S & A & F & E \end{bmatrix}$$

vi $$\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 26 & 51 & 30 & 35 & 58 & 23 & 39 & 58 \\ 46 & 84 & 55 & 66 & 89 & 37 & 59 & 89 \end{bmatrix}$$
$$= \begin{bmatrix} 6 & 18 & 5 & 4 & 27 & 9 & 19 & 27 \\ 14 & 15 & 20 & 27 & 4 & 5 & 1 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} F & R & E & D & & I & S & \\ N & O & T & & D & E & A & D \end{bmatrix}$$

c **i** $$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 & 4 & 7 & 2 & 3 & 8 & 1 \\ 6 & 0 & 5 & 8 & 9 & 3 & 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & 1 & 9 & 15 & 11 & 6 & 8 & 8 \\ 15 & 1 & 14 & 23 & 20 & 9 & 8 & 15 \end{bmatrix}$$

ii $a = 1, b = 1, c = 1, d = 2$

$\det(B) = ad - bc$

$= 1 \times 2 - 1 \times 1$

$= 2 - 1 = 1$

Inverse matrix is $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$.

$$\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 7 & 10 & 8 & 13 & 12 & 0 & 12 & 12 \\ 8 & 19 & 9 & 19 & 20 & 0 & 16 & 21 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & 7 & 7 & 4 & 0 & 8 & 3 \\ 1 & 9 & 1 & 6 & 8 & 0 & 4 & 9 \end{bmatrix}$$

d Answers will vary. Teacher to supervise, other students to check and decode.

e Answers will vary. Teacher to supervise, other students to check and decode.

Financial mathematics investigation

4 Answers will vary for this investigation. Students could begin by considering their likely starting salary for their chosen profession and then consider cost of living to ascertain how much they can afford to spend on a car. They can then research costs of cars and then different finance options offered by banks by looking online.
To work out depreciation, students can research the second hand market and work out how much different cars depreciate.
As an extention, they could look at the value of the car and the value of the remaining loan over time.

5 **a** Copy the values into a spreadsheet and then fill down until the value (A_n) is zero. You should have thirteen rows (Row 12 has a value of \$26 200 and Row 13 has a value of −\$8700.

b To include interest of 1.5%, create a new column with the formula:
$= \text{C2} + \text{C2} * 1.5/100 - 2400 - 2500 * \text{A3}$
You will now require 14 rows until you get a negative value.

c In order to pay the loan back in less than 15 years, use trial and error. You should find that the answer is around 3%

Linear relations and modelling investigation

6 Let S = small, M = medium and L = large.

$\frac{1}{4}$ hectare = 2500 m^2

$5S + 3M + 2L = 2500$
$M = S + 100$
$L = M + 200$

Substitute M and L into $5S + 3M + 2L = 2500$

$$5S + 3(S + 100) + 2(M + 200) = 2500$$

$$5S + 3S + 300 + 2M + 400 = 2500$$

$$8S + 700 + 2M = 2500$$

Substitute $M = S + 100$

$$8S + 700 + 2(S + 100) = 2500$$

$$8S + 700 + 2S + 200 = 2500$$

$$10S + 900 = 2500$$

$$10S = 1600$$

$$\therefore S = 160$$

$M = S + 100 = 160 + 100 = 260$

$L = M + 200 = 260 + 200 = 460$

Small block is 160 m^2, medium block is 260 m^2 and large block is 460 m^2.

Chapter 7 – Investigating relationships between two numerical variables

Solutions to NTT 7A

3 EV: *time on social media*
RV: *sleep*

4 Follow the steps in Example 4 to construct the scatterplot.

Solutions to Exercise 7A

1 **a** Definition based.
EV = *age*, RV = *diameter*

b Definition based.
EV = *weeks*, RV = *weight loss*

c Definition based.
EV = *age*, RV = *price*

d Definition based.
EV = *hours*, RV = *amount of gas*

e Definition based.
EV = *balls bowled*, RV = *runs*

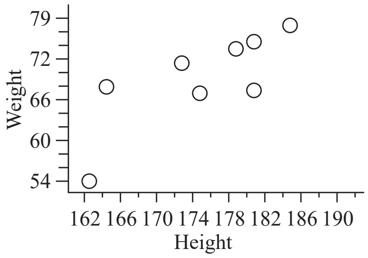
2 The EV and RV should be numerical variables, if a scatterplot were to be constructed. Alternatives **C** and **D** satisfy the criteria. However, it makes no sense to compare the *weights* of 12 oranges and *lengths* of 9 bananas. Thus, **D.**

3 **a** EV (horizontal axis) = *hours of sleep*, RV (vertical axis) = *number of mistakes*

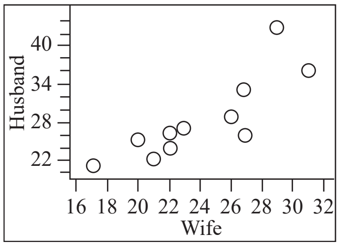
b Count the number of points on the scatterplot: 12 people.

c Read from scatterplot: 9 hours of sleep and 6 mistakes

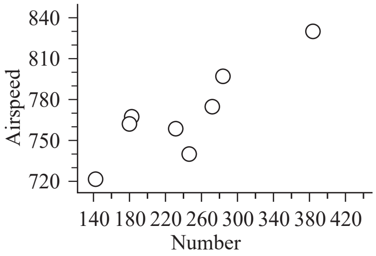
4 Enter the data into your calculator and follow the instructions on page 405 (TI) or 406(CASIO) to construct a scatterplot.



5 Enter the data into your calculator and follow the instructions on page 405 (TI) or 406(CASIO) to construct a scatterplot.

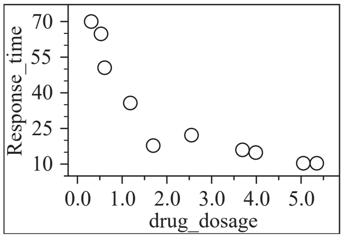


6 Enter the data into your calculator and follow the instructions on page 405 (TI) or 406(CASIO) to construct a scatterplot.



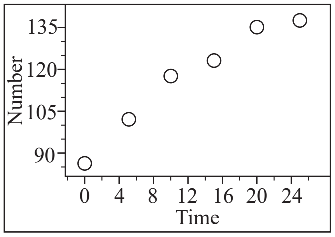
7 **a** The explanatory variable is the *drug dosages*

b



8 **a** The explanatory variable is *time*.

b



Solutions to NTT 7B

5 Use the scatterplots in Example 5 as a guide.

a downward trend = negative association

b no clear trend = no association

c upward trend = positive association

d upward trend = positive association

6 Use the wording in Example 6 as a guide.

a negative: "Temperature tends to be lower at greater heights above sea level"

b positive: "People who spend more years studying tend to earn higher salaries"

7 Use the scatterplots in Example 7 as a guide.

8 Use the scatterplots in Example 8 as a guide.

Solutions to Exercise 7B

1 Use the scatterplots in Example 5 as a guide.

2 Use the scatterplots in Example 7 as a guide.

3 Use the scatterplots in Example 8 as a guide.

4 **a** Positive. People who exercise more tend to have higher levels of fitness.

b Negative. People who run faster in a marathon tend to take less time to complete the run.

5 **a** Strong positive linear association. Those who spent more time studying for the exam tended to get higher marks.

b Strong negative linear association. The price of older cars tended to be lower as the age of the cars increased.

c Moderate positive linear association. Daughters from taller mothers also tended to be taller.

d Strong positive non-linear association. Performance level tended to increase with time spent practising for the first eight hours, after which performance appeared to level off.

e Moderate negative linear association. Students tended to score lower on a test when the temperature was higher.

f Strong positive linear association. Older husbands tended to have older wives.

6 **a** Enter the data into your calculator and follow the instructions on page 405 (TI) or 406 (CASIO) to construct a scatterplot.

b There is a strong, positive, non-linear association between the diameter of a circle and its area.

c Create the new variable and then follow the instructions on page 405 (TI) or 406 (CASIO) to construct a scatterplot.

d There is a strong, positive, linear association between the square of the diameter of a circle and its area.

e The affect has been to change the form of the association from non-linear to linear.

Solutions to NTT 7C

9 Use the scatterplots on page 420 as a guide.

10 Follow the calculator instructions on page 423 (TI) or 424 (CASIO) to calculate the value of *r*.

11 Using the table on page

a $0.75 \leq 0.807 \leq 1 \Rightarrow$ strong, positive

b $-1 \leq -0.818 \leq 0.75 \Rightarrow$ strong, negative

c $-0.25 \leq 0.224 \leq 0.25 \Rightarrow$ no association

d $-0.75 \leq -0.667 \leq -0.25 \Rightarrow$ moderate, negative

12 No we cannot say its is causal.

Solutions to Exercise 7C

1 The data should be linearly related and numerical.

2 Using the reference plots on pg. 420

a Visual comparison makes it look most like $r \approx 0.9$.

b Looks a bit stronger than the -0.501 plot, maybe $r \approx -0.6$

c Looks a bit stronger than the 0.767 plot, maybe $r \approx 0.8$

d Looks a bit similar in strength to the -0.767 plot, but positive, maybe $r \approx 0.7$

3 Using the reference plots on pg. 420

a Visual comparison makes it look most like $r \approx 0.9$.

b The plots still exhibit a positive linear relationship, but not as strong as (a), therefore it is $r \approx 0.7$

c The plots slope roughly down, showing a moderate linear relationship, therefore it is the bottom middle box. $r \approx -0.6$

d Plots are widely spread, showing no associations.Therefore $r \approx 0$

4 Follow the calculator instructions on page 423 (TI) or 424 (CASIO) to calculate the value of r.

5 Follow the calculator instructions on page 423 (TI) or 424 (CASIO) to calculate the value of r.

6 Follow the calculator instructions on page 423 (TI) or 424 (CASIO) to calculate the value of r.

7 Follow the calculator instructions on page 423 (TI) or 424 (CASIO) to calculate the value of r.

8 **a** $r = 0.250$, no linear relationship

b $r = -0.305$, weak negative linear relationship

c $r = -0.851$, strong negative linear relationship

d $r = 0.333$, weak positive linear relationship

e $r = 0.952$, strong positive linear relationship

f $r = -0.740$, moderate negative linear relationship

g $r = 0.659$, moderate positive linear relationship

h $r = -0.240$, no linear relationship

i $r = -0.484$, weak moderate linear relationship

j $r = 0.292$, weak positive linear relationship

k $r = 1$, perfect positive linear relationship

l $r = -1$, perfect negative linear relationship

9 Since the number of bars and the number of school teachers will both increase with the size of the city, then size of the city is likely to explain this correlation. This would be an example of a common response.

10 No; possible confounding variables include the general wealth of a country, level of post-natal care, and literary rates.

11 **a** EV: *hours* or the number of hours spent gambling
RV: *amount* or amount spent gambling

b Enter the data into your calculator and follow the instructions on page 405 (TI) or 406(CASIO) to construct a scatterplot.

c Follow the calculator instructions on page 423 (TI) or 424 (CASIO) to calculate the value of $r = 0.922$

d Strong positive linear association: Those who gambled for longer tended to spend more on gambling.

12 **a** Either variable could be the EV.

b Enter the data into your calculator and follow the instructions on page 405 (TI) or 406(CASIO) to construct a scatterplot.

c Follow the calculator instructions on page 423 (TI) or 424 (CASIO) to calculate the value of $r = 0.894$

d Strong positive linear association with an outlier(Other Oceania countries). Those countries with high percentages of males with eye disease also tended to have a high percentage of females with eye disease.

13 **a** **i** Enter the data into your calculator and follow the instructions on page 405 (TI) or 406(CASIO) to construct a scatterplot.

ii Follow the calculator instructions on page 423 (TI) or 424 (CASIO) to calculate the value of $r = 0.297$

b **i** Enter the data into your calculator and follow the instructions on page 405 (TI) or 406(CASIO) to construct a scatterplot.

ii Follow the calculator instructions on page 423 (TI) or 424 (CASIO) to calculate the value of $r = 0.411$

c **i** Enter the data into your calculator and follow the instructions on page 405 (TI) or 406(CASIO) to construct a scatterplot.

ii Follow the calculator instructions on page 423 (TI) or 424 (CASIO) to calculate the value of $r = 0.954$

d When educational level is not taken into consideration there is only a weak correlation between salary and years of employment. However, when the two groups are considered separately there is a moderate correlation between salary and years of employment for those with a secondary education, and a strong correlation between salary and years of employment for those with a tertiary education. For this group, those with higher years of employment did tend to earn higher salaries.

Solutions to NTT 7D

13 **a** *exercise*=0 is the y-intercept: *breath*=20

b The lines passes through the points (0, 20) and (8,40) ⇒ slope=$\frac{20}{8} = 2.5$

c Using the rule $y = a + bx \Rightarrow \mathit{breath} = 20 + 2.5 \times \mathit{exercise}$

14 There are many possible points that could be chosen. The line passes through (70, 68) and (90, 84).

$b = \frac{rise}{run} = \frac{84 - 68}{90 - 70} = \frac{16}{20} = 0.8$

Using the point (70, 68)

$\therefore 68 = a + 0.8 \times 70 \Rightarrow a = 68 - 56 = 12$

$\therefore y = 12 + 0.8 \times x \Rightarrow weight = 12 + 0.8 \times weeks$

15 $b = r \times \frac{s_y}{s_x} = 0.700 \times 3.402.30 = 1.035$

$a = \bar{y} - b\bar{x} = 24.5 - 1.035 \times 15.2 = 8.768$

$\therefore y = 8.77 + 1.03x$

Solutions to Exercise 7D

1 **a** yes, the variables are both numerical and the association is linear.

b no, the variables are both numerical but the association is non-linear.

2 **a** The line passes through (0, 20) and (20, 80)

$a = y - intercept = 20$

$$b = \frac{rise}{run} = \frac{80 - 20}{20 - 0} = \frac{60}{20} = 3$$

$\therefore y = 20.0 + 3.00x$

b The line passes through (0, 50) and (20, 32)

$a = y - intercept = 50$

$$b = \frac{rise}{run} = \frac{32 - 50}{20 - 0} = \frac{-18}{20} = -0.9$$

$\therefore y = 50.0 - 0.900x$

3 The line passes through (20, 10) and (80, 70)

$$b = \frac{rise}{run} = \frac{70 - 10}{80 - 20} = \frac{60}{60} = 1.0$$

Using the point (20, 10)

$\therefore 10 = a + 1.0 \times 20 \Rightarrow a = 10 - 20 = -10$

$\therefore y = -10 + 1.0x$

4 Enter the data into your calculator and follow the instructions on page 437 (TI) or 438 (CASIO) to find $r = 0.818$.

5 The line passes through (750, 100) and (3500, 325)

$$b = \frac{rise}{run} = \frac{325 - 100}{3500 - 750} = \frac{225}{2750} = 0.082$$

Using the point (750, 100)

$\therefore 100 = a + 0.08 \times 750 \Rightarrow a = 100 - 60 = 40$

$\therefore y = 40 + 0.08x$

$\therefore time = 40 + 0.08 \times distance$

6 The line passes through (0, 160) and (25, 120)

$a = y - intercept = 160$

$b = \frac{rise}{run} = \frac{120 - 160}{25 - 0} = \frac{-40}{25} = -1.6$

$\therefore y = 160.0 - 1.6x$

$\therefore$ *infant mortality rate* = 160 − 1.6× *female literacy rate*

7 The line passes through (0, 50) and (15, 90)

$a = y - intercept = 50.0$

$b = \frac{rise}{run} = \frac{90 - 50}{15 - 0} = \frac{40}{15} = 2.67$

$\therefore y = 50.0 + 2.7x$

$\therefore$ *Heart weight* =50 + 2.7× *Body weight*

8 The line passes through (5, 6) and (11, 3)

$b = \frac{rise}{run} = \frac{3 - 6}{11 - 5} = \frac{-3}{6} = -0.50$

Using the point (5, 6)

$\therefore 6 = a - 0.50 \times 5 \Rightarrow a = 6 + 2.5 = 8.5$

$\therefore y = 8.5 - 0.50x$

$\therefore$ *number of errors* =8.5 − 0.50× *number of practice sessions*

9 The line passes through (17.5, 27) and (19.5, 30)

$b = \frac{rise}{run} = \frac{30 - 27}{19.5 - 17.5} = \frac{3}{2} = 1.5$

Using the point (17.5, 27)

$\therefore 27 = a + 1.5 \times 17.5 \Rightarrow a = 27 - 26.25 = 0.75$

$\therefore y = 0.75 + 1.5x$

$\therefore$ *forearm length* = 0.75 + 1.5×*wrist circumference*

10 $b = r \times \frac{s_y}{s_x} = 0.8 \times \frac{20.5}{6.8} = 2.412$

$a = \bar{y} - b\bar{x} = 123.5 - 2.412 \times 115.0 = -153.88$

$a = -153.88,\ b = 2.41$

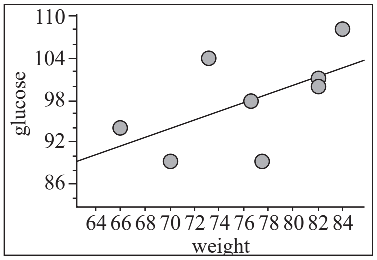
11 $b = r \times \frac{s_y}{s_x} = -0.6 \times \frac{1.70}{2.50} = -0.408$

$a = \bar{y} - b\bar{x} = 15.7 + 0.408 \times 18 = 23.044$

$a = 23.0,\ b = -0.408$

12

a, b Follow the instructions on page 405 (TI) or 406 (CASIO) to construct the scatterplot. Follow the instructions on page 437 (TI) or 438 (CASIO) fit a least squares lines to the data.

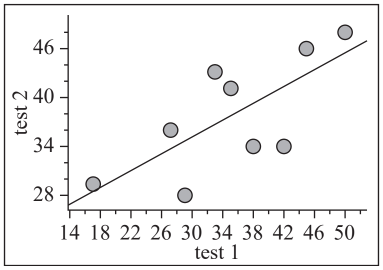


c Using CAS calculator: *glucose* = 50.8 + 0.616 × *weight*.

d Using CAS calculator: $r = 0.570$.

13

a, b Follow the instructions on page 405 (TI) or 406 (CASIO) to construct the scatterplot. Follow the instructions on page 437 (TI) or 438 (CASIO) fit a least squares lines to the data.

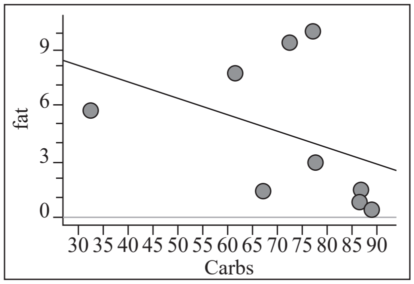


c Using CAS calculator: *test* 2 = 19.6 + 0.515 × *test 1*.

d Using CAS calculator: $r = 0.722$.

14

a, b Follow the instructions on page 405 (TI) or 406 (CASIO) to construct the scatterplot. Follow the instructions on page 437 (TI) or 438 (CASIO) fit a least squares lines to the data.

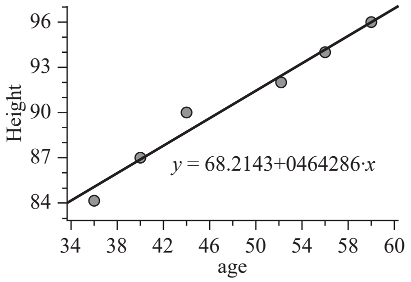


c Using CAS calculator: $fat = 10.7 - 0.088 \times carbs$.

d Using CAS calculator: $r = -0.396$.

15

a, b Follow the instructions on page 405 (TI) or 406 (CASIO) to construct the scatterplot. Follow the instructions on page 437 (TI) or 438 (CASIO) fit a least squares lines to the data.



c Using CAS calculator: $height = 68.2 + 0.464 \times age$.

d Using CAS calculator: $r = 0.985$.

a Follow the instructions on page 405 (TI) or 406 (CASIO) to construct the scatterplot.

b Follow the instructions on page 437 (TI) or 438 (CASIO) fit a least squares lines to the data.

c $arm\ span = 45.6 + 0.720 \times height$

d $r = 0.903$

16 Since $b = r \times \frac{s_y}{s_x}$ we can rearrange this equation to give $r = b \times \frac{s_x}{s_y}$
From the equation given we can see that $b = 0.312$
$\therefore r = 0.312 \times \frac{4.29}{1.69} = 0.792$

17 **a** Follow the instructions on page 405 (TI) or 406 (CASIO) to construct the scatterplot.

b Follow the instructions on page 437 (TI) or 438 (CASIO) fit a least squares lines to the data.
$a = -0.484$ b = 0.795

c $area = -0.484 + 0.795 \times diameter^2$

d $area = \frac{\pi}{4} \times diameter^2 = 0.785 \times diameter^2$

Solutions to NTT 7E

16 $chores = 8.0 - 0.30 \times work$

a **i** From the equations intercept = $a = 8.00$

ii On average, students who spend no time in part-time work will spend 8 hours doing household chores.

b **i** From the equations slope= $b = -0.3$

ii On average, the number of hours spent doing household chores decreases by 0.3 hours for each extra hour of spent in part time work.

17 $chores = 8.0 - 0.30 \times work$

a $work = 2 \Rightarrow chores = 8.0 - 0.30 \times 2 = 7.4$ hours, interpolating

b $work = 10 \Rightarrow chores = 8.0 - 0.30 \times 10 = 5$ hours, extrapolating

Solutions to Exercise 7E

1 **a** The EV is the x-variable which is *age*, The RV is the y-variable which is *height*

b From the equation $a = 69$ $b = 0.50$

c **i** 0.5 cm, 1 month

ii 0 months, 69 cm.

2 **a** By definition, interpolating

b By definition, extrapolating

3 **a** $48.1 + 2.20 \times 5 = 59$

b $48.1 + 2.20 \times 8 = 66$

4 **a** **i** The intercept is 37 650.

ii The price of the car, when new (time = 0), is predicted to be $37 650.

b **i** The slope is –4200.

ii The slope indicates that the price of a used car decreases by $4200 each year

c Let $age = 5$:
$price = 37650 - 4200 \times 5 = \$16650$

5 **a** **i** The intercept is 40.

ii The intercept indicates that the flavour rating of yoghurt with zero fat content is 40.

b **i** The slope is 2.0.

ii The flavour rating of a yoghurt increases by 2.0 for each 1% increase in fat content.

c Let $calories = 30$:
$flavour\ rating = 40 + 2.0 \times 30 = 100$

6 **a** $height = 72 + 0.4 \times age$

$age = 40$ months

$$\begin{aligned} height &= 72 + 0.4 \times 40 \\ &= 72 + 16 \\ &= 88 \text{ cm} \end{aligned}$$

This is interpolation, as the age falls within the range given (36 and 60 months).

b $age = 55$ months old

$$\begin{aligned} height &= 72 + 0.4 \times 55 \\ &= 72 + 22 \\ &= 94 \text{ cm} \end{aligned}$$

This is interpolation, as the age falls within the range given (36 and 60 months).

c $age = 70$ months old

$$\begin{aligned} height &= 72 + 0.4 \times 70 \\ &= 72 + 28 \\ &= 100 \text{ cm} \end{aligned}$$

This is extrapolation, as the age falls outside the range given (36 and 60 months).

7 **a** $cost = 175 + 5.8 \times number\ of\ meals$

$number\ of\ meals = 0$ (no meals)

$$\begin{aligned} cost &= 175 + 5.8 \times 0 \\ &= 175 + 0 \\ &= \$175 \end{aligned}$$

This is extrapolation, as the number of meals falls outside the range given (25 and 100 meals).

b $number\ of\ meals = 60$

$$\begin{aligned} cost &= 175 + 5.8 \times 60 \\ &= 175 + 348 \\ &= \$523 \end{aligned}$$

This is interpolation, as the number of meals falls within the range given (25 and 100 meals).

c $number\ of\ meals = 89$

$$\begin{aligned} cost &= 175 + 5.8 \times 89 \\ &= 175 + 516.2 \\ &= \$691.20 \end{aligned}$$

This is interpolation, as the number of meals falls within the range given (25 and 100 meals).

8 **a** *daughter's height* = 18.3 + 0.91× *mother's height*

mother's height = 168 cm

daughter's height = 18.3 + 0.91 × 168

= 18.3 + 152.88

= 171.18 cm

This is interpolation, as the height falls within the range given (150 and 180 cm).

b *mother's height* = 196

daughter's height = 18.3 + 0.91 × 196

= 18.3 + 178.36

= 196.66 cm

This is extrapolation, as the height falls within the range given (150 and 180 cm).

c *mother's height* = 155 cm

daughter's height = 18.3 + 0.91 × 155

= 18.3 + 141.05

= 159.35 cm

This is interpolation, as the height falls within the range given (150 and 180 cm).

9 **a** **i** The intercept is 15.7.

ii The intercept predicts that students who obtained a zero mark in exam 1 obtained a mark of 15.7 for exam 2.

b **i** The slope is 0.650.

ii The slope indicates that exam 2 marks increase by 0.650 for each additional mark obtained on exam 1.

c Let *exam 1* = 20:

exam 2 = 15.7 + 0.650 ×20 = 29

10 **a** **i** The intercept is 51.

ii The intercept does not have a meaningful interpretation, as it is impossible for an adult to have a weight of zero.

b **i** The slope is 0.62.

ii The slope indicates that the blood glucose level of an adult increases by 0.62 mg/100 mL for each kilogram increase in weight.

c Let *weight* = 75:
glucose = 51 + 0.62 × 75
= 97.5 mg/100mL

11 We first need to decide which is the EV and which is the RV. Since we think that student score would depend on the staff:student ratio then
EV=*staff:student ratio* =*x*
RV=*student score* = *y*

$$b = r \times \frac{s_y}{s_x} = -0.651 \times \frac{12.013}{4.128} = -1.894$$

$$a = \bar{y} - b\bar{x} = 71.669 + 1.894 \times 13.404 = 97.063$$

$\therefore$ *student score* = 97.063 − 1.894× *staff:student ratio*
When *staff:student ratio*=15, *student score* = 97.063 − 1.894 × 15 = 68.7

12 **a** **i** *salary 000's* = 55.916 + 0.840× *years employed*

ii Intercept: On average, new employees with secondary level education are paid $55,916 per year.
Slope: On average, the salary for employees with secondary level education increases by $840 per year.

iii When *years employed*=5, *salary 000s* = 55.916 + 0.840 × 5
= 60.116 = $60 100 to the nearest $100.

b **i** *salary* = 57.956 + 3.361× *years employed*

ii Intercept: On average, new employees with tertiary level education are paid $57,956 per year. Slope: On average, the salary for employees with secondary level education increases by $3361 per year.

iii When *years employed*=5, *salary 000s* = 57.956 + 3.361 × 5
= 74.761 = $74 800 to the nearest $100.

c Employees with secondary and tertiary levels of education are paid on average similar salaries when they commence employment. However, the salaries for those with tertiary level education increase much more each year on average than those of employees with secondary level education.

d Both are interpolating.

Solutions to Skills Check Questions

1 See Examples 1, 2 and 3

2 Follow the instructions on page 405 (TI) or 406 (CASIO) to construct the scatterplot.

3 See Examples 5, 7 and 9. There is a moderate, positive, linear relationship between a person's height and their foot length.

4 See plots on page 420 for comparison. $r \approx 0.7$
Follow the instructions on page 423 (TI) or 424 (CASIO) to calculate $r = 0.657$

5 Use the table on page 425 to classify the value of r as moderate

6 No, we can only say that increasing sales of umbrellas are associated with increasing number of traffic accidents.

7 The line passes through (0, 10 000) and (10, 60 000)

$$a = 10\,000 \; b = \frac{rise}{run} = \frac{60\,000 - 10\,000}{10 - 0} = \frac{50\,000}{10} = 5000$$

$$\therefore y = 10\,000 + 5000 \times x \Rightarrow kms = 10,000 + 5000\times \textit{ age of car}$$

8 Enter the data from Skills Check Question 2 into your calculator and follow the instructions on page 437 (TI) or 438 (CASIO) to calculate the regression equation.
$\textit{foot length} = -31.27 + 0.34\times \textit{ height}$

9 Intercept: -31.27. This has no meaningful interpretation. Slope: On average feet are is 0.34cm longer for each additional 1 cm increase in height.

10 **a** 23.1cm, interpolating.

b 2.7cm, extrapolating.

Solutions to Chapter Review Multiple-Choice Questions

1 Definition based. Both variables must be numerical. **D**

2 Definition based. Strong negative. **B**

3 Definition based. Weak positive. **D**

4 Definition based. Strong positive. **B**

5 Definition based. No relationship. **C**

6 Definition based. Strong, non-linear. **E**

7 Negatively related means that as one variable tends to increase, the other tends to decrease.
If infant mortality tends to decrease, then birth weight tends to increase. **D**

8 For **A** it is quadratic, **C** is perfectly linear with an outlier, and **D** is exponential. Therefore only **E** is appropriate to calculate the value r. **E**

9 There is a strong positive correlation however it is not perfect. **D**

10 There is a strong negative correlation however it is not perfect. **A**

11 Definition based. $r = -0.32$.
Correlation is 'weak negative' when $-0.25 \le r < -0.5$ **B**

12 Intercept $a = 110$

slope $b = \dfrac{-110}{100} = -1.1$

$\Rightarrow y = 110 - 1.10x$ **A**

13 $b = r \times \dfrac{s_y}{s_x} = 0.500 \times \dfrac{12.5}{2.40} = 2.604$

$a = \bar{y} - b \times \bar{x} = 13.668$

$\Rightarrow y = 13.7 - 2.60x$ **D**

14 Use CAS calculator: $r = 0.8077$. **E**

15 Use CAS calculator:
expenditure = -42.84 +0.482 × *income* **C**

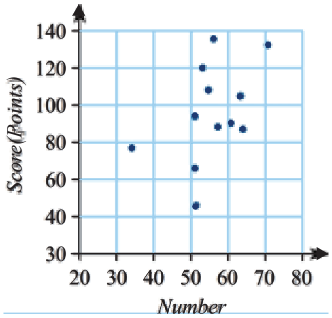
16 *expenditure* = 40 +0.1 × 600
= $100 **C**

17 As the gradient is 0.10, for every extra $1 earned, a person will on average spend 10 cents more on entertainment. **B**

Solutions to Chapter Review Short-Answer Questions

1 a The response variable is *score*.

b



c The scatterplot suggests a weak, positive, linear association between the variables *number* and *score*.

2 a The EV is *serves* and the RV is *cost*.

b Using the points (0,500) and (20, 900)

Intercept $a = 500$

$$\text{slope } b = \frac{900 - 500}{20 - 0} = \frac{400}{20} = 20.0$$

$\Rightarrow$ $\mathit{cost} = 500 + 20.0 \times \mathit{serves}$

3 a Using CAS calculator, $r = 0.9271$.

b Using CAS calculator,

$\mathit{distance} = 3.50 + 0.553 \times \mathit{time}$.

4 height = EV $= x$, weight = RV $= y$

$$b = r \times \frac{s_y}{s_x} = 0.75 \times \frac{10.8}{9.3} = 0.871$$

$$a = \bar{y} - b \times \bar{x} = 65.9 - 0.871 \times 174.5$$

$$= -86.1$$

$$\Rightarrow y = -86.1 + 0.871x$$

5 a The explanatory variable is *magnesium content*.

b The slope of the regression line is 7.3. This indicates that the *taste score* tends to increase by 7.3 for each time the *magnesium content* increases by 1 mg/litre.

c Let *magnesium content* = 16:

$\mathit{tast\ score} = -22 + 7.3 \times 16$

$= 94.8$ mg/L

6 a Using CAS calculator,

$\mathit{errors} = 14.9 - 0.533 \times \mathit{time}$

b Using CAS calculator, $r = -0.94$.

Solutions to extended-response questions

1 **a** EV: *played*
RV: *weekly sales*

b Follow the instruction pn page 405 (TI) or page 406 (CASIO) to construct the scatterplot.

c Use CAS calculator: $r = 0.9458$ to 4 decimal places

d From the scatterplot in part **b**, the strength is strong, the form is linear and the direction is positive.

e Using CAS calculator:
$\textit{weekly sales} = 293 + 74.3 \times \textit{played}$

f The slope is 74.3. This indicates that the number of downloads increases, on average, by 74.3 for each additional time the song is played on the radio in the previous week.
The intercept is 293. This indicates that 293 downloads of a song are predicted if it is not played on the radio in the previous week.

g Let *played* = 100:

$$\begin{aligned}\textit{weekly sales} &= 293 + 74.3 \times 100\\ &= 7732\end{aligned}$$

h The value substituted (100) is outside of the range of data supplied, so we are extrapolating.

2 **a** EV: *hours*
RV: *score*

b Follow the instruction on page 405 (TI) or page 406 (CASIO) to construct the scatterplot.

c Using CAS calculator, $r = 0.9375$.

d From the scatterplot in part **b**, the strength is strong, the form is linear and the direction is positive.

e Using CAS calculator:
$\textit{score} = 12.3 + 0.930 \times \textit{hours}$

f The slope is 0.930. This indicates that the test score of learner drivers, on average,

increases by 0.93 marks when instruction time increases by one hour.
The intercept is 12.3. This indicates that the test scores of learner drivers who received no instruction prior to taking the test received a score of 12.3 marks.

g Let *hours* = 10:
$score = 12.3 + 0.930 \times 10$
$= 22$

3 **a** Using the points (0,25) and (80,80)
Intercept $a = 25.0$
slope $b = \frac{80 - 25}{80 - 0} = \frac{55}{80} = 0.688$ *exam mark*= 25 + 0.69× *assignment mark*

b The slope is 0.69. This indicates that, on average, exam marks increase by 0.7 for each additional mark obtained on the assignment.
The intercept is 25. This indicates that those who scored 0 on the assignment scored 25 marks on the final exam (or equivalent)

c Let *assignment mark* = 50:
$exam\ mark = 25 + 0.69 \times 50$
$= 60$

d The prediction falls well within the range of data provided (i.e. interpolation). This suggests that the prediction is reliable.

e No we can only say that students who scored well on the assignment also tended to score well on the exam.

Chapter 8 – Graphs and networks

Solutions to 8A Now Try This Questions

1 a The graph has 4 points (A, B, C and D), therefore the graph has 4 vertices.

b The graph has 6 lines that connect the points, therefore the graph has 6 edges.

c Vertex A has two edges attached to it, therefore deg(A)=2

d Vertex D has three edges attached to it (two connecting to other vertices and one loop that connects the vertex to itself). The loop contributes two degrees to a vertex, therefore deg(D)=4

e *Two methods are possible for determining the sum of degrees for a graph.*
Method 1: Double the total number of edges for this graph. This graph has a total of 6 edges, therefore the sum of degrees for this graph is 12 ($6 \times 2 = 12$).
Method 2: Find the degree of each vertex and then find the sum. For this graph deg(A)=2, deg(B)=4, deg(C)=2 and deg(D)=4, therefore the sum of degrees for this graph is 12 ($2 + 4 + 2 + 4 = 12$).

Solutions to Exercise 8A

1 a i The graph has 5 points (A, B, C, D and E), therefore the graph has 5 vertices.

ii The graph has 6 lines that connect the points, therefore the graph has 6 edges.

iii The graph does not have any edges that connect a vertex to itself, therefore the graph has 0 loops.

iv Vertex A has two edges attached to it, therefore deg(A)=2.

v Vertex E has three edges attached to it, therefore deg(E)=3.

vi Determine the degree of each vertex; deg(A)=2, deg(B)=3, deg(C)=3, deg(D)=1 and deg(E)=3. Of all the vertices in the graph, vertices B, C, D and E have an odd degree (odd number of edges attached to them), therefore the graph has 4 odd vertices.

vii From the list above, only vertex A had an even degree (even number of edges attached to it), therefore the graph has 1 even vertex.

b i The graph has 4 points (A, B, C and D), therefore the graph has 4 vertices.

ii The graph has 6 lines that connect the points and 1 line that connects vertex *B* to itself, therefore the graph has 7 edges.

iii The graph has 1 edge that connects a vertex to itself, therefore the graph has 1 loop.

iv Vertex *B* has 4 edges connecting it to other vertices and 1 edge connecting it to itself (this loop will contribute 2 degrees to a vertex), therefore deg(*B*)=6.

v Vertex *D* has 2 edges attached to it, therefore deg(*D*)=2.

vi Determine the degree of each vertex; deg(*A*)=3, deg(*B*)=6, deg(*C*)=3 and deg(*D*)=2. Of all the vertices in the graph, vertices *A* and *C* have an odd degree (odd number of edges attached to them), therefore the graph has 2 odd vertices.

vii From the list above, vertices *B* and *D* have an even degree (even number of edges attached to them), therefore the graph has 2 even vertices.

c **i** The graph has 4 points (*A*, *B*, *C* and *D*), therefore the graph has 4 vertices.

ii The graph has 7 lines that connect the points, therefore the graph has 7 edges.

iii The graph does not have any edges that connect a vertex to itself, therefore the graph has 0 loops.

iv Vertex *A* has 3 edges connecting it to other vertices, therefore deg(*A*)=3.

v Vertex *C* has 3 edges attached to it, therefore deg(*C*)=3.

vi Determine the degree of each vertex; deg(*A*)=3, deg(*B*)=3, deg(*C*)=4 and deg(*D*)=4. Of all the vertices in the graph, vertices *A* and *B* have an odd degree (odd number of edges attached to them), therefore the graph has 2 odd vertices.

vii From the list above, vertices *C* and *D* have an even degree (even number of edges attached to them), therefore the graph has 2 even vertices.

d **i** The graph has 8 points (*A*, *B*, *C*, *D*, *E*, *F*, *G* and *H*), therefore the graph has 8 vertices.

ii The graph has 12 lines that connect the points and 2 lines that connect two vertices to themselves, therefore the graph has 14 edges.

iii The graph has 2 edges that connect a vertex to itself, therefore the graph has 2 loops.

iv Vertex *C* has 3 edges connecting it to other vertices and 1 edge connecting it to itself (this loop will contribute 2 degrees to a vertex), therefore deg(*C*)=5.

v Vertex *F* has 3 edges attached to it, therefore deg(*F*)=3.

vi Determine the degree of each vertex; deg(*A*)=3, deg(*B*)=3, deg(C)=5, deg(D)=5, deg(E)=3, deg(F)=3, deg(G)=3 and deg(H)=3. All vertices in the graph have an odd degree (odd number of edges attached to them), therefore the graph has 8 odd vertices.

vii From the list above, no vertices have an even degree (even number of edges attached to them), therefore the graph has 0 even vertices.

2 The sum of the degrees of a graph
= 2 × the total number of edges

a $2 \times 5 = 10$

b $2 \times 3 = 6$

c $2 \times 1 = 2$

3 Yes, as each edge must start and end at a vertex. Each edge contributes two degrees overall to a graph, because it is connected to two vertices (or one vertex twice in the case of a loop); therefore the sum of degrees would be twice the number of edges.

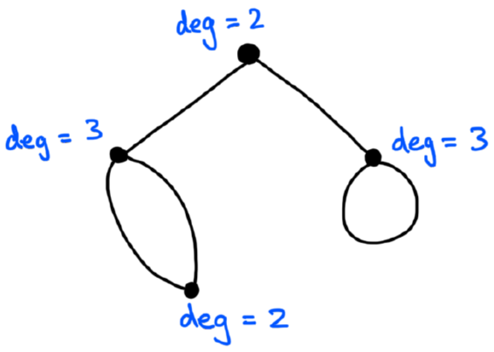
4 a Originally deg(A)=2. A loop contributes 2 to the degree of a vertex. Adding a loop to vertex A will increase the degree by 2, so now deg(A)=4.

b Adding 1 loop to the graph adds 1 edge to the graph. Although the loop contributes 2 to the degree of a vertex, it is only 1 line, increasing the total number of edges for the graph by 1.

5 The total number of edges for this graph is 7.
The sum of the degrees of a graph
= 2 × the total number of edges
$= 2 \times 7 = 14$
B

6 The sum of the degrees of a graph
= 2 × the total number of edges
For Graph 1:
Sum = $2 \times 8 = 16$
For Graph 2:
Sum = $2 \times 8 = 16$
C

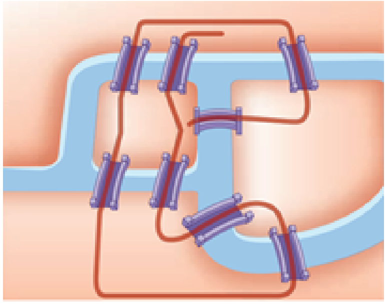
7



From the annotated graph above, it can be seen that two vertices have an odd degree. ***Note: a loop contributes 2 to the degree of a vertex and multiple edges have no unique impact on the degree of a vertex.***
C

8 A loop is an edge that connects a vertex to itself. In this graph there are multiple edges that may appear as loops, however each edge connects one vertex to another; there are 6 different edges in this graph.
B

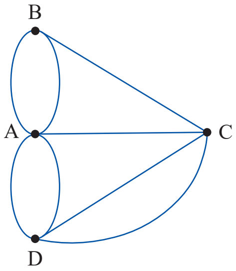
9 a i This is done using trial and error.
Here is an example:



ii Draw 4 points: *A*, *B*, *C* and *D*. Now, using the result in part i, we can see (by following the path) that:

B links to *A*
A links to *D*
D links to *C*
C links to *D*
D links to *A*
A links to *B*
B links to *C*
C links to *A*

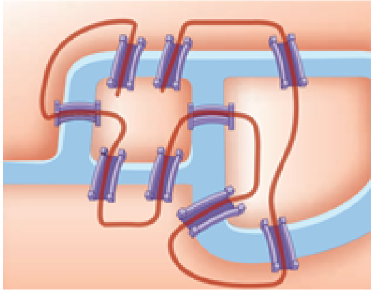
The result is:



iii We can see that vertices *A* and *B* are of odd-degree.

iv There are multiple ways to complete this task; an example would be *B*-*C*-*D*-*C*-*A*-*D*-*A*-*B*-*A*.

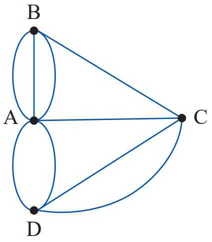
b i This is done using trial and error.
Here is an example:



ii Using the method in part a, we can see that by following the path,

A links to *B*
B links to *C*
C links to *D*
D links to *C*
C links to *A*
A links to *D*
D links to *A*
A links to *B*
B links to *A*

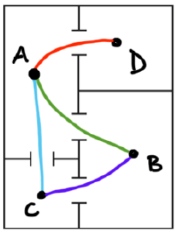
The result is:

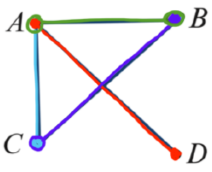


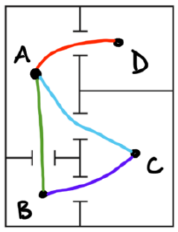
iii We can see that all vertices are of even-degree.

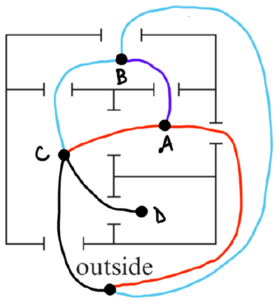
iv There are many ways to complete the task; an example would be *B*-*C*-*D*-*A*-*D*-*C*-*A*-*B*-*A*-*B*.

10 *Two solutions possible*

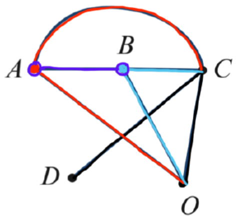


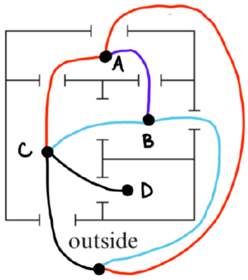






11 *Two solutions possible*





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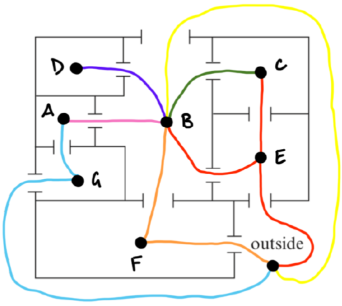
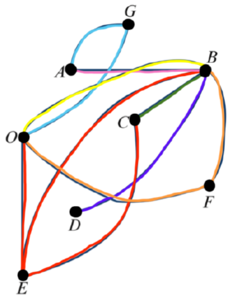




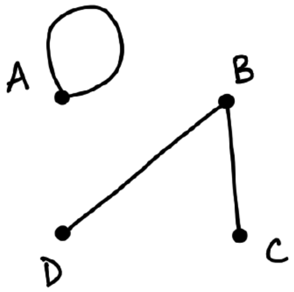
Solutions to 8B Now Try This Questions

2 **a** Graphs 1, 2 and 3 are connected. In each graph, every vertex is connected to every other vertex, either directly or indirectly. There are no isolated vertices in each graph.

b If the edge between vertices A and D was removed, vertex A would be isolated from the other vertices and could not be connected directly, nor indirectly to the other vertices. If the edge between vertices D and C was removed, the vertices C and B would not be connected to the other vertices directly, nor indirectly. Therefore the bridges are AD and DC *(Note: the order of the letters does not matter, as this is merely a method of naming the undircted edge that exists between the named vertices).*

c Isomorphic graphs contain identical information. Graphs 1 and 2 have an edge between vertices C and D. Graph 3 does not have an edge between those vertices. All other information (number of vertices, number of edges, connections between vertices) is identical for Graphs 1 and 2, therefore they are isomorphic graphs.

3 Draw a dot for each vertex, and label A to D. Starting with row A, there is a '1' in column A, therefore a loop exists at vertex A, connecting it with itself. Row B contains a '1' in columns C and D, so one edge must be drawn connecting B to C and B to D. Row C contains a '1' in column B, however this has already been accounted for. Row D contains a '1' in column B, however this has already been accounted for.



Solutions to Exercise 8B

1 Graph A, every vertex is connected to every other vertex, directly or indirectly.
Graph B, vertex C is isolated from the other vertices and is not connected to the other vertices.
Graph C, although vertices D and E are connected to each other, they are no connected to the other vertices in the graph.
Graph D, similar to Graph A, every vertex is connected to every other vertex, directly or indirectly.
Graph E, similar to Graph B, although vertices B and C are connected to each other, they are no connected to the other vertices in the graph.
Graph F, similar to Graph A, every vertex is connected to every other vertex, directly or indirectly.

2 **a** If the edge connecting vertices *B* and *D* was removed, vertex *D* would not be connected to the other vertices, directly or indirectly. Every other edge, if removed, would not result in one or more vertices being isolated from the other vertices. Therefore, for this graph one bridge exists: *BD*.

b If the edge connecting vertices *A* and *B* was removed, vertex *A* would be isolated from the other vertices. Likewise, if the edge connecting vertices *B* and *C* was removed, vertex *C* would be isolated from the other vertices. Therefore, for this graph two bridges exist: *AB* and *BC*.

c If the edge connecting vertices *W* and *X* was removed, vertices *U*, *W*, *V* would be isolated from vertices *X* and *Z*. Likewise if the edge connecting vertices *W* and *V* was removed, the vertex *V* would be isolated from the other vertices. Therefore, for this graph two bridges exist: *WX* and *WV*.

3 **a** Graph 2 has an edge connecting vertices *C* and *B* directly; graphs 1 and 3 do not have a similar edge connecting vertices *B* and *C*. All other information is identical between Graphs 1 and 3, therefore they are isomorphic graphs.

b Graphs 2 and 3 have multiple edges between edges *B* and *C*; graph 1 only has one edge between these vertices. All other information is identical between Graphs 2 and 3, therefore they are isomorphic graphs.

c Graph 3 has multiple edges between vertices *B* and *C*; graphs 1 and 2 only have one edge connecting vertices *B* and *C*. All other information is identical between Graphs 1 and 2, therefore they are isomorphic graphs.

d The vertices of Graph 1 have degrees of 4, 2 and 3 (two vertices have a degree of 3). The vertices of Graph 2 have simialr degrees. The vertices of Graph 3 have degrees of 4 (two vertices have a degree of 4) and 2 (two vertices have a degree of 2). Graphs 1 and 2 are isomorphic graphs.

e Graphs 1, 2 and 3 have fives vertices of similar degrees: 4, 3 (two vertices with degree 3) and 2 (two vertices with degree 2). Graphs1, 2 and 3 have multiple edges between one vertex of degree 4 and one vertex of degree 3. In Graphs 2 and 3, the vertex with degree 3, that has multiple edges with the vertex of degree 4, is also connected to a vertex of degree 2. In Graph 1, the vertex with degree 3, that has multiple edges with the vertex of degree 4, is connected to a vertex of degree 3. Graphs 2 and 3 are isomorphic graphs.

4 For each matrix, the number of rows and columns corresponds to the number of vertices the graph has. It should be a square matrix. Work through, row by row, to ensure no information in lost in the transfer from the graph to the matrix. A loop contributes ’1’ to the matrix.

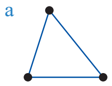
a $\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 1 \\ B & 1 & 0 & 1 \\ C & 1 & 1 & 0 \end{array}$

b $\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 1 & 0 \\ B & 1 & 0 & 1 & 1 \\ C & 1 & 1 & 0 & 1 \\ D & 0 & 1 & 1 & 0 \end{array}$

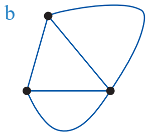
c $\begin{array}{c|ccc} & X & Y & Z \\ \hline X & 0 & 1 & 2 \\ Y & 1 & 1 & 1 \\ Z & 2 & 1 & 0 \end{array}$

5 Many solutions possible. Examples:

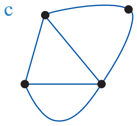
a



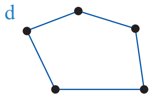
b



c

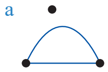


d

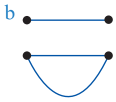


6 Many solutions possible. Examples:

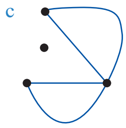
a



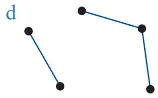
b



c



d

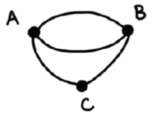


7 If a graph has 4 vertices, then a minimum of 3 edges must be present in order for the graph to be connected. If 2 or less edges were used, at least one vertex would be isolated.

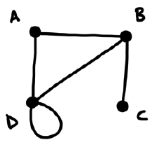
8 Many solutions possible. One example is:

9 For each matrix, the number of rows and columns corresponds to the number of vertices the graph has. Work through, row by row of the matrix, to ensure no information in lost in the transfer from the matrix to the graph. A loop contributes ’1’ to the matrix.

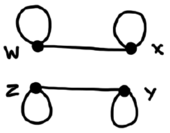
a



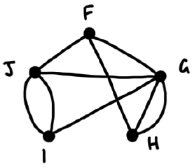
b



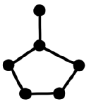
c



d



10 For a graph with these conditions, a subgraph must exist using 5 edges to connect 5 vertices with no bridges, then the final 6th vertex is connected to this subgraph by sixth edge, which is a bridge, in order to ensure the minumum number of edges are used. Overall **6 edges are required**.

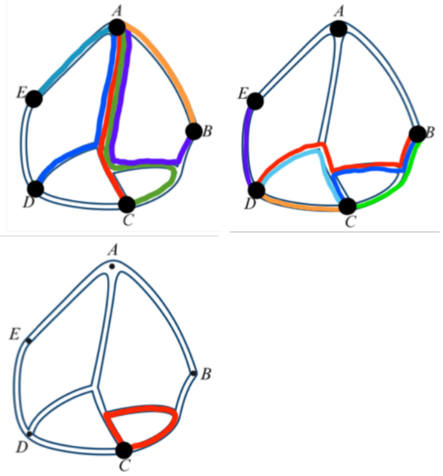


11 a This connected graph has 4 vertices, therefore the minimum number of edges required to remain connected is 3 (4 – 1 = 3, *Note: this situation is different to Question 10 as there are no restrictions on the number of bridges that can exist*). The graph has 6 edges, so **3 edges can be removed**, leaving 3 edges connecting the 4 vertices.

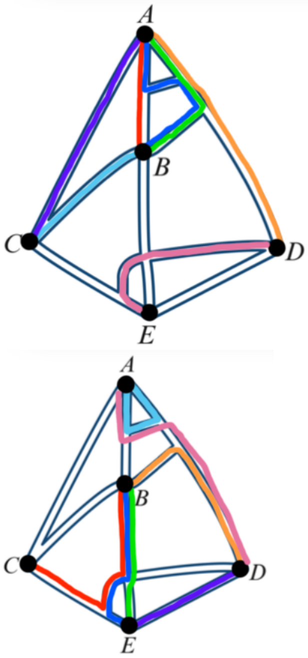
b This connected graph has 5 vertices, therefore the minimum number of edges required to remain connected is 4 (5 – 1 = 4). The graph has 10 edges, so **6 edges can be removed**, leaving 4 edges connecting the 5 vertices.

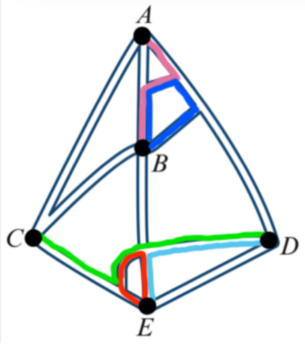
c This connected graph has 6 vertices, therefore the minimum number of edges required to remain connected is 5 (6 – 1 = 5). The graph has 15 edges (*Note: a quick method of counting the number of edges is to find the sum of degrees for the graph and divide by 2*), so **10 edges can be removed**, leaving 5 edges connecting the 6 vertices.

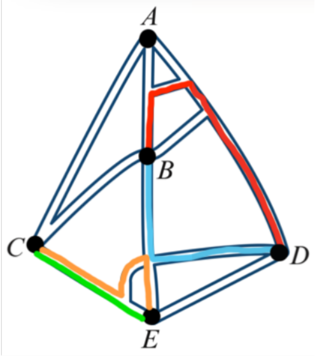
12 a By inspection, identify all possible paths between each of the towns. Consider forks in the road and the possibility of a loop. Below is a breakdown of all possible paths:



b By inspection, identify all possible paths between each of the towns. Consider forks in the road and the possibility of a loop. Below is a breakdown of all possible paths:



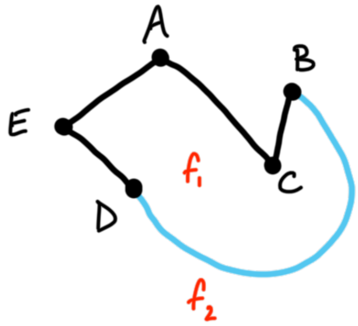

A
B
C
D
E


A
B
C
D
E

Solutions to 8C Now Try This Questions

4 To redraw the graph in planar form, no edges can intersect, except at the vertices. Always make sure the redrawn graph contains the same information as the original graph. In this case, the edge between vertices *A* and *D* will be redrawn, but it will still connect these two vertices.

6 To veritfy Euler's formula, the number of faces must be identified. The number of faces can only be identified when the graph is drawn in planar form (no edges can intersect, except at the vertices).



$v = 5$, $e = 5$, $f = 2$ *Note: do not forget to count the surrounding space as one face.*

$v + f = e + 2$

$5 + 2 = 5 + 2$

$10 = 10$

Therefore, Euler's formula is verified.

7 $f = 5$, $e = 8$

$v + f = e + 2$

$v + 5 = 8 + 2$

$v + 5 = 10$

$v = 10 - 5$

$v = 5$

Therefore, the graph has 5 vertices.

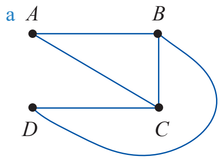
Solutions to Exercise 8C

1 Graphs *A*, *B*, *D* and *F* do not have any edges intersecting, except at the vertices. These are the graphs drawn in planar form. Graphs *C* and *E* can be redrawn in planar form.

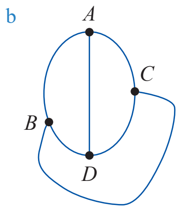
2 *In each case, the graphs must be redrawn without any edges intersecting, except at the vertices. For the more difficult graphs, the vertices may be moved from their original position, however care must be taken in order to preserve the information of the graph and not alter the connections between the vertices..*

Many solutions are possible.

