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VCE GENERAL MATHEMATICS

Units 1 & 2

2ND EDITION

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Linear relations and equations

CHAPTER 1

LESSON 1A

Linear equations

The key skills you will learn in this lesson are:

1. Identifying linear relations
2. Substituting values into linear relations
3. Transposing linear relations

VCAA key knowledge points:

"substitution into, and transposition of linear relations, such as scale conversion"

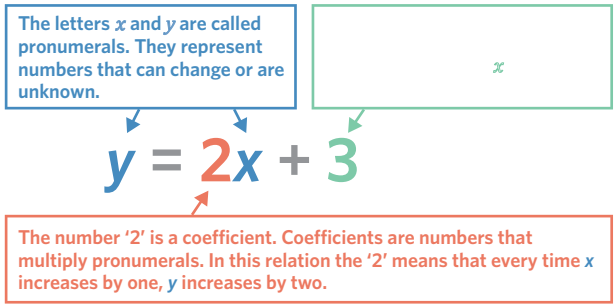
"solution of linear equations, including literal linear equations"

The relationship between two quantities is said to be linear when the quantities change in response to each other by a set amount. This set amount is called the gradient or slope of the equation.

The features of a linear equation are shown to the right.

There are two types of pronumerals: constants and variables.

- Constants keep the same value at all times (e.g. π, which is approximately 3.14).
- Variables can change their value (e.g. x and y in a linear equation).



1. Identifying linear relations

Determining whether an equation is linear or not involves examining the equation to make sure that all of the terms are linear.

Linear terms include:

- constants (such as $+6$ or -3)
- variables with constant coefficients (such as $2x$ or $-5y$)

Non-linear terms include:

- variables raised to a power (such as x^2, $y^{-3.5}$)
- roots of variables (such as $\sqrt{x}$, $\sqrt[3]{\ }$)
- variables being multiplied together (such as xy)
- trigonometric or logarithmic functions

The worked example below includes examples of common expressions that cannot be in a linear equation.

WORKED EXAMPLE 1 *(1 mark)*

Is the equation $y = 2x + 1$ linear?

SOLUTION

Step 1 Are there any variables raised to a power? E.g. a^2, $b^{-1.5}$ **No.**

Step 2 Are there any roots (square roots or otherwise) of a variable? E.g. $\sqrt{c}$ $\sqrt[3]{d}$ **No.**

Step 3 Are there any sets of two (or more) variables multiplied together? E.g. $e \times f$ **No.**

Step 4 Are there any numbers raised to the power of a variable? E.g. 2^g **No.**

Step 5 Are there any trigonometric or logarithmic terms? E.g. sin, cos, tan, $\log_e$, $\log_{10}$, ln, etc. **No.**

Step 6 If the answer to any of the above questions is 'yes', then the equation is **not** linear.

Since the answer to all of the above questions is 'no', the equation $y = 2x + 1$ is linear.

NOTE: There are other ways to have non-linear equations but they are beyond the scope of this course.

2. Substituting values into linear relations

Substitution is the replacement of one term with another. In linear relations, substitution is when a pronumeral is replaced with a number. The terms 'let' and 'when' are often used to indicate that a variable is going to be substituted with a particular value in the following calculation.

Example

'Let $x = 0$', or 'When $a = 5$'.

To substitute a pronumeral with a number, replace the pronumeral with the necessary value and enclose it in brackets.

WORKED EXAMPLE 2 *(1 mark)*

For the equation $y = 2x + 1$, find y when $x = 3$.

SOLUTION

Step 1 Replace the pronumeral with the necessary value and enclose it in brackets.

$y = 2(3) + 1$

Step 2 Evaluate the equation.

$y = 2(3) + 1$

$y = 2 \times 3 + 1$

$y = 6 + 1$

$y = 7$

3. Transposing linear relations

To transpose something means to rearrange it. Equations are often transposed to find the value of a variable. This is done by 'moving' everything except the desired variable to the other side of the equals sign. The variable by itself is called the subject of the equation. Making a variable the subject of an equation allows the calculation of its valueusing substitution.

Transposing an equation can be thought of as 'undoing' part or all of the equation. The parts of an equation that are not numbers or pronumerals are called 'operators'. To 'undo' an operator, the inverse operator, or 'opposite' operator is required. These are shown in the table below.

Operator	$+$	$-$	$\times$	$\div$
Inverse Operator	$-$	$+$	$\div$	$\times$

WORKED EXAMPLE 3 *(1 mark)*

Transpose the equation $y = 2x + 1$ to make x the subject.

SOLUTION

Step 1 List, in order, the operations that transform the desired variable into its current form in the equation.

In this case, that is x into $2x + 1$.

The first operation is the one 'closest' to x, $\times 2$. The next operation is $+ 1$.

$\times 2, + 1$.

Step 2 Write down the inverses of the operations written in step 1, but in the opposite order.

$-1, \div 2$.

Step 3 Apply these transformations, in the order written in step 2, to both sides of the equation.

$$y = 2x + 1$$
$$y - 1 = 2x + 1 - 1$$
$$y - 1 = 2x$$
$$(y - 1) \div 2 = 2x \div 2$$
$$\frac{1}{2}(y - 1) = \frac{2x}{2}$$
$$\frac{1}{2}(y - 1) = x$$

We have transposed the equation for x. x is now the subject of the equation.

NOTE: It is common to write the subject on the left-hand side of the equation. This is not a rule, however, and it is perfectly fine to write it on the right-hand side. You should be comfortable using equations written either way.

Questions 1A *Linear equations*

Refresher question

Q1. $N = 2 + 7 \times 3$. What is N?

1. Identifying linear relations

Q2. Consider the equation $y = 2x - z + 23$.

Skill

a) Do any of the following expressions appear in the equation?

- **i** Are there any variables raised to a power? E.g. a^2, $b^{-1.5}$.
- **ii** Are there any roots (square roots or otherwise) of a variable? E.g. $\sqrt{c}$ $\sqrt[3]{d}$.
- **iii** Are there any sets of two (or more) variables multiplied together? E.g. $e \times f$.
- **iv** Are there any numbers raised to the power of a variable? E.g. 2^g.
- **v** Are there any trigonometric or logarithmic terms? E.g. sin, cos, tan, $\log_e$, $\log_{10}$, etc.

b) Is the equation $y = 2x - z + 23$ linear?

Q3. Are the following equations linear?

Skill

NOTE: All of the pronumerals in this question are variables.

a) $P = a - 3b + 3c - d$

b) $i = \sqrt{j} + k$

c) $x^3 = 5y + 2z$

d) $P = 2l + 2w$

e) $b = ac + 5$

f) $f = 3^x + 6$

Check your understanding

Q4. Determine whether the following equations are linear.

Skill

a) $p = q + 7^2$

b) $C = 2\pi r$

NOTE: π is a constant. It is approximately 3.14.

2. Substituting values into linear relations

Q5. $a = 2n + 3$

Skill

a) Substitute $n = 2$ into the equation above by filling in the box below:

$a = 2 \times \square + 3$

b) Using your answer from part a), find a.

Q6. Follow the instructions for the following equations:

Skill

a) $p = q - 4$

- **i** Without solving or simplifying, substitute $q = 1$ into the equation above.
- **ii** Using your answer from part i, find p.

b) $f = \frac{1}{3}x + 7$

i Without solving or simplifying, substitute $x = 9$ into the equation above.

ii Using your answer from part i, find f.

c) $h = -2.3t + 11$

i Without solving or simplifying, substitute $t = 1.7$ into the equation above.

ii Using your answer from part i, find h.

Q7. Follow the instructions for the following equations:

Skill

a) $y = 5x - 4$

Find y when $x = 3$.

b) $a = 3b - 2c$

Find a when $b = 2$ and $c = 8$.

c) $p = -\frac{1}{4}(q + 3r)$

Find p when $q = 5$ and $r = -2$.

Q8. Answer the following questions.

Application

a) The number of goals, g, Vanessa kicks in an Australian Rules Football match is given by the equation $g = \frac{1}{3}w + 2$, where w is number of Weet-Bix she ate for breakfast. If she eats 12 Weet-Bix for breakfast, how many goals will she kick?

b) The equation $T = \frac{2}{3}m$ tells us how long in minutes, T, it takes to watch a video that is m minutes long, if you are watching at 1.5× speed. How long will it take to watch a 5.3-minute-long Edrolo video at this speed? Give your answer correct to 1 decimal place.

c) The equation to convert temperatures in Fahrenheit, F, to Celsius, C, is $C = \frac{5}{9}(F - 32)$. Convert 100°F to Celsius. Give your answer correct to the nearest whole number.

Check your understanding

Q9. Daisy and Beppe's doona is 210 cm long. This means that the area, A, covered by their doona, in cm, is given by the linear relation $A = 210w$, where w is the width, in cm, of their current doona.

Application

Daisy and Beppe's dog, Barnesworth, always gets on the bed in the morning and takes up a lot of the doona. They decide to get a doona that is 30 cm wider than their current one.

Write down an equation for the area, A, covered by their new doona in terms of w.

3. Transposing linear relations

Q10. Consider the relation $y = \frac{1}{3}x + 5$.

Skill

a) List, in order, the operations that transform x into $\frac{1}{3}x + 5$.

b) List, in order, the operations that transform $\frac{1}{3}x + 5$ into x.

c) Transpose the relation $y = \frac{1}{3}x + 5$ to make x the subject.

Q11. Transpose each of the following linear relations to make the indicated pronumeral the subject.

Skill

a) $m = 2n + 4$

Transpose for n.

b) $y = 42 - 3x$

Transpose for x.

c) $\frac{x + c}{g} = 5$

Transpose for c.

d) $\frac{3q + 6w - 66}{p} = 66$

Transpose for w.

Q12. Answer the following questions:

Application

a) The equation that converts a distance in miles, M, to kilometres, K, is $K = 1.6M$.

Find the equation that converts a distance in kilometres to miles.

b) The length of time, t, in minutes, that Jane and her dog Bowie spend at the local park is given by $t = 30 + 5d$, where d is the number of other dogs at the park.

Her husband, Des, wants to deduce the number of dogs Jane and Bowie encounter from the amount of time they spend at the park.

Rearrange $t = 30 + 5d$ to make d the subject to find the required equation for Des.

Check your understanding

Q13. The equation to convert temperatures in Fahrenheit, F, to Celsius, C, is $C = \frac{5}{9}(F - 32)$.

Application

Show that the equation to convert Celsius to Fahrenheit is $\frac{9}{5}C + 32 = F$.

Joining it all together

Q14. Follow the instructions for the following equations:

Skill
3 marks

a) For the equation $A = \frac{1}{2}bh$, find h when $b = 5$ and $A = 15$. (1 mark)

b) For the equation $\frac{12}{a} - \left(3c + b\right) = 33$, find c when $b = 6$, and $a = 4$. (1 mark)

c) For the equation $\frac{q}{r} + 8 = 3t$, find r when $q = 2$ and $t = \frac{3}{2}$. Give your answer correct to two decimal places. (1 mark)

Q15. The cost, C, in dollars, of hiring an UberX is $C = 2 + 1.15d + 0.35t$, where d is the distance travelled in km, and t is the time taken in minutes.

Application
3 marks

a) Find the cost of hiring an UberX to travel 15 km if the trip took 12 minutes. Give your answer correct to two decimal places. (1 mark)

b) Angelo hired an UberX to travel 10 km from the Night Cat in Fitzroy to his home in Reservoir. If the total cost was $20.85, how long did the trip take? Give your answer correct to the nearest minute. (2 marks)

Q16. In most branches of science, temperature is measured in Kelvin, K. The equation to convert between Kelvin and Celsius is $K = C + 273$.

Application
4 marks

The equation to convert temperatures in Fahrenheit, F, to Celsius, C, is $C = \frac{5}{9}(F - 32)$.

NOTE: Temperatures in Kelvin are written **without** the degree symbol.

a) If the temperature is 18 degrees Fahrenheit, what is the temperature in Kelvin? Give your answer correct to the nearest degree Kelvin. (2 marks)

b) If the temperature is 298 Kelvin, what is the temperature in Fahrenheit? (2 marks)

Q17. Consider the equation $p - q = \frac{1}{r}(12s + t)$.

Skill
1 mark

Show that $r = \frac{12s + t}{p - q}$.

Q18. Explain why the equation $\frac{y}{x} = 4$ is linear.

Skill
2 marks

Questions from multiple lessons

Q19.

Difficulty:

Year 10
1 mark

Jonty found a bag of marbles, and wants to start collecting more.

There are 80 marbles in the bag he found. Each week, he plans to collect two more.

This can be represented by the following equation:

$M = 80 + 2w$

Jonty now has 128 marbles.

How many weeks has Jonty been collecting marbles for?

A. 14

B. 24

C. 48

D. 64

E. 128

VCAA 2017NH Exam 1 Module 4: Graphs and relations Q2 – Adapted

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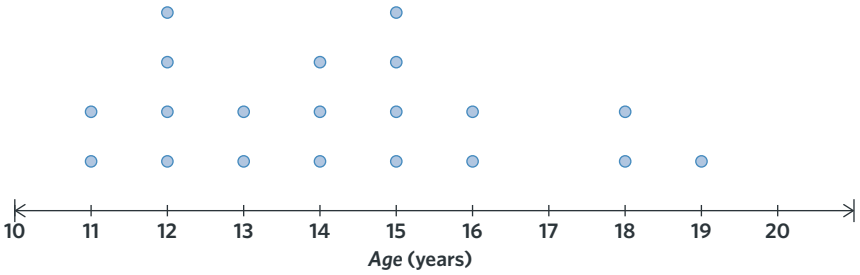
Q20.

Difficulty:

Year 10
1 mark

The following dot plot displays the *age*, in years, of 20 members of a Shawn Mendes fan club.



The percentage of members that are 15 years of age is

A. 4%

B. 5%

C. 20%

D. 24%

E. 25%

VCAA 2018 Exam 1 Data analysis Q1 – Adapted

Q21.

Difficulty:

Year 10
2 marks

Markus owns a local supermarket that sells exotic fruits.

He sells lychees and dragonfruit.

All lychees are sold for the same price and all dragonfruit are sold for the same price.

Let L represent the selling price of each lychee and D represent the selling price of each dragonfruit.

One customer, Alan, purchased 267 lychees and 41 dragonfruits and paid $484.40.

A linear relation representing Alan's purchase is $267L + 41D = 484.40$.

a) A second customer, Mary, purchased 129 lychees and 26 dragonfruits and paid $258.80. Write down a linear relation representing Mary's purchase. (1 mark)

b) The selling price of each dragonfruit in Markus' shop is $4.00. What is the selling price of each lychee? (1 mark)

VCAA 2019NH Exam 2 Module 4: Graphs and relations Q1a,b – Adapted

LESSON 1B

Tables and recursion

The key skills you will learn in this lesson are:

1. Constructing a table of values
2. Recursion relationships

VCAA key knowledge points:

"construction of tables of values from a given formula"

"linear relations defined recursively and simple applications of these relations"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.18; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Constructing a table of values

For any equation, a table of values can be constructed to compare the variables at different points. This can be done by substituting each value needed into the formula by hand, or by using a CAS calculator. A table of values is helpful for seeing the pattern set out by the equation to understand the equation with more clarity.

WORKED EXAMPLE 4 *(1 mark)*

The cost to hire a catering company for an event is given by the formula: $c = 50n + 200$, where c is the total cost and n is the number of people at the event.

Construct a table of values in intervals of 10 from $n = 10$ to $n = 100$.

SOLUTION

Step 1 Draw a table with 2 rows and 11 columns. (We are looking at $n = 10$ to $n = 100$, in increments of 10, which is a total of 10 points, and means we need 10 columns. One column is also needed for the labels.).

Label the rows n and $c(\$)$. Write the given values from 10 to 100 in row n.

n	10	20	30	40	50	60	70	80	90	100
$c(\$)$										

Step 2 Substitute $n = 10$ into the formula to get $c = 50 \times 10 + 200 = 700$. Enter it into the table.

Repeat this for $n = 20$ and the rest of the n values.

n	10	20	30	40	50	60	70	80	90	100
$c(\$)$	700	1200	1700	2200	2700	3200	3700	4200	4700	5200

WORKED EXAMPLE 5 *(1 mark)*

The formula to convert inches to centimetres is given below.

$$C = 2.54 \times I$$

Find the values of C for I values 1 to 10.

SOLUTION: TI-NSPIRE

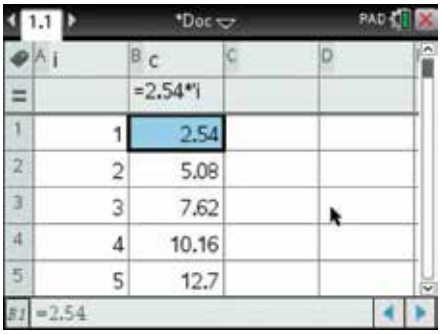
Step 1 Open a new page by pressing ctrl + doc and press 4 ('Add Lists & Spreadsheet').

Step 2 In the very top row, label column A as 'i' and column B as 'c' (for inches and centimetres). Enter the I values 1 to 10 in the first column.

Step 3 In the grey cell in the second column type the formula '=2.54×i'.

Press enter.

A box prompts you to choose 'Column Reference' or 'Variable Reference'. Select 'Variable Reference'. Press 'OK'.



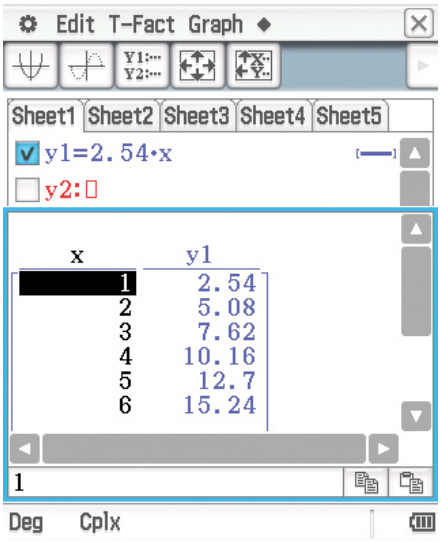
SOLUTION: CASIO CLASSPAD

Step 1 From the menu, tap Graph&Table.

Step 2 In the 'y1' line, type the formula '2.54x' and press EXE.

Step 3 Tap the Table Input icon and set the start value as 1, the end value as 10, and the step as 1.

Step 4 Tap the icon to display the table.



2. Recursion relationships

Recursion is a method of defining a number sequence that follows a linear pattern.

This means that the difference between each number and the next is always the same.

This difference is known as the common difference.

A recurrence relation calculates the next term in a sequence based on the previous term.

Recurrence relations for linear patterns can all be written in the form: $t_n = t_{n-1} + d, t_0 = a$, where a is the first term of the sequence, and d is the common difference.

To find the next term in the sequence, use the equation $t_1 = t_0 + d$.

Then the next term is found by using the equation $t_2 = t_1 + d$ and so on.

Example

Consider the number sequence 4, 6, 8, 10, 12,...

A recurrence relation for this sequence is $t_n = t_{n-1} + 2, t_0 = 4$.

WORKED EXAMPLE 6 *(2 marks)*

For this number sequences given below, find a recurrence relation.

9, 7, 5, 3, 1,...

SOLUTION

Step 1 Identify the first term.

The first term in this sequence is 9.

This means that $a = 9$.

Step 2 Determine d, the difference between each term.

Do this by finding second term minus first term in the sequence..

$7 - 9 = -2$

This means that $d = -2$.

✓ *1 mark for finding the values of a and d.*

Step 3 Write the equation.

$t_n = t_{n-1} - 2, t_0 = 9$

✓ *1 mark for writing the recurrence relation.*

Questions 1B *Tables and recursion*

Refresher question

Q1. Consider the following equation.

$y = -2x + 3$

Find the value of y when $x = 4$.

1. Constructing a table of values

Q2. Skill

A bakery sells croissants in multiples of five. The price, p, of the croissants is given by the equation $p = 1.5n + 2$, where n is the number of croissants being bought.

Complete the table of values.

n	5	10	15	20	25	30	35	40	45
p(\$)	9.5	17	24.5	32					

Q3. Skill

The amount of daily rainfall in mm, r, can be roughly estimated from the maximum daily temperature in °C, t, using the following equation:

$r = -0.2t + 5$

a) Complete the table of values.

t(°C)	10	11	12	13	14	15	16	17	18	19	20
r(mm)	3.0	2.8									

b) According to this table, what is the maximum temperature when 1.6 mm of rainfall is expected?

Q4. Application

The number of chocolate bars made per hour in a factory, C, is given by the equation $C = 100 + 150n$, where n is the number of workers on shift that day.

Construct a table of values that shows the number of chocolate bars made per hour with 5, 10, 15, ..., 45 workers.

Check your understanding

Q5. Application

Ella currently has \$350 in her savings account and is planning on depositing \$65 each week into the account.

The total amount of money in her account, M, after w weeks is given by $M = 350 + 65w$.

She is hoping to buy return flights to Japan for \$800 soon.

a) Construct a table of values showing the amount of money in Ella's account for the first ten weeks (include $w = 0$).

b) After how many weeks will Ella have saved at least \$500?

c) Ella wants to book her flights after 6 weeks of saving. Will she be able to afford them at this time?

2. Recursion relationships

Q6. Skill

The first term in a linear sequence is 7.

Each value after this is three more than the term before.

a) What is the value of a?

b) What is the value of d?

c) Replace a and d with their values in the following equation to write a recurrence relation to describe this: $t_n = t_{n-1} + d, t_0 = a$.

Q7. Skill

For the following number sequences, find a recurrence relation.

a) 5, 9, 13, 17, 21,...

b) 20, 16, 12, 8, 4,...

c) $-7, -4, -1, 2, 5,...$

Q8. Skill

A recurrence relation for a sequence is given by the formula $t_n = t_{n-1} + 6.5, t_0 = 2$.

Write the first four values of the sequence.

Q9. Skill

Which of the following number sequences can **not** be represented using a recurrence relation of the form $t_n = t_{n-1} + d, t_0 = a$?

A. 2, 7, 12, 17, 22,...

B. 2, 4, 8, 16, 32,...

C. 29, 23, 17, 11, 5,...

D. 0.1, 0.7, 1.3, 1.9, 2.5,...

Check your understanding

Q10. Application

Gen is an avid knitter. She knits a scarf every day for a week, beginning on Monday.

She gets faster with practice, so every day it takes her four fewer minutes to finish knitting.

On Monday it takes her 83 minutes.

a) Write a recurrence relation to describe this.

b) Find how long it will take Gen to knit a scarf on Tuesday, Wednesday, and Thursday.

Joining it all together

Q11. A linear recurrence relation is described by the formula $t_n = t_{n-1} + 13, t_0 = 5$.

Skill
1 mark

Complete this table of values to show the first 8 values of the sequence.

n	0	1	2	3	4	5	6	7
t_n	5							

Q12. Alicia is on holiday and is spending \$50 per day. On the first day, she has \$620 in her bank account.

Skill
3 marks

a) Define a recurrence relation for the daily balances of Alicia's bank account.

b) Use a table of values to show that on day eight of her holiday, Alicia has \$270 left.

Q13. The volume, V, of a cylinder is given by the formula $V = \pi r^2 \times h$, where r is its radius and h is its height.

Application
2 marks

Raymond's Rainwater Tanks makes cylindrical tanks with a radius of 1 metre and of varying heights.

a) Construct a table of values that shows the volumes of these tanks ranging from 1 metre to 3 metres in height. Use intervals of 0.2 metres and give values correct to one decimal place.

b) Linda wants a rainwater tank for her backyard that is no taller than 1.8 metres and has a capacity of at least 5.5 m^3. Does Raymond have a tank that is suitable?

Q14. Frankie is trying to get fit, so she decides to start running once a week.

Application
2 marks

She plans to run 2 km more each week than the previous week. In week 1 she runs 3 km.

a) Construct a table of values that shows the distance of Frankie's runs for the first eight weeks.

b) If she continues on this trend, in what week will Frankie be able to run further than a half marathon?

NOTE: A half marathon is approximately 21.1 km.

Q15. A ball is thrown according to the equation $h = -(t-5)^2 + 26$, where h is the height in metres of the ball above the ground and t is the time in seconds after it is thrown.

Application
2 marks

a) Construct a table of values showing the height of the ball each second for the first 10 seconds (starting at $t = 0$).

b) At what time(s) is the ball 17 metres above the ground?

Questions from multiple lessons

Q16.

Difficulty:

Year 10
1 mark

Artemis starts up a t-shirt business. He opens with 100 t-shirts, and each week he will order 65 more. The total number of t-shirts that Artemis has ordered can be determined from the rule $N = 100 + 65w$, where w represents the number of weeks the store has been open for.

According to this rule, the total number of t-shirts ordered will be

A. 100 after 1 week.

B. 130 after 2 weeks.

C. 650 after 10 weeks.

D. 975 after 15 weeks.

E. 1075 after 15 weeks.

VCAA 2007 Exam 1 Module 3: Graphs and relations Q2 – Adapted

Q17.

Difficulty:

Year 10
1 mark

A laptop was purchased for $4000 using a loan scheme.

A deposit of $200 was paid.

The balance will be repaid with 20 monthly repayments of $250.

The total amount of interest charged is

A. $1200

B. $1000

C. $4000

D. $5000

E. $0

VCAA 2015 Exam 1 Module 4: Business-related mathematics Q5 – Adapted

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Q18.

Difficulty:

Year 10
2 marks

Within the central district of Paris there is a linear relationship between the price of a coffee in euros (€), p, and the distance, in km, from the Eiffel Tower, d. This relationship is given by the equation $3d = 26 - 4p$.

a) Transpose this linear relation to make p the subject. (1 mark)

b) What is the price of a coffee at the Louvre, 4.5 km away from the Eiffel Tower? Give your answer to the nearest cent. (1 mark)

LESSON 1C

Developing formulae from words

The key skills you will learn in this lesson are:

1. Forming linear equations
2. Solving linear equations

VCAA key knowledge point:

"developing formulas from word descriptions and substitution of values into formulas and evaluation"

1. Forming linear equations

Often, mathematical problems require interpreting a situation and setting up a linear equation in order to find the solution.

When making a linear equation to describe a situation it is helpful to ask yourself the following questions:

- What things in the situation can change?
  - *These will be the variables. We use pronumerals to represent them.*
- How do the things that change relate to each other?
  - *This will be the coefficient of one the variables.*
- Is there anything apart from the coefficient that does not change?
  - *This will be represented by plus or minus a number, also known as a constant.*

WORKED EXAMPLE 7 (*2 marks*)

Amanda wants to keep track of how much vibraphone practice she does for the rest of the year.

She will do three hours of vibraphone practice every day.

a) Write a linear equation that gives the total number of hours, H, for which she will have practiced after d days.

b) Amanda remembers that she has already done 200 hours of vibraphone practice this year. Rewrite the equation from part a) to include this new information.

SOLUTION

a) Write a linear equation that gives the total number of hours, H, she will have practiced after d days.

Step 1 Identify the things in the situation that can change.

These are the variables. They are represented by the pronumerals d and H.

Step 2 Identify how the things that can change relate to each other.

The question states that Amanda will do three hours of practice every day.

In other words, the total number of hours spent practicing is three times the number of days: $H = 3d$.

Step 3 Identify any other elements that do not change.

There is nothing apart from the coefficients that do not change.

This means there is no 'plus or minus something' in the equation.

Step 4 Write the equation.

$H = 3d$

b) Amanda remembers that she has already done 200 hours of vibraphone practice this year. Rewrite the equation from part a) to include this new information.

Step 1 and **Step 2** are the same.

Step 3 Identify any other elements that do not change.

The 200 hours of practice Amanda has done is not going to change as she does more practice. Any more hours of practice she does will be in addition to the 200 hours already done. This means that there will be a $+ 200$ in the equation.

Step 4 Write the equation.

$H = 200 + 3d$

2. Solving linear equations

There are three main steps involved in solving a linear equation:

- identifying what the pronumerals represent
- substituting in the given value
- calculating the value of the variable in question.

WORKED EXAMPLE 8 *(1 mark)*

The number of hours, H, of vibraphone practice Amanda will have done after d days from today is given by the equation $H = 200 + 3d$.

How many hours of practice will Amanda have done after 10 days?

SOLUTION

Step 1 Identify what the pronumerals represent.

H is the total number of hours.

d is the number of days.

Step 2 Substitute in the necessary value.

The question asks for the total number of hours practiced after 10 days.

This means that $d = 10$.

Step 3 Calculate the value of the variable in question.

$H = 200 + 3 \times 10$

$= 200 + 30$

$= 230$

Step 4 Write your answer.

After ten days, Amanda will have done a total of 230 hours of vibraphone practice this year.

Questions 1C *Developing formulae from words*

Refresher question

Q1. $2 \times A = 8$

What is A?

1. Forming linear equations

Q2. Skill

Daisy and Beppe take their dog, Barnesworth, for a walk every morning.

They walk at 80 metres per minute.

The linear equation below gives the distance they've travelled in metres, D, after t minutes of walking.

Fill in the box to complete the equation.

$D = \square \times t$

Q3. Skill

Every year Jane buys 2 new pets from an animal rescue shelter.

Write a linear equation that gives the numbers of pets, p, that Jane will have after t years.

Q4. Skill

Rashpal has an extensive collection of funk music on vinyl. He currently owns 350 vinyls and buys two new records each month.

Write a linear equation that gives the number of vinyls, V, that Rashpal will own after M months.

Q5. Skill

A water company charges their customers yearly.

The bill includes a flat fee of \$534.68 per year plus 0.47 cents for each litre of water used.

Write a linear equation that gives the cost of the bill, C, in dollars, if a customer used L litres of water last year.

Q6. Skill

Xi uses a prepaid data package on his smart tablet. Each month he buys \$30 worth of credit.

Each gigabyte of mobile data he uses costs \$2 in credit.

Write a linear equation that gives the amount of credit, C, Xi has left for the current month after he has used g gigabytes of mobile data.

Check your understanding

Q7. Skill

Use the following equation to complete the paragraph below.

$E = 20 + 30h$

Iris works part-time as a fitness coach for a local football club.

She charges a flat callout-fee of \$ _____ with an hourly rate of \$ _____.

2. Solving linear equations

Q8. Skill

The amount of money, A, in dollars, in the swear jar at EdroloHQ for each swear word said, s, is given by the linear equation below.

$A = 2s$

How much money will be in the swear jar after the staff have sworn 35 times?

Q9. Skill

Julius is hanging out his washing. He wants to use the least number of pegs possible to hang out his t-shirts.

He developed a linear equation that gives the number of pegs, n, needed to hang out t t-shirts:

$n = 1 + t$

How many pegs does Julius need to hang out all 23 of his t-shirts?

Check your understanding

Q10. Application

The cost, C, in dollars, of hiring an UberSELECT to travel a distance of d kilometres is given by the linear equation below.

$C = 3.65 + 2.25d$

a) How much would it cost to hire an UberSELECT to travel 6 kilometres?

b) Angelo wants to catch an UberSELECT from Revolver Upstairs in Prahran to his home in Reservoir, 17 km away. He only has \$40 in his bank account.

Can he afford to hire the UberSELECT?

c) The minimum cost of hiring an UberSELECT is \$10. What is the minimum distance you would have to travel in an UberSELECT in order to be charged more than the minimum fare?

Give your answer in kilometres, correct to one decimal place.

Joining it all together

Q11. Skill 2 marks

Fatima wants to go overseas after she finishes high school. Fatima deposits $10 per week into her bank account. Fatima's grandparents have also deposited a gift of $500 into her account.

a) Write a linear equation that gives the amount of money in the account, A, after w weeks.

b) How much money will be in the account after 24 weeks?

Q12. Skill 2 marks

Every morning Sergeant Steve makes the new recruits do five more sit-ups than they did the day before.

If the new recruits did 30 sit-ups this morning, how many will Sergeant Steve make them do in nine days' time?

Q13. Skill 2 marks

A clothing store pays its staff $15 per hour plus a commission of $5 for each sale they make.

Each shift at the store is 6 hours long. On one shift, a staff member make 17 sales.

How much money did this staff member make on that shift?

Q14. Skill 2 marks

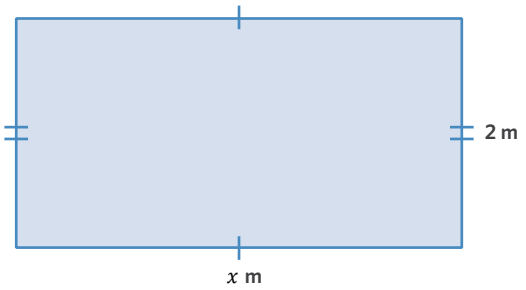
A small rural community of 20 people has a 500 000 litre water tank. The tank is currently full.

Each day, each member of the community uses 300 litres of water.

Assuming the tank doesn't get refilled at all, how many litres of water will be left in the tank after 30 days?

Q15. Skill 2 marks

Using this diagram:



a) Find the perimeter of the of the rectangle when $x = 3.5$.

b) Find the value of x if the perimeter is 20 m.

Q16. Application 2 marks

A parking inspector's job is to find cars that have been parked longer than allowed and to issue fines to the drivers of those cars.

Assuming that one in every five cars is parked for longer than allowed, how many cars must the parking inspector check in order to issue 17 fines?

Q17. Application 2 marks

Captain Raymond Holt is making scrambled eggs. He whisks four eggs in a measuring jug. The measuring jug weighs 0.618 kilograms. The total weight of the measuring jug and the four eggs is 858 grams.

Excluding the shell, what is the average weight, in grams, of an egg?

Q18. Skill 2 marks

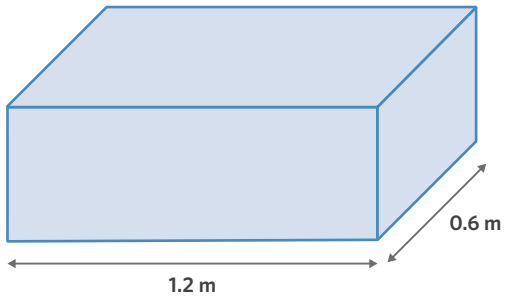
Joe has to write a 1500-word essay. Unfortunately, it is due in four and a half hours and he has not started. It will take Joe half an hour to plan the essay and then he will write 350 words per hour.

a) Joe believes he can finish the essay in time. Show that Joe is incorrect.

b) What is the minimum whole number of words Joe must write per hour in order to complete the essay in time?

Q19. Charlie is building a planter box for their vegetable garden. Their planter box, shown below, will have a volume of 360 litres. 1 m^3 is equal to 1000 litres.

Skill
1 mark



Show that the planter box will be 0.5 metres deep.

VCAA question

Q20. A phone company charges a fixed, monthly line rental fee of \$28 and \$0.25 per call.

1 mark

Let n be the number of calls that are made in a month. Let C be the monthly phone bill, in dollars.

The equation for the relationship between the monthly phone bill, in dollars, and the number of calls is

A. $C = 28 + 0.25n$ **B.** $C = 28n + 0.25$ **C.** $C = n + 28.25$

D. $C = 28(n + 0.25)$ **E.** $C = 0.25(n + 28)$

2016 VCAA Exam 1, Module 4: Graphs and relations. Q2

Questions from multiple lessons

Q21. As part of her new year resolutions, Sarah decides to read every month from January to December for one year.

Difficulty:

Year 10
1 mark

Each month she counts the number of pages that she has read.

- In January, she reads 12 pages of a book.
- In February, she reads 18 pages.
- In March, she reads 24 pages.
- In April, she reads 30 pages.

The number of pages she reads each month continues to increase according to this pattern.

The number of pages she reads in September is

A. 48

B. 54

C. 60

D. 66

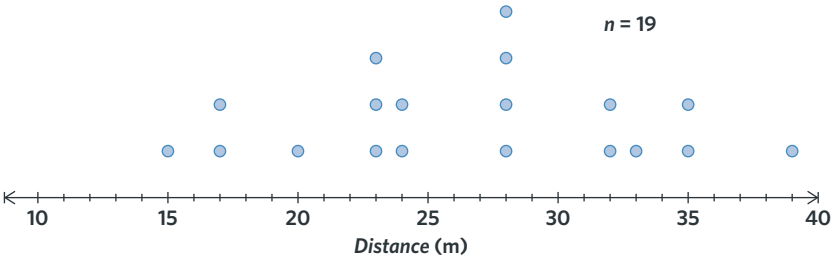
E. 72

VCAA 2014 Exam 1 Module 1: Number patterns Q1 – Adapted

Q22. The dot plot below displays the *distance*, in metres, of a football kicked by each student in a Year 7 class.

Difficulty:

Year 10
1 mark



The median *distance* is

A. 23 m

B. 34 m

C. 28 m

D. 25.5 m

E. 26 m

VCAA 2018 Exam 1 Data analysis Q2 – Adapted

Q23. Corey decides to set himself a challenge in regards to the number of push-ups he can do every night before he falls asleep.

Difficulty:

Year 10
2 marks

- On the 1st night, he does 42 push-ups.
- On the 2nd night, he does 60 push-ups.
- On the 3rd night, he does 78 push-ups.

The number of push-ups he does each night forms the terms of an arithmetic sequence, as shown below. This sequence continues for each night for a week.

42, 60, 78…

a) Write down a calculation which shows that the common difference for this sequence is 18. (1 mark)

b) How many push-ups will Corey do on the 6th night? (1 mark)

VCAA 2015 Exam 2 Module 1: Number patterns Q1 a,b – Adapted

LESSON 1D

Simultaneous equations – numerical and graphical solutions

VCAA key knowledge point:

"numerical [and] graphical ... solutions of simultaneous linear equations in two variables"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.18; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

The key skills you will learn in this lesson are:

1. Finding the point of intersection numerically
2. Reading the point of intersection off a graph
3. Finding the point of intersection using CAS

Simultaneous equations are a set of equations with two or more unknowns which have a common solution (a solution that can satisfy all of the equations simultaneously).

The solution to a pair of simultaneous equations is equal to the point of intersection between the two lines.

1. Finding the point of intersection numerically

The point of intersection is the point at which two lines meet. To find the point of intersection numerically, fill in a table of values for a small amount of x values. The point of intersection is found when for, two identical x values, the two corresponding y values are the same.

NOTE: This method will only work if the coordinates of the point of intersection are whole numbers.

WORKED EXAMPLE 9 *(2 marks)*

Two lines are given by the equations:

$y_1 = 2x + 1$

$y_2 = x + 3$

Use a table of values from $x = -2$ to $x = 2$ to find the point of intersection of the two lines.

SOLUTION

Step 1 Set up a table with three rows and six columns and label accordingly.

x	−2	−1	0	1	2
y_1					
y_2					

Step 2 Fill in the first value.

Substitute $x = -2$ into $y_1 = 2x + 1$ to find the first value.

$y_1 = 2 \times (-2) + 1$

$y_1 = -3$

x	−2	−1	0	1	2
y_1	−3				
y_2					

Step 3 Repeat for the rest of the values.

x	−2	−1	0	1	2
y_1	−3	−1	1	3	5
y_2	1	2	3	4	5

✓ *1 mark for completing the table*

Step 4 Find the column in which the y values are the same.

x	−2	−1	0	1	**2**
y_1	−3	−1	1	3	**5**
y_2	1	2	3	4	**5**

The y values are both 5 when $x = 2$.

Therefore, the point of intersection is (2,5).

✓ *1 mark for writing the point of intersection*

2. Reading the point of intersection off a graph

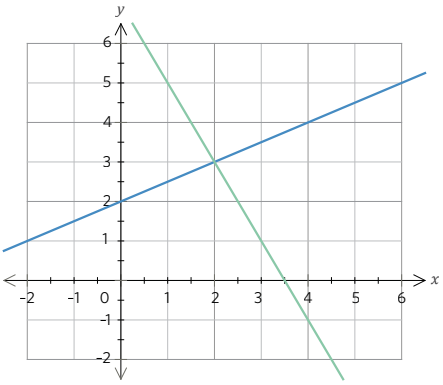
It is also possible to find the point of intersection visually from a graph. To find the point of intersection, locate where the two lines cross then find the coordinates of the point.

If the lines never meet, then there is no point of intersection and there is no solution to the simultaneous equations. This happens only when the lines are parallel.

WORKED EXAMPLE 10 *(1 mark)*

Here is a graph displaying two linear equations.

Find the point of intersection.

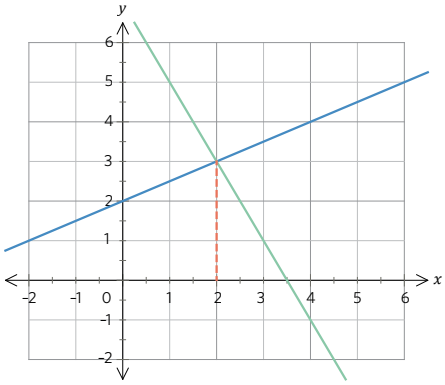


SOLUTION

Step 1 Locate the point of intersection.

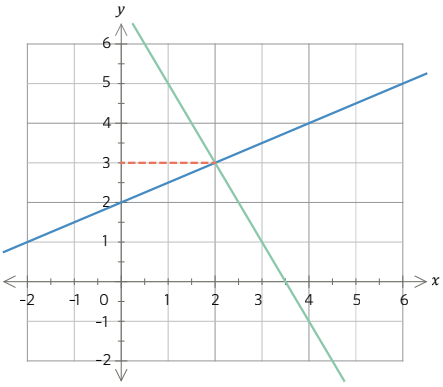
This is the point at which the lines cross.

Step 2 Draw a line down (or up) from the point to find the x coordinate.



The x coordinate is 2.

Step 3 Draw a line left (or right) from the point to find the y coordinate.



The y coordinate is 3.

Step 4 Write the answer. The point of intersection is (2,3).

3. Finding the point of intersection using CAS

Another way of finding the point of intersection is by plotting a graph of the lines on CAS.

This method will be much more accurate than the other methods when the point of intersection is not a whole number.

WORKED EXAMPLE 11 *(1 mark)*

Two lines have the equations $y_1 = 3x + 1$ and $y_2 = -2x + 4$.

Use a CAS calculator to find the point of intersection graphically.

SOLUTION: TI-NSPIRE

Step 1 Open a new page by pressing ctrl + doc ▾ and press 2 ('Add Graphs').

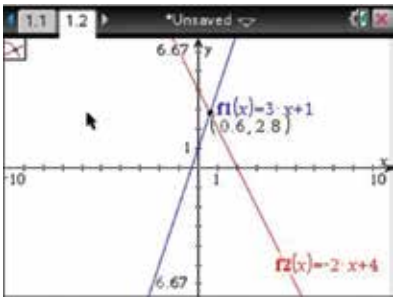
Step 2 Enter the first equation as 'f1(x)=3x+1' and press enter.

Step 3 Press tab and then enter the second equation as 'f2(x)=−2x+4' and press enter.

Step 4 Press menu, press 6 ('Analyze Graph'), press 4 ('Intersection').

Use the mouse to first click anywhere to the left of the point of intersection and then click anywhere to the right of the point of intersection.

The solution will appear.



The coordinates of the point of intersection are $x = 0.6$ and $y = 2.8$.

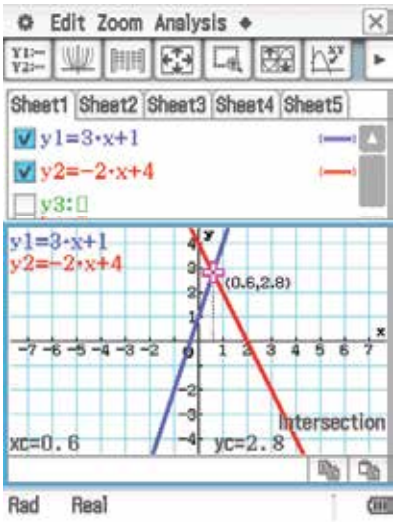
SOLUTION: CASIO CLASSPAD

Step 1 From the menu, tap Graph&Table

Step 2 In row 'y1' enter the first equation '3x+1' and press EXE. In row 'y2' enter the first equation '−2x+4' and press EXE.

Step 3 Tap ⊹ to graph the equations.

Step 4 Tap Analysis then G-solve then Intersect. The solution will appear..



The coordinates of the point of intersection are $x = 0.6$ and $y = 2.8$.

Questions 1D *Simultaneous equations – numerical and graphical solutions*

Refresher question

Q1. The equation of a line is $y = 3x + 4$. Find the value of y when $x = 2$.

1. Finding the point of intersection numerically

Q2. Skill

Below is the table of values for the equations of two lines. Determine the point of intersection of the two lines.

x	−3	−2	−1	0	1	2	3
y_1	−9	−6	−3	0	3	6	9
y_2	−5	−3	−1	1	3	5	7

Q3. Skill

Two lines have the equations $y_1 = 2x + 2$ and $y_2 = 4x - 2$.

Use a table of values from $x = -2$ to $x = 2$ to find the point of intersection.

Check your understanding

Q4. Skill

Henry and Katie are both saving their pocket money to buy the latest iPhone. At x weeks after they start saving, the amount of money in dollars, y, they each have is given by the following equations:

Henry: $y = 20x + 35$

Katie: $y = 25x + 15$

a) In the first six weeks (from $x = 0$ to $x = 6$), when will they have the same amount of savings?

b) How much money do they each have at this point?

Q5. Skill

The equations of two lines are $y_1 = 4x - 7$ and $y_2 = -2x + 5$.

Using a table of values, show that the point of intersection of the two lines occurs at $(2, 1)$.

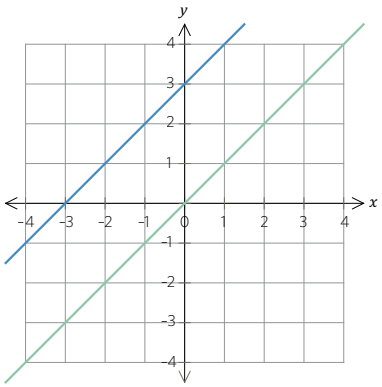
2. Reading the point of intersection off a graph

Q6. Skill

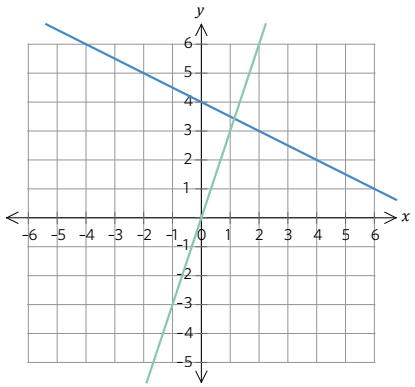
Consider the 4 pairs of simultaneous equations represented in the graphs below.

Which of the pairs will have no solution?

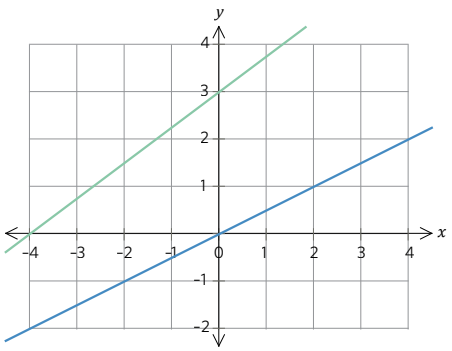
A.



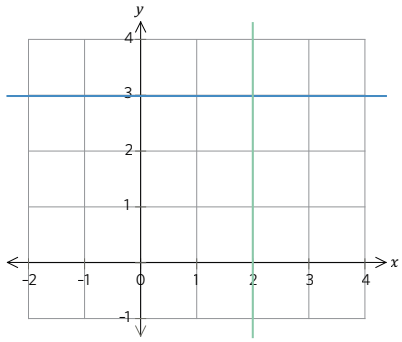
B.



C.



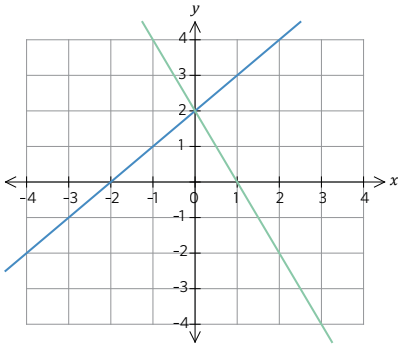
D.



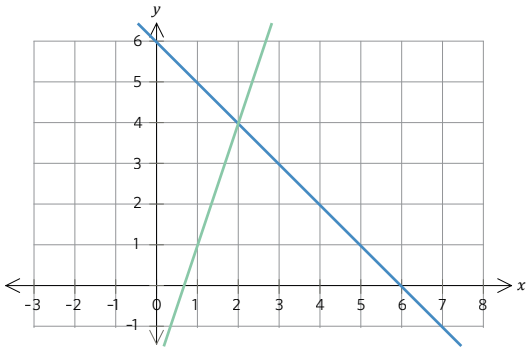
Q7. Skill

State the coordinates of the point of intersection for each of the following graphs:

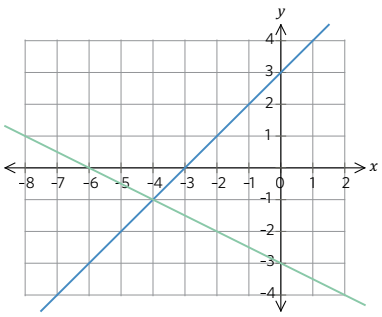
a)



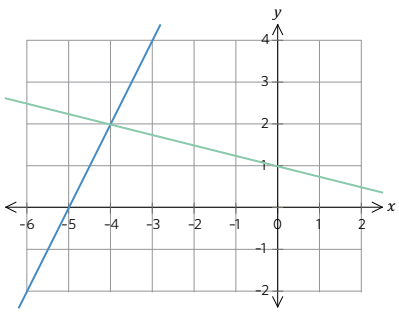
b)



c)



d)

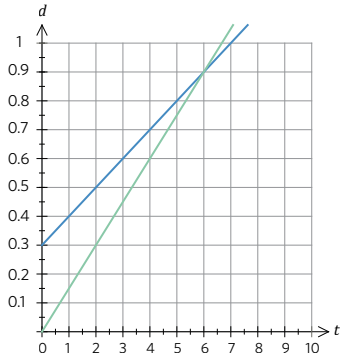


Check your understanding

Q8. Application

Allison and Beatrice are running a race along a straight track. Beatrice received a handicap so she starts ahead of Allison.

The positions of the runners are shown in the graph below where d is the distance, in kilometres, from the start of the track and t is the time, in minutes, after the beginning of the race. If the race starts at 9:45am, at what time does Allison catch up to Beatrice?



3. Finding the point of intersection using CAS

Q9. Skill

Using a calculator, find the point of intersection for each of the following pairs of lines. Give answers correct to two decimal places where necessary.

a) $y_1 = -5x + 1$ and $y_2 = 3x + 3$

b) $y_1 = 3x + 6$ and $y_2 = -2x - 2$

c) $y_1 = -2x - 1$ and $y_2 = 6x + 5$

d) $y_1 = 4x - 4$ and $y_2 = -5x + 2$

Joining it all together

Q10. Skill 1 mark

Two lines intersect at $y = 5$. The equations of the two lines are $y_1 = 2x + 1$ and $y_2 = -x + a$.

Find the value of a.

Q11. Application 1 mark

Jenny and Kendra both work part-time at a burger shop. Let the number of hours Jenny works per week be j and the number of hours Kendra works per week be k. The number of hours they both work must satisfy the following equations.

$j = -k + 20$

$j = 2k + 2$

Find j, and k.

Q12. Application 1 mark

Charlotte is going to the supermarket to buy some snacks. She knows that a bag of chips is one and a half times the price of a bag of popcorn, and that the total cost of a bag of chips and a bag of popcorn is \$6.25. This is shown in the equations below.

$c = 1.5p$

$c + p = 6.25$

Find the price of one bag of chips.

Questions from multiple lessons

Q13.

Difficulty:

1 mark

Harry is a very successful musician. His yearly salary has two components. He receives a yearly fixed base wage of $100 000 and an extra $5000 for each concert he plays.

Let n be the number of concerts he plays in a year.

Let S be his total yearly salary.

What is the equation that models the relationship between Harry's yearly salary and the number of concerts he plays?

A. $S = 100\ 000n + 5000$

B. $S = 5000n + 100\ 000$

C. $S = 5000(n + 100\ 000)$

D. $S = 100\ 000(n + 5000)$

E. $S = n + 105\ 000$

VCAA 2016 Exam 1 Module 4: Graphs and relations Q2 – Adapted

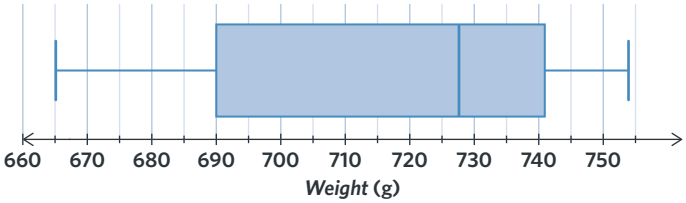
Q14.

Difficulty:

Year 10
1 mark

Niall and Louis are holidaying in Australia and decide to visit the zoo.

The following boxplot displays the *weight*, in grams, of 83 meerkats at Werribee Zoo.



The percentage of these 83 meerkats with a weight greater than 690 g is closest to

A. 15% **B.** 25% **C.** 50%

D. 75% **E.** 85%

VCAA 2017 Exam 1 Data analysis Q1 – Adapted

Q15.

Difficulty:

2 marks

One Direction are selling tickets to their last concert before they disband.

There are two types of tickets, adult and child.

The revenue is then divided into two major spendings, venue costs and staff wages.

They are expressed as percentages of the ticket prices in the table below.

	Adult	Child
Venue costs	40%	30%
Staff wages	50%	40%

a) A group of friends decided to buy tickets together. They spent $880 on adult tickets and $360 on child tickets. Calculate the amount, in dollars, of the group of friends' ticket purchases that went towards the venue costs of the concert. (1 mark)

b) A different group of friends purchased tickets, and by doing so contributed $402 towards staff wages. They purchased $180 worth of child tickets. How much money did they spend on adult tickets? (1 mark)

VCAA 2011 Exam 2 Module 3: Graphs and relations Q1a,b – Adapted

LESSON 1E

Simultaneous equations – algebraic solutions

The key skills you will learn in this lesson are:

1. Substitution
2. Elimination

VCAA key knowledge point:

"...algebraic solutions of simultaneous linear equations in two variables"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.18; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

Simultaneous equations can be solved algebraically using two methods: substitution and elimination.

Solving involves finding the values of the variables, normally x and y, at the point of intersection of the equations.

Each pair of simultaneous equations can be solved by both substitution and elimination, however one method is often more efficient depending on the equations.

This chapter will focus on how and when to use substitution and elimination.

1. Substitution

Solving simultaneous equations using the substitution method involves a variable from one equation being substituted into the other.

Substitution is useful when one or both of the equations are in the form: $y = ax + b$ *or* $x = ay + b$, where a and b are constants.

This is because having an equation written with x or y as the subject means we can do the substitution right away; no transposing is needed.

Example

Some pairs of equations that could be solved by the substitution method are:

$y = 2x + 3$ and $x + y = 9$

$x = y - 4$ and $x = -2y + 9$

$y - 2x = 7$ and $x = 3 - 4y$

WORKED EXAMPLE 12 *(4 marks)*

Solve the following pairs of simultaneous equations using substitution:

a) $y = 5 - 3x$ and $4x + 3y = 5$ **b)** $x = y + 7$ and $x = 4y - 2$

SOLUTION

a) $y = 5 - 3x$ and $4x + 3y = 5$

Step 1 Write the two equations on separate lines and label them equation 1 and equation 2.

$y = 5 - 3x$ ①

$4x + 3y = 5$ ②

Step 2 Substitute equation 1 into equation 2.

We have chosen to substitute y because in equation 1, y is on its own so it is easy to substitute.

$y = 5 - 3x$ ①

$4x + 3y = 5$ ②

Substitute ① into ②.

$4x + 3(5 - 3x) = 5$

Step 3 Expand, then solve for x.

$4x + 15 - 9x = 5$

$4x - 9x = 5 - 15$

$-5x = -10$

$x = \frac{-10}{-5}$

$x = 2$

✓ *1 mark for finding x value*

Step 4 To find y, substitute $x = 2$ into one of the equations. Both equations will work, but choosing the simpler one makes for easier calculations.

Substitute $x = 2$ into ①

$y = 5 - 3$ ②

$y = 5 - 6$

$y = -1$

✓ *1 mark for finding y value*

Step 5 Write the answer.

$x = 2, y = -1$

b) $x = y + 7$ and $x = 4y - 2$

Step 1 Write the two equations on separate lines and label them equation 1 and equation 2.

$x = y + 7$ ①

$x = 4y - 2$ ②

Step 2 Since both equations are expressions for x, let them equal each other.

$y + 7 = 4y - 2$

Step 3 Solve for y.

$y + 7 = 4y - 2$

$y - 4y = -2 - 7$

$-3y = -9$

$y = \frac{-9}{-3}$

$y = 3$

✓ *1 mark for finding y value*

Step 4 To find x, substitute $y = 3$ into one of the equations. Both equations will work, but choosing the simpler one makes for easier calculations.

Substitute $y = 3$ into ①

$x = 3 + 7$

$x = 10$

✓ *1 mark for finding x value*

Step 5 Write the answer.

$x = 10, y = 3$

2. Elimination

The method of elimination requires the equations to be added or subtracted in a way that eliminates one of the variables.

Elimination is useful when the equations are in the form $ax + by = c$, where a, b and c are constants.

That is, both variables are on the same side of the equals sign in both equations.

Example

Some pairs of equations that could be solved by the substitution method are:

$3x + y = 2$ and $4x - 5y = -1$

$y - 4x = 3$ and $5x + 3y = 11$

$2x + 2y = 16$ and $8y - 2x = -4$

WORKED EXAMPLE 13 *(4 marks)*

Solve the following pairs of simultaneous equations using elimination:

a) $3x + 2y = 7$ and $-x - 2y = -1$

b) $2x + 3y = 2$ and $3x + 4y = 4$

SOLUTION

a) $3x + 2y = 7$ and $-x - 2y = -1$

Step 1 Write the two equations on separate lines and label them equation 1 and equation 2.

$3x + 2y = 7$ ①

$-x - 2y = -1$ ②

Step 2 Choose a variable to be eliminated.

Since the coefficients for y are equal but opposite, if the two equations are added, y will be eliminated. This means choosing to eliminate y will be the easier option.

Step 3 Add the equations and simplify.

Add the left-hand sides and the right-hand sides of the equations together.

$$\begin{array}{rl} & (1) + (2) \\ & 3x + 2y = 7 \\ + & -x - 2y = -1 \\ \hline & 2x + 0 = 6 \end{array}$$

So, $2x = 6$

Step 4 Solve for x.

$$2x = 6$$
$$x = \frac{6}{2}$$
$$x = 3$$

✓ *1 mark for finding x value*

Step 5 To find y, substitute $x = 3$ into one of the equations. Either will work, but try to pick the simpler one.

Substitute $x = 3$ into ①

$$3\textcircled{3} + 2y = 7$$
$$9 + 2y = 7$$
$$2y = 7 - 9$$
$$2y = -2$$
$$y = \frac{-2}{2}$$
$$y = -1$$

✓ *1 mark for finding y value*

Step 6 Write the answer.

$x = 3, y = -1$

b) $2x + 3y = 2$ and $3x + 4y = 4$

Step 1 Write the two equations on separate lines and label them equation 1 and equation 2.

$2x + 3y = 2$ ①

$3x + 4y = 4$ ②

Step 2 Choose a variable to be eliminated.

The coefficients for x and y are not equal so neither variable can be eliminated directly. Choose x to eliminate since the coefficients are smaller than those for y. This will result in easier calculations later.

Step 3 Look at the coefficient in each equation for the variable chosen to be eliminated. Find the lowest common multiple of these, and work out how they should be multiplied to reach this amount.

The coefficients of x are 2 and 3. The lowest common multiple is 6.

$2 \times 3 = 6$

$3 \times 2 = 6$

Step 4 Multiply each equation as determined in the previous step and label the new equations 3 and 4.

① $\times$ 3

$6x + 9y = 6$ ③

② $\times$ 2

$6x + 8y = 8$ ④

Step 5 Since the coefficients of x are positive in both equation 3 and 4, subtract one equation from the other and simplify.

③ − ④

$$\begin{array}{r} 6x + 9y = 7 \\ -\ (6x + 8y = 8) \\ \hline 0 + y = -2 \end{array}$$

So, $y = -2$

✓ *1 mark for finding y value*

Step 6 To find x, substitute $y = -2$ into one of the equations and solve.

Substitute $y = -2$ into ①

$$2x + 3(-2) = 2$$
$$2x - 6 = 2$$
$$2x = 2 + 6$$
$$2x = 8$$
$$x = \frac{8}{2}$$
$$x = 4$$

✓ *1 mark for finding x value*

Step 7 Write the answer.

$x = 4, y = -2$

Questions 1E *Simultaneous equations – algebraic solutions*

Refresher question

Q1. Consider the equation $4x + 3y = 17$.

Given that $x = 2$, find the value of y.

1. Substitution

Q2. Solve the following pairs of simultaneous equations using substitution.

Skill

a) $x = y + 1$ and $x + y = 7$

b) $x = 7 - y$ and $x = y - 3$

Q3. Use the method of substitution to solve the following pairs of simultaneous equations.

Skill

a) $x = 2y + 3$ and $2x - 3y = 7$

b) $4x + 3y = -5$ and $y = x - 4$

c) $x = 7y - 5$ and $x = 3y + 3$

d) $y = 14 - 4x$ and $y = x - 16$

Check your understanding

Q4. Consider the following linear equations:

Skill

$y = 3x - 2$

$2y - 4x = 2$

Which of the following is the point of intersection of the two equations?

A. $(-3, 7)$ **B.** $(7, -3)$ **C.** $(3, 7)$ **D.** $(7, 3)$

Q5. The following pair of simultaneous equations were given in a Year 11 maths test.

Skill

$3x + 3y = 6$ ①

$y = 2x + 5$ ②

a) One student decided to solve the equations using the method of elimination. Explain why it would have been more appropriate to use substitution to solve these equations.

b) Find the solution using the method of substitution.

2. Elimination

Q6. Use the method of elimination to solve the following pairs of simultaneous equations.

Skill

a) $2x + y = 8$ and $3x - y = 2$

b) $3x + 2y = 9$ and $4x - 2y = -2$

c) $x + 4y = 3$ and $x - 5y = -6$

d) $4x - 3y = 6$ and $6x - 3y = 12$

Q7. Consider the following pair of simultaneous equations.

Skill

$2x - 2y = 10$ ①

$4x + 3y = 6$ ②

a) Solve the equations by eliminating x first.

b) Solve the equations by eliminating y first.

Q8. Solve the following pairs of equations using the method of elimination.

Skill

a) $x + 3y = 11$ and $7y - 2x = 4$

b) $2x + 5y = 9$ and $4x + 6y = 6$

c) $3x - 4y = 20$ and $x + 7y = -10$

d) $7x + 3y = 16$ and $2x + 2y = 8$

Check your understanding

Q9. Skill

Monica and Niamh wanted to stock up on spreads, so they went together to the supermarket. Monica bought two jars of Vegemite and four jars of Nutella for \$33, and Niamh bought five jars of Vegemite and three jars of Nutella for \$40.50.

This is shown in the equations below.

$2V + 4N = 33$

$5V + 3N = 40.5$

Use the method of elimination to find the cost of a jar of Vegemite and a jar of Nutella.

Joining it all together

Q10. Skill 6 marks

For the following pairs of simultaneous equations, write whether the pair would best be solved using the method of substitution or elimination.

NOTE: You do not need to solve any of them.

a) $y + 2x = 3$ and $x = 4 - 5y$

b) $5y - 2x = 1$ and $y = 3 - x$

c) $2x - y = 4$ and $x + y = 3$

d) $x = 7 - 3y$ and $x = 1 + 2y$

e) $x - 4y = 2$ and $3y - x = -1$

f) $7y - 2x = 5$ and $3x + 4y = 11$

Q11. Application 3 marks

Mary's Marvellous Milkshakes sells milkshakes in two different sizes: small and large.

Mary looked at her sales records from Monday and Tuesday this week.

She found that on Monday, she sold 10 small milkshakes and 12 large milkshakes and received a total of \$73. On Tuesday, she sold 20 small milkshakes and 16 large milkshakes and received a total of \$114.

This information can be shown in the following pair of simultaneous equations.

Monday: $10s + 12l = 73$

Tuesday: $20s + 16l = 114$

a) Would the method of elimination or substitution be more appropriate in solving these simultaneous equations? (1 mark)

b) Solve the equations using your chosen method to find the price of each size milkshake. (2 marks)

Q12. Application 2 marks

Zeke is an avid baker and is currently working on perfecting his creme brulée recipe. His scales are broken at the moment so he is trying to figure out the weight of a cup of flour and a cup of sugar using algebra.

He knows that one cup of sugar weighs 40 grams less than two cups of flour, and that the total weight of two cups of sugar and five cups of flour is one kilogram.
This information is shown in the equations below.

$s = 2f - 40$

$2s + 5f = 1000$

Solve the equations using the most appropriate method to find the weights of a cup of flour and a cup of sugar.

Questions from multiple lessons

Q13.

Difficulty:

1 mark

Link wants to manage the amount of money that he spends each week on hairspray. He begins with \$150 that he will spend purely on hairspray over a 12-week period. Each week, he plans to spend \$12.50.

Let h_n be the amount of hairspray money Link has left after week n.

A recurrence relation that can be used to model this situation is

A. $h_0 = 12.5 \quad h_{n+1} = 150 - 12.5$

B. $h_0 = 12.5 \quad h_{n+1} = h_n - 150$

C. $h_0 = 150 \quad h_{n+1} = h_n - 12.5\,h_n$

D. $h_0 = 150 \quad h_{n+1} = h_n - 12.5$

E. $h_0 = 150 \quad h_{n+1} = 1 - 0.125\,h_n$

VCAA 2014 Exam 1 Module 1: Number patterns Q4 – Adapted

Q14.

Difficulty:

1 mark

The transaction details for a loan established at the start of November 2018 are shown in the table below.

The table is incomplete.

Date	Details	Loan	Payment	Balance
01.11.2018	Loan established	\$1000.00		\$–1000.00
15.11.2018	Repayment		\$50.00	\$–950.00
20.11.2018	Borrow	\$70.00		\$–1020.00
21.11.2018	Repayment		\$120.00	\$–900.00
25.11.2018	Borrow	\$180.00		\$–1080.00
26.11.2018	Borrow	\$200.00		\$–1280.00
29.11.2018	Repayment		\$540	\$–740.00
30.11.2018	Interest			\$–745.33

Interest is calculated and paid monthly on the maximum accumulated debt for that month.

The annual rate of interest paid on this account is closest to

A. 5%

B. 0.04%

C. 4%

D. 1.25%

E. 1%

VCAA 2015 Exam 1 Module 4: Business-related mathematics Q6 – Adapted

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Q15.

Difficulty:

Year 10

2 marks

Anita sells second hand t-shirts at a local market. The revenue, in dollars, that she makes from selling n second hand t-shirts is given by

$$revenue = 24n$$

The cost, in dollars, of buying these second hand t-shirts, and cleaning them before sale, is given by

$$cost = 15.5n + 14$$

a) What is the selling price of Anita's second hand t-shirts? (1 mark)

b) Anita sold 18 second hand t-shirts last Saturday. How much profit did she earn? Profit is calculated by subtracting cost from revenue. (1 mark)

VCAA 2017NH Exam 2 Module 4: Graphs and relations Q2 – Adapted

LESSON 1F

Practical linear equations

The key skills you will learn in this lesson are:

1. Solving simultaneous equations on CAS
2. Application of simultaneous equations

VCAA key knowledge point:

"use of linear equations, including simultaneous linear equations in two variables, and their application to solve practical problems"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.18; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Solving simultaneous equations on CAS

We can algebraically solve simultaneous equations efficiently on CAS. One big advantage of using CAS is it can solve a system of simultaneous equations with more than two unknowns; this can be a lengthy process to do by hand.

WORKED EXAMPLE 14 *(1 mark)*

Use CAS to find the values of x and y that satisfy the following set of simultaneous equations.

$3x + y = 2$ (1) $\quad$ $2x + y = -7$ (2)

SOLUTION: TI-NSPIRE

Step 1 Open a new page by pressing ctrl + doc and press 1 ('Add Calculator').

Step 2 Press menu, then press 3 ('Algebra'), 7 ('Solve System of Equations'), and then press 2 ('Solve System of Linear Equations'). A box will appear.

Step 3 Set the 'Number of equations' to 2 and enter the variables as x,y. Press enter.

Step 4 Enter equation (1) into the top box provided.
Enter equation (2) into the box below.
Press enter.

Step 5 Read the answer.

$x = 9$ and $y = -25$.

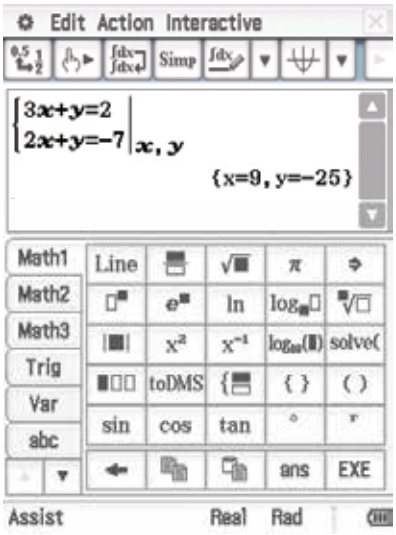
SOLUTION: CASIO CLASSPAD

Step 6 From the menu, tap Main.

Step 7 Press the keyboard button and then tap the simultaneous equations icon.

Step 8 Enter equation (1) into the top box. Enter equation (2) into the box below.
Enter the variables x, y in the last box.

Step 9 Press EXE. The answer will appear.
$x = 9$ and $y = -25$.



2. Application of simultaneous equations

Simultaneous equations can be used to solve real-world problems with multiple unknowns.

To solve a question using simultaneous equations, first identify and define the variables, set out the equations, and then solve.

WORKED EXAMPLE 15 *(2 marks)*

Tickets for a Justin Bieber concert were \$110 for A-reserve seats and \$85 for B-reserve seats.

Annie and her friends bought 12 tickets and spent a total of \$1095. How many B-reserve tickets did they buy?

SOLUTION

Step 1 Identify and define the variables.

In this case, the variables are the number of the A-reserve and B-reserve tickets bought.

Let a be the number of A-reserve tickets and b be the number of B-reserve tickets they bought.

Step 2 Convert the information given into two equations using the chosen variables.

The total number of tickets bought was 12, so the sum of the number of A-reserve tickets and the number of B-reserve tickets will be 12:

$a + b = 12$ ①

The total amount spent was \$1095 on a \$110 tickets and b \$85 tickets:

$110a + 85b = 1095$ ②

✓ *1 mark for writing equations*

Step 3 Solve the equations simultaneously to find the values of a and b.

$a = 3, b = 9$

Step 4 Step 4. Write the answer.

Annie and her friends bought 9 B-reserve tickets.

✓ *1 mark for writing answer*

WORKED EXAMPLE 16 *(2 marks)*

Seven years ago, Viola's age was half of her brother Sebastian's age.

In three years' time, Viola's age will be three quarters of Sebastian's age.

What is her current age?

SOLUTION

Step 1 Identify and define the variables.

The variables are Viola and Sebastian's current ages.

Let v be Viola's current age and s be Sebastian's current age.

Step 2 Convert the information given into two equations using the chosen variables.

Viola's age seven years ago was half of Sebastian's age seven years ago:

$v - 7 = \frac{1}{2}(s - 7)$ ①

Viola's age in three years will be three quarters of Sebastian's age in three years:

$v + 3 = \frac{3}{4}(s + 3)$ ②

✓ *1 mark for writing equations*

Step 3 Solve the equations simultaneously to find the values of v and s.

The equations can be put directly into CAS in this form. To solve by hand, they would need to first be rearranged.

$v = 12, s = 17.$

Step 4 Write the answer.

Viola is currently 12 years old.

✓ *1 mark for writing answer*

Questions 1F *Practical linear equations*

Refresher question

Q1. Let Stuart's age be s. Write an expression for the following:

a) Stuart's age nine years from now.

b) Stuart's age four years ago.

1. Solving simultaneous equations on CAS

Q2. Skill

Use CAS to solve the following pairs of simultaneous equations:

a) $4x - y = 16$ and $2y - 3x = 3$

b) $2y - 7 = 3x + 4$ and $5x - y = 12$

c) $-7y + 8x = 52$ and $2x = 5y$

Q3. Skill

Use CAS to solve the following sets of simultaneous equations:

a) $10x - 3y = 3.3$ and $y = 3x - 0.9$

b) $3.5x = 10y + 5.65$ and $7y + 0.4 = 2x$

Check your understanding

Q4. Application

The perimeter of a rectangle is 60 cm. The length of the rectangle is four times the width. This information is shown in the equations below.

$2l + 2w = 60$

$l = 4w$

Use CAS to find the length and width of the rectangle.

2. Applications of simultaneous equations

Q5. Skill

Jason buys 3 notebooks and 4 pens for \$11.60 and Keiran buys 6 notebooks and 2 pens for \$14.80.

Let n be the cost of a notebook and p be the cost of a pen. Represent this information in a pair of simultaneous equations.

Q6. Skill

A group went out for Daniel's 10^{th} birthday party. They first went bowling, and then went to see a movie. The prices, in dollars, of bowling and movie tickets for children and adults are shown in the following equations.

Let c be the number of children and a be the number of adults.

Bowling: $12c + 16a = 156$

Movie: $11c + 15a = 144$

a) What is the price of an adult movie ticket?

b) What is the price of a children's movie ticket?

c) How many children and adults were in the group?

Q7. Skill

Saskia was at a Vinnies op shop when there was a sale on. All skirts were $3.50 and all tops were $2. Saskia spent $35 and bought 13 pieces of clothing (all of which were either tops or skirts).

a) Represent this information in two simultaneous equations. Let s be the number of skirts Saskia bought and t be the number of tops she bought.

b) How many tops did Saskia buy?

Q8. Skill

Amelia and India went to their favourite gourmet doughnut shop and stocked up on their two favourite flavours: cinnamon and Oreo. Amelia bought four cinnamon doughnuts and two Oreo doughnuts for $17.80. India bought six cinnamon doughnuts and nine Oreo doughnuts (she was very hungry) for $48.90.

a) Write down two simultaneous equations to present this information. Let c be the price of a cinnamon doughnut and o be the price of an Oreo doughnut.

b) What is the price of an Oreo doughnut?

Check your understanding

Q9. Application

Dana is six years older than Ben. In two years' time, her age will be one and a half times Ben's age.

Show that Dana is currently 16 years old.

Joining it all together

Q10. Skill 2 marks

At McDonald's, it costs $1.50 to upgrade from a small to a large cheeseburger meal. On Friday, McDonald's sold 130 small and 160 large cheeseburger meals for $1748.

a) Display this information as a pair of simultaneous equations.

b) What is the price of a small cheeseburger meal?

Q11. Application 2 marks

Michelle is making a stir-fry and adds three carrots and one head of broccoli, weighing a total of 580 grams. She also knows from prior experience that five carrots weigh 60 grams less than three heads of broccoli. What is the weight of one carrot?

Q12. Application 2 marks

Katya is 6 years younger than Ginger. In four years, her age will be four fifths of Ginger's age.

What is Katya's current age?

Q13. Application 3 marks

Alex and Danni both work part-time as waitresses.

The number of hours they each worked on Monday and Tuesday this week along with the total amount they were both paid is shown in the table below.

	Alex	Danni	Total amount paid
Monday	4	6	$168
Tuesday	7	3	$166.5

Show that Alex gets paid $16.50 per hour.

Q14. Application 2 marks

The registration fee for a charity fun run is $24 for adults and $16 for children. 345 people registered this year and the charity received $7280 in revenue.

How many adults participated?

Q15. Five years ago, Ellie's age was two thirds of Angelica's age.

Application
3 marks

In seven years, Ellie's age will be five sixths of Angelica's age.

What is the age difference between Ellie and Angelica?

Q16. A rectangle has a perimeter of 70 cm.

Application
3 marks

The length of the rectangle is two and a half times the width.

What is the area of the rectangle?

VCAA question

Q17. The total playing time of three CDs and four DVDs is 690 minutes.

1 mark

The total playing time of five CDs and seven DVDs is 1192 minutes.

All of the CDs have the same playing time as each other and all of the DVDs have the same playing time as each other.

Let x be the playing time of a CD.

Let y be the playing time of a DVD.

The set of simultaneous linear equations that can be solved to find the playing time of a CD and the playing time of a DVD is

A. $4x + 3y = 690$
$7x + 5y = 1192$

B. $3x + 4y = 690$
$5x + 7y = 1192$

C. $3x + 5y = 690$
$4x + 7y = 1192$

D. $3x + 4y = 1192$
$5x + 7y = 690$

E. $4x + 3y = 1192$
$7x + 5y = 690$

2009 VCAA Exam 1, Module 3: Graphs and relations. Q4

Questions from multiple lessons

Q18. A group of friends went shopping at Lush for bath products.

Difficulty:

1 mark

- Isobel bought four bath bombs and one face mask for $44.00.
- Jackson bought two bath bombs and two face masks for $43.00.
- Klaudia bought three bath bombs and four face masks.

How much did Klaudia spend in total?

A. $72.00

B. $73.00

C. $78.50

D. $81.00

E. $86.00

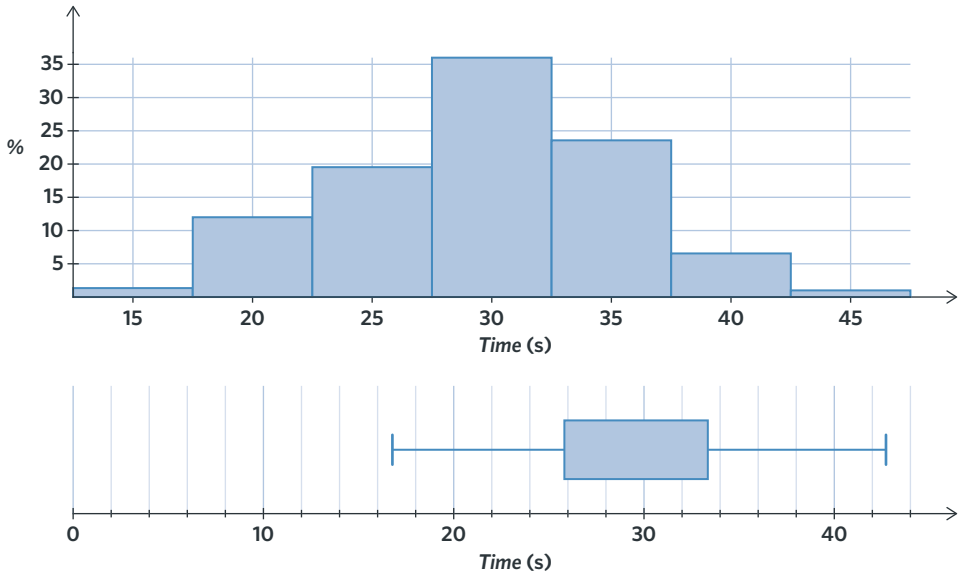
VCAA 2017 Exam 1 Module 4: Graphs and relations Q6 – Adapted

Q19.

Difficulty:

Year 10
1 mark

The histogram and boxplot shown below both display the distribution of the *time*, in seconds, of 150 TV advertisements.



The shape of the distribution of *time* for the advertisements is best described as

A. approximately symmetric with outliers.

B. perfectly symmetric with no outliers.

C. positively skewed with outliers.

D. approximately symmetric with no outliers.

E. negatively skewed with no outliers.

VCAA 2019NH Exam 1 Data analysis Q1 – Adapted

Q20.

Difficulty:

2 marks

A local cinema charges $20 per ticket.

They have to pay $60 a day to rent the venue and spend an average of $4 per ticket sold to pay their employees.

Let n be the number of tickets sold per day at the cinema.

a) Write an expression for the profit that the cinema will make in terms of n (1 mark)

The cinema wants to make at least $373 of profit per day.

b) Determine the minimum number of tickets they need to sell per day. (1 mark)

VCAA 2013 Exam 2 Module 3: Graphs and relations Q3 a,b – Adapted

AOS 2: Arithmetic and number

Computation and practical arithmetic

CHAPTER 2

LESSON 2A

Order of operations and directed numbers

The key skills you will learn in this lesson are:

1. Directed numbers
2. Order of operations

VCAA key knowledge point:

"review of computation: order of operations, directed numbers, scientific notation, estimation, exact and approximate answers, rounding correct to a given number of decimal places or significant figures"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Directed numbers

Directed numbers involve operations with negative numbers.

Directed numbers are regular numbers (e.g. 1, 2, 3) with a direction attached (e.g. $-1, -2, -3, +1, +2, +3$).

Positive and negative are opposite directions.

Numbers with the same value but opposite direction are said to be opposites.

For example, positive four is the opposite of negative four. Adding them equals zero.

Positive and negative numbers can be added or subtracted. When this happens, first simplify the expression (see table).

Notice that adding or subtracting negative numbers has the opposite effect to adding or subtracting the same positive number.

Signs	Examples	Operation
$+ \quad +$	$10 + (+5)$ becomes $10 + 5$	Addition
$- \quad -$	$10 - (-5)$ becomes $10 + 5$	Addition
$+ \quad -$	$10 + (-5)$ becomes $10 - 5$	Subtraction
	$10 - (+5)$ becomes $10 - 5$	Subtraction

dividing with negative numbers can affect the sign of the answer. This information can be seen in the

	Examples – multiplying	Examples – dividing	Answer
	$10 \times (+3)$ becomes 10×3	$\frac{10}{(+3)}$ becomes $\frac{10}{3}$	Positive
	$(-10) \times (-3)$ becomes 10×3	$\frac{(-10)}{(-3)}$ becomes $\frac{10}{3}$	Positive
	$10 \times (-3)$ becomes -10×3	$\frac{10}{(-3)}$ becomes $-\frac{10}{3}$	Negative
	$(-10) \times (+3)$ becomes -10×3	$\frac{(-10)}{(+3)}$ becomes $-\frac{10}{3}$	Negative

WORKED EXAMPLE 1 *(2 marks)*

a) Show $+3$ and -2 on a number line.

b) Evaluate $-2 + 7 - (-3)$

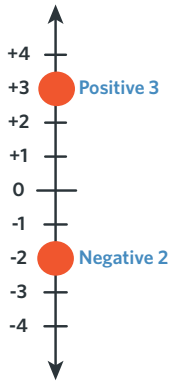
SOLUTION

a) Show $+3$ and -2 on a number line.

Step 1 Draw a number line, ensuring there is space for $+3$ and -2 to be included.

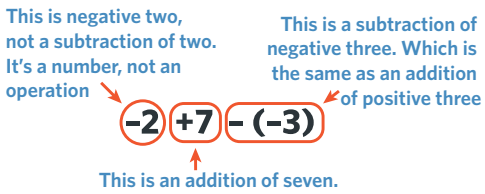
The number line can be drawn vertically or horizontally.

Step 2 Count up three units from zero to reach $+3$, and two units below zero to reach -2. Mark these points.



b) Evaluate $-2 + 7 - (-3)$

Step 1 Examine the expression.



Step 2 The $-(-3)$ is simplified to $+3$.

$= -2 + 7 + 3$

Step 3 The operations are performed.

$= 8$

2. Order of operations

Applying operations in different orders can give different results.

Example

Look at $3 \times 5 + 2$

Doing the multiplication first, we get $15 + 2$. Which gives an answer of 17.

Doing the addition first, we get 3×7, which leads to an answer of 21.

It is important to know which operations to apply first. This can be remembered using the acronym BIDMAS, also known as BODMAS. The order of the letters in this acronym signifies the order operations should be completed in.

BIDMAS	Order	Meaning	Example
Brackets	First	Perform operations inside brackets first	$(1 + 3) \times 2 = 4 \times 2$
Indices (Order)	Second	Perform squares or square roots	$2 + 3^2 = 2 + 9$
Division/Multiplication	Third	Perform division and/or multiplication	$3 + 5 \times 2 = 3 + 10$
Addition/Subtraction	Fourth	Perform addition and/or subtraction	$9 + 3 = 12$

WORKED EXAMPLE 2 *(2 marks)*

Simplify the following:

a) $2 + 3(4^2 - 12)$

b) $\frac{(12 - 4)}{(5 \times 2 - 6)}$

SOLUTION

a) $2 + 3(4^2 - 12)$

Step 1 Start with the brackets first. There is one set of brackets here, (4^2-12). Within this set of brackets, we start the BIDMAS process again, at B. There are no brackets inside the brackets so we move to the next letter in BIDMAS, I, and calculate the squares and square roots.

$2 + 3(4^2 - 12)$

$= 2 + 3(16 - 12)$

Step 2 Within the brackets there are no squares left, and no multiplication or division operations, so we progress to the AS letters, addition and subtraction.

$= 2 + 3(4)$

Step 3 The bracket has no operations in it, so we can expand it. The number 3 outside the bracket implies that 3 is being multiplied by whatever is inside the brackets, which in this case, is 4.

$= 2 + 12$

Step 4 The operations are performed.

$= 14$

b) $\frac{(12 - 4)}{(5 \times 2 - 6)}$

Step 1 There are two sets of brackets. There are no squares or square roots, so we do multiplication first, in this case the operation is in the denominator bracket.

$\frac{(12 - 4)}{(5 \times 2 - 6)}$

$= \frac{(12 - 4)}{(10 - 6)}$

Step 2 There is no multiplication or division left so we do addition or subtraction, and then remove the brackets.

$= \frac{(8)}{(4)}$

Step 3 The operations are performed, which leads to the answer.

$= \frac{8}{4}$

$= 2$

Questions 2A *Order of operations and directed numbers*

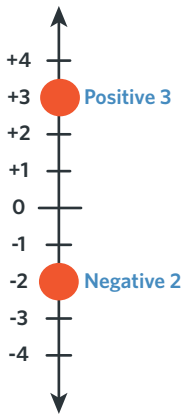
Refresher question

Q1. Answer the following questions:

a) By counting the spaces between +3 and −2 on the number line to the right find the difference between these values.

b) Which of the following is the same as 5(6)?

- 56
- $5 + 6$
- 5×6



1. Directed numbers

Q2. Evaluate

Skill

a) $12 + (+4)$

b) $12 + (-4)$

Q3. Evaluate

Skill

a) $6 + 13 + (-4)$

b) $14 - 10 - (-3)$

c) $-10 + (-5) - (-12)$

Check your understanding

Q4. Evaluate

Skill

$-12.6 - (-10.3) + 13.5 + (-2.1)$

Q5. Tom delivers newspapers every morning. Today Tom did the following:

Application

- Delivered ten newspapers on Flinders Street
- Delivered six newspapers on Collins Street
- Picked up 20 newspapers from the newsagent
- Delivered 15 newspapers on Lonsdale Street.

When Tom returned home there were eight newspapers in his bag.

How many did Tom start with?

2. Order of operations

Q6. Evaluate

Skill

a) $1 + 3 \times 2$

b) $2(10)$

c) $2 \times (3 + 1) - 5$

d) $2 + 6(3 + 1)$

Q7. Evaluate

Skill

a) $3 \times 5 + 12 - \frac{6}{2}$

b) $\frac{(40 + 8)}{(5 \times 2 - 2)}$

c) $(2(10 - 5) + 1) + 13 \times 2$

Check your understanding

Q8. Omar never makes a mistake. On the last question of a maths test, Omar gets an answer of 23. Which of the following three expressions was Omar asked to simplify?

Application

A. $\frac{(4^2 + 10(3^2 - 8))}{2^2 + 9}$

B. $\frac{(5 \times 6)}{(7 - 4)} + 5(2^2 - 1) - 2$

C. $5(10 - 3) - \frac{(3 \times 2^2)}{2}$

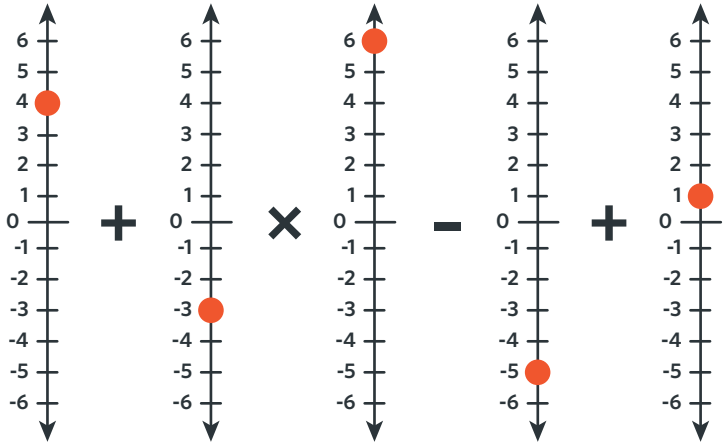
Q9. Application

Maggie runs a fruit shop. At the start of the day she has 70 plums in stock. Read the cartoon below. Does Maggie have enough plums to give her mum? Show your working.

Joining it all together

Q10. Skill 2 marks

Write an expression to accompany the visual representation of directed numbers below, and evaluate the expression.



Q11. Skill 5 marks

Evaluate the following expressions and find which one has the highest value.

a) $(6 + (-3)) \times 2 + 5 - (-10)$

b) $12 + (-5) \times 2 - (-3)$

c) $\dfrac{(50 - (-10))}{(2 \times (-3) + 3 \times 4)}$

d) $(-5) \times (-3) + \dfrac{4}{2} + (-5)$

Q12. Application 1 mark

Luis has \$50. He spends \$10.50 on lunch for himself, and then buys his three best friends a can of soft drink each from the canteen. At lunch Luis finds a watch in the schoolyard worth \$200. After school he goes to the shops and gets refunded \$24.99 for a shirt he returns to a store.

Afterwards, Luis meets up with two classmates to buy their friend a JB-Hifi voucher for \$90 for his birthday. They split the cost evenly. While at the shops, he buys some flowers for his mum for \$19.95. At the end he has \$1.94 left.

How much does a can of soft drink at the canteen cost?

Questions from multiple lessons

Q13.
Difficulty:
1 mark

Elizabeth and Ash are buying souvenirs from overseas for their friends and family in Australia. A particular store sells small and large fridge magnets. The equations below describe the amount of money Elizabeth and Ash each spent on fridge magnets, where s is the price of small fridge magnets and l is the price of large fridge magnets.

Elizabeth: $3s + 4l = 21.80$

Ash: $6s + 3l = 24.60$

Use your calculator to draw a graph of the two lines and find the point of intersection. Use the point of intersection to determine the price of a small fridge magnet.

A. \$2.40

B. \$3.60

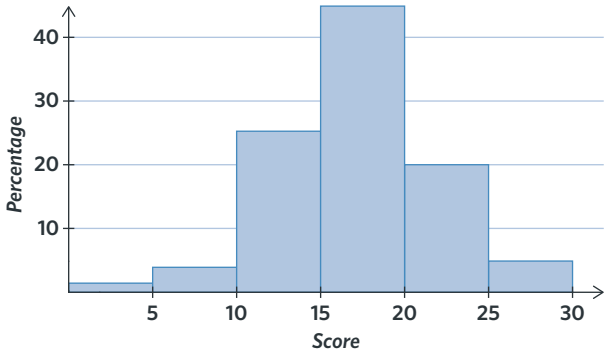
C. \$3.80

D. \$2.20

E. \$2.60

Q14.
Difficulty:
Year 10
1 mark

The histogram below displays the distribution of *scores*, out of 30, on a spelling test for 80 primary school students.



The number of students with a *score* between 10 and 15 is closest to

A. 36

B. 20

C. 25

D. 16

E. 15

VCAA 2019NH Exam 1 Data analysis Q2 – Adapted

Q15.
Difficulty:
4 marks

Boston and Alana are renting games and movies from Blockbuster. Boston rents two games and seven movies for \$46.40 while Alana rents four games and five movies for \$49.60. Let g represent the cost of renting a game at Blockbuster and m represent the cost of renting a movie at Blockbuster.

a) Write down two simultaneous equations that represent this information. (2 marks)

b) Use your calculator to find the cost of a game rental at Blockbuster. (1 mark)

c) Lauren goes to Blockbuster and rents six games and three movies. How much money did she spend? (1 mark)

LESSON 2B

Scientific notation

The key skills you will learn in this lesson are:

1. Converting to scientific notation
2. Converting from scientific notation

VCAA key knowledge point:

"review of computation: order of operations, directed numbers, scientific notation, estimation, exact and approximate answers, rounding correct to a given number of decimal places or significant figures"

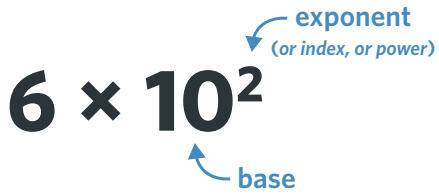
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016–2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

Scientific notation is a method used to make very small or very large numbers easier to work with. A number written in scientific notation is of the form; $x \times 10^y$ where $1 \leq x < 10$ and y is an integer (a positive or negative whole number). Here, '10' is known as the base, and 'y' is known as the exponent.

An example of a number written in scientific notation is 5×10^{-2}. Here, $1 \leq 5 < 10$ and -2 is an integer.

Example

In 6×10^2, 10 is the base and 2 is the exponent.



1. Converting to scientific notation

To convert a numeral to scientific notation we must do the following:

1. Place a decimal point after the first non zero digit.
2. Count the number of places the decimal point has moved from its original position.

   This number becomes the exponent on the base 10.
3. Find whether the decimal point has moved to the left or right.

   If it has moved to the left the exponent is positive. If it has moved to the right the exponent is negative.

Thinking about scientific notation

For × 10

- the number on the exponent increases by one
- the decimal point moves to the right by one place (another 0 appears at the end).

Expression	Scientific notation	Number
7 × 10	7×10^1	70
7 × 10 × 10	7×10^2	700
7 × 10 × 10 × 10	7×10^3	7000
7 × 10 × 10 × 10 × 10	7×10^4	70000

For ÷ 10

- the number on the exponent decrease by one
- the decimal point moves to the left by one place.

Expression	Scientific notation	Number
7 ÷ 10	7×10^{-1}	0.7
7 ÷ 10 ÷ 10	7×10^{-2}	0.07
7 ÷ 10 ÷ 10 ÷ 10	7×10^{-3}	0.007
7 ÷ 10 ÷ 10 ÷ 10 ÷ 10	7×10^{-4}	0.0007

WORKED EXAMPLE 3 *(1 mark)*

Convert 946 000 to scientific notation.

SOLUTION

Step 1 Write the number and put the decimal point after the first non-zero digit.

9.46000

Step 2 Find the direction and number of places that the decimal point moved.

It moved five places to the left.

Step 3 Write the answer excluding the trailing zeros (a sequence of zeros which have no other numbers after it).

9.46×10^5

To check the answer multiply 9.46 by 10^5. The decimal point will move to the right by five places, giving 946 000. This matches the number in the question, so the answer is correct.

2. Converting from scientific notation

To convert from scientific notation to a numeral we must do the following:

1. Write the number excluding the '$\times 10^y$'
2. Move the decimal point according to the exponent of the base 10. The decimal point moves to the right if the exponent is positive and moves to the left if the exponent is negative.
3. Include trailing zeros if the decimal point moves to the right and leading zeros if the decimal point moves to the left.

WORKED EXAMPLE 4 *(1 mark)*

Convert 3.58×10^{-4} to numerals.

SOLUTION

Step 1 Write the number excluding the '$\times 10^y$'

3.58

Step 2 As the exponent of 10 is -4, move the decimal point four places to the left. Include zeros as placeholders as you go.

Step 3 Write the answer, excluding the original decimal point.

0.000358

Questions 2B *Scientific notation*

Refresher question

Q1. What is 10^{-5} in numerals?

1. Converting to scientific notation

Q2. What is 200 expressed in scientific notation?

Skill

Q3. Convert the following to scientific notation.

Skill

a) 32 815 **b)** 8 234 400 **c)** 0.000782 **d)** 0.000000038492

Check your understanding

Q4. Which of these is **not** written in scientific notation?

Skill

A. 5.8×10^{-6} **B.** 0.9×10^{3} **C.** 3.7823×10^{2} **D.** 1.003×10^{-8}

Q5. The distance around an athletics track is 40000 cm.

Application

What is the distance, in centimetres, of half the track written in scientific notation?

2. Converting from scientific notation

Q6. What is 9×10^{-3} in numerals?

Skill

Q7. Convert the following to numerals.

Skill

a) 3.54×10^{5} **b)** 4.92×10^{-3} **c)** 8.7408×10^{-5}

Check your understanding

Q8. Which number is the smallest?

Skill

A. 5.834×10^{0} **B.** 2.31×10^{3} **C.** 4.234×10^{-7} **D.** 1.6×10^{-5}

Q9. A giraffe is 5.283706×10^{3} mm tall.

Application

What is its height in millimetres written in numerals?

Joining it all together

Q10. The radius of a tennis ball is 0.033 metres.

Application
2 marks

What is its radius in kilometres written in scientific notation?

Q11. A student makes a mistake when converting 0.0000000027 to scientific notation and gets an answer of 2.7×10^{-8}.

Application
1 mark

What should the answer actually be?

Q12. Which number is **not** correctly converted to scientific notation?

Skill
1 mark

	Numerals	Scientific notation
A.	0.000000739	7.39×10^{-7}
B.	4.673	4.673×10^{0}
C.	2 749 800	2.7498×10^{6}
D.	348.940000	3.4894×10^{-2}

Questions from multiple lessons

Q13. The following table of values is governed by the equation $y = -2x - 18$.

Difficulty:

Year 10
1 mark

x	−2	3	b	5	1
y	−14	a	−12	−28	c

What are the values of a, b and c?

A. $a = -12, b = -3, c = -24$

B. $a = -24, b = -12, c = -3$

C. $a = -24, b = -3, c = -20$

D. $a = -20, b = -12, c = -3$

E. $a = -12, b = -24, c = -20$

Q14. The cost of a VPN subscription is $60.00 per year plus GST.

Difficulty:

1 mark

The cost of a VPN subscription for three years, including GST, is

A. $180.00

B. $186.00

C. $198.00

D. $240.00

E. $264.00

VCAA 2014 Exam 1 Recursion and financial modelling Q4 – Adapted

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Q15. Riley is travelling around Central America. She is spending exactly $170 per day. At the start of her trip, she has $7100 in her bank account.

Difficulty:

2 marks

a) Define a recurrence relation, using B_n, B_{n-1} and B_0, that represents the daily balance of Riley's bank account. (1 mark)

b) Use a table of values to show that Riley has $5910 in her bank account after one week. (1 mark)

LESSON 2C

Estimation, types of answers and rounding

The key skills you will learn in this lesson are:

1. Rounding
2. Significant figures
3. Estimation

VCAA key knowledge points:

"review of computation: order of operations, directed numbers, scientific notation, estimation, exact and approximate answers, rounding correct to a given number of decimal places or significant figures"

"efficient mental and by-hand estimation and computation in relevant contexts"

"effective use of technology for computation"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016–2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Rounding

Rounding is writing a number in a shorter form that is an estimate of the actual number.

This means that rounding a number decreases its accuracy.

Rounding makes numbers easier to read, or makes calculations simpler.

Rules for rounding:

- if the digit being rounded is between 1–4, round it down.
- if the digit being rounded is between 5–9, round it up.

Examples

By rounding to three decimal places:

One third, written as 0.3333333... becomes 0.333

Pi, written as 3.1415926535..., becomes 3.142

WORKED EXAMPLE 5 *(2 marks)*

a) Round 4.6 to the nearest whole number.

b) Round 62 to the nearest ten.

SOLUTION

a) Round 4.6 to the nearest whole number.

Step 1 Consider the nearest whole numbers to 4.6. They are 4 and 5.

Step 2 Look at which one is closer to 4.6. The number 5 is closer to 4.6 (compared to 4).

Therefore, 4.6 rounded to the nearest whole number is 5.

b) Round 62 to the nearest ten.

Step 1 Work out which multiples of ten are either side of 62. In this case, it is 60 and 70.

Step 2 Decide which of 60 and 70 is closer to 62.

60 is closer to 62.

Therefore 62 rounded to the nearest ten is equal to 60.

2. Significant figures

A significant figure is a digit in a number that contributes to the value of the number with certainty. In other words, this digit was originally part of the number, and isn't the result of rounding.

Rules for finding significant figures:

- The first significant figure is always the first non-zero digit, looking from left to right.
- All non-zero digits are significant.
- All zeros in between significant figures are significant.
- All zeros **after** a decimal point that are on the right side of a significant figure are significant.
- All other zeros are not significant.

Example

In the following examples, **red** digits are significant, and **blue** digits are not.

153 Each digit is non-zero, so 153 has three significant figures.

100 The two zeros in 100 are not significant. Therefore, only the 1 is significant.

1070 The 1 and the 7 are significant. The first 0 is between the 1 and the 7, so is significant.

The zero on the right is not significant.

0.00470 The 4 and the 7 are significant. The zero on the right is after a decimal point and to the right of significant digits, therefore is significant.

The other zeros are not significant.

WORKED EXAMPLE 6 *(2 marks)*

Find the number of significant figures in the following numbers.

a) 1030

b) 016.040

SOLUTION

a) 1030

Step 1 For 1030

The non-zero digits are 1 and 3. These are significant.

Step 2 Determine which zeros are significant.

One zero is between two non-zero numbers, therefore it is significant . The significant figures are 1, 0 and 3, as shown here:

1030

Therefore, 1030 has three significant figures.

b) 016.040

Step 1 Examine the number: 016.040

There is a zero before the first non-zero digit before the decimal place, we can remove it.

We are left with 16.040

Step 2 The non-zero digits are 1, 6 and 4. These are significant.

Step 3 Determine which zeros are significant.

The zeros at the end of the number is significant because it is after the decimal place.

One zero is between two non-zero numbers, therefore it is significant .

The significant figures are 1, 6, 0, 4 and 0, as shown here:

16.040

Therefore, 16.040 has five significant figures.

3. Estimation

Estimation is using rounding to make a calculation simpler, at the expense of accuracy.

Estimation can be used to quickly check if an answer looks correct, or if a rough answer is needed.

An estimation is an approximation of the exact value.

WORKED EXAMPLE 7 *(1 mark)*

Estimate the area of a rectangular swimming pool of dimensions 7.8 metres and 14.2 metres.

SOLUTION

Step 1 Decide an appropriate amount to round the numbers.

Here, rounding to the nearest whole number seems appropriate.

Step 2 Round the numbers to the nearest whole number.

7.8 becomes 8

14.2 becomes 14

Step 3 Calculate the area using $Area = length \times width$.

7.8×14.2

$\approx 8 \times 14$

$\approx 112\ m^2$

NOTE: the 'approximately equal' sign, $\approx$, is used.

Step 4 Interpret the result.

The area of the pool is approximately $112\ m^2$.

Questions 2C *Estimation, types of answers and rounding*

Refresher question

Q1. How many decimal places does the number 8.125 have?

1. Rounding

Q2. Skill

a) Round 3.7 to the nearest whole number.

b) Which of the following is equal to 343 rounded to the nearest ten? 340 or 350.

c) Which of the following is equal to 2.835 rounded to two decimal places? 2.83 or 2.84.

Q3. Skill

a) Round 182.341 to two decimal places.

b) Round 8,256 to the nearest hundred.

c) Round 3.0068 to two decimal places.

Check your understanding

Q4. Application

Natalia lives 2.7 kilometres from school, and walks to school and back every day.

How far does Natalia walk along this route every day?

Round your final answer to the nearest kilometre.

Q5. Application

A butcher rounds meat to the nearest hundred grams and sells it at that price.

If the scale reads 0.652 kg and the meat is $8 per kg, how much will the customer have to pay?

2. Significant figures

Q6. Skill

How many significant figures are in the following number?

25

Q7. Skill

How many significant figures are in the following numbers?

a) 103.2

b) 2580

c) 0.0105

Check your understanding

Q8. Skill

Which of these does **not** have exactly three significant figures?

A. 10 500 **B.** 0.0236

C. 20.50 **D.** 37.0

Q9. Application

Ms. Tao asked four of her students to write one number each that had the same amount of significant figures as the number of letters in their name.

Which students were right? (More than one student got it right).

Jake: 4030, Emily: 2.0155, An: 65 000, Annabelle: 4.0010013

3. Estimation

Q10. Skill

Estimate the cost to buy two pens at 2.05 each.

Use rounding to the nearest whole number.

Q11. Skill

a) Estimate the area of a desk with dimensions 1.8×3.1 metres.

b) Estimate the total amount of time required to paint the walls of a square room if it took 3.9 hours for the first wall.

c) Estimate the weight of 5 bottles if one weighs 495 grams.

Check your understanding

Q12. Application

Three friends are stuck on a deserted island. The only thing to eat there is coconuts. If there are 89 coconuts, estimate how long they can survive, if they must eat three each per day.

Q13. Application

There are 32 minutes left in class, and Aarav still has 9 questions left to complete. If he doesn't finish them in time, the teacher will give him lots of homework. Roughly estimate how much time Aarav has per question left if he wants to have no homework.

Joining it all together

Q14. Skill 3 marks

Estimate the answer to the following expressions. Then calculate the answer and round to three significant figures. Find the difference between the estimations and the answers.

a) 3.9×11.1

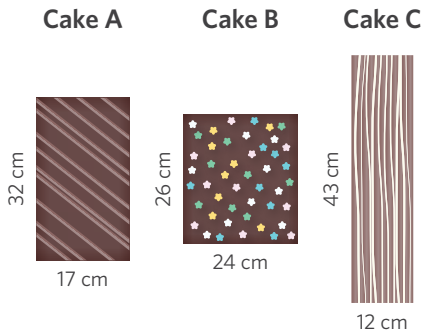
b) 22×49

c) 0.48×6.04

Q15. Elena is buying a cake for her best friend's birthday to eat with their classmates in maths class, but she's worried most cakes will be too small. She knows the cake needs an area of at least 600 cm^2.

Application
5 marks

A diagram of the three cakes at the cake shop is below.



a) Without doing any calculations, guess which one has the biggest area.

b) Use rounding, to the nearest five centimetres, to estimate an approximate area for each cake.

c) Calculate the actual area of each cake.

d) Was your guess from a) correct?

e) Which cakes are big enough to feed the class?

Q16. Ruby, Ethan and Ishara are trying to estimate the area of their school.

Application
4 marks

The school grounds are a rectangle with dimensions 207.85 m by 322.37 m, and there are 400 students.

Here are their results:

Name	Ruby	Ethan	Ishara
Estimate (m^2)	67,200	60,000	66,976

Before calculating the estimated area, the students performed the following rounding of the dimensions:

- One student used rounding to the nearest metre.
- One student used rounding to the nearest 10 metres.
- One student used rounding to the nearest 100 metres.

a) Who used which method?

b) Which methods estimated higher than the actual answer, and which methods estimated lower?

c) Who got closest to the actual answer?

d) Estimate the area of your school.

Questions from multiple lessons

Q17.

Difficulty:

Year 10
1 mark

Which of the following expressions has a value of 1?

A. $\frac{(-10) \times (-4)}{8} + (-2)^2$

B. $\frac{6(4-3)^2}{(8-6)(7+(-2)\times 5)}$

C. $3((9-2)+(-6))+(-7)\times 2$

D. $\frac{9-(-3)}{4} - \frac{1}{2}(11+(-7))$

E. $\frac{(-3)\times(5-(-1))}{(4-1)^2}$

Q18.

Difficulty:

Year 10
1 mark

Lachlan is working on his fitness so he went for nine runs last week. The total distance he ran was 42.3 km. The mean distance of these runs is

A. 4.23 km

B. 4.67 km

C. 4.7 km

D. 4.8 km

E. 4.92 km

VCAA 2019NH Exam 1 Data Analysis Q3 – Adapted

Q19.

Difficulty:

Year 10
2 marks

Rafael is asked to evaluate the expression $\frac{-\left(\frac{3}{4}+12\right)\times 4}{21-38}$

a) Rafael's working out is shown below. Which line does the first mistake appear on? (1 mark)

Line 1: $\frac{-(\frac{3}{4}+12)\times 4}{21-38} = \frac{-(\frac{3}{4}+\frac{48}{4})\times 4}{-17}$

Line 2: $= \frac{-(\frac{51}{4})\times 4}{-17}$

Line 3: $= \frac{51}{-17}$

Line 4: $= -3$

b) Evaluate the expression correctly. (1 mark)

LESSON 2D

Magnitude

The key skills you will learn in this lesson are:

1. Order of magnitude
2. Logarithms
3. Logarithmic scale

VCAA key knowledge point:

"orders of magnitude, units of measure that range over multiple orders of magnitude and their use and interpretation, and the use and interpretation of log to base 10 scales, such as the Richter scale"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016–2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Order of magnitude

Orders of magnitude are used to describe the approximate size of a number. They relate to the powers of ten, such as 10^1 10^2 and 10^3. The order of magnitude of a number is given by the exponent on the base 10, after the number has been converted to scientific notation.

Example

5.79×10^5 has an order of magnitude of 5.

When comparing the order of magnitude of two values:

- A difference of one in the order of magnitude means that one value is $10^1 = 10$ times larger than the other.
- A difference of two in the order of magnitude means that one value is $10^2 = 100$ times larger than the other.

WORKED EXAMPLE 8 *(1 mark)*

The length of an ant is 0.6 cm and the length of a front yard is 600 cm.

How many orders of magnitude do these two lengths differ by?

SOLUTION

Step 1 Find how many times larger 600 cm is than 0.6 cm.

$\frac{600}{0.6} = 1000$

Step 2 Express this as an exponent with base 10.

$1000 = 10^3$

Step 3 Write out the answer using the exponent on the base 10.

The two lengths differ by 3 orders of magnitude.

2. Logarithms

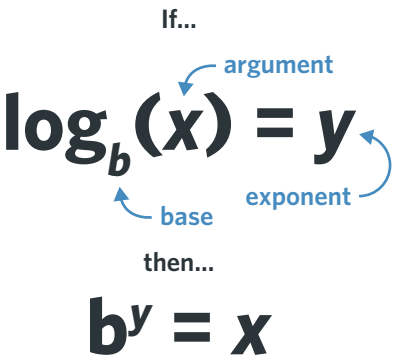
Logarithms, commonly referred to as logs, are a mathematical operation. Each logarithm has a base, an argument, and an exponent.

This topic focuses on logs with a base of 10. This is given by the formula $\log_{10}(x) = y$ where $x = 10^y$.

When we are given y and asked to find x, we can find x by raising 10 to the power of the value, y.

Often logarithms of base 10 are expressed without the subscript 10. For example, $\log(100) = 2$. Sometimes the logarithm of a number is not a whole number and cannot be solved by hand so a calculator may be used.

NOTE: Logarithms can have bases other than 10 but these are not covered in this subject.



WORKED EXAMPLE 9 *(1 mark)*

What is $\log_{10}(0.0001)$?

SOLUTION

Step 1 Convert 0.0001 so it is in the form 10^y.

10^{-4}

Step 2 Find the value of y (the power). This is the answer.

-4

3. Logarithmic scale

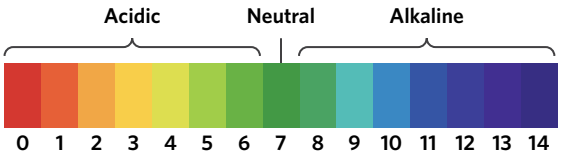
Logarithmic scales are scales which use logarithms of base 10 to plot data which has a large range.

Two examples of logarithmic scales are:

- the Richter scale.
- the pH scale.

The Richter scale measures the amplitudes of earthquakes, which are often very large, and converts them to smaller numbers using the base 10 logarithm. This is done by finding the value of $\log_{10}(\textit{amplitude of earthquake})$, known as the magnitude of the earthquake.

The pH scale ranges from 0 to 14 and measures how acidic or how alkaline a solution is. A solution with a pH of 7 is considered to be neutral. Anything that has a pH of less than 7 is acidic whereas anything that has a pH of more than 7 is alkaline. As the pH scale uses a base 10 logarithmic scale, an increase by 1 in pH means a solution is 10 times as alkaline, and a decrease by one in pH means a solution is 10 times as acidic.



The pH scale

WORKED EXAMPLE 10 *(1 mark)*

How many times larger is an earthquake of magnitude 8.5 on the Richter Scale compared to an earthquake of magnitude 3.2? Write your answer correct to the nearest whole number.

SOLUTION

Step 1 Find the amplitude of an earthquake of magnitude 8.5 on the Richter Scale.

$10^{8.5} = 316\,227\,766.02$

Step 2 Find the amplitude of an earthquake of magnitude 3.2 on the Richter Scale.

$10^{3.2} = 1585.89$

Step 3 Divide the amplitude of an earthquake of magnitude 8.5 on the Richter Scale by the amplitude of an earthquake of magnitude 3.2 on the Richter Scale.

$$\frac{316\,227\,766.016\ldots}{1585.893\ldots} = 199\,526.23$$

Step 4 Write out the answer correct to the nearest whole number.

An earthquake of magnitude 8.5 on the Richter Scale is 199 526 times larger than an earthquake of magnitude 3.2 on the Richter Scale.

Questions 2D *Magnitude*

Refresher questions

Q1. How many times larger is 3000 than 30?

Q2. Use your calculator to find the value of $\log_{10}(10\,000)$.

1. Order of magnitude

Q3. Skill
What is the order of magnitude of the following numbers?

a) $4367 = 4.367 \times 10^3$ **b)** $0.04367 = 4.367 \times 10^{-2}$

Q4. Skill
How many orders of magnitude do these following values differ by?

a) 780 and 78 000 000 **b)** 0.1 and 0.001 **c)** 0.0024 and 24 000

Check your understanding

Q5. Application
The diameter of the earth is 13 000 000 m. Find the order of magnitude of the earth's diameter by first converting to scientific notation.

Q6. Application
The difference in the order of magnitude of the height of Mount Everest and the height of an ant hill is 5.

The height of Mount Everest is 9000 m. The height of the ant hill $= 9 \times 10^x$ cm.

a) Fill out this table

	Height (m)	Height (cm)	Height (cm) in scientific notation	Order of magnitude
Mount Everest	9000			
Ant hill			9×10^x	

b) What is the value of x?

2. Logarithms

Q7. Skill
Use your calculator to find the value of $\log_{10}(36\,250)$ correct to two decimal places.

Q8. Skill
Without using your calculator answer the following:

a) What is $\log_{10}(1000)$? **b)** What is $\log_{10}(1)$? **c)** What is $\log_{10}(0.00001)$?

Check your understanding

Q9. Skill
Use $\log_{10}(x) = y$ and $x = 10^y$ to fill in the boxes for the following values and find the value of x.

$\log_{10}(\square) = \square$ and $\square = 10^{\square}$

a) $x = ?$ and $y = -2.4$ **b)** $x = ?$ and $y = 0.006545$

3. Logarithmic scale

Q10. Application

How many times more acidic is orange juice than water if water has a pH of 7 and orange juice has a pH of 3?

Q11. Application

The table below shows the $\log_{10}$(weight) values of a number of different animals.

Weight is in kg.

Animal	$\log_{10}$(weight)
Blue Whale	5.3
African elephant	3.7
Moose	2.9
Crocodile	3
Dolphin	2
Orangutan	1.8
Gorilla	2.3
Humming bird	−2.7
Pygmy rabbit	−0.4
Cat	0.6

What are the actual weights of these animals in kilograms? Write your answers to two decimal places where necessary.

a) Dolphin

b) Pygmy rabbit

c) Blue whale

Check your understanding

Q12. Application

An unknown solution is 1000 times more alkaline than coffee, which has a pH of 5.

Which of the following solutions is it most likely to be?

NOTE: A high pH indicates a solution is alkaline. A low pH indicates a solution is acidic.

A. Tomato juice (pH of 4)

B. Seawater (pH of 8)

C. Lemon juice (pH of 2)

D. Soapy water (pH of 12)

Q13. Application

An earthquake in Japan was 2000 times stronger than an earthquake the same day in Australia. If the one in Australia had a magnitude of 4.3 on the Richter scale, what was the magnitude of the one in Japan? Give your answer in terms of the Richter scale, correct to one decimal place.

Joining it all together

Q14. If $\log_{10}(x) = 0.435$, which of the following expressions is equal to x?

Skill
1 mark

A. -0.362

B. $\log(0.435)$

C. $10^{0.435}$

D. 0.435^{10}

Q15. Which of the values below have the biggest difference in their orders of magnitudes?

Skill
1 mark

A. 50 cm and 500 km

B. 7.2 cm and 7200 m

C. 8 mm and 8000 m

D. 0.023 g and 230 kg

Q16. Without using a calculator, find which of the following statements is **not** true.

Skill
1 mark

A. $\log(1) > \log(0.487)$

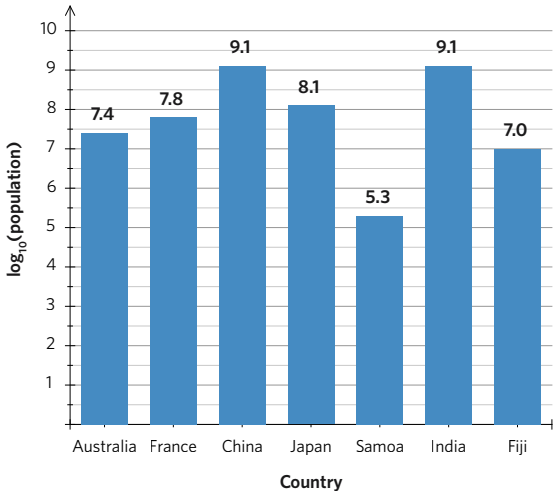
B. $\log(63) > \log(0.097)$

C. $\log(0.42) < \log(0.33)$

D. $\log(3.8) < \log(50)$

Q17.

Application
3 marks



a) Use the graph above to find the difference in population between China and Australia. (2 marks)

b) There is a mistake on the graph. The population of Fiji is shown as ten times more than it should be. What should the actual value on the graph have been? (1 mark)

Q18. Two students are drawing a map of their school. They measured the length of their oval to be 120 m.

Application
1 mark

The length of the piece of paper they are drawing their map on is 30 cm.

How should they scale the length of the oval to ensure it fits on the piece of paper?

A. Increase by an order of magnitude of 1

B. Increase by an order of magnitude of 3

C. Decrease by an order of magnitude of 3

D. Decrease by an order of magnitude of 1

Questions from multiple lessons

Q19.

Difficulty:

Year 10
1 mark

At a berry picking farm, the price of strawberries is determined by rounding the weight of the fruit to the nearest ten grams and multiplying by the price per gram. If Dave picks 416 grams worth of strawberries, and the price of strawberries is \$0.011 per gram, how much does Dave pay?

A. \$4.62

B. \$4.58

C. \$4.51

D. \$4.57

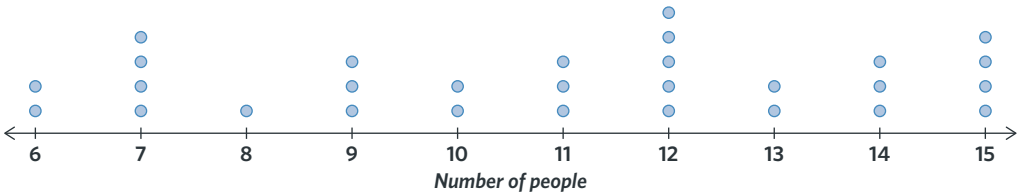
E. \$4.16

Q20.

Difficulty:

Year 10
1 mark

The following dot plot shows the number of people in 29 different running groups.



The median number of people is

A. 10.5

B. 11

C. 11.5

D. 12

E. 12.5

VCAA 2019NH Exam 1 Data Analysis Q4 – Adapted

Q21.

Difficulty:

Year 10
2 marks

A water bottle contains 0.00327 litres of liquid.

a) What is 0.00327 converted to scientific notation? (1 mark)

b) If 1.893×10^2 millilitres of water is added to the bottle, how much liquid does the bottle now contain? Give your answer in millilitres. (1 mark)

LESSON 2E

Percentages and percentage change

The key skills you will learn in this lesson are:

1. Convert to a percentage
2. Convert from a percentage
3. Using percentages
4. Percentage change

VCAA key knowledge point:

"use of ratios and proportions, and percentages and percentage change to solve practical problems"

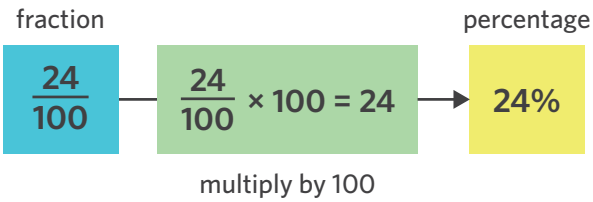
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016–2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Convert to a percentage

Percentages are another type of fraction. More precisely, they are a fraction of 100, which has been multiplied by 100.

Fractions and decimals can be converted to percentages by multiplying them by 100 and simplifying the result.

Example:



WORKED EXAMPLE 11 *(1 mark)*

A bottle of water is three eighths full. Express this as a percentage.

SOLUTION

Step 1 Write the fraction or decimal.

$\frac{3}{8}$

Step 2 Multiply it by 100.

$\frac{3}{8} \times 100$

Step 3 Simplify.

$\frac{300}{8}$

$= \frac{75}{2}$

$= 37.5$

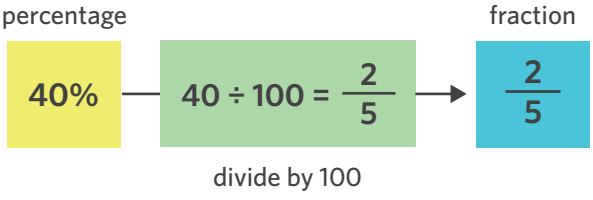
Step 4 Interpret the number.

The bottle of water is 37.5% full.

2. Convert from a percentage

Converting percentages back to fractions or decimals is done by dividing the percentage by 100 and simplifying.

Example:



WORKED EXAMPLE 12 *(2 marks)*

a) Express 22.5% as a decimal.

b) Convert 65% to a fraction.

SOLUTION

a) Express 22.5% as a decimal.

Step 1 Write the percentage.

22.5%

Step 2 Divide it by 100.

When converting to a decimal move the decimal place two spots left.

If there is no decimal place assume it is to the right of the number.

So $22.5\% = 0.225$

b) Convert 65% to a fraction.

Step 1 Write the percentage.

65%

Step 2 Divide it by 100.

To convert to a fraction, write the percentage number with '100' written as a denominator.

$\frac{65}{100}$

Step 3 Simplify where possible.

$= \frac{13}{20}$

So $65\% = \frac{13}{20}$

3. Using percentages

Percentages can be used to express a fraction of a larger quantity.

This comes up in daily conversation as: "Soft drink is 9.6% sugar", "My phone only has 12% battery" or "70% of Australia is classified as desert".

WORKED EXAMPLE 13 *(1 mark)*

If a loaf of garlic bread has 12 slices, and Annie ate 25% of the loaf, how many slices did she eat?

SOLUTION

Step 1 Write the percentage as a decimal or a fraction.

$\frac{25}{100}$

Step 2 Multiply this with the full amount.

$\frac{25}{100} \times 12$

Step 3 Simplify.

$= \frac{300}{100}$

$= 3$

Step 4 Interpret the number.

Annie ate three slices.

4. Percentage change

Percentages can also be used to express a change in the difference between two values. An investor might say "The value of my stocks rose 3% today", and a researcher might say "82% more Aussie children play rugby league than rugby union".

Percentage change can be thought of as change compared to the original value.

$$Percentage\ change = \frac{Difference}{Original\ value} \times 100$$

Although percentage change can be negative, it is normally written in a sentence as positive and next to a word that indicates direction. For example, 'The value of my car decreased 30%'.

WORKED EXAMPLE 14 *(1 mark)*

Zahra is 1.72 m tall. Yvonne is 1.54 m tall. What percent taller is Zahra than Yvonne?

SOLUTION

Step 1 Calculate the difference.

$1.72 - 1.54$

$= 0.18$

Step 2 Choose which value to compare to the other.

We need to compare Zahra's height to Yvonne's height.

This means Yvonne's height becomes the 'original value'.

Step 3 Substitute these values into the equation and solve.

$$\textit{Percentage change} = \frac{0.18}{1.54} \times 100$$
$$= 11.69\%$$

Step 4 Interpret the value.

Zahra is 11.69% taller than Yvonne.

Questions 2E *Percentages and percentage change*

Refresher question

Q1. Can percentages be more than 100?

1. Convert to a percentage

Q2. Skill

What is $\frac{1}{2}$ expressed as a percentage? Choose one of the following answers.

A. 55%

B. 70%

C. 50%

D. 45%

Q3. Skill

a) What is $\frac{4}{5}$ as a percentage?

b) What is 0.32 as a percentage?

c) What is $\frac{1}{15}$ as a percentage? Express your answer to two decimal places.

Check your understanding

Q4. Application

Bert says his water bottle is two fifths full.

Express this as a percentage.

Q5. Application

Sam has completed six elevenths of their homework. Charlie has completed four sevenths of their homework.

a) Convert both of these fractions to percentages, rounded to two decimal places.

b) Who is further ahead?

2. Convert from a percentage

Q6. Skill

What is 30% expressed as a fraction?

A. $\frac{3}{5}$ **B.** $\frac{3}{12}$ **C.** $\frac{10}{3}$ **D.** $\frac{3}{10}$

Q7. Skill

a) What is 79% expressed as a decimal?

b) What is 35% expressed as a fraction?

c) What is 62.5% expressed as a fraction?

Check your understanding

Q8. Application

Out of a circular block of cheese, there are two pieces left. One is 24% of the original size, and the other is 11% of the original size. How much cheese is left in total? Express as a fraction.

Q9. Application

In a class, students were asked which subjects they liked out of: history, chemistry, woodwork and English. The results were:

	History	Chemistry	Woodwork	English
26% of students	✓	✓		
31% of students		✓	✓	
15% of students	✓		✓	
18% of students	✓			
?% of students				✓

a) What percentage of students only like English?

b) What amount of the students like history? Express as a fraction.

3. Using percentages

Q10. Skill

If a block of chocolate has 30 pieces, how many pieces are there in 60% of the chocolate?

A. 18 **B.** 14 **C.** 20 **D.** 6

Q11. Skill

a) What is 35% of 260?

b) What is 2% of 15?

c) What is 99% of 103?

Check your understanding

Q12. Application

Cormac has read 12% of a book with 425 pages.

How many pages has Cormac read so far?

Q13. Application

Barbara has drunk 73% of a 2.5 L water bottle.

How much water is left, in mL?

4. Percentage change

Q14. Skill

What is the difference between 5.32 and 3.87?

A. 1.35 **B.** 1.45 **C.** 1.85 **D.** 1.25

Q15. Skill

For the following calculate the percentage change, correct to two decimal places. Write your answer as a sentence.

a) From 12.3 to 14.8.

b) From 218 to 134.

c) From 0.34 to 0.37.

Check your understanding

Q16. Skill

Calculate the percentage change, correct to two decimal places, between 0.76 and 3.25.

Write your answer as a sentence.

Q17. Application

Amelia's phone is almost flat, on 18%. She charges it up for an hour, and it increases by 230%.

What is the current battery status of the phone?

Joining it all together

Q18. Application 3 marks

Goran is assembling a flagpole. It comes in three pieces, a pole, a flag-holder, and a cap on the top, that are connected end-to-end.

The pole is 9 metres long, the flag-holder is 80 cm long, and the metal cap at the top is 25 mm tall.

Assuming there is no overlap between parts when connected, express each part as a percentage of the total height, to two decimal places.

Q19. Application 2 marks

A bucket is placed under a leak in the ceiling of a house while the owner goes on holiday.

It has a volume of 25 L. Currently it is one quarter full. One day there is a thunderstorm, and it fills up by 12 L.

The next day it rains again, and the bucket fills up an extra 10% of its total capacity. On the third day there is no rain, but on the fourth day 5000 mL of rain drips through the crack.

At the end of the fourth day, has the bucket overflowed yet?

Q20. Application 4 marks

When examining the rates of Australian children participating in different sports, the Australian Sports Commission found the information in the table below.

a) If there are 1 200 000 5–8-year-olds in Australia, how many 5–8-year-olds do swimming?

b) If there are 920 000 12–14-year-olds in Australia, how many 12–14-year-olds don't do swimming?

c) There are 510 000 0–4-year-olds who do swimming. How many more 5–8-year-olds do swimming than 0–4-year-olds?

d) Express your answer from part c) as a percentage of the number of 0–4-year-olds who do swimming. State your answer as a sentence, with numbers rounded to two decimal places.

Age group	Percentage of children who swim
0–4	33.7%
5–8	43.6%
9–11	30.8%
12–14	29.0%

Questions from multiple lessons

Q21.
Difficulty:
1 mark

Clem needs 0.00076 kg of aluminium hydroxide for a science experiment. Convert this weight to scientific notation.

A. 76×10^{5} kg

B. 76×10^{-5} kg

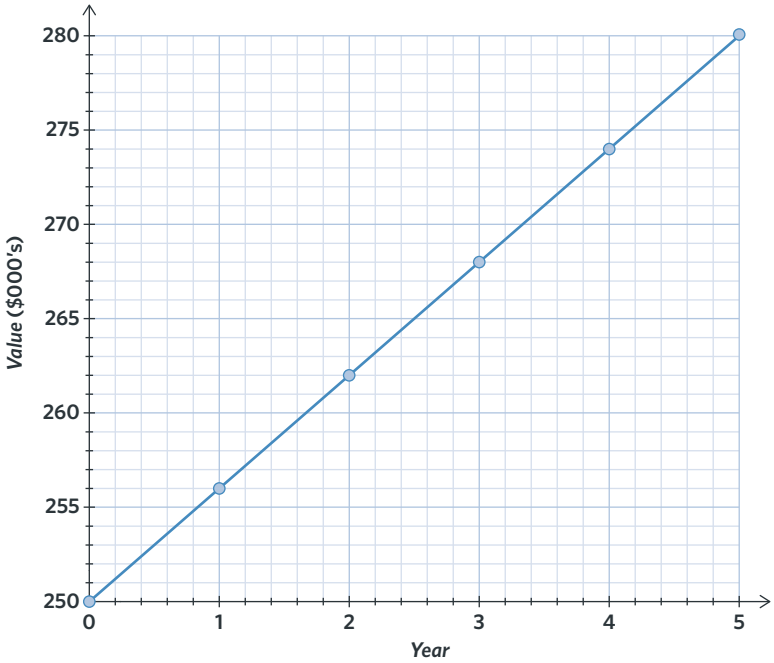
C. 7.6×10^{4} kg

D. 7.6×10^{-4} kg

E. $-5 \times 10^{7.6}$ kg

Q22.
Difficulty:
1 mark

The graph shows the value of an investment over a period of five years.



A different account was opened under the same growth conditions as the initial one but for 10 years.

After 10 years, this investment has grown by $30 000.

The initial amount invested in this account is

A. $306 000

B. $360 000

C. $294 000

D. $125 000

E. $250 000

VCAA 2013 Exam 1 Module 4: Business-related mathematics Q7 – Adapted

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Q23.
Difficulty:
Year 10
2 marks

While Dylan is on holiday in Japan, two earthquakes occur. The first measures 3.4 on the Richter scale and the second measures 2.7 on the Richter scale. Note that the Richter scale is a logarithmic scale.

a) How many times stronger was the first earthquake than the second? Give your answer correct to one decimal place. (1 mark)

b) What is the order of magnitude of the difference between the strengths of the two earthquakes? (1 mark)

LESSON 2F

Ratios and proportions

The key skills you will learn in this lesson are:

1. Ratios
2. Proportions

VCAA key knowledge point:

"use of ratios and proportions, and percentages and percentage change to solve practical problems"

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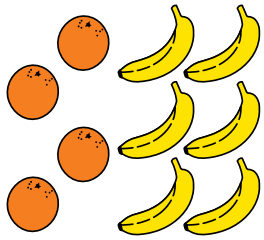
1. Ratios

A ratio is a way to compare two or more quantities to each other. Ratios are often used in cooking, to communicate how much of one ingredient is used compared to another.

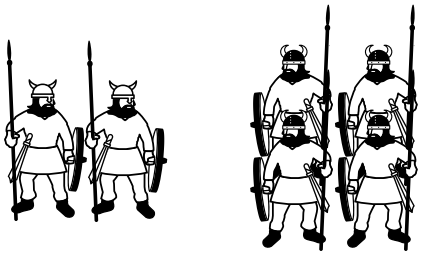
The order is important in ratios. The ratio 1 : 2 is different to 2 : 1. The units being compared can be different, but each unit must be stated.

Example

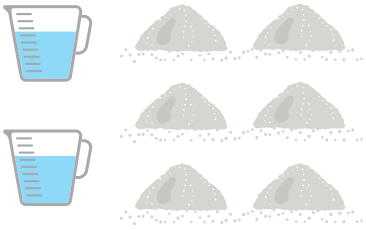
'The ratio of oranges to bananas is 2 : 3.'



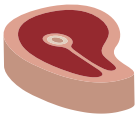
'The enemy outnumbers us two to one.'



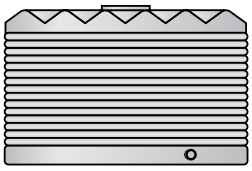
'Add three parts of sugar for each part of water.'



'For every gram of meat produced 20 L of water is used.'



200 g



4000 L

WORKED EXAMPLE 15 *(2 marks)*

a) Calculate the ratio of teachers to students if there are 15 teachers in the school and 375 students.

b) A recipe for a cake uses 100 g of flour, 50 g of sugar and 200 mL of water. Write this as a ratio.

SOLUTION

a) Calculate the ratio of teachers to students if there are 15 teachers in the school and 375 students.

Step 1 Set up the ratio. Here we'll use:

teachers : *students*

Step 2 Fill in the numbers for each quantity into the ratio.

teachers : *students*
15 : 375

Step 3 Look for the highest common factor in the numbers in the ratio, and divide by it. Here the highest common factor is 15, so divide each number in the ratio by 15.

15 ÷ 15 = 1 ⟲ **15 : 375** / **1 : 25** ⟳ 375 ÷ 15 = 25

Step 4 Interpret. 'Teachers' is 1, and 'students' is 25. Therefore there are 25 students for every 1 teacher.

b) A recipe for a cake uses 100 g of flour, 50 g of sugar and 200 mL of water. Write this as a ratio.

Step 1 Set up the ratio. We'll use flour : sugar : water.

Step 2 Fill in the numbers for each quantity into the ratio.

flour : *sugar* : *water*
100 : 50 : 200

Step 3 Look for the highest common factor in the numbers in the ratio, and divide by it. Here the largest common factor is 50, so divide by 50.

100 : 50 : 200
100 ÷ 50 = 2 | 50 ÷ 50 = 1 | 200 ÷ 50 = 4
2 : 1 : 4

Step 4 Interpret, remembering to include units.

'Flour' is 2, 'sugar' is 1 and 'water' is 4. Therefore for every one gram of sugar, two grams of flour and four millilitres of water must be added.

2. Proportions

When two ratios are equal to each other, they are 'in proportion'.

Often it is important to keep things in proportion.

Example

On the left there are two cups of flour, and one cup of sugar, at ratio 2 : 1. On the right, there are four cups of flour, and two cups of sugar, at ratio 2 : 1.

Therefore, both groups of ingredients are in proportion with each other.

WORKED EXAMPLE 16 *(1 mark)*

Are the following ratios in proportion?

10 : 5 and **30 : 10.**

SOLUTION

Step 1 Simplify the ratios to compare them.

÷ 5 both numbers ⟲ **10 : 5** / **2 : 1** | **30 : 10** / **3 : 1** ⟳ ÷ 10 both numbers

Step 2 Compare the two ratios. If they are the same they are said to be in proportion.

We have ratios of 2 : 1 and 3 : 1. These ratios are not in proportion.

WORKED EXAMPLE 17 *(1 mark)*

Rochelle is making eight servings of scones. The recipe says that for two servings of scones, she should use five grams of salt. Using proportions, work out how much salt should be added to Rochelle's scones.

SOLUTION

Step 1 Write the ratio involving salt and servings. It is two servings to five grams of salt.

2 : 5

Step 2 Multiple or divide the ratio to reach the desired quantity.

She wants to make eight servings, which is four times as much as two servings, so multiply by four.

8 : 20

Step 3 For eight servings, Rochelle needs to use 20 grams of salt.

Questions 2F *Ratios and proportions*

Refresher question

Q1. Simplify the fraction $\frac{3}{15}$.

1. Ratios

Q2. Skill

There are 12 dogs and 4 cats in a room. Fill in the missing number in the ratio of dogs to cats.

☐ : 1

Q3. Skill

a) Find a ratio for the number of pencils to pens in a shop if there are 24 pencils and 40 pens.

b) In a hotel on a particular day there are 180 rooms and 270 guests. Find the ratio of rooms to guests.

c) Find a ratio for the number of students in government schools to those in Catholic schools in Victoria, if there are 615 000 students in government schools and 205 000 in Catholic schools.

Check your understanding

Q4. Application

One Monday morning in a cafe, 20 customers used reusable coffee cups and 80 used take-away cups. Find this as a ratio. If on Tuesday 50 customers use reusable coffee cups, how many can be expected to want take-away cups if the ratio stays the same?

2. Proportions

Q5. Skill

Are the following ratios in proportion?

2 : 4 and 1 : 2

Q6. Are the following ratios in proportion?

Skill

a) 10 : 3 and 40 : 12

b) 1 : 19 and 5 : 85

c) 7 : 11 and 98 : 154

Check your understanding

Q7. A standard landscape photograph has a width to height ratio of 3 : 2. Ned wants to blow up one of his photos to put on the wall. He's chosen a canvas that's 80 cm by 60 cm. Will the photo nicely fit the canvas, or will it have to be stretched or cropped to fit?

Application

Joining it all together

Q8. The ratio of 'Yes' voters to 'No' voters in the 2017 postal vote was roughly 3 : 2. If roughly 5 million people voted 'No', roughly how many people voted 'Yes'?

Application
1 mark

Q9. Three jars contain only red and green jelly beans. They are filled with the following ratios of *red* : *green*.

Application
2 marks

If there are 100 jelly beans in each jar,

a) Which jar has the most green jelly beans?

b) How many green jelly beans does the jar from a) have?

Q10. The number of people in Melbourne is approximately 4.5 million. The number of houses is 1.8 million. Find the ratio of houses to people.

Application
2 marks

Assuming Newcastle has the same ratio of houses to people, find how many houses are in Newcastle assuming the population is 500 000.

Q11. Kato is baking a red velvet cake.

Application
2 marks

The ratio of mL of milk to the number of eggs to grams of sugar is 125 : 1 : 160.

Kato has half a litre of milk, 4 eggs and half a kilo of sugar. Kato wants to make 4 batches . One batch uses one egg. Does she have enough ingredients?

Q12. In Melbourne, roughly 66 000 people walk to work, 330 000 people take public transport and 1.419 million people drive to work. Express this as a ratio.

Application
2 marks

In 2051, Melbourne's population is set to hit 8 million. Assuming the proportion of people using different modes of transport is the same in 2051, if 600 000 people take public transport to work in 2051, how many will drive to work?

Questions from multiple lessons

Q13.

Difficulty:

1 mark

The Richter scale is a logarithmic scale used to measure the amplitude of earthquakes using the following formula.

$magnitude = \log_{10}(amplitude)$

In 2011, two major earthquakes occurred.

New Zealand experienced an earthquake with a magnitude of 6.3.

Japan experienced an earthquake with a magnitude of 9.0.

How many times stronger was the amplitude of the Japan earthquake compared to the amplitude of the New Zealand earthquake? Give your answer correct to one decimal place.

A. 1.4

B. 2.7

C. 199.5

D. 501.2

E. 1995.3

Q14.

Difficulty:

1 mark

Scientists are studying the relationship between the variables *clothing size* (extra small, small, medium, large, extra large) and *height* (in cm).

These variables are

A. ordinal and numerical respectively.

B. ordinal and nominal respectively.

C. nominal and numerical respectively.

D. numerical and nominal respectively.

E. both numerical.

VCAA 2019NH Exam 1 Data Analysis Q8 – Adapted

Q15.

Difficulty:

3 marks

David Attenborough is producing a documentary on an aardvark population in Namibia.

His team monitored the population over a three-year period.

At the beginning of monitoring, the population contained 65 aardvark cubs and 232 fully-grown aardvarks.

a) What percentage of the aardvarks are cubs? Give your answer correct to the nearest percent. (1 mark)

b) After the first year of monitoring, eleven of the aardvarks passed away. Express this change as a percentage of the original population, correct to one decimal place. (1 mark)

c) From the beginning of monitoring to the end of the three-year period, the aardvark population increased by 24%. What is the current population, correct to the nearest whole number? (1 mark)

LESSON 2G

The unitary method

The key skills you will learn in this lesson are:

1. Finding the number of units
2. Finding the amount for one unit
3. Finding the amount for many units

VCAA key knowledge point:

"the unitary method and its use to make comparisons and solve practical problems involving ratio and proportion"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

The unitary method is used to find a quantity for a large number of identical terms, such as the cost of 1500 apples, or the time taken to write 350 letters. It works by first calculating the cost or time taken for one item, and then multiplying this by the total number of items in question.

1. Finding the number of units

The first part of the unitary method is finding how many 'units' there are.

Some examples of units are:

- number of people
- number of litres
- number of minutes.

A unit is the thing you are counting or measuring.

To find the number of units, use:

number of units in one thing × *number of things*

Example

How many seconds in 20 minutes?

Number of seconds per minute is 60 and number of minutes is 20.

$60 \times 20 = 1200$ seconds

WORKED EXAMPLE 18 *(1 mark)*

A school bought new tables for four classrooms. Each classroom got 25 new tables.

How many new tables did the school buy?

SOLUTION

Step 1 Ask two questions:

How many tables are in one classroom?

25

How many classrooms are there?

4

Step 2 Multiply those numbers:

$25 \times 4 = 100$ tables

Step 3 Write the answer.

The school bought 100 tables.

2. Finding the amount for one unit

When the total number of units is known, it is possible to find the amount for one unit.

The 'amount' often refers to:

- cost
- weight
- time.

To find the amount for one unit use:

amount per unit = *total amount* ÷ *number of units*

Example

Five sandwiches cost $30. What is the cost of one sandwich?

The total amount is $30, and the total number of sandwiches is 5.

$30 \div 5 = 6$

Each sandwich costs $6.

WORKED EXAMPLE 19 *(1 mark)*

A school spent $3000 when they bought 100 tables. What is the cost for one table?

SOLUTION

Step 1 Ask two questions:

What is the total amount?

$3000

What is the number of units?

100

Step 2 Divide the amount by the number of units to get the answer:

$3000 \div 100 = 30$

Step 3 Write the answer.

One table costs $30.

3. Finding the amount for many units

Once the amount for one unit is known, it is possible to find the amount for any number of units.

This method is useful because it takes known data and turns it into a future prediction.

This is useful for many situations, for example:

- The use of past fuel consumption to estimate how much the fuel for a road trip will cost.
- The owner of a restaurant finds how much a burger costs to make, to decide how much to charge per burger.

To find the amount for many units, use:

amount for one unit $\times$ *new number of units*

WORKED EXAMPLE 20 *(1 mark)*

A school will buy another 230 tables at $30 for one table. How much will this cost?

SOLUTION

Step 1 Ask two questions:

What is the cost for one table?

$30

How many tables will the school buy?

230

Step 2 Multiply those numbers to get the answer:

$30 \times 230 = 6900$

Step 3 Write the answer.

It will cost $6900 to buy 230 tables.

Questions 2G *The unitary method*

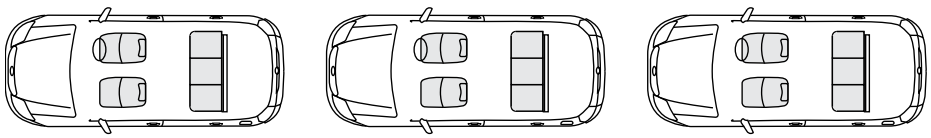
Refresher question

Q1. How many grams are there in a kilogram?

1. Finding the number of units

Q2. There are five seats in each of these three cars. How many seats are there in total?

Skill



Q3. How many:

Skill

a) minutes are there in 24 hours?

b) grams are there in 4.5 kg?

Check your understanding

Q4. Each day your friend gives you the same amount of stickers. After seven days you have 21 stickers.

Skill

How many stickers did you get each day?

2. Finding the amount for one unit

Q5. Find the following:

Skill

a) Ricardo buys 14 snake lollies for $2.10. How much does one snake lolly cost?

b) A dozen eggs weigh 600 g. How much does one egg weigh?
NOTE: A dozen is 12.

c) Joe reads 16 pages of his novel for English homework. It takes him 25 minutes in total. How long does it take him to read one page? Give your answer in minutes rounded to 1 decimal place.

Check your understanding

Q6. Ingrid says she can read a page of a novel in 1.2 minutes.

Skill

How long does it take her to read 16 pages?

3. Finding the amount for many units

Q7. Find the following:

Skill

a) You buy five tickets to a concert. The tickets are $15 each.

How much does it cost in total?

b) An Olympic-sized swimming pool holds 2500 kilolitres of water and water costs $2.70 per kilolitre.

What is the cost of filling up the pool?

Check your understanding

Q8. You want to buy new wheels and laces for your pair of roller skates. There are four wheels and one lace on each skate. How much will it cost you in total?

Application

The wheels cost $8 each and the laces cost $2 each.

Joining it all together

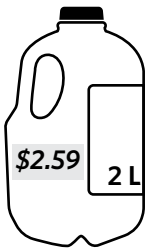
Q9. For this two litre bottle of milk:

Application
3 marks

a) How many mL of milk are there?

b) How much does one mL cost, in cents, rounded to two decimal places?

c) How much does one cup cost, in cents, rounded to two decimal places?



Q10. At the fruit store you buy seven oranges for $2.31.

Application
1 mark

How much would it cost to buy 25 oranges?

Follow these steps:

- Find the cost of one orange.
- Find the cost of 25 oranges.

Q11. On Patrick's last hiking trip he packed 1080 g of food for four days.

Application
1 mark

For his next hiking trip he wants to take enough food for 17 meals in total.

If on his last trip he had three meals a day, how many grams of food should he take? Assume each meal is the same weight.

Follow these steps:

- Find the number of meals for his last trip.
- Find the weight of each meal.
- Find the total weight of the meals for his next trip.

Q12. You watch two seasons of your favourite TV show without a break. Each season has eight episodes.

Application
1 mark

It takes four hours and 48 minutes.

The next day you watch 3 episodes of the show.

Which of the following is **not** true?

A. It takes more than 50 minutes.

B. It takes less than 58 minutes.

C. It takes more than 55 minutes.

Q13. Susan cycles to school. How long would it take her to cycle from Melbourne to Sydney at the same pace? Give your answer to the nearest hour.

Application
1 mark

Trip	Distance (km)	Time
Home–School	8	30 minutes
Melbourne–Sydney	878	**?**

Q14. Agnes is selling raffle tickets as a fundraiser for the local footy club.

Application
1 mark

She has five books of tickets, and each book has 100 tickets. If she sells all the tickets she will raise $750.

Show that she raises less than $260 if she sells 173 tickets.

Q15. Five cubby house kits weigh 580 kg. A hardware store hires a truck to transport these kits.

Application
2 marks

The truck can carry up to 2240 kg.

a) What is the maximum number of kits that the truck can carry?

b) How many trucks would the hardware company need to hire to transport 150 kits?

Questions from multiple lessons

Q16. At Sinead's favourite vegan cafe, a small smoothie is 260 mL and a regular smoothie is 350 mL.

Difficulty:

1 mark

The increase in volume if Sinead sizes up from a small to a regular smoothie, as a percentage of the small smoothie's volume, is closest to

A. 25.7%

B. 31.4%

C. 34.6%

D. 38.5%

E. 42.3%

Q17. The following stem plot displays the hourly wage, in dollars, of 35 hospitality workers.

Difficulty:

Year 10
1 mark

Key: 13 | 5 = \$13.50 $n = 35$

13	5
14	2 7
15	0 3 6 8 8
16	3 4 5 9
17	0 0 6 7 7 7
18	4 8
19	5 5 8 9
20	0 2 9
21	2 5 5 6
22	0
23	1 4 8

What is the median hourly wage?

A. \$17.60

B. \$17.65

C. \$17.70

D. \$18.05

E. \$18.40

VCAA 2018NH Exam 1 Data analysis Q1 – Adapted

Q18. The floor plan of a Brighton mansion has a scale of 1:500.

Difficulty:

2 marks

Thus, every 1 centimetre on the map represents a distance on land of 500 centimetres.

a) On the floor plan, the distance from the front fence to the back fence of the property is 14.2 cm. What is the actual distance in centimetres? (1 mark)

b) The actual distance between the tennis court and the swimming pool is 5205 cm. What is this distance on the floor plan? (1 mark)

AOS 2: Arithmetic and number

Financial arithmetic

CHAPTER
3

LESSON 3A

Financial percentage change

The key skills you will learn in this lesson are:

1. Percentage
2. Percentage change
3. Discounts and mark-ups

VCAA key knowledge point:

"percentage increase and decrease applied to various financial contexts such as the price to earnings ratios of shares and percentage dividends, determining the impact of inflation on costs and the spending power of money over time, calculating percentage mark-ups and discounts, and calculating GST"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Percentage

A percentage is any number that represents a proportion out of 100. Percentages can be used to compare two values by expressing one value relative to another value.

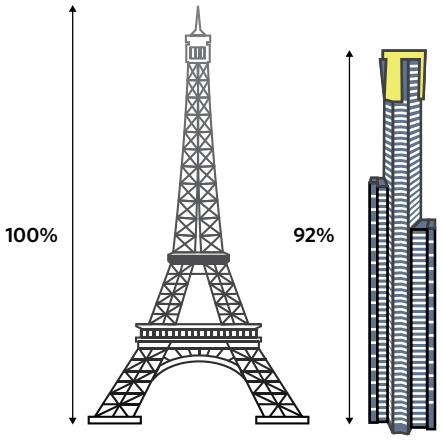
Example

The Eureka tower is 92% the height of the Eiffel tower.

There are various formulas for working with percentages. Here V_1 is one value and V_2 is another value.

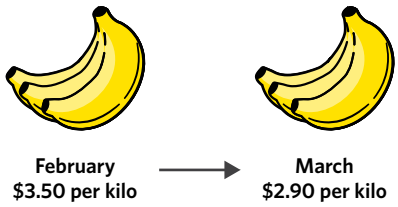
The formula to calculate V_2 as a percentage of V_1 is

$$Percentage = \frac{V_2}{V_1} \times 100$$



WORKED EXAMPLE 1 *(1 mark)*

What is the price of bananas per kilo in March compared to the price in February, as a percentage?



SOLUTION

Step 1 Choose a formula that expresses percentage.

$$Percentage = \frac{V_2}{V_1} \times 100$$

Step 2 Find the required terms in the equation. Here we compare March to February.

So February is V_1 and March is V_2.

$V_1 = 3.50$

$V_2 = 2.90$

Step 3 Substitute these into the formula. Calculate the answer.

$$Percentage = \frac{2.90}{3.50} \times 100 = 82.86$$

Step 4 Interpret the result.

The price of bananas in March is 82.86% of the price in February.

2. Percentage change

Percentages can be used to express the change between two values.

This is done by looking at the difference between two values.

For example, using the height of the Eiffel tower as reference, the Eureka tower is 8% (equal to $\frac{8}{100}$) shorter.

This is a percentage change of 8%.

Example

One formula for expressing change as a percentage is

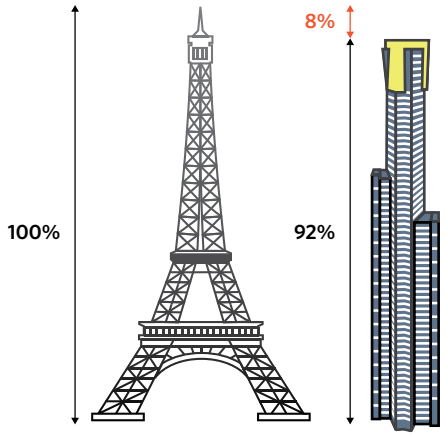
$$Percentage\ change = \frac{Change}{Original\ value} \times 100$$

If V_1 and V_2 are known, the following formula can be used.

$$Percentage\ change = \frac{V_2 - V_1}{V_1} \times 100$$

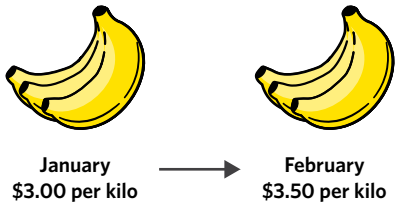
Here:

- V_1 is original value and V_2 is final value.
- $V_2 - V_1$ is equal to the change.



To find the original value, V_1, or the final value, V_2, when the change expressed as a percentage is known, use the following formulae.

Finding the original value, V_1	Finding the final value, V_2
$V_1 = V_2 \times \frac{100}{100 + Percentage\ change}$	$V_2 = V_1 \times \frac{100 + Percentage\ change}{100}$



This is a percentage change of 16.67%.
In February, the price has increased by 16.67%

WORKED EXAMPLE 2 *(2 marks)*

Find the following percentage changes:

a) an increase in the price of bananas from \$2.90 per kilo to \$3.05 per kilo.

b) an increase in the price of mushrooms from \$9.50 per kilo to \$12 per kilo as a percentage.

SOLUTION

a) an increase in the price of bananas from \$2.90 per kilo to \$3.05 per kilo.

Step 1 Choose a formula that expresses change as a percentage.

$$Percentage\ change = \frac{Change}{Original\ value} \times 100$$

Step 2 Find the required terms in the equation.

$$Change = 3.05 - 2.90 = 0.15$$

$$Original\ value = 2.90$$

Step 3 Substitute these into the formula. Calculate the answer.

$$Percentage\ change = \frac{0.15}{2.90} \times 100$$

$$Percentage\ change = 5.17$$

Step 4 Interpret the result.

The price of bananas has increased 5.17% from \$2.90 to \$3.05.

b) an increase in the price of mushrooms from \$9.50 per kilo to \$12 per kilo as a percentage.

Step 1 Choose a formula that expresses change as a percentage.

$$Percentage\ change = \frac{V_2 - V_1}{V_1} \times 100$$

Step 2 Find the required terms in the equation, V_1 and V_2.

$V_1 = 9.5$

$V_2 = 12$

Step 3 Substitute these into the formula. Calculate the answer.

$$Percentage\ change = \frac{12 - 9.5}{9.5} \times 100$$

$$Percentage\ change = 26.32$$

Step 4 Interpret the result.

The price of mushrooms has increased 26.32% from \$9.50 to \$12.00.

3. Discounts and mark-ups

Discounts and mark-ups are changes in price that can be expressed as percentages. The change in price can either be found in dollars and then converted to a percentage, or vice-versa.

$$Price\ change = Original\ price \times \frac{Percentage\ change}{100}$$

Discounts

Discounts are any kind of price **decrease**, and are often seen during sales.

$$Final\ price = Original\ price - Price\ change$$

Example

A discount of \$2 applies to apples in a shop, normally sold at \$4.50 per kilo. The new price is \$2.50 per kilo.

Mark-ups

Mark-ups are any type of price **increase**, often referring to shops increasing the prices of their stock to make a profit.

$$Final\ price = Original\ price + Price\ change$$

Example

A milk bar buys Chupa Chups for \$0.60 and sells them for \$1. This is a mark-up of \$0.40.

WORKED EXAMPLE 3 *(1 mark)*

A store buys hats for \$22 and sells them at a 50% markup. How much do they sell for?

SOLUTION

Step 1 Identify what we are looking for.

We know that the original price is \$22, and the mark-up amount is 50%.

We need to find *Price change*. The formula we need is:

$$Price\ change = Original\ price \times \frac{Percentage\ change}{100}$$

Step 2 Substitute the known values into the formula to find the price change.

$$Price\ change = 22 \times \frac{50}{100}$$

$$Price\ change = 11$$

Step 3 Find the new price.

This is a mark-up, so we use:

$$Final\ price = Original\ price + Price\ change$$

$$Final\ price = 22 + 11$$

$$Final\ price = 33$$

Step 4 Write the answer.

The hats sell for \$33.

Questions 3A *Financial percentage change*

Refresher question

Q1. If I have 10 apples and 3 are red, what percentage are red?

1. Percentage

Q2. Andy is 150 cm tall and Bertha is 165 cm tall.

Skill

a) Use this formula to find Andy's height as a percentage of Bertha's height to one decimal place:

$$Percentage = \frac{Andy's\ height}{Bertha's\ height} \times 100$$

b) Complete this sentence:

Andy is ☐ % the height of Bertha.

c) Find Bertha's height as a percentage of Andy's height.

Check your understanding

Q3. Which of these has the biggest percentage discount?

Application

Photo by Haythem Gataa on Unsplash

2. Percentage change

Q4. The following question has been partially answered. Look at the working out below, and fill in the missing numbers.

Skill

$10.50 is marked up to $13.35. What is this increase as a percentage?

$$Percentage\ change = \frac{V_2 - V_1}{V_1} \times \square$$
$$= \frac{13.35 - 10.50}{10.50} \times \square$$
$$= \frac{2.85}{10.50} \times \square$$
$$= 0.2714 \times \square$$
$$Percentage\ change = 27.14$$

This increase as a percentage is ☐ %.

Q5. Express the following changes as a percentage and indicate whether the percentage represents an increase or a decrease.

Application

a) The price of avocados was $8 per kilo last month but has increased to $12 per kilo this month.

b) Monday's maximum temperature was 23 degrees Celsius and Tuesday's maximum temperature was 16 degrees Celsius. Give your answer correct to two decimal places.

c) Bianca's height last year was 1.53 m and this year is 1.60 m. Give your answer correct to two decimal places.

Check your understanding

Q6. Application
Last year at Village Cinemas, the price of a movie ticket for a student was $15. This year the price went up 13.33%. What is the price of a student ticket this year? Round your answer to two decimal places.

Q7. Application
In Round 1, Ken's cricket team scored 124 runs. In Round 2, they scored 36 percent less than Round 1. In Round 3, they scored 22 percent more than in Round 2. Calculate their score in Round 3. At each step round the score to the nearest run.

3. Discounts and mark-ups

Q8. Skill
A dress that normally sells for $75 is marked down by 15%. What is the discounted price, rounded to the nearest cent? Use the working below to help you.

$$Price\ change = Original\ price \times \frac{Percentage\ change}{100}$$

$$= \$75 \times \frac{15}{100}$$

$$Price\ change = \$11.25$$

$$Final\ price = Original\ price - Price\ change$$

$$Final\ price = \square - \square$$

$$Final\ price = \square$$

Q9. Application
For each of the following, find

i. the price change **ii.** the new price

a) A T-shirt bought with an original price of $12 is marked up by 20%.

b) A giant chocolate bar is usually $25, but it currently has a discount of 45%.

c) String lights with an original price of $12.50 are marked up 98%.

Check your understanding

Q10. Application
To deal with having too many hats in stock, Cotton On has decided to discount the hats by $5. All hats sell for $14.95 normally. What is the discount as a percentage, correct to two decimal places?

Q11. Application
Clara wants to buy a suit at Beaut Suits. Normally suits at Beaut Suits are priced at $199. Beaut Suits have a range of monthly specials, as seen below.

Clara decides to buy two suits in October. How much is the total discount per suit, in dollars, when buying two suits in October, compared to the original price?

Joining it all together

Q12. Application 1 mark

A large pizza at Joey's Pizza is 20% more expensive than a small pizza. If a large pizza costs $18 and a medium pizza costs $16, how much does a small pizza cost? Round your answer to two decimal places.

Q13. Application 1 mark

A film editor takes the director's cut of a film and cuts down the footage by 27%, into the film's final version. If the final version goes for 118 minutes and there originally was six hours of footage, what was the length of the director's cut of the footage, to the nearest minute?

Q14. Application 1 mark

Calculate the following discounts as a percentage and state which one is the largest.

- $18 off a watch worth $100
- $4 off a DVD worth $12
- $2 off a block of chocolate worth $5
- $8 off a book worth $25

Q15. Application 1 mark

Four people, Will, Xanthe, Yan and Zach, are participating in a silent auction for a signed cricket bat. The reserve price is $1200. Will offers 10% more than the reserve, Xanthe offers $99 more than the reserve, and Yan offers 25% more than the reserve. Zach knows how much Yan will offer and offers $50 more. Who won and how much was the winning bid?

Q16. Application 1 mark

Matthew wants to sell his bike to make some money. Matthew's friend offers to buy it for half of the original price. Matthew's teacher wants to buy it for her son, and offers $80. Matthew gets a better offer from Dee, whose offer is 20% higher than the offer from Matthew's friend. Matthew accepts this offer, for $108. What was the original price?

Q17. Application 1 mark

Three people, An, Boris and Celina, are participating in an auction for a maths textbook from the 1920s. The following table represents the increasing bids in the auction. Boris won the auction with a bid of $170. What was the starting bid?

Bid #	An	Boris	Celina
1st	+$5		
2nd			+10%
3rd	+$5		
4th		+$6	
5th			+20%
6th	+$18		
7th			+10%
Winning bid		+$5	

VCAA question

Q18. 1 mark

An investment property was purchased for $600 000. Over a 10-year period, its value increased to $850 000. The increase in value, as a percentage of the purchase price, is closest to

A. 4.2%

B. 25.0%

C. 29.4%

D. 41.7%

E. 70.6%

VCAA 2015 Exam 1, Module 4: Business-related mathematics. Q2

Questions from multiple lessons

Q19.

Difficulty:

Year 10
1 mark

A beanie store is currently having a Boxing Day sale. The price of a beanie is usually $9.80. If there is a 20% discount on all beanies, how much is the price of a beanie reduced by?

A. $1.96

B. $7.84

C. $2.00

D. $7.80

E. $2.04

VCAA 2015 Exam 1 Module 4: Business-related mathematics Q1 – Adapted

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Q20.

Difficulty:

Year 10
1 mark

The following stem plot displays the number of *hours driven* for twenty Year 12 students on their L's.

The modal number of *hours driven* is

A. 36

B. 44

C. 65.5

D. 66

E. 88

VCAA 2016 Exam 1 Data analysis Q3 – Adapted

Key: 2 | 8 = 28 hours

2	8
3	6 6
4	1 4 4 7
5	8 9
6	5 6 6 6 6 9
7	8
8	8 8
9	1
10	6

Q21.

Difficulty:

2 marks

A hydrotherapy session costs $800.

Concession card holders are offered a discount of $120 off the full price.

a) Write down the discount for concession card holders as a percentage of the full price. (1 mark)

Those that do not have concession cards also have to pay an extra $60 per hour that they spend in the hydrotherapy pool after their session.

b) John does not have a concession card and spends an extra five hours in the pool. What is the total he has to pay? (1 mark)

VCAA 2015 Exam 1 Module 4: Business-related mathematics Q1 a,b – Adapted

LESSON 3B

Financial percentage change applications

The key skills you will learn in this lesson are:

1. GST
2. Dividends and shares
3. Percentage applications

VCAA key knowledge point:

"percentage increase and decrease applied to various financial contexts such as the price to earnings ratios of shares and percentage dividends, determining the impact of inflation on costs and the spending power of money over time, calculating percentage markups and discounts, and calculating GST"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. GST

The Goods and Services Tax, or GST, is a 10% tax applied to most sales in Australia.

Businesses making more than $75 000 per year need to include GST in their final price.

Throughout this lesson:

- Q will indicate a price including GST
- P will indicate a price not including GST.

To calculate the GST included in a price, use the following formula.

$$\text{GST} = \frac{Q}{11}$$

To find the GST that needs to be added to a price non-inclusive of GST, calculate 10% of the price.

$$\text{GST} = \frac{P}{10}$$

To find the GST-inclusive price from a price non-inclusive of GST, use either formula below.

$$Q = \frac{P \times 11}{10}$$

$$Q = P + \text{GST}$$

WORKED EXAMPLE 4 *(2 marks)*

a) Calculate the GST included in a $15 000 car.

b) Calculate the GST-inclusive price of a lamp if it sells for $120 non-inclusive of GST.

SOLUTION

a) Calculate the GST included in a $15 000 car.

Step 1 We have Q and are looking for GST. Find the relevant formula.

$$\text{GST} = \frac{Q}{11}$$

Step 2 Substitute 15000 into the equation.

$$\text{GST} = \frac{15000}{11}$$

Step 3 Calculate.

$$\text{GST} = \$1363.64$$

b) Calculate the GST-inclusive price of a lamp if it sells for $120 non-inclusive of GST.

Step 1 Find the formula to get Q from P.

$$Q = \frac{P \times 11}{10}$$

Step 2 Substitute 120 into the equation.

$$Q = \frac{120 \times 11}{10}$$

Step 3 Calculate.

$$Q = \$132$$

2. Dividends and shares

Shares are units of ownership in a company. Public companies issue shares to raise money for their business. In exchange, the people who buy shares, called shareholders or investors, usually receive a share of the profit made by the company each year, called a dividend.

The dividend received by shareholders can be related to the profit by the following formula.

$$Dividend\ per\ share = \frac{Total\ dividend\ paid}{Number\ of\ shares\ in\ company}$$

Some useful formulas exist to evaluate the performance of a company. The price to earnings ratio (PE ratio) measures the amount paid for shares compared to the dividend returned. Investors prefer a low PE ratio. An 'average' PE ratio is around 15−25.

$$Price\ to\ earnings\ ratio = \frac{Share\ price\ per\ share}{Dividend\ per\ share}$$

Percentage dividend also measures cost and benefit. A high percentage dividend means the shareholder receives a high dividend for each share owned.

$$Percentage\ dividend = \frac{Dividend}{Share\ price} \times 100$$

WORKED EXAMPLE 5 *(1 mark)*

Calculate and interpret the PE ratio for The Lean Green Bean company, which has a current share price of $3.65 and paid a dividend of $0.45 last year.

SOLUTION

Step 1 Use the PE ratio formula.

$$Price\ to\ earnings\ ratio = \frac{Share\ price\ per\ share}{Dividend\ per\ share}$$

Step 2 Substitute the share price and most recent dividend into the equation.

$$Price\ to\ earnings\ ratio = \frac{3.65}{0.45}$$

Step 3 Calculate.

$$Price\ to\ earnings\ ratio = 8.11$$

Step 4 Interpret. This ratio is lower than average, which is good for investors who want a high dividend each year.

3. Percentage applications

As seen in lesson 3A, percentages are often used to express a change. In this section we will apply percentage change to everyday situations.

WORKED EXAMPLE 6 *(2 marks)*

The price of gold in Australia in 2010 was about $40 per gram.

a) If the price increased by 20% to 2015, what was the price in 2015?

b) If the price was $55 per gram in 2018, what was the percentage change from 2010 to 2018?

SOLUTION

a) If the price increased by 20% to 2015, what was the price in 2015?

Step 1 We are searching for V_2, the new value. Find the formula in lesson 3A.

$$V_2 = V_1 \times \frac{100 + Percentage\ (Change\ from\ V_1\ to\ V_2)}{100}$$

Step 2 Substitute the values.

$$V_2 = 40 \times \frac{100 + 20}{100}$$

Step 3 Calculate.

$$V_2 = \$48\ per\ gram$$

b) If the price was \$55 per gram in 2018, what was the percentage change from 2010 to 2018?

Step 1 Find the correct formula. We have V_1 and V_2 and want the percentage change.

$$\textit{Percentage change } (\textit{from } V_1 \textit{ to } V_2) = \frac{V_2 - V_1}{V_1} \times 100$$

Step 2 Substitute the values.

$V_1 = 40$

$V_2 = 55$

$$\textit{Percentage change } (\textit{from } V_1 \textit{ to } V_2) = \frac{55 - 40}{40} \times 100$$

Step 3 Calculate.

$$\textit{Percentage change } (\textit{from } V_1 \textit{ to } V_2) = 37.5$$

The percentage change from V_1 to V_2 is 37.5%.

Questions 3B *Financial percentage change applications*

Refresher question

Q1. Fill in the gap: GST is a ☐ % tax added to goods and services in Australia.

1. GST

Q2. Skill

What is the price of a watch including GST (Q) if the price without GST (GST-exclusive price, P) is \$80?

Finish this working to find Q.

$P = 80$

$Q = \frac{P \times 11}{10}$

$= \frac{80 \times 11}{10}$

$Q = \$$ ☐

Q3. Skill

Find the GST-inclusive price (Q) of the following items.

a) Shoes with a GST-exclusive price of \$180.

b) A landscaper who quotes \$350 for a job, excluding GST.

c) A Blu-ray movie bought for \$14.50, excluding GST.

Q4. Skill

Find the GST included in the following items rounded to 2 decimal places:

a) A hat that sells for \$45 in a store.

b) A bicycle that sells for \$175 in a store.

c) An album purchased online for \$16.99.

Check your understanding

Q5. Application

A pair of Marvel socks costs \$20, including GST. A pair of DC socks costs \$18.50, excluding GST.

Calculate the retail price of the DC socks and determine which socks are more expensive.

2. Dividends and shares

Q6. Skill

If the total dividend given out by a company is $1000, and there are 500 company shares owned by shareholders, find the dividend per share.

Q7. Skill

Calculate the price to earnings (PE) ratio for the following companies, to two decimal places.

a) Woolworths has a share price of $27.90 and a dividend per share of $1.25.

b) Cochlear has a share price of $194.06 and a dividend per share of $4.31.

c) Origin Energy has a dividend per share of $0.59 and a share price of $9.75.

Check your understanding

Q8. Skill

What is the share price of Qantas if they have a dividend of $0.60 and a PE ratio of 9.67? Round your answer to two decimal places.

Q9. Skill

Which of the following two companies have the higher share price?

Bega Cheese: Dividend of $0.41, PE ratio of 17.3

Inghams Group: Dividend of $0.28, PE ratio of 13.3

3. Percentage applications

Q10. Skill

Fill in the blank.

Calculate the percentage change in the price of pears increasing from $3 per kilo to $5 per kilo.

Original Price = 3

New Price = 5

$$\text{Percentage change} = \frac{\text{Change}}{\text{Original amount}} \times 100$$

$$= \frac{5-3}{3} \times \square$$

$$= 66.67$$

The percentage change is $\square$ %

Q11. Skill

Find the percentage change in the following scenarios:

a) A cafe increasing its milkshakes from $4 to $5.

b) An item that is discounted from $20 to $14.

c) A dance school increasing class fees from $30 to $35.

Check your understanding

Q12. Application

Two weeks ago the price of petrol was $1.40 per litre. In the week after, the price increased by 10%. The week after that, it dropped by 5%.

What is the current price of petrol? Give your answer to two decimal places.

Q13. Application

The average price of a can of coke increased 300% from 1970 to 1990. From 1990 until now, it has increased 150%. Currently the average price at a corner shop is $2.50.

What was the average price in 1970?

Joining it all together

Q14. Find the total cost of the three items in the table below including GST.

Application
1 mark

If Xavier has a twenty percent discount card, how much will buying the three items cost?

Item	Cost (exc. GST)
Xbox One X	$450
TV	$1500
Soundbar	$120

Q15. Company X has an overall profit of $1 million, a share price of $2.80 and a dividend of $0.25. Company Y has an overall profit of $5 million, a share price of $12.25 and a dividend of $0.35.

Application
2 marks

Assume all profit was given as dividends. Calculate the number of shares and the percentage dividend for the two companies.

Which company looks more attractive to investors?

Q16. At Mt Dandenong High School, the year 7 English class has 32 students, and the maths class has 16 students. After one week of classes, 10 students in the English class change into the maths class.

Application
2 marks

Represent the change in both classes as a percentage.

Q17. Who spent the most money today? Kathy hired an electrician at $85 an hour (exc. GST) for three hours. Linda bought 100 shares in a company that last year paid dividends per share of $0.10 and has a PE ratio of 35.

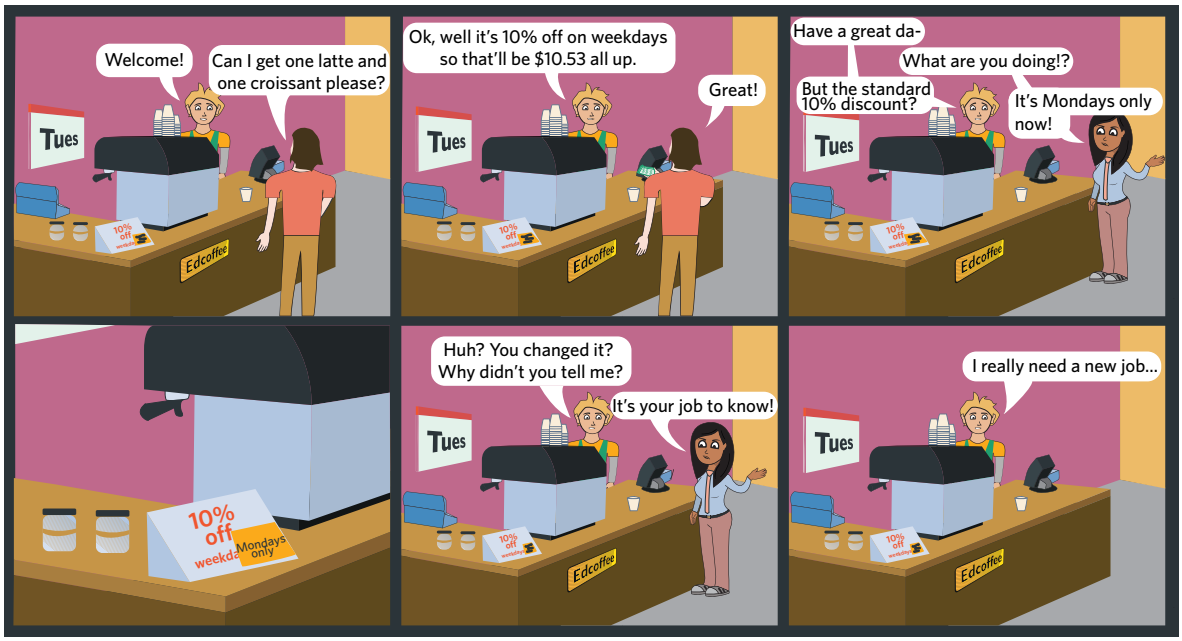
Application
2 marks

Q18. Sergey has a 10% off voucher. If Sergey buys some products with GST, does he pay less than, equal to or more than the GST-exclusive price?

Application
1 mark

Q19. Eric works at a cafe. Read the following cartoon and calculate the actual amount Eric should've charged the customer.

Application
1 mark



Q20. Application 2 marks

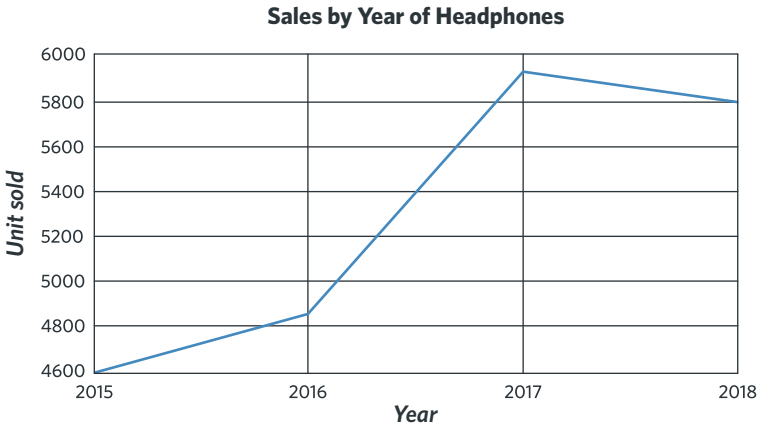
Bangin Fones has one product. They sell headphones for $120 each. The cost to make each one is $35. The total amount of money paid to staff in 2018 was $260 000.

a) Look at the graph below, and calculate the amount of profit made in 2018.

20% of the profit is reinvested into the company and the rest is paid in dividends.

The total number of shares is 350 000.

b) What is the dividend per share, to the nearest cent?



Q21. Application 3 marks

Thomson Reservoir in Melbourne can hold 1.068 million ML of water. At the end of March one year, it has 636,100 ML. It loses about 750 ML every day in April due to water use in Melbourne. Round your answers to two decimal places for this question.

NOTE: One megalitre (ML) is equal to 10^6 litres.

a) Calculate the percentage change in amount of water stored, relative to the reservoir's total capacity, over the entire month of April, assuming no rain. (1 mark)

b) A day of rain results in 900 million litres of water entering the reservoir. If it rains every Sunday and Tuesday in April, and the first day of the month is a Sunday, calculate the percentage change in amount of water stored, relative to the reservoir's total capacity. (2 marks)

VCAA question

Q22. 2 marks

Jane and Michael have started a business that provides music at parties. The business charges customers $88 per hour. The $88 per hour includes a 10% goods and services tax (GST).

a) Calculate the amount of GST included in the $88 hourly rate.

After six months of regular work, Jane and Michael decided to increase the hourly rate they charge by 12.5%.

b) Calculate the new hourly rate (including GST).

2015 Exam 2, Module 4: Business-related mathematics. Q1a, Q1c

Questions from multiple lessons

Q23.

Difficulty:

1 mark

A rare painting was bought for $140 000 in 2012.

In 2019, it sold at auction for $189 000.

What is the increase in value, as a percentage of the original purchase price?

A. 5.0%

B. 25.9%

C. 35.0%

D. 42.1%

E. 74.1%

VCAA 2015 Exam 1 Module 4: Business-related mathematics Q2 – Adapted

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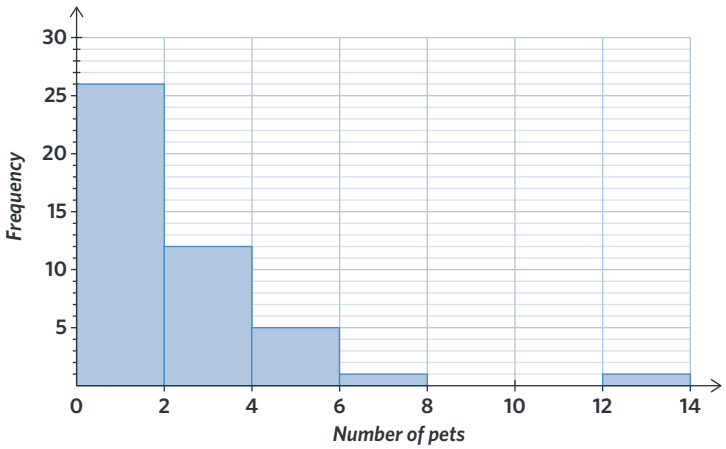
Q24.

Difficulty:

Year 10
1 mark

The following histogram shows the distribution of the number of pets in different households.



Using this histogram, the percentage of these 45 households with fewer than two pets is closest to

A. 26%

B. 42%

C. 58%

D. 44%

E. 57%

VCAA 2016 Exam 1 Data analysis Q6 – Adapted

Q25.

Difficulty:

Year 10
3 marks

A house was purchased in Eltham for $750 000. A 20% deposit was paid.

a) Calculate the deposit. (1 mark)

b) Determine the amount that the homeowners still owe after the deposit is paid. (1 mark)

The price of $750 000 included 10% GST.

c) Calculate the price of the house before GST was added. Give your answer correct to the nearest dollar. (1 mark)

VCAA 2012 Exam 2 Module 4: Business-related mathematics Q1 a,bi,c – Adapted

LESSON 3C

Simple interest

The key skills you will learn in this lesson are:

1. Introduction to simple interest
2. Simple interest in practice
3. Working with balance sheets

VCAA key knowledge points:

"applications of simple interest and compound interest"

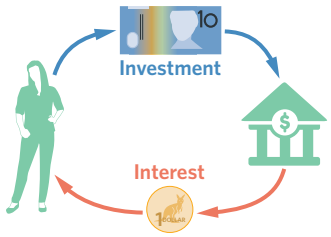
"cash flow in common savings and credit accounts including interest calculation"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Introduction to simple interest

When money is loaned to a person or organisation this loan will usually have to be repaid with interest. Interest is money paid in return for having a loan.

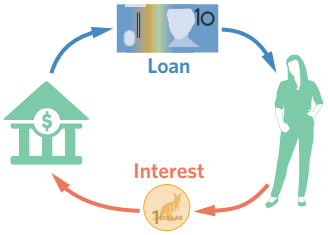
Earning interest



Investing money with a financial institution (such as a bank or superannuation fund) can be thought of as giving a loan to an organisation.

Banks use the money deposited in their accounts to make their own investments. The banks then pay the account holders interest for letting them use their money.

Paying interest



Banks and other organisations like to lend money to people. In this case, the person with the loan will pay interest. This is one way banks and other organisations make money.

The amount of interest paid depends on three things:

- The amount borrowed or loaned, called the principal.
- The interest rate, which is the percentage of the principal paid each time period.
- The number of time periods.

In simple interest scenarios, the interest charged is kept separate to the principal. This means that the principal does not change as interest is paid.

The formula for simple interest (I) is:

$I = \frac{Prn}{100}$, where:

- P is the principal.
- r is the interest rate per time period, expressed as a percentage.
- n is the number of time periods.

Example

If the monthly interest rate is 1%, the loan lasts half a year, and the interest is paid monthly, then $r = 1$ and $n = 6$.

WORKED EXAMPLE 7 *(1 mark)*

Marge deposited $500 in a savings account that pays simple interest at a rate of 4% per annum. How much interest will she have earned after three years?

SOLUTION

Step 1 Write out the simple interest formula and assign values to the variables.

$I = \frac{Prn}{100}$

$P = 500$ because Marge deposited \$500.

$r = 4$ because the interest rate is 4%.

$n = 3$ because the amount of time is 3 years.

Step 2 Substitute in the values.

$I = \frac{500 \times 4 \times 3}{100}$

Step 3 Compute the value of I.

$I = 60$

Step 4 Interpret the result.

The account earned \$60 in interest in three years.

2. Simple interest in practice

Total value of an investment or loan

The total value of an investment or loan, A, is given by the formula:

$$A = P + I$$

This formula can also be written as:

$$A = P\left(1 + \frac{rn}{100}\right)$$

Working with different time periods

Interest rates are usually stated as a yearly rate, also called 'per annum' or 'p.a.', but the interest is often paid more frequently, such as monthly or quarterly. In financial mathematics the following estimates are used when dealing with different time periods:

1 Year =	12 Months
	4 Quarters
	52 Weeks
	365 Days

If you are not familiar with converting units, please refer to Lesson 2G.

Using the simple interest formula to find the rate, period, or principal

To find the rate, period, or principal the simple interest formula needs to be transposed to make r, P, or n the subject.

Here are the transposed formulae:

Principal: $P = \frac{100I}{rn}$ or $P = \frac{A}{1 + \frac{rn}{100}}$

Rate: $r = \frac{100I}{Pn}$ or $r = \frac{100}{n}\left(\frac{A}{P} - 1\right)$

Number of periods: $n = \frac{100I}{Pr}$ or $n = \frac{100}{r}\left(\frac{A}{P} - 1\right)$

WORKED EXAMPLE 8 *(1 mark)*

Marge deposited \$500 in a savings account that pays simple interest at a rate of 4% per annum.

Currently there is \$860 in the account. For how long has the money been in the account?

METHOD 1: TRANSPOSING

Step 1 Write out the simple interest formula that includes the total value of an investment.

$A = P\left(1 + \frac{rn}{100}\right)$

Step 2 Transpose the equation to make the number of periods, n, the subject.

$n = \frac{100}{r}\left(\frac{A}{P} - 1\right)$

Step 3 Substitute in the given values.

$n = \frac{100}{4}\left(\frac{860}{500} - 1\right)$

Step 4 Compute the value of n.

$n = 18$

Step 5 Interpret the result.

The money has been in the savings account for 18 years.

METHOD 2: USING A FORMULA

Step 1 Identify the required formula.

We need to find the number of periods so we can use either $n = \frac{100I}{Pr}$ or $n = \frac{100}{r}\left(\frac{A}{P} - 1\right)$. However, we don't know the interest, I, but we do know the final amount, A. So we use $n = \frac{100}{r}\left(\frac{A}{P} - 1\right)$.

Step 2 Substitute in the given values.

$n = \frac{100}{4}\left(\frac{860}{500} - 1\right)$

Step 3 Compute the value of n.

$n = 18$

Step 4 Interpret the result.

The money has been in the savings account for 18 years.

METHOD 3: SOLVING USING CAS

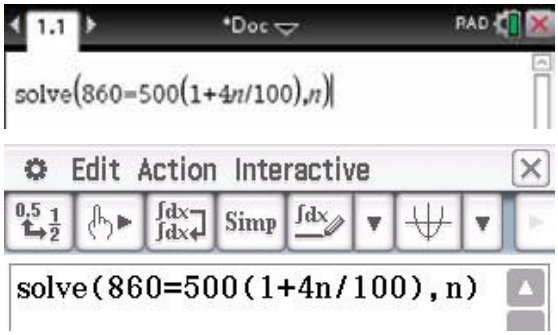
Step 1 Substitute the given values into the simple interest formula that includes the total value of an investment, $A = P\left(1 + \frac{rn}{100}\right)$, and type this equation into your calculator.

$860 = 500\left(1 + \frac{4n}{100}\right)$

Step 2 Use the solve function to find the value of n.

$n = 18$

The money has been in the savings account for 18 years.



3. Working with balance sheets

The money that enters and exits a bank account is often summarised in a balance sheet.

One way to calculate the interest for a period is by using the smallest amount of money in the account during that time.

WORKED EXAMPLE 9 *(1 mark)*

The transaction details for a savings account of the month of October 2016 are shown in the table below.
NOTE: 'Brought forward' refers to the balance from the last day of the previous month being carried over to the first day of the current month.

Date	Details	Deposit	Withdrawal	Balance
01 Oct. 2016	Brought forward			320.00
09 Oct. 2016	Income	400.00		720.00
17 Oct. 2016	ATM withdrawal		150.00	570.00

Interest is calculated and paid monthly on the minimum balance for that month. The interest rate for the account is 3% per annum. How much interest will the account earn for October 2016?

SOLUTION

Step 1 Write out the simple interest formula.

$I = \frac{Prn}{100}$

Step 2 Identify the minimum balance for the month. This acts as the principal.

$320.00

Step 3 Convert r into a monthly rate expressed as a decimal or fraction.

$r = 3\%$ p.a

$r = \frac{3}{12} = 0.25$

Step 4 Substitute in the given values.

$I = \frac{320 \times 0.25 \times 1}{100}$

Step 5 Compute the value of I.

$I = 0.8$

Step 6 Interpret the result.

The account earned 80 cents in interest in October 2016.

Questions 3C *Simple interest*

Refresher question

Q1. **a)** Express 3% as:

i a fraction.

ii a decimal.

b) How many:

i months are there in a year?

ii weeks are there in a month?

iii months are there in a quarter?

1. Introduction to simple interest

Q2. Mr Barefoot has $100 in a bank account that pays simple interest at a rate of 2% per annum.

Skill

a) Find an expression for the amount of interest that Mr Barefoot will earn in a year by filling in the missing numbers in the equation below.

$I = \frac{\square \times \square \times \square}{100}$

b) How much interest will Mr Barefoot have earned in a year?

Q3. Kai takes out a loan of $750 that charges simple interest at a monthly rate of 1.5%.

Skill

Find the amount of interest charged after:

a) one month.

b) three months.

c) seven months.

Check your understanding

Q4. Skill

Luisa and Jay have a farm in Fish Creek, Victoria. They want to buy a herd of goats. The total cost of the goats and associated expenses (transportation, vaccines, fencing, etc) comes to $1500. In January 2018 they opened a savings account with Buffalo Bank that pays simple interest at a monthly rate of 3%. At that time, they made a deposit of $900.

a) How much interest will Luisa and Jay have earned by January 2020?

b) If Luisa and Jay use the initial $900 deposit and the interest earned, will they be able to afford the goats and the associated expenses by January 2020?

2. Simple interest in practice

Q5. Skill

Transpose the simple interest formula, $A = P\left(1 + \frac{rn}{100}\right)$, to make

a) the number of periods, n, the subject.

b) the interest rate, r, the subject.

c) the principal, P, the subject.

Q6. Skill

Cath and Dave deposit $10 000 into an account that pays simple interest at a rate of 14% per annum on the condition that they do not withdraw any money from the account. Cath and Dave are very impatient people, however. How much interest will they have earned if they withdraw their money after:

a) half a year? Give your answer in dollars, correct to the nearest cent.

b) a quarter? Give your answer in dollars, correct to the nearest cent.

c) a month? Give your answer in dollars, correct to the nearest cent.

Q7. Skill

Answer the following questions.

a) Maria invests $500 in a bank account that pays simple interest. In 3 years' time there is $542 in the account. Assuming Maria did not deposit or withdraw any money from the account, calculate the yearly interest rate as a percentage.

b) Just over 10 years ago Iris found a large sum of cash in a dumpster. She reported it to the police but it went unclaimed; she was able to keep it. Exactly 10 years ago she deposited it in a savings account that pays simple interest at a rate of 10% per annum. Iris has since earned $39 000 in interest. How much money did she find?

c) Bill from the Gully takes out a personal loan with Easy Loans. He borrows $5000 at an interest rate of 156% per annum. Assuming Bill makes no repayments, how much will he owe in one year's time?

d) Some time ago, David used one of the toll roads around Melbourne without an e–TAG. At the time, he was charged $14 dollars. He refuses to pay the fine. Each month since, he has been charged 50% of the original fee. He currently owes $63.

i How much interest has David been charged?

ii For how many months has David been avoiding paying the fine?

Q8. Application

Laura borrows $2000 with a monthly simple interest rate of 15%. She has to repay the total value of the loan in one year's time. In six months' time she has saved $1500 and decides to deposit it in a savings account that pays simple interest with the idea of earning enough interest to pay back the loan.

What is the lowest monthly interest rate that would allow her to repay her loan with the deposited money? Give your answer as a percentage, correct to two decimal places.

Check your understanding

Q9. Application

Beppe and Daisy are saving up to buy a tenth birthday present for their dog, Barnesworth. They deposit $1500 into a savings account that pays simple interest at a rate of 2.8% per annum.

a) Find an expression for the total interest earned after Y years.

b) They realise that the interest is actually paid monthly. Find an expression for the total interest earned after M months.

Q10. Skill

Starting with the formula $A = P + I$, show that the total value of a loan or investment, A, can be found using the formula $A = P\left(1 + \frac{rn}{100}\right)$.

3. Working with balance sheets

Q11. Skill

Below is Ayesha's bank statement for the month of February. Her account pays out simple interest each month at a rate of 3% per annum, calculated on the minimum monthly balance. She wants to know how much interest she earned.

Date	Details	Deposit	Withdrawal	Balance
1 Feb. 2019	Carried forward			2300.00
3 Feb. 2019	Valentine's Day present		50.00	2250.00
7 Feb. 2019	Income	150.00		2400.00
21 Feb. 2019	Income	150.00		2550.00
27 Feb. 2019	Concert tickets		320.00	2230.00

a) What is the minimum balance?

b) How much interest was earned? Give your answer in dollars, correct to the nearest cent.

Q12. Skill

a) The balance sheet below shows Joey's cash flow for the month of March.

How much money in total did Joey spend?

Date	Details	Deposit	Withdrawal	Balance
01 April 2018	Brought forward			60.00
03 April 2018	Income	50.00		110.00
04 April 2018	ATM withdrawal		40.00	70.00
05 April 2018	Lunch		10.00	60.00
08 April 2018	Netflix		15.00	45.00
09 April 2018	Brunch		35.00	10.00
12 April 2018	Coffee		5.00	5.00
21 April 2018	Income	50.00		55.00
26 April 2018	Pocket money	25.00		80.00
30 April 2018	Dinner		75.00	5.00

b) Grace uses a bank account that pays simple interest monthly at a rate of 2.8% per annum calculated on the monthly minimum balance. The balance sheet below shows her bank balance for the month of May.

How much interest will she earn for the month of May? Give your answer in dollars, correct to the nearest cent.

Date	Details	Deposit	Withdrawal	Balance
1 May 2018	Brought forward			220.00
03 May 2018	Tutoring pay	80.00		300.00
04 May 2018	Income for April	2400.00		2700.00
07 May 2018	Loan repayment		300	2400.00
08 May 2018	ATM withdrawal		300	2100.00
21 May 2018		100.00		2200.00
22 May 2018	Bills and rent		2000.00	200.00
31 May 2018	Tutoring pay	40.00		240.00

Q13. Application

Ted Landers doesn't trust the banks so he has been keeping his own balance sheet. However, he has forgotten to include interest. He knows that his bank is supposed to pay him simple interest on the lowest account balance for the calendar month at a rate of 1.2% per month. The interest is paid into a separate account.

How much money did Ted earn in interest over the three months covered by the balance sheet below? Give your answer in dollars, correct to the nearest cent.

Date	Details	Deposit	Withdrawal	Balance
27 Aug. 2018	Opening balance			9500.00
2 Sep. 2018	Income	250.00		9750.00
13 Sep. 2018	Income	800.00		10550.00
24 Sep. 2018	Church renovation		1500.00	9050.00
7 Oct. 2018	Income	320.00		9370.00
13 Oct. 2018	Income	120.00		9490.00
27 Oct. 2018	Church renovation		500.00	8990.00
31 Oct. 2018	Closing balance			8990.00

Check your understanding

Q14. Application

Yuri keeps his savings in a bank account that pays simple interest of 0.44% per month on the minimum monthly balance.

Yuri earned $1.21 in interest last month.

What was the minimum balance for that month?

Joining it all together

Q15.
Skill
4 marks

a) A simple interest investment of $1300 has a rate of 5% per annum. How much interest is earned in 5 years?

b) A simple interest investment with a simple interest rate of 2% per annum earned $870 in ten years. What was the principal?

c) A simple interest investment of $400 earned $100 in interest in four years. What is the annual interest rate?

d) A simple interest investment of $800 with a rate of 4.5% per annum earned a total of $400 in interest. For how long did the investment last? Give your answer correct to the nearest year.

Q16.
Application
5 marks

Nozomi wants to take out a loan of $1000 to buy a new graphics card. She is considering three options: The Dollar Dealer, Crazy Cash, and Legal Loans. They all offer simple interest loans.

a) The Dollar Dealer asks that she pay back twice the amount she originally borrowed in two years' time. What is the interest rate per annum offered by The Dollar Dealer?

b) Crazy Cash offers an annual interest rate of 60%. They also state that the loan must be repaid in 20 months' time. What will the total value of the loan be at that time?

c) Legal Loans offer a monthly interest rate of 0.01 times the value of the principal. Their loan must be paid off in one years' time.

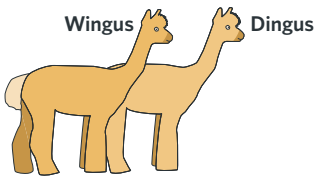
What will the total value of the loan be at that time?

d) **i** Nozomi want to pay back the least amount of money possible. Which loan provider should she **not** choose?

ii Nozomi is worried about the financial strain of repaying the debt. Which option should she choose in order to give herself the longest time to pay back the money owing?

Q17.
Application
3 marks

Wingus and Dingus opened a savings account together on the first of November, 2018. The transaction details for their savings account of the month of November 2018 are shown in the table below.



The table is incomplete.

Date	Details	Deposit	Withdrawal	Balance
01 Nov. 2018	Opening balance			2000.00
09 Nov. 2018	Income	300.00		2300.00
17 Nov. 2018	ATM withdrawal		500.00	1800.00
23 Nov. 2018	Income	300.00		2100.00
30 Nov. 2018	Interest	45.00		

Interest is calculated and paid monthly on the minimum balance for that month.

a) What is the closing balance?

b) What is the quarterly interest rate if interest is calculated on the minimum balance for that month?

c) Wingus and Dingus want to use the interest from their savings account to pay their monthly internet bill which is $75. What is the smallest amount of money they need to keep in their bank account at all times in order to do this?

Q18. The balance sheet below shows the cash flow of George's bank account for a three–month period in 2018.

Skill
3 marks

Date	Details	Deposit	Withdrawal	Balance
01 May 2018	Brought forward			250 000.00
08 May 2018	Trust Fund	25 000.00		275 000.00
22 May 2018	Trust Fund	25 000.00		300 000.00
27 May 2018	New car		50 000.00	250 000.00
05 Jun. 2018	Trust Fund	25 000.00		275 000.00
15 Jun. 2018	New Yacht		200 000.00	75 000.00
19 Jun. 2018	Trust Fund	25 000.00		100 000.00
03 Jul. 2018	Trust Fund	25 000.00		125 000.00
17 Jul. 2018	Trust Fund	25 000.00		150 000.00
31 Jul. 2018	Trust Fund	25 000.00		175 000.00

The monthly payments the bank makes to George are calculated as a simple percentage of the monthly minimum balance and are paid into a different account.

George earned $231.25 in interest for the month of June. Interest is calculated on the minimum balance for that month.

Show that the annual interest rate is 3.7%.

VCAA question

Q19. The transaction details for a savings account of the month of August 2014 are shown in the table below.

1 mark

The table is incomplete.

Date	Details	Deposit	Withdrawal	Balance
01 Aug. 2014	Brought forward			5120.50
10 Aug. 2014	Purchase		250.00	4870.50
17 Aug. 2014	Cheque	1000.00		5870.50
30 Aug. 2014	Interest			5885.72

Interest is calculated and paid monthly on the minimum balance for that month.

The annual rate of interest paid on this account is closest to:

A. 3.10%

B. 3.11%

C. 3.57%

D. 3.75%

E. 14.9%

VCAA 2015 Exam 1, Module 4: Business-related mathematics. Q6

Questions from multiple lessons

Q20.

Difficulty:

Year 10
1 mark

A billboard in New York City costs $10 320 to hire for the promotion of concert tickets.

The total amount earned selling tickets is $86 000.

The $10 320 charge as a percentage of the amount earned from selling tickets is

A. 0.10% **B.** 0.12% **C.** 10.0%

D. 12.0% **E.** 8.33%

VCAA 2014 Exam 1 Module 4: Business-related mathematics Q2 – Adapted

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Q21.

Difficulty:

1 mark

The table below shows the *height* in centimetres of a group of 14 students selected from a cohort of 130 students.

***height* (cm)**	122.6	162.3	183.7	152.1	142.5	156.6	162.5	172.2	125.2	123.2	162.5	152.9	172.5	159.6

The mean, $\bar{x}$, and the standard deviation, s_x, of the *height* for this sample of students are closest to

A. $\bar{x} = 18.33, s_x = 153.6$

B. $\bar{x} = 19.03, s_x = 153.6$

C. $\bar{x} = 19.72, s_x = 153.6$

D. $\bar{x} = 153.6, s_x = 18.33$

E. $\bar{x} = 153.6, s_x = 19.03$

VCAA 2017 Exam 1 Data analysis Q3 – Adapted

Q22.

Difficulty:

Year 10
2 marks

Carl has just opened his business selling backpacks made out of recycled plastic.

The business charges customers $140 per backpack.

This price of $140 per backpack includes GST.

a) Calculate the amount of GST included in the price of the backpack. Round your answer to the nearest cent. (1 mark)

b) A customer purchased 10 backpacks. Calculate the total amount Carl was paid for this purchase. Give your answer correct to the nearest dollar. (1 mark)

VCAA 2015 Exam 2 Module 4: Business-related mathematics Q 1a-c – Adapted

LESSON 3D

Compound interest

The key skills you will learn in this lesson are:

1. Introduction to compound interest
2. Compound interest in practice
3. Inflation

VCAA key knowledge points:

"percentage increase and decrease applied to various financial contexts such as the price to earnings ratios of shares and percentage dividends, determining the impact of inflation on costs and the spending power of money over time, calculating percentage mark-ups and discounts, and calculating GST"

"applications of simple interest and compound interest"

"compound interest investments and loans"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Introduction to compound interest

Compound interest differs from simple interest because the interest is added to the principal amount at regular periods over the course of the loan or investment. This means that the amount of interest paid or charged increases each period.

The total value of a compound-interest investment or loan is given by:

$A = P\left(1 + \frac{r}{100}\right)^n$, where

- A is the total value of the loan or investment.
- P is the principal.
- r is the interest rate per compounding period, expressed as a percentage.
- n is the number of compounding periods.

WORKED EXAMPLE 10 (3 marks)

Johnny Two-Hats opens a savings account that pays interest at a rate of 5% per annum, compounding monthly.

At this time he deposits $2000 into the account.

What will be the balance of the account be after two years?

SOLUTION

Step 1 Write down the compound interest equation.

$A = P\left(1 + \frac{r}{100}\right)^n$

Step 2 Convert the interest rate to a rate per compounding period to find r.

5 p.a. $= \frac{5}{12}$ per month

$r = \frac{5}{12}$ or $r = 5 \ / \ 12$

NOTE: To make your calculations accurate, do not use a decimal approximation at this stage.

✓ *1 mark for calculating monthly interest rate*

Step 3 Find the number of compounding periods to find n.

The loan is for two years, and the account compounds monthly.

2 years is 24 months, so 24 compounding periods.

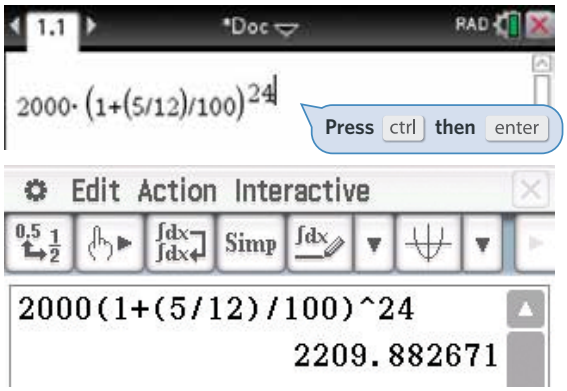
$n = 24$

✓ *1 mark for calculating number of periods*

Step 4 Substitute the given values into the compound interest equation.

$A = 2000\left(1 + \frac{(5 \ / \ 12)}{100}\right)^{24}$

Step 5 Calculate the total value of the investment.



$A = 2209.88$

After two years, the balance of Johnny Two-Hats' savings account is $2209.88.

✓ *1 mark for calculating the balance of savings account*

2. Compound interest in practice

The total amount of interest charged on a compound-interest loan can be found using the following formula:

$$I = A - P$$

Which can be expressed as:

$$I = P\left(\left(1 + \frac{r}{100}\right)^n - 1\right)$$

To find the values of quantities other than the total value or amount of interest of the investment or loan, either transpose the compound interest formula, solve it using a CAS calculator, or use the equations below.

Principal:

$$P = \frac{A}{\left(1 + \frac{r}{100}\right)^n} \text{ or } P = \frac{I}{\left(\left(1 + \frac{r}{100}\right)^n - 1\right)}$$

Interest rate per compounding period:

$$r = 100\left(\left(\frac{A}{P}\right)^{1/n} - 1\right) \text{ or } r = 100\left(\left(\frac{I}{P}\right)^{1/n} - 1\right)$$

The equation to find the number of periods uses logarithms other than $\log_{10}$, and is therefore outside the scope of the course. However, it can be found by using the solve function on CAS.

It is important to note that, unless otherwise specified, interest is paid at the end of a compounding period.

WORKED EXAMPLE 11 *(4 marks)*

Hats are becoming more popular, and Johnny Two-Hats is worried that two hats aren't enough to make him stand out any more.

He decides to:

- buy a third hat, which costs $320
- change his name to Johnny Three-Hats, which costs $185.

Johnny Two-Hats wants to use the interest earned from a savings account to pay for his new hat and name change. His opening balance will be $2210.

a) If his savings account will pay interest that compounds monthly, what is the minimum annual interest rate that Johnny Two-Hats' savings account must have in order for him to afford the hat and the name change in one year's time?
Give your answer correct to two decimal places. *(2 marks)*

b) The highest rate he could find was 8% per annum, compounding monthly.
For how many months must Johnny Two-Hats wait for his new hat and name change?
His savings account pays interest at the end of each month. *(2 marks)*

SOLUTION

a) If his savings account will pay interest that compounds monthly, what is the minimum annual interest rate that Johnny Two-Hats' savings account must have in order for him to afford the hat and the name change in one year's time? Give your answer correct to two decimal places.

Step 1 Write down the required equation.

$$r = 100\left(\left(\frac{I}{P} + 1\right)^{1/n} - 1\right)$$

Step 2 Find the number of compounding periods.

One year is 12 months.

$n = 12$

Step 3 Substitute the given values into the compound interest equation.

$$r = 100\left(\left(\frac{320 + 185}{2210} + 1\right)^{1/12} - 1\right)$$

Step 4 Calculate the interest rate per compounding period.

$r = 1.73...\%$

✓ *1 mark for calculating the interest rate per compounding period*

Step 5 Convert the interest rate per compounding period into an annual interest rate.

$r_{annual} = compounding\ periods\ per\ year \times r$

$r_{annual} = 12 \times r = 20.757...\%$

$r_{annual} = 20.76\%$

✓ *1 mark for calculating the annual interest rate*

Johnny Two-Hats needs to find a savings account that pays interest at a rate of at least 20.76% per annum.

b) The highest rate he could find was 8% per annum, compounding monthly.
For how many months must Johnny Two-Hats wait for his new hat and name change? His savings account pays interest at the end of each month.

Step 1 Convert the interest rate to a rate per compounding period.

8 % p.a. and 12 compounding periods per year

$r = \frac{8}{12} = \frac{2}{3}$ or $r = 2 / 3$

✓ *1 mark for calculating the interest rate per compounding period*

Step 2 Find the necessary equation.

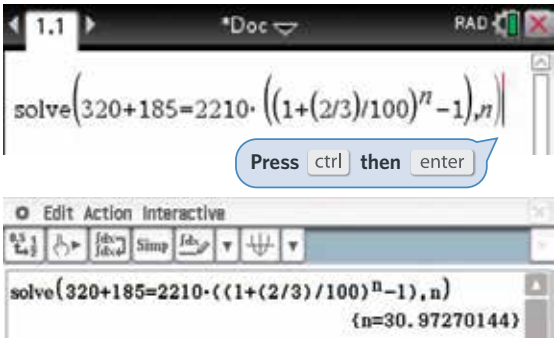
The question gives us the total amount of interest needed so it will be easiest to use the following equation:

$$I = P\left(\left(1 + \frac{r}{100}\right)^{n} - 1\right)$$

Step 3 Substitute the given values into the compound interest equation.

$$320 + 185 = 2210\left(\left(1 + \frac{2/3}{100}\right)^{n} - 1\right)$$

Step 4 Use your CAS to solve for the number of periods.



$n = 30.9727$

✓ *1 mark for calculating the interest rate per compounding period*

Step 5 Round up to the next whole number.

It will take Johnny Two-Hats 31 months to save up for the new hat and name change.

3. Inflation

Generally, the prices of goods and services increase year to year. Inflation is a measure of how these costs change over time. When prices are increasing, a set amount of money will be "worth" less in the future than it is now. This is often referred to as the purchasing power of money: what can a given amount of money buy?

Inflation is most often expressed as the yearly percentage increase in the price of goods and services. This is called the inflation rate.

Inflation is calculated in the same way as a compound interest investment that compounds yearly.

The purchasing power of money can also increase. This is called deflation. The mathematics of deflation is outside the scope of this course.

WORKED EXAMPLE 12 *(1 mark)*

The hat Johnny Two-Hats wants to buy costs $320. He calculates that it will take him three years to save up enough money to buy the hat.

Unfortunately, Johnny Two-Hats forgot to take inflation into account.

If the inflation rate over the next three years is 4.5% per annum, how much will the hat cost in three years' time?

SOLUTION

Step 1 Write down the compound interest equation.

$A = P\left(1 + \frac{r}{100}\right)^n$

Step 2 Substitute the given values into the compound interest equation.

$A = 320\left(1 + \frac{4.5}{100}\right)^3$

Step 3 Calculate the value of the hat.

$A = 365.17316$

In three years' time, the hat will cost $365.17.

Questions 3D *Compound interest*

Refresher question

Q1. **a)** $y = \frac{1}{2}$

What is $\frac{y}{100}$?

b) $x = 3^3$

What is x?

1. Introduction to compound interest

Q2. Skill

Anna invests $1000 in a savings account that pays interest at a monthly rate of 0.5%, compounding monthly, for 8 months.

a) Substitute the given values into the compound interest formula by filling in the three boxes.

$A = \square\left(1 + \frac{\square}{100}\right)^{\square}$

b) Using your result from part a), compute the value of the compound interest investment, to the nearest cent.

Q3. Skill

a) Eugene takes out a personal loan of $7500. The lender charges interest at a rate of 13.5% p.a. compounding monthly. How much will Eugene owe in five years' time?

b) Maya invests $2750 in a savings account that pays interest at a yearly rate of 2.8%, compounding monthly. What will Maya's savings account balance be in four years' time?

c) Gustave collects Magic: The Gathering cards. Gustave's collection is currently worth $5500, it grows in value at a rate of 4% p.a., compounding quarterly. How much will Gustave's collection be worth in three years' time?

Check your understanding

Q4. Skill

Xander and Yuri both invest $1000 in savings accounts that pay interest at a monthly rate of 0.5%. The account Xander uses pays simple interest. The account that Yuri uses pays interest that compounds monthly. In one years' time, how much larger will Yuri's account balance be than Xander's?

Q5. Application

Quinn and Tobi each invest $10 000 in a savings account. Quinn's account pays interest at a rate of 2.8% p.a. compounding monthly. Tobi's account pays interest at a rate of 3% p.a. compounding yearly.

Whose savings account will have the greater balance after two years?

2. Compound interest in practice

Q6. Skill

Gregory opens a savings account that pays interest compounding monthly.
He deposits $2000 into this account.

a) Write the compound interest formula in which the interest rate per compounding period, r, is the subject.

b) Using the equation from part a), what is the minimum monthly interest rate that would make the balance greater than $2050 in 11 months' time? Give your answer as a percentage correct to two decimal places.

Q7. Skill

a) The total value of a compound-interest loan of $300 is now $550. How much interest has been charged?

b) Bill invests $500 in a savings account that pays interest that compounds monthly. After 18 months he has $510.91 in his account. What is the monthly interest rate? Give your answer as a percentage correct to two decimal places.

c) Three years ago Marta took out a home loan. The interest rate on the loan is 4.6% per annum compounding monthly. The value of the loan is now $286 918. What was the principal of Marta's loan? Give your answer correct to the nearest dollar.

d) Some time ago, Abdul took out a loan of $1000 that compounds quarterly with a yearly rate of 6.3%. The total value of the loan is now $1455.07. How many years ago did Abdul take out the loan?

e) Cristina takes out a loan of $750. The interest charged on the loan compounds weekly. In 23 weeks' time Cristina will owe a total of $1300. What is the weekly interest rate? Give your answer as a percentage correct to one decimal place.

f) Amanda takes out a $5000 dollar loan that charges interest which compounds quarterly at a rate of 11% per annum. In how many compounding periods will Amanda owe over $1000 in interest alone?

Check your understanding

Q8. Application

a) After making an unwise financial decision, Maria owes $400 000. If she originally borrowed $270 000 through a quarterly-compounding-interest loan four years ago, what was the yearly interest rate of the loan? Give your answer as a percentage correct to one decimal place.

b) Priya wants to buy a rare houseplant that costs $1700. To do this, Priya opens a savings account that has an interest rate of 8% p.a. compounding weekly. Priya wants to buy the plant in 80 weeks' time. How much does Priya need to deposit in order to achieve this?

c) Madi wants to buy a new glockenspiel that costs $6500. She invests $4000 in a savings account that pays interest at a rate of 5% p.a. compounding monthly. For how long must she keep the money in the account in order to afford the glockenspiel? Round your answer to the nearest month.

Q9. Skill

Dr Grey is comparing two savings accounts. Both have an interest rate of 4% per annum.
One pays simple interest while the other pays interest that compounding annually.

Create a table that compares the **interest** that would be earned by each account over three years if the opening balance were $50 000.

3. Inflation

Q10. Skill

a) The public transport ticketing system in Melbourne, myki, had a daily concession price of $4.10 in 2017. If the average rate of inflation is 2%, and the myki fare price rises with inflation, find the formula to calculate the cost of a daily concession myki fare in 2027 by filling out the boxes in the formula below.

$$A = \square\left(1 + \frac{\square}{100}\right)^{\square}$$

b) Using your result from part a), calculate the value of a daily concession myki fare in 2027.

Q11. Skill

a) In 1980 the minimum wage was $129.50 per week. If the average inflation rate is 5% per annum, what was the equivalent weekly wage in 2018?

b) If the inflation rate is 4% per annum, how much money would you need in 2050 to match the purchasing power of $20 in 2020?

c) In 1951 the inflation rate reached an all-time high of 23.9% per annum. If the inflation rate had remained at this value, how much money would you need in 2019 to match the purchasing power of $1 in 1951?

Q12. Application

a) In 1967 the minimum hourly wage was $1.00. In 2017 it was $18.29. Assuming that this rise is due entirely to inflation, estimate the average inflation rate over these 50 years. Give your answer as a percentage correct to one decimal place.

b) Bruce is always saying that when he was 10 years old you could buy a Paddle Pop ice cream for 50 cents. In 2018 the price of a Paddle Pop was $2.50. If the average annual rate of inflation is 4.2%, in what year was Bruce born? Assume that the price of a Paddle Pop increased only due to inflation. Give your answer to the nearest year.

Check your understanding

Q13. Application

Karl Marx has deep ideological issues with for-profit corporations, especially financial institutions.

He decided to bury his life savings, $20 000, in cash in his backyard instead of investing it.

If the average annual rate of inflation is 3%, how much will Karl have lost in 30 years' time due to inflation alone?

Joining it all together

Q14. Application 5 marks

Beppe and Daisy have a combined HECS-HELP debt of $64 000.

The value of the debt increases yearly with inflation.

a) If the rate of inflation remains at 2% per annum what will be the value of the HECS-HELP debt in 20 years' time? (1 mark)

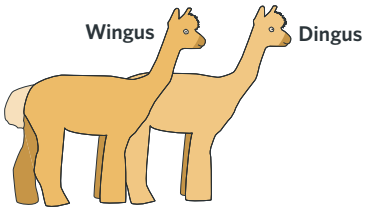
b) Beppe and Daisy have found a long-term savings account that offers an interest rate of 5.6% per annum. The interest compounds monthly. What amount do they need to deposit in order to use the balance to pay off their HECS-HELP debt in 20 years' time? (1 mark)

c) In five years' time, Beppe and Daisy's dog, Barnesworth, wins Most Poorly Behaved Dog at the Darebin City Council Dog Show. Using the prize money and the money in their long-term savings account at the time, they are able to pay off their HECS-HELP debt.

How much money did Barnesworth win? Give your answer correct to the nearest dollar. (3 marks)

Q15.
Application
3 marks

The alpacas Wingus and Dingus have recently been shorn.



They each sell their wool for $50.

a) Wingus deposits his earnings in a savings account. The interest compounds monthly. After one year, the balance of the account is $56.43.

What is the monthly interest rate? Give your answer as a percentage correct to one decimal place. (1 mark)

b) Dingus also deposits his interest in a savings account that pays compound interest. The account's interest rate is 8% per annum. After one year the balance of Dingus' savings account is $54.16.

Show that Dingus' savings account compounds weekly. (2 marks)

Q16.
Skill
3 marks

Luisa and Jay have a farm in Fish Creek, Victoria, and are saving up to buy a new tractor that costs $35 000. They deposit $20 000 in a savings account that pays compound interest at a rate of 6% p.a., compounding quarterly.

Show that the amount of interest earned during the 5^{th} year is given by

$$20000\left(1+\frac{6}{400}\right)^{20}-20000\left(1+\frac{6}{400}\right)^{16}.$$

Q17.
Application
9 marks

In 1975, the average Australian yearly wage was $7618 and the average house price in Melbourne was $19 800.

a) How many average Australian yearly wages would it cost to buy an average house in Melbourne in 1975? Give your answer correct to one decimal place. (1 mark)

b) In 2015 the average Australian wage was $72 000. Assuming that this increase is due entirely to inflation, calculate the yearly inflation rate over this period. Give your answer as a percentage correct to one decimal place. (1 mark)

c) If the price of housing increased at the same rate as the average wage, calculate the average house price in Melbourne in 2015. Use your rounded answer from part b). Give your answer correct to the nearest dollar. (1 mark)

d) In 2015 the average house price in Melbourne was actually $615 100.
How many average Australian yearly wages would it cost to buy an average house in Melbourne in 2015? Give your answer correct to one decimal place. (1 mark)

e) The increase in the average house price of Melbourne between 1975 and 2015 can be calculated by the compound interest in a similar way to inflation.
If the average house price of Melbourne compounds quarterly, find the yearly rate at which it increases. Give your answer as a percentage correct to one decimal place. (1 mark)

f) Using your results from parts b) and e), create a table that compares the average yearly wage and the average house price in Melbourne over the years 2020–2025 if they continue to increase as they have been. Round all table entries to the nearest dollar. (3 marks)

g) How many average Australian yearly wages will it cost to buy an average house in Melbourne in 2025? Give your answer correct to one decimal place. (1 mark)

VCAA questions

Q18. 1 mark

Alex sends a bill to his customers after repairs are completed. If a customer does not pay the bill by the due date, interest is charged. Alex charges interest after the due date at the rate of 1.5% per month on the amount of an unpaid bill. The interest on this amount will compound monthly.

Alex sent Marcus a bill of $200 for repairs to his car. Marcus paid the full amount one month after the due date.

How much did Marcus pay?

VCAA 2017 Exam 2, Core module. Q6a

Q19. 1 mark

Anthony invested $15 000 in an account. It earned r% interest per annum, compounding monthly.

The amount of interest that is earned in the third year of the investment is given by

A. $1500(1 + \frac{r}{1200})^3 - 15000(1 + \frac{r}{1200})^2$

B. $1500(1 + \frac{r}{1200})^{36} - 15000(1 + \frac{r}{1200})^{24}$

C. $1500(1 + \frac{r}{100})^3 - 15000(1 + \frac{r}{100})^2$

D. $1500(1 + \frac{r}{100})^{36} - 15000(1 + \frac{r}{100})^{24}$

E. $1500(1 + \frac{r}{1200})^4 - 15000(1 + \frac{r}{1200})^3$

VCAA 2011 Exam 1, Module 4: Business-related mathematics. Q7

Questions from multiple lessons

Q20. Difficulty: Year 10 1 mark

As part of a promotion, a new customer is offered to have his new air conditioner professionally installed in his home for only $200. However, he must have it installed within the week that he purchases it.

Next week, the charge will increase by 5.2%.

The charge next week will be

A. $200.00 **B.** $210.00 **C.** $210.40

D. $215.60 **E.** $189.60

VCAA 2014 Exam 1 Module 4: Business-related mathematics Q1 – Adapted

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Q21. Difficulty: 1 mark

The variables *car type* (1: Tesla, 2: Mustang, 3: Other) and *number plate type* (custom, auto-generated) are

A. both nominal variables.

B. both ordinal variables.

C. a numerical variable and a categorical variable respectively.

D. a nominal variable and an ordinal variable respectively.

E. an ordinal variable and a nominal variable respectively.

VCAA 2017 Exam 1 Data analysis Q7 – Adapted

Q22. Difficulty: 2 marks

Claire decides to borrow $7000 to purchase a car.

a) If she decides to borrow $7000 from the bank, she will pay interest at a rate of 4.3% per month. Calculate the interest she will pay in the first month. (1 mark)

b) If she borrows from the dealer's finance company, she will pay $8.75 in interest per month. Calculate the annual simple interest rate charged. (1 mark)

VCAA 2009 Exam 2 Module 4: Business-related mathematics Q2 a,b – Adapted

LESSON 3E

Purchasing options

The key skills you will learn in this lesson are:

1. Cash, credit and debit cards
2. Hire purchase agreements
3. Personal loans

VCAA key knowledge point:

"comparison of purchase options including cash, credit and debit cards, personal loans, and time payments (hire purchase)"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Cash, credit and debit cards

The three most common ways to pay for something are with cash or with a credit or debit card.

Cash

- Physical currency, such as notes and coins.

Debit cards

- Linked directly to a bank account.
- Allows withdrawal of cash from an automatic teller machine (ATM) and payment for goods and services by EFTPOS.

Credit cards

- Credit is another term for a loan.
- When using a credit card, purchases are made with the bank's money, which must be repaid every month by the owner.

Some shops have a minimum amount for using cards, or an added fee, due to EFTPOS costs. In contrast, some businesses offer discounts for cash purchases, because EFTPOS fees aren't required.

Different credit card providers have different rules for their use and calculation of interest. The following definitions will apply throughout this book:

- **Credit limit:** The maximum amount that can be borrowed at once.
- **Billing period:** Normally one month. At the same time each month the bank will send a statement to the account owner, which details all purchases within that period, and states when the bill needs to be paid, known as the payment date.
- **Interest-free grace period:** The period between receiving the statement and the payment date is known as the interest-free grace period. Paying back the amount owed by the payment date results in no interest being charged.
- **Interest:** If a purchase is not paid back by the payment date, interest is owed to the bank. The interest is compounded daily from when the purchase was made.

After the payment date, the amount of money owed on a purchase made through a credit card, A, is given by:

$$A = P \times \left(1 + \frac{r}{365 \times 100}\right)^n$$

where P is the purchase price, r is the interest rate per annum, and n is the number of days since the purchase was made.

Example

Kirollos' credit card has an interest rate of 20% per annum. He made a \$1200 purchase with the card on December 28, and the interest free grace period for that payment ended on January 15. If he pays it back on January 18, the amount he needs to pay is:

$r = 20$

$n =$ number of days between December 28 and January 18

$n = (31 - 28) + 18$

$n = 21$

$A = 1200 \times \left(1 + \frac{20}{365 \times 100}\right)^{21}$

$A = 1213.88$

WORKED EXAMPLE 13 *(1 mark)*

Esther's credit card has an interest rate of 15% p.a. If Esther spends $769 using the card on July 5 and pays it back on August 12, two days after the interest-free period ends, how much does she owe?

SOLUTION

Step 1 Determine if the payment was made during or after the interest-free period.

Esther paid it back after the interest-free period.

Step 2 Calculate the number of days since the purchase.

NOTE: July has 31 days.

$n = (31 - 5) + 12$

$n = 26 + 12$

$n = 38$

Step 3 Use the formula to find the amount she owes.

$A = 769 \times \left(1 + \frac{15}{365 \times 100}\right)^{38}$

$A = 781.10$

Esther owes the bank $781.10.

2. Hire purchase agreements

A hire purchase agreement involves a customer paying a deposit on an item (that is a portion of the total price), and then making small, regular payments until the item is paid off. Making payments in this way results in the customer paying more money overall. This extra money can be expressed as a flat rate of interest, r_f

Flat rate of interest

The flat rate of interest is the amount of interest paid as a percentage of the amount owed from purchase, converted to a yearly rate.

The equations to calculate the repayment agreement according to a flat rate of interest are shown in the table below:

Symbol	Definition	Equation
P_0	Original price	
t	Time period (in years)	
A	Total amount of money paid	$A = deposit + (payment \times number\ of\ payments)$
I	Total interest paid	$I = A - P_0$
P	Amount owed from purchase	$P = P_0 - deposit$
r_f	Flat rate of interest	$r_f = \frac{I}{Pt} \times 100$

Example

Hala enters a hire purchase agreement for a surfboard that sells for $750 in a shop. She pays a deposit of $200 up front, and then makes monthly payments of $120 for 6 months.

Original price is:

$P_0 = 750$

The total amount Hala pays is:

$A = 200 + (6 \times 120)$

$A = 920$

The total interest paid is:

$I = A - P_0$

$I = 920 - 750$

$I = 170$

The amount owed from purchase is:

$P = P_0 - deposit$

$P = 750 - 200$

$P = 550$

The time period needs to be given in years:

$t = 0.5$

The flat rate of interest is:

$r_f = \frac{I}{Pt} \times 100$

$r_f = \frac{170}{550 \times 0.5} \times 100$

$r_f = 61.8\%$

WORKED EXAMPLE 14 *(1 mark)*

KayMarket stocks a sofa bed with a sale price of $800. The sofa bed can be bought through a hire purchase agreement with an $80 deposit and ten monthly payments of $80.

What is the flat rate of interest charged under the hire purchase agreement?
Give your answer as a percentage to one decimal place.

SOLUTION

Step 1 Calculate the total payment for the purchase of the sofa bed.

$A = 80 + (10 \times 80)$

$A = 880$

Step 2 Calculate the amount of interest charged.

$I = A - P_0$

$I = 880 - 800$

$I = 80$

Step 3 Calculate the amount owed from the purchase (the principal).

$P = P_0 - \text{deposit}$

$P = 800 - 80$

$P = 720$

Step 4 Express the time of the agreement in years

$t = \frac{10}{12}$

$t = \frac{5}{6}$

Step 5 Substitute the values of I, P and t into the flat rate of interest formula.

$r_f = \frac{I}{Pt} \times 100$

$r_f = \frac{80}{720 \times \frac{5}{6}} \times 100$

$r_f = 13.3\%$

The flat rate of interest is 13.3%.

3. Personal loans

A personal loan is a compound interest loan of about $5000 and $50 000, often used to pay for large personal expenses such as cars and renovations.

Main features of a personal loan:

- Payments are made every period (usually a month) to pay off the loan. These payments are equal in size for each period.
- Compound interest is charged. It is calculated at the end of each period, before the payment is made. The payment made at the end of each period is always bigger than the interest that is charged. The interest rate for personal loans is usually less than it is for credit cards.
- Personal loans have a set term (duration).

A table is often used to keep track of the loan over time.

NOTE: Repayments pay for the accumulated interest of the period first. After the interest is paid off, the leftover from the repayment amount then contributes to repaying the remaining debt. Hence, the repayment each period is equal to the sum of the interest paid and the debt repaid in that period.

Example

Below are the details of the first two monthly repayments of $200 made against a monthly compounding personal loan of $10 000, with an interest rate of 12% p.a. (1% per month).

Payment number	Balance owed before payment ($)	Repayment ($)	Interest paid ($)	Debt repaid ($)	Balance owed after payment ($)
1	10 000.00	200.00	100.00	100.00	9900.00
2	9900.00	200.00	99.00	101.00	9799.00

The interest of the first month was $100 and the second was $99. The interest amount will continue to decrease each month.

Due to the complex nature of personal loans, this course will examine unusually short-term loans or only the first few repayments of longer-term loans.

WORKED EXAMPLE 15 *(2 marks)*

Bronson takes out a personal loan of $6500 at an interest rate of 14.5% p.a. compounding monthly. Bronson makes monthly repayments of $230. After making his third repayment:

a) What will be the balance of Bronson's loan?

b) How much interest will Bronson have paid?

SOLUTION

a) What will be the balance of Bronson's loan?

Step 1 Create a table, and enter the value of balance owed before the payment along with the value of payment number one.

Bronson borrowed $6500. The repayment is $230 each month.

Payment number	Balance owed before payment ($)	Repayment ($)	Interest paid ($)	Debt repaid ($)	Balance owed after payment ($)
1	6500.00	230.00			
2					
3					

Step 2 Calculate the 'interest paid' and 'debt repaid'.

This question requires the monthly interest rate, so $r = \frac{14.5}{12} = 1.2083$.

$$Interest\ paid = \frac{interest\ rate}{100} \times balance\ owed\ before\ payment$$

$$Interest\ paid = \frac{1.2083}{100} \times 6500 = 78.54$$

$$Debt\ repaid = repayment\ amount - interest\ paid$$

$$Debt\ repaid = 230 - 78.54 = 151.46$$

Payment number	Balance owed before payment ($)	Repayment ($)	Interest paid ($)	Debt repaid ($)	Balance owed after payment ($)
1	6500.00	230.00	78.54	151.46	
2					
3					

Step 3 Find the 'balance owed after payment'. Enter this value in both the 'balance owed after payment' column in the current row, and in the 'balance owed before payment' in the following row.

$$Balance\ owed\ after\ payment = balance\ owed\ before\ payment - debt\ repaid$$

$$Balance\ owed\ after\ payment = 6500 - 151.46 = 6348.54$$

Payment number	Balance owed before payment ($)	Repayment ($)	Interest paid ($)	Debt repaid ($)	Balance owed after payment ($)
1	6500.00	230.00	78.54	151.46	6348.54
2	6348.54				

Step 4 Repeat steps 2 and 3 until the third row is filled.

Payment number	Balance owed before payment ($)	Repayment ($)	Interest paid ($)	Debt repaid ($)	Balance owed after payment ($)
1	6500.00	230.00	78.54	151.46	6348.54
2	6348.54	230.00	76.71	153.29	6195.25
3	6195.25	230.00	74.86	155.14	6040.11

Step 5 Interpret:

After making his third repayment, the balance of Bronson's loan will be $6040.11.

b) How much interest will Bronson have paid?

Step 1 Add all of the values for 'interest paid'.

Total interest paid $= 78.54 + 76.71 + 74.86 = 230.11$

After making his third repayment, Bronson will have paid $230.11 in interest

Step 2 Interpret:

After making this third repayment, Bronson will have paid $230.11 in interest.

Questions 3E *Purchasing options*

Refresher question

Q1. Determine:

a) 2×0.9

b) 2×0.9^2

1. Cash, credit and debit cards

Q2. Skill

A cafe has a $0.50 surcharge on all EFTPOS transactions of less than ten dollars.

The sale price of an egg and bacon roll is $8.20.

How much would it cost to buy an egg and bacon roll using a debit card?

Q3. Skill

For each of the following, give your answer in dollars correct to the nearest cent.

a) A local bakery offers a five percent discount on purchases made with cash. The sale price of a dozen nutella doughnuts is $30.

How much would it cost to buy a dozen nutella doughnuts with cash, correct to the nearest cent?

b) A dumpling restaurant has a 1.2% surcharge on all EFTPOS transactions. The Scully family ate at this restaurant last night for dinner. Their bill was $62.50.

How much would it cost the Scully family to pay the bill using a debit card?

Q4. Application

Rosa's credit card has a ten-day interest-free grace period, and a billing period that ends on the last day of a calendar month. The card has an interest rate of 15% p.a.

On July 12 last year, Rosa purchased a vintage Australia Post bicycle for $350 using her credit card. If Rosa didn't make the repayment until 26 October, what was the total amount she had to pay for the bike? Give you answer in dollars, correct to the nearest cent.

Check your understanding

Q5. Skill

On September 1, Evelyn uses her credit card to pay an editor $4500 to help her finish her upcoming book. On November 16, long after the interest-free period finishes, Evelyn pays her credit card balance of $4657.77.

What is the annual interest rate of Evelyn's credit card?

Give your answer as a percentage correct to one decimal place.

2. Hire purchase agreements

Q6. Skill

Use the following information about a hire purchase agreement to determine the total interest the customer paid.

- Normal price: $800
- Deposit paid: $350
- Monthly repayment: $100
- Number of repayments: 7

Q7. Skill

State the total interest charged, and the flat rate of interest for the following scenarios. Round all relevant answers to two decimal places.

a) Frankie buys a bassoon through a hire purchase agreement from Would Wind Sailing and Music Supplies. The bassoon usually costs $4200. Under the agreement, Frankie pays a deposit of $420 and makes ten monthly repayments of $399.

b) Iris needs to purchase a new screen printing press to start up her custom t-shirt business. The model she wants to buy costs $1600. Iris chooses to buy the screen printing press through a hire purchase agreement. She pays a deposit of $200 and makes 15 fortnightly repayments of $110.

Check your understanding

Q8. Application

Two hire purchase agreements of the same item both result in the same amount of interest being paid overall. One has a term of nine months. The other has a term of 18 months.

Are the flat rates of interest for each agreement the same, or different? If they are different, which rate is higher?

Q9. Application

Mei has entered a hire purchase agreement to buy a road bike, normally valued at $2500. She pays a deposit of $750, and makes monthly payments of $320. If the flat interest rate on the bike is 6.8%, how many payments does the agreement last for?

3. Personal loans

Q10. Skill

Carl is trying to work out what the balance of his loan will be after interest is added for this month. The current balance of the loan is $75 000, and the interest is 8% per annum.

Fill in the missing numbers in his working out, rounding answers to the nearest cent.

interest rate = 8% per annum

$$\text{interest rate per month} = \frac{8}{12}$$

$$= 0.67\%$$

balance owed = 75 000

$$\text{interest} = \frac{\text{interest rate}}{100} \times \text{balance}$$

$$\text{interest} = \frac{0.67}{100} \times 75\,000$$

interest = ☐

balance after interest = balance owed + interest

balance after interest = ☐

Q11. Skill

The table below details the the first four monthly repayments of $600 made against a personal loan of $15 000 with an interest rate of 11% per annum, compounding monthly.

Complete the blank entries in the table below.

Payment number	Balance owed before payment ($)	Repayment ($)	Interest paid ($)	Debt repaid ($)	Balance owed after payment ($)
1	15 000.00	600.00	137.50	462.50	14 537.50
2	14 537.50	600.00		466.74	
3	14 070.76		128.98		13 599.74
4		600.00			13 124.40

Q12. Skill

For each of the following problems, give your answer in dollars correct to the nearest cent.

a) Madeline takes out a personal loan for $10 000 to help Miss Clavel pay for renovations to her house in Paris. The loan charges interest at a rate of 12.6% p.a. compounding monthly. Madeline makes monthly repayments of $500.

What is the balance of the loan after Madeline makes three repayments, correct to the nearest cent?

b) Hans takes out a personal loan of $7500. The loan charges interest at a rate of 15% p.a. compounding monthly. Hans makes monthly repayments of $450.

What is the balance of the loan after Hans makes five repayments, correct to the nearest cent?

Check your understanding

Q13. Skill

Which of the following statements about personal loans is **not** true?

A. Personal loans charge compound interest.

B. Repayments must be greater than the interest charged in order to pay off the loan.

C. The interest is calculated for the period after the repayment has been made.

D. A personal loan is generally between $5000 and $50 000.

E. The term of a loan can be reduced by increasing repayment amount.

Joining it all together

Q14. Skill 1 mark

Which of the following is an advantage of a credit card?

A. Low interest rate

B. No credit limit

C. Fixed payments

D. Interest-free grace period

Q15. Skill 1 mark

Which of the following is an advantage of a personal loan?

A. Fixed repayments

B. Interest rates generally lower than credit card

C. More popular than hire purchase agreements

D. Requires a deposit

Q16.

Application
3 marks

Gordon needs to buy a new oven for his restaurant. The model he wants costs $6500.

His favourite cookware store currently has the model available through a hire purchase agreement with no deposit, but 7 monthly payments of $975.

Gordon's friend, Nigella, offers him a personal loan of $6500 at an interest rate of 15% p.a. compounding monthly. This loan is to be repaid in six months with equal monthly repayments of $1131.22.

Gordon wants to spend the least amount of money possible.

a) Which option should Gordon choose? (2 marks)

b) How much will he save by choosing this option? (1 mark)

VCAA question

Q17.

1 mark

Nathan bought a $2500 bedroom suite on a contract that involves no deposit and an interest-free loan for period of 48 months.

He has to pay an initial set-up fee of $25.

In addition, he pays an administration fee of $3.95 per month.

The total amount that nathan will have to pay in fees for the entire 48 months, as a percentage of the original price of $2500, is closest to

A. 1.6%

B. 4.0%

C. 7.6%

D. 8.5%

E. 8.6%

VCAA 2011 Exam 1, Module 4: Business-related mathematics. Q4

Questions from multiple lessons

Q18.

Difficulty:

1 mark

Jackson invests $12 000 for 200 days. If his interest compounds daily at a rate of 4.5% per annum, what is his investment worth after 200 days? Round your answer to the nearest dollar.

A. $80 821 398

B. $79 880 235

C. $25 368

D. $12 300

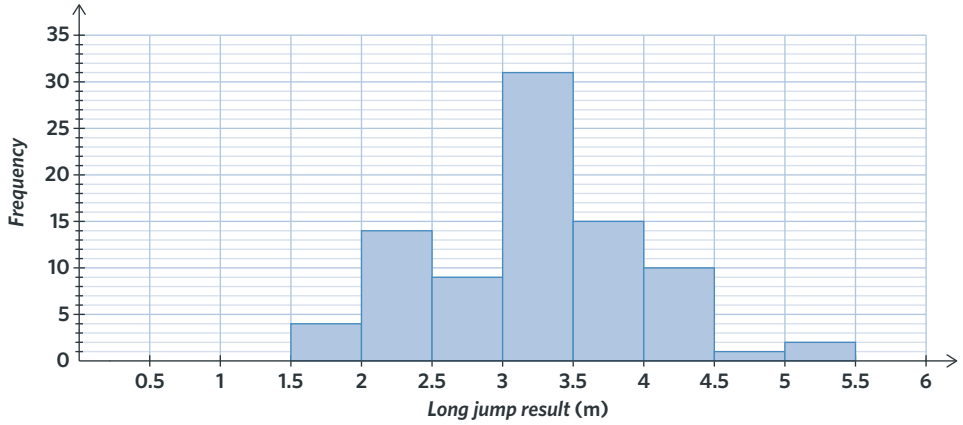
E. $12 296

VCAA 2014 Exam 1 Module 4: Business-related mathematics Q3 – Adapted

Q19. The following histogram displays the long jump results of a year level of 86 students.

Difficulty:

Year 10
1 mark



The interquartile range for this distribution is closest to

A. 0.5 m

B. 0.75 m

C. 1 m

D. 1.25 m

E. 1.5 m

VCAA 2018NH Exam 1 Data analysis Q4 – Adapted

Q20. A small emergency fund of $14 000 was invested in an account for eight years.

Difficulty:

After eight years of earning interest, the value of this investment was $15 783.77.

2 marks

a) How much interest was earned during the eight years of this investment? (1 mark)

b) Interest on the account had been calculated and paid monthly. What was the annual rate of interest for this investment? Write your answer correct to one decimal place. (1 mark)

VCAA 2014 Exam 2 Module 4: Business-related mathematics Q3 a,b – Adapted

AOS 3: Discrete mathematics

Matrices

CHAPTER
4

LESSON 4A

Introduction to matrices

The key skills you will learn in this lesson are:

1. Identifying matrix properties
2. Displaying information with matrices

VCAA key knowledge points:

"use of matrices to store and display information that can be presented in a rectangular array of rows and columns such as databases and links in social and road networks"

"types of matrices (row, column, square, zero and identity) and the order of a matrix"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Identifying matrix properties

A matrix is a rectangular array of numbers arranged into rows and columns.

Various features of matrices are described below.

Example

Consider the matrix A:

$$A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 9 & 6 & 7 & 8 \end{bmatrix}$$

Rows and columns

Rows are horizontal and are numbered from top to bottom.

Rows

$$\begin{matrix} 1 \\ 2 \end{matrix} \begin{bmatrix} 4 & 2 & 1 & 3 \\ 9 & 6 & 7 & 8 \end{bmatrix}$$

Columns are vertical and are numbered from left to right.

Columns

$$\begin{matrix} & 1 & 2 & 3 & 4 \end{matrix}$$
$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 9 & 6 & 7 & 8 \end{bmatrix}$$

Matrix A has 2 rows and 4 columns.

Order

The size of a matrix is known as its order. It is written in the form:

number of rows × *number of columns*

The order of matrix A is 2×4. It is called a 'two-by-four' matrix.

Element

Each number in a matrix is called an element of that matrix.

The element in row i and column j of matrix A can be written as a_{ij}

Element a_{23} in matrix A is 7. It is in the second row and the third column.

$$\begin{matrix} & & 3 & \\ 2 & \end{matrix} \begin{bmatrix} 4 & 2 & 1 & 3 \\ 9 & 6 & 7 & 8 \end{bmatrix}$$

Special types of matrices

Matrix type	Description	Examples
Row matrix	A row matrix is a matrix with one row and any number of columns.	$\begin{bmatrix}10 & 5\end{bmatrix}$ and $\begin{bmatrix}4 & 4 & 8 & 8 & 3\end{bmatrix}$
Column matrix	A column matrix is a matrix with one column and any number of rows.	$\begin{bmatrix}2\\14\\6\\9\end{bmatrix}$ and $\begin{bmatrix}1\\1\end{bmatrix}$
Square matrix	A square matrix is a matrix where the number of rows is equal to the number of columns.	$\begin{bmatrix}5.2 & 1\\9 & 16\end{bmatrix}$ and $\begin{bmatrix}1 & 2 & 3 & 4\\5 & 6 & 7 & 8\\9 & 10 & 11 & 12\\13 & 14 & 15 & 16\end{bmatrix}$
Zero matrix	A zero matrix is a matrix of any size in which all elements are 0.	$\begin{bmatrix}0\end{bmatrix}$ and $\begin{bmatrix}0 & 0\\0 & 0\\0 & 0\end{bmatrix}$
Identity matrix	An identity matrix is a square matrix in which all elements on the leading diagonal are 1 and the rest of the elements are 0. The leading diagonal of a matrix is the diagonal line from the top left corner of the matrix to the bottom right corner.	$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$ and $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

WORKED EXAMPLE 1 *(2 marks)*

Consider the matrix $B = \begin{bmatrix}9 & 8\\2 & 1\\6 & 3\\4 & 7\end{bmatrix}$.

a) What is the order of matrix B?

b) What is element b_{21}?

SOLUTION

a) What is the order of matrix B?

Step 1 To find order, first count the rows.

1 $\begin{bmatrix}9 & 8\end{bmatrix}$
2 $\begin{bmatrix}2 & 1\end{bmatrix}$
3 $\begin{bmatrix}6 & 3\end{bmatrix}$
4 $\begin{bmatrix}4 & 7\end{bmatrix}$

There are four rows.

Step 2 Count the columns.

1 2

$\begin{bmatrix}9 & 8\\2 & 1\\6 & 3\\4 & 7\end{bmatrix}$

There are two columns.

Step 3 Write down the order.

The order is written as
number of rows × *number of columns*.

∴ The order of matrix B is 4×2.

b) What is element b_{21}?

Step 1 For element b_{21}, locate row 2.

$\begin{bmatrix}9 & 8\\2 & 1\\6 & 3\\4 & 7\end{bmatrix}$ 2

Step 2 Locate column 1.

1

$\begin{bmatrix}9 & 8\\2 & 1\\6 & 3\\4 & 7\end{bmatrix}$ 2

Step 3 Write down the element.

Element b_{21} is 2.

WORKED EXAMPLE 2 *(1 mark)*

Consider the matrix $X = \begin{bmatrix} 7 & 4 & 1 & 2 \\ 9 & 6 & 3 & 0 \end{bmatrix}$.

Store matrix X in your calculator and use it to find x_{14}.

SOLUTION: TI-NSPIRE

Step 1 Open a new page by pressing ctrl + doc, then press 1 ('Add Calculator').

Step 2 Press and use the arrows to find the matrix icon. Press enter.

Set the number of rows to 2 and the number of columns to 4. Press enter.

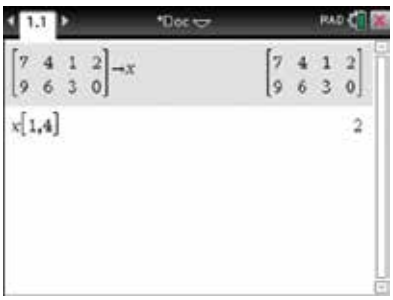
Step 3 Enter the elements of the matrix into each box, using tab to go between boxes, or using the cursor to select elements.

Once the matrix is complete, move the cursor to the right of the matrix and then press ctrl + var then type 'x' to store the matrix as x.

Press enter.

Step 4 To display element x_{14}, type 'x[1,4]' and press enter.

The answer will appear.



SOLUTION: CLASSPAD

Step 1 From the menu, tap Main and press then 'Keyboard'.

Step 2 Tap Math2, tap, then tap twice to create a 2 × 4 matrix.

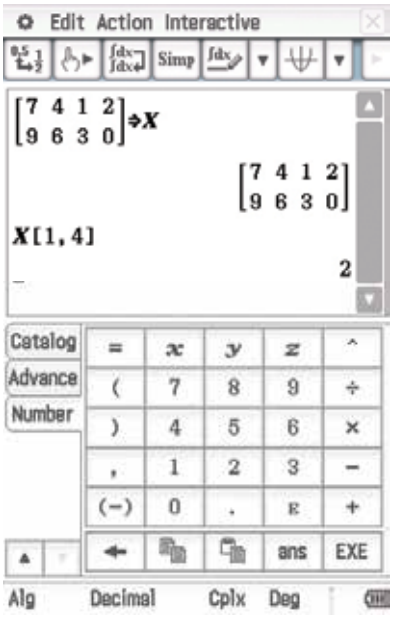
Step 3 Enter the corresponding elements of the matrix into each box, using the stylus to go between boxes.

Once the matrix is complete, click the right side of the matrix, tap and type 'X'.

Press EXE to store the matrix as X.

Step 4 To locate x_{14}, type 'X[1,4]' (square brackets are in Math3) and press EXE.

Element x_{14} will appear.



2. Displaying information with matrices

Matrices can be used to display many types of numerical information, such as the results of a survey, or sales data.

Example

The following table shows the number of milkshakes, coffees and doughnuts sold at a cafe on Monday and Tuesday.

	Monday	Tuesday
Milkshakes	17	12
Coffees	34	38
Doughnuts	19	22

This information can also be displayed in matrix form:

$$\begin{matrix} & \text{M} & \text{T} \\ \text{M} \\ \text{C} \\ \text{D} \end{matrix} \begin{bmatrix} 17 & 12 \\ 34 & 38 \\ 19 & 22 \end{bmatrix}$$

WORKED EXAMPLE 3 *(5 marks)*

The favourite sports of Year 11 and Year 12 students at Edrolo High are shown in matrix M below.

$$\text{M} = \begin{matrix} & \text{Year 11} & \text{Year 12} \\ \text{Soccer} \\ \text{Basketball} \\ \text{Netball} \end{matrix} \begin{bmatrix} 32 & 45 \\ 53 & 29 \\ 16 & 21 \end{bmatrix}$$

a) What information does element m_{32} represent? *(1 mark)*

b) How many Year 12 students are there at Edrolo High? *(1 mark)*

c) What is the most popular sport amongst Year 11 students? *(1 mark)*

d) Create a column matrix showing the total number of students that prefer each sport.

Hence, find the most popular sport overall. *(2 marks)*

SOLUTION

a) What information does element m_{32} represent?

Step 1 Locate row 3.

$$\begin{matrix} & \text{Year 11} & \text{Year 12} \\ \text{Soccer} \\ \text{Basketball} \\ \text{Netball} \end{matrix} \begin{bmatrix} 32 & 45 \\ 53 & 29 \\ 16 & 21 \end{bmatrix}$$

Row 3 is the row labelled 'Netball'.

Step 2 Locate column 2.

$$\begin{matrix} & \text{Year 11} & \text{Year 12} \\ \text{Soccer} \\ \text{Basketball} \\ \text{Netball} \end{matrix} \begin{bmatrix} 32 & 45 \\ 53 & 29 \\ 16 & 21 \end{bmatrix}$$

Column 2 is the column labelled 'Year 12'.

Step 3 Interpret the element.

The element m_{32} is 21. It represents the 21 Year 12 students who chose netball as their favourite sport.

b) How many Year 12 students are there at Edrolo High?

Step 1 Locate the Year 12 column.

$$\begin{matrix} & \text{Year 11} & \text{Year 12} \\ \text{Soccer} \\ \text{Basketball} \\ \text{Netball} \end{matrix} \begin{bmatrix} 32 & 45 \\ 53 & 29 \\ 16 & 21 \end{bmatrix}$$

Step 2 Find the sum of the values in the Year 12 column.

$45 + 29 + 21 = 95$

There are 95 students in Year 12.

c) What is the most popular sport amongst Year 11 students?

Step 1 Locate the Year 11 column.

$$\begin{matrix} \\ \text{Soccer} \\ \text{Basketball} \\ \text{Netball} \end{matrix} \begin{matrix} \begin{matrix} \text{Year 11} & \text{Year 12} \end{matrix} \\ \begin{bmatrix} 32 & 45 \\ 53 & 29 \\ 16 & 21 \end{bmatrix} \end{matrix}$$

Step 2 Find the highest value in the Year 11 column.

53 is the highest value.

This is in the Basketball row.

∴ Basketball is the most popular sport amongst Year 11 students.

d) Create a column matrix showing the total number of students that prefer each sport. Hence, find the most popular sport overall.

Step 1 Create and label a 3 × 1 matrix.

$$\begin{matrix} \\ \text{Soccer} \\ \text{Basketball} \\ \text{Netball} \end{matrix} \begin{matrix} \text{Total} \\ \begin{bmatrix} \\ \\ \\ \end{bmatrix} \end{matrix}$$

Step 2 Add the Year 11 and Year 12 values for each sport.

$$\begin{matrix} \\ \text{Soccer} \\ \text{Basketball} \\ \text{Netball} \end{matrix} \begin{matrix} \text{Total} \\ \begin{bmatrix} 32 + 45 \\ 53 + 29 \\ 16 + 21 \end{bmatrix} \end{matrix} = \begin{matrix} \\ \text{Soccer} \\ \text{Basketball} \\ \text{Netball} \end{matrix} \begin{bmatrix} 77 \\ 82 \\ 37 \end{bmatrix}$$

✓ *1 mark for correct column matrix*

Step 3 Find the highest value.

The highest value is 82. This is in the Basketball row.

∴ Basketball is the most popular sport.

✓ *1 mark for writing most popular sport*

Questions 4A *Introduction to matrices*

Refresher question

Q1. How many elements are in a 2 × 2 matrix?

1. Identifying matrix properties

Q2. Which of the following is a 3 × 4 matrix?

Skill

A. $\begin{bmatrix} 4 & 3 & 3 & 4 & 3 & 3 \\ 3 & 4 & 3 & 3 & 4 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 7 & 8 \\ 2 & 1 & 3 \\ 6 & 2 & 9 \\ 5 & 4 & 8 \end{bmatrix}$

C. $\begin{bmatrix} 7 & 6 & 5 & 4 \\ 0 & 2 & 8 & 1 \\ 9 & 5 & 6 & 3 \end{bmatrix}$

D. [12]

Q3. For matrix M below, write each of the following elements.

Skill

$$\begin{bmatrix} 12 & 3 & 2 & 1 \\ 7 & 9 & 16 & 5 \\ 10 & 11 & 4 & 7 \\ 6 & 8 & 18 & 0 \end{bmatrix}$$

a) m_{14}

b) m_{34}

c) m_{23}

d) m_{42}

Q4. Write the order of each of the following matrices.

Skill

a) $\begin{bmatrix} 2 & 1 & 44 & 6 & 17 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 4 \\ 15 & 2 \\ 6 & 11 \end{bmatrix}$

c) $\begin{bmatrix} 6 \\ -4 \end{bmatrix}$

d) $\begin{bmatrix} 2 & 33 & 15 & -3 \\ 19 & 6 & 7 & 8 \\ 4 & -55 & 71 & 5 \end{bmatrix}$

Q5. Consider matrix N.

Skill

$$N = \begin{bmatrix} 6 & -9 & 15 \\ 4 & 2 & 11 \\ 3 & 10 & 7 \\ -1 & 8 & -4 \end{bmatrix}$$

a) What is the order of matrix N?

b) Find element n_{32}.

c) Find the sum of column 3.

Check your understanding

Q6. Create the following matrices:

Skill

a) A zero row matrix containing 7 elements.

b) An identity matrix with 3 rows.

Q7. $A = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$ $B = [0]$

Skill

$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$E = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $F = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$G = [1]$

Write down which of the above matrices are:

a) zero matrices

b) row matrices

c) column matrices

d) square matrices

e) identity matrices

2. Displaying information with matrices

Q8. The table below shows the amount of milk (M), flour (F), and sugar (S) needed to make pancakes (P) and cupcakes (C). All measurements are in cups.

Application

	Pancakes	Cupcakes
Milk	2	1.5
Flour	1.5	2
Sugar	0.5	1

Fill in the matrix below to display this information.

$$\begin{array}{c} \\ M \\ F \\ S \end{array}\begin{array}{c} \begin{array}{cc} P & C \end{array} \\ \begin{bmatrix} & \\ & \\ & \end{bmatrix} \end{array}$$

Q9. Application

The following table displays the highest, lowest, and median marks awarded for a maths test in four Year 11 classes.

	Highest (%)	Lowest (%)	Median (%)
Class A	97	34	71
Class B	89	40	68
Class C	100	29	65
Class D	94	42	70

Which of the following matrices does not display information that is in the table?

A.
$$\begin{array}{c} \\ \text{Highest} \\ \text{Lowest} \\ \text{Median} \end{array}\begin{array}{c} \begin{array}{c} C \end{array} \\ \begin{bmatrix} 100 \\ 29 \\ 65 \end{bmatrix} \end{array}$$

B.
$$\begin{array}{c} \\ \text{Lowest} \\ \text{Highest} \\ \text{Median} \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 34 & 40 & 29 & 42 \\ 97 & 89 & 100 & 94 \\ 71 & 68 & 65 & 70 \end{bmatrix} \end{array}$$

C.
$$\begin{array}{c} \text{Median} \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 71 & 68 & 65 & 70 \end{bmatrix} \end{array}$$

D.
$$\begin{array}{c} \text{Highest} \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 94 & 100 & 89 & 97 \end{bmatrix} \end{array}$$

Check your understanding

Q10. Application

Alex has three cats and two dogs. Bella has one cat and one dog. Catherine has two cats and no dogs. Display this information in a 2×3 matrix.

Joining it all together

Q11. Application 4 marks

A group of children were asked their favourite flavour of ice cream between vanilla, strawberry, and mint choc chip. The results are shown below.

Gender	Vanilla	Strawberry	Mint choc chip
Boys	9	12	21
Girls	15	11	17

a) Display this information in a 2×3 matrix.

b) Create a column matrix showing the number of boys and girls that chose mint choc chip.

c) Create a row matrix showing the ice cream choices of the boys only.

d) Find the sum of the elements in the row matrix in part c) and interpret the result.

Q12. Application 5 marks

Three media classes were asked in a survey what their favourite TV show was out of Friends, Grey's Anatomy, and RuPaul's Drag Race (F, G, R respectively). Matrix X displays the results of the survey separated into classes A, B, and C.

$$X = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \end{array} \begin{array}{c} \begin{array}{ccc} \text{F} & \text{G} & \text{R} \end{array} \\ \begin{bmatrix} 7 & 7 & 12 \\ 8 & 9 & 10 \\ 8 & 11 & 6 \end{bmatrix} \end{array}$$

a) How many people in Class A chose Friends?

b) What information does x_{23} represent?

c) Which element represents the number of people in Class C that chose Grey's Anatomy?

d) How many people are in each of the three classes?

e) What is the most popular TV show overall?

Q13. Application 2 marks

India likes to plan her dinners so she doesn't have to eat the same thing two nights in a row. She uses the matrix below to decide what she will have for dinner based on what she had last night. The four meals she can make are pasta (P), risotto (R), stir-fry (S), and casserole (C).

$$\text{Tonight} \begin{array}{c} \\ \text{P} \\ \text{R} \\ \text{S} \\ \text{C} \end{array} \begin{array}{c} \text{Last night} \\ \begin{array}{cccc} \text{P} & \text{R} & \text{S} & \text{C} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

a) If India has stir-fry on Tuesday, what will she have on Wednesday, Thursday and Friday?

b) India's friend, Jason, has recently started planning his dinners the same way, but using a different matrix. Below are his meals for the past week.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Stir-fry	Casserole	Risotto	Pasta	Stir-fry	Casserole	Risotto

Fill in the matrix below based on Jason's meal choices.

$$\text{Tonight} \begin{array}{c} \\ \text{P} \\ \text{R} \\ \text{S} \\ \text{C} \end{array} \begin{array}{c} \text{Last night} \\ \begin{array}{cccc} \text{P} & \text{R} & \text{S} & \text{C} \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

VCAA question

Q14. 1 mark

The order of matrix X is 3×2. The elements of matrix X are given below.

