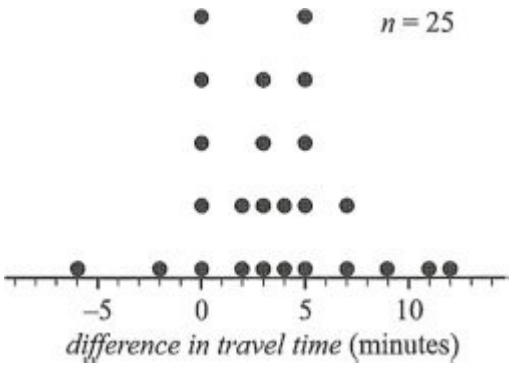
Question 1 / 103

Use the following information to answer Questions 2 and 3.

The dot plot below displays the *difference in travel time* between the morning peak and the evening peak travel times for the same journey on 25 days.



Question 2 / 103

[VCAA 2018 FM (85%)]

The percentage of days when there was five minutes *difference in travel time* between the morning peak and the evening peak travel times is

---

A.

0%

B.

5%

C.

20%

D.

25%

E.

28%

Question 3 / 103

[VCAA 2018 FM (85%)]
The median *difference in travel time* is

A. 3.0 minutes.

B. 3.5 minutes.

C. 4.0 minutes.

D. 4.5 minutes.

E. 5.0 minutes.

Question 4 / 103

Use the following information to answer Questions 5 7.

The pulse rates of a population of Year 12 students are approximately normally distributed with a mean of 69 beats per minute and a standard deviation of 4 beats per minute.

Question 5 / 103

[VCAA 2018 FM (86%)]
A student selected at random from this population has a standardised pulse rate of $z=-2.5$. This student's actual pulse rate is

---

A.

59 beats per minute.

B.

63 beats per minute.

C.

65 beats per minute.

D.

73 beats per minute.

E.

79 beats per minute.

Question 6 / 103

[VCAA 2018 FM (68%)]
Another student selected at random from this population has a standardised pulse rate of $z=-1$. The percentage of students in this population with a pulse rate greater than this student is closest to

---

A. 2.5%

B. 5%

C. 16%

D. 68%

E. 84%

Question 7 / 103

[VCAA 2018 FM (69%)]
A sample of 200 students was selected at random from this population.

The number of these students with a pulse rate of less than 61 beats per minute or greater than 73 beats per minute is closest to

---

A.

19

B.

37

C.

64

D.

95

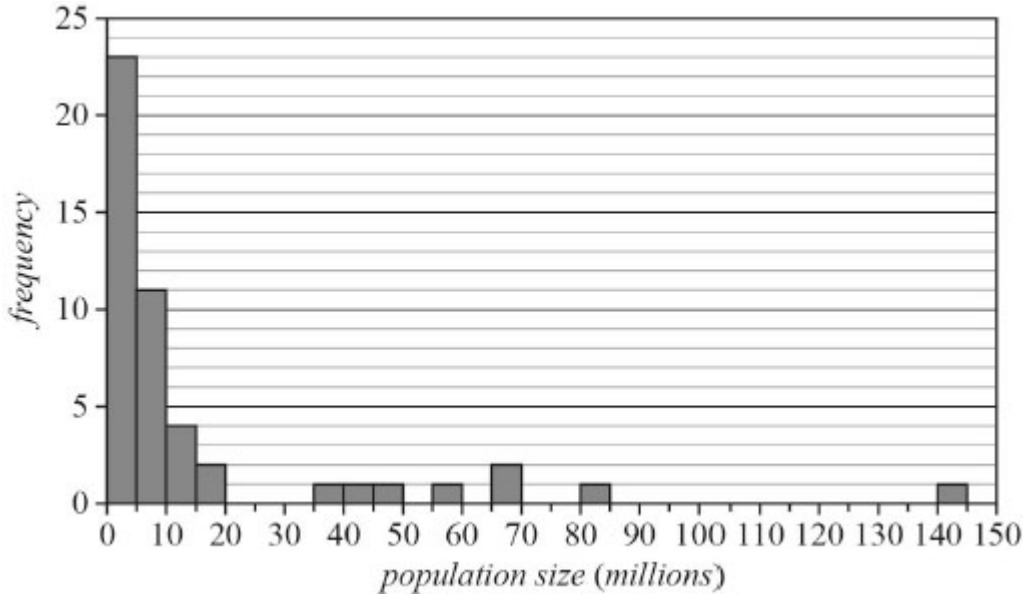
E.

190

Question 8 / 103

Use the following information to answer Questions 9–10.

The histogram below shows the distribution of the *population size* of 48 countries in 2018.



Question 9 / 103

[VCAA 2019 FM (92%)]
The number of these countries with a *population size* between 5 million and 20 million people is

---

A. 11

B. 17

C. 23

D. 34

E. 35

Question 10 / 103

[VCAA 2019 FM (86%)]
The shape of this histogram is best described as

A. positively skewed with no outliers.

B. positively skewed with outliers.

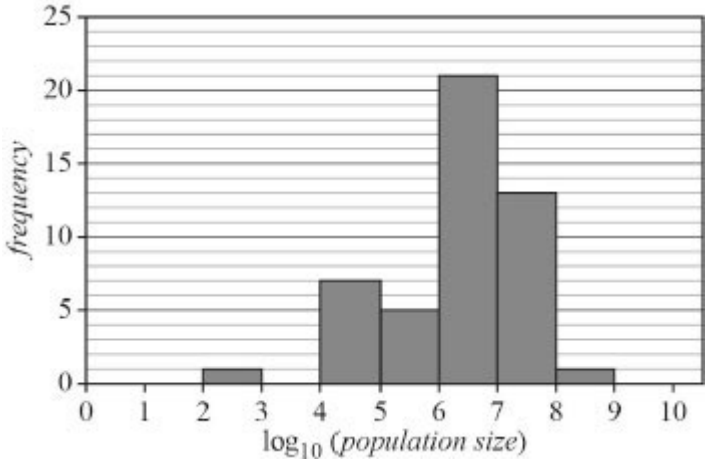
C. approximately symmetric.

D. negatively skewed with no outliers.

E. negatively skewed with outliers.

Question 11 / 103

[VCAA 2019 FM (71%)]
The histogram below shows the *population size* for these 48 countries plotted on a log10 scale.



Based on this histogram, the number of countries with a *population size* that is less than 100 000 people is

---

A. 1

B. 5

C. 7

D. 8

E. 48

Question 12 / 103

Use the following information to answer Questions 13 and 14.

The stem plot below shows the distribution of mathematics *test scores* for a class of 23 students.

key : 4|2 = 42 $n = 23$

4	0 1 4 4
5	2 7 9 9 9
6	5 6 8 8 9 9
7	0 0 5 6 7 8
8	5 9

Question 13 / 103

[VCAA 2019 FM (96%)]
For this class, the range of *test scores* is

A. 22

B. 44

C. 45

D. 49

E. 89

Question 14 / 103

[VCAA 2019 FM (87%)]
For this class, the interquartile range (IQR) of *test scores* is

---

A. 14.5

B. 17.5

C. 18

D. 24

E. 49

Question 15 / 103

[VCAA 2019 FM (79%)]
The time taken to *travel* between two regional cities is approximately normally distributed with a mean of 70 minutes and a standard deviation of 2 minutes.

The percentage of *travel* times that are between 66 minutes and 72 minutes is closest to

---

A. 2.5%

B. 34%

C. 68%

D. 81.5%

E. 95%

Question 16 / 103

[VCAA 2019 FM (72%)]
The volume of a cup of soup served by a machine is normally distributed with a mean of 240 mL and a standard deviation of 5 mL. A fast-food store used this machine to serve 160 cups of soup. The number of these cups of soup that are expected to contain less than 230 mL of soup is closest to

---

A.

3

B.

4

C.

8

D.

26

E.

156

Question 17 / 103

Use the following information to answer Questions 18 20.

The times between successive nerve impulses (time), in milliseconds, were recorded.

Table 1 shows the mean and the five-number summary calculated using 800 recorded data values.

Table 1

	Time **(milliseconds)**
Mean	220
Minimum value	10
First quartile (Q1)	70
Median	150
Third quartile (Q3)	300
Maximum value	1380

Data: adapted from P Fatt and B Katz, 'Spontaneous subthreshold activity at motor nerve endings', *The Journal of Physiology*, 117, 1952, pp. 109-128

Question 18 / 103

[VCAA 2020 FM (98%)]
The difference, in milliseconds, between the mean *time* and the median *time* is

---

A. 10

B. 70

C. 150

D. 220

E. 230

Question 19 / 103

[VCAA 2020 FM (73%)]
Of these 800 times, the number of times that are longer than 300 milliseconds is closest to

---

A.

20

B.

25

C.

75

D.

200

E.

400

Question 20 / 103

[VCAA 2020 FM (59%)]
The shape of the distribution of these 800 times is best described as

---

A.

approximately symmetric.

B.

positively skewed.

C.

positively skewed with one or more outliers.

D.

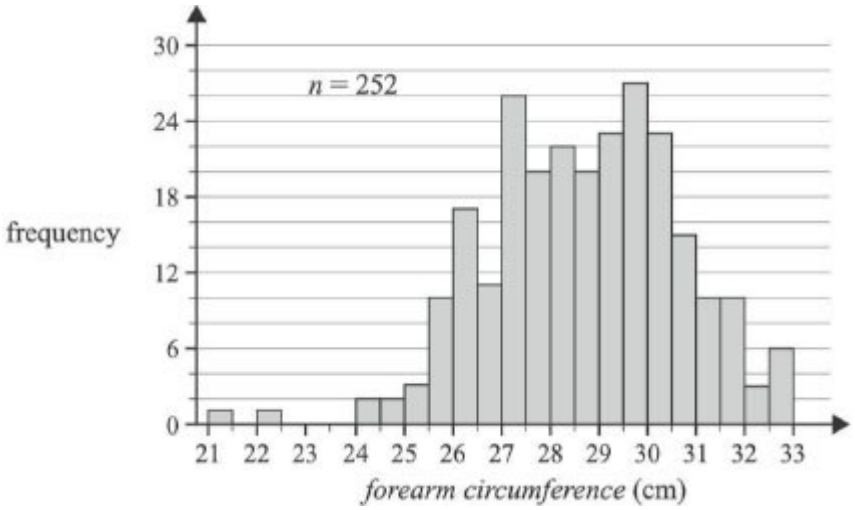
negatively skewed.

E.

negatively skewed with one or more outliers.

Question 21 / 103

[VCAA 2020 FM (50%)]
The histogram below shows the distribution of the *forearm circumference*, in centimetres, of 252 men. Assume that the *forearm circumference* values were all rounded to one decimal place.



The third quartile (Q3) for this distribution could be

A.

29.3

B.

29.8

C.

30.3

D.

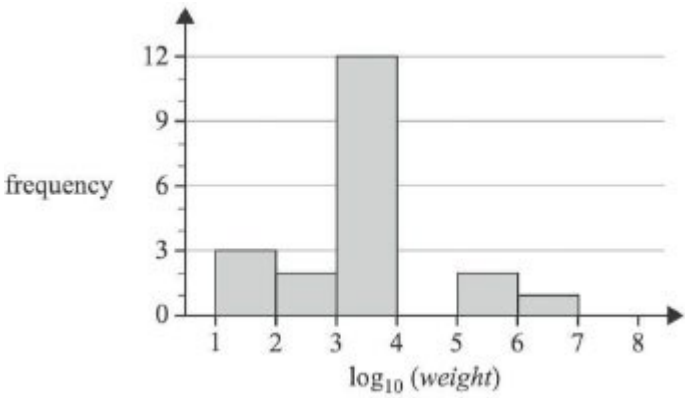
30.8

E.

31.3

Question 22 / 103

[VCAA 2020 FM (78%)]
The histogram below shows the distribution of *weight*, in grams, for a sample of 20 animal species. The histogram has been plotted on a log10 scale.



The percentage of these animal species with a *weight* of less than 10 000 g is

---

A.

17%

B.

70%

C.

75%

D.

80%

E.

85%

Question 23 / 103

[VCAA 2020 FM (72%)]
A percentage segmented bar chart would be an appropriate graphical tool to display the association between *month of the year* (January, February, March, etc.) and the

---

A. *monthly average rainfall* (in millimetres).

B. *monthly mean temperature* (in degrees Celsius).

C. *annual median wind speed* (in kilometres per hour).

D. *monthly average rainfall* (below average, average, above average).

E. *annual average temperature* (in degrees Celsius).

Question 26 / 103

[VCAA 2020 FM (77%)]
The lifetime of a certain brand of light globe, in hours, is approximately normally distributed. It is known that 16% of the light globes have a lifetime of less than 655 hours and 50% of the light globes have a lifetime that is greater than 670 hours.

The mean and the standard deviation of this normal distribution are closest to

---

A.

mean = 655 hours standard deviation = 10 hours

B.

mean = 655 hours standard deviation = 15 hours

C.

mean = 670 hours standard deviation = 10 hours

D.

mean = 670 hours standard deviation = 15 hours

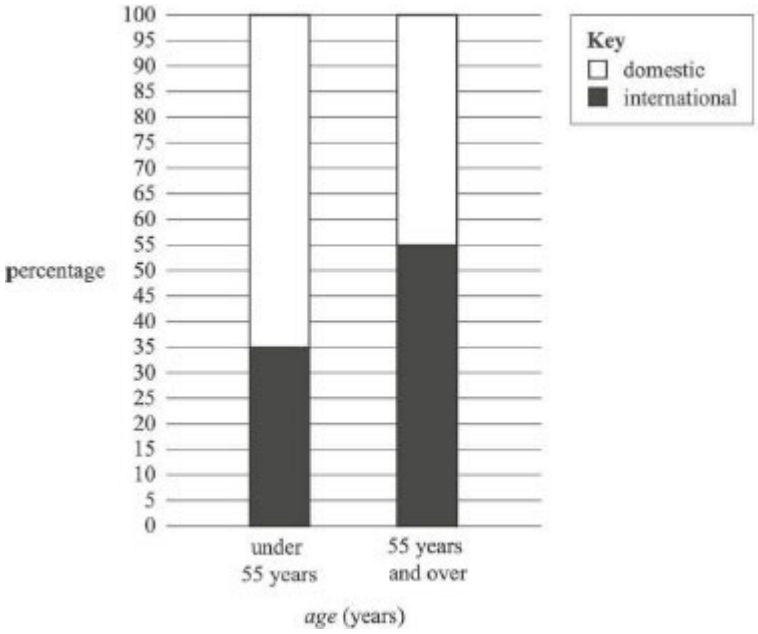
E.

mean = 670 hours standard deviation = 20 hours

Question 27 / 103

Use the following information to answer Questions 28 to 30.

The percentaged segmented bar chart below shows the age (under 55 years, 55 years and overof visitors at travel convention, segmented by *preferred travel destination* (domestic, international).



Question 28 / 103

[VCAA 2021 FM (70%)]
The variables *age* (under 55 years, 55 years and over) and *preferred travel destination* (domestic, international) are

---

A. both categorical variables.

B. both numerical variables.

C. a numerical variable and a categorical variable respectively.

D. a categorical variable and a numerical variable respectively.

E. a discrete variable and a continuous variable respectively.

Question 29 / 103

[VCAA 2021 FM (82%)]
The data displayed in the percentaged segmented bar chart supports the contention that there is an association between *preferred travel destination* and *age* because

---

A.

more visitors favour international travel.

B.

35% of visitors under 55 years favour international travel.

C.

45% of visitors 55 years and over favour domestic travel.

D.

65% of visitors under 55 years favour domestic travel while 45% of visitors 55 years and over favour domestic travel.

E.

the percentage of visitors who prefer domestic travel is greater than the percentage of visitors who prefer international travel.

Question 30 / 103

[VCAA 2021 FM (50%)]
The results could also be summarised in a two-way frequency table.

Which one of the following frequency tables could match the percentaged segmented bar chart?

A.

	Age	
Preferred travel destination	**Under 55 years**	**55 years and over**
domestic	91	90
international	49	110

	Age	
Preferred travel destination	**Under 55 years**	**55 years and over**
Total	140	200

B.

	Age	
Preferred travel destination	**Under 55 years**	**55 years and over**
domestic	65	35
international	45	55
Total	110	90

C.

	Age	
Preferred travel destination	**Under 55 years**	**55 years and over**
domestic	35	55
international	65	45
Total	100	100

D.

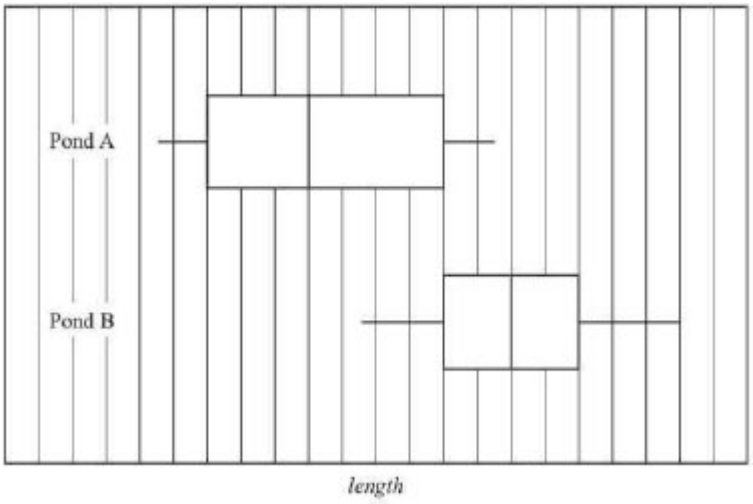
	Age	
Preferred travel destination	**Under 55 years**	**55 years and over**
domestic	50	70
international	100	50
Total	150	120

E.

	Age	
Preferred travel destination	**Under 55 years**	**55 years and over**
domestic	71	39
international	29	61
Total	100	100

Question 31 / 103

[VCAA 2021 FM (60%)]
The boxplots below show the distribution of the length of fish caught in two different ponds, Pond A and Pond B.



Based on the boxplots above, it can be said that

A. 50% of the fish caught in Pond A are the same length as the fish caught in Pond B.

B. 50% of the fish caught in Pond B are longer than all of the fish caught in Pond A.

C. 50% of the fish caught in Pond B are shorter than all of the fish caught in Pond A.

D. 75% of the fish caught in Pond A are shorter than all of the fish caught in Pond B.

E. 75% of the fish caught in Pond B are longer than all of the fish caught in Pond A.

Question 32 / 103

[VCAA 2021 FM (63%)]
The stem plot below shows the *height*, in centimetres, of 20 players in a junior football team.

key: 14|2 = 142 cm $n = 20$

14	2	2	4	7	8	8	9	
15	0	0	1	2	5	5	6	8
16	0	1	1	2				
17	9							

A player with a height of 179 cm is considered an outlier because 179 cm is greater than

---

A. 162 cm

B. 169 cm

C. 172.5 cm

D. 173 cm

E. 175.5 cm

Question 33 / 103

[VCAA 2021 FM (51%)]
The relationship between *resting pulse rate*, in beats per minute, and *age group* (15–20 years, 21–30 years, 31–50 years, over 50 years) is best displayed using

---

A. a histogram.

B. a scatterplot.

C. parallel boxplots.

D. a time series plot.

E. a back-to-back stem plot.

Question 34 / 103

Use the following information to answer Questions 35 and 36.

800 participants auditioned for a stage musical. Each participant was required to complete a series of ability tests for which they received an overall score.

The overall scores were approximately normally distributed with a mean score of 69.5 points and a standard deviation of 6.5 points.

Question 35 / 103

[VCAA 2021 FM (54%)]
Only the participants who scored at least 76.0 points in the audition were considered successful. Using the 68–95–99.7% rule, how many of the participants were considered unsuccessful?

A. 127

B. 128

C. 272

D. 672

E. 673

Question 36 / 103

[VCAA 2021 FM (67%)]
To be offered a leading role in the stage musical, a participant must achieve a standardised score of at least 1.80.

Three participants' names and their overall scores are given in the table below.

Participant	**Overall score**
Amy	81.5
Brian	80.5
Cherie	82.0

Which one of the following statements is true?

A.

Only Amy was offered a leading role.

B.

Only Cherie was offered a leading role.

C.

Only Brian was not offered a leading role.

D.

Both Brian and Cherie were offered leading roles.

E.

All three participants were offered leading roles.

Question 37 / 103

[VCAA 2021 FM (71%)]
The heights of females living in a small country town are normally distributed:

- 16% of the females are more than 160 cm tall.
- 2.5% of the females are less than 115 cm tall.

The mean and the standard deviation of this female population, in centimetres, are closest to

---

A.

mean = 135 standard deviation = 15

B.

mean = 135 standard deviation = 25

C.

mean = 145 standard deviation = 15

D.

mean = 145 standard deviation = 20

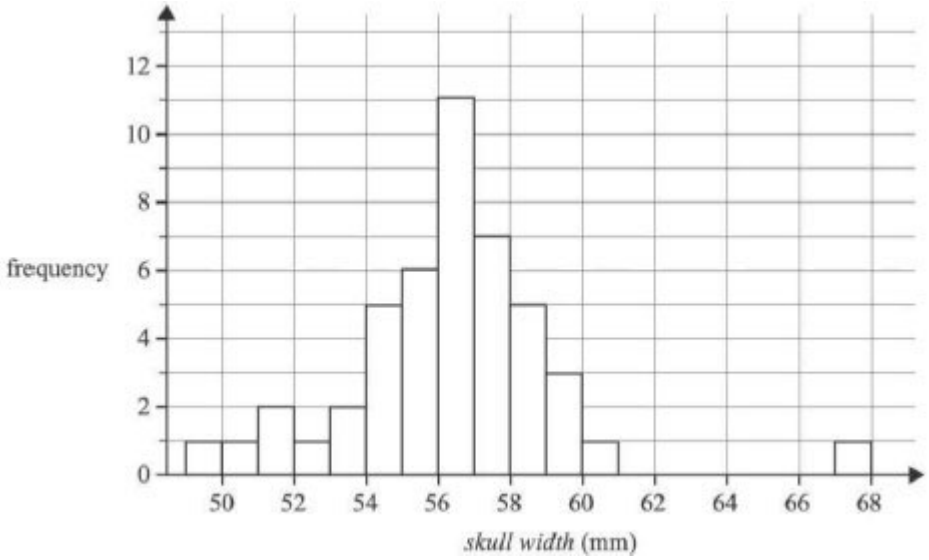
E.

mean = 150 standard deviation = 10

Question 38 / 103

Use the following information to answer Questions 39 41.

The histogram below displays the distribution of *skull width*, in millimetres, for 46 female possums.



Data: adapted from DB Lindenmayer et al., 'Morphological variation among populations of the mountain brushtail possum, *Trichosurus caninus* Ogilby (Phalangeridae: Marsupialia)', *Australian Journal of Zoology*, 43(5), 1995, p. 453

Question 39 / 103

[VCAA 2022 FM (36%)]
The shape of the distribution is best described as

A. negatively skewed.

B. approximately symmetric.

C. negatively skewed with a possible outlier.

D. positively skewed with a possible outlier.

E. approximately symmetric with a possible outlier.

Question 40 / 103

[VCAA 2022 FM (85%)]
The percentage of the 46 possums with a *skull width* of less than 55 mm is closest to

---

A.

12%

B.

26%

C.

39%

D.

61%

E.

74%

Question 41 / 103

[VCAA 2022 FM (55%)]
The third quartile (Q3) for this distribution, in millimetres, could be

---

A.

55.8

B.

56.2

C.

56.9

D.

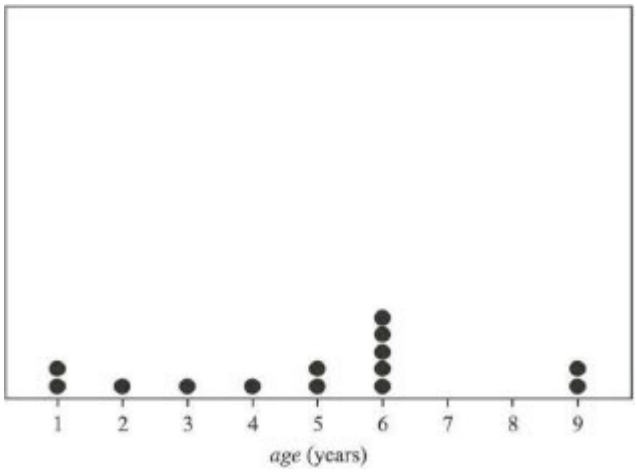
57.7

E.

58.3

Question 42 / 103

[VCAA 2022 FM (74%)]
The *age*, in years, of a sample of 14 possums is displayed in the dot plot below



The mean and the standard deviation of *age* for this sample of possums are closest to

A. mean = 4.25 standard deviation = 2.6

B. mean = 4.8 standard deviation = 2.4

C. mean = 4.8 standard deviation = 2.5

D. mean = 4.9 standard deviation = 2.4

E. mean = 4.9 standard deviation = 2.5

Question 43 / 103

[VCAA 2022 FM (76%)]
The possum population of a large city park is 2498.

The body lengths of this species of possum are known to be approximately normally distributed with a mean of 88 cm and a standard deviation of 4 cm.

Using the 68–95–99.7% rule, the number of possums in this park with a body length between 84 cm and 96 cm is closest to

---

A. 2036

B. 2043

C. 2047

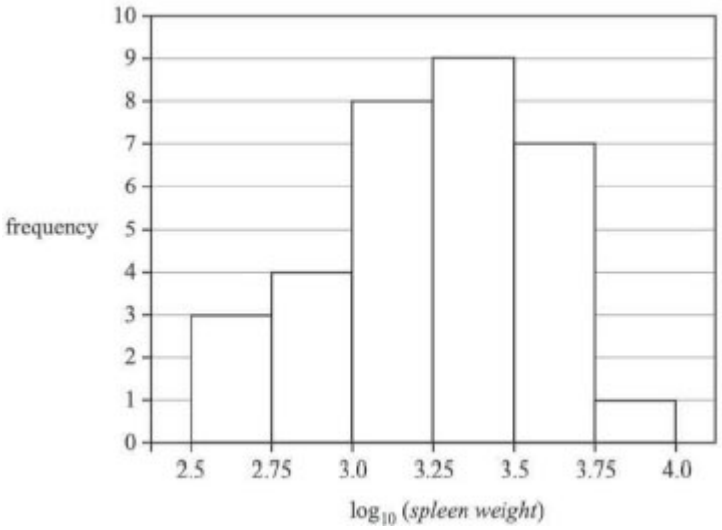
D. 2105

E. 2156

Question 44 / 103

[VCAA 2022 FM (75%)]
The histogram below displays the distribution of *spleen weight* for a sample of 32 seals.

The histogram has a log10 scale.



Data: adapted from CL Stewardson et al., 'Gross and microscopic visceral anatomy of the male Cape fur seal, *Arctocephalus pusillus pusillus* (Pinnipedia: Otariidae), with reference to organ size and growth', *Journal of Anatomy, 195, 1999, p. 240*

The number of seals in this sample with a spleen weight of 1000 g or more is

---

A.

7

B.

8

C.

17

D.

25

E.

27

Question 45 / 103

[VCAA 2018 FM (71%)]
Data was collected to investigate the association between the following two variables:

- *age* (29 and under, 30–59, 60 and over)
- *uses public transport* (yes, no)

Which one of the following is appropriate to use in the statistical analysis of this association?

A. a scatterplot

B. parallel box plots

C. a least squares line

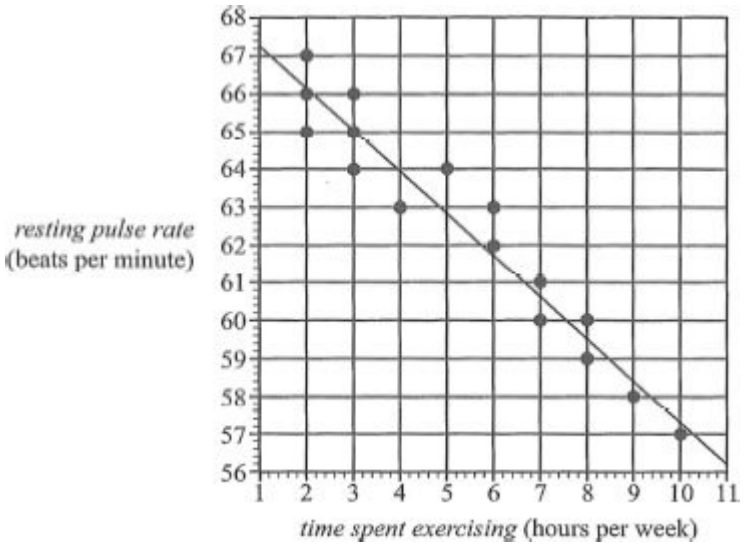
D. a segmented bar chart

E. the correlation coefficient r

Question 46 / 103

Use the following information to answer Questions 47 49.

The scatterplot below displays the *resting pulse rate*, in beats per minute, and the *time spent exercising*, in hours per week, of 16 students. A least squares line has been fitted to the data.



Question 47 / 103

[VCAA 2018 FM (73%)]
Using this least squares line to model the association between *resting pulse rate* and *time spent exercising*, the residual for the student who spent four hours per week exercising is closest to

---

A.

−2.0 beats per minute.

B.

−1.0 beats per minute.

C.

−0.3 beats per minute.

D.

1.0 beats per minute.

E.

2.0 beats per minute.

Question 48 / 103

[VCAA 2018 FM (46%)]
The equation of this least squares line is closest to

A. resting pulse rate=67.2−0.91×time spent exercising

B. resting pulse rate=67.2−1.10×time spent exercising

C. resting pulse rate=68.3−0.91×time spent exercising

D. resting pulse rate=68.3−1.10×time spent exercising

E. resting pulse rate=67.2+1.10×time spent exercising

Question 49 / 103

[VCAA 2018 FM (45%)]
The coefficient of determination is 0.8339.

The correlation coefficient r is closest to

A. −0.913

B. −0.834

C. −0.695

D. 0.834

E. 0.913

Question 50 / 103

[VCAA 2018 FM (51%)]
In a study of the association between a person's *height*, in centimetres, and *body surface area*, in square metres, the following least squares line was obtained.

body surface area=−1.1+0.019×height

Which one of the following is a conclusion that can be made from this least square line?

A. An increase of 1 m^2 in *body surface area* is associated with an increase of 0.019 cm in *height*.

B. An increase of 1 cm in *height* is associated with an increase of 0.019 m^2 in *body surface area*.

C. The correlation coefficient is 0.019

D. A person's *body surface area*, in square metres, can be determined by adding 1.1 cm to their *height*.

E. A person's *height*, in centimetres, can be determined by subtracting 1.1 from their *body surface area*, in square metres.

Question 51 / 103

[VCAA 2018 FM (58%)]
Freya uses the following data to generate the scatterplot below.

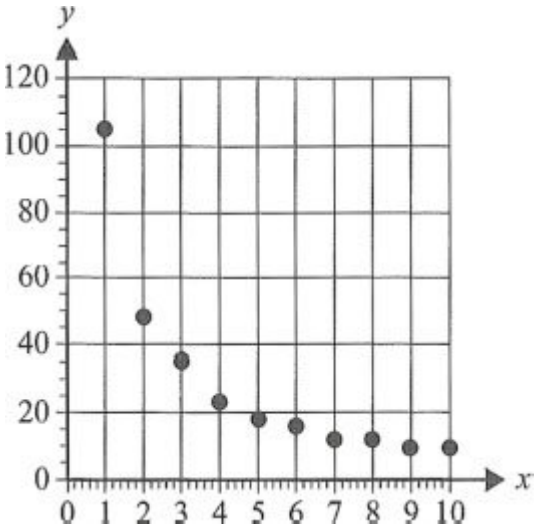
x	1	2	3	4	5	6	7	8	9	10
y	105	48	35	23	18	16	12	12	9	9

The scatterplot shows that the data is non-linear.

To linearise the data, Freya applies a reciprocal transformation to the variable y.

She then fits a least squares line to the transformed data.

With x as the explanatory variable, the equation of this least squares line is closest to



A. $\frac{1}{y}$=−0.0039+0.012x

B. $\frac{1}{y}$=−0.025+1.1x

C. $\frac{1}{y}$=7.8−0.082x

D. y=45.3+59.7×$\frac{1}{x}$

E. y=59.7+45.3×$\frac{1}{x}$

Question 52 / 103

[VCAA 2018 FM (54%)]
A log10(y) transformation was used to linearise a set of non-linear bivariate data.

A least squares line was then fitted to the transformed data.

The equation of this least squares line is

$\log10(y)=3.1-2.3x$

This equation is used to predict the value of y when $x=1.1$

The value of y is closest to

A. −0.24

B. 0.57

C. 0.91

D. 1.6

E. 3.7

Question 53 / 103

[VCAA 2018 FM (49%)]
The statistical analysis of a set of bivariate data involving variables x and y resulted in the information displayed in the table below.

Mean	$x=27.8$	$y=33.4$
Standard deviation	$sx=2.33$	$sy=3.24$
Equation of the least squares line	$y=-2.84+1.31x$	

Using this information, the value of the correlation coefficient r for this set of bivariate data is closest to

A. 0.88

B. 0.89

C. 0.92

D. 0.94

E. 0.97

Question 54 / 103

[VCAA 2018 FM (42%)]
A least squares line is fitted to a set of bivariate data.

Another least squares line is fitted with response and explanatory variables reversed.

Which one of the following statistics will **not** change in value?

A. the residual values

B. the predicted values

C. the correlation coefficient r

D. the slope of the least squares line

E. the intercept of the least squares line

Question 55 / 103

[VCAA 2019 FM (63%)]
Percy conducted a survey of people in his workplace. He constructed a two–way frequency table involving two variables.

One of the variables was *attitude towards shorter working days* (for, against).

The other variable could have been

A.

age (in years).

B.

sex (male, female).

C.

height (to the nearest centimetre).

D.

income (to the nearest thousand dollars).

E.

time spent travelling to work (in minutes).

Question 56 / 103

Use the following information to answer Questions 57 and 58.

A least squars line is used tm del the relationship between the monthly *average temperature* and *latitude* recorded at seven different weather stations. The equation of the least squares line is found to be

average temperature=42.9842−0.877447×latitude

Question 57 / 103

[VCAA 2019 FM (67%)]
When the numbers in this equation are correctly rounded to three significant figures, the equation will be

---

A. average temperature=42.984−0.877×latitude

B. average temperature=42.984−0.878×latitude

C. average temperature=43.0−0.878×latitude

D. average temperature=42.9−0.878×latitude

E. average temperature=43.0−0.877×latitude

Question 58 / 103

[VCAA 2019 FM (41%)]
The coefficient of determination was calculated to be 0.893743

The value of the correlation coefficient, rounded to three decimal places, is

---

A. −0.945

B. 0.898

C. 0.806

D. 0.898

E. 0.945

Question 59 / 103

[VCAA 2019 FM (37%)]
A study was conducted to investigate the effect of drinking *coffee* on sleep.

In this study, the amount of *sleep*, in hours, and the amount of *coffee* drunk, in cups, on a given day were recorded for a group of adults.

The following summary statistics were generated.

	***Sleep* (hours)**	***Coffee* (cups)**
Mean	7.08	2.42
Standard deviation	1.12	1.56
Correlation coefficient (r)	−0.770	

On average, for each additional cup of *coffee* drunk, the amount of *sleep*

A.

decreased by 0.55 hours.

B.

decreased by 0.77 hours.

C.

decreased by 1.1 hours.

D.

increased by 1.1 hours.

E.

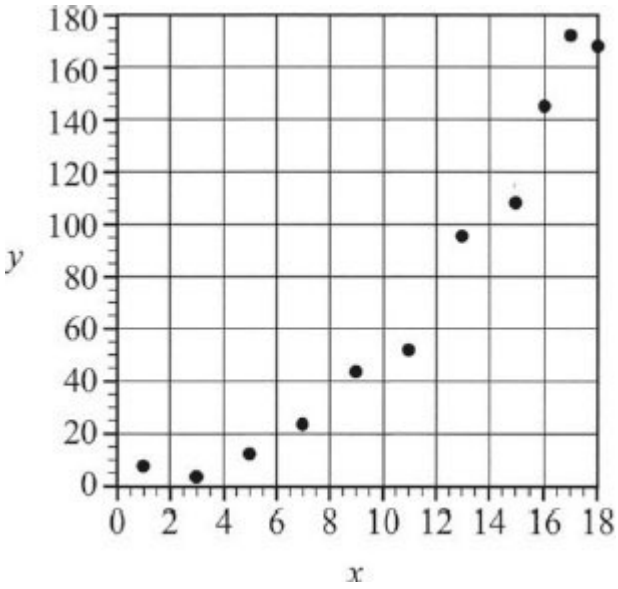
increased by 2.3 hours.

Question 60 / 103

[VCAA 2019 FM (57%)]
The table below shows the values of two variables x and y.

The associated scatterplot is also shown.

The explanatory variable is x.



x	y
1	7.6
3	3.4
5	12.1
7	23.4
9	43.6
11	51.8
13	95.4
15	108
16	145

17	172
18	168

The scatterplot is non-linear.

A squared transformation applied to the variable x can be used to linearise the scatterplot.

The equation of the least squares line fitted to the linearised data is closest to

---

A. $y=-1.34+0.546x$

B. $y=-1.34+0.546x^2$

C. $y=3.93-0.00864x^2$

D. $y=34.6-10.5x$

E. $y=34.6-10.5x^2$

Question 61 / 103

Use the following information to answer Questions 62–64.

The data in Table 2 was collected in a study of the association between the variables *frequency of nightmares* (low, high) and *snores* (no, yes).

Table 2

Frequency of nightmares	***Snores***		**Total**
	No	**Yes**	
low	80	58	138
high	11	12	23
Total	91	70	161

Data: adapted from RA Hicks and J Bautista, 'Snoring and nightmares', *Perceptual and Motor Skills*, l October 1993, <https//doi.org10.246pms.1993.77.2.433 >

Question 62 / 103

[VCAA 2020 FM (57%)]
The variables in this *study, frequency of nightmares* (low, high) and *snores* (no, yes), are

---

A.

ordinal and nominal respectively.

B.

nominal and ordinal respectively.

C.

both numerical.

D.

both ordinal.

E.

both nominal.

Question 63 / 103

[VCAA 2020 FM (95%)]
The percentage of participants in the study who did not snore is closest to

---

A.

42.0%

B.

43.5%

C.

49.7%

D.

52.2%

E.

56.5%

Question 64 / 103

[VCAA 2020 FM (70%)]
Of those people in the study who did snore, the percentage who have a high frequency of nightmares is closest to

---

A. 7.5%

B. 17.1%

C. 47.8%

D. 52.2%

E. 58.0%

Question 65 / 103

[VCAA 2020 FM (34%)]
A least squares line of the form y=a+bx is fitted to a scatterplot.

Which one of the following is always true?

A.

As many of the data points in the scatterplot as possible will lie on the line.

B.

The data points in the scatterplot will be divided so that there are as many data points above the line as there are below the line.

C.

The sum of the squares of the shortest distances from the line to each data point will be a minimum.

D.

The sum of the squares of the horizontal distances from the line to each data point will be a minimum.

E.

The sum of the squares of the vertical distances from the line to each data point will be a minimum.

Question 66 / 103

[VCAA 2020 FM (74%)]
In a study, the association between the *number of tasks* completed on a test and the *time* allowed for the test, in hours, was found to be non-linear.

The data can be linearised using a log10 transformation applied to the variable *number of tasks*. The equation of the least squares line for the transformed data is

$\log_{10}(number\ of\ tasks)$=1.160+0.03617×time

This equation predicts that the *number of tasks* completed when the *time* allowed for the test is three hours is closest to

---

A. 13

B. 16

C. 19

D. 25

E. 26

Question 67 / 103

[VCAA 2021 FM (71%)]
Oscar walked for nine consecutive *days*. The *time*, in minutes, that Oscar spent walking on each day is shown in the table below.

Day	1	2	3	4	5	6	7	8	9
Time	46	40	45	34	36	38	39	40	33

A least squares line is fitted to the data.

The equation of this line predicts that on day 10 the time Oscar spends walking will be the same as the time he spent walking on

A. day 3

B. day 4

C. day 6

D. day 8

E. day 9

Question 68 / 103

[VCAA 2021 FM (40%)]
The table below shows the *weight*, in kilograms, and the *height*, in centimetres, of 10 adults.