Examples:

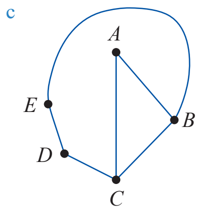


a

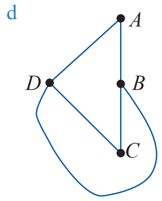


b

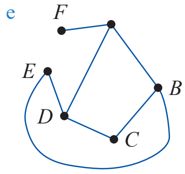
c



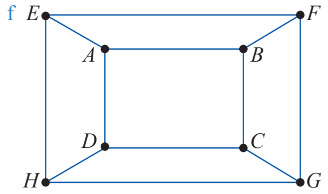
d



e



f



3 **a** **i** $v = 4, e = 4, f = 2$

ii $v + f = e + 2$
$4 + 2 = 4 + 2$
$6 = 6$

b **i** $v = 7, e = 9, f = 4$

ii $v + f = e + 2$
$7 + 4 = 9 + 2$
$11 = 11$

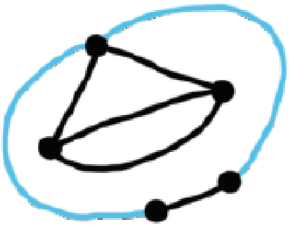
c **i** $v = 7, e = 12, f = 7$

ii $v + f = e + 2$
$7 + 7 = 12 + 2$
$14 = 14$

d **i** $v = 7, e = 10, f = 5$

ii $v + f = e + 2$
$7 + 5 = 10 + 2$
$12 = 12$

e *Must redraw graph in planar form*



i $v = 5, e = 7, f = 4$

ii $v + f = e + 2$
$5 + 4 = 7 + 2$
$9 = 9$

f *Must redraw graph in planar form*. If the second dot from the top is moved to the bottom of the diagram, the faces can easily be identified.

i $v = 4, e = 6, f = 4$

ii $v + f = e + 2$
$4 + 4 = 6 + 2$
$8 = 8$

4 **a** $v = 4, e = 4$
$v + f = e + 2$
$4 + f = 4 + 2$
$4 + f = 6$
$f = 6 - 4$
$f = 2$

b $e = 3, f = 2$
$v + f = e + 2$

$v + 2 = 3 + 2$
$v + 2 = 5$
$v = 5 - 2$
$v = 3$

c $v = 3, f = 3$
$v + f = e + 2$
$3 + 3 = e + 2$
$6 = e + 2$
$e = 6 - 2$
$e = 4$

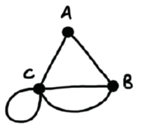
d $e = 6, f = 4$
$v + f = e + 2$
$v + 4 = 6 + 2$
$v + 4 = 8$
$v = 8 - 4$
$v = 4$

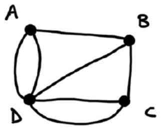
e $v = 4, e = 6$
$v + f = e + 2$
$4 + f = 6 + 2$
$4 + f = 8$
$f = 8 - 4$
$f = 4$

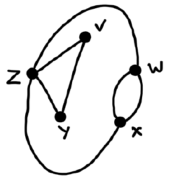
f $v = 6, e = 11$
$v + f = e + 2$
$6 + f = 11 + 2$
$6 + f = 13$
$f = 13 - 6$
$f = 7$

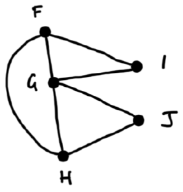
g $v = 10, f = 11$
$v + f = e + 2$
$10 + 11 = e + 2$
$21 = e + 2$
$e = 21 - 2$
$e = 19$

5 *For each, a graph must be drawn, based on the information given in the matrices, to determine the number of faces for each graph.*

a


b


c


d


6 Before adding extra edges:
$v = 6, e = 9$
$v + f = e + 2$
$6 + f = 9 + 2$
$6 + f = 11$
$f = 11 - 6$
$f = 5$
With the addition of 3 edges, $e = 12$
$v + f = e + 2$
$6 + f = 12 + 2$
$6 + f = 14$
$f = 14 - 6$
$f = 8$
Therefore, the number of faces increased by 3
D

7 $f = 4$
$v + f = e + 2$
$v + 4 = e + 2$
Of all the options available, only one holds true for the formula above:
$v = 5, e = 7$
$v + 4 = e + 2$

$5 + 4 = 7 + 2$
$9 = 9$
E

8 Sum of degrees = $4 + 4 + 4 + 3 + 3 = 18$
The sum of the degrees
= 2 × the total number of edges.
Therefore, total number of edges
= The sum of the degrees ÷ 2
$e = 18 \div 2 = 9, v = 5$
$v + f = e + 2$
$5 + f = 9 + 2$
$5 + f = 11$
$f = 11 - 5$
$f = 6$
A

9 **B** is not true; when redrawn in planar form the graph has six faces (five enclosed by the edges and 1 including the surrounding space). Faces are only identified when the graph is drawn in planar form; if it is not, it must be redrawn first.

10 The first and fourth statements are true. The second statement is false; there are only 4 edges and if the graph had multiple edges, the graph could not be connected as at least on vertex would be isolated. The third statement is false; it is impossible to draw a graph with 5 vertices, 4 edges and all vertices having an even degree.
C

11 Let x = number of vertices or edges (same number)
$v = e = x$
$v + f = e + 2$
$x + f = x + 2$
$f = x - x + 2$
$f = 2$
B

12 Let x = number of vertices or faces (same number)
$v = f = x$
$v + f = e + 2$
$x + x = e + 2$
$2x = e + 2$
$e = 2x - 2$
Interpret as: The number of edges is equal to two less than twice the number of vertices (or faces, as they are the same number).
D

Solutions to 8D Now Try This Questions

8 This walk repeats the vertex B. A trail may repeat vertices, however a path does not repeat vertices, so this walk is a **trail**.

9 This walk repeats the vertex C. A circuit may repeat vertices, however a cycle does not repeat vertices, so this walk is a **circuit**.

Solutions to Exercise 8D

1 a i The edge $K - M$ is repeated. A trail does not repeat edges, so this walk is **not a trail**.

ii There are no repeated edges, so this walk is a **trail**.

iii The edge $K - L$ is repeated. A trail does not repeat edges, so this walk is **not a trail**.

b i There are no repeated edges and no repeated vertices, so this walk is a **path**.

ii There are no repeated edges and no repeated vertices, so this walk is a **path**.

iii There are no repeated edges, but the vertex K is repeated, so this walk is **not a path**.

c i The walk starts and ends at the same vertex, however the edge $K - L$ is repeated, so this walk is **not a circuit**.

ii The walk starts and ends at the same vertex, there are no repeated edges, the vertex K is repeated, so this walk is a **circuit**.

iii The walk starts and ends at the same vertex, however the edge $K - G$ is repeated, so this walk is **not a circuit**.

d i The walk starts and ends at the same vertex, there are no repeated edges and there are no repeated vertices, so this walk is a **cycle**.

ii The walk starts and ends at the same vertex, however the edge $K - G$ is repeated, so this walk is **not a cycle**.

iii The walk starts and ends at the same vertex, there are no repeated edges and there are no repeated vertices, so this walk is a **cycle**.

2 a The walk starts and ends at the same vertex, so it could be a circuit or a cycle. The walk has no repeated edge and no repeated vertex, so the walk is a **cycle**.

b The walk starts and ends at the same vertex, so it could be a circuit or a cycle. The walk has no repeated edge, but the vertex D is repeated, so the walk is a **circuit**.

c The walk starts and ends at different vertices, so it is not a circuit, nor a cycle. The walk has no repeated edge, but the vertex *B* is repeated, so is it a **walk** only.

d The walk starts and ends at different vertices, so it is not a circuit, nor a cycle. The walk has no repeated edge and no repeated vertex, so this walk is a **path**.

e The walk starts and ends at the same vertex, so it could be a circuit or a cycle. The walk has no repeated edge, but the vertex *B* is repeated, so the walk is a **circuit**.

f The walk starts and ends at different vertices, so it is not a circuit, nor a cycle. The walk has no repeated edge and no repeated vertex, so this walk is a **path**.

3 a The walk starts and ends at different vertices, so it is not a circuit, nor a cycle. The walk has no repeated edge, but the vertex *B* is repeated, so the walk is a **trail**.

b The walk starts and ends at the same vertex, so it could be a circuit or a cycle. The walk has no repeated edge and no repeated vertex, so the walk is a **cycle**.

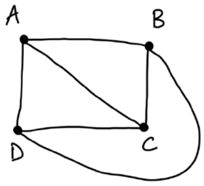
c The walk starts and ends at different vertices, so it is not a circuit, nor a cycle. The walk has no repeated edge and no repeated vertex, so this walk is a **path**.

d The walk starts and ends at different vertices, so it is not a circuit, nor a cycle. The walk has no repeated edge, but the vertex *B* is repeated, so the walk is a **trail**.

e The walk starts and ends at the same vertex, so it could be a circuit or a cycle. The walk has no repeated edge and no repeated vertex, so the walk is a **cycle**.

f The walk starts and ends at the same vertex, so it could be a circuit or a cycle. The walk has no repeated edge, but the vertex *B* is repeated, so the walk is a **circuit**.

4 a Draw the graph that is represented by this matrix.

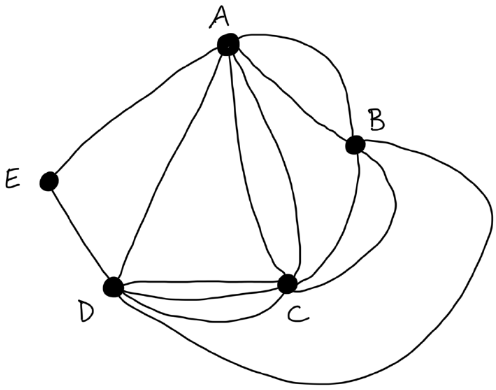


i A path is a walk that does not repeat any vertices. By inspection of the graph, the possible paths from Town *A* to Town *D* are:
$A - D$
$A - B - D$
$A - C - D$
$A - B - C - D$
$A - C - B - D$
There are **five** paths from Town *A* to Town *D*.

ii A cycle must start and finish at the same vertex and not repeat any vertices. Starting at Town *A* these are the possible cycles:
$A - B - C - A$
$A - B - D - A$
$A - B - C - D - A$
$A - B - D - C - A$
$A - C - B - A$
$A - C - D - A$
$A - C - B - D - A$

A – C – D – B – A
A – D – C – A
A – D – B – A
A – D – B – C – A
A – D – C – B – A
There are **12** different cycles possible.

b Draw the graph that is represented by this road map.



i A path is a walk that does not repeat any vertices. By inspection of the road map, the possible paths from Town *A* to Town *D* are:
A – D
A – B – D Note: this path is counted ***twice*** *as there are two different ways you can travel from Town A to Town B.*
A – C – D Note: this path is counted ***six times*** *as there are two different ways you can travel from Town A to Town C and three different ways you can travel from Town C to Town D.*
A – E – D
A – B – C – D Note: this path is counted ***twelve times*** *for the same reasons given above.*
There are **22** paths from Town *A* to Town *D*.

ii A cycle must start and finish at the same vertex and not repeat any vertices. Starting at Town *A* these are the possible cycles:
A – B – A Note: counted twice because multiple edges between Town A and Town B.
A – B – C – A Note: counted eight times, see graph above.
A – B – C – D – A Note: counted twelve times, see graph above.
A – B – C – D – E – A Note: counted twelve times, see graph above.
A – C – B – A Note: counted eight times.
A – C – D – A Note: counted six times.
A – C – D – E – A Note: counted six times.
A – C – B – D – A Note: counted four times.
A – C – B – D – E – A Note: counted four times.
A – D – B – A Note: counted twice.
A – D – C – A Note: counted six times.
A – D – E – A
A – D – C – B – A Note: counted twelve times.
A – E – D – A
A – E – D – B – A Note: counted twice.
A – E – D – C – A Note: counted six times.
A – E – D – B – C – A Note: counted four times.
A – E – D – C – B – A Note: counted twelve times.

Solutions to 8E Now Try This Questions

10 a i Neither: 4 vertices have an *odd* degree.

b i Both: zero *odd* vertices.

ii $A - B - C - D - B - D - A$
Note: There are multiple answers to this question.

c i Eulerian trail: two *odd* vertices only.

ii $A - D - E - D - C - B - C$
Note: There are multiple answers to this question.

Solutions to Exercise 8E

1 a Both: zero *odd* vertices.

b Neither: all vetices have an *odd* degree.

c Eulerian trail: two vertices have an *odd* degree.

d Eulerian trail: two vertices have an *odd* degree.

e Both: zero *odd* vertices.

f Eulerian trail: two vertices have an *odd* degree.

g Both: zero *odd* vertices.

h Eulerian trail: two vertices have an *odd* degree.

i Neither: 4 vertices have an *odd* degree.

2 a The walk decribed is an *Eulerian circuit*. All vertices have an *even* degree, therefore it is possible for the inspector to carry out the inspection decribed.

b $A - B - C - B - E - C - D - E - A$
Note: There are multiple answers to this question.

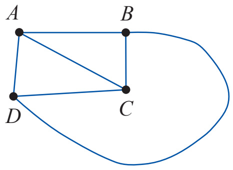
3 a The walk described is an Eulerian circuit. Vertices F and H have an odd degree. For an Eulerian circuit, all vertices must have an *even* degree, so this walk is not possible.

b The walk described is an Eulerian trail. Only two vertices have an odd degree; vertices F and H. This walk is possible. One example is $H - A - B - C - H - G - F - E - D - C - F$. *Note: There are multiple Eulerian trails possible for this question.*

4 a The walk described is an Eulerian circuit. All vertices have an *even* degree, so this walk is possible.

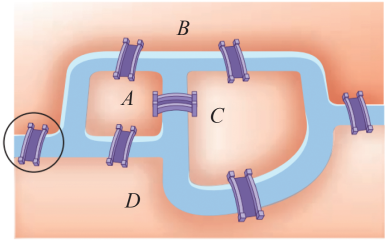
b $K - M - G - K - E - G - D - E - S - K$. *Note: There are multiple Eulerian circuits possible for this question.*

5 a

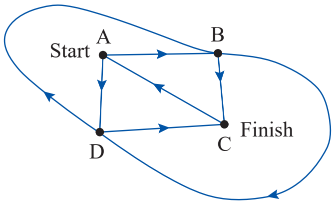


b There are more than 2 odd vertices, which means that you cannot traverse each path only once.

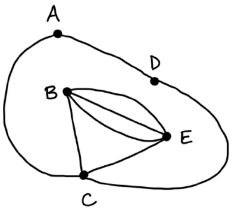
c **i** A possible solution is shown:



ii

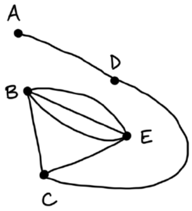


iii The bridges can now be crossed only once in a single walk because an eulerian trail now exists. The graph has two odd vertices and the rest are even. See graph in part **ii** (starting from A and finishing at C).



6 **a** The degree of each vertex should be found to easilty conclude whether an Eulerian trail, Eulerian circuit, both or neither exist for this graph. The graph can be drawn to conclude the degree of each vertex, however the sum of each row or the sum of each column would provide the same answers: deg(A)=2, deg(B)=4, deg(C)=4, deg(D)=2, deg(E)=4.
All vertices of this graph have an even degree, therefore both an Eulerian trail and an Eulerian circuit exist for this graph.

b Yes, if those changes took place, the answer to part **a** would change. With the changes of those values in the matrix, the edge between vertices A and C is removed; as reflected the graph below.



The removal of this edge results in the degree of each of these vertices decreasing;
deg(A)=1, deg(C)=3,
Note: the degrees of vertices B, D, E *are unchanged.*
There are now two vertices with an odd degree. With the removal of the edge between vertices A and C only an Eulerian trail exists for this graph and not an Eulerian circuit.

7 All seven vertices in this graph have an odd degree. With the addition of each new edge, the degrees of two vertices will increase by 1 (as every additional edge must start at one vertex and finish at another) thus creating two vertices with an even degree. For an Eulerian circuit, two or zero vertices must have an odd degree. If **two edges** were added to the graph, to two different pairs of vertices, there will only be two vertices remaining with an odd degree, enabling an Eulerian circuit to exist.

8 Of the five vertices in this graph, four of them have an odd degree. For an Eulerian trail to exist, two or zero vertices must have an odd degree. If an edge connecting two vertices, which both have an odd degree, their degrees would decrease by 1 each leaving only two vertices with an odd degree and thus enabling an Eulerian trail to exist. There are 5 edges than connect two vertices of an odd degree, therefore this can be done **5 different ways**.

Solutions to 8F Now Try This Questions

11 **a** **i** Yes, starting at vertex A it is possible to walk to all other vertices without repeating any vertices, ending at any vertex. One example is $A - D - E - F - B - C$.

ii Yes, starting at vertex A it is possible to walk to all other vertices, starting and ending at the same vertex, without repeating any vertices. One example is $A - D - E - F - B - C - A$.

b **i** Starting at vertex A, it is impossible to walk to every vertex without repeating a vertex.

Solutions to Exercise 8F

1 There are many possible answers to these problems. Some examples:

a *A-F-G-B-C-H-E-D*

b *F-A-B-C-D-E-H-G*

2 There are many possible answers to these problems. Some examples:

a *A-B-C-D-E-F-A*

b *A-B-C-D-E-A*

c *A-G-B-C-D-E-F-A*

d *A-D-G-E-H-I-F-C-B-A*

e No Hamiltonian cycle possible.

f *A-E-F-G-H-D-C-B-A*

3 **a** By inspection, this is not possible.

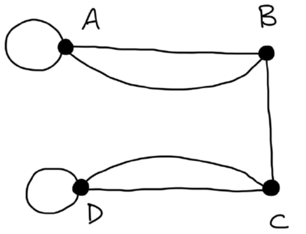
b This is possible. By inspection, the route *C-D-E-B-A* works. The mathematical name is hamiltonian path.

c This is possible. By inspection, the route *E-A-B-C-D-E* works. The mathematical name is hamiltonian cycle.

4 **a** This is possible. By inspection, the route *K-M-T-L-S-E-D-G-K* works. The mathematical name is hamiltonian cycle.

b This is possible. By inspection, the route *D-E-S-L-T-M-G-K* works. The mathematical name is hamiltonian path.

5 **a** To identify the number of faces, the graph represented by this matrix must be drawn.



This graph has **fives faces**; two loops, two pairs of multiple edges and the surrounding space.

b If the edge between vertices B and C was removed, the graph would not be connected, as vertices C and D would not be connected to the other vertices directly or indirectly, so the bridge connects vertices **B and C**.

c To determine whether an Eulerian trail, Eulerian circuit, both or neither exist, the degrees of each vertex must be found:
deg(A)=4, deg(B)=3,
deg(C)=3, deg(D)=4.
For an Eulerian trail there must be two or zero vertices with an odd degree. For an Eulerian circuit, all vertices must have an even degree. This graph will have an **Eulerian trail** only.

d For a Hamiltonian path, all vertices must be visited and no vertices can be repeated. One example is $A - B - C - D$. For a Hamiltonian cycle, the walk must start and finish at the same vertex, visit all vertices and not repeat any vertices; this is not possible. For this graph, only a **Hamiltonian path** is possible.

6 **a** For a Hamiltonian path to exist, the walk must include all vertices and none are repeated. This can be completed by adding an edge between A and B, followed by B and C, then C and D and finally E and F; in total a minimum of **5** edges must be added for a Hamiltonian path to exist ($A - B - C - D - E - F$).

b The Hamiltonian cycle is a Hamiltonian path, except the walk must start and finish at the same vertex. The walk described above in part a. ($A - B - C - D - E - F$) can be used as all vertices were visited and none were repeated, with the addition of another edge between F and A closing the cycle using the minimum number of edges. The Hamiltonian cycle $A - B - C - D - E - F - A$ exixts if **6** edges are added.

7 **a** The original graph has 9 vertices and 14 edges. For a Hamiltonian path to exist between 9 vertices, using the minimum number of edges, only 8 edges would be required;
a possible Hamiltonian path is $A - B - C - F - D - I - H - E - G$.
Therefore **6 edges must be removed** in order to leave 8 edges to create the Hamiltonian path given above.

b The original graph has 9 vertices and 14 edges. One possible Hamiltonian cycle is $A - B - C - F - D - I - H - G - E - A$ as this walk visits all vertices, does not repeat any vertices and starts/finishes at the same vertex. 9 edges are required for this Hamiltonian cycle. Therefore **5 edges must be removed** in order to leave 9 edges to create the Hamiltonian cycle given above.

8 **a** No, although all vertices in a connected graph can be reached by another directly or indirectly, this may require the repeating of a vertex to achieve such a Hamiltonian path.

b No, to achieve a Hamiltonian cycle, there must be at least one vertex

with an even degree for the cycle
to start and end at the same vertex

with no repeated vertices or edges.

Solutions to 8G Now Try This Questions

12 **a** The edge connecting Stratmoore to Osburn has a weight of 9. The weights are measured in kilometres, so the distance from Stratmoore to Osburn is 9 kilometres.

b The walk described is Stratmoore - Melville - Osburn - Kenton. The distance from Stratmoore to Melville is 8km. The distance from Melville to Osburn is 5km. The distance from Osburn to Kenton is 11km. The total distance of this walk is 8 + 5 + 11 = 24km.

13 Possible paths from W to F and their corresponding durations are:
$W - G1 - F$ for a duration of $8 + 8 = 16$ minutes
$W - B - F$ for a duration of $9 + 12 = 21$ minutes
$W - T - G2 - F$ for a duration of $10 + 6 + 10 = 26$ minutes
By inspection, the shortest route from W to F is $W - G1 - F$.

Solutions to Exercise 8G

1 Possible paths from A to E and their corresponding durations are:
$A - B - D - E$ for a duration of $4 + 6 + 3 = 13$ hours
$A - C - D - E$ for a duration of $3 + 5 + 3 = 11$ hours
By inspection, the shortest path from A to E is 11 hours.

2 Possible paths from A to D and their corresponding distances are:
$A - B - D$ for a distance of $20 + 15 = 35$ metres
$A - B - C - D$ for a distance of $20 + 10 + 20 = 50$ metres
$A - E - D$ for a distance of $35 + 25 = 60$ metres
$A - E - C - D$ for a distance of $35 + 15 + 20 = 70$ metres
By inspection, the shortest path from A to D is 35 metres.

3 Possible paths from B to D and their corresponding costs are:
$B - C - D$ for a cost of $7 + 10 = \$17$
$B - E - D$ for a cost of $6 + 5 = \$11$
$B - C - D$ for a cost of $7 + 10 = \$17$
$B - A - D$ for a cost of $2 + 4 = \$6$
By inspection, the shortest path from B to D is \$6.

4 Possible paths from B to F and their corresponding durations are:
$B - A - F$ for a duration of $5 + 3 = 8$ minutes
$B - C - G - F$ for a duration of $6 + 4 + 6 = 14$ minutes
$B - G - F$ for a duration of $3 + 6 = 9$ minutes
By inspection, the shortest path from B to F is 8 minutes.

5 Possible paths from A to I and their corresponding distances are:
$A - B - D - H - I$ for a distance of $4 + 8 + 10 + 8 = 30$ kilometres
$A - B - E - H - I$ for a distance of $4 + 9 + 12 + 8 = 33$ kilometres

$A - E - H - I$ for a distance of
$8 + 12 + 8 = 28$ kilometres
$A - C - F - G - I$ for a distance of
$4 + 10 + 4 + 8 = 26$ kilometres
$A - E - F - G - I$ for a distance of
$8 + 6 + 4 + 8 = 26$ kilometres
By inspection, the shortest path from A to I is 26 kilometres.

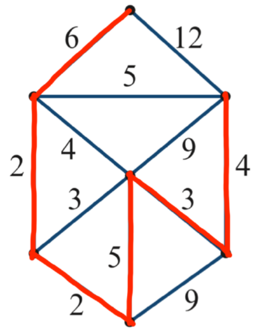
6 Possible paths from A to B and their corresponding distances are:
$8 + 3 + 12 + 2 = 25$ km
$11 + 3 + 3 + 2 = 19$ km
$11 + 15 + 14 + 8 = 48$ km
By inspection, the shortest path from A to B has a length of 19 km.

7 Possible paths from *Home* to *School* and their corresponding durations are:
$20 + 37 + 26 = 83$ minutes
$47 + 10 + 26 = 83$ minutes
$47 + 20 + 17 = 84$ minutes
$30 + 8 + 60 = 98$ minutes
$30 + 8 + 26 + 17 = 81$ minutes
By inspection, the shortest path from *Home* to *School* has a duration of 81 minutes.

8 Possible paths from *Home* to *Work* and their corresponding durations are:
$7 + 4 + 9 + 5 = 25$ minutes
$7 + 2 + 9 + 4 + 3 = 25$ minutes
$4 + 4 + 9 + 4 + 3 = 24$ minutes
$2 + 4 + 13 + 4 + 3 = 26$ minutes
$2 + 16 + 4 + 3 = 25$ minutes
By inspection, the shortest path from *Home* to *Work* has a duration of 24 minutes.

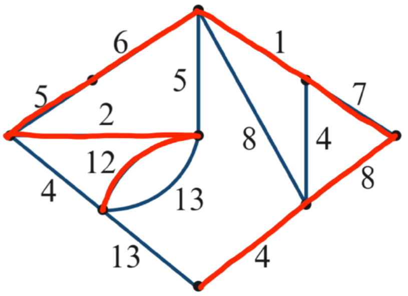
9 Possible routes starting from A, passing through all other checkpoints and finishing at A with their corresponding durations are:
$A - B - C - F - E - D - A$ for a duration of $11 + 15 + 9 + 8 + 11 + 10 = 64$ minutes
$A - B - C - E - F - D - A$ for a duration of $11 + 15 + 7 + 8 + 12 + 10 = 63$ minutes
By inspection, the route with the shortest completion time would be
$A - B - C - E - F - D - A$

10 **a**



The length of the shortest Hamiltonian path is
$2 + 2 + 5 + 3 + 6 + 4 = 22$.

b



The length of the shortest Hamiltonian path is
$12 + 2 + 5 + 6 + 1 + 7 + 8 + 4 = 45$

11 10a - No, 10b - Yes; all Eulerian trails in a graph will have the same total weighting because all edges are covered, so it is unnecessary to determine the length of the shortest one.

Solutions to 8H Now Try This Questions

15 Start with vertex A. The smallest weighted edge from vertex A is to B, with a weight of 2. Look at vertices A and B. The smallest weighted edge from either vertex A or vertex B not already used in the tree is from B to E, with weight 1. Look at vertices A, B and E. The next smallest weighted edge from A, B or E not already used in the tree is from E to F, with weight 2. The next smallest, unused edge from vertices A, B, E and F is from F to G with weight 2. The next smallest, unused edge from vertices A, B, E, F and G is from B to C **AND** from E to D both with weights of 3. All vertices have been included in the graph; this is the minimum spanning tree. The next smallest, unused edge is from E to H with weight 4, followed by the edge from H to I with weight 2. The length of the minimum spanning tree for this graph is $2 + 1 + 2 + 2 + 3 + 3 + 4 + 2 = 19$.

INCLUDE GRAPH

16 The edge with the smallest weight is from B to E with a weight of 1. From the remaining edges, smallest weight is 2: the edges from A to B, from E to F, from F to G and from H to I. From the remaining edges, smallest weight is 3: the edges from B to C and from E to D. From the remaining edges, the smallest weight is 4: the edge from E to H, however the edge from C to D is not included as these vertices have already been included in the tree. All vertices have been included in the graph; this is the minimum spanning tree. The length of the minimum spanning tree for this graph is $1 + 2 + 2 + 2 + 2 + 3 + 3 + 4 = 19$.

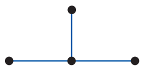
17 **INCLUDE GRAPHS**

Solutions to Exercise 8H

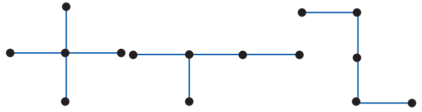
1 **a** No. of edges = no. of vertices – 1
= 15 – 1
= 14

b No. of vertices = no. of edges + 1
= 5 + 1
= 6

c Examples:



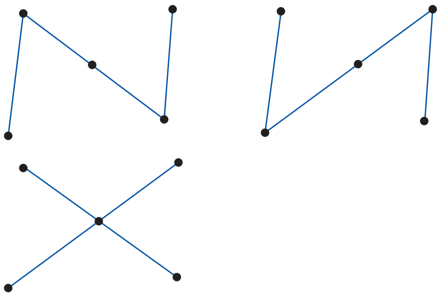
d Examples:



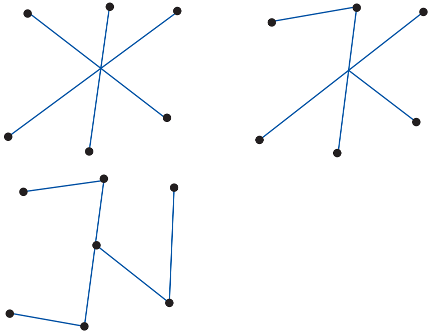
2 A spanning tree has n vertices and $(n – 1)$ edges.
$n = 8$ vertices, $(n – 1) = 7$ edges

3 **A** ($v = 8$, $e = 7$), **B** ($v = 8$, $e = 7$), and **D** ($v = 8$, $e = 7$).
The other networks do not fit the definition of a spanning tree.

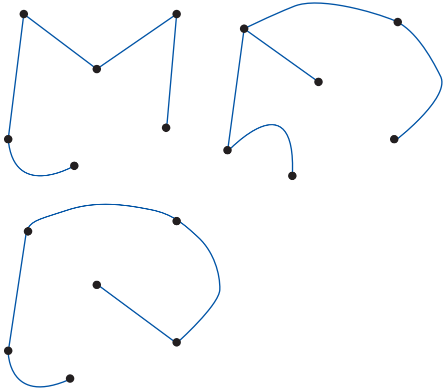
4 **a** Examples:

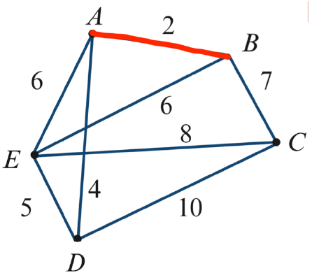


b Examples:

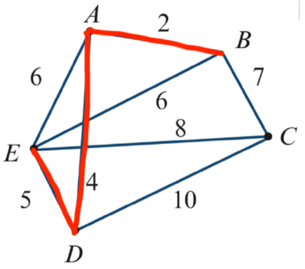


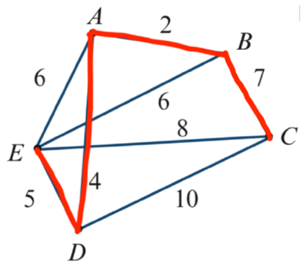
c Examples:



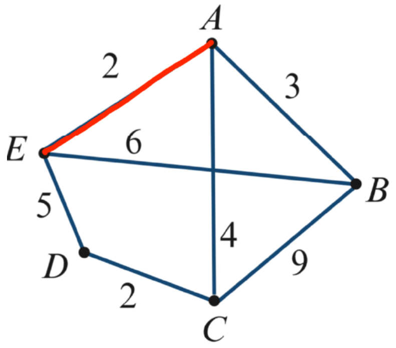


5 **a**

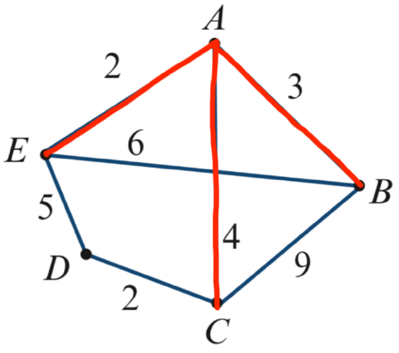


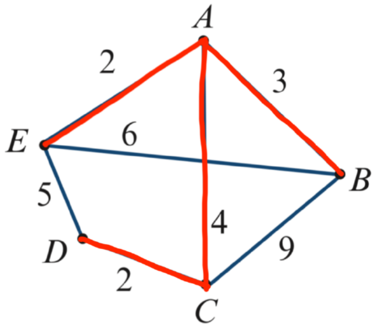


Length of minimum spanning tree= 2 + 4 + 5 + 7 = 18



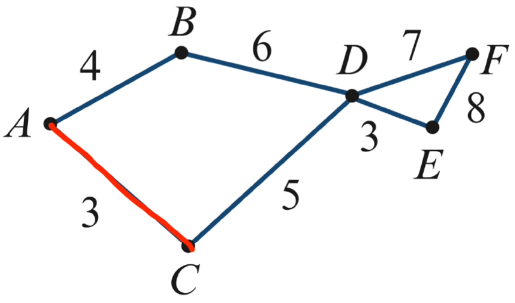
b

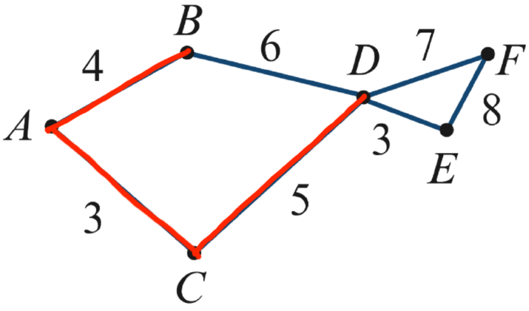


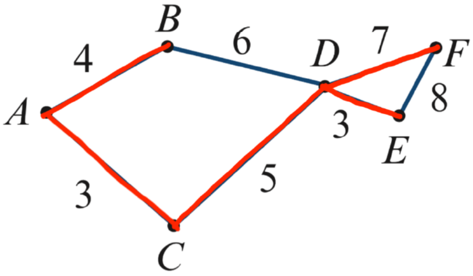


Length of minimum spanning tree= 2 + 3 + 4 + 2 = 11

c

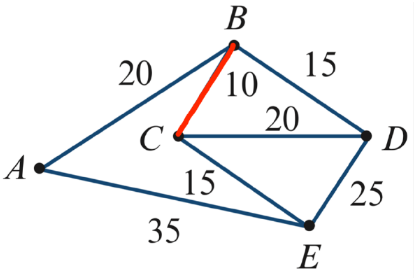


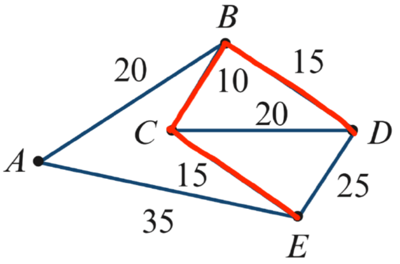


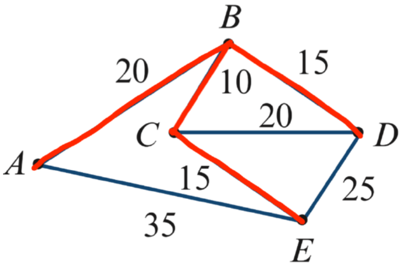


Length of minimum spanning tree= 3 + 4 + 5 + 3 + 7 = 22

6 a

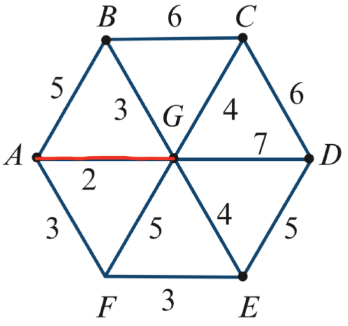


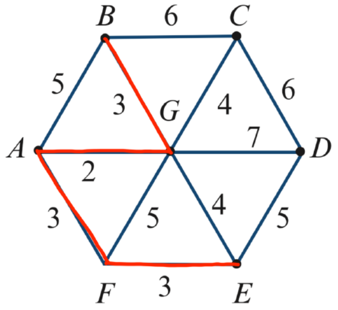


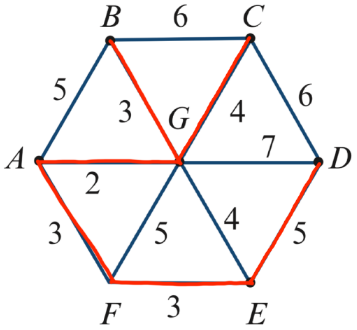


Length of minimum spanning tree= 10 + 15 + 15 + 20 = 60

b

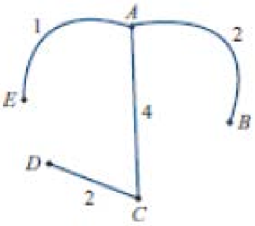




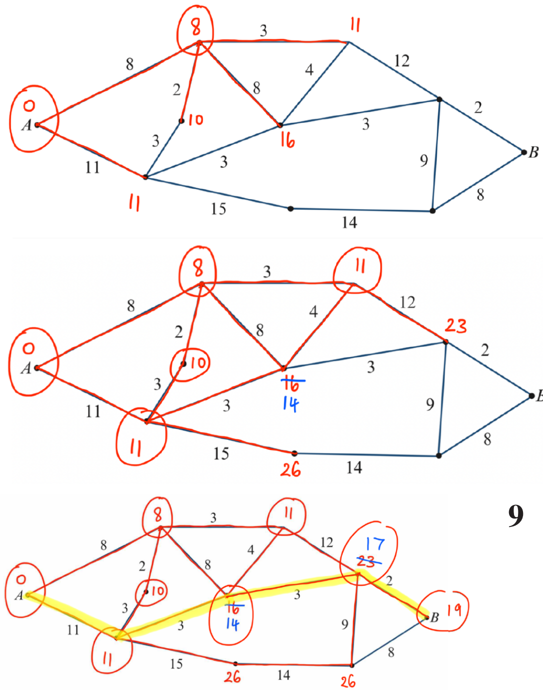


Length of minimum spanning tree= 2 + 3 + 3 + 3 + 4 + 5 = 20

c Length of tree = 9

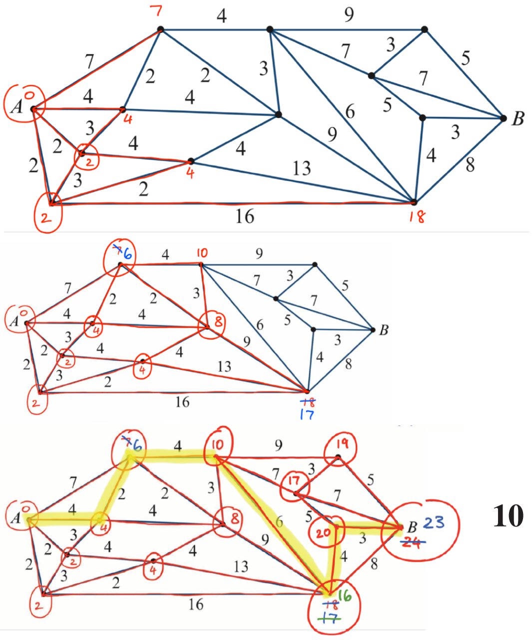


7 a



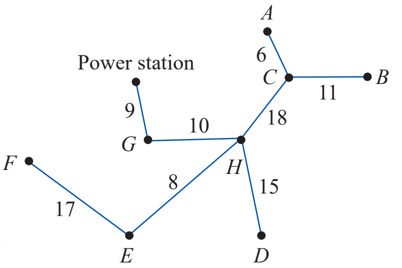
The length of the shortest path from *A* to *B* is 19.

b

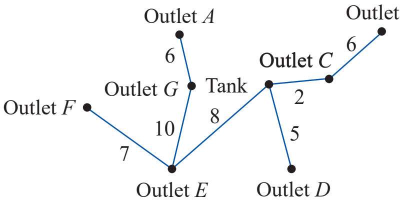


The length of the shortest path from *A* to *B* is 23.

8 Minimum length of cable = 94 km

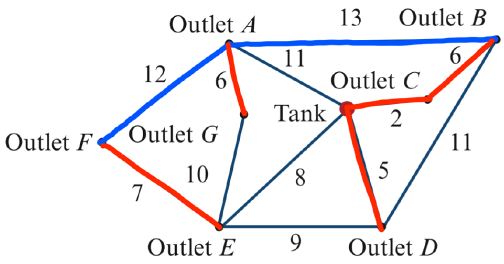


9 a



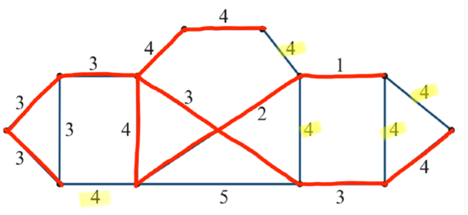
Minimum length of pipe = 44 m

b



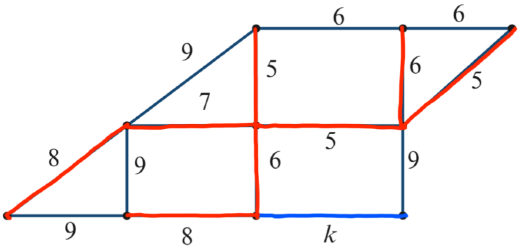
New minimum length of pipe
= 12 + 13 + 6 + 6 + 2 + 5 + 7 = 51,
therefore new pipe is **7 metres longer**.

10



Above is an example of one minimum spanning tree for this graph. Highlighted in yellow are the edges with weight 4 not included. In total **5** edges of weight 4 were not included in the minimum spanning tree.

11



Above is an example of one minimum spanning tree for this graph, where k was included in the beginning.

The total weight = 58

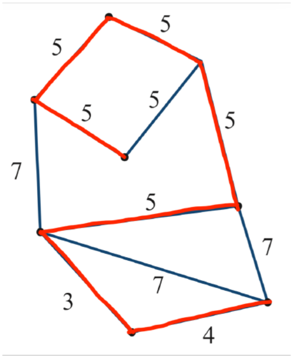
$k + 6 + 5 + 5 + 5 + 6 + 7 + 8 + 8 = 58$

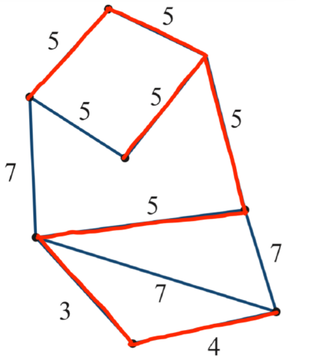
$k + 50 = 58$

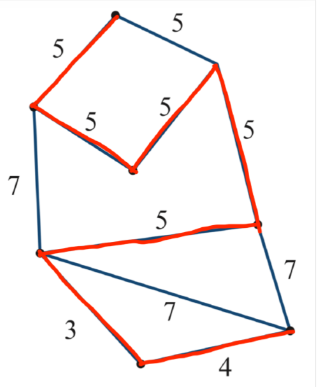
$k = 58 - 50$

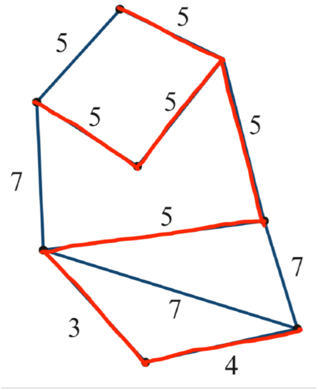
$k = 8.$

12 4 minimum spanning trees are possible (each having the same weight). Here are the four:









Solutions to Skills Checklist Questions

1 Vertices are the points of a graph; this graph has 5 vertices (A, B, C, D and E). Edges are the lines of a graph, connecting vertices to each other or a vertex to itself; this graph has 8 edges (AB, AE, BD, BE, CD, DE, loop at C, loop at E). A loop is an edge that connects a vertex to itself; there are 2 loops (vertex C has a loop, vertex E has a loop).

2 The degree of a vertex is the number of edges connected to a vertex; a loop contributes 2 to the degree of a vertex.
Vertex A has 2 edges connected to it; deg(A)=2
Vertex B has 3 edges connected to it; deg(B)=3
Vertex C has 2 edges connected to it and one of those edges is a loop; deg(C)=3
Vertex D has 3 edges connected to it; deg(D)=3
Vertex E has 4 edges connected to it and one of those edges is a loop; deg(E)=5

3 A graph is classified as a connected graph if there is a path for all vertices, directly or undirectly, to every other vertex. There should be no isolated vertices. For this graph, this is true; each vertex is connected to every other vertex, directly or indirectly. A bridge is a single edge in a connected graph that, if removed, leaves the graph disconnected. For this graph, there is a bridge between vertices C and D; if the edge between C and D was removed, the vertex C would be isolated from vertices A, B, D, E because there would be no direct, nor indirect connection to these other vertices.

4 The first graph is not isomorphic to the original graph, as there is no edge between vertices B and E. The second graph is isomorphic to the original graph; although the vertices are positioned differently, the second graph here contains the same information as the original graph (all vertices are connected in the same way).

5 **need matrix**

6 The original graph is planar, however it is not drawn in planar form. The graph can be redrawn in planar form, where no lines are intersecting, except at the vertices.
need graph

7 The number of faces can only be determined after the graph is redrawn in planar form. **need graph** Faces are the enclosed space between vertices and edges. The infinite surrounding space is always counted as a face for all graphs. The graph has 4 faces.

8 $v = 3, e = 5, f = 4$
$v + f = e + 2$
$3 + 4 = 5 + 2$
$7 = 7$ Therefore Euler's formula is verified.

9 The walk starts and finishes at the same vertex, therefore it could be a circuit or a cycle. There is no repeated edge, nor vertex, so the walk is a **cycle**.

10 An Eulerian trail is a walk that must include every edge just once. An Eulerian trail exists if two or zero vertices have an odd degree; if two, the Eulerian trail can start at one of the vertices with an odd degree. There are two vertices with an odd degree in this graph, vertices M and O. Many answers exist. Here are two:
$M-S-R-M-N-O-P-Q-R-O$
$O-R-S-M-R-Q-P-O-N-M$.

11 To have an Eulerian circuit, all vertices must have an even degree. The graph has fives vertices of an even degree (S, R, Q, P, N) and two vertices of an odd degree (M, O), therefore an Eulerian circuit is not possible for this graph.

12 A Hamiltonian path is a walk that must include every vertex just once. It is only identified through inspection. Many answers exist. Here are two:
$Q-R-S-M-N-O-P$
$P-Q-R-O-N-M-S$.

13 A Hamiltonian cycle is a walk that must include every vertex just once, starting and finishing at the same vertex. Many answers exist. Here are two:
$Q-P-O-N-M-S-R-Q$
$M-N-O-P-Q-R-S-M$.

14 **see graph**

15 **see graph**

Solutions to Chapter Review Multiple-Choice Questions

1 The graph has 6 dots (A, B, C, D, E, F), therefore the graph has 6 vertices **C**

2 The graph has 9 lines $(AB, AF(\times 2), BC, BD, BE, CD, DE, EF)$, therefore the graph has 9 edges. **E**

3 Vertex B is connected through 4 edges, therefore $\deg(B) = 4$. **D**

4 Consider the degree of all vertices in the graph:
$\deg(A) = 3$, $\deg(B) = 4$
$\deg(C) = 2$, $\deg(D) = 3$
$\deg(E) = 3$, $\deg(F) = 3$.
Vertices B and C are the only vertices with an even degree. **B**

5 Option **B** has 7 edges.
The sum of the degrees
$= 2 \times$ the total number of edges.
$= 2 \times 7 = 14$ **B**

6 The graph described by the matrix has 4 vertices (A, B, C, D) therefore eliminate options A and B. The matrix shows that vertex A should only have two edges connected to it; one edge connecting to vertex B and the other connecting to vertex D; option **D** is the only option that reflects this. **D**

7 The matrix that represents the graph should only include the following:
In row A, there should only be one '1' in column C (vertex A only has one edge and it is connected to vertex C) and zeroes for all other columns.
In row B, there should only be one '1' in column C (vertex B only has one edge and it is connected to vertex C) and zeroes for all other columns.
In row C, the number '1' should appear three times in columns A, B, D as the vertex C is connected to the other three vertices.
In row D, the number '1' should appear twice; once in column C because vertex D is connected to vertex C and once in column D as there is a loop at vertex D. *Note: a loop only contributes 1 to a matrix.*
A

8 The sequence of vertices represents a walk only, because vertices and edges are repeated. **A**

9 The sequence of vertices represents a circuit, as it starts and finishes at the same vertex (D) and does not repeat an edge. *Note: a circuit may repeat a vertex.* **D**

10 The sequence of vertices represents a trail but not a circuit as it starts and finishes at different vertices and does not repeat an edge. **B**

11 The sequence of vertices represents a cycle as it starts and finishes at the same vertex (D) and does not repeat any edges, nor vertices. **D**

12 This graph has 13 edges.
The sum of the degrees
$= 2 \times$ the total number of edges.
$= 2 \times 13 = 26.$ **E**

13 The graph is in planar form (no lines intersecting, except at the vertices), so: $v = 9$, $e = 13$, $f = 6$
Note: do not forget to count the surrounding space.. **B**

14 $v + f = e + 2$
This formula can be rearranged by subtracting e from both sides to give:
$v - e + f = 2$ **B**

15 $v = 10$, $f = 5$
$v + f = e + 2$
$10 + 5 = e + 2$
$15 = e + 2$
$e = 15 - 2$
$e = 13$ **D**

16 Only option C gives the correct exlcusion in the identification of an Eulerian circuit only. **C**

17 An Eulerian circuit exists if all vertices have an even degree. **B**

18 For an Eulerian trail to exist, but not an Eulerian circuit, there must be only two vertices with an odd degree; this is true for option B only. **B**

19 For an Eulerian circuit to exist, all vertices of a connected graph must have an even degree; this is true for option C only. *Note: although all vertices in option D have an even degree, this is not a connected graph.* **C**

20 For an Eulerian circuit to exist, all vertices of a connected graph must have an even degree; this is true for option E only. **E**

21 For an Eulerian trail to exist, there must be two or zero vertices with an *odd* degree. The original graph had four vertices with an odd degree (A, C, D, E). If an edge was added between two of these vertices, the degrees of each vertex would increase by 1. With the newly added edge these vertices would have an even degree, thus leaving only two vertices with an odd degree. Option B is the only option that adds an edge to two vertices with an odd degree; adding an edge between vertices A and D would increase the degree of each vertex to 4, leaving only two vertices with an odd degree (C and E) thus enabling an Euleriant trail to exist. **B**

22 Possible paths from F to B and their corresponding lengths are:
$F - C - B$ for a length of
$6 + 13 = 19$
$F - E - C - B$ for a length of
$2 + 3 + 13 = 18$
$F - E - D - A - B$ for a length of
$2 + 3 + 3 + 7 + 7 = 22$
By inspection, the shortest path from F to B has a length of 18. **B**

23 A Hamiltonian cycle starts and finishes at the same vertex and visits all vertices with no repeated vertices. Option C is the only walk that matches the criteria. *Note: Option A with the sequence $F - E - D - F$ is a cycle, however to be classified as a Hamiltonian cycle all vertices must be visited.* **C**

24 A graph has an Eulerian circuit if all the vertices have an even degree. A Hamiltonian cycle starts and finishes at the same vertex and visits all vertices with no repeated vertices. Option A is the only graph that matches the criteria. *Note: Although all the vertices in the graph of Option B have an even degree, it is impossible to meet the criteria of a Hamiltonian cycle.* **A**

25 A tree is a connected graph that contains no cycles, multiple edges or loops. A tree with n vertices has $n - 1$ edges. Option B is the only graph that matches this criteria. Options A and D contain multiple edges, option C contains a cycle and option E contains a loop. **B**

26 Options A, D, E are incorrect as they contain cycles. Option B contains an edge between vertices 2 and 4, although the original graph does not have this edge. **C**

27 Each campsite can be represented as a vertex and each walking track an edge. To visit each vertex, start and finish at the same vertex without repeating any vertices - this is describing a Hamiltonian cycle. There was no condition stating that every walking track should be followed, therefore neither an Eulerian trail, nor an Eulerian circuit are applicable (walking every edge). A minimum spanning tree may reach every vertex in the shortest length possible, however it does not take into account the return to the starting vertex. **D**

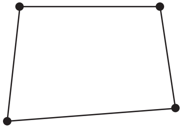
28 To reach every campsite (vertex, as classified above) with the shortest length of pipe possible is an example of a minimum spanning tree. There is no mention of a route to follow to start and return at a specific point, nor condititons such as not repeating edges or vertices, therefore trails, circuits, paths, or cycles would not be applicable for this problem. **E**

29 To reach every campsite (vertex, as classified above), start and finish at the tip while also visiting each campsite only once - this is describing a Hamiltonian cycle. There was no condition stating that every walking track should be followed, therefore neither an Eulerian trail, nor an Eulerian circuit are applicable (walking every edge). A minimum spanning tree may reach every vertex in the shortest length possible, however it does not take into account the return to the starting vertex. **D**

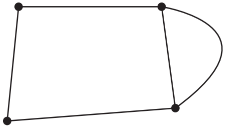
Solutions to Chapter Review Short-Answer Questions

1 Many answers are possible. Some examples are given below.

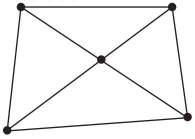
a Example of a connected graph with four vertices, four edges and two faces.



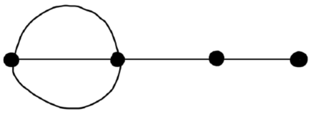
b Example of a connected graph with four vertices, five edges and three faces.



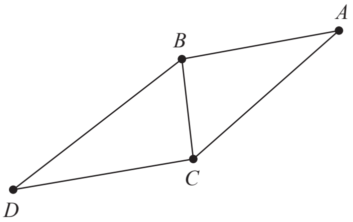
c Example of a connected graph with five vertices, eight edges and five faces.



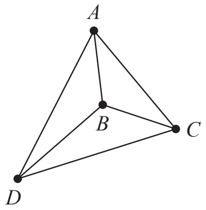
d Example of a connected graph with four vertices, four edges, two faces and two bridges



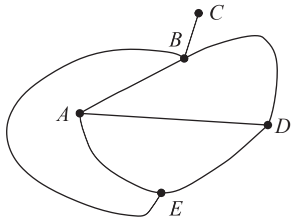
2 a



b



c



3 a Since 3 edges meet at vertex *C*, the degree of vertex *C* is 3.

b There are 2 odd vertices, namely *B* and *C*.
There are 2 even vertices, namely *A* and *D*.

c There are 4 possible eulerian paths, paths that include every edge just once and do not finish at the vertex at which they started.

B–A–C–B–D–C
or *B–A–C–D–B–C*
or *B–C–A–B–D–C*
or *B–C–D–B–A–C*

4 *A* connects to *B, C* and *D* – so in column *A*, there should be a '1' where it meets with rows *B*, *C* and *D*.

B connects to *A, D* and *C* – so in column *B*, there should be a '1' where it meets with rows *A*, *D* and *C*.

C connects to *A*, *B* and *D* – so in column *C*, there should be a ‘1’ where it meets with rows *A*, *B* and *D*.

D connects to *A*, *B* and *C* – so in column *D*, there should be a ‘1’ where it meets with rows *A*, *B* and *C*

The resulting matrix is as shown below: **see matrix**

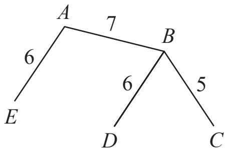
5 **a** Since 4 edges meet at vertex *C*, the degree of vertex *C* is 4.

b There are no odd vertices.
There are 5 even vertices, namely *A*, *B*, *C*, *D* and *E*.

c There are 42 possible eulerian circuits, paths that include every edge just once and finish at the vertex at which they started.