$x_{11} = 1 + 1$ $\quad$ $x_{12} = 1 + 2$

$x_{21} = 2 + 1$ $\quad$ $x_{22} = 2 + 2$

$x_{31} = 3 + 1$ $\quad$ $x_{32} = 3 + 2$

The matrix X is

A. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ **B.** $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}$ **C.** $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ **D.** $\begin{bmatrix} 1 & 2 \\ 3 & 3 \\ 4 & 4 \end{bmatrix}$ **E.** $\begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \end{bmatrix}$

Adapted from VCAA 2014 Exam 1, Module 6: Matrices. Q6

Questions from multiple lessons

Q15.

Difficulty:

1 mark

The number of music festivals (F), concerts (C) and gigs (G) that Heidi (H), Ingrid (I), Joey (J), Kathy (K) and Leo (L) attended last summer is given in matrix M.

$$M = \begin{matrix} \\ \text{F} \\ \text{C} \\ \text{G} \end{matrix} \begin{matrix} \begin{matrix} \text{H} & \text{I} & \text{J} & \text{K} & \text{L} \end{matrix} \\ \begin{bmatrix} 3 & 2 & 0 & 0 & 6 \\ 1 & 5 & 4 & 7 & 0 \\ 8 & 3 & 2 & 1 & 8 \end{bmatrix} \end{matrix}$$

The element in row i and column j in matrix M is m_{ij}.

The element m_{23} is the number of

A. gigs Joey attended last summer

B. gigs Ingrid attended last summer

C. concerts Kathy attended last summer

D. concerts Joey attended last summer

E. concerts Ingrid attended last summer

VCAA 2015 Exam 1 Module 6: Matrices Q1 – Adapted

Q16.

Difficulty:

1 mark

Research is being conducted into trees in Victorian forests. The focus is on two variables:

- *Height* (*metres*) (less than 20, 20-40, more than 40)
- *Type of tree* (Eucalpyt, Mountain Ash, Redwood)

These variables are:

A. a numerical variable and a categorical variable respectively

B. an ordinal variable and nominal variable respectively

C. both ordinal variables

D. a nominal variable and ordinal variable respectively

E. both nominal variables

VCAA 2017 Exam 1 Data analysis Q7 – Adapted

Q17.

Difficulty:

3 marks

Cadrolo (C), Edlindt (E), and Ferred Rocholo (F) are three popular brands of chocolate. Woolworths (W) and IGA (I) are supermarkets that sell all three brands of chocolate.

The cost, in dollars, for one block of each of these brands of chocolate, in each supermarket, is shown in matrix B below.

$$B = \begin{matrix} \\ \text{C} \\ \text{E} \\ \text{F} \end{matrix} \begin{matrix} \begin{matrix} \text{W} & \text{I} \end{matrix} \\ \begin{bmatrix} 2.59 & 2.39 \\ 5.99 & 6.25 \\ 3.89 & 3.99 \end{bmatrix} \end{matrix}$$

a) What is the cost of one block of Edlindt at Woolworths? (1 mark)

b) Write down the order of matrix B (1 mark)

c) What does the element b_{12} represent? (1 mark)

VCAA Exam 2 Module 1: Matrices Q1 – Adapted

LESSON 4B

Addition and subtraction of matrices

The key skills you will learn in this lesson are:

1. Adding and subtracting matrices
2. Finding the unknown element

VCAA key knowledge point:

"matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using technology as applicable"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Adding and subtracting matrices

When adding matrices, we add the elements in the same position of each matrix.

When subtracting matrices, we subtract the elements in the same position of each matrix.

Matrices must be of the same order to be added together or subtracted from each other.

If the matrices in an addition or subtraction operation are of the same order, the expression is **defined**. If the matrices are not of the same order, the expression is **not defined**.

WORKED EXAMPLE 4 *(2 marks)*

If $A = \begin{bmatrix} 2 & 0 \\ 6 & 4 \\ 1 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 3 \\ 2 & 9 \\ 4 & 2 \end{bmatrix}$ find:

a) $A + B$

b) $A - B$

SOLUTION

a) $A + B$

Step 1 Write the calculation.

$$\begin{bmatrix} 2 & 0 \\ 6 & 4 \\ 1 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 3 \\ 2 & 9 \\ 4 & 2 \end{bmatrix}$$

Step 2 Add the elements which are in the same position in the two matrices.

$$\begin{bmatrix} 2+8 & 0+3 \\ 6+2 & 4+9 \\ 1+4 & 7+2 \end{bmatrix}$$

Step 3 Perform the operations to find the answer.

$$\begin{bmatrix} 10 & 3 \\ 8 & 13 \\ 5 & 9 \end{bmatrix}$$

b) $A - B$

Step 1 Write the calculation.

$$\begin{bmatrix} 2 & 0 \\ 6 & 4 \\ 1 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 3 \\ 2 & 9 \\ 4 & 2 \end{bmatrix}$$

Step 2 Subtract the elements which are in the same position in the two matrices.

$$\begin{bmatrix} 2-8 & 0-3 \\ 6-2 & 4-9 \\ 1-4 & 7-2 \end{bmatrix}$$

Step 3 Perform the operations to find the answer.

$$\begin{bmatrix} -6 & -3 \\ 4 & -5 \\ -3 & 5 \end{bmatrix}$$

2. Finding the unknown element

To find the value of an unknown element in a matrix we must find the position of the element in the matrix. The value of the unknown element can be found by identifying the elements in the same position, and writing an equation which relates these elements.

WORKED EXAMPLE 5 *(1 mark)*

What is the value of a?

$$\begin{bmatrix} 1 & a \\ -3 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 2 & 15 \end{bmatrix}$$

SOLUTION

Step 1 Find the position of element a.

a is in the first row and the second column.

$$\begin{bmatrix} 1 & (a) \\ -3 & 7 \end{bmatrix} + \begin{bmatrix} 6 & (3) \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 7 & (1) \\ 2 & 15 \end{bmatrix}$$

Step 2 Find all the elements in the same position – the first row and the second column.

Step 3 Write an equation.

$a + 3 = 1$

Step 4 Solve the equation algebraically or by using the solve function on your CAS to find the value of a.

$a = -2$

Questions 4B *Addition and subtraction of matrices*

Refresher question

Q1. $x + 6 = 2$. What is the value of x?

1. Adding and subtracting matrices

Q2. Skill

If $A = \begin{bmatrix} -5 & 8 \\ 3 & 10 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 9 \\ 11 & 7 \end{bmatrix}$, $C = \begin{bmatrix} -4 & 5 \\ -6 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 0 & -7 \\ -2 & 12 \end{bmatrix}$, find the following.

a) $A + B$

b) $B - C$

c) $C - A + D$

Check your understanding

Q3. Skill

$A = \begin{bmatrix} 3 & -7 & 12 \\ 0 & 16 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -1 \\ -3 & 8 \\ 9 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -14 & 2 \\ 10 & 11 \\ -8 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 6 & -16 \\ 3 & 4 & 7 \\ -8 & 17 & 9 \end{bmatrix}$, $E = \begin{bmatrix} 5 & 10 \\ -6 & 7 \end{bmatrix}$

$F = \begin{bmatrix} 28 & 3 & -7 \\ 7 & 15 & 0 \end{bmatrix}$ and $G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, which of the following is **not** defined?

Choose all answers that apply.

A. $A + B$ **B.** $B - C$ **C.** $E + G$

D. $F + A$ **E.** $D - C$

Q4. Application

The matrices below show the number of small and large blueberry, banana and chocolate muffins sold at two stores in a day. Construct a matrix that shows the total number of each type of muffin sold that day across the two stores.

	Blueberry	Banana	Chocolate
Small	15	11	17
Large	21	16	19

Store 1

	Blueberry	Banana	Chocolate
Small	19	20	22
Large	25	23	27

Store 2

2. Finding the unknown element

Q5. Skill

If $\begin{bmatrix} 0 & 7 \\ 17 & 9 \end{bmatrix} + \begin{bmatrix} 3 & -5 \\ -8 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ x & 13 \end{bmatrix}$, what is the value of x?

Q6. Skill

Find the unknown values.

a) $\begin{bmatrix} 6 & a & 1 \\ 9 & 0 & -8 \end{bmatrix} + \begin{bmatrix} 13 & 9 & 11 \\ 2 & 4 & -b \end{bmatrix} = \begin{bmatrix} 19 & 16 & 12 \\ 11 & 4 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 7 \\ -c \\ 14 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 5 \\ d \\ 11 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 6 \\ -8 \end{bmatrix}$

c) $\begin{bmatrix} 16 & -7 & 3 \\ -5 & 13 & 8 \\ 6 & 10 & e \end{bmatrix} + \begin{bmatrix} 7 & -2 & g \\ f & -3 & 12 \\ 16 & 0 & -7 \end{bmatrix} = \begin{bmatrix} 9 & -5 & 5 \\ -6 & 16 & -4 \\ -10 & 10 & -7 \end{bmatrix}$

Check your understanding

Q7. Application

A student was conducting a survey about the types of pets owned by students in two classes. He recorded his data in the matrices on the right, but some of the data got smudged out. What are the missing values?

Class 1		Class 2		Total	
7	Dog	6	Dog	c	Dog
5	Cat	7	Cat	12	Cat
a	Rabbit	2	Rabbit	4	Rabbit
0	Guinea Pig	3	Guinea Pig	3	Guinea Pig
1	Bird	2	Bird	3	Bird
10	None	b	None	16	None

Joining it all together

Q8. Skill, 1 mark

What is the missing matrix?

$\begin{bmatrix} 6 & 0 \\ -9 & 3 \\ 2 & 8 \end{bmatrix} - ? = \begin{bmatrix} 3 & 7 \\ -4 & 5 \\ -6 & 4 \end{bmatrix}$

Q9. Application, 1 mark

The total number of students who have brown, blue, hazel and green eyes in three Year 11 classes is represented in the matrix below. Which could **not** have been the number of brown, blue, hazel and green eyes in the three individual classes?

37	Brown
18	Blue
11	Hazel
3	Green

A.

Class 11A:		Class 11B:		Class 11C:	
10	Brown	13	Brown	14	Brown
9	Blue	5	Blue	4	Blue
3	Hazel	3	Hazel	5	Hazel
1	Green	2	Green	0	Green

B.

Class 11A:		Class 11B:		Class 11C:	
12	Brown	13	Brown	12	Brown
6	Blue	7	Blue	5	Blue
2	Hazel	3	Hazel	6	Hazel
1	Green	1	Green	1	Green

C.

Class 11A:		Class 11B:		Class 11C:	
13	Brown	10	Brown	14	Brown
7	Blue	9	Blue	2	Blue
3	Hazel	3	Hazel	5	Hazel
1	Green	1	Green	1	Green

D.

Class 11A:		Class 11B:		Class 11C:	
14	Brown	11	Brown	12	Brown
2	Blue	10	Blue	6	Blue
5	Hazel	5	Hazel	2	Hazel
1	Green	1	Green	1	Green

Q10. Application 2 marks

The weight (kg) and body fat percentage (%) of four people were recorded before and after taking part in an exercise program.

$$\text{Before: } \begin{matrix} \text{Alex} & \text{Pauline} & \text{Zac} & \text{Kelly} \\ \begin{bmatrix} 77 & 56 & 72 & 61 \\ 24 & 22 & 18 & 27 \end{bmatrix} & \begin{matrix} \text{Weight (kg)} \\ \text{Body fat (\%)} \end{matrix} \end{matrix} \qquad \text{After: } \begin{matrix} \text{Alex} & \text{Pauline} & \text{Zac} & \text{Kelly} \\ \begin{bmatrix} 76 & 57 & 74 & 59 \\ 19 & 20 & 16 & 24 \end{bmatrix} & \begin{matrix} \text{Weight (kg)} \\ \text{Body fat (\%)} \end{matrix} \end{matrix}$$

a) Calculate the matrix which shows the change in each person's weight and body fat percentage after taking part in the exercise program.

b) Who lost the most weight?

VCAA question

Q11. 1 mark

If $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ 8 & d \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 8 & 11 \end{bmatrix}$ then d is equal to

A. -11 **B.** -10 **C.** 7

D. 10 **E.** 11

VCAA 2008 Exam 1, Module 6: Matrices. Q1

Questions from multiple lessons

Q12. Difficulty: 1 mark

Jane recently started working at a grocery store on weekends. The number of hours she works on Saturday and Sunday over three consecutive weeks (W1, W2 and W3) is shown in the matrix below.

$$\begin{matrix} & \text{W1} & \text{W2} & \text{W3} \\ \text{Saturday} & \begin{bmatrix} 4 & 7 & 5 \\ 5 & 7 & 8 \end{bmatrix} \\ \text{Sunday} & \end{matrix}$$

How many hours did Jane work on Saturday in week 3?

A. 7 **B.** 5 **C.** 8

D. 4 **E.** 13

VCAA 2017 Exam 1 Module 1: Matrices Q1 – Adapted

Q13. Difficulty: 1 mark

Find the first five terms of the recurrence relation below.

$B_0 = -1, \quad B_{n+1} = -3B_n + 4$

A. $-1, 3, 7, 11, 15...$ **B.** $-1, 7, -17, 55, -161...$ **C.** $-1, 1, 7, 25, 79...$

D. $7, -17, 55, -161, 487...$ **E.** $-1, 3, -9, 27, -81...$

VCAA 2016 Exam 1 Recursion and financial modelling Q17 – Adapted

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Q14. Difficulty: 3 marks

The number of child (C), student (S) and adult (A) tickets sold for a particular movie is shown in the matrix N below.

$$N = \begin{matrix} \begin{bmatrix} 36 \\ 34 \\ 49 \end{bmatrix} & \begin{matrix} \text{C} \\ \text{S} \\ \text{A} \end{matrix} \end{matrix}$$

a) What is the order of matrix N? (1 mark)

b) What is the element $n_{3\,1}$? (1 mark)

c) What is the sum of the elements in the column matrix, and what does it represent? (1 mark)

VCAA 2016 Exam 2 Module 1: Matrices Q1 – Adapted

LESSON 4C

Multiplication and matrices 1

The key skills you will learn in this lesson are:

1. Scalar multiplication
2. Addition and subtraction with scalar multiplication

VCAA key knowledge point:

"matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using technology as applicable"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Scalar multiplication

There are two types of multiplication with matrices: scalar multiplication and the matrix product.

Scalar multiplication is the multiplication of a matrix by a number.

This number is known as a scalar, because it 'scales' the values inside the matrix.

To calculate the answer, multiply each element in the matrix by the scalar.

WORKED EXAMPLE 6 *(1 mark)*

If $A = \begin{bmatrix} 6 & -7 \\ 0 & 3 \end{bmatrix}$, find $3A$.

SOLUTION

Step 1 Write the equation in matrix form.

$3A$ means 3 times A.

$3A = 3\begin{bmatrix} 6 & -7 \\ 0 & 3 \end{bmatrix}$

Step 2 Multiply every element inside the matrix by 3.

$= \begin{bmatrix} 3 \times 6 & 3 \times (-7) \\ 3 \times 0 & 3 \times 3 \end{bmatrix}$

Step 3 Complete the operations to find the answer.

$= \begin{bmatrix} 18 & -21 \\ 0 & 9 \end{bmatrix}$

WORKED EXAMPLE 7 *(1 mark)*

Use CAS to evaluate $2.6\begin{bmatrix} 10.2 & 5.1 \\ 3.9 & -4.7 \end{bmatrix}$.

SOLUTION: TI-NSPIRE

Step 1 Go to the calculator screen within a document. Type the scalar, 2.6, then press multiply '×'.

Step 2 Push menu, then 7 for 'Matrix & Vector', 1 for 'Create' and 1 for 'Matrix ...' .

Enter '2' for the number of rows and '2' for the number of columns. Press OK.

Step 3 Enter 10.2, 5.1, 3.9 and −4.7 in their respective places in the matrix.

You can use the up, down, left and right buttons or the cursor to navigate around the matrix.

Step 4 Press ctrl then enter to calculate the answer in decimal form.

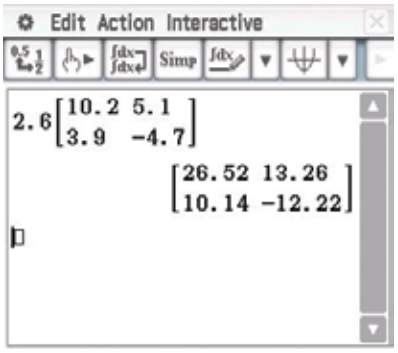
SOLUTION: CASIO CLASSPAD

Step 1 Using the stylus, touch Main. Type the scalar, 2.6, then press multiply '×'.

Step 2 Under the Math2 section of the keyboard, push (add row) and (add column) once each to create a 2 by 2 matrix.

Step 3 Use the stylus to tap on different elements in the matrix and enter the relevant numbers.

Step 4 Ensure the calculator is on Decimal mode, not Standard mode, and tap EXE.



2. Addition and subtraction with scalar multiplication

In matrices the order of operations (BIDMAS or BODMAS) applies to scalar multiplication, matrix addition and and matrix subtraction.

WORKED EXAMPLE 8 *(1 mark)*

If $A = \begin{bmatrix} 6 & -7 \\ 0 & 3 \end{bmatrix}$, and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, find $2A - 3B$.

SOLUTION

Step 1 Write $2A - 3B$ in matrix form.

$$2A - 3B = 2\begin{bmatrix} 6 & -7 \\ 0 & 3 \end{bmatrix} - 3\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

Step 2 Multiply each element inside the matrices with the multiplier (scalar) outside it.

$$= \begin{bmatrix} 2 \times 6 & 2(-7) \\ 2 \times 0 & 2 \times 3 \end{bmatrix} - \begin{bmatrix} 3 \times 1 & 3 \times 3 \\ 3(-2) & 3 \times 5 \end{bmatrix}$$

Step 3 Complete the operations within each matrix.

$$= \begin{bmatrix} 12 & -14 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix}$$

Step 4 Do the subtraction operation.

$$= \begin{bmatrix} 9 & -23 \\ 6 & -9 \end{bmatrix}$$

Questions 4C *Multiplication and matrices 1*

Refresher question

Q1. Find the value of the following:

a) $2 \times 5 + 3 \times 7$

b) $7 \times 1 - 4 \times (-2)$

1. Scalar multiplication

Q2. Skill

Find the missing number in the following scalar multiplication operation.

$$2\begin{bmatrix}4\\5\end{bmatrix} = \begin{bmatrix}2 \times 4\\2 \times 5\end{bmatrix} = \begin{bmatrix}8\\\textbf{?}\end{bmatrix}$$

Q3. Skill

Evaluate the matrix below.

$$3\begin{bmatrix}2 & 3\\1 & 2\end{bmatrix}.$$

Q4. Skill

Evaluate the following expressions.

a) $-2\begin{bmatrix}4 & -1\\2 & 3\end{bmatrix}$

b) $10\begin{bmatrix}4 & -2\\7 & 3\end{bmatrix}$

c) $0.25\begin{bmatrix}12 & 4\\16 & 40\end{bmatrix}$

Check your understanding

Q5. Skill

For the following, find the value of x.

a) $x\begin{bmatrix}1 & 3\\2 & 1\end{bmatrix} = \begin{bmatrix}2 & 6\\4 & 2\end{bmatrix}$

b) $x\begin{bmatrix}-2 & 5\\3 & 1\end{bmatrix} = \begin{bmatrix}-18 & 45\\27 & 9\end{bmatrix}$

2. Addition and subtraction with scalar multiplication

Q6. Skill

Complete the matrix equation below by filling in the missing element.

$$\begin{bmatrix}-3 & 6\\0 & 4\end{bmatrix} + \begin{bmatrix}2 & -4\\7 & -3\end{bmatrix} = \begin{bmatrix}-1 & 2\\7 & \textbf{?}\end{bmatrix}$$

Q7. Skill

Evaluate the following expressions.

a) $8\,[11 \quad 15 \quad -19] - \frac{1}{2}\,[210 \quad 68 \quad 130]$

b) $2\begin{bmatrix}1\\-3\\4\end{bmatrix} + 3\begin{bmatrix}-1\\2\\2\end{bmatrix}$

c) $3\begin{bmatrix}4 & 7\\-3 & 5\end{bmatrix} - 4\begin{bmatrix}4 & -5\\3 & 2\end{bmatrix}$

Q8. Evaluate the following expressions without using CAS.

Skill

a) $4\begin{bmatrix}-2 & 2\\ 1 & 3\end{bmatrix}+2\begin{bmatrix}4 & -3\\ -1 & 5\end{bmatrix}$

b) $-3\begin{bmatrix}-2 & 2\\ 1 & 3\end{bmatrix}+2\begin{bmatrix}4 & -3\\ -1 & 5\end{bmatrix}$

c) $-3\begin{bmatrix}-2 & 2\\ 1 & 3\end{bmatrix}-2\begin{bmatrix}4 & -3\\ -1 & 5\end{bmatrix}$

Q9. Evaluate the following expressions using CAS.

Skill

a) $2\begin{bmatrix}3 & 2 & 4\\ 1 & -1 & -2\end{bmatrix}-3\begin{bmatrix}-1 & 0 & 2\\ 4 & -2 & 3\end{bmatrix}$

b) $0.5\begin{bmatrix}6 & 4 & -2\\ 3 & 8 & -5\end{bmatrix}-\begin{bmatrix}-2 & 4 & 1\\ 1 & 0 & -1\end{bmatrix}$

Check your understanding

Q10. Due to an increase in demand, the price for Edrolo merchandise will double.

Application

The original prices, in matrix form, were:

$$\begin{bmatrix}\textit{hoodie} & \textit{tracksuit pants}\\ \textit{cap} & \textit{tote bag}\end{bmatrix}=\begin{bmatrix}\$70 & \$55\\ \$20 & \$10\end{bmatrix}$$

Which expressions will give the matrix with the new prices? There may be more than one correct option.

Expression 1

$$0.5\begin{bmatrix}\$70 & \$55\\ \$20 & \$10\end{bmatrix}$$

Expression 2

$$2\begin{bmatrix}\$70 & \$55\\ \$20 & \$10\end{bmatrix}$$

Expression 3

$$\begin{bmatrix}\$70 & \$55\\ \$20 & \$10\end{bmatrix}+\begin{bmatrix}\$70 & \$55\\ \$20 & \$10\end{bmatrix}$$

Joining it all together

Q11. Which of the following does not equal $\begin{bmatrix}2\\ 3\end{bmatrix}$?

Skill
1 mark

A. $\frac{2}{3}\begin{bmatrix}6\\ 12\end{bmatrix}-2\begin{bmatrix}1\\ 2.5\end{bmatrix}$

B. $\frac{1}{2}\begin{bmatrix}4\\ 6\end{bmatrix}$

C. $3\begin{bmatrix}0\\ 0\end{bmatrix}+3\begin{bmatrix}\frac{2}{3}\\ 1\end{bmatrix}$

D. $2\begin{bmatrix}\frac{1}{3}\\ \frac{1}{2}\end{bmatrix}-4\begin{bmatrix}-\frac{1}{3}\\ -1\end{bmatrix}$

Q12. Solve the following equation for x and y.

Skill
2 marks

$$2\begin{bmatrix}1 & 3\\ x & 4\end{bmatrix}-3\begin{bmatrix}y & 2\\ 3 & 4\end{bmatrix}=\begin{bmatrix}5 & 0\\ -3 & -4\end{bmatrix}$$

Q13. Which of the following problems cannot be simplified down to a single matrix?

Skill
1 mark

A. $0.27\begin{bmatrix}2.2 & 3.5 & -2.8\\ 37 & 8.3 & -0.5\end{bmatrix} - \begin{bmatrix}0 & 4 & 3\\ 6 & 0 & 0\end{bmatrix}$

B. $-2.3\begin{bmatrix}-4\\ 3\\ 21\end{bmatrix} + 3\begin{bmatrix}2 & -7\\ 0.4 & 3.3\\ -1 & 0.78\end{bmatrix}$

C. $\begin{bmatrix}3 & 4 & 6\\ -5 & 0.1 & 3\\ 4 & -0.6 & 0.33\end{bmatrix} - \begin{bmatrix}-0.1 & 2 & 0.1\\ 0.2 & 1 & 0\\ 4 & 3.2 & 43\end{bmatrix}$

D. $45\begin{bmatrix}-3 & 1.4\end{bmatrix} - 5.6\begin{bmatrix}3.1 & -2\end{bmatrix}$

Q14. Gabrielle and Hussain are racing to finish their classwork correctly. The final problem for the lesson is below. Look at their working. Who finished all the problems correctly first?

Application
1 mark

$$1.5\begin{bmatrix}4 & -3 & 2\\ 2 & -1 & 0\end{bmatrix} - 2\begin{bmatrix}3 & 1.5 & -2\\ -0.5 & 3 & 4.5\end{bmatrix}$$

Gabrielle:

$$= \begin{bmatrix}1.5\times 4 & 1.5\times -3 & 1.5\times 2\\ 1.5\times 2 & 1.5\times -1 & 1.5\times 0\end{bmatrix} - \begin{bmatrix}2\times 3 & 2\times 1.5 & 2\times -2\\ 2\times -0.5 & 2\times 3 & 2\times 4.5\end{bmatrix}$$

$$= \begin{bmatrix}6 & -4.5 & 3\\ 3 & -1.5 & 0\end{bmatrix} - \begin{bmatrix}6 & 3 & -4\\ -1 & 6 & 9\end{bmatrix}$$

$$= \begin{bmatrix}0 & -7.5 & 7\\ 2 & -7.5 & -9\end{bmatrix}$$

Hussain:

$$= \begin{bmatrix}1.5\times 4 & 1.5\times -3 & 1.5\times 2\\ 1.5\times 2 & 1.5\times -1 & 1.5\times 0\end{bmatrix} - \begin{bmatrix}2\times 3 & 2\times 1.5 & 2\times -2\\ 2\times -0.5 & 2\times 3 & 2\times 4.5\end{bmatrix}$$

$$= \begin{bmatrix}6 & -4.5 & 3\\ 3 & -1.5 & 0\end{bmatrix} - \begin{bmatrix}6 & 3 & -4\\ -1 & 6 & 9\end{bmatrix}$$

$$= \begin{bmatrix}0 & -7.5 & 7\\ 4 & -7.5 & -9\end{bmatrix}$$

VCAA question

Q15. $3\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix} + 2\begin{bmatrix}-1 & 0\\ 2 & -7\end{bmatrix}$ equals

1 mark

A. $\begin{bmatrix}4 & 3\\ 4 & -5\end{bmatrix}$

B. $6\begin{bmatrix}1 & 1\\ 2 & -4\end{bmatrix}$

C. $\begin{bmatrix}4 & 3\\ 4 & 2\end{bmatrix}$

D. $5\begin{bmatrix}1 & 1\\ 2 & -4\end{bmatrix}$

E. $\begin{bmatrix}3 & 6\\ 7 & 4\end{bmatrix}$

VCAA 2009 Exam 1, Module 6: Matrices. Q1

Questions from multiple lessons

Q16.

Difficulty:

1 mark

Consider the following matrix equation.

$$2 \times \begin{bmatrix} 6 & -4 \\ 3 & 0 \end{bmatrix} - M = \begin{bmatrix} 5 & 1 \\ 4 & -1 \end{bmatrix}$$

Which of the matrices below is M?

A. $\begin{bmatrix} 1 & -5 \\ -1 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 3 & -3 \\ 1 & 3 \end{bmatrix}$

C. $\begin{bmatrix} -3 & 3 \\ -1 & -3 \end{bmatrix}$

D. $\begin{bmatrix} 7 & -9 \\ 2 & 1 \end{bmatrix}$

E. $\begin{bmatrix} -7 & 9 \\ -2 & -1 \end{bmatrix}$

VCAA 2018NH Exam 1 Module 1: Matrices Q2 – Adapted

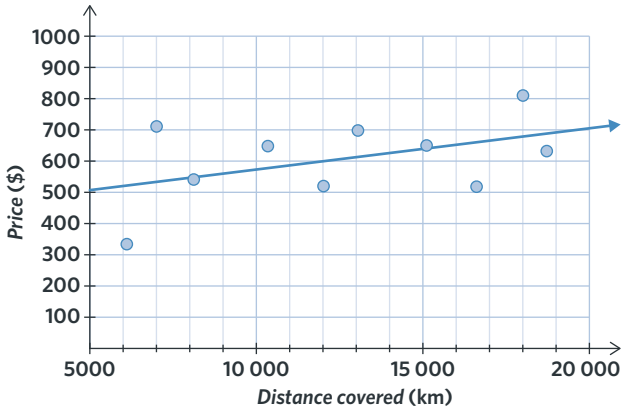
Q17.

Difficulty:

1 mark

The following scatterplot shows the *distance covered* (km) and *price* ($) per ticket for 10 international flights.

A least squares regression line has been added to the scatterplot with *distance covered* as the explanatory variable.



The equation of the regression line is closest to

A. *distance covered* $= 507 + 0.987 \times$ *price*

B. *distance covered* $= 507 + 0.013 \times$ *price*

C. *distance covered* $= 441 + 0.013 \times$ *price*

D. *price* $= 507 + 0.013 \times$ *distance covered*

E. *price* $= 441 + 0.013 \times$ *distance covered*

VCAA 2017 Exam 1 Data Analysis Q8 – Adapted

Q18.

Difficulty:

3 marks

The cost, in dollars, for some Apple products at a tech shop are shown in matrix M below.

$$M = \begin{bmatrix} 1000 \\ 450 \\ 1250 \\ 800 \end{bmatrix} \begin{matrix} \text{iPhone} \\ \text{iPad} \\ \text{MacBook} \\ \text{AppleWatch} \end{matrix}$$

a) What is the cost of a MacBook? (1 mark)

b) What is the order of matrix M? (1 mark)

c) The products have recently been discounted. The new price matrix, N, can be found from the equation below.

$N = 0.75 \times M$

Evaluate N (1 mark)

VCAA 2018 Exam 2 Module 1: Matrices Q1 – Adapted

LESSON 4D

Multiplication and matrices 2

The key skills you will learn in this lesson are:

1. Finding whether a matrix multiplication operation is defined
2. Multiplication of matrices

VCAA key knowledge point:

"matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using technology as applicable"

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1. Finding whether a matrix multiplication operation is defined

When multiplying matrices, the order of the matrices determines whether or not the matrices can be multiplied together.

If A and B can be multiplied together, we say AB is defined. If not, we say AB is undefined.

AB is defined when the number of columns in the matrix on the left equals the number of rows in the matrix on the right. In words, if the 'width' of A and the 'height' of B are the same, then AB is defined.

If A is an $m \times n$ matrix, and B is a $p \times q$ matrix, then matrix AB is defined if $n = p$.

AB will be of order $m \times q$.

WORKED EXAMPLE 9 *(2 marks)*

A is the 3×2 matrix $\begin{bmatrix} 3 & -12 \\ 9 & 4 \\ -7 & 6 \end{bmatrix}$ and B is the 5×3 matrix $\begin{bmatrix} 2 & 3 & -11 \\ 4 & 0 & -5 \\ 3 & -2 & 7 \\ 2 & -5 & 0 \\ 1 & 1 & 6 \end{bmatrix}$.

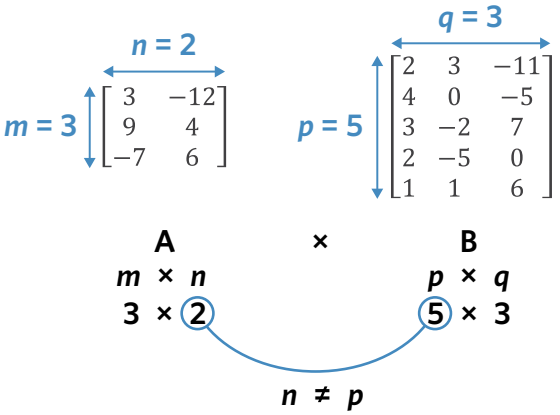
a) Is AB defined?

b) Is BA defined?

SOLUTION

a) Is AB defined?

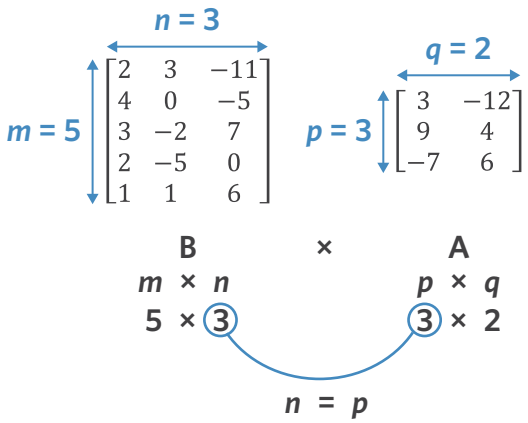
Step 1 Look for m, n, p, and q. Here, A is the left matrix and B is the right.



Step 2 Here, $n = 2$ and $p = 5$. As $n \neq p$, AB is not defined.

b) Is BA defined?

Step 1 BA will have different values of m, n, p, and q. Look for the new values.



Step 2 Here, $n = 3$ and $p = 3$. As $n = p$, BA is defined.

2. Multiplication of matrices

Once a matrix product is known to be defined, the multiplication can begin. Matrix multiplication is not commutative, meaning that for two matrices A and B, AB generally doesn't equal BA.

In matrix multiplication, the rows of the left matrix are multiplied with the columns of the right matrix. This process is outlined below for the multiplication of two 2×2 matrices.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

WORKED EXAMPLE 10 *(2 marks)*

a) Find $\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix} \times \begin{bmatrix} 8 & 3 \\ 4 & -5 \end{bmatrix}$.

b) If A is $\begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix}$ and B is $\begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix}$, calculate AB.

SOLUTION

a) Find $\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix} \times \begin{bmatrix} 8 & 3 \\ 4 & -5 \end{bmatrix}$.

Step 1 Check if the product is defined.

Looking at the matrices, $n = 2$ and $p = 2$. Since these are the same, we can continue.

Step 2 Find the order of the product.

The product will be of order $m \times q$ (number of rows in first matrix by number of columns in second matrix).

It will be a 2×2 matrix.

Step 3 Find the expression for calculating the first element.

$$\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix} \times \begin{bmatrix} 8 & 3 \\ 4 & -5 \end{bmatrix} = \begin{bmatrix} 1 \times 8 \times 3 \times 4 & \_\_\_\_\_ \\ \_\_\_\_\_ & \_\_\_\_\_ \end{bmatrix}$$

Notice that the elements from the rows in the first matrix are multiplied by the elements in the column of the second matrix and these results are added.

Step 4 Find expressions for all elements.

$$\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix} \times \begin{bmatrix} 8 & 3 \\ 4 & -5 \end{bmatrix} = \begin{bmatrix} 1 \times 8 \times 3 \times 4 & 1 \times 3 \times 3 \times -5 \\ -2 \times 8 \times 2 \times 4 & -2 \times 3 \times 2 \times -5 \end{bmatrix}$$

Step 5 Perform the operations for each element to find the answer.

$$\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix} \times \begin{bmatrix} 8 & 3 \\ 4 & -5 \end{bmatrix} = \begin{bmatrix} 20 & -12 \\ -8 & -16 \end{bmatrix}$$

b) If A is $\begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix}$ and B is $\begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix}$, calculate AB.

Step 1 Check AB is defined.

Looking at the matrices, $n = 3$ and $p = 3$. We can continue.

Step 2 Find the order of the product.

The product will be of order $m \times q$ (number of rows in A by number of columns in B).

AB will be a 2×2 matrix.

Step 3 Find the expression for calculating the first element.

$$\begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2 \times (-1) + 3 \times 0 + (-1) \times 4 & \_\_\_\_\_ \\ \_\_\_\_\_ & \_\_\_\_\_ \end{bmatrix}$$

Notice that the elements from the rows in matrix A are multiplied by the elements in the column of matrix B and these results are added.

Step 4 Find expressions for all elements.

$$\begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2 \times (-1) + 3 \times 0 + (-1) \times 4 & 2 \times (2) + 3 \times 3 + (-1) \times 1 \\ 4 \times (-1) + 0 \times 0 + (-5) \times 4 & 4 \times (2) + 0 \times 3 + (-5) \times 1 \end{bmatrix}$$

Step 5 Perform the operations to each element to find the answer.

$$AB = \begin{bmatrix} -6 & 12 \\ -24 & 3 \end{bmatrix}$$

WORKED EXAMPLE 11 *(1 mark)*

Use CAS to calculate the following matrix product.

$$\begin{bmatrix} 1.6 & 2.3 \\ -4.2 & 0.3 \end{bmatrix} \times \begin{bmatrix} 2.1 & 0.5 & -0.8 \\ 1.1 & 0 & -2.2 \end{bmatrix}$$

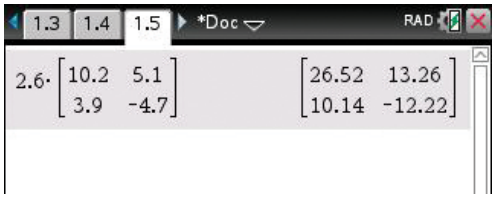
SOLUTION: TI-NSPIRE

Step 1 From the home screen, select 'Calculator' to open a new page. Press menu, then 7 for 'Matrix & Vector', 1 for 'Create' and 1 for 'Matrix...'. Enter two rows and two columns for the matrix on the left. Press OK.

Step 2 Enter the matrix values, 1.6, 2.3, −4.2, and 0.3, into the appropriate spots.

Step 3 Move to the right of this matrix, and repeat steps 1 and 2 for the matrix on the right.

Step 4 Press ctrl then enter to calculate the answer in decimal form.



SOLUTION: CASIO CLASSPAD

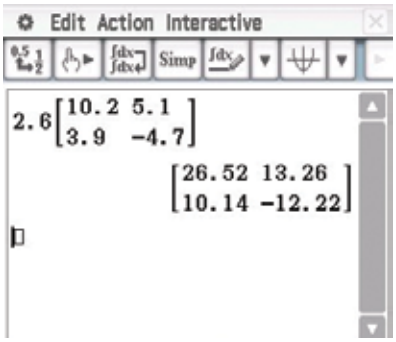
Step 1 Using the stylus, tap Main. Tap and open the keyboard.

Step 2 Under the Math2 section of the keyboard, push (add row) and (add column) once each to create a 2×2 matrix. Tap to the right of the matrix, and tap once and twice to create the second matrix.

Step 3 Tap the top-left entry in the first matrix. Enter 1.6 there.

Enter all the other elements into their appropriate places.

Step 4 Ensure the calculator is in Decimal mode, not Standard mode, and tap EXE.



Questions 4D *Multiplication and matrices 2*

Refresher question

Q1. What is the order of the following matrix?

$$\begin{bmatrix} 1 & -2 & 6 & 4 \\ 3 & 5 & 1 & 1 \end{bmatrix}$$

1. Finding whether an operation is defined

Q2. Skill

The matrix product BA is defined.

If matrix A has two rows, how many columns does matrix B have?

Q3. Skill

Is the matrix product AB defined for the following pairs of A and B?

If so, what will be the order of matrix product AB?

a) $A = \begin{bmatrix} 2 & 5 \\ -2 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 4 \\ 0 & -1 \end{bmatrix}$

b) $A = \begin{bmatrix} 2 & 1 & 5 \\ -2 & 6 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 5 \\ -9 & 4 & 2 \\ 3 & -3 & 4 \end{bmatrix}$

c) $A = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ -4 & -2 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

Check your understanding

Q4. Skill

If $A = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$, is $2A \times 3B$ defined?

If so, what will be the order of the matrix product?

2. Multiplication of matrices

Q5. Look at the working below. Where is the mistake and what is the mistake?

Skill

$$\begin{bmatrix}2 & 1\\3 & 1\end{bmatrix} \times \begin{bmatrix}4\\-2\end{bmatrix} = \begin{bmatrix}2 \times 4 + 1 \times (-2)\\3 \times 4 + 1 \times (-2)\end{bmatrix} \quad \text{line 1}$$

$$= \begin{bmatrix}8 + -2\\10 + -2\end{bmatrix} \quad \text{line 2}$$

$$= \begin{bmatrix}6\\8\end{bmatrix} \quad \text{line 3}$$

Q6. Evaluate the following without using CAS.

Skill

a) $\begin{bmatrix}1 & 4\\5 & 3\end{bmatrix} \times \begin{bmatrix}2 & 1\\2 & 3\end{bmatrix}$

b) $\begin{bmatrix}2 & -1\\-4 & 1\end{bmatrix} \times \begin{bmatrix}10 & -2\\2 & 6\end{bmatrix}$

c) $\begin{bmatrix}2 & 2\\1 & -1\end{bmatrix} \times \begin{bmatrix}1 & \frac{1}{2} & 1\\2 & 1 & -\frac{1}{2}\end{bmatrix}$

Q7. Evaluate using CAS.

Skill

If A is $\begin{bmatrix}1 & 4 & -2 & 1\\3 & 0 & -4 & -2\\2 & 5 & -3 & 1\end{bmatrix}$ and B is $\begin{bmatrix}-3\\4\\2\\0\end{bmatrix}$, find AB.

Q8. Which of the following matrix products do **not** equal $\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix}$?

Skill

A. $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \times \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}$

B. $\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix} \times \begin{bmatrix}1 & 1\\-2 & 1\end{bmatrix}$

C. $\begin{bmatrix}0 & 1\\-1 & 1\end{bmatrix} \times \begin{bmatrix}1 & 1\\2 & 1\end{bmatrix}$

D. $\begin{bmatrix}2 & 1\\1 & 0\end{bmatrix} \times \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Joining it all together

Q9. If matrix B has 3 rows and 4 columns, and the matrix product AB is defined, how many columns must matrix A have?

Skill
1 mark

Q10. If A is a $1 \times n$ matrix and B is a $p \times 1$ matrix, and $n = p$, then the matrix product AB is a 1×1 matrix, containing a single number. Which of the following products is highest in value?

Skill
1 mark

A. $\begin{bmatrix}1 & 4 & -3 & 7\end{bmatrix} \times \begin{bmatrix}2\\-2\\3\\1\end{bmatrix}$

B. $\begin{bmatrix}2 & 5 & -2\end{bmatrix} \times \begin{bmatrix}1\\1\\1\end{bmatrix}$

C. $\begin{bmatrix}10 & 12\end{bmatrix} \times \begin{bmatrix}-2\\3\end{bmatrix}$

D. $\begin{bmatrix}-1 & 1 & -1 & 1 & -1\end{bmatrix} \times \begin{bmatrix}-4\\-4\\2\\5\\-3\end{bmatrix}$

Q11. Skill 6 marks

If $A = \begin{bmatrix} 2 & -4 & 6 \\ 9 & -11 & 2 \\ 0 & 14 & -7 \end{bmatrix}$ $B = \begin{bmatrix} -7 & 0 \\ 12 & 2 \\ 5 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} -2 & 5 & -1 \\ 13 & -4 & 0 \end{bmatrix}$, find whether the following products are defined, and if so, calculate the product.

a) AB

b) A^2

c) AC

d) CA

e) BC

f) BA

Q12. Application 1 mark

Youssef and Maya's teacher wrote the following question on the board.

$$\begin{bmatrix} a & b \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 7 & 10 \end{bmatrix}$$

Youssef got an answer of $a = 4, b = -2$.

Maya got an answer of $a = -5, b = 1$.

Who is correct?

VCAA question

Q13. 1 mark

Matrix A has three rows and two columns.

Matrix B has four rows and three columns.

Matrix C = B × A has:

A. two rows and three columns.

B. three rows and two columns.

C. three rows and three columns.

D. four rows and two columns.

E. four rows and three columns.

VCAA 2013 Exam 1, Module 6: Matrices. Q2

Questions from multiple lessons

Q14. The table below shows information about two matrices, P and Q.

Difficulty:

1 mark

Matrix	Order	Rule
P	3×3	$p_{ij} = 5i - 2j$
Q	3×3	$q_{ij} = 3i - 2j$

The element in row i and column j of matrix P is p_{ij}.

The element in row i and column j of matrix Q is q_{ij}.

The difference $P - Q$ is

A. $\begin{bmatrix} 3 & 1 & -1 \\ 8 & 6 & 4 \\ 13 & 11 & 9 \end{bmatrix}$ **B.** $\begin{bmatrix} 2 & 2 & 2 \\ 4 & 4 & 4 \\ 6 & 6 & 6 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \\ 7 & 5 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 4 & 6 \\ 2 & 4 & 6 \\ 2 & 4 & 6 \end{bmatrix}$ **E.** $\begin{bmatrix} 2 & 4 & 2 \\ 4 & 2 & 6 \\ 2 & 6 & 2 \end{bmatrix}$

VCAA 2017 Exam 1 Module 1: Matrices Q6 – Adapted

Q15. Mattie wants to open up her own fashion studio. She will take out a loan of \$160 000 with interest charged at a rate of 2.6% per annum, compounding monthly.

Difficulty:

1 mark

Each month, Mattie will only pay the interest charged for that month.

After 30 months, the amount Mattie will owe is

A. \$84 900

B. \$122 250

C. \$149 600

D. \$160 000

E. \$170 400

VCAA 2017 Exam 1 Recursion and financial modelling Q19 – Adapted

---

Q16. A local game shop sells Jenga (J), Pictionary (P), Monopoly (M) and Scrabble (S). Matrix N contains the number of sales of each game in the past month.

Difficulty:

2 marks

$$N = \begin{bmatrix} 58 \\ 14 \\ 167 \\ 115 \end{bmatrix} \begin{matrix} \text{J} \\ \text{P} \\ \text{M} \\ \text{S} \end{matrix}$$

a) Write down the order of matrix N (1 mark)

b) The price of each game is contained within Matrix P.

$$P = \begin{matrix} \text{J} & \text{P} & \text{M} & \text{S} \end{matrix} \begin{bmatrix} 29 & 13 & 25 & 34 \end{bmatrix}$$

Calculate the matrix product $R = P \times N$ (1 mark)

VCAA 2016 Exam 2 Module 1: Matrices Q1 – Adapted

LESSON 4E

Applications of matrices

The key skills you will learn in this lesson are:

1. Using matrices to count
2. Business applications

VCAA key knowledge point:

"use of matrices, including matrix products and powers of matrices, to model and solve problems, for example costing or pricing problems, and squaring a matrix to determine the number of ways pairs of people in a network can communicate with each other via a third person"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Using matrices to count

Matrices can be used to add many quantities at once, such as the total numbers of different products sold at a shop in a week.

To do this, a matrix is multiplied by a single-row or single-column matrix of ones, called a vector.

It is important to note that if the vector is left of the matrix it must be a row, and if it is right of the matrix it must be a column. This ensures the matrix product is defined.

Example

Such a multiplication may look like this.

Vector is on the left so it is a row

$$\begin{bmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1}\end{bmatrix} \times \begin{bmatrix}23 & 25 & 21\\ 13 & 15 & 12\\ 42 & 39 & 45\\ 12 & 11 & 9\end{bmatrix}$$

Vector is on the right so it is a column

$$\begin{bmatrix}1 & 4 & -2\\ 5 & 5 & 3\\ -4 & 2 & -1\end{bmatrix} \times \begin{bmatrix}\mathbf{1}\\ \mathbf{1}\\ \mathbf{1}\end{bmatrix}$$

WORKED EXAMPLE 12 *(1 mark)*

The following matrix shows the number of students who have travelled to different cities in Australia in classes 11A, 11B and 11C at a school.

Calculate how many students in the three classes have been to Adelaide.

	Adelaide	Sydney	Hobart	Darwin
Class A	12	10	6	7
Class B	13	9	4	8
Class C	10	10	5	4

SOLUTION

Step 1 Looking at the matrix, we want to sum the elements for Adelaide.

	Adelaide	Sydney	Hobart	Darwin
Class A	**12**	10	6	7
Class B	**13**	9	4	8
Class C	**10**	10	5	4

Step 2 The information we need is in a column, so we need to multiply by a row.

The column has 3 elements, so the row must also have 3 elements.

$$[1 \quad 1 \quad 1] \times \begin{matrix} \text{Adelaide} & \text{Sydney} & \text{Hobart} & \text{Darwin} \\ \begin{bmatrix} \mathbf{12} & 10 & 6 & 7 \\ \mathbf{13} & 9 & 4 & 8 \\ \mathbf{10} & 10 & 5 & 4 \end{bmatrix} & \begin{matrix} \text{Class A} \\ \text{Class B} \\ \text{Class C} \end{matrix} \end{matrix}$$

Step 3 Multiply the matrices. Note that the multiplication sums the data for all the cities.

$$\begin{matrix} \text{Adelaide} & \text{Sydney} & \text{Hobart} & \text{Darwin} \\ [1 \times \mathbf{12} + 1 \times \mathbf{13} + 1 \times \mathbf{10} & 1 \times 10 + 1 \times 9 + 1 \times 10 & 1 \times 6 + 1 \times 4 + 1 \times 5 & 1 \times 7 + 1 \times 8 + 1 \times 4] \end{matrix}$$

$$= \begin{matrix} \text{Adelaide} & \text{Sydney} & \text{Hobart} & \text{Darwin} \\ [35 & 29 & 15 & 19] \end{matrix}$$

Step 4 The matrix shows that 35 students from the three classes have been to Adelaide.

WORKED EXAMPLE 13 *(1 mark)*

The following matrix shows how many boys and girls in a school are in each year level.

Find the total number of boys in the school.

$$\begin{matrix} & \text{Year 7} & \text{Year 8} & \text{Year 9} & \text{Year 10} & \text{Year 11} & \text{Year 12} \\ \text{Boys} & \begin{bmatrix} 280 & 285 & 358 & 235 & 203 & 194 \\ 278 & 287 & 246 & 242 & 208 & 198 \end{bmatrix} & \\ \text{Girls} & & & & & & \end{matrix}$$

SOLUTION

Step 1 The data on boys' enrolment figures is a row.

$$\begin{matrix} & \text{Year 7} & \text{Year 8} & \text{Year 9} & \text{Year 10} & \text{Year 11} & \text{Year 12} \\ \text{Boys} & \begin{bmatrix} \mathbf{280} & \mathbf{285} & \mathbf{358} & \mathbf{235} & \mathbf{203} & \mathbf{194} \\ 278 & 287 & 246 & 242 & 208 & 198 \end{bmatrix} & \\ \text{Girls} & & & & & & \end{matrix}$$

Step 2 As the data we want is a row, we multiply it by a column on the right.

$$\begin{matrix} & \text{Year 7} & \text{Year 8} & \text{Year 9} & \text{Year 10} & \text{Year 11} & \text{Year 12} \\ \text{Boys} & \begin{bmatrix} \mathbf{280} & \mathbf{285} & \mathbf{358} & \mathbf{235} & \mathbf{203} & \mathbf{194} \\ 278 & 287 & 246 & 242 & 208 & 198 \end{bmatrix} & \\ \text{Girls} & & & & & & \end{matrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

← As the row has six elements in it, the column has six ones.

Step 3 Calculate. Note that this operation will also find the total number of girls.

$$\begin{matrix} \text{Boys} \\ \text{Girls} \end{matrix} \begin{bmatrix} \mathbf{280} \times 1 + \mathbf{285} \times 1 + \mathbf{358} \times 1 + \mathbf{235} \times 1 + \mathbf{203} \times 1 + \mathbf{194} \times 1 \\ 278 \times 1 + 287 \times 1 + 246 \times 1 + 242 \times 1 + 208 \times 1 + 198 \times 1 \end{bmatrix}$$

$$= \begin{matrix} \text{Boys} \\ \text{Girls} \end{matrix} \begin{bmatrix} 1555 \\ 1459 \end{bmatrix}$$

Step 4 Each row in the product matrix corresponds to the same row in the original matrix.
The number of boys at the school is 1555.

2. Business applications

Matrices can be used to calculate revenue or profit, by using similar operations to the examples above.

The difference is that instead of multiplying by a matrix of ones, the matrix will contain the prices of items.

Example

The left matrix contains information for the number of bread rolls, pies and drinks sold by two bakeries.

The right matrix holds the prices of each item. It works because the numbers of bread rolls sold will be multiplied by the price of a bread roll, and so on.

$$\begin{matrix} \\ \text{Bakery 1} \\ \text{Bakery 2} \end{matrix} \begin{matrix} \begin{matrix} \text{Bread rolls} & \text{Pies} & \text{Drinks} \end{matrix} \\ \begin{bmatrix} 130 & 56 & 34 \\ 142 & 63 & 45 \end{bmatrix} \end{matrix} \times \begin{matrix} \begin{bmatrix} 0.5 \\ 4 \\ 3.5 \end{bmatrix} \\ \begin{matrix} \text{Cost of a bread roll} \\ \text{Cost of a pie} \\ \text{Cost of a drink} \end{matrix} \end{matrix}$$

The example below is incorrect because there are four columns in the left matrix, but only three prices in the right matrix. The product is not defined.

$$\begin{matrix} \\ \text{Shop 1} \\ \text{Shop 2} \end{matrix} \begin{matrix} \begin{matrix} \text{Nike shoes} & \text{Adidas shoes} & \text{Converse shoes} & \text{Tom's shoes} \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 & 3 \\ 5 & 3 & 6 & 8 \end{bmatrix} \end{matrix} \times \begin{bmatrix} 110 \\ 105 \\ 85 \end{bmatrix} \leftarrow \textbf{Only three prices listed here}$$

WORKED EXAMPLE 14 *(2 marks)*

Assume Hoyts and Village sell movie tickets for the same price: $22 for adults, $16 for children, and $18.50 for concessions. The matrix below shows how many tickets of each type were sold on a particular Saturday.

Create a price matrix and use matrix multiplication to work out which cinema had a higher revenue.

SOLUTION

$$\begin{matrix} \\ \text{Village} \\ \text{Hoyts} \end{matrix} \begin{matrix} \begin{matrix} \text{Adult} & \text{Child} & \text{Conc.} \end{matrix} \\ \begin{bmatrix} 2467 & 1378 & 895 \\ 2388 & 1432 & 910 \end{bmatrix} \end{matrix}$$

Step 1 There are three ticket types, so the price matrix will have 3 elements.

As we want to sum the information in rows, the price matrix is a column.

$$\begin{matrix} \\ \text{Village} \\ \text{Hoyts} \end{matrix} \begin{matrix} \begin{matrix} \text{Adult} & \text{Child} & \text{Conc.} \end{matrix} \\ \begin{bmatrix} 2467 & 1378 & 895 \\ 2388 & 1432 & 910 \end{bmatrix} \end{matrix} \times \begin{matrix} \begin{bmatrix} 22 \\ 16 \\ 18.5 \end{bmatrix} \\ \begin{matrix} \text{Price of adult ticket} \\ \text{Price of child ticket} \\ \text{Price of concession ticket} \end{matrix} \end{matrix}$$

✓ *1 mark for correct price matrix*

Step 2 Perform the multiplication.

$$\begin{matrix} \\ \text{Village} \\ \text{Hoyts} \end{matrix} \begin{bmatrix} 92\,879.5 \\ 92\,283 \end{bmatrix}$$

Step 3 The top row of the product matrix contains data relevant to the top row in the information matrix.

Village had a higher revenue on that particular Saturday.

✓ *1 mark for stating Village had a higher revenue on Saturday*

Questions 4E *Applications of matrices*

Refresher question

Q1. A is a 3×2 matrix. B is a 2×4 matrix. What order will AB be?

1. Using matrices to count

Q2. Skill

Matrix A indicates the numbers of students with pets in two classes. Matrix B can be multiplied with A to find the total number of students with pets in Class B. Should the multiplication be AB or BA?

$$\begin{matrix} & \text{Dogs} & \text{Cats} & \text{Birds} \\ \text{Class A} \\ \text{Class B} \end{matrix} \begin{bmatrix} 8 & 5 & 3 \\ 7 & 6 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

A **B**

Q3. Skill

Matrix A indicates what each student in each class had for dinner last night.

$$\begin{matrix} & \text{Class A} & \text{Class B} & \text{Class C} \\ \text{Pasta} \\ \text{Curry} \\ \text{Salad} \\ \text{Other} \end{matrix} \begin{bmatrix} 8 & 3 & 6 \\ 5 & 7 & 6 \\ 6 & 2 & 5 \\ 4 & 10 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

A **B** **C**

a) What does matrix product AB represent?

b) Calculate AB to find how many students ate salad for dinner yesterday.

c) For matrices A and C to be multiplied together, matrix C needs to be on the left.

What does product CA represent?

Check your understanding

Q4. Application

A primary school class is studying averages, and to do so they went outside for 30 minutes each day last week to count the number of different coloured cars driving past. Ali recorded the following data. It is displayed in matrix form below. This matrix will be called A.

$$\begin{matrix} & \text{White} & \text{Black} & \text{Red} \\ \text{Mon} \\ \text{Tue} \\ \text{Wed} \\ \text{Thu} \\ \text{Fri} \end{matrix} \begin{bmatrix} 35 & 26 & 15 \\ 40 & 24 & 16 \\ 38 & 21 & 18 \\ 33 & 25 & 15 \\ 41 & 26 & 17 \end{bmatrix}$$

A

a) What does AB represent, if $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$?

b) What does CA represent, if $C = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$?

2. Business applications

Q5. A bag shop sells school bags, handbags and wallets. The sale price of each is in matrix P.

Skill

The number of each item sold on a particular day is stored in matrix S.

$$\begin{matrix} \text{School bags} \\ \text{Handbags} \\ \text{Wallets} \end{matrix} \begin{bmatrix} 40 \\ 30 \\ 20 \end{bmatrix} \quad \begin{matrix} \text{School bags} & \text{Handbags} & \text{Wallets} \end{matrix} \begin{bmatrix} 3 & 6 & 2 \end{bmatrix}$$

P **S**

a) Does matrix product PS or SP represent the amount of money received on that day?

b) Find the amount of money received on that day.

Q6. Sausages, burgers and cans of drink are sold at a sausage sizzle.

Skill

Robert and Dhanuka are the two students taking payments.

The numbers of items sold by each of them is written in the following matrix S.

$$\begin{matrix} \\ \text{Robert} \\ \text{Dhanuka} \end{matrix} \begin{matrix} \text{Sausages} & \text{Burgers} & \text{Drinks} \end{matrix} \begin{bmatrix} 56 & 24 & 26 \\ 62 & 18 & 32 \end{bmatrix}$$

S

a) Use the matrix $[1 \quad 1]$ to calculate how many sausages were sold.

b) The cost of each item, \$2 for sausages, \$3 for burgers, and \$1.50 for drinks, is provided in the following matrix P. Calculate matrix SP.

$$P = \begin{bmatrix} 2 \\ 3 \\ 1.5 \end{bmatrix}$$

c) State how much money was collected by Dhanuka.

Check your understanding

Q7. Noah always buys the same three items for lunch from the canteen, just in different quantities.

Application

The following matrix L indicates the number of each item bought over Monday and Tuesday of a school week.

Assume Monday is the top row. The price of each item is given by the matrix P.

$$L = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \quad P = \begin{bmatrix} 1 \\ 2 \\ 1.5 \end{bmatrix}$$

a) Use matrix multiplication to calculate the total number of items bought on Monday.

b) Use matrix multiplication to calculate the amount of money spent on Tuesday.

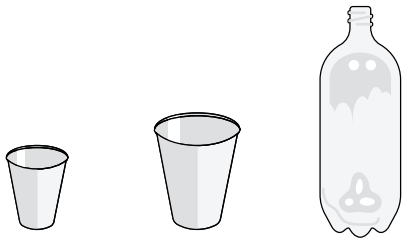
Joining it all together

Q8. Application 3 marks

Breanna is running a lemonade stand over the weekend to raise money for charity. She sells small cups for $0.50, large cups for $1, and bottles for $2.

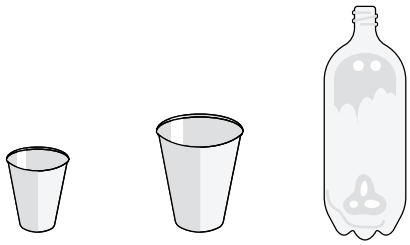
The stand's sales data for the two days is given in the following diagram.

Saturday



43 small cups **24 large cups** **10 bottles**

Sunday



36 small cups **26 large cups** **7 bottles**

a) Put all of this information into two matrices, so that total revenue can later be calculated. (1 mark)

b) Find the product of these two matrices, and use the result to state the total amount of money made on Saturday. (2 marks)

Q9. Application 5 marks

A shop at Mt Buller sells hats, gloves, scarves and socks to visitors. The number of each item sold during the first week of the winter school holidays is shown below in matrix M.

The cost and sale price of each item is also listed.

$$\begin{matrix} & \text{Hats} & \text{Gloves} & \text{Scarves} & \text{Socks} \\ \text{Mon} & 18 & 26 & 40 & 15 \\ \text{Tue} & 20 & 23 & 38 & 12 \\ \text{Wed} & 24 & 29 & 47 & 17 \\ \text{Thu} & 19 & 24 & 38 & 14 \\ \text{Fri} & 21 & 25 & 39 & 13 \end{matrix}$$

M

Item	Cost to buy	Sale price
Hats	$9	$20
Gloves	$7.50	$15
Scarves	$8.50	$20
Socks	$4	$10

a) Calculate the total sales of each item in the week, by multiplying $[1 \quad 1 \quad 1 \quad 1 \quad 1]$ with M. (1 mark)

b) Create a profit matrix, P, and use matrix multiplication to find the profit made on Monday. (2 marks)

c) Multiply matrix product M × P by another matrix to find this week's average profit per day. (2 marks)

VCAA question

Q10. 4 marks

A manufacturer sells three products, A, B, and C, through outlets at two shopping centres, Eastown (E) and Noxland (N).

The number of units of each product sold per month through each shop is given by the matrix Q, where

$$Q = \begin{matrix} \\ E \\ N \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 2500 & 3400 & 1890 \\ 1765 & 4588 & 2456 \end{bmatrix} \end{matrix}$$

a) Write down the order of matrix Q.

The matrix P, shown below, gives the selling price, in dollars of products A, B, C.

$$P = \begin{matrix} \begin{bmatrix} 14.50 \\ 21.60 \\ 19.20 \end{bmatrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} \end{matrix}$$

b) **i** Evaluate the matrix M, where $M = QP$.

ii What information do the elements of matrix M provide?

iii Explain why the matrix PQ is **not** defined.

VCAA 2006 Exam 2, Module 6: Matrices. Q1

Questions from multiple lessons

Q11. Difficulty:

1 mark

Sandra loves running and is training for her first marathon in one month's time. The distances of her three favourite tracks, Albert Park Lake, The Tan, and Princes Park, are shown in the table below.

Albert Park Lake	4.70 km
The Tan	3.83 km
Princes Park	3.21 km

In her training, Sandra will run the Albert Park Lake 7 times, The Tan 19 times, and Princes Park 13 times.

Which of the following matrix products will result in a matrix that contains the total distance Sandra will run in her training, in kilometres?

A. $\begin{bmatrix} 4.70 \\ 3.83 \\ 3.21 \end{bmatrix} \begin{bmatrix} 7 & 19 & 13 \end{bmatrix}$

B. $\begin{bmatrix} 4.70 \\ 3.83 \\ 3.21 \end{bmatrix} \begin{bmatrix} 7 \\ 19 \\ 13 \end{bmatrix}$

C. $\begin{bmatrix} 4.70 & 3.83 & 3.21 \end{bmatrix} \begin{bmatrix} 7 \\ 19 \\ 13 \end{bmatrix}$

D. $\begin{bmatrix} 4.70 & 3.83 & 3.21 \end{bmatrix} \begin{bmatrix} 7 & 19 & 13 \end{bmatrix}$

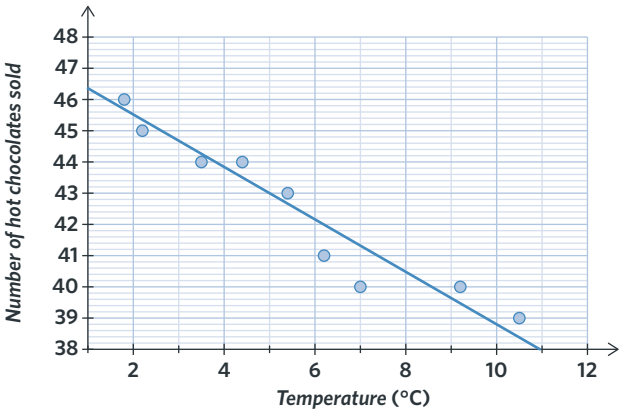
E. $\begin{bmatrix} 4.70 & 3.83 & 3.21 \end{bmatrix} \begin{bmatrix} 7 & 0 & 0 \\ 0 & 19 & 0 \\ 0 & 0 & 13 \end{bmatrix}$

VCAA 2017NH Exam 1 Module 1: Matrices Q2 – Adapted

Q12. The scatterplot below displays the *number of hot chocolates sold*, and the *temperature*, in degrees Celsius, of nine cafes for a week in April in Toronto. A least squares line has been fitted to the data.

Difficulty:

1 mark



The equation of this least squares line is closest to

A. $number\ of\ hot\ chocolates = 46.4 - 0.84 \times temperature$

B. $number\ of\ hot\ chocolates = 47.2 - 0.91 \times temperature$

C. $number\ of\ hot\ chocolates = 46.4 + 0.84 \times temperature$

D. $number\ of\ hot\ chocolates = 47.8 - 0.91 \times temperature$

E. $number\ of\ hot\ chocolates = 47.2 - 0.84 \times temperature$

VCAA 2018 Exam 1 Data Analysis Q8 – Adapted

Q13. At the cinema, there are three sizes of popcorn: small, medium and large.

Difficulty:

3 marks

The number of each size sold last month and their individual prices are shown in the following table.

Size	Number sold	Cost
Small	120	$4.50
Medium	380	$8.00
Large	710	$11.50

a) The column matrix Q displays the cost of each popcorn.

$$Q = \begin{bmatrix} 4.50 \\ 8.00 \\ 11.50 \end{bmatrix} \begin{matrix} \text{Small} \\ \text{Medium} \\ \text{Large} \end{matrix}$$

What is the order of matrix Q? (1 mark)

b) Matrix R contains the number of each size sold in the cinema.

$$R = \begin{matrix} \text{Small} & \text{Medium} & \text{Large} \end{matrix} \begin{bmatrix} 120 & 380 & 710 \end{bmatrix}$$

i Determine the matrix product RQ (1 mark)

ii Explain what the matrix product RQ represents. (1 mark)

VCAA 2017NH Exam 2 Module 1: Matrices Q1 – Adapted

LESSON 4F

Matrices and networks

The key skills you will learn in this lesson are:

1. Networks
2. Two-step paths

VCAA key knowledge points:

"use of matrices to store and display information that can be presented in a rectangular array of rows and columns such as databases and links in social and road networks"

"use of matrices, including matrix products and powers of matrices, to model and solve problems, for example costing or pricing problems, and squaring a matrix to determine the number of ways pairs of people in a network can communicate with each other via a third person"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Networks

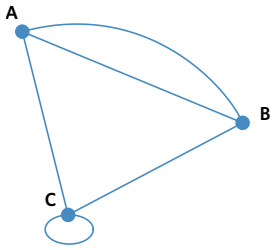
Matrices can be used to display information about connections in networks. Networks (sometimes called graphs) are diagrams made up of points, known as vertices, and lines connecting them, known as edges.

In such matrices, each element indicates how many direct paths there are between two points. These paths are called one-step paths because they indicate that it takes one-step to move between the two points.

Matrices used in graph and network problems are always square, and symmetrical about the leading diagonal. These types of matrices are useful for practical situations such as modelling internet packet pathways and networks of people.

Example

The network of roads between towns A, B and C can be represented in matrix form:



$$\begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \end{array}\begin{array}{c} \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

In the matrix:

- a '2' indicates two towns have **two** direct roads between them. In this example there are two roads between towns A and B.
- a '1' indicates two towns have **one** direct road between them. In this example there is one path connecting towns A and C, as well as towns B and C.
- a '0' indicates two towns have **no** direct roads between them. In this example town A has no direct roads back to itself. The same applies to town B.

NOTE: The loop on vertex C is represented by a 1 in Row C and Column C because C is connected to C by one-step.

Properties of a network matrix:

- It is a square matrix – same number of rows and columns.
- It is symmetrical about the leading diagonal.

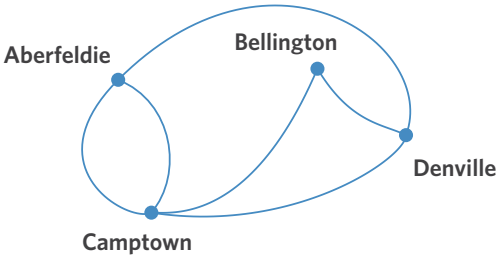
$$\begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \end{array}\begin{array}{c} \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

3

WORKED EXAMPLE 15 *(1 mark)*

The road links between four small country towns are shown in the following map.

Display the road links in a matrix.



SOLUTION

Step 1 We need to determine the size of the matrix. There are four towns involved, so the matrix will be 4×4. Each element that involves two cities directly connected by a path will be a number (indicating the number of connections). All others will be '0'.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

Step 2 Fill in the first column.

From Aberfeldie:

- there is **no** direct road back to Aberfeldie – put a 0 in row A
- there is **no** direct road to Bellington – put a 0 in row B
- there are **two** roads direct to Camptown – put a 2 in row C
- there is **one** road direct to Denville – put a 1 in row D.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & & & \\ 0 & & & \\ 2 & & & \\ 1 & & & \end{bmatrix} \end{array}$$

Step 3 Repeat for the rest of the columns.

Use the road links from Bellington, Camptown and Denville to fill in the next three columns.

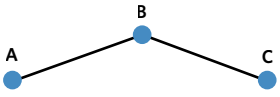
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

2. Two-step paths

If a matrix of a network is squared, it indicates the number of two-step paths between two points (or vertices) in the network. These paths are called two-step paths because they indicate that it takes two-steps to move between the two points.

Example

Three towns A, B and C are connected by roads, as described by the following matrix.



$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Squaring the matrix gives the number of two-step paths between the towns.

The squared matrix is:

	A	B	C
A	1	0	1
B	0	2	0
C	1	0	1

This tells us:

	A	B	C
A	1	0	1
B	0	2	0
C	1	0	1

There is **one way** to leave A and come back to A in two steps.

There is **one way** to leave C and come back to C in two steps.

There is **one way** to move between A and C in two steps.

There are **two ways** to leave B and come back to B in two steps.

There are **no ways** to move between A and B or C and B in two steps.

WORKED EXAMPLE 16 *(3 marks)*

Below is a map of places in Victoria and the paths between them.

Construct a matrix to reflect the map.

How many two-step paths are there between Melbourne and Shepparton?

SOLUTION

Step 1 We need to determine the size of the matrix. There are six places involved, so the matrix will be 6×6. We'll call this matrix A. Each element that involves two places directly connected by a path will be '1', all others will be '0'.

	Ballarat	Bendigo	Geelong	Melbourne	Seymour	Shepparton
Ballarat	0	1	1	1	0	0
Bendigo	1	0	0	1	0	1
Geelong	1	0	0	1	0	0
Melbourne	1	1	1	0	1	0
Seymour	0	0	0	1	0	1
Shepparton	0	1	0	0	1	0

✓ *1 mark for finding the matrix*

Step 2 We are looking for two-step paths, so calculate the square of the matrix, A^2.

	Ballarat	Bendigo	Geelong	Melbourne	Seymour	Shepparton
Ballarat	3	1	1	2	1	1
Bendigo	1	3	2	1	2	0
Geelong	1	2	2	1	1	0
Melbourne	2	1	1	4	0	2
Seymour	1	2	1	0	2	0
Shepparton	1	0	0	2	0	2

✓ *1 mark for squaring the matrix*

Step 3 Use the matrix to write the answer.

	Ballarat	Bendigo	Geelong	Melbourne	Seymour	Shepparton
Ballarat	3	1	1	2	1	1
Bendigo	1	3	2	1	2	0
Geelong	1	2	2	1	1	0
Melbourne	2	1	1	4	0	**2**
Seymour	1	2	1	0	2	0
Shepparton	1	0	0	**2**	0	2

Melbourne and Shepparton have 2 two-step paths.

✓ *1 mark for the correct answer*

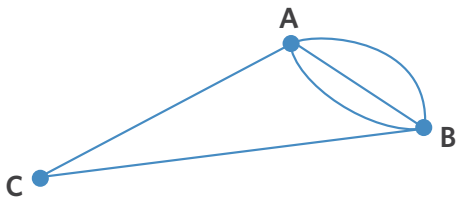
Questions 4F *Matrices and networks*

Refresher question

Q1. In the network to the right, how many connections exist between:

a) A and C

b) A and B



1. Networks

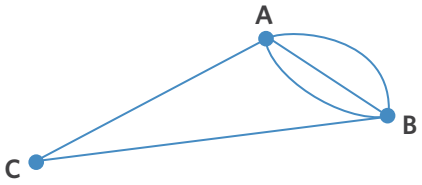
Q2. Skill

Look at the following matrix representing a network, indicating one-step paths. What does the circled number indicate?

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & \textcircled{1} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

Q3. Skill

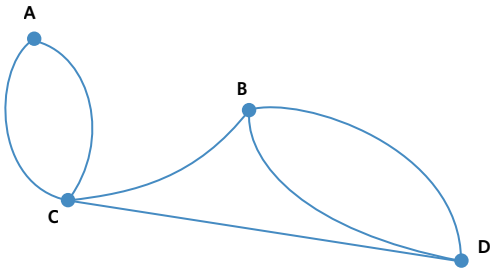
The matrix below represents the one-step paths of the following network. Fill in the missing elements.



$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & \_ & \_ \\ 3 & \_ & \_ \\ 1 & \_ & \_ \end{bmatrix} \end{array}$$

Q4. Application

Below is a map showing the trails between four different lookout points in a national park.



Display these trails in matrix form.

Q5. Skill

The matrix below represents the connections between vertices A, B, C, D and E in a network.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Construct a network based on this matrix.

Check your understanding

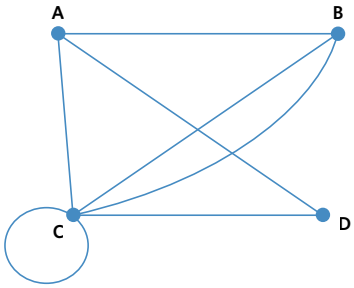
Q6. Below is a matrix representing the connections between four points on a graph.

Skill

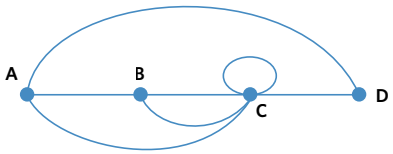
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Which of the following graphs **could not represent this matrix**?

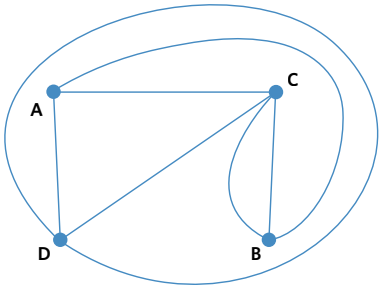
A.



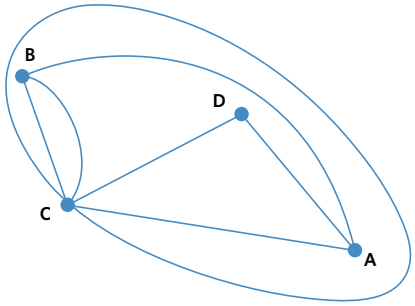
B.



C.



D.



2. Two-step paths

Q7. This is a network matrix, X.

Skill

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix} \end{array}$$

a) How many one-step connections are there:

i between A and B **ii** between A and C

b) Use your calculator to find X^2.

c) Use X^2 to find how many two-step connections exist:

i between A and B **ii** between A and C

Q8. The network matrix below reveals the connections between five people at a party, Annabelle, Ben, Christian, Denise and Ernie. A '1' indicates two people know each other, a '0' indicates they haven't met before.

Application

a) How many people does Ben know?

b) Draw the network of this matrix.

c) Find the square of the matrix. How many people can introduce Annabelle to Ernie?

In other words, how many two-step connections are between A and E?

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Check your understanding

Q9. Application

The following map shows the paths of Internet sea cables from Singapore to Los Angeles, via Hong Kong, Tokyo and Guam.

Source: Satellite imagery ©2018 Google

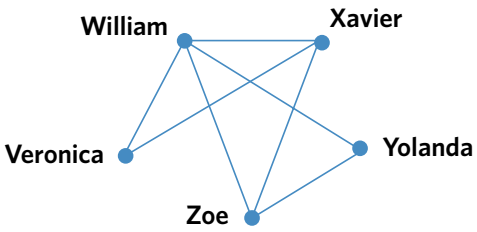
a) Put this information into a network matrix.

b) How many two step paths are there from Hong Kong to Los Angeles?

Joining it all together

Q10. Application 6 marks

Below is a diagram showing connections on Facebook between a group of five students: Veronica, William, Xavier, Yolanda and Zoe. A line between two people means that they are friends on Facebook.



a) Display this information in a matrix.

b) The leading diagonal of the matrix should be made up of zeros. What is the reason for this?

c) Find the sum of each row of the matrix.

d) Use your answer to c) to ident

e) Use your calculator to square

f) How many ways can Xavier an

Q11. Application 6 marks

Helena has driven on a lot of highw but her eyesight has deteriorated like driving on new roads. She is lo cities and wants to choose a city th Assume driving to Hobart can be d Melbourne and taking the Spirit o Bass Strait.

Source: Satellite imagery ©2018 Google

a) Write a 6×6 network matrix, M, to summarise the connections in the map. (1 mark)

b) By calculating and examining $M + M^2$, find which city is connected to every other city by a path of two steps or less. (2 marks)

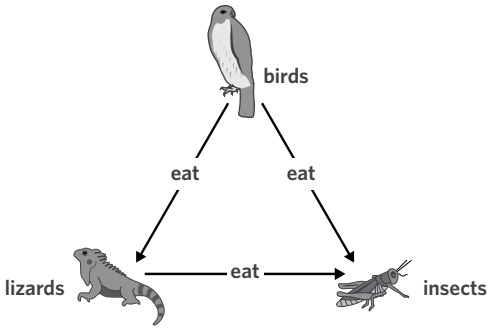
c) If Helena realises that the highway from Adelaide to Sydney is safe for her to drive on, and should therefore be included in M, how does the expression $M + M^2$ and your answer to b) change? (2 marks)

d) Give some reasons using 'steps' between cities is not the best method of finding which cities are central. (1 mark)

VCAA question

Q12. 2 marks

The diagram below shows the feeding paths for insects (I), birds (B) and lizards (L). The matrix E has been constructed to represent the information in this diagram. In matrix E, a '1' is read as 'eat' and a '0' is read as 'do not eat'.



$$E = \begin{matrix} \\ \text{I} \\ \text{B} \\ \text{L} \end{matrix} \begin{matrix} \begin{matrix} \text{I} & \text{B} & \text{L} \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

a) Referring to insects, birds or lizards:

i what does the '1' in column B, row L, of matrix E indicate?

ii what does the row of zeros in matrix E indicate?

VCAA 2011 Exam 2, Module 6: Matrices. Q1

Questions from multiple lessons

Q13.

Difficulty:

1 mark

Element x_{ij} is the element in row i and column j of matrix X.

The elements in matrix X are determined by the rule $x_{ij} = i + 3j$.

Which of the following matrices **cannot** be matrix X?

A. $[4]$

B. $\begin{bmatrix} 4 & 7 & 10 & 13 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 7 & 10 \\ 5 & 8 & 11 \\ 6 & 9 & 12 \end{bmatrix}$

D. $\begin{bmatrix} 4 & 7 \\ 5 & 8 \end{bmatrix}$

E. $\begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix}$

VCAA 2017NH Exam 1 Module: 1 Matrices Q4 – Adapted

Q14. Consider the following recurrence relation.

Difficulty:

$t_0 = -10, \quad t_{n+1} = 2t_n + 11$

1 mark

Which term of the sequence generated by this relation is the first to be positive?

A. t_1 **B.** t_2 **C.** t_3

D. t_4 **E.** t_5

VCAA 2018NH Exam 1 Recursion and financial modelling Q18 – Adapted

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Q15. The four most popular plants sold at a plant shop in Brunswick are the succulent (S), cactus (C), fern (F), and monstera (M).

Difficulty:

4 marks

Matrix P_{2019} shows the sale price of each of the plants, in dollars.

$$P_{2019} = \begin{bmatrix} 15 \\ 20 \\ 30 \\ 40 \end{bmatrix} \begin{matrix} \text{S} \\ \text{C} \\ \text{F} \\ \text{M} \end{matrix}$$

a) What is the order of matrix P_{2019}? (1 mark)

b) Nick buys a cactus and a monstera plant. How much does Nick have to pay? (1 mark)

c) The table below shows the number of each plant sold in the past week.

Plant	Number sold
Succulent	22
Cactus	24
Fern	17
Monstera	31

Write down a matrix that, when multiplied by matrix P_{2019}, would give the total revenue from these plant sales for the past week. (1 mark)

d) The shop decides to implement a 20% price increase at the start of 2020 due to increased demand.

Fill in the box below with a scalar so the product is the matrix of 2020 plant prices. (1 mark)

$P_{2020} = \square \times P_{2019}$

VCAA 2018NH Exam 2 Module 1: Matrices Q1 – Adapted

LESSON 4G

The inverse matrix

The key skills you will learn in this lesson are:

1. The identity matrix
2. Finding inverse matrices
3. Using inverse matrices to solve problems

VCAA key knowledge point:

"inverse matrices and their applications including solving a system of simultaneous linear equations"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.19; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. The identity matrix

An identity matrix, usually denoted by I, is a square matrix of any size with a diagonal line of 1s from the top left hand corner of the matrix to the bottom right hand corner, and with 0s in all other positions.

The matrices below are all identity matrices.

[1] $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ $\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$ $\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}$

The identity matrix acts like the number 1 in the multiplication of regular numbers as a matrix will remain unchanged when it is multiplied by an identity matrix.

That is for matrix A, $AI = IA = A$.

However, the normal rules of matrix multiplication apply, so identity matrices of the appropriate order must be used.

Example

$$\begin{bmatrix}8&-3&1\\2&8&10\end{bmatrix} \times \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}8&-3&1\\2&8&10\end{bmatrix}$$

$2 \times ③$ $③ \times 3$

and

$$\begin{bmatrix}1&0\\0&1\end{bmatrix} \times \begin{bmatrix}8&-3&1\\2&8&10\end{bmatrix} = \begin{bmatrix}8&-3&1\\2&8&10\end{bmatrix}$$

$2 \times ②$ $② \times 3$

WORKED EXAMPLE 17 *(1 mark)*

What is $\begin{bmatrix}2&-4\\5&3\end{bmatrix} \times \begin{bmatrix}1&0\\0&1\end{bmatrix}$?

SOLUTION

Step 1 Write out the calculation for each element.

$$\begin{bmatrix}2\times1+(-4\times0) & 2\times0+(-4\times1)\\ 5\times1+3\times0 & 5\times0+3\times1\end{bmatrix}$$

Step 2 Calculate the result for each element.

$$\begin{bmatrix}2&-4\\5&3\end{bmatrix}$$

The product is the same as the original matrix.

2. Finding inverse matrices

The inverse matrix of a matrix A, denoted by A^{-1}, is a matrix such that $A \times A^{-1} = A^{-1} \times A = I$.

Only square matrices have inverse matrices, however in this course, only the inverse of a 2×2 square matrix will be found by hand.

The product of $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ and its inverse $\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$ is the identity matrix, which acts like the number 1 in multiplication.

$$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To find the inverse of a matrix $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, follow the following steps:

Step 1 Multiply X by $\frac{1}{ad - bc}$

$$\frac{1}{ad - bc}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Step 2 Swap a and d

$$\frac{1}{ad - bc}\begin{bmatrix} d & b \\ c & a \end{bmatrix}$$

Step 3 Multiply b and c by -1

$$\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$X^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The term $ad - bc$ is known as the determinant.

If the determinant is 0, the inverse matrix does not exist as $\frac{1}{0}$ is undefined.

WORKED EXAMPLE 18 *(2 marks)*

a) If $X = \begin{bmatrix} 3 & -7 \\ 10 & 1 \end{bmatrix}$, what is X^{-1}?

b) If $Y = \begin{bmatrix} 2 & 3 & -7 \\ 0 & 4 & 10 \\ 12 & -8 & 1 \end{bmatrix}$, find Y^{-1} using a calculator.

SOLUTION

a) If $X = \begin{bmatrix} 3 & -7 \\ 10 & 1 \end{bmatrix}$, what is X^{-1}?

Step 1 Multiply X by $\frac{1}{ad - bc}$

$$\frac{1}{(3 \times 1) - (-7 \times 10)}\begin{bmatrix} 3 & -7 \\ 10 & 1 \end{bmatrix} = \frac{1}{73}\begin{bmatrix} 3 & -7 \\ 10 & 1 \end{bmatrix}$$

Step 2 Swap a and d.

$\frac{1}{73}\begin{bmatrix} 3 & -7 \\ 10 & 1 \end{bmatrix}$ becomes $\frac{1}{73}\begin{bmatrix} 1 & -7 \\ 10 & 3 \end{bmatrix}$

Step 3 Multiply b and c by -1.

$$= \frac{1}{73}\begin{bmatrix} 1 & -7 \times -1 \\ 10 \times -1 & 3 \end{bmatrix} = \frac{1}{73}\begin{bmatrix} 1 & 7 \\ -10 & 3 \end{bmatrix}$$

$$X^{-1} = \frac{1}{73}\begin{bmatrix} 1 & 7 \\ -10 & 3 \end{bmatrix}$$

b) If $Y = \begin{bmatrix} 2 & 3 & -7 \\ 0 & 4 & 10 \\ 12 & -8 & 1 \end{bmatrix}$, find Y^{-1} using a calculator.

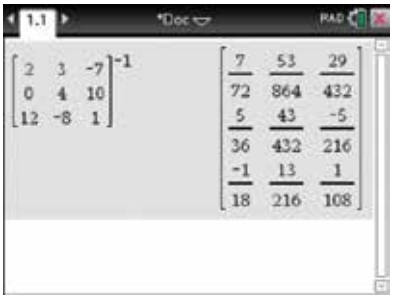
SOLUTION: TI-NSPIRE

Step 1 Open a new page by pressing ctrl + doc and press 1 ('Add Calculator').

Step 2 Press ▫ and use the arrows to find the matrix icon ▦. Press enter. Set the number of rows as 3 and the number of columns as 3. Press enter.

Step 3 Enter the elements of the matrix into each box, using the arrows to go between boxes.

Step 4 Use the arrows to move outside the matrix and raise the matrix to the power of -1.
Press enter. The answer will appear.



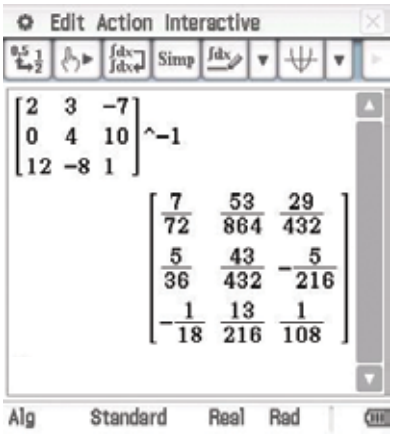
SOLUTION: CASIO CLASSPAD

Step 1 From the menu, tap √α Main and press keyboard.

Step 2 Tap Math2, tap ▦, then tap ▦ once and ▦ once to create a 3×3 matrix.

Step 3 Enter the elements of the matrix into each box, using the arrows to go between boxes.

Step 4 Use the arrows to move outside the matrix and raise the matrix to the power of -1.
Press EXE. The answer will appear.



3. Using inverse matrices to solve problems

One matrix cannot be divided by another. However, inverse matrices can be utilised to overcome this problem. To find the unknown matrix X in the equation $AX = B$, the following is done:

Step 1 Multiply both sides by A^{-1}.
$A^{-1}AX = A^{-1}B$

Step 2 Simplify $A^{-1}A$ to I.
$IX = A^{-1}B$

Step 3 Simplify IX to X.
$X = A^{-1}B$

Step 4 Find the product of A^{-1} and B to find the unknown matrix X.

It is important to note than when both sides were multiplied by the equation by A^{-1}, the A^{-1} was on the left on both sides. Matrix multiplication must always come from the same direction on both sides of a matrix equation.

Simultaneous equations in matrix form

The pair of simultaneous equations $ax + by = e$ and $cx + dy = f$ can be converted to a matrix equation as follows.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}.$$

This is because $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$ expanded is $\begin{bmatrix} ax + by \\ cx + dy \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$, which is equivalent to our original equations of $ax + by = e$ and $cx + dy = f$.

WORKED EXAMPLE 19 *(1 mark)*

If $\begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -5 \end{bmatrix}$, what are the values of x and y?

SOLUTION

Step 1 Multiply both sides by $\begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1}$ from the left.

$$\begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 13 \\ -5 \end{bmatrix}$$

Step 2 Simplify $\begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}$ to the identity matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 13 \\ -5 \end{bmatrix}$$

Step 3 Simplify $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ to $\begin{bmatrix} x \\ y \end{bmatrix}$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 13 \\ -5 \end{bmatrix}$$

Step 4 Find $\begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1}$.

$$\begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1} = \frac{1}{9 \times 3 - (8 \times -1)} \begin{bmatrix} 3 & -8 \\ 1 & 9 \end{bmatrix} = \frac{1}{35} \begin{bmatrix} 3 & -8 \\ 1 & 9 \end{bmatrix} = \begin{bmatrix} \frac{3}{35} & \frac{-8}{35} \\ \frac{1}{35} & \frac{9}{35} \end{bmatrix}$$

Step 5 Multiply $\begin{bmatrix} \frac{3}{35} & \frac{-8}{35} \\ \frac{1}{35} & \frac{9}{35} \end{bmatrix}$ and $\begin{bmatrix} 13 \\ -5 \end{bmatrix}$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{35} & \frac{-8}{35} \\ \frac{1}{35} & \frac{9}{35} \end{bmatrix} \times \begin{bmatrix} 13 \\ -5 \end{bmatrix} = \begin{bmatrix} \frac{79}{35} \\ \frac{-32}{35} \end{bmatrix}$$

Step 6 Find the values of x and y.

$x = \frac{79}{35}$ and $y = \frac{-32}{35}$

Questions 4G *The inverse matrix*

Refresher questions

Q1. What is $\begin{bmatrix} 9 & -5 \\ 0 & 8 \end{bmatrix} \times \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$?

Q2. Use your calculator to find $\begin{bmatrix} 2 & -8 & 3 \\ 12 & 5 & -7 \\ 8 & 6 & 15 \end{bmatrix} \times \begin{bmatrix} 13 & 0 & 4 \\ 6 & -8 & -6 \\ 11 & -9 & 3 \end{bmatrix}$.

1. The identity matrix

Q3. Skill

a) What is $\begin{bmatrix} 7 & 2 \\ -1 & -9 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$?

b) What is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 7 & 2 \\ -1 & -9 \end{bmatrix}$?

c) What do you notice about the answers to part a) and b)?

Q4. Skill

Find the product of the following matrices, and use the result to determine if they are inverses of each other.

a) $\begin{bmatrix} -2 & -9 \\ 3 & 16 \end{bmatrix}$ and $\frac{1}{5}\begin{bmatrix} -16 & -9 \\ 3 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 0 & -6 \\ 7 & 8 \end{bmatrix}$ and $\begin{bmatrix} \frac{4}{21} & \frac{1}{7} \\ \frac{-1}{6} & 0 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 1 \\ -4 & 8 \end{bmatrix}$ and $\frac{1}{7}\begin{bmatrix} 2 & \frac{-1}{4} \\ 1 & 3 \end{bmatrix}$

Check your understanding

Q5. Skill

Which of the following is not an identity matrix?

A. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

C. $[1]$

D. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. Finding inverse matrices

Q6. Skill

What is the value of the determinant of $\begin{bmatrix} 3 & -2 \\ 6 & 1 \end{bmatrix}$?

NOTE: The determinant is $ad - bc$ for a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Q7. Skill

Find the inverse of the following matrices.

a) $\begin{bmatrix} 3 & -8 \\ 10 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 12 & 7 \\ -9 & -3 \end{bmatrix}$

c) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

Check your understanding

Q8. Skill

Find the inverse of matrix A below and show that $A \times A^{-1}$ is the identity matrix.

$A = \begin{bmatrix} 12 & 4 \\ -2 & 9 \end{bmatrix}$

3. Using inverse matrices to solve problems

Q9. Skill

Find the values of x and y for the following.

a) $\begin{bmatrix} 2 & 0 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$

b) $\begin{bmatrix} 7 & -3 \\ 15 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \end{bmatrix}$

c) $\begin{bmatrix} -8 & 10 \\ 11 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ -8 \end{bmatrix}$

Q10. Skill

Use the fact that equations $ax + by = e$ and $cx + dy = f$ correspond to the matrix equation, $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$ to solve the following.

a) If $2x + 5y = 15$ and $x - 7y = -2$, find the values of x and y using matrices.

b) If $-3x + 8y = 18$ and $5x - 4y = -2$, find the values of x and y using matrices.

c) If $2x + 3y = 7$ and $-5x + y = -2$, find the values of x and y using matrices.

Check your understanding

Q11. Application

a) The total cost of 3 apples and 4 pears was \$18, and the total cost of 5 apples and 3 pears was \$19. Create a matrix equation representing this situation.

b) From the matrix equation found in part **a**, find the cost of one apple.

Joining it all together

Q12. Skill, 1 mark

Which of these matrices does not have an inverse?

A. $\begin{bmatrix} 3 & -6 \\ 2 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 5 & -10 \\ 2 & -4 \end{bmatrix}$

C. $\begin{bmatrix} 7 & 11 \\ 12 & 3 \end{bmatrix}$

D. $\begin{bmatrix} -1 & 10 \\ -2 & 12 \end{bmatrix}$

Q13. Skill, 2 marks

If $\begin{bmatrix} 2 & -3 \\ 3 & -5 \end{bmatrix}$ and $\begin{bmatrix} 5 & -3 \\ 3 & b \end{bmatrix}$ are inverses of each other, find the value of b.

Q14. Application, 2 marks

A student finds the inverse of a matrix to be $\begin{bmatrix} 3 & -7 \\ 2 & 14 \end{bmatrix}$.

However, they forgot to include $\frac{1}{determinant}$ in front of the matrix.

What should the inverse matrix have been?

Q15. Two families went to Luna Park on the weekend. The first family had two adults and two children and the second family had one adult and three children. They spent $180 and $170 on tickets respectively.

Application
3 marks

a) Fill in the missing information in the matrix below where a is the cost of an adult ticket and c is the cost of a child ticket in dollars, and determine the cost of an adult ticket and a child ticket. (2 marks)

$$\begin{bmatrix} & 2 \\ 1 & \end{bmatrix}\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 180 \\ \end{bmatrix}$$

b) The families below each brought different amounts of money on a trip to Luna Park. Which family could not afford tickets to the park? (1 mark)

A. One adult and four children who brought $220

B. Five children who brought $210

C. Two adults and three children who have $215

D. Three adults and three children who brought $275

VCAA question

Q16. Consider the following matrix A.

1 mark

$$A = \begin{bmatrix} 3 & k \\ -4 & -3 \end{bmatrix}$$

A is equal to its inverse A^{-1} for a particular value of k.

The value of k is

A. -4 **B.** -2 **C.** 0

D. 2 **E.** 4

VCAA 2011 Exam, 1 Module 6: Matrices. Q8

Questions from multiple lessons

Q17. The matrix product $\begin{bmatrix} 12 & 3 & 9 \end{bmatrix} \times \begin{bmatrix} 3 \\ 15 \\ 6 \end{bmatrix}$ is equal to

Difficulty:

1 mark

A. $3 \times \begin{bmatrix} 4 & 3 & 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 15 \\ 6 \end{bmatrix}$ **B.** $\begin{bmatrix} 36 \\ 45 \\ 54 \end{bmatrix}$ **C.** $\begin{bmatrix} 576 \end{bmatrix}$

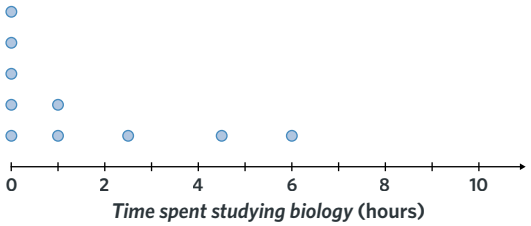
D. $9 \times \begin{bmatrix} 4 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$ **E.** $3 \times \begin{bmatrix} 4 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$

VCAA 2018 Exam 1 Module 1: Matrices Q2 – Adapted

Q18. Sofia recorded how long she spent studying biology each of the last 10 nights. The results are displayed in the following dot plot.

Difficulty:

1 mark



Correct to one decimal place, the mean and standard deviation of the time Sofia spent studying are

A. $\overline{x} = 2.2, \quad s_x = 1.5$

B. $\overline{x} = 1.5, \quad s_x = 2.0$

C. $\overline{x} = 1.5, \quad s_x = 2.2$

D. $\overline{x} = 3.0, \quad s_x = 2.2$

E. $\overline{x} = 2.2, \quad s_x = 3.0$

VCAA 2018 Exam 1 Data analysis Q5 – Adapted

Q19. One realm has four kingdoms: the North (N), the Vale (V), the Iron Islands (I) and Dorne (D).

Difficulty:

3 marks

The number of knights in each kingdom is shown in the matrix, K, below:

$$K = \begin{bmatrix} 52 \\ 29 \\ 20 \\ 38 \end{bmatrix} \begin{matrix} \text{N} \\ \text{V} \\ \text{I} \\ \text{D} \end{matrix}$$

a) What is the order of matrix K? (1 mark)

b) How many knights are there in either the Vale or the Iron Islands? (1 mark)

The table below shows the cost of armour for one knight per kingdom.

Kingdom	Cost per knight ($)
The North	15
The Vale	29
The Iron Islands	11
Dorne	23

c) Write down a matrix that could be multiplied by K to give a total cost to armour all knights. (1 mark)

VCAA 2018NH Exam 2 Module 1: Matrices Q1 – Adapted

AOS 3: Discrete mathematics

Graphs and networks

CHAPTER 5

LESSON 5A

Introduction to graphs and networks

The key skills you will learn in this lesson are:

1. Vertices, edges, and degrees
2. Types of graphs
3. Adjacency matrices

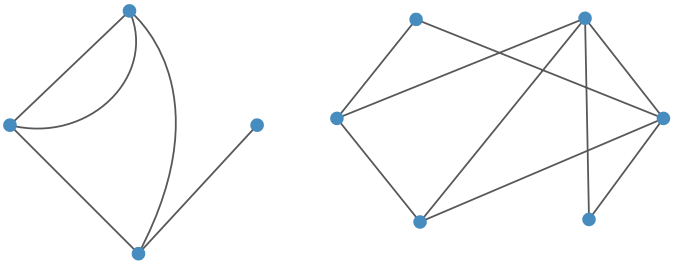
VCAA key knowledge point:

"Introduction to the notations, conventions and representations of types and properties of graphs, including edge, loop, vertex, the degree of a vertex, isomorphic and connected graphs and the adjacency matrix"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

This chapter is all about graphs that model networks and connections between points.

Below are two examples.



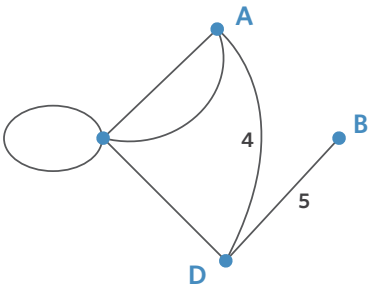
1. Vertices, edges, and degrees

The most important characteristics of graphs are vertices and edges.

Vertices (singular: vertex) are the name we give to points on the graph, usually represented by a dot or a circle.

Edges are the lines that connect two points. Edges that cross over are assumed to be separate and
ching. An edge that returns to the same vertex is called a **loop**.

four vertices (A, B, C, and D), and six edges (1, 2, 3, 4, 5, 6).



f a vertex is equal to the number of edges connected to it. Each loop on a vertex
two connections.

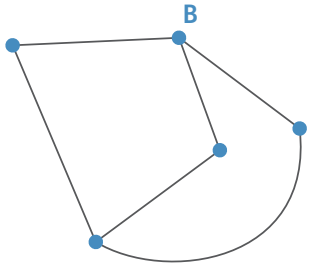
raph above, vertex A and vertex D both have a degree of three, vertex B has a degree of one,
as a degree of five.

2 marks)

For t, find:

a) th s and edges.

b) the .



SOLUTION

a) Find the number of vertices and edges.

Step 1 Count the vertices.

Vertices are the dots, points or circles. There are five vertices.

Step 2 Count the edges.

The edges are the lines. There are six edges in this graph.

b) Find the degree of vertex B.

Step 1 Count the edges connected to vertex B.

There are three edges connected, so the degree of vertex B is 3.

2. Types of graphs

There are many types of graphs.

These types include connected, disconnected, simple and complete graphs.

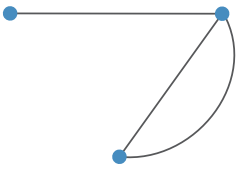
Connected and disconnected graphs

Connected Graphs

Connected graphs are graphs where all of the vertices can be reached from all of the other vertices by moving along the edges.

Example

This is a connected graph

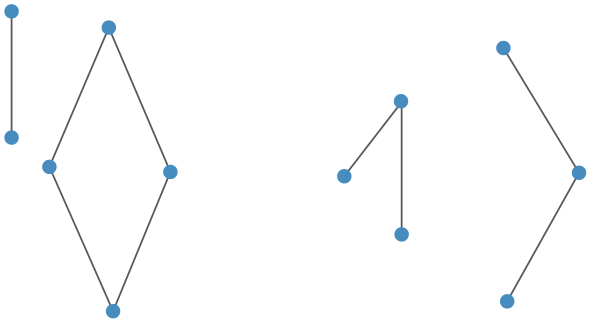


Disconnected graphs

Disconnected graphs are graphs that aren't connected.

Examples

These are both disconnected graphs.



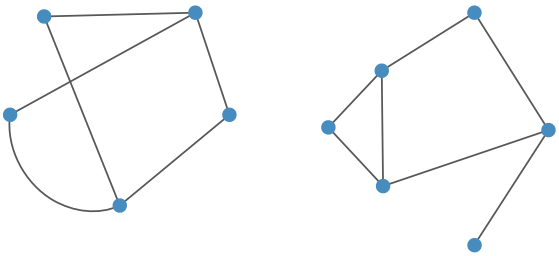
Simple and complete graphs

Simple Graphs

Simple graphs are graphs where each pair of vertices have a maximum of one connection between them.

Examples

These two graphs are simple graphs.

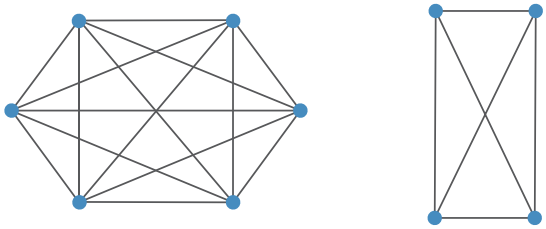


Complete graphs

Complete graphs are simple graphs in which each vertex is directly connected to every other vertex.

Examples

These two graphs are complete graphs.



All graphs can be classified as either connected or disconnected, but not all graphs can be classified as simple or complete.

If a graph is disconnected, it can't be complete.

Isomorphic

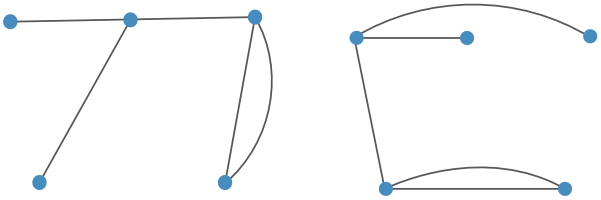
Two graphs that have the same number of vertices and edges, with each vertex connected in the same way, are called an isomorphic pair.

They will be in the same categories as each other. If one of the pair is simple, the other must be simple.

If one is complete, the other must be complete, and so on.

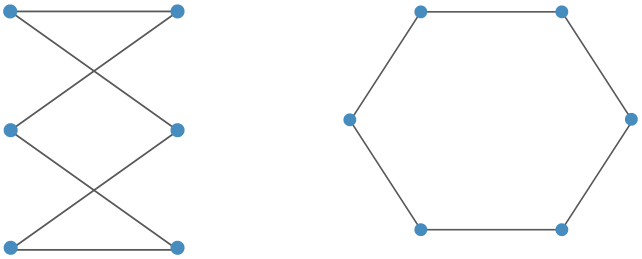
Example

These two graphs are an isomorphic pair.



Sometimes it is extremely hard to tell that some pairs of graphs are isomorphic.

The following two graphs are also an isomorphic pair, but look nothing alike!

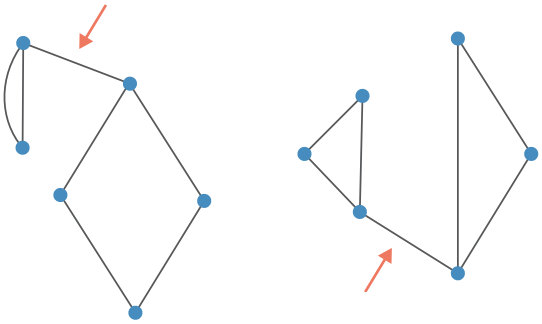


Graphs with bridges

A bridge is an edge that is preventing a graph from becoming disconnected. If a bridge is removed, the graph would become disconnected.

Example

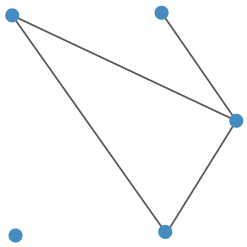
The orange arrows indicate the bridges in these graphs.



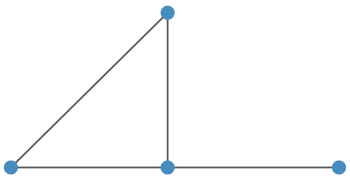
WORKED EXAMPLE 2 *(2 marks)*

Choose from the following terms to classify the following graphs: connected, disconnected, simple, complete.

a)



b)



SOLUTION

a)

Step 1 Find whether the graph is connected or disconnected.

The bottom left vertex is not connected to any other vertices, so the graph is disconnected.

Step 2 Find whether the graph is simple, complete, or neither.

There is a maximum of one connection between each vertex (either zero or one). This means the graph is simple.

b)

Step 1 Find whether the graph is connected or disconnected.

You can get from any vertex to any other vertex by moving along the edges.

Therefore, this graph is connected.

Step 2 Find whether the graph is simple, complete, or neither.

For the graph to be complete, every vertex needs to have a direct line to each other vertex.

However, the vertex on the right doesn't have an edge connecting it to all of the other vertices. Therefore, the graph isn't complete.

For the graph to be simple, there must be a maximum of one edge between each pair of vertices. This is true.

Therefore, the graph is simple.

3. Adjacency matrices

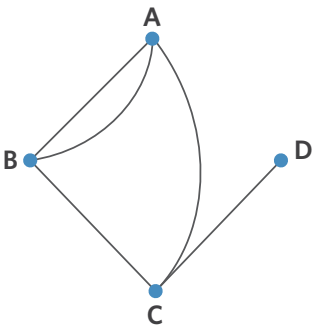
Adjacency matrices are used to describe a graph simply. Each element in an adjacency matrix represents the number of edges between the vertices of the respective row and column.

Adjacency matrices are:

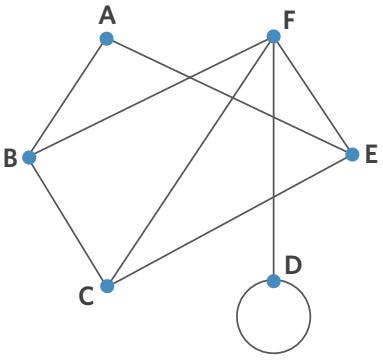
- square matrices that have the same number of rows as the graph has vertices.
- symmetrical, because if A is connected to C, then C must be connected to A.

Example

Here are two graphs and their adjacency matrices.



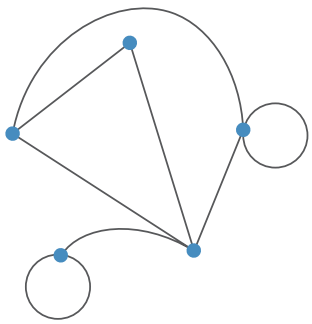
	A	B	C	D
A	0	2	1	0
B	2	0	1	0
C	1	1	0	1
D	0	0	1	0



	A	B	C	D	E	F
A	0	1	0	0	1	0
B	1	0	1	0	0	1
C	0	1	0	0	1	1
D	0	0	0	1	0	1
E	1	0	1	0	0	1
F	0	1	1	1	1	0

WORKED EXAMPLE 3 *(1 mark)*

Write an adjacency matrix for the graph shown to the right.



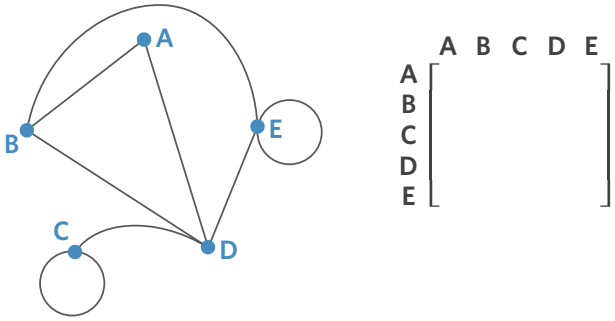
SOLUTION

Step 1 Determine the appropriate dimensions of the adjacency matrix.

There are five vertices, so the matrix needs to have five rows and five columns.

Step 2 Label the vertices and matrix rows and columns.

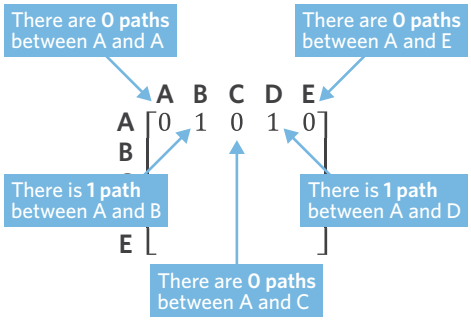
There are five vertices, so we will label them *A* to *E*. Label the rows and columns on the matrix with the same letters.



$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} \end{array}$$

Step 3 Begin finding the values in the matrix.

Start with row A.



$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} \end{array}$$

Step 4 As the matrix is symmetric, you can copy the entries from row 1 into the rest of column 1.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & & & & \\ 0 & & & & \\ 1 & & & & \\ 0 & & & & \end{bmatrix} \end{array}$$

Step 5 Repeat for the rest of the columns and rows in the matrix.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Step 6 Check the matrix is symmetrical.

Although we already have our answer, we can do a quick check by drawing a line down the diagonal and making sure the matrix is symmetric.

If it isn't symmetric we've made a mistake.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Questions 5A *Introduction to graphs and networks*

Refresher question

Q1. What is the order of the following matrix?

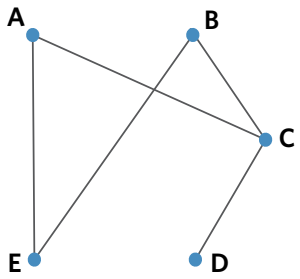
$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

1. Vertices, edges, faces and degrees

Q2. Skill

The graph to the right has its vertices labelled.

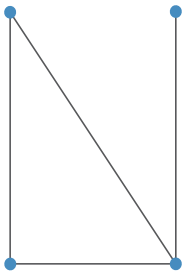
How many vertices are there?



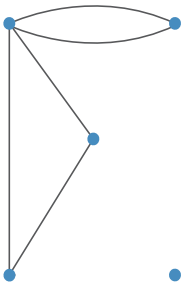
Q3. Skill

For each of the following graphs, state the number of vertices and edges.

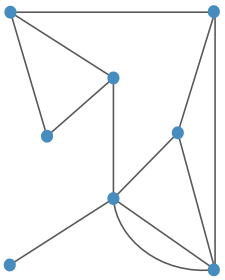
a)



b)

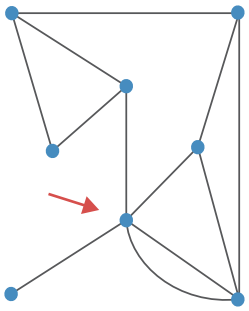


c)



Q4. Skill

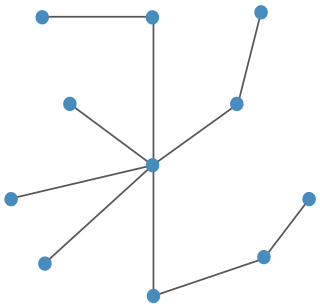
What is the degree of the vertex indicated by the arrow?



Check your understanding

Q5. Skill

State the number of vertices and edges in this graph.

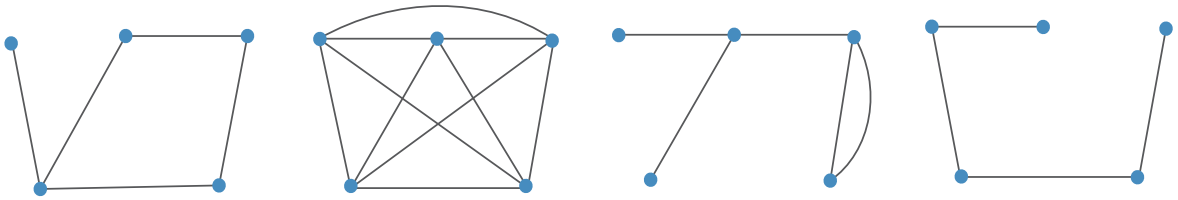


2. Types of graphs

Q6. Skill

Graph A is simple, Graph B is complete, and Graph C is neither simple or complete.

a) Is Graph D simple, complete, or neither? **b)** Explain why Graph C is **not** simple.



A B C D

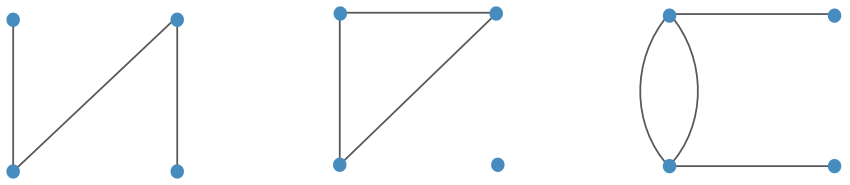
Q7. Skill

For the following sets of graphs:

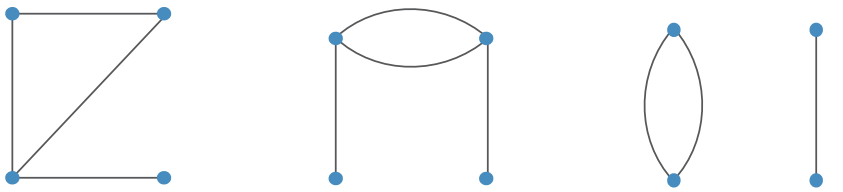
i List all the graphs in each of these categories: simple, complete, connected, disconnected.

ii Identify which two in each set are isomorphic.

a)

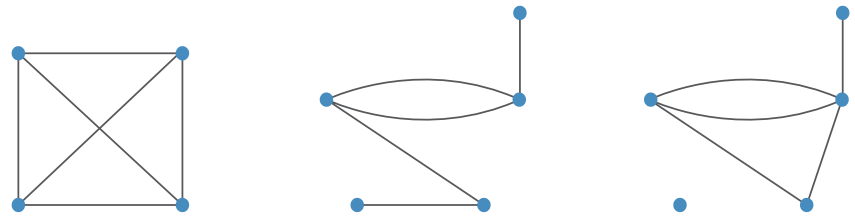


A B C

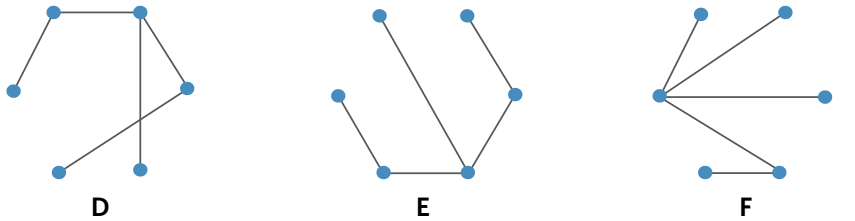


D E F

b)

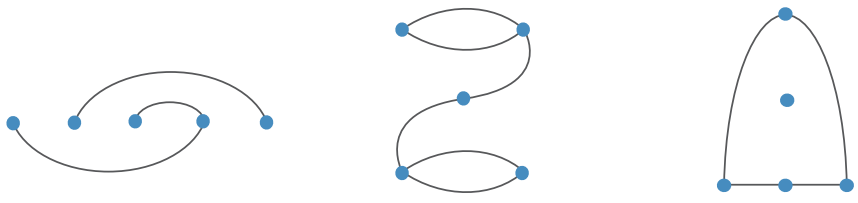


A B C

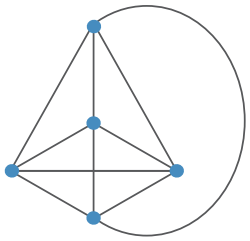


D E F

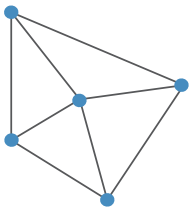
c)



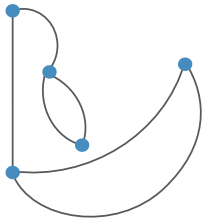
A B C



D



E



F

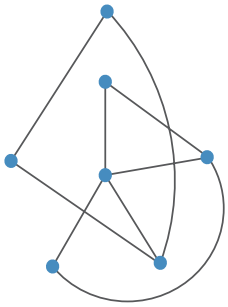
Check your understanding

Q8. Draw a complete graph with six vertices.

Skill

Q9. Which edge in the graph shown is a bridge?

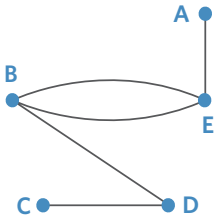
Skill



3. Adjacency matrices

Q10. Below is a graph and its adjacency matrix. Fill in the missing entry in the adjacency matrix.

Skill



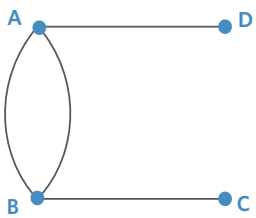
	A	B	C	D	E
A	0	0	0	0	1
B	0	0	0	1	2
C	0	0	0	1	0
D	0	1	1	0	0
E	1	?	0	0	0

Q11. Write an adjacency matrix for each of the following graphs.

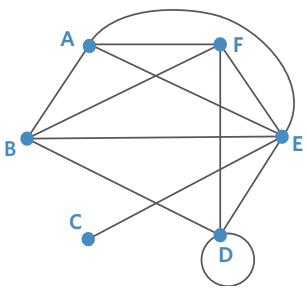
Skill

The vertices are labelled for you.

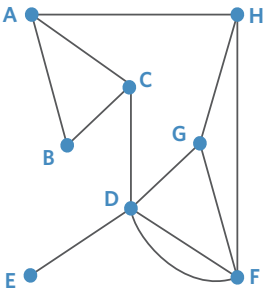
a)



b)



c)



Check for understanding

Q12. Look at Question 11, part c). Which two vertices have the most edges between them?

Skill

Q13. Skill

Draw a graph that would have the adjacency matrix to the right.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

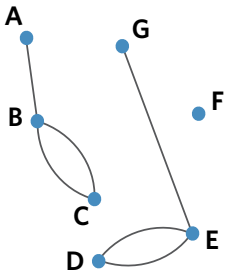
Joining it all together

Q14. Skill 1 mark

Make the graph below connected by adding in two edges.

Update the adjacency matrix, M, to include this new information.

NOTE: There are many possible answers.



$$M = \begin{array}{c} \\ \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \\ \mathbf{D} \\ \mathbf{E} \\ \mathbf{F} \\ \mathbf{G} \end{array} \begin{array}{c} \begin{array}{ccccccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} & \mathbf{E} & \mathbf{F} & \mathbf{G} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Q15. Skill 2 marks

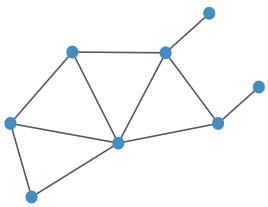
How many vertices and edges are in the graph described by the following adjacency matrix?

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

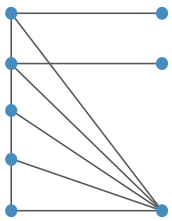
Q16. Application 1 mark

Which of the following graphs is **not** isomorphic with the graph of Australia below?

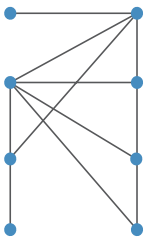
A.



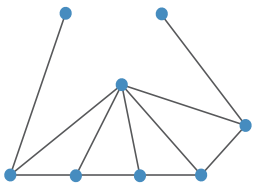
B.



C.

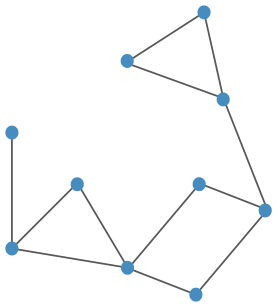


D.



Q17.
Skill
1 mark

How many bridges are in the graph shown to the right?



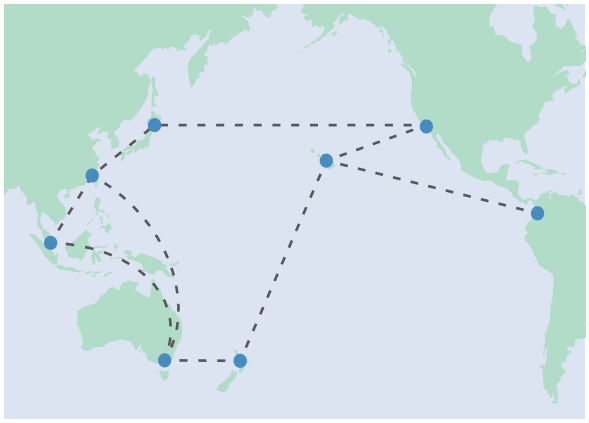
Q
Application
3 marks

graph of shipping routes in the
cific ocean.

W rtices represent?

Wh edges represe t?

Can a ship travel along all of the edges without taking the sam ge twice?



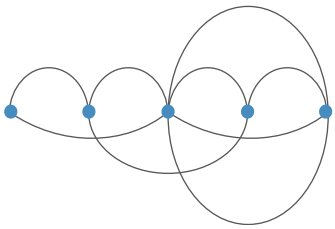
Q19.
Application
1 mark

Here is an adjacency matrix, X, for a network of roads.

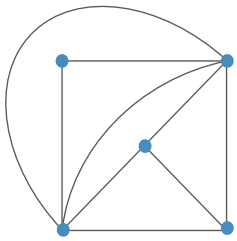
Draw a graph that matches adjacency matrix X. Which one of the graphs below forms an isomorphic pair with the graph you have drawn?

$$X = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 1 & 1 \\ 1 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

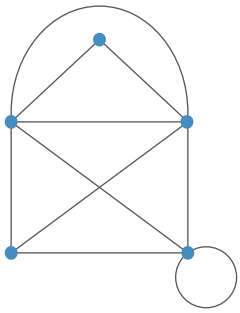
A.



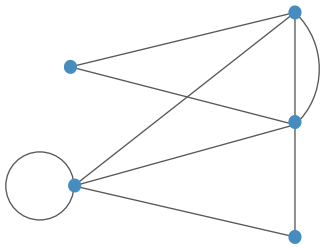
B.



C.



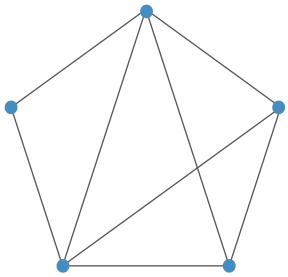
D.



VCAA questions

Q20.

1 mark



The number of edges in the graph above is

A. 5

B. 7

C. 8

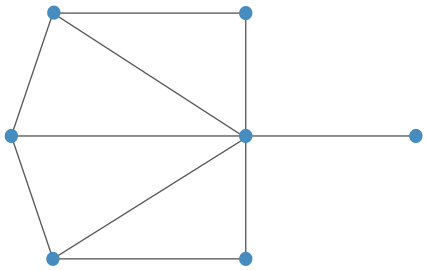
D. 10

E. 11

VCAA 2010 Exam 1 Module 5: Networks and decision mathematics. Q2

Q21.

1 mark



In the network shown, the number of vertices of even degree is

A. 2

B. 3

C. 4

D. 5

E. 6

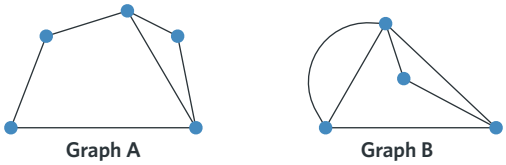
VCAA 2011 Exam 1 Module 5: Networks and decision mathematics. Q1

Questions from multiple lessons

Q22. Consider Graphs A and B shown below.

Difficulty:

1 mark



The sum of the degrees of the vertices of Graph B is

A. two less than the sum of the degrees of the vertices of graph A.

B. one less than the sum of the degrees of the vertices of graph A.

C. equal to the sum of the degrees of the vertices of graph A.

D. one more than the sum of the degrees of the vertices of graph A.

E. two more than the sum of the degrees of the vertices of graph A.

VCAA 2017 Exam 1 Module 2: Networks and decision mathematics Q2 – Adapted

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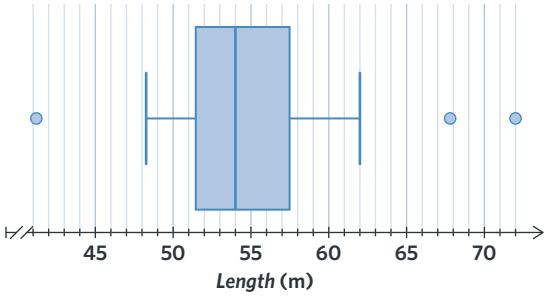
Q23. The length of 189 dragons, in metres, was recorded in the following boxplot.

Difficulty:

1 mark



The five-number summary for the length of these 189 dragons is closest to

A. 41.2, 51.4, 54, 57.4, 72

B. 41.2, 51.4, 54, 62, 67.8

C. 48.2, 51.4, 54, 57.4, 67.8

D. 48.2, 51.4, 54, 57.4, 62

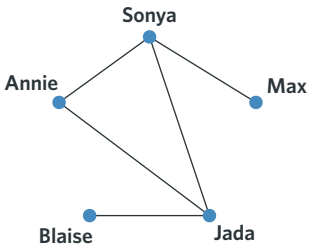
E. 51.4, 54, 56.4, 62, 72

VCAA 2017 Exam 1 Data analysis Q2 – Adapted

Q24. Five colleagues are connected by different friendships, represented by edges in the following graph.

Difficulty:

2 marks



The adjacency matrix also shows the different friendships between the five colleagues.

$$\begin{array}{c} \begin{matrix} S & M & J & B & A \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & x & 0 \\ 1 & 0 & 0 & y & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{matrix} S \\ M \\ J \\ B \\ A \end{matrix} \end{array}$$

a) Explain the meaning of a zero in the adjacency matrix in the context of friendships. (1 mark)

b) What are the values of x and y in the adjacency matrix? (1 mark)

VCAA 2010 Exam 2 Module 5: Networks and decision mathematics Q1 – Adapted

LESSON 5B

Euler's formula for planar graphs

The key skills you will learn in this lesson are:

1. Planar graphs
2. Euler's formula

VCAA key knowledge point:

"description of graphs in terms of faces (regions), vertices and edges and the application of Euler's formula for planar graphs"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Planar graphs

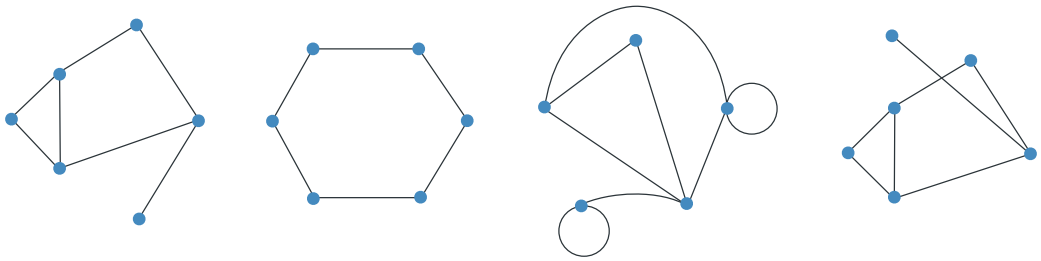
All graphs must either be planar or non-planar.

Planar

If a graph can be drawn so that none of its edges overlap with each other, we say the graph is 'planar'. A graph drawn like this is said to be drawn in 'planar form'.

Example

The following graphs are planar. None of their edges intersect.



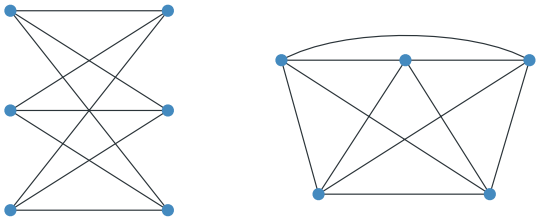
NOTE: Planar graphs can be drawn in non-planar form. The last graph is isomorphic with the first (and therefore planar), but is drawn in non-planar form.

Non-planar

If the graph cannot be drawn without overlapping edges, it is said to be 'non-planar'.

Example

The following graphs are non-planar. They have at least one edge that intersects other edges and cannot be drawn without the edges intersecting.

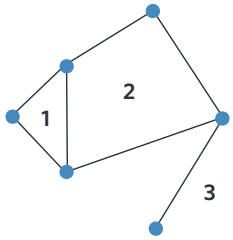


Faces

The enclosed areas found in planar graphs are called faces. The area outside a planar graph also counts as a face.

Example

This graph has three faces.



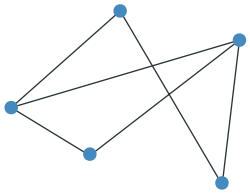
Re-drawing graphs to determine if they are planar

Sometimes a graph may appear to be non-planar. In this case the graph may be redrawn. If it is possible to redraw the graph with no overlapping edges, then the graph is planar.

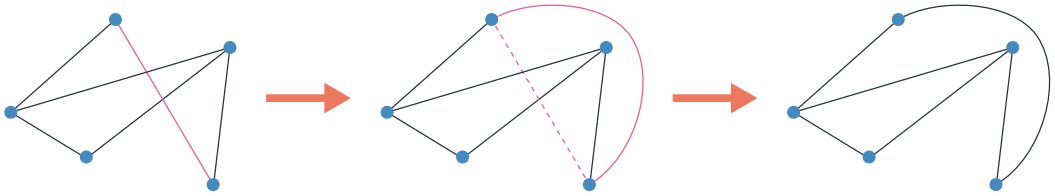
Re-drawing a graph can be done by moving edges or vertices, as long as the connections are the same as the original graph.

Example

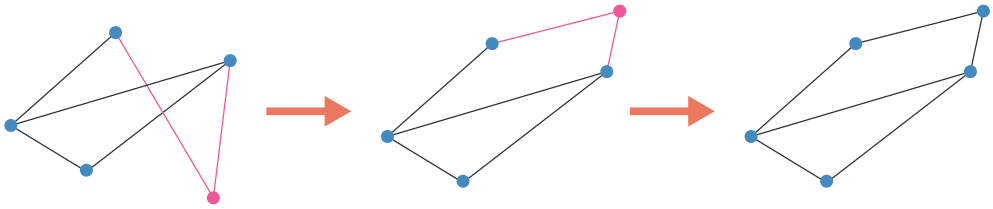
This graph can be redrawn to show that it is planar.



- By moving an edge:



- By moving a vertex:

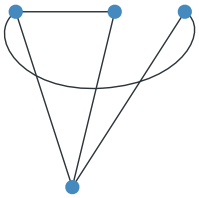


WORKED EXAMPLE 4 *(2 marks)*

Find:

a) whether the following graph is planar.

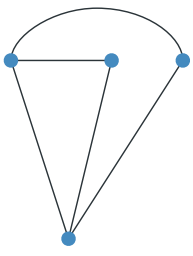
b) how many faces this graph has.



SOLUTION

a) Find whether the following graph is planar.

Step 1 Try re-drawing the graph so that the edges that were overlapping no longer overlap.

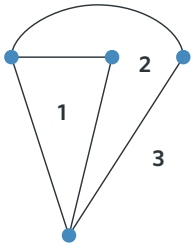


Step 2 If the graph can be drawn with no intersecting edges, you can declare it as planar. If it is impossible to draw the graph without intersecting edges, it is non-planar.

Our graph has no intersecting lines, so it is planar.

b) Find how many faces this graph has.

Step 1 Use the re-drawn graph to count the number of faces, remembering to include the area outside the graph.



This graph has three faces.

2. Euler's formula

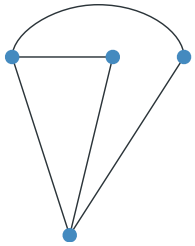
Leonhard Euler was a Swiss mathematician who studied graphs intensely. He found that there is a relationship between the number of edges (E), the number of faces (F) and the number of vertices (V) for all connected planar graphs. The relationship is:

$$V - E + F = 2$$

This relationship is known as Euler's formula for planar graphs.

Example

For this connected planar graph:



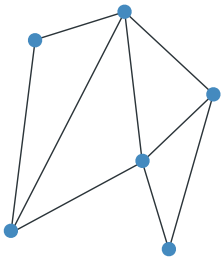
There are four vertices ($V = 4$), five edges ($E = 5$), and three faces ($F = 3$).

Putting these values into Euler's formula:

$$V - E + F = 2$$
$$4 - 5 + 3 = 2$$
$$2 = 2$$

As expected, the formula works for this graph.

WORKED EXAMPLE 5 *(2 marks)*



a) Use Euler's formula to calculate the number of edges in the graph on the right, and check that the answer is correct.

b) Use Euler's formula to calculate the number of vertices in a connected planar graph with seven edges and four faces.

SOLUTION

a) Use Euler's formula to calculate the number of edges in the graph on the right, and check that the answer is correct.

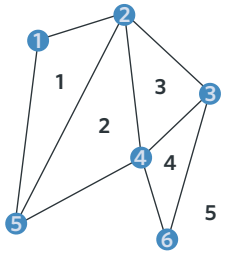
Step 1 Check if the graph is planar and connected because Euler's law only applies to connected planar graphs.

All the vertices are connected, and no edges overlap, so the graph is connected and planar.

Step 2 Count the vertices and faces of the graph.

There are six vertices and five faces. It isn't necessary to label them like we've done here.

So $V = 6$ and $F = 5$.



Step 3 Apply the formula and solve to find the missing quantity and interpret.

$$V - E + F = 2$$
$$6 - E + 5 = 2$$
$$11 - E = 2$$
$$E = 9$$

Euler's formula tells us that the graph has nine edges.

Step 4 Check.

Counting the edges on the graph gives nine also. Therefore the answer is correct.

b) Use Euler's formula to calculate the number of vertices in a connected planar graph with seven edges and four faces.

Step 1 Write what we know.

There are seven edges and four faces.

So $E = 7$ and $F = 4$.

Step 2 Apply the formula, solve to find the missing quantity and interpret.

$V - E + F = 2$

$V - 7 + 4 = 2$

$V - 3 = 2$

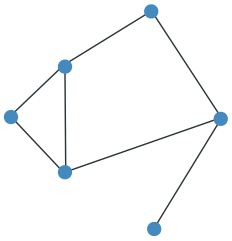
$V = 5$

Euler's formula tells us that the graph has five vertices.

Questions 5B *Euler's formula for planar graphs*

Refresher question

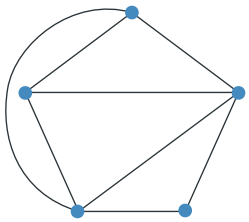
Q1. How many edges does the following graph have?



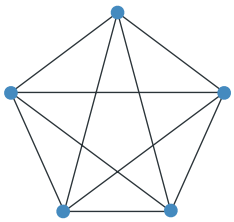
1. Planar graphs

Q2. A is planar, B is non-planar. What is C?

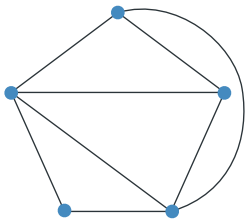
Skill



A



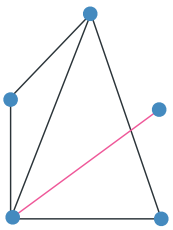
B



C

Q3. Redraw the pink edge so this graph is drawn in planar form.

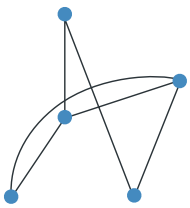
Skill



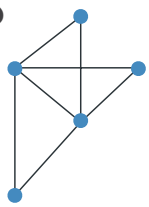
Q4. Draw the following graphs in planar form.

Skill

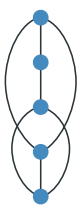
a)



b)



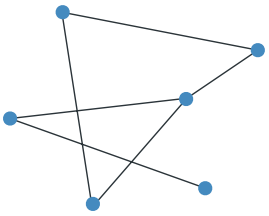
c)



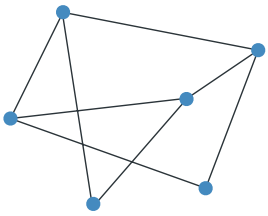
Q5. Are the following graphs planar or non-planar?

Skill

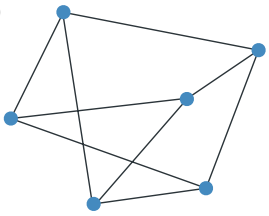
a)



b)



c)

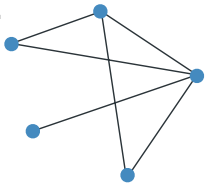


Check your understanding

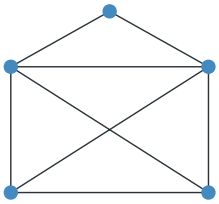
Q6. Which of the following graphs is non-planar?

Skill

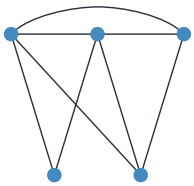
A.



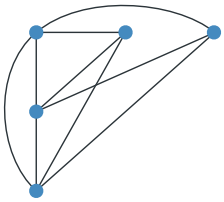
B.



C.



D.

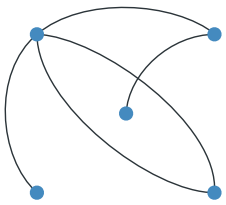


Q7. Using what you know about planar graphs:

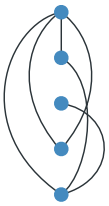
Skill

a) Draw Graph A and Graph B in planar form.

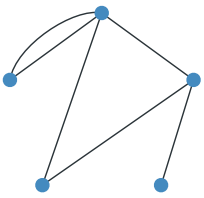
b) Which graph from part a) is isomorphic with Graph C?



Graph A



Graph B



Graph C

2. Euler's formula

Q8. A connected planar graph has six vertices and three faces. Use Euler's formula to calculate the number of edges in the graph.

Skill

Q9. A graph has five vertices and ten edges. How many faces will this graph have if it is planar and connected?

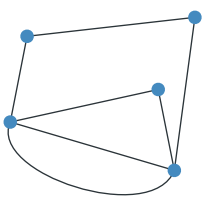
Skill

Q10. The following graphs are connected and planar.

Skill

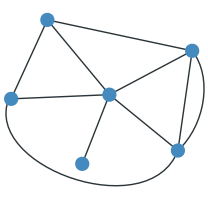
The first two graphs have the number of vertices and edges given. Use Euler's formula to find the number of faces in each graph.

a)



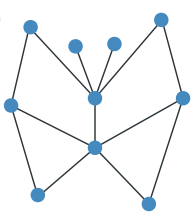
$V = 5$
$E = 7$

b)



$V = 6$
$E = 10$

c)

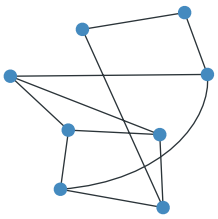


Check your understanding

Q11. Skill

a) By redrawing the graph in planar form, show that Euler's formula can be applied to this graph.

b) How many faces does the graph have?



$V = 8$
$E = 11$

Joining it all together

Q12. Skill 2 marks

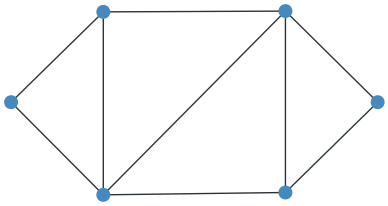
Draw a planar version of the graph given by the following adjacency matrix.

$$\begin{bmatrix} 0 & 2 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Q13. Skill 2 marks

a) Use Euler's formula to calculate the number of faces in the following graph.

b) Find the degree of each vertex and sum these numbers. How does the total number of edges compare to this sum?



Q14. Application 2 marks

Angel drew the graph below based on Melbourne's freeway system.

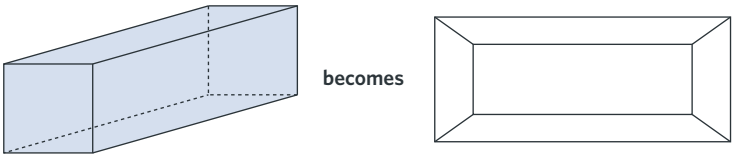
Source: Map data ©2018 Google

a) How many vertices, edges, and faces are there?

b) Show that Euler's formula holds for Angel's graph.

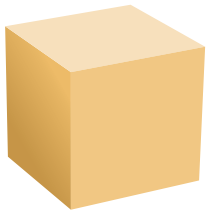
Q15. Some 3D shapes can be drawn as 2D planar graphs. Each 2D graph should have the same number of vertices and edges as the 3D shape. An example is below.

Skill
3 marks

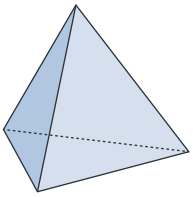


Draw the following 3D shapes as 2D planar graphs.

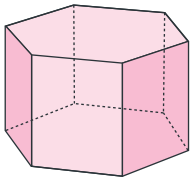
a)



b)



c)

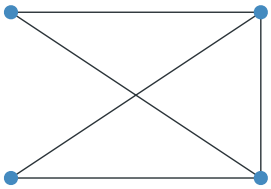


VCAA question

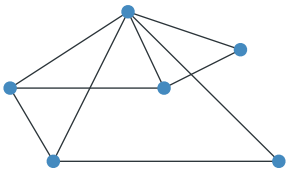
Q16. Euler's formula, relating vertices, faces and edges, does **not** apply to which one of the following graphs?

1 mark

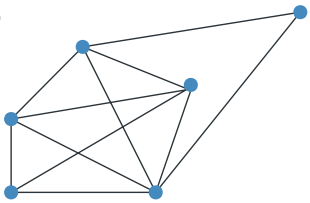
A.



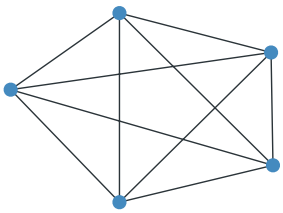
B.



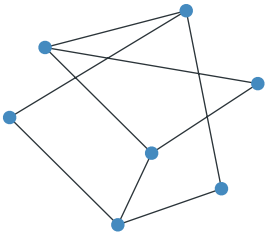
C.



D.



E.



VCAA 2006 Exam 1 Module 5: Networks and decision mathematics. Q8

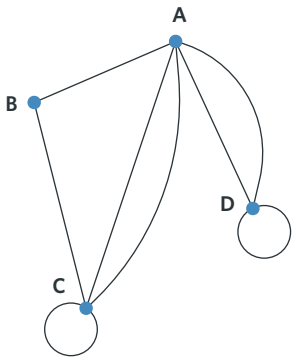
Questions from multiple lessons

Q17. A graph and its corresponding adjacency matrix, with some elements missing, is shown below.

Difficulty:

1 mark

	A	B	C	D
A	–	–	–	2
B	–	0	1	–
C	–	1	–	0
D	2	–	0	–



Of the nine missing elements, there are

A. four '0's, three '1's and two '2's.

B. three '0's, four '1's and two '2's.

C. three '0's, five '1's and one '2'.

D. four '0's, four '1's and one '2'.

E. three '0's, three '1's and two '2's.

VCAA 2017 Exam 1 Module 2: Networks and decision mathematics Q3 – Adapted

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Q18.

Difficulty:

1 mark

Are the variables *exam grade* (high distinction, distinction, pass, fail) and *time spent studying* (less than 5 hours, 5 to 10 hours, more than 10 hours) nominal, ordinal, discrete or continuous?

A. The variables are both nominal.

B. The variables are both ordinal.

C. The variables are nominal and ordinal respectively.

D. The variables are ordinal and discrete respectively.

E. The variables are nominal and discrete respectively.

VCAA 2016 Exam 1 Data analysis Q2 – Adapted

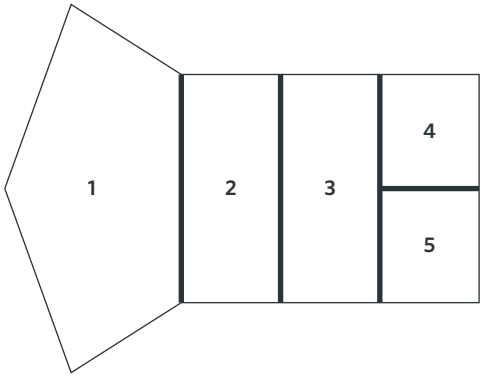
Q19.

Difficulty:

2 marks

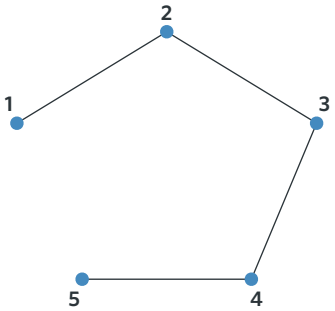
Paulo wants to design a new, unique house. He has decided to build it according to the following diagram, where each of the rooms is labelled 1 to 5.

The bold lines represent the walls between two rooms.



In the graph below, the five rooms are represented as vertices.

The edges of the graph represent the walls between two rooms.



One of the edges is missing from this graph.

a) Draw the complete graph. (1 mark)

b) With the missing edge included, what is the sum of the degrees of the vertices of the graph? (1 mark)

VCAA 2018NH Exam 2 Module 2: Networks and decision mathematics Q1 – Adapted

LESSON 5C

Connected graphs

The key skills you will learn in this lesson are:

1. Walks, trails and paths
2. Circuits and cycles

VCAA key knowledge point:

"connected graphs: walks, trails, paths, cycles and circuits with practical applications"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

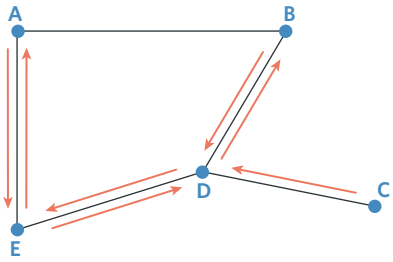
1. Walks, trails and paths

Walks

A walk is a route through a graph, moving from vertex to vertex along the edges that connect them. A walk is written as a list of all of the vertices travelled to in the order that they are visited.

Example

The walk *CDBDEAED* looks like this:



Trails

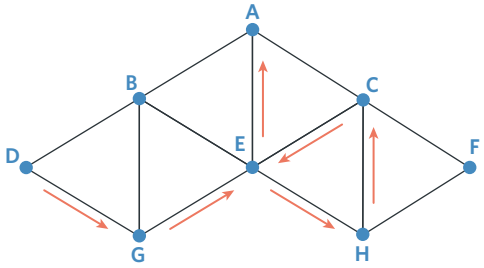
A trail is a walk that does not repeat any edges.

Paths

A path is a trail that does not repeat any vertices, with the exception that it may start and finish at the same vertex.

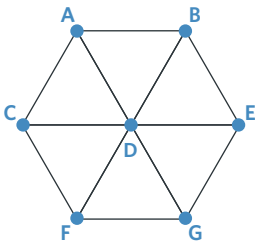
Example

The walk *DGEHCEA* is a trail but is not a path because the vertex E is visited more than once.



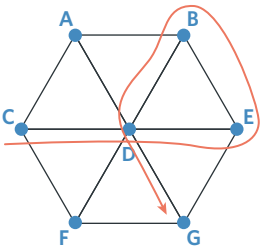
WORKED EXAMPLE 6 *(1 mark)*

For the graph to the right, which one of the terms, trail or path, best describes the route *CDEBDG*?



SOLUTION

Step 1 Trace the route on the graph.



Step 2 Check whether any edges are repeated.

No edges are repeated. This means that the route is a trail.

Step 3 Check whether any vertices are repeated.

The vertex *D* is repeated. This means that the route is not a path.

Step 4 Write down the term that best describes the route.

The route *CDEBDG* is a trail.

2. Circuits and cycles

Circuit

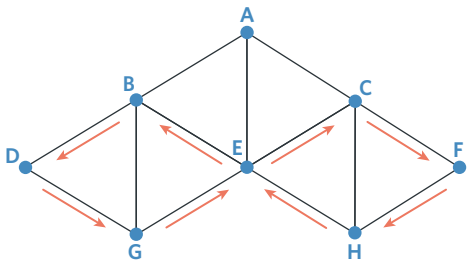
A circuit is a trail that starts and finishes at the same vertex.

Cycle

A cycle is a path that starts and finishes at the same vertex.

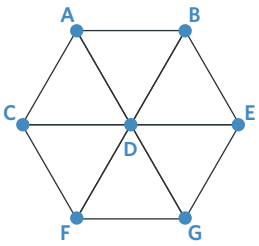
Example

The walk *DGECFHEBD* is a circuit but is not a cycle.



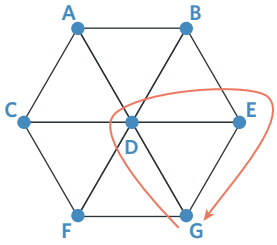
WORKED EXAMPLE 7 *(1 mark)*

For the graph below, which of the terms, circuit, cycle or both, best describes the route *GDEG*?



SOLUTION

Step 1 Trace the route on the graph.



Step 2 Check whether the route starts and ends at the same vertex.

The route starts and ends at *G*. This means that the route may be a circuit or cycle.

Step 3 Check whether any edges are repeated.

No edges are repeated. This means that the route is a circuit.

Step 4 Check whether any vertices are repeated, (excluding the starting vertex).

No vertex is repeated (other than the starting vertex). This means that the route is a cycle.

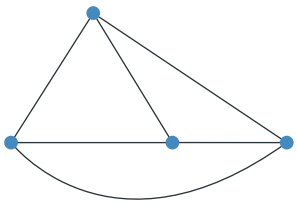
Step 5 Write down the terms that describe the route.

The route *GDEG* is a circuit, and a cycle.

Questions 5C *Connected graphs*

Refresher question

Q1. How many vertices and edges does the following network have?



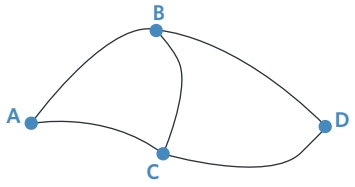
1. Walks, trails, and paths

Q2. Skill

For the network shown,

a) draw the trail *DBCAB*.

b) draw the path *ACBD*.



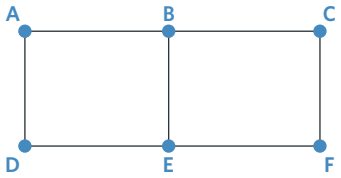
Q3. Skill

For the graph shown, list all of the terms, walk, trail, or path, that describe the following routes?

a) *EDABEFCB*

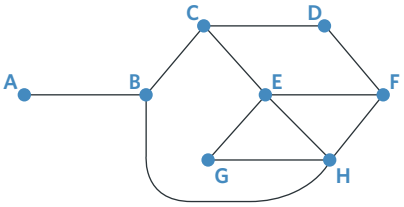
b) *DABEFCBED*

c) *ABCFEB*



Q4. Skill

For the following network find a path that starts at *E* and visits every vertex.



Check your understanding

Q5. Skill

By removing one edge from the graph from Question 4 it is possible to find a trail that starts at *F* and uses every edge.

List all edges that can be removed.

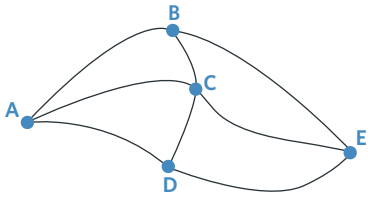
2. Circuits and cycles

Q6. Skill

For the network shown,

a) draw the circuit *ABCDECA*.

b) draw the cycle *ECBE*.



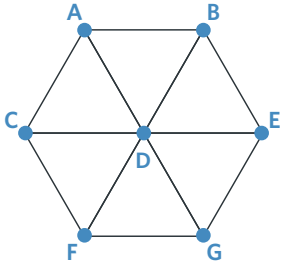
Q7. Skill

For the graph shown, list all of the terms (walk, circuit, or cycle) that describe the following routes.

a) *ADFCDA*

b) *DEGDFCDABD*

c) *BEDFCAB*

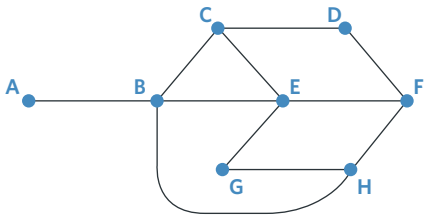


Q8. For the following network,

Skill

a) How many edges does the longest possible circuit have?

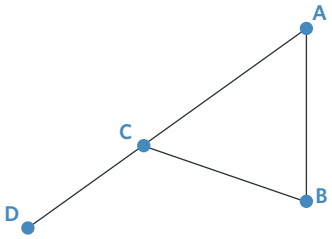
b) Find a cycle that visits the greatest number of vertices.



Check your understanding

Q9. For the graph below, explain why the route *ABA* is not a cycle.

Skill

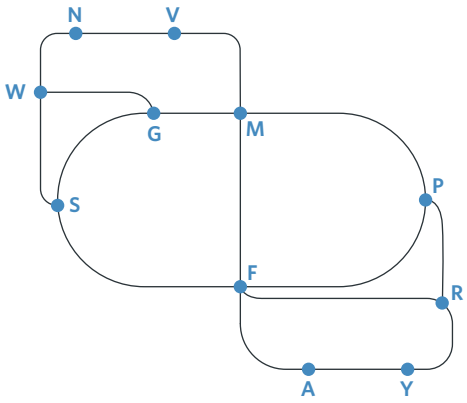


Joining it all together

Q10. The Victorian State Government is currently upgrading the City Loops and surrounding train lines. The network diagram below shows the plan for the new city loop.

Application
4 marks

The stations are North Melbourne, *N*; Parkville, *V*; West Melbourne, *W*; Flagstaff, *G*; Melbourne Central, *M*; Southern Cross, *S*; Parliament, *P*; Flinders Street, *F*; Richmond, *R*; Anzac, *A*; and South Yarra, *Y*.



a) Find a cycle that starts at Flagstaff and visits every station. (1 mark)

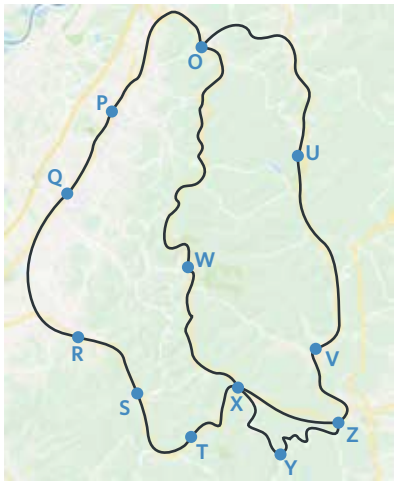
b) A child boarded a train without their parents. Two stations later they realised their mistake and got off the train. They are now at Flagstaff Station, *G*.

List all possible stations at which they may have boarded the train. (2 marks)

c) The train that the child boarded follows a cycle around all of the stations shown in the network diagram. Use you answer from part a) to identify the two stations at which the child may have boarded the train. (1 mark)

Q11. Application 8 marks

Graeme, Bill, and Tim are going to Germany. Their friend Horst Jankowski recommends that they take a walk in the Black Forest, in the country's south west. He gives them the following map of the region with his must-see locations marked with the letters *O*, *P*, *Q*, *R*, *S*, *T*, *U*, *V*, *W*, *Y*, *X*, and *Z*.



Source: Map data ©2018 Google

a) Draw a graph to represent the possible ways of travelling between the marked locations. (1 mark)

b) Graeme wants to visit each location only once.

i List the five locations from which they can leave in order to achieve this. (2 marks)

ii List all terms, walk, trail, path, circuit, or cycle, that describe these routes. (1 mark)

c) Bill wants to travel each road exactly once.

i Is this possible? (1 mark)

ii List all terms (walk, trail, path, circuit, or cycle) that describe this route. (1 mark)

d) Tim wants to finish where they started.

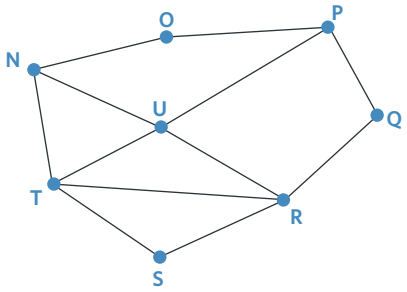
i Is it possible to travel every road once, ending at the starting location? (1 mark)

ii The local government of the Black Forest is planning to construct another road that will connect *W* to either *P*, *T*, *U*, or *Z*. Which road would not allow Tim, Graeme, and Bill plan a cycle that visits every location? *WZ*, *WT*, *WU*, or *WP*? (1 mark)

VCAA question

Q12. VCAA 5 marks

The city of Robville contains eight landmarks denoted as vertices N to U on the network diagram below. The edges of this network represent the roads that link the eight landmarks.



a) Write down the degree of vertex U (1 mark)

b) Steven wants to visit each landmark, but drive along each road only once. He will begin his journey at landmark N (2 marks)

i At which landmark must he finish his journey?

ii Regardless of which route Steven decides to take, how many of the landmarks (including those at the start and finish) will he see on exactly two occasions?

c) Cathy decides to visit each landmark only once. (2 marks)

i Suppose she starts at S, then visits R and finishes at T. Write down the order Cathy will visit the landmarks.

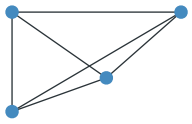
ii Suppose Cathy starts at S, then visits R but does not finish at T. List three different ways that she can visit the landmarks.

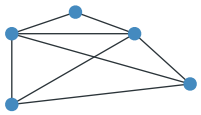
VCAA 2009 Exam 2 Module 5: Networks and decision mathematics. Q3

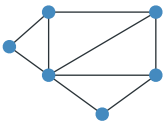
Questions from multiple lessons

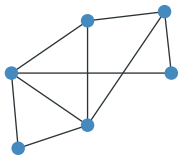
Q13. Difficulty: 1 mark

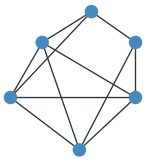
Which one of the following graphs is **not** a planar graph?

A.


B.


C.


D.


E.


VCAA 2018 Exam 1 Module 2: Networks and decision mathematics Q6 – Adapted

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Q14. Difficulty: 1 mark

Pauline deposited \$3500 into a savings account with an interest rate of 2.8% per annum, compounding annually. Which one of the following recurrence relations can be used to determine the amount in the savings account, S_n, after n years?

A. $S_0 = 3500, \quad S_{n+1} = S_n + 98$

B. $S_0 = 3500, \quad S_{n+1} = 9.8 \times S_n$

C. $S_0 = 3500, \quad S_{n+1} = 2.8 \times S_n$

D. $S_0 = 3500, \quad S_{n+1} = 1.028 \times S_n$

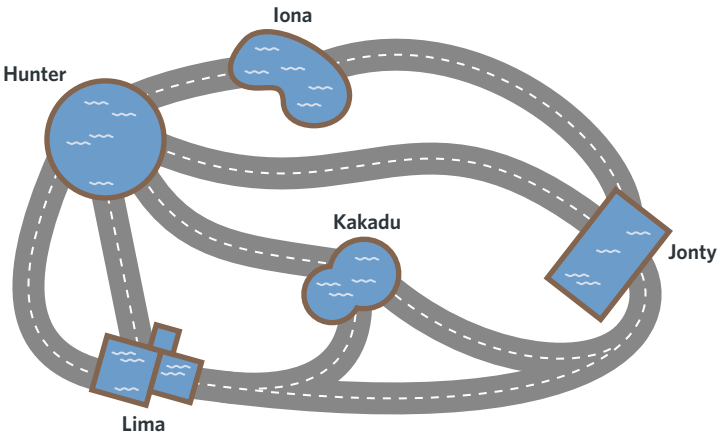
E. $S_0 = 3500, \quad S_{n+1} = S_n + 2.8$

VCAA 2017NH Exam 1 Recursion and financial modelling Q18 – Adapted

Q15. Shown below is a map of the roads between five public pools; Hunter (H), Iona (I), Jonty (J), Kakadu (K), and Lima (L).

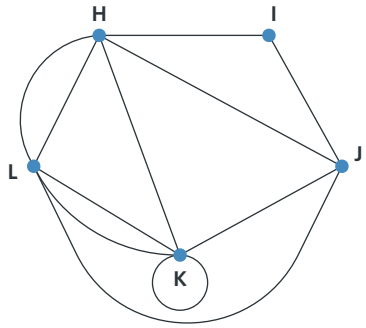
Difficulty:

3 marks



a) Alejandro is currently at Iona pool. Which two pools can he visit travelling only along one road? (1 mark)

b) The map has been converted into the following graph. However, the graph is missing an edge.



i Which two vertices is the missing edge between? (1 mark)

ii What does the loop at K represent in the context of travel from Kakadu pool? (1 mark)

VCAA 2016 Exam 2 Module 2: Networks and decision mathematics Q1 – Adapted

LESSON 5D

Weighted graphs

The key skills you will learn in this lesson are:

1. Interpreting a weighted graph
2. Finding the shortest path

VCAA key knowledge point:

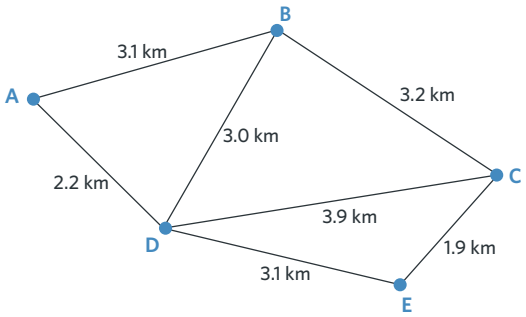
"weighted graphs and networks, and an introduction to the shortest path problem (solution by inspection only) and its practical application"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Interpreting a weighted graph

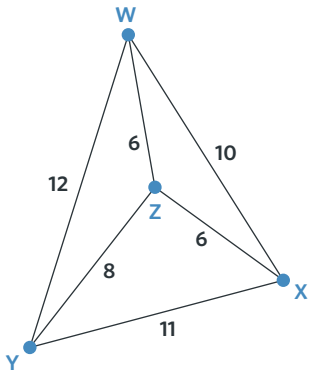
Weighted graphs are graphs with a number attached to each edge. This number indicates the length of the edge, and often represents the distance between two locations or the time taken to travel along the edge.

The weighted graph to the right shows the distance in kilometres between five locations, A, B, C, D, and E.



WORKED EXAMPLE 8 *(1 mark)*

The distances between W, X, Y and Z in kilometres are shown in the graph below.



What is the total length of the path $YZWX$?

SOLUTION

Step 1 Find the distances between Y and Z, Z and W and W and X.

Y and Z: 8 km

Z and W: 6 km

W and X: 10 km

Step 2 Add all the distances together.

$8 + 6 + 10 = 24$

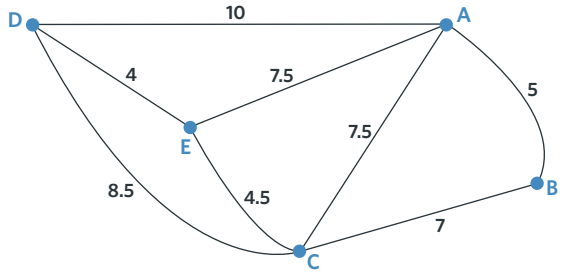
The path is 24 km long.

2. Finding the shortest path

Weighted graphs are very useful in real life situations and can be used to determine the shortest distance or the least amount of time needed to travel between two locations. The shortest path can be found by identifying all the likely shortest routes from one location to another, and comparing their lengths.

WORKED EXAMPLE 9 *(1 mark)*

What is the shortest route from B to D?



SOLUTION

Step 1 Write out all the likely shortest routes from B to D.

$BCED$

BCD

BAD

$BAED$

Step 2 Find the length of all the routes.

$BCED = 7 + 4.5 + 4 = 15.5$

$BCD = 7 + 8.5 = 15.5$

$BAD = 5 + 10 = 15$

$BAED = 5 + 7.5 + 4 = 16.5$

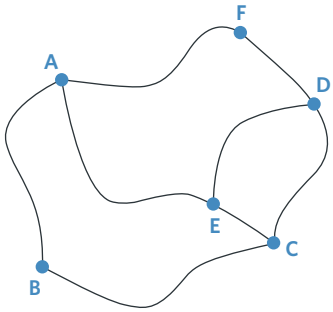
Step 3 Identify the shortest route.

The shortest route is BAD.

Questions 5D *Weighted graphs*

Refresher question

Q1. Identify a path from D to F that passes through all vertices exactly once.



1. Interpreting a weighted graph

Q2. The weighted graph shows the distance between five country towns A, B, C, D, and E in kilometres.

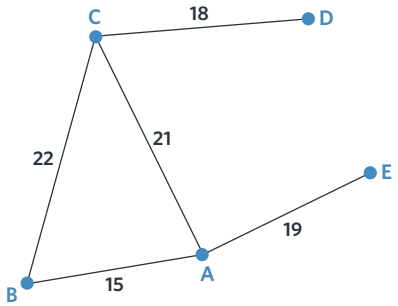
Skill

Find the distances of the following paths between the towns.

a) C to D

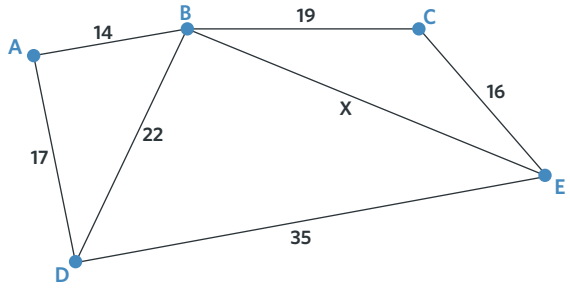
b) E to A to C

c) B to C to A to E



Q3. Skill

The weighted graph below shows the time taken to walk between five locations.
If it takes 59 minutes to walk the path *DABE*, what is the value of x?



Check your understanding

Q4. Application

Sabrina, Mary, Jessica and Victoria are four friends who all live in the same suburb. Sabrina lives three minutes away from Mary and 5 minutes away from Jessica.

Mary and Jessica are four minutes away from each other. Victoria lives two minutes away from Mary and three minutes away from Sabrina.

Draw a weighted diagram to represent the situation.

2. Finding the shortest path

Q5. Skill

The weighted graph below shows the time taken in minutes to walk between seven locations.

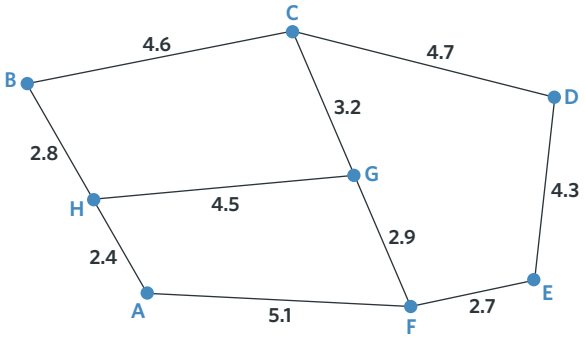
Walking along which of the following paths would take the shortest amount of time to get from *B* to *E*?

A. *BCGFE*

B. *BCDE*

C. *BHGFE*

D. *BHAFE*



Q6. Skill

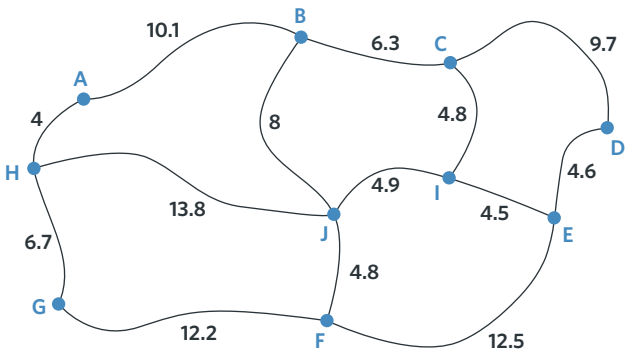
The weighted graph below shows the distance, in kilometres, of cycling trails in a park.

What is the distance of the shortest path between the following locations?

a) *H* and *E*

b) *A* and *F*

c) *G* and *D*

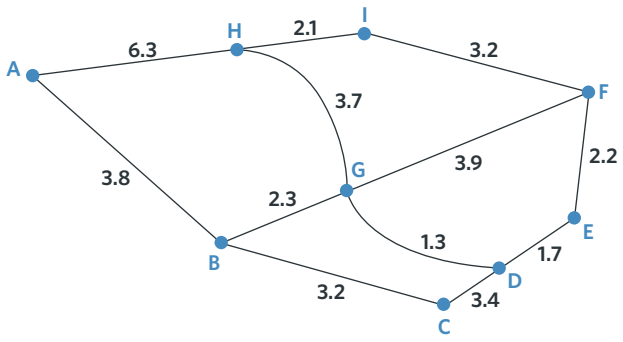


Check your understanding

Q7. Skill

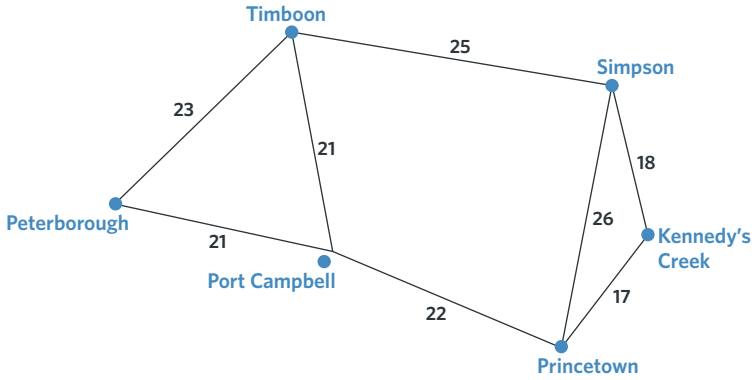
The numbers in the graph below represent distance in metres.

What is the shortest path from A to E, while avoiding vertex G? What is the distance of this path?



Q8. Application

Bailey is on holiday in Timboon and wants to visit a few nearby towns, as shown in the graph. If the numbers represent the distance between towns in kilometres, what is the distance of the path that starts at Timboon, travels through each town exactly once and ends at Kennedy's Creek?

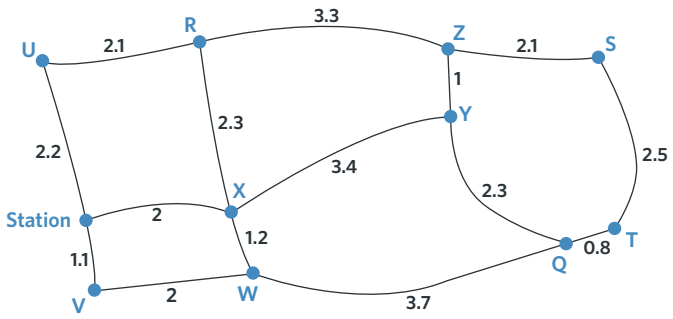


Joining it all together

Q9. Application 2 marks

Angus is travelling from Glen Waverley railway station to his home located at Y. He usually goes from the station to X to Y but the intersection at X is blocked off due to roadworks. The numbers represent distance in kilometres.

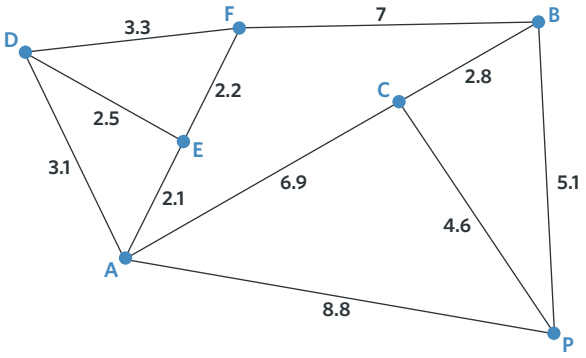
How much further does Angus have to travel compared to his usual route due to the roadworks, assuming that he takes the shortest path?



Q10. Application 1 mark

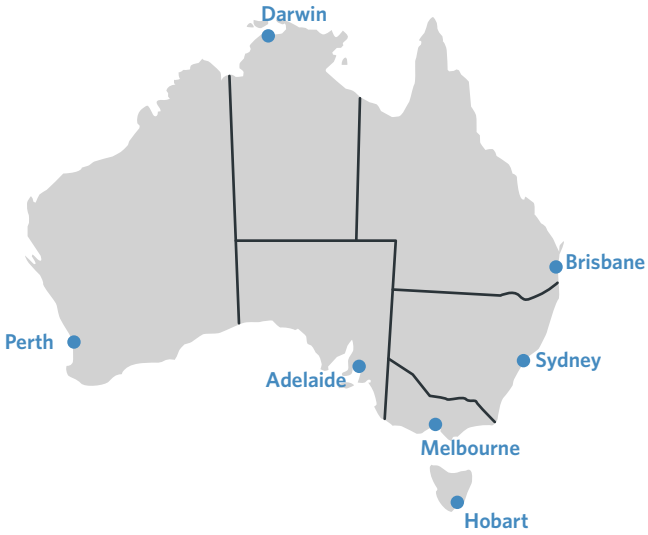
A postman leaves the post office located at point *P* and makes deliveries to each vertex on the weighted graph below before returning to the post office. The numbers represent distance in kilometres.

What is the shortest route he can take?



Q11. Application 3 marks

The capital cities of Australia are shown on the map below.



The table represents the distance between these cities in kilometres. Blank entries in the table indicate no direct path between the cities.

	Adelaide	Brisbane	Darwin	Hobart	Melbourne	Perth	Sydney
Adelaide			2620		650	2130	
Brisbane			2850				730
Darwin	2620	2850				2660	3150
Hobart					600	3010	1060
Melbourne	650			600			710
Perth	2130		2660	3010			
Sydney		730	3150	1060	710		

a) Draw a weighted graph to represent this situation. (2 marks)

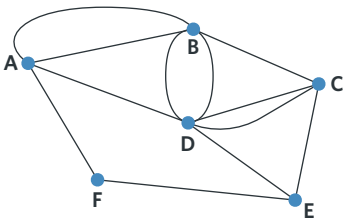
b) What is the shortest cycle beginning at Melbourne that passes through all the cities exactly once? (1 mark)

Questions from multiple lessons

Q12.

Difficulty:

1 mark



For the graph above, which one of the following is **not** a path?

A. *DEFAB*

B. *DCBAF*

C. *DBAFE*

D. *DABCE*

E. *DACEF*

VCAA 2018 Exam 1 Module 2: Networks and decision mathematics Q4 – Adapted

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Q13.

Difficulty:

Year 10
1 mark

The histogram below shows the distribution of *weight*, in kilograms, of 28 cats in an animal shelter.

The most common interval of *weight* for the cats is

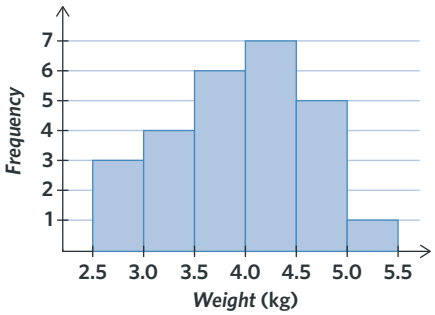
A. greater than or equal to 2.5 kg and less than 3.0 kg.

B. greater than or equal to 3.0 kg and less than 3.5 kg.

C. greater than or equal to 3.5 kg and less than 4.0 kg.

D. greater than or equal to 4.0 kg and less than 4.5 kg.

E. greater than or equal to 5.0 kg and less than 5.5 kg.



VCAA 2018NH Exam 1 Data analysis Q3 – Adapted

Q14.

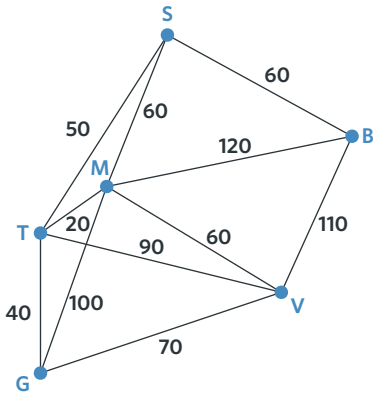
Difficulty:

3 marks

Train routes connect different cities in Spain.

Some of the cities are Madrid (M), Barcelona (B), Toledo (T), San Sebastian (S), Valencia (V), and Granada (G).

The graph below gives the cost, in dollars, of train travel along these routes.



Sophie is currently staying in Barcelona (B) and she wants to travel to Granada (G).

a) Sophie considers travelling by train along the route Barcelona (B) – Madrid (M) – Granada (G).
How much will she have to pay? (1 mark)

b) If Sophie takes the cheapest route from Barcelona (B) to Granada (G), which other town(s) will she pass through? (1 mark)

c) Complete Euler's formula by replacing v, f, and e with their respective values. The answer should be in the form $v + f = e + 2$. (1 mark)

VCAA 2017 Exam 2 Module 2: Networks and decision mathematics Q1 – Adapted

LESSON 5E

Trees, spanning trees and Prim's algorithm

The key skills you will learn in this lesson are:

1. Trees and spanning trees
2. Minimum spanning trees and Prim's algorithm

VCAA key knowledge point:

"Trees and minimum spanning trees, Prim's algorithm, and their use to solve practical problems."

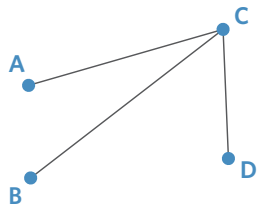
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Trees and spanning trees

A tree is a simple connected graph that contains no loops, circuits or duplicate edges. A tree is often part of a larger graph. The number of edges in a tree is always one less than the number of vertices.

$$number\ of\ edges = number\ of\ vertices - 1$$

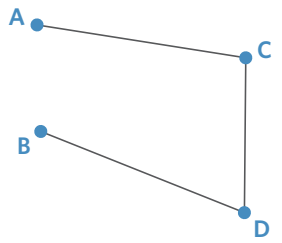
Example



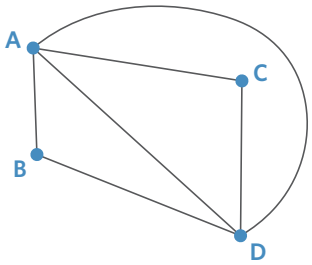
is a tree. It contains four vertices and three edges.

A spanning tree is a tree that contains all of the vertices in a larger graph, hence 'spanning' the entire graph.

Example

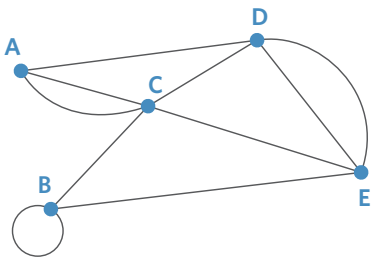


is a spanning tree of



WORKED EXAMPLE 10 *(1 mark)*

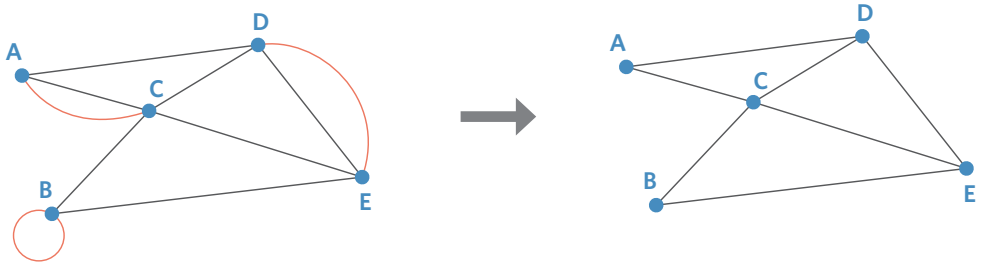
Draw a spanning tree for the graph below.



SOLUTION

Step 1 Remove any loops or duplicate edges.

There is a loop on vertex B. There are duplicate edges between vertices A and C and between vertices D and E.



Step 2 Remove the necessary number of remaining edges.

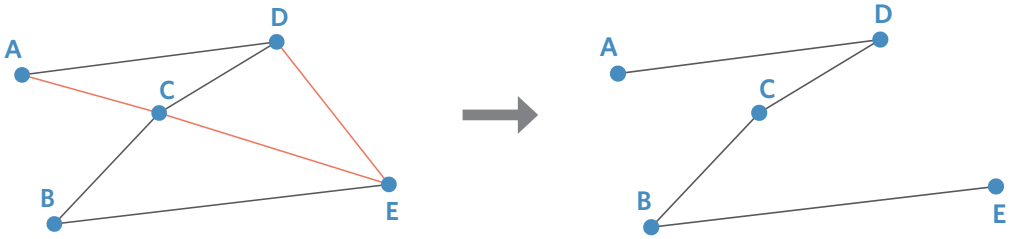
The number of edges in a tree is always one less than the number of vertices. In this graph there are 5 vertices (A,B,C,D and E).

$number\ of\ edges = number\ of\ vertices - 1$

$number\ of\ edges = 5 - 1 = 4$

Since the graph has five vertices, the tree must have four edges.

There are currently seven edges, so we have to remove three more while keeping the graph connected.



This is one of many possible spanning trees for this graph.

2. Minimum spanning trees and Prim's algorithm

A minimum spanning tree is the spanning tree of a weighted graph with the minimum total weight possible.

NOTE: There may be multiple minimum spanning trees for one graph.

Example

The total weight of a minimum spanning tree could indicate the minimum length of pipe needed to connect five water systems.

Prim's algorithm:

Prim's algorithm is a method for finding the minimum spanning tree for a graph.

The algorithm follows the steps below:

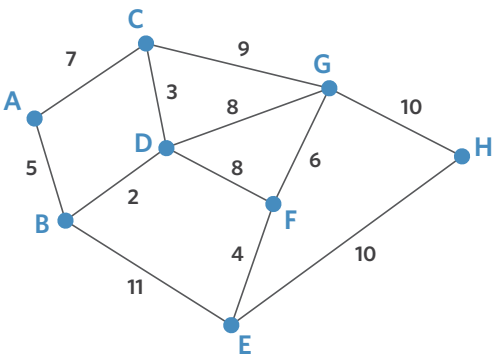
Step 1 Start at any vertex. Look at the edges that connect to this vertex and choose the edge with the lowest weight. If there are two edges of the same weight, pick either.

Step 2 There are now two vertices and one edge selected. Look at the edges that connect to either of these vertices and choose the one with the lowest weight that connects to a new vertex.

Step 3 Repeat this process until all vertices are connected.

WORKED EXAMPLE 11 *(1 mark)*

Use Prim's algorithm to find the minimum spanning tree for the graph below.



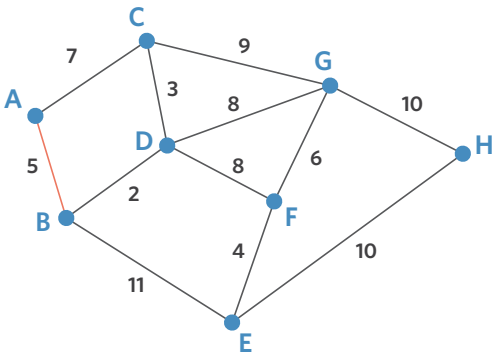
SOLUTION

Step 1 Choose a starting vertex and choose an edge.

We are going to start at vertex *A*.

There are two edges that connect to vertex *A*, with weights 7 and 5.

5 is the lowest value, so select the edge going to vertex *B*.

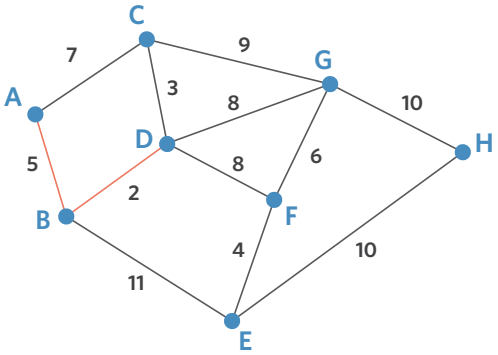


Step 2 Choose the next edge from either vertex *A* or *B*.

From vertex *A*, there is one edge with a weight of 7.

From vertex *B*, there are two edges with weights 2 and 11.

2 is the lowest value, so select the edge from vertex *B* to *D*.



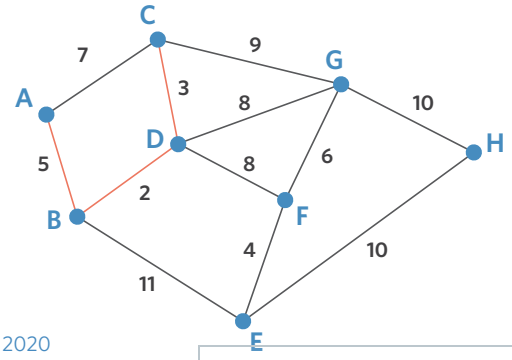
Step 3 Choose the next edge from either vertex *A*, *B* or *D*.

From vertex *A*, there is one edge with a weight of 7.

From vertex *B*, there is one edge with a weight of 11.

From vertex *D*, there are three edges with weights 3, 8 and 8.

3 is the smallest value. Select the edge connecting vertices *D* and *C*.



Step 4 Choose the next edge from either vertex *A*, *B*, *C* or *D*.

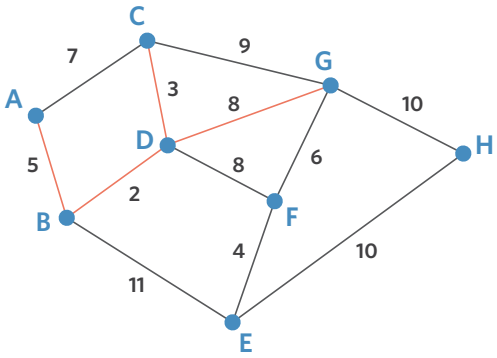
From vertex *B*, there is one edge with a weight of 11.

From vertex *C*, there is one edge with a weight of 9.

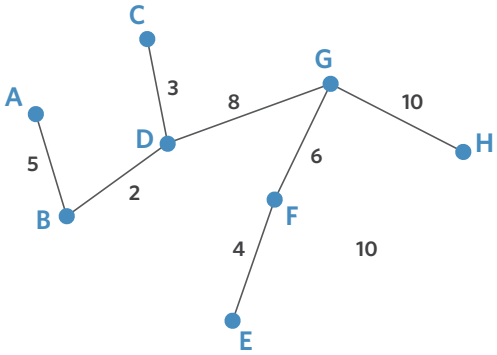
From vertex *D*, there are two edges with weights 8 and 8.

NOTE: Although the edge connecting vertices *A* and *C* has the lowest weight (7), it does not connect to a new vertex, so it cannot be part of the tree.

8 is the smallest value. Select either the edge connecting vertices *D* and *G* or *D* and *F*. Both choices are valid.



Step 5 Continue the process until all vertices are connected.



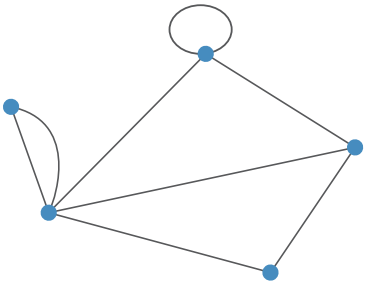
This is a minimum spanning tree for this graph. It has a total weight of 38.

NOTE: There are other possible minimum spanning trees for this graph.

Questions 5E *Trees, spanning trees and Prim's algorithm*

Refresher question

Q1. State the number of edges and vertices present in the graph below.



1. Trees and spanning trees

Q2. Skill

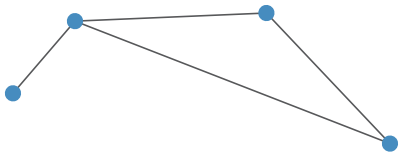
a) A tree has seven edges. How many vertices does it have?

b) Draw a tree with six vertices.

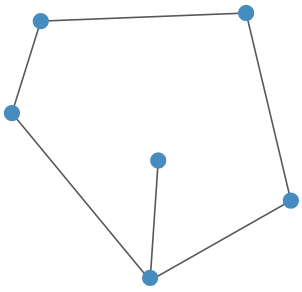
Q3. Skill

Which of the following graphs is a tree?

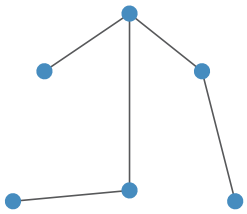
A.



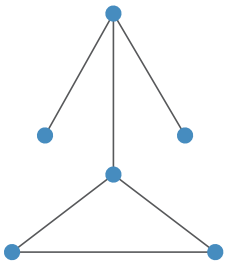
B.



C.



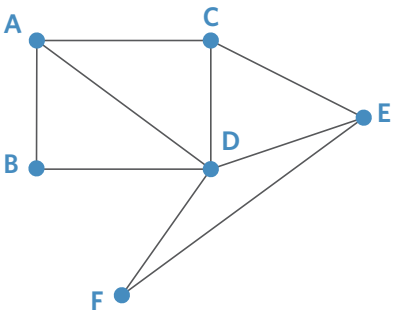
D.



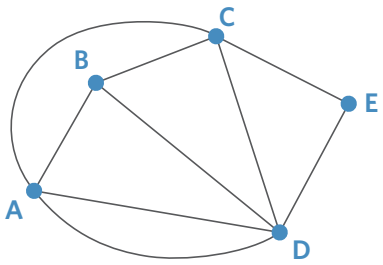
Q4. Skill

For the following graphs, draw three different spanning trees.

a)



b)

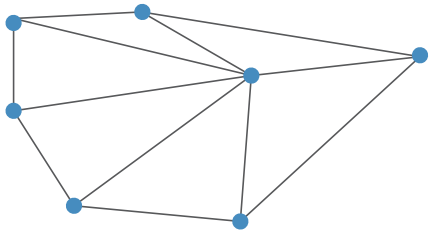


Check your understanding

Q5. Application

Sally the software engineer is setting up a computer network and is trying to connect all of the computers using the least number of wires possible.

To the right is a graph in which each vertex represents a computer and each edge is a possible connection path where Sally can place a wire.



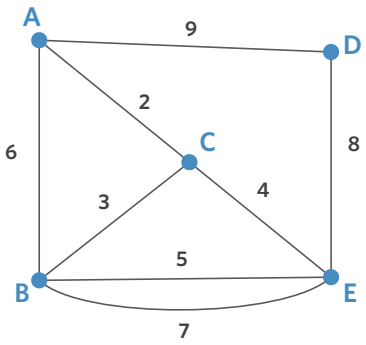
a) What is the least number of wires Sally can use?

b) Draw two possible layouts of the network.

2. Minimum spanning trees and Prim's algorithm

Q6. Skill

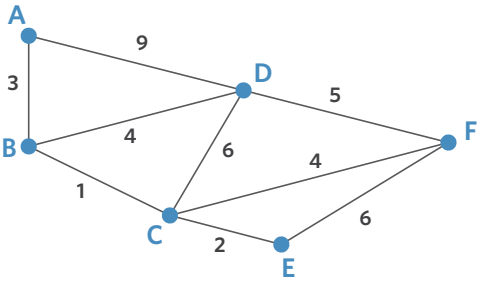
Draw the minimum spanning tree for the following graph.



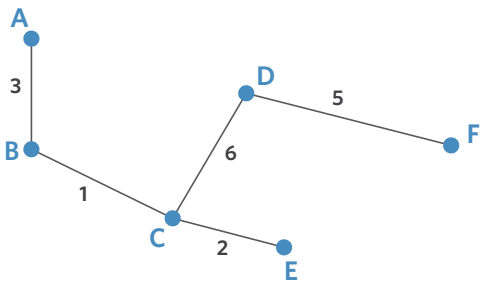
Q7. Skill

Consider the graph to the right.

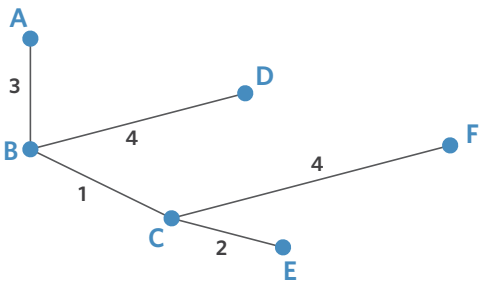
Which of the following trees is the minimum spanning tree for this graph?



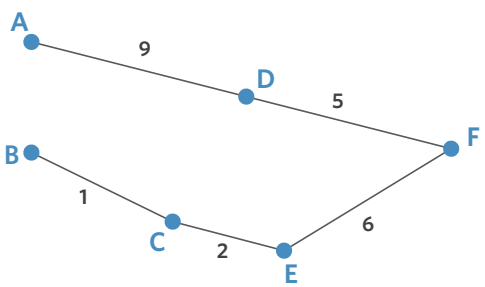
A.



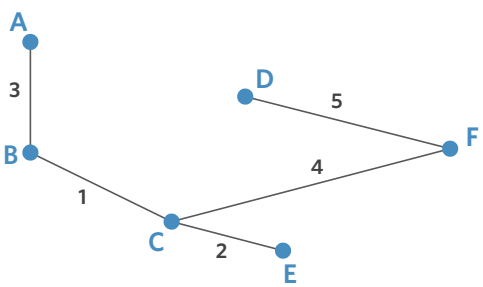
B.



C.

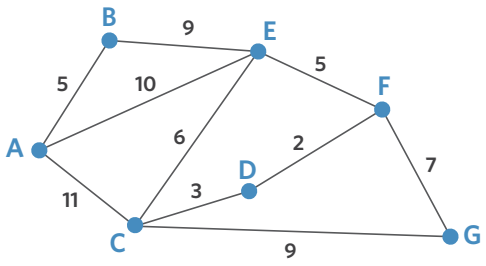


D.



Q8. Skill

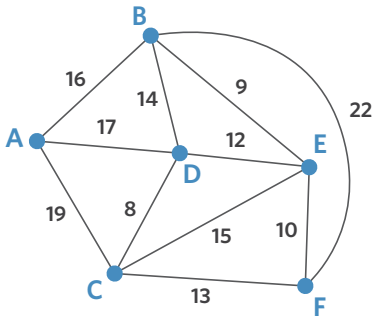
The total weight of the minimum spanning tree for the graph on the right is 31. Draw the minimum spanning tree.



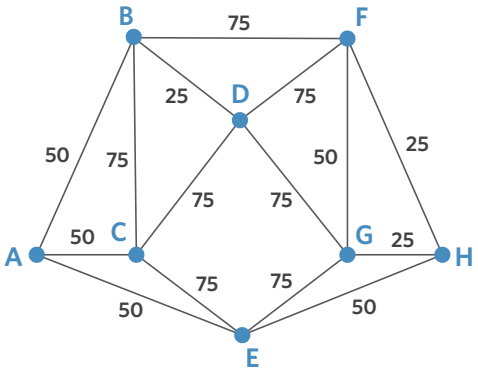
Q9. Skill

Use Prim's algorithm to find the minimum spanning tree for each of the following graphs.

a)



b)



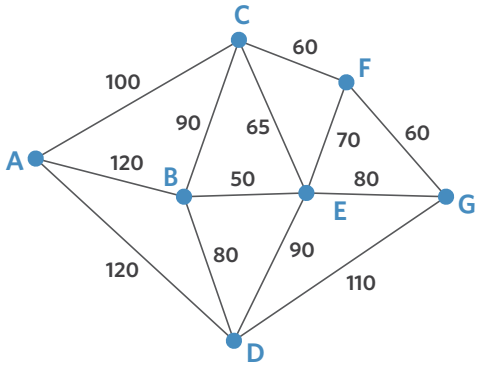
Check your understanding

Q10. Application

Underground tunnels are being built between seven different gold mines (A, B, C, D, E, F and G) so that they are all accessible directly or indirectly from every other mine. The possible tunnel routes between each mine and their lengths in metres are shown below.

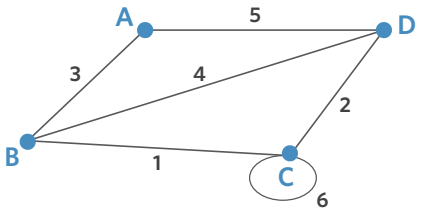
a) Draw the tunnel layout that would allow for the shortest total distance of tunnels, while keeping all of the mines connected.

b) In this layout, which mine has the most tunnels connected to it?



Joining it all together

Q11. Skill 3 marks



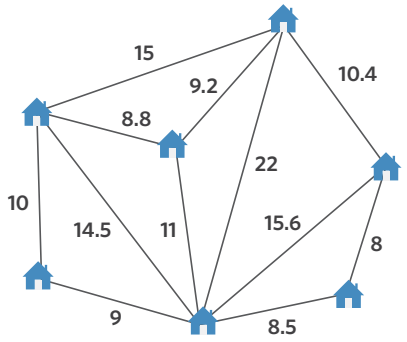
a) How many different spanning trees are there for this graph?

b) Draw the minimum spanning tree and state its total weight.

c) Draw the spanning tree with the maximum total weight and state its weight.

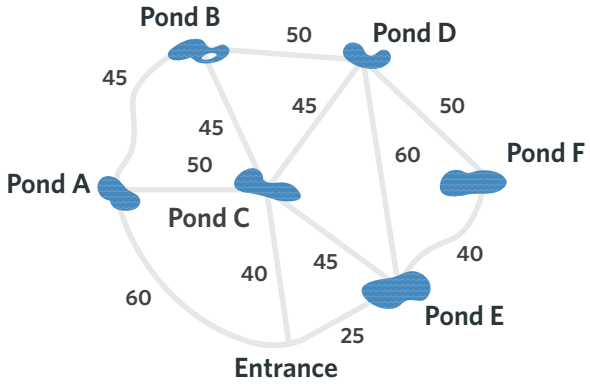
Q12. Application 1 mark

An electricity company is trying to connect the electricity for seven bungalows at a resort in Bora Bora. The amount of electric cable in metres needed to connect the bungalows is shown in the graph below. What is the minimum amount of cable needed?



Q13. Application 2 marks

A Japanese garden is being built in a park and the council wants to pave paths so that the six small ponds are accessible by path from the entrance. The distances in metres of the possible paths are shown in the graph below.

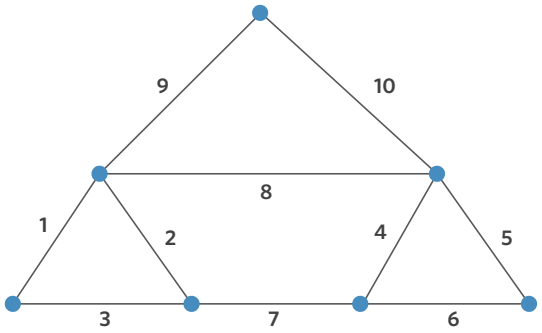


a) The council wants to minimise the length of the paths to minimise cost. Draw the path layout with the minimum total length that reaches every pond.

b) The cost to pave the paths is $120 per metre in length. How much will the council have to pay?

VCAA question

Q14. 1 mark



The minimal spanning tree for the network above will include the edge that has a weight of

A. 3 **B.** 6 **C.** 8 **D.** 9 **E.** 10

VCAA 2006 Exam 1 Module 5: Networks and decision mathematics. Q4

Questions from multiple lessons

Q15.

Difficulty:

1 mark

The following graph is made of seven isolated vertices. The minimum number of edges that must be added to the graph to form a tree is

A. 1

B. 5

C. 6

D. 7

E. 8

VCAA 2018 Exam 1 Module 2: Networks and decision mathematics Q1 – Adapted

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Q16.

Difficulty:

1 mark

Lachie is passionate about dogs.

Recently, he made his way to the local dog park and took the weights, heights and lengths of the first 13 dogs he could find, and recorded them in the following table.

Height (cm)	Weight (kg)	Length (cm)
58.2	25.8	55.1
20.1	9.7	22.5
38.4	15.9	32.9
23.9	12.1	28.0
61.2	31.9	70.5
34.1	19.1	30.1
42.1	18.1	38.1
50.1	21.1	51.3
29.1	8.4	34.7
32.7	13.8	29.1
38.1	17.1	35.0
35.9	19.2	38.1
40.1	20.1	41.1

The mean, $\overline{x}$, and the standard deviation, s_x, for the height of these dogs, in centimetres, are closest to

A. $\overline{x} = 38.8 \quad s_x = 12.08$

B. $\overline{x} = 38.8 \quad s_x = 12.06$

C. $\overline{x} = 38.7 \quad s_x = 12.08$

D. $\overline{x} = 38.7 \quad s_x = 12.06$

E. $\overline{x} = 41.8 \quad s_x = 11.18$

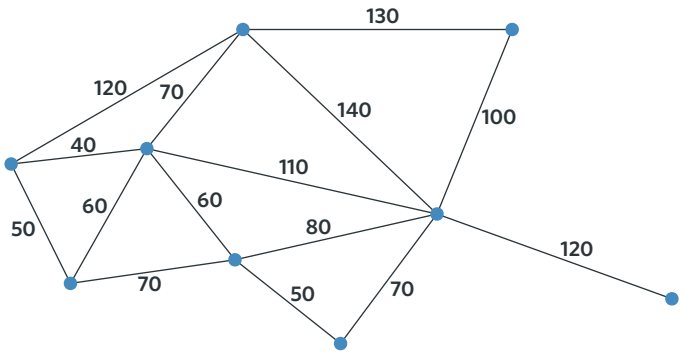
VCAA 2018NH Exam 1 Data analysis Q7 – Adapted

Q17.

Difficulty:

2 marks

A small country has nine towns. They are represented as vertices in the following graph. The edges represent possible power lines that could be built between the towns, with the numbers representing the distance, in kilometres, between them.



These power lines will create a connected graph. The country is trying to lower costs, so the shortest length of power line will be used.

a) What is the mathematical term that describes the graph represented by these power lines? (1 mark)

b) Draw the graph that represents these power lines. (1 mark)

VCAA 2017 Exam 2 Module 2: Networks and decision mathematics Q3 – Adapted

AOS 3: Discrete mathematics

Number patterns and recursion

CHAPTER 6

LESSON 6A

Introduction to sequences

The key skills you will learn in this lesson are:

1. Introduction to sequences
2. Notation
3. Plotting sequences

VCAA key knowledge points:

"the concept of a sequence as a function"

"use of a first-order linear recurrence relation to generate the terms of a number sequence"

"tabular and graphical display of sequences"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016–2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Introduction to sequences

A sequence is a set of numbers, known as terms, written from left to right.

Sequences with patterns

If a sequence has a pattern, such as a repeated addition or multiplication, we can predict the next term. Sequences with repeated addition are called **arithmetic** and sequences with repeated multiplication are called **geometric**.

The repeated operation is called the rule of the sequence.

Example

The following sequences involve patterns.

3, 6, 9, 12, 15, 18, 21

Rule: addition of three

2, 4, 8, 16, 32, 64, 128

Rule: multiplication by two

Sequences without patterns

Random sequences have no pattern.

Example

The following sequences are random sequences.

4, 12, −3, 5, 6, −1, 2, 2, −4

10, −4, 7, 5, 2, 8, −2, 0, 1

Finding the next term

By finding the pattern between terms in a sequence, it is possible to predict the next term.

Example

The pattern in this sequences is an addition of two:

13, 15, 17, 19, 21, ...

So the next term is 23.

WORKED EXAMPLE 1 *(2 marks)*

Do the following sequences have an addition rule, a multiplication rule, or are they random?

a) 2, 5, 8, 11, 14, 17, 20

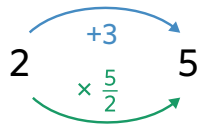
b) −3, −1, 5, 2, 7, −2, −6, 0

SOLUTION

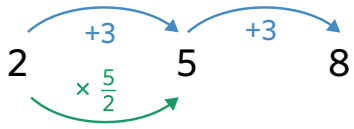
a) 2, 5, 8, 11, 14, 17, 20

Step 1 Look for any relationship between the terms.

The first two terms are 2 and 5. To go from the first to the second term, we can add 3, or multiply by $\frac{5}{2}$.

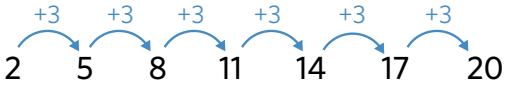


Step 2 The second term is 5, and the third term is 8. Does adding 3 or multiplying by $\frac{5}{2}$ transform 5 into 8?



Adding 3 to 5 results in 8.

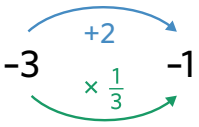
Step 3 Continue testing the rule for the other terms.



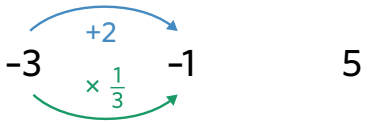
It satisfies all of the terms, so we can say the rule of the sequence is an addition of 3.

b) −3, −1, 5, 2, 7, −2, −6, 0

Step 1 The two first terms are −3 and −1. To get from −3 to −1, we can add 2, or multiply by $\frac{1}{3}$.



Step 2 Compare the second and third terms.



The second term is −1, and the third term is 5. Adding 2 to −1, we get 1. Multiplying −1 by $\frac{1}{3}$ we get $-\frac{1}{3}$. Neither of these operations work. The pattern is random.

NOTE: There may be a more complicated relationship, but based on the question we can assume this is a random sequence.

WORKED EXAMPLE 2 *(2 marks)*

Find the next term in the sequence:

a) −3, 1, 5, 9, 13

b) 1, 3, 9, 27

SOLUTION

a) −3, 1, 5, 9, 13

Step 1 Find the pattern.

$-3 + 4 = 1$

$1 + 4 = 5$

$5 + 4 = 9$

$9 + 4 = 13$

The pattern is that 4 is added each time.

Step 2 Use the pattern to determine the next term.

$13 + 4 = 17$

The next term is 17.

b) 1, 3, 9, 27

Step 1 Find the pattern.

$1 \times 3 = 3$

$3 \times 3 = 9$

$9 \times 3 = 27$

The pattern is that the term is multiplied by 3 each time.

Step 2 Use the pattern to determine the next term.

$27 \times 3 = 81$

The next term is 81.

2. Notation

There is specific notation used for sequences, which makes it easier to communicate which term is which.

The first value	The second value	The third value	and so on
t_0	t_1	t_2	...

Using 't' is quite common, but other letters can be used too, as long as it stays the same throughout the sequence.

The general form of a sequence is:

$t_0, t_1, t_2, t_3, t_4, ...$

Occasionally a sequence will start at t_1.

WORKED EXAMPLE 3 *(1 mark)*

A sequence is 4, 9, 14, 19, 24, 29, 34, with $t_0 = 4$. What is t_3?

SOLUTION

Step 1 The term on the left is t_0. Label the other terms up to t_3.

4, 9, 14, 19, 24, 29, 34

t_0 t_1 t_2 t_3

Step 2 Write the answer as an equation.

$t_3 = 19$

3. Plotting sequences

Graphing terms of a sequence can help us quickly identify if a sequence is arithmetic or not.

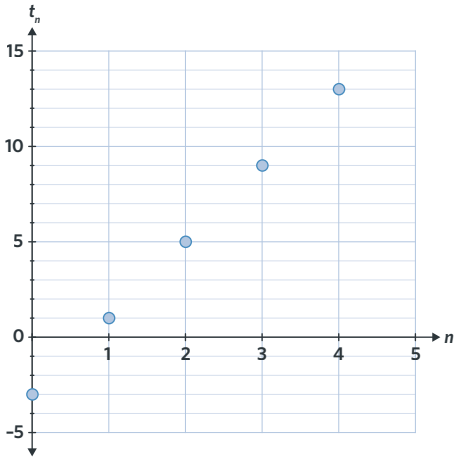
Generally n is placed along the x–axis, with t_n on the y–axis.

If the points form a straight line, the sequence is arithmetic.

Example

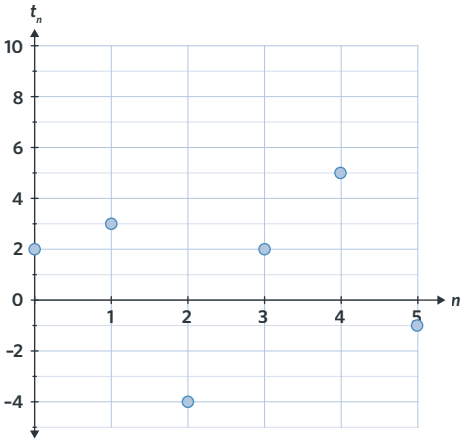
A graph of the sequence –3, 1, 5, 9, 13:

The line is straight, because this sequence is arithmetic.



A graph of a non–arithmetic sequence:

The sequence is 2, 3, –4, 2, 5, –1.



WORKED EXAMPLE 4 *(1 mark)*

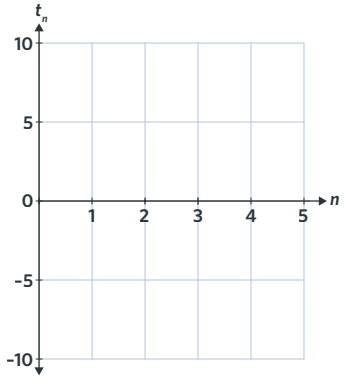
Draw a graph of the following sequence and state whether it is an arithmetic sequence or not.

6, 3, 0, –3, –6, –9.

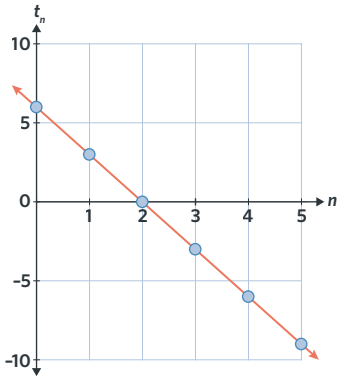
SOLUTION

Step 1 Choose to plot t_n from –10 to 10.

Choose to plot n from 0 to 5.



Step 2 Draw the sequence as a series of points and connect with a line.



Step 3 The line is straight, so we can conclude this is an arithmetic sequence.

WORKED EXAMPLE 5 *(1 mark)*

Draw a graph of the following sequence and state whether it is an arithmetic sequence.

–3, 1, 5, 9, 13

SOLUTION: TI-NSPIRE

Step 1 Open a new spreadsheet by pressing ctrl + doc and press 4 ('Add Lists & Spreadsheet').

Step 2 Scroll to the A box at the top of column 1, push 'n', then enter.

Scroll right to box B, and push 't', then enter.

Step 3 Scroll down to A1 and type '1', then enter.

Move to A2 and push '=a1+1'. Push enter.

Scroll up so cell A2 is highlighted.

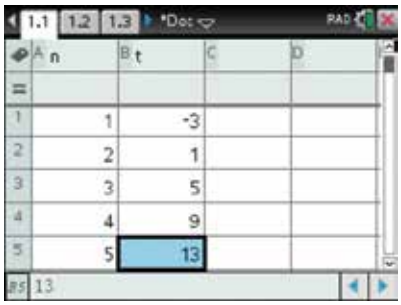
Push menu, then 3 for 'Data' and 3 again for 'Fill'.

Use the cursor highlight cells A2 to A5. Push enter.

The values of n are complete.

Step 4 Scroll to B1. Type the value for t_1, '–3', and push enter.

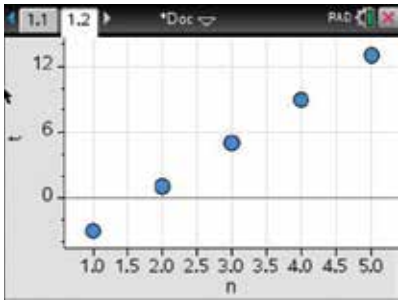
Enter the rest of the sequence into column B.



Press ctrl + doc and press 5 ('Add Data & Statistics').

At the bottom of the screen select 'Click to add variable' and choose 'n'.

On the left of the screen, select 'Click to add variable' and choose 't'.



The line is straight, so this sequence is arithmetic.

SOLUTION: CASIO CLASSPAD

Step 1 Using the stylus, select menu and then Spread-sheet.

In cell A1, type 1. Tap EXE.

In A2 type '=a1+1', and push EXE

Tap cell A2, then tap edit, fill, fill range.

Tap in the range box and type 'A2:A5'.

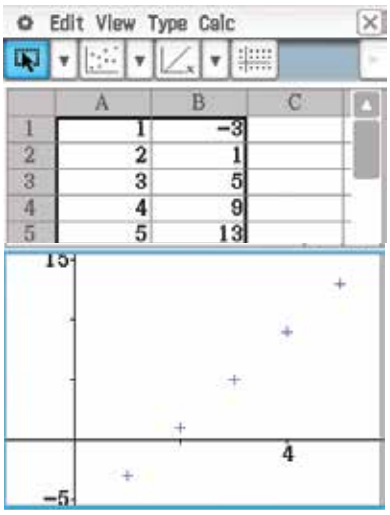
This creates the n values.

Step 2 In B1, type the value for t_1, '–3', and push EXE.

Enter the rest of the sequence into column B.

Step 3 Highlight all of the data, from A1 to B5. Tap.

In the graph screen, tap the down arrow and choose the scatter plot.



The line is straight, so this sequence is arithmetic.

Questions 6A *Introduction to sequences*

Refresher question

Q1. What is the next number in the sequence?

2, 4, 6, 8, 10, ?

1. Introduction to sequences

Q2. Skill

The following sequence has a rule of an addition of 5 between terms. What is the next number in the sequence?

3, 8, 13, ?

Q3. Skill

For each sequence below:

i Does the sequence have a rule or is it random?

ii If the sequence has a rule, what is the rule?

a) –4, 3, 10, 17, 24, 31 **b)** 13, 9, 8, –4, 0, –1, 8 **c)** 128, 64, 32, 16, 8

Q4. Skill

Find the pattern in the following sequences, and use the pattern to find the next term.

a) 2, 4, 8, 16 **b)** –4, 4, 12, 20, 28 **c)** 10, 4, –2, –8, –14

Check your understanding

Q5. Skill

Find the rule of this arithmetic sequence, and fill in the missing numbers:

11, ?, 5, 2, ?

Q6. Skill

Does the following sequence have a rule, or is it random? If it has a rule, what is the rule?

0.5, 1, 2, 5, 10, 16, 32, 36

2. Notation

Q7. Examine the table below, which shows a sequence with a rule of +2. What is the value of t_2?

Skill

t_0	t_1	t_2	t_3	t_4
7	9	11	13	15

Q8. Examine the following sequences, which begin at t_0.

Skill

a) State the value of t_0, t_2, and t_6.

3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43

b) State the value of t_0, t_3, and t_8.

16, 9, 2, –5, –12, –19, –26, –33, –40, –47, –54

c) State the value of t_0, t_4, and t_9.

–66, –55, –44, –33, –22, –11, 0, 11, 22, 33, 44

Check your understanding

Q9. Examine the following random sequence. What term, from t_0 to t_{10}, is the largest?

Skill

–2, 5, 3, 1, –4, –2, 4, 9, 2, –3, –2

Q10. The daily maximum temperature over a week in Melbourne in May is given in the table below.

Application

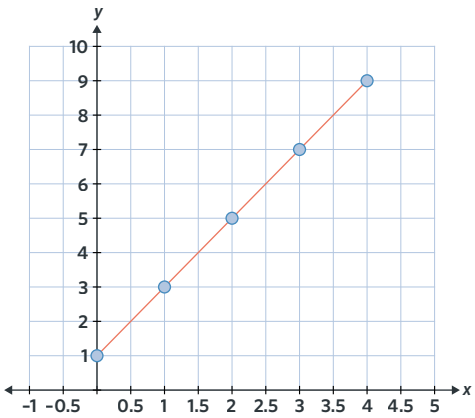
Which day, from D_1 to D_7, was the hottest?

D_n	D_1	D_2	D_3	D_4	D_5	D_6	D_7
Temperature in Celsius	16°	16°	19°	20°	19°	18°	13°

3. Plotting sequences

Q11. A plot of a sequence is given below. Is this sequence arithmetic?

Skill



Q12. Draw the plots the following sequences by hand (without using your calculator).

Skill

Is the sequence arithmetic?

a) 10, 6, 2, –2, –6

b) 2, 3, 4, 6, 7

c) –3, 2, 7, 12, 17

Q13. Skill

Create the plots the following sequences on your calculator.

Based on the calculator graph, is the sequence arithmetic?

a) 3, 5, 7, 9, 11 **b)** 4, 2, 0, –3, –6

Check your understanding

Q14. Application

Paul is the manager of an Italian restaurant. He is trying to roster staff for the coming week, and looks at the average number of guests visiting on Thursdays to Sundays. The average number of guests on Thursdays is 20, Fridays is 32, Saturdays is 54, and Sundays is 42.

Given that the number of guests from Thursday to Sunday is represented by t_1 to t_4, plot this information and find if it is an arithmetic sequence.

Q15. Application

The number of tourists visiting Wilson's Promontory in the weeks leading up to Christmas is shown in the table below. Plot the sequence. Is this an arithmetic sequence?

Week	December 1–7	December 8–14	December 15–21	December 22–28
t_n	t_1	t_2	t_3	t_4
Number of tourists	1350	1700	2050	2400

Joining it all together

Q16. Skill 1 mark

Which of the following arithmetic sequences has the highest t_5 value? Assume they all start at t_0.

A. 0, 4, 8, 12, 16, 20 **B.** 52, 44, 36, 28, 20, 12

C. –40, –24, –8, 8, 24, 40 **D.** –1, 4, 9, 14, 19, 24, 29

Q17. Skill 2 marks

Find the pattern in the following arithmetic sequence. Use this to work out t_4.

t_0	t_1	t_2	t_3	t_4
–12	5	22	39	?

Q18. Skill 2 marks

Find the pattern in the following arithmetic sequence. Use this to work out t_4.

t_0	t_1	t_2	t_3	t_4
253	185	117	49	?

Q19. Application 3 marks

A local football club opened in 2016. The number of members of the club every year after its inception in 2016 is stated below in the table. The number of members in 2016 is represented by m_0

Year	**Number of members (m_n)**
2016	340
2017	420
2018	500
2019	580

a) Plot this information.

b) What type of sequence is this?

c) Assuming the sequence continues, in what year will the club first reach 1000 members?

Q20. Nadia bought her car two years ago, when she first got her learner permit. She bought it for $4000.

Application
2 marks

After a year, the car was revalued at $3250. Her insurance company inspected the car today and valued it at $2500.

a) Plot this information, using v_0 as the price when Nadia bought the car.

b) Assuming the arithmetic sequence continues, in how many years will it be worth $1000?

VCAA question

Q21. Paul went running every morning from Monday to Sunday for one week.

1 mark

On Monday, Paul ran 1.0 km.

On Tuesday, Paul ran 1.5 km.

On Wednesday, Paul ran 2.0 km.

The number of kilometres that Paul ran each day continued to increase according to this pattern.

The number of kilometres that Paul ran on Thursday is

A. 2.5

B. 3.0

C. 3.5

D. 4.0

E. 5.0

VCAA 2014 Exam 1 Module 1: Number patters. Q1

Questions from multiple lessons

Q22. The first five terms in a geometric sequence are: 32, 16, 8, 4, 2,...

Difficulty:

What is the next term in this sequence?

Year 10
1 mark

A. -2

B. -1

C. 0

D. 1

E. 2

VCAA 2015 Exam 1 Module 1: Number patterns Q1 – Adapted

Q23. A least squares line in the form $y = a + bx$ is fitted to a set of bivariate data.

Difficulty:

For this set of data, $\overline{x} = 4.30$, $\overline{y} = 5.12$, $s_x = 2.94$, $s_y = 1.98$ and $a = -2.8$.

1 mark

What is the value of b correct to one decimal place?

A. -1.7

B. -1.8

C. -1.9

D. 1.8

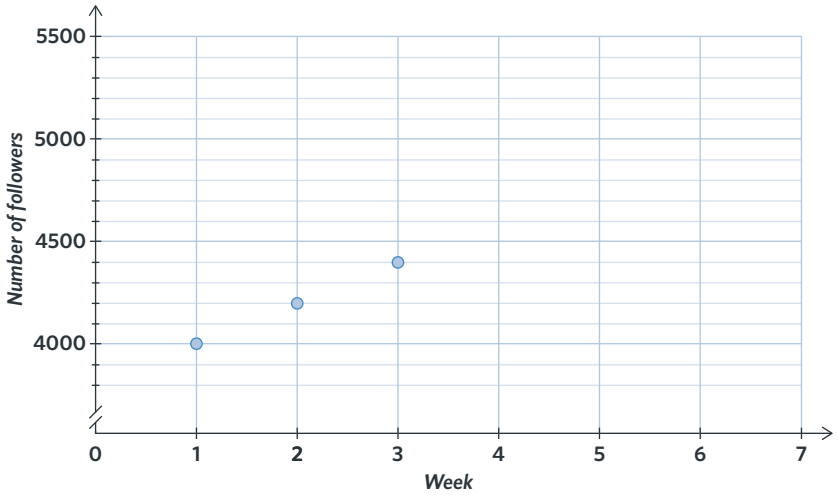
E. 1.9

VCAA 2019NH Exam 1 Data Analysis Q9 – Adapted

Q24. Hector is a rising TikTok star and his number of followers increases weekly at a constant rate. The graph below shows his number of followers in the first four weeks of 2020.

Difficulty:

Year 10
2 marks



a) How many followers does Hector gain each week? (1 mark)

b) How many followers will Hector have in week 6? (1 mark)

VCAA 2011 Exam 2 Recursion and financial modelling Q1 a,b – Adapted

LESSON 6B

Generating arithmetic sequences

The key skills you will learn in this lesson are:

1. Finding recurrence relations
2. Generating terms

VCAA key knowledge points:

"generation of an arithmetic sequence using a recurrence relation, tabular and graphical display; and the rule for the n^{th} term of an arithmetic sequence and its evaluation"

"use of a recurrence relation to model and analyse practical situations involving discrete linear growth or decay such as a simple interest loan or investment, the depreciating value of an asset using the unit cost method; and the rule for the value of a quantity after n periods of linear growth or decay and its use"

1. Finding recurrence relations

A formula that links one term in a sequence to the next is called a recurrence relation.

For arithmetic sequences, recurrence relations are of the form:

$$t_{n+1} = t_n + d,\ t_0 = a$$

Here, d is the common difference between terms, and a is the first value in the sequence. The term t_{n+1} is the next term in the sequence, and t_n is the current value. The term t_{n-1} represents the previous term in a sequence.

Example

An arithmetic sequence is given by $t_{n+1} = t_n + 4$ where $t_0 = 7$.

The average price of petrol in dollars over the last five years is given by $p_{n+1} = p_n + 0.03$ where $p_0 = 1.41$.

WORKED EXAMPLE 6 *(2 marks)*

Find the recurrence relation for the following arithmetic sequences.

a) –8, –5, –2, 1, 4, 7...

b) 43, 35, 27, 19, 11, 3...

SOLUTION

a) –8, –5, –2, 1, 4, 7...

Step 1 Find the starting value of the sequence.

The starting value is always on the left. Here it is –8. Write this as a.

$a = -8$

Step 2 Find the common difference, d.

The common difference is the difference between each pair of adjacent terms. Choose two terms in the sequence that are next to each other and find the difference between them. It is usually easiest to use t_0 and t_1.

$$\begin{aligned} d &= t_1 - t_0 \\ &= -5 - (-8) \\ &= -5 + 8 \\ &= 3 \end{aligned}$$

Step 3 Write the recurrence relation using a and d.

$t_{n+1} = t_n + d,\ t_0 = a$

$t_{n+1} = t_n + 3,\ t_0 = -8$

b) 43, 35, 27, 19, 11, 3...

Step 1 Find the starting value of the sequence.

As always, the starting value is always the value on the left. Here is it 43. Write this as a.

$a = 43$

Step 2 Find the common difference, d.

$d = t_1 - t_0$

$= 35 - 43$

$= -8$

Step 3 Write the recurrence relation using a and d.

$t_{n+1} = t_n + d, t_0 = a$

$t_{n+1} = t_n - 8, t_0 = 43$

2. Generating terms

It is easy to calculate future values of a sequence when given a recurrence relation. By using the recurrence relation and substituting the first value, the second value can be found. The second value can be used to find the third, and so on.

Example

A sequence is given by $t_{n+1} = t_n + 3, t_0 = 10$. Future values can be calculated simply, as shown below.

Let $n = 0$.

$t_1 = t_0 + 3$

$t_1 = 10 + 3$

$t_1 = 13$

Let $n = 1$.

$t_2 = t_1 + 3$

$t_2 = 13 + 3$

$t_2 = 16$

WORKED EXAMPLE 7 *(1 mark)*

Calculate t_3 for the sequence $t_{n+1} = t_n - 4, t_0 = 11$.

SOLUTION

Step 1 Work out which values we need to find and which values we have.

We need to find t_3 and we only have t_0 at present. First we'll need to find t_1, use t_1 to find t_2, and use t_2 to find t_3.

Step 2 Use the recurrence relation to calculate the desired values.

Let $n = 0$.

$t_1 = t_0 - 4$

$t_1 = 11 - 4$

$t_1 = 7$

Let $n = 1$.

$t_2 = t_1 - 4$

$t_2 = 7 - 4$

$t_2 = 3$

Let $n = 2$.

$t_3 = t_2 - 4$

$t_3 = 3 - 4$

$t_3 = -1$

Questions 6B *Generating arithmetic sequences*

Refresher question

Q1. What is the difference between the following pairs of numbers?

a) 1, 3

b) 4, 12

1. Finding recurrence relations

Q2. Skill

What is the common difference in the following arithmetic sequence?

1, 4, 7, 10, 13...

Q3. Skill

Write down the recurrence relations that represent these sequences.

a) 3, 5, 7, 9, 11...

b) 19, 12, 5, –2, –9...

c) –13, –7, –1, 5, 11...

Check your understanding

Q4. Application

The number of students in a new dance class is 6 in the first week, 9 in the second week, 12 in the third week, and 15 in the fourth week.

Write the number of students in the class every week as a recurrence relation, where S_0 is the number of students in the first week.

Q5. Application

The number of people buying phones from Joan's Fones, on Tuesday and Wednesday, is given by $C_1 = 10$ and $C_2 = 12$.

Assuming the number of customers buying phones is an arithmetic sequence, state the number of customers that bought phones on Monday, and write the recurrence relation.

2. Generating terms

Q6. Skill

If $t_0 = 4$ and the common difference between terms is 3, what is t_1?

Q7. Skill

Find t_3 for the following arithmetic sequences, using the recurrence relations.

a) $t_{n+1} = t_n + 6, t_0 = 3$

b) $t_{n+1} = t_n - 7, t_0 = 16$

c) $t_{n+1} = t_n - 14, t_0 = -2$

Check your understanding

Q8. Application

The number of cups of lemonade Jackson sells at his lemonade stand on day 1 is 23. The next day he sells two more. Assuming the sales form an arithmetic sequence, state how many cups are sold on day 5.

Q9. Application

The population of Australia in 1900 was around 3.76 million people. The population of Australia over the years 1900–1904 can be roughly modelled by using the recurrence relation:

$$P_{n+1} = P_n + 65\,000, P_0 = 3\,760\,000$$

What was Australia's population in 1904?

Joining it all together

Q10. Which of these arithmetic sequences has term t_4 not equal to 15?

Skill
1 mark

A. $t_{n+1} = t_n + 2, t_0 = 5$ **B.** $t_{n+1} = t_n - 8, t_0 = 47$

C. $t_{n+1} = t_n + 7, t_0 = -13$ **D.** $t_{n+1} = t_n - 3, t_0 = 27$

Q11. Ava's height, in metres, from 12 years of age to 17 years of age is modelled by $h_{n+1} = h_n + 0.05$, $h_0 = 1.25$. Show that at age 17 she is 1.50 metres tall.

Application
1 mark

Q12. Charlotte heard that human hair grows according to the equation $h_{n+1} = h_n + 0.08$, where h is the length of hair in centimetres, and n is the number of days. She measures her hair on the third of April and finds it is 23.84 cm long. One week later she measures it again.

Application
1 mark

Use the recurrence relation to work out how long her hair will be.

Q13. The area of forest across the world from 2013 to 2015 is listed in this table.

Application
3 marks

Year	Area of forest (million hectares)
2013	4065
2014	4032
2015	3999

a) Assuming this information is an arithmetic sequence, using 2013 as the start of the sequence, represent this information as a recurrence relation.

Use F_n to represent the area of forest in year n.

b) Use the recurrence relation to calculate the amount of forest left in 2016.

c) Find the first year the area of forest in the world will drop below 3.9 billion hectares.

Q14. The increase in average global temperature predictions for the decades leading up to 2050 is shown in the table on the right.

Application
3 marks

Decade	Average global temperature compared to 1900–2000
2010s	+0.8°
2020s	+0.975°
2030s	+1.15°
2040s	+1.325°

a) Write the average global temperature compared to 1900–2000 each decade, T_n, as a recurrence relation. Take T_0 to represent the 2010s.

b) In what decade will the average global temperature have risen by more than 2° for the first time?

c) Sea level is also projected to rise over the coming century. One scientist created the following model, where S_n is the increase in sea level in millimetres, in decade n. The 2010s are represented by $n = 0$. According to this model, in what decade will increase in sea level pass 30 cm?

$S_{n+1} = S_n + 36, S_0 = 88$

VCAA question

Q15. Kai commenced a 12-day program of daily exercise. The time, in minutes, that he spent exercising on each of the first four days of the program is shown in this table.

1 mark

Day number	1	2	3	4
Time (minutes)	15	19	23	27

If this pattern continues, the total time (in minutes) that Kai will have spent exercising after 12 days is

A. 59 **B.** 180 **C.** 354 **D.** 444 **E.** 468

Adapted from VCAA 2008 Exam 1 Module 1: Number patterns. Q6

Questions from multiple lessons

Q16.

Difficulty:

1 mark

Lorraine has started a small business selling homemade pottery.

Her business's monthly profit forms an arithmetic sequence.

The following table shows her profit in months 1 to 3.

Month number	Profit ($)
1	170
2	215
3	260

If this pattern continues, what will Lorraine's profit be in month seven?

A. $380

B. $395

C. $415

D. $440

E. $485

VCAA 2009 Exam 1 Module 1: Number patterns Q2 – Adapted

Q17.

Difficulty:

1 mark

A least squares regression line is fitted to the following data.

x	−1.2	2.7	3.2	−0.9	7.3	4.7	−9.4	2.3
y	3.4	8.3	7.4	2.5	5.2	8.3	−0.6	1.5

What is the equation of the line? Round all values to two decimal places.

A. $y = 4.21 + 0.53x$

B. $y = 3.42 + 0.44x$

C. $y = 3.97 + 0.48x$

D. $y = 4.58 + 0.67x$

E. $y = 4.95 + 0.32x$

VCAA 2018NH Exam 1 Data analysis Q8 – Adapted

Q18.

Difficulty:

Year 10

3 marks

A large umbrella factory wants to determine the number of umbrellas they can produce each hour.

- In the 1st hour, they produced 30 000 umbrellas.
- In the 2nd hour, they produced 32 000 umbrellas.
- In the 3rd hour, they produced 34 000 umbrellas.

The number of umbrellas produced in each hour continues in this pattern for 10 hours.

The amount of stock produced in each hour forms the terms of an arithmetic sequence, as shown below.

30 000, 32 000, 34 000...

a) Show that the common difference for this sequence is 2000. (1 mark)

b) How many umbrellas will be produced during the 8th hour? (1 mark)

c) In total, how many umbrellas can be produced in the first five hours? (1 mark)

VCAA 2015 Exam 2 Module 1: Number patterns Q1 a-c – Adapted

LESSON 6C

Applications of arithmetic sequences

The key skills you will learn in this lesson are:

1. Defining and using the formula to find the value of the term after *n* periods
2. Applications of the formula
3. Interest and depreciation

VCAA key knowledge points:

"generation of an arithmetic sequence using a recurrence relation, tabular and graphical display; and the rule for the n^{th} term of an arithmetic sequence and its evaluation"

"use of a recurrence relation to model and analyse practical situations involving discrete linear growth or decay such as a simple interest loan or investment, the depreciating value of an asset using the unit cost method; and the rule for the value of a quantity after *n* periods of linear growth or decay and its use"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016–2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Defining and using the formula to find the value of the term after *n* periods

For an arithmetic sequence, there is a formula that gives the value of any term in the sequence. This is useful if we want to find a specific term far along in the sequence, without having to find each of the preceding terms first.

An expression for the value in the sequence after *n* periods is given by $t_n = a + nd$, where a is the first term (t_0) of the sequence and d is the common difference.

It is important to note that *n* represents how many periods have passed. For example, if an arithmetic sequence is used to describe the balance of a bank account earning simple interest on a monthly basis:

- $a = t_0$ is the 1^{st} term which would represent the opening balance of the account
- t_1 is the value after one month (i.e. the 2nd term)

To find the 15^{th} term in the sequence, we need to solve for t_{14} so we would substitute $n = 14$ into the equation.

This is summarised in the table below:

$n = 0$	$n = 1$	$n = 2$	$n = 3$	...	$n = n$
t_0	t_1	t_2	t_3	...	t_n
after 0 periods	after 1 period	after 2 periods	after 3 periods	...	after *n* periods
initial/1^{st} term	2^{nd} term	3^{rd} term	4^{th} term	...	$(n + 1)^{th}$ term

WORKED EXAMPLE 8 *(2 marks)*

For the following number sequences, find an expression for the value of the term after *n* periods.

a) 3, 8, 13, 18, 23, ... **b)** 21, 18.5, 16, 13.5, 11, ...

SOLUTION

a) 3, 8, 13, 18, 23, ...

Step 1 Define the values of a and d.

a is the initial value.

$a = 3$

d is the difference between consecutive values.

$d = 8 - 3$

$d = 5$

Step 2 Substitute the values of a and d into the formula, then simplify.

$t_n = a + nd$

$t_n = 3 + n \times 5$

$\therefore t_n = 3 + 5n$

b) 21, 18.5, 16, 13.5, 11, ...

Step 1 Define the values of a and d.

a is the initial value.

$a = 21$

d is the difference between consecutive values.

$d = 18.5 - 21$

$d = -2.5$

NOTE: d is negative when each value is lower than the one before.

Step 2 Substitute the values of a and d into the formula, then simplify.

$t_n = a + nd$

$t_n = 21 + n \times (-2.5)$

$\therefore t_n = 21 - 2.5n$

WORKED EXAMPLE 9 *(2 marks)*

Given the formula $t_n = 8 + 6n$, find t_5 and t_{25}.

SOLUTION

Step 1 Substitute $n = 5$ into the formula and solve.

$t_n = 8 + 6n$

$t_5 = 8 + 6 \times 5$

$t_5 = 38$

✓ *1 mark for finding t_5*

Step 2 Substitute $n = 25$ into the formula and solve.

$t_n = 8 + 6n$

$t_{25} = 8 + 6 \times 25$

$t_{25} = 158$

✓ *1 mark for finding t_{25}*

Step 3 Write the answer.

$t_5 = 38$ and $t_{25} = 158$.

2. Applications of the formula

The basic formula $t_n = a + nd$ is used to find the value of the term after n periods (t_n), which is the value of the $(n + 1)^{th}$ term.

However, in some cases this value is given and the question requires one of the other components to be found. This can be done by using these formulae:

Finding	Use the formula
the initial value, a	$a = t_n - nd$
the common difference, d	$d = \frac{t_n - a}{n}$
the term number, n	$n = \frac{t_n - a}{d}$

NOTE: These are found by transposing (rearranging) the original formula.

WORKED EXAMPLE 10 *(1 mark)*

At the beginning of a weight-loss program, Sam weighed 96 kg. After the 15th week, he weighed 84 kg.

Assuming his weight loss was linear, how much weight did Sam lose each week?

SOLUTION

Step 1 Write down the known values from the question.

The question states that after the first week, Sam weighed 96 kg. This means that $a = 96$.

After the 15th week, Sam weighed 84 kg.

Hence, $n = 15$ and $t_{15} = 84$.

Step 2 Identify the unknown value.

We know the values of a, t_n, and n. The only unknown value is d. d is the difference between each term, which relates to the amount of weight Sam lost each week.

Step 3 Substitute the known values into the formula for d and solve.

$$d = \frac{t_n - a}{n}$$

$$d = \frac{84 - 96}{15}$$

$$d = \frac{-12}{15}$$

$$d = -0.8$$

Step 4 Interpret the result.

The common difference is –0.8.

This means that each term in the sequence is 0.8 less than the previous term.

∴ Sam lost 0.8 kg each week.

3. Interest and depreciation

Interest

Simple interest is calculated as a fraction of the initial amount of money invested, so the amount of interest remains constant as the balance changes. The amount of interest earned/paid per time period is given by $i = \frac{Pr}{100}$ where P is the principal (initial amount invested), and r is the interest rate per time period (expressed as a percentage). Note that i is used instead of I as i is the interest earned/paid in a **single** time period, whilst I is the total interest earned/paid over n time periods.

The balance of a simple interest account can be modelled as an arithmetic sequence, where the common difference, d, represents the interest earned/paid each time period (i.e. $d = i$), and the initial value, a is the principal (i.e. $a = P$). t_n is the balance of the account after n periods.

Depreciation

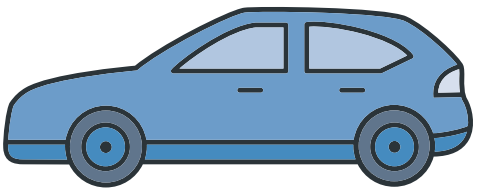
Unit cost depreciation is a type of depreciation in which the value of an item reduces linearly as it is used. For example, the value of a second-hand car depreciates by the same amount each kilometre driven. The point at which the value of an item has depreciated so much that it is equal to 0 is known as the write-off value.

The value of the item can be modelled as an arithmetic sequence, where the common difference, d, represents the amount of depreciation per unit, and the initial value, a, represents the initial value of the item. t_n is the value of the item after n units of usage.

Note that d is always negative because the item loses value as units are used.

Example

Consider a car purchased for $8000 that loses $0.30 of value for every kilometre driven:



$t_n = 8000 - 0.3n$

After being driven 10 000 km, the value will be $5000:

$t_{10\,000} = 8000 - 0.3 \times 10\,000 = 5000$

The car will be written off after driving approximately 26 667 km:

$t_n = 0 = 8000 - 0.3n$

$0.3n = 8000$

$n = 26\,667$

WORKED EXAMPLE 11 *(2 marks)*

Ash puts $5000 into an account earning simple interest at a rate of 1.2% per quarter.

What will be the balance of the account after six years?

SOLUTION

Step 1 Find the value of the interest earned each quarter, i.

$$i = \frac{Pr}{100}$$

$$i = \frac{5000 \times 1.2}{100}$$

$$i = 60$$

Step 2 Find the values of a and d.

d is the difference between each term. In this case it is the amount of interest, so $d = i$.

$$d = 60$$

a is the initial term, which is the initial investment into the account, so $a = P$.

$$a = 5000$$

Step 3 Set up an equation.

Substitute the values of a and d into the equation $t_n = a + nd$ and simplify.

$$t_n = 5000 + 60n$$

✓ *1 mark for setting up equation*

Step 4 Determine the number of time periods, n.

$$n = 6 \times 4$$

$$n = 24$$

Step 5 Substitute $n = 24$ into the equation.

$$t_n = 5000 + 60n$$

$$t_{24} = 5000 + 60 \times 24$$

$$t_{24} = 6440$$

∴ The account balance after six years will be \$6440.

✓ *1 mark for substituting values*

WORKED EXAMPLE 12 *(2 marks)*

Miles bought a second-hand car for \$7000. If it loses \$0.25 of value for every kilometre driven, what will be its value after being driven 12 000 kilometres?

SOLUTION

Step 1 Find the values of a and d.

d is the amount of value lost per unit of use.

Since the value is depreciating, d is negative.

$$d = -0.25$$

a is the initial value

$$a = 7000$$

Step 2 Set up an equation.

Substitute the values of a and d into the equation $t_n = a + nd$

$$t_n = 7000 + n \times (-0.25)$$

$$t_n = 7000 - 0.25n$$

✓ *1 mark for setting up equation*

Step 3 Substitute $n = 12\,000$ into the equation.

$$t_n = 7000 - 0.25n$$

$$t_{12\,000} = 7000 - 0.25 \times 12\,000$$

$$t_{12\,000} = 4000$$

∴ The value after 12 000 km will be \$4000.

✓ *1 mark for substituting values*

Questions 6C *Applications of arithmetic sequences*

Refresher question

Q1. Which of the following is an arithmetic sequence?

A. 0, 4, –7, 3, 29, ...

B. 1, 3, 9, 27, 81, ...

C. 0, 2, 4, 6, 8, ...

D. 1, 1, 2, 3, 5, ...

1. Defining and using a formula for the value of the term after *n* periods

Q2. Skill

For each of the following sequences, state the values of a and d.

a) 2, 5, 8, 11, 14, ...

b) 7, 5, 3, 1, –1, ...

c) 15, 20, 25, 30, 35, ...

Q3. Skill

For the following number sequences, find an expression for the value of the term after n periods by:

- Writing the value of a and d
- Substituting these values into $t_n = a + nd$
- Simplifying

a) –5, –3, –1, 1, 3, ...

b) 11, 7, 3, –1, –5, ...

c) 9, 12.5, 16, 19.5, 23, ...

Q4. Skill

Consider the following sequence.

24, 21.5, 19, 16.5, 14, ...

a) Find an expression for the value of the term after n periods in the sequence.

b) What is the value of the 15^{th} term in the sequence?

Q5. Skill

Find the required term for each of the following sequences.

a) 1, 4, 7, 10, 13, ... Find t_{10}

b) 22, 28, 34, 40, 46, ... Find t_{23}

c) 13, 3, –7, –17, –27, ... Find t_{41}

Check your understanding

Q6. Application

Oliver bought an apple tree that grows two new apples every day. On the first day, Oliver has 12 apples.

Define an expression, in simplified form, for the number of apples, A_n, Oliver will have after n days.

Q7. Skill

A recurrence relation for a sequence is given by the formula $t_{n+1} = t_n - 11, t_0 = 161$.

Find an expression for the value of the term after n periods.

2. Applications of the formula

Q8. Skill

An arithmetic sequence has an initial value of 33 and a common difference of –6.

Which term will have a value of –51?

Q9. Application

The price of mini golf is $16 for the first hour and $11 for every hour after the first.

How much would it cost to play mini golf for seven hours?

Q10. Application

A plant grew at a linear rate of four centimetres per month. 12 months after planting, it was 72 centimetres tall.

How tall was it after the first month?

Q11. Application

Last Monday, Dion started walking dogs every afternoon to save money for a new laptop. On Monday night, he had a total of $210. On Thursday night the following week, he had $485. Assume that Dion makes the same amount of money from dog walking each day.

a) How much money does he make each day?

b) The laptop Dion wants is $800.
After how many days of dog walking will he be able to afford the laptop?

Check for understanding

Q12. Application

Amy is trying to grow out her hair to donate it to charity. She measured it today and found it was 30 cm long from root to tip. Exactly three weeks ago, her hair was 28.65 cm long. In order to donate to charity, it must be at least 40 cm long.

Assuming linear hair growth, in how many full weeks from today will Amy's hair be long enough to donate?

3. Interest and depreciation

Q13. Application

Amanda deposits $120 into a simple interest bank account that pays 4% per annum.

What will the balance in the account be after seven years?

Q14. Application

$4000 is invested in a simple interest account for ten years. It pays 3.6% per annum.

At the end of this period, what is the account balance?

Q15. Application

Jenny recently bought a new car and set up the following equation to show its value in dollars based on the number of kilometres driven, n.

$V_n = 13\,500 - 0.3n$

a) What was the initial value of the car?

b) By how much does the value depreciate per kilometre driven?

Q16. Application

A printing machine at Officeworks has an initial value of $2000.

The value is set to depreciate 0.2 cents per page printed.

What will the value of the machine be after it has printed 500 000 pages?

Check your understanding

Q17. Application

Adrian deposits a sum of money into a simple interest investment account. Shown in the table below is the principal invested, and the account balance after one and two years.

Year	Account balance
Principal	$7200
1	$7596
2	$7992

a) What is the simple interest rate per annum?

b) What will the account balance be after eight years?

Joining it all together

Q18. Application 2 marks

Daphne has a serious coffee addiction, so she is trying to reduce the amount of coffee she drinks. Each week, she is planning to drink three less coffees than she did the previous week. In the first week, she drinks 18 coffees.

a) Define an expression, in simplified form, for the amount of coffee Daphne drinks, C_n, after n weeks.

b) In what week will she drink no coffee?

Q19. Application 3 marks

Alexis deposits $3500 into an account that earns simple interest at a rate of 1.1% per quarter.

a) Set up and simplify an equation representing the situation as an arithmetic sequence, where n is the number of quarters since the initial deposit.

b) After how many quarters will Alexis have earned over a thousand dollars in interest?

c) What will be the account balance after ten years?

Q20. Application 2 marks

Alicia is on holiday and is spending $50 per day. At the beginning of the first day, she had $670 in her bank account.

a) Define an expression for the balance, b_n, of Alicia's bank account after the n^{th} day of her holiday.

b) At the end of today, Alicia has $20 left. What day of the holiday is it?

Q21. Application 2 marks

Vincent just bought a car for $12 000 that will lose $0.35 of value for every kilometre driven.

Vincent is an Uber driver, and as a result he drives 350 kilometres every week.

a) What will be the value of the car after 40 weeks?

b) After how many weeks will the car be effectively written off?

Q22. Application 4 marks

Melinda is a social geographer and has recently been studying population growth.

She has created the table below with estimated world populations for the years 2015, 2016, and 2017.

Assume that population growth remains linear from 2010 to 2040.

Year	Population (millions)
2015	7383
2016	7466
2017	7549

a) By how much does the world's population increase each year?

b) What was the estimated world population in 2012?

c) In what year will the world's population be over 8 billion people?

d) In what year will the world's population be over 9 billion people?

VCAA question

Q23. 2 marks

In a concert hall there are 28 seats in row one, 29 seats in row two, 30 seats in row three, and so on. The number of seats in successive rows of the concert hall form the terms of an arithmetic sequence.

a) How many seats are in row 10?

b) There are 70 seats in the last row of the concert hall.
How many rows of seats does the concert hall contain?

VCAA 2009 Exam 2 Module 1: Number patterns. Q1a,b

Questions from multiple lessons

Q24.

Difficulty:

1 mark

Jacob wants to display a complete collection of ancient Mayan coins in an exhibition in four years.

Each year, he collects 200 more ancient coins than the previous year.

His team of archaeologists predict that there are 4200 coins to collect.

The number of coins that Jacob will need to collect in the first year, if he is to finish collecting the coins in four years, is

A. 700

B. 725

C. 750

D. 775

E. 800

VCAA 2014 Exam 1 Module 1: Number patterns Q5 – Adapted

Q25.

Difficulty:

1 mark

The following table shows the *diameter* and *weight* of ten watermelons.

***Diameter* (cm)**	30.6	29.7	35.1	26.8	30.4	29.3	27.9	33.4	34.5	28.3
***Weight* (kg)**	11.8	11.5	12.4	10.7	11.3	12.9	11.6	12.6	12.2	11.3

Find the value of the Pearson correlation coefficient, r, between *diameter* and *weight*, correct to two decimal places.

A. -0.65

B. -0.54

C. 0.54

D. 0.55

E. 0.65

VCAA 2018NH Exam 1 Data analysis Q9 – Adapted

Q26.

Difficulty:

3 marks

A car rental company has a special promotion for 2020.

Customers are charged based on the whole number of kilometres they travel each day.

- The first kilometre costs $8.
- Each kilometre after this costs 25 cents less than the previous one.
- Once the cost of travel reaches zero, any additional travel within that day is free.

a) How much does the eighth kilometre cost? (1 mark)

b) What is the total cost of travel for 10 kilometres? (1 mark)

A customer paid the maximum charge for travel in a day.

c) What is the minimum number of kilometres that this customer could have travelled? (1 mark)

VCAA 2011 Exam 2 Module 1: Number patterns Q2 a-c – Adapted

LESSON 6D

Generating geometric sequences

The key skills you will learn in this lesson are:

1. Finding recurrence relations
2. Generating terms

VCAA key knowledge points:

"generation of a geometric sequence using a recurrence relation and its tabular or graphical display; and the rule for the nth term and its evaluation"

"use of a recurrence relation to model and analyse practical situations involving geometric growth or decay such as the growth of a compound interest loan, the reducing height of a bouncing ball, reducing balance depreciation; and the rule for the value of a quantity after n periods of geometric growth or decay and its use"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Finding recurrence relations

A geometric sequence is a sequence where each number is obtained by multiplying the previous value in the sequence by a fixed number.

This number is known as the common ratio. Below is a geometric sequence with a common ratio of 3.

2, 6, 18, 54, 162 ...

The common ratio can be found by dividing a term in the geometric sequence by the previous term. In the geometric sequence above, the common ratio is found by dividing 6 by 2 which equals 3.

The common ratio can be found using any two consecutive terms.

For example, $\frac{54}{18}$ also equals 3.

A geometric sequence can be represented by a recurrence relation in the form $t_{n+1} = Rt_n$, $t_0 = a$ where R is the common ratio and a is the initial value of the sequence.

The recurrence relation for the geometric sequence above is $t_{n+1} = 3t_n$, $t_0 = 2$.

WORKED EXAMPLE 13 *(1 mark)*

What is the recurrence relation for the following geometric sequence?

4, 8, 16, 32, 64...

SOLUTION

Step 1 Find the common ratio.

$R = \frac{64}{32} = \frac{32}{16} = \frac{16}{8} = \frac{8}{4} = 2$

Step 2 Find the initial value of the sequence.

4

Step 3 Write out the recurrence relation in the form $t_{n+1} = Rt_n$, $t_0 = a$.

$t_{n+1} = 2t_n$, $t_0 = 4$

2. Generating terms

The subsequent value of a geometric sequence can be found by multiplying the prior term by the common ratio, R.

WORKED EXAMPLE 14 *(1 mark)*

Find the value of t_3 for the following recurrence relation.

$t_{n+1} = 3\ t_n\ \ t_0 = 2$

SOLUTION

Step 1 Find the value of t_1.

$t_1 = 3\ t_0 = 3 \times 2 = 6$

Step 2 Find the value of t_2.

$t_2 = 3\ t_1 = 3 \times 6 = 18$

Step 3 Find the value of t_3.

$t_3 = 3\ t_2 = 3 \times 18 = 54$

$t_3 = 54$

Questions 6D *Generating geometric sequences*

Refresher question

Q1. What must 2 be multiplied by to get 10?

1. Finding recurrence relations

Q2. Skill

What is the common ratio for the following geometric sequence?

4, 12, 36, 108, 324 ...

Q3. Skill

Find the recurrence relation for the following sequences in the form $t_{n+1} = R\,t_n\ ,\ t_0 = a$.

a) 3, 15, 75, 375, 1875...

b) $-162, 54, -18, 6, -2$ ...

c) $\frac{1}{4}\ \frac{5}{4}\ \frac{25}{4}\ \frac{125}{4}\ \frac{625}{4}$ ...

Check your understanding

Q4. Skill

Which of the following is not a geometric sequence?

A. $\frac{1}{2}$, 1, 2, 4, 8 ...

B. $4, -4, 4, -4, 4$ ...

C. 2, 10, 50, 250, 1250 ...

D. 3, 6, 9, 12, 15 ...

Q5. Application

The profits of a company double every month. If p_n is the profit of the company in dollars after n months, and $p_0 = 1500$, find a recurrence relation that represents this situation.

2. Generating terms

Q6. Skill

What is the common ratio for the geometric sequence represented by the following recurrence relation?

$t_{n+1} = -\frac{2}{3}t_n\ \ t_0 = 9$

Q7. Skill

Find t_4 for the following recurrence relations.

a) $t_{n+1} = 4t_n\ \ t_0 = 2$

b) $t_{n+1} = \frac{1}{6}t_n\ \ t_0 = 84$

c) $t_{n+1} = -3t_n\ \ t_2 = \frac{1}{3}$

Check your understanding

Q8. Skill

For the recurrence relation $t_{n+1} = \frac{1}{7}t_n$, what is the value of t_2 if $t_5 = \frac{1}{98}$?

Q9. Application

The population of rabbits in a forest is given by $p_{n+1} = 3p_n$ where p_n is the population of rabbits after n years. If the population of rabbits was 52 in 2015, what would the population be in 2017?

Joining it all together

Q10. Application 1 mark

The height of a bouncing ball is given by $h_{n+1} = 0.6h_n$ where h_n is the height in centimetres the ball bounces to on the n^{th} bounce. If the ball bounces to a height of 10 cm on the 5^{th} bounce, what height did it bounce to on the 1^{st} bounce correct to one decimal place?

Q11. Application 4 marks

a) The table below shows the number of bacteria after a certain amount of time.

Given that the number of bacteria follows a geometric sequence and b_n is the number of bacteria after n hours, find the value of x and thus find a recurrence relation to describe the situation. (2 marks)

b) Using the recurrence relation, how many bacteria would there be after 6 hours? (1 mark)

c) After how many hours will the number of bacteria first exceed 1000? (1 mark)

Time (hours)	Number of bacteria
0	7
1	x
2	28

Q12. Application 1 mark

If the air pressure decreases by 12% for every rise of 1000 metres (m) in altitude, find a recurrence relation representing this situation in the form $p_{n+1} = Rp_n$ where p_n is the air pressure in kilopascals (kpa) at an altitude of $1000n$ and $p_0 = 101.325$.

Q13. Application 2 marks

The population of cats on a farm is given by $C_{n+1} = 2C_n$, $C_0 = 2$ where C_n is the number of cats after n years and the population of mice is given by $M_{n+1} = \frac{1}{3}M_n$, $M_0 = 54$ where M_n is the number of mice after n years.

a) After how many years will the population of cats be greater than the population of mice?

b) After how many years will there be no mice? **NOTE:** No mice is when $M_n < 1$.

VCAA question

Q14. 1 mark

In 2008, there are 800 bats living in a park. After 2008, the number of bats living in the park is expected to increase by 15% per year. Let B_n represent the number of bats living in the park n years after 2008. A recurrence relation that can be used to determine the number of bats living in the park n years after 2008 is

A. $B_n = 1.15B_{n-1} - 800$ $B_0 = 2008$

B. $B_n = B_{n-1} + 1.15 \times 800$ $B_0 = 2008$

C. $B_n = B_{n-1} - 1.15 \times 800$ $B_0 = 800$

D. $B_n = 0.15B_{n-1}$ $B_0 = 800$

E. $B_n = 1.15B_{n-1}$ $B_0 = 800$

Adapted from VCAA 2008 Exam 1 Module 1: Number patterns. Q4

Questions from multiple lessons

Q15.
Difficulty:
1 mark

Beatrice has decided to pursue a career as an Instagram influencer so she is currently trying to build her following.

On day 1, Beatrice has 200 Instagram followers.

Each day thereafter, Beatrice has 20 more followers than she did the day before.