***Weight* (kg)**	***Height* (cm)**
59	173
67	180
69	184
84	195
64	173
74	180
76	192
56	169
58	164
66	180

A least squares line is fitted to the data. The least squares line enables an adult's *weight* to be predicted from their *height*. The number of times that the predicted value of an adult's *weight* is greater than the actual value of their *weight* is

A. 3

B. 4

C. 5

D. 6

E. 7

Question 71 / 103

[VCAA 2022 FM (56%)]
Using the equation of the least squares line, the predicted *spleen weight* of a 30-month-old seal, in grams, is closest to

---

A. 3

B. 511

C. 772

D. 957

E. 1192

Question 72 / 103

Use the following information to answer Questions 73 75.

Table 1 summarises the results of a study that compared the effectiveness of individual and group instruction *(instructional method)* when training future basketball referees.

Table 1

	Instructional method	
Test grade	**Individual**	**Group**
A (85% or above)	10	18
B (75–84%)	35	30
C (65–74%)	30	24
D (50–64%)	28	48
E (less than 50%)	12	6
Total	115	126

In this table, *test grade* is the response variable and *instructional method* is the explanatory variable.

Question 73 / 103

[VCAA 2022 FM (67%)]
The variables *test grade* (A, B, C, D, E) and *instructional method* (individual, group) are

---

A.

a numerical and a categorical variable respectively.

B.

both nominal variables.

C.

a nominal and an ordinal variable respectively.

D.

both ordinal variables.

E.

an ordinal and a nominal variable respectively.

Question 74 / 103

[VCAA 2022 FM (56%)]
Of the students who received an A grade, the percentage who were instructed individually is closest to

---

A.

9%

B.

22%

C.

36%

D.

56%

E.

64%

Question 75 / 103

[VCAA 2022 FM (86%)]
To become a qualified referee, a grade of A or B on the test is required. Those who receive a C, a D or an E will not qualify.

Using column percentages, a new two-way percentage frequency table is constructed from the data in Table 1.

In this new table, *qualified to be a referee* (yes, no) is the response variable and *instructional method* (individual, group) is the explanatory variable.

Which one of the following tables correctly displays the data from Table 1?

A.

	Instructional method (%)	
Qualified to be a referee	**Individual**	**Group**
yes (A or B grade)	35	38
no (C, D or E grade)	65	62

B.

	Instructional method (%)	
Qualified to be a referee	**Individual**	**Group**
yes (A or B grade)	39	38
no (C, D or E grade)	61	62

C.

	Instructional method (%)	
Qualified to be a referee	**Individual**	**Group**
yes (A or B grade)	39	43
no (C, D or E grade)	61	57

D.

	Instructional method (%)	
Qualified to be a referee	**Individual**	**Group**
yes (A or B grade)	61	62
no (C, D or E grade)	39	38

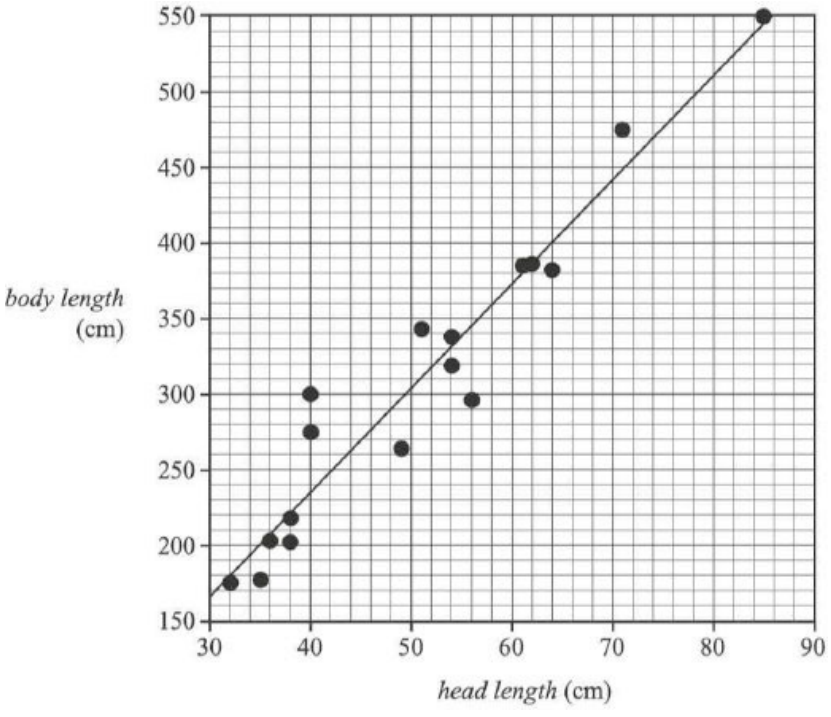
E.

	Instructional method (%)	
Qualified to be a referee	**Individual**	**Group**
yes (A or B grade)	65	57
no (C, D or E grade)	35	43

Question 76 / 103

Use the following information to answer Questions 77 79.

The scatterplot below displays the *body length*, in centimetres, of 17 crocodiles, plotted against their *head length*, in centimetres. A least squares line has been fitted to the scatterplot. The explanatory variable is *head*



Data: adapted from Data and Story Library (DASL), 'Crocodile_lengths',
<https:/dasatadescripion.com/datafile/crocodile -lengths/>

Question 77 / 103

[VCAA 2022 FM (58%)]
The equation of the least squares line is closest to

A. head length=−40+7×body length

B. body length=−40+7×head length

C. head length=168+7×bodylength

D. body length=168−40×head length

E. body length=7+168×head length

Question 78 / 103

[VCAA 2022 FM (51%)]
The median *head length* of the 17 crocodiles, in centimetres, is closest to

---

A.

49

B.

51

C.

54

D.

300

E.

345

Question 79 / 103

[VCAA 2022 FM (48%)]
The correlation coefficient r is equal to 0.963.

The percentage of variation in *body length* that is **not** explained by the variation in *head length* is closest to

---

A.

0.9%

B.

3.7%

C.

7.3%

D.

92.7%

E.

96.3%

Question 80 / 103

[VCAA 2018 FM (80%)]
The table below shows the monthly profit, in dollars, of a new coffee shop for the first nine months of 2018.

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.
Profit ($)	2890	1978	2402	2456	4651	3456	2823	2678	2345

Using four-mean smoothing with centring, the smoothed profit for May is closest to

A. $2502

B. $3294

C. $3503

D. $3804

E. $4651

Question 81 / 103

[VCAA 2018 FM (51%)]
The quarterly sales figures for a large suburban garden centre, in millions of dollars, for 2016 and 2017 are displayed in the table below.

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2016	1.73	2.87	3.34	1.23
2017	1.03	2.45	2.05	0.78

Using these sales figures, the seasonal index for Quarter 3 is closest to

---

A.

1.28

B.

1.30

C.

1.38

D.

1.46

E.

1.48

Question 82 / 103

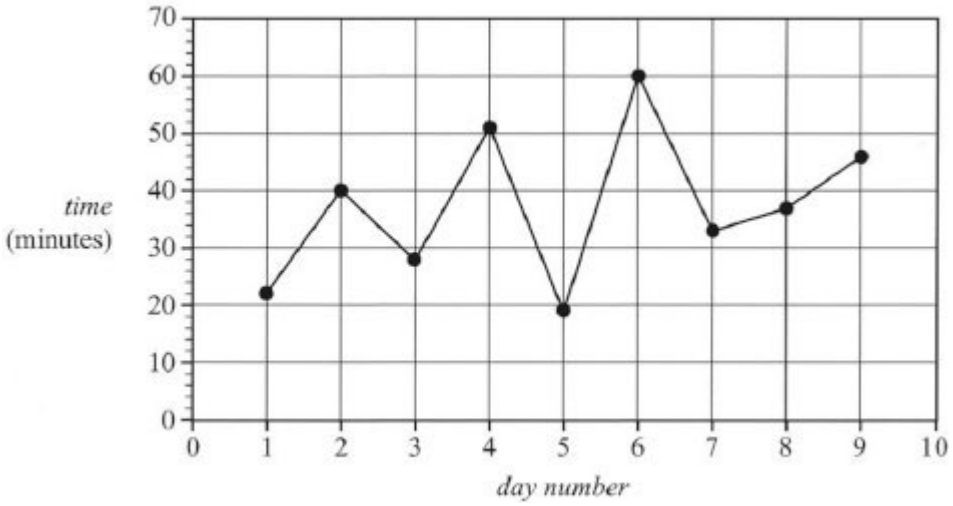
Use the following information to answer Questions 83 and 84.

The *time*, in minutes, that Liv ran each day was recorded for nine days.

These times are shown in the table below.

Day number	1	2	3	4	5	6	7	8	9
***Time* (minutes)**	22	40	28	51	19	60	33	37	46

The time series plot below was generated from this data.



Question 83 / 103

[VCAA 2019 FM (45%)]
Both three-median smoothing and five-median smoothing are being considered for this data. Both of these methods result in the same smoothed value on *day number*

A.

3

B.

4

C.

5

D.

6

E.

7

Question 84 / 103

[VCAA 2019 FM (69%)]
A least squares line is to be fitted to the time series plot shown on the previous page.

The equation of this least squares line, with *day number* as the explanatory variable, is closest to

---

A.

day number=23.8+2.29×time

B.

day number=28.5+1.77×time

C.

time23.8+1.77×day number

D.

time23.8+2.29×day number

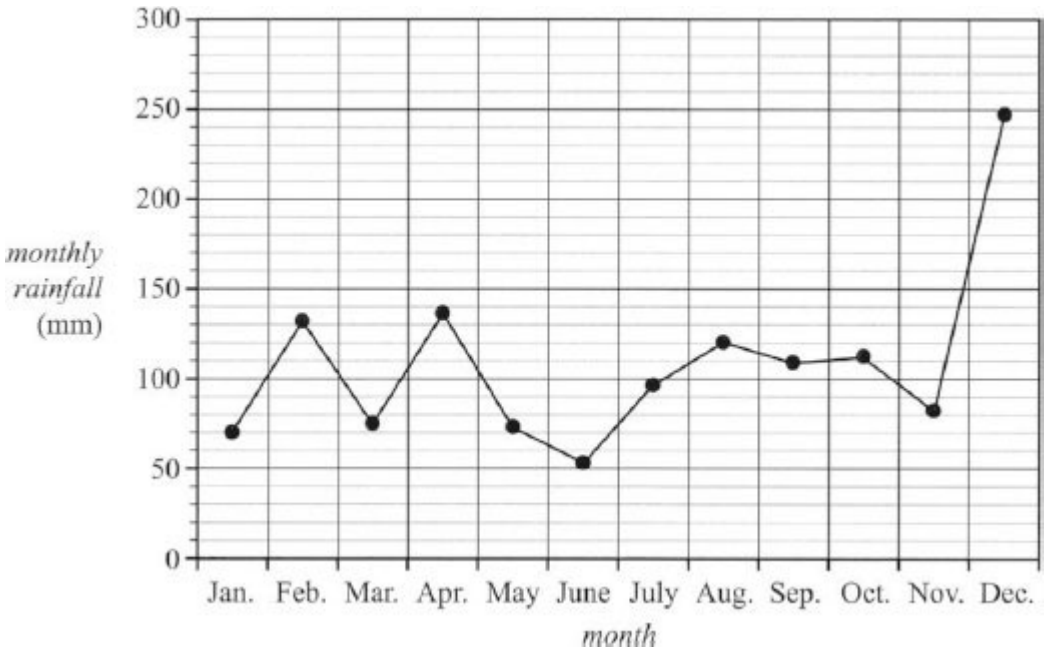
E.

time=8.+1.77×day number

Question 85 / 103

Use the following information to answer Questions 86 and 87.

The time series plot below shows the *monthly rainfall* at a weather station, in millimetres, for each *month* in 2017.



Question 86 / 103

[VCAA 2019 FM (37%)]
The median *monthly rainfall* for 2017 was closest to

A. 53 mm

B. 82 mm

C. 96 mm

D. 103 mm

E. 111 mm

Question 87 / 103

[VCAA 2019 FM (74%)]
If seven-mean smoothing is used to smooth this time series plot, the number of smoothed data points would be

---

A.

3

B.

5

C.

6

D.

8

E.

10

Question 88 / 103

Use the following information to answer Questions 89 and 90.

Table 3 below shows the long-term mean rainfall, in millimetres, recorded at a weather station, and the associated long-term seasonal indices for each month of the year.

The long-term mean rainfall for December is missing.

Table 3

	Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**July**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Mean rainfall(mm)	51.9	52.3	52.8	66.6	87.1	69.4	73.0	79.9	82.6	82.4	81.1	
Seasonal index	0.728	0.734	0.741	0.934	1.222	0.973	1.024	1.121	1.159	1.156	1.138	1.012

Question 89 / 103

[VCAA 2020 FM (43%)]
To correct the rainfall in March for seasonality, the actual rainfall should be, to the nearest per cent

---

A.

decreased by 26%

B.

increased by 26%

C.

decreased by 35%

D.

increased by 35%

E.

increased by 74%

Question 90 / 103

[VCAA 2020 FM (67%)]
The long-term mean rainfall for December is closest to

A.

64.7 mm

B.

65.1 mm

C.

71.3 mm

D.

76.4 mm

E.

82.0 mm

Question 90 / 103

[VCAA 2020 FM (67%)]
The long-term mean rainfall for December is closest to

A.

64.7 mm

B.

65.1 mm

C.

71.3 mm

D.

76.4 mm

E.

82.0 mm

Question 91 / 103

Use the following information to answer Questions 92 and 93.

Table 4 below shows the monthly rainfall for 2019, in millimetres, recorded at a weather station, and the associated long-term seasonal indices for each month of the year.

	Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**June**	**July**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
Monthly rainfall (mm)	18.4	17.6	46.8	23.6	92.6	77.2	80.0	86.8	93.8	55.2	97.3	69.4
Seasonal index	0.728	0.734	0.741	0.934	1.222	0.973	1.024	1.121	1.159	1.156	1.138	1.072

Data: adapted from © Commonwealth of Australia 2020, Bureau of Meteorology, <www.om.gov.au/ >

Question 92 / 103

[VCAA 2020 FM (88%)]
The deseasonalised rainfall for May 2019 is closest to

A.

71.3 mm

B.

75.8 mm

C.

86.1 mm

D.

88.1 mm

E.

113.0 mm

Question 93 / 103

[VCAA 2020 FM (73%)]
The six-mean smoothed monthly rainfall with centring for August 2019 is closest to

---

A.

67.8 mm

B.

75.9 mm

C.

81.3 mm

D.

83.4 mm

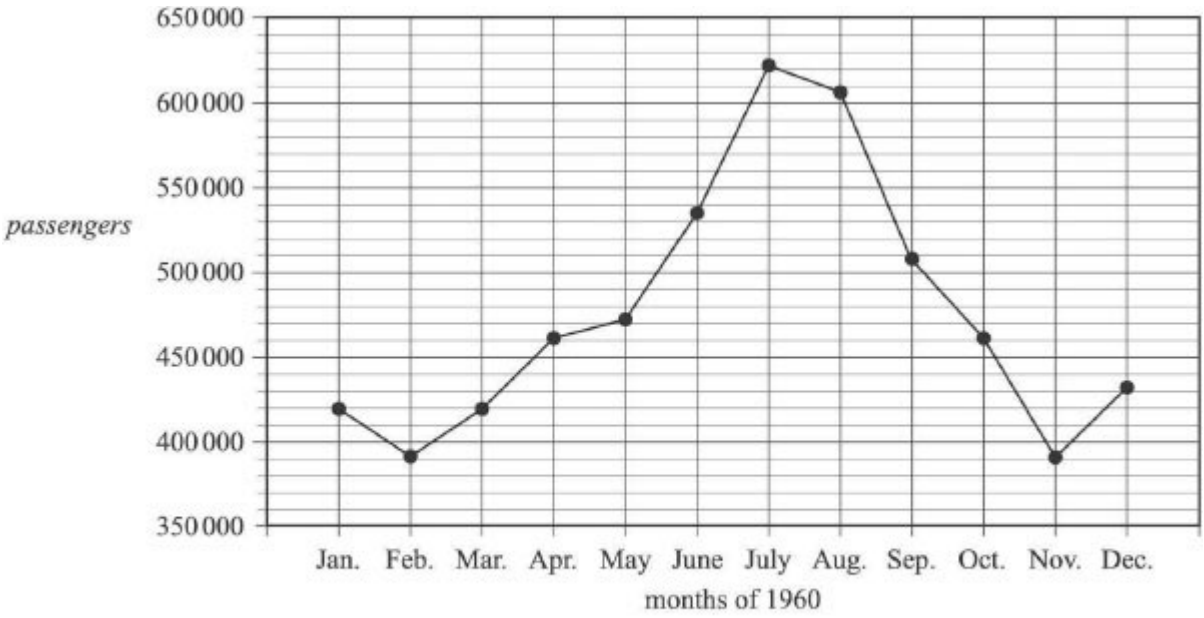
E.

86.4 mm

Question 94 / 103

Use the following information to answer Questions 95 and 96.

The time series plot below displays the number of airline passengers, in thousands, each month during the period January to December 1960.



Data: GEP Box and GM Jenkins, *Time Series Analysis: Forecasting and Control*, Holden-Day, San Francisco, 1970, p. 531

Question 95 / 103

[VCAA 2020 FM (45%)]
During 1960, the median number of monthly airline *passengers* was closest to

---

A. 461 000

B. 465 000

C. 471 000

D. 573 000

E. 621 000

Question 96 / 103

[VCAA 2020 FM (57%)]
During the period January to May 1960, the total number of airline *passengers* was 2 160 000. The five-mean smoothed number of passengers for March 1960 is

---

A.

419 000

B.

424 000

C.

430 000

D.

432 000

E.

434 000

Question 96 / 103

[VCAA 2020 FM (57%)]
During the period January to May 1960, the total number of airline *passengers* was 2 160 000. The five-mean smoothed number of passengers for March 1960 is

---

A. 419 000

B. 424 000

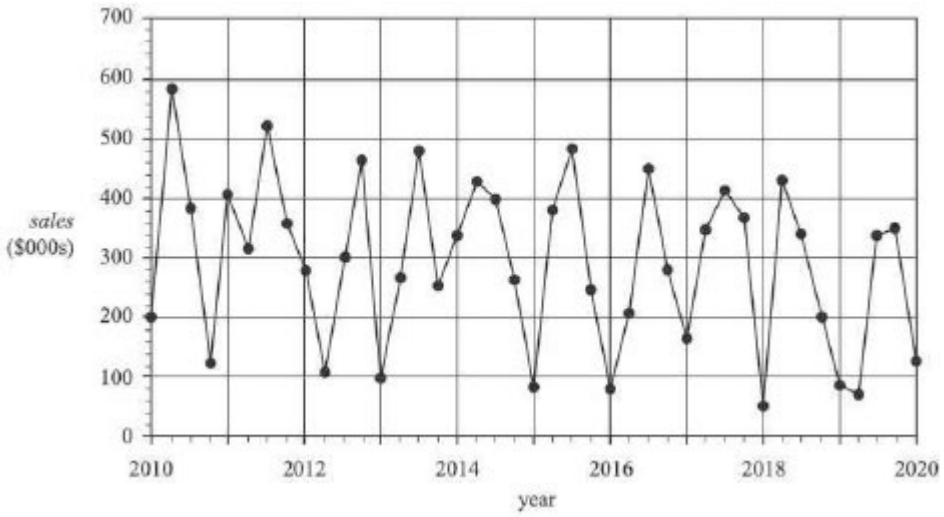
C. 430 000

D. 432 000

E. 434 000

Question 97 / 103

[VCAA 2021 FM (34%)]
Thetim series plot below shows the quarterly *sales*, in thousands of dollars, of a small business for the years 2010 to 2020.



The time series plot is best described as having

A.

seasonality only.

B.

irregular fluctuations only.

C.

seasonality with irregular fluctuations.

D.

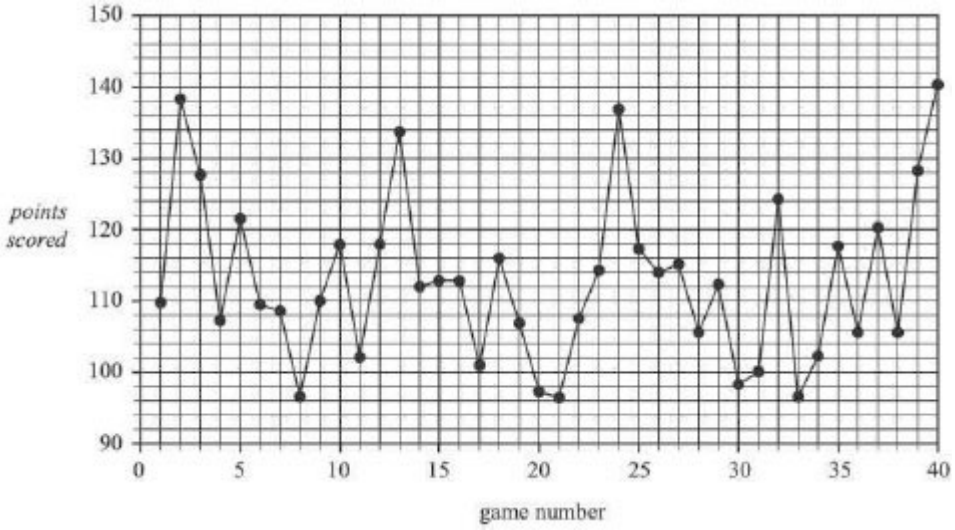
a decreasing trend with irregular fluctuations.

E.

a decreasing trend with seasonality and irregular fluctuations.

Question 98 / 103

[VCAA 2021 FM (53%)]
The time series plot below shows the *points scored* by a basketball team over 40 games.



The nine-median smoothed *points scored* for game number 10 is closest to

---

A.

102

B.

108

C.

110

D.

112

E.

117

Question 99 / 103

[VCAA 2021 FM (48%)]
A garden centre sells garden soil.

The table below shos the daily *quantity* of garden soil sold, in cubic metres, over a one-week period.

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Quantity **(m^3)**	234	186				346	346

The *quantity* of garden soil sold on Wednesday, Thursday and Friday is not shown.

The five-mean smoothed *quantity* of garden soil sold on Thursday is 206 m^3.

The three-mean smoothed *quantity* of garden soil sold on Thursday, in cubic metres, is

A. 143

B. 166

C. 206

D. 239

E. 403

Question 100 / 103

[VCAA 2021 FM (59%)]
The table below shows the number of visitors to an art gallery during the summer, autumn, winter and spring quarters for the years 2017 to 2019.

The quarterly average is also shown for each of these years.

Season	**2017**	**2018**	**2019**
summer quarter	29 685	25 420	31 496
autumn quarter	27 462	23 320	29 874
winter quarter	25 564	21 097	27 453
spring quarter	26 065	22 897	28 149
Quarterly average	27 194.0	23 183.5	29 243.0

The seasonal index for summer is closest to

A. 1.077

B. 1.081

C. 1.088

D. 1.092

E. 1.096

Question 101 / 103

[VCAA 2021 FM (52%)]
The number of visitors to a regional animal park is seasonal.

Data is collected and deseasonalised before a least squares line is fitted.

The equation of the least squares line is

deseasonalised number of visitors =2349−198.5×month number

where *month number* 1 is January 2020.

The seasonal indices for the 12 months of 2020 are shown in the table below.

Month number	1	2	3	4	5	6	7	8	9	10	11	12
Seasonal index	1.10	1.25	1.15	0.95	0.85	0.75	0.80	0.85	0.95	1.10	1.15	1.10

The actual number of visitors predicted for February 2020 was closest to

---

A. 1562

B. 1697

C. 1952

D. 2245

E. 2440

Question 102 / 103

[VCAA 2022 FM (66%)]
The daily number of cups of coffee sold by a food truck over a three-week period is shown in the table below.

Week	Mon	Tue	Wed	Thu	Fri	Sat	Sun
1	123	91	120	176	145	99	187
2	98	104	145	163	134	128	206
3	125	134	128	187	156	102	179

The six-mean smoothed number of cups of coffee, with centring, sold on Thursday in Week 2 is closest to

A. 127

B. 138

C. 147

D. 155

E. 163

Question 103 / 103

[VCAA 2022 FM (40%)]
The seasonal index for sales of sunscreen in summer is 1.25

To correct for seasonality, the actual sunscreen sales for summer should be

---

A. reduced by 20%

B. reduced by 25%

C. reduced by 80%

D. increased by 20%

E. increased by 25%

Question 1 / 25

[VCAA 2018 FM]

Table 1

City	***Congestion level***	***Size***	***Increase in travel time*** **(minutes per day)**
Belfast	high	small	52
Edinburgh	high	small	43
London	high	large	40
Manchester	high	large	44
Brighton and Hove	high	small	35
Bournemouth	high	small	36
Sheffield	medium	small	36
Hull	medium	small	40
Bristol	medium	small	39
Newcastle-Sunderland	medium	large	34
Leicester	medium	small	36
Liverpool	medium	large	29
Swansea	low	small	30
Glasgow	low	large	34
Cardiff	low	small	31
Nottingham	low	small	31
Birmingham-Wolverhampton	low	large	29

City	***Congestion level***	***Size***	***Increase in travel time* (minutes per day)**
Leeds-Bradford	low	large	31
Portsmouth	low	small	27
Southampton	low	small	30
Reading	low	small	31
Coventry	low	small	30
Stoke-on-Trent	low	small	29

Data: TomTom International BV, <www.tmtom.coen_gb/trafficindex >

The data in Table 1 relates to the impact of traffic congestion in 2016 on travel times in 23 cities in the United Kingdom (UK).

The four variables in this data set are:

- *city* – name o city
- *congestion level* – traffic congestion level (high, medium, low)
- *size* – size of city (large, small)
- *increase in travel time* – increase in travel time due to traffic congestion (minutes per day).

a. How many variables in this data set are categorical variables?

[1 mark (0.7)]

b. How many variables in this data set are ordinal variables?

[1 mark (0.7)]

c. Name the large UK cities with a medium level of traffic congestion.

[1 mark (0.8)]

d. Use the data in Table 1 to complete the following two-way frequency table, Table 2.

[2 marks (1.9)]

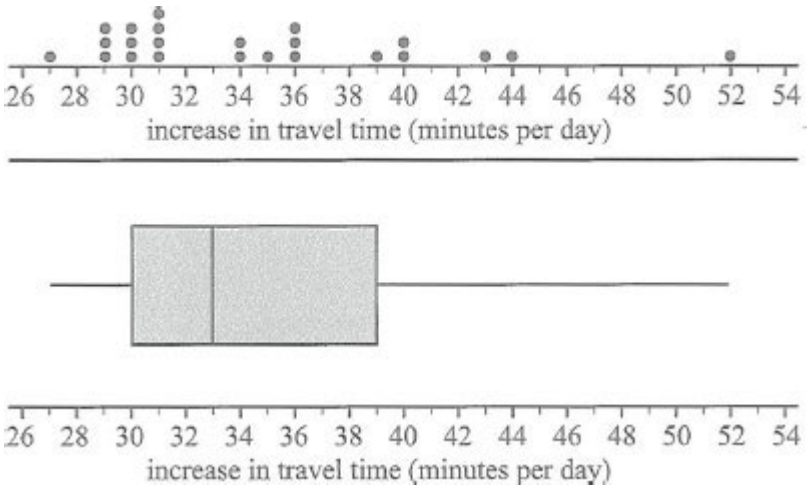
Table 2

	City size	
Congestion level	**Small**	**Large**
high	4	
medium		
low		
Total	16	

e. What percentage of the small cities have a high level of traffic congestion?

[1 mark (0.8)]

Traffic congestion can lead to an increase in travel times in cities. The dot plot and boxplot below both show the increase in travel time due to traffic congestion, in minutes per day, for the 23 UK cities.



f. Describe the shape of the distribution of the increase in travel time for the 23 cities.

[1 mark (0.9)]

g. The data value 52 is below the upper fence and is not an outlier.

Determine the value of the upper fence.

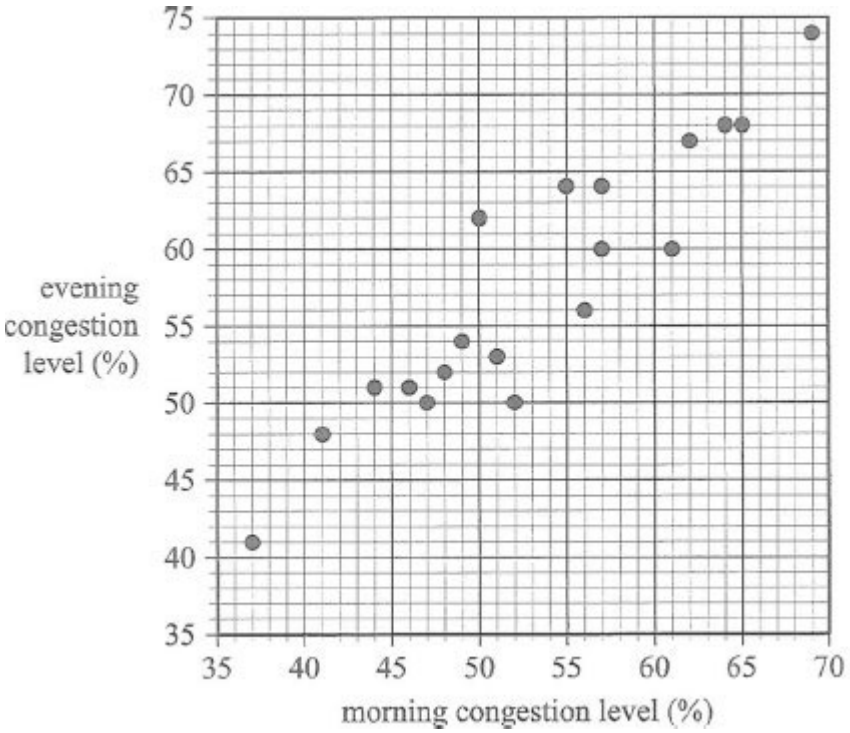
[1 mark (0.8)]

Question 2 / 25

[VCAA 2018 FM]
The congestion level in a city can also be recorded as the percentage increase in travel time due to traffic congestion in peak periods (compared to non-peak periods).

This is called the percentage congestion level.

The percentage congestion levels for the morning and evening peak periods for 19 large cities are plotted on the scatterplot below.



a. Determine the median percentage congestion level for the morning peak period and the evening peak period.

Write your answers in the appropriate boxes provided below.

Median percentage congestion level for morning peak period ____ %

Median percentage congestion level for evening peak period ____ %

[2 marks (1.1)]

A least squares line is to be fitted to the data with the aim of predicting evening congestion level from morning congestion level.

The equation of this line is

evening congestion level=8.48+0.922×morning congestion level.

b. Name the response variable in this equation.

[1 mark (0.9)]

c. Use the equation of the least squares line to predict the evening congestion level when the morning congestion level is 60%.

[1 mark (0.8)]

d. Determine the residual value when the equation of the least squares line is used to predict the evening congestion level when the morning congestion level is 47%.

Round your answer to one decimal place.

[2 marks (1.1)]

e. The value of the correlation coefficient r is 0.92

What percentage of the variation in the evening congestion level can be explained by the variation in the morning congestion level?

Round your answer to the nearest whole number.

[1 mark (0.6)]

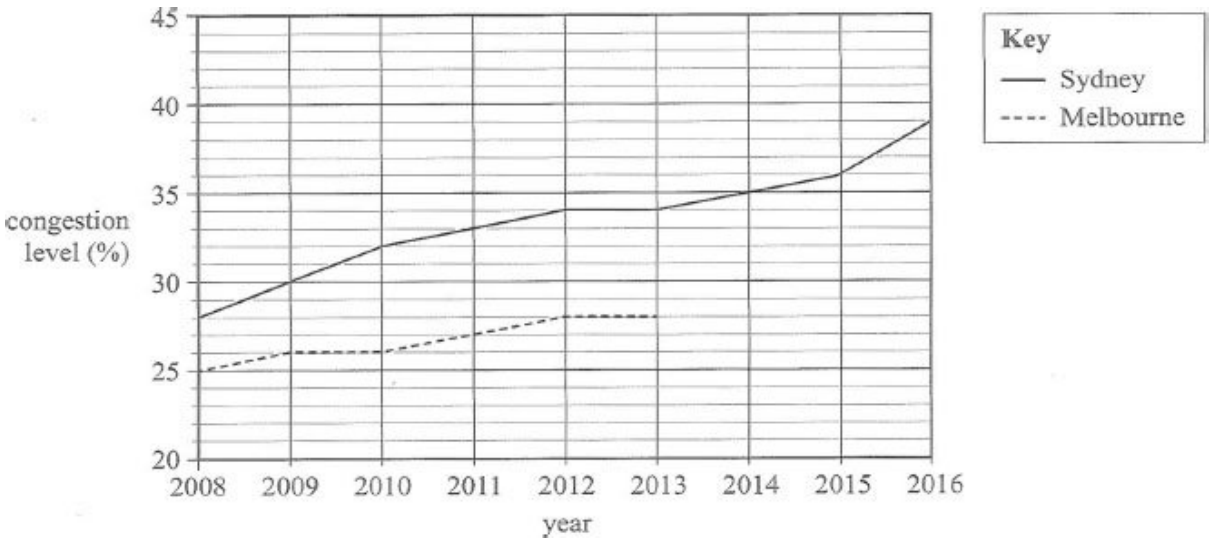
Question 3 / 25

[VCAA 2018 FM]
Table 3 shows the yearly average traffic congestion levels in two cities, Melbourne and Sydney, during the period 2008 to 2016. Also shown is a time series plot of the same data.

The time series plot for Melbourne is incomplete.

Table 3

	Congestion level (%)								
Year	2008	2009	2010	2011	2012	2013	2014	2015	2016
Melbourne	25	26	26	27	28	28	28	29	33
Sydney	28	30	32	33	34	34	35	36	39



Data: TomTom International BV, <www.tmtom.com/en_gb/traflicindex >

a. Use the data in Table 3 to complete the **time series plot above** for Melbourne.

[1 mark (0.9)]

b. A least squares line is used to model the trend in the time series plot for Sydney. The equation is

congestion level=−2280+1.15×year

i. Draw this least squares line on the **time series plot above.**

[1 mark (0.5)]

ii. Use the equation of the least squares line to determine the average rate of increase in percentage congestion level for the period 2008 to 2016 in Sydney.

Write your answer in the box provided below.

	% per year

[1 mark (0.4)]

iii. Use the least squares line to predict when the percentage congestion level in Sydney will be 43%.

[1 mark (0.8)]

The yearly average traffic congestion level data for Melbourne is repeated in Table 4 below.

Table 4

	Congestion level (%)								
Year	2008	2009	2010	2011	2012	2013	2014	2015	2016
Melbourne	25	26	26	27	28	28	28	29	33

c. When a least squares line is used to model the trend in the data for Melbourne, the ntercept o this line is approximately -1514.75556

Round this value to four significant figures.

[1 mark (0.7)]

d. Use the data in Table 4 to determine the equation of the least squares line that can be used to model the trend in the data for Melbourne. The variable *year* is the explanatory variable.

Write the values of the intercept and the slope of this least squares line in the appropriate boxes provided below. Round both values to four significant figures.

$$congestion\ level = \square + \square \times year$$

[2 marks (1.4)]

e. Since 2008, the equations of the least squares lines for Sydney and Melbourne have predicted that future traffic congestion levels in Sydney will always exceed future traffic congestion levels in Melbourne.

Explain why, quoting the values of appropriate statistics.

[2 marks (0.4)]

Question 4 / 25

[VCAA 2019 FM]

Table 5

Table 5 shows the *day number* and the *minimum temperature*, in degrees Celsius, for 15 consecutive days in May 2017.

Table 5

Day number	***Minimum temperature* (°C)**
1	12.7
2	11.8
3	10.7
4	9.0
5	6.0
6	7.0
7	4.1
8	4.8
9	9.2
10	6.7
11	7.5
12	8.0
13	8.6
14	9.8
15	7.7

Data: Australian Government, Bureau of Meteorology, <www.om.gov.au >

a. Which of the two variables in this data set is an ordinal variable?

[1 mark (0.9)]

The incomplete ordered stem plot below has been constructed using the data values for days 1 to 10.

key: 4|1 = 4.1 $n = 15$

minimum temperature (°C)

4	1 8
5	
6	0 7
7	0
8	
9	0 2
10	7
11	8
12	7

b. Complete the **stem plot above** by adding the data values for days 11 to 15.

[1 mark (0.9)]

c. The ordered stem plot below shows the maximum temperature, in degrees Celsius, for the same 15 days.

key: $9|2 = 9.2$ $\quad n = 15$

maximum temperature (°C)

9	2
10	
11	5 6
12	2 5
13	5 5 7
14	9 9
15	0 2 5 6
16	0

Data: Australian Government, Bureau of Meteorology, <www.om.gov.au >

Use this stem plot to determine

i. the value of the first quartile (Q1)

[1 mark (0.8)]

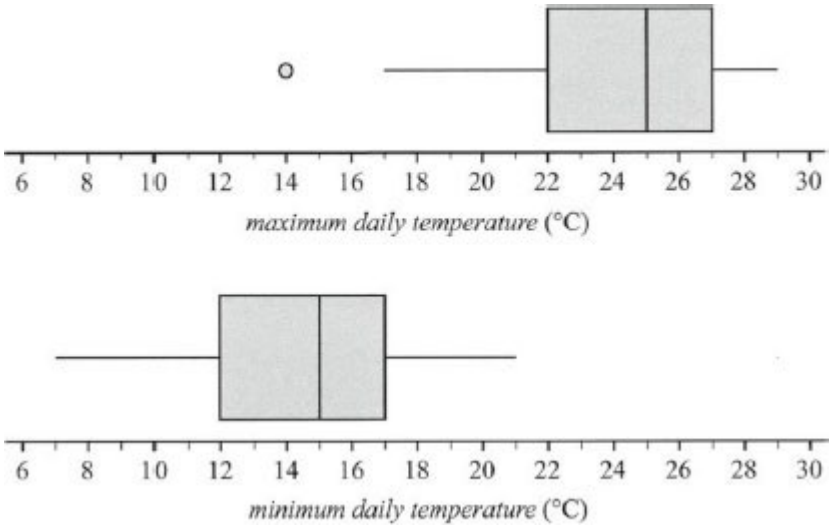
ii. the percentage of days with a maximum temperature higher than 15.3°C.

[1 mark (0.9)]

Question 5 / 25

[VCAA 2019 FM]

The parallel boxplots below show the *maximum daily temperature* and *minimum daily temperature*, in degrees Celsius, for 30 days in November 2017.



Data: Australian Government, Bureau of Meteorology, <www.om.gov.au >

a. Use the information in the boxplots to complete the following sentences.

For November 2017

i. the interquartile range for the *minimum daily temperature* was ____ °C [1 mark (0.9)]

ii. the median value for *maximum daily temperature* was ____ °C higher than the median value for *minimum daily temperature*. [1 mark (0.9)]

iii. the number of days on which the *maximum daily temperature* was less than the median value for *minimum daily temperature* was ____ [1 mark (0.8)]

b. The *temperature difference* between the *minimum daily temperature* and the *maximum daily temperature* in November 2017 at this location is approximately normally distributed with a mean of 9.4°C and a standard deviation of 3.2°C.

Determine the number of days in November 2017 for which this *temperature difference* is expected to be greater than 9.4°C.

[1 mark (0.6)]

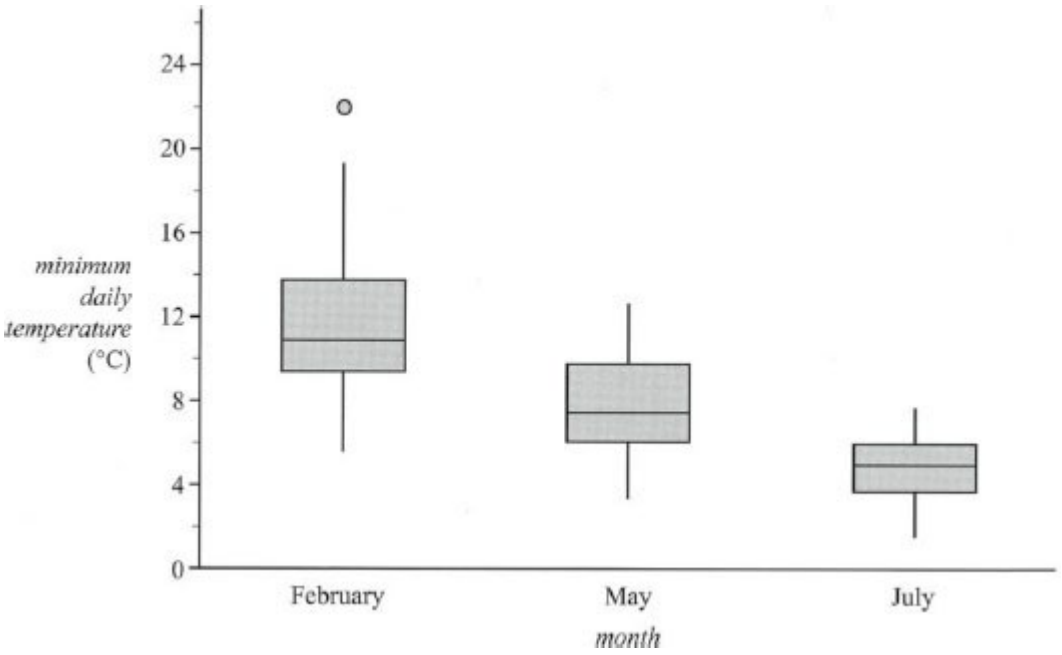
Question 6 / 25

[VCAA 2019 FM]
The five-number summary for the distribution of *minimum daily temperature* for the months of February, May and July in 2017 is shown in Table 6.