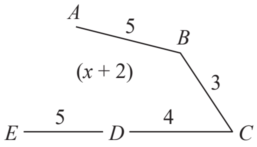
A–B–A–D–C–B–E–C–D–E–A
or *A–B–A–D–C–B–E–D–C–E–A*
or *A–B–A–D–C–D–E–B–C–E–A*
or *A–B–A–D–C–D–E–C–B–E–A*
or *A–B–A–D–C–E–B–C–D–E–A*
or *A–B–A–D–C–E–D–C–B–E–A*
or *A–B–A–D–E–B–C–D–C–E–A*
or *A–B–A–D–E–C–D–C–B–E–A*
or *A–B–A–E–D–C–B–E–C–D–A*
or *A–B–A–E–D–C–E–B–C–D–A*
or *A–B–C–D–A–B–E–C–D–E–A*
or *A–B–C–D–A–E–D–C–E–B–A*
or *A–B–C–D–C–E–B–A–E–D–A*
or *A–B–C–D–C–E–D–A–E–B–A*
or *A–B–C–D–E–A–B–E–C–D–A*
or *A–B–C–D–E–B–A–D–C–E–A*
or *A–B–C–D–E–B–A–E–C–D–A*
or *A–B–C–D–E–C–D–A–B–E–A*
or *A–B–C–D–E–C–D–A–E–B–A*
or *A–B–C–E–D–C–D–A–B–E–A*
or *A–B–C–E–D–C–D–A–E–B–A*
or *A–B–E–A–B–C–D–C–E–D–A*
or *A–B–E–A–B–C–D–E–C–D–A*
or *A–B–E–A–B–C–E–D–C–D–A*
or *A–B–E–A–D–C–D–E–C–B–A*
or *A–B–E–A–D–C–E–D–C–B–A*
or *A–B–E–A–D–E–C–D–C–B–A*
or *A–B–E–C–B–A–D–C–D–E–A*
or *A–B–E–C–B–A–E–D–C–D–A*
or *A–B–E–C–D–A–B–C–D–E–A*
or *A–B–E–C–D–A–E–D–C–B–A*
or *A–B–E–C–D–C–B–A–E–D–A*
or *A–B–E–C–D–E–A–B–C–D–A*
or *A–B–E–C–D–E–A–D–C–B–A*
or *A–B–E–D–A–B–C–D–C–E–A*
or *A–B–E–D–A–E–C–D–C–B–A*
or *A–B–E–D–C–B–A–D–C–E–A*
or *A–B–E–D–C–B–A–E–C–D–A*
or *A–B–E–D–C–D–A–B–C–E–A*
or *A–B–E–D–C–D–A–E–C–B–A*
or *A–B–E–D–C–E–A–B–C–D–A*
or *A–B–E–D–C–E–A–D–C–B–A*

6 The minimum spanning tree is:



The length of the minimum spanning tree is 6 + 7 + 6 + 5 = 24

7 Find the minimum spanning tree:



a The shortest path between *A* and *D* is *A*-*E*-*D*. The length is 6 + 5 = 11 km.

b The length of the minimum spanning tree is 5 + 3 + 4 + 5 = 17 km.

Solutions to Chapter Review Extended-Response Questions

1 **a** The statement 'the network of tracks is planar' means that no edges intersect, except at vertices.

b For the network above, there are 9 vertices (v), 14 edges (e) and 7 faces (f). For a connected planar graph, Euler's formula states $v - e + f = 2$.

$$\begin{aligned} \text{LHS} &= v - e + f \\ &= 9 - 14 + 7 \\ &= 2 \\ &= \text{RHS} \end{aligned}$$

Therefore Euler's rule for this network holds.

c Use trial and error:

There is only one route that works, i.e. *C8–C1–C2–C3–C4–C5–P.*

This is a distance of 160 + 120 + 130 + 110 + 80 + 150 = 750.

The shortest distance the ranger will have to walk is 750 metres.

d

Vertex	*Degree*	*Odd / Even*
P	2	Even
C1	4	Even
C2	4	Even
C3	2	Even
C4	4	Even
C5	4	Even
C6	2	Even
C7	2	Even
C8	4	Even

There are 9 vertices of even degree, and no vertices of odd degree.

e **i** Since the network is connected and all vertices are of even degree, an eulerian circuit exists, which means that a path exists that includes every edge just once, starting and finishing at the same vertex. Hence, it is possible for the ranger to inspect each of the tracks, starting and finishing at the Park Office.

ii You should arrive at the correct answer using trial and error. A systematic approach will determine all of the possible paths from the Park Office returning to the Park Office without traversing an edge more than once, although there are too many possible answers to find efficiently without the aid of a computer package.

P–C1–C2–C3–C4–C1–C8–C2–C4–C5–C6–C8–C7–C5–P
P–C1–C2–C3–C4–C1–C8–C2–C4–C5–C7–C8–C6–C5–P
P–C1–C2–C3–C4–C1–C8–C6–C5–C4–C2–C8–C7–C5–P
P–C1–C2–C3–C4–C1–C8–C6–C5–C7–C8–C2–C4–C5–P

P–C1–C2–C3–C4–C1–C8–C7–C5–C4–C2–C8–C6–C5–P
P–C1–C2–C3–C4–C1–C8–C7–C5–C6–C8–C2–C4–C5–P

P–C1–C2–C3–C4–C2–C8–C1–C4–C5–C6–C8–C7–C5–P
P–C1–C2–C3–C4–C2–C8–C1–C4–C5–C7–C8–C6–C5–P
P–C1–C2–C3–C4–C2–C8–C6–C5–C4–C1–C8–C7–C5–P
P–C1–C2–C3–C4–C2–C8–C6–C5–C7–C8–C1–C4–C5–P
P–C1–C2–C3–C4–C2–C8–C7–C5–C4–C1–C8–C6–C5–P
P–C1–C2–C3–C4–C2–C8–C7–C5–C6–C8–C1–C4–C5–P

P–C1–C2–C3–C4–C5–C6–C8–C1–C4–C2–C8–C7–C5–P
P–C1–C2–C3–C4–C5–C6–C8–C2–C4–C1–C8–C7–C5–P
P–C1–C2–C3–C4–C5–C7–C8–C1–C4–C1–C8–C6–C5–P
P–C1–C2–C3–C4–C5–C7–C8–C2–C4–C1–C8–C6–C5–P

P–C1–C2–C4–C1–C8–C2–C3–C4–C5–C6–C8–C7–C5–P
P–C1–C2–C4–C1–C8–C2–C3–C4–C5–C7–C8–C6–C5–P
P–C1–C2–C4–C1–C8–C6–C5–C4–C3–C2–C8–C7–C5–P
P–C1–C2–C4–C1–C8–C6–C5–C7–C8–C2–C3–C4–C5–P
P–C1–C2–C4–C1–C8–C7–C5–C4–C3–C2–C8–C6–C5–P
P–C1–C2–C4–C1–C8–C7–C5–C6–C8–C2–C3–C4–C5–P

P–C1–C2–C4–C3–C2–C8–C1–C4–C5–C6–C8–C7–C5–P
P–C1–C2–C4–C3–C2–C8–C1–C4–C5–C7–C8–C6–C5–P
P–C1–C2–C4–C3–C2–C8–C6–C5–C4–C1–C8–C7–C5–P
P–C1–C2–C4–C3–C2–C8–C6–C5–C7–C8–C1–C4–C5–P
P–C1–C2–C4–C3–C2–C8–C7–C5–C4–C1–C8–C6–C5–P
P–C1–C2–C4–C3–C2–C8–C7–C5–C6–C8–C1–C4–C5–P

P–C1–C2–C4–C5–C6–C8–C1–C4–C3–C2–C8–C7–C5–P
P–C1–C2–C4–C5–C6–C8–C2–C3–C4–C1–C8–C7–C5–P
P–C1–C2–C4–C5–C7–C8–C1–C4–C3–C2–C8–C6–C5–P
P–C1–C2–C4–C5–C7–C8–C2–C3–C4–C1–C8–C6–C5–P

P–C1–C2–C8–C1–C4–C2–C3–C4–C5–C6–C8–C7–C5–P
P–C1–C2–C8–C1–C4–C2–C3–C4–C5–C7–C8–C6–C5–P
P–C1–C2–C8–C1–C4–C3–C2–C4–C5–C6–C8–C7–C5–P
P–C1–C2–C8–C1–C4–C3–C2–C4–C5–C7–C8–C6–C5–P

P–C1–C2–C8–C6–C5–C4–C2–C3–C4–C1–C8–C7–C5–P
P–C1–C2–C8–C6–C5–C4–C3–C2–C4–C1–C8–C7–C5–P
P–C1–C2–C8–C6–C5–C7–C8–C1–C4–C2–C3–C4–C5–P
P–C I–C2–C8–C6–C5–C7–C8–C1–C4–C3–C2–C4–C5–P

P–C1–C2–C8–C7–C5–C4–C2–C3–C4–C1–C8–C6–C5–P
P–C1–C2–C8–C7–C5–C4–C3–C2–C4–C1–C8–C6–C5–P
P–C1–C2–C8–C7–C5–C6–C8–C1–C4–C2–C3–C4–C5–P
P–C I–C2–C8–C7–C5–C6–C8–C1–C4–C3–C2–C4–C5–P

P–C1–C4–C2–C1–C8–C2–C3–C4–C5–C6–C8–C7–C5–P
P–C1–C4–C2–C1–C8–C2–C3–C4–C5–C7–C8–C6–C5–P
P–C1–C4–C2–C1–C8–C6–C5–C4–C3–C2–C8–C7–C5–P
P–C1–C4–C2–C1–C8–C6–C5–C7–C8–C2–C3–C4–C5–P
P–C1–C4–C2–C1–C8–C7–C5–C4–C3–C2–C8–C6–C5–P
P–C1–C4–C2–C1–C8–C7–C5–C6–C8–C2–C3–C4–C5–P

P–C1–C4–C2–C3–C4–C5–C6–C8–C1–C2–C8–C7–C5–P
P–C1–C4–C2–C3–C4–C5–C6–C8–C2–C1–C8–C7–C5–P
P–C1–C4–C2–C3–C4–C5–C7–C8–C1–C2–C8–C6–C5–P
P–C1–C4–C2–C3–C4–C5–C7–C8–C2–C1–C8–C6–C5–P

P–C1–C4–C2–C8–C1–C2–C3–C4–C5–C6–C8–C7–C5–P
P–C1–C4–C2–C8–C1–C2–C3–C4–C5–C7–C8–C6–C5–P
P–C1–C4–C2–C8–C6–C5–C4–C3–C2–C1–C8–C7–C5–P
P–C1–C4–C2–C8–C6–C5–C7–C8–C1–C2–C3–C4–C5–P
P–C1–C4–C2–C8–C7–C5–C4–C3–C2–C1–C8–C6–C5–P
P–C1–C4–C2–C8–C7–C5–C6–C8–C1–C2–C3–C4–C5–P

P–C1–C4–C3–C2–C1–C8–C2–C4–C5–C6–C8–C7–C5–P
P–C1–C4–C3–C2–C1–C8–C2–C4–C5–C7–C8–C6–C5–P
P–C1–C4–C3–C2–C1–C8–C6–C5–C4–C2–C8–C7–C5–P
P–C1–C4–C3–C2–C1–C8–C6–C5–C7–C8–C2–C4–C5–P
P–C1–C4–C3–C2–C1–C8–C7–C5–C4–C2–C8–C6–C5–P
P–C1–C4–C3–C2–C1–C8–C7–C5–C6–C8–C2–C4–C5–P

P–C1–C4–C3–C2–C4–C5–C6–C8–C1–C2–C8–C7–C5–P
P–C1–C4–C3–C2–C4–C5–C6–C8–C2–C1–C8–C7–C5–P
P–C1–C4–C3–C2–C4–C5–C7–C8–C1–C2–C8–C6–C5–P
P–C1–C4–C3–C2–C4–C5–C7–C8–C2–C1–C8–C6–C5–P

P–C1–C4–C3–C2–C8–C1–C2–C4–C5–C7–C8–C6–C5–P
P–C1–C4–C3–C2–C8–C1–C2–C4–C5–C6–C8–C7–C5–P
P–C1–C4–C3–C2–C8–C6–C5–C4–C2–C1–C8–C7–C5–P
P–C1–C4–C3–C2–C8–C6–C5–C7–C8–C1–C2–C4–C5–P
P–C1–C4–C3–C2–C8–C7–C5–C4–C2–C1–C8–C6–C5–P
P–C1–C4–C3–C2–C8–C7–C5–C6–C5–C7–C8–C6–C5–P

P–C1–C4–C5–C6–C8–C1–C2–C3–C4–C2–C8–C7–C5–P
P–C1–C4–C5–C6–C8–C1–C2–C4–C3–C2–C8–C7–C5–P
P–C1–C4–C5–C6–C8–C2–C3–C4–C2–C1–C8–C7–C5–P
P–C1–C4–C5–C6–C8–C2–C4–C3–C2–C1–C8–C7–C5–P

P–C1–C4–C5–C7–C8–C1–C2–C3–C4–C2–C8–C6–C5–P
P–C1–C4–C5–C7–C8–C1–C2–C4–C3–C2–C8–C6–C5–P
P–C1–C4–C5–C7–C8–C2–C3–C4–C2–C1–C8–C6–C5–P
P–C1–C4–C5–C7–C8–C2–C4–C3–C2–C1–C8–C6–C5–P

P–C1–C8–C2–C1–C4–C2–C3–C4–C5–C6–C8–C7–C5–P
P–C1–C8–C2–C1–C4–C2–C3–C4–C5–C7–C8–C6–C5–P
P–C1–C8–C2–C1–C4–C3–C2–C4–C5–C6–C8–C7–C5–P
P–C1–C8–C2–C1–C4–C3–C2–C4–C5–C7–C8–C6–C5–P

P–C1–C8–C2–C3–C4–C1–C2–C4–C5–C6–C8–C7–C5–P
P–C1–C8–C2–C3–C4–C1–C2–C4–C5–C7–C8–C6–C5–P
P–C1–C8–C2–C3–C4–C2–C1–C4–C5–C6–C8–C7–C5–P
P–C1–C8–C2–C3–C4–C2–C1–C4–C5–C7–C8–C6–C5–P

P–C1–C8–C2–C4–C1–C2–C3–C4–C5–C6–C8–C7–C5–P
P–C1–C8–C2–C4–C1–C2–C3–C4–C5–C7–C8–C6–C5–P
P–C1–C8–C2–C4–C3–C2–C1–C4–C5–C6–C8–C7–C5–P
P–C1–C8–C2–C4–C3–C2–C1–C4–C5–C7–C8–C6–C5–P

P–C1–C8–C6–C5–C4–C1–C2–C3–C4–C2–C8–C7–C5–P
P–C1–C8–C6–C5–C4–C1–C2–C4–C3–C2–C8–C7–C5–P
P–C1–C8–C6–C5–C4–C2–C1–C4–C3–C2–C8–C7–C5–P
P–C1–C8–C6–C5–C4–C2–C3–C4–C1–C2–C8–C7–C5–P
P–C1–C8–C6–C5–C4–C3–C2–C1–C4–C2–C8–C7–C5–P
P–C1–C8–C6–C5–C4–C3–C2–C4–C1–C2–C8–C7–C5–P

P–C1–C8–C6–C5–C7–C8–C2–C1–C4–C2–C3–C4–C5–P
P–C1–C8–C6–C5–C7–C8–C2–C1–C4–C3–C2–C4–C5–P
P–C1–C8–C6–C5–C7–C8–C2–C3–C4–C1–C2–C4–C5–P
P–C1–C8–C6–C5–C7–C8–C2–C3–C 4–C2–C1–C4–C5–P
P–C1–C8–C6–C5–C7–C8–C2–C4–C1–C2–C3–C4–C5–P

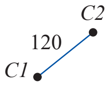
P–C1–C8–C6–C5–C7–C8–C2–C4–C3–C2–C1–C4–C5–P

P–C1–C8–C7–C5–C4–C1–C2–C3–C4–C2–C8–C6–C5–P
P–C1–C8–C7–C5–C4–C1–C2–C4–C3–C2–C8–C6–C5–P
P–C1–C8–C7–C5–C4–C2–C1–C4–C3–C2–C8–C6–C5–P
P–C1–C8–C7–C5–C4–C2–C3–C4–C1–C2–C8–C6–C5–P
P–C1–C8–C7–C5–C4–C3–C2–C1–C4–C2–C8–C6–C5–P
P–C1–C8–C7–C5–C4–C3–C2–C4–C1–C2–C8–C6–C5–P

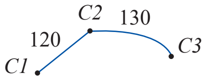
P–C1–C8–C7–C5–C6–C8–C2–C1–C4–C2–C3–C4–C5–P
P–C1–C8–C7–C5–C6–C8–C2–C1–C4–C3–C2–C4–C5–P
P–C1–C8–C7–C5–C6–C8–C2–C3–C4–C1–C2–C4–C5–P
P–C1–C8–C7–C5–C6–C8–C2–C3–C4–C2–C1–C4–C5–P
P–C1–C8–C7–C5–C6–C8–C2–C4–C1–C2–C3–C4–C5–P
P–C1–C8–C7–C5–C6–C8–C2–C4–C3–C2–C1–C4–C5–P

Each of the above paths can be traversed from left-to-right or from right-to-left, making a total of 240 possible routes that the ranger can choose from!

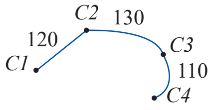
f The minimum length of track that will need to be gravelled to ensure that all camp sites and the Park Office are accessible along a gravelled track can be found by determining the minimum spanning tree for the above network. Use Prim's algorithm for finding a minimum spanning tree. Choose any vertex as the starting vertex, say *C1*. Choose the edge from this vertex with the lowest weight, i.e., the edge of weight 120 connecting *C1* and *C2*. We now have two vertices and one edge.



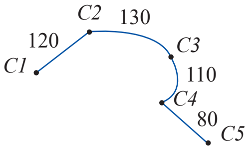
Next, inspect the two vertices *C1* and *C2*. Choose the edge with the lowest weight provided it does not form a cycle, i.e., the edge of weight 130 connecting *C2* and *C3*. We now have three vertices and two edges.



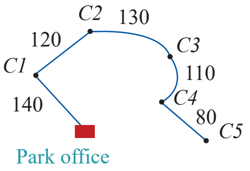
Next, inspect the three vertices *C1*, *C2* and *C3*. Choose the edge with the lowest weight provided it does not form a cycle, i.e., the edge of weight 110 connecting *C3* and *C4*. We now have four vertices and three edges.



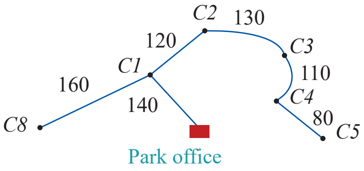
Next, inspect the four vertices *C1*, *C2*, *C3* and *C4*. Choose the edge with the lowest weight provided it does not form a cycle, i.e., the edge of weight 80 connecting *C4* and *C5*. We now have five vertices and four edges.



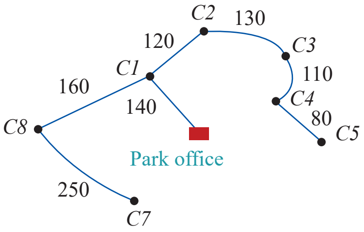
Next, inspect the five vertices *C1, C2, C3, C4* and *C5*. Choose the edge with the lowest weight provided it does not form a cycle, i.e., the edge of weight 140 connecting *C1* and the Park Office. We now have six vertices and five edges.



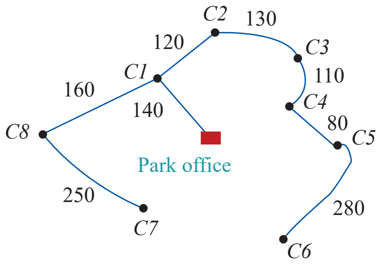
Next, inspect the six vertices *C1*, C2, C3, *C4*, *C5* and the Park Office. Choose the edge with the lowest weight provided it does not form a cycle, i.e., the edge of weight 160 connecting *C1* and *C8*.
Note: We can't choose the edge of weight 150 connecting *C5* and the Park Office as it would form a cycle.
We now have seven vertices and six edges.



Next, inspect the seven vertices *C1*, *C2*, *C3, C4*, C5, *C8* and the Park Office. Choose the edge with the lowest weight provided it does not form a cycle, i.e., the edge of weight 250 connecting *C7* and *C8*. We now have eight vertices and seven edges.



Next, inspect the eight vertices *C1, C2*, C3, *C4*, C5, C7, *C8* and the Park Office. Choose the edge with the lowest weight provided it does not form a cycle, i.e., the edge of weight 280 connecting *C5* and *C6*. We now have all nine vertices and the required eight edges.



The minimum length of track to be gravelled
= 120 + 130 + 110 + 80 + 140 + 160 + 250 + 280
= 1270

The minimum length of track to be gravelled is 1270 metres.

g **i** The ranger wishes to visit each camp site exactly once, starting and finishing at the Park Office. A path through a graph that passes through each vertex exactly once, starting and finishing at the same vertex, is called a hamiltonian cycle.

ii There are many possible answers. One is provided here. The ranger could take the path *P–C1–C2–C3–C4–C5–C6–C8–C7* and, if a track was laid from *C7* to the Park Office, the ranger could then return directly from *C7* to the Park Office. He would have visited all of the camp sites exactly once during his trip.

iii There are four possible routes the ranger could take, with a new track from *C7* to the Park Office, to ensure that he visits each camp site exactly once before returning to the Park Office. They are listed here.

P–C1–C2–C3–C4–C5–C6–C8–C7–P
or, in reverse, *P–C7–C8–C6–C5–C4–C3–C2–C1–P*

or *P–C1–C4–C3–C2–C8–C6–C5–C7–P*
or, in reverse, *P–C7–C5–C6–C8–C2–C3–C4–C1–P*

2 **a** Use trial and error.
One (shortest) route is: Nhill-Dimboola-Horsham-Murtoa-Donald
Length is 40 + 36 + 21 + 38 = 135 km

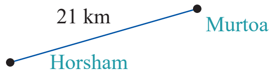
b Count the number of vertices, edges and faces:
$v = 8$
$e = 12$
$f = 6$
$v - e + f = 8 - 12 + 6 = 2$
Hence, $v - e + f = 2$, as required.

c **i** This cannot be done (i.e. the network does not have an eulerian circuit) as there are two odd vertices.

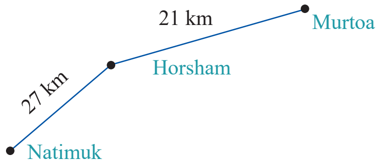
ii By inspection, a route that works is:
Horsham–Murtoa–Stawell–Horsham–Warracknabeal–Donald–Murtoa–Warracknabeal–Dimboola–Nhill–Natimuk–Horsham–Dimboola
The distance is 21 + 41 + 74 + 49 + 57 + 38 + 53 + 38 + 40 + 82 + 27 + 36 = 556 km

iii An example would be the one in **ii**, Horsham–Murtoa–Stawell–Horsham–Warracknabeal–Donald–Murtoa–Warracknabeal–Dimboola–Nhill–Natimuk–Horsham–Dimboola

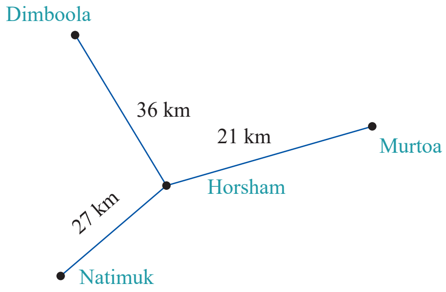
d This can be done by finding the minimum spanning tree.
If we start at Horsham, the link with the least weight is the one to Murtoa:



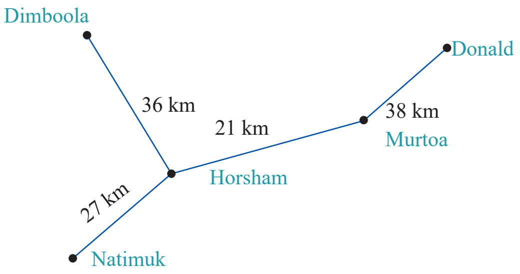
Now, inspect the links connecting to vertices Horsham and Murtoa.
The one with minimum weight is the one connecting Horsham to Natimuk (27 km)



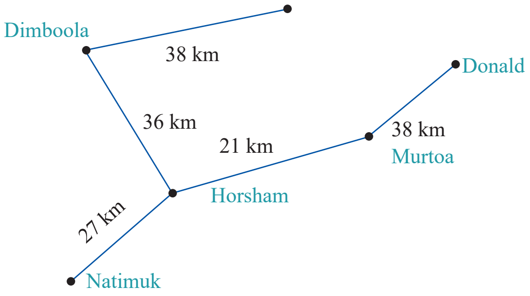
Now, inspect the links connecting to vertices Natimuk, Horsham and Murtoa.
The one with minimum weight is the one connecting Horsham to Dimboola (36 km)



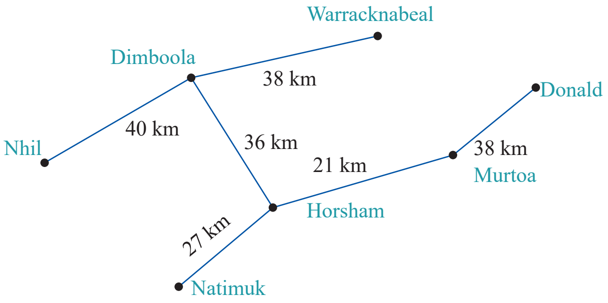
Now, inspect the links connecting to vertices Dimboola, Natimuk, Horsham and Murtoa.
The one(s) with minimum weight are the ones connecting:
Dimboola to Warracknabeal (38 km); and
Murtoa to Donald (38 km).
We will pick the one connecting Murtoa to Donald (algorithm states that any can be used)



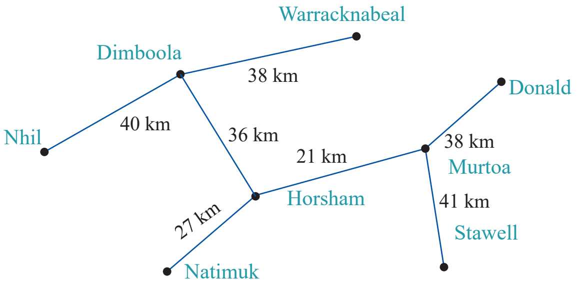
Now, inspect the links connecting to vertices Dimboola, Natimuk, Horsham, Murtoa and Donald.
The one with minimum weight is the one connecting Dimboola to Warracknabeal (38 km)



Now, inspect the links connecting to vertices Dimboola, Natimuk, Horsham, Murtoa, Donald and Warracknabeal.
The one with minimum weight is the one connecting Dimboola to Nhill (40 km)



Now, inspect the links connecting to vertices Dimboola, Natimuk, Horsham, Murtoa, Donald, Warracknabeal and Nhill.
The one with minimum weight is the one connecting Murtoa to Stawell (41 km)



The minimum spanning tree has now been established.
The sum of the weights is 40 + 38 + 36 + 27 + 21 + 38 + 41 = 241 km

Therefore, 241 km of cable is required.

e It was established in part **c** that it is possible to travel on every road, starting from Horsham and finishing at Dimboola. Hence, to finish at Horsham, the engineer will need to travel on the Dimboola – Horsham road.

Chapter 9 – Variation

Solutions to 9A Now Try This Questions

1 $y \propto x$

$y = kx$

When $x = 2,\ y = 10$

So $10 = 2k$

$$k = 5$$

$$\therefore y = 5x$$

When $x = 4,\ y = 5(4) = 20$

When $y = 100,\ 100 = 5x$

$$100 \div 5 = 5x \div 5$$

$$\therefore x = 20$$

Complete the table.

x	2	4	6	20
y	10	20	30	100

2 $y \propto x^2$

$y = kx^2$

When $x = 4,\ y = 32$

So $32 = (4)^2 k$

$$32 = 16k$$

$$k = 2$$

$$\therefore y = 2x^2$$

When $x = 2,\ y = 2(2)^2 = 8$

When $y = 288,\ 288 = 2(x^2)$

$$144 = x^2$$

$$\therefore x = 12$$

Complete the table.

x	2	4	6	12
y	8	32	72	288

3 **a** $d \propto t$

$d = kt$

When $t = 2,\ d = 190$

$$190 = 2k$$

$$k = 95$$

$$\therefore d = 95t$$

When $t = 3,\ d = 95(3)$

$$= 285$$

Car has travelled 285 kms

b When $d = 500$,

$$500 = 95t$$

$$\frac{500}{95} = \frac{95t}{95}$$

$$\therefore t = 5.26$$

0.26 of one hour = 0.26 of 60 mins

$0.26 \times 60 = 15.6 \approx 16$ mins

Time taken is approximately 5 hours 16 mins.

Solutions to Exercise 9A

1 **a** $y = 3x$

Substitute values for x into equation.

When $x = 6,\ y = 3 \times 6$

$$= 18$$

Substitute y value into equation

When $y = 24,\ 24 = 3x$

$$24 \div 3 = 3x \div 3$$

$$\therefore x = 8$$

x	2	4	6	8
y	6	12	18	24

b $y = 4x$

Substitute values for x into equation.

When $x = 1$, $y = 4 \times 1$

$$= 4$$

When $x = 2$, $y = 4 \times 2$

$$= 8$$

Substitute y value into equation

When $y = 12$, $12 = 4x$

$$12 \div 4 = 4x \div 4$$

$$\therefore x = \frac{12}{4} = 3$$

x	0	1	2	3
y	0	4	8	12

2 **a** $y = kx$

b $y = kx^2$

c $y = x^5$

d $a = kb$

e $z = kw$

f $y = k\sqrt{x}$

3 **a** $m \propto n$

b $y \propto x^2$

c $y \propto \sqrt{x}$

d $s \propto t^2$

4 **a** $y \propto x$

$y = kx$

Substitute corresponding values for x and y and solve for k.

When $x = 3$, $y = 21$

$$21 = 3k$$

$$k = \frac{21}{3} = 7$$

Equation is thus $y = 7x$

When $x = 4$, $y = 7 \times 4 = 28$

When $y = 84$, $84 = 7x$

$$\therefore x = \frac{84}{7} = 12$$

x	3	4	7	12
y	21	28	49	84

b $y \propto x$

$y = kx$

Substitute corresponding values for x and y and solve for k.

When $x = 4$, $y = 2$

$$2 = 4k$$

Divide both sides by 4

$$\frac{2}{4} = k$$

$$\therefore k = \frac{1}{2}$$

Equation is thus $y = \frac{1}{2}x$

Substitute x value into equation to find y.

When $x = 14$, $y = \frac{1}{2}(14) = 7$

Substitute y value into equation to find x.

When $y = 10$, $10 = \frac{1}{2}x$

$$\therefore x = 20$$

x	4	9	14	20
y	2	4.5	7	10

c $y \propto x^2$

$y = kx^2$

Substitute corresponding values for x and y and solve for k.

When $x = 2$, $y = 8$

$$8 = k(2^2)$$

$$8 = 4k$$

Divide both sides by 4

$$\therefore k = 2$$

Equation is thus $y = 2x^2$

Substitute $x = 6$ to find y.

$$y = 2 \times 6^2$$
$$= 2 \times 36$$
$$= 72$$

Substitute $y = 128$ to find x.

$$128 = 2x^2$$
$$64 = x^2$$
$$\therefore x = 8$$

x	2	4	6	8
y	8	32	72	128

5 $y \propto x$

$y = kx$

When $y = 42$, $x = 7$

$$42 = 7k$$
$$\therefore\ k = 42 \div 7 = 6$$

So $y = 6x$

a When $x = 9$, $y = 6 \times 9$
$$= 54$$

b When $y = 102$, $102 = 6x$
$$\therefore x = 102 \div 6$$
$$= 17$$

6 $M \propto n^2$

$M = kn^2$

When $M = 48$, $n = 4$

$$48 = k \times 4^2$$
$$48 = 16k$$
$$\therefore k = 48 \div 16 = 3$$

So $M = 3n^2$

a When $n = 10$, $M = 3 \times 10^2$
$$= 300$$

b When $M = 90$, $90 = 3n^2$
$$n^2 = 30$$
$$\therefore n = \sqrt{30}$$

$n = 5.5$, correct to one dec. place.

7 $A \propto h$

$A = kh$

When $A = 60$, $h = 10$

$$60 = 10k$$
$$\therefore k = 60 \div 10 = 6$$

So $A = 6h$

a When $h = 12$, $A = 6 \times 12$
$$= 72 \text{ cm}^2$$

b When $A = 120$, $120 = 6h$
$$\therefore h = 120 \div 6$$
$$= 20 \text{ cm}$$

8 $C \propto W$

$C = kW$

When $C = 15.60$, $W = 4$

$$15.60 = 4k$$
$$\therefore k = 15.60 \div 4 = 3.90$$

So $C = 3.9W$

a When $W = 6$, $C = 3.9 \times 6$
$$= \$23.40$$

b When $C = 25$, $25 = 3.9W$
$$\therefore W = 25 \div 3.9$$
$$= 6.4 \text{ kg}$$

9 $d \propto t$

$d = kt$

When $d = 330$, $t = 3$

$$330 = 3k$$
$$\therefore k = 330 \div 3 = 110$$

So $d = 110t$

a When $d = 500,\ 500 = 110t$

$$\therefore t = 500 \div 110$$
$$= 4.55 \text{ hours}$$

(0.55hours = $0.55 \times 60 = 33$ mins)
4.55 hours = 4 hours 33 mins.

b **i** When $t = 5$

$$d = 110 \times 5$$
$$= 550 \text{ km}$$

ii When $t = 90$ mins = 1.5 hours,

$$d = 110 \times 1.5$$
$$= 165 \text{ km}$$

10 $W \propto L^2$

$W = kL^2$

When $L = 20,\ W = 18$

$$18 = k \times 20^2$$
$$18 = 400k$$
$$\therefore k = 18 \div 400 = 0.045$$

So $W = 0.045L^2$

If the area is 225, then $L = \sqrt{225}$

$L = 15$

$$W = 0.045 \times 15^2$$
$$= 0.045 \times 225$$
$$= 10.125 \text{ kg}$$

11 $d \propto \sqrt{h}$

$d = k\sqrt{h}$

When $h = 1.8,\ d = 4.8$

$$4.8 = k \times \sqrt{1.8}$$
$$\therefore k = 4.8 \div \sqrt{1.8}$$
$$k = \frac{4.8}{\sqrt{1.8}}$$

If the 1.8m person climbed a 4m tower, the height, above sea level, would be 4 + 1.8 = 5.8 m.

$$d = \frac{4.8}{\sqrt{1.8}}\sqrt{h}$$
$$d = \frac{4.8}{\sqrt{1.8}} \times \sqrt{5.8}$$
$$= 8.62 \text{ m}$$

Solutions to 9B Now Try This Questions

4 $y \propto \frac{1}{x}$

$y = \frac{k}{x}$

When $x = 2,\ y = 1$

$1 = \frac{k}{2}$

$k = 2$

Thus $y = \frac{2}{x}$

When $x = 10$

$y = \frac{2}{10} = 0.2$

When $y = 0.1$

$0.1 = \frac{2}{x}$

$x = \frac{2}{0.1}$

$\therefore x = 20$

x	2	4	5	10	20
y	1	0.5	0.4	0.2	0.1

5 $t \propto \frac{1}{r}$

$t = \frac{k}{r}$

When $t = 2,\ r = 1200$

$2 = \frac{k}{1200}$

$\therefore k = 2400$

So $t = \frac{2400}{r}$

When $r = 2000$

$t = \frac{2400}{2000} = 1.2$

It will take 1.2 hours

1.2 hours = 1 hour 12 mins.

Solutions to Exercise 9B

1 **a** $y = \frac{20}{x}$

When $x = 5,\ y = \frac{20}{5} = 4$

When $y = 2,\ 2 = \frac{20}{x}$

$2x = 20$

$\therefore x = 10$

x	2	4	5	10
y	10	5	4	2

b $y = \frac{5}{x}$

When $x = 2,\ y = \frac{5}{2} = 2.5$

When $x = 4,\ y = \frac{5}{4} = 1.25$

When $y = 1,\ 1 = \frac{5}{x}$

$\therefore x = 5$

x	1	2	4	5
y	5	2.5	1.25	1

2 **a** $y = \frac{k}{x}$

b $y = \frac{k}{x^2}$

c $y = \frac{k}{x^3}$

d $m = \frac{k}{n}$

e $z = \frac{k}{w}$

f $y = \dfrac{k}{\sqrt{x}}$

3 a $A \propto \dfrac{1}{r}$

b $y \propto \dfrac{1}{x^2}$

c $y \propto \dfrac{1}{\sqrt{x}}$

d $m \propto \dfrac{1}{n^3}$

e $s \propto \dfrac{1}{t}$

4 a $y \propto \dfrac{1}{x}$

$y = \dfrac{k}{x}$

When $y = 10,\ x = 1$

$10 = \dfrac{k}{1}$

$\therefore k = 10$

Note: Multiplying x by y gives k value in inverse variation.

Thus $y = \dfrac{10}{x}$

When $x = 4,\ y = \dfrac{10}{4} = 2.5$

When $y = 1,\quad 1 = \dfrac{10}{x}$

$\therefore x = 10$

x	1	2	4	10
y	10	5	2.5	1

b $y \propto \dfrac{1}{x}$

$y = \dfrac{k}{x}$

When $x = 2,\ y = 1$

$1 = \dfrac{k}{2}$

$\therefore k = 2$

Thus $y = \dfrac{2}{x}$

When $x = 10,\ y = \dfrac{2}{10} = \dfrac{1}{5}$

When $y = \dfrac{1}{15},\ \dfrac{1}{15} = \dfrac{2}{x}$

$\therefore x = 30$

x	2	4	10	30
y	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{15}$

c $y \propto \dfrac{1}{x}$

$y = \dfrac{k}{x}$

When $x = 1,\ y = 1$

$1 = \dfrac{k}{1}$

$\therefore k = 1$

Thus $y = \dfrac{1}{x}$

When $x = 4,\ y = \dfrac{1}{4}$

When $y = \dfrac{1}{5},\quad \dfrac{1}{5} = \dfrac{1}{x}$

$\therefore x = 5$

x	1	2	4	5
y	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{5}$

d $y \propto \dfrac{1}{x}$

$y = \dfrac{k}{x}$

When $x = 0.5,\ y = 1$

$1 = \dfrac{k}{0.5}$

$\therefore k = 0.5$

Thus $y = \dfrac{0.5}{x}$

When $x = 5,\ y = \frac{0.5}{5} = 0.1$

When $y = 0.25,\ 0.25 = \frac{0.5}{x}$

$\therefore x = 2$

x	0.5	1	2	5
y	1	0.5	0.25	0.1

5 $y \propto \frac{1}{x}$

$y = \frac{k}{x}$

When $x = 5,\ y = 20$

$20 = \frac{k}{5}$

$\therefore k = 100$

Thus $y = \frac{100}{x}$

a When $x = 10,\ y = \frac{100}{10} = 10$

b When $y = 50,\quad 50 = \frac{100}{x}$

$\therefore x = 2$

6 $a \propto \frac{1}{b}$

$a = \frac{k}{b}$

When $a = 1, b = 2$

$1 = \frac{k}{2}$

$\therefore k = 2$

Thus $a = \frac{2}{b}$

a When $b = 4,\ a = \frac{2}{4} = \frac{1}{2}$

b When $a = \frac{1}{8},\quad \frac{1}{8} = \frac{2}{b}$

$\therefore b = 16$

7 $a \propto \frac{1}{b}$

$a = \frac{k}{b}$

When $a = 5,\ b = 2$

$5 = \frac{k}{2}$

$\therefore k = 10$

Thus $a = \frac{10}{b}$

a When $b = 4,\ a = \frac{10}{4} = 2.5$

b When $a = 1,\quad 1 = \frac{10}{b}$

$\therefore b = 10$

8 $I \propto \frac{1}{R}$

$I = \frac{k}{R}$

When $I = 3,\ R = 80$

$3 = \frac{k}{80}$

$\therefore k = 240$

Thus $I = \frac{240}{R}$

When $R = 100,\quad I = \frac{240}{100}$

$\therefore I = 2.4$ amps

9 $t \propto \frac{1}{n}$

$t = \frac{k}{n}$

When $n = 5,\ t = 40$

$40 = \frac{k}{5}$

$\therefore k = 200$

Thus $t = \frac{200}{n}$

When $n = 8, \quad t = \frac{200}{8}$

$\therefore t = 25$ mins

10 Let L = length, F = frequency vibration

$L \propto \frac{1}{F}$

$L = \frac{k}{F}$

When $L = 32.5, \; F = 400$

$32.5 = \frac{k}{400}$

$\therefore k = 13\,000$

Thus $L = \frac{13\,000}{F}$

When $L = 24.2, \; 24.2 = \frac{13\,000}{F}$

$\therefore F = 537$ cycles/sec

11 $V \propto \frac{1}{P}$

$V = \frac{k}{P}$

When $V = 22.5, \; P = 1.9$

$22.5 = \frac{k}{1.9}$

$\therefore k = 42.79$

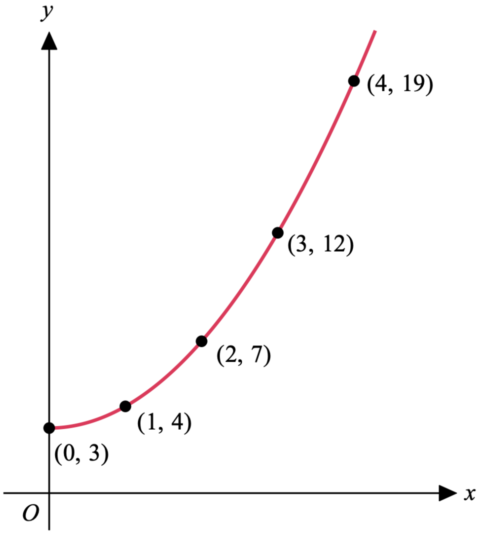
Thus $V = \frac{42.75}{P}$

When $V = 15, \quad 15 = \frac{42.75}{P}$

$\therefore P = 2.85 \text{ kg/cm}^2$

Solutions to 9C Now Try This Questions

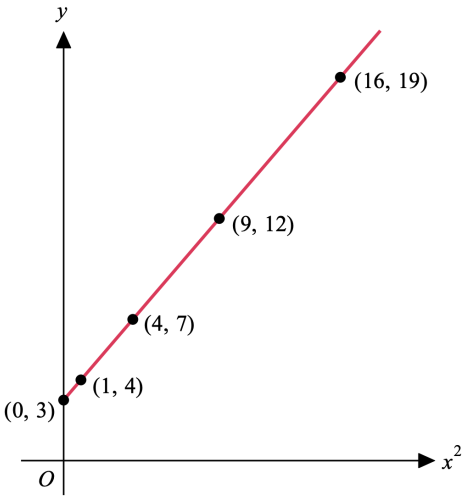
6 Plot corresponding x and y values on a graph.



This is clearly non-linear.
Square all x values.

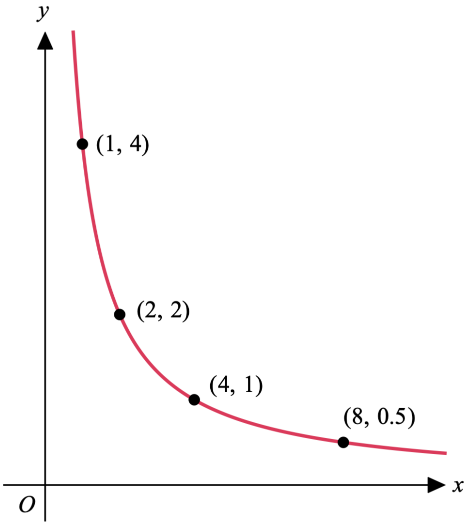
x	0	1	2	3	4
x^2	0	1	4	9	16
y	3	4	7	12	19

Plot corresponding x^2 and y values on a graph. Remember to label horizontal axis as x^2.



The graph is a straight line, so the graph has been linearised with the x^2 transformation.

7 Plot corresponding x and y values on a graph.

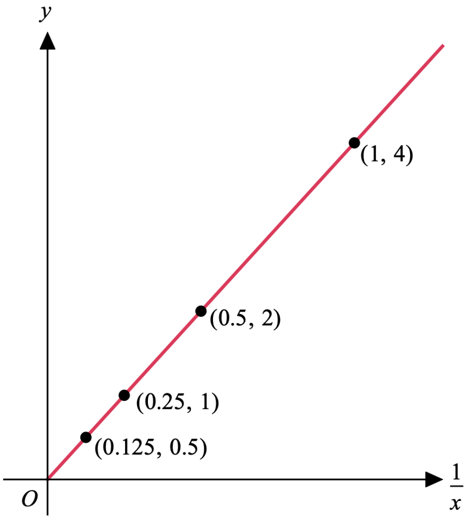


This is clearly non-linear.

Find the reciprocal $\left(\frac{1}{x}\right)$ of all x values.

x	1	2	4	8
$\frac{1}{x}$	1	0.5	0.25	0.125
y	4	2	1	0.5

Plot corresponding $\frac{1}{x}$ and y values on a graph. Remember to label horizontal axis as $\frac{1}{x}$.



The graph is a straight line, so the

graph has been linearised with the $\frac{1}{x}$ transformation.

8 Use a CAS calculator to plot graph of y against x^2. A linear graph is given.

9 Use a CAS calculator to plot graph of y against $\frac{1}{x}$. A linear graph is given.

Solutions to Exercise 9C

1 **a** In direct variation, if the values of x increase, the values of y increase.

b Direct variation graphs are straight (or linear) lines that pass through the origin.

c In inverse variation, as the values of x increase, the values of y decrease.

2 **a** As x increases, y decreases, so it is inverse variation.

b As x increases, y increases, so it is direct variation.

3 **a** As x increases, y increases so it is direct variation. It also passes through the origin.

b As x increases, y decreases, so it is inverse variation.

4 **a** Square each x value to give x^2. Write the reciprocal, $\frac{1}{x}$ for the last row in the table.

x	2	4	6	8
x^2	4	16	36	64
$\frac{1}{x}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$

b Square each x value to give x^2. Write the reciprocal, $\frac{1}{x}$ for the last row in the table.

x	10	20	30	40
x^2	100	400	900	1600
$\frac{1}{x}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{30}$	$\frac{1}{40}$

5 In both (i) and (ii), as x increases, y increases so they both must be direct variation.
In (iii), as x increases, y decreases so it is inverse variation, $y \propto \frac{1}{x}$.

a (ii) direct, $y \propto x$

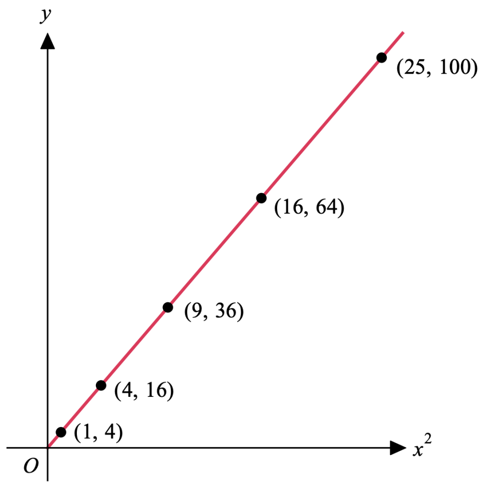
b (iii) inverse, $y \propto \frac{1}{x}$

c (i) direct, $y \propto x^2$

6 Square all x values.

x	1	2	3	4	5
x^2	1	4	9	16	25
y	4	16	36	64	100

Plot corresponding x^2 and y values on a graph, labelling horizontal axis as x^2.

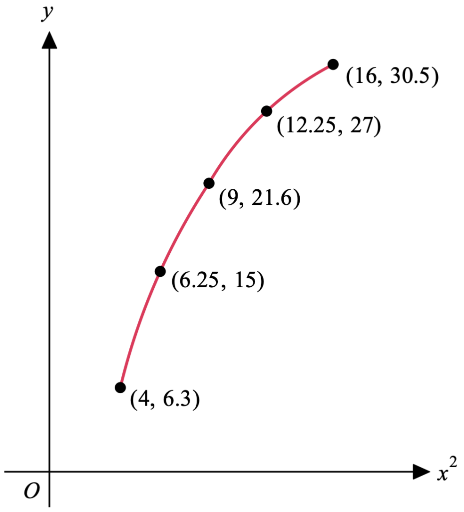


The graph is a straight line so it has been linearised with the x^2 transformation.

7 Square all x values.

x	2	2.5	3	3.5	4
x^2	4	6.25	9	12.25	16
y	6.3	15	21.6	27	30.5

Plot corresponding x^2 and y values on a graph, labelling horizontal axis as x^2.

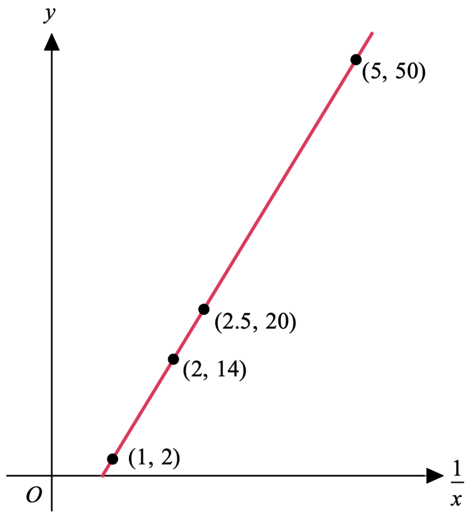


No, it is not linear.

8 Find the reciprocal $\frac{1}{x}$ of all x values.

x	0.2	0.4	0.5	1
$\frac{1}{x}$	5	2.5	2	1
y	50	20	14	2

Plot corresponding $\frac{1}{x}$ and y values on a graph, labelling the horizontal axis as $\frac{1}{x}$.



Yes it is linear.

9 Use CAS calculator. Yes it is linear.

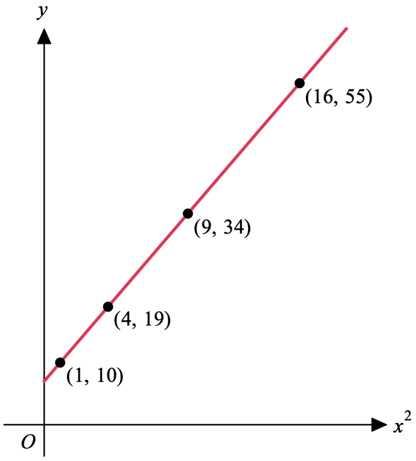
10 Use CAS calculator. Yes it is linear.

11 **a** Since the y values are increasing as x increases, it will be direct variation. You would use the x^2 transformation.

b Square all x values.

x	1	2	3	4
x^2	1	4	9	16
y	10	19	34	55

Plot corresponding x^2 and y values on a graph, labelling horizontal axis as x^2.



Yes it is linear.

12 $t \propto \frac{1}{n}$

$t = \frac{k}{n}$

When $t = 2, n = 1$

$2 = \frac{k}{1}$

$\therefore k = 2$

Thus $t = \frac{2}{n}$

Note: question gives table in minutes.

$k = 2$ hours $= 120$ mins

$t = \frac{120}{n}$

a

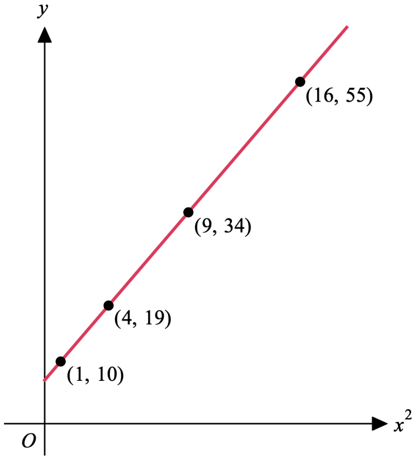
n	1	2	3	4	5	6
t	120	60	40	30	24	20

b As x increases, y decreases so it is an inverse variation. You would use $\frac{1}{x}$ transformation.

c Find the reciprocal of each n value.

n	1	2	3	4	5	6
$\frac{1}{n}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
t	120	60	40	30	24	20

Plot the graph of corresponding $\frac{1}{n}$ and t values with $\frac{1}{n}$ on the horizontal axis.



Solutions to 9D Now Try This Questions

10 10 000 = 10^4

log(10 000) = log(10^4) = 4

11 Use CAS calculator and enter log(245)

log(245) = 2.3891...

Round to one decimal place.

log(245) ≈ 2.4 to one d.p.

12 Use CAS calculator to evaluate $10^{2.8517}$.

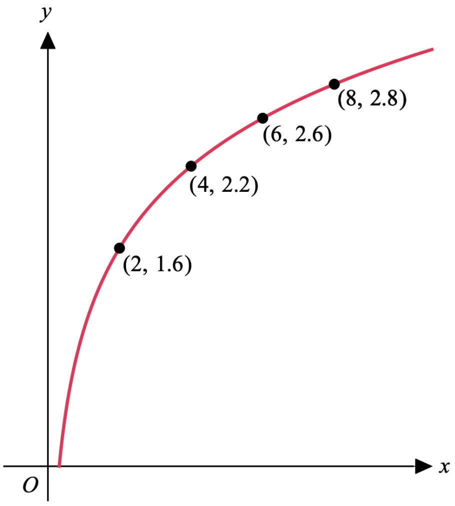
$10^{2.8517}$ = 710.7223...

≈ 710.7 to one decimal place.

13 35 500 = 3.55 × 10^4

The power of 10 is 4, so the order of magnitude of 35 500 is 4.

14 Plot corresponding x and y values on a graph.

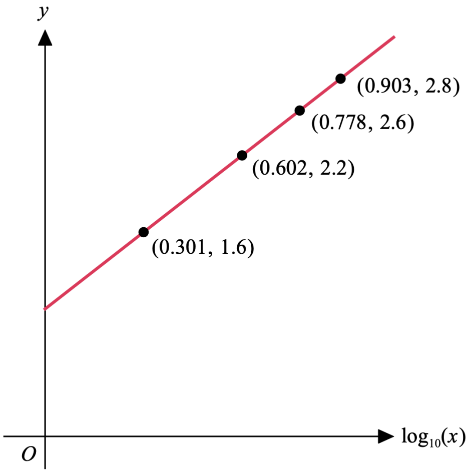


This is clearly non-linear.

Find $\log_{10}(x)$ of all x values.

x	2	4	6	8
$\log_{10}(x)$	0.301	0.602	0.778	0.903
y	1.6	2.2	2.6	2.8

Plot corresponding $\log_{10}(x)$ and y values on a graph. Remember to label horizontal axis as $\log_{10}(x)$.



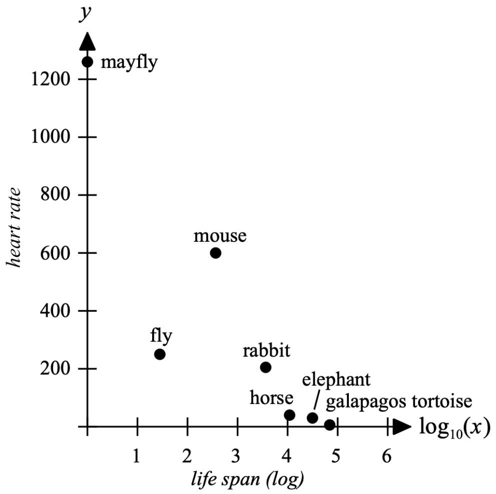
The graph is a straight line,

15 Use a CAS calculator to plot graph of y against $\log_{10}(x)$. A linear graph is given.

16 Find the logarithm of each animal's lifespan.

animal	lifespan	log	h.b./min
mayfly	1	0	1260
fly	28	1.45	250
mouse	365	2.56	600
rabbit	36 50	3.56	205
horse	10 959	4.04	40
elephant	31 390	4.50	30
G tortoise	69 350	4.84	6

Draw a graph by plotting the logarithms of the animal's life span on the horizontal axis and the heartbeat/minute on the vertical axis.



17 logarithm of weight of elephant = 6.70
logarithm of weight of tree kangaroo = 3.90
$6.70 - 3.90 = 2.8$
So an elephant is $10^{2.8} = 630.957... \approx 631$ times heavier than a tree kangaroo.

Solutions to Exercise 9D

1 **a** $100 = 10^2$

b $1000 = 10^3$

c $10 = 10^1$

d $1 = 10^0$

e $10\ 000 = 10^4$

2 **a** 5

b 8

c 0

d 9

e 4.5

3 **a** $10^{1.5} = 31.6227... = 31.62$

b $10^{2.2} = 158.4893... = 158.49$

c $10^{3.8} = 6309.5734... = 6309.57$

d $10^{0.7} = 5.0118... = 5.01$

e $10^{6.9} = 7\ 943\ 282.347$

$\approx 7\ 943\ 282.35$

4 **a** 4

b 33

c 100

5 **a** $1000 = 10^3$

$\log(10^3) = 3$

b $1\ 000\ 000 = 10^6$

$\log(10^6) = 6$

c $10\ 000\ 000 = 10^7$

$\log(10^7) = 7$

d $1 = 10^0$

$\log(10^0) = 0$

e $10 = 10^1$

$\log(10^1) = 1$

6 **a** $\log(300) = 2.4771\ldots$

≈ 2.477

b $\log(5946) = 3.7742\ldots$

≈ 3.774

c $\log(10390) = 4.0166\ldots$

≈ 4.017

d $\log(7.25) = 0.8603\ldots$

≈ 0.860

7 **a** $10^{2.5} = 316.2277\ldots$

≈ 316.23

b $10^{1.5} = 31.6227\ldots$

≈ 31.62

c $10^{0.5} = 3.1622\ldots$

≈ 3.16

d $10^0 = 1$

8 **a** $46\,000 = 4.6 \times 10^4$

Order of magnitude is 4.

b $559 = 5.59 \times 10^2$

Order of magnitude is 2.

c $3\,000\,000\,000 = 3.0 \times 10^9$

Order of magnitude is 9.

d 4.21×10^{12}

Order of magnitude is 12.

e $600\,000\,000\,000 = 6.0 \times 10^{11}$

Order of magnitude is 11.