Let F_n be the number of followers Beatrice has on day n.

A recurrence relation that can be used to model this is

A. $F_{n+1} = 1.20\,F_n, \quad F_1 = 200$

B. $F_{n+1} = 1.10\,F_n, \quad F_1 = 200$

C. $F_{n+1} = F_n + 20, \quad F_1 = 200$

D. $F_{n+1} = F_n + 10, \quad F_1 = 200$

E. $F_{n+1} = F_n + 0.10, \quad F_1 = 200$

VCAA 2014 Exam 1 Module 1: Number patterns Q4 – Adapted

Q16.
Difficulty:
1 mark

The following histogram displays the time taken in seconds for a group of primary school students to run 800 metres.

The shape of this distribution can be described as

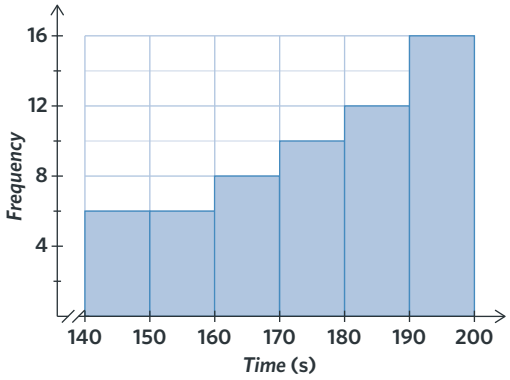
A. symmetric.

B. approximately symmetric.

C. approximately normal.

D. negatively skewed.

E. positively skewed.



VCAA 2017NH Exam 1 Data analysis Q1 – Adapted

Q17.
Difficulty:
2 marks

An investment of $1 000 000 was put into a savings account modelled by the following recurrence relation, where V_n is the value of the investment after n years.

$V_0 = 1\ 000\ 000, \quad V_{n+1} = 1.06 \times V_n$

a) What is the balance of the savings account five years after the investment was made? Give your answer correct to the nearest cent. (1 mark)

b) After how many years will the value of the investment first exceed $1 500 000? (1 mark)

VCAA 2012 Exam 2 Module 1: Number patterns Q3 a,b – Adapted

LESSON 6E

Applications of geometric sequences

The key skills you will learn in this lesson are:

1. Defining and using the formula for the n^{th} term
2. Applications of the formula

VCAA key knowledge points:

"generation of a geometric sequence using a recurrence relation and its tabular or graphical display; and the rule for the n^{th} term and its evaluation"

"use of a recurrence relation to model and analyse practical situations involving geometric growth or decay such as the growth of a compound interest loan, the reducing height of a bouncing ball, reducing balance depreciation; and the rule for the value of a quantity after n periods of geometric growth or decay and its use"

1. Defining and using the formula for the n^{th} term

Like arithmetic sequences, there is a formula to calculate term n of a geometric sequence. Using the formula allows the term n to be found without having to calculate all of the previous terms. The formula is $t_n = t_0 R^n$ where t_0 is the first term in the sequence, and R is the common ratio between terms.

Example

The sequence $\frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, 16\ldots$ has the formula $t_n = \frac{1}{4} \times 2^n$. The initial value is $\frac{1}{4}$.

$t_{10} = \frac{1}{4} \times 2^{10} = 256$

$t_{13} = \frac{1}{4} \times 2^{13} = 2048$

WORKED EXAMPLE 15 *(2 marks)*

Find the formula that allows any term to be found for the geometric sequence $\frac{3}{2}, 1, \frac{2}{3}, \frac{4}{9}\ldots$, and find t_5.

SOLUTION

Step 1 Find the initial value, and the common ratio.

The initial value is the first term, $\frac{3}{2}$.

The common ratio is equal to t_1 divided by t_0.

$R = \frac{t_1}{t_0} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

So $t_0 = \frac{3}{2}$ and $R = \frac{2}{3}$.

Step 2 Substitute these values into the equation $t_n = t_0 R^n$.

$t_n = \frac{3}{2} \times \left(\frac{2}{3}\right)^n$

✓ *1 mark for finding the formula*

Step 3 To find t_5, let $n = 5$.

$t_5 = \frac{3}{2} \times \left(\frac{2}{3}\right)^5$

$t_5 = \frac{16}{81}$

✓ *1 mark for finding t_5*

2. Applications of the formula

Geometric sequences can be useful for modelling compound interest, the height of a bouncing ball, or a reduction in bank balance. $R < 1$ in models that involve a decrease in value, such as the height of a bouncing ball, and $R > 1$ in models that involve an increase in value, such as compound interest. If each term has the same value, $R = 1$. To convert a percentage to the common ratio, the formula $R = \left(1 + \frac{Percentage}{100}\right)$ can be used.

Example

Balance of a savings account: $B_n = D \times 1.1^n$, where B_n is the balance at month n, and D is the initial deposit.

Height of a bouncing ball: $H_n = H_0 \times \left(\frac{3}{4}\right)^n$, where H_n is the height of bounce n, and H_0 is the height the ball was dropped from.

WORKED EXAMPLE 16 *(2 marks)*

A savings account is opened with an initial deposit of $2000. Interest is paid monthly at a rate of 4% per month. If the balance at month n is given by B_n, find a model for B_n in terms of n. Assuming no withdrawals are made from the account, what will the balance be at the end of the fifth month, to the nearest cent?

SOLUTION

Step 1 Find the initial value, and the common ratio.

The initial value is $B_0 = 2000$.

R is given as a percentage, so we use $R = \left(1 + \frac{Percentage}{100}\right)$.

$$R = \left(1 + \frac{Percentage}{100}\right)$$

$$R = \left(1 + \frac{4}{100}\right)$$

$$R = 1.04$$

So $B_0 = 2000$ and $R = 1.04$.

Step 2 Substitute these values into the equation $t_n = t_0 R^n$.

Using B_n instead of t_n, we have $B_n = 2000 \times 1.04^n$.

✓ *1 mark for finding the formula*

Step 3 To find B_5, let $n = 5$.

$$B_5 = 2000 \times 1.04^5$$

$$B_5 = 2433.31$$

✓ *1 mark for calculating B_5*

Questions 6E *Applications of geometric sequences*

Refresher question

Q1. If $x = 0.9$, then what is the value of $10x$ and $10x^2$?

1. Defining and using the formula for the n^{th} term

Q2. Skill

Jeremy is experimenting with the model $t_n = 0.05 \times 3^n$. Finish his working out to find t_6

$t_n = 0.05 \times 3^n$

Let $n = 6$

$t_6 = 0.05 \times 3^6$

$t_6 = 0.05 \times 729$

$t_6 =$ [ ]

Q3. Skill
Find formulas for t_n for the following sequences in terms of n. Use the formulas to find t_8. Round your answers to two decimal places.

a) 2, 1.98, 1.9602, 1.940598, ...

b) 3, 6, 12, 24, ...

c) $1, \frac{4}{5}, \frac{16}{25}, \frac{64}{125}, \ldots$

d) $2, -6, 18, -54, 162, \ldots$

Check your understanding

Q4. Skill
A sequence is defined by the formula $t_n = 6.5 \times 0.9^n$. State the initial value of the sequence, and the common ratio between terms. Is the sequence increasing or decreasing?

Q5. Application
Two students, Mandeep and Sandra, are fighting over who is best at maths. They agree to determine who is the best by racing to find t_{10} of the sequence 10, 7.5, 5.625, ... They each use different formulas to reach the answer. Who will win the race?

Mandeep: $t_{n+1} = 0.75t_n$, $t_0 = 10$

Sandra: $t_n = 10 \times 0.75^n$

2. Applications of the formula

Q6. Application
Caroline is trying to calculate how much money will be in her savings account at Christmas. She opened the account in May. The formula for the balance in the account, B_n, during month n after May, is given below.

$B_n = 800 \times 1.05^n$

Use this formula to find the amount of money in the account at Christmas, to the nearest cent.

Q7. Application
Three employees, Quentin, Sahar, and Terence are given different salary packages in their first year of work. Use the information below to find an equation to represent each person's salary, Q_n S_n, and T_n, in terms of year n.

NOTE: Use $n = 1$ as the first year, meaning R will be raised to the power of $n - 1$.

a) Quentin's starting salary is \$38 000 and increases 3.1% each year.

b) Sahar's starting salary is \$35 000 and increases 4% each year.

c) Terence's starting salary is \$40 000 and increases 2% each year.

d) Who is on the highest salary during their tenth year?

Check your understanding

Q8. Application
The following table shows the height of a table tennis ball dropped from a height of one metre.

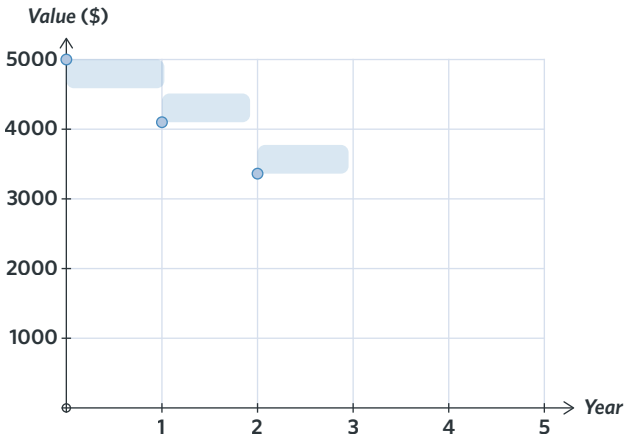
Bounce #	0	1
Height (cm)	100	45

a) Assuming the height of each bounce is a geometric sequence, find a formula for the height, H_n, of bounce n.

b) What height will the ball reach on bounce 3? Round your answer to two decimal places.

Q9. Application

The following graph shows the depreciation of the value of a car over time. The data points form a geometric sequence.



a) Use the graph to find a formula for the car's value, V_n, at year n.

b) Use the formula to find the first year the car will be worth less than $2000. Only look at whole values of n.

Joining it all together

Q10. Skill 2 marks

a) A geometric sequence has $t_0 = 6.21$ and $t_2 = 18.63$. Given that $\frac{t_2}{t_0} = R^2$, find R and state a formula for t_n in terms of n. Assume all the terms in this sequence are positive.

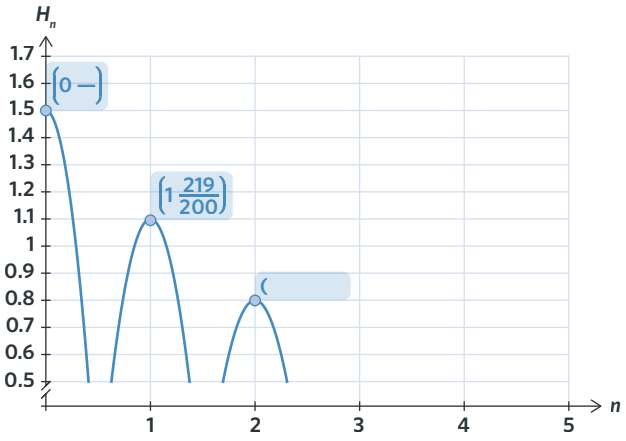
b) A different sequence has $t_1 = 8.918$ and $t_3 = 7.3849958$. Find a formula for the sequence. Assume all the terms in this sequence are positive.

Q11. Application 10 marks

Luka is examining the performance of different bouncy balls.

a) Use the data below to find an equation for the height of bounce n for each of the bouncy balls below. (3 marks)

BIG-Bounce Ball (BB_n): The following graph shows the first two bounces of a BIG-Bounce ball, after being dropped from a height of one and a half metres.



BouncePRO Ball (BP_n): When dropped from a height of three metres, this ball reaches 2.7648 metres on the second bounce.

BallTECH Ball (BT_n): The following table gives some results for this ball.

Bounce #	4	6
Height (m)	0.8192	0.524288

b) Luka takes all three balls and drops them from a five-metre diving platform, onto the concrete below.

Use the models to estimate how high each ball will bounce on bounces 1 and 10. Don't forget to change the BB_0 BP_0, and BT_0 terms. Round all answers to two decimal places. (3 marks)

c) Which ball is the bounciest? (1 mark)

d) Which of the following doesn't fit the model for the BIG-Bounce ball with $BB_0 = 1.2$? (1 mark)

A. Bounce 6 has a height of approximately 18.16 cm.

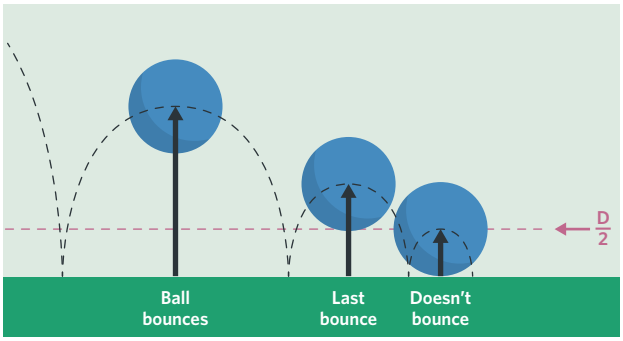
B. Bounce 2 has a height of 0.73948 m.

C. Bounce 3 has a height of approximately 46.68 cm.

D. Bounce 1 has a height of 0.876 m.

e) The formulas found in part a indicate the height the centre of the ball reaches above the ground. Thus, if the diameter of the ball is D, the ball stops bouncing when the height drops below $\frac{D}{2}$

If the BouncePRO ball has a diameter of 6 cm, how many times does it bounce when it is dropped from a height of two metres? (2 marks)



Q12.

Application
4 marks

Isabelle and Nurul are trying to buy a SNES console, which is now a collector's item. It costs about $1000 to buy one. They open a savings account at different banks in the hope of saving enough money. Isabelle deposits $390, and Nurul deposits $400.

Both accounts have compound interest. Isabelle's account interest is 5% per month, paid monthly, and Nurul's account is 15% per quarter, paid quarterly.

a) Write an equation to describe the amount of money in Isabelle's account at month m I_m (1 mark)

b) Write an equation to describe the amount of money in Nurul's account at quarter q N_q (1 mark)

c) Whose account will reach $1000 first? (2 marks)

VCAA question

Q13.

1 mark

The first term of a geometric sequence is 9.

The third term of this sequence is 121.

The second term of this sequence could be:

A. -65 **B.** -33 **C.** 56 **D.** 65 **E.** 112

VCAA 2008 Exam 1 Module 1: Number patterns. Q5

Questions from multiple lessons

Q14.

Difficulty:

1 mark

Calliope is a Year 12 student commencing her study plan for her VCE exams.

Calliope studied for 150 minutes in the first week and will successively increase the amount of time she spends studying by 15 minutes each week. Hence, in the second week, she studied for 165 minutes and in the third week, she studied for 180 minutes.

For how long will Calliope study in the 13th week of the study plan?

A. 315 minutes

B. 330 minutes

C. 345 minutes

D. 350 minutes

E. 365 minutes

VCAA 2012 Exam 1 Module 1: Number patterns Q3 – Adapted

Q15.

Difficulty:

1 mark

A new doughnut store just opened up in Melbourne. The store manager wants to investigate the association between the *number of doughnuts sold* and the *day of the week*. These variables are

A. a numerical variable and an ordinal variable respectively.

B. a numerical variable and a nominal variable respectively.

C. an ordinal and numerical variable respectively.

D. both numerical variables.

E. both categorical variables.

VCAA 2017NH Exam 1 Data analysis Q4 – Adapted

Q16.

Difficulty:

4 marks

The projected koala population is represented in the following recurrence relation, where K_n is the projected population at the beginning of year n.

$K_{n+1} = 0.8\,K_n \qquad K_{2020} = 85\ 000$

a) What is the projected population of koalas at the beginning of 2020? (1 mark)

b) What is the projected percentage decrease of koala populations from year to year? (1 mark)

c) Show that the projected population of koalas at the beginning of 2021 is 68 000. (1 mark)

d) What is the projected change in population of koalas during the year 2021? (1 mark)

VCAA 2014 Exam 2 Module 1: Number patterns Q1a-d – Adapted

LESSON 6F

Fibonacci and similar sequences

The key skills you will learn in this lesson are:

1. Terms of Fibonacci and similar sequences
2. Fibonacci and similar sequences using second-order recurrence relations

VCAA key knowledge points:

"generation of the Fibonacci and similar sequences using a recurrence relation, tabular and graphical display"

"use of Fibonacci and similar sequences to model and analyse practical situations"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Terms of Fibonacci and similar sequences

In 1202, Italian mathematician Leonardo Fibonacci published a book containing a very influential set of numbers, that were later called the Fibonacci sequence.

The Fibonacci sequence is:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55...

where each term is found by adding the previous two terms. Although the Fibonacci sequence always begins with two ones, there are many Fibonacci-related sequences that begin with two different terms.

One example is the Lucas sequence, which starts with the terms one and three.

The Lucas sequence is:

1, 3, 4, 7, 11, 18, 29, 47, 76, 123...

WORKED EXAMPLE 17 *(1 mark)*

What is the next term in the sequence below?

2, 6, 8, 14, 22, 36...

SOLUTION

Step 1 Recognise the sequence is Fibonacci-related.

Each term is the sum of the previous two terms.

$2 + 6 = 8$

$6 + 8 = 14$

$8 + 14 = 22$

$14 + 22 = 36$

Step 2 Find the sum of the last two terms.

$22 + 36 = 58$

The next term in the sequence is 58.

2. Fibonacci and similar sequences using second-order recurrence relations

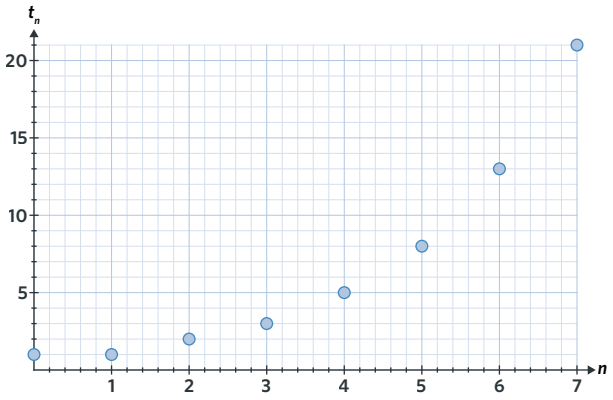
Sequences related to the Fibonacci sequence can be described by the second-order recurrence relation:

$t_n = t_{n-2} + t_{n-1}$

where t_0 is the first term of the sequence and t_1 is the second term of the sequence.

The recurrence relation is described as being second-order because it depends on the two previous terms in the sequence. All the previous recurrence relations in this chapter are first-order relations as they only depend on the immediately preceding term of the sequence.

The graph of Fibonacci-related sequences usually form a smooth curve, as shown below:



WORKED EXAMPLE 18 *(1 mark)*

A sequence is described the recurrence relation $t_n = t_{n-2} + t_{n-1}$.

Find t_5, given that $t_0 = -1$ and $t_1 = -1$.

SOLUTION

Step 1 Find t_2

$t_2 = t_0 + t_1$

$t_2 = -1 + (-1) = -2$

Step 2 Repeat step 1 to find t_3 t_4 and t_5

$t_3 = t_1 + t_2$

$t_3 = -1 + (-2) = -3$

$t_4 = t_2 + t_3$

$t_4 = -2 + (-3) = -5$

$t_5 = t_3 + t_4$

$t_5 = -3 + (-5) = -8$

The fifth term of the sequence is –8.

Questions 6F *Fibonacci and similar sequences*

Refresher question

Q1. If $t_n = t_{n-1} + 3$ and $t_0 = 2$, what is the value of t_1?

1. Terms of Fibonacci and similar sequences

Q2. Skill

Which calculation would give the next number in the Fibonacci sequence below?

1, 1, 2, 3, 5, 8, ?

A. $3 + 8$ **B.** $5 + 8$ **C.** $3 + 5$ **D.** $8 - 5$

Q3. Skill

What is the next term in each of the sequences below?

a) 1, 3, 4, 7, 11, 18, 29...

b) –1, 2, 1, 3, 4, 7, 11...

c) –4, –5, –9, –14, –23, –37, –60...

Check your understanding

Q4. Application

The increase in the population of cats on a farm every month follows a Fibonacci-related sequence. The population of cats after two months is three, after three months is x and after six months is 18.

What is the value of x?

2. Fibonacci and similar sequences using second-order recurrence relations

Q5. Skill

What are the values of t_0 and t_1 in the sequence below?

2, –3, –1, –4, –5, –9, –14...

Q6. Skill

A sequence is described by the recurrence relation $t_n = t_{n-2} + t_{n-1}$. Find t_5 given that:

a) $t_0 = 1$ and $t_1 = 2$

b) $t_0 = 8$ and $t_1 = -5$

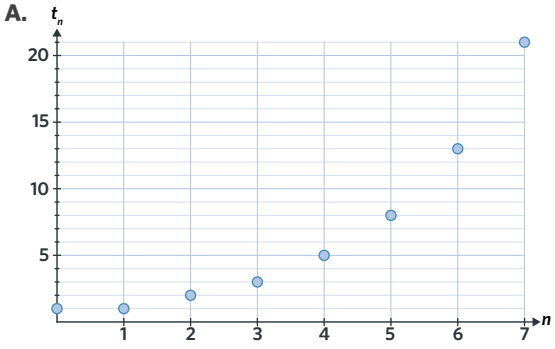
c) $t_1 = -4$ and $t_2 = 2$

Check your understanding

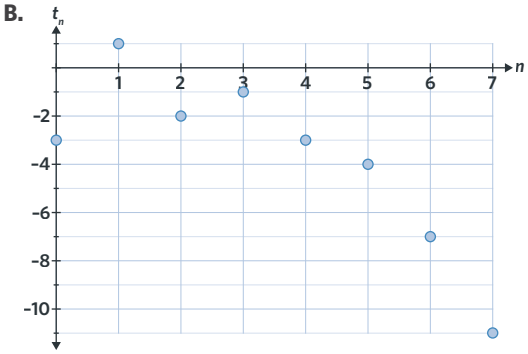
Q7. Skill

Which of the graphs below does not show a Fibonacci-related sequence?

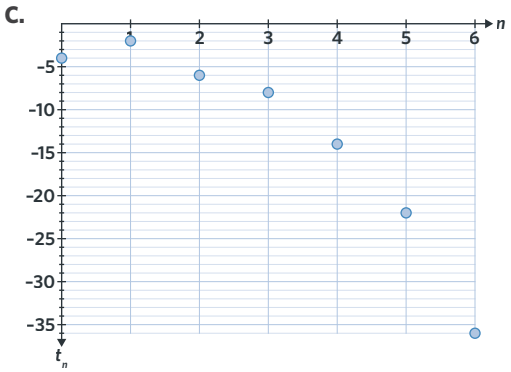
A.



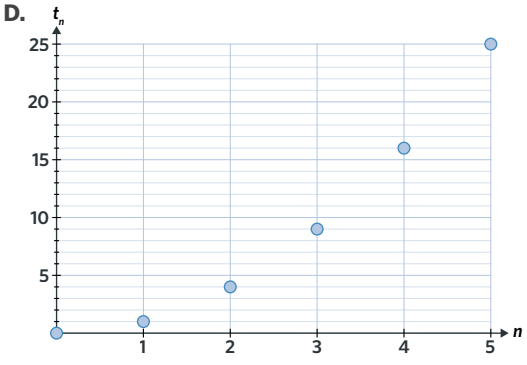
B.



C.



D.



Q8. Skill

For which Fibonacci-related sequence will t_6 be the greatest?

A. $t_2 = 2$ and $t_3 = 4$

B. $t_0 = -1$ and $t_1 = 5$

C. $t_2 = 6$ and $t_3 = 7$

D. $t_0 = 2$ and $t_1 = 3$

Joining it all together

Q9. Skill 1 mark

If t_3 and t_5 of a Fibonacci-related sequence are –3 and 8 respectively, what is the value of t_4?

Q10. Which of the following is not an expression for t_6?

Skill
1 mark

A. $t_3 + 2t_4$ **B.** $t_2 + t_3 + t_5$ **C.** $2t_3 + t_5$ **D.** $t_7 - t_5$

Q11. The population of rabbits at a farm after n months, P_n, is described by the Fibonacci-related sequence in the table below:

Application
3 marks

n	0	1	2	3	4
P_n	2	4	6	10	16

a) Find a recurrence relation for P_n

b) How many rabbits were on the farm to begin with?

c) What will the population of rabbits be after half a year?

Questions from multiple lessons

Q12. A sequence can be generated using the following recurrence relation.

Difficulty:

1 mark

$a_{n+1} = 5 \times a_n \quad a_0 = 3$

An expression for the value in the sequence after n periods is given by

A. $a_n = 3 + 5n$

B. $a_n = 5 + 3n$

C. $a_n = 3 \times 5^n$

D. $a_n = 5 \times n^3$

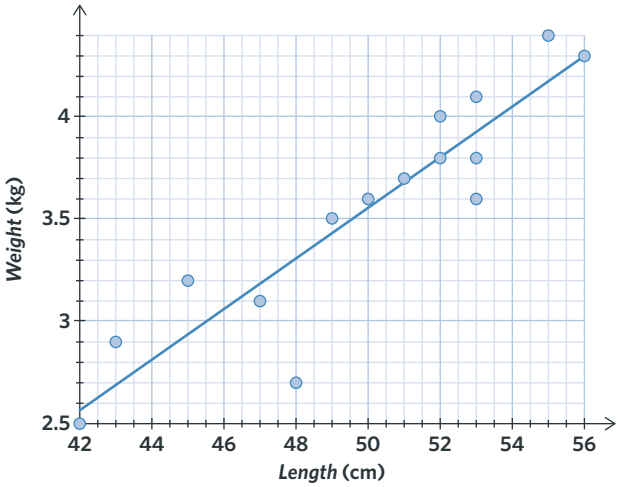
E. $a_n = 5 \times 3^n$

VCAA 2013 Exam 1 Module 1: Number patterns Q5 – Adapted

Q13. The following scatterplot displays the *length* and *weight* of a group of newborn babies. A least squares regression line has been fitted to the data, as shown.

Difficulty:

1 mark



The equation of the line is closest to

A. *length* $= 2.60 + 0.12 \times$ *weight*

B. *weight* $= 2.60 + 0.12 \times$ *length*

C. *length* $= 0.12 + 2.60 \times$ *weight*

D. *length* $= -2.60 + 0.12 \times$ *weight*

E. *weight* $= -2.60 + 0.12 \times$ *length*

VCAA 2017NH Exam 1 Data analysis Q10 – Adapted

Q14.

Difficulty:

3 marks

A cosmetics studio needs to predict the number of eye shadow palettes that they are going to sell so they can produce the units accordingly.

In the first month, they are predicted to sell 20 000 units of eyeshadow.

Each subsequent month, they expect to sell 5% more units than the previous month.

a) How many units are they expected to sell in the third month? (1 mark)

b) Determine the difference between the units expected to be sold in the second and fifth month. Round your answer to the nearest whole number. (1 mark)

c) Determine the total number of units they expect to sell in the first six months. Round your answer to the nearest whole number. (1 mark)

VCAA 2011 Exam 2 Module 1: Number patterns Q3 a-c – Adapted

AOS 4: Geometry, measurement and trigonometry

Shape and measurement

CHAPTER 7

LESSON 7A

Review of units of measurement

The key skills you will learn in this lesson are:

1. Definition of angle, length, area, volume and capacity
2. Units of angle, length, area, volume and capacity
3. Conversion of units

VCAA key knowledge point:

"review of units of measurement of length, angle, area, volume and capacity"

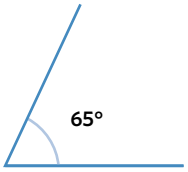
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

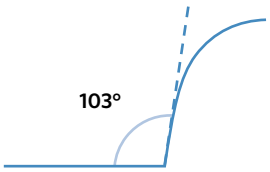
1. Definition of angle, length, area, volume and capacity

Five of the most important characteristics when studying shapes are angle, length, area, volume, and capacity.

Angle

Angle is the difference in direction between two lines. It can be anywhere from 0° to 360°.

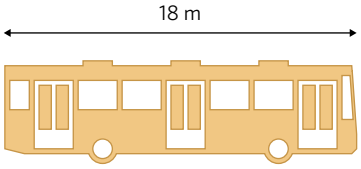


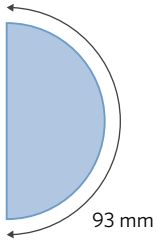


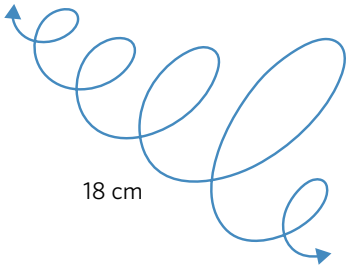
Length

Length tells us how long something is.

Length can measure distance in a straight line or along a curve.

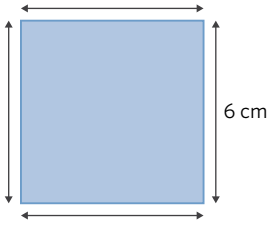




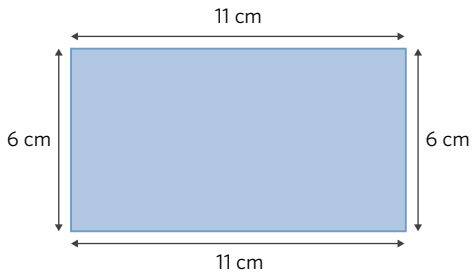


P ce around a shape, is a length.

F square, the perimeter is the sum of all four side lengths.



+ 6

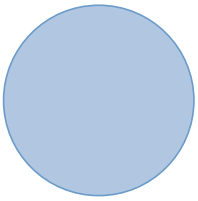


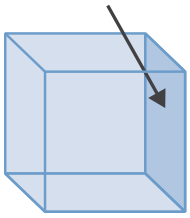
Perimeter = 11 + 6 + 11 + 6
= 34 cm

Area

Area measures the amount of space inside a shape.

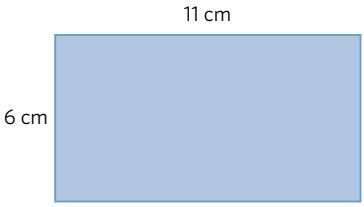
A line does not have area, but a square does. Area is a two-dimensional unit.





For rectangles, area is found by multiplying two one-dimensional quantities together, the length and the width.

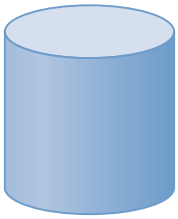
Area = length²
= 6²
= 36 cm²

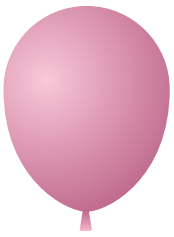


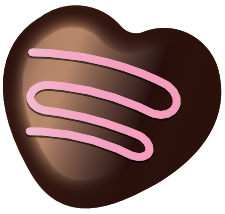
Area = length × width
= 11 × 6
= 66 cm²

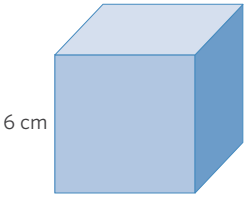
Volume

Volume is the amount of three-dimensional space taken up by an object.

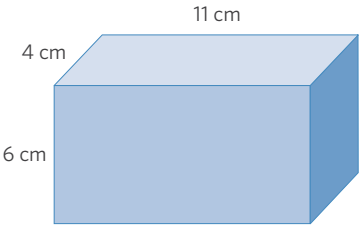








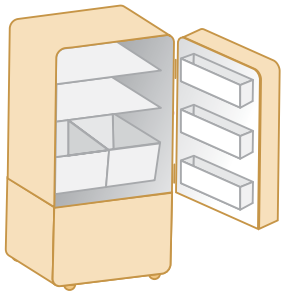
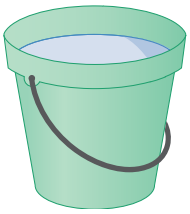
Volume = length³
= 6³
= 216 cm³



Volume = length × width × depth
= 11 × 6 × 4
= 264 cm³

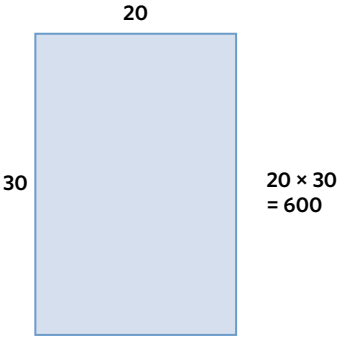
Capacity

Capacity is similar to volume, but instead is the amount of three-dimensional space an object can hold.

WORKED EXAMPLE 1 *(1 mark)*

Jerry is measuring a piece of paper 20 cm by 30 cm. His working is below. Is Jerry measuring angle, length, area, volume or capacity?



SOLUTION

Step 1 The piece of paper is flat. The values of length and width are used in the calculation, suggesting Jerry is calculating something two-dimensional.

Length is one-dimensional, and volume and capacity are three-dimensional, so Jerry isn't measuring these.

Step 2 Looking at the working, two lengths are being multiplied together, so we know Jerry is measuring area.

2. Units of angle, length, area, volume and capacity

Angles

Angles are measured in degrees (°) or radians (^{c}), but only degrees are used in this book.

Length

Length has one dimension, and is often measured in millimetres (mm), centimetres (cm), metres (m) or kilometres (km).

1 mm = 0.1 cm = 0.001 m

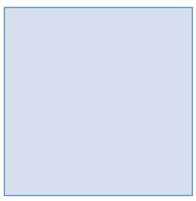
10 mm = 1 cm = 0.01 m

1000 mm = 100 cm = 1 m

Area

Area is two-dimensional. Thus, the common units used for area are mm^2, cm^2, m^2, and km^2.

1 mm²



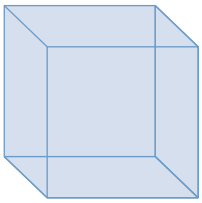
1 cm²

$1\ mm^2 = 0.01\ cm^2$
$1\ cm^2 = 100\ mm^2$

Volume

Volume is three-dimensional. Commonly used units for volume are mm^3, cm^3, m^3, mL and L.

1 mm³



1 cm³

$1\ mm^3 = 0.01\ cm^3$
$1\ cm^3 = 100\ mm^3$

Capacity

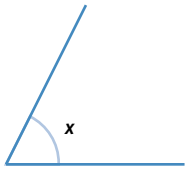
Capacity has the same units as volume, but is often measured in litres (L).

One litre is equal to 1000 cm^3.

WORKED EXAMPLE 2 *(2 marks)*

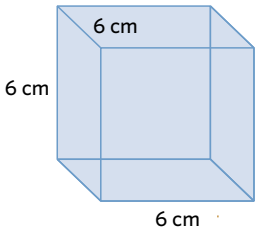
What are the units in the following calculations?

a)



80
x = 80

b)



6 × 6 × 6
x = 216

SOLUTION

a)

Step 1 This is looking at the difference in direction of two lines, and therefore is an angle. In general maths we only use degrees, so choose degrees.

b)

Step 1 There are three quantities being multiplied together. Just as $6 \times 6 \times 6$ equals 6^3, $cm \times cm \times cm = cm^3$. The unit is cm^3.

3. Conversion of units

To convert measurements in different units, multiply the by the conversion factor, given below.

$$conversion\ factor = \frac{C_{old\ unit}}{C_{new\ unit}}$$

Length		
Name	**Unit**	**C**
millimetre	mm	0.001
centimetre	cm	0.01
metre	m	1
kilometre	km	1000

Area		
Name	**Unit**	**C**
square millimetre	mm^2	0.000001
square centimetre	cm^2	0.0001
square metre	m^2	1
square kilometre	km^2	1000000
hectare	h	10000

Volume and Capacity		
Name	**Unit**	**C**
cubic millimetres	mm^3	0.000000001
cubic centimetres	cm^3	0.000001
cubic metre	m^3	1
cubic kilometre	km^3	1000000000
millilitre	mL	0.000001
litre	L	0.001
megalitre	ML	1000

WORKED EXAMPLE 3 *(1 mark)*

Convert 245 cm to m.

SOLUTION

Step 1 The value of C for centimetres is 0.01 and for metres is 1.

$C_{old} = 0.01$

$C_{new} = 1$

Step 2 Multiply the measurement by the conversion factor.

$245 \times \frac{C_{old}}{C_{new}}$

$= 245 \times \frac{0.01}{1}$

$= 2.45$

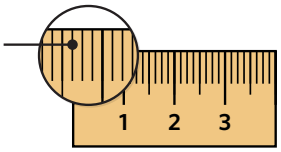
Step 3 Add the unit.

$= 2.45$ m

Questions 7A *Review of units of measurement*

Refresher question

Q1. What do these markings represent on a ruler?



1. Definition of angle, length, area, volume and capacity

Q2. If you are multiplying two lengths together you are finding:

Skill

A. Length **B.** Area **C.** Angle **D.** Volume

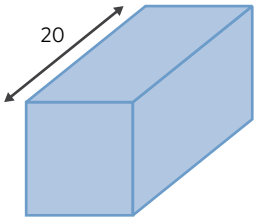
Q3. Which calculations below are finding:

Skill

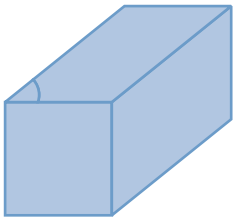
a) Angle **b)** Length **c)** Area

d) Volume **e)** Capacity

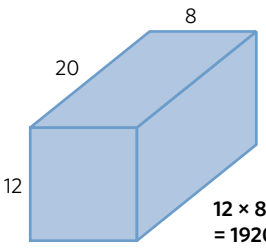
A.



B.

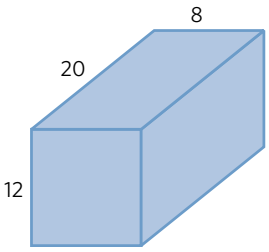


C.



12 × 8 × 20 = 1920

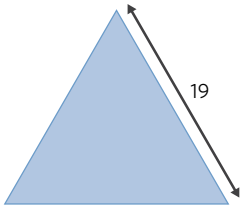
D.



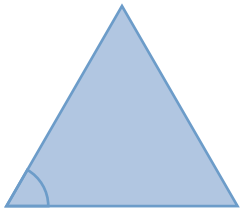
12 × 8 = 96

E.

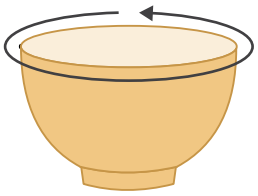
F.



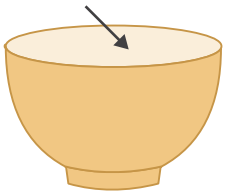
G.



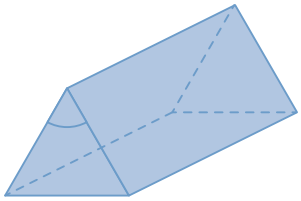
H.



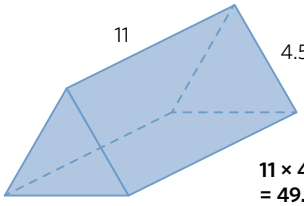
I.



J.

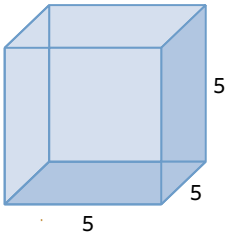


K.



11 × 4.5 = 49.5

L.



5 × 5 × 5 = 125

Check your understanding

Q4. What is being calculated here?

Skill

A. Area

B. Length

C. Volume

D. Capacity

7 × 12 = 84

Q5. Which of the following calculations gives the volume of the prism?

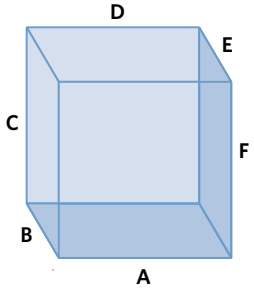
Skill

A. $B \times D \times F$

B. $A \times D \times E$

C. $C \times B \times E$

D. $A \times B \times E$



2. Units of angle, length, area, volume and capacity

Q6. Which of the following is a unit of volume?

Skill

A. mm^2

B. cm^2

C. cm^3

D. km

Q7. What are the missing units in the following calculations?

Skill

a) 6 cm × 20 cm = 120 ☐

b) 1.2 m × 6 m × 3 m = 21.6 ☐

c) 130 mm × 5 mm = 650 ☐

Check your understanding

Q8. What are the missing units in the following calculations?

Skill

a) 14 m × 9 ☐ × 7.5 m = 945 m^3

b) 0.03 mm × 2 mm × 0.67 ☐ = 0.0402 mm^3

3. Conversion of units

Q9. How many millimetres are in a centimetre?

Skill

Q10. Convert:

Skill

a) 12 L to cm^3

b) 4.32 km to m

c) 0.005 m^2 to mm^2

Check your understanding

Q11. What is 23.45 km in mm?

Skill

Q12. The capacity of a standard bath is around 378 L. What is this in cm^3?

Application

Joining it all together

Q13. The HTC Droid DNA was the first mobile phone to have a full HD (1080 × 1920 pixel) screen. The dimensions of the screen were 11.07 cm by 6.23 cm.

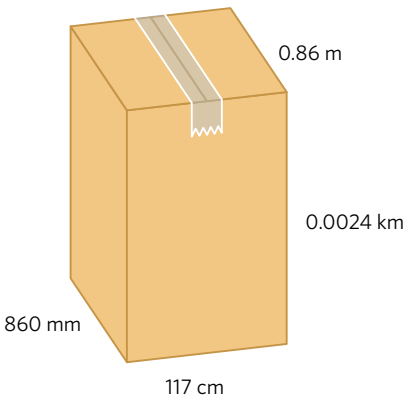
Application
2 marks

a) What was the area of the screen? Give your answer in cm^2, rounded to two decimal places.

b) What was the area taken up by one pixel? Give your answer in cm^2, rounded to two significant figures.

Q14. Find the volume of the box below in m^3, rounded to two decimal places.

Application
1 mark



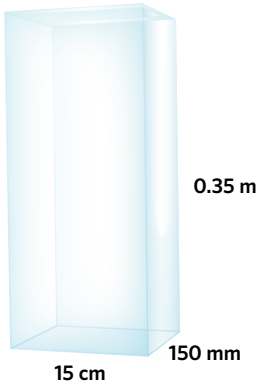
Q15. An Olympic swimming pool is a rectangle of dimensions 50 m by 25 m. If the pool is two metres deep, calculate the amount of water in the swimming pool, in litres.

Application
2 marks

Q16. Leroy, Matias and Nada were trying to calculate the capacity of a rectangular vase. Who calculated the capacity incorrectly?

Application
1 mark

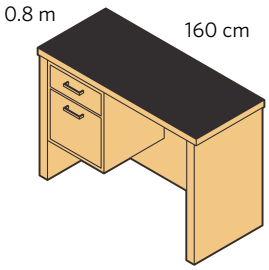
Leroy	$15 \times 15 \times 35 = 7875\ cm^3$
Matias	$150 \times 150 \times 350 = 7\ 875\ 000\ mm^3$
Nada	$0.15 \times 0.15 \times 0.35 = 0.0007875\ m^3$



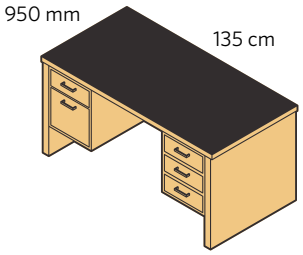
Q17. Look at the following diagrams of desks.

Application
2 marks

Desk A:



Desk B:



a) What quantity is possible to calculate using the measurements within each of the diagrams?

b) Which desk is larger?

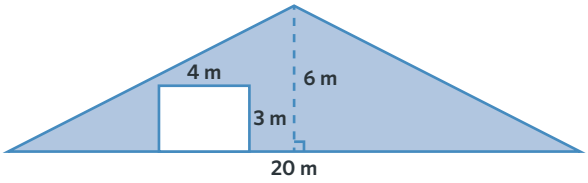
Questions from multiple lessons

Q18.

Difficulty:

Year 10
1 mark

Consider the following diagram.



The shaded area, in square meters, is

A. 120 m^2

B. 108 m^2

C. 60 m^2

D. 48 m^2

E. 12 m^2

VCAA 2016 Exam 1 Module 3: Geometry and measurement Q1 – Adapted

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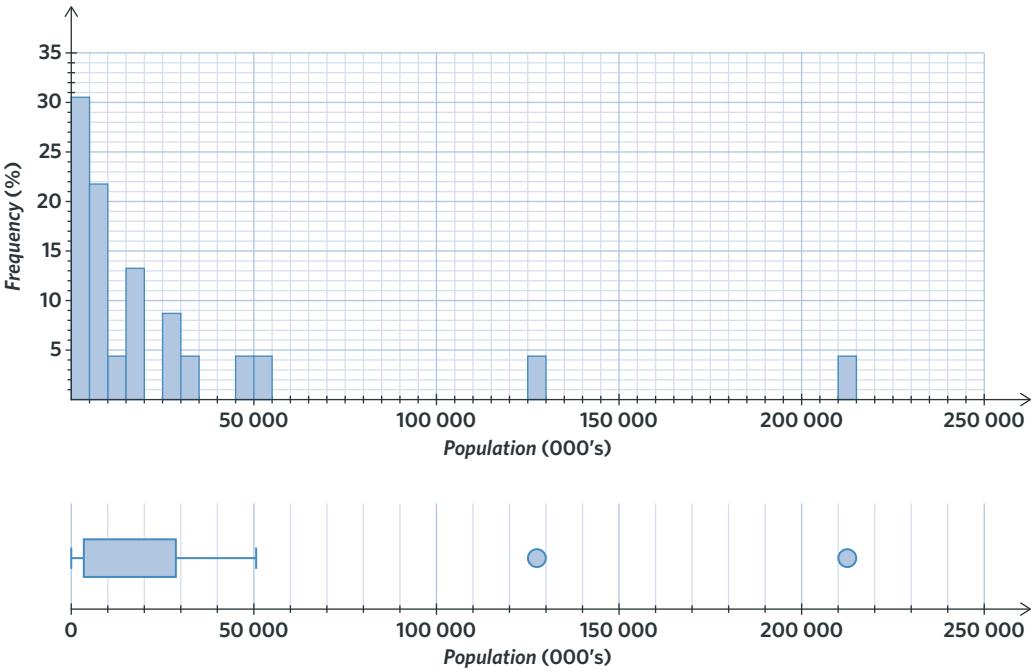
Q19.

Difficulty:

Year 10
1 mark

The histogram and boxplot shown both display the distribution of the *population* of 23 Central and South American countries.



The shape of the distribution for the countries' *population* is best described as

A. approximately symmetric with no outliers.

B. approximately symmetric with outliers.

C. positively skewed with outliers.

D. negatively skewed with outliers.

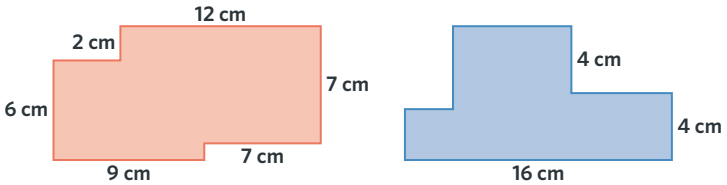
E. positively skewed with no outliers.

VCAA 2019NH Exam 1 Data analysis Q1 – Adapted

Q20.

Difficulty:

Year 10
2 marks



NOTE: All angles within both shapes are right angles.

a) What is the perimeter of the orange shape? (1 mark)

b) What is the difference between the perimeters of the two shapes? (1 mark)

LESSON 7B

Pythagoras' theorem – 2D

The key skills you will learn in this lesson are:

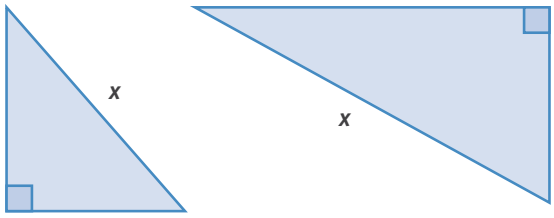
1. Finding the length of a hypotenuse
2. Finding an unknown side length

VCAA key knowledge point:

"Pythagoras' theorem in two dimensions, and simple examples in three dimensions, and application to practical problems"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

A right-angled triangle is a triangle in which there is one angle of 90° (a right angle). This angle is shown geometrically using a small square inside the angle, as seen below.

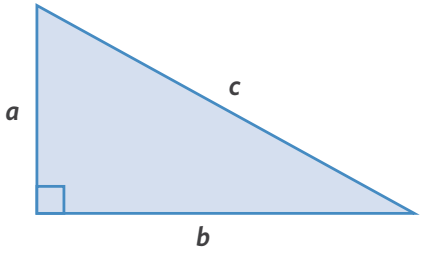


The longest side in a right-angled triangle is known as the hypotenuse. The hypotenuse is marked by an x in the triangles above. The hypotenuse is always opposite the right angle.

Pythagoras' theorem is a rule that states that for every right-angled triangle:

$$a^2 + b^2 = c^2$$

where c is the length of the hypotenuse and a and b are the other two side lengths (a and b are interchangeable).



Example

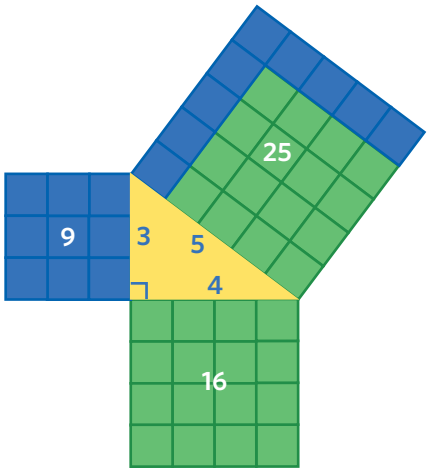
Consider a right-angled triangle with side lengths 3 and 4, and a hypotenuse of length 5.

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

These side lengths satisfy Pythagoras' theorem.

This can be shown visually:



1. Finding the length of a hypotenuse

Pythagoras' theorem is often used to find the length of a hypotenuse when the other two side lengths are known.

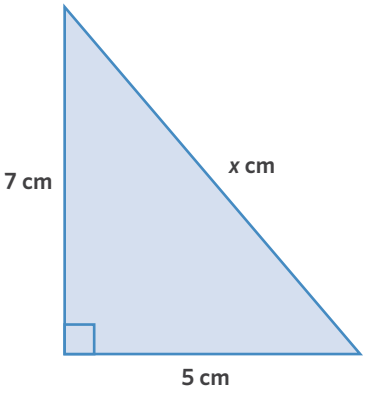
Pythagoras' theorem can be rearranged to $c = \sqrt{a^2 + b^2}$ to find the length of the hypotenuse.

WORKED EXAMPLE 4 *(1 mark)*

For a right-angled triangle with a height of 7 cm and a width of 5 cm, find the length of the hypotenuse correct to two decimal places.

SOLUTION

Step 1 Draw the triangle.



Step 2 Rearrange the formula to give the value of c.

$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

Step 3 Substitute the values of a and b into this formula.

c is the hypotenuse (the side opposite the right angle) and a and b are the other sides, so let $a = 5$ and $b = 7$.

$c = \sqrt{a^2 + b^2}$

$c = \sqrt{5^2 + 7^2}$

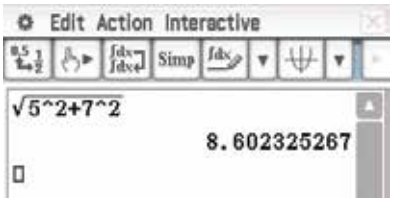
Step 4 Find the decimal answer using your calculator.

TI-NSPIRE

Type in expression then press ctrl + enter to get decimal answer.

CASIO CLASSPAD

Type in the expression then press EXE to get decimal answer.



Step 5 Write the answer to two decimal places.

$c = 8.6023\ldots$

The length of the hypotenuse is 8.60 cm.

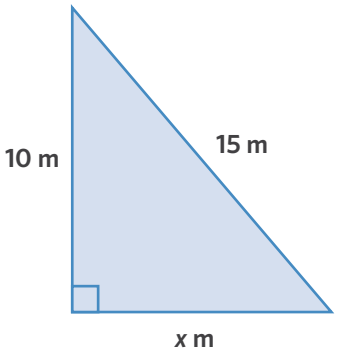
2. Finding an unknown side length

If the hypotenuse is known along with one of the other sides, the last unknown side can be found using Pythagoras' theorem.

Pythagoras' theorem can be rearranged so the length of the unknown side is given by the formula $a = \sqrt{c^2 - b^2}$ or $b = \sqrt{c^2 - a^2}$ (a and b are interchangeable).

WORKED EXAMPLE 5 *(1 mark)*

Find the value of x in the triangle below correct to two decimal places.



SOLUTION

Step 1 Rearrange the formula to give the value of a.

$c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$

$a = \sqrt{c^2 - b^2}$

Step 2 Substitute the values of b and c into this formula.

c is the hypotenuse (the side opposite the right angle), so $c = 15$.

b is the other known side, so $b = 10$.

$a = \sqrt{15^2 - 10^2}$

Step 3 Use your calculator to find the answer and write the answer.

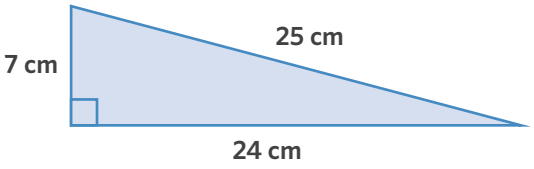
$a = 11.1803 \ldots$

The value of x is 11.18.

Questions 7B *Pythagoras' theorem – 2D*

Refresher question

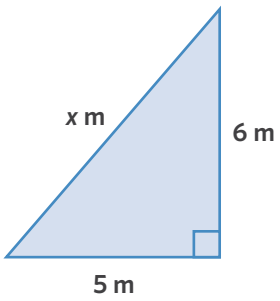
Q1. What is the length of the hypotenuse in the triangle below?



1. Finding the length of a hypotenuse

Q2. Skill

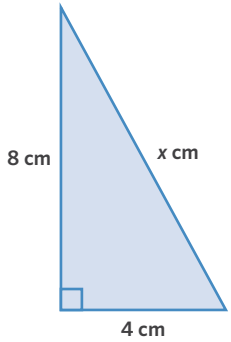
Use the formula $c = \sqrt{a^2 + b^2}$ to find the length of the hypotenuse in the triangle below correct to two decimal places.



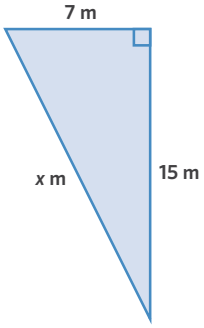
Q3. Skill

Find the length of the hypotenuse for the following triangles. Give answers correct to two decimal places.

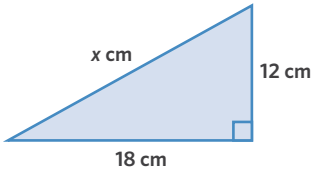
a)



b)



c)



Q4. Skill
A right-angled triangle has height 7 cm and width 10 cm. What is the length of its hypotenuse correct to two decimal places?

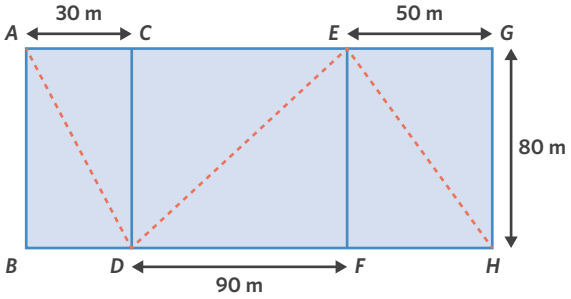
Check your understanding

Q5. Application
A new flying fox is being set up at Camp Edrolo. The flying fox will start on a wooden platform 12 metres above ground and will end on the ground, 100 metres horizontally from the bottom of the platform.

The rope for the flying fox will stretch from the top of the platform to the end point on the ground.

What length of rope will be needed, correct to two decimal places?

Q6. Application
Lucy started at point A and walked to point D, then to point E and finished at point H, as shown below. How far did Lucy walk in total correct to the nearest metre?

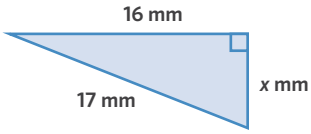


2. Finding an unknown side length

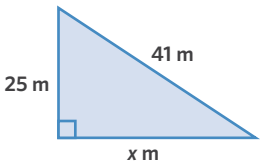
Q7. Skill
Use $a = \sqrt{c^2 - b^2}$ or $b = \sqrt{c^2 - a^2}$ to find the unknown side length in the triangles below.

Give answers correct to two decimal places.

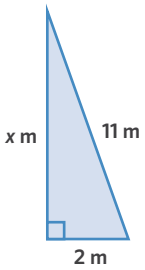
a)



b)

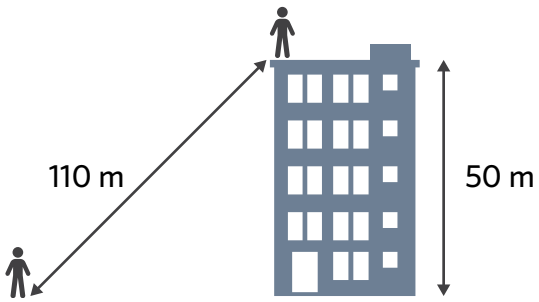


c)



Q8. Skill
A right-angled triangle has a height of 9 millimetres and a hypotenuse with a length of 17 millimetres. What is the width of the triangle correct to two decimal places?

Q9. Application
Adam is on top of a 50 metre-tall building and can see his friend, James, on the ground. He is at a distance of 110 metres from James. What is the distance between James and the base of the building, correct to two decimal places?



Check your understanding

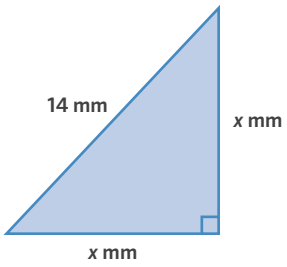
Q10. Skill

When both side lengths are the same it means that $a = b$.
In these cases, Pythagoras' theorem can be written as:

$a^2 + a^2 = c^2$.

With some rearranging, this gives the formula: $a = \sqrt{\frac{c^2}{2}}$

Use this formula to find the value of x correct to one decimal place.



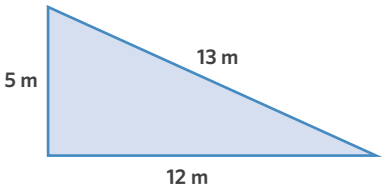
Q11. Application

While taking off, an aeroplane travels in a straight line until it reaches an altitude of 8 km. At this point, the plane has travelled a total of 50 km. What horizontal distance has the plane covered in this time, correct to three decimal places?

Joining it all together

Q12. Skill 2 marks

Show, using Pythagoras' theorem, that the triangle to the right is right-angled.

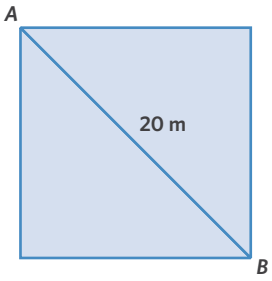


Q13. Application 1 mark

Jim is doing an orienteering course. He walks three kilometres north from the starting point. He then turns right and walks six kilometres east. How far will he have to walk to get directly back to the start, correct to three decimal places?

Q14. Skill 1 mark

If the diagonal length from A to B in the square below is 20 cm, what is the area of the square correct to the nearest cm^2?

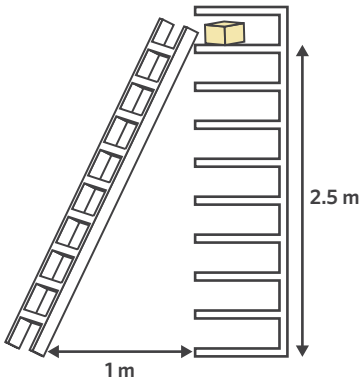


Q15. Application 2 marks

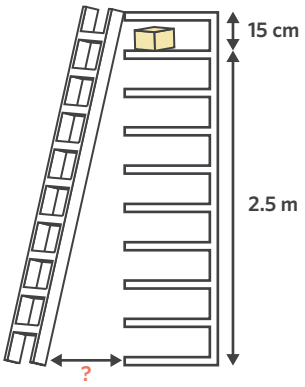
Allen needs to get something from a high shelf in his garage, so he gets out his ladder.

The shelf is 2.5 metres high and he positions the ladder 1 metre from the base of the shelf.

a) If the ladder reaches exactly to the shelf, calculate the length of the ladder in centimetres correct to two decimal places.



b) Allen now needs to put something back on the very top shelf. This shelf is 15 centimetres higher than the last one. Using your rounded value from part a) for the length of the ladder, calculate the distance between the bottom of the ladder and the base of the shelf. Give your answer in centimetres correct to one decimal place.



VCAA question

Q16. A soccer goal is 7.4 metres wide.

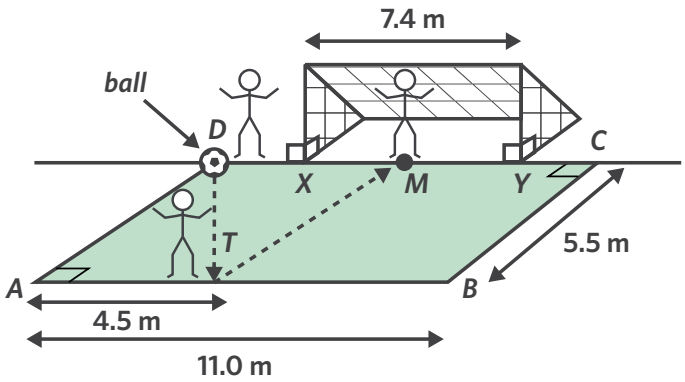
1 mark

A rectangular region $ABCD$ is marked out directly in front of the goal.

In this rectangular region, $AB = DC = 11.0$ metres and $AD = BC = 5.5$ metres.

The goal line XY lies on DC and M is the midpoint of both DC and XY.

David kicks the ball from point D in a straight line to Tara. Tara is standing near point T on the line AB, a distance of 4.5 metres from point A. Tara then kicks the ball from point T in a straight line to the midpoint of the goal line at M.



The total distance that the ball will travel in moving from point D to T to M is closest to

A. 5.5 m

B. 12.1 m

C. 12.5 m

D. 12.7 m

E. 12.9 m

VCAA 2010 Exam 1, Module 2: Geometry and Trigonometry. Q6

Questions from multiple lessons

Q17.

Difficulty:

Year 10
1 mark

A cartographer is sketching a map. The scale used for the map is 1:100 000.

On the map, a distance of 10 km will be represented by

A. 1000 cm

B. 1 cm

C. 100 cm

D. 10 cm

E. 0.1 cm

VCAA 2013 Exam 1 Module 2: Geometry and trigonometry Q5 – Adapted

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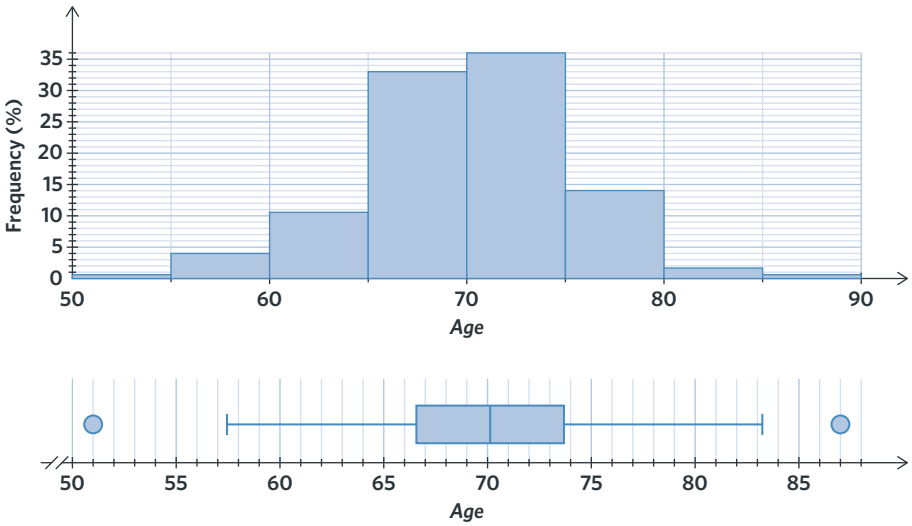
Q18.

Difficulty:

Year 10
1 mark

The following histogram and boxplot display the distribution for the *age* of 200 residents at a retirement village in Nepal.



The number of people between the ages of 65 and 70 years is closest to

A. 70

B. 66

C. 35

D. 33

E. 21

VCAA 2019NH Exam 1 Data analysis Q2 – Adapted

Q19.

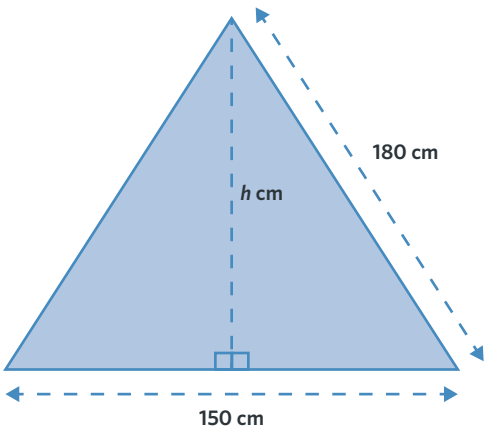
Difficulty:

Year 10
2 marks

The front view of a tent is shown.

a) Show that the height, h, of the tent is 163.6 cm, correct to the nearest millimetre. (1 mark)

b) Convert 163.6 cm into km. (1 mark)



LESSON 7C

Pythagoras' theorem – 3D

The key skills you will learn in this lesson are:

1. Pythagoras' theorem in three dimensions
2. Applications of Pythagoras' theorem in three dimensions

VCAA key knowledge point:

"Pythagoras' theorem in two dimensions, and simple examples in three dimensions, and application to practical problems"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.20; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

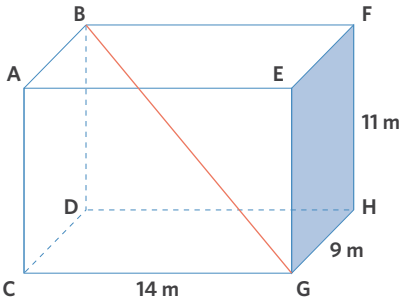
1. Pythagoras' theorem in three dimensions

In many cases, a missing side length or diagonal in a three-dimensional object can be found using Pythagoras' theorem. To do so, split the problem up into separate steps, and draw a diagram for each one.

WORKED EXAMPLE 6 *(2 marks)*

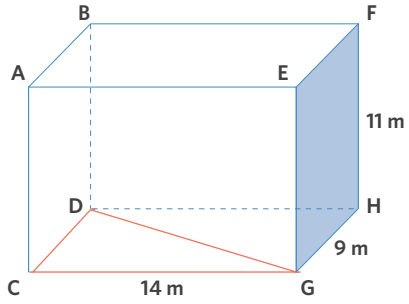
A rectangular prism has length 14 metres, width 9 metres, and height 11 metres.

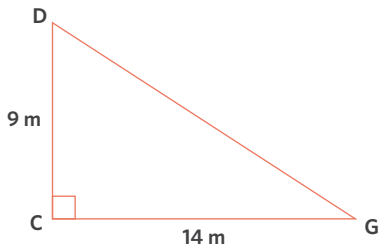
Find the length of the diagonal BG correct to two decimal places.



SOLUTION

Step 1 Draw the triangle CDG and find the length of DG.





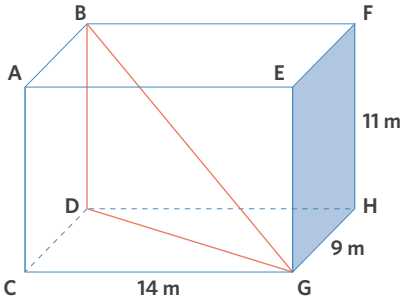
$a^2 + b^2 = c^2$

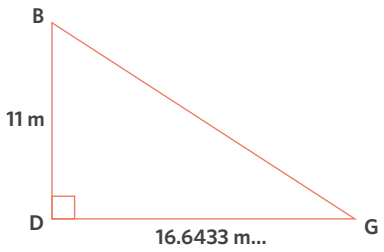
$DG = \sqrt{9^2 + 14^2}$

$DG = 16.6433\ldots$

✓ *1 mark for finding length of side DG*

Step 2 Draw the triangle BDG and find the length of BG.





$BG = \sqrt{11^2 + 16.6433\ldots^2}$

$BG = 19.9499\ldots$

The length of BG is 19.95 metres.

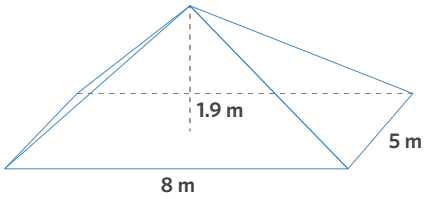
✓ *1 mark for finding length of side BG*

2. Applications of Pythagoras' theorem in three dimensions

This method can also be applied in many real-life situations. For example, Pythagoras' theorem can be used to determine whether something can fit inside an object, or for measurements used in building and planning.

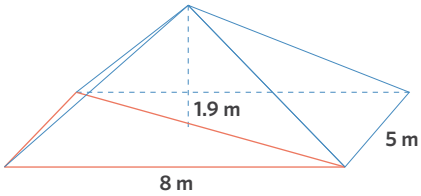
WORKED EXAMPLE 7 *(2 marks)*

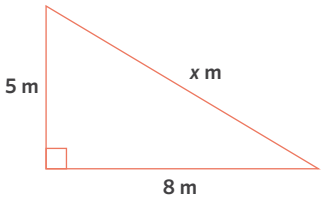
The roof of a house is in the shape of a rectangular-based pyramid with a width of 5 metres, a length of 8 metres and a height of 1.9 metres. What is the distance from the top of the roof to each corner correct to two decimal places?



SOLUTION

Step 1 Find the length of the base diagonal.





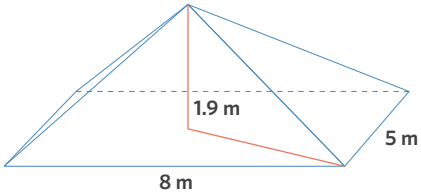
$a^2 + b^2 = c^2$

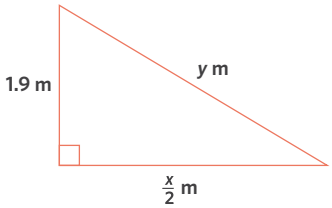
$x = \sqrt{8^2 + 5^2}$

$x = 9.4339\ldots$

✓ *1 mark for finding the length of the base diagonal*

Step 2 Find the length from the top to the corner.





The base of this triangle is half the length of the hypotenuse that we found in step 1.

$\frac{x}{2} = \frac{9.4339\ldots}{2} = 4.7169\ldots$

$a^2 + b^2 = c^2$

$y = \sqrt{1.9^2 + 4.7169\ldots^2}$

$y = 5.0852\ldots$

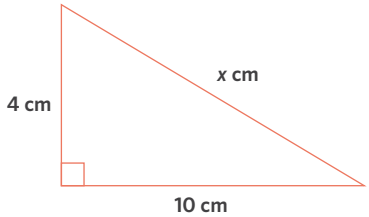
∴ The distance from the top of the roof to each corner is 5.09 metres.

✓ *1 mark for finding the distance from the top of the roof to each corner*

Questions 7C *Pythagoras' theorem – 3D*

Refresher question

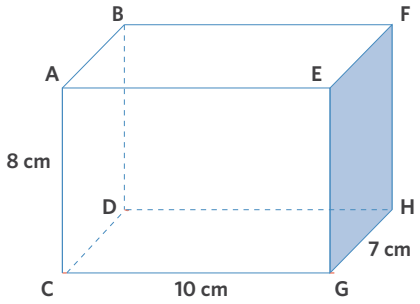
Q1. Find the value of x in the triangle on the right, correct to one decimal place.



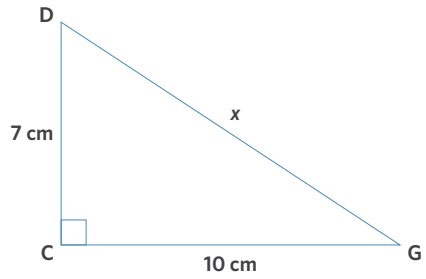
1. Pythagoras' theorem in three dimensions

Q2. Consider the rectangular prism to the right.

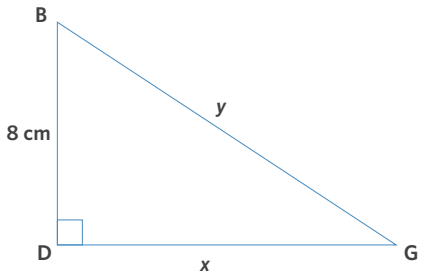
Skill



a) Find the length of the diagonal from vertex D to G correct to three decimal places. The right-angled triangle has been drawn for you below.

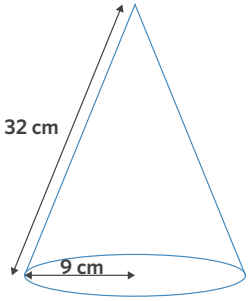


b) Using your value from part **a)**, find the length of the diagonal between vertices B and G correct to two decimal places. The right-angled triangle has been drawn for you below.



Q3. Find the vertical height of the cone below correct to two decimal places.

Skill

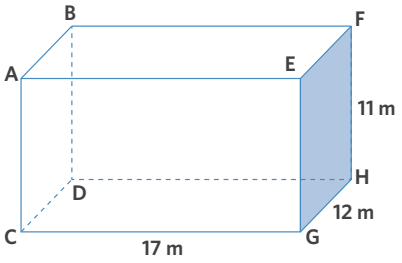


Q4. In the rectangular prism below, find, correct to two decimal places:

Skill

a) the length of the diagonal from B to E.

b) the length of the diagonal from C to F.



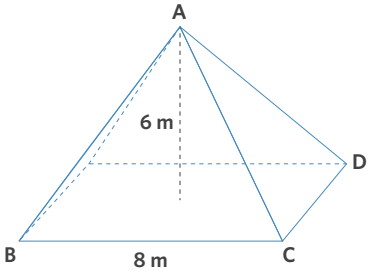
Q5. A square-based pyramid has a width of 8 metres and a height of 6 metres.

Skill

Find:

a) the length of the diagonal from B to D correct to two decimal places.

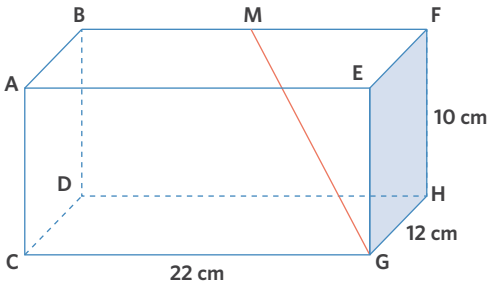
b) the length of the edge AB correct to two decimal places.



Check your understanding

Q6. Skill

Consider the rectangular prism below with length 22 cm, height 10 cm and width 12 cm. M is the midpoint between points B and F. What is the length of the line from point G to M correct to one decimal place?

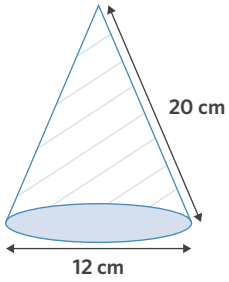


2. Applications of Pythagoras' theorem in three dimensions

Q7. Application

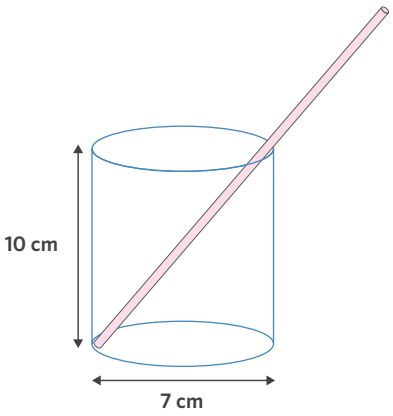
For his eighth birthday party, Sammy bought party hats for his friends. One of the hats is shown below.

What is the vertical height of the hat correct to two decimal places?



Q8. Application

A straw is placed diagonally in a cylindrical cup so that exactly half of the straw is inside the cup. The cup has a height of 10 centimetres and a width of 7 centimetres. What is the length of the straw correct to one decimal place?



Q9. Application

James has a rectangular prism-shaped pencil case with a width of nine centimetres, a height of six centimetres, and a length of 20 centimetres. He has a pencil that is 23 centimetres long. Will it fit inside his pencil case?

Q10. Application

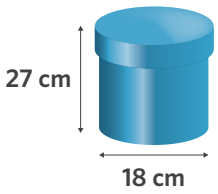
Lucy is decorating for a party so decides to hang streamers across the room. She wants to hang one streamer across the room from the top corner of the room to the opposite bottom corner. If the room measures 3 metres by 2.5 metres by 4 metres, how long must the streamer be correct to two decimal places?

Check your understanding

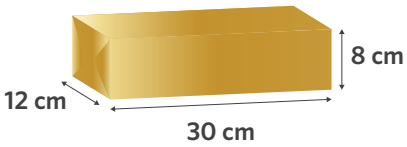
Q11. Application

Sammy's friend, Alex, knows that he loves Harry Potter so he gets him a replica wand for his birthday. The wand is 33 centimetres long. Alex is looking for a gift box that the wand will fit inside. Which of the boxes below will be big enough?

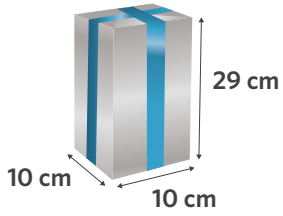
Box A



Box B



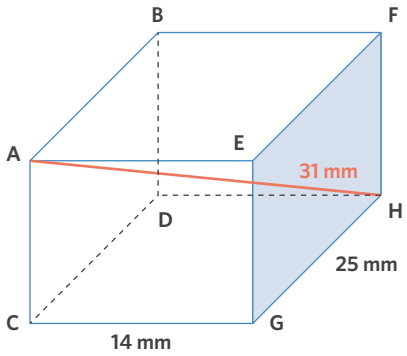
Box C



Joining it all together

Q12. Skill 2 marks

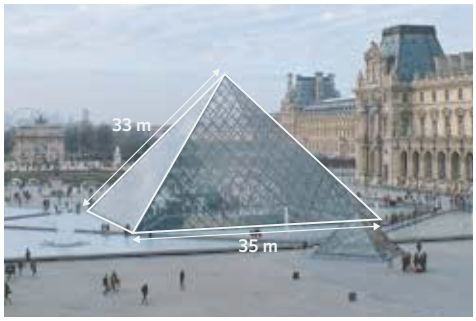
The length of the diagonal between vertices A and H in the rectangular prism below is 31 millimetres. What is the height of the prism correct to two decimal places?



Q13. Application 2 marks

The Pyramide du Louvre is a glass and metal square-based pyramid outside the Louvre in Paris.

The base of the pyramid has side lengths of 35 metres and the distance from the top of the pyramid to each corner is 33 metres. What is the vertical height of the pyramid correct to two decimal places?



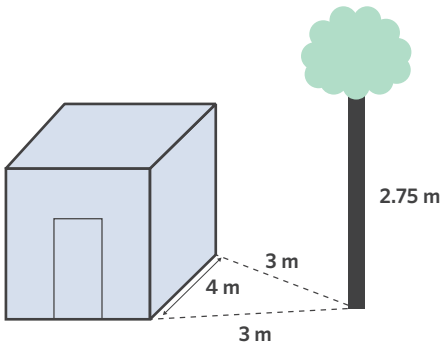
Source: Photo by Irene Ledyaeva on Unsplash

Q14. Application 2 marks

A tree standing next to a garden shed falls down due to strong wind, and hits the wall of the shed.

The base of the tree is three metres from two of the corners of the shed, and the shed is four metres wide.

If the tree is 2.75 metres tall, what is the maximum height above the ground at which the tree could have hit the wall, correct to one decimal place?



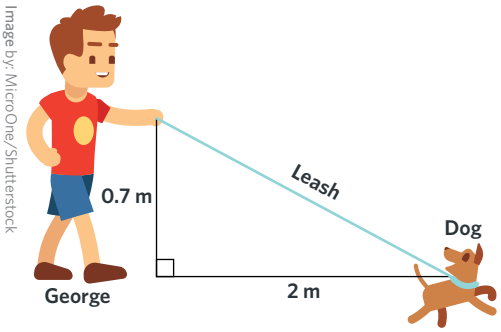
Questions from multiple lessons

Q15.

Difficulty:

Year 10
1 mark

George decides to walk his dog on a leash as shown in the following diagram.



The length of the leash, in metres, is closest to

A. 2.1 m

B. 2.2 m

C. 2.7 m

D. 3.5 m

E. 4.5 m

VCAA 2018 Exam 1 Module 3: Geometry and measurement Q1 – Adapted

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Q16.

Difficulty:

1 mark

Hugo would like to purchase a new yacht. He will establish a loan from the bank for $1 200 000 with interest charged at a rate of 4.9% per annum, compounding monthly.

Each month, Hugo will only pay for the interest charged that month.

After 7 months, the amount that he still owes is closest to

A. $1 166 117

B. $844 199

C. $1 677 295

D. $1 200 000

E. $1 234 723

VCAA 2017 Exam 1 Recursion and financial modelling Q19 – Adapted

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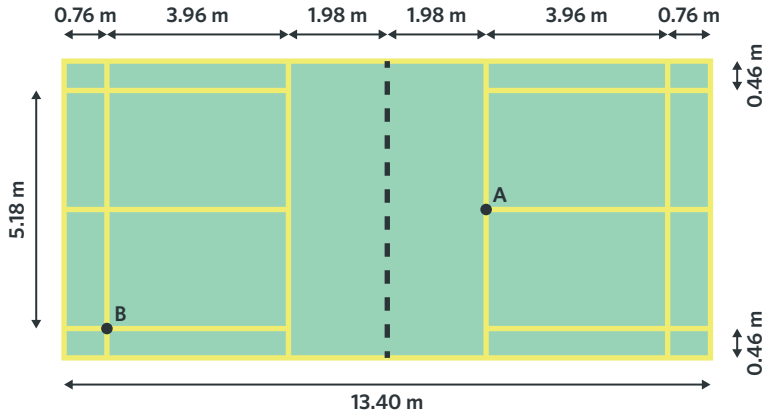
Q17.

Difficulty:

Year 10
2 marks

Boris has a badminton court in his backyard. Shown is a diagram of Boris' badminton court. Assume that all intersecting lines meet at right angles.



Boris stands at point *A* and serves to point *B*.

a) What is the straight-line distance, in metres, between point *A* and point *B*?
Round your answer to two decimal places. (1 mark)

b) Boris serves at a height of 2.0 m directly above point *A*.
Assume that the ball travels in a straight line to the ground at point *B*.
What is the straight-line distance, in metres, that the ball travels?
Round your answer to one decimal place. (1 mark)

VCAA 2018 Exam 1 Module 3: Geometry and measurement Q3 a,b – Adapted

LESSON 7D

Quadrilaterals and circles

The key skills you will learn in this lesson are:

1. Quadrilaterals
2. Circles

VCAA key knowledge point:

"perimeter and areas of triangles (including the use of Heron's formula), quadrilaterals, circles and composite shapes and practical applications"

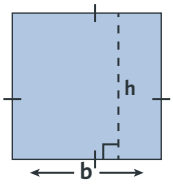
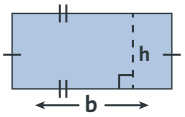
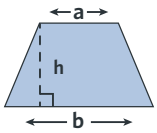
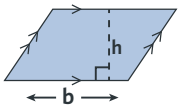
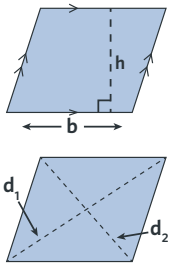
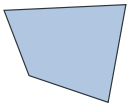
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21;

1. Quadrilaterals

A quadrilateral is any shape that has four sides and four corners. Squares and rectangles are quadrilaterals, but there are other types of quadrilaterals too.

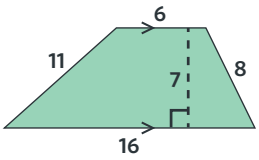
The perimeter is the distance around the edges of any shape, and is found by finding the lengths of each side and adding them together.

The area of quadrilaterals can be found using the formulas given below.

Name	Distinguishing features	Example	Formula for area
Square	Two pairs of parallel sides all the same length, at right angles		$area = b \times h = b^2$
Rectangle	Two pairs of parallel sides all at right angles		$area = b \times h$
Trapezium	One pair of parallel sides		$area = \frac{(a + b)}{2} \times h$
Parallelogram	Two pairs of parallel sides		$area = b \times h$
Rhombus	Two pairs of parallel sides all the same length		$area = b \times h$ or $area = \frac{d_1 \times d_2}{2}$
Irregular	No parallel sides		None

WORKED EXAMPLE 8 *(2 marks)*

Find the area and perimeter of the following quadrilateral. All measurements are in centimetres.



SOLUTION

Step 1 We need to identify the type of quadrilateral. There is one pair of parallel sides in this shape, so we know it is a trapezium.

Step 2 Apply the relevant formula.

$$area = \frac{(a + b)}{2} \times h$$

Step 3 Sides a and b are the parallel sides, so $a = 6$ and $b = 16$.

Further, $h = 7$.

Step 4 Calculate.

$$area = \frac{(6 + 16)}{2} \times 7$$
$$= 11 \times 7$$
$$= 77 \text{ cm}^2$$

✓ *1 mark for calculating area*

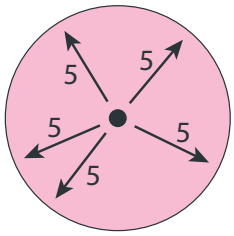
Step 5 The perimeter is found by adding the lengths of the sides together.

$$perimeter = 11 + 6 + 8 + 16$$
$$= 41 \text{ cm}$$

✓ *1 mark for calculating perimeter*

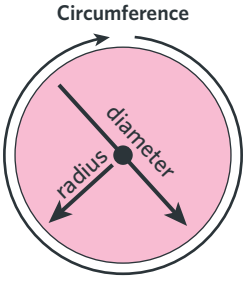
2. Circles

A circle is a curve where each point is the same distance from the centre. In the circle below, every part of the curve is 5 centimetres from the centre point.

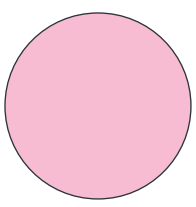


Important properties of a circle:

- The diameter of a circle (d) is the length from one side of the circle to the other, through the centre.
- The radius of a circle (r) is the distance from any point on a circle's edge to its centre.
- The circumference (C) is the distance around a circle.
- Pi (π) is an extremely important number when dealing with circles. It is approximately equal to 3.14159.

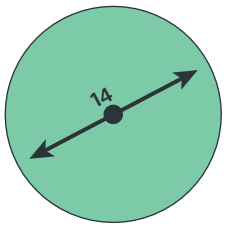


The area and circumference (perimeter) of circles can be found using the formulas given below.

	$area = \pi \times r^2$ $= \pi r^2$	$C = 2 \times \pi \times r$ $= 2\pi r$ $C = \pi \times d$ $= \pi d$

WORKED EXAMPLE 9 *(2 marks)*

Find the circumference and area of the following circle, rounded to two decimal places.



SOLUTION

Step 1 Write the formula for circumference.

$C = \pi d$

Step 2 Substitute in the diameter and calculate.

$C = \pi \times 14$

$= 43.98$ cm

✓ *1 mark for calculating circumference*

Step 3 Write the formula for area.

$area = \pi r^2$

Step 4 The radius, r, is half of the diameter.

$14 \times \frac{1}{2} = 7$

So $r = 7$. Calculate the area.

$area = \pi \times 7^2$

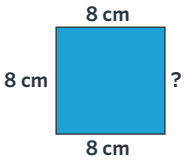
$= 153.94$ cm^2

✓ *1 mark for calculating area*

Questions 7D *Quadrilaterals and circles*

Refresher question

Q1. What is the length of the unknown side on the square?



1. Quadrilaterals

Q2. Fill in the missing numbers in the following working.

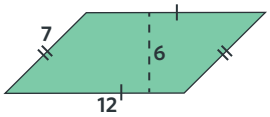
$Perimeter = 12 + 12 + 7 + 7$

$= \square$ cm

$area = \text{b} \times \text{h}$

$= 12 \times 6$

$= \square$ cm^2

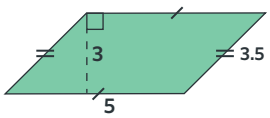


Q3. Skill

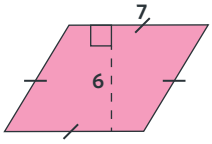
Find the area and perimeter for each of the following shapes.
Assume all measurements are in centimetres.

a)

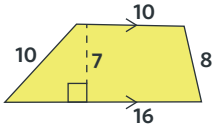
b)



c)



d)

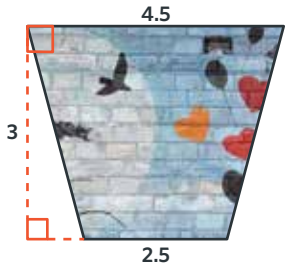


Check your understanding

Q4. Application

Derek is painting the following mural on a school wall. It is a trapezium of parallel sides 2.5 metres and 4.5 metres, with a height of 3 metres.

What is the total painted area?



Q5. Application

Below is a diagram of a park. All measurements are in kilometres. What is the total area of the park,

2. Circles

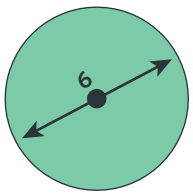
Q6. Skill

Fill in the **three** missing numbers in the following working to find the circumference (C) and area. Round your answers to two decimal places.

$C = \pi \times d$

$C = \pi \times 6$

$C = \square$ cm



$r = \frac{d}{2}$

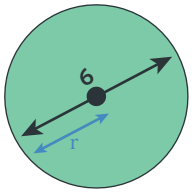
$r = \frac{\square}{2}$

$r = 3$

$area = \pi \times r^2$

$area = \pi \times 3^2$

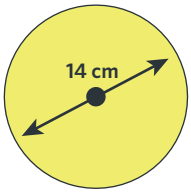
$area = \square$ cm^2



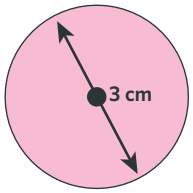
Q7. Skill

Find the area and circumference, correct to two decimal places, for each of the following circles.

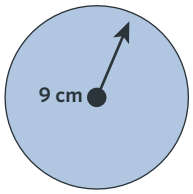
a)



b)



c)

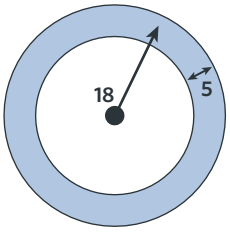


Check your understanding

Q8. Application

The following diagram shows the blueprint of a new water pipe, to supply water to a small town. The blue part is the pipe, and the white part is where the water flows. To supply the town with enough water, the area the water flows through needs to be at least 500 cm^2.

Is the pipe big enough? All measurements are in centimetres.

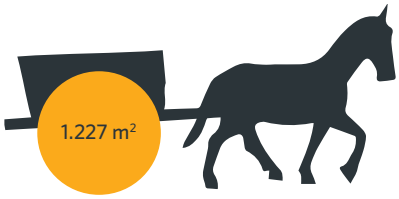


Q9. Application

The wheel on the carriage below has an area of 1.227 m^2.
Round your answers for this question to two decimal places.

a) What is the diameter of the wheel?

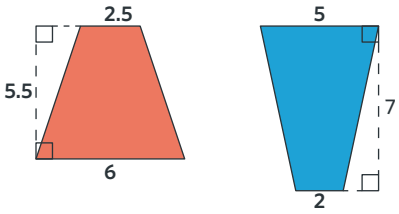
b) Using your answer from part a, find how far the cart moves forward for each turn of the wheel.



Joining it all together

Q10. Skill 1 mark

Show that the area of the blue trapezium is larger than the area of the red trapezium.
All measurements are in centimetres.



Q11. Application 1 mark

The distance around the earth at the equator is 40 075 km.

Assuming the equator is a perfect circle and the Earth a perfect sphere, calculate the radius of the Earth to the nearest kilometre.

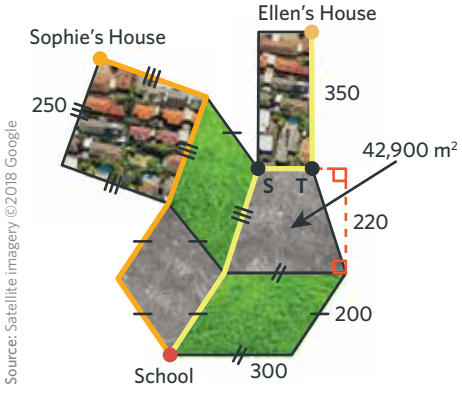
Q12. Application 2 marks

Two prep students, Sophie and Ellen, are fighting about who lives closest to school.

The following map shows their paths home.

a) Use the given area of a paddock to calculate the length of path *ST*.

b) Who has a shorter walk to school?

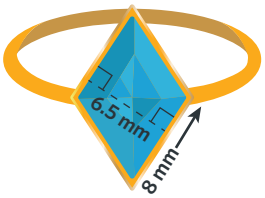


Q13. Andy bought Sam the commitment ring below, and Sam is anxious to be able to tell their friends how big the ring is!

Application
2 marks

a) By modelling the diamond as a circle, find how much area the diamond takes up, to two decimal places.

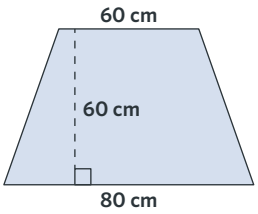
b) Sam's friend Alex was also given a ring recently. They are bickering about who has the largest ring. Find the area of the sapphire in Alex's ring and declare whose ring is bigger.



VCAA question

Q14. The following graphic shows the top of a table in the shape of a trapezium.

1 mark



The area of the tabletop, in square centimetres, is

A. 200

B. 260

C. 4200

D. 4800

E. 288 000

VCAA 2015 Exam 1, Module 2: Geometry and Trigonometry. Q1

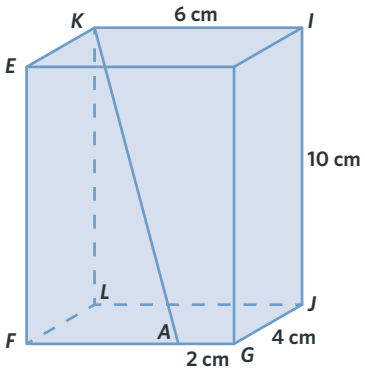
Questions from multiple lessons

Q15. The following graphic shows a juice box that is 6 cm long, 4 cm wide and 10 cm high.

Difficulty:

Year 10
1 mark

A straw sits inside the box. One end of the straw sits at A and the other end sits at K.



The point A lies on the line FG at a distance of 2 cm from G.

The length of the straw, in centimetres, is closest to

A. 10.77 cm

B. 10.95 cm

C. 11.49 cm

D. 12.33 cm

E. 13.42 cm

VCAA 2014 Exam 1 Module 2: Geometry and trigonometry Q5 – Adapted

Q16. The sum of the ages of a family of twelve people is 312 years.

Difficulty:

The mean age of members in this family is

Year 10
1 mark

A. 26

B. 25

C. 16.5

D. 15.5

E. 27

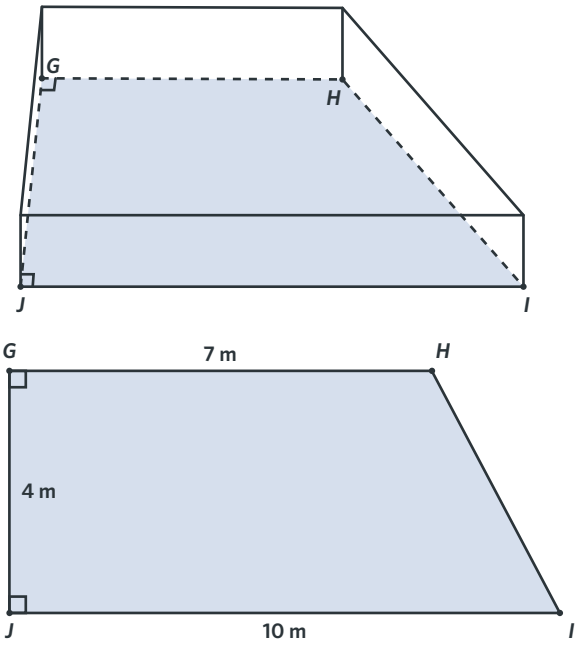
VCAA 2019NH Exam 1 Data analysis Q3 – Adapted

Q17. A fence around the perimeter of a plant nursery is in the shape of a trapezium.

Difficulty:

The fence, and the base, *GHIJ*, are shown below.

2 marks



$GH = 7$ m, $IJ = 10$ m, $GJ = 4$ m.

a) What is the area of the base of the plant nursery?
Write your answer in square metres. (1 mark)

b) What is the perimeter of the plant nursery?
Write your answer in metres, correct to one decimal place. (1 mark)

VCAA 2014 Exam 2 Module 2: Geometry and trigonometry Q1a,b – Adapted

LESSON 7E

Triangles

The key skills you will learn in this lesson are:

1. Perimeter of a triangle
2. Area of a triangle
3. Area using Heron's formula

VCAA key knowledge point:

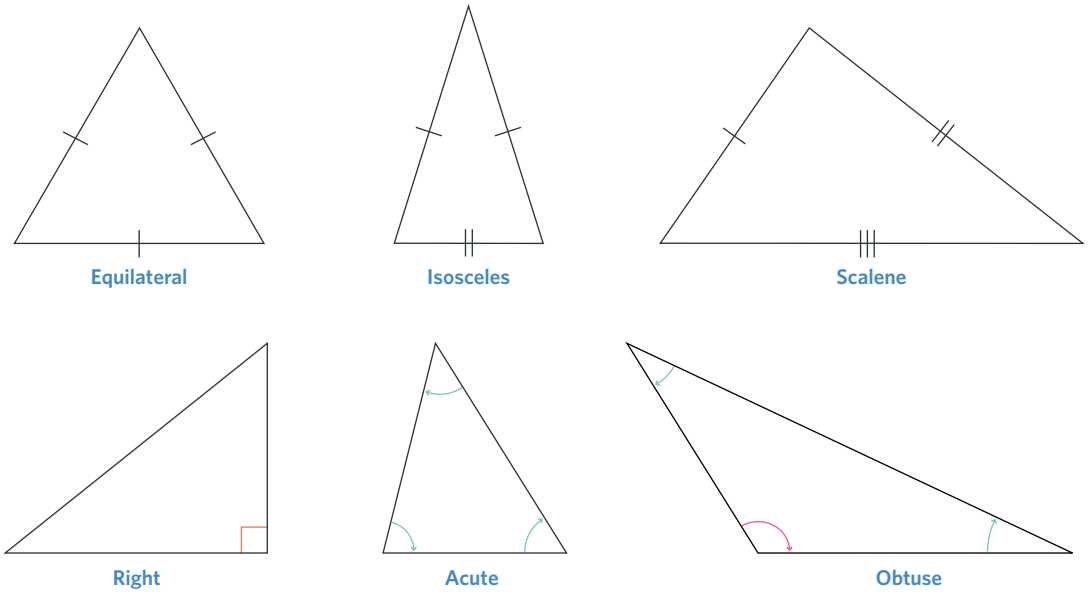
"perimeter and areas of triangles (including the use of Heron's formula), quadrilaterals, circles and composite shapes and practical applications"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016–2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

A triangle is any shape with three edges and three vertices.

This lesson will show how to calculate the perimeter and area of different types of triangles.

Types of triangles

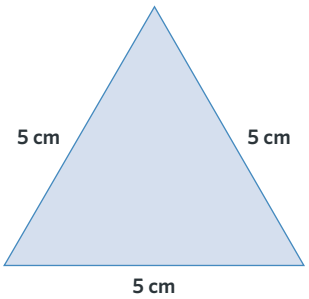


1. Perimeter of a triangle

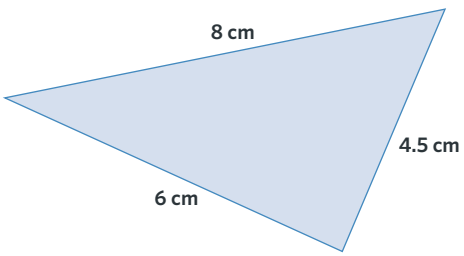
As seen in 7A and 7D, the perimeter of a shape is the length around its edges.

The perimeter of a triangle (P) is found by finding the lengths of each of the three sides and adding them together.

Example



Perimeter = 5 + 5 + 5
= 15 cm

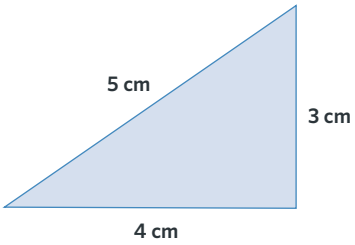


Perimeter = 6 + 8 + 4.5
= 18.5 cm

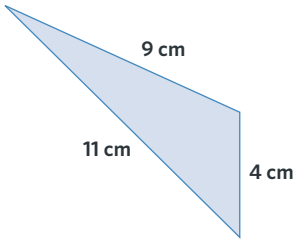
WORKED EXAMPLE 10 *(2 marks)*

Find the perimeter of the following triangles.

a)



b)



SOLUTION

Step 1 The three sides are 5 cm, 4 cm, and 3 cm long.

Step 2 Add up the length of the sides to find the perimeter.

$P = 5 + 4 + 3$

$= 12$ cm

Step 1 The three sides are 11 cm, 9 cm, and 4 cm long.

Step 2 Add up the length of the sides to find the perimeter.

$P = 11 + 9 + 4$

$= 24$ cm

2. Area of a triangle

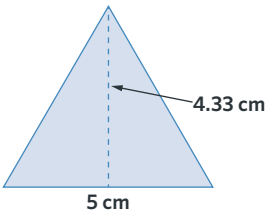
There are a few different ways to work out the area of a triangle (A).

If the length of one side (called the 'base') is known, as well as the 'height' of the triangle (measured perpendicular to the base), then the following formula can be used.

$$Area = \frac{1}{2} \times base \times height$$

$$= \frac{1}{2} \times b \times h$$

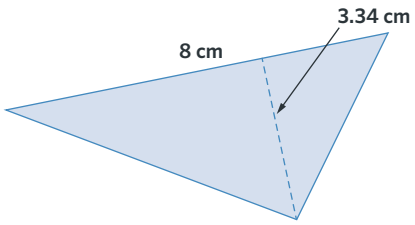
Example



$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 5 \times 4.33$$

$$= 10.825 \text{ cm}^2$$



$$\text{Area} = \frac{1}{2} \times b \times h$$

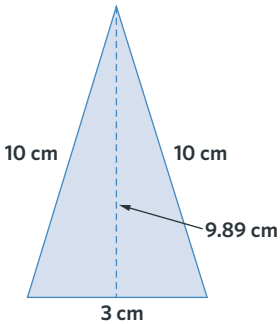
$$= \frac{1}{2} \times 8 \times 3.34$$

$$= 13.36 \text{ cm}^2$$

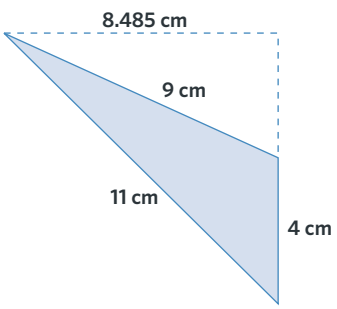
WORKED EXAMPLE 11 *(2 marks)*

Find the area of the following triangles.

a)

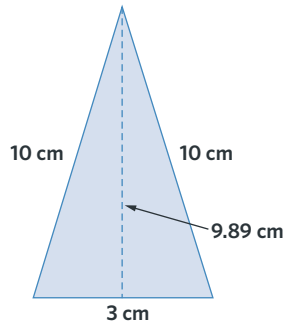


b)



SOLUTION

a)



Step 1 We need a base and a height. Three sides, 10 cm, 10 cm, and 3 cm, and one height, 9.89 cm, are given.

Use the side that is perpendicular to the height, 3 cm.

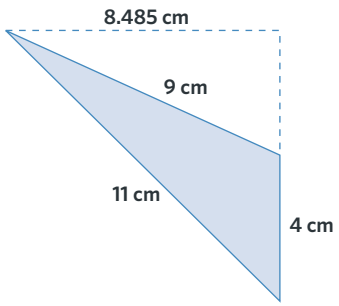
Step 2 Use the formula to find the area.

$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 3 \times 9.89$

$= 14.835 \text{ cm}^2$

b)



Step 1 In this problem the height given is perpendicular to the side of length 4 cm.

Step 2 Use the formula to find the area.

$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4 \times 8.485$

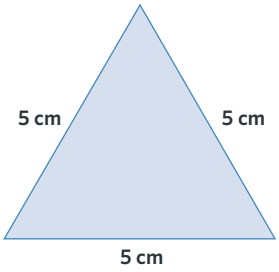
$= 16.97 \text{ cm}^2$

3. Area using Heron's formula

If three sides of a triangle are known, but the height isn't, Heron's formula can be used to calculate the area.

$A = \sqrt{S(S-a)(S-b)(S-c)}$ where $S = \frac{Perimeter}{2}$ and a, b, c are the three side lengths.

Example



$Perimeter = 5 + 5 + 5$

$= 15 \text{ cm}$

$S = \frac{P}{2}$

$= 7.5$

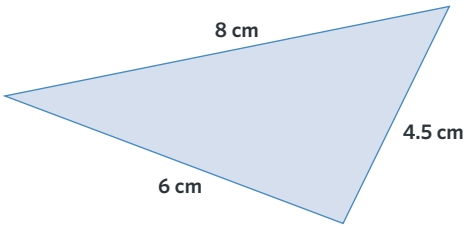
$A = \sqrt{S(S-a)(S-b)(S-c)}$

$A = \sqrt{7.5(7.5-5)(7.5-5)(7.5-5)}$

$A = \sqrt{7.5 \times 2.5 \times 2.5 \times 2.5}$

$A = \sqrt{117.1875}$

$A = 10.825 \text{ cm}^2$



$Perimeter = 6 + 8 + 4.5$

$= 18.5 \text{ cm}$

$S = \frac{P}{2}$

$= 9.25$

$A = \sqrt{S(S-a)(S-b)(S-c)}$

$A = \sqrt{9.25(9.25-8)(9.25-6)(9.25-4.5)}$

$A = \sqrt{9.25(1.25)(3.25)(4.75)}$

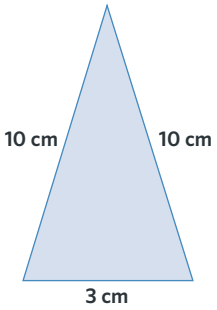
$A = \sqrt{178.496}$

$A = 13.36 \text{ cm}^2$

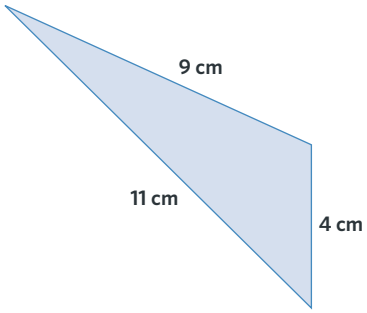
WORKED EXAMPLE 12 *(2 marks)*

Find the area of the following triangles using Heron's formula.

a)



b)



SOLUTION

Step 1 Calculate the perimeter.

$P = 10 + 10 + 3$

$= 23$ cm

Step 2 Find S.

$S = \frac{P}{2}$

$S = 11.5$

Step 3 Substitute S and the three sides, a, b and c, into Heron's formula.

$A = \sqrt{S(S-a)(S-b)(S-c)}$

$A = \sqrt{11.5(11.5-10)(11.5-10)(11.5-3)}$

$A = \sqrt{11.5(1.5)(1.5)(8.5)}$

$A = \sqrt{219.9375}$

$A = 14.83$ cm^2

Step 1 Calculate the perimeter.

$P = 11 + 9 + 4$

$= 24$ cm

Step 2 Find S.

$S = \frac{P}{2}$

$S = 12$

Step 3 Substitute S and the three sides, a, b and c, into Heron's formula.

$A = \sqrt{S(S-a)(S-b)(S-c)}$

$A = \sqrt{12(12-11)(12-9)(12-4)}$

$A = \sqrt{12(1)(3)(8)}$

$A = \sqrt{288}$

$A = 16.97$ cm^2

Questions 7E *Triangles*

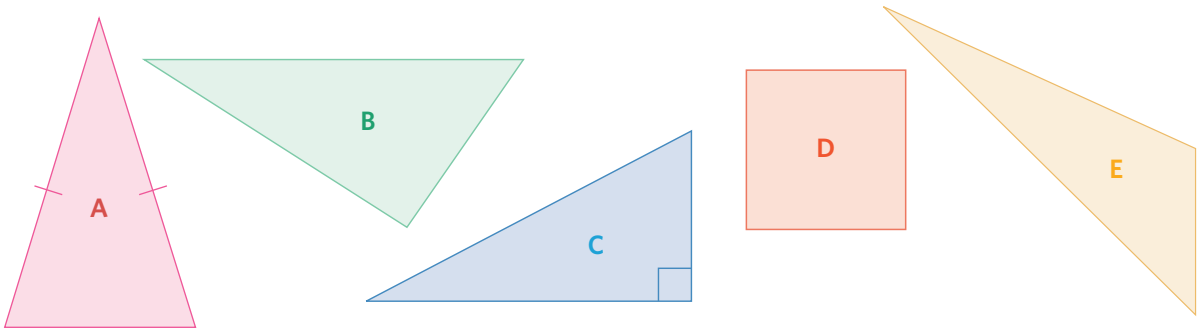
Refresher question

Q1. Which of the following is:

a) a right-angled triangle?

b) an isosceles triangle?

c) an obtuse triangle?



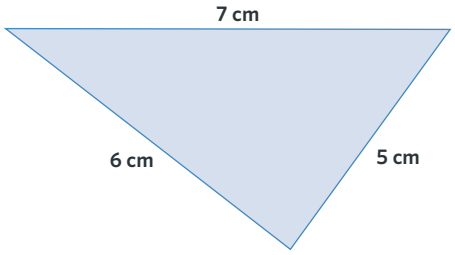
1. Perimeter of a triangle

Q2. Skill

Fill in the missing number in the working out below.

Perimeter = 7 + 6 + 5

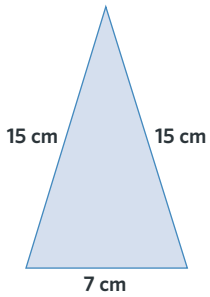
= ☐ *cm*



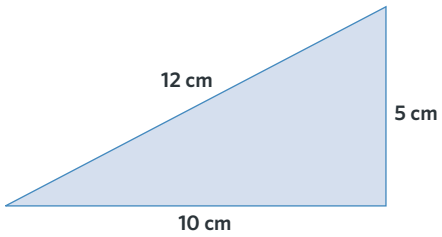
Q3. Skill

Calculate the perimeter of the following triangles.

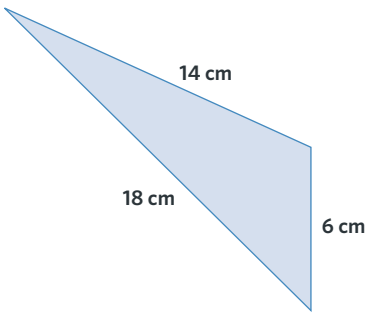
a)



b)



c)

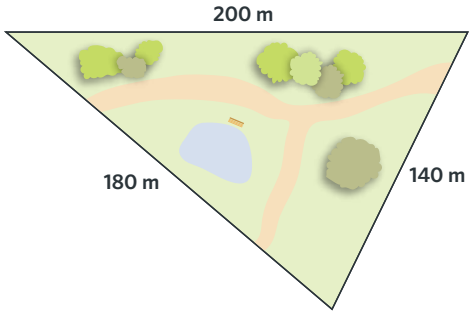


Check your understanding

Q4. Application

Every day, Dion walks two laps of the local park.

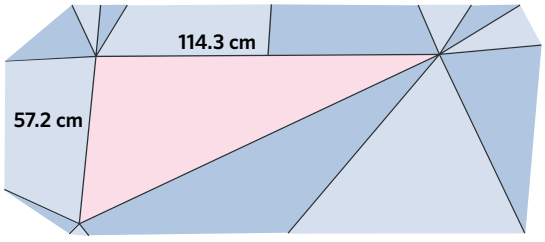
Calculate the perimeter of the park, and how far Dion walks around the park each day.



Q5. Application

The triangles that cover the buildings in Federation Square have a perimeter of 299.2 cm.

If the two shorter sides are 57.2 cm and 114.3 cm, what is the length of the third side?

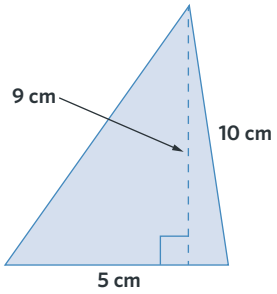


2. Area of a triangle

Q6. Skill

Fill in the missing number in the working out below.

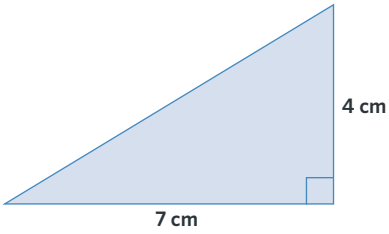
$$\text{Area} = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times 5 \times 9$$
$$= \square \text{ cm}^2$$



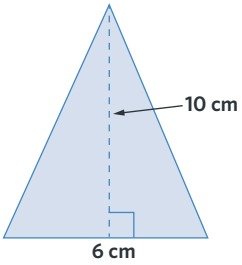
Q7. Skill

Find the areas of the following triangles using the formula $A = \frac{1}{2} \times b \times h$.

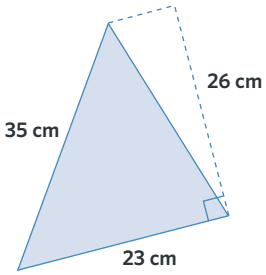
a)



b)



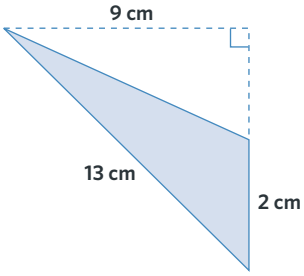
c)



Check your understanding

Q8. Skill

Find the area of the following triangle.

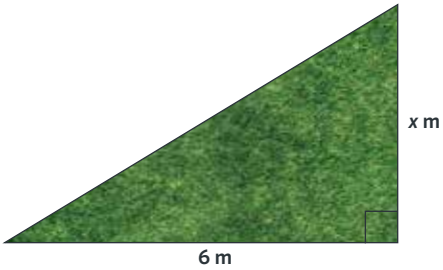


Q9. Application

Gary is a landscaper. He has been contracted to buil a lawn covering an area of nine square metres.

A diagram of the garden is here.

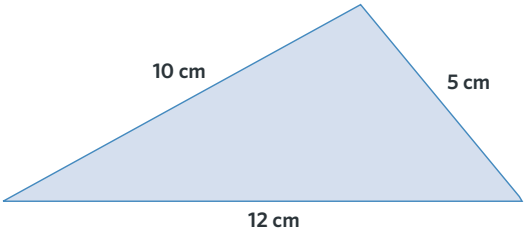
What length must side x be?



3. Area using Heron's formula

Q10. Skill

Look at the following application of Heron's formula and fill in the missing number, assuming it is rounded to two decimal places.



$Perimeter = 10 + 5 + 12$

$= 27 \text{ cm}$

$S = \frac{P}{2}$

$= 13.5$

$A = \sqrt{S(S - a)(S - b)(S - c)}$

$A = \sqrt{13.5(13.5 - 10)(13.5 - 5)(13.5 - 12)}$

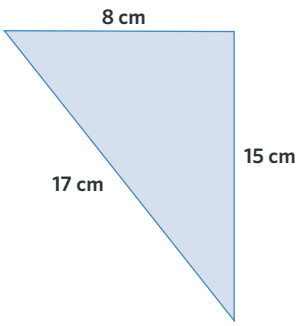
$A = \sqrt{13.5(3.5)(8.5)(1.5)}$

$A = \square \text{ cm}^2$

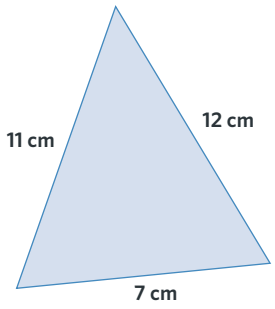
Q11. Skill

Use Heron's formula to find the areas of the following triangles, rounded to two decimal places.

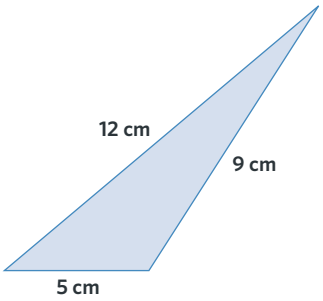
a)



b)



c)



Check your understanding

Q12. Application

A pool triangle is equilateral, and has a side length of approximately 28 cm.

Use Heron's formula to calculate the area inside the triangle to two decimal places.

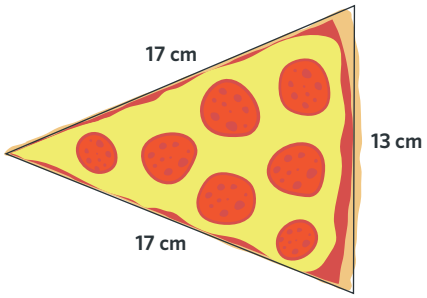
Q13. Application

One slice of a large pizza at Duncan's Pumpin Pizzas can be modelled by the triangle given here.

Round answers in this question to two decimal places.

a) What is the size of one slice, in cm^2?

b) Use your rounded answer from part a to find the area of an entire large pizza, given each pizza has eight slices.

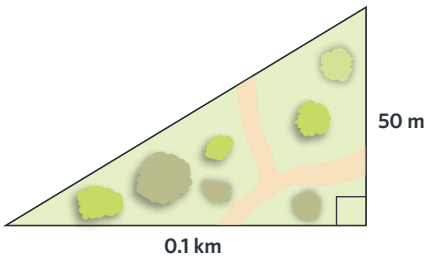


Joining it all together

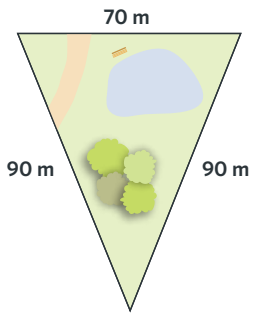
Q14. Application 1 mark

Use the methods studied in this chapter to calculate the area of the largest park, rounded to the nearest square metre.

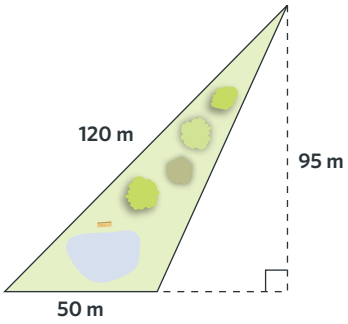
A.



B.

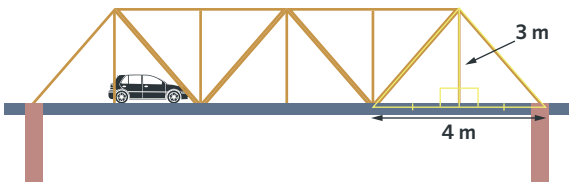


C.



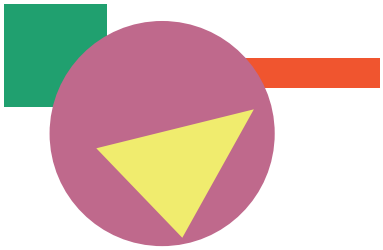
Q15. Application 1 mark

Two triangle segments are shown in yellow on the truss bridge below. Assuming each segment has the same area, what is the area of one triangle segment?



Q16. Application 3 marks

Otis is painting an abstract mural on the wall of his bathroom. One section of the mural is shown below. The yellow triangle has side lengths 60 cm, 90 cm, and 100 cm, and the circle has radius 70 cm.



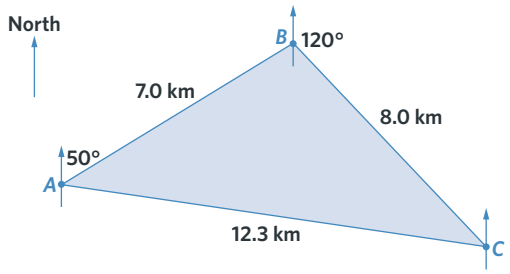
a) What is the area of the triangle, to two decimal places? (1 mark)

b) Find the area of this section that requires purple paint. Round your answer to two decimal places. (2 marks)

VCAA question

Q17. 1 mark

An orienteering course is triangular in shape and is marked by three points, A, B and C, as shown in the diagram below.



In this course, B is 7.0 km from A, C is 8.0 km from B and A is 12.3 km from C.

The area (in km^2) enclosed by this course is closest to:

A. 21

B. 24

C. 25

D. 26

E. 28

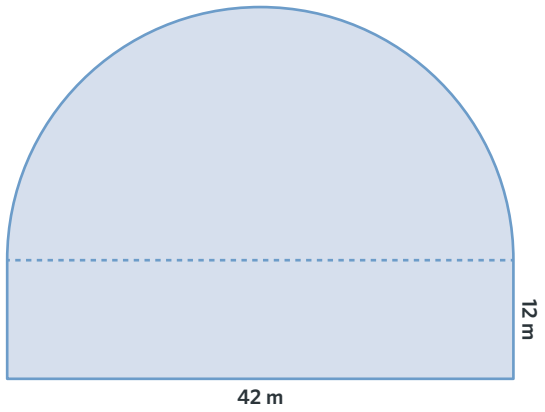
VCAA 2008 Exam 1, Module 2: Geometry and Trigonometry. Q3

Questions from multiple lessons

Q18. Difficulty:

Year 10
1 mark

A dairy farmer is planning on expanding his farm. There is a plot of land for sale and he plans to purchase it. It consists of a rectangular base and a semicircle. A diagram of the plot of land is included below.



He wants to construct a fence around the perimeter. The distance of fence that he needs is closest to

A. 132 m

B. 66 m

C. 198 m

D. 104 m

E. 42 m

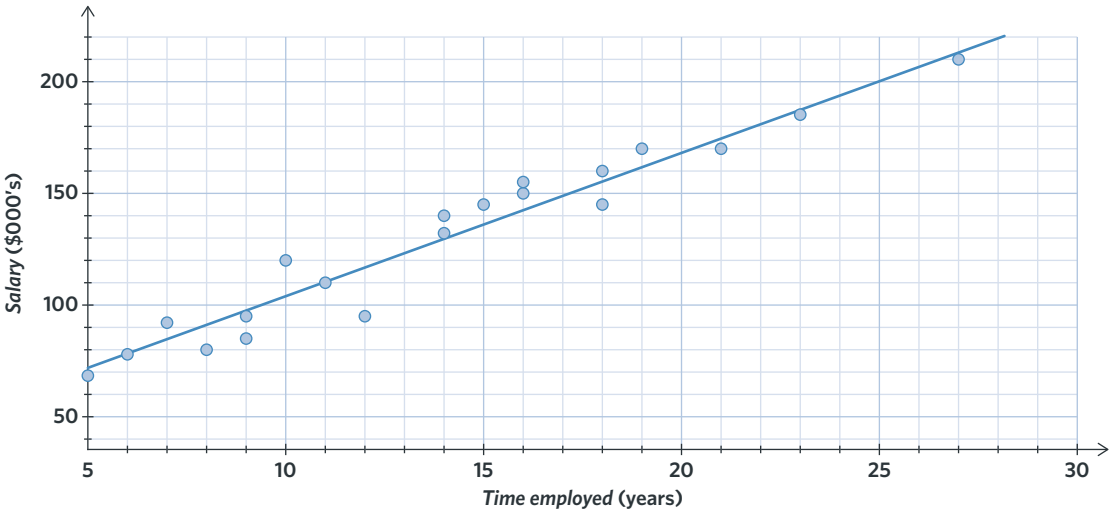
VCAA 2015 Exam 1 Module 2: Geometry and trigonometry Q2 – Adapted

Q19.

Difficulty:

1 mark

The following scatterplot shows the *salary* and the *time employed*, in years, of 20 employees at a company. A least squares line has been fitted to the data.



The equation of this least squares regression line is closest to

A. *time employed* $= 40 + 4.5 \times$ *salary*

B. *time employed* $= 72 + 6.4 \times$ *salary*

C. *salary* $= 40 + 4.5 \times$ *time employed*

D. *salary* $= 72 + 4.5 \times$ *time employed*

E. *salary* $= 40 + 6.4 \times$ *time employed*

VCAA 2018 Exam 1 Data analysis Q8 – Adapted

Q20.

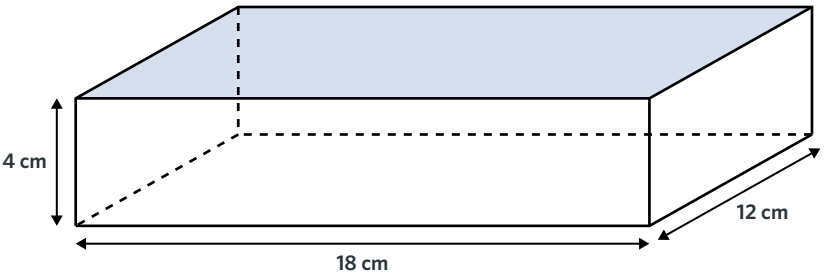
Difficulty:

Year 10
2 marks

A new eye shadow palette is currently in production.

The box is in the shape of a rectangular prism.

The diagram below shows the dimensions of the palette.



The lid of the palette is shaded above.

a) What is the area of the lid when it is closed on the box? (1 mark)

b) What is the total surface area of the palette? (1 mark)

VCAA 2017 Exam 2 Module 3: Geometry and measurement Q1ai-aii – Adapted

LESSON 7F

Composite shapes

The key skills you will learn in this lesson are:

1. Perimeter of composite shapes
2. Area of composite shapes

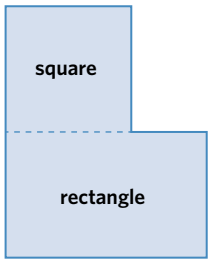
VCAA key knowledge point:

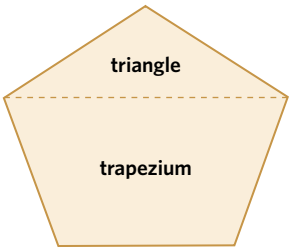
"perimeter and areas of triangles (including the use of Heron's formula), quadrilaterals, circles and composite shapes and practical applications"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Perimeter of composite shapes

A shape made up of two or more simple shapes is known as a composite shape. Below are some examples of composite shapes.

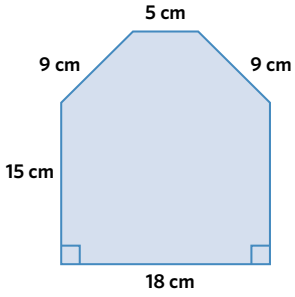




Finding the perimeter of a composite shape is the same as finding the perimeter of a simple shape. The perimeter of a composite shape can be found by finding the lengths of each side of the shape, and adding them together.

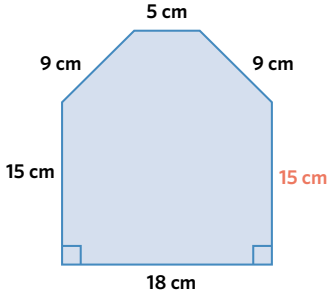
WORKED EXAMPLE 13 *(1 mark)*

Find the perimeter of the shape to the right.



SOLUTION

Step 1 Find missing side lengths. In the shape above the length of one side is missing. The length of the missing side is 15 cm.



Step 2 Find the sum of the lengths of all the sides.

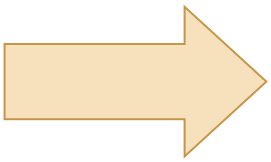
$15 + 9 + 5 + 9 + 15 + 18 = 71$

The perimeter of the shape is 71 cm.

2. Area of composite shapes

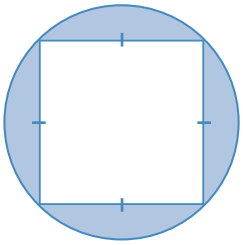
A composite shape can be either made up of two or more shapes added together, or one shape taken out of another shape.

The area of a composite shape made up of two or more shapes, like the one below, can be calculated by finding the sum of the area of the individual shapes.



Area = *Area of rectangle* + *Area of triangle*

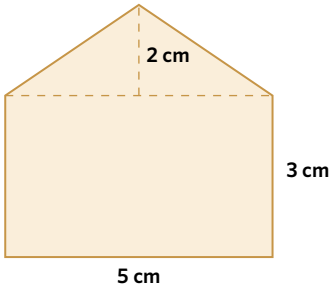
The area of a composite shape made up of one shape taken out of another shape, like the shaded region below, can be calculated by subtracting the area of the inner shape from the area of the outer shape.



Area = *Area of circle* − *Area of square*

WORKED EXAMPLE 14 *(1 mark)*

Find the area of the shape to the right.



SOLUTION

Step 1 Find the area of the triangle.

$Area\ of\ triangle = \frac{1}{2} \times b \times h = \frac{1}{2} \times 5 \times 2 = 5$

Step 2 Find the area of the rectangle.

$Area\ of\ rectangle = b \times h = 5 \times 3 = 15$

Step 3 Add the two areas.

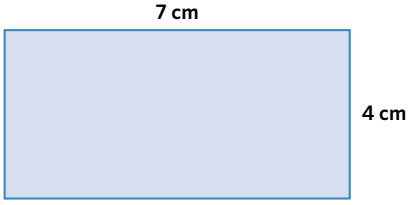
$Total\ area = 5 + 15 = 20$

The area of the shape is 20 cm^2.

Questions 7F *Composite shapes*

Refresher question

Q1.



What is:

a) The area of this rectangle?

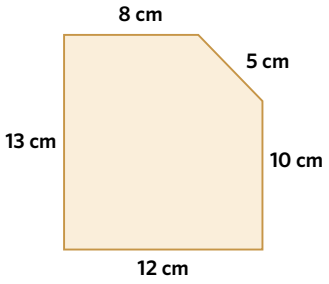
b) The perimeter of this rectangle?

1. Perimeter of composite shapes

Q2. Skill

What is the value of x in the working out below?

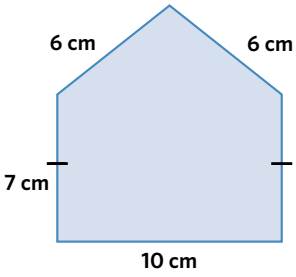
$$\text{Perimeter} = 8 + x + 10 + 12 + 13$$
$$= 48 \text{ cm}$$



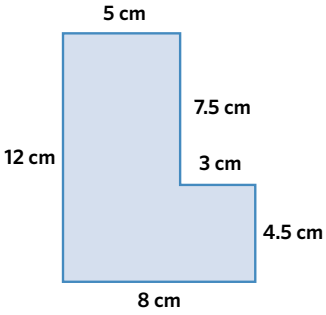
Q3. Skill

Find the perimeter of the composite shapes below correct to two decimal places where appropriate.

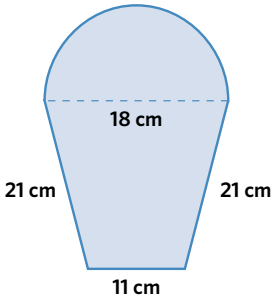
a)



b)



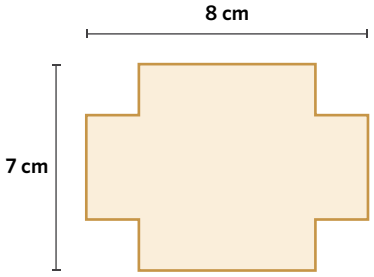
c)



Check your understanding

Q4. Skill

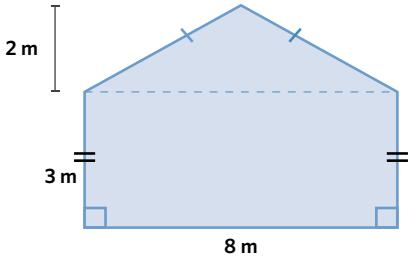
Find the perimeter of the shape to the right.



Q5. Application

The shape of the front of a house is shown to the right.

Find the perimeter of the house correct to two decimal places.

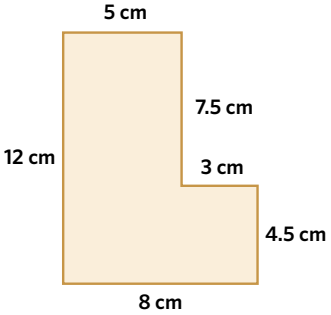


2. Area of composite shapes

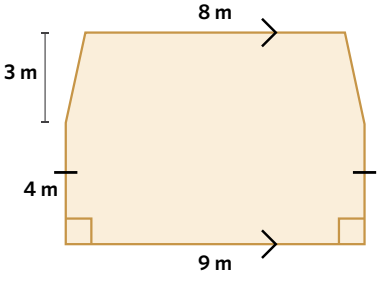
Q6. Skill

Find the area of the composite shapes below correct to one decimal place where appropriate.

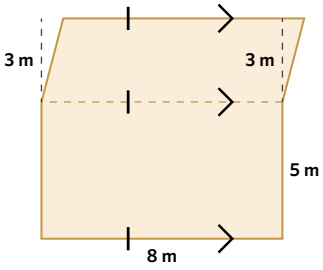
a)



b)



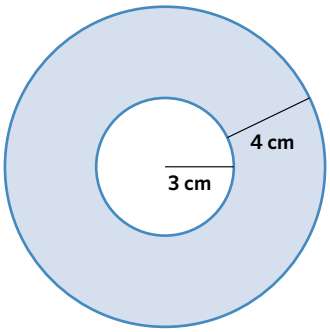
c)



Check your understanding

Q7. What is the area of the shaded region on the right, correct to two decimal places?

Skill



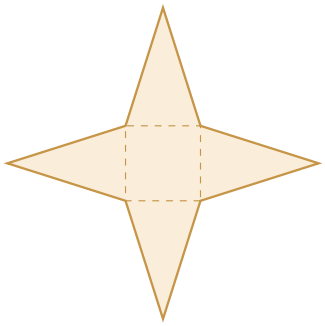
Q8. A rectangular piece of paper with side lengths 21 cm and 25 cm has a semicircle of radius 20 mm cut out of it. What is the remaining area of the piece of paper correct to one decimal place?

Application

Q9. This shape has an area of 32 m^2. The square in the centre has sides that are 2 m long.

What is the height of the triangles, assuming all four triangles are identical?

Skill

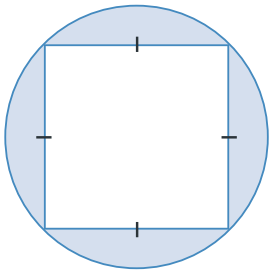


Joining it all together

Q10. A square with side lengths 2 cm fits in a circle with radius 1.414 cm.

Find the area of the shaded region to the right correct to two decimal places.

Skill
1 mark



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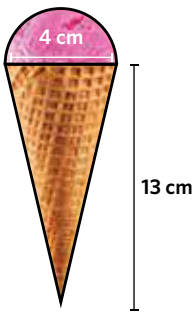
ow, to the nearest whole number, given that
ngle and the ice cream on top is in the shape

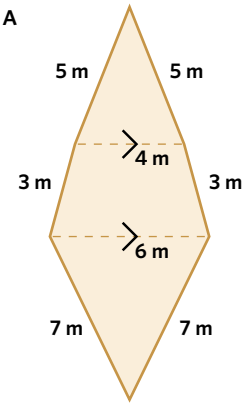
Application
1 mark



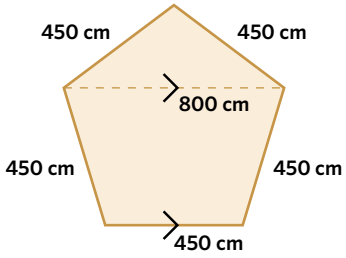
Q12. Which shape has a greater area, and what is its area correct to two decimal places?

Skill
2 marks

Shape A



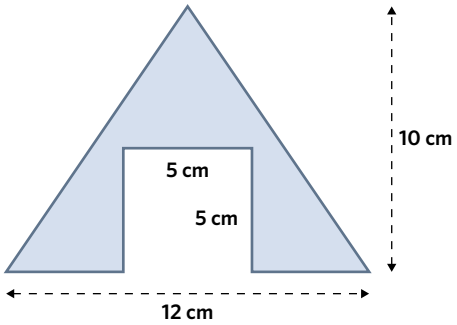
Shape B



VCAA question

Q13. Consider the diagram below.

1 mark



The shaded area, in square centimetres, is

A. 35 **B.** 45 **C.** 60 **D.** 85 **E.** 95

VCAA 2016 Exam 1, Module 3: Geometry and measurement. Q1

Questions from multiple lessons

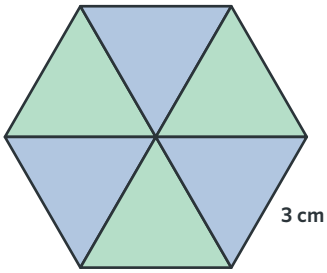
Q14. A helicopter landing pad is hexagonal and has a side length of three metres.

Difficulty:

1 mark

The landing pad can be divided into six equilateral triangles.

Find the green area of the landing pad, in square metres, correct to one decimal place.



A. 3.9 m^2

B. 11.7 m^2

C. 14.7 m^2

D. 15.2 m^2

E. 15.6 m^2

VCAA 2015 Exam 1 Module 2: Geometry and trigonometry Q5 – Adapted

Q15. A sequence of numbers can be generated by the recurrence relation below.

Difficulty:

1 mark

$V_0 = 23, \quad V_{n+1} = 4 \times V_n$

What is the value of V_7?

A. 94 208

B. 1 507 328

C. 5888

D. 376 832

E. 23 552

VCAA 2019NH Exam 1 Recursion and financial modelling Q17 – Adapted

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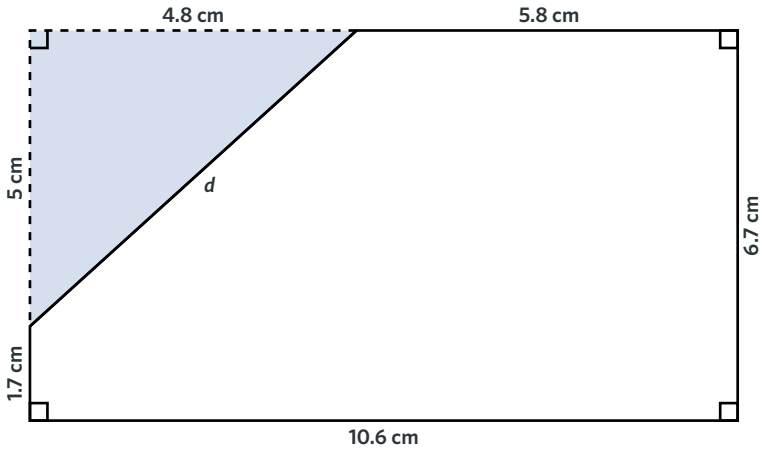
Q16. Kyle intends to build a toy house for his daughter using a piece of cardboard as a base.

Difficulty:

3 marks

The unshaded area in the diagram below represents the cardboard piece.



a) Show that the length d is 6.9 cm, rounded to one decimal place. (1 mark)

b) Using $d = 6.9$, calculate the perimeter, in centimetres, of this cardboard piece. (1 mark)

c) Calculate the area of this piece of cardboard. Round your answer to the nearest whole number. (1 mark)

VCAA 2017NH Exam 2 Module 3: Geometry and measurement Q1 a-c – Adapted

LESSON 7G

Volumes

The key skills you will learn in this lesson are:

1. Volume of prisms and cylinders
2. Volume of pyramids and cones
3. Volume of spheres
4. Volume of composite objects

VCAA key knowledge point:

"volumes and surface areas of solids (spheres, cylinders, pyramids and prisms and their composites) and practical applications"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

Volume is the amount of three-dimensional space taken up by an object. Volume is usually measured in mm^3, cm^3, and m^3, but can also be measured in mL and L. Recall that:

1 mL is 1 cm^3.

1 litre is 1000 cm^3.

1000 litres is 1 m^3.

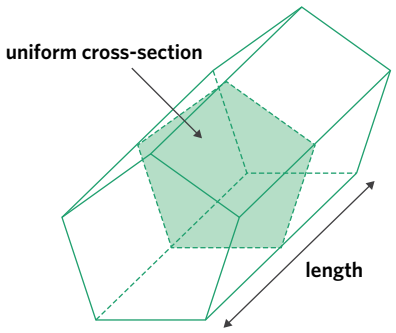
1. Volume of prisms and cylinders

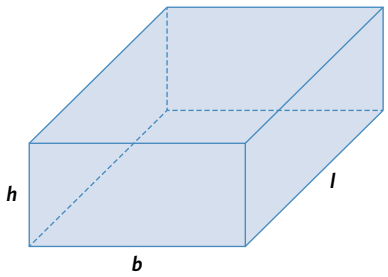
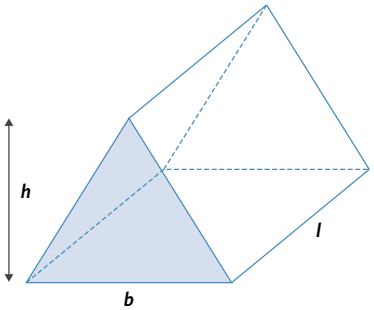
A prism is a three-dimensional object made up of planar faces that has a uniform cross-section across its entire length. Its volume is calculated by taking the area of the uniform cross-section and multiplying it by the object's length.

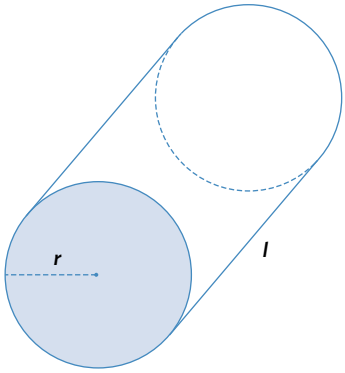
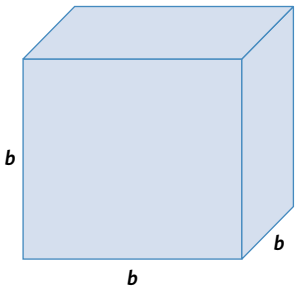
volume = area of cross section × length

Although a cylinder is not technically considered a prism as it has a rounded face, its volume is calculated the same way.

Some common formulas are given below:

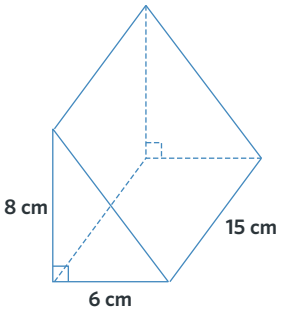


Name	Example	Area of cross-section	Volume
Rectangular prism		$a = bh$	$v = bhl$
Triangular prism		$a = \frac{1}{2}bh$	$v = \frac{1}{2}bhl$

Name	Example	Area of cross-section	Volume
Cylinder		$a = \pi r^2$	$v = \pi r^2 l$
Cube		$a = b^2$	$v = b^3$

WORKED EXAMPLE 15 *(1 mark)*

Calculate the volume of the triangular prism to the right.



SOLUTION

Step 1 Calculate the area of the uniform cross-section.

$a = \frac{1}{2}bh$

$a = \frac{1}{2} \times 6 \times 8$

$a = 24\text{cm}^2$

Step 2 Multiply the area of the cross-section with the length of the prism.

$v = \textit{area of uniform cross section} \times \textit{length}$

$v = 24 \times 15$

$v = 360$

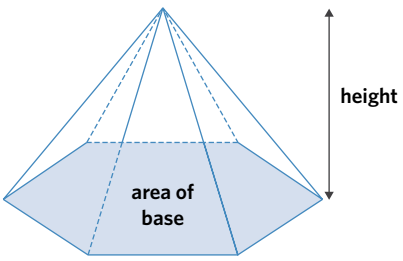
$\therefore$ The volume is 360 cm^3.

2. Volume of pyramids and cones

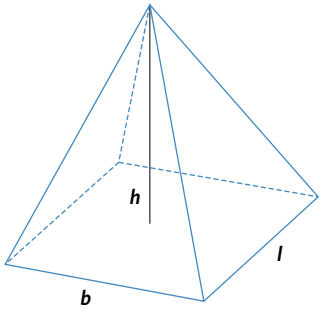
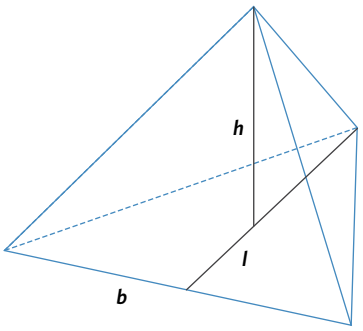
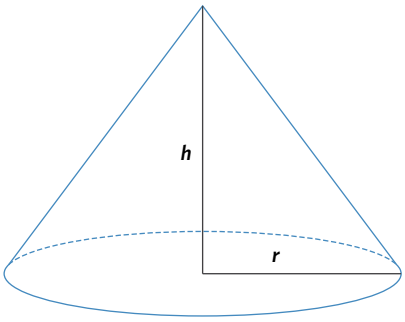
A pyramid is an object with a flat base and a number of triangular sides that meet at a point at the top, known as the apex.

The volume of a pyramid is given by $v = \frac{1}{3} \times$ area of base $\times$ height.

A cone is similar to a pyramid as it has a flat base and tapers to a point at the top, although it does not have triangular sides. However, its volume can still be calculated using the same formula.



Some common formulas are given below:

Name	Example	Area of base	Volume
Rectangular pyramid		$a = bl$	$v = \frac{1}{3}blh$
Triangular pyramid		$a = \frac{1}{2}bl$	$v = \frac{1}{6}blh$
Cone		$a = \pi r^2$	$v = \frac{1}{3}\pi r^2 h$

WORKED EXAMPLE 16 *(1 mark)*

Calculate the volume of a cone with a radius of 1.2 metres and a height of 4.1 metres correct to two decimal places.

SOLUTION

Step 1 Write down the known values.

$r = 1.2$

$h = 4.1$

Step 2 Substitute these values into the formula and solve.

$v = \frac{1}{3}\pi r^2 h$

$v = \frac{1}{3} \times \pi \times 1.2^2 \times 4.1$

$v = 6.1826 \ldots$

$\therefore$ The volume is 6.18 m^3.

3. Volume of spheres

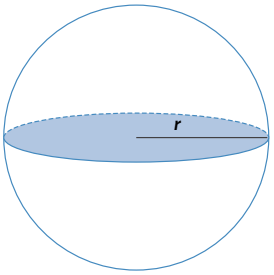
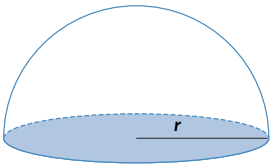
A sphere is a perfectly round object where every point on the surface is equidistant from the centre.

The volume of a sphere is given by the formula:

$v = \frac{4}{3}\pi r^3$

The volume of a hemisphere is half the volume of a sphere:

$$v = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{4}{6}\pi r^3 = \frac{2}{3}\pi r^3$$

Name	Example	Volume
Sphere		$v = \frac{4}{3}\pi r^3$
Hemisphere		$v = \frac{2}{3}\pi r^3$

WORKED EXAMPLE 17 *(1 mark)*

A bouncy ball has a diameter of four centimetres. What is the volume of the ball correct to two decimal places?

SOLUTION

Step 1 Calculate the radius.

The radius is half of the diameter.

$r = \frac{d}{2}$

$r = \frac{4}{2} = 2$

The radius is 2 cm.

Step 2 Substitute the radius into the formula and solve.

$v = \frac{4}{3}\pi r^3$

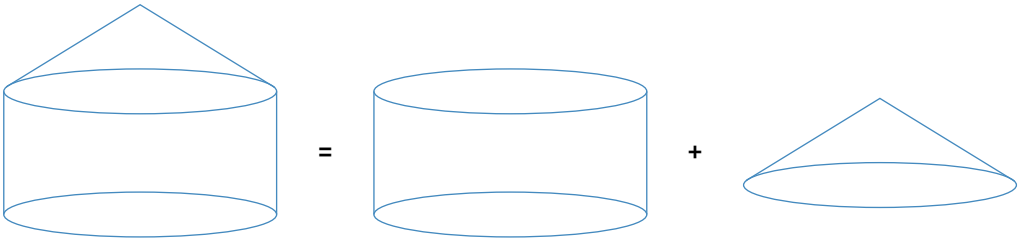
$v = \frac{4}{3} \times \pi \times 2^3$

$v = 33.5103 \ldots$

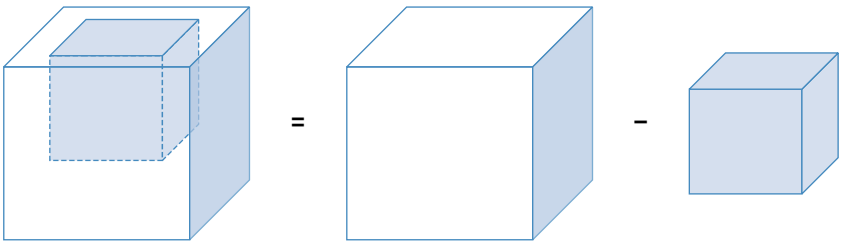
$\therefore$ The volume is 33.51 cm^3.

4. Volume of composite objects

If an object is comprised of two or more shapes, simply find the volume of each shape and add them together for the total volume.



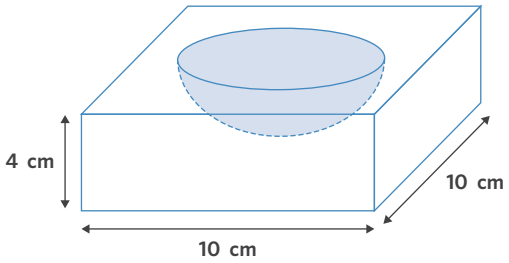
If an object is comprised of a shape with a smaller shape taken out, find the volume of each shape and subtract the smaller volume from the larger volume.



WORKED EXAMPLE 18 *(2 marks)*

A block of wood has had a hemisphere with a radius of three centimetres carved out of it, as shown below.

What is the volume of the remaining block of wood correct to two decimal places?



SOLUTION

Step 1 Find the volume of the rectangular prism.

$v = bhl$

$v = 10 \times 10 \times 4$

$v = 400$

Step 2 Find the volume of the hemisphere

$v = \frac{2}{3}\pi r^3$

$v = \frac{2}{3}\pi \times 3^3$

$v = 56.5486 \ldots$

✓ *1 mark for finding the volume of the hemisphere*

Step 3 Subtract the volume of the hemisphere from the volume of the rectangular prism.

$400 - 56.5486 \ldots = 343.4513 \ldots$

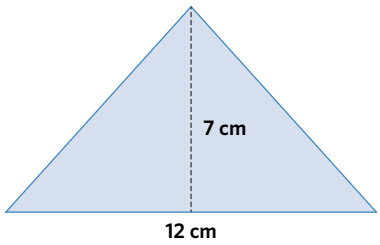
∴ The block of wood has a volume of 343.45 cm^3.

✓ *1 mark for finding the remaining volume of the block of wood*

Questions 7G *Volumes*

Refresher questions

Q1. Find the area of the triangle on the right.



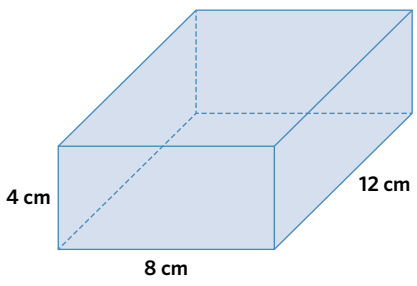
Q2. What is 2000 litres expressed in m^3?

1. Volume of prisms and cylinders

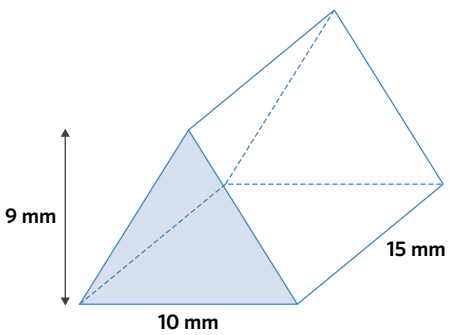
Q3. Calculate the volume of each of the following prisms correct to the nearest whole number where necessary.

Skill

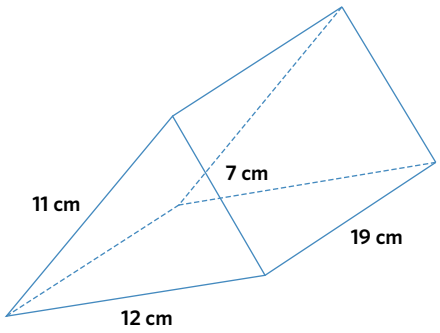
a)



b)



c)



Q4. Skill

a) A cube has a side length of nine millimetres. What is its volume?

b) A cylinder has a height of 12 centimetres and a radius of 13 centimetres. What is its volume correct to two decimal places?

Q5. Application

A cylindrical rainwater tank is two metres tall and 1.5 metres wide. What volume of water can it hold correct to the nearest litre?

Q6. Application

A rectangular swimming pool is 25 metres long, 8 metres wide, and has a constant depth throughout. The pool has a volume of 300 000 litres. What is the depth of the pool in metres?

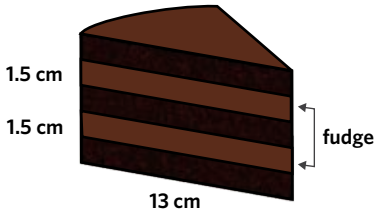
Check your understanding

Q7. Application

For Eloise's birthday, she buys a cylindrical chocolate fudge cake. It has a radius of 13 centimetres and a height of 11 centimetres.

a) If the cake is split evenly between eight people, what volume of cake does each person receive correct to two decimal places?

b) Eloise notices that the cake has two layers of fudge inside it, each 1.5 centimetres thick, as shown to the right. How much fudge does each person receive in their serving correct to the nearest mL?

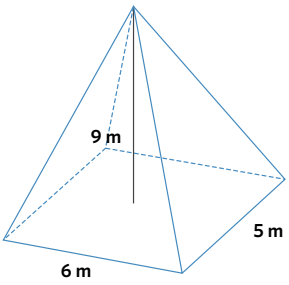


2. Volume of pyramids and cones

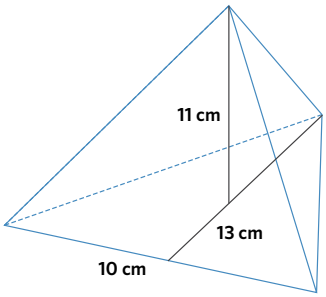
Q8. Skill

Calculate the volume of each of the following pyramids correct to two decimal places where necessary.

a)



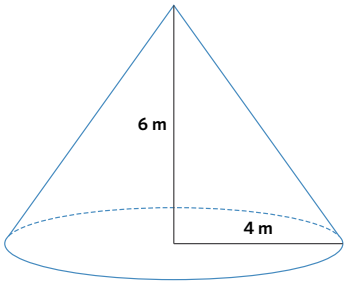
b)



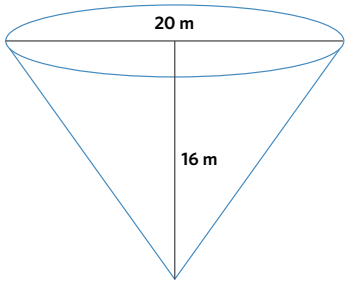
Q9. Calculate the volume of each of the cones below correct to two decimal places.

Skill

a)

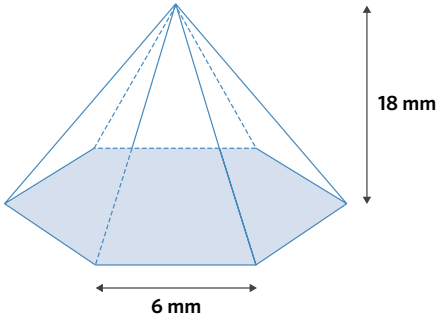


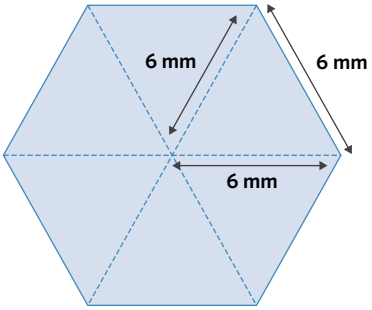
b)



Q10. A hexagonal pyramid has a height of 18 millimetres and a side length of 6 millimetres. The hexagonal base can be split up into six identical equilateral triangles, as shown below. What is the volume of the pyramid correct to one decimal place?

Skill





Check your understanding

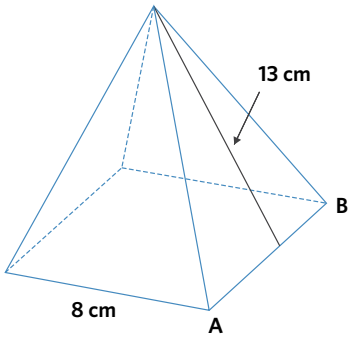
Q11. The Pyramid of Khafre in Egypt is a square-based pyramid with a volume of 2 115 072 m^3. The square base of the pyramid has side lengths of 216 metres. What is the height of the pyramid?

Application

Q12. A square-based pyramid has a base length of 8 cm and a slant height of 13 cm.

Skill

The slant height is the distance from the apex to the midpoint between corners *A* and *B*. Using this information, find the volume of the pyramid correct to one decimal place.

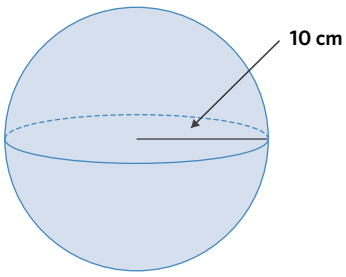


3. Volume of spheres

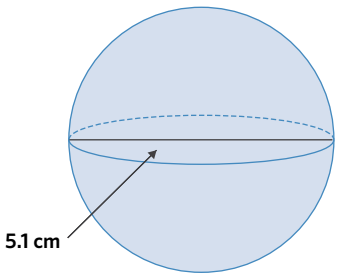
Q13. Find the volume of the spheres below correct to two decimal places.

Skill

a)



b)



Q14. Skill

a) A sphere has a circumference of 15 cm. What is its volume correct to two decimal places?

b) A sphere has a volume of 177 cm^3. What is its diameter, correct to two decimal places?

Q15. Application

Mary made scale models of the Earth and Venus for her science class.

Her model of Earth had a radius of 4 cm, and her model of Venus had a volume of 230 cm^3.

Assuming her models are correct, which planet is larger: Earth or Venus?

Q16. Application

A grapefruit is cut in half. If its radius is five centimetres, what is the volume of one half correct to two decimal places?

Check your understanding

Q17. Application

The radius measured from the inside of a particular basketball is 11.5 centimetres.

a) What volume of air does the basketball contain when pumped up? Give your answer in millilitres correct to two decimal places.

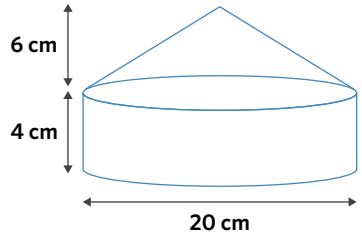
b) The basketball is made of rubber 5 millimetres thick so the ball at its widest point is 24 centimetres wide. What volume of rubber was used to make the ball? Give your answer in cm^3 correct to one decimal place.

4. Volume of composite objects

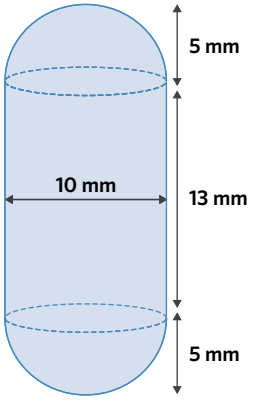
Q18. Skill

Find the volume of each of the following objects correct to two decimal places.

a)

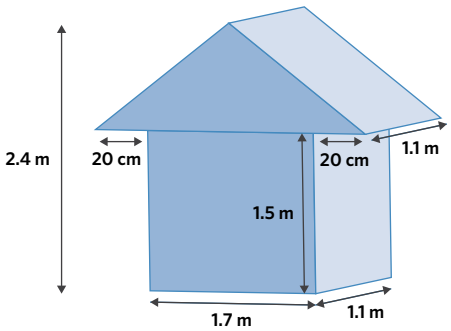


b)



Q19. Application

A treehouse is comprised of a rectangular prism and a triangular prism, as shown below.

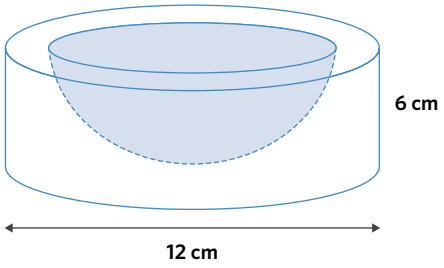


What is the volume of the treehouse in m^3 correct to three decimal places?

Check your understanding

Q20. Application

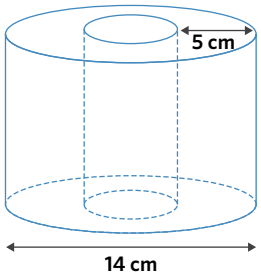
A glass bowl is in the shape of a cylinder with a hemispherical hole in the top. The cylinder is six centimetres in height and 12 centimetres at its widest point. The radius of the hemisphere is one centimetre less than the radius of the cylinder. What is the volume of glass that makes up the bowl correct to two decimal places?



Q21. Application

The volume of the roll of toilet paper below is 1700 cm^3.

What is its height correct to two decimal places?



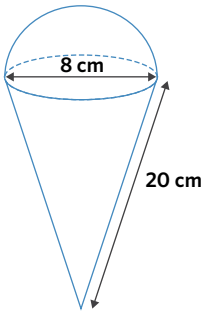
Joining it all together

Q22. Application 2 marks

A choc top ice cream is comprised of a cone and a hemisphere, as shown below. The cone at its widest point is 8 centimetres, and the length of the side of the cone is 20 centimetres. Give answers correct to two decimal places.

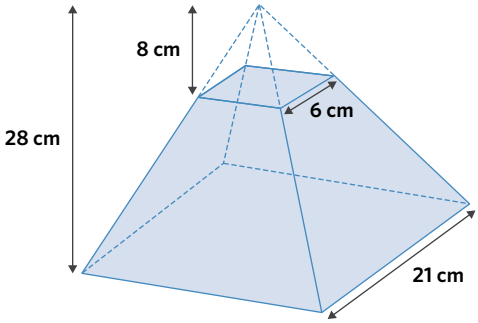
a) What is the total height of the choc top?

b) What is the total volume of the choc top?



Q23. Skill 1 mark

What is the volume of the shaded area in the square pyramid below?



Q24.
Application
2 marks

A rectangular box containing 12 chocolate truffles is four centimetres internally in height. Each truffle in the box is spherical and has a radius of 1.5 centimetres. The box fits exactly four truffles along its length and exactly three truffles along its width.

a) What is the total volume of chocolate in the box correct to the nearest mL?

b) What is the volume of empty space in the box correct to the nearest cm^3?

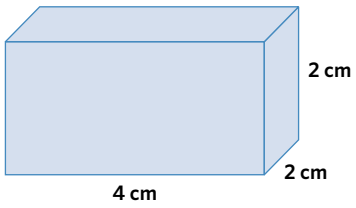
Q25.
Application
3 marks

250 mL of water is poured into a cylindrical drinking glass, which fills 65% of its capacity.

The glass has an inner diameter of 6 centimetres.

a) What is the inner height of the glass correct to one decimal place? (1 mark)

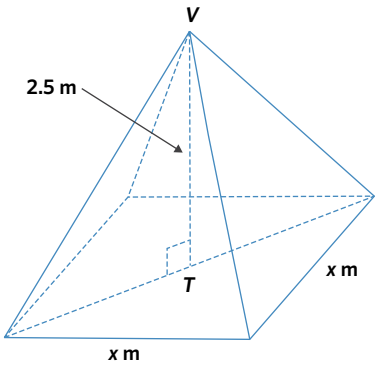
b) 3 ice cubes are added to the glass. The ice cubes are in the shape of rectangular prisms, with measurements as shown below.



To what percent of its capacity is the glass now full, correct to one decimal place? (2 marks)

VCAA question

Q26.
2 marks



A concrete square pyramid with volume 1.8 m^3 sits on the flat top of the hill.

The length of the square base of the pyramid is x metres. The height of the pyramid, VT, is 2.5 metres.

Find the value of x, correct to two decimal places.

VCAA 2010 Exam 2, Module 2: Geometry and Trigonometry. Q3

Questions from multiple lessons

Q27.

Difficulty:

1 mark

The following diagram shows a rectangular-based right pyramid, *HIJKL*.

In this pyramid, $JK = IK = HK = LK = 36$ km, $IJ = HL = 15$ km, and $HI = JL = 25$ km.

The height, *KP*, of the pyramid is closest to

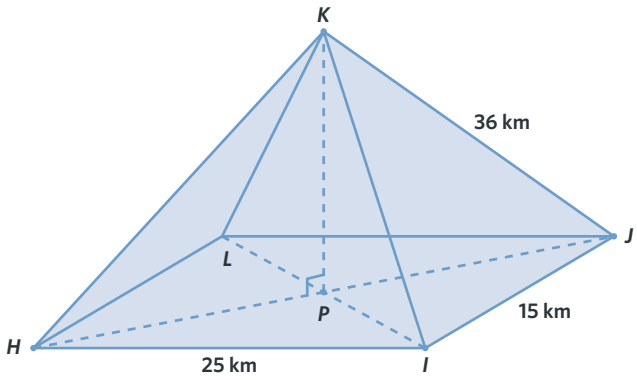
A. 25.0 km

B. 32.7 km

C. 36.0 km

D. 21.1 km

E. 32.9 km



VCAA 2015 Exam 1 Module 2: Geometry and trigonometry Q7 – Adapted

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Q28.

Difficulty:

1 mark

The ATARs of a sample of eight students were recorded and are shown in the following table.

ATAR	78.05	93.65	88.20	61.40	82.85	49.95	98.10	79.00

The mean, $\overline{x}$, and standard deviation, s_x, of the ATARs for this sample are closest to

A. $\overline{x} = 78.9, \quad s_x = 16.2$

B. $\overline{x} = 78.9, \quad s_x = 15.1$

C. $\overline{x} = 78.9, \quad s_x = 15.7$

D. $\overline{x} = 15.1, \quad s_x = 78.9$

E. $\overline{x} = 15.7, \quad s_x = 78.9$

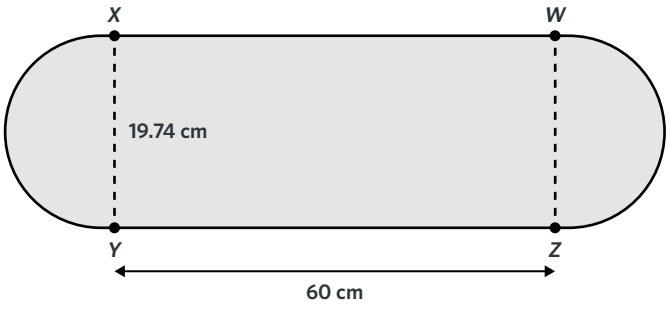
VCAA 2017 Exam 1 Data Analysis Q3 – Adapted

Q29.

Difficulty:

2 marks

Jack bought a new skateboard and drew the following diagram.



The skateboard is comprised of a rectangle, *WXYZ* and two semi-circles, *WZ* and *XY*. The width of the rectangle is 60 cm and the perimeter of the skateboard is 182 cm.

a) Show that the diameter of the semi-circle *XY* is 19.74 cm, correct to two decimal places. (1 mark)

b) Calculate the area of the skateboard, correct to the nearest square centimetre. (1 mark)

VCAA 2013 Exam 2 Module 2: Geometry and trigonometry Q3 a,b – Adapted

LESSON 7H

Surface area

The key skills you will learn in this lesson are:

1. Surface area of solids with planar faces
2. Surface area of solids with curved faces
3. Surface area of composite shapes

VCAA key knowledge point:

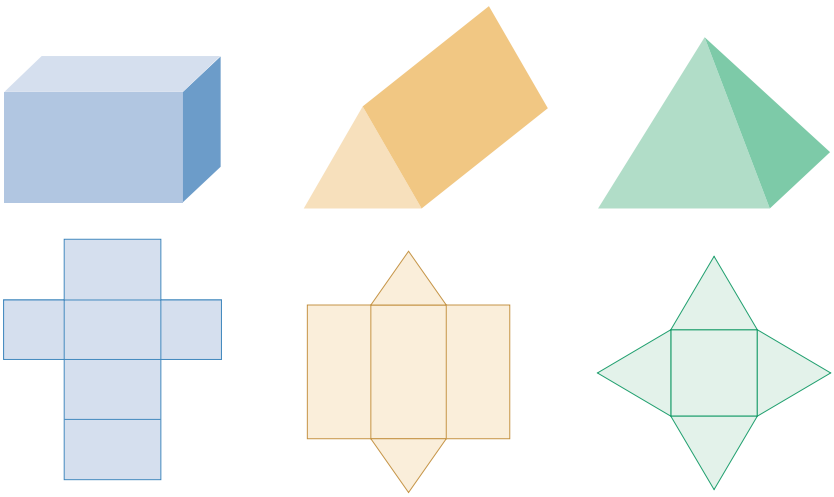
"volumes and surface areas of solids (spheres, cylinders, pyramids and prisms and their composites) and practical applications"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Surface area of solids with planar faces

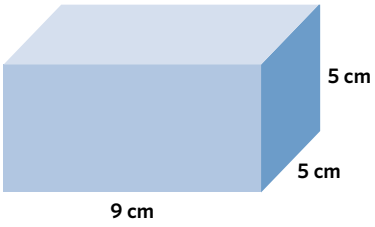
The surface area of a three dimensional solid is the total area of all the surfaces of the object. The surface area of a planar shape, a shape which only has flat faces, can be found by finding the area of each face of the shape, and adding these areas together.

Another way of thinking about this is that, the surface area of a three dimensional solid can be found by finding the area of the net of the solid. The net of the solid is the two dimensional shape that is formed when a three dimensional object is unfolded. Below are examples of the nets of some three dimensional shapes.



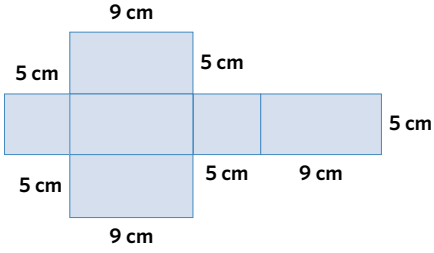
WORKED EXAMPLE 19 *(1 mark)*

What is the surface area of the rectangular prism below?



SOLUTION

Step 1 Identify all the faces by inspection or by drawing the net.



There are **two 5 cm × 5 cm** and **four 5 cm × 9 cm** rectangular faces.

Step 2 Find the sum of the area of the faces.

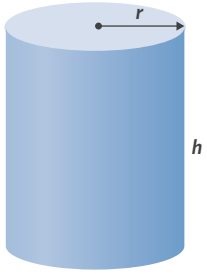
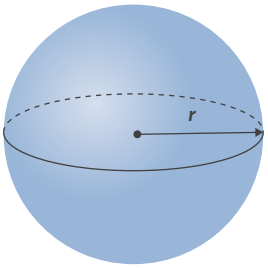
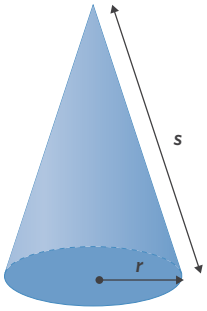
$SA = 2(5 \times 5) + 4(5 \times 9) = 230$

The surface area of the shape is 230 cm^2.

2. Surface area of solids with curved faces

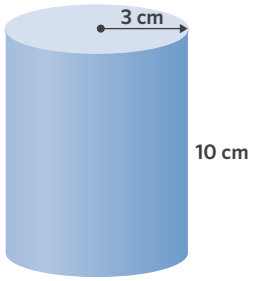
Some three dimensional solids have curved faces such as cylinders, spheres and cones.

The surface area of these shapes can be calculated by adding the area of any planar faces with the area of the curved face. The formulas for the surface area of these shapes are given in the table below.

Name	Shape	Formula for surface area
Cylinder		$SA = \text{area of two circles} + \text{area of curved side}$ $SA = 2\pi r^2 + 2\pi rh$
Sphere		$SA = \text{area of curved side}$ $SA = 4\pi r^2$
Cone		$SA = \text{area of circle} + \text{area of curved side}$ $SA = \pi r^2 + \pi rs$

WORKED EXAMPLE 20 *(1 mark)*

Find the surface area of the shape to the right correct to two decimal places.



SOLUTION

Step 1 Identify the relevant formula.

The formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$.

Step 2 Find the values of r and h.

$r = 3$

$h = 10$

Step 3 Substitute the values of r and h into the equation.

$SA = 2 \times \pi \times 3^2 + 2 \times \pi \times 3 \times 10 = 245.04$

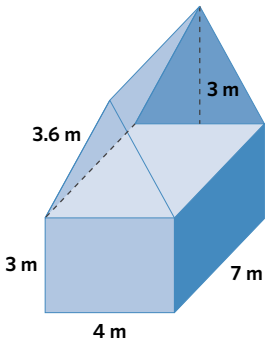
The surface area of the shape is 245.04 cm^2.

3. Surface area of composite shapes

The surface area of a composite shape can be found by adding the area of the faces on the outer surface of the shape. It is important not to include the area of any internal surfaces in the total surface area.

WORKED EXAMPLE 21 *(1 mark)*

What is the surface area of the shape to the right?



SOLUTION

Step 1 Identify all the faces on the outer part of the shape.

There are two 7 m × 3 m, two 3 m × 4 m, one 7 m × 4 m and two 7 m × 3.6 m rectangular faces. There are two triangular faces with base 4 m and height 3 m.

Step 1 Add the area of the faces.

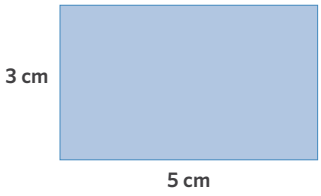
$SA = 2(7 \times 3) + 2(3 \times 4) + (7 \times 4) + 2(7 \times 3.6) + 2\left(\frac{1}{2} \times 4 \times 3\right) = 156.4$

The surface area of the shape is 156.4 m^2.

Questions 7H *Surface area*

Refresher questions

Q1. What is the area of the rectangle to the right?



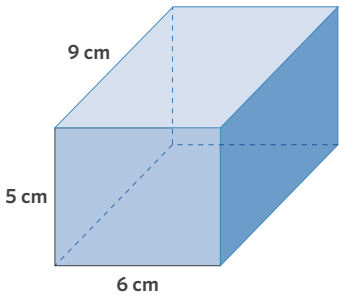
Q2. What is the area of a circle with a radius of 4 mm, correct to one decimal place?

1. Surface area of solids with planar faces

Q3. Skill

What is the value of x in the working out below?

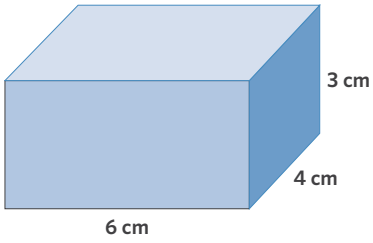
$SA = 2(5 \times 6) + 2(5 \times x) + 2(6 \times 9)$



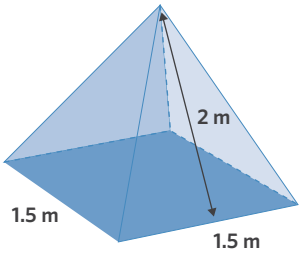
Q4. Find the surface area of the shapes below. Round your answer to two decimal places where appropriate.

Skill

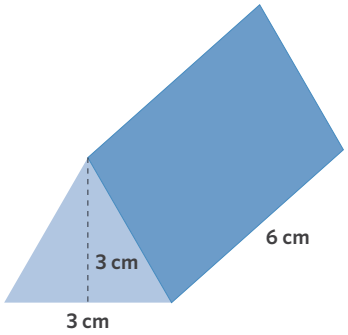
a)



b)



c)



Check your understanding

Q5. The surface area of a triangular pyramid is 87.6 cm^2. If the base of the pyramid is an equilateral triangle with side lengths of 6 cm, what is the height of the other three triangles that make up the pyramid, correct to the nearest centimetre?

Skill

Q6. The Great Pyramid of Giza is approximately in the shape of a square based pyramid. It is 139 m tall and has a length and width of 230 m. What is the surface area of the Great Pyramid of Giza, excluding the base, correct to one decimal place?

Application

2. Surface area of solids with curved faces

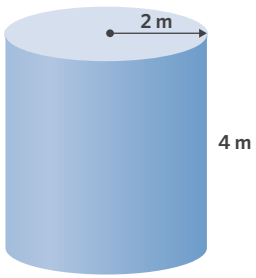
Q7. What is the surface area of a sphere with radius 2 cm, correct to the nearest whole number?

Skill

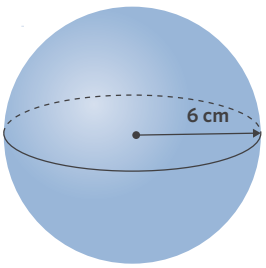
Q8. Find the surface area of the shapes below, correct to two decimal places.

Skill

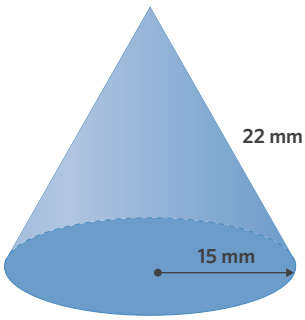
a)



b)



c)



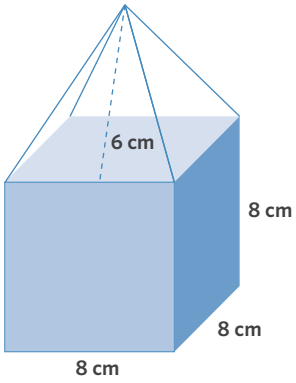
Check your understanding

Q9. Application
Six basketballs need to be made for a basketball tournament. If the diameter of a basketball is 24 cm, what is the total area of material needed to cover the basketballs, correct to the nearest square centimetre?

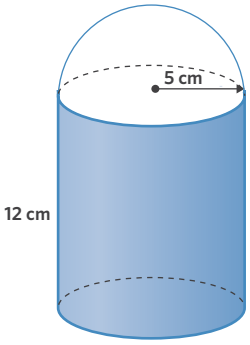
3. Surface area of composite shapes

Q10. Skill
Find the surface area of the shapes below, correct to two decimal places where appropriate.

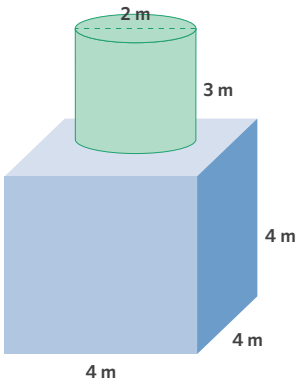
a)



b)



c)

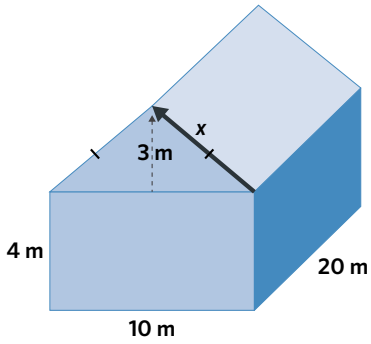


Check your understanding

Q11. Application
A new house is made up of a rectangular prism and triangular prism as shown below. The roof has a vertical height of 3 m.

a) What is the length of side x to two decimal places?

b) If the entire exterior of the house needs to be painted, including the roof but not including the base, what is the total surface area that needs to be painted, correct to the nearest m^2?



Q12. Application
An expensive perfume bottle is in the shape of a cube with a cylindrical pump of height 2 cm and radius of 0.6 cm.

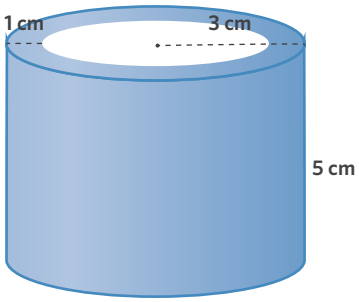
If the perfume bottle holds 125 ml of liquid, what is the total surface area of the perfume bottle including the pump, correct to the nearest cm^2?

NOTE: 1 ml $=$ 1 cm^3

Joining it all together

Q13. Skill 1 mark

Which of the following answers is the surface area of the hollow tube on the right closest to?



A. 125 cm^2

B. 188 cm^2

C. 126 cm^2

D. 189 cm^2

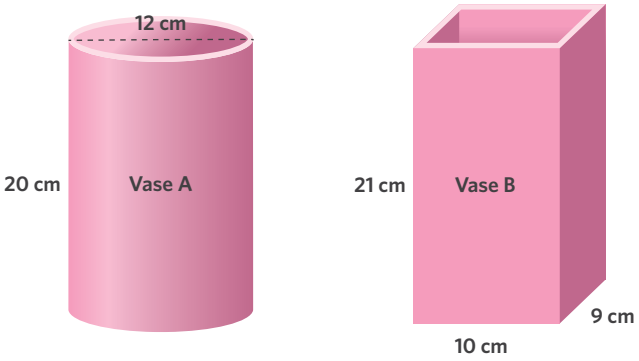
Q14. Skill 1 mark

The surface area of a sphere is 13000 cm^2.

What is its radius in metres, correct to two decimal places?

Q15. Application 2 marks

Find the external surface area of the two vases below, correct to two decimal places where appropriate, and determine which vase has a larger external surface area.



Q16. Application 2 marks

The walls and ceiling of a rectangular room, with width 650 cm and length 800 cm, need to be painted. 1 m^2 requires 0.3 L of paint and 46.05 L of paint is needed in total.

a) What is the height of the room in centimetres?

b) If there is a window of width 1.74 m and height 1 m in the room, what is the height of the room, correct to the nearest centimetre?

VCAA question

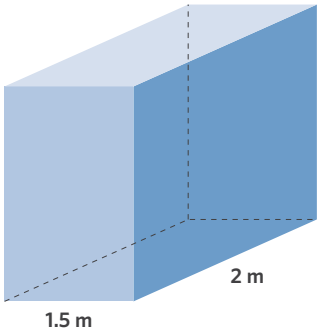
Q17. 1 mark

The rectangular box shown in this diagram is closed at the top and the bottom.

It has a volume of 6 m^3.

The base dimensions are 1.5 m × 2 m.

The total surface area of this box is



A. 10 m^2

B. 13 m^2

C. 13.5 m^2

D. 20 m^2

E. 27 m^2

VCAA 2006 Exam 1, Module 2: Geometry and Trigonometry. Q6

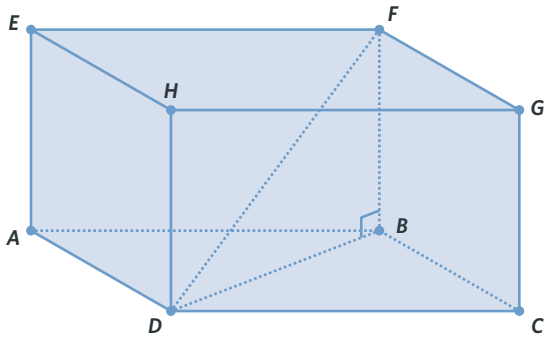
Questions from multiple lessons

Q18.

Difficulty:

1 mark

The following diagram is a rectangular prism with a square base, *ABCD*.



The diagonal of the prism, *DF*, is 10 cm.

The height of the prism, *BF*, is 6 cm.

What is the volume of the prism?

A. 36 cm^3

B. 100 cm^3

C. 384 cm^3

D. 192 cm^3

E. 523 cm^3

VCAA 2013 Exam 1 Module 2: Geometry and trigonometry Q9 – Adapted

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Q19.

Difficulty:

1 mark

Zubyn and Cyrus are basketball teammates. The following table shows the average number of points scored by Zubyn and the number of assists created by Cyrus in 10 basketball games.

Points scored by Zubyn	28	19	15	22	8	9	15	33	18	10
Assists created by Cyrus	13	8	6	16	3	1	6	12	5	2

Find the value of the Pearson correlation coefficient, *r*, between *points scored by Zubyn* and *assists created by Cyrus*, correct to two decimal places.

A. 0.85

B. −0.46

C. 0.92

D. −0.85

E. 0.46

VCAA 2018NH Exam 1 Data analysis Q9 – Adapted

Q20.

Difficulty:

Year 10

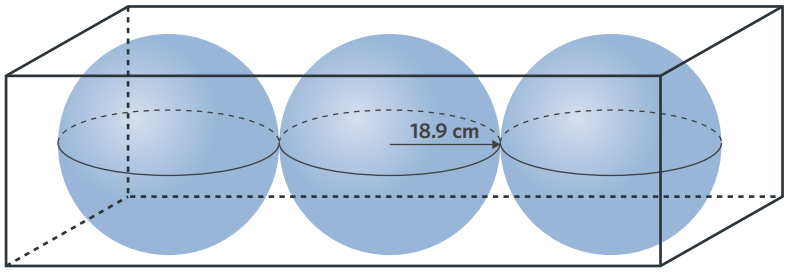
2 marks

A beach ball is spherical in shape and has a radius of 18.9 cm.

Assume that the surface of the beach ball is smooth.

a) What is the surface area of the beach ball? Round your answer to the nearest whole number. (1 mark)

Beach balls are now sold in a rectangular box that contains three identical beach balls, as shown.



b) What is the minimum length, in centimetres, of the box? (1 mark)

VCAA 2016 Exam 2 Module 3: Geometry and measurement Q1 a,b – Adapted

LESSON 7I

Similar figures and scale factor

The key skills you will learn in this lesson are:

1. Similar figures
2. Similar triangles
3. Linear scale factor

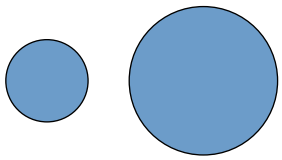
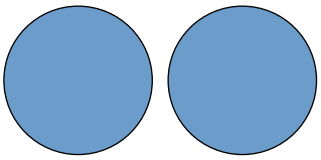
VCAA key knowledge point:

"similar figures including the mathematical conditions for similarity of two-dimensional shapes, and the linear scale factor and its extension to areas and volumes"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

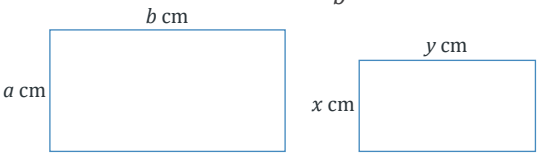
1. Similar figures

Two objects are considered similar figures if they are exactly the same shape, but different sizes. If two objects are the same shape and size they are congruent. All circles are either similar or congruent as they are exactly the same shape.

Similar	Congruent
Same shape Different sizes 	Same shape Same size 

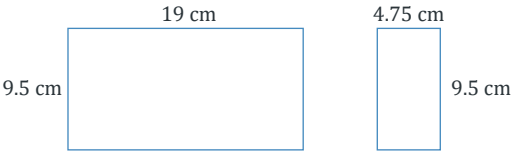
Two dimensional shapes are similar when all their corresponding angles are equal, and their corresponding side lengths are proportional. Proportional means that the length of all corresponding sides are in the same ratio.

In the diagram below if $\frac{x}{a} = \frac{y}{b}$ then the two rectangles are similar.



WORKED EXAMPLE 22 *(1 mark)*

Are the following rectangles similar?



SOLUTION

Step 1 Check if all internal angles are equal.

Yes, all internal angles in a rectangle are right angles.

Step 2 Calculate the ratio for the shorter side of the rectangle.

$\frac{4.75}{9.5} = 0.5$

Step 3 Calculate the ratio for the longer side of the rectangle.

$\frac{9.5}{19} = 0.5$

Step 4 Compare the two ratios.

The two ratios are the same meaning that the two rectangles are similar.

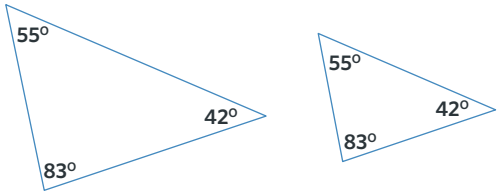
2. Similar triangles

Although all corresponding angles are equal and all corresponding side lengths are proportional in a pair of similar triangles, not all angles and side lengths need to be known to determine if two triangles are similar.

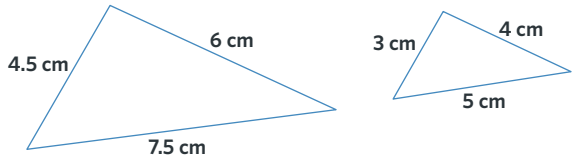
Two triangles will be similar if one of the following conditions is met:

1. All angles are equal. Referred to as AAA (Angle-Angle-Angle).

   NOTE: all internal angles in a triangle add up to 180°.

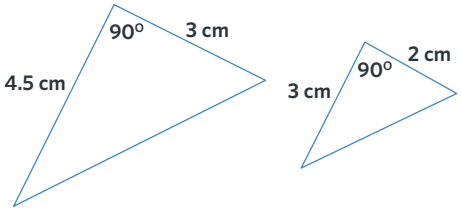


2. All corresponding side lengths are proportional. Referred to as SSS (Side-Side-Side).



$$\frac{3}{4.5} = 0.67$$
$$\frac{4}{6} = 0.67$$
$$\frac{5}{7.5} = 0.67$$

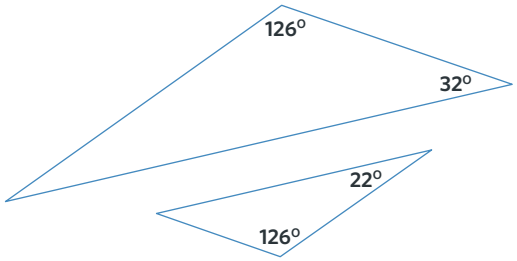
3. Two side lengths are proportional and the angle between them, the included angle, is equal. Referred to as SAS (Side-Angle-Side).



$$\frac{3}{4.5} = 0.67$$
$$\frac{2}{3} = 0.67$$

WORKED EXAMPLE 23 *(1 mark)*

Is the following pair of triangles similar?



SOLUTION

Step 1 Identify what condition could be used to test for similarity.

AAA, as there are two angles known in both triangles.

Step 2 Work out the missing angle in the first triangle.

$180° - 126° - 32° = 22°$

Step 3 Work out the missing angle in the second triangle.

$180° - 126° - 22° = 32°$

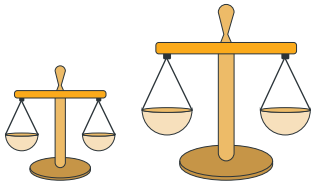
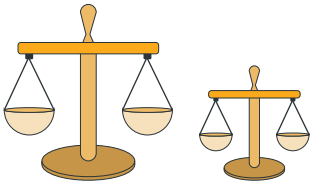
Step 4 Compare the angles present in both triangles.

Both triangles have angles of 126°, 32° and 22°. As all angles in the two triangles are equal, the condition AAA is satisfied and they are similar.

3. Linear scale factor

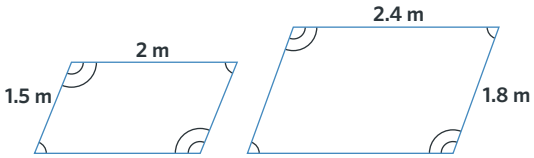
The ratio of corresponding sides in a pair of similar figures is referred to as the scale factor. The scale factor, k, is always calculated by the side length of the image (the shape which has increased or decreased in size), divided by the side length of the original object.

$$k = \frac{Length\ of\ image}{Length\ of\ original}$$

Linear scale factor, k		
Larger than 1 Image larger than the original	Smaller than 1 Image smaller than the original	Equals 1 Image the same size as the original
		

WORKED EXAMPLE 24 *(1 mark)*

Calculate the linear scale factors for the similar figures below.



Step 1 Identify the corresponding side lengths.

Side length of image $= 2.4$ m, *Side length of object* $= 2$ m.

Side length of image $= 1.8$ m, *Side length of object* $= 1.5$ m.

Step 2 Calculate the linear scale factor using the formula $k = \frac{Length\ of\ image}{Length\ of\ original}$.

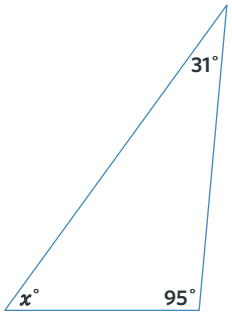
$$k = \frac{2.4}{2} = \frac{1.8}{1.5} = 1.2$$

The linear scale factor is 1.2.

Questions 7I *Similar figures and scale factor*

Refresher question

Q1. What is the value of x in the diagram below?



1. Similar figures

Q2. What are the values of a and b in the working out below?

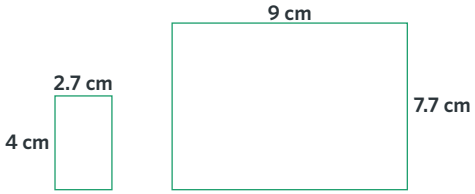
Skill

$\frac{3}{4} = 0.75$ and $\frac{a}{8} = b$

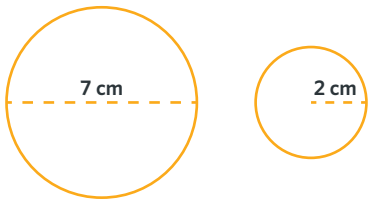
Q3. Are the following pairs of shapes similar?

Skill

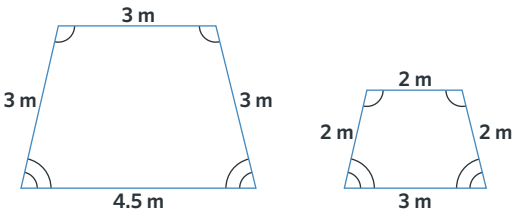
a)



b)



c)



Check your understanding

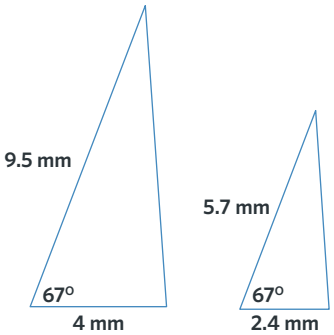
Q4. A circle with a radius of 2.5 cm increased in size so that its circumference became 8π cm. What factor did the radius increase by?

Skill

2. Similar triangles

Q5. Would you use AAA, SSS or SAS to work out if the two triangles below are similar?

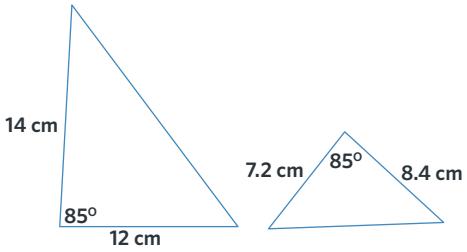
Skill



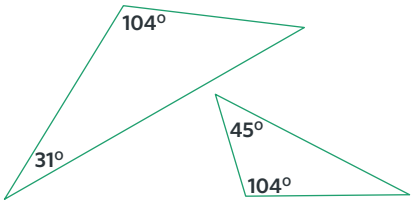
Q6. Are the following pairs of triangles similar?

Skill

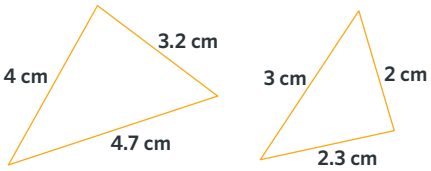
a)



b)



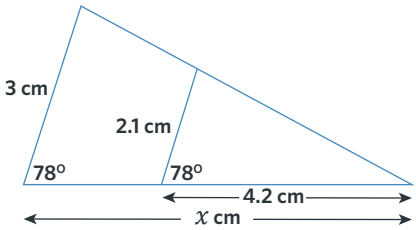
c)



Check your understanding

Q7. What is the value of x in the diagram below?

Skill



Q8. Which of the following conditions would not be sufficient to work out if two triangles were similar?

Skill

A. Two sides labelled in two right-angled triangles.

B. Three sides labelled in both triangles.

C. Two sides and one non-included angle labelled in both triangles.

D. Two angles labelled in both triangles.

3. Linear scale factor

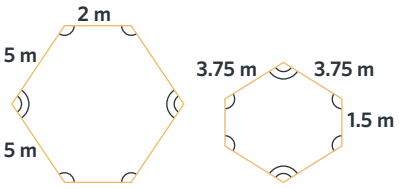
Q9. If the linear scale factor for a pair of objects is 0.65, is the image smaller or larger than the original object?

Skill

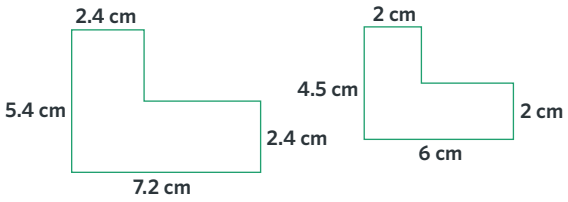
Q10. Calculate the linear scale factors for the similar figures below. Round your answer to two decimal places where appropriate.

Skill

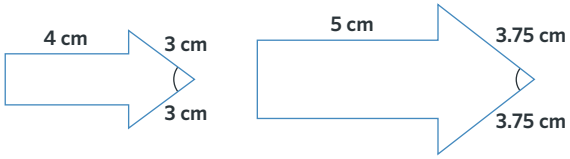
a)



b)



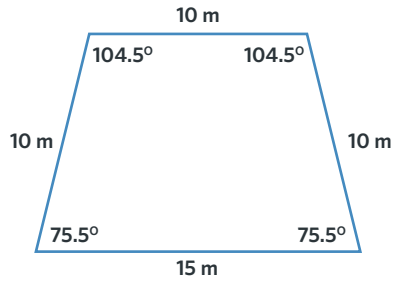
c)



Check your understanding

Q11. Application
A rectangular image with length 3.5 cm and width 2 cm was enlarged to a rectangle with length 6.3 cm and width 3.6 cm. What scale factor was it enlarged by?

Q12. Skill
If the figure below was reduced using a scale factor of 0.8, draw the resulting shape, labelling all side lengths and internal angles.



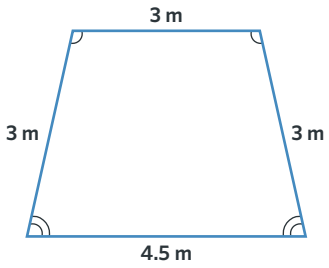
Joining it all together

Q13. Application 2 marks
A map of a town is made by reducing the actual distance between locations by a scale factor of 15 000.

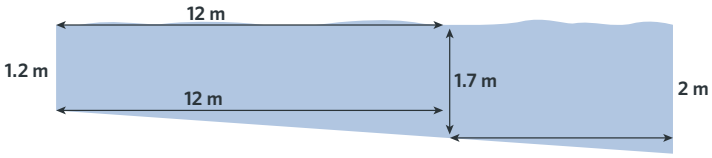
a) If the distance between the post office and the library is 6 cm on the map, what is the actual distance between the two locations in metres?

b) If the actual distance between the library and the train station is 3 km, what is the distance between the two locations on the map in centimetres?

Q14. Application 1 mark
Gabby wants to make a two dimensional outline of the trapezium below but he only has 8.1 cm of string. If Gabby wants to make the biggest model possible, what scale factor should he use for the model?



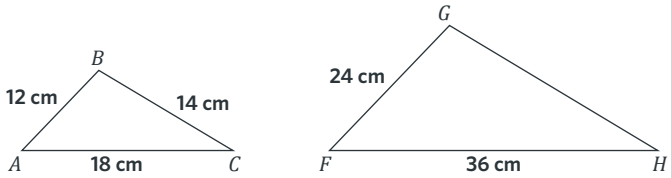
Q15. Application 1 mark
The side view of the swimming pool below shows a depth of 1.2 m at the shallow end and a depth of 2 m at the deep end. What is the length of the swimming pool?



VCAA question

Q16. The two triangles, ABC and FGH, are similar.

1 mark



The length GH is

A. 14 cm

B. 24 cm

C. 26 cm

D. 28 cm

E. 32 cm

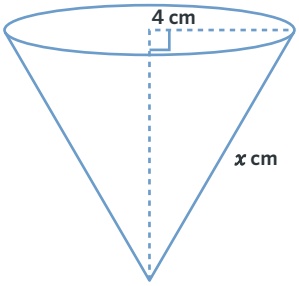
VCAA 2009 Exam 1, Module 2: Geometry and Trigonometry. Q1

Questions from multiple lessons

Q17. A cone has a radius of 4 cm, a slant edge of x cm, and a volume of 120 cm^3.

Difficulty:

1 mark



Its surface area, including the base, can be found using the rule $surface\ area = \pi r(r + x)$.

The total surface area of this cone is closest to

A. 125 cm^2

B. 129 cm^2

C. 140 cm^2

D. 145 cm^2

E. 153 cm^2

VCAA 2018 Exam 1 Module 3: Geometry and measurement Q8 – Adapted

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Q18. Consider the recurrence relation below,

Difficulty:

1 mark

$W_0 = 12\ 000, \quad W_{n+1} = W_n + 180$

This recurrence relation could be used to model a

A. simple interest investment of \$12 000 with an interest rate of 0.015% per period.

B. simple interest investment of \$12 000 with an interest rate of 1.5% per period.

C. simple interest investment of \$12 000 with an interest rate of 15% per period.

D. compound interest investment of \$12 000 with an interest rate of 1.5% per period.

E. compound interest loan of \$12 000 with an interest rate of 1.5% per period.

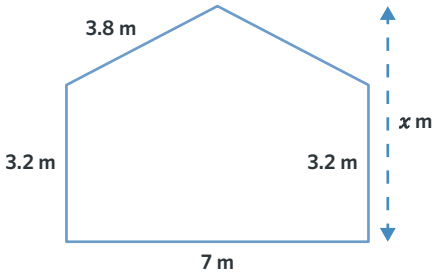
VCAA 2017NH Exam 1 Recursion and financial modelling Q19 – Adapted

Q19.

Difficulty:

3 marks

Dimitri wants to build a greenhouse in the backyard of his mansion. The front of the greenhouse is shown.



The walls of the greenhouse are 3.2 m high.

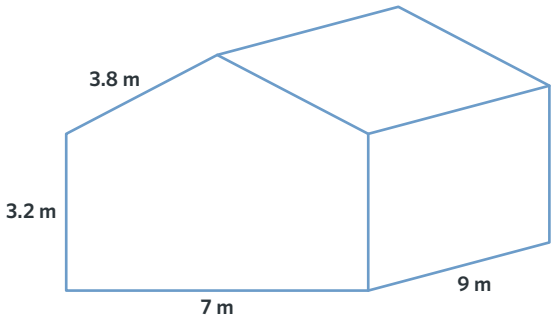
The front of the greenhouse is 7 m wide.

The sloping edges of the roof are 3.8 m long.

The total height of the greenhouse is x m.

a) Show that x is 4.68, correct to two decimal places. (1 mark)

b) The greenhouse is in the shape of a prism, as shown.



All of the surfaces of the greenhouse are to be made of glass, excluding the base.

Correct to the nearest square metre, how much glass will Dimitri need? (2 marks)

VCAA 2015 Exam 2 Module 2: Geometry and trigonometry Q3 – Adapted

LESSON 7J

Similarity of solids and scale factor

The key skills you will learn in this lesson are:

1. Area scale factor
2. Volume scale factor

VCAA key knowledge point:

"similarity of solids and the application of linear scale factor k > 0 to scale lengths, surface areas and volumes with practical applications."

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Area scale factor

If two similar figures have a length scale factor of k, the area scale factor will be k^2.

$$k^2 = \frac{Area\ of\ image}{Area\ of\ original}$$

To find the area of the image, the formula can be rearranged as

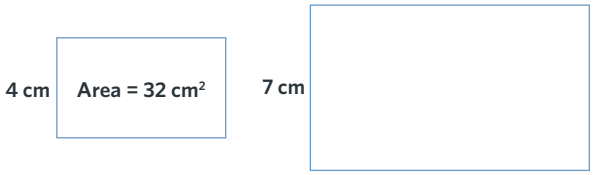
$$Area\ of\ image = k^2 \times Area\ of\ original$$

For a three-dimensional object, surface area follows the same pattern as area. To find the surface area of an image, use the formula

$$Surface\ area\ of\ image = k^2 \times Surface\ area\ of\ original$$

WORKED EXAMPLE 25 (1 mark)

Given that the two rectangles below are similar, find the area of the larger rectangle.



SOLUTION

Step 1 Find the length scale factor, k.

$$k = \frac{Length\ of\ image}{Length\ of\ original}$$

In this case, the image is the large rectangle with a height of 7 cm, and the original is the small rectangle with a height of 4 cm.

$$k = \frac{7}{4}$$

Step 2 Calculate k^2.

$$k^2 = \left(\frac{7}{4}\right)^2 = \frac{7^2}{4^2}$$

$$k^2 = \frac{49}{16}$$

Step 3 Calculate the area of the larger rectangle.

$$Area\ of\ image = k^2 \times Area\ of\ original$$

$$Area\ of\ image = \frac{49}{16} \times 32$$

$$Area\ of\ image = 98$$

$\therefore$ The area of the larger rectangle is 98 cm^2.

2. Volume scale factor

If two similar figures have a length scale factor of k, the volume scale factor will be k^3.

$$k^3 = \frac{Volume\ of\ image}{Volume\ of\ original}$$

To find the volume of the image, the formula is

$$Volume\ of\ image = k^3 \times Volume\ of\ original.$$

WORKED EXAMPLE 26 *(1 mark)*

In Alex's chemistry class, there are two similarly-shaped beakers. Alex knows that the larger beaker has a capacity of 500 mL, but she has forgotten the capacity of the smaller beaker.

If the large beaker is 15 centimetres tall and the small beaker is 12 centimetres tall, calculate the capacity of the smaller beaker.

SOLUTION

Step 1 Find the length scale factor, k.

$$k = \frac{Length\ of\ image}{Length\ of\ original}$$

In this case, the image is the small beaker with a known length of 12 cm, and the original is the large beaker with a known length of 15 cm.

$$k = \frac{12}{15}$$

Step 2 Calculate k^3.

$$k^3 = \left(\frac{12}{15}\right)^3 = \frac{12^3}{15^3}$$

$$k^3 = \frac{1728}{3375}$$

Step 3 Calculate the volume of the smaller beaker.

The volume or the larger beaker is 500 mL.

$$Volume\ of\ image = k^3 \times Volume\ of\ original$$

$$Volume\ of\ image = \frac{1728}{3375} \times 500$$

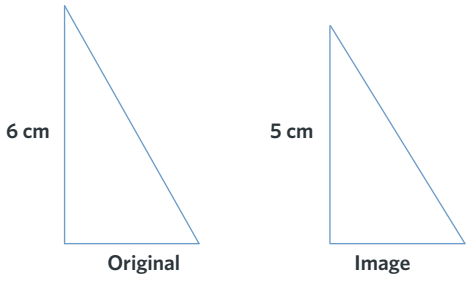
$$Volume\ of\ image = 256$$

∴ The volume of the smaller beaker is 256 mL.

Questions 7J *Similarity of solids and scale factor*

Refresher question

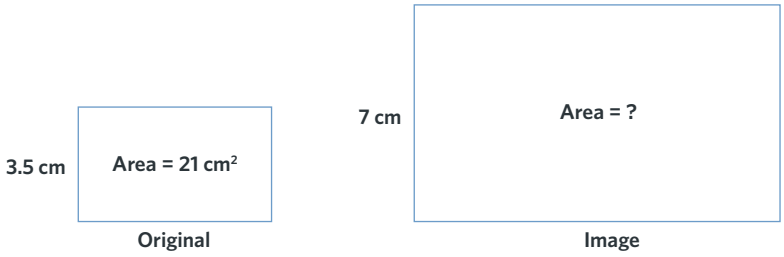
Q1. Find the length scale factor, k, for the similar triangles below.



1. Area scale factor

Q2. For these rectangles:

Skill



a) Find k.

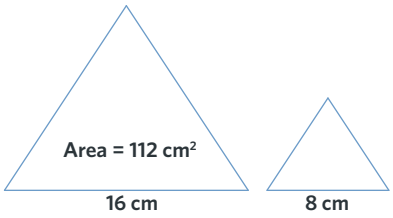
b) Find k^2.

c) Find the area of the image using: $Area\ of\ image = k^2 \times Area\ of\ original$.

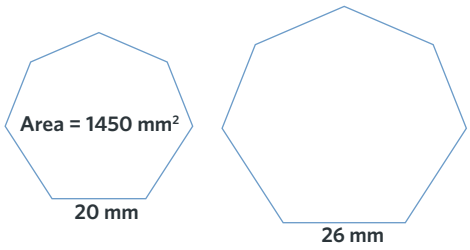
Q3. Skill

Find the unknown area for each of the pairs of similar shapes below, correct to two decimal places where necessary.

a)



b)



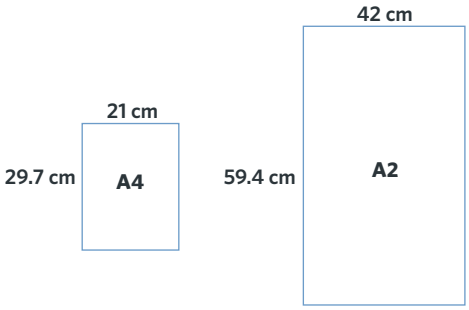
c)

Q4. Application

Below are the dimensions of A4 and A2 pieces of paper. A sheet of A4 paper has an area of 623.7 cm².

a) Show that the shapes of A4 and A2 paper are similar.

b) Find k^2, and use it to calculate the area of an A2 piece of paper.



Q5. Skill

Answer the following questions. Drawing a diagram may help.

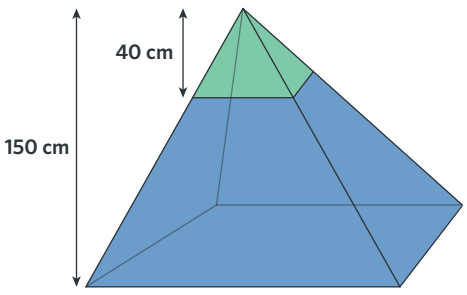
a) Triangle A has width W, vertical height H, and area A. Triangle B has a width of $4W$ and a vertical height of $4H$. Which of the following is an expression for the area of triangle B: $4A$ $8A$, $12A$, $16A$, or $32A$?

b) Cube A has a surface area of 150 cm². Cube B has side lengths that are three times the side lengths of cube A. What is the surface area of cube B?

Check your understanding

Q6. Application

Bella is an avid hiker and her favourite tent is in the shape of a pyramid.



The surface area of the green portion of the tent is 3000 cm².

Find the surface area of the blue portion of the tent, excluding the base.

Q7. Application

Angelica is an architecture student and has to make a model of a building she designed for an assignment. She makes a model with a height of 18 centimetres and a surface area of 980 cm^2. The actual building will have a surface area of 310 000 cm^2.

What will the actual height of the building be, correct to the nearest centimetre?

2. Volume scale factor

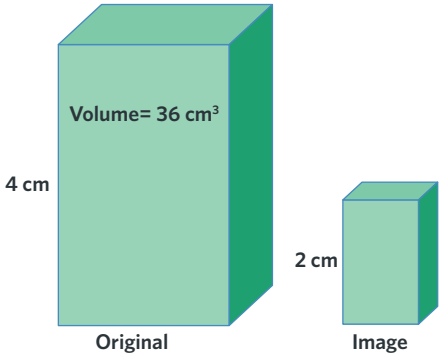
Q8. Skill

For these rectangular prisms:

a) Find k.

b) Find k^3.

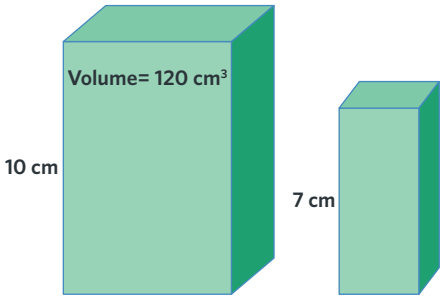
c) Find the volume of the image using: *Volume of image* = $k^3 \times$ *Volume of original*.



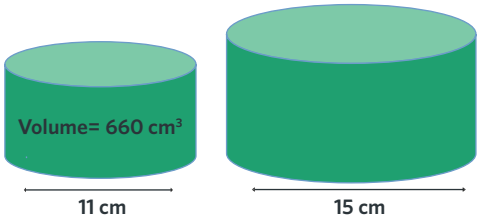
Q9. Skill

Find the unknown volume for each of the pairs of similar shapes below correct to two decimal places.

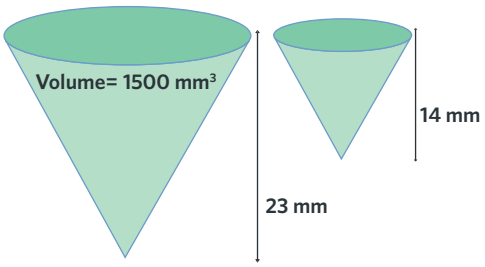
a)



b)



c)



Q10. Skill

Cube A has a side length of L and a volume of V. Cube B has a side length of $2L$.

Show that the volume of cube B is $8V$.

Q11. Application

A soccer ball has a diameter of 22 centimetres and a volume of 5575.28 cm^3. A tennis ball has a volume of 143.79 cm^3.

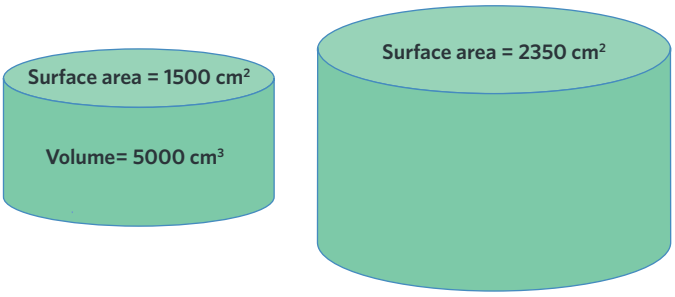
a) Find k^3, and use this to find k, given that the tennis ball is treated as the image and the soccer ball as the original object. Give your value of k correct to three decimal places.

b) Using your value of k, find the diameter of a tennis ball correct to one decimal place.

Check your understanding

Q12. Given that the cylinders below are similar, find the volume of the larger cylinder correct to one decimal place.

Skill



Q13. Caleb's Cardboard Container Company have three different sizes of similarly shaped cardboard boxes; small, medium and large. Let the width and volume of the small box be w and v respectively.

Application

a) The width of the large box is four times the width of the small box. Express the volume of the large box in terms of v.

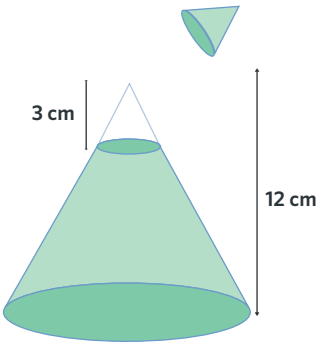
b) The volume of the medium box is eight times the volume of the small box. Express the width of the medium box in terms of w.

c) If the medium box has a volume of 64 000 cm^3, what is the volume of the large box?

Joining it all together

Q14. The top three centimetres are removed from the tip of a cone.

Skill
1 mark



What percentage of the volume of the cone remains, correct to one decimal place?

Q15. India is going on holiday, so she buys a travel-size bottle of her favourite perfume. She has the full-size bottle of the same perfume and notices that the two bottles are similarly shaped, and the height of the travel-size bottle is half the height of the full-size bottle.

Application
1 mark

If the full-size perfume bottle can hold 100 mL, how much perfume is in the travel-size bottle?

Q16. The Eiffel Tower in Paris, France is 324 metres tall and has a total surface area of 220 000 m^2. In Las Vegas, there is a scaled-down replica of the Eiffel Tower with a height of 165 metres.

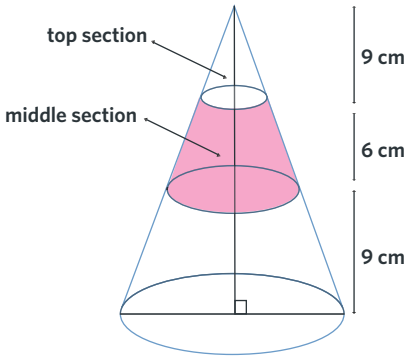
Application
1 mark

What is the surface area of the Eiffel Tower replica in Las Vegas correct to the nearest square metre?

VCAA questions

Q17. The middle section of a cone is shaded, as shown in the diagram below.

1 mark



The surface area of the unshaded top section of the cone is 180 cm^2.

The surface area of the middle section of the cone, in square centimetres, is

A. 80

B. 120

C. 300

D. 320

E. 500

VCAA 2014 Exam 1, Module 2: Geometry and Trigonometry. Q9

Q18. A cafe sells two sizes of cupcakes with a similar shape.

1 mark

The large cupcake is 6 cm wide at the base and the small cupcake is 4 cm wide at the base.

The price of a cupcake is proportional to its volume. If the large cupcake costs $5.40, then the small cupcake will cost

A. $1.60

B. $2.32

C. $2.40

D. $3.40

E. $3.60

VCAA 2013 Exam 1, Module 2: Geometry and Trigonometry. Q4

Questions from multiple lessons

Q19.

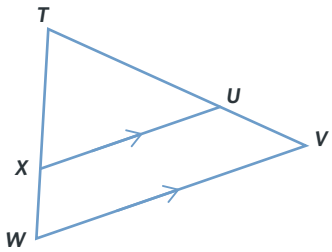
Difficulty:

1 mark

In triangle TVW, point X lies on side TW and point U lies on side TV.

The lines UX and VW are parallel.

The length of TU is 8 cm, the length of UV is 4 cm and the length of UX is 10 cm.



The length of VW is

A. 20 cm

B. 18 cm

C. 15 cm

D. 12 cm

E. 10 cm

VCAA 2013 Exam 1 Module 2: Geometry and trigonometry Q6 – Adapted

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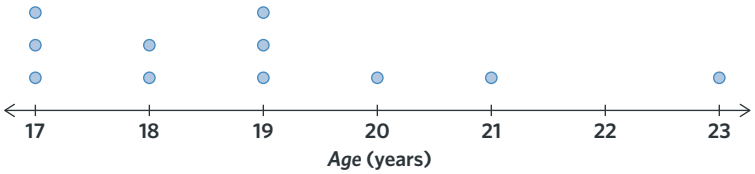
Q20.

Difficulty:

1 mark

The following dot plot displays the age, in years, of eleven Mathletics speed demons.



Correct to one decimal place, the mean and standard deviation of the *age* of the Mathletics speed demons are

A. $\overline{x} = 18.9 \quad s_x = 1.8$

B. $\overline{x} = 1.9 \quad s_x = 18.9$

C. $\overline{x} = 18.5 \quad s_x = 1.4$

D. $\overline{x} = 1.4 \quad s_x = 18.5$

E. $\overline{x} = 18.9 \quad s_x = 1.9$

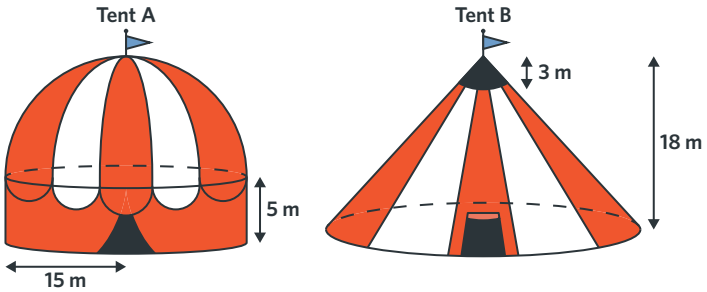
VCAA 2008 Exam 1 Data analysis Q5 – Adapted

Q21.

Difficulty:

3 marks

A travelling circus has two tents in which it holds shows: tent A and tent B.



Tent A is comprised of a cylinder and a hemisphere. It has a radius of 15 metres and the cylinder has a height of 5 metres.

a) Calculate the volume of tent A. Write your answer correct to the nearest m^3. (2 marks)

Tent B is in the shape of a cone with a height of 18 metres. The top three metres of the tent are blocked off to store lighting equipment, as shown.

b) What percentage of the volume of the tent is not blocked off? Give your answer correct to two decimal places? (1 mark)

VCAA 2009 Exam 2 Module 2: Geometry and trigonometry Q4 – Adapted

AOS 4: Geometry, measurement and trigonometry

Applications of trigonometry

CHAPTER
8

LESSON 8A

Review of trigonometry

The key skills you will learn in this lesson are:

1. Finding the unknown side length
2. Finding the unknown angle

VCAA key knowledge point:

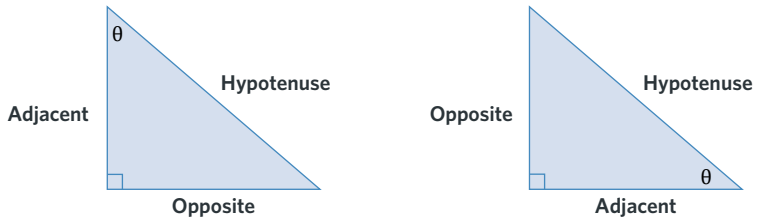
"review of the use of trigonometric ratios for sine, cosine and tangent to find the length of an unknown side or the size of an unknown angle in a right-angled triangle"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Finding the unknown side length

The three sides of a right-angled triangle are called the hypotenuse, the opposite side and the adjacent side. The hypotenuse is the longest side of a right-angled triangle, and is always directly opposite the right-angle. The location of the opposite side and the adjacent side depend on the location of the angle θ (theta). As the names suggest, the opposite side is the located opposite to θ, and the adjacent side is located adjacent to θ.

Below are two triangles with the hypotenuse, opposite side and adjacent side labelled, in reference to the angle θ.



The trigonometric ratios sin, cos and tan can be used to find the length of an unknown side or the size of an unknown angle in a right-angled triangle. The ratios are:

- $\sin(\theta) = \frac{Opposite}{Hypotenuse}$
- $\cos(\theta) = \frac{Adjacent}{Hypotenuse}$
- $\tan(\theta) = \frac{Opposite}{Adjacent}$

and are often remembered by the phrase SOH CAH TOA.

To find the length of an unknown side, determine which trigonometric ratio should be applied, and substitute the value of the known side and the known angle into the equation.

WORKED EXAMPLE 1 *(1 mark)*

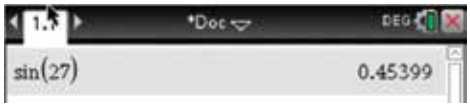
Use a calculator to find the value of sin(27°) correct to two decimal places (ensure your calculator is in degrees mode).

SOLUTION: TI-NSPIRE

Step 1 Open a new page by pressing ctrl + doc and press 1 ('Add Calculator').

Step 2 Press trig and, using the arrows, select the sin icon sin. Press enter. Type 27.

Step 3 Press ctrl + enter. The answer will appear.

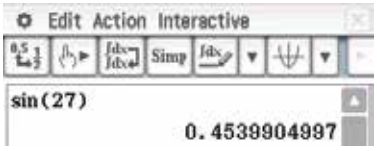


SOLUTION: CASIO CLASSPAD

Step 1 From the menu, tap Main and press Keyboard.

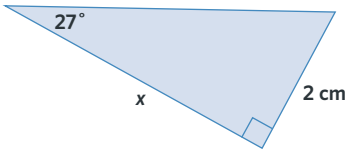
Step 2 Tap sin. Type 27.

Step 3 Press EXE. The answer will appear.



WORKED EXAMPLE 2 *(1 mark)*

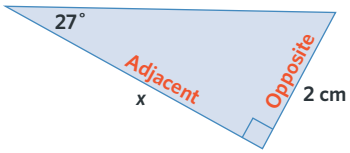
Find the value of x in this triangle, correct to two decimal places.



SOLUTION

Step 1 Identify which trigonometric ratio to use.

2 cm is the opposite side as it is opposite the known angle, and x is the adjacent side as it is adjacent the known angle.



Use tan as $\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$.

Step 2 Substitute the values into the equation.

$\tan(27°) = \frac{2}{x}$

Step 3 Rearrange the equation to find x.

$x \tan(27°) = 2$

$x = \frac{2}{\tan(27°)}$

Step 4 Use your calculator to find the value of x.

$x = 3.93$

The value of x is 3.93 cm.

2. Finding the unknown angle

The trigonometric ratios can also be used to find the size of an unknown angle in a right-angled triangle. To do so, determine which trigonometric ratio should be applied, and substitute the values of the two known sides into the equation.

To find the value of theta, the inverse of the trigonometric function must be used. The inverse trigonometric functions are $\sin^{-1}$ $\cos^{-1}$, and $\tan^{-1}$, where:

- $\sin^{-1}(\sin(\theta)) = \theta$
- $\cos^{-1}(\cos(\theta)) = \theta$
- $\tan^{-1}(\tan(\theta)) = \theta$

NOTE: The –1 is not a power, and is just used to denote the inverse.

Below is an example of how to find the value of the unknown angle θ, when the opposite side and hypotenuse are known.

$$\sin(\theta) = \frac{Opposite}{Hypotenuse}$$

$$\sin^{-1}(\sin(\theta)) = \sin^{-1}\left(\frac{Opposite}{Hypotenuse}\right)$$

$$\theta = \sin^{-1}\left(\frac{Opposite}{Hypotenuse}\right)$$

WORKED EXAMPLE 3 *(1 mark)*

Use a calculator to find the value of $\sin^{-1}(0.7)$ correct to two decimal places (ensure your calculator is in degrees mode).

SOLUTION: TI-NSPIRE

Step 1 Open a new page by pressing ctrl + doc and press 1 ('Add Calculator').

Step 2 Press trig and use the arrows to find the inverse sin icon $\sin^{-1}$. Press enter. Type 0.7.

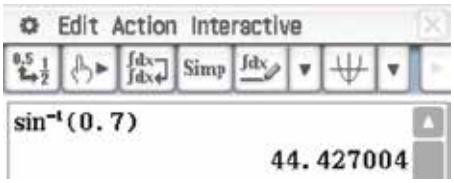
Step 3 Press enter. The answer will appear.

SOLUTION: CASIO CLASSPAD

Step 1 From the menu, tap Main and press Keyboard.

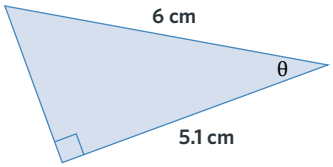
Step 2 Tap Trig and $\sin^{-1}$. Type 0.7.

Step 3 Press EXE. The answer will appear.



WORKED EXAMPLE 4 *(1 mark)*

Find the value of θ in this triangle, correct to two decimal places.



SOLUTION

Step 1 Identify which trigonometric ratio to use.

5.1 cm is the adjacent side as it is adjacent the unknown angle, and 6 cm is the hypotenuse.

Use cos as $\cos(\theta) = \frac{Adjacent}{Hypotenuse}$

Step 2 Substitute the values into the equation.

$\cos(\theta) = \frac{5.1}{6}$

Step 3 Rearrange the equation to find θ.

$\cos^{-1}(\cos(\theta)) = \cos^{-1}\left(\frac{5.1}{6}\right)$

$\theta = \cos^{-1}\left(\frac{5.1}{6}\right)$

Step 4 Use your calculator to find the value of θ.

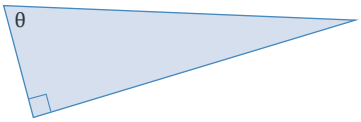
$\theta = 31.79$

The value of θ is 31.79°.

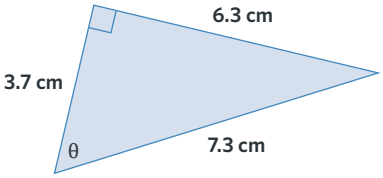
Questions 8A *Review of trigonometry*

Refresher questions

Q1. Label this triangle with opposite, adjacent and hypotenuse, with reference to the angle θ.



Q2. What is the length of the adjacent side in this triangle, with reference to the angle θ?



1. Finding the unknown side length

Q3. Skill

Fill in the blanks.

$\sin(\theta) = \frac{\square}{Hypotenuse}$

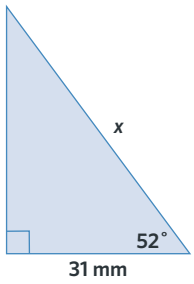
$\cos(\theta) = \frac{\square}{Hypotenuse}$

$\tan(\theta) = \frac{\square}{\square}$

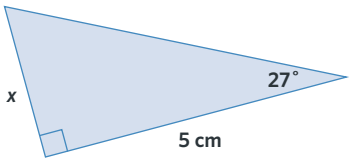
Q4. Skill

Find the value of x in the triangles below, correct to two decimal places.

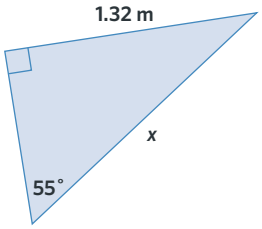
a)



b)



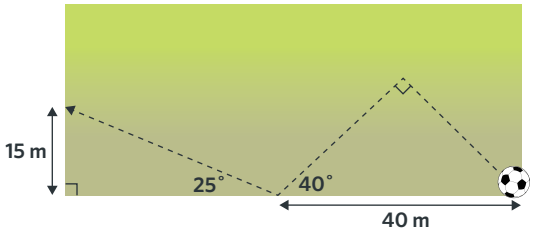
c)



Check your understanding

Q5. Application

A soccer ball travels along the dotted path as shown in this diagram. What is the total distance travelled by the soccer ball, correct to the nearest metre?



2. Finding the unknown angle

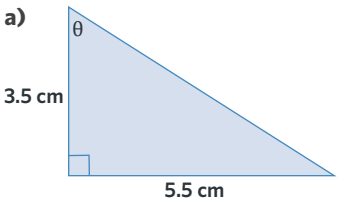
Q6. Skill

If $\sin(\theta) = 0.43$, what is the value of θ, correct to the nearest degree?

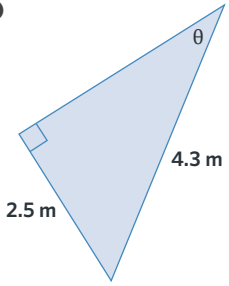
Q7. Skill

Find the value of θ in the triangles below, correct to two decimal places.

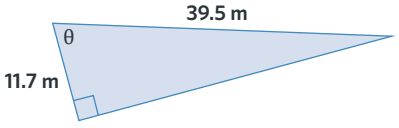
a)



b)



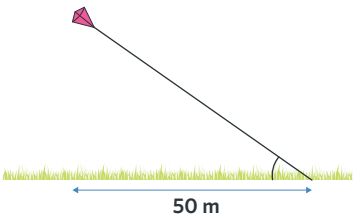
c)



Check your understanding

Q8. Application

A student is flying a kite on a windy day. The string of the kite is 70 m long and the kite flies at a horizontal distance of 50 m away from the student. What angle does the string of the kite form with the ground, correct to the nearest degree?



Q9. Application

A bird swoops, in a straight path, from a tree with a height of four metres to a worm on the ground. If the path the bird takes makes an angle of 30° with the ground, what is the total distance travelled by the bird?

Joining it all together

Q10. Application 2 marks

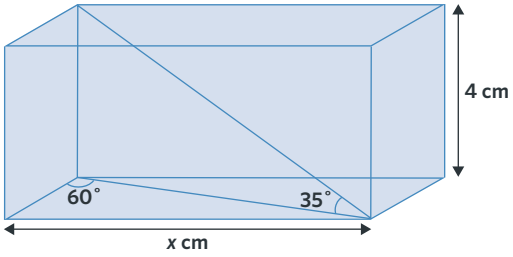
A three metre ladder leans against the wall such that the angle between the ladder and the wall is 30°.

a) How far away is the base of the ladder from the wall?

b) The ladder slips 20 cm down the wall. What is the angle between the ladder and the wall, correct to the nearest degree?

Q11. Skill 2 marks

Find the value of x in this rectangular prism, correct to two decimal places.



Q12. Application 2 marks

A student worked out that an angle in a right-angled triangle was 47°.

However, instead of using $\tan(\theta) = \frac{Opposite}{Adjacent}$, they mistakenly used $\tan(\theta) = \frac{Adjacent}{Opposite}$.

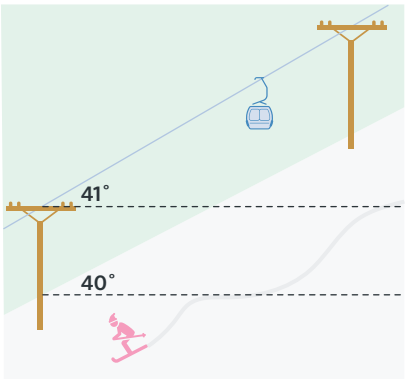
If the length of the opposite side was 3.7 cm, what should the angle have actually been?

Q13. Application 2 marks

A ski slope has an incline of 40° whereas the ski lift has an incline of 41°.

a) Over a horizontal distance of 100 metres, how many more metres are travelled by making a descent in the ski lift compared to skiing directly down the slope? Round your answer to two decimal places.

b) Over what horizontal distance will the ski lift rise two metres more than the slope? Round your answer to two decimal places.



VCAA question

Q14. 2 marks

A farmer owns a flat allotment of land in the shape of triangle ABC shown below.

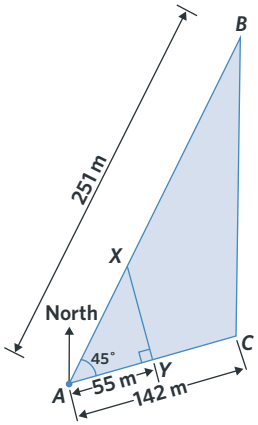
Boundary AB is 251 metres. Boundary AC is 142 metres.
Angle BAC is 45°.

A straight track, XY, runs perpendicular to the boundary AC. Point Y is 55 m from A along the boundary AC.

a) Determine the size of angle AXY.

b) Determine the length of AX. Write your answer, in metres, correct to one decimal place.

VCAA 2006 Exam 2 Module 2: Geometry and Trigonometry. Q1



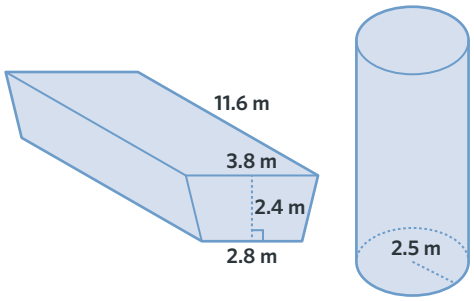
Questions from multiple lessons

Q15.

Difficulty:

1 mark

A farmer uses a specific truck to transport his grain from the farm to a silo. The grain fills a trailer in the shape of a trapezoid prism with the dimensions as shown.



The farmer's silo is cylindrical in shape with a radius of 2.5 m. Assuming that the grain will sit flat in the silo, the height at which exactly one trailer-load of grain would sit within the silo is closest to

A. 5.4 m

B. 5.1 m

C. 4.5 m

D. 3.9 m

E. 4.7 m

VCAA 2010 Exam 1 Module 2: Geometry and trigonometry Q9 – Adapted

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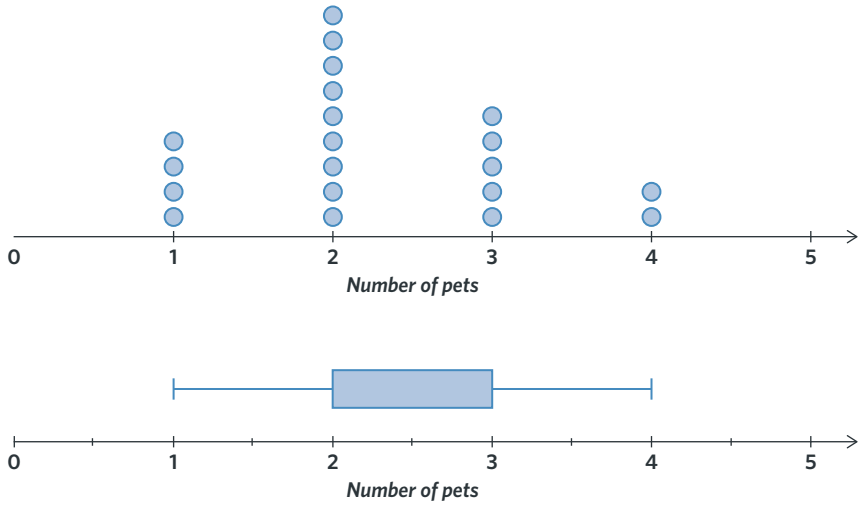
Q16.

Difficulty:

Year 10
1 mark

The following dot plot and boxplot display the number of pets owned by 20 zookeepers.



What is the median number of pets for this group of zookeepers?

A. 1

B. 2

C. 2.5

D. 3

E. 4

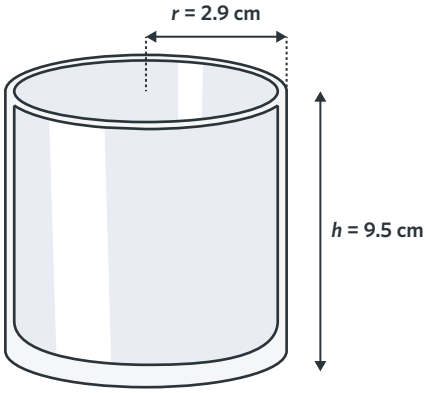
VCAA 2019NH Exam 1 Data analysis Q4 – Adapted

Q17.

Difficulty:

2 marks

Blaise runs a bar, and is looking at purchasing new glasses. The glass model he is looking at is cylindrical in shape with a radius of 2.9 cm and a height of 9.5 cm, as shown.



a) What is the diameter, in centimetres, of the glass? (1 mark)

b) What is the surface area of the outside of the glass, including the bottom?
Write your answer in square centimetres, correct to one decimal place. (1 mark)

VCAA 2018 Exam 2 Module 3: Geometry and measurement Q1 a,b – Adapted

LESSON 8B

Angles of elevation and depression

The key skills you will learn in this lesson are:

1. Angles of elevation
2. Angles of depression

VCAA key knowledge point:

"application of the trigonometry of right-angled triangles to solve practical problems including the use of angles of elevation and depression, and the use of three figure (true) bearings in navigation"

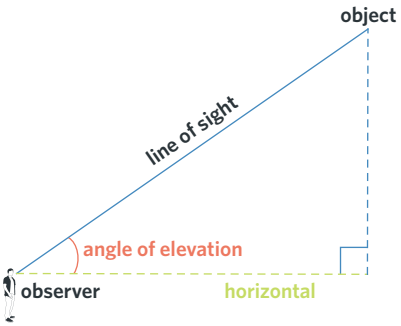
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

Angles of elevation and depression measure the angle between the horizontal and the line of sight to an object.

The horizontal is the line parallel to the ground, beginning at the point of the observer, and the line of sight is the straight line created between the observer and the object.

1. Angles of elevation

The angle of elevation is used when the object being observed is above the horizontal (when looking up at something).



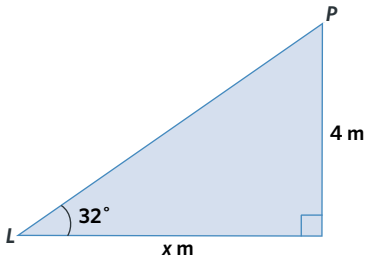
WORKED EXAMPLE 5 *(1 mark)*

Lauren is standing in a park and sees a ringtail possum four metres up a tree. The angle of elevation from Lauren to the possum is 32°.

What is the horizontal distance between Lauren and the tree, correct to the nearest centimetre?

SOLUTION

Step 1 Draw a diagram.



Step 2 Set up an equation.

We know the angle of elevation (32°) and the length of the side opposite the angle (4 m).

We are looking for the length of the side adjacent to the angle (x).

Hence, we can use tan to find the missing side length.

$$\tan(\theta) = \frac{opposite}{adjacent}$$

$$\tan(32°) = \frac{4}{x}$$

Step 3 Solve the equation.

$$\tan(32°) = \frac{4}{x}$$

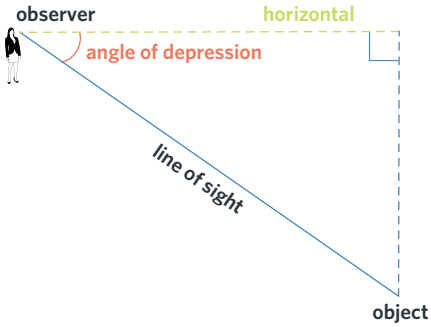
$$x = \frac{4}{\tan(32°)}$$

$$x = 6.4013\ldots$$

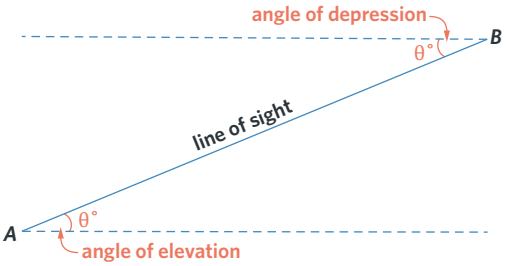
∴ The horizontal distance between Lauren and the tree is 6.40 metres.

2. Angles of depression

The angle of depression is used when the object being observed is below the horizontal (when looking down at something).



Angles of elevation and depression are alternate angles. Hence, the angle of elevation of point B from A will always be equal to the angle of depression of point A from B.



angle of elevation of B from A = angle of depression of A from B

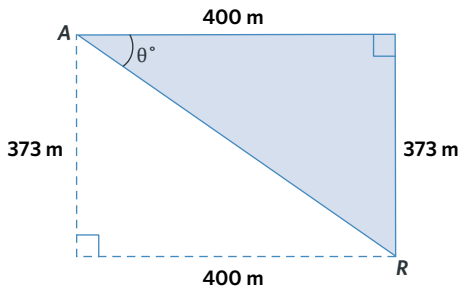
WORKED EXAMPLE 6 *(1 mark)*

Andy is standing on the observation deck of the Empire State Building, which is 373 metres above ground level. Andy can see his favourite restaurant on the street below, which he knows is 400 metres from the base of the building.

What is the angle of depression from Andy to the restaurant, correct to two decimal places?

SOLUTION

Step 1 Draw a diagram.



Step 2 Set up an equation.

We are looking for the angle θ in the triangle above.

We know the length of the side opposite the angle (373 m) and the side adjacent to the angle (400 m).

Hence, we can use tan to find the angle.

$$\tan(\theta) = \frac{opposite}{adjacent}$$

$$\tan(\theta) = \frac{373}{400}$$

Step 3 Solve the equation.

$$\tan(\theta) = \frac{373}{400}$$

$$\theta = \tan^{-1}\left(\frac{373}{400}\right)$$

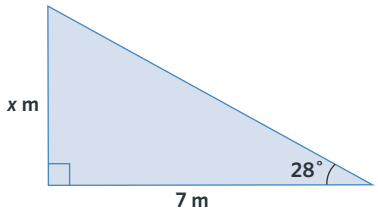
$$\theta = 42.9995\ldots$$

∴ The angle of depression from Andy to the restaurant is 43.00°.

Questions 8B *Angles of elevation and depression*

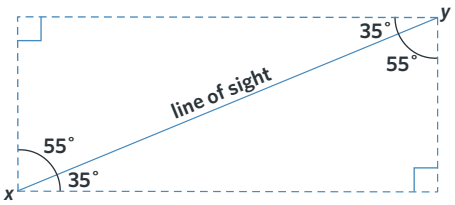
Refresher question

Q1. Find the value of x in this triangle, correct to two decimal places.



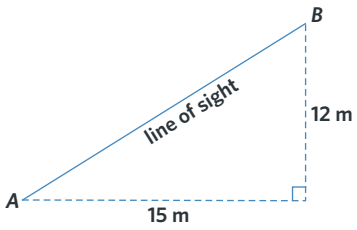
1. Angles of elevation

Q2. Skill
What is the angle of elevation, in degrees, of y from x in this diagram?

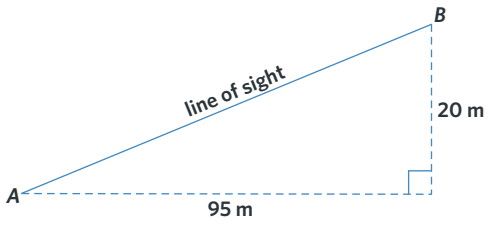


Q3. Skill
For each of the following diagrams, find the angle of elevation, in degrees, of point B from point A, correct to two decimal places.

a)



b)



Q4. Application
Sam is standing in a field when he spots a UFO in the sky.
The UFO is 120 metres from him horizontally and is 45 metres above the ground.

What is the angle of elevation of the UFO from Sam, in degrees, correct to two decimal places?

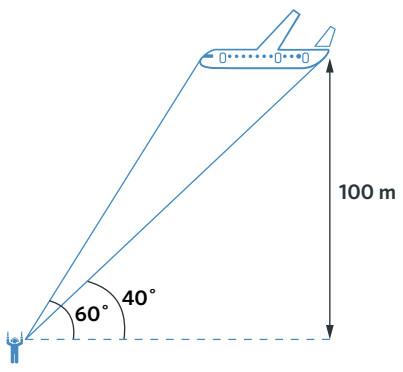
Q5. Application
Mason is on holiday in Dubai and decides to visit the tallest structure in the world, the Burj Khalifa. He stands 270 metres from the base of the tower and the angle of elevation of the top of the tower from where he is standing is 72°.

What is the vertical height of the Burj Khalifa, in metres, correct to two decimal places?

Check your understanding

Q6. Application
An airport control officer is standing on the runway and sees an aeroplane facing in his direction. From where he is standing, the angle of elevation of the front of the plane is 60°, and the angle of elevation of the back of the plane is 40°.

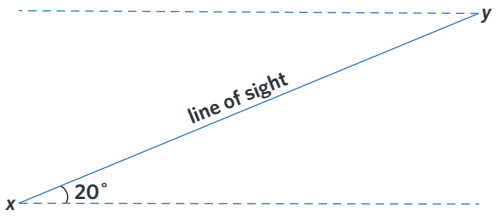
Given that the plane is flying parallel to the ground at an altitude of 100 metres, what is the length of the aeroplane, in metres, correct to one decimal place?



2. Angles of depression

Q7. Consider the diagram below.

Skill

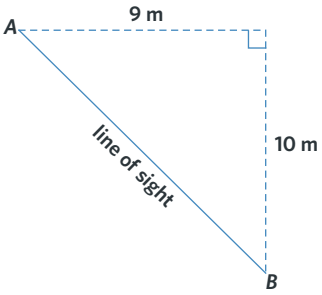


What is the angle of depression, in degrees, of point x from point y?

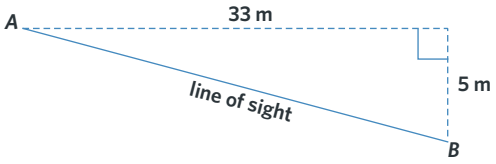
Q8. Calculate the angle of depression of point B from point A in each of the diagrams below, in degrees, correct to two decimal places.

Skill

a)



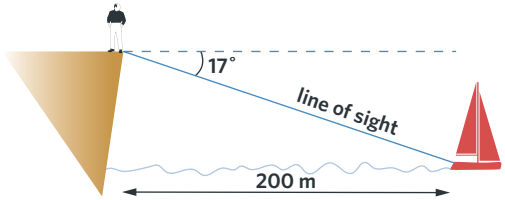
b)



Q9. A person standing on the edge of a cliff can see a boat, at an angle of depression of 17°.

Application

Using the diagram below, calculate the height of the cliff correct to two decimal places.



Q10. Angie is attending a Beyoncé concert but could only afford tickets at the very back of the arena. Her seat is 200 metres horizontally from the centre of the stage, and is 20 metres higher than the stage.

Application

If Beyoncé stands in the centre of the stage, what is the angle of depression, in degrees, of where Beyoncé stands from Angie's seat, correct to one decimal place?

Check your understanding

Q11. Gordon is sitting in his treehouse 3.2 metres above the ground when he sees a kookaburra's nest in a nearby tree. The nest is 2.55 metres above the ground and the angle of depression of the nest from Gordon is 10°.

Application

What is the horizontal distance between the two trees, correct to the nearest centimetre?

Q12. Lauren is in a hot air balloon, flying at a height of 300 metres, and has spotted both her house and her school. Her angle of depression of her house is 65°, and her angle of depression of her school is 22°.

Application

Calculate the shortest possible distance between Lauren's house and school, in metres, correct to one decimal place.

Joining it all together

Q13. Application 2 marks

A zipline is being constructed through a jungle in Thailand. The zipline will start from an eight-metre platform and finish on the ground. The length of rope needed for the zipline is 170 metres.

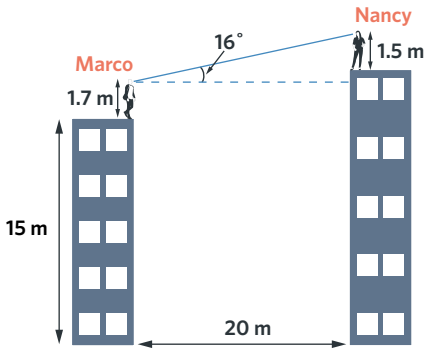
a) When standing on the starting platform, what will be the angle of depression to the bottom of the zipline, in degrees, correct to two decimal places?

b) What will be the horizontal distance spanned by the zipline, correct to the nearest centimetre?

Q14. Application 2 marks

Marco is standing on the roof of a 15-metre-tall building, and he is 1.7 metres tall. He can see his friend Nancy, who is 1.5 metres tall, on top of a nearby building, at an angle of elevation of 16°. The buildings are 20 metres apart.

How tall is the second building, in metres, correct to one decimal place, taking into account the heights of Marco and Nancy?



VCAA question

Q15. 2 marks

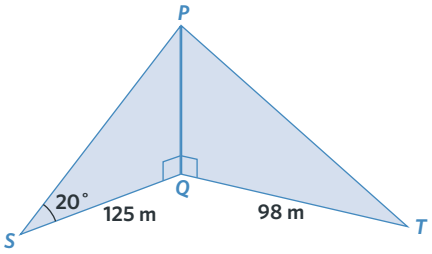
The allotment of land contains a communications tower, PQ.

Points S, Q and T are situated on level ground.

From S the angle of elevation of P is 20°.

Distance SQ is 125 metres.

Distance TQ is 98 metres.



a) Determine the height, PQ, of the communications tower.

Write your answer, in metres, correct to one decimal place.

b) Determine the angle of depression of T from P.

Write your answer, in degrees, correct to one decimal place.

VCAA 2006 Exam 2 Module 2: Geometry and Trigonometry. Q2

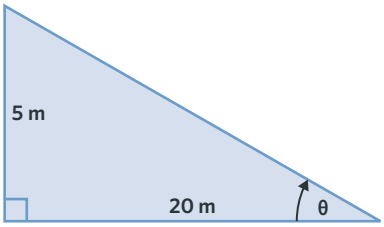
Questions from multiple lessons

Q16.

Difficulty:

Year 10
1 mark

An escalator spans a horizontal distance of 20 m and a vertical distance of 5 m, as shown.



The angle that the escalator makes with the ground is closest to

A. 15.0°

B. 14.4°

C. 14.0°

D. 75.5°

E. 60.0°

VCAA 2017NH Exam 1 Module 3: Geometry and measurement Q2 – Adapted

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Q17.

Difficulty:

1 mark

Ash borrows $6500 which she has to repay fully in a lump sum payment after two years.

She has a choice of one of the following five loans, with the given interest rates and compounding periods:

- Loan 1 – 9.70% per annum, compounding monthly
- Loan 2 – 9.72% per annum, compounding monthly
- Loan 3 – 9.74% per annum, compounding monthly
- Loan 4 – 9.76% per annum, compounding quarterly
- Loan 5 – 9.78% per annum, compounding quarterly

The loan that will cost Ash the least amount of money is

A. Loan 1.

B. Loan 2.

C. Loan 3.

D. Loan 4.

E. Loan 5.

VCAA 2018 Exam 1 Recursion and financial modelling Q19 – Adapted

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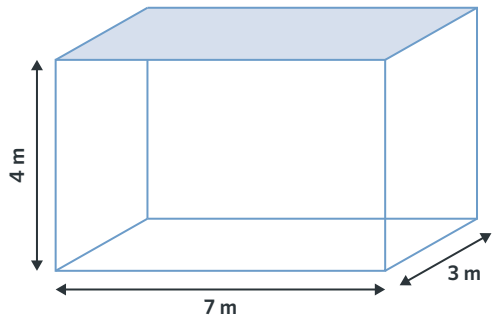
Q18.

Difficulty:

2 marks

An engineer is designing the size of a water tank to distribute water to a remote community. He has decided that the tank will be 7 m wide, 3 m long and 4 m tall.



On his drafts of the water tank, a 1 cm line is used to represent a length of 1 m.

a) What is the scale factor, k, used in his drafts? (1 mark)

The shaded lid in the diagram to the right has an area of 21 m^2.

b) What is the area of the lid in his drafts? Give your answer in m^2. (1 mark)

VCAA 2008 Exam 2 Module 2: Geometry and trigonometry Q1b – Adapted

LESSON 8C

Bearings

The key skills you will learn in this lesson are:

1. Finding bearings
2. Plotting bearings
3. Applications of bearings

VCAA key knowledge point:

"application of the trigonometry of right-angled triangles to solve practical problems including the use of angles of elevation and depression, and the use of three figure (true) bearings in navigation"

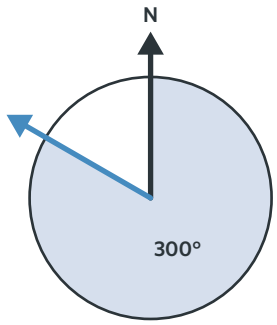
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Finding bearings

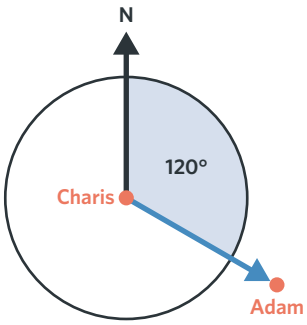
Three-figure bearings, also called true bearings, are used to show the direction of one object from another object (the reference object). They are often used in sailing and orienteering.

A bearing is measured clockwise from north. All bearings, including angles less than 100°, still written with three digits, such as 045°.

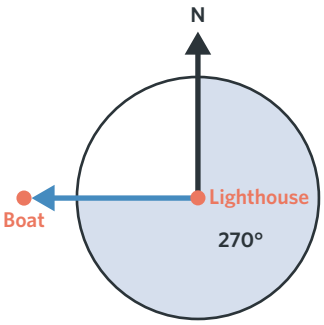
Example



A bearing of 300°.



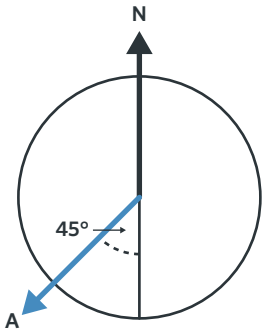
Adam is on a bearing of 120 degrees from Charis.



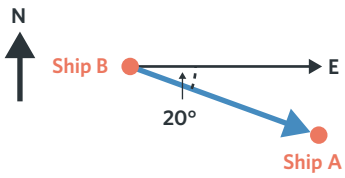
The boat is on a bearing of 270 degrees from the lighthouse.

WORKED EXAMPLE 7 *(2 marks)*

a) What is the bearing of A from the centre of the circle?