The associated boxplots are shown below the table.

Table 6. Five-number summary for *minimum daily temperature*

Month	Minimum	Q1	Median	Q3	Maximum
February	5.9	9.5	10.9	13.9	22.2
May	3.3	6.0	7.5	9.8	12.7
July	1.6	3.7	5.0	5.9	7.7



Data: Australian Government, Bureau of Meteorology, <www.om.gov.au >

Explain why the information given above supports the contention that *minimum daily temperature* is associated with the *month*. Refer to the values of an appropriate statistic in your response.

[2 marks (1.0)]

Question 7 / 25

[VCAA 2019 FM]
The relative humidity (%) at 9 am and 3 pm on 14 days in November 2017 is shown in Table 7 below.

Table 7

Relative humidity (%)	
9 am	**3 pm**
100	87
99	75
95	67
63	57
81	57
94	74
96	71
81	62
73	53
53	54
57	36
77	39
51	30
41	32

Data: Australian Government, Bureau of Meteorology, <www.om.gov.au >

A least squares line is to be fitted to the data with the aim of predicting the relative humidity at 3 pm *(humidity 3 pm)* from the relative humidity at 9 am *(humidity 9 am)*.

a. Name the explanatory variable.

[1 mark (0.9)]

b. Determine the values of the intercept and the slope of this least squares line.

Round both values to three significant figures and write them in the appropriate boxes provided.

humidity 3 pm = ☐ + ☐ × *humidity 9 am*

[1 mark (0.6)]

c. Determine the value of the correlation coefficient for this data set.

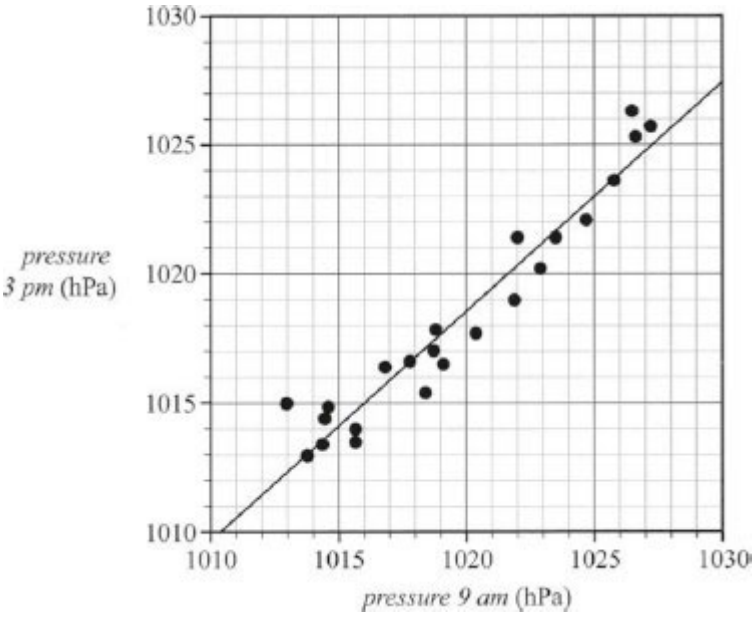
Round your answer to three decimal places.

[1 mark (0.8)]

Question 8 / 25

[VCAA 2019 FM]

The scatterplot below shows the atmospheric pressure, in hectopascals (hPa), at 3 pm *(pressure 3 pm)* plotted against the atmospheric pressure, in hectopascals, at 9 am *(pressure 9 am)* for 23 days in November 2017 at a particular weather station.



Data: Australian Government, Bureau of Meteorology. <www.om.gov.au >

A least squares line has been fitted to the scatterplot as shown.

The equation of this line is

pressure 3 pm=111.4+0.8894×pressure 9 am

a. Interpret the slope of this least squares line in terms of the atmospheric pressure at this weather station at 9 am and at 3 pm.

[1 mark (0.5)]

b. Use the equation of the least squares line to predict the atmospheric pressure at 3 pm when the atmospheric pressure at 9 am is 1025 hPa.

Round your answer to the nearest whole number.

[1 mark (0.9)]

c. Is the prediction made in **part b.** an example of extrapolation or interpolation?

[1 mark (0.9)]

d. Determine the residual when the atmospheric pressure at 9 am is 1013 hPa.

Round your answer to the nearest whole number.

[1 mark (0.4)]

e. The mean and the standard deviation of *pressure 9 am* and *pressure 3 pm* for these 23 days are shown in Table 8 below.

Table 8

	Pressure 9 am	***Pressure 3 pm***
Mean	1019.7	1018.3
Standard deviation	4.5477	4.1884

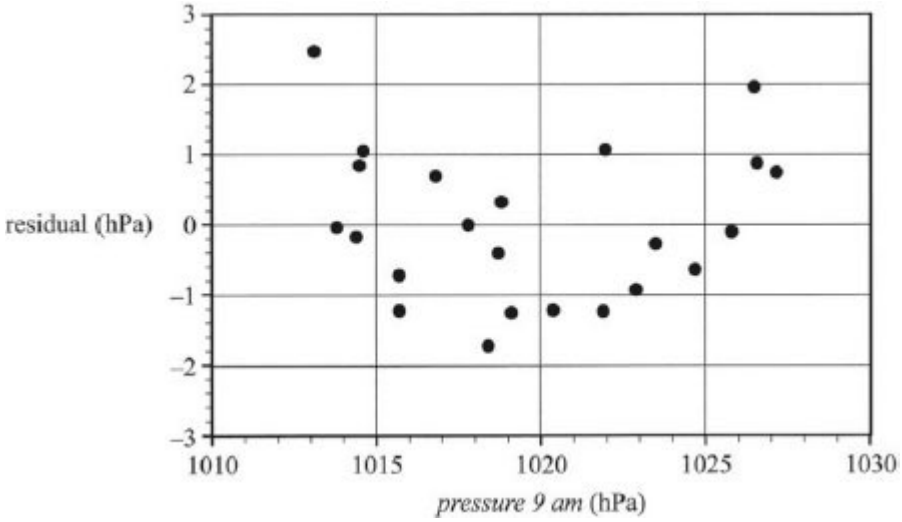
i. Use the equation of the least squares line and the information in Table 8 to show that the correlation coefficient for this data, rounded to three decimal places, is r=0.966

[1 mark (0.4)]

ii. What percentage of the variation in *pressure 3 pm* is explained by the variation in *pressure 9 am*? Round your answer to one decimal place.

[1 mark (0.5)]

f. The residual plot associated with the least squares line is shown below.



i. The residual plot above can be used to test one of the assumptions about the nature of the association between the atmospheric pressure at 3 pm and the atmospheric pressure at 9 am. What is this assumption?

[1 mark (0.4)]

ii. The residual plot above does not support this assumption. Explain why.

[1 mark (0.4)]

Question 9 / 25

[VCAA 2019 FM]
The total rainfall, in millimetres, for each of the four seasons in 2015 and 2016 is shown in Table 9 below.

Table 9

	Total rainfall (mm)			
Year	**Summer**	**Autumn**	**Winter**	**Spring**
2015	142	156	222	120
2016	135	153	216	96

a. The seasonal index for winter is shown in Table 10 below.

Use the values in Table 9 to find the seasonal indices for summer, autumn and spring.

Write your answers in Table 10, rounded to two decimal places.

Table 10

	Summer	**Autumn**	**Winter**	**Spring**
Seasonal index			1.41	

[2 marks (0.9)]

b. The total rainfall for each of the four seasons in 2017 is shown in Table 11 below. **Table 11**

Table 11

	Total rainfall (mm)			
Year	**Summer**	**Autumn**	**Winter**	**Spring**
2017	141	156	262	120

Use the appropriate seasonal index from Table 10 to deseasonalise the total rainfall for winter in 2017. Round your answer to the nearest whole number.

[1 mark (0.6)]

Question 10 / 25

[VCAA 2020 FM]
Body mass index (BMI), in kilograms per square metre, was recorded for a sample of 32 men and displayed in the ordered stem plot below.

key: 21 | 6 = 21.6 $n = 32$

21	6	9	9				
22	1	2	5	6			
23	0	1	4	6	6	7	8
24	4	5	6	7	7	9	
25	6	8					
26	1	7	9				
27	3	7					
28	2						
29	1	8					
30	4						
31	1						

a. Describe the shape of the distribution.

[1 mark (0.9)]

b. Determine the median BMI for this group of men.

[1 mark (0.7)]

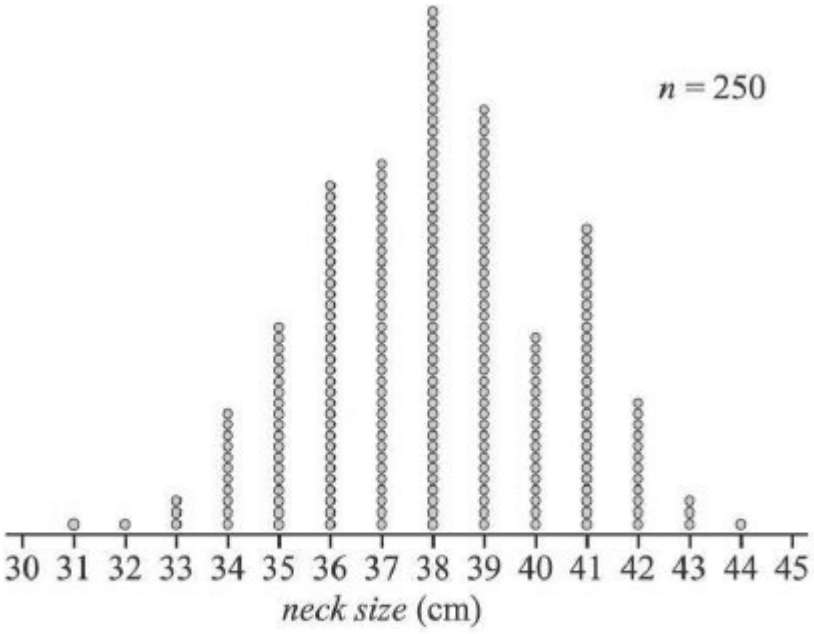
c. People with a BMI of 25 or over are considered to be overweight.

What percentage of these men would be considered to be overweight?

[1 mark (0.8)]

Question 11 / 25

[VCAA 2020 FM]
The *neck size*, in centimetres, of 250 men was recorded and displayed in the dot plot below.



Data: RW Johnson, 'Fitting percentage of body fat to simple body measurements', *Journal of Statistics Education*, 4:1, 1996, <https://doi.org/10.1080/10691898.1996.11910505>

a. Write down the modal *neck size*, in centimetres, for these 250 men.

[1 mark (0.9)]

b. Assume that this sample of 250 men has been drawn at random from a population of men whose *neck size* is normally distributed with a mean of 38 cm and a standard deviation of 2.3 cm.

i. How many of these 250 men are **expected** to have a *neck size* that is more than three standard deviations above or below the mean?

Round your answer to the nearest whole number.

[1 mark (0.4)]

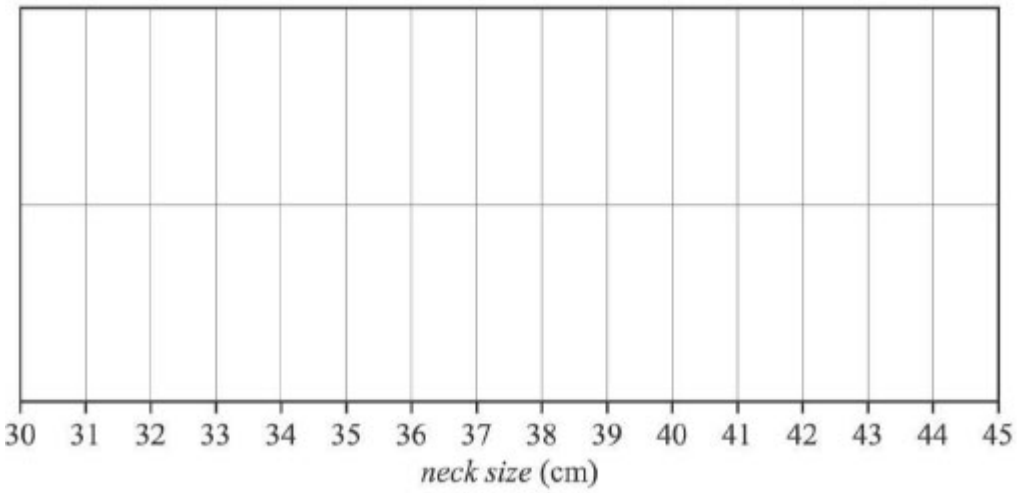
ii. How many of these 250 men **actually** have a *neck size* that is more than three standard deviations above or below the mean?

[1 mark (0.4)]

c. The five-number summary for this sample of neck sizes, in centimetres, is given below.

Minimum	First quartile (Q1)	Median	Third quartile (Q3)	Maximum
31	36	38	39	44

Use the five-number summary to construct a boxplot, showing any outliers if appropriate, on the grid below.



[2 marks (1.1)]

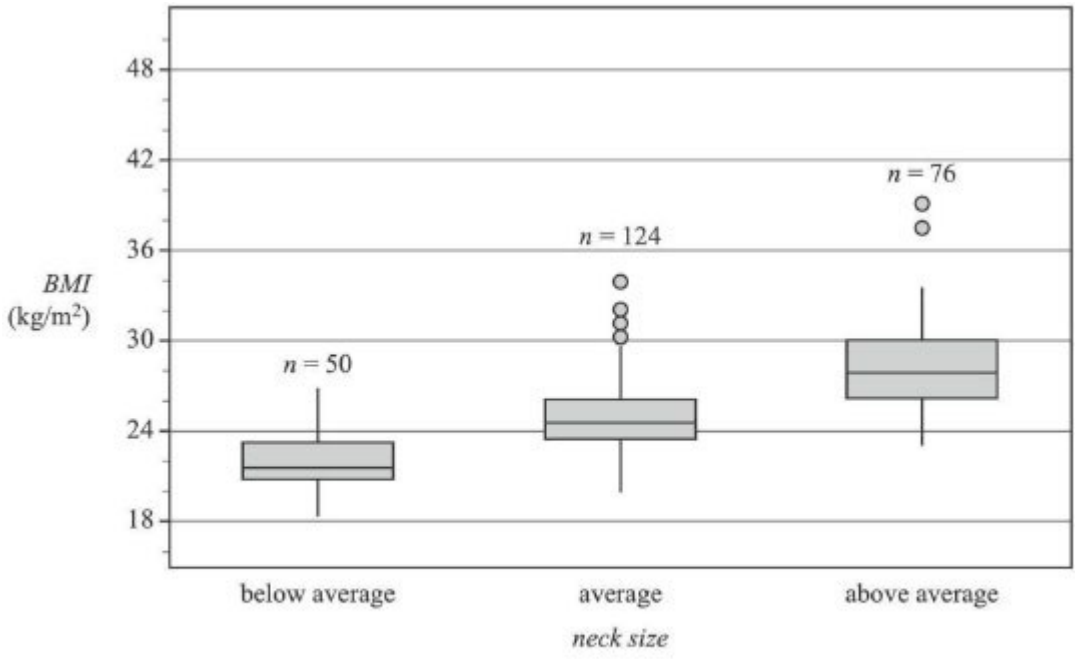
Question 12 / 25

[VCAA 2020 FM]
In a study of the association between BMI and *neck size*, 250 men were grouped by *neck size* (below average, average and above average) and their BMI recorded.

Five-number summaries describing the distribution of BMI for each group are displayed in the table below along with the group size.

The associated boxplots are shown below the table.

		BMI (kg/m²)				
Neck size	**Group size**	**Min.**	**Q1**	**Median**	**Q3**	**Max.**
below average	50	18.l	20.6	21.6	23.2	26.8
average	124	19.8	23.4	24.6	26.0	33.9
above average	76	23.l	26.25	28.1	29.95	39.1



Data: RW Johnson, 'Fitting percentage of body fat to simple body measurements', *Journal of Statistics Education*, 4:1, 1996, <https://doi.org/10.1080/10691898.1996.11910505>

a. What percentage of these 250 men are classified as having a below average neck size?

[1 mark (0.9)]

b. What is the interquartile range (IQR) of BMI for the men with an average neck size?

[1 mark (0.9)]

c. People with a BMI of 30 or more are classified as being obese.

Using this criterion, how many of these 250 men would be classified as obese? Assume that the BMI values were all rounded to one decimal place.

[1 mark (0.2)]

d. Do the boxplots support the contention that BMI is associated with *neck size*? Refer to the values of an appropriate statistic in your response.

[2 marks (1.0)]

Question 13 / 25

[VCAA 2020 FM]
The *age*, in years, *body density*, in kilograms per litre, and *weight*, in kilograms, of a sample of 12 men aged 23 to 25 years are shown in the table below.

Age **(years)**	***Body density*** **(kg/litre)**	***Weight*** **(kg)**
23	1.07	70.1
23	1.07	90.4
23	1.08	73.2
23	1.08	85.0
24	1.03	84.3
24	1.05	95.6
24	1.07	71.7
24	1.06	95.0
25	1.07	80.2
25	1.09	87.4
25	1.02	94.9
25	l.09	65.3

a. For these 12 men, determine

i. their median age, in years

[1 mark (1.0)]

ii. the mean of their *body density*, in kilograms per litre.

[1 mark (0.8)]

b. A least squares line is to be fitted to the data with the aim of predicting *body density* from *weight*.

i. Name the explanatory variable for this least squares line.

[1 mark (0.8)]

ii. Determine the slope of this least squares line.

Round your answer to three significant figures.

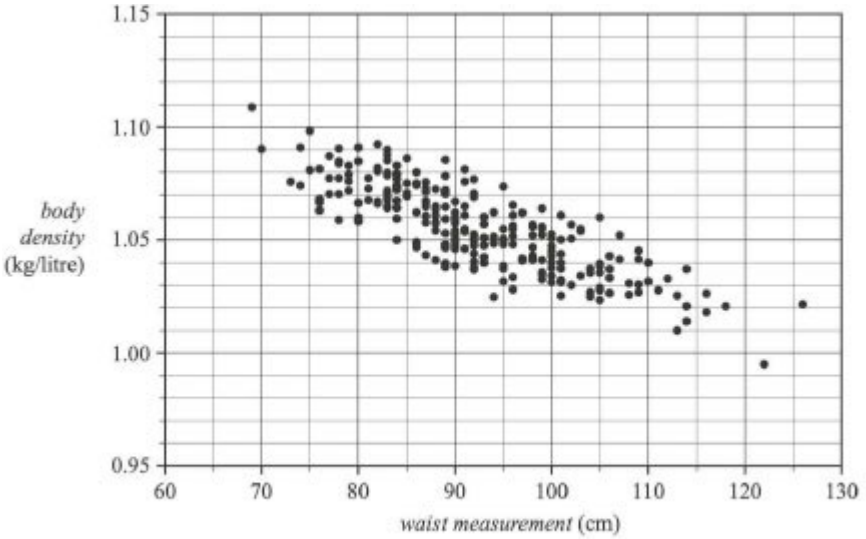
[1 mark (0.3)]

c. What percentage of the variation in *body density* can be explained by the variation in *weight*? Round your answer to the nearest percentage.

[1 mark (0.5)]

Question 14 / 25

[VCAA 2020 FM]
The scatterplot below shows *body density*, in kilograms per litre, plotted against *waist measurement*, in centimetres, for 250 men.



Data: RW Johnson, 'Fitting percentage of body fat to simple body measurements', *Journal of Statistics Education*, 4:1, 1996, <https://doi.org/10.1080/10691898.1996.11910505>

When a least squares line is fitted to the scatterplot, the equation of this line is

body density=1.195−0.001512×waist measurement

a. Draw the graph of this least squares line on the **scatterplot above.**

[1 mark (0.4)]

b. Use the equation of this least squares line to predict the *body density* of a man whose *waist measurement* is 65 cm. Round your answer to two decimal places.

[1 mark (0.7)]

c. When using the equation of this least squares line to make the prediction in **part b.**, are you extrapolating or interpolating?

[1 mark (0.4)]

d. Interpret the slope of this least squares line in terms of a man's *body density* and *waist measurement*.

[1 mark (0.4)]

e. In this study, the body density of the man with a waist measurement of 122 cm was 0.995 kg/litre.

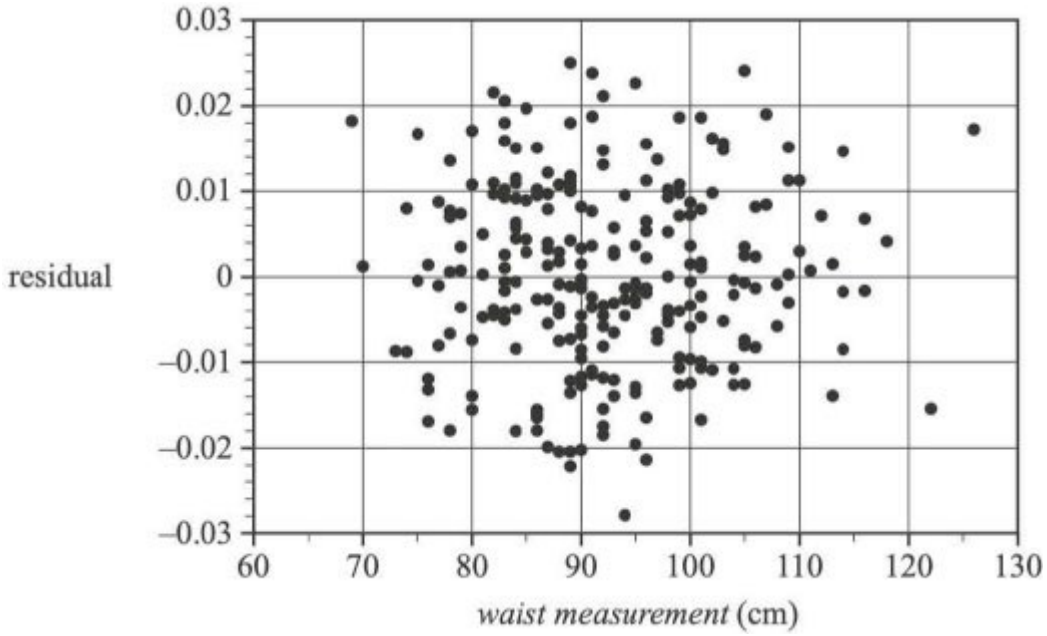
Show that, when this least squares line is fitted to the scatterplot, the residual, rounded to two decimal places, is −0.02.

[1 mark (0.5)]

f. The coefficient of determination for this data is 0.6783 Write down the value of the correlation coefficient r. Round your answer to three decimal places.

[1 mark (0.3)]

g. The residual plot associated with fitting a least squares line to this data is shown below.



Does this residual plot support the assumption of linearity that was made when fitting this line to this data? Briefly explain your answer.

[1 mark (0.6)]

Question 15 / 25

[VCAA 2020 FM]
The table below shows the *mean age*, in years, and the *mean height*, in centimetres, of 648 women from seven different age groups.

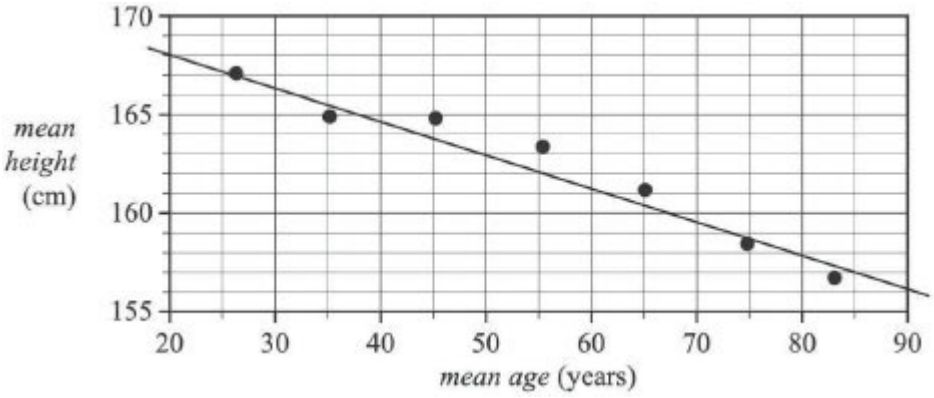
	Age group						
	Twenties	**Thirties**	**Forties**	**Fifties**	**Sixties**	**Seventies**	**Eighties**
***Mean age* (years)**	26.3	35.2	45.2	55.3	65.1	74.8	83.1
***Mean height* (cm)**	167.1	164.9	164.8	163.4	161.2	158.4	156.7

Data: J Sorkin et al., 'Longitudinal change in height of men and women: Implications for interpretation of the body mass index', *American Journal of Epidemiology*, vol. 150, no. 9, 1999, p. 971

a. What was the difference, in centimetres, between the *mean height* of the women in their twenties and the *mean height* of the women in their eighties?

[1 mark (0.9)]

A scatterplot displaying this data shows an association between the *mean height* and the *mean age* of these women. In an initial analysis of the data, a line is fitted to the data by eye, as shown.



b. Describe this association in terms of strength and direction.

[1 mark (0.5)]

c. The line on the scatterplot passes through the points (20, 168) and (85, 157).

Using these two points, determine the equation of this line.

Write the values of the intercept and the slope in the appropriate boxes below.

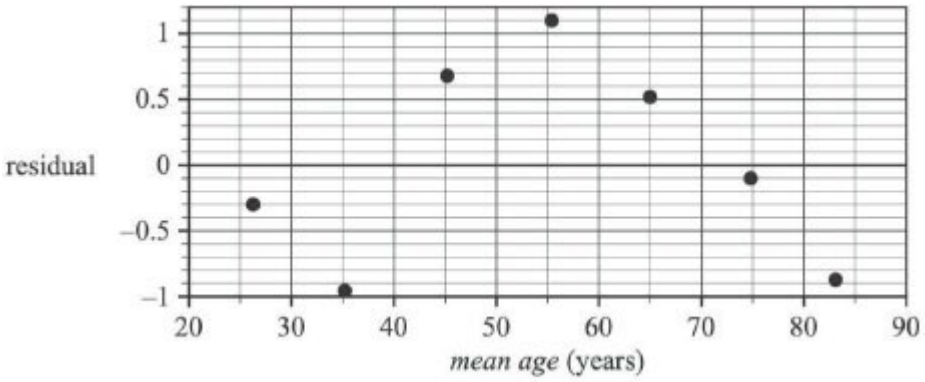
Round your answers to three significant figures.

mean height = ☐ + ☐ × *mean age*

[1 mark (0.2)]

d. In a further analysis of the data, a least squares line was fitted.

The associated residual plot that was generated is shown below.



The residual plot indicates that the association between the *mean height* and the *mean age* of women is non-linear.

The data presented in the table at the start of this question is repeated below.

It can be linearised by applying an appropriate transformation to the variable *mean age*.

***Mean age* (years)**	26.3	35.2	45.2	55.3	65.1	74.8	83.1
***Mean height* (cm)**	167.1	164.9	164.8	163.4	161.2	158.4	156.7

Apply an appropriate transformation to the variable *mean age* to linearise the data.

Fit a least squares line to the transformed data and write its equation below.

Round the values of the intercept and the slope to four significant figures.

[2 marks (0.6)]

Question 16 / 25

[VCAA 2021 FM]
In the sport of heptathlon, athletes compete in seven events. These events are the 100 m hurdles, high jump, shot-put, javelin, 200 m run, 800 m run and long jump.

Fifteen female athletes competed to qualify for the heptathlon at the Olympic Games.

Their results for three of the heptathlon events – high jump, shot-put and javelin – are shown in Table 12.

Table 12

Athlete number	High jump (metres)	Shot-put (metres)	Javelin (metres)
1	1.76	15.34	41.22
2	1.79	16.96	42.41
3	1.83	13.87	46.53
4	1.82	14.23	40.62
5	1.87	13.78	45.64
6	1.73	14.50	42.33
7	1.68	15.08	40.88
8	1.82	13.13	39.22
9	1.83	14.22	42.51
10	1.87	13.62	42.75
11	1.87	12.01	38.12
12	1.80	12.88	42.65
13	1.83	12.68	45.68
14	1.87	12.45	41.32

Athlete number	High jump (metres)	Shot-put (metres)	Javelin (metres)
15	1.78	11.31	42.88

a. Write down the number of numerical variables in Table 12.

[1 mark (0.5)]

b. Complete Table 13 below by calculating the mean height jumped for the high jump, in metres, by the 15 athletes. Write your answer in the space provided in the table.

[1 mark (0.9)]

Table 13

Statistic	High jump (metres)	Shot-put (metres)
mean		13.74
standard deviation	0.06	1.43

c. In shot-put, athletes throw a heavy spherical ball (a shot) as far as they can.

Athlete number six, Jamilia, threw the shot 14.50 m.

Calculate Jamilia's standardised score (z).

Round your answer to one decimal place.

[1 mark (0.8)]

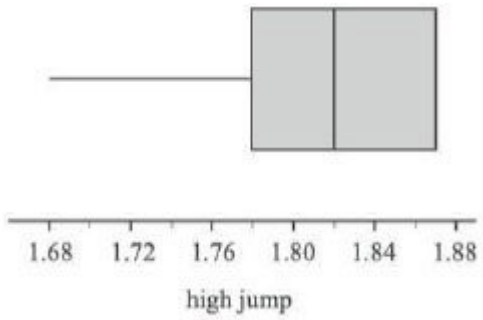
d. In the qualifying competition, the heights jumped in the high jump are expected to be approximately normally distributed.

Chara's jump in this competition would give her a standardised score of $z=-1.0$

Use the 68–95–99.7% rule to calculate the percentage of athletes who would be expected to jump higher than Chara in the qualifying competition.

[1 mark (0.6)]

e. The boxplot below was constructed to show the distribution of high jump heights for all 15 athletes in the qualifying competition.



Explain why the boxplot has no whisker at its upper end.

[1 mark 0.5)]

f. For the javelin qualifying competition (refer to Table 12), another boxplot is used to display the distribution of athletes' results.

An athlete whose result is displayed as an outlier at the upper end of the plot is considered to be a potential medal winner in the event.

What is the minimum distance that an athlete needs to throw the javelin to be considered a potential medal winner?

[2 marks (0.6)]

Question 17 / 25

[VCAA 2021 FM]
The two running events in the heptathlon are the 200 m run and the 800 m run. The times taken by the athletes in these two events, *time200* and *time800*, are linearly related.

When a least squares line is fitted to the data, the equation of this line is found to be

time800=0.03931+5.2756×time200

a. Round the values for the intercept and the slope to three significant figures. Write your answers in the boxes provided.

[1 mark (0.5)]

$time800 =$ ☐ $+$ ☐ $\times time200$

b. The mean and the standard deviation for each variable, *time200* and *time800*, are shown in the table below.

Statistic	***Time200* (seconds)**	***Time800* (seconds)**
Mean	24.6492	136.054
standard deviation	0.96956	8.2910

The equation of the least squares line is

time800=0.03931+5.2756×time200

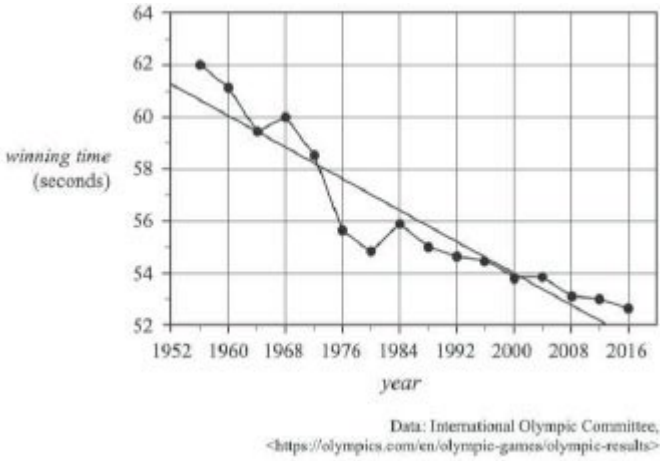
Use this information to calculate the coefficient of determination as a percentage. Round your answer to the nearest percentage.

[2 marks (0.6)]

Question 18 / 25

[VCAA 2021 FM]

The time series plot on the next page shows the *winning time*, in seconds, for the women's 100 m freestyle swim plotted against *year*, for each year that the Olympic Games were held during the period 1956 to 2016. A least squares line has been fitted to the plot to model the decreasing trend in the *winning time* over this period.



Data: International Olympic Committee,
<https://olympics.com/en/olympic-games/olympic-results>

The equation of the least squares line is

winning time=357.1−0.1515×year

The coefficient of determination is 0.8794

a. Name the explanatory variable in this time series plot.

[1 mark (0.9)]

b. Determine the value of the correlation coefficient (r).

Round your answer to three decimal places.

[1 mark (0.2)]

c. Write down the average decrease in *winning time*, in seconds per year, during the period 1956 to 2016.

[1 mark (0.3)]

d. The predicted *winning time* for the women's 100 m freestyle in 2000 was 54.10 seconds.

The actual *winning time* for the women's 100 m freestyle in 2000 was 53.83 seconds.

Determine the residual value in seconds.

[1 mark (0.7)]

e. The following equation can be used to predict the *winning time* for the women's 100 m freestyle in the future.

winning time=357.1−0.1515×year

i. Show that the predicted *winning time* for the women's 100 m freestyle in 2032 is 49.252 seconds.

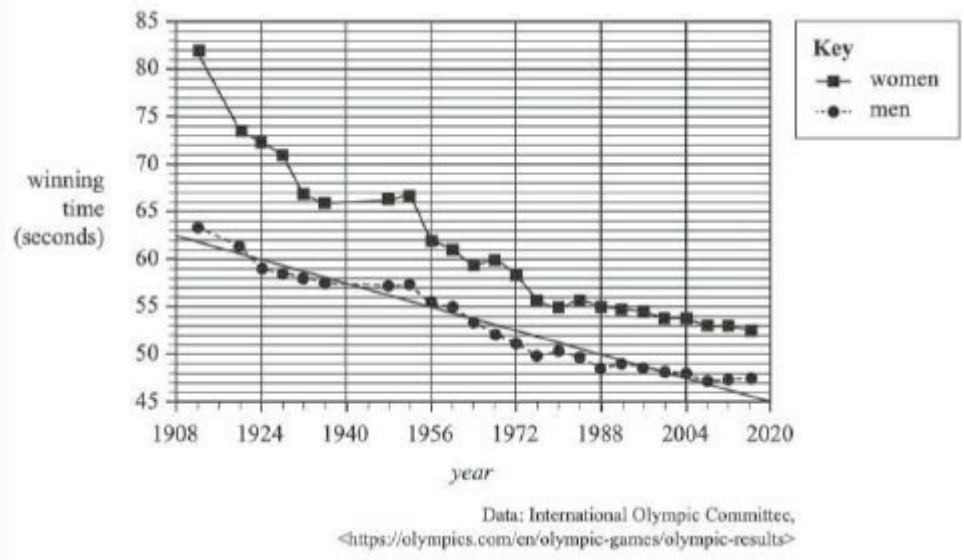
[1 mark (0.9)]

ii. What assumption is being made when this equation is used to predict the *winning time* for the women's 100 m freestyle in 2032?

[1 mark (0.2)]

Question 19 / 25

[VCAA 2021 FM]
The time series plot below shows that the winning time for both men and women in the 100 m freestyle swim in the Olympic Games has been decreasing during the period 1912 to 2016.



Least squares lines are used to model the trend for both men and women.

The least squares line for the men's winning time has been drawn on the time series plot above.

The equation of the least squares line for men is

winning time men=356.9−0.1544×year

The equation of the least squares line for women is

winning time women=538.9−0.2430×year

a. Draw the least squares line for *winning time women* on the time series plot above.

[1 mark (0.5)]

b. The difference between the women's predicted winning time and the men's predicted winning time can be calculated using the formula

difference=winning time women−winning time men

Use the equations of the least squares lines above and the formula above to calculate the *difference* predicted for the 2024 Olympic Games.

Round your answer to one decimal place.

[2 marks (1.5)]

c. The Olympic Games are held every four years. The next Olympic Games will be held in 2024, then 2028, 2032 and so on.

In which **Olympic year** do the two least squares lines predict that the winning time for women will first be faster than the winning time for men in the 100 m freestyle?

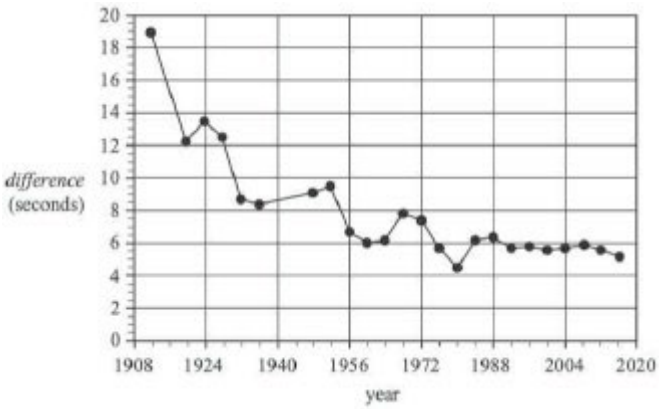
[2 marks (0.9)]

Question 20 / 25

[VCAA 2021 FM]
A method for predicting future time differences in the 100 m freestyle swim is to use the formula

difference=winning time women−winning time men

The resulting data and time series plot are shown below. The plot is clearly non-linear.



Year	Difference (seconds)
1912	18.8
1920	12.2
1924	13.4
1928	12.4
1932	8.6
1936	8.3
1948	9.0
1952	9.4
1956	6.6

Year	Difference (seconds)
1960	6.0
1964	6.1
1968	7.8
1972	7.4
1976	5.7
1980	4.4
1984	6.1
1988	6.3
1992	5.6
1996	5.8
2000	5.5
2004	5.7
2008	5.9
2012	5.5
2016	5.1

Note: No Olympic Games were held in 1916, 1940 and 1944.

a. Apply a reciprocal transformation to the variable *difference* to linearise the data. Fit a least squares line to the transformed data and write its equation below.

Round the values of the intercept and the slope to four significant figures.

[2 marks (0.5)]

b. Use the equation from **part a.** to predict, in seconds, the *difference* between the women's and men's winning times in the year 2032.

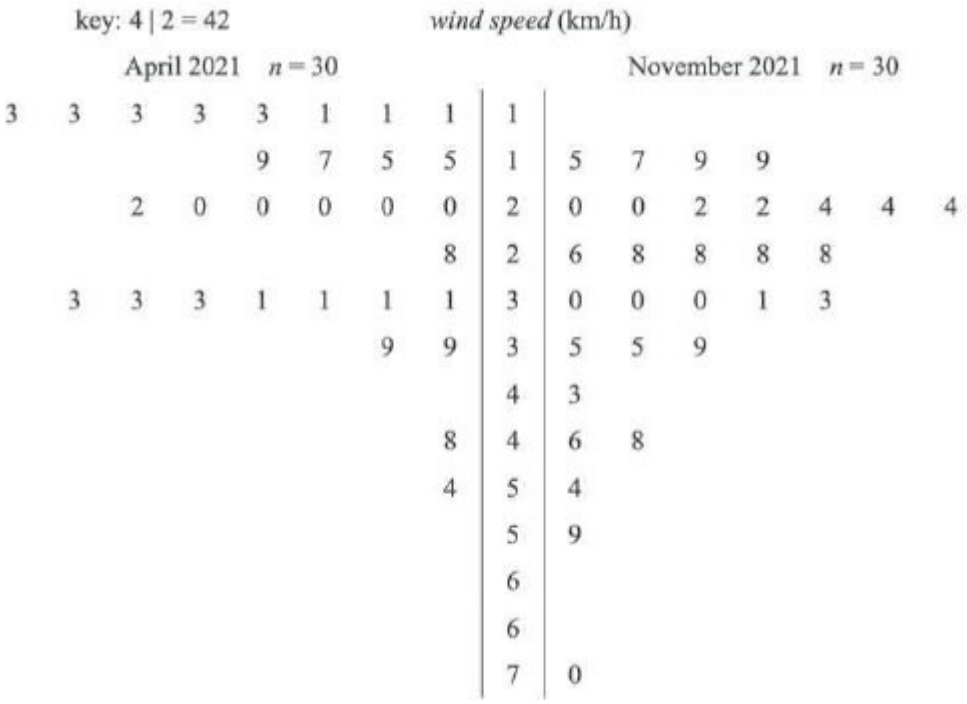
Round your answer to one decimal place.