9 $2 \times 4000 = 8000$

$8000 = 8.0 \times 10^3$

Order of magnitude is 3.

10 $35\,000 \times 8 = 280\,000$

$280\,000 = 2.8 \times 10^5$

Order of magnitude is 5.

11 **a** $9 \times 1000 = 9000$ screws

$9000 = 9.0 \times 10^3$

Order of magnitude is 3.

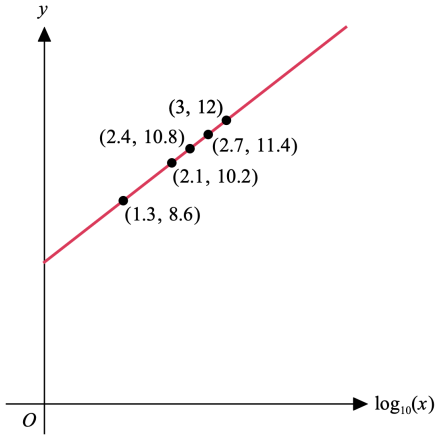
b $90 = 9.0 \times 10^1$

Order of magnitude is 1.

12 Find $\log_{10}(x)$ of all x values.

x	20	125	250	500	1000
$\log_{10}(x)$	1.30	2.10	2.40	2.70	3
y	8.6	10.2	10.8	11.4	12

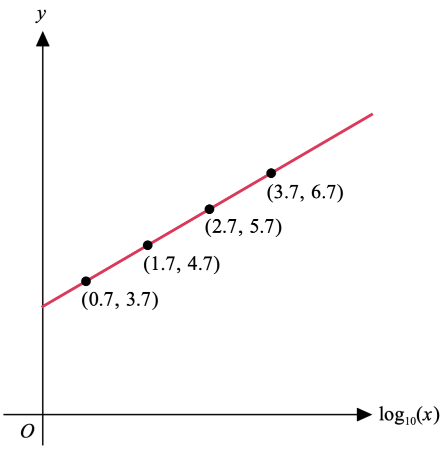
Plot corresponding $\log_{10}(x)$ and y values on a graph, labelling horizontal axis as $\log_{10}(x)$.



Yes, plotting the values of $\log_{10}(x)$ and y is linear.

13 Find $\log_{10}(x)$ of all x values.

Plot corresponding $\log_{10}(x)$ and y values on a graph, labelling horizontal axis as $\log_{10}(x)$.



Yes, plotting the values of $\log_{10}(x)$ and y is linear.

14 For shrew, body weight is represented by:

$\log(2.5) = 0.3979\ldots$

≈ 0.4

For tree kangaroo, body weight is represented by:

$\log(8000) = 3.9030$

≈ 3.9

$3.9 - 0.4 = 3.5$

$10^{3.5} = 3162.27\ldots$

≈ 3000

Weight of tree kangaroo is 3000 times heavier than shrew.

15 Diameter $= 2 \times 69\,910$

$= 139\,820$

$= 1.3982 \times 10^5$

The order of magnitude is 5.

16 $10^{2.3} = 199.5262...$

$= 199.53$ kg, correct to 2 d.p.

Solutions to 9E Now Try This Questions

18 $k = \text{Slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Using (9, 46) and (16, 81),

$$k = \text{slope} = \frac{81 - 46}{16 - 9} = \frac{35}{7} = 5$$

$$y = kx^2 + c$$

$$y = 5x^2 + c$$

Find the y-intercept (c). The graph cuts the y-axis at 1. So $c = 1$.

Thus $y = 5x^2 + 1$.

19 $k = \text{Slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Using (0.5, 4.5) and (1, 7),

$$k = \text{slope} = \frac{7 - 4.5}{1 - 0.5} = \frac{2.5}{0.5} = 5$$

$$y = \frac{k}{x} + c$$

$$y = \frac{5}{x} + c$$

To find the value of c choose a known value for x and y and substitute into the equation $y = \dfrac{5}{x} + c$.

Using (1,7), $7 = \dfrac{5}{1} + c$

$$7 = 5 + c$$

$$\therefore c = 2$$

Thus $y = \dfrac{5}{x} + 2$

20 $k = \text{Slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Using (1, 25) and (2, 35),

$$k = \text{slope} = \frac{35 - 25}{2 - 1} = \frac{10}{1} = 10$$

$$y = k\log_{10}(x) + c$$

$$\therefore y = 10\log_{10}(x) + c$$

To find the value of c choose a known value for x and y and substitute into the equation $y = 10\log_{10}(x) + c$.

Using (10, 25), $25 = 10\log_{10}(10) + c$

$$25 = 10 \times 1 + c$$

$$25 = 10 + c$$

$$\therefore c = 15$$

Thus $y = 10\log_{10}(x) + 15$.

Solutions to Exercise 9E

1 **a** From the graph, the y-intercept is 4.

$\text{Slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Using (0, 4) and (3, 13)

$$\text{Slope} = \frac{13 - 4}{3 - 0} = \frac{9}{3} = 3$$

b From the graph, the y-intercept is 2.

$\text{Slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Using (0,2) and (3,14)

$$\text{Slope} = \frac{14 - 2}{3 - 0} = \frac{12}{3} = 4$$

2 $y = 3x^2 + 7$

a Read from the equation. Slope is 3

b y-intercept is 7

3 $y = \frac{2}{x} - 5$

a Read from equation. Slope = 2

b y intercept = −5

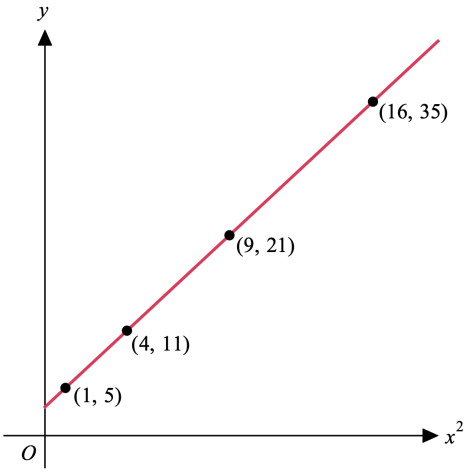
4 $y = 4\log_{10}(x) + 3$

a Read from equation. Slope = 4

b y-intercept = 3

5

x	1	2	3	4
x^2	1	4	9	16
y	5	11	21	35



Use any 2 points to find slope of line. Using (1, 5) and (4, 11),

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{11 - 5}{4 - 1} = \frac{6}{3} = 2$$

$\therefore y = 2x^2 + c$

To find c, use any pair of corresponding values for x^2 and y.

Using (4, 11), $11 = 2(4) + c$

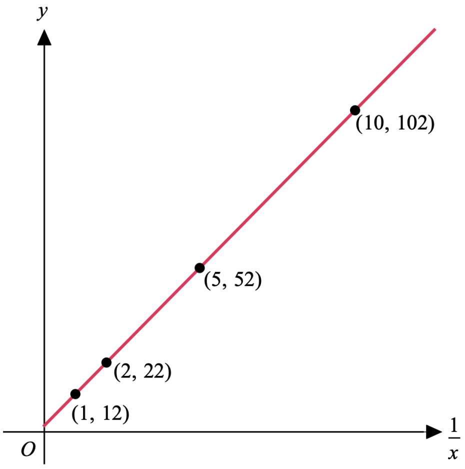
$11 = 8 + c$

$\therefore c = 3$

The equation of line is $y = 2x^2 + 3$.

6

x	0.1	0.2	0.5	1
$\frac{1}{x}$	10	5	2	1
y	102	52	22	12



Use any 2 points to find slope of line. Using (1, 12) and (2, 22),

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{22 - 12}{2 - 1} = \frac{10}{1} = 10$$

$\therefore y = \frac{10}{x} + c$

To find c, use any pair of corresponding values for $\frac{1}{x}$ and y.

Using (1, 12), $12 = \frac{10}{1} + c$

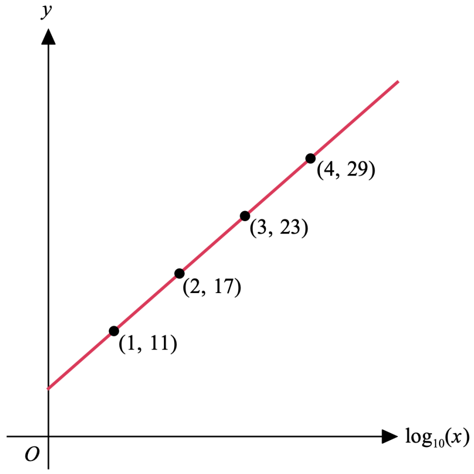
$12 = 10 + c$

$\therefore c = 2$

The equation of line is $y = \frac{10}{x} + 2$.

7

x	10	100	1000	10000
$\log_{10}(x)$	1	2	3	4
y	11	17	23	329



Use any 2 points to find slope of line.
Using (1, 11) and (2, 17),

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{17 - 11}{2 - 1} = \frac{6}{1} = 6$$

$\therefore y = 6\log_{10}(x) + c$

To find c, use any pair of corresponding values for $\log_{10}(x)$ and y.

Using (2, 17), $17 = 6(2) + c$

$17 = 12 + c$

$\therefore c = 5$

The equation of line is
$y = 6\log_{10}(x) + 5$.

8 Square each t value.

t	1	5	10	20
t^2	1	25	100	400
P	15	135	510	2010

Equation will be of the form:
$P = kt^2 + c$ where k is the slope.

Use any 2 points to find slope of line.

Using (1, 15) and (25, 135),

$$k = \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{135 - 15}{25 - 1} = \frac{120}{24} = 5$$

$\therefore P = 5t^2 + c$

To find c, use any pair of corresponding values for t^2 and y.
Using (25, 135), $135 = 5(25) + c$

$135 = 125 + c$

$\therefore c = 10$

The rule that allows population, P, to be predicted from time, t, is
$P = 5t^2 + 10$.

9 **a** Find the reciprocal $\frac{1}{v}$ of each v value.

v	60	80	100
$\frac{1}{v}$	0.017	0.0125	0.01
t	3.9	2.9	2.3

Equation will be of the form:
$t = \frac{k}{v} + c$ where k is the slope.
Use any 2 points for $\frac{1}{v}$ and t to find slope of line.
Using (0.01, 2.3) and (0.0125, 2.9),

$$k = \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2.9 - 2.3}{0.0125 - 0.01}$$

$$= \frac{0.6}{0.0025} = 240$$

$\therefore P = \frac{240}{v} + c$

To find c, use any pair of corresponding values for v and t.

Using (60, 3.9), $3.9 = \frac{240}{60} + c$

$3.9 = 4 + c$

$\therefore c = -0.1$

The rule that describes the time taken to travel, t and the average speed, v is $t = \frac{240}{v} - 0.1$.

b $t = \frac{240}{v} - 0.1$

$t = 2$ hours, 6 mins $= 2\frac{6}{60}$ hours

$= 2.1$ hours

Substitute $t = 2.1$ into $t = \frac{240}{v} - 0.1$ and solve for v.

$2.1 = \frac{240}{v} - 0.1$

$2.2 = \frac{240}{v}$

$\therefore v = 109.09 = 109$ km/hr

10 Find $\log_{10}(t)$ of each v value.

t	1	10	100	500	1000
$\log_{10}(t)$	0	1	2	2.70	3
N	15	115	215	285	315

Equation will be of the form:
$N = k\log_{10}(t) + c$ where k is the slope.

Use any 2 points for $\log_{10}(t)$ and y to find slope of line.
Using (0, 15) and (1, 115),

$$k = \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{115 - 15}{1 - 0} = \frac{100}{1} = 100$$

$\therefore N = 100\log_{10}(t) + c$

To find c, use any pair of corresponding values for $\log_{10}(t)$ and N.
Using (0, 15), $15 = 100 \times 0 + c$

$\therefore c = 15$

$\therefore N = 100\log_{10}(t) + 15$

Solutions to Skills Checklist Questions

1 $d \propto t$

$d = kt$

When $t = 1, d = 95$

$95 = k(1)$

$k = 95$

2 $F \propto \dfrac{1}{d}$

$F = \dfrac{k}{d}$

When $d = 10, F = 10$

$10 = \dfrac{k}{10}$

$k = 10 \times 10$

$\therefore k = 100$

3 Since y is decreasing as x increases, the graph shows inverse variation.

4

x	1	2	5
x^2	1	4	25
y	12	27	132

Draw a graph on CAS with x^2 on horizontal axis and y on vertical axis. Graph produced is a linear graph (straight line) so it has been linearised.

5

x	0.5	0.2	0.1
$\frac{1}{x}$	2	5	10
y	5	35	85

Draw a graph on CAS with $\frac{1}{x}$ on horizontal axis and y on vertical axis. Graph produced is a linear graph (straight line) so it has been linearised.

6 $\log_{10}(592) = 2.772... \approx 2.77$ correct to 2 d.p.

7 $478\,000 = 4.78 \times 10^5$

The order of magnitude is 5.

8 For an elephant, body weight of 5 000 000 is represented by:

$\log(5\,000\,000) = 6.6989\ldots$

≈ 6.7

For a monkey, body weight of 5 000 is represented by:

$\log(5\,000) = 3.689...$

≈ 3.7

$6.7 - 3.7 = 3.0$

$10^3 = 1000$

An elephant is 1000 times heavier than a monkey.

9 $10^{2.1567} = 143.449... \approx 143.4$

10

x	10	100	1000
$\log_{10}(x)$	1	2	3
y	3	6	9

Draw a graph on CAS with $\log_{10}(x)$ on horizontal axis and y on vertical axis. Graph produced is a linear graph (straight line) so it has been linearised.

11 Use any pair of corresponding points of x^2 and y to find slope of line.

(1, 23) and (100, 320)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{320 - 23}{100 - 1} = \frac{297}{99} = 3$$

$y = 3x^2 + c$

To find c, use any pair of corresponding values for x^2 and y.

Using $(1, 23)$, $\quad 23 = 3(1) + c$

$$23 = 3 + c$$

$$\therefore c = 20$$

The equation of line is $y = 3x^2 + 20$.

Solutions to Chapter Review Multiple-Choice Questions

1 $\quad y \propto x$

$y = kx$

Choose a value for x and its corresponding y-value and substitute in equation $y = kx$.

Using $(3, 12)$, $\quad 12 = 3k$

$\therefore k = 4$ **C**

2 $\quad y \propto x$

$y = kx$

Choose a value for x and its corresponding y-value and substitute in equation $y = kx$.

Using $(2, 20)$, $\quad 20 = 2k$

$\therefore k = 10$ **C**

3 $\quad y \propto \frac{1}{x}$

$y = \frac{k}{x}$

Choose a value for x and its corresponding y-value and substitute in equation $y = \frac{k}{x}$.

Using $(1, 3)$, $\quad 3 = \frac{k}{1}$

$\therefore k = 3$ **E**

4 $\quad y \propto x^2$

$y = kx$

Choose a value for x and its corresponding y-value and substitute in equation $y = kx^2$.

Using $(6, 12)$, $\quad 12 = k(6^2)$

$$36k = 12$$

$\therefore k = \frac{12}{36} = \frac{1}{3}$ **C**

5 $\quad y \propto \frac{1}{x}$

$y = \frac{k}{x}$

Choose a value for x and its corresponding y-value and substitute in equation $y = \frac{k}{x}$.

Using $\left(2, \frac{1}{4}\right)$, $\quad \frac{1}{4} = \frac{k}{2}$

$$\frac{1}{4} \times 2 = \frac{k}{2} \times 2$$

$$\frac{2}{4} = k$$

$\therefore k = \frac{1}{2}$ **A**

6 $\quad a \propto b$

$a = kb$

When $a = 18$, $b = 3$

$18 = 3k$

$\therefore k = 6$

Thus $a = 6b$

When $b = 7$, $\; a = 6 \times 7$

$= 42$ **E**

7 $a \propto b^2$

$a = kb^2$

When $a = 32$, $b = 2$

$32 = k(2^2)$

$32 = 4k$

$\therefore k = 8$

Thus $a = 8b^2$

$$\begin{aligned} \text{When } b = 4, a &= 8 \times 4^2 \\ &= 8 \times 16 \\ &= 128 \end{aligned}$$

E

8 $p \propto \dfrac{1}{q}$

$p = \dfrac{1}{q}$

When $p = \dfrac{1}{3}$, $q = 3$

$\dfrac{1}{3} = \dfrac{k}{3}$

$\therefore k = 1$

Thus $p = \dfrac{1}{q}$

When $p = 1$, $\quad 1 = \dfrac{1}{q}$

$\therefore q = 1$

D

9 $y = kx^2 + c$

Square all x-values.

x	1	2	3	4
x^2	1	4	9	16
y	4	22	52	94

k = slope.

To find k, use any two x^2 values with their corresponding y-values.

Using (1, 4) and (4, 22),

$$\begin{aligned} k = \text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{22 - 4}{4 - 1} = \frac{18}{3} = 6 \end{aligned}$$

Thus $y = 6x^2 + c$

To find c, select any $\dfrac{1}{x}$ value with its corresponding y-value and substitute into $y = 6x^2 + c$.

$$\begin{aligned} \text{Using (4, 22),} \quad 22 &= 6(4) + c \\ 22 &= 24 + c \\ \therefore c &= -2 \end{aligned}$$

So $k = 6$, $c = -2$. **B**

10 $y = \dfrac{k}{x} + c$

Find the reciprocal of all x-values.

x	1	2	4	5
$\frac{1}{x}$	1	0.5	0.25	0.2
y	6	3.5	2.25	2

k = slope.

To find k, use any two $\dfrac{1}{x}$ values with their corresponding y-values.

Using (1, 6) and (0.5, 3.5),

$$\begin{aligned} k = \text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3.5 - 6}{0.5 - 1} = \frac{-2.5}{-0.5} = 5 \end{aligned}$$

Thus $y = \dfrac{5}{x} + c$

To find c, select any x value with its corresponding y-value and substitute into $y = \dfrac{5}{x} + c$.

$$\begin{aligned} \text{Using (5, 2),} \quad 2 &= \frac{5}{x} + c \\ 2 &= \frac{5}{5} + c \\ 2 &= 1 + c \\ \therefore c &= 1 \end{aligned}$$

So $k = 5$, $c = 1$. **A**

11 $y = k\log_{10}(x) + c$

Find $\log_{10}(x)$ for all x-values.

x	1	10	100	1000
$\log_{10}(x)$	0	1	2	3
y	50	350	650	950

k = slope.

To find k, use any two $\log_{10}(x)$ values with their corresponding y-values.

Using (0, 50) and (1, 350),

$$k = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{350 - 50}{1 - 0} = \frac{300}{1} = 300$$

Thus $y = \frac{5}{x} + c$

To find c, select any $\log_{10}(x)$ value with its corresponding y-value and substitute into $y = 300\log_{10}(x) + c$.

Using (2, 650),

$650 = 300 \times 2 + c650 \quad = 600 + c$

$\therefore c = 50$

So $k = 300,\ c = 50$. **E**

Solutions to Chapter Review Short-Answer Questions

1 $a \propto b$

$a = kb$

When $a = 8,\ b = 2$

$8 = 2k$

$\therefore k = 4$

a $a = 4b$

When $b = 50$,

$a = 4 \times 50$

$= 200$

b $a = 4b$

When $a = 60$,

$60 = 4b$

$\therefore b = \frac{60}{4} = 15$

2 $y \propto x^2$

$y = kx^2$

When $y = 108,\ x = 6$

$108 = k(6^2)$

$108 = 36k$

$\therefore k = \frac{108}{16} = 3$

a $y = 3x^2$

When $x = 4$,

$y = 3(4^2)$

$= 48$

b $a = 4b$

When $y = 90$,

$90 = 3x^2$

$30 = x^2$

$\therefore x = \sqrt{30} = 5.48$

3 $y \propto \frac{1}{x}$

$y = \frac{k}{x}$

When $y = 3,\ x = 2$

$3 = \frac{k}{2}$

$\therefore k = 6$

a $y = \frac{6}{x}$

When $x = \frac{1}{2}$,

$y = 6 \div \frac{1}{2}$

$= 6 \times 2$

$= 12$

b $y = \frac{6}{x}$

When $y = \frac{2}{3}, \frac{2}{3} = \frac{6}{x}$

Cross-multiplying gives:

$2 \times x = 3 \times 6$

$\therefore x = 9$

4 **a** $d \propto t^2$

$d = kt^2$

When $d = 78.56,\ t = 4$

$78.56 = k(4^2)$

$78.56 = 16k$

$\therefore k = \frac{78.56}{16} = 4.91$

b $d = 4.1t^2$

c When $t = 10,\ d = 4.91(10)^2$

$= 4.91 \times 100$

$= 491$ metres

d When $d = 19.64$,

$19.64 = 4.91t^2$

$\frac{19.64}{4.91} = t^2$

$t^2 = 4$

$\therefore t = \sqrt{4} = 2$ secs

5 Let t = time taken
and v = average speed

$t \propto \frac{1}{v}$

$t = \frac{k}{v}$

When $t = 4,\ v = 30$

$4 = \frac{k}{30}$

$\therefore k = 4 \times 30 = 120$

Thus $t = \frac{120}{v}$

When $v = 50,\ t = \frac{120}{50}$

$= 2.4$ hours

0.4 hours $= 0.4 \times 60$ mins

$= 24$ mins.

Thus time taken is 2.4 hours
or 2 hrs 24 mins.

6 $V \propto I$

$V = kI$

When $V = 24,\ I = 6$

$24 = 6k$

$\therefore k = 4$

Thus $V = 4I$

When $V = 72$,

$72 = 4I$

$\therefore I = \frac{72}{4} = 18$

Current is 18 amps.

7 Let N = number of square tiles
and L = side length of tile

$N \propto \frac{1}{L^2}$

$N = \frac{k}{L^2}$

When $N = 2016,\ L = 0.4$

$$2016 = \frac{k}{(0.4)^2}$$

$$k = 2016 \times (0.4)^2$$

$$\therefore k = 322.56$$

Thus $N = \dfrac{322.56}{L^2}$

When $L = 0.3$,

$$N = \frac{322.56}{(0.3)^2}$$

$$= 3584 \text{ tiles}$$

8 Let t = time
and v = volume

$$t \propto \frac{1}{v}$$

$$t = \frac{k}{v}$$

When $t = 45,\ v = 22$

$$45 = \frac{k}{22}$$

$$k = 45 \times 22$$

$$\therefore k = 990$$

Thus $t = \dfrac{990}{v}$

When $t = 30$,

$$30 = \frac{990}{v}$$

$$\therefore v = 33$$

Flow rate is 33 litres/min

9 $6 - 3 = 3$
$10^3 = 1000$
An earthquake of magnitude 6 is 1000 times stronger than an earthquake of magnitude 3.

Solutions to Chapter Review Written-Response Questions

1 Let M = mass and d = diameter.

$$M \propto d^2$$

$$M = kd^2$$

When $M = 5,\ d = 10$

$$5 = k(10)^2$$

$$5 = 100k$$

$$k = \frac{5}{100} = 0.05$$

So $M = 0.05d^2$

a Diameter of second sphere is 14 cm.

When $d = 14$, $M = 0.05(14)^2$

$$= 0.05 \times 196$$

$$= 9.80 \text{ kg}$$

b Mass of third sphere is 6 kg.

When $M = 6,\ 6 = 0.05d^2$

$$\frac{6}{0.05} = d^2$$

$$d^2 = 120$$

$$d = \sqrt{120} = 10.95 \text{ cm}$$

2 $t \propto \frac{1}{n}$

$t = \frac{k}{n}$

When $n = 6,\ t = 20$

$20 = \frac{k}{6}$

$\therefore k = 120$

So $t = \frac{120}{n}$

a When $n = 4$, $t = \frac{120}{4}$

$= 30$ days

b When $t = 15$, $15 = \frac{120}{n}$

$n = \frac{120}{15}$

$\therefore n = 8$ painters

3 **a** $V \propto \frac{1}{P}$

$V = \frac{k}{P}$

When $V = 43.5,\ P = 2.8$

$43.5 = \frac{k}{2.8}$

$k = 43.5 \times 2.8$

$\therefore k = 121.8$

So $V = \frac{121.8}{P}$

b $V = \frac{121.8}{P}$

When $V = 12.7$, $12.7 = \dfrac{121.8}{P}$

$$\therefore P = \frac{121.8}{12.7}$$

$$= 9.59 \text{ kg/cm}^2$$

4 **a** $w \propto \dfrac{1}{d}$

$w = \dfrac{k}{d}$

When $d = 6$, $w = 500$

$500 = \dfrac{k}{6}$

$\therefore k = 500 \times 6 = 3000$

So $w = \dfrac{3000}{d}$

b When $d = 5$, $w = \dfrac{3000}{5}$

$$= 600\text{kg}$$

c When $d = 9$, $w = \dfrac{3000}{9} = 333.333....$

$$\approx 333.33 \text{ kg}$$

5 **a** As the values for P are increasing, the values for V are decreasing so it is inverse variation.

$P \propto \dfrac{1}{V}$

$P = \dfrac{k}{V}$

Choose any value for V and its corresponding P value and substitute into equation $P = \dfrac{k}{V}$ to find k.

Using (8, 18), $8 = \dfrac{k}{18}$

$$\therefore k = 8 \times 18 = 144$$

A possible equation relating P and V is $P = \dfrac{144}{V}$

b $P = \dfrac{144}{V}$

i When $P = 72$, $72 = \dfrac{144}{v}$

$$\therefore V = 2 \text{ units}$$

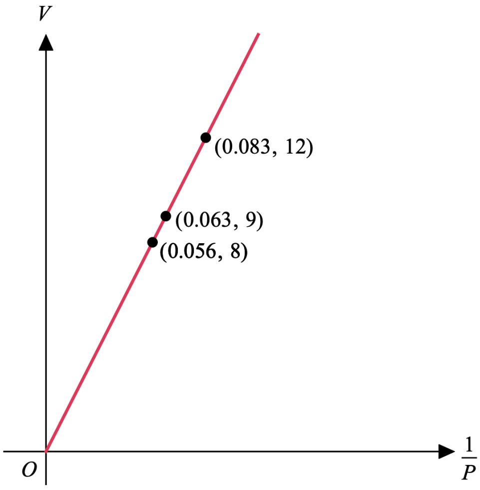
ii When $V = 3$, $P = \frac{144}{3}$

$\therefore P = 48$ units

c Find the reciprocal of all P-values.

Pressure P	12	16	18
$\frac{1}{P}$	$\frac{1}{12}$	$\frac{1}{16}$	$\frac{1}{18}$
Volume V	12	9	8

Plot corresponding $\frac{1}{P}$ and V values on graph ensuring that horizontal axis is labelled as $\frac{1}{P}$.



Chapter 10 – Shape and measurement

Solutions to 10A Now Try This Questions

1 Round 57 642 to the nearest thousand.
Look at the digit to the right of the thousands. It is a 6.
As it is 5 or more, increase the thousands by one, making 58 thousand.
The digits to the right all become zero, 58 000.
So 57 642 to the nearest thousand is 58 000.

2 Round 43.632 697 to two decimal places.
For two decimal places, count two places to the right of the decimal point and look at the digit to the right (2).
As 2 is not '5 or more' do not increase the second decimal place.
So 43.632 697 to two decimal places is 43.63

3 Write in scientific notation

a 670 000
Place a decimal point to the right of the first non-zero digit.
6.70 000
Count the number of places the decimal point needs to move and whether it is to the left or right.
To move the decimal point 5 places to the right, we need to multiply by 10^5.
Write your answer.
670 000
$= 6.7 \times 10^5$

b 0.000 006
Place a decimal point to the right of the first non-zero digit.
6.0
Count the number of places the decimal point needs to move and whether it is to the left or right.
To move the decimal point 6 places to the right, we need to multiply by 10^{-6}.
Write your answer.
0.000 006
$= 6.0 \times 10^{-6}$

4 Write a scientific notation number as a basic numeral.

a 4.231×10^6
4.231 by 10^6 means that the decimal point needs to be moved 6 places to the right.
Move the decimal point 6 places to the right and write your answer.
Zeroes will need to be used as places holders.
4.231×10^6
= 4 231 000

b 8.2×10^{-4}
8.2 by 10^{-4} means that the decimal point needs to be moved 4 places to the left.
Move the decimal point 4 places to the right and write your answer.
Zeroes will need to be used as places holders.
8.2×10^{-4}
= 0.00082

5 Round to 3 significant figures.

a 57.892 607

To write in scientific notation, put a decimal point after the first non-zero digit, and multiply by the required power of 10.

57.892 607

$= 5.7892607 \times 10^1$

To round to the third significant figure, check if the fourth digit is 5 or more.

The fourth digit (9) is greater than 5, so the third digit must be increased by 1 (so it changes from 8 to 9).

5.79×10^1

$= 57.9$

b 0.000 471 68

To write in scientific notation, put a decimal point after the first non-zero digit, and multiply by the required power of 10.

0.000 471 68

$= 4.7168 \times 10^{-4}$

To round to the third significant figure, check if the fourth digit is 5 or more.

The fourth digit (6) is greater than 5, so the third digit must be increased by 1 (so it changes from 1 to 2).

4.72×10^{-4}

$= 0.000\ 472$

Solutions to Exercise 10A

1 Count the number of digits to the right of the decimal point.

a 3

b 2

c 3

d 2

e 4

2 If the number to the right of the decimal point is 5 or greater, increase the whole number by 1.

a 4

b 12

c 67

d 557

3 The size of the positive power of 10 tells you the number of shifts the decimal point must move to the right.

a 3

b 5

c 4

4 The size of the negative power of 10 tells you the number of shifts the decimal point must move to the left.

a 3

b 1

c 2

5 a 482
4**8**2
Since the number (8) to the right of the hundreds is greater than 5, round up.
Answer is 500.

b 46 770
46 **7**7**0**

Since the number (7) to the right of the hundreds is greater than 5, round up.
Answer is 46 800.

c 79 399
79 3**9**9
Since the number (9) to the right of the hundreds is greater than 5, round up.
Answer is 79 400.

d 313.4
3**1**3.4
Since the number (1) to the right of the hundreds is less than 5, leave the 3 unchanged.
Answer is 300.

6 a $689.**7**9
Since the number (7) to the right of the dollar units is greater than 5, round up.
Answer is $690.

b $20.**4**5
Since the number (4) to the right of the dollar units is not greater than 5, leave the 0 unchanged.
Answer is $20.

c $927.**5**8
Since the number (5) to the right of the dollar units is 5, round up.
Answer is $928.

d $13.**5**0
Since the number (5) to the right of the dollar units is 5, round up.
Answer is $14.

7 a 3.185×0.49 (2)
Use CAS calculator
$= 1.5605$
The number after “6” is “0”, which is less than 5, so leave “6” unchanged.
$= 1.56$ correct to 2 dec. places.

b $0.064 \div 2.536$ (3)
Use CAS calculator
$= 0.02523\ldots$
The number after “5” is “2”, which is less than 5, so leave “5” unchanged.
$= 0.025$ correct to 3 dec. places.

c 0.474×0.0693 (2)
Use CAS calculator
$= 0.0328482$
The number after “3” is “2”, which is less than 5, so leave “3” unchanged.
$= 0.03$ correct to 2 dec. places.

d $12.943 \div 6.876$ (4)
Use CAS calculator
$= 1.882344\ldots$
The number after “3” is “4”, which is less than 5, so leave “3” unchanged.
$= 1.8823$ correct to 4 dec. places.

8 a $\sqrt{7^2 + 14^2}$
Use CAS calculator
$= 15.6524\ldots$
The number after “5” is “2”, which is less than 5 so leave “5” unchanged.

= 15.65 correct to 2 dec. places.

b $\sqrt{(3.9)^2 + (2.6)^2}$
Use CAS calculator
= 4.687…
The number after “8” is “7”, which is greater than 5, so round up.
= 4.69 correct to 2 dec. places.

c $\sqrt{48.71^2 - 29^2}$
Use CAS calculator
= 39.136…
The number after “3” is “6”, which is greater than 5, so round up.
= 39.14 correct to 2 dec. places.

9 Set up your CAS calculator to display answers in scientific notation.

a 792 000
$= 7.92 \times 10^5$

b 14 600 000
$= 1.46 \times 10^7$

c 500 000 000 000
$= 5.0 \times 10^{11}$

d 0.000 009 8
$= 9.8 \times 10^{-6}$

e 0.145 697
$= 1.45697 \times 10^{-1}$

f 0.000 000 000 06
$= 6.0 \times 10^{-11}$

g 2 679 886
$= 2.679\ 886 \times 10^6$

h 0.0087
$= 8.7 \times 10^{-3}$

10 **a** 5.3467×10^4
Decimal point will move 4 places to the right.
Answer is 53 467.

b 3.8×10^6
Decimal point will move 6 places to the right.
Answer is 3 800 000.

c 7.89×10^5
Decimal point will move 5 places to the right.
Answer is 789 000.

d 9.21×10^{-3}
Decimal point will move 3 places to the left.
Answer is 0.009 21.

e 1.03×10^{-7}
Decimal point will move 7 places to the left.
Answer is 0.000 000 103.

f 2.907×10^6
Decimal point will move 6 places to the right.
Answer is 2 907 000.

g 3.8×10^{-12}
Decimal point will move 12 places to the left.
Answer is 0.000 000 000 003 8.

h 2.1×10^{10}
Decimal point will move 10 places to the right.
Answer is 21 000 000 000.

11 **a** 6 000 000 000 000 000 000 000 000
$= 6.0 \times 10^{24}$

b 40 000 000
$= 4.0 \times 10^{7}$

c 0.000 000 000 1
$= 1.0 \times 10^{-10}$

d 150 000 000
$= 1.5 \times 10^{8}$

12 **a** 4.8736 (2)
To round to 2 significant figures, check if the third digit (7) is 5 or greater.
It is, so round up.
Answer is 4.9

b 0.078 74 (3)
Write in scientific notation.
$= 7.874 \times 10^{-2}$
To round to the third significant figure, check if the fourth digit (4) is 5 or greater.
It is not, so retain the three digits.
$= 7.87 \times 10^{-2}$
= 0.0787
Answer is 0.0787

c 1506.862 (5)
Write in scientific notation.
$= 1.506\ 862 \times 10^{3}$
To round to the fifth significant figure, check if the sixth digit (6) is 5 or greater.
It is, so round up.
$= 1.5069 \times 10^{3}$
= 1506.9
Answer is 1506.9.

d 5.**5**23 (1)
To round to one significant figure, check if the second digit (5) is 5 or greater.
It is, so round up.
Answer is 6.

13 **a** $4.3968 \times 0.000\ 7453\ 8$ (2)
Use CAS calculator and answer shown is 3.2703398E – 3 which means
3.2703398×10^{-3}
To round to 2 significant figures, check if the third digit (7) is 5 or greater.
It is, so round up.
3.3×10^{-3}
= 0.0033
The answer is 0.0033

b $0.61135 \div 4.1119$ (5)
Use CAS calculator
$= 1.48678266 \times 10^{-1}$
Give answer to 5 sig. figs.
To round to five significant figures, check if the sixth digit (8) is 5 or greater.
It is, so round up.
1.4868×10^{-1}
= 0.14868
The answer is 0.148 68

c $3.4572 \div 0.0109$ (3)
Use CAS calculator
$= 3.1717431 \times 10^{2}$
To round to three significant figures, check if the fourth digit (1) in 3.171 is 5 or greater.
It is not, so leave the 7 in 3.17 unchanged.
$= 3.17 \times 10^{2}$
= 317
The answer is 317

d $50\ 042 \times 0.0067$ (3)
Use CAS calculator
$= 3.352814 \times 10^{2}$
To round to three significant figures,

check if the fourth digit (2) is 5 or greater.
It is not, so leave the 5 in 3.35 unchanged.
$= 3.35 \times 10^2$
= 335
The answer is 335

14 To round 35.8997 to three decimal places count three digits to the right of the decimal point and look at the fourth digit (7). As 7 is "5 or greater than 5", round up.
The rounding up carries over to the 8 which rounds up to a 9, making '900' in the three decimal places.
Answer is 35.900

15 Write each number as a basic numeral.
A 0.787 **B** 2.0 **C** 0.05
D 0.00321 **E** 0.67
From the basic numeral versions it is clear that the smallest number is:
D 0.00321

16 To find significant figures, first express the number in scientific notation. Count the required number of digits as significant figures and round up if the next digit is 5 or more.

To express in the required number of decimal places, count the required number of digits to the right of the decimal point and look at the next digit. If it is 5 or greater, round up.

a **i** 421.389
$= 4.21389 \times 10^2$
$= 4.2 \times 10^2$
= 420 to two significant figures

ii 421.389
= 421.39 to two decimal places

b **i** 64.031
$= 6.4031 \times 10^1$
$= 6.40 \times 10^1$
= 64.0 to three significant figures

ii 64.031
= 64.031 to three decimal places

c **i** 5090.0493
$= 5.0900493 \times 10^3$
$= 5.09 \times 10^3$
= 5090 to three significant figures

ii 5090.0493
= 5090.049 to three decimal places

d **i** 70.549
$= 7.0549 \times 10^1$
$= 7.1 \times 10^1$
= 71 to two significant figures

ii 70.549
= 70.55 to two decimal places

e **i** 0.4573
$= 4.573 \times 10^{-1}$
$= 4.6 \times 10^{-1}$
= 0.46 to two significant figures

ii 0.4573
= 0.46 to two decimal places

f **i** 0.405
$= 4.05 \times 10^{-1}$
$= 4.1 \times 10^{-1}$
= 0.41 to two significant figures

ii 0.405
= 0.41 to two decimal places

Solutions to 10B Now Try This Questions

6 By Pythagoras' theorem

$c^2 = a^2 + b^2$

$c^2 = 5^2 + 8^2$

$c = \sqrt{5^2 + 8^2}$

c = 9.43398

= 9.43 m

7 By Pythagoras' theorem

$7^2 = 5.2^2 + x^2$

$x^2 = 7^2 - 5.2^2$

$x = \sqrt{7^2 - 5.2^2}$

= 4.68615

= 4.7 cm

8 x = length of rope

By Pythagoras' theorem

$x^2 = 5^2 + 6^2$

$x^2 = 25 + 36$

$x^2 = 61$

x = 7.810

= 7.8 m

9 **a** In triangle *ABC*

$AC^2 = AB^2 + BC^2$

$= 8^2 + 9^2$

$= 145$

$AC = \sqrt{145}$

=12.04 cm

=12.0 cm

b In triangle *ACD*

$AD^2 = AC^2 + CD^2$

$= 145 + 6^2$

$AD = \sqrt{181}$

=13.45 cm

=13.5 cm

Solutions to Exercise 10B

1 **a** Pythagoras' theorem

$c^2 = a^2 + b^2$

where c is the length of the hypotenuse (the side opposite the right-angle) and a and b are the lengths of the other two sides.

b $a = 5$ and $b = 7$

c Substitution into $c^2 = a^2 + b^2$ gives:

$c^2 = 5^2 + 7^2$

d $c^2 = 25 + 49$

$c^2 = 74$

$c = \sqrt{74}$

= 8.6 to one decimal place.

2 **a** $a = 2.5$, $b = 4.2$

$c^2 = a^2 + b^2$

$= (2.5)^2 + (4.2)^2$

$c = \sqrt{2.5^2 + 4.2^2}$

= 4.8877. . .

= 4.9 cm, correct to 1 d.p.

b $a = 54$, $b = 63.2$

$h^2 = a^2 + b^2$

$= (54)^2 + (63.2)^2$

$h = \sqrt{54^2 + 63.2^2}$

= 83.127. . .

= 83.1 cm, correct to 1 d.p.

c $a^2 + b^2 = c^2$

$x^2 + 10^2 = 26^2$

$x = \sqrt{26^2 - 10^2}$

= 24 mm

d $a^2 + b^2 = c^2$
$y^2 + (2.3)^2 = (3.3)^2$
$y = \sqrt{(3.3)^2 - (2.3)^2}$
$= 2.366\ldots$
$= 2.4$ mm

3 a $a^2 + b^2 = c^2$
$k^2 + (15.7)^2 = (22.3)^2$
$y = \sqrt{(22.3)^2 - (15.7)^2}$
$= 15.836\ldots$
$= 15.8$ mm

b $a = 3.9, b = 6.3$
$x^2 = a^2 + b^2$
$= (3.9)^2 + (6.3)^2$
$x = \sqrt{(3.9)^2 + (6.3)^2}$
$= 7.409\ldots$
$= 7.4$ cm

c $a = 4.5, b = 4.5$
$v^2 = a^2 + b^2$
$= (4.52 + (4.5)2$
$x = \sqrt{(4.5)^2 + (4.5)^2}$
$= 6.363\ldots$
$= 6.4$ cm

d $a^2 + b^2 = c^2$
$a^2 + (158)^2 = (212)^2$
$a = \sqrt{(212)^2 - (158)^2}$
$= 141.350\ldots$
$= 141.4$ mm

4 $c = 3.2, a = 1.4$
$a^2 + b^2 = c^2$
$(1.4)^2 + b^2 = (3.2)^2$
$b = \sqrt{(3.2)^2 - (1.4)^2}$
$= 2.877\ldots$
$= 2.9$ m

5 $a = 1.5, b = 3.5$
$a^2 + b^2 = c^2$
$(1.5)^2 + (3.5)^2 = c^2$
$c = \sqrt{(1.5)^2 + (3.5)^2}$
$= 3.807\ldots$
$= 3.8$ m

6 $c = 5.5, a = 1.5$
$a^2 + b^2 = c^2$
$a^2 + (1.5)^2 = (5.5)^2$
$a = \sqrt{(5.5)^2 - (1.4)^2}$
$= 5.318\ldots$
$= 5.3$ m

7 $a = 42, b = 25$
$a^2 + b^2 = c^2$
$(42)^2 + (25)^2 = c^2$
$c = \sqrt{(42)^2 + (25)^2}$
$= 48.877\ldots$
$= 48.88$ km

8 $c^2 = a^2 + b^2$
$= (25)^2 + (100)^2$
$c = \sqrt{25^2 + 100^2}$
$= 103.077\ldots$
$= 103$ m

9 a $AC^2 = AB^2 + BC^2$

b $AC^2 = 9^2 + 5^2$

c $AC^2 = 81 + 25$
$AC^2 = 106$
$AC = \sqrt{106}$
$= 10.2956\ldots$ cm

d $AD^2 = AC^2 + CD^2$

e $AD^2 = 106 + 7^2$

f $AD^2 = 106 + 49$
$AD^2 = 155$
$AD = \sqrt{155}$
$= 12.4$ cm

10 **a** $AB = 3$, $BC = 3$, let $c = AC$
$c^2 = a^2 + b^2$
$= 3^2 + 3^2$
$c = \sqrt{9 + 9}$
$= \sqrt{18}$
$= 4.2426\ldots$
$AC = 4.243$ cm

b $AC = 4.243$, $CG = 3$, let $c = AG$
(Note: from **a,** $AC^2 = 18$)
$c^2 = a^2 + b^2$
$= 18 + 3^2$
$c = \sqrt{27}$
$= 5.196\ldots$
$AG = 5.20$ cm

11 **a** $AD = 4$, $AB = 10$, let $c = DB$
$c^2 = a^2 + b^2$
$= 42 + 102$
$c = \sqrt{4^2 + 10^2}$
$= 10.770\ldots$
$DB = 10.77$ cm

b $DH = 5$, $DB = 10.77$, let $c = BH$
$c^2 = a^2 + b^2$
$= 5^2 + (10.77)^2$
$c = \sqrt{5 + (10.77)^2}$
$= 11.874\ldots$
$BH = 11.87$ cm

c $AD = 4$, $DH = 5$, let $c = AH$
$c^2 = a^2 + b^2$
$= 42 + 52$
$c = \sqrt{4^2 + 5^2}$
$= 6.403\ldots$
$AH = 6.40$ cm

12 **a** $a = 12$, $b = 25$
$s^2 = a^2 + b^2$
$= 12^2 + 25^2$
$s = \sqrt{12^2 + 25^2}$
$= 27.730\ldots$
$= 27.73$ mm

b $a = 42$, $b = 96$
$s^2 = a^2 + b^2$
$= 42^2 + 96^2$
$s = \sqrt{42^2 + 96^2}$
$= 104.785\ldots$
$= 104.79$ mm

13 $c = 10$, $a = 3$
$a^2 + b^2 = c^2$
$3^2 + h^2 = 10^2$
$h = \sqrt{10^2 - 3^2}$
$= 9.539\ldots$
$= 9.54$ cm

14 **a** **i** $a = 6$, $b = 6$
$c^2 = a^2 + b^2$
$= 62 + 62$
$c = \sqrt{36 + 36}$
$= \sqrt{72}$
$= 8.485\ldots$
$= 8.5$ cm

ii $c = 10$, $a = 4.25$
(half the diagonal length of the base)
$a^2 + b^2 = c^2$
$(425)^2 + b^2 = 10^2$
$b = \sqrt{(10)^2 - (4.25)^2}$
$= 9.051\ldots$
$= 9.1$ cm

b **i** $a = 7.5$, $b = 7.5$
$c^2 = a^2 + b^2$
$= (7.5)^2 + (7.5)^2$
$c = \sqrt{(7.5)^2 + (7.5)^2}$
$= 10.606\ldots$
$= 10.6$ cm

ii $c = 6.5$, $a = 5.3$

(half the diagonal length of the base)

$a^2 + b^2 = c^2$

$(5.3)^2 + b^2 = 6.5^2$

$b = \sqrt{(6.5)^2 - (5.3)^2}$

$= 3.762...$

$= 3.8$ cm

15 $a = 8$, $b = 15$

$c^2 = a^2 + b^2$

$= 8^2 + 15^2$

$c = \sqrt{8^2 + 15^2}$

$= 17$ cm

16 Length of base diagonal: $a = 20$, $b = 12$

$c^2 = a^2 + b^2$

$= 20^2 + 12^2$

$c = \sqrt{20^2 + 12^2}$

$= \sqrt{544}$

$= 23.323...$

Length of internal diagonal:

$a = 23.3$ or $\sqrt{544}$, $b = 10$

$c^2 = a^2 + b^2$

$= (\sqrt{544})^2 + 10^2$

$c = \sqrt{544 + (10)^2}$

$= 25.377...$

$= 25$ cm pencil

Alternatively:

Length $= \sqrt{20^2 + 12^2 + 10^2}$

$= 25.377...$

$= 25$ cm pencil

17 For base diagonal, $a = 45$, $b = 50$

$c^2 = a^2 + b^2$

$= 45^2 + 50^2$

$c = \sqrt{45^2 + 50^2}$

$= 67.268...$

For internal diagonal, $a = 67.27$, $b = 140$

$c^2 = a^2 + b^2$

$= (67.27)^2 + (140)^2$

$c = \sqrt{(67.27)^2 + (140)^2}$

$= 155.323...$

Yes, broom of length 145 cm would fit.

Alternatively:

Length $= \sqrt{45^2 + 50^2 + 145^2}$

$= 155.322...$

Yes, a 145 cm broom would fit.

18 For base diagonal, $a = 8$, $b = 10$

$c^2 = a^2 + b^2$

$= 8^2 + 10^2$

$c = \sqrt{8^2 + 10^2}$

$= \sqrt{164}$

$= 12.806...$

For internal diagonal,

$a = 12.806$ or $\sqrt{164}$, $b = 12$

$c^2 = a^2 + b^2$

$= (\sqrt{164})^2 + (12)^2$

$c = \sqrt{164 + 12^2}$

$= 17.549...$

$= 17.55$ m

Alternatively:

Length $= \sqrt{8^2 + 10^2 + 12^2}$

$= 17.549...$

$= 17.55$ m

19 By Pythagoras' theorem

$15^2 = x^2 + (2x)^2$

$225 = 5x^2$

$x^2 = 45$

$x = \sqrt{45}$

$x = 6.7$

20 **a** In triangle WXZ by Pythagoras' theorem

$$(XZ)^2 = (XW)^2 + (WZ)^2$$
$$= a^2 + b^2 \qquad (1)$$

b In triangle XYZ by Pythagoras' theorem

$$(XY)^2 = (XZ)^2 + (ZY)^2 \qquad (2)$$

c Use equation (1) to substitute for $(XZ)^2$ and $(ZY)^2 = c^2$ into equation (2).

$$(XY)^2 = a^2 + b^2 + c^2$$
$$XY = \sqrt{a^2 + b^2 + c^2}$$

Solutions to 10C Now Try This Questions

10 Draw a rectangle to fit the shape given in the 'Now try this question'.
In a rectangle opposite sides are equal in length. So the sides opposite the 6 cm side will have a total length of 6 cm.
Similarly, the sides opposite the 8 cm side will have a total length of 8 cm.

$$\begin{aligned}\text{Perimeter} &= 2(8+6)\\ &= 28 \text{ cm}\end{aligned}$$

11

$$\begin{aligned}\text{Perimeter} &= 3+3+3+3\\ &= 12 \text{ m}\end{aligned}$$

12

$$\begin{aligned}\text{Area of trapezium} &= \frac{1}{2}(a+b)h\\ &= \frac{1}{2}(10+14)\times 8\\ &= 96 \text{ cm}^2\end{aligned}$$

13 Using Heron's formula for the area of a triangle.

$A = \sqrt{s(s-a)(s-b)(s-c)}$

where a, b, c are the side lengths and s is the semi-perimeter.

$$\begin{aligned}s &= \frac{1}{2}(a+b+c)\\ &= \frac{1}{2}(4+5+6) \quad = 7.5 \text{ m}\end{aligned}$$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = 7.5, a = 4, b = 5, c = 6$

$A = \sqrt{7.5(7.5-4)(7.5-5)(7.5-6)}$

$= 9.9 \text{ m}^2$

14

$$\begin{aligned}\text{Area of wall} &= 3 \text{ m} \times 9 \text{ m}\\ &= 27 \text{ m}^2\end{aligned}$$

2 litres covers 6 m^2
Divide by 2.
1 litres covers 3 m^2
Multiple by 9.
9 litres covers 27 m^2

15 **a**

$$\begin{aligned}A &= \frac{1}{2}(a+b)h\\ &= \frac{1}{2}(10+7)(4)\\ &= 34 \text{ m}^2\end{aligned}$$

b Draw a right-angled triangle with the sloping edge as its hypotenuse.
The triangle made is a 3,4,5 triangle.
Its hypotenuse is 5 m long.
Its height is 4 m
The shortest side is 3 m long.

$$\begin{aligned}\text{Perimeter} &= 5+7+4+10\\ &= 26 \text{ m}\end{aligned}$$

16 **a**

$$\begin{aligned}C &= 2\pi r\\ &= 2\pi \times 12\\ &= 75.4 \text{ m}\end{aligned}$$

b

$$\begin{aligned}A &= \pi r^2\\ &= \pi(12)^2\\ &= 452.4 \text{ m}^2\end{aligned}$$

Solutions to Exercise 10C

1 **a**

$$\begin{aligned}\text{Area of } ACEF &= 12 \times 7\\ &= 84 \text{ cm}^2\end{aligned}$$

b

$$\begin{aligned}\text{Length of side } BC &= 7-3\\ &= 4 \text{ cm}\end{aligned}$$

$$\begin{aligned}\text{Length of side } CD &= 12-8\\ &= 4 \text{ cm}\end{aligned}$$

c Area of triangle $BCD = \frac{1}{2}(4 \times 4)$
$= 8 \text{ cm}^2$

d Area ABDEF $= 84 - 8$
$= 76 \text{ cm}^2$

2 **a** $C = 2\pi r$
$= 2\pi \times 7$
$= 43.98 \text{ cm}$

b $A = \pi r^2$
$= \pi(7)r^2$
$= 153.94 \text{ cm}^2$

3 **a** **i** $P = 4l$
$= 4(15)$
$= 60 \text{ cm}$

ii $A = l^2$
$= (15)^2$
$= 225 \text{ cm}^2$

b **i** $P = 2l + 2w$
$= 2(7.9) + 2(3.3)$
$= 15.8 + 6.6$
$= 22.4 \text{ cm}$

ii $A = l \times w$
$= 7.9 \times 3.3$
$= 26.1 \text{ cm}^2$

c **i** $P = b + h + d$
$= 104 + 78 + 130$
$= 312 \text{ cm}$

ii $A = 0.5bh$
$= 0.5(104)(78)$
$= 4056 \text{ cm}^2$

d **i** $P = 2l + 2w$
$= 2(15) + 2(7)$
$= 30 + 14$
$= 44 \text{ cm}$

ii $A = l \times h$
$= 15 \times 5$
$= 75 \text{ cm}^2$

4 **a** $A = \frac{1}{2}bh$
$= \frac{1}{2}(15.2)(7.4)$
$= 56.2 \text{ cm}^2$

b $A = \frac{1}{2}(a + b)h$
$= \frac{1}{2}(4.2 + 4.8)(3.7)$
$= \frac{1}{2}(9.0)(3.7)$
$= 16.7 \text{ m}^2$

c $A = l \times h$
$= (15.7)(6.6)$
$= 103.6 \text{ cm}^2$

d $A = \frac{1}{2}bh$
$= \frac{1}{2}(15.7)(9.4)$
$= 73.8 \text{ cm}^2$

e Divide the shape into 2 rectangles.
Rectangle 1:
$A = l \times w$
$= (2) \times (6.5)$
$= 13\text{cm}^2$
Rectangle 2:
$A = l \times w$
$= (7.5) \times (2)$
$= 15 \text{ cm}^2$
$A_{total} = 13 + 15$
$= 28 \text{ cm}^2$

f $A = \frac{1}{2}bh$
$= \frac{1}{2}(6.9)(10.4)$
$= 35.9 \text{ cm}^2$

g $A = \frac{1}{2}(a + b)h$

$= \frac{1}{2}(1.8 + 5)(8.8)$

$= \frac{1}{2}(6.8)(8.8)$

$= 29.9 \text{ m}^2$

h $A = l \times h$

$= (2.5)(12.5)$

$= 31.3 \text{ m}^2$

i $A = \sqrt{s(s - a)(s - b)(s - c)}$

$s = \frac{1}{2}(10 + 10 + 10)$

$= 15$

$A =$

$\sqrt{15(15 - 10)(15 - 10)(15 - 10)}$

$= \sqrt{15(5)(5)(5)}$

$= 43.3 \text{ cm}^2$

j $A = \sqrt{s(s - a)(s - b)(s - c)}$

$s = \frac{1}{2}(11 + 9 + 7)$

$= 13.5$

$A =$

$\sqrt{13.5(13.5 - 11)(13.5 - 9)(13.5 - 7)}$

$= \sqrt{15(2.5)(4.5)(6.5)}$

$= 31.4 \text{ m}^2$

5 $A = \frac{1}{2}(a + b)h$

$= \frac{1}{2}(1.5 + 2.5)(50)$

$= \frac{1}{2}(4)(50)$

$= 100 \text{ m}^2$

6 $A = \frac{1}{2}(a + b)h$

$= \frac{1}{2}(550 + 1400)(65)$

$= \frac{1}{2}(1950)(65)$

$= 63\,375 \text{ m}^2$

7 $A = l \times w$

$= (1.6) \times (4)$

$= 6.4 \text{ m}^2$

Tiles measure 0.4 m × 0.4 m = 0.16 m^2

Number of tiles = 6.4 ÷ 0.16

= 40

8 $A = l \times w$

$= 12 \times 3$

$= 36 \text{ m}^2$

Amount of paint = 36 ÷ 9

= 4 litres

9 **a** Total Area = area of 5 squares

$= 5 \times l^2$

$= 5 \times (1)^2$

$= 5 \text{ cm}^2$

b Total Area = area of trapezium + area of square

$= \frac{1}{2}(a + b)h + l^2$

$= \frac{1}{2}(12 + 7)8 + (7)^2$

$= 125 \text{ cm}^2$

10 Total area = area of rectangle + area of trapezium

$= l \times w + \frac{1}{2}(a + b)h$

$= (6.5) \times (4) + \frac{1}{2}(4 + 3.5)(1.3)$

$= 30.875$

$= 30.88 \text{ m}^2$, correct to 2 d.p.