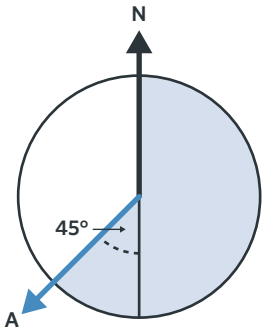
b) Find the bearing of ship A from ship B.



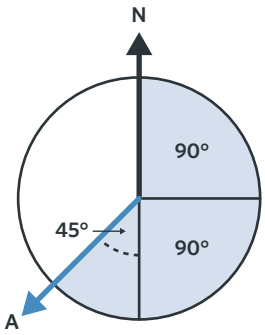
SOLUTION

a) What is the bearing of A from the centre of the circle?

Step 1 Identify the angle of the bearing.



Step 2 Break the bearing into smaller parts of 90 or 180 degrees.



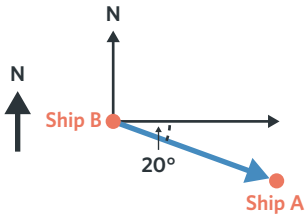
Step 3 Add up the smaller angles to find the bearing.

$A = 90 + 90 + 45$

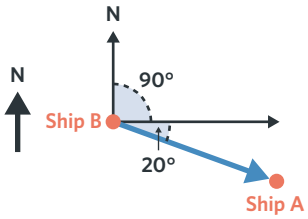
$= 225°$

b) Find the bearing of ship A from ship B.

Step 1 Draw a north arrow on the reference object, ship B.



Step 2 Break the bearing into smaller angles.



Step 3 Add up the angles.

Bearing (A from B) $= 90 + 20$

$= 110°$

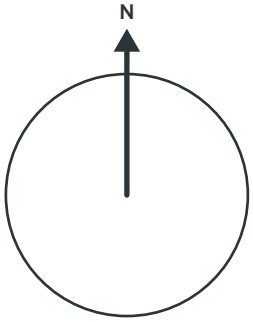
2. Plotting bearings

Bearings can be plotted by rotating the appropriate number of degrees clockwise from north and marking any point on this angle.

Bearings that describe the position of one object from another, usually written as 'Object A is on a bearing of 290° from Object B', are plotted by drawing object B first, then measuring object A's position relative to object B.

WORKED EXAMPLE 8 *(2 marks)*

a) Draw a bearing of 190° on the following diagram.



b) Draw a diagram of towns A and B, if town B is on a bearing of 340° from town A.

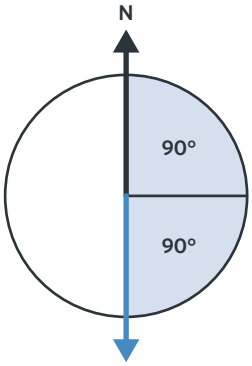
SOLUTION

a) Draw a bearing of 190° on the following diagram.

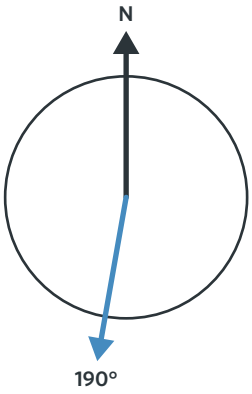
Step 1 Break down the angle into 90 degree parts.

$190° = 90° + 90° + 10°$

Step 2 There are two 90 degree segments, so move two quarter circles clockwise from north.



Step 3 Move clockwise a further ten degrees, then mark this point.



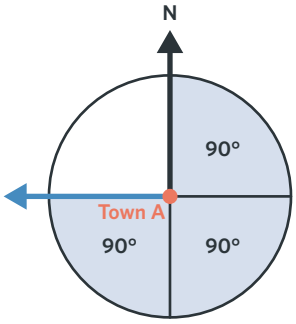
b) Draw a diagram of towns A and B, if town B is on a bearing of 340° from town A.

Step 1 Town B is being measured relative to town A, so town A is the reference point. Draw a north arrow on town A.

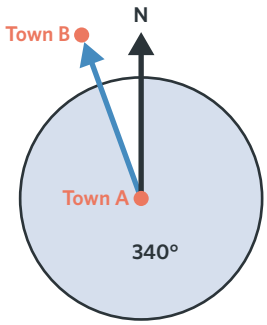
Step 2 Break the angle into 90 degree parts.

$340° = 90° + 90° + 90° + 70°$

Step 3 There are three 90 degree segments, so move three quarter-circles clockwise from north.



Step 4 Move clockwise 70 degrees further, then mark town B.



3. Applications of bearings

Bearings can be used with distances to work out precise locations of different objects. Most applications will involve trigonometry.

WORKED EXAMPLE 9 *(2 marks)*

The bearing of Sydney from Melbourne is 053.92°, and the bearing of Adelaide from Melbourne is 297.48°. Assume the earth is flat for these questions.

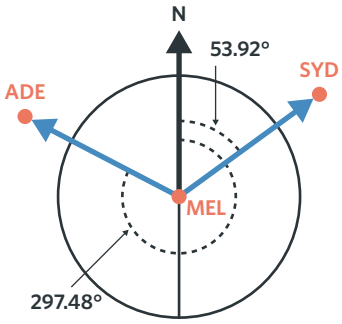
a) What is the angle Adelaide-Melbourne-Sydney?

b) If Sydney is 420 km north of Melbourne, what is the distance between Sydney and Melbourne, to the nearest kilometre?

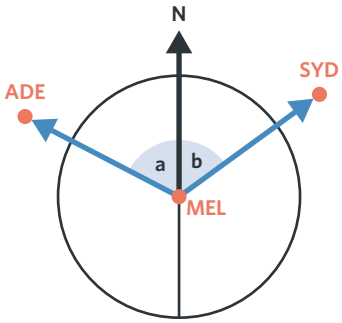
SOLUTION

a) What is the angle Adelaide-Melbourne-Sydney?

Step 1 Draw a diagram of the cities.



Step 2 Identify the desired angle, split it into segments, and calculate the segments.



$a = 360 - 297.48$

$= 62.52°$

$b = 53.92°$

Step 3 Add the segments together.

$\text{ADE-MEL-SYD} = a + b$

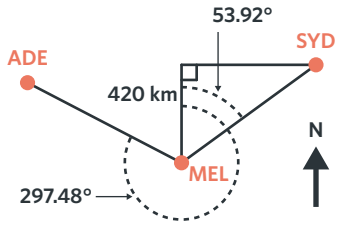
$= 62.52 + 53.92$

$= 116.44°$

The angle Adelaide-Melbourne-Sydney is 116.44°.

b) If Sydney is 420 km north of Melbourne, what is the distance between Sydney and Melbourne, to the nearest kilometre?

Step 1 Add the new information to the diagram, and add a horizontal line to create a right-angled triangle.



Step 2 Use trigonometry to find the missing length. Here we want to find the hypotenuse. We already know the length of the adjacent side, so we will use cosine.

$$\cos(\theta) = \frac{adjacent}{hypotenuse}$$

$$\cos(53.92) = \frac{420}{Distance\ Mel\text{-}Syd}$$

$$Distance\ Mel\text{-}Syd = \frac{420}{\cos(53.92)}$$

$$Distance\ Mel\text{-}Syd = 713 \text{ km}$$

Questions 8C *Bearings*

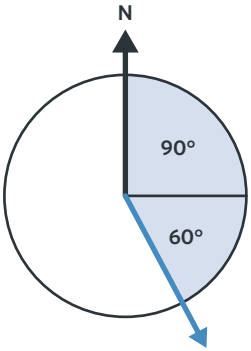
Refresher question

Q1. Label the directions E, S and W on the compass.

1. Finding bearings

Q2. Find the value of the following bearing.

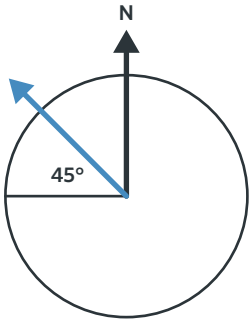
Skill



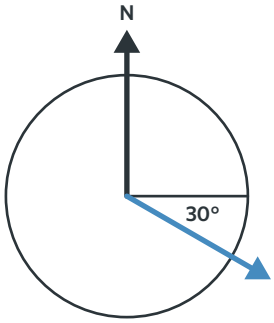
Q3. What are the following bearings?

Skill

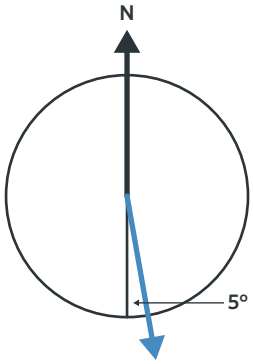
a)



b)



c)

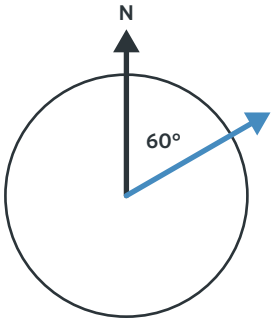


Check your understanding

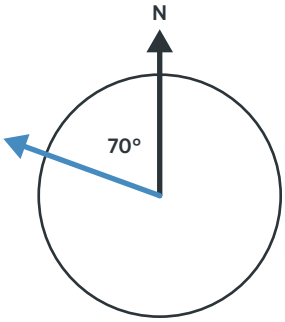
Q4. Which of the following shows a bearing of 200°?

Skill

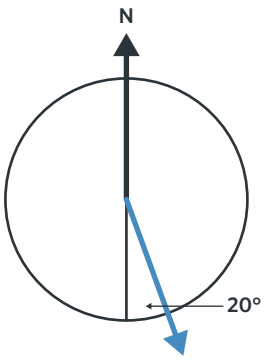
A.



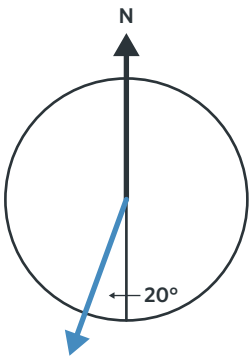
B.



C.

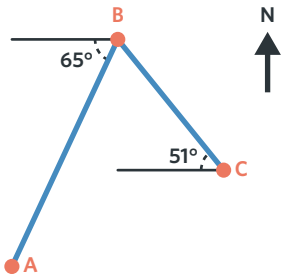


D.



Q5. Application

Three points A, B, and C are different campsites in a national park. What is the bearing of C from B?

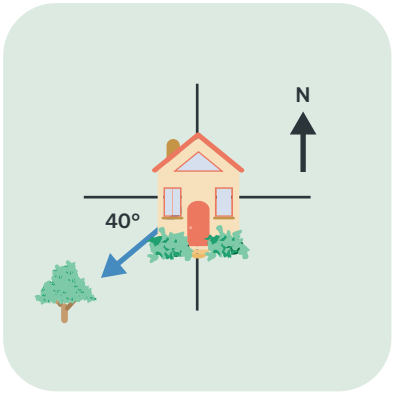


2. Plotting bearings

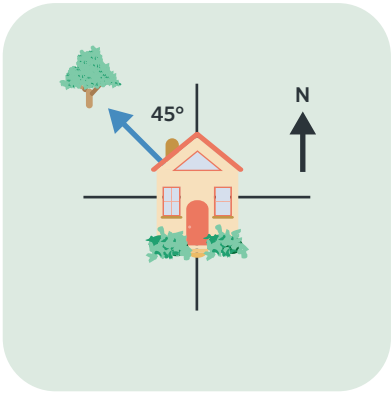
Q6. Skill

A gum tree is planted at a bearing of 315° from a house. Which of the following diagrams shows the tree in the correct position?

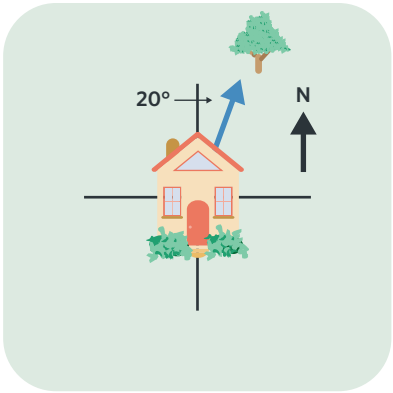
A.



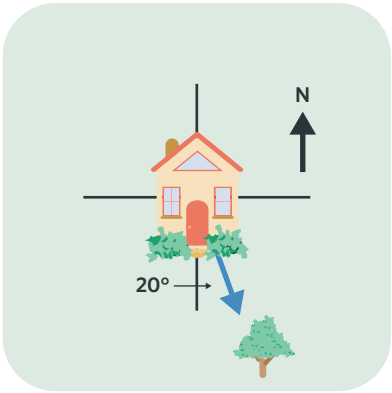
B.



C.



D.



Q7. Skill

Plot the following.

a) A bearing of 140°

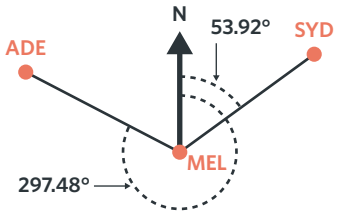
b) A bearing of 300°

c) The position of Tim, who is on a bearing of 190° from Celine

Check your understanding

Q8. Application

A new city called Edroloton is going to be built at a bearing of 35.3° from Adelaide (ADE). Plot Edroloton on the following diagram.



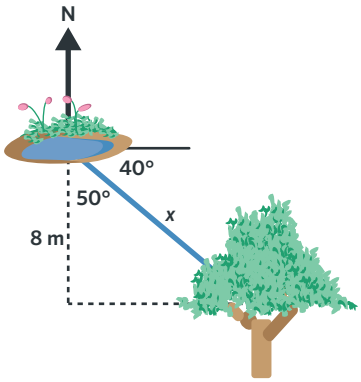
Q9. Application

Yan is trying to make the perfect backyard. She currently has a tree, a cubby house, a veggie garden, and a basketball ring, as seen here. She wants to add an inflatable pool at a bearing of 290° from the veggie garden. Plot the inflatable pool on the diagram.

3. Applications of bearings

Q10. Skill

Which of the following calculates the distance between the tree and the pond?



A

$$\cos(\theta) = \frac{adjacent}{hypotenuse}$$

$$\cos(50°) = \frac{8}{x}$$

$$x = \frac{8}{\cos(50°)}$$

$$x = 12.45 \text{ m}$$

B

$$\cos(\theta) = \frac{adjacent}{hypotenuse}$$

$$\cos(40°) = \frac{8}{x}$$

$$x = \frac{8}{\cos(40°)}$$

$$x = 10.44 \text{ m}$$

C

$$\sin(\theta) = \frac{opposite}{hypotenuse}$$

$$\sin(45°) = \frac{8}{x}$$

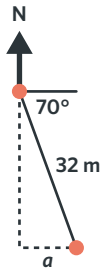
$$x = \frac{8}{\sin(45°)}$$

$$x = 11.31 \text{ m}$$

Q11. Calculate the distances a, b, and c in the following diagrams. Round your answers to two decimal places.

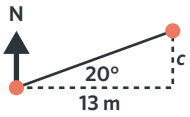
Skill

a)



b)

c)



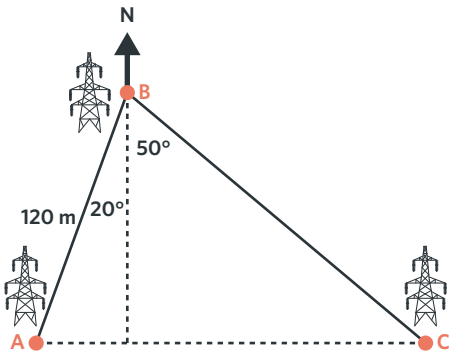
Check your understanding

Q12. Riders on a cycling route travel from point A to point B on a bearing of 180° for 13 kilometres, then turn and travel to point C on a bearing of 270° for 6 kilometres. Draw this scenario to find the bearing of C from A, correct to two decimal places.

Application

Q13. Three transmission towers A, B, and C are situated as shown below. C is directly east of A. What is the distance between A and C, rounded to the nearest metre?

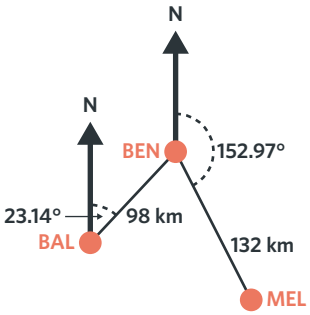
Application



Joining it all together

Q14. The distances between Ballarat and Bendigo, and Bendigo and Melbourne, are shown here on the diagram. For the following questions give all answers to two decimal places.

Application
4 marks



a) What is the angle Ballarat-Bendigo-Melbourne?

b) What is the bearing of Bendigo from Melbourne?

c) How many kilometers is Bendigo North of Melbourne?

d) If a new town called Mathswick is built at a bearing of 40° from Melbourne, what will the angle Bendigo-Melbourne-Mathswick be?

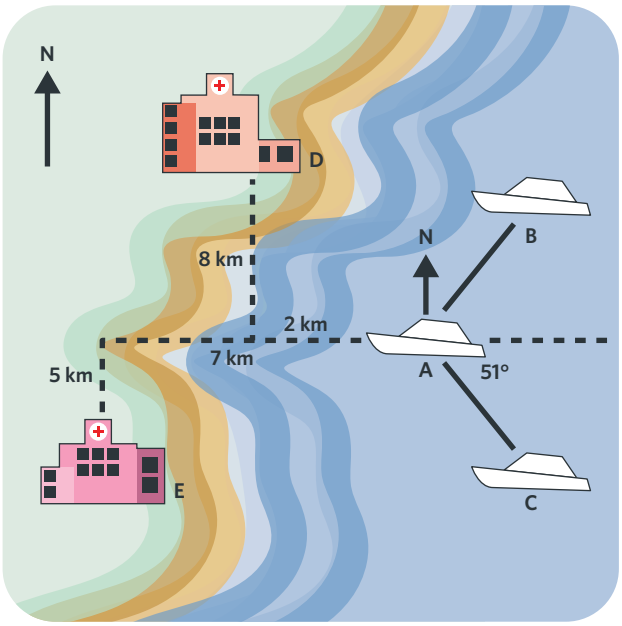
Q15. Three ships are sailing in the sea, as shown.

Application
3 marks

a) Ship B is 2.5 km east of ship A, and 4 km north of ship A. Show that angle BAC, rounded to the nearest degree, is 109°.

b) Ship A hits an iceberg. If ship C is 3.5 km south of ship A, how far away is the closest ship to ship A? Round your answer to two decimal places.

c) Find bearing of the closest hospital from ship A, and the distance between them. Round your answers to two decimal places.



VCAA question

Q16. Two hikers, Anton and Beth, walk in different directions from the same camp.

1 mark

Beth walks for 12 km on a bearing of 135° to a picnic ground.

Anton walks for 6 km on a bearing of 045° to a lookout tower.

On what bearing (to the nearest degree) should Anton walk from the lookout tower to meet Beth at the picnic ground?

A. 063° **B.** 108° **C.** 153° **D.** 162° **E.** 180°

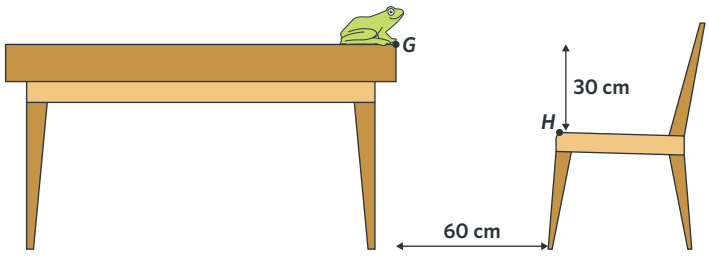
VCAA 2008 Exam 1 Module 2: Geometry and trigonometry. Q9

Questions from multiple lessons

Q17.

Difficulty:

1 mark



A frog wants to jump from point *G* on a table to point *H* on a chair.

The table is 30 cm taller than the chair.

The horizontal distance between the table and the chair is 60 cm.

The angle of depression of point *H* from point *G*, is closest to

A. 63°

B. 27°

C. 65°

D. 80°

E. 25°

VCAA 2014 Exam 1 Module 2: Geometry and trigonometry Q6 – Adapted

Q18.

Difficulty:

Year 10
1 mark

The variables *typing speed* (in words per minute) and *language* (English, French, German) are

A. both numerical.

B. both categorical.

C. an ordinal variable and a nominal variable respectively.

D. a numerical variable and a nominal variable respectively.

E. a numerical variable and an ordinal variable respectively.

VCAA 2019NH Exam 1 Data analysis Q8 – Adapted

Q19.

Difficulty:

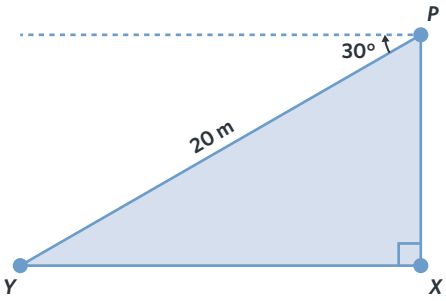
Year 10
2 marks

April attempts archery for the first time by firing arrows from point *P*.

She misfires her first attempt. The arrow travels directly downwards, landing at point *X*.

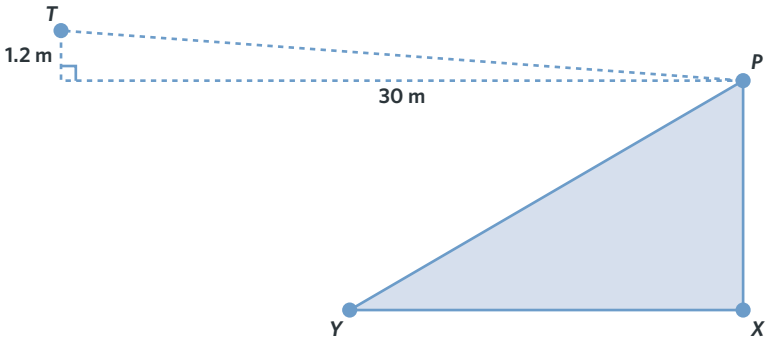
The second arrow travels for 20 m with an angle of depression of 30°, landing at point *Y*.

The diagram to the right shows the positions of the two arrows after they have landed in relation to the point in which April fired the arrows from.



a) How far apart, in metres, are the two arrows? Round your answer to two decimal places. (1 mark)

The target, *T*, is 30 m away horizontally and 1.2 m vertically from point *P*.



b) What is the angle of elevation from *P* to the target? Round your answer to the nearest degree. (1 mark)

VCAA 2016 Exam 2 Module 3: Geometry and measurement Q2 a,b – Adapted

LESSON 8D

The sine rule

The key skills you will learn in this lesson are:

1. Finding an unknown side
2. Finding an unknown angle
3. The ambiguous case of the sine rule

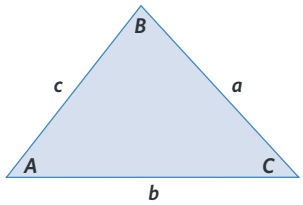
VCAA key knowledge point:

"the sine rule (including the ambiguous case) and cosine rule (as a generalisation of Pythagoras' theorem) and their application to solving practical problems requiring the solution of non-right angled triangles"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21;

Below is the standard way of labelling a non-right-angled triangle. Side lengths are denoted by the lowercase letters a, b, and c while angles are represented by the uppercase letters A, B, and C.

The sides are interchangeable, as long as the angle directly opposite each side is denoted by the same letter.



The sine and cosine rules can be used to find both unknown sides and unknown angles in non-right-angled triangles. This lesson will focus on the sine rule.

1. Finding an unknown side

The sine rule states that in the triangle ABC:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

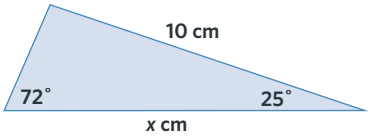
In simpler terms, it states that the ratio between a side and the sine value of its opposite angle will be equal for all three sides of a triangle.

The information needed to find an unknown side length is:

- the value of its opposite angle.
- one other side length and its opposite angle.

WORKED EXAMPLE 10 *(2 marks)*

Find the value of x in this triangle, correct to two decimal places.



SOLUTION

Step 1 Calculate the magnitude of the angle opposite x.

The interior angles of a triangle add up to 180°.

$x = 180° - 72° - 25° = 83°$

✓ *1 mark for finding the first angle*

Step 2 Substitute the values into the sine rule.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

Side x is opposite the angle with a magnitude of 83°. Let $a = x$ and $A = 83°$.

The side of length 10 cm is directly opposite the angle of magnitude 72°. Let $b = 10$ and $B = 72°$.

$$\frac{x}{\sin(83°)} = \frac{10}{\sin(72°)}$$

Step 3 Solve the equation for x.

BY HAND

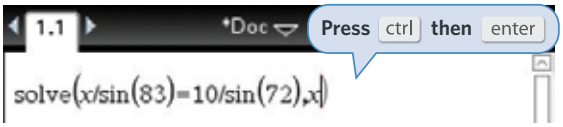
$$\frac{x}{\sin(83°)} = \frac{10}{\sin(72°)}$$

$$x = \sin(83°) \times \frac{10}{\sin(72°)}$$

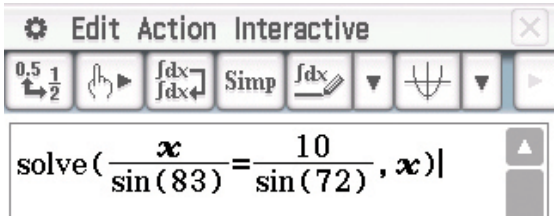
$$x = 10.4362\ldots$$

$\therefore$ The value of x is 10.44 cm.

TI-NSPIRE



CASIO CLASSPAD



✓ *1 mark for finding the value of x*

2. Finding an unknown angle

In order to find an unknown angle, the sine rule can be 'flipped' so the equation will be easier to solve:

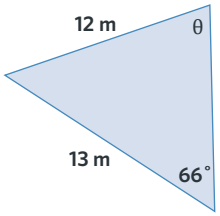
$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

The information needed to find an unknown angle is:

- the length of the side directly opposite the angle.
- one other side length and its opposite angle.

WORKED EXAMPLE 11 *(1 mark)*

Find the angle θ in this triangle correct to two decimal places.



SOLUTION

Step 1 Substitute the values into the 'flipped' sine rule.

The side of length 13 m is opposite the angle θ. Let $a = 13$ and $A = \theta$.

The side of length 12 m is opposite the angle of magnitude 66°. Let $b = 12$ and $B = 66°$.

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

$$\frac{\sin(\theta)}{13} = \frac{\sin(66°)}{12}$$

Step 2 Solve the equation for θ.

Here we solve by hand, but you can also use the solve function on CAS.

$$\frac{\sin(\theta)}{13} = \frac{\sin(66°)}{12}$$

$$\sin(\theta) = 13 \times \frac{\sin(66°)}{12}$$

$$\theta = \sin^{-1}\left(13 \times \frac{\sin(66°)}{12}\right)$$

$$\theta = 81.7591\ldots°$$

$\therefore$ The value of θ is 81.76°.

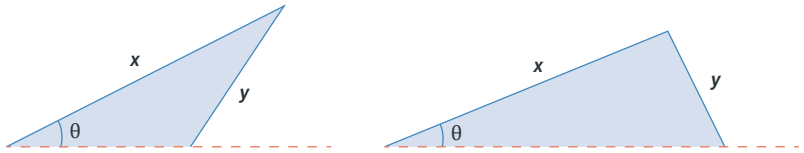
3. The ambiguous case of the sine rule

In some cases, there can be two possible triangles that both satisfy the given information.
When two sides and one non-included angle are known (SSA), and the following conditions for a non-right angled triangle are met, there may be two possible triangles:

- The known angle is acute.
- The shorter of the two known sides is opposite the known angle.

Consider a triangle in which two side lengths (x and y) and one angle that is NOT the included angle (θ) are known. The shortest length is y and it is opposite θ.

There are two possible triangles that could be drawn, as shown below.

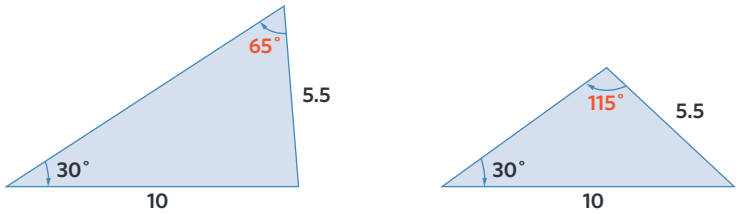


Example

A triangle has side lengths 10, 5.5, and unknown.

The same triangle has an angle of 30° between the side of length 10 and the unknown side.

There are two possible triangles. One with an acute angle (65°) and one with an obtuse angle (115°) between the short side and the unknown side.



NOTE: The angles between the short side and the unknown side in the two triangles sum to 180°:

$65° + 115° = 180°$

On a calculator

When calculating the angle between the shortest side and the unknown side in the ambiguous case, the calculator will only give one angle, the acute angle. To find the obtuse angle in the second triangle, subtract the acute angle from 180°.

NOTE: If the obtuse angle plus the original angle sum to more than 180° the second triangle (with the obtuse angle) does not exist.

Example

From the triangles in the previous example:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

$$\frac{\sin(x°)}{10} = \frac{\sin(30°)}{5.5}$$

Gives $x° = \sin^{-1}\left(10 \times \frac{\sin(30°)}{5.5}\right) = 65°$

To find the angle in the second triangle:

$180° - 65° = 115°$

NOTE: $\sin(65°) = 0.906$ and $\sin(115°) = 0.906$

WORKED EXAMPLE 12 *(4 marks)*

Consider the following information about triangle *ABC*:

- Side *AB* has a length of 9 centimetres.
- Side *BC* has a length of 4 centimetres.
- Angle *BAC* has a magnitude of 22°.

a) Draw all possible triangles that satisfy this information. *(2 marks)*

b) Find all possible values of the angle *ACB* correct to two decimal places. *(2 marks)*

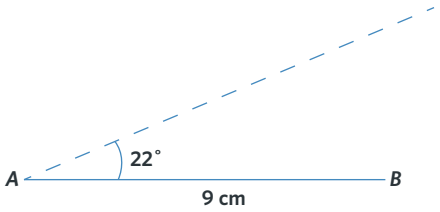
SOLUTION

a) Draw all possible triangles that satisfy this information.

Step 1 Choose a side and angle to draw first.

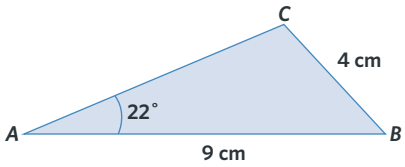
Always start by drawing the longer known side and the known angle.

Hence, start by drawing side *AB* and angle *BAC*.



Point *C* will lie somewhere along the dotted line.

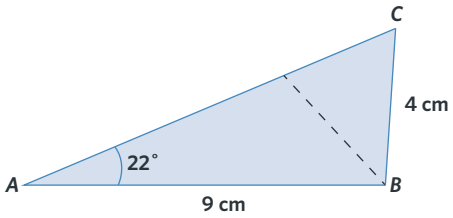
Step 2 Draw side *BC* so that the angle *ABC* is acute.



✓ *1 mark for drawing the first triangle*

Step 3 Draw side *BC* so that the angle *ABC* is obtuse.

In the second possible triangle, *C* is further up the dotted line so that angle *ABC* is obtuse.

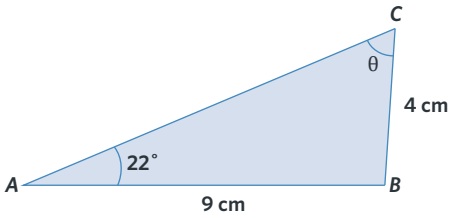


✓ *1 mark for drawing the second triangle*

b) Find all possible values of the angle *ACB*, correct to two decimal places.

Step 1 Substitute the values into the 'flipped' sine rule.

Since there are two possible triangles, there will be two possible values of the angle *ACB*. The calculator will give the value of the acute angle.



$$\frac{\sin(\theta)}{9} = \frac{\sin(22°)}{4}$$

$$\theta = \sin^{-1}\left(9 \times \frac{\sin(22°)}{4}\right)$$

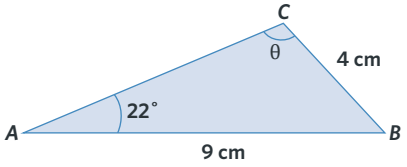
$$\theta = 57.4438\ldots$$

The first possible value is 57.44°.

✓ *1 mark for finding the first angle*

Step 2 Find the obtuse angle.

As we can see in the first triangle from part a), the angle *ACB* can also be obtuse.



To find the obtuse angle, subtract the value of θ from 180°.

$180 - 57.4438\ldots = 122.5561\ldots$

✓ *1 mark for finding the second angle*

∴ The two possible values are 57.44° and 122.56°.

Questions 8D *The sine rule*

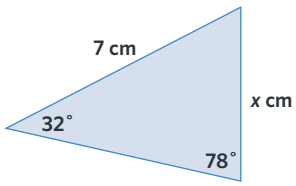
Refresher question

Q1. Calculate the value of $\sin^{-1}(0.75)$ correct to two decimal places.

1. Finding an unknown side

Q2. Skill

Consider this triangle.



a) Fill in the blank spaces in the sine rule below.

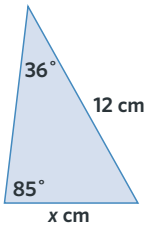
$$\frac{x}{\sin\square^\circ} = \frac{\square}{\sin\square^\circ}$$

b) Find the value of x to two decimal places. Include units.

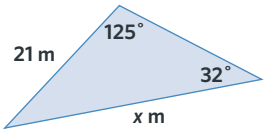
Q3. Skill

Use the sine rule to find the value of x in each of the following triangles, correct to two decimal places.

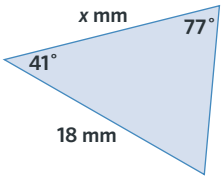
a)



b)

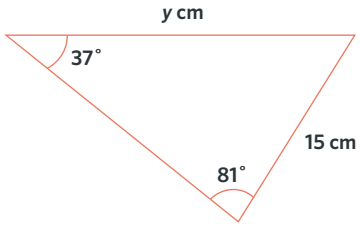


c)



Q4. Skill

Which of the following equations can be used to find the value of y in this triangle?



A. $\frac{y}{81^\circ} = \frac{5}{37^\circ}$

B. $\frac{y}{\sin(37^\circ)} = \frac{5}{\sin(81^\circ)}$

C. $y = 15 \times \frac{\sin(37^\circ)}{\sin(81^\circ)}$

D. $y = \frac{15 \times \sin(81^\circ)}{\sin(37^\circ)}$

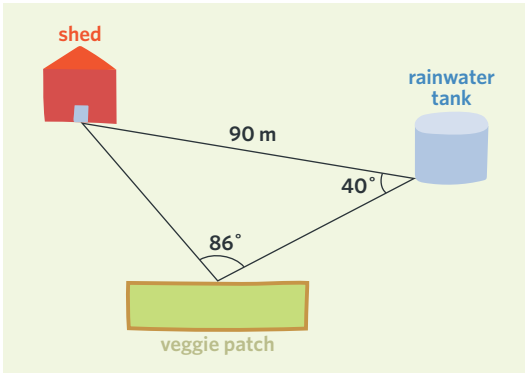
E. $\frac{y}{\sin(81^\circ)} = \frac{\sin(37^\circ)}{15}$

Check your understanding

Q5. Application

Charlie took some measurements of his garden at his farm and drew them on a map.

Show that the distance, correct to the nearest metre, between the shed and the veggie patch is 58 metres.



Q6. Application

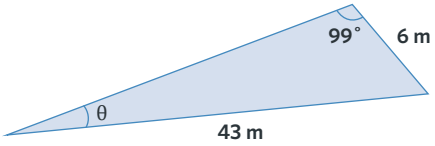
During a whale-watching expedition, passengers on boat A spot a humpback whale at a bearing of 137° from them. A second boat, boat B, is 400 metres due south of boat A. Passengers on boat B can see the whale on a bearing of 19°.

What is the direct distance between the whale and each of the boats? Give your answers in metres correct to the nearest centimetre.

2. Finding an unknown angle

Q7. Skill

Consider this triangle.



a) Fill in the blank spaces in the following working:

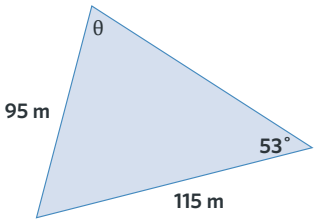
$$\theta = \sin^{-1}\left(\square \times \frac{\sin\square^\circ}{\square}\right)$$

b) Find the value of θ to one decimal place.

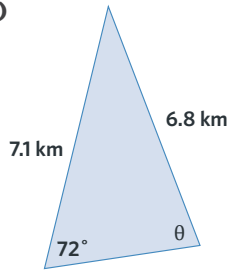
Q8. Skill

Use the sine rule to find the value of the unknown angle θ in each of the triangles below, correct to two decimal places.

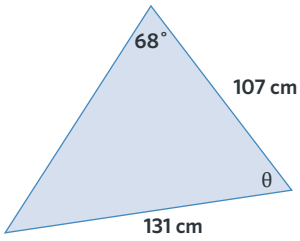
a)



b)

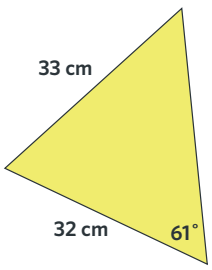


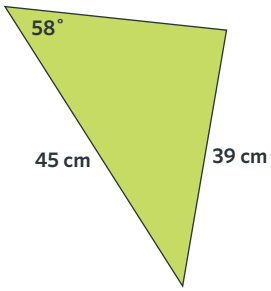
c)



Q9. Skill

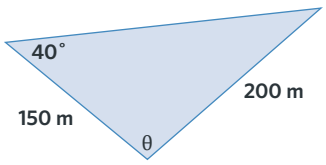
Find all the missing angles in the triangles below. Give all values correct to two decimal places.





Check your understanding

Q10. Skill



Show that the value of θ can be calculated using the equation $\theta = 140° - \sin^{-1}\left(\frac{3 \times \sin(40°)}{4}\right)$.

3. The ambiguous case of the sine rule

Q11. Skill

In an ambiguous case of the sine rule, the sine rule has been used to find the magnitude of the angle between a the short known angle and the unknown angle. The angle is called *ABC*.

The calculator returns a value of 61°. What is the other possible magnitude of the angle *ABC*?

Q12. Skill

In the triangle *ABC*, side *AC* has length of 9 cm, side *BC* has a length of 12 cm, and angle *ABC* has a magnitude of 32°.

a) Draw the two possible triangles that satisfy this information.

b) Find the two possible values of the angle *BAC*, each to two decimal places.

c) Find the two possible values of the angle *ACB*, each to two decimal places.

Check your understanding

Q13. Skill

Triangle *ABC* satisfies the following properties:

- Side *AB* has a length of 20 cm.
- Angle *ACB* has a magnitude of 40°.
- Side *AC* has a length of 30 cm.

Calculate all possible lengths of the side *BC*, correct to one decimal place.

Q14. Skill

Consider the triangle *ABC* that has:

- side *AB* of length 13 cm.
- side *AC* of length 15 cm.
- angle *ACB* of magnitude 44°.

Which one of the following angles, correct to the nearest degree, could **not** be another angle in triangle *ABC*?

A. 9°

B. 53°

C. 83°

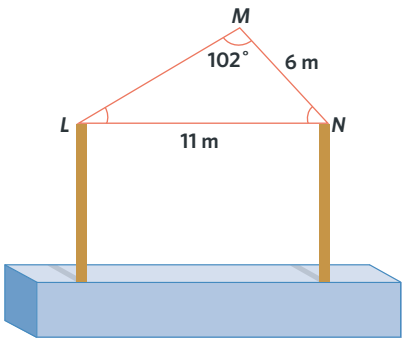
D. 97°

E. 127°

Joining it all together

Q15. Application 4 marks

The design for a new shade cloth being built for the local skate park is shown on the right, but some information is missing.



a) From the given information, which angle should be calculated first using the sine rule?

b) Calculate the magnitude of the angle chosen in a), correct to two decimal places.

c) Find the magnitude of the last unknown angle, correct to two decimal places.

d) Calculate the length of the remaining unknown side correct to two decimal places.

Q16. Application 4 marks

In an orienteering course, Point A is 200 metres directly south of point B. A third point, point C, is at a bearing of 36° from point A and 68° from point B.

a) Using the sine rule, show that the direct distance between points B and C is 221.84 metres, correct to the nearest centimetre. (2 marks)

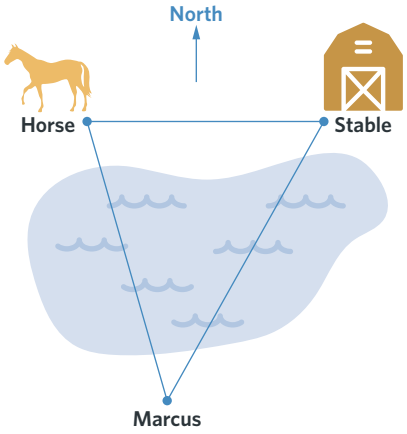
b) A fourth point, point D, is directly north of point C and at a bearing of 21° from point B.

Find the direct distance between points C and D, correct to the nearest metre. (2 marks)

VCAA question

Q17. 1 mark

Marcus is on the opposite side of a large lake from a horse and its stable. The stable is 150 m directly east of the horse. Marcus is on a bearing of 170° from the horse and on a bearing of 205° from the stable.



The straight-line distance, in metres, between Marcus and the horse is closest to:

A. 45

B. 61

C. 95

D. 192

E. 237

VCAA 2016 Exam 1 Module 3: Geometry and Measurement. Q6

Questions from multiple lessons

Q18. Difficulty:

1 mark

A lost dog runs directly west from its kennel and then directly south. It was found on a bearing of 226° from its kennel, exactly 4 km away. The distance, in km, that the dog travelled directly west is closest to

A. 2.9 km

B. 3.0 km

C. 1.2 km

D. 6.7 km

E. 2.6 km

VCAA 2018NH Exam 1 Module 3: Geometry and measurement Q2 – Adapted

Q19.

Difficulty:

Year 10
1 mark

The following stem plot displays the number of teabags consumed each day in the Edrolo office over a period of 31 days.

Key: 0 | 1 = 1 $n = 31$

0	1 2
0	5 6 8 8 9
1	0 1 1 3 4 4
1	6 7 8 9 9 9
2	2 2 3 4 4
2	5 5 6 7
3	0 1
3	6

What is the median number of teabags consumed?

A. 17.5

B. 18

C. 18.5

D. 19

E. 22

VCAA 2018NH Exam 1 Data analysis Q1 – Adapted

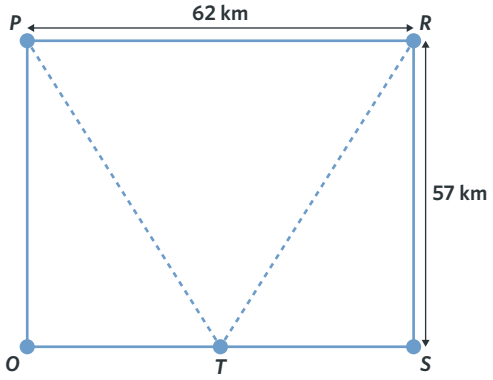
Q20.

Difficulty:

Year 10
2 marks

A dairy farm is trying to grow different types of grass to determine which one yields the best milk from their cows. They have decided to divide their rectangular farm into triangles.

On their plot, *OPRS*, they mark point *T* directly halfway between *O* and *S*. They then mark line segments between *PT* and *RT* as shown below.



a) Find the length, in km, of *ST* (1 mark)

b) Calculate the angle *SRT*.
Write your angle in degrees, correct to one decimal place. (1 mark)

VCAA 2007 Exam 2 Module 2: Geometry and trigonometry Q1 a,b – Adapted

LESSON 8E

The cosine rule

The key skills you will learn in this lesson are:

1. Finding an unknown side length
2. Finding an unknown angle

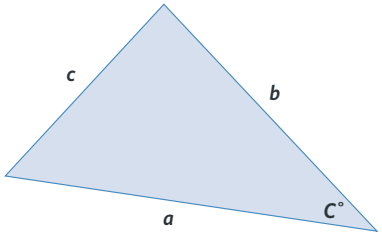
VCAA key knowledge point:

"the sine rule (including the ambiguous case) and cosine rule (as a generalisation of Pythagoras' theorem) and their application to solving practical problems requiring the solution of non-right angled triangles"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21;

1. Finding an unknown side length

To find the unknown side length c in a triangle with side lengths a and b and internal angle C as shown below, the cosine rule can be used.



The cosine rule states that:

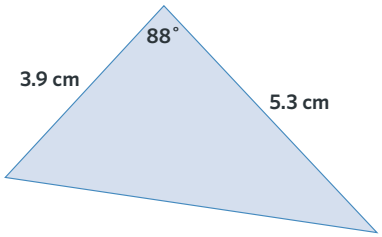
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

where C is the angle opposite the unknown side and a and b are the two known side lengths. Sides a and b are interchangeable.

The cosine rule can be used to find the missing side length in a triangle when two sides and the included angle between them are known.

WORKED EXAMPLE 13 *(1 mark)*

Find the length of the unknown side in the triangle below, correct to one decimal place.



SOLUTION

Step 1 Identify the known values.

$a = 3.9$

$b = 5.3$

$C = 88°$

NOTE: It does not matter which side length is assigned a or b.

Step 2 Substitute the values into the formula.

$c^2 = a^2 + b^2 - 2ab\cos(C)$

$c^2 = 3.9^2 + 5.3^2 - 2 \times 3.9 \times 5.3 \times \cos(88°)$

Step 3 Solve for c.

$c^2 = 41.857...$

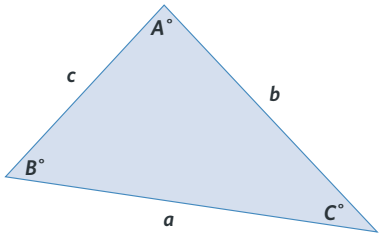
$c = \sqrt{41.857...} = 6.5$

The length of the unknown side is 6.5 cm.

2. Finding an unknown angle

The cosine rule can also be used to find an unknown angle in a non–right angled triangle when all three side lengths are known.

To find the unknown angle C in a triangle with side lengths a, b and c as shown below,



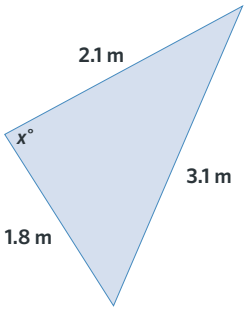
the cosine rule can be rearranged to:

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

where c is the side opposite the unknown angle and a and b are the two known side lengths. Sides a and b are interchangeable.

WORKED EXAMPLE 14 *(1 mark)*

Find the value of x in the triangle below, correct to the nearest degree.



SOLUTION

Step 1 Identify what values are present.

$a = 1.8$

$b = 2.1$

$c = 3.1$

$C = x$

NOTE: It does not matter which side length is assigned a or b.

NOTE: c is the side opposite the unknown angle.

Step 2 Substitute the values into the formula.

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(x) = \frac{1\,8^2 + 2\,1^2 - 3\,1^2}{2 \times 1.8 \times 2.1}$$

Step 3 Solve for x.

$\cos(x) = -0.259\ldots$

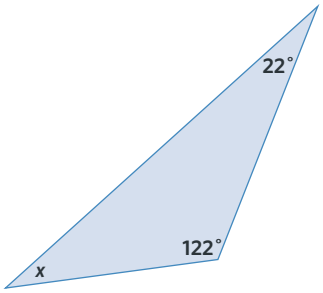
$x = \cos^{-1}(-0.259\ldots) = 105°$

The value of x is 105°.

Questions 8E *The cosine rule*

Refresher question

Q1. What is the value of x in the diagram below?



1. Finding an unknown side length

Q2. Skill

A triangle has side lengths of 3 cm and 5 cm, and an angle of 80° between the two sides.

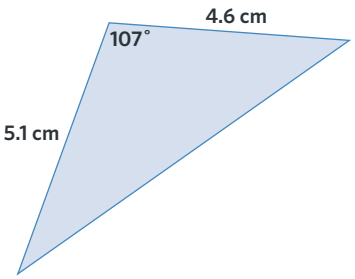
Fill in the blanks and find the length of the missing side, correct to two decimal places.

$$c^2 = 3^2 + 5^2 - 2 \times \square \times 5 \times \cos\left(\square\right)$$

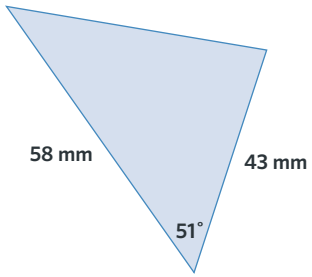
Q3. Skill

Find the length of the unknown side in the triangles below, correct to two decimal places.

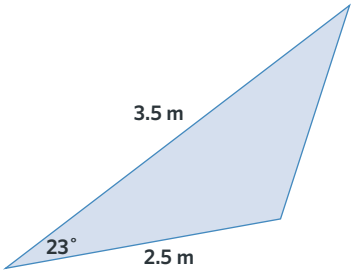
a)



b)



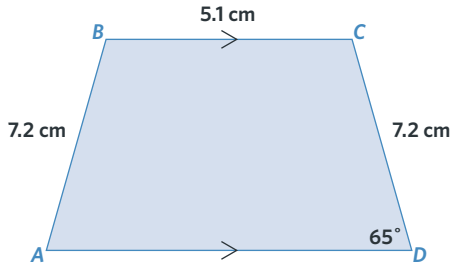
c)



Check your understanding

Q4. Skill

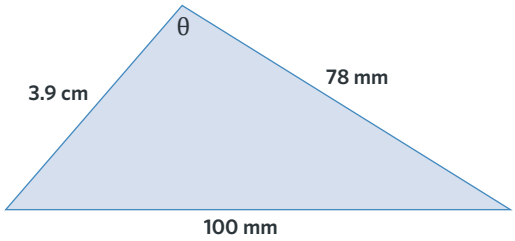
What is the length of the diagonal AC in the trapezium below, correct to one decimal place?



2. Finding an unknown angle

Q5. Skill

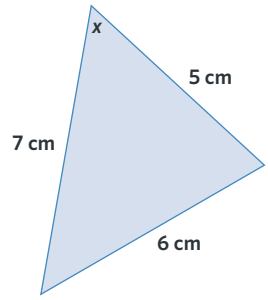
To find θ in the triangle below, what should the value of c be in the formula $\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$?



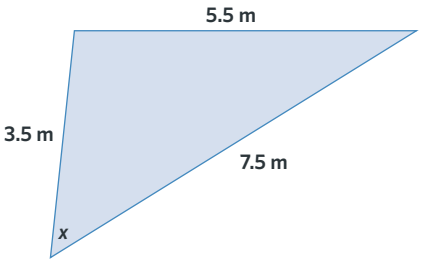
Q6. Skill

Find the value of x in the triangles below, correct to two decimal places.

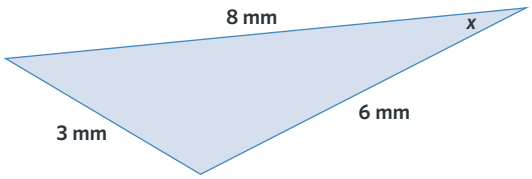
a)



b)



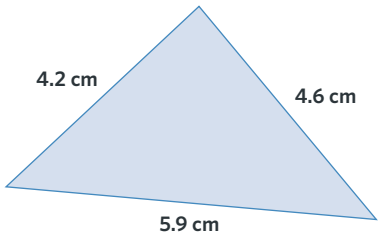
c)



Check your understanding

Q7. Skill

What is the value of the smallest angle in the triangle on the right, correct to two decimal places?



Q8. Skill

A student finds one of the angles in a non–right angled triangle to be 105°. The length of the side opposite to the angle is 5.3 cm and the length of the other two sides are x cm and $2x$ cm.

What is the value of x, correct to two decimal places?

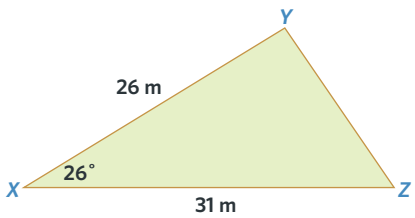
Joining it all together

Q9. Application 2 marks

As part of your studies of the cosine rule, your maths class is designing a garden bed for the school yard.

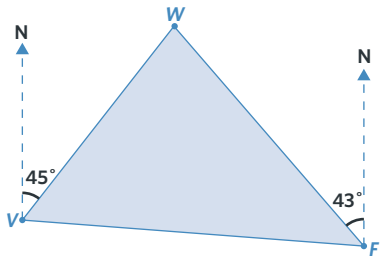
a) What is the distance between points Y and Z? Give your answer to one decimal place.

b) Using you rounded answer from a), find the value of the angle $\angle XZY$. Give your answer to the nearest degree.



Q10. Application 2 marks

Two friends, Victoria (V) and Felicity (F), are trying to find each other in the bush. The diagram below shows their positions relative to a waterfall, W.



If Victoria is 600m away from the waterfall and Felicity is 0.75 km away from the waterfall, how far away are they from each other, correct to the nearest metre?

Q11. Application 2 marks

Two ships can be seen from a lighthouse. Ship A is on a bearing of 048° and is 2.3 km away. Ship B is on a bearing of 326° and is 3.2 km away.

a) What is the distance between the two ships, correct to two decimal places?

b) What is the bearing of Ship A from Ship B, correct to the nearest degree?

Q12. Application 2 marks

Bailey goes for a run around her local park. She first jogs on a bearing of 063° for 2.8 km. She then turns 87° to her right and runs at a constant speed of 2.5 m/s for 13 minutes.

a) What is the shortest distance back to the starting point, in kilometres, correct to one decimal place?

b) What is the bearing from the starting point to Bailey, correct to the nearest degree?

VCAA question

Q13. 1 mark

The points M, N and P form the vertices of a triangular course for a yacht race.

$MN = MP = 4$ km

The bearing of N from M is 070°.

The bearing of P from M is 180°.

Three people perform different calculation to determine the length of NP in kilometres.

- **Graeme:** $NP = \sqrt{16 + 16 - 2 \times 4 \times 4 \times \cos(110°)}$
- **Shelley:** $NP = 2 \times 4 \times \cos(35°)$
- **Tran:** $NP = \dfrac{4 \times \sin(110°)}{\sin(35°)}$

The correct length of NP would be found by:

A. Graeme only

B. Tran only

C. Graeme and Shelley only

D. Graeme and Tran only

E. Graeme, Shelley and Tran.

VCAA 2007 Exam 1 Module 2: Geometry and Trigonometry. Q9

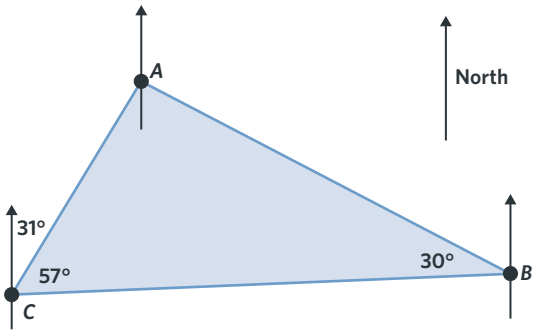
Questions from multiple lessons

Q14.

Difficulty:

1 mark

The course for an orienteering competition is triangular in shape and is marked by three flags, A, B and C.



The bearing of A from B is

A. 298°

B. 334°

C. 268°

D. 149°

E. 096°

VCAA 2018 Exam 1 Module 3: Geometry and measurement Q4 – Adapted

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Q15.

Difficulty:

1 mark

For the past eight years, the value of a vase has increased by 5% each year.

This year, the vase's value is $165 000.

Eight years ago, its value was closest to

A. $109 464

B. $108 257

C. $111 678

D. $130 253

E. $248 711

VCAA 2013 Exam 1 Module 4: Business-related mathematics Q6 – Adapted

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Q16.

Difficulty:

2 marks

August trains at a local football field. Her house is located 400 m north and 357 m east of the field.

a) What distance will August have to walk if she was to walk in a straight line from her home to the football field? Round your answer to the nearest metre. (1 mark)

b) What is the three-figure bearing of the football field from August's home? Round your answer to the nearest degree. (1 mark)

VCAA 2017 Exam 2 Module 3: Geometry and measurement Q2b – Adapted

LESSON 8F

Sufficient information

The key skills you will learn in this lesson are:

1. Sufficient information in a right-angled triangle
2. Sufficient information in a non-right-angled triangle

VCAA key knowledge point:

"sets of sufficient information to determine a triangle"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

When given sufficient information about a triangle, it means that there is enough information to find all the side lengths and angles of that triangle.

1. Sufficient information in a right-angled triangle

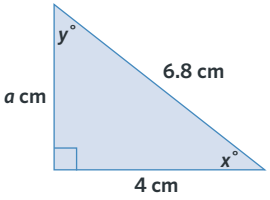
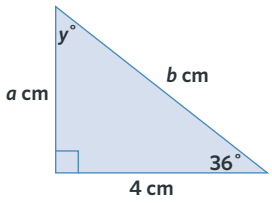
To have sufficient information about a right-angled triangle, the following information is needed:

- The length and position of two sides

  or

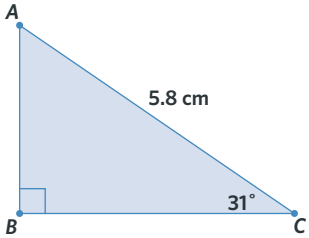
- The length and location of one side and the size and location of one angle (other than the right angle).

Given the above information, all sides with unknown lengths, and all angles in a right-angled triangle can be found, according to this table:

Information given	Example	How to find missing information	Setting up the first line of working
Two side lengths		Side length: Pythagoras Angle: Trigonometry	Side length: $a^2 + 4^2 = 6.8^2$ Angle: $\cos(x°) = \frac{4}{6.8}$ $\sin(y°) = \frac{4}{6.8}$
One side length and one angle		Side length: Trigonometry Angle: Angle sum of a triangle	Side length: $\cos(36°) = \frac{4}{b}$ $\tan(36°) = \frac{a}{4}$ Angle: $y = 180 - 90 - 36$

WORKED EXAMPLE 15 *(3 marks)*

Find and label all missing side lengths and angles in the triangles below, correct to one decimal place where appropriate.



SOLUTION

Step 1 What information do we know?

The information given is one side length and one angle.

Look at the table for how to find missing information.

Step 2 Find the length of BC using trigonometry.

$\cos(31°) = \frac{BC}{5.8}$

$BC = \cos(31°) \times 5.8$

$BC = 4.971 \ldots$

$BC = 5.0$

BC is 5.0 cm

✓ *1 mark for finding the correct length*

Step 3 Find the length of AB using trigonometry.

$\sin(31°) = \frac{AB}{5.8}$

$AB = \sin(31°) \times 5.8$

$AB = 2.987 \ldots$

$AB = 3.0$

AB is 3.0 cm

✓ *1 mark for finding the correct length*

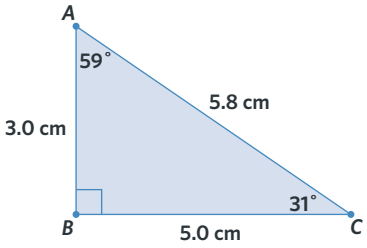
Step 4 Find the size of $\angle BAC$ using the angle sum of a triangle.

$\angle BAC = 180 - 90 - 31 = 59$

$\angle BAC = 59°$

✓ *1 mark for finding the correct angle*

Step 5 Label all side lengths and angles in the triangle.



2. Sufficient information in a non-right-angled triangle

As discussed in 8D, when two side lengths are known in a triangle, as well as the acute angle opposite the shorter known side, there are two possible triangles that could be drawn.

Therefore, to have sufficient information about a non-right-angled triangle given two side lengths and a non-included angle, either the given angle must be obtuse, or the known angle must be opposite the longer known side.

NOTE: A non-included angle is an angle that does not lie between the two given side lengths.

To have sufficient information about a non-right-angled triangle, the following information is needed:

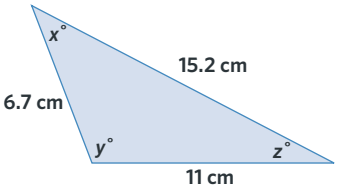
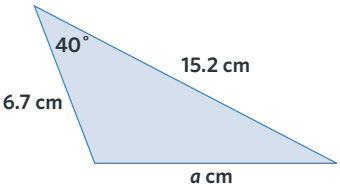
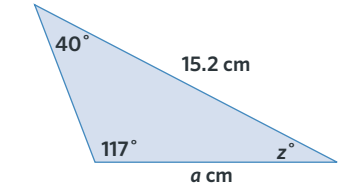
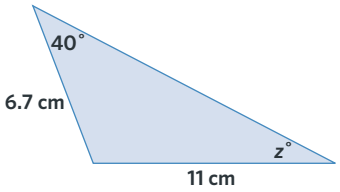
- Three side lengths

  or
- Two side lengths and an included angle

  or
- The length and location of one side and two angles

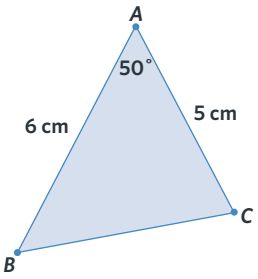
  or
- Two side lengths and a non-included angle (where the given angle is obtuse or if the known angle is opposite the longer known side).

Given the above information, all sides with unknown lengths and all angles in a triangle can be found, according to this table:

Information given	Example	How to find missing information	Setting up the first line of working
Three side lengths		Angle: Cosine rule	Angle: $\cos(x°) = \frac{6.7^2 + 15.2^2 - 11^2}{2 \times 6.7 \times 15.2}$ $\cos(y°) = \frac{6.7^2 + 11^2 - 15.2^2}{2 \times 6.7 \times 11}$ $\cos(z°) = \frac{11^2 + 15.2^2 - 6.7^2}{2 \times 11 \times 15.2}$
Two side lengths and the included angle		Side length: Cosine rule	Side length: $a^2 = 6.7^2 + 15.2^2 - 2 \times 6.7 \times 15.2 \times \cos(40°)$
One side length and two angles		Side length: Sine rule Angle: Angle sum of a triangle	Side length: $\frac{15.2}{\sin(117°)} = \frac{a}{\sin(40°)}$ Angle: $z = 180 - 117 - 40$
Two side lengths and a non-included angle (where the given angle is obtuse or the known angle is opposite the longer known side)		Angle: Sine rule	Angle: $\frac{11}{\sin(40°)} = \frac{6.7}{\sin(z°)}$

WORKED EXAMPLE 16 *(3 marks)*

Find and label all missing side lengths and angles in the triangle below, correct to one decimal place where appropriate.



SOLUTION

Step 1 What information do we know?

The information given is two side lengths and the included angle.

Look at the table for how to find missing information.

Step 2 Find the length of BC using the cosine rule.

$BC^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \times \cos(50°)$

$BC = \sqrt{6^2 + 5^2 - 2 \times 6 \times 5 \times \cos(50°)}$

$BC = 4.736...$

$BC = 4.7$

$BC = 4.7$ cm

✓ *1 mark for finding the correct length*

Step 3 Find the size of $\angle ABC$ using the sine rule.

NOTE: The cosine rule would also work.

$$\frac{4.736\ldots}{\sin(50°)} = \frac{5}{\sin(\angle ABC)}$$

$$\sin(\angle ABC) = \frac{5 \times \sin(50°)}{4.736\ldots}$$

$$\angle ABC = \sin - 1\left(\frac{5 \times \sin(50°)}{4.736\ldots}\right)$$

$$\angle ABC = 53.968\ldots$$

$$\angle ABC = 54.0°$$

✓ *1 mark for finding the correct angle*

Step 4 Find the size of $\angle ACB$ using the angle sum of a triangle.

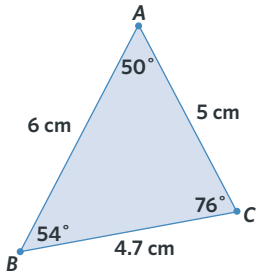
NOTE: The sine rule and the cosine rule would also work.

$$\angle BAC = 180 - 50 - 54 = 76$$

$$\angle ACB = 76°$$

✓ *1 mark for finding the correct angle*

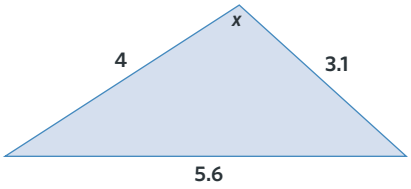
Step 5 Label all side lengths and angles in the triangle.



Questions 8F *Sufficient information*

Refresher question

Q1. Which of the following statements are true for this triangle? There may be more than one true statement.



A. Three side lengths are known.

B. One angle is known.

C. The included angle between 4 and 3.1 is known.

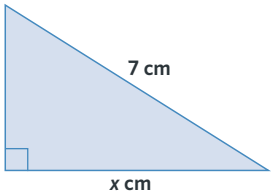
D. This triangle has sufficient information to find the missing information.

E. One side length and two angles are known.

1. Sufficient information in a right-angled triangle

Q2. Skill

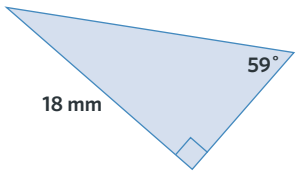
What other information is needed to find the value of x in this triangle?



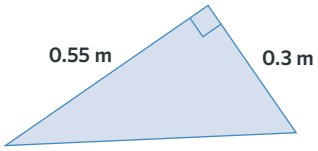
Q3. Find and label all missing side lengths and angles in the triangles below, correct to one decimal place where appropriate.

Skill

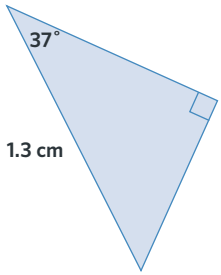
a)



b)



c)



Check your understanding

Q4. Why is it **not** possible to determine the size of a right-angled triangle when all three internal angles are known?

Skill

Q5. The following steps are given to draw a triangle. After which step is there sufficient information to draw the triangle?

Skill

Step 1 The triangle is a right-angled triangle.

Step 2 One side length of the triangle is 1 cm.

Step 3 The hypotenuse of the triangle is 2 cm.

Step 4 One internal angle of the triangle is 30°.

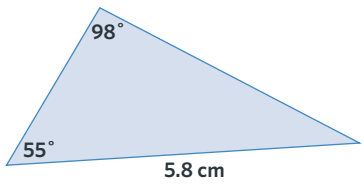
Step 5 The other side length of the triangle is $\sqrt{3}$ cm.

2. Sufficient information in a non-right-angled triangle

Q6. Find and label all missing side lengths and angles in the triangles below, correct to one decimal place where appropriate.

Skill

a)



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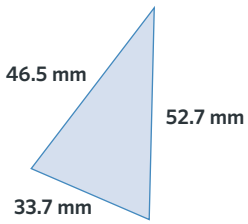
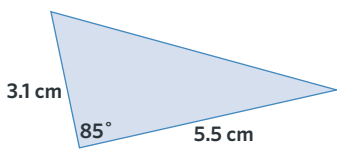


c)

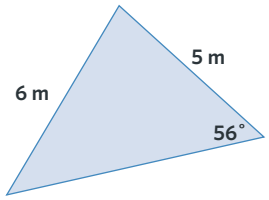


Check your understanding

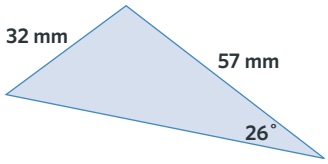
Q7. In which of the triangles below could you **not** find all missing side lengths and angles?

Skill

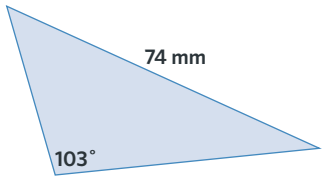
A.



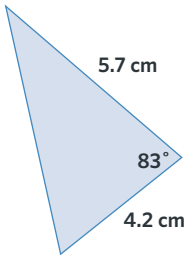
B.



C.



D.



Q8. A give way sign on the side of the road is an isosceles triangle.

Application

If one of the sides is 50 cm, and one of the angles is 70°, draw four possible shapes of the sign. Missing side lengths and angles do not need to be labelled.

Joining it all together

Q9. Is it possible to determine a triangle given the following information?

Skill
3 marks

a) Three angles

b) Three side lengths

c) Two side lengths and one included angle

Q10. The length of two sides of a triangle are 4 cm and 7 cm.
What could **not** be the length of the third side?

Skill
1 mark

A. 4 cm

B. 2 cm

C. 10 cm

D. 8 cm

Q11. A teacher asks her students to draw a triangle with side lengths 4.6 cm and 5.4 cm and a non-included angle of 56°. What are the three possible triangles the students could have drawn?

Application
3 marks

Questions from multiple lessons

Q12.

Difficulty:

1 mark

The triangle XYZ has the following properties:

- Side XY has a length of 6 cm.
- Side YZ has a length of 10 cm.
- Angle XZY has a magnitude of 32°.

Which of the following angles, correct to the nearest degree, could **not** be another angle in triangle XYZ?

A. 30°

B. 62°

C. 86°

D. 94°

E. 118°

VCAA 2017 Exam 1 Module 3: Geometry and measurement Q7 – Adapted

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Q13.

Difficulty:

1 mark

An investigation was carried out to determine the association between the *number of cars owned* and *income level* (low, medium, high) of a group of people. These variables are

A. a numerical variable and an ordinal variable respectively.

B. a numerical variable and a nominal variable respectively.

C. a nominal and a numerical variable respectively.

D. both numerical variables.

E. both categorical variables.

VCAA 2017NH Exam 1 Data analysis Q4 – Adapted

Q14.

Difficulty:

2 marks

Martin stands at E and throws two sticks across a park to play fetch with his dog.

The boundaries of the park are 30 m by 58 m.

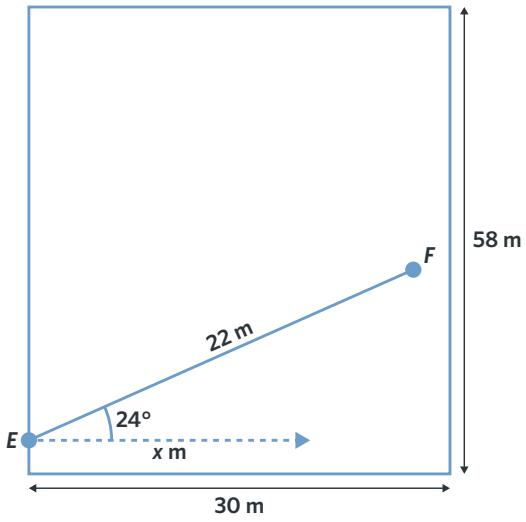
The first stick hits the ground at point F, 22 m away from point E.

The second stick hits the ground at point G.

Point G is x metres from point E.

Point G is 14 metres from point F.

The angle FEG is 24°.



a) Determine two possible values for the angle EGF. Round your answers to the nearest degree. (1 mark)

b) If point G is within the park, what is the value of x? Write your answer to the nearest metre. (1 mark)

VCAA 2018 Exam 2 Module 3: Geometry and measurement Q3c – Adapted

LESSON 8G

Area of a triangle

The key skills you will learn in this lesson are:

1. Finding the area
2. Applications of the formula

VCAA key knowledge point:

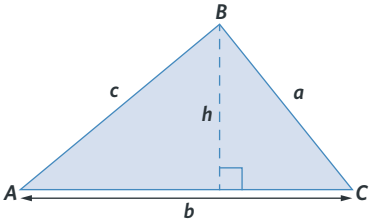
"area of a triangle using the rule $\text{Area} = \frac{1}{2}ab\sin(C)$"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21;

1. Finding the area

The basic formula for the area of a triangle is $\text{Area} = \frac{1}{2} \times b \times h$. However, in some cases, the perpendicular height (h) is not known. Trigonometry may then be used to find the area of the triangle.

Consider triangle ABC below.



This formula can be used to find the area when h is unknown:

$$Area = \frac{1}{2}ab\sin(C)$$

This formula can be used to find the area of a triangle when:

- any two sides and their included angle are known.

Formula explanation

Understanding where this formula comes from is not required for the course, however it can be useful to know this information. Here is an explanation of the formula:

Trigonometric rules state that in the right-angled triangle to the right of h, $\sin(C) = \frac{h}{a}$. Solving for h, this equation becomes $h = a \times \sin(C)$. This expression can then be substituted into the basic formula for area:

$$Area = \frac{1}{2} \times b \times h$$

$$Area = \frac{1}{2} \times b \times (a \times \sin(C))$$

$$Area = \frac{1}{2} \times a \times b \times \sin(C)$$

WORKED EXAMPLE 17 *(1 mark)*

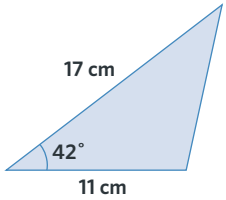
Consider the triangle with the following properties:

- One side has a length of 17 centimetres.
- Another side has a length of 11 centimetres.
- The included angle has a magnitude of 42°.

What is the area of the triangle, correct to two decimal places?

SOLUTION

Step 1 Draw a diagram.



Step 2 Assign values to the variables.

a and b are side lengths and C is their included angle.

Let the sides be $a = 17$, $b = 11$, and the angle be $C = 42°$.

Step 3 Substitute these values into the formula and solve.

$$\text{Area} = \frac{1}{2}ab\sin(C)$$

$$\text{Area} = \frac{1}{2} \times a \times b \times \sin(C)$$

$$\text{Area} = \frac{1}{2} \times 17 \times 11 \times \sin(42°)$$

$$\text{Area} = 62.5637 \ldots$$

$\therefore$ The area of the triangle is 62.56 cm^2.

2. Applications of the formula

In some cases, the area is given and either an unknown side or angle can be found using this formula.

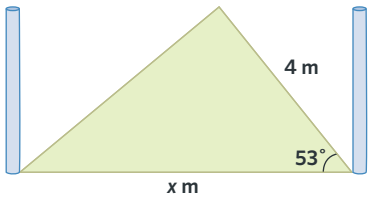
To find an unknown side, the formula can be rearranged to $a = \dfrac{\text{Area}}{\frac{1}{2}b\sin(C)}$

To find an unknown angle, the formula can be rearranged to $C = \sin^{-1}\left(\dfrac{\text{Area}}{\frac{1}{2}ab}\right)$.

Using the unknown angle formula will result in the calculator returning an acute angle. If the angle is obtuse, subtract the returned value from 180° to calculate the actual magnitude.

WORKED EXAMPLE 18 *(1 mark)*

A triangular patch of a sports oval with an area of 9.5 m^2 has been marked off between two goal-posts.



What is the distance, x, between the two goal-posts, correct to two decimal places?

SOLUTION

Step 1 Write down the known values.

a and b are side lengths and C is their included angle.

Therefore, $a = x$, $b = 4$, $C = 53°$, and Area $= 9.5$.

Step 2 Use the equation for a to solve for a.

$$a = \frac{\text{Area}}{\frac{1}{2}b\sin(C)}$$

$$a = \frac{9.5}{\frac{1}{2} \times 4 \times \sin(53°)}$$

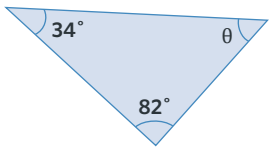
$$a = 5.9476 \ldots$$

$\therefore$ The distance between the two goal-posts is 5.95 metres.

Questions 8G *Area of a triangle*

Refresher question

Q1. What is the value of θ in this triangle?



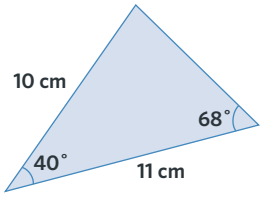
1. Area of a triangle

Q2. Skill

Consider the following triangle and the formula for area of a triangle, $Area = \frac{1}{2}ab\sin(C)$.

Fill in the blank spaces in the working below:

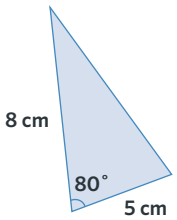
$Area = \frac{1}{2} \times 11 \times \square \times \sin \square\,°$



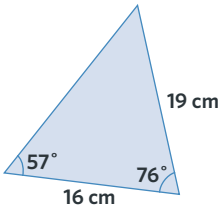
Q3. Skill

Find the area of each of the triangles below using the formula $Area = \frac{1}{2}ab\sin(C)$. Give answers correct to two decimal places.

a)



b)



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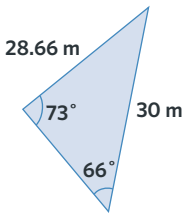


Q4. Skill

Triangle *XYZ* has the following properties:

- Side *XY* has a length of 21 cm.
- Side *YZ* has a length of 27.4 cm.
- Angle *XYZ* has a magnitude of 38°.
- Angle *YXZ* has a magnitude of 92°.

Draw the triangle and find the area of the triangle *XYZ*, correct to two decimal places.

Check your understanding

Q5. Application

Brodie is fencing off a triangular area of her farm that will be used for harvesting in the future. She starts the fence by first walking 160 metres at a bearing of 215°. She then continues the fence for 205 metres at a bearing of 78°. After this, she walks back to the starting point to create the triangle.

What is the area contained by the fence, correct to one decimal place?

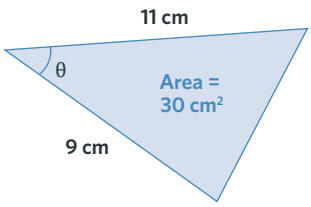
2. Applications of the formula

Q6. Skill

Consider the triangle on the right and the rearranged formula for finding an angle, $C = \sin^{-1}\left(\frac{\text{Area}}{\frac{1}{2}ab}\right)$.

Fill in the blank spaces in the working below.

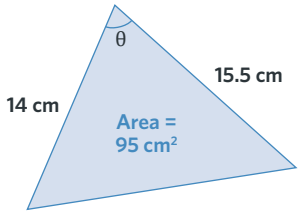
$\theta = \sin^{-1}\left(\frac{\square}{\frac{1}{2} \times 9 \times \square}\right)$



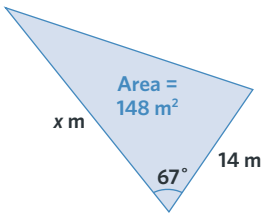
Q7. Find the unknown value in each of the triangles below, correct to two decimal places.

Skill

a)



b)



Q8. Consider the triangle ABC with an area of 412 m^2. In triangle ABC, side AB has a length of 21 metres, and angle BAC has a magnitude of 40°.

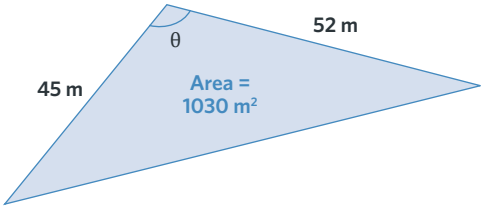
Skill

What is the length of the side AC, in metres, correct to the nearest centimetre?

Check your understanding

Q9. Find the value of θ in this triangle correct to one decimal place, given that the angle θ is obtuse.

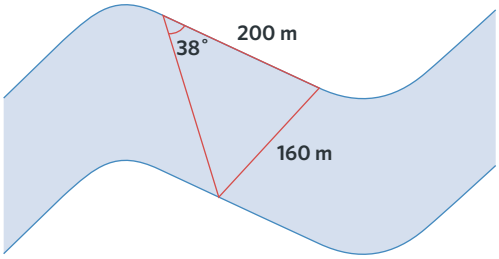
Skill



Q10. A triangular section of the Yarra River with an area of 13 250 m^2 has been fenced off to allow for a waterskiing showcase at Melbourne's annual Moomba Festival.

Application

What was the total length of fence used, correct to one decimal place?

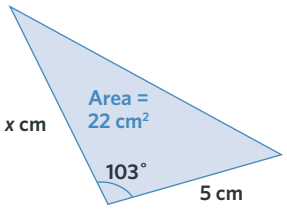


Joining it all together

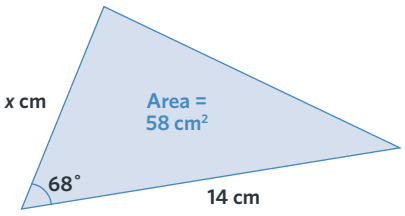
Q11. In which of the triangles below is the value of x the greatest?

Skill
1 mark

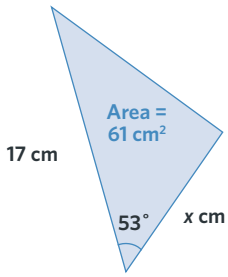
A.



B.



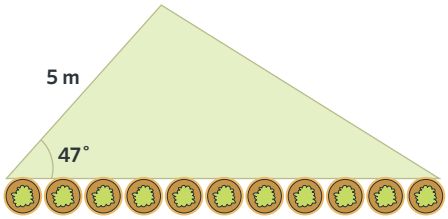
C.



Q12.

Application
2 marks

Elias is working on his garden and wants to line the front edge of his garden with pot plants. He took some measurements and drew a rough map of his garden design (not to scale), as shown.



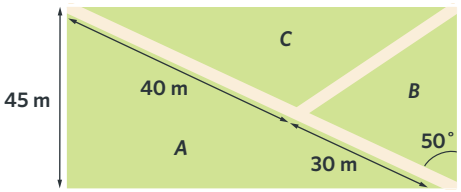
Elias knows that the garden has a total area of 25 m^2. Each pot plant has a diameter of 45 cm.

What is the maximum number of pot plants he can fit along the front edge of his garden?

Q13.

Application
2 marks

A rectangular park contains two gravel paths; one running diagonally through it and another path branching off of it, that separate the grass into three triangles (labelled A, B, and C).



The area of triangle A is 1206.4 m^2.

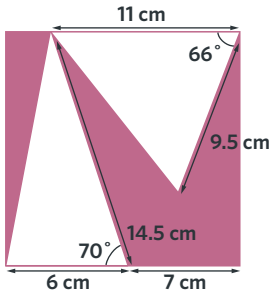
a) What is the area of triangle B, correct to the nearest square metre?

b) What is the area of triangle C, correct to the nearest square metre?

Q14.

Application
2 marks

Andrej is re-tiling his bathroom floor and chose to use square tiles with a geometric pattern. One of the tiles is shown here.



Show that the percentage of the tile that is purple is 47.6%, correct to one decimal place.

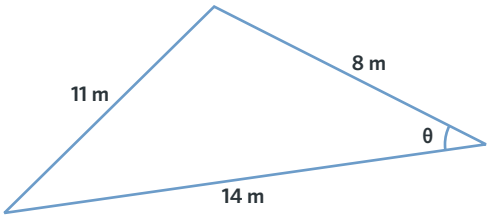
Questions from multiple lessons

Q15.

Difficulty:

1 mark

Consider the following triangle.



The angle θ could be found using

A. $\cos(\theta) = \frac{8}{11}$

B. $\cos(\theta) = \frac{8}{14}$

C. $\cos(\theta) = \frac{8^2 + 14^2 - 11^2}{2 \times 8 \times 14}$

D. $\cos(\theta) = \frac{8^2 + 14^2 + 11^2}{2 \times 8 \times 14}$

E. $\cos(\theta) = \frac{8^2 - 14^2 + 11^2}{2 \times 8 \times 11}$

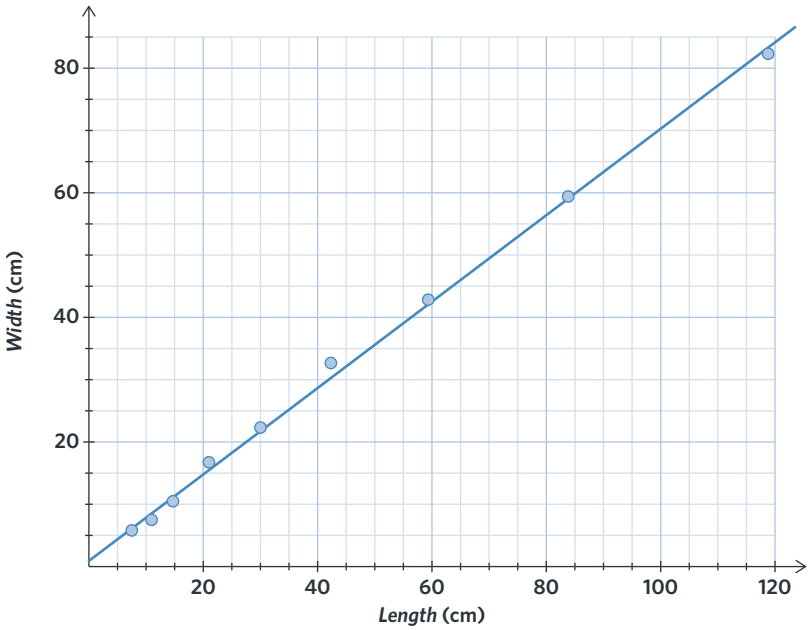
VCAA 2015 Exam 1 Module 2: Geometry and trigonometry Q3 – Adapted

Q16.

Difficulty:

1 mark

The scatterplot below displays the *length* and *width* of a number of different picture frames. A least squares regression line has been fitted to the data, as shown.



The equation of the line is closest to

A. $length = 1.25 + 0.69 \times width$

B. $width = 1.25 + 0.85 \times length$

C. $width = 1.25 + 0.69 \times length$

D. $length = 1.25 + 0.85 \times width$

E. $width = -1.25 + 0.69 \times length$

VCAA 2017NH Exam 1 Data analysis Q10 – Adapted

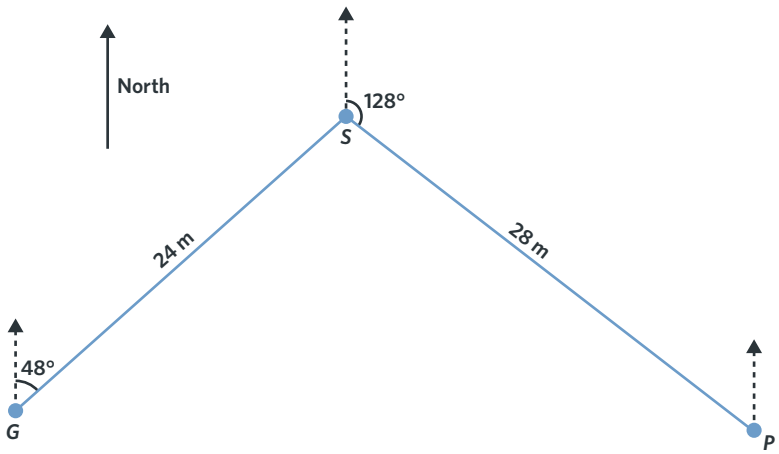
Q17.

Difficulty:

3 marks

During footy training, Grundy kicks the ball 24 m on a bearing of 048° from *G* to Steele at *S*, who then kicks it 28 m on a bearing of 128° to Pendles at *P*.

The path of the football is shown in the diagram below.



a) Pendles then kicks the ball back to Grundy. Use the cosine rule to find the distance of Pendles' kick. Round your answer to the nearest metre. (2 marks)

b) What is the bearing of Pendles from Grundy? Round your answer to the nearest degree. (1 mark)

VCAA 2016 Exam 2 Module 3: Geometry and measurement Q4 a,b – Adapted

AOS 5: Graphs of linear and non-linear relations

Linear graphs and models

CHAPTER 9

LESSON 9A

Linear graphing review

The key skills you will learn in this lesson are:

1. Plotting a line by creating a table of values
2. Plotting a line using x and y-intercepts
3. Plotting a line using a calculator
4. Plotting horizontal and vertical lines

VCAA key knowledge point:

"review of linear functions and graphs"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

A linear graph is any graph of the form $y = mx + c$. The variables are often represented by x and y, but other letters can also be used. Knowing two points on the line, or one point and the gradient, is enough to calculate the equation of the line.

Example

The line $y = 2x + 1$ is a linear graph.

Plotting a linear graph allows us to understand the relationship between two variables. There are three commonly used ways to plot linear graphs: using a table, using the intercepts and using a calculator. Horizontal and vertical lines are also types of linear graphs. These graphs will be of the form $y = c$ or $x = d$, respectively.

1. Plotting a line by creating a table of values

A table of values contains a row of x values, with the corresponding y values below them. Filling out a table of values makes it easy to plot a linear graph. Each pair of values represents a point on the line.

NOTE: After plotting, each point is written in brackets, with the x value first. If $y = 5$ when $x = 2$, this is written as $(2, 5)$.

WORKED EXAMPLE 1 *(2 marks)*

Plot $y = 2x + 1$ by creating a table of values for $x = -1, 0, 1, 2$ and 3.

SOLUTION

Step 1 Set up a table of values for x and y over the appropriate values. Write the x values into the table.

x	-1	0	1	2	3
y					

Step 2 Substitute the x values into the equation.

Here we substitute $x = -1, 0, 1, 2$ and 3 into $y = 2x + 1$.

Let $x = -1$

$y = 2(-1) + 1$

$y = -2 + 1$

$y = -1$

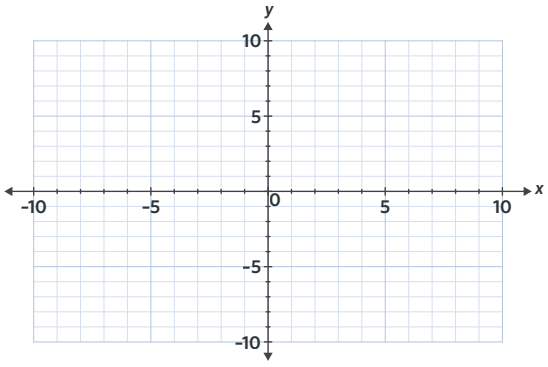
Repeat for the remaining x values.

Step 3 Enter the y values into the table.

x	-1	0	1	2	3
y	-1	1	3	5	7

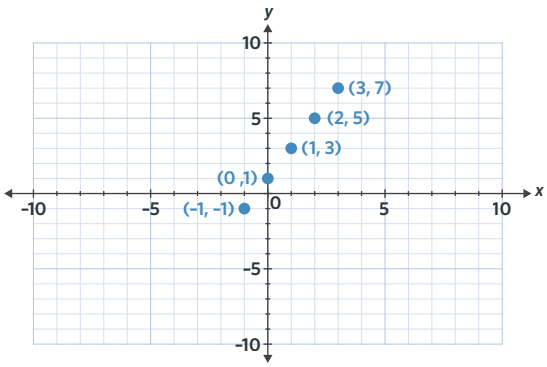
✓ *1 mark for completing the table of values*

Step 4 Draw and label a set of x and y axes. Ensure all values in the table are covered.



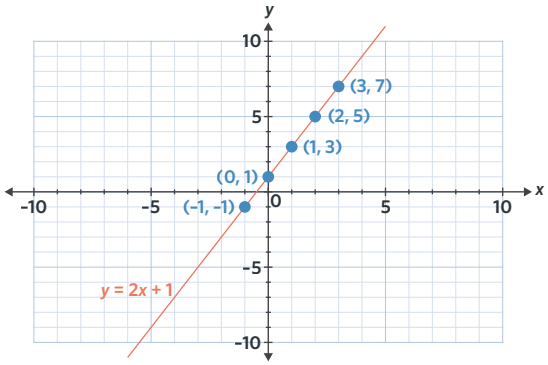
Step 5 Plot each pair of values from the table as a dot.

The set of points here is (–1, –1), (0, 1), (1, 3), (2, 5), and (3, 7).



Step 6 Connect the dots into a straight line.

Label the line $y = 2x + 1$.



✓ *1 mark for drawing the line*

2. Plotting a line using x and y-intercepts

The x-intercept of a line is the point where the line crosses the x-axis. As $y = 0$ for every point along the x-axis, the x-intercept takes the form $(x, 0)$.

The y-intercept is the point where the line crosses the y-axis, and takes the form $(0, y)$.

Linear graphs can be drawn by finding the x and y-intercepts, drawing a line connecting them, and then extending that line.

WORKED EXAMPLE 2 *(2 marks)*

Plot $y = 2x + 1$ by finding its x and y-intercepts.

SOLUTION

Step 1 Find the x-intercept by finding the x value when $y = 0$.

Substitute $y = 0$ into $y = 2x + 1$ and find x by rearranging or using the solve function on your CAS. Here we rearrange.

$y = 2x + 1$. Let $y = 0$.

$0 = 2x + 1$

$2x = -1$

$x = -\frac{1}{2}$

The x-intercept is $-\frac{1}{2}$

Step 2 Find the y-intercept by finding the y value when $x = 0$.

Substitute $x = 0$ into $y = 2x + 1$.

$y = 2x + 1$. Let $x = 0$.

$y = 2(0) + 1$

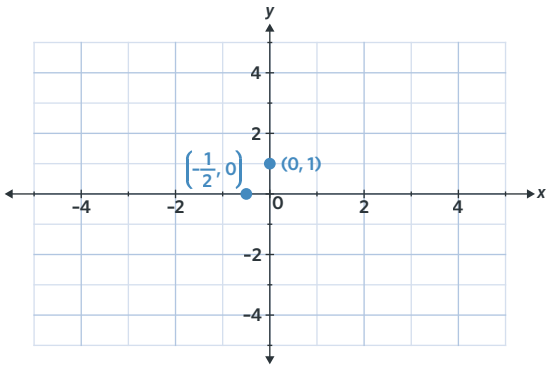
$y = 0 + 1$

$y = 1$

The y-intercept is 1.

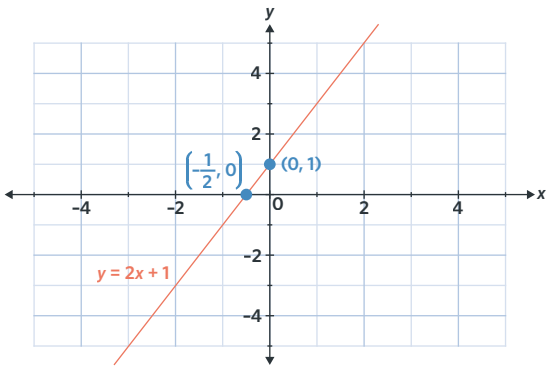
✓ *1 mark for finding the x and y-intercepts*

Step 3 Draw a set of x and y axes, and plot the x and y-intercepts on the axes with dots.



Step 4 Connect the two dots with a straight line and extend that line outwards.

Label the line $y = 2x + 1$.



✓ *1 mark for drawing the line*

3. Plotting a line using a calculator

It is easy to plot a linear equation of the form $y = mx + c$ on a calculator. Linear equations not in this form, such as $3x - y = 7$, need to be rearranged into the form $y = mx + c$ before they can be plotted on a calculator.

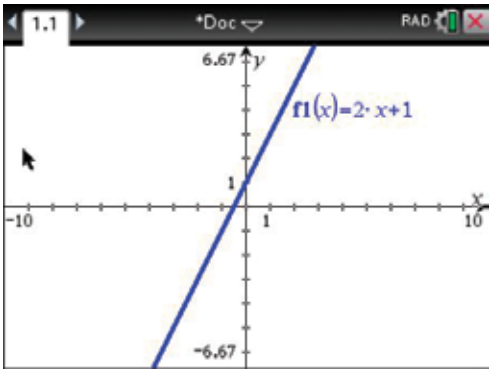
WORKED EXAMPLE 3 *(1 mark)*

Use your calculator to plot $y = 2x + 1$.

SOLUTION: TI-NSPIRE

Step 1 Open a new page by pressing ctrl + doc and press 2 ('Add Graphs').

Step 2 On the first line, type '$2x + 1$' and press enter. The graph will appear.

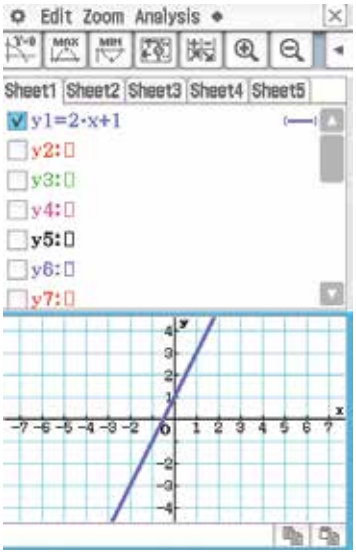


SOLUTION: CASIO CLASSPAD

Step 1 Select Graph& Table.

Step 2 On the first line, type '$2x + 1$' and tick the box next to the equation.

Step 3 Tap to plot the graph.



4. Plotting horizontal and vertical lines

Horizontal lines share the same y value at every point, so they are written as only a function of y.

Vertical lines share the same x value at every point, so they are written as only a function of x.

Example

$y = 2$, $y = -5$, and $y = 13$ are three examples of horizontal lines.

$x = 3$, $x = -10$, and $x = 8$ are three examples of vertical lines.

WORKED EXAMPLE 4 *(2 marks)*

Plot the following lines.

a) $y = 2$

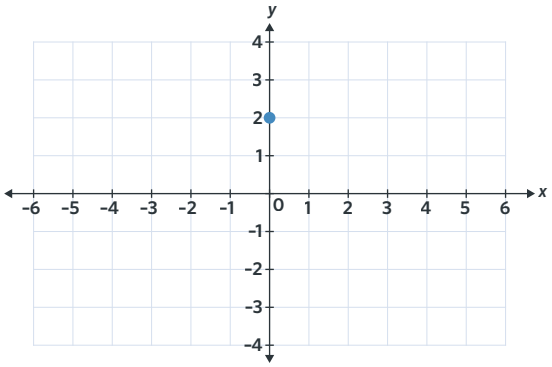
b) $x = -4$

SOLUTION

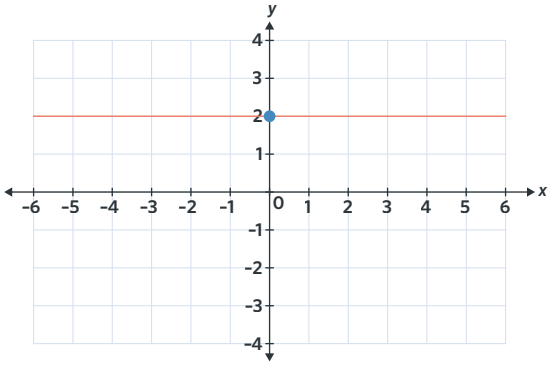
a) $y = 2$

Step 1 Mark the y-intercept.

The y-intercept is 2, so mark that on a graph.



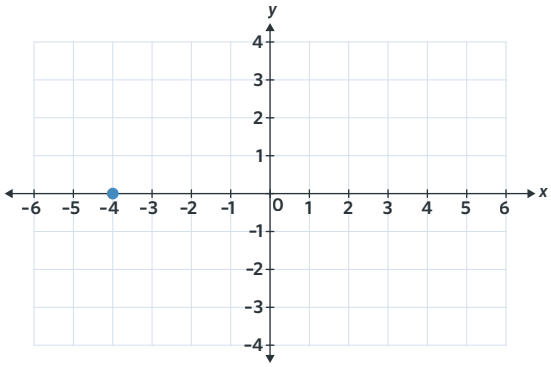
Step 2 Draw the rest of the line extending out horizontally from the y-intercept.



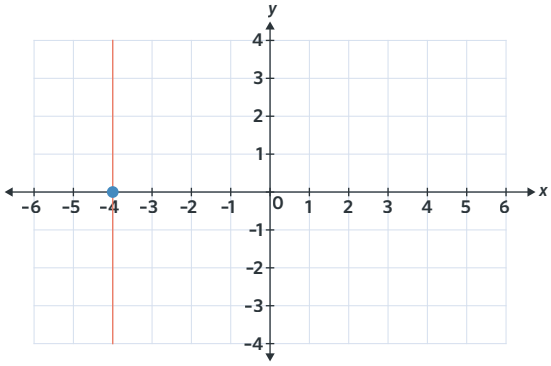
b) $x = -4$

Step 1 Mark the x-intercept.

The x-intercept is –4, so mark that on a graph.



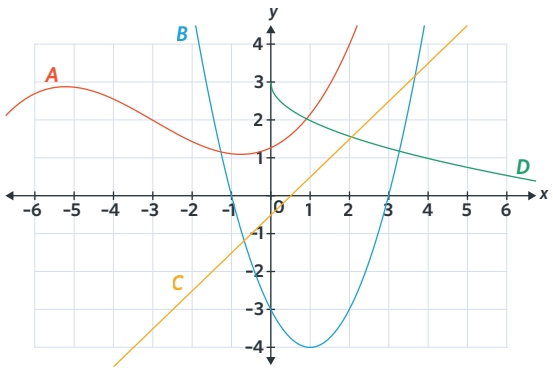
Step 2 Draw the rest of the line extending out vertically from the x-intercept.



Questions 9A *Linear graphing review*

Refresher question

Q1. Which of the following graphs is linear?

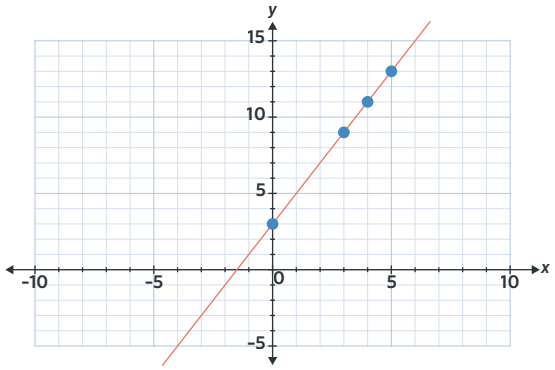


1. Plotting a line by creating a table of values

Q2. Skill

The line $y = 3 + 2x$ is plotted here. The four points drawn on the graph are given by the coordinates (0, ?), (3, ?), (?, 11), and (?, 13).

Find the missing coordinates and write the points completely.



Q3. Skill

a) Create a table of values for the line $y = 5x$ for the values $x = 0, 1, 2, 3, 4$.

b) Create a table of values for the line $y = 2x + 5$ for the values $x = 0, 1, 2, 3, 4$.

Q4. Skill

Plot the line $y = 1 - 3x$ by first creating a table of values with $x = 0, 1, 2, 3, 4$.

Q5. Skill

Plot the line $y = -0.5x + 1$ by finding the values of y when $x = 0, 1$.

Check your understanding

Q6. Skill

Create a table of values for the equation $3x + 1 = y$ over the values $x = -1, 0, 1, 2$.

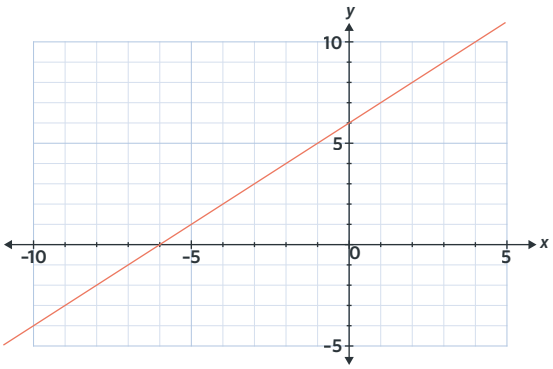
Q7. Skill

Plot the line $10 - 5x = y$ by first creating a table of values for $x = 0, 1, 2$.

2. Plotting a line using x and y-intercepts

Q8. Skill

What are the x and y-intercepts of the line shown on this graph?



Q9. Skill

Plot $y = 6 - 4x$ by connecting the x-intercept, (1.5, 0), to the y-intercept, (0, 6).

Q10. Skill

a) Plot the line $y = 7 + x$ by finding its x and y-intercepts.

b) Plot the line $y = \frac{1}{3}x + 2$ by finding its x and y-intercepts.

c) Plot the line $y = 7 - \frac{1}{2}x$ by finding its x and y-intercepts.

Check your understanding

Q11. Plot the line $6x - 7 = y$, and mark its x and y-intercepts.

Skill

3. Plotting a line using a calculator

Q12. Use a calculator to plot the following lines.

Skill

a) $y = 7x + 2$

b) $y = -20x + 2$

Check your understanding

Q13. Rearrange the following equations into the form $y = mx + c$, and use a calculator to plot them.

Skill

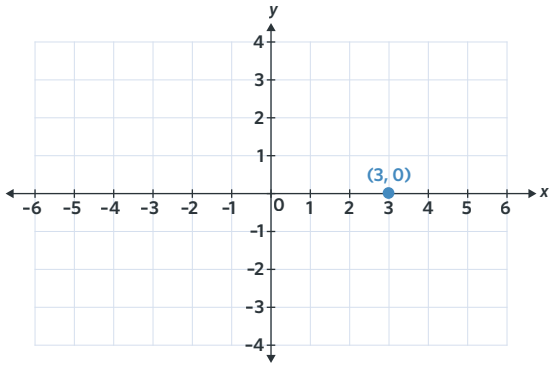
a) $-5x + 7y = 2$

b) $2y - 30x = 9$

4. Plotting horizontal and vertical lines

Q14. Use the x-intercept of the following vertical line to plot the graph $x = 3$.

Skill



Q15. Plot the following lines.

Skill

a) $y = 1$ **b)** $x = 5$ **c)** $y = -3$

Check your understanding

Q16. State which one of the following lines:

Skill

i has an x-intercept, and which has a y-intercept.

ii is vertical, and which is horizontal.

$y = 23$ $x = -38$

Joining it all together

Q17. Plot the line $y - \frac{1}{3}x = 1$ by generating a table of values for $x = -2, -1, 0, 1$.

Skill
2 marks

Q18. Use a calculator to plot the line $3x - y = 1$, and label the x and y-intercepts.

Skill
2 marks

Q19. Alison is comparing ski slopes around her town. She has written the following equations to represent the slopes at two ski resorts, where h is the height of the slope in metres and d is the horizontal distance in metres along the slope.

Application
5 marks

Sunrise Ridge: $h = 1125 - \frac{9}{4}d$

Cumulus Peak: $h = 760 - \frac{4}{3}d$

a) Use a table of values for $d = 0, 100, 200$ to plot the slope at Sunrise Ridge. Put h on the y-axis and d on the x-axis. Scale the d-axis from 0 to 600.

b) Use the method of intercepts to plot the slope at Cumulus Peak on the same graph.

c) Which slope is the highest?

d) Which slope is steepest?

e) Which slope covers the most ground horizontally?

VCAA question

Q20. A line passes through the points $(-1, 1)$ and $(3, 5)$. Another point that lies on this line is

1 mark

A. $(0, 1)$

B. $(1, 3)$

C. $(2, 6)$

D. $(3, 4)$

E. $(4, 7)$

VCAA 2014 Exam 1, Module 3: Graphs and relations. Q4

Questions from multiple lessons

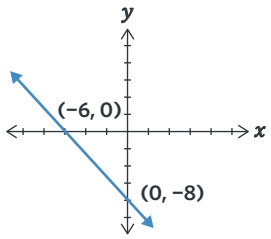
Q21. A straight line has an x-intercept at $x = 8$ and a y-intercept at $y = 6$.

Difficulty:

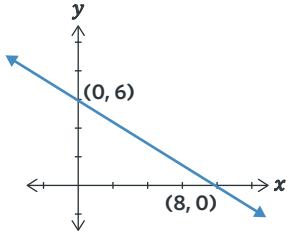
Year 10
1 mark

Plot the line.

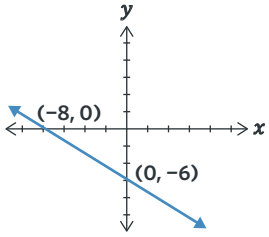
A.



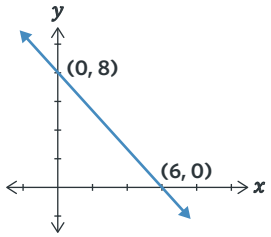
B.



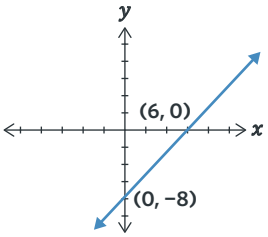
C.



D.



E.



VCAA 2018 Exam 1 Module 4: Graphs and relations Q1 – Adapted

Q22.

Difficulty:

Year 10
1 mark

The number of minutes spent driving to work on a particular day was recorded for a group of 25 people. The results are shown in the following stem plot.

Key: 1 | 1 = 11 minutes

1	5 6 7 7
2	2 5 6
3	2 7 8 8
4	1 1 2 6 9
5	0 1 3 7 8
6	2 4 5 7

The percentage of these people that spent less than 50 minutes driving to work is

A. 16%

B. 32%

C. 34%

D. 64%

E. 66%

VCAA 2018NH Exam 1 Data analysis Q2 – Adapted

Q23.

Difficulty:

Year 10
2 marks

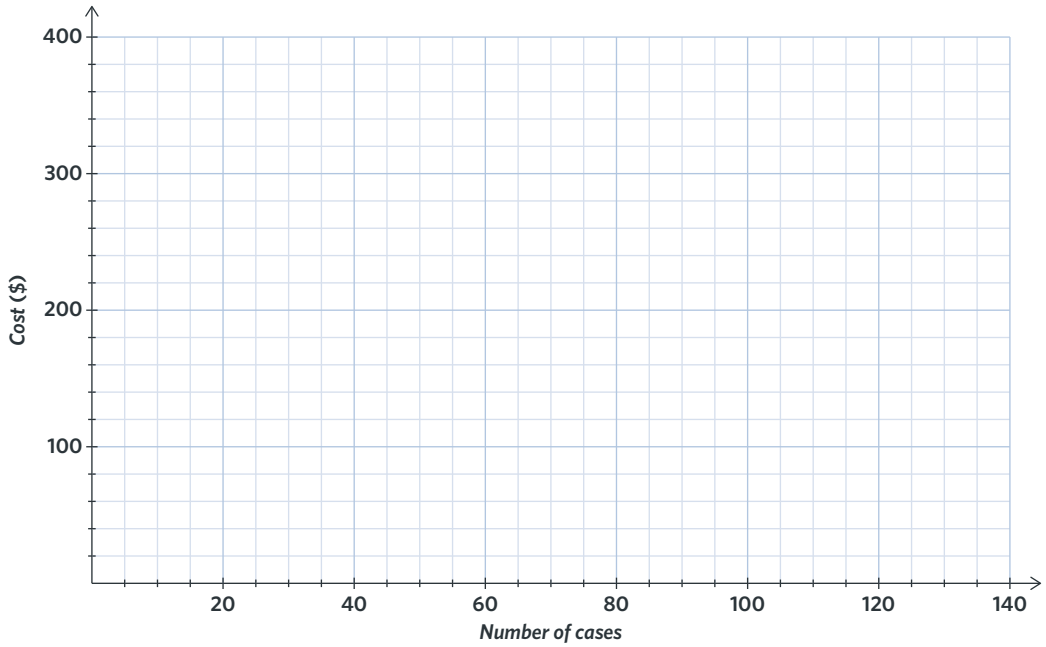
A phone company is keeping records on the cost of manufacturing and selling phone cases.

The *cost*, in dollars, of producing a certain *number of cases* can be found using the equation

$$cost = 200 + 1.25 \times number\ of\ cases$$

a) How many cases are produced if the *cost* is \$325? (1 mark)

b) Copy the following set of axes and sketch the relationship between *cost* and the *number of cases* produced. (1 mark)



VCAA 2016 Exam 2 Module 4: Graphs and relations Q2 a,b – Adapted

LESSON 9B

Linear equation review

The key skills you will learn in this lesson are:

1. Gradient of a line
2. Equation of a line
3. Finding the equation of a line
4. Finding the equation of a line using CAS

VCAA key knowledge point:

"the concept of a linear model and its specification"

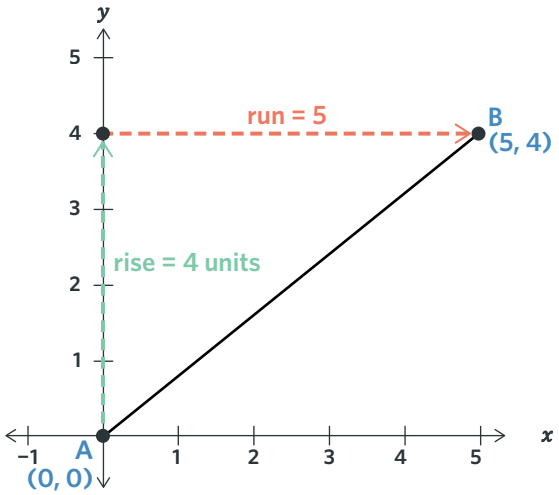
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Gradient of a line

The gradient (sometimes called the 'slope') of a line represents its steepness, and is equal to the change in y for every one unit increase in x.

It is commonly expressed as 'rise over run'. For two points $(x_1\ y_1)$ and $(x_2\ y_2)$, the vertical change, known as the change in y or the 'rise', is equal to $y_2 - y_1$. The horizontal change, known as the change in x or the 'run', is equal to $x_2 - x_1$.

$$\text{Gradient} = \frac{Rise}{Run} = \frac{Change\ in\ y}{Change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

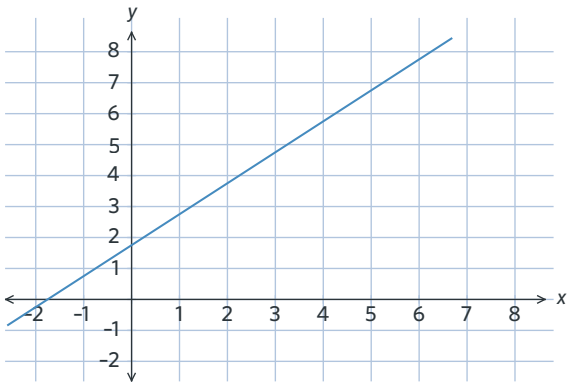


Example

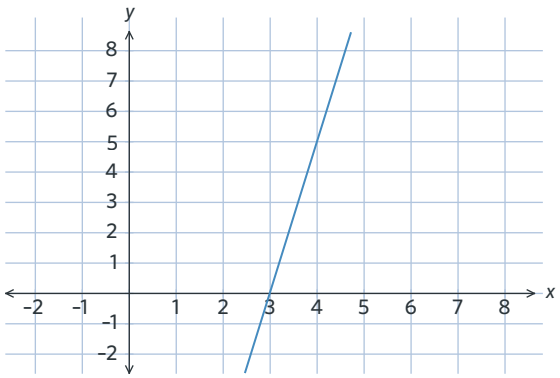
The graph to the right has a gradient of $\frac{4}{5}$.

Types of gradients

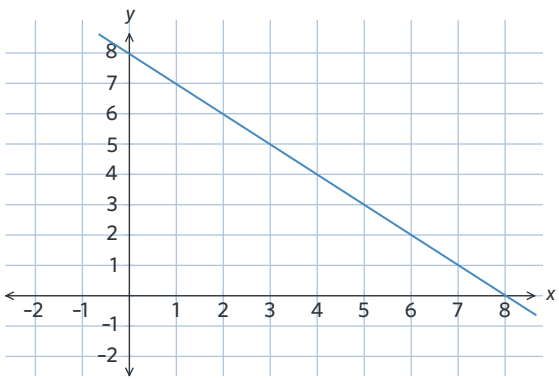
Positive gradient



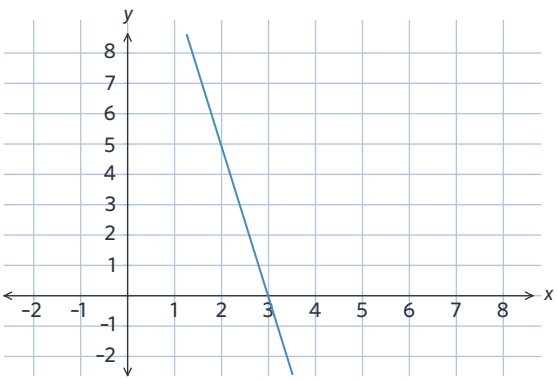
Steep positive gradient



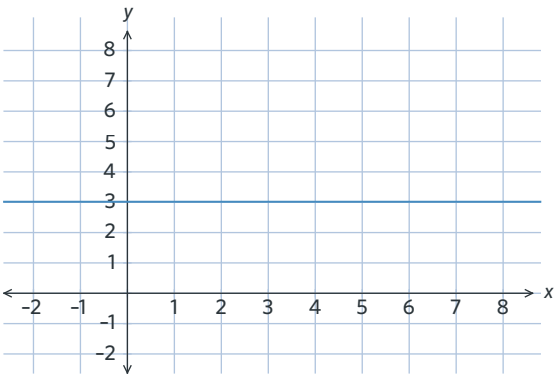
Negative gradient



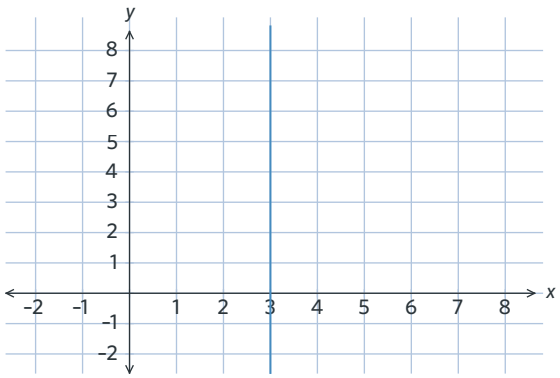
Steep negative gradient



Zero gradient



Undefined gradient



WORKED EXAMPLE 5 *(1 mark)*

Find the slope of the line that goes through the points (1, 2) and (5, 7).

Method 1: Using the gradient formula

Step 1 State x_1 x_2 y_1 and y_2.

We'll choose (1, 2) to be $(x_1\ y_1)$ and (5, 7) to be $(x_2\ y_2)$.

So $x_1 = 1, y_1 = 2, x_2 = 5$ and $y_2 = 7$.

Step 2 Substitute these values into the gradient formula.

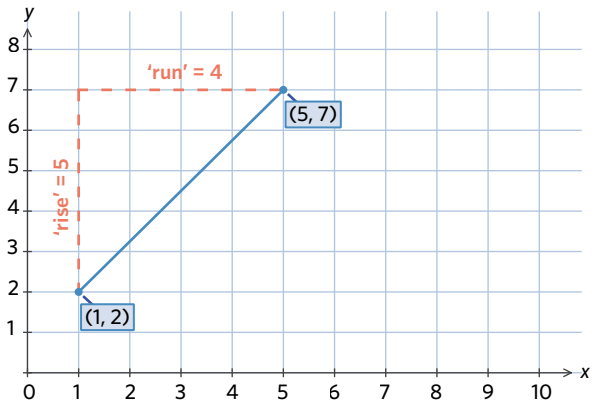
$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient} = \frac{7 - 2}{5 - 1}$$

$$\text{Gradient} = \frac{5}{4}$$

Method 2: Drawing the points

Step 1 Draw a set of axes, plot the points on the graph and find the 'rise' and the 'run'.



Step 2 Write the gradient using 'rise over run'.

$$\text{Gradient} = \frac{Rise}{Run} = \frac{5}{4}$$

Step 3 Check whether the gradient is positive or negative. Here the gradient is positive, so:

$$\text{Gradient} = \frac{5}{4}$$

2. Equation of a line

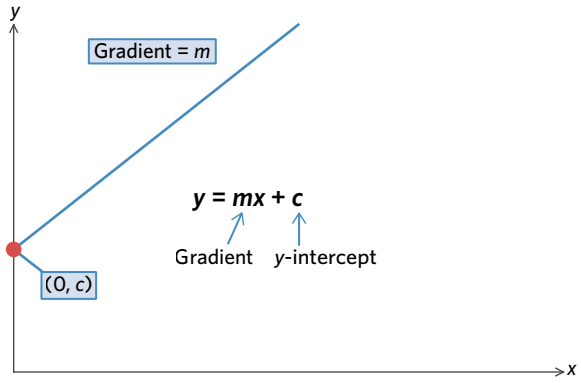
The standard equation of a line is

$$y = mx + c$$

In this form, m is the gradient of the line, and c is the y-intercept.

The equation of a line can be quickly written if the gradient and y-intercept of the line are known.

Conversely, if the equation of a line is known, the gradient and y-intercept can be quickly found.



WORKED EXAMPLE 6 *(1 mark)*

Determine the y-intercept and gradient for the straight line $y = -6 + 9x$.

Step 1 Rearrange the equation to the standard form $y = mx + c$.

$y = 9x - 6$

Step 2 Identify the gradient.

Since $m = 9$, the gradient is 9.

Step 3 Identify the y-intercept.

Since $c = -6$, the y-intercept is -6.

3. Finding the equation of a line

When the y-intercept and gradient of a line are unknown, the line's equation can still be determined if either:

- one point on the line and the gradient are known

  or

- any two points on the line are known.

This can be done using the formula:

$$y - y_1 = m(x - x_1)$$

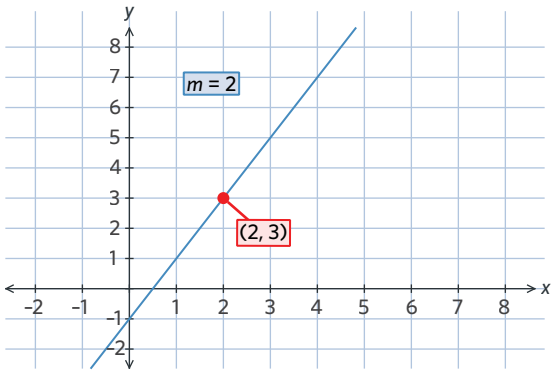
In this equation, m is the gradient of the line, and x_1 and y_1 are the values from a point on the line $(x_1 \ y_1)$.

The variables x and y represent the variables in the final equation, and are therefore not substituted with other values.

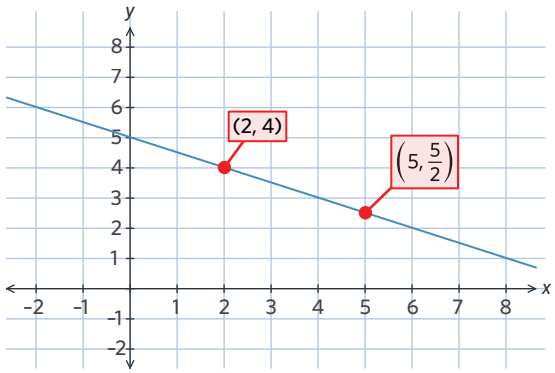
WORKED EXAMPLE 7 *(2 marks)*

Find the equation of each of the following lines.

a)



b)



SOLUTION

a)

Step 1 Write the formula $y - y_1 = m(x - x_1)$. Check if x_1 y_1, and m are known.

We know the point (2, 3) is on the line, and the gradient is 2.

So $x_1 = 2, y_1 = 3$, and $m = 2$.

Step 2 Substitute these numbers in and solve for y.

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x - 2)$

$y - 3 = 2x - 4$

$y = 2x - 1$

b)

Step 1 Write the formula $y - y_1 = m(x - x_1)$. Check if x_1 y_1, and m are known.

We know the point (2, 4) is on the line, but we don't know the gradient.

Use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to find the gradient.

Step 2 Find the gradient.

$x_1 = 2$ $y_1 = 4$ $x_2 = 5$ and $y_2 = \frac{5}{2} = 2.5$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{2.5 - 4}{5 - 2}$

$m = \frac{-1.5}{3}$

$m = -\frac{1}{2}$

Step 3 Substitute the gradient and one point into the formula.

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{2}(x - 2)$

$y - 4 = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x + 5$

4. Finding the equation of a line using CAS

WORKED EXAMPLE 8 *(1 mark)*

Find the equation of the line that passes through (5, 2) and (8, 10) using values rounded to two decimal places.

SOLUTION: TI-NSPIRE

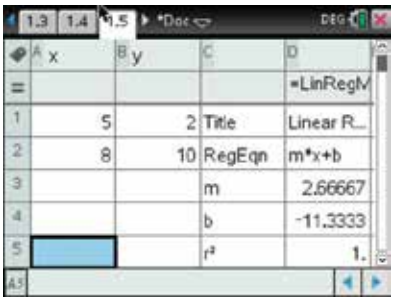
Step 1 Press 'Home', then 'List & Spreadsheets'.
Label list A as 'x' and list B as 'y'. Enter the data.

Step 2 Type '5' into cell A1 and '2' into cell B1, then type '8' into cell A2 and '10' into cell B2.

Step 3 Press menu, select '4: Statistics', then '1: Stat Calculations', and click on '3: Linear Regression (mx+b)'.

Step 4 Under 'X List', select 'x, and under 'Y List' select 'y. Press OK.

Step 5 The gradient is given by m and the y-intercept is given by b.
$m = 2.67$ and $b = -11.33$



Step 6 Write the equation.
$y = 2.67x - 11.33$

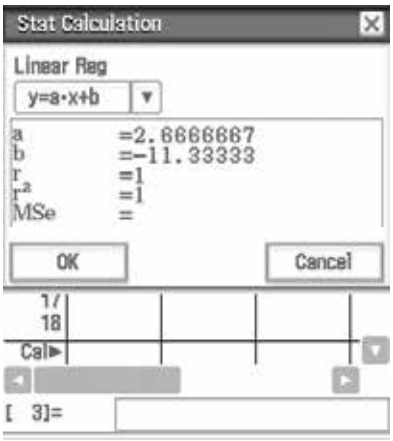
SOLUTION: CASIO CLASSPAD

Step 1 Select Statistics from the home screen, and rename two of the lists 'x' and 'y'.

Step 2 Tap the 'Calc' menu, then select 'Regression' and 'Linear Reg'.

Step 3 Under 'XList' choose 'main\x' and under 'YList' choose 'main\y'.
Push 'OK'. The results will appear.

Step 4 The gradient is given by a and the y-intercept is given by b.
$a = 2.67$ and $b = -11.33$

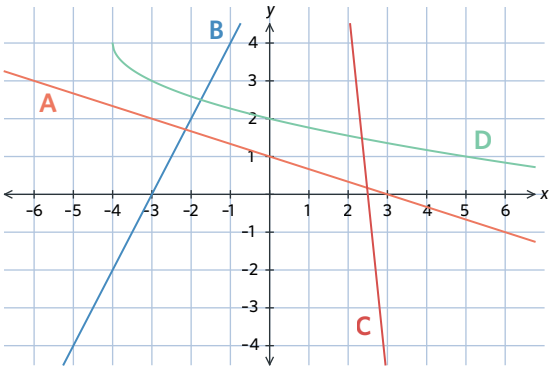


Step 5 Write the equation.
$y = 2.67x - 11.33$

Questions 9B *Linear equation review*

Refresher question

Q1. Which one of these graphs does not represent a linear equation?



1. Gradient of a line

Q2. Skill

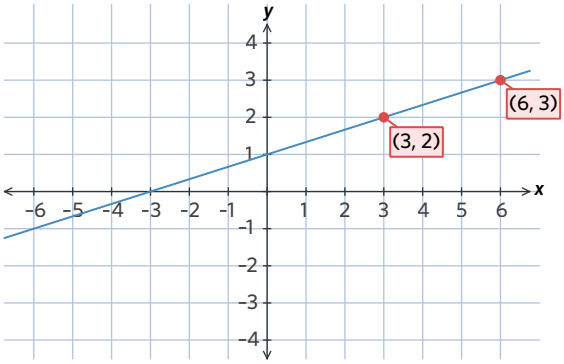
Tamara is trying to calculate the gradient of the following line.

Finish the last line of working for her.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3 - 2}{6 - 3}$$

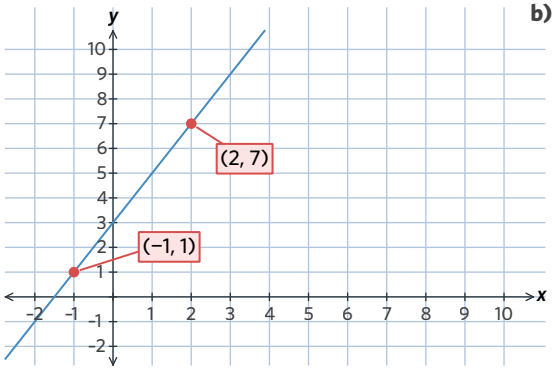
$$m =$$



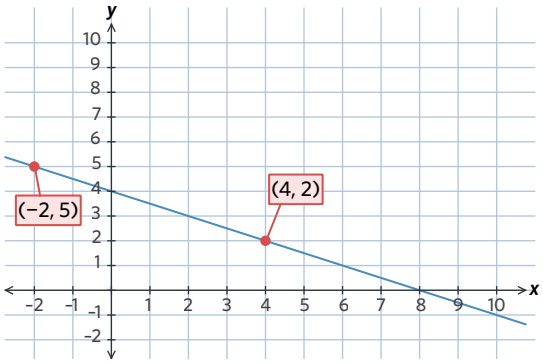
Q3. Skill

Calculate the gradients of the following lines.

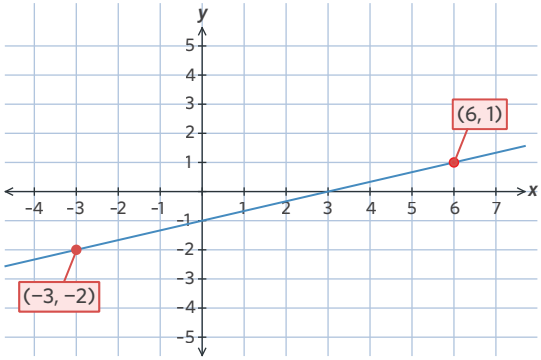
a)



b)



c)



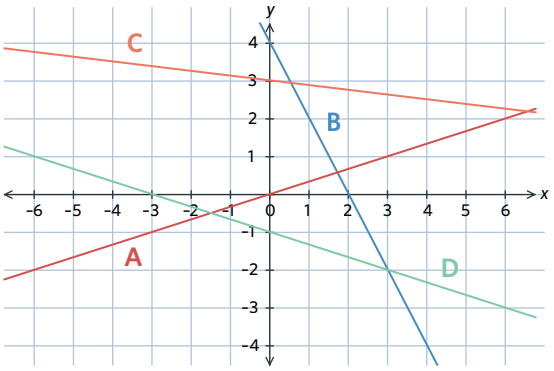
Check your understanding

Q4. A line goes through $(-1, 3)$ and $(3, 19)$. What is the gradient of this line?

Skill

Q5. Which of the following lines has a positive gradient?

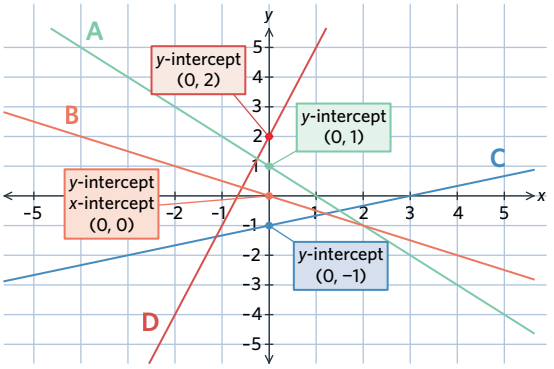
Skill



2. Equation of a line

Q6. All of the graphs below have their y-intercepts shown. Which one has a y-intercept of 1?

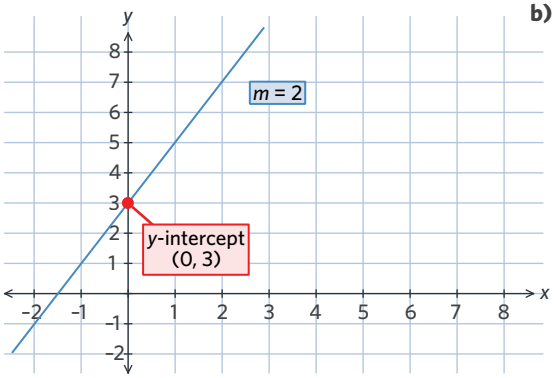
Skill



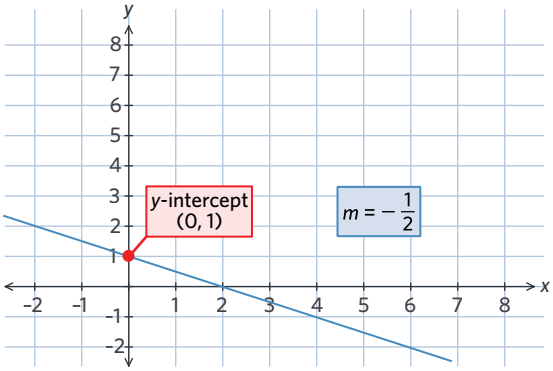
Q7. Write equations for the following lines.

Skill

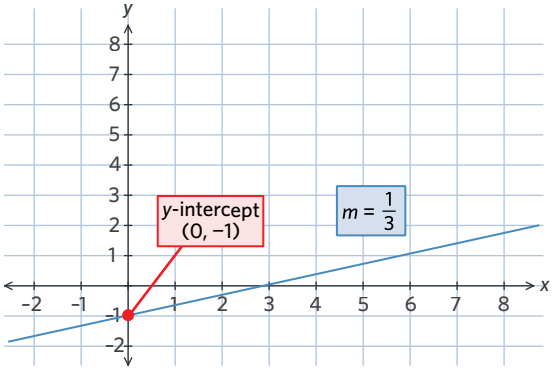
a)



b)



c)

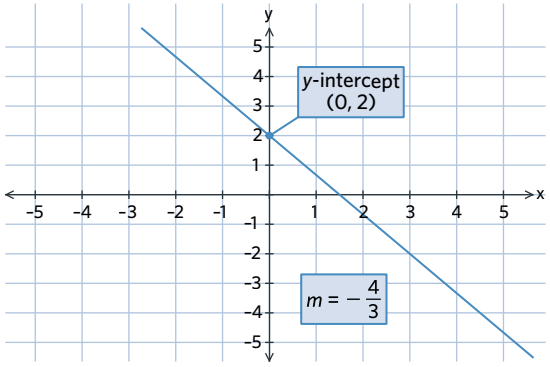


Check your understanding

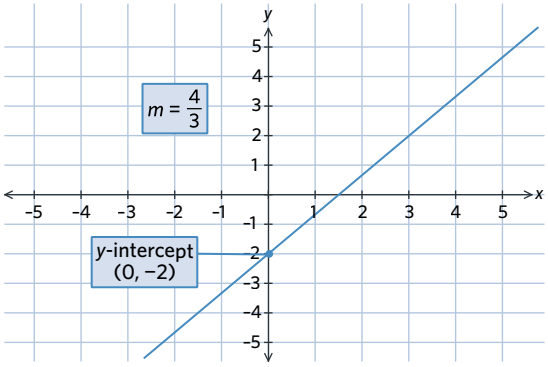
Q8. Which of the following lines has the equation $y = \frac{4}{3}x + 2$?

Skill

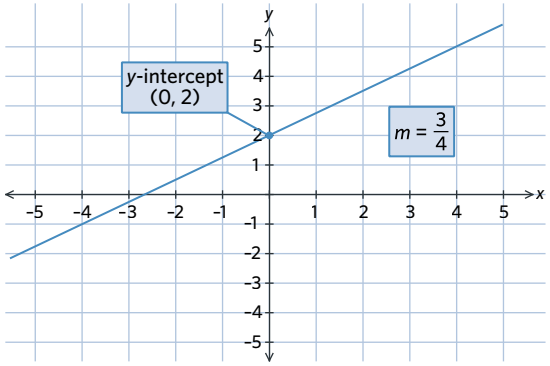
A.



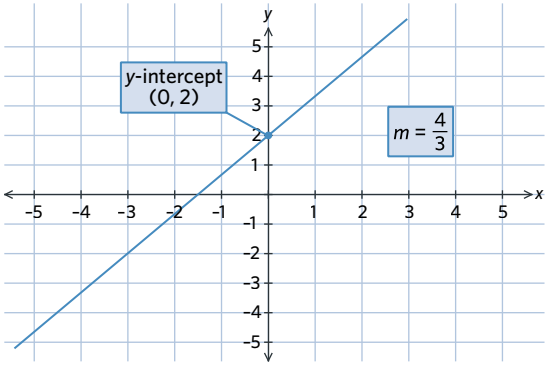
B.



C.



D.



Q9. Application

Bilal is baking a cake. The height of the cake in centimetres, y, x minutes after entering the oven is given by the line $y = 0.5x + 6$.

What does the y-intercept represent here?

3. Finding the equation of a line

Q10. Skill

Charlie is calculating the equation of the line below. Finish the working out for them.

One point: $\left(3, \frac{13}{2}\right) = (3, 6.5)$

Gradient: $\frac{3}{2} = 1.5$

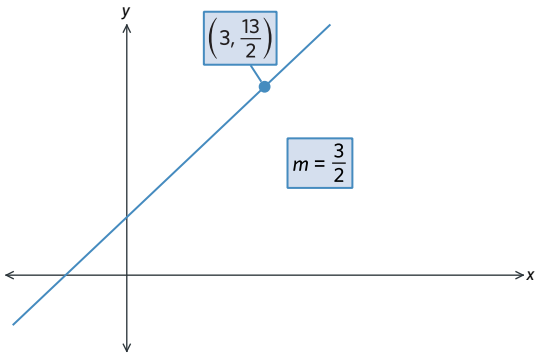
$y - y_1 = m(x - x_1)$

$y - 6.5 = 1.5(x - 3)$

$y - 6.5 = 1.5x - 4.5$

$y = 1.5x - 4.5 + 6.5$

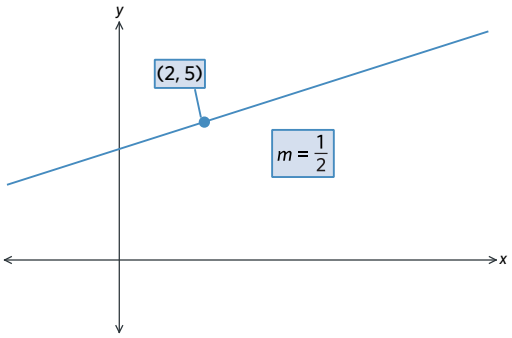
$y =$



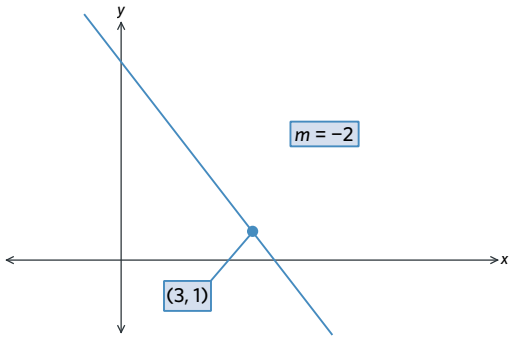
Q11. Find the equations of the following lines.

Skill

a)



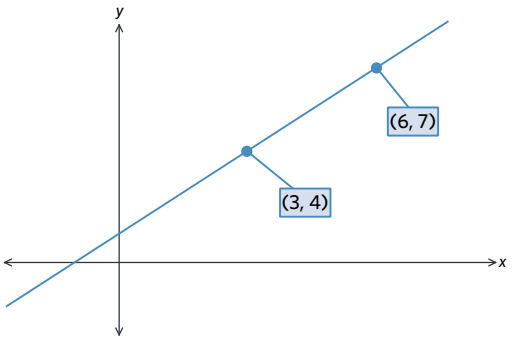
b)



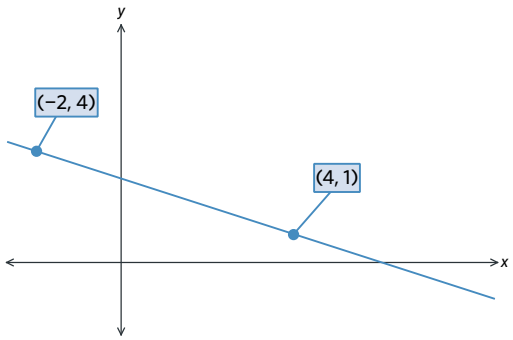
Q12. Find the equations of the following lines.

Skill

a)



b)



Check your understanding

Q13. An old bucket filled with rainwater cracked and began leaking. The amount of water in the bucket in litres, W, with time in minutes, t, is given by $W = 16 - 2t$.

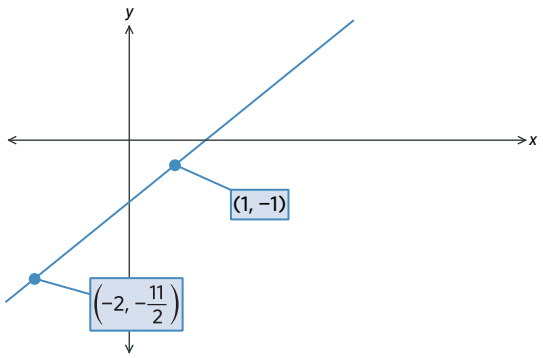
Application

a) Five minutes later after the bucket cracked, how much water was in the bucket?

b) How much water was in the bucket when it cracked?

Q14. Find the equation of the following line.

Skill



4. Finding the equation of a line using CAS

Q15. Determine the equation of the following lines using a calculator.

Skill

a) A line that goes through the points (1, 5) and (4, 11).

b) A line that goes through the points (3, −2) and (7, −5).

Q16. Determine the equation of the following lines using a calculator.

Skill

a) A line with a y-intercept of $\frac{9}{2}$ that goes through the point $(3, -1.5)$.

b) A line that passes through the point $(-2, -3.5)$ and has an x-intercept of 12.

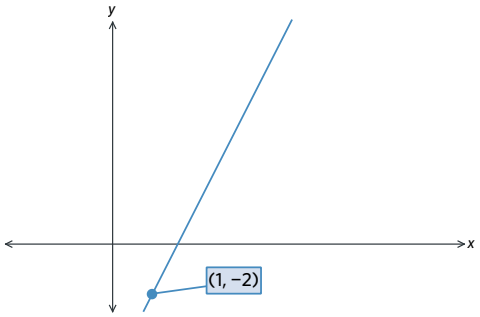
Joining it all together

Q17. For which of the following lines is it possible to determine an equation?

Skill
1 mark

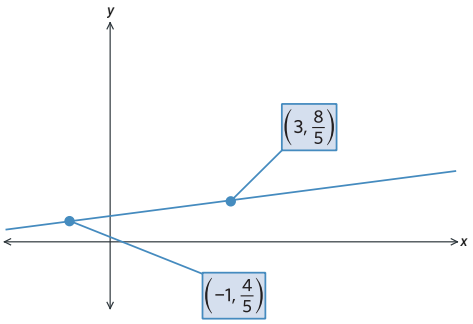
NOTE: There may be more than one answer.

A.



B. A line that has a y-intercept of 2 and a gradient of 10.

C.



D. A line that goes through the point $(5.5, 5.5)$.

E. A line with gradient $\frac{1}{6}$ and an x-intercept of $\frac{1}{5}$

F. A line that goes through the point $(12, 10)$ and the origin.

Q18. Jim lives near a lake and is sick of the ducks leaving their droppings all over his lawn.

Application
6 marks

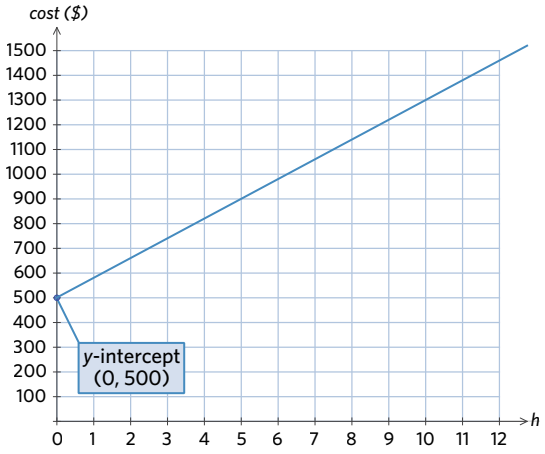
He is looking at some quotes to build a fence to keep the ducks out. Company quotes consist of a fixed materials fee, plus an hourly labour fee.

Use C to represent the cost in dollars to build the fence if it takes h hours to build.

a) The quote from The Low Expense Fence Company is plotted here.

Their hourly rate is $80.

What is the equation of this line?



b) The quote from Your Mate's Gates says materials will cost $800, and their hourly rate is $50.

Plot the quote on the graph.

c) Jim expects the job will take 11 hours. Which company will provide the cheapest price?

After constructing the fence, Jim remembers ducks can fly. Use the following cartoon to answer the rest of this question.

d) The paper in Jim's hand is another quote from the Low Expense Fence Company, and details the cost to install netting to keep the ducks out. Use the only two points visible on the paper to find The Low Expense Fence Company's hourly rate for netting.

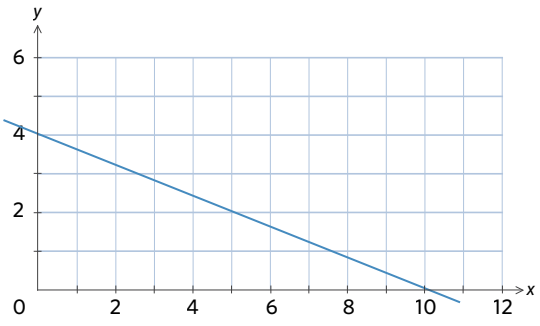
e) What is the fixed materials fee for the netting?

f) Jim thinks that installing the netting will take thirteen hours. What will be the total cost?

VCAA question

Q19. 1 mark

A straight line is graphed to the right.



An equation for this line is:

A. $2x + 5y = 20$

B. $10x + 4y = 20$

C. $2x - 5y = 20$

D. $5x + 2y = 20$

E. $4x - 10y = 20$

Adapted from VCAA 2015 Exam 1, Module 3: Graphs and relations. Q4

Questions from multiple lessons

Q20.

Difficulty:

Year 10
1 mark

The equation of the line that passes through the points (3, 1) and (3, 8) is

A. $x = 3$

B. $y = 8$

C. $y = 8x$

D. $y = 3x + 1$

E. $y = 8x + 3$

VCAA 2017 Exam 1 Module 4: Graphs and relations Q1 – Adapted

Q21.

Difficulty:

1 mark

Which one of these sequences could **not** be generated from the following Fibonacci-related recurrence relation?

$t_0 = 1, \quad t_n = t_{n-2} + t_{n-1}$

A. 1, 3, 4, 7, 11...

B. $1, -2, -1, -3, -4$...

C. 1, 5, 6, 11, 17...

D. $1, -1, 0, -1, 0$...

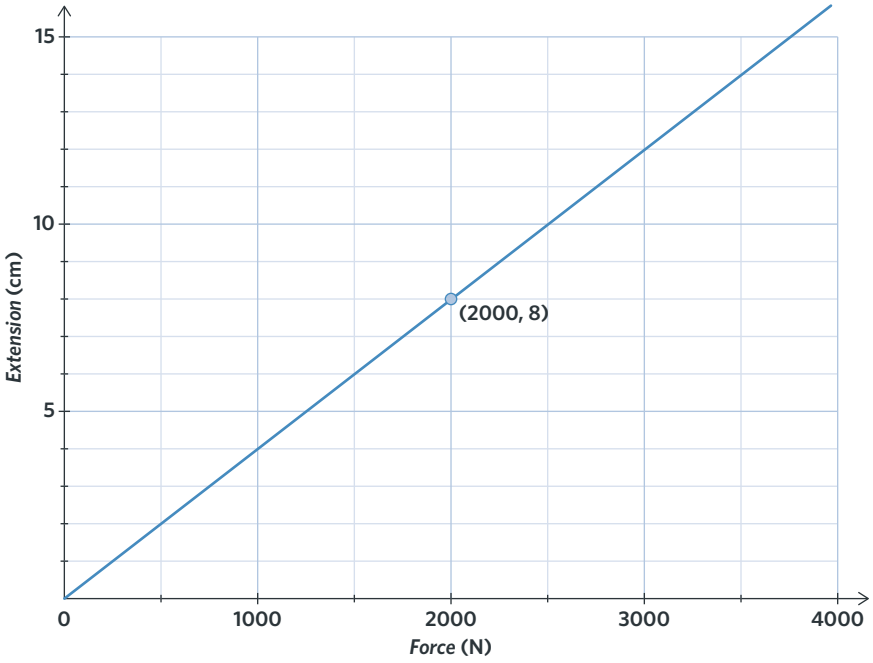
E. 1, 2, 3, 5, 8...

Q22.

Difficulty:

Year 10
2 marks

The extension of a spring is the distance it is stretched by, in centimetres, when a force is applied. The following graph shows the relationship between *force*, in Newtons (N) and *extension* (cm) for a linear spring.



The relationship between *force* and *extension* is given by the equation $extension = \frac{force}{k}$

a) Show that $k = 250$. (1 mark)

b) Natalia stretched a spring by 20 cm.
Using the equation above, calculate the force applied, in Newtons, when she stretched this spring. (1 mark)

VCAA 2018 Exam 2 Module 4: Graphs and relations Q2 a,b – Adapted

LESSON 9C

Constructing linear models

The key skills you will learn in this lesson are:

1. Constructing linear equations
2. Domain of linear models

VCAA key knowledge points:

"the concept of a linear model and its specification"

"the construction of a linear model to represent a practical situation including domain of application"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Constructing linear equations

Linear equations are often used to represent real life situations where there is a constant change over a period of time. The amount of water in a sink when filling it up, the fare of a taxi ride, and the amount of fuel left in a car tank can all be represented by linear equations.

A linear model can be written in the form:

$$y = mx + c$$

For linear models, m, is the rate of change, and c is the initial value.

Examples of rates of change include 60 km/h, 12 litres per minute and \$2 per day. The rate of change will be positive for an increase in value and negative for a decrease in value. If the rate of change is positive, m is positive, and if the rate of change is negative, m is negative.

Example

If a ski slope initially had 100 cm of snow, and 5 cm melts each day, the linear equation to describe the situation would be $S = -5n + 100$ where S is the depth of snow in centimetres after n days. Notice that the rate of change, -5, is negative because S represents a decrease.

WORKED EXAMPLE 9 *(1 mark)*

Construct an equation to describe the amount of air in a balloon, A (cm^3), after t seconds, given that the balloon initially had 200 cm^3 of air and fills up at a rate of 50 cm^3 per second.

SOLUTION

Step 1 Find the initial amount of air in the balloon.

200 cm^3

Step 2 Find the rate of change of the amount of air in the balloon.

The balloon fills up by 50 cm^3 per second

Step 3 Represent the information in a linear equation.

The linear equation will be in the form $A = mt + c$ where c is the initial value, and m is the rate of change.

$c = 200$ and $m = 50$

$A = 50t + 200$

2. Domain of linear models

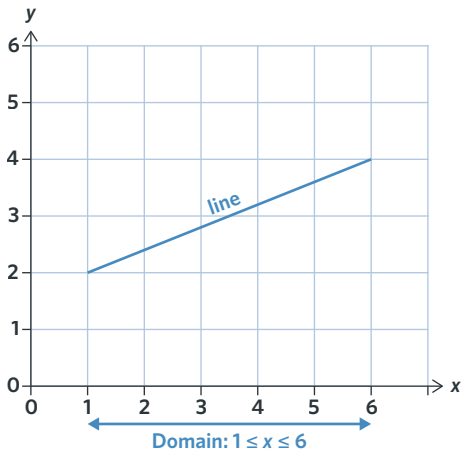
Sometimes linear models are only applicable over certain values. For example, for the equation $S = 100 - 5n$, which describes the depth of snow in centimetres after n days, the value of n cannot be greater than 20, otherwise the depth of snow would be negative. Therefore $0 \leq n \leq 20$. This is known as the domain of the equation, and represents all the possible values of the independent variable.

In the equation $y = mx + c$ the independent variable is x.

The domain of a graph can also be determined by looking at the spread of values on the x-axis.

Example

For this line the domain is $1 \leq x \leq 6$ because those are the values covered in the horizontal direction.



WORKED EXAMPLE 10 *(1 mark)*

The balloon described in the previous worked example will burst if it has more than 5000 cm^3 of air in it. Find the domain of the equation.

SOLUTION

Step 1 Find when the balloon will have 5000 cm^3 of air in it by substituting in $A = 5000$.

$5000 = 50t + 200$

$4800 = 50t$

$t = 96$

Step 2 Find the domain of the equation.

t must be greater than or equal to 0 as time cannot be negative, and less than or equal to 96 as the balloon will burst.

The domain of the equation is $0 \leq t \leq 96$.

Questions 9C *Constructing linear models*

Refresher questions

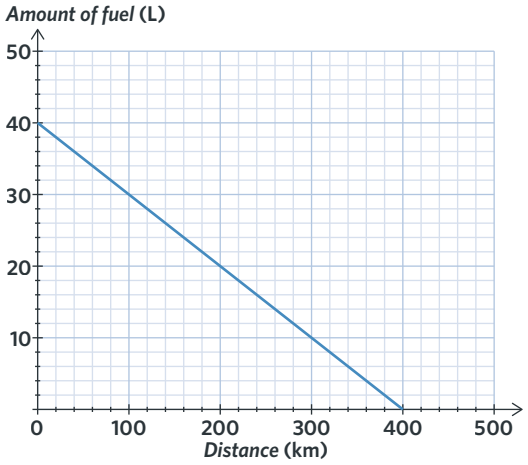
Q1. What is the gradient of the line $y = -4x + 3$?

Q2. Find the gradient of the line which passes through the coordinates $(2, 7)$ and $(6, -1)$.

1. Constructing linear equations

Q3. Skill

The following graph shows the amount of fuel in the tank of a car after driving a certain distance. What was the amount of fuel left in the tank after 100 km had been driven?



Q4. Skill

Construct an equation to describe:

a) The number of birds in a population, B, after n years, given that there were initially 10 birds and the number of birds increases by 3 birds per year.

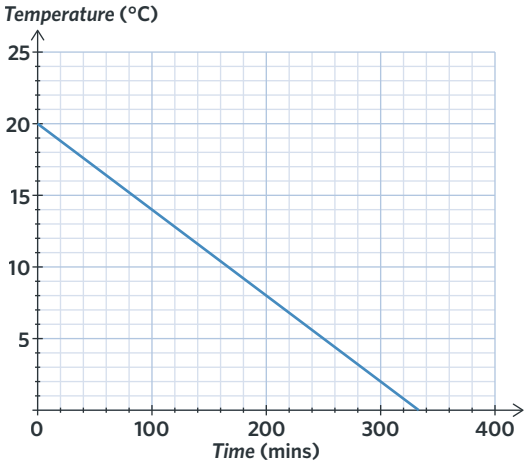
b) The height of a bamboo plant, H (m), after t days, given that the plant was initially 1.5 metres tall and grows at a rate of 0.6 metres per day.

c) The amount of water in a rainwater tank, W (L), after t minutes, given that the tank was initially empty and fills at a rate of 1.6 litres per minute.

Check your understanding

Q5. Skill

The following graph shows the temperature of an ice block cooling in the freezer, T (°C), after t minutes. The ice block cools at a rate of −0.06°C per minute. Find the equation of the graph.



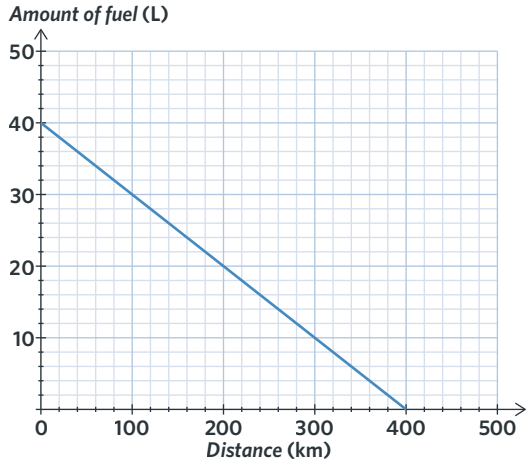
Q6. Skill

An inflatable swimming pool was filled with water at a constant rate. After two minutes the amount of water in the swimming pool was 200 L. After five minutes the amount of water in the swimming pool was 380 L. How many litres of water were in the swimming pool to begin with?

2. Domain of linear models

Q7. Skill

The following graph shows the amount of fuel in the tank of a car after driving a certain distance. What is the domain of the graph?



Q8. Skill

Find the domain of the equations in question 4 given that:

a) The population of birds no longer increases when there are 28 birds.

b) The bamboo plant stops growing after it reaches a height of 20.1 metres.

c) The rainwater tank can hold a maximum of 4000 litres of water.

Check your understanding

Q9. Application

A cup of coffee cools according to the equation $T = -1.5t + 80$, where T (°C) is the temperature of the coffee and t is the time in minutes. Find the domain of the equation, given that the temperature of the coffee does not change after it reaches 20°C, the temperature of the room.

Joining it all together

Q10. Application 1 mark

The table below shows the amount of money, M ($), in a student's bank account after n weeks.

n	1	2	3	4
M	1200	1600	2000	2400

Find an equation to describe the amount of money in the student's bank account.

Q11. Application 2 marks

The number of people on a tram, T, after n stops is given by the equation $T = 30 - 4n$.

a) How many people got off the tram in the first three stops?

b) After how many stops would there be less than eight people on the tram?

Q12. Application 2 marks

Dian is trying to find a mechanic to fix her car. Mechanic A charges a call-out fee of $70 and $55 for every hour of work, whereas Mechanic B charges a call-out fee of $85 and $50 for every hour of work. If it will take four hours to fix her car, which mechanic would be cheaper to hire?

Q13. Application 4 marks

A car hiring company charges an initial fee of $40 as well as a cost of $70 per day to hire a car for the first week. After the first week it costs $50 per day to hire the car.

a) Construct a linear equation describing the total cost, C ($), to hire the car for n days during the first week.

b) How much would it cost to hire the car for one week?

c) Construct a linear equation describing the total cost, C ($), to hire the car for m days after the first week.

d) How much would it cost to hire the car for 15 days?

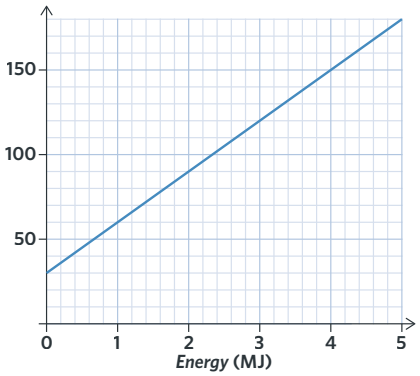
VCAA question

Q14. 1 mark

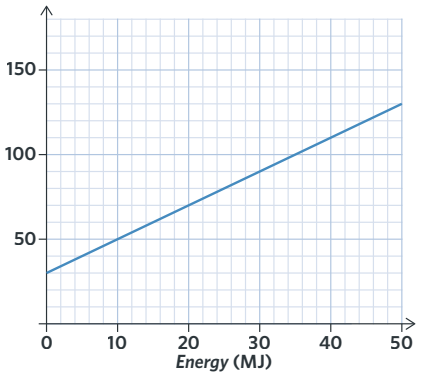
In one month, an energy company charges a $30 service fee plus a supply charge of two cents per megajoule (MJ) of energy used.

The graph that best models this situation is:

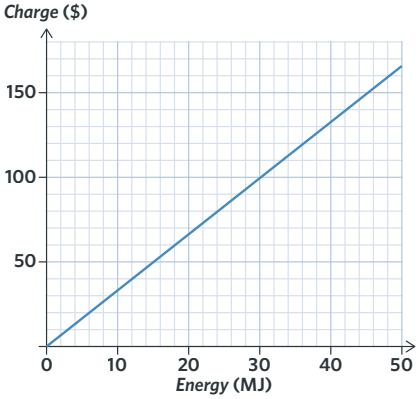
A.



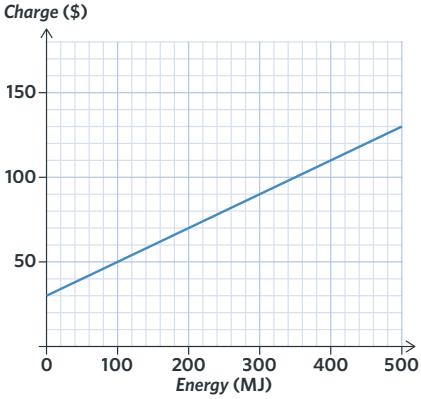
B.



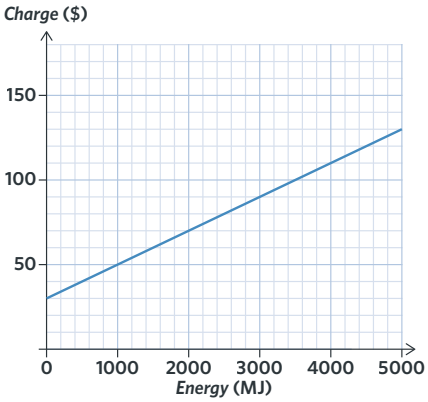
C.



D.



E.



VCAA 2013 Exam 1, Module 3: Graphs and relations. Q6

Questions from multiple lessons

Q15.

Difficulty:

1 mark

A straight line is shown.

An equation for this line is

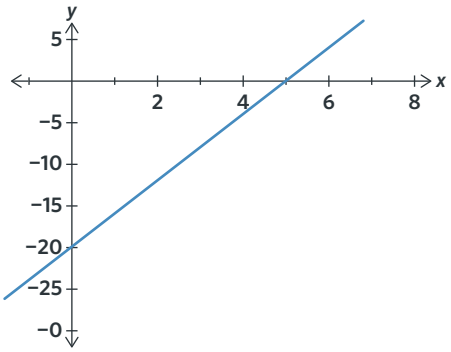
A. $y = 20x + 5$

B. $y = 5x - 20$

C. $y = 4x - 20$

D. $x = 20y + 5$

E. $x = 5y - 20$



VCAA 2015 Exam 1 Module 3: Graphs and relations Q4 – Adapted

Q16.

Difficulty:

1 mark

An investigation was carried out to determine the association between the variables *length of a movie* (less than 1 hour, 1–2 hours, over 2 hours) and *popularity* (low, medium, high). These variables are

A. both numerical variables.

B. both nominal variables.

C. both ordinal variables.

D. a numerical and ordinal variable respectively.

E. an ordinal and nominal variable respectively.

VCAA 2018NH Exam 1 Data analysis Q5 – Adapted

Q17.

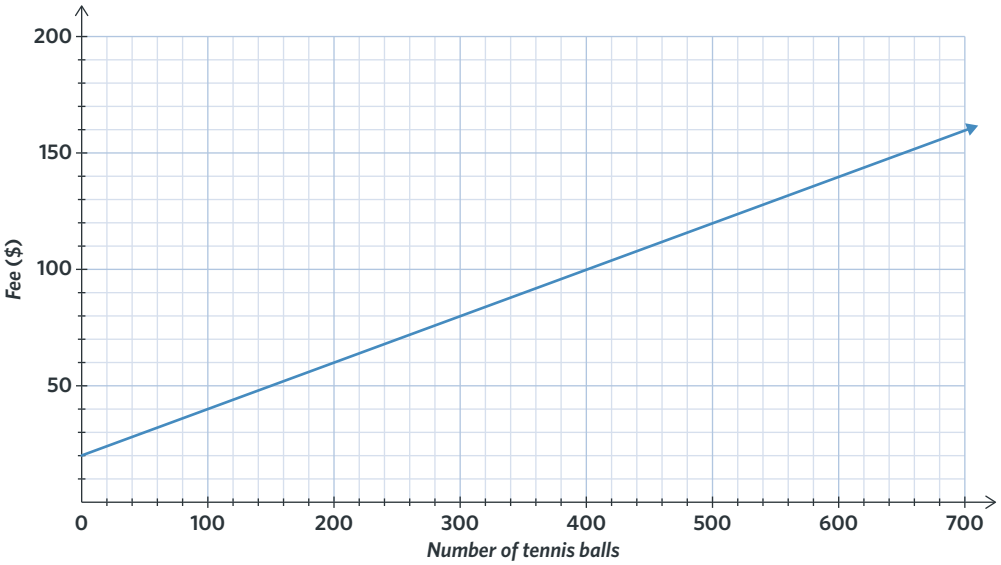
Difficulty:

2 marks

A company wants to sell tennis balls.

Hopkins and Hoppers are two factories that produce tennis balls and have given the company two different offers that consist of a fixed amount and a charge per tennis ball made.

The following graph shows the offer made by Hopkins.



a) Complete the following sentence.
Hopkins charges a fixed amount of $_____ and a charge per tennis ball of $_____. (1 mark)

b) Hopkins and Hoppers both charge a fee of $100 for 400 tennis balls.

Hoppers' charge per tennis ball is $0.15.

Draw a linear relation representing the fee charged by Hoppers for the number of tennis balls on the same graph as the linear relation that represents the fee charged by Hopkins. (1 mark)

VCAA 2019NH Exam 2 Module 4: Graphs and relations Q3 a,b – Adapted

LESSON 9D

Fitting linear models by eye

The key skills you will learn in this lesson are:

1. Drawing the line of best fit for a graph
2. Finding the equation of the line of best fit for a graph

VCAA key knowledge point:

"fitting a linear model to data by using the equation of a line fitted by eye"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

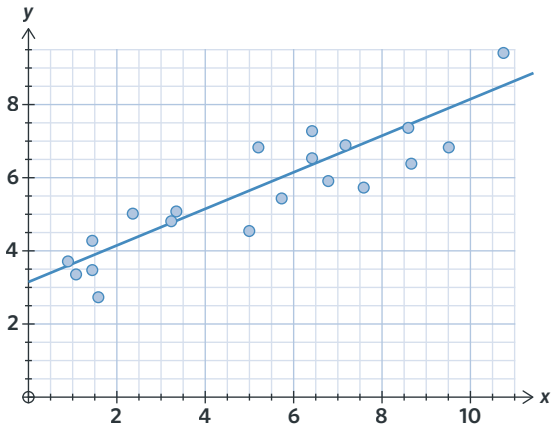
1. Drawing the line of best fit for a graph

Many real life relationships are approximately linear, and can be represented by a linear equation. A linear equation that shows the general trend of a data set is called a line of best fit. Lines of best fit are drawn so that approximately half the data points lie below the line, and half above.

Calculators can find the optimum line of best fit that best represents the data. However, when finding the line of best fit by eye, drawing the optimum line of best fit is very hard. In exams, marks are awarded for all lines of best fit that are approximately correct.

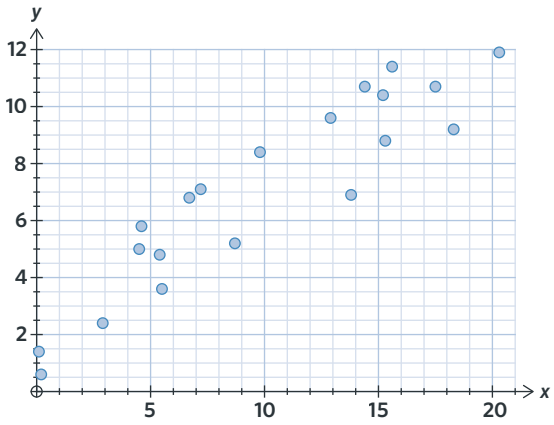
Example

The scatterplot below shows a line of best fit drawn for the data. The line follows the general trend of the data and approximately half the data points lie below the line, and half above.



WORKED EXAMPLE 11 *(1 mark)*

Draw a line of best fit for the following scatterplot.



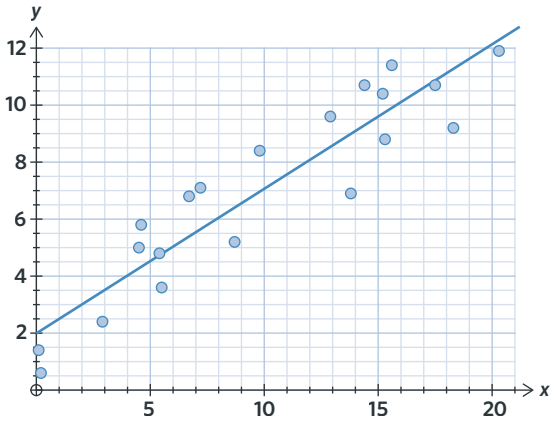
SOLUTION

Step 1 Determine the general trend in the data.

As the x-values increase, the y-values increase, so the line of best fit must have a positive gradient.

Step 2 Use a ruler to draw a line of best fit so that approximately half the data points lie below the line, and half above.

NOTE: As stated above, although calculators can show us the most accurate line of best fit, lines of best fit drawn by hand are expected to vary slightly. Any answers similar to this one will be given full marks in an exam.



2. Finding the equation of the line of best fit for a graph

The equation of the line of best fit for a graph can be found by calculating the gradient and y-intercept of the line, using two points that lie on the line.

The gradient can be found by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

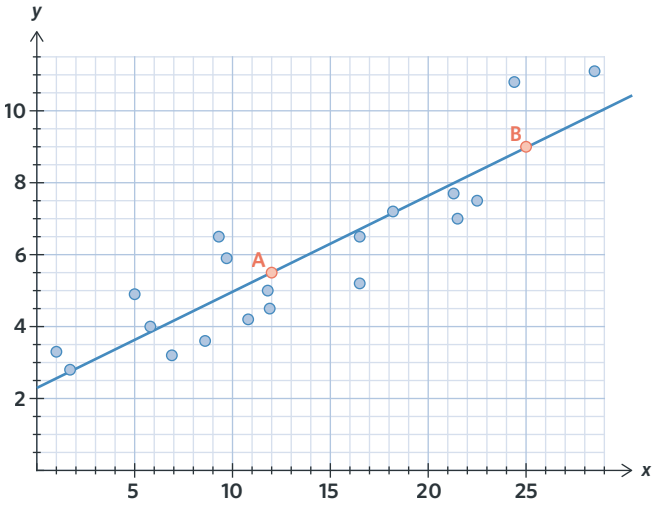
The y-intercept can be found by substituting one point into the equation:

$$y = mx + c$$

and finding the value of c.

WORKED EXAMPLE 12 *(2 marks)*

Find the equation of the line of best fit below, using the points labelled A and B.



SOLUTION

Step 1 Find the coordinates of point A.

(12, 5.5)

Step 2 Find the coordinates of point B.

(25, 9)

Step 3 Find the gradient of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5.5}{25 - 12} = \frac{3.5}{13} = \frac{7}{26}$$

✓ *1 mark for finding the gradient of the line*

Step 4 Find the y-intercept of the line.

Substitute the point $(25, 9)$ into the equation $y = mx + c$.

$$9 = \frac{7}{26} \times 25 + c$$

$$9 = \frac{175}{26} + c$$

$$c = 9 - \frac{175}{26}$$

$$c = \frac{59}{26}$$

Step 5 Substitute the values of m and c into the equation.

$$y = \frac{7}{26}x + \frac{59}{26}$$

The equation of the line of best fit is $y = \frac{7}{26}x + \frac{59}{26}$.

✓ *1 mark for finding the equation*

Questions 9D *Fitting linear models by eye*

Refresher question

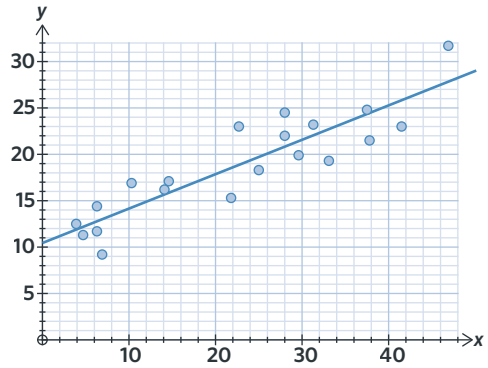
Q1. What is the gradient of the line which passes through the points $(2, 7)$ and $(-4, 12)$?

1. Drawing the line of best fit for a graph

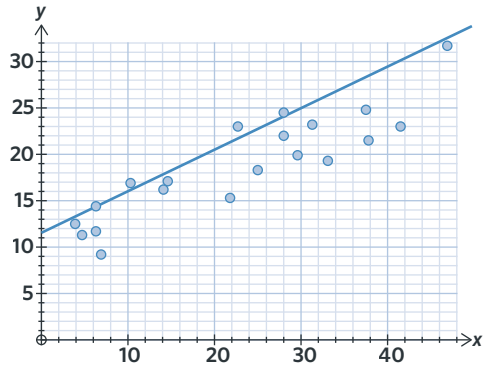
Q2. Which of the following is the most appropriate line of best fit?

Skill

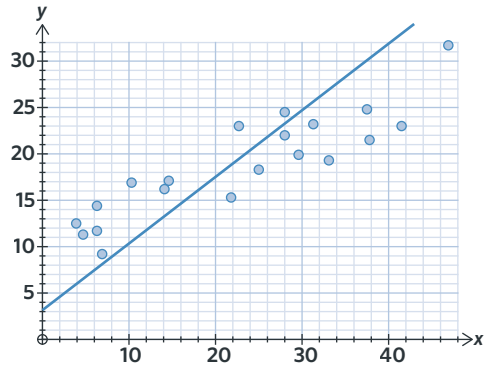
A.



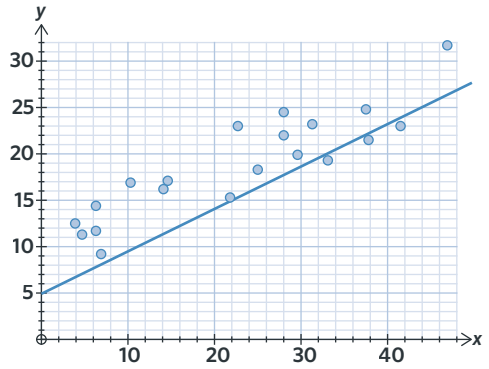
B.



C.



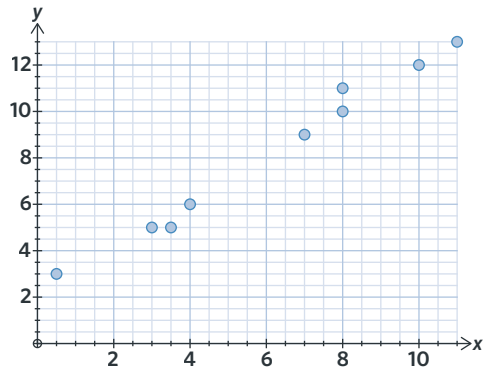
D.



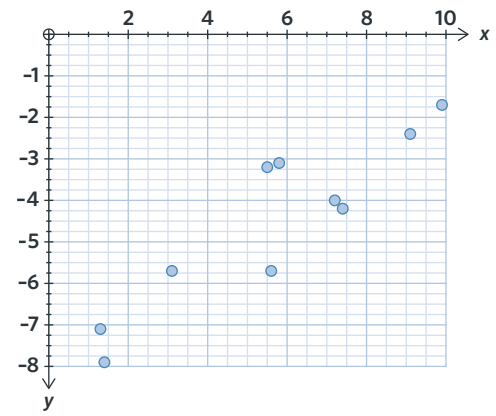
Q3. Draw a line of best fit for each of the following scatterplots.

Skill

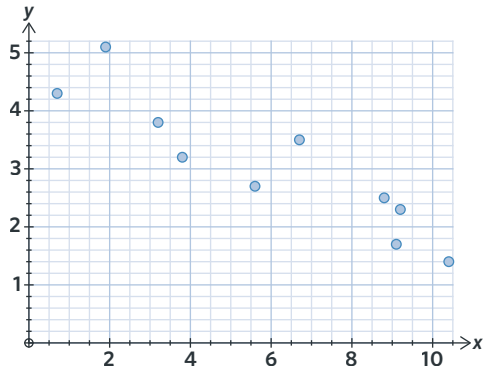
a)



c)



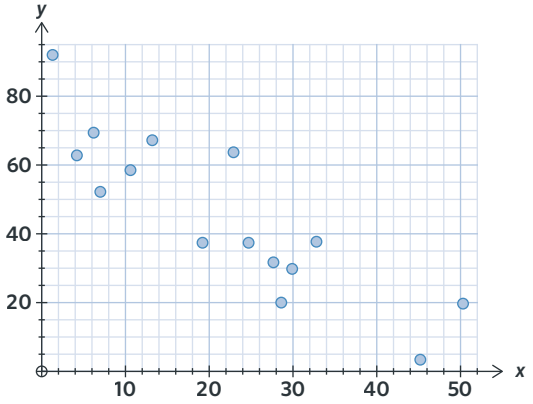
b)



Check your understanding

Q4. Application

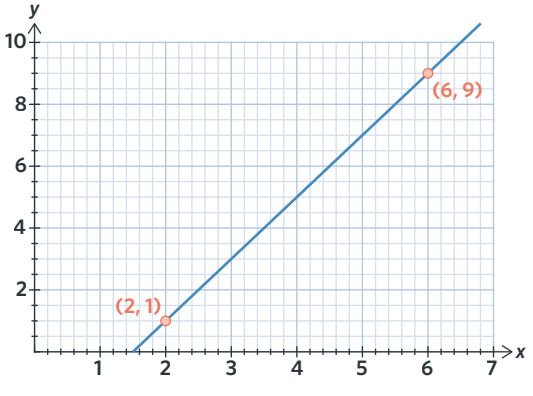
Two students are asked to draw a line of best fit for the data below. Michael draws the line through the coordinates (0, 80) and (50, 20), whereas Holly draws the line through the coordinates (10, 60) and (40, 20). Who drew the more appropriate line of best fit?



2. Finding the equation of the line of best fit for a graph

Q5. Skill

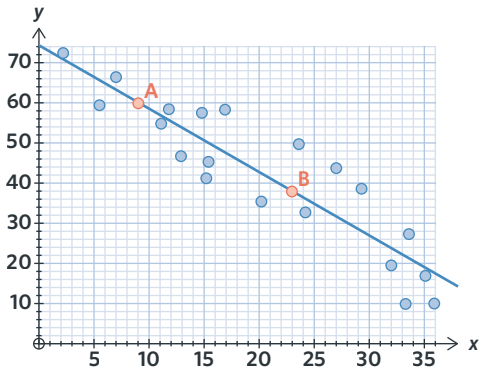
Find the equation of the following line, using the two labelled points.



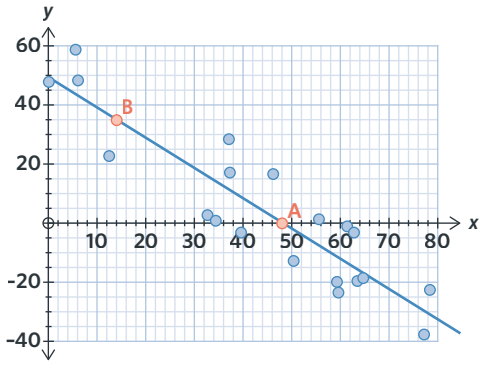
Q6. Find the equations of the lines of best fit below, using the points labelled A and B.

Skill

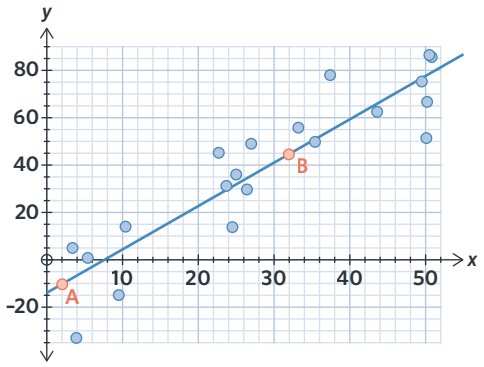
a)



b)



c)



Check your understanding

Q7. The speed, S (km/hr), of a car after t seconds has the line of best fit $S = 6.2t + 7$. Which of the following coordinates could not have been used to find the line of best fit?

Application

A. (2, 19.4) and (0.5, 10.1)

B. (0.7, 11.34) and (3.9, 31.18)

C. (8.5, 59.7) and (2.1, 20.02)

D. (3.3, 20.46) and (6.3, 39.06)

Q8. Which of the following equations is the most appropriate line of best fit for the following scatterplot?

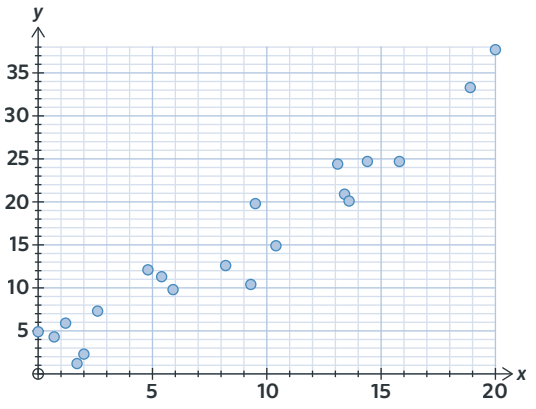
Skill

A. $y = 2x$

B. $y = 2 + 1.1x$

C. $y = 2 + 1.6x$

D. $y = 5 - 0.6x$



Joining it all together

Q9. The table below shows the height and weight of one particular child during their first six years of life.

Application
4 marks

Height (cm)	50	77	87	95	102	108	116
Weight (kg)	4	9	13	15	17	19	22

a) Draw a scatterplot representing the data, with height on the x-axis, and weight on the y-axis.

b) If the line of best fit passes through the points (50, 3) and (90, 14), draw the line of best fit on the scatterplot, and calculate the equation of the line, using h as height and w as weight.

c) Using the line of best fit, find how much the child weighed when they were 100 cm tall. Round your answer to the nearest gram.

d) Using the line of best fit, find how tall the child was when they weighed 30 kg. Round your answer to the nearest millimetre.

Q10. Charlie makes a mistake while working out the equation of the line of best fit for some data. Charlie chooses two points on the line, however he mistakenly uses the coordinate (3, 7) instead of (3, 9). He works out the equation of the line to be $y = 2x + 1$.

Application
2 marks

If the x coordinate of the other point he used was -1, what should the equation of the line have been?

Questions from multiple lessons

Q11. Hamish is a carpenter.

Difficulty:

He charges a call-out fee of \$140, plus \$90 for each hour of work.

1 mark

The equation that represents the total cost, \$$C$, Hamish charges, for t hours of carpentry is

A. $C = 230t$

B. $C = 90t$

C. $C = 140 + 90t$

D. $C = 140t$

E. $C = 90 + 140t$

VCAA 2018 Exam 1 Module 4: Graphs and relations Q2 – Adapted

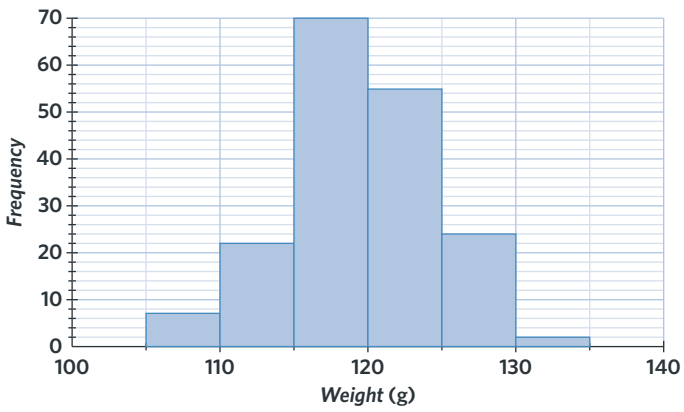
Q12. The histogram to the right shows the distribution of the *weight* of 180 ducks, in grams.

Difficulty:

Year 10
1 mark

Ducks weighing 125 grams or higher are adopted as pets.

The percentage of ducks adopted is closest to



A. 13%

B. 14%

C. 17%

D. 24%

E. 26%

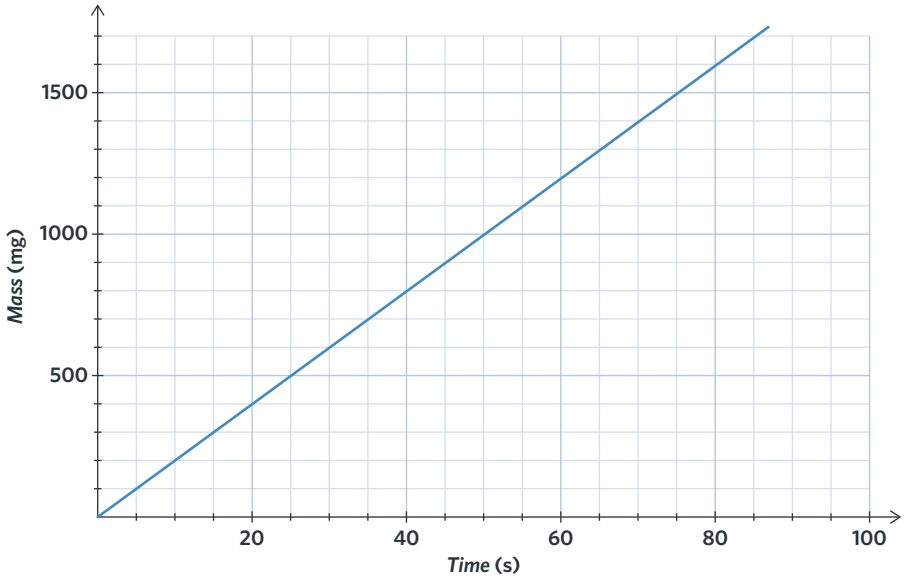
VCAA 2017NH Exam 1 Data analysis Q2 – Adapted

Q13. A researcher is trying to determine the time taken for different masses of an unknown compound to completely break down and dissolve in strong acid.

Difficulty:

Year 10
2 marks

The graph below displays the relationship between *time* in seconds and *mass* in mg.



a) Lara has an unknown mass of the compound. She does not have a scale so she decides to test and see how long it takes to completely dissolve. It dissolves in 60 seconds.
How much of the compound does she have? (1 mark)

b) The slope of this graph is the rate at which the compound dissolves per second.
How much can she dissolve in one second? (1 mark)

VCAA 2015 Exam 2 Module 3: Graphs and relations Q2 – Adapted

LESSON 9E

Linear model interpolation

The key skills you will learn in this lesson are:

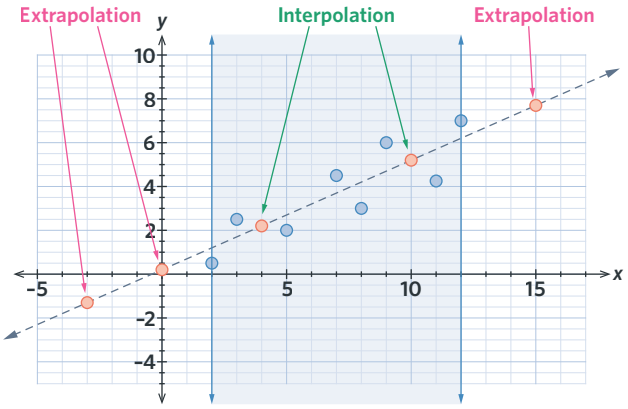
1. Interpolation using graphs
2. Interpolation using formulas

VCAA key knowledge point:

"the interpretation of the parameters of a linear model and its use to make predictions, including the issues of interpolation and extrapolation"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

Interpolation and extrapolation are processes that use existing data to make predictions about new data points. The difference between them is that interpolation works within the domain of the data. This means interpolation predicts points that have x values between the left-most and right-most data points. Extrapolation works outside the domain of the data, meaning it predicts points with x values greater than any data point or lower than any data point. This lesson will focus on interpolation.



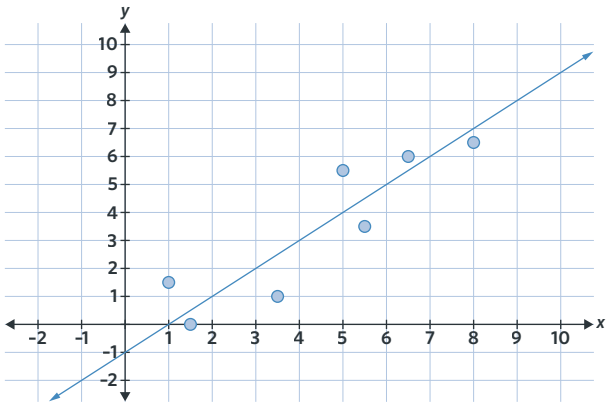
In the graph to the left, interpolation uses the existing data points (**blue**) to predict the new data points (**orange**) within the domain. Extrapolation uses the existing data points to predict the new data points (**orange**) outside the domain.

1. Interpolation using graphs

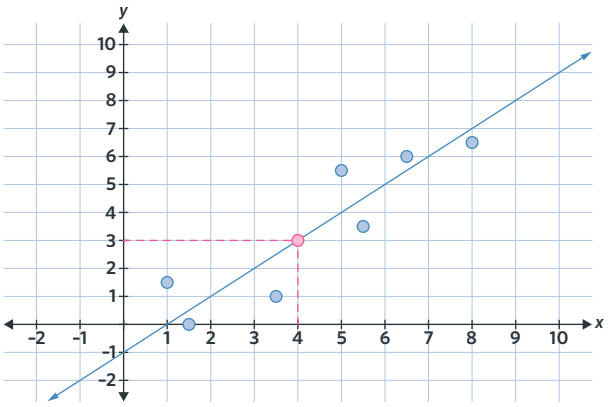
Interpolation can be performed visually by using the line of best fit to predict the values of new data points. If a line of best fit isn't given, one can be drawn by hand.

Example

Look at the graph below. We have the points (1, 1.5), (1.5, 0), (3.5, 1), (5, 5.5), (5.5, 3.5), (6.5, 6), and (8, 6.5). A line of best fit is drawn, representing the points.



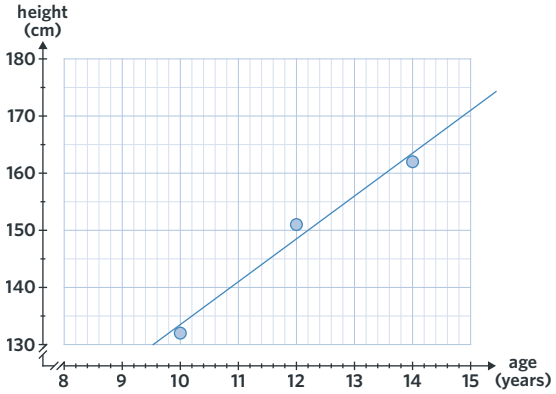
The line of best fit can be used to make an educated guess about the y-value when $x = 4$. In this case, y should be roughly equal to 3, as shown in **pink**.



WORKED EXAMPLE 13 *(1 mark)*

Below is a plot of Clara's height, measured on her 10th, 12th, and 14th birthdays.

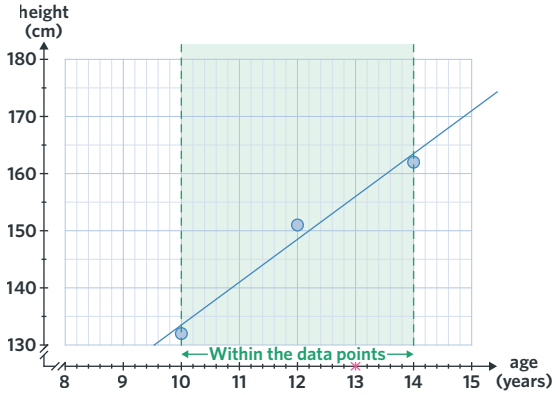
Based on this data, use interpolation to estimate her height on her 13th birthday.



SOLUTION

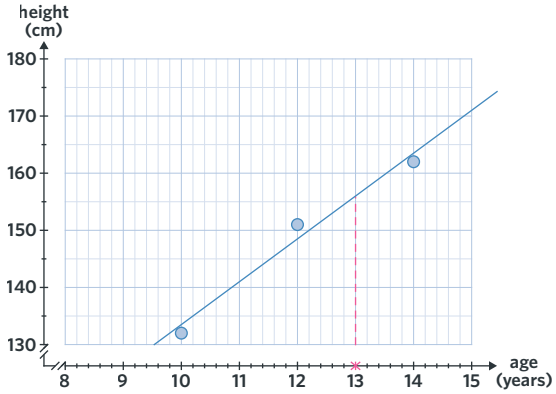
Step 1 Check the desired data point is within the domain of the existing data.

The data points cover values from $x = 10$ to $x = 14$. The data point we want is at $x = 13$, which is within the existing data points.

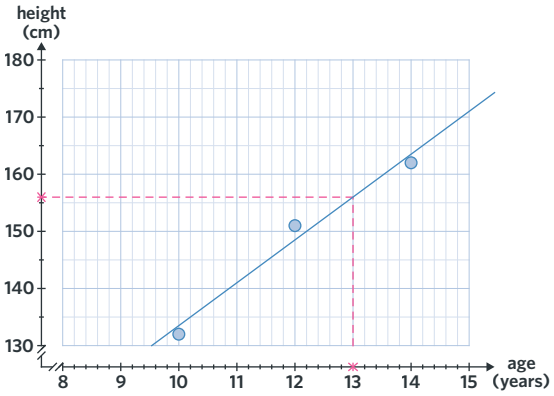


Step 2 Draw a vertical line up from the desired point's x coordinate to the line of best fit.

This line goes up from $x = 13$.

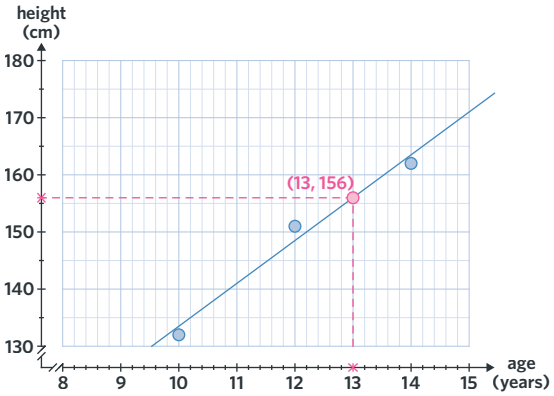


Step 3 Draw a line horizontally from the line of best fit to the y-axis.



Step 4 Find the y-intercept with this line, state the new data point and interpret this information.

The horizontal line crosses the y-axis at $y = 156$. The interpolated data point is (13, 156). We have estimated that when Clara was 13 years old, she was 156 centimetres tall.



2. Interpolation using formulas

Interpolation can also be performed using the formula of a line of best fit. The x values of the desired points are substituted into the formula to find the desired y values.

Example

For the line of best fit $y = 3x - 4$, where the data point furthest to the left is $x = -2$ and the data point furthest to the right is $x = 5$, we can use interpolation to find any point between $x = -2$ and $x = 5$.

If we wanted to find the data point where $x = 3$, substitute $x = 3$ into the formula. This gives us $y = 5$, and we can write the interpolated data point as $(3, 5)$.

WORKED EXAMPLE 14 *(1 mark)*

During a rocket launch, the following data points were noted. At $t = 0$ the rocket was stationary, and at $t = 10$ the rocket was travelling at 150 metres per second. This data can be modelled by the formula below, which gives v, the speed of the rocket, t seconds after launch. Estimate the speed of the rocket eight seconds after launch.

$v = 15t$

SOLUTION

Step 1 Check the desired data point is within the domain of the existing data.

The data point we want is at $t = 8$. The data points in the model are at $t = 0$ and $t = 10$, so our data point is within the existing data.

Step 2 Substitute the information from the new data point into the formula.

Here we let $t = 8$ and substitute this into the formula.

$v = 15 \times 8$

Step 3 Solve for the missing variable, state the data point, and interpret the prediction.

$v = 15 \times 8$

$v = 120$

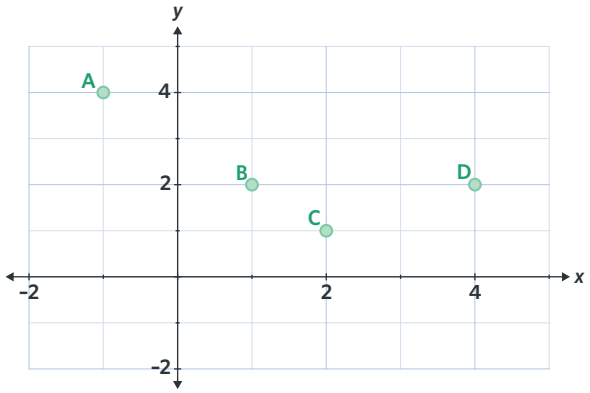
The interpolated data point is $(8, 120)$.

The estimated speed of the rocket eight seconds after launch is 120 metres per second.

Questions 9E *Linear model interpolation*

Refresher question

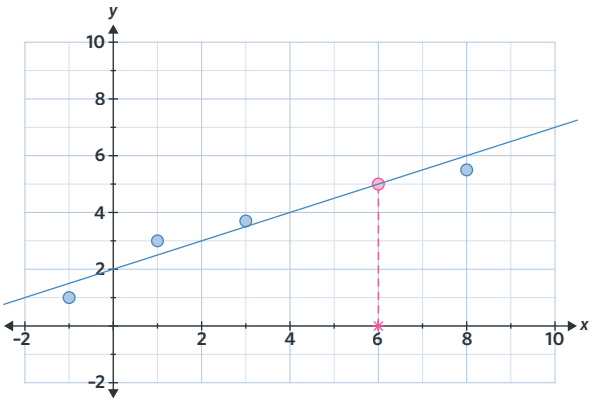
Q1. Which of the following points is at $(2, 1)$?



1. Interpolation using graphs

Q2. Skill

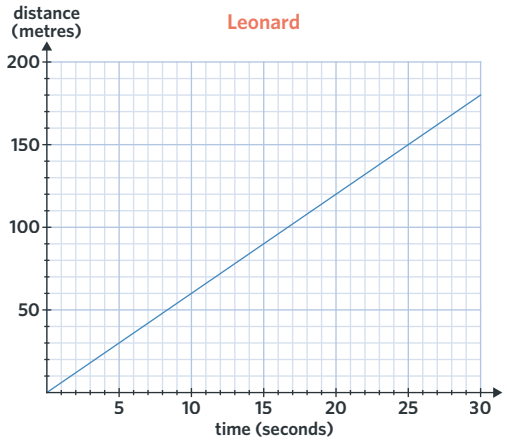
The following graph shows four data points and a line of best fit. Use interpolation to estimate the value of y when $x = 6$. The pink dot is there to guide you.



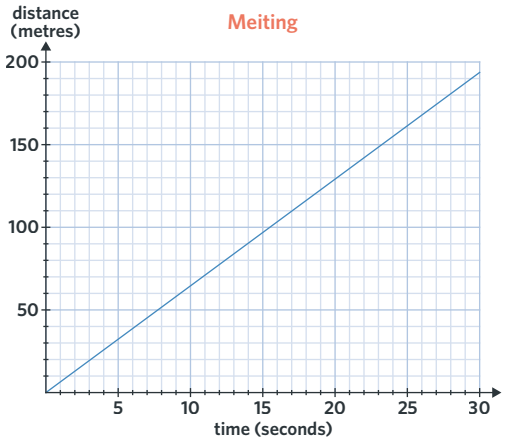
Q3. Skill

Three students, Leonard, Meiting, and Nina, are participating in a 30-second running race. The following three graphs show the lines of best fit for their distances from the starting line versus time for each student, based on their starting and finishing positions. Nina was given a twenty metre head-start because she is a year younger than the others. Find how far each student was from the start line after twenty seconds.

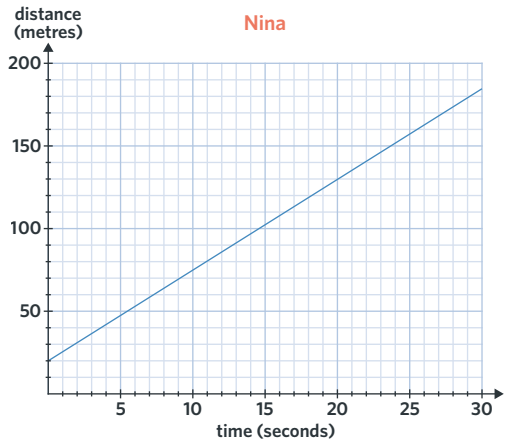
a)



b)



c)

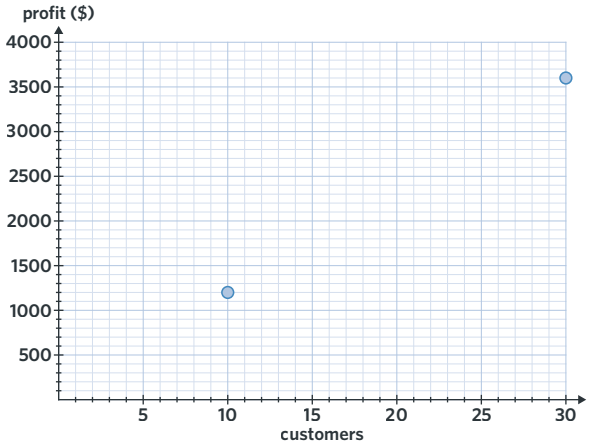


d) Who was furthest from the start line after 20 seconds?

Check your understanding

Q4. Skill

A store manager noticed that when ten customers came into her store in a day, a profit of $1200 was made, and when 30 customers came into her store in a day, a profit of $3600 was made. This information is shown below. Draw a line of best fit between the points, and use interpolation to estimate how many customers need to enter the store in a day to make a $3000 profit.



Q5. Application

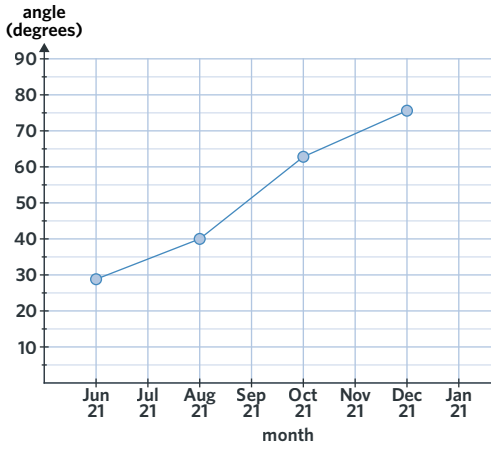
The highest point of the sun in the sky in Melbourne on the 21st day of June, August, October, and December is given in this table. This data has been used to construct the graph below.

Date	Highest elevation of Sun above horizon
June 21	28.8°
August 21	40.0°
October 21	62.8°
December 21	75.6°

a) Use interpolation to estimate the highest elevation of the sun on July 21, September 21, and November 21. Round your answers to the nearest 5 degrees.

b) On which of these three days is the sun the highest?

c) Why can't interpolation be used to estimate the highest elevation on January 21?



2. Interpolation using formulas

Q6. Skill

Six days after it was planted, Harry's bamboo plant was 60 cm tall, and fifteen days after it was planted, it was 150 cm tall. The line of best fit for these two points is given below, where h is the height in centimetres, and d is the number of days since the bamboo was planted.

$$h = 10d$$

Harry is trying to interpolate how tall it was after nine days. Complete the working out below to find the height after nine days.

Nine days: $d = 9$
Left-most point: $x = 6$
Right-most point: $x = 15$
Is $d = 9$ within the data points? Yes
$h = 10 \times d$
Let $d = 9$
$h = 10 \times 9$
$h =$ ☐ cm

Q7. Application

Caitlin completed ten practice exam papers the night before her maths exam, and got 100%. Hannah did not complete any practice exam papers the night before the exam, and got 30%. The line of best fit using these data points is below, where S is the exam score as a percentage and p is the number of practice exam papers.

$$S = 30 + 7p$$

Assuming the relationship between practice exam papers and maths exams holds true for the rest of the class, estimate the scores of the following students if:

a) Anna completed four practice exam papers?

b) Zephyr completed six practice exam papers?

c) Jessica completed two practice exam papers?

d) If a score of 50% means a pass, what is the minimum number of practice exam papers that need to be completed to pass the exam?

e) Do you think it is reasonable to interpolate exam scores based on the number of exam papers completed the night before the exam?

Check your understanding

Q8. Application

Class 11C's teacher, Mr. Yu, has been studying pets and their owners. The formula of the line of best fit that best models his data is below. It relates the number of pets a student has, p, with the number of hours that student spends outside every day, T. The data includes students with zero pets up to students with five pets.

$$T = 1.5 + p$$

a) Mr. Yu then asked some of his students to use the number of pets they have to see how much time the model expected them to spend outside every day. Which student interpolated their data incorrectly?

A. Dionisia: 2 pets, 3.5 hours outside

B. Tim: 1 pet, 2.5 hours outside

C. Rhyannon: 4 pets, 5.5 hours outside

D. Michael: 3 pets, 5.5 hours outside

b) Matthew has 6 pets. Can he use the model to interpolate how long he spends outside every day?

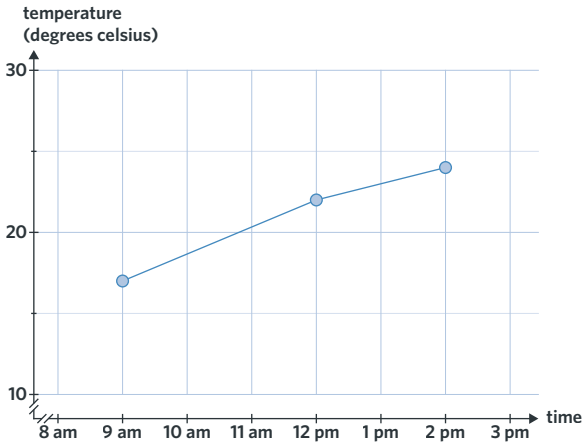
Joining it all together

Q9. Skill 2 marks

The following chart shows the relationship between three measurements of temperature throughout a day.

a) Can interpolation be used to estimate the temperature at 10 am?

b) Can interpolation be used to estimate the temperature at 3 pm?



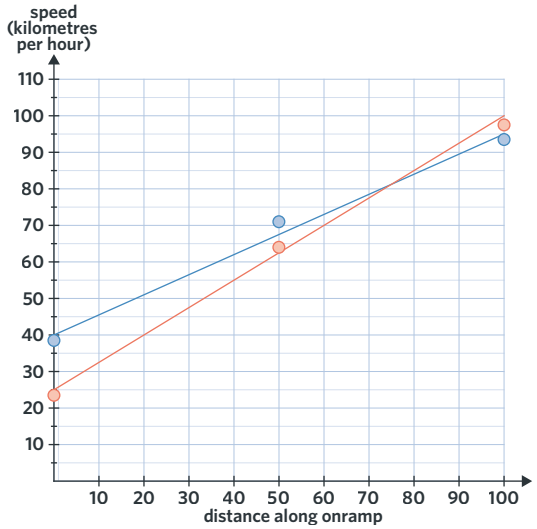
Q10.

Application
3 marks

A Ford Falcon and a Holden Commodore entered the Monash Freeway via a 100-metre freeway onramp. Their speeds at ten, fifty, and ninety metres along the onramp were recorded. The data was plotted, and the formulas for lines of best fit are below. F is the Ford's speed, H is the Holden's speed, and d is the distance along the onramp. F and H are measured in kilometres per hour, and d is in metres.

$$F = 40 + 0.55d$$

$$H = 25 + 0.75d$$



a) Use the formulas for line of best fit to estimate the speeds of the cars 30 metres along the onramp.

b) Use the formulas for line of best fit to estimate the speeds of the cars 80 metres along the onramp.

c) Use the graph to find at what distance along the onramp they were travelling at the same speed.

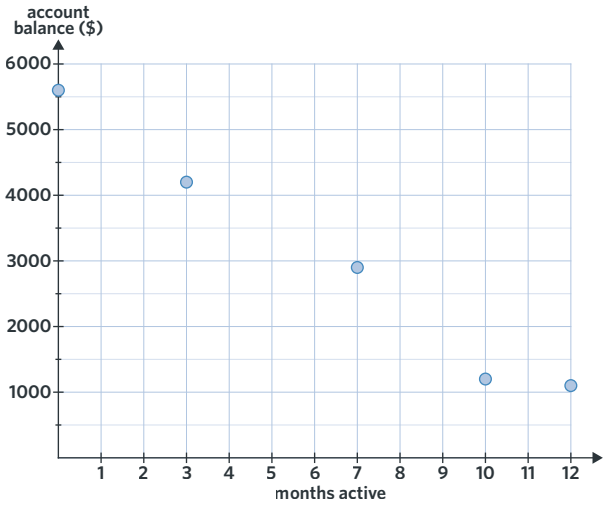
Q11.

Application
6 marks

On January 1 last year, Arina opened a savings account, A, and an everyday spending account, B. Arina put \$1000 in account A and \$5600 in account B when she opened them. She didn't know if the bank paid interest monthly, quarterly, or yearly, so she didn't check the balance of account A again until one year later, at which point it had \$3600. She calculated a line of best fit for this information. A represents the amount of money in account A after m months.

$$A = 1000 + 200m,\ 0 \leq m \leq 12$$

Arina checked the balance in account B at four other times during the year, as seen in the graph.



a) State whether the balance of each account is increasing or decreasing. (1 mark)

b) Given the line of best fit for account B goes through the points (5, 3500) and (10, 1500) plot the line of best fit on the graph. (1 mark)

c) Find a formula for the line of best fit. Use B to represent the balance in account B after m months. (1 mark)

d) Use the line of best fit formulas to interpolate the balance of each account after 9 months. (2 marks)

e) By plotting another line on the graph, find the time when both accounts contained the same amount of money. (1 mark)

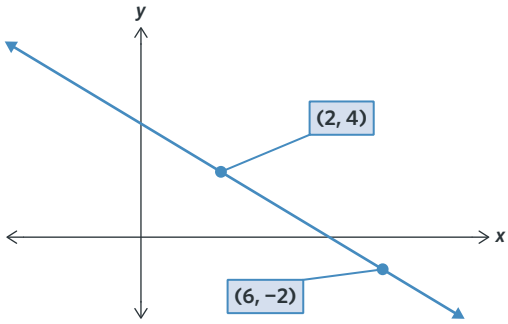
Questions from multiple lessons

Q12.

Difficulty:

1 mark

A line passes through the points $(2, 4)$ and $(6, -2)$, as shown.



What are the coordinates of the line's x-intercept?

A. $(0, 7)$

B. $(4.5, 0)$

C. $(0, 2)$

D. $\left(\frac{11}{3}, 0\right)$

E. $\left(\frac{14}{3}, 0\right)$

VCAA 2016 Exam 1 Module 4: Graphs and relations Q3 – Adapted

Q13.

Difficulty:

1 mark

Kelly wants to buy a show dog and deposits $2000 into a savings account with an interest rate of 1.8% per annum, compounding monthly. Which one of the following recurrence relations can be used to determine the amount in the savings account, S_n, after n months?

A. $S_0 = 2000, \quad S_{n+1} = S_n + 36$

B. $S_0 = 2000, \quad S_{n+1} = 1.018 \times S_n$

C. $S_0 = 2000, \quad S_{n+1} = 1.0015 \times S_n$

D. $S_0 = 2000, \quad S_{n+1} = 1.8 \times S_n$

E. $S_0 = 2000, \quad S_{n+1} = S_n + 3$

VCAA 2017NH Exam 1 Recursion and financial modelling Q18 – Adapted

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Q14.

Difficulty:

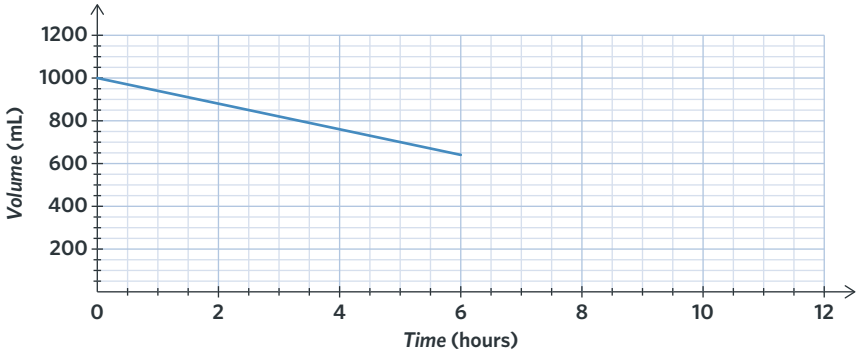
3 marks

A patient is being administered an IV drip.

When the bag was first administered, it contained 1000 mL of saline solution.

After six hours, there is 640 mL of saline solution left.

The volume of saline solution remaining in the bag followed a linear trend as shown in the following graph.



a) Determine the equation of the line shown in the graph. (2 marks)

b) Assume this linear trend continues. How much longer will it take for the IV drip to be completely administered? Give your answer in hours and minutes. (1 mark)

VCAA 2006 Exam 2 Module 3: Graphs and relations Q2 a,b – Adapted

LESSON 9F

Linear model extrapolation

The key skills you will learn in this lesson are:

1. Extrapolation using formulas
2. Extrapolation using graphs

VCAA key knowledge point:

"the interpretation of the parameters of a linear model and its use to make predictions, including the issues of interpolation and extrapolation"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

The general process of extrapolation is identical to that of interpolation, except that extrapolation estimates points outside the domain of the data.

1. Extrapolation using formulas

If the formula for a line of best fit is known, it is relatively straightforward to extrapolate the data to new data points. If the formula for the line of best fit isn't known, it will have to be determined by hand.

As the line of best fit is created to minimise error for the points in the original data set, estimating points outside the data set (extrapolation) is much less accurate than interpolation.

Example

The line of best fit for the tail length of baby red kangaroos in millimetres, t, versus their age in days, d, is given by the following formula. The model was created using kangaroos that were between 20 and 100 days old.

$$t = d + 5$$

Using extrapolation, a 120 day-old kangaroo should have a tail length of 125 millimetres. A 365 day-old kangaroo should have a tail length of 370 millimetres.

WORKED EXAMPLE 15 *(2 marks)*

The line of best fit for some data on the tail length of baby swamp wallabies, t, measured in millimetres, versus age in days, d, is given below. The wallabies in the data set were between 20 and 100 days old.

$$t = 0.5d + 5$$

Use the line of best fit to extrapolate the tail length of a baby swamp wallaby at 120 days and 200 days old.

SOLUTION

Step 1 Substitute the desired values into the formula.

Let $d = 120$.

$t = 0.5 \times 120 + 5$

$t = 65$ mm

Let $d = 200$.

$t = 0.5 \times 200 + 5$

$t = 105$ mm

✓ *1 mark for calculating the values*

Step 2 Interpret.

We have estimated a swamp wallaby's tail will be 65 mm long at day 120.

We have estimated a swamp wallaby's tail will be 105 mm long at day 200.

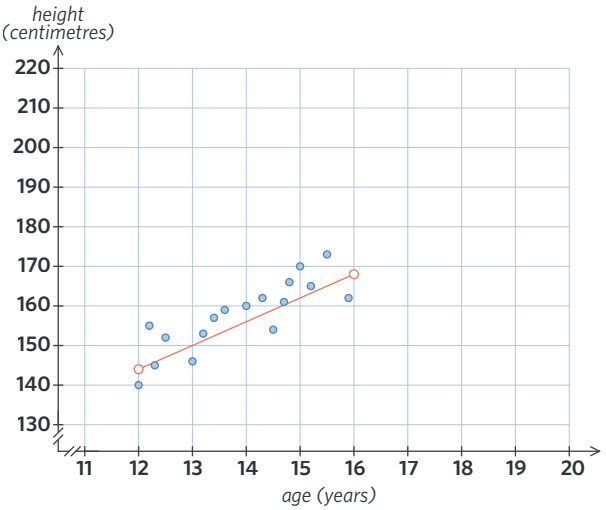
✓ *1 mark for interpreting the values*

2. Extrapolation using graphs

Extrapolation can be done using graphs in two ways. The first is to extend the line of best fit on a graph, and read the values of data points off the graph. The second is to take the line of best fit drawn on a graph, and determine its formula.

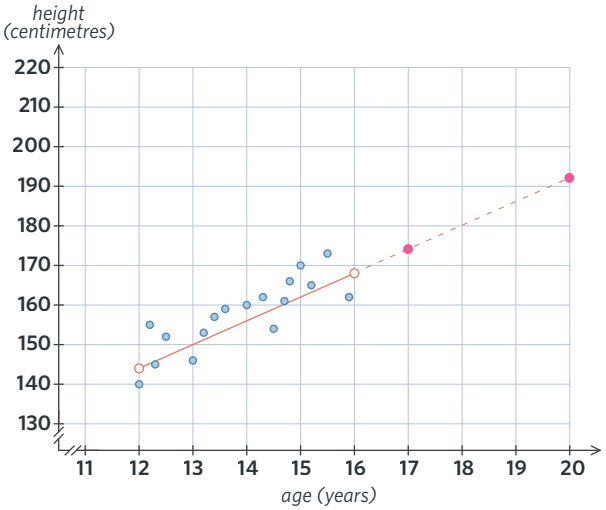
Example

The line of best fit for the relationship between age and height is given below, for a group of students between 12 and 16 years old at a school. Blue dots represent each student measured, and the red line is the line of best fit.



Extrapolating the data to age 17, we find the average height is 174 cm.

Extrapolating the data to age 20, we find the average height is 192 cm. This value is very inaccurate; the actual average height for 20-year-olds is around 169 cm.



WORKED EXAMPLE 16 (2 marks)

This worked example relates to the information above on average student height in a school.

a) Use the graph to estimate the average height of 11-year-olds.

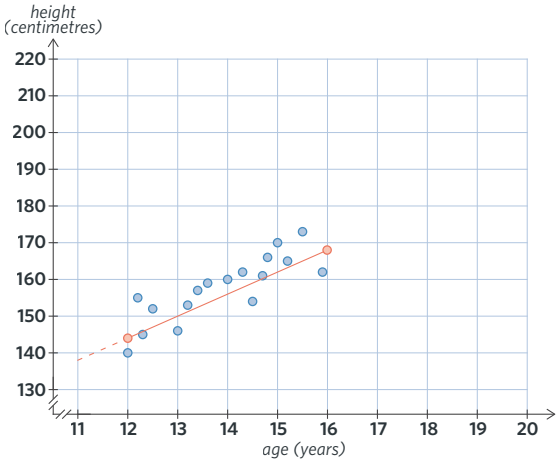
b) Use a formula for the line of best fit to estimate the average height of 11-year-olds.

SOLUTION

a) Use the graph to estimate the average height of 11-year-olds.

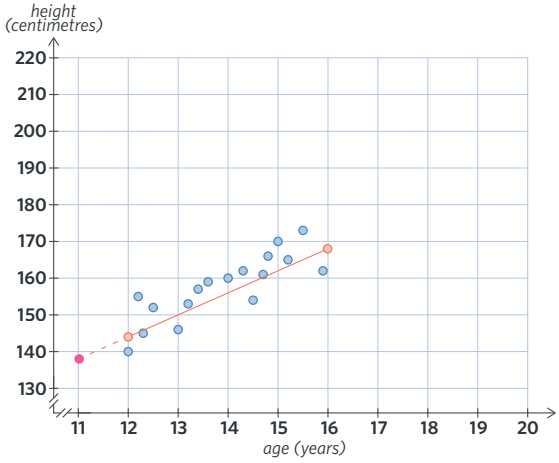
Step 1 Extend the line of best fit towards the desired point.

Here, we extend it towards 11 on the x-axis.



Step 2 Mark the data point on the line of best fit, and find the relevant y-value.

The y-value of the point is 138.



Step 3 Interpret.

The average height of 11-year-olds is 138 cm.

b) Use a formula for the line of best fit to estimate the average height of 11-year-olds.

Step 1 Pick two points on the line of best fit, and use them to write the line of best fit equation.

We'll use the start and end points, (12, 144) and (16, 168).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{168 - 144}{16 - 12}$$

$$m = \frac{24}{4}$$

$$m = 6$$

$$y - y_1 = m(x - x_1)$$

$$y - 144 = 6(x - 12)$$

$$y = 6x - 72 + 144$$

$$y = 6x + 72$$

Step 2 Substitute the x-value of the desired point into the formula.

Here we substitute $x = 11$.

$y = 6x + 72$

$y = 6 \times 11 + 72$

$y = 138\text{cm}$

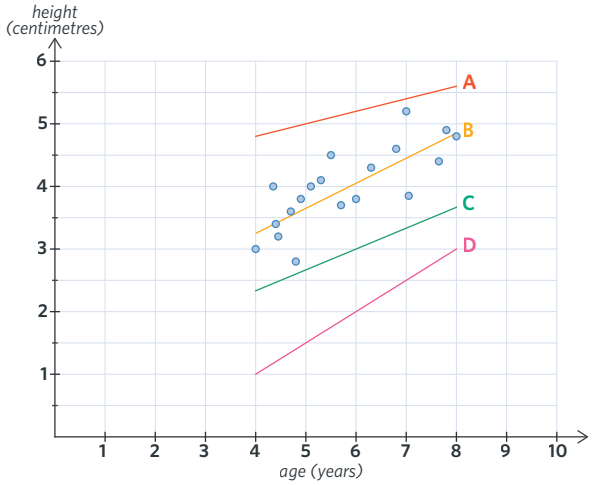
Step 3 Interpret.

The average height of 11-year-olds is 138 cm.

Questions 9F *Linear model extrapolation*

Refresher question

Q1. Which of the following lines best represent the data?



1. Extrapolation using formulas

Q2. Skill

The line of best fit of a data set is given by the following formula.

$$h = 3t + 2, 1 \leq t \leq 5$$

Helene wanted to take the line of best fit and extrapolate it to the point $t = 6$. Fill in the missing answer in her working out.

Let $t = 6$.
$h = 3t + 2$
$h = 3 \times 6 + 2$
$h =$ ☐

Q3. Skill

a) The line of best fit of a data set is given by $h = 5t - 3$, $2 \leq t \leq 4$. The data points range from $t = 2$ to $t = 4$. Use extrapolation to find the point where $t = 6$.

b) The line of best fit of a data set is given by $y = 2x + 7$, $-1 \leq x \leq 3$. The data points range from $x = -1$ to $x = 3$. Use extrapolation to find the point where $x = -2$.

c) The line of best fit of a data set is given by $z = 35d - 9$, $0 \leq d \leq 6$. The data points range from $d = 0$ to $d = 6$. Use extrapolation to find the point where $d = 11$.

Check your understanding

Q4. Application

Nicholas is recording the growth of bean sprouts, in centimetres, over their first ten days after sprouting. Unfortunately, on day 10, he spilt water all over the data in his notebook while watering the beans. The only two data points left visible are (2, 1.5) and (8, 6.5). Time is on the x-axis.

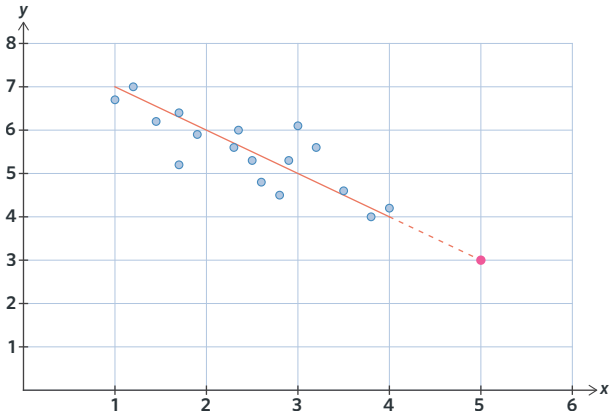
a) Find a line of best fit for the length of the sprouts in centimetres, l, in terms of their age in days, d.

b) Use your answer from part **a)** to extrapolate the length of the sprouts at 1 day old, 10 days old, and 20 days old, rounded to two decimal places. Comment on the accuracy of your answers.

2. Extrapolation using graphs

Q5. Skill

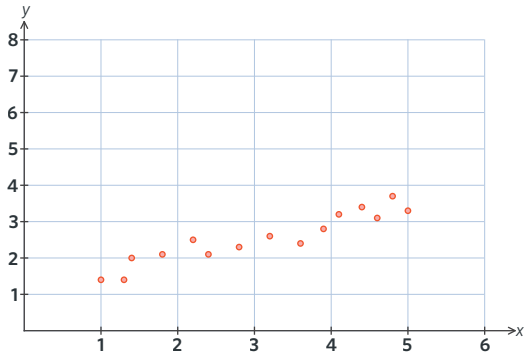
The line of best fit for the following data set has been drawn on the graph, and extrapolated to $x = 5$. Use the graph to find the value of y when $x = 5$.



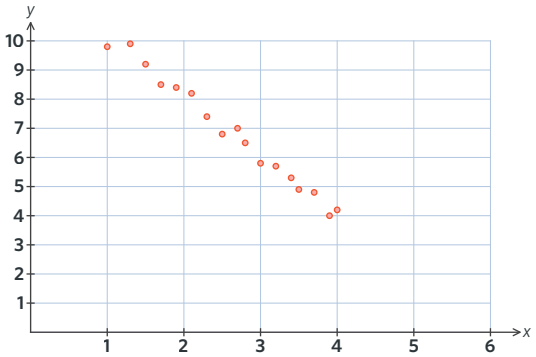
Q6. Skill

Draw a line of best fit on the following graphs. Extrapolate the line of best fit to the given x value, and state the point.

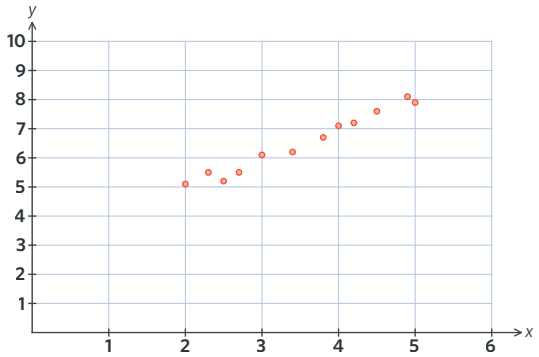
a) $x = 6$



b) $x = 5$



c) $x = 1$



Check your understanding

Q7. Application

Maya is a courier based in Bendigo. She loves statistics, and records the average driving speed of each delivery she makes. Below is a table of the average driving speed compared to the distance of the delivery.

Delivery distance (km)	16	21	28	45	53	62	75	80	100	120
Average speed (km/h)	45	58	52	66	58	67	81	68	82	87

a) Plot these points on a chart, ensuring the x-axis covers the values 0 at 200 and the y-axis covers the values 0 to 120.

b) Draw a line of best fit, and extrapolate Maya's average speeds over distances of 150 and 200 kilometres.

c) Comment on the plausibility of these answers.

Joining it all together

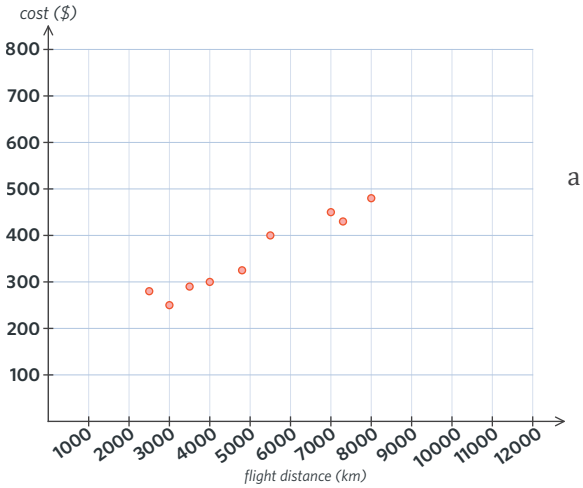
Q8. Application 3 marks

The following plot shows the relationship between flight distance and flight cost for various flights.

Vincent wants to use this data to estimate the cost of a 6000 km flight, 9000 km flight, and a 12 000 km flight.

a) Which of these points can be found by interpolation and which can be found by extrapolation? (1 mark)

b) Comment on the difference in accuracy when estimating these three data points. Refer to interpolation and extrapolation in your answer. (Don't actually perform the calculations.) (2 marks)



a

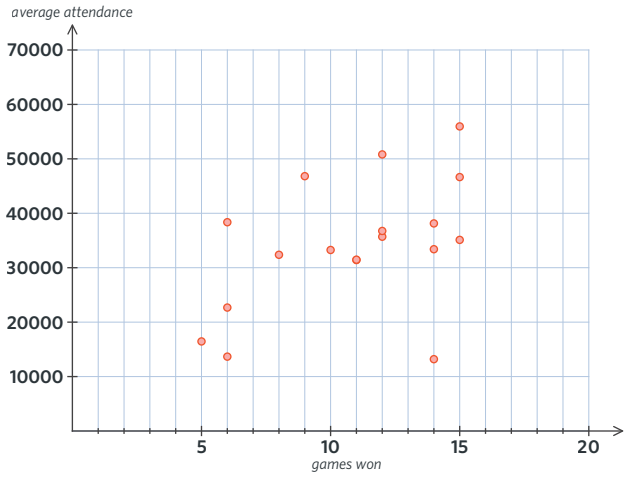
Q9. Application 3 marks

The following chart shows the average attendance figures at home games, A, plotted against the number of wins in the season, w for the 2017 AFL season.

a) Draw a line of best fit using the points (5, 26500) and (15, 39500).

b) Use the two points to find the equation of the line of best fit.

c) Use the equation to extrapolate the average attendance at home games if a team had won 18 games.



Q10. Application 4 marks

Employees at a small restaurant were surveyed on their happiness at work each shift over a week. The ratio of customers to staff working each shift was also recorded. The responses are tabled below.

	Mon PM	Tue PM	Wed PM	Thu PM	Fri PM	Sat AM	Sat PM	Sun AM
Average employee happiness on shift	8.5	8	9	7.5	4.5	8	6.5	7
Ratio of customers to staff	2:1	3:1	5:2	4:1	10:1	9:2	8:1	11:2

a) Plot this information.

b) Draw the line of best fit on the model, which goes through (2, 9) and (10, 5).

c) What is the equation of this line of best fit?

d) Use interpolation and extrapolation to find the expected happiness on shift, when the ratio of customers to staff is 1:1, and 9:1.

Q11. Application 3 marks

Tareq and Ariel are studying the effect of temperature on the number of ice creams sold at a local milk bar. For this question use I as the number of ice-creams sold and T as the temperature in Celsius.

a) Tareq used his line of best fit to estimate the information in the following table. What must the equation of his line of best fit be?

Temperature (°C)	25	32	40
Ice-creams sold	35	56	80

Ariel's line of best fit for her data is below.

$I = 2.5T - 30$

b) Whose model predicts the higher number of ice-creams to be sold on a 30 degree day?

c) Tareq collected his data on days between 12 and 28 degrees Celsius, and Ariel collected her data on days ranging from 15 to 35 degrees Celsius. Whose model is likely to be most accurate for the calculations in part **b)**?

Questions from multiple lessons

Q12. Difficulty:

1 mark

A line of best fit has been fitted to a set of data as shown.

Which point(s) could not have been estimated using interpolation?

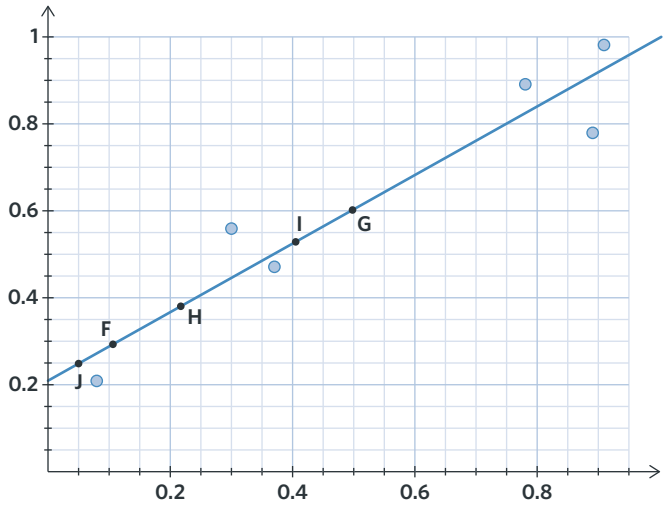
A. J, I, G

B. J

C. J, G

D. F, H

E. F, H, I, G



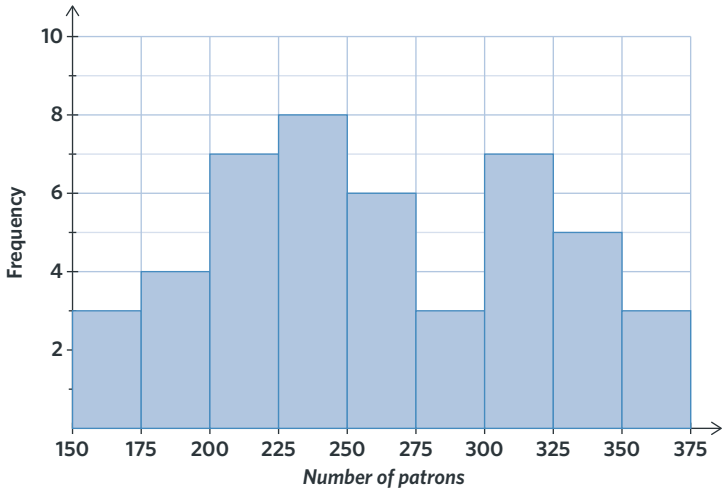
Q13.

Difficulty:

Year 10
1 mark

Angelo's favourite hangout spot is Revolver Upstairs in Prahran. Each night, he counts the number of patrons inside the establishment at a particular time.

The following histogram displays his results over a period of 46 days.



The median number of patrons is

A. at least 200 but less than 225.

B. at least 225 but less than 250.

C. at least 250 but less than 275.

D. at least 275 but less than 300.

E. at least 300 but less than 325.

VCAA 2017NH Exam 1 Data analysis Q3 – Adapted

Q14.

Difficulty:

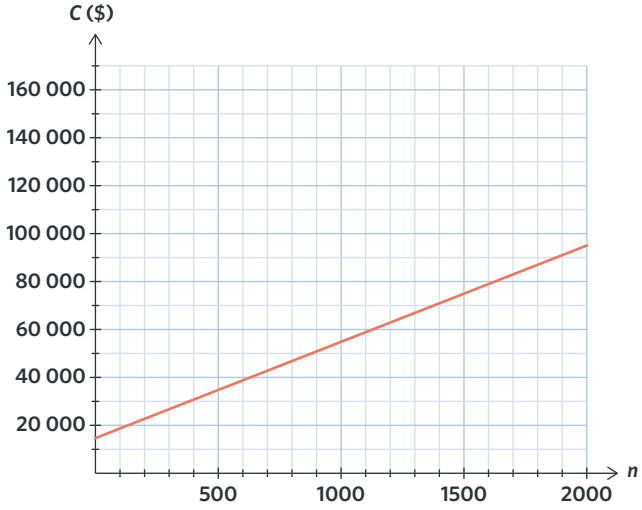
3 marks

Melbourne label Crumprolo makes fashionable, yet practical, laptop bags for high school students.

The cost, C, in dollars, of producing n laptop bags is given by $C = 40n + 15\ 000$.

The revenue, R, in dollars, from selling n laptop bags is given by $R = 80n$.

The cost, C, for the production of n laptop bags is shown in the following graph.



a) Draw the revenue equation line, $R = 80n$, on the same graph as the given cost equation. (1 mark)

b) What profit will Crumprolo make if they sell 1500 laptop bags?
NOTE: $Profit = Revenue - Cost$ (2 marks)

VCAA 2014 Exam 2 Module 3: Graphs and relations Q2 a,b – Adapted

LESSON 9G

Piecewise linear models

The key skills you will learn in this lesson are:

1. Sketching piecewise linear models
2. Functions of piecewise linear models

VCAA key knowledge point:

"use of piecewise linear (line segment) graphs to model and analyse practical situations"

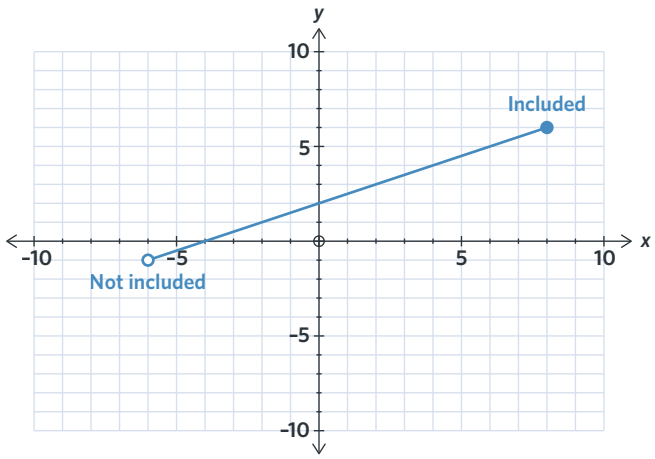
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Sketching piecewise linear models

Piecewise linear models, also known as line segment graphs, are graphs made up of two or more linear equations. Piecewise models are always continuous, meaning there are no breaks in the graph.

Each segment of a piecewise linear function will have a domain, with any endpoints marked by a circle. An open circle means the point is not included in the line, whereas a closed circle means the point is included. Each point of the function is only included once in the domain.

In the graph on the right, the point $(-6, -1)$ is not included in the line, whereas the point $(8, 6)$ is included. The domain of the graph is $-6 < x \leq 8$.



WORKED EXAMPLE 17 *(2 marks)*

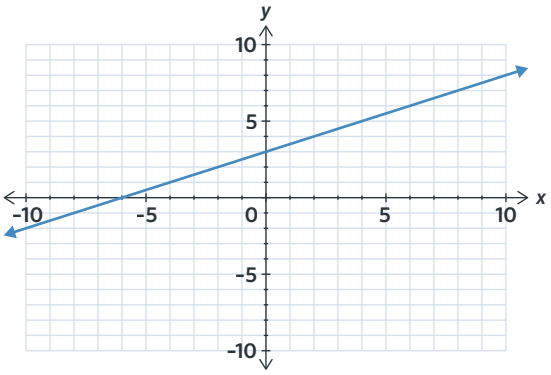
Sketch the following piecewise linear model.

$y = 0.5x + 3,\ -9 \leq x < 2$

$y = -2x + 8,\ 2 \leq x < 7$

SOLUTION

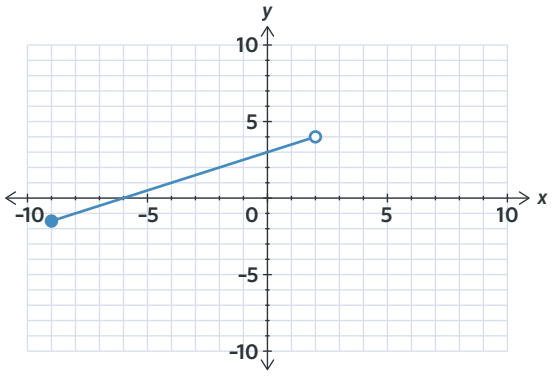
Step 1 Draw the line $y = 0.5x + 3$.



Step 2 Add in the endpoints.

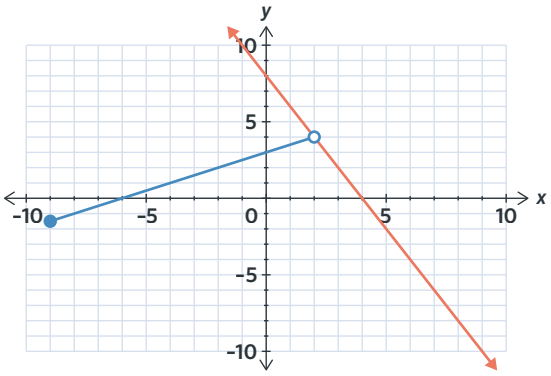
The x-coordinate -9 is included whereas the x-coordinate 2 is not included.

Draw a closed circle at $x = -9$ and an open circle at $x = 2$.



✓ *1 mark for sketching the first line and the correct domain*

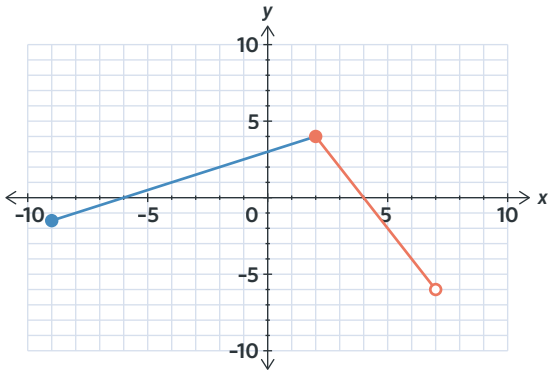
Step 3 Draw the line $y = -2x + 8$.



Step 4 Add in the endpoints.

The x-coordinate 2 is included whereas the x-coordinate 7 is not included.

Draw a closed circle at $x = 2$ and an open circle at $x = 7$.



✓ *1 mark for sketching the second line and the correct domain*

2. Functions of piecewise linear models

The function of a piecewise linear model can be found by calculating the equation of each of the line segments that make up the graph, along with each of their domains.

The gradient can be found by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The y-intercept is found by substituting one point into the equation:

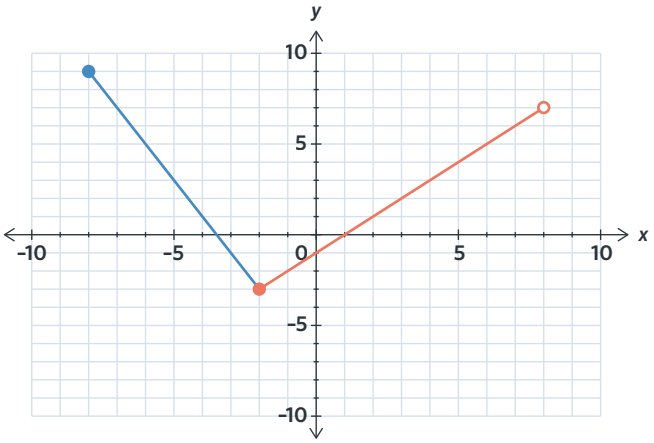
$$y = mx + c$$

and finding the value of c.

If an endpoint is an open circle, use the greater than ($>$) or less than ($<$) sign for the domain. If an endpoint is a closed circle, use the less than or equal to sign ($\leq$) or the greater than or equal to sign ($\geq$) for the domain.

WORKED EXAMPLE 18 *(2 marks)*

What is the function for the following piecewise linear model?



SOLUTION

Step 1 Find the equation of the blue line.

The line passes through the points $(-7, 7)$ and $(-3, -1)$.

$$m = \frac{-1 - 7}{-3 - (-7)} = \frac{-8}{4} = -2$$

So we have $y = -2x + c$.

Substitute the point $(-3, -1)$ into the equation.

$-1 = -2 \times -3 + c$

$-1 = 6 + c$

$c = -7$

The **blue** line is $y = -2x - 7$.

Step 2 Find the domain of the blue line.

The x-coordinate -8 is included whereas the x-coordinate -2 is not included.

The domain is $-8 \leq x < -2$

✓ *1 mark for calculating equation and domain of blue line*

Step 3 Find the equation of the red line.

The line passes through the points $(2, 1)$ and $(5, 4)$.

$$m = \frac{4 - 1}{5 - 2} = \frac{3}{3} = 1$$

So we have $y = x + c$.

Substitute the point $(2, 1)$ into the equation.

$1 = 2 + c$

$c = -1$

The **orange** line is $y = x - 1$

Step 4 Find the domain of the red line.

The x-coordinate -2 is included whereas the x-coordinate 8 is not included.

The domain is $-2 \leq x < 8$.

Step 5 Write the function of the piecewise linear model.

$y = -2x - 7, \ -8 \leq x < -2$

$y = x - 1, \ -2 \leq x < 8$

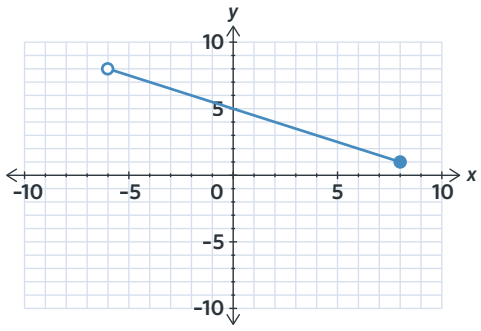
✓ *1 mark for calculating equation and domain of red line*

Questions 9G *Piecewise linear models*

Refresher question

Q1. **a)** What is the equation of the following graph?

b) What is the domain of the graph?



1. Sketching piecewise linear models

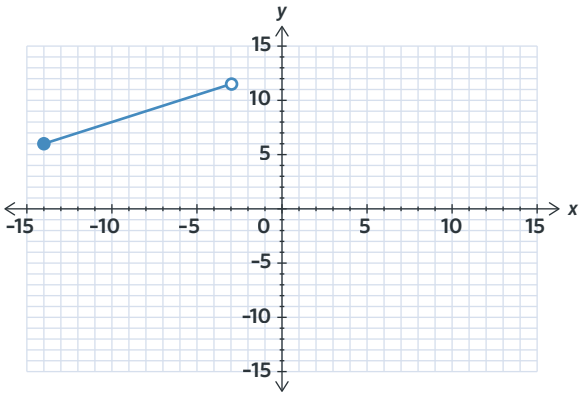
Q2. Skill

A piecewise linear graph is defined by the following function:

$y = 0.5x + 13, -14 \leq x < -3$

$y = -1.5x + 7, -3 \leq x < 12$

Part of the graph is shown below. Sketch the other part of the graph on the same axis.



Q3. Sketch the following piecewise linear models.

Skill

a) $y = 2x + 6, x < 0$

$y = -3x + 6, x \geq 0$

b) $y = -0.5x - 2, x < -2$

$y = x + 1, -2 \leq x < 5$

$y = -2x + 16, x \geq 5$

c) $y = 2.5x + 2, 0 \leq x < 6$

$y = -2x + 29, 6 \leq x < 12$

$y = -0.5x + 11, 12 \leq x < 20$

Check your understanding

Q4. A car is travelling into the city, which is 23 km away. The car travels 15 km in 20 minutes before stopping at a petrol station for 10 minutes. It takes another 12 minutes to complete the journey.

Application

Represent this information in a graph where D is the distance away from the city in kilometres, and t is the time in minutes.

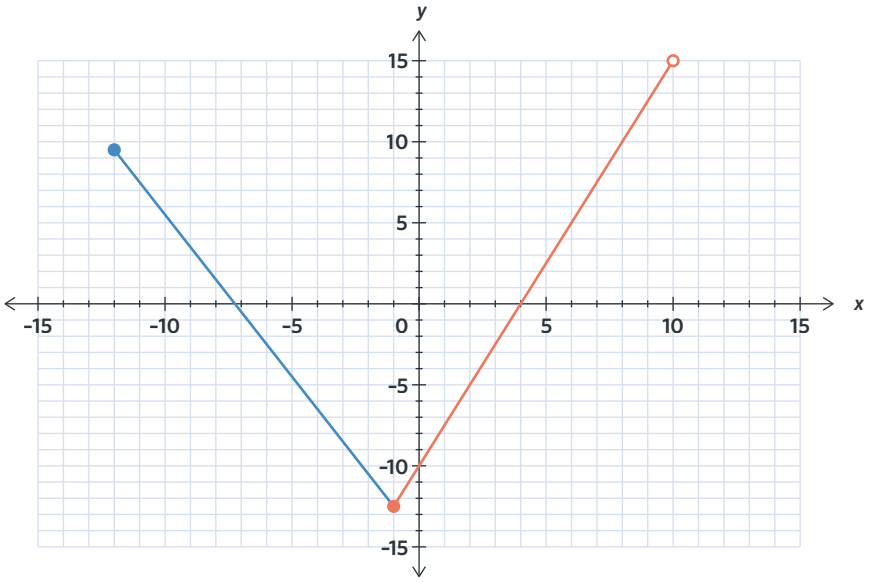
2. Functions of piecewise linear models

Q5. Fill in the missing information for the following graph.

Skill

$y = -\square x - 14.5, -12 \leq x < \square$

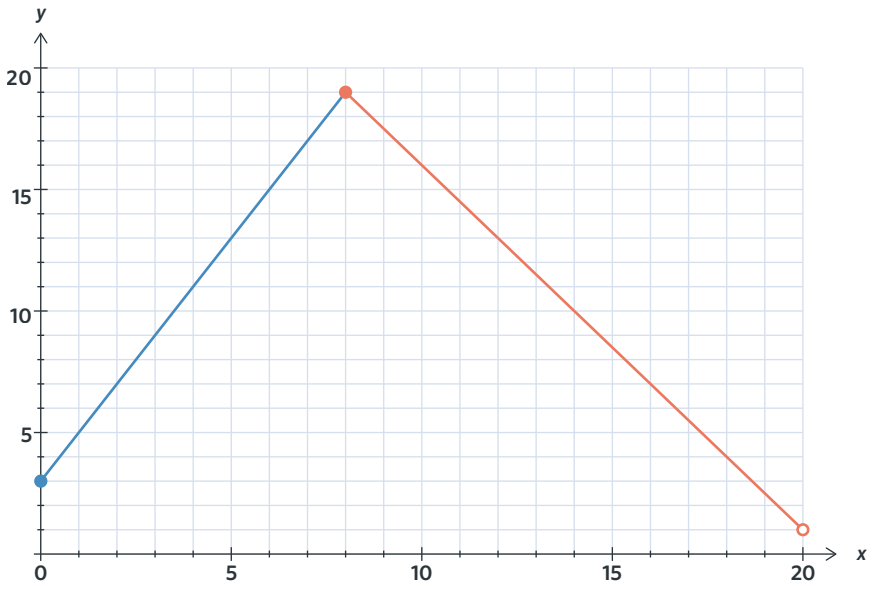
$y = 2.5x - \square, \square \leq x < \square$



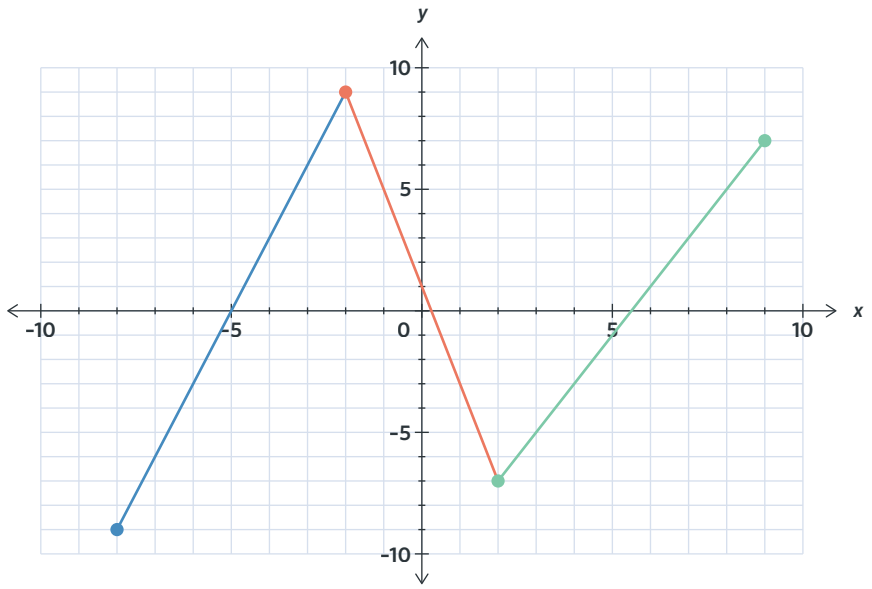
Q6. What is the function for each of the following piecewise linear models?

Skill

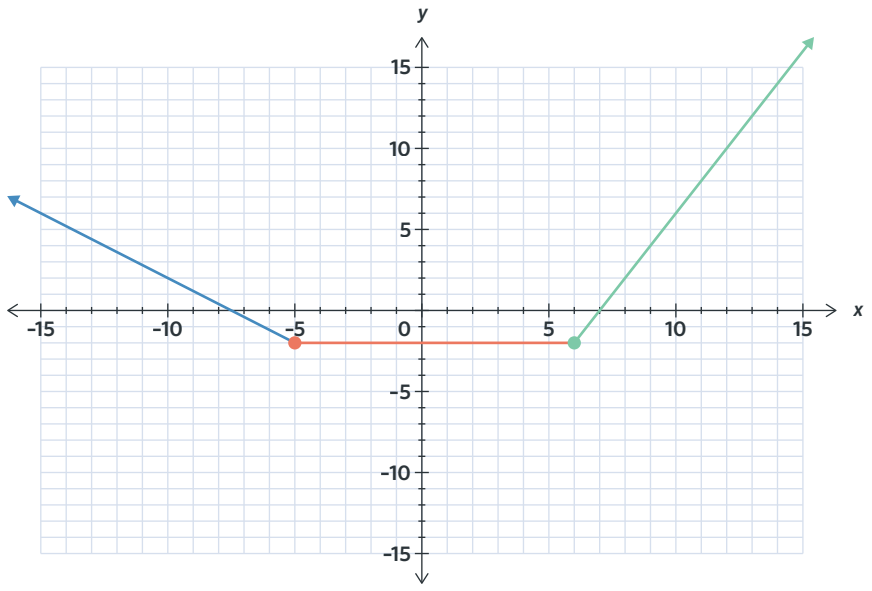
a)



b)



c)



Check your understanding

Q7. Application

Find the function of the piecewise linear model described in Question 4.

Q8. Skill

The function of a piecewise linear model is:

$y = x + 5,\ -8 \leq x < -2$

$y = -1.5x,\ -2 \leq x < a$

$y = 2x - 10.5,\ a \leq x < 7$

What is the value of a?

Joining it all together

Q9. Application 1 mark

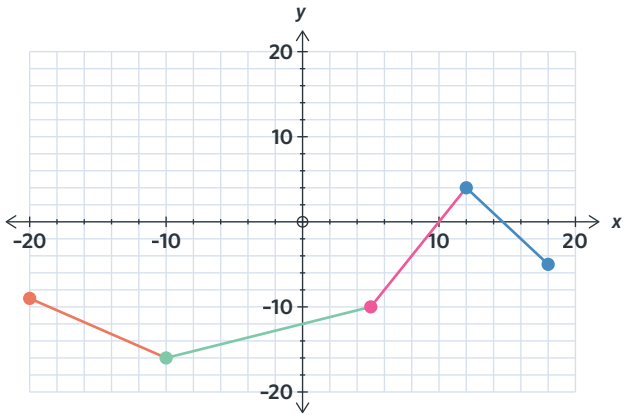
Consider the piecewise linear model below. Which of the following equations is not part of the graph?

A. $y = 2x - 20, 5 \leq x < 12$

B. $y = -0.7x - 23, -20 \leq x < -10$

C. $y = -1.5x + 22, 12 \leq x \leq 18$

D. $y = 0.4x + 12, -10 \leq x < 5$



Q10. Application 4 marks

Alex is comparing the costs of two electricians. Andrew charges a call-out fee of \$90: \$70 per hour for the first four hours of work, and \$65 per hour after that. Bob charges a call-out fee of \$80: \$75 per hour for the first three hours of work, and \$60 per hour after that.

a) Define a piecewise linear function to describe the cost, C (\$), of hiring Andrew for n hours.

b) Define a piecewise linear function to describe the cost, C (\$), of hiring Bob for n hours.

c) Which electrician would be cheaper to hire for three hours?

d) Which electrician would be cheaper to hire for six hours?

Q11. Skill 2 marks

A piecewise model is made up of two linear graphs. The equation of one of the graphs is $y = 3.5x + 2, -4 \leq x < 6$.

What are two possible rules for the second graph, if its gradient is -2?

Q12. Application 3 marks

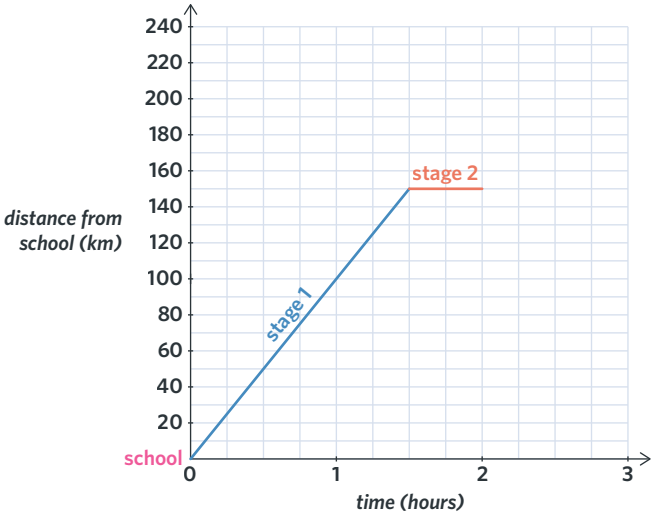
Indianna is walking her dog. It takes her six minutes to walk 0.6 km to the park. She stays at the park for fifteen minutes before walking back home. She takes the same route back home, but it takes her four minutes longer.

Define a function to describe the situation, where D is her distance from home, in metres, after t minutes.

VCAA question

Q13. The distance-time graph below shows the first two stages of a bus journey from a school to a camp.

5 marks



a) At what constant speed, in kilometres per hour, did the bus travel during stage 1 of the journey?

b) For how many minutes did the bus stop during stage 2 of the journey?

The third stage of the journey is missing from the graph. During stage 3, the bus continued its journey to the camp at a constant speed of 60 km/h for one hour.

c) Draw a line segment on the graph above to represent stage three of the journey.

d) Find the average speed of the bus over the three hours. Write your answer in kilometres per hour.