[1 mark (0.2)]

Question 21 / 25

[VCAA 2022 FM]

The back-to-back stem plot below displays the distribution of daily maximum *wind speed*, in kilometres per hour, recorded at a weather station in April and November 2021.



a. For April 2021, determine

i. the median *wind speed*, in kilometres per hour

[1 mark (0.9)]

ii. the percentage of days for which the *wind speed* was less than 25 km/h.

[1 mark (0.8)]

b. The five-number summary for *wind speed* in November 2021 is given below.

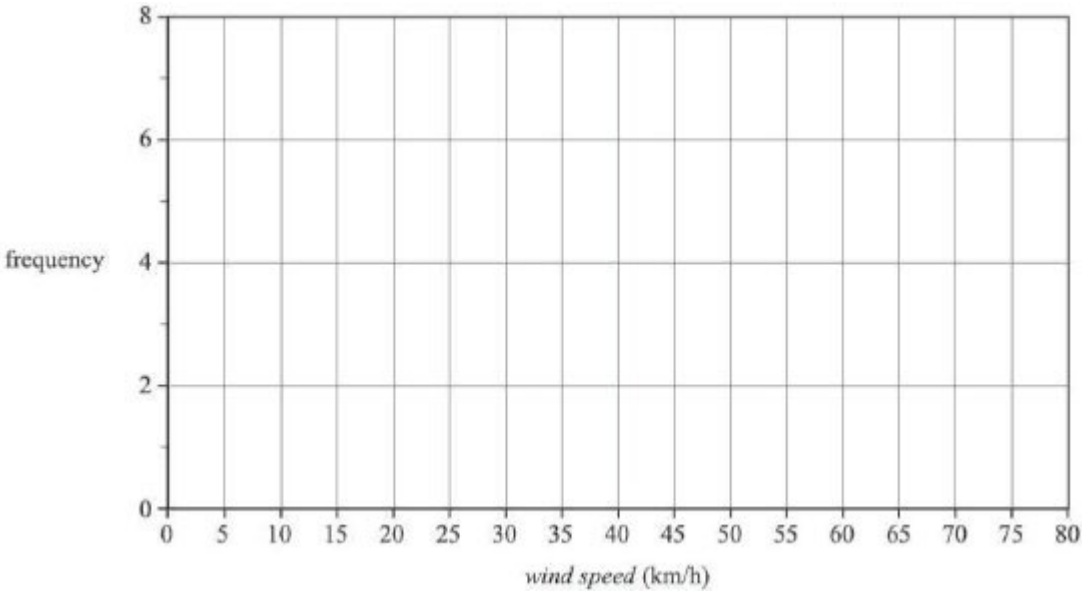
Minimum	***First quartile* (Q1)**	***Median***	***Third quartile* (Q3)**	***Maximum***
15	22	28	35	70

The *wind speeds* for November are to be used to construct a boxplot.

Show that the *wind speeds* of 59 km/h and 70 km/h would appear as outliers in this boxplot.

[2 marks (1.5)]

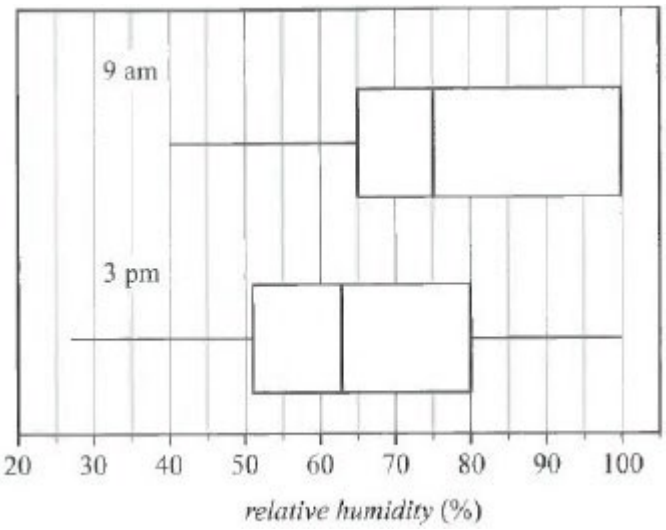
c. On the grid below, use the data from the stem plot on page 134 to construct a histogram that displays the distribution of *wind speed* for November 2021. Use class intervals of widths of five, starting at 15 km/h.



[2 marks (1.3)]

Question 22 / 25

[VCAA 2022 FM]
The boxplots below show the distribution of *relative humidity* (%) at 9 am and 3 pm at a weather station for the 30 days of November 2021.



a. i. Determine whether the *relative humidity* in November is more variable at 9 am or 3 pm.

[1 mark (0.6)]

ii. Write down the percentage of days in November for which the *relative humidity* at 3 pm was less than 80%.

[1 mark (0.7)]

iii. Complete the five-number summaries for the distributions of *relative humidity* at 9 am and 3 pm by writing the missing values in the table below.

[1 mark (0.9)]

Time	***Minimum***	***First quartile* (Q1)**	***Median***	***Third quartile* Q3)**	***Maximum***
9 am		65	75		
3 pm	27	51	63		

b. Do the boxplots support the contention that there is an association between relative humidity and time of day? Refer to the values of an appropriate statistic in your answer.

[2 marks (1.1)]

Question 23 / 25

[VCAA 2022 FM]
The table below displays data for seven weather-related variables for the first eight days of December 2021.

Day number	***Minimum temperature (°C)***	***Maximum temperature (°C)***	***Rainfall (mm)***	***Maximum wind speed (km/h)***	***Direction of maximum wind speed***	***Temperature 9 am (°C)***
1	19.4	28.3	0	35	ENE	22.9
2	17.6	29.7	1.0	35	WSW	24.2
3	7.6	16.5	11.6	26	WSW	12.7
4	7.5	15.9	0	30	WSW	10.9
5	5.7	19.0	0.2	24	ESE	10.4
6	9.9	23.8	0	39	NE	17.8
7	***11.0***	11.9	0	22	SSW	11.7
8	6.5	14.2	0	28	ESE	9.5

Data: Commonwealth of Australia 2022, Bureau of Meteorology, www.om.gov.au

a. Write down

i. the number of numerical variables in the table.

[1 mark (0.6)]

ii. the median *rainfall*, in millimetres, for the eight-day period.

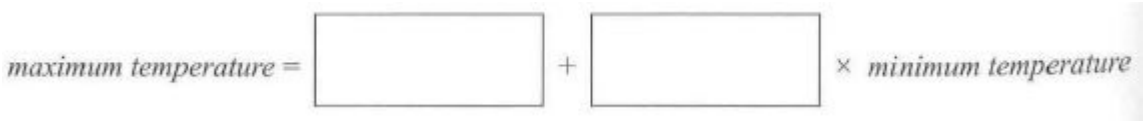
[1 mark (0.7)]

b. A least sqaes line is used to model the association between daily *maximum temperature* and daly *minimum temperature*.

With unrounded values for the intercept and slope, the equation of the least squares line is

maximum temperature=9.235946…+1.002493…×minimum temperature

Round the intercept and slope to four significant figures. Write your answers in the boxes provided.



[1 mark (0.6)]

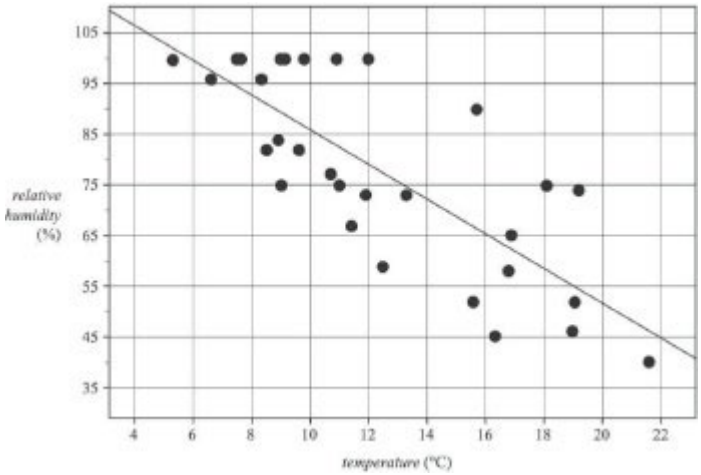
c. Calculate the coefficient of determination for the association between the dily *maximum temperature* and the aily *minimum temperature*. Write down its value as a percentage, rounded to one decimal place.

[1 mark (0.5)]

Question 24 / 25

[VCAA 2022 FM]
The scatterplot below shows the *relative humidity* (%) at 9 am plotted against the *temperature* (°C) at 9 am for the 30 days of November 2021.

A least squares line has been fitted to the scatterplot.



The equation of the least squares line is

relative humidity=120.1−3.417×temperature

The coefficient of determination is 0.6073.

a. The equation of the least squares line can be used to predict *relative humidity* at 9 am from the *temperature* at 9 am.

Name the explanatory variable.

[1 mark (1.0)]

b. Describe the association between *relative humidity* and *temperature* in terms of strength and direction.

[1 mark (0.3)]

c. Interpret the slope of the least squares line in terms of the variables *relative humidity* and *temperature*.

[1 mark (0.5)]

d. On the day when the *temperature* at 9 am was 16.3 °C, the *relative humidity* was 45.0%.

The following least squares line can be used to predict the relative humidity at 9 am on this day.

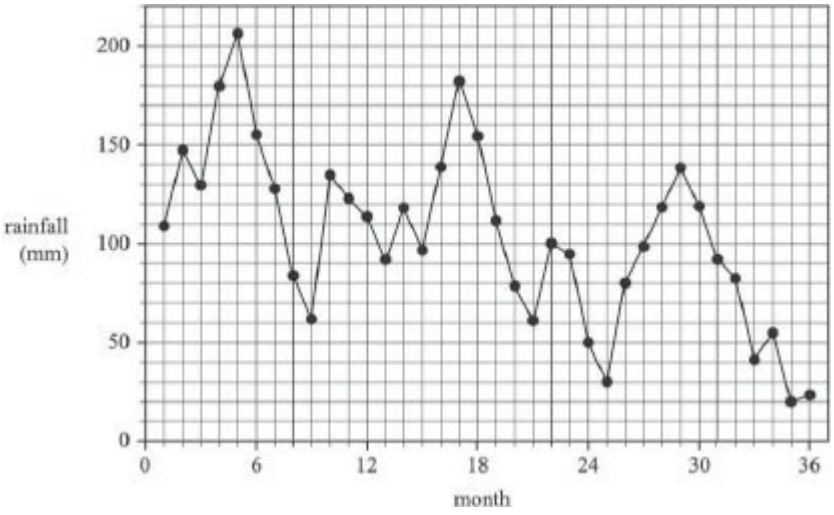
relative humidity=120.1−3.417×temperature

Calculate the residual value. Round your answer to one decimal place.

[2 marks (1.3)]

Question 25 / 25

[VCAA 2022 FM]
The time series plot below shows the monthly rainfall, in millimetres, recorded at a weather station over 36-month period.



a. The time series plot contains irregular fluctuations.

Give **two** other descriptions of the pattern in this time series plot.

[1 mark (0.4)]

b. Write down the value of the five-median smoothed rainfall for month 20.

[1 mark (0.6)]

c. The data for months 1 to 12 of the time series plot is shown in the table below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Month number	1	2	3	4	5	6	7	8	9	10	11	12
Rainfall (mm)	109.0	147.2	129.6	179.6	206.2	155.2	127.6	83.8	61.8	134.6	122.8	113.6

i. Calculate the nine-mean smoothed rainfall for month 7. Round your answer to one decimal place.

[1 mark (0.6)]

ii. What would be the number of points in the smoothed plot if nine-mean smoothing is applied for the full 36-month period?

[1 mark (0.4)]

Question 1 / 45

Use the following information to answer Questions 2 and 3.

The value of an annuity investment, in dollars, after n years, Vn, can be modelled by the recurrence relation shown below.

V_0=46 000, V_{n+1}=1.0034 V_n+500

Question 2 / 45

[VCAA 2018 FM (79%)]
What is the value of the regular payment added to the principal of this annuity investment?

A.

$34.00

B.

$156.40

C.

$466.00

D.

$500.00

E.

$656.40

Question 4 / 45

[VCAA 2018 FM (54%)]
Daniel borrows $5000, which he intends to repay fully in a lump sum after one year.

The annual interest rate and compounding period for five different compound interest loans are given below:

- Loan I – 12.6% per annum, compounding weekly
- Loan II – 12.8% per annum, compounding weekly
- Loan III – 12.9% per annum, compounding weekly
- Loan IV – 12.7% per annum, compounding quarterly

- Loan V – 13.2% per annum, ompounding quarterly

When fully repaid, the loan that will cost Daniel the least amount of money is

---

A. Loan I.

B. Loan II.

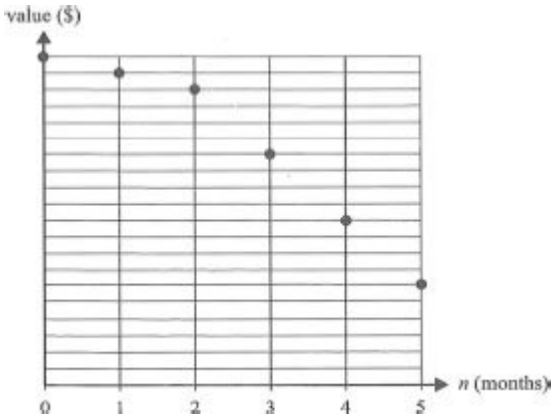
C. Loan III.

D. Loan IV.

E. Loan V.

Question 5 / 45

[VCAA 2018 FM (39%)]
The graph below shows the value, Vn, of an asset as it depreciates over a period of five months.



Which one of the following depreciation situations does this graph best represent?

A. flat rate depreciation with a decrease in depreciation rate after two months

B. flat rate depreciation with an increase in depreciation rate after two months

C. unit cost depreciation with a decrease in units used per month after two months

D. reducing balance depreciation with an increase in the rate of depreciation after two months

E. reducing balance depreciation with a decrease in the rate of depreciation after two months

Question 6 / 45

[VCAA 2018 FM (59%)]
Which one of the following recurrence relations could be used to model the value of a perpetuity investment, Pn, after n months?

A. $P_0=120\,000,\ P_{n+1}=1.0029\times P_n-356$

B. $P_0=180\,000,\ P_{n+1}=1.0047\times P_n-846$

C. $P_0=210\,000,\ P_{n+1}=1.0071\times P_n-1534$

D. $P_0=240\,000,\ P_{n+1}=0.0047\times P_n-2232$

E. $P_0=250\,000,\ P_{n+1}=0.0085\times P_n-2125$

Question 7 / 45

[VCAA 2018 FM (29%)]
Adam has a home loan with a present value of $175 260.56

The interest rate for Adam's loan is 3.72% per annum, compounding monthly.

His monthly repayment is $3200.

The loan is to be fully repaid after five years.

Adam knows that the loan cannot be exactly repaid with 60 repayments of $3200.

To solve this problem, Adam will make 59 repayments of $3200. He will then adjust the value of the final repayment so that the loan is fully repaid with the 60th repayment.

The value of the 60th repayment will be closest to

A. $368.12

B. $2831.88

C. $3200.56

D. $3557.09

E. $3568.12

Question 7 / 45

[VCAA 2018 FM (29%)]
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The value of the 60th repayment will be closest to

A. $368.12

B. $2831.88

C. $3200.56

D. $3557.09

E. $3568.12

Question 9 / 45

[VCAA 2018 FM (45%)]
Mariska plans to retire from work 10 years from now.

Her retirement goal is to have a balance of $600 000 in an annuity investment at that time.

The present value of this annuity investment is $265 298.48, on which she earns interest at the rate of 3.24% per annum, compounding monthly.

To make this investment grow faster, Mariska will add a $1000 payment at the end of every month.

Two years from now, she expects the interest rate of this investment to fall to 3.20% per annum, compounding monthly. It is expected to remain at this rate until Mariska retires.

When the interest rate drops, she must increase her monthly payment if she is to reach her retirement goal.

The value of this new monthly payment will be closest to

A.

$1234

B.

$1250

C.

$1649

D.

$1839

E.

$1854

Question 10 / 45

[VCAA 2019 FM (68%)]
Consider the recurrence relation shown below.

$A_0=3, A_{n+1}=2A_n+4$

The value of A_3 n the sequence generated by this rcurrence relation is given by

---

A.

$2\times3+4$

B.

$2\times4+4$

C.

$2\times10+4$

D.

$2\times24+4$

E.

$2\times52+4$

Question 11 / 45

[VCAA 2019 FM (66%)]
The value of a compound interest investment, in dollars, after n years, Vn, can be modelled by the recurrence relation shown below.

$V_0=100\,000$, $V_{n+1}=1.01\,V_n$

The interest rate, per annum, for this investment is

A. 0.01%

B. 0.101%

C. 1%

D. 1.01%

E. 101%

Question 12 / 45

[VCAA 2019 FM (76%)]
Geoff purchased a computer for $4500. He will depreciate the value of his computer by a flat rate of 10% of the purchase price per annum.

A recurrence relation that Geoff can use to determine the value of the computer after n years, Vn, is

---

A. $V_0=4500$, $V_{n+1}=V_n-450$

B. $V_0=4500$, $V_{n+1}=V_n+450$

C. $V_0=4500$, $V_{n+1}=0.9V_n$

D. $V_0=4500$, $V_{n+1}=1.1\,V_n$

E. $V_0=4500$, $V_{n+1}=0.1\,(V_n-450)$

Question 13 / 45

[VCAA 2019 FM (58%)]
Consider the following amortisation table for a reducing balance loan.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	300 000.00
1	1050.00	900.00	150.00	299 850.00
2	1050.00	899.55	150.45	299 699.55
3	1050.00	899.10	150.90	299 548.65

The annual interest rate for this loan is 3.6%. Interest is calculated immediately before each payment. For this loan, the repayments are made

---

A.

weekly.

B.

fortnightly.

C.

monthly.

D.

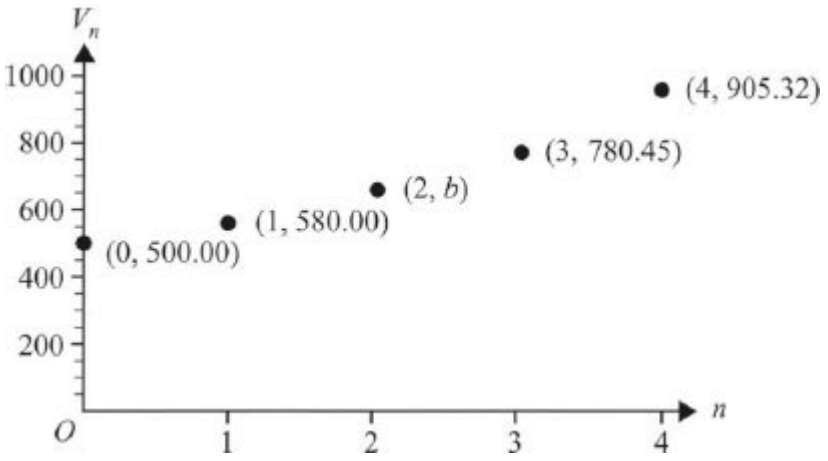
quarterly.

E.

yearly.

Question 14 / 45

[VCAA 2019 FM (54%)]
The graph below shows the value, in dollars, of a compound interest investment after n compounding periods, Vn, for a period of four compounding periods.



The coordinates of the point where n=2 are (2, b). The value of b is

---

A.

660.00

B.

670.00

C.

672.80

D.

678.40

E.

685.60

Question 15 / 45

[VCAA 2019 FM (53%)]
A machine is purchased for $30 000.

It produces 24 000 items each year.

The value of the machine is depreciated using a unit cost method of depreciation.

After three years, the value of the machine is $18 480.

A rule for the value of the machine after n units are produced, Vn, is

---

A. $Vn=0.872n$

B. $Vn=24\,000n-3840$

C. $Vn=30\,000-24\,000n$

D. $Vn=30\,000-0.872n$

E. $Vn=30\,000-0.16n$

Question 16 / 45

[VCAA 2019 FM (38%)]
Joseph borrowed $50 000 to buy a new car.

Interest on this loan is charged at the rate of 7.5% per annum, compounding monthly.

Joseph will fully repay this loan with 60 monthly repayments over five years.

Immediately after the 59th repayment is made, Joseph still owes $995.49

The value of his final repayment, to the nearest cent, will be

---

A. $995.49

B. $998.36

C. $1001.71

D. $1001.90

E. $1070.15

Question 17 / 45

[VCAA 2019 FM (51%)]
Millie invested $20 000 in an account at her bank with interest compounding monthly.

After one year, the balance of Millie's account was $20 732.

The difference between the rate of interest per annum used by her bank and the effective annual rate of interest for Millie's investment is closest to

---

A. 0.04%

B. 0.06%

C. 0.08%

D. 0.10%

E. 0.12%

Question 18 / 45

[VCAA 2020 FM (85%)]
The following recurrence relation can generate a sequence of numbers.

T0=10, Tn+1=Tn+3

The number 13 appears in this sequence as

A. T1

B. T2

C. T3

D. T10

E.

T13

Question 19 / 45

[VCAA 2020 FM (55%)]
An asset is purchased for $2480.

The value of this asset after n time periods, Vn, can be determined using the rule

$V_n = 2480 + 45n$.

A recurrence relation that also models the value of this asset after n time periods is

---

A. $V_0 = 2480, \quad V_{n+1} = V_n + 45n$

B. $V_n = 2480, \quad V_{n+1} = V_n + 45n$

C. $V_0 = 2480, \quad V_{n+1} = V_n + 45$

D. $V_1 = 2480, \quad V_{n+1} = V_n + 45$

E. $V_n = 2480, \quad V_{n+1} = V_n + 45$

Question 20 / 45

[VCAA 2020 FM (80%)]
Consider the following four recurrence relations representing the value of an asset after n years, Vn.

- $V_0 = 20000, \quad V_{n+1} = V_n + 2500$
- $V_0 = 20000, \quad V_{n+1} = V_n - 2500$
- $V_0 = 20000, \quad V_{n+1} = 0.875V_n$
- $V_0 = 20000, \quad V_{n+1} = 1.125V_n - 2500$

How many of these recurrence relations indicate that the value of an asset is depreciating?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 21 / 45

[VCAA 2020 FM (80%)]
Manu invests $3000 in an account that pays interest compounding monthly.

The balance of his investment after n months, Bn, can be determined using the recurrence relation

$B_0=3000$, $B_{n+1}=1.0048\times B_n$

The total interest earned by Manu's investment after the first five months is closest to

A. $57.60

B. $58.02

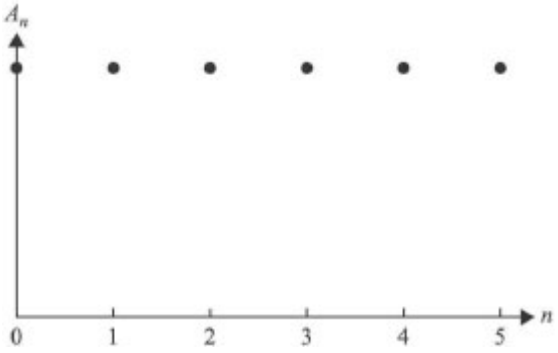
C. $72.00

D. $72.69

E. $87.44

Question 22 / 45

[VCAA 2020 FM (79%)]
The graph below represents the value of an annuity investment, An, in dollars, after n time periods.



A recurrence relation that could match this graphical representation is

A. $A_0=200\,000$, $A_{n+1}=1.015A_n-2500$

B. $A_0=200\,000$, $A_{n+1}=1.025A_n-5000$

C. $A_0=200\,000$, $A_{n+1}=1.03A_n-5500$

D. $A_0=200\,000$, $A_{n+1}=1.04A_n-6000$

E. $A_0=200\,000$, $A_{n+1}=1.05A_n-8000$

Question 23 / 45

[VCAA 2020 FM (20%)]
Ray deposited $5000 in an investment account earning interest at the rate of 3% per annum, compounding quarterly.

A rule for the balance, Rn, in dollars, after n years is given by

---

A. Rn=5000×0.03^n

B. Rn=5000×1.03^n

C. Rn=5000×0.034^n

D. Rn=5000×1.0075^n

E. Rn=5000×1.00754^n

Question 24 / 45

[VCAA 2020 FM (77%)]
Gen invests $10 000 at an interest rate of 5.5% per annum, compounding annually.

After how many years will her investment first be more than double its original value?

A. 12

B. 13

C. 14

D. 15

E. 16

Question 25 / 45

[VCAA 2020 FM (58%)]
The nominal interest rate for a loan is 8% per annum.

When rounded to two decimal places, the effective interest rate for this loan is **not**

A. 8.33% per annum when interest is charged daily.

B. 8.32% per annum when interest is charged weekly.

C. 8.31% per annum when interest is charged fortnightly.

D. 8.30% per annum when interest is charged monthly.

E. 8.24% per annum when interest is charged quarterly.

Question 26 / 45

[VCAA 2020 FM (54%)]
The value of a van purchased for $45000 s depreciated by k% per annum using the reducing balance method.

After three years of this depreciation, it is then depreciated in the fourth year under the unit cost method at the rate of 15 cents per kilometre.

The value of the van after it travels 30 000 km in this fourth year is $26 166.24

The value of k is

A. 9

B. 12

C. 14

D. 16

E. 18

Question 27 / 45

[VCAA 2020 FM (41%)]
Twenty years ago, Hector invested a sum of money in an account earning interest at the rate of 3.2% per annum, compounding monthly.

After 10 years, he made a one-off extra payment of $10 000 to the account.

For the next 10 years, the account earned interest at the rate of 2.8% per annum, compounding monthly.

The balance of his account today is $686 904.09

The sum of money Hector originally invested is closest to

A. $355 000

B. $370 000

C. $377 000

D. $384 000

E. $385 000

Question 28 / 45

[VCAA 2021 FM (83%)]
The following recurrence relation can generate a sequence of numbers.

$L0=37, \quad Ln+1=Ln + C$

The value of L2 is 25.

The value of C is

A. −6

B. −4

C. 4

D. 6

E. 37

Question 29 / 45

Use the following information to answer Questions 30 and 31.

Deepa invests $500000 in an annuity that provides an annual payment of $44970.55. Interest is calculated annually.

The first five lines of the amortisation table are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	500 000.00
1	44 970.55	20 000.00	24 970.55	475 029.45
2	44 970.55	19 001.18	25 969.37	449 060.08
3	44 970.55	17 962.40		422 051.93
4	44 970.55	16 882.08	28 088.47	393 963.46

Question 30 / 45

[VCAA 2021 FM (93%)]
The principal reduction associated with payment number 3 is

A. $17 962.40

B. $25 969.37

C. $27 008.15

D. $28 088.47

E. $44 970.55

Question 31 / 45

[VCAA 2021 FM (44%)]
The number of years, in total, for which Deepa will receive the regular payment of $44 970.55 is closest to

---

A.

12

B.

15

C.

16

D.

18

E.

20

Question 32 / 45

[VCAA 2021 FM (44%)]
Sammy purchased a boat for $72 000.

The value of the boat is depreciated each year by 10% using the reducing balance method.

In the third year, the boat will depreciate in value by 10% of

---

A. $47 239.20

B. $52 488.00

C. $58 320.00

D. $64 800.00

E. $72 000.00

Question 33 / 45

[VCAA 2021 FM (40%)]
Enrico invests $3000 in an account that pays interest compounding monthly.

After four years, the balance of the account is $3728.92

The effective annual interest rate for this investment, rounded to two decimal places, is

---

A.

5.45%

B.

5.52%

C.

5.56%

D.

5.59%

E.

5.60%

Question 34 / 45

[VCAA 2021 FM (56%)]
Joanna deposited $12 000 in an investment account earning interest at the rate of 2.8% per annum, compounding monthly.

She would like this account to reach a balance of $25 000 after five years.

To achieve this balance, she will make an extra payment into the account each month, immediately after the interest is calculated.

The minimum value of this payment is closest to

A. $113.85

B. $174.11

C. $580.16

D. $603.22

E. $615.47

Question 35 / 45

[VCAA 2021 FM (31%)]
Bimal has a reducing balance loan.

The balance, in dollars, of the loan from month to month, Bn, is modelled by the recurrence relation below.

$B_0 = 450\,000, \qquad B_{n+1} = R \times B_n - 2633$

Given that the loan will be fully repaid in 20 years, the value of R is closest to

---

A. 1.003

B. 1.0036

C. 1.03

D. 1.036

E. 1.36

Question 37 / 45

[VCAA 2022 FM (44%)]
A sequence of numbers is generated by the recurrence relation shown below.

$R_0=2,\ R_{n+1}=2-R_n$

The value of R_2 is

A. -4

B. -2

C. 0

D. 2

E. 4

Question 38 / 45

Use the following information to answer Questions 39 and 40.

The balance of a loan, V_n, in dollars, after n months is modelled by the recurrence relation.

$V_0=400000,\ V_{n+1}=1.003V_n-2024$

Question 39 / 45

[VCAA 2022 FM (78%)]
The balance of the loan first falls below $398 000 after how many months?

A. 1

B. 2

C. 3

D. 4

E. 5

Question 40 / 45

[VCAA 2022 FM (47%)]
With a small change to the final payment, the loan is expected to be repaid in full in

---

A. 25 years.

B. 26 years.

C. 28 years.

D. 29 years.

E. 30 years.

Question 41 / 45

[VCAA 2022 FM (65%)]
Nidhi owns equipment that is used for 10 hours per day for all 365 days of the year.

The value of the equipment is depreciated by Nidhi using the unit cost method.

The value of the equipment, En, in dollars, after n years can be modelled by the recurrence relation

$E_0=100\,000$, $E_{n+1}=E_n-5475$

The value of the equipment is depreciated by

A.

$1.50 per hour.

B.

$10 per hour.

C.

$15 per hour.

D.

$1.50 per day.

E.

$10 per day.

Question 42 / 45

[VCAA 2022 FM (33%)]
Consider the following four statements regarding nominal and effective interest rates as they apply to compound interest investments and loans:

- An effective interest rate is the same as a nominal interest rate if interest compounds annually.
- Effective interest rates increase as the number of compounding periods per year increases.
- A nominal rate of 12% per annum is equivalent to a nominal rate of 1% per month.
- An effective interest rate can be lower than a nominal interest rate.

How many of these four statements are true?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 43 / 45

[VCAA 2022 FM (58%)]
Tim deposited $6000 into an investment account earning compound interest calculated monthly.

A rule for the balance, Tn, in dollars, after n years is given by $T_n = 6000 \times 1.00312^n$.

Let Rn be a new recurrence relation that models the balance of Tim's account after n months.

This recurrence relation is

A. $R_0 = 6000$, $R_{n+1} = R_n + 18$

B. $R_0 = 6000$, $R_{n+1} = R_n + 36$

C. $R_0 = 6000$, $R_{n+1} = 1.003R_n$

D. $R_0 = 6000$, $R_{n+1} = 1.0036R_n$

E. $R_0 = 6000$, $R_{n+1} = 1.0036R_n$

Question 44 / 45

[VCAA 2022 FM (49%)]
Li invests $4000 for five years at 3.88% per annum, compounding annually.

Joseph invests a sum of money for five years, which earns simple interest paid annually.

Let Jn be the value, in dollars, of Joseph's investment after n years.

The two investments will finish at the same value, rounded to the nearest cent, if

Joseph's investment is modelled by which one of the following recurrence relations?

A. J0=2000, Jn+1=Jn+467.72

B. J0=2500, Jn+1=Jn+367.72

C. J0=3000, Jn+1=Jn+317.72

D. J0=3500, Jn+1=Jn+267.72

E. J0=4000, Jn+1=Jn+467.72

Question 45 / 45

[VCAA 2022 FM (32%)]
On 1 January 2020, Dion invested $10 500 into an investment account paying compound interest of 0.52% quarterly.

At the end of each quarter, after the interest was credited, Dion added an additional amount of money.

Let Dn represent the additional amount, in dollars, added at the end of quarter n.

This additional amount per quarter is modelled by the recurrence relation

D1=C,Dn+1=Dn

The balance of Dion's investment account on 1 January 2022 was $12700.95. The value of C is

A. $71.69

B. $215.55

C. $260.22

D. $270.15

E. $275.12

Question 1 / 18

[VCAA 2018 FM]
Julie deposits some money into a savings account that will pay compound interest every month. The balance of Julie's account, in dollars, after n months, Vn, can be modelled by the recurrence relation shown below.

$V_0=12000, V_{n+1}=1.0062\,V_n$

a. How many dollars does Julie initially invest?

[1 mark (1.0)]

b. Recursion can be used to calculate the balance of the account after one month.

i. Write down a calculation to show that the balance in the account after one month, V1, is $12074.40

[1 mark (0.9)]

ii. After how many months will the balance of Julie's account first exceed $12 300?

[1 mark (0.8)]

c. A rule of the form $V_n=a\times b^n$ can be used to determine the balance of Julie's account after n months.

i. Complete this rule for Julie's investment after n months by writing the appropriate numbers in the boxes provided below.

$$\text{balance} = \square \times \square^{n}$$

[1 mark (0.8)]

ii. What would be the value of n if Julie wanted to determine the value of her investment after three years?

[1 mark (0.7)]

Question 2 / 18

[VCAA 2018 FM]
After three years, Julie withdraws $14000 from her account to purchase a car for her business. For tax purposes, she plans to depreciate the value of her car using the reducing balance method. The value of Julie's car, in dollars, after n years, Cn, can be modelled by the recurrence relation shown below.

$C0=14000, Cn+1=R\times Cn$

a. For each of the first three years of reducing balance depreciation, the value of R is 0.85. What is the annual rate of depreciation in the value of the car during these three years?

[1 mark (0.6)]

b. For the next five years of reducing balance depreciation, the annual rate of depreciation in the value of the car is changed to 8.6%.

What is the value of the car eight years after it was purchased?

Round your answer to the nearest cent.

[2 marks (0.8)]

Question 3 / 18

[VCAA 2018 FM]
Julie has retired from work and has received a superannuation payment of $492 800.

She has two options for investing her money.

Option 1

Julie could invest the $492800 in a perpetuity. She would then receive $887.04 each fortnight for the rest of her life.

a. At what annual percentage rate is interest earned by this perpetuity?

[1 mark (0.4)]

Option 2

Julie could invest the $492 800 in an annuity, instead of a perpetuity.

The annuity earns interest at the rate of 4.32% per annum, compounding monthly.

The balance of Julie's annuity at the end of the first year of investment would be $480 242.25

b. **i.** What monthly payment, in dollars, would Julie receive?

[1 mark (0.5)]

ii. How much interest would Julie's annuity earn in the second year of investment? Round your answer to the nearest cent.

[2 marks (0.3)]

Question 4 / 18

[VCAA 2019 FM]
Phil is a builder who has purchased a large set of tools.

The value of Phil's tools is depreciated using the reducing balance method.

The value of the tools, in dollars, after n years, Vn, can be modelled by the recurrence relation shown below.

V0=60000, Vn+1= 0.9Vn

a. Use recursion to show that the value of the tools after two years, V2, is $48600.

[1 mark (0.8)]

b. What is the annual percentage rate of depreciation used by Phil?

[1 mark (0.7)]

c. Phil plans to replace these tools when their value first falls below $20 000.

After how many years will Phil replace these tools?

[1 mark (0.8)]

d. Phil has another option for depreciation. He depreciates the value of the tools by a flat rate of 8% of the purchase price per annum.

Let Vn be the value of the tools after n years, in dollars.

Write down a recurrence relation, in terms of V0, Vn+1, and Vn, that could be used to model the value of the tools using this flat rate depreciation.

[1 mark (0.5)]

Question 5 / 18

[VCAA 2019 FM]
Phil invests $200 000 in an annuity from which he receives a regular monthly payment.

The balance of the annuity, in dollars, after n months, An, can be modelled by the recurrence relation

$A_0 = 200\,000$, $A_{n+1} = 1.0035 \times A_n - 3700$

a. What monthly payment does Phil receive?

[1 mark (0.7)]

b. Show that the annual percentage compound interest rate for this annuity is 4.2%.

[1 mark (0.3)]

At some point in the future, the annuity will have a balance that is lower than the monthly payment amount.

c. What is the balance of the annuity when it first falls below the monthly payment amount?

Round your answer to the nearest cent.

[1 mark (0.4)]

d. If the payment received each month by Phil had been a different amount, the investment would act as a simple perpetuity.

What monthly payment could Phil have received from this perpetuity?

[1 mark (0.4)]

Question 6 / 18

[VCAA 2019 FM]
Phil would like to purchase a block of land.

He will borrow $350 000 to make this purchase.

Interest on this loan will be charged at the rate of 4.9% per annum, compounding fortnightly.

After three years of equal fortnightly repayments, the balance of Phil's loan will be $262 332.33

a. What is the value of each fortnightly repayment Phil will make?

Round your answer to the nearest cent.

[1 mark (0.3)]

b. What is the total interest Phil will have paid after three years?

Round your answer to the nearest cent.

[1 mark (0.2)]

c. Over the next four years of his loan, Phil will make monthly repayments of $3517.28 and will be charged interest at the rate of 4.8% per annum, compounding monthly.

Let Bn be the balance of the loan n months after these changes apply.

Write down a recurrence relation, in terms of B0, Bn+1, and Bn, that could be used to model the balance of the loan over these four years.

[2 marks (0.7)]

Question 7 / 18

[VCAA 2020 FM]
Samuel owns a printing machine.

The printing machine is depreciated in value by Samuel using flat rate depreciation.

The value of the machine, in dollars, after n years, Vn, can be modelled by the recurrence relation

$V_0 = 120\,000, V_{n+1} = V_n - 15\,000$

a. By what amount, in dollars, does the value of the machine decrease each year?

[1 mark (0.9)]

b. Showing recursive calculations, determine the value of the machine, in dollars, after two years.

[1 mark (0.7)]

c. What annual flat rate percentage of depreciation is used by Samuel?

[1 mark (0.7)]

d. The value of the machine, in dollars, after n years, Vn, could also be determined using a rule of the form $V_n = a + bn$.

Write down this rule for Vn.

[1 mark (0.4)]

Question 8 / 18

[VCAA 2020 FM]
Samuel has a reducing balance loan.

The first five lines of the amortisation table for Samuel's loan are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	320 000.00
1	1600.00	960.00	640.00	319 360.00
2	1600.00	958.08	641.92	318 718.08
3	1600.00	956.15		318 074.23
4	1600.00			

nterst is calculated monthly and Samuel makes monthly payments of $1600.

Interest is charged on this loan at the rate of 3.6% per annum.

a. Using the values in the amortisation table

i. calculate the principal reduction associated with payment number 3.

[1 mark (0.8)

ii. calculate the balance of the loan after payment number 4 is made.

[1 mark (0.4)]

b. Let Sn be the balance of Samuel's loan after n months.

Write down a recurrence relation, in terms of S0, Sn+1 and Sn, that could be used to model the month-to-month balance of the loan.

[1 mark (0.3)]

Question 9 / 18

[VCAA 2020 FM]
Samuel opens a savings account.

Let Bn be the balance of this savings account, in dollars, **n** months after it was opened.

The month-to-month value of Bn can be determined using the recurrence relation shown below.

$B_0=5000$, $B_{n+1}=1.003B_n$

a. Write down the value of B4, the balance of the savings account after four months. Round your answer to the nearest cent.

[1 mark (0.6)]

b. Calculate the monthly interest rate percentage for Samuel's savings account.

[1 mark (0.4)]

c. After one year, the balance of Samuel's savings account, to the nearest dollar, is $5183.

If Samuel had deposited an additional $50 at the end of each month immediately after the interest was added, how much extra money would be in the savings account after one year? Round your answer to the nearest dollar.

[1 mark (0.3)]

Question 10 / 18

[VCAA 2020 FM]
Samuel now invests $500 000 in an annuity from which he receives a regular monthly payment.