11 **a** Total area of playground

= area of rectangle + area of trapezium

$= l \times w + \frac{1}{2}(a + b)h$

$= 8 \times 15 + \frac{1}{2}(15 + 7)(12)$

$= 252 \text{ m}^2$

b Area of lawn = total area of rectangle – area of playground
$= 35 \times 15 - 252$
$= 273\ \text{m}^2$

12 a i $C = 2\pi r$
$= 2\pi(5) = 31.4$ cm

ii $A = \pi r^2$
$= \pi(5)^2 = 78.5\ \text{cm}^2$

b i $C = 2\pi r$
$= 2\pi(8.5) = 53.4$ cm

ii $A = \pi r^2$
$= \pi(8.5)^2 = 227\ \text{cm}^2$

13 a i $P = \pi r + D$
$= \pi(5) + (10)$
$= 15.71 + 10$
$= 25.71$ cm

ii $A = \frac{1}{2}\pi r^2$
$= \frac{1}{2}\pi(5)^2$
$= 39.27\ \text{cm}^2$

b i $P = \pi r + 2l + w$
$= \pi(14) + 2(495) + (28)$
$= 43.98 + 990 + 28$
$= 1061.98$ mm

ii $A = \frac{1}{2}\pi r^2 + l \times w$
$= \frac{1}{2}\pi(14)^2 + (495) \times (28)$
$= 307.88 + 13\,860$
$= 14\,167.88\ \text{mm}^2$

c i $P = r + r + \frac{1}{2}\pi r$
$= 57 + 57 + \frac{1}{2}\pi(57)$
$= 203.54$ cm

ii $A = \frac{1}{4}\pi r^2$
$= \frac{1}{4}\pi(57)^2$
$= 2551.76\ \text{cm}^2$

d i $P = r + r + 1\frac{1}{2}\pi r$
$= 8 + 8 + 1\frac{1}{2}\pi(8)$
$= 53.70$ mm

ii $A = \frac{3}{4}\pi r^2$
$= \frac{3}{4}\pi(57)^2$
$= 150.80\ \text{mm}^2$

14 a Find large area:
$A = \pi r^2$
$= \pi(11.5)^2$
$= 415.48\ \text{cm}^2$
Find small area:
$A = \pi r^2$
$= \pi(4.8)^2$
$= 72.38\ \text{cm}^2$
Shaded area = large area – small area
$= 415.48 - 72.38$
$= 343.1\ \text{cm}^2$

b Area of square:
$A = l^2$
$= (12.75)^2$
$= 162.56\ \text{m}^2$
Area of circle:
$A = \pi r^2$
$= \pi(6.375)^2$
$= 127.68$
Shaded area = area of square – area of circle
$= 162.56 - 127.68$
$= 34.9\ \text{m}^2$

15 a $P = 2l + 2\pi r$
$= 2(400) + 2\pi(40)$
$= 800 + 251.33$
$= 1051.33$ m

b $A = \pi r^2 + (l \times w)$

$= \pi(40)^2 + ((400) \times (80))$

$= 5026.55 + 32\,000$

$= 37\,026.55 \text{ m}^2$

16 Area of annulus = area of large circle – area of smaller circle

$= \pi r^2$ (large circle) – πr^2 (small circle)

$= \pi(12.5)^2 - \pi(10)^2$

$= 176.7145\ldots$

For both sides to be painted, multiply by 2.

There are 3 rings so multiply this answer by 3.

Thus $176.715 \times 2 \times 3 = 1060.29 =$ 1060 m^2

17 Radius of large circle = $(7 \div 2) + 1.2 =$ 4.7

Area of large circle = πr^2

$= \pi(4.7)^2$

Radius of small circle = $(7 \div 2) = 3.5$

Area of small circle = $\pi r^2 = \pi(3.5)^2$

Thus area of annulus = area of path

$= \pi(4.7)^2 -$

$\pi(3.5)^2$

$= 30.912\ldots$

$= 30.91 \text{ m}^2$

18 Area of the square = 12×12

$= 144 \text{ cm}^2$

Finding area of the equilateral triangle

Using Heron's rule:

$s = \dfrac{12 + 12 + 12 + 12}{2}$

$= 18$

Area = $\sqrt{s(s-a)(s-b)(s-c)}$

$=$

$\sqrt{18(18-12)(18-12)(18-12)}$

$= \sqrt{3888}$

$= 62.3538$

Fraction of the triangle in the square

$= \dfrac{62.3538}{144}$

$= 0.433$

19 Distance from the centre of the sphere to the centre of the slice

$= 30 - 10$

$= 20$ cm

Let r = radius of the slice

By Pythagoras' theorem

$30^2 = 20^2 + r^2$

$r = \sqrt{500}$

$= 22.3607$

Circumference of the circle

$= 2\pi r$

$= 2\pi(22.3607)$

$=140.5$ cm

Solutions to 10D Now Try This Questions

17 $\theta = 52°, \quad r = 18 \text{ cm}$

$$s = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 18 \times 52}{180}$$
$$= 16.3 \text{ cm}$$

18 $r = 34 \text{ cm} \quad \theta = 73°$

$$A = \frac{\pi r^2 \theta}{360}$$
$$A = \frac{\pi \times 34^2 \times 73}{360}$$
$$= \frac{\pi \times 1156 \times 73}{360}$$
$$= 736.4 \text{ cm}^2$$

Solutions to Exercise 10D

1 **a** $\theta = 90°$

$$\text{Fraction} = \frac{90}{360} = \frac{1}{4}$$

b $\theta = 270°$

$$\text{Fraction} = \frac{270}{360} = \frac{3}{4}$$

c $\theta = 30°$

$$\text{Fraction} = \frac{30}{360} = \frac{1}{12}$$

d $\theta = 120°$

$$\text{Fraction} = \frac{120}{360} = \frac{1}{3}$$

e $\theta = 60°$

$$\text{Fraction} = \frac{60}{360} = \frac{1}{6}$$

f $\theta = 150°$

$$\text{Fraction} = \frac{150}{360} = \frac{5}{12}$$

2 **a** $\theta = 45°, r = 10$

$$S = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 10 \times 45}{180}$$
$$= 7.85 \text{ cm}$$

b $\theta = 60°, r = 10$

$$S = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 10 \times 60}{180}$$
$$= 10.47 \text{ cm}$$

c $\theta = 150°, r = 10$

$$S = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 10 \times 150}{180}$$
$$= 26.18 \text{ cm}$$

d $\theta = 135°, r = 10$

$$S = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 10 \times 135}{180}$$
$$= 23.56 \text{ cm}$$

3 **a** $r = 15, \theta = 50°$

$$S = \frac{\pi \times r \times \theta}{180}$$

$$= \frac{\pi \times 15 \times 50}{180}$$
$$= 13.09 \text{ cm}$$

b $r = 20, \theta = 15°$
$$S = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 20 \times 15}{180}$$
$$= 5.24 \text{ cm}$$

c $r = 30, \theta = 150°$
$$S = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 30 \times 150}{180}$$
$$= 78.54 \text{ cm}$$

d $r = 16, \theta = 135°$
$$S = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 16 \times 135}{180}$$
$$= 37.70 \text{ cm}$$

e $r = 40, \theta = 175°$
$$S = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 40 \times 175}{180}$$
$$= 122.17 \text{ cm}$$

f $r = 30, \theta = 210°$
$$S = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 30 \times 210}{180}$$
$$= 109.96 \text{ cm}$$

4 $r = 3, \theta = 58°$
$$P = r + r + \frac{\pi \times r \times \theta}{180}$$
$$= 3 + 3 + \frac{\pi \times 3 \times 58}{180}$$
$$= 9.0 \text{ cm}$$

5 **a** $r = 10, \theta = 45°$
$$A = \frac{\pi \times r^2 \times \theta}{360}$$
$$= \frac{\pi \times 10^2 \times 45}{360}$$
$$= 39.3 \text{ cm}^2$$

b $r = 10, \theta = 60°$
$$A = \frac{\pi \times r^2 \times \theta}{360}$$
$$= \frac{\pi \times 10^2 \times 60}{360}$$
$$= 52.4 \text{ cm}^2$$

c $r = 10, \theta = 150°$
$$A = \frac{\pi \times r^2 \times \theta}{360}$$
$$= \frac{\pi \times 10^2 \times 150}{360}$$
$$= 130.9 \text{ cm}^2$$

d $r = 10, \theta = 135°$
$$A = \frac{\pi \times r^2 \times \theta}{360}$$
$$= \frac{\pi \times 10^2 \times 135}{360}$$
$$= 117.8 \text{ cm}^2$$

6 **a** $r = 10, \theta = 150°$
$$A = \frac{\pi \times r^2 \times \theta}{360}$$
$$= \frac{\pi \times 10^2 \times 150}{360}$$
$$= 130.90 \text{ cm}^2$$

b $r = 40, \theta = 35°$
$$A = \frac{\pi \times r^2 \times \theta}{360}$$
$$= \frac{\pi \times 40^2 \times 35}{360}$$
$$= 488.69 \text{ cm}^2$$

c $r = 45, \theta = 150°$

$$A = \frac{\pi \times r^2 \times \theta}{360}$$

$$= \frac{\pi \times 45^2 \times 150}{360}$$

$= 2650.72 \text{ cm}^2$

d $r = 16, \theta = 300°$

$$A = \frac{\pi \times r^2 \times \theta}{360}$$

$$= \frac{\pi \times 16^2 \times 300}{360}$$

$= 670.21 \text{ cm}^2$

e $r = 50, \theta = 108°$

$$A = \frac{\pi \times r^2 \times \theta}{360}$$

$$= \frac{\pi \times 50^2 \times 108}{360}$$

$= 2356.20 \text{ cm}^2$

f $r = 30, \theta = 210°$

$$A = \frac{\pi \times r^2 \times \theta}{360}$$

$$= \frac{\pi \times 30^2 \times 210}{360}$$

$= 1649.34 \text{ cm}^2$

7 **a** $r = 30, \theta = ?, S=50$

$$S = \frac{\pi \times r \times \theta}{180}$$

$$50 = \frac{\pi \times 30 \times \theta}{180}$$

$$50 = \frac{\pi \times \theta}{6}$$

$$\theta = \frac{50 \times 6}{\pi}$$

$\theta = 95.49°$

b $r = 30, \theta = ?, S=25$

$$S = \frac{\pi \times r \times \theta}{180}$$

$$25 = \frac{\pi \times 30 \times \theta}{180}$$

$$25 = \frac{\pi \times \theta}{6}$$

$$\theta = \frac{25 \times 6}{\pi}$$

$\theta = 47.75°$

Compare parts **a** and **b**.

Halving the arc length gives half the angle.

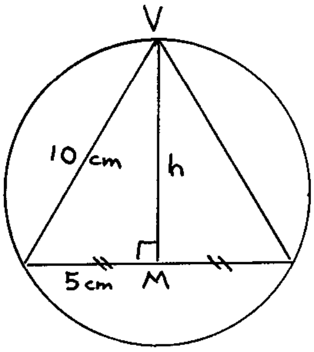
8 Draw a line from the vertex, V, of the triangle to the midpoint , M, of the opposite side.

Let h = height of triangle

By Pythagoras' theorem

$10^2 = 5^2 + h^2$

$h = 8.66025$ cm



Area of triangle

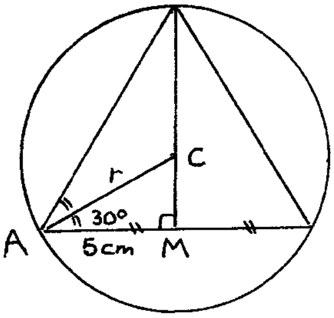
$= \frac{1}{2}$ base × height

$= \frac{1}{2} \times 10 \times 8.66025$

$= 43.3013 \text{ cm}^2$

Draw a line from the centre C to the point A.

In the right-angled triangle ACM, the angle CAM is 30°.



$$\cos 30° = \frac{5}{r}$$

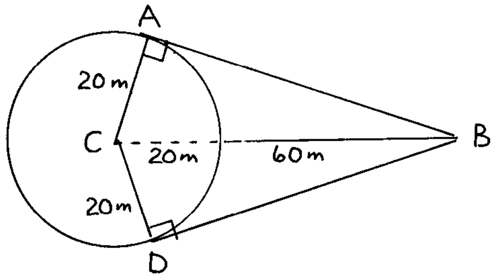
$$r = \frac{5}{\cos 30°}$$
$$= 5.7735 \text{ cm}$$

Area of circle
$= \pi r^2$
$= \pi\ (5.7735)^2$
$= 104.7198 \text{ cm}^2$

Area of required region
$= 104.7198 - 43.3013$
$= 61.42 \text{ cm}^2$

9 **a** $C = 2\pi r$
$= 2\pi \times 20$
$= 125.66 \text{ m}$

b The diagram shows the view of the tank and the position of the observer, *B*, from above.



$$\sin\angle ABC = \frac{20}{80}$$
$\angle ABC = 14.4775°$
$\angle ACB = 90° - \angle ABC$
$= 75.5225°$

$\angle ACD = 2 \times \angle ACB$
$\angle ACD = 2 \times 75.5225°$
$= 151.045°$

$$\text{Arc length} = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 20 \times 151.045}{180}$$
$$= 152.72 \text{ cm}$$
$$\text{Percentage} = \frac{52.72}{2\pi r} \times 100$$
$$= \frac{52.72}{2 \times \pi \times 20} \times 100$$
$$= 41.96\ \%$$

10 **a** $r = 4 \text{ m}$
12:10 p.m. to 12:35 p.m. = 25 min
$$\theta = \frac{25}{60} \times 360$$
$$= 150°$$
$$\text{Arc length} = \frac{\pi \times r \times \theta}{180}$$
$$= \frac{\pi \times 4 \times 150}{180}$$
$$= 10.47 \text{ cm}$$

b $r = 4,\ \theta = 150°$
$$\text{Area of sector} = \frac{\pi \times r^2 \times \theta}{360}$$
$$= \frac{\pi \times 4^2 \times 150}{360}$$
$$= 20.94 \text{ cm}^2$$

Solutions to 10E Now Try This Questions

19 $l = 4\text{ m} \quad w = 3\text{ m} \quad h = 2\text{ m}$

$$\begin{aligned} V &= lwh \\ &= 4 \times 3 \times 2 \\ &= 24\text{ m}^3 \end{aligned}$$

20 $r = 18\text{ cm} \quad h = 24\text{ cm}$

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 18^2 \times 24 \\ &= 24\,429.0\text{ cm}^3 \end{aligned}$$

21 To find the volume of the triangular prism, find the area of the triangular end and multiply by the length of the prism.

Using Heron's formula to find the area of the triangle.

Side lengths:

$a = 5\text{ m} \quad b = 8\text{ m} \quad c = 10\text{ m}$

Semi-perimeter:

$$\begin{aligned} s &= \frac{1}{2}(a + b + c) \\ &= \frac{1}{2}(5 + 8 + 10) \\ &= 11.5\text{ m} \end{aligned}$$

Area of triangle:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\begin{aligned} A &= \\ &\sqrt{11.5(11.5-5)(11.5-8)(11.5-10)} \\ &= 19.81\text{ m}^2 \end{aligned}$$

Volume = area of cross-section × length

$$\begin{aligned} \text{Volume} &= 19.81 \times 14 \\ &= 277.3\text{ m}^3 \end{aligned}$$

22 $r = 9\text{ cm} \quad h = 30\text{ cm}$

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 9^2 \times 30 \\ &= 7634.07\text{ cm}^3 \end{aligned}$$

$1000\text{ cm}^3 = 1$ litre

Divide cm^3 by 1000 to convert into litres.

$$\begin{aligned} \text{Capacity} &= 7.63407\text{ litres} \\ &= 7.6\text{ litres} \end{aligned}$$

23 $h = 3\text{ m} \quad r = 2.5\text{ m}$

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi\,(2.5)^2 \times 3 \\ &= 19.6\text{ m}^3 \end{aligned}$$

24 $r = 45\text{ cm}$

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 45^3 \\ &= 381\,703.5\text{ cm}^3 \end{aligned}$$

Solutions to Exercise 10E

1 **a**
$$\begin{aligned} V &= l^3 \\ &= (5)^3 \\ &= 125\text{ cm}^2 \end{aligned}$$

b
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(17.5)^2(51) \\ &= 49\,067.8\text{ cm}^3 \end{aligned}$$

c
$$\begin{aligned} V &= l \times w \times h \\ &= (15.6) \times (12.5) \times (18.9) \\ &= 3685.5\text{ cm}^3 \end{aligned}$$

d
$$\begin{aligned} V &= \frac{1}{2}bh \times l \\ &= \frac{1}{2}(14)(12.7) \times (35.8) \\ &= 3182.6\text{ mm}^3 \end{aligned}$$

e $V = \frac{1}{2}(a + b)h \times l$

$= \frac{1}{2}(20 + 58)(10) \times (75)$

$= 390 \times 75$

$= 29\ 250 \text{ cm}^3$

f $V = \frac{1}{2}bh \times l$

$= \frac{1}{2}(0.48)(0.5) \times (2.5)$

$= 0.3 \text{ m}^3$

g $V = \pi r^2 h$

$= \pi(13.5)^2(11.8)$

$= 6756.2 \text{ cm}^3$

h $V = \pi r^2 h$

$= \pi(2)^2(3.8)$

$= 47.8 \text{ m}^3$

i $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(18)^2(28)$

$= 9500.18 \text{ cm}^3$

j $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(2.5)^2(2.5)$

$= 16.36 \text{ m}^3$

k $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(3.8)^3$

$= 229.85 \text{ mm}^3$

l $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(12)^3$

$= 7238.23 \text{ cm}^3$

2 $V = \pi r^2 h$

$= \pi(3)^2(15)$

$= 424.115\ldots$

$= 424 \text{ cm}^3$ to the nearest cm^3

3 $V = l \times w \times h$

$= (5.5) \times (7.5) \times (12.5)$

$= 515.625$

$= 516 \text{ cm}^3$

4 **a** $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(3.5)^2(12)$

$= 153.94 \text{ cm}^3$

b $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(7.9)^2(10.8)$

$= 705.84 \text{ m}^3$

c $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(3.3)^2(9.03)$

$= 102.98 \text{ cm}^3$

d $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(6.76)^2(30.98)$

$= 1482.53 \text{ cm}^3$

5 **a** $V = \frac{4}{3}\pi r$

$= \frac{4}{3}\pi(3.5)^3$

$= 179.59 \text{ cm}^3$

b $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(14)^3$

$= 11\ 494.04 \text{ cm}^3$

c $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(2)^3$

$= 33.51 \text{ cm}^3$

6 **a** $V = \frac{4}{3}\pi r^3 \times \frac{1}{2}$

$= \frac{4}{3}\pi(16)^3 \times \frac{1}{2}$

$= 8578.64 \text{ cm}^3$

b $V = \frac{4}{3}\pi r^3 \times \frac{1}{2}$

$= \frac{4}{3}\pi(15.42)^3 \times \frac{1}{2}$
$= 7679.12 \text{ cm}^3$

7 $V = l \times w \times h$
$= 50 \times 20 \times 24$
$= 24\,000 \text{ cm}^3$
$= 24\,000 \text{ mL}$
$= 24\,000 \div 1\,000 \text{ L}$
$= 24 \text{ L}$

8 **a** $V = \pi r^2 h$
$= \pi(14)^2(33)$
$= 20\,319.82 \text{ cm}^3$

b $20\,319.82 \div 1000 = 20.319$
20 litres of paint.

9 First find height of triangle using Pythagoras:
$a^2 + b^2 = c^2$
$(\frac{4.5}{2})^2 + b^2 = (4.5)^2$
$b = \sqrt{(4.5)^2 - (2.25)^2}$
$= 3.897\ldots$
$= 3.9$
$V = \frac{1}{2}bh \times l$
$= \frac{1}{2}(4.5)(3.9) \times 26$
$= 228.15$
$= 228 \text{ cm}^3$

10 $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(5)^2(15)$
$= 392.699\ldots$
$= 393 \text{ cm}^3$

11 $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(1.7)^2(2.6)$
$= 7.868\ldots$
$= 7.87 \text{ m}^3$

12 $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(1.4)^2(10)$
$= 20.525\ldots$
$= 20.53 \text{ cm}^3$
$= 20.53 \text{ mL}$
$= 20.53 \div 1\,000 \text{ L}$
$= 0.02$ litres

13 Volume of cone before truncated:

$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi\,(25)^2(32)$
$= 20943.95102 \text{ cm}^3$

Volume of small cone removed:

$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi\,(16)^2(10)$
$= 2680.82573 \text{ cm}^3$

Thus volume of truncated cone is:

$20943.951 - 2680.826$
$= 18263.125$
$= 18263.13 \text{ cm}^3$, correct to 2 dec. pl.

14 Volume of cone before truncated:

$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi\,(5)^2(30)$
$= 785.398\ldots \text{ cm}^3$

Volume of small cone removed.
To find r of small cone, use ratios.
$\frac{5}{30} = \frac{r}{5}$
$\frac{5}{30} \times 5 = \frac{r}{5} \times 5$
$r = \frac{5}{6}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \left(\frac{5}{6}\right)^2(5)$

$= 3.636\ldots$

Thus volume of truncated cone is:

$785.40 - 3.64 = 781.76$ cm^3

Capacity = 782 mL

15 $V = \frac{4}{3}\pi r^3 \times \frac{1}{2}$

$= \frac{4}{3}\pi(19)^3 \times \frac{1}{2}$

$= 14\,365.5$ cm^3

$14\,365.5 \div 1000 = 14.36$

The bowl can hold approximately 14 litres of fluid.

16 For the rectangular prism let:

a = width

b = depth

c = height

a Front face area:

$ac = 10$

Side area:

$bc = 15$

Top area:

$ab = 6$

$$(ac)(bc)(ab) = 10 \times 15 \times 6$$
$$a^2b^2c^2 = 900$$
$$abc = \sqrt{900}$$
$$abc = 30$$

The volume is 30 m^3

b Front face area:

$ac = X$ m^2

Side area:

$bc = Y$ m^2

Top area:

$ab = Z$ m^2

$$(ac)(bc)(ab) = X \times Y \times Z$$
$$a^2b^2c^2 = X \times Y \times Z$$
$$abc = \sqrt{X \times Y \times Z}$$

The volume of the rectangular prism

is $\sqrt{X \times Y \times Z}$ m^3

17 First find the volume of cylinder using:

$V = \pi r^2 h$

Use circumference $C = 2\ \pi r$ to find r.

$53.4 = 2\ \pi\ r$

$r = \dfrac{53.4}{2\pi}$

Vol cylinder: $V = \pi(\dfrac{53.4}{2\pi})^2(10.8)$

$= 2450.74\ \text{m}^3$

Vol cone: $V = \dfrac{1}{3}\pi r^2 h$, where

$h = 15.3 - 10.8 = 4.5$

$V = \dfrac{1}{3}\pi(\dfrac{53.4}{2\pi})^2(4.5) = 340.38\ \text{m}^3$

Vol. Cylinder + Vol. cone

$= 2450.74 + 340.38$

$= 2791.12$

$= 2791\ \text{m}^3$ to the nearest m^3

18 Submerging a sphere with a volume of 500 m^3 has the same effect on the water level as adding 500 m^3 of water.

V = volume of water added

A = area of the cylinder base

h = increase of the water level

$V = A \times h$

$500 = 250 \times h$

$h = 2$ cm

The water level rose by 2 cm.

Solutions to 10F Now Try This Questions

25 $h = 54$ m

Side of the square base = 38 m

Area of base $= 38^2$

$= 1444 \text{ m}^2$

$$V = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$= \frac{1}{3} \times 1444 \times 54$$

$$= 25\,992 \text{ m}^3$$

26 Area of base = 320 m^2

Height = 24 m

$$V = \frac{1}{3} \times \text{area of base} \times \text{height}$$

$$= \frac{1}{3} \times 320 \times 24$$

$$= 2560 \text{ m}^3$$

Solutions to Exercise 10F

1 **a** $V = \frac{1}{3} l^2 \times h$

$= \frac{1}{3}(4)^2 \times (5)$

$= 26.67 \text{ cm}^3$

b $V = \frac{1}{3} l \times w \times h$

$= \frac{1}{3}(7) \times (12) \times (15)$

$= 420.00 \text{ m}^3$

c $V = \frac{1}{3} A \times h$

$= \frac{1}{3}(16) \times (4.5)$

$= 24.00 \text{ m}^3$

d $V = \frac{1}{3} l^2 \times h$

$= \frac{1}{3}(6.72)^2 \times (4.56)$

$= 68.64 \text{ cm}^3$

2 $V = \frac{1}{3} l^2 \times h$

$= \frac{1}{3}(220)^2 \times (105)$

$= 1\,694\,000 \text{ m}^3$

3 **a** Volume of cone:

$V = \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \pi (3.2)^2 (5.8)$

$= 62.19 \text{ cm}^3$

Volume of cylinder:

$V = \pi r^2 h$

$= \pi (3.2)^2 (8.5)$

$= 273.44 \text{ cm}^3$

Volume of composite

= volume of cone + volume of cylinder

$= 62.19 + 273.44$

$= 335.6 \text{ cm}^3$

b Volume of pyramid:

$V = \frac{1}{3} l^2 \times h$

$= \frac{1}{3}(3.5)^2 \times (5.8)$

$= 23.68 \text{ cm}^3$

Volume of cube:

$V = l^3$

$= (3.5)^3$

$= 42.875 \text{ cm}^3$

Volume of composite
= volume of pyramid + volume of cube
= 23.68 + 42.875
= 66.6 cm^3

4 Volume of pyramid before truncation

$= \frac{1}{3}l^2 \times h$

$= \frac{1}{3}(20)^2 \times 30$

$= 4000$ cm^3

Volume of small pyramid removed

$= \frac{1}{3}l^2 \times h$

$= \frac{1}{3}(5)^2 \times (7.5)$

$= 62.5$ cm^3

Volume of truncated pyramid
= 4000 – 62.5
= 3937.5 cm^3

Solutions to 10G Now Try This Questions

27 Area of each triangular face

$= \frac{1}{2} \times \text{base} \times \text{perpendicular height}$

$= \frac{1}{2} \times 12 \times 15$

$= 90\text{ cm}^2$

Area of square base

$= 12 \times 12$

$= 144\text{ cm}^2$

Total surface area $= 4 \times 90 + 144$

$= 504\text{ cm}^2$

28 h = 45 m   r = 30 m

Surface area of a cylinder

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(30)^2 + 2\pi \times 30 \times 45$

$= 14\,137.2\text{ m}^2$

29 r = 20 cm

$SA = 4\pi r^2$

$= 4\pi \times 20^2$

$= 5026.6\text{ cm}^2$

Solutions to Exercise 10G

1 **a** Total of the top and bottom areas

$= 2 \times (20 \times 10)$

$= 400\text{ cm}^2$

b Total area of the two side faces

$= 2 \times (5 \times 10)$

$= 100\text{ cm}^2$

c Total area of the front and back faces

$= 2 \times (20 \times 5)$

$= 200\text{ cm}^2$

d Total area $= 400 + 100 + 200$

$= 700\text{ cm}^2$

2 **a** $SA = 2(l \times w) + 2(l \times h) + 2(h \times w)$

$= 2(10 \times 13) + 2(10 \times 20) + 2(20 \times 13)$

$= 260 + 400 + 520$

$= 1180\text{ cm}^2$

b $SA = 4(\frac{1}{2}bh) + l^2$

$= 4(\frac{1}{2}(4)(3)) + (4)^2$

$= 24 + 16$

$= 40\text{ m}^2$

c $SA = 4(\frac{1}{2}bh) + l^2$

$= 4(\frac{1}{2}(10.5)(13)) + (10.5)^2$

$= 273 + 110.25$

$= 383.3\text{ cm}^2$

d $SA = (bh) + (l \times w) + 2(l \times d)$

$= ((6)(8.5)) + ((20) \times (6)) + 2((20) \times (9))$

$= 51 + 120 + 360$

$= 531\text{ cm}^2$

e Length of hypotenuse (d):

$d^2 = a^2 + b^2$
$= (10)^2 + (25)^2$
$= 100 + 625$
$= 725$
$d = 26.92$ cm

$SA = (bh) + (l \times w) + (l \times d)$
$+ (l \times h)$
$= (25)(10) + ((30) \times (26.92)) +$
$(30) \times (10)$
$= 250 + 750 + 807.8 + 300$
$= 2107.8$ cm^2

f Area of front:

$A = (l \times w) + (\frac{1}{2}bh)$
$= (5 \times 2.9) + (\frac{1}{2}(5)(3.1))$
$= 14.5 + 7.75$
$= 22.25$

$SA = 2(\text{area of front}) + 2(l \times h) +$
$2(l \times d) + (l \times b)$
$= 2(22.25) + 2(7$
$\times 2.9) + 2(7 \times 4)$
$+ (7 \times 5)$
$= 44.5 + 40.6 + 56 + 35$
$= 176.1$ m^2

3 **a** $SA = 2\pi r(r + h)$
$= 2\pi(9)((9) + (45))$
$= 18\pi(54)$
$= 3053.63$ cm^2

b $SA = 2\pi r(r + h)$
$= 2\pi(5.5)((5.5) + (7))$
$= 11\pi(12.5)$
$= 431.97$ cm^2

c $SA = \pi r^2 + \pi rs$
$= \pi(0.95)^2 + \pi(0.95)(1.52)$
$= 2.84 + 4.54$
$= 7.37$ m^2

d $SA = 4\pi r^2$
$= 4\pi(1.4)^2$
$= 24.63$ m^2

e $SA = 2\pi r^2 + \pi r^2$
$= 2\pi(5)^2 + \pi(5)^2$
$= 157.08 + 78.54$
$= 235.62$ m^2

f $SA = \pi r^2 + \pi rs - \pi rs$ (small cone removed) $+ \pi r^2$ (area of flat top)
$= \pi(4.5)^2 + \pi(4.5)(6) - \pi(1.5)(q)$
$+ \pi(1.5)^2$
$= 146.084\ldots$
$= 146.08$ m^2

4 $SA = 4\pi r^2 \times 100$
$= 4\pi(3.5)^2 \times 100$
$= 15\,394$ cm^2

5 **a** $SA = \pi rs$ (do not need base)

Use Pythagoras' theorem to find s.

$s^2 = (9.5)^2 + (35)^2$
$s = \sqrt{9.5^2 + 35^2}$

$SA = \pi rs$
$= \pi \times 9.5 \times \sqrt{9.5^2 + 35^2}$
$= 1082.3747\ldots$

So for 10 hats: 10×1082.37 cm^2
$= 10823.7$ cm^2
$= 10823.7 \div 100^2$ m^2
$= 1.08$ m^2

b $C = 2\pi r$

For 10 hats, tinsel required is:

$10 \times 2\pi \times 9.5$
$= 596.90\ldots$ cm
$= 5.969\ldots$ m
$= 6$ m (to the nearest metre)

6 Radius of the semi-circle

$S = 20$ cm

Length of the semi-circle

$= \frac{1}{2}(2\pi r)$

$= \frac{1}{2}(2\pi S)$

$= \pi S$

$= 20\pi$ cm (1)

The length of the semi-circle forms the circumference of the base of the cone.

Let R = radius of the base of the cone.

Circumference of the cone base

$= 2\pi R$ (2)

Equating equations (1) and (2)

$$20\pi = 2\pi R$$

$$R = 10 \text{ cm}$$

When curved to form a cone, the radius of the semi-circle becomes the sloping edge S of the cone.

The height of the cone = h cm

By Pythagoras' theorem:

$$S^2 = h^2 + R^2$$

$$20^2 = h^2 + 10^2$$

$$h^2 = 300$$

$$h = 17.3 \text{ cm}$$

The cone is 17.3 cm tall.

Solutions to 10H Now Try This Questions

30 **a** Scale factor of sides

= length in second shape divided by corresponding length in first shape

$= \frac{15}{3}$

$= 5$

b Scale factor of areas

= scale factor of sides squared

$= 5^2$

$= 25$

31 The corresponding lengths of similar triangles are in the same ratio.

$$\frac{x}{33} = \frac{18}{27}$$

$$x = \frac{33 \times 18}{27}$$

$x = 22$ cm

Solutions to Exercise 10H

1 **a** **i** $\frac{9}{3} = \frac{6}{2} = \frac{3}{1} = 3$

$k = 3$

ii $\frac{9 \times 6}{3 \times 2} = \frac{54}{6} = \frac{9}{1}$

$k^2 = 9$

b **i** $\frac{12}{6} = \frac{10}{5} = \frac{2}{1} = 2$

$k = 2$

ii $\frac{\frac{1}{2} \times 10 \times 12}{\frac{1}{2} \times 5 \times 6} = \frac{60}{15} = \frac{4}{1} = 4$

$k^2 = 4$

2 **a** Similar: $\frac{9}{3} = \frac{30}{10} = \frac{3}{1}$

$k = 3$

b Similar: $\frac{8}{4} = \frac{7}{3.5} = \frac{2}{1} = 2$

$k = 2$

c Not similar.

$\frac{5}{3} \neq \frac{1.5}{0.5}$

3 **a** $k = \frac{6}{2} = 3$

b $\frac{9}{x} = \frac{6}{2}$

$\frac{9}{x} = 3$

$3x = 9$

$x = 3$

c $\frac{\frac{1}{2} \times 9 \times 6}{\frac{1}{2} \times 3 \times 2} = \frac{27}{3} = \frac{9}{1}$

Ratio of areas is $\frac{9}{1}$.

$k^2 = 9$

4 The ratio of the sides is $\frac{8}{4} = \frac{2}{1}$

So the ratio of the areas is $\left(\frac{2}{1}\right)^2 = \frac{4}{1}$

Area of Rectangle B = 28 × 4 = 112 cm^2

5 The new photo will have lengths 12 × 3 and 8 × 3.

The photo will be 36 cm by 24 cm

$A = 36 \times 24 = 864$ cm^2

Alternative solution:

If the ratio of the lengths is 3:1 then the ratio of the areas will be 9:1. Area of original photo is $12 \times 8 = 96\text{ cm}^2$

Thus, the area of the new photo is $96 \times 9 = 864\text{ cm}^2$

6 Scale factor $= \frac{25}{15} = \frac{5}{3} = 1.67$

7 **a** $\frac{500\,000}{1} = \frac{x}{7.2}$

$$\begin{aligned} x &= 500\,000 \times 7.2 \\ &= 3\,600\,000\text{ cm} \\ &= 3\,600\,000 \div 100 \div 1000\text{ km} \\ &= 36\text{ km} \end{aligned}$$

b $15\text{ km} = 15 \times 1000 \times 100\text{ cm}$
$= 1\,500\,000\text{ cm}$

$$\frac{500\,000}{1} = \frac{1\,500\,000}{x}$$

$$x = \frac{1\,500\,000}{500\,000}$$

$x = 3$ cm

Solutions to 10I Now Try This Questions

32 Compare corresponding side ratios:

$\frac{18}{6} = 3$

$\frac{15}{5} = 3$

The triangles have a corresponding included angle of 43°

They have two pairs of corresponding sides in the same ratio and the included corresponding angles are equal. (SAS)

Solutions to Exercise 10I

1 **a** SSS: the corresponding sides are in the same ratio, $\frac{3}{1}$.

b AA: the corresponding angles are equal.

c SAS or SAS or SSS: the corresponding angles are equal and corresponding sides are in the same ratio, $\frac{2}{1}$.

2 **a** Triangles are in the ratio $\frac{3}{1}$.

Therefore side x is 3 × 9 cm = 27 cm

side y is 3 × 10 cm = 30 cm

b Triangles are in the ratio $\frac{1}{2}$.

Therefore side x is $\frac{1}{2} \times 52$ m = 26 m

side y is 1/2 × 48 m = 24 m

3 **a** The longest side on the first triangle is 8 cm, so the triangles are in the ratio $\frac{7}{1}$.

$k = 7$

b Corresponding side with 5 cm is

7 × 5 cm = 35 cm

Corresponding side with 4 cm is

7 × 4 cm = 28 cm

c $P = a + b + c$

$= 28 + 35 + 56$

$= 119$ cm

4 **a** The sun is at the same angle; therefore the shadows will have the same angles. AA.

b Scale factor is $\frac{1}{2}$.

c Tree is 2 × 1 m = 2 m tall.

5 Scale factor is $\frac{4.5}{3} = \frac{1.5}{1}$; hence 1.5 × 1.2 m = 1.8 m.

John is 1.8 m tall.

6 Scale factor is $\frac{3}{1}$.

$A = 8 \text{ cm}^2 \times (3)^2$

$= 72 \text{ cm}^2$

7 Scale factor, $k = \frac{7}{2}$.

Thus scale factor for area is $k^2 = \frac{49}{4}$

$A = 2.4 \times \frac{49}{4}$

$= 29.4 \text{ cm}^2$

Solutions to 10J Now Try This Questions

33 The scale factor for the lengths is 7.

$k = 7$

A scale factor of lengths of k implies a scale factor of k^3 for volumes.

The scale factor for the volumes

$= k^3$

$= 7^3$

$= 343$

Solutions to Exercise 10J

1 When all dimensions are multiplied by a scale factor of k, the volume is multiplied by a scale factor of k^3.

$k = 3$

$V = k^3$

$= (3)^3$

$= 27$ times larger than the smaller tank.

2 a $\dfrac{\text{Radius of 2nd}}{\text{Radius of 1st}} = \dfrac{16}{4} = \dfrac{4}{1}$ ie. 1:4

b $k = \dfrac{4}{1}$

$V = k^3$

$= \left(\dfrac{4}{1}\right)^3$

$= \dfrac{64}{1}$ ie. 1:64

3 $k = \dfrac{3}{1}$

$V = k^3$

$= \left(\dfrac{3}{1}\right)^3$

$= \dfrac{27}{1}$ ie. 1:27

4 a $\dfrac{\text{Height of A}}{\text{Height of B}} = \dfrac{45}{x} = \dfrac{5}{1}$

$x = 45 \div 5 = 9$ cm

b $k = \dfrac{5}{1}$

$V = k^3$

$= \left(\dfrac{5}{1}\right)^3$

$= \dfrac{125}{1}$ ie. 1:125

5 a scaled up

b $k = \dfrac{3}{1}$

$V = k^3$

$= \left(\dfrac{3}{1}\right)^3$

$= \dfrac{27}{1}$

c Volume of small cone = 120 cm^3

Volume of large cone $= 120 \times \left(\dfrac{3}{1}\right)^3$

$= 120 \times \dfrac{27}{1}$

$= 3240$ cm^3

6 a Scale factor, $k = \dfrac{4}{3}$

Let x be height of small cylinder.

$\dfrac{x}{8} = \dfrac{3}{4}$

$x = 6$ cm

b ratio of volumes 27 : 64 or

$\left(\dfrac{3}{4}\right)^3 = \dfrac{27}{64}$

7 a $V = \dfrac{1}{3}lwh$

$16 = \dfrac{1}{3} \times 4 \times 4 \times h$

$h = 3$ cm

b ratio of volumes , $k^3 = \frac{1024}{16} = 64$

$k^3 = 64$

$k = \sqrt[3]{16}$

$= 4$

Height of small pyramid = 3 cm (from **a**)

Let h = height of larger pyramid.

Ratio of heights : $\frac{h}{3} = 4$

$h = 12$ cm

Let b = length of base

Ratio of bases : $\frac{b}{4} = 4$

$b = 16$ cm

The height is 12 cm and the base is 16 cm.

8 **a** Scale factor, $k = \frac{6}{12} = \frac{1}{2}$

Scale factor for area, $k^2 = (\frac{1}{2})^2 = (\frac{1}{4})$

Ratio of the surface areas is 4 : 1, ie. $\frac{1}{4}$

b Scale factor for volume,

$k^3 = (\frac{1}{2})^3 = (\frac{1}{8})$

Ratio of the volumes is 8 : 1, ie. $\frac{1}{8}$

Solutions to Skills Checklist Questions

1 Count two places to the right of the decimal point and look at the digit to the right (9).

As 9 is ’5 or more’, increase the digit of the second decimal place by one (0 becomes 1).

307.509

= 307.51 to 2 decimal places.

2 Write the number in scientific notation:

Put a decimal point after the first non-zero digit and multiply by the required of 10.

307.509

$= 3.07509 \times 10^2$.

As the third digit is ’5 or more’, increase the second digit by 1 (so 0 changes to 1).

$3.1 \times 10^2 = 310$

3 By Pythagoras’ theorem

$11^2 = x^2 + 8^2$

$121 = x^2 + 64$

$x^2 = 57$

$\mathrm{x} = 7.5$ cm

4 Consider the 10×10 square on the base of the cube.

Let d be the diagonal along the base.

By Pythagoras’ theorem

$10^2 + 10^2 = d^2$

$d^2 = 200$

Consider the right-angled triangle along the diagonal of the base and up the vertical edge v of the cube.

Call its hypotenuse h.

By Pythagoras’ theorem

$d^2 + v^2 = h^2$

$200 + 10^2 = h^2$

$h^2 = 300$

$\mathrm{h} = \sqrt{300}$

$\mathrm{h} = 17.3$ cm

OR

$h = \sqrt{x^2 + y^2 + z^2}$

$h = \sqrt{300}$

$= 17.3$ cm

5 Perimeter = 6 + 7 + 8

= 21 cm

Using Heron’s formula

First, find the semi-perimeter, s.

$s = \frac{1}{2}(6 + 7 + 8)$

= 10.5 cm

Area

$= \sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{10.5(10.5 - 6)(10.5 - 7)(10.5 - 8)}$

= 20.3331

$= 20 \text{ cm}^2$

6 Form a right-angled triangle with the slant side as the hypotenuse. The other sides will be 4 m and 3 m long.

Slant side $= \sqrt{3^2 + 4^2}$

= 5 m

Perimeter = 3 + 4 + 6 + 5

= 18 m

Area $= \frac{1}{2}(a + b)h$

$= \frac{1}{2}(6 + 3)4$

$= \frac{1}{2}(9)4$

$= 18 \text{ m}^2$

7 Radius, $r = 7$ cm

Circumference

$C = 2\pi r$

$= 2\pi \times 7$

$= 44.0$ cm

Area

$A = \pi r^2$

$= \pi(7)^2$

$= 153.9 \text{ cm}^2$

8 $r = 10, \quad \theta = 76°$

$s = \dfrac{\pi r\theta}{180}$

$= \dfrac{\pi(10)(76)}{180}$

$= 13.3$ m

9 $r = 10, \quad \theta = 76°$

$A = \dfrac{\pi r^2\theta}{360}$

$= \dfrac{\pi(10)^2(76)}{360}$

$= 66.3 \text{ m}^2$

10 Cross-section area

$= \dfrac{1}{2}$ base × height

$= \dfrac{1}{2}(44) \times (28)$

$= 616 \text{ mm}^2$

Volume

$= 616 \times 52$

$= 32\,032 \text{ mm}^3$

11 Volume

= Area of base × height

$= 600 \times 45$

$=27\,000 \text{ cm}^3$

$1000 \text{ cm}^3 = 1$ litre

Capacity

= 27 litres

12 Volume of a cone

$= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\pi(18)^2(30)$

$= 3240\pi$

$=10\,178.8 \text{ cm}^3$

13 r = 15 cm

Volume of a hemi-sphere

$= \dfrac{1}{2} \times \dfrac{4}{3}\pi r^3$

$= \dfrac{2}{3}\pi r^3$

$= \dfrac{2}{3}\pi 15^3$

$= 7068.6 \text{ cm}^3$

14 Volume of a pyramid

$= \dfrac{1}{3}$ area of base × height

Area of base $= 58 \times 58 \text{ m}^2$

Height = 48 m

Volume

$= \dfrac{1}{3}$ area of base × height

$= \dfrac{1}{3}(58)^2 \times 48$

$= 53\,824 \text{ m}^3$

15 Total surface area

$= 2\,(2 \times 3 + 2 \times 4 + 3 \times 4)$

$= 52 \text{ m}^2$

16 Radius: $r = 26$ cm

Sloping edge: $s = 39$ cm

Surface area of a cone

$= \pi r^2 + \pi rs$

$= \pi(26)^2 + \pi(26)(39)$

$= 5309.3 \text{ cm}^2$

17 Ratios of corresponding sides:

$\frac{6}{8} = \frac{9}{12} = \frac{3}{4}$

Corresponding sides are in the same ratio.

Scale factor, $k =$

$\frac{a \text{ length of the second shape}}{\text{corresponding length of first shape}}$

$= \frac{6}{8}$

$= \frac{3}{4}$

18 From question 17.

$k = \frac{3}{4}$

$k^2 = \frac{9}{16}$

19 Scale factor, $k =$

$\frac{a \text{ length of the second shape}}{\text{corresponding length of first shape}}$

$= \frac{16}{12}$

$= \frac{4}{3}$

$x \times \frac{4}{3} = 12$

$x = 12 \times \frac{3}{4}$

$= 9$ cm

20 Check if the ratio of corresponding sides are equal.

$\frac{3}{6} = \frac{5}{10} = \frac{6}{12}$

$= \frac{1}{2}$

The ratio of corresponding sides are equal, SSS, so the triangles are similar.

21 The ratio of the corresponding sides

$= \frac{10}{6} = \frac{15}{9} = \frac{20}{12}$

$= \frac{5}{3}$

The ratio of the corresponding sides are equal, so the solids are similar.

The scale factor, $k = \frac{5}{3}$

Solutions to Chapter Review Multiple-Choice Questions

1 Look at the digit in the third decimal place (5). It is 5 or greater, so round up.

3.90 **C**

2 Look at the digit after the hundreds (7).

It is greater than 5, so round up.

4700 **D**

3 Multiplying 5.21 by 10^5 means that the decimal point needs to be moved 5 places right. Use zeroes as place holders.

$5.21 \times 10^5 = 521\ 000$ **D**

4 Place a decimal point to the right of the first non-zero digit.
4.8
The decimal point needs to move 3 places to the left
from 4.8 to make 0.0048
4.8 must be multiplied by 10^{-3}.
$0.0048 = 4.8 \times 10^{-3}$ **B**

5 Look at the third digit (0). It is not greater than or equal to 5, so don't round up. Giving 28 000. **D**

6 $0.03069 = 3.069 \times 10^{-2}$
Look at the third digit (6). It is greater than 5, so round up.
$3.1 \times 10^{-2} = 0.031$ **E**

7 A zero as the last digit does not count as a significant figure.
C

8 Pythagoras' theorem: $a^2 + b^2 = c^2$
Only E fits the theorem:
$15^2 + 20^2 = 25^2$ **E**

9 Pythagoras' theorem: $a^2 + b^2 = c^2$
$12^2 + 16^2 = c^2$
$c = \sqrt{16^2 + 12^2}$
$c = 20$ **C**

10 Pythagoras' theorem: $c^2 - a^2 = b^2$
$56^2 - 50^2 = b^2$
$b = \sqrt{56^2 - 50^2}$
$b = 25.22$ **B**

11 $P = A + B + C$
$= 20 + 30 + 45$
$= 95$ cm **C**

12 $P = 2L + 2W$
$= 2(12) + 2(8)$
$= 40$ cm **D**

13 $C = \pi D$
$= \pi \times 12$
$= 37.7$ m **B**

14 $A = l \times h$
$= 12 \times 9$
$= 108 \text{ cm}^2$ **C**

15 $A = \pi r^2$
$= \pi \times (3)^2$
$= 28.27 \text{ cm}^2$ **C**

16 Arc length, $s = \dfrac{\pi r \theta}{180}$
$= \dfrac{\pi \times 5.4 \times 49}{180}$
$= 4.618$ m
$= 4.6$ m
C

17 Sector area, $A = \dfrac{\pi r^2\theta}{360}$

$= \dfrac{\pi \times 34^2 \times 110}{360}$

$= 1109.68\ \text{cm}^2$

$= 1110\ \text{cm}^2$

B

18 $V = l^3$

$= (5)^3$

$= 125\ \text{cm}^3$ **C**

19 $V = l \times w \times h$

$= 11 \times 5 \times 6$

$= 330\ \text{cm}^3$ **D**

20 $V = \dfrac{4}{3}\pi r^3$

$= \dfrac{4}{3} \times \pi \times (16)^3$

$= \dfrac{4}{3} \times \pi \times 4096$

$= 17\,157.28\ \text{mm}^3$ **E**

21 $V = \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \pi \times (6)^2 \times (8)$

$= 301.59\ \text{cm}^3$ **A**

22 $V = \pi r^2 h$

$= \pi \times (3)^2 \times (4)$

$= \pi \times 9 \times 4$

$= 113.10\ \text{m}^3$ **D**

23 Using Pythagoras' theorem in ADC:

$a^2 + b^2 = c^2$

$x^2 + 6^2 = (\text{AC})^2$ (1)

Using Pythagoras' theorem in ACB:

$a^2 + b^2 = c^2$

$x^2 + (\text{AC})^2 = 10^2$ (2)

Substitute $(\text{AC})^2 = x^2 + 6^2$ from (1) into (2)

$x^2 + x^2 + 6^2 = 10^2$

$2x^2 + 36 = 100$

$2x^2 + 36 - 36 = 100 - 36$

$2x^2 = 64$

$2x^2 \div 2 = 64 \div 2$

$x^2 = 32$

$x = \sqrt{32}$

$= 5.66$ **C**

24

$\dfrac{y}{4} = \dfrac{25}{5}$

$\dfrac{y}{4} \times 4 = \dfrac{25}{5} \times 4$

$y = 20$ **C**

25 The metre length ratio is 4 : 1

So volume ratio is $4^3 : 1^3 = 64 : 1$ **E**

Solutions to Chapter Review Short-Answer Questions

1 **a** Place a decimal point to the right of the first non-zero digit.
2.945
The decimal point needs to move 3 places to the right:
from 2.945 to make 2945
2.945 must be multiplied by 10^3.
$2945 = 2.945 \times 10^3$

b Place a decimal point to the right of the first non-zero digit.
5.7
The decimal point needs to move 2 places to the left:
from 5.7 to make 0.057
5.7 must be multiplied by 10^{-2}.
$0.057 = 5.7 \times 10^{-2}$

c Place a decimal point to the right of the first non-zero digit.
3.69
The decimal point needs to move 5 places to the right:
from 3.69 to make 369 000
3.69 must be multiplied by 10^5.
$369\,000 = 3.69 \times 10^5$

d Place a decimal point to the right of the first non-zero digit.
8.509
The decimal point needs to move 2 places to the right:
from 8.509 to make 850.9
8.509 must be multiplied by 10^2.
$850.9 = 8.509 \times 10^2$

2 **a** 7.5×10^3
Multiplying 7.5 by 10^3 means that the decimal point needs to be moved 3 places to the right.
$7.5 \times 10^3 = 7500$
Use zeroes as place holders.

b 1.07×10^{-3}
Multiplying 1.07 by 10^{-3} means that the decimal point needs to be moved 3 places to the left.
$1.07 \times 10^{-3} = 0.00107$
Use zeroes as place holders.

c 4.56×10^{-1}
Multiplying 4.56 by 10^{-1} means that the decimal point needs to be moved 1 place to the left.
$4.56 \times 10^{-1} = 0.456$
Use zeroes as place holders.

3 **a** Write 8.916 to two significant figures.
Look at the third digit (1)
It is not 5 or greater, so don’t round up the second digit(9).
8.9

b Write 0.0589 to two significant figures.
Write in scientific notation.
$0.0589 = 5.89 \times 10^{-2}$
The third digit (9) is 5 or greater so round up the second digit (8) to 9.
0.059

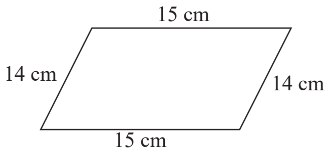
c Write 809 to one significant figure.
Write in scientific notation.
$809 = 8.09 \times 10^2$
The second digit (0) is not 5 or greater, so leave the first digit (8) unchanged.
800

4 a To write 7.145 to two decimal places, count two decimal place to the right of the decimal point and look at the digit to the right (5).
As 5 is "5 or more", increase the digit in the second decimal place (4) by one.
7.15

b To write 598.241 to one decimal place, count one decimal place to the right of the decimal point and look at the digit to the right (4).
As 4 is not "5 or more", do not round up.
598.2

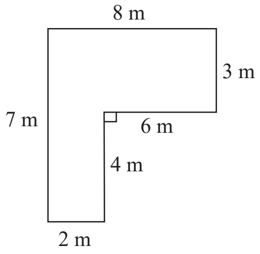
c To write 4.0789 to three decimal places, count three decimal place to the right of the decimal point and look at the digit to the right (9).
As 9 is "5 or more", increase the digit in the third decimal place (8) by one.
4.079

5 a



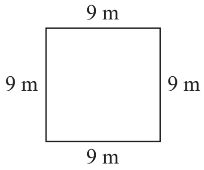
Perimeter = 15 + 14 + 15 + 14 = 58
The perimeter of the parallelogram is 58 cm.

b



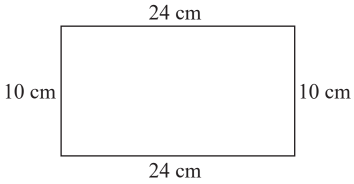
Perimeter = 8 + 3 + 6 + 4 + 2 + 7 = 30
The perimeter of the shape is 30 m.

6



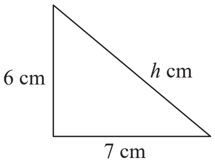
Perimeter = 9 + 9 + 9 + 9 = 36
The perimeter of the square is 36 cm.

7



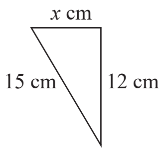
Perimeter = 24 + 10 + 24 + 10 = 68
The perimeter of the rectangle is 68 cm.

8 a



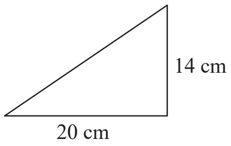
Using Pythagoras' theorem,
$h^2 = 7^2 + 6^2 = 85$
$h = \sqrt{85} = 9.22$, correct to 2 d.p.
The length of the unknown side of the triangle is 9.22 cm, correct to 2 decimal places.

b



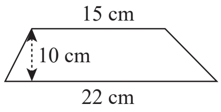
Using Pythagoras' theorem,
$x^2 + 12^2 = 15^2$
$x^2 + 144 = 225$
$x^2 = 225 - 144$
$= 81$
$x = \sqrt{81}$
$= 9$
The length of the unknown side of the triangle is 9 cm.

9 a



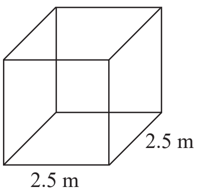
Area of triangle = $\frac{1}{2}bh$
where $b = 20$, $h = 14$
$= \frac{1}{2} \times 20 \times 14$
$= 140$
The area of the triangle is 140 cm^2.

b



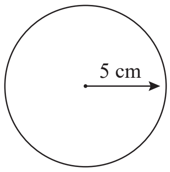
Area of trapezium = $\frac{1}{2}(a + b)h$
where $a = 15$, $b = 22$, $h = 10$
$= \frac{1}{2} \times (15 + 22) \times 10$
$= 185$
The area of the trapezium is 185 cm^2.

10



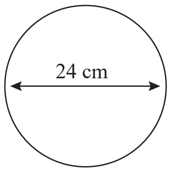
Surface area of cube =
6 × area of each square face
$= 6 \times 2.5 \times 2.5$
$= 37.5$
The surface area of the cube is 37.5 cm^2.

11 a



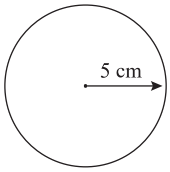
Circumference of circle = $2\pi r$
where $r = 5$
$= 2 \times \pi \times 5$
$= 31.42$, correct to 2 decimal places
The circumference of the circle is 31.42 cm, correct to 2 decimal places.

b



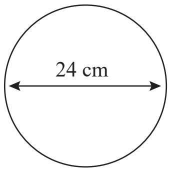
Circumference of circle = $2\pi r$
where $r = 12$
$= 2 \times \pi \times 12$
$= 75.40$, correct to 2 decimal places
The circumference of the circle is 75.40 cm, correct to 2 decimal places.

12 a



Area of circle = πr^2
where $r = 5$
$= \pi \times 5^2$
$= 78.54$, correct to 2 decimal places
The area of the circle is 78.54 cm^2, correct to 2 decimal places.

b



Area of circle = πr^2

where $r = 12$

$= \pi \times 12^2$

= 452.39, correct to 2 decimal places

The area of the circle is 452.39 cm^2, correct to 2 decimal places.