The distance, D km, of the bus from the school t hours after departure is given by:

$$D = \begin{cases} 100t & 0 \leq t \leq 1.5 \\ 150 & 1.5 \leq t \leq 2 \\ 60t + k & 2 \leq t \leq 3 \end{cases}$$

e) Determine the value of k.

VCAA 2013 Exam 2, Module 3: Graphs and relations. Q1

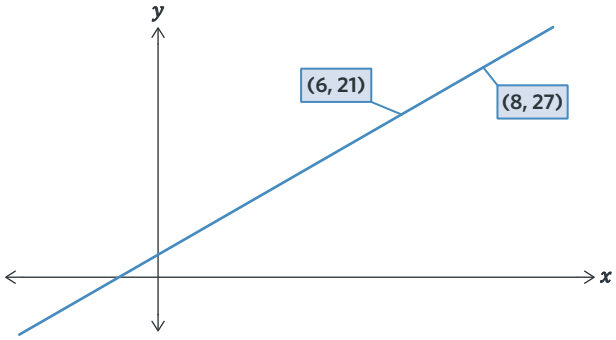
Questions from multiple lessons

Q14. The following graph shows a straight line that passes through the points (6, 21) and (8, 27).

Difficulty:

1 mark

The coordinates of the point where the line crosses the x-axis is



A. $(-2, 0)$

B. $(-1, 0)$

C. $(1, 0)$

D. $(2, 0)$

E. $(0, 3)$

VCAA 2017NH Exam 1 Module 4: Graphs and relations Q3 – Adapted

Q15.

Difficulty:

1 mark

The following two-way frequency table displays the *favourite soft drink* (Coke, Sprite, Fanta) and *sex* (male, female) of 149 people.

The percentage of females who chose Sprite as their favorite soft drink is closest to

A. 55%

B. 23%

C. 43%

D. 37%

E. 45%

		Sex	
		Male	**Female**
Favourite soft drink	**Coke**	33	26
	Sprite	25	35
	Fanta	13	17
Total		71	78

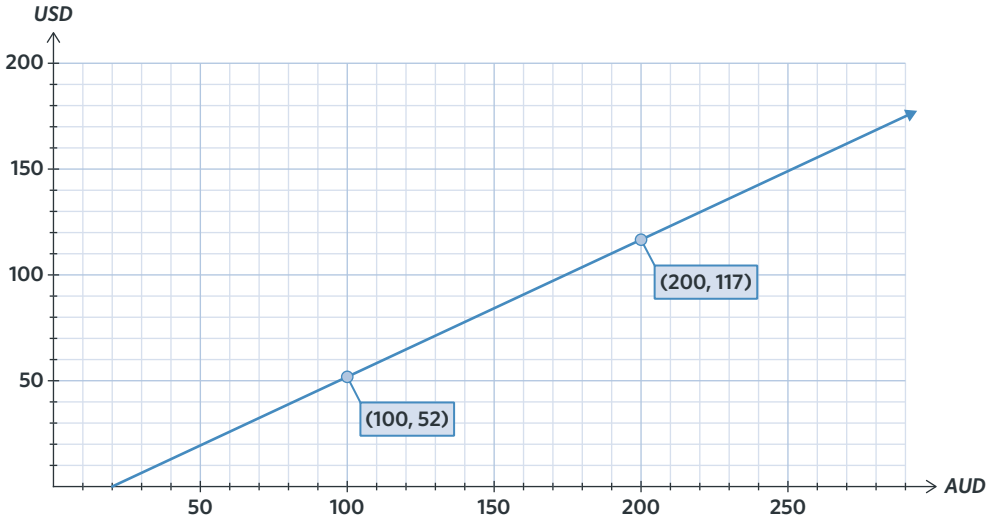
VCAA 2016 Exam 1 Data analysis Q1 – Adapted

Q16.

Difficulty:

3 marks

The graph displays the relationship between *USD* and *AUD* at a currency exchange agency.



The points (100, 52) and (200, 117) are labelled.

The equation for the relationship between the US dollar and the AU dollar is

$USD = 0.65 \times AUD + k$

a) Use the point (200, 117) to show that the value of k is -13. (1 mark)

b) Determine the coordinates of the horizontal axis intercept. (1 mark)

c) Interpret the horizontal axis intercept in the context of the question. (1 mark)

VCAA 2015 Exam 2 Module 3: Graphs and relations Q3 – Adapted

AOS 5: Graphs of linear and non-linear relations

Inequalities and linear programming

CHAPTER 10

LESSON 10A

Linear inequalities in one variable

The key skills you will learn in this lesson are:

1. Inequalities on a number line
2. Inequalities on a set of axes

VCAA key knowledge point:

"linear inequalities in one and two variables and their graphical representation"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Inequalities on a number line

Introduction to inequalities

Inequalities are mathematical expressions that show how one quantity is greater or less than another.

There are two types of inequalities: those that exclude the limit (**exclusive**) and those that include the limit (**inclusive**).

Exclusive inequalities	
$>$	Greater than
$<$	Less than

Inclusive inequalities	
$\geq$	Greater than or equal to
$\leq$	Less than or equal to

It is useful to remember that the 'pointed' side of an inequality refers to the smaller quantity.

When a quantity has both an upper and lower limit, an inequality is placed on either side of the pronumeral.

Example

According to the Australian Bureau of Statistics, the term 'Baby Boomer' describes someone born between the years 1946 and 1964, inclusive.

The year of birth, y, of a Baby Boomer is given by the inequality $1946 \leq y \leq 1964$.

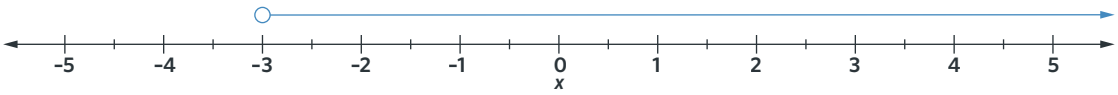
Representing inequalities on a number line

To represent an inequality on a number line, mark the inclusive limits with a solid circle ● and the exclusive ones with an empty circle ○ and then draw a line over all the allowed values.

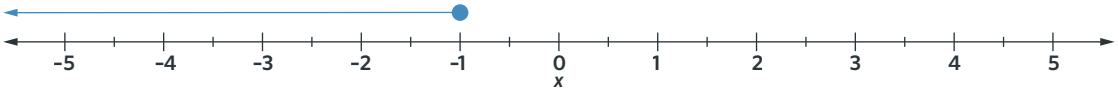
If there is no upper or lower limit continue the line to the appropriate end of the number line and draw an arrow on the end.

Examples

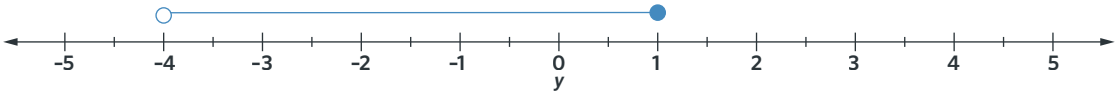
$x > -3$



$x \leq -1$



$-4 < y \leq 1$



Transposing inequalities

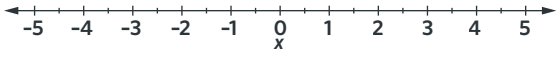
Transposing inequalities is the same as transposing linear equations with **one exception**: multiplying or dividing by a negative number changes the direction of the inequality.

WORKED EXAMPLE 1 *(1 mark)*

Display the inequality $x < -1$ on a number line.

SOLUTION

Step 1 Draw a number line.



Step 2 Find the limits and determine whether they are inclusive or exclusive.

The limit is -1.

The inequality is given by the symbol $<$.
This means the limit is 'less than'.

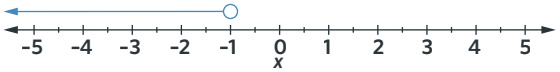
The limit is exclusive.

It will be represented by an empty circle ◯ on or above the number line.

Step 3 Mark the limits and allowed values on or above the number line.

The inequality is given by the symbol $<$.
This means the limit is 'less than'.

The line will point towards the values that are lower than -1.



WORKED EXAMPLE 2 *(1 mark)*

Transpose the following inequality, making y the subject.

$3 < 2y + 1 < 5$

SOLUTION

Step 1 Subtract 1 from all sides.

$3 - 1 < 2y + 1 - 1 < 5 - 1$

$2 < 2y < 4$

Step 2 Divide by 2, and write the answer.

$\frac{2}{2} < \frac{2y}{2} < \frac{4}{2}$

$1 < y < 2$

The answer is $1 < y < 2$.

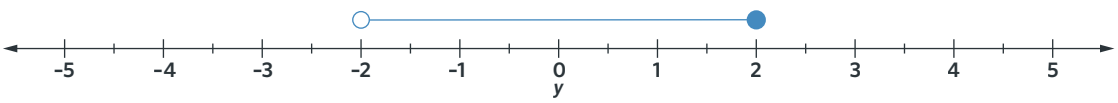
2. Inequalities on a set of axes

Inequalities can also be represented on a set of axes.

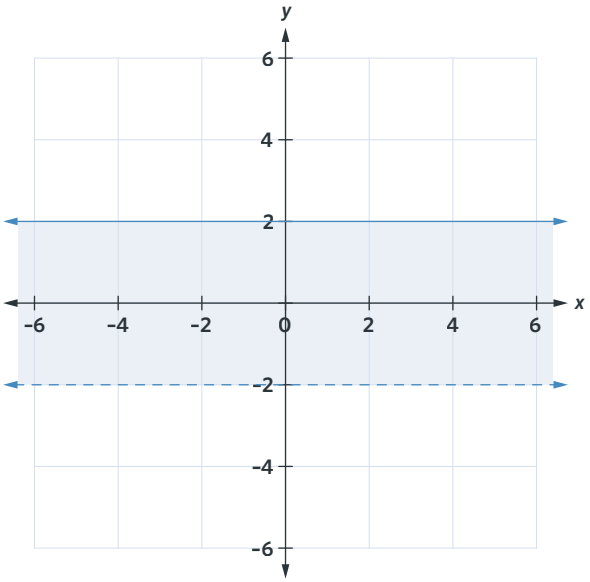
When representing inequalities on a set of axes, inclusive limits are denoted by a solid line and exclusive limits by a dashed line. On a set of axes the accepted values are denoted by a shaded area.

Example

The inequality $-2 < y \leq 2$ can be represented on a number line as:



And on a set of axes as:



The differences between representing inequalities on a set of axes and a number line are summarised in the table below.

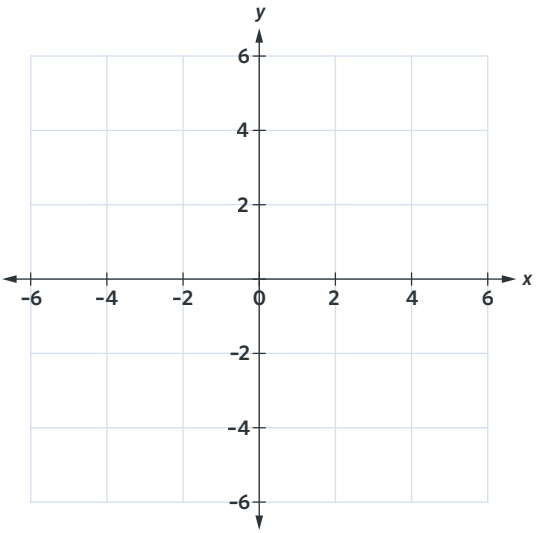
	Number line	Set of axes
Included		
Excluded		
Acceptable values		

WORKED EXAMPLE 3 *(1 mark)*

Display the inequality $x < -1$ on a set of axes.

SOLUTION

Step 1 Draw a set of axes.

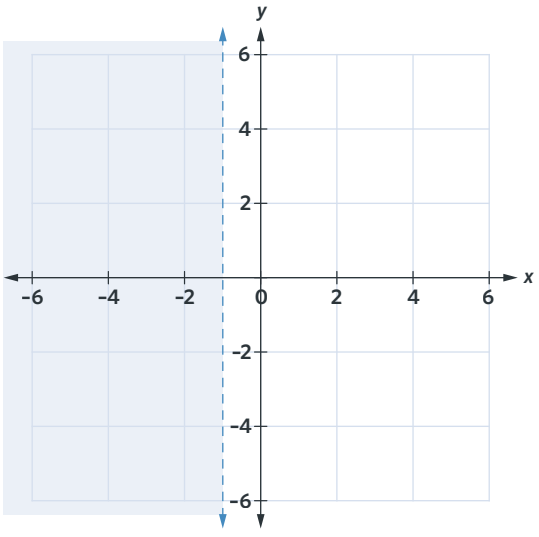


Step 2 Find the limits and determine whether they are inclusive or exclusive.

The limit is -1, exclusive.

This means it will be represented by a dashed line on the set of axes.

Step 3 Mark the limits and allowed values on the set of axes.



WORKED EXAMPLE 4 *(1 mark)*

Display the inequality $x \geq 0$ on a set of axes.

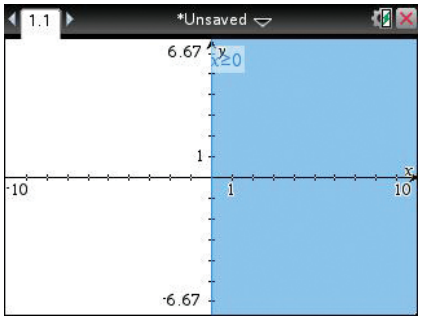
SOLUTION: TI-NSPIRE

Step 1 Open a new set of axes by pressing ctrl + doc then 2 ('Add Graphs').

Step 2 Press del, then 5 ('≥').

Then replace the 'y' with 'x'.

Step 3 Type '0' after the '≥' symbol and then press enter.



SOLUTION: CASIO CLASSPAD

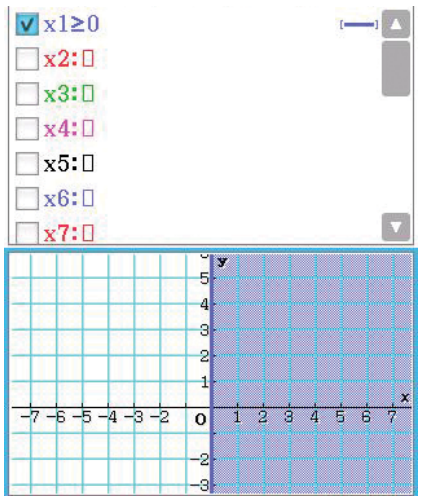
Step 1 Using the stylus, select menu and then Graph& Table.

Tap the down arrow y= and select x≥ from the drop down menu.

Step 2 In the first row, labelled 'x1', type '0'.

Then check the box to the left by tapping it with the stylus.

Step 3 Tap the ⊬ button to graph the inequality.



Questions 10A *Linear inequalities in one variable*

Refresher question

Q1. **a)** Complete the following inequalities by filling in the boxes with either $<$ or $>$.

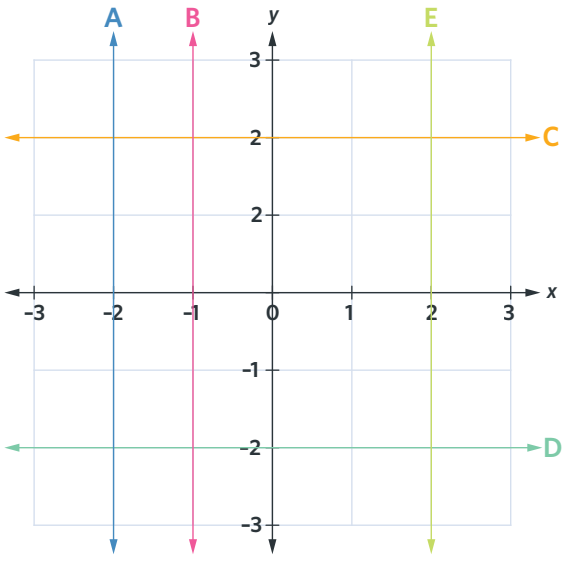
i $2 \square 0$

ii $-3 \square -2$

b) Which of these lines is:

i $x = 2$

ii $y = -2$



1. Inequalities on a number line

Q2. Skill

True or false: a closed circle on a number line indicates that the value is included.

Q3. Skill

Display the following inequalities on a number line.

a) $x \leq 5$

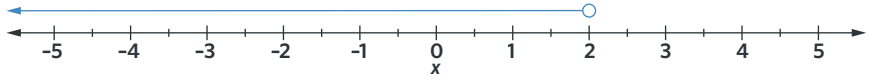
b) $y > -3$

c) $\frac{3}{2} < x \leq \frac{5}{2}$

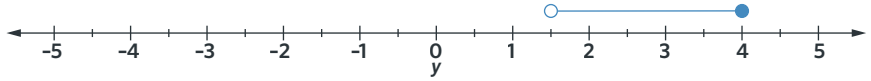
Q4. Skill

Express the information shown on each of the number lines below as an inequality.

a)



b)



Q5. Skill

Display the following inequalities on separate number lines.

a) $y - 3 < 3$

b) $2x \geq -8$

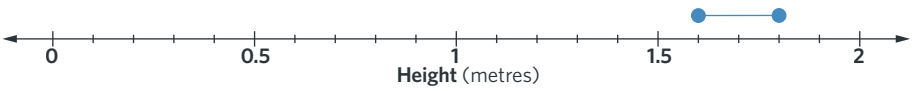
c) $1 \leq x + 5 < 3$

d) $-3 < 2y - 1 \leq 5$

Check your understanding

Q6. Application

The Edrolo Theatre Company is holding auditions for their upcoming play. The allowed height for the lead role is displayed on the number line below.



Which of the options below correctly completes the following sentence?

The actor playing the lead role must be _____ metres tall but _____ metres.

A. more than 1.6; no taller than 1.8

B. at least 1.5; shorter than 1.8

C. at least 1.6; shorter than 1.8

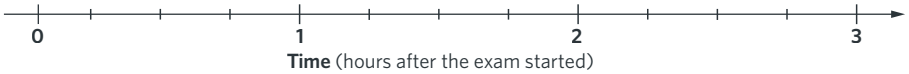
D. more than 1.6; no taller than 1.8

E. at least 1.6; no taller than 1.8

Q7. Application

Beppe's last physics exam at university went for three hours. The students were not allowed to leave the examination venue in the first half hour or in the last 15 minutes of the exam. Beppe left the exam early.

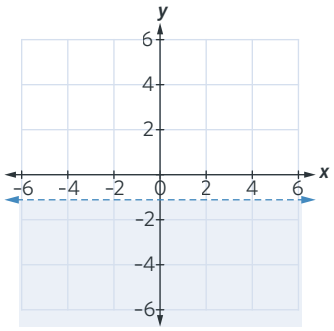
On the number line below, display the period of time during which Beppe could have left the exam. Assume that the limits on this number line are exclusive.



2. Inequalities on a set of axes

Q8. Skill

True or false: the dotted horizontal line on this set of axes indicates that the y-value is included.



Q9. Skill

Display the following inequalities on separate sets of axes.

a) $x > -3$

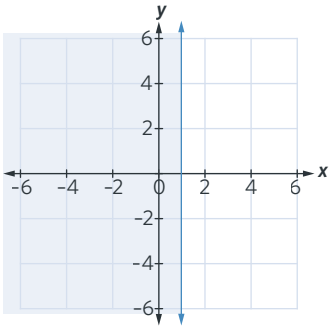
b) $y \leq 1$

c) $-3 \leq y < 2$

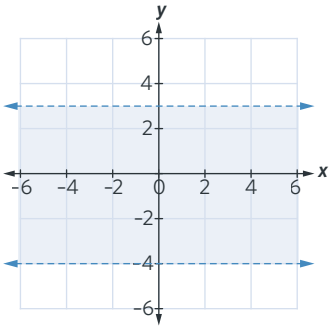
Q10. Skill

Express the information shown on each set of axes below as an inequality.

a)



b)



Q11. Skill

Display the following inequalities on a set of axes.

a) $-\frac{3}{2} > 0.5y$

b) $1 < x + 2 \leq 3$

c) $\frac{1}{2} \leq y \leq \frac{5}{2}$

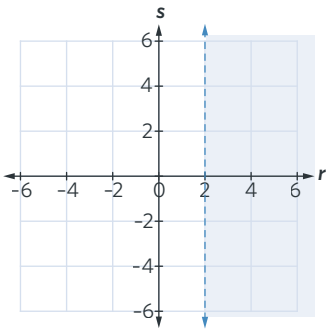
d) $0 \leq 3(y + 1) < 9$

Check your understanding

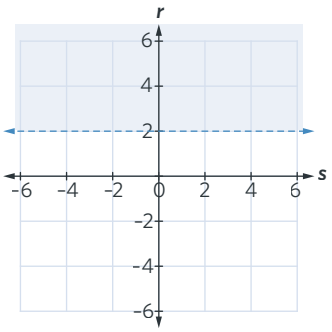
Q12. Skill

Which of the following is **not** a valid representation of the inequality $r > 2$?

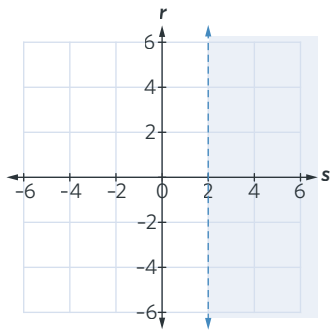
A.



B.



C.



D. All of the above

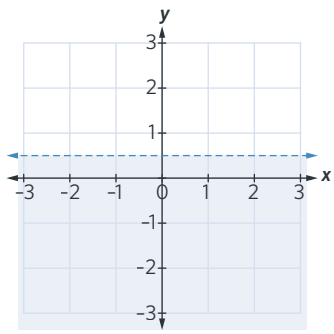
E. None of the above

Joining it all together

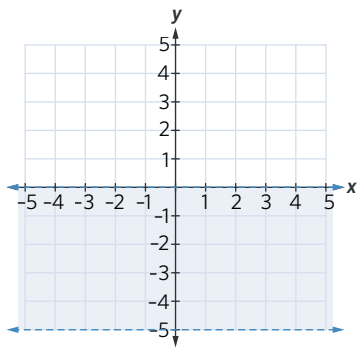
Q13. Express the following inequalities on a number line.

Skill
3 marks

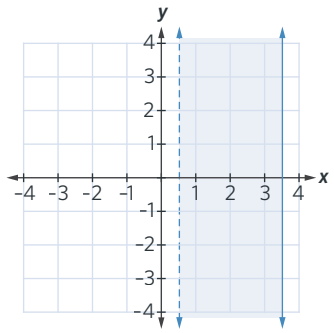
a)



b)



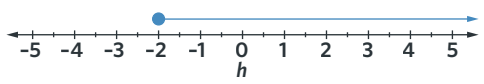
c)



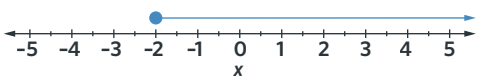
Q14. Which of following representations of the inequality $h \geq -2$ is incorrect?

Skill
1 mark

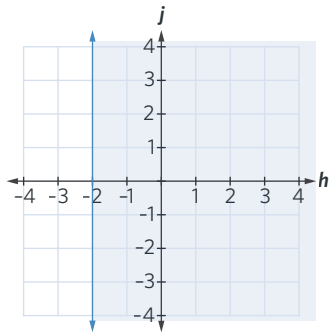
A.



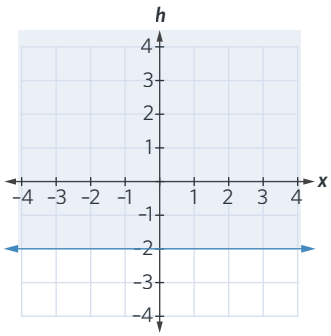
C.



B.



D.



E. None of the above

Q15. Buycycles 'r' Us sells fully assembled bicycles that weigh at least 7 kg but less than 11 kg.

Application
2 marks

The wheels Buycycles 'r' Us use each weigh 1 kg.

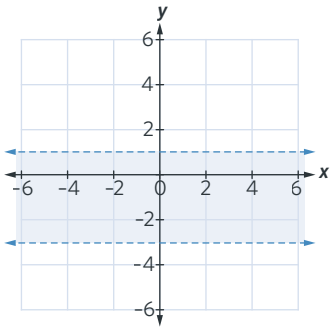
Jessie buys a bicycle from Buycycles 'r' Us, but wants to attach her own wheels.

On a number line, display the possible weight of Jessie's new bicycle without its wheels.

The number line does not need to include zero.

Q16. Show that the inequality shown on the set of axes below is $-2 < 3y + 7 < 10$.

Skill
2 marks



Questions from multiple lessons

Q17. Junior swimmers competed at the Junior World Championship in Budapest, Hungary in 2019.

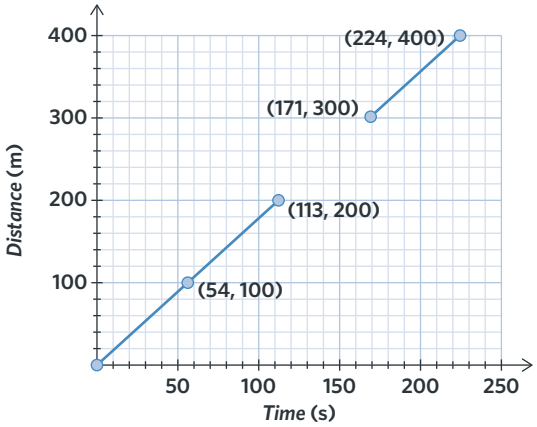
Difficulty:

The order in which the USA's team of four swimmers completed the 4×100 m medley relay is shown.

1 mark

Order	First	Second	Third	Fourth
Name	William	Josh	Torri	Gretchen

The following line segment graph represents the incomplete progression of the race.
Torri's segment is missing.



The speed that Torri swam at is closest to

A. 1.5 m/s

B. 1.5 km/h

C. 1.7 m/s

D. 1.8 m/s

E. 1.8 m/s

VCAA 2018 Exam 1 Module 4: Graphs and relations Q4 – Adapted

Q18. The number of minutes spent eating breakfast on a particular day was recorded for a group of 20 high school students. The results are shown in the following stem plot.

Difficulty:

1 mark

Key: 0 | 6 = 6 minutes

0	6 7 7 8 8 9
1	0 0 0 1 3 7 8
2	2 2 4 5 7
3	1 3

The percentage of these people that spent more than 10 minutes eating breakfast is

A. 45%

B. 30%

C. 55%

D. 40%

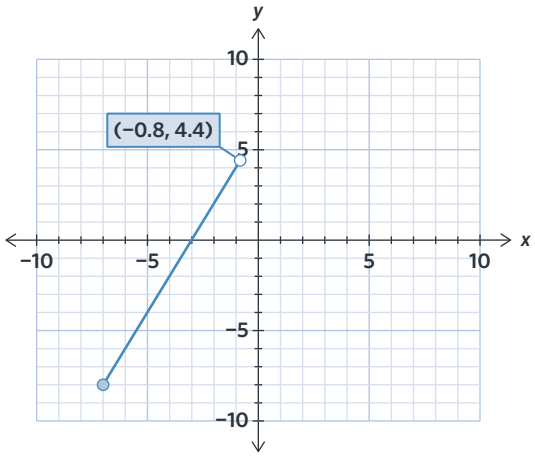
E. 70%

VCAA 2018NH Exam 1 Data Analysis Q2 – Adapted

Q19. One section of a piecewise linear model is shown in the following graph.

Difficulty:

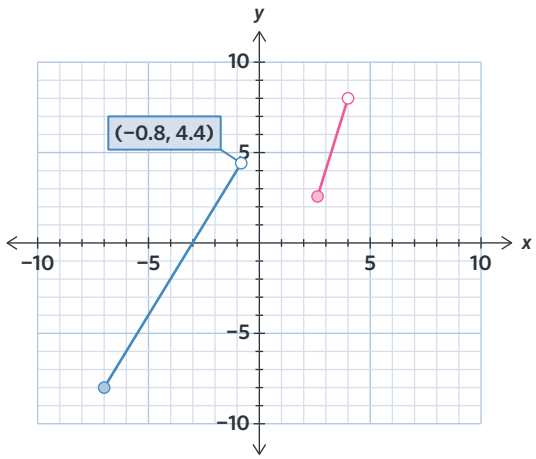
3 marks



The equation of the line is $y = mx + 6$, where $-7 \leq x < -0.8$.

a) What is the value of m? (1 mark)

b) Another section of the piecewise linear model has the equation $y = 4x - 8$, where $\frac{8}{3} \leq x < 4$. This section is displayed in pink.



Determine the equation of the section that connects the blue and pink sections of the piecewise linear model. You do not need to give the domain. (2 marks)

LESSON 10B

Linear inequalities in two variables

The key skills you will learn in this lesson are:

1. Testing points
2. Representing inequalities in two variables on a set of axes

VCAA key knowledge point:

"linear inequalities in one and two variables and their graphical representation"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Testing points

Inequalities are also used to show a relationship between two variables.

Example

Edrolo is buying all their employees a second computer monitor to improve productivity.

Edrolo needs to buy **at least** as many monitors, y, as they have employees, x.

The inequality that describes this situation is $y \geq x$.

Testing points

Testing points is a method of trialling specific pairs of numbers (known as points or ordered pairs) to see if they satisfy an inequality. These points are written using the standard notation:

(x-coordinate, y-coordinate).

This is done by substituting the necessary values into the inequality and determining if the expression is true or false.

WORKED EXAMPLE 5 *(1 mark)*

Edrolo is buying all their employees a second monitor to improve productivity.

Edrolo needs to buy **at least** as many monitors, y, as they have employees, x.

The inequality that describes this situation is $y \geq x$.

Edrolo currently has 50 employees and they buy 45 monitors.

Does this point (50, 45) satisfy the inequality $y \geq x$?

SOLUTION

Step 1 Substitute the point (50, 45) into the inequality.

$45 \geq 50$

Step 2 Check whether the expression is true.

$45 \geq 50$ is false.

Step 3 Interpret the result.

This means the point (50, 45) does not satisfy the inequality.

2. Representing inequalities in two variables on a set of axes

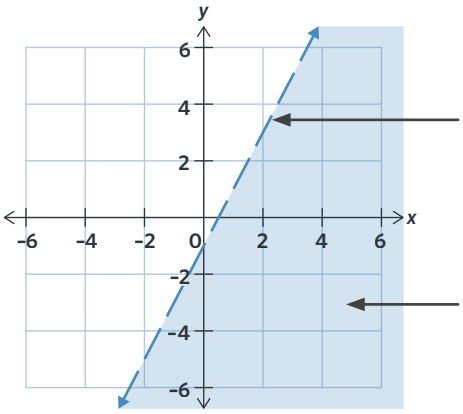
Representing inequalities in two variables on a set of axes is done in the same way as representing inequalities in one variable. The key difference is that the linear equation will involve both x and y.

When drawing inequalities on a set of axes:

1. Determine whether the inequality is inclusive or exclusive and sketch the line.
   - For expressions with $\leq$ or $\geq$ sketch a solid line.
   - For expressions with $<$ or $>$ sketch a dashed line.
2. Shade the region associated with the expression.

Example

The inequality $y < 2x - 1$ is represented on the set of axes below.



A dotted line along $y = 2x - 1$

The region 'less than' the line is shaded.

When expressions do not have y along on the left hand side the expression can be rearranged to make sketching easier. This is the same as rearranging an equation (except if multiplying or dividing by -1, this will change the direction of the inequality sign).

Example

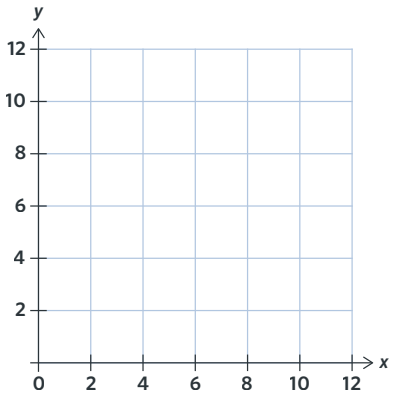
The expression $2y - 10 \leq 4x$ can be rearranged:

$$2y - 10 \textbf{ + 10} \leq 4x \textbf{ + 10}$$
$$2y \leq 4x + 10$$
$$2y \textbf{ ÷ 2} \leq (4x + 10) \textbf{ ÷ 2}$$
$$y \leq 2x + 5$$

WORKED EXAMPLE 6 *(1 mark)*

Edrolo is buying all their employees a second monitor to improve productivity. Edrolo needs to buy **at least** as many monitors, y, as they have employees, x. The inequality that describes this situation is $y \geq x$.

Represent this inequality on the following set of axes.



SOLUTION

Step 1 Determine whether the inequality is inclusive or exclusive.

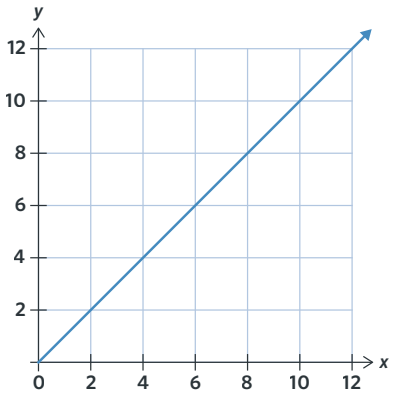
The inequality includes $\geq$, the 'greater than or equal to' symbol.

The inequality is inclusive.

This means the limit will be represented by a solid line.

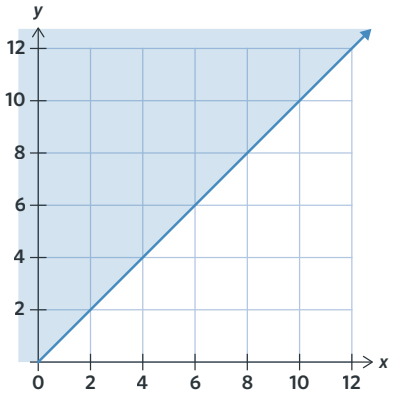
Step 2 Plot the line using a solid or dashed line as required.

Here we plot $y = x$ as a solid line.



Step 3 Shade in the appropriate region.

The region we are looking for is $y \geq x$.
The region we need to shade in is the part **above** the line, where y is greater than x.



WORKED EXAMPLE 7 *(1 mark)*

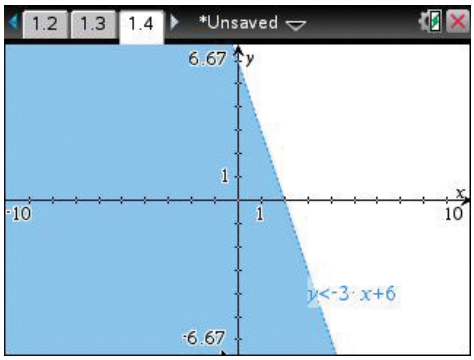
Represent the inequality $y < -3x + 6$ on a set of axes.

SOLUTION: TI-NSPIRE

Step 1 Open a new set of axes by pressing ctrl + doc then 2 ('Add Graphs').

Step 2 Press del, then 2 ('<').

Step 3 Type '$-3x + 6$' after the '<' symbol and then press enter.



SOLUTION: CASIO CLASSPAD

Step 1 Using the stylus, select menu and then Graph& Table.

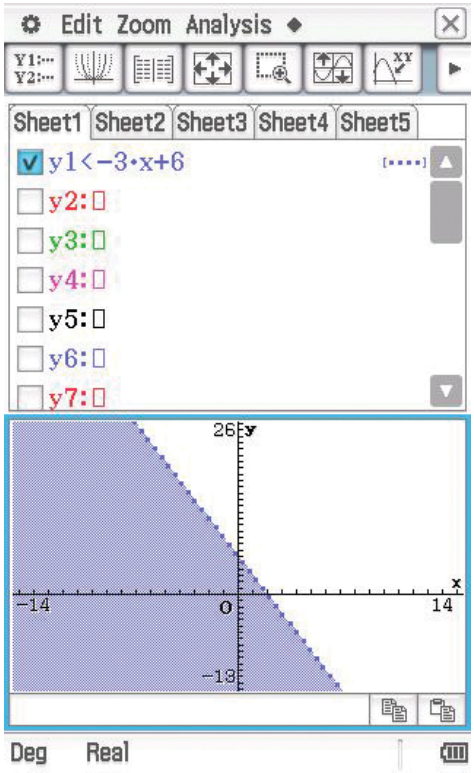
Tap the down arrow y= and select y< from the drop down menu.

Step 2 In the first row, labelled '$y1$', type '$-3x + 6$'.

Then check the box to the left by tapping it with the stylus.

Step 3 Tap the button to graph the inequality.

Step 4 Tap 'Zoom', then tap 'Auto'.



Questions 10B *Linear inequalities in two variables*

Refresher question

Q1. **a)** What is the y-coordinate of the point $(1, 0)$?

b) Plot the equation $y = 2x$ on a set of axes.

1. Testing points

Q2. Skill

Complete the following questions:

a) Substitute the point $(2, 1)$ into the inequality $y > x - 2$ and simplify.

b) Is the expression you found in part a) true or false?

Q3. Skill

For each of the following, test whether the point satisfies the inequality.

a) $(1, 3)$ $\quad y \geq 2x + 1$

b) $(2, 0)$ $\quad y + 2x < 3$

c) $(-3, 2)$ $\quad 3y - x > 0$

Q4. Skill

The point $(2, 3)$ satisfies the inequality:

A. $y > x + 1$ **B.** $-y + x > 0$ **C.** $2y + x < 5$

D. $2y \geq x + 1$ **E.** $x \leq 2y - 5$

Check your understanding

Q5. Application

In many Victorian hospital wards there must be at least one nurse rostered on for every four patients.

a) In one such ward there are eight patients. What is the minimum number of nurses that must be rostered on?

b) In another such hospital ward there are 36 patients. What is the minimum number of nurses that must be rostered on?

c) Write down the inequality that gives the required number of nurses, n, in a ward with p patients.

d) In another such hospital ward there are 21 patients and five nurses currently on duty. Are there enough nurses rostered on?

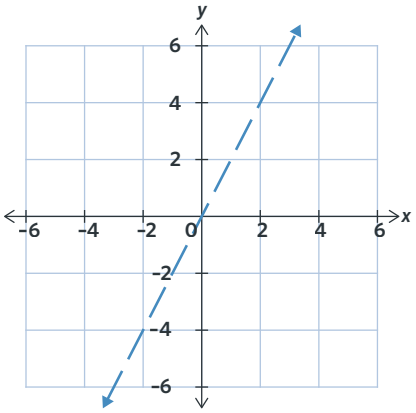
2. Representing inequalities in two variables on a set of axes

Q6. Skill

The inequality $y > 2x$ is represented on the following set of axes.

The representation is incomplete.

Complete the graph by shading the accepted area.



Q7. Skill

Represent each of the following inequalities on a separate set of axes.

a) $y < 2x - 1$ **b)** $y \geq -x + 2$ **c)** $y - x > 1$

Q8. Skill

Represent each of the following inequalities on a separate set of axes.

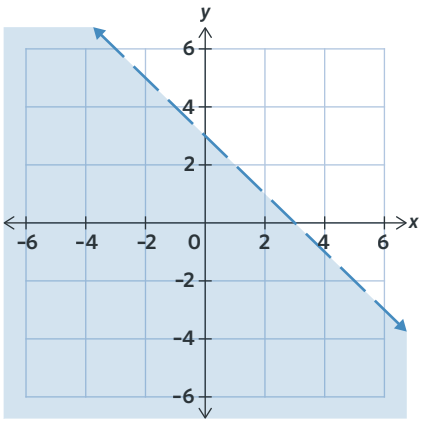
a) $2y + 6x \leq 6$ **b)** $0.5y - 3x > -1.5$ **c)** $3y + 9x \geq 18$

Check your understanding

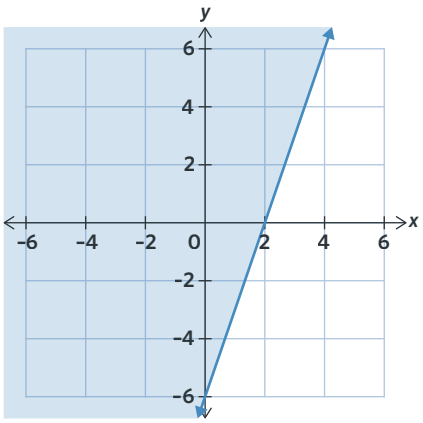
Q9. Skill

State the inequality that defines the following graphs.

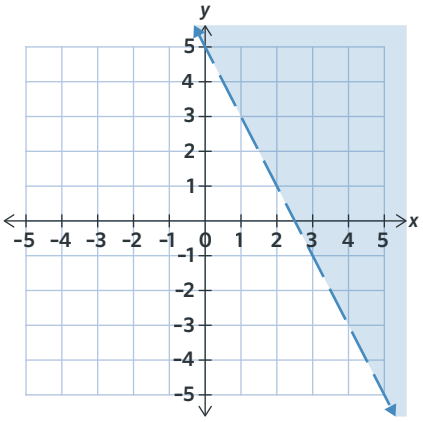
a)



b)

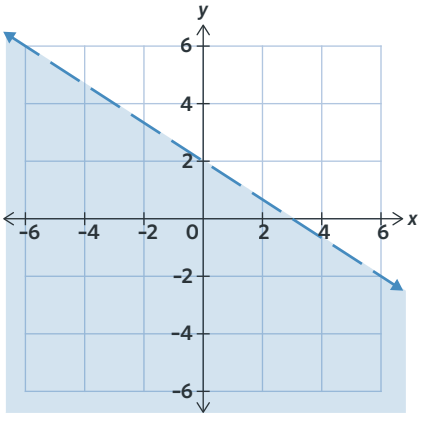


c)



Q10. Skill

Which of the inequalities below is represented on the following graph?



A. $y > -\frac{2}{3}x + 2$ **B.** $3y - 2x < 12$ **C.** $y < \frac{2}{3}x - 3$

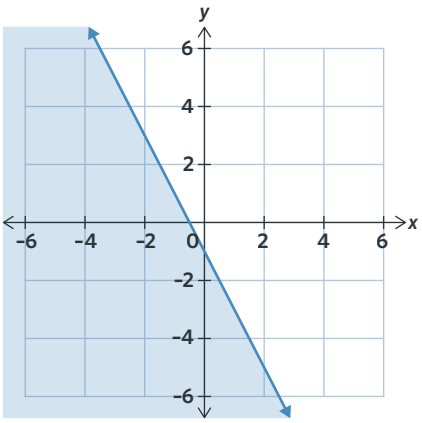
D. $\frac{3}{2}y + x < 3$ **E.** $2y + 3x > 12$

Joining it all together

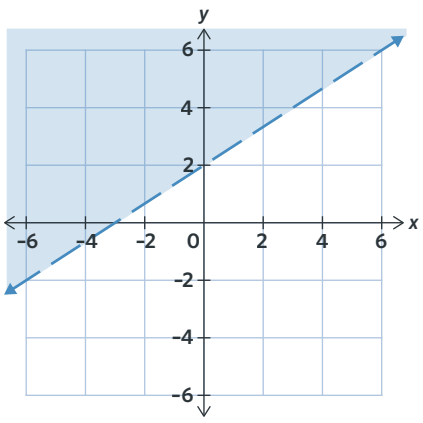
Q11. For each of the following, determine whether the point satisfies the inequality shown on the set of axes.

Skill
2 marks

a) $(2\ -4)$



b) $(-1, 3)$



Q12. A local high school is organising an overseas study tour for some of its students. There must be at least one teacher for every ten students.

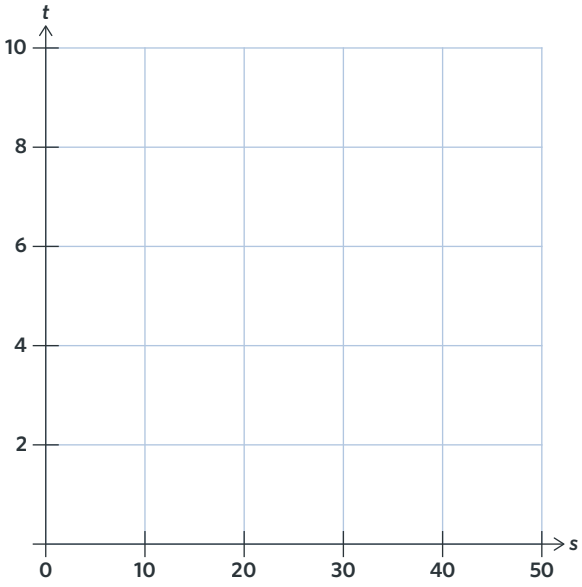
Application
3 marks

Let:

- t be the number of teachers.
- s be the number of students.

a) Write down an inequality in terms of s and t that describes this situation.

b) Graph the inequality from part **a)** on the following set of axes.



c) What is the minimum number of teachers that are needed to take 30 students overseas?

VCAA question

Q13. 1 mark

For an overnight school excursion there must be at least one teacher for every 15 students.

Let:

- x be the number of students.
- y be the number of teachers.

The inequality that describes this situation is:

A. $y \geq 15$ **B.** $x \leq 15$ **C.** $y \geq 15x$

D. $y \geq \frac{x}{15}$ **E.** $x \geq \frac{y}{15}$

Adapted from VCAA 2015 Exam 1, Module 3: Graphs and relations: Q5

Questions from multiple lessons

Q14. Difficulty: 1 mark

A music festival makes sure to have at least three all female acts for every five all male acts.

Let m represent the number of all male acts, and f represent the number of all female acts.

The relationship between the number of all male and all female acts can be represented by the inequality

A. $3m \geq 5f$

B. $3f \geq 5m$

C. $3m \leq 5f$

D. $3f \leq 5m$

E. $m \geq f$

VCAA 2019NH Exam 1 Module 4: Graphs and relations Q4 – Adapted

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Q15. Difficulty: 1 mark

Zoe measured the height and weight of nine koalas she saved during the 2019/20 bushfires. They are displayed in the following table.

Height (cm)	78	92	80	81	72	75	78	84	74
Weight (kg)	4.2	9.9	7.2	5.4	4.8	5.5	5.0	8.2	7.6

The mean, $\bar{x}$, and the standard deviation, s_x, for the weight of these koalas, in kilograms, are closest to

A. $\bar{x} = 6.28, \quad s_x = 1.79$

B. $\bar{x} = 6.28, \quad s_x = 1.90$

C. $\bar{x} = 6.42, \quad s_x = 1.90$

D. $\bar{x} = 6.42, \quad s_x = 1.79$

E. $\bar{x} = 6.53, \quad s_x = 1.79$

VCAA 2018NH Exam 1 Data analysis Q7 – Adapted

Q16. EdrofficeWorks is a start-up stationery producer.

Difficulty:

3 marks

Let x be the number of pens EdrofficeWorks produces each day.

Let y be the number of pencils EdrofficeWorks produces each day.

A constraint on the production by EdrofficeWorks is given by

Inequality 1: $x \geq 50$

a) Explain the meaning of Inequality 1 in terms of the given variables. (1 mark)

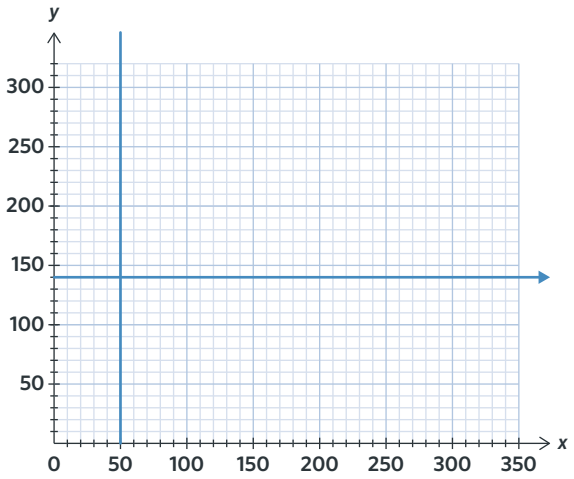
Each day, EdrofficeWorks produces at least a pencils, but can only produce up to 300 pens and pencils combined.

These constraints may be written as

Inequality 2: $y \geq a$

Inequality 3: $x + y \leq 300$

The graphs of $x = 50$ and $y = a$ are shown on the right.



b) State the value of a (1 mark)

c) Plot the line $x + y = 300$ on the given graph. (1 mark)

VCAA 2010 Exam 2 Module 3: Graphs and relations Q3 a,b – Adapted

LESSON 10C

Feasible region

The key skills you will learn in this lesson are:

1. Sketching feasible regions
2. Worded problems involving feasible regions

VCAA key knowledge point:

"the concepts of feasible region, constraint and objective function in the context of solving a linear programming problem"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Sketching feasible regions

In many cases, multiple linear inequalities must be solved simultaneously. By graphing all of the linear inequalities on the same set of axes, the region that satisfies all of the inequalities can be located. This region is known as the feasible region.

The feasible region is the region in which every point within it is a possible solution to the problem.

Like with inequalities, points on the boundary of the feasible region are included if the line is solid ($\leq$ or $\geq$), and points on the boundary are not included if the line is dotted ($<$ or $>$).

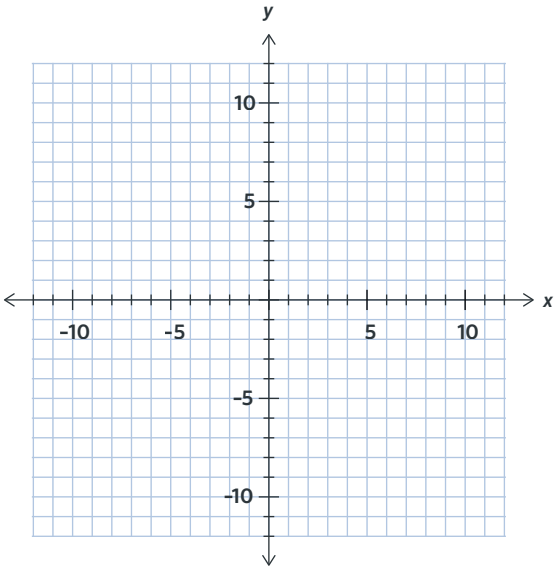
WORKED EXAMPLE 8 *(1 mark)*

Sketch the feasible region defined by the following inequalities:

$x \geq -3$ $x < 5$ $y > -6$ $x + y \leq 8$

SOLUTION

Step 1 Draw and label the axes.

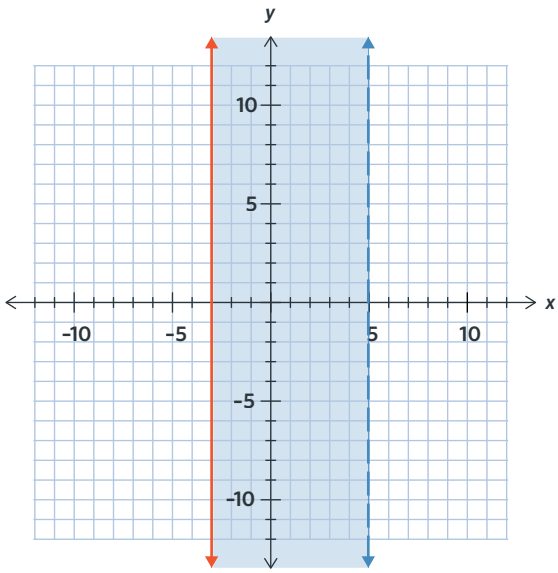


Step 2 Sketch the first two inequalities.

Since these equalities only involve x, they will be vertical lines on the plane.

The inequality $x \geq -3$ will be a solid line while the inequality $x < 5$ will be a dotted line.

The required region is the region between these two lines.

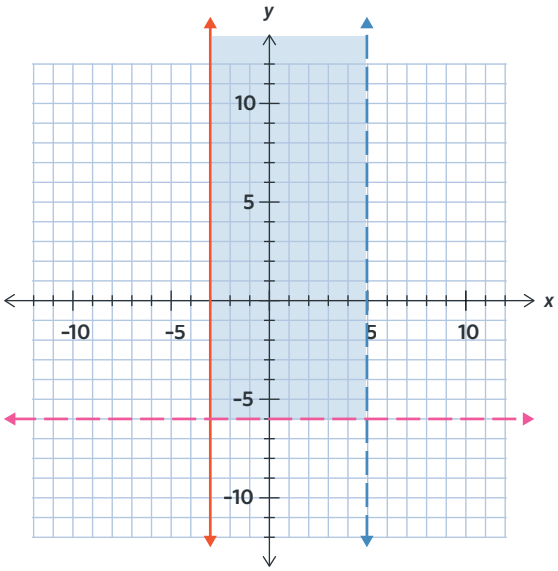


Step 3 Sketch the third inequality.

Since this inequality only involves y, it will be a horizontal line on the plane.

The inequality $y > -6$ is exclusive so will be shown by a dotted line.

The accepted y-values are those **greater** than –6, so the required region will be the area above the line.



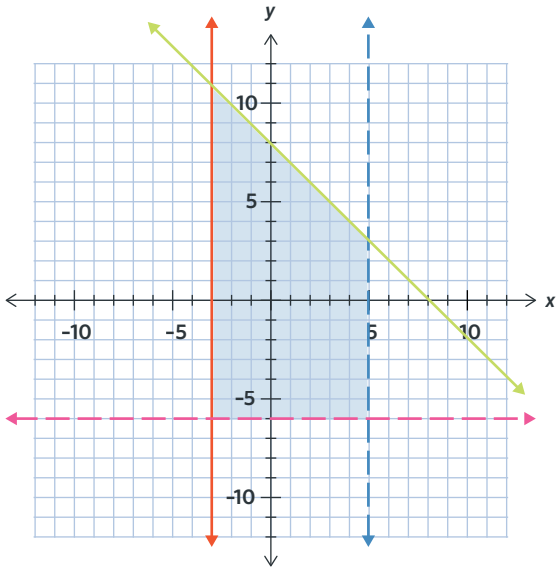
Step 4 Sketch the last inequality and shade the feasible region.

It might help to rearrange the inequality for y.

$x + y \leq 8$

$y \leq -x + 8$

This gives a line with gradient –1 that has a y-intercept of 8.



2. Worded problems involving feasible regions

Problems which involve multiple linear inequalities are known as linear programming problems. The inequalities in these problems are known as constraints. These will be covered in depth in the next lessons.

To solve a worded problem involving feasible regions, linear inequalities must first be deduced from the context of the question.

WORKED EXAMPLE 9 *(2 marks)*

Katie is a seamstress and in order to meet demand, she knows she must make at least five dresses and seven skirts each week. It takes her three hours to sew a dress and two hours to sew a skirt. She works at least 30 hours but less than 50 hours per week.

a) Express the information given in four linear inequalities. Let x be the number of dresses made per week and y be the number of skirts made per week. Graph the inequalities and shade in the feasible region.

b) Katie just received an order for fourteen skirts and eight dresses. Will she be able to fill this order in one week? Show your answer by plotting the point on the graph.

SOLUTION

a) Graph the inequalities and shade in the feasible region.

Step 1 Define the first two inequalities.

Katie must sew at least five dresses per week:

$x \geq 5$

Katie must sew at least seven skirts per week:

$y \geq 7$

Step 2 Define the last two inequalities.

The other two inequalities are slightly less straightforward.

We know that Katie must work at least 30 but less than 50 hours per week.

We also know that it takes Katie three hours to make a dress and two hours to make a skirt.

Hence,

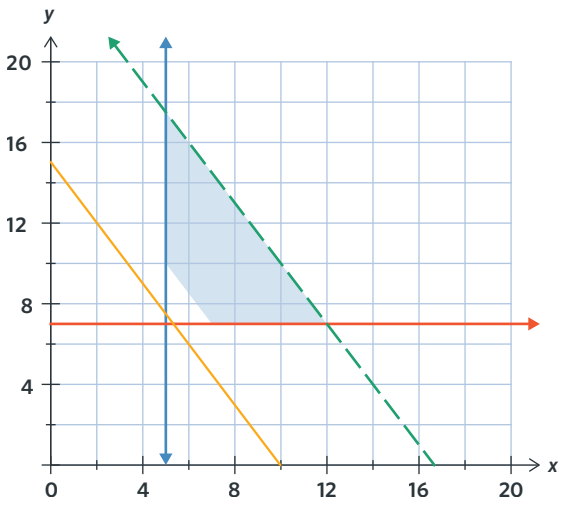
$3x + 2y \geq 30$

$3x + 2y < 50$

Step 3 Graph the inequalities and shade the feasible region.

The first three inequalities are inclusive so will be plotted using solid lines.

The fourth inequality is exclusive so will be plotted using a dotted line.



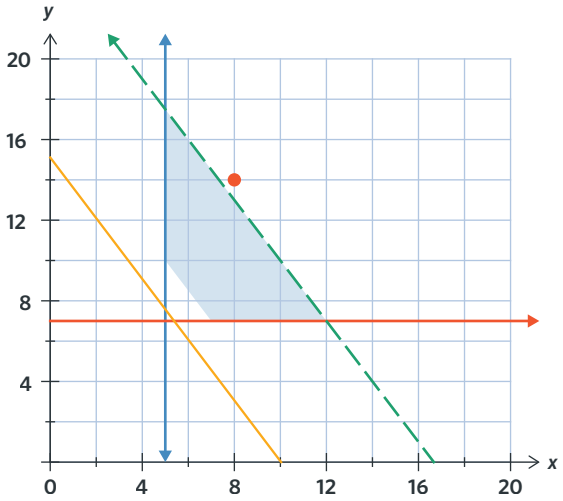
b) Will Katie be able to fill this order in one week?

Step 1 Write the given information as coordinates.

The order requires 14 skirts and 8 dresses. Therefore, $y = 14$ and $x = 8$.

The point is (8, 14).

Step 2 Plot the point on the graph.



As seen on the graph, the point is outside the feasible region. Hence, Katie will not be able to fill this order in one week.

Questions 10C *Feasible region*

Refresher question

Q1. Rearrange the inequality $2y + 3x > 24$ for y.

1. Sketching feasible regions

Q2. Skill

A feasible region is defined by the following three inequalities:

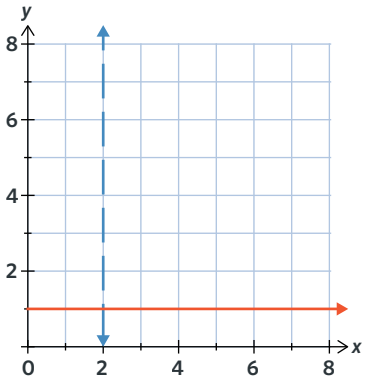
Inequality 1: $x > 2$

Inequality 2: $y \geq 1$

Inequality 3: $x + y \leq 7$

The lines of inequalities 1 and 2 have been plotted on the graph.

Sketch inequality 3 onto the graph and then shade the feasible region.



Q3. Skill

Sketch each of the feasible regions bounded by the inequalities below:

a) $x \geq 0$
$y \geq 0$
$x + y \leq 6$
$y - x \leq 2$

b) $x > -10$
$y < 5$
$x + y \leq 10$

c) $x < 16$
$y \leq 16$
$x + y \geq 8$
$2y - x \leq 22$

Q4. Skill

Consider a feasible region defined by the inequalities;

$x > -2$ $y < 12$ $x + y < 15$ $x \leq y + 3$

Identify whether each of the following points is inside or outside of the feasible region.

a) $(4, 5)$ **b)** $(6, -2)$ **c)** $(1, 12)$ **d)** $(8, 5)$ **e)** $(6, 9)$

Check your understanding

Q5. Skill

The feasible region in the following graph is bounded by four inequalities:

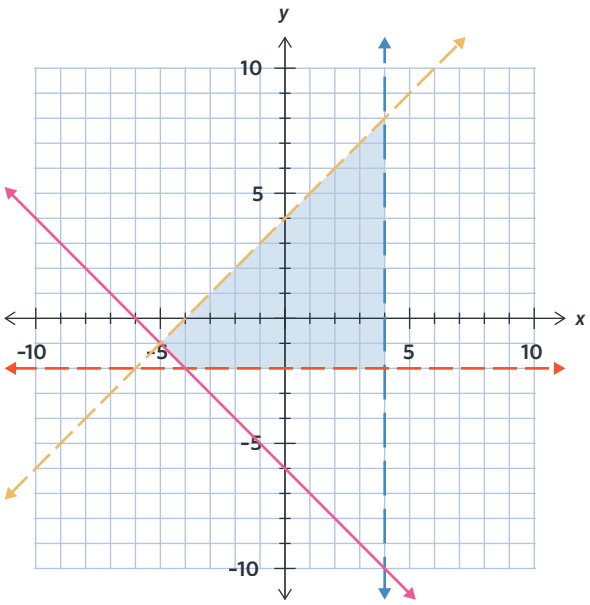
Inequality 1: $x < 4$

Inequality 2: $y > -2$

Inequality 3: $x + y \geq -6$

Inequality 4: ?

What is the equation of inequality 4?



2. Worded problems involving feasible regions

Q6. Application

Represent each of the following scenarios as a linear inequality using the variables provided.

a) In order to purchase alcohol in Australia, you must be at least 18 years of age. Let a be age.

b) A modelling agency requires their female models to be at least 175 but less than 185 centimetres tall. Let h be height.

c) A cafe in Melbourne has only enough coffee beans to sell 120 coffees a day. The cafe sells two types of coffees: espressos and cappuccinos. Let e be the number of espressos sold and c be the number of cappuccinos sold.

Q7. Application

Liv works at an animal shelter, caring for stray cats and dogs. For every dog, there are at least two cats. The shelter always has less than 40 cats due to lack of equipment. The maximum capacity of the shelter is 55 animals.

Let c be the number of cats and d be the number of dogs in the shelter at any given time.

a) Express the above information in three linear inequalities.

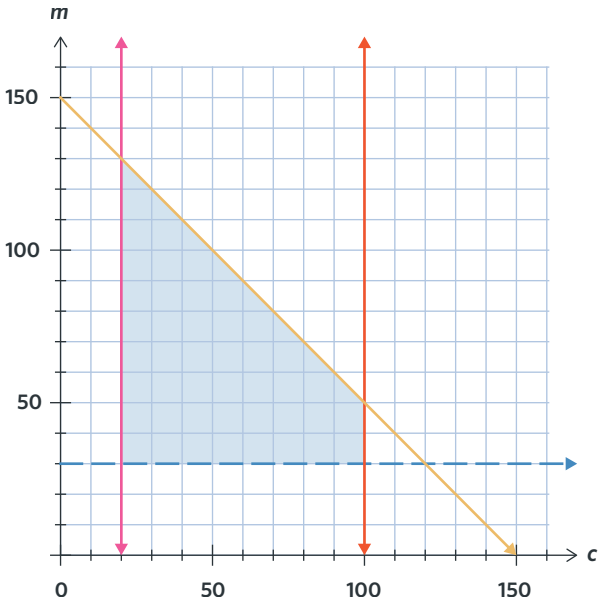
b) Plot the feasible region with c on the x-axis and d on the y-axis.

Q8. Application

A French bakery sells both macaroons and croissants. Let the number of macaroons sold be m and the number of croissants sold be c.

The feasible region for the sales of their pastries each day is shown on the graph.

Interpret, in the context of the situation, each of the inequalities on the graph.



Check your understanding

Q9. Application

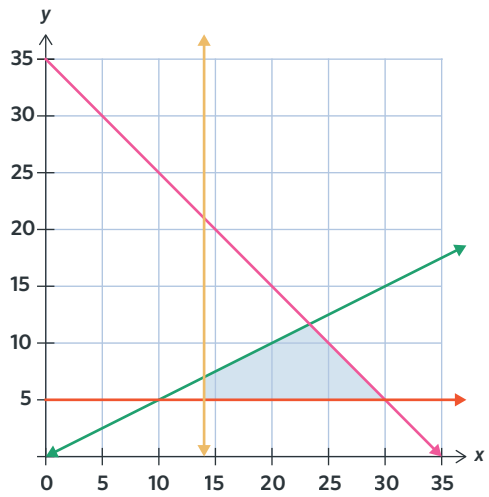
David is currently in Year 12 and is studying for his exams. His two hardest subjects are Chemistry and Geography, so he is dividing his time between the two.

David has decided that for each hour of Geography study he does, he will do at least two hours of study for Chemistry. He wants to spend a minimum of five hours per week on Geography and a minimum of 14 hours per week on Chemistry. David wants to study for no more than 35 hours each week.

Let x be the number of hours spent studying Geography and y be the number of hours spent studying Chemistry each week.

In which of the following graphs does the shaded area show the feasible region defined by these constraints?

A.



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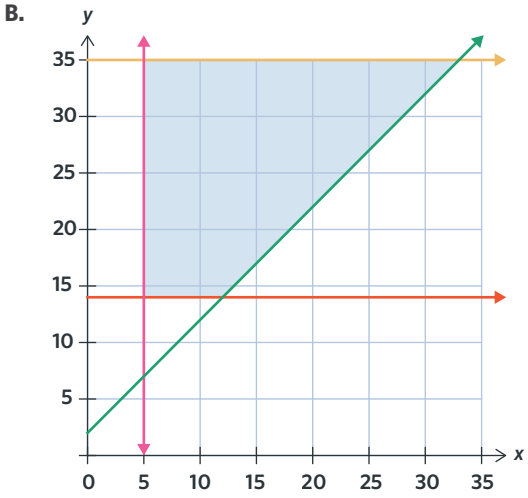
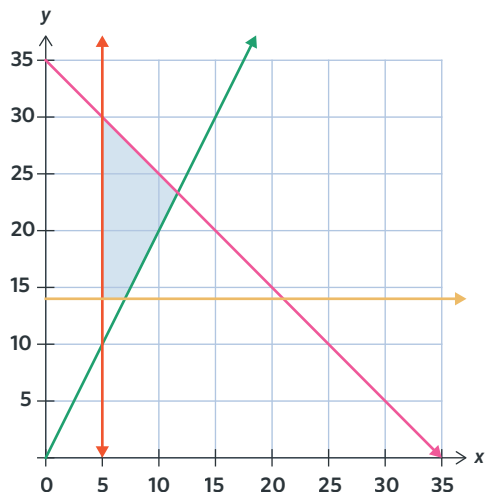
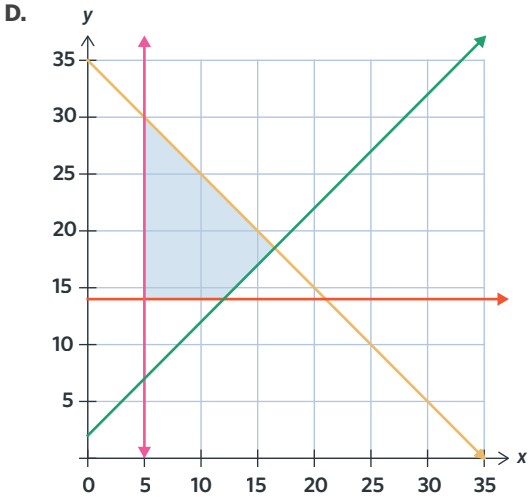


C.



D.



Joining it all together

Q10. Application 2 marks

A roller coaster at Luna Park has certain height and weight requirements.

Each passenger must be at least 120 centimetres tall to ride the roller coaster, and no heavier than 100 kg.

Assume that a person's weight, y (kg), can be related to their height, x (cm), by the inequality $y \geq \frac{3}{5}x - 30$.

a) Sketch the feasible region that satisfies these conditions.

b) According to these conditions, what is the maximum height a person can be to ride the roller coaster, correct to one decimal place?

Q11. For which of the following sets of linear inequalities is there no feasible region?

Skill
1 mark

A. $x > 0$, $y > 0$, and $x + y > 20$

B. $x \leq 6$, $y < -3$, and $y + x > 2$

C. $x \geq -5$, $y \leq -10$, and $y - x < -30$

D. $x < -1$, $y > 11$, and $x - y > 3$

Q12. Amy is taking a gap year and is saving up to travel. She has two part-time jobs; as a swim instructor and a receptionist.

Application
2 marks

Her contract as a swim instructor requires her to teach at least 10 hours and at most 24 hours per week. As a receptionist, Amy must work a minimum of 8 hours but less than 25 hours per week. She wants to work no more than 40 hours each week.

Let x be the number of hours per week spent working as a swim instructor and y be the number of hours per week spent working as a receptionist.

a) Sketch the feasible region.

b) Is it possible for Amy to work 22 hours as a swim instructor and 16 hours as a receptionist in one week while satisfying all of the conditions? Plot the point on the graph to check your answer.

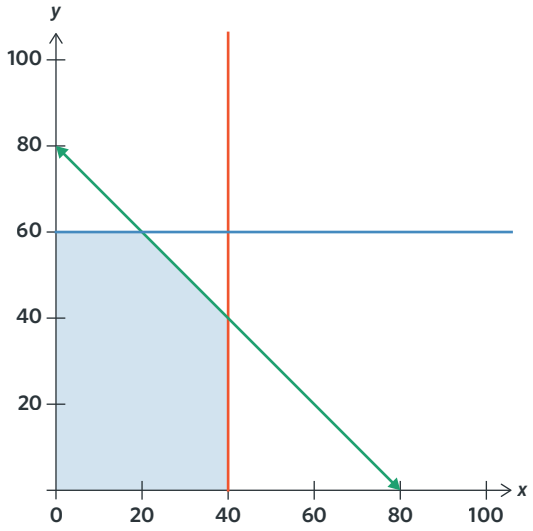
VCAA questions

Q13. In a linear programming problem involving animal management on a farm:

1 mark

- x represents the number of cows on the farm.
- y represents the number of sheep on the farm.

The feasible region (with boundaries included) for the problem is indicated by the shaded region on the following diagram.



One of the constraints defining the feasible region indicates that:

A. there must be 20 cows and 60 sheep.

B. there must be 40 cows and 40 sheep.

C. the number of sheep cannot exceed 40.

D. the number of cows must be at least 60.

E. the total number of cows and sheep cannot exceed 80.

VCAA 2006 Exam 1 Module 3: Graphs and relations. Q7

Q14. The four inequalities below were used to construct the feasible region for a linear programming problem.

1 mark

$x \geq 0$ $\quad y \geq 0$ $\quad x + y \leq 9$ $\quad y \leq \frac{1}{2}x$

A point that lies within this feasible region is:

A. $(4, 4)$ **B.** $(5, 3)$ **C.** $(6, 2)$ **D.** $(6, 4)$ **E.** $(7, 3)$

VCAA 2006 Exam 1, Module 3: Graphs and relations. Q9

Questions from multiple lessons

Q15.

Difficulty:

Year 10
1 mark

A music school requires at least one music teacher for every eight students.

Let x be the number of music teachers and y be the number of students.

Which one of the following inequalities represents this situation?

A. $y \leq \frac{x}{8}$

B. $y \geq \frac{x}{8}$

C. $y \leq \frac{8}{x}$

D. $y \geq 8x$

E. $y \leq 8x$

VCAA 2017 Exam 1 Module: 4 Graphs and relations Q5 – Adapted

Q16.

Difficulty:

1 mark

Consider the following recurrence relation.

$M_0 = 50\ 000, \quad M_{n+1} = M_n + 3250$

Which of the following scenarios could this recurrence relation be used to model?

A. A simple interest investment of \$50 000 with an annual interest rate of 3.25%.

B. A simple interest investment of \$50 000 with an annual interest rate of 6.5%.

C. A simple interest investment of \$5000 with an annual interest rate of 65%.

D. A compound interest investment of \$50 000 with an annual interest rate of 3.25%.

E. A compound interest investment of \$50 000 with an interest rate of 6.5%.

VCAA 2017 Exam 1 Recursion and financial modelling Q19 – Adapted

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Q17.

Difficulty:

2 marks

A juice company produces two flavours; blackcurrant and apple.

Let x be the number of blackcurrant juices sold each day.

Let y be the number of apple juices sold each day.

Each day, a maximum of 400 juices can be sold.

The inequalities below represent the constraints on the number of juices that can be sold each day.

Constraint 1: $x \geq 0$

Constraint 2: $y \geq 0$

Constraint 3: $x + y \leq 400$

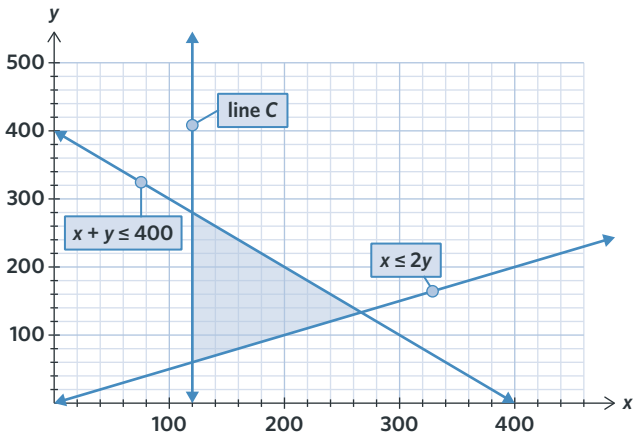
Constraint 4: $x \leq 2y$

a) Interpret Constraint 4 in terms of the number of blackcurrant and apple juices sold in one day. (1 mark)

There is another constraint, Constraint 5, on the number of each drink that can be sold each day.

Constraint 5 is bounded by Line *C*, shown in the following graph.

The shaded region of the graph contains the points that satisfy Constraints 1 to 5.



b) Write down the inequality that represents Constraint 5. (1 mark)

VCAA 2016 Exam 2 Module 4: Graphs and Relations Q3 a,b – Adapted

LESSON 10D

Linear programming corner-point principle

The key skills you will learn in this lesson are:

1. Finding the coordinates of the vertices of a feasible region
2. Finding the minimum or maximum solution

VCAA key knowledge point:

"use of the corner-point principle to determine the optimal solution/s of a linear programming problem"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Finding the coordinates of the vertices of a feasible region

The coordinates of the corner points of a feasible region can be found by sketching a graph of the linear inequalities that make up the feasible region, and determining where each of the boundaries intersect. These coordinates are extremely important when working out the minimum or maximum solution of an objective function, as discussed later in the lesson.

WORKED EXAMPLE 10 *(3 marks)*

Find the coordinates of the vertices of the feasible region defined by the following constraints.

$y \geq 0.5x - 6y \leq 2x + 6y \leq -3x + 7$

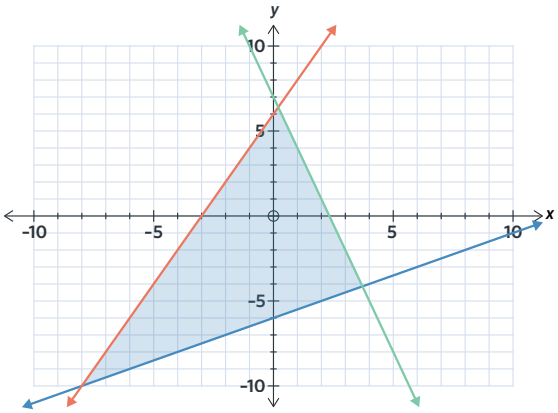
SOLUTION

Step 1 Sketch the feasible region.

$y \geq 0.5x - 6$

$y \leq 2x + 6$

$y \leq -3x + 7$



Step 2 Find the point of intersection between the lines $y = 0.5x - 6$ and $y = 2x + 6$.

$0.5x - 6 = 2x + 6$

$1.5x = -12$

$x = -8$

Substitute $x = -8$ into $y = 0.5x - 6$.

$y = (0.5 \times -8) - 6$

$y = -4 - 6$

$y = -10$

Point of intersection: $(-8, -10)$

✓ *1 mark for finding the point of intersection*

Step 3 Find the point of intersection between the lines $y = 0.5x - 6$ and $y = -3x + 7$.

$0.5x - 6 = -3x + 7$

$3.5x = 13$

$x = \frac{13}{3.5}$

$x = \frac{26}{7}$

Substitute $x = \frac{26}{7}$ into $y = 0.5x - 6$.

$y = \left(0\,5 \times \frac{26}{7}\right) - 6$

$y = \frac{13}{7} - 6$

$y = -\frac{29}{7}$

Point of intersection: $\left(\frac{26}{7}, -\frac{29}{7}\right)$

✓ *1 mark for finding the point of intersection*

Step 4 Find the point of intersection between the lines $y = 2x + 6$ and $y = -3x + 7$.

$2x + 6 = -3x + 7$

$5x = 1$

$x = \frac{1}{5}$

Substitute $x = \frac{1}{5}$ into $y = 2x + 6$.

$y = \left(2 \times \frac{1}{5}\right) + 6$

$y = \frac{2}{5} + 6$

$y = \frac{32}{5}$

Point of intersection: $\left(\frac{1}{5}\ \frac{32}{5}\right)$

The coordinates are $(-8, -10)$, $\left(\frac{26}{7}, -\frac{29}{7}\right)$ and $\left(\frac{1}{5}\ \frac{32}{5}\right)$.

✓ *1 mark for finding the point of intersection*

2. Finding the minimum or maximum solution

The objective function is a function including the variables of a linear programming problem. Often an objective function needs to be minimised or maximised. This could be to minimise resources used or to maximise profit. It is impossible to test every point in the feasible region, therefore the corner-point principle is utilised.

The corner-point principle states that the minimum or maximum solution of an objective function will lie on one of the vertices of the feasible region.

To find the minimum or maximum solution, the coordinate of each vertex of the feasible region is substituted into the objective function, as shown in the worked example below.

WORKED EXAMPLE 11 *(2 marks)*

If the coordinates of the vertices of a feasible region are $(-8, -10)$, $\left(\frac{26}{7}\ \frac{29}{7}\right)$ and $\left(\frac{1}{5}\ \frac{32}{5}\right)$, what are the minimum and maximum solutions of the objective function $A = -5x + 7y$, correct to one decimal place?

SOLUTION

Step 1 Substitute the point $(-8, -10)$ into A.

$A = (-5 \times -8) + (7 \times -10)$

$A = 40 - 70$

$A = -30$

Step 2 Substitute the point $\left(\frac{26}{7}\ -\frac{29}{7}\right)$ into A.

$A = \left(-5 \times \frac{26}{7}\right) + \left(7 \times -\frac{29}{7}\right)$

$A = -\frac{130}{7} - 29$

$A = -\frac{333}{7} = -47.6$

Step 3 Substitute the point $\left(\frac{1}{5}\ \frac{32}{5}\right)$ into A.

$A = \left(-5 \times \frac{1}{5}\right) + \left(7 \times \frac{32}{5}\right)$

$A = -1 + \frac{224}{5}$

$A = 43.8$

✓ *1 mark for substituting the points into equations*

Step 4 Find the minimum and maximum solutions.

The minimum solution is -47.6 and the maximum solution is 43.8.

✓ *1 mark for identifying the minimum and maximum solution*

Questions 10D *Linear programming corner-point principle*

Refresher question

Q1. What are the coordinates of the point of intersection of the two lines below?

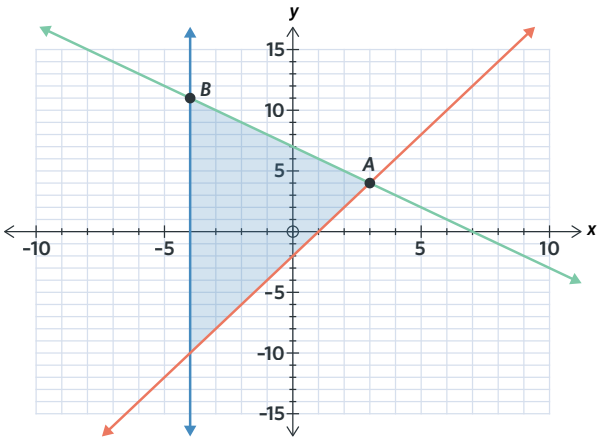
$y = -3x + 4$

$y = 5x - 6$

1. Finding the coordinates of the vertices of a feasible region

Q2. Skill

What are the coordinates of points A and B?



Q3. Skill

Find the coordinates of the vertices of the feasible region defined by the constraints below, correct to one decimal place where appropriate.

a) $y \geq -2$
$y \leq 1.5x + 4$
$y \leq -2x + 6$

b) $y \geq 0$
$x \geq -1$
$y \leq 0.5x + 5$
$y \leq -2x + 9$

c) $y \leq -2x$
$x \geq -4$
$y \geq 3x - 6$
$y \geq 0.5x - 4$

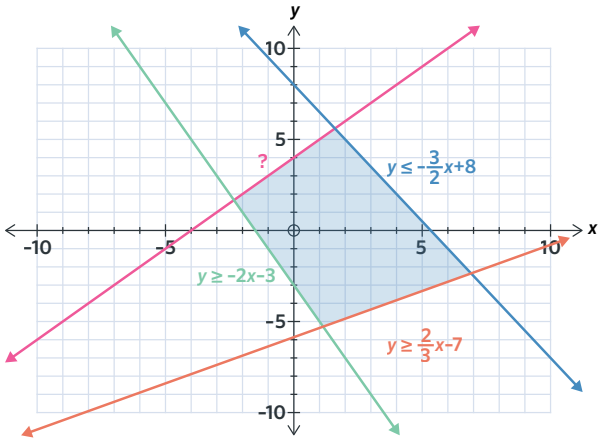
Check your understanding

Q4. Skill

Using this feasible region:

a) Find the missing inequality for the **pink** line.

b) Find the coordinates of the vertices of the shaded feasible region.



2. Finding the minimum or maximum solution

Q5. Skill

If the coordinates of the vertices of a feasible region are $(1, 2)$, $(3, -4)$, and $(-2, -3)$, what are the minimum and maximum solutions of the objective function $A = 3x + 4y$?

Q6. Skill

For all the feasible regions described in question 3, determine the minimum and maximum solutions of the objective function $A = 3x - 2y$.

Check your understanding

Q7. Application

A dog grooming salon grooms small dogs and large dogs. On a particular day:

i more than two but less than twenty-five small dogs were groomed.

ii more than two but less than twenty large dogs were groomed.

iii no more than forty dogs were groomed.

a) Set up the inequalities for i, ii and iii using S to represent number of small dogs groomed and L represent number of large dogs groomed.

b) If the salon charges \$60 to groom a small dog and \$75 to groom a large dog, what is the maximum revenue the salon could have made that day?

Q8. Skill

Consider the feasible region described by the constraints:

$y \geq 2x - 7$ $x \geq -2$ $y \leq -x + 8$ $y \geq -4x - 10$

Which objective function has the greatest maximum?

A. $2x - 3y$ **B.** $-x + 4y$ **C.** $3x - 5y$ **D.** $x + 6y$

Joining it all together

Q9. Application 1 mark

The shape of a field can be described by the constraints:

$x \leq 3$ $x \geq -1$ $y \geq -2x - 3$ $y \leq x + 5$

If one unit on the cartesian plane represents four metres, what is the area of the field?

Q10. Application 4 marks

A bakery sells blueberry muffins and banana bread. Let M represent the number of blueberry muffins sold and let B represent the number of banana bread slices sold. On a particular day, the number of blueberry muffins and banana bread sold can be represented by the following inequalities:

Inequality 1: $B \geq 0$

Inequality 2: $B + M \leq 40$

Inequality 3: $B \leq 3M$

Inequality 4: $B + 2M \leq 60$

a) Describe the meaning of inequality 3.

b) Draw the feasible region with M on the x-axis and B on the y-axis.

c) If blueberry muffins sell for \$3 and banana bread sells for \$2, find an equation to describe the revenue made by the bakery, $R(\$)$.

d) What is the maximum revenue that could have been made on that day?

Q11. Skill 1 mark

Three of the four constraints that make up a feasible region are:

$y \geq 2x - 8$ $y \geq -1$ $y \geq -3x - 4$

If the maximum and minimum solutions for the objective function $B = 10x - 2y$ are -49.6 and 47.2 respectively, what is the other inequality that makes up the feasible region?

Questions from multiple lessons

Q12.

Difficulty:

1 mark

A feasible region has been created using the three inequalities below.

$x < 14$

$y < 8$

$x + y > 18$

A point that lies within this feasible region is

A. $(15, 4)$

B. $(7, 12)$

C. $(10, 13)$

D. $(12, 7)$

E. $(12, 6)$

VCAA 2018 Exam 1 Module 4: Graphs and relations Q3 – Adapted

Q13.

Difficulty:

1 mark

The variables *weight* (less than 1 kg, 1–2 kg, over 2 kg) and *size* (small, medium, large) are

A. both numerical variables.

B. a numerical and ordinal variable respectively.

C. both nominal variables.

D. an ordinal and nominal variable respectively.

E. both ordinal variables.

VCAA 2018NH Exam 1 Data analysis Q5 – Adapted

Q14.

Difficulty:

2 marks

Antonio has decided to paint his bedroom pink.

He has a tin of red paint and a tin of white paint.

Let x be the amount of red paint used, in litres.

Let y be the amount of white paint used, in litres.

To cover the entire room, he will require at least five litres and at most seven litres of paint.

He will have to mix the red and white paint to acquire his favourite shade of pink. The amount of white paint used must be at least twice the amount of red paint used.

This information can be represented by Inequalities 1, 2 and 3.

Inequality 1: $x + y \geq 5$

Inequality 2: $x + y \leq 7$

Inequality 3: $y \geq 2x$

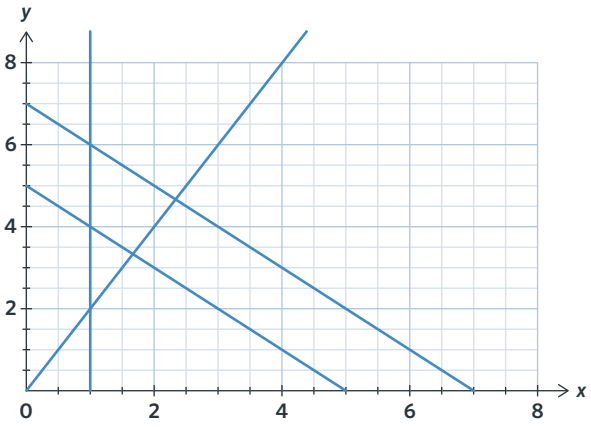
There is another constraint given by

Inequality 4: $x \geq 1$

a) Describe Inequality 4 in terms of the amount of paint required. (1 mark)

b) The following graph shows the lines that represent the boundaries of Inequalities 1 to 4.

Recreate the graph and shade the region that contains the points that satisfy these inequalities. (1 mark)



VCAA 2015 Exam 2 Module 3: Graphs and relations Q5 a,b – Adapted

LESSON 10E

Applications of linear programming

The key skill you will learn in this lesson is:

1. Finding optimum points using the sliding line method

VCAA key knowledge point:

"formulation and graphical solution of linear programming problems involving two variables"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.21; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Finding optimum points using the sliding line method

- The sliding line method provides another way to determine the maximum or minimum point of an objective function within a feasible region. It works by taking the objective function, transposing it so it is in terms of y, and sliding the line up and down the y-axis until it intersects one of the corners of the feasible region.

The intersection in which the objective function is highest up the y-axis can be rearranged to find the maximum value of the objective function. The intersection in which the objective function is lowest down the y-axis can be rearranged to find the minimum value of the objective function.

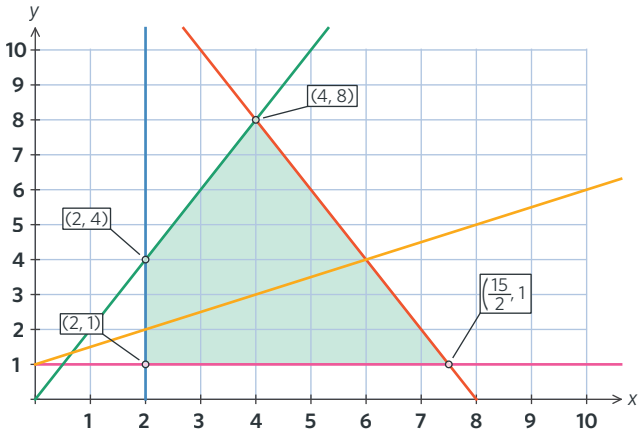
Example

The following feasible region is given by:

- $y \geq 1$
- $x \geq 2$
- $y \leq -2x + 16$
- $y \leq 2x$

For the objective function $P = -x + 2y$, we can rearrange the equation to find $y = \frac{1}{2}x + \frac{P}{2}$. This is plotted in orange, using $P = 2$.

In the equation $y = \frac{1}{2}x + \frac{P}{2}$ $\frac{P}{2}$ is the y-intercept. Finding the maximum value of P within the feasible region is done by finding the maximum y-intercept the line can reach while staying in the feasible region. This is shown in the second graph.

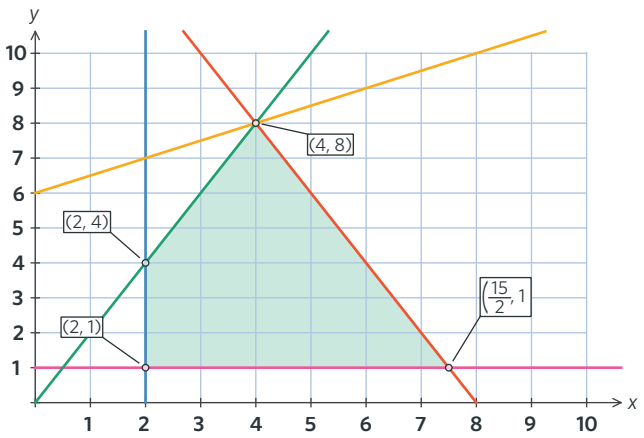


In this graph, we see the maximum y-intercept that can be obtained using a line of the form $y = \frac{1}{2}x + \frac{P}{2}$ is 6. This occurs for the corner at $(4, 8)$. P can be found by remembering that $\frac{P}{2}$ is equal to the y-intercept.

$$\frac{P}{2} = 6$$

$$P = 12$$

So the maximum value of $P = -x + 2y$ in the feasible region is 12.



WORKED EXAMPLE 12 *(2 marks)*

Xue is buying apples, x, and bananas, y, at a stall for her classmates. She knows that five students only eat apples and three students only eat bananas.

Her teacher asked her to buy between 30 and 50 pieces of fruit such that there are at least twice as many bananas as apples. The cost of one apple is \$0.80 and one banana is \$1.20.

What is the cheapest way to satisfy all of the demands?

SOLUTION

Step 1 Convert the written information to inequalities and equations.

"Five students only eat apples" $\rightarrow x \geq 5$

"Three students only eat bananas" $\rightarrow y \geq 3$

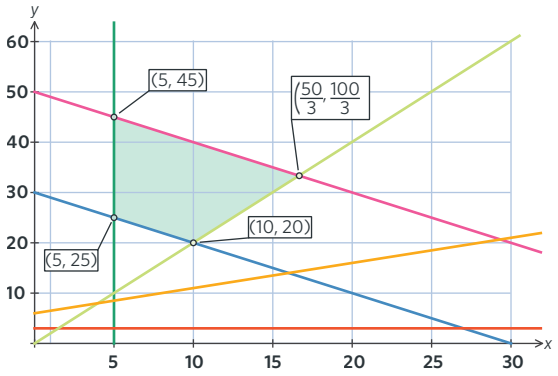
"Buy between 30 and 50 pieces of fruit" $\rightarrow x + y \geq 30$ and $x + y \leq 50$

"At least two times the number of bananas as apples" $\rightarrow y \geq 2x$

"The cost of one apple is \$0.80 and one banana is \$1.20." $C = 0.8x + 1.2y$

NOTE: The last equation is the objective function.

Step 2 Plot the feasible region.



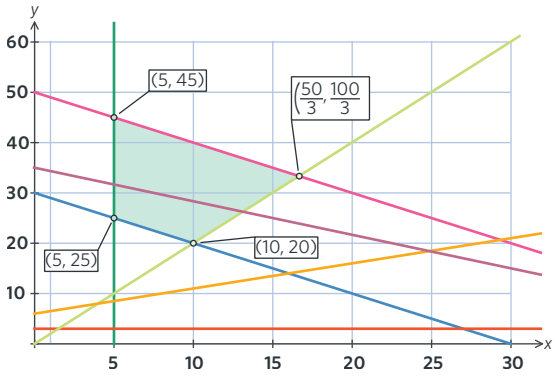
✓ *1 mark for plotting the feasible region*

Step 3 Transpose the objective function to make y the subject.

$$C = 0.8x + 1.2y$$

$$1.2y = -0.8x + C$$

$$y = -\frac{2}{3}x + \frac{5}{6}C$$



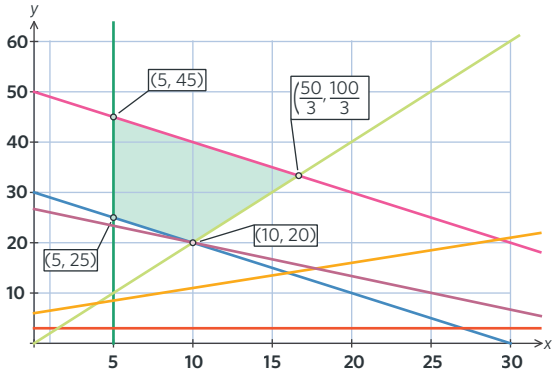
Step 4 Examine the equation for how a minimum value of C will affect the equation.

The y-intercept is $\frac{5}{6}C$. The minimum value of C will result in the minimum y-intercept for the feasible region.

Step 5 Move the line up or down to maximise or minimise C.

Here we move the line down to reach the minimum y-intercept while still remaining within the feasible region.

We find the point $(10, 20)$ results in the minimum y-intercept.



Step 6 Find the value of C by using the y-intercept, if it can be read off the graph, or by using the point found in step 5.

We know the y-intercept equals $\frac{5}{6}C$, but the y-intercept is hard to read off the graph. So we'll use the formula.

$$C = 0.8x + 1.2y$$

$$C = 0.8 \times 10 + 1.2 \times 20$$

$$C = 32$$

The cheapest arrangement is to buy 10 apples and 20 bananas, which costs \$32.

✓ *1 mark for correct numbers of apples and bananas*

Questions 10E *Applications of linear programming*

Refresher question

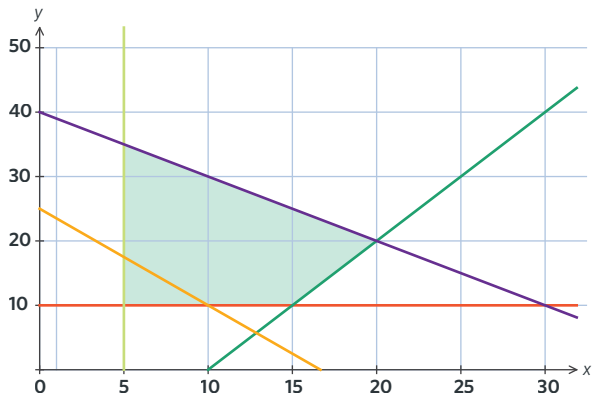
Q1. Which of the following is greater than $\frac{3}{5}$?

A. $\frac{1}{2}$ **B.** $\frac{2}{3}$ **C.** $\frac{3}{6}$ **D.** $\frac{3}{5}$

1. Finding optimum points using the sliding line method

Q2. Skill

Astrid has already plotted the feasible region, and transposed the objective equation to be in terms of y, $y = -\frac{3}{2}x + 2P$. Move the orange line up and down to determine which corner point will give the largest value of P, and state P. Plot the graph of the feasible region and the orange line in this position.



Q3. Skill

The following constraints apply to x and y.

$x \geq 4$ $y \geq 8$ $x + y \leq 20$ $y \geq 2x - 6$

For each of the following objective equations, state the maximum value in the feasible region, and the x and y values for which the maximum occurs.

a) $P = -3x + y$

b) $C = 3x + 2y$

For each of the following objective equations, state the minimum value in the feasible region, and the x and y values for which the minimum occurs.

c) $S = x + 2y$

d) $T = -x + 3y$

Check your understanding

Q4. Application

Bruce is playing tennis tournaments around Victoria to try to win some money. He has already entered six tournaments this year, and won two of them. He plans to enter between 14 and 18 tournaments for the year. He expects to win at least one tournament for every three he doesn't win, but also expects to win no more than 12 tournaments.

Use x to represent tournaments that Bruce didn't win, and y to represent tournaments he won.

a) Use this information to plot a feasible region.

b) If the average prize money for winning a tournament is \$500, and the entry fee is \$50, use the sliding line method to find the maximum amount of profit he can make this year, W. Assume there is no prize money for coming second, third, etc.

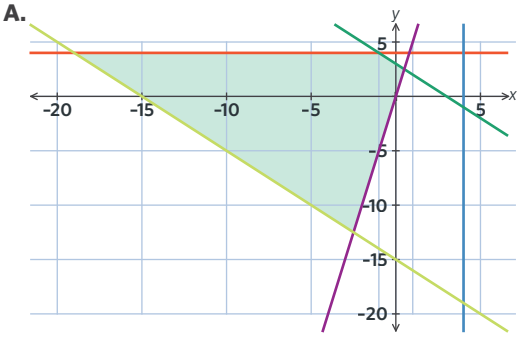
Joining it all together

Q5. Match the feasible region to the set of constraints.

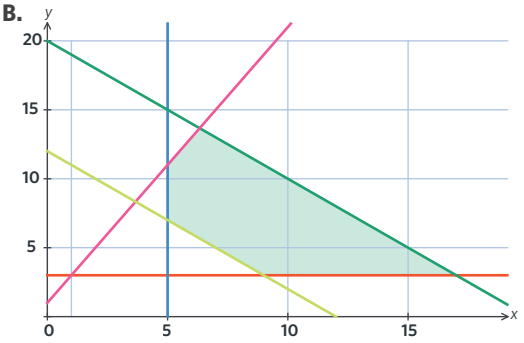
Skill
4 marks

Set 1	Set 2	Set 3	Set 4
$y \geq 3$ $x \geq 5$ $x + y \geq 12$ $x + y \leq 20$ $y \leq 2x + 1$	$y \geq 6$ $x \geq 3$ $x + y \geq 13$ $x + y \leq 25$ $y \geq \frac{3}{2}x + 1$	$y \leq 4$ $x \leq 4$ $x + y \geq -15$ $x + y \leq 3$ $y \leq 5x$	$y \leq 2$ $x \leq -2$ $x + y \geq -10$ $x + y \leq 0$ $y \leq \frac{1}{5}x + 1$

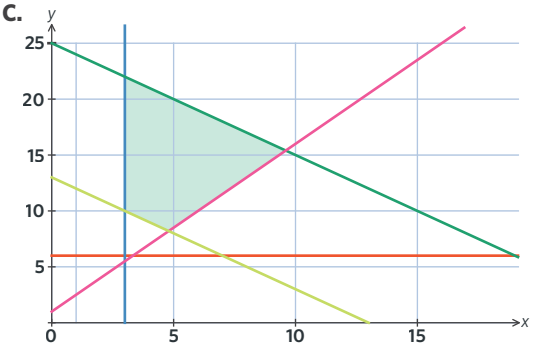
A.



B.



C.



D.

Q6. Class 11D at school are running a bake sale to raise money for the Good Friday Appeal. The class is trying to work out the optimum numbers of muffins, x, and brownies, y, to bake. They expect to bake at least 60 muffins and 80 brownies. The class feels brownies are easier to make, so they plan to bake at least 50% more brownies than muffins. They expect to bake between 180 and 300 items altogether.

Application
5 marks

a) Draw this information as a feasibility region. (2 marks)

b) If they plan to sell muffins for \$2.50 and brownies for \$1.50, what is the maximum amount of money they can raise? Give the number of muffins and brownies sold for this scenario. (1 mark)

c) The class realises they also have a time constraint. Between them they have 11 hours in which they can bake items. They expect on average it takes four minutes to make a muffin, and two minutes to make a brownie. Write this information as an inequality. (1 mark)

d) Using your answer in part c, find the new number of muffins and brownies baked to maximise profit. Find the corresponding profit. (1 mark)

Q7.
Application
4 marks

Marc runs a hair salon. On Saturday mornings his employee, Tina, works by herself. On average it takes 10 minutes to do an express cut, and 30 minutes to do a salon cut. Marc wants at least three of each cut to be done every day. On Saturdays the salon will book at least twice as many express cuts as salon cuts, and they never do more than a total of 16 haircuts.

a) Write an equation for the amount of time Tina will work, T, based on the number of express cuts, x, and salon cuts, y. Assume there are no gaps between appointments.

b) What is the minimum amount of time Tina will work this Saturday? Use the sliding line method to find your answer.

c) Write an equation for the amount of revenue made on Saturday, R, if express cuts cost \$15 and salon cuts cost \$40.

d) Assuming the salon is only open from 9 am–1 pm on Saturdays, what is the maximum possible amount of revenue that can be made?

VCAA question

Q8.
2 marks

Cheapstar Airlines wishes to find the optimum number of flights per day on two of its most popular routes: Alberton to Bisley and Alberton to Crofton.

Let:

- x be the number of flights per day from Alberton to Bisley
- y be the number of flights per day from Alberton to Crofton

The table shows the constraints on the number of flights per day and the number of crew per flight.

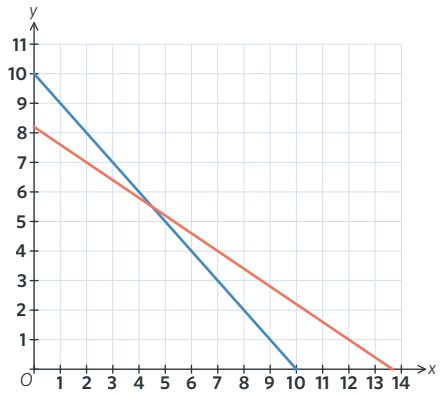
	Alberton to Bisley	Alberton to Crofton	Maximum per day	Constraint
Number of flights per day	x	y	10	$x + y \leq 10$
Number of crew per flight	3	5	41	$3x + 5y \leq 41$

The lines $x + y = 10$ and $3x + 5y = 41$ are graphed on the right.

A profit of \$1300 is made on each flight from Alberton to Bisley and a profit of \$2100 is made on each flight from Alberton to Crofton.

Determine the maximum total profit that Cheapstar Airlines can make per day from these flights.

Adapted from VCAA 2009 Exam 2, Module 3: Graphs and relations. Q4



Questions from multiple lessons

Q9.
Difficulty:
1 mark

The constraints of a linear programming problem are given by the following set of inequalities.

$x + y \leq 12$

$y \geq x - 4$

$x \geq 0$

$y \geq 0$

The coordinates of the points that define the boundaries of the feasible region for this linear programming problem are

A. $(0, 0), (0, 12), (8, 4), (12, 0)$

B. $(0, 0), (4, 0), (8, 4), (12, 0)$

C. $(0, 0), (0, 4), (4, 8), (12, 0)$

D. $(0, 0), (4, 0), (4, 8), (12, 0)$

E. $(0, 0), (0, 12), (4, 0), (8, 4)$

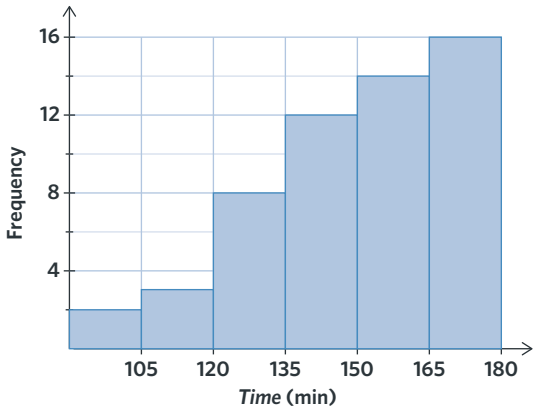
VCAA 2014 Exam 1 Module 3: Graphs and relations Q8 – Adapted

Q10.

Difficulty:

1 mark

The following histogram displays the time taken in minutes for a group of university students to complete a three-hour exam.



The shape of this distribution can be described as

A. approximately symmetric.

B. approximately normal.

C. negatively skewed.

D. positively skewed.

E. symmetric.

VCAA 2017NH Exam 1 Data analysis Q1 – Adapted

Q11.

Difficulty:

3 marks

A local cafe sells cappuccinos and lattes.

Let x be the number of cappuccinos sold each day.

Let y be the number of lattes sold each day.

Each cappuccino costs \$4 and each latte costs \$3.50.

The cafe sets constraints in order to meet its daily quota.

Constraint 1: $x \geq 10$

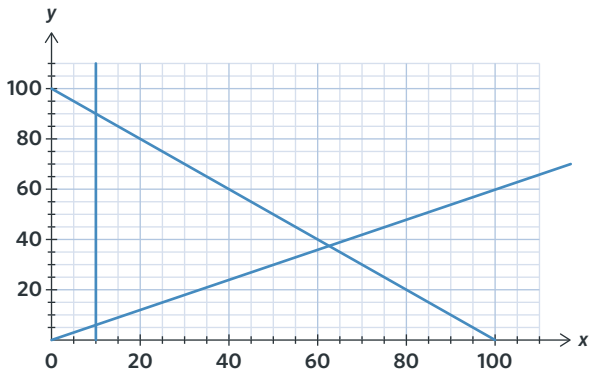
Constraint 2: $5y \geq 3x$

Constraint 3: $4x + 3.5y \geq 200$

Constraint 4: $x + y \leq 100$

a) Explain the meaning of constraint 3 in terms of the revenue acquired in selling cappuccinos and lattes. (1 mark)

The lines $x = 10$, $3x = 5y$, and $x + y = 100$ are shown.



b) Draw the line $4x + 3.5y = 200$ on the graph. (1 mark)

c) What is the maximum number of cappuccinos that can be sold each day? (1 mark)

VCAA 2012 Exam 2 Module 3: Graphs and relations Q3 a,c,d – Adapted

AOS 5: Graphs of linear and non-linear relations

Variation

CHAPTER
11

LESSON 11A

Variation using a graphical approach

The key skills you will learn in this lesson are:

1. Identifying direct, inverse and joint variation
2. Drawing graphs of direct, inverse and joint variation

VCAA key knowledge point:

"numerical, graphical and algebraic approaches to direct, inverse and joint variation"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Identifying direct, inverse and joint variation

When two variables are proportional, it can be said that they vary with each other, and a change in one variable results in a change in the other variable.

The proportionality sign, $\propto$, indicates that two variables are proportional.

An equation relating the two variables can be found by replacing the proportionality sign with '$= k$', where k, called the constant of proportionality, is the ratio between the two variables.

There are three types of variation: direct, inverse and joint.

Direct variation

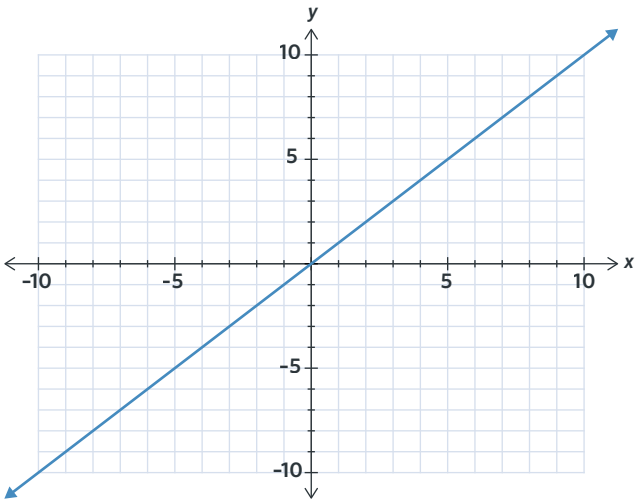
When two variables vary directly, an increase in one variable results in an increase in the other variable.

If y varies directly with x:

$$y \propto x$$

$$y = kx$$

The graph of the relationship between x and y is a straight line through the origin with gradient k.
The following is a graph showing $y = kx$ where $k = 1$.



Inverse variation

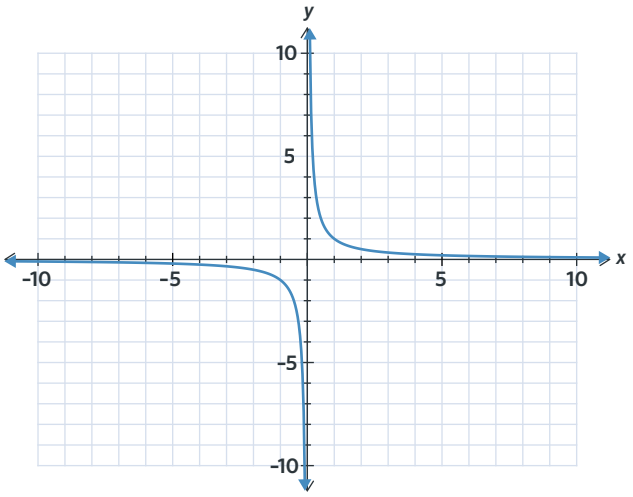
When two variables vary inversely, an increase in one variable results in a decrease in the other variable.

If y varies inversely with x:

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

The graph of the relationship between x and y is a hyperbola. The following is a graph showing $y = \frac{k}{x}$ where $k = 1$.



Joint variation

Joint variation is when one variable is affected by two or more variables. Below are some of the many examples of joint variation.

If y varies directly with the square of x and inversely with z:

$$y \propto \frac{x^2}{z}$$

$$y = k\frac{x^2}{z}$$

If y varies directly with x and directly with the square root of z:

$$y \propto x\sqrt{\ }$$

$$y = kx\sqrt{\ }$$

If y varies directly with the cube of z and inversely with the square root of x:

$$y \propto \frac{z^3}{\sqrt{\ }}$$

Drawing a graph with three or more variables is outside the scope of this course, but it is possible to draw the graph if enough variables are replaced by constants such that only two variables remain.

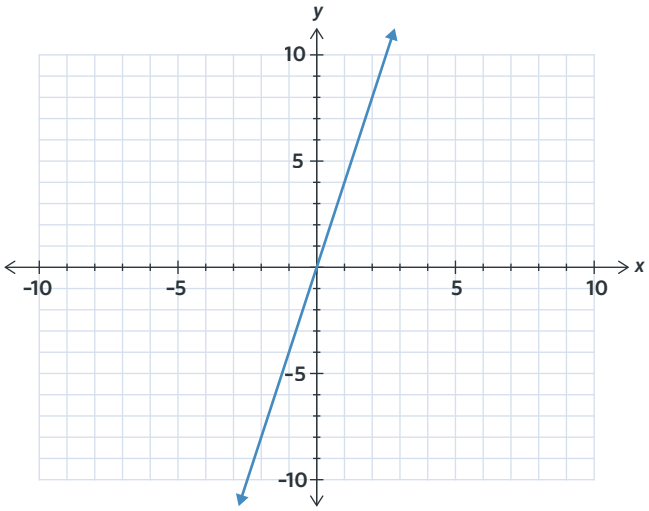
Below is a graph showing $y = kx\sqrt{\ }$ where $z = 4$ and $k = 2$.

$y = kx\sqrt{\ }$ can be simplified by substituting in the values of $z = 4$ and $k = 2$:

$$y = 2x\sqrt{\ }$$

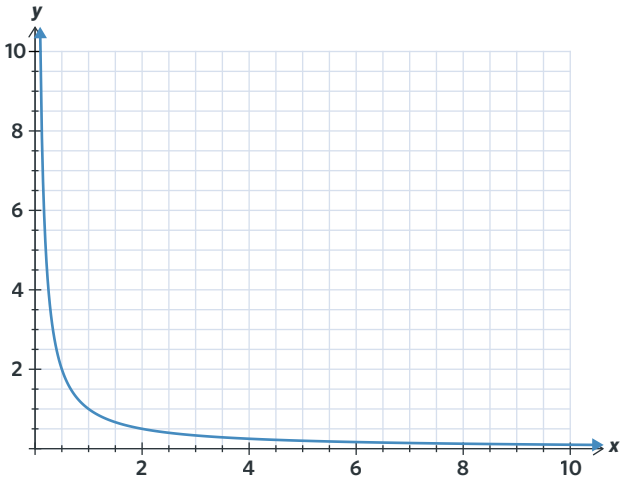
$$y = 2x \times 2$$

$$y = 4x$$



WORKED EXAMPLE 1 *(1 mark)*

Does the graph below show direct, inverse or joint variation?



SOLUTION

Step 1 Identify the shape of the graph.

The graph is in the shape of a hyperbola.

Step 2 Determine what type of variation the shape of the graph corresponds to.

Inverse variation results in a graph in the shape of a hyperbola.

The graph shows inverse variation.

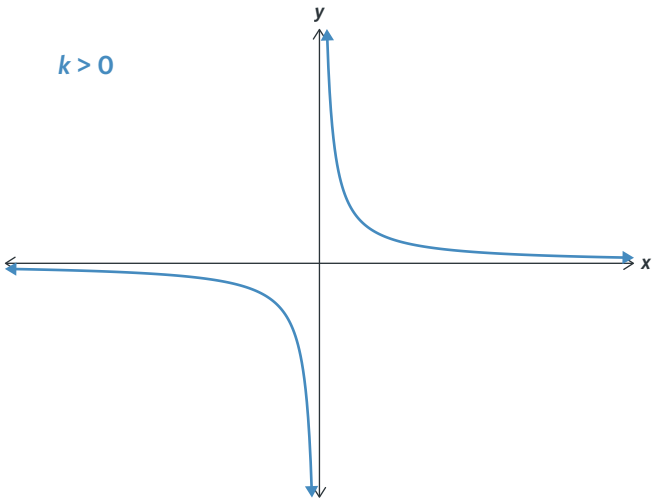
2. Drawing graphs of direct, inverse and joint variation

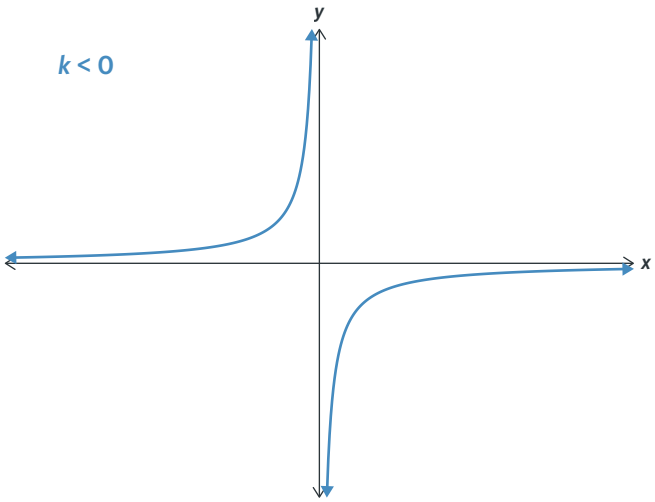
Direct variation

If two variables vary directly, the resulting graph is a straight line through the origin with gradient k. Refer to lesson 9A for revision on how to graph linear equations.

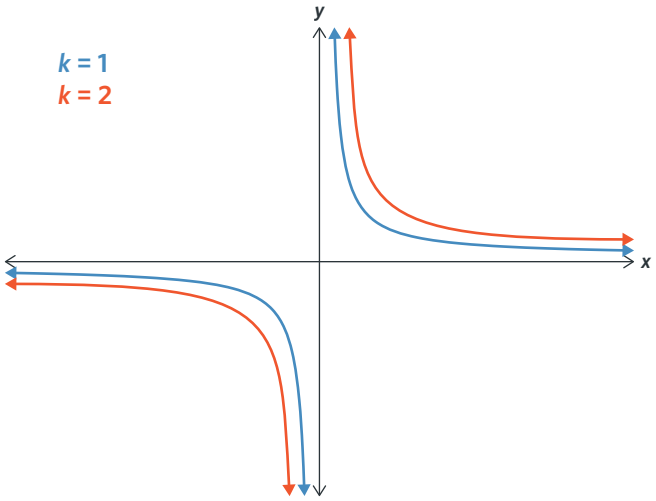
Inverse variation

If two variables vary inversely, the resulting graph is a hyperbola. The hyperbola will be in either of the two forms below, depending on whether the constant of proportionality is negative or positive.





Larger values of k will bring the graph away from the axes as shown below:



Joint variation

If two or more variables vary jointly, the resulting graph can only be drawn if one or more variables are replaced by a constant.

WORKED EXAMPLE 2 *(1 mark)*

If y varies directly with x and $k = 0.4$, draw a graph to represent the relationship between y and x.

SOLUTION

Step 1 Identify whether the two variables vary directly, inversely or jointly.

y varies directly with x.

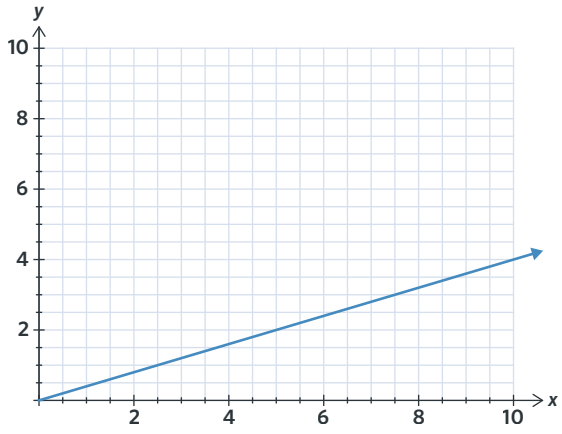
$y \propto x$

$y = kx$

Step 2 Substitute the value of k into the equation.

$y = 0.4x$

Step 3 Draw the graph of $y = 0.4x$.



Questions 11A *Variation using a graphical approach*

Refresher question

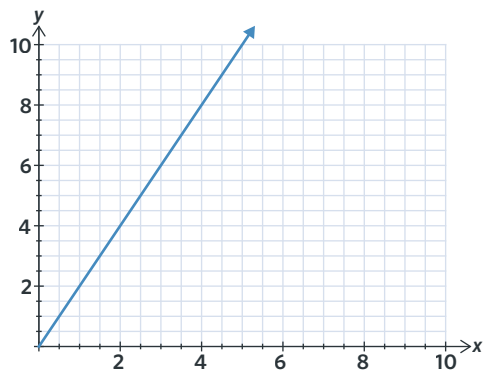
Q1. Draw the graph of $y = -x$.

1. Identifying direct, inverse and joint variation

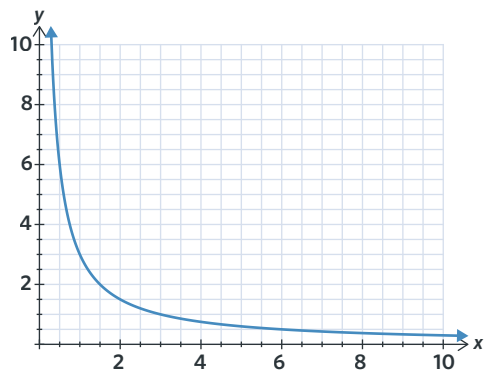
Q2. Do the graphs below show direct, inverse or joint variation?

Skill

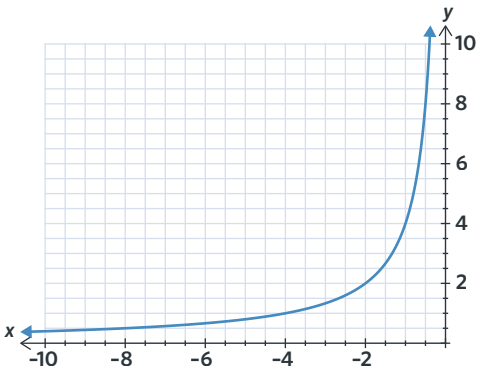
a)



b)



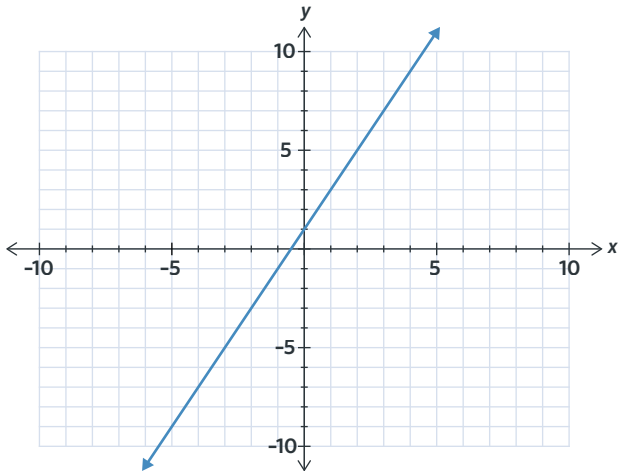
c)



Check your understanding

Q3. Skill

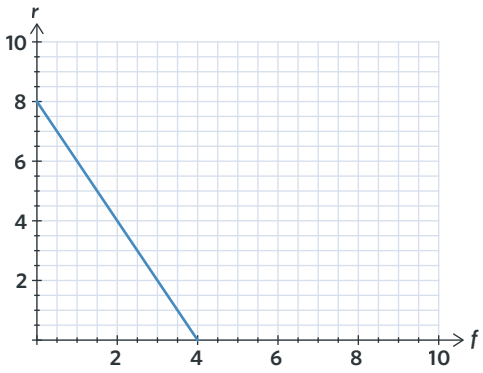
In the following graph, does y vary directly with x?



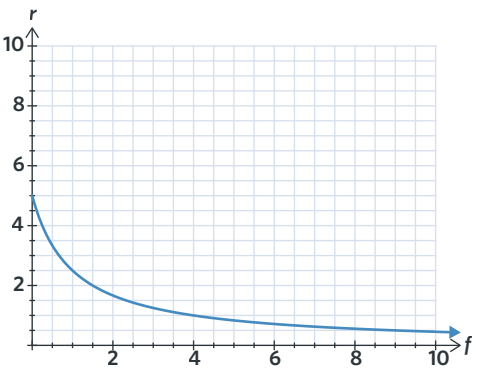
Q4. Application

The population of rabbits in a town, r, is inversely proportional to the population of foxes, f. Which graph could represent the relationship between foxes and rabbits?

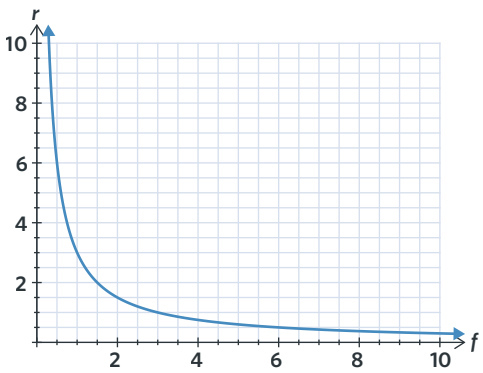
A.



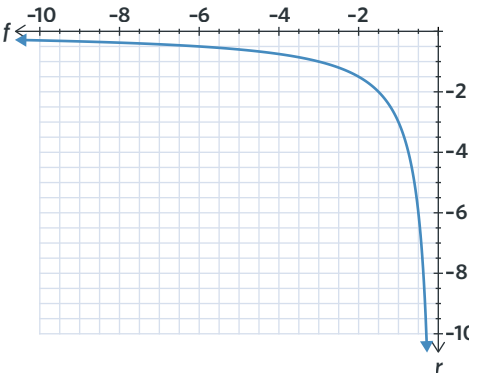
B.



C.



D.



2. Drawing graphs of direct, inverse and joint variation

Q5. Skill

If $y \propto x$ and $k = 0.3$, draw a graph of the relationship between y and x.

Q6. Skill

Draw a graph to represent the following relationships:

a) y varies directly with x, and $k = -3$.

b) y varies inversely with x, and $k = 5$.

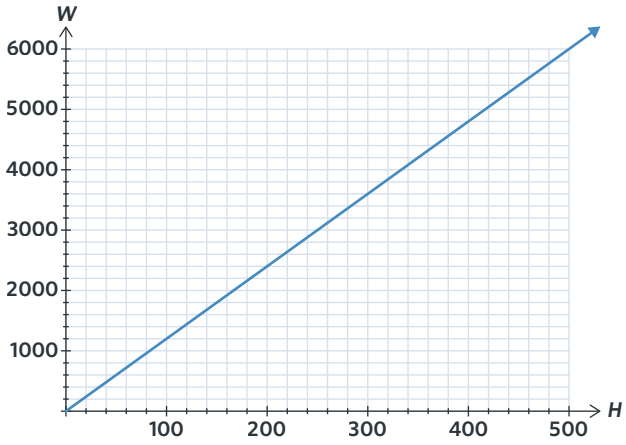
c) y varies directly with the square of z and directly with x, and $k = 0.5$ and $z = 2$.

Check your understanding

Q7. Application

The weight of an elephant in kilograms, W, varies directly with the height of the elephant in centimetres, H, as shown in the following graph.

Find an equation to describe the relationship between W and H.



Q8. Application

If $V \propto \frac{T}{P}$, where V is volume in litres, T is temperature in Kelvin and P is pressure in kilopascals, draw a graph when:

a) $k = 2$ and $T = 300$

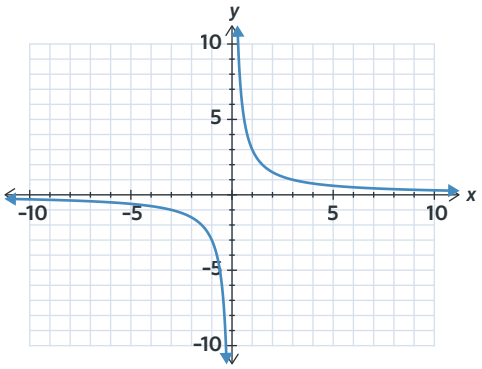
b) $k = 10$ and $P = 100$

Joining it all together

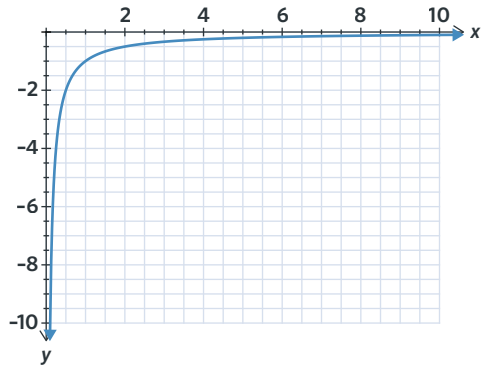
Q9. Skill 1 mark

Which of the graphs below does not show inverse variation?

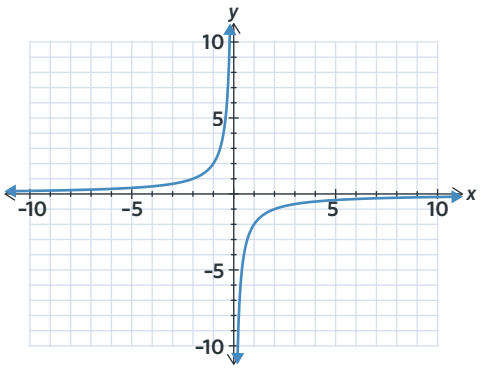
A.



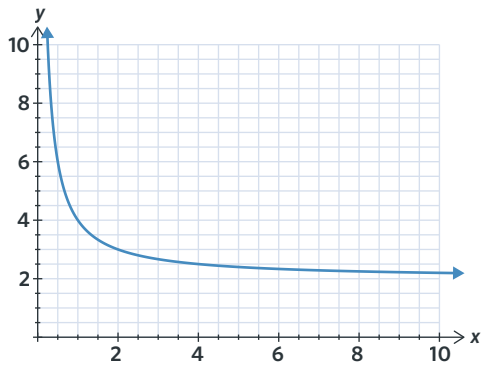
B.



C.

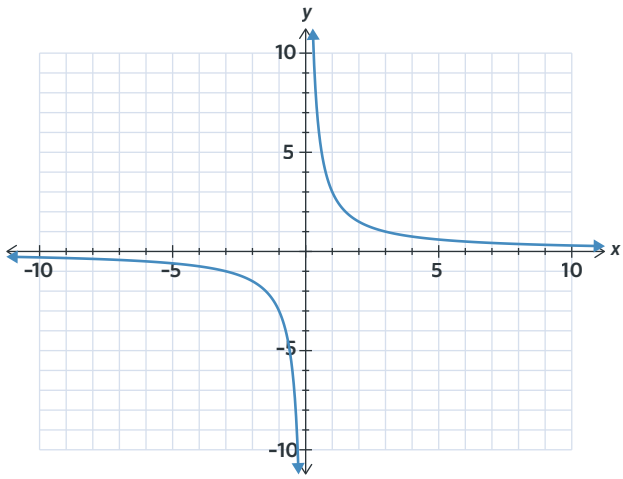


D.



Q10. What is the constant of proportionality for the following graph?

Skill
1 mark



Q11. The time taken to travel from Geelong to Melbourne is inversely proportional to average speed, as shown in the table below.

Skill
4 marks

Average speed (km/h)	100	75	50
Time (h)	0.75	1	1.5

a) Find a rule to describe the time taken to travel from Geelong to Melbourne, t (h), in terms of average speed, s (km/h).

b) Draw a graph of the rule in part **a**.

c) If it took Jenny one hour and twenty-four minutes to travel from Geelong to Melbourne, what was her average speed, correct to two decimal places?

d) If Jenny's brother Jeremy travelled at an average speed of 20 m/s, how long did it take him to travel from Geelong to Melbourne correct to the nearest minute?

Questions from multiple lessons

Q12. Which of the given points does not satisfy the following inequality?

Difficulty:

$-2y \geq 3x + 14$

Year 10
1 mark

A. $(-6, 2)$ **B.** $(-4, -8)$ **C.** $(-2, -3)$

D. $(-3, -5)$ **E.** $(-12, 2)$

Q13. Mary would like to purchase a new iPhone. She will establish a loan from her parents for \$1000 with interest charged at a rate of 3.3% per annum, compounding monthly.

Difficulty:

Each month, Mary will only pay for the interest charged that month.

1 mark

After 10 months, the amount that she still owes is closest to

A. \$1000

B. \$1383.58

C. \$1027.84

D. \$972.84

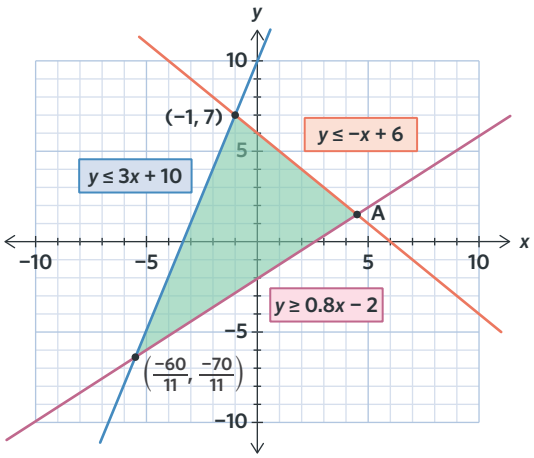
E. \$714.93

VCAA 2017 Exam 1 Recursion and financial modelling Q19 – Adapted

Q14. A feasible region is shown.

Difficulty:

2 marks



a) Determine the coordinates of point A (1 mark)

b) What is the maximum solution for the objective function $C = -2x + 7y$? (1 mark)

LESSON 11B

Variation using a numerical and algebraic approach

The key skills you will learn in this lesson are:

1. Finding the rule for direct, inverse and joint variation
2. Applying the rule for direct, inverse and joint variation

VCAA key knowledge point:

"numerical, graphical and algebraic approaches to direct, inverse and joint variation"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Finding the rule for direct, inverse and joint variation

There are multiple ways to describe the relationship between two or more variables. The table below shows a few examples of how relationships can be described, alongside their corresponding rules.

Rule	Phrasing
$y \propto x$	y is directly proportional to x y varies directly with x
$y \propto \frac{1}{x}$	y is inversely proportional to x y varies inversely with x y varies directly with $\frac{1}{x}$
$y \propto \frac{x^2}{z}$	y is directly proportional to the square of x and inversely proportional to z y varies directly with the square of x and inversely with z y varies directly with x^2 and $\frac{1}{z}$

Once the relationship between the variables is established, the proportionality sign can be replaced by the constant of proportionality, '$= k$'.

WORKED EXAMPLE 3 *(1 mark)*

y varies inversely with the square root of x and directly with z. Find a rule relating y with x and z, in terms of k.

SOLUTION

Step 1 Find the relationship between y and x.

y varies inversely with the square root of x.

$$y \propto \frac{1}{\sqrt{\ }}$$

Step 2 Find the relationship between y and z.

y varies directly with z.

$$y \propto z$$

Step 3 Write y in terms of both x and z.

$$y \propto \frac{z}{\sqrt{\ }}$$

Step 4 Replace the proportionality sign with the constant of proportionality.

$$y = \frac{kz}{\sqrt{\ }}$$

The rule is $y = \frac{kz}{\sqrt{\ }}$.

2. Applying the rule for direct, inverse and joint variation

When the rule relating two or more variables is known, unknown values can be found by replacing all other variables with given values. This is shown in the following worked example.

WORKED EXAMPLE 4 *(1 mark)*

If $y \propto \frac{z}{\sqrt{\ }}$ and $k = 2$, find the value of x when $y = 3$ and $z = 1$.

SOLUTION

Step 1 Replace the proportionality sign with the constant of proportionality.

$$y = \frac{kz}{\sqrt{\ }}$$

$$y = \frac{2z}{\sqrt{\ }}$$

Step 2 Substitute $y = 3$ and $z = 1$ into the equation.

$$3 = \frac{2 \times 1}{\sqrt{\ }}$$

$$3\sqrt{\ } = 2$$

$$\sqrt{\ } = \frac{2}{3}$$

$$x = \frac{4}{9}$$

The value of x is $\frac{4}{9}$.

Questions 11B *Variation using a numerical and algebraic approach*

Refresher question

Q1. If $y = 2x$, what is the value of y when $x = 3$?

1. Finding the rule for direct, inverse and joint variation

Q2. Skill

In which of the following equations does y vary directly with x?

A. $y = 3x + 1$

B. $y = \frac{-2}{x}$

C. $2x = -0.75y$

D. $y = -x + 4$

Q3. Skill

Write rules for the following statements, in terms of k.

a) y is inversely proportional to x and directly proportional to z.

b) y varies directly with x^2 and inversely with the cube root of z.

c) y varies directly with $\frac{1}{x}$ and inversely with z.

Check your understanding

Q4. Application

The kinetic energy, E, of an object is directly proportional to its mass, m, and the square of its velocity, v. If the constant of proportionality is $\frac{1}{2}$, find an equation relating E with m and v.

Q5. Skill

Which of the following rules is not the same as the others?

A. y varies directly with $\frac{1}{x^2}$ and inversely to the square root of $\frac{1}{z}$.

B. x^2 varies inversely with y and directly with z.

C. z varies directly with x^4 and directly with the square of y.

D. y is directly proportional to $\sqrt{z}$ and inversely proportional to the square of x.

2. Applying the rule for direct, inverse and joint variation

Q6. Skill

If $y = 3x^3$, find the value of y when $x = -2$.

Q7. Skill

Fill in the boxes below:

a) y varies directly with x^2

$k = 4$

$x =$ ☐ when $y = 1$

b) y varies inversely with x

$k =$ ☐

$y = 5$ when $x = -2$

c) $y \propto \frac{x^2}{z}$

$k =$ ☐

$y = 2$ when $x = 3$ and $z = -\frac{1}{2}$

Check your understanding

Q8. Application

The cost of a bunch of grapes varies directly with its weight.

A bunch of grapes weighing 0.5 kg costs \$5.50.

a) Find a rule relating the cost C (\$) with weight w (kg).

b) How much would a bunch of grapes weighing 700 grams cost?

Q9. Skill

What type of variation is shown in the table below?

x	1	2	3	4
y	-4	-2	$-\frac{4}{3}$	-1

Joining it all together

Q10. Application 3 marks

The weight of a pole vaulting pole varies directly with its length. The rule connecting weight, W (kg), and length, L (m) is $W = 0.7L$

a) What is the constant of proportionality?

b) How much would a 4.5 m long pole weigh in kilograms?

c) How many centimetres long is a pole weighing 2100 grams?

Q11. Skill 1 mark

In which of the following tables is y not directly proportional to x?

A.

x	0	1	2	3	4
y	0	3	6	9	12

B.

x	3	4	5	6	7
y	3	4	5	6	7

C.

x	2	4	6	8	10
y	6	10	14	18	22

D.

x	-3	-2	-1	0	1
y	6	4	2	0	-2

Q12. Application 1 mark

Mary is working on a maths problem in class, but she is shortsighted and cannot see the board clearly. She can see that y varies inversely with $\sqrt{z}$ and directly with x raised to a certain power, but she cannot see what the power is.

If $k = -3$, and $x = 4$ when $y = -\frac{3}{4}$ and $z = 64$, what is the power on x?

VCAA question

Q13. 1 mark

The point (2, 20) lies on the graph of $y = \frac{k}{x}$ as shown here.

The value of k is

A. 5

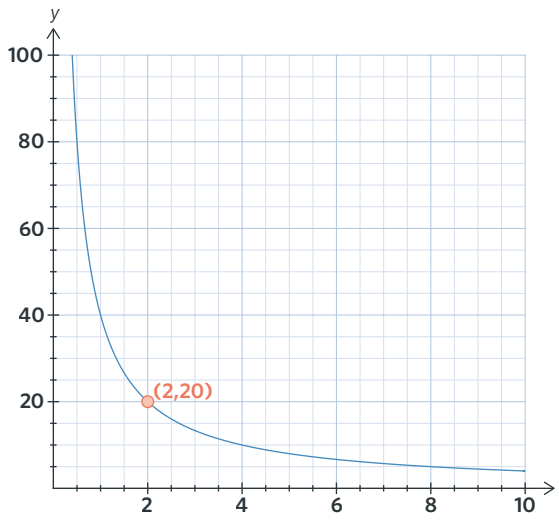
B. 10

C. 20

D. 40

E. 80

Adapted fromVCAA 2014 Exam 1, Module 3: Graphs and relations. Q3



Questions from multiple lessons

Q14. Difficulty:

1 mark

The following graph displays inverse variation between x and y.

What is the constant of proportionality?

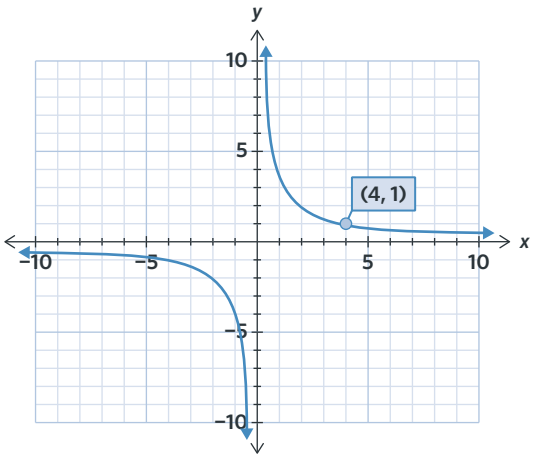
A. 8

B. 4

C. 2

D. 1

E. 0.5

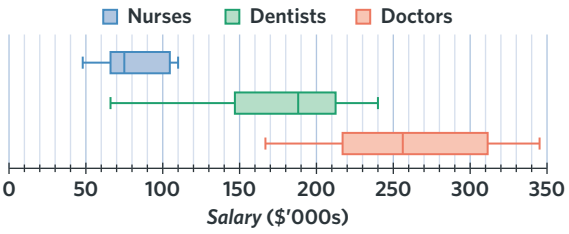


Q15. Difficulty:

1 mark

Research was conducted into the relationship between the occupation of medical professionals and their yearly salary. The salaries of a group of nurses, dentists and doctors were collected.

The results were then displayed using parallel boxplots.



Which of the following statements is **false**?

A. The nurses have the least variable salary.

B. The professional with the lowest salary is a nurse.

C. More than 50% of the doctors have a higher salary than all of the dentists and nurses.

D. At least 50% of the dentists have a lower salary than all of the doctors.

E. The doctors are the highest-paid in terms of median salary.

VCAA 2018NH Exam 1 Data analysis Q6 – Adapted

Q16. A carpet store recently received an order of new carpets which need to be measured before they can be sold. The table of values below shows the relationship between the width of a particular style of carpet, and the area it takes up.

Difficulty:

Year 10
2 marks

***Width* (m)**	2	3	5	6
***Area* (m^2)**	5	7.5	12.5	15

a) The relationship between *width* and *area* is linear. Find the equation of the line in the form $area = m \times width + c$ (1 mark)

b) Draw the graph that shows the relationship between *width* and *area*, with *area* on the vertical axis. (1 mark)

LESSON 11C

Transforming data

The key skills you will learn in this lesson are:

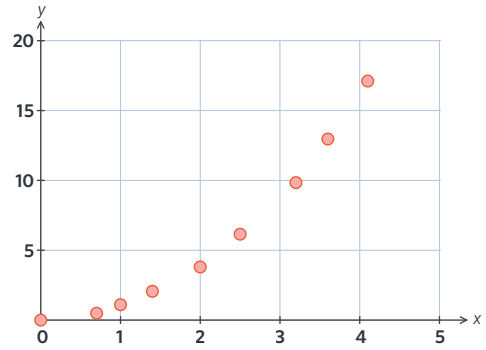
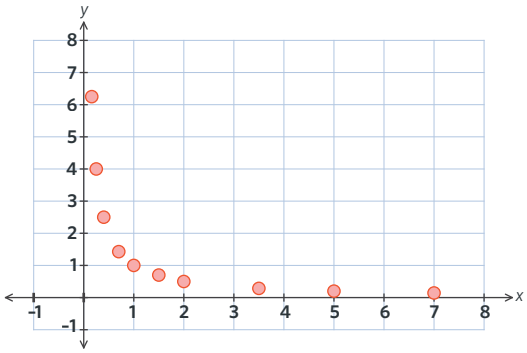
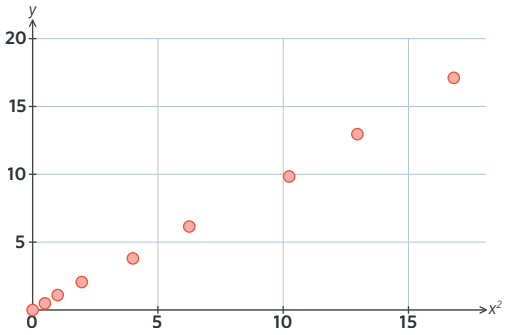
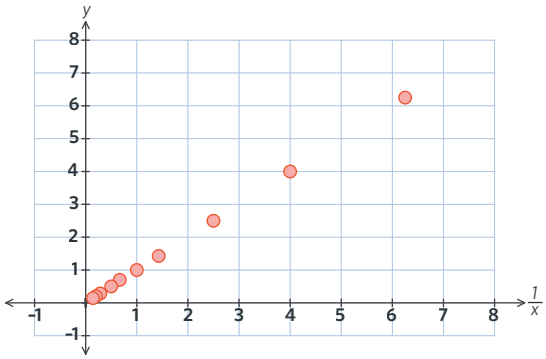
1. Transforming data with x^2
2. Transforming data with $\frac{1}{x}$
3. Using CAS to transform data
4. Deciding which transformation to perform

VCAA key knowledge point:

"transformation of data to linearity to establish relationships between variables, for example y and x^2, or y and $\frac{1}{x}$"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

The transformation of data is used to take non-linear relationships between two variables and make them linear. Having data in linear form makes it easy to model relationships and easy to perform interpolation and extrapolation. The x^2 transformation and the $\frac{1}{x}$ transformation will be covered in this chapter.

x^2 **transformation**	$\frac{1}{x}$ **transformation**
Used on data with a **quadratic** shape.	Used on data with a **hyperbolic** shape.
	
Results in linear data...	Results in linear data...
	

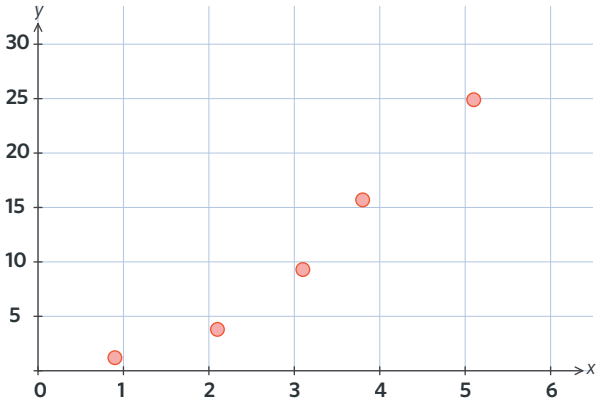
1. Transforming data with x^2

For many quadratic relationships, transforming data using x^2 will result in a linear plot. The process is to take the original values of x, use them to find the values of x^2, and then plot x^2 against y. If the plot is approximately linear, the relationship can be modelled by $y = kx^2$. The constant k can be found using any point either on the graph or on a line of best fit drawn on the data. If the plot isn't linear, the transformation isn't appropriate.

Example

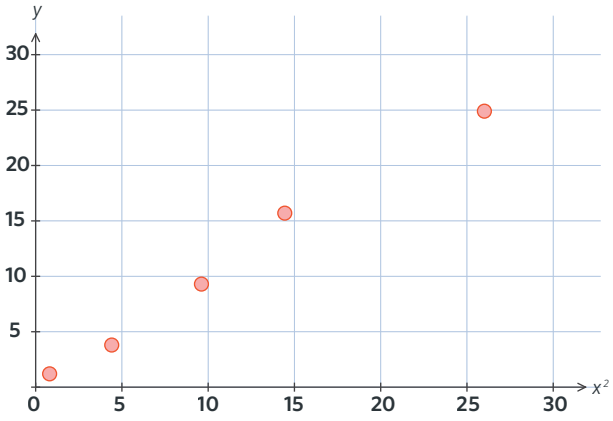
Here are some data points. They look quadratic, as y is increasing more and more quickly.

x	0.9	2.1	3.1	3.8	5.1
y	1.2	3.8	9.3	15.7	24.9



Using the x values to find x^2, we can plot y against x^2. The result is approximately linear, showing there is direct variation between x^2 and y. The data can be modelled by $y = kx^2$.

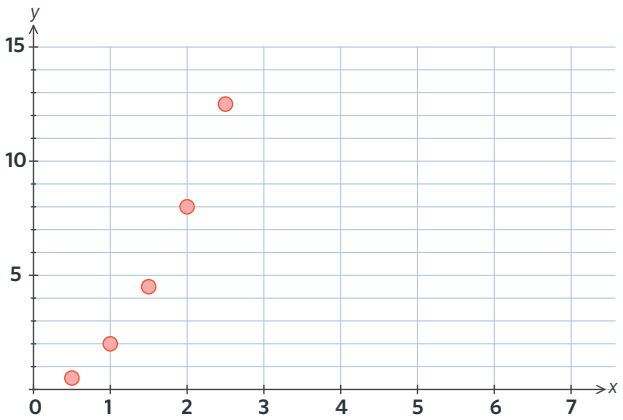
x	0.9	2.1	3.1	3.8	5.1
x^2	0.81	4.41	9.61	14.44	26.01
y	1.2	3.8	9.3	15.7	24.9



WORKED EXAMPLE 5 *(2 marks)*

Apply an x^2 transformation on the following data. Plot the result and state the relationship if possible.

x	0.5	1	1.5	2	2.5
y	0.5	2	4.5	8	12.5

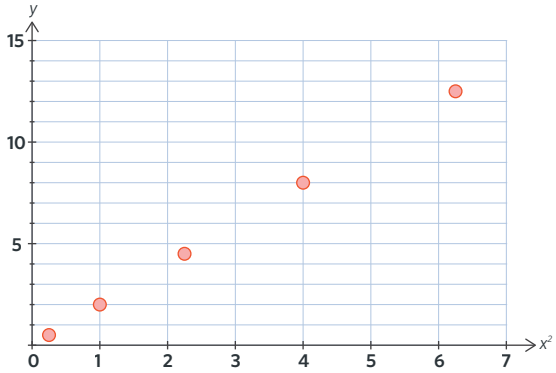


SOLUTION

Step 1 Use the x values to find the values of x^2.

x	0.5	1	1.5	2	2.5
x^2	0.25	1	2.25	4	6.25
y	0.5	2	4.5	8	12.5

Step 2 Plot the values of y against x^2.



✓ *1 mark for plotting the graph*

Step 3 Write the general formula for the relationship.

From the graph, we see y is linear with x^2.
The relationship takes the form $y = kx^2$.

Step 4 Use a point on the graph to find k.

We'll use $(1, 2)$.

$y = kx^2$

$2 = k \times 1^2$

$2 = k \times 1$

$k = 2$

The relationship between x and y is $y = 2x^2$.

✓ *1 mark for identifying the relationship*

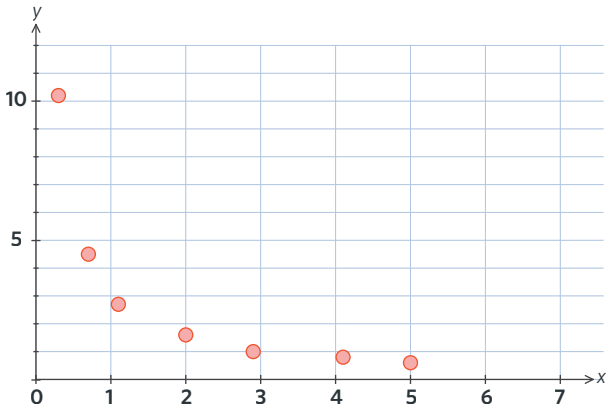
2. Transforming data with $\frac{1}{x}$

When dealing with data with a hyperbolic shape, it is best to transform data using $\frac{1}{x}$. The process is the same as above, except that the values of $\frac{1}{x}$ are calculated, and then y is plotted against $\frac{1}{x}$.

Example

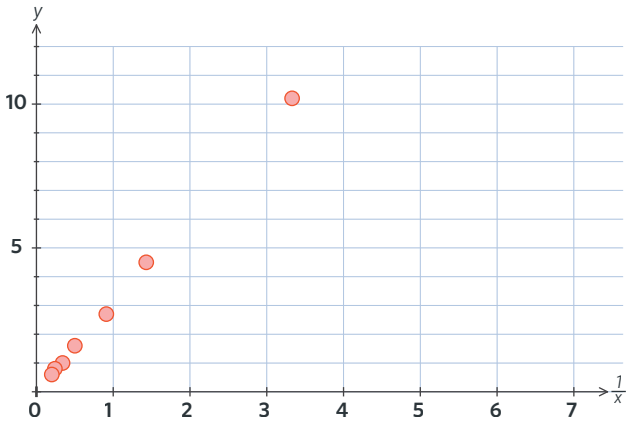
Here are some data points. They look roughly hyperbolic; the change in y decreases as x increases.

x	0.3	0.7	1.1	2.0	2.9	4.1	5.0
y	10.2	4.5	2.7	1.6	1.0	0.8	0.6



The x values are used to find $\frac{1}{x}$. The plot of y against $\frac{1}{x}$ is below. The result is approximately linear, so there is direct variation between y and $\frac{1}{x}$. The data can be modelled by $y = \frac{k}{x}$.

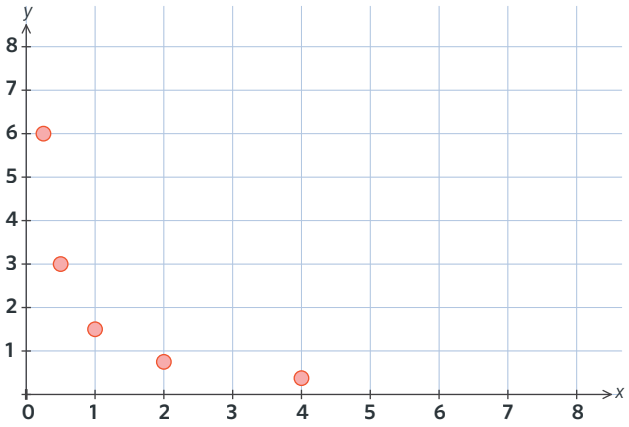
x	0.3	0.7	1.1	2.0	2.9	4.1	5.0
$\frac{1}{x}$	3.33	1.43	0.91	0.5	0.34	0.24	0.2
y	10.2	4.5	2.7	1.6	1.0	0.8	0.6



WORKED EXAMPLE 6 (2 marks)

Apply a $\frac{1}{x}$ transformation on the following data. Plot the result and state the relationship if possible.

x	0.25	0.5	1	2	4
y	6	3	1.5	0.75	0.375

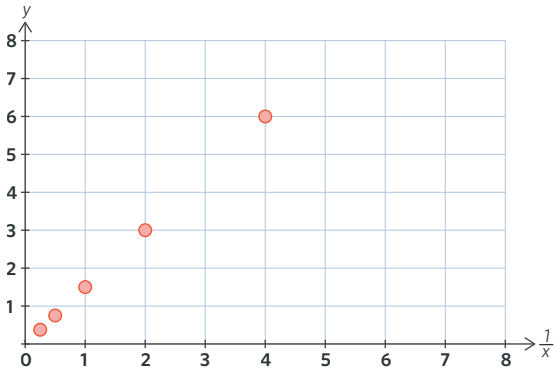


SOLUTION

Step 1 Use the x values to find the values of $\frac{1}{x}$

x	0.25	0.5	1	2	4
$\frac{1}{x}$	4	2	1	0.5	0.25
y	6	3	1.5	0.75	0.375

Step 2 Plot the values of y against $\frac{1}{x}$



✓ *1 mark for plotting the graph*

Step 3 Write the general formula for the relationship.

From the graph, we see y is linear with $\frac{1}{x}$.

The relationship takes the form $y = \frac{k}{x}$.

Step 4 Use a point on the graph to find k.

Let's use $(1, 1.5)$.

$y = \frac{k}{x}$

$1.5 = \frac{k}{1}$

$k = 1.5$

The relationship between x and y is $y = \frac{1.5}{x}$, which is $y = \frac{3}{2x}$.

✓ *1 mark for identifying the relationship*

3. Using CAS to transform data

Calculators can be used to test the linearity of transformed data.

WORKED EXAMPLE 7 (1 mark)

Use CAS to see if an x^2 or $\frac{1}{x}$ transformation is appropriate for the following data.

x	1	2	3	4	5
y	0.6	4.3	13.2	30.5	67

SOLUTION: TI-NSPIRE

Step 1 From the home screen, select . Label list A as x, list B as y, and list C as $x2$.

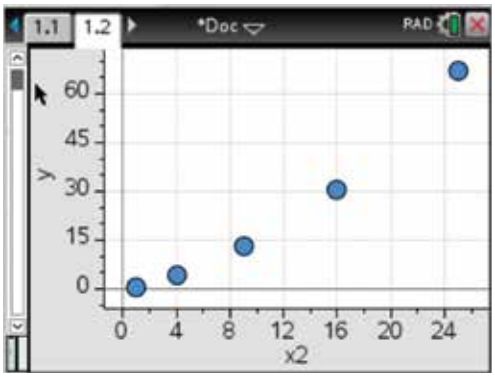
Step 2 Type the data into the x and y columns.

Step 3 In the row above row one of column $x2$, type '=x^2'. When prompted, select 'Variable Reference' and click 'OK'.

Step 4 Push 'menu', then go to '3: Data' and '9: Graph'. Click on the y-axis and select 'y'. If the x-axis doesn't already say '$x2$', click on it and select '$x2$'.

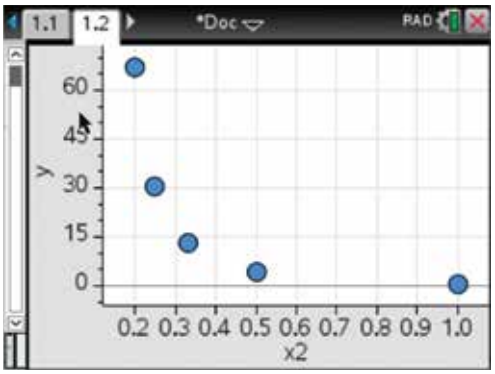
Step 5 Examine the plot. If the data is linear, we can model the data using $y = kx^2$.

The data is not linear.



Step 6 Repeat steps 2–6 for the $\frac{1}{x}$ transform, using '=1/x' instead of '=x^2'.

This plot also isn't linear.



Neither a transformation of x^2 or a transformation of $\frac{1}{x}$ is appropriate.

SOLUTION: CASIO CLASSPAD

Step 1 From the menu, select Spread-sheet.

Step 2 Enter the x data in column A, and the corresponding y data in column C.

Step 3 In cell B1, type '=a1^2'. Tap on cell B1 again, then click 'Edit', 'Fill', 'Fill Range' and change the range to B1:B5. Tap 'OK'.

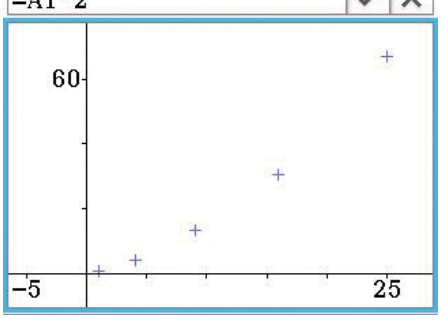
Step 4 Using the stylus, select all of the data from B1 to C5. Tap the icon, and change the style to .

Step 5 Examine the plot. If the data is linear, we can model the data using $y = kx^2$.

The data is not linear.

	A	B	C
1	1	1	0.6
2	2	4	4.3
3	3	9	13.2
4	4	16	30.5
5	5	25	67
6			

=A1^2

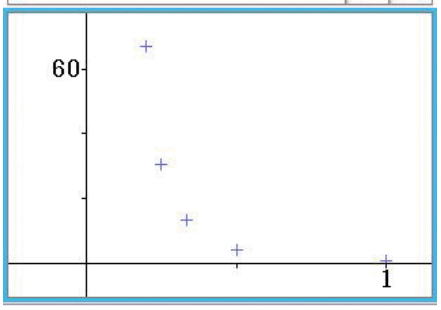


Step 6 Repeat steps 2–6 for the $\frac{1}{x}$ transform, using '=1/a1' instead of '=a1^2'.

This plot also isn't linear.

	A	B	C
1	1	1	0.6
2	2	0.5	4.3
3	3	0.33333	13.2
4	4	0.25	30.5
5	5	0.2	67
6			

67



Neither a transformation of x^2 or a transformation of $\frac{1}{x}$ is appropriate.

4. Deciding which transformation to perform

When given a set of data, it will be unclear at first as to which type of transformation will suit the data. The choice can be made easier by plotting the data to observe its shape. Data with a parabolic shape will be best suited to an x^2 transformation.

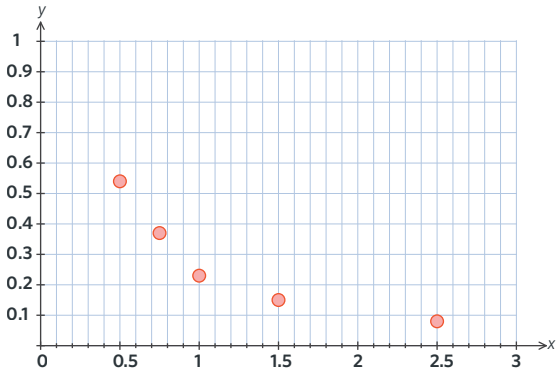
WORKED EXAMPLE 8 *(2 marks)*

Find which transformation, x^2 or $\frac{1}{x}$, is appropriate for the following data.

x	0.5	0.75	1	1.5	2	2.5
y	0.54	0.37	0.23	0.15	0.11	0.08

SOLUTION

Step 1 Plot the data.



Step 2 Examine the data. Does it look parabolic or hyperbolic?

The data here looks hyperbolic.

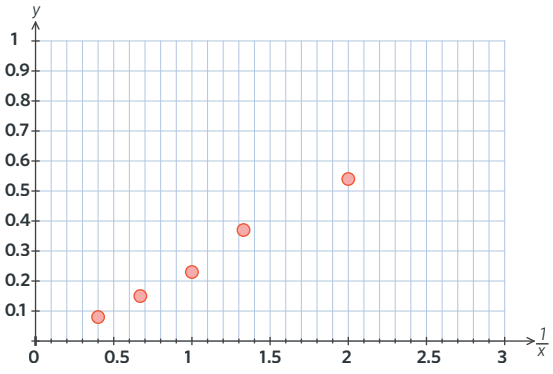
Step 3 If the data is parabolic, use an x^2 transformation. If it's hyperbolic, use a $\frac{1}{x}$ transformation.

We will use a $\frac{1}{x}$ transformation.

Step 4 Add a row to the table of values, and fill in the transformed data.

x	0.5	0.75	1	1.5	2	2.5
$\frac{1}{x}$	2	1.33	1	0.67	0.5	0.4
y	0.54	0.37	0.23	0.15	0.11	0.08

Step 5 Plot y against the transformed data.



✓ *1 mark for plotting the graph*

Step 6 Decide if the transformed data has a linear relationship with y.

The data here looks roughly linear. Thus, the transformation $\frac{1}{x}$ is appropriate.

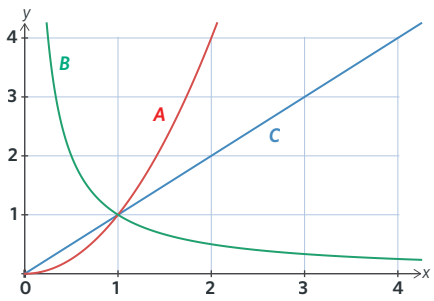
If a relationship is required, state the data can be modelled by $y = \frac{k}{x}$.

✓ *1 mark for choosing the right information*

Questions 11C *Transforming data*

Refresher question

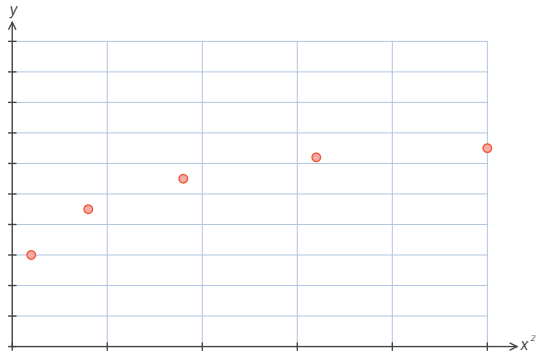
Q1. Which line is linear, which is parabolic and which is hyperbolic?



1. Transforming data with x^2

Q2. Skill

Shiran has transformed some data. Does it look like the transformation made the data linear?



Q3. Skill

Transform the following data sets using x^2 and plot the result. Are the transformations appropriate?

a)

x	1	2	3	4	5	6
y	1.2	7.8	27.8	66	119	220

b)

x	1	2	3	4	5	6
y	0.9	4.3	9.5	15.3	24.3	34.8

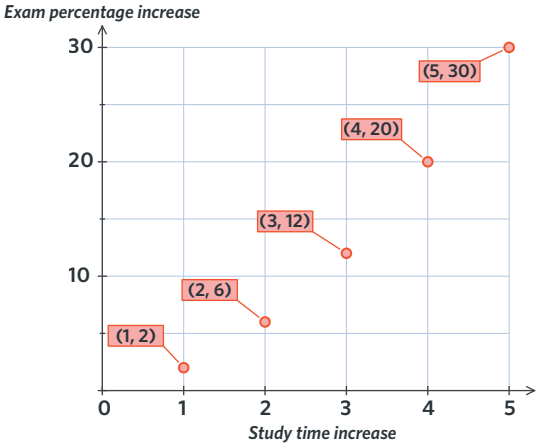
c)

x	1	1.5	2	2.5	3	3.5
y	2.2	5.6	11.7	22.5	37	54

Check your understanding

Q4. Application

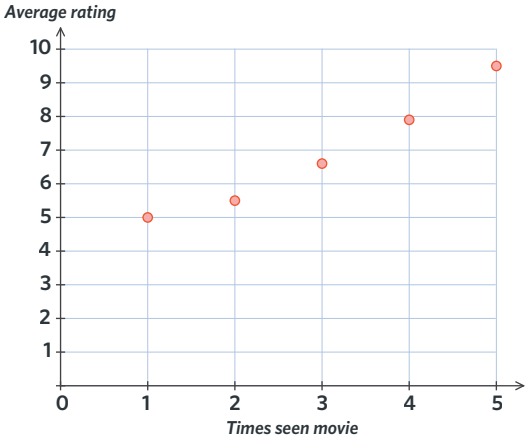
The following plot shows the improvement in exam score compared to the increase in study time for five students. Perform an x^2 transformation on the data. Plot the result and state whether the transformation is appropriate.



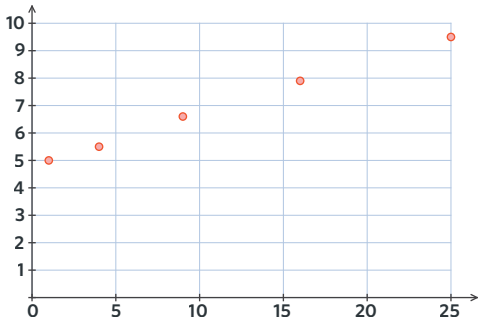
Q5. Application

A teacher asked their students to name their favourite and least favourite movies. Students were then asked to state the number of times they had seen each movie, and rate it out of 10. The results are here.

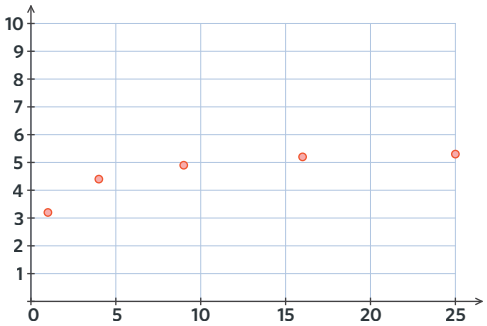
Given that the data resembles a quadratic relationship with $k > 0$, what is the likely result of an x^2 transformation?



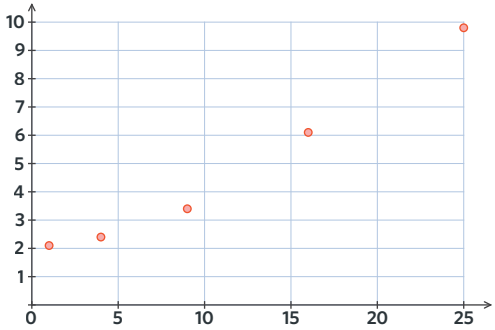
A.



B.



C.



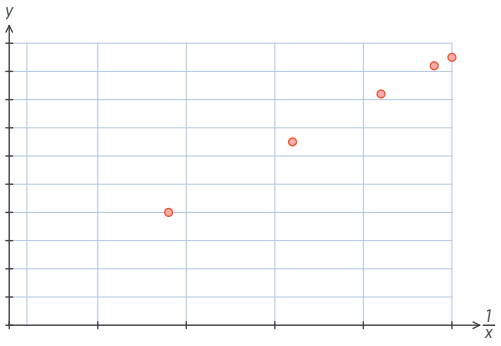
2. Transforming data with $\frac{1}{x}$

Q6. Skill

Mel used a $\frac{1}{x}$ transformation on a data set.

A plot of y against $\frac{1}{x}$ is shown here.

Is the transformation appropriate?



Q7. Skill

Transform the following data sets using $\frac{1}{x}$ and plot the result. Are the transformations appropriate?

a)

x	0.25	0.5	1	2	3	4
y	38	19	9	5.2	3.4	2.4

b)

x	0.25	0.5	1	2	3	4
y	1.5	2.1	3	4.3	5.1	6

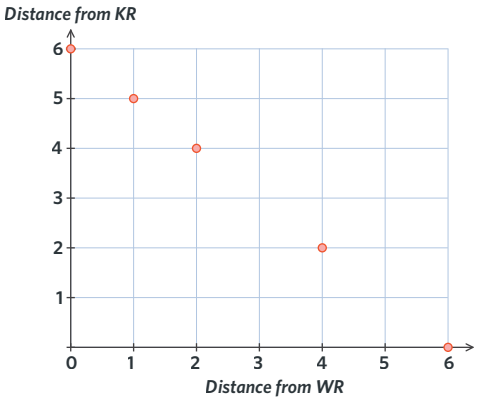
c)

x	1	2	3	4	5
y	0.5	2	4.5	7.5	12

Check your understanding

Q8. Application

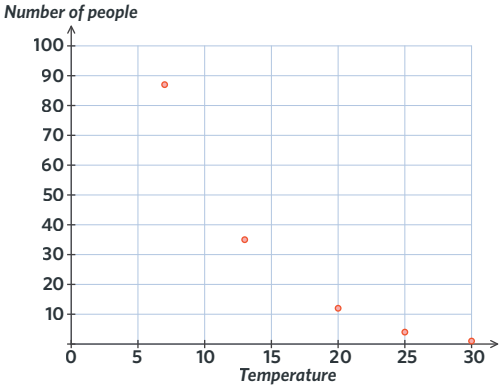
Wilma is walking from Wye River to Kennett River along the Great Ocean Road. Her distances from Wye River and Kennett River at various times are plotted on the right. She wants to show a direct relationship. Perform a $\frac{1}{x}$ transformation, ignoring the point $(0, 6)$, and plot the result. Was the transformation appropriate?



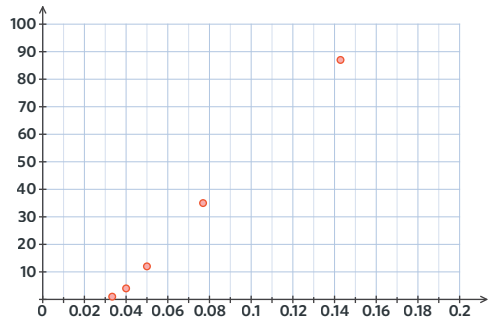
Q9. Application

Chai recorded data on the daily temperature and the number of people she sees wearing scarves at various points during the year. This data is graphed on the right.

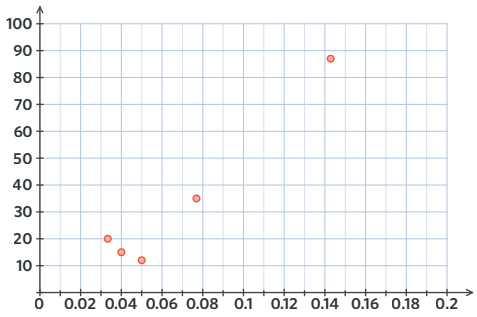
Given that the data resembles a hyperbolic relationship with $k > 0$, what is the likely result of a $\frac{1}{x}$ transformation on this data?



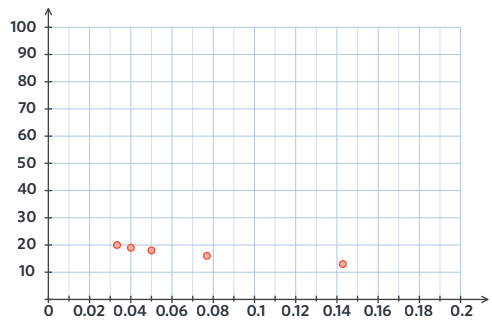
A.



B.



C.



3. Using CAS to transform data

Q10. Skill

This data has already been transformed using x^2. Plot y against x^2 on a calculator and state whether the transformation linearised the data.

x	0.5	1	1.5	2	2.5
x^2	0.25	1	2.25	4	6.25
y	0.17	0.7	1.5	2.8	4

Q11. Skill

Use CAS to see if an x^2 transformation, a $\frac{1}{x}$ transformation, or neither are effective in linearising the following data.

a)

x	1	2	3	4	5
y	0.25	1	2.25	4	6.25

b)

x	1	2	3	4	5
y	0.125	1	3.5	8	15.5

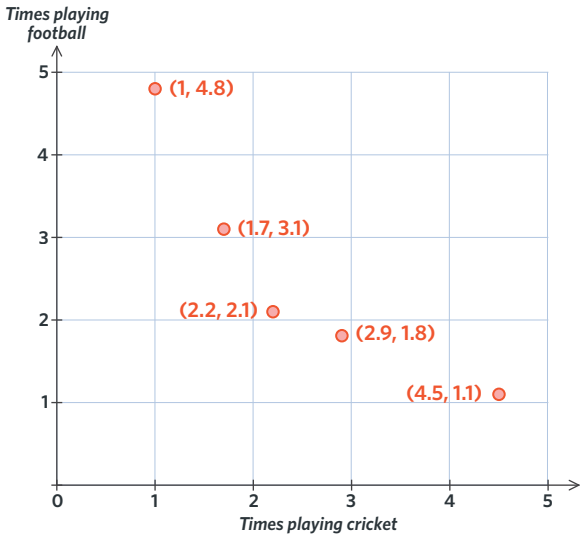
c)

x	1	2	3	4	5
y	5	2.5	1.6	1.3	1

Check your understanding

Q12. Application

Mr Nadan wanted to see if there was a preference of playing cricket or football at different times of the year. He asked five students to record the number of times a week they would play each sport over a year. Five data points representing the averages for different months are here. Use CAS to perform a $\frac{1}{x}$ transformation on the following data. Is it effective at linearising the data?

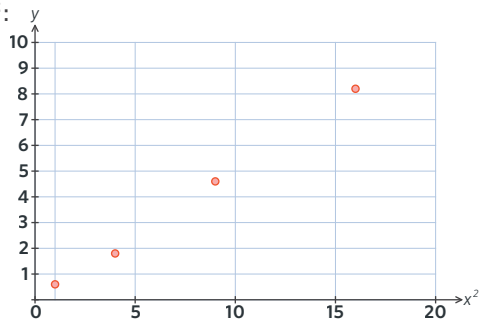


4. Deciding which transformation to perform

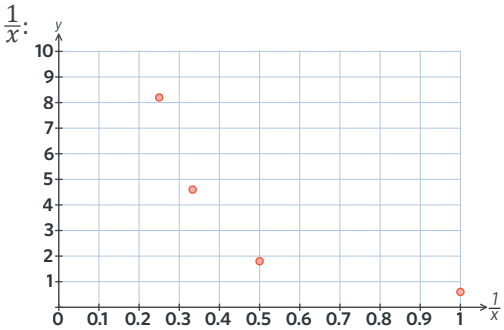
Q13. Skill

Below are the results of an x^2 and a $\frac{1}{x}$ transformation on a data set. Which transformation has been more effective at linearising the data?

x^2:



$\frac{1}{x}$:

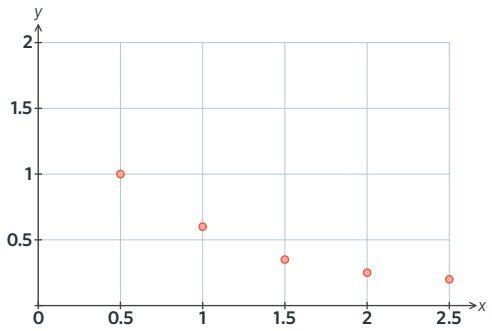


Q14. Each data set below has a table of values and a plot. Use these to decide which transformation to use, then transform the data and plot the result.

Skill

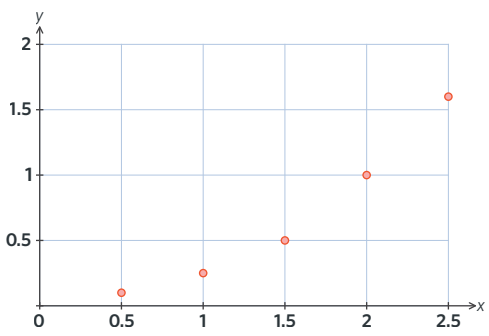
a)

x	0.5	1	1.5	2	2.5
y	1	0.6	0.35	0.25	0.2



b)

x	0.5	1	1.5	2	2.5
y	0.1	0.25	0.5	1	1.55

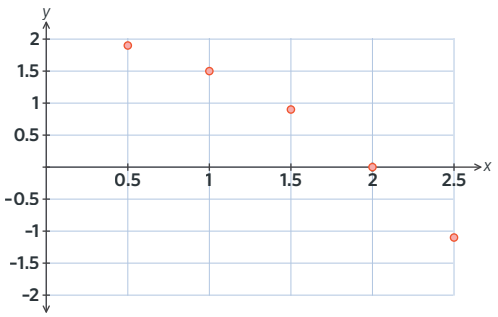


Check your understanding

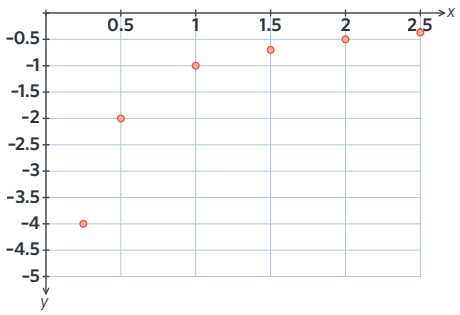
Q15. Suggest transformations for the following graphs.

Skill

a)



b)



Q16. Yasmin has been investigating the risk of climbing Mt Kosciuszko in winter, as the temperature approaches zero. The higher the temperature, the more the snow and ice will melt in the sun. The data is given in the table below.

Application

Temperature (°C)	-4	-3	-2	-1	-0.7	-0.5
Risk	3	4	5	7	8	10

a) Plot the data, using temperature on the y-axis.

b) Choose an appropriate transformation. Perform the transformation and plot the result.

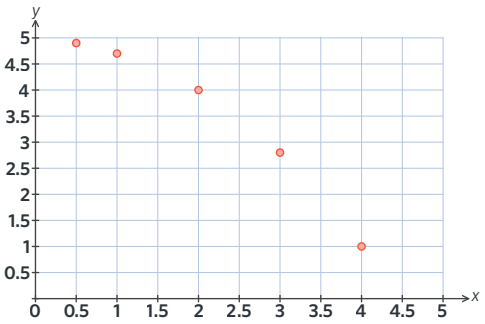
c) Use the point where the temperature is –4, to model the relationship between temperature, T, and risk, R.

Joining it all together

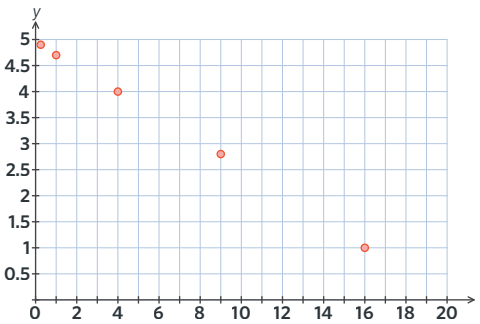
Q17. Skill 2 marks

Below is a set of data. For each part below, state which graph is the x^2 transformation and which is the $\frac{1}{x}$ transformation for that data set.

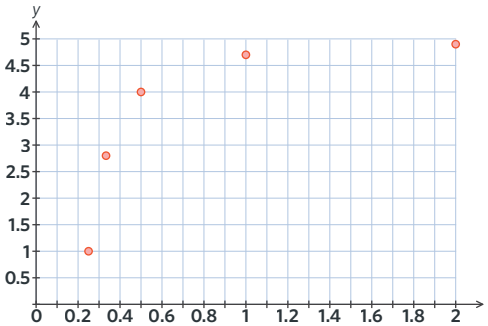
a) Data set:



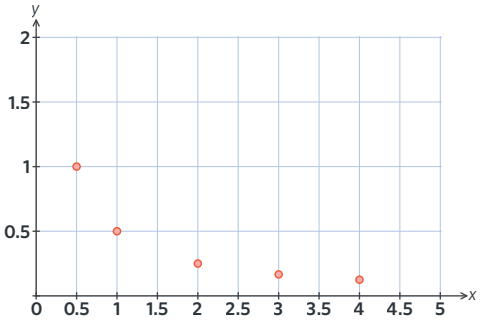
Graph A



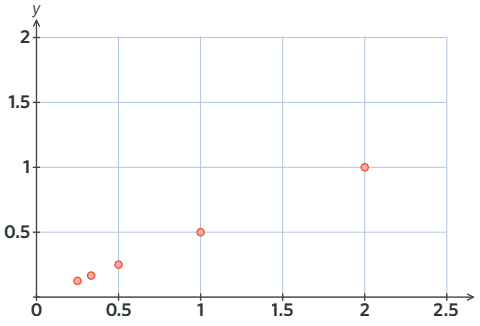
Graph B



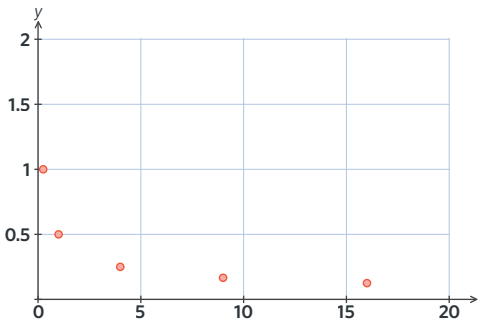
b) Data set:



Graph A



Graph B



Q18. Skill 2 marks

Choose a transformation for the following data, plot the transformed data, then use the data point where $y = 1.5$ to find an equation to model the data.

x	0.5	1	1.5	2	2.5
y	0.15	0.7	1.5	2.6	4.2

Q19. Application 4 marks

Robin is collecting data on athletes. Different types of runners were interviewed about the distance they run, D, in kilometres, and the speed at which they run, S, in kilometres per hour. The data is below.

Distance (km)	0.5	0.6	0.8	1	1.5
Speed (km/h)	25	20	14	12	8

a) Use a $\frac{1}{x}$ transformation to linearise the data. Take D to be on the x-axis.

b) Plot the transformed data.

c) Using the data point of the runner who runs the furthest, find a model for the data using S and $\frac{1}{D}$.

d) Use the model found in part **c)** to interpolate how fast a 1200 metre runner would run.

Questions from multiple lessons

Q20. Difficulty: 1 mark

Given that y is inversely proportional to the square root of x and directly proportional to z, what is the value of k if, at a point, $x = 64$, $y = 1$ and $z = 4$?

A. 16

B. 4

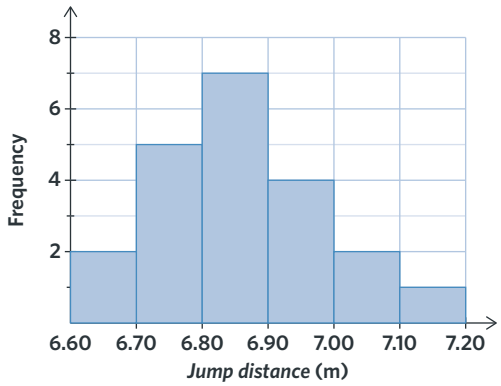
C. 2

D. $\frac{1}{2}$

E. $\frac{1}{4}$

Q21. Difficulty: Year 10 1 mark

The following histogram shows the long jump distance of 21 female long jumpers at a qualifying event for the Olympics.



The qualifying female long jump standard for the Olympics is 6.80 m.

What percentage of the participating female long jumpers qualified for the Olympics? Round your answer to the nearest whole number.

A. 33%

B. 14%

C. 55%

D. 86%

E. 67%

VCAA 2017NH Exam 1 Data analysis Q2 – Adapted

Q22. Difficulty: 2 marks

y varies inversely with the square of x and directly with the square root of z.

a) Find a rule relating y with x and z in terms of k (1 mark)

b) If $k = -0.6$, find the value of z when $x = 3$ and $y = -5$. (1 mark)

LESSON 11D

Modelling with $kx^2 + c$

The key skills you will learn in this lesson are:

1. Recognising quadratic relationships
2. Solving for the unknown constants of $y = kx^2 + c$

VCAA key knowledge point:

"modelling of given non-linear data using the relationships $y = kx^2 + c$ and $y = \frac{k}{x} + c$ where $k > 0$"

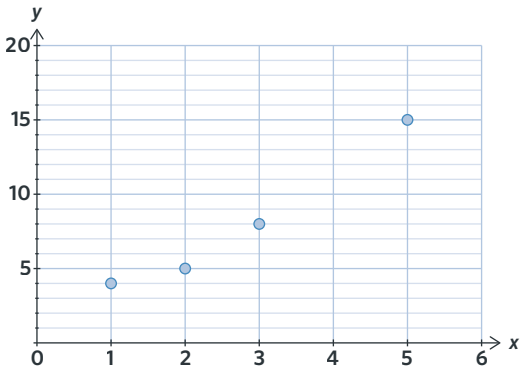
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

Continuing on from the relationship $y = kx^2$ used in lesson 11C, the formula $y = kx^2 + c$ can be used to model even more relationships.

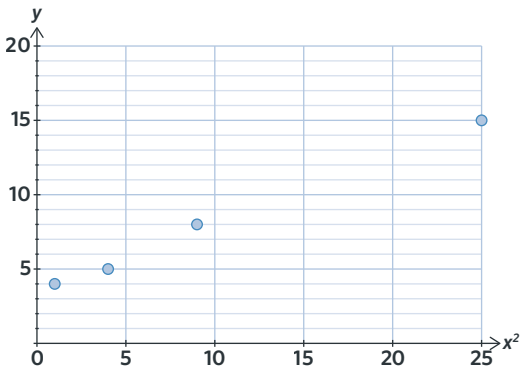
1. Recognising quadratic relationships

The model $y = kx^2 + c$ fits almost all quadratic relationships. However, before the values of k and c are found, it is essential to make sure the data has a quadratic nature. This is best done by transforming the data and seeing if the result is linear. If the result is linear, the model will fit the data. The gradient of the line is equal to k, and the y-intercept is equal to c.

Example

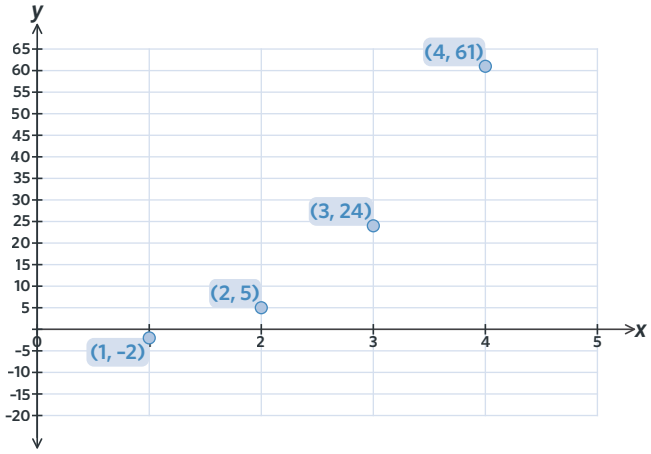


An x^2 transformation is performed on the data above, resulting in the plot below. This result is linear, so it can be concluded that the model $y = kx^2 + c$ fits the data.



WORKED EXAMPLE 9 *(1 mark)*

Can the following data be represented by the model $y = kx^2 + c$?



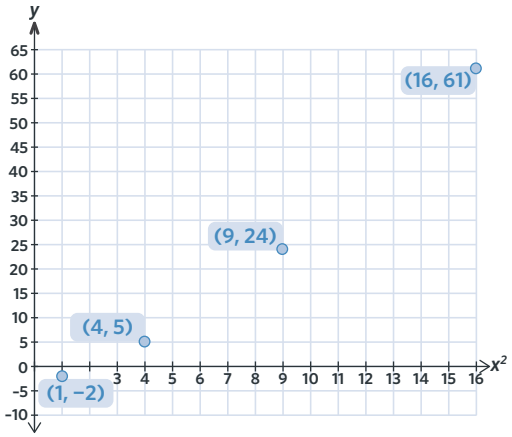
SOLUTION

Step 1 Transform the data using x^2.

We'll use a table.

x	1	2	3	4
x^2	1	4	9	16
y	−2	5	24	61

Step 2 Plot y against x^2.



Step 3 Examine the data for linearity.

The data isn't linear, so this data set can't be modelled by $y = kx^2 + c$.

2. Solving for the unknown constants of $y = kx^2 + c$

If the transformed data is linear, the model can be used for the data. There are two methods for finding k and c:

- Choose two data points and substitute those into the model, to create two simultaneous equations. Then solve these for k and c.

  or

- Draw a line of best fit onto the plot of transformed data, and find the gradient and y-intercept of the line. Then k equals the gradient, and c equals the y-intercept.

The latter method will be used predominantly in this lesson.

WORKED EXAMPLE 10 *(1 mark)*

The following data can be represented by $y = kx^2 + c$. Find k and c and state the equation.

x	1	2	3	4
y	8	12.5	22	37

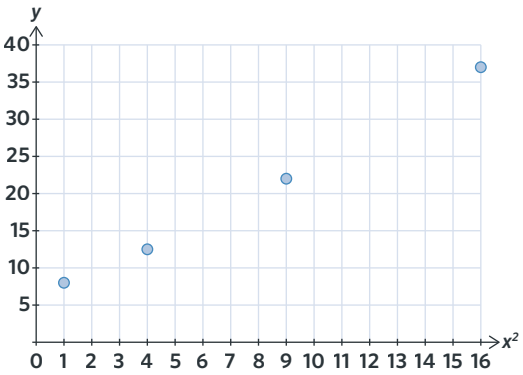
SOLUTION

Step 1 Transform the data using x^2.

We'll use a table.

x	1	2	3	4
x^2	1	4	9	16
y	8	12.5	22	37

Step 2 Plot y against x^2.



Step 3 Draw a line of best fit, and state the gradient and y-intercept.

This line of best fit has a y-intercept of 5. The gradient is calculated using the formula below.

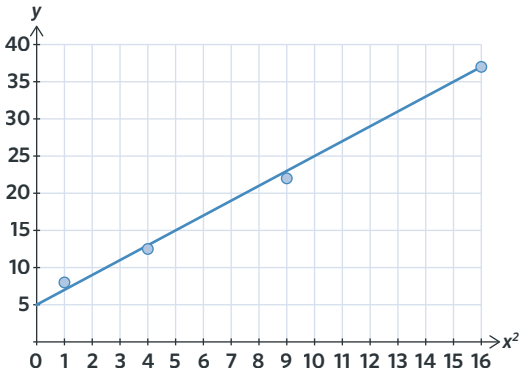
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Choose two points on the line of best fit to calculate the gradient m. In this case, one is the y-intercept $(0, 5)$ and the other can be any point on the line, so we'll choose $(5, 15)$. We have:

$$m = \frac{15 - 5}{5 - 0}$$

$$m = \frac{10}{5}$$

$$m = 2$$



Step 4 Write the equation.

k is equal to the gradient 2, and c is equal to the y-intercept 5.

$$y = kx^2 + c$$

$$y = 2x^2 + 5$$

Questions 11D *Modelling with $kx^2 + c$*

Refresher question

Q1. If $x = 3$, which of these is equal to 9?

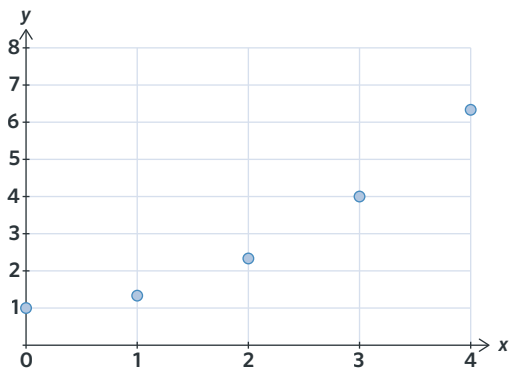
A. x^3 **B.** $4x$ **C.** $x + 5$ **D.** x^2

1. Recognising quadratic relationships

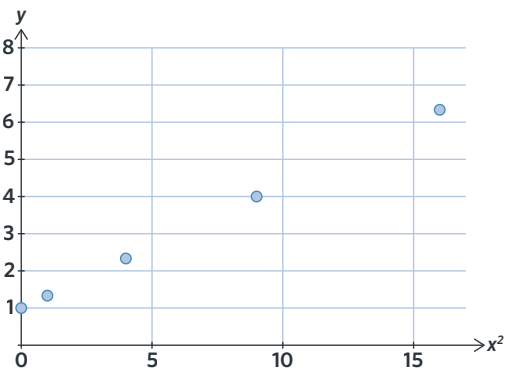
Q2. The following data has been transformed using x^2. Can it be modelled accurately using $y = kx^2 + c$?

Skill

Original data:



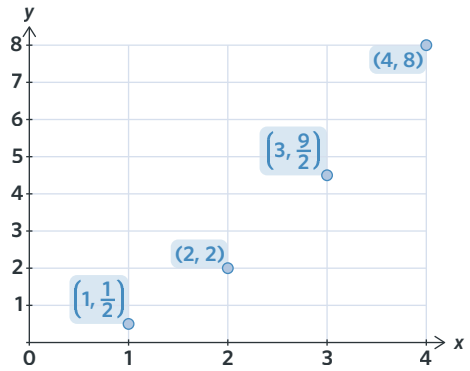
Transformed data:



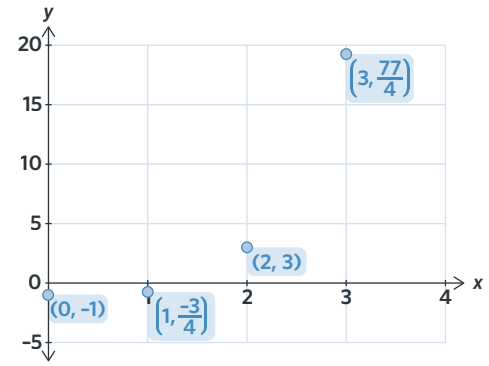
Q3. Transform the following data sets using x^2 to see if they can be modelled accurately using $y = kx^2 + c$.

Skill

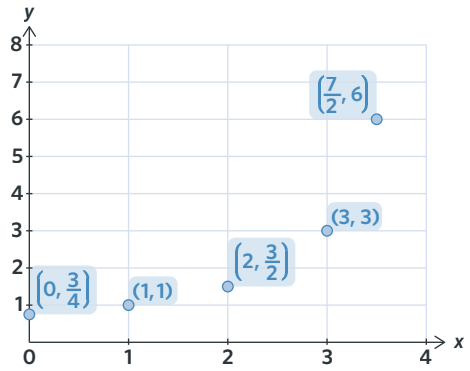
a)



c)



b)



Check your understanding

Q4. Application

Doug collected some data on his friends' houses. He compared the width of each house, w, to its floor area, A. The results are below.

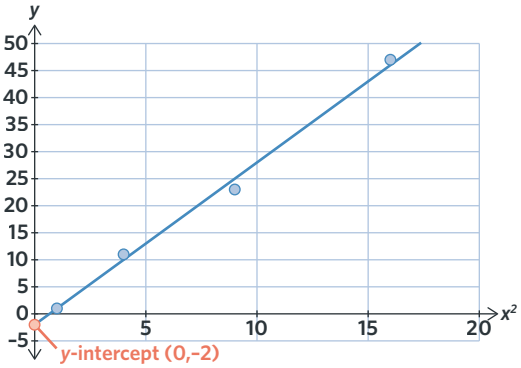
Width (m)	9	10	11.5	13
Area (m²)	145	170	220	275

Decide if the model $A = kw^2 + c$ can be used to accurately model this data.

2. Solving for the unknown constants of $y = kx^2 + c$

Q5. Skill

The data below can be modelled by the formula $y = kx^2 + c$. The value of k is 3. Find c.



Q6. Skill

The following data sets can be modelled by $y = kx^2 + c$. Find k and c and state the equation for each. In each question, k and c are either whole numbers or relatively simple fractions.

a)

x	0	1	2	3
y	0.9	1.5	3.1	5.4

b)

x	1	2	3	4
y	−3.1	3.2	13.1	26.8

c)

x	1	2	3	4
y	−5.3	−3.4	−0.1	4.7

Check your understanding

Q7. Application

Camila has been recording the number of people who go to the beach near her house on days of varying temperatures. Her data can be modelled by $N = kT^2 + c$, where N is the number of people at the beach on a day with a maximum temperature of T°C. Find k and c and state the equation.

T	12	20	27	35
N	53	106	170	272

Joining it all together

Q8. Application 4 marks

Hasti and Gabriel were studying the amount of energy, E, it takes to stop soccer balls travelling at different speeds, s. Find if their data sets can be modelled by $E = ks^2 + c$ and if so, find the relevant values of k and c, and state the equation.

Hasti:

s (m/s)	2	5	7	10
E (J)	1	7	11	17

(2 marks)

Gabriel:

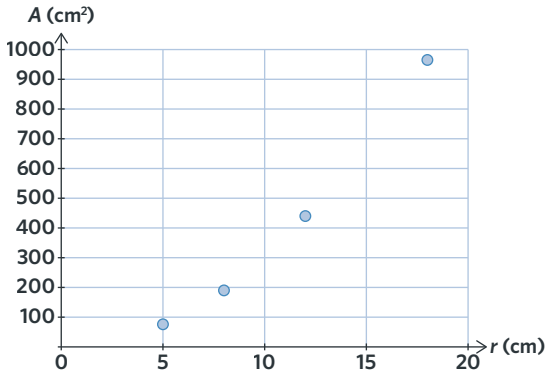
s (m/s)	2	5	7	10
E (J)	1	6	12	26

(2 marks)

Q9. Application 3 marks

A machine makes circular placemats for dinner tables. The machine is quite old, and the placemats aren't created as a perfect circle anymore. Some data on the radius of four placemats, r, and the area of those placemats, A, is below.

r (cm)	5	8	12	18
A (cm²)	76	190	440	965



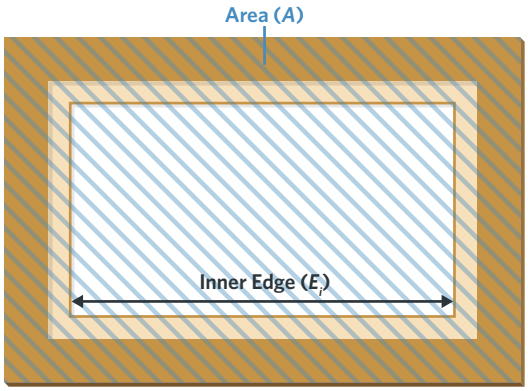
a) Can a $A = kr^2 + c$ model fit the data well? If so, find the model.

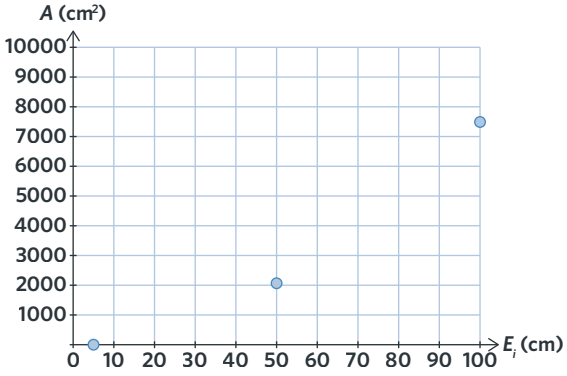
b) What would be the area of a placemat with radius 15 cm made by this machine? Round your answer to two decimal places.

c) Two new placemats were measured. One had a radius of 10 cm and an area of 303 cm², and the other had a radius of 6.5 cm and an area of 145 cm². Which one does not fit the model?

Q10. Application 3 marks

Zoe measured the width of the inner edge, E_i, of three landscape photo frames and compared these to their total area, A. A plot of her findings is below. In this question round all relevant answers to two decimal places.





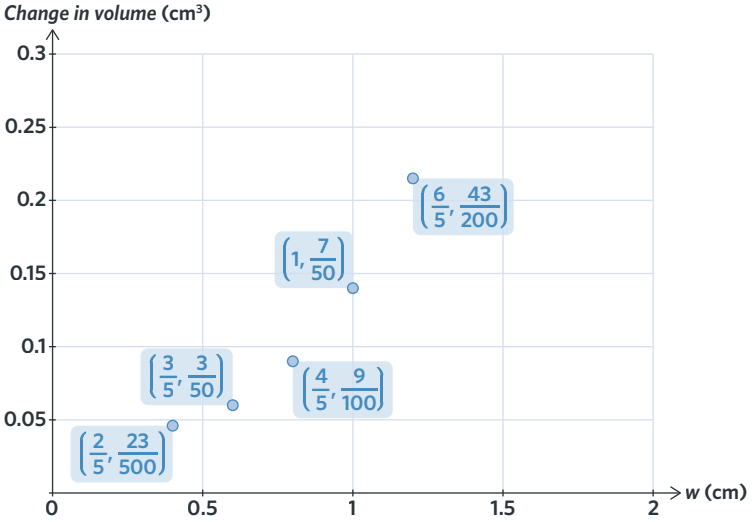
E_i (cm)	5	50	100
A (cm²)	147	2066	7489

a) Transform the data into a model of form $A = kE_i^2 + c$ using the left most and right most points to find the slope. Round k and c to two decimal places. (2 marks)

b) Use the model to estimate the area of a frame with an inner edge of 70 cm, to two decimal places. (1 mark)

Q11. Application 2 marks

Mahesh is measuring the increase in volume, V, as water becomes ice, for ice cubes of varying widths, w. He has fit the data to a $V = kw^2 + c$ model, getting values of $k = \frac{1}{8}$ and $c = \frac{1}{40}$.



w (cm)	0.4	0.6	0.8	1.0	1.2
V (cm²)	0.046	0.06	0.09	0.14	0.215

a) Test Mahesh's model for the following values of w:

$w = 0.4$ $w = 0.6$ $w = 0.8$ $w = 1.0$ $w = 1.2$

Does the model seem to fit the data?

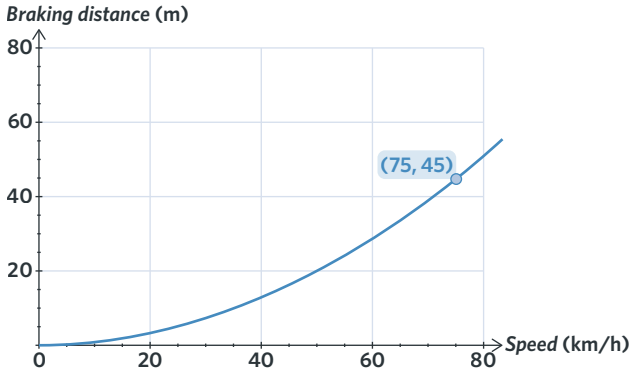
b) Test the model once more for the more distant point, $w = 3$. According to the data, $V = 2.74$ when $w = 3$.

Does the model still fit?

VCAA question

Q12. 1 mark

The graph below shows the braking distance, in metres, of a car at different speeds, in kilometres per hour. The coordinates of a point on the graph are also shown.



The relationship between *braking distance* and *speed* can be modelled by an equation of the form

$braking\ distance = k \times (speed)^2$

Using this model, the braking distance, in metres, when the speed is 60 km/h is:

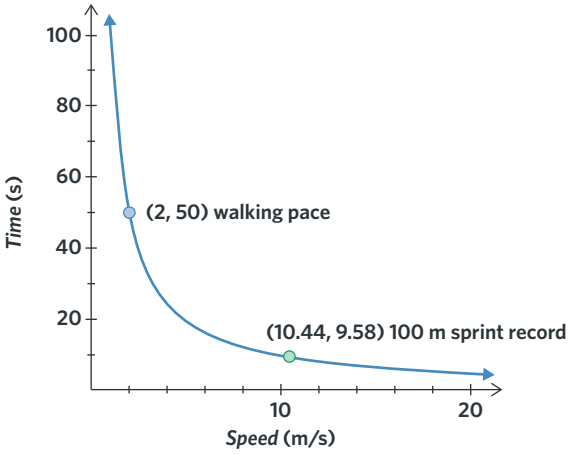
A. 24.0 **B.** 28.8 **C.** 30.0 **D.** 32.2 **E.** 48.5

VCAA 2015 Exam 1, Module 3: Graphs and relations. Q6

Questions from multiple lessons

Q13. Difficulty: 1 mark

The graph to the right shows the relationship between time t, in seconds, taken to travel 100 m at an average speed of s m/s. Also labelled is the average walking pace and the 100 m sprint record.



Which statement is false?

A. As average speed increases, the time taken to travel 100 m decreases.

B. It will take 50 seconds to travel 100 m when travelling at 2 m/s.

C. The relationship between time and average speed is linear.

D. To complete the 100 m sprint in 9.58 seconds, the sprinter must have been travelling at an average speed of 10.44 m/s.

E. A formula that relates time and speed over a distance of 100 m is $time = \frac{100}{speed}$ $s > 0$.

VCAA 2008 Exam 1 Module 3: Graphs and relations Q3 – Adapted

Q14. A sequence can be generated using the following recurrence relation.

Difficulty:

1 mark

$T_{n+1} = T_n + 3, \quad T_0 = -1$

What are the first four terms of the sequence?

A. $-1, -4, -7, -10$

B. $-1, 2, 5, 8$

C. $-1, -3, -9, -27$

D. $1, 4, 7, 10$

E. $2, 5, 8, 11$

VCAA 2016 Exam 1 Recursion and financial modelling Q17 – Adapted

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Q15. A car rental company, Canoe, uses the two equations below to calculate the total cost of a rental car.

Difficulty:

2 marks

The *rental cost*, in dollars, for a given amount of days of car rental, is calculated using the equation

$$rental\ cost = 96 + 32 \times days$$

The **additional** *youth charge*, in dollars, for drivers under 25, is calculated using the equation

$$youth\ charge = m \times (25 - age)^2$$

Kris is 21 and will rent a car from Canoe for 15 days.

He will pay $960, in total, for the rental car.

Determine the value of m (2 marks)

VCAA 2009 Exam 2 Module 3: Graphs and relations Q3 – Adapted

LESSON 11E

Modelling with $\frac{k}{x} + c$

The key skills you will learn in this lesson are:

1. Recognising inverse relationships
2. Solving for the unknown constants k and c

VCAA key knowledge point:

"modelling of given non-linear data using the relationships $y = kx^2 + c$ and $y = \frac{k}{x} + c$ where $k > 0$"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

If doubling one variable halves the other, this is known as an inverse relationship. As either becomes very large, the other becomes very close to zero. A relationship of this type can be modelled as $y = \frac{k}{x} + c$.

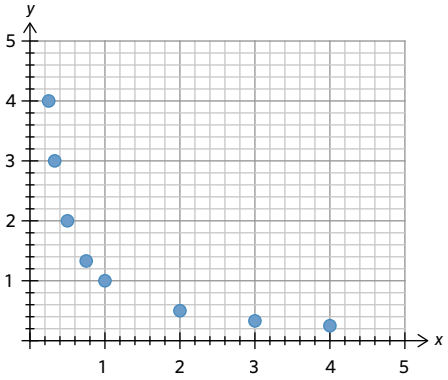
1. Recognising inverse relationships

Almost all hyperbolic relationships are expressed by the model $y = \frac{k}{x} + c$.

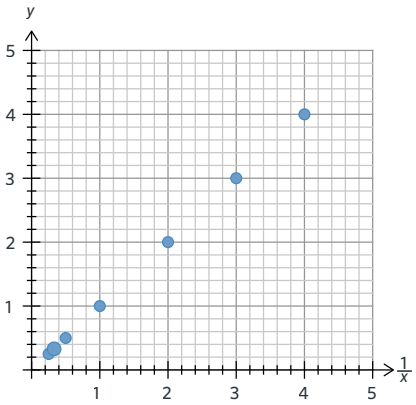
However, before the values of k and c are found, it is essential to make sure the data follows a hyperbolic relationship.

The easiest way to identify whether the data fits the model of $y = \frac{k}{x} + c$ is to linearise the data, and ensure that the result follows a linear relationship.

Example



A $\frac{1}{x}$ transformation can be applied to the above graph, resulting in a perfectly linear relation.

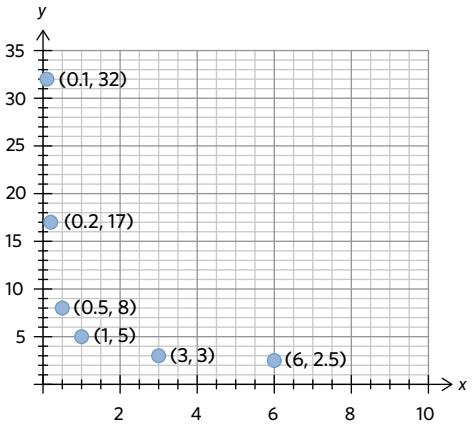


This shows the data has an inverse relationship, and can be modelled by $y = \frac{k}{x} + c$.

A detailed explanation of the transformation is shown in the worked example below.

WORKED EXAMPLE 11 *(1 mark)*

Can the following data be modelled by the equation $y = \frac{k}{x} + c$?

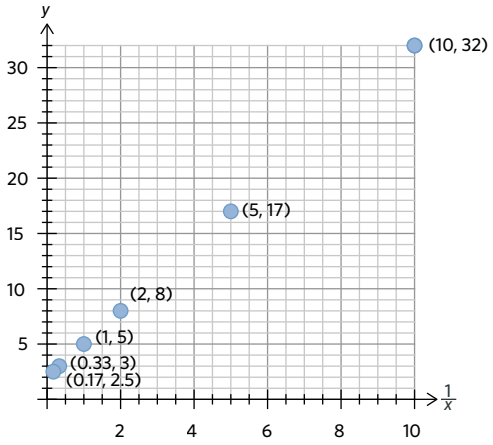


SOLUTION

Step 1 Create a table containing the data and find the corresponding $\frac{1}{x}$ values.

x	0.1	0.2	0.5	1	3	6
y	32	17	8	5	3	2.5
$\frac{1}{x}$	10	5	2	1	0.33	0.17

Step 2 Plot y against $\frac{1}{x}$



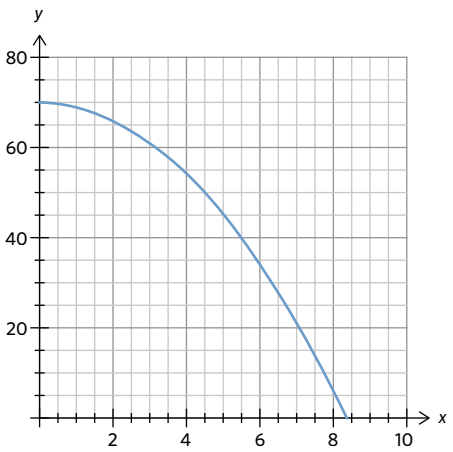
Step 3 Examine the data for linearity.

The data is perfectly linear, therefore can be modelled by the equation $y = \frac{k}{x} + c$.

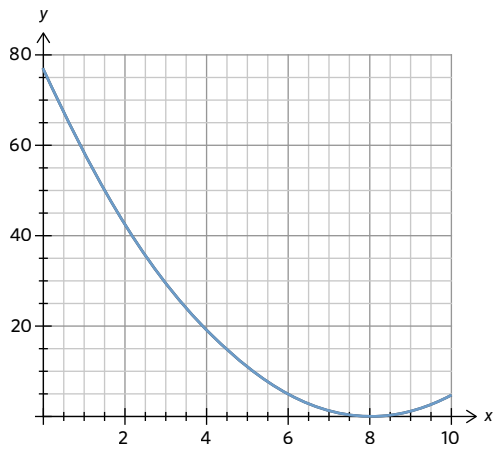
WORKED EXAMPLE 12 *(1 mark)*

Which of the following is inversely proportional and can be modelled by the equation $y = \frac{k}{x} + c$?

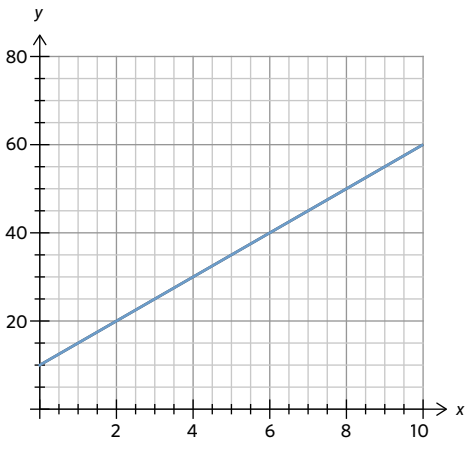
A.



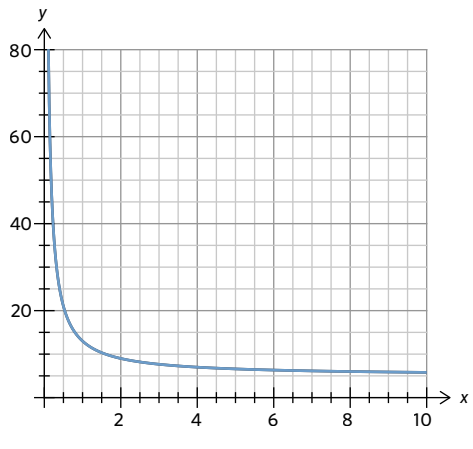
B.



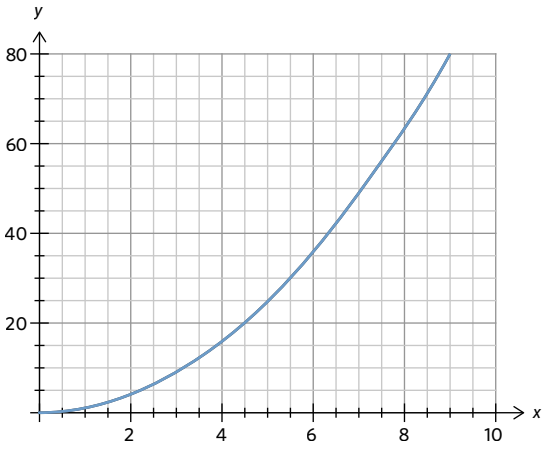
C.



D.



E.



SOLUTION

Step 1 Search for a downward trend. Only A, B and D have a downward trend.

Step 2 Look for the distinctive hyperbola shape: a graph that begins steep and ends almost flat, that is all one smooth curve.

A becomes steeper as the x-variable increases, so it cannot be a hyperbola.

B begins to curve upwards, so it cannot be a hyperbola.

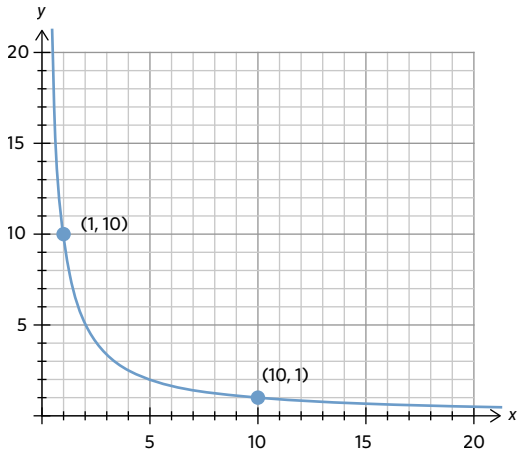
D begins steep, then gradually plateaus, and is a smooth curve throughout.
The hyperbola must be D.

2. Solving for the unknown constants k and c

In the equation for inverse proportionality, k can be any value greater than zero, and c can take on any value. In some examples, k or c will be unknown, and the rule of the function must be found. To find the rule, the known x and y values can be entered to solve for the unknown variables.

However, if the data has already been linearised, the value of k is simply the gradient and c is the y-intercept.

Example



The rule for the graph is $y = \frac{10}{x}$

WORKED EXAMPLE 13 *(1 mark)*

Given that this table varies according to the function $y = \frac{k}{x} + c$, find the value of k if $c = 2$.

x	1	2	3	4	5
y	7.0	4.5	3.7	3.3	3.0

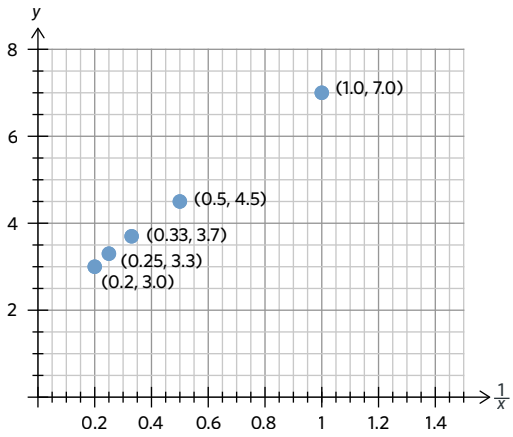
SOLUTION

Method 1: Transforming the data

Step 1 Transform the data using a table.

x	1	2	3	4	5
$\frac{1}{x}$	1.00	0.50	0.33	0.25	0.20
y	7.0	4.5	3.7	3.3	3.0

Step 2 Plot y against $\frac{1}{x}$.



Step 3 Draw a line of best fit, and calculate the gradient, k, using two points.

(1.0, 7.0)

(0.5, 4.5)

$$k = \frac{y_2 - y_1}{x_2 - x_1}$$

$$k = \frac{7.0 - 4.5}{1 - 0.5}$$

$$k = \frac{2.5}{0.5}$$

$$k = 5$$

Method 2: Substituting a point

Step 1 Choose one point on the hyperbola.

(1, 7)

Step 2 Substitute in the point and c to generate an equation.

$$7 = \frac{k}{1} + 2$$

Step 3 Solve the equation.

$$7 = k + 2$$

$$k = 5$$

Questions 11E *Modelling with* $\frac{k}{x} + c$

Refresher question

Q1. Jorge must drive from Melbourne to Sandy Point. He was planning on travelling the whole distance at 80km/h but wants to arrive earlier. Must he increase or decrease his speed?

1. Recognising inverse relationships

Q2. Application Josie is having her birthday party soon, and she has bought 50 slices of cake but does not know how many people are going to arrive. Below is the number of slices of cake there will be per person depending on the number of people who come.

a) Fill in the blank spaces in the table.

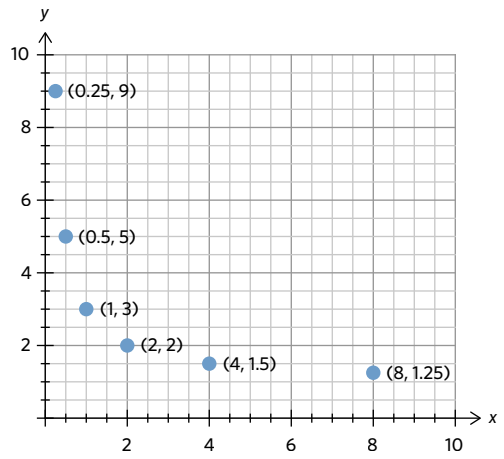
b) Is this an example of inverse variation? Why/why not?

Number of guests	1		10		50
Slices per person	50	10		2	1

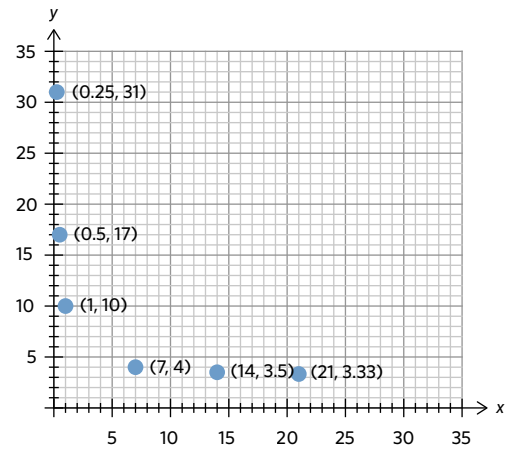
Q3. By transforming the following data, state whether an inverse relationship is present.

Skill

a)



b)



Q4. Which of the following tables shows an inverse relationship that can be modelled by $y = \frac{k}{x} + c$?

Skill

A.

x	1	2	3	4	5
y	4	13	28	49	126

B.

x	7	8	9	10	11	12
y	4.75	5.00	5.25	5.50	5.75	6.00

C.

x	0.5	1.0	1.5	2.0	2.5	3.0
y	4.00	2.00	1.33	1	0.8	0.67

D.

x	10	11	12	13	14	15
y	5	4	3	2	1	0

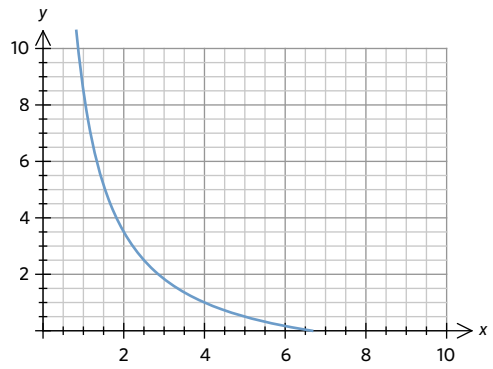
E.

x	10	20	30	40	50	60
y	49	46	41	34	25	14

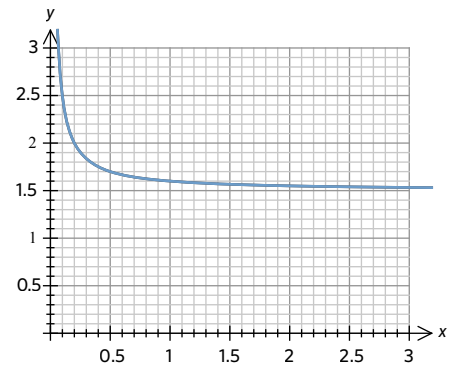
Q5. Which of the following graphs does not represent an inverse relationship that can be modelled by $y = \frac{k}{x} + c$?

Skill

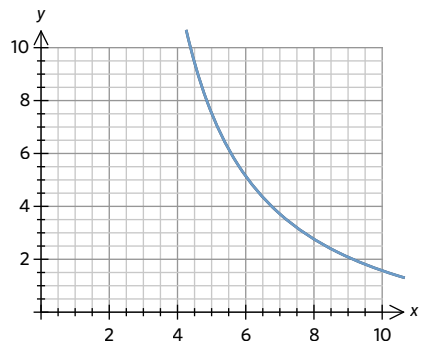
A.



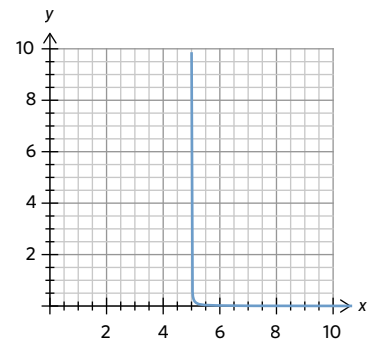
B.



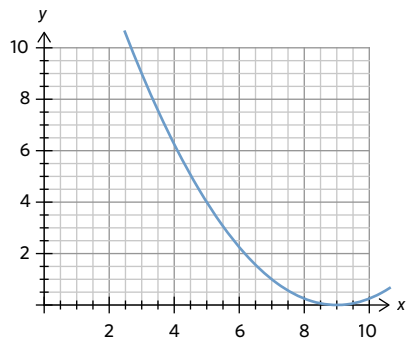
C.



D.



E.



Check your understanding

Q6. Skill

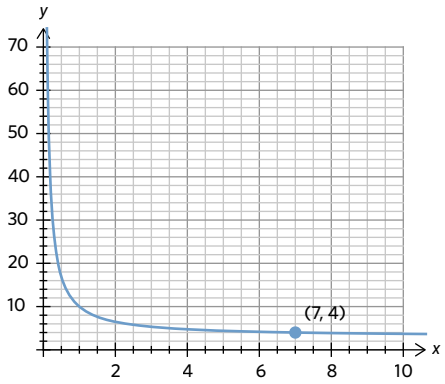
Plot a graph of the inverse relation that can be modelled by $y = \frac{k}{x} + c$ given that it passes through the points $(2, 1)$, $(8, 0.25)$ and $(0.5, 4)$.

Do not find the equation of the line.

2. Solving for the unknown constants k and c

Q7. Skill

Given that $c = 3$, find the equation for the following inverse relationship by substituting the given point into the equation $y = \frac{k}{x} + c$.



Q8. Application

Ernestine works at a pet shop and notices something strange. She notices there is an inverse relationship between how much a puppy weighs, and how likely they are to be bought.

She places her findings in the table below:

Weight of puppy (grams)	300	400	500	600	700
Probability of being bought	0.300	0.225	0.180	0.150	0.129

Given that this relationship can be described by the equation

$$probability\ of\ being\ bought = \frac{k}{weight\ of\ puppy} + c$$

and that $c = 0$, calculate the value of k.

Q9. Application

The average price of bread sold by a bakery in a day has an inverse relationship with the amount of bread sold that day.

Price of bread ($)	2	5	7.5	10	15
Number of loaves sold	32	14	10	8	6

a) From the table, derive an equation in the form $number\ of\ loaves\ sold = \frac{k}{price\ of\ bread} + c$ given that $k = 60$.

b) Using the equation, estimate the number of loaves sold in a day if the average price of a loaf was \$1.

c) Given 154 loaves were sold in a week, calculate the price of a loaf of bread that week, assuming the price didn't change all week.

Check your understanding

Q10. Application

The velocity of the planets **squared** varies inversely with their distance to the sun (AU) as shown in the following table.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Mean distance from Sun (AU)	0.39	0.72	1.00	1.53	5.20	9.54	19.18	30.06
Velocity (km/s)	47.97	35.30	29.95	24.22	13.14	9.70	6.84	5.46
Velocity² (km/s)	2300.77	1246.25	897.3	586.47	172.56	94.06	46.78	29.85

a) Find the equation that relates the velocity **squared** and the distance to the sun in the terms of $velocity\ squared = \frac{k}{distance\ from\ sun}$

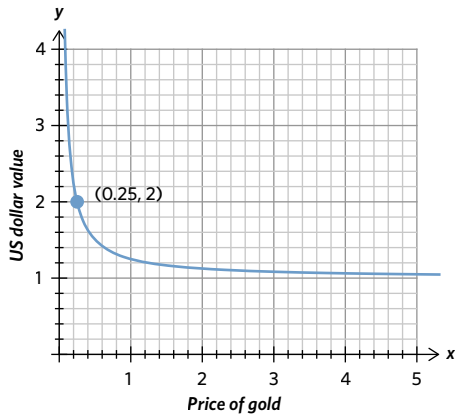
Give the value of k to one decimal place.

b) Using the equation, predict the distance from the sun of a planet that has a velocity of 42.28 km/s. Round your answer to one decimal place.

Joining it all together

Q11. Application 2 marks

As the US dollar decreases in value compared to other currencies, the price of gold is known to rise. Similarly, as the US dollar increases, the price of gold always falls, as shown by the graph below using arbitrary units:



a) Find the equation that relates the price of gold to the US dollar value, given that $c = 1$.

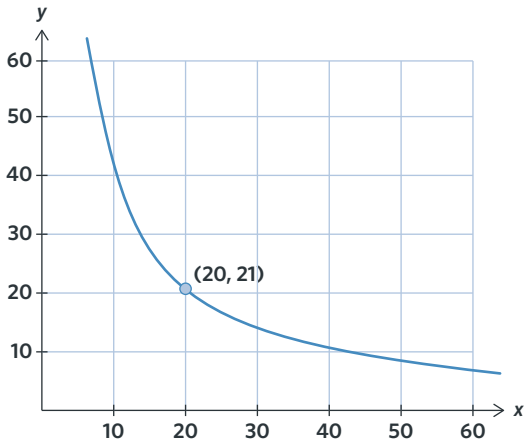
b) Using this equation, predict the value of the US dollar when the price of gold is 0.125.

Questions from multiple lessons

Q12. Difficulty:

1 mark

The point (20, 21) lies on the graph of $y = \frac{k}{x}$ as shown below.



The value of k is

A. 1.05

B. 21

C. 1

D. 420

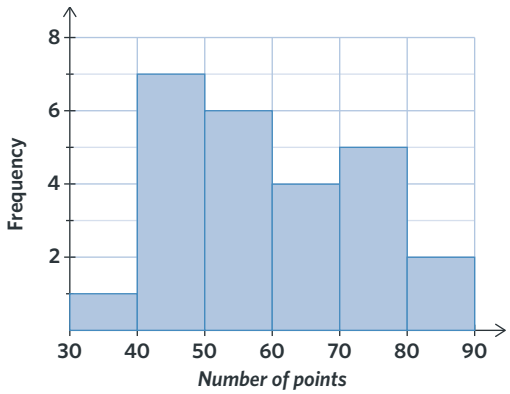
E. 20

VCAA 2017 Exam 1 Module 4: Graphs and relations Q3 – Adapted

Q13. Difficulty:

1 mark

The number of points scored by 25 different teams in their first game of a basketball tournament is displayed in the following histogram.



The median number of points scored by a team in their first game of the tournament is

A. at least 40 but less than 50.

B. at least 50 but less than 60.

C. at least 60 but less than 70.

D. at least 70 but less than 80.

E. at least 80 but less than 90.

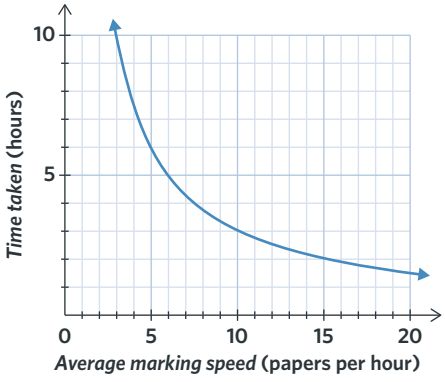
VCAA 2017NH Exam 1 Data analysis Q3 – Adapted

Q14. Difficulty:

3 marks

George is an English teacher. He needs to mark as many exam papers as he can so that he can release his students' results on time.

The following graph shows the relationship between the *average marking speed* (papers per hour) and the *time taken* (in hours) to finish marking a whole class.



Assume that every class has the same number of students.

a) For Class 11C, George marked at an average speed of 15 papers per hour.

For Class 11D, George marked at an average speed of 6 papers per hour because of sleep deprivation.

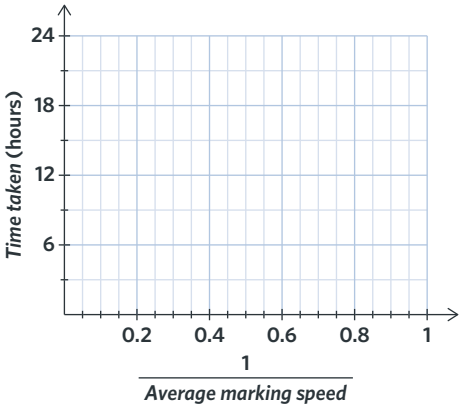
How much longer did it take George to finish marking papers for 11D compared to the time he took for 11C? (1 mark)

The equation for the relationship between *average marking speed* and *time taken* is in the form

$$time\ taken = \frac{k}{average\ marking\ speed}$$

b) Find the value of k (1 mark)

c) Use the axes below to draw a graph of the relationship between *time taken* and $\frac{1}{average\ marking\ speed}$ (1 mark)



VCAA 2017NH Exam 2 Module 4: Graphs and relations Q1 b,c – Adapted

LESSON 11F

Modelling with logarithms

The key skills you will learn in this lesson are:

1. Recognising logarithmic relationships
2. Solving for the unknown constants of $y = a\log_{10}(x) + c$

VCAA key knowledge point:

"modelling of data using the logarithmic function $y = a\log_{10}(x) + c$ where $a > 0$"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

Logarithms can be used to express very small or very large numbers. There can be better understood through the following expression.

$$y = \log_{10}(x) \Leftrightarrow x = 10^y$$

Logarithmic functions are modelled by the formula $y = a\log_{10}(x) + c$, where a and c are constants. Note that logarithms can have other numerical bases rather than 10, but in this topic only 10 is used for simplicity.

1. Recognising logarithmic relationships

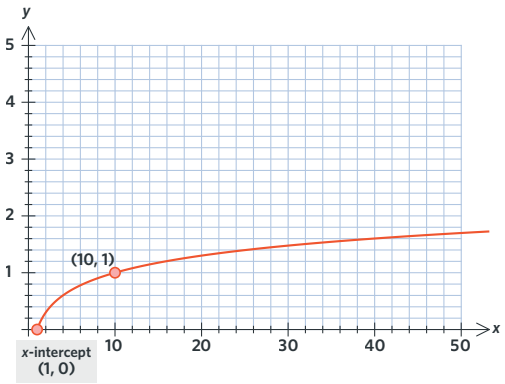
Logarithmic functions can be identified by an initially steep gradient, that gradually plateaus as the value of x increases. When $a > 0$, a logarithmic functions will have a positive slope for all values of x.

Example

Here is a log graph with the corresponding table of values.

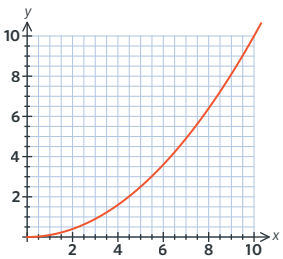
$$y = \log_{10}(x)$$

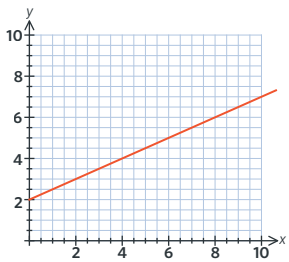
x	1	10	20	30	40	50
y	0	1	1.30	1.48	1.60	1.70

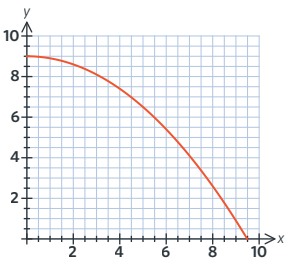


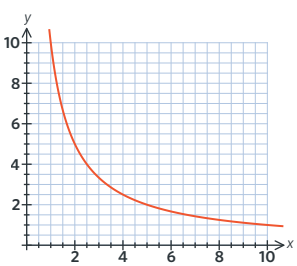
WORKED EXAMPLE 14 *(1 mark)*

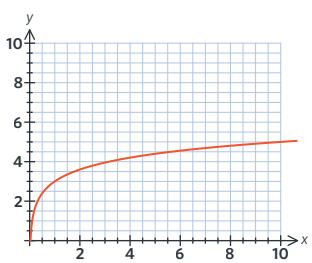
Which of the following graphs shows a logarithmic function that can be modelled by the equation $a\log_{10}(x) + c$, where $a > 0$?

A.


B.


C.


D.


E.


SOLUTION

Step 1 As $a > 0$, look for a graph that has a positive slope for all values of x.

A, B and E have a positive slope for all values of x.

Step 2 Look for the general form of a logarithm – the gradient should begin steep and gradually plateau as x values increase.

A gets increasingly steeper, so it cannot be logarithmic.

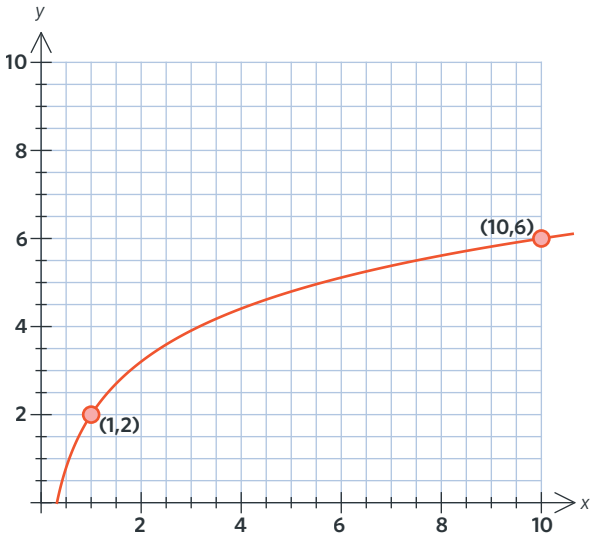
B maintains the same slope, so it cannot be logarithmic.

E begins with a steep, positive gradient and gradually plateaus. Therefore, E must be the logarithmic function.

2. Solving for the unknown constants of $y = a\log_{10}(x) + c$

The general form of a logarithmic function is $y = a\log_{10}(x) + c$. Here, a and c are constants and can take on any value, however in this course only values of a greater than zero are used. In some examples, a or c are unknown, and the rule of the function must be found. To find the rule, the known x and y values can be substituted to solve for the unknown variables.

Example



By substituting in (1, 2) and (10, 6), the following equations are found:

$2 = a\log_{10}(1) + c$

$6 = a\log_{10}(10) + c$

Solve for a and c simultaneously to get $a = 4$ and $c = 2$.

Therefore, the equation of the graph is $y = 4\log_{10}(x) + 2$.

WORKED EXAMPLE 15 *(1 mark)*

Given that this table varies according to the function $y = a\log_{10}(x) + c$, find the value of a if $c = 2$.

x	1	10	100	1000	10 000
y	2	5	8	11	14

SOLUTION

Step 1 Choose one point on the table.

(10, 5)

Step 2 Substitute the point into the given equation.

$5 = a\log_{10}(10) + 2$

Step 3 Solve for a.

$5 = a \times 1 + 2$

$a = 3$

Questions 11F *Modelling with logarithms*

Refresher question

Q1. $y = 8 \log_{10}(x) + 5$

What are a and c in the above equation?

1. Recognising logarithmic relationships

Q2. Skill

The area covered by bacteria on a plate following a treatment with antibiotics is shown in the table below:

Incubation time (hours)	1	5	10	15	20
Area covered (cm²)	0	4.89	7.00	8.23	9.11

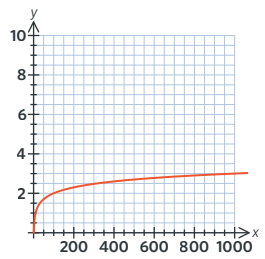
Plot the points on a graph, with 'incubation hours' on the x-axis and 'area covered' on the y-axis, and connect the points.

Is this a logarithmic function?

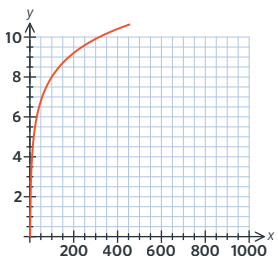
Q3. Skill

Which of the following graphs does not represent a logarithmic function?

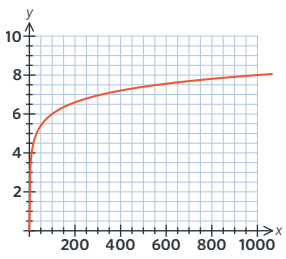
A.



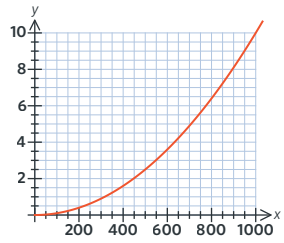
B.



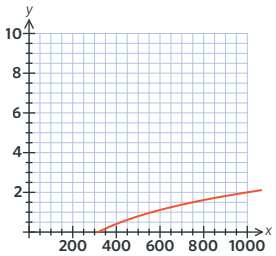
C.



D.



E.



Q4. Skill

Which of the following tables can be modelled by $y = a \log_{10}(x) + c$ where $a > 0$?

A.

x	2	4	6	8	10
y	7	19	39	67	103

B.

x	2	4	6	8	10
y	96	84	64	36	0

C.

x	2	4	6	8	10
y	14	18	22	26	30

D.

x	2	4	6	8	10
y	2.44	9.76	21.96	39.04	61.00

E.

x	2	4	6	8	10
y	1.51	3.01	3.89	4.52	5.00

Check your understanding

Q5. Application

Genya has been tracking the number of deer that live in the forest near his home for 30 years. He has noticed that the deer population follows a noticeable pattern, but he accidentally missed an entry. The following table shows Genya's incomplete record of the deer.

Number of days	1	10	100	1000	10 000
Number of deer	0	300	600		1200

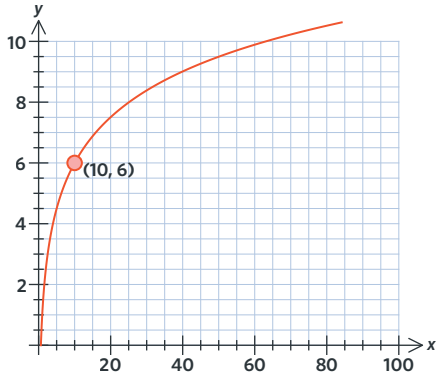
a) Complete the table above. Do not find the rule.

b) Can the deer population be modelled by a logarithmic function? Why or why not?

2. Solving for the unknown constants of $y = a\log_{10}(x) + c$

Q6. Skill

Given that $c = 1$, find the equation for the following relationship by substituting the given point into the equation $y = a\log_{10}(x) + c$.



Q7. Application

Elephants in Serengeti National Park have their population recorded yearly, as shown in the table below:

Year (*Y*)	1	2	3	4
Number of Elephants (*E*)	1000.000	1150.515	1238.561	1301.030

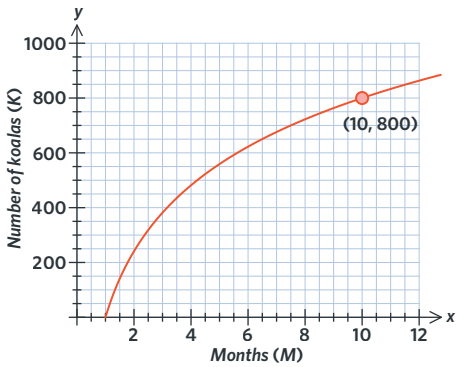
NOTE: Some years the rangers believe they may have double counted, so half an elephant is counted.

a) Given that $E = a\log_{10}(Y) + c$, estimate the value of a to the nearest whole number.

b) Using the rule, find the number of elephants in the Serengeti National Park after 10 years.

Q8. Application

The following graph shows the growth of a population of koalas in Wilson's Promontory National Park in months, given they were introduced at month 1.



a) The model $K = a\log_{10}(M)$ can accurately model the population of koalas in Wilson's Promontory National Park. Find the value of a.

b) Using the model, find number of koalas in Wilson's Promontory National Park after two years to the nearest whole number.

Check your understanding

Q9. Application

The volume of water within a water tank being filled is recorded in the following table.

Number of minutes	60	90	120	150
Volume of water in litres (V)	0	10.57	18.06	23.88

a) By first converting minutes into hours, find the equation $V = a\log_{10}(H) + c$, given that $a = 60$.

b) Find the volume of the tank at 50 hours to two decimal places.

c) If the tank has a total volume of 120 L, how long does it take to fill completely?

Joining it all together

Q10. Application 4 marks

The value of an action figure modelled from a science fiction film increases according to the logarithmic function $y = 15\log_{10}(x) + 10$, where y is the value of the action figure, in dollars, and x is the number of years from 1977.

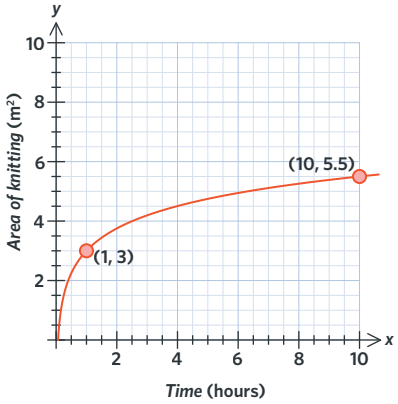
a) By finding three points on the function, plot the graph of price of action figure, y, against number of years, x. (2 marks)

b) Find the value of the action figure in 2019 to the nearest cent. (1 mark)

c) In what year did four action figures cost \$100? (1 mark)

Q11. Application 3 marks

Margaret has recently noticed that her knitting speed slows considerably as she progresses along a blanket. She decided she would record the area of the blanket knitted in square metres and compare it to the time in hours she spent knitting. The data is shown in the graph on the right.



a) Find the rule for the area of the blanket knitted in terms of the hours spent knitting, in the form of $area\ of\ blanket = a\log_{10}(hours\ spend\ knitting) + c$.

b) Margaret's husband John enjoys knitting as well, but is significantly slower than his wife. His knitting follows the equation: $area\ of\ blanket = 1.5\log_{10}(hours\ spent\ knitting) + 2$. How much larger is the area of Margaret's blanket compared to John's after 5 hours? Give your answer to two decimal places.

c) How much longer does it take John to knit 3 square metres of blanket? Give your answer to the nearest minute.

Questions from multiple lessons

Q12.

Difficulty:

1 mark

Johnny's job requires him to transport racehorses during the spring carnival. The following graph shows the distance Johnny travels each hour throughout an entire day.

What was the average speed that Johnny was travelling over the entire day, including the time he spent resting?

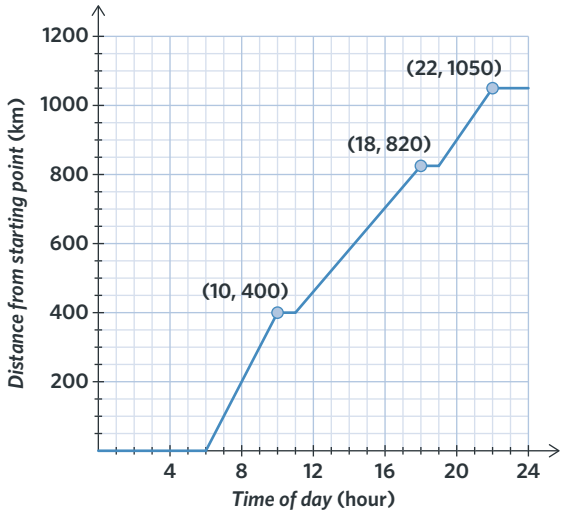
A. 30.50 km/h

B. 43.75 km/h

C. 51.25 km/h

D. 75.00 km/h

E. 105.00 km/h



VCAA 2014 Exam 1 Module 3: Graphs and relations Q5 – Adapted

Q13.

Difficulty:

1 mark

A least squares regression line is fitted to the following data.

x	−10.2	2.3	−1.3	−0.9	1.3	4.7	−8.7	−2.3
y	13.6	−1.8	−1.2	2.5	2.1	−3.8	7.8	3.9

What is the equation of the line? Round all numbers to two decimal places.

A. $y = 0.96 - 1.02x$ **B.** $y = 1.02 - 0.96x$ **C.** $y = 1.03 - 0.95x$

D. $y = 0.95 - 1.03x$ **E.** $y = -1.02 + 0.96x$

VCAA 2018NH Exam 1 Data analysis Q8 – Adapted

Q14.

Difficulty:

2 marks

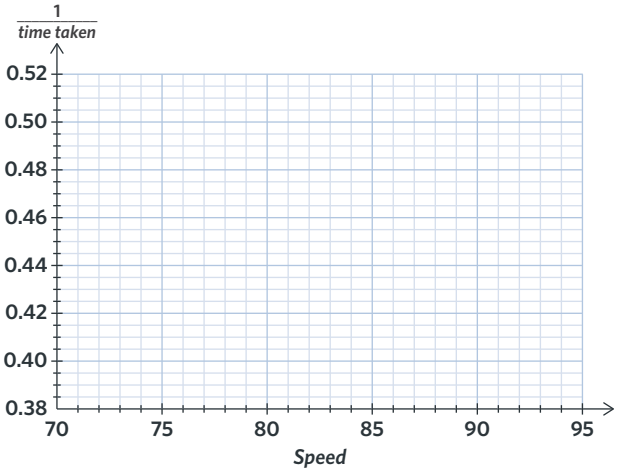
The time taken to travel between two train stations varies inversely with the train's speed. The *speed* (km/h) and the *time taken* (minutes) of a number of different trains during the day is recorded in the table below.

speed	90.00	85.71	78.26	75.00	72.00
time taken	2.0	2.1	2.3	2.4	2.5
$\frac{1}{\textit{time taken}}$					

A $\frac{1}{\textit{time taken}}$ transformation is performed to linearize the data.

a) Transform the data by completing the table of values. Round all values to two decimal places. (1 mark)

b) Plot the transformed data using the axes below. (1 mark)



AOS 6: Statistics

Investigating and comparing data distributions

CHAPTER 12

LESSON 12A

Types of data

The key skills you will learn in this lesson are:

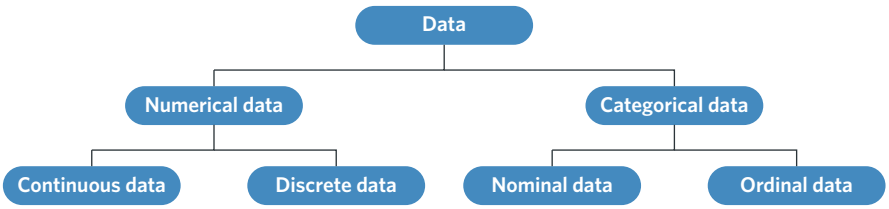
1. Categorical data
2. Numerical data

VCAA key knowledge point:

"types of data, including categorical (nominal or ordinal) or numerical (discrete or continuous)"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

Data can be classified into two broad categories: numerical data and categorical data. These can then both be divided into two subgroups, as shown below.



1. Categorical data

Data that is organised into several categories or groups is known as categorical data.

Categorical data can be classified in two ways: nominal and ordinal.

Ordinal variables are categorical variables that exist in a logical order, for example, drink size: small, medium, large.

Nominal variables do not have a logical order or hierarchy. For example, hair colour.

Example

The rating of a film from one star to five stars is an ordinal variable, as it naturally assumes an order.

The genre of film (action, comedy, drama, thriller) is nominal, as it does not logically assume an order.

WORKED EXAMPLE 1 *(1 mark)*

A poll asked the question 'Ferrari makes the best cars. Do you agree?' with the possible responses 'Strongly agree', 'Agree', 'Neutral', 'Disagree', or 'Strongly disagree'.

Are the responses ordinal or nominal?

SOLUTION

Step 1 Identify whether there is a logical order for the responses.

It makes sense for them to start and end with either strongly agree and strongly disagree, and for neutral to be in the middle.

Therefore, there is a logical order.

Step 2 Classify the variable as ordinal or nominal.

The responses are ordinal.

2. Numerical data

Data that is represented by a quantity is known as numerical data. Numerical data can be classified in two ways: discrete and continuous.

Discrete data can be counted. It exists only as separate numerical values, and is not on a continuous scale. The number of students in a class is an example of discrete data.

Continuous data is measured, for example, distance, time, or weight. It exists on a continuous scale.

Example

The distance from home to school is a continuous variable, as it is measured and exists on a continuous scale.

The number of steps it takes to walk to school is discrete, as they are counted and can only exist as a whole number.

WORKED EXAMPLE 2 *(1 mark)*

The maximum temperature each day this year is being recorded. Is maximum temperature a discrete or continuous variable?

SOLUTION

Step 1 Identify whether the variable is measured or counted.

Maximum temperature is a measured quantity; it can be measured to varying degrees of accuracy.

Step 2 Classify the variable as discrete or continuous.

Maximum temperature is a continuous variable.

Questions 12A *Types of data*

Refresher question

Q1. Are the number of dogs in a dog park counted or measured?

1. Categorical data

Q2. Skill

Students in a class are each asked what pets they have. This data is categorical.

Is it ordinal or nominal?

Q3. Skill

Classify the following categorical data as either ordinal or nominal.

a) Favourite flavour of ice cream

b) How often a student walks to school (never, once a month, once a week, daily)

c) Skill level (beginner, intermediate, advanced, expert)

d) Favourite brand of clothing

Check your understanding

Q4. Skill

Data is collected from a local university inquiring about the ages of their students. The possible answers are 14–17, 18–21, 22–25, and 26+.

Classify the data as ordinal or nominal data.

2. Numerical data

Q5. Each member of the audience at a concert is asked their age in years. This data is numerical.
Skill
Is it discrete or continuous?

Q6. Classify the following numerical data as either discrete or continuous.
Skill

a) The number of people attending a football match

b) The distance from earth to the moon

c) The temperature of a pool

d) The number of children in a classroom

Check your understanding

Q7. The least time taken for Usain Bolt to run 100 metres is 9.58 seconds. Is time taken a discrete or continuous variable?
Application

Joining it all together

Q8. Classify the following as either ordinal, nominal, discrete or continuous.
Skill
4 marks

a) The weight of a bag of apples

b) The different brands of apples

c) The classification of apples as small, medium or large

d) The number of apples in a bag

Q9. Classify the following as either categorical or numerical data.
Skill
4 marks

a) The number of breeds of cat

b) The ranking of difficulty of maths questions from one to five

c) A postcode

d) Women's jean sizes

VCAA questions

Q10. The variables, number of moths (less than 250, 250–500, more than 500), and trap type (sugar, scent, light) are:
1 mark

A. both nominal variables.

B. both ordinal variables.

C. a numerical variable and a categorical variable respectively.

D. a nominal variable and an ordinal variable respectively.

E. an ordinal variable and a nominal variable respectively.

VCAA 2017 Exam 1 Core: Data analysis. Q7

Q11. 1 mark

The following table shows the data collected from a sample of seven drivers who entered a supermarket car park. The variables in the table are:

- *Distance* – the distance that each driver travelled to the supermarket from their home
- *Sex* – the sex of the driver (female, male)
- *Number of children* – the number of children in the car
- *Type of car* – the type of car (sedan, wagon, other)
- *Postcode* – the postcode of the driver's home.

Distance (km)	Sex (F = female, M = male)	Number of children	Type of car (1 = sedan, 2 = wagon, 3 = other)	Postcode
4.2	F	2	1	8148
0.8	M	3	2	8147
3.9	F	3	2	8146
5.6	F	1	3	8245
0.9	M	1	3	8148
1.7	F	2	2	8147
2.5	M	2	2	8145

The number of categorical variables in this data set is

A. 0 **B.** 1 **C.** 2

D. 3 **E.** 4

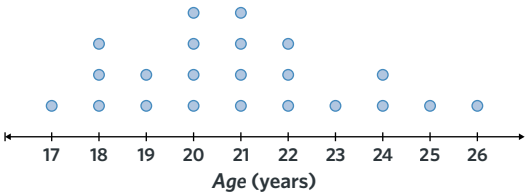
VCAA 2014 Exam 1, Core: Data analysis. Q4

Questions from multiple lessons

Q12. Difficulty:

Year 10
1 mark

The following dot plot displays the *age*, in years, of the 22 players in a university football team.



The percentage of players that are 20 years of age is closest to

A. 9% **B.** 14% **C.** 15%

D. 18% **E.** 25%

VCAA 2018 Exam 1 Data analysis Q1 – Adapted

Q13. Difficulty:

1 mark

A sequence of numbers can be generated using the recurrence relation below.

$V_0 = 1, \quad V_{n+1} = V_n + 6$

What is the value of V_4?

A. 1 **B.** 7 **C.** 13

D. 19 **E.** 25

VCAA 2019NH Exam 1 Recursion and financial modelling Q17 – Adapted

Q14.

Difficulty:

3 marks

Fruit	*Seed location*	*Size*	*Price* ($ per kg)
Apple	Internal	Medium	$4.99
Apricot	Internal	Medium	$2.30
Avocado	Internal	Medium	$5.20
Banana	Internal	Medium	$3.49
Blackcurrant	Internal	Small	$7.25
Blueberry	Internal	Small	$6.95
Boysenberry	Internal	Small	$6.99
Cherry	Internal	Small	$8.99
Cranberry	Internal	Small	$6.99
Fig	Internal	Medium	$5.49
Grape	Internal	Small	$5.89
Grapefruit	Internal	Large	$9.49
Jackfruit	Internal	Large	$12.49
Kiwifruit	Internal	Medium	$4.99
Kumquat	Internal	Small	$5.99
Lime	Internal	Small	$1.99
Lychee	Internal	Small	$14.99
Mango	Internal	Medium	$6.99
Orange	Internal	Medium	$2.99
Peach	Internal	Medium	$10.49
Pineapple	Internal	Large	$3.49
Pomegranate	Internal	Medium	$7.99
Raspberry	Internal	Small	$4.99
Strawberry	External	Small	$2.49
Watermelon	Internal	Large	$1.99

The data in the table displays information for 25 different types of fruit.

The four variables in this data set are:

- *Fruit* – type of fruit
- *Seed location* – where seeds are found (external, internal)
- *Size* – size of the fruit (small, medium, large)
- *Price* – the average prices for these fruits (dollars per kg)

a) How many variables in this data set are categorical variables? (1 mark)

b) How many variables in this data set are ordinal variables? (1 mark)

c) Name the large fruits that cost more than $3 per kg. (1 mark)

VCAA 2018 Exam 2 Data analysis Q1 a-c – Adapted

LESSON 12B

Categorical data distributions

The key skills you will learn in this lesson are:

1. Frequency tables
2. Bar charts
3. The mode

VCAA key knowledge point:

"display and description of categorical data distributions using frequency tables and bar charts; and the mode and its interpretation"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Frequency tables

A frequency table consists of two columns. One column lists the different categories of the data set, and the other lists the frequency that the categories occur at.

A third column, known as the 'tally' column, can be used to help find the frequency, as shown in the worked example below.

Example

Anna, Joey, and Clary were playing a game of tag and decided to count how many times they were tagged. The results are shown in the frequency table below.

Person	Frequency
Anna	2
Joey	4
Clary	3

The frequency table shows that Anna was tagged two times, Joey was tagged four times and Clary was tagged three times.

WORKED EXAMPLE 3 *(1 mark)*

A class of students were asked about their favourite ice cream flavours. Their answers were:

Chocolate, Strawberry, Chocolate, Chocolate, Vanilla, Chocolate, Strawberry, Chocolate, Chocolate, Strawberry, Strawberry, Vanilla, Chocolate, Chocolate, Chocolate

Display this information in a frequency table.

SOLUTION

Step 1 Set up a table with the categories in the first column, the tally in the second, and the frequency in the third.