The balance of the annuity, in dollars, after n months, An, can be modelled by a recurrence relation of the form

$A_0 = 500\,000$, $A_{n+1} = kA_n - 2000$

a. Calculate the balance of this annuity after two months if $k = 1.0024$

[1 mark (0.6)]

b. Calculate the annual compound interest rate percentage for this annuity if $k = 1.0024$

[1 mark (0.5)]

c. For what value of k would this investment act as a simple perpetuity?

[1 mark (0.4)]

Question 11 / 18

[VCAA 2020 FM]
Later, Samuel took out a new reducing balance loan.

The interest rate for this loan was 4.1% per annum, compounding monthly.

The balance of the loan after four years of monthly repayments was $329 587.25

The balance of the loan after seven years of monthly repayments was $280 875.15

Samuel will continue to make the same monthly repayment.

To ensure the loan is fully repaid, to the nearest cent, the required final repayment will be lower.

In the first seven years, Samuel made 84 monthly repayments.

From this point on, how many more monthly repayments will Samuel make to fully repay the loan?

[2 marks (0.4)]

Question 12 / 18

[VCAA 2021 FM]
Sienna invests $420000 in a perpetuity from which she will receive a regular monthly payment of $1890.

The perpetuity earns interest at the rate of 5.4% per annum.

a. Determine the total amount, in dollars, that Sienna will receive after one year of monthly payments.

[1 mark (0.6)]

b. Write down the value of the perpetuity after Sienna has received one year of monthly payments.

[1 mark (0.5)]

c. Let Sn be the value of Sienna's perpetuity after n months.

Complete the recurrence relation, in terms of S0, Sn+1 and Sn, that would model the value of this perpetuity over time. Write your answers in the boxes provided.

$S_0 =$ [ ] , $S_{n+1} =$ [ ] $\times S_n - 1890$

[1 mark (0.4)]

Question 13 / 18

[VCAA 2021 FM]
Sienna owns a coffee shop. A coffee machine, purchased for $12 000, is depreciated in value using the unit cost method.

The rate of depreciation is $0.05 per cup of coffee made.

The recurrence relation that models the year-to-year value, in dollars, of the coffee machine is

M0=12 000, Mn+1=Mn−1440

a. Calculate the number of cups of coffee that the machine produces per year.

[1 mark (0.6)]

b. The recurrence relation above could also represent the value of the coffee machine depreciating at a flat rate.

What annual flat rate percentage of depreciation is represented?

[1 mark (0.5)]

c. Complete the rule below that gives the value of the coffee machine, Mn, in dollars, after n cups have been produced. Write your answers in the boxes provided.

$M_n =$ [ ] $+$ [ ] $\times n$

[1 mark (0.2)]

Question 14 / 18

[VCAA 2021 FM]
For renovations to the coffee shop, Sienna took out a reducing balance loan of $570 000 with interest calculated fortnightly.

The balance of the loan, in dollars, after n fortnights, Sn, can be modelled by the recurrence relation

$S_0 = 570\,000$, $S_{n+1} = 1.001\,S_n - 1193$

a. Calculate the balance of this loan after the first fortnightly repayment is made.

[1 mark (0.7)]

b. Show that the compound interest rate for this loan is 2.6% per annum.

[1 mark (0.3)]

c. For the loan to be fully repaid, to the nearest cent, Sienna's final repayment will be a larger amount. Determine this final repayment amount.

Round your answer to the nearest cent.

[1 mark (0.2)]

Question 15 / 18

[VCAA 2021 FM]
Sienna invests $152 431 into an annuity from which she will receive a regular monthly payment of $900 for 25 years. The interest rate for this annuity is 5.1% per annum, compounding monthly.

a. Let Vn be the balance of the annuity after n monthly payments. A recurrence relation written in terms of V0, Vn+1 and Vn can model the value of this annuity from month to month.

Showing recursive calculations, determine the value of the annuity after two months. Round your answer to the nearest cent.

[2 marks (0.7)]

b. After two years, the interest rate for this annuity will fall to 4.6%. To ensure that she will still receive the same number of $900 monthly payments, Sienna will add an extra one-off amount into the annuity at this time. Determine the value of this extra amount that Sienna will add. Round your answer to the nearest cent.

[1 mark (0.1)]

Question 16 / 18

[VCAA 2022 FM]
Pina owns workplace equipment, which she depreciates in value using flat rate depreciation. The value of the equipment, in dollars, after n years, Vn, can be determined using the rule

$V_n = 200000 - 12500n$

a. Determine V1, the value of the equipment after one year.

[1 mark (1.0)]

b. After how many years will the equipment first have a value of zero?

[1 mark (1.0)]

c. The value of the equipment, in dollars, after n years, Vn, can also be modelled by a recurrence relation. Write this recurrence relation in terms of V0, Vn+1 and Vn.

[1 mark (0.6)]

d. Using Pina's depreciation model, the value of the equipment decreases by a fixed percentage of its original value each year.

Alternatively, the value of the equipment could have been depreciated by a fixed percentage of its current value each year.

What name is given to this type of depreciation?

[1 mark (0.5)]

Question 17 / 18

[VCAA 2022 FM]
Pina invests $540000 in an annuity paying 3% interest per annum, compounding monthly. Her annuity will provide a monthly payment of $5214.28 for 10 years.

Four lines of the amortisation tale for Pina's annuity are shown below.

The information for payment number 3 is missing.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	540000.00
1	5214.28	1350.00	3864.28	536135.72
2	5214.28	1340.34	3873.94	532261.78
3				

a. What is the value of payment number 3?

[1 mark (0.9)]

b. Calculate the interest associated with payment number 3. Round your answer to the nearest cent.

[1 mark (0.5)]

c. Let Pn be the balance, in dollars, of Pina's annuity after n months.

I Write a recurrence relation, in terms of P0, Pn+1 and Pn, that can model this balance from month to month.

[1 mark (0.4)]

d. If Pina had invested the original $540 000 annuity as a simple perpetuity, what monthly payment would she have drawn?

[1 mark (0.5)]

Question 18 / 18

[VCAA 2022 FM]
To purchase additional workplace equipment, Pina took out a reducing balance loan of $580 000 with interest calculated monthly.

The balance of the loan, in dollars, after n months, Ln, can be modelled by the recurrence relation

$L_0 = 580\,000,\ L_{n+1} = 1.002 \times L_n - 3045.26$

a. Showing recursive calculations, determine the balance of the loan after two months. Round your answer to the nearest cent.

[1 mark (0.5)]

b. Determine the annual compound interest rate for this loan.

[1 mark (0.5)]

c. The final repayment that is required to fully pay off the loan is smaller than all other repayments by an amount less than one dollar. Determine this small amount in dollars, rounded to the nearest cent.

[1 mark (0.3)]

d. The original recurrence relation

$L_0 = 580\,000,\ L_{n+1} = 1.002 \times L_n - 3045.26$

models the loan being fully repaid in a certain number of years.

Using a different multiplication factor (other than 1.002) the loan would be fully repaid one year sooner.

Determine this multiplication factor. Round your answer to four decimal places.

[1 mark (0.2)]

[VCAA 2018 FM (78%)]
Which one of the following matrices has a determinant of zero?

A. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2 \\ -3 & 6 \end{bmatrix}$

D. $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$

E. $\begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix}$

Question 2 / 39

[VCAA 2018 FM (68%)]

The matrix product $\begin{pmatrix} 4 & 2 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 2 \\ 8 \end{pmatrix}$

is equal to

A. [144]

B. $\begin{pmatrix} 16 \\ 24 \\ 40 \end{pmatrix}$

C. $4 \times \begin{pmatrix} 1 & 2 & 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 12 \\ 8 \end{pmatrix}$

D. $2 \times \begin{pmatrix} 2 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix}$

E. $4 \times \begin{pmatrix} 2 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix}$

Question 3 / 39

[VCAA 2018 FM (78%)]
Five people, India (I), Jackson (J), Krishna (K), Leanne (L) and Mustafa (M), competed in a table tennis tournament. Each competitor played every other competitor once only.

Each match resulted in a winner and a loser.

The matrix below shows the tournament results.

$$\begin{array}{cc} & \text{loser} \\ & \begin{array}{ccccc} I & J & K & L & M \end{array} \\ \begin{array}{c} I \\ J \\ \text{winner } K \\ L \\ M \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

A '1' in the matrix shows that the competitor named in that row defeated the competitor named in that column.

For example, the '1' in the fourth row shows that Leanne defeated Jackson.

A '0' in the matrix shows that the competitor named in that row lost to the competitor named in that column.

There is an error in the matrix. The winner of one of the matches has been incorrectly recorded as a '0'.

This match was between

A. India and Mustafa.

B. India and Krishna.

C. Krishna and Leanne.

D. Leanne and Mustafa.

E. Jackson and Mustafa.

Question 4 / 39

[VCAA 2018 FM (69%)]
Matrix P is a 4×4 permutation matrix. Matrix W is another matrix such that the matrix product P×W is defined. This matrix product results in the entire first and third rows of matrix W being swapped. The permutation matrix P is

---

A. $\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

[1000010010000001]

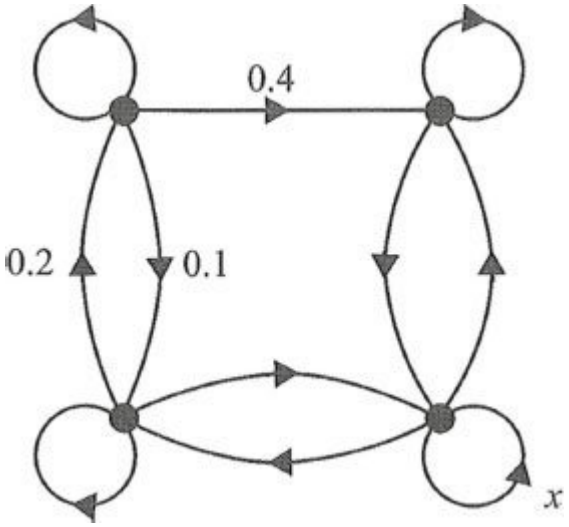
Question 5 / 39

[VCAA 2018 FM (81%)]
A transition matrix, V, is shown below.

$$V = \begin{array}{c} \\ L \\ T \\ F \\ M \end{array} \begin{array}{c} \begin{array}{cccc} \multicolumn{4}{c}{this\ month} \\ L & T & F & M \end{array} \\ \begin{bmatrix} 0.6 & 0.6 & 0.2 & 0.0 \\ 0.1 & 0.2 & 0.0 & 0.1 \\ 0.3 & 0.0 & 0.8 & 0.4 \\ 0.0 & 0.2 & 0.0 & 0.5 \end{bmatrix} \end{array} \begin{array}{l} L \\ T \\ F \\ M \end{array} \ next\ month$$

The transition diagram below has been constructed from the transition matrix V.

The labelling in the transition diagram is not yet complete.



The proportion for one of the transitions is labelled x.

The value of x is

A. 0.2

B. 0.5

C. 0.6

D. 0.7

E. 0.8

Question 6 / 39

[VCAA 2018 FM (28%)]
A study of the antelope population in a wildlife park has shown that antelope regularly move between three locations, east (E), north (N) and west (W).

Let An be the state matrix that shows the population of antelope in each location n months after the study began.

The expected population of antelope in each location can be determined by the matrix recurrence rule

$$A_{n+1} = TA_n - D$$

Where:

$$\begin{matrix} & this\ month & & \\ & E & N & W \\ \end{matrix}$$
$$\begin{matrix} \begin{bmatrix} 0.4 & 0.2 & 0.2 \\ 0.3 & 0.6 & 0.3 \\ 0.3 & 0.2 & 0.5 \end{bmatrix} & \begin{matrix} E \\ N\ next\ month \\ W \end{matrix} \end{matrix}$$

And

$$D = \begin{bmatrix} 50 \\ 50 \\ 50 \end{bmatrix} \begin{matrix} E \\ N \\ W \end{matrix}$$

The state matrix, A3, below shows the population of antelope three months after the study began.

$$A_3 = \begin{bmatrix} 1616 \\ 2800 \\ 2134 \end{bmatrix} \begin{matrix} E \\ N \\ W \end{matrix}$$

The number of antelope in the **west** (W) location two months after the study began, as found in the state matrix A2, is closest to

---

A. 2060

B. 2130

C. 2200

D. 2240

E. 2270

Question 7 / 39

[VCAA 2018 FM (46%)]
A public library organised 500 of its members into five categories according to the number of books each member borrows each month.

These categories are

J=no books borrowed per monthK=one book borrowed per monthL=two books borrowed per monthM=three books borrowed per monthN=four or more books borrowed per mon th

The transition matrix, T, below shows how the number of books borrowed per month by the members is expected to change from month to month.

$$T = \begin{matrix} & \text{this month} \\ & J \quad K \quad L \quad M \quad N \\ \text{next month} \begin{matrix} J \\ K \\ L \\ M \\ N \end{matrix} & \begin{bmatrix} 0.1 & 0.2 & 0.2 & 0 & 0 \\ 0.5 & 0.2 & 0.3 & 0.1 & 0.3 \\ 0.3 & 0.3 & 0.4 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.1 & 0.6 & 0.3 \\ 0 & 0.1 & 0 & 0.2 & 0.5 \end{bmatrix} \end{matrix}$$

In the long term, which category is expected to have approximately 96 members each month?

A. J

B. K

C. L

D. M

E. N

Question 8 / 39

[VCAA 2019 FM (87%)]
Consider the following four matrix expressions.

$\begin{bmatrix} 8 \\ 12 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 8 \\ 12 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 8 & 0 \\ 12 & 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 0 \\ 12 & 0 \end{bmatrix}$

How many of these four matrix expressions are defined?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 9 / 39

[VCAA 2019 FM (68%)]
There are two rides called The Big Dipper and The Terror Train at a carnival.

The cost, in dollars, for a child to ride on each ride is shown in the table below.

Ride	**Cost ($)**
The Big Dipper	7
The Terror Train	8

Six children ride once only on The Big Dipper and once only on The Terror Train.

The total cost of the rides, in dollars, for these six children can be determined by which one of the following calculations.

A. $[\mathbf{6}] \times [\mathbf{7} \quad \mathbf{8}]$

B. $[\mathbf{6}] \times \begin{bmatrix} \mathbf{7} \\ \mathbf{8} \end{bmatrix}$

C. $[\mathbf{6} \quad \mathbf{6}] \times [\mathbf{7} \quad \mathbf{8}]$

D. $[\mathbf{6} \quad \mathbf{6}] \times \begin{bmatrix} \mathbf{7} \\ \mathbf{8} \end{bmatrix}$

E. $\begin{bmatrix} \mathbf{7} \\ \mathbf{8} \end{bmatrix} \times [\mathbf{6} \quad \mathbf{6}]$

Question 10 / 39

[VCAA 2019 FM (59%)]

Consider the matrix P, where $P = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$

The element in row i and column j of matrix P is p_{ij}.

The elements in matrix P are determined by the rule

A. $p_{ij} = 4 - j$

B. $p_{ij} = 2i + 1$

C. $p_{ij} = i + j + 1$

D. $p_{ij} = i + 2j$

E. $p_{ij} = 2i - j + 2$

Question 11 / 39

[VCAA 2019 FM (56%)]
Stella completed a multiple-choice test that had 10 questions.

Each question had five possible answers A, B, C, D, and E.

For question number one, Stella chose the answer E.

Stella choose each of the nine remaining answers, in order, by following the transition matrix, T, below.

$$T = \begin{array}{c} \\ \\ \\ \\ \\ \end{array} \begin{array}{c} \text{this question} \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \\ D \\ E \end{array} \text{next question}$$

What answer did Stella choose for question number six?

A. A

B. B

C. P

D. D

E. E

Question 12 / 39

[VCAA 2019 FM (39%)]
A water park is open from 9 am until 5 pm. There are three activities, the pool (P), the slide (S) and the water jets (W), at the water park.

Children have been found to change their activity at the water park each half hour, as shown in the transition matrix, T, below.

$$\begin{array}{c} \text{this month} \\ \begin{array}{ccc} P & S & W \end{array} \\ \begin{bmatrix} 0.8 & 0.2 & 0.4 \\ 0.05 & 0.6 & 0.1 \\ 0.15 & 0.2 & 0.5 \end{bmatrix} \end{array} \begin{array}{l} P \\ S \\ W \end{array} \text{next month}$$

A group of children has come to the water park for the whole day.

The percentage of these children who are expected to be at the slide (S) at closing time is closest to

A. 14%

B. 20%

C. 24%

D. 25%

E. 62%

Question 13 / 39

[VCAA 2019 FM (54%)]
The communication matrix below shows the direct paths by which messages can be sent between two people in a group of six people, U to Z.

$$\begin{array}{cc} & reciever \\ & \begin{array}{cccccc} U & V & W & X & Y & Z \end{array} \\ sender & \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{array}{c} U \\ V \\ W \\ X \\ Y \\ Z \end{array} \end{array}$$

A '1' in the matrix shows that the person named in that row can send a message directly to the person named in that column. For example, the '1' in row 4, column 2 shows that X can send a message directly to V.

In how many ways can Y get a message to W by sending it directly to one other person?

A. 0

B. 1

C. 2

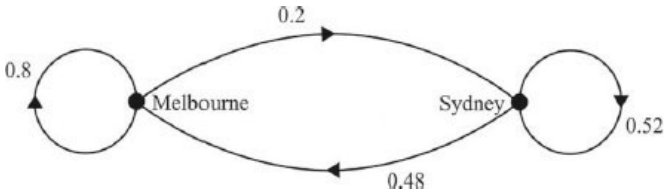
D. 3

E. 4

Question 14 / 39

[VCAA 2019 FM (15%)]
An airline parks all its planes at Sydney airport or Melbourne airport overnight.

The transition diagram below shows the change in the location of the planes from night to night.



There is always m planes parked at Melbourne airport.

There is always s planes parked at Sydney airport.

Of the planes parked at Melbourne airport on Tuesday night, 12 had been parked at Sydney airport on Monday night.

How many planes does the airline have?

A. 25

B. 37

C. 62

D. 65

E. 85

Question 15 / 39

[VCAA 2020 FM (83%)]
The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ is an example of

A. a binary matrix.

B. an identity matrix.

C. a triangular matrix.

D. a symmetric matrix.

E. a permutation matrix.

Question 16 / 39

[VCAA 2020 FM (79%)]

Matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 1 & 0 \\ 4 & 5 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 2 & 0 & 3 & 1 \\ 4 & 5 & 2 & 0 \end{bmatrix}$

Matrix Q=A×B.

The element in row i and column j of matrix Q is q_{ij}.

Element q_{41} is determined by the calculation

A. 0×0+3×5

B. 1×1×2×0

C. 1×2+2×4

D. 4×1+5×0

E. 4×2+5×4

Question 17 / 39

[VCAA 2020 FM (64%)]
Matrices P and W are defined below.

$$P = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad W = \begin{bmatrix} A \\ S \\ T \\ O \\ R \end{bmatrix}$$

If $P^n = W \times \begin{bmatrix} A \\ S \\ T \\ O \\ R \end{bmatrix}$, the value of n could be

---

A. 1

B. 2

C. 3

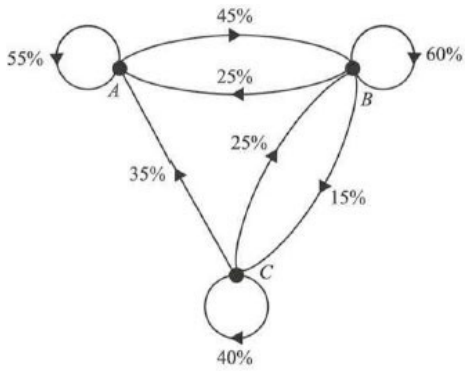
D. 4

E. 5

Question 18 / 39

[VCAA 2020 FM (90%)]
Ina particular supermarket, the three top-selling magazines are *Angel (A), Bella (B)* and *Crystal (C)*.

The transition diagram below shows the way shoppers at this supermarket change their magazine choice from week to week.



A transition matrix that provides the same information as the transition diagram is

A.

$$\begin{matrix} & this\ week & \\ A & B & C \\ \begin{bmatrix} 0.55 & 0.7 & 0.35 \\ 0.7 & 0.6 & 0.4 \\ 0.35 & 0.4 & 0.4 \end{bmatrix} & & \begin{matrix} A \\ Bnext\ week \\ C \end{matrix} \end{matrix}$$

B.

$$\begin{matrix} & this\ week & \\ A & B & C \\ \begin{bmatrix} 0.55 & 0.6 & 0.25 \\ 0.45 & 0.15 & 0.35 \\ 0 & 0.25 & 0.4 \end{bmatrix} & & \begin{matrix} A \\ Bnext\ week \\ C \end{matrix} \end{matrix}$$

C.

$$\begin{matrix} & this\ week & \\ A & B & C \\ \begin{bmatrix} 0.55 & 0.25 & 0.35 \\ 0.45 & 0.6 & 0.25 \\ 0 & 0.15 & 0.4 \end{bmatrix} & & \begin{matrix} A \\ Bnext\ week \\ C \end{matrix} \end{matrix}$$

D.

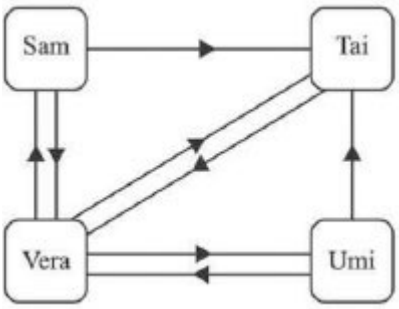
$$\begin{matrix} & this\ week & \\ A & B & C \\ \begin{bmatrix} 0.55 & 0.25 & 0.35 \\ 0.45 & 0.6 & 0.25 \\ 0.35 & 0.15 & 0.4 \end{bmatrix} & & \begin{matrix} A \\ Bnext\ week \\ C \end{matrix} \end{matrix}$$

E.

$$\begin{matrix} & this\ week & \\ A & B & C \\ \begin{bmatrix} 0.55 & 0.25 & 0 \\ 0.45 & 0.6 & 0.25 \\ 0 & 0.15 & 0.4 \end{bmatrix} & & \begin{matrix} A \\ Bnext\ week \\ C \end{matrix} \end{matrix}$$

Question 19 / 39

[VCAA 2020 FM (86%)]
The diagram below shows the direct communication links that exist between Sam (S), Tai (T), Umi (U) and Vera (V). For example, the arrow from Umi to Vera indicates that Umi can communicate directly with Vera.



A communication matrix can be used to convey the same information. In this matrix:

- a '1' indicates that a direct communication link exists between a sender and a receiver
- a '0' indicates that a direct communication link does not exist between a sender and a receiver.

The communication matrix could be

A.

$$\begin{matrix} & & reciever & \\ & S & T & U & V \\ sender\ S & 0 & 1 & 0 & 1 \\ T & 0 & 0 & 0 & 1 \\ U & 0 & 1 & 0 & 1 \\ V & 1 & 0 & 1 & 0 \end{matrix}$$

B.

$$\begin{matrix} & & reciever & \\ & S & T & U & V \\ sender\ S & 0 & 1 & 0 & 1 \\ T & 1 & 0 & 0 & 1 \\ U & 0 & 1 & 0 & 1 \\ V & 1 & 1 & 1 & 0 \end{matrix}$$

C.

$$\begin{matrix} & & reciever & \\ & S & T & U & V \\ sender\ S & 0 & 1 & 0 & 1 \\ T & 0 & 0 & 0 & 1 \\ U & 0 & 1 & 0 & 0 \\ V & 1 & 1 & 1 & 0 \end{matrix}$$

D.

$$\begin{array}{c} \\ sender \end{array} \begin{array}{c} reciever \\ \begin{array}{cccc} S & T & U & V \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} S \\ T \\ U \\ V \end{array}$$

E.

$$\begin{array}{c} \\ sender \end{array} \begin{array}{c} reciever \\ \begin{array}{cccc} S & T & U & V \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 2 & 2 & 2 & 0 \end{bmatrix} \end{array} \begin{array}{l} S \\ T \\ U \\ V \end{array}$$

Question 20 / 39

[VCAA 2020 FM (73%)]
The element in row i and column j of matrix M is m_{ij}.

M *is a* 3×3 matrix. It is constructed using the rule $m_{ij} = 3i + 2j$.

M is

A. $\begin{bmatrix} \mathbf{5} & \mathbf{7} & \mathbf{9} \\ \mathbf{7} & \mathbf{9} & \mathbf{11} \\ \mathbf{11} & \mathbf{13} & \mathbf{15} \end{bmatrix}$

B. $\begin{bmatrix} \mathbf{5} & \mathbf{7} & \mathbf{9} \\ \mathbf{8} & \mathbf{10} & \mathbf{12} \\ \mathbf{11} & \mathbf{13} & \mathbf{15} \end{bmatrix}$

C. $\begin{bmatrix} \mathbf{5} & \mathbf{7} & \mathbf{10} \\ \mathbf{8} & \mathbf{10} & \mathbf{13} \\ \mathbf{11} & \mathbf{13} & \mathbf{16} \end{bmatrix}$

D. $\begin{bmatrix} \mathbf{5} & \mathbf{8} & \mathbf{11} \\ \mathbf{7} & \mathbf{10} & \mathbf{13} \\ \mathbf{9} & \mathbf{12} & \mathbf{15} \end{bmatrix}$

E. $\begin{bmatrix} \mathbf{5} & \mathbf{8} & \mathbf{11} \\ \mathbf{8} & \mathbf{11} & \mathbf{14} \\ \mathbf{11} & \mathbf{14} & \mathbf{17} \end{bmatrix}$

Question 21 / 39

[VCAA 2020 FM (38%)]
A small shopping centre has two coffee shops: Fatima's (F) and Giorgio's (G).

The percentage of coffee-buyers at each shop changes from day to day, as shown in the transition matrix T.

$$T = \begin{matrix} & \begin{matrix} today \\ F \quad G \end{matrix} \\ \begin{matrix} \\ tomorrow \end{matrix} & \begin{matrix} F \\ G \end{matrix} \begin{bmatrix} 0.85 & 0.35 \\ 0.15 & 0.65 \end{bmatrix} \end{matrix}$$

On a particular Monday, 40% of coffee-buyers bought their coffees at Fatima's.

The matrix recursion relation $S_{n+1} = TS_n$ is used to model this situation.

The percentage of coffee-buyers who are expected to buy their coffee at Giorgio's on Friday of the same week is closest to

---

A. 31%

B. 32%

C. 34%

D. 45%

E. 68%

Question 22 / 39

[VCAA 2020 FM (57%)]
The table below shows information about three matrices: A, B and C.

Matrix	Order
A	2×4
B	2×3
C	3×4

The transpose of matrix A, for example, is written as AT.

What is the order of the product $C^T \times (A^T \times B)^T$?

A.

2×3

B.

3×4

C.

4×2

D.

4×3

E.

4×4

Question 23 / 39

[VCAA 2020 FM (41%)]
Consider the matrix recurrence relation below.

$S_0 = \begin{bmatrix} 30 \\ 20 \\ 40 \end{bmatrix}, S_{n+1} = TS_n$ where $T = \begin{bmatrix} j & 0.3 & l \\ 0.2 & m & 0.3 \\ 0.4 & 0.2 & n \end{bmatrix}$

Matrix T is a regular transition matrix.

Given the information above and that $S_1 = \begin{bmatrix} 42 \\ 28 \\ 20 \end{bmatrix}$, which one of the following is true?

A. $\boldsymbol{m > l}$

B. $\boldsymbol{j + l = 0.7}$

C. $\boldsymbol{j = n}$

D. $\boldsymbol{j > m}$

E. $\boldsymbol{l = m + n}$

Question 24 / 39

[VCAA 2020 FM (63%)]
Five copetitors, Andy (A), Brie (B), Cleo (C), Della (D) and Eddie (E), participate in a darts tournament.

Each competitor plays each of the other competitors once only, and each match results in a winner and a loser.

The matrix below shows the results of this darts tournament.

There are still two matches that need to be played.

.

$$\begin{matrix} & & & winner & & \\ A & B & C & D & E & \\ \lceil 0 & \dots & 0 & 1 & 0 \rceil & A \\ \dots & 0 & 1 & 0 & 1 & B \\ 1 & 0 & 0 & \dots & 1 & C \ loser \\ \lfloor 0 & 1 & \dots & 0 & 0 \rfloor & D \\ 1 & 0 & 0 & 1 & 0 & E \end{matrix}$$

A '1' in the matrix shows that the competitor named in that row defeated the competitor named in that column.

For example, the '1' in row 2, column 3 shows that Brie defeated Cleo.

A '…' in the matrix shows that the competitor named in that row has not yet played the competitor named in that column.

The winner of this darts tournament is the competitor with the highest sum of their one-step and two-step dominances.

Which player, by winning their remaining match, will ensure that they are ranked first by the sum of their one-step and two-step dominances?

A. Andy

B. Brie

C. Cleo

D. Della

E. Eddie

Question 25 / 39

[VCAA 2021 FM (82%)]

If matrix $M = \begin{bmatrix} 3 & 2 \\ 8 & 9 \\ 13 & 7 \end{bmatrix}$, then its transpose, M^T, is

A. $\begin{bmatrix} 2 & 3 \\ 9 & 8 \\ 7 & 13 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 9 & 7 \\ 3 & 8 & 13 \end{bmatrix}$

C. $\begin{bmatrix} 7 & 9 & 2 \\ 13 & 8 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 3 & 8 & 13 \\ 2 & 9 & 7 \end{bmatrix}$

E. $\begin{bmatrix} 13 & 8 & 3 \\ 7 & 9 & 2 \end{bmatrix}$

Question 26 / 39

[VCAA 2021 FM (73%)]

Every Friday, the same number of workers from a large office building regularly purchase their lunch from one of two locations: the deli, D, or the cafe, C.

It has been found that:

- of the workers who purchase lunch from the deli on one Friday, 65% will return to purchase from the deli on the next Friday

- of the workers who purchase lunch from the cafe on one Friday, 55% will return to purchase from the cafe on the next Friday.

A transition matrix that can be used to describe this situation is

A.

$$T = \begin{matrix} \\ D \\ C \end{matrix} \begin{matrix} \begin{matrix} this\ Friday \\ D \end{matrix} & \begin{matrix} \\ C \end{matrix} \\ \begin{bmatrix} 0.55 & 0.35 \\ 0.45 & 0.65 \end{bmatrix} & \end{matrix} \begin{matrix} \\ D \\ C \end{matrix} next\ Friday$$

B.

$$T = \begin{matrix} \\ D \\ C \end{matrix} \begin{matrix} \begin{matrix} this\ Friday \\ D \end{matrix} & \begin{matrix} \\ C \end{matrix} \\ \begin{bmatrix} 0.65 & 0.45 \\ 0.45 & 0.55 \end{bmatrix} & \end{matrix} \begin{matrix} \\ D \\ C \end{matrix} next\ Friday$$

C.

$$T = \begin{matrix} \\ D \\ C \end{matrix} \begin{matrix} \begin{matrix} this\ Friday \\ D \end{matrix} & \begin{matrix} \\ C \end{matrix} \\ \begin{bmatrix} 0.65 & 0.55 \\ 0.45 & 0.55 \end{bmatrix} & \end{matrix} \begin{matrix} \\ D \\ C \end{matrix} next\ Friday$$

D.

$$T = \begin{matrix} \\ D \\ C \end{matrix} \begin{matrix} \begin{matrix} this\ Friday \\ D \end{matrix} & \begin{matrix} \\ C \end{matrix} \\ \begin{bmatrix} 0.65 & 0.45 \\ 0.35 & 0.55 \end{bmatrix} & \end{matrix} \begin{matrix} \\ D \\ C \end{matrix} next\ Friday$$

E.

$$T = \begin{matrix} \\ D \\ C \end{matrix} \begin{matrix} \begin{matrix} this\ Friday \\ D \end{matrix} & \begin{matrix} \\ C \end{matrix} \\ \begin{bmatrix} 0.65 & 0.55 \\ 0.35 & 0.45 \end{bmatrix} & \end{matrix} \begin{matrix} \\ D \\ C \end{matrix} next\ Friday$$

Question 27 / 39

[VCAA 2021 FM (73%)]
Ramon and Norma are names that contain the same letters but in a different order.

The permutation matrix that can change [RAMON] into [NORMA] is

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A. $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

Question 28 / 39

[VCAA 2021 FM (45%)]
A is a 7×7 matrix.

B is a 10×7 matrix.

Which one of the following matrix expressions is defined?

A. AB−2B

B. $A(BA)^{-1}$

C. AB^2

D. A^2−BA

E. A(B^T)

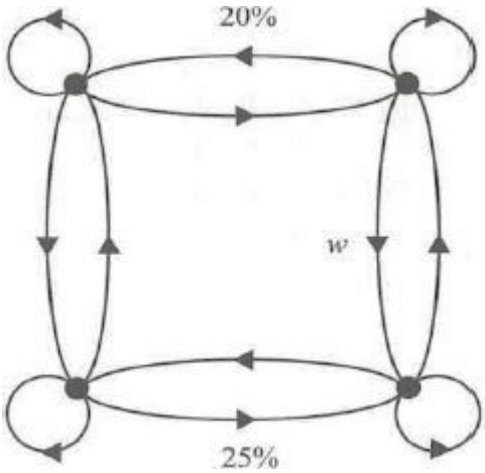
Question 29 / 39

[VCAA 2021 FM (66%)]
A fitness centre offers four different exercise classes: aerobics (A), boxfit (B), cardio (C) and dance (D).

A customer's choice of fitness class is expected to change from week to week according to the transition matrix P, shown below.

$$P = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0.65 & 0 & 0.2 & 0.1 \\ 0 & 0.65 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.7 & 0 \\ 0.15 & 0.25 & 0 & 0.6 \end{bmatrix} \end{array} \begin{array}{l} \\ A \\ B \\ C \\ D \end{array} \begin{array}{l} \\ \\ next\ week \\ \\ \end{array}$$
$$this\ week$$

An equivalent transition diagram has been constructed below, but the labelling is not complete.



The proportion for one of the transitions is labelled w.

The value of w is

A. 10%

B. 15%

C. 20%

D. 25%

E. 30%

Question 30 / 39

[VCAA 2021 FM (40%)]
The matrix Sn+1 is determined from the matrix Sn using the recurrence relation

$S_{n+1} = T \times S_n - C$, where

$$T = \begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.3 & 0.8 & 0.2 \\ 0.1 & 0.1 & 0.5 \end{bmatrix}, S_0 = \begin{bmatrix} 21 \\ 51 \\ 31 \end{bmatrix}, S_1 = \begin{bmatrix} 24.0 \\ 54.3 \\ 20.7 \end{bmatrix}$$

and C is a column matrix.

Matrix S2 is equal to

A. $S_2 = \begin{bmatrix} 23.04 \\ 55.78 \\ 16.18 \end{bmatrix}$

B. $S_2 = \begin{bmatrix} 25.34 \\ 56.28 \\ 17.38 \end{bmatrix}$

C. $S_2 = \begin{bmatrix} 26.04 \\ 54.78 \\ 18.18 \end{bmatrix}$

D. $S_2 = \begin{bmatrix} 28.34 \\ 55.28 \\ 19.38 \end{bmatrix}$

E. $S_2 = \begin{bmatrix} 29.04 \\ 53.78 \\ 20.18 \end{bmatrix}$

Question 31 / 39

[VCAA 2021 FM (29%)]
A new colony of endangered marsupials is established on a remote island. For one week, the marsupials can feed from only one of three feeding stations: A, B or C. On Monday, 50% of the marsupials were observed feeding at station A and 50% were observed feeding at station B. No marsupials were observed feeding at station C.

The marsupials are expected to change their feeding stations each day this week according to the transition matrix T.

$$T = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} today & & \\ A & B & C \end{array} \\ \begin{bmatrix} 0.4 & 0.1 & 0.2 \\ 0.2 & 0.5 & 0.2 \\ 0.4 & 0.4 & 0.6 \end{bmatrix} \end{array} \begin{array}{l} \\ A \\ Btomorrow \\ C \end{array}$$

Let S_n represent the state matrix showing the percentage of marsupials observed feeding at each feeding station n days after Monday of this week.

The matrix recurrence rule $S_{n+1} = TS_n$ is used to model this situation.

From Tuesday to Wednesday, the percentage of marsupials who are **not** expected to change their feeding location is

A. 44.5%

B. 45%

C. 50%

D. 51.5%

E. 52%

Question 32 / 39

Use the following information to answer Questions 33 and 34.

A bike rental business rents road bikes (R) and mountain bikes (M) in three sizes: child (C), junior (J) and adult (A). Matrix B shows the daily rental cost, in dollars, for each type of bike. The element in row i and column j in matrix B is by.

$$B = \begin{matrix} \\ C \\ J \\ A \end{matrix} \begin{matrix} \begin{matrix} R & M \end{matrix} \\ \begin{bmatrix} 80 & 95 \\ 110 & 120 \\ 120 & 135 \end{bmatrix} \end{matrix}$$

Question 33 / 39

[VCAA 2022 FM (83%)]
The daily cost of renting an adult mountain bike is shown in element A.

A. b12

B. b21

C. b23

D. b31

E. b32

Question 34 / 39

[VCAA 2022 FM (68%)]
On Sundays, the business increases the daily rental price for each type of bike by 10%. To determine the rental cost for each type of bike on a Sunday, which one of the following matrix calculations needs to be completed?

A. 0.01B

B. 0.1B

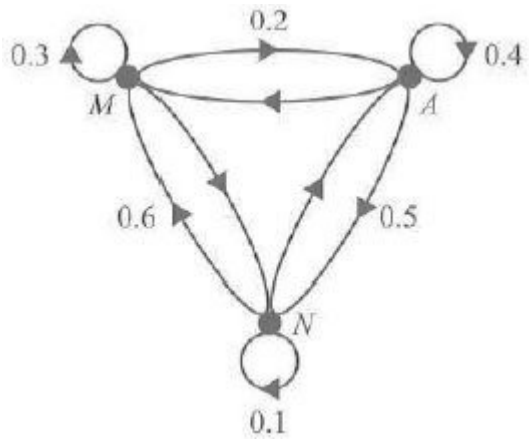
C. 1.01B

D. 1.1B

E. 11B

Question 35 / 39

[VCAA 2022 FM (71%)]
Each day, members of a swim centre can choose to attend a morning session (M), an afternoon session (A) or no session (N). The transition diagram right shows the transition from day to day. The transition diagram is incomplete.



Which one of the following transition matrices represents this transition diagram?

A.

$$\begin{matrix} & this\ day & \\ M & A & N \\ \begin{bmatrix} 0.2 & 0.1 & 0.2 \\ 0.3 & 0.5 & 0.3 \\ 0.5 & 0.4 & 0.6 \end{bmatrix} & \begin{matrix} M \\ A \\ N \end{matrix} & \begin{matrix} \\ next\ day \\ \\ \end{matrix} \end{matrix}$$

B.

$$\begin{matrix} & this\ day & \\ M & A & N \\ \begin{bmatrix} 0.3 & 0.1 & 0.6 \\ 0.5 & 0.4 & 0.3 \\ 0.2 & 0.5 & 0.1 \end{bmatrix} & \begin{matrix} M \\ A \\ N \end{matrix} & \begin{matrix} \\ next\ day \\ \\ \end{matrix} \end{matrix}$$

C.

$$\begin{matrix} & this\ day & \\ M & A & N \\ \begin{bmatrix} 0.3 & 0.1 & 0.6 \\ 0.2 & 0.4 & 0.3 \\ 0.5 & 0.5 & 0.1 \end{bmatrix} & \begin{matrix} M \\ A \\ N \end{matrix} & \begin{matrix} \\ next\ day \\ \\ \end{matrix} \end{matrix}$$

D.

$$\begin{matrix} & this\ day & \\ M & A & N \\ \begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.1 & 0.4 & 0.7 \\ 0.6 & 0.1 & 0.1 \end{bmatrix} & \begin{matrix} M \\ A \\ N \end{matrix} & \begin{matrix} \\ next\ day \\ \\ \end{matrix} \end{matrix}$$

E.

$$\begin{matrix} & this\ day & \\ M & A & N \\ \begin{bmatrix} 0.3 & 0 & 0.6 \\ 0.2 & 0.4 & 0 \\ 0 & 0.5 & 0.1 \end{bmatrix} & \begin{matrix} M \\ A \\ N \end{matrix} & \begin{matrix} \\ next\ day \\ \\ \end{matrix} \end{matrix}$$

Question 36 / 39

[VCAA 2022 FM (87%)]
The communication matrix below shows the communication links between five people: Steph (S), Tran (7), Ursula (U), Vinh (V) and Wanda (W).

$$\begin{array}{cc} & sender \\ & \begin{array}{ccccc} S & T & U & V & W \end{array} \\ \begin{array}{c} S \\ T \\ sender\ U \\ V \\ W \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

In this matrix:

- the '1' in row S, column T indicates that Steph can communicate directly with Tran
- the '0' in row V, column W indicates that Vinh cannot communicate directly with Wanda.