13 **a** Surface area of can = $2\pi r^2 + 2\pi rh$

where $r = 3.5$, $h = 13.5$

$= 2 \times \pi \times (3.5)^2 + 2 \times \pi \times 3.5 \times 13.5$

= 373.849…

= 373.85 cm^2

b For outside label, surface area = $2\pi rh$

$= 2 \times \pi \times 3.5 \times 13.5$

= 296.880…

= 296.88 cm^2

For 100 cans paper required:

$= 296.88 \times 100$

= 29 688 cm^2

$= 29\,688 \div 100^2\ \text{m}^2$

= 2.97 m^2

c Volume of can = $\pi r^2 h$

$= \pi \times (3.5)^2 \times 13.5$

= 519.54 cm^3

Capacity of can = 519.54 mL

$= 519.54 \div 1000$ L

= 0.51954 L

= 0.52 L, correct to 2 decimal places.

14 Volume of swimming pool = $\pi r^2 h$

where $r = 2.25$, $h = 2$

$V = \pi \times (2.25)^2 \times 2 = 31.8086\ldots\ \text{m}^3$

Capacity of swimming pool:

$= 31.8086 \times 1000$ L

= 31808.6

= 31 809 L

15 **a** Surface Area of earth = $4\pi r^2$

where $r = 6400$

$= 4 \times \pi \times 6400^2$

= 514 718 540 km^2

b Volume of earth = $\frac{4}{3}\pi r^3$

where $r = 6400$

$= \frac{4}{3}\pi \times 6400^3$

$= 1.09806\ldots \times 10^{12}$

$= 1.098 \times 10^{12}\ \text{km}^3$

16 **a** Volume of oil can = $\frac{1}{3}\pi r^2 h$

where $r = 6$, $h = 10$

$= \frac{1}{3}\pi \times 6^2 \times 10$

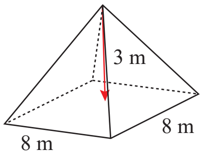
= 376.991…

= 376.99 cm^3

b Capacity of oil can = 376.99 mL

= 377 mL

17



Find length of diagonal on base:

Use Pythagoras' theorem $a^2 + b^2 = c^2$ where $a = 8$, $b = 8$ and c is length of diagonal.

$c^2 = 8^2 + 8^2$

$c^2 = 64 + 64 = 128$

$c = \sqrt{128}$

Find length of slant edge:

Use Pythagoras' theorem $a^2 + b^2 = c^2$ where

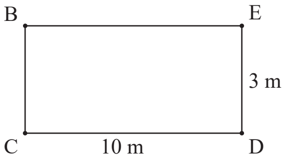
$a = \frac{1}{2}$ length base diagonal $= \frac{\sqrt{128}}{2}$,

$b = 3$ and c is length of slant edge.

$$c^2 = (\frac{\sqrt{128}}{2})^2 + 3^2$$

$$c = \sqrt{\frac{128}{4} + 9} = 6.403\ldots = 6.4 \text{ m}$$

18 **a**

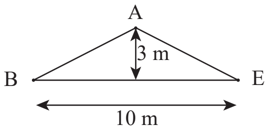


Area of rectangle $BCDE = 3 \times 10$

$= 30$

The area of the rectangle $BCDE$ is 30 m^2.

b



Area of triangle $ABE = \frac{1}{2}\ bh$

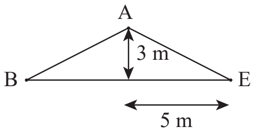
where $b = 10$, $h = 3$

$= \frac{1}{2} \times 10 \times 3$

$= 15$

The area of the triangle ABE is 15 m^2.

c



Using Pythagoras' theorem,

$AE^2 = 3^2 + 5^2$

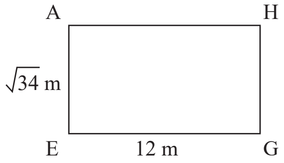
$= 34$

$AE = \sqrt{34}$

$= 5.83$, correct to 2 d.p.

The length of AE is 5.83 m, correct to 2 decimal places.

d



Area of rectangle $AEGH = lw$

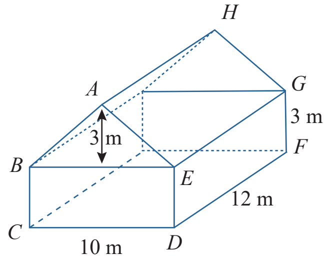
where $l = \sqrt{34}$, $w = 12$

$= \sqrt{34} \times 12$

$= 69.97$, correct to 2 decimal places

The area of the rectangle $AEGH$ is 69.97 m^2, correct to 2 decimal places.

e



Total Surface Area of solid

$= 2 \times (\text{area } ABE + \text{area } AEGH + \text{area } BCDE + 12 \times 3) + 12 \times 10$

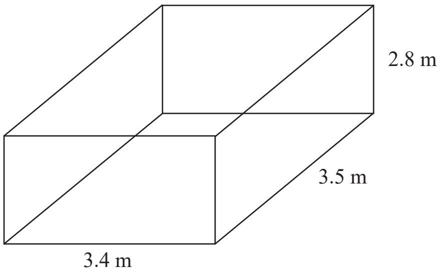
$= 2 \times (15 + 12\sqrt{34} + 30 + 36) + 120$

$= 2 \times (81 + 12\sqrt{34}) + 120$

$= 421.94$, correct to 2 d.p.

The total area of the solid is 421.94 m^2, correct to 2 decimal places.

19



Volume of rectangular prism $= lwh$

where $l = 3.5$, $w = 3.4$, $h = 2.8$

$= 3.5 \times 3.4 \times 2.8$

$= 33.32$

The volume of the rectangular prism is 33.32 m^3.

20 Length ratio is 2 : 1

So area ratio is $2^2 : 1^2$

$= 4 : 1$

Area will increase by a factor of 4.

21 Length ratio is $\frac{1}{2} : 1$

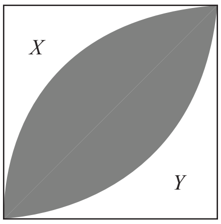
So area ratio is $(\frac{1}{2})^2 : 1^2$

$= \frac{1}{4} : 1$

Area will decrease by $\frac{1}{4}$.

22 **a** Perimeter of shaded region
$= 2 (\frac{1}{4}$ circumference of a circle)
$= 2 \times \frac{1}{4} \times 2\pi r$ where $r = 16$
$= 2 \times \frac{1}{4} \times 2 \times\pi \times 16$
$= 50.265\ldots$
$= 50.27$ cm

b



Area of square $= l^2$ where $l = 16$
$= 16^2$
$= 256$ cm^2
Area of $\frac{1}{4}$ circle $= \frac{1}{4}\pi r^2$ where $r = 16$
$= \frac{1}{4}\pi \times 16^2$
$= 201.06$ cm^2
Thus area X on diagram $= 256 - 201.06$
$= 54.94$ cm^2
Area Y on diagram $= 54.94$ cm^2
Area X + Area Y = 109.88 cm^2
Required shaded area $= 256 - 109.88$
$= 146.12$ cm^2

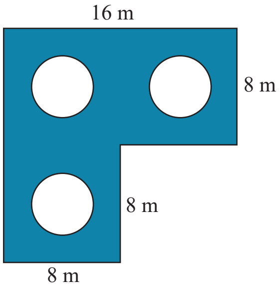
23 Length of large semi-circle $= \frac{1}{2}$ circumference of circle with radius 8 m
$= \frac{1}{2} \times 2\pi r$ where $r = 8$ cm
$= \pi \times 8$
$= 8\pi$
$= 25.13$ cm

Length of small semi-circle $= \frac{1}{2}$ circumference of circle with diameter $(16 \div 4) = 4$ cm
$= \frac{1}{2} \times 2\pi r$ where $r = 2$ cm
$= \pi \times 2$
$= 2\pi$
There are 4 small semi-circles so total distance $= 2\pi \times 4$
$= 8\pi$
$= 25.13$ cm

Distances are the same. They are both 25.13 cm.

Solutions to Chapter Review Written-Response Questions

1



a The area of each circular flower bed, A_f m^2, is given by:

$$\begin{aligned} A_f &= \pi r^2 \\ &= \pi \times 2^2 \\ &= 4\pi \end{aligned}$$

The area of the whole garden, A m^2, is given by:

$$\begin{aligned} A &= 16^2 - 8^2 \\ &= 192 \end{aligned}$$

The area of the lawn, A_l m^2, is given by:

$$\begin{aligned} A_l &= A - 3 \times A_f \\ &= 192 - 3 \times 4\pi \\ &= 154.30, \text{ correct to 2 decimal places} \end{aligned}$$

The area to be mown is 154.30 m^2.

b The circumference of each flower bed, C m, is given by:

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi \times 2 \\ &= 4\pi \end{aligned}$$

The perimeter of the lawn, P m, is given by:

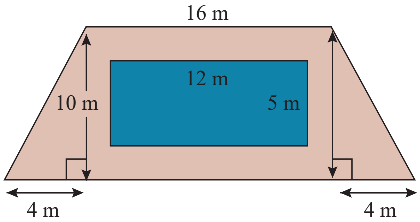
$$P = 16 + 16 + 8 + 8 + 8 + 8 = 64$$

The length of the edges to be trimmed, L m, is given by:

$$\begin{aligned} L &= P + 3 \times C \\ &= 64 + 3 \times 4\pi \\ &= 101.70, \text{ correct to 2 decimal places} \end{aligned}$$

The length of the edges to be trimmed is 101.70 m.

2



a Using Pythagoras' theorem:
$AD^2 = 4^2 + 10^2 = 116$
$AD = 10.77$, correct to 2 decimal places

Similarly, $BC = 10.77$.

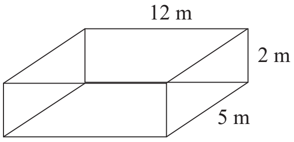
The length of fencing needed to surround the timber decking, L m, is given by:
$L = 16 + 10.77 + 4 + 16 + 4 + 10.77$
$= 61.54$

The length of fencing needed to surround the timber decking is 61.54 m, correct to 2 decimal places.

b The area of the trapezium, A m^2, is given by:
$A = \frac{1}{2} \times (16 + 24) \times 10 = 200$
The area of the pool, A_p m^2, is given by:
$A_p = 12 \times 5 = 60$

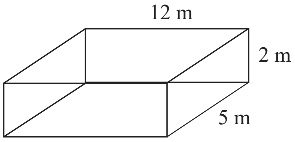
The area of the timber decking, A_t m^2, is given by:
$A_t = A - A_p = 200 - 60 = 140$
The area of timber decking required is 140 m^2.

c



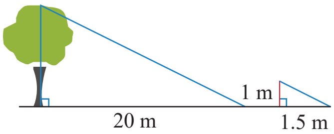
The volume of the pool, V m^3, is given by:
$V = 12 \times 5 \times 2 = 120$ m^3

d



The surface area to be painted, S m^2, is given by:
S = area of base + area of 4 sides
$= 12 \times 5 + 12 \times 2 + 12 \times 2 + 5 \times 2 + 5 \times 2$
$= 128$
The surface area to be painted is 128 m^2.

3



Triangles *ABC* and *A'B'C'* are similar.

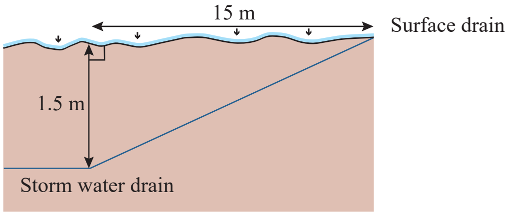
$$\frac{\text{Height of } \Delta ABC}{\text{Height of } \Delta A'B'C} = \frac{\text{Base of } \Delta ABC}{\text{Base of } \Delta A'B'C}$$

$$\frac{h}{1} = \frac{20}{1.5}$$

$$h = 13.33$$

The height of the tree is 13.33 m, correct to 2 decimal places.

4



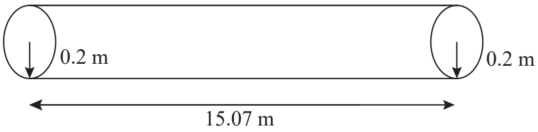
a Using Pythagoras' theorem:

$AB^2 = (1.5)^2 + 15^2 = 227.25$

$AB = 15.07$, correct to 2 decimal places

The length of water pipe required to connect the surface drain to the storm water drain is 15.07 m, correct to 2 decimal places.

b



The radius of the water pipe is 20 cm, or 0.2 m.

The volume of the water pipe, V m^3, is given by:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times (0.2)^2 \times 15.07 \\ &= 1.89, \text{ correct to 2 decimal places} \end{aligned}$$

The volume of the water pipe is 1.89 m^3, correct to 2 decimal places.

5 a

$$\begin{aligned} \text{length ratio} &= \frac{3.5}{2.5} \text{ or } 2.5 : 3.5 \\ &= \frac{7}{5} \text{ or } 5 : 7 \\ &= 1.4 \text{ or } 5 : 7 \\ &= 1.4 \text{ or } 1 : 1.4 \end{aligned}$$

$$\begin{aligned} \text{area ratio} &= 1.4^2 \\ &= 1.96 \text{ or } 1 : 1.96 \end{aligned}$$

b

$$\text{volume ratio} = 1.4^3$$
$$= 2.744 \text{ or } 1 : 2.744$$

c

$$V = AL$$
$$= \frac{1}{2} \times 5 \times 2.5 \times 10$$
$$= 62.5 \text{ cm}^3$$
$$\approx 63 \text{ cm}^3$$

6 Volume of rectangular prism, $V = lwh$ where $l = 8h$ and $w = 4h$

$= (8h)(4h)h$

$= 32h^3 \quad (1)$

Use triangle ABC to find length of diagonal AC

Use Pythagoras' theorem, $a^2 + b^2 = c^2$ where $a = 8h$, $b = 4h$ and $c = AC$

$(AC)^2 = (8h)^2 + (4h)^2 = 64h^2 + 16h^2$

$AC = \sqrt{80h^2}$

Use Pythagoras' theorem with triangle ACG to find value of h.

$a^2 + b^2 = c^2$ where $a = AC = \sqrt{80h^2}$,

$b = CG = h$ and $c = AG = 36$

$(AG)^2 = (AC)^2 + (CG)^2$

$36^2 = (\sqrt{80h^2})^2 + h^2$

$36^2 = 80h^2 + h^2$

$36^2 = 81h^2$

$\sqrt{36^2} = \sqrt{81h^2}$

$36 = 9h$

$h = \frac{36}{9}$

$h = 4$

Thus $V = 32h^3$ from (1)

$= 2048 \text{ cm}^3$

7 Ratio of volumes = 420 : 120

Scale factor of vol. $= \frac{420}{120} = \frac{7}{2} = 3.5$

Thus scale factor for length is $\sqrt[3]{3.5}$

The height of a similar cone will therefore be $28.4 \div \sqrt[3]{3.5}$

= 18.705…

= 18.71 cm , correct to 2 d.p.

8 **a** Total distance = 2 × 101 + circumference of circle with radius (63 ÷ 2)

$= 202 + 2\pi\,(31.5)$

= 399.92…

= 400 m to the nearest m.

b From part **a,** competitor in inside lane runs 400 m.
Competitor in second lane runs distance
= 202 + circumference of circle with radius (65 ÷ 2)
= 202 + 2π (32.5)
= 406.20…
= 406 m to the nearest metre.

Competitor in third lane runs distance
= 202 + circumference of circle with radius (67 ÷ 2)
= 202 + 2π (33.5)
= 412.48…
= 412 m to the nearest metre.

Competitor in fourth lane runs distance
= 202 + circumference of circle with radius (69 ÷ 2)
= 202 + 2π (34.5)
= 418.76…
= 419 m to the nearest metre.

Competitor in fifth lane runs distance
= 202 + circumference of circle with radius (71 ÷ 2)
= 202 + 2π (35.5)
= 425.05…
= 425 m to the nearest metre

Competitor in sixth lane runs distance
= 202 + circumference of circle with radius (73 ÷ 2)
= 202 + 2π (36.5)
= 431.33…
= 431 m to the nearest metre.

c The starting point for each runner should be 6 metres apart except for runners in 3^{rd} and 4^{th} lanes where distance should be 7 m.

Chapter 11 – Applications of trigonometry

Solutions to 11A Now Try This Questions

1 a The side is opposite the angle θ.
It is called the **opposite** side.

b The side is between the angle θ and the right-angle.
It is called the **adjacent** side.

c The side is opposite the right-angle.
It is called the **hypotenuse**.

2 Make sure your calculator is in **degree** mode.

a Choose the **tan** button on your CAS calculator.
Type **28** and press **enter** or **EXE**, to get 0.531709
Look at the fourth decimal place.
It is **7**, which is greater than 5, so increase the third decimal place by 1 to make it **2**.
0.532

b Choose the **cos** button on your CAS calculator.
Type **43** and press **enter** or **EXE**, to get 0.731354
Look at the fourth decimal place.
It is **3**, which is not 5 or greater than 5, so the third decimal place stays at **1**.
0.731

c Choose the **sin** button on your CAS calculator.
Type **62.8** and press **enter** or **EXE**, to get 0.889416
Look at the fourth decimal place.
It is **4**, which is not 5 or greater than 5, so the third decimal place stays at **9**.
0.889

Solutions to Exercise 11A

1 a The 8 cm side is opposite angle θ, so it is the **opposite** side.

b The 15 cm side is the longest side (and it is opposite the right-angle), so it is the **hypotenuse**.

c The 17 cm side is between θ and the right-angle. It is called the **adjacent** side.

2 a The 45 m side is opposite angle θ, so it is the **opposite** side.

b The 28 m side is between θ and the right-angle. It is called the **adjacent** side.

c The 53 m side is the longest side (and it is opposite the right-angle), so it is the **hypotenuse**.

3 a $\sin\theta = \frac{opp}{hyp} = \frac{21}{29}$

b $\cos\theta = \frac{adj}{hyp} = \frac{20}{29}$

c $\tan\theta = \dfrac{opp}{adj} = \dfrac{21}{20}$

4 h = hypotenuse; o = opposite; a = adjacent

a $h = 13$; $o = 5$; $a = 12$

b $h = 10$; $o = 6$; $a = 8$

c $h = 17$; $o = 8$; $a = 15$

5 **a** $\sin\theta = \dfrac{o}{h} = \dfrac{5}{13}$

$\cos\theta = \dfrac{a}{h} = \dfrac{12}{13}$

$\tan\theta = \dfrac{o}{a} = \dfrac{5}{12}$

b $\sin\theta = \dfrac{o}{h} = \dfrac{6}{10} = \dfrac{3}{5}$

$\cos\theta = \dfrac{a}{h} = \dfrac{8}{10} = \dfrac{4}{5}$

$\tan\theta = \dfrac{o}{a} = \dfrac{6}{8} = \dfrac{3}{4}$

c $\sin\theta = \dfrac{o}{h} = \dfrac{8}{17}$

$\cos\theta = \dfrac{a}{h} = \dfrac{15}{17}$

$\tan\theta = \dfrac{o}{a} = \dfrac{8}{15}$

6 Use your calculator in DEGREE mode to evaluate the trigonometric ratios.

a $\sin 27° = 0.4540$

b $\cos 43° = 0.7314$

c $\tan 62° = 1.8807$

d $\cos 79° = 0.1908$

e $\tan 14° = 0.2493$

f $\sin 81° = 0.9877$

g $\cos 17° = 0.9563$

h $\tan 48° = 1.1106$

7 $\cos\theta = \dfrac{adj}{hyp} = \dfrac{20}{29}$

Let $adj = 20$, hyp = 29
By Pythagoras' theorem:

$$
\begin{aligned}
(adj)^2 + (opp)^2 &= (hyp)^2\\
20 + (opp)^2 &= 29^2\\
(opp)^2 &= 441\\
opp &= 21\\
\sin\theta &= \frac{opp}{hyp}\\
&= \frac{21}{29}
\end{aligned}
$$

8 $\sin\theta = \dfrac{opp}{hyp} = \dfrac{9}{41}$

Let $opp = 9$, hyp = 41
By Pythagoras' theorem:

$$
\begin{aligned}
(adj)^2 + (opp)^2 &= (hyp)^2\\
(adj)^2 + (9)^2 &= (41)^2\\
(adj)^2 &= 1600\\
adj &= 40\\
\tan\theta &= \frac{opp}{adj}\\
&= \frac{9}{40}
\end{aligned}
$$

Solutions to 11B Now Try This Questions

3 The sides involved are the opposite and the adjacent , so use tan θ.

$$\tan\theta = \frac{opp}{adj}$$

$\theta = 42°$, opp = x, adj = 64

$$\tan 42° = \frac{x}{64}$$

$$\begin{aligned} x &= 64 \tan 42° \\ &= 57.6 \end{aligned}$$

4 The sides involved are the opposite and the hypotenuse , so use sin θ.

$$\sin\theta = \frac{opp}{hyp}$$

$\theta = 47°$, opp = 19, hyp = x

$$\sin 47° = \frac{19}{x}$$

$$x \times \sin 47° = 19$$

$$\begin{aligned} x &= \frac{19}{\sin 47°} \\ &= 25.979 \\ &= 26.0 \end{aligned}$$

Solutions to Exercise 11B

1 **a** The 36 cm side is between the given angle and the right-angle. It is called the **adjacent** side.

b The side x is opposite the given angle so it is called the **opposite** side.

c The tan ratio uses the adjacent and opposite sides so use:

$$\tan\theta = \frac{opp}{adj}$$

d Use $\theta = 47°$, $opp = x$, $adj = 36$

$$\tan 47° = \frac{x}{36}$$

e $\tan 47° = \frac{x}{36}$

$$\begin{aligned} x &= 36 \times \tan 47° \\ &= 38.61 \end{aligned}$$

2 **a** The side 19 cm long is opposite the given angle so it is called the **opposite** side.

b The side x is opposite the right-angle and is called the **hypotenuse**.

c The sin ratio uses the opposite and adjacent sides so use:

$$\sin\theta = \frac{opp}{adj}$$

d Use $\theta = 39°$, $opp = 19$, $adj = x$

$$\sin 39° = \frac{19}{x}$$

e $x = \frac{19}{\sin 39°}$

f $x = 30.2\text{cm}$

3 **a** x = opposite

$h = 31, \theta = 42$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 42^\circ = \frac{x}{31}$$

$$x = 31 \times \sin 42^\circ$$

$$= 20.74$$

b x = adjacent

$h = 26, \theta = 37$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 37^\circ = \frac{x}{26}$$

$$x = 26 \times \cos 37^\circ$$

$$= 20.76$$

c x = opposite

$a = 58, \theta = 29$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 29^\circ = \frac{x}{58}$$

$$x = 58 \times \tan 29^\circ$$

$$= 32.15$$

d x = adjacent

$h = 22, \theta = 68$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 68^\circ = \frac{x}{22}$$

$$x = 22 \times \cos 68^\circ$$

$$= 8.24$$

e x = opposite

$a = 16, \theta = 59$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 59^\circ = \frac{x}{16}$$

$$x = 16 \times \tan 59^\circ$$

$$= 26.63$$

f x = opposite

$h = 9, \theta = 57$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 57^\circ = \frac{x}{9}$$

$$x = 9 \times \sin 57^\circ$$

$$= 7.55$$

4 **a** x = hypotenuse

$a = 58, \theta = 42$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 42^\circ = \frac{58}{x}$$

$$x = \frac{58}{\cos 42^\circ}$$

$$= 78.05$$

b x = hypotenuse

$o = 22, \theta = 59$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 59^\circ = \frac{22}{x}$$

$$x = \frac{22}{\sin 59^\circ}$$

$$= 25.67$$

c x = adjacent

$o = 8, \theta = 43$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 43^\circ = \frac{8}{x}$$

$$x = \frac{8}{\tan 43^\circ}$$

$$= 8.58$$

d x = hypotenuse

$o = 49,\ \theta = 63$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 63^\circ = \frac{49}{x}$$

$$x = \frac{49}{\sin 63^\circ}$$

$= 54.99$

e x = hypotenuse

$a = 19,\ \theta = 27$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 27^\circ = \frac{19}{x}$$

$$x = \frac{18}{\cos 27^\circ}$$

$= 21.32$

f x = adjacent

$o = 12,\ \theta = 46$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 46^\circ = \frac{12}{x}$$

$$x = \frac{12}{\tan 46^\circ}$$

$= 11.59$

5 a x = adjacent

$h = 16,\ \theta = 37$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 37^\circ = \frac{x}{16}$$

$x = 16 \times \cos 37^\circ$

$= 12.8$

b x = hypotenuse

$a = 21,\ \theta = 42$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 42^\circ = \frac{21}{x}$$

$$x = \frac{21}{\cos 42^\circ}$$

$= 28.3$

c x = opposite

$h = 47,\ \theta = 55$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 55^\circ = \frac{x}{47}$$

$x = 47 \times \sin 55^\circ$

$= 38.5$

d x = hypotenuse

$o = 59,\ \theta = 48$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 48^\circ = \frac{59}{x}$$

$$x = \frac{59}{\sin 48^\circ}$$

$= 79.4$

e x = opposite

$a = 20,\ \theta = 39$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 39^\circ = \frac{x}{20}$$

$x = 20 \times \tan 39^\circ$

$= 16.2$

f x = adjacent

$o = 14,\ \theta = 43$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 43^\circ = \frac{14}{x}$$

$$x = \frac{14}{\tan 43^\circ}$$

$= 15.0$

6 $\sin 32^\circ = \dfrac{y}{23}$

$y = 12.1881$

$\sin 65^\circ = \dfrac{y}{x}$

$x = \dfrac{y}{\sin 65^\circ}$

$\quad = 13.4$

7 $\cos 46^\circ = \dfrac{a}{10}$

$a = 6.94658$

$\sin 46^\circ = \dfrac{b}{10}$

$b = 7.1934$

$\tan 32^\circ = \dfrac{b}{c}$

$c = \dfrac{b}{\tan 32^\circ}$

$\quad = 11.5118$

$z = a + c$

$\quad = 18.5$

Solutions to 11C Now Try This Questions

5 **a** $\cos\theta = 0.6847$

$\theta = \cos^{-1}(0.6847)$

Select $\cos^{-1}$ and type 0.6847

$\theta = 46.787982°$

$\theta = 46.79°$

b $\tan\theta = 7.5509$

$\theta = \tan^{-1}(7.5509)$

Select $\tan^{-1}$ and type 7.5509

$\theta = 82.45596°$

$\theta = 82.46°$

c $\sin\theta = 0.2169$

$\theta = \sin^{-1}(0.2169)$

Select $\sin^{-1}$ and type 0.2169

$\theta = 12.52702°$

$\theta = 52.53°$

6 Adjacent and hypotenuse are involved.

Use $\cos\theta = \dfrac{adj}{hyp}$

adj = 8 hyp = 14

$\cos\theta = \dfrac{8}{14}$

$\theta = \cos^{-1}\dfrac{8}{14}$

Select $\cos^{-1}$ and type $8 \div 14$

$\theta = 55.150095°$

$\theta = 55.15°$

Solutions to Exercise 11C

1 **a** $\cos\theta = 0.4867$

$\theta = \cos^{-1} 0.4867$

$= 60.9°$

b $\tan\theta = 0.6384$

$\theta = \tan^{-1} 0.6384$

$= 32.6°$

c $\sin\theta = 0.3928$

$\theta = \sin^{-1} 0.3928$

$= 23.1°$

2 **a** The side 28 cm long is opposite the right-angle so it is called the **opposite** side.

b The 25 cm side is between the given angle and the right-angle and is called the **adjacent** side.

c The cos ratio uses the adjacent and hypotenuse sides so use:

$$\cos\theta = \frac{adj}{hyp}$$

d $$\cos\theta = \frac{25}{28}$$

e $\cos\theta = \dfrac{25}{28}$

$\theta = \cos^{-1}\dfrac{25}{28}$

$= 26.8°$

3 **a** $\sin\theta = 0.4817$

$\theta = \sin^{-1} 0.4817$

$= 28.8°$

b $\cos\theta = 0.6275$

$\theta = \cos^{-1} 0.6275$

$= 51.1°$

c $\tan\theta = 0.8666$

$\theta = \tan^{-1} 0.8666$

$= 40.9°$

d $\sin\theta = 0.5000$

$\theta = \sin^{-1} 0.5000$

$= 30.0°$

e $\tan\theta = 1.0000$

$\theta = \tan^{-1} 1.0000$

$= 45.0°$

f $\cos\theta = 0.7071$

$\theta = \cos^{-1} 0.7071$

$= 45.0°$

g $\sin\theta = 0.8660$

$\theta = \sin^{-1} 0.8660$

$= 60.0°$

h $\tan\theta = 2.5000$

$\theta = \tan^{-1} 2.5000$

$= 68.2°$

i $\cos\theta = 0.8383$

$\theta = \cos^{-1} 0.8383$

$= 33.0°$

4 a $h = 30, o = 16$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{16}{30} = 0.5333$$

$\theta = \sin^{-1} 0.5333$

$= 32.2°$

b $h = 47, a = 24$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{24}{47} = 0.5106$$

$\theta = \cos^{-1} 0.5106$

$= 59.3°$

c $o = 7, a = 13$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{7}{12} = 0.5385$$

$\theta = \tan^{-1} 0.5385$

$= 28.3°$

d $h = 16, a = 9$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{9}{16} = 0.5625$$

$\theta = \cos^{-1} 0.5625$

$= 55.8°$

e $o = 19, a = 18$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{19}{18} = 1.0555$$

$\theta = \tan^{-1} 1.0555$

$= 46.5°$

f $h = 48, o = 36$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{36}{48} = 0.7500$$

$\theta = \sin^{-1} 0.7500$

$= 48.6°$

g $o = 4, a = 3$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{4}{3} = 1.3333$$

$\theta = \tan^{-1} 1.3333$

$= 53.1°$

h $h = 90, o = 77$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{77}{90} = 0.8555$$

$\theta = \sin^{-1} 0.8555$

$= 58.8°$

i $h = 13, a = 12$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{12}{13} = 0.9231$$

$$\theta = \cos^{-1} 0.9231$$

$$= 22.6°$$

5 **a** $h = 5, o = 3$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{3}{5} = 0.6000$$

$$\theta = \sin^{-1} 0.6000$$

$$= 36.9°$$

b $h = 13, o = 12$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{12}{13} = 0.9231$$

$$\theta = \sin^{-1} 0.9231$$

$$= 67.4°$$

c $h = 17, o = 8$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{8}{17} = 0.4706$$

$$\theta = \sin^{-1} 0.4706$$

$$= 28.1°$$

6 $\tan \angle BDC = \frac{5}{7}$

$$\angle BDC = 35.54°$$

$$= 180° - \angle BDC$$

$$= 144.5°$$

7 $\sin \angle BDC = \frac{19}{24}$

$$\angle BDC = 52.3415°$$

$$\theta = 90° - \angle ADB$$

$$= 90° - (90° - \angle BDC)$$

$$= \angle BDC$$

$$= 52.3°$$

Solutions to 11D Now Try This Questions

7 Opposite and hypotenuse are involved.

Use $\sin\theta = \dfrac{opp}{hyp}$

$\theta = 35°, \quad \text{opp} = rise, \quad \text{hyp} = 200$

$$\sin 35° = \frac{rise}{200}$$

$$200 \times \sin 35° = rise$$

$$rise = 114.715$$

$$= 115 \text{ m}$$

8 Opposite and hypotenuse are involved.

Use $\sin\theta = \dfrac{opp}{hyp}$

$\text{opp} = 2 \quad \text{hyp} = 3.5$

$$\sin\theta = \frac{2}{3.5}$$

$$\theta = \sin^{-1}\frac{2}{3.5}$$

Select $\sin^{-1}$ and type $2 \div 3.5$

$$\theta = 34.8499°$$

$$\theta = 35°$$

Solutions to Exercise 11D

1 **a** The 28 m side is between the given angle and the right-angle. It is called the **adjacent** side.

b The side x is opposite the given angle so it is called the **opposite** side.

c The tan ratio uses the adjacent and opposite sides so use:

$$\tan\theta = \frac{opp}{adj}$$

d Use $\theta = 40°$, $opp = x$, $adj = 28$

$$\tan 40° = \frac{x}{28}$$

e $\tan 40° = \dfrac{x}{28}$

$$x = 28 \times \tan 40°$$

$$= 23.5 \text{ m}$$

2 **a** The 35 m side is opposite the right-angle. It is called the **hypotenuse**.

b The side 10 m long is opposite the given angle so it is called the **opposite** side.

c The sin ratio uses the opposite and hypotenuse so use:

$$\sin\theta = \frac{opp}{hyp}$$

d Use $opp = 10$, $hyp = 35$

$$\sin\theta = \frac{10}{35}$$

e $\sin\theta = \dfrac{10}{35} = 0.2857$

$$\theta = \sin^{-1} 0.2857$$

$$= 16.6°$$

3 **a** The 11 km side is between the given angle and the right-angle. It is called the **adjacent** side.
The x km side is opposite the right-angle. It is called the **hypotenuse**.

b The cos ratio uses the adjacent and hypotenuse sides so use:

$$\cos\theta = \frac{adj}{hyp}$$

c

$$\cos 28^\circ = \frac{11}{x}$$

d

$$x \times \cos 28^\circ = 11$$

$$x = \frac{11}{\cos 28^\circ}$$

$$= 12.5 \text{ km}$$

4 $a = 6$, $\theta = 47$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 47^\circ = \frac{x}{6}$$

$$x = 6 \times \tan 47^\circ$$

$$= 6.43 \text{ m}$$

5 $h = 3$, $a = 2.8$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{2.8}{3} = 0.9333$$

$$\theta = \cos^{-1} 0.9333$$

$$= 21.04^\circ$$

6 x = opposite
$a = 30$, $\theta = 28$

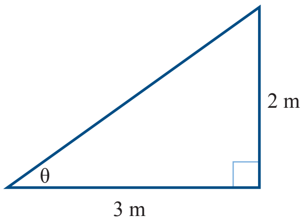
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 28^\circ = \frac{x}{30}$$

$$x = 30 \times \tan 28^\circ$$

$$= 16 \text{ m wide}$$

7 **a**



b $o = 2$, $a = 3$

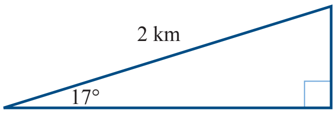
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{2}{3} = 0.6667$$

$$\theta = \tan^{-1} 0.6667$$

$$= 33.7^\circ$$

8 **a**



b a = adjacent
$h = 2$, $\theta = 17$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 17^\circ = \frac{a}{2}$$

$$a = 2 \times \cos 17^\circ$$

$= 1.91$ km horizontal from its take-off point

$h = 2$, $\theta = 17$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 17^\circ = \frac{x}{2}$$

$$x = 2 \times \sin 17^\circ$$

$$= 20.74 \text{ m}$$

$= 0.58$ km above the ground

9 $h = 3, a = 1$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$= \frac{1}{3} = 0.3333$$

$$\theta = \cos^{-1} 0.3333$$

$$= 70.5°$$

10 x = opposite

$h = 200, \theta = 23$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 23° = \frac{x}{200}$$

$$x = 200 \times \sin 23°$$

$= 78.1$ m deep

11 In triangle PQR

$$\tan 38° = \frac{PR}{24}$$

$PR = 18.751$ m

In triangle SPR

$$\sin 65° = \frac{PR}{SP}$$

$$SP = \frac{PR}{\sin 65°}$$

$$SP = \frac{18.751}{\sin 65°} = 20.7 \text{ m}$$

The cable needed is 20.7 m long

12 In triangle ABC

$$\tan 60° = \frac{AC}{200}$$

$AC = 346.41$ m

In triangle ACD

$$\tan\theta = \frac{AC}{300}$$

$$\theta = 49.1°$$

The spotlight should be at an angle of 49.1°

13 This question is unusual because no right-angled triangles are shown. Perhaps you would like to consider where a useful right-angled triangle could be drawn before reading the solution below.

****

Draw a vertical line down from B to the horizontal.
Draw a horizontal line from G to meet the vertical line at H.

In triangle BGH:

$\angle BGH = 30°$

$$\cos 30° = \frac{HG}{20}$$

$$HG = 20 \cos 30°$$

$=17.3205$ m

Also, $\sin 30° = \frac{BH}{20}$

$$BH = 20 \sin 30°$$

$= 10$ m

In triangle PGH:

$$\tan 65° = \frac{PH}{HG}$$

$$PH = HG \times \tan 65°$$

$$= 17.3205 \times \tan 65°$$

$= 37.1439$ m

Also, $PB = PH - BH$

$$= 37.1439 - 10$$

$$= 27.1439$$

$= 27.1$ m

The height of the pole is 27.1 m

Solutions to 11E Now Try This Questions

9 $\tan 47^\circ = \dfrac{\text{cliff}}{200}$

$$\begin{aligned}\text{cliff} &= 200\tan 47^\circ\\ &= 214.4737\\ &= 214\text{ m}\end{aligned}$$

10 $\tan 18^\circ = \dfrac{400}{x}$

$$\begin{aligned}x &= \frac{400}{\tan 18^\circ}\\ &= 1231.07\\ &= 1321\text{ m}\end{aligned}$$

11 **a** $\sin 47^\circ = \dfrac{h}{70}$

$$\begin{aligned}h &= 70\sin 47^\circ\\ &= 51.1948\\ &= 51\text{ m}\end{aligned}$$

b $\sin\angle BAC = \dfrac{51.1948}{150}$

$\angle BAC = 20^\circ$

Solutions to Exercise 11E

1 **a** Angle A is the angle of depression, so $A = 38^\circ$

b Angle B is the angle of elevation looking from point Q.
= Angle A, the angle of depression
= 38°

c Angle $C = 90^\circ -$ Angle A
$= 52^\circ$

d Angle B is the angle of elevation looking from point Q.
= Angle A, the angle of depression
= 38°

2 **a** The side x is opposite the given angle so it is called the **opposite** side.
The 20 m side is between the given angle and the right-angle. It is called the **adjacent** side.

b The tan ratio uses the opposite and adjacent sides so use:

$$\tan\theta = \frac{opp}{adj}$$

c Use $\theta = 52^\circ$, $opp = x$, $adj = 20$

$$\tan 52^\circ = \frac{x}{20}$$

d $\tan 52^\circ = \dfrac{x}{20}$

$$\begin{aligned}x &= 20\times\tan 52^\circ\\ &= 25.6\text{ m}\end{aligned}$$

3 **a** The angle of depression from T is the angle swept down from the horizontal to view the rabbit at point R.
The angle of depression is

$$\angle RTV = 40^\circ$$

b The tan ratio uses the opposite and adjacent sides so use:

$$\tan\theta = \frac{opp}{adj}$$

c Notice that $\angle RTV = \angle TRS$
Use $\angle TRS = 40^\circ$, $opp = x$, $adj = 20$

$$\tan 40^\circ = \frac{45}{d}$$

d Multiply both sides by d

$$d\times\tan 40^\circ = 45$$

Divide both sides by $\tan 40^\circ$

$$d = \frac{45}{\tan 40°}$$
$$= 53.6 \text{ m}$$

4 x = opposite

$a = 300, \theta = 54$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 54° = \frac{x}{300}$$
$$x = 300 \times \tan 54°$$
$$= 413 \text{ m high}$$

5 x = adjacent

$o = 3000, \theta = 15$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 15° = \frac{3000}{x}$$
$$x = \frac{3000}{\tan 15°}$$
$$= 11\,196 \text{ m from the airport}$$

6 $o = 100, a = 400$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{100}{400} = 0.2500$$
$$\theta = \tan^{-1} 0.2500$$
$$= 14°$$

7 **a** x = opposite

$h = 50, \theta = 63$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\sin 63° = \frac{x}{50}$$
$$x = 50 \times \tan 63°$$
$$= 44.6 \text{ m}$$

b $h = 75, o = 44.6$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$= \frac{44.6}{75} = 0.5947$$
$$\theta = \sin^{-1} 0.5947$$
$$= 36°$$

8 **a** x = adjacent

$o = 45, \theta = 52$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 52° = \frac{45}{x}$$
$$x = \frac{45}{\tan 52°}$$
$$= 35 \text{ m away}$$

b y = adjacent

$o = 45$, $\theta = 35$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$\tan 35° = \dfrac{45}{y}$

$y = \dfrac{45}{\tan 35°}$

= 64 m away

c Distance = $y - x$

= 64 – 35

= 29 m between man and boat

9 a $\tan 35° = \dfrac{BE}{40}$

$BE = 40 \times \tan 35°$

= 28.01 m

$\tan \angle BDE = \dfrac{BE}{15}$

$\tan \angle BDE = \dfrac{28.01}{15}$

$\angle BDE = \tan^{-1}(\dfrac{28.01}{15})$

= 61.8°

b $\tan 30° = \dfrac{BE}{EC}$

$EC = \dfrac{BE}{\tan 30°}$

$= \dfrac{28.01}{\tan 30°}$

= 48.52 m

$DC = EC - ED$

= 48.52 – 15

= 33.5 m

Solutions to 11F Now Try This Questions

12 $3 \times 90° + 40°$
$= 310°$

13 **a** $\sin 35° = \dfrac{x}{20}$
$x = 20 \sin 35°$

b Standing at Q, face North and then turn clockwise to face P.
The angle swept is:
$= 180° - 35°$
$= 145°$

Solutions to Exercise 11F

1 A **three-figure bearing** measures the angle swept out clockwise from North to face the required direction.

a We always start facing North, so no angle needs to be swept out to face in the direction of North.
The tree-figure bearing of North is 000°

b To face in the direction of East we need to rotate 90° clockwise from North.
The tree-figure bearing of East is 090°.

c To face in the direction of South we need to rotate 180° clockwise from North.
The tree-figure bearing of South is 180°.

d To face in the direction of West we need to rotate 270° clockwise from North.
The tree-figure bearing of West is 270°.

2 **a** Required clockwise rotation from North
$= 90° - 30° = 60°$
The tree-figure bearing is 060°

b Required clockwise rotation from North
$= 90° + 40° = 130°$
The tree-figure bearing is 130°

c Required clockwise rotation from North
= 180° + 20° = 200°
The tree-figure bearing is 200°

d Required clockwise rotation from North
= 360° – 10° = 350°
The tree-figure bearing is 350°

3 a The 12 km side is opposite the right-angle. It is the **hypotenuse**. The side *HC* is opposite the given angle. It is the **opposite** side.

b The opposite side and the hypotenuse are the sides used in part **a**. So use
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

c x = opposite, h =12, θ = 60
$\sin 60° = \dfrac{x}{12}$

d $\sin 60° = \dfrac{x}{12}$
$x = 12 \times \sin 60°$
$= 10.4$ km
The driver must walk 10.4 km

4 a A = N 25° E
= 0 + 25 = 025°

b B = S 70° E
= 180 – 70 = 110°

c C = S 30° W
= 180 + 30 = 210°

d D = N 80° W
= 270 – 10 = 280°

5 a 90° – 65° = 25°

b River-to-B = opposite = x
$h = 18$, $\theta = 25$
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
$\sin 25° = \dfrac{x}{18}$
$x = 18 \times \sin 25°$
$= 7.61$ km

6 a $o = 2$, $a = 3$
$\sin\theta = \dfrac{\text{opposite}}{\text{adjacent}}$
$= \dfrac{2}{3} = 0.6667$
$\theta = \tan^{-1} 0.6667$
$= 33.7°$
Bearing = 270° – 33.7°
= 236.3°
= 236°

b Bearing = 90° – 33.7°
= 56.3°
= 056°

7 a AB = opposite
$h = 20$, $\theta = 40$
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$
$\sin 40° = \dfrac{AB}{20}$
$AB = 20 \times \sin 40°$
$= 12.9$ km

b BP = adjacent
$h = 20$, $\theta = 40$
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$
$\cos 40° = \dfrac{BP}{20}$
$BP = 20 \cos 40°$
$= 15.3$ km

c $BC = 30 - AB$
$= 30 - (12.9)$

$= 17.1$ km

d $o = 15.4$, $a = 17.1$

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$= \dfrac{15.4}{17.1} = 0.9006$

$\theta = \tan^{-1} 0.9006$

$= 42°$

e Bearing = 180○ − θ

$= 180° - 42°$

$= 138°$

$h^2 = a^2 + b^2$

$= (17.1)^2 + (15.4)^2$

$= 292.41 + 1237.16$

$= 529.57$

$\sqrt{h^2} = \sqrt{529.57}$

$h = 23.0$ km

8 $\tan\theta = \dfrac{20 - 8}{21 - 15}$

$= \dfrac{12}{6}$

$= 2$

$\theta = \tan^{-1} 2$

$= 63.4°$

The bearing of the point *E* from the point *D*, to the nearest degree, is 063°

9 **a** The surveyor walked 5 km from *B* to *P* on a bearing of 310°.

Draw compass axes at *B* and rotate clockwise from north:
90° + 90° + 90° + 40° = 310°
Draw a line representing 5 km from *B* to *P*.
Draw compass axes at *P*.
To find the bearing she needed to walk from *P* back to *B*, start by facing north and rotate clockwise until facing *B*.
The angle swept out = 90° + 40° *
= 130°
* (the alternate angle to the 40° at *B*) The surveyor needed to walk on a bearing of 130° to return from *P* to *B*.

b The surveyor returned to *B* and walked 4 km from *B* to *Q* on a bearing of 060°.

At *B* rotate 60° clockwise from north.
Draw a line representing 4 km from *B* to *Q*.
Draw compass axes at *Q*.
To find the bearing she needed to walk from *Q* back to *B*, start by facing north and rotate clockwise until facing *B*.
The angle swept out
= 90° + 90° + 60° *
= 240°
* (the alternate angle to the 60° at *B*) The surveyor needed to walk on a bearing of 240° to return from *Q* to *B*.

Solutions to 11G Now Try This Questions

14 $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 47°}{6} = \frac{\sin B}{5}$

$\sin B = \frac{5\sin 47°}{6}$

$B = \sin^{-1}(\frac{5\sin 47°}{6})$

$= 37.5506°$

$= 37.6°$

15 $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin 48°} = \frac{10}{\sin 47°}$

$a = \frac{10\sin 48°}{\sin 47°}$

$a = 10.16$

$= 10.2$

16 $\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{12} = \frac{\sin 35°}{7}$

$\sin A = \frac{12\sin 35°}{7}$

$A = 79.506°$

$= 79.51°$

$\angle BAC = 79.51°$

$\angle BA'A = 79.506°$

$\angle BA'C = 180° - 79.506°$

$= 100.49°$

17 $P + 36° + 40° = 180°$

$P = 180° - 76°$

$= 104°$

$\frac{r}{\sin R} = \frac{p}{\sin P}$

$\frac{r}{\sin 36°} = \frac{100}{\sin 104°}$

$r = \frac{100\sin 36°}{\sin 104°}$

$a = 60.5779$

$= 60.58$ m

Solutions to Exercise 11G

1 The pattern of each of the three forms of the sine rule:
A side divided by the sine of its opposite angle
= Another side divided by the sine of its opposite angle

a $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{b}{\sin B} = \frac{c}{\sin C}$

b $\frac{p}{\sin P} = \frac{q}{\sin Q}$

$\frac{p}{\sin P} = \frac{r}{\sin R}$

$\frac{r}{\sin R} = \frac{q}{\sin Q}$

2 a To find side b we need $\frac{b}{\sin B}$ as part of the rule.
As we are given:

$A = 110°$ and $a = 21$ cm

use $\frac{a}{\sin A}$ as the other side of the rule.

So use:

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

b $A = 110°, \quad a = 21, \quad B = 33°$

$$\frac{b}{\sin 33°} = \frac{21}{\sin 110°}$$

c $$b = \sin 110° \times \frac{21}{\sin 110°}$$
$$= 12.2 \text{ cm}$$

3 **a** To find angle C we need $\frac{c}{\sin C}$ as part of the rule.
As we are also given:
$A = 120°$ and $a = 12$ cm
use $\frac{a}{\sin A}$ as the other side of the rule.

So use:

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

b $A = 120°, \quad a = 12, \quad c = 7$

$$\frac{7}{\sin C} = \frac{12}{\sin 120°}$$

c When the unknown, such as $\sin C$, is in the denominator the equation is easier to solve if each fraction is flipped.
The original denominators become numerators and vice versa. Flip the fractions.

$$\frac{\sin C}{7} = \frac{\sin 120°}{12}$$

d $$\sin C = 7 \times \frac{\sin 120°}{12}$$
$$C = \sin^{-1}(7 \times \frac{\sin 120°}{12})$$
$$= 30.3°$$

4 **a** $a = 15, b = 14, c = 13$

b $a = 19, b = 18, c = 21$

c $a = 31, b = 34, c = 48$

5 **a** $A + B + C = 180°$
$C = 180 - 70 - 60$
$= 50°$

b $A + B + C = 180°$
$A = 180 - 120 - 20$
$= 40°$

c $A + B + C = 180°$

$B = 180 - 40 - 35$
$= 105°$

6 **a** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\dfrac{a}{\sin 40°} = \dfrac{8}{\sin 60°}$

$a = \dfrac{8}{\sin 60°} \times \sin 40°$

$= \dfrac{8}{0.8660} \times 0.6428$

$= 5.94$

b $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\dfrac{b}{\sin 50°} = \dfrac{15}{\sin 72°}$

$b = \dfrac{15}{\sin 72°} \times \sin 50°$

$= \dfrac{15}{0.9511} \times 0.7660$

$= 12.08$

c $\dfrac{c}{\sin 110°} = \dfrac{24}{\sin 30°}$

$c = \dfrac{24 \sin 110°}{\sin 30°}$

$= 45.11$

d $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\dfrac{17}{\sin A} = \dfrac{16}{\sin 70°}$

$\dfrac{\sin A}{17} = \dfrac{\sin 70°}{16}$

$\sin A = \dfrac{\sin 70°}{16} \times 17$

$= 0.9984$

$A = \sin^{-1} 0.9984$

$= 86.8°$

e $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\dfrac{26}{\sin B} = \dfrac{37}{\sin 95°}$

$\dfrac{\sin B}{26} = \dfrac{\sin 95°}{37}$

$\sin B = \dfrac{\sin 95°}{37} \times 26$

$= 0.7000$

$B = \sin^{-1} 0.7000$

$= 44.4°$

f $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{21}{\sin C} = \dfrac{47}{\sin 115°}$

$\dfrac{\sin C}{21} = \dfrac{\sin 115°}{47}$

$\sin C = \dfrac{\sin 115°}{47} \times 21$

$= 0.4049$

$C = \sin^{-1} 0.4049$

$= 23.9°$

7 **a** $a = 12, A = 100°, b = 8$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\dfrac{\sin B}{8} = \dfrac{\sin 100°}{12}$

$\sin B = \dfrac{\sin 100°}{12} \times 8$

$= 0.6565$

$B = \sin^{-1} 0.6565$

$= 41.0°$

b $a = 14, A = 110°, c = 12$

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{\sin C}{12} = \dfrac{\sin 110°}{14}$

$\sin C = \dfrac{\sin 110°}{14} \times 12$

$= 0.8055$

$C = \sin^{-1} 0.8055$

$= 53.7°$

c $a = 17, c = 21, C = 115^\circ$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{\sin A}{17} = \frac{\sin 115^\circ}{21}$$

$$\sin A = \frac{\sin 115^\circ}{21} \times 17$$

$$= 0.7337$$

$$A = \sin^{-1} 0.7337$$

$$= 47.2^\circ$$

d $b = 25, c = 32, C = 80^\circ$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin B}{25} = \frac{\sin 80^\circ}{32}$$

$$\sin B = \frac{\sin 80^\circ}{32} \times 25$$

$$= 0.7694$$

$$B = \sin^{-1} 0.7694$$

$$= 50.3^\circ$$

8 a $a = 28, A = 103^\circ, B = 43^\circ$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{b}{\sin 43^\circ} = \frac{28}{\sin 103^\circ}$$

$$b = \frac{28}{\sin 103^\circ} \times \sin 43^\circ$$

$$= \frac{28}{0.9744} \times 0.6820$$

$$= 19.60$$

b $B = 61^\circ, c = 33, C = 70^\circ$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 61^\circ} = \frac{33}{\sin 70^\circ}$$

$$b = \frac{33}{\sin 70^\circ} \times \sin 61^\circ$$

$$= \frac{15}{0.9397} \times 0.8746$$

$$= 30.71$$

c $A = 39^\circ, b = 44, B = 30^\circ$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 39^\circ} = \frac{44}{\sin 30^\circ}$$

$$a = \frac{44}{\sin 30^\circ} \times \sin 39^\circ$$

$$= \frac{44}{0.5000} \times 0.6293$$

$$= 55.38$$

d $a = 88, A = 72^\circ, C = 47^\circ$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{c}{\sin 47^\circ} = \frac{88}{\sin 72^\circ}$$

$$c = \frac{88}{\sin 72^\circ} \times \sin 88^\circ$$

$$= \frac{88}{0.9511} \times 0.7314$$

$$= 67.67$$

9 a $a = 26, A = 108^\circ, b = 21$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{\sin B}{21} = \frac{\sin 108^\circ}{26}$$

$$\sin B = \frac{\sin 108^\circ}{26} \times 21$$

$$= 0.7682$$

$$B = \sin^{-1} 0.7682$$

$$= 50.2^\circ$$

$$A + B + C = 180^\circ$$

$$C = 180 - 108 - 50.2$$

$$= 21.8^\circ$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{c}{\sin 21.8^\circ} = \frac{26}{\sin 108^\circ}$$

$$c = \frac{26}{\sin 108^\circ} \times \sin 21.8^\circ$$

$$= \frac{26}{0.951} \times 0.3714$$

$$= 10.16$$

b $a = 19, A = 120°, c = 14$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{\sin C}{14} = \frac{\sin 120°}{19}$$

$$\sin C = \frac{\sin 120°}{19} \times 14$$

$$= 0.6381$$

$$C = \sin^{-1} 0.6381$$

$$= 39.7°$$

$$A + B + C = 180°$$

$$B = 180 - 120 - 39.7$$

$$= 20.3°$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{b}{\sin 20.35°} = \frac{19}{\sin 120°}$$

$$b = \frac{19}{\sin 120°} \times \sin 20.35°$$

$$= \frac{19}{0.8660} \times 0.3469$$

$$= 7.63$$

c $A = 31°, b = 94, B = 112°, C = 37°$

where $C = 180° - (112°+31°) = 37°$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{c}{\sin 37°} = \frac{94}{\sin 112°}$$

$$c = \frac{94}{\sin 112°} \times \sin 37°$$

$$= \frac{94}{0.9272} \times 0.6018$$

$$= 61.01$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 31°} = \frac{61.01}{\sin 37°}$$

$$a = \frac{61.01}{\sin 37°} \times \sin 31°$$

$$= \frac{61.01}{0.6018} \times 0.5150$$

$$= 52.22$$

d $a = 40, A = 71°, B = 55°, C = 54°$

where $C = 180° - (71°+55°) = 54°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{c}{\sin 54°} = \frac{40}{\sin 71°}$$

$$c = \frac{40}{\sin 71°} \times \sin 54°$$

$$= \frac{40}{0.9445} \times 0.8090$$

$$= 34.23$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{b}{\sin 55°} = \frac{40}{\sin 71°}$$

$$b = \frac{40}{\sin 71^\circ} \times \sin 55^\circ$$
$$= \frac{40}{0.9445} \times 0.8192$$
$$= 34.65$$

10 $a = 60, A = 105^\circ, B = 39^\circ$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{b}{\sin 39^\circ} = \frac{60}{\sin 105^\circ}$$
$$b = \frac{60}{\sin 105^\circ} \times \sin 39^\circ$$
$$= \frac{60}{0.94659} \times 0.6293$$
$$= 39.09$$

11 $a = 65, A = 112^\circ, c = 48$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{\sin C}{48} = \frac{\sin 112^\circ}{65}$$
$$\sin C = \frac{\sin 112^\circ}{65} \times 48$$
$$= 0.6847$$
$$C = \sin^{-1} 0.6847$$
$$= 43.2^\circ$$

12 $p = 70, P = 85^\circ, R = 45^\circ$

$$\frac{p}{\sin P} = \frac{r}{\sin R}$$
$$\frac{r}{\sin 45^\circ} = \frac{70}{\sin 85^\circ}$$
$$r = \frac{70}{\sin 85^\circ} \times \sin 45^\circ$$
$$= \frac{70}{0.9962} \times 0.7071$$
$$= 49.69$$

13 $A = 47^\circ, B = 59^\circ, c = 41, C = 74^\circ$

$$\frac{a}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 59^\circ} = \frac{41}{\sin 74^\circ}$$
$$b = \frac{41}{\sin 74^\circ} \times \sin 59^\circ$$
$$= \frac{41}{0.9613} \times 0.8572$$
$$= 36.59$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 47^\circ} = \frac{41}{\sin 74^\circ}$$
$$a = \frac{41 \sin 47^\circ}{\sin 74^\circ}$$
$$= 31.19$$

14 $a = 60, b = 100, B = 130^\circ$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin A}{60} = \frac{\sin 130^\circ}{100}$$
$$\sin A = \frac{60 \sin 130^\circ}{100}$$
$$A = \sin^{-1}\left(\frac{60 \sin 130^\circ}{100}\right)$$
$$= 27.4^\circ$$

$$A + B + C = 180^\circ$$
$$C = 180^\circ - 27.4^\circ - 130^\circ$$
$$= 22.6^\circ$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{\sin 22.637^\circ} = \frac{100}{\sin 130^\circ}$$
$$c = \frac{100 \sin 22.637^\circ}{\sin 130^\circ}$$
$$= 50.24$$

15 $P = 130^\circ, Q = 30^\circ, r = 69, R = 20^\circ$

$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$
$$\frac{q}{\sin 30^\circ} = \frac{69}{\sin 20^\circ}$$

$$q = \frac{69}{\sin 20^\circ} \times \sin 30^\circ$$
$$= \frac{69}{0.3420} \times 0.5000$$
$$= 100.87$$

$$\frac{p}{\sin P} = \frac{r}{\sin R}$$
$$\frac{p}{\sin 130^\circ} = \frac{69}{\sin 20^\circ}$$
$$p = \frac{69}{\sin 20^\circ} \times \sin 130^\circ$$
$$= \frac{69}{0.3420} \times 0.7660$$
$$= 154.54$$

16 **a** $A = 35^\circ, c = 8, a = 5$

$$\begin{aligned} \frac{\sin C}{c} &= \frac{\sin A}{a} \\ \frac{\sin C}{8} &= \frac{\sin 35^\circ}{5} \\ \sin C &= \frac{8 \sin 35^\circ}{5} \\ C &= \sin^{-1}\left(\frac{8 \sin 35^\circ}{5}\right) \\ &= 66.5953^\circ \\ \angle BCA &= 66.60^\circ \end{aligned}$$

b
$$\begin{aligned} \angle BC'C &= \angle BCA \\ &= 66.60^\circ \end{aligned}$$

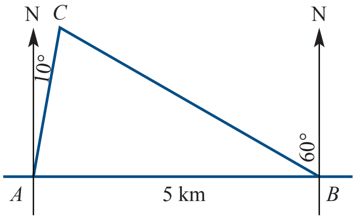
c
$$\begin{aligned} \angle AC'B &= 180^\circ - \angle BC'C \\ &= 180^\circ - 66.60^\circ \\ &= 113.40^\circ \end{aligned}$$

d 66.60°, 113.40°

17 $A = 30^\circ$, c = 7, a = 4

$$\begin{aligned} \frac{\sin C}{c} &= \frac{\sin A}{a} \\ \frac{\sin C}{7} &= \frac{\sin 30^\circ}{4} \\ \sin C &= \frac{7 \sin 30^\circ}{4} \\ C &= \sin^{-1}\left(\frac{7 \sin 30^\circ}{4}\right) \\ C &= 61.04^\circ \\ C' &= 180 - C \\ &= 180 - 61.04^\circ \\ &= 118.96^\circ \end{aligned}$$

18



$A = 80^\circ, B = 30^\circ, c = 5, C = 70^\circ$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 80^\circ} = \frac{5}{\sin 70^\circ}$$
$$a = \frac{5}{\sin 70^\circ} \times \sin 80^\circ$$
$$= \frac{5}{0.9397} \times 0.9848$$
$$= 5.24$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 30^\circ} = \frac{5}{\sin 70^\circ}$$
$$b = \frac{5}{\sin 70^\circ} \times \sin 30^\circ$$
$$= \frac{5}{0.9397} \times 0.5000$$
$$= 2.66 \text{ km}$$

19 $A = 30°, B = 135°, c = 150, C = 15°$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 135°} = \frac{150}{\sin 15°}$$

$$b = \frac{150}{\sin 15°} \times \sin 135°$$

$$= \frac{150}{0.2588} \times 0.7071$$

$$= 409.81 \text{ m}$$

20 **a** $A = 90° - 44° = 46°$

$C = 342° - 270° = 72°$

$B = 180° - (46° + 72°) = 62°$

$b = 25$

c = distance of naval ship from the island

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{c}{\sin 72°} = \frac{25}{\sin 62°}$$

$$c = \frac{25 \sin 72°}{\sin 62°}$$

$$= 26.93 \text{ km}$$

a = distance of second ship from the island

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 46°} = \frac{25}{\sin 62°}$$

$$a = \frac{25 \sin 46°}{\sin 62°}$$

$$= 20.37 \text{ km}$$

b

$$\text{Time } (h) = \frac{\text{Distance } (km)}{\text{Constant speed } (km/h)}$$

$$= \frac{20.37}{15}$$

$$= 1.36 \text{ h}$$

$$= 1 \text{ h } 22 \text{ m}$$

21 $A = 70°, B = 60°, c = 80, C = 50°$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 60°} = \frac{80}{\sin 50°}$$

$$b = \frac{80}{\sin 50°} \times \sin 60°$$

$$= \frac{80}{0.7660} \times 0.8660$$

$= 90.44$ km (airport A to plane)

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 70°} = \frac{80}{\sin 50°}$$

$$a = \frac{80}{\sin 50°} \times \sin 70°$$

$$= \frac{80}{0.7660} \times 0.9397$$

$= 98.13$ km (airport B to plane)

a Therefore airport A is closer to the plane.

b 90.44 km

c

1525 litres needed for 100 km

$\frac{1525}{100}$ litres needed for 1 km

$\frac{1525}{100} \times 90.44$ litres needed for 90.44 km

1379.21 litres needed for 90.44 km

There is *just* enough fuel

22 $\dfrac{c}{\sin \angle BCA} = \dfrac{b}{\sin \angle ABC}$

$\dfrac{16}{\sin \angle BCA} = \dfrac{17}{\sin 50°}$

$\dfrac{\sin \angle BCA}{16} = \dfrac{\sin 50°}{17}$

$\sin \angle BCA = \dfrac{16 \times \sin 50°}{17}$

$\angle BCA = 46.1357°$

$\angle ACD = 180° - 46.1357°$

$= 133.864°$

$\angle CDA = 180° - (133.864° + 30°)$

$= 16.1357°$

$\dfrac{AD}{\sin \angle ACD} = \dfrac{17}{\sin \angle CDA}$

$AD = 44.1$ cm

23 a $A = 30°$ $a = 4.5$ $c = 10$

$\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\sin C = \dfrac{c \sin A}{a}$

$= \dfrac{10 \sin 30°}{4.5}$

$= 1.1111$

$C = \sin^{-1}(1.1111)$

$=$ undefined

Impossible triangle

Alternatively, draw a triangle with
$A = 30°$ $a = 4.5$ $c = 10$
and show that when:
A is acute and a < c sinA
no triangle can be made.

b $C = 40°$ $b = 12$ $c = 8.5$

$\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\sin B = \dfrac{b \sin C}{c}$

$= \dfrac{12 \sin 40°}{8.5}$

Use the **solve** feature on your CAS calculator, where angle B is x.
For example:

$\text{solve}(\sin(x) = \dfrac{12 \sin(40)}{8.5}, x) \mid 0 \le x \le 180$

Gives: $x = 65.157$ or $x = 114.84$
So: $B = 65.157°$ or $B = 114.84°$

Two possible solutions, so this is an **ambiguous** case.