Favourite flavour	Tally	Frequency
Chocolate		
Strawberry		
Vanilla		

Step 2 Count the number of students who preferred chocolate, making a mark in the tally for every student counted. Repeat for strawberry and vanilla.

Favourite flavour	Tally	Frequency
Chocolate	𝍸 ||||	
Strawberry	||||	
Vanilla	||	

Step 3 Count the tallies for each flavour individually, and record them in the frequency column.

Favourite flavour	Tally	Frequency
Chocolate	𝍸 ||||	9
Strawberry	||||	4
Vanilla	||	2

2. Bar charts

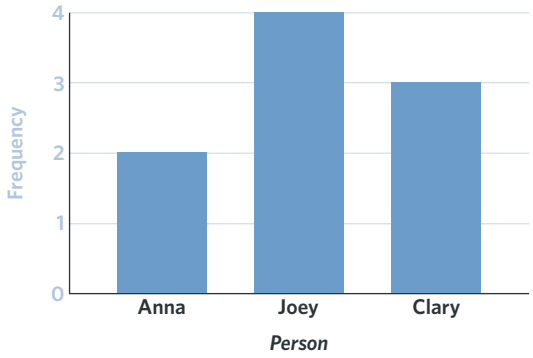
A bar chart is a visual representation of categorical data.

A bar chart shows the categories of a data set on the horizontal axis, and the frequency of the categories on the vertical axis.

Each category is given a separate bar. The height of each bar represents the frequency of that category, and the bars are drawn with a gap between them.

Example

The data from Anna, Joey, and Clary's game of tag is shown in the bar chart below.



WORKED EXAMPLE 4 *(1 mark)*

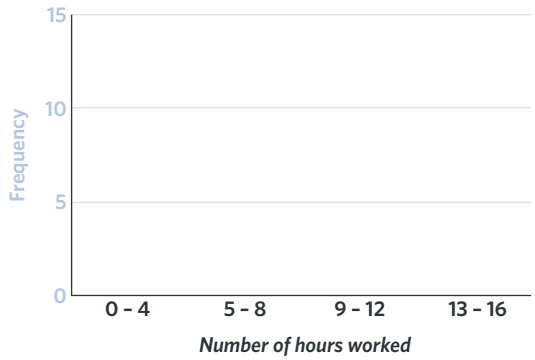
Dr. Barras always works an exact amount of hours. Over a month, the number of hours he worked each day was recorded. This information is summarised in the frequency table below. From the frequency table, construct a bar chart.

Number of hours worked	Frequency
0–4	4
5–8	15
9–12	8
13–16	2

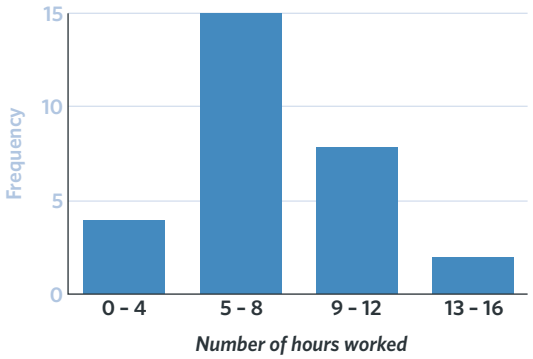
SOLUTION

Step 1 Set up the axes.

Draw the vertical axis from 0 to 15, and draw the horizontal axis such that there is a small gap between each category. Label each axis appropriately.



Step 2 Match the height of each individual bar to the frequency written in the table.



3. The mode

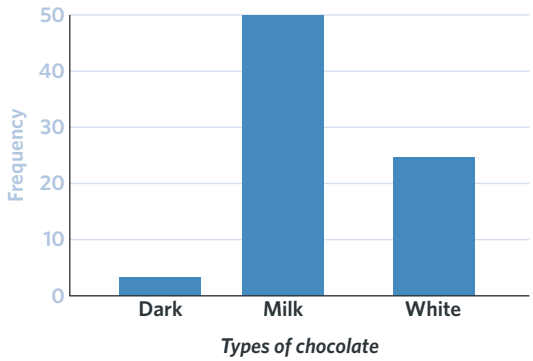
The mode is the category that occurs the most frequently. It can also be called the modal category. In a bar chart, the mode is the tallest bar. There can be multiple modal categories of a data set, if two or more categories equally occur the most.

Example

In Anna, Joey, and Clary's game of tag, it is clear Joey was tagged the most times, because his bar is taller than the others. Therefore, he is the modal category.

WORKED EXAMPLE 5 *(1 mark)*

Students in a class were surveyed and asked about their favourite type of chocolate. The results are shown in the following bar chart.



Which is the most popular type of chocolate?

SOLUTION

Step 1 Identify what the question is asking for.

The question refers to the 'most popular' type of chocolate. We are looking for the modal category.

Step 2 Identify the modal category.

The tallest bar in the bar chart is 'Milk'. Therefore, milk chocolate is the most popular type of chocolate.

Questions 12B *Categorical data distributions*

Refresher question

Q1. Students in a class were surveyed about the pets they own.

If three students own a cat, seven own a dog and two own a guinea pig, which pet is most popular?

1. Frequency tables

Q2. A group of 20 people were asked to name their favourite sport. The following data was collected.

Skill

Sport	Frequency
Football	11
Netball	6
Basketball	3

How many people chose netball as their favourite sport?

Q3. Skill

In an office, the eye colour of each person was collected. The results were as follows:

Blue, Blue, Brown, Brown, Brown, Green, Hazel, Brown, Blue, Brown, Green, Brown, Green, Blue, Hazel, Brown, Brown, Blue, Hazel, Brown

Eye Colour	Tally	Frequency

a) Fill in the frequency table above.

b) How many people in the office have blue eyes?

Q4. Skill

The preferred movie genre of 25 people is shown below. The categories were abbreviated as A (Action), C (Comedy), D (Drama), R (Romance), and T (Thriller).

A, C, D, D, A, A, R, R, T, R, C, C, A, T, R, C, A, T, R, C, A, A, T, R, C

Create a frequency table that displays this information.

Q5. Skill

A group of 12 people were asked about their favourite genre of music. The results are shown in the following table.

Create a frequency table that displays the information from the table.

Person	Favourite genre
A	Pop
B	Classical
C	Rock
D	Classical
E	Pop
F	Pop
G	Rock
H	Pop
I	Pop
J	Rock
K	Pop
L	Rock

Check your understanding

Q6. Skill

A group of people were asked about their favourite pizza topping. The results of their responses are shown in this frequency table.

Pizza topping	Frequency
Margherita	9
Hawaiian	4
Meat Lovers	2
Vegetarian	4

a) How many people were surveyed?

b) What percentage of people like vegetarian pizza the most? Round your answer to the nearest whole number.

2. Bar charts

Q7. Skill

The favourite holiday destination of 15 people is shown in the following frequency table.

Favourite holiday destination	Frequency
Beach	8
Snow	3
City	4

Create a bar chart that displays this information.

Q8. Skill

A cafe recorded how many of each type of beverage was sold in one morning. A total of 200 beverages were sold. The data is shown in this frequency table.

Type of beverage	Frequency
Coffee	38
Tea	46
Juice	83
Water	?

a) What is the missing frequency value?

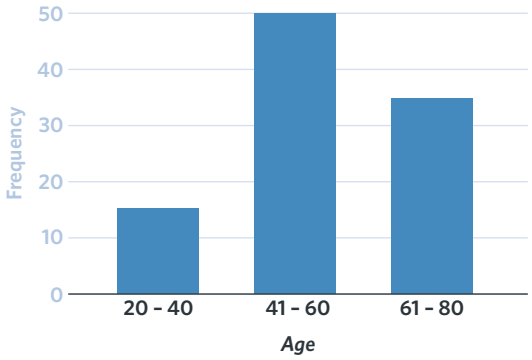
b) Create a bar chart that displays the information from the table, including the missing value from part a).

Check your understanding

Q9. Skill

A small university recorded the age of each member of its teaching staff. The results are shown in this bar chart.

Use the bar chart to create a frequency table.



Q10. Skill

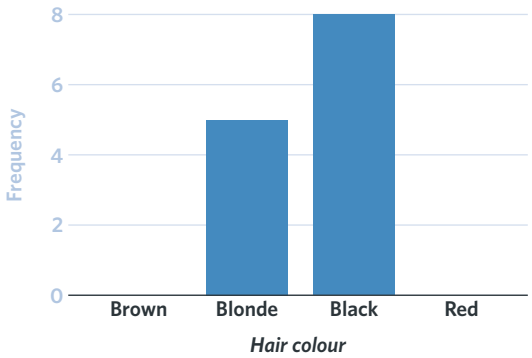
A class of 25 students was surveyed about their hair colour, and the following bar chart was created from the data collected.

The bar chart is incomplete. In the class five times the number of students who have red hair have brown hair.

a) Complete the bar chart.

b) Use the complete bar chart to create a frequency table.

c) What percentage of the class have red hair?

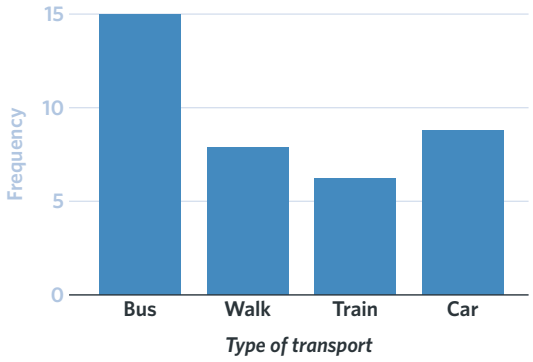


3. The mode

Q11. Skill

A class was asked what type of transport they use to get to school. The results are displayed in the bar chart below.

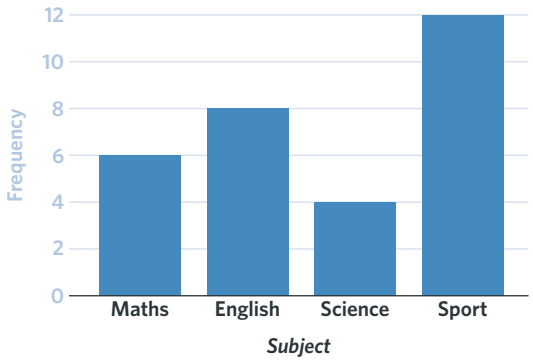
Which type of transportation is the most popular?



Q12. Skill

Thirty students were asked about their favourite subject. The results are shown in the following bar chart.

Which subject is the mode?



Q13. Skill

A group of 1100 people were asked which city they were born in. The results are shown in the following frequency table.

City	Frequency
Melbourne	492
Sydney	279
Canberra	

a) What is the missing frequency value?

b) Where were the most people born?

Check your understanding

Q14. Skill

A group of 53 mothers was asked how many children they have. The results are shown in the following incomplete frequency table.

Number of children	Frequency
1	8
2	18
3	
4	

There were twice as many mothers who have three children as there were mothers who have four children.

a) Complete the frequency table above.

b) What is the modal number of children?

Joining it all together

Q15. A group of 20 people were asked about their favourite type of tea. The results were:

Skill
2 marks

Black, Peppermint, Chamomile, Black, Black, Peppermint, Chamomile, Black, Peppermint, Black, Peppermint, Chamomile, Black, Peppermint, Black, Chamomile, Black, Peppermint, Black, Peppermint.

a) Create a frequency table that displays this information.

b) Use the frequency table to create a bar chart.

Q16. A group of 24 people was asked about their favourite Australian animal. The results of the survey are shown in the following table:

Application
3 marks

Person	Favourite animal
A	Koala
B	Emu
C	Koala
D	Kangaroo
E	Wombat
F	Emu
G	Wombat
H	Emu
I	Koala
J	Wombat
K	Kangaroo
L	Wombat

Person	Favourite animal
M	Koala
N	Wombat
O	Emu
P	Wombat
Q	Wombat
R	Koala
S	Kangaroo
T	Koala
U	Wombat
V	Emu
W	Emu
X	Wombat

a) Create a frequency table displaying this information.

b) Construct a bar chart displaying this information.

c) Which animal is the most popular?

Q17. A concert hall was filled with 25 000 people for a concert by the Melbourne Symphony Orchestra. The people who attended the concert were asked to fill out a survey as to their favourite instrument family. In the survey 17.5% responded brass, 24.8% responded woodwind, 6.3% responded percussion and the rest responded strings.

Application
4 marks

a) Strings is the favourite family for what percentage of people?

b) Use the data to construct a frequency table.

c) From the frequency table, construct a bar chart.

d) What is the most popular instrument family?

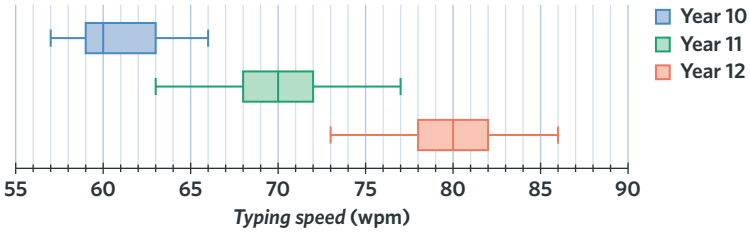
Questions from multiple lessons

Q18.

Difficulty:

1 mark

The parallel boxplots below display the distribution of typing speed, in words per minute, for three different year levels; Year 10, Year 11 and Year 12.



Which one of the following is **not** true?

A. The slowest typer is a Year 10.

B. Year 10s have the least variable typing speed.

C. At least 75% of Year 11s type faster than any Year 10.

D. In terms of the median typing speed, Year 12s are the fastest typers.

E. The fastest typer is a Year 11.

VCAA 2018NH Exam 1 Data analysis Q6 – Adapted

Q19.

Difficulty:

1 mark

The first four terms of a sequence are 3, 6, 12, 24...

Which of the following recurrence relations could generate the above sequence?

A. $T_0 = 2, \quad T_{n+1} = 3T_n$

B. $T_0 = 3, \quad T_{n+1} = 2T_n$

C. $T_0 = 3, \quad T_{n+1} = 3T_n$

D. $T_0 = 3, \quad T_{n+1} = T_n + 2$

E. $T_0 = 3, \quad T_{n+1} = T_n + 3$

VCAA 2017 Exam 1 Recursion and financial modelling Q19 – Adapted

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Q20.

Difficulty:

Year 10
2 marks

Kevin wants to get feedback on the dinner he made for a group of 16 friends. He asked them all to specify whether they found the meal rancid, unpalatable, mediocre, appetising or divine.

Their responses are recorded below.

Unpalatable	*Mediocre*	*Unpalatable*	*Divine*
Rancid	*Rancid*	*Mediocre*	*Appetising*
Divine	*Mediocre*	*Mediocre*	*Mediocre*
Divine	*Appetising*	*Divine*	*Divine*

Use the data to

a) Complete the frequency table to the right. (1 mark)

b) Determine the percentage of Kevin's friends who rated Kevin's dinner as 'mediocre'. (1 mark)

Rating	Frequency
Rancid	
Unpalatable	
Mediocre	
Appetising	
Divine	
Total	

VCAA 2008 Exam 2 Data analysis Q1 – Adapted

LESSON 12C

Displaying numerical data

The key skills you will learn in this lesson are:

1. Grouped data
2. Histograms
3. Dot plots
4. Stem plots

VCAA key knowledge point:

"display and description of numerical data distributions in terms of shape, centre and spread using histograms, stem plots (including back-to-back stem plots) and dot plots and choosing between plots"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Grouped data

Organising data into groups, or intervals, can make it easier to interpret. This is especially true when a data set is large, or the data is continuous.

There are guidelines for separating data into groups.

- Around 5–10 groups should be selected.
- Groups should have no gaps in between them.
- Groups should be chosen such that each data point fits into a group.
- Groups should not overlap.
- Groups should be the same size.

Usually groups have sizes that are easy to comprehend, such as 5, 10, 100, or 1000 units.

The way the groupings are organised will depend on whether the data is discrete or continuous.

Example

For groups (intervals) of size five:

Discrete data	
Group (interval)	Values in the group
0–4	0, 1, 2, 3, 4
5–9	5, 6, 7, 8, 9
10–14	10, 11, 12, 13, 14
etc.	etc.

Continuous data	
Group (interval)	Values in the group
0–<5	0 to 5, not including 5
5–<10	5 to 10, not including 10
10–<15	10 to 15, not including 15
etc.	etc.

Frequency chart

A frequency chart is used to communicate grouped data by displaying the number of data points that fall within each group of values. The group with the highest frequency is called the **modal interval**.

Example

Number of paintings within homes	Frequency	
0–4	3	← 3 homes have 0 to 4 paintings within them
5–9	2	
10–14	6	← Highest frequency
15–19	5	
20–24	3	
25–29	4	

Modal interval →

WORKED EXAMPLE 6 *(2 marks)*

A class sat a 30-mark maths quiz. The following data set contains the scores, out of 30, of individual students in the class.

Create a grouped frequency table of the data.

12	18	11	25	10	5	28	27	8	14	3	19	20
11	1	19	28	22	29	15	4	14	18	16	20	21

SOLUTION

Step 1 Decide how the data should be grouped.

Splitting the data into groups of 5 marks is suitable.

Step 2 Draw a frequency table and put the groups of values in the left column.

Score of quiz	Frequency
0–4	
5–9	
10–14	
15–19	
20–24	
25–29	

✓ *1 mark for appropriately grouping data*

Step 3 Count the number of values that fall within each group and write this in the frequency column.

Score on quiz	Frequency
0–4	3
5–9	2
10–14	6
15–19	6
20–24	4
25–29	5

✓ *1 mark for completing the grouped frequency table*

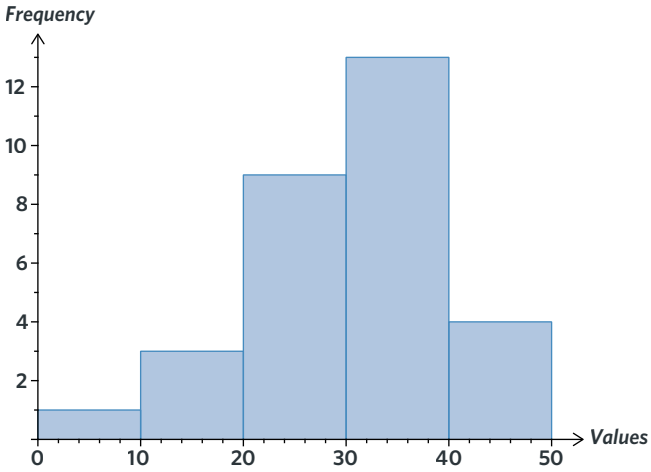
2. Histograms

Histograms are very similar to bar charts, but there are some key differences. Each column within a histogram represents a group of values, whilst the height of each column represents the frequency of each group (interval).

As histograms represent the frequency of intervals of data, they are used to show grouped continuous data, or large numbers of discrete values.

NOTE: There is no gap between the columns. This is because the columns represent intervals that have no gap between them. For example, there is no gap between 0–<10 and 10–<20.

Example



WORKED EXAMPLE 7 *(2 marks)*

Use the scores from the previous worked example to:

a) Create a histogram.

b) Find how many students got less than 50% on the quiz.

12	18	11	25	10	5	28	27	8	14	3	19	20
11	1	19	28	22	29	15	4	14	18	16	20	21

SOLUTION: BY HAND

a) Create a histogram.

Step 1 Create a grouped frequency table.

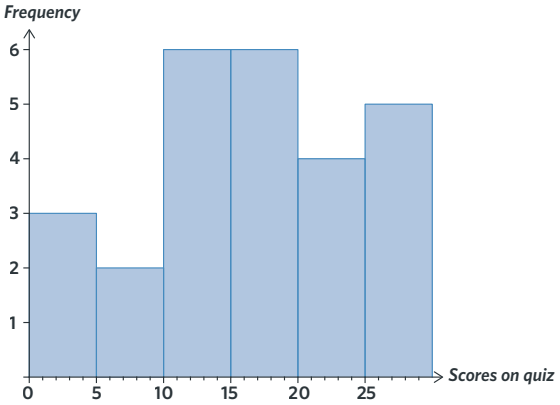
NOTE: This was done in the previous example.

Step 2 Create the graph.

Mark the first value of each group on the x-axis (0, 5, 10, etc.).

Draw vertical columns between each mark with a height equal to the groups' frequency.

Label the x-axis with the title 'scores on quiz' and the y-axis with 'frequency'.



b) Find how many students got less than 50% on the quiz.

Step 1 Find out what score is equal to 50% on the quiz.

There are 30 marks so find 50% of 30.

$\frac{50}{100} \times 30 = 15$

Less than 50% is a score less than 15.

Step 2 Find out how many scores were less than 15.

Look at the histogram.

Columns 1, 2 and 3 show the scores less than 15. Add the heights of the columns to find the total amount.

$3 + 2 + 6 = 11$

Step 3 Interpret.

There were 11 students that scored less than 50%.

SOLUTION: TI-NSPIRE

a) Create a histogram.

Step 1 Open a new page by pressing ctrl + doc and select 'Add Lists & Spreadsheets'.

Step 2 Name a list 'scores'. Starting from row 1, enter data into the list titled 'scores'.

Step 3 Press ctrl + doc and select 'Add Data & Statistics'.

Step 4 Move the cursor to the x-axis and click 'Click to add variable'.

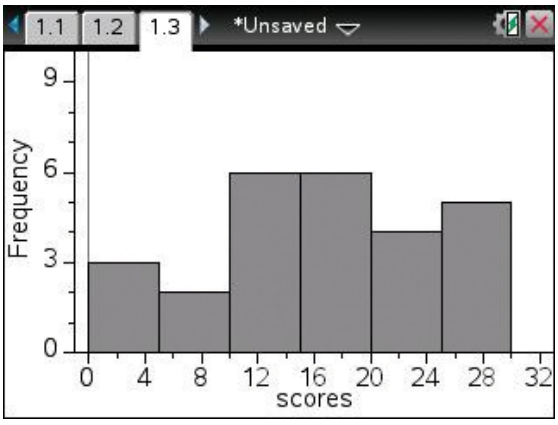
Step 5 Select 'scores'.

Step 6 Press menu, select 'Plot Type' → 'Histogram'.

Step 7 To adjust the column width and the starting point, move the cursor over the histogram bars and press ctrl + menu. Select 'Bin Settings' → 'Equal Bin Width'.

Step 8 Set the column width to 5 by changing 'Width' to 5.

Step 9 Set the starting point to 0 by changing 'Alignment' to 0.



b) Find how many students got less than 50% on the quiz.

Please refer to 'Solution: By hand' for this part of the solution.

SOLUTION: CASIO CLASSPAD

a) Create a histogram.

Step 1 From the main menu, tap Statistics.

Step 2 Press keyboard and select abc.

Rename 'list1' to 'scores'.

Step 3 Starting from row 1, enter data into the list titled 'scores'.

Step 4 Configure the settings of the graph by tapping .

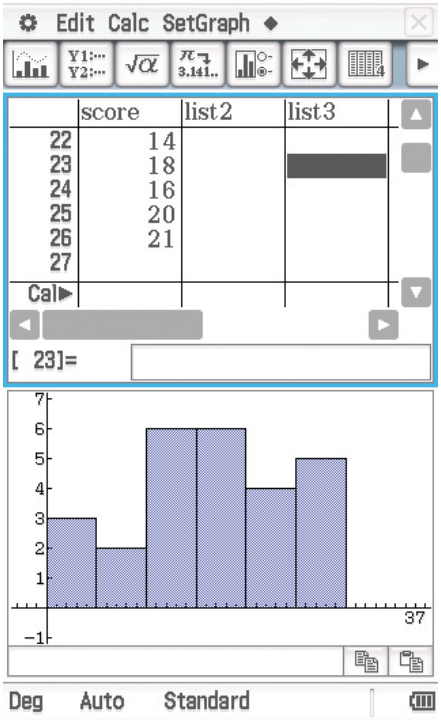
Step 5 Create a histogram by changing 'Type' to 'Histogram'.

Step 6 Specify the data set by changing 'XList' to 'main\scores'.

Tap set to confirm.

Step 7 Tap in the icon bar to plot the histogram.

Step 8 Set the starting point and column width by changing 'HStart' to 0 and by changing 'HStep' to 5.



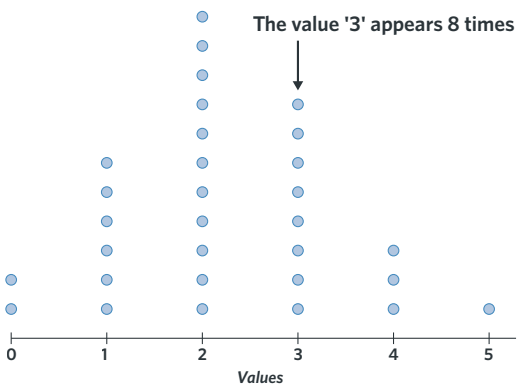
b) Find how many students got less than 50% on the quiz.

Please refer to 'Solution: By hand' for this part of the solution.

3. Dot plots

Dot plots use stacked dots to convey the frequency of data values. Dot plots are best at conveying discrete numerical data or categorical data, but can also be used to display continuous numerical data. This can be achieved by grouping data and displaying the frequency of values within that group with the stacked dots (much like a histogram).

Example



WORKED EXAMPLE 8 *(4 marks)*

The following data represents the number of pets owned by residents of George St in Fitzroy.

0	1	0	2	2	4	1	0	0	1	2	0	1
1	5	0	2	4	1	0	0	0	2	1	0	1

a) Create a frequency table of this data.

b) Create a dot plot representing the data.

c) What is the modal number of pets?

d) How many residents have one pet?

SOLUTION

a) Create a frequency table of this data.

Step 1 Find the frequencies for each data value.

As the data is discrete, no grouping is necessary.

Number of Pets	Frequency
0	10
1	8
2	5
3	0
4	2
5	1

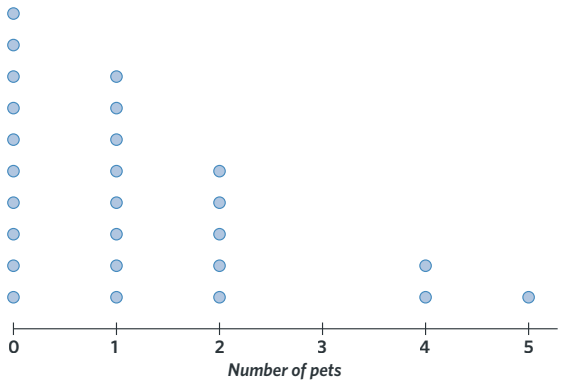
b) Create a dot plot representing the data.

Step 1 Create the dot plot.

Draw a number line that covers the relevant data values.

Draw dots above each value in accordance with the frequency of that value.

Label the plot with the title 'number of pets'.



c) What is the modal number of pets?

Step 1 Look at the dot plot. Which stack has the most dots?

The number 0 has the most dots.

Step 2 Interpret.

The modal number of pets is 0.

d) How many residents have one pet?

Step 1 Read the frequency table or the dot plot.

Frequency table: Look for '1' in the 'number of pets' column and the corresponding frequency in the right column. For **number of pets = 1**, the frequency is 8.

Dot plot: Find the value of 1 for **number of pets**. Count the number of dots above the value 1. There are 8 dots.

Step 2 Interpret.

There are eight residents with one pet.

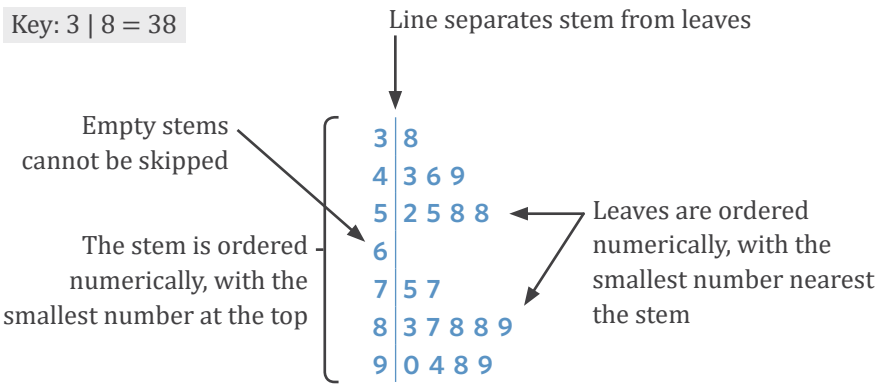
4. Stem plots

Stem plots (or stem-and-leaf plots) display numerical data by grouping data points according to their largest unit (called a stem), and displaying individual values through their smallest unit (called a leaf). Each stem plot has a key that explains how to translate the stem and leaf information into a number.

Example

This is a stem-and-leaf plot.

Key: 3 | 8 = 38



Back-to-back stem plots can be used to compare two sets of data. In these plots the two data sets share a stem, one of the data sets connects to the left of the stem and the other connects to the right.

Example

This is a back-to-back stem-and-leaf plot.

Key: 3 | 8 = 38

Leaves are still ordered numerically, with the smallest number nearest the stem →

7 6 5 2	3	8
9 7 7 6 4 1	4	3 6 9
9 7 4 3 0	5	2 5 8 8
8 5	6	
6	7	5 7
4	8	3 7 8 8 9
	9	0 4 8 9

Stem plots are best at conveying ungrouped data. Stem plots should not be used if the range of data is large, or if the data values are large, as it becomes difficult to display the data concisely. On the other hand, if the data takes few values, a stem plot might be unnecessary.

WORKED EXAMPLE 9 *(1 mark)*

A local swimming squad record their personal best 50-metre freestyle times. The following data set contains the personal best times (in seconds) of all the individual members.

43 38 52 41 47 32 39 45 40 28 49 30 43

Create a stem plot using this data.

SOLUTION

Step 1 Create the stem by finding all the different 'tens' values.

2	
3	
4	
5	

Step 2 List the 'units' of each value off the stem that matches the 'tens' of the value. Remember to create a key.

Key: 2 | 8 = 28 seconds

2	8
3	0 2 8 9
4	0 1 3 3 5 7 9
5	2

Questions 12C *Displaying numerical data*

Refresher question

Q1. Fill out the table by placing the following data values into the correct group.

7 13 5 0 11 9 15 18 6 2 7 16 10

Group	Values
0–4	
5–9	
10–14	
15–19	

1. Grouped data

Q2. Consider the following data set:

24	31	36	26	55	41	59	47	40	21	43	22	60
21	67	52	42	59	33	28	21	30	18	39	40	62

a) How many of these numbers are between 10 and 19?

b) Fill out this grouped frequency table.

Group	Frequency
10–19	
20–29	
30–39	
40–49	
50–59	
60–69	

c) Find the modal interval (the group with the highest frequency) and state its frequency.

Q3. Application

A group of 26 students took part in a survey where they were asked how many hours of exercise they complete weekly. The following data set shows the results.

4	8	14	3	7	9	6	19	10	4	6	0	2
7	11	7	8	6	21	15	12	5	3	6	2	5

a) Using groups of 5 hours, create a grouped frequency chart.

b) How many students exercise less than 5 hours per week?

c) What percentage of students exercise less than 5 hours per week? Give your answer to one decimal place.

Check your understanding

Q4. Application

The following frequency graph displays the heights of high-rise buildings in Melbourne's CBD.

Height (m)	Frequency
50–<100	96
100–<150	78
150–<200	27
200–<250	8
250–<300	5
Total	214

a) Grant owns the entire top floor of a building. His floor is exactly 200 metres above the ground. What number of Melbourne high-rises does Grant overlook?

b) Part of Grant's suite is an outdoor balcony. He is concerned that other luxury dwellers who live on the same level or above him can invade his privacy whilst he is out sunbathing. How many other high-rise buildings can peer onto Grant and his outdoor balcony?

2. Histograms

Q5. Use the following data set to answer the questions below.

Skill

11	27	29	20	27	10	22	11	9	16	5	24	10
12	3	17	1	25	34	21	16	13	7	23	29	17

a) Create a histogram with intervals of 5 units, starting with interval 0–4.

b) What is the frequency of values of within the interval 10–14?

Q6. The girths of trees (in cm) in a local park are as follows:

Application

15	110	76	22	84	18	94	11	45	72	53	81	118
26	18	104	111	63	38	23	14	65	103	89	31	16

a) Create a histogram with intervals of 20 cm.

b) The local council are looking to build a treehouse in one of the larger trees. The council estimates that a tree of girth more than a metre is necessary to hold up multiple children. How many trees are suitable?

c) All the trees with a girth of less than 20 cm were planted within the last year. What percentage of trees have been within the park for more than a year? Give your answer to one decimal place.

Check your understanding

Q7. A group of 100 athletes from different disciplines had their resting heart rate measured. The following frequency table details the results.

Application

Heart rate (bpm)	Frequency
30–<50	32
50–<70	43
70–<90	15
90–<110	8
110–<130	2

a) Create a histogram using this chart.

b) Which intervals contain more than 25% of the athletes?

c) A resting heart rate less than 90 bpm is considered healthy. How many athletes are considered unhealthy?

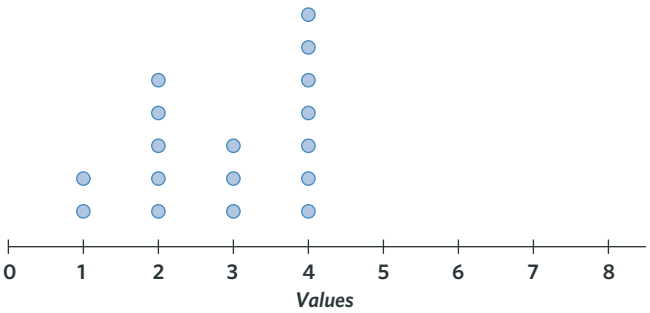
3. Dot plots

Q8. Use the following data set to answer the questions below.

Skill

5	4	5	4	3	5	2	5	4	3	6	2	4
3	1	1	2	4	2	4	6	4	2	8	5	5

a) The following dotplot has been filled out for values 1–4. Complete the dotplot by filling out values 5–8.



b) What is the modal value?

c) How many data values are equal to or less than 3?

Check your understanding

Q9. Application

Every day, Mr Robinson counts how many students in his General Maths class look at their phones at some point instead of doing questions. The data he has collected after 36 days is displayed below in a frequency chart.

a) Help Mr Robinson visually display the effect of phones on his students by creating a dot plot based upon his data.

b) On what percentage of days do none of Mr Robinson's students look at their phones, to one decimal place?

Number of students on phone	Frequency
0	7
1	5
2	13
3	6
4	3
5	1
6	0
7	1

4. Stem plots

Q10. Skill

Use the following data set to answer the questions below.

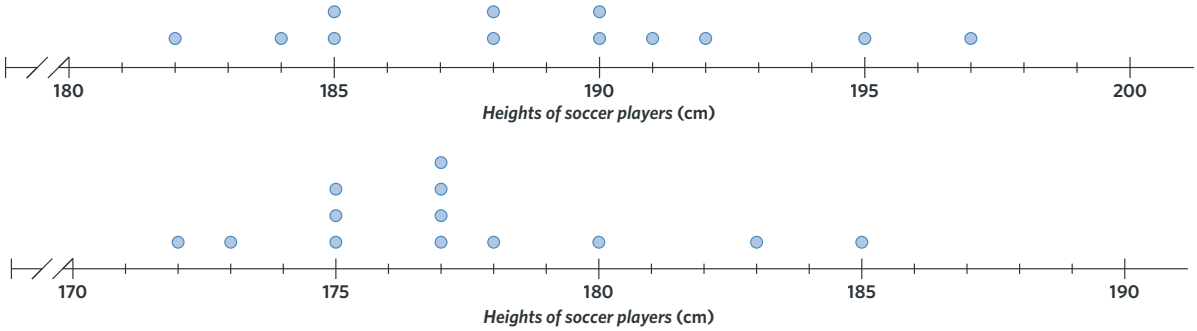
45	28	73	38	75	27	39	47	76	89	63	75	37
37	19	37	39	49	57	26	75	55	49	20	70	63

a) Create a stem plot using this data.

b) How many values are **greater than** 70?

Q11. Application

The following dot plots display the heights (in cm) of the local basketball and soccer teams.



a) Turn this data into a back-to-back stem plot, using the given stem:

17
18
19

b) What percentage of basketball players are taller than 190 cm? Give your answer to one decimal place.

c) How many soccer players are taller than the shortest basketball player?

d) What percentage of basketball players are taller than the tallest soccer player? Give your answer to one decimal place.

Check your understanding

Q12. Application

The following stem plot displays the maximum temperature in Melbourne for each day in June 2018.

Key: 10 | 6 = 10.6°C

10	6
11	2
12	0 5 6 8
13	1 1 1 3 5 6 6 8
14	0 1 1 2 4 5 5
15	0 1 3 3 6
16	6
17	6 8
18	
19	1

a) Louis wears pants only when the maximum temperature for the day is lower than 12.5°C. On how many days did he wear pants?

b) Denise, on the other hand, always wears pants, but only wears a jumper when it is less than 15°C. On how many days did Denise **not** wear a jumper?

Joining it all together

Q13. Skill 5 marks

Use the following set of data to answer the questions below.

13	38	32	27	24	22	20	14	11	26	36	31	28
11	15	23	17	34	39	39	37	14	16	20	21	28

a) Create a grouped frequency table using this data with appropriate intervals. (2 marks)

b) Create a histogram based on the grouped frequency table. (1 mark)

c) Create a grouped dot plot based on the grouped frequency table. (1 mark)

d) Create a stem plot based on this data. (1 mark)

Q14. Application 2 marks

Harry surveyed his whole neighbourhood, and collected data on how many children are in each family.

a) Explain why a stem plot is **not** a suitable display of Harry's data?

b) Which display, out of a dot plot or histogram, would be most suitable to this kind of data? Explain why.

Q15. Application 2 marks

A sports statistician wants to display the heights (rounded to the nearest millimetre) of every Olympian at the 2016 Rio Games on a single chart or graph.

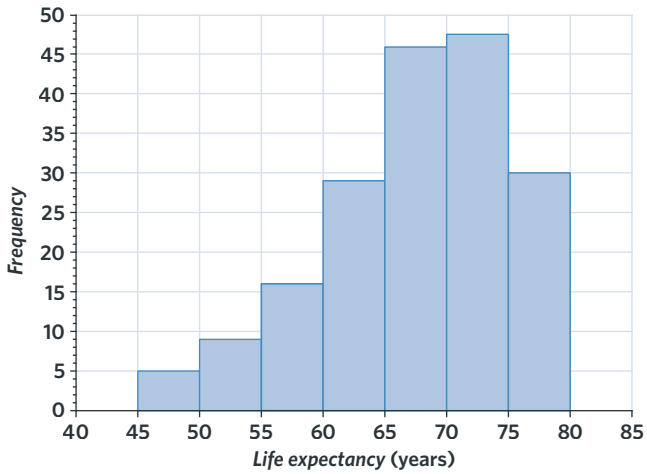
a) Would a dot plot, stem plot or histogram be most suitable? Explain why.

b) What would be the best way of displaying data that compared the heights of male and female javelin throwers? Explain why.

VCAA questions

Q16. The histogram below shows the distribution of life expectancy of people for 183 countries.

3 marks



a) For this distribution, the modal interval is ☐.

b) In how many of these countries is life expectancy less than 55 years?

c) In what percentage of these 183 countries is life expectancy between 75 and 80 years? Write your answer correct to one decimal place.

VCAA 2015 Exam 2, Core: Data analysis. Q1

Q17. 1 mark

This ordered stem plot shows the percentage of homes connected to broadband internet for 24 countries in 2007.

Key: 1 | 6 = 16%

1	
1	6 7
2	0 1 1 3 4 4
2	5 7 8 9
3	0 0 1 1 1 2 2 3
3	5 7 8 8
4	

The number of these countries with more than 22% of homes connected to broadband internet in 2007 is:

A. 4

B. 5

C. 19

D. 20

E. 22

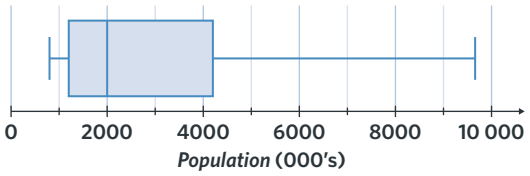
VCAA 2013 Exam 1, Core: Data analysis. Q1

Questions from multiple lessons

Q18. Difficulty:

Year 10
1 mark

The following boxplot displays the *population*, in thousands of people, of the 65 most populated African cities.



The percentage of these 65 cities with a population greater than 2 000 000 is closest to

A. 15%

B. 25%

C. 50%

D. 75%

E. 85%

VCAA 2017 Exam 1 Data analysis Q1 – Adapted

Q19.

Difficulty:

1 mark

What is the sequence generated from the following recurrence relation?

$T_0 = 4, \quad T_{n+1} = -5 \times T_n$

A. $4, -1, -6, -11, -16...$

B. $4, 9, 14, 19, 24...$

C. $4, 20, 100, 500, 2500...$

D. $4, -20, -100, -500, -2500...$

E. $4, -20, 100, -500, 2500...$

VCAA 2017NH Exam 1 Recursion and financial modelling Q17 – Adapted

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Q20.

Difficulty:

Year 10
3 marks

Type of coffee	*Size*	*Caffeine level*	*Price*
Cappuccino	Medium	Medium	$3.50
Mocha	Large	Low	$4.50
White mocha	Large	Low	$4.99
Dark mocha	Large	Low	$4.99
Espresso	Small	High	$4.99
Latte	Medium	Medium	$4.50
Cold brew	Medium	Medium	$5.50
Cold brew latte	Medium	Medium	$5.50
Iced americano	Large	High	$6.50
Iced cappuccino	Medium	Medium	$4.50
Iced mocha	Large	Low	$5.50
Iced white mocha	Large	Low	$5.99
Iced dark mocha	Large	Low	$5.99
Iced espresso	Small	High	$5.99
Iced latte	Medium	Medium	$5.50

a) Use the data in the table to complete the following two-way frequency table. (2 marks)

		Coffee size		
		Small	**Medium**	**Large**
Caffeine level	**Low**			
	Medium			
	High			
Total				

b) What percentage of large coffees had a high *caffeine level*? (1 mark)

VCAA 2018 Exam 2 Data analysis Q1 d,e – Adapted

LESSON 12D

Describing numerical data

The key skills you will learn in this lesson are:

1. Shape of a distribution
2. Location and spread of distributions

VCAA key knowledge point:

"display and description of numerical data distributions in terms of shape, centre and spread using histograms, stem plots (including back-to-back stem plots) and dot plots and choosing between plots"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Shape of a distribution

The shape of a distribution is the way the data is arranged. The shape can be described in terms of symmetry and skew.

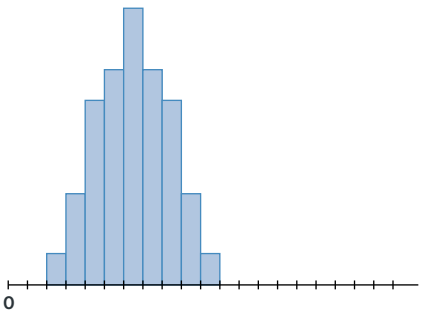
It is important to remember that shape has nothing to do with where the data is located along the horizontal axis.

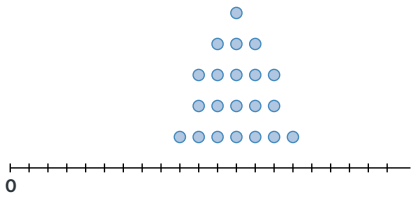
Symmetry

A **perfectly symmetrical** distribution is mirrored around the centre of the data. A distribution is **approximately symmetrical** if it is close to mirrored around the centre of the data.

Example

A perfectly symmetrical distribution:

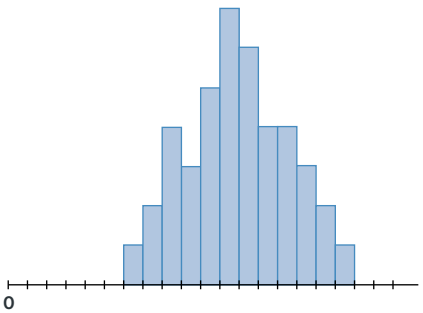


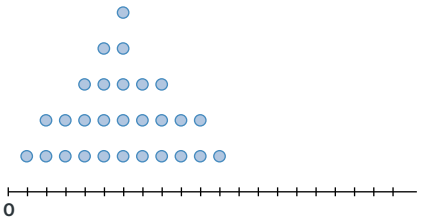


3	9
4	2 3 8
5	2 5 6 8 9
6	3 6 9
7	6

Example

An approximately symmetrical distribution:





4	1 6
5	3 5 6 9
6	2 3 5 8 9
7	2 4 8
8	5 6

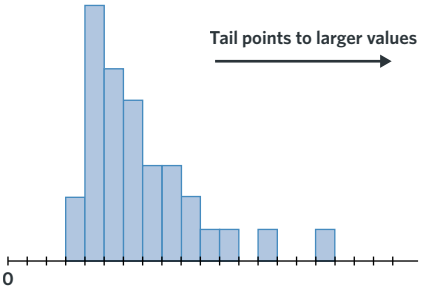
Skew

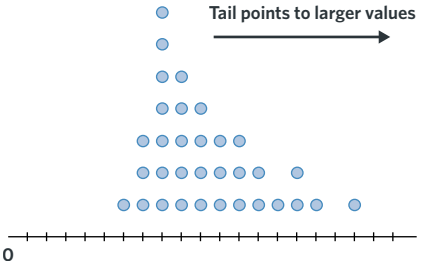
A skew occurs when one side of a distribution is weighted more heavily than the other.

- A **positive skew** is when the left side of a distribution is weighted more than the right. There is a 'tail' to the distribution that points to the right (positive).
- A **negative skew** is when the right side of a distribution is weighted more than the left. There is a 'tail' to the distribution that points to the left (negative).

Example

Positive skew

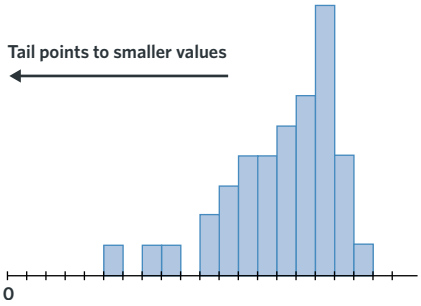


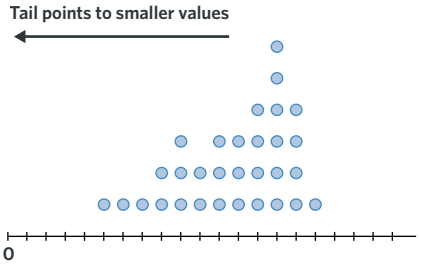


3	5
4	1 3 4 4 6 9
5	0 3 7 8
6	2 5 6
7	1
8	4 8
9	1

Tail points to larger values

Negative skew



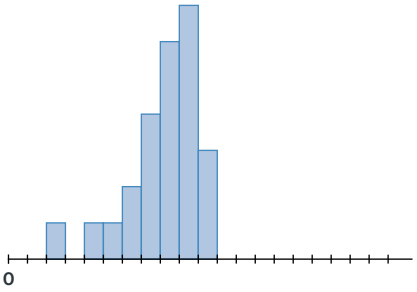


2	6
3	8
4	
5	7
6	2 3 9
7	0 2 5 8
8	1 3 4 6 8 9
9	1

Tail points to smaller values

WORKED EXAMPLE 10 *(1 mark)*

Describe the shape of the following distribution.



SOLUTION

Step 1 Is the distribution symmetrical, skewed or a mixture of both?

It is skewed.

Step 2 Is the distribution positively or negatively skewed?

The tail of the distribution points to the left, so the distribution is negatively skewed.

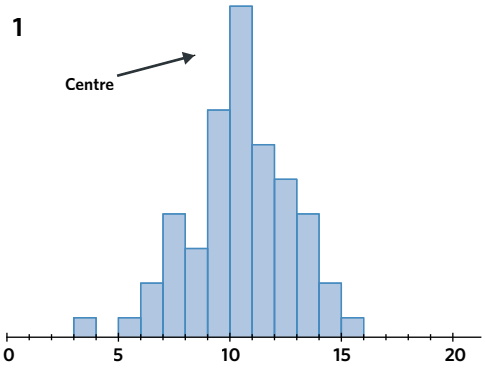
2. Location and spread of distributions

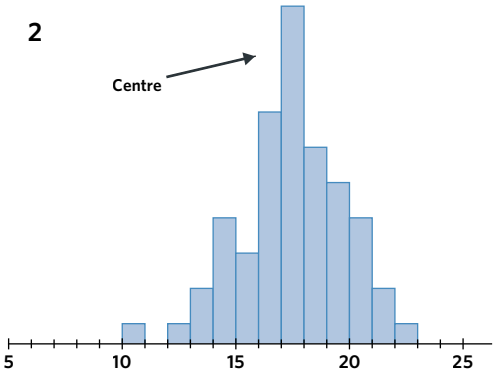
Location

The position of the data values within a distribution is known as the location. Location is found by estimating the centre of the data: the value, or group, in which there are approximately the same amount of data values that are greater than, and less than itself. Two distributions share a similar location if their centres are approximately the same, whist they differ in location if their approximate centres are different.

Example

These two distributions are shaped identically, but differ in location. The second histogram has a centre that is more positive.



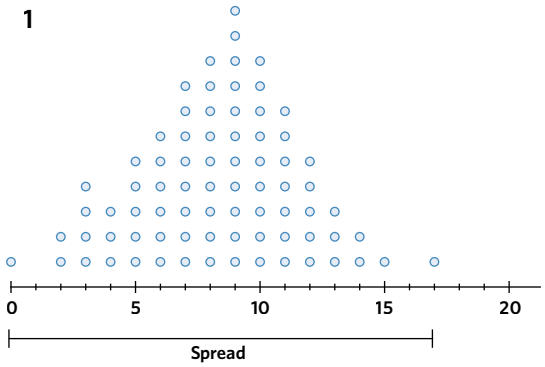


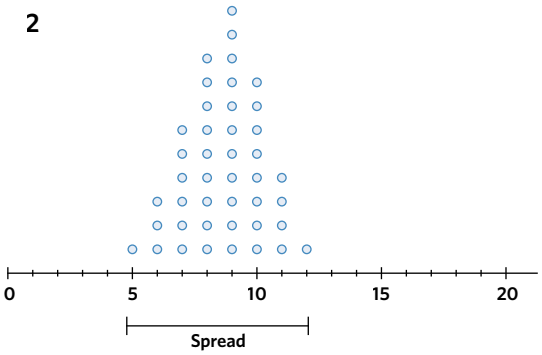
Spread

The spread of a distribution describes how variable a data set is. A distribution that contains values over a large number of intervals has a larger spread than a distribution that has values that are clustered together. Two distributions have a similar spread if they contain values over the same number of intervals.

Example

The data displayed in the first distribution has a larger spread than the data displayed in the second distribution.

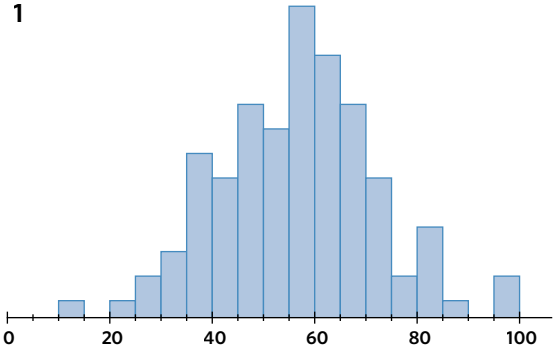




NOTE: Spread and location are **relative** to other distributions.

WORKED EXAMPLE 11 *(2 marks)*

Compare the centre and spread of the following two distributions.



2

2	9
3	2 4
3	5 6 8 8
4	0 1 2 2 4
4	5 6 6 7 8 9 9
5	0 1 3 4 4
5	5 8 9
6	0

SOLUTION

Step 1 Compare the centre.

Distribution 1 is centred around 55–60. Distribution 2 is centred around 45. Therefore, distribution 1 tends to have larger values.

✓ *1 mark for correct comparison of centre*

Step 2 Compare the spread.

Distribution 1 has a larger spread than distribution 2. It takes values from 10–100, whilst distribution 2 takes values from 29–60.

✓ *1 mark for correct comparison of spread*

Questions 12D *Describing numerical data*

Refresher question

Q1. Are these shapes symmetrical along the given axis?

a)

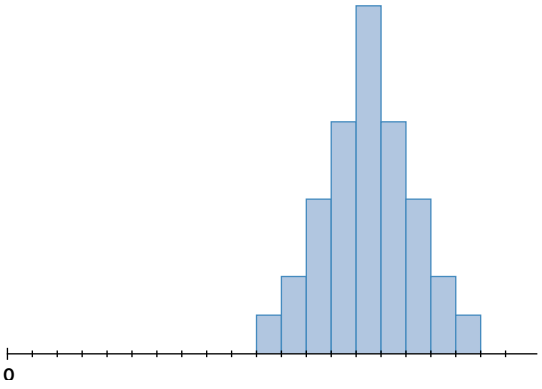
b)

1. Shape of distributions

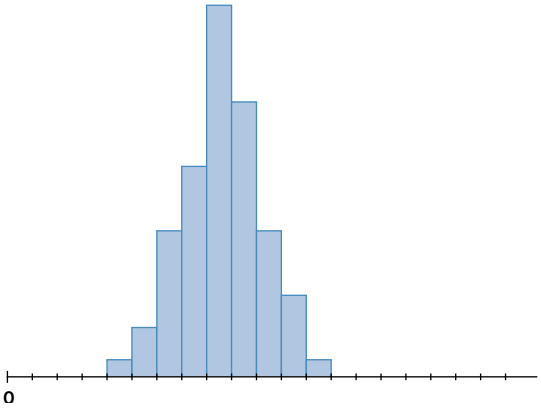
Q2. Are the following distributions perfectly symmetrical, approximately symmetrical or asymmetrical?

Skill

a)



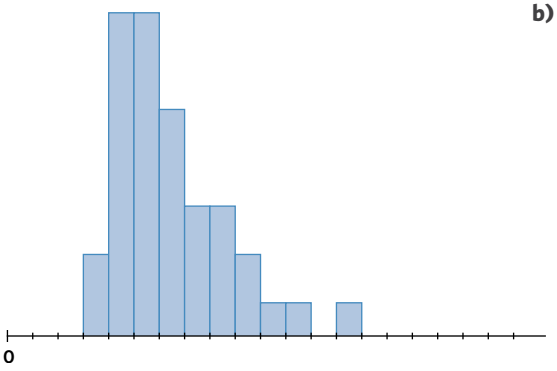
b)



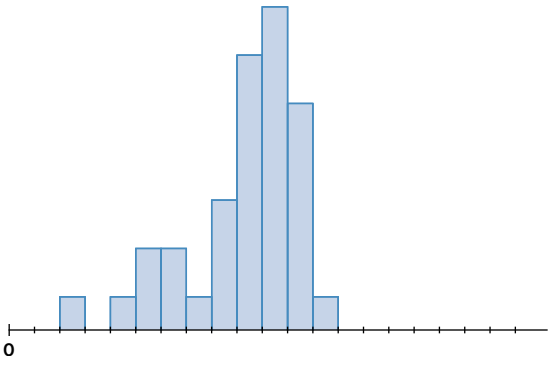
Q3. Are the following histograms positively or negatively skewed?

Skill

a)



b)



Q4. Explain how a distribution's shape can be described as positively skewed, even if all of its data points are negative.

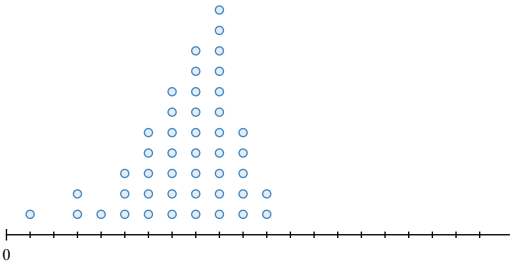
Skill

Check your understanding

Q5. Describe the shape of the following distributions.

Skill

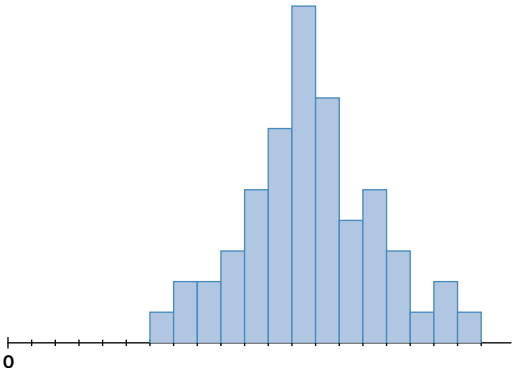
a)



b)

4	4 6
5	2 4 5 8
6	1 3 5 7 7 9
7	0 2 3 8
8	0 4

c)



d)

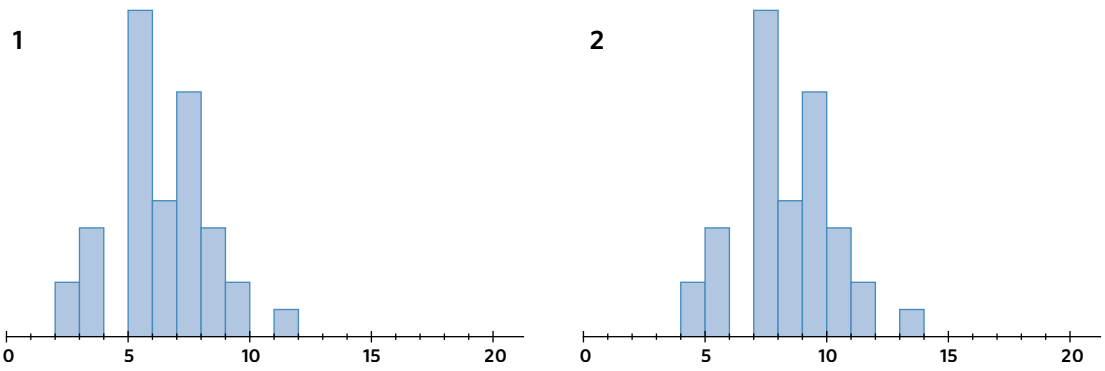
13	3 9
14	0 2 4 5 7 9
15	0 1 4 6 8
16	2 6 7
17	1 6 9
18	5 8
19	
20	4
21	1

2. Location and spread of distributions

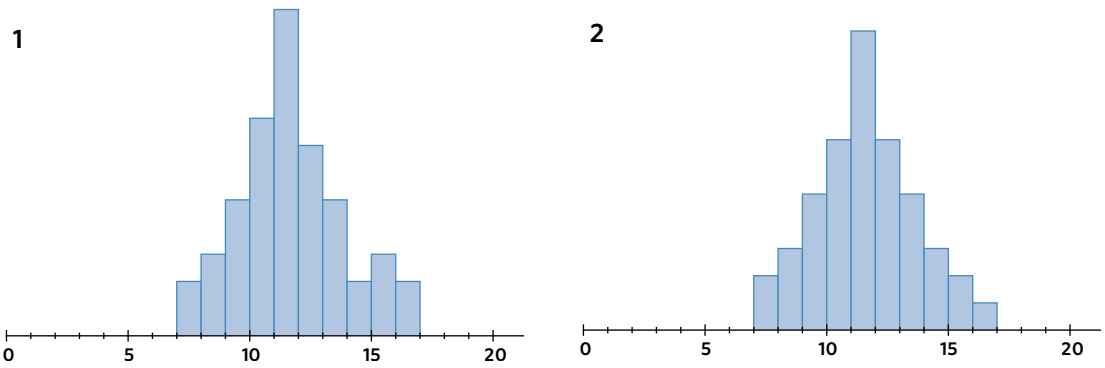
Q6. Compare the locations of the following pairs of histograms.

Skill

a)



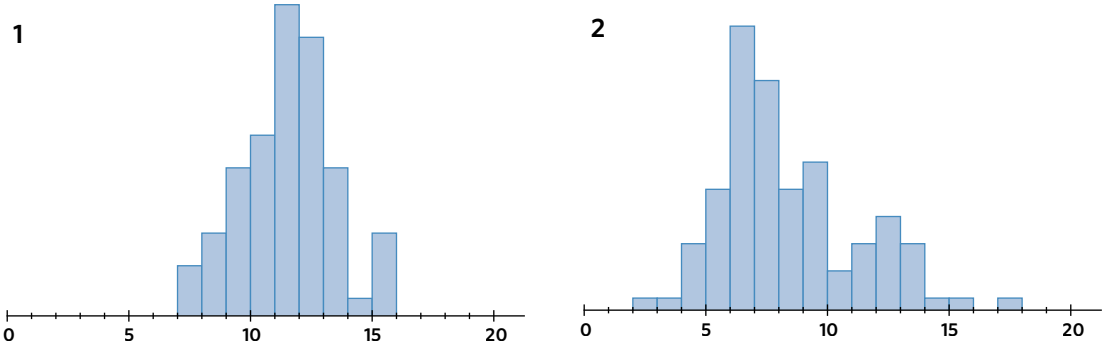
b)



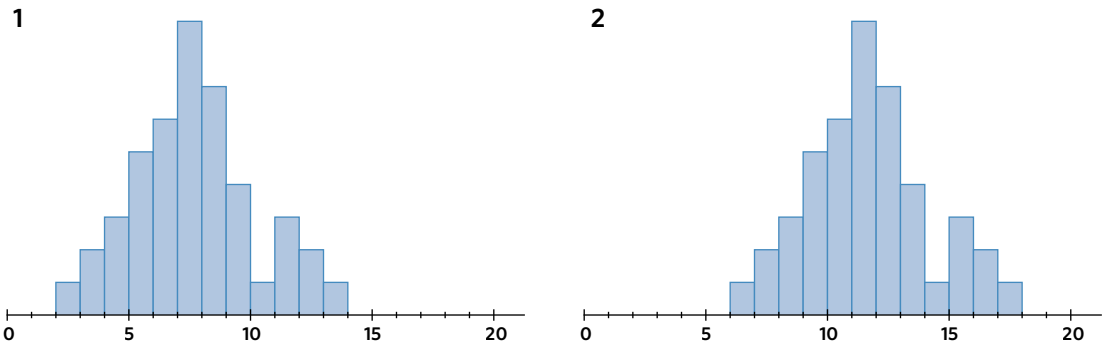
Q7. Compare the spread of the following pairs of histograms.

Skill

a)



b)

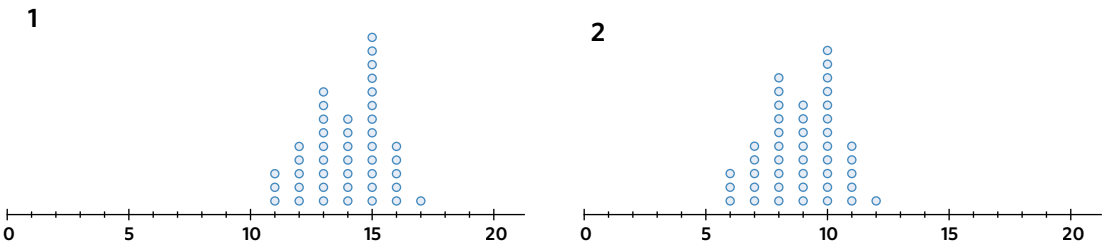


Check your understanding

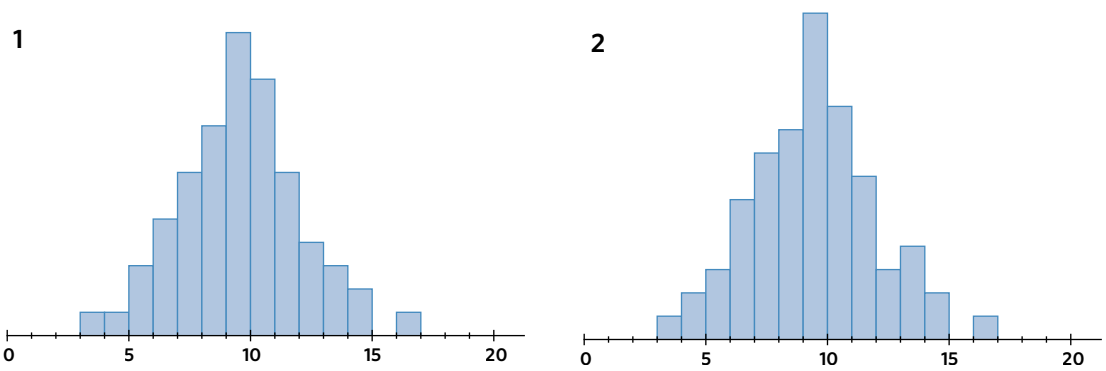
Q8. Do the following pairs of distributions differ in spread, location, both, or neither?

Skill

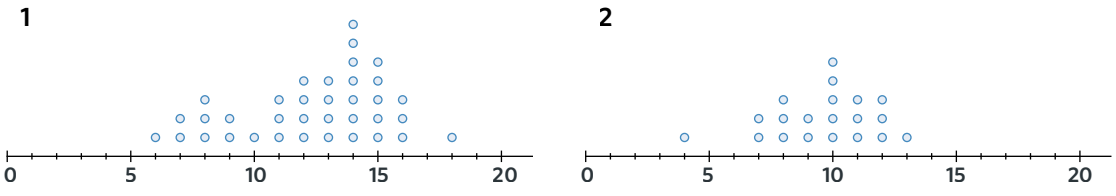
a)



b)



c)



d) **1**

17	8
18	3 6 9
19	
20	6
21	4 4 5 9
22	1 8
23	1 4 6 7 8 9
24	0 6 9
25	2 7

2

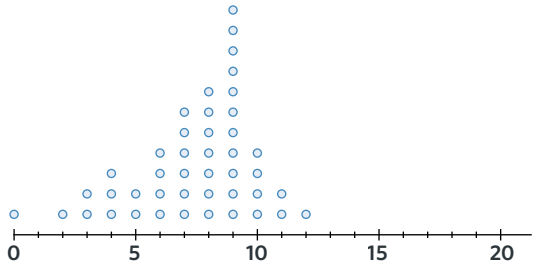
30	8
31	3 6 9
32	
33	6
34	4 4 5 9
35	1 8
36	1 4 6 7 8 9
37	0 6 9
38	2 7

Joining it all together

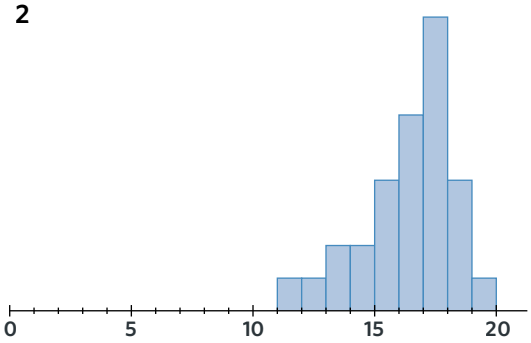
Q9. Compare the following pairs of distributions by referencing their shape, location and spread.

Skill
8 marks

a) **1**



2

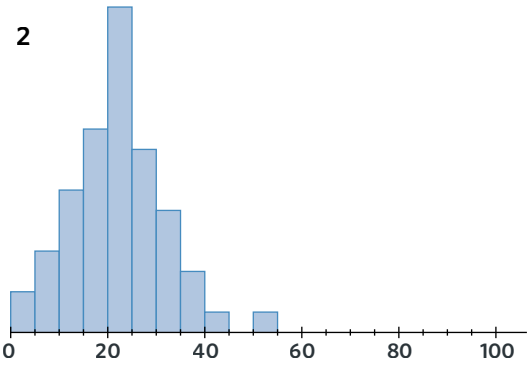


b)

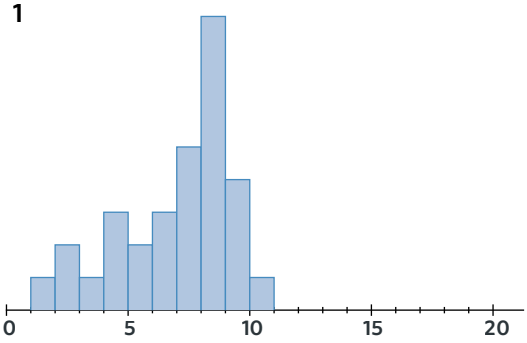
1

1	9
2	2 3 4 4
2	5 6 6 7 8 8 9
3	0 1 2 2 3 4 4 4
3	5 5 6 6 6 7 7 8 9 9
4	0 0 0 1 1 3 4
4	5 5 6 8 9
5	0 3 3
5	6

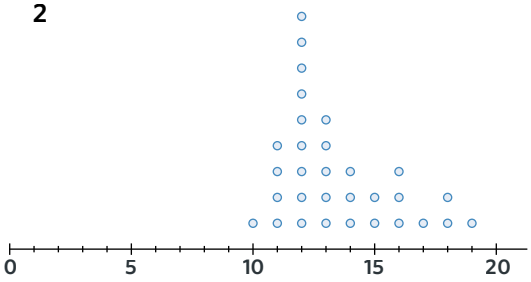
2



c) **1**



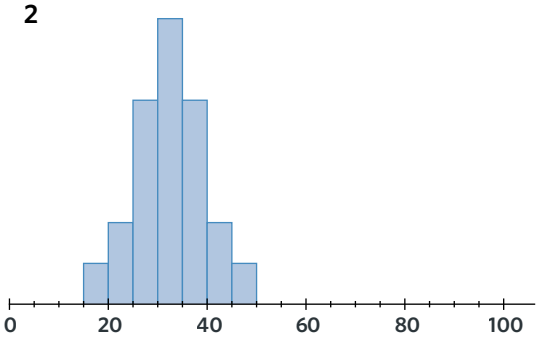
2



d) **1**

2	4
2	5 6 7 9
3	0 0 1 1 2 3 3 3 4 4
3	6 6 6 7 8 8 9 9
4	0 1 1 2 2 3 4
4	6 7 7 8 9 9
5	1 2 2 3 4
5	5 6 8
6	0 4
6	6

2

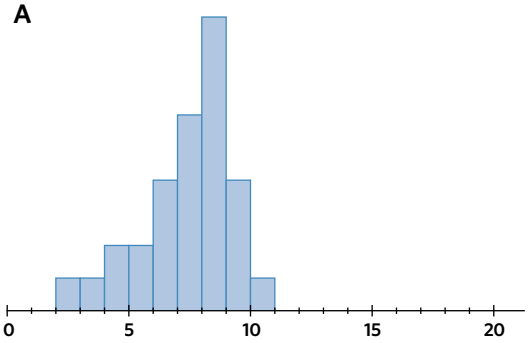


Q10. Fill out the following table referencing distributions A–F.

Skill
4 marks

Description	Distribution
Largest spread	
Largest centre value	
Skewed (positively or negatively)	
Symmetrical (perfectly or approximately)	

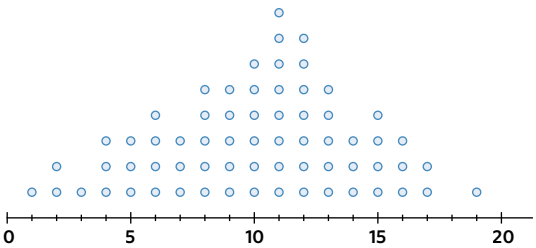
A



B

5	1
6	2 6 9
7	4 5 7 8
8	3 7
9	0

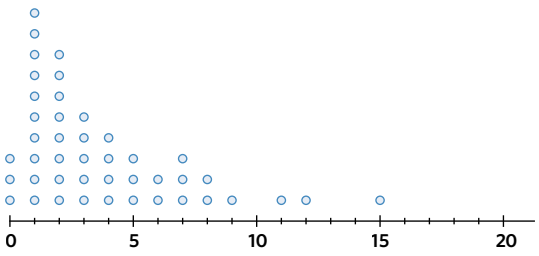
C



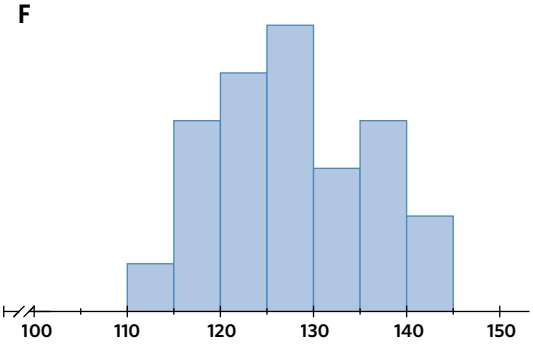
D

8	3
8	
9	1
9	6 8
10	0 4
10	6 7 8
11	1 2 2 3 4
11	5 7

E



F



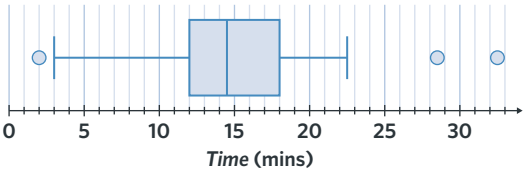
Questions from multiple lessons

Q11.

Difficulty:

1 mark

The *time*, in minutes, taken for students to get from school to the local fish and chips shop was recorded and is displayed in the following boxplot.



The five-number summary for the *time* taken to get to to the fish and chips shop is

A. 2, 11, 14.5, 18, 22.5

B. 2, 12, 14.5, 18, 32.5

C. 3, 11, 14.5, 18, 22.5

D. 3, 12, 14.5, 18, 22.5

E. 3, 12, 15, 18, 22.5

VCAA 2017 Exam 1 Data analysis Q2 – Adapted

Q12.

Difficulty:

1 mark

Gonzalo is saving up for a trip to Machu Picchu. He invests $4000 in a savings account with an interest rate of 4.4% per annum, compounding quarterly. Correct to the nearest cent, how much interest will Gonzalo earn in three years?

A. $528.00

B. $551.57

C. $557.91

D. $561.14

E. $563.33

VCAA 2015 Exam 1 Module 4: Business-related mathematics Q4 – Adapted

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Q13.

Difficulty:

3 marks

A group of 20 tourists visiting the Sydney Opera House were asked which country they were from. The results are displayed in the following frequency table.

Country	Frequency
New Zealand	3
China	8
USA	5
UK	2
Japan	x

a) What is the value of x in the frequency table above? (1 mark)

b) Create a bar chart using the data from the frequency table. (2 marks)

LESSON 12E

Median and mean

The key skills you will learn in this lesson are:

1. Finding the mean and median
2. Finding the mean and median from tables and charts
3. Identifying the most appropriate measure of centre

VCAA key knowledge point:

"measures of centre and spread and their use in summarising numerical data distributions, including use of and calculation of the sample summary statistics, median, mean, range, interquartile range (IQR) and standard deviation; and choosing between the measures of centre and spread"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016–2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

The mean and median are both known as measures of centre, because they help determine the central position of the data set. They give an indication of a typical value from the data set.

1. Finding the mean and median

Mean

The mean is one way to measure the centre of a data set. The numbers in a data set are known as **elements**.

The mean is calculated using the formula:

$$Mean = \frac{The\ sum\ of\ all\ elements\ in\ the\ data\ set}{The\ total\ number\ of\ elements\ in\ the\ data\ set}$$

The mean can also be written as:

$\bar{x} = \frac{\sum x}{n}$, where:

- $\bar{}$ represents the mean of the data set.
- x represents each element in the data set.
- $\sum$ (Greek letter sigma) means 'the sum of'.
- n is the number of elements in the data set.

Example

Find the mean of the data set {3, 5, 4} using both formulae.

First formula	Second formula
$Mean = \frac{3+5+4}{3} = \frac{12}{3} = 4$	$\sum x = 3+5+4 = 12$, $n = 3$ so $\bar{} = \frac{12}{3} = 4$

Therefore, the mean is 4 for this data set.

Median

The median is the **middle** value of a data set. To find the median, the data set is first rearranged into ascending order, from smallest to largest, then one of the following formulas is used:

- If n is **odd**, the median is the located at the $\left(\frac{n+1}{2}\right)^{th}$ position
- If n is **even**, the median is the average of the two values either side of the $\left(\frac{n+1}{2}\right)^{th}$ position.
  - For example, if there are 20 values in a data set, $\left(\frac{n+1}{2}\right) = 10.5$
  - On either side of 10.5 we have the 10^{th} and 11^{th} values.
  - The median would be the average of these two values.

NOTE: n is the number of elements in the data set.

WORKED EXAMPLE 12 *(2 marks)*

For the data set {43, 252, 465, 859, 245, 492, 453, 48, 239, 4}, calculate:

a) the mean.

b) the median.

SOLUTION

a) For the data set {43, 252, 465, 859, 245, 492, 453, 48, 239, 4}, calculate the mean.

Step 1 To calculate the mean, firstly calculate the sum of the elements in the data set.

$$\sum x = 43 + 252 + 465 + 859 + 245 + 492 + 453 + 48 + 239 + 4 = 3100$$

Step 2 Count the total number of elements in the data set.

There are 10 elements in this data set.

$n = 10$

Step 3 Use the formula $\bar{} = \frac{\sum x}{n}$

This divides the sum by the total number of elements in the data set.

$$\bar{} = \frac{\sum x}{n} = \frac{3100}{10} = 310$$

Therefore, the mean is 310.

b) For the data set {43, 252, 465, 859, 245, 492, 453, 48, 239, 4}, calculate the median.

Step 1 To calculate the median, firstly rearrange the data set from smallest to largest.

{4, 43, 48, 239, 245, 252, 453, 465, 492, 859}

Step 2 Identify whether the data set has an even or odd number of elements.

There are 10 elements, so there are an even number of elements. As there are an even number of elements, we must find the average of the $\frac{n}{2}^{th}$ and $\frac{n}{2} + 1^{th}$ elements.

Step 3 Use the formula $\frac{n}{2}$ and $\frac{n}{2} + 1$ to find the position of the two desired values.

$n = 10$

$$\frac{n}{2} = \frac{10}{2} = 5$$

$$\frac{n}{2} + 1 = \frac{10}{2} + 1 = 5 + 1 = 6$$

Therefore, the values we need are located at the 5^{th} and 6^{th} positions.

Step 4 Find the average of these two values.

The 5^{th} and 6^{th} positions of the data set are shown:

{4, 43, 48, 239, **245**, **252**, 453, 465, 492, 859}

$$\text{Average} = \frac{245 + 252}{2} = 248.5$$

Therefore, the median is 248.5.

2. Finding the mean and median from tables and charts

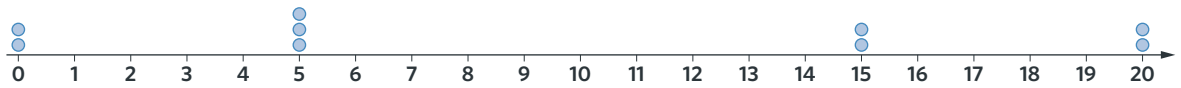
For different tables and charts, the mean and median can be calculated either by listing the data or writing the data in a frequency table.

Listing the data

For dot plots and stem-and-leaf plots, listing the elements in the data set will help in making calculations of mean and median

Dot plots

Example



This dot plot represents the data set: {0, 0, 5, 5, 5, 15, 15, 20, 20}.

Stem-and-leaf plots

Example

Key: 5 | 1 = 5.1

1	2 5
2	3
3	6
4	5
5	1 4

This stem-and-leaf plot represents the data set: {1.2, 1.5, 2.3, 3.6, 4.5, 5.1, 5.4}.

Frequency tables

Mean

The mean can be calculated from a frequency table. To do this, the frequency table has an extra column added, column 'xf', where the values from column x are multiplied with the values from the 'Frequency f' column.

Then this formula is used:

$$\bar{x} = \frac{\sum xf}{\sum f}$$

- $\bar{x}$ is the mean of the data set.
- $\sum f$ is the sum of the frequency column.
- $\sum xf$ is the sum of the xf column.

Example

Find the mean of this frequency table.

x	Frequency f	xf
4	**3**	$4 \times 3 = \mathbf{12}$
5	**6**	$5 \times 6 = \mathbf{30}$
12	**10**	$12 \times 10 = \mathbf{120}$
16	**9**	$16 \times 9 = \mathbf{144}$

$\sum f = 3 + 6 + 10 + 9 = 28$

$\sum xf = 12 + 30 + 120 + 144 = 306$

$$\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{306}{28} = 10.9$$

Median

To find the median from a frequency table, a cumulative frequency column is added to the frequency table. To calculate values in this column, add the frequency value of the current row to the cumulative frequency value of the row above.

The following formula is then applied to find the position of the median within the table.

$$\text{Position of median} = \frac{n + 1}{2}$$

- n is the sum of the frequency column
- If n is **odd**, the median is the located at the $\left(\frac{n + 1}{2}\right)^{\text{th}}$ position
- If n is even, the median is the average of the two values either side of the $\left(\frac{n + 1}{2}\right)^{\text{th}}$ position

Example

Find the median of this frequency table.

x	Frequency f	Cumulative frequency cf
4	3	3
6	6	3 + 6 = 9
12	10	9 + 10 = 19
16	9	19 + 9 = 28

$n = \mathbf{28}$

Position of median $= \frac{\mathbf{28} + 1}{2} = \frac{29}{2} = 14.5$

The median will be the average of the values either side of position 14.5. That is, the 14th and 15th values.

Using the cumulative frequency, we can look for these values:

- First row: contains the 1st to 3rd values – so the median is not in the first row.
- Second row: contains the 4th to 9th values – so the median is not in the second row.
- Third row: contains 10th to 19th values – both the 14th and 15th values are from the third row, so both values are 12.

Therefore, the median is 12.

Interval frequency table

An interval frequency table is used to find mean and median of histograms.

For an interval frequency table, an additional midpoint column is added. The midpoint refers to the value that is exactly halfway between each interval.

To find the midpoint:

$$Midpoint = \frac{lower\ bound + upper\ bound}{2}$$

This midpoint column takes the place of the x data points in the calculation of xf

Example

Find the mean of this frequency table.

Interval	Midpoint x	Frequency f	xf
0–<5	2.5	**3**	$2.5 \times 3 = 7.5$
5–<10	7.5	**4**	$7.5 \times 4 = 30$
10–<15	12.5	**12**	$12.5 \times 12 = 150$

$\mathbf{\sum f = 3 + 4 + 12 = 19}$

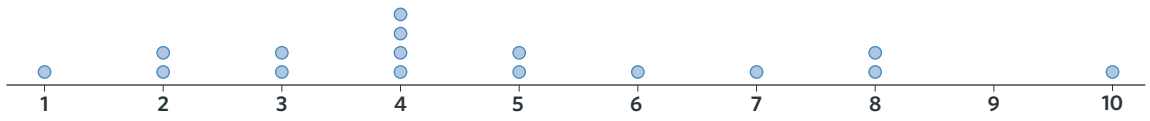
$\mathbf{\sum xf = 7.5 + 30 + 150 = 187.5}$

$x = \frac{\mathbf{\Sigma xf}}{\mathbf{\Sigma f}} = \frac{\mathbf{187.5}}{\mathbf{19}} = 9.87$

The mean is 9.87.

WORKED EXAMPLE 13 *(1 mark)*

Find the mean for the following dot plot.



SOLUTION

Step 1 Write the individual data points from the dot plot to complete a data set.

{1, 2, 2, 3, 3, 4, 4, 4, 4, 5, 5, 6, 7, 8, 8, 10}.

Step 2 Add all of the values in the data set.

$$\sum x = 1 + 2 + 2 + 3 + 3 + 4 + 4 + 4 + 4 + 5 + 5 + 6 + 7 + 8 + 8 + 10 = 76$$

Step 3 Find the number of elements in the data set.

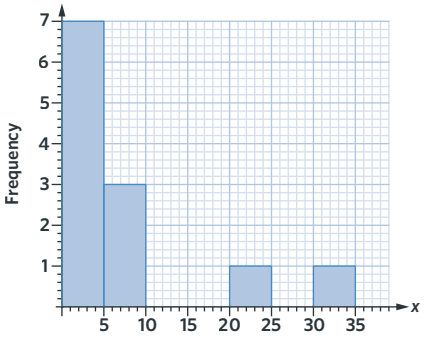
There are 16 elements in the data set, so $n = 16$.

Step 4 Substitute these values into the formula $\bar{} = \frac{\sum x}{n}$ to find the mean.

$$\bar{} = \frac{76}{16} = 4.75$$

WORKED EXAMPLE 14 *(2 marks)*

Calculate the mean for this histogram.



SOLUTION

Step 1 Convert the histogram to an interval frequency table.

Looking at the histogram, the first interval (0–<5) has a frequency of 7, the second interval (5–<10) has a frequency of 3, and so on. The frequency table can be constructed using these values. Leave the second and fourth columns empty.

Interval		Frequency f	
0–<5		7	
5–<10		3	
20–<25		1	
30–<35		1	

Step 2 Because this is an interval table, the second column becomes a midpoint column. These values are found by averaging the bounds:

First row: $\frac{0 + 5}{2} = 2.5$

Second row: $\frac{5 + 10}{2} = 7.5$, and so on.

Interval	Midpoint x	Frequency f	
0–<5	2.5	7	
5–<10	7.5	3	
20–<25	22.5	1	
30–<35	32.5	1	

Step 3 Multiply the midpoint column x with the frequency column f to produce corresponding xf values.

Interval	Midpoint x	Frequency f	xf
0–<5	2.5	7	$2.5 \times 7 = 17.5$
5–<10	7.5	3	$7.5 \times 3 = 22.5$
20–<25	22.5	1	$22.5 \times 1 = 22.5$
30–<35	32.5	1	$32.5 \times 1 = 32.5$

✓ *1 mark for completing the table*

Step 4 Sum the totals for both the frequency f column and the xf column.

$$\sum f = 7 + 3 + 1 + 1 = 12$$

$$\sum xf = 17.5 + 22.5 + 22.5 + 32.5 = 95$$

Step 5 Apply the formula $\bar{} = \frac{\sum xf}{\sum f}$ by substituting in the answers from the previous step.

$$\bar{} = \frac{\sum xf}{\sum f} = \frac{95}{12} = 7.92$$

Therefore, the mean of the above histogram is 7.92.

✓ *1 mark for finding the mean*

3. Identifying the most appropriate measure of centre

To decide which measure of centre is most appropriate for different data sets, it is important to understand the difference between the properties of the median and mean.

Outliers

The mean

The mean is greatly affected by extreme values that are unlike the rest of the data set. These extreme values are called outliers.

A large outlier will increase the value of the mean.

A small outlier will decrease the value of the mean.

Example

Consider the data set {3, 5, 4}.

- The mean of this data set is 4.

Compare this to the data set {3, 5, 4, 200}.

- 200 is an outlier, a value significantly larger than the rest of the data set.
- The mean of this data set is: $Mean = \frac{3 + 5 + 4 + 200}{4} = 53$.

The mean has increased considerably from 4 to 53 with the addition of the outlier 200.

The median

The median is affected minimally by an outlier. This is because the median refers to the middle value, and therefore does not include the numerical value of the outlier.

Example

Consider the same data set, {3, 5, 4}.

- The median of this data set is 4.

Compare this to {3, 5, 4, 200}.

- Ordered data set {3, 4, 5, 200}
- The central value is shared by 4 and 5.
- $Median = \frac{4 + 5}{2} = 4.5$

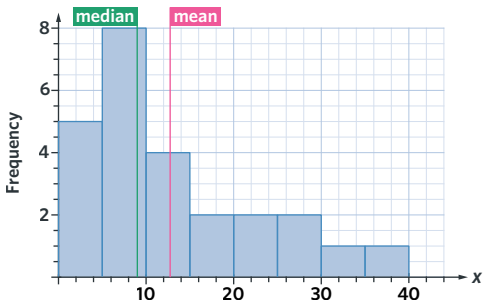
The outlier has only slightly changed the median. Hence, the median is a more appropriate measure of centre when there are outliers present.

Skewed data

The distribution of a data set is said to be skewed when the data set is not symmetrical around the mean, and the curve appears distorted to the left or right. The median is the more appropriate measure of centre than the mean when representing skewed data.

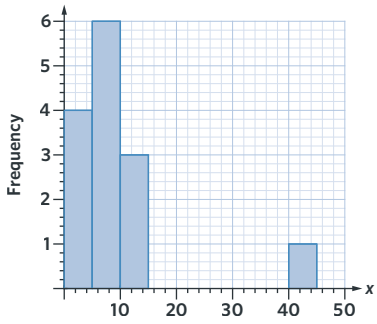
Example

For this graph the mean is 12.6 and the median is 9. In this case the median is a better measure of centre. This can been seen by looking at how the median lines up with the tallest part of the histogram.



WORKED EXAMPLE 15 *(1 mark)*

Which measure of centre is the most appropriate for the following histogram?



SOLUTION

Step 1 Does the data contain an outlier or is it skewed?

Yes, there is an outlier at the 40–<45 interval.

Step 2 Recall the properties of the mean and median.

The mean is affected by an outlier.

Step 3 Make a conclusion and decide which measure of centre is the most appropriate for the chosen data.

The median is the most appropriate measure of centre, since the mean will be affected by the outlier.

Questions 12E *Median and mean*

Refresher question

Q1. How many elements are in the following data set?

{4, 5, 324, 234, 5, 5, 2, 6, 53, 53}

1. Finding the median and mean

Q2. Skill

Find the average of 493 and 194, correct to one decimal place.

Q3. Skill

Find the median of the following data set.

{65, 90, 120}

Q4. Application

a) Abby recently received her exam results for Semester 1. Her results are shown below.

Mathematics	English	Geography	History	Chemistry
66%	95%	40%	79%	10%

Calculate her mean score for the Semester 1 exam period.

b) Betty also received her exam results for Semester 1. Her results are shown below:

Mathematics	English	Geography	History	Chemistry
34%	20%	88%	51%	86%

Calculate her median score for the Semester 1 exam period.

c) Betty asked to review her exam scores, and found a mistake. Instead of 34%, she actually scored 100% in Mathematics.

Find her new mean and median scores after this amazing achievement.

Check your understanding

Q5. Application

The mean price of the new Pineapple Watch across 3 different suppliers is $543.
Use this information and the table below to calculate the price at which Supplier *X* sells this watch.

Supplier *X*	Supplier *Y*	Supplier *Z*
?	$694	$345

Q6. Application

The data set for the weight in kilograms of a group of puppies is shown below.

{10.4, 5.6, 6.5, 8.0, 5.3, 3.2, 7.9}

Show that the mean weight for this group of puppies is higher than their median weight.

2. Finding the mean and median from tables and charts

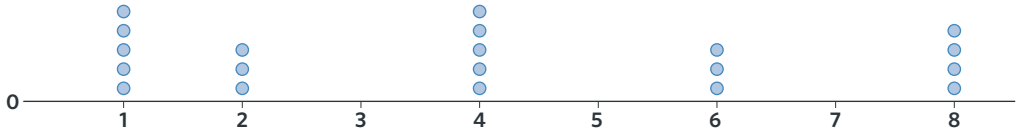
Q7. Skill

Construct a frequency table for the following data, and without doing any calculations, write how you would find the mean from the table, step-by-step.

{5, 4, 5, 5, 5, 7, 8, 8, 9, 10}

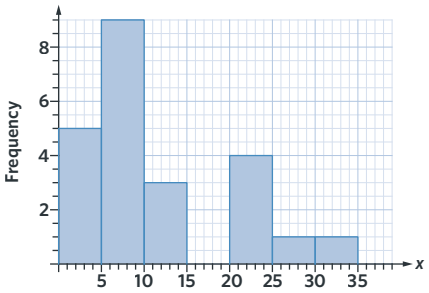
Q8. Skill

Find the median value from the dot plot below.



Q9. Skill

Find the mean value of the histogram below, correct to 2 decimal places.



Q10. Skill

Find the mean and median values from the stem-and-leaf plot below, correct to 2 decimal places.

Key: 1 | 3 = 1.3

1	3 7 8
2	1 5
3	5 7
4	3 4 5
5	4 6
6	1 5
7	2 5 7
8	9
9	3

Check your understanding

Q11. Which of these two frequency tables has the higher mean?

Skill

x	Frequency f
4	11
5	9
9	9
15	1

Interval	Frequency f
0–<5	7
5–<10	15
10–<15	6
15–<20	2

Q12. Players of a claw game at a carnival try to grab as many chocolate bars as possible using the claw, within five minutes. The number of chocolate bars grabbed by players is displayed on a frequency table below.

Application

Re-create this table with:

- a midpoint column
- a cumulative frequency column.

Use the new table to show that the median number of chocolate bars grabbed was five.

Interval	Frequency f
0–<2	6
2–<4	3
4–<6	8
6–<8	2
8–<10	1

3. Identifying the most appropriate measure of centre

Q13. Which of the following statements is true?

Skill

A. The mean is always smaller than the median when the distribution is skewed.

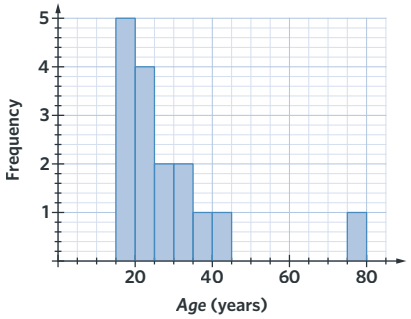
B. The mean of a dot plot can be found, but not the median.

C. Outliers can have a bigger effect on the mean than the median.

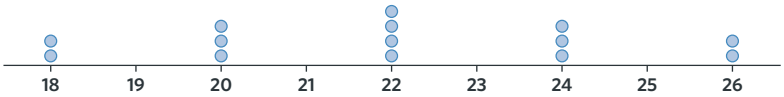
D. In a histogram, the mean is always larger than the median.

Q14. **a)** Study the following histogram, which shows the different ages for a group of 27 L-plate drivers. Which measure of centre represents this data set best and why?

Application



b) Study this dot plot for a group of 14 P-plate drivers. Explain why both measures of centre could be appropriate for this data set.



c) Clarke, a new driver, had her speed recorded below. During her latest driving session she was in a rush and accidentally drove at 170 km/h. Inspect the following data and make a conclusion on the measure of centre that would be affected the most and why?

{50, 40, 30, 70, 60, 170}

Check your understanding

Q15. Skill

The data set below is given as part of a Year 11 General Maths test.

{169, 4, 6, 7, 8, 11, 3, 5, 6, 14}

One student, Finn, believes that the median is the most appropriate measure of centre. Is he correct? Explain your answer.

Q16. Application

The amount of money Derek earns from his part-time job on a monthly basis is as follows.
Which measure of centre is the most **inappropriate** choice for this data set, if there is an inappropriate choice at all? Explain your answer.

{300, 300, 300, 300, 300, 300, 300, 300, 300, 300, 300, 300}

Joining it all together

Q17. Application 4 marks

Leon is performing a chemistry experiment with five test tubes, labelled *A* to *E*.

a) The pH value of each chemical is shown in the table below. Show that the mean pH level is 6.8 and that it is lower than the median. (2 marks)

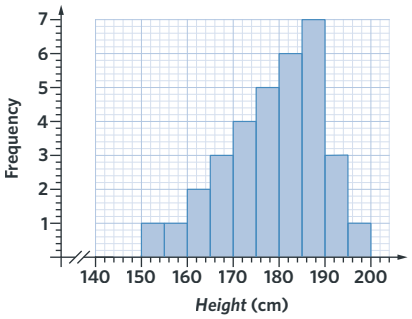
Test tube	*A*	*B*	*C*	*D*	*E*
pH	3	4	7	9	11

b) Leon accidentally poured too much acid into test tube *F*. Test tube *F* now has a pH of 2. To increase the mean overall pH of the test tubes to 7, what pH should test tube *G* be? (2 marks)

Test tube	*A*	*B*	*C*	*D*	*E*	*F*	*G*
pH	3	4	7	9	11	2	?

Q18. The heights of 33 students in a Year 11 cohort (in centimetres) is recorded in a histogram, shown below.

Application
6 marks



a) Convert the histogram to a frequency table and find the mean height of the students, correct to 2 decimal places. (2 marks)

b) Which measure of centre is the most appropriate for this data set and why? (2 marks)

c) Without doing any calculations, what would be a possible value of the median so that the distribution becomes symmetrical? Explain why you chose this value. (2 marks)

VCAA question

Q19. The following stem plot shows the areas in square kilometres, of 27 suburbs of a large city.

1 mark

Key: 1 | 6 = 1.6 km^2

1	5 6 7 8
2	1 2 3 4 6 8 9 9
3	0 1 1 2 2 8 9
4	0 4 7
5	0 1
6	1 9
7	
8	4

The median area of these suburbs, in square kilometres, is:

A. 3.0

B. 3.1

C. 3.5

D. 30.0

E. 30.5

VCAA 2014 Exam 1, Core: Data Analysis. Q1

Questions from multiple lessons

Q20. Difficulty:

1 mark

The following two-way frequency table displays the *preferred beverage* (coffee, tea, juice) and *age* (under 40 years, 40 years or over) of 83 people.

		Age	
		Under 40 years	**40 years or over**
Preferred beverage	**Coffee**	18	21
	Tea	9	16
	Juice	14	5
Total		41	42

The percentage of people under 40 years old who chose tea as their preferred beverage is closest to

A. 32% **B.** 21% **C.** 78%

D. 22% **E.** 11%

VCAA 2016 Exam 1 Data analysis Q1 – Adapted

Q21. Difficulty:

1 mark

A sequence is defined by the following recurrence relation.

$T_{n+1} = -3T_n \quad T_0 = -\frac{1}{3}$

Which term in the sequence is the first to be greater than 10?

A. T_1 **B.** T_2 **C.** T_3

D. T_4 **E.** T_5

VCAA 2018NH Exam 1 Recursion and financial modelling Q18 – Adapted

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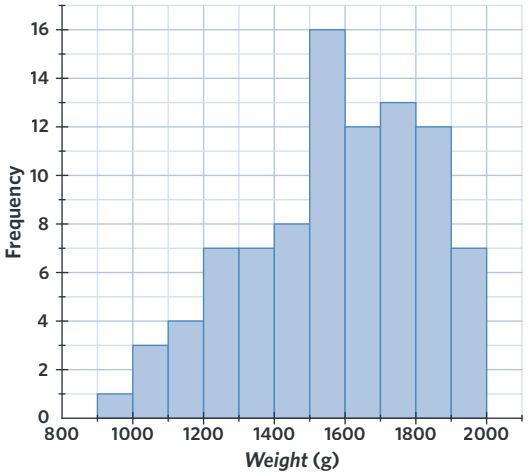
Q22. Difficulty:

Year 10
3 marks

The histogram below shows the distribution of the weight, in grams, of potatoes purchased on a particular day at a supermarket in Ireland. There were 90 customers that day.



a) Describe the shape of the histogram. (1 mark)

b) Determine the number of customers that purchased 1800 g or more of potatoes. (1 mark)

c) Determine the percentage of customers that purchased between 1000 g and 1400 g of potatoes. Round your answer to one decimal place. (1 mark)

VCAA 2007 Exam 2 Data analysis Q1 – Adapted

LESSON 12F

Range, IQR and standard deviation

The key skills you will learn in this lesson are:

1. Finding the range
2. Finding the quartiles
3. Finding the interquartile range (IQR)
4. Finding the standard deviation

VCAA key knowledge point:

"measures of centre and spread and their use in summarising numerical data distributions, including use of and calculation of the sample summary statistics, median, mean, range, interquartile range (IQR) and standard deviation; and choosing between the measures of centre and spread"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

The range, interquartile range (IQR) and standard deviation are measures of spread. They are used to find out how "spread out" the values in a data set are.

1. Finding the range

Finding the range is the simplest way to measure the spread of a data set. The range of a data set is the difference between the largest number and the smallest number in the data set.

Range = Largest number – Smallest number

WORKED EXAMPLE 16 *(1 mark)*

Find the range of the data set shown below.

{4, 55, 6, 77, 87, 34, 56, 23, 11, 23, 4, 6, 94, 23, 123, –3, –6, –2}

SOLUTION

Step 1 Find the largest number in the data set.

The largest number is 123.

Step 2 Find the smallest number in the data set.

The smallest number is −6.

Step 3 Subtract the smallest number from the largest number.

$123 - (-6) = 129$

Therefore, the range of the data set is 129.

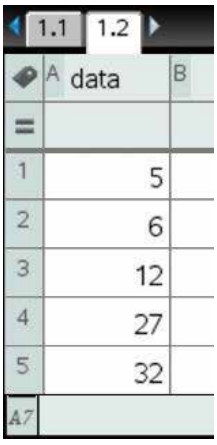
WORKED EXAMPLE 17 *(1 mark)*

Find the largest and smallest data values of the data set {5, 6, 12, 27, 32}.

SOLUTION: TI-NSPIRE

Step 1 Open a new spreadsheet page by pressing ctrl + doc and press 4 ('Lists and Spreadsheets').

Step 2 Enter the data set into a list under the A column. Give the column a name in box containing A. Here we call it 'data'.



Step 3 Press menu then 4 ('Statistics') then 1 ('Stat Calculations') then 1 ('One Variable Statistics...'). Press enter to set 'Num of lists' to 1.

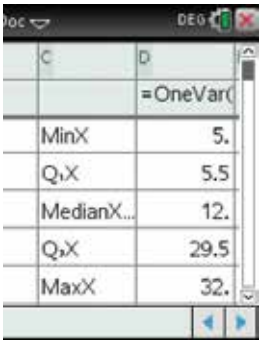
Step 4 In the X1 list, use ▶ to select the title of the column you created previously. In this case, it will be 'data'.

Step 5 Press enter to see the summary statistics for the data set. Use the up and down arrows to see the full list of results.

Step 6 Scroll down and read off the maximum and minimum values.

Minimum is MinX, so 5.

Maximum is MaxX, so 32.



SOLUTION: CASIO CLASSPAD

Step 1 From the menu, select Statistics.

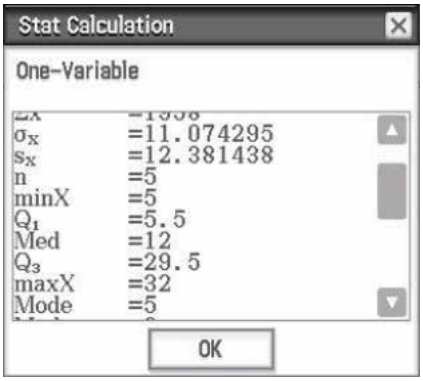
Step 2 Type the values in column A.

Step 3 Tap 'Calc', then 'One-Variable'. Check that 'XList:' is 'list1' and 'Freq:' is '1'.

Tap 'OK'.

Scroll down. The smallest data value is given by 'minX' and the largest data value is given by 'maxX'.

Here minX is 5 and maxX is 32.



2. Finding the quartiles

Quartiles

A more precise way to find the spread of a data set is to divide it using the interquartile range (IQR). Before this can be done, the data needs to be split into quartiles.

Looking at the data placed in ascending order:

The first quartile (Q_1) is the value 25% of the way along the rearranged data set.

The second quartile (Q_2) is the median value, 50% of the way along the rearranged data set.

The third quartile (Q_3) is the value 75% of the way along the rearranged data set.

To find the quartiles:

Step 1 Arrange the data into ascending order – from smallest to largest.

Step 2 Find Q_2 by finding the median of the data set.

Step 3 Split the data in half.

Step 4 Find Q_1 by finding the median of the first half of the data.

Step 5 Find Q_3 by finding the median of the second half of the data.

WORKED EXAMPLE 18 *(1 mark)*

Find Q_1 Q_2, and Q_3 of the following data set.

{3, 50, 34, 2, 34, 21, 4, 5, 7, 4, 3, 12, 4, 8, 10}

SOLUTION

Step 1 Rearrange the data set from smallest to largest.

{2, 3, 3, 4, 4, 4, 5, 7, 8, 10, 12, 21, 34, 34, 50}

Step 2 Calculate the median (Q_2) of the data set. This was covered last lesson.

Position of median $\left(\frac{n+1}{2}\right) = \frac{15+1}{2}$

$= 8$th element

{2, 3, 3, 4, 4, 4, 5, 7, 8, 10, 12, 21, 34, 34, 50}

Therefore, Q_2 is 7.

Step 3 For a data set with an odd number of elements, we split the data into halves at the median, removing the median value.

split

{2, 3, 3, 4, 4, 4, 5 | 8, 10, 12, 21, 34, 34, 50}

This becomes {2, 3, 3, 4, 4, 4, 5} and {8, 10, 12, 21, 34, 34, 50}.

Step 4 Find the median for the first half of the data set. This the first quartile, Q_1:

Position of median $\left(\frac{n+1}{2}\right) = \frac{7+1}{2}$

$= 4$th element

{2, 3, 3, 4, 4, 4, 5}

Therefore, Q_1 is 4.

Step 5 Find the median for the second half of the data set. This is the third quartile, Q_3:

Position of median $\left(\frac{n+1}{2}\right) = \frac{7+1}{2}$

$= 4$th element

{8, 10, 12, 21, 34, 34, 50}

Therefore, Q_3 is 21.

In some cases, there are an even number of elements in the original data set. When this happens, the data will be split differently into the first and second halves.

WORKED EXAMPLE 19 *(1 mark)*

Find Q_1, Q_2, and Q_3 of the following data set.

{2, 5, 6, 8, 8, 10, 12, 14, 15, 16}

SOLUTION

Step 1 Calculate the median (Q_2) of the data set, remembering to take the average of the values that share the centre.

Position of median $\left(\frac{n+1}{2}\right) = \frac{10+1}{2}$

$= 5.5$th element

{2, 5, 6, 8, **8**, **10**, 12, 14, 15, 16}

The 5th element is 8 and the 6th element is 10.

Average $= \frac{8+10}{2} = 9$

Therefore, Q_2 is 9.

Step 2 For a data set with an even number of elements, split the data set into two halves, between the two values that share the centre.

The two values that share the centre are 8 and 10.

split

{2, 5, 6, 8, **8**, | **10**, 12, 14, 15, 16}

This becomes {2, 5, 6, 8, 8} and {10, 12, 14, 15, 16}.

Step 3 Find the median for the first half of the data set. This the first quartile, Q_1:

Position of median $\left(\frac{n+1}{2}\right) = \frac{5+1}{2}$

$= 3$rd element

{2, 5, 6, 8, 8}

Therefore, $Q_1 = 6$.

Step 4 Find the median for the second half of the data set. This the third quartile, Q_3:

Position of median $\left(\frac{n+1}{2}\right) = \frac{5+1}{2}$

$= 3$rd element

{10, 12, 14, 15, 16}

Therefore, $Q_3 = 14$.

WORKED EXAMPLE 20 *(1 mark)*

Find Q_1, Q_2, and Q_3 of the following data set: {5, 6, 12, 27, 32}.

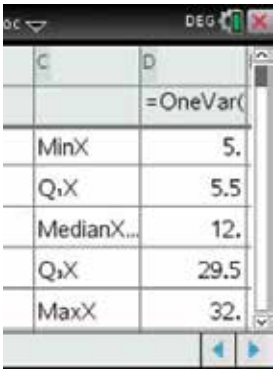
SOLUTION: TI-NSPIRE

Step 1 Using the same spreadsheet page from the previous example, find the values for Q_1 Q_2 and Q_3

Q_1 is 'Q_1X' on the calculator, so 5.5.

Q_2 is 'MedianX...' on the calculator, so 12.

Q_3 is 'Q_3X' on the calculator, so 29.5.



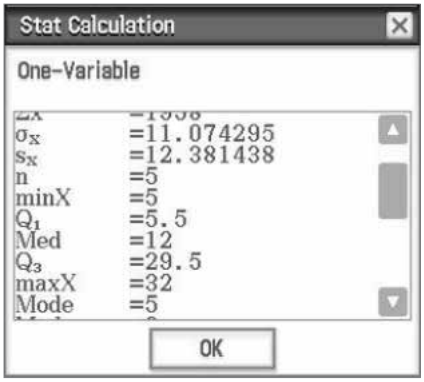
SOLUTION: CASIO CLASSPAD

Step 1 Use the steps from the previous example to obtain the same list of statistics.

Q_1 is 'Q_1', so 5.5.

Q_2 is 'Med', so 12.

Q_3 is 'Q_3', so 29.5.

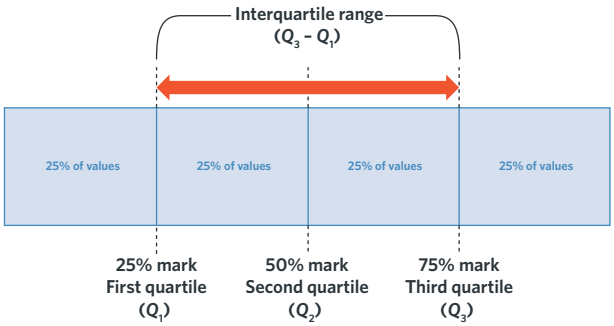


3. Finding the interquartile range (IQR)

The interquartile range is also referred to as the middle 50% of data values. This is because by subtracting the lower quartile (25%) from the upper quartile (75%), the middle 50% of data values are obtained.

To find the interquartile range (IQR), subtract the first quartile from the third quartile.

IQR = third quartile (Q_3) – first quartile (Q_1)



WORKED EXAMPLE 21 *(2 marks)*

Find the interquartile range of the following data sets.

a) {3, 50, 34, 2, 34, 21, 4, 5, 7, 4, 3, 12, 4, 8, 10}

b) {2, 5, 6, 8, 8, 10, 12, 14, 15, 16}

SOLUTION

a) {3, 50, 34, 2, 34, 21, 4, 5, 7, 4, 3, 12, 4, 8, 10}

Step 1 From the previous worked example we know that $Q_1 = 4$ and $Q_3 = 21$.

Step 2 Find the interquartile range (IQR)

IQR = third quartile (Q_3) – first quartile (Q_1)

IQR = 21 – 4

IQR = 17

The IQR is 17.

b) {2, 5, 6, 8, 8, 10, 12, 14, 15, 16}

Step 1 From the previous worked example we know that $Q_1 = 6$ and $Q_3 = 14$.

Step 2 Find the interquartile range (IQR)

IQR = third quartile (Q_3) – first quartile (Q_1)

IQR = 14 – 6

IQR = 8

The IQR is 8.

4. Finding the standard deviation

To find the standard deviation, the variance s^2 must be calculated first. The variance is a measure of how much values in the data set deviate from the mean.

The standard deviation is simply the square root of the variance, shown in the formula below.

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

s refers to the standard deviation

$\bar{x}$ is the sample mean – the mean of the data set.

n is the number of elements in the data set.

WORKED EXAMPLE 22 *(1 mark)*

Find the standard deviation of the dot plot below, to 2 decimal places.

SOLUTION

Step 1 Extract the data points from the dot plot by studying each x value and its corresponding frequency.

{1, 1, 1, 2, 3, 3, 4, 9}

Step 2 Find the mean of the data set.

$$\bar{x} = \frac{1 + 1 + 1 + 2 + 3 + 3 + 4 + 9}{8}$$

$$= 3$$

Step 3 Subtract the mean from every single value in the data set. Write the answers as a new data set.

{–2, –2, –2, –1, 0, 0, 1, 6}

Step 4 Square every value in the new data set.

{4, 4, 4, 1, 0, 0, 1, 36}

Step 5 Add these values together.

$4 + 4 + 4 + 1 + 0 + 0 + 1 + 36 = 50$

Step 6 Divide the result by $n - 1$, that is, the number of elements in the data set subtracted by 1. The answer is known as the variance.

There are 8 elements in the data set.

$n = 8 - 1 = 7$

$\frac{50}{7} = 7.143$

Step 7 Take the square root of the variance to find the standard deviation.

$s = \sqrt{7.143} = 2.67$ (to 2 decimal places)

Therefore, the standard deviation for the data set is 2.67.

For frequency tables, stem plots and dot plots, first extract the data points from the graph to form a data set, as shown in the worked example, and follow the same steps to find the standard deviation.

WORKED EXAMPLE 23 *(1 mark)*

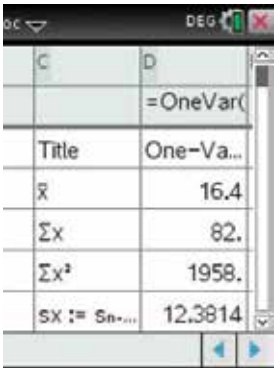
Calculate the standard deviation to two decimal places of the data set {5, 6, 12, 27, 32}.

SOLUTION: TI-NSPIRE

Step 1 Using the same spreadsheet page from the previous example, find the values for the standard deviation to two decimal places.

Standard deviation is $sx := s_{n-1}$ ... on the calculator, so 12.3814 ...

To two decimal places, this is 12.38.

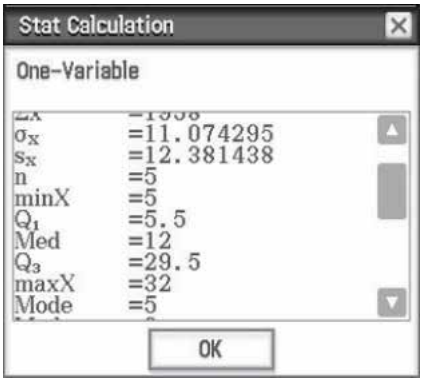


SOLUTION: CASIO CLASSPAD

Step 1 Use the steps from the previous examples to obtain the same list of statistics.

Standard deviation is s_x on the calculator, so 12.381438.

To two decimal places, this is 12.38.



Questions 12F *Range, IQR and standard deviation*

Refresher question

Q1. Out of these four data sets, which one is the most spread out?

A. {3, 4}

B. {4, 5, 6, 7}

C. {1, 4, 10, 20}

D. {1, 100, 654, 1000}

1. Finding the range

Q2. Skill

Subtract the smallest value in this data set from the largest.

{4, 999, 43, 56, 23, 54, 2, 5, 75, 2}

Q3. Skill

Find the range for the following data set.

{2345, 2457, 24 543, 643, 2345, 243, 3425, 234}

Q4. Find the range of this stem-and-leaf plot.

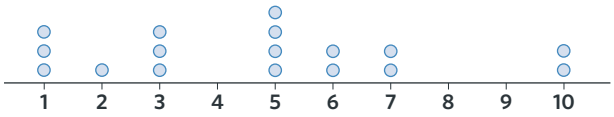
Skill

Key: 2 | 5 = 2.5

2	5
3	1 4
4	2 5 8
5	1 6 9
6	1 2 5
7	6

Q5. Find the range of this dot plot.

Skill



Check your understanding

Q6. Tim counted the number of times he got fined on public transport each month last year. The results are shown in the table below. Show that the range for this set of data is 11.

Application

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
3	4	2	5	1	5	3	7	3	9	2	12

Q7. Murphy and Jasper have been recording their heights for the past 10 years. The range of Jasper's heights is 0.24 m. Show that the range of Murphy's heights is greater than Jasper's, and find Jasper's current height.

Application

Murphy:

1	2	3	4	5	6	7	8	9	10
1.23 m	1.30 m	1.41 m	1.50 m	1.64 m	1.70 m	1.77 m	1.89 m	1.97 m	2.01 m

Jasper:

1	2	3	4	5	6	7	8	9	10
1.31 m	1.33 m	1.35 m	1.37 m	1.40 m	1.41 m	1.45 m	1.46 m	1.51 m	?

2. Finding the quartiles

Q8. For the data set {3, 6, 7, 9, 13, 14, 14}:

Skill

a) What is the median (Q_2)?

b) Use the first half of the data {3, 6, 7} to find the median (Q_1) of the first half of the data.

c) Use the second half of the data {13, 14, 14} to find the median (Q_3) of the second half of the data.

Q9. For the data set {1, 2, 5, 5, 7, 9, 9} what are the values of Q_1 Q_2 and Q_3?

Skill

Q10. For the data set {12, 13, 15, 18, 18, 20} what are the values of Q_1 Q_2 and Q_3?

Skill

Q11. For the stem-and-leaf plot:

Skill

a) List the data from the plot, in ascending order.

b) What is the median (Q_2)?

c) List the first half of the data and find Q_1

d) List the second half of the data and find Q_3

Key: 1 | 2 = 12

1	2 4
2	5 9
3	3
4	1

3. Finding the interquartile range

Q12. Skill

What does the interquartile range refer to?

A. The difference between the second quartile and the fourth quartile

B. The spread of the middle 50% of data values

C. The upper 25% of data values

D. The difference between the largest and smallest value of the data set

---

Q13. Skill

a) Find the IQR for this stem-and-leaf plot.

b) A mistake was made in the stem plot. The highlighted value has been replaced by 2.4. Find the new IQR for the stem plot.

Key: 2 | 5 = 2.5

2	5
3	3 **5**
4	2 5 8
5	6 9
6	2 2

Check your understanding

Q14. Application

In the Amazing Chef cooking competition, competitors were given a score out of 10 based on several dishes.

a) The results for one chef, Kaiden, is show below. Find the IQR of his results.

Jam donuts	Spaghetti	Caesar salad	Cheesecake	Pizza	Rice
1.4	9.9	4.5	7.8	3.4	10.0

b) Another contestant, Emily, has her results tabulated below. She is yet to be judged for her last dish. What score must she get to have an IQR of 2.2?

Jam donuts	Spaghetti	Caesar salad	Cheesecake	Pizza	Rice
5.4	7.8	6.2	5.4	4.9	?

4. Finding the standard deviation

Q15. Skill

If a data set has a variance of 49, find the standard deviation of the data set.

Q16. Application

The amount of money Mary spends on online shopping over one week is recorded below.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
$20	$23	$12	$5	$10	$12	$30

a) Find the variance of this data set.

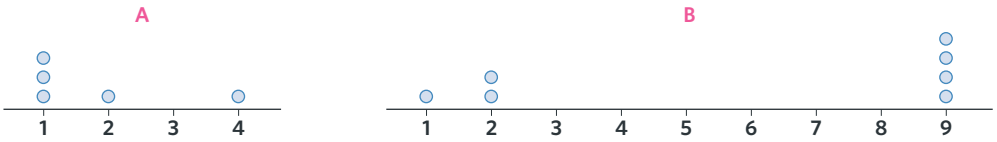
b) Find the standard deviation of this data set, correct to two decimal places.

c) On the following Monday, it was Mary's birthday. Mary decided to celebrate by spending $100 on a gift for herself. What is the new standard deviation for her 8-day shopping spree, correct to two decimal places?

Check your understanding

Q17. Skill

Two dot plots are shown below. Without doing any calculations, state which dot plot has the higher standard deviation and why.



Q18. Application

Nicky is working at Coles. She weighs groceries on the scales, and the weights in kg are recorded over the course of an hour.

4.5	6.9	2.4	8.1	2.5	2.6

Nicky claims that the variance for these weights is 30.75, and the standard deviation is 5.45.

Where did Nicky go wrong in her calculations, and what are the correct answers, correct to two decimal places?

Joining it all together

Q19. Skill 1 mark

Find the IQR for the frequency table below.

x	Frequency f
4	3
5	2
12	4
15	2

Q20. Application 8 marks

a) Amy is a YouTuber and records daily vlogs. If her shortest vlog is 340 seconds, and the range of the length of her vlogs is 2.4 minutes, what is the length of her longest vlog, in seconds? (2 marks)

b) Amy records a new set of vlogs that showcase her trip to China. Show that the IQR of this data set is lower than that of her daily vlogs, and circle the daily vlog(s) that Amy could adjust to ensure that her IQR for the China series is higher. (3 marks)

Daily vlogs (in minutes):

11.2	11.5	12.4	15.6	20.4

China vlogs (in minutes):

10.3	10.6	10.4	11.3	12.5

c) Now Amy is YouTube famous. She decides to play advertisements for revenue. She receives revenue if the standard deviation of the length of her advertisements is higher than 1 minute.

Use CAS to find the minimum length her next advertisement x must be in order to receive revenue, to 3 decimal places. (3 marks)

Advertisements (in minutes):

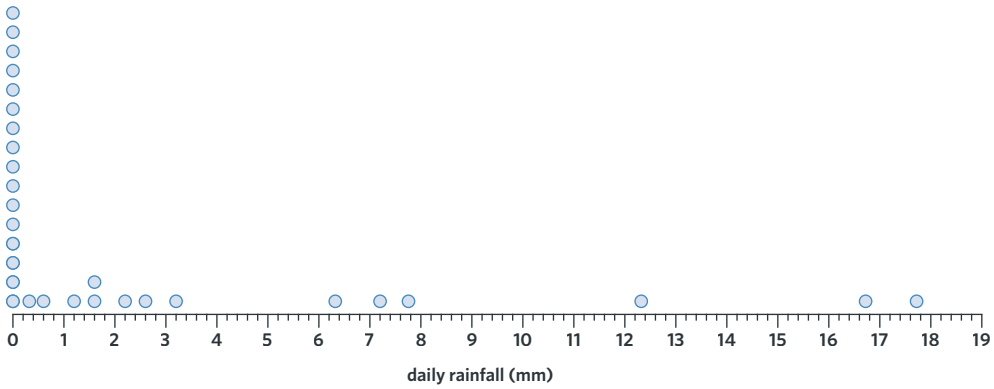
1.3	1.1	0.8	0.9	1.0	x

VCAA question

Q21.
1 mark

The dot plot below shows the distribution of daily rainfall, in millimetres, at a weather station for 30 days in September.

What is the value of the data point on the dot plot below that corresponds to the third quartile?



Adapted from VCAA 2014 Exam 1, Core: Data Analysis. Q1

Questions from multiple lessons

Q22.
Difficulty:
1 mark

A manager is investigating the relationship between the time it takes for employees to complete tasks and the quality of work.

The variables *time taken* and *quality* (below satisfactory, satisfactory, good, excellent) are

A. both numerical.

B. both categorical.

C. a discrete variable and a nominal variable respectively.

D. a continuous variable and an ordinal variable respectively.

E. a continuous variable and a nominal variable respectively.

VCAA 2016 Exam 1 Data analysis Q2 – Adapted

Q23.
Difficulty:
Year 10
1 mark

Due to a faulty zipper, the price of a pair of jeans has been reduced by 60%. If the original price of the jeans was $45, how much has the price been reduced by?

A. $18

B. $19

C. $22

D. $23

E. $27

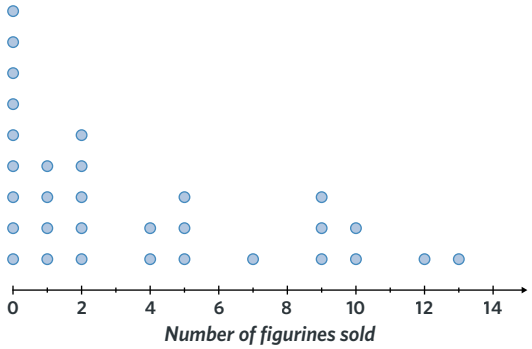
VCAA 2015 Exam 1 Module 4: Business-related mathematics Q1 – Adapted

Q24. Horacio has just opened his start-up business selling custom pet figurines.

Difficulty:

4 marks

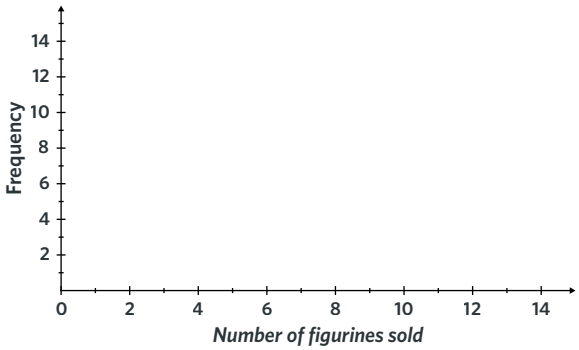
The following dot plot shows the number of figurines sold each day over a period of 31 days.



a) Identify the number of days on which Horacio made no sales. (1 mark)

b) Find the percentage of days on which **more than** 10 figurines were sold. Give your answer correct to one decimal place. (1 mark)

c) Copy the following axis and construct a histogram displaying the distribution of *number of figurines sold* for the 31-day period. Intervals should start at zero and have a width of two. (2 marks)



VCAA 2016 Exam 2 Data analysis Q1c-d – Adapted

LESSON 12G

Boxplots and outliers

The key skills you will learn in this lesson are:

1. Five-number summaries
2. Outliers
3. Boxplots

VCAA key knowledge point:

"the five-number summary and the boxplot as its graphical representation and display, including the use of the lower fence ($Q_1 - 1.5 \times IQR$) and upper fence ($Q_3 + 1.5 \times IQR$) to identify possible outliers"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Five-number summaries

The five-number summary is five values that give key information about a set of data and its distribution. The summary is as follows:

- Minimum value
- First quartile (Q_1)
- Median (Q_2)
- Third quartile (Q_3)
- Maximum value.

Example

The five-number summary for the data set 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 is:

- Minimum: 1
- First quartile: 3
- Median: 5.5
- Third quartile: 8
- Maximum: 10

WORKED EXAMPLE 24 *(1 mark)*

Create a five-number summary for the following data set:

18, 16, 21, 4, 24, 17, 34, 19, 15, 10, 14.

SOLUTION

Step 1 Rewrite the data in ascending order:

4, 10, 14, 15, 16, 17, 18, 19, 21, 24, 34.

Step 2 Identify the minimum and the maximum values.

Minimum: 4

Maximum: 34

Step 3 Find the position of the median (Q_2) using the formula $\left(\frac{n+1}{2}\right)$, where n is the number of elements in the data set.

$\frac{11+1}{2} = 6$

4, 10, 14, 15, 16, **17**, 18, 19, 21, 24, 34

The median is the sixth element of the data set, which is 17.

Step 4 To find Q_1 and Q_3, we split the data into two halves at the median. Further, if the data set has an **odd** number of elements, the median is removed before splitting the data.

We have an odd number of elements, so the median, 17, is removed.

split

4, 10, 14, 15, 16 | 18, 19, 21, 24, 34

Step 5 Think of Q_1 and Q_3 as the medians of the lower half and upper half respectively.

Find the Q_1 and Q_3 by finding the medians of each half:

4, 10, **14**, 15, 16 and 18, 19, **21**, 24, 34

Step 6 Write the values as the five-number summary:

Minimum: 4

First quartile: 14

Median: 17

Third quartile: 21

Maximum: 34

SOLUTION: TI-NSPIRE

Step 1 Press ctrl + doc and select 'Add Lists & Spreadsheets'.

Step 2 Starting from row 1, enter data into the first column.

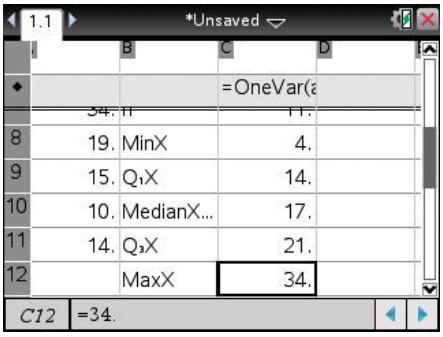
Step 3 Press menu.

Step 4 Select Statistics → Stat Calculations → One Variable Statistics.

Step 5 Press OK to confirm one variable statistics for one data set only.

Step 6 Specify the data set by entering 'a[]' in 'X1 List'.

Step 7 Press OK to exit this window and generate the statistics.



MinX = Minimum

Q_1X = Q_1

MedianX = Median

Q_3X = Q_3

MaxX = Maximum

SOLUTION: CASIO CLASSPAD

Step 1 From the main menu, tap Statistics.

Step 2 Starting from row 1, enter the data into list1.

Step 3 Tap Calc → One Variable.

Step 4 Specify the data set by keeping 'XList' as list1.

Step 5 Scroll down until you find the five-number summary statistics.

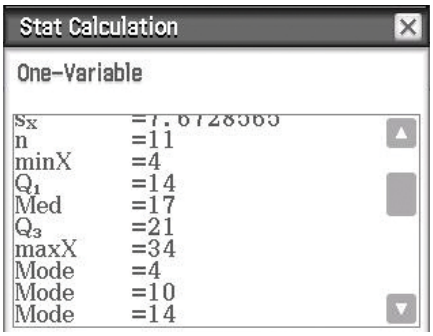
minX = Minimum

$Q_1 = Q_1$

Med = Median

$Q_3 = Q_3$

maxX = Maximum



2. Outliers

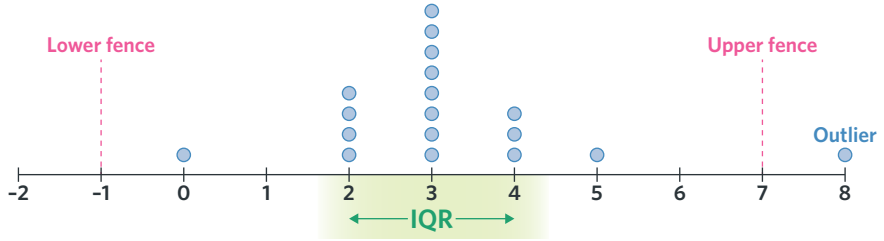
Each data set has an upper and a lower fence. They are used to determine which points are outliers, and can be found using the following formulae. Any points outside the fences are classified as outliers.

$Lower\ fence = Q_1 - (1.5 \times IQR)$

$Upper\ fence = Q_3 + (1.5 \times IQR)$

Outliers can still be considered as the minimum or maximum value.

Example



WORKED EXAMPLE 25 *(2 marks)*

Identify the outliers in the following data set:

75, 82, 87, 76, 81, 83, 82, 25, 83, 79, 105, 85, 78.

SOLUTION

Step 1 Rewrite the data in ascending order:

25, 75, 76, 78, 79, 81, 82, 82, 83, 83, 85, 87, 105.

Step 2 Find the five-number summary.

Minimum: 25

First quartile (Q_1): 77

Median (Q_2): 82

Third quartile (Q_3): 84

Maximum: 105

✓ *1 mark for stating the five-number summary*

Step 3 Find the *IQR*.

$IQR = Q_3 - Q_1 = 84 - 77 = 7$

Step 4 Find the lower fence.

$Lower\ fence = Q_1 - (1.5 \times IQR)$

$Lower\ fence = 77 - (1.5 \times 7)$

$Lower\ fence = 66.5$

Step 5 Find the upper fence.

$Upper\ fence = Q_3 + (1.5 \times IQR)$

$Upper\ fence = 84 + (1.5 \times 7)$

$Upper\ fence = 94.5$

Step 6 Determine if any elements in the data set lie below the lower fence or above the upper fence.

25, | 75, 76, 78, 79, 81, 82, 82, 83, 83, 85, 87, | 105.

$Lower\ fence = 66.5$ $Upper\ fence = 94.5$

The outliers are 25 and 105.

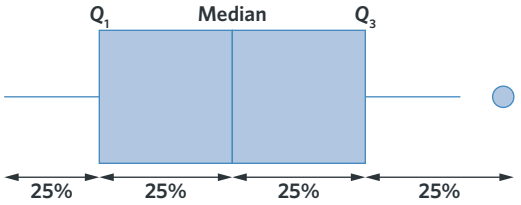
✓ *1 mark for stating the outliers*

3. Boxplots

A boxplot is a graphical representation of a set of numerical data. It contains the five-number summary and any outliers.

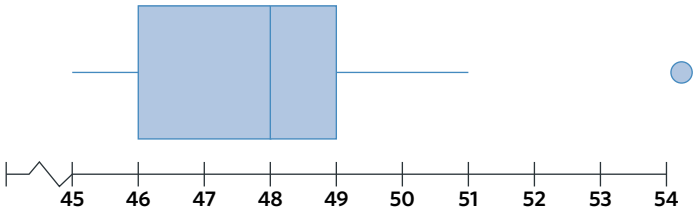
A boxplot consists of a central box and whiskers. If outliers exist in the data set, they are shown as separate dots on the number line. The central box represents the interquartile range. The left and right borders of the box represent Q_1 and Q_3, and the vertical line in the centre of the box represents the median. The whiskers extend to the most extreme value that is not an outlier.

The points between the minimum and Q_1, Q_1 and the median, the median and Q_3, and Q_3 and the maximum all represent 25% of the data.



Example

The local pub recently hosted a speed dating function for singles over 40.
The age of the participants was recorded and is shown in the box plot below.



From the box plot, it is evident that 50% of participants were between 46 and 49, and 75% were below 49.

WORKED EXAMPLE 26 *(1 mark)*

Use CAS to construct a boxplot using the following data:

1, 5, 5, 6, 7, 7, 7, 8, 8, 9.

SOLUTION: TI-NSPIRE

Step 1 Press ctrl + doc and select 'Add Lists & Spreadsheets'.

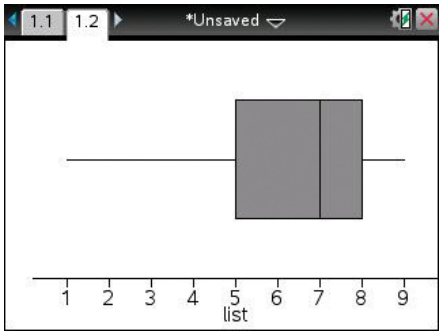
Step 2 Name a column 'list'. Starting from row 1, enter data into the column titled 'list'.

Step 3 Press ctrl + doc and select 'Add Data and Statistics'.

Step 4 Move the cursor to the x-axis and click 'Click to Add Variable'.

Step 5 Select 'list'.

Step 6 Select menu → Plot Type → Box Plot



SOLUTION: CASIO CLASSPAD

Step 1 From the main menu, tap Statistics and rename list1 as 'list'.

Step 2 Starting from row 1, enter data into the column titled 'list'.

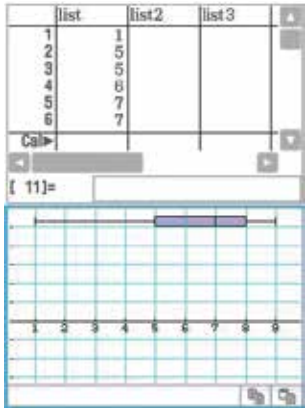
Step 3 Configure the settings of the graph by tapping in the icon bar near the top of the screen.

Step 4 Create a boxplot by changing 'Type' to 'MedBox'.

Step 5 Specify the data set by changing 'XList' to 'main\list'.

Step 6 Tick the 'Show Outliers' box. Tap set to confirm.

Step 7 Tap in the icon bar to plot the boxplot.



Questions 12G *Boxplots and outliers*

Refresher question

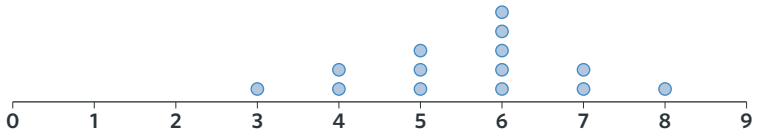
Q1. The birth year of several World War II veterans was surveyed, but one mistake was made in taking down the years. Which one of the following numbers does not fit with the data?

1919, 1921, 1923, 1918, 1915, 191, 1916, 1922.

1. Five-number summaries

Q2. Skill

Construct a five-number summary for the dot plot below.



Q3. Skill

Construct a five-number summary for the stem plot below.

Key: 3 | 9 = 39

0	4
1	0 3 5 8
2	2 6 9
3	6 6
4	3
5	7

Q4. Skill

The shoe sizes of a small group of people were recorded. The results are shown in this frequency table.

Construct a five-number summary from the data in the frequency table.

Shoe size	Frequency
7	3
8	5
9	3
10	2

Check your understanding

Q5. Skill

The five-number summary shown below was constructed from six pieces of data. Given that the original data consisted of whole numbers only, state the data.

Minimum: 6 Q_1: 6 Median: 6.5 Q_3: 8 Maximum: 9

2. Outliers

Q6. Skill

Employees in an office were asked how many hours they work in a week. The following data shows 15 responses. Without calculating the lower and upper fence, guess the one outlier.

40, 41, 43, 42, 39, 42, 40, 38, 40, 42, 19, 41, 40, 39, 40.

Q7. Skill

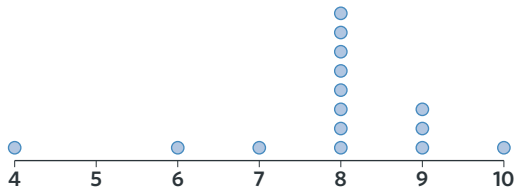
Using upper and lower fences, identify any outliers from the following data sets:

a) 5, 5, 6, 6, 6, 7, 7, 8, 8, 10, 14

b) 41, 43, 48, 48, 49, 49, 50, 50, 51, 51, 55, 59

Q8. Using upper and lower fences, identify any outliers from the following dot plot:

Skill



Check your understanding

Q9. Tamara's new job at the zoo involves weighing the wombat joeys immediately after they are born. An important aspect of this job is to note any outliers. This is to ensure that the joeys that weigh too much or too little can receive special care. After recording their weights, she displays the data in the stem plot shown to the right. Identify the outliers in this stem plot.

Application

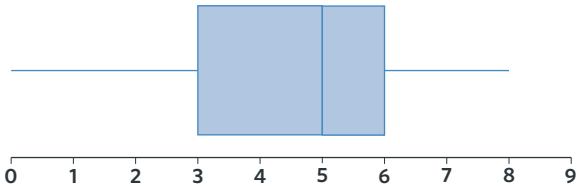
Key: 5 | 1 = 5.1

0	9
1	8 9
2	0 0 1 1 2 3
3	1
4	0

3. Boxplots

Q10. Write down the five-number summary from the following boxplot.

Skill



Q11. A class of 23 students was asked to rank Beyonce's music out of ten. The results are shown below:

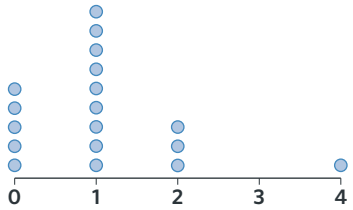
Skill

1, 4, 5, 5, 6, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 9, 9, 9, 10, 10, 10, 10, 10.

Use a CAS to construct a boxplot from the data and include outliers.

Q12. A group of 18 people was surveyed about how many pets they own. The results are shown in the dot plot below.

Skill



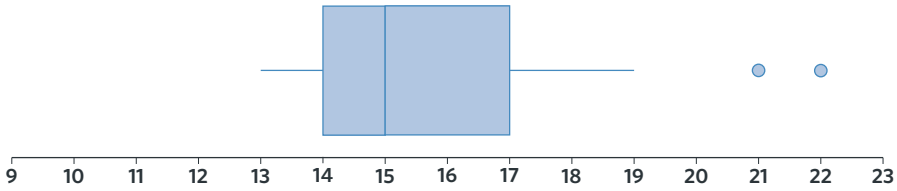
a) Using CAS, construct a boxplot from the data.

b) Use the boxplot to find the interquartile range.

Check your understanding

Q13. The maximum temperature over 24 days in July in Melbourne is shown in the boxplot below.

Skill



Between which two temperatures do exactly 6 days of data lie?

A. 13 and 19 degrees

B. 17 and 19 degrees

C. 14 and 19 degrees

D. 17 and 22 degrees

E. 14 and 17 degrees

Joining it all together

Q14. The handspan of 23 basketball players was recorded in centimetres in this table.

Application
3 marks

Hand span (cm)	Frequency
19	1
20	0
21	0
22	1
23	4
24	6
25	8
26	2
27	1

a) Use a CAS calculator to construct a boxplot.

b) Identify any outliers.

c) Construct a five-number summary.

Q15. The Biology exam mark of each of Mr Moses' thirteen students is listed below:

Application
3 marks

66, 73, 47, 91, 67, 72, 65, 71, 61, 70, 68, 70, 71.

a) Construct a five-number summary that describes the biology results.

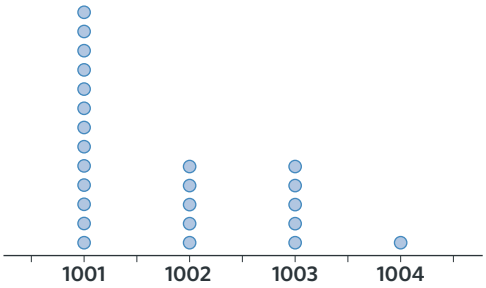
b) What is the interquartile range?

c) Mr Moses made a mistake. The student who got 72 should actually have scored 82. Does this change the interquartile range? If so, what should it be?

VCAA questions

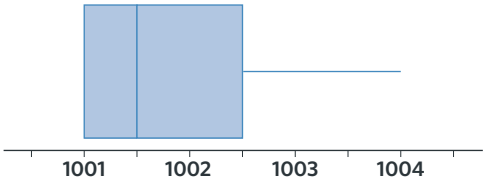
Q16. A dot plot for a set of data is shown below.

1 mark

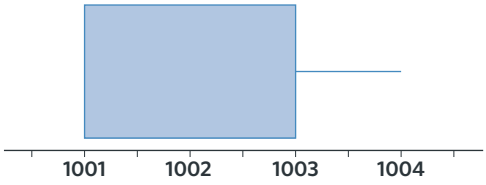


Which one of the following boxplots would best represent the dot plot above?

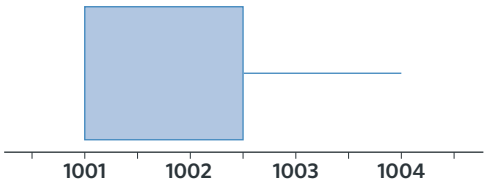
A.



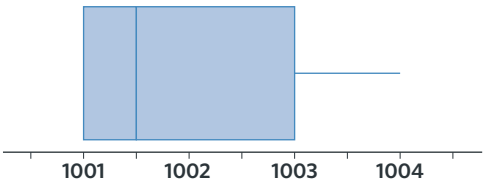
B.



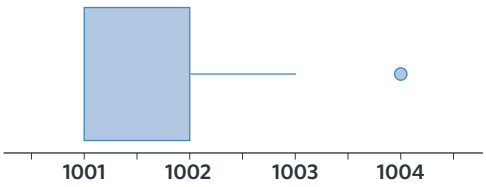
C.



D.



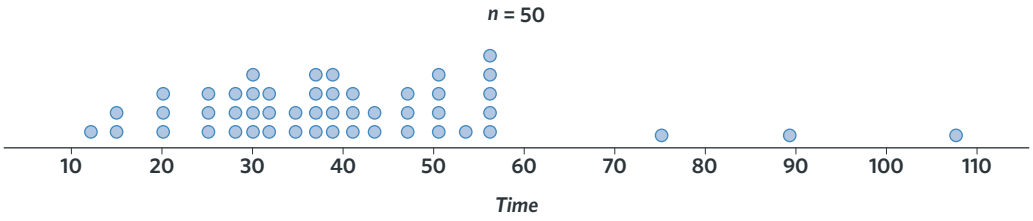
E.



VCAA 2015 Exam 1, Core: Data analysis. Q8

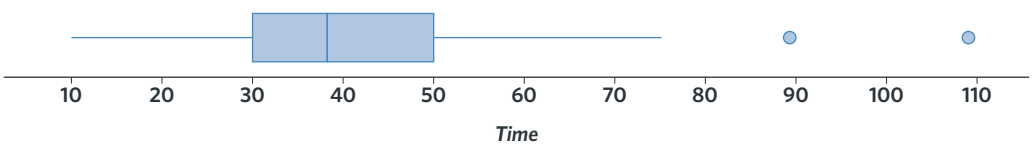
Q17. 1 mark

The dot plot below shows the distribution of the time, in minutes, that 50 people spent waiting to get help from a call centre.

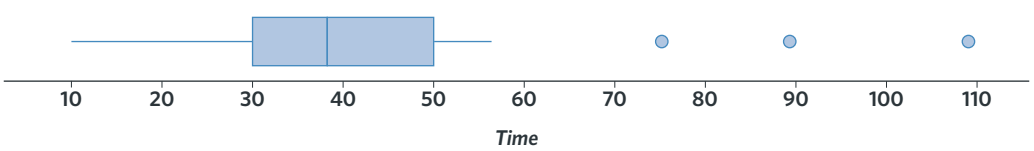


Which of the following boxplots best represents the data?

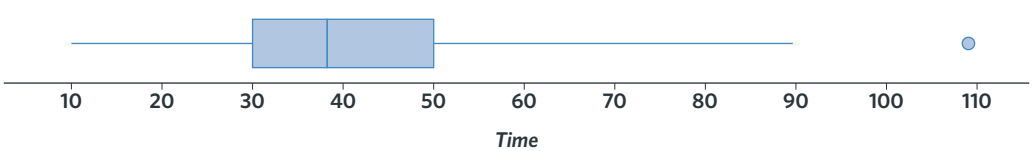
A.



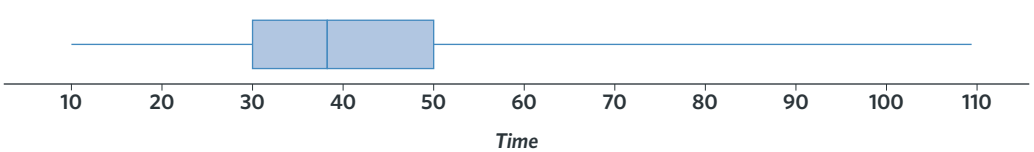
B.



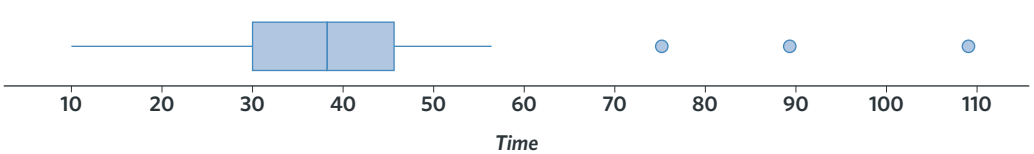
C.



D.



E.



VCAA 2014 Exam 1, Core: Data analysis. Q6

Questions from multiple lessons

Q18. Difficulty:

Year 10
1 mark

The weights of 22 cavoodles are shown in the following stem plot.

Key: 10 | 5 = 10.5 kg

Weight

10	5 8 9
11	0 2 2 3
11	6 6 7 9 9
12	0 1 1 1 3 4
12	5 8 8
13	2

The modal weight is

A. 11.0 kg

B. 11.6 kg

C. 11.9 kg

D. 12.1 kg

E. 12.8 kg

VCAA 2016 Exam 1 Data analysis Q3 – Adapted

Q19. Difficulty:

1 mark

Sonya borrows $10 000 which she will repay fully in a lump sum payment after eighteen months.

She has a choice of one of the following five loans, with the given interest rates and compounding periods:

- Loan 1 – 11.30% per annum, compounding monthly
- Loan 2 – 11.35% per annum, compounding monthly
- Loan 3 – 11.39% per annum, compounding monthly
- Loan 4 – 11.40% per annum, compounding quarterly
- Loan 5 – 11.45% per annum, compounding quarterly

The loan that will cost Sonya the least amount of money is

A. loan 1. **B.** loan 2. **C.** loan 3.

D. loan 4. **E.** loan 5.

VCAA 2018 Exam 1 Recursion and financial modelling Q19 – Adapted

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Q20. Difficulty:

4 marks

The following back-to-back stemplot displays the distribution of the *height*, in cm, of 21 jockeys and 21 hockey players, separated by the *type of athlete* they are.

Key: 17 | 4 = 174 cm

Jockeys	*Height* (cm)	Hockey players
	13	
9 9 8 8 8 8 6 5 5 4 2	14	
5 4 4 3 2 1	15	
5 4 2	16	
	17	4 5 5 7 7 7 8 9 9 9
	18	0 1 2 3 3 4 5 7 9
	19	1 3

a) Which variable, *height* or *type of athlete*, is a categorical variable? (1 mark)

b) Write down the modal *height*(s), in cm, of the hockey players. (1 mark)

c) Use the information in the back-to-back stem plot to find the values of x and y in the table below. (2 marks)

		Height (cm)				
		Minimum	Q_1	Median	Q_3	Maximum
Type of athlete	Jockeys	137	x	149	154	165
	Hockey players	174	177	180	184.5	y

VCAA 2017 Exam 2 Data analysis Q2 a-c – Adapted

LESSON 12H

Comparing distributions

The key skills you will learn in this lesson are:

1. Back-to-back stem plots
2. Parallel boxplots
3. Interpreting data

VCAA key knowledge point:

"use of back-to-back stem plots or parallel boxplots, as appropriate, to compare the distributions of a single numerical variable across two or more groups in terms of centre (median) and spread (IQR and range), and the interpretation of any differences observed in the context of the data"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Back-to-back stem plots

Back-to-back stem plots can be used to compare two sets of data. Specifically, the centre and the spread of the data can be compared.

To compare the centres of each set of data the median is used. To compare median values, use the words 'higher' or 'lower'.

To compare the spread of each set of data, the interquartile range (IQR) and range are used. When the data contains outliers, the range will not give an accurate representation of the data, so only IQR is used. To compare IQR and range, use the words 'larger' or 'smaller'.

The data set with a larger spread is 'more variable.'

Example

Data from 20 house sales in Melbourne was compared to 20 house sales in Adelaide.

The **median** house price in Melbourne was \$720 000 compared to \$610 000 in Adelaide. Melbourne had a **higher** median house price than Adelaide.

The **range** of sale prices is Melbourne was \$190 000 compared to \$110 000 in Adelaide. Melbourne had a **larger** range of sale prices compared to Adelaide. The sale price in Melbourne was **more variable** than Adelaide.

WORKED EXAMPLE 27 *(3 marks)*

The test scores of two classes were recorded in this stem and leaf plot.

Key: 2 | 7 = 27%

Class A		Class B
6 6 2 1	4	4
7 6 5	5	9
3 2 1	6	0
7 5	7	6 8
0	8	0 0 1
	9	5 7 8 9 9

Compare these values for Class A and Class B:

a) Median

b) Interquartile range (IQR)

c) Range

SOLUTION

a) Median

Step 1 Find the position of the median for Class A.

$\left(\frac{n+1}{2}\right) = \frac{13+1}{2} = 7$

The median is the 7th value: 57.

Step 2 Find the position of the median for Class B.

$\left(\frac{n+1}{2}\right) = \frac{13+1}{2} = 7$

The median is the 7th value: 80.

Step 3 Compare the medians of the two classes.

The median of Class A is 57, and the median of Class B is 80. Class B has a higher median than Class A.

b) Interquartile range (IQR)

Step 1 Calculate the first and third quartiles for Class A.

The median divides the data into two equal halves of six.

Q_1 is the average of the 3rd and 4th values.

Q_3 is the average of the 10th and 11th values.

Step 2 Calculate the IQR for Class A.

$IQR = Q_3 - Q_1$

$IQR = 69 - 46$

$IQR = 23$

Step 3 Calculate the first and third quartiles for Class B.

$Q_1 = 68$

$Q_3 = 97.5$

Step 4 Calculate the IQR for Class B.

$IQR = 97.5 - 68$

$IQR = 29.5$

Step 5 Compare the IQR of the two classes.

The IQR of Class A is 23, and the IQR and of Class B is 29.5. Class B has a larger IQR than Class A.

c) Range

Step 1 Write the maximum and minimum values for each class.

Class A: maximum value is 80, minimum value is 41.

Class B: maximum value is 99, minimum value is 44.

Step 2 Calculate the range for each class

Class A: $Range = 80 - 41 = 39$

Class B: $Range = 99 - 44 = 55$

Step 3 Compare the range of the two classes.

The range of Class A is 39, and the range of Class B is 55. Class B has a larger range than Class A.

NOTE: Class B has a larger IQR and range. As such, the test results for Class B are more variable than those of Class A.

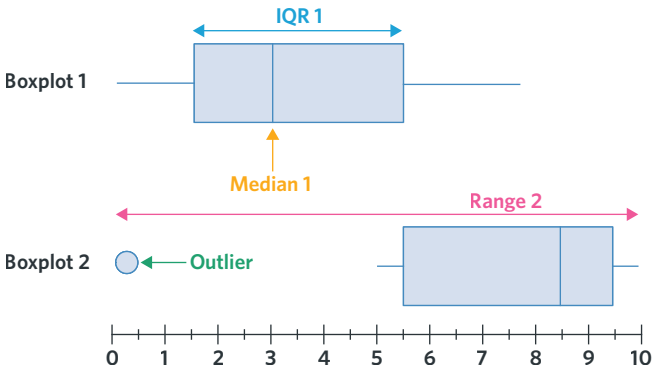
2. Parallel boxplots

A parallel boxplot is a plot containing two or more boxplots, plotted one above another. Parallel boxplots are useful for large spreads of data, and are capable of displaying more than two sets of data. In contrast, back-to-back stem plots are more detailed, but can only display two sets of data.

Parallel boxplots display the median, range and interquartile range of multiple sets of data. This makes comparison between sets of data relatively simple.

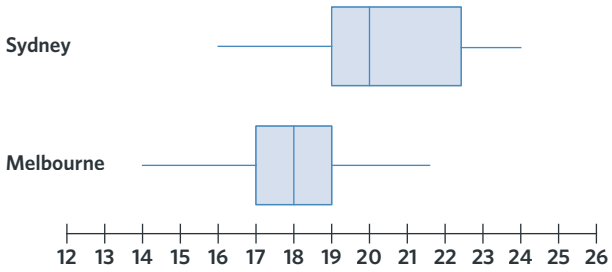
Parallel boxplots can be used to compare data in the same way as back-to-back stem plots: the centres can be compared using median, and the spreads can be compared using the IQR and range. If the data contains an outlier, the outlier is excluded when finding the range.

Example



WORKED EXAMPLE 28 *(3 marks)*

The maximum temperature of every day in May last year was recorded in Sydney and Melbourne. The results are shown in this parallel boxplot.



For this parallel boxplot compare:

a) Median

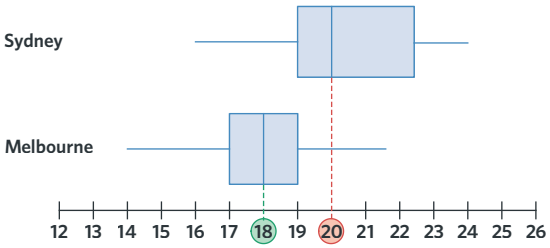
b) Interquartile range (IQR)

c) Range

SOLUTION

a) Median

Step 1 Find the medians for Sydney and Melbourne by looking at the plot.



The median for Sydney is 20 and for Melbourne is 18.

Step 2 Compare.

Sydney has a higher median than Melbourne.

b) Interquartile range (IQR)

Step 1 Find Q_1 and Q_3 for the Sydney boxplot by looking at the plot.

$Q_1 = 19$

$Q_3 = 23$

Step 2 Calculate the IQR.

$IQR = Q_3 - Q_1$

$IQR = 4$

Step 3 Find Q_1 and Q_3 for the Melbourne boxplot by looking at the plot.

$Q_1 = 17$

$Q_3 = 19$

Step 4 Calculate the IQR.

$IQR = Q_3 - Q_1$

$IQR = 2$

Step 5 Compare the two.

$4 > 2$

Sydney has the larger IQR.

c) Range

Step 1 Find the maximum and minimum values for Sydney and use these to find the range.

Highest is 24, lowest is 16.

$Range = 24 - 16$

$Range = 8$

Step 2 Find the maximum and minimum values for Melbourne and use these to find the range.

Highest is 21.8, lowest is 14.2.

$Range = 21.8 - 14.2$

$Range = 7.6$

Step 3 Compare.

Sydney had a larger range of temperatures in May last year than Melbourne.

3. Interpreting data

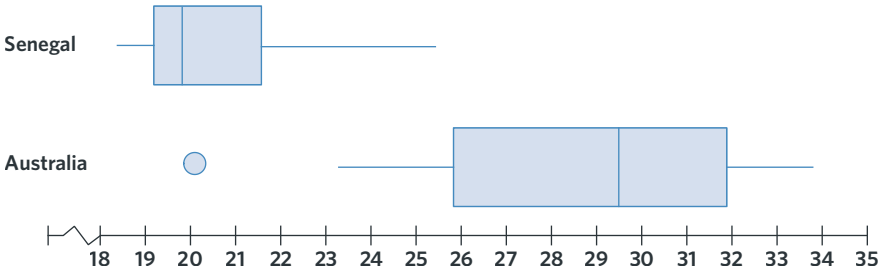
To interpret the data in a back-to-back stem plot or a parallel boxplot, examine the data sets for any differences between them, and make a general comment about what is being shown.

The type of data will indicate what type of comparison you will make. For example:

- For data about prices interpret which data group is cheaper or more expensive.
- For data about height interpret which data group is taller or shorter.
- For data about length interpret which data group is longer or shorter.

Example

A random sample of 100 women from Australia and Senegal were asked about their age at marriage. The data was recorded and is displayed in the parallel boxplot below:



An interpretation of this data is that women in Senegal generally get married at a younger age than women in Australia.

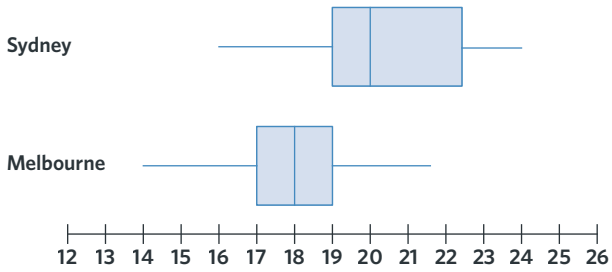
WORKED EXAMPLE 29 *(2 marks)*

Interpret the data from the earlier examples in this lesson.

a) Key: 2 | 7 = 27%

Class A		Class B
6 6 2 1	4	4
7 6 5	5	9
3 2 0	6	0
7 5	7	6 8
0	8	0 0 1
	9	5 7 8 9 9

b)



SOLUTION

a)

Step 1 Recognise what the data is about.

The data is about test scores.

Step 2 Compare.

In general Class B had higher test scores than Class A.

b)

Step 1 Recognise what the data is about.

The data is about daily maximum temperatures in May.

Step 2 Compare.

In general Sydney had higher maximum daily temperatures in May last year compared to Melbourne.

Questions 12H *Comparing distributions*

Refresher question

Q1. Desdemona and Cordelia recorded how many coffees they drank every day for a week.

Desdemona: 2, 3, 2, 1, 3, 1, 0.

Cordelia: 4, 6, 5, 7, 5, 4, 3.

Who consistently drank more coffee over the week?

1. Back-to-back stem plots

Q2. Skill

A number of men and women were asked how many litres of milk they normally drink in a year. The results are shown in this back-to-back stem plot.

Key: 1 | 5 = 15 litres

Women		Men
9 9 6 4 1	0	1 4
8 7 4 3 2 1	1	6
2 2	2	8
	3	9
	4	2 2 4 5 6 7
1	5	1 2 5 6

a) What is the smallest value for women?

b) What is the greatest value for men?

c) Which group has a larger range?

d) Which group has the higher median?

Q3. Skill

Fernando and Lucas are in a competition to see who can score the most points in one season of basketball.

This boxplot displays the number of points each player scored in each game of the season.

Compare these values for Fernando and Lucas:

Key: 3 | 2 = 32 points

Fernando		Lucas
4 2	0	2 2
9	0	6 8 9
	1	2 2
8 8 7	1	6
4 4	2	
6	2	8

a) Median

b) Interquartile range (IQR)

c) Range

Q4. Skill

The following back-to-back stem plot shows sales figures for individuals at two small companies over one day.

Compare these values for Company A and Company B:

Key: 4 | 5 = $4500

Company A		Company B
5	0	5 7 8
2	1	2 7 8 9
9 1	2	2 5 5 6
9 5 5	3	1
6 6 5 3 1 1	4	
	5	8

a) Median

b) Interquartile range (IQR)

c) Range

Check your understanding

Q5. Skill

The following data shows the ages of people from the UK and the USA on a bus tour around Japan:

UK: 29, 28, 31, 18, 27, 17, 22, 24, 26, 23, 28, 19, 25, 22, 26.

USA: 18, 21, 19, 18, 22, 20, 24, 18, 23, 34, 24, 21, 25, 19, 21.

a) Construct a back-to-back stem plot from the data.

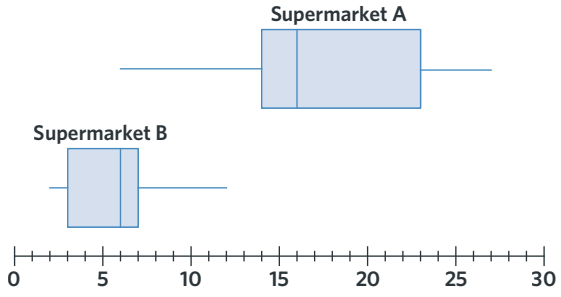
b) Find the range of each set of data.

c) Use the range to find whether the people from the UK or the USA had the most variable ages.

2. Parallel boxplots

Q6. Skill

These parallel boxplots show the different prices of gouda cheese at two supermarkets.

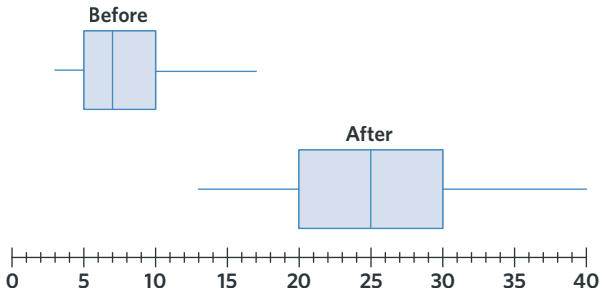


a) Which supermarket has the higher median price of gouda cheese?

b) Which supermarket has the larger spread, as indicated by IQR?

Q7. Skill

150 people participated in a fitness course over the summer. The number of push ups they could do before and after the course is recorded in this parallel boxplot.



For these parallel boxplots compare:

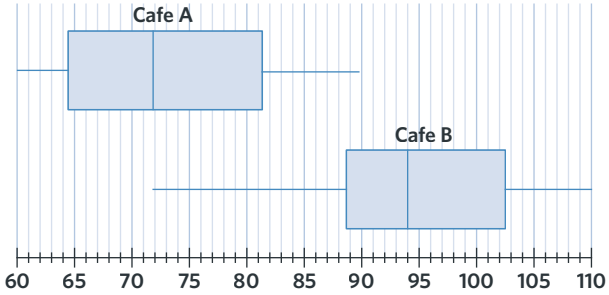
a) Median

b) Interquartile range (IQR)

c) Range

Q8. Skill

The number of coffees sold each day in two cafes over a period of two weeks is shown here.



For this parallel boxplot compare:

a) Median

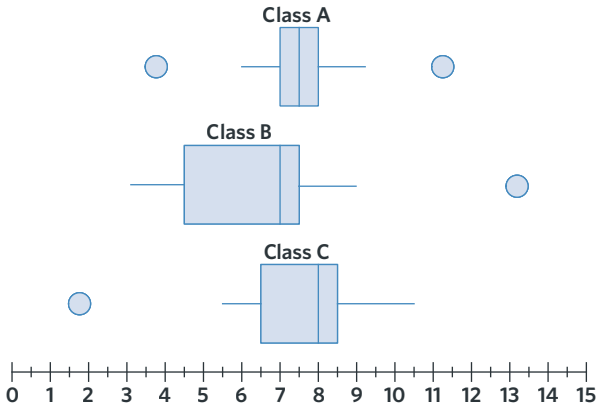
b) Interquartile range (IQR)

c) Range

Check your understanding

Q9. Application

Three classes sat the same test on basic multiplication. The time taken to sit the test for the students in each class is shown in this parallel boxplot.



Which class had:

a) The highest median?

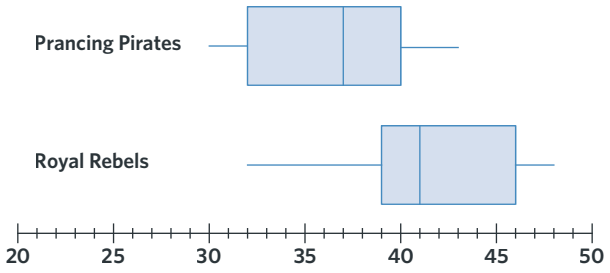
b) The smallest interquartile range?

c) Which class had the least variable times (excluding outliers)?

Explain your answer.

Q10. Skill

The final score of two netball teams in every match of an eleven game season is recorded in this parallel boxplot.



Which of the following statements is not true?

A. At least 75% of the Prancing Pirates' scores lie below Royal Rebels' median.

B. The Prancing Pirates scored less than or equal to 37 in at least six games.

C. The Royal Rebels scored more than 46 in three games.

D. The Royal Rebels scored more than or equal to 41 in at least six games.

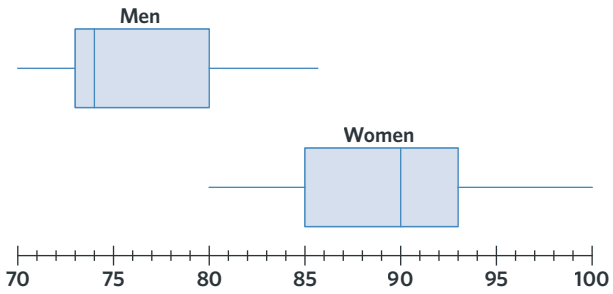
E. The median of the Prancing Pirates is 37.

3. Interpreting data

Q11. Skill

The following parallel boxplot displays the ages of residents in a retirement village.

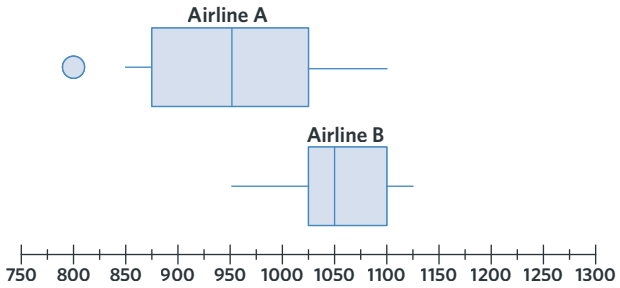
Interpret this data.



Q12. Skill

The following parallel boxplot displays the prices of one-way economy flights from Melbourne to Los Angeles from two different airlines.

Interpret this data by comparing median price and variability (using IQR).



Check your understanding

Q13. Application

Here is the data again from Question 2. Men and women were asked how many litres of milk they normally drink in a year. The results are shown in this back-to-back stem plot.

Key: 1 | 5 = 15 litres

Women		Men
9 9 6 4 1	0	1 4
8 7 4 3 2 1	1	6
2 2	2	8
	3	9
	4	2 2 4 5 6 7
1	5	1 2 5 6

a) How many people were surveyed?

b) Explain why this statement is incorrect: "This data proves that men drink more milk than women."

Joining it all together

Q14. Application 4 marks

The following data shows the number of hours per week 45 children across three year levels spent studying:

Year 10: 12, 6, 15, 25, 2, 17, 8, 21, 13, 19, 26, 11, 9, 3, 16

Year 11: 10, 16, 22, 17, 9, 18, 21, 28, 32, 19, 23, 13, 21, 20, 15.

Year 12: 25, 29, 35, 21, 3, 31, 28, 25, 17, 28, 30, 26, 34, 29, 30.

a) Construct a parallel box plot. (3 marks)

b) Using the information from the boxplots, explain why year level is associated with the number of hours per week spent studying. (1 mark)

Q15. Application 5 marks

The following back-to-back stem plot displays the exam mark obtained by each student in two geography classes.

Key: 3 | 8 = 38%

Class A		Class B
	4	5
9 5 3 2 1	5	8
5 4 4 3 1 1	6	1 4 7
7 6 4 1	7	1 1 3 4 7 8
5 1	8	2 2 3 4 7 9
9	9	1

For this data:

a) Compare the medians.

b) Compare the interquartile ranges (IQRs).

c) Compare the ranges.

d) Which class had the more variable result? Explain your answer.

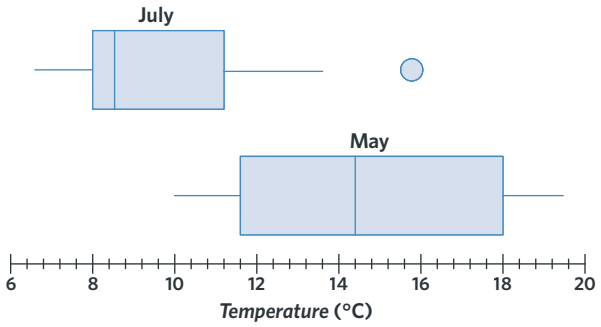
e) Interpret the data.

VCAA question

Q16. 1 mark

A weather station records daily maximum temperatures.

The boxplots below display the distribution of maximum daily temperature for the months of May and July.



Using the information from the boxplots, explain why the maximum daily **temperature** is associated with the **month** of the year. Quote the values of the appropriate statistics in your response.

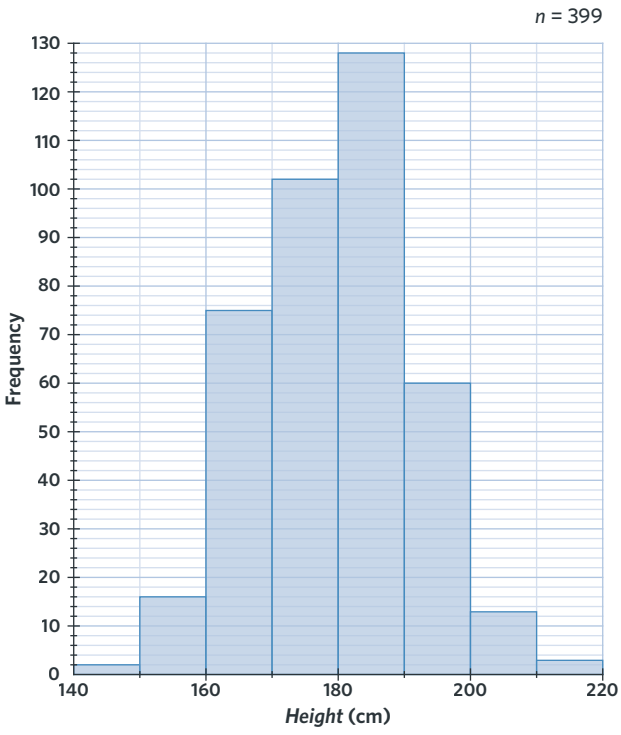
Adapted from VCAA 2016 Exam 2, Core: Data analysis. Q2b.iii

Questions from multiple lessons

Q17. Difficulty:

Year 10 1 mark

The following histogram shows the distribution of the heights of Australian Olympians at the 2012 Olympics.



Using this histogram, the percentage of these 399 Australian Olympians that are 190 cm or taller is closest to

A. 17%

B. 19%

C. 76%

D. 81%

E. 83%

VCAA 2016 Exam 1 Data analysis Q6 – Adapted

Q18.

Difficulty:

Year 10
1 mark

Kourtney goes to her favourite Japanese restaurant, Nobu, and orders a meal for $56. Kourtney then tips her waiter $5. The $5 tip as a percentage of the price of the meal is closest to

A. 0.09%

B. 0.89%

C. 1.12%

D. 8.93%

E. 11.20%

VCAA 2014 Exam 1 Module 4: Business-related mathematics Q2 – Adapted

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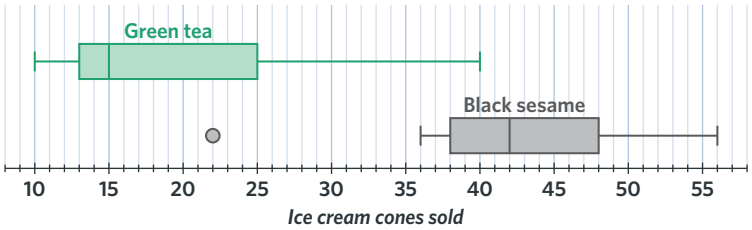
Q19.

Difficulty:

2 marks

The parallel boxplots display the number of *ice cream cones sold* each hour for two flavours; black sesame and green tea.



a) Describe the shapes of the distribution of the number of *ice cream cones sold* (including any outliers) for green tea and black sesame. (1 mark)

b) Determine the value of the lower fence for the black sesame boxplot. (1 mark)

VCAA 2016 Exam 2 Data analysis Q2b – Adapted

AOS 6: Statistics

Investigating relationships between two numerical variables

CHAPTER 13

LESSON 13A

Scatterplots

The key skills you will learn in this lesson are:

1. Response and explanatory variables
2. Scatterplots

VCAA key knowledge points:

"response and explanatory variables"

"scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction, form and strength"

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1. Response and explanatory variables

When investigating the relationship between two variables, it is important to find out whether the values of one variable affect, or explain, the values of the other. One of the variables is labelled as the explanatory variable and the other as the response variable according to the following definition:

The explanatory variable can affect, or explain, the values of the response variable.

The response variable is expected to change as a result of the explanatory variable.

Example

The relationship between the frequency of watering plants and plant height.

It is reasonable to expect that the frequency of watering will have some effect on the growth of a plant. Would it be reasonable to say the height of the plants controls how often they are watered? No. Therefore the frequency of watering is the explanatory variable and the plant height is the response variable.

WORKED EXAMPLE 1 *(1 mark)*

Data is collected to investigate the relationship between the income of households and the number of days they spend on holiday per year. Identify the response and explanatory variables.

SOLUTION

Step 1 Think about which variable can explain the values of the other.

The number of days spent on holiday cannot explain a higher or lower income.

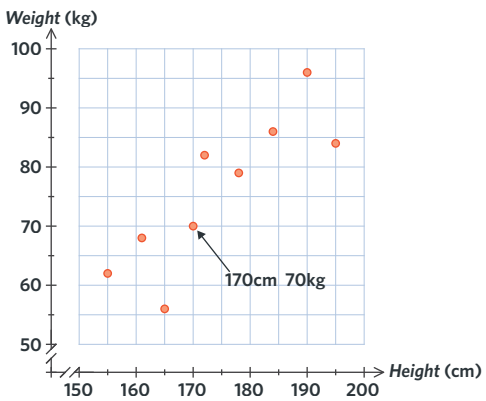
An increased income allows a household to go on holiday as they have more spare money.

Step 2 Identify the explanatory and response variables.

Income can explain the number of days spent on holiday. Therefore 'household income' is the explanatory variable and the 'number of days spent on holiday' is the response variable.

2. Scatterplots

A scatterplot is a type of graph that displays the relationship between two variables. Both variables must be quantifiable; categorical data cannot be represented. It is convention to use the horizontal axis (x-axis) for the explanatory variable and the vertical axis (y-axis) for the response variable. A dot is used to signify a single data entry, and is placed at the point where its value aligns with both variables.



WORKED EXAMPLE 2 *(2 marks)*

Create a scatterplot using the following data. This worked example follows on from the previous one.

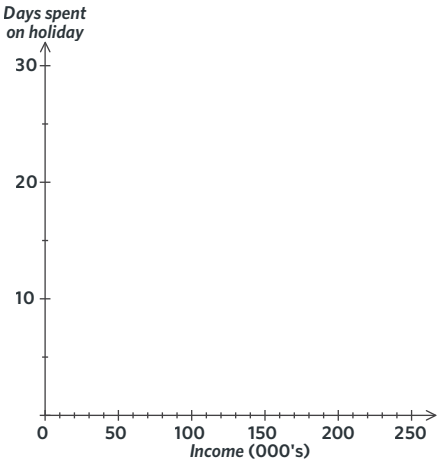
Income ($ 000's)	67	245	36	32	89	122	96	45	75	103
Days spent on holiday	12	28	5	4	10	23	18	13	16	20

SOLUTION: BY HAND

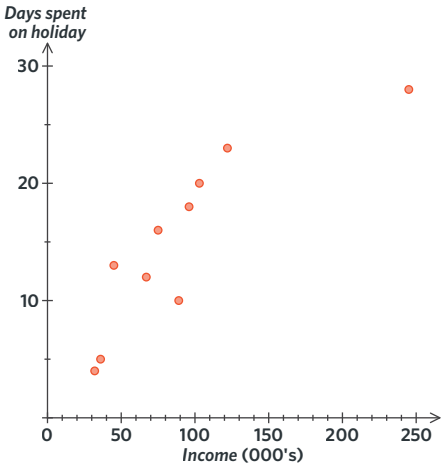
Step 1 Determine the explanatory and response variables.

As seen in the previous worked example, 'household income' is the explanatory variable and 'days spent on holiday' is the response variable.

Step 2 Draw an x-axis and label it as the explanatory variable. Then draw a y-axis and label it as the response variable. Add tick marks and appropriately scale the axes.



Step 3 Label each data point as a dot, in the position that represents their value.



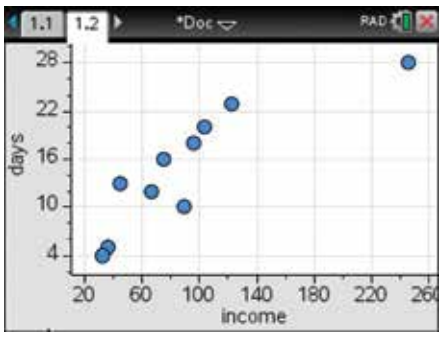
SOLUTION: TI-NSPIRE

Step 1 Open a new page by pressing ctrl + doc, then 4 ('Add Lists & Spreadsheets').

Step 2 Name column A as the explanatory variable 'income' and column B as the response variable 'days', and enter the data into the subsequent columns.

Step 3 Press ctrl + i and press 5 ('Add Data & Statistics').

Step 4 Use the mouse to move to the horizontal axis and click on 'Click to add variable'.
Use the arrows to scroll down and select the explanatory variable 'income'. Click on the vertical axis and select the response variable 'days'. The completed scatterplot will appear.

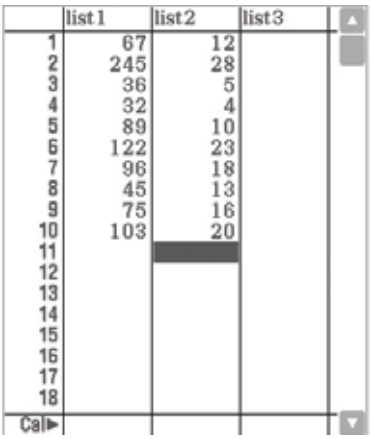


Step 5 If the values are not all visible, press menu, press 5 ('Window/Zoom'), then press 4 ('Zoom - Out') to adjust the display.

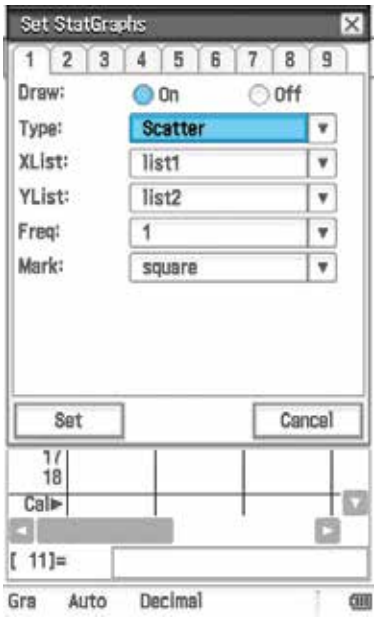
SOLUTION: CASIO CLASSPAD

Step 1 Open the Statistics application.

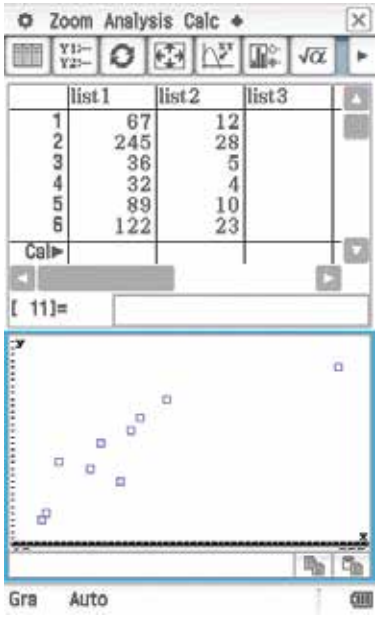
Step 2 Enter the values of the explanatory variable (income) under 'list1' and the values of the response variable (days) under 'list2'.



Step 3 Press , choose the following settings and then press set.



Step 4 Press and the scatterplot will appear below.



Step 5 Press Resize to change between a half-screen graph and a full-screen graph.

Questions 13A *Scatterplots*

Refresher question

Q1. Could the following variables be related?

a) The number of balloons at a circus and the happiness of the children attending.

b) The amount of balloons at a circus and the heights of the clowns.

1. Response and explanatory variables

Q2. Application

Identify the response variable and the explanatory variable in the following situations and explain your choices.

a) The relationship between the daily maximum temperature and the number of people at the local swimming pool.

b) The relationship between age and the amount of time spent exercising daily.

c) The relationship between workers' pay rate per hour and their surveyed level of happiness.

Check your understanding

Q3. Application

Explain the faults in the following statements.

a) When investigating the relationship between the number of dogs in the neighbourhood to the average hours of sleep received by residents, the response variable is the number of dogs.

b) When investigating the relationship between the height of the waves at Bells Beach and the number of surfers at Bondi, the explanatory variable is the height of the waves.

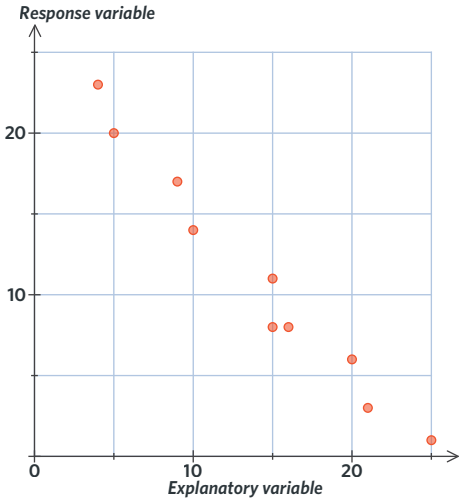
2. Scatterplots

Q4. Skill

a) Complete the scatterplot by including these data points:

Explanatory variable	13	20	22	11	18
Response variable	15	5	6	12	9

b) For a certain data point in this set, the value of the explanatory variable is 16. What is the value of the response variable at this point?



Q5. Application

For which of the following pairs of variables would it be appropriate to construct a scatterplot?

A. The colour and heights of cats

B. The amount of time spent studying and subsequent results on a maths test

C. Phone models and their battery life

D. People's age and whether they wear glasses or not

E. Number of cinema tickets sold for different movies

Q6. Application

Students at a local secondary school must wear blazers regardless of the temperature. The table below shows the number of students who disobeyed this policy over the last 2 weeks of the school year, alongside the temperature on each of the days.

Day	M	T	W	Th	F	M	T	W	Th	F
Students not wearing a blazer	12	17	24	118	17	10	8	15	33	28
Temperature (°C)	23	24	26	33	25	21	20	23	27	26

Create a scatterplot, using temperature as the explanatory variable and the number of disobeying students as the response variable.

Check your understanding

Q7. Application

The owners of a pub receive constant noise complaints from neighbours. They decide to record the number of patrons at their pub and the noise level every night, in order to understand how to restrict the noise. They collected data over an entire month, and the following table shows the average results for each day of the week.

Night	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Patrons	34	46	24	35	95	113	32
Noise level (dB)	74	85	62	71	104	112	54

a) Create a scatterplot with the number of patrons being the explanatory variable and the noise level being the response variable.

b) The council sets a maximum noise level of 98 decibels for the pub. On which days is this regulation currently broken?

c) The owners set a limit of 100 patrons in the pub at once. Based on the data at hand, are the owners going to continue to break council regulations? Explain your answer.

Joining it all together

Q8. Application 2 marks

Students were surveyed on how long they spent working on their history projects. Despite being adamant that quality is more important than quantity, Ms Riosa compared this data with the number of words contained within each project. The data for eight students is displayed in the following table.

Time spent (hours)	15	12	2	7	10	13	22	3
Words written	1230	1106	709	1058	976	1362	2589	542

a) Identify the explanatory and response variables.

b) Create a scatterplot using this data.

Q9. Application 2 marks

Josh is frustrated that the Wi-Fi in his room is very weak. He decides to record the internet speed in the centre of every room of his house, and also the distance from the room to the router. The data he collected is displayed in the following table.

Room	Living room	Balcony	Kitchen	Bedroom	Office	Bathroom
Internet speed (mbps)	14	9	11	6	23	17
Distance to modem (m)	10	18	13	22	3	7

Create a scatterplot using this data.

Q10. Application 2 marks

Malachy records the social media followers and average likes received on his and his friends' social media accounts. The following table is a display of the data he collected.

Friends	Malachy	Jack	Blaise	Nathan	Antoine	Zahir	Scott	Max	Julian
Followers	804	52	89	153	185	128	76	141	61
Likes	193	14	22	48	67	30	17	36	18

a) Create a scatterplot on your CAS using this data.

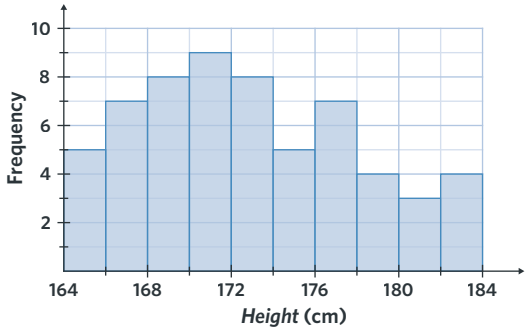
b) Identify an outlier, disregard it, and resketch the scatterplot by hand.

Questions from multiple lessons

Q11. The following histogram displays the *height*, in centimetres, of 75 competitors in an E-Sports tournament.

Difficulty:

Year 10
1 mark



The *height* for this sample of competitors is most frequently

A. greater than or equal to 168 cm and less than 170 cm.

B. greater than or equal to 170 cm and less than 172 cm.

C. greater than or equal to 172 cm and less than 174 cm.

D. greater than or equal to 174 cm and less than 176 cm.

E. greater than or equal to 176 cm and less than 178 cm.

VCAA 2018NH Exam 1 Data analysis Q3 – Adapted

Q12. A house was purchased for $800 000 with a deposit of $50 000.

Difficulty:

The balance will be completely repaid with 85 monthly repayments of $10 000.

Year 10
1 mark

The total amount of interest charged is

A. $10 000

B. $50 000

C. $100 000

D. $850 000

E. $900 000

VCAA 2015 Exam 1 Module 4: Business-related mathematics Q5 – Adapted

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Q13. The number of movies 13 different people watched on Netflix in a month is displayed below.

Difficulty:

9 8 4 3 16 5 4 7 3 2 0 4 10

3 marks

a) Create a five-number summary from the data above. (1 mark)

b) Determine if there are any outliers in the data. If there are outliers, what are they? (1 mark)

c) Draw a boxplot for the data. (1 mark)

LESSON 13B

Correlation

The key skill you will learn in this lesson is:

1. Finding the relationship between variables

VCAA key knowledge point:

"scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction, form and strength"

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1. Finding the relationship between variables

Correlation is the measure of the strength of the relationship between the explanatory and response variables. However, before discussing correlation, it is important to find whether or not a relationship between the explanatory and response variables actually exists.

Other terms for correlation include "association" or "relationship".

Example

Has a relationship	No relationship
The temperature and the amount of ice cream sold.	The amount of ice cream sold and the number of plants owned.
The length of hair and the amount of conditioner used.	The length of hair and the number of apples eaten.
The amount of time spent watching films and the ability to quote them.	The amount of time spent watching movies and the number of siblings.

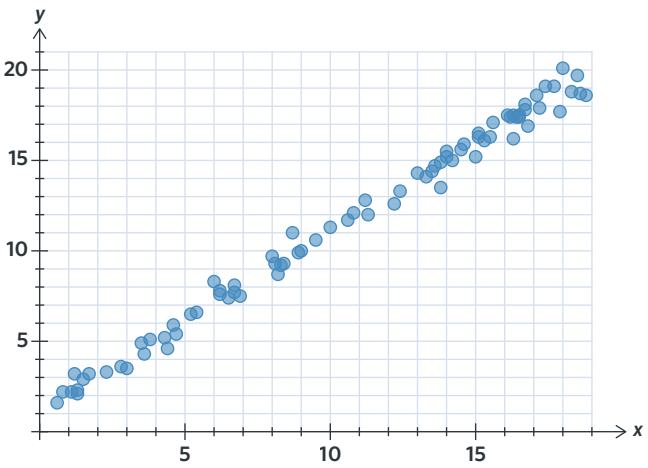
The correlation between two variables is defined by 3 factors:

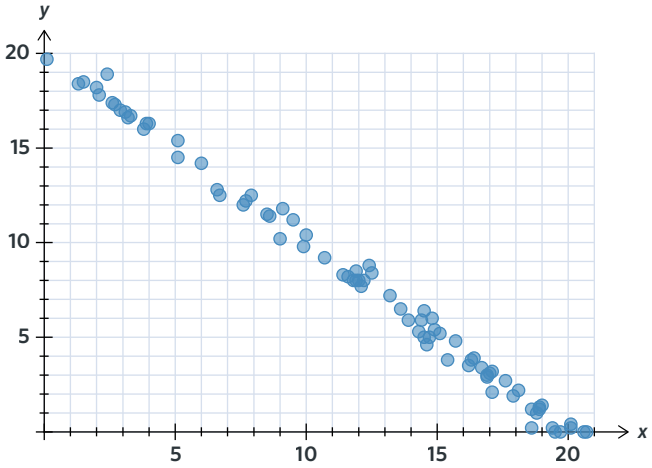
- Direction
- Strength
- Form

The direction of correlation

If the points follow an upwards trend, from the lower left corner to the upper right corner of the graph, a **positive relationship** exists between the variables.

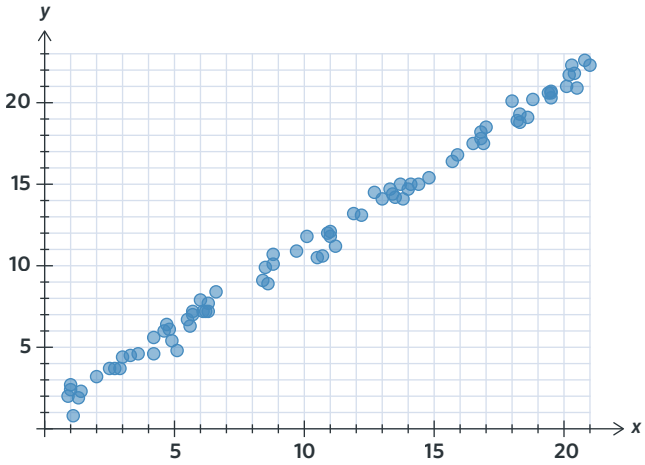
If the points follow a downwards trend, from the upper left corner to the lower right corner of the graph, a **negative relationship** exists between the variables.

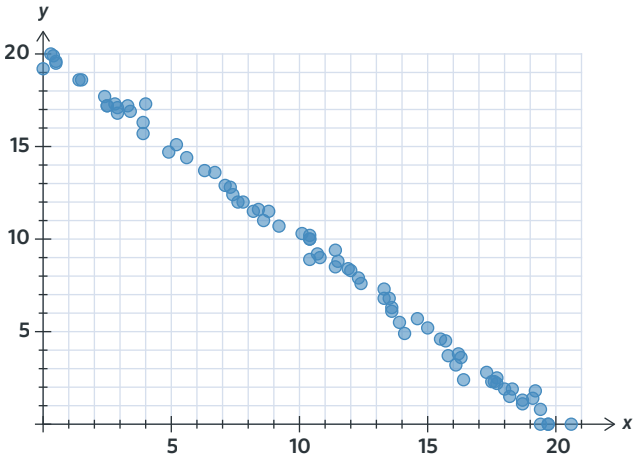




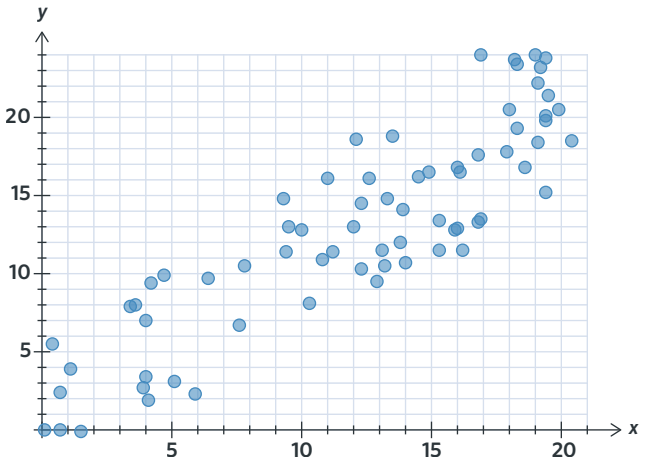
The strength of correlation

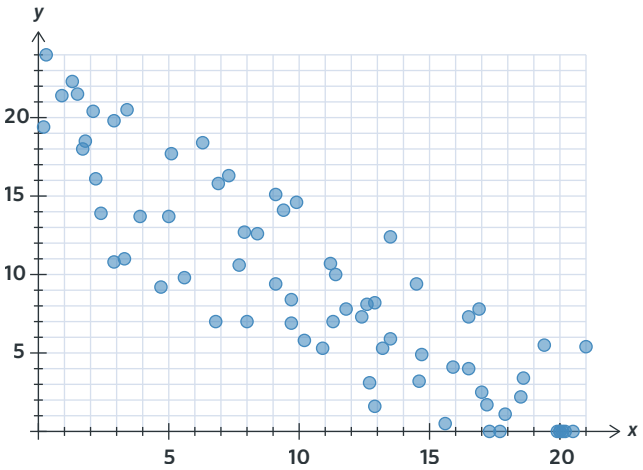
If the points follow the same line and are packed closely together, then a **strong relationship** exists between the two variables.



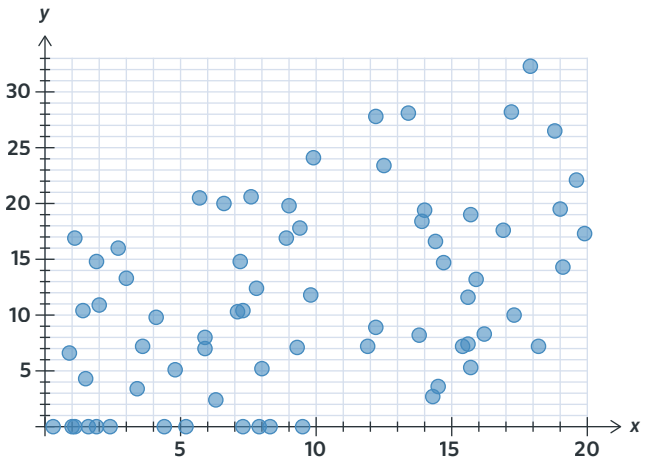


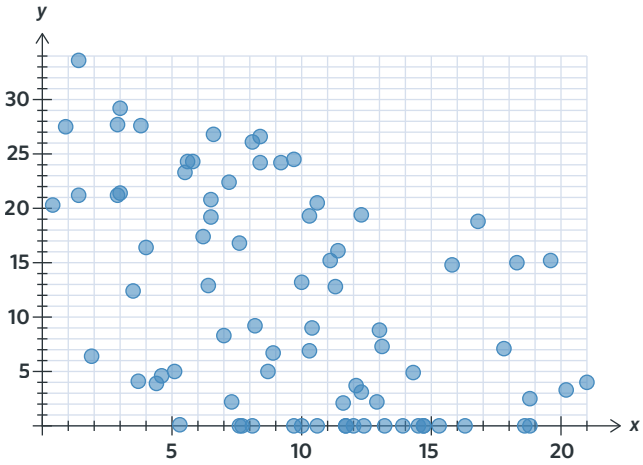
If the values are more spread out across the plot, but roughly follow the same line, then a **moderate relationship** exists between the two variables.



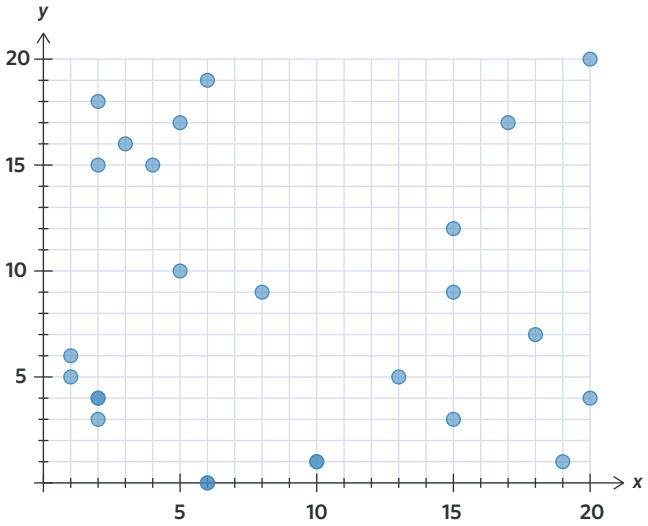


If the values are scattered across the plot, and a pattern exists but is barely visible, then a **weak relationship** exists between the two variables.



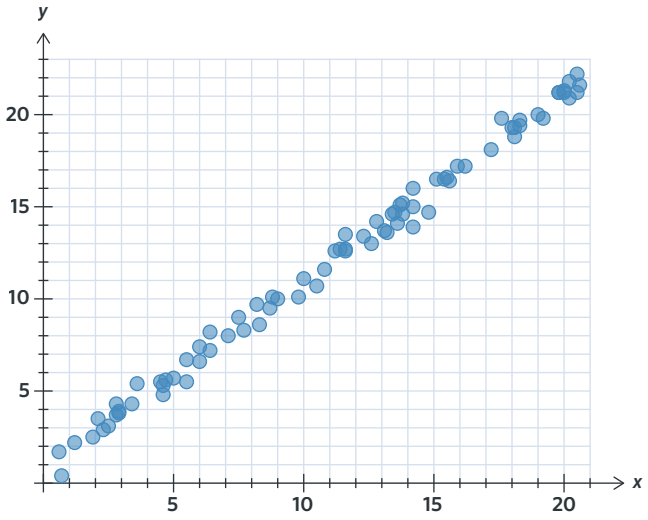


If the values follow no pattern at all – scattered across the graph randomly, then there is **no relationship** between the variables. The explanatory and response variables have no effect on each other.



The form of correlation

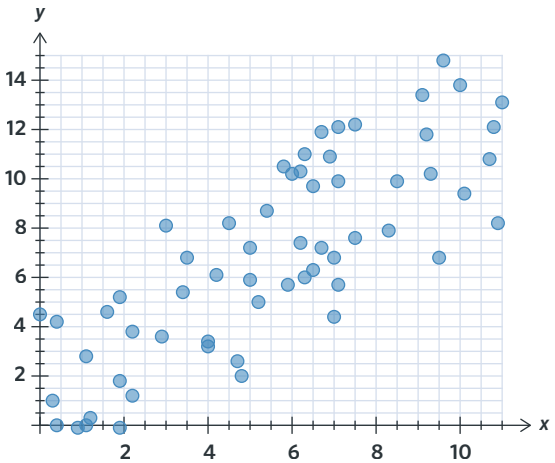
If the values follow a straight line, then the relationship between the variables is **linear**.



If the values follow a curved line, then the relationship is **non-linear**.

WORKED EXAMPLE 3 *(1 mark)*

Describe the correlation of the following scatterplot in terms of strength, direction and form.



SOLUTION

Step 1 Identify the direction of the correlation.

The values are following an upwards trend from left to right.

Therefore, the direction of the correlation is positive.

Step 2 Find the strength of the correlation.

Every value is following the same straight line.

The values are packed together with medium density.

Therefore, the strength of the correlation is moderate.

Step 3 Identify the form of the correlation.

The values follow a straight line.

Therefore the form of correlation is linear.

Step 4 Summarise your answers and make a conclusion.

The correlation between the two variables is moderate, positive, and linear.

Questions 13B *Correlation*

Refresher question

Q1. Is there a relationship between the following variables?

a) Playing video games and level of intelligence

b) Smoking regularly and having good health

c) Drink driving and being in car accidents

1. Identifying the relationship between variables

Q2. Application

The relationship between exercise and health is:

A. strong and positive.

B. weak and negative.

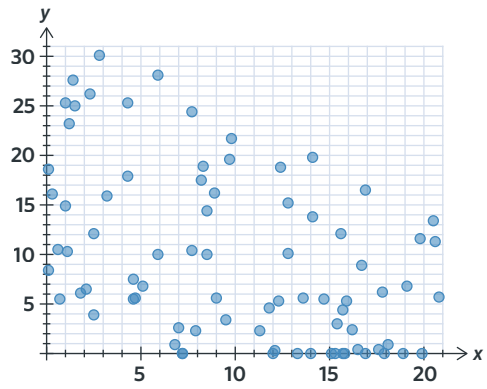
C. weak and positive.

D. weak and negative.

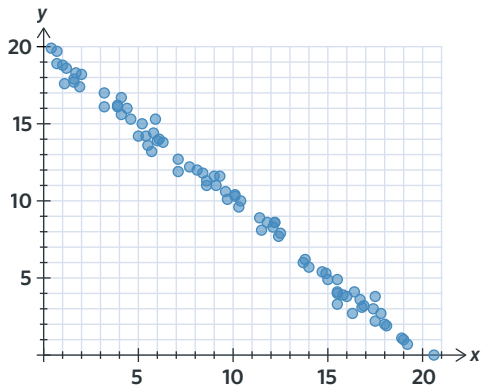
Q3. Skill

Describe the correlation of each of the scatterplots shown below, in terms of strength, direction and form.

a)



b)



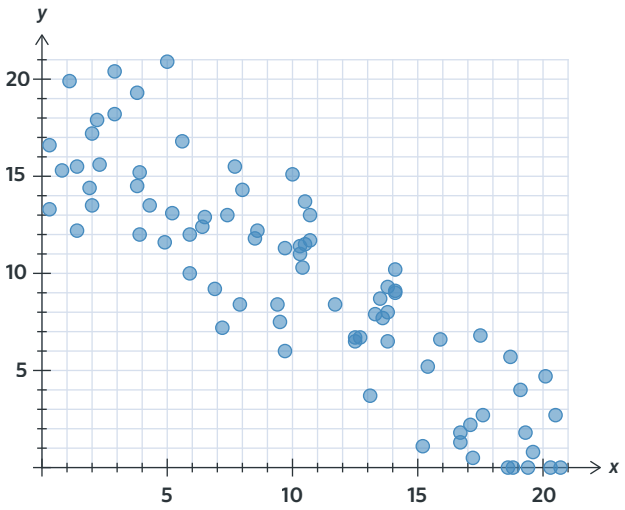
Q4. Application

Annie is observing the association between an individual's average driving speed in kilometres per hour and the length of their hair in centimetres. Draw an estimate of this association using a scatterplot, remembering to label the axes with appropriate titles.

Q5. Skill

After studying the following scatterplot for some time, Garen concluded that the correlation is weak, positive and non-linear.

Is Garen's conclusion correct? Why or why not?



Q6. Skill

Which of the following statements is false?

A. The explanatory variable is represented on the x-axis of a scatterplot and the response variable is represented on the y-axis.

B. A change in the explanatory variable will always cause a change in the response variable.

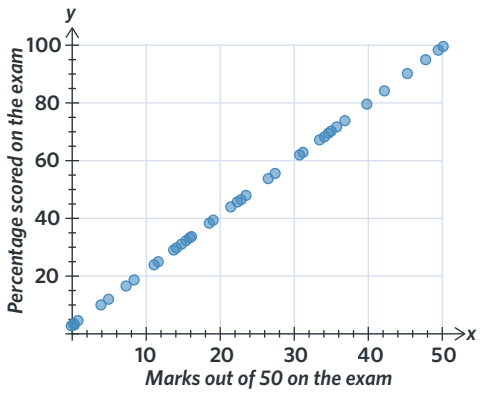
C. There can be more than one explanatory variable that affects the response variable.

D. It is possible for no relationship to exist between explanatory and response variables in terms of correlation.

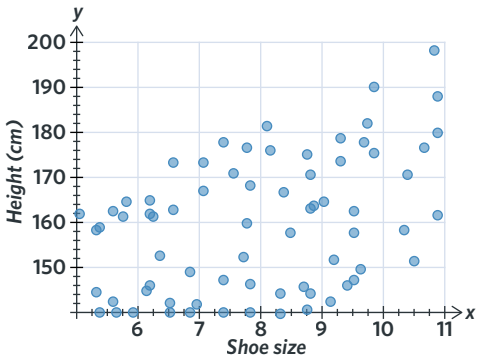
Q7. Application

Describe how the correlation seen in each scatterplot relates to the corresponding explanatory and response variables.

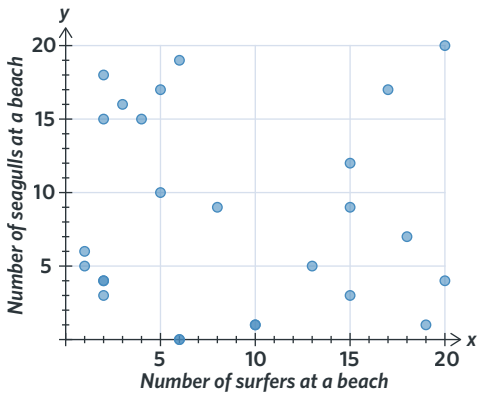
a)



b)



c)



Check your understanding

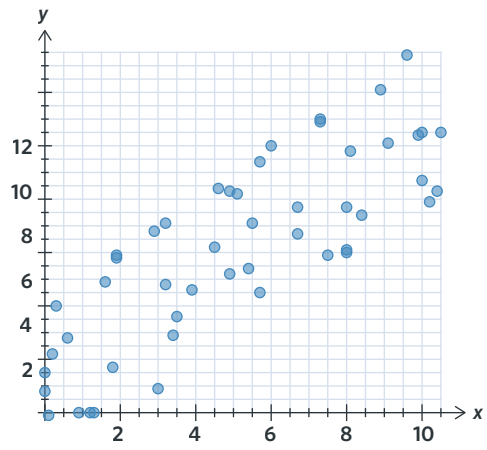
Q8. Application

John is organising a party in the North. The more attendees over 18 years old, the more cars there are in the parking lot. State the explanatory and response variables, and describe the direction of the correlation between them.

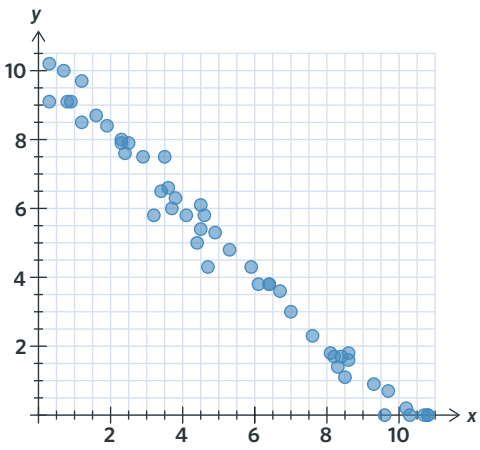
Q9. Skill

Which of the graphs below has the strongest relationship between x and y?

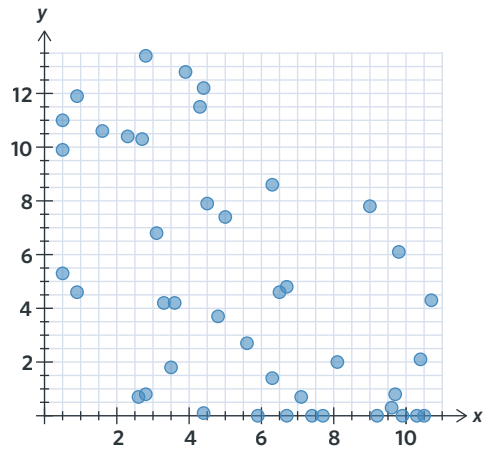
A.



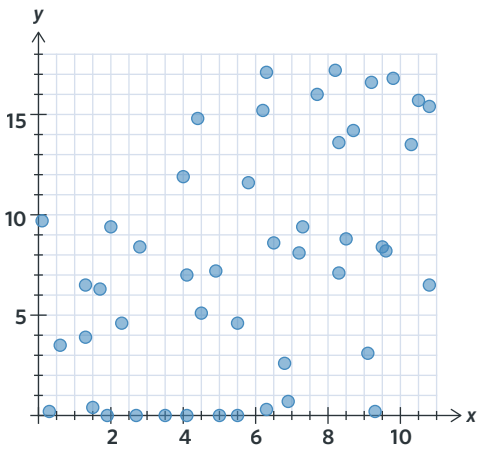
B.



C.



D.



Joining it all together

Q10. Application 4 marks

Quinn is participating in a tetris tournament.

a) Construct a scatterplot that accurately portrays the relationship between her opponent's experience in years (up to 10) and Quinn's respective win rate (%), with appropriate labels. Assume that there is a moderate negative correlation between the two variables, and that Quinn has about five years' experience of playing tetris. (2 marks)

b) Quinn progresses to the final round. Her last opponent is known to have 9 years of tetris experience. Predict whether the matchup will be favourable for Quinn, with reference to the scatterplot you created previously. (2 marks)

VCAA question

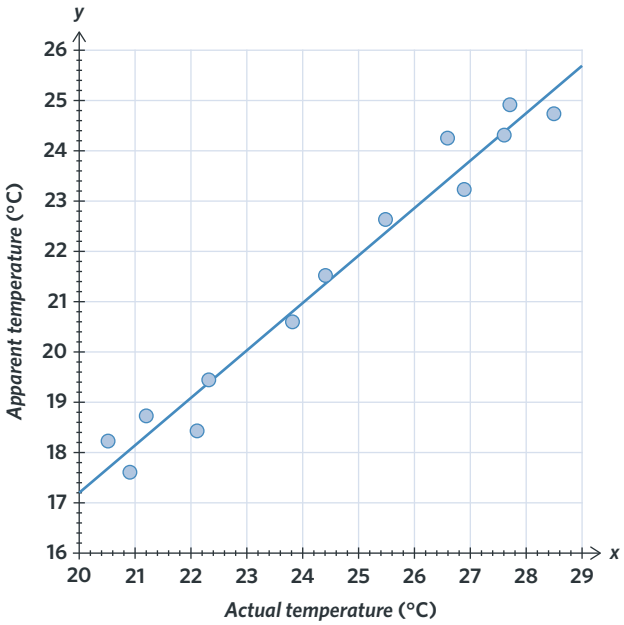
Q11. 1 mark

The data in the table below shows a sample of actual temperatures and apparent temperatures recorded at a weather station. A scatterplot of the data is also shown.

The data will be used to investigate the association between variables **apparent temperature** and **actual temperature**.

Use the scatterplot to describe the association between **apparent temperature** and **actual temperature** in terms of strength, direction and form.

Apparent temperature (°C)	Actual temperature (°C)
24.7	28.5
24.3	27.6
24.9	27.7
23.2	26.9
24.2	26.6
22.6	25.5
21.5	24.4
20.6	23.8
19.4	22.3
18.4	22.1
17.6	20.9
18.7	21.2
18.2	20.5



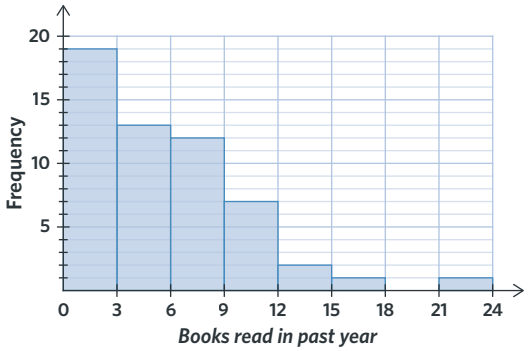
VCAA 2016 Exam 2, Core: Data analysis. Q3

Questions from multiple lessons

Q12. Difficulty:

1 mark

The following histogram displays the number of books read in the past year by 55 surveyed students.



The interquartile range for this distribution is

A. 2 books.

B. 3 books.

C. 4 books.

D. 6 books.

E. 8 books.

VCAA 2018NH Exam 1 Data analysis Q4 – Adapted

Q13. Difficulty:

1 mark

The first five terms of a sequence are $-17, -8, 1, 10, 19...$

Which of the following could be the recurrence relation that generates the above sequence?

A. $T_0 = -26, \quad T_{n+1} = \frac{1}{2}T_n$

B. $T_0 = -17, \quad T_{n+1} = -2T_n$

C. $T_0 = -17, \quad T_{n+1} = T_n - 9$

D. $T_0 = -26, \quad T_{n+1} = 10T_n$

E. $T_0 = -17, \quad T_{n+1} = T_n + 9$

VCAA 2017 Exam 1 Recursion and financial modelling Q19 – Adapted

Q14. A group of male and female powerlifters were surveyed on the number of hours they spend at the gym each week. Their responses are recorded in the following back-to-back stem plot.

Difficulty:

2 marks

Key: 1 | 0 = 10

Number of hours

Men						Women					
				0		4					
		8	5	5	0	6	8	8	9		
4	3	3	3	2	1	0	0	1	2	2	4
8	8	7	6	5	1	7	7	9			
	2	2	1	0	2	0	0	1			
				5	2						

a) Which group has a larger range? (1 mark)

b) What is the difference in the median of the two groups? (1 mark)

LESSON 13C

The Pearson correlation coefficient

The key skills you will learn in this lesson are:

1. Correlation and causality
2. Interpreting Pearson's correlation coefficient
3. Finding Pearson's correlation coefficient with CAS

VCAA key knowledge point:

"the Pearson correlation coefficient *r*, calculation and interpretation, and correlation and causation"

Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Correlation and causality

Recall from last lesson that correlation is a measure of how two variables are related. Causation, on the other hand, indicates that one event is the result of another event. It is important to realise that causation is not the same as correlation. Two variables A and B may be related to one another, but that does not necessarily mean A causes B.

In fact, the presence of a relationship between A and B may be due to a number of different reasons:

A third variable

Example

As ice-cream sales increase, the number of people at the beach increases.

So do ice-cream sales cause the number of people at the beach to increase?

No, this relationship is likely caused by a third variable — hot weather.

- Hot weather results in more people buying ice-cream.
- Hot weather results in more people going to the beach.

Confounding variables

Example

The less a student attends lectures, the higher the score they get on the exam.

So does attending less lectures cause the student to do better on the exam?

No, this relationship could be caused by a hidden factor.

- The student could watch the lectures online because the lecturer speaks too fast and the student is unable to take notes. By watching the lectures online, the student is able to pause the lecture and not miss any information.

Coincidence

Example

The more clothes an individual owns, the more hours they work per week.

So does owning more clothes cause the individual to work more?

No, but a data set might show a relationship like this due to coincidence.

It is only by chance there is a correlation between these two variables.

WORKED EXAMPLE 4 *(1 mark)*

The more time an athlete spent warming up, the worse they performed in a 100 metre race. Give a reason why this occurs.

SOLUTION

Step 1 Consider the possibility of a third variable influencing the results.

The third variable must influence both time spent warming up and the athletes performance in the race.

Time cannot be influenced by another variable, therefore a third variable does not exist for this case.

Step 2 Consider the possibility of a confounding variable.

A confounding variable is that the longer the athlete spent warming up, the more tired they became, and the worse they performed in the 100 metre race.

Step 3 Consider the possibility of coincidence.

If no solutions were reached in steps 1 and 2, with no evidence to support a causal relationship between the two variables, then the correlation in question is likely due to pure chance.

In this case, a confounding variable was found.

Step 4 Make a conclusion by summarising the above answers.

A plausible explanation is that the longer the athlete spent warming up, the more tired they became, and the worse they performed in the 100 metre race.

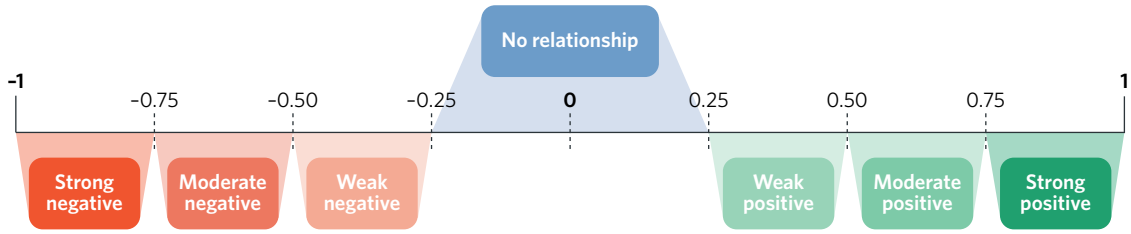
2. Interpreting Pearson's correlation coefficient

Recall that in the last chapter, terms such as strong, moderate, and weak were used to describe the strength of the relationship between two variables.

Pearson's correlation coefficient is a numerical measure of the strength of the linear relationship between two variables, denoted by r.

The properties of r are outlined below:

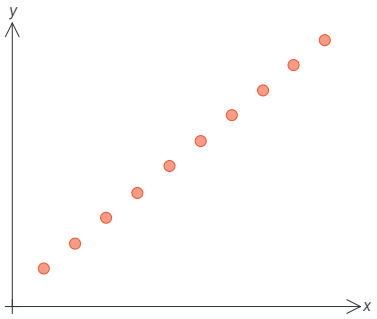
1. r must be between -1 and 1 $(-1 \leq r \leq 1)$
2. If r is positive, the direction of the relationship is positive.
3. If r is negative, the direction of the relationship is negative.
4. The closer r is to -1 or 1, the stronger the strength of the relationship. Likewise, the closer r is to 0, the weaker the strength of the relationship.



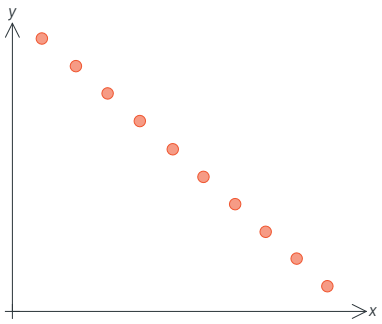
r-value	Type of relationship
$r = -1$	Perfect negative linear relationship
$-1 < r \leq -0.75$	Strong negative linear relationship
$-0.75 < r \leq -0.50$	Moderate negative linear relationship
$-0.50 < r \leq -0.25$	Weak negative linear relationship
$-0.25 < r \leq 0.25$	No relationship
$0.25 \leq r < 0.50$	Weak positive linear relationship
$0.50 \leq r < 0.75$	Moderate positive linear relationship
$0.75 \leq r < 1$	Strong positive linear relationship
$r = 1$	Perfect positive linear relationship

This new information can be applied to scatterplots and an r-value can be assigned to measure the strength of the relationship between variables.

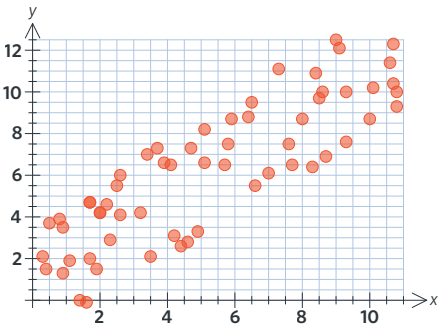
$r = 1$



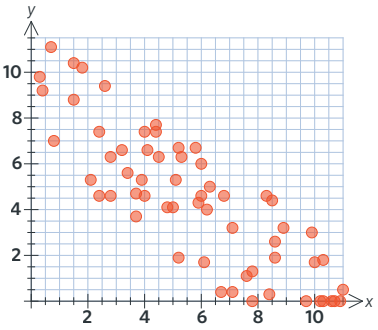
$r = -1$



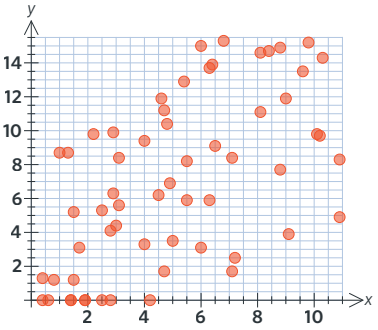
$r = 0.75$



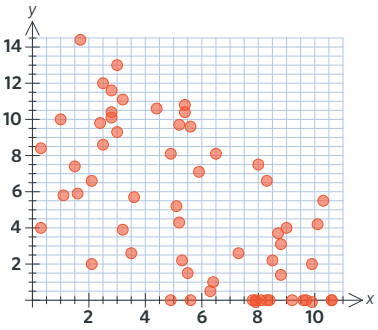
$r = -0.75$



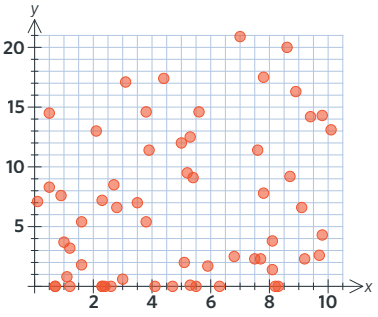
$r = 0.50$



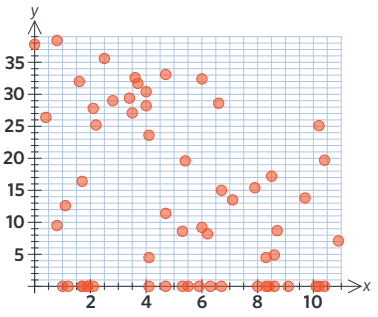
$r = -0.50$



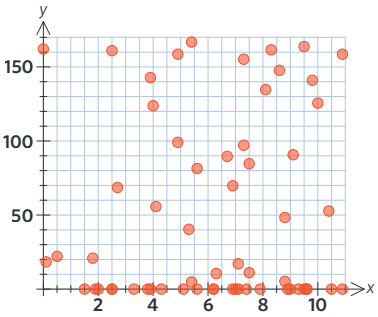
$r = 0.25$



$r = -0.25$



$r = 0$



WORKED EXAMPLE 5 *(1 mark)*

Bailey observed the relationship between the number of cars that passed his house and the number of events in his town that day. He found that $r = 0.78$. Explain what this value represents in terms of strength, direction, and form.

SOLUTION

Step 1 Determine the value of r, between -1 and 1, and refer to the diagram above for the approximate strength and direction of the correlation.

Between 0.75 and 1, a strong, positive relationship exists.

0.78 is between 0.75 and 1, therefore $r = 0.78$ also has a strong positive relationship.

Step 2 Recall that r measures the strength of linear relationships.

Therefore, the correlation is linear.

Step 3 Summarise the relationship by linking the above results together.

The correlation between the number of cars that pass Bailey's house on a Saturday and the number of events in his town that Saturday is strong, linear and positive.

3. Finding Pearson's correlation coefficient with CAS

Pearson's correlation coefficient r can be calculated by hand, using the formula:

$$r = \frac{1}{n-1}\sum\left(\frac{x-\bar{x}}{s_x}\right)\left(\frac{y-\bar{y}}{s_y}\right)$$

- n: number of data points
- x: an x-value
- y: a y-value
- $\bar{x}$: mean of the x-values
- $\bar{y}$: mean of the y-values
- s_x: standard deviation of the x-values
- s_y: standard deviation of the y-values

However, for General Mathematics, r will be found only using the calculator.

It is important to remember that r can only be calculated if the correlation is linear.

r is also heavily influenced by the presence of outliers in small data sets.

WORKED EXAMPLE 6 *(1 mark)*

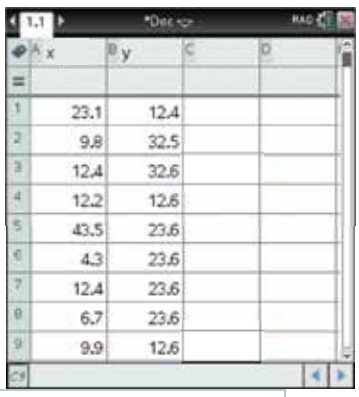
Calculate the Pearson correlation coefficient of the following data set.

x	23.1	9.8	12.4	12.2	43.5	4.3	12.4	6.7	9.9
y	12.4	32.5	32.6	12.6	23.6	23.6	23.6	23.6	12.6

SOLUTION: TI-NSPIRE

Step 1 Open a new spreadsheet page by pressing ctrl + doc, then 4 ('Lists and Spreadsheets').

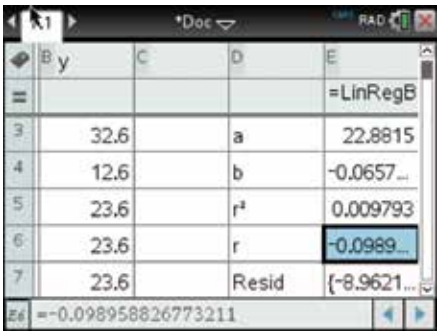
Step 2 In the top row, label column A as x and column B as y. Enter the data into the corresponding x and y columns as shown on the right.



Step 3 Press menu then 4 ('Statistics') then 1 ('Stat calculations') and then 4 ('Linear Regression (a+bx)').

Step 4 In the 'X List' and 'Y List' entries, select the titles of the columns you created previously.
In this case, it will be x and y. This generates the screen below.

Step 5 Press enter to yield a list of values for the data set. Scroll down to find the value of r, and read off the answer.



We find the r-value is -0.098959. Rounded to 2 decimal places, this is -0.10.

SOLUTION: CASIO CLASSPAD

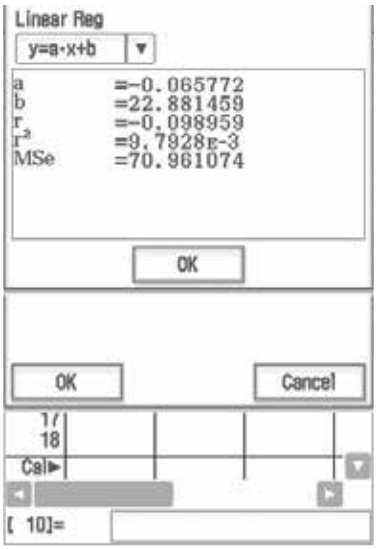
Step 1 From the menu, select Statistics.

Step 2 Input the x-values into 'list1' and the y-values into 'list2'.

	list1	list2	list3
1	23.1	12.4	
2	9.8	32.5	
3	12.4	32.6	
4	12.2	12.6	
5	43.5	23.6	
6	4.3	23.6	
7	12.4	23.6	
8	6.7	23.6	
9	9.9	12.6	
10			
11			
12			
13			
14			
15			
16			
17			
18			
Cal▶			

Step 3 Using the stylus, Select 'Calc', then 'Regression', and finally 'Linear Regression'.

Step 4 Press EXE. This produces the screen below. Read off the r-value from the answer screen.



We find the r-value is -0.098959. Rounded to 2 decimal places, this is -0.10.

Questions 13C *The Pearson correlation coefficient*

Refresher question

Q1. Describe the strength of the relationship between the number of cars on the road and the number of clouds in the sky.

A. Strong

B. Moderate

C. Non-existent

1. Correlation and causality

Q2. Skill
If there is a strong linear relationship between two variables, does this mean one variable causes the other?

Q3. Application
Michael notices the more popular a television show is, the more unfavourable reviews it has. Which of the following options could be a reason why this relationship exists?

A. The more popular a television show is, the more disliked it is.

B. The more popular a television show is, the more liked it is.

C. The more popular a television show is, the more reviews it has, both favourable and unfavourable.

D. The more popular a television show is, the more favourable reviews it has.

Q4. Application
The more cheeseburgers sold, the greater the height of the tide that evening. Why might this correlation exist?

A. A third variable

B. A confounding variable

C. Selling more cheeseburgers causes the tide to rise

D. Coincidence

Check your understanding

Q5. Application
A study claims that the sweeter the snack, the more energy it contains. Jeremy decided to test this claim by eating very sweet candy. However, Jeremy felt tired immediately after.
What is a possible confounding variable?

2. Interpreting Pearson's correlation coefficient

Q6. Skill
The Pearson correlation coefficient must lie between what values?

A. 1 and 10

B. 0 and 1

C. -1 and 1

D. 0 and 10

Q7. Skill
Describe the strength, direction and form of the following correlation coefficients.

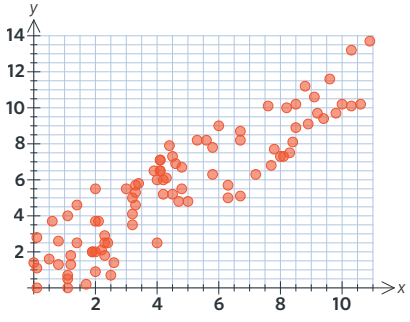
a) $r = 0.5$

b) $r = 0.69$

c) $r = -0.20$

Check your understanding

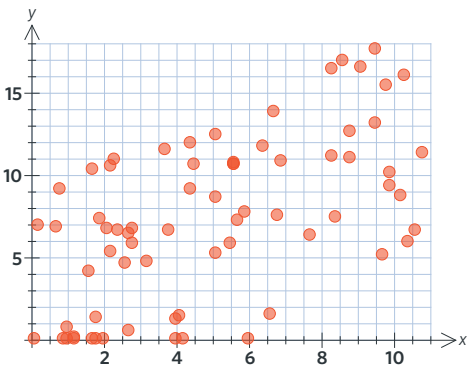
Q8. Application
For the scatter plot on the right, Travis estimated that the correlation coefficient must be around 0.8. Is he correct? Why or why not?



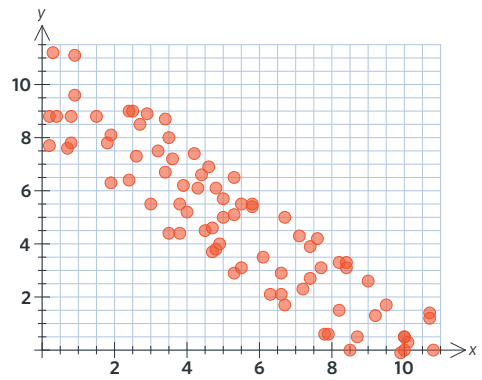
Q9. Skill

a) Predict which scatterplot will have the larger correlation coefficient.

A.



B.



b) Estimate the approximate r values for each scatterplot.

3. Finding Pearson's correlation coefficient with CAS

Q10. Skill

What is Pearson's correlation coefficient a measure of?

A. The strength of the non-linear relationship between two variables

B. The strength of the linear relationship between two variables

C. The form of the relationship between two variables

D. The impact of one variable on another

Q11. Skill

Using a calculator, find Pearson's correlation coefficient r for the following data sets, correct to two decimal places.

a)

x	24.1	43.1	12.2	23.1	86.4	32.4	0.9	1.1	4.5
y	12.4	32.5	65.6	101.3	109.2	107.8	23.4	23.6	12.6

b)

Income per annum y	100 000	123 600	280 900	350 000	193 230	80 125	56 010
University GPA x	6.5	5.4	6.7	7.0	3.4	2.5	3.2

c)

Number of hours spent exercising per week	23.0	25.6	40.1	32.5	11.0	8.6	15.4	10.0
Average number of hours of sleep per night	8.2	9.2	15.6	12.6	6.3	5.4	7.0	8.9

Check your understanding

Q12. Application

Sam is researching a disease called Whitescale. He conducted a trial with eight subjects, and observed that the weaker the host's body, the more severe the symptoms of Whitescale.

Strength of host's body (0–10)	1.2	9.8	7.6	2.4	4.5	5.8	6.4	0.1
Severity of Whitescale (0–10)	9.7	0.4	4.3	8.8	7.1	6.5	5.3	9.3

a) Calculate the Pearson correlation coefficient for this data set, correct to two decimal places, and verify Sam's observation.

b) A new subject, Jora, contracted Whitescale with a severity level of 10. Sam found that Jora did not have any physical strength. Sam decided to conduct the trial again, now with 9 subjects, including Jora. Without doing any calculations, predict whether the value of r for this second trial will have greater or less **strength** than the first trial.

c) Explain the reason for your prediction in part b).

Joining it all together

Q13. At a fast food restaurant, the following poster is displayed on the wall.

Application
3 marks

a) Predict the Pearson correlation coefficient value for the scatterplot and explain why you chose that value, incorporating strength, direction and form in your answer.

b) Matt, the chef at the restaurant, measured the size of his patties after he cooked them and seven customers were asked to rate their satisfaction with the meal on a scale from 0 to 20. The following data set was obtained.

Diameter of burger patty (cm)	15.7	20.3	16.7	18.4	1.2	10.0	18.3
Customer satisfaction level (0-20)	19.0	20.0	16.2	18.5	0	11.1	19.0

Find the Pearson correlation coefficient, r.

c) Is the above an example of cause or correlation? Explain your reasoning.

VCAA question

Q14. The table shows the hourly rate of pay earned by 10 employees in a company in 1990 and 2010.

1 mark

The value of the correlation coefficient, r, for this set of data is closest to

A. 0.74

B. 0.86

C. 0.92

D. 0.93

E. 0.96

VCAA 2013 Exam 1, Core: Data analysis. Q8

Employee	**Hourly rate of pay ($)**	
	1990	**2010**
Ben	9.53	17.02
Lani	9.15	16.71
Freya	8.88	15.10
Jill	8.60	15.93
David	7.67	14.40
Hong	7.96	13.32
Stuart	6.42	15.40
Mei Lien	11.86	19.79
Tim	14.64	23.38
Simon	15.31	25.11

Questions from multiple lessons

Q15.

Difficulty:

1 mark

A frozen drinks store is monitoring their sales over the summer. They notice that the higher the temperature on a particular day, the more drinks that are sold.

State the explanatory and response variables in this situation and the direction of correlation between the variables.

A. Explanatory variable: *temperature*
Response variable: *drinks sold*
Positive correlation

B. Explanatory variable: *drinks sold*
Response variable: *temperature*
Positive correlation

C. Explanatory variable: *temperature*
Response variable: *drinks sold*
Negative correlation

D. Explanatory variable: *drinks sold*
Response variable: *temperature*
Negative correlation

E. No relationship

Q16.

Difficulty:

1 mark

A rare coin was bought for $4500 in 2009.

In 2019, it sold at auction for $6000.

What is the increase in value, as a percentage of the original purchase price? Round your answer to one decimal place.

A. 3.3%

B. 25.0%

C. 33.3%

D. 66.7%

E. 75.0%

VCAA 2015 Exam 1 Module 4: Business-related mathematics Q2 – Adapted

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Q17.

Difficulty:

2 marks

Sonya is starting a 30-day skipping program. She records the *time spent skipping* (mins) per day over the first six days, as well as the number of *calories burned*.

***Time spent skipping* (mins)**	5	7	3	8	12	4
Calories burned	51	72	30	78	122	39

a) Create a scatterplot using the data above. (1 mark)

b) Describe the correlation in terms of strength, direction and form. (1 mark)

LESSON 13D

Least squares regression

The key skills you will learn in this lesson are:

1. Least squares regression model
2. Interpretation of regression lines

VCAA key knowledge point:

"use of the least squares line to model an observed linear association and the interpretation of its intercept and slope in the context of the data"

1. Least squares regression model

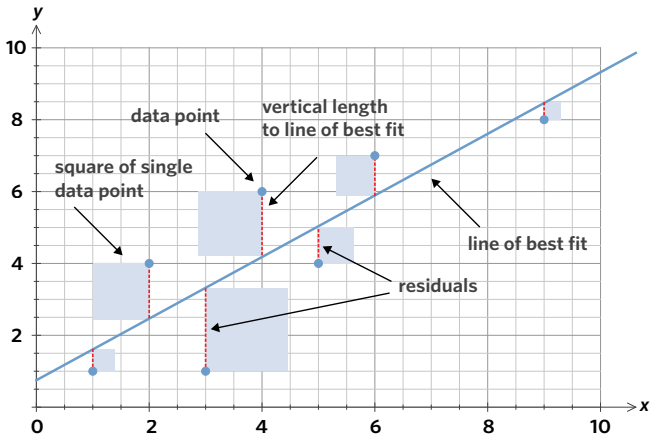
The least squares regression model builds on the 'line of best fit by eye' covered in chapter 9.

Using the mean, standard deviation and Pearson's correlation coefficient for the two variables, the process of least squares regression creates a very accurate line of best fit for the data.

It works by minimising the total area of squares that have side lengths equal to the vertical distance between the points and the line, hence the name 'least squares regression model'.

The difference between the data points and the line of best fit are known as the residuals.

Outliers must be discarded for the model to be effective.



The line that creates the minimum summed area of the squares is the line of best fit.

Least squares regression by hand

The line of best fit is in the form:

$$y = a + bx$$

where x is the explanatory variable and y is the response variable.

The slope b is given by:

$$b = \frac{r s_y}{s_x}$$

The intercept a is given by:

$$a = \bar{y} - b\bar{x}$$

- s_x and s_y are the standard deviation of variable x (explanatory) and variable y (response).
- $\bar{x}$ and $\bar{y}$ are the mean of variable x (explanatory) and variable y (response).
- r is the Pearson's correlation coefficient for the two variables.

Once a and b are found, they are substituted into the first equation, creating a linear equation. Then x and y are replaced with the explanatory and response variables respectively.

Calculating the line of best fit using the least squares regression model by hand is not necessary in this course, but computing it using CAS is a required skill.

Least squares regression using technology

WORKED EXAMPLE 7 *(1 mark)*

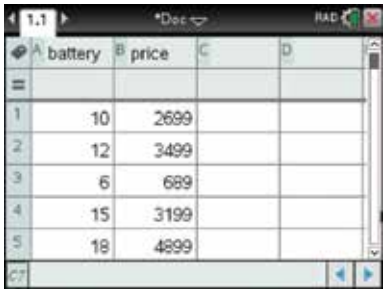
Calculate the line of best fit, in terms of the explanatory and response variables, using the following data. Use 'battery life' as the explanatory variable. Give all values to 3 decimal places.

Battery life (hours)	10	12	6	15	18	14	8	10
Price of laptop ($)	2699	3499	689	3199	4899	2399	1299	1499

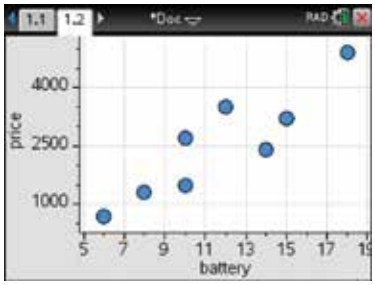
SOLUTION: TI-NSPIRE

Step 1 Open a new page by pressing ctrl + doc, then 4 ('Add Lists & Spreadsheets').

Step 2 Label column A as the explanatory variable 'battery' and column B as the response variable 'price', and enter the data into the subsequent columns.

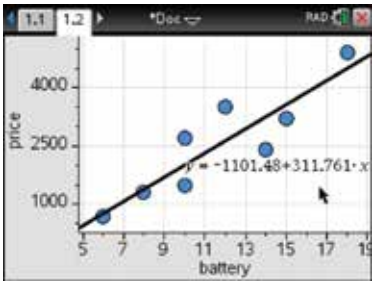


Step 3 Press ctrl + i and press 5 ('Add Data & Statistics'). Add the explanatory variable 'battery' to the horizontal axis and the response variable 'price' to the vertical axis. A scatter plot should appear.



Step 4 Press menu, then 'Analyze', 'Regression', 'Show linear (a+bx)' to display the regression line on the scatterplot.

The equation of the regression line will be displayed alongside the line.



Step 5 Write the equation, and replace x and y with 'battery life' and 'price' respectively.

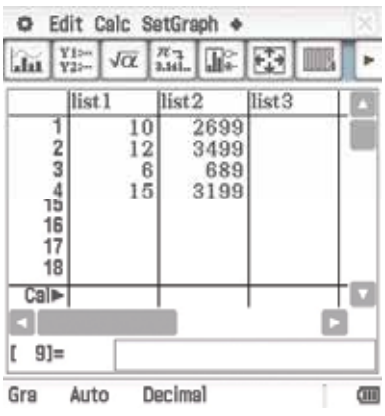
$y = -1101.475 + 311.761x$

$price = -1101.475 + 311.761 \times battery\ life$

SOLUTION: CASIO CLASSPAD

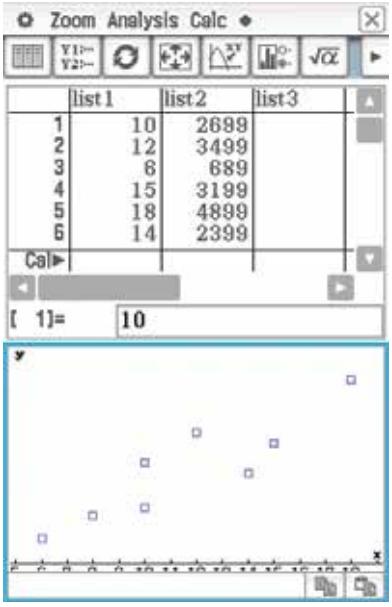
Step 1 Open the Statistics application.

Step 2 Enter the values of battery life under 'list 1' and the values of price under 'list 2'.



Step 3 Press , choose the following settings and then press set.

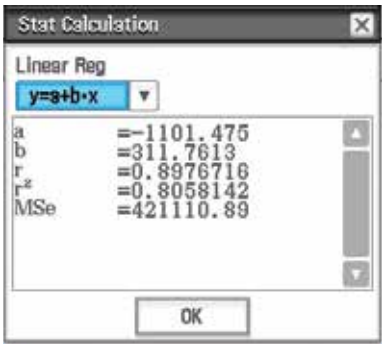
Step 4 Press and a scatterplot will appear.



Step 5 Tap 'Calc', 'Regression', 'Linear Reg', and a 'Set Calculation' screen will appear. Press OK.

A 'Stat Calculation' screen will appear, showing the values of a and b

Remember to change the 'Linear Reg' into $y = a + bx$ form.



Step 6 Round the values of a and b to three decimal places, write the equation of the regression line, and substitute in the variables.

$a = -1101.475$

$b = 311.761$

$price = -1101.475 + 311.761 \times battery\ life$

Step 7 Press OK to plot the regression line on the scatterplot previously created.

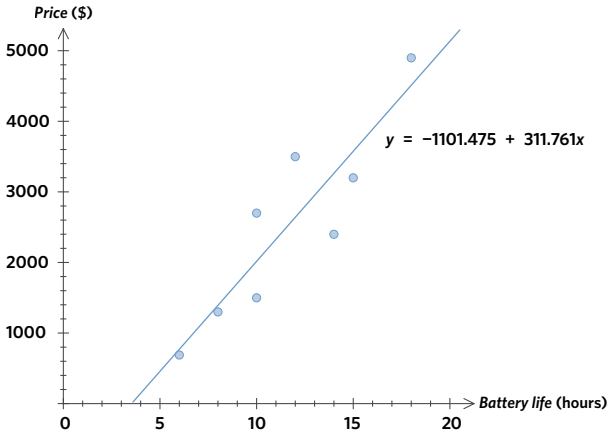
2. Interpretation of regression lines

When creating a regression line $y = a + bx$, the values of a and b provide information about the data.

- The value of b predicts the change in the response variable for a change of one unit in the explanatory variable, otherwise known as the slope.
- The value of a predicts the value of the response variable when the explanatory variable is equal to zero, otherwise known as the y-intercept.

WORKED EXAMPLE 8 *(1 mark)*

Using the data from the previous example, find how much it will cost a customer to buy a phone with an extra hour of battery life.



SOLUTION

Step 1 Find the equation of the regression line.

Previously worked out: $price = -1101.475 + 311.761 \times battery\ life$

Step 2 Recognise the key information in the question.

The question is asking about the slope of the regression line and therefore the value of b

The change in the value of the vertical axis (price) when the horizontal axis (battery life) increases by one unit is the slope b of the regression line.

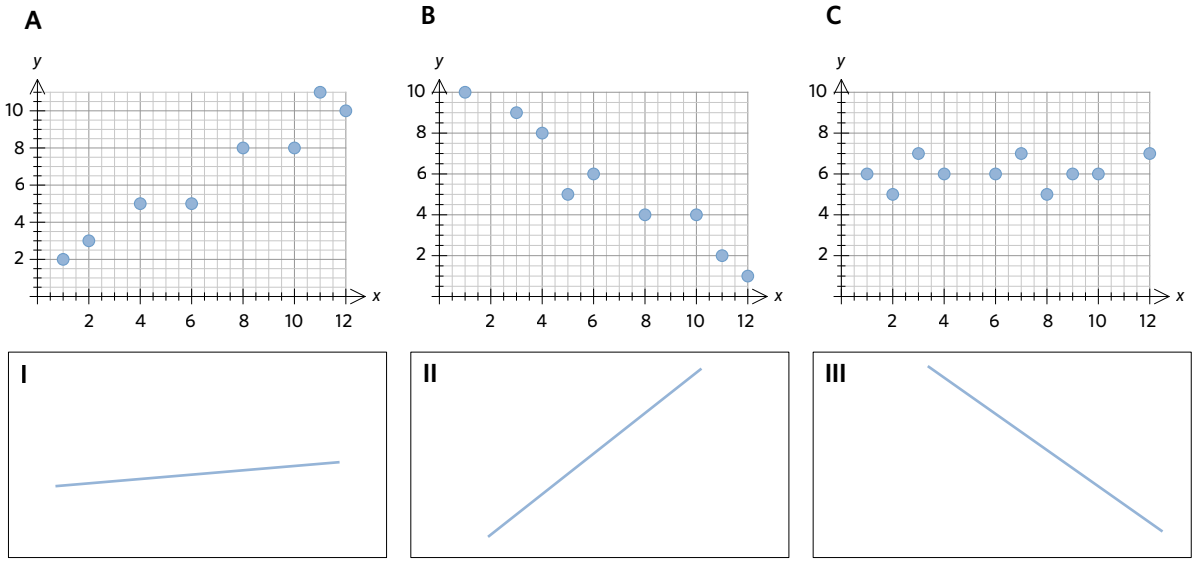
Step 3 Interpret the regression line.

The slope b of the regression line is 311.761. Therefore a consumer will pay $311.76 for an extra hour of battery life.

Questions 13D *Least squares regression*

Refresher question

Q1. Match the distribution to the line that best conveys its trend.



1. Least squares regression model

Q2. Use the following data to answer question 2.

Skill

x	14	8	3	10	17	16	12	4	6	11
y	13	7	2	11	15	17	13	5	6	10

a) Create a scatterplot using this data.

b) Estimate a regression line in terms of x and y

c) Find the actual regression line, using the least squares model, in terms of x and y.
Give all values within your answer to three decimal places.

Q3. The driver of an ice cream van wants to compare the maximum temperature of each day for a week during summer to his number of sales.

Application

Max temperature (°C)	26	29	34	28	31	42	36
Sales	28	53	94	50	76	153	107

a) Find the regression line, using the least squares model, in terms of x and y.
Give all values within your answer to three decimal places.

b) Find the regression line, using the least squares model, in terms of the two variables.
Give all values within your answer to three decimal places.

Check your understanding

Q4. A principal aims to collect data regarding her pupils' interests while at school. She picks ten random students for a survey, and asks how many hours they spend doing art work and maths work throughout a particular week. The following table displays the data she collected.

Application

Hours of maths work completed	0	8	4	15	3	7	10	2	0	12
Hours of art work completed	7	2	5	16	6	3	0	9	1	1

a) Create a scatterplot with this data, using hours of maths work completed as the explanatory variable.

b) Calculate the regression line of the data, using the least squares model, in terms of the variables.
Give all values within your answer to three decimal places.

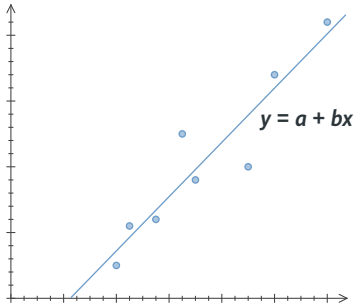
c) There are two outliers present in this data. Recalculate the regression line, in terms of the variables, after removing the two outliers. Give all values within your answer to three decimal places.

2. Interpretation of regression lines

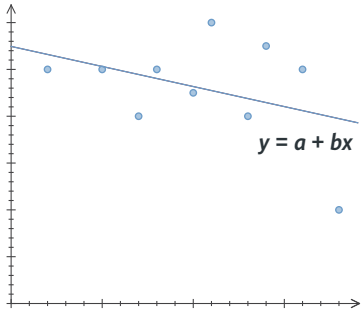
Q5. Identify, for each of the regression lines below, whether the values of a and b will be positive or negative.

Skill

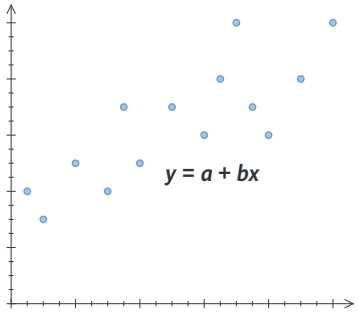
a)



b)



c)

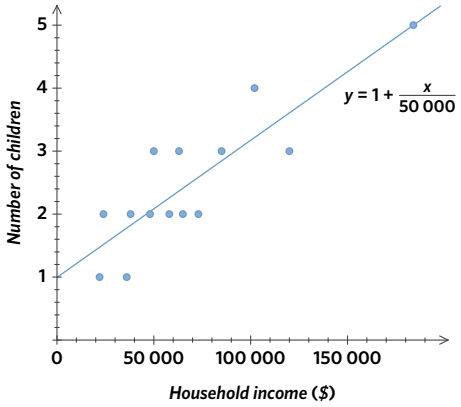


Q6. Application

The following scatterplot displays the relationship between a household's income and the number of children within the household.

a) Give the regression line in terms of the two variables with the value of b in decimal form.

b) According to the regression line, how many children would a household without an income have?



Check your understanding

Q7. Application

The regression line of the relationship between an amateur football team's average number of players attending training and their total season wins is:

$total\ wins = 4 + 0.5 \times training\ attendance$

a) One particular team decides training isn't worth the time, and cancels it altogether. According to the regression line, how many wins would they expect to have throughout the season?

b) How many wins does each player add to the tally, on average, purely by turning up to training each week?

Joining it all together

Q8. Application 3 marks

The following data displays the share (%) of transport that cyclists take up in a sample of European cities, and the pollution index of those cities. The pollution index rates cities out of 100 based on how polluted their air is. A pollution index of 100 means the city is extremely polluted.

Cyclists' share (%)	5	2	13	3	1	35	2	32
Pollution index	65.92	59.92	42.23	65.14	67.17	21.30	53.51	30.06

a) Find the regression line, in terms of the variables, using the least squares regression model. Give all values within your answer to three decimal places.

b) According to the regression line, what would be the pollution index of a European city without any cyclists? Give your answer to three decimal places.

c) According to the regression line, what is the change in the pollution index for every extra percent of cyclists' share within a European city? Give your answer to three decimal places.

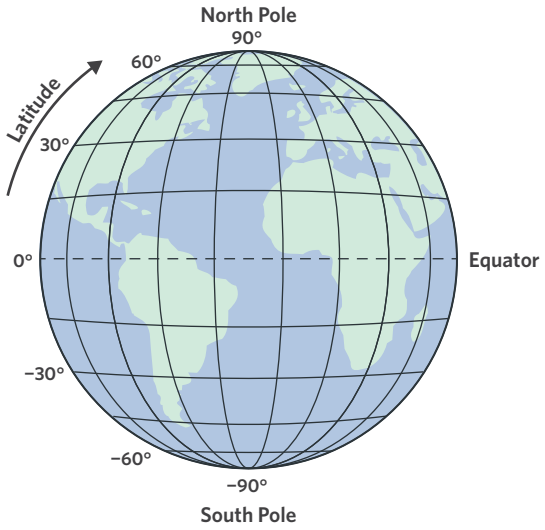
Q9. Application 3 marks

The following data displays the average daily maximum temperatures for the major cities in Australia, and the degrees latitude (South) that the cities are situated.

Latitude is a measure of distance from the equator, but it is measured in angles.

Latitude has a value of 0° at the equator, 90°N at the north pole and 90°S at the south pole.

The higher the magnitude of the latitude, the further away the location is from the equator.



City	Sydney	Melbourne	Brisbane	Adelaide	Perth	Hobart	Darwin	Canberra
Average daily maximum temperature (°C)	22.3	20.1	25.3	22.1	24.5	17.2	32.1	19.7
Latitude (°S)	33.87	37.81	27.47	34.93	31.95	42.82	12.46	35.28

a) Find the regression line, in terms of the variables, using the least squares regression model. Give all the values within your answer to three decimal places.

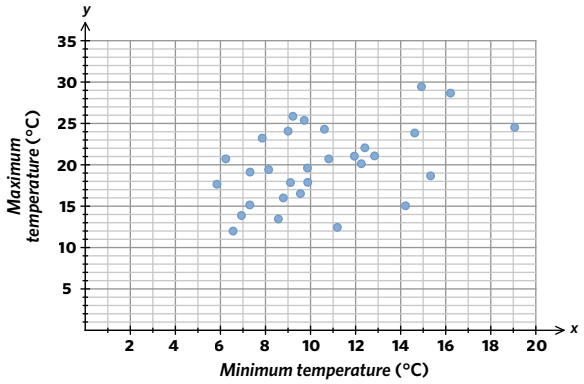
b) According to the regression line, what is the change in temperature for every degree of latitude travelled south? Give your answer to three decimal places.

c) According to the regression line, what would the average daily temperature be at the equator? Give your answer to three decimal places.

VCAA questions

Q10. 2 marks

The maximum temperature and the minimum temperature at this weather station on each of the 30 days in November 2011 are displayed in the scatterplot below.



The correlation coefficient for this data set is $r = 0.630$.

The equation of the least square regression line for this data set is
$maximum\ temperature = 13 + 0.67 \times minimum\ temperature$

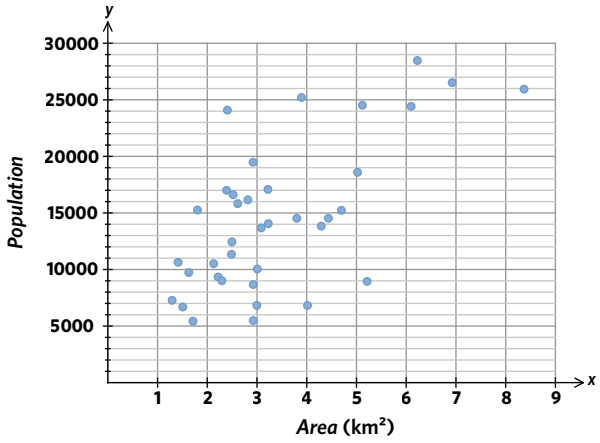
a) Interpret the vertical intercept of the least squares regression line in terms of maximum temperature and minimum temperature.

b) Interpret the slope of the least squares regression line in terms of maximum temperature and minimum temperature.

Adapted from VCAA 2012 Exam 2, Core. Q2 b,d

Q11. 2 marks

The scatterplot below shows the population and area (in square kilometres) of a sample of inner suburbs of a large city.



The equation of the least squares regression line for the data in the scatterplot is
$population = 5530 + 2680 \times area$

Interpret the slope of this least squares regression line in terms of the variables 'area' and 'population'.

Adapted from VCAA 2014 Exam 2, Core. Q2c

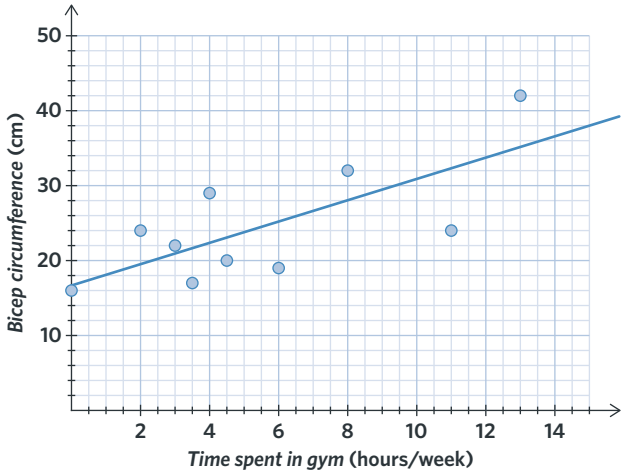
Questions from multiple lessons

Q12.

Difficulty:

1 mark

The following scatterplot and least squares regression line show the relationship between the time spent in the gym per week and bicep circumference.



By inspection, the value of the correlation coefficient (r) for this data is closest to

A. 0.98

B. 0.73

C. 0.16

D. −0.45

E. −0.83

VCAA 2010 Exam 1 Data analysis Q7 – Adapted

Q13.

Difficulty:

1 mark

Kingsley invests $25 000 at 1.5% per annum with interest compounding monthly.

The value of Kingsley's investment at the end of the fifth month is closest to

A. $25 157

B. $25 218

C. $26 932

D. $45 051

E. $50 284

VCAA 2014 Exam 1 Module 4: Business-related mathematics Q8 – Adapted

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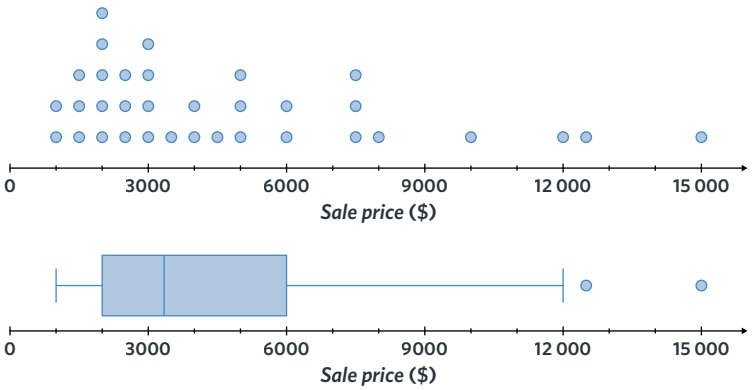
Q14.