Ursula needs to communicate with Steph.

The sequence of communication links that will successfully allow Ursula to communicate with Steph is

---

A. U-T-S

B. U-W-S

C. U-T-W-S

D. U–V–T–S

E. U–W–T–S

Question 37 / 39

[VCAA 2022 FM (71%)]
Matrix E is a 2×2 matrix. Matrix F is a 2×3 matrix. Matrix G is a 3×2 matrix.

Matrix H is a 3×3 matrix.

Which one of the following matrix products could have an inverse?

A. EF

B. FH

C. GE

D. GF

E. HG

Question 38 / 39

[VCAA 2022 FM (42%)]
Matrix K is a permutation matrix.

$$K = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$ K=[10000]

Matrix M is a column matrix that is multiplied once by matrix K to obtain matrix P.

When matrix M is multiplied by matrix K, the element m_{31} moves to element

---

A. p_{11}

B. p_{21}

C. p_{31}

D. p_{41}

E. p_{51}

Question 39 / 39

[VCAA 2022 FM (31%)]
Two types of computers – laptops (L) and desktops (D) – can be erviced by Henry (H), Irvine (I) or Jean (J). Matrix N shows the time, in minutes, it takes each person to service a laptop and a desktop.

$$N = \begin{matrix} \\ H \\ I \\ J \end{matrix} \begin{matrix} \begin{matrix} L & D \end{matrix} \\ \begin{bmatrix} 18 & 8 \\ 10 & 17 \\ 12 & 9 \end{bmatrix} \end{matrix}$$

Matrix Q shows the number of laptops and desktops in four different departments: marketing (M), advertising (A), publishing (P) and editing (E).

$$N = \begin{matrix} \\ M \\ A \\ P \\ E \end{matrix} \begin{matrix} \begin{matrix} L & D \end{matrix} \\ \begin{bmatrix} 6 & 8 \\ 4 & 7 \\ 5 & 5 \\ 10 & 12 \end{bmatrix} \end{matrix}$$

A calculation that determines the total time that it would take each of Henry, Irvine or Jean, working alone, to service all the laptops and desktops in all four departments is

---

A.

$$[1 \quad 1 \quad 1 \quad 1] \times (Q \times N^T)$$

B.

$$(Q \times N^T) \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

C.

$$(N \times Q^T) \times Q$$

D.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times N \times Q^T$$

E.

$$[1 \quad 1 \quad 1 \quad 1] \times Q \times N^T \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Question 1 / 18

[VCAA 2018 FM]
A toll road is divided into three sections, E, F and G. The cost, in dollars, to drive one journey on each section is shown in matrix C right.

$$C = \begin{bmatrix} 3.58 \\ 2.22 \\ 2.87 \end{bmatrix} \begin{matrix} E \\ F \\ G \end{matrix}$$

a. What is the cost of one journey on section G?

[1 mark (1.0)]

b. Write down the order of matrix C.

[1 mark (1.0)]

c. One day Kim travels once on section E and twice on section G. His total toll cost for this day can be found by the matrix product M×C. Write down the matrix M.

[1 mark (0.7)]

Question 2 / 18

[VCAA 2018 FM]
The Westhorn Council must prepare roads for expected population changes in each of three locations: main town (M), villages (V) and rural areas (R).

The population of each of these locations in 2018 is shown in matrix P_{2018} right.

$$P_{2019} = \begin{bmatrix} 2100 \\ 1800 \\ 1700 \end{bmatrix} \begin{matrix} M \\ V \\ R \end{matrix}$$

The expected annual change in population in each location is shown in the table below.

Location	main town	villages	rural areas
Annual change	increase by 4%	decrease by 1 %	decrease by 2%

a. Write down matrix P_{2019}, which shows the expected population in each location in 2019.

[1 mark (0.6)]

b. The expected population in each of the three locations in 2019 can be determined from the matrix product P_{2019}=F×P_{2018} where F is a diagonal matrix. Write down matrix F.

[1 mark (0.4)]

Question 3 / 18

[VCAA 2018 FM]
The Hiroads company has a contract to maintain and improve 2700 km of highway.

Each year sections of highway must be graded (G), resurfaced (R) or sealed (S).

The remaining highway will need no maintenance (N) that year.

Let Sn be the state matrix that shows the highway maintenance schedule for the nth year after 2018.

The maintenance schedule for 2018 is shown in matrix S_0 below.

$$S_0 = \begin{bmatrix} 700 \\ 400 \\ 200 \\ 1400 \end{bmatrix} \begin{matrix} G \\ R \\ S \\ N \end{matrix}$$

The type of maintenance in sections of highway varies from year to year, as shown in the transition matrix, T, below.

$$T = \begin{matrix} \\ \\ \\ \\ next\ year \end{matrix} \begin{matrix} \begin{matrix} this\ year \\ G \quad R \quad S \quad N \end{matrix} \\ \begin{bmatrix} 0.2 & 0.1 & 0 & 0.2 \\ 0.1 & 0.1 & 0 & 0.2 \\ 0.2 & 0.1 & 0.2 & 0.1 \\ 0.5 & 0.7 & 0.8 & 0.5 \end{bmatrix} \end{matrix} \begin{matrix} G \\ R \\ S \\ N \end{matrix}$$

a. Of the length of highway that was graded (G) in 2018, how many kilometres are expected to be resurfaced (R) the following year?

[1 mark (0.5)]

b. Show that the length of highway that is to be graded (G) in 2019 is 460 km by writing the appropriate numbers in the boxes below.

$$\square \times 700 + \square \times 400 + \square \times 200 + \square \times 1400 = 460$$

[1 mark (0.8)]

The state matrix describing the highway maintenance schedule for the nth year after 2018 is given by

$$S_{n+1} = TS_n$$

c. Complete the state matrix, S_1, below for the highway maintenance schedule for 2019 (one year after 2018).

$$S_1 = \begin{bmatrix} 460 \\ \_\_\_ \\ \_\_\_ \\ 1490 \end{bmatrix} \begin{matrix} G \\ R \\ S \\ N \end{matrix}$$

[1 mark (0.8)]

d. In 2020, 1536 km of highway is expected to require no maintenance (N).

Of these kilometres, what percentage is expected to have had no maintenance (N) in 2019? Round your answer to one decimal place.

[1 mark (0.2)]

e. In the long term, what percentage of highway each year is expected to have no maintenance (N)? Round your answer to one decimal place.

[1 mark (0.4)]

Question 4 / 18

[VCAA 2018 FM]
Beginning in the year 2021, a new company will take over maintenance of the same 2700 km highway with a new contract. Let Mn be the state matrix that shows the highway maintenance schedule of this company for the nth year after 2020.

The maintenance schedule for 2020 is shown in matrix M0 below.

For these 2700 km of highway, the matrix recurrence relation shown below can be used to determine the number of kilometres of this highway that will require each type of maintenance from year to year.

$$M_{n+1} = TM_n + B \quad \text{where}$$

$$M_0 = \begin{bmatrix} 500 \\ 400 \\ 300 \\ 1500 \end{bmatrix} \begin{matrix} G \\ R \\ S \\ N \end{matrix}$$

$$T = \begin{matrix} \\ \\ \\ \\ \text{next year} \end{matrix} \begin{matrix} \text{this year} \\ G \quad R \quad S \quad N \end{matrix} \begin{bmatrix} 0.2 & 0.1 & 0.0 & 0.2 \\ 0.1 & 0.1 & 0.0 & 0.2 \\ 0.2 & 0.1 & 0.2 & 0.1 \\ 0.5 & 0.7 & 0.8 & 0.5 \end{bmatrix} \begin{matrix} G \\ R \\ S \\ N \end{matrix}$$

$$B = \begin{bmatrix} k \\ 20 \\ 10 \\ -60 \end{bmatrix}$$

a. Write down the value of k in matrix B.

[1 mark (0.4)]

b. How many kilometres of highway are expected to be graded (G) in the year 2022?

[1 mark (0.3)]

Question 5 / 18

[VCAA 2019 FM]
The car park at a theme park has three areas, A, B and C. The number of empty (E) and full (F) parking spaces in each of the three areas at 1 pm on Friday are shown in matrix Q below.

$$Q = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} E & F \\ \end{matrix} \begin{bmatrix} 70 & 50 \\ 30 & 20 \\ 40 & 40 \end{bmatrix} area$$

a. What is the order of matrix Q?

[1 mark (1.0)]

b. Write down a calculation to show that 110 parking spaces are full at 1 pm.

Drivers must pay a parking fee for each hour of parking. Matrix P, below, shows the hourly fee, in dollars, for a car parked in each of the three areas.

$$P = \begin{matrix} \\ area \end{matrix} \begin{matrix} A & B & C \\ \end{matrix} \begin{bmatrix} 1.30 & 3.50 & 1.80 \end{bmatrix}$$

[1 mark (0.8)]

c. The total parking fee, in dollars, collected from these 110 parked cars if they were parked for one hour is calculated as follows.

$P \times L = [207.00]$ where matrix L is a 3×1 matrix.

Write down matrix L.

[1 mark (0.7)]

The number of whole hours that each of the 110 cars had been parked was recorded at l pm. Matrix R, below, shows the number of cars parked for one, two, three or four hours in each of the areas A, B and C.

$$R = \begin{matrix} \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{matrix} A & B & C \\ \end{matrix} \begin{bmatrix} 3 & 1 & 1 \\ 6 & 10 & 3 \\ 22 & 7 & 10 \\ 19 & 2 & 26 \end{bmatrix} area$$

d. Matrix R^T is the transpose of matrix R. Find the matrix R^T below.

[1 mark (0.8)]

e. Explain what the element in row 3, column 2 of matrix R^T represents.

[1 mark (0.6)]

Question 6 / 18

[VCAA 2019 FM]
The theme park has four locations, Air World (A), Food World (F), Ground World (G) and Water World (W).

The number of visitors at each of the four locations is counted every hour.

By 10 am on Saturday the park had reached its capacity of 2000 visitors and could take no more visitors.

The park stayed at capacity until the end of the day.

The state matrix, S0, below, shows the number of visitors at each location at 10 am on Saturday.

$$S_0 = \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix}$$

a. What percentage of the park's visitors were at Water World (W) at 10 am on Saturday?

[1 mark (0.9)]

Let Sn be the state matrix that shows the number of visitors expected at each location n hours after 10 am on Saturday.

The number of visitors expected at each location n hours after 10 am on Saturday can be determined by the matrix recurrence relation below.

$$S_0 = \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix}, \quad S_{n+1} = T \times S_n \quad \text{where } T = \begin{matrix} \\ A \\ F \\ G \\ W \end{matrix} \begin{matrix} \begin{matrix} A & F & G & W \end{matrix} \\ \begin{bmatrix} 0.1 & 0.2 & 0.1 & 0.2 \\ 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix} \end{matrix} \begin{matrix} \\ A \\ F \\ G \\ W \end{matrix} \quad next\ hours$$

b. Complete the state matrix, S1, below to show the number of visitors expected at each location at 11 am on Saturday.

$$S_1 = \begin{bmatrix} — \\ — \\ 300 \\ — \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix}$$

[1 mark (0.9)]

c. Of the 300 visitors expected at Ground World (G) at 11 am, what percentage was at either Air World (A) or Food World (F) at 10 am?

[1 mark (0.3)]

d. The proportion of visitors moving from one location to another each hour on Sunday is different from Saturday.

Matrix V, below, shows the proportion of visitors moving from one location to another each hour after 10 am on Sunday.

$$V = \begin{matrix} \\ A \\ F \\ G \\ W \end{matrix} \overset{\overset{this\ hour}{\begin{matrix} A & F & G & W \end{matrix}}}{\begin{bmatrix} 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix}} \begin{matrix} A \\ F \\ G \\ W \end{matrix} next\ hour$$

Matrix V is similar to matrix T but has the first two rows of matrix T interchanged.

The matrix product that will generate matrix V from matrix T is V=M×T where matrix M is a binary matrix.

Write down matrix M.

[1 mark (0.2)]

Question 7 / 18

[VCAA 2019 FM]
On Sunday, matrix V is used when calculating the expected number of visitors at each location every hour after 10 am. It is assumed that the park will be at its capacity of 2000 visitors for all of Sunday.

Let L_0 be the state matrix that shows the number of visitors at each location at 10 am on Sunday.

The number of visitors expected at each location at 11 am on Sunday can be determined by the matrix product

$$V \times L_0 \quad \text{where} \quad L_0 = \begin{bmatrix} 500 \\ 600 \\ 500 \\ 400 \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix} \quad \text{and} \quad V = \begin{matrix} \\ A \\ F \\ G \\ W \end{matrix} \overset{\text{this hour}}{\begin{bmatrix} A & F & G & W \\ 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix}} \text{next hours}$$

a. Safety restrictions require that all four locations have a maximum of 600 visitors.

Which location is expected to have more than 600 visitors at 11 am on Sunday?

[1 mark (0.8)]

b. Whenever more than 600 visitors are expected to be at a location on Sunday, the first 600 visitors can stay at that location and all others will be moved directly to Ground World (G).

State matrix Rn contains the number of visitors at each location n hours after 10 am on Sunday, after the safety restrictions have been enforced.

Matrix R1 can be determined from the matrix recurrence relation

$$R_0 = \begin{bmatrix} 500 \\ 600 \\ 500 \\ 400 \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix}, R_1 = V \times R_0 + B_1$$

where matrix B1 shows the required movement of visitors at 11 am.

i. Determine the matrix B1.

[1 mark (0.3)]

ii. State matrix R2 can be determined from the new matrix rule

$R_2 = V \times R_1 + B_2$where matrix B_2 shows the required movement of visitors at 12 noon.

Determine the state matrix R_2.

[1 mark (0.1)]

Question 8 / 18

[VCAA 2020 FM]
The three major shopping centres in a large city, Eastmall (E), Grandmall (G) and Westmall (W), are owned by the same company.

The total number of shoppers at each of the centres at 1.00 pm on a typical day is shown in matrix V.

$$V = \begin{matrix} E & G & W \end{matrix} \\ \begin{bmatrix} 2300 & 2700 & 2200 \end{bmatrix}$$

a. Write down the order of matrix V.

[1 mark (1.0)]

Each of these centres has three major shopping areas: food (F), clothing (C) and merchandise (M).

The proportion of shoppers in each of these three areas at 1.00 pm on a typical day is the same at all three centres and is given in matrix P below.

$$P = \begin{bmatrix} 0.48 \\ 0.27 \\ 0.25 \end{bmatrix} \begin{matrix} F \\ C \\ M \end{matrix}$$

b. Grandmall's management would like to see 700 shoppers in its merchandise area at 1.00 pm.

If this were to happen, how many shoppers, in total, would be at Grandmall at this time?

[1 mark (0.5)]

c. The matrix Q=P×V is shown below. Two of the elements of this matrix are missing.

$$Q = \begin{matrix} E & G & W \end{matrix} \\ \begin{bmatrix} 1104 & \_\_\_ & 1056 \\ 621 & \_\_\_ & 594 \\ 575 & 675 & 550 \end{bmatrix} \begin{matrix} F \\ C \\ M \end{matrix}$$

i. Complete **matrix Q above** by filling in the missing elements.

(*Answer on matrix Q above*.)

[1 mark (0.9)]

ii. The element in row i and column j of matrix Q is q_{ij}.

What does the element q_{23} represent?

[1 mark (0.7)]

The average daily amount spent, in dollars. by each shopper in each of the three areas at Grandmall in 2019 is shown in matrix A_{2019} below.

$$A_{2019} = \begin{bmatrix} 21.30 \\ 34.00 \\ 14.70 \end{bmatrix} \begin{matrix} F \\ C \\ M \end{matrix} =$$

On one day, 135 shoppers spent the average daily amount on food, 143 shoppers spent the average daily amount on clothing and 131 shoppers spent the average daily amount on merchandise.

d. Write a matrix calculation, using matrix A_{2019}, showing that the total amount spent by all these shoppers is $9663.20.

[1 mark (0.6)]

e. In 2020, the average daily amount spent by each shopper was expected to change by the percentage shown in the table below.

Area	food	clothing	merchandise
Expected change	increase by 5%	decrease by 15%	decrease by 1 %

The average daily amount, in dollars, expected to be spent in each area in 2020 can be determined by forming the matrix product

$$A_{2020} = K \times A_{2019}$$

Write down matrix K.

[1 mark (0.2)]

Question 9 / 18

[VCAA 2020 FM]
An offer to buy the Westmall shopping centre was made by a competitor.

One market research project suggested that if the Westmall shopping centre were sold, each of the three centres (Westmall, Grandmall and Eastmall) would continue to have regular shoppers but would attract and lose shoppers on a weekly basis.

Let Sn be the state matrix that shows the expected number of shoppers at each of the three centres n weeks after Westmall is sold.

A matrix recurrence relation that generates values of Sn is

$$S_{n+1} = T \times S_n$$

where

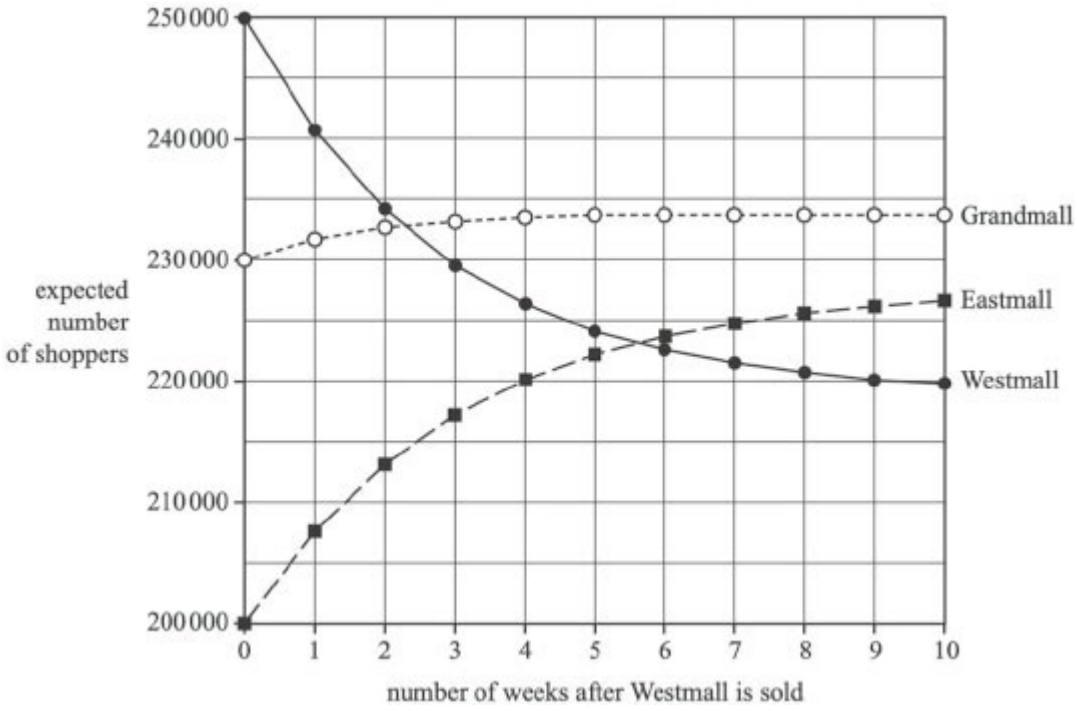
$$T = \begin{array}{c} \\ W \\ G \\ E \end{array} \begin{array}{c} \begin{array}{ccc} \text{this week} & & \\ W & G & E \end{array} \\ \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.79 & 0.10 \\ 0.08 & 0.12 & 0.80 \end{bmatrix} \end{array} \begin{array}{l} W \\ G \\ E \end{array} \text{ next week,} \qquad S_0 = \begin{bmatrix} 250\,000 \\ 230\,000 \\ 200\,000 \end{bmatrix} \begin{array}{l} W \\ G \\ E \end{array}$$

a. Calculate the state matrix, S1, to show the expected number of shoppers at each of the three centres one week after Westmall is sold.

$$S_1 = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix} \begin{array}{l} W \\ G \\ E \end{array}$$

[1 mark (0.8)]

Using values from the recurrence relation on page 160, the graph below shows the expected number of shoppers at Westmall, Grandmall and Eastmall for each of the 10 weeks after Westmall is sold.



b. What is the difference in the expected weekly number of shoppers at Westmall from the time Westmall is sold to 10 weeks after Westmall is sold?

Give your answer correct to the nearest thousand.

[1 mark (0.6)]

c. Grandmall is expected to achieve its maximum number of shoppers sometime between the fourth and the tenth week after Westmall is sold.

Write down the week number in which this is expected to occur.

[1 mark (0.3)]

d. In the long term, what is the expected weekly number of shoppers at Westmall? Round your answer to the nearest whole number.

[1 mark (0.4)]

Question 10 / 18

[VCAA 2020 FM]

A second market research project also suggested that if the Westmall shopping centre were sold, each of the three centres (Westmall, Grandmall and Eastmall) would continue to have regular shoppers but would attract and lose shoppers on a weekly basis. Let Rn be the state matrix that shows the expected number of shoppers at each of the three centres n weeks after Westmall is sold.

A matrix recurrence relation that generates values of R_n is $R_{n+1} = TR_n + B$, where

$$T = \begin{array}{c} \\ W \\ G \\ E \end{array} \begin{array}{c} \begin{array}{ccc} \text{this week} & & \\ W & G & E \end{array} \\ \begin{bmatrix} 0.78 & 0.13 & 0.10 \\ 0.12 & 0.82 & 0.10 \\ 0.10 & 0.05 & 0.80 \end{bmatrix} \end{array} \begin{array}{l} W \\ G \\ E \end{array} \text{next week}, \qquad B = \begin{bmatrix} -400 \\ 700 \\ 500 \end{bmatrix} \begin{array}{l} W \\ G \\ E \end{array}$$

The matrix R2 is the state matrix that shows the expected number of shoppers at each of the three centres in the second week after Westmall is sold.

$$R_2 = \begin{bmatrix} 239 & 60 \\ 250 & 540 \\ 192 & 900 \end{bmatrix} \begin{array}{l} W \\ G \\ E \end{array}$$

a. Determine the expected number of shoppers at Westmall in the third week after it is sold.

[1 mark (0.5)]

b. Determine the expected number of shoppers at Westmall in the first week after it is sold.

[1 mark (0.2)]

Question 11 / 18

[VCAA 2021 FM]
Elena imports three brands of olive oil: Carmani (C), Linelli (L) and Ghana (O).

The number of 1 litre bottles of these oils sold in January 2021 is shown in matrix J below.

$$J = \begin{bmatrix} 2800 \\ 1700 \\ 2400 \end{bmatrix} \begin{matrix} C \\ L \\ O \end{matrix}$$

a. What is the order of matrix J?

[1 mark (1.0)]

b. Elena expected that in February 2021 the sales of all three brands of olive oil would increase by 5%. She multiplied matrix J by a scalar value, k, to determine the expected volume of sales for February. Write down the value of the scalar k.

[1 mark (0.5)]

Question 12 / 18

[VCAA 2021 FM]
The main computer system in Elena's office has broken down.

The five staff members, Alex (A), Brie (B), Chai (C), Dex (D) and Elena (E), are having problems sending information to each other.

Matrix M below shows the available communication links between the staff members.

$$M = \begin{array}{c} \\ \text{sender} \end{array} \begin{array}{c} \begin{array}{ccccc} \multicolumn{5}{c}{\textit{receiver}} \\ A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \\ D \\ E \end{array}$$

In this matrix:

- the '1' in row A, column B indicates that Alex can send information to Brie
- the '0' in row D, column C indicates that Dex cannot information to Chai.

a. Which two staff members can send information directly to each other?

[1 mark (0.9)]

b. Elena needs to send documents to Chai.

What is the sequence of communication links that will successfully get the information from Elena to Chai?

[1 mark (0.9)]

c. Matrix M2 below is the square of matrix M and shows the number of two-step communication links between each pair of staff members.

$$M^2 = \begin{array}{c} \\ \text{sender} \end{array} \begin{array}{c} \begin{array}{ccccc} \multicolumn{5}{c}{\textit{receiver}} \\ A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \\ D \\ E \end{array}$$

Only one pair of individuals has two different two-step communication links. List each two-step communication link for this pair.

[1 mark (0.2)]

Question 13 / 18

[VCAA 2021 FM]
A market research study of shoppers showed that the buying preferences for the three olive oils, Carmani (C), Linelli (L) and Ohana (O), change from month to month according to the transition matrix T below.

$$T = \begin{array}{c} \\ C \\ L \\ O \end{array} \begin{array}{c} \begin{array}{ccc} \text{this month} & & \\ C & L & O \end{array} \\ \begin{bmatrix} 0.85 & 0.10 & 0.05 \\ 0.05 & 0.80 & 0.05 \\ 0.10 & 0.10 & 0.90 \end{bmatrix} \end{array} \begin{array}{l} C \\ L \\ O \end{array} \quad \text{next month}$$

The initial state matrix S0 below shows the number of shoppers who bought each brand of olive oil in July 2021.

$$S_0 = \begin{bmatrix} 3200 \\ 2000 \\ 2800 \end{bmatrix} \begin{matrix} C \\ L \\ O \end{matrix}$$

Let Sn represent the state matrix describing the number of shoppers buying each brand n months after July 2021.

a. How many of these 8000 shoppers bought a different brand of olive oil in August 2021 from the brand bought in July 2021?

[1 mark (0.3)]

b. Using the rule $S_{n+1} = T \times S_n$, complete the matrix S_1 below.

[1 mark (0.8)]

$$S_1 = \begin{bmatrix} 3060 \\ \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix} \begin{matrix} C \\ L \\ O \end{matrix}$$

c. Consider the shoppers who were expected to buy Carmani olive oil in August 2021. What percentage of these shoppers also bought Carmani olive oil in July 2021? Round your answer to the nearest percentage.

[1 mark (0.2)]

d. Write a calculation that shows that Ohana olive oil is the brand bought by 50% of these shoppers in the long run.

[1 mark (0.2]

e. Further research suggests more shoppers will buy olive oil in the coming months. A rule to model this situation is $R_{n+1} = T \times R_n + B$, where Rn represents the state matrix describing the number of shoppers n months after July 2021.

$$T = \begin{array}{c} \\ C \\ L \\ O \end{array} \begin{array}{c} \text{this month} \\ \begin{bmatrix} 0.85 & 0.10 & 0.05 \\ 0.05 & 0.80 & 0.05 \\ 0.10 & 0.10 & 0.90 \end{bmatrix} \end{array} \begin{array}{c} \\ C \\ L \\ O \end{array} \quad \text{next month,} \quad B = \begin{bmatrix} 200 \\ 100 \\ k \end{bmatrix} \begin{array}{c} C \\ L \\ O \end{array}, \quad R_0 = \begin{bmatrix} 3200 \\ 2000 \\ 2800 \end{bmatrix} \begin{array}{c} C \\ L \\ O \end{array}$$

k represents the extra number of shoppers expected to buy Ohana olive oil each month.

If $R_2 = \begin{bmatrix} 3333 \\ 2025 \\ 3642 \end{bmatrix}$ what is the value of k?

[1 mark (0.2)]

Question 14 / 18

[VCAA 2021 FM]
Five staff members in Elena's office played a round-robin video game tournament, where each employee played each of the other employees once. In each game there was a winner and a loser.

A table of their one-step and two-step dominances was prepared to summarise the results.

Staff member	One-step dominance	Two-step dominance
Ike (I)	3	5
Joelene (J)	3	4
Katie (K)	1	1
Leslie (L)	1	2
Mikki (M)	2	4

Consider the results matrix shown below.

A '1' in this matrix shows that the player named in that row defeated the player named in that column.

A '0' in this matrix shows that the player named in that row lost to the player named in that column.

Use all of the information provided to complete the results matrix.

winner \ loser	I	J	K	L	M
I	0	—	—	—	—
J	—	0	—	—	—
K	0	0	0	1	0
L	—	—	—	0	—
M	—	—	—	—	0

[2 marks (0.4)]

Question 15 / 18

The following table represents a study of a particular population of marsupials, which has been divided into four age groups. The table gives the birth rate, survival rate and the initial population for each age group.

Age group (years)	Birth rate	Survival rate	Number of females (in 2022)
0	0	0.90	520
1	0.55	0.85	2130
2	0.50	0.50	960
3	0.45	0	70

a. Construct a Leslie matrix for the marsupial population.

b. Use the Leslie matrix to predict the marsupial population in 2030.

c. Estimate the long-term annual growth rate of the population, expressing your answer as a percentage correct to 1 decimal place.

Question 16 / 18

[VCAA 2022 FM]
Matrix C shows the nightly cost, in dollars, of three types of family accommodation at a ski resort: hostel (H), motel (M) and apartment (A).

$$C = \begin{matrix} H & M & A \end{matrix} \\ \begin{bmatrix} 80 & 140 & 270 \end{bmatrix}$$

The Dwyer family is planning to stay at the ski resort for five nights.

a. Complete the matrix equation below to show the cost of staying at each type of accommodation **for** five nights.

$$[\_\_\_\_\_] \times [80 \quad 140 \quad 270] = [\_\_\_\_ \_\_\_\_ \_\_\_\_]$$

[1 mark (1.0)]

b. The family has decided to stay at the motel for two nights and in an apartment for three nights.

Write down the column matrix A for which the product matrix CA gives the total accommodation cost for the five nights.

$$A = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix}$$

[1 mark (0.6)]

Question 17 / 18

[VCAA 2022 FM]
Students from a nearby school spend one school term skiing at the resort. One year, the school term begins on 16 July and ends on 22 September. The resort has three different types of ski runs, each of which is classified as beginner (B), intermediate (I) or advanced (A). Each day, the students use one of the three types of ski runs.

Matrix T below contains the proportion of students who are expected to change their choice of ski run from day to day.

$$T = \begin{array}{c} \\ B \\ I \\ A \end{array} \begin{array}{c} \begin{array}{ccc} \textit{this day} & & \\ B & I & A \end{array} \\ \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.3 & 0.7 & 0.2 \\ 0 & 0.2 & 0.8 \end{bmatrix} \end{array} \begin{array}{l} \\ B \\ I \\ A \end{array} \textit{next day}$$

Let Sn be the matrix that shows the number of students who choose each type of ski run n days after 16 July.

Matrix S0 below shows the number of skiers who chose each type of ski run on 16 July.

$$S_0 = \begin{bmatrix} 210 \\ 190 \\ 80 \end{bmatrix} \begin{matrix} B \\ I \\ A \end{matrix}$$

a. How many skiers are expected to choose the same ski run on 17 July?

[1 mark (0.6)]

b. Consider the skiers who are expected to choose the advanced ski run on 17 July.

What percentage of these skiers also chose the advanced ski run on 16 July?

Round your answer to the nearest whole number.

[1 mark (0.3)]

c. What is the maximum number of students expected to ski the intermediate ski run on any one day? Round your answer to the nearest whole number.

[1 mark (0.2)]

Question 18 / 18

[VCAA 2022 FM]
At the end of the school term, the students vote for who they want to captain the ski team for the next ski season. Three students – Ali, Lee and Max – have been nominated for captain.

Seven days before the end of the school term, the students were asked who they planned to vote for.

The following table shows each candidate and the number of students who plan to vote for each candidate at the start of the seven days.

Candidate	***Number of people who plan to vote for the candidate***
Ali	160
Lee	140
Max	180

Each day all students are asked who they plan to vote for.

It is expected that students may change the candidate they plan to vote for each day over the seven days as follows. Some percentages are missing.

- 40% of students who plan to vote for Ali one day will plan to vote for Ali the next day.
- 30% of students who plan to vote for Ali one day will plan to vote for Lee the next day.
- 50% of students who plan to vote for Lee one day will plan to vote for Lee the next day.
- 30% of students who plan to vote for Lee one day will plan to vote for Max the next day.
- 50% of students who plan to vote for Max one day will plan to vote for Max the next day.
- 20% of students who plan to vote for Max one day will plan to vote for Lee the next day.

a. How many students plan to vote for Ali after one day?

[1 mark (0.3)]

b. Max decides to withdraw his nomination after one day.

Matrix T below shows the proportion of students who change their preferred candidate from one day to the next for the remaining six days after Max withdraws his nomination.

$$T = \begin{array}{c} \\ B \\ I \\ A \end{array} \begin{array}{c} \begin{array}{ccc} \text{this day} & & \\ B & I & A \end{array} \\ \begin{bmatrix} 0.5 & 0.7 & 0.5 \\ 0.5 & 0.3 & 0.5 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} B \\ I \\ A \end{array} \text{ next day}$$

At the end of the seven days, who is expected to become captain of the ski team and how many votes will this person receive?

Round your answer to the nearest whole number.

[2 marks (0.8)]

Question 1 / 43

[VCAA 2018 FM (92%)]
Consider the graph with five isolated vertices shown below.

To form a tree, the minimum number of edges that must be added to the graph is

---

A. 1

B. 4

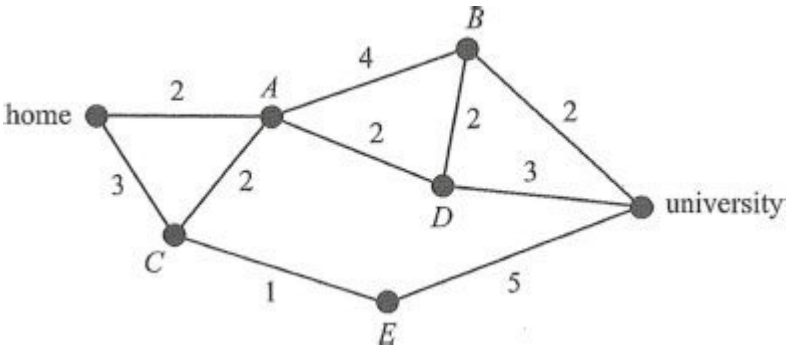
C. 5

D. 6

E. 10

Question 2 / 43

[VCAA 2018 FM (68%)]
Niko drives from his home to university. The network below shows the distances, in kilometres, along a series of streets connecting Niko's home to the university. The vertices A,B,C,D and E represent the intersection of these streets.



The shortest path for Niko from his home to the university could be found using

---

A. a minimum cut.

B. Prim's algorithm.

C. Dijkstra's algorithm.

D. critical path analysis.

E. the Hungarian algorithm.

Question 3 / 43

[VCAA 2018 FM (76%)]
A planar graph has five faces. This graph could have

A. eight vertices and eight edges.

B. six vertices and eight edges.

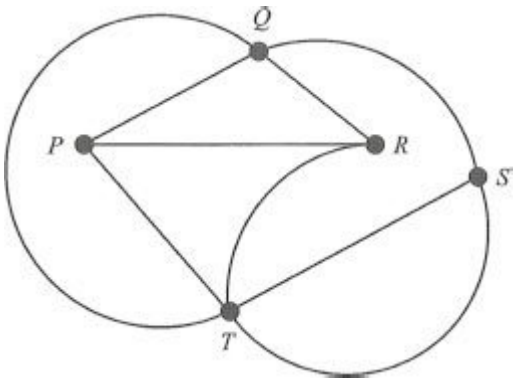
C. eight vertices and five edges.

D. eight vertices and six edges.

E. five vertices and eight edges.

Question 4 / 43

[VCAA 2018 FM (88%)]
Consider the graph right. Which one of the following is **not** a path for this graph?



A. PRQTS

B. PQRTS

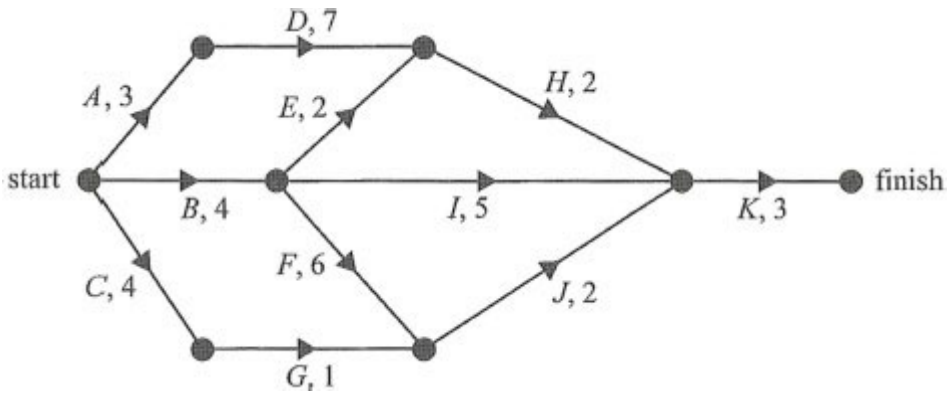
C. PRTSQ

D. PTQSR

E. PTRQS

Question 5 / 43

[VCAA 2018 FM (43%)]
The directed network below shows the sequence of 11 activities that are needed to complete a project. The time, in weeks, that it takes to complete each activity is also shown.



How many of these activities could be delayed without affecting the minimum completion time of the project?

A. 3

B. 4

C. 5

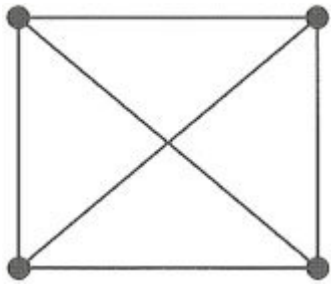
D. 6

E. 7

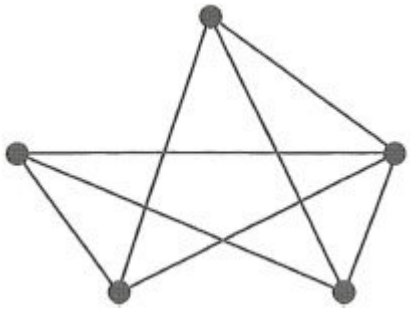
Question 6 / 43

[VCAA 2018 FM (41%)]
Which one of the following graphs is **not** a planar graph?

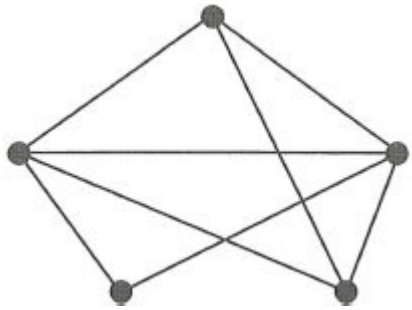
A.



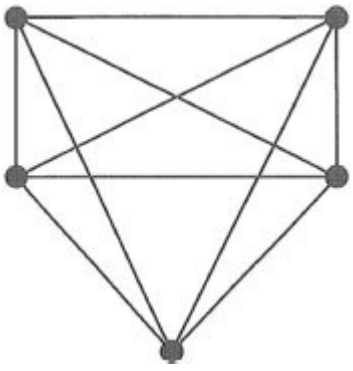
B.



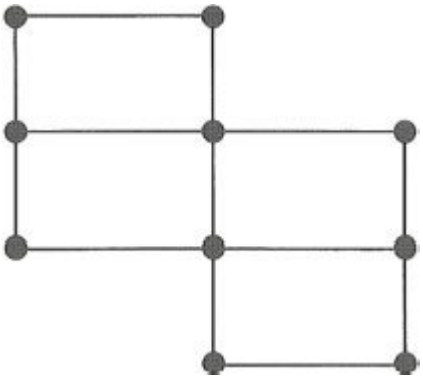
C.



D.



E.



Question 7 / 43

[VCAA 2018 FM (49%)]
A project requires nine activities (A–I) to be completed. The duration, in hours, and the immediate predecessor(s) of each activity are shown in the table below.

Acivity	**Duration (hours)**	**Immediate predecessor(s)**
A	4	–
B	3	A
C	7	A
D	2	A
E	5	B
F	2	C
G	4	E,F
H	5	D
I	3	G,H

The minimum completion time for this project, in hours, is

A. 14

B. 19

C. 20

D. 24

E. 35

Question 8 / 43

[VCAA 2018 FM (24%)]
Annie, Buddhi, Chuck and Dorothy work in a factory.

Today each worker will complete one of four tasks, 1, 2, 3 and 4.

The usual completion times for Annie, Chuck and Dorothy are shown in the table below.