The ambiguous case occurs when two sides and a non-included angle are given and for example, b, c and C are such that:

$$b \sin C < c < b$$

Alternative method: Show that the given values meet these conditions and draw a diagram using:

$C = 40°$ $\quad b = 12 \quad c = 8.5$

to demonstrate how two triangles are possible.

c $B = 50° \qquad a = 8 \qquad b = 9$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\sin A = \frac{a \sin B}{b}$$

$$= \frac{8 \sin 50°}{9}$$

Use the **solve** feature on your CAS calculator, where angle A is x.

For example:

$$\text{solve}(\sin(x) = \frac{8\sin(50)}{9}, x) \mid 0 \le x \le 180$$

Gives: $x = 42.916$ or $x = 137.084$

So: $A = 42.916°$ or $A = 137.084°$

But using the second solution:

$A + B = 137.084° + 50°$

$= 187.084° > 180°$

So the second result must be rejected.

There is **only one possible triangle**.

Alternatively, draw a triangle with

$B = 50° \quad a = 8 \quad b = 9$

and show that when

B is acute and $b > a > a \sin 50°$

only one triangle can be drawn.

Solutions to 11H Now Try This Questions

18 $b^2 = a^2 + c^2 - 2ac\cos B$

$= 23^2 + 21^2 - 2(23)(21)\cos 31°$

$= 141.976$

$b = 11.9154$

$= 11.92$

19 $c^2 = a^2 + b^2 - 2ab\cos C$

$3^2 = 7^2 + 8^2 - 2(7)(8)\cos C$

$112\cos C = 104$

$\cos C = \dfrac{104}{112}$

$C = \cos^{-1}\dfrac{104}{112}$

$= 21.8°$

20 **a** $c^2 = a^2 + b^2 - 2ab\cos C$

$9^2 = 8^2 + 11^2 - 2(8)(11)\cos C$

$176\cos C = 104$

$\cos C = \dfrac{104}{176}$

$C = \cos^{-1}\dfrac{104}{176}$

$= 53.8°$

b 270° + 54°

324°

21 **a** 48° – 28°

= 20°

b $a^2 = b^2 + c^2 - 2bc\cos A$

$= 20^2 + 15^2 - 2(20)(15)\cos 20°$

$a^2 = 61.1844$

a = 7.822

= 7.82 km

Solutions to Exercise 11H

1 The pattern of the cosine rule:
The square of one side equals the sum of the squares of the other two sides, minus twice their product, times the cosine of the angle between them.

a $a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

b $x^2 = y^2 + z^2 - 2yz\cos X$

$y^2 = x^2 + z^2 - 2xz\cos Y$

$z^2 = x^2 + y^2 - 2xy\cos Z$

2 **a** To find side c use:

$c^2 = a^2 + b^2 - 2ab\cos C$

b $C = 38°$, $a = 31$, $b = 45$

$c^2 = 31^2 + 45^2 - 2(31 \times 45)\cos 38°$

c $c^2 = 961 + 2025 - 2790 \times 0.7880$

$c^2 = 787.45$

$\sqrt{c^2} = \sqrt{787.45}$

$c = 28.1$ cm

3 **a** The angle Y occurs in this form of the cosine rule:

$y^2 = x^2 + z^2 - 2xz\cos Y$

b Substitute in the values:

$y = 28$, $x = 37$, $z = 49$

$28^2 = 37^2 + 49^2 - 2(37 \times 49)\cos Y$

c Use your calculator to tidy up the numbers

$784 = 3770 - 3626\cos Y$

d $3626\cos Y = 3770 - 784$

$3626\cos Y = 2986$

$\cos Y = \dfrac{2986}{3626}$

$\cos Y = 0.8235$

e Use the inverse cosine, $\cos^{-1}$, feature on your CAS calculator to find angle Y.

$Y = \cos^{-1} 0.8235$

$= 34.6°$

4 a $a = 51, b = 37, C = 46°$

$c^2 = a^2 + b^2 - 2ab \cos C$

$= 51^2 + 37^2 - 2(51 \times 37) \cos 46°$

$= 2601 + 1369 - 3774 \times 0.6947$

$= 1348.36$

$\sqrt{c^2} = \sqrt{1348.36}$

$c = 36.72$

b $a = 30, b = 58, C = 55°$

$c^2 = a^2 + b^2 - 2ab \cos C$

$= 30^2 + 58^2 - 2(30 \times 58) \cos 55°$

$= 900 + 3364 - 3480 \times 0.5736$

$= 2267.872$

$\sqrt{c^2} = \sqrt{2267.872}$

$c = 47.62$

c $a = 21, c = 24, B = 30°$

$b^2 = a^2 + c^2 - 2ac \cos B$

$= 21^2 + 24^2 - 2(21 \times 24) \cos 30°$

$= 441 + 576 - 1008 \times 0.8660$

$= 144.05$

$\sqrt{b^2} = \sqrt{144.05}$

$b = 12.00$

d $b = 18, c = 25, A = 35°$

$a^2 = b^2 + c^2 - 2bc \cos A$

$= 18^2 + 25^2 - 2(18 \times 25) \cos 35°$

$= 324 + 625 - 900 \times 0.8192$

$= 211.72$

$\sqrt{a^2} = \sqrt{211.72}$

$a = 14.55$

e $a = 41, b = 60, C = 27°$

$c^2 = a^2 + b^2 - 2ab \cos C$

$= 41^2 + 60^2 - 2(41 \times 60) \cos 27°$

$= 1681 + 3600 - 4920 \times 0.8910$

$= 897.25$

$\sqrt{c^2} = \sqrt{897.25}$

$c = 29.95$

f $a = 12, c = 17, B = 42°$

$b^2 = a^2 + c^2 - 2ac \cos B$

$= 12^2 + 17^2 - 2(12 \times 17) \cos 42°$

$= 144 + 289 - 408 \times 0.7431$

$= 129.80$

$\sqrt{b^2} = \sqrt{129.80}$

$b = 11.39$

5 $a = 27, b = 22, C = 40°$

$c^2 = a^2 + b^2 - 2ab \cos C$

$= 27^2 + 22^2 - 2(27 \times 22) \cos 40°$

$= 729 + 484 - 1188 \times 0.7660$

$= 302.94$

$\sqrt{c^2} = \sqrt{302.94}$

$c = 17.41$

6 $a = 18, c = 15, B = 110°$

$b^2 = a^2 + c^2 - 2ac \cos B$

$= 18^2 + 15^2 - 2(18 \times 15) \cos 110°$

$= 324 + 225 - 540 \times -0.3420$

$= 733.68$

$\sqrt{b^2} = \sqrt{733.68}$

$b = 27.09$

7 $b = 42, c = 38, A = 80°$

$a^2 = b^2 + c^2 - 2bc \cos A$

$= 42^2 + 38^2 - 2(42 \times 38) \cos 80°$

$= 1764 + 1444 - 3192 \times 0.1736$

$= 2653.72$

$\sqrt{a^2} = \sqrt{2653.72}$

$a = 51.51$

8 a $a = 5, b = 9, c = 7$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{9^2 + 7^2 - 5^2}{2 \times 9 \times 7}$$

$$= \frac{105}{126} = 0.8333$$

$A = \cos^{-1} 0.8333$

$= 33.6°$

b $a = 16, b = 12, c = 11$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{11^2 + 12^2 - 16^2}{2 \times 12 \times 11}$$
$$= \frac{9}{264} = 0.0341$$
$$A = \cos^{-1} 0.0341$$
$$= 88.0°$$

c $a = 14, b = 8, c = 9$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{8^2 + 9^2 - 14^2}{2 \times 8 \times 9}$$
$$= \frac{-51}{144} = -0.3542$$
$$A = \cos^{-1} -0.3542$$
$$= 110.7°$$

d $a = 13, b = 10, c = 8$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{10^2 + 8^2 - 13^2}{2 \times 10 \times 8}$$
$$= \frac{-5}{160} = -0.0313$$
$$A = \cos^{-1} -0.0313$$
$$= 91.8°$$

e $a = 14, b = 11, c = 9$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{11^2 + 9^2 - 14^2}{2 \times 11 \times 9}$$
$$= \frac{6}{198} = 0.0303$$
$$A = \cos^{-1} 0.0303$$
$$= 88.3°$$

f $a = 12, b = 6, c = 8$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{6^2 + 8^2 - 12^2}{2 \times 6 \times 8}$$
$$= \frac{-44}{96} = -0.4583$$
$$A = \cos^{-1} -0.4583$$
$$= 117.3°$$

9 $a = 31, b = 47, c = 52$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$= \frac{31^2 + 52^2 - 47^2}{2 \times 31 \times 52}$$
$$= \frac{1456}{3324} = 0.4516$$
$$B = \cos^{-1} 0.4516$$
$$= 63.2°$$

10 $a = 66, b = 29, c = 48$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$= \frac{66^2 + 29^2 - 48^2}{2(66)(29)}$$
$$= \frac{263}{348}$$
$$C = \cos^{-1} \frac{263}{348}$$
$$= 40.9°$$

11 The smallest angle is between the two longest sides.

$a = 120, b = 90, c = 105$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$= \frac{120^2 + 105^2 - 90^2}{2 \times 120 \times 105}$$
$$= \frac{17\,325}{25\,200} = 0.6875$$
$$B = \cos^{-1} 0.6875$$
$$= 46.6°$$

12 The smallest angle is between the two longest sides.

$a = 5, b = 7, c = 9$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{7^2 + 9^2 - 5^2}{2 \times 7 \times 9}$$
$$= \frac{105}{126} = 0.8333$$
$$A = \cos^{-1} 0.8333$$
$$= 33.6°$$

13 Label the port as point C

$a = 20, b = 18, C = 60°$

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ &= 20^2 + 18^2 - 2(20)(18)\cos 60° \\ &= 364 \\ c &= \sqrt{364} \\ &= 19.1 \text{ km} \end{aligned}$$

14 **a**

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ 9^2 &= 14^2 + 12^2 - 2(14)(12)\cos A \\ \cos A &= \frac{9^2 - 14^2 - 12^2}{-2(14)(12)} \\ &= \frac{37}{48} \\ A &= \cos^{-1}\left(\frac{37}{48}\right) \\ &= 39.6° \end{aligned}$$

b Bearing of point C from point A – from diagram, it is seen that the solution is:

$$\begin{aligned} 270° + A &= 270° + 39.6° \\ &= 310° \end{aligned}$$

15 **a** $A = 180° - 40° - 80°$

$= 60°$

b $b = 49, c = 27, A = 60°$

$a^2 = b^2 + c^2 - 2bc \cos A$

$= 49^2 + 27^2 - 2(49 \times 27) \cos 60°$

$= 2401 + 729 - 2646 \times 0.5000$

$= 1807$

$\sqrt{a^2} = \sqrt{1807}$

$a = 42.51$ km

16 $a = 5$, $c = 8$, $B = 40°$

$b^2 = a^2 + c^2 - 2ac \cos B$

$= 5^2 + 8^2 - 2(5 \times 8) \cos 40°$

$= 25 + 64 - 80 \times 0.7660$

$= 27.72$

$\sqrt{b^2} = \sqrt{27.72}$

$b = 5.26$ km

17 In triangle CDE

$c^2 = d^2 + e^2 - 2de \cos C$

$11^2 = 8^2 + 5^2 - 2(8)(5) \cos C$

$121 = 64 + 25 - 80 \cos C$

$80 \cos C = 89 - 121$

$\cos C = \frac{-32}{80}$

$= -0.4$

In triangle ABC

$AB^2 = AC^2 + BC^2 - 2(AC)(BC) \cos C$

$= 7^2 + 6^2 - 2(7)(6) \times -0.4$

$= 85 + 84 \times -0.4$

$= 118.6$

$AB = 10.8904$

$= 10.9$ m

Solutions to 11I Now Try This Questions

22 $Area = \frac{1}{2}\, base \times height$

$= \frac{1}{2}\,(8)\,(4)$

$= 16.0 \text{ m}^2$

23 $Area = \frac{1}{2} pr \sin Q$

$= \frac{1}{2}(7)(4)\sin 70°$

$= 14 \sin 70°$

$= 13.1557$

$= 13.2 \text{ cm}^2$

24 $Semi - perimeter$

$s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(8 + 7 + 9)$

$= 12$ m

$Area$

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-8)(12-7)(12-9)}$

$= \sqrt{12(4)(5)(3)}$

$= 26.8328$

$= 26.8 \text{ m}^2$

Solutions to Exercise 11I

1 **a** The perpendicular height is measured from the horizontal to the highest point of the triangle.
The perpendicular height is 6 m

b The base is the horizontal part of the triangle. The length of the base is 5 m

c $Area = \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 5 \times 6$

$= 15 \text{ m}^2$

2 **a** Imagine the line AB has been rotated so that it is horizontal. The perpendicular height is measured perpendicular to AB to the highest point of the triangle.
The perpendicular height is 4 m

b The base is the horizontal part of the triangle after the rotation.
The length of the base is 7 m

c $Area = \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 7 \times 4$

$= 14 \text{ m}^2$

3 **a** Use the angle between the two given sides.
The angle between the 9 cm side and the 12 cm is 46°

b $Area = \frac{1}{2}bc \sin A$

$= \frac{1}{2}(9 \times 12) \sin 46°$

$= \frac{1}{2}(108) \times 0.7193$

$= 38.8 \text{ cm}^2$

4 a $b = 17, h = 12$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(17 \times 12)$

$= \frac{1}{2}(204) = 102 \text{ cm}^2$

b $b = 8, h = 10$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(8 \times 10)$

$= \frac{1}{2}(80) = 40 \text{ cm}^2$

c $b = 8, h = 6$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(8 \times 6)$

$= \frac{1}{2}(48) = 24 \text{ cm}^2$

5 a $b = 8, c = 10, A = 140°$

$A = \frac{1}{2}bc \sin A$

$= \frac{1}{2}(8 \times 10) \sin 140°$

$= \frac{1}{2}(80) \times 0.6428$

$= 25.7 \text{ cm}^2$

b $b = 11, c = 12, A = 80°$

$A = \frac{1}{2}bc \sin A$

$= \frac{1}{2}(11 \times 12) \sin 80°$

$= 64.9973$

$= 65.0 \text{ cm}^2$

c $a = 12, c = 5, B = 120°$

$A = \frac{1}{2}ac \sin B$

$= \frac{1}{2}(12 \times 5) \sin 120°$

$= \frac{1}{2}(60) \times 0.8660$

$= 26.0 \text{ cm}^2$

d $a = 11, c = 6, B = 85°$

$A = \frac{1}{2}ac \sin B$

$= \frac{1}{2}(11 \times 6) \sin 85°$

$= \frac{1}{2}(66) \times 0.9962$

$= 32.9 \text{ cm}^2$

6 a $a = 7, b = 11, c = 15$

$s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(7 + 11 + 15)$

$= \frac{1}{2}(33) = 16.5$

$A = \sqrt{s(s - a)(s - b)(s - c)}$

$=$

$\sqrt{16.5(16.5 - 7)(16.5 - 11)(16.5 - 15)}$

$= \sqrt{1293.19} = 36.0 \text{ km}^2$

b $a = 7, b = 4, c = 5$

$s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(7 + 4 + 5)$

$= \frac{1}{2}(16) = 8$

$A = \sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{8(8 - 7)(8 - 4)(8 - 5)}$

$= \sqrt{96} = 9.8 \text{ m}^2$

c $a = 6, b = 8, c = 9$

$s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(6 + 8 + 9)$

$= \frac{1}{2}(23) = 11.5$

$A = \sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{11.5(11.5 - 6)(11.5 - 8)(11.5 - 9)}$

$= \sqrt{553.44} = 23.5 \text{ cm}^2$

7 **a** Given three sides, use Heron's rule.
So: **iv**

b Given two sides *a* and *c* and the included angle *B* use: Area = $\frac{1}{2}$ *ac* sin*B*.
So: **iii**

c Given the length of the base *AB* and the height of the triangle use:
Area = $\frac{1}{2}$ base × height
So: **i**

d Given two sides *b* and *c* and find the included angle *A* use:
Area = $\frac{1}{2}$ *bc* sin*A*.
So: **ii**

8 **a** $b = 5, h = 4$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(5 \times 4)$

$= \frac{1}{2}(20) = 10 \text{ cm}^2$

b $b = 9, c = 10, A = 32°$

$A = \frac{1}{2}bc \sin A$

$= \frac{1}{2}(9 \times 10) \sin 32°$

$= \frac{1}{2}(90) \times 0.5299$

$= 23.8 \text{ cm}^2$

c $a = 8, b = 16, c = 17$

$s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(8 + 16 + 17)$

$= \frac{1}{2}(41) = 20.5$

$A = \sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{20.5(20.5 - 8)(20.5 - 16)(20.5 - 17)}$

$= \sqrt{4035.94} = 63.5 \text{ cm}^2$

d $a = 9, b = 11, c = 12$

$s = \frac{1}{2}(a + b + c)$

$= \frac{1}{2}(9 + 11 + 12)$

$= \frac{1}{2}(32) = 16$

$A = \sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{16(16 - 9)(16 - 11)(16 - 12)}$

$= \sqrt{2240} = 47.3 \text{ m}^2$

e $b = 12, h = 5$

$A = \frac{1}{2}bh$

$= \frac{1}{2}(12 \times 5)$

$= \frac{1}{2}(60) = 30 \text{ m}^2$

f $b = 8, c = 8, A = 110°$

$$A = \frac{1}{2}bc\sin A$$
$$= \frac{1}{2}(8 \times 8)\sin 110°$$
$$= \frac{1}{2}(64) \times 0.9397$$
$$= 30.1 \text{ m}^2$$

9 $b = 28, h = 16$

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(28 \times 16)$$
$$= \frac{1}{2}(448) = 224 \text{ cm}^2$$

10 $r = 42, s = 57, T = 70°$

$$A = \frac{1}{2}rs\sin T$$
$$= \frac{1}{2}(42 \times 57)\sin 70°$$
$$= \frac{1}{2}(2394) \times 0.9397$$
$$= 1124.8 \text{ cm}^2$$

11 $a = 16, b = 19, c = 23$

$$s = \frac{1}{2}(a + b + c)$$
$$= \frac{1}{2}(16 + 19 + 23)$$
$$= \frac{1}{2}(58) = 29$$
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{29(29-16)(29-19)(29-23)}$$
$$= \sqrt{22\,620} = 150.4 \text{ km}^2$$

12 $b = 100, h = 35$

$$A = \frac{1}{2}bh \times 2$$
$$= \frac{1}{2}(100 \times 35) \times 2$$
$$= 3500 \text{ cm}^2$$

13 **a** Base = 3, height = 4

$$\text{Area} = \frac{1}{2}\text{base} \times \text{height}$$
$$= \frac{1}{2}(3)(4)$$
$$= 6 \text{ m}^2$$

b $a = 2, b = 5, c = 5$

$$s = \frac{1}{2}(a + b + c)$$
$$= \frac{1}{2}(2 + 5 + 5)$$
$$= 6 \text{ m}$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{6(6-2)(6-5)(6-5)}$$
$$= \sqrt{24}$$
$$= 4.9 \text{ m}^2$$

c $a = 4, b = 4, c = 4$

$$s = \frac{1}{2}(a + b + c)$$
$$= \frac{1}{2}(4 + 4 + 4)$$
$$= 6 \text{ m}$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{6(6-4)(6-4)(6-4)}$$
$$= \sqrt{48}$$
$$= 6.9 \text{ m}^2$$

14 **a** $b = 8, d = 9, A = 70°$

$$A = \frac{1}{2}bd\sin A$$
$$= \frac{1}{2}(8 \times 9)\sin 70°$$
$$= \frac{1}{2}(72) \times 0.9397$$
$$= 33.83 \text{ km}^2$$

b $b = 6.76, d = 6, C = 100°$

$A = \frac{1}{2}bd \sin C$

$= \frac{1}{2}(6.76 \times 6) \sin 100°$

$= \frac{1}{2}(40.56) \times 0.9848$

$= 19.97 \text{ km}^2$

c Total area = $ABD + BCD$

$= 33.83 + 19.97$

$= 53.80 \text{ km}^2$

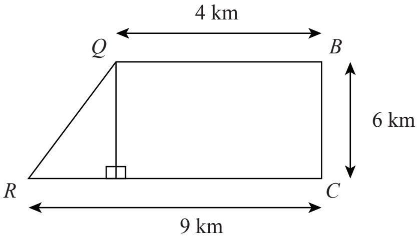
15 a i Area of PAQ:

$$A_{PAQ} = \frac{1}{2}bh$$

$$= \frac{1}{2} \times 8 \times 3$$

$$= 12 \text{ km}^2$$

ii



Area of triangle:

$$A_T = \frac{1}{2}bh$$

$$= \frac{1}{2}(9 - 4)(6)$$

$$= 15 \text{ km}^2$$

Area of square:

$$A_S = lw$$

$$= 6 \times 4$$

$$= 24 \text{ km}^2$$

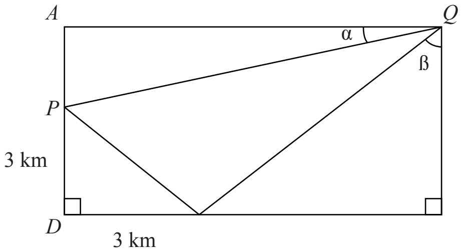
Area of $QBCR$:

$$A_{QBCR} = A_T + A_S$$

$$= 15 + 24$$

$$= 39 \text{ km}^2$$

iii



Area of PDR:

$$A_{PDR} = \frac{1}{2}bh$$

$$= \frac{1}{2} \times 3 \times 3$$

$$= 4.5 \text{ km}^2$$

Area of PRQ:

Distance PR:

$$\begin{aligned} L_{PR} &= \sqrt{3^2 + 3^2} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \text{ km} \end{aligned}$$

Distance RQ:

$$\begin{aligned} L_{RQ} &= \sqrt{(9-4)^2 + 6^2} \\ &= \sqrt{25 + 36} \\ &= \sqrt{61} \text{ km} \end{aligned}$$

Distance PQ:

$$\begin{aligned} L_{PQ} &= \sqrt{3^2 + 8^2} \\ &= \sqrt{9 + 64} \\ &= \sqrt{73} \text{ km} \end{aligned}$$

Using Heron's rule:

$$\begin{aligned} s &= \frac{1}{2}(a+b+c) \\ s &= \frac{1}{2}(L_{PR} + L_{RQ} + L_{PQ}) \\ &= \frac{1}{2}\left(3\sqrt{2} + \sqrt{61} + \sqrt{73}\right) \\ &\approx 10.298 \end{aligned}$$

$$\begin{aligned} A_{PQR} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{s\,(s-L_{PR})\,(s-L_{RQ})\,(s-L_{PQ})} \\ &= \sqrt{\begin{gathered} 10.298 \times \left(10.298 - 3\sqrt{2}\right) \times \\ \left(10.298 - \sqrt{61}\right) \times \\ \left(10.298 - \sqrt{73}\right) \end{gathered}} \\ &\approx 16.495 \text{ km}^2 \end{aligned}$$

Area of PQRD:

$$\begin{aligned} A_{PQRD} &= A_{PDR} + A_{PQR} \\ &= 4.5 + 16.495 \\ &= 20.995 \approx 21 \text{ km}^2 \end{aligned}$$

b

$$\begin{aligned} \angle PQR &= 90^\circ - \alpha - \beta \\ \tan\alpha &= \frac{3}{8} \\ \alpha &= \tan^{-1}\left(\frac{3}{8}\right) \\ \alpha &\approx 20.56^\circ \\ \tan\beta &= \frac{5}{6} \\ \beta &= \tan^{-1}\left(\frac{5}{6}\right) \\ \beta &\approx 39.81^\circ \\ \angle PQR &= 90^\circ - 20.56^\circ - 39.81^\circ \\ &\approx 29.6^\circ \end{aligned}$$

16 Area $ABC = \frac{1}{2}$ base × height

$$\begin{aligned} &= \frac{1}{2}(180)(54) \\ &= 4860 \text{ m}^2 \end{aligned}$$

Area $ACD = \frac{1}{2}$ base × height

$$\begin{aligned} &= \frac{1}{2}(180)(42) \\ &= 3780 \text{ m}^2 \end{aligned}$$

Area $ABCD$ = Area ABC + Area ACD

$$\begin{aligned} &= 4860 + 3780 \\ &= 8640 \text{ m}^2 \end{aligned}$$

17 **i** Use Area $= \frac{1}{2}$ base × height

$$= \frac{1}{2}(12)(5)$$

$$= 30 \text{ square units}$$

ii Use Area $= \frac{1}{2}ab\sin C$

$$= \frac{1}{2}(12)(13)(\frac{5}{13})$$

$$= 30 \text{ square units}$$

iii Use Heron's rule:

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{1}{2}(a+b+c)$

$$s = \frac{1}{2}(a+b+c)$$

$$= \frac{1}{2}(5+12+13)$$

$$= 15$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15(15-5)(15-12)(15-13)}$$

$$= \sqrt{15(10)(3)(2)}$$

$$= \sqrt{900}$$

$$= 30 \text{ square units}$$

18 Rotate the triangle clockwise so that $BC = 7$ is its base.

$$\text{Area} = \frac{1}{2} \text{ base} \times \text{height}$$

$$= \frac{1}{2} \times 7 \times \text{height}$$

The height will be a maximum when θ is 90°.

Solutions to Skills Checklist Questions

1 The 33 cm side is directly opposite the given angle. It's called the **opposite** side.
The 56 cm side is **adjacent** to the given angle (and it's not the hypotenuse). It runs from the given angle to the right angle.
The 65 cm side is **hypotenuse**. It is the longest side and is opposite the right angle.

2 $$\cos\theta = \frac{adj}{hyp} = \frac{56}{65}$$

$$\sin\theta = \frac{opp}{hyp} = \frac{33}{65}$$

$$\tan\theta = \frac{opp}{adj} = \frac{33}{56}$$

3 Make sure your calculator is in **DEGREE** mode.
$\cos 27^\circ = 0.8910$
$\sin 58^\circ = 0.8480$
$\tan 73^\circ = 3.2709$

4 x is *opposite* 28°
The side 20 units long is the *hypotenuse*.
So use:

$$\sin\theta = \frac{opp}{hyp}$$

5 $$\sin 28^\circ = \frac{x}{20}$$
$$x = 20\sin 28^\circ = 9.4$$

6 $$\cos\theta = 0.7431$$
$$\theta = \cos^{-1}(0.7431) = 42.0^\circ$$

7 $3 = opposite$
$7 = adjacent$
$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta = \frac{3}{7}$$
$$\theta = \tan^{-1}\frac{3}{7} = 23.2^\circ$$

8 See diagram in the Answers section of the textbook.

9 Tree $= opposite$
Shadow $= adjacent$
$$\tan\theta = \frac{opp}{adj}$$
$$\tan 34^\circ = \frac{Tree}{23}$$
$$\text{Tree} = 23\tan 34^\circ = 15.5\text{ m}$$

10 See diagram in the Answers section of the textbook.
The angle of depression, 28°, is swept from the horizontal down to the boat.
The angle of elevation is swept up from the horizontal to the cliff-top.
The angle of elevation is equal to the angle of depression.

11 $adjacent = x$

$opposite = 50$

$\theta = 28°$

$\tan\theta = \frac{opp}{adj}$

$\tan 28° = \frac{50}{x}$

$x = \frac{50}{\tan 28°}$

$= 94.04$

$= 94$ m

12 See diagram in the Answers section of the textbook.

A bearing of 050° is swept clockwise from north for 50°.

13 See diagram for Q12 in the Answers section of the textbook.

The side opposite the 50° is the distance to the highway.

$opposite = x$

$hypotenuse = 3$

$\theta = 50°$

$\sin\theta = \frac{opp}{hyp}$

$\sin 50° = \frac{x}{3}$

$x = 3\sin 50°$

$= 2.3$ km

14 $B = 115°, \quad b = 27, \quad c = 24$

$\frac{\sin C}{c} = \frac{\sin B}{b}$

$\frac{\sin C}{24} = \frac{\sin 115°}{27}$

$\sin C = \frac{24\sin 115°}{27}$

$= 0.8056$

$C = \sin^{-1}(0.8056)$

$= 53.7°$

15 $A = 30°, \quad C = 110°, \quad c = 49$

$\frac{a}{\sin A} = \frac{c}{\sin C}$

$\frac{a}{\sin 30°} = \frac{49}{\sin 110°}$

$a = \frac{49\sin 30°}{\sin 110°}$

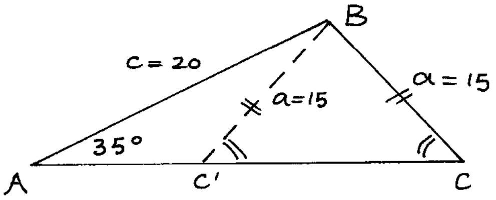
$= 26.07$

$= 26$ km

16 $A = 35°, \quad a = 15, \quad c = 20$

Given two sides and an angle not between the two given sides, there may be two possible triangles that satisfy the given information.

The diagram shows that there are two triangles, ABC and ABC', that satisfy the given information.



In triangle ABC

$a = 15, \quad A = 35°, \quad c = 20$

$\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin C}{20} = \frac{\sin 35°}{15}$

$\sin C = \frac{20\sin 35°}{15}$

$= 0.7648$

$C = \sin^{-1}(0.7648)$

$= 49.9°$

In isosceles triangle BCC'

$\angle BC'C = \angle BCC'$

$= 49.9\ °$

$\angle AC'B + \angle BC'C = 180°$

$\angle AC'B + 49.9° = 180°$

$\angle AC'B = 130.1°$

Possible values for angle C are 49.9° and 130.1°

17 When given three sides, the cosine rule can be used to find the required angle.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

18 $B = 70°, \quad a = 8, \quad c = 10$

$$\begin{aligned} b^2 &= a^2 + c^2 - 2ac\cos B \\ &= 8^2 + 10^2 - 2(8)(10)\cos 70° \\ &= 109.276 \\ b &= 10.5 \end{aligned}$$

19 a = 21, b = 23, c = 26

$$\begin{aligned} \cos C &= \frac{a^2 + b^2 - c^2}{2ab} \\ &= \frac{21^2 + 23^2 - 26^2}{2(21)(23)} \\ &= 0.3043 \\ C &= \cos^{-1}(0.3043) \\ &= 72.3° \end{aligned}$$

20 When given three sides, Heron's rule can be used to find the area of a triangle.

21 $A = 47°, \quad b = 29, \quad c = 31$

When given two sides b, c, and the angle A between the two sides, use

$$\begin{aligned} Area &= \frac{1}{2}(b)(c)\sin A \\ &= \frac{1}{2}(29)(31)\sin 47° \\ &= 328.7 \text{ cm}^2 \end{aligned}$$

Solutions to Chapter Review Multiple-Choice Questions

1

$$\begin{aligned} \sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{12}{13} \end{aligned}$$

C

2

$$\begin{aligned} &\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \\ &\cos 36° \times \text{hypotenuse} = \text{adjacent} \\ &\cos 36° \times 24 = x \\ &x = 24\cos 36° \end{aligned}$$

C

3

$$\begin{aligned} &\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \\ &\tan 62° \times \text{adjacent} = \text{opposite} \\ &\tan 62° \times 17 = x \\ &x = 17\tan 62° \end{aligned}$$

B

4

$$\begin{aligned} &\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \\ &\text{hypotenuse} = \frac{\text{opposite}}{\sin\theta} \\ &x = \frac{95}{\sin 46°} \end{aligned}$$

E

5

$$\begin{aligned} &\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \\ &\text{adjacent} = \frac{\text{opposite}}{\tan\theta} \\ &x = \frac{20}{\tan 43°} \end{aligned}$$

B

6

$$\begin{aligned} \cos\theta &= \frac{\text{adjacent}}{\text{hypotenuse}} \\ \cos\theta &= \frac{15}{19} \\ \theta &= \cos^{-1}\left(\frac{15}{19}\right) \end{aligned}$$

A

7

$$\begin{aligned} &\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{8}{10} = 0.8 \\ &\theta = \sin^{-1} 0.8 \\ &= 53.1° \end{aligned}$$

D

8 Read from diagram:
$90° + 30° = 120°$ **C**

9 Read from diagram:
$180° + 30° = 210°$ **E**

10 $b = 8$, $B = 40°$, $c = 9$ (2 sides and an opposite angle, so use the sine rule.)

$$\begin{aligned} \frac{\sin C}{c} &= \frac{\sin B}{b} \\ \frac{\sin C}{9} &= \frac{\sin 40°}{8} \\ \sin C &= \frac{9\sin 40°}{8} \qquad \textbf{B} \\ C &= \sin^{-1}\left(\frac{9\sin 40°}{8}\right) \\ &= 46.3° \end{aligned}$$

11 $A = 50°$, $a = 12$, $C = 100°$ (2 angles and one side, so use the sine rule.)

$$\begin{aligned} \frac{c}{\sin C} &= \frac{a}{\sin A} \\ \frac{c}{\sin 100°} &= \frac{12}{\sin 50°} \qquad \textbf{D} \\ c &= \frac{12\sin 100°}{\sin 50°} \end{aligned}$$

12 $A = 60°$, $a = 5$, $b = 3$ (2 sides and an opposite angle, so use the sine rule.)

$$\begin{aligned} \frac{\sin B}{b} &= \frac{\sin A}{a} \\ \frac{\sin B}{3} &= \frac{\sin 60°}{5} \qquad \textbf{B} \\ \sin B &= \frac{3\sin 60°}{5} \end{aligned}$$

13 $a = 21$, $b = 29$, $C = 47°$ (2 sides and the angle between them, so use the cosine rule.)

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ &= 21^2 + 29^2 - 2(21)(29)\cos 47° \qquad \textbf{E} \\ c &= \sqrt{21^2 + 29^2 - 2(21)(29)\cos 47°} \end{aligned}$$

14 $A= x\circ$, $a = 6$, $b = 7$, $c = 5$ (three sides, so use cosine rule)

$$\begin{aligned} \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \qquad \textbf{B} \\ \cos x° &= \frac{7^2 + 5^2 - 6^2}{2(7)(5)} \end{aligned}$$

15 $a = 13$, $b = 14$, $c = 12$ (3 sides, so use the cosine rule.)

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \qquad \textbf{D}$$

16
$$\begin{aligned} Area &= \frac{1}{2}bh \\ &= \frac{1}{2}(12 \times 9) \\ &= 54 \text{ cm}^2 \qquad \textbf{B} \end{aligned}$$

17 $b = 7$, $c = 10$, $A = 115°$

$$\begin{aligned} Area &= \frac{1}{2}bc\sin A \\ &= \frac{1}{2}(7 \times 10)\sin 115° \\ &= \frac{1}{2}(70) \times 0.9063 \\ &= 31.72 \text{ cm}^2 \qquad \textbf{B} \end{aligned}$$

18 $a = 6$, $b = 9$, $c = 11$ (To find the area of a triangle given three sides use Heron's rule)

$$\begin{aligned} s &= \frac{1}{2}(a + b + c) \\ &= \frac{1}{2}(6 + 9 + 11) \qquad \textbf{C} \\ &= 13 \\ \text{Area} &= \sqrt{s(s - a)(s - b)(s - c)} \\ &= \sqrt{13(13 - 6)(13 - 9)(13 - 11)} \end{aligned}$$

19
$$\begin{aligned} s &= \frac{1}{2}(a + b + c) \\ &= \frac{1}{2}(23 + 19 + 17) \\ &= \frac{1}{2}(59) = 29.5 \\ A &= \sqrt{s(s - a)(s - b)(s - c)} \\ &= \sqrt{29.5(29.5 - 23)(29.5 - 19)(29.5 - 17)} \\ &= \sqrt{25\,167.19} \\ &= 158.6m^2 \qquad \textbf{B} \end{aligned}$$

Solutions to Chapter Review Short-Answer Questions

1 The sides involved to find the length of x are the opposite and the hypotenuse, so use sin of the angle.

$\sin 39° = \frac{x}{57}$

$x = 57 \times \sin 39°$

$= 35.87$, correct to 2 decimal places

The length of x, correct to 2 decimal places, is 35.87 cm.

2 The sides involved to find the length of the hypotenuse are the adjacent and the hypotenuse, so use cos of the angle.

$$\begin{aligned}\cos 28° &= \frac{104}{\text{hypotenuse}}\\ \text{hypotenuse} &= \frac{104}{\cos 28°}\\ &= 117.79\end{aligned}$$

The length of the hypotenuse, correct to 2 decimal places, is 117.79 cm.

3 First make the units consistent.

2 m = 200 cm.

The sides involved to find the value of θ are the opposite and the adjacent, so use tan of the angle.

$\tan\theta = \frac{15}{200} = 0.075$

$\theta = 4°$

The angle of slope is 4°, to the nearest degree.

4 a

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{72}{97}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{65}{72}$

So the given ratios of cos θ and tan θ would be achieved in a right-angled triangle with:

adjacent = 72 , opposite = 65 and hypotenuse = 97

or multiplies of all those sides.

b $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{65}{97}$

5 Use the form of the sine rule:

$$\begin{aligned}\frac{b}{\sin B} &= \frac{c}{\sin C}\\ \frac{b}{\sin 29°} &= \frac{17}{\sin 36°}\\ b &= \frac{17}{\sin 36°} \times \sin 29°\\ &= 14.02\end{aligned}$$

The length of side b, correct to 2 decimal places, is 14.02 cm.

6 Use the form of the sine rule:

$$\begin{aligned}\frac{a}{\sin A} &= \frac{c}{\sin C}\\ \frac{28}{\sin 51°} &= \frac{35}{\sin C}\\ \sin C &= 35 \div \frac{28}{\sin 51°}\\ &= 35 \times \frac{\sin 51°}{28}\\ &= 0.9714\\ C &= 76.3°\end{aligned}$$

The angle C is 76.3°, correct to 1 decimal place.

Just using a = 28, $A = 51°$, c = 35 gives an ambiguous case with a possible value, $C' = 180° - C = 103.7°$.

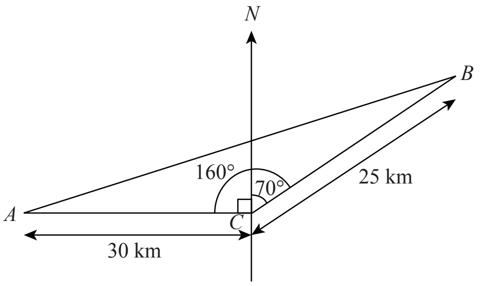
However, the diagram shows $C < 90°$.

7 The smallest angle will be A, the angle opposite the smallest side length.
To find A, use the form of the cosine rule:

$$\begin{aligned}\cos A &= \frac{b^2 + c^2 - a^2}{2bc}\\ &= \frac{23^2 + 17^2 - 15^5}{2 \times 23 \times 17}\\ &= 0.7583\\ A &= 40.7°\end{aligned}$$

The smallest angle, A, is 40.7°, correct to 1 decimal place.

8



The distance of the car from its starting point, represented by the line AB in the above triangle, can be determined using the cosine rule.

$AB^2 = 30^2 + 25^2 - 2 \times 30 \times 25 \cos$
160°
$= 2934.54$
$AB = 54.17$

The car is 54.17 km away from its starting point, correct to 2 decimal places.

9 Area of triangle = $bc \sin A$
$= \frac{1}{2} \times 60 \times 60 \sin 25°$
$= 760.7$
The area of cloth needed for the flag is 760.7 cm^2, correct to 1 decimal place.

10 Area of triangle = $\frac{1}{2} bc \sin A$
$= \frac{1}{2} \times 8 \times 8 \sin 60°$
$= 27.7$
The area of the equilateral triangle is 27.7 m^2, correct to 1 decimal place.

Solutions to Chapter Review Written-Response Questions

1 **a** The sides involved to find the width of the river are the opposite AT and the adjacent AB (=100 m), so use tan of the angle 27°.

$\tan 27° = \frac{AT}{100}$
$AT = 100 \times \tan 27°$
$= 50.95$, correct to 2 decimal places
The river was 50.95 m wide, to 2 decimal places.

b The sides involved to find the distance from point B to the base of the tree T are the adjacent *AB* and the hypotenuse *BT*, so use cos of the angle 27°.

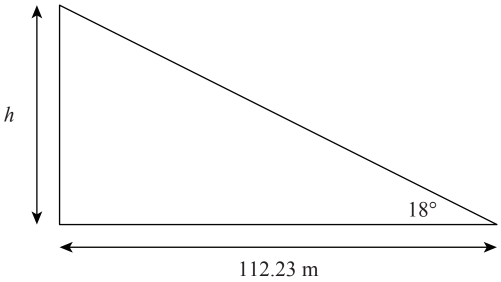
$$\cos 27^\circ = \frac{100}{BT}$$

$$BT = \frac{100}{\cos 27^\circ}$$

= 112.23, correct to 2 decimal places

The distance from point *B* to the tree is 112.23 m, to 2 decimal places.

c



The sides involved to find the height of the tree, *h*, are the opposite and the adjacent *BT*, so use tan of the angle 18°.

$$\tan 18^\circ = \frac{h}{112.23}$$

$h = 112.23 \times \tan 18^\circ$

= 36.47, correct to 2 decimal places

The height of the tree is 36.47 m, to 2 decimal places.

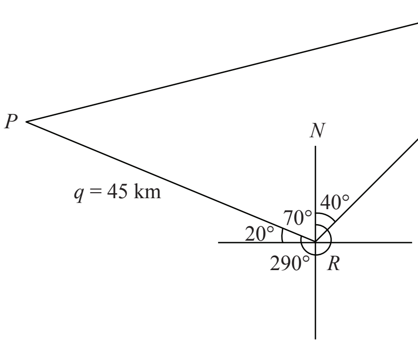
2 **a** From the diagram:

$\angle PRQ$

$= 70° + 40°$

$= 110°$

The angle between the directions of yacht *P* and yacht *Q* is 110°.



b The distance between yacht *P* and yacht *Q* is represented by the side length *PQ*.

Using the cosine rule,

$PQ^2 = 45^2 + 54^2 - 2 \times 45 \times 54 \cos 110°$

$= 6603.22$

$PQ = 81.26$

The yachts are 81.26 km apart, correct to 2 decimal places.

3 **a** Area of a triangular face $= \frac{1}{2}bh$

$= \frac{1}{2} \times 100 \times 120$

$= 6000$

The area of each triangular face of the pyramid is 6000 m^2.

The total surface area of the pyramid is

$4 \times 6000\ m^2 = 24\ 000\ m^2$.

b 1 kg of gold covers 0.5 m^2

2 kg of gold covers 1 m^2

24 000 × 2 kg of gold covers 24 000 m^2

48 000 kg of gold covers 24 000 m^2

It will take 48 000 kg of gold to cover the surface of the pyramid.

c 48 000 × \$62 500 = \$3 000 000 000

It would cost \$3 000 million to cover the surface of the pyramid with gold.

Chapter 12 – Revision of Unit 2

Solutions to Multiple-choice questions

Chapter 7: Investigating relationships between two numerical variables

1 *age* and *reaction time* are both numerical variables. **E**

2 weak, linear, negative is the best of these options, it is not strong. **A**

3 Positive association means the variables tend to increase together, so we can say that as the demand for the handbags tends to increase, the price of the handbags tends to increase. **C**

4 The association is strong and negative so A is the best choice. **A**

5 Using the table on page 425, this is classified as moderate, positive. **C**

6 Using the points $(0, 40)$ and $(18, 65)$, intercept is $a = 40$,

$$\text{slope is } b = \frac{65 - 40}{18 - 0} = \frac{25}{18} = 1.39.$$

Therefore the equation is:

starting salary =
40 + 1.4 × *years education* **E**

7 $$b = r \times \frac{s_y}{s_x} = -0.6 \times \frac{6.4}{3.2} = -1.2$$

$$\begin{aligned} a &= \bar{y} - b \times \bar{x} \\ &= 63.3 - (-1.2) \times 48.7 \\ &= 121.74 \end{aligned}$$

$\therefore y = 122 - 1.2x$ **D**

8 Enter the data into your calculator and follow the instructions on page 423 (TI) or 424 (CASIO) to calculate the value of $r = 0.8456$. **D**

9 Enter the data into your calculator and follow the instructions on page 437 (TI) or 438 (CASIO) to find the regression equation.

body fat = −6.40 + 0.481 × *age* **B**

10 *weight* = 61.32 + 1.057 × *body fat*

When *body fat* = 20, *weight* = 61.32 + 1.057 × 20 = 82.46% **C**

11 slope = 1.057 means that on average there is an increase of 1.057 kg in weight for each 1 percentage increase in body fat. **E**

12 $48.9034 \approx 48.90$

$0.0428681 \approx 0.04287$

Therefore the equation becomes:

hearing test score =
48.90 − 0.04287 × *age* **C**

Chapter 8: Graphs and networks

13 A spanning tree connects all vertices in a graph with the minimum number of edges. If a graph has 6 vertices, the spanning tree for the graph will have 5 edges (If a tree has n vertices, it will have $n - 1$

edges; 6 – 1 = 5). **C**

14 *Two methods are possible for determining the sum of degrees for a graph.*
Method 1: Double the total number of edges for this graph. This graph has a total of 6 edges, therefore the sum of degrees for this graph is 12 (6 × 2 = 12).
Method 2: Find the degree of each vertex and then find the sum. For this graph the degrees of each vertex are 1, 3, 3, 3, 2 therefore the sum of degrees for this graph is 12 (1 + 3 + 3 + 3 + 2 = 12). **E**

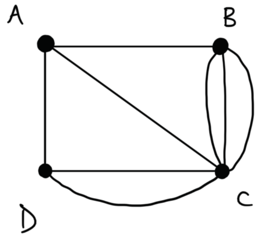
15 If a tree has n vertices, it will have $n - 1$ edges; 10 – 1 = 9. **B**

16 Starting at A and finishing at A, there are 4 different paths of length 3:
$A - B - C - A$ (× 2)
$A - C - B - A$ (× 2)
Note: in this question, 3 is not referring to a distance, it is referring to how many edges are travelled in each path. **E**

17 For an Eulerian circuit to exist, *all* vertices must have an *even* degree. The vertices C and D are the only vertices with an *odd* degree. Adding an edge between vertices C and D would increase the degree of each of those vertices by 1, resulting in all vertices in the graph having an even degree. **E**

18 A Hamiltonian cycle must start and finish at the same vertex and pass through each vertex exactly once. Options **C** and **D** are incorrect because they do not start and finish at the same vertices. Option **A** is incorrect because it includes an edge from U to Q. Option **B** is incorrect because it repeats the vertex T. Option **E** is the only walk that satifies all conditions for a Hamiltonian Cycle. **E**

19 Construct the graph represented by the matrix:



For an Eulerian trail, two or zero vertices must have an *odd degree*.
The degrees of each vertex are:
$\deg(A) = 3$, $\deg(B) = 4$
$\deg(C) = 6$, $\deg(D) = 3$
As there are two vertices with an *odd degree*, an Eulerian trail exists.
An Eulerian trail can begin and end at the vertices with an odd degree; vertices A and D. **C**

20 A graph that is connected and planar will follow the conditions of Euler's formula: $v + f = e + 2$
$v = 8, f = 8;$

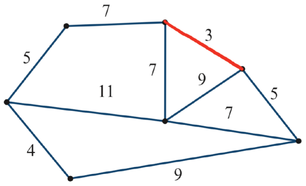
$$\begin{aligned} v + f &= e + 2 \\ 8 + 8 &= e + 2 \\ 16 &= e + 2 \\ 16 - 2 &= e \\ e &= 14 \end{aligned}$$

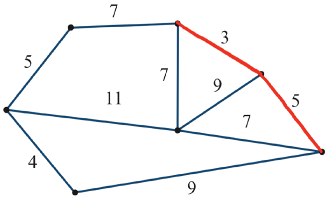
E

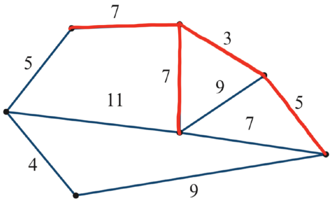
21 A graph that is connected and planar will follow the conditions of Euler's formula: $v + f = e + 2$
$v = 6;$
$6 + f = e + 2$
Only one option gives a true statement; $e = 10, f = 6$.