Difficulty:

2 marks

Nick and Vickie are artists. Together they sold 34 artworks last financial year. The distribution of the sale prices of these artworks is shown in the dot plot and boxplot.



a) Describe the shape of the distribution of the sale prices of the 34 artworks. (1 mark)

b) Determine the value of the lower fence. (1 mark)

VCAA 2018 Exam 2 Data analysis Q1 – Adapted

LESSON 13E

Predictions and limitations of models

The key skills you will learn in this lesson are:

1. Interpolation and extrapolation
2. Limitations of regression line predictions

VCAA key knowledge point:

"use of the model to make predictions and identify limitations of extrapolation"

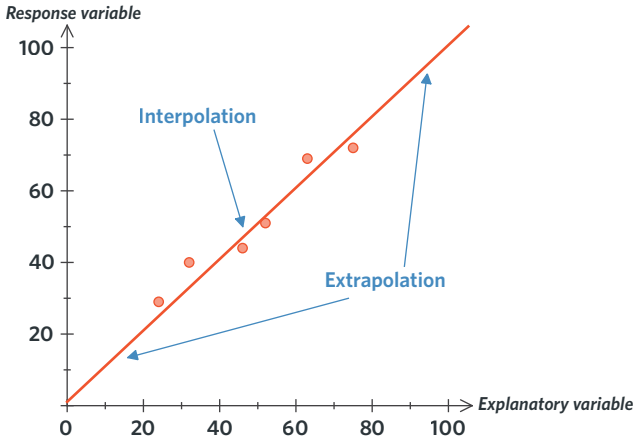
Mathematics Area of Study key knowledge points derived from VCE Mathematics Study Design 2016-2020 p.22; © The Victorian Curriculum and Assessment Authority (VCAA). Used with permission.

1. Interpolation and extrapolation

When a regression line is found, predictions can be made by substituting values of the explanatory variable or response variable into the equation.

Interpolation is the process of predicting values that are within the domain of the data (between the smallest and largest value of the explanatory variable). This was previously covered in lesson 9D.

Extrapolation is the process of predicting values that are outside the domain of the data (smaller than the smallest value or larger than the largest value of the explanatory variable). This was previously covered in lesson 9E.



WORKED EXAMPLE 9 *(2 marks)*

When producing between 0 and 8000 products, a factory's costs take the equation:

$costs = 4500 + 3 \times products\ produced$

a) The factory spent \$34 500. How many products did it produce?

b) Was this prediction made using interpolation or extrapolation?

SOLUTION

a) The factory spent \$34 500. How many products did it produce?

Step 1 Substitute \$34 500 into the equation.

$34\,500 = 4500 + 3 \times products\ produced$

Step 2 Find the number of products produced by the factory.

$34\,500 = 4500 + 3 \times products\ produced$

$30\,000 = 3 \times products\ produced$

$products\ produced = \frac{30\,000}{3}$

$products\ produced = 10\,000$

b) Was this predication made using interpolation or extrapolation?

Step 1 Is the predicted value within or outside the domain of the data?

This regression line is based upon data values between 0 and 8000 products produced. The prediction was that 10 000 products would have been produced if the cost was \$34 500. This prediction is outside the domain of the data.

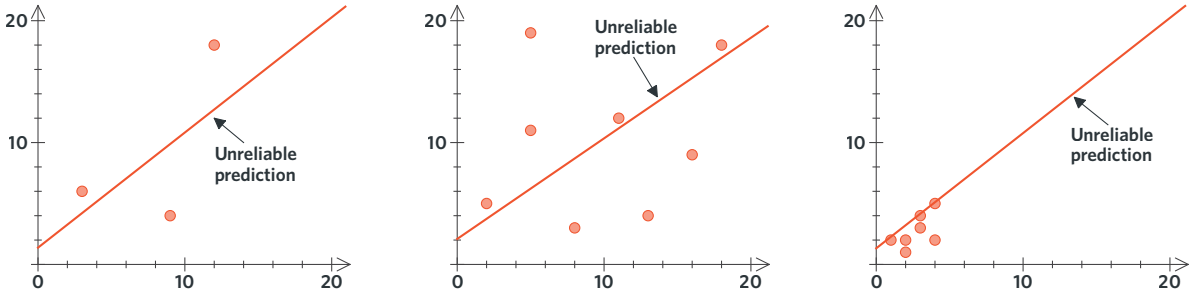
Step 2 Determine whether the prediction was made using interpolation or extrapolation.

Because the predicted value is outside the domain of the data, the prediction was made using extrapolation.

2. Limitations of regression line predictions

Predictions based upon regression lines are not always accurate.

- A regression line based upon a small sample of data is unreliable as the data can contain biases or lack diversity. The larger a data set is, the less likely it is that these problems will occur.
- A regression line based upon data with a Pearson's correlation coefficient between -0.5 and 0.5 will provide little insight into the relationship between variables, as a prediction based upon a weak trend cannot be reliable.
- When extrapolating, predictions may not be accurate as they are outside the domain of data, and therefore are based on a trend that is not proven to continue over all possible values. Extrapolating can provide some level of insight if relatively close to the domain of the data set.



WORKED EXAMPLE 10 *(1 mark)*

Interpolation was used to predict a point within a data set of 1000 values. The data has a Pearson's correlation coefficient value of 0.39. Why might this prediction be unreliable?

SOLUTION

Step 1 Check for sample size.

1000 participants is a large sample size, and therefore reliable.

Step 2 Check for correlation.

The value of r is 0.39. This is a weak positive correlation and therefore makes the prediction unreliable.

Step 3 Check that the prediction is within, or close to the domain of the data.

This is an example of interpolation, which is reliable.

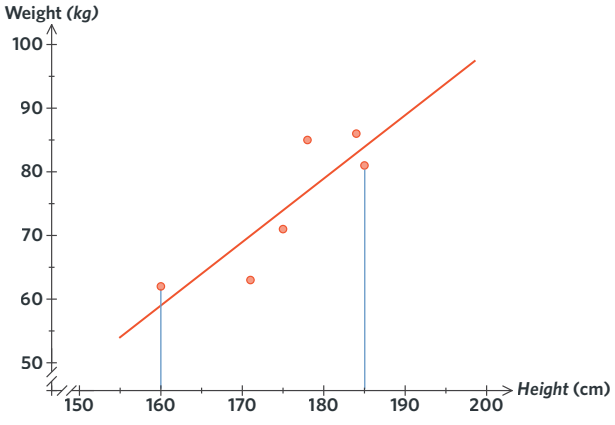
Step 4 Summarise.

The prediction may be unreliable due to the weak correlation between the variables.

Questions 13E *Predictions and limitations of models*

Refresher question

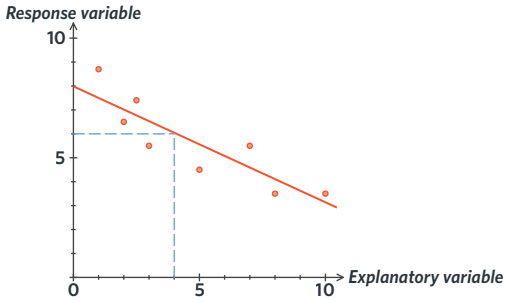
Q1. What is the domain of the following data set?



1. Interpolation and extrapolation

Q2. Skill

Predict the value of the response variable from the following regression line, given the value of the explanatory variable is four.



Q3. Skill

Predict the value of the explanatory variable from the following regression line, to three decimal places, given the value of the response variable is ten.

$response\ variable = 2.605 + 0.626 \times explanatory\ variable$

Q4. Application

An artist collects data on the amount of time, in hours, he spends on each individual artwork, and the artworks' individual selling prices, in dollars. He finds the regression line of his data, between 10 and 100 hours, to be:

$selling\ price = 3621 + 62 \times time\ spent$

a) Predict the selling price of an artwork that took 35 hours to create, and determine whether this prediction used interpolation or extrapolation.

b) An artwork sold for $10 000. Determine whether the artwork took over or under 100 hours to create, and also determine whether this prediction used interpolation or extrapolation.

Check your understanding

Q5. Application

Emilie thinks that her results on school tests, as a percentage, might be linked to the time she spends on Netflix, in hours, the night before the test. She finds that the trend has the regression line, between 0 and 8 hours, of:

$results = 97.2 - 4.8 \times hours\ spent\ on\ Netflix$

a) Predict Emilie's test result if she spends 4 hours on Netflix the night before a test, and determine whether this prediction used interpolation or extrapolation.

b) Emilie fell sick and spent 14 hours and 45 minutes on Netflix one night. Predict Emilie's result on the maths test the next day, and determine whether this prediction used interpolation or extrapolation.

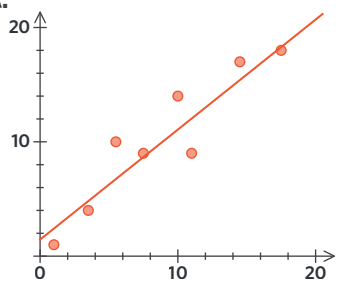
c) Emilie got her English test back and scored 75.6%. Estimate how many hours and minutes Emilie spent on Netflix the night before the English test, and determine whether this estimation used interpolation or extrapolation.

2. Limitations of regression line predictions

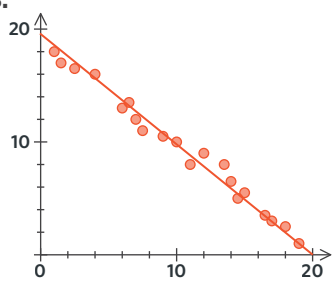
Q6. Skill

From which of the following regression lines would predictions be most reliable?

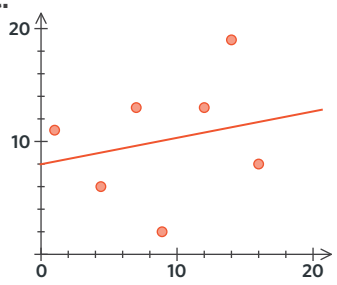
A.



B.



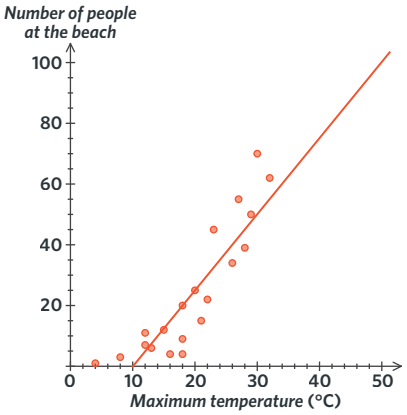
C.



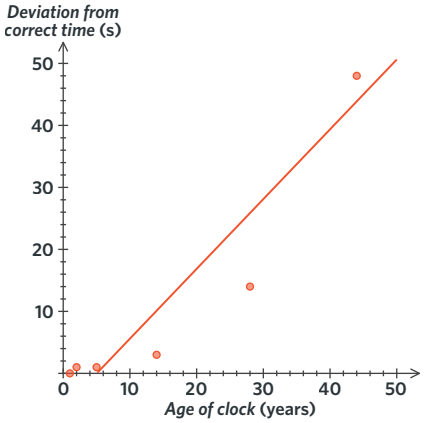
Q7. Explain the limitations of the following predictions:

Application

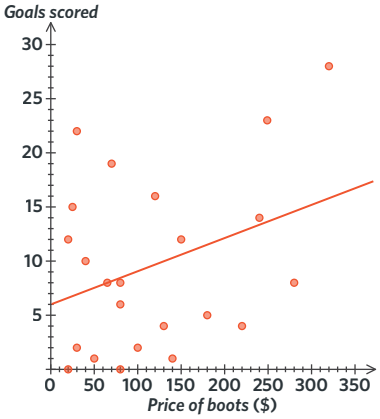
a) There should be 86 people at the beach on a day in which the maximum temperature is 44°C.



b) A clock that has deviated 30 seconds from the correct time should be 36 years old.

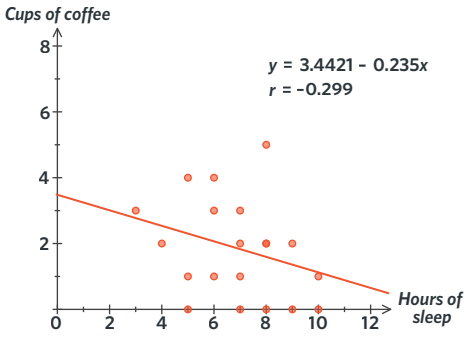


c) A player with $180 boots should score 11 goals.



Q8. The regression line on the right is used to interpolate the number of cups of coffee a person drinks by the hours of sleep they got the night before. Explain whether this prediction would be reliable or not.

Application



Check your understanding

Q9. The following regression line describes the average time in which headphones last, in days, in relation to their price, in dollars. The data consisted of 500 different headphones, between $10 and $200, and the value of the Pearson's correlation coefficient of the data was 0.78.

Application

$duration\ of\ headphone\ functionality = 33 + 3 \times price$

Are there any limitations to predicting the price of headphones that last two years? Explain your answer.

Joining it all together

Q10. Application 4 marks

The following regression line details the number of surfers at two Australian beaches in relation to the speed of the wind, in metres per second. The data was collected every day of a week during summer, with the speed of the wind varying from 0 m/s to 20 m/s.

Surfers Paradise: $number\ of\ surfers = 3 + 4 \times speed\ of\ wind$ $r = 0.82$

Bondi Beach: $number\ of\ surfers = 1 + 5 \times speed\ of\ wind$ $r = 0.79$

a) Predict the speed of the wind given there are 71 surfers at Surfers Paradise. Determine whether the prediction used interpolation or extrapolation.

b) A hurricane has winds of 32 m/s. Predict how many surfers would endure those conditions at Surfers Paradise and Bondi Beach. Find the difference between the two.

c) Using the results from part b), explain how extrapolating data can give unreliable predictions.

d) With reference to the sample diversity of this data, explain how making predictions based upon a small sample size can be unreliable.

Q11. Application 3 marks

The following regression line displays the relationship between the box office revenue and budgets of the highest grossing movies, in dollars.

$box\ office\ revenue = 58\ 000\ 000 + 2.6 \times movie\ budget$

The data is based upon 200 movies, with budgets between $20 million and $200 million. The value of the Pearson's correlation coefficient of the data was 0.43.

a) Predict the box office revenue of a movie that had a budget of $225 million, and determine whether the prediction used interpolation or extrapolation.

b) Predict the budget of a movie that has a $348 million box office revenue, to the nearest dollar, and determine whether the prediction used interpolation or extrapolation.

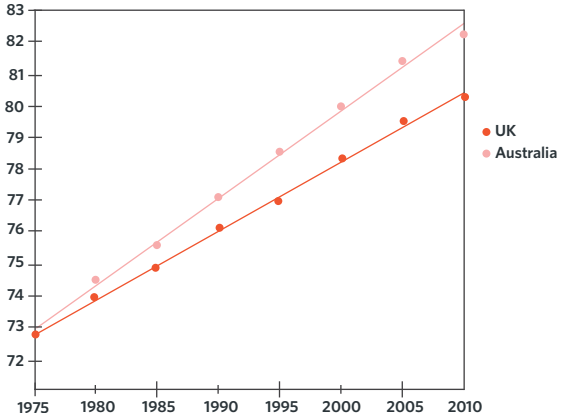
c) Are there any limitations of this regression line, regarding its ability to interpolate values? If so, explain.

VCAA question

Q12. 3 marks

To investigate the difference in life expectancies between residents of Australia and the UK, least squares regression lines were fitted to data from the period between 1975 and 2010.

The results are shown on the right.



The equations of the least squares regression lines are as follows.

Australia:
$life\ expectancy = -451.7 + 0.2657 \times year$

UK:
$life\ expectancy = -350.4 + 0.2143 \times year$

a) Use these equations to predict the difference between the life expectancies of Australia and the UK in 2030. Give your answer correct to the nearest year. (2 marks)

b) Explain why this prediction may be of limited reliability. (1 mark)

Adapted from VCAA 2015 Exam 2, Core. Q5bi,bii

Questions from multiple lessons

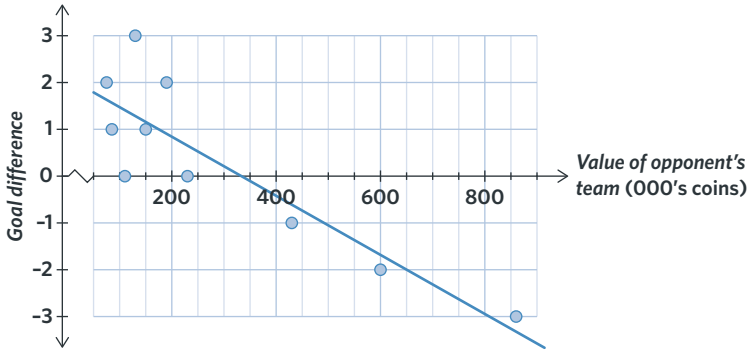
Q13.

Difficulty:

1 mark

Olajide plays FIFA Ultimate Team, a video game based on the sport of soccer. He records the coin value (an in-game currency) of his last 10 opponents' teams, along with the goal difference of the games.

The results can be seen in the following scatterplot. A least squares regression line has been added with *value of opponent's team* as the explanatory variable.



The equation of the regression line is closest to

A. $goal\ difference = 2.11 - 0.0000063 \times value\ of\ opponent's\ team$

B. $goal\ difference = 1.81 - 0.0000063 \times value\ of\ opponent's\ team$

C. $value\ of\ opponent's\ team = 2.11 - 0.0000063 \times goal\ difference$

D. $value\ of\ opponent's\ team = 1.81 + 0.0000063 \times goal\ difference$

E. $value\ of\ opponent's\ team = 1.81 - 0.0000063 \times goal\ difference$

VCAA 2017 Exam 1 Data analysis Q8 – Adapted

Q14.

Difficulty:

1 mark

Each year, the value of a rare Italian violin increases by 5.5%.

In 2019, the violin has a value of $170 500.

In 2017, the value of the violin was closest to

A. $151 790 **B.** $152 261 **C.** $153 186

D. $153 649 **E.** $189 771

VCAA 2013 Exam 1 Module 4: Business-related mathematics Q6 – Adapted

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Q15.

Difficulty:

2 marks

The table below shows the number of *cricket players at training* and the *temperature* on the day of those training sessions.

Temperature (°C)	22	32	18	19	28	34	23	19	20
Cricket players at training	31	25	42	40	28	17	30	38	33

a) Determine the equation of the least squares regression line that can be used to predict the number of *cricket players at training* from the *temperature*. The equation should be in the form

$$cricket\ players\ at\ training = a + b \times temperature$$

Give values correct to two decimal places. (1 mark)

b) There is also a relationship between the number of *patrons at the cricketers' bar* and the *temperature*. The equation of the least squares line is

$$patrons\ at\ the\ cricketers'\ bar = -33.46 + 3.27 \times temperature$$

Interpret the slope of the regression line in terms of the variables *patrons at the cricketers' bar* and *temperature* (1 mark)

VCAA 2017 Exam 2 Data analysis Q3a,bii – Adapted

Answers

1A

1 $N = 23$

2 **a** **i** No **ii** No **iii** No **iv** No **v** No

b Yes

3 **a** Yes **b** No **c** No **d** Yes **e** No **f** No

4 **a** Yes **b** Yes

5 **a** $a = 2 \times (2) + 3$ **b** $a = 7$

6 **a** **i** $p = (1) - 4$ **ii** $p = -3$

b **i** $f = \frac{1}{3}(9) + 7$ **ii** $f = 10$

c **i** $h = -2.3(1.7) + 11$

c **ii** $h = 7.09$

7 **a** $y = 11$ **b** $a = -10$ **c** $p = \frac{1}{4}$

8 **a** 6 **b** 3.5 minutes **c** 38°C

9 $A = 210(w + 30)$

10 **a** $\div 3, + 5$ or $\times \frac{1}{3}, + 5$

b $- 5, \times 3$ **c** $3(y - 5) = x$ or $3y - 15 = x$

11 **a** $n = \frac{1}{2}(m - 4)$ or $n = \frac{1}{2}m - 2$

b $x = \frac{1}{3}(42 - y)$ or $x = 14 - \frac{y}{3}$

c $c = 5g - x$

d $w = \frac{1}{6}(66p - 3q + 66)$ or $w = 11p - 0.5q + 11$

12 **a** $M = 0.625K$

b $d = \frac{1}{5}(t - 30)$ or $0.2t - 6 = d$

13 $C = \frac{5}{9}(F - 32)$

$\frac{9}{5}C = F - 32$

$\frac{9}{5}C + 32 = F$

14 **a** $A = 6$ **b** $c = -12$ **c** $r = -0.57$

15 **a** $C = 23.45$ **b** $t = 21$

16 **a** $K = 265\text{K}$

b $F = 77°\text{F}$

17 $p - q = \frac{1}{r}(12s + t)$

$r(p - q) = 12s + t$

$r = \frac{12s + t}{p - q}$

18 $y = 4x$

The value of y increases by a set amount, four, every time x increases by one.

Questions from multiple lessons

19 *Graphs and relations*

Decoding the question

Keywords: equation, 'how many weeks'

Relevant lesson: 1A

We need to substitute values into the given linear equation to find an unknown value, in this case the number of weeks.

Information we are given

- There are 80 marbles in the bag Jonty found.
- Each week, Jonty plans to collect 2 more.
- $M = 80 + 2w$
- Jonty now has 128 marbles.

Using the theory

Jonty now has 128 marbles.

Substitute $M = 128$ into the given equation and solve for w

$128 = 80 + 2w$

$48 = 2w$

$w = 24$

Jonty has been collecting marbles for 24 weeks.

Answer

B

✓1 mark for the correct answer

20 *Data analysis*

Decoding the question

Keywords: dot plot, percentage

Relevant lessons: 2E, 12C

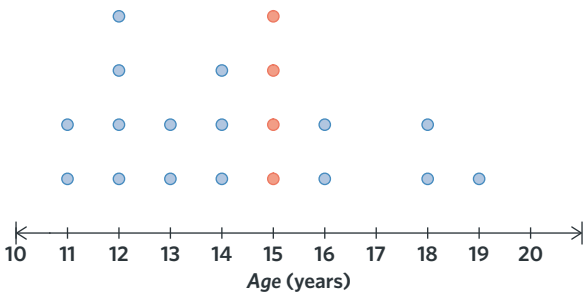
We must count the number of members that are 15 years of age and convert this frequency to a percentage frequency value.

Information we are given

- The given dot plot
- There are 20 members.

Using the theory

There are 4 members with an *age* of 15 years, as highlighted below.



We can convert this to a percentage using the formula

$$percentage\ frequency = \frac{frequency}{total\ frequency} \times 100$$

$$percentage\ frequency = \frac{4}{20} \times 100$$

$$percentage\ frequency = 20\%$$

$\therefore$ 20% of the members are 15 years of age.

Answer

C

✓ *1 mark for the correct answer*

21a ***Graphs and relations***

Decoding the question

Keyword: linear relation

Relevant lessons: 1A, 1E, 1F

We need to determine the linear equation that represents Mary's purchase.

Information we are given

- She purchased 129 lychees.
- She purchased 26 dragonfruits.
- She paid \$258.80.
- L represents the price for each lychee.
- D represents the price for each dragonfruit.

Using the theory

The linear relation that represents Mary's purchase will be in the form:

$total\ price = number\ of\ lychees \times cost\ per\ lychee + number\ of\ dragonfruits \times cost\ per\ dragonfruit.$

Substitute the following into the equation.

$total\ price = 258.80$

$number\ of\ lychees = 129$ and $cost\ per\ lychee = L$

$number\ of\ dragonfruits = 26$ and $cost\ per\ dragonfruit = D$

The equation is $258.80 = 129L + 26D$

Answer

$258.80 = 129L + 26D$

✓ *1 mark for the correct answer*

21b ***Graphs and relations***

Decoding the question

Keywords: linear relation, selling price

Relevant lessons: 1A, 1E, 1F

We need to use the given information to determine the cost of each lychee.

Information we are given

- A linear relation representing Alan's purchase is $267L + 41D = 484.40$.
- The selling price of each dragonfruit is \$4.00.

Using the theory

The relation for Alan's purchase is $267L + 41D = 484.40$. The equation found in part **a** could also be used.

Since the selling price for each dragonfruit is \$4.00, substitute $D = 4$ into $267L + 41D = 484.40$ and solve for L

$267L + 41(4) = 484.40$

$267L = 484.40 - 164$

$L = \frac{484.40 - 164}{267}$

$L = 1.20$

The selling price for each lychee is \$1.20.

Answer

\$1.20

✓ *1 mark for the correct answer*

1B

1 $y = -5$

2

n	5	10	15	20	25	30	35	40	45
p (\$)	9.5	17	24.5	32	39.5	47	54.5	62	69.5

3 **a**

t (°C)	10	11	12	13	14	15	16	17	18	19	20
r (mm)	3.0	2.8	2.6	2.4	2.2	2.0	1.8	1.6	1.4	1.2	1.0

b 17°C

4

n	5	10	15	20	25	30	35	40	45
C	850	1600	2350	3100	3850	4600	5350	6100	6850

5 **a**

w	0	1	2	3	4	5	6	7	8	9	10
M (\$)	350	415	480	545	610	675	740	805	870	935	1000

b 3 weeks **c** No

6 **a** 7 **b** 3

c $t_n = t_{n-1} + 3, t_0 = 7$

7 **a** $t_n = t_{n-1} + 4, t_0 = 5$

b $t_n = t_{n-1} - 4, t_0 = 20$

c $t_n = t_{n-1} + 3, t_0 = -7$

8 2, 8.5, 15, 21.5

9 B

10 **a** $t_n = t_{n-1} - 4, t_0 = 83$

b 79 minutes, 75 minutes, 71 minutes

11

n	0	1	2	3	4	5	6	7
t_n	5	18	31	44	57	70	83	96

12 **a** $t_n = t_{n-1} - 50, t_0 = 620$

b

n (day)	1	2	3	4	5	6	7	8
t_n (amount, $)	620	570	520	470	420	370	320	270

Alicia has $270 on day 8.

13 **a**

h (m)	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
V (m³)	3.1	3.8	4.4	5.0	5.7	6.3	6.9	7.5	8.2	8.8	9.4

b Yes

14 **a**

Week	1	2	3	4	5	6	7	8
Distance (km)	3	5	7	9	11	13	15	17

b Week 11

15 **a**

t (s)	0	1	2	3	4	5	6	7	8	9	10
h (m)	1	10	17	22	25	26	25	22	17	10	1

b 2 seconds and 8 seconds

Questions from multiple lessons

16 *Graphs and relations*

Decoding the question

Keywords: total number, rule

Relevant lesson: 1A

This question requires us to substitute values into the given linear equation to determine which of the scenarios is correct.

Information we are given

- $N = 100 + 65w$
- N is the total number of t-shirts ordered.
- w is the number of weeks the store has been open for.

Using the theory

Substitute the number of weeks for w into the given equation for each multiple-choice option and compare to the number of weeks to determine which one satisfies the rule.

A 100 after 1 week.
$100 = 100 + 65(1)$
$100 \neq 165$
This option is incorrect.

B 130 after 2 weeks.
$130 = 100 + 65(2)$
$130 \neq 230$
This option is incorrect.

C 650 after 10 weeks.
$650 = 100 + 65(10)$
$650 \neq 750$
This option is incorrect.

D 975 after 15 weeks.
$975 = 100 + 65(15)$
$975 \neq 1075$
This option is incorrect.

E 1075 after 15 weeks.
$1075 = 100 + 65(15)$
$1075 = 1075$
This option is correct.

Answer

E

✓1 mark for the correct answer

17 *Recursion and financial modelling*

Decoding the question

Keywords: deposit, balance, repayments, interest

Relevant lesson: 3C

This question requires us to determine the amount of interest paid for a laptop by calculating the amount of money paid through a loan scheme.

Information we are given

- The laptop costs $4000.
- A deposit of $200 was paid.
- The balance will be repaid with 20 monthly repayments of $250.

Using the theory

Calculate the principal after the deposit has been made.

$principal = 4000 - 200$

$principal = 3800$

Calculate the total amount of money repaid.

$total\ repayment = number\ of\ monthly\ repayments \times value\ of\ monthly\ repayment$

$total\ repayment = 20 \times 250$

$total\ repayment = \$5000$

Calculate the amount of interest paid, by subtracting the principal, P, from the total repayment, A

$I = A - P$

$I = 5000 - 3800$

$I = 1200$

Answer

A

✓1 mark for the correct answer

18a *Graphs and relations*

Decoding the question

Keywords: linear, equation, transpose

Relevant lesson: 1A

We must use algebra to transpose the equation so that p is on the left hand side of the equal sign, without a coefficient, and the rest of the equation is on the right hand side of the equal sign.

Information we are given

- $3d = 26 - 4p$

Using the theory

We have to move p to the left hand side of the equal sign and then manipulate all other variables, coefficients and constants so that p is alone.

Add $4p$ to both sides.

$4p + 3d = 26$

Subtract $3d$ from both sides.

$4p = -3d + 26$

Divide both sides by 4.

$p = \frac{-3d}{4} + 6.5$

Answer

$p = \frac{-3d}{4} + 6.5$

✓ *1 mark for the correct answer*

18b ***Graphs and relations***

Decoding the question

Keywords: linear, equation, 'what is the price'

Relevant lesson: 1A

We need to substitute the known distance value into the equation to find the price of a coffee at the Louvre.

Information we are given

- $3d = 26 - 4p$
- The Louvre is 4.5 km away from the Eiffel Tower.

Using the theory

We can either use the equation given in the question or the transposed equation found in part **a**.

In this solution we will use the equation given in the question.

Substitute $d = 4.5$ into the equation $3d = 26 - 4p$ and solve for p

$3 \times 4.5 = 26 - 4p$

$13.5 = 26 - 4p$

$4p = 26 - 13.5$

$4p = 12.5$

$p = \frac{12.5}{4} = 3.125$

NOTE: The value of p can also be found using the solve function on your calculator.

The price of a coffee at the Louvre is 3.13 €, correct to the nearest cent.

Answer

3.13 €

✓ *1 mark for the correct answer*

1C

1 4

2 80

3 $p = 2t$

4 $V = 350 + 2M$

5 $C = 534.68 + 0.0047L$

6 $C = 30 - 2g$

7 20, 30

8 \$70

9 24

10 **a** \$17.15 **b** No **c** 2.9 kilometres

11 **a** $A = 500 + 10w$ **b** \$740

12 75

13 \$175

14 320 000 litres

15 **a** 11 metres **b** 8

16 85

17 60 grams

18 **a** $H = 0.5 + \frac{1}{350} \times W$, where H is the number of hours it will take Joe to write an essay of W words.

$H = 0.5 + \frac{1}{350} \times 1500$

$H = 4.79$ hours

$4.79 > 4.5$

Joe cannot finish his essay in time.

$\therefore$ Joe is incorrect

b 375

19 $V = lwd$, where V is the volume, in m^3, of a planter box that is l metres long, w metres wide, and d metres deep.

360 litres $= \frac{360}{1000}\ m^3 = 0.36\ m^3$

$0.36 = 1.2 \times 0.6 \times d$

$d = \frac{0.36}{1.2 \times 0.6}$

$\therefore d = 0.5$ m

20 A

Questions from multiple lessons

21 ***Recursion and financial modelling***

Decoding the question

Keywords: increase, pattern

Relevant lesson: 1B

This question requires us to use an arithmetic sequence to determine the number of pages Sarah will read in September.

Information we are given

- In January, she reads 12 pages.
- In February, she reads 18 pages.
- In March, she reads 24 pages.
- In April, she reads 30 pages.

Using the theory

Find the increase in the number of pages per month.

$difference = number\ of\ pages\ in\ February - number\ of\ pages\ in\ January$

$difference = 18 - 12$

$difference = 6$

Every month, Sarah will read 6 more pages than the previous one.

Find the number of pages Sarah will read every month from May until September.

$number\ of\ pages\ in\ May = number\ of\ pages\ in\ April + 6$
$number\ of\ pages\ in\ May = 30 + 6$
$number\ of\ pages\ in\ May = 36$

$number\ of\ pages\ in\ June = number\ of\ pages\ in\ May + 6$
$number\ of\ pages\ in\ June = 36 + 6$
$number\ of\ pages\ in\ June = 42$

$number\ of\ pages\ in\ July = number\ of\ pages\ in\ June + 6$
$number\ of\ pages\ in\ July = 42 + 6$
$number\ of\ pages\ in\ July = 48$

$number\ of\ pages\ in\ August = number\ of\ pages\ in\ July + 6$
$number\ of\ pages\ in\ August = 48 + 6$
$number\ of\ pages\ in\ August = 54$

$number\ of\ pages\ in\ September = number\ of\ pages\ in\ August + 6$
$number\ of\ pages\ in\ September = 54 + 6$
$number\ of\ pages\ in\ September = 60$

Answer

C

✓1 mark for the correct answer

22 ***Data analysis***

Decoding the question

Keywords: dot plot, median

Relevant lesson: 12E

This question requires us to analyse the dot plot to find the median value.

Information we are given

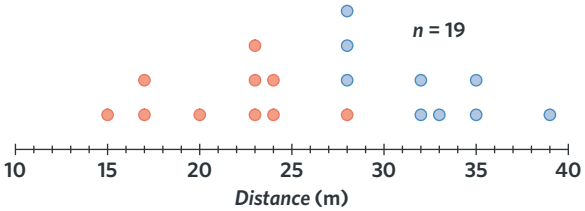
- The given dot plot
- $n = 19$

Using the theory

The median is the middle value of a data set and is located in the $\left(\frac{n+1}{2}\right)^{th}$ position. We are given $n = 19$.

$\left(\frac{19+1}{2}\right) = \left(\frac{20}{2}\right) = 10$

The median will be the 10^{th} number in the data set. The first 10 numbers are highlighted below.



The 10^{th} value is located above a distance of 28 m.

We can check that this is the right answer by counting the number of values above and below the data point. In this case, there are 9 data points above and below the median, meaning we have selected the correct point.

Answer

C

✓1 mark for the correct answer

23a ***Recursion and financial modelling***

Decoding the question

Keywords: show that, arithmetic sequence, common difference, calculation

Relevant lesson: 1B

We need to use the arithmetic sequence to prove that the common difference is 18.

Information we are given

- The given arithmetic sequence
- The common difference is 18.

Using the theory

To find the common difference, find the difference between two consecutive terms in the arithmetic sequence.

$common\ difference = term_2 - term_1$

Substitute $term_2 = 60$ and $term_1 = 42$ into the equation.

$common\ difference = 60 - 42$

$common\ difference = 18$

NOTE: A 'show that' style of question requires us to obtain the desired value or equation without using it in the working out. In this case, we cannot use the common difference to find the next term in the sequence and subsequently prove that it is the common difference.

Answer

$common\ difference = 60 - 42 = 18$

✓1 mark for the correct answer

23b ***Recursion and financial modelling***

Decoding the question

Keywords: arithmetic sequence, how many

Relevant lesson: 1B

We need to use the arithmetic sequence and the common difference to determine the number of push-ups Corey will do on the 6th night.

Information we are given

- The given arithmetic sequence.
- The common difference is 18.

Using the theory

Calculate the number of push-ups Corey will do each night starting from night 4 up until night 6.

For day 4:
number of push-ups on night 4
= *number of push-ups on night* 3 + *common difference*
number of push-ups on night 4 = 78 + 18
number of push-ups on night 4 = 96

For day 5:
number of push-ups on night 5
= *number of push-ups on night* 4 + *common difference*
number of push-ups on night 5 = 96 + 18
number of push-ups on night 5 = 114

For day 6:
number of push-ups on night 6
= *number of push-ups on night* 5 + *common difference*
number of push-ups on night 6 = 114 + 18
number of push-ups on night 6 = 132

Corey will do 132 push-ups on the 6th night.

Answer

132

✓ *1 mark for the correct answer*

1D

1 10

2 (1, 3)

3 (2, 6)

4 **a** Week 4 **b** $115

5

x	1	2	3
y_1	−3	1	5
y_2	3	1	−1

At $x = 2$, $y_1 = y_2 = 1$. Therefore, the pair of simultaneous equations has a solution at (2, 1).

6 A

7 **a** (0, 2) **b** (2, 4)
c (−4, −1) **d** (−4, 2)

8 9:51 am

9 **a** (−0.25, 2.25) **b** (−1.6, 1.2)
c (−0.75, 0.5) **d** (0.67, −1.33)

10 7

11 $j = 14$, $k = 6$

12 $3.75

Questions from multiple lessons

13 ***Graphs and relations***

Decoding the question

Keywords: fixed base wage, equation

Relevant lesson: 1C

We must represent this situation using a linear equation in terms of S and n

Information we are given

- Harry receives a yearly fixed base wage of $100 000.
- He receives an extra $5000 for each concert he plays.
- n is the number of concerts he plays in a year.
- S is his total yearly salary.

Using the theory

The variables given are n, the number of concerts Harry plays, and S, Harry's total yearly salary.

His salary, S, increases by $5000 for every additional concert played, so the coefficient of n will be 5000.

The constant in this situation is the fixed base wage of $100 000. This will be represented as '+ 100 000' in the equation.

Hence, the equation to represent this relationship is $S = 5000n + 100\ 000$.

Answer

B

✓ *1 mark for the correct answer*

14 ***Data analysis***

Decoding the question

Keywords: boxplot, percentage

Relevant lesson: 12G

We must use our knowledge of boxplots and quartiles to deduce the percentage of weights greater than 690 g.

Information we are given

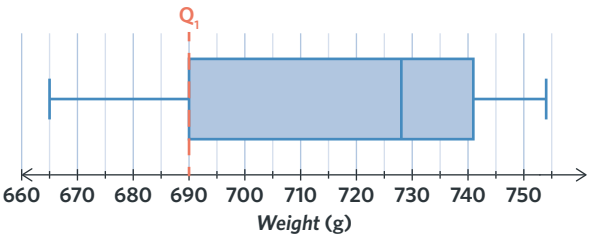
- The given boxplot

Using the theory

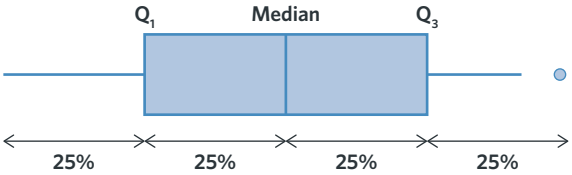
First, find the value of 690 g on the boxplot.

690 g lies on the left border of the box. The left border represents the first quartile, Q_1

Therefore, $Q_1 = 690$.



We know that 25% of the data lies below Q_1 as shown below.



Hence, the rest of the data must lie above Q_1
This remaining percentage is $100\% - 25\% = 75\%$

$\therefore$ 75% of meerkats have a weight greater than 690 g.

Answer

D

✓1 mark for the correct answer

15a ***Graphs and relations***

Decoding the question

Keywords: amount, percentage

Relevant lessons: 1A, 1C

We need to use a linear equation and our knowledge of percentages to determine the amount of money that the group of friends contributed towards venue costs.

Information we are given

- The given table
- The group of friends spent \$880 on adult tickets and \$360 on child tickets.

Using the theory

In regards to the profit received, the table details that 40% of the money received for adult tickets and 30% for child tickets is allocated towards venue costs.

Write an equation based on this information.

$venue\ costs = 0.4 \times adult\ tickets + 0.3 \times child\ tickets$

Substitute $adult\ tickets = 880$ and $child\ tickets = 360$ into this equation.

$venue\ costs = 0.4 \times 880 + 0.3 \times 360$

$venue\ costs = 352 + 108$

$venue\ costs = 460$

Answer

\$460

✓1 mark for the correct answer

15b ***Graphs and relations***

Decoding the question

Keywords: how much, percentage

Relevant lessons: 1A, 1C

We need to use a linear equation and the values we are given to find how much money was spent on adult tickets by the group of friends in question.

Information we are given

- The given table
- The group of friends contributed \$402 in staff wages.
- The group of friends purchased \$180 worth of child tickets.

Using the theory

The table details that 50% of the money received for adult tickets and 40% for child tickets is allocated to staff wages.

Write an equation based on this information.

$staff\ wages = 0.5 \times adult\ tickets + 0.4 \times child\ tickets$

Substitute $staff\ wages = 402$ and $child\ tickets = 180$.

$402 = 0.5 \times adult\ tickets + 0.4 \times 180$

$0.5 \times adult\ tickets = 402 - 0.4 \times 180$

$adult\ tickets = \frac{402 - 0.4 \times 180}{0.5}$

$adult\ tickets = 660$

Answer

\$660

✓1 mark for the correct answer

1E

1 3

2 **a** $x = 4, y = 3$ **b** $x = 2, y = 5$

3 **a** $x = 5, y = 1$ **b** $x = 1, y = -3$

c $x = 9, y = 2$ **d** $x = 6, y = -10$

4 $(3, 7)$

5 **a** Equation (2) is in the form $y = mx + c$ therefore substitution is appropriate.

b $x = -1, y = 3$

6 **a** $x = 2, y = 4$ **b** $x = 1, y = 3$

c $x = -1, y = 1$ **d** $x = 3, y = 2$

7 **a** $x = 3, y = -2$ **b** $x = 3, y = -2$

8 **a** $x = 5, y = 2$ **b** $x = -3, y = 3$

c $x = 4, y = -2$ **d** $x = 1, y = 3$

9 Vegemite: \$4.50, Nutella: \$6.00

10 **a** Substitution **b** Substitution

c Elimination **d** Substitution

e Elimination **f** Elimination

11 **a** Elimination **b** Small: \$2.50, Large: \$4.00

12 Sugar: 200 g, Flour: 120 g

Questions from multiple lessons

13 ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation

Relevant lesson: 1B

We need to interpret the information to produce a recurrence relation that models the provided scenario.

Information we are given

- Link begins with \$150.
- Link plans to spend \$12.50 each week.

Using the theory

Recurrence relations for linear patterns are written in the form $t_0 = a$, $t_{n+1} = t_n + d$, where a is the initial value and d is the common difference. In this case, we are using h instead of t

As Link begins with \$150, this is the first term of the sequence. Hence, $a = 150$. Since he spends \$12.50 each week, the amount of hairspray money he has left decreases by \$12.50 each time. Hence, $d = -12.5$.

Therefore, a recurrence relation that can be used to model Link's situation is

$h_0 = 150 \qquad h_{n+1} = h_n - 12.5$

Answer

D

✓ *1 mark for the correct answer*

14 ***Recursion and financial modelling***

Decoding the question

Keywords: transaction details, table, interest, monthly, annual rate of interest.

Relevant lesson: 3C

This question requires us to use details in an amortisation table to find the annual interest rate.

Information we are given

- The given table
- Interest is calculated monthly on the maximum accumulated debt balance.

Using the theory

First, calculate the amount of interest that is added to the loan account.

$Interest = A - P$

$Interest = \$745.33 - \$740$

$Interest = \$5.33$

The interest formula is given by $I = \frac{Prn}{100}$. Transpose this equation to have r as the subject and substitute the known values. Since interest is calculated on the monthly maximum accumulated debt, the principal value is 1280.00.

$I = \frac{Prn}{100}$

$r = \frac{I}{Pn} \times 100$

$r = \frac{5.33}{1 \times 1280.00} \times 100$

$r = 0.416...\%$

The monthly interest rate needs to be converted to an annual interest rate.

$r = 0.416...\% \times 12$

$r = 4.99...\%$ p.a.

$r \approx 5\%$ p.a.

The annual rate of interest for this loan is closest to 5% p.a.

Answer

A

✓ *1 mark for the correct answer*

15a ***Graphs and relations***

Decoding the question

Keywords: revenue, price

Relevant lesson: 1C

We need to use our understanding of revenue to find the price of Anita's second-hand t-shirts.

Information we are given

- $revenue = 24n$

Using the theory

Revenue is the total amount of money collected, disregarding any expenses.

For a business that sells one item, like Anita's second-hand t-shirt stall, revenue is given by the simple equation $price \times number\ of\ items\ sold$

$price \times number\ of\ items\ sold = 24n$

We are told that the number of second-hand t-shirts sold is represented by n

$\therefore price = 24$

Answer

\$24

✓ *1 mark for the correct answer*

15b ***Graphs and relations***

Decoding the question

Keywords: revenue, cost, profit

Relevant lesson: 1C

We need to find the difference between the revenue and cost of selling 18 second-hand t-shirts, in order to find the amount of profit Anita earned.

Information we are given

- $revenue = 24n$
- $cost = 15.5n + 14$
- Anita sold 18 second-hand t-shirts.

Using the theory

Profit is equal to the difference between revenue and cost.

$profit = revenue - cost$

Substitute $revenue = 24n$ and $cost = 15.5n + 14$ into $profit = revenue - cost$

$profit = 24n - (15.5n + 14)$

$profit = 8.5n - 14$

Since Anita sold 18 second-hand t-shirts, substitute $n = 18$ into $profit = 8.5n - 14$.

$profit = 8.5 \times 18 - 14$

$profit = 139$

Anita earned \$139 of profit.

Answer

\$139

✓*1 mark for the correct answer*

1F

1 **a** $s + 9$ **b** $s - 4$

2 **a** $x = 7, y = 12$ **b** $x = 5, y = 13$

c $x = 10, y = 4$

3 **a** $x = 0.6, y = 0.9$ **b** $x = 7.9, y = 2.2$

4 Length: 24 cm, Width: 6 cm

5 $3n + 4p = 11.6, 6n + 2p = 14.8$

6 **a** \$15 **b** \$11

c 9 children, 3 adults

7 **a** $s + t = 13, 3.5s + 2t = 35$

b 7

8 **a** $4c + 2o = 17.8, 6c + 9o = 48.9$

b \$3.70

9 $d = b + 6$

$d + 2 = 1.5(b + 2)$

$b = 10$

$d = 16$

10 **a** $l = s + 1.5, 130s + 160l = 1748$

b \$5.20

11 One carrot weighs 120 grams.

12 Katya is currently 20 years old.

13 $4a + 6d = 168$

$7a + 3d = 166.5$

$14a + 6d = 333$

$a = 16.5$

Alex gets paid \$16.50 per hour.

14 220 adults participated

15 The age difference is 4 years.

16 250 cm^2

17 B

✓*1 mark for the correct answer*

Questions from multiple lessons

18 ***Graphs and relations***

Decoding the question

Keyword: total

Relevant lessons: 1E, 1F

We must use simultaneous equations to calculate the price of a bath bomb and a face mask in order to find the total amount spent by Klaudia.

Information we are given

- Isobel bought four bath bombs and one face mask for \$44.00.
- Jackson bought two bath bombs and two face masks for \$43.00.
- Klaudia bought three bath bombs and four face masks.

Using the theory

First, represent Isobel and Jackson's purchases as a pair of simultaneous equations.

Let b be the number of bath bombs bought and f be the number of face masks bought.

Isobel: $4b + f = 44$ ①

Jackson: $2b + 2f = 43$ ②

The simultaneous equations can be solved using the elimination method.

② × 2:

$4b + 4f = 86$ ③

③ − ① :

$4b + 4f - (4b + f) = 86 - 44$

$3f = 42$

$f = 14$

Substitute $f = 14$ into ① and solve for b

$4b + 14 = 44$

$4b = 30$

$b = 7.5$

NOTE: Simultaneous equations can also be solved using a calculator. See video solution for a button-by-button demonstration.

Hence, a bath bomb costs \$7.50 and a face mask costs \$14.00.

Klaudia bought three bath bombs and four face masks.

The total amount Klaudia spent is
$3 \times 7.5 + 4 \times 14 = 78.5$

∴ Klaudia spent \$78.50.

Answer

C

✓ *1 mark for the correct answer*

19 ***Data analysis***

Decoding the question

Keywords: histogram, boxplot, distribution

Relevant lessons: 12D, 12G

We need to analyse the given graphs to determine their shape and the best way to describe them.

Information we are given

- The given histogram
- The given boxplot

Using the theory

The shape of a distribution refers to the way the data is arranged and can be described in terms of symmetry and skew.

A perfectly symmetrical distribution is mirrored around the centre of the data. A distribution is approximately symmetrical if it is close to being mirrored around the centre of the data. By looking at the histogram, we can tell that the data is weighted to the centre with the distribution tailing off roughly equally on both sides, suggesting an approximately symmetrical distribution. As the data either side of the centre doesn't match up exactly, the distribution is not perfectly symmetrical.

To check for outliers, we can look at the boxplot. Outliers are represented by a dot either side of the box plot. As there are no dots on either side of the boxplot, we can conclude that there are no outliers.

The distribution can be best described as approximately symmetric with no outliers.

Answer

D

✓ *1 mark for the correct answer*

20a ***Graphs and relations***

Decoding the question

Keywords: expression, profit

Relevant lesson: 1F

This question requires us to determine the cost and revenue functions, and, subsequently, the profit function.

Information we are given

- The cinema charges \$20 per ticket.
- They have to pay \$60 a day to rent the venue and spend an average of \$4 per ticket sold to pay their employees.

Using the theory

The revenue function is the amount of money they receive from customers. Since they charge \$20 per ticket, the revenue function is $revenue = 20 \times n$

The cost function is the expenses that the cinema has. In this case, it is a flat rate of \$60 a day for rent and \$4 per ticket, for wages.

Hence, the cost function is $cost = 60 + 4 \times n$

The profit is the difference between the cost and revenue functions.

$profit = revenue - cost$

$profit = 20n - (60 + 4n)$

$profit = 20n - 60 - 4n$

$profit = 16n - 60$

The expression for the profit the cinema will make is $profit = 16n - 60$.

Answer

$profit = 16n - 60$

✓ *1 mark for the correct answer*

20b ***Graphs and relations***

Decoding the question

Keywords: profit, minimum

Relevant lesson: 1F

We need to use the profit function and determine the minimum number of tickets the cinema needs to sell in order to reach \$373 in profit.

Information we are given

- The cinema wants to make a profit of at least \$373 per day.

Using the theory

From part **a**, the profit function is $profit = 16n - 60$.

Substitute $profit = 373$ into the equation and solve for n

$373 = 16n - 60$

$16n = 373 + 60$

$n = \frac{373 + 60}{16}$

$n = 27.0625$

Since we cannot have a decimal number of tickets, we need a whole number answer. Usually, we would round down to 27, however, we need to consider the context of the question.

The cinema wants a minimum of \$373 profit, meaning their profit can be greater than \$373 but not less.

If they sell 27 tickets,

$profit = 16 \times 27 - 60$

$profit = 372$

They will only make \$372 profit.

Therefore, a minimum of 28 tickets need to be sold per day.

Answer

28 tickets

✓1 mark for the correct answer

2A

1 **a** 5 **b** 5×6

2 **a** 16 **b** 8

3 **a** 15 **b** 7 **c** -3

4 9.1

5 19

6 **a** 7 **b** 20

c 3 **d** 26

7 **a** 24 **b** 6 **c** 37

8 B

9 No, there are 11 plums left.

10 $4 + (-3) \times 6 - (-5) + 1 = -8$

11 **a** 21 **b** 5

c 10 **d** 12

Expression 'a' has the highest value

12 \$4.20

Questions from multiple lessons

13 *Graphs and relations*

Decoding the question

Keywords: equation, graph, point of intersection

Relevant lesson: 1D

We must use a calculator to plot the two simultaneous equations and determine the point of intersection. The coordinates of the point of intersection will be the prices of the small and large fridge magnets.

Information we are given

- $3s + 4l = 21.80$
- $6s + 3l = 24.60$

Using the theory

Rearrange the equations so that s or l is the subject. We will rearrange the equations for s

Elizabeth:

$3s + 4l = 21.80$

$3s = -4l + 21.80$

$s = -\frac{4}{3}l + \frac{21.80}{3}$

Ash:

$6s + 3l = 24.60$

$6s = -3l + 24.60$

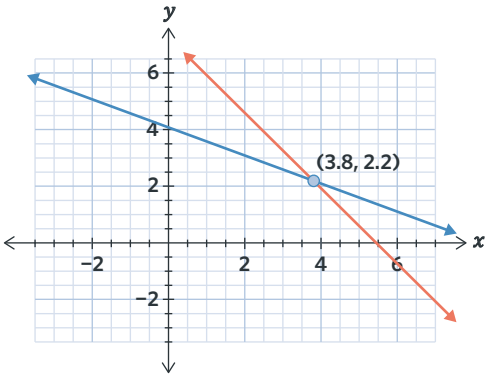
$s = -0.5l + 4.1$

Replace s with y and l with x.

Elizabeth: $y = -\frac{4}{3}x + \frac{21.80}{3}$

Ash: $y = -0.5x + 4.1$

Use a calculator to graph the two lines and find the point of intersection.



The point of intersection is (3.8, 2.2).

Since s , the price of a small fridge magnet, is represented by y, the price of a small fridge magnet is the y-coordinate of the point of intersection.

Hence, the price of a small fridge magnet is \$2.20.

See video solution for a button-by-button demonstration.

Answer

D

✓1 mark for the correct answer

14 *Data analysis*

Decoding the question

Keywords: histogram, number of students, distribution

Relevant lesson: 12C

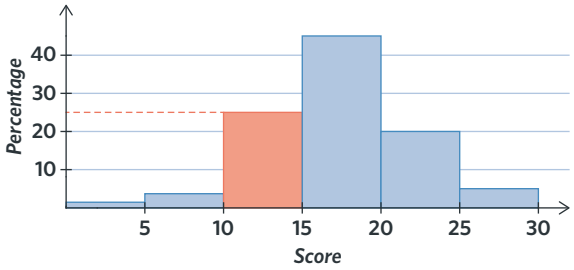
This question requires us to interpret the graph in terms of the number of students that fall within a particular interval.

Information we are given

- The given histogram
- 80 students participated in the spelling test.

Using the theory

First, we will use the histogram to find the percentage of students with a *score* between 10 and 15. This is shown in the following diagram.



We can see that this correlates to a percentage between 20 and 30, but we can't be completely sure. This means we will have to calculate both percentages out of 80 to find a range which will include the answer.

20% of 80:

$\frac{20}{100} \times 80 = 16$

30% of 80:

$\frac{30}{100} \times 80 = 24$

This means that the number of students that received a *score* between 10 and 15 is roughly in the middle of the values 16 and 24. Looking at our options, B is the only option between these two values.

$\therefore$ The number of students with a *score* between 10 and 15 is closest to 20.

Answer

B

✓1 mark for the correct answer

15a ***Graphs and relations***

Decoding the question

Keywords: simultaneous equations, represent

Relevant lesson: 1F

We must convert the information provided into two simultaneous equations.

Information we are given

- Boston rents two games and seven movies for \$46.40.
- Alana rents four games and five movies for \$49.60.
- g represents the cost of renting a game.
- m represents the cost of renting a movie.

Using the theory

The total amount of money spent at Blockbuster on rental games and movies can be found using the formula:

$number\ of\ games\ rented \times price\ of\ game\ rental + number\ of\ movies\ rented \times price\ of\ movie\ rental = total\ spent$

We know that g represents the cost of renting a game and m represents the cost of renting a movie, hence

$number\ of\ games\ rented \times g + number\ of\ movies\ rented \times m = total\ spent$

Boston rents two games and seven movies for \$46.40. Substitute $number\ of\ games\ rented = 2$, $number\ of\ movies\ rented = 7$ and $total\ spent = 46.4$ into the equation.

$2g + 7m = 46.4$

Alana rents four games and five movies for \$49.60. Substitute $number\ of\ games\ rented = 4$, $number\ of\ movies\ rented = 5$ and $total\ spent = 49.6$ into the equation.

$4g + 5m = 49.6$

These are the two simultaneous equations that represent the given information.

Answer

$2g + 7m = 46.4$

$4g + 5m = 49.6$

✓1 mark for the first correct equation
✓1 mark for the second correct equation

15b ***Graphs and relations***

Decoding the question

Keywords: simultaneous equations, cost

Relevant lesson: 1F

We must solve the simultaneous equations found in part **a** to find the value of g; the cost of renting a game.

Information we are given

- The simultaneous equations found in part **a**
- g represents the cost of renting a game.

Using the theory

We must solve the simultaneous equations found in part **a**.

$2g + 7m = 46.4$

$4g + 5m = 49.6$

Enter the equations into the 'solve simultaneous equations' function on your calculator.

The calculator should give:

$g = 6.4$

$m = 4.8$

Hence, the cost of renting a game is \$6.40.

See video solution for a button-by-button demonstration.

Answer

\$6.40

✓1 mark for the correct answer

15c *Graphs and relations*

Decoding the question

Keywords: simultaneous equations, cost

Relevant lesson: 1F

We must use the values of g and m to find the cost of renting six games and three movies.

Information we are given

- The values of g and m found in part **b**
- Lauren rented six games and three movies.

Using the theory

Recall from part **a** that the general formula for finding the total spent at Blockbuster was

$number\ of\ games\ rented \times g$
$+\ number\ of\ movies\ rented \times m = total\ spent$

Recall from part **b** that $g = 6.4$ and $m = 4.8$.

Substitute $g = 6.4$ and $m = 4.8$ into the formula:

$number\ of\ games\ rented \times 6.4$
$+\ number\ of\ movies\ rented \times 4.8 = total\ spent$

Lauren rented six games and three movies.
Substitute $number\ of\ games\ rented = 6$ and $number\ of\ movies\ rented = 3$ into the formula to find the total spent.

$6 \times 6.4 + 3 \times 4.8 = total\ spent$

$total\ spent = 52.8$

Answer

\$52.80

✓1 mark for the correct answer

2B

1 0.00001

2 2×10^2

3 **a** 3.2815×10^4 **b** 8.2344×10^6
c 7.82×10^{-4} **d** 3.8492×10^{-8}

4 B

5 2×10^4

6 0.009

7 **a** 354 000 **b** 0.00492
c 0.000087408

8 C

9 5283.706

10 3.3×10^{-5} km

11 2.7×10^{-9}

12 D

Questions from multiple lessons

13 *Graphs and relations*

Decoding the question

Keywords: table, equation, values

Relevant lesson: 1B

We must substitute the numbers in the table into the given equation to find the values of a, b and c

Information we are given

- The given table
- $y = -2x - 18$

Using the theory

Find the value of a:

Substitute $x = 3$ and $y = a$ into the equation $y = -2x - 18$ and solve for a

$a = -2 \times 3 - 18$

$a = -6 - 18$

$a = -24$

Find the value of b:

Substitute $y = -12$ and $x = b$ into the equation $y = -2x - 18$ and solve for b

$-12 = -2b - 18$

$2b = -6$

$b = -3$

Find the value of c:

Substitute $x = 1$ and $y = c$ into the equation $y = -2x - 18$ and solve for c

$c = -2 \times 1 - 18$

$c = -2 - 18$

$c = -20$

$\therefore a = -24, b = -3, c = -20$

Answer

C

✓1 mark for the correct answer

14 *Recursion and financial modelling*

Decoding the question

Keywords: cost, per year, GST

Relevant lesson: 3B

This question requires us to calculate the total cost of a subscription including GST.

Information we are given

- The cost of a VPN subscription is \$60.00 per year plus GST.

Using the theory

Calculate the total cost for the subscription without GST.

$subscription = 60.00 \times 3$

$subscription = 180.00$

Find the total cost for the subscription with GST:

$Q = \frac{P \times 11}{10}$

$Q = \frac{180 \times 11}{10}$

$Q = 198.00$

The total price including GST for a VPN subscription of three years is $198.00.

Answer

C

✓ *1 mark for the correct answer*

15a ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation, represents

Relevant lesson: 1B

We need to use our knowledge of recurrence relations to model the balance of Riley's bank account.

Information we are given

- Riley spends exactly $170 per day.
- Riley has $7100 at the start of her trip.
- The relation must be in terms of B_n B_{n-1} and B_0

Using the theory

Using the terms B_n B_{n-1} and B_0, the recurrence relation must be in the form $B_n = B_{n-1} + d$, $B_0 = a$, where d is the common difference and a is the initial term.

As Riley's bank account balance will reduce by $170 each day, d must be -170.

As Riley's bank account balance was initially $7100, a must be 7100.

Substitute $d = -170$ and $a = 7100$ into $B_n = B_{n-1} + d$, $B_0 = a$

$B_n = B_{n-1} - 170, \quad B_0 = 7100$

Answer

$B_n = B_{n-1} - 170, \quad B_0 = 7100$

✓ *1 mark for the correct answer*

15b ***Recursion and financial modelling***

Decoding the question

Keywords: table of values, 'show that'

Relevant lesson: 1B

We must create a table of values that shows the daily balance of Riley's bank account from $n = 0$ (initial balance) to $n = 7$ (balance after one week).

Information we are given

- Riley spends exactly $170 per day.
- Riley has $7100 at the start of her trip.
- Riley has $5910 in her bank account after one week.

Using the theory

Create a table with two rows (one for n and one for B_n) from $n = 0$ to $n = 7$. The table is shown below.

n	0	1	2	3	4	5	6	7
B_n								

We know that B_0 is 7100.

Now we must find the values B_1 to B_7

$B_1 = B_0 - 170$

$B_1 = 7100 - 170 = 6930$

$B_2 = B_1 - 170$

$B_2 = 6930 - 170 = 6760$

Repeat this until B_7 is found.

$B_7 = B_6 - 170$

$B_7 = 6080 - 170 = 5910$

Put all the values from B_0 to B_7 into the table as seen below.

n	0	1	2	3	4	5	6	7
B_n	7100	6930	6760	6590	6420	6250	6080	5910

As the value of B_7 is 5910 in the table, it is proven that Riley has $5910 in her bank account after a week.

Answer

n	0	1	2	3	4	5	6	7
B_n	7100	6930	6760	6590	6420	6250	6080	5910

✓ *1 mark for the correct answer*

2C

1 3

2 **a** 4 **b** 340 **c** 2.84

3 **a** 182.34 **b** 8300 **c** 3.01

4 5 km

5 $5.60

6 2

7 **a** 4 **b** 3 **c** 3

8 C

9 Emily and An

10 $4

11 **a** $\approx 6\ m^2$ **b** $\approx$ 16 hours **c** $\approx$ 2.5 kg

12 10 days (answers close to this are correct)

13 3 minutes per question (answers close to this are correct)

14 **a** Our estimate was 44.

The actual value is 43.3, a difference of 0.7.

b Our estimate was 1000.

The actual value is 1078, a difference of 78.

c Our estimate was 3.

The actual value is 2.90, a difference of 0.1.

15 **a** This is a guess (no right/wrong answer)

b Cake A: 450 cm^2, Cake B: 625 cm^2, Cake C: 450 cm^2

c Cake A: 544 cm^2, Cake B: 624 cm^2, Cake C: 516 cm^2

d Yes or no

e Cake B

16 **a** Ruby: rounding to the nearest ten metres.

Ethan: rounding to the nearest 100 metres.

Ishara: rounding to the nearest metre.

b Actual value: 67 004.60 m^2. Rounding to the nearest ten metres estimated above the actual value, rounding to the nearest 100 metres and rounding to the nearest metre estimated below the actual value.

c Closest estimate: Ishara

d Compare your answer to other students

Questions from multiple lessons

17 ***Computation and practical arithmetic***

Decoding the question

Keyword: value

Relevant lesson: 2A

We must evaluate each option using BIDMAS in order to identify which expression has a value of 1.

Information we are given

- The given expressions

Using the theory

Simplify each expression.

A $\frac{(-10) \times (-4)}{8} + (-2)^2$

$= \frac{40}{8} + 4$

$= 5 + 4$

$= 9$

B $\frac{6(4-3)^2}{(8-6)(7+(-2)\times 5)}$

$= \frac{6 \times 1^2}{2 \times (7+(-10))}$

$= \frac{6}{2 \times (-3)}$

$= \frac{6}{-6}$

$= -1$

C $3((9-2)+(-6))+(-7)\times 2$

$= 3(7-6)+(-14)$

$= 3 \times 1 - 14$

$= 3 - 14$

$= -11$

D $\frac{9-(-3)}{4} - \frac{1}{2}(11+(-7))$

$= \frac{9+3}{4} - \frac{1}{2}(11-7)$

$= \frac{12}{4} - \frac{1}{2} \times 4$

$= 3 - 2$

$= 1$

E $\frac{(-3) \times (5-(-1))}{(4-1)^2}$

$= \frac{(-3) \times (5+1)}{3^2}$

$= \frac{(-3) \times 6}{3^2}$

$= \frac{-18}{9}$

$= -2$

Answer

D

✓1 mark for the correct answer

18 ***Data analysis***

Decoding the question

Keywords: total, mean

Relevant lesson: 12E

We must use the information given to calculate the mean.

Information we are given

- Total distance run was 42.3 km.
- He went on nine runs.

Using the theory

The mean can be found using the formula

$mean = \frac{sum\ of\ all\ elements}{number\ of\ elements}$

In this case, we have been given both the sum of all elements and the number of elements.

The sum of all elements is the total distance run. This is 42.3 km.

The number of elements is the number of runs Lachlan went on. This is 9.

Hence, substitute $sum\ of\ all\ elements = 42.3$ and $number\ of\ elements = 9$ into $mean = \frac{sum\ of\ all\ elements}{number\ of\ elements}$

$mean = \frac{42.3}{9}$

$mean = 4.7$

The mean distance of these runs is 4.7 km.

Answer

C

✓1 mark for the correct answer

19a ***Computation and practical arithmetic***

Decoding the question

Keywords: evaluate, expression, mistake

Relevant lesson: 2A

We need to inspect Rafael's working out in order to find the line in which he made his first mistake.

Information we are given

- The expression $\frac{-\left(\frac{3}{4}+12\right)\times 4}{21-38}$
- The given working out

Using the theory

Look through each line for changes made from the previous line and make sure that the calculations were correct.

Line 1:

12 changed to $\frac{48}{4}$ Correct

$21-38$ was calculated to be -17 Correct

Line 2:

$\frac{3}{4}+\frac{48}{4}$ was calculated to be $\frac{51}{4}$ Correct

Line 3:

$-\left(\frac{51}{4}\right)\times 4$ was calculated to be 51 Incorrect

$-\left(\frac{51}{4}\right)\times 4$ should have been calculated to be -51.

The first mistake was made on line 3.

Answer

Line 3

✓*1 mark for the correct answer*

19b ***Computation and practical arithmetic***

Decoding the question

Keywords: evaluate, expression

Relevant lesson: 2A

We need to use BIDMAS to evaluate the expression.

Information we are given

- The expression is $\frac{-\left(\frac{3}{4}+12\right)\times 4}{21-38}$

Using the theory

Using BIDMAS, the first step is to evaluate the brackets.

$\frac{3}{4}+12=\frac{3}{4}+\frac{48}{4}=\frac{51}{4}$

$\frac{-\left(\frac{3}{4}+12\right)\times 4}{21-38}=\frac{-\frac{51}{4}\times 4}{21-38}$

Evaluate the numerator.

$-\left(\frac{51}{4}\right)\times 4=-51$

Evaluate the denominator.

$21-38=-17$

$\frac{-\left(\frac{51}{4}\right)\times 4}{21-38}=\frac{-51}{-17}=\frac{51}{17}=3$

Answer

3

✓*1 mark for the correct answer*

2D

1 100

2 4

3 **a** 3 **b** -2

4 **a** 5 **b** 2 **c** 7

5 7

6 **a**

	Height (m)	Height (cm)	Height (cm) in scientific notation	Order of magnitude
Mount Everest	9000	900 000	9×10^{5}	5
Ant hill	0.09	9	9×10^{x}	0

b 0

7 4.56

8 **a** 3 **b** 0 **c** -5

9 **a** 0.004 **b** 1.015

10 10 000

11 **a** 100 kg **b** 0.40 kg **c** 199 526.23 kg

12 B

13 7.6

14 C

15 D

16 C

17 **a** 1 233 806 547 **b** 6

18 C

Questions from multiple lessons

19 ***Computation and practical arithmetic***

Decoding the question

Keywords: rounding, multiplying

Relevant lesson: 2C

We must round 416 grams to the nearest ten grams and multiply this number by 0.011.

Information we are given

- Dave picks 416 grams worth of strawberries.
- The price of strawberries is \$0.011 per gram.
- The price of strawberries is determined by rounding the weight of the fruit to the nearest ten grams, and multiplying by the price per gram.

Using the theory

First, round the weight of the strawberries to the nearest ten grams.

416 is between 410 and 420.

The last digit of 416 is between 5–9, so we round up to 420.

Now multiply 420 by the price per gram.

$price = 420 \times 0.011 = 4.62$

Dave pays $4.62.

Answer

A

✓1 mark for the correct answer

20 ***Data analysis***

Decoding the question

Keywords: dot plot, median

Relevant lesson: 12E

We must determine the position of the median and locate it on the given dot plot.

Information we are given

- The given dot plot
- There are 29 running groups.

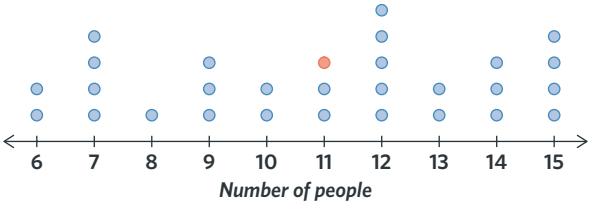
Using the theory

The median is located in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position. Since there are 29 different running groups, $n = 29$.

Position of the median: $\frac{29+1}{2} = \frac{30}{2} = 15$

The median is in the 15^{th} position.

The 15^{th} value is highlighted red in the dot plot below.



The 15^{th} value is 11.

Answer

B

✓1 mark for the correct answer

21a ***Computation and practical arithmetic***

Decoding the question

Keyword: scientific notation

Relevant lesson: 2B

We must write 0.00327 in scientific notation.

Information we are given

- A water bottle contains 0.00327 litres of liquid.

Using the theory

A number written in scientific notation is in the form $x \times 10^y$, where x is the base and y is the exponent.

Write the number and place the decimal point after the first non-zero digit. This will be the base.

0003.27

The base is 3.27.

Find the direction and the number of places that the decimal point moved in the last step. This will be the exponent.

0.003.27

The decimal place moved three places to the right.

The exponent is −3.

0.00327 written in scientific notation is 3.27×10^{-3}

Answer

3.27×10^{-3}

✓1 mark for the correct answer

21b ***Computation and practical arithmetic***

Decoding the question

Keyword: added

Relevant lesson: 2B

We must ensure both numbers are written as numerals in millilitres before adding them to find how much liquid the bottle contains.

Information we are given

- 1.893×10^2 millilitres of water is added to the bottle.

Using the theory

First, convert 0.00327 litres to millilitres.

Since there are 1000 millilitres in one litre, multiply 0.00327 by 1000.

$0.00327 \times 1000 = 3.27$

$0.00327 \text{ L} = 3.27 \text{ mL}$

Next, convert 1.893×10^2 millilitres to numerals.

Write the number excluding '$\times 10^2$'

1.893

As the exponent of 10 is 2, move the decimal place two places to the right.

1.89.3

$1.893 \times 10^2 \text{ mL} = 189.3 \text{ mL}$

Sum 3.27 mL and 189.3 mL to find how much liquid the bottle contains.

$3.27 \text{ mL} + 189.3 \text{ mL} = 192.57 \text{ mL}$

The bottle contains 192.57 mL of liquid.

Answer

192.57 mL

✓1 mark for the correct answer

2E

1 Yes

2 C

3 **a** 80% **b** 32% **c** 6.67%

4 40%

5 **a** Sam: 54.55%. Charlie: 57.14%.

b Charlie

6 D

7 **a** 0.79 **b** $\frac{7}{20}$ **c** $\frac{5}{8}$

8 $\frac{7}{20}$

9 **a** 10% **b** $\frac{59}{100}$

10 A

11 **a** 91 **b** 0.3 **c** 101.97

12 51 pages

13 675 mL

14 B

15 **a** A change from 12.3 to 14.8 is an increase of 20.33%.

b A change from 218 to 134 is a decrease of 38.53%.

c A change from 0.34 to 0.37 is an increase of 8.82%.

16 A change from 0.76 to 3.25 is an increase of 327.63%.

17 59.4%

18 Pole: 91.60%

Flag: 8.14%

Cap: 0.25%

19 Yes. 750 mL has overflowed from the bucket.

20 **a** 523 200

b 653 200

c 13 200

d 2.59% more 5–8 year olds do swimming compared to 0–4 year olds.

Questions from multiple lessons

21 ***Computation and practical arithmetic***

Decoding the question

Keywords: convert, scientific notation

Relevant lesson: 2B

We must convert 0.00076 kg to scientific notation.

Information we are given

- The weight of aluminium hydroxide is 0.00076 kg.

Using the theory

A number written in scientific notation is in the form $x \times 10^y$ where $1 \leq x < 10$.

First, place a decimal point after the first non-zero digit (7) and count the number of places the decimal point has moved from its original position.

$0.0007.6$

The decimal point moved 4 places. Hence the exponent of the base 10 will be 4.

The decimal point moved to the right, so the exponent (4) of the base 10 will be negative.

0.00076 kg written in scientific notation is 7.6×10^{-4} kg

Answer

D

✓*1 mark for the correct answer*

22 ***Recursion and financial modelling***

Decoding the question

Keywords: investment, value, growth

Relevant lesson: 3C

We need to use the given graph to determine the principal of another investment under the same conditions.

Information we are given

- The given graph
- Another investment was made under the same growth conditions.
- The time period for the other investment is 10 years.
- After 10 years, the other investment has grown by \$30 000.

Using the theory

The graph shows that the first investment is growing at a steady rate of \$6000 per year. This indicates that this is a simple interest account.

Calculate the yearly increase as a percentage of the initial value of the investment.

$$percentage = \frac{6000}{250\ 000} \times 100$$

$$percentage = 2.4\%$$

The interest rate is 2.4% per annum.

Use the simple interest formula $I = \frac{Prn}{100}$ to find the principal of the investment. Substitute $I = 30\ 000$, $r = 2.4$ and $n = 10$ into the formula and solve for P.

$$30\ 000 = \frac{P(2.4 \times 10)}{100}$$

$$3\ 000\ 000 = P(24)$$

$$P = \frac{3\ 000\ 000}{24}$$

$$P = 125\ 000$$

The initial investment value is \$125 000.

Answer

D

✓*1 mark for the correct answer*

23a ***Computation and practical arithmetic***

Decoding the question

Keyword: logarithmic scale

Relevant lesson: 2D

We must find the difference in strength between the two earthquakes, taking into account the logarithmic scale.

Information we are given

- The first earthquake measures 3.4 on the Richter scale.
- The second earthquake measures 2.7 on the Richter scale.
- The Richter scale is a logarithmic scale.

Using the theory

The Richter scale is a logarithmic scale using the base 10 logarithm.

$magnitude\ on\ Richter\ scale = \log_{10}(amplitude\ of\ earthquake)$

This can be rearranged to give the following:

$amplitude\ of\ earthquake = 10^{magnitude\ on\ Richter\ scale}$

Therefore, the actual amplitude of an earthquake can be found by raising 10 to the power of the magnitude of the earthquake on the Richter scale.

Find the amplitude of the earthquake of magnitude 3.4 on the Richter scale by raising 10 to the power of 3.4.

$10^{3.4} = 2511.886...$

Find the amplitude of the earthquake of magnitude 2.7 on the Richter scale by raising 10 to the power of 2.7.

$10^{2.7} = 501.187...$

Divide the amplitude of the first earthquake by the second to find the difference in strength of the two earthquakes.

$\frac{2511.886...}{501.187...} = 5.011...$

The first earthquake was 5.0 times stronger than the second.

Answer

5.0

✓1 mark for the correct answer

23b ***Computation and practical arithmetic***

Decoding the question

Keyword: order of magnitude

Relevant lessons: 2B, 2D

We must find the order of magnitude by converting the difference between the strengths of the two earthquakes into scientific notation.

Information we are given

- The first earthquake measures 3.4 on the Richter scale.
- The second earthquake measures 2.7 on the Richter scale.
- The Richter scale is a logarithmic scale.

Using the theory

The order of magnitude is given by the exponent on the base 10, after the number has been converted into scientific notation.

From part **a**, the difference in strengths of the two earthquakes was 5.0.

Write 5.0 in scientific notation.

$5.0 = 5.0 \times 10^0$

The exponent on the base 10 is 0.

Therefore, the order of magnitude of the difference between the strengths of the two earthquakes is 0.

Answer

0

✓1 mark for the correct answer

2F

1 $\frac{1}{5}$

2 The ratio of dogs to cats is 3 : 1.

3 **a** The ratio of pencils to pens is 3 : 5.

b The ratio of rooms to guests is 2 : 3.

c The ratio of students at government schools to Catholic schools is 3 : 1.

4 200

5 Yes

6 **a** Yes **b** No **c** Yes

7 No, it won't fit. It will have to be cropped or stretched.

8 7.5 million

9 **a** Jar 1 **b** 13 green jelly beans

10 The ratio of houses to people is 2 : 5.

There should be approximately 200 000 homes in Newcastle.

11 No. She doesn't have enough sugar and milk.

12 The ratio of people walking, to catching public transport, to driving to work is 2 : 10 : 43.

In 2051, 2.580 million people will drive to work.

Questions from multiple lessons

13 ***Computation and practical arithmetic***

Decoding the question

Keywords: logarithmic scale, magnitude, amplitude, 'how many times stronger'

Relevant lesson: 2D

We must use the given formula to calculate the proportional difference in amplitude between the two earthquakes.

Information we are given

- $magnitude = \log_{10}(amplitude)$
- The New Zealand earthquake had a magnitude of 6.3.
- The Japan earthquake had a magnitude of 9.0.

Using the theory

Calculate the amplitude of each earthquake using the formula $magnitude = \log_{10}(amplitude)$

New Zealand:
$6.3 = \log_{10}(amplitude)$
$amplitude = 10^{6.3}$
$amplitude = 1\ 995\ 262.31...$

Japan:
$9.0 = \log_{10}(amplitude)$
$amplitude = 10^{9.0}$
$amplitude = 1\ 000\ 000\ 000$

Divide the amplitude of the Japan earthquake by the amplitude of the New Zealand earthquake.

$\frac{1\ 000\ 000\ 000}{1\ 995\ 262.31...} = 501.187...$

Hence, the Japan earthquake was 501.2 times stronger than the New Zealand earthquake.

NOTE: The correct answer can also be found by first calculating the difference in magnitude, and then applying the formula $magnitude = \log_{10}(amplitude)$ once.

Answer

D

✓ *1 mark for the correct answer*

14 ***Data analysis***

Decoding the question

Keywords: variables, numerical, ordinal, nominal

Relevant lesson: 12A

We must categorise each of the variables as either numerical, ordinal or nominal.

Information we are given

- *clothing size* (extra small, small, medium, large, extra large)
- *height* (in cm)

Using the theory

First, we must classify each variable as numerical or categorical. Numerical variables represent a quantity and can be counted or measured. Categorical variables usually represent a quality or attribute and can always be broken into groups or categories.

Height is measured and has not been given with a set of categories, so it must be numerical.

Clothing size has been presented with a set of categories, so it must be categorical.

Categorical variables can be either ordinal or nominal. Ordinal variables are those with a hierarchy or order among the categories. Nominal variables cannot be ordered. The categories of *clothing size* are ordinal as they have a natural progression from extra small to extra large.

Hence, *clothing size* is ordinal and *height* is numerical.

Answer

A

✓ *1 mark for the correct answer*

15a ***Computation and practical arithmetic***

Decoding the question

Keyword: percentage

Relevant lesson: 2E

We must calculate the number of cubs as a percentage of the total number of aardvarks in the population.

Information we are given

- There are 65 aardvark cubs and 232 fully-grown aardvarks.

Using the theory

First, express the number of aardvark cubs as a fraction over the total number of aardvarks.

$\frac{65}{65 + 232} = \frac{65}{297}$

Multiply this by 100 to convert to a percentage.

$\frac{65}{297} \times 100 = 21.8855...$

Correct to the nearest percent, 22% of the aardvarks are cubs.

Answer

22%

✓ *1 mark for the correct answer*

15b ***Computation and practical arithmetic***

Decoding the question

Keywords: change, percentage

Relevant lesson: 2E

We must calculate the number of aardvarks that passed away as a percentage change of the total number of aardvarks in the population.

Information we are given

- At the beginning of monitoring, there were 65 aardvark cubs and 232 fully-grown aardvarks.
- 11 aardvarks passed away.

Using the theory

We can calculate the percentage change using the following formula.

$percentage\ change = \frac{difference}{original\ value} \times 100$

The difference is -11, as eleven aardvarks have passed away.

The original value is the total number of aardvarks:
$65 + 232 = 297$

$percentage\ change = \frac{-11}{297} \times 100$

$percentage\ change = -3.703...$

The negative value indicates a decrease.

Answer

3.7% decrease

✓1 mark for the correct answer

15c ***Computation and practical arithmetic***

Decoding the question

Keywords: increased, current population

Relevant lesson: 2E

We must calculate the current aardvark population using the given percentage change value.

Information we are given

- At the beginning of monitoring, there were 65 aardvark cubs and 232 fully-grown aardvarks.
- The population has increased by 24%.

Using the theory

An increase of 24% means that the current population will be 124% of the original population.

First, convert this to a decimal by dividing by 100.

$\frac{124}{100} = 1.24$

Now, multiply the original population by 1.24 to find the current population.

$current\ population = (65 + 232) \times 1.24$

$current\ population = 297 \times 1.24$

$current\ population = 368.28$

Correct to the nearest whole number, the current population is 368 aardvarks.

Answer

368 aardvarks

✓1 mark for the correct answer

2G

1 1000

2 15 seats

3 **a** 1440 minutes **b** 4500 grams

4 3 stickers

5 **a** 15 cents **b** 50 grams **c** 1.6 minutes

6 19.2 minutes

7 **a** \$75 **b** 6750

8 \$68

9 **a** 2000 mL **b** 0.13 cents **c** 32.38 cents

10 \$8.25

11 1530 g

12 C

13 55 hours

14 $Number\ of\ tickets = 5 \times 100$

$Number\ of\ tickets = 500$

$Price\ of\ one\ ticket = \frac{750}{500}$

$Price\ of\ one\ ticket = \$1.50$

$profit\ selling\ 173\ tickets = 173 \times 1.5$

$profit\ selling\ 173\ tickets = 259.5$

$259.5 < 260$

Therefore Agnes raises less than \$260 from selling 173 tickets.

15 **a** 19 kits **b** 8 trucks

Questions from multiple lessons

16 ***Computation and practical arithmetic***

Decoding the question

Keywords: increase, percentage

Relevant lesson: 2E

We must first find the difference between the two volumes and then convert this to a percentage of the small volume.

Information we are given

- A small smoothie is 260 mL.
- A regular smoothie is 350 mL.

Using the theory

We can use the formula for percentage change to find the percentage difference between sizes.

$$percentage\ change = \frac{difference}{original\ value} \times 100$$

The difference can be calculated as $350 - 260 = 90$ mL.

As we are looking for the difference as a percentage of the small smoothie's volume, the original value will be 260 mL.

Substitute $difference = 90$ and $original\ value = 260$ into $percentage\ change = \frac{difference}{original\ value} \times 100$.

$percentage\ change = \frac{90}{260} \times 100 = 34.615...$

This is closest to 34.6%.

Answer

C

✓1 mark for the correct answer

17 *Data analysis*

Decoding the question

Keywords: stem plot, median

Relevant lessons: 12C, 12E

We can locate the median value from the stem plot using the formula $\frac{n+1}{2}$

Information we are given

- The given stem plot
- $n = 35$

Using the theory

The median is the middle value in a dataset. In a dataset with n elements, it is located in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position.

Substitute $n = 35$ into $\frac{n+1}{2}$ to find the position of the median.

$\frac{35+1}{2} = 18$

Hence, the median is the 18^{th} value in the stem plot. This value is highlighted.

Key: 13 | 5 = \$13.50 $n = 35$

13	5
14	2 7
15	0 3 6 8 8
16	3 4 5 9
17	0 0 6 7 7 **7**
18	4 8
19	5 5 8 9
20	0 2 9
21	2 5 5 6
22	0
23	1 4 8

The median hourly wage is \$17.70.

Answer

C

✓1 mark for the correct answer

18a *Computational and practical arithmetic*

Decoding the question

Keywords: scale, actual distance

Relevant lesson: 2F

We can represent the scale of the floor plan as a fraction and multiply this by the given distance to find the actual distance.

Information we are given

- The floor plan has a scale of 1:500.
- The distance on the floor plan is 14.2 cm.

Using the theory

Express the scale of the floor plan as a fraction relative to the distance on the floor plan and the actual distance.

$\frac{\textit{distance on floor plan}}{\textit{actual distance}} = \frac{1}{500}$

Isolate the *actual distance*

$\textit{actual distance} = \textit{distance on floor plan} \div \frac{1}{500}$

$\textit{actual distance} = \textit{distance on floor plan} \times 500$

Substitute $\textit{distance on floor plan} = 14.2$ into $\textit{actual distance} = \textit{distance on floor plan} \times 500$ to find the actual distance.

$\textit{actual distance} = 14.2 \times 500$

$\textit{actual distance} = 7100$

The actual distance is 7100 cm.

Answer

7100 cm

✓1 mark for the correct answer

18b *Computational and practical arithmetic*

Decoding the question

Keywords: scale, distance

Relevant lesson: 2F

We can represent the scale of the floor plan as a fraction and multiply this by the given distance to find the distance on the floor plan.

Information we are given

- The floor plan has a scale of 1:500.
- The actual distance is 5205 cm.

Using the theory

Express the scale of the floor plan as a fraction relative to the distance on the floor plan and the actual distance.

$\frac{\textit{distance on floor plan}}{\textit{actual distance}} = \frac{1}{500}$

Isolate the *distance on floor plan*

$\textit{distance on floor plan} = \textit{actual distance} \times \frac{1}{500}$

Substitute $\textit{actual distance} = 5205$ into $\textit{distance on floor plan} = \textit{actual distance} \times \frac{1}{500}$ to find the distance on the floor plan.

$\textit{distance on floor plan} = 5205 \times \frac{1}{500}$

$\textit{distance on floor plan} = 10.41$

The distance on the floor plan is 10.41 cm.

Answer

10.41 cm

✓1 mark for the correct answer

3A

1 30%

2 **a** 90.9% **b** 90.9 **c** 110%

3 Croissant – 44.4% (Note: Car – 12.5%)

4 100, 100, 100, 100, 27.14

5 **a** 50% increase **b** 30.43% decrease

c 4.58% increase

6 $17.00

7 96 runs

8 $63.75

9 **a** **i** $2.40

ii $14.40

b **i** $11.25

ii $13.75

c **i** $12.25

ii $24.75

10 33.44%

11 $64

12 $15

13 162 minutes

14 18% discount on the watch, 33.33% discount on the DVD, 40% discount on the chocolate, 32% discount on the book.

The chocolate has the biggest percentage discount.

15 Zach won the auction with a bid of $1550.

16 $180

17 $85

18 D

Questions from multiple lessons

19 ***Recursion and financial modelling***

Decoding the question

Keywords: price, reduced

Relevant lesson: 3A

We must find 20% of $9.80 to determine the price reduction for a beanie.

Information we are given

- The price of beanies is usually $9.80.
- There is a 20% discount on all beanies.

Using the theory

Find the price reduction when a 20% discount is applied to the beanie using the following formula:

$$discount = original\ price \times \frac{percentage\ change}{100}$$

Substitute in $original\ price = 9.80$ and $percentage\ change = 20$ into the equation.

$$discount = 9.80 \times \frac{20}{100}$$

$$discount = 1.96$$

The price of a beanie is reduced by $1.96.

Answer

A

✓*1 mark for the correct answer*

20 ***Data analysis***

Decoding the question

Keywords: stem plot, modal

Relevant lesson: 12C

The word 'modal' tells us that we need to find the most frequently occurring number in the stem plot.

Information we are given

- The given stem plot

Using the theory

The modal hours driven is given by the most frequently occurring number of hours driven. This can be found by looking at stem '6'. There are four leaves with the number '6' corresponding to this stem. Referring to our key, this means that four students have driven 66 hours.

∴ The modal number of hours driven is 66.

Answer

D

✓*1 mark for the correct answer*

21a ***Recursion and financial modelling***

Decoding the question

Keywords: express, percentage

Relevant lesson: 3A

We need to express $120 as a percentage of $800.

Information we are given

- The full price is $800.
- The concession price is $120 off the full price.

Using the theory

The percentage can be found using the following formula:

$$percentage\ off\ original\ price = \frac{amount\ off\ original\ price}{original\ price} \times 100$$

Substitute $amount\ off\ original\ price = 120$ and $original\ price = 800$ into the equation.

$$percentage\ off\ original\ price = \frac{120}{800} \times 100$$

$$percentage\ off\ original\ price = 15\%$$

Concession cardholders receive 15% off the full price.

Answer

15%

✓*1 mark for the correct answer*

21b ***Recursion and financial modelling***

Decoding the question

Keywords: total, pay

Relevant lesson: 1C

We need to determine the total cost for a full-fee payer who spends an extra 5 hours in the pool.

Information we are given

- A hydrotherapist pool costs a flat rate of $800 per session.
- Full-fee payers have to pay an extra $60 per hour that they spend in the pool.
- John does not have a concession card.
- He spends an extra five hours in the pool.

Using the theory

Determine the cost function.

John is charged a flat rate of $800 and an extra $60 per hour that he spends there.

The function for this scenario is $cost = 800 + 60 \times t$ where t is the time in hours.

He spends an extra five hours in the pool. Substitute $t = 5$ into the equation and determine his cost.

$cost = 800 + 60 \times 5$

$cost = 1100$

It will cost him $1100.

Answer

$1100

✓ *1 mark for the correct answer*

3B

1 10

2 $88

3 **a** $198 **b** $385 **c** $15.95

4 **a** $4.09 **b** $15.91 **c** $1.54

5 The retail price of the DC socks is $20.35. The DC socks are more expensive.

6 $2 per share

7 **a** 22.32 **b** 45.03 **c** 16.53

8 $5.80

9 Bega Cheese, at $7.09, has a higher share price than Inghams Group, at $3.72.

10 The missing numbers are '100' and '66.67'.

11 **a** 25% increase

b 30% decrease

c 16.67% increase

12 $1.46 per litre

13 $0.25

14 Price including GST: $2277

Discounted price: $1821.60

15 Company X has 4 million shares, and a percentage dividend of 8.93%.

Company Y has 14.28 million shares, and a percentage dividend of 2.86%.

Company X is more attractive to investors.

16 The English class has dropped 31.25%.

The Maths class has increased 62.5%.

17 Kathy spent $280.50. Linda spent $350. Therefore, Linda spent more money today.

18 Less than

19 $11.70

20 **a** $233 000 **b** $0.53

21 **a** 2.11% decrease **b** 1.35% decrease

22 **a** $8 **b** $99

Questions from multiple lessons

23 ***Recursion and financial modelling***

Decoding the question

Keywords: increase in value, percentage

Relevant lesson: 3A

We must first find the difference between the two values and then represent this as a percentage of the original value.

Information we are given

- The painting was bought for $140 000.
- The painting then sold at auction for $189 000.

Using the theory

The percentage change can be found using the following formula:

$$percentage\ change = \frac{change}{original\ value} \times 100$$

In this case, the original value of the painting was $140 000 and the change in value was $189 000 − $140 000 = $49 000.

Substitute these values into the formula.

$$percentage\ change = \frac{49\ 000}{140\ 000} \times 100$$

$percentage\ change = 35\%$

There was a 35% increase in value.

Answer

C

✓ *1 mark for the correct answer*

24 ***Data analysis***

Decoding the question

Keywords: histogram, percentage

Relevant lessons: 2E, 12C

We need to analyse the histogram to find the percentage of households with fewer than two pets.

Information we are given

- The given histogram
- The sample size is 45.

Using the theory

The number of households with fewer than two pets is the frequency of the leftmost column in the histogram. The frequency in a histogram is given by the height of the column.

Hence, 26 households have fewer than two pets.

Represent this as a percentage of the total number of households surveyed using the formula $percentage = \frac{number\ of\ households\ with\ fewer\ than\ two\ pets}{total\ number\ of\ households} \times 100$.

Substitute $number\ of\ households\ with\ fewer\ than\ two\ pets = 26$ and $total\ number\ of\ households = 45$.

$percentage = \frac{26}{45} \times 100$

$percentage = 57.77...\%$

$percentage \approx 58\%$

58% of households in the sample own fewer than two pets.

Answer

C

✓*1 mark for the correct answer*

25a ***Recursion and financial modelling***

Decoding the question

Keyword: deposit

Relevant lesson: 3A

This question requires us to determine the deposit for the house.

Information we are given

- The house was purchased for $750 000.
- A 20% deposit was paid.

Using the theory

To find the value of the deposit, find 20% of $750 000.

$deposit = percentage \times 750\ 000$

$deposit = \frac{20}{100} \times 750\ 000$

$deposit = 150\ 000$

The deposit was $150 000.

Answer

$150 000

✓*1 mark for the correct answer*

25b ***Recursion and financial modelling***

Decoding the question

Keywords: deposit, value, owe

Relevant lessons: 3A, 3B

This question requires us to determine the principal value after paying the deposit.

Information we are given

- The house was purchased for $750 000.
- A 20% deposit was paid.

Using the theory

From part **a**, the deposit paid is $150 000.

Find the principal value by subtracting the deposit from the purchase value.

$principal\ value = purchase\ value - deposit$

$principal\ value = 750\ 000 - 150\ 000$

$principal\ value = 600\ 000$

The homeowners still owe $600 000 after paying the deposit.

Answer

$600 000

✓*1 mark for the correct answer*

25c ***Recursion and financial modelling***

Decoding the question

Keywords: GST, included, calculate

Relevant lessons: 3B

This question requires us to determine the price of the house without GST.

Information we are given

- The price inclusive of GST is $750 000.

Using the theory

Determine the amount of GST included in the price using the formula $GST = \frac{Q}{11}$

Substitute $Q = 750\ 000$ into the equation.

$GST = \frac{750\ 000}{11}$

$GST = 68\ 181.8181...$

$GST \approx 68\ 182$

Determine the price without GST.

$price\ without\ GST = price\ with\ GST - GST$

$price\ without\ GST = 750\ 000 - 68\ 182$

$price\ without\ GST = 681\ 818$

The price of the house before GST was added was $681 818, correct to the nearest dollar.

Answer

$681 818

✓*1 mark for the correct answer*

3C

1 **a** **i** $\frac{3}{100}$

ii 0.03

b **i** 12

ii 4

iii 3

2 **a** $I = \frac{100 \times 2 \times 1}{100}$ **b** $2.00

3 **a** $11.25 **b** $33.75 **c** $78.75

4 **a** $648.00 **b** Yes

5 **a** $n = \frac{100}{r}\left(\frac{A}{P} - 1\right)$ **b** $r = \frac{100}{n}\left(\frac{A}{P} - 1\right)$

c $\frac{A}{\left(1 + \frac{rn}{100}\right)} = P$

6 **a** $700.00 **b** $350.00 **c** $116.67

7 **a** 2.8% **b** $39 000 **c** $12 800

d **i** $49.00

ii 7

8 45.56%

9 **a** $I = 42Y$ **b** $I = 3.5M$

10 $A = P + I$

$I = \frac{Prn}{100}$

$A = P + \frac{Prn}{100}$

$A = P\left(1 + \frac{rn}{100}\right)$

11 **a** $2230.00 **b** $5.58

12 **a** $180.00 **b** $0.47

13 $330.48

14 $275.00

15 **a** $325.00 **b** $4350.00 **c** 6.25%

d 11 years

16 **a** 50% **b** $2000.00 **c** $2200.00

d **i** Legal Loans

ii The Dollar Dealer

17 **a** $2145.00 **b** 7.5% **c** $3000.00

18 $I = Prn$

$r_{monthly} = \frac{I}{Pn}$

$r_{monthly} = \frac{231.25}{75000 \times 1}$

$r_{annual} = r_{monthly} \times 12$

$r_{annual} = \frac{231.25 \times 12}{75000}$

$r_{annual} = 0.037$

$r_{annual} = 3.7\%$

19 D

Questions from multiple lessons

20 ***Recursion and financial modelling***

Decoding the question

Keyword: percentage

Relevant lesson: 3A

This question requires us to determine the cost of hiring the billboard as a percentage of the total amount earned from selling tickets.

Information we are given

- The billboard costs $10 320 to hire.
- The musician earned a total of $86 000 selling tickets.

Using the theory

Calculate the cost of hiring the billboard as a percentage of the total amount earned from selling tickets.

$percentage = \frac{10\ 320}{86\ 000} \times 100$

$percentage = 12\%$

Answer

D

✓*1 mark for the correct answer*

21 ***Data analysis***

Decoding the question

Keywords: sample, mean, standard deviation, table

Relevant lessons: 12E, 12F

This question requires us to perform one-variable statistics analysis to determine the mean and standard deviation of the sample.

Information we are given

- The given table
- The data is of 14 students from a sample of 130 students.

Using the theory

Enter the values into a spreadsheet on the graphics calculator and perform one-variable statistics analysis.

The values for the mean, $\bar{x}$ and standard deviation, s_x are 153.60 and 19.03 respectively.

See video solution for a button-by-button demonstration.

Answer

E

✓*1 mark for the correct answer*

22a ***Recursion and financial modelling***

Decoding the question

Keywords: calculate, amount, included, GST

Relevant lesson: 3B

This question requires us to determine the amount of GST included in the price.

Information we are given

- The business charges customers \$140 per backpack.
- The price of \$140 per backpack includes GST.

Using the theory

To calculate the amount of GST included in a product, use the formula $GST = \frac{Q}{11}$ where Q is the price including GST.

Substitute $Q = 140$ into the equation.

$GST = \frac{140}{11}$

$GST = 12.7272\ldots$

$GST \approx \$12.73$

Answer

\$12.73

✓1 mark for the correct answer

22b ***Recursion and financial modelling***

Decoding the question

Keywords: total amount

Relevant lesson: 3B

This question requires us to determine the amount of money the company receives as revenue from the sale of 10 backpacks after subtracting GST.

Information we are given

- The business charges customers \$140 per backpack.
- The price of \$140 per backpack includes GST.
- A customer purchased 10 backpacks.

Using the theory

From part **a**, we know that the amount of GST per backpack is \$12.73.

Calculate the revenue received for one backpack.

$revenue\ for\ one\ backpack = price - GST$

$revenue\ for\ one\ backpack = 140 - 12.73$

$revenue\ for\ one\ backpack = 127.27$

Calculate the revenue for 10 backpacks.

$revenue = 127.27 \times 10$

$revenue = 1272.70$

$revenue \approx 1273$

Hence, Carl receives \$1273 as revenue, correct to the nearest dollar.

Answer

\$1273

✓1 mark for the correct answer

3D

1 **a** $\frac{1}{200}$ **b** 27

2 **a** $A = 1000\left(1 + \frac{0.5}{100}\right)^8$

b \$1040.71

3 **a** \$ 14 674.84 **b** \$3075.51 **c** \$6197.54

4 \$1.68

5 Tobi's account will have a greater balance.

6 **a** $r = 100\left(\left(\frac{A}{P}\right)^{\frac{1}{n}} - 1\right)$

b 0.22%

7 **a** \$250 **b** 0.12% **c** \$250 000

d 6 years ago **e** 2.4%

f 7 compounding periods

8 **a** 9.9% **b** \$1503.27 **c** 117 months

9

Year	1	2	3
Simple	\$2000.00	\$2000.00	\$2000.00
Compound	\$2000.00	\$2080.00	\$2163.20

10 **a** $A = 4.1\left(1 + \frac{2}{100}\right)^{10}$

b \$5.00

11 **a** \$826.92 **b** \$64.87 **c** \$2 132 302.59

12 **a** 6.0% **b** 1969

13 \$28 545.25

14 **a** \$ 95 100.63 **b** \$31 110.36

c Amount in long-term savings: \$41 136.30

HECS-HELP loan: \$70 661.17

Barnesworth won \$29 525

15 **a** 1.0%

b There are 52 weeks in a year.

So $A = 50\left(1 + \frac{8}{5200}\right)^{52}$

$A = 54.16$

16 Amount of interest earnt is $I = A - P$

So the amount of interest earnt in a particular year is the balance from the end of the previous year minus the new balance:

Interest earnt in fifth year
= value at end of fifth year – value at end of fourth year.

$P = 20000$, $r = 0.06/4$ and $n = 4 \times 5 = 20$ for the fifth year and $n = 4 \times 4 = 16$ for the fourth year.

Interest earnt in fifth year

$= 20000\left(1 + \frac{0.06}{4}\right)^{20} - 20000\left(1 + \frac{0.06}{4}\right)^{16}$

17 **a** 2.6 average Australian yearly wages

b 5.8% **c** \$188 839

d 8.5 average Australian yearly wages

e 8.7%

f

	Average Wage ($)	Average Melbourne House Price ($)
2020	95 447	945 886
2021	100 983	1 030 902
2022	106 840	1 123 559
2023	113 036	1 224 544
2024	119 592	1 334 606
2025	126 529	1 454 560

g 11.5 average Australian yearly wages

18 $203

19 B

Questions from multiple lessons

20 ***Recursion and financial modelling***

Decoding the question

Keywords: charge, increase

Relevant lesson: 3A

This question requires us to calculate the increased cost of an air conditioner installation given a percentage change figure.

Information we are given

- The cost this week is $200.
- The cost next week is increased by 5.2%

Using the theory

To find the increase in cost, find 5.2% of $200.

5.2% expressed as a decimal is 0.052.

$cost\ increase = 5.2\,\% \times 200$

$cost\ increase = 0.052 \times 200$

$cost\ increase = 10.4$

To find the total cost, add the increase in cost to the initial amount.

$total\ cost\ to\ install = 200 + 10.4$

$total\ cost\ to\ install = 210.4$

The charge next week will be $210.40.

Answer

C

✓ *1 mark for the correct answer*

21 ***Data analysis***

Decoding the question

Keywords: variables, nominal, ordinal, numerical, categorical

Relevant lesson: 12A

We need to classify the given variables.

Information we are given

- *Car type* (1: Tesla, 2: Mustang, 3: Other)
- *Number plate type* (custom, auto-generated)

Using the theory

While the variable *car type* is classified by numbers, these numbers represent different car brands, separating them into categories. Hence, *car type* is a categorical variable. This eliminates option C. Furthermore, the car brands do not have a natural order, making it a nominal variable. This eliminates options B and E.

The variable *number plate type* is classified into two different categories; 'custom' or 'auto-generated'. Hence, it is a categorical variable. As there is no logical order to these categories, it is nominal.

Therefore, both variables are nominal.

Answer

A

✓ *1 mark for the correct answer*

22a ***Recursion and financial modelling***

Decoding the question

Keyword: simple interest rate

Relevant lesson: 3C, 3D

We need to use our understanding of interest rates to determine the interest Claire is charged in the first month.

Information we are given

- Claire decides to borrow $7000 to purchase a car.
- She will pay interest at a rate of 4.3% per month.

Using the theory

The type of interest has not been specified as either simple or compound. However, since we are only looking for the interest charged in the first month, this value is the same for both types of interest.

The formula for the amount of interest paid for a simple interest agreement is $I = \frac{Prn}{100}$

$P = 7000$ as this is the principal of the loan.

$r = 4.3$ as the interest rate is 4.3% per month.

$n = 1$ as we are calculating the interest accumulated in one month.

Substitute $P = 7000$, $r = 4.3$ and $n = 1$ into $I = \frac{Prn}{100}$

$I = \frac{7000 \times 4.3 \times 1}{100}$

$I = 301$

∴ In the first month, Claire will pay $301 in interest.

Answer

$301

✓ *1 mark for the correct answer*

22b *Recursion and financial modelling*

Decoding the question

Keyword: simple interest rate

Relevant lesson: 3C

We need to determine the annual simple interest rate that Claire is charged if she decides to loan from the car dealer's finance company.

Information we are given

- Claire decides to borrow \$7000 to purchase a car.
- She will pay \$8.75 in interest per month.

Using the theory

The formula for the amount of interest paid for a simple interest agreement is $I = \frac{Prn}{100}$

Substitute $I = 8.75$, $P = 7000$ and $n = 1$ and solve for r

$8.75 = \frac{7000 \times r \times 1}{100}$

$r = \frac{8.75 \times 100}{7000}$

$r = 0.125\%$ per month

Convert the monthly interest rate to a yearly interest rate by multiplying by 12:

$yearly\ interest\ rate = monthly\ interest\ rate \times 12$

$yearly\ interest\ rate = 0.125 \times 12$

$yearly\ interest\ rate = 1.5\%$

∴ The interest rate is 1.5% p.a.

Answer

1.5%

✓1 mark for the correct answer

3E

1 **a** 1.8 **b** 1.62

2 \$8.70

3 **a** \$28.50 **b** \$63.25

4 \$365.58

5 16.6%

6 \$250

7 **a** Interest: \$210

$r = 6.67\%$

b Interest: \$250

$r = 30.95\%$

8 Different, the 9 month agreement has a higher rate.

9 6 payments

10 Interest = \$502.50

Balance after interest = \$75 502.50

11

Payment number	Principal owed before payment (\$)	Amount paid (\$)	Interest paid (\$)	Debt repaid (\$)	Principal owed after payment (\$)
1	15 000.00	600.00	137.50	462.50	14 537.50
2	14 537.50	600.00	133.26	466.74	14 070.76
3	14 070.76	600.00	128.98	471.02	13 599.74
4	13 599.74	600.00	124.66	475.34	13 124.40

12 **a** \$8802.51 **b** \$5673.66

13 C

14 D

15 B

16 **a** Gordon should choose Nigella's loan.

b He will save \$37.68.

17 E

Questions from multiple lessons

18 *Recursion and financial modelling*

Decoding the question

Keywords: investment, interest, compounds daily

Relevant lesson: 3D

We must find the value of the investment using the compound interest formula.

Information we are given

- Jackson invests \$12 000 for two years.
- Interest compounds daily at a rate of 4.5% per annum.

Using the theory

Find the interest rate per compounding period

$r = \frac{yearly\ interest\ rate}{365}$

$r = \frac{4.5}{365}$

$r = 0.0123...\%$

The compound interest formula states that $A = P\left(1 + \frac{r}{100}\right)^n$ where A is the total value of the investment, P is the principal, r is the interest rate per compounding period and n is the number of compounding periods.

Substitute $P = 12\ 000$, $r = 0.0123...$ and $n = 200$ into the formula.

$A = 12\ 000\left(1 + \frac{0.0123...}{100}\right)^{200}$

$A = 12\ 299.549...$

$A \approx 12\ 300$

His investment is worth \$12 300 after 200 days, correct to the nearest dollar.

Answer

D

✓1 mark for the correct answer

19 *Data analysis*

Decoding the question

Keywords: histogram, interquartile range

Relevant lesson: 12F

We must identify the values of Q_1 and Q_3 from the histogram to calculate the interquartile range.

Information we are given

- The given histogram
- The sample contains 86 students.

Using the theory

First, we must find the values of Q_1 and Q_3

There are 86 data values in the sample. Hence, there are 43 values above and below the median.

The first quartile, Q_1, is the median of the first half of the data. It is the value in the $\frac{43+1}{2} = 22^{\text{nd}}$ position of the first half of the data. The 22^{nd} value lies in the interval from 2.5 m to 3 m.

The midpoint of this interval is 2.75 m. Hence, $Q_1 = 2.75$.

The third quartile, Q_3, is the median of the second half of the data. It is the value in the $\frac{43+1}{2} = 22^{\text{nd}}$ position of the second half of the data, which is the data point in the 65^{th} position overall. The 65^{th} value lies in the interval from 3.5 m to 4 m.

The midpoint of this interval is 3.75 m. Hence, $Q_3 = 3.75$.

The interquartile range is given by the formula $IQR = Q_3 - Q_1$

Substitute $Q_3 = 3.75$ and $Q_1 = 2.75$ into the formula.

$IQR = 3.75 - 2.75$

$IQR = 1$

Therefore, the interquartile range is 1 m.

Answer

C

✓ *1 mark for the correct answer*

20a *Recursion and financial modelling*

Decoding the question

Keywords: interest, investment

Relevant lesson: 3D

This question requires us to calculate the interest earned in eight years.

Information we are given

- \$14 000 was invested for eight years.
- At the end of eight years, the investment is worth \$15 783.77.

Using the theory

Interest can be calculated using the formula:
$interest = final\ value - initial\ value$

Substitute $final\ value = 15\ 783.77$ and $initial\ value = 14\ 000.00$ into the formula.

$interest = 15\ 783.77 - 14\ 000$

$interest = 1783.77$

The amount of interest earned in eight years is \$1783.77.

Answer

\$1783.77

✓ *1 mark for the correct answer*

20b *Recursion and financial modelling*

Decoding the question

Keywords: annual rate of interest, investment

Relevant lesson: 3D

This question requires us to determine the yearly interest rate for a compounding interest scenario.

Information we are given

- \$14 000 was invested for eight years.
- At the end of eight years, the investment is worth \$15 783.77.
- Interest on the account had been calculated and paid monthly.

Using the theory

The compound interest formula for interest compounding monthly is $A = P\left(1 + \frac{r}{1200}\right)^n$, where r is the interest rate per annum and n is the number of months.

Substitute $A = 15\ 783.77$, $P = 14\ 000$, $n = 8 \times 12$ into the formula.

$15\ 783.77 = 14\ 000 \times \left(1 + \frac{r}{1200}\right)^{96}$

Use the solve function on the calculator to determine r.

$r = 1.499...\%$

$r \approx 1.5\%$

See video solution for a button-by-button demonstration.

Answer

1.5% p.a.

✓ *1 mark for the correct answer*

4A

1 4

2 C

3 **a** 1 **b** 7
c 16 **d** 8

4 **a** 1×5 **b** 3×2
c 2×1 **d** 3×4

5 **a** 4×3 **b** 10 **c** 29

6 **a** $[0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$

b $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7 **a** B, F **b** A, B, G

c B, E, F, G **d** B, C, D, G

e C, G

8 $\begin{array}{c} \\ \text{M} \\ \text{F} \\ \text{S} \end{array} \begin{array}{c} \begin{array}{cc} \text{P} & \text{C} \end{array} \\ \begin{bmatrix} 2 & 1.5 \\ 1.5 & 2 \\ 0.5 & 1 \end{bmatrix} \end{array}$

9 D

10 $\begin{array}{c} \\ \text{Cats} \\ \text{Dogs} \end{array} \begin{array}{c} \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix} \end{array}$

11 **a** $\begin{array}{c} \\ \text{Boys} \\ \text{Girls} \end{array} \begin{array}{c} \begin{array}{ccc} \text{V} & \text{S} & \text{M} \end{array} \\ \begin{bmatrix} 9 & 12 & 21 \\ 15 & 11 & 17 \end{bmatrix} \end{array}$ **b** $\begin{array}{c} \\ \text{Boys} \\ \text{Girls} \end{array} \begin{array}{c} \\ \begin{bmatrix} 21 \\ 17 \end{bmatrix} \end{array}$

c $\begin{array}{c} \begin{array}{ccc} \text{V} & \text{S} & \text{M} \end{array} \\ [9 \ 12 \ 21] \end{array}$

d 42. It means that a total of 42 boys were surveyed.

12 **a** 7

b 10 people in Class B chose RuPaul's Drag Race.

c x_{32}

d A: 26

B: 27

C: 25

e RuPaul's Drag Race

13 **a** Wednesday: risotto

Thursday: casserole

Friday: pasta

b $\begin{array}{c} \\ \text{Tonight} \\ \\ \\ \\ \end{array} \begin{array}{c} \text{Last night} \\ \begin{array}{cccc} \text{P} & \text{R} & \text{S} & \text{C} \end{array} \\ \begin{array}{c} \text{P} \\ \text{R} \\ \text{S} \\ \text{C} \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$

14 E

Questions from multiple lessons

15 ***Matrices***

Decoding the question

Keyword: element

Relevant lesson: 4A

The question asks us to use our knowledge of element labelling to find what element m_{23} represents within matrix M

Information we are given

- The given matrix
- The element in row i and column j in matrix M is m_{ij}
- The element in question is m_{23}

Using the theory

Element m_{23} is the element in row 2 and column 3.

Row 2 represents the number of times each person attended a concert last summer.

Column 3 represents the number of times Joey attended each type of music event last summer.

$\therefore$ The element m_{23} represents the number of concerts Joey attended last summer.

Answer

D

✓*1 mark for the correct answer*

16 ***Data analysis***

Decoding the question

Keywords: variables, nominal, ordinal, numerical, categorical

Relevant lesson: 1A

The topic that links these keywords is 'data types'.

NOTE: 'respectively' means 'in the order already mentioned'.

Information we are given

- *Height* (metres) (less than 20, 20-40, more than 40)
- *Type of tree* (Eucalpyt, Mountain Ash, Redwood)

Using the theory

Height (metres) places data points into one of three categories/groups so it is a categorical variable. We can get more specific by checking if the categories have a logical order. In this case, they do - from smallest to the largest heights - thus this is an ordinal categorical variable.

NOTE: Just because these categories involve numbers, it doesn't mean that it is a numerical variable.

Type of tree does not use numbers so it cannot be a numerical variable. It places data points into one of three categories/groups so it is also a categorical variable.
As before, we can get more specific by checking if the categories have a logical order. In this case, they do not – there is no inherent way to rank Eucalypt, Mountain Ash and Redwood – thus this is a nominal categorical variable.

Answer

B

✓*1 mark for the correct answer*

17a *Matrices*

Decoding the question

Keyword: cost

Relevant lesson: 4A

This question asks you to inspect the given matrix to find the cost of one block of Edlindt at Woolworths.

Information we are given

- The given matrix
- All values are in dollars.

Using the theory

The elements in matrix B represent the cost of one block of each brand of chocolate.

The elements in the row labelled 'E' are the cost of one block of Edlindt at different stores.

The elements in the column labelled 'W' are the cost of one block of each brand of chocolate at Woolworths.

The intersection of row E and column W is the cost of one block of Edlindt at Woolworths.

The value of this element is 5.99.

Answer

$5.99

✓1 mark for the correct answer

17b *Matrices*

Decoding the question

Keyword: order

Relevant lesson: 4A

This question asks for the order of matrix B

Information we are given

- The given matrix

Using the theory

The order of a matrix is expressed as
number of rows × *number of columns*

There are 3 rows and 2 columns in matrix B

Hence, the order of matrix B is 3×2.

Answer

3×2

✓1 mark for the correct answer

17c *Matrices*

Decoding the question

Keywords: element, represent

Relevant lesson: 4A

By locating the given element in the matrix, we will find what it represents.

Information we are given

- The given matrix

Using the theory

The element $b_{1\,2}$ is in the first row and second column of the matrix.

The first row is the price of one Cadrolo block.

The second column is the price of the chocolates at IGA.

Answer

The price of one block of Cadrolo chocolate at IGA.

✓1 mark for the correct answer

4B

1 -4

2 **a** $\begin{bmatrix} -2 & 17 \\ 14 & 17 \end{bmatrix}$

b $\begin{bmatrix} 7 & 4 \\ 17 & 4 \end{bmatrix}$

c $\begin{bmatrix} 1 & -10 \\ -11 & 5 \end{bmatrix}$

3 A, E

4
$$\begin{matrix} \\ \text{Small} \\ \text{Large} \end{matrix} \begin{matrix} \begin{matrix} \text{Blueberry} & \text{Banana} & \text{Chocolate} \end{matrix} \\ \begin{bmatrix} 34 & 31 & 39 \\ 46 & 39 & 46 \end{bmatrix} \end{matrix}$$

5 9

6 **a** $a = 7, b = -12$ **b** $c = 3, d = 8$

c $e = 0, f = -1, g = 2$

7 $a = 2, b = 6, c = 13$

8 $\begin{bmatrix} 3 & -7 \\ -5 & -2 \\ 8 & 4 \end{bmatrix}$

9 D

10 **a**
$$\begin{matrix} \\ \text{Change in weight (kg)} \\ \text{Change in body fat (\%)} \end{matrix} \begin{matrix} \begin{matrix} \text{Alex} & \text{Pauline} & \text{Zac} & \text{Kelly} \end{matrix} \\ \begin{bmatrix} -1 & 1 & 2 & -2 \\ -5 & -2 & -2 & -3 \end{bmatrix} \end{matrix}$$

b Kelly

11 D

Questions from multiple lessons

12 *Matrices*

Decoding the question

Keyword: matrix

Relevant lesson: 4A

The question asks us for the element that represents the number of hours worked on Saturday during week 3.

Information we are given

- The given matrix

Using the theory

Saturday is represented by the first row. Week 3 is represented by the third column ($W3$).

The element in the first row and third column is 5, circled below.

$$\begin{array}{r} \\ \text{Saturday} \\ \text{Sunday} \end{array}\begin{array}{c} \begin{array}{ccc} \text{W1} & \text{W2} & \text{W3} \end{array} \\ \begin{bmatrix} 4 & 7 & \textcircled{5} \\ 5 & 7 & 8 \end{bmatrix} \end{array}$$

∴ Jane worked 5 hours on Saturday in week 3.

Answer

B

✓1 mark for the correct answer

13 ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation, terms

Relevant lesson: 6B

The first term in the recurrence relation is $B_0 = -1$.
Use the rule $B_{n+1} = -3B_n + 4$ to find the next four terms.

Information we are given

- $B_0 = -1,\ B_{n+1} = -3B_n + 4$

Using the theory

$B_0 = -1,\ B_{n+1} = -3B_n + 4$

The first term is $B_0 = -1$, so we can already eliminate option D.

Substitute $n = 0$ into $B_{n+1} = -3B_n + 4$ to find the second term:

$B_1 = -3B_0 + 4 = (-3) \times (-1) + 4 = 3 + 4 = 7$

Substitute $n = 1$ into $B_{n+1} = -3B_n + 4$ to find the third term:

$B_2 = -3B_1 + 4 = (-3) \times 7 + 4 = -21 + 4 = -17$

Substitute $n = 2$ into $B_{n+1} = -3B_n + 4$ to find the fourth term:

$B_3 = -3B_2 + 4 = (-3) \times (-17) + 4 = 51 + 4 = 55$

Substitute $n = 3$ into $B_{n+1} = -3B_n + 4$ to find the fifth term:

$B_4 = -3B_3 + 4 = (-3) \times 55 + 4 = -165 + 4 = -161$

∴ The first five terms of the recurrence relation are:
$-1, 7, -17, 55, -161$.

Answer

B

✓1 mark for the correct answer

14a ***Matrices***

Decoding the question

Keywords: order, matrix

Relevant lesson: 4A

To find the order of matrix N, the number of rows and columns of the matrix need to be found.

Information we are given

- The given matrix

Using the theory

The order of a matrix is written in the form:
number of rows × *number of columns*

There are 3 rows and 1 column.

∴ The order of matrix N is 3×1.

Answer

3×1

✓1 mark for the correct answer

14b ***Matrices***

Decoding the question

Keyword: element

Relevant lesson: 4A

We need to find the element $n_{3\,1}$ in the matrix.

Information we are given

- The given matrix
- Element $n_{1\,2}$

Using the theory

$n_{3\,1}$ represents the element in row 3 and column 1 of matrix N

$$N = \begin{array}{c} \text{Column 1} \\ \begin{bmatrix} 3 & \text{C} \\ 3 & \text{S} \\ 49 & \text{A} \end{bmatrix} \end{array} \text{Row 3}$$

The element in row 3 and column 1 of the matrix is 49.

Answer

49

✓1 mark for the correct answer

14c ***Matrices***

Decoding the question

Keywords: sum, elements, matrix, represent

Relevant lesson: 4A

Find the sum of the elements by adding all the numbers in the matrix, and determine what it represents.

Information we are given

- The given matrix

Using the theory

The elements in the matrix are 36, 34 and 49.

$$N = \begin{bmatrix} 36 \\ 34 \\ 49 \end{bmatrix} \begin{matrix} \text{C} \\ \text{S} \\ \text{A} \end{matrix}$$

Add the elements together:

$36 + 34 + 49 = 119$

36, 34 and 49 represent the number of child, student and adult tickets respectively, sold for the movie. Hence, the sum of these three numbers is the total number of tickets sold for the movie.

Answer

119, the total number of tickets sold for the movie.

✓1 mark for the correct answer

4C

1 **a** 31 **b** 15

2 10

3 $\begin{bmatrix} 6 & 9 \\ 3 & 6 \end{bmatrix}$

4 **a** $\begin{bmatrix} -8 & 2 \\ -4 & -6 \end{bmatrix}$ **b** $\begin{bmatrix} 40 & -20 \\ 70 & 30 \end{bmatrix}$

c $\begin{bmatrix} 3 & 1 \\ 4 & 10 \end{bmatrix}$

5 **a** 2 **b** 9

6 1

7 **a** $[-17 \quad 86 \quad -217]$ **b** $\begin{bmatrix} -1 \\ 0 \\ 14 \end{bmatrix}$

c $\begin{bmatrix} -4 & 41 \\ -21 & 7 \end{bmatrix}$

8 **a** $\begin{bmatrix} 0 & 2 \\ 2 & 22 \end{bmatrix}$ **b** $\begin{bmatrix} 14 & -12 \\ -5 & 1 \end{bmatrix}$

c $\begin{bmatrix} -2 & 0 \\ -1 & -19 \end{bmatrix}$

9 **a** $\begin{bmatrix} 9 & 4 & 2 \\ -10 & 4 & -13 \end{bmatrix}$

b $\begin{bmatrix} 5 & -2 & -2 \\ 0.5 & 4 & -1.5 \end{bmatrix}$

10 Expression 2 and Expression 3

11 D

12 $x = 3, y = -1$

13 B

14 Hussain

15 A

Questions from multiple lessons

16 *Matrices*

Decoding the question

Keyword: matrix equation

Relevant lessons: 4B, 4C

We must rearrange and evaluate the matrix equation to find the value of the unknown matrix M

Information we are given

- The given matrix

Using the theory

$2 \times \begin{bmatrix} 6 & -4 \\ 3 & 0 \end{bmatrix}$ can first be simplified using scalar multiplication. Each element in the matrix is multiplied by 2.

$$2 \times \begin{bmatrix} 6 & -4 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 2 \times 6 & 2 \times (-4) \\ 2 \times 3 & 2 \times 0 \end{bmatrix} = \begin{bmatrix} 12 & -8 \\ 6 & 0 \end{bmatrix}$$

Hence, the equation is now

$$\begin{bmatrix} 12 & -8 \\ 6 & 0 \end{bmatrix} - M = \begin{bmatrix} 5 & 1 \\ 4 & -1 \end{bmatrix}$$

Add M to both sides of the equation.

$$\begin{bmatrix} 12 & -8 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ 4 & -1 \end{bmatrix} + M$$

Subtract $\begin{bmatrix} 5 & 1 \\ 4 & -1 \end{bmatrix}$ from both sides of the equation.

$$M = \begin{bmatrix} 12 & -8 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 1 \\ 4 & -1 \end{bmatrix}$$

$$M = \begin{bmatrix} 12 - 5 & -8 - 1 \\ 6 - 4 & 0 - (-1) \end{bmatrix}$$

$$M = \begin{bmatrix} 7 & -9 \\ 2 & 1 \end{bmatrix}$$

NOTE: M can also be found by directly substituting this equation into the solve function on your calculator.

Answer

D

✓1 mark for the correct answer

17 *Data analysis*

Decoding the question

Keywords: explanatory variable, least squares regression line

Relevant lesson: 13D

In this question, we must evaluate which of the given equations would best represent the regression line on the scatterplot.

Information we are given

- *Distance covered* is the explanatory variable.
- The given scatterplot

Using the theory

As stated in the question, *distance covered* is the explanatory variable (x) so *price* must be the response variable (y).

A least squares regression equation is of the form $y = a + bx$, where a is the y-intercept, and b is the gradient. After substituting in the variables, the equation becomes: $price = a + b \times distance\ covered$

In options A, B, and C, *price* is the explanatory variable. Hence, the correct option must be either D or E.

The only difference between options D and E is the value of the y-intercept.

A common error would be to assume that the y-intercept of the regression line was 507 by reading directly from the graph. However, the horizontal axis begins at 5000, not 0, so the y-intercept cannot be read off of the graph. The y-intercept must be less than 507 as the gradient is positive. Thus, the y-intercept must be closest to 441.

$\therefore$ The regression equation must be closest to $price = 441 + 0.013 \times distance\ covered$

Answer

E

✓1 mark for the correct answer

18a ***Matrices***

Decoding the question

Keywords: cost, matrix

Relevant lesson: 4A

This question asks us to inspect the given matrix to find the cost of a MacBook.

Information we are given

- The given matrix
- Values are in dollars.

Using the theory

The elements in matrix M represent the cost of each of the products. The number next to the label 'MacBook' will be the cost of a MacBook. This number is 1250.

Answer

$1250

✓1 mark for the correct answer

18b ***Matrices***

Decoding the question

Keyword: order

Relevant lesson: 4A

We must find the number of rows and columns in matrix M to determine its order.

Information we are given

- The given matrix

Using the theory

The order of a matrix is expressed as $number\ of\ rows \times number\ of\ columns$

Matrix M has 4 rows and 1 column.

Hence, the order of matrix M is 4×1.

Answer

4×1

✓1 mark for the correct answer

18c ***Matrices***

Decoding the question

Keywords: discounted, new price matrix, equation

Relevant lesson: 4C

We must use scalar multiplication to find the discounted price matrix N

Information we are given

- The given matrix
- $N = 0.75 \times M$

Using the theory

The new price matrix, N, can be found using the equation $N = 0.75 \times M$. The matrix M undergoes scalar multiplication of 0.75, so each element in M must be multiplied by 0.75.

$$N = 0.75 \times \begin{bmatrix} 1000 \\ 450 \\ 1250 \\ 800 \end{bmatrix} = \begin{bmatrix} 0.75 \times 1000 \\ 0.75 \times 450 \\ 0.75 \times 1250 \\ 0.75 \times 800 \end{bmatrix} = \begin{bmatrix} 750.0 \\ 337.50 \\ 937.50 \\ 600.00 \end{bmatrix}$$

Answer

$$N = \begin{bmatrix} 750.0 \\ 337.50 \\ 937.50 \\ 600.00 \end{bmatrix}$$

✓1 mark for the correct answer

4D

1 2×4

2 Two columns

3 **a** No

b Yes, the product will be a 2×3 matrix.

c Yes, the product will be a 3×1 matrix.

4 Yes, the product will be a 2×4 matrix.

5 The mistake is on line 2. The mistake is 3×4 was simplified to 10, it should have been simplified to 12.

6 **a** $\begin{bmatrix} 10 & 13 \\ 16 & 14 \end{bmatrix}$

b $\begin{bmatrix} 18 & -10 \\ -38 & 14 \end{bmatrix}$

c $\begin{bmatrix} 6 & 3 & 1 \\ -1 & -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$

7 $\begin{bmatrix} 9 \\ -17 \\ 8 \end{bmatrix}$

8 B

9 Three columns.

10 C

11 **a** $\begin{bmatrix} -32 & -26 \\ -185 & -28 \\ 133 & 49 \end{bmatrix}$ **b** $\begin{bmatrix} -32 & 120 & -38 \\ -81 & 113 & 18 \\ 126 & -252 & 77 \end{bmatrix}$

c AC is not defined **d** $\begin{bmatrix} 41 & -61 & 5 \\ -10 & -8 & 70 \end{bmatrix}$

e $\begin{bmatrix} 14 & -35 & 7 \\ 2 & 52 & -12 \\ -49 & 37 & -5 \end{bmatrix}$ **f** BA is not defined.

12 Youssef

13 D

Questions from multiple lessons

14 ***Matrices***

Decoding the question

Keywords: matrices, element, row i and column j

Relevant lesson: 4A

This question is asking us to construct a matrix using the provided rules that are based on each element's row and column position.

Information we are given

- The order of matrix P is 3×3.
- The order of matrix Q is 3×3.
- $p_{ij} = 5i - 2j$
- $q_{ij} = 3i - 2j$

Using the theory

First, recognise that each element in the matrix $P - Q$ can be generated by summing the formulas that generate each element in matrix P and Q

$(p - q)_{ij} = p_{ij} - q_{ij}$

This means that the formula of each element in matrix $P - Q$ can be found:

$(p - q)_{ij} = (5i - 2j) - (3i - 2j)$

Evaluating:

$(p - q)_{ij} = 5i - 2j - 3i - (-2j)$

$(p - q)_{ij} = (5 - 3)i - (2 - 2)j$

$(p - q)_{ij} = 2i$

The elements within the matrix $P - Q$ are equal in value to the row number multiplied by two.

$$P - Q = \begin{bmatrix} 1 \times 2 & 1 \times 2 & 1 \times 2 \\ 2 \times 2 & 2 \times 2 & 2 \times 2 \\ 3 \times 2 & 3 \times 2 & 3 \times 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 4 & 4 \\ 6 & 6 & 6 \end{bmatrix}$$

A slightly longer but also valid method would have been to individually generate matrices P and Q and then subtract Q from P

$$P = \begin{bmatrix} 3 & 1 & -1 \\ 8 & 6 & 4 \\ 13 & 11 & 9 \end{bmatrix} \quad Q = \begin{bmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \\ 7 & 5 & 3 \end{bmatrix}$$

$$P - Q = \begin{bmatrix} 3 - 1 & 1 - (-1) & -1 - (-3) \\ 8 - 4 & 6 - 2 & 4 - 0 \\ 13 - 7 & 11 - 5 & 9 - 3 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 4 & 4 \\ 6 & 6 & 6 \end{bmatrix}$$

Answer

B

✓ *1 mark for the correct answer*

15 ***Recursion and financial modelling***

Decoding the question

Keywords: loan, 'only pay the interest charged'

Relevant lesson: 3E

We need to find the value of an interest-only loan two and a half years into the future, given the present value of the loan.

Information we are given

- The present value of the loan is $160 000.
- Each month, Mattie will only pay the interest charged for that month.

Using the theory

A loan increases in value through compound interest. The value of a loan decreases when repayments are made.

Every month, Mattie's repayment is equal to the amount of interest that was charged for that month. Hence, the value of the loan stays constant.

The value of the loan was initially $160 000, hence the value of the loan will be $160 000 in 30 months time.

Answer

D

✓ *1 mark for the correct answer*

16a *Matrices*

Decoding the question

Keyword: order

Relevant lesson: 4A

We must determine the order of matrix N

Information we are given

- The given matrix

Using the theory

The order of a matrix is expressed as
number of rows × *number of columns*

There are 4 rows and 1 column.

Hence, the order of matrix N is 4×1.

Answer

4×1

✓1 mark for the correct answer

16b *Matrices*

Decoding the question

Keyword: matrix product

Relevant lesson: 4D

We need to calculate the matrix product R by solving the matrix multiplication $P \times N$

Information we are given

- $R = P \times N$
- The given matrix N
- The given matrix P

Using the theory

Multiplying the matrices:

$$P \times N = \begin{bmatrix} 29 & 13 & 25 & 34 \end{bmatrix} \times \begin{bmatrix} 58 \\ 14 \\ 167 \\ 115 \end{bmatrix}$$

$$= [(29 \times 58) + (13 \times 14) + (25 \times 167) + (34 \times 115)]$$

$$= [1682 + 182 + 4175 + 3910]$$

$$= [9949]$$

NOTE: This calculation can be performed on your calculator.

Answer

$[9949]$

✓1 mark for the correct answer

4E

1 3×4

2 AB

3 **a** The total number of students that had each type of dinner.

b 13

c The number of students in each class.

4 **a** The total number of cars recorded on each day.

b The average number of cars of each colour counted by Ali in 30 mins each day.

or

One fifth of the total number of cars of each colour counted.

5 **a** SP **b** \$340

6 **a** 118

b $\begin{bmatrix} 223 \\ 226 \end{bmatrix}$ **c** \$226

7 **a** $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$

The total number of items eaten on Monday is 6.

b $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 1.5 \end{bmatrix} = \begin{bmatrix} 9.5 \\ 8 \end{bmatrix}$

\$8 was spent on Tuesday.

8 **a** $\begin{bmatrix} 43 & 24 & 10 \\ 36 & 26 & 7 \end{bmatrix}$ and $\begin{bmatrix} 0.5 \\ 1 \\ 2 \end{bmatrix}$

or

$\begin{bmatrix} 0.5 & 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 43 & 36 \\ 24 & 26 \\ 10 & 7 \end{bmatrix}$

b $\begin{bmatrix} 65.5 \\ 58 \end{bmatrix}$

The total amount of money made on Saturday was \$65.50.

9 **a** 102 hats, 127 gloves, 202 scarves and 71 socks.

b $P = \begin{bmatrix} 18 & 26 & 40 & 15 \\ 20 & 23 & 38 & 12 \\ 24 & 29 & 47 & 17 \\ 19 & 24 & 38 & 14 \\ 21 & 25 & 39 & 13 \end{bmatrix} \times \begin{bmatrix} 11 \\ 7.5 \\ 11.5 \\ 6 \end{bmatrix} = \begin{bmatrix} 943 \\ 901.5 \\ 1124 \\ 910 \\ 945 \end{bmatrix}$

The profit made on Monday was \$943.

c $\begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix} \times \begin{bmatrix} 943 \\ 901.5 \\ 1124 \\ 910 \\ 945 \end{bmatrix} = [964.7]$

The average profit was \$964.70.

10 **a** 2×3

b **i** $M = \begin{bmatrix} 145\,978.00 \\ 171\,848.50 \end{bmatrix}$

ii Total revenue from selling products A, B and C at Eastown and Noxland.

c The number of columns in P $\neq$ number of rows in Q

Questions from multiple lessons

11 *Matrices*

Decoding the question

Keywords: matrix multiplication, total cost

Relevant lesson: 4E

In this question, we must find which of the given matrix products would show the total distance of Sandra's runs.

Information we are given

- Sandra will run the Albert Park Lake 7 times, The Tan 19 times, and Princes Park 13 times.
- Albert Park Lake: 4.70 km, The Tan: 3.82 km, Princes Park: 3.21 km

Using the theory

A matrix showing the total distance of Sandra's runs should be a 1×1 matrix as it will only contain one element; a value representing the total distance.

Hence, identify if each matrix product is defined and if so, its order.

If X is a $m \times n$ matrix and Y is a $p \times q$ matrix, then XY is defined if $n = p$ (if the number of columns in X is equal to the number of rows in Y). XY will be of order $m \times q$

A The first matrix is 3×1 and the second matrix is 1×3. The number of columns in the first matrix (1) is equal to the number of rows in the second matrix (1), so the multiplication is defined. $m = 3$ and $q = 3$, so the product will be a 3×3 matrix.

B The number of columns (1) in the first matrix is not equal to the number of rows in the second matrix (3), so the multiplication is not defined.

C The first matrix is 1×3 and the second matrix is 3×1. The number of columns in the first matrix (3) is equal to the number of rows in the second matrix (3), so the multiplication is defined. $m = 1$ and $q = 1$, so the product will be a 1×1 matrix.

D The number of columns (3) in the first matrix is not equal to the number of rows in the second matrix (3), so the multiplication is not defined.

E The first matrix is 1×3 and the second matrix is 3×3. The number of columns in the first matrix (3) is equal to the number of rows in the second matrix (3), so the multiplication is defined. $m = 1$ and $q = 3$, so the product will be a 1×3 matrix.

The only matrix product that is defined and has the order 1×1 is option C.

Answer

C

✓*1 mark for the correct answer*

12 *Data analysis*

Decoding the question

Keywords: least squares line, equation, scatterplot

Relevant lesson: 13D

We must find the equation of the least squares regression line using information from the given graph.

Information we are given

- The given graph

Using the theory

The equation of the least squares line is in the form $y = a + bx$, where x is the explanatory variable, y is the response variable, b is the gradient and a is the y-intercept.

First, we will calculate the gradient with the formula $b = \frac{y_2 - y_1}{x_2 - x_1}$

We will need to read two points off the line. They could be any two points, but in this solution we have chosen (1, 46.4) and (10, 38.8).

Substitute these into the equation:

$b = \frac{38.8 - 46.4}{10 - 1}$

$b = -0.84$

Now solve for a by substituting in $b = -0.84$ and the point (1, 46.4) into $y = a + bx$.

$46.4 = a + (-0.84 \times 1)$

$46.4 = a - 0.84$

$a = 47.2$

Substitute in the values of a and b, as well as x, the explanatory variable (*temperature*), and y, the response variable (*number of hot chocolates*), into $y = a + bx$.

$number\ of\ hot\ chocolates = 47.2 - 0.84 \times temperature$

A common error is to read the value of the y-intercept off the graph, but the leftmost x-value on the graph is 1, not 0.

Answer

E

✓*1 mark for the correct answer*

13a *Matrices*

Decoding the question

Keywords: matrix, order

Relevant lesson: 4A

We need to use the number of rows and columns to find the order of the given matrix Q

Information we are given

- The given matrix Q

Using the theory

The order of a matrix is written as
number of rows × *number of columns*

Matrix Q has three rows and one column.

Hence, its order must be 3×1.

Answer

3×1

✓1 mark for the correct answer

13bi ***Matrices***

Decoding the question

Keyword: matrix product

Relevant lesson: 4D

This question is asking for the matrix product RQ

Information we are given

- The given matrix Q
- The given matrix R

Using the theory

The matrix product can be found using a calculator or by hand.

By hand:

$$\begin{bmatrix}120 & 380 & 710\end{bmatrix} \times \begin{bmatrix}4.50\\8.00\\11.50\end{bmatrix}$$

$$= [(120 \times 4.50) + (380 \times 8.00) + (710 \times 11.50)]$$

$$RQ = [(540) + (3040) + (8165)]$$

$$RQ = [11\,745]$$

As the answer is a matrix product, it must be presented as a 1×1 matrix, not just a number.

Answer

$RQ = [11\,745]$

✓1 mark for the correct answer

13bii ***Matrices***

Decoding the question

Keywords: matrix product, represents

Relevant lesson: 4E

This question is asking for the information represented by the matrix product RQ

Information we are given

- Matrix Q contains the prices of each popcorn size.
- Matrix R contains the number of each size sold last month.

Using the theory

The resulting matrix product from part **b** is a 1×1 matrix. The multiplication process multiplied the price for each popcorn size by the number of each size sold.

$$\begin{bmatrix}120 & 380 & 710\end{bmatrix} \times \begin{bmatrix}4.50\\8.00\\11.50\end{bmatrix}$$

$$= [(120 \times 4.50) + (380 \times 8.00) + (710 \times 11.50)]$$

For example, as highlighted above, the price for a small popcorn ($4.50) was multiplied by the number of small popcorn sold (120). This calculated the revenue gained from the small popcorn sales last month. This process was repeated with each popcorn size.

The matrix multiplication process then calculated the sum of each of these three values. Hence, the value in matrix RQ is the total revenue collected from the popcorn sales last month.

Answer

The revenue of all popcorn sales in the last month.

✓1 mark for the correct answer

4F

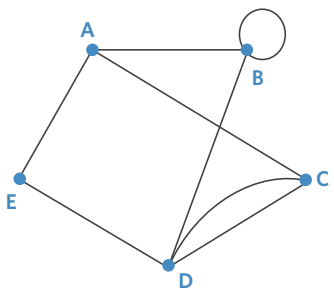
1 **a** 1 **b** 3

2 The circled number indicates there is a one–step path from A to C.

3
$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c}\begin{array}{ccc}A & B & C\end{array}\\\begin{bmatrix}0 & 3 & 1\\3 & 0 & 1\\1 & 1 & 0\end{bmatrix}\end{array}$$

4
$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c}\begin{array}{cccc}A & B & C & D\end{array}\\\begin{bmatrix}0 & 0 & 2 & 0\\0 & 0 & 1 & 2\\2 & 1 & 0 & 1\\0 & 2 & 1 & 0\end{bmatrix}\end{array}$$

5



6 C

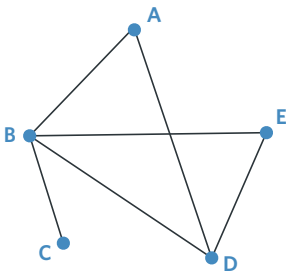
7 **a** **i** 1 **ii** 2

b $\begin{bmatrix}5 & 6 & 3\\6 & 10 & 2\\3 & 2 & 13\end{bmatrix}$

c **i** 6 **ii** 3

8 **a** Four people

b



c $\begin{bmatrix} 2 & 1 & 1 & 1 & 2 \\ 1 & 4 & 0 & 2 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 \\ 2 & 1 & 1 & 1 & 2 \end{bmatrix}$

2

9 **a**

	S	H	T	G	L
S	0	1	0	0	0
H	1	0	1	1	0
T	0	1	0	0	1
G	0	1	0	0	1
L	0	0	1	1	0

NOTE: The order of the rows and columns can vary.

b 2

10 **a**

	V	W	X	Y	Z
V	0	1	1	0	0
W	1	0	1	1	1
X	1	1	0	0	1
Y	0	1	0	0	1
Z	0	1	1	1	0

b No one can be friends with themselves on Facebook so these values are all 0.

c $V = 2$, $W = 4$, $X = 3$, $Y = 2$, $Z = 3$

d William

e

	V	W	X	Y	Z
V	2	1	1	1	2
W	1	4	2	1	2
X	1	2	3	2	1
Y	1	1	2	2	1
Z	2	2	1	1	3

f 2

11 **a**

	A	B	C	H	M	S
A	0	0	0	0	1	0
B	0	0	0	0	0	1
C	0	0	0	0	1	1
H	0	0	0	0	1	0
M	1	0	1	1	0	0
S	0	1	1	0	0	0

b Canberra

$M^2 =$

	A	B	C	H	M	S
A	1	0	1	1	0	0
B	0	1	1	0	0	0
C	1	1	2	1	0	0
H	1	0	1	1	0	0
M	0	0	0	0	3	1
S	0	0	0	0	1	2

$M + M^2 =$

	A	B	C	H	M	S
A	1	0		1	1	0
B	0	1		0	0	1
C	1	1		1	1	1
H	1	0		1	1	0
M	1	0		1	3	1
S	0	1	1	0	1	2

c $M + M^2 =$

	A	B	C	H	M	S
A		1		1	1	1
B		1		0	0	1
C		1		1	1	1
H		0		1	1	0
M		0		1	3	2
S	1	1	1	0	2	3

Adelaide is also connected to every city by a path of two steps or less.

d Steps don't take into account the distance between cities. For example, the distance between Canberra to Sydney, 286 km, is treated as the same as Adelaide to Sydney, which is 1374 km.

12 **a** **i** Birds eat lizards.

ii Birds, lizards and insects do not eat birds.

Questions from multiple lessons

13 ***Matrices***

Decoding the question

Keywords: matrix, element, row, column, 'determined by the rule'

Relevant lesson: 4A

The question is asking us to identify which matrix does not fit the element rule $x_{ij} = i + 3j$

Information we are given

- x_{ij} is the element in row i and column j of matrix X
- $x_{ij} = i + 3j$

Using the theory

x_{ij} is the element in row i and column j of matrix X

As we move down a column in matrix X, the value of the elements will increase by 1 each time, as the coefficient of i in the matrix element rule is 1.

As we move across a row, from left to right, in matrix X the value of the elements will increase by 3 each time, as the coefficient of j in the matrix element rule is 3.

These are the two constraints of matrix X

For the element $x_{1\,1}$, substitute $i = 1$ and $j = 1$ into the element rule $x_{i\,j} = i + 3j$

$x_{1\,1} = (1 \times 1) + (3 \times 1) = 4$

This is true for each option.

Now we apply the two constraints established previously on the rest of the elements.

As we go across row 1, each element should be 3 larger than the one before, starting from 4. This holds true in every option.

As we go down column 1, each element should be 1 larger than the one before, starting from 4. This holds true in every option except for option E.

Answer

E

✓1 mark for the correct answer

14 ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation, term, positive

Relevant lessons: 6B, 6D

We must use the given recurrence relation to generate a sequence and identify which term is the first to be positive.

Information we are given

- $t_0 = -10, \quad t_{n+1} = 2t_n + 11$

Using the theory

Substitute $t_0 = -10$ into the recurrence relation $t_{n+1} = 2t_n + 11$ to find the value of t_1

$t_1 = 2t_0 + 11$

$t_1 = 2 \times (-10) = -20 + 11 = -9$

Repeat the process for the rest of the sequence until a positive value is generated.

$t_2 = 2t_1 + 11$

$t_2 = 2 \times (-9) + 11 = -18 + 11 = -7$

$t_3 = 2t_2 + 11$

$t_3 = 2 \times (-7) + 11 = -14 + 11 = -3$

$t_4 = 2t_3 + 11$

$t_4 = 2 \times (-3) + 11 = -6 + 11 = 5$

The first positive value in the sequence is t_4

Answer

D

✓1 mark for the correct answer

15a ***Matrices***

Decoding the question

Keyword: order

Relevant lesson: 4A

We must inspect matrix P_{2019} to find its order.

Information we are given

- The given matrix

Using the theory

The order of a matrix is expressed as
number of rows × *number of columns*

There are 4 rows and 1 column.

Hence, the order of matrix P_{2019} is 4×1.

Answer

4×1

✓1 mark for the correct answer

15b ***Matrices***

Decoding the question

Keyword: how much

Relevant lesson: 4A

We must identify the price of a cactus and a monstera plant in the matrix and calculate their sum.

Information we are given

- The given matrix
- Values are in dollars.
- Nick buys a cactus and a monstera plant.

Using the theory

We can find the price of each of the plants by identifying their elements in P_{2019}

The value in row 'C' is 20. Hence, the price of a cactus is \$20.

The value in row 'M' is 40. Hence, the price of a monstera is \$40.

Calculate the sum.

$20 + 40 = 60$

Answer

\$60

✓1 mark for the correct answer

15c ***Matrices***

Decoding the question

Keywords: 'number of each plant sold', total revenue

Relevant lesson: 4D

We must find a matrix that, when multiplied by matrix P_{2019}, will produce a matrix containing the total revenue.

Information we are given

- The given matrix
- The given table

Using the theory

The question asks us to find the total revenue from the past week. As the total is a singular value, the matrix product must be a 1×1 matrix.

The order of matrix P_{2019} is 4×1.

To produce a 1×1 matrix, P_{2019} must be pre-multiplied by a 1×4 matrix.

The values within the 1×4 matrix must be the number of each plant sold in the last week for the matrix product to be the revenue for the last week.

The number of each plant sold is given in the table. As the plants in the table are in the same order as those in matrix P_{2019}, the values can be copied directly into a 1×4 matrix.

Answer

$\begin{bmatrix} 22 & 24 & 17 & 31 \end{bmatrix}$

✓ *1 mark for the correct answer*

15d ***Matrices***

Decoding the question

Keywords: '20% price increase', scalar, product

Relevant lessons: 2E, 4C

We must identify a scalar that, when multiplied by P_{2019} will cause each element to increase by 20%.

Information we are given

- The given skeleton equation

Using the theory

Multiplication by a scalar causes every element in a matrix to be multiplied by the scalar.

The price of each plant must increase by 20%.

Hence, the 2020 prices must be 120% of the 2019 prices.

Divide 120 by 100 to convert from a percentage to a scalar.

$\frac{120}{100} = 1.2$

Answer

1.2

✓ *1 mark for the correct answer*

4G

1 $\begin{bmatrix} 22 & -44 \\ 8 & -16 \end{bmatrix}$

2 $\begin{bmatrix} 11 & 37 & 65 \\ 109 & 23 & -3 \\ 305 & -183 & 41 \end{bmatrix}$

3 **a** $\begin{bmatrix} 7 & 2 \\ -1 & -9 \end{bmatrix}$

b $\begin{bmatrix} 7 & 2 \\ -1 & -9 \end{bmatrix}$

c They are the same.

4 **a** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

They are inverses of each other.

b $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

They are inverses of each other.

c $\begin{bmatrix} 1 & \frac{9}{28} \\ 0 & \frac{25}{7} \end{bmatrix}$

They are not inverses of each other.

5 B

6 15

7 **a** $\frac{1}{92}\begin{bmatrix} 4 & 8 \\ -10 & 3 \end{bmatrix}$

b $\frac{1}{27}\begin{bmatrix} -3 & -7 \\ 9 & 12 \end{bmatrix}$

c $\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$

8 $A^{-1} = \frac{1}{116}\begin{bmatrix} 9 & -4 \\ 2 & 12 \end{bmatrix}$

$A \times A^{-1} = \begin{bmatrix} 12 & 4 \\ -2 & 9 \end{bmatrix} \times \frac{1}{116}\begin{bmatrix} 9 & -4 \\ 2 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

9 **a** $x = 5, y = \frac{9}{2}$

b $x = \frac{5}{12}, y = \frac{-49}{36}$

c $x = \frac{-4}{7}, y = \frac{-6}{7}$

10 **a** $x = 5, y = 1$

b $x = 2, y = 3$

c $x = \frac{13}{17}, y = \frac{31}{17}$

11 **a** $\begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} a \\ p \end{bmatrix} = \begin{bmatrix} 18 \\ 19 \end{bmatrix}$ **b** \$2

12 B

13 $b = -2$

14 $\frac{1}{56}\begin{bmatrix} 3 & -7 \\ 2 & 14 \end{bmatrix}$

15 **a** $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 180 \\ 170 \end{bmatrix}$

$a = 50, c = 40$

The cost of an adult ticket is \$50, the cost of a child ticket is \$40.

b C

16 D

Questions from multiple lessons

17 *Matrices*

Decoding the question

Keyword: matrix product

Relevant lessons: 4C, 4D

The question is asking us to use our knowledge of the multiplication of matrices to determine which of the options is equal to the given matrix product.

Information we are given

- The given matrix product

Using the theory

Find the matrix product.

$$\begin{bmatrix}12 & 3 & 9\end{bmatrix} \times \begin{bmatrix}3\\15\\6\end{bmatrix} = [(12 \times 3) + (3 \times 15) + (9 \times 6)]$$
$$= [36 + 45 + 54]$$
$$= [135]$$

The matrix product simplifies to a 1×1 matrix, hence option B is not correct.

$[576] \neq [135]$, hence option C is not correct.

Work through the remaining options to find the answer.

A $3 \times \begin{bmatrix}4 & 3 & 9\end{bmatrix} \times \begin{bmatrix}1\\15\\6\end{bmatrix}$
$= 3 \times [(4 \times 1) + (3 \times 15) + (9 \times 6)] = 3 \times [103]$
$= [309] \neq [135]$

D $9 \times \begin{bmatrix}4 & 1 & 3\end{bmatrix} \times \begin{bmatrix}1\\5\\2\end{bmatrix}$
$= 9 \times [(4 \times 1) + (1 \times 5) + (3 \times 2)] = 9 \times [15]$
$= [135]$

E $3 \times \begin{bmatrix}4 & 1 & 3\end{bmatrix} \times \begin{bmatrix}1\\5\\2\end{bmatrix}$
$= 3 \times [(4 \times 1) + (1 \times 5) + (3 \times 2)] = 3 \times [15]$
$= [45] \neq [135]$

Hence, the answer is D.

NOTE: All calculations within this solution can be completed on your calculator.

Alternative Solution:

Every number within the two matrices in the given matrix product is a multiple of 3. Hence, 3 can be removed from each of the matrices, as shown below.

$$\begin{bmatrix}12 & 3 & 9\end{bmatrix} \times \begin{bmatrix}3\\15\\6\end{bmatrix} = 3 \times \begin{bmatrix}4 & 1 & 3\end{bmatrix} \times 3 \times \begin{bmatrix}1\\5\\2\end{bmatrix}$$

The two 3's can be multiplied by each other, giving

$$9 \times \begin{bmatrix}4 & 1 & 3\end{bmatrix} \times \begin{bmatrix}1\\5\\2\end{bmatrix}$$

Answer

D

✓1 mark for the correct answer

18 *Data analysis*

Decoding the question

Keywords: dot plot, mean, standard deviation

Relevant lessons: 12C, 12E, 12F

We need to list the data points and use our calculators to find the mean and standard deviation.

Information we are given

- The given dot plot

Using the theory

List the value of the data points in a spreadsheet in your calculator and use the 'one-variable statistics' function. Remember to include the zeros.

The calculator should return, correct to one decimal place, $\bar{x} = 1.5$ and $s_x = 2.2$.

NOTE: Make sure to use the value of s_x instead of σ_x as we are finding the standard deviation of a sample, not a population.

See video solution for button-by-button demonstration.

Answer

C

✓1 mark for the correct answer

19a *Matrices*

Decoding the question

Keyword: order

Relevant lesson: 4A

We must find the number of rows and columns in matrix K to determine its order.

Information we are given

- The given matrix

Using the theory

The order of a matrix is expressed as
number of rows $\times$ *number of columns*

Matrix K has 4 rows and 1 column.

Hence, the order of matrix K is 4×1.

Answer

4×1

✓1 mark for the correct answer

19b *Matrices*

Decoding the question

Keywords: how many, either

Relevant lesson: 4E

This question asks us to inspect the given matrix to find how many knights live in either the Vale or the Iron Islands.

Information we are given

- The given matrix

Using the theory

Each element of the matrix K represents the population of knights of a different kingdom.

From the matrix, we can determine the population of knights of the Vale and the Iron Islands.

Vale: 29 knights

Iron Islands: 20 knights

The question specifies the population of either the Vale or the Iron Islands, so their two separate populations must be summed.

$29 + 20 = 49$

Answer

49 knights

✓1 mark for the correct answer

19c *Matrices*

Decoding the question

Keywords: matrix, multiplied, 'total cost to armour all knights'

Relevant lesson: 4E

This question asks us to create an equation, that when multiplied with the given equation, provides a 1×1 matrix with the total cost of all armour.

Information we are given

- The given matrix
- The given table

Using the theory

Let our new matrix be called L

The product matrix $L \times K$ gives a total cost, therefore it will only have one element. The order of $L \times K$ will be 1×1.

As matrix K is a 4×1 matrix, the order of matrix L must be 1×4 for the product matrix to be defined and have an order of 1×1.

The order of the information in the matrix must correspond to the order of K, so The North must be the first element, The Vale the second, The Iron Islands the third, and Dorne the fourth.

Answer

$[15 \quad 29 \quad 11 \quad 23]$

✓1 mark for the correct answer

5A

1 3×3

2 5

3 **a** Vertices: 4, Edges: 4

b Vertices: 5, Edges: 5

c Vertices: 8, Edges: 12

4 5

5 Vertices: 11, Edges: 10

6 **a** Simple

b There are two edges joining the same two vertices.

7 **a** **i** Simple: A, B, D
Complete: None
Connected: A, C, D, E
Disconnected: B, F

ii Isomorphic pair: C and E

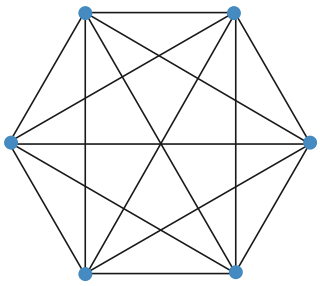
b **i** Simple: A, D, E, F
Complete: A
Connected: A, B, D, E, F
Disconnected: C

ii Isomorphic pair: D and E

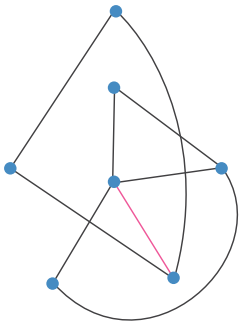
c **i** Simple: A, C, D, E
Complete: D
Connected: B, D, E, F
Disconnected: A, C

ii Isomorphic pair: B and F

8 Although many different arrangements are possible, this is the standard complete graph with 6 vertices.



9



10 The missing entry is 2.

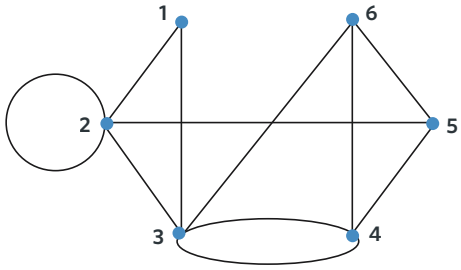
11 **a** $\begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

b $\begin{bmatrix} 0 & 1 & 0 & 0 & 2 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$

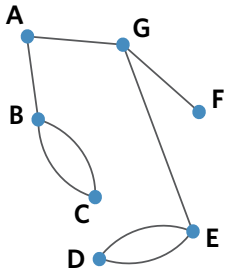
c $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$

12 Vertices D and F have the most edges between them.

13 There are many possible graphs. The graph drawn with vertices arranged as a hexagon is below.



14 The graph is disconnected, with vertices A, B, and C in one part, D, E, and G in another part, and vertex F by itself. Any two lines that join these parts all together are valid. The following is an example of a solution.



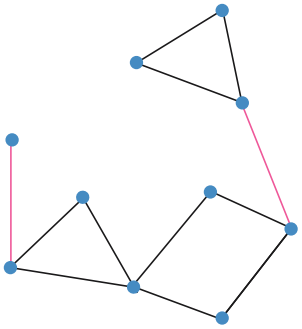
$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

15 Vertices: 5

Edges: 10

16 D

17 2



18 The vertices represent shipping ports. The edges represent the paths ships can take.

The ships cannot traverse all the paths without going over the same path twice.

19 D

20 C

21 B

Questions from multiple lessons

22 ***Networks and decision mathematics***

Decoding the question

Keywords: sum, degrees, vertices

Relevant Lesson: 5A

We must find the degree of each vertex in each graph, calculate the sum for each graph and then compare these values.

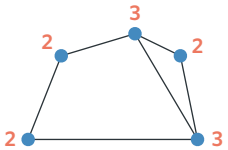
Information we are given

- The given graphs

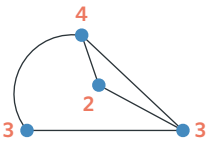
Using the theory

The degree of a vertex is equal to the number of edges connected to it.

The degrees of each vertex can be found by counting the number of times an edge attaches to it. These are shown below.



Graph A



Graph B

Calculate the sum of the degrees of the vertices for each graph.

Graph A: $2 + 2 + 2 + 3 + 3 = 12$

Graph B: $3 + 3 + 2 + 4 = 12$

Calculate the difference.

$12 - 12 = 0$

∴ The sum of the degrees of the vertices of Graph B is equal to the sum of the degrees of the vertices of Graph A.

Answer

C

✓1 mark for the correct answer

23 ***Data analysis***

Decoding the question

Keywords: boxplot, five-number summary

Relevant lesson: 12G

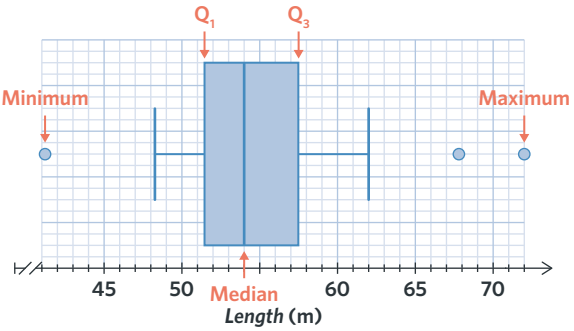
We need to read features of the given boxplot to deduce the five-number summary of the data.

Information we are given

- The given boxplot

Using the theory

The five-number summary's location on the boxplot is shown below.



Hence, the five-number summary, reading from left to right, is: 41.2, 51.4, 54, 57.4, 72.

NOTE: Outliers should be included in the five-number summary.

Answer

A

✓1 mark for the correct answer

24a ***Networks and decision mathematics***

Decoding the question

Keywords: explain, adjacency matrix

Relevant lesson: 5A

We need to use our knowledge of adjacency matrices, along with the context given in the question, to determine the meaning of a zero in the adjacency matrix.

Information we are given

- The given adjacency matrix
- The adjacency matrix shows the different friendships between the five peers.

Using the theory

Each element in the adjacency matrix represents the number of edges between the vertices of the respective row and column. An edge represents a friendship between the two colleagues represented by the vertices.

∴ A zero in the adjacency matrix means that the two colleagues, represented by the vertices, are not friends.

Answer

A zero in the adjacency matrix means that the two colleagues are not friends.

✓1 mark for the correct answer

24b ***Networks and decision mathematics***

Decoding the question

Keywords: value, adjacency matrix

Relevant lesson: 5A

We need to refer back to the graph to find the values of x and y.

Information we are given

- The given graph
- The given adjacency matrix

Using the theory

x is in row M and column B

The value of x will be the number of edges between vertices M and B on the graph.

There are no edges between M and B

$x = 0$

y is in row J and column B

The value of y will be the number of edges between vertices J and B on the graph.

There is one edge between J and B

$y = 1$

Answer

$x = 0, y = 1$

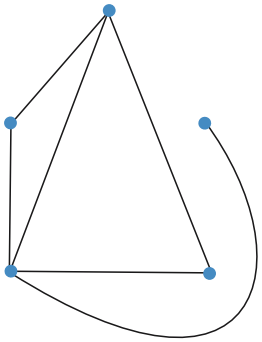
✓1 mark for the correct answer

5B

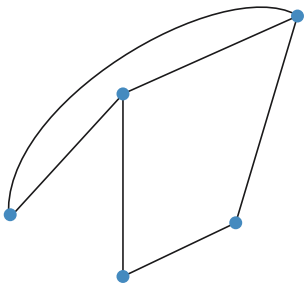
1 7

2 Planar

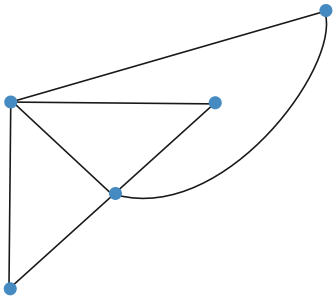
3 One possible answer is below.



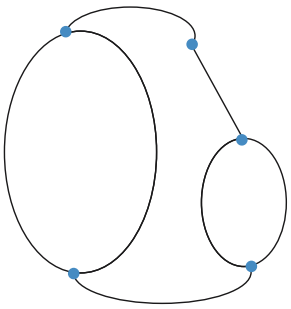
4 **a** One possible answer is below.



b One possible answer is below.



c One possible answer is below.



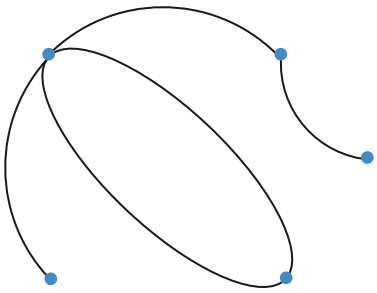
5 **a** Planar

b Planar

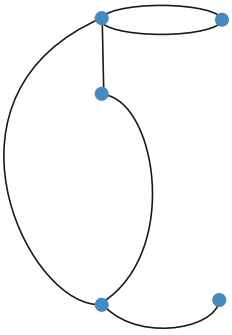
c Non−planar

6 D

7 **a** A is



B is



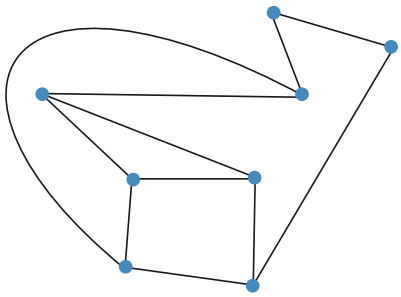
b B

8 7

9 7

10 **a** 4 **b** 6 **c** 5

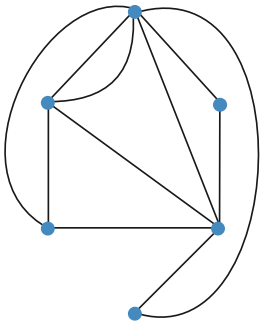
11 **a**



The graph is connected and planar, so Euler's formula applies.

b 5

12



13 **a** 5

b The sum of the degrees is 18. The sum of vertices' degrees is double the number of edges.

14 **a** Vertices: 13

Edges: 15

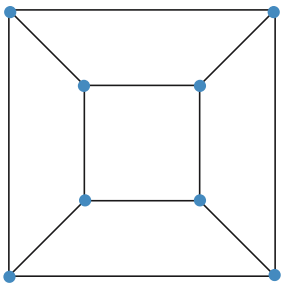
Faces: 4

b $V + F = E + 2$

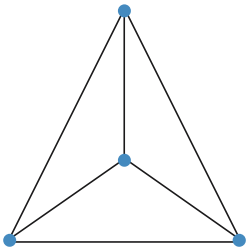
$13 + 4 = 15 + 2$

$LHS = RHS$

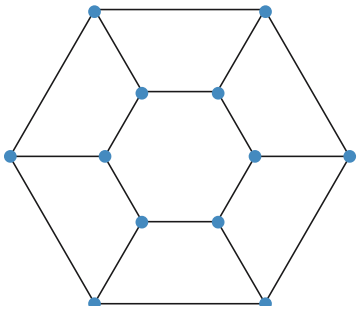
15 **a**



b



c



16 D

Questions from multiple lessons

17 ***Networks and decision mathematics***

Decoding the question

Keywords: adjacency matrix, 'missing elements'

Relevant lesson: 5A

We need to determine the missing elements in the adjacency matrix by finding the number of edges between the vertices of the respective rows and columns.

Information we are given

- The given graph
- The given adjacency matrix

Using the theory

The first row is the number of edges between vertex *A* and each other vertex.

There are 0 edges with itself, 1 edge with *B*, 2 edges with *C* and 2 edges with *D*

Hence, the values in the first row from left to right will be 0, 1, 2, 2.

Fill out the rest of the rows in the same manner.
The matrix should look as follows:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The missing elements are three '0's, four '1's and two '2's.

Answer

B

✓*1 mark for the correct answer*

18 ***Data analysis***

Decoding the question

Keywords: variables, 'nominal, ordinal, discrete or continuous'

Relevant lesson: 12A

This question asks us to identify what type of data *exam grade* and *time spent studying* are.

Information we are given

- *Exam grade* (high distinction, distinction, pass, fail)
- *Time spent studying* (less than 5 hours, 5 to 10 hours, more than 10 hours)

Using the theory

We must first identify if the variables *exam grade* (high distinction, distinction, pass, fail) and *time spent studying* (less than 5 hours, 5 to 10 hours, more than 10 hours) are categorical or numerical variables. If they are categorical variables, determine if they are nominal or ordinal variables. If they are numerical variables, determine if they are discrete or continuous variables.

Exam grade is a categorical variable as it has four categories: high distinction, distinction, pass or fail. As the categories within the variable *exam grade* can be logically ordered from best to worst grade, or vice versa, it is an ordinal variable.

Time spent studying is a categorical variable as it has three categories: less than 5 hours, 5 to 10 hours, more than 10 hours. As the categories within the variable *time spent studying* can be logically ordered from least time spent studying to most time studying or vice versa, it is an ordinal variable.

∴ The variables are both ordinal.

Answer

B

✓*1 mark for the correct answer*

19a ***Networks and decision mathematics***

Decoding the question

Keywords: 'missing edge', vertices, graph, draw

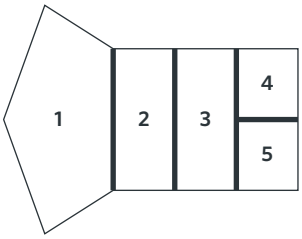
Relevant lesson: 5A

By comparing the diagram to the provided graph, we will identify the missing edge, and redraw the complete graph.

Information we are given

- The given diagram
- The given graph
- Each vertex represents a room.
- Each edge represents a wall between two rooms.

Using the theory



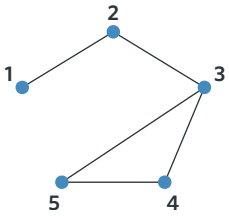
From the diagram, we can see there are walls connecting:

- Rooms 1 & 2
- Rooms 2 & 3
- Rooms 3 & 4
- Rooms 3 & 5
- Rooms 4 & 5

On the diagram there is a wall between room 3 and room 5, yet on the graph there is no edge between vertex 3 and vertex 5.

Redraw the graph, but add an edge that connects vertices 3 and 5.

Answer



✓1 mark – Correct edge

19b ***Networks and decision mathematics***

Decoding the question

Keywords: sum, degrees, vertices

Relevant lesson: 5A

By finding the degree of each vertex of our new graph and adding them together, we can find the sum of the degree of the vertices.

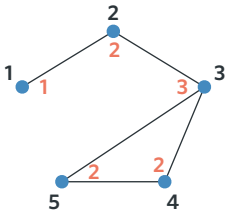
Information we are given

- The graph from part **a**

Using the theory

The degree of a vertex is equal to the number of edges connected to it.

The degrees of each vertex can be found by counting the number of times an edge attaches to it. These are shown in red on the graph below.



The sum of these degrees is:

$1 + 2 + 3 + 2 + 2 = 10$

Answer

10

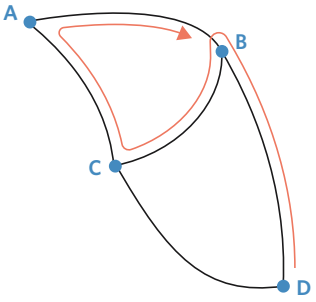
✓1 mark for the correct answer

5C

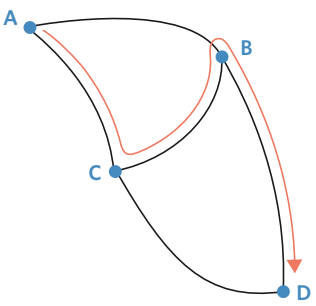
1 Vertices: 4

Edges: 6

2 **a**



b



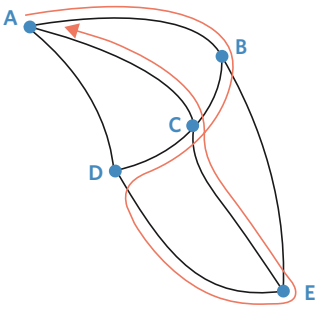
3 **a** Walk and trail **b** Walk

c Walk and trail

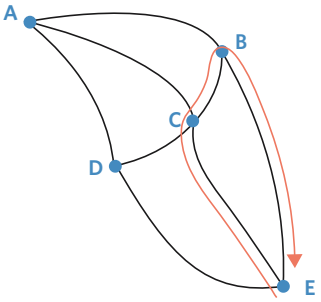
4 *EGHFDCBA*

5 *AB* or *BC*

6 **a**



b



7 **a** Walk

b Walk and circuit

c Walk, circuit, and cycle

8 **a** 8

b *BCDFEGHB* or *BHGEFDCB*

NOTE: You may start at any of the vertices as long as you follow one of the orders listed above.

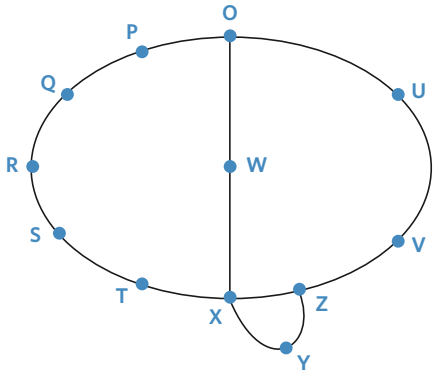
9 The route *ABA* is neither a circuit nor a cycle because it repeats the edge *AB*. This means that the route is not a circuit. Therefore, it cannot be a cycle.

10 **a** *GSFAYRPMVNWG* or *GWNVMPRYAFSG*

b *N*, *V*, *S*, *P*, *F* and *W*

c *N* or *F*

11 **a**



b **i** *P*, *T*, *U*, *W*, *Y*

ii Walk, Trail, Path

c **i** Yes

ii Walk, Trail

d **i** No

ii *WZ*

12 **a** 4

b **i** *P*

ii 5

c **i** *SRQPONUT*

ii Any three of *SRQPUTNO* *SRQPONTU* *SRTUNOPQ*, or *SRUTNOPQ*

Questions from multiple lessons

13 ***Networks and decision mathematics***

Decoding the question

Keywords: planar graph, not

Relevant lesson: 5A

The graph that is **not** a planar graph can be determined by finding the graph that cannot be drawn without overlapping edges.

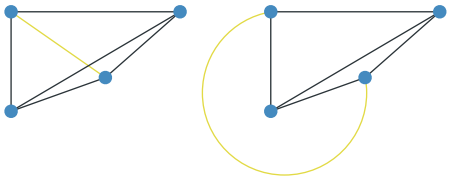
Information we are given

- The given graphs

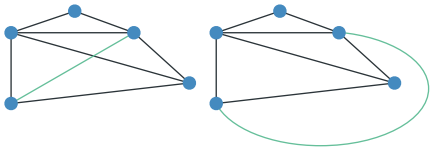
Using the theory

A planar graph is a graph that can be drawn with no overlapping edges. By redrawing certain edges in the graphs in options A, B, C, D and E, we can determine which graphs are planar and can be drawn without overlapping edges, and which graph is non-planar.

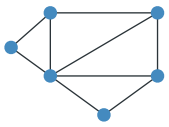
A The yellow edge can be redrawn so that the graph is in planar form.



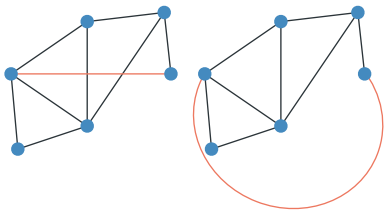
B The green edge can be redrawn so that the graph is in planar form.



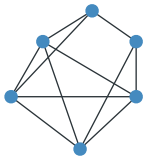
C The graph is already in planar form.



D The red edge can be redrawn so that the graph is in planar form.



E The graph cannot be drawn without overlapping edges, so it is not a planar graph.



Answer

E

✓*1 mark for the correct answer*

14 ***Recursion and financial modelling***

Decoding the question

Keywords: compounding, recurrence relation

Relevant lessons: 3D, 6D

The recurrence relation for compound interest can be used to determine the amount in the savings account, S_n after n years.

Information we are given

- Pauline deposited $3500.
- Interest compounds annually at a rate of 2.8% per annum.

Using the theory

Since we are dealing with compound interest, the recurrence relation is in the form

$S_0 = a \quad S_{n+1} = R \times S_n$

S_0 is the initial deposit, which is \$3500.

The interest rate of 2.8% per annum can be converted to the common ratio using the formula:

$R = 1 + \frac{r}{100}$, where r is the interest rate expressed as a percentage.

Substitute $r = 2.8$ into $R = 1 + \frac{r}{100}$

$R = 1 + \frac{2.8}{100} = 1.028$

$\therefore S_0 = 3500, \quad S_{n+1} = 1.028 \times S_n$

Answer

D

✓1 mark for the correct answer

15a ***Networks and decision mathematics***

Decoding the question

Keyword: 'only one road'

Relevant lesson: 5C

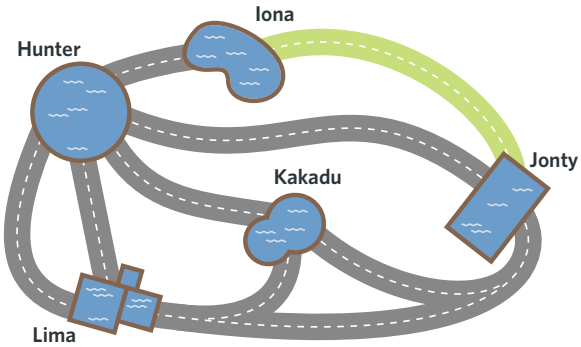
We must use the map to find the two pools directly accessible from Iona using only one road.

Information we are given

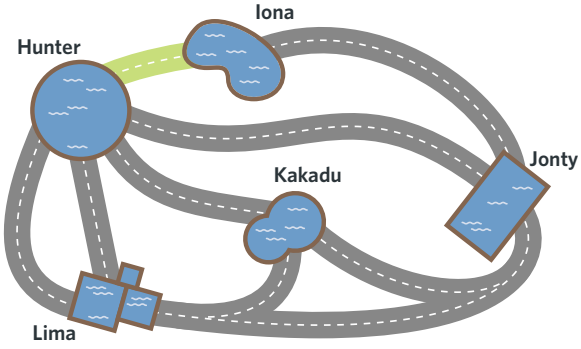
- The given map

Using the theory

There is one road between Iona and Jonty, as shown below.



There is one road between Iona and Hunter, as shown below.



Answer

Jonty and Hunter

✓1 mark for the correct answer

15bi ***Networks and decision mathematics***

Decoding the question

Keywords: graph, vertices, 'missing an edge'

Relevant lesson: 5C

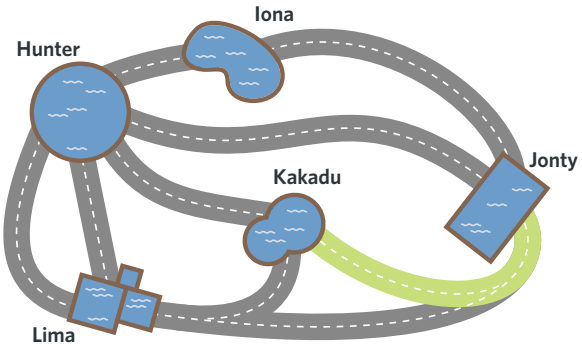
We must associate the connections on the map with the edges on the graph in order to find the missing edge.

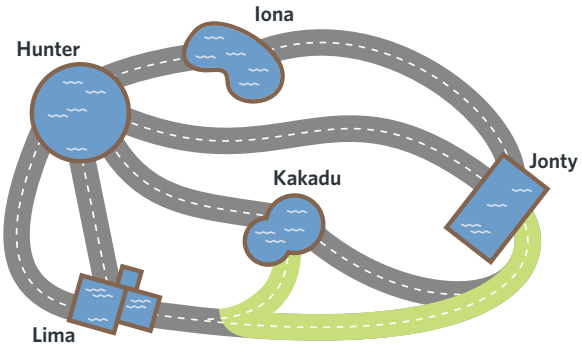
Information we are given

- The given graph

Using the theory

The following graphics show the two ways to travel directly between Jonty and Kakadu, without passing any other pools.



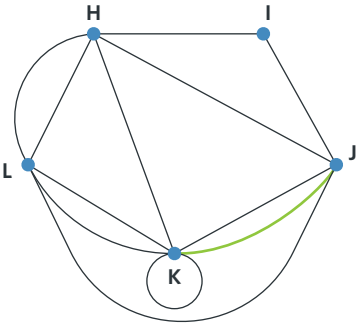


Hence, on the graph, there should be two edges connecting K and J.

However, there is only one edge between J and K on the given graph.

The missing edge is between J and K.

The complete graph is shown below.



Answer

J and *K*

✓1 mark for the correct answer

15bii ***Networks and decision mathematics***

Decoding the question

Keywords: loop, represent

Relevant lesson: 5C

We must explain the significance of a loop on the graph in the context of the given situation.

Information we are given

- The given graph

Using the theory

A loop is an edge that connects a vertex back to itself. It indicates that one can travel from one vertex and return back to the same vertex without passing any other vertices.

In this case, there is a loop on vertex *K*. This means that people at Kakadu can travel from, and return to the pool without passing any of the other pools.

Answer

People at Kakadu can travel from the pool and return without passing any of the other pools.

✓1 mark – Correct explanation

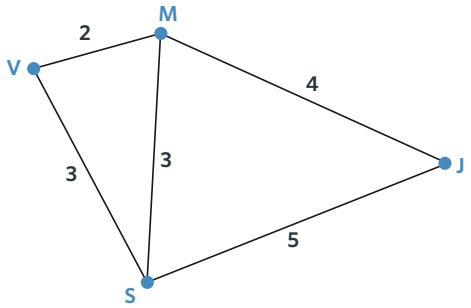
5D

1 *DECBAF*

2 **a** 18 km **b** 40 km **c** 62 km

3 28

4



5 C

6 **a** 23.2 km **b** 22.6 km **c** 29.3 km

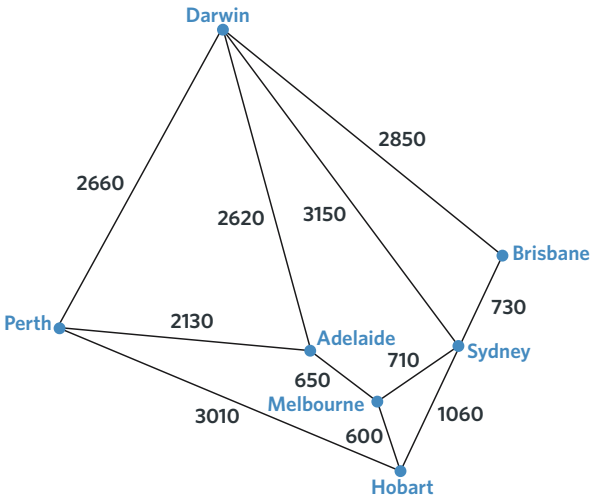
7 *ABCDE*, 12.1 m

8 110 km

9 3.2 km

10 *PCBFEDAP* or *PADEFBCP*

11 **a**



b Melbourne, Hobart, Sydney, Brisbane, Darwin, Perth, Adelaide, Melbourne

or

Melbourne, Adelaide, Perth, Darwin, Brisbane, Sydney, Hobart, Melbourne

Questions from multiple lessons

12 ***Networks and decision mathematics***

Decoding the question

Keywords: graph, path

Relevant lesson: 5C

By recalling the definition of the path we will go through the options on the graph and establish which does not fit the definition.

Information we are given

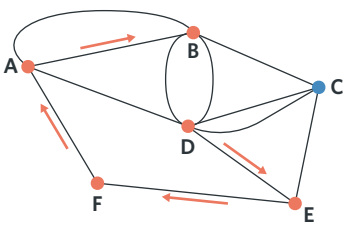
- The given graph

Using the theory

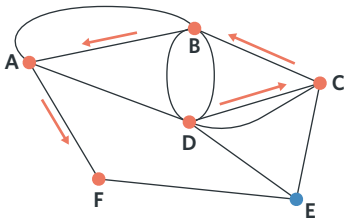
A path does not repeat any vertices or edges, with the exception that it may start and finish at the same vertex.

To find which of the options is not a path, we will work through each option until we find one that does not fit the criteria.

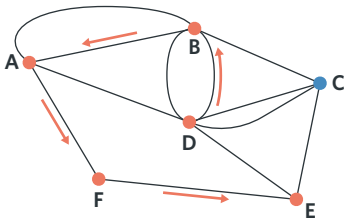
A This is a path as it does not repeat any vertices or edges, as seen in the diagram.



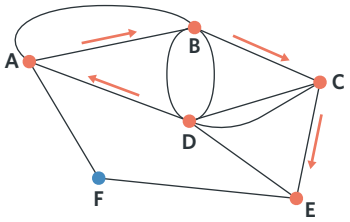
B This is a path as it does not repeat any vertices or edges, as seen in the diagram.



C This is a path as it does not repeat any vertices or edges, as seen in the diagram.



D This is a path as it does not repeat any vertices or edges, as seen in the diagram.



E is not a path as there is no edge between A and C.

Answer

E

✓1 mark for the correct answer

13 ***Data analysis***

Decoding the question

Keywords: histogram, most common

Relevant lesson: 12C

We must determine the *weight* interval with the highest frequency by examining the histogram.

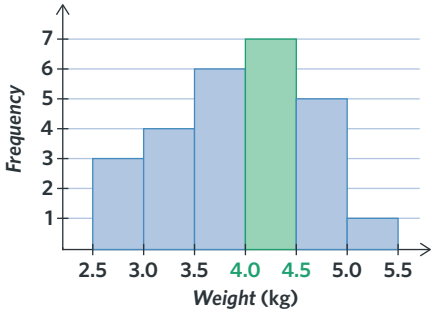
Information we are given

- The given histogram

Using the theory

The *weight* interval with the highest frequency is the tallest bar.

The tallest bar is highlighted green in the following histogram.



The *weight* interval this bar corresponds to is shown on the x-axis. The lower bound value is 4.0 and the upper bound value is 4.5.

∴ The most common interval of *weight* for the cats is greater than or equal to 4.0 kg and less than 4.5 kg.

Answer

D

✓1 mark for the correct answer

14a ***Networks and decision mathematics***

Decoding the question

Keywords: graph, cost

Relevant lesson: 5D

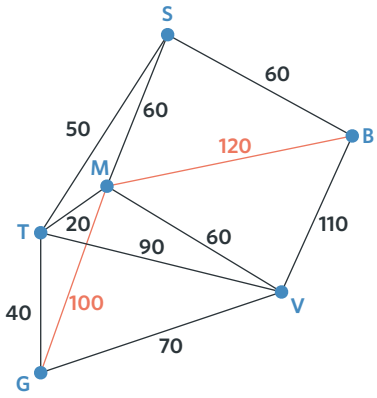
We must use the given weighted graph and the route provided to find the sum of the weights associated to the required edges to calculate the total cost of the proposed route.

Information we are given

- The given graph
- The proposed route: Barcelona (B) – Madrid (M) – Granada (G)

Using the theory

The following proposed route is highlighted in red.



The total cost is the sum of the two weights, which are the numbers in red associated with each edge.

$total\ cost = 100 + 120 = 220$

Answer

$220

✓1 mark for the correct answer

14b ***Networks and decision mathematics***

Decoding the question

Keywords: graph, cost, cheapest route

Relevant lesson: 5D

By finding all practical routes from Barcelona to Granada and assessing their respective weights, we can find the cheapest route for Sophie.

Information we are given

- The given graph

Using the theory

First, we will write out all the practical routes from Barcelona to Granada and find their respective costs.

$BVG = 110 + 70 = 180$

$BMG = 120 + 100 = 220$

$BMTG = 120 + 20 + 40 = 180$

$BSMG = 60 + 60 + 100 = 220$

$BSTG = 60 + 50 + 40 = 150$

The cheapest route will have the smallest sum.

The smallest sum above is 150, for *BSTG*.

This means Sophie will pass through San Sebastian and Toledo.

Answer

San Sebastian and Toledo.

✓1 mark for the correct answer

14c ***Networks and decision mathematics***

Decoding the question

Keywords: Euler's formula, graph

Relevant lesson: 5B

By finding the number of vertices and edges, we can solve for the number of faces and complete Euler's formula.

Information we are given

- The given graph
- Euler's formula $v + f = e + 2$

Using the theory

There are 6 vertices.

There are 11 edges.

Solve for the number of faces by substituting in $v = 6$ and $e = 11$ into Euler's formula.

$6 + f = 11 + 2$

$f = 7$

Substitute all values back into Euler's formula as we must give our answer in this form.

Answer

$6 + 7 = 11 + 2$

✓1 mark for the correct answer

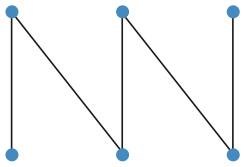
5E

1 Vertices: 5

Edges: 8

2 **a** 8

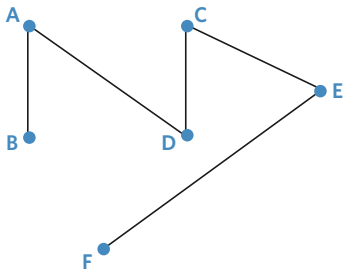
b

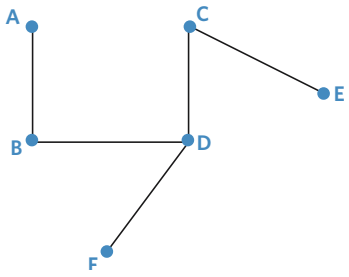


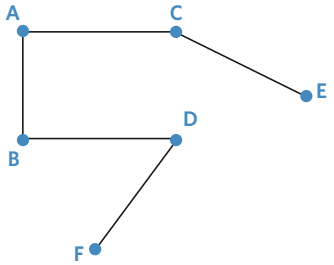
NOTE: There are many possible answers.

3 C

4 **a**

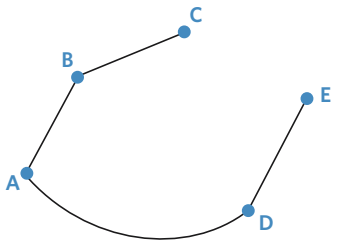


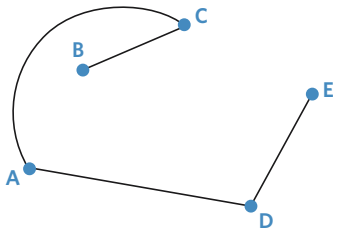


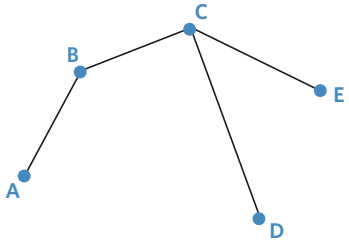


NOTE: There are many possible answers.

b



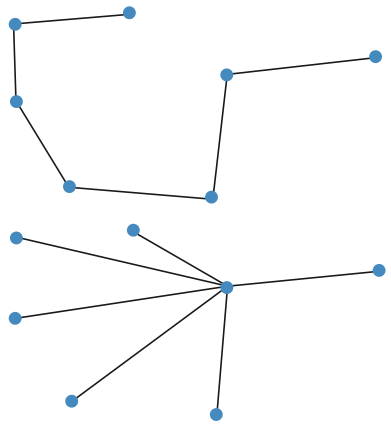




NOTE: There are many possible answers.

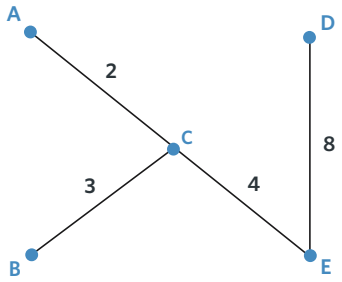
5 **a** 6

b



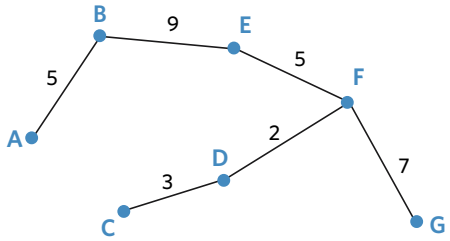
NOTE: There are many possible answers.

6 B

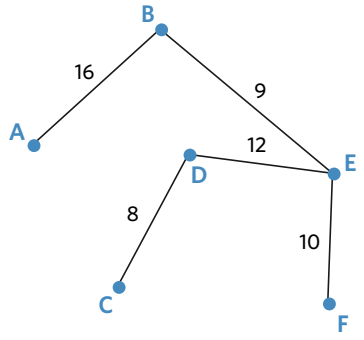


7 B

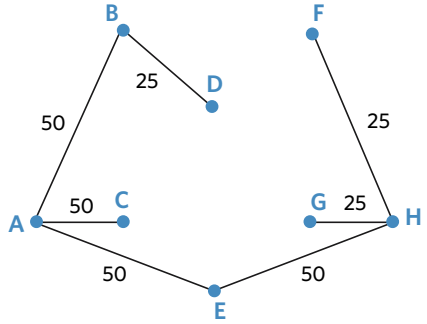
8



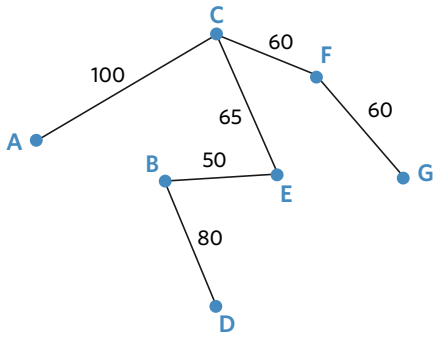
9 **a**



b



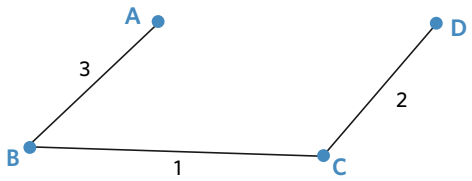
10 **a**



b C

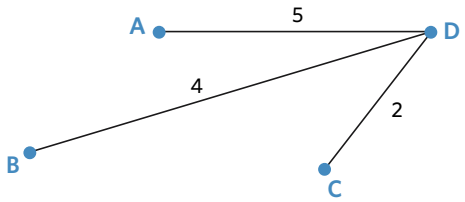
11 **a** 8

b



Total weight is 6.

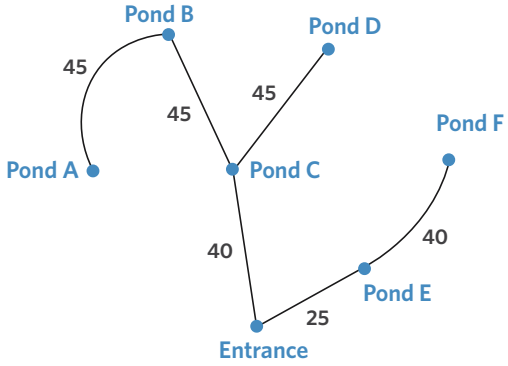
c



Total weight is 11.

12 53.5 m

13 **a**



b $28 800

14 D

Questions from multiple lessons

15 ***Networks and decision mathematics***

Decoding the question

Keywords: 'isolated vertices', 'minimum number of edges', tree

Relevant lesson: 5E

Using the definition of trees, we need to find the minimum number of edges to connect all vertices.

Information we are given

- The location of seven isolated vertices

Using the theory

Trees are a special type of connected graph which use as few edges as possible to connect a group of vertices.

In a tree, the number of edges is one less than the number of vertices.

The formula is $e = v - 1$.

Substitute in $v = 7$.

$e = 7 - 1$.

$e = 6$

$\therefore$ 6 edges are needed.

Answer

C

✓ *1 mark for the correct answer*

16 ***Data analysis***

Decoding the question

Keywords: standard deviation, mean

Relevant lessons: 12E, 12F

We will use the calculator to find the standard deviation and mean of the height of the 13 dogs.

Information we are given

- The given table

Using the theory

On your calculator, open a spreadsheet, enter the height data into a column and use the one variable statistic calculations.

The values should be $\bar{x} = 38.8$ and $s_x = 12.08$.

See video solution for a button-by-button demonstration.

Answer

A

✓ *1 mark for the correct answer*

17a ***Networks and decision mathematics***

Decoding the question

Keywords: 'mathematical term', connected graph, shortest length

Relevant lesson: 5E

We need to recall the mathematical term that describes a connected graph with minimal weight.

Information we are given

- The given weighted graph
- The graph we are describing is a connected graph.
- The graph we are describing has minimal weight.

Using the theory

In order for the graph to have minimal weight, as few edges as possible should be used. The term that describes a connected graph which uses the fewest number of edges is a 'spanning tree'. However, there may be multiple spanning trees. The spanning tree with the lowest total weight is called a 'minimal spanning tree'.

Answer

Minimal spanning tree

✓ *1 mark for the correct answer*

17b ***Networks and decision mathematics***

Decoding the question

Keywords: draw, connected graph, shortest length

Relevant lesson: 5E

We need to use Prim's Algorithm to draw the minimal spanning tree of this weighted graph.

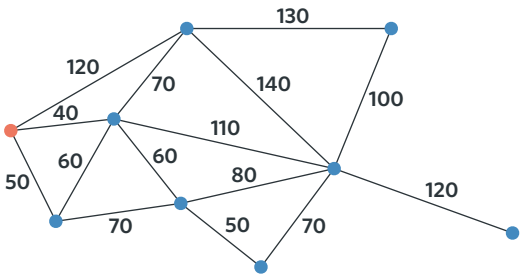
Information we are given

- The given weighted graph
- The graph we are drawing is a minimal spanning tree.

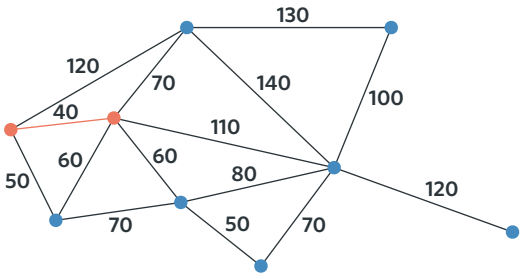
Using the theory

The minimal spanning tree can be found using Prim's algorithm.

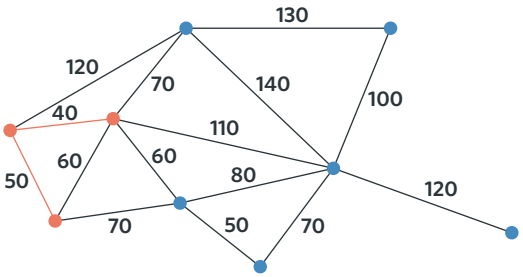
First, select any vertex. In this solution, we will start on the left.



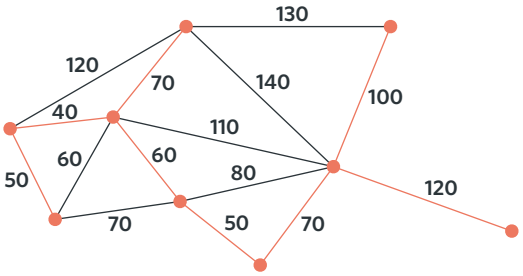
Then select the edge with the lowest weight that is connected to the vertex.



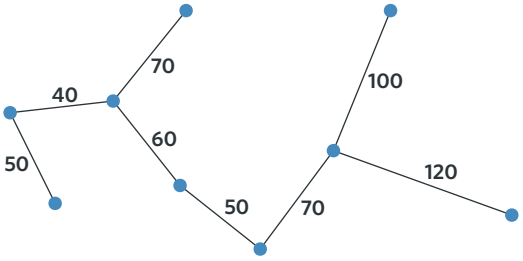
Two vertices are now connected. Look at all the edges that are connected to either vertex. Select the edge with the lowest weight that will connect a new vertex to the tree.



Continue this until every vertex is included.



Answer



✓1 mark for the correct answer

6A

1 12

2 18

3 **a** **i** Has a rule

ii Add 7

b **i** No rule – random

c **i** Has a rule

ii Divide by 2 or multiply by $\frac{1}{2}$

4 **a** Pattern: multiply by 2, Next term: 32

b Pattern: add 8, Next term: 36

c Pattern: subtract 6, Next term: −20

5 11, 8, 5, 2, −1

6 Random

7 11

8 **a** $t_0 = 3$ $t_2 = 11$ $t_6 = 27$

b $t_0 = 16$ $t_3 = -5$ $t_8 = -40$

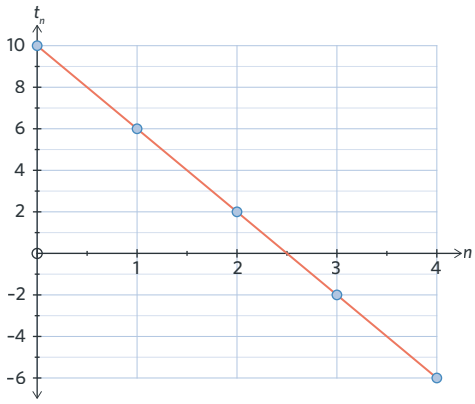
c $t_0 = -66$ $t_4 = -22$ $t_9 = 33$

9 t_7 is the largest.

10 D_4 was the hottest.

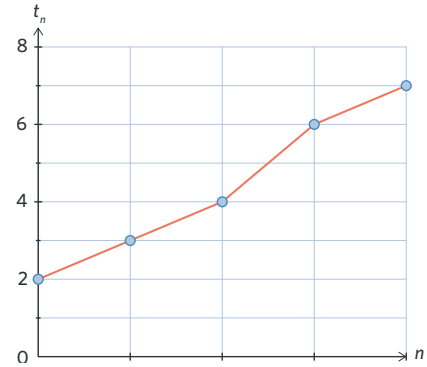
11 Yes

12 **a**



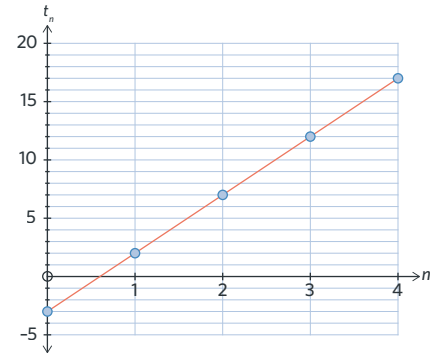
The plot makes a straight line, so this sequence is arithmetic.

b



The plot doesn't make a straight line, so this sequence isn't arithmetic.

c

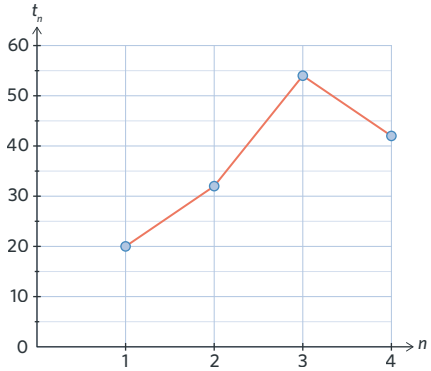


The plot makes a straight line, so this sequence is arithmetic.

13 **a** The plot makes a straight line, so this sequence is arithmetic.

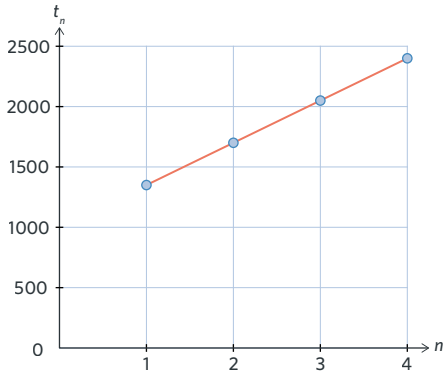
b The plot doesn't make a straight line, so this sequence isn't arithmetic.

14



This is not an arithmetic sequence.

15



This is an arithmetic sequence.

16 C

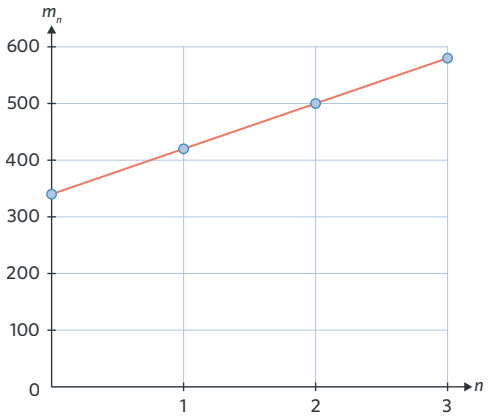
17 Pattern: add 17

$t_5 = 56$

18 Pattern: subtract 68

$t_5 = -19$

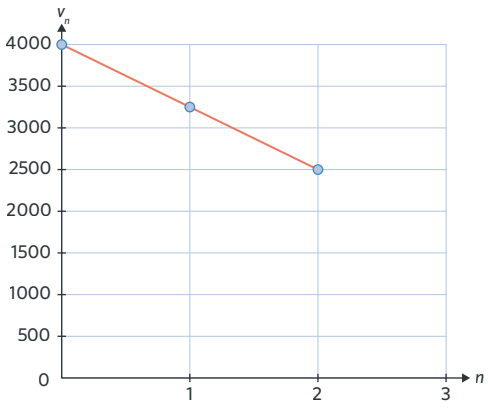
19 **a**



b Arithmetic sequence.

c Year 2025

20 **a**



b $v_3 = 1750$

$v_4 = 1000$

In two years from now.

21 A

Questions from multiple lessons

22 ***Recursion and financial modelling***

Decoding the question

Keywords: geometric sequence, next term

Relevant lesson: 6A

We need to analyse the given geometric sequence in order to determine the next term.

Information we are given

- The first five terms of the sequence are 32, 16, 8, 4, 2.
- It is a geometric sequence.

Using the theory

The question specifies that this is a geometric sequence. Hence, we know that the pattern follows a repeated multiplication or division and there is no need to focus on a potential addition or subtraction.

Looking at the first two terms of the sequence, in order to get from 32 to 16, we must divide by two. To check if this is the appropriate repeated division, divide each term by the next to see if this applies to all the numbers in the sequence.

$16 \div 8 = 2$

$8 \div 4 = 2$

$4 \div 2 = 2$

The rule for this sequence is to divide by two.

Hence, the next term in this sequence can be found by dividing the previous term by two.

$next\ term = previous\ term \div 2$

$next\ term = 2 \div 2$

$next\ term = 1$

Answer

D

✓1 mark for the correct answer

23 ***Data analysis***

Decoding the question

Keywords: least squares line, bivariate data

Relevant lesson: 13D

We must use the relevant formula and the given information to find the value of b

Information we are given

- $y = a + bx$
- $\overline{x} = 4.30$
- $\overline{y} = 5.12$
- $s_x = 2.94$
- $s_y = 1.98$
- $a = -2.8$

Using the theory

We can use the formula $a = \bar{y} - b\bar{x}$ to find the value of b

Substitute in $\bar{x} = 4.30$, $\bar{y} = 5.12$ and $a = -2.8$.

$-2.8 = 5.12 - b \times 4.30$

$4.30b = 7.92$

$b = 1.841...$

The value of b is 1.8 correct to one decimal place.

Answer

D

✓1 mark for the correct answer

24a ***Recursion and financial modelling***

Decoding the question

Keywords: how many, gain, each week

Relevant lesson: 6A

We need to determine the weekly increase in Hector's follower count using the given graph.

Information we are given

- The given graph
- Hector's followers increase weekly at a constant rate.

Using the theory

From looking at the graph, we can see that in week 1, Hector has 4000 followers. Looking across to week 2, the number of followers is two intervals higher. As each interval represents 100 followers, Hector's followers have increased by 200. As we know that the number of followers he has increases at a constant rate, we don't need to check with any other data points.

Answer

200 followers

✓1 mark for the correct answer

24b ***Recursion and financial modelling***

Decoding the question

Keywords: constant rate, how many

Relevant lesson: 6A

We can use our answer from part **a** to find Hector's number of followers in week 6.

Information we are given

- The given graph

Using the theory

From part **a**, we know that Hector gains followers at a constant rate of 200 followers per week.

We can follow this pattern until we reach week 6.

In week 1, Hector has 4000 followers.

In week 2, Hector has $4000 + 200 = 4200$ followers.

In week 3, Hector has $4200 + 200 = 4400$ followers.

In week 4, Hector has $4400 + 200 = 4600$ followers.

In week 5, Hector has $4600 + 200 = 4800$ followers.

In week 6, Hector has $4800 + 200 = 5000$ followers.

Answer

5000 followers

✓1 mark for the correct answer

6B

1 **a** 2 **b** 8

2 3

3 **a** $t_{n+1} = t_n + 2, \quad t_0 = 3$

b $t_{n+1} = t_n - 7, \quad t_0 = 19$

c $t_{n+1} = t_n + 6, \quad t_0 = -13$

4 $S_{n+1} = S_n + 3, \quad S_0 = 6$

5 8 customers bought phones.

$C_{n+1} = C_n + 2, \quad C_0 = 8$

6 $t_1 = 7$

7 **a** $t_3 = 21$ **b** $t_3 = -5$ **c** $t_3 = -44$

8 31 cups

9 4 020 000

10 A

11 $h_{n+1} = h_n + 0.05, \quad h_0 = 1.25.$

The year she is 17 is h_5

$h_1 = h_0 + 0.05$

$= 1.30$

$h_2 = h_1 + 0.05$

$= 1.35$

$h_3 = h_2 + 0.05$

$= 1.40$

$h_4 = h_3 + 0.05$

$= 1.45$

$h_5 = h_4 + 0.05$

$= 1.50$

12 24.40 cm

13 **a** $F_{n+1} = F_n - 33, F_0 = 4065$

b 3966 million hectares

c 2019

14 **a** $T_{n+1} = T_n + 0.175, T_0 = 0.8$

b 2080s **c** 2070s

15 D

Questions from multiple lessons

16 *Recursion and financial modelling*

Decoding the question

Keyword: arithmetic sequence

Relevant lessons: 6A, 6B

We must use the information in the table to deduce the common difference between terms. We can then use this to find the seventh term in the sequence.

Information we are given

- The given table
- Lorraine's monthly profit forms an arithmetic sequence.

Using the theory

First, we need to find the common difference. The common difference is the difference between each pair of adjacent terms.

Using the profit from months 1 and 2:

$d = 215 - 170 = 45$

Hence, the profit increases by \$45 each month.

We can use this common difference to find the profit each month until month seven.

Month 4: $260 + 45 = 305$

Month 5: $305 + 45 = 350$

Month 6: $350 + 45 = 395$

Month 7: $395 + 45 = 440$

$\therefore$ The profit in month seven will be \$440.

Answer

D

✓*1 mark for the correct answer*

17 *Data analysis*

Decoding the question

Keywords: least squares regression line, equation

Relevant lesson: 13D

We must use a calculator to find the equation of the least squares regression line.

Information we are given

- The given table

Using the theory

Enter the data into the calculator spreadsheet and find the equation of the least squares regression line.

The equation of the least squares regression line is $y = 3.97 + 0.48x$.

See video solution for a button-by-button demonstration.

Answer

C

✓*1 mark for the correct answer*

18a *Recursion and financial modelling*

Decoding the question

Keywords: 'show that', common difference, arithmetic sequence

Relevant lesson: 6B

We need to prove that the common difference is 2000 using the given sequence.

Information we are given

- The given sequence
- In the 1st hour, they produced 30 000 umbrellas.
- In the 2nd hour, they produced 32 000 umbrellas.
- In the 3rd hour, they produced 34 000 umbrellas.

Using the theory

To find the common difference, calculate the difference between two consecutive terms in an arithmetic sequence.

It can be found using the following formula:

$common\ difference = t_1 - t_0$

Substitute $t_1 = 32\ 000$ and $t_0 = 30\ 000$ into the formula.

$common\ difference = 32\ 000 - 30\ 000$

$\therefore common\ difference = 2000$

Answer

$common\ difference = 32\ 000 - 30\ 000 = 2000$

✓*1 mark – Correct method*

18b *Recursion and financial modelling*

Decoding the question

Keywords: 8th hour, 'how many'

Relevant lesson: 6B

We need to determine the 8th term in the sequence.

Information we are given

- The given sequence.
- The amount of stock produced in each hour forms the terms of an arithmetic sequence.
- The common difference is 2000.

Using the theory

The first term in the sequence is 30 000. This is our initial value, a

The common difference is 2000, hence $d = 2000$.

A recurrence relation that can be used to model this situation follows the form $t_0 = a \quad t_{n+1} = t_n + d$
Substitute $a = 30\ 000$ and $d = 2000$ into the relation.

$t_0 = 30\ 000, \quad t_{n+1} = t_n + 2000$

Since t_0 represents the number of umbrellas produced in the first hour, we need to find t_7 for the 8th hour.

We are given the values for the first three terms already. So $t_0 = 30\ 000$, $t_1 = 32\ 000$ and $t_2 = 34\ 000$.

$t_3 = t_2 + 2000$
$t_3 = 34\ 000 + 2000$
$t_3 = 36\ 000$

$t_4 = t_3 + 2000$
$t_4 = 36\ 000 + 2000$
$t_4 = 38\ 000$

$t_5 = t_4 + 2000$
$t_5 = 38\ 000 + 2000$
$t_5 = 40\ 000$

$t_6 = t_5 + 2000$
$t_6 = 40\ 000 + 2000$
$t_6 = 42\ 000$

$t_7 = t_6 + 2000$
$t_7 = 42\ 000 + 2000$
$t_7 = 44\ 000$

∴ In the 8th hour, the factory will produce 44 000 umbrellas.

Answer

44 000 umbrellas

✓1 mark for the correct answer

18c ***Recursion and financial modelling***

Decoding the question

Keywords: total, 'first five hours'

Relevant lesson: 6B

We need to find the sum of the first five terms of the sequence.

Information we are given

- The given sequence

Using the theory

From part **b**, we determined the first five terms already.

$t_0 = 30\ 000, t_1 = 32\ 000, t_2 = 34\ 000, t_3 = 36\ 000$ and $t_4 = 38\ 000$.

Find the sum of the first five terms.

$sum = t_0 + t_1 + t_2 + t_3 + t_4$

$sum = 30\ 000 + 32\ 000 + 34\ 000 + 36\ 000 + 38\ 000$

$sum = 170\ 000$

∴ A total of 170 000 umbrellas are produced during the first five hours.

Answer

170 000 umbrellas

✓1 mark for the correct answer

6C

1 C

2 **a** $a = 2, d = 3$

b $a = 7, d = -2$

c $a = 15, d = 5$

3 **a** $t_n = 2n - 5$

b $t_n = 11 - 4n$

c $t_n = 9 + 3.5n$

4 **a** $t_n = 24 - 2.5n$

b -11

5 **a** 31 **b** 160 **c** -397

6 $A_n = 12 + 2n$

7 $t_n = 161 - 11n$

8 15th term

9 $82

10 28 cm

11 **a** $27.50 **b** 23 days

12 23 weeks

13 $153.60

14 $5440

15 **a** $13 500 **b** $0.30

16 $1000

17 **a** 5.5% **b** $10 368

18 **a** $C_n = 18 - 3n$ **b** Week 7

19 **a** $t_n = 3500 + 38.5n$ **b** 26 quarters

c $5040

20 **a** $b_n = 670 - 50n$ **b** Day 13

21 **a** $7100 **b** 98 weeks

22 **a** 83 million **b** 7134 million people

c 2023 **d** 2035

23 **a** 37 **b** 43

Questions from multiple lessons

24 ***Recursion and financial modelling***

Decoding the question

Keyword: '200 more ancient coins than the previous year'

Relevant lessons: 6B, 6C

This question requires us to determine the initial value of a recurrence relation.

Information we are given

- Jacob wants to exhibit a complete collection of ancient Mayan coins in an exhibition in four years.
- Each year, he collects 200 more coins than the previous one.
- His team of archaeologists predict that there are 4200 coins to collect.

Using the theory

This scenario can be modelled using an arithmetic recurrence relation in the form $V_0 = a, V_{n+1} = V_n + d$

As the number of coins increases by 200 each year, $d = 200$. Let the number of coins collected in the first year be x. In this case, we will use V_1 instead of V_0 to represent the initial value (number of coins collected in the first year), so that V_2 represents the second year, and so on.

Hence, the recurrence relation that models the number of coins collected each year is given by
$V_1 = x, \quad V_{n+1} = V_n + 200.$

Use this recurrence relation to determine how many coins are collected each year.

$V_1 = x$

$V_2 = V_1 + 200$
$V_2 = x + 200$

$V_3 = V_2 + 200$
$V_3 = (x + 200) + 200$
$V_3 = x + 400$

$V_4 = V_3 + 200$
$V_4 = (x + 400) + 200$
$V_4 = x + 600$

At the end of four years, the **total** number of ancient coins that Jacob must collect is 4200 so we must find the sum of the four terms.

$4200 = V_1 + V_2 + V_3 + V_4$

$4200 = (x) + (x + 200) + (x + 400) + (x + 600)$

$4200 = 4x + 1200$

$4x = 4200 - 1200$

$x = \frac{4200 - 1200}{4}$

$x = 750$

∴ Jacob must collect 750 coins in the first year.

Answer

C

✓1 mark for the correct answer

25 ***Data analysis***

Decoding the question

Keyword: Pearson correlation coefficient

Relevant lesson: 13C

We must enter the data into the calculator and use its statistics functions to find the value of r

Information we are given

- The given table

Using the theory

Enter the data into the calculator spreadsheet and use the two-variable statistics functions to calculate the value of r. The value of r is 0.65 correct to two decimal places.

See video solution for a button-by-button demonstration.

Answer

E

✓1 mark for the correct answer

26a ***Recursion and financial modelling***

Decoding the question

Keywords: how much, cost

Relevant lessons: 6B, 6C

We need to determine the value of the 8th term in a sequence.

Information we are given

- Customers are charged based on the whole number of kilometres they travel each day.
- The first kilometre costs \$8.
- Each kilometre after this costs 25 cents less than the previous one.
- Once the cost of travel reaches zero, any additional travel within that day is free.

Using the theory

Since the cost per kilometre decreases by 25 cents with each consecutive kilometre travelled, this scenario is an arithmetic sequence with a common difference, d, of -0.25.

The first kilometre costs \$8, so this is our initial value, a

We can find the rule for the value of the term after n iterations using the formula:

$t_n = a + n \times d.$

Substitute $a = 8$ and $d = -0.25$ into the equation.

$t_n = 8 + n \times (-0.25)$

$t_n = 8 - 0.25n$

As the first term is denoted t_0, the 8th term is t_7
Substitute $n = 7$ into the equation.

$t_7 = 8 - 0.25(7)$

$t_7 = 8 - 1.75$

$t_7 = 6.25$

Therefore, the 8th kilometre will cost \$6.25.

Answer

\$6.25

✓1 mark for the correct answer

26b ***Recursion and financial modelling***

Decoding the question

Keyword: total cost

Relevant lesson: 6C

We need to find the sum of the first 10 terms in the sequence to determine the total cost of travelling 10 km.

Information we are given

- Customers are charged based on the whole number of kilometres they travel each day.

- The first kilometre costs $8.
- Each kilometre after this costs 25 cents less than the previous one.
- Once the cost of travel reaches zero, any additional travel within that day is free.

Using the theory

From part **a**, the formula for the value of the term after n iterations in this scenario is $t_n = 8 - 0.25n$

We need to find the sum of the terms from t_0 to t_9 to find the total cost of travelling 10 km.

We know that the first kilometre costs $8, hence $t_0 = 8$.

Find t_1:
$t_1 = 8 - 0.25(1)$
$t_1 = 7.75$

Find t_2:
$t_2 = 8 - 0.25(2)$
$t_2 = 7.5$

Find t_3:
$t_3 = 8 - 0.25(3)$
$t_3 = 7.25$

Find t_4:
$t_4 = 8 - 0.25(4)$
$t_4 = 7$

Find t_5:
$t_5 = 8 - 0.25(5)$
$t_5 = 6.75$

Find t_6:
$t_6 = 8 - 0.25(6)$
$t_6 = 6.5$

Find t_7:
$t_7 = 8 - 0.25(7)$
$t_7 = 6.25$

Find t_8:
$t_8 = 8 - 0.25(8)$
$t_8 = 6$

Find t_9:
$t_9 = 8 - 0.25(9)$
$t_9 = 5.75$

The total cost can be found by calculating the sum of the first 10 terms.

$total\ cost = t_0 + t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 + t_8 + t_9$

$total\ cost = 8 + 7.75 + 7.5 + 7.25 + 7 + 6.75 + 6.5 + 6.25 + 6 + 5.75$

$total\ cost = 68.75$

Therefore, the total cost to travel 10 km is $68.75.

Answer

$68.75

✓1 mark for the correct answer

26c ***Recursion and financial modelling***

Decoding the question

Keyword: total cost

Relevant lesson: 6C

We need to find the maximum cost of travel each day and subsequently, the minimum number of kilometres travelled to incur the maximum total cost.

Information we are given

- Customers are charged based on the whole number of kilometres they travel each day.
- The first kilometre costs $8.
- Each kilometre after this costs 25 cents less than the previous one.
- Once the cost of travel reaches zero, any additional travel within that day is free.

Using the theory

From part **a**, we determined that the equation for the value of the term after n iterations was $t_n = 8 - 0.25n$

We first need to find the number of kilometres travelled before additional travel is free. Therefore, we need to find when $t_n = 0$.

$0 = 8 - 0.25n$

$0.25n = 8$

$n = \frac{8}{0.25}$

$n = 32$

This means that the 33rd km travelled is free (as the initial value was denoted t_0), so a minimum of 32 km needs to be travelled to incur the maximum charge.

NOTE: 33 km is not the correct answer. Since the cost of travel is free including and after the 33rd km, the cost to travel 32 km and 33 km is the same. The question asks for the **minimum** distance travelled to incur the maximum cost.

Answer

32 km

✓1 mark for the correct answer

6D

1 5

2 3

3 **a** $t_{n+1} = 5t_n,\ t_0 = 3$ **b** $t_{n+1} = -\frac{1}{3}t_n,\ t_0 = -162$

c $t_{n+1} = 5t_n,\ t_0 = \frac{1}{4}$

4 D

5 $p_{n+1} = 2p_n \quad p_0 = 1500$

6 $-\frac{2}{3}$

7 **a** 512 **b** $\frac{7}{108}$ **c** 3

8 $\frac{7}{2}$

9 468

10 77.2 cm

11 **a** $x = 14$

$b_{n+1} = 2_n \quad b_0 = 7$

b 448 **c** 8 hours

12 $p_{n+1} = 0.88p_n \quad p_0 = 101.325$

13 **a** 2 years **b** 4 years

14 E

Questions from multiple lessons

15 *Recursion and financial modelling*

Decoding the question

Keyword: recurrence relation

Relevant lesson: 6B

We must identify which of the given recurrence relations could model Beatrice's number of followers.

Information we are given

- Beatrice has 200 followers on day 1.
- Each day, Beatrice has 20 more followers than she did the day before.

Using the theory

First, we must identify whether the sequence is arithmetic or geometric.

Since Beatrice's number of followers increases by a fixed amount each day, F_n increases linearly, so the sequence must be arithmetic.

An arithmetic sequence is given by $F_{n+1} = F_n + d$ where d is the common difference between terms in an arithmetic sequence. Beatrice gains 20 followers each day. Hence, $d = 20$.

The recurrence relation can therefore be written as $F_{n+1} = F_n + 20$.

On day 1, Beatrice has 200 followers. Hence, $F_1 = 200$.

$\therefore F_{n+1} = F_n + 20, \quad F_1 = 200$

Answer

C

✓ *1 mark for the correct answer*

16 *Data analysis*

Decoding the question

Keywords: histogram, distribution, shape

Relevant lesson: 12D

We must look at the data displayed in the graph to determine the shape of the distribution.

Information we are given

- The given histogram

Using the theory

There are more data values on the right side of the histogram than there are on the left. Therefore the distribution cannot be approximately normal or symmetric and options A, B, and C are incorrect.

Since the right side of the distribution is weighted more than the left and the tail of the distribution points to the left, the distribution is negatively skewed.

Answer

D

✓ *1 mark for the correct answer*

17a *Recursion and financial modelling*

Decoding the question

Keywords: balance, savings account, investment

Relevant lessons: 3D, 6D

We need to use the recurrence relation to determine the value of the investment after five years.

Information we are given

- $V_0 = 1\,000\,000 \quad V_{n+1} = 1.06 \times V_n$

Using the theory

The value of the investment after five years is denoted V_5

We can use the recurrence relation to find the values from V_0 to V_5

$V_1 = 1.06 \times 1\,000\,000$

$V_1 = 1\,060\,000$

$V_2 = 1.06 \times 1\,060\,000$

$V_2 = 1\,123\,600$

$V_3 = 1.06 \times 1\,123\,600$

$V_3 = 1\,191\,016$

$V_4 = 1.06 \times 1\,191\,016$

$V_4 = 1\,262\,476.96$

$V_5 = 1.06 \times 1\,262\,476.96$

$V_5 = 1\,338\,225.5776$

Therefore, the value of the investment after five years is \$1 338 225.58.

Answer

\$1 338 225.58

✓ *1 mark for the correct answer*

17b *Recursion and financial modelling*

Decoding the question

Keywords: value, exceed

Relevant lessons: 3D, 6D

We can use the given recurrence relation to determine when the balance will reach \$1 500 000.

Information we are given

- $V_0 = 1\ 000\ 000 \quad V_{n+1} = 1.06 \times V_n$

Using the theory

Determine the next terms in the sequence using the recurrence relation:

$V_0 = 1\ 000\ 000 \quad V_{n+1} = 1.06 \times V_n$

$V_0 = 1\ 000\ 000$

$V_1 = 1.06 \times 1\ 000\ 000 = 1\ 060\ 000$

$V_2 = 1.06 \times 1\ 060\ 000 = 1\ 123\ 600$

$V_3 = 1.06 \times 1\ 123\ 600 = 1\ 191\ 016$

$V_4 = 1.06 \times 1\ 191\ 016 = 1\ 262\ 476.96$

$V_5 = 1.06 \times 1\ 262\ 476.96 = 1\ 338\ 225.5776$

$V_6 = 1.06 \times 1\ 338\ 225.5776 = 1\ 418\ 519.11226$

$V_7 = 1.06 \times 1\ 418\ 519.11226 = 1\ 503\ 630.25899$

V_7 represents the value of the investment after seven years.

Therefore, the balance exceeds \$1 500 000 after seven years.

Answer

7 years

✓1 mark for the correct answer

6E

1 $10x = 9$

$10x^2 = 8.1$

2 $t_6 = 36.45$

3 **a** $t_n = 2 \times 0.99^n$

$t_8 = 1.85$

b $t_n = 3 \times 2^n$

$t_8 = 768$

c $t_n = \left(\frac{4}{5}\right)^n$

$t_8 = 0.17$

d $t_n = 2 \times (-3)^n$

$t_8 = 13\ 122$

4 Initial value is 6.5. Common ratio is 0.9. The sequence is decreasing.

5 Sandra will win. She is using a formula for t_n, whereas Mandeep is using a recursion formula. Mandeep has to do eight calculations to get from t_2 to t_{10}. Sandra just needs to perform one calculation.

6 \$1125.68

7 **a** $Q_n = 38000 \times (1.031)^{n-1}$

b $S_n = 35000 \times (1.04)^{n-1}$

c $T_n = 40000 \times (1.02)^{n-1}$

d Quentin

8 **a** $H_n = 100 \times (0.45)^n$

b 9.11 cm

9 **a** $V_n = 5000 \times (0.82)^n$

b Year 5

10 **a** $R = \sqrt{\ }$

$t_n = 6.21 \times (\sqrt{\ })^n$

b $t_n = 9.8 \times (0.91)^n$

11 **a** $BB_n = \frac{3}{2} \times 0.73^n$

$BP_n = 3 \times 0.96^n$

$BT_n = 2 \times 0.8^n$

b

Ball	Bounce 1	Bounce 10
BIG–Bounce	3.65 m	0.21 m
BouncePRO	4.8 m	3.32 m
BallTECH	4 m	0.54 m

c The BouncePRO ball.

d B **e** 102 bounces

12 **a** $I_m = 390 \times (1.05)^m$

b $N_q = 400 \times (1.15)^q$

c Isabelle will reach \$1000 first.

13 B

Questions from multiple lessons

14 ***Recursion and financial modelling***

Decoding the question

Keywords: successively increase, how long

Relevant lesson: 6C

We can use the arithmetic sequence formula $t_n = a + nd$ to calculate the amount of time Calliope spent studying in week 13.

Information we are given

- Calliope studied for 150 minutes in the first week.
- Time spent studying increases by 15 minutes each week.

Using the theory

Since Calliope's time spent studying increases by a constant amount each week, it can be modelled by an arithmetic sequence.

An expression for the value in an arithmetic sequence after n periods is $t_n = a + nd$, where a is the first term of the sequence and d is the common difference.

In this case, the first term of the sequence is 150 and the common difference is 15.

Substitute $a = 150$ and $d = 15$ into $t_n = a + nd$

$\therefore t_n = 150 + 15n$

To find the time spent studying in the 13th week, we need to solve this equation for t_{12} (due to the first term being denoted t_0)

$t_{12} = 150 + 15 \times 12$

$t_{12} = 150 + 180$

$t_{12} = 330$

Answer

B

✓1 mark for the correct answer

15 ***Data analysis***

Decoding the question

Keyword: variables

Relevant lesson: 12A

We need to determine what type of data the given variables are.

Information we are given

- *Number of doughnuts sold*
- *Day of the week*

Using the theory

First, categorise each variable as either numerical or categorical.

Numerical variables represent a quantity and can be counted or measured. As *number of doughnuts sold* is a variable that is counted, this is a numerical variable.

Categorical variables represent a quality or attribute and are broken down into groups or categories. *Day of the week* is a categorical variable as it has the categories Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday. Categorical variables can be either nominal or ordinal variables.

Nominal variables are categorical variables that can be grouped but not ordered. As there is no logical hierarchy within the days of the week, this is a nominal variable.

The variables *number of doughnuts sold* and *day of the week* are a numerical variable and a nominal variable respectively.

Answer

B

✓1 mark for the correct answer

16a ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation, how many

Relevant lesson: 6D

We need to determine the initial value of the recurrence relation.

Information we are given

- $K_{n+1} = 0.8\,K_n \quad K_{2020} = 85\ 000$
- K_n is the projected population at the beginning of year n

Using the theory

K_{2020} represents the projected population of koalas at the beginning of 2020.

$K_{2020} = 85\ 000$

∴ The projected population of koalas at the beginning of 2020 is 85 000.

Answer

85 000

✓1 mark for the correct answer

16b ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation, percentage

Relevant lessons: 6D, 6E

We need to use our knowledge of recurrence relations to determine the projected percentage decrease of koala populations from year to year.

Information we are given

- $K_{n+1} = 0.8\,K_n \quad K_{2020} = 85\ 000$

Using the theory

The recurrence relation given is in the form

$K_{n+1} = R \times K_n$ where $R = 1 + \frac{percentage}{100}$

In the recurrence relation, $K_{n+1} = 0.8 \times K_n$

Substitute $R = 0.8$ into $R = 1 + \frac{percentage}{100}$ and solve for *percentage*

$0.8 = 1 + \frac{percentage}{100}$

$\frac{percentage}{100} = -0.2$

$percentage = -20\%$

The percentage is negative as the koala population decreases each month.

Therefore, the projected percentage decrease of koala populations from year to year is 20%.

Answer

20%

✓1 mark for the correct answer

16c ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation, show that

Relevant lesson: 6D

We need to prove that at the beginning of 2013, the population of koalas is projected to be 68 000.

Information we are given

- $K_{n+1} = 0.8K_n \quad K_{2020} = 85\ 000$
- K_n is the projected population at the beginning of year n

Using the theory

We need to find the value of K_{2021} as it represents the projected population of koalas at the beginning of 2021.

$K_{2021} = 0.8 \times K_{2020}$

$K_{2021} = 0.8 \times 85\ 000$

$K_{2021} = 68\ 000$

∴ The projected population of koalas at the beginning of 2021 is 68 000.

Answer

$K_{2021} = 0.8 \times 85\ 000 = 68\ 000$

✓1 mark for the correct working

16d ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation, 'change in population'

Relevant lesson: 6D

We need to use the recurrence relation to find the difference between K_{2022} and K_{2021}

Information we are given

- $K_{n+1} = 0.8K_n \quad K_{2020} = 85\ 000$
- K_n is the projected population at the beginning of year n

Using the theory

The projected change in population of koalas over 2021 can be found using the following equation.

projected change during $2021 = K_{2022} - K_{2021}$

From part **c**, we know that the population is projected to be 68 000 at the beginning of 2021.

We can use the recurrence relation to find the projected population at the beginning of 2022.

$K_{2022} = 0.8 \times K_{2021}$

$K_{2022} = 0.8 \times 68\ 000$

$K_{2022} = 54\ 400$

Substitute $K_{2021} = 68\ 000$ and $K_{2022} = 54\ 400$ into *projected change during* $2021 = K_{2022} - K_{2021}$

projected change during $2021 = 54\ 400 - 68\ 000$

projected change during $2021 = -13\ 600$

∴ The projected change in the koala population is a reduction of 13 600 koalas.

Answer

Reduction of 13 600

✓1 mark for the correct answer

6F

1 5

2 B

3 **a** 47 **b** 18 **c** −97

4 4

5 $t_0 = 2$

$t_1 = -3$

6 **a** 13 **b** −1 **c** −2

7 D

8 B

9 11

10 C

11 **a** $P_n = P_{n-2} + P_{n-1}$ **b** 2

c 42

Questions from multiple lessons

12 ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation, expression, after n periods

Relevant lessons: 6B, 6D, 6E

We must first identify the pattern in the sequence generated by the given recurrence relation. This will allow us to determine an expression that gives the value of the sequence after n periods.

Information we are given

- $a_{n+1} = 5 \times a_n \quad a_0 = 3$

Using the theory

First, identify whether the given recurrence relation is arithmetic or geometric.

The recurrence relation for an arithmetic sequence involves a common difference (a fixed number is added or subtracted) while the recurrence relation for a geometric sequence involves a common ratio (the term is multiplied or divided by a fixed number).

In the given recurrence relation, a_n is multiplied by 5. Hence, the sequence is geometric and has a common ratio of 5.

An expression for the value in a geometric sequence after n periods is $t_n = t_0 \times R^n$, where t_0 is the initial term and R is the common ratio.

Substitute $t_0 = 3$, $R = 5$, and $t = a$ into $t_n = t_0 \times R^n$ to find the expression for this sequence.

∴ $a_n = 3 \times 5^n$

Answer

C

✓1 mark for the correct answer

13 ***Data analysis***

Decoding the question

Keywords: least squares regression, equation

Relevant lesson: 13D

This question requires us to use the given graph to determine the equation of the least squares regression line that has been fitted to the data.

Information we are given

- The given graph

Using the theory

A least squares regression line is in the form $y = a + bx$, where y is the response variable and x is the explanatory variable.

We know that the response variable is the *weight* of the baby as it is located on the y-axis and the explanatory variable is *length* as it is located on the x-axis. Therefore, the equation is in the form $weight = a + b \times length$ Options A, C and D can be ruled out.

We must decide if the answer is B or E by determining the value of a, which is the y-intercept. In the given graph, neither the x-axis nor the y-axis starts at the origin. Therefore, the point at $y = 2.6$, where the line appears to be crossing the y-axis is not the y-intercept. As a result, we can eliminate Option B because we know that $a \neq 2.60$.

Therefore, the answer must be E.

Answer

E

✓*1 mark for the correct answer*

14a ***Recursion and financial modelling***

Decoding the question

Keyword: how many

Relevant lesson: 6D

We need to use a recurrence relation to determine the third term in the sequence.

Information we are given

- In the first month, they sold 20 000 units of eyeshadow.
- Each month after this, they expect to sell 5% more than the previous month.

Using the theory

The number of cosmetic palettes sold grows by a percentage rate, not a constant value. Hence it can be represented by a recurrence relation in the form

$$P_{n+1} = P_n \times \left(1 + \frac{r}{100}\right) \quad P_1 = initial\ value$$

Note that in this case, our initial value is P_1, rather than P_0, because in the first month, 20 000 units are expected to be sold and we want to count from there.

Substitute $P_1 = 20\ 000$ and $r = 5$ into the equation.

$P_{n+1} = P_n \times 1.05, \quad P_1 = 20\ 000$

$P_2 = 1.05 \times P_1$

$P_2 = 1.05 \times 20\ 000$

$P_2 = 21\ 000$

$P_3 = 1.05 \times P_2$

$P_3 = 1.05 \times 21\ 000$

$P_3 = 22\ 050$

$\therefore$ In the third month, they expect to sell 22 050 units.

Answer

22 050 units

✓*1 mark for the correct answer*

14b ***Recursion and financial modelling***

Decoding the question

Keyword: difference

Relevant lesson: 6D

We need to use our recurrence relation from part **a** to find the difference between the second and fifth terms of the sequence.

Information we are given

- In the first month, they sold 20 000 units of eyeshadow.
- Each month after this, they expect to sell 5% more than the previous month.

Using the theory

The difference between the predicted sales of the second and fifth month can be found by subtracting P_2 from P_5

From part **a** we determined the recurrence relation that models this scenario is $P_{n+1} = P_n \times 1.05, \quad P_1 = 20\ 000$. We also found the first three terms.

$P_1 = 20\ 000$

$P_2 = 21\ 000$

$P_3 = 22\ 050$

We can now use the recurrence relation to find P_4 and P_5

$P_4 = 1.05 \times P_3$

$P_4 = 1.05 \times 22\ 050$

$P_4 = 23\ 152.50$

$P_5 = 1.05 \times P_4$

$P_5 = 1.05 \times 231\ 152.50$

$P_5 = 24\ 310.125$

Subtract P_2 from P_5 to find the difference in predicted sales.

$difference = P_5 - P_2$

$difference = 24\ 310.125 - 21\ 000$

$difference = 3310.125$

$difference \approx 3310$

Answer

3310 units

✓1 mark for the correct answer

14c ***Recursion and financial modelling***

Decoding the question

Keyword: total

Relevant lesson: 6D

We need to use the recurrence relation to find the sum of the first six terms of the sequence.

Information we are given

- In the first month, they sold 20 000 units of eyeshadow.
- Each month after this, they expect to sell 5% more than the previous month.

Using the theory

From part **a** we determined the recurrence relation that models this scenario is $P_{n+1} = P_n \times 1.05, \quad P_1 = 20\ 000$. We have also found the first five terms.

$P_1 = 20\ 000$

$P_2 = 21\ 000$

$P_3 = 22\ 050$

$P_4 = 23\ 152.50$

$P_5 = 24\ 310.12...$

We need to determine the value of P_6

$P_6 = 1.05 \times P_5$

$P_6 = 1.05 \times 24\ 310.125$

$P_6 = 25\ 525.63...$

Find the sum of the first six terms.

$sum = P_1 + P_2 + P_3 + P_4 + P_5 + P_6$

$sum = 20\ 000 + 21\ 000 + 22\ 050 + 23\ 152.50 + 24\ 310.12... + 25\ 525.63...$

$sum = 136\ 038.25...$

$sum \approx 136\ 038$

Answer

136 038 units

✓1 mark for the correct answer

7A

1 Millimetres

2 B

3 **a** B, G, J **b** A, F, H **c** D, K **d** C, L **e** E, I

4 A

5 A

6 C

7 **a** cm^2 **b** m^3 **c** mm^2

8 **a** m **b** mm

9 10

10 **a** 12 000 cm^3 **b** 4320 m **c** 5000 mm^2

11 23 450 000 mm

12 378 000 cm^3

13 **a** 68.97 cm^2 **b** 0.000033 cm^2/pixel

14 2.41 m^2

15 2500 m^3, 2 500 000 L

16 Nada

17 **a** The area of the top of each desk can be calculated using the measurements provided.

b Desk B

Questions from multiple lessons

18 ***Geometry and measurement***

Decoding the question

Keyword: area

Relevant lessons: 7F, 8G

This question requires us to determine the area of a composite shape.

Information we are given

- The given diagram

Using the theory

Find the area of the entire triangle.

$A_T = \frac{1}{2} \times base \times height$

$A_T = \frac{1}{2} \times 20 \times 6$

$A_T = 60\ m^2$

Find the area of the unshaded rectangle.

$A_R = length \times width$

$A_R = 4 \times 3$

$A_R = 12\ m^2$

Find the area of the shaded region.

$A_{shaded} = A_T - A_R$

$A_{shaded} = 60 - 12$

$A_{shadead} = 48$

The area of the shaded region is 48 m^2.

Answer

D

✓1 mark for the correct answer

19 *Data analysis*

Decoding the question

Keywords: histogram, boxplot, distribution, shape

Relevant lesson: 12A

This question requires us to describe the distribution of a given set of data.

Information we are given

- The given histogram
- The given boxplot

Using the theory

Looking at the histogram, it can be seen that the data is skewed. The tail of the data is pointing right, hence it is positively skewed. This eliminates options A, B and D and leaves us with options C and E.

A boxplot displays outliers as dots. There are two dots on the boxplot, indicating the existence of two outliers.

∴ The distribution is positively skewed with outliers.

Answer

C

✓1 mark for the correct answer

20a *Geometry and trigonometry*

Decoding the question

Keyword: perimeter

Relevant lesson: 7A

We need to calculate the unknown side lengths in order to find the perimeter of the orange shape.

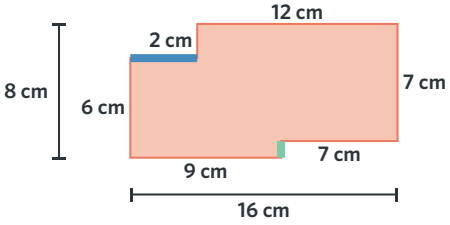
Information we are given

- The given diagram

Using the theory

We can use the known side lengths to find the total length and width of the shape. The two horizontal lengths on the bottom of the shape can be added to find the total length of the shape; 16 cm. The two vertical lengths on the left side of the shape can be added to find the total width of the shape; 8 cm.

The unknown side lengths are highlighted blue and green.



The blue side length must be $16 - 12 = 4$ cm.

The green side length must be $8 - 7 = 1$ cm.

The total perimeter is
$6 + 2 + 12 + 7 + 7 + 9 + 4 + 1 = 48$ cm.

Answer

48 cm

✓1 mark for the correct answer

20b *Geometry and trigonometry*

Decoding the question

Keywords: perimeter, difference

Relevant lesson: 7A

We need to find the perimeter of the blue shape in order to find the difference in perimeter between the two shapes.

Information we are given

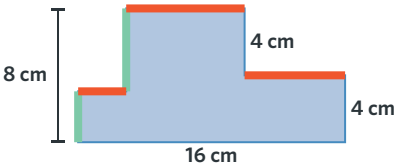
- The given diagram

Using the theory

First, we must find the perimeter of the blue shape.

We can use the known side lengths to find the total length and width of the shape. The length of the shape is given as the bottom length spans the entire shape; 16 cm. The two vertical lengths on the right side of the shape can be added to find the total width of the shape; 8 cm.

The unknown side lengths are highlighted in red and green.



The length of the blue shape is 16 cm, so the red lengths must add up to 16 cm.

The width of the blue shape is 8 cm, so the green lengths must add up to 8 cm.

The perimeter of the blue shape is
$4 + 4 + 8 + 16 + 16 = 48$ cm.

The perimeter of the orange shape is 48 cm from part **a**.

The difference in perimeter between the orange and blue shape is 0 cm.

Answer

0 cm

✓1 mark for the correct answer

7B

1 25 cm

2 7.81 m

3 **a** 8.94 cm **b** 16.55 m **c** 21.63 cm

4 12.21 cm

5 100.72 m

6 300 m

7 **a** 5.74 mm **b** 32.50 m **c** 10.82 m

8 14.42 mm

9 97.98 m

10 9.9 mm

11 49.356 km

12 Pythagoras' theorem states that $a^2 + b^2 = c^2$ for right-angled triangles.

Let $a = 5$, $b = 12$, and $c = 13$.

$a^2 + b^2 = c^2$

$5^2 + 12^2 = 13^2$

$25 + 144 = 169$

$LHS = RHS$

$\therefore$ Triangle is right-angled

13 6.708 km

14 200 cm^2

15 **a** 269.26 cm **b** 47.7 cm

16 D

Questions from multiple lessons

17 ***Geometry and measurement***

Decoding the question

Keywords: scale, represent

Relevant lesson: 7I

This question requires us to use the given scale to find the scaled distance when the actual distance is 10 km.

Information we are given

- The scale is 1:100 000

Using the theory

First, convert 10 km using the given scale.

$$scaled\ distance = \frac{10}{100\ 000}$$

$scaled\ distance = 0.0001$

Hence, the scaled distance is 0.0001 km.

Since the options are given in cm, this length must be converted to cm.

We know that 1 km can be converted to 100 000 cm.

$scaled\ distance = 100\ 000 \times 0.0001$ cm

$scaled\ distance = 10$ cm

Answer

D

✓1 mark for the correct answer

18 ***Data analysis***

Decoding the question

Keywords: distribution, histogram, boxplot

Relevant lesson: 12C

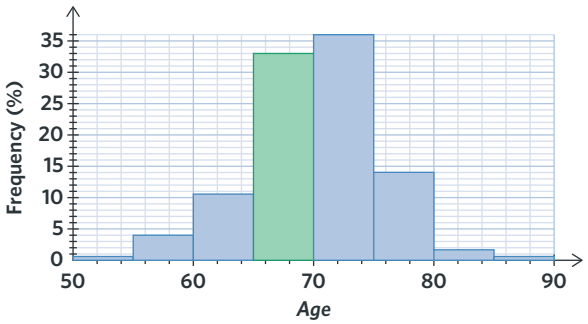
This question requires us to identify the percentage frequency from the histogram for a given interval and use this to find the number of people.

Information we are given

- The given histogram
- The given boxplot
- There are 200 residents.

Using the theory

The interval between 65 and 70 years has been highlighted below on the histogram we were given.



This interval has a percentage frequency of 33%.
To determine the number of residents between the age of 65 and 70, find 33% of 200.

$0.33 \times 200 = 66$

$\therefore$ 66 residents

NOTE: This answer could not be determined from the boxplot.

Answer

B

✓1 mark for the correct answer

19a ***Geometry and measurement***

Decoding the question

Keyword: show that

Relevant lesson: 7B

We must use Pythagoras' theorem and the measurements in the diagram to show that $h = 163.6$.

Information we are given

- The given diagram
- The height is 163.6 cm, correct to the nearest millimetre.

Using the theory

Since the front of the tent can be split into two right-angled triangles, we will use Pythagoras' theorem on one of the right-angled triangles to find h

Pythagoras' theorem states that for a right-angled triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse. In this case, we already know the length of the hypotenuse and are looking for an unknown side length.

To do this, we can rearrange the formula to give the value of a

$a^2 = c^2 - b^2$

$a = \sqrt{c^2 - b^2}$

Substitute $c = 180$ and $b = \frac{150}{2} = 75$, and $a = h$ into $a = \sqrt{c^2 - b^2}$

$h = \sqrt{180^2 - 75^2}$

$h = 163.6306...$

Answer

The height of the tent is 163.6 cm, correct to the nearest millimetre.

✓ *1 mark for the correct working*

19b ***Geometry and measurement***

Decoding the question

Keyword: convert

Relevant lesson: 7A

We can convert from cm to km by first finding the conversion factor between the two units.

Information we are given

- The height is 163.6 cm.

Using the theory

To convert a measurement into a different unit, we can multiply by the conversion factor, given by $conversion\ factor = \frac{C_{old\ unit}}{C_{new\ unit}}$.

The C values can be found in the table in 7A. The necessary C values are shown below.

Length		
Name	**Unit**	**C**
Millimetre	mm	0.001
Centimetre	cm	0.01
Metre	m	1
Kilometre	km	1000

$C_{old\ unit} = 0.01$

$C_{new\ unit} = 1000$

$conversion\ factor = \frac{0.01}{1000}$

Multiply 163.6 cm by the conversion factor.

$\frac{0.01}{1000} \times 163.6 = 0.001636$

Answer

0.001636 km

✓ *1 mark for the correct answer*

7C

1 10.8 cm

2 **a** 12.207 cm **b** 14.59 cm

3 30.71 cm

4 **a** 20.81 m **b** 23.54 m

5 **a** 11.31 m **b** 8.25 m

6 19.1 cm

7 19.08 cm

8 24.4 cm

9 No, the longest diagonal is 22.74 cm.

10 5.59 m

11 Box B

12 11.83 mm

13 21.83 m

14 1.6 m

Questions from multiple lessons

15 ***Geometry and measurement***

Decoding the question

Keyword: length

Relevant lesson: 7B

This question requires us to use Pythagoras' theorem to determine the unknown length.

Information we are given

- The given diagram

Using the theory

We can determine the length of the leash using Pythagoras' theorem.

Pythagoras' theorem states that $c = \sqrt{a^2 + b^2}$, where c is the length of the hypotenuse.

Substitute $c = length\ of\ leash$, $a = 2$ and $b = 0.7$ into the equation.

$length\ of\ leash = \sqrt{2^2 + 0.7^2}$

$length\ of\ leash = \sqrt{4.49}$

$length\ of\ leash = 2.11896...$

∴ The length of the leash is closest to 2.1 m.

Answer

A

✓ *1 mark for the correct answer*

16 ***Recursion and financial modelling***

Decoding the question

Keywords: loan, interest, compounding monthly

Relevant lesson: 6C

We must determine the amount Hugo owes 7 months after establishing a loan if he only pays back the interest charged each month.

Information we are given

- Jason establishes a loan for $1 200 000.
- Interest is charged at a rate of 4.9% per annum, compounding monthly.
- Each month, Hugo will only pay for the interest charged that month.

Using the theory

Since Hugo only pays back the interest charged each month, the balance of the loan does not change. Hence, the amount that he still owes after 7 months is equal to the initial balance of the loan, which is $1 200 000.

Answer

D

✓1 mark for the correct answer

17a ***Geometry and measurement***

Decoding the question

Keyword: straight-line distance

Relevant lesson: 7B

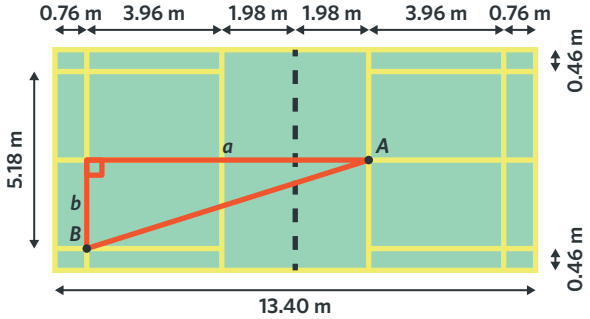
This question requires us to use Pythagoras' theorem to determine the unknown distance between A and B.

Information we are given

- The given diagram

Using the theory

The length of AB can be found by applying Pythagoras' theorem to the triangle shown below.



$a = 3.96 + 1.98 + 1.98 = 7.92$

$b = \frac{5.18}{2} = 2.59$

Using Pythagoras' theorem:

$c = \sqrt{a^2 + b^2}$

$AB = \sqrt{a^2 + b^2}$

$AB = \sqrt{(7.92)^2 + (2.59)^2}$

$AB = 8.332...$

∴ The straight-line distance between A and B, correct to two decimal places, is 8.33 m.

Answer

8.33 m

✓1 mark for the correct answer

17b ***Geometry and measurement***

Decoding the question

Keywords: directly above, straight-line distance

Relevant lessons: 7B, 7C

This question requires us to use Pythagoras' theorem in three dimensions to determine the unknown distance.

Information we are given

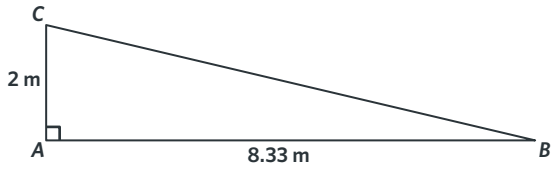
- The given diagram
- Boris serves at a height of 2.0 m directly above point A.

Using the theory

From part **a**, we know that the horizontal straight-line distance between A and B is 8.33 m.

The serve occurs 2 m directly above point A.

The diagram below illustrates this scenario with point C representing the position that the serve occurs.



The straight-line distance that the ball travels is equal to the distance between B and C in the diagram above. This length can be found using Pythagoras' theorem.

$c = \sqrt{a^2 + b^2}$

$BC = \sqrt{8.33^2 + 2^2}$

$BC = 8.56...$

∴ The straight-line distance of Boris' serve, correct to one decimal place, is 8.6 m.

Answer

8.6 m

✓1 mark for the correct answer

7D

1 8 cm

2 38, 72

3 **a** Perimeter : 32 cm
Area: 48 cm^2

b Perimeter: 17 cm
Area: 15 cm^2

c Perimeter: 28 cm
Area: 42 cm^2

d Perimeter: 44 cm
Area: 91 cm^2

4 10.5 m^2

5 2.42 km^2

6 18.85, 6, 28.27

7 **a** $C = 43.98$ cm, $A = 153.94$ cm^2

b $C = 9.42$ cm, $A = 7.07$ cm^2

c $C = 56.55$ cm, $A = 254.47$ cm^2

8 Yes

9 **a** 1.25 m **b** 3.93 m

10 $Area(red) = \frac{2.5 + 6}{2} \times 5.5$

$Area(red) = 23.375 \text{ cm}^2$

$Area(blue) = \frac{5 + 2}{2} \times 7$

$Area(blue) = 24.5 \text{ cm}^2$

Therefore, the blue trapezium is bigger.

11 6378 km

12 **a** ST = 90 m **b** Ellen

13 **a** 50.27 mm^2

b 52 mm^2

Alex's ring is bigger.

14 C

Questions from multiple lessons

15 ***Geometry and measurement***

Decoding the question

Keyword: length

Relevant lesson: 7C

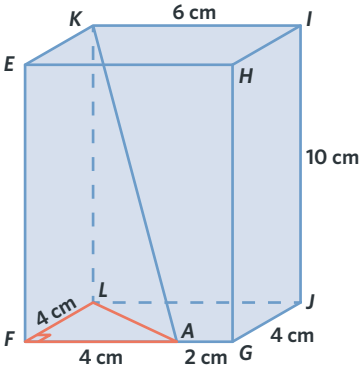
This question requires us to apply Pythagoras' theorem in three dimensions to find the unknown length.

Information we are given

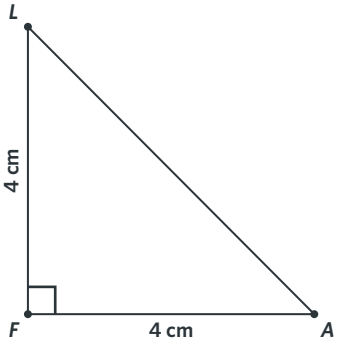
- The given diagram

Using the theory

We can first find the length of *AL*. In order to do this, we must look at the triangle *LAF*



Draw the triangle and label the known lengths of the sides, as shown below.



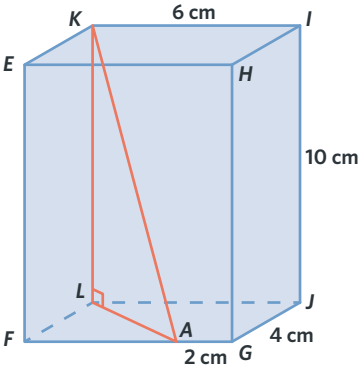
The length of *AL* can be calculated using Pythagoras' theorem.

$AL = \sqrt{AF^2 + FL^2}$

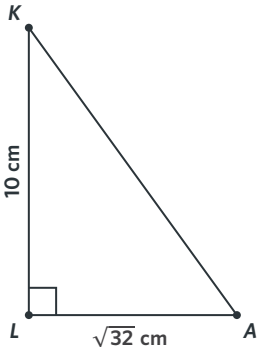
$AL = \sqrt{4^2 + 4^2}$

$AL = \sqrt{32}$

Now that length *AL* has been found, the triangle *AKL* can be used to find the length of the straw. The specified triangle is outlined below.



Draw the triangle and label the lengths of the sides, as shown below.



The length of *AK* can be found using Pythagoras' theorem.

$AK = \sqrt{10^2 + (\sqrt{32})^2}$

$AK = \sqrt{132}$

$AK = 11.489\ldots$

∴ The length of the straw is closest to 11.49 cm.

Answer

C

✓*1 mark for the correct answer*

16 ***Data analysis***

Decoding the question

Keywords: total, mean

Relevant lesson: 12E

This question requires us to determine the mean of the given data set.

Information we are given

- There are twelve family members.
- The total age of the family is 312 years.

Using the theory

The mean can be found using the formula

$$mean = \frac{sum\ of\ all\ elements\ in\ the\ data\ set}{total\ number\ of\ elements\ in\ the\ data\ set}$$

In this case, the sum of the elements is 228 and the number of elements is 12.

$mean = \frac{228}{12}$

$mean = 26$

Answer

A

✓1 mark for the correct answer

17a *Geometry and measurement*

Decoding the question

Keywords: trapezium, area, base

Relevant lesson: 7D

This question requires us to determine the area of a trapezium.

Information we are given

- The given diagram
- $GH = 7$ m
- $IJ = 10$ m
- $GJ = 4$ m

Using the theory

The area of a trapezium is given by $A = \frac{a + b}{2} \times h$ where a and b are parallel sides.

In this case, $a = GH = 7$, $b = IJ = 10$ and $h = GJ = 4$.

NOTE: a and b are interchangeable.

$A = \frac{7 + 10}{2} \times 4$

$A = 8.5 \times 4$

$A = 34$

∴ The area of the plant nursery is 34 m^2.

Answer

34 m^2

✓1 mark for the correct answer

17b *Geometry and measurement*

Decoding the question

Keyword: perimeter

Relevant lessons: 7B, 7D

This question requires us to determine the perimeter of a trapezium.