	Task 1	**Task 2**	**Task 3**	**Task 4**
Annie	7	3	8	2
Buddhi	k	k	3	k
Chuck	5	6	9	2
Dorothy	4	8	5	3

Buddhi takes 3 minutes for Task 3. He takes k minutes for each other task.

Today the factory supervisor allocates the tasks as follows:

- Task 1 to Dorothy
- Task 2 to Annie
- Task 3 to Buddh
- Task 4 to Chuck

This allocation will achieve the minimum total completion time if the value of k is at least

A. 0

B. 1

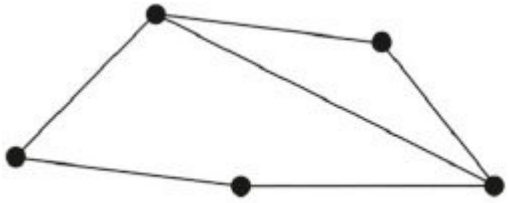
C. 2

D. 3

E. 4

Question 9 / 43

[VCAA 2019 FM (85%)]
In the graph shown to the right, the sum of the degrees of the vertices is



A. 5

B. 6

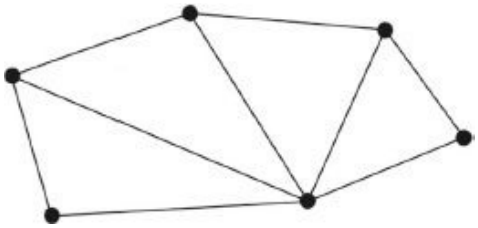
C. 10

D. 11

E. 12

Question 10 / 43

[VCAA 2019 FM (64%)]
Consider the graph below.



The minimum number of extra edges that are required so that an Eulerian circuit is possible in this graph is

---

A. 0

B. 1

C. 2

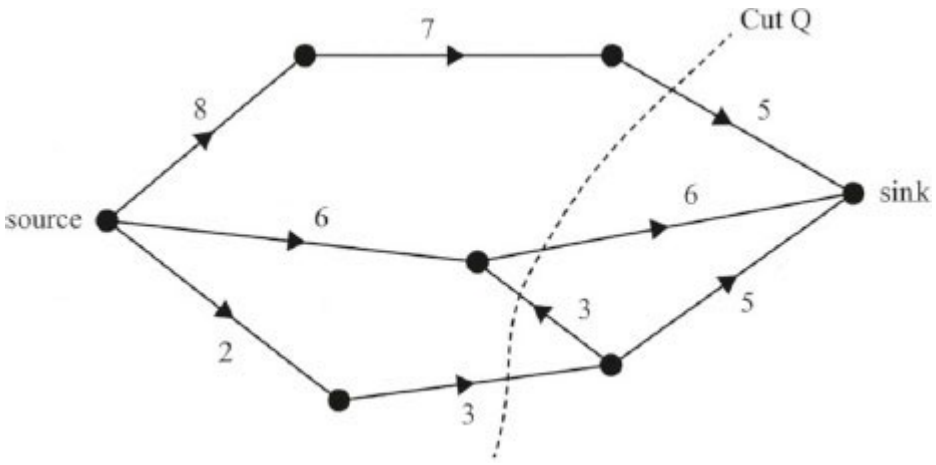
D. 3

E. 4

Question 11 / 43

[VCAA 2019 FM (85%)]
The flow of water through a series of pipes is shown in the network below.

The numbers on the edges show the maximum flow through each pipe in litres per minute.



The capacity of Cut Q, in litres per minute, is

A. 11

B. 13

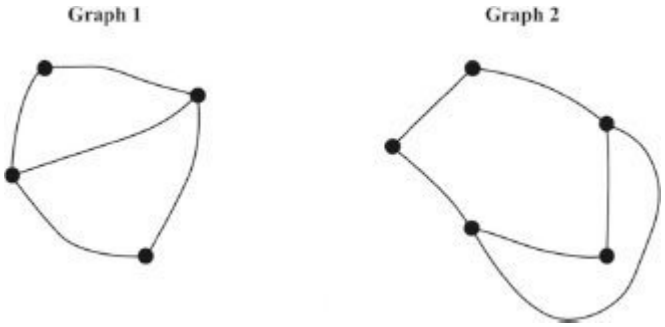
C. 14

D. 16

E. 17

Question 12 / 43

[VCAA 2019 FM (71%)]
Two graphs, labelled Graph 1 and Graph 2, are shown below.



Which one of the following statements is **not** true?

A. Graph 1 and Graph 2 are isomorphic.

B. Graph 1 has five edges and Graph 2 has six edges.

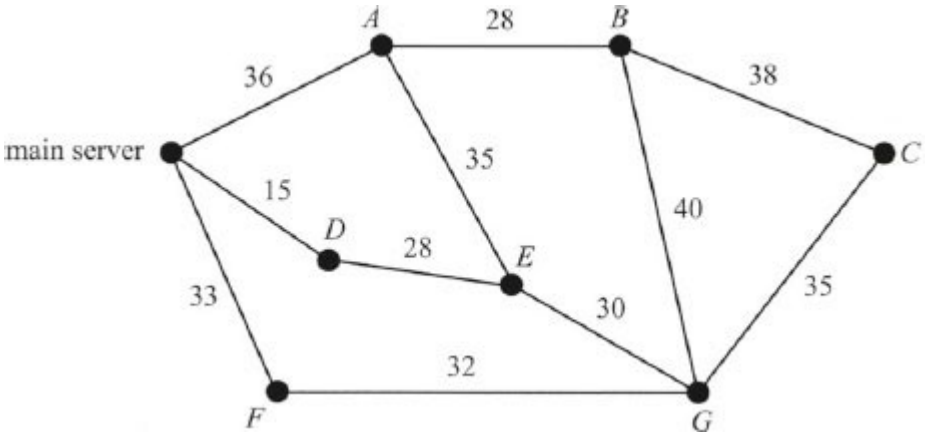
C. Both Graph 1 and Graph 2 are connected graphs.

D. Both Graph 1 and Graph 2 have three faces each.

E. Neither Graph 1 nor Graph 2 are complete graphs.

Question 13 / 43

[VCAA 2019 FM (75%)]
The following diagram shows the distances, in metres, along a series of cables connecting a main server to seven points, A to G, in a computer network.



The minimum length of cable, in metres, required to ensure that each of the seven points is connected to the main server directly or via another point is

---

A. 175

B. 203

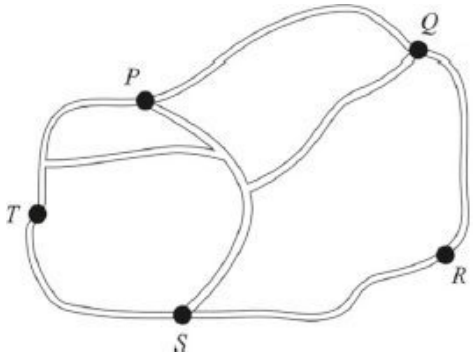
C. 208

D. 221

E. 236

Question 14 / 43

[VCAA 2019 FM (39%)]
The map below shows all the road connections between five towns, P,Q,R,S and T.



The road connections could be represented by the adjacency matrix

A.

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \end{array} \begin{array}{c} \begin{array}{ccccc} P & Q & R & S & T \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 2 & 2 \\ 3 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

B.

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \end{array} \begin{array}{c} \begin{array}{ccccc} P & Q & R & S & T \end{array} \\ \begin{bmatrix} 1 & 2 & 0 & 2 & 2 \\ 2 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

C.

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \end{array} \begin{array}{c} \begin{array}{ccccc} P & Q & R & S & T \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 2 & 2 \\ 3 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

D.

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \end{array} \begin{array}{c} \begin{array}{ccccc} P & Q & R & S & T \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 2 & 2 \\ 2 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 2 \\ 2 & 1 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

E.

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \end{array} \begin{array}{c} \begin{array}{ccccc} P & Q & R & S & T \end{array} \\ \begin{bmatrix} 1 & 2 & 0 & 2 & 2 \\ 2 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Question 15 / 43

[VCAA 2019 FM (42%)]

A project involves nine activities, A to I. The immediate predecessor(s) of each activity is shown in the table below.

Acivity	**Immediate predecessor(s)**
A	–
B	A
C	A
D	B
E	B,C
F	D
G	D
H	E,F
I	G,H

A directed network for this project will require a dummy activity. The dummy activity will be drawn from the end of

---

A. actvity B to the starof activity C.

B. actvity B to the starof activity E.

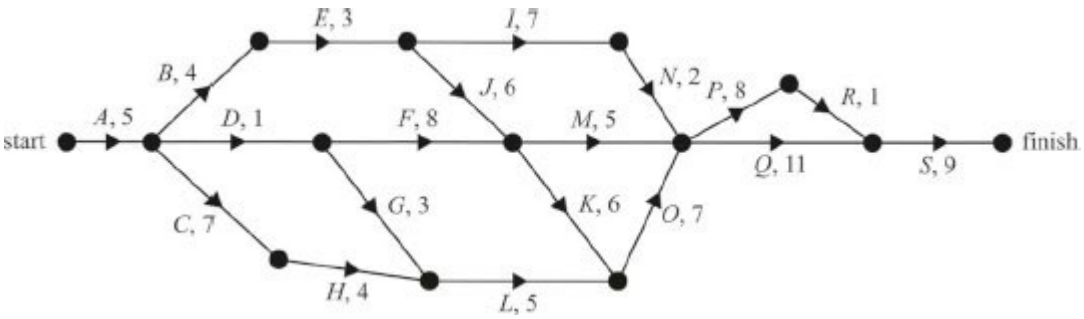
C. actvity D to the star f activity E.

D. actvity E to the starof activity H.

E. actvity E to the starof activity F.

Question 16 / 43

[VCAA 2019 FM (31%)]
The directed network below shows the sequence of activities, A to S, that is required to complete a manufacturing process. The time taken to complete each activity, in hours, is also shown.



The number of activities that have a float time of 10 hours is

A. 0

B. 1

C. 2

D. 3

E. 4

Question 17 / 43

[VCAA 2020 FM (89%)]
A connected planar graph has seven vertices and nine edges.

The number of faces that this graph will have is

A. 1

B. 2

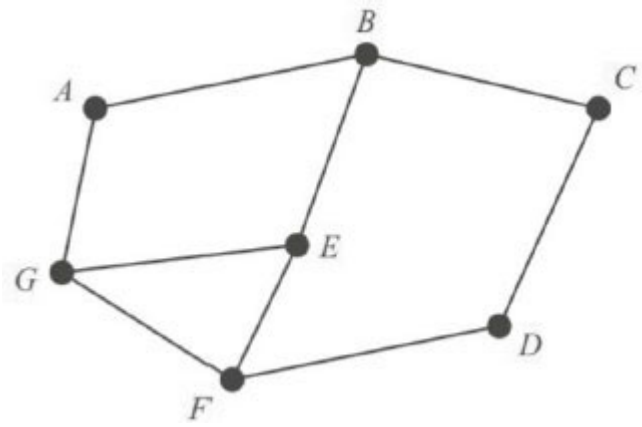
C. 3

D. 4

E. 5

Question 18 / 43

[VCAA 2020 FM (93%)]
Consider the graph below.



Which one of the following is **not** a Hamiltonian cycle for this graph?

A. ABCDFEGA

B. BAGEFDCB

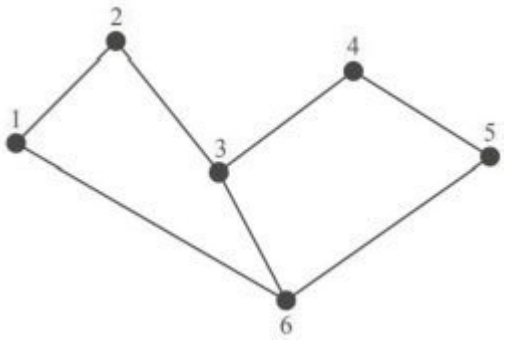
C. CDFEGABC

D. DCBAGFED

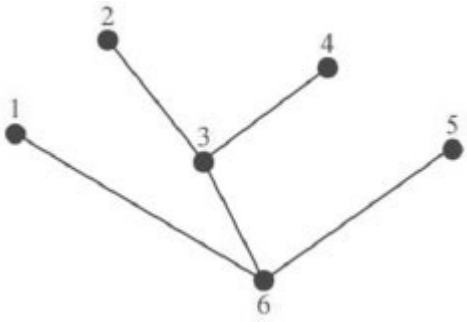
E. EGABCDFE

Question 19 / 43

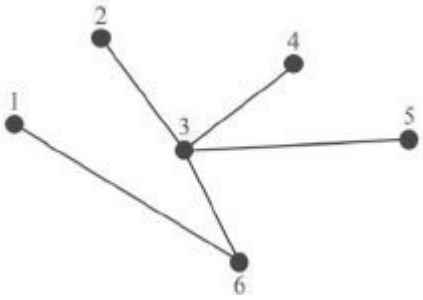
[VCAA 2020 FM (81%)]
Which one of the following is not a spanning tree for the network right?



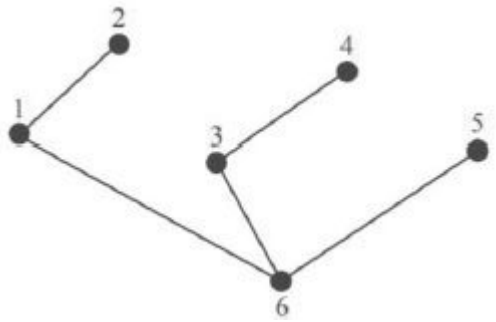
A.



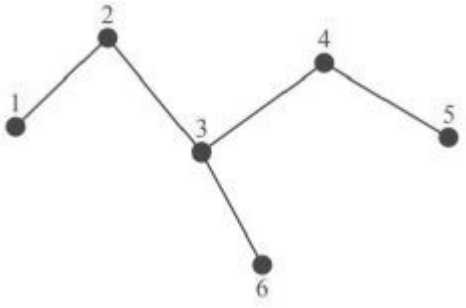
B.



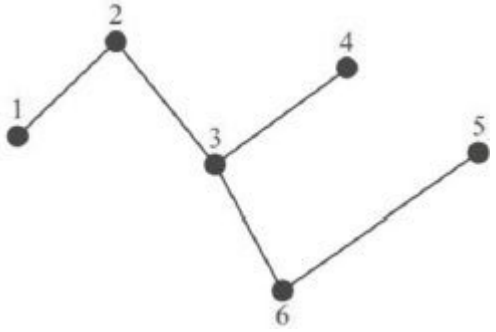
C.



D.



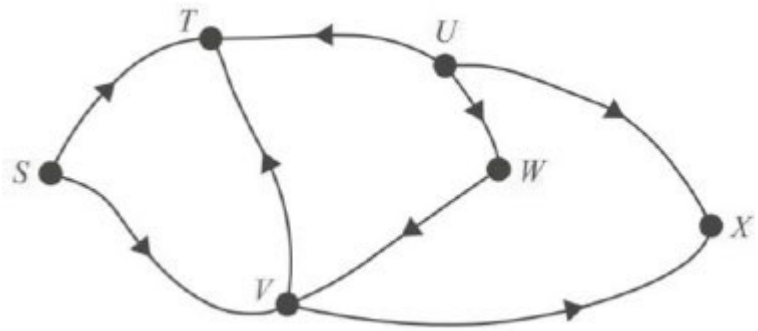
E.



Question 20 / 43

[VCAA 2020 FM (39%)]
The directed graph below represents a series of one-way streets.

The vertices represent the intersections of these streets.



The number of vertices that can be reached from **S** is

A. 1

B. 2

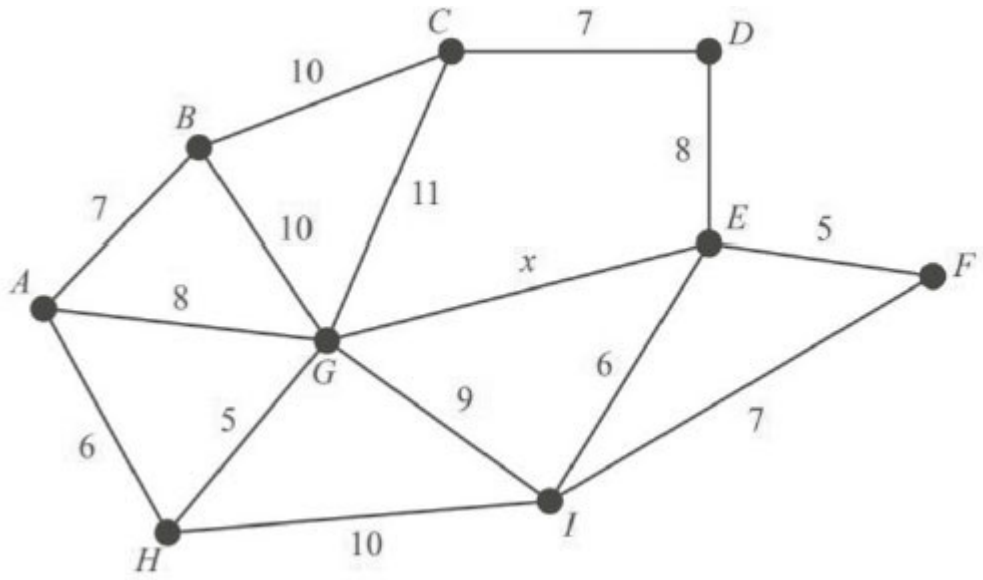
C. 3

D. 4

E. 5

Question 21 / 43

[VCAA 2020 FM (56%)]
The network below shows the distances, in metres, between camp sites at a camping ground that has electricity. The vertices A to I represent the camp sites.



The minimum length of cable required to connect all the camp sites is 53 m.

The value of x, in metres, is at least

A. 5

B. 6

C. 8

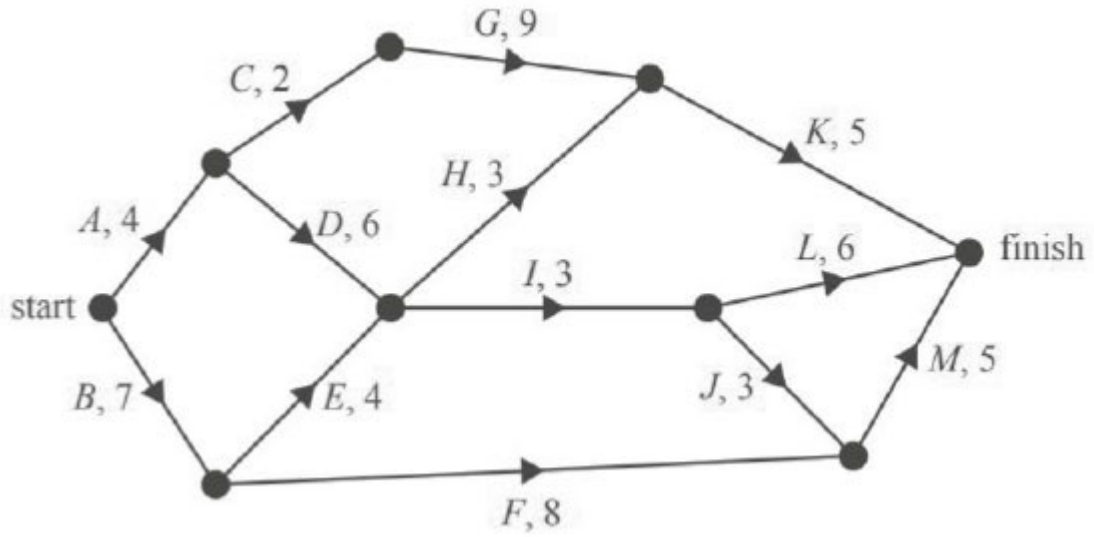
D. 9

E. 11

Question 22 / 43

[VCAA 2020 FM (61%)]
The activity network below shows the sequence of activities required to complete a project.

The number next to each activity in the network is the time it takes to complete that activity, in days.



The minimum completion time for this project, in days, is

A. 18

B. 19

C. 20

D. 21

E. 22

Question 23 / 43

[VCAA 2020 FM (11%)]
Four friends go to an ice-cream shop.

Akiro chooses chocolate and strawberry ice cream.

Doris chooses chocolate and vanilla ice cream.

Gohar chooses vanilla ice cream.

Imani chooses vanilla and lemon ice cream.

This information could be presented as a graph.

Consider the following four statements:

- The graph would be connected.
- The graph would be bipartite.
- The graph would be planar.
- The graph would be a tree.

How many of these four statements are true?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 24 / 43

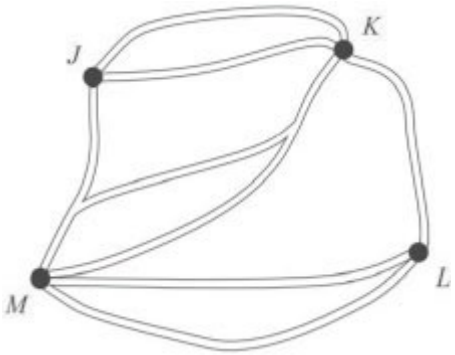
[VCAA 2020 FM (75%)]
The adjacency matrix right shows the number of pathway connections between four landmarks: J, K, L, and M.

$$\begin{array}{c} \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{cccc} J & K & L & M \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 2 \\ 3 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 \\ 2 & 2 & 2 & 0 \end{bmatrix} \end{array}$$

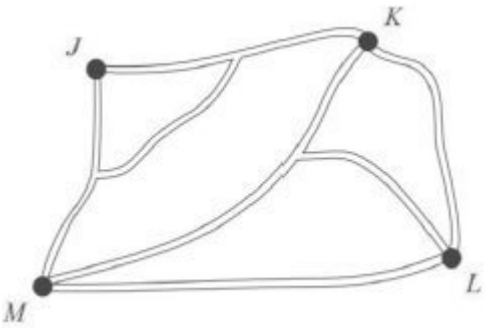
A network of pathways that could be represented by the adjacency matrix is

---

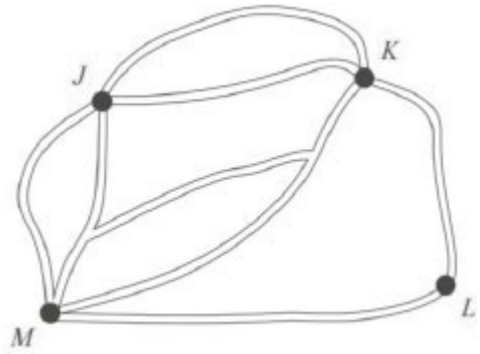
A.



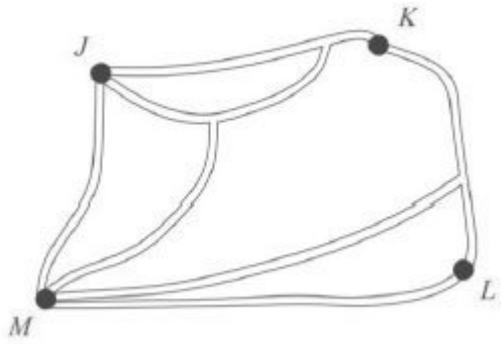
B.



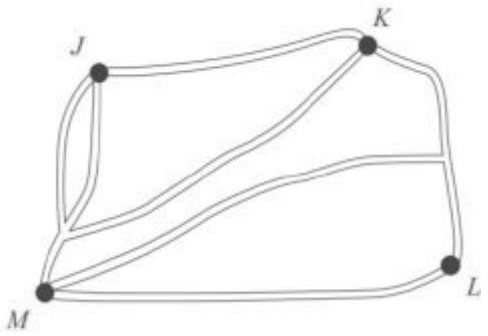
C.



D.

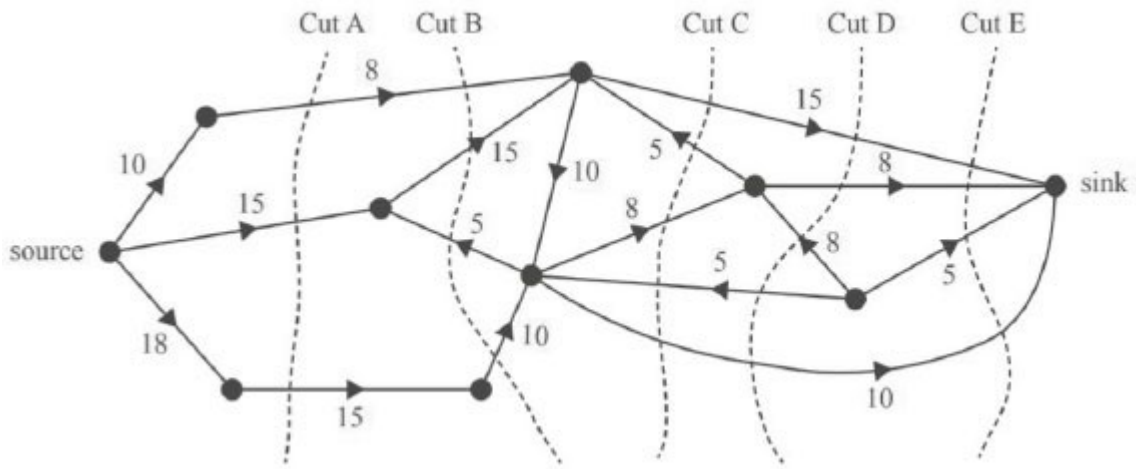


E.



Question 25 / 43

[VCAA 2020 FM (60%)]
The flow of liquid through a series of pipelines, in litres per minute, is shown in the directed network below.



Five cuts labelled A to E are shown on the network.

The number of these cuts with a capacity equal to the maximum flow of liquid from the source to the sink, in litres per minute, is

---

A. 1

B. 2

C. 3

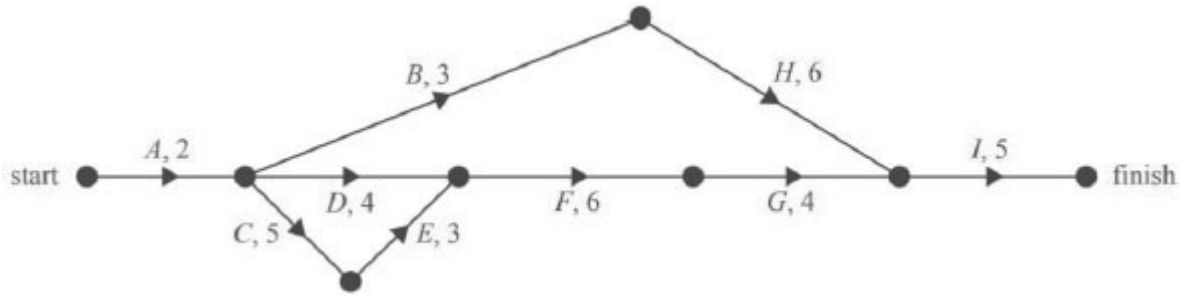
D. 4

E. 5

Question 26 / 43

[VCAA 2020 FM (24%)]
The directed network below shows the sequence of activities, A to I, that is required to complete an office renovation.

The time taken to complete each activity, in weeks, is also shown.



The project manager would like to complete the office renovation in less time.

The project manager asks ll theworkers assigned to activity H to also work on actvity F.

Ti will reduc the completion time of activity F to three weeks.

The work‹s assigned to activity H cannot work oboth activity H and activity F at the same time. No other activity times will be changed.

This change to the network will result in a change to the completion time of the office renovation. Which one of the following is correct?

A. The copltin time will be ed ed by one week if activity F is completed before actvity H is started.

B. The copltin time will be reduced by three weeks if activity F is completed before actvity H is started.

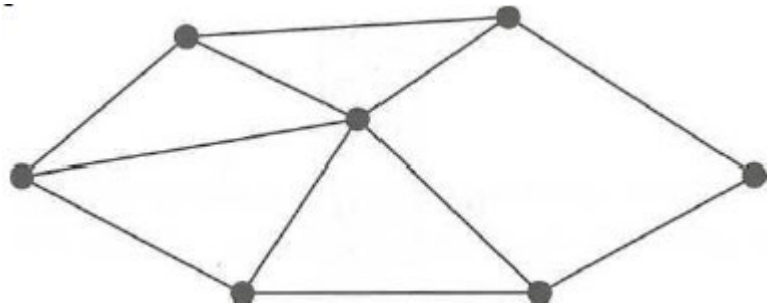
C. The copltin time will be ed ed by one week if activity H is completed before actvity F is started.

D. The copletion time will be reduced by three week if activity H is completed before actvity F is started.

E. The copletion tme will be ine ed by three weeks if activity H is completed before activity F *is* started.

Question 27 / 43

[VCAA 2021 FM (91%)]
Consider the graph below.



The number of vertices with a degree of 3 is

A. 1

B. 2

C. 3

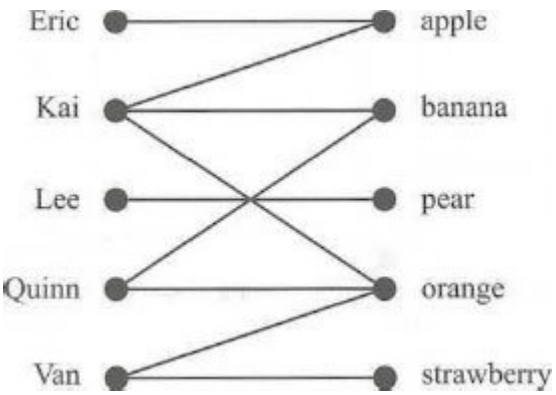
D. 4

E. 5

Question 28 / 43

[VCAA 2021 FM (95%)]
Five friends ate fruit for morning tea.

The bipartite graph below shows which types of fruit each friend ate.



Which one of the following statements is **not** true

A. Only Lee ate pear.

B. Eric and Kai each ate apple.

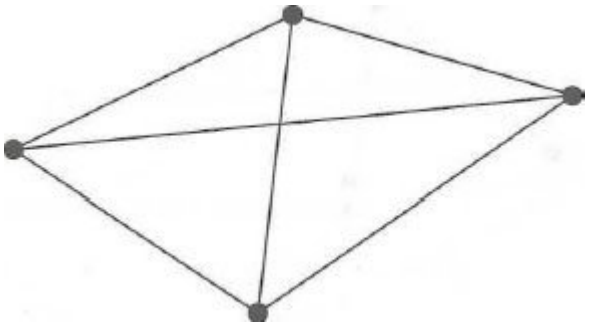
C. Van ate only strawberry.

D. Quinn and Kai each ate banana.

E. Orange was the most eaten type of fruit.

Question 29 / 43

[VCAA 2021 FM (34%)]
Consider the graph below.



The number of faces is

A. 2

B. 3

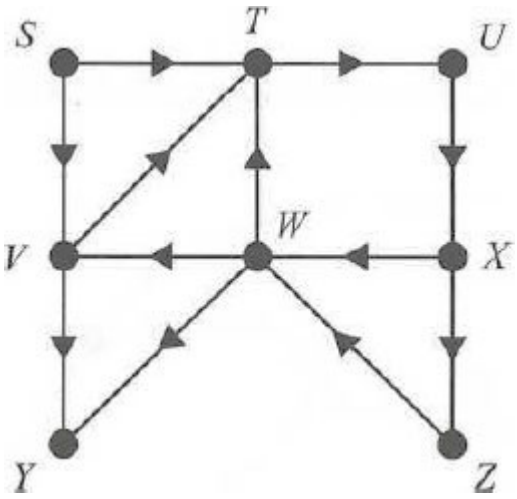
C. 4

D. 5

E. 6

Question 30 / 43

Consider the directed network below.



The number of vertices that **cannot** be reached from X is

A. 1

B. 2

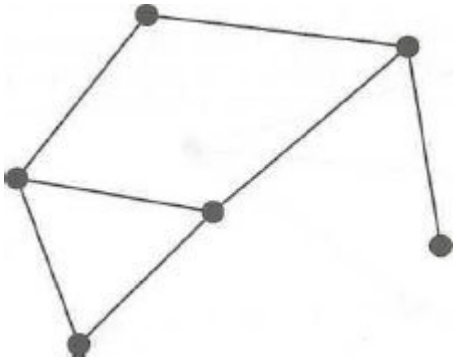
C. 3

D. 4

E. 5

Question 31 / 43

[VCAA 2021 FM (17%)]



Consider the following five statements about the graph above:

- The graph is planar.

- The graph contains a cycle.
- The graph contains a bridge.
- The graph contains an Eulerian trail.
- The graph contains a Hamiltonian path.

How many of these statements are true?

A. 1

B. 2

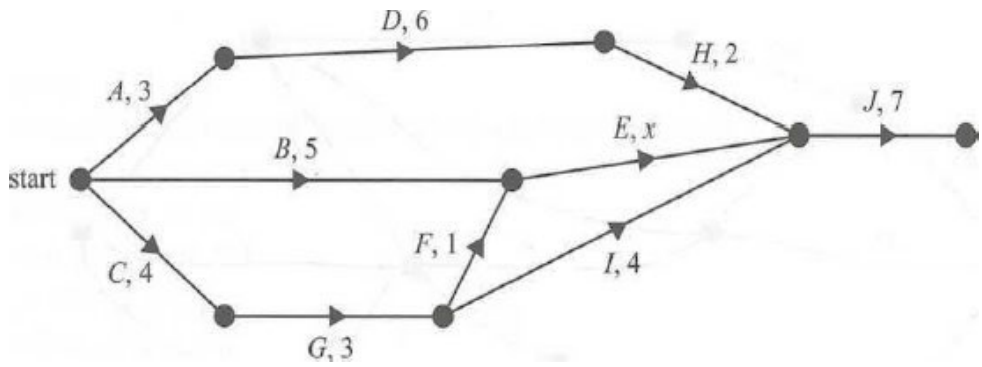
C. 3

D. 4

E. 5

Question 32 / 43

[VCAA 2021 FM (46%)]
The directed graph below shows the sequence of activities required to complete a project. The time taken to complete each activity, in hours, is also shown.



The minimum completion time for this project is 18 hours.

The time taken to mplete activity E is labelled x.

The maximum value of x is

A. 2

B. 3

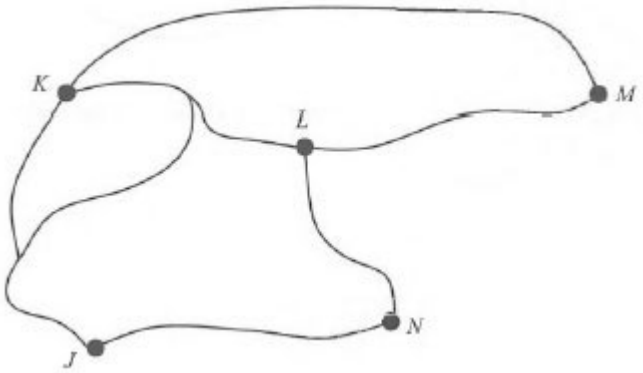
C. 4

D. 5

E. 6

Question 33 / 43

[VCAA 2021 FM (33%)]
The network below shows the pathways between five buildings: J, K, L, M and N.



An adjacency matrix for this network is formed. The number of zeros in this matrix is

---

A. 8

B. 9

C. 10

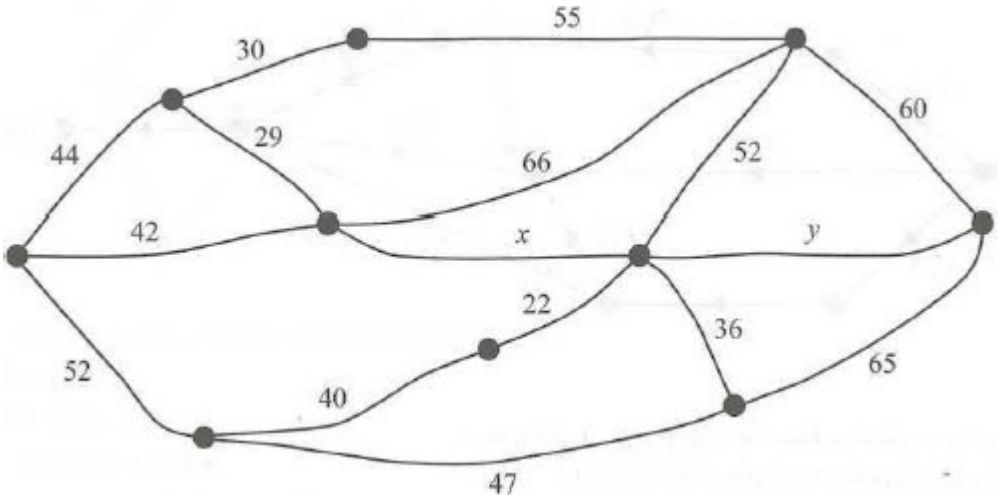
D. 11

E. 12

Question 34 / 43

[VCAA 2021 FM (48%)]
A network of roads connecting towns in an alpine region is shown below.

The distances between neighbouring towns, represented by the vertices, are given in kilometres.



The region receives a large snowfall, leaving all roads between the towns closed to traffic. To ensure each town is accessible by car from every other town, some roads will be cleared.

The minimum total length of road, in kilometres, that needs to be cleared is

---

A.

361 if x=50 and y=55

B.

361 if x=50 and y=60

C.

366 if x=55 and y=55

D.

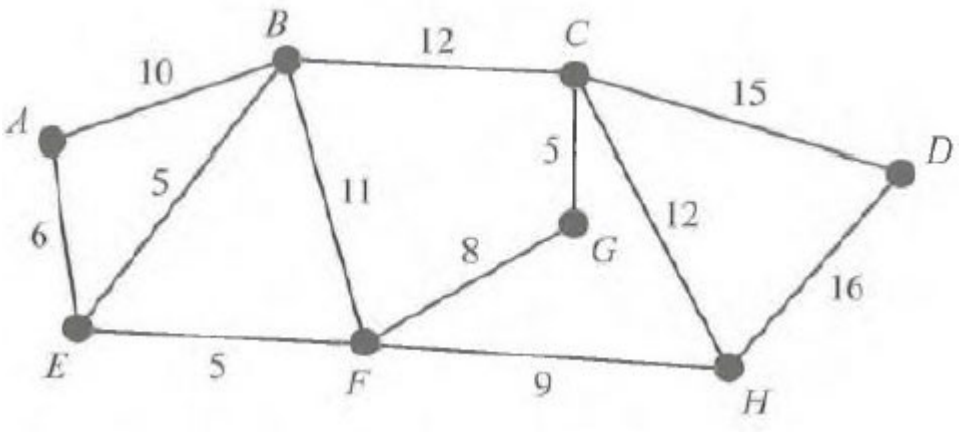
366 if x=55 and y=60

E.

371 if x=55 and y=65

Question 35 / 43

[VCAA 2022 FM (80%)]
The network below shows the distances, in kilometres, along a series of roads. The vertices A,B,C,D,E,F,G and H represent the intersections of these roads.



Prim's algorithm can be used to find the

A. critical path.

B. shortest path.

C. minimum cut.

D. minimum allocation.

E. minimum spanning tree.

Question 36 / 43

[VCAA 2022 FM (49%)]
The map below shows seven countries within Central America.

A network diagram was drawn with seven vertices to represent each of the countries on the map of Central America. Edges were drawn to represent a border shared between two countries. The number of edges that this network has is

---

A. 5

B. 6

C. 7

D. 8

E. 9

Question 37 / 43

[VCAA 2022 FM (39%)]
An athletics club needs to select one team of four athletes. The team is required to have one long jump, one high jump, one shot put and one javelin competitor. The following table shows the best distances, in metres, for each athlete for each event.

Athlete	**Long jump (m)**	**High jump (m)**	**Shot put (m)**	**Javelin (m)**
Eve	4.8	1.7	13.1	40.9
Harsha	4.8	1.6	13.9	39.5
Shona	5.1	1.8	14.4	41.2
Taylor	4.8	1.7	12.8	39.8

The athletics club will allocate each athlete to one event to maximise the total distance that the team jumps and throws. Which allocation of athlete to event must occur to maximise the total distance

A.

long jump	high jump	shot put	javelin
Shona	Harsha	Eve	Taylor

B.

long jump	high jump	shot put	javelin
Shona	Taylor	Harsha	Eve

C.

long jump	high jump	shot put	javelin
Eve	Harsha	Taylor	Shona

D.

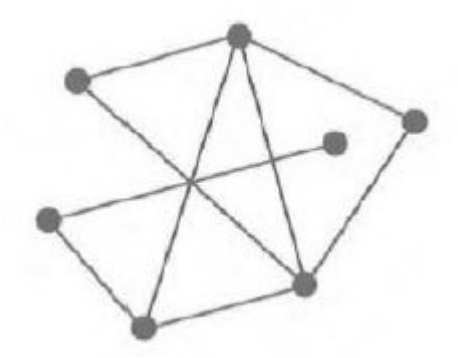
long jump	high jump	shot put	javelin
Harsha	Taylor	Shona	Eve

E.

long jump	high jump	shot put	javelin
Harsha	Taylor	Eve	Shona

Question 38 / 43

[VCAA 2022 FM (21%)]
Consider the graph right.