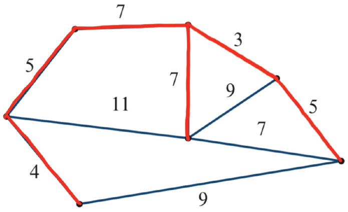
$6 + 6 = 10 + 2$
$12 = 12$ **D**

22 Either Prim's or Kruskal's algorithm can be used to find the minimum spanning tree. Here, Prim's algorithm was used:





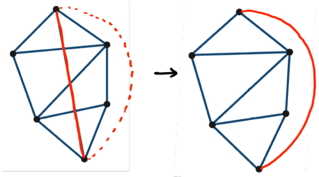




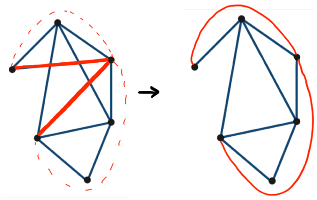
Weight of minimum spanning tree
$= 3 + 5 + 7 + 7 + 5 + 4 = 31$

C

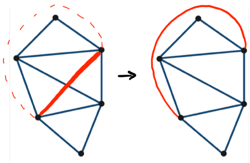
23 Going through each option, attempt to redraw each graph with no edges crossing (except at the vertices).
Graph 1 can be redrawn with no edges crossing, therefore it is planar.



Graph 2: Is connected and has no edges crossing, therefore it is planar (does not need to be redrawn).
Graph 3 can be redrawn with no edges crossing, therefore it is planar.



Graph 4 can be redrawn with no edges crossing, therefore it is planar.



E

24 Going through line by line of each matrix, you can deduce which one correctly represents the graph.
Vertex *A* has 1 edge to vertex *B* and 1 edge to vetex *D*; options **C** and **D** are incorrect. Vertex *B* has 1 edge to vertex *A*, 1 edge to vertex *C* and 1 edge to vertex *D*; option **B** is incorrect. Vertex *C* has 1 edge to vertex *B* and 2 edges to vertex *D*; option **E** is incorrect. **A**

Chapter 9: Variation

25 $m \propto n$
$m = kn$
Substitute $m = 9$, $n = 4$

$9 = 4k$
$\therefore k = \frac{9}{4}$ **A**

26 $y \propto \frac{1}{x}$

$y = \frac{k}{x}$

Substitute $y = 14$, $x = 2$

$14 = \frac{k}{2}$

$\therefore k = 28$

So $y = \frac{28}{x}$

When $x = 7$,

$y = \frac{28}{7} = 4$ **B**

27 $x \propto \frac{1}{y}$

$x = \frac{k}{y}$

If y is multiplied by 5 then the right hand side of this equation becomes $\frac{k}{5y}$.

To keep the equation balanced the left hand side would also be divided by 5.

$\frac{x}{5} = \frac{k}{5y}$

Thus x would be divided by 5. **D**

28 $A \propto L$

$A = kL$

Substitute $A = 14$, $L = 2.4$

$14 = 2.4k$

$\therefore k = \frac{14}{2.4} = 5.833$

So $A = 5.833L$

Find L when $A = 18$.

$18 = 5.833L$

$\therefore L = \frac{18}{5.833} = 3.08589$

$L = 3.086$ correct to 3 dec. pl. **A**

29 $A \propto \frac{1}{D}$

$A = \frac{k}{D}$

Substitute $A = 1.25$, $L = 2$

$1.25 = \frac{k}{2}$

$\therefore k = 2.5$

So $A = \frac{2.5}{D}$

Find A when $D = 3$,

$A = \frac{2.5}{3} = 0.833$

$A = 0.83\text{m}^2$ **C**

30 $y = kx^2 + c$

Select 2 points and substitute into the equation.

$(1, 4.5)$ $4.5 = k + c$

$(2, 9)$ $9 = 4k + c$

Use a CAS calculator to solve the two simultaneous equations.

$k = 1.5$, $c = 3$ **B**

31 $y = \frac{k}{x} + c$

Select 2 points and substitute into the equation.

$(1, 7.5)$ $7.5 = k + c$

$(2, 4.5)$ $4.5 = \frac{k}{2} + c$

Use a CAS calculator to solve the two simultaneous equations.

$k = 6$, $c = 1.5$ **D**

32 $y = k\log_{10}(x) + c$

Select 2 points and substitute into the equation.

$(1, 5)$ $5 = k\log_{10}(1) + c$

$5 = k(0) + c$

$\therefore c = 5$ (Note: $\log_{10} 1 = 0$)

$(10, 105)$ $105 = k\log_{10}(10) + c$

Substitute $c = 5$

$105 = k + 5$ (Note: $\log_{10} 10 = 1$)
$k = 100, \quad c = 5$ **D**

33 y-intercept = 0
Use (0, 0) and (2, 4) to find the slope.

$$\text{slope} = \frac{4-0}{2-0} = 2$$

Note that the axes are labelled as y and x^2.
$\therefore y = 2x^2$ **B**

34 $P \propto \dfrac{1}{V}$

$$P = \frac{k}{V}$$

Substitute $P=80$, $V = 60$

$$80 = \frac{k}{60}$$

$\therefore k = 4800$

$$\text{So } P = \frac{4800}{V}$$

When $V = 80$,

$$P = \frac{4800}{80} = 60$$ **A**

Chapter 10: Measurement, scale and similarity

35 To round 3978.6249 to 3 significant figures write it in scientific notation.
$3978.6249 = 3.9786249 \times 10^3$
Check if the fourth digit (8) is 5 or more.
It is, so the third digit (7) must be increased by 1 (so the 7 changes to 8).
$\therefore 3.98 \times 10^3$ **E**

36 By Pythagoras' theorem

$$\begin{aligned} 17^2 &= x^2 + 14^2 \\ x^2 &= 17^2 - 14^2 \\ &= 289 - 196 \\ &= 93 \\ x &= 9.6\text{m} \end{aligned}$$ **C**

37 In triangle DAB:

$$\begin{aligned} DB^2 &= AD^2 + AB^2 \\ &= 7^2 + 6^2 \end{aligned}$$

In triangle EDB:

$$\begin{aligned} EB^2 &= ED^2 + DB^2 \\ &= 8^2 + 7^2 + 6^2 \\ &= 149 \\ EB &= \sqrt{149} \\ &= 12.2066 \\ &= 12 \text{ m} \end{aligned}$$

OR use

$$\begin{aligned} d &= \sqrt{a^2 + b^2 + c^2} \\ &= \sqrt{6^2 + 7^2 + 8^2} \\ &= 12 \text{ m} \end{aligned}$$ **C**

38 Volume $= \pi r^2 h$

$$\begin{aligned} &= \pi\left(\frac{35}{2}\right)^2 (45) \\ &= 43\ 295 \text{ cm}^3 \end{aligned}$$

Capacity = 43 295 ml
= 43 Litres **B**

39 Volume of large pyramid

$$= \frac{1}{3}(\text{area of base})(\text{height})$$

$$= \frac{1}{3}\,(30)^2(20)$$

$$= 6000 \text{ m}^3$$

Volume of small pyramid

$$= \frac{1}{3}(\text{area of base})(\text{height})$$

$$= \frac{1}{3}\,(15)^2(10)$$

$$= 750 \text{ m}^3$$

Volume of truncated pyramid

$= 6000 - 750\ \text{m}^3$

$= 5250\ \text{m}^3$ **B**

40 Volume of hemi-sphere

$$= \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$= \frac{2}{3}\pi(25)^3$$

$= 32\ 725\ \text{cm}^3$ **C**

41 Surface area of a cone

$= \pi r^2 + \pi r s$

$= \pi r(r + s)$

$= \pi(5)(5 + 13)$

$= 282.7\ \text{cm}^2$ **B**

42 $\frac{x}{7} = \frac{2}{3}$

$$x = \frac{7 \times 2}{3}$$

$= 4.7$ m **A**

43 The height of the small cone as a fraction of the height of the large cone:

Height, $k = \frac{1}{3}$

The volume of the small cone as a fraction of the large cone:

$$\text{Volume, } k^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

The fraction of the remaining of the large cone:

$1 - \frac{1}{27} = \frac{26}{27}$ **E**

Chapter 11: Applications of trigonometry

44 $\sin 34^\circ = \frac{x}{19}$

$x = 19 \sin 34^\circ$

$= 10.6$ **A**

45 $\cos \theta = \frac{31}{52}$

$$\theta = \cos^{-1}\left(\frac{31}{52}\right)$$

$= 53.4^\circ$ **D**

46 $\tan 25^\circ = \frac{32}{\text{adj}}$

$$\text{adj} = \frac{32}{\tan 25^\circ}$$

$= 68.6242$

$$\tan 50^\circ = \frac{\text{opp}}{\text{adj}} = \frac{\text{opp}}{68.6242}$$

$\text{opp} = 68.6242 \times \tan 50^\circ$

$= 81.7832$

Height $= 32 + 81.7832$

$= 114$ m (nearest metre) **E**

47 Using the cosine rule:

$x^2 = 7^2 + 9^2 - 2(7)(9) \cos 20^\circ$

$= 11.5987$

$= 3.405$

$= 3.4$ **A**

48 $\angle GPH = 90^\circ - 50^\circ$

$= 40^\circ$

$\angle PGH = 50^\circ - 15^\circ$

$= 35^\circ$

Using the sine rule:

$$\frac{GH}{\sin 40^\circ} = \frac{20}{\sin 35^\circ}$$

$$GH = \frac{20 \sin 40^\circ}{\sin 35^\circ}$$

$= 22.41$

$= 22$ km **D**

49 Describe $\angle DEF$ as $\angle E$.

Using the cosine rule:

$$e^2 = d^2 + f^2 - 2df\cos E$$

$$16^2 = 14^2 + 17^2 - 2(14)(17)\cos E$$

$$476\cos E = 229$$

$$\cos E = 0.48109$$

$$E = 61.2°$$

D

50 $\angle CPD = 110° - 50°$

$= 60°$

Describe $\angle CPD$ as $\angle P$.

Using the cosine rule:

$$p^2 = c^2 + d^2 - 2cd\cos P$$

$$= 32^2 + 25^2 - 2(32)(25)\cos 60°$$

$$= 849$$

$$p = 29.1376$$

$$= 29 \text{ km}$$

The ships are 29 km apart, to the nearest kilometre. **C**

51 In the isosceles triangle DBC, $\angle DBC = \angle DCB$

So $\angle DBC = 52°$

Two angles on a straight line add to 180°.

$$\angle ABD + \angle DBC = 180°$$

$$\angle ABD + 52° = 180°$$

$$\angle ABD = 180° - 52°$$

$$= 128°$$

D

52 Using Heron's rule,

Semi-perimeter

$$s = \frac{1}{2}(p + q + r)$$

$$= \frac{1}{2}(48 + 72 + 72)$$

$$= 96$$

Area

$$= \sqrt{s(s-p)(s-q)(s-r)}$$

$$= \sqrt{96(96-48)(96-72)(96-72)}$$

$$= 1629.17$$

$$= 1629 \text{ m}^2$$

D

53 $\text{Area} = \frac{1}{2}ef\sin D$

$$= \frac{1}{2}(21)(16)\sin 25°$$

$$= 76.27$$

$$= 76 \text{ cm}^2$$

A

Solutions to Written-response questions

Chapter 7: Investigating relationships between two numerical variables

1 **a** EV = *revised*, RV = *exam score*

b Enter the data into your calculator and follow the instructions on page 405 (TI) or page 406 (CASIO) to construct the scatterplot.

c Follow the instructions on page 423 (TI) or page 424 (CASIO) to find $r = 0.894$.

d There is a strong, positive, linear relationship between exam score and the number of revision sessions.

e Follow the instructions on page 437 (TI) or page 438 (CASIO) to find the equation of the least squares regression line.

exam score = 57.03 + 2.98× *revised*

f Intercept: On average, students who do no revision sessions will score 57 on the examination. Slope: On average, a student's score on the examination will increase by 3 marks for each additional revision session.

g *exam score* = 57.03 + 2.98 × 12 = 92.79

h unreliable as we are extrapolating.

2 **a** Enter the data into your calculator and follow the instructions on page 405 (TI) or page 406 (CASIO) to construct the scatterplot.

b $r = 0.787$

c There is a strong, negative, linear relationship between time spent playing games and time spent reading.

d Follow the instructions on page 437 (TI) or page 438 (CASIO) to find the equation of the least squares regression line.
reading = 9.87 − 0.340 × *games*

e Intercept: On average, students who spend no time playing games will read for 9.87 hours. Slope: On average, students will read for 0.34 hours less for each additional hour they spend playing games.

f *reading* = 9.87 − 0.340 × 10 = 6.47 hours

g reliable as we are interpolating.

3 **a** EV: *length*, RV: *weight*

b There is a strong, positive, linear relationship between length and weight.

c $b = r \times \frac{s_y}{s_x} = 0.937 \times \frac{209.206}{3.594} = 54.543$

$a = \bar{y} - b \times \bar{x} = 617.829 - 54.543 \times 30.306 = -1035.15$

weight = −1035.15 + 54.54× *length*

d Intercept: no meaningful interpretation, Slope: On average, fish will weigh 54.54 grams more for each additional cm in length.

e *weight* = −1035.15 + 54.54 × 50 = 1691.85 gm

f unreliable as we are extrapolating.

4 **a** EV: *advertising*, RV: *customers*

b There is a strong, positive linear relationship between the amount spent on advertising and the number of customers.

c Using the points (0, 50) and (7500,800),
intercept: $a = 50$
slope: $b = \dfrac{800 - 50}{7500 - 0} = \dfrac{750}{7500} = 0.1$
customers = 50.0 + 0.1× *advertising*

d intercept = \$50

e Since *customers* = 50.0 + 0.1× *advertising*, to increase the number of customers by 1, the advertising will need to increase by \$10.

f If it costs \$2 to attract a customer we know that $b \times 2 = 1 \Rightarrow b = 0.5$
customers = 50.0 + 0.5× *advertising*

Chapter 8: Graphs and networks

5 **a** Vertex *D* has 1 edge to vertex *C*, 1 edge to vertex *E* and a loop. A loop contirbutes 2 to the degree of a vertex, therefore the degree of vertex *D* is 4.

b A walk that follows every edge is either an Eulerian trail or an Eulerian circuit. If the walk also finishes at the starting point it is considered an Eulerian circuit.

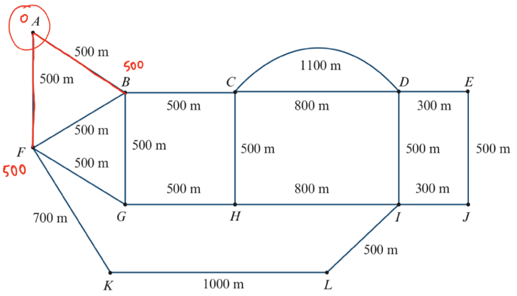
c An Eulerian circuit is possible if the degree of every vertex is *even*. Vertices *B* and *E* have an *odd* degree, therefore an Eulerian circuit is not possible.

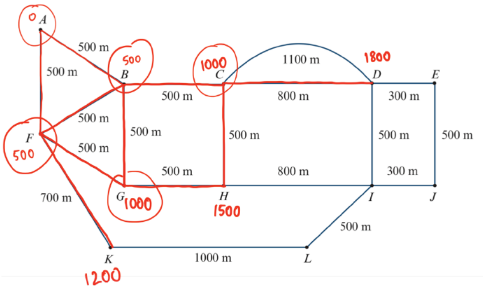
d The walk described is an Eulerian circuit; this is only possible if all vertices have an *even* degree. Two vertices have an *odd* degree; vertices *B* and *E*. If another edge was added between vertices *B* and *E*, then all vertices would have an *even* degree and an Eulerian circuit would be possible. Adding an extra edge to this graph between vertices *B* and *E* simulates walking along the existing edge between *B* and *E* twice, resulting in a possible Eulerian circuit.

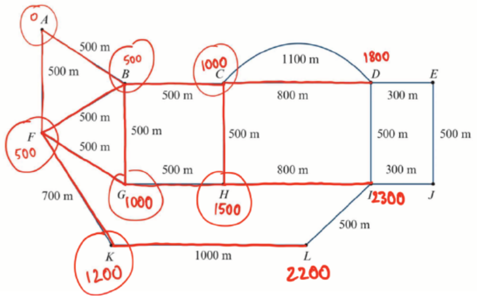
e Row *A* column *A* refers to the connection vertex *A* has with itself; as there is no loop in the graph at vertex *A*, this value must be **zero**. Row *A* column *B* refers to the connections between vertices *A* and *B*; there is one edge between vertices *A* and *B* so the value must be **1**. Row *B* column *A* refers to the connections between vertices *B* and *A*; as previously stated, this value must be **1**. Row *D* column *D* refers to the connection vertex *D* has with itself; there is a loop at vertex *D*, so this value must be **1**. Row *E* column *D* refers to the connections between vertices *E* and *D*; there is one edge between vertices *E* and *D* therefore the value must be **1**.

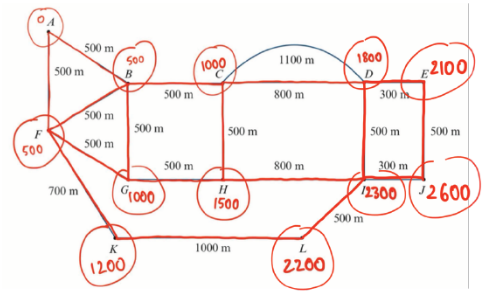
$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 0 & 0 & 1 \\ B & 1 & 0 & 1 & 0 & 1 \\ C & 0 & 1 & 0 & 1 & 0 \\ D & 0 & 0 & 1 & 1 & 1 \\ E & 1 & 1 & 0 & 1 & 0 \end{array}$$

6 a Using Dijkstra's algorithm, the shortest path between vertices A and J can be determined:









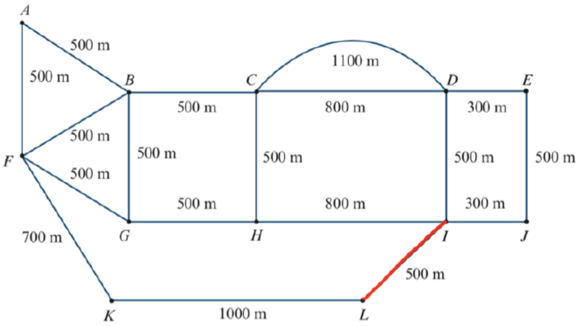
2600 m.

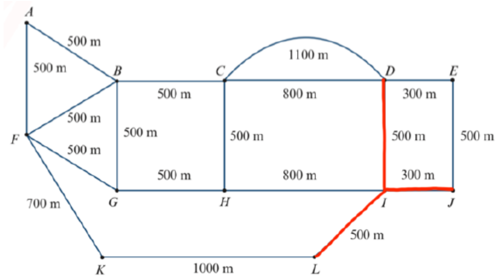
b i The walk described is an Eulerian circuit. For an Eulerian circuit, all vertices must have an *even* degree, so find the degree of each vertex; deg(A) = 2, deg(B) = 4, deg(C) = 4, deg(D) = 4, deg(E) = 2, deg(F) = 4, deg(G) = 3, deg(*H*) = 3, deg(*I*) = 4, deg(*J*) = 2, deg(*K*) = 2, deg(*L*) = 2. The vertices *G* and *H* have an *odd* degree, therefore **no**, the postie cannot complete the walk described (an Eulerian circuit).

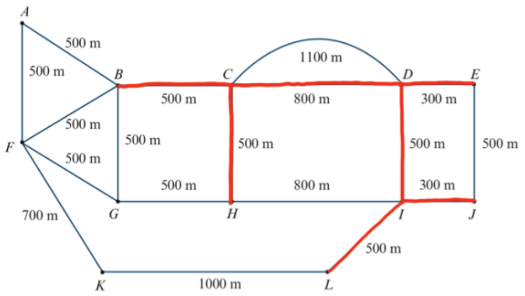
ii The walk described in an Eulerian trail. For an Eulerian trail, exactly two or zero vertices must have an *odd* degree. From part **i** above, only two vertices have an odd degree; vertices *G* and *H*. **Yes**, the postie can complete the walk described (an Eulerian trail), starting at *G* and finishing at *H* (starting and finishing at the two vertices with an odd degree).

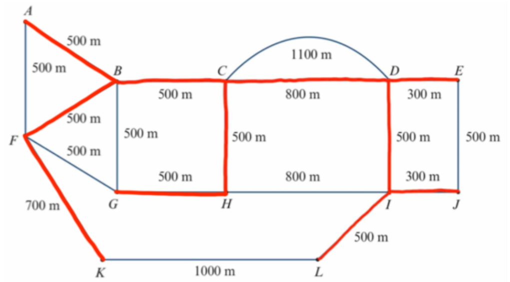
iii The walk described is an Eulerian circuit. Although this is not possible without walking along an edge more than once, the walk can be completed if an Eulerian trail was completed and the edge connecting the starting vertex *G* and finishing vertex *H* was repeated. The edge between vertices *G* and *H* has a weight of 500m. The sum of the weights for all edges in this graph is 10 500 m, therefore the total distance travelled by the postie for the walk described will be 10500 + 500 = **11000 m**.

c The connecting cables between all nodes in the network describes a minimum spanning tree. As the ’exchange’ is located at vertex *L*, Prim’s algorithm must start at this vertex:









The sum of all highlighted edges is **5600 m**.

Chapter 9: Variation

7 **a** $d \propto t^2$

$d = kt^2$

Substitute $d = 20$ and $t = 2$.

$20 = 4k$

$\therefore k = 5$

$d = 5t^2$

b **i** When $t = 5$, $d = 5 \times 5^2$

$d = 5 \times 25 = 125$ m

ii Half the depth is $125 \div 2 = 62.5$.

Find t when $d = 62.5$.

$62.5 = 5t^2$

$t^2 = 12.5$

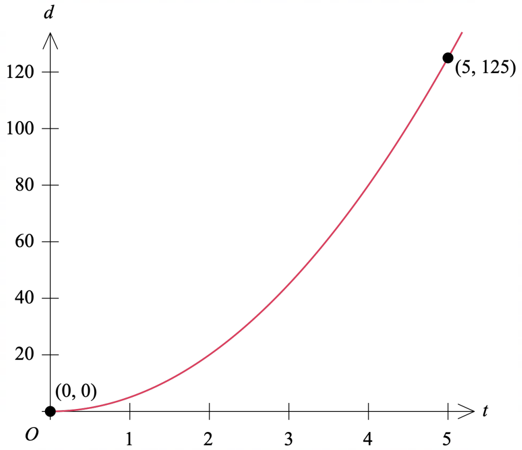
$\therefore t = 3.5355$

$t = 3.54$ secs

c The graph of $d = kt^2$ will be a parabola.

When $t = 0,\ d = 0$ so the graph will go through the origin.

Select another point on the graph. When $t = 5,\ d = 125$ so the point (5, 125) will also be on the graph.



Alternatively the graph can be sketched on the CAS calculator.

8 **a** $t \propto \frac{1}{d}$

$t = \frac{k}{d}$

Substitute $t = 60$, $d = 10$.

$60 = \frac{k}{10}$

$\therefore k = 600$

$t = \frac{600}{d}$

b $T \propto d^2$

$T = kd^2$

Substitute $T = 120$, $d = 10$.

$120 = 100k$

$\therefore k = \frac{120}{100} = 1.2$

$T = 1.2d^2$

c Find t when $d = 30$.

$t = \frac{600}{d}$

$t = \frac{600}{30}$

$\therefore t = 20$ minutes

Find T when $d = 30$.

$T = 1.2(30)^2$

$T = 1.2 \times 900$

$T = 1080$ mins

d $t = \frac{600}{d}$

At $t = 40$, $40 = \frac{600}{d}$

$\therefore d = \frac{600}{40} = 15$ mL

e $T = 1.2d^2$

When $d = 15$,

$T = 1.2(15)^2 = 270$ minutes

Chapter 10: Measurement, scale and similarity

9 a Copy the diagram given in the textbook.
Draw a vertical line to form a rectangle with opposite sides of 9 m and 7 m.
A triangle with sides of 9 m, 5 m and 3 m has also been formed.

Area of rectangle = $9 \times 7 = 63$ m^2

Determine the area of the triangle using Heron's formula.
Semi-perimeter:

$$\begin{aligned} s &= \frac{1}{2}(a + b + c) \\ &= \frac{1}{2}(3 + 5 + 7) \\ &= 7.5 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Area of triangle} &= \sqrt{s(s - a)(s - b)(s - c)} \\ &= \sqrt{7.5(7.5 - 3)(7.5 - 5)(7.5 - 7)} \\ &= \sqrt{7.5(4.5)(2.5)(0.5)} \\ &= 6.4952 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Total area} &= 63 + 6.4952 \\ &= 69.4952 \\ &= 69.5 \text{ m}^2 \end{aligned}$$

b Volume = area × depth

$$\begin{aligned} &= 69.4952 \times \frac{20}{100} \\ &= 13.899 \\ &= 13.9 \text{ m}^3 \end{aligned}$$

c Cost = 13.899×60
= \$833.90

d Perimeter = $3 + 5 + 18 + 7$
= 33 m
Cost = 9×33
= \$297.00

10 a $C = 2\pi r$

i $12 = 2\pi r$

$$r = \frac{12}{2\pi}$$

$r = 1.91$ m

$= 1.9$ m

ii $30 = 2\pi r$

$$r = \frac{15}{\pi}$$

$$r = 4.77465 \text{ m}$$

$$= 4.8 \text{ m}$$

b Area $= \pi(R^2 - r^2)$

$$= \pi(4.77465^2 - 1.90986^2)$$

$$= \pi \times 19.1497$$

$$= 60.16$$

$$= 60.2 \text{ m}^2$$

11 **a** Volume of water = Area of mouth × rainfall

Area of mouth $= \pi r^2$

$$= \pi(40)^2$$

$$= 1600\pi$$

$$= 5026.55 \text{ mm}^2$$

Volume of water $= 5026.55 \times 12$

$$= 60\,318.6 \text{ mm}^3 \left(= 60.3\text{cm}^3\right)$$

b By similar triangles, as the height is halved the radius will be halved.

$h = 60 \quad r = 20$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(20)^2(60)$$

$$= 8000\pi$$

$$= 25\,132.7 \text{ mm}^3 \left(= 25.1\text{cm}^3\right)$$

c Volume of water $= 8000\pi$ (from part b)

Area of the mouth $= 1600\pi$ (from part a)

Volume of water = Area of the mouth × rainfall

$$8000\pi = 1600\pi \times \text{rainfall}$$

$$\text{rainfall} = \frac{8000\pi}{1600\pi}$$

$$= 5 \text{ mm}$$

12 **a** 1 cm : 5000 cm

1 cm : 50 m

8 cm : 400 m

10 cm : 500 m

Area $= 400 \times 500 \text{ m}^2$

$= 200\,000 \text{ m}^2$

$= 20$ hectares

b 1 cm : 5000 cm

1 cm : 50 m

3 cm : 150 m

Use 3 cm diameter to represent 150 metres.

Chapter 11: Applications of trigonometry

13 **a** $\angle$ ABD $= 180° - 27°$

$= 153°$

b $\angle ADB = 180° - (20° + 153°)$

$= 180° - 173°$

$= 7°$

c Using the sine rule:

$$\frac{AD}{\sin 153°} = \frac{7000}{\sin 7°}$$

$$AD = \frac{7000 \times \sin 153°}{\sin 7°}$$

$$= 26076.56 \text{ m}$$

d $\sin 20° = \dfrac{DC}{26\,076.56}$

$DC = 8918.71$

$= 8919$ m

14 **a** In $\triangle PRS$, $\angle PRS = 60°$

Using the cosine rule:

$PS^2 = 1^2 + 3^2 - 2(1)(3)\cos 60°$

$= 7$

$PS = \sqrt{7}$

$= 2.64575$

$= 2.6$ units

b Using the sine rule in $\triangle QPS$

$$\frac{\sin(\angle QPS)}{2} = \frac{\sin 60°}{\sqrt{7}}$$

$$\angle QPS = \sin^{-1}\left(\frac{2\sin 60°}{\sqrt{7}}\right)$$

$$= 40.9°$$

c Finding the area using half base times height.

$$\text{Area} = \frac{1}{2}sp\sin Q$$
$$= \frac{1}{2}(3)(2)\sin 60°$$
$$= 2.59808$$
$$= 2.6 \text{ sq units}$$

Finding the area using Heron's rule.
Let $a = 3, b = 3, c = \sqrt{7}$
Semi-perimeter,

$$s = \frac{a+b+c}{2}$$
$$= \frac{3+2+\sqrt{7}}{2}$$
$$= 3.82288$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Where $a = 3,\ b = 3,\ c = \sqrt{7}, s = 3.82288$
Area = 2.59808
= 2.6 sq units

15 **a** Using the sine rule:

$$\frac{DC}{\sin 80°} = \frac{30}{\sin 50°}$$
$$DC = \frac{30\sin 80°}{\sin 50°}$$
$$= 38.5673$$
$$= 38.6 \text{ m}$$

b $\angle BDC = 180° - (80° + 50°)$
$= 50°$
$= \angle DCB$
So $\triangle BCD$ is isosceles.
Therefore, $BC = DB$
$= 30$ m

c Using the cosine rule:

$$b^2 = a^2 + d^2 - 2ad\cos B$$
$$= 30^2 + 30^2 - 2(30)(30)\cos B$$
$$= 2112.5667$$
$$b = 45.9627$$
$$= 46.0 \text{ m}$$

d $AB = BD = 30$ m

$\triangle ABD$ is isosceles.

$$
\begin{aligned}
&\angle DAB = \angle ADB \\
&\angle ABD = 100^\circ \\
&\angle DAB + \angle ADB + \angle ABD = 180^\circ \\
&\angle DAB + \angle ADB + 100^\circ = 180^\circ \\
&\angle DAB + \angle ADB = 80^\circ \\
&2 \times \angle DAB = 80^\circ \\
&\angle DAB = 40^\circ
\end{aligned}
$$

16 a Using the sine rule in $\triangle ABC$:

$$
\begin{aligned}
\frac{\sin(\angle CAB)}{12} &= \frac{\sin 40^\circ}{8} \\
\sin(\angle CAB) &= \frac{12 \sin 40^\circ}{8} \\
&= 0.96418 \\
\angle CAB &= \sin^{-1}(0.96418) \\
&= 74.6186^\circ \\
&= 74.6^\circ
\end{aligned}
$$

b $\triangle CAA'$ is isosceles.

So $\angle CA'A = \angle CAB$

$= 74.6^\circ$

$\angle CA'B = 180^\circ - 74.6^\circ$

$= 105.4^\circ$

c $\angle A = 74.6^\circ$

$\angle B = 40^\circ$

$\angle C = 180^\circ - (74.6^\circ + 40^\circ)$

$= 65.4^\circ$

d $\angle A' = \angle CA'B$

$= 105.4^\circ$ (from **b**)

$\angle B = 40^\circ$

$\angle C = 180^\circ - (\angle A' + \angle B)$

$= 180^\circ - (105.4^\circ + 40^\circ)$

$= 34.6^\circ$

e Use sine rule in $\triangle ABC$

$$\begin{aligned}\frac{AB}{\sin\angle C} &= \frac{AC}{\sin\angle B}\\ \frac{AB}{\sin 65.4^\circ} &= \frac{8}{\sin 40^\circ}\\ AB &= \frac{8\sin 65.4^\circ}{\sin 40^\circ}\\ &= 11.3\text{ cm}\end{aligned}$$

f Use sine rule in $\triangle A'BC$

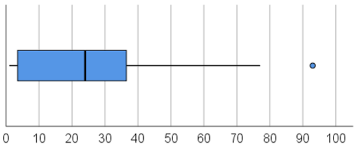
$$\begin{aligned}\frac{A'B}{\sin A'CB} &= \frac{8}{\sin 40^\circ}\\ \frac{A'B}{\sin 34.6^\circ} &= \frac{8}{\sin 40^\circ}\\ AB' &= \frac{8\sin 34.6^\circ}{\sin 40^\circ}\\ &= 7.1\text{ cm}\end{aligned}$$

Solutions to Investigations

1 Cricket Captains

a **i** Tests played key: 1|7 = 17

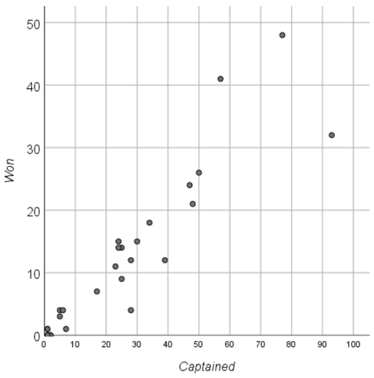
0 | 1 1 1 1 1 2 2 5 5 6 7
1 | 7
2 | 3 4 4 5 5 8 8
3 | 0 4 9
4 | 7 8
5 | 0 7
6 |
7 | 7
8 |
9 | 3



ii The distribution is positively skewed with an outlier. The median number of tests played by these captains is $M = 24$. This means that 50% of the captains played 24 or less tests. The number of tests captained is quite variable with 50% captaining between 4 tests ($Q_1 = 3.5$) and 36 tests ($Q_3 = 36.5$). There is one outlier who captained 93 tests.

iii Allan Border

b i



$r \approx 0.8$

ii There is a strong, positive association between the number of matches played and the number of matches won.

iii $r = 0.992$

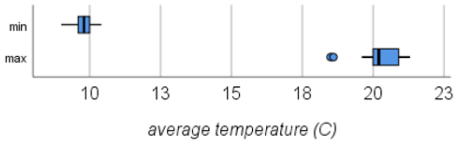
iv $won = -0.150 + 0.487 \times played$

v slope = 0.487. On average, the number of matches won by a captain increases by 0.487 for each additional match played.

vi Border: Played 93, predicted to win 45.1, residual = -13.1
Waugh: Played 57, predicted to win 27.6, residual = 13.5
Ponting: Played 77, predicted to win 37.3, residual = 10.7
Waugh, as the large positive residual indicates his performance as the highest above average.

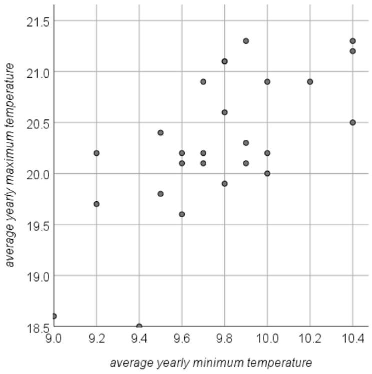
2 Global Warming

a i



ii As expected, we can see from the boxplots that the average minimum temperature is clearly much lower (median = 9.8) than the average maximum temperature (median = 20.2). The spread of the average minimum temperature ($IQR = 0.4$) is also less that the spread of the average maximum temperature ($IQR = 0.9$). There are two outliers for average maximum temperature, at 18.5 C and 18.6 C. These occurred in 1995 and 1996 respectively.

b i



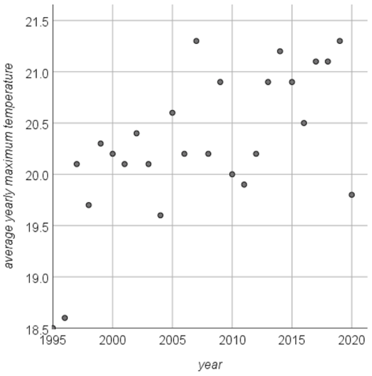
ii There is a moderate, positive, linear relationship between the average minimum temperature and the average maximum temperature.

iii $r = 0.683$

iv *average maximum temperature* = 6.907 + 1.371 × *average minimum temperature*

v Intercept = 6.907. On average, when the average minimum temperature is zero, the average maximum temperature is 6.9 C. Slope = 1.371. On average, the average maximum temperature is 1.4 C higher for each additional 1 C increase in average minimum temperature.

c i



ii There is a moderate, positive, linear relationship between average maximum temperature and year.

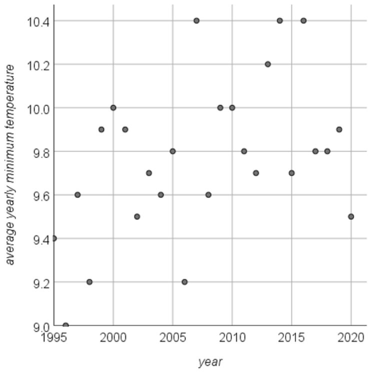
iii *average maximum temperature* = −101.115 + 0.060 × *year*

iv Intercept = −101.115: No meaningful interpretation. Slope = 0.060: On average, the average maximum temperature is 0.06 C higher each year.

v 20.4

vi extrapolating

d **i**



ii There is a weak, positive, linear relationship between average maximum temperature and year.

iii *average minimum temperature* = −32.097 + 0.021 × *year*

iv Intercept = −32.097: No meaningful interpretation. Slope = 0.020: On average, the average minimum temperature is 0.02 C higher each year.

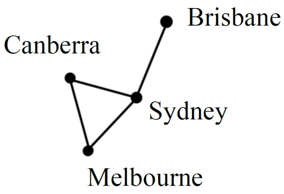
v 10.4

vi extrapolating

e The analyses show that there has been a steady increase in both average minimum temperature and average maximum temperature over the years 1995 to 2020. The increase is more pronounced for average maximum temperature, which has been increasing on average 0.06 C each year, compared to the average minimum temperature which has been increasing on average 0.02 C each year. To further investigate we could look other variables such as maximum/minimum temperature, number of days in each year over/under a certain temperature (e.g. 35 C/10 C), average number of hours of sunlight each year, rainfall (minimum and maximum).

3 Road Trip

a **i**



ii $v = 4$, $e = 4$, $f = 2$;

$v + f = e + 2$

$4 + 2 = 4 + 2$

$6 = 6$

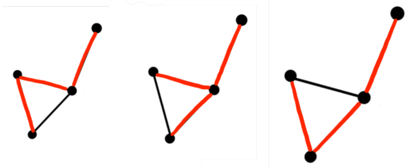
iii Sum of degrees = Number of edges × 2

= 4 × 2 = 8

iv Yes, for this graph one brirdge exists between Sydney and Brisbane. If this edge was removed, the vertex representing Brisbane would be isolated from the rest of the graph and therefore the graph would be disconnected.

v Nick and Maria's walk can only be described as a walk, because edges and vertices are repeated in their journey.

vi A spanning tree must connect all vertices and if the graph contains n vertices, it must only use $n - 1$ edges. There are 4 vertices, so only 3 edges can be used. There are three spanning trees possible:



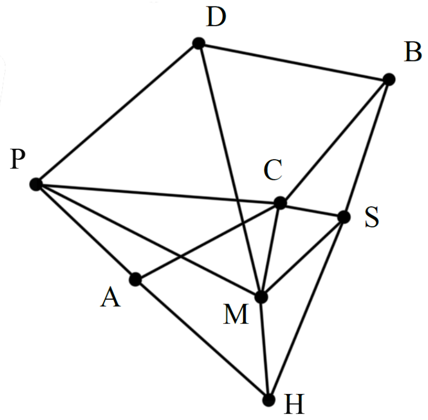
vii On their previous trip, Nick and Maria travelled from Melbourne to Sydney, followed by Brisbane, back to Sydney, across to Canberra and returning to Melbourne. The total distance travelled was 705 + 750 + 750 + 236 + 469 = 2910 km.

viii On their previous trip, Nick and Maria travelled from Melbourne to Sydney, followed by Brisbane, back to Sydney, across to Canberra and returning to Melbourne. The total time spent flying on their trip was 80 + 90 + 90 + 45 + 65 = 370 minutes (6 hours and 10 minutes).

b **i** Using the matrix outlining flight paths, the following routes could potentially be taken:

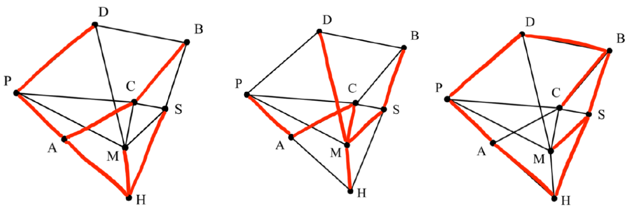
- Route 1: $M - S - H - A - P - D - B - C - M$
- Route 2: $M - D - B - S - C - P - A - H - M$
- Route 3: $M - P - D - B - C - A - H - S - M$

ii

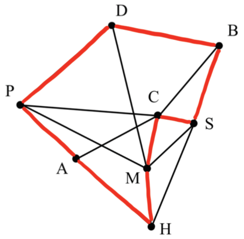


iii No, this graph is not planar. The graph cannot be redrawn without edges crossing, therefore the number of faces cannot be determined and Euler's formula cannot be verified.

iv



v For this graph, there are multiple cycles possible, therefore Nick and Maria could leave from **any** of the capital cities, fly to each other capital city and return to the same city they originally started their journey from. Below is an example of one cycle they could follow, starting at any city, flying to all other cities and returning to their starting city:

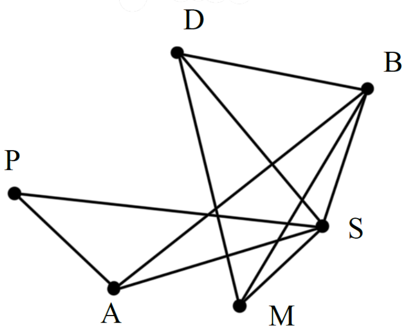


Some potential journies:

- $M - H - A - P - D - B - S - C - M$
- $S - B - D - P - A - H - M - C - S$
- $P - A - H - M - C - S - B - D - P$

c **i** Three routes Nick and Maria could take could be:

- $M - S - B - A - P$
- $M - D - S - A - P$
- $M - B - D - S - P$



ii It was decided that the distance of each flight between the capital cities was to be invesitgated. A table summarising the distance travelled by each flight is included below:

From	To	Distance (km)
Adelaide	Brisbane	1618
Adelaide	Perth	2114
Adelaide	Sydney	1162
Brisbane	Darwin	2849
Brisbane	Melbourne	1380
Brisbane	Sydney	750
Darwin	Melbourne	3131
Darwin	Sydney	3151
Melbourne	Sydney	705
Perth	Sydney	3274

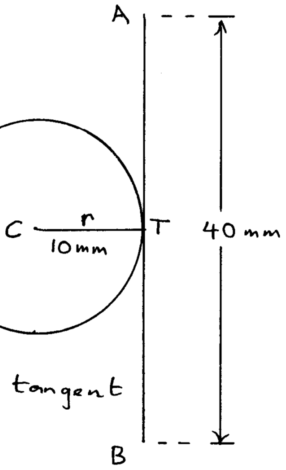
A number of different routes, where the starting city was Melbourne, the final city was Perth and 3 other cities were visited were investigated.

- $M - D - B - S - P = 10004$
- $M - S - B - A - P = 5187$
- $M - D - B - A - P = 9712$
- $M - B - S - A - P = 5406$

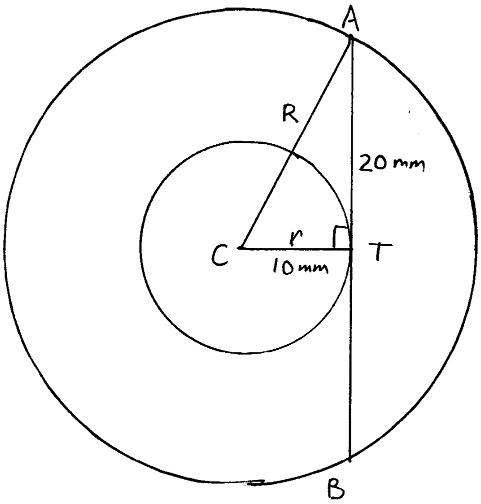
Using the graph contructed in **part i** above, THE route with the shortest distance travelled was $Melbourne - Sydney - Brisbane - Adelaide - Perth$ with a total distance of 5187 km travelled.

4 Gold Medal

a For example:
Draw what is to be the inner circle, a circle centre C, with a radius of 10 mm.
Let r = 10 mm.
To satisfy the required condition, draw a tangent AB, 40 mm long touching the "inner circle" at T.
AB = 40 mm



Use a compass to draw an outer circle, radius *R*, that passes through the end points *A* and *B* of the tangent.



In the right-angled triangle *CAT*, the length *AT* is half the length of the tangent *AB*.
AT = 20 mm

b Using Pythagoras' theorem in the right-angled triangle *CAT*.

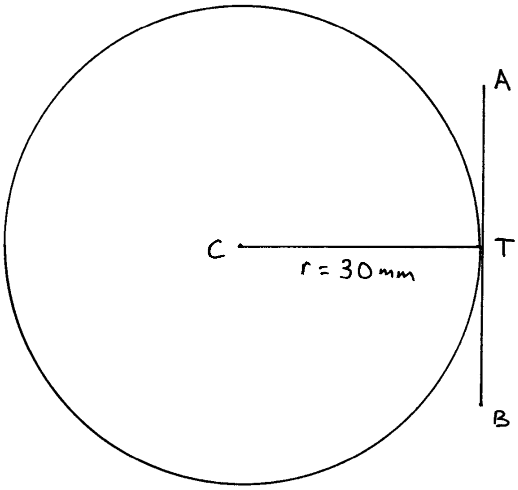
$R^2 = r^2 + 20^2$
$R^2 = 10^2 + 20^2$
$R^2 = 500$
$R = \sqrt{500}$
$= 22.36$ mm

The inner circle radius is 10 mm.
The outer circle radius is 22.36 mm.

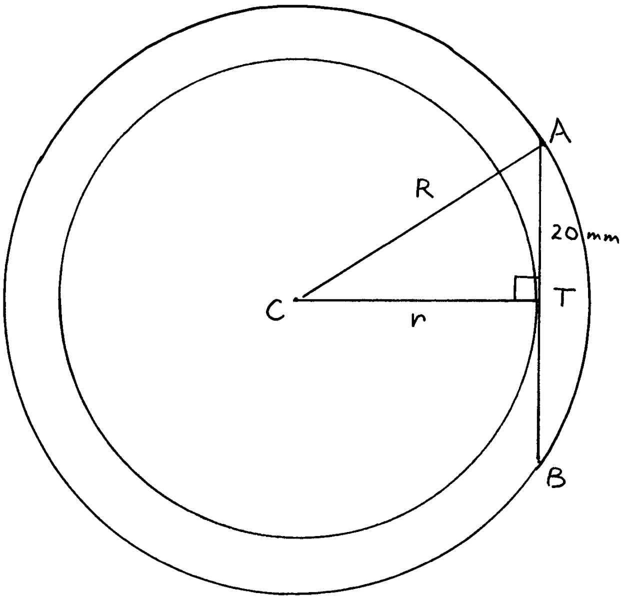
c Area of the gold annulus
= Area of the large circle − Area of the small circle

$= \pi R^2 - \pi r^2$
$= \pi(500) - \pi(100)$
$= 400\pi \text{ mm}^2$

d Let the radius of the inner circle be 30 mm.
$r = 30$ mm
Draw the tangent AB 40 mm long and touching the inner circle at T.
$AB = 40$ mm



Use a compass to draw an outer circle radius R, that passes through the end points A, B of the tangent.



e Using Pythagoras' theorem in the right-angled triangle CAT.
$R^2 = r^2 + 20^2$
$R^2 = 30^2 + 20^2$
$R^2 = 900 + 400$

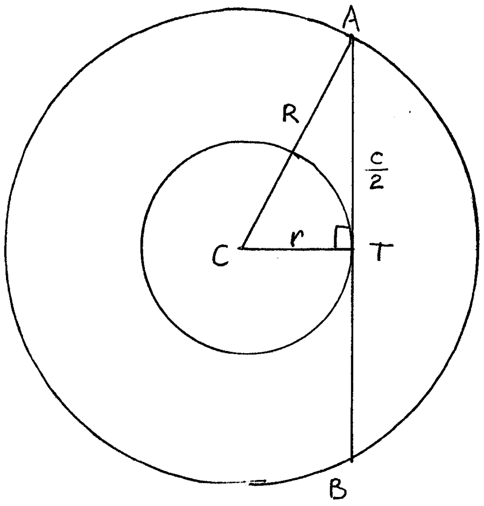
$R^2 = 1300$
$R = \sqrt{1300}$
$= 36.056$ mm

The inner circle radius is 30 mm.
The outer circle radius is 36.056 mm.

Area of the gold annulus
= Area of the large circle − Area of the small circle
$= \pi R^2 - \pi r^2$
$= \pi(1300) - \pi(900)$
$= 400\pi$ mm^2

f Even though each annulus had different values for the radius of the inner and the outer circles they had the same area of gold, 400π mm^2.
It seems that the area of the annulus is determined by the length of the chord *AB*, which was 40 mm long, and does not depend on the radius of each circle. A general rule is needed for the area of the annulus when the length of the chord is constant.
See part **g**.

g Let the length of the chord *AB* be *c* mm.



Using Pythagoras' theorem in the right-angled triangle *CAT*
$R^2 = r^2 + \left(\frac{c}{2}\right)^2$

Area of the gold annulus
= Area of the large circle − Area of the small circle
$= \pi R^2 - \pi r^2$
$= \pi\left(r^2 + \left(\frac{c}{2}\right)^2\right) - \pi r^2$
$= \pi r^2 + \pi\left(\frac{c}{2}\right)^2 - \pi r^2$

$$= \pi\left(\frac{c^2}{4}\right)$$

$$= \frac{\pi c^2}{4}$$

When the length of the chord is c mm the area of the annulus will be given by

$$\text{Area of annulus} = \frac{\pi c^2}{4}\text{ mm}^2.$$

In the investigation the length of the chord was 40 mm.
Substitute $c = 40$ into

$$\begin{aligned}\text{Area of annulus} &= \frac{\pi c^2}{4}\\ &= \frac{\pi(40)^2}{4}\\ &= \frac{1600\pi}{4}\\ &= 400\pi\text{ mm}^2\end{aligned}$$

So when the length of the chord is 40 mm the area of the annulus will always be 400π mm^2.

The radius of the inner cirle r, and the radius of the outer circle R do not appear in the area rule:

$$\text{Area of annulus} = \frac{\pi c^2}{4}$$

So the area of the annulus only depends on c, the length of the chord.

5 Awning Angles

a **i** The angles along the ray of sunlight add to 180°.

$$\begin{aligned}60° + 90° + \angle BAC &= 180°\\ \angle BAC &= 30°\end{aligned}$$

$$\begin{aligned}\text{In } \triangle ABC,\ \tan 30° &= \frac{BC}{3}\\ BC &= 3\tan 30°\\ &= 1.7\text{ m (to one decimal place)}\end{aligned}$$

ii In triangle ABC the ray of sunlight hits the vertical window at an angle of 60°. Therefore $\angle BCA = 60°$.

The angles in triangle ABC add to 180°

$$\angle BAC + 50° + 60° = 180°$$
$$\angle BAC = 70°$$

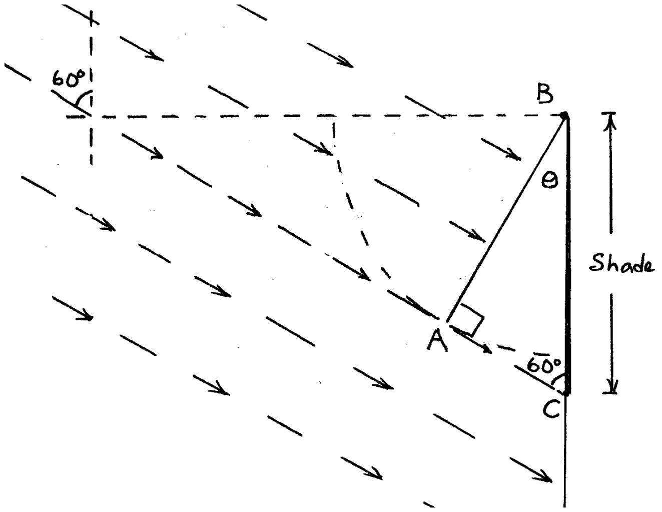
Using the Sine rule in triangle ABC:

$$\frac{BC}{\sin 70°} = \frac{3}{\sin 60°}$$
$$BC = \frac{3 \sin 70°}{\sin 60°}$$
$$= 3.3 \text{ m (to one decimal place)}$$

b Method 1

The awning will create the longest length of shade when it sweeps furtherest into the parallel rays of sunlight.

This occurs when the awning is perpendicular to the rays of sunlight.



For maximum shade the awning is perpendicular to the rays of sunlight, so $\angle BAC = 90°$.

The sun's rays are 60° from the vertical, so the rays strike the vertical window at 60°. Therefore $\angle ACB = 60°$.

Angles in a triangle add to 180°.

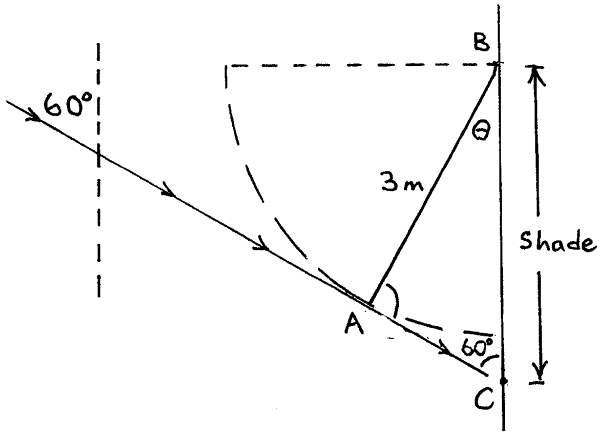
$$\angle BAC + \angle ACB + \angle ABC = 180°$$
$$90° + 60° + \theta = 180°$$
$$\theta = 30°$$

When $\theta = 30°$, the awning makes an angle of 30° with the window and the shade is at a maximum length.

Method 2

The awning makes an angle θ with the window. So $\angle ABC = \theta$

The sunlight strikes the vertical window at an angle of 60°. So $\angle ACB = 60°$



Angles in a triangle add to 180°.

$\theta + 60° + \angle BAC = 180°$

$\angle BAC = 180° - 60° - \theta$

$\angle BAC = 120° - \theta$

The length of the shade is given by side *BC*

Using the Sine rule in triangle *ABC*:

$$\frac{BC}{\sin(120° - \theta)} = \frac{3}{\sin 60°}$$

$$BC = \frac{3\sin(120° - \theta)}{\sin 60°}$$

$$= \frac{3\sin(120° - \theta)}{\frac{\sqrt{3}}{2}}$$

$$= 2\sqrt{3}\sin(120° - \theta)$$

To find the maximum value of *BC* set up a table of values for θ from 0° to 50° in steps of 10°. Then refine your search using θ from 28° to 32° in steps of 1°.

θ	*BC*
0°	3.000
10°	3.255
20°	3.411
30°	3.464
40°	3.411
50°	3.255

θ	*BC*
27°	3.4594
28°	3.4620
29°	3.4636
30°	**3.4641**
31°	3.4636
32°	3.4620

From the table the maximum value of *BC* occurs when θ equals 30°.

$BC = 2\sqrt{3}\sin(120° - 30°)$

$= 2\sqrt{3}\sin(90°)$

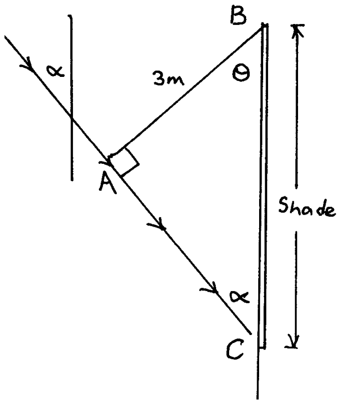
$= 3.5$ m (to one decimal place)

Angle *BAC* is the angle the awning makes with the ray of sunlight.

$$\begin{aligned}\angle BAC &= 120° - \theta \\ &= 120° - 30° \\ &= 90°\end{aligned}$$

This confirms **Method 1** which said the maximum shade would occur when the awning was perpendicular to the rays of sunlight.

c From part **b** the maximum length of shade occurs when the awning is perpendicular to the rays of sunlight.
Let the angle of the sun from the vertical be α degrees.
The awning is at θ degrees from the window.



From the diagram:

$90° + \theta + \alpha = 180°$

$\theta + \alpha = 90°$

$\theta = 90° - \alpha$

For maximum shade the awning should have its angle θ from the vertical equal to $90° - \alpha$.
For example, when the sun's rays are at 10° from the vertical, the awning should be at 80° from the wall for maximum shade.

To find the maximum length of shade when the sun's rays are α from the vertical, referring to the diagram above:

$$\sin\alpha = \frac{3}{shade}$$

$$shade = \frac{3}{\sin\alpha}$$

In the case given in this investigation, the sun's rays were 60° from the vertical. To find the maximum length of shade, put $\alpha = 60°$.

$$\begin{aligned}shade &= \frac{3}{\sin 60°} \\ &= 3.4641 \text{ m} \\ &= 3.5 \text{ m (to one decimal place)}\end{aligned}$$

This confirms the result found in part **b**.