The number of edges that need to be removed for this graph to be planar is

---

A. 0

B. 1

C. 2

D. 3

E. 4

Question 39 / 43

[VCAA 2022 FM (54%)]
A connected graph consists of five vertices and four edges.

Which one of the following statements is **not** true?

A. The graph could be a tree.

B. The graph could be planar.

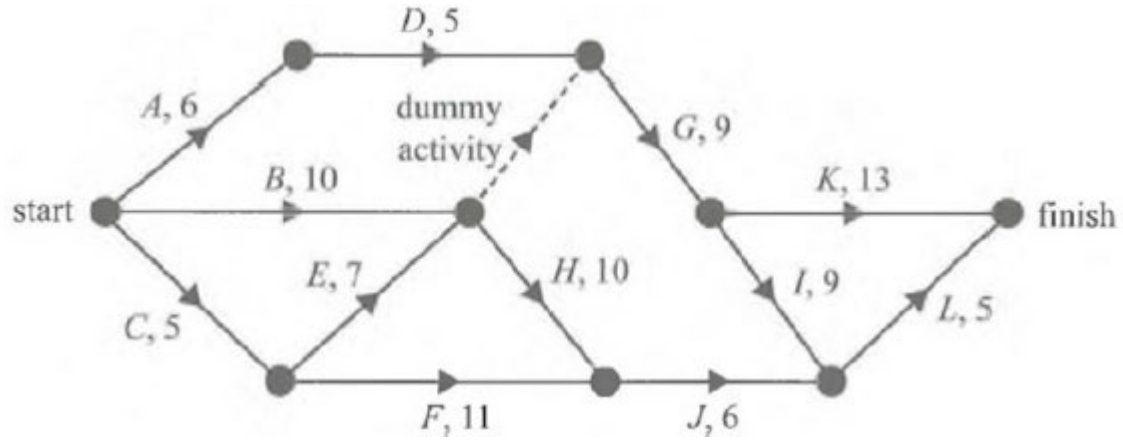
C. The graph could be bipartite.

D. The graph could contain a path.

E. The graph could contain a cycle.

Question 40 / 43

[VCAA 2022 FM (48%)]
A landscaping project has 12 activities. The network below gives the time, in hours, that it takes to complete each activity.



The earliest sart time,in hours, for activity G is

A. 10

B. 11

C. 12

D. 13

E. 14

Question 41 / 43

Use the following information to answer Questions 42 and 43.

A project involves 11 activities, A to K. The table below shows the earliest start time and duration, in days, for each activity. The immediate predecessor(s) of each activity is also shown.

Acivity	Earliest start time	Duration	Immediate predecessor(s)
A	0	6	–
B	0	7	–
C	6	10	A
D	6	7	A
E	7	8	B
F	15	2	D,E
G	15	2	E
H	17	3	G
I	20	6	C,F,H
J	17	5	G
K	26	2	I,J

Question 42 / 43

[VCAA 2022 FM (49%)]
A directed network for this project will require a dummy activity.

The dummy activity will be drawn from the end of

A. actvity A to the starof activity D.

B. actvity E to the starof activity F.

C. actvity F to the starof activity I.

D. actvity G to the star f activity H.

E. actvity I to the star of activity J.

Question 43 / 43

[VCAA 2022 FM (34%)]
When this project is completed in the minimum time, the sum of all the float times, in days, will be

---

A. 0

B. 16

C.18

D. 20

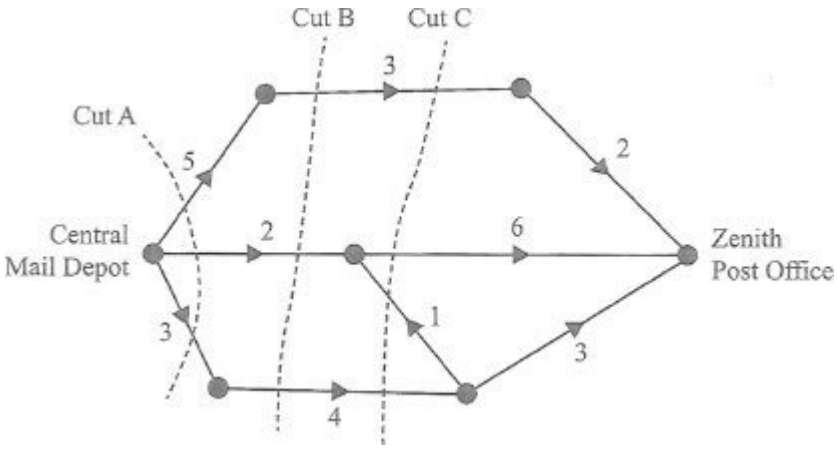
E. 28

Question 1 / 19

[VCAA 2018 FM]
The graph below shows the possible number of postal deliveries each day between the Central Mail Depot and the Zenith Post Office.

The unmarked vertices represent other depots in the region.

The weighting of each edge represents the maximum number of deliveries that can be made each day.



a. Cut A, shown on the graph, has a capacity of 10.

Two other cuts are labelled as Cut B and Cut C.

i. Write down the capacity of Cut B.

[1 mark (0.9)]

ii. Write down the capacity of Cut C.

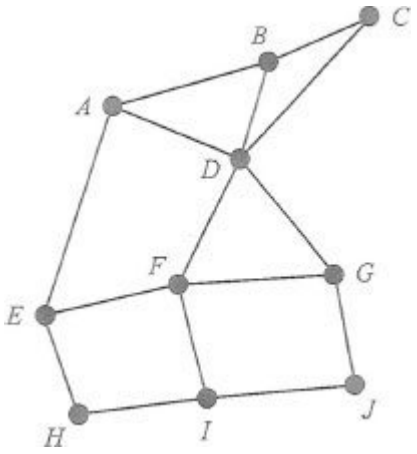
[1 mark (0.8)]

b. Determine the maximum number of deliveries that can be made each day from the Central Mail Depot to the Zenith Post Office.

[1 mark (0.3)]

Question 2 / 19

[VCAA 2018 FM]
In one area of the town of Zenith, a postal worker delivers mail to 10 houses labelled as vertices A to J on the graph below.



a. Which one of the vertices on the graph has degree 4?

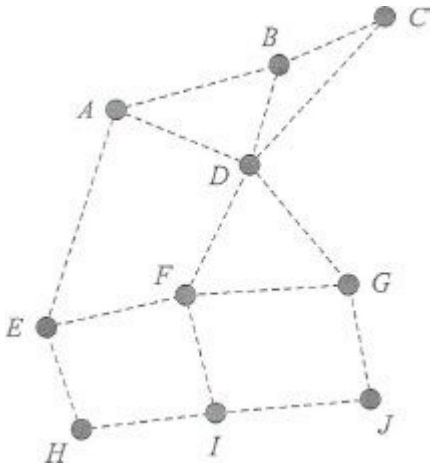
[1 mark (1.0)]

For this graph, an Eulerian trail does not currently exist.

b. For an Eulerian trail to exist, what is the minimum number of extra edges that the graph would require?

[1 mark (0.6)]

c. The postal worker has delivered the mail at F and will continue her deliveries by following a Hamiltonian path from F. Draw in a possible Hamiltonian path for the postal worker on the diagram below.



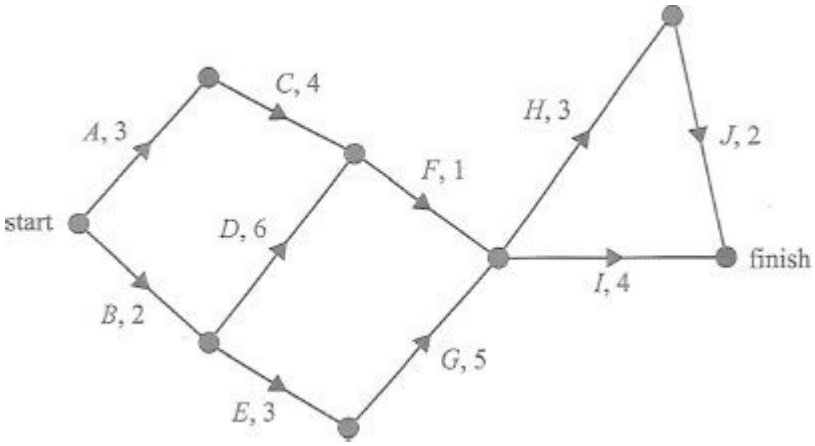
[1 mark (0.6)]

Question 3 / 19

[VCAA 2018 FM]
At the Zenith Post Office all computer systems are to be upgraded.

This project involves 10 activities, A to J.

The directed network below shows these activities and their completion times, in hours.



a. Detrmine he ealies startingme, in hours, for activity I.

[1 mark (0.7)]

b. The minimum completion time for the project is 15 hours. Write down the critical path.

[1 mark (0.8)]

c. Two of the activities have a float time of two hours. Write down these two activities.

[1 mark (0.5)]

d. For the next upgrade, the same project will be repeated but one extra activity will be added.

This activity has a duration of one hour, an earliest starting time of five hours and a latest starting time of 12 hours.

Complete the following sentence by filling in the boxes provided.

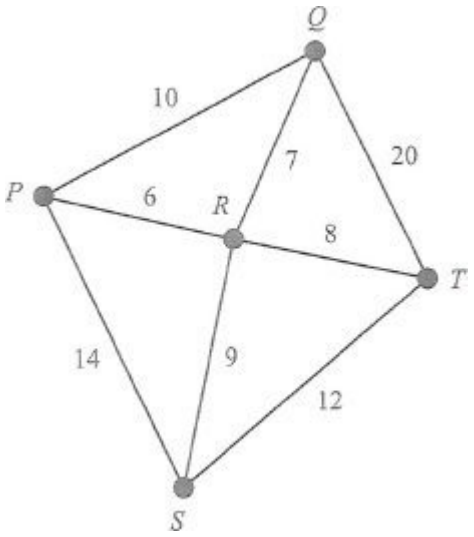
[1 mark (0.3)]

The extra activity could be represented on the network above by a directed edge from the end of activity ☐ to the star f activity ☐

Question 4 / 19

[VCAA 2018 FM]
Parcel deliveries are made between five nearby towns, P to T.

The roads connecting these five towns are shown on the graph below. The distances, in kilometres, are also shown.



A road inspector will leave from town P to check all the roads and return to town P when the inspection is complete. He will travel the minimum distance possible.

a. How many roads will the inspector have to travel on more than once?

[1 mark (0.3)]

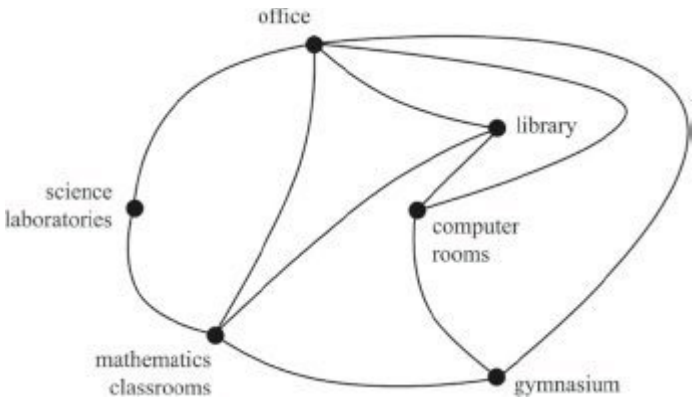
b. Determine the minimum distance, in kilometres, that the inspector will travel.

[1 mark (0.2)]

Question 5 / 19

[VCAA 2019 FM]

Fencedale High School has six buildings. The network below shows these buildings represented by vertices. The edges of the network represent the paths between the buildings.



a. Which building in the school can be reached directly from all other buildings?

[1 mark (1.0)]

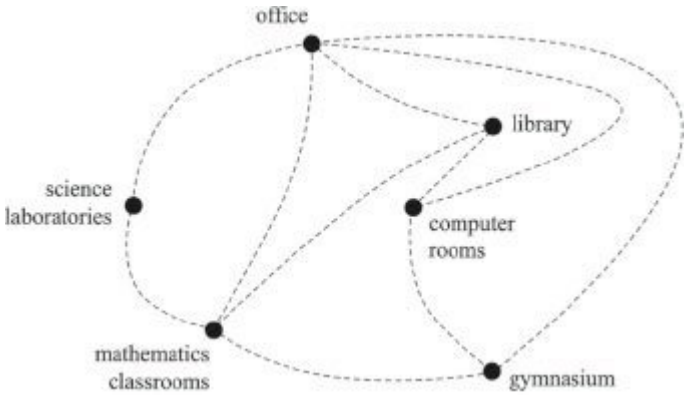
b. A school tour is to start and finish at the office, visiting each building only once.

i. What is the mathematical term for this route?

[1 mark (0.7)]

ii. Draw in a possible route for this school tour on the diagram below.

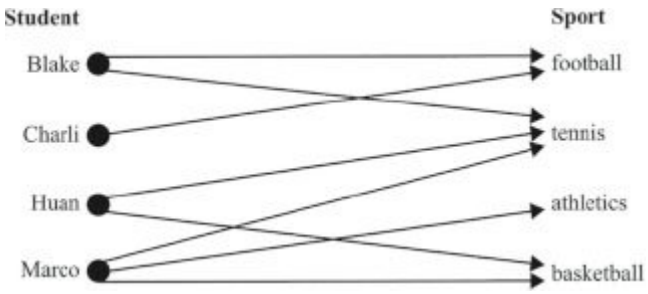
[1 mark (1.0)]



Question 6 / 19

[VCAA 2019 FM]
Fencedale High School offers students a choice of four sports, football, tennis, athletics and basketball.

The bipartite graph below illustrates the sports that each student can play.



Each student will be allocated to only one sport.

a. Complete the table below by allocating the appropriate sport to each student.

[1 mark (1.0)]

b. The school medley relay team consists of four students, Anita, Imani, Jordan and Lola.

Student		**Sport**
Blake		
Charli		
Huan		
Marco		

b. The school medley relay team consists of four students, Anita, Imani, Jordan and Lola.

The medley relay race is a combination of four different sprinting distances: 100 m, 200 m, 300 m and 400 m, run in that order.

The following table shows the best time, in seconds, for each student for each sprinting distance.

	Best time for each sprinting distance (seconds)			
Student	**100 m**	**200 m**	**300 m**	**400 m**
Anita	13.3	29.6	61.8	87.1
Imani	14.5	29.6	63.5	88.9
Jordan	13.3	29.3	63.6	89.1
Lola	15.2	29.2	61.6	87.9

The school will allocate each student to one sprinting distance in order to minimise the total time taken to complete the race. To which distance should each student be allocated?

Write your answers in the table below.

[2 marks (1.3)]

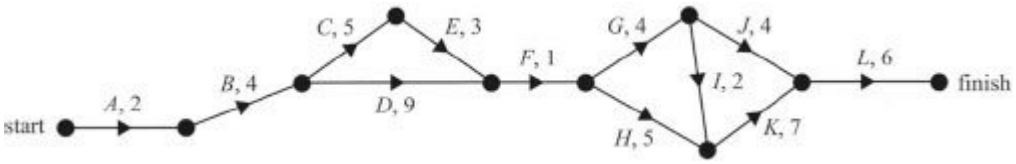
Student	**Sprinting distance (m)**
Anita	
lmani	
Jordan	
Lola	

Question 7 / 19

[VCAA 2019 FM]
Fencedale High School is planning to renovate its gymnasium.

This project involves 12 activities, A to L.

The directed network below shows these activities and their completion times, in weeks.



The minimum completion time for the project is 35 weeks.

a. How many activities are on the critical path?

[1 mark (0.6)]

b. Detrmine the læst srt time of activity E.

[1 mark (0.5)]

c. Which activity has the longest float time?

[1 mark (0.5)]

It is possible to reduce the completion time for activities C, D, G, H and K by employing more workers.

d. The completion time for each of these five activities can be reduced by a maximum of two weeks.

What is the minimum time, in weeks, that the renovation project could take?

[1 mark (0.3)]

e. The reduction in completion time for each of these five activities will incur an additional cost to the school.

The table below shows the five activities that can have their completion times reduced and the associated weekly cost, in dollars.

Acivity	Weekly cost ($)
C	3000
D	2000
G	2500
H	1000
K	4000

The completion time for each of these five activities can be reduced by a maximum of two weeks.

Fencedale High School requires the overall completion time for the renovation project to be reduced by four weeks at minimum cost.

Complete the table below, showing the reductions in individual activity completion times that would achieve this.

Acivity	Reduction in completion time (0, 1 or 2 weeks)
C	
D	
G	
H	
K	

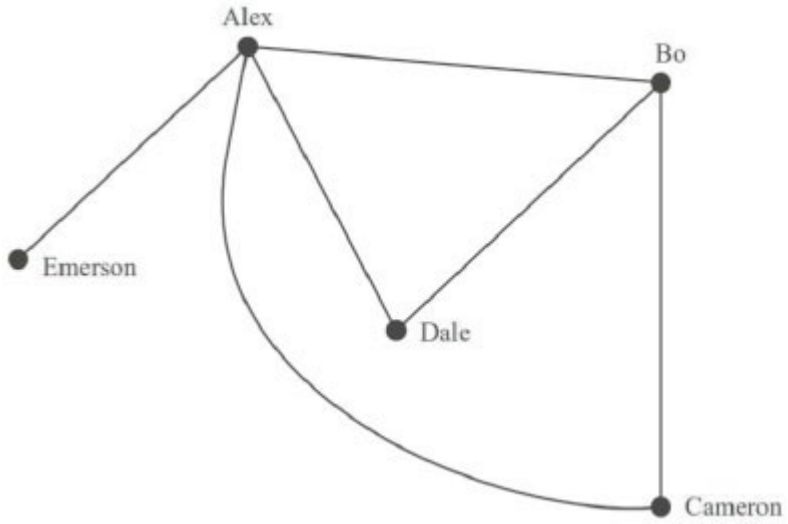
[2 marks (0.2)]

Question 8 / 19

[VCAA 2020 FM]
The Sunny Coast Cricket Club has five new players join its team: Alex, Bo, Cameron, Dale and Emerson.

The graph below shows the players who have played cricket together before joining the team.

For example, the edge between Alex and Bo shows that they have previously played cricket together.



a. How many of these players had Emerson played cricket with before joining the team?

[1 mark (0.9)]

b. Who had played cricket with both Alex and Bo before joining the team?

[1 mark (0.6)]

c. During the season, another new player, Finn, joined the team. Finn had not played cricket with any of these players before. Represent this information on the graph above. (*Answer on the graph above*.)

[1 mark (0.9)]

Question 9 / 19

[VCAA 2020 FM]
A cricket team has 11 players who are each assigned to a batting position.

Three of the new players, Alex, Bo and Cameron, can bat in position 1, 2 or 3.

The table below shows the average scores, in runs, for each player for the batting positions 1, 2 and 3.

		Batting position		
		1	**2**	**3**
Player	**Alex**	22	24	24
	Bo	25	25	21
	Cameron	24	25	19

Each player will be assigned to one batting position.

To which position should each player be assigned to maximise the team's score? Write your answer in the table below.

Player	**Batting position**
Alex	
Bo	
Cameron	

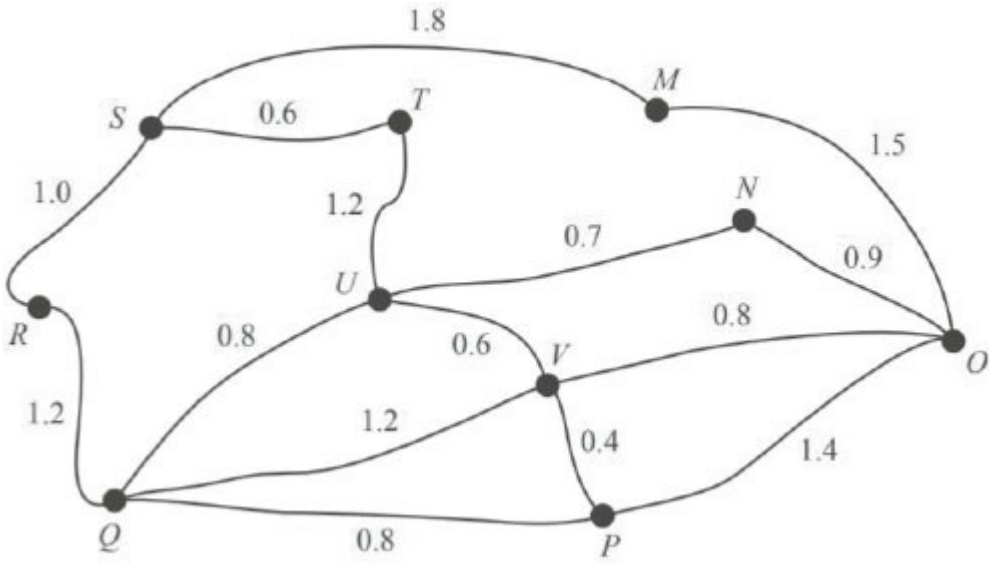
[1 mark (0.6)]

Question 10 / 19

[VCAA 2020 FM]
A local fitness park has 10 exercise stations: M to V.

The edges on the graph below represent the tracks between the exercise stations.

The number on each edge represents the length, in kilometres, of each track.



The Sunny Coast cricket coach designs three different training programs, all starting at exercise station S.

Training program number	**Training details**
1	The team must run to exercise station O.
2	The team must run along all tracks just once.
3	The team must visit each exercise station and return to exercise station S.

a. What is the shortest distance, in kilometres, covered in training program 1?

[1 mark (0.6)]

b. **i.** What mathematical term is used to describe training program 2?

[1 mark (0.7)]

ii. At which exercise station would training program 2 finish?

[1 mark (0.6)]

c. To complete training program 3 in the minimum distance, one track will need to be repeated. Complete the following sentence by filling in the boxes provided.

This track is between exercise station ☐ and exercise station ☐

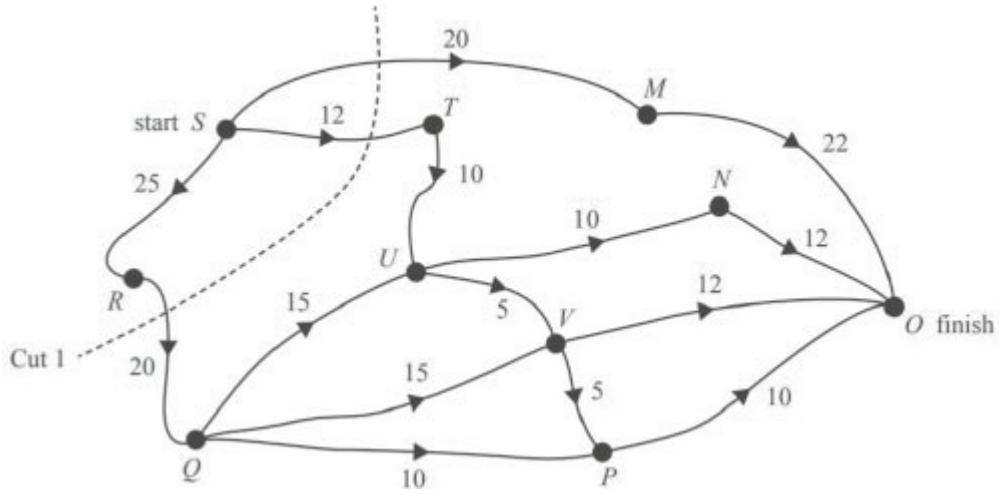
[1 mark (0.2)]

Question 11 / 19

[VCAA 2020 FM]
Training program 1 has the cricket team starting from exercise station S and running to exercise station O.

For safety reasons, the cricket coach has placed a restriction on the maximum number of people who can use the tracks in the fitness park.

The directed graph below shows the capacity of the tracks, in number of people per minute.



a. How many different routes from S to O are possible?

[1 mark (0.3)]

When considering the possible flow of people through this network, many different cuts can be made.

b. Determine the capacity of Cut 1, shown above.

[1 mark (0.7)]

c. What is the maximum flow from S to O, in number of people per minute?

[1 mark (0.3)]

Question 12 / 19

[VCAA 2020 FM]

The Sunny Coast cricket clubroom is undergoing a major works project. This project involves nine activities: A to I. The table below shows the earliest start time (EST) and duration, in months, for each activity. The immediate predecessor(s) is also shown. The duration r activity C is missing.

Acivity	EST	Duration	Immediate predecessor(s)
A	0	2	–
B	0	5	–
C	5		A,B
D	7	7	C
E	7	9	C
F	5	3	B
G	14	4	D
H	8	9	F
I	18	2	E,G,H

The information in the table above can be used to complete a directed network.

This network will require a dummy activity.

a. Complete the following sentence by filling in the boxes provided.

[1 mark (0.3)]

This dummy activity could be drawn as a directed edge from the end of

actvity ☐ to the startof activity ☐

b. What is the duration, in months, of activity C?

[1 mark (0.5)]

c. Name the four activities that have a float time.

[1 mark (0.2)]

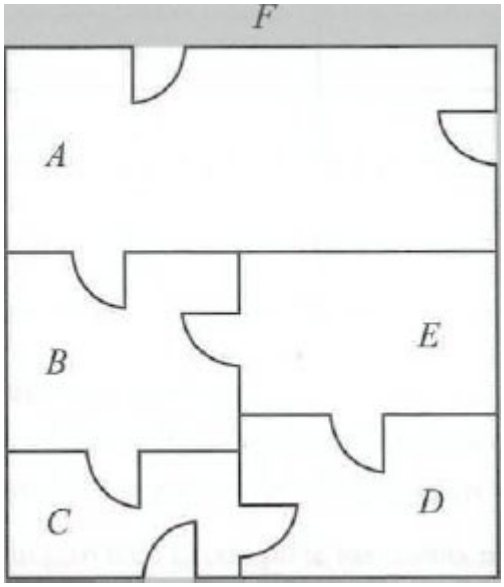
d. The project is to be crashed by reducing the completion time of one activity only. What is the minimum time, in months, that the project can be completed in?

[l mark (0.1)]

Question 13 / 19

[VCAA 2021 FM]
Maggie's house has five rooms, A, B, C, D and E, and eight doors.

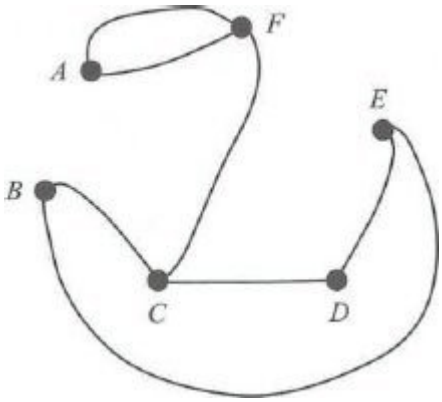
The floor plan of these rooms and doors is shown below. The outside area, F, is shown shaded on the floor plan.



The floor plan is represented by the graph below.

On this graph, vertices represent the rooms and the outside area. Edges represent direct access to the rooms through the doors.

One edge is missing from the graph.



a. On the graph above, draw the missing edge.

[1 mark (1.0)]

b. What is the degree of vertex E?

[1 mark (0.9)]

c. Maggie hires a cleaner to clean the house.

It is possible for the cleaner to enter the house from the outside area, F, and walk through each room only once, cleaning each room as he goes and finishing in the outside area, F.

i. Complete the following to show **one** possible route that the cleaner could take.

[1 mark (0.9)]

ii. What is the mathematical term for such a journey?

[1 mark (0.6)]

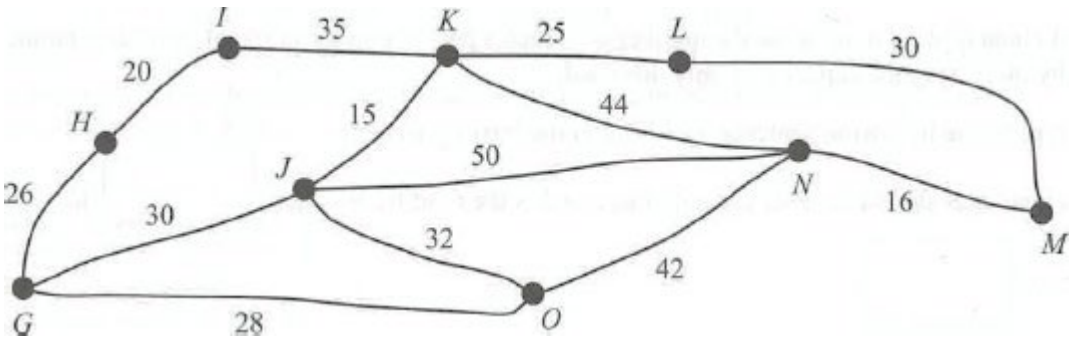
Question 14 / 19

[VCAA 2021 FM]
George lives in Town G and Maggie lives in Town M.

The diagram below shows the network of main roads between Town G and Town M.

The vertices G, H, I, J, K, L, M, N and O represent towns.

The edges represent the main roads. The numbers on the edges indicate the distances, in kilometres, between adjacent towns.



a. What is the shortest distance, in kilometres, between Town G and Town M?

[1 mark (0.9)]

b. George plans to travel to Maggie's house. He will pass through all the towns shown above.

George plans to take the shortest route possible.

Which town will George pass through twice?

[1 mark (0.7)]

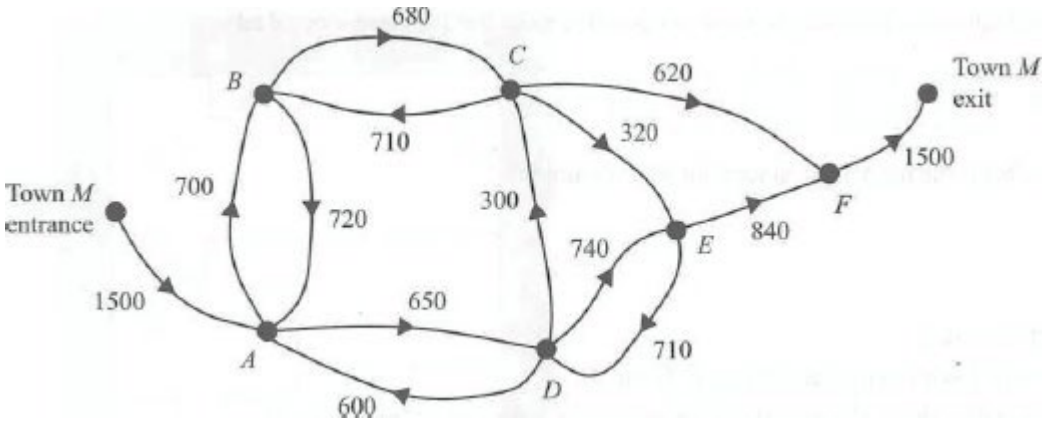
Question 15 / 19

[VCAA 2021 FM]
The network diagram below shows the local road network of Town M.

The numbers on the edges indicate the maximum number of vehicles per hour that can travel along each road in this network.

The arrows represent the permitted direction of travel.

The vertices A, B, C, D, E and F represent the intersections of the roads.



a. Determine the maximum number of vehicles that can travel from the entrance to the exit per hour.

[1 mark (0.3)]

b. The local council plans to increase the number of vehicles per hour that can travel from the entrance to the exit by increasing the capacity of only one road.

i. Complete the following sentence by filling in the boxes provided. The road that should have its capacity increased is the road from vertex ☐ to vertex ☐.

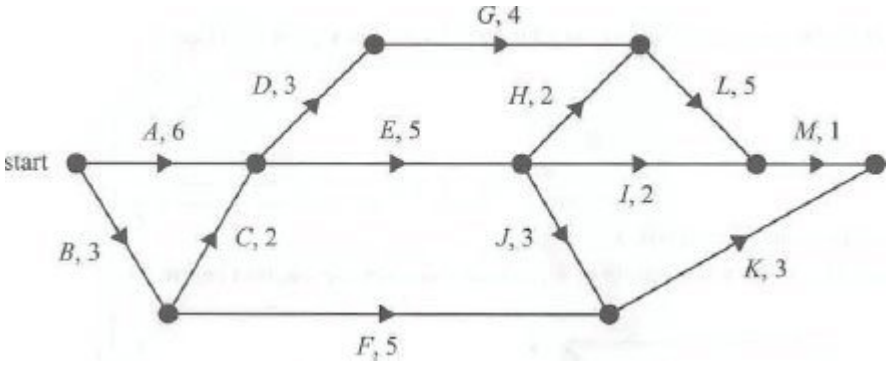
[1 mark (0.4)]

ii. What should be the minimum capacity of this road to maximise the flow of vehicles from the entrance to the exit?

[1 mark (0.1)]

Question 16 / 19

[VCAA 2021 FM]
Roadworks planned by the local council require 13 activities to be completed. The network below shows these 13 activities and their completion times in weeks.



a. What is th earlies start tie, in weeks, of activity K?

[1 mark (0.5)]

b. How many of these activities have zero float time?

[1 mark (0.2)]

c. It is possible to reduce the completion time for activities A, E, F, L and K.

The reduction in completion time for each of these five activities will incur an additional cost.

The table below shows the five activities that can have their completion time reduced and the associated weekly cost, in dollars.

Acivity	**Weekly cost ($)**
A	140 000
E	100 000
F	100 000
L	120 000
K	80 000

The completion time for each of these five activities can be reduced by a maximum of two weeks.

The overall completion time for the roadworks can be reduced to 16 weeks.

What is the minimum cost, in dollars, of this change in completion time?

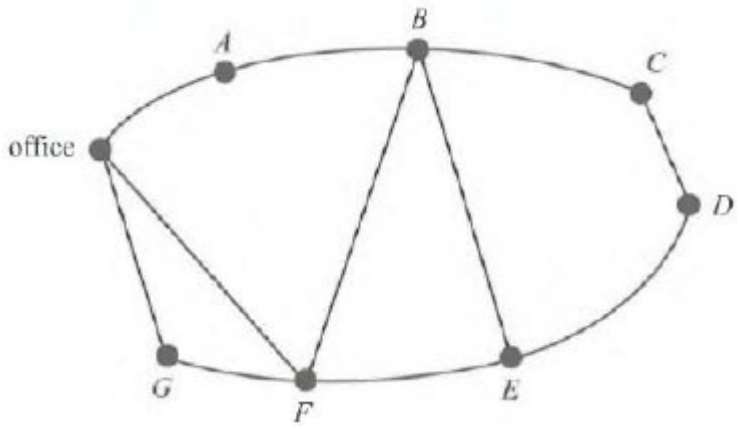
[1 mark (0.1)]

Question 17 / 19

[VCAA 2022 FM]
Joe owns a holiday park with seven cabins.

The diagram below shows a network of roads in the holiday park.

Joe's office is labelled as a vertex on the network. The other vertices –A, B, C, D, E, F and G – represent the cabins.



In the morning, Joe leaves his office and visits each cabin once only before returning to the office.

a. Write down a route that Joe could follow.

[1 mark (0.8)]

b. Later in the day, Joe will leave his office and travel along each road once only to check the road conditions.

i. Which vertex will Joe finish on?

[1 mark (0.8)]

ii. What is the mathematical term for this route?

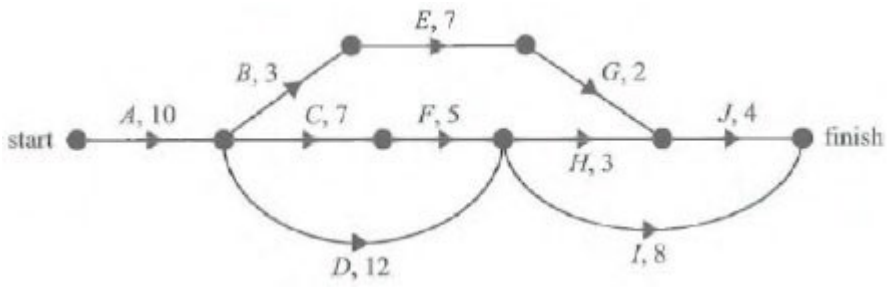
[1 mark (0.7)]

Question 18 / 19

[VCAA 2022 FM]
Joe plans to add a new cabin to the holiday park.

This project requires 10 activities to be completed.

The network below shows these 10 activities and their completion times in weeks.



a. How many of these activities have two immediate predecessors?

[1 mark (0.5)]

b. What is the minimum completion time, in weeks, for this project?

[1 mark (0.6)]

c. It is possible to reduce the completion time of two activities.

One activity can have its completion time decreased by two weeks and another activity can have its completion time decreased by one week.

These two changes result in the minimum completion time being reduced by three weeks.

Complete the table below, showing the two activities that could have their completion times reduced and the reduction in individual activity completion time that would achieve the three-week reduction.

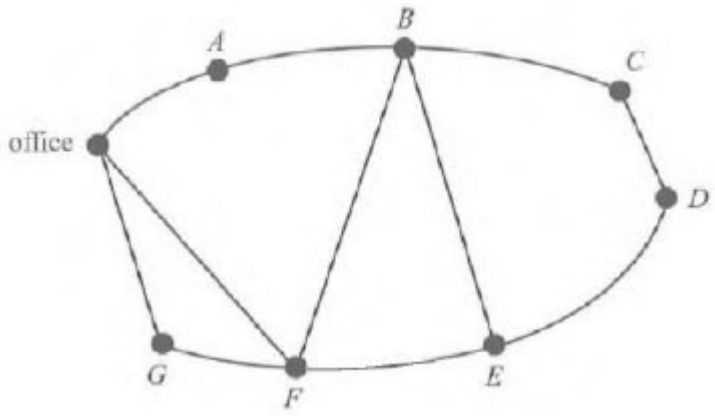
Acivity	**Reduction in completion time (1 week or 2 weeks)**

d. The holiday park has had some changes to its roads to accommodate the new cabin.

The adjacency matrix below shows road connections between the office and each cabin. The new cabin, H, is included in the matrix.

	office	A	B	C	D	E	F	G	H
office	0	1	0	0	0	0	1	1	0
A	1	0	1	0	0	0	0	0	0
B	0	1	0	1	0	1	1	0	0
C	0	0	1	0	1	0	0	0	0
D	0	0	0	1	0	1	0	0	1
E	0	0	1	0	1	0	1	0	1
F	1	0	1	0	0	1	0	1	0
G	1	0	0	0	0	0	1	0	0
H	0	0	0	0	1	1	0	0	1

On the diagram below, add the new cabin, H, and any additional roads to the network.

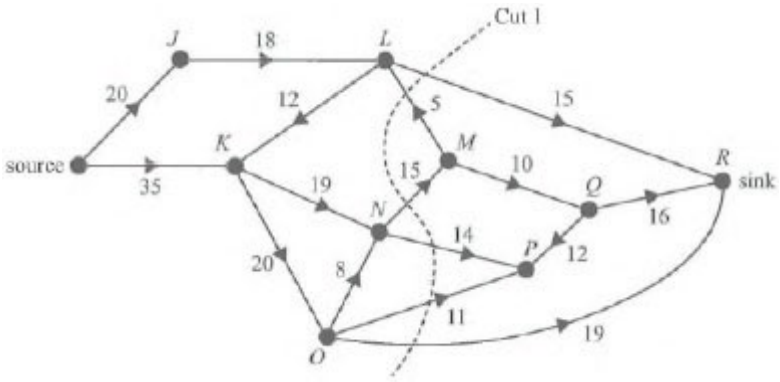


[2 marks (1.8)]

Question 19 / 19

[VCAA 2022 FM]

A series of pipelines is installed to allow for the flow of stormwater from the holiday park. The capacity of these pipes, in litres per minute, is shown in the directed network below.



When considering the possible flow of stormwater through this network, many different cuts can be made.

a. Determine the capacity of Cut 1, shown above.

[1 mark (0.6)]

b. What is the maximum flow of stormwater, in litres per minute, from the source to the sink?

[1 mark (0.4)]

Joe would like to increase the maximum flow through this network.

The maximum flow through this network may be increased either by reversing the direction of flow through one pipe or by increasing the capacity of one pipe.

c. The direction of flow is reversed through one pipe.

Complete the following sentence by filling in the boxes provided.

The pipe that should have its flow reversed to cause the largest increase in flow from source to sink is the pipe that runs from vertex ☐ to vertex ☐.

[1 mark (0.5)]

d. The capacity of one pipe is increased.

Complete the following sentences by filling in the boxes provided.

The pipe that should have its capacity increased to cause the largest increase in flow from source to sink is the pipe that runs from vertex ☐ to vertex ☐.

Its new capacity, in litres per minute, should be at least ☐.

[1 mark (0.02)]