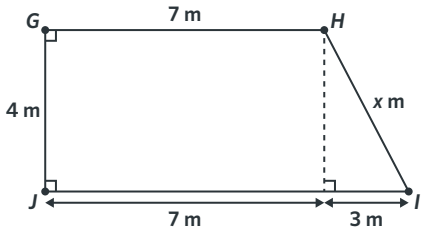
Information we are given

- The given diagram
- $GH = 7$ m
- $IJ = 10$ m
- $GJ = 4$ m

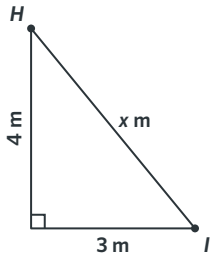
Using the theory

In order to find the perimeter, we must know the lengths of all the edges.

The diagram below shows all the known lengths, along with the unknown length labelled x.



First, calculate the unknown length using Pythagoras' theorem. The triangle that will help us identify its value is displayed with the appropriate measurements.



Using Pythagoras' theorem:

$x = \sqrt{4^2 + 3^2}$

$x = \sqrt{25}$

$x = 5$

Hence, the length of the unknown side is 5 m.

The perimeter is the sum of all edges.

$perimeter = 10 + 7 + 5 + 4$

$perimeter = 26$

∴ The perimeter of the plant nursery is 26.0 m.

Answer

26.0 m

✓1 mark for the correct answer

7E

1 **a** C **b** A **c** E

2 18

3 **a** 37 cm **b** 27 cm **c** 38 cm

4 P = 520 m

Dion walks 1040 m each day.

5 127.7 cm

6 22.5

7 **a** 14 cm^2 **b** 30 cm^2 **c** 299 cm^2

8 9 cm^2

9 3 m

10 24.54

11 **a** 60.00 cm^2 **b** 37.95 cm^2 **c** 20.40 cm^2

12 339.48 cm^2

13 **a** 102.10 cm^2 **b** 816.80 cm^2

14 Park B is the largest, at 2902 m^2.

15 3 m^2

16 **a** 2666.34 cm^2 **b** 12 727.46 cm^2

17 D

Questions from multiple lessons

18 ***Geometry and measurement***

Decoding the question

Keywords: perimeter, semi-circle, rectangular base

Relevant lessons: 7D, 7F

This question requires us to calculate the perimeter of the given composite shape.

Information we are given

- The given diagram

Using the theory

The perimeter of the shape will be given by the circumference of the semi-circle and the perimeter of the rectangle excluding one side length.

Calculate the perimeter of the semi-circle.

The circumference of a circle is given by $2\pi r$. Hence, the perimeter of a semicircle is πr

The radius of the semi-circle is half of the length of the rectangle that is adjoining it.

$r = \frac{42}{2} = 21$

$perimeter = \pi \times 21$

$perimeter = 65.97...$

Calculate the perimeter of the rectangle. Ensure to take into account only the sides that are exposed.

$perimeter = 42 + 12 + 12$

$perimeter = 66$

Calculate the total perimeter.

$total\ perimeter = 65.97... + 66$

$total\ perimeter = 131.97...$

∴ The minimum length of fence required for the plot of land is closest to 132 m.

Answer

A

✓ *1 mark for the correct answer*

19 ***Data analysis***

Decoding the question

Keywords: scatterplot, least squares regression line, equation

Relevant lesson: 9D

The scatterplot must be analysed in order to determine the equation for the regression line.

Information we are given

- The given scatterplot and a regression line

Using the theory

A least squares regression line is given in the form $y = a + bx$, where x is the explanatory variable and y is the response variable. Looking at the scatterplot, we can see that *time employed* is on the x-axis, meaning it is the explanatory variable, and *salary* is on the y-axis, meaning it is the response variable. This means that the equation must be in the form $salary = a + b \times time\ employed$. As options A and B are of the form $time\ employed = a + b \times salary$, they can be eliminated.

Next, find the value of a, which is the y-intercept. Looking at the graph, the y-intercept appears to be at 72. However, this graph begins at $x = 5$, not $x = 0$. This means that the y-intercept will actually be less than 72. Option D can be eliminated.

Lastly, use a point on the graph to find the gradient, b It appears to go through the point (11, 110). Substitute $salary = 110$ and $time\ employed = 12$ into the equation $salary = 40 + b \times time\ employed$ and solve for b

$110 = 40 + b \times 11$

$11b = 110 - 40$

$b = \frac{110 - 40}{11}$

$b = 6.3636...$

$b \approx 6.4$

∴ The equation of the least squares regression line is closest to $salary = 40 + 6.4 \times time\ employed$

Answer

E

✓ *1 mark for the correct answer*

20a ***Geometry and measurement***

Decoding the question

Keywords: surface area, square centimetres, rectangular prism

Relevant lessons: 7D, 7H

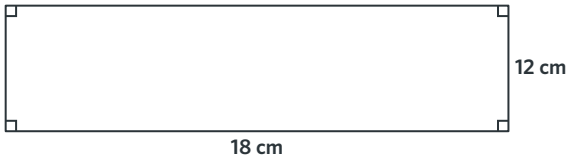
This question requires us to determine the surface area for one side of a 3D shape.

Information we are given

- The given diagram

Using the theory

The lid is the shaded rectangle with dimensions 18 cm by 12 cm, as shown below.



$A = l \times w$

$A = 18 \times 12$

$A = 216$

∴ The surface area of the lid is 216 cm^2.

Answer

216 cm^2

✓1 mark for the correct answer

20b ***Geometry and measurement***

Decoding the question

Keywords: total surface area, rectangular prism

Relevant lessons: 7D, 7H

This question requires us to calculate the total surface area of the rectangular prism.

Information we are given

- The given diagram

Using the theory

The surface area of a rectangular prism can be found by adding up the surface area of all the faces. In this rectangular prism, there are two 12 × 18 cm faces, two 4 × 12 cm faces and two 4 × 18 cm faces.

$TSA = 2 \times (12 \times 18) + 2 \times (4 \times 12) + 2 \times (4 \times 18)$

$TSA = 432 + 96 + 144$

$TSA = 672$

∴ The total surface area is 672 cm^2.

Answer

672 cm^2

✓1 mark for the correct answer

7F

1 **a** 28 cm^2 **b** 22 cm

2 5

3 **a** 36 cm **b** 40 cm **c** 81.27 cm

4 30 cm

5 22.94 m

6 **a** 73.5 cm^2 **b** 61.5 m^2 **c** 64 m^2

7 125.66 cm^2

8 518.7 cm^2

9 7 m

10 2.28 cm^2

11 32 cm^2

12 Shape A: 42.28 m^2

Shape B: 34.16 m^2

Shape A is larger.

13 A

Questions from multiple lessons

14 ***Geometry and measurement***

Decoding the question

Keywords: hexagonal, equilateral, area

Relevant lesson: 7E

We can use Heron's formula to find the area of each triangle within the hexagon.

Information we are given

- The given diagram
- The side length of the hexagon is 3 m.
- The hexagon can be divided into six equilateral triangles.

Using the theory

The area of each triangle can be found using Heron's formula.

Heron's formula states that $A = \sqrt{S(S-a)(S-b)(S-c)}$, where a, b, and c are the side lengths and $S = \frac{a+b+c}{2}$

First, find S

Since the triangles are equilateral and the hexagon has a side length of 3 m, all sides of the triangles must also be 3 m in length.

$S = \frac{3+3+3}{2} = 4.5$

Substitute $a = 3$, $b = 3$, $c = 3$, and $S = 4.5$ into $A = \sqrt{S(S-a)(S-b)(S-c)}$

$A = \sqrt{4.5(4.5-3)(4.5-3)(4.5-3)}$

$A = 3.897\ldots$

There are three green triangles, so multiply this area by 3.

$3.897\ldots \times 3 = 11.691\ldots$

∴ The total green area is 11.7 m^2, correct to one decimal place.

Answer

B

✓1 mark for the correct answer

15 ***Recursion and financial modelling***

Decoding the question

Keywords: sequence, recurrence relation

Relevant lesson: 6D

This question requires us to find V_7 using the given recurrence relation.

Information we are given

- $V_0 = 23, \quad V_{n+1} = 4 \times V_n$

Using the theory

Use the recurrence relation $V_0 = 23, V_{n+1} = 4 \times V_n$ to find terms V_1 to V_7

$V_0 = 23$

$V_1 = 4 \times 23 = 92$

$V_2 = 4 \times 92 = 368$

$V_3 = 4 \times 368 = 1472$

$V_4 = 4 \times 1472 = 5888$

$V_5 = 4 \times 5888 = 23\ 552$

$V_6 = 4 \times 23\ 552 = 94\ 208$

$V_7 = 4 \times 94\ 208 = 376\ 832$

Answer

D

✓*1 mark for the correct answer*

16a ***Geometry and measurement***

Decoding the question

Keywords: show that, length

Relevant lesson: 7B

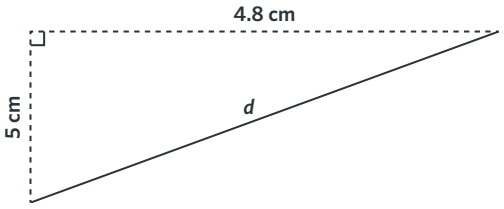
This question requires us to use Pythagoras' theorem to show that the length d is 6.9 cm.

Information we are given

- The given diagram

Using the theory

The length d can be obtained by using the right-angled triangle displayed below.



Using Pythagoras' theorem,

$d = \sqrt{5^2 + 4.8^2}$

$d = \sqrt{48.04}$

$d = 6.9310...$

$\therefore$ The length of side d is 6.9 cm correct to one decimal place.

Answer

$d = \sqrt{5^2 + 4.8^2} \approx 6.9$

✓*1 mark for the correct method*

16b ***Geometry and measurement***

Decoding the question

Keyword: perimeter

Relevant lessons: 7D, 7E

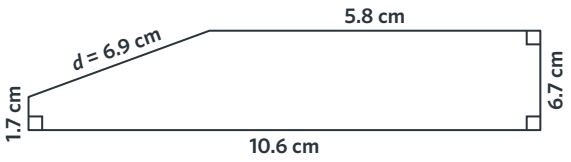
This question requires us to calculate the perimeter of the shape.

Information we are given

- The given diagram

Using the theory

The perimeter of the cardboard can be calculated using the unshaded diagram, as illustrated below.



The perimeter is the sum of all the lengths of the sides of the shape.

$perimeter = 6.9 + 5.8 + 6.7 + 10.6 + 1.7$

$perimeter = 31.7$

$\therefore$ The perimeter of the cardboard piece is 31.7 cm.

Answer

31.7 cm

✓*1 mark for the correct answer*

16c ***Geometry and measurement***

Decoding the question

Keyword: area

Relevant lessons: 7D, 7E

This question requires us to calculate the area of the piece of cardboard.

Information we are given

- The given diagram

Using the theory

The cardboard piece is composed of a rectangle with a triangle cut out. The total area of the cardboard piece (A_{total}) can be calculated by subtracting the area of the triangle (A_T) from the area of the rectangle (A_R)

$A_R = l \times w$
$A_R = 10.6 \times 6.7$
$A_R = 71.02$

$A_T = \frac{1}{2} \times b \times h$
$A_T = \frac{1}{2} \times 5 \times 4.8$
$A_T = 12$

$A_{total} = A_R - A_T$
$A_{total} = 71.02 - 12$
$A_{total} = 59.02$

∴ The total area of the piece of cardboard is 59 cm², correct to the nearest whole number.

Answer

59 cm²

✓1 mark for the correct answer

7G

1 42 cm²

2 2 m³

3 **a** 384 cm³ **b** 675 mm³ **c** 721 cm³

4 **a** 729 mm³ **b** 6371.15 cm³

5 3534 L

6 1.5 m

7 **a** 730.03 cm³ **b** 199 mL

8 **a** 90 m³ **b** 238.33 cm³

9 **a** 100.53 m³ **b** 1675.52 m³

10 561.2 mm³

11 136 m

12 263.9 cm³

13 **a** 4188.79 cm³ **b** 69.46 cm³

14 **a** 56.99 cm³ **b** 6.97 cm

15 Earth

16 261.80 cm³

17 **a** 6370.63 mL **b** 867.6 cm³

18 **a** 1884.96 cm³ **b** 1544.62 mm³

19 3.845 m³

20 416.78 cm³

21 12.03 cm

22 **a** 23.60 cm **b** 462.37 cm³

23 4020 cm³

24 **a** 170 mL **b** 262 cm³

25 **a** 13.6 cm

b New volume: 298 mL

Percent: 77.5%

26 1.47 m

Questions from multiple lessons

27 *Geometry and measurement*

Decoding the question

Keywords: rectangular-based right pyramid, height

Relevant lesson: 7C

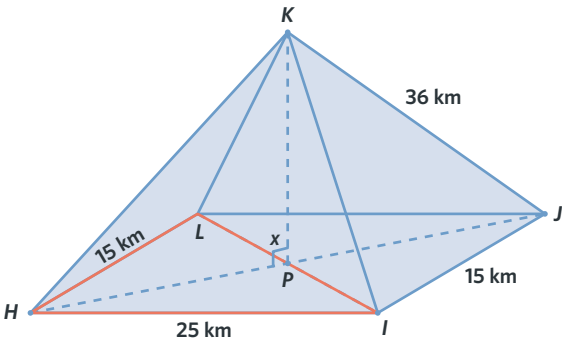
The question requires us to use Pythagoras' theorem to calculate an unknown length.

Information we are given

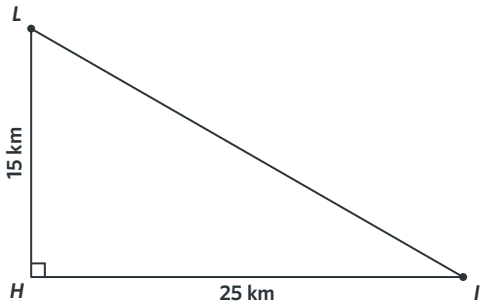
- The given diagram
- $JK = IK = HK = LK = 36$ km
- $HI = JL = 25$ km

Using the theory

First, the length of *IL* (or *HJ*) must be determined. This is done in order to be able to find the value *LP* (or *HP*, *IP* or *JP*), which will allow us to determine *KP*. *IL* can be found using the triangle *HIL* as shown below.



Draw the triangle again with the appropriate measurements as shown below.



The length for *IL* can be calculated using Pythagoras' theorem

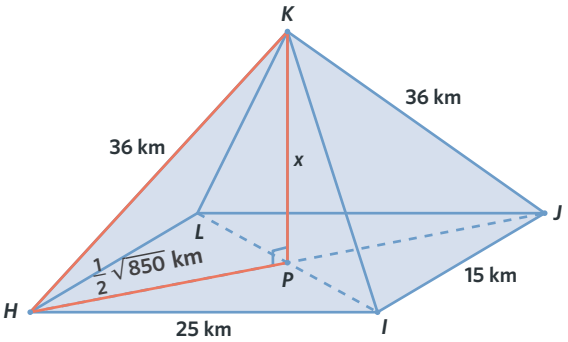
$IL = \sqrt{25^2 + 15^2}$

$IL = \sqrt{850}$

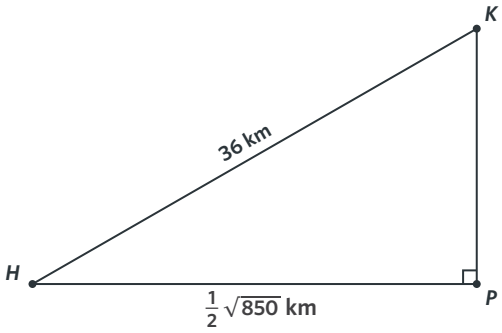
IP is half the length of *IL*

$IP = HP = LP = JP = \frac{\sqrt{850}}{2}$

To find the length for *KP*, we need to look at the triangle *HKP* which is outlined in the following diagram.



Draw the triangle again with the appropriate measurements as shown below.



The length for KP can be calculated using Pythagoras' theorem

$HK^2 = HP^2 + KP^2$

$KP^2 = HK^2 - HP^2$

$KP = \sqrt{HK^2 - HP^2}$

$KP = \sqrt{36^2 - \left(\frac{\sqrt{850}}{2}\right)^2}$

$KP = 32.916...$

The height of the pyramid is closest to 32.9 km.

Answer

E

✓1 mark for the correct answer

28 ***Data analysis***

Decoding the question

Keywords: mean, standard deviation, sample

Relevant lesson: 12F

We must use the calculator to calculate the mean and standard deviation of a sample.

Information we are given

- The given table

Using the theory

Enter the data into a spreadsheet on your calculator and use the one-variable statistics to find the required values.

The calculator will return values of $\bar{x} = 78.9$ and $s_x = 16.2$.

NOTE: As we are calculating the standard deviation of a sample, we use the value of s_x, rather than σ_x. σ_x is the standard deviation of a population.

See the video solution for a button-by-button demonstration.

Answer

A

✓1 mark for the correct answer

29a ***Geometry and measurement***

Decoding the question

Keywords: rectangle, semi-circle, perimeter, diameter

Relevant lessons: 7D, 7F

We must use the given perimeter to show that the diameter of the semi-circle is 19.74 cm.

Information we are given

- The given diagram
- The width of the rectangle is 60 cm.
- The perimeter of the skateboard is 182 cm.

Using the theory

The perimeter of the skateboard will be the sum of the two horizontal lengths, WX and YZ, plus the perimeter of the semi-circles XY and WZ. We know that the perimeter is 182 cm.

The two horizontal lengths are 60 cm each.

Since the two semi-circles are equal in dimensions, the perimeter of the two semi-circles will be equal to the circumference of one whole circle. The circumference of a circle is given by the formula $C = \pi d$, where d is the diameter.

We can now set up an equation for the total perimeter of the skateboard.

$182 = 2 \times 60 + \pi d$

Solve for d to find the diameter.

$\pi d + 120 = 182$

$\pi d = 62$

$d = \frac{62}{\pi}$

$d = 19.7352...$

$\therefore$ The diameter is 19.74 cm, correct to two decimal places.

Answer

$\pi d + 120 = 182$

✓1 mark for the correct answer

29b ***Geometry and measurement***

Decoding the question

Keywords: rectangle, semi-circle, area

Relevant lessons: 7D, 7F

We can calculate the area of the rectangle and semi-circles separately to find the total area of the skateboard.

Information we are given

- The given diagram
- The width of the rectangle is 60 cm.
- The diameter of the semi-circles is 19.74 cm, correct to two decimal places.

Using the theory

First, calculate the area of the rectangle.
The area of a rectangle is given by $A = l \times w$

Substitute $l = 19.74$ and $w = 60$ into the formula.

$A = 19.74 \times 60$

$A = 1184.4$

Now, calculate the area of the semi-circles.

As there are two semi-circles of equal dimensions, we can just calculate the area of one full circle. The area of a circle is given by $A = \pi r^2$

The radius of a circle is half the length of its diameter. Hence, the radius must be $\frac{1}{2} \times 19.74 = 9.87$ cm.

Substitute $r = 9.87$ into the formula.

$A = \pi \times 9.87^2$

$A = 306.0442...$

Calculate the sum of these areas.

$1184.4 + 306.0442... = 1490.4442...$

∴ The area is 1490 cm^2, correct to the nearest square centimetre.

Answer

1490 cm^2

✓1 mark for the correct answer

7H

1 15 cm^2

2 50.3 mm^2

3 9

4 **a** 108 cm^2 **b** 8.25 m^2 **c** 67.25 cm^2

5 8 cm

6 82 986.3 m^2

7 50 cm^2

8 **a** 75.40 m^2 **b** 452.40 cm^2 **c** 1743.58 mm^2

9 10 857 cm^2

10 **a** 416 cm^2 **b** 612.61 cm^2 **c** 114.85 m^2

11 **a** 5.83 m **b** 503 m^2

12 158 cm^2

13 B

14 0.32 m

15 Vase A: 867.08 cm^2 Vase B: 888 cm^2

Vase B has a larger external surface area.

16 **a** 350 cm **b** 356 cm

17 D

Questions from multiple lessons

18 ***Geometry and measurement***

Decoding the question

Keywords: rectangular prism, square base, diagonal, height, volume

Relevant lessons: 7C, 7G

This question requires us to use Pythagoras' theorem to determine the volume of a given rectangular prism with missing side lengths.

Information we are given

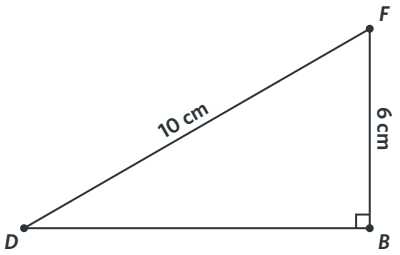
- The given diagram
- The rectangular prism has a square base.
- The diagonal of the prism, *DF*, is 10 cm.
- The height of the prism, *BF*, is 6 cm.

Using the theory

In order to calculate the volume of a square-based rectangular prism, we need the length and height.

First, calculate the length of the diagonal *BD*. This can be done using the triangle *BDF* and Pythagoras' theorem.

The triangle *BDF* is shown below with side lengths labelled.



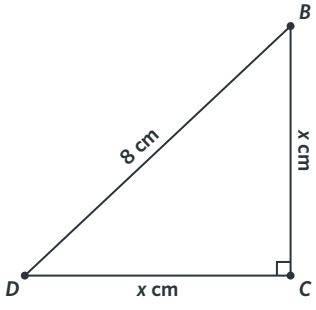
$BD = \sqrt{DF^2 - BF^2}$

$BD = \sqrt{10^2 - 6^2}$

$BD = \sqrt{64}$

$BD = 8$

We can now find the length of the prism using triangle *BCD*. The triangle along with its known lengths is shown below.



Since the base is a square, sides BC and CD must be equal. Hence, we can use Pythagoras' theorem to find the value of x.

$BD^2 = BC^2 + CD^2$

$8^2 = x^2 + x^2$

$2x^2 = 64$

$x^2 = 32$

$x = \sqrt{32}$

Hence, the length and width of the prism are both $\sqrt{32}$ cm.

Calculate the volume of the prism.

$V = l \times w \times h$

$V = \sqrt{32} \times \sqrt{32} \times 6$

$V = 192$

∴ The volume of the prism is 192 cm^3.

Answer

D

✓1 mark for the correct answer

19 ***Data analysis***

Decoding the question

Keyword: Pearson correlation coefficient

Relevant lesson: 13C

We must enter the data into the calculator and use its statistics functions to find the value of r

Information we are given

- The given table

Using the theory

Enter the data into the calculator spreadsheet and use the two-variable statistics analysis function to calculate the value of r

The value of r is 0.85 correct to two decimal places.

See video solution for a button-by-button demonstration.

Answer

A

✓1 mark for the correct answer

20a ***Geometry and measurement***

Decoding the question

Keywords: surface area, ball

Relevant lesson: 7H

This question requires us to determine the surface area of a spherical ball.

Information we are given

- The beach ball has a radius of 18.9 cm.

Using the theory

The formula for the surface area of a sphere is $SA = 4\pi r^2$

Substitute $r = 18.9$ into the formula.

$SA = 4\pi \times 18.9^2$

$SA = 4488.833...$

∴ The surface area of the beach ball, to the nearest whole number, is 4489 cm^2.

Answer

4489 cm^2

✓1 mark for the correct answer

20b ***Geometry and measurement***

Decoding the question

Keyword: minimum length

Relevant lesson: 7G

The question requires us to determine the minimum length of a box if it contains three beach balls.

Information we are given

- The given diagram
- Each beach ball has a radius of 18.9 cm.

Using the theory

Calculate the diameter of the sphere.

$diameter = 2 \times radius$

$diameter = 2 \times 18.9$

$diameter = 37.8$

The diameter is the maximum length of the sphere, so we can now use the diameter to find the maximum length of three beach balls.

$length = 3 \times 37.8$

$length = 113.4$

Hence, for a box to fit all three beach balls as the diagram displays, it must be at least 113.4 cm long.

Answer

113.4 cm

✓1 mark for the correct answer

7I

1 54

2 $a = 6$, $b = 0.75$

3 **a** No **b** Yes **c** Yes

4 1.6

5 SAS

6 **a** Yes **b** Yes **c** No

7 6

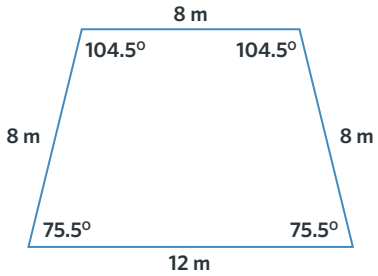
8 C

9 Smaller

10 **a** 0.75 **b** 0.83 **c** 1.25

11 1.8

12



13 **a** 900 m **b** 20 cm

14 0.006

15 19.2 m

16 D

Questions from multiple lessons

17 ***Geometry and measurement***

Decoding the question

Keywords: cone, slant edge, volume, surface area

Relevant lesson: 7G

We must first use the given measurements to calculate the value of x. We can then substitute x into the given formula to find the total surface area.

Information we are given

- The radius is 4 cm.
- The volume is 120 cm^3.
- The slant edge is x cm.
- $surface\ area = \pi r(r + x)$

Using the theory

The volume of a cone is given by formula $V = \frac{1}{3}\pi r^2 h$

We already know the values of V and r

The value of h can be expressed using Pythagoras' theorem.

$h^2 + 4^2 = x^2$

$\therefore h = \sqrt{x^2 - 4^2}$

Substitute $V = 120$, $r = 4$, and $h = \sqrt{x^2 - 4^2}$ into the formula $V = \frac{1}{3}\pi r^2 h$ and use the solve function on your calculator to solve for x.

$120 = \frac{1}{3} \times \pi \times 4^2 \times \sqrt{x^2 - 4^2}$

$x = 8.2032...$

Now that we know the value of x, we can calculate the surface area using the given formula.

Substitute $r = 4$ and $x = 8.2032...$ into
$surface\ area = \pi r(r + x)$

$surface\ area = \pi \times 4(4 + 8.2032...)$

$surface\ area = 153.3509...$

$\therefore$ The total surface area is closest to 153 cm^2.

Answer

E

✓1 mark for the correct answer

18 ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation, model

Relevant lessons: 3C, 3D, 6B

This question requires us to interpret the context of a recurrence relation.

Information we are given

- $W_0 = 12\ 000, \quad W_{n+1} = W_n + 180$

Using the theory

The provided sequence follows the form $t_0 = a\ t_{n+1} = t_n + d$, so it is an arithmetic sequence. Arithmetic sequences model simple interest as the balance of the investment increases by a set amount each period. Hence, options D and E can be eliminated.

Calculate 180 as a percentage of 12 000 to find the interest rate.

$$percentage = \frac{common\ difference}{initial\ value} \times 100$$

$$percentage = \frac{180}{12\ 000} \times 100$$

$percentage = 1.5\%$

$\therefore$ The recurrence relation could model a simple interest investment of \$12 000 with an interest rate of 1.5% per period.

Answer

B

✓1 mark for the correct answer

19a ***Geometry and measurement***

Decoding the question

Keywords: total height, show that

Relevant lesson: 7B

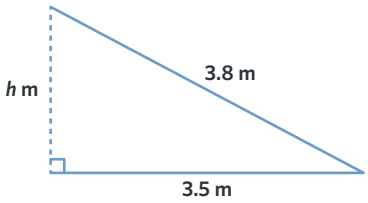
We must use Pythagoras' theorem to find the height of the roof in order to find the total height of the greenhouse.

Information we are given

- The given diagram
- The walls are 3.2 m high.
- The front is 7 m wide.
- The sloping edges are 3.8 m long.
- The total height is x m.

Using the theory

First, we will find the height of the roof.



Pythagoras' theorem states that in a right-angled triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse.

Substitute $a = h$, $b = 3.5$, and $c = 3.8$ and solve for h

$h^2 + 3.5^2 = 3.8^2$

$h = \sqrt{3\,8^2 - 3.5^2}$

$h = 1.4798...$

x is the total height of the greenhouse.

$x = h + 3.2$

$x = 1.4798... + 3.2$

$x \approx 4.68$

Answer

$x = \sqrt{3\,8^2 - 3.5^2} + 3.2 = 4.68$

✓1 mark for the correct method

19b ***Geometry and measurement***

Decoding the question

Keywords: surfaces, excluding the base

Relevant lessons: 7F, 7H

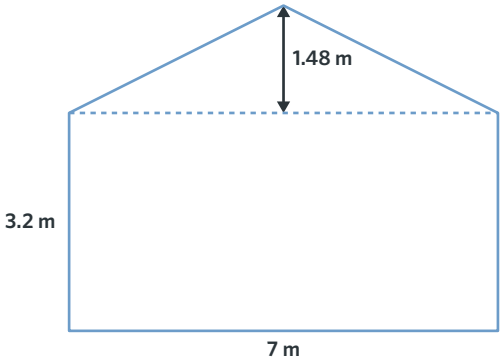
We must find the surface area of the greenhouse, excluding the base.

Information we are given

- The given diagram
- $x = 4.68$

Using the theory

We can first find the area of the front and back of the greenhouse. Each side is a composite shape made up of a rectangle and a triangle.



The area of the triangle is $\frac{1}{2} \times 1.48 \times 7$.

The area of the rectangle is 3.2×7.

Hence, the area of both the front and back is given by:

$2\left(\frac{1}{2} \times 1.48 \times 7 + 3.2 \times 7\right) = 55.16$

The two roof panels and two side walls are all rectangles. Their area can be found using the formula $A = b \times h$

Roof: $2 \times 3.8 \times 9 = 68.4$

Walls: $2 \times 3.2 \times 9 = 57.6$

$TSA = 55.16 + 68.4 + 57.6 = 181.16$

∴ The total area of glass needed is 181 m^2.

Answer

181 m^2

✓1 mark for the finding the area of the front of the greenhouse
✓1 mark for the correct answer

7J

1 $k = \frac{5}{6}$ or $k = 0.833$

2 **a** 2 **b** 4 **c** 84 cm^2

3 **a** 28 cm^2 **b** 2450.5 mm^2 **c** 158.06 cm^2

4 **a** All angles are equal as they are both rectangles.

Short side: $\frac{42}{21} = 2$

Long side: $\frac{59.4}{29.7} = 2$

∴ The rectangles are similar.

b 2494.8 cm^2

5 **a** $16A$ **b** 1350 cm^2

6 39 187.5 cm^2

7 320 cm

8 **a** 0.5 **b** 0.125 **c** 4.5 cm^3

9 **a** 41.16 cm^3 **b** 1673.55 cm^3 **c** 338.29 mm^3

10 $k = \frac{\textit{length of image}}{\textit{length of original}} = \frac{2L}{L} = 2$

$k^3 = 2^3 = 8$

$\textit{volume of image} = k^3 \times \textit{volume of original} = 8 \times V$

$\therefore \textit{volume of image} = 8V$

11 **a** 0.295 **b** 6.5 cm

12 9804.7 cm^3

13 **a** $64v$ **b** $2w$ **c** 512 000 cm^3

14 98.4%

15 12.5 mL

16 57 056 m^2

17 D

18 A

Questions from multiple lessons

19 *Geometry and measurement*

Decoding the question

Keywords: length, parallel

Relevant lesson: 7I

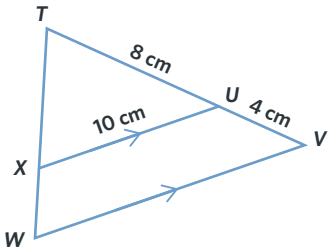
The question requires us to use our knowledge of similar figures to calculate the length of an unknown side.

Information we are given

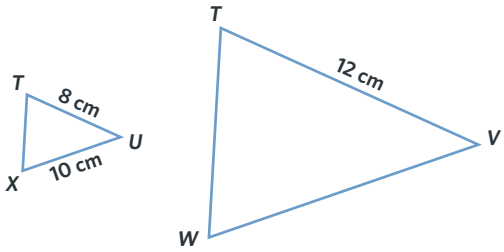
- The given diagram
- The lines *UX* and *VW* are parallel.
- The length of *TU* is 8 cm.
- The length of *UV* is 4 cm.
- The length of *UX* is 10 cm.

Using the theory

First, label the diagram with the appropriate lengths. This is shown below.



This shape is comprised of two triangles, *TUX* and *TVW* Since sides *UX* and *VW* are parallel, these two triangles are similar. The two triangles are illustrated separately below.



By comparing the sides *TU* and *TV*, the linear scale factor, *k*, can be determined.

$$k = \frac{length\ of\ image}{length\ of\ original}$$

$$k = \frac{TV}{TU}$$

$$k = \frac{12}{8}$$

$$k = 1.5$$

Find the length of *VW* by applying the linear scale factor.

$$VW = k \times UX$$

$$VW = 1.5 \times 10$$

$$VW = 15$$

∴ The length of *VW* is 15 cm.

Answer

C

✓ *1 mark for the correct answer*

20 *Data analysis*

Decoding the question

Keywords: dotplot, mean, standard deviation

Relevant lessons: 12C, 12E, 12F

We need to list the data points and use our calculators to find the mean and standard deviation.

Information we are given

- The given dot plot

Using the theory

List the value of the data points in a spreadsheet on your calculator and use the 'one-variable statistics' function.

The calculator should return, correct to one decimal place, $\overline{x} = 18.9$ and $s_x = 1.9$.

NOTE: Make sure to use the value of s_x instead of σ_x as we are finding the standard deviation of a sample, not a population.

See video solution for button-by-button demonstration.

Answer

E

✓ *1 mark for the correct answer*

21a *Geometry and measurement*

Decoding the question

Keywords: cylinder, hemisphere, radius, height, volume

Relevant lesson: 7G

This question requires us to calculate the volume of a composite object.

Information we are given

- The given diagram

Using the theory

The formula for the volume of a cylinder is $V = \pi r^2 h$

$$V_{cylinder} = \pi \times 15^2 \times 5$$

$$V_{cylinder} = 1125\pi$$

$$V_{cylinder} = 3534.29...$$

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$

Hence, the formula for the volume of a hemisphere is $V = \frac{2}{3}\pi r^3$

$$V_{hemisphere} = \frac{2}{3} \times \pi \times 15^3$$

$$V_{hemisphere} = 2250\pi$$

$$V_{hemisphere} = 7068.58...$$

Add the volume of the cylinder and hemisphere to calculate the total volume.

$V_{total} = V_{cylinder} + V_{hemisphere}$

$V_{total} = 3534.29... + 7068.58...$

$V_{total} = 10\ 602.87...$

∴ The volume of tent A is 10 603 m³.

Answer

10 603 m³

✓1 mark for the correct value for hemisphere volume
✓1 mark for the correct value for cylinder and final volume

21b ***Geometry and measurement***

Decoding the question

Keywords: percentage, volume

Relevant lesson: 7G

This question requires us to find the percentage of volume removed using the volume scale factor.

Information we are given

- Tent B is a cone with a height of 18 metres.
- The top three metres of the tent are blocked off.

Using the theory

The blocked off area and the entire tent are similar shapes.

Find the length scale factor. Let the entire tent be the original and the blocked off area be the image.

$k = \frac{length\ of\ image}{length\ of\ original}$

$k = \frac{3}{18}$

$k = \frac{1}{6}$

Find the volume scale factor.

$k^3 = \left(\frac{1}{6}\right)^3$

$k^3 = \frac{1}{216}$

This means that the volume of blocked off section is $\frac{1}{216}$ of the volume of the tent. Hence, the volume of the tent that is not blocked off is $1 - \frac{1}{216} = \frac{215}{216}$ of the volume of the entire tent.

Multiply $\frac{215}{216}$ by 100 to convert this to a percentage.

$\frac{215}{216} \times 100 = 99.537...\%$

∴ 99.54% of the volume of the tent is not blocked off.

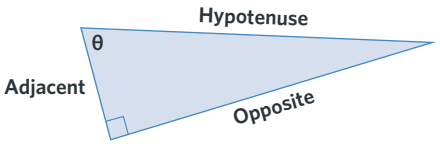
Answer

99.54%

✓1 mark for the correct answer

8A

1



2 3.7 cm

3 $\sin(\theta) = \frac{Opposite}{Hypotenuse}$

$\cos(\theta) = \frac{Adjacent}{Hypotenuse}$

$\tan(\theta) = \frac{Opposite}{Adjacent}$

4 **a** 50.35 mm **b** 2.55 cm **c** 1.61 m

5 92 m

6 25°

7 **a** 57.53° **b** 35.55° **c** 72.77°

8 44°

9 8 m

10 **a** 1.5 m **b** 37°

11 4.95

12 43°

13 **a** 1.96 m **b** 66.25 m

14 **a** 45° **b** 77.8 m

Questions from multiple lessons

15 ***Geometry and measurement***

Decoding the question

Keywords: trapezoid prism, cylinder, radius, height

Relevant lessons: 7D, 7G

We need to find the volume of the trapezoid prism and use this value to determine the height of a cylinder with the same volume, given its radius.

Information we are given

- The given diagram
- The trailer is in the shape of a trapezoid prism.
- The silo is in the shape of a cylinder.
- The silo has a radius of 2.5 m.

Using the theory

First, we must find the volume of the trailer which is in the shape of a trapezoid prism. The volume will be the area of the trapezium face multiplied by its length.

The area of the trapezium can be found by the formula $A = \frac{a + b}{2} \times h$, where a and b are the lengths of the parallel sides and h is the height.

Substitute $a = 3.8$, $b = 2.8$ and $h = 2.4$ into $A = \frac{a + b}{2} \times h$

$A = \frac{3.8 + 2.8}{2} \times 2.4$

$A = 7.92$

Multiply the area of the trapezium by the length of the trapezoid prism.

$V = A \times length$

$V = 7.92 \times 11.6$

$V = 91.872$

The formula for the volume of a cylinder is $V = \pi r^2 h$
We know the volume that the grain fills, along with the radius of the silo.

Substitute $V = 91.872$ and $r = 2.5$ into $V = \pi r^2 h$ and solve for h

$91.872 = \pi \times 2.5^2 \times h$

$h = \frac{91.872}{\pi \times 2.5^2}$

$h = 4.679...$

∴ The height at which one trailer-load of grain would sit within the silo is closest to 4.7 m.

Answer

E

✓1 mark for the correct answer

16 ***Data analysis***

Decoding the question

Keyword: median

Relevant lessons: 12C, 12E

We need to use the $\frac{n + 1}{2}$ rule to determine the median.

Information we are given

- The given dot plot
- The given boxplot
- There are 20 zookeepers in the group.

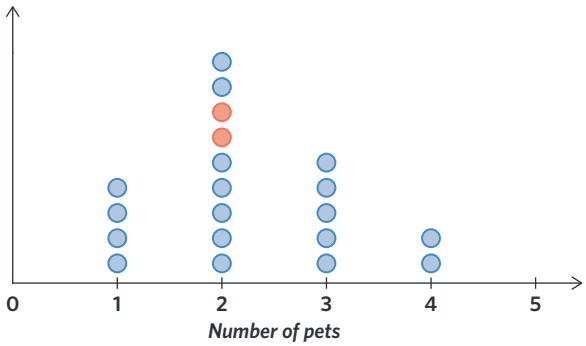
Using the theory

The median on a boxplot is represented by a line in the box. As there is no middle line in the given boxplot, the median will be equal to either Q_1 or Q_3. Since we don't know whether the median is equal to Q_1 or Q_3 on the boxplot, we will use the dot plot to determine the median.

The median can be found in the $\left(\frac{n + 1}{2}\right)^{th}$ position of the data points. Substitute $n = 20$ into the formula.

$\frac{20 + 1}{2} = 10.5$

The median will be located in between the 10th and the 11th positions, which can be calculated by finding the average of these values. They are highlighted in the following dot plot.



The value of both the 10th and the 11th data points is 2.

∴ The median number of pets owned is 2.

Answer

B

✓1 mark for the correct answer

17a ***Geometry and measurement***

Decoding the question

Keywords: diameter, in centimetres

Relevant lesson: 7D

We can use the measurements provided to find the diameter of the glass.

Information we are given

- The glass is cylindrical in shape.
- The glass has a radius of 2.9 cm and a height of 9.5 cm.

Using the theory

The diameter (d) of a circle is the length from one side of the circle to the other, through the centre. It is twice the length of the radius (r). Hence, $d = 2r$

Substitute $r = 2.9$ into $d = 2r$

$d = 2 \times 2.9$

$d = 5.8$

∴ The diameter of the glass is 5.8 cm.

Answer

5.8 cm

✓1 mark for the correct answer

17b ***Geometry and measurement***

Decoding the question

Keywords: surface area, 'including the bottom'

Relevant lesson: 7H

We need to recall the formula for calculating the surface area of a cylinder, and remember to subtract the area of one of the circles due to the open top of the glass.

Information we are given

- The glass is cylindrical in shape.
- The glass has a radius of 2.9 cm and a height of 9.5 cm.

Using the theory

As the glass is cylindrical, we will need the formula for the surface area of a cylinder.

This is given as
$SA = area\ of\ two\ circles + area\ of\ curved\ side$
also written as $SA = 2\pi r^2 + 2\pi rh$

However, as the top of the glass is exposed, we will only require the bottom circle of the cylinder. Hence, we must subtract the area of one circle from the formula.

$SA = 2\pi r^2 + 2\pi rh - \pi r^2$

$SA = \pi r^2 + 2\pi rh$

We have been given $r = 2.9$ and $h = 9.5$. Substitute these values into the formula to find the surface area.

$SA = \pi \times 2.9^2 + 2\pi \times 2.9 \times 9.5$

$SA = 199.522...$

∴ The surface area is 199.5 cm^2, correct to one decimal place.

Answer

199.5 cm^2

✓1 mark for the correct answer

8B

1 $x = 3.72$

2 35°

3 **a** 38.66° **b** 11.89°

4 20.56°

5 830.97 m

6 61.4 m

7 20°

8 **a** 48.01° **b** 8.62°

9 61.15 m

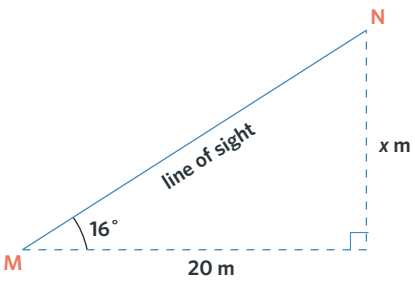
10 5.7°

11 3.69 m

12 602.6 m

13 **a** 2.70° **b** 169.81 m

14



$x = 5.7349...$

The height of the building is 20.9 m.

15 **a** 45.5 m **b** 24.9°

Questions from multiple lessons

16 *Geometry and measurement*

Decoding the question

Keyword: angle

Relevant lesson: 8A

This question requires us to apply trigonometric skills to determine the unknown angle.

Information we are given

- The given diagram

Using the theory

The given triangle displays lengths for the opposite and adjacent sides of the triangle, so the tangent function must be used to solve for the unknown angle.

$\tan(\theta) = \frac{opposite}{adjacent}$

$\tan(\theta) = \frac{5}{20} = \frac{1}{4}$

$\theta = \tan^{-1}\left(\frac{1}{4}\right)$

$\theta = 14.036...$

∴ The angle is closest to 14.0°.

Answer

C

✓1 mark for the correct answer

17 *Recursion and financial modelling*

Decoding the question

Keywords: loan, interest rate, compounding period, lump sum

Relevant lesson: 3D

We must determine which loan will cost Ash the least amount of money, taking into account the different interest rates and compounding periods.

Information we are given

- Ash borrows $6500.
- Ash will fully repay the loan in a lump sum payment after two years.
- The interest rates and compounding periods of each loan.

Using the theory

Since different compounding periods result in different amounts of interest being added, we can only directly compare the loans that have the same compounding periods.

Loans 1, 2 and 3 all compound monthly and have annual interest rates of 9.70%, 9.72% and 9.74% respectively. Loan 1 has the lowest annual interest rate of 9.70%, so out of the loans that compound monthly, loan 1 will cost Ash the least amount of money.

Loans 4 and 5 both compound quarterly and have annual interest rates of 9.76% and 9.78% respectively. Loan 4 has the lower annual interest rate of 9.76%, so out of the loans that compound quarterly, loan 4 will cost Ash the least amount of money.

Now we must compare loans 1 and 4 using the compound interest formula $A = P\left(1 + \frac{r}{100}\right)^n$

Loan 1:

Since interest compounds monthly, $r = \frac{9.70}{12}$ and $n = 2 \times 12 = 24$.

Substitute $P = 6500$, $r = \frac{9.70}{12}$ and $n = 24$ into the compound interest formula.

$$A = 6500\left(1 + \frac{\left(\frac{9.70}{12}\right)}{100}\right)^{24}$$

$A = 7885.47\ldots$

Loan 4:

Since interest compounds quarterly, $r = \frac{9.76}{4}$ and $n = 2 \times 4 = 8$.

Substitute $P = 6500$, $r = \frac{9.76}{4}$ and $n = 8$ into the compound interest formula.

$$A = 6500\left(1 + \frac{\left(\frac{9.76}{4}\right)}{100}\right)^{8}$$

$A = 7882.60\ldots$

$7882.60\ldots < 7885.47\ldots$

∴ Loan 4 will cost Ash the least amount of money.

Answer

D

✓1 mark for the correct answer

18a ***Geometry and measurement***

Decoding the question

Keyword: scale factor

Relevant lesson: 7I

This question requires us to determine the scale factor the engineer uses in his plans.

Information we are given

- The given diagram
- A 1 cm line is used to represent a length of 1 m.

Using the theory

As all units of measurement need to be the same to calculate scale factors, we need to convert 1 metre into centimetres. There are 100 centimetres in a metre.

The formula for the scale factor is given as $k = \frac{length\ of\ image}{length\ of\ original}$. Substitute $length\ of\ image = 1$ and $length\ of\ original = 100$ to find the scale factor.

$k = \frac{1}{100}$

$k = 0.01$

Answer

$k = 0.01$

✓1 mark for the correct answer

18b ***Geometry and measurement***

Decoding the question

Keyword: area

Relevant lesson: 7J

This question requires us to apply scale factors in the context of areas.

Information we are given

- The given diagram
- The lid's area is 21 m^2.

Using the theory

From part **a**, the scale factor was determined to be 0.01. To use this scale factor with areas, it must be converted to k^2

$k^2 = 0.01^2$

$k^2 = 0.0001$

Calculate the plan area.

$A_{image} = k^2 \times A_{original}$

$A_{image} = 0.0001 \times 21$

$A_{image} = 0.0021$ m^2

∴ The area of the lid on the plan is 0.0021 m^2.

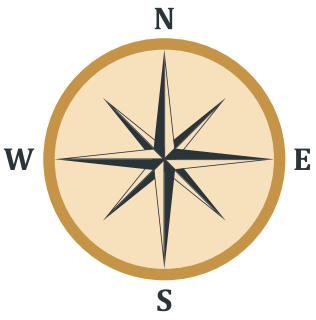
Answer

0.0021 m^2

✓1 mark for the correct answer

8C

1



2 150°

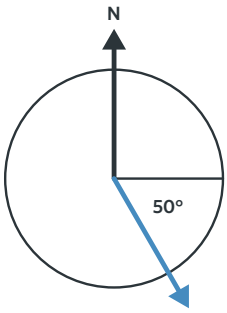
3 **a** 315° **b** 120° **c** 175°

4 D

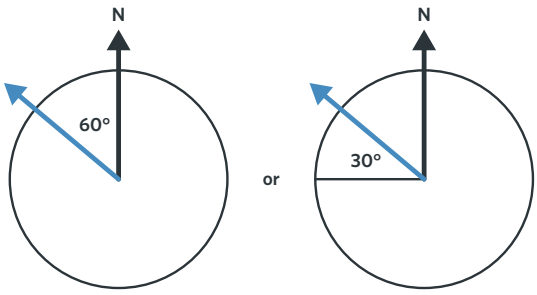
5 141°

6 B

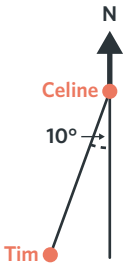
7 **a**



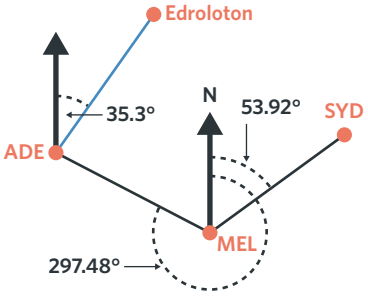
b



c



8



9

10 A

11 **a** 10.94 m **b** 16.56 cm **c** 4.73 m

12 204.78°

13 175 m

14 **a** 50.17° **b** 332.97°

c 117.58 km **d** 67.03°

15 **a** Bearing of B from A = 57.99°

Angle = 51 + 57.99

= 109°, after rounding to the nearest degree.

b 4.50 km

c 8.25 km away, at bearing 345.96°.

16 D

Questions from multiple lessons

17 *Geometry and trigonometry*

Decoding the question

Keywords: horizontal distance, angle of depression

Relevant lessons: 8A, 8B

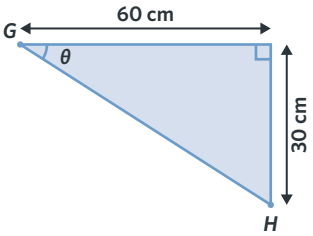
This question requires us to use trigonometric functions to calculate the angle of depression.

Information we are given

- The given diagram
- The table is 30 cm taller than the chair.
- The horizontal distance between the table and the chair is 60 cm.

Using the theory

Draw a right-angled triangle using the horizontal and vertical distances between the table and chair. Make sure that the triangle includes the angle of depression from point G to H



The known side lengths are the opposite and adjacent sides, so we must use the tangent formula to find the angle of depression.

$$\tan(\theta) = \frac{opposite}{adjacent}$$

$$\tan(\theta) = \frac{30}{60} = \frac{1}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 26.565...°$$

∴ The angle of depression of point H from point G is closest to 27°.

Answer

B

✓*1 mark for the correct answer*

18 ***Data analysis***

Decoding the question

Keyword: variables

Relevant lesson: 12A

This question requires us to classify the variables given.

Information we are given

- *Typing speed* (in words per minute)
- *Language* (English, French, German)

Using the theory

First, we must classify each variable as numerical or categorical. Numerical variables represent a quantity and can be counted or measured. Categorical variables usually represent a quality or attribute and can always be broken into groups or categories.

Typing speed is measured and has not been given a set of categories, so it must be numerical.

Language has been presented with a set of categories, so it must be categorical.

Categorical variables can be either ordinal or nominal. Ordinal variables are those with a hierarchy or order among the categories. Nominal variables cannot be ordered. The categories of *language* are nominal as they do not follow a logical order or natural progression.

Hence, *typing speed* is numerical and *language* is nominal.

Answer

D

✓1 mark for the correct answer

19a ***Geometry and measurement***

Decoding the question

Keywords: angle of depression, distance

Relevant lessons: 8A, 8B

This question requires us to apply trigonometric functions to determine the unknown length.

Information we are given

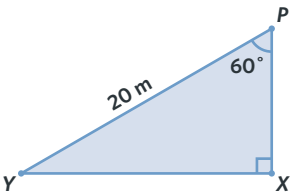
- The given diagram

Using the theory

First, we can find the angle *XPY*

As *XP* is a vertical line, *XPY* will be equal to 90° minus the angle of depression.

$90° - 30° = 60°$



The distance between the two arrows, *XY*, can now be calculated by using the sine function.

$\sin(\theta) = \frac{opposite}{hypotenuse}$

$\sin(60°) = \frac{XY}{20}$

$XY = 20 \times \sin(60°)$

$XY = 17.3205...$

∴ The distance between the two arrows is 17.32 m, correct to two decimal places.

Answer

17.32 m

✓1 mark for the correct answer

19b ***Geometry and measurement***

Decoding the question

Keyword: angle of elevation

Relevant lessons: 8A, 8B

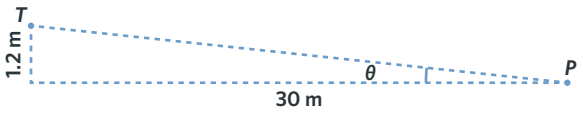
This question requires us to apply trigonometric functions to find an unknown angle.

Information we are given

- The given diagram

Using the theory

The angle of elevation from *P* to the top of the wall is marked θ



The two known lengths are the opposite and the adjacent sides, so we must use the tangent function to determine the angle.

$\tan(\theta) = \frac{opposite}{adjacent}$

$\tan(\theta) = \frac{1.2}{30}$

$\theta = \tan^{-1}\left(\frac{1.2}{30}\right)$

$\theta = 2.29...°$

∴ The angle of elevation is 2°, correct to the nearest degree.

Answer

2°

✓1 mark for the correct answer

8D

1 48.59°

2 **a** $\frac{x}{\sin(32°)} = \frac{7}{\sin(78°)}$ **b** $x = 3.79$ cm

3 **a** 7.08 cm **b** 32.46 m **c** 16.31 mm

4 D

5 $\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$

$\frac{x}{\sin(40°)} = \frac{90}{\sin(86°)}$

$x = \sin(40°) \times \frac{90}{\sin(86°)}$

$x = 57.9921...$

∴ The distance between the shed and the veggie patch is 58 metres.

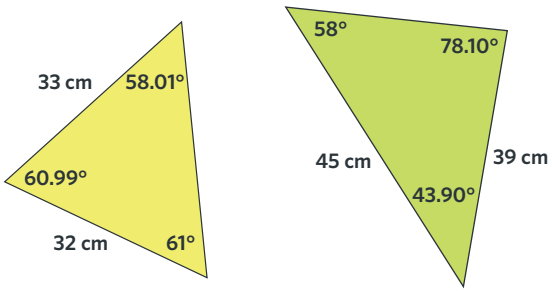
6 Distance from boat A: 147.49 m
Distance from boat B: 308.96 m

7 **a** $\theta = \sin^{-1}\left(6 \times \frac{\sin(99°)}{43}\right)$

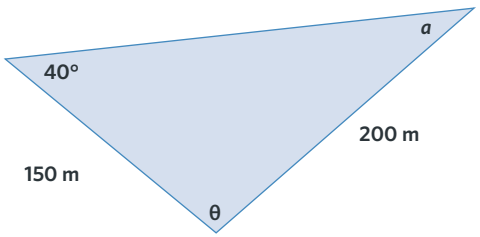
b $\theta = 7.9°$

8 **a** 75.19° **b** 83.22° **c** 62.77°

9



10 Let α be the third angle, as shown below:



$\frac{\sin(\alpha)}{150} = \frac{\sin(40°)}{200}$

$\alpha = \sin^{-1}\left(150 \times \frac{\sin(40°)}{200}\right)$

Simplify the fraction:

$\alpha = \sin^{-1}\left(\frac{3 \times \sin(40°)}{4}\right)$

Interior angles of a triangle add up to 180°, so:

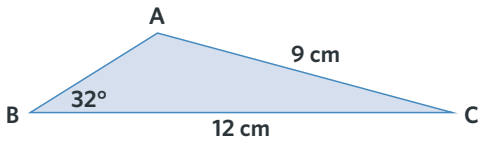
$\theta + \alpha + 40° = 180°$

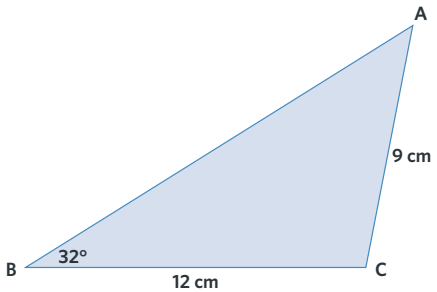
$\theta = 180° - 40° - \alpha$

$\therefore \theta = 140° - \sin^{-1}\left(\frac{3 \times \sin(40°)}{4}\right)$

11 119°

12 **a**





b 44.96° and 135.04°

c 12.96° and 103.04°

13 17.7 cm and 28.3 cm

14 D

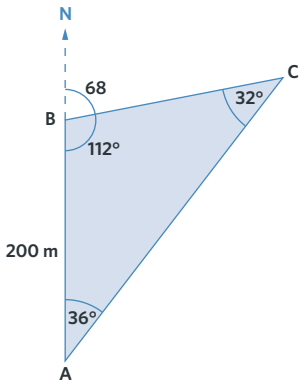
15 **a** Angle *MLN*

b 32.24°

c 45.76°

d 8.06 m

16 **a**



Let x be the distance between points B and C.

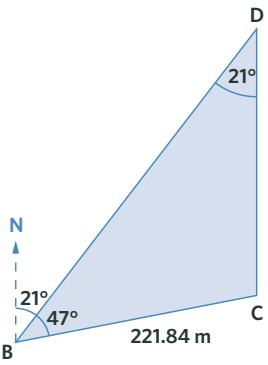
$\frac{x}{\sin(36°)} = \frac{200}{\sin(32°)}$

$x = \sin(36°) \times \frac{200}{\sin(32°)}$

$x = 221.8395...$

∴ The distance between points B and C is 221.84 metres.

b



453 m

17 E

Questions from multiple lessons

18 *Geometry and measurement*

Decoding the question

Keywords: bearing, distance

Relevant lessons: 8A, 8C

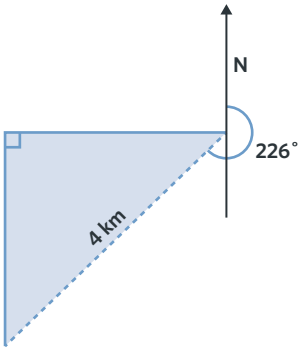
This question requires us to use our knowledge on bearings and trigonometric functions to determine an unknown length.

Information we are given

- The lost dog ran directly west from its kennel and then directly south.
- It was found on a bearing of 226° from its kennel.
- It was found exactly 4 km away.

Using the theory

First draw a diagram that represents the information given.



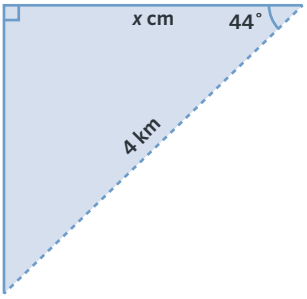
Find the value of an interior angle of the triangle.

West is at a bearing of 270° from north.

$interior\ angle = 270° - 226°$

$interior\ angle = 44°$

We now have a right-angled triangle with a known side length and known angle as shown. The distance the dog travelled directly west is labelled x.



We can use the cosine function to determine the unknown length.

$$\cos(\theta) = \frac{adjacent}{hypotenuse}$$

$$\cos(44°) = \frac{x}{4}$$

$$x = 4 \times \cos(44°)$$

$$x = 2.877...$$

∴ The distance that the dog travelled west is closest to 2.9 km.

Answer

A

✓1 mark for the correct answer

19 ***Data analysis***

Decoding the question

Keywords: stem plot, median

Relevant lessons: 12C, 12E

We must locate the median value from the stem plot using the formula $\frac{n+1}{2}$

Information we are given

- The given stem plot
- $n = 31$

Using the theory

The median is the middle value in a dataset. In a dataset with n elements, it is located in the $\left(\frac{n+1}{2}\right)^{th}$ position.

Substitute $n = 31$ into $\frac{n+1}{2}$ to find the position of the median.

$$\frac{31+1}{2} = 16$$

Hence, the median is the 16^{th} value in the stem plot. This value is circled.

Key: 0 | 1 = 1 $n = 31$

0	1	2				
0	5	6	8	8	9	
1	0	1	1	3	4	4
1	6	7	(8)	9	9	9
2	2	2	3	4	4	
2	5	5	6	7		
3	0	1				
3	6					

The median number of teabags consumed is 18.

Answer

B

✓1 mark for the correct answer

20a ***Geometry and measurement***

Decoding the question

Keyword: length

Relevant lesson: 7A

We need to determine the length of an unknown edge.

Information we are given

- The given diagram
- Point T is directly halfway between O and S

Using the theory

We know that T is exactly halfway between O and S

The length of OS is 62 km.

Hence, the length of ST must be half of 62 km.

$$\frac{1}{2} \times 62 = 31$$

∴ The length of ST is 31 km.

Answer

31 km

✓1 mark for the correct answer

20b ***Geometry and measurement***

Decoding the question

Keyword: angle

Relevant lesson: 8A

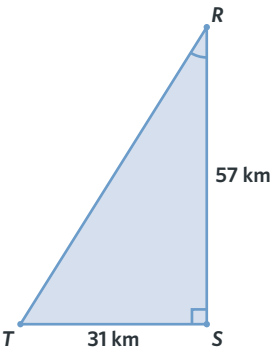
We need to use trigonometric functions to determine the unknown angle SRT

Information we are given

- The given diagram

Using the theory

From part **a**, the length of ST is 31 km.
Draw the triangle SRT



The known lengths are the adjacent and opposite sides, so use the tangent function to calculate the unknown angle.

$\tan(\angle SRT) = \frac{opposite}{adjacent}$

$\tan(\angle SRT) = \frac{31}{57}$

$\angle SRT = \tan^{-1}\left(\frac{31}{57}\right)$

$\angle SRT = 28.539...°$

∴ The angle SRT is 28.5°, correct to one decimal place.

Answer

28.5°

✓1 mark for the correct answer

8E

1 36°

2 5.37 cm

3 **a** 7.80 cm **b** 45.54 mm **c** 1.55 m

4 10.4 cm

5 100

6 **a** 57.12° **b** 43.23° **c** 18.57°

7 45.08°

8 2.16

9 **a** 13.7 m **b** 56°

10 944 m

11 **a** 3.67 km

b 108°

12 **a** 3.5 km

b 97°

13 E

Questions from multiple lessons

14 ***Geometry and measurement***

Decoding the question

Keywords: bearing, angle

Relevant lesson: 8C

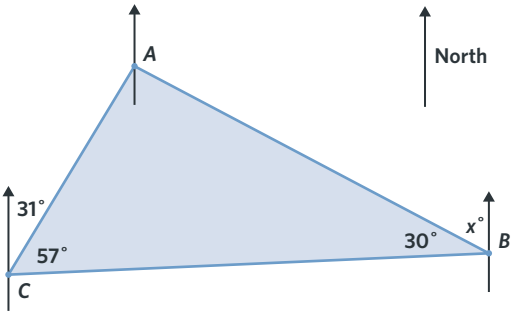
This question requires us to apply our knowledge of bearings and alternate angles to determine the bearing of A from B

Information we are given

- The given diagram

Using the theory

The angle that we need to determine is labelled x.



Using supplementary angles, the angle x can be determined as shown below.

$x + 30° + 31° + 57° = 180°$

$x = 180° - 30° - 31° - 57°$

$x = 62°$

The bearing of A from B is equal to 360° minus 62°.

$360° - 62° = 298°$

∴ The bearing of A from B is 298°.

Answer

A

✓1 mark for the correct answer

15 ***Recursion and financial modelling***

Decoding the question

Keywords: increase, value

Relevant lesson: 3D

This question requires us to determine the original value of the vase eight years ago.

Information we are given

- The percentage increase in value is 5% per year for the past eight years.
- This year, the vase's value is $165 000.

Using the theory

We can model this scenario using the compound interest formula $A = P \times \left(1 + \frac{r}{100}\right)^n$, where A is the value of the vase after n years, P is the initial value, and r is the growth rate.

Substitute the values $n = 8, A = 165\ 000$, and $r = 5$ into the equation.

$165\ 000 = \mathrm{P} \times \left(1 + \frac{5}{100}\right)^8$

$P = \frac{165\ 000}{1.05^8}$

$P = 111\ 678.494\ldots$

∴ The original value of the vase is closest to $111 678.

Answer

C

✓1 mark for the correct answer

16a *Geometry and measurement*

Decoding the question

Keywords: distance, straight line

Relevant lesson: 7B

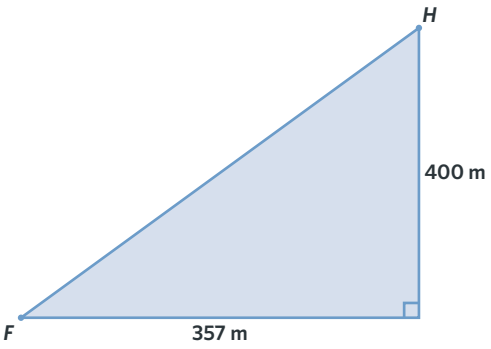
This question requires us to use Pythagoras' theorem to calculate the unknown length.

Information we are given

- Her house is located 400 m north and 357 m east of the field.

Using the theory

A diagram depicting the position of April's house relative to the field is shown below.



This triangle is right-angled as the known distances are in terms compass directions. Hence, Pythagoras' theorem can be used to determine the length of FH

$FH = \sqrt{400^2 + 357^2}$

$FH = \sqrt{287\ 449}$

$FH = 536.1427\ldots$

∴ August has to walk 536 m from her home to get to the football field.

Answer

536 m

✓1 mark for the correct answer

16b *Geometry and measurement*

Decoding the question

Keyword: three-figure bearing

Relevant lessons: 8A, 8C

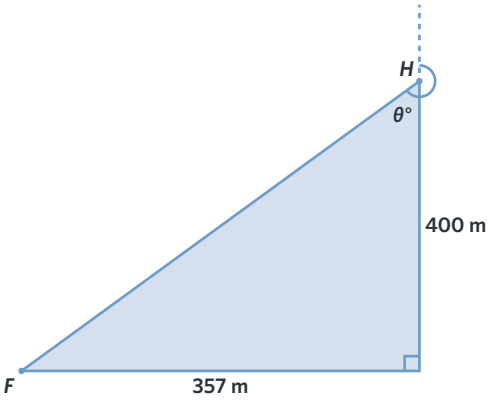
This question requires us to find the bearing of the football field from August's home using trigonometry.

Information we are given

- Her house is located 400 m north and 357 m east of the field.

Using the theory

A diagram showing her journey, along with the angle of the three-figure bearing, is shown in the diagram below. The three-figure bearing will be 180° plus θ°



To find the value of θ the tangent must be used, as the triangle is right-angled and the lengths displayed are the opposite and adjacent edges.

$\tan(\theta) = \frac{opposite}{adjacent}$

$\tan(\theta) = \frac{357}{400}$

$\theta = \tan^{-1}\left(\frac{357}{400}\right)$

$\theta = 41.74\ldots°$

$bearing = 180° + 41.74\ldots°$

$bearing = 221.74\ldots°$

∴ The three-figure bearing, correct to the nearest degree, of the football field from August's home is 222°.

Answer

222°

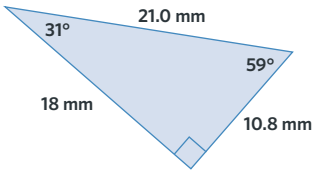
✓1 mark for the correct answer

8F

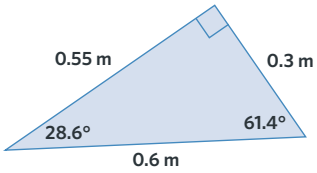
1 A, D

2 A side length or an angle

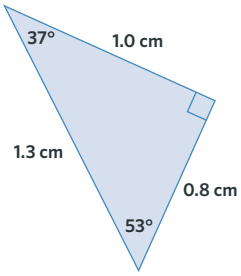
3 **a**



b



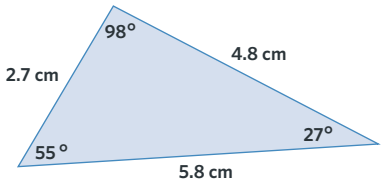
c



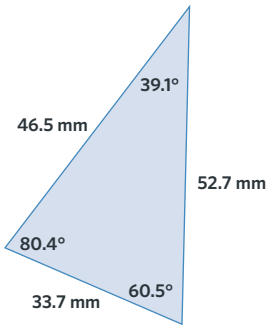
4 The size of the triangle cannot be determined. At least one side length is needed to determine a triangle. If only angles are known, the triangle can be scaled to any size.

5 Step 3

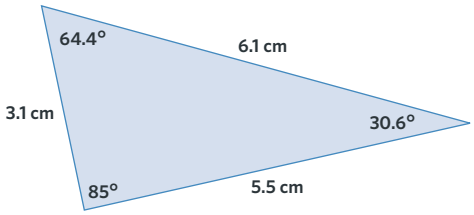
6 **a**



b

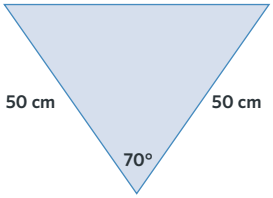


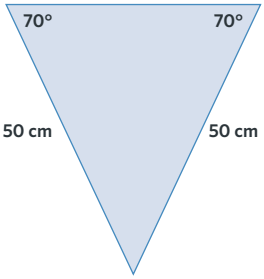
c

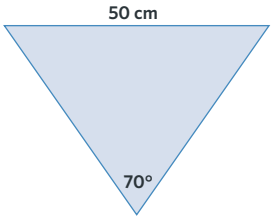


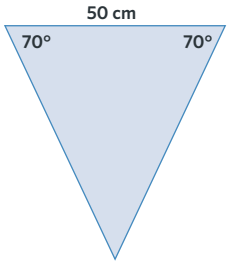
7 C

8





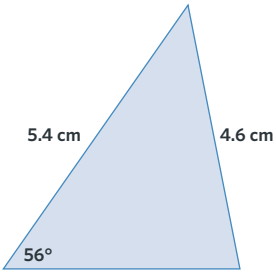


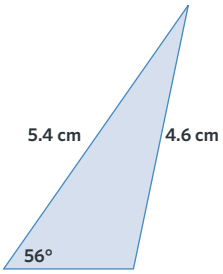


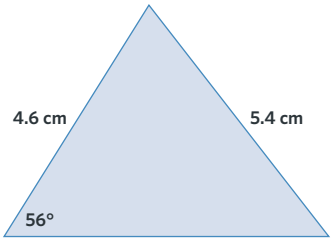
9 **a** No **b** Yes **c** Yes

10 B

11







Questions from multiple lessons

12 ***Geometry and measurement***

Decoding the question

Keywords: triangle, angles, 'could not'

Relevant lesson: 8D

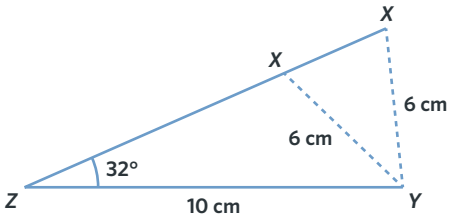
This is an ambiguous case of the sine rule where two side lengths are known and the shorter known side (XY) is opposite the acute angle ($\angle XZY$). We must find all angles in the two possible triangles for XYZ

Information we are given

- Side XY has a length of 6 cm.
- Side YZ has a length of 10 cm.
- Angle XZY has a magnitude of 32°.

Using the theory

Draw the triangle XYZ. There are two possible ways to draw the triangle, as this is an ambiguous case of the sine rule.



Find both possible values of the angle *YXZ* using the sine rule.

$\frac{10}{\sin(\angle YXZ)} = \frac{6}{\sin(32°)}$

$\angle YXZ = \sin^{-1}\left(10 \times \frac{\sin(32°)}{6}\right)$

The calculator will return the value of the acute angle.

$\angle YXZ = 62.03...°$

To find the second possible value of $\angle YXZ$, subtract this from 180°.

$180° - 62.03...° = 117.96...°$

The interior angles of a triangle must add up to 180°. Since we know the angle of *XZY* and the two possible angles of *YXZ*, we can now find the two possible values of angle *ZYX*

$180° - 32° - 62.03...° = 85.96...°$

$180° - 32° - 117.96...° = 30.03...°$

The possible angles are 62°, 118°, 86°, and 30°.

$\therefore$ 94° could **not** be an angle in triangle *XYZ*

Answer

D

✓1 mark for the correct answer

13 ***Data analysis***

Decoding the question

Keyword: variables

Relevant lesson: 12A

We need to determine the data types of the given variables.

Information we are given

- *Number of cars owned*
- *Income level* (low, medium, high)

Using the theory

First, categorise each variable as either numerical or categorical.

Numerical variables represent a quantity and can be counted or measured. As *number of cars owned* is a variable that can be counted and has not been given with a set of categories, it is a numerical variable.

Categorical variables represent a quality or attribute and are broken down into groups or categories. *Income level* is a categorical variable as it has the categories 'low', 'medium' and 'high'. Categorical variables can be either nominal or ordinal variables. Ordinal variables are categorical variables that can be ordered. As there is a logical order to the variable *income level* (low, medium, high), it is an ordinal variable.

The variables *number of cars owned* and *income level* (low, medium, high) are a numerical variable and an ordinal variable respectively.

Answer

A

✓1 mark for the correct answer

14a ***Geometry and measurement***

Decoding the question

Keywords: angle, possible values

Relevant lesson: 8D

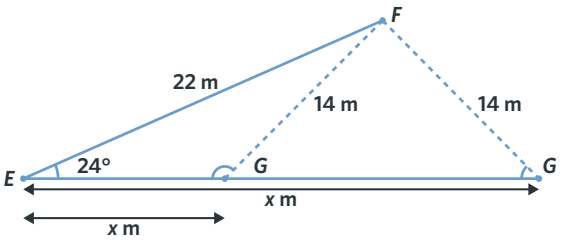
This question requires us to use the sine rule to determine the possible values for the angle *EGF*

Information we are given

- The given diagram
- Point *G* is 14 metres from point *F*

Using the theory

There are two scenarios for the possible values of $\angle EGF$ as shown.



The sine rule states that $\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$

Substitute the appropriate values into the formula.

$\frac{\sin(\angle EGF)}{22} = \frac{\sin(24°)}{14}$

$\sin(\angle EGF) = \frac{22 \times \sin(24°)}{14}$

$\angle EGF = \sin^{-1}\left(\frac{22 \times \sin(24°)}{14}\right)$

$\angle EGF = 39.729...°$

$\angle EGF \approx 40°$

However, this is an ambiguous case of the sine rule, so we must consider the possibility that $\angle EGF$ is an obtuse angle.

To calculate the other possible value of $\angle EFG$, subtract the answer from 180°.

$\angle EGF = 180° - 40°$

$\angle EGF = 140°$

∴ The two possible values for $\angle EGF$ are 40° and 140°.

Answer

40° and 140°

✓1 mark for the correct answer

14b ***Geometry and measurement***

Decoding the question

Keywords: 'within the park', value

Relevant lesson: 8D

This question requires us to use the sine rule to determine an unknown length.

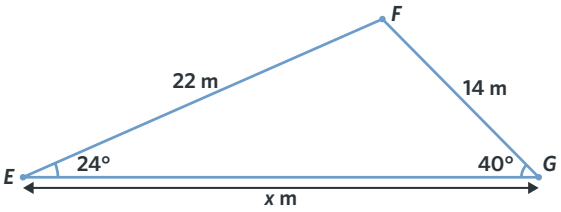
Information we are given

- The given diagram
- Point G is within the park.

Using the theory

If point G is within the park, then the length of x must be less than 30 m. From part **a**, we determined that $\angle EGF$ can be either 40° or 140°.

For $\angle EGF = 40°$:



As we know the angle sum of a triangle is 180°, $\angle EFG$ can be calculated.

$\angle EFG = 180° - 24° - 40°$

$\angle EFG = 116°$

Using the sine rule:

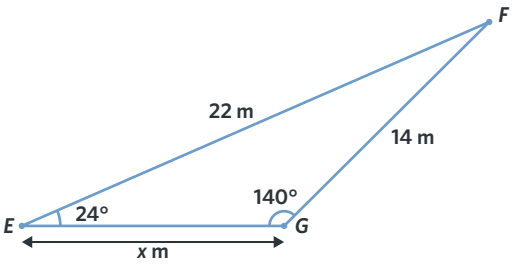
$\frac{\sin(116°)}{x} = \frac{\sin(24°)}{14}$

$x = \frac{14 \times \sin(116°)}{\sin(24°)}$

$x = 30.9...$

$30.9... > 30$, hence it is outside the boundary.

For $\angle EGF = 140°$:



As we know the angle sum of a triangle is 180°, $\angle EFG$ can be calculated.

$\angle EFG = 180° - 24° - 140°$

$\angle EFG = 16°$

Using the sine rule:

$\frac{\sin(16°)}{x} = \frac{\sin(24°)}{14}$

$x = \frac{14 \times \sin(16°)}{\sin(24°)}$

$x = 9.48...$

$9.48... < 30$, hence it is inside the boundary.

∴ The value of x, to the nearest metre, is 9 m.

Answer

9 m

✓1 mark for the correct answer

8G

1 64°

2 $Area = \frac{1}{2} \times 11 \times 10 \times \sin(40°)$

3 **a** 19.70 cm^2 **b** 147.48 cm^2 **c** 282.04 m^2

4 177.13 cm^2

5 11 184.8 m^2

6 $\theta = \sin^{-1}\left(\frac{30}{\frac{1}{2} \times 9 \times 11}\right)$

7 **a** $\theta = 61.11°$ **b** $x = 22.97$

8 61.04 m

9 118.3°

10 575.2 m

11 A

12 30

13 **a** 517 m^2 **b** 689 m^2

14 Area of left white triangle:

$Area = \frac{1}{2} \times 6 \times 14.5 \times \sin(70°) = 40.8766...$

Area of right white triangle:

$Area = \frac{1}{2} \times 9.5 \times 11 \times \sin(66°) = 47.7327...$

Total white area: $40.8766... + 47.7327... = 88.6093...$

Total area of tile: $Area = 13 \times 13 = 169$

Total purple area: $169 - 88.6093... = 80.3906...$

Percentage of purple area:

$\frac{80.3906...}{169} \times 100 = 47.5684...$

∴ The percentage of the tile that is purple is 47.6%.

Questions from multiple lessons

15 *Geometry and measurement*

Decoding the question

Keywords: lengths, angle

Relevant lesson: 8E

Since all three side lengths are known, the magnitude of angle θ can be found using the cosine rule.

Information we are given

- The given diagram

Using the theory

The cosine rule states that $\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$, where c is the side opposite the unknown angle C.

In the given triangle, the side opposite angle θ has a length of 11 m. Hence, $c = 11$. The other two sides are interchangeable as a and b

Substitute $a = 8$, $b = 14$, $c = 11$, and $C = \theta$ into the cosine rule.

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(\theta) = \frac{8^2 + 14^2 - 11^2}{2 \times 8 \times 14}$$

Answer

C

✓1 mark for the correct answer

16 *Data analysis*

Decoding the question

Keywords: least squares regression, equation, scatter plot

Relevant lesson: 13D

This question requires us to use the given graph to determine the equation of the least squares regression line has been fitted to the data.

Information we are given

- The given graph

Using the theory

A least squares regression line is in the form $y = a + bx$, where y is the response variable and x is the explanatory variable.

We know that the response variable is the *width* of the frame as it is located on the y-axis and the explanatory variable is *length* as it is located on the x-axis. Therefore, the equation can be written in the form $width = a + b \times length$

Next, determine the value of a, which is the y-intercept. The line crosses the y-axis at approximately 1. Looking at the provided options, $a = 1.25$.

Now, determine the value of b by finding the gradient of the line using two points off the graph. We will use the points (20, 15) and (100, 70) in this solution. Note that the gradient found will be an estimate, but it should be accurate enough for us to determine the correct option.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

Let $(x_1\ y_1) = (20, 15)$ and $(x_2\ y_2) = (100, 70)$

$$b = \frac{70 - 15}{100 - 20}$$

$$b = 0.6875$$

Out of the provided options, the equation of the line is closest to $width = 1.25 + 0.69 \times length$

Answer

C

✓1 mark for the correct answer

17a *Geometry and measurement*

Decoding the question

Keywords: cosine rule, distance

Relevant lesson: 8E

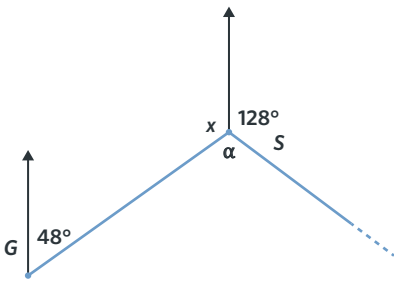
We must first determine the unknown angle GSP and then use the cosine rule to find the unknown length GP

Information we are given

- The given diagram

Using the theory

Before we can use the cosine rule, we need to determine the angle GSP. The diagram below outlines the two angles we need to determine around point S, x and α. Angle GSP is equal to α



The angle of 48° at G and the angle x add up to 180° as they are supplementary angles. Find the value of x.

$$180° = x + 48°$$

$$x = 180° - 48°$$

$$x = 132°$$

The angles α, x and 128° add up to 360°. Use these to determine the value of α

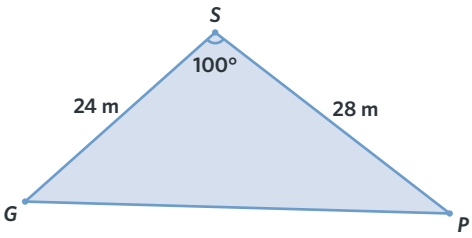
$$360° = \alpha + x + 128°$$

$$360° = \alpha + 132° + 128°$$

$$\alpha = 360° - 132° - 128°$$

$$\alpha = 100°$$

The triangle GSP, with the known lengths and internal angles, is shown.



The cosine rule states that $c = \sqrt{a^2 + b^2 - 2ab \times \cos(C)}$ where C is the angle opposite length c

Substitute $c = GP$, $a = 24$, $b = 28$ and $C = 100°$ into the equation and solve for length GP

$GP = \sqrt{24^2 + 28^2 - 2 \times 24 \times 28 \times \cos(100°)}$

$GP = 39.91...$

∴ Pendles kicked the ball 40 m, rounded to the nearest metre.

Answer

40 m

✓1 mark for the correct angle GSP
✓1 mark for the correct answer

17b ***Geometry and measurement***

Decoding the question

Keyword: bearing

Relevant lessons: 8C, 8D

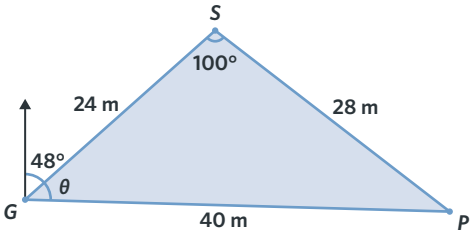
We can determine the bearing of P from G using the sine rule.

Information we are given

- The given diagram

Using the theory

To determine the bearing of P from G, we can find the angle SGP and add it to the bearing of S from G. We can do this using the sine rule or cosine rule. In this solution, we will use the sine rule.



The sine rule is given as $\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$, where A and B are the angles opposite the side lengths a and b respectively. Substitute $a = 28$, $b = 40$, $B = 100$ and $A = \theta$, and solve for θ

$\frac{28}{\sin(\theta)} = \frac{40}{\sin(100°)}$

$\sin(\theta) = \frac{28 \times \sin(100°)}{40}$

$\theta = \sin^{-1}\left(\frac{28 \times \sin(100°)}{40}\right)$

$\theta = 43.57...°$

The bearing will be the sum of angle θ and the bearing of S from G

$bearing = 48° + 43.57...°$

$bearing = 91.57...°$

∴ The bearing of Pendles from Grundy, to the nearest degree, is 92°.

Answer

92°

✓1 mark for the correct answer

9A

1 C

2 (0, 3) (3, 9) (4, 11) (5, 13)

3 **a**

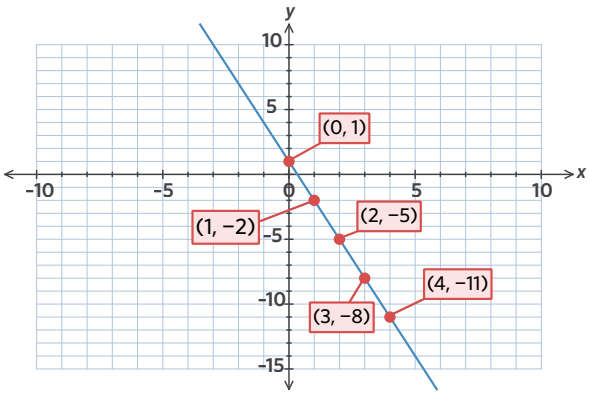
x	0	1	2	3	4
y	0	5	10	15	20

b

x	0	1	2	3	4
y	5	7	9	11	13

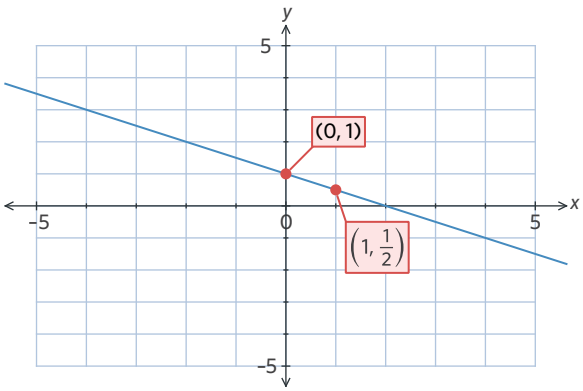
4

x	0	1	2	3	4
y	1	−2	−5	−8	−11



5 When $x = 0$, $y = 1$

When $x = 1$, $y = 0.5$

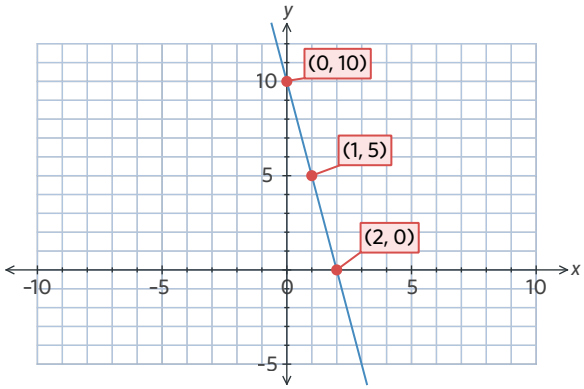


6

x	−1	0	1	2
y	−2	1	4	7

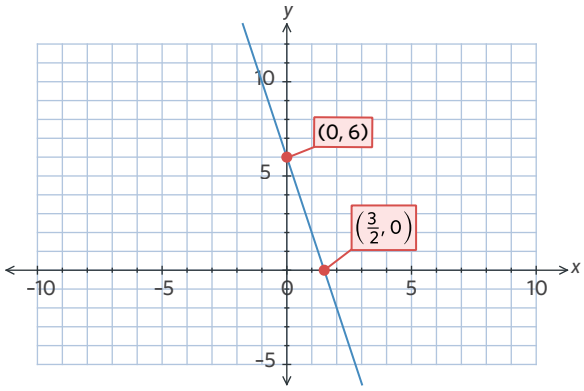
7

x	0	1	2
y	10	5	0

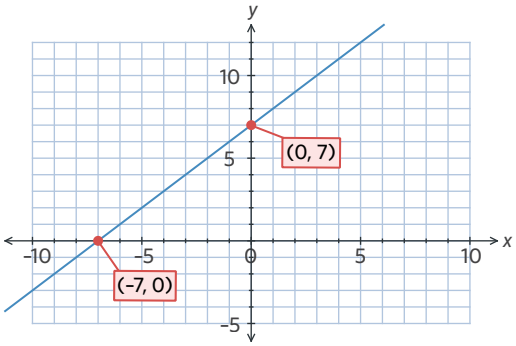


8 x-intercept: $(-6, 0)$ y-intercept: $(0, 6)$

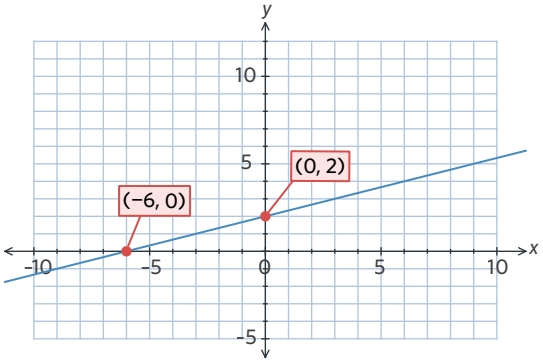
9



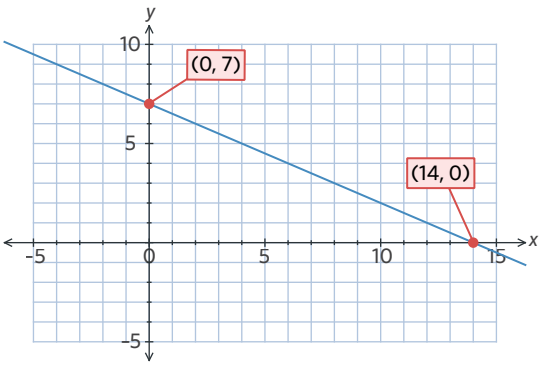
10 **a**



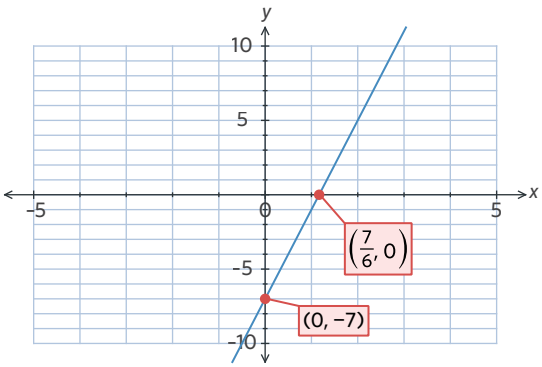
b



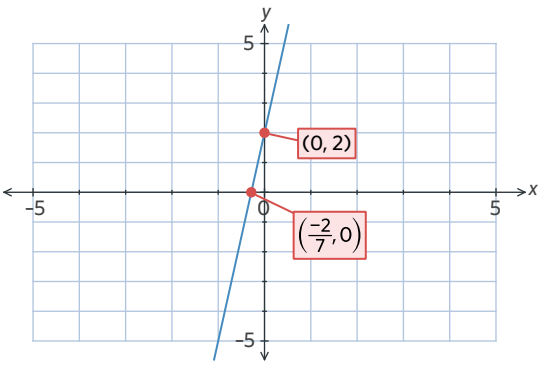
c



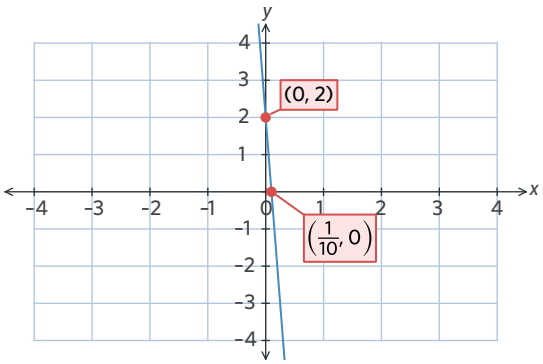
11



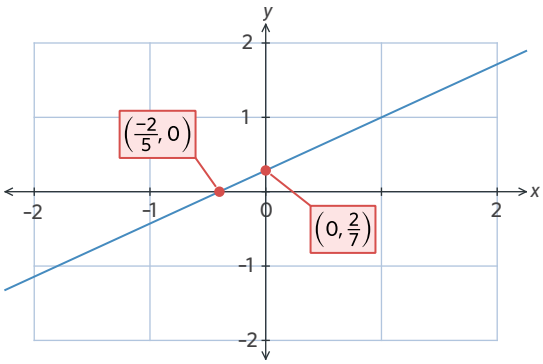
12 **a**



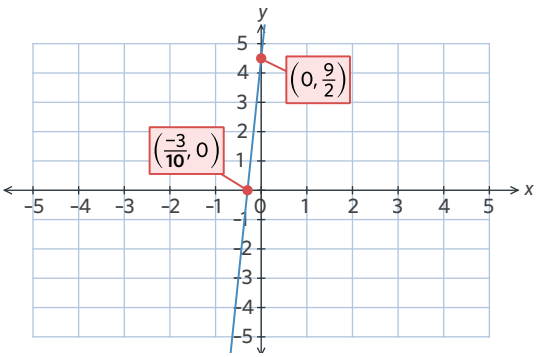
b



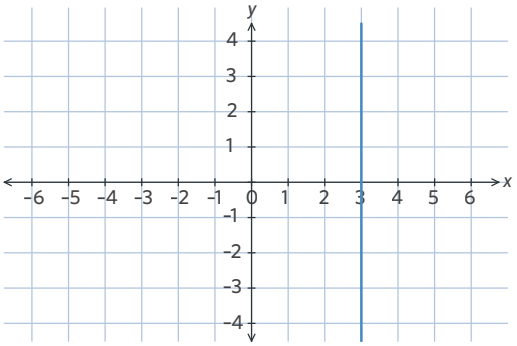
13 **a** $y = \frac{5}{7}x + \frac{2}{7}$



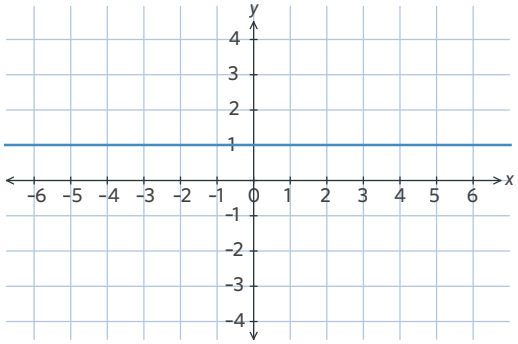
b $y = 15x + \frac{9}{2}$



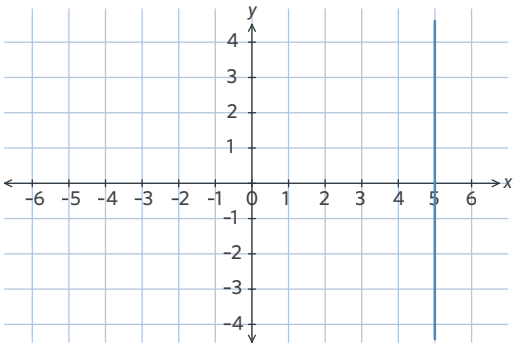
14



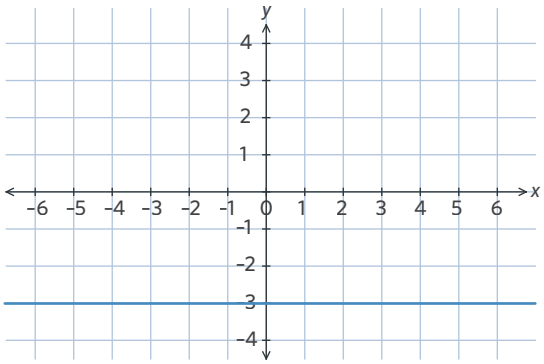
15 **a**



b



c

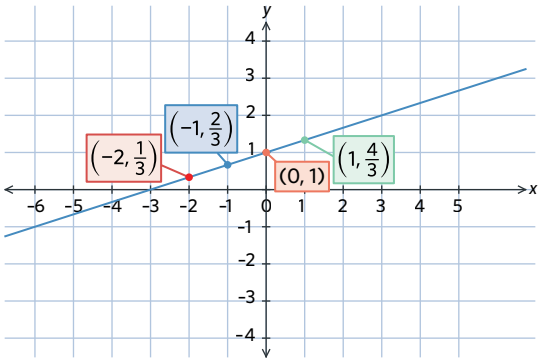


16 **i** $x = -38$ has an x-intercept and $y = 23$ has a y-intercept.

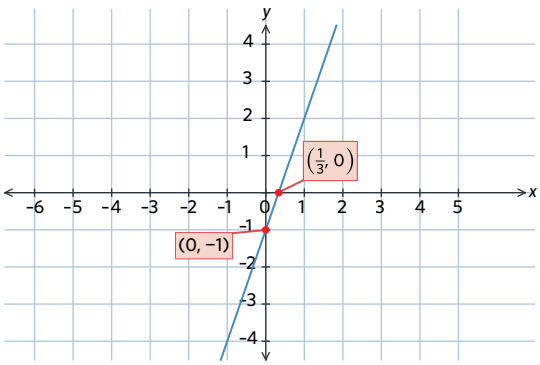
ii $x = -38$ is vertical, and $y = 23$ is horizontal.

17

x	−2	−1	0	1
y	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$

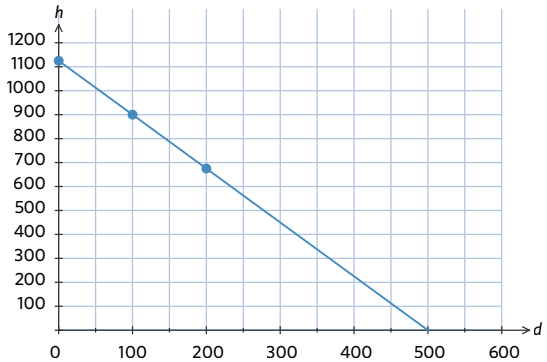


18

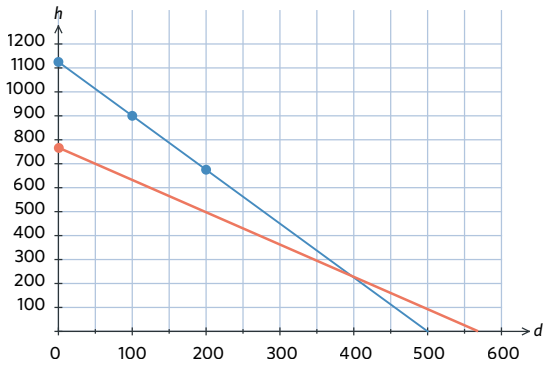


19 **a**

d	0	100	200
h	1125	900	675



b y-intercept: 760, x-intercept: 570



c Sunrise Ridge **d** Sunrise Ridge

e Cumulus Peak

20 B

Questions from multiple lessons

21 *Graphs and relations*

Decoding the question

Keywords: straight line, intercept, plot

Relevant lesson: 9A

We need to use the given intercepts to plot the line.

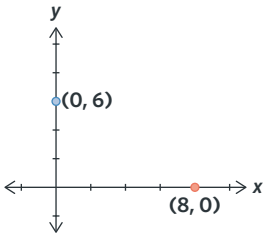
Information we are given

- A line has an x-intercept at $x = 8$ and a y-intercept at $y = 6$.
- The line is linear.

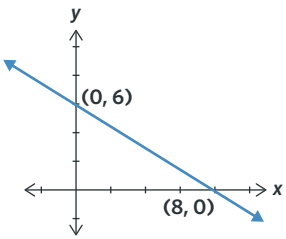
Using the theory

A line with an x-intercept of 8 crosses the x-axis at $(8, 0)$ and a y-intercept of 6 crosses the y-axis at $(0, 6)$.

Plot the points on a graph.



Since the line is linear, it is a straight line that goes through these two points.



Answer

B

✓1 mark for the correct answer

22 *Data analysis*

Decoding the question

Keywords: stem plot, percentage

Relevant lesson: 12C

We must count the number of values lower than 50 minutes and represent this number as a percentage of the total number of values.

Information we are given

- The given stem plot
- The sample contains 25 people.

Using the theory

Find the number of values less than 50. These are all the numbers highlighted red.

Key: 1 | 1 = 11 minutes

1	5 6 7 7
2	2 5 6
3	2 7 8 8
4	1 1 2 6 9
5	0 1 3 7 8
6	2 4 5 7

There are 16 values less than 50.

Represent this as a percentage of the total number of values.

$$percentage = \frac{number\ of\ values}{total\ number\ of\ values} \times 100$$

$$percentage = \frac{16}{25} \times 100$$

$$percentage = 64\%$$

$\therefore$ 64% of the people spent less than 50 minutes driving to work.

Answer

D

✓1 mark for the correct answer

23a *Graphs and relations*

Decoding the question

Keywords: how many, equation

Relevant lesson: 1A

We need to determine the number of cases produced using the given equation.

Information we are given

- $Cost = 200 + 1.25 \times number\ of\ cases$
- The cost is \$325.

Using the theory

The equation states that
$cost = 200 + 1.25 \times number\ of\ cases$

Substitute $cost = 325$ into the equation and solve for $number\ of\ cases$

$325 = 200 + 1.25 \times number\ of\ cases$

$125 = 1.25 \times number\ of\ cases$

$number\ of\ cases = \frac{125}{1.25}$

$number\ of\ cases = 100$

Answer

100 cases

✓1 mark for the correct answer

23b ***Graphs and relations***

Decoding the question

Keywords: equation, grid, relationship

Relevant lesson: 9A

We need to graph the equation that models the number of cases produced on the given set of axes.

Information we are given

- The given grid
- $Cost = 200 + 1.25 \times number\ of\ cases$

Using the theory

The horizontal axis spans from 0 to 140. Determine and plot the coordinates for $number\ of\ cases = 0$ and $number\ of\ cases = 140$ and draw a line to connect the points.

At $number\ of\ cases = 0$:

Substitute $number\ of\ cases = 0$ into $cost = 200 + 1.25 \times number\ of\ cases$

$cost = 200 + 1.25 \times (0)$

$cost = 200$

Plot the point (0, 200) on the graph.

At $number\ of\ cases = 140$:

Substitute $number\ of\ cases = 140$ into $cost = 200 + 1.25 \times number\ of\ cases$

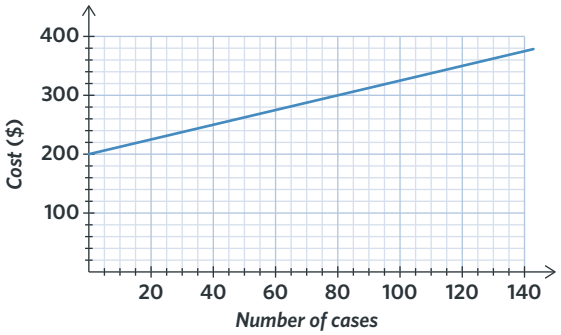
$cost = 200 + 1.25 \times (140)$

$cost = 375$

Plot the point (140, 375) on the graph and draw a line to connect the two points.

The straight line going through the two points represents the equation $cost = 200 + 1.25 \times number\ of\ cases$

Answer



✓1 mark for the correct graph

9B

1 D

2 $\frac{1}{3}$

3 **a** 2 **b** $-\frac{1}{2}$ **c** $\frac{1}{3}$

4 4

5 A

6 A

7 **a** $y = 2x + 3$ **b** $y = -\frac{1}{2}x + 1$

c $y = \frac{1}{3}x - 1$

8 D

9 The y-intercept represents the height of the batter (cake) before it is placed in the oven.

10 $y = 1.5x + 2$

11 **a** $y = \frac{1}{2}x + 4$ **b** $y = -2x + 7$

12 **a** $y = x + 1$ **b** $y = -\frac{1}{2}x + 3$

13 **a** 6 L **b** 16 L

14 $y = \frac{3}{2}x - \frac{5}{2}$

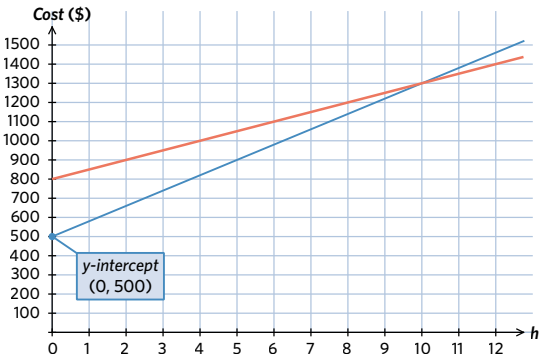
15 **a** $y = 2x + 3$ **b** $y = -0.75x + 0.25$

16 **a** $y = -2x + 4.5$ **b** $y = 0.25x - 3$

17 B, C, E, F

18 **a** $C = 500 + 80h$

b



c Your Mate's Gates **d** $90

e $400 **f** $1570

19 A

Questions from multiple lessons

20 ***Graphs and relations***

Decoding the question

Keywords: equation, 'passes through the points'

Relevant lesson: 9B

We need to use the provided points to determine the equation of a line.

Information we are given

- The line passes through the points (3, 1) and (3, 8).

Using the theory

Looking at the given coordinates, the x-coordinate is the same for both. Hence, the line that goes through both these points must be a vertical line. The x-coordinate is 3.

$\therefore$ The line passes through the points (3, 1) and (3, 8) is $x = 3$.

Answer

A

✓1 mark for the correct answer

21 ***Recursion and financial modelling***

Decoding the question

Keywords: Fibonacci, sequence, recurrence relation

Relevant lesson: 6F

We must find the option that does not fit the given Fibonacci-related recurrence relation.

Information we are given

- $t_0 = 1, \quad t_n = t_{n-2} + t_{n-1}$

Using the theory

We need to test each option against the given Fibonacci-related recurrence relation. For Fibonacci-related sequences we have to take the values of t_0 and t_1 as given, hence we can only test the values of t_2 t_3 and t_4 for each option.

A 1, 3, 4, 7, 11...

$t_0 = 1$ and $t_1 = 3$

$t_2 = t_1 + t_0 = 3 + 1 = 4$

$t_3 = t_2 + t_1 = 4 + 3 = 7$

$t_4 = t_3 + t_2 = 7 + 4 = 11$

Hence, option A fits the Fibonacci-related recurrence relation.

B 1, −2, −1, −3, −4...

$t_0 = 1$ and $t_1 = -2$

$t_2 = t_1 + t_0 = -2 + 1 = -1$

$t_3 = t_2 + t_1 = -1 + -2 = -3$

$t_4 = t_3 + t_2 = -3 + -1 = -4$

Hence, option B fits the Fibonacci-related recurrence relation.

C 1, 5, 6, 11, 17...

$t_0 = 1$ and $t_1 = 5$

$t_2 = t_1 + t_0 = 5 + 1 = 6$

$t_3 = t_2 + t_1 = 6 + 5 = 11$

$t_4 = t_3 + t_2 = 11 + 6 = 17$

Hence, option C fits the Fibonacci-related recurrence relation.

D 1, −1, 0, −1, 0...

$t_0 = 1$ and $t_1 = -1$

$t_2 = t_1 + t_0 = -1 + 1 = 0$

$t_3 = t_2 + t_1 = 0 + -1 = -1$

$t_4 = t_3 + t_2 = -1 + 0 = -1$

Hence, option D does not fit the Fibonacci-related recurrence relation.

E 1, 2, 3, 5, 8...

$t_0 = 1$ and $t_1 = 2$

$t_2 = t_1 + t_0 = 2 + 1 = 3$

$t_3 = t_2 + t_1 = 3 + 2 = 5$

$t_4 = t_3 + t_2 = 5 + 3 = 8$

Hence, option E fits the Fibonacci-related recurrence relation.

Answer

D

✓1 mark for the correct answer

22a ***Graphs and relations***

Decoding the question

Keywords: show that, graph

Relevant lesson: 9B

This question requires us to prove $k = 250$ given an equation and a point on a graph.

Information we are given

- The given graph
- $k = 250$

Using the theory

From the graph, we can read off the given point (2000, 8).

We are told that the equation of the line is

$extension = \frac{force}{k}$

Substitute $force = 2000$ and $extension = 8$.

$8 = \frac{2000}{k}$

$k = \frac{2000}{8}$

$k = 250$

Answer

$k = \frac{2000}{8} = 250$

✓1 mark for the correct method

22b ***Graphs and relations***

Decoding the question

Keywords: equation, calculate

Relevant lesson: 9B

This question requires us to use the linear equation to calculate the force required to cause the spring to extend by 20 cm.

Information we are given

- $extension = \frac{force}{k}$
- $k = 250$

Using the theory

The equation is $extension = \frac{force}{k}$

Substitute $extension = 20$ into the equation.

$20 = \frac{force}{250}$

$force = 250 \times 20$

$force = 5000$

∴ The amount of force applied is 5000 N.

Answer

5000 N

✓1 mark for the correct answer

9C

1 -4

2 -2

3 30 L

4 **a** $B = 10 + 3n$ **b** $H = 1.5 + 0.6t$

c $W = 1.6t$

5 $T = -0.06t + 20$

6 80 L

7 $0 \leq Distance \leq 400$

8 **a** $0 \leq n \leq 6$ **b** $0 \leq t \leq 31$

c $0 \leq t \leq 2500$

9 $0 \leq t \leq 40$

10 $M = 800 + 400n$

11 **a** 12 **b** 6

12 Mechanic A: \$290, Mechanic B: \$285

It would be cheaper to hire mechanic B.

13 **a** $C = 40 + 70n$ **b** \$530

c $C = 530 + 50m$ **d** \$930

14 E

Questions from multiple lessons

15 ***Graphs and relations***

Decoding the question

Keywords: straight line, equation

Relevant lesson: 9B

We must apply our knowledge of linear graphs to deduce an equation for the line.

Information we are given

- The given graph
- The line is straight.

Using the theory

Linear graphs are generally presented in the form $y = mx + c$, where c is the y-intercept and m is the gradient.

Looking at the graph, we can see that the line passes through the y-axis at $y = -20$. Hence, $c = -20$.

We can also see the line passes through the x-axis at $x = 5$.

Substitute $(5, 0)$ into the equation $y = mx - 20$ to find the gradient.

$0 = m(5) - 20$

$20 = 5m$

$m = 4$

∴ The equation of the line is $y = 4x - 20$.

Answer

C

✓1 mark for the correct answer

16 ***Data analysis***

Decoding the question

Keyword: variables

Relevant lesson: 12A

We need to determine the data types of the given variables.

Information we are given

- *Length of a movie* (less than 1 hour, 1–2 hours, over 2 hours)
- *Popularity* (low, medium, high)

Using the theory

Firstly, categorise each variable as either numerical or categorical, then determine whether they are nominal, ordinal, discrete or continuous.

The variable *length of a movie* on its own would classify as numerical. However, in this example, it is broken into three categories (less than 1 hour, 1-2 hours, over 2 hours). Hence, it is a categorical variable. Since the categories less than 1 hour, 1-2 hours and over 2 hours follow a logical order, *length of a movie* is an ordinal variable.

NOTE: Just because a variable contains numbers, this doesn't immediately classify it as a numerical variable.

The variable *popularity* in this situation is broken into three categories (low, medium, high) so it is also a categorical variable. Since the categories low, medium and high follow a logical order, *popularity* is also an ordinal variable.

Answer

C

✓1 mark for the correct answer

17a ***Graphs and relations***

Decoding the question

Keywords: complete, fixed amount, charge per tennis ball, graph

Relevant lessons: 9A, 9B, 9C

We need to determine the equation of the given graph and interpret its meaning in the context of the question.

Information we are given

- The given graph
- The factory charges a fee that consists of a fixed amount and a charge per number of tennis balls produced.

Using the theory

Number of tennis balls is on the x-axis and *fee* is on the y-axis. Hence, the equation for this line will be in the form $fee = m \times number\ of\ tennis\ balls + c$, where c is the y-intercept and represents the fixed amount, and m is the gradient and represents the charge per tennis ball produced.

The line intersects the y-axis at 20, meaning that $c = 20$.

Hence, the fixed amount is \$20.

The line goes through the point $(400, 100)$. Substitute $c = 20$, $fee = 100$ and $number\ of\ tennis\ balls = 400$ into the equation $fee = m \times number\ of\ tennis\ balls + c$ and solve for m

$100 = 20 + m \times 400$

$400m = 100 - 20$

$m = \frac{100 - 20}{400}$

$m = 0.20$

Hence, the charge per tennis ball is \$0.20.

Answer

Hopkins charges a fixed amount of \$20 and a charge per tennis ball of \$0.20.

✓1 mark for the correct answer

17b ***Graphs and relations***

Decoding the question

Keywords: charge, fee, linear relation, representing, graph

Relevant lessons: 9A, 9B

We need to determine the equation for Hoppers' offer and sketch it on the given graph.

Information we are given

- Hopkins and Hoppers both charge a fee of \$100 for 400 tennis balls.
- Hoppers' charge per tennis ball is \$0.15.

Using the theory

The linear equation is in the form $fee = m \times number\ of\ tennis\ balls + c$ where c is the y-intercept and represents the fixed amount, and m is the gradient and represents the charge per tennis ball produced.

Since Hoppers' charge per tennis ball is \$0.15, $m = 0.15$.

Hoppers also charge \$100 for 400 tennis balls.

Substitute $m = 0.15$, $fee = 100$ and $number\ of\ tennis\ balls = 400$ into the equation $fee = m \times number\ of\ tennis\ balls + c$ and solve for c

$100 = c + 0.15 \times 400$

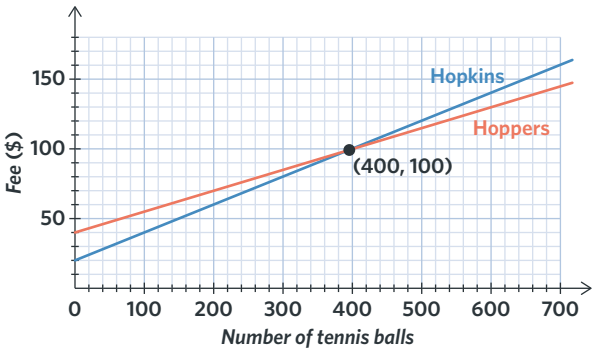
$c = 100 - 0.15 \times 400$

$c = 40$

Hence, the equation that represents Hopkins' charge is $fee = 0.15 \times number\ of\ tennis\ balls + 40$.

The y-intercept is 40. We know that the line will go through the points $(400, 100)$ and $(0, 40)$, so plot these points on the given graph and draw a line to connect them. Ensure to extend the line beyond $number\ of\ tennis\ balls = 400$.

Answer



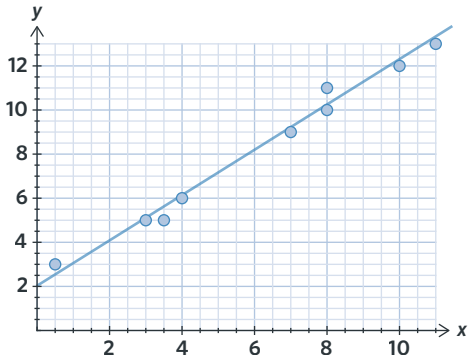
✓1 mark for the correct answer

9D

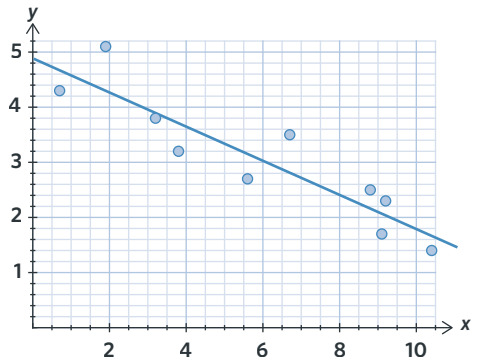
1 $-\frac{5}{6}$

2 A

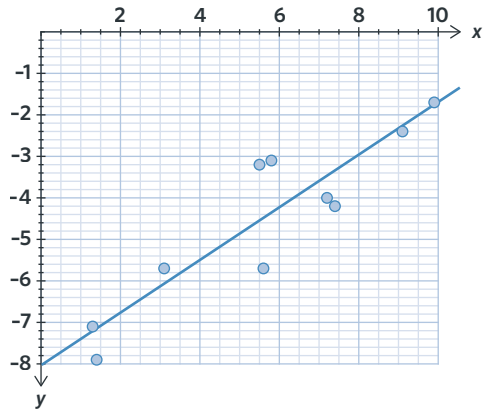
3 **a**



b



c



4 Holly

5 $y = 2x - 3$

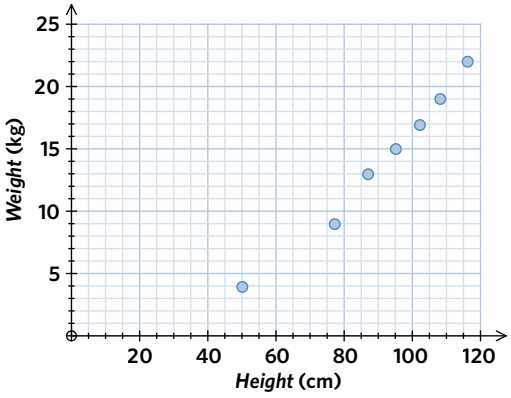
6 **a** $y = -\frac{11}{7}x + \frac{519}{7}$ **b** $y = -\frac{35}{34}x + \frac{840}{17}$

c $y = \frac{11}{6}x - \frac{41}{3}$

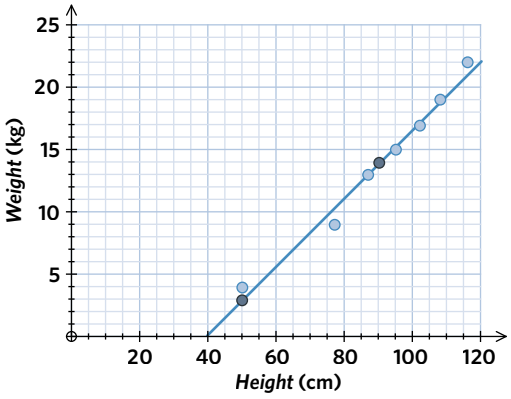
7 D

8 C

9 **a**



b



$w = \frac{11}{40}h - \frac{43}{4}$

c 16 750 g **d** 1482 mm

10 Other coordinate: $(-1, -1)$

$y = \frac{5}{2}x + \frac{3}{2}$

Questions from multiple lessons

11 *Graphs and relations*

Decoding the question

Keywords: equation, 'represents', 'total cost'

Relevant lesson: 1C

We need to find the equation that represents the worded situation.

Information we are given

- Hamish charges a call-out fee of $140.
- Hamish charges $90 for each hour of work.

Using the theory

First, identify the fixed and variable components of the total cost.

The fixed component is the call-out fee of $140.

The variable component is the additional fee of $90 per hour since this changes according to the number of hours of work.

Linear models are of the form $y = mx + c$ where m represents the variable component and c represents the fixed component.

Hence, $y = 90x + 140$.

Rewrite the equation in terms of the given variables C and t

$C = 90t + 140$

Answer

C

✓1 mark for the correct answer

12 *Data analysis*

Decoding the question

Keywords: histogram, distribution, percentage

Relevant lesson: 12C

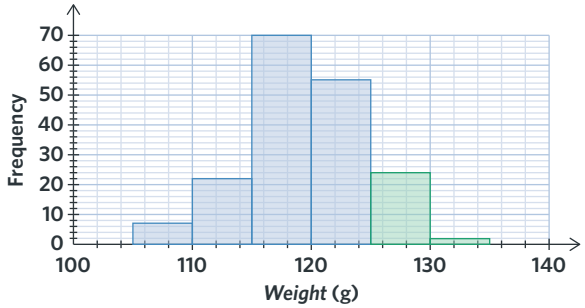
This question requires us to use the histogram to determine the percentage of ducks adopted.

Information we are given

- The given histogram
- There are 180 ducks in the sample.

Using the theory

Determine the number of ducks that weigh 125 g or more. The required columns are highlighted in the following histogram.



$number\ of\ ducks\ adopted = 24 + 2$

$number\ of\ ducks\ adopted = 26$

Hence, 26 ducks are adopted.

Represent this as a percentage of the total number of ducks.

$$percentage = \frac{number\ of\ ducks\ adopted}{total\ number\ of\ ducks} \times 100$$

$$percentage = \frac{26}{180} \times 100$$

$$percentage = 14.444...\%$$

∴ The percentage of ducks adopted is closest to 14%.

Answer

B

✓1 mark for the correct answer

13a ***Graphs and relations***

Decoding the question

Keywords: graph, unknown mass

Relevant lesson: 9B

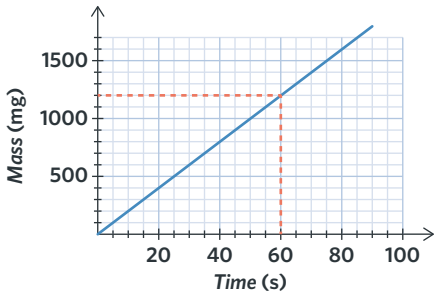
This question requires us to read the y-coordinate of a given point from the graph.

Information we are given

- The given graph
- The mass dissolves in 60 seconds.

Using the theory

Use the vertical grid lines provided on the graph to move vertically from the horizontal axis at the point $time = 60$ until the grid line intersects with the line. At the intersection point, use the horizontal grid lines to find the *mass* value at this point.



∴ 1200 mg of the compound will dissolve in 60 seconds.

Answer

1200 mg

✓1 mark for the correct answer

13b ***Graphs and relations***

Decoding the question

Keyword: slope

Relevant lesson: 9B

This question requires us to determine the slope of the graph.

Information we are given

- The given graph

Using the theory

The slope of the graph will tell us the rate at which the compound is dissolved.

From part **a**, we determined that 1200 mg of the compound will dissolve in 60 seconds.

Looking at the graph, we can see that it passes through the origin.

Use the two points (0, 0) and (60, 1200) to find the gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1200 - 0}{60 - 0}$$

$$m = \frac{1200}{60}$$

$$m = 20$$

The slope is 20.

∴ 20 mg of the compound is dissolved each second.

Answer

20 mg

✓1 mark for the correct answer

9E

1 C

2 5

3 **a** 120 metres **b** 130 metres

c 130 metres **d** Meiting and Nina

4 25 customers

5 **a** July 21: 35°

September 21: 50°

November 21: 70°

b November 21

c January 21 is outside the domain, and so interpolation can't be used.

6 90

7 **a** 58% **b** 72%

c 44% **d** 3 practice exam papers

e No, two data points are not sufficient to reasonably determine a relationship between variables.

8 **a** Michael **b** No

9 **a** Yes **b** No

10 **a** Ford Falcon: 56.5 kph

Holden Commodore: 47.5 kph

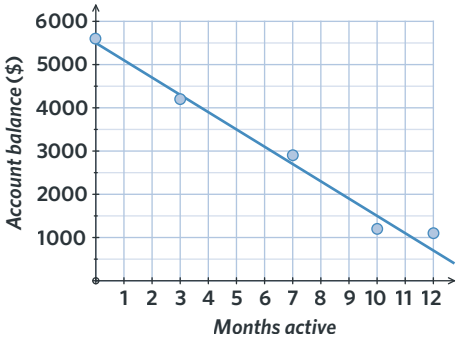
b Ford Falcon: 84 kph

Holden Commodore: 85 kph

c 75 metres

11 **a** Account A is increasing. Account B is decreasing.

b



c $B = 5500 - 400m$

d Account A: \$2800 Account B: \$1900

e At 7.5 months

Questions from multiple lessons

12 *Graphs and relations*

Decoding the question

Keywords: line, coordinates, x-intercept

Relevant lesson: 9B

We must use the given coordinates to calculate the equation of the line in order to find the x-intercept.

Information we are given

- The line passes through the points $(2, 4)$ and $(6, -2)$.

Using the theory

Linear graphs are of the form $y = mx + c$, where m is the gradient and c is the y-intercept.

The gradient can be found using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Using the given coordinates $(2, 4)$ and $(6, -2)$, substitute $y_2 = -2$, $y_1 = 4$, $x_2 = 6$ and $x_1 = 2$ into the equation.

$m = \frac{(-2) - 4}{6 - 2} = \frac{-6}{4} = -\frac{3}{2}$

Substitute one of the points into $y = -\frac{3}{2}x + c$ and solve for c

We are going to use $(2, 4)$.

$4 = -\frac{3}{2} \times 2 + c$

$c = 7$

Hence, the equation of the line is $y = -\frac{3}{2}x + 7$.

The x-intercept occurs when $y = 0$, so substitute $y = 0$ into $y = -\frac{3}{2}x + 7$ and solve for x.

$0 = -\frac{3}{2}x + 7$

$\frac{3}{2}x = 7$

$x = 7 \times \frac{2}{3}$

$x = \frac{14}{3}$

$\therefore$ The coordinate of the x-intercept is $\left(\frac{14}{3}, 0\right)$

Answer

E

✓1 mark for the correct answer

13 *Recursion and financial modelling*

Decoding the question

Keywords: compounding, recurrence relation

Relevant lesson: 6D

The recurrence relation can be determined by finding the common ratio and initial value of the geometric sequence.

Information we are given

- Kelly deposited \$2000 into a savings account.
- Interest compounds monthly at a rate of 1.8% per annum.

Using the theory

Since we are dealing with compound interest, the recurrence relation is in the form $S_0 = a$ $S_{n+1} = R \times S_n$ where a is the initial value and R is the common ratio.

a is equal to the initial deposit, which is \$2000.

The yearly interest rate of 1.8% is equivalent to a monthly interest rate of $\frac{1.8\%}{12} = 0.15\%$.

The monthly interest rate can be converted to the common ratio using the formula $R = 1 + \frac{percentage}{100}$

Substitute $percentage = 0.15$ into the formula.

$R = 1 + \frac{0.15}{100} = 1.0015$

$\therefore S_0 = 2000, \quad S_{n+1} = 1.0015 \times S_n$

Answer

C

✓1 mark for the correct answer

14a *Graphs and relations*

Decoding the question

Keywords: equation, line

Relevant lesson: 9B

We need to determine the equation of the line given in the graph.

Information we are given

- The given graph
- The bag had 1000 mL of saline solution when first administered.
- After six hours, there was 640 mL of saline solution left.

Using the theory

Determine two coordinates that lie on the graph.

At $time = 0$, $volume = 1000$. The coordinate $(0, 1000)$ lies on the graph.

At $time = 6$, $volume = 640$. The coordinate $(6, 640)$ lies on the graph.

The variable *time* is on the horizontal axis and *volume* is on the vertical axis. Hence, the equation of the line will be in the form $volume = m \times time + c$ where m is the gradient and c is the y-intercept.

Since the initial volume is 1000 mL, we know that the y-intercept is 1000, hence $c = 1000$.

The gradient can be found using $m = \frac{y_2 - y_1}{x_2 - x_1}$

Substitute the coordinates $(0, 1000)$ and $(6, 640)$ into the formula.

$m = \frac{1000 - 640}{0 - 6}$

$m = \frac{360}{-6}$

$m = -60$

∴ The equation for the graph is $volume = -60 \times time + 1000$.

Answer

$volume = -60 \times time + 1000$

✓1 mark for the correct gradient and y-intercept
✓1 mark for the correct variables

14b ***Graphs and relations***

Decoding the question

Keywords: graph, linear trend

Relevant lesson: 9B

This question requires us to determine the amount of time left until the volume reaches zero.

Information we are given

- The given graph

Using the theory

From part **a**, the equation of the graph was $volume = -60 \times time + 1000$. When the IV drip is completely administered, the amount of saline solution left in the bag is zero. Hence, substitute $volume = 0$ into the equation and solve for *time*

$0 = -60 \times time + 1000$

$60 \times time = 1000$

$time = \frac{1000}{60}$

$time = 16.66...$

This means that the IV will take a total of 16 hours and 40 minutes to be completely administered. However, the question asks how much **longer** it will take. Since it has already been six hours, subtract this from the total amount of time.

∴ It will take another 10 hours and 40 minutes.

Answer

10 hours and 40 minutes

✓1 mark for the correct answer

9F

1 B

2 20

3 **a** $(6, 27)$ **b** $(-2, 3)$ **c** $(11, 376)$

4 **a** $l = \frac{5}{6}d - \frac{1}{6}$

b Day 1: 0.67 cm

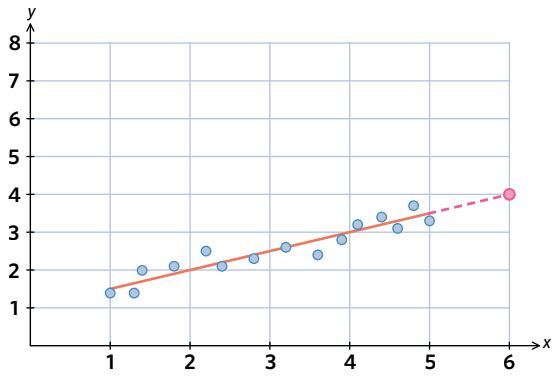
Day 10: 8.17 cm

Day 20: 16.50 cm

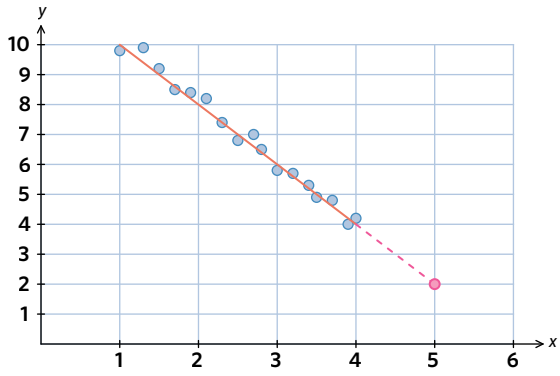
The estimates for days 1 and 10 are relatively accurate, because they are close to the original data. The estimate for day 20 is probably very inaccurate.

5 3

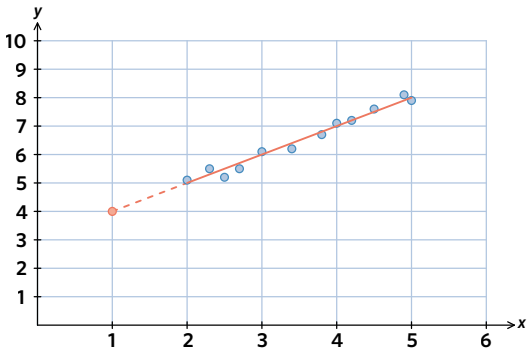
6 **a** $(6, 4)$



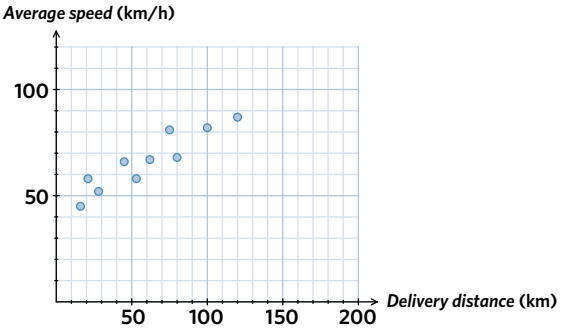
b $(5, 2)$



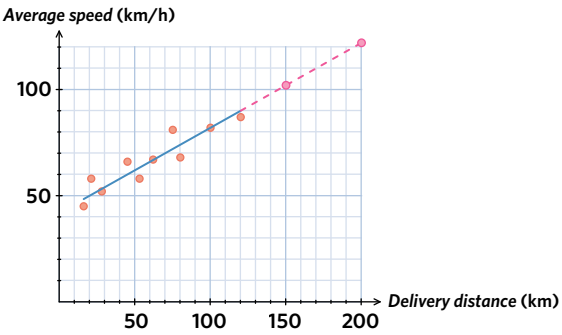
c (1, 4)



7 a



b



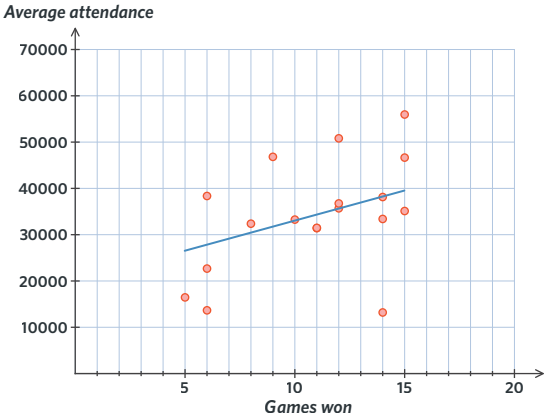
The average speed for a delivery of 150 km will be about 102 km/h. The average speed for a delivery of 200 km will be about 122 km/h.

c The average speed for a 150 km delivery, 102 km/h, is plausible, as the top speed limit in Victoria is 110 km/h. The average speed for a 200 km delivery, 122 km/h, is not realistic. There is no way any vehicle observing speed laws in Victoria could maintain such an average. This is an example of answers becoming more inaccurate when extrapolating points far away from the data set.

8 a The point involving 6000 km flights involves interpolation. The 9000 and 12000 km flights involve extrapolation, because they are outside the domain of the data set.

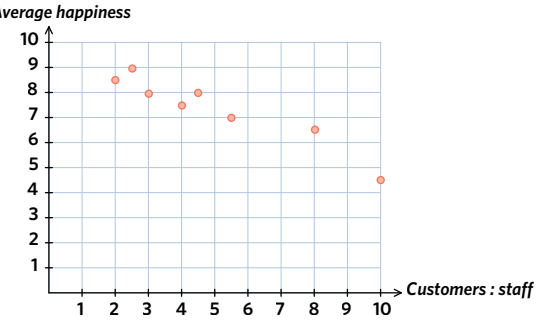
b Interpolation is generally more accurate than extrapolation. Estimating the price of a 6000 km flight would be quite accurate. Estimating the price of a 9000 km flight would reasonably accurate, because this point is not far outside the domain of the data set. However, we would expect an estimation on the price of a 12000 km flight to be very inaccurate, because it is far outside the domain of the data set.

9 a

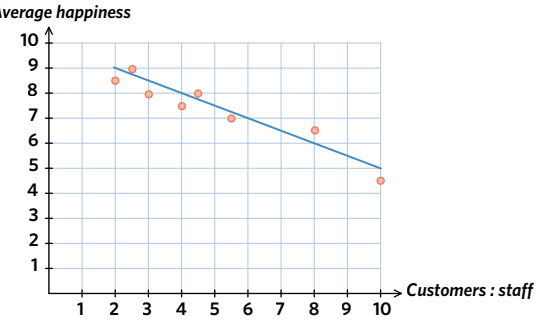


b $A = 20\,000 + 1300w$ **c** 43 400

10 a



b



c $y = -\frac{x}{2} + 10$

d The expected happiness at a ratio of 9:1 is 5.5. The expected happiness at a ratio of 1:1 is 9.5.

11 a $I = 3T - 40$

b Tareq's model predicts a higher number of ice-creams sold.

c Ariel's model is more likely to be accurate. The point of interest, $T = 30$, is within the domain of her model, so she can interpolate the point. Tareq has to extrapolate the point.

Questions from multiple lessons

12 *Graphs and relations*

Decoding the question

Keywords: line of best fit, data, interpolation

Relevant lesson: 9E

We must analyse the given graph to determine which of the points was not estimated using interpolation.

Information we are given

- The given graph

Using the theory

Interpolation is used to predict values within the domain of the existing data points. The x-coordinate of the leftmost data point is approximately 0.08. The x-coordinate of the rightmost data point is approximately 0.91.

The x-coordinate for points F, H, I and G are all between 0.08 and 0.91.

The x-coordinate for point J is approximately 0.05, which is outside the domain of the existing data points.

$\therefore$ Point J could not have been estimated using interpolation.

Answer

B

✓1 mark for the correct answer

13 ***Data analysis***

Decoding the question

Keywords: histogram, median

Relevant lesson: 12E

We must locate the median value from the histogram using the $\frac{n+1}{2}$ rule.

Information we are given

- The given histogram
- Angelo's observations span a period of 46 days.

Using the theory

In a data set with n elements, the median is located in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position.

Substitute $n = 46$ into $\frac{n+1}{2}$

$\frac{46+1}{2} = 23.5$

The median is the average of the 23rd and 24th values.

Calculate the cumulative frequency of each column of the histogram.

Interval	Frequency	Cumulative frequency
150–<175	3	3
175–<200	4	7
200–<225	7	14
225–<250	8	22
250–<275	6	28

The 23rd and 24th values are both in the interval 250–<275.

$\therefore$ The median must be at least 250 but less than 275.

Answer

C

✓1 mark for the correct answer

14a ***Graphs and relations***

Decoding the question

Keywords: draw, graph, equation

Relevant lesson: 9C

This question requires us to plot a function on the same graph as the given function.

Information we are given

- The given graph
- $R = 80n$

Using the theory

The easiest way to plot a straight line is to plot two points that lie on the line and connect them. In this solution, we will plot $n = 0$ and $n = 2000$.

For $n = 0$:

$R = 80 \times 0$

$R = 0$

Hence, the point (0, 0) lies on the line.

For $n = 2000$:

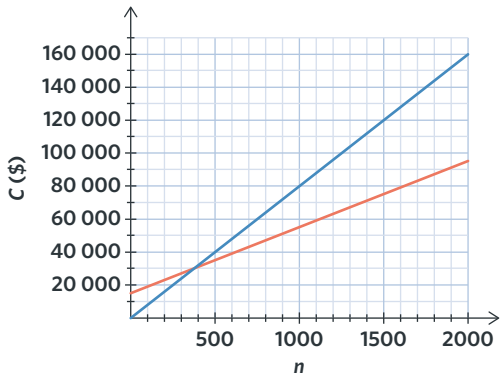
$R = 80 \times 2000$

$R = 160\ 000$

Hence, the point (2000, 160 000) lies on the line.

Plot these two points on the graph and draw a line to connect them.

Answer



✓1 mark for the correct graph

14b ***Graphs and relations***

Decoding the question

Keywords: graph, profit

Relevant lesson: 9C

This question requires us to calculate the profit when given the cost and revenue functions.

Information we are given

- $C = 40n + 15\ 000$
- $R = 80n$
- The company is expecting to sell 1500 laptop bags.

Using the theory

Calculate the revenue if the company sells 1500 laptop bags.

$R = 80n$

$R = 80 \times 1500$

$R = 120\ 000$

Calculate the cost if the company produces 1500 laptop bags.

$C = 40n + 15\ 000$

$C = 40 \times 1500 + 15\ 000$

$C = 75\ 000$

Find the profit by subtracting the cost from the revenue.

$profit = 120\ 000 - 75\ 000$

$profit = 45\ 000$

Hence, the company will make a profit of \$45 000 if they sell 1500 laptop bags.

Answer

\$45 000

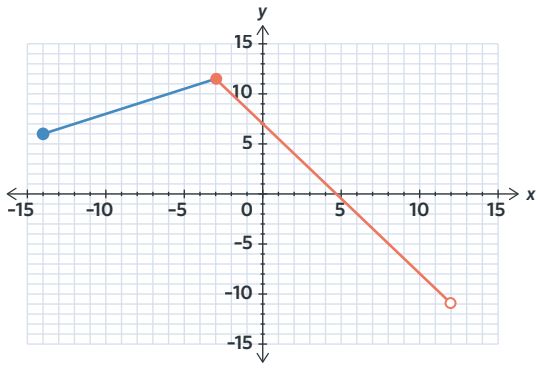
✓1 mark for the correct cost and revenue values
✓1 mark for the correct profit value

9G

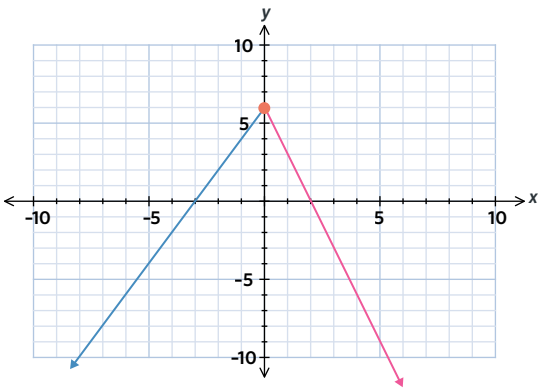
1 **a** $y = -0.5x + 5$

b $-6 < x \leq 8$

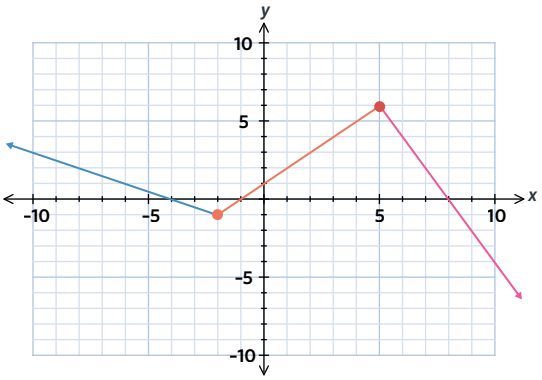
2



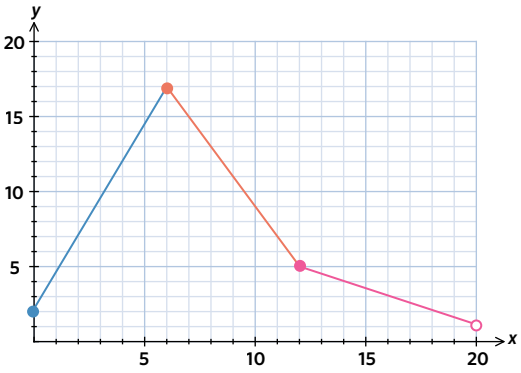
3 **a**



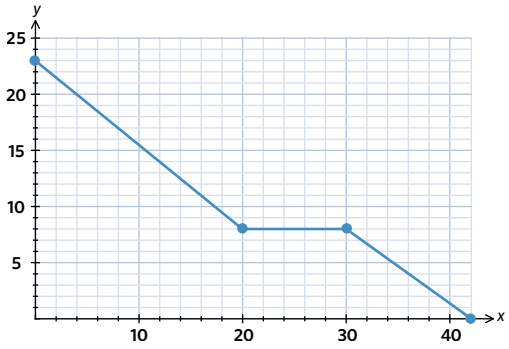
b



c



4



5 $y = -2x - 14.5, -12 \leq x < -1$

$y = 2.5x - 10, -1 \leq x < 10$

6 **a** $y = 2x + 3, 0 \leq x < 8$

$y = -1.5x + 31, 8 \leq x < 20$

b $y = 3x + 15, -8 \leq x < -2$

$y = -4x + 1, -2 \leq x < 2$

$y = 2x - 11, 2 \leq x \leq 9$

c $y = -0.8x - 6, x < -5$

$y = -2, -5 \leq x < 6$

$y = 2x - 14, x \geq 6$

7 $D = -\frac{3}{4}t + 23, 0 \leq t \leq 20$

$D = 8, 20 < t \leq 30$

$D = -\frac{2}{3}t + 28, 30 < t \leq 42$

8 3

9 D

10 **a** $C = 90 + 70n, 0 \leq n \leq 4$

$C = 370 + 65(n - 4)\ \ n > 4$ or
$C = 110 + 65n, n > 4$

b $C = 80 + 75n, 0 \leq n \leq 3$

$C = 305 + 60(n - 3) \quad n > 3$ or

$C = 125 + 60n, n > 3$

c Andrew

d Bob

11 $y = -2x + 35$

$y = -2x - 20$

12 $D = 100t, 0 \leq t < 6$

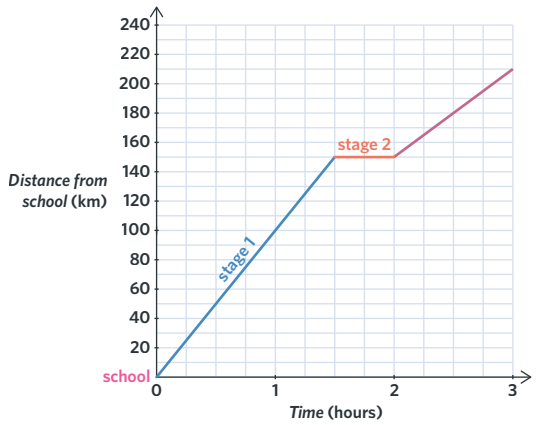
$D = 600, 6 \leq t < 21$

$D = -60t + 1860, 21 \leq t < 31$

NOTE: It is not important whether the inequality signs are $<$ or $\leq$.

13 **a** 100 km/h **b** 30 minutes

c



d 70 km/h **e** $k = 30$

Questions from multiple lessons

14 *Graphs and relations*

Decoding the question

Keywords: straight line, coordinates, graph

Relevant lesson: 9B

This question requires us to determine the equation of the line in order to find its x-intercept.

Information we are given

- The given graph
- The line passes through the points (6, 21) and (8, 27).

Using the theory

Linear graphs are of the form $y = mx + c$, where m is the gradient and c is the y-intercept.

First, determine the gradient of the line using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Using the given coordinates (6, 21) and (8, 27), substitute $y_2 = 27$, $y_1 = 21$, $x_2 = 8$ and $x_1 = 6$ into the equation.

$m = \frac{27 - 21}{8 - 6} = \frac{6}{2} = 3$

Hence, $y = 3x + c$

Substitute the point (27, 8) into the formula $y = 3x + c$ and solve for c

$27 = 3 \times 8 + c$

$27 = 24 + c$

$c = 3$

Hence, $y = 3x + 3$.

To find the x-intercept, substitute $y = 0$ into $y = 3x + 3$ and solve for x.

$0 = 3x + 3$

$3x = -3$

$x = -1$

∴ The x-intercept occurs at $(-1, 0)$.

Answer

B

✓1 mark for the correct answer

15 *Data analysis*

Decoding the question

Keywords: two-way frequency table, percentage

Relevant lessons: 2E, 12B

We must calculate the number of females who chose Sprite as their favourite soft drink as a percentage of the total number of females surveyed.

Information we are given

- The given table

Using the theory

The percentage can be found using the following formula:

$$percentage = \frac{number\ of\ females\ who\ chose\ Sprite}{total\ number\ of\ females} \times 100$$

The number of females who chose Sprite as their favourite soft drink is 35.

The total number of females surveyed is 78.

Substitute these values into the formula.

$percentage = \frac{35}{78} \times 100$

$percentage = 44.871...\%$

∴ The percentage of females who chose Sprite as their favourite soft drink is closest to 45%.

NOTE: A common error is to calculate the percentage using the total number of people rather than the total number of females.

Answer

E

✓1 mark for the correct answer

16a *Graphs and relations*

Decoding the question

Keywords: point, show that, value, equation

Relevant lesson: 9B

We need to substitute the given point into the equation to prove that $k = -13$.

Information we are given

- The given graph
- $USD = 0.65 \times AUD + k$
- The point $(200, 117)$ lies on the graph.

Using the theory

Substitute the point $(200, 117)$ into the equation $USD = 0.65 \times AUD + k$ and solve for k

$117 = 0.65 \times 200 + k$

$k = 117 - 0.65 \times 200$

$k = -13$

Answer

$k = 117 - 0.65 \times 200 = -13$

✓1 mark for the correct method

16b *Graphs and relations*

Decoding the question

Keywords: intercept, horizontal axis, equation

Relevant lesson: 9B

We need to use the equation and given k value to determine the horizontal axis intercept.

Information we are given

- The given graph
- $USD = 0.65 \times AUD + k$
- $k = -13$

Using the theory

The equation, including the k value of -13, is $USD = 0.65 \times AUD - 13$.

Since AUD is on the horizontal axis, substitute $USD = 0$ into the equation to solve for the horizontal axis intercept.

$0 = 0.65 \times AUD - 13$

$0.65 \times AUD = 13$

$AUD = \frac{13}{0.65}$

$AUD = 20$

∴ The horizontal axis intercept occurs at $(20, 0)$

Answer

$(20, 0)$

✓1 mark for the correct answer

16c *Graphs and relations*

Decoding the question

Keywords: interpret, intercept

Relevant lesson: 9C

We need to determine the meaning of the horizontal axis intercept in the context of the question.

Information we are given

- The graph displays the relationship between *USD* and *AUD* at a currency exchange agency.
- $USD = 0.65 \times AUD - 13$

Using the theory

In part **b**, we found the horizontal axis intercept occurs when $AUD = 20$. This means that when \$20 *AUD* is converted to *USD* from this currency exchange agency, \$0 *USD* is received. For every dollar after the first \$20 *AUD* exchanged, \$0.65 *USD* is received. Hence, the currency exchange agency must charge a \$20 flat rate charge or commission.

Answer

There is a \$20 commission for any conversion of *AUD* into *USD*.

✓1 mark for the correct answer

10A

1 **a** **i** $2 > 0$

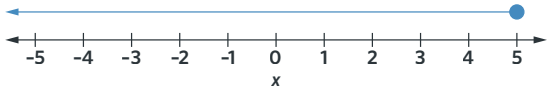
ii $-3 < -2$

b **i** E

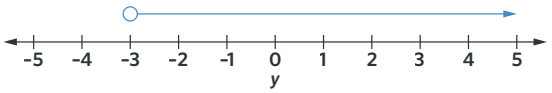
ii D

2 True

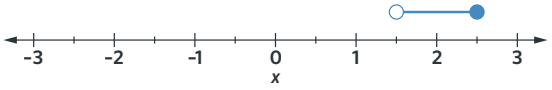
3 **a**



b

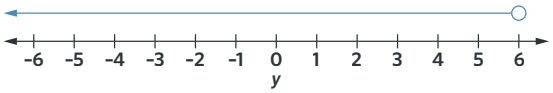


c



4 **a** $x < 2$ **b** $1.5 < y \leq 4$

5 **a**



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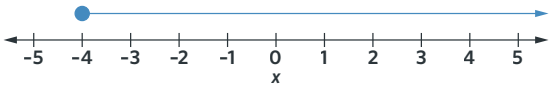

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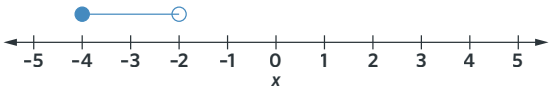

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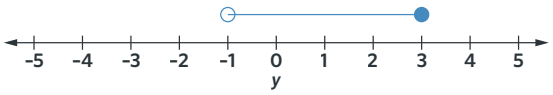
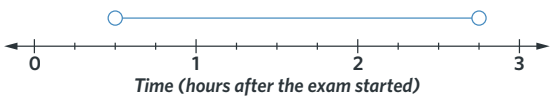


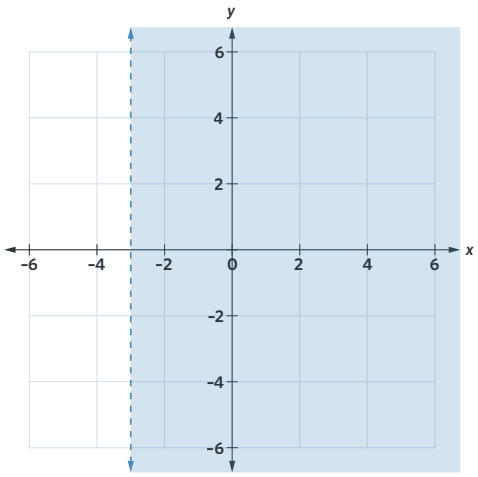
6 E

7



8 False

9 **a**



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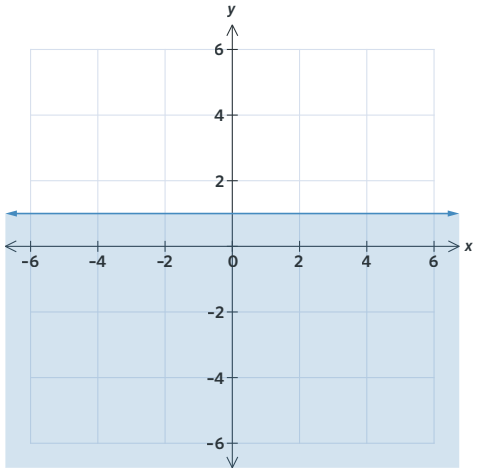

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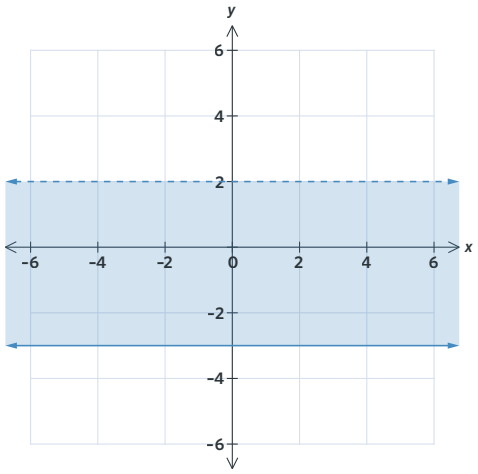
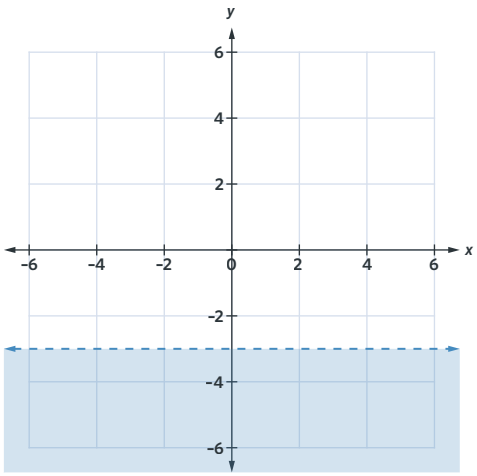


10 **a** $x \leq 1$

b $-4 < y < 3$

11 **a**



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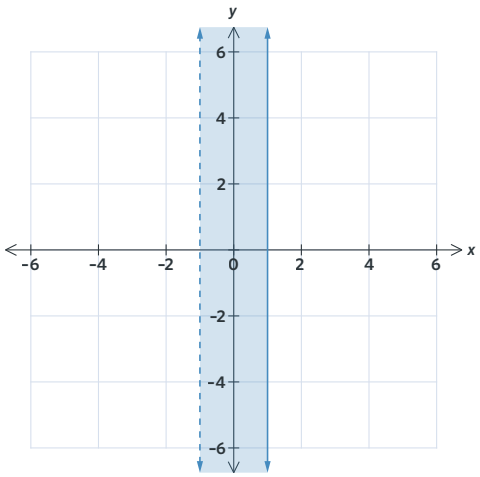

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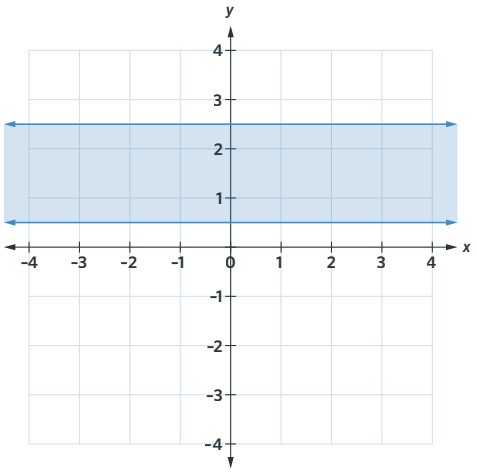

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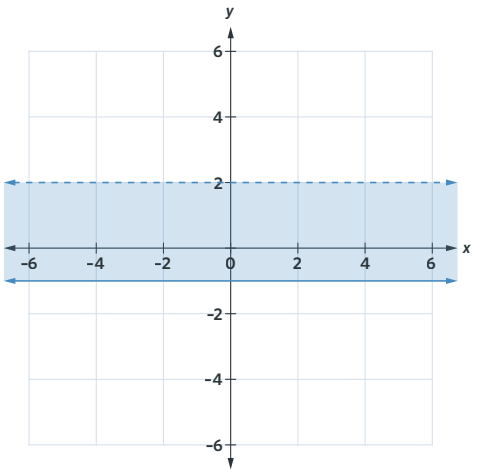
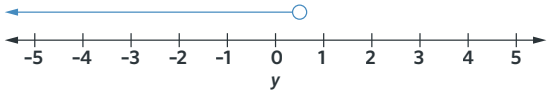
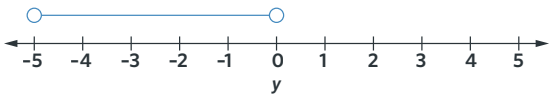


12 C

13 **a**



b



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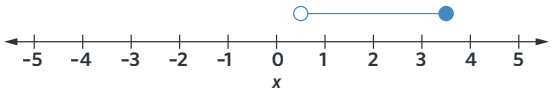
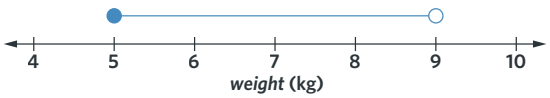


14 C

15



16 The inequality shown on the set of axes is: $-3 < y < 1$

Questions from multiple lessons

17 *Graphs and relations*

Decoding the question

Keywords: line segment graph, speed

Relevant lessons: 9B, 9G

This question requires us to use the given graph to determine the speed Torri swam in her leg of the race.

Information we are given

- The given graph
- The given table

Using the theory

Torri's speed can be determined by calculating the slope for her leg of the race.

From the graph, we can see that Torri's part would extend from (113, 200) to (171, 300).

The slope of the line can be calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Using the points (113, 200) and (171, 300), substitute $y_2 = 300$, $y_1 = 200$, $x_2 = 171$ and $x_1 = 113$ into $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{300 - 200}{171 - 113}$

$m = \frac{100}{58}$

$m = 1.724...$

$m \approx 1.7$

To determine the units, we can look at the axes of the given graph. Time is expressed in seconds and distance is expressed in metres. Since speed is expressed as a function in relation to distance over time, the units must be metres per second.

∴ Torri's speed is closest to 1.7 m/s.

Answer

C

✓1 mark for the correct answer

18 *Data analysis*

Decoding the question

Keywords: stem plot, percentage

Relevant lesson: 12C

We must count the number of values greater than 10 minutes and represent this as a percentage of the total number of values.

Information we are given

- The given stem plot
- The sample contains 20 people.

Using the theory

Find the number of values greater than 10. These are the numbers highlighted below.

Key: 0 | 6 = 6 minutes

0	6 7 7 8 8 9
1	0 0 0 1 3 7 8
2	2 2 4 5 7
3	1 3

There are 11 values larger than 10. Represent this as a percentage of the total number of values.

$percentage = \frac{frequency}{total\ frequency} \times 100$

$percentage = \frac{11}{20} \times 100$

$percentage = 55\%$

∴ 55% of students spent more than 10 minutes eating breakfast.

Answer

C

✓1 mark for the correct answer

19a *Graphs and relations*

Decoding the question

Keywords: piecewise linear model, graph, equation

Relevant lessons: 9B, 9G

We must find the value of m using the coordinates of the point displayed on the graph and the given equation.

Information we are given

- The given graph
- $y = mx + 6$ where $-7 \leq x < -0.8$

Using the theory

Substitute the point $(-0.8, 4.4)$ into the equation $y = mx + 6$ and solve for m

$4.4 = m \times (-0.8) + 6$

$4.4 - 6 = -0.8m$

$-1.6 = -0.8m$

$m = \frac{-1.6}{-0.8}$

$m = 2$

Answer

2

✓1 mark for the correct answer

19b ***Graphs and relations***

Decoding the question

Keywords: piecewise linear model, graph, equation

Relevant lessons: 9B, 9G

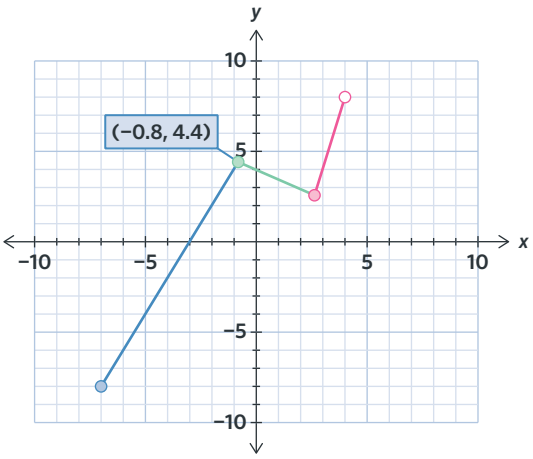
We need to determine the endpoints of the missing section and use them to find its equation.

Information we are given

- The given graph
- $y = 4x - 8$ where $\frac{8}{3} \leq x < 4$

Using the theory

The missing section of the model is drawn in green in the following graph.



To determine the equation of the line, we require the coordinates of two points.

The left endpoint is $(-0.8, 4.4)$.

The right endpoint will be the left endpoint of the pink line. Since the domain of the pink line is $\frac{8}{3} \leq x < 4$, the left endpoint of the pink line is located at $x = \frac{8}{3}$

Substitute $x = \frac{8}{3}$ into $y = 4x - 8$ to work out the y-coordinate of the endpoint.

$y = 4 \times \frac{8}{3} - 8$

$y = \frac{32}{3} - 8$

$y = \frac{8}{3}$

Hence, the two endpoints of the missing section of the model are $(-0.8, 4.4)$ and $\left(\frac{8}{3}\ \frac{8}{3}\right)$

Let $y = mx + c$ be the equation of the missing section of the model.

Determine the value of m using the formula

$m = \frac{y_2 - y_1}{x_2 - x_1}$

Let $(x_1\ y_1) = (-0.8, 4.4)$ and $(x_2\ y_2) = \left(\frac{8}{3}\ \frac{8}{3}\right)$

$m = \frac{\frac{8}{3} - 4.4}{\frac{8}{3} - (-0.8)}$

$m = -0.5$

Hence, the equation becomes $y = -0.5x + c$

We can now find the value of c by substituting $(-0.8, 4.4)$ into the equation $y = -0.5x + c$

$4.4 = -0.5 \times -0.8 + c$

$4.4 = 0.4 + c$

$c = 4$

Substitute $c = 4$ into the equation $y = -0.5x + c$

$y = -0.5x + 4$

Answer

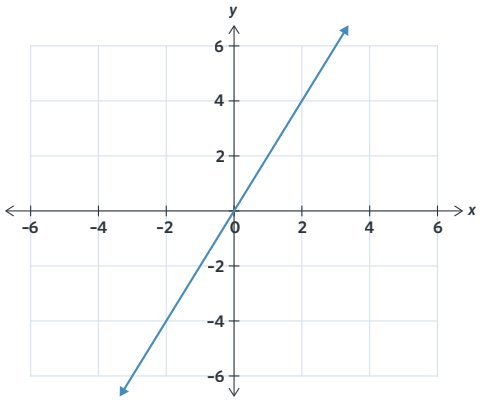
$y = -0.5x + 4$

✓1 mark for finding the endpoint coordinates
✓1 mark for the correct equation

10B

1 **a** 0

b



2 **a** $1 > 0$

b True

3 **a** Yes

b No

c Yes

4 D

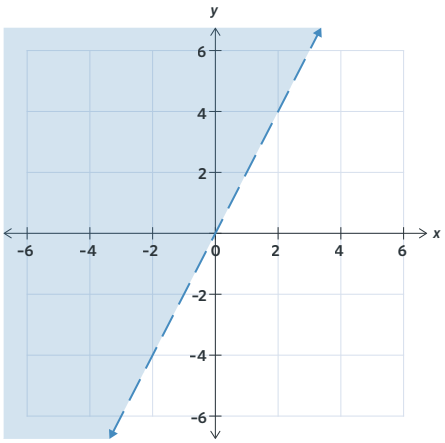
5 **a** 2

b 9

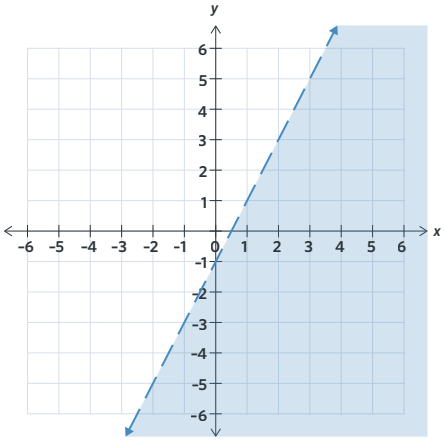
c $n \geq \frac{1}{4}p$ or $4n \geq p$

d No

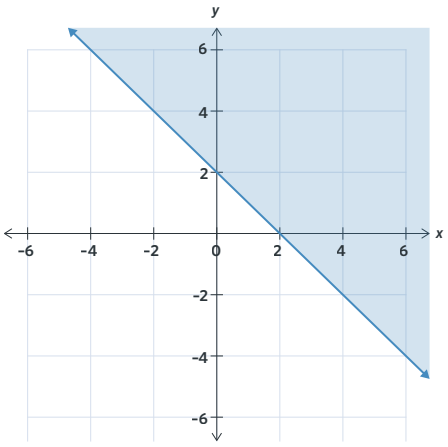
6



7 **a**



b



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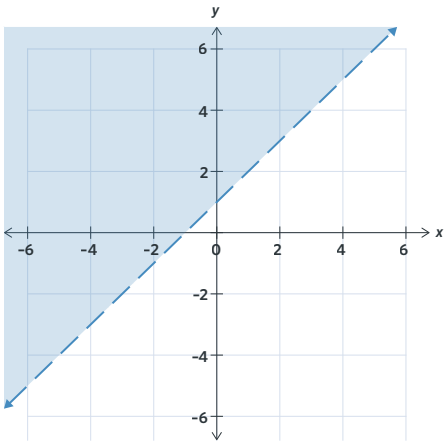
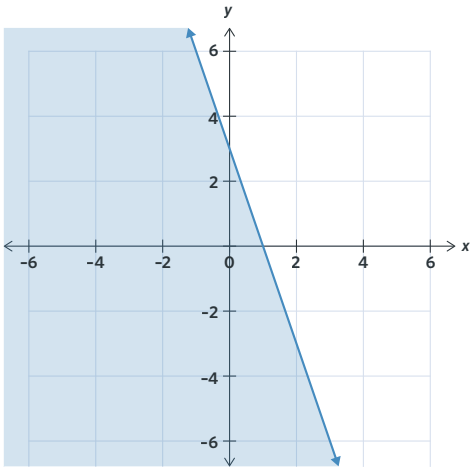


8 **a**



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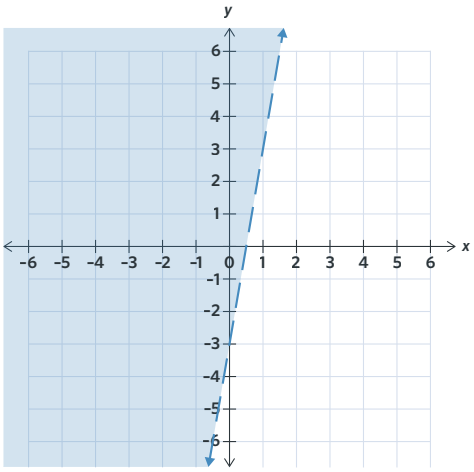

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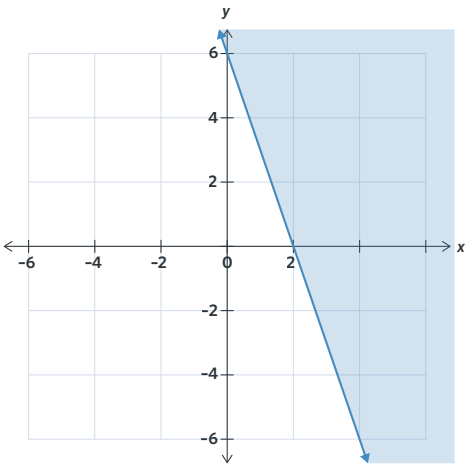


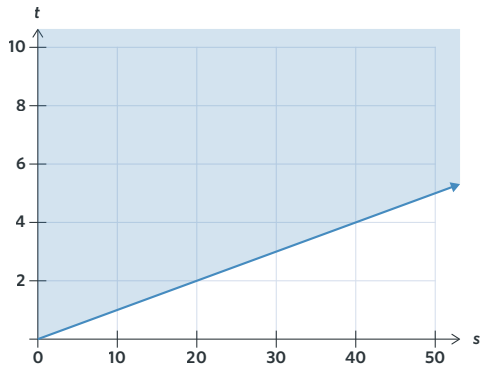
9 **a** $y < -x + 3$ **b** $y \geq 3x - 6$ **c** $y > -2x + 5$

10 D

11 **a** No **b** Yes

12 **a** $t \geq \frac{1}{10}s$ or $10t \geq s$

b



c 3

13 D

Questions from multiple lessons

14 *Graphs and relations*

Decoding the question

Keywords: relationship, inequality, 'at least'

Relevant lesson: 10B

We must find the inequality that represents the worded situation.

Information we are given

- There are at least three all female acts for every five all male acts.
- m represents the number of all male acts.
- f represents the number of all female acts.

Using the theory

First, consider the ratio of three female acts to five male acts.

$f : m$

$3 : 5$

Represent this as an equation.

$\frac{f}{m} = \frac{3}{5}$

$f = \frac{3}{5}m$

$5f = 3m$

The phrase "at least three female acts for every five male acts" requires the sign $\geq$ since females acts (f) are on the left-hand side of the equation and male acts (m) are on the right.

This means that the expression becomes $5f \geq 3m$

This can also be written as $3m \leq 5f$

Answer

C

✓1 mark for the correct answer

15 *Data analysis*

Decoding the question

Keywords: standard deviation, mean

Relevant lessons: 12E, 12F

We will use the calculator to find the standard deviation and mean of the weight of the nine koalas.

Information we are given

- The given table

Using the theory

Open a spreadsheet on your calculator. Enter the weight data into a column and use the one variable statistics calculations to find the mean and standard deviation of a sample.

The values should be $\bar{} = 6.42$ and $s_x = 1.90$.

See video solution for a button-by-button demonstration.

Answer

C

✓1 mark for the correct answer

16a *Graphs and relations*

Decoding the question

Keywords: meaning, inequality

Relevant lesson: 10A

This question requires us to interpret a given inequality of one variable in the context of the problem.

Information we are given

- x is the number of pens produced each day.
- Inequality 1: $x \geq 50$

Using the theory

x is the number of pens produced per day.

The $\geq$ sign means greater or equal to.

$\therefore$ The inequality $x \geq 50$ indicates that the number of pens produced per day must be greater or equal to 50.

Answer

At least 50 pens are produced per day.

✓1 mark for the correct answer

16b *Graphs and relations*

Decoding the question

Keywords: state, value

Relevant lessons: 9A, 9B

This question requires us to determine an unknown equation when given its graph.

Information we are given

- The given graph
- $y = a$

Using the theory

There are two lines plotted on the graph; $x = 50$ and $y = a$

The horizontal line must be the $y = a$ graph.

The horizontal line has a y-intercept of 140, hence the equation of the line is $y = 140$.

$\therefore a = 140$

Answer

$a = 140$

✓1 mark for the correct answer

16c ***Graphs and relations***

Decoding the question

Keywords: graph, draw

Relevant lessons: 9A, 9B

This question requires us to plot a line on a given set of axes.

Information we are given

- The given graph
- $x + y = 300$

Using the theory

Transpose $x + y = 300$ into graphical form.

$y = 300 - x$

To find the y-intercept, set $x = 0$.

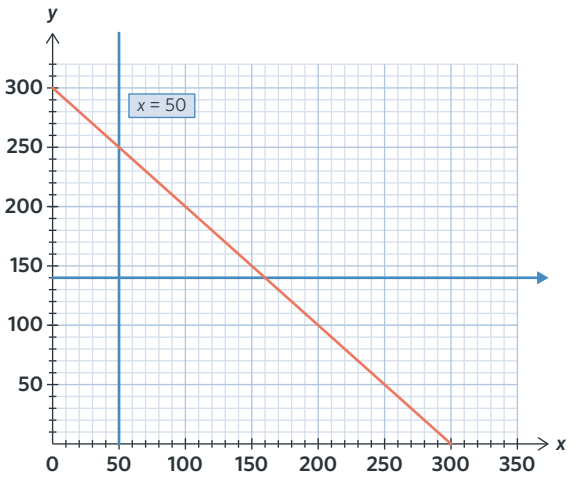
$y = 300$

To find the x-intercept, set $y = 0$.

$x = 300$

Plot the two intercepts and draw a straight line between them. This is the graph of $x + y = 300$.

Answer

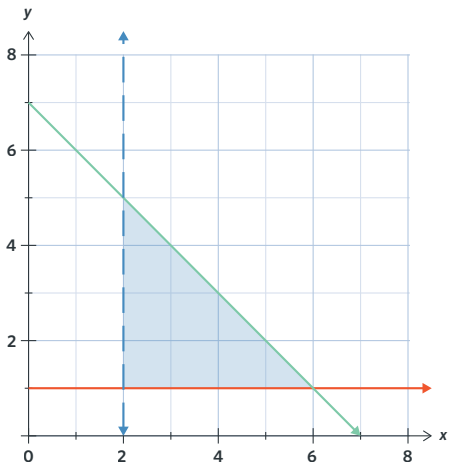


✓1 mark for the correct answer

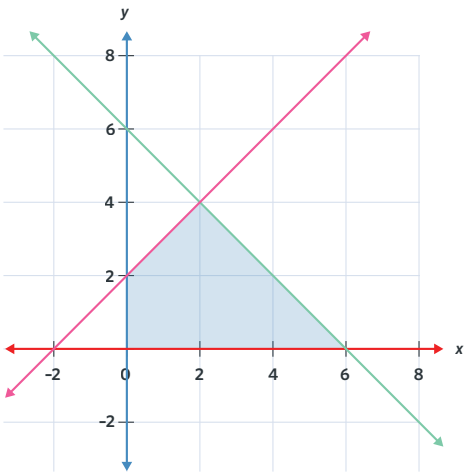
10C

1 $y > 12 - \frac{3}{2}x$

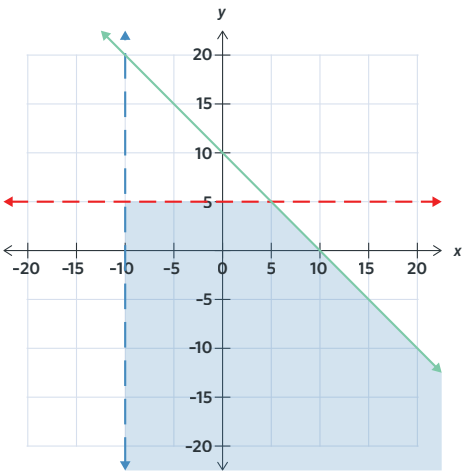
2



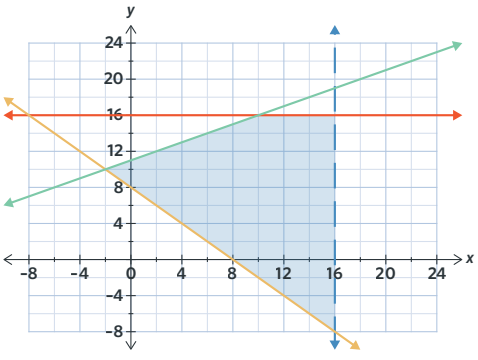
3 **a**



b



c



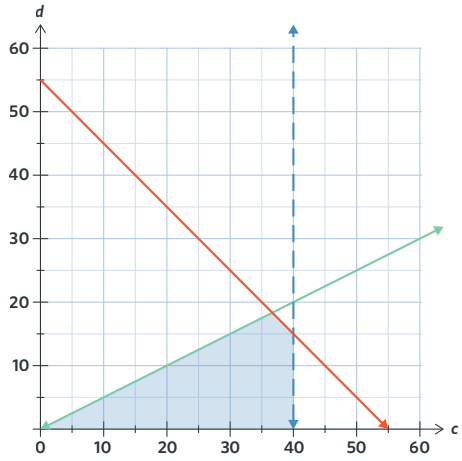
4 **a** Inside **b** Outside **c** Outside
d Inside **e** Outside

5 $y < x + 4$ or $y - x < 4$

6 **a** $a \geq 18$ **b** $175 \leq h < 185$
c $e + c \leq 120$

7 **a** $c \geq 2d$
$c < 40$
$c + d \leq 55$

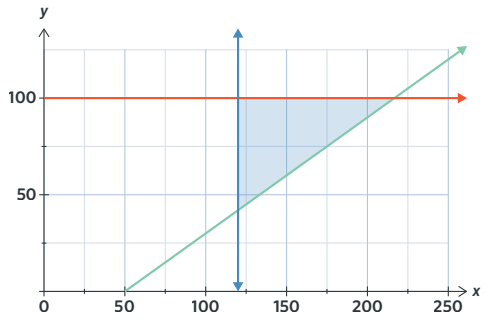
b



8 More than 30 macarons must be sold each day.
The number of croissants to be sold each day is between 20 and 100, inclusive.
The total number of pastries sold cannot exceed 150.

9 C

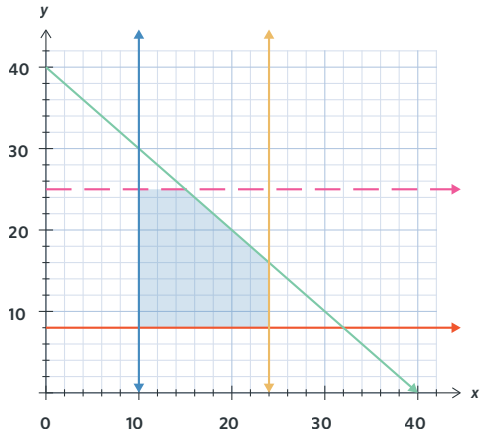
10 **a**



b 216.7 cm

11 D

12 **a**



b Yes

13 E

14 C

Questions from multiple lessons

15 ***Graphs and relations***

Decoding the question

Keyword: inequalities

Relevant lesson: 10B

We must find an inequality to describe the situation where one music teacher is required for every eight students, in terms of x and y.

Information we are given

- At least one music teacher is required for every eight students.
- x is the number of music teachers.
- y is the number of students.

Using the theory

First, consider the ratio of one music teacher to every eight students.

$x : y$

$1 : 8$

Represent this as an equation.

$\frac{y}{x} = \frac{8}{1}$

$y = 8x$

The phrase "at least one music teacher for every eight students" requires the $\leq$ sign since music teachers are on the right-hand side of the equation and students are on the left. This means that the expression becomes $y \leq 8x$.

Answer

E

✓*1 mark for the correct answer*

16 ***Recursion and financial modelling***

Decoding the question

Keywords: recurrence relation, model

Relevant lessons: 3C, 6B

The given recurrence relation must be analysed in order to determine the potential scenario that it could be used to model.

Information we are given

- $M_0 = 50\,000, \quad M_{n+1} = M_n + 3250$

Using the theory

First, we will determine whether the relation models a simple interest or compound interest investment.

The recurrence relation increases by a flat rate of \$3250 each period. This means it doesn't increase by a percentage of the current value, and instead must continually increase by a percentage of the principal investment. Hence, the recurrence relation is modelling a simple interest investment. Options D and E can be eliminated.

$M_0 = 50\ 000$ shows that the initial value of the investment is \$50 000. Option C can be eliminated.

To find the annual interest rate, represent 3250 as a percentage of 50 000.

$$percentage = \frac{3250}{50\ 000} \times 100$$

$$percentage = 6.5\%$$

∴ The recurrence relation could be used to model a simple interest investment of \$50 000 with an annual interest rate of 6.5%.

Answer

B

✓1 mark for the correct answer

17a ***Graphs and relations***

Decoding the question

Keywords: interpret, constraint, in terms of

Relevant lessons: 10B, 10C

We need to interpret the given constraint in the context of the question.

Information we are given

- x is the number of blackcurrant juices sold in one day.
- y is the number of apple juices sold in one day.
- Constraint 4: $x \leq 2y$

Using the theory

Constraint 4 states that $x \leq 2y$.

If this was an equality, it could be written as $x = 2y$, which means that the value of x will always be twice that of y. Hence, the inequality $x \leq 2y$ means that x will always be less than or equal to twice that of y.

As x represents blackcurrant juices and y represents apple juices, the number of blackcurrant juices sold in a day will always be less than or equal to twice the number of apple juices sold.

Answer

The number of blackcurrant juices sold in a day must be no more than twice the number of apple juices.

✓1 mark for the correct answer

17b ***Graphs and relations***

Decoding the question

Keywords: constraint, represents, inequality

Relevant lessons: 10B, 10C

We need to determine the inequality shown in the graph.

Information we are given

- The given graph
- Constraint 5 is bounded by line C

Using the theory

First, ignore any inequality properties and just consider the line. At every point on Line C, the x-coordinate is 120. Hence, the equation of line C is $x = 120$. As line C is an inequality, we need to determine the appropriate sign to apply in this situation.

The feasible region lies on the right-hand side of this inequality, so the two signs that can be used are $\geq$ and $>$. As the inequality is drawn with a solid line rather than a dotted one, the $\geq$ sign is the appropriate sign to use.

∴ The inequality is $x \geq 120$.

Answer

$x \geq 120$

✓1 mark for the correct answer

10D

1 $(1.25, 0.25)$ or $\left(\frac{5}{4}\ \frac{1}{4}\right)$

2 A: $(3, 4)$

B: $(-4, 11)$

3 **a** $(-4, -2), (4, -2), (0.6, 4.9)$

b $(-1, 0), (4.5, 0), (-1, 4.5), (1.6, 5.8)$

c $(-4, 8), (-4, -6), (1.2, -2.4), (0.8, -3.6)$

4 **a** $y \leq x + 4$

b $\left(\frac{-7}{3}, \frac{5}{3}\right), \left(\frac{3}{2}, -6\right), \left(\frac{8}{5}, \frac{28}{5}\right), \left(\frac{90}{13}, \frac{-31}{13}\right)$

5 Maximum: 11

Minimum: −18

6 **a** Minimum: −8

Maximum: 16

b Minimum: −12

Maximum: 13.5

c Minimum: −28

Maximum: 9.6

7 **a** **i** $2 < S < 25$

ii $2 < L < 20$

iii $S + L \leq 40$

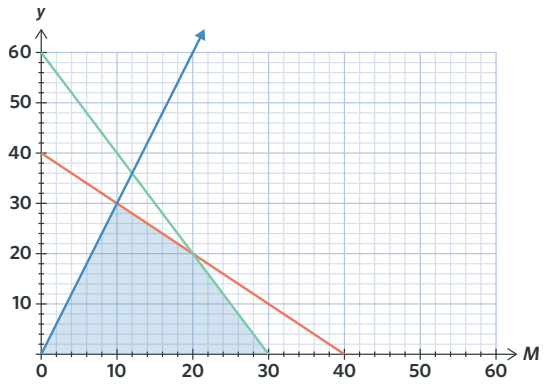
b \$2685

8 D

9 704 m^2

10 **a** The number of pieces of banana bread sold was less than or equal to three times the number of blueberry muffins sold.

b



c $R = 3M + 2B$ **d** $100

11 $y \leq -0.5x + 5$

Questions from multiple lessons

12 *Graphs and relations*

Decoding the question

Keywords: feasible region, inequalities, within

Relevant lesson: 10C

This question requires us to identify which of the options fit within the feasible region.

Information we are given

- $x < 14$
- $y < 8$
- $x + y > 18$

Using the theory

This question can be solved without graphing the feasible region and simply checking if each point fits all three inequalities provided.

$(15, 4)$ does not lie within the feasible region as $x < 14$ and this point has $x = 15$. Option A can be eliminated.

$(7, 12)$ and $(10, 13)$ do not lie within the feasible region as $y < 8$ and these points have $y = 12$ and $y = 13$ respectively. Both options B and C can be eliminated.

$(12, 7)$ does lie within the feasible region as all inequalities have been fulfilled. $12 < 14$, $7 < 8$ and $12 + 7 = 19 > 18$. Option D lies in the feasible region.

To make sure, we will still look at option E.

$(12, 6)$ doesn't lie within the feasible region. $x < 14$ and $y < 8$ are satisfied, however, $x + y > 18$ is not. Replacing x and y with this point gives us $12 + 6 = 18$ which is not greater than 18. Option E can be eliminated.

NOTE: Graphing each of the three inequalities to observe the feasible region is another way to solve this question.

Answer

D

✓1 mark for the correct answer

13 *Data analysis*

Decoding the question

Keyword: variables

Relevant lesson: 12A

We need to categorise the given variables as either numerical, nominal or ordinal.

Information we are given

- *Weight* (less than 1 kg, 1–2 kg, over 2 kg)
- *Size* (small, medium, large)

Using the theory

Firstly, categorise each variable as either numerical or categorical. If a variable is categorical, determine whether it is nominal or ordinal.

The variable *weight* on its own would classify as numerical. However, in this example, it is broken into three categories (less than 1 kg, 1–2 kg, over 2 kg), so it is a categorical variable. Since the categories 'less than 1 kg', '1–2 kg' and 'over 2 kg' follow a logical order, *weight* is an ordinal variable.

NOTE: Just because a variable contains numbers, it doesn't immediately classify as a numerical variable.

The variable *size* is broken into three categories (small, medium, large), so it is also a categorical variable. Since the categories 'small', 'medium' and 'large' follow a logical order, *size* is an ordinal variable.

The variables *weight* (less than 1 kg, 1–2 kg, over 2 kg) and *size* (small, medium, large) are both ordinal variables.

Answer

E

✓1 mark for the correct answer

14a *Graphs and relations*

Decoding the question

Keywords: describe, inequality, in terms of

Relevant lesson: 10B

We need to explain the inequality in terms of the given variables.

Information we are given

- x is the amount of red paint used, in litres.
- y is the amount of white paint used, in litres.
- Inequality 4: $x \geq 1$

Using the theory

The $\geq$ sign means greater than or equal to. Hence, x has to be greater than or equal to 1.

As x represents the amount of red paint used, in litres, the amount of red paint used must be at least one litre.

Answer

Antonio requires at least one litre of red paint.

✓1 mark for the correct answer

14b *Graphs and relations*

Decoding the question

Keywords: line, boundary, inequality, region

Relevant lesson: 10C

We need to interpret the inequalities to identify the feasible region on the provided graph.

Information we are given

- Inequality 1: $x + y \geq 5$
- Inequality 2: $x + y \leq 7$
- Inequality 3: $y \geq 2x$
- Inequality 4: $x \geq 1$
- The given graph

Using the theory

Inequality 1: $x + y \geq 5$

Since the total amount of paint used must be greater than or equal to 5 litres, the feasible region will lie above this line.

Inequality 2: $x + y \leq 7$

Since the total amount of paint used must be less than or equal to 7 litres, the feasible region will lie below this line.

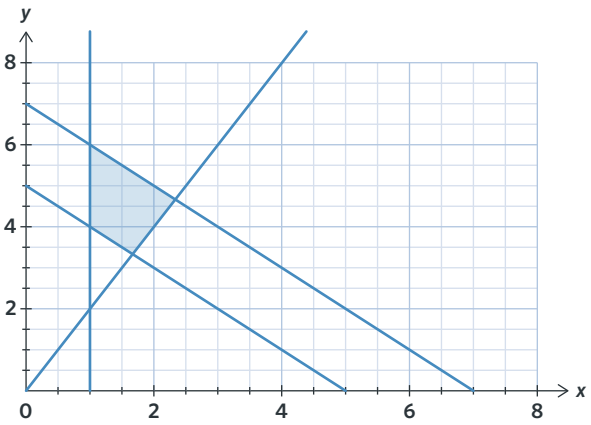
Inequality 3: $y \geq 2x$

Since the amount of white paint must be greater than or equal to twice the amount of red paint, the feasible region will lie above this line.

Inequality 4: $x \geq 1$

Since the amount of red paint used must be greater than or equal to 1 litre, the feasible region will lie to the right of this line.

Answer

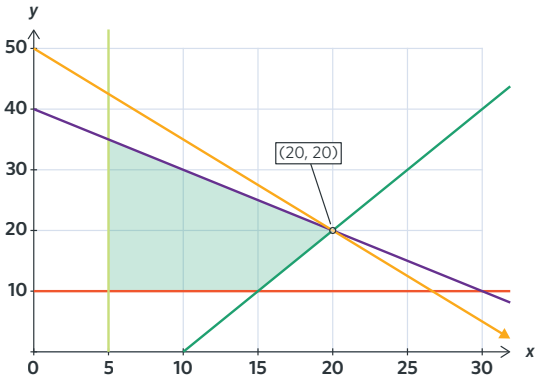


✓1 mark for the correct answer

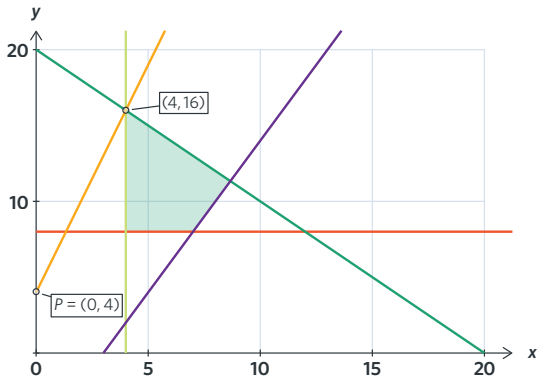
10E

1 B

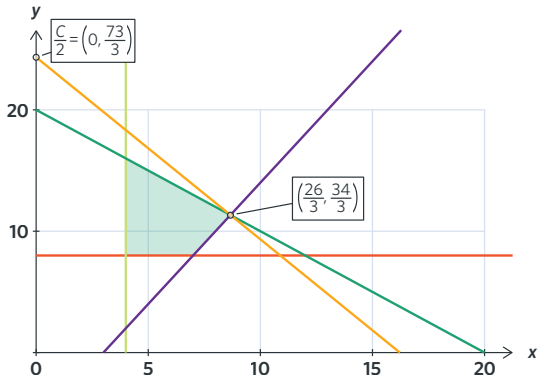
2 $P = 25$



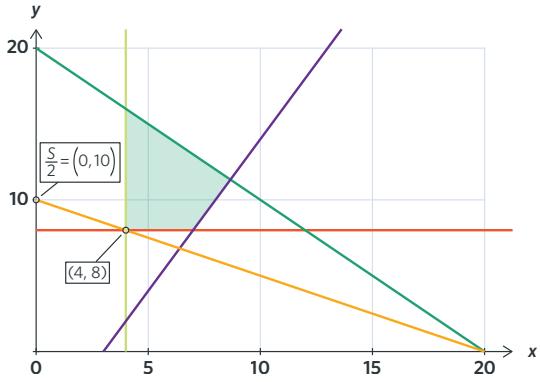
3 **a** The maximum is $P = 4$ which occurs at $(4, 16)$



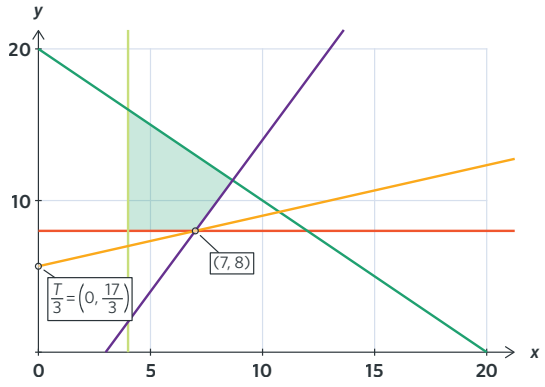
b The maximum is $C = \frac{146}{3}$ which occurs at $\left(\frac{26}{3}, \frac{34}{3}\right)$



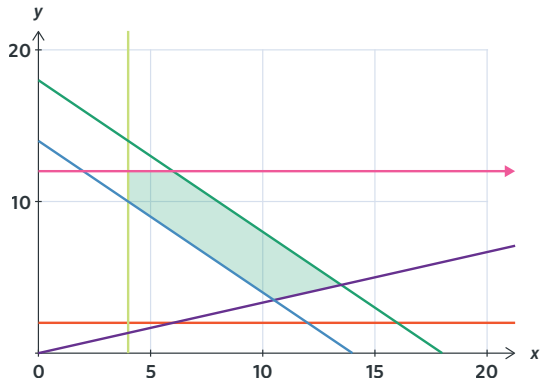
c The minimum is $S = 20$ which occurs at $(4, 8)$



d The minimum is $T = 17$ which occurs at $(7, 8)$



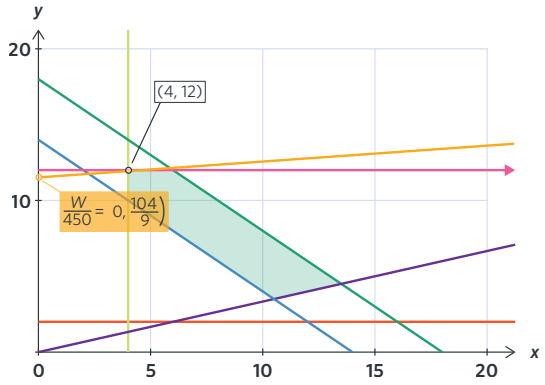
4 a



b $W = 500y - 50(x + y)$

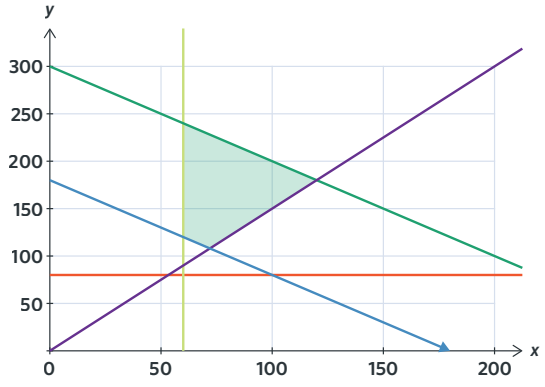
$y = \frac{1}{9}x + \frac{1}{450}W$

The maximum amount of money is $5200.

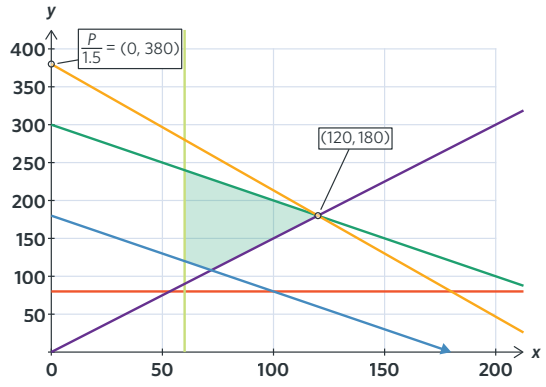


5 Set 1: B, Set 2: C, Set 3: A, Set 4: D

6 a

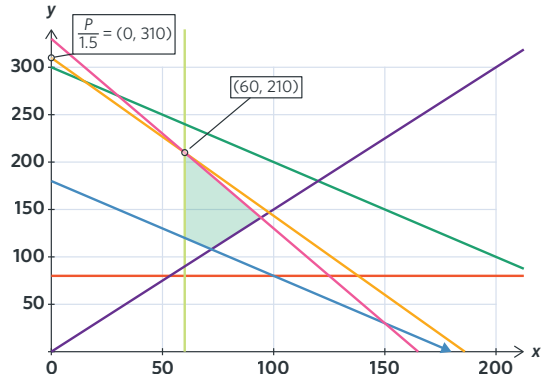


b 120 muffins, 180 brownies, $P = 570$



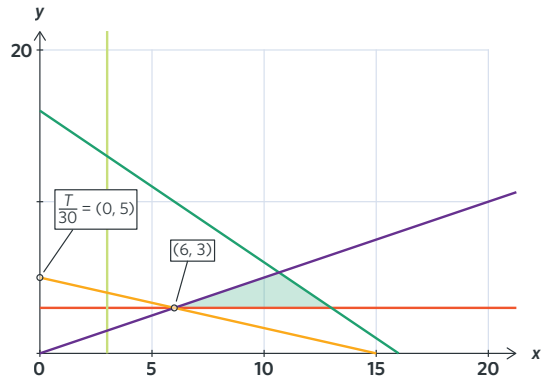
c $4x + 2y \leq 660$ or $2x + y \leq 330$

d 60 muffins, 210 brownies, $P = 465$



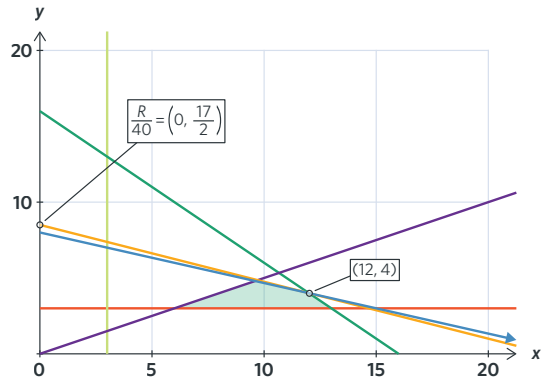
7 a $T = 10x + 30y$

b Minimum value of T is 150 minutes. Tina will work a minimum of 2.5 hours.



c $R = 15x + 40y$

d 12 express cuts, 4 salon cuts. $R = 340$



8 The objective function was $P = 1300x + 2100y$.
Substituting (2, 7) gave the maximum profit of $17 300.

Questions from multiple lessons

9 ***Graphs and relations***

Decoding the question

Keywords: linear programming, inequalities, boundaries, feasible region

Relevant lessons: 10C, 10D

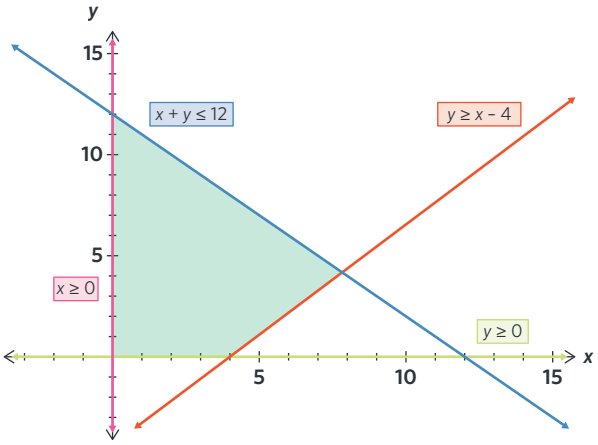
We must plot the set of inequalities to find the coordinates that define the boundary of the feasible region.

Information we are given

- $x + y \leq 12$
- $y \geq x - 4$
- $x \geq 0$
- $y \geq 0$

Using the theory

Plot all inequalities on your calculator or by hand.



The vertices of the feasible region are:

The origin, (0, 0).

The intersection of $x \geq 0$ and $x + y \leq 12$.
To find this point, substitute $x = 0$ into $x + y = 12$.
$0 + y = 12$
$y = 12$
The point is (0, 12).

The intersection of $y \geq 0$ and $y \geq x - 4$.
To find this point substitute $y = 0$ into $y = x - 4$.
$0 = x - 4$
$x = 4$
The point is (4, 0).

The intersection of $x + y \leq 12$ and $y \geq x - 4$.
To find this point, substitute $y = x - 4$ into $x + y = 12$.
$x + x - 4 = 12$
$2x = 16$
$x = 8$

Substitute $x = 8$ into $x + y = 12$.
$8 + y = 12$
$y = 4$

The point is (8, 4).

The coordinates of the points that define the boundaries of the feasible region are (0, 0), (0, 12), (4, 0), (8, 4).

Answer

E

✓1 mark for the correct answer

10 ***Data analysis***

Decoding the question

Keywords: histogram, distribution, shape

Relevant lesson: 12D

We must look at the data displayed in the graph to determine the shape of the distribution.

Information we are given

- The given histogram

Using the theory

There are substantially more data values on the right side of the histogram than there are on the left. Therefore, the distribution cannot be approximately normal or symmetric and options A, B, and E are incorrect.

Since the bulk of the data is on the right and the tail of the distribution points to the left, the distribution is negatively skewed.

Answer

C

✓1 mark for the correct answer

11a ***Graphs and relations***

Decoding the question

Keywords: explain, in terms of

Relevant lesson: 10B

We need to interpret the inequality in terms of the given variables.

Information we are given

- x is the number of cappuccinos sold each day.
- y is the number of lattes sold each day.
- Each cappuccino costs $4 and each latte costs $3.50.
- Constraint 3: $4x + 3.5y \geq 200$

Using the theory

The equation $4x + 3.5y$ represents the total revenue earned from selling cappuccinos and lattes. This is because the coefficients 4 and 3.5 are the prices of cappuccinos and lattes respectively, and x and y represent the number of cappuccinos and lattes sold respectively.

For $4x + 3.5y \geq 200$, the sign $\geq$ means greater than or equal to, hence the total revenue earned selling coffees has to be greater than or equal to 200.

Answer

The revenue earned selling cappuccinos and lattes must be at least \$200 each day.

✓1 mark for the correct answer

11b *Graphs and relations*

Decoding the question

Keywords: graph, line, draw

Relevant lesson: 10B

We need to plot the line $4x + 3.5y = 200$ onto the given graph.

Information we are given

- The given graph

Using the theory

Determine the x-intercept and y-intercept of the given line.

The x-intercept occurs when $y = 0$, so substitute $y = 0$ into the equation and solve for x.

$4x + 3.5 \times 0 = 200$

$4x = 200$

$x = \frac{200}{4}$

$x = 50$

The point $(50, 0)$ lies on the graph.

The y-intercept occurs when $x = 0$, so substitute $x = 0$ into the equation and solve for y.

$4 \times 0 + 3.5y = 200$

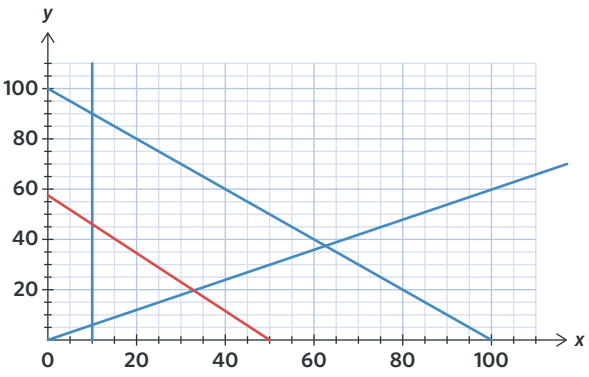
$3.5y = 200$

$y = \frac{400}{7}$

The point $\left(0, \frac{400}{7}\right)$ lies on the graph.

Plot these two points on the graph and draw a line to connect them.

Answer



✓1 mark for the correct answer

11c *Graphs and relations*

Decoding the question

Keyword: maximum

Relevant lesson: 10C

We need to isolate the feasible region on the graph and determine the maximum number of cappuccinos sold each day.

Information we are given

- The given graph
- Constraint 1: $x \geq 10$
- Constraint 2: $5y \geq 3x$
- Constraint 3: $4x + 3.5y \geq 200$
- Constraint 4: $x + y \leq 100$
- x is the number of cappuccinos sold each day.

Using the theory

Find the feasible region.

Inequality 1: $x \geq 10$

Values of x must be greater or equal to 10, hence the feasible region is to the right of the line $x = 10$.

Inequality 2: $5y \geq 3x$

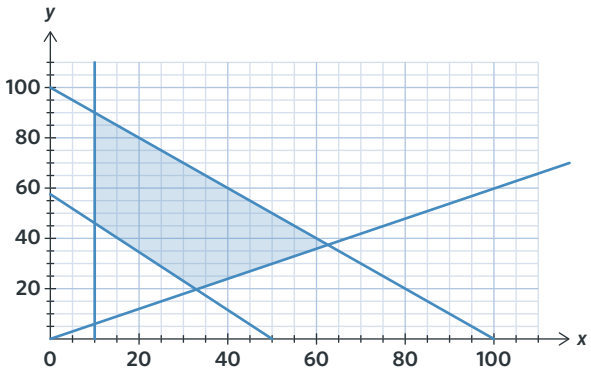
Isolating y gives us $y \geq \frac{3}{5}x$. As a greater or equal to sign follows y, the feasible region is above the line $5y = 3x$.

Inequality 3: $4x + 3.5y \geq 200$

Isolating y gives us $y \geq \frac{1}{3.5}(200 - 4x)$. As a greater or equal to sign follows y, the feasible region is above the line $4x + 3.5y = 200$.

Inequality 4: $x + y \leq 100$

Isolating y gives us $y \leq 100 - x$. As a less than or equal to sign follows y, the feasible region is below the line $x + y = 100$.



x represents the number of cappuccinos sold each day, so we are looking for the largest value of x inside the feasible region. This is the corner point where the lines $3x = 5y$ and $x + y = 100$ intersect.

Consider the two equations $3x = 5y$ and $x + y = 100$. Previously, we isolated the y in the equation $x + y = 100$, and obtained $y = 100 - x$.

Substitute $y = 100 - x$ into $3x = 5y$ and solve for x.

$3x = 5(100 - x)$

$3x = 500 - 5x$

$8x = 500$

$x = 62.5$

It is not possible to sell half a cappuccino, so we must round down to 62. If we were to round up to 63, the point would be outside the feasible region.

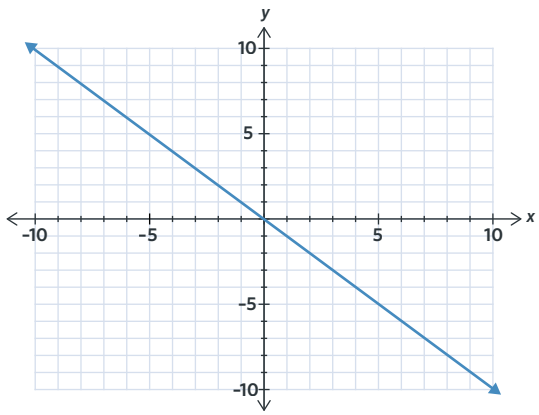
Answer

62 cappuccinos

✓1 mark for the correct answer

11A

1

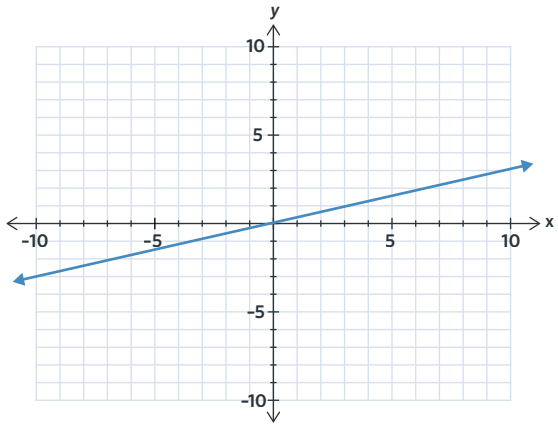


2 **a** Direct **b** Inverse **c** Inverse

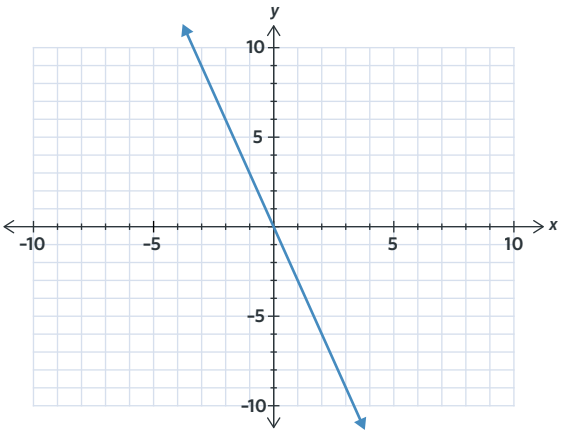
3 No

4 C

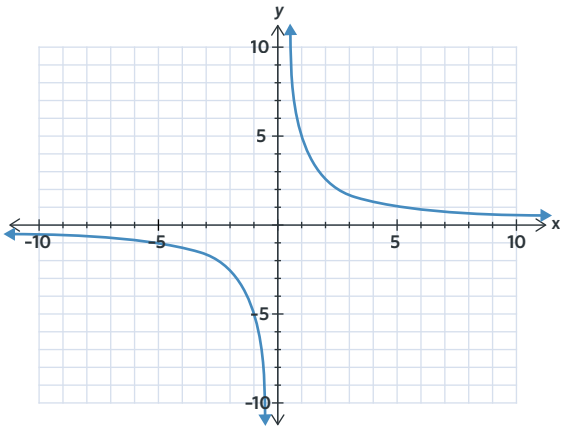
5



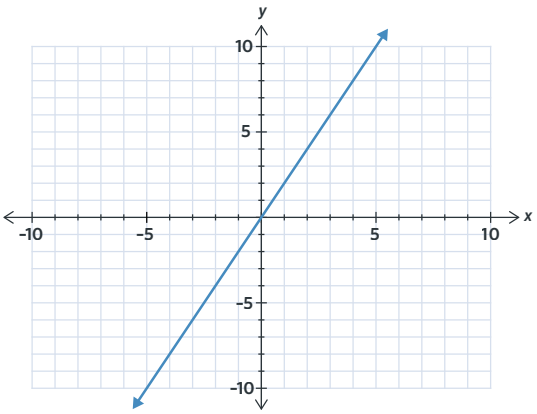
6 **a**



b

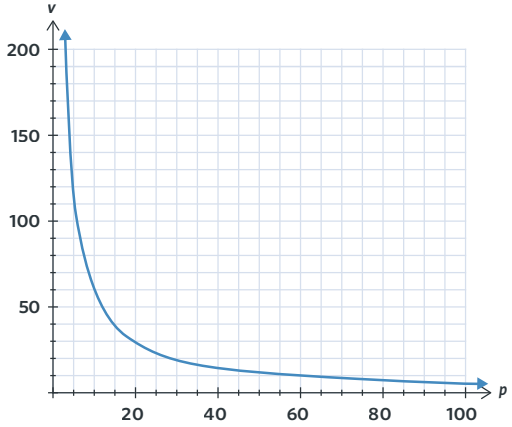


c

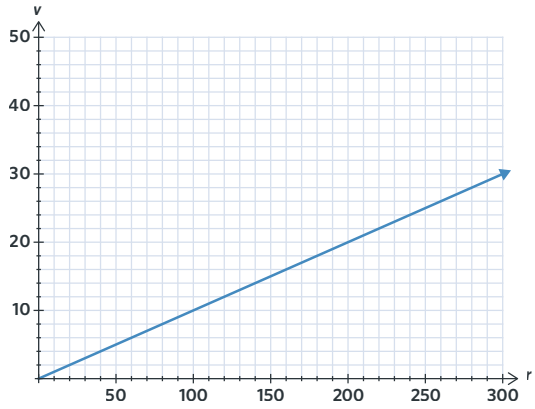


7 $W = 12H$

8 **a**



b

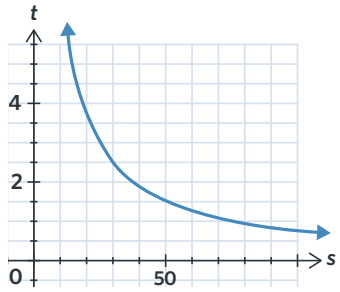


9 D

10 3

11 **a** $t = \frac{75}{s}$

b



c 53.57 km/h **d** 63 minutes

Questions from multiple lessons

12 *Graphs and relations*

Decoding the question

Keywords: points, inequality

Relevant lesson: 10B

We must test all the given points to determine which one does not satisfy the inequality.

Information we are given

- $-2y \geq 3x + 14$

Using the theory

Substitute each of the given points for x and y to determine if they satisfy the inequality $-2y \geq 3x + 14$.

A $(-6, 2)$
$-2 \times 2 \geq 3 \times -6 + 14$
$-4 \geq -18 + 14$
$-4 \geq -4$
True

B $(-4, -8)$
$-2 \times -8 \geq 3 \times -4 + 14$
$16 \geq -12 + 14$
$16 \geq 2$
True

C $(-2, -3)$
$-2 \times -3 \geq 3 \times -2 + 14$
$6 \geq -6 + 14$
$6 \geq 8$
Not true

D $(-3, -5)$
$-2 \times -5 \geq 3 \times -3 + 14$
$10 \geq -9 + 14$
$10 \geq 5$
True

E $(-12, 2)$
$-2 \times 2 \geq 3 \times -12 + 14$
$-4 \geq -36 + 14$
$-4 \geq -22$
True

$\therefore$ The point $(-2, -3)$ does not satisfy the inequality $-2y \geq 3x + 14$.

Answer

C

✓1 mark for the correct answer

13 *Recursion and financial modelling*

Decoding the question

Keywords: loan, interest, compounding monthly

Relevant lesson: 6C

We must determine the amount Mary owes 10 months after establishing a loan if she only pays back the interest charged each month.

Information we are given

- Mary establishes a loan for $1000.
- Interest is charged at a rate of 3.3% per annum, compounding monthly.
- Each month, Mary will only pay for the interest charged that month.

Using the theory

Since Mary only pays back the interest charged each month, the balance of the loan does not change. Hence, the amount that she still owes after 10 months is equal to the initial balance of the loan, which is $1000.

Answer

A

✓1 mark for the correct answer

14a *Graphs and relations*

Decoding the question

Keywords: feasible region, coordinates

Relevant lesson: 10D

We must determine the coordinates of A by finding the point of intersection between two lines.

Information we are given

- The given graph

Using the theory

To find the coordinates of A, find the point of intersection between the lines $y = 0.8x - 2$ and $y = -x + 6$ using the substitution method.

$0.8x - 2 = -x + 6$

$0.8x + x = 6 + 2$

$1.8x = 8$

$x = \frac{40}{9}$

Substitute $x = \frac{40}{9}$ into $y = -x + 6$.

$y = -\frac{40}{9} + 6$

$y = \frac{14}{9}$

∴ The coordinates of point A are $\left(\frac{40}{9}\ \frac{14}{9}\right)$

Answer

$\left(\frac{40}{9}\ \frac{14}{9}\right)$

✓1 mark for the correct answer

14b *Graphs and relations*

Decoding the question

Keywords: feasible region, maximum solution, objective function

Relevant lesson: 10D

We must find the maximum solution of the objective function using the corner-point principle.

Information we are given

- The given graph

Using the theory

The corner-point principle states that the minimum or maximum solution of an objective function will lie on one of the vertices of the feasible region. Therefore, to find the maximum solution, we must substitute each corner point into the objective function $C = -2x + 7y$.

For $\left(\frac{40}{9}\ \frac{14}{9}\right)$:

$C = -2 \times \frac{40}{9} + 7 \times \frac{14}{9}$

$C = \frac{-80}{9} + \frac{98}{9}$

$C = \frac{18}{9}$

$C = 2$

For $(-1, 7)$:

$C = -2 \times (-1) + 7 \times 7$

$C = 2 + 49$

$C = 51$

For $\left(\frac{-60}{11}\ \frac{-70}{11}\right)$:

$C = -2 \times \frac{-60}{11} + 7 \times \frac{-70}{11}$

$C = \frac{120}{11} - \frac{490}{11}$

$C = \frac{-370}{11}$

$C = -33.63...$

$51 > 2 > -33.63...$

∴ The maximum solution of the objective function is 51.

Answer

51

✓1 mark for the correct answer

11B

1 6

2 C

3 **a** $y = \frac{kz}{x}$ **b** $y = \frac{kx^2}{\sqrt[3]{z}}$ **c** $y = \frac{k}{xz}$

4 $E = \frac{1}{2}mv^2$

5 B

6 -24

7 **a** $x = \frac{1}{2}$ or $x = -\frac{1}{2}$

b $k = -10$

c $k = \frac{-1}{9}$

8 **a** $C = 11w$ **b** \$7.70

9 Inverse variation

10 **a** 0.7 **b** 3.15 kg **c** 300 cm

11 C

12 $\frac{1}{2}$

13 D

Questions from multiple lessons

14 *Graphs and relations*

Decoding the question

Keywords: graph, inverse variation, constant of proportionality

Relevant lessons: 11A, 11B

We must use the coordinates displayed in the given graph to find the constant of proportionality.

Information we are given

- The given graph
- The graph displays inverse variation.

Using the theory

The graph displays inverse variation.

$y \propto \frac{1}{x}$

Replace the proportionality sign with the constant of proportionality.

$y = \frac{k}{x}$

Substitute the point $(4, 1)$ into $y = \frac{k}{x}$ and solve for k

$1 = \frac{k}{4}$

$k = 4$

Answer

B

✓1 mark for the correct answer

15 ***Data analysis***

Decoding the question

Keywords: parallel boxplots, false

Relevant lessons: 12G, 12H

We need to analyse the boxplots to determine which of the statements is false.

Information we are given

- The given parallel boxplots

Using the theory

Option A states that nurses have the least variable salary. Nurses' salaries have both the smallest interquartile range and the smallest range out of the three occupations.

Hence, option A is true.

Option B states that the medical professional with the lowest salary is a nurse. The minimum salary (leftmost data point) for each occupation is approximately:

Nurse: $48 000

Dentist: $66 000

Doctor: $167 000

Hence, option B is true.

Option C states that more than 50% of the doctors have a higher salary than all of the dentists and nurses. For this statement to be true, the median salary for a doctor would have to be greater than the maximum salary of a dentist and the maximum salary of a nurse. The median salary for a doctor is approximately $256 000. The maximum salaries for a nurse and dentist are approximately $110 000 and $240 000 respectively.

Hence, option C is true.

Option D states that at least 50% of the dentists have a lower salary than all of the doctors. For this statement to be true, the median salary for a dentist would have to be less than the minimum salary for a doctor. The median dentist salary is approximately $188 000 and the minimum doctor salary is approximately $167 000.

Hence, option D is false.

Option E states that the doctors are the highest-paid in terms of median salary. The median salary for each occupation are approximately:

Nurse: $73 000

Dentist: $188 000

Doctor: $256 000

Hence, option E is true.

Answer

D

✓1 mark for the correct answer

16a ***Graphs and relations***

Decoding the question

Keywords: table of values, equation, line, response variable

Relevant lesson: 9B

We must use the values in the table to find the equation of the line.

Information we are given

- The given table
- The relationship between *width* and *area* is linear.
- The form of the linear equation is $area = m \times width + c$

Using the theory

We are told that the linear equation is in the form $area = m \times width + c$

Find m using the following formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

Let $(x_1 \ y_1) = (2, 5)$ and $(x_2 \ y_2) = (3, 7.5)$

$m = \frac{7.5 - 5}{3 - 2} = \frac{2.5}{1} = 2.5$

Substitute $m = 2.5$ into the equation $area = m \times width + c$

$area = 2.5 \times width + c$

Substitute a point from the table into $area = 2.5 \times width + c$. We will use the point $(2, 5)$.

$5 = 2.5 \times 2 + c$

$5 = 5 + c$

$c = 0$

Substitute $c = 0$ into the equation $area = 2.5 \times width + c$

$area = 2.5 \times width$

Answer

$area = 2.5 \times width$

✓1 mark for the correct answer

16b *Graphs and relations*

Decoding the question

Keywords: table of values, equation, line, response variable, graph, axes

Relevant lesson: 11A

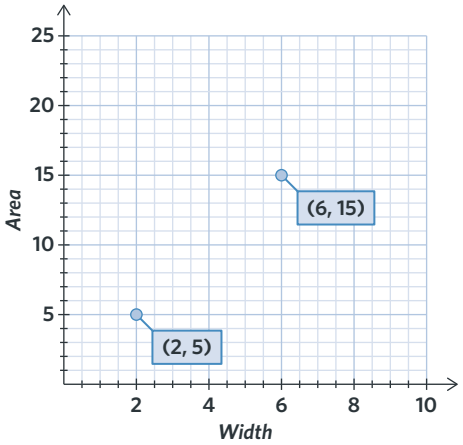
We must plot two points to draw the graph that shows the relationship between *width* and *area*

Information we are given

- The given table
- The relationship between *width* and *area* is linear.
- *Area* is on the vertical axis.

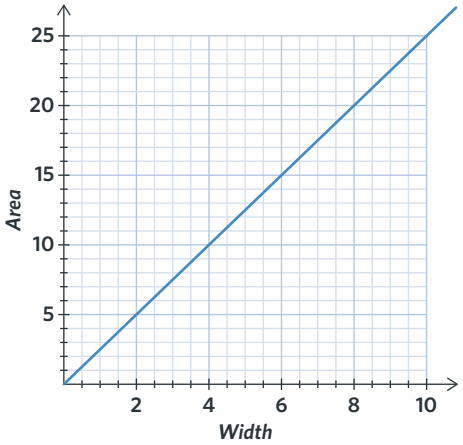
Using the theory

Plot two points from the given table of values. In this solution we will use the points (2, 5) and (6, 15).



Connect the two points with a line. This will be the graph that represents the relationship between *width* and *area*

Answer



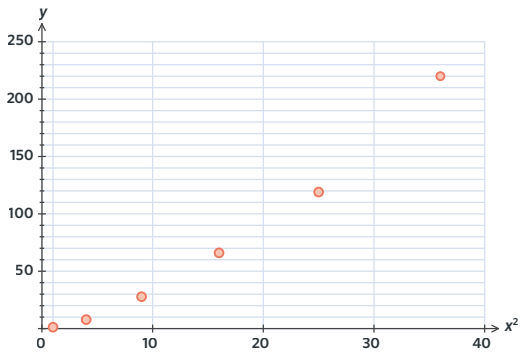
✓1 mark for the correct answer

11C

1 Linear: C, Quadratic: A, Hyperbolic: B

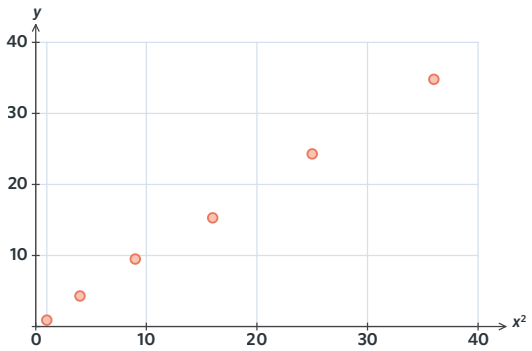
2 No, as the graph isn't linear.

3 **a**



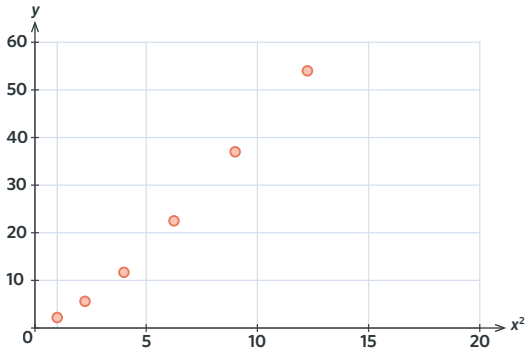
The plot isn't linear, so the transformation isn't appropriate.

b



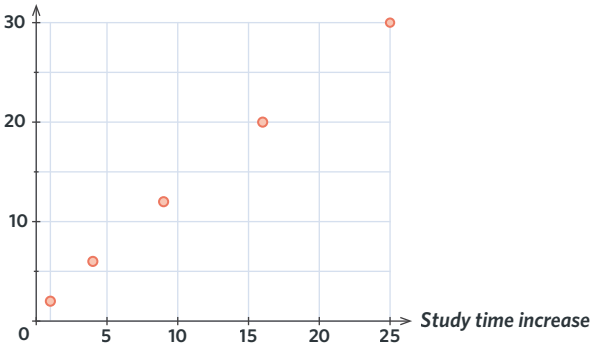
The plot looks linear.
This is an appropriate transformation.

c



The plot isn't linear, so this transformation isn't appropriate.

4 *Exam percentage increase*

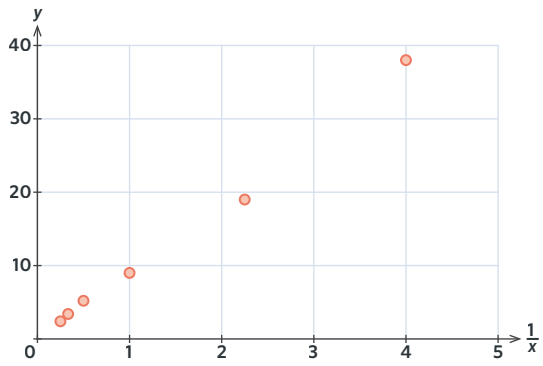


The plot looks linear, so the transformation is appropriate.

5 A

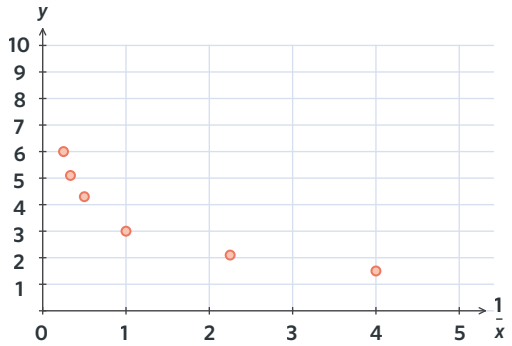
6 Yes

7 **a**



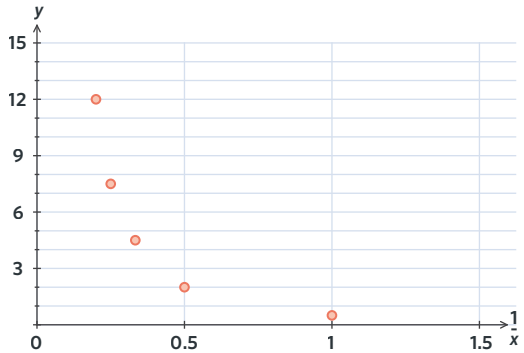
The transformation is appropriate.

b



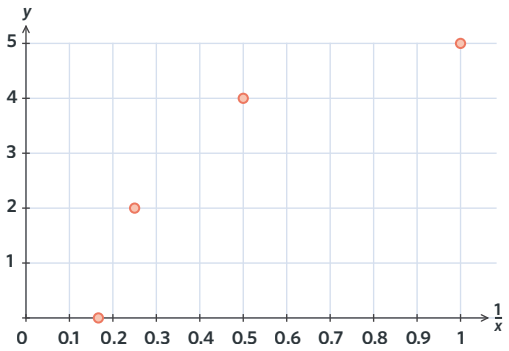
The transformation isn't appropriate.

c



The transformation isn't appropriate.

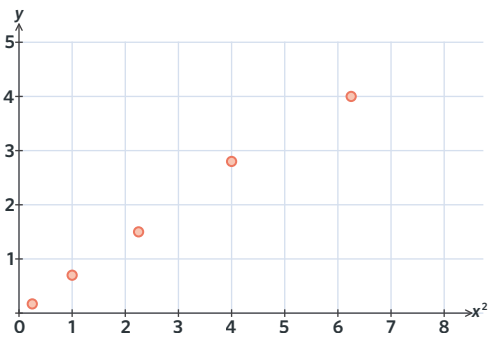
8



The transformation isn't appropriate.

9 A

10



The transformation linearised the data well.

11 **a** The x^2 transformation linearises the data well.

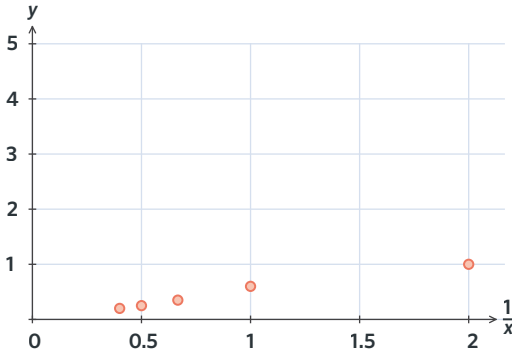
b Neither transformation linearises the data.

c The $\frac{1}{x}$ transformation linearises the data well.

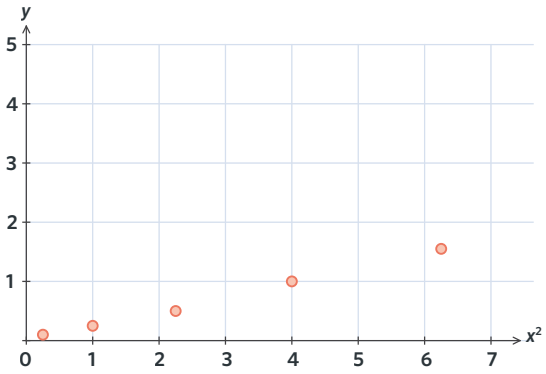
12 Yes, it linearises the data well.

13 The x^2 transformation was much more effective.

14 **a** The data can be linearised using a $\frac{1}{x}$ transformation.



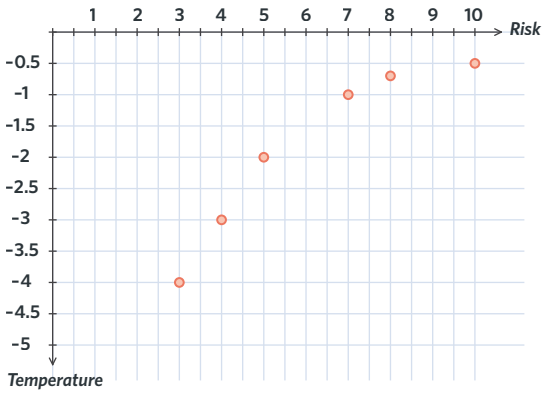
b The data can be linearised using an x^2 transformation.



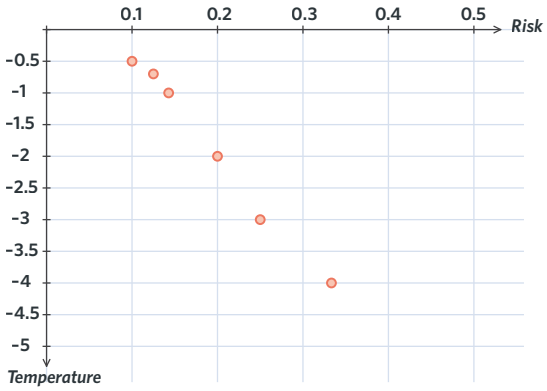
15 **a** An x^2 transformation would work well for this graph.

b A $\frac{1}{x}$ transformation would work well for this graph.

16 **a**



b The $\frac{1}{x}$ transformation works well for this graph.



c $T = -\frac{12}{R}$

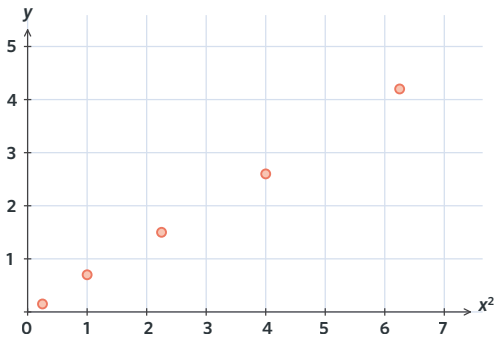
17 **a** x^2 transformation: Graph A

$\frac{1}{x}$ transformation: Graph B

b x^2 transformation: Graph B

$\frac{1}{x}$ transformation: Graph A

18

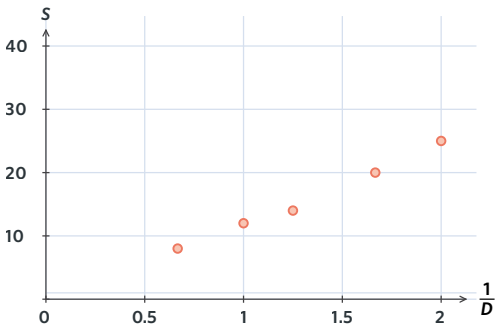


$y = \frac{2}{3}x^2$

19 **a**

Distance	0.5	0.6	0.8	1	1.5
$\frac{1}{\text{Distance}}$	2	$\frac{5}{3}$	$\frac{5}{4}$	1	$\frac{2}{3}$
Speed (km/h)	25	20	14	12	8

b



c Using $(1.5, 8)$...

$S = \frac{k}{D}$

$8 = \frac{k}{1.5}$

$k = 12$

So we have $S = \frac{12}{D}$

d $D = 1.2$

$S = \frac{12}{1.2}$

$S = 10$

A 1.2 kilometre runner would run at about 10 km/h.

Questions from multiple lessons

20 *Graphs and relations*

Decoding the question

Keywords: inversely proportional, directly proportional

Relevant lesson: 11B

We must first find a rule relating y with x and z, in terms of k, then substitute in the provided values to find the value of k

Information we are given

- y is inversely proportional to the square root of x.
- y is directly proportional to z.
- $x = 64$
- $y = 1$
- $z = 4$

Using the theory

First, we need to find a rule relating y with x and z, in terms of k

y is inversely proportional to the square root of x:

$y \propto \frac{1}{\sqrt{\ }}$

y is directly proportional to z:

$y \propto z$

y is inversely proportional to the square root of x and directly proportional to z:

$y \propto \frac{z}{\sqrt{\ }}$

Replace the proportionality sign with the constant of proportionality.

$y = \frac{kz}{\sqrt{\ }}$

Substitute $x = 64$, $y = 1$ and $z = 4$ into $y = \frac{kz}{\sqrt{\ }}$

$1 = \frac{k \times 4}{\sqrt{64}}$

$k = \frac{\sqrt{64}}{4}$

$k = 2$

Answer

C

✓1 mark for the correct answer

21 ***Data analysis***

Decoding the question

Keywords: histogram, distribution, percentage

Relevant lesson: 12C

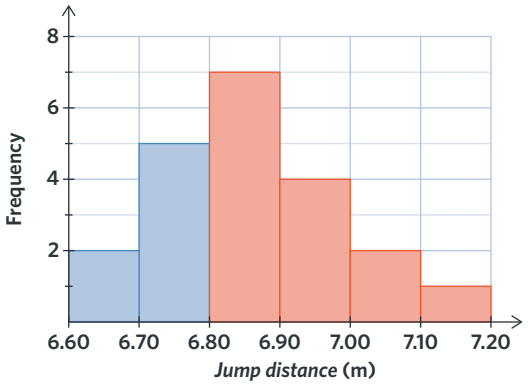
This question requires us to use the given histogram to determine the percentage of female long jumpers who jumped over 6.80 m.

Information we are given

- The given histogram
- There are 21 long jumpers in the sample.

Using the theory

Determine the number of long jumpers who jumped over 6.80 m, and thus have qualified for the Olympics. This is the sum of the frequencies of the columns shaded in orange in the histogram below.



$number\ qualified = 7 + 4 + 2 + 1 = 14$

Calculate the number of female long jumpers who qualified for the Olympics as a percentage of the number of participants.

$$percentage = \frac{number\ qualified}{number\ of\ participants} \times 100$$

$$percentage = \frac{14}{21} \times 100$$

$$percentage = 66.6\ldots$$

$\therefore$ 67% of the long jumpers have qualified for the Olympics.

Answer

E

✓1 mark for the correct answer

22a ***Graphs and relations***

Decoding the question

Keywords: varies, inversely, directly, rule

Relevant lesson: 11B

We must find the rule for y in terms of x, z and k

Information we are given

- y varies inversely with the square of x.
- y varies directly with the square root of z.

Using the theory

y varies inversely with the square of x:

$$y \propto \frac{1}{x^2}$$

y varies directly with the square root of z:

$$y \propto \sqrt{\ }$$

y varies inversely with the square of x and directly with the square root of z:

$$y \propto \frac{\sqrt{z}}{x^2}$$

Replace the proportionality sign with the constant of proportionality, k

$$y = \frac{k\sqrt{z}}{x^2}$$

Answer

$$y = \frac{k\sqrt{z}}{x^2}$$

✓1 mark for the correct answer

22b ***Graphs and relations***

Decoding the question

Keywords: varies, value

Relevant lesson: 11B

We must find the value of z by substituting the given values into the rule from part **a**.

Information we are given

- $k = -0.6$
- $x = 3$
- $y = -5$

Using the theory

From part **a**, the expression is $y = \frac{k\sqrt{z}}{x^2}$

Substitute $k = -0.6$, $x = 3$ and $y = -5$ into $y = \frac{k\sqrt{z}}{x^2}$ and solve for z.

$$-5 = \frac{-0.6\sqrt{z}}{3^2}$$

$$\sqrt{\ } = \frac{-5 \times 3^2}{-0.6}$$

$$z = 5625$$

Answer

5625

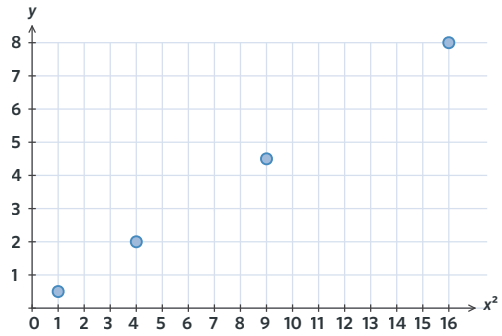
✓1 mark for the correct answer

11D

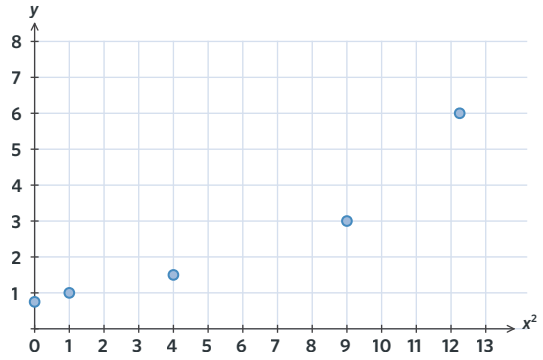
1 D

2 Yes

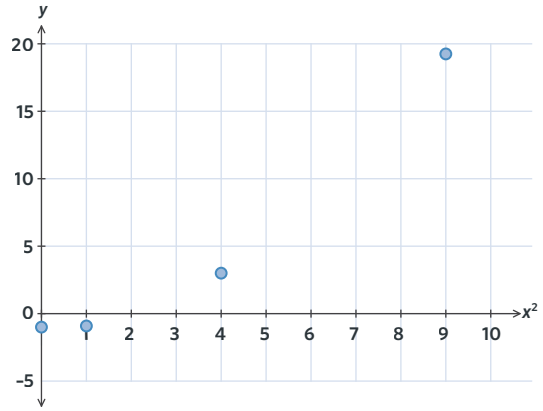
3 **a** Yes, the model can be used for this data set.



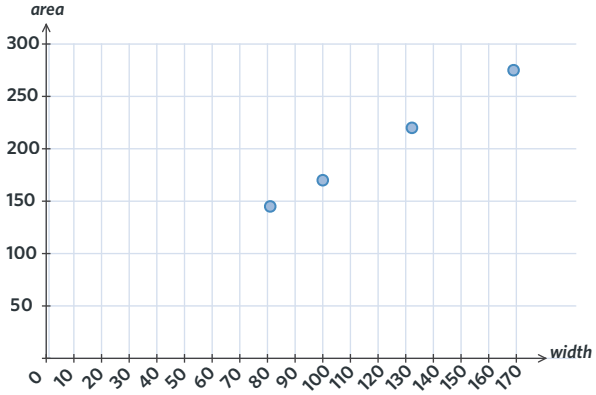
b No, the model can't be used for this data set.



c No, the model can't be used for this data set.



4 Yes, the model will fit this data set.

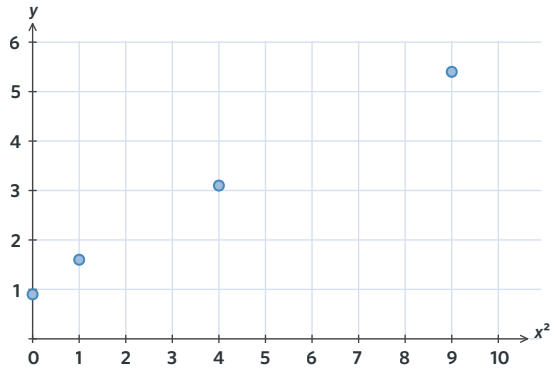


5 $c = -2$

6 **a** $k = \frac{1}{2}$

$c = 1$

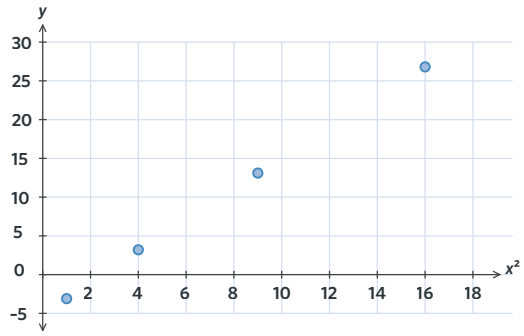
$y = \frac{x^2}{2} + 1$



b $k = 2$

$c = -5$

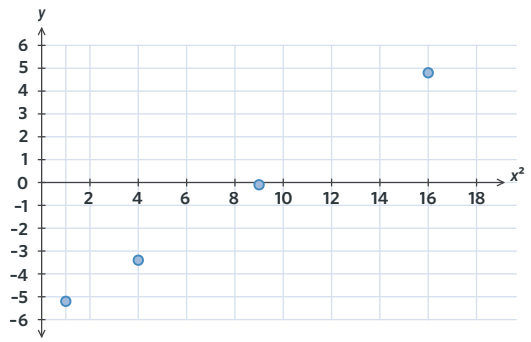
$y = 2x^2 - 5$



c $k = \frac{2}{3}$

$c = -6$

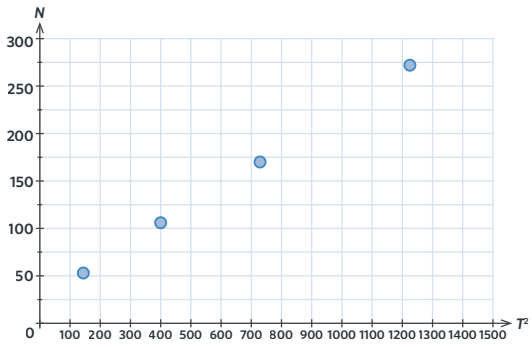
$y = \frac{2}{3}x^2 - 6$



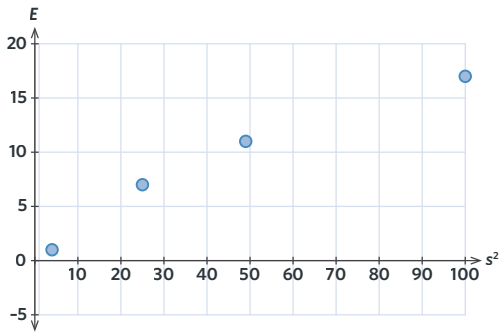
7 $k = \frac{1}{5}$

$c = 25$

$N = \frac{T^2}{5} + 25$



8 Hasti's data set cannot be modelled by $E = ks^2 + c$

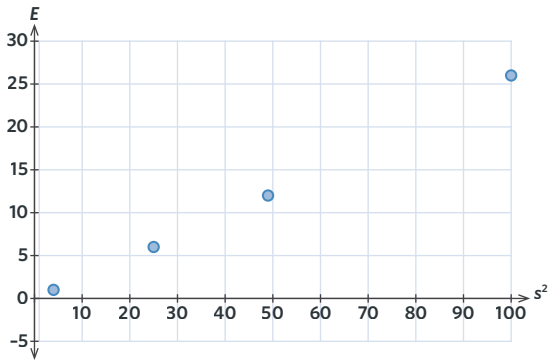


Gabriel's data set can be modelled by $E = ks^2 + c$

$k = \frac{1}{4}$

$c = 0$

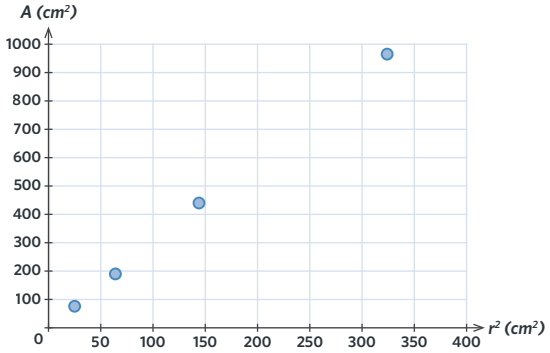
$E = \frac{s^2}{4}$



9 **a** $k = 3$

$c = 0$

$A = 3r^2$



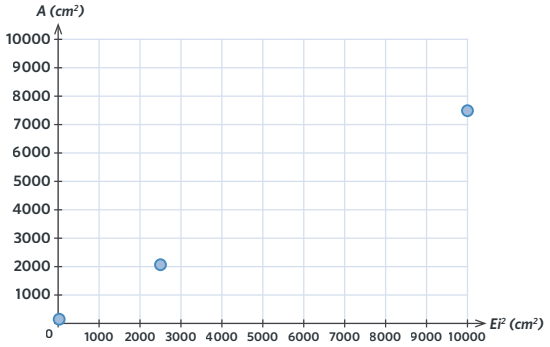
b 675 cm²

c The placemat with radius 6.5 cm does not fit the model.

10 **a** $k = 0.74$

$c = 128.60$

$A = 0.74E_i^2 + 128.60$



b 3754.60 cm²

11 **a**

w (cm)	0.4	0.6	0.8	1.0	1.2
V (cm³)	0.046	0.06	0.09	0.14	0.215
Model Value of V (cm³)	0.045	0.07	0.105	0.15	0.205

The model fits the data accurately.

b According to the model, when $w = 3$, $V = 1.15 \neq 2.74$.

The model clearly does not fit values around $w = 3$.

12 B

Questions from multiple lessons

13 *Graphs and relations*

Decoding the question

Keywords: graph, relationship, false

Relevant lesson: 11A

We need to determine which one of the given statements is false.

Information we are given

- The given graph

Using the theory

For the relationship to be linear, time and speed must have a direct relationship. The relationship between time and speed is inversely proportional. Hence, C is false.

Another way of finding out whether a relationship is linear is by looking at the graph. A linear relationship is always represented by a straight line. The line in the given graph is not straight.

Answer

C

✓ *1 mark for the correct answer*

14 ***Recursion and financial modelling***

Decoding the question

Keywords: sequence, recurrence relation

Relevant lesson: 6B

We must use the given recurrence relation to generate the first four terms of the sequence.

Information we are given

- $T_{n+1} = T_n + 3$
- $T_0 = -1$

Using the theory

We can use the recurrence relation to generate the sequence term by term.

We are given the first term as $T_0 = -1$.

Substitute $T_0 = -1$ into $T_{n+1} = T_n + 3$ to find T_1

$T_1 = T_0 + 3 = -1 + 3 = 2$

Substitute $T_1 = 2$ into $T_{n+1} = T_n + 3$ to find T_2

$T_2 = T_1 + 3 = 2 + 3 = 5$

Substitute $T_2 = 5$ into $T_{n+1} = T_n + 3$ to find T_3

$T_3 = T_2 + 3 = 5 + 3 = 8$

$\therefore$ The first four terms of the sequence are -1, 2, 5 and 8.

Answer

B

✓ *1 mark for the correct answer*

15 ***Graphs and relations***

Decoding the question

Keywords: cost, equation, value

Relevant lesson: 11B

We need to substitute the known values into the given formulas to find the value of m

Information we are given

- $Cost = 96 + 32 \times days$
- $Youth\ charge = m \times (25 - age)^2$
- Kris is 21 years old.
- Kris will rent a car from Canoe for 15 days.
- Kris will pay \$960 in total.

Using the theory

Kris will rent the car for 15 days. Substitute $days = 15$ into the equation $cost = 96 + 32 \times days$ and determine the cost.

$cost = 96 + 32 \times (15)$

$cost = 96 + 480$

$cost = 576$

The rental cost is \$576.

Kris is under 25 years old, hence he will pay a youth charge on top of the usual cost of renting a car.

Determine the youth charge using the formula $youth\ charge = total\ cost - rental\ cost$

Substitute $total\ cost = 960$ and $rental\ cost = 576$.

$youth\ charge = 960 - 576$

$youth\ charge = 384$

Substitute $youth\ charge = 384$ and $age = 21$ into $youth\ charge = m \times (25 - age)^2$ and solve for m

$384 = m \times (25 - 21)^2$

$m = \frac{384}{(25-21)^2}$

$m = \frac{384}{16}$

$m = 24$

Answer

24

✓ *1 mark for the correct youth charge value*
✓ *1 mark for the correct value of m*

11E

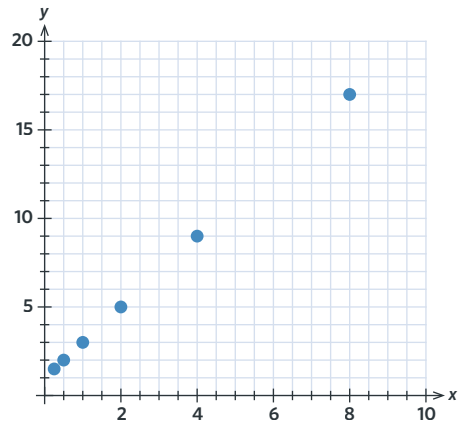
1 Increase

2 **a**

Number of people	1	5	10	25	50
Slices per person	50	10	5	2	1

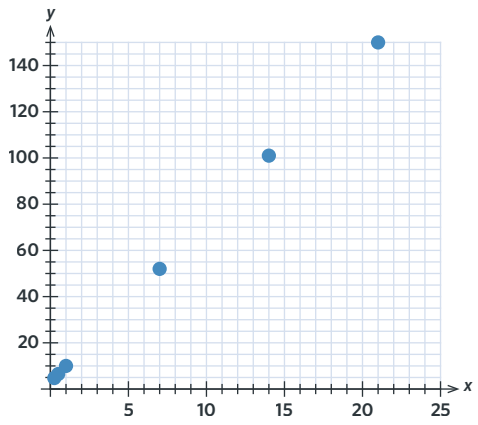
b Yes, as the relationship can be represented by $Slices\ per\ person = \frac{50}{Number\ of\ people}$

3 **a**



Inverse relationship is present

b

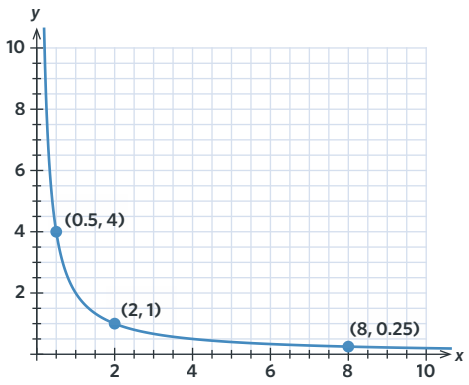


Inverse relationship is present

4 C

5 E

6



7 $y = \frac{7}{x} + 3$

8 $k = 90$

9 **a** $number\ of\ loaves\ sold = \frac{60}{price\ of\ bread} + 2$

b 62 loaves **c** $3

10 **a** $velocity\ squared = \frac{897.3}{distance\ from\ the\ sun}$

b 0.5 AU

11 **a** $US\ dollar\ value = \frac{0.25}{Price\ of\ gold} + 1$

b $3

Questions from multiple lessons

12 ***Graphs and relations***

Decoding the question

Keywords: point, value

Relevant lesson: 11B

This question requires us to use the provided coordinates to find the value of k in the equation $y = \frac{k}{x}$

Information we are given

- The point (20, 21) lies on the graph of $y = \frac{k}{x}$

Using the theory

The provided point (20, 21) allows us to substitute the values $x = 20$ and $y = 21$ into the equation $y = \frac{k}{x}$ to find k

$21 = \frac{k}{20}$

$k = 21 \times 20$

$k = 420$

Answer

D

✓1 mark for the correct answer

13 ***Data analysis***

Decoding the question

Keywords: histogram, median

Relevant lesson: 12E

We must find the median interval by referring to the histogram.

Information we are given

- The given histogram
- There are 25 teams.

Using the theory

In a data set with n values, the median is located in the $\left(\frac{n+1}{2}\right)^{th}$ position.

Since there are 25 teams, substitute in $n = 25$.

$\frac{25+1}{2} = 13$

Hence, the median is in the 13th position. We can find the median by determining the number of teams that scored points within each interval, and keeping track of the cumulative frequency.

Interval	Frequency	Cumulative frequency
30–<40	1	1
40–<50	7	8
50–<60	6	14

Since the cumulative frequency is greater than 13 for the interval 50–<60, the 13th position is located in the 50–<60 interval.

∴ The median number of points scored by a team in their first game of the tournament is at least 50 but less than 60.

Answer

B

✓1 mark for the correct answer

14a ***Graphs and relations***

Decoding the question

Keywords: average speed, graph, 'how much longer'

Relevant lesson: 13E

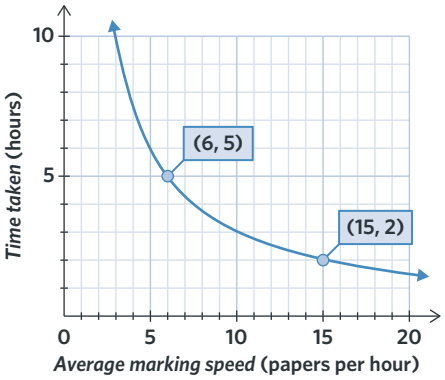
We need to find the difference between the time taken to mark papers at a speed of 15 papers per hour and 6 papers per hour.

Information we are given

- The given graph

Using the theory

Label the points on the curve where *average marking speed* $= 15$ and *average marking speed* $= 6$ and determine the *time taken* to mark at each respective speed. This is shown on the graph below.



It took 2 hours to mark papers for 11C and 5 hours for 11D.

Find the difference.

$difference = 5 - 2$

$difference = 3$

∴ George took an extra 3 hours to mark papers for class 11D compared to class 11C.

Answer

3 hours

✓1 mark for the correct answer

14b ***Graphs and relations***

Decoding the question

Keywords: equation, relationship, form

Relevant lesson: 11E

We need to determine the value of the constant of proportionality, k

Information we are given

- The given graph
- The equation for the relationship between *average marking speed* and *time* is in the form $average\ marking\ speed = \frac{k}{time}$

Using the theory

From part **a**, we determined two coordinates that lie on the graph, (2, 15) and (5, 6).

Using the point (5, 6), substitute $average\ marking\ speed = 6$ and $time = 5$ into the equation $average\ marking\ speed = \frac{k}{time}$

NOTE: The point (2, 15) could also be used here.

$6 = \frac{k}{5}$

$k = 6 \times 5$

$k = 30$

Answer

$k = 30$

✓1 mark for the correct answer

14c ***Graphs and relations***

Decoding the question

Keywords: axes, draw a graph, relationship

Relevant lesson: 11E

We need to draw the graph for the variables *average marking speed* and $\frac{1}{time}$ using the provided axes.

Information we are given

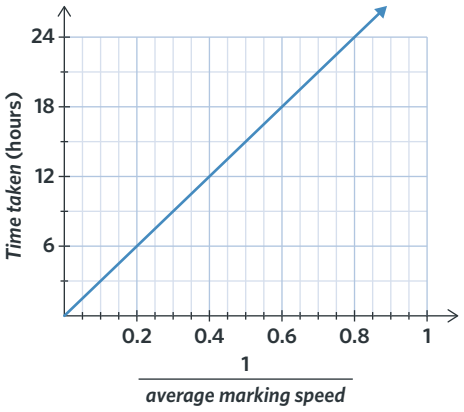
- The given graph
- $time\ taken = \frac{k}{average\ marking\ speed}$

Using the theory

In part **b** we determined that $k = 30$.
Hence, the relationship between *time taken* and *average marking speed* is $time\ taken = 30 \times \frac{1}{average\ marking\ speed}$

As the axes provided has $\frac{1}{average\ marking\ speed}$ on the horizontal axis, the graph will be linear and have a gradient of 30. As there is no constant, the line goes through the origin.

Answer

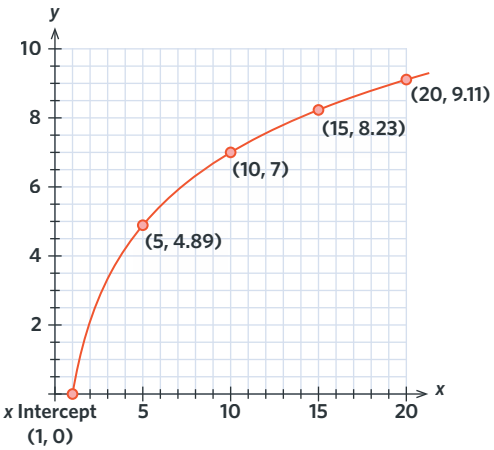


✓1 mark for the correct answer

11F

1 $a = 8, c = 5$

2



Yes

3 D

4 E

5 **a**

Number of days	1	10	100	1000	10 000
Number of deer	0	300	600	900	1200

b Yes. The deer population is increasing, but the rate of increase slows as the number of days increases. This fits the description of a logarithmic function.

6 $y = 5\log_{10}(x) + 1$

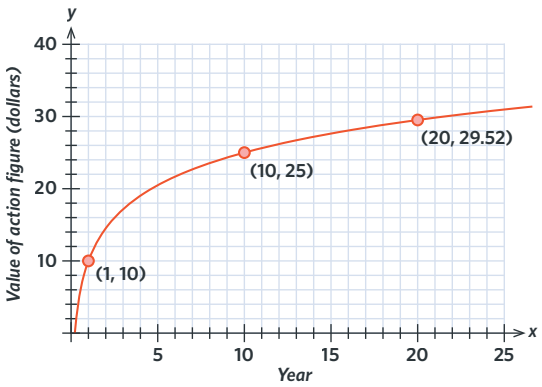
7 **a** 500 **b** 1500

8 **a** 800 **b** 1104

9 **a** $V = 60\log_{10}(H)$ **b** 101.94 litres

c 100 hours

10 **a**



b $34.35 **c** 1987

11 **a** *area of blanket* $= 2.5\log_{10}(\textit{hours spent knitting}) + 3$

b 1.70 m^2

c 3 hours 38 minutes

Questions from multiple lessons

12 ***Graphs and relations***

Decoding the question

Keywords: distance, average speed

Relevant lesson: 9G

We need to study the graph to find the total distance travelled, and use this to find the average speed.

Information we are given

- The given graph
- We are finding the average speed over the entire day.

Using the theory

The formula for speed is $speed = \frac{distance}{time}$

We are finding the average speed over the entire day, so the time will be 24 hours.

As the vertical axis represents *distance from starting point*, and the function is never downwards sloping, the total distance travelled in the day will be the highest point that the piecewise function reaches. Reading off the graph, this is 1050 km.

Substitute $time = 24$ and $distance = 1050$ into $speed = \frac{distance}{time}$

$speed = \frac{1050}{24}$

$speed = 43.75$

∴ Johnny's average speed over the entire day was 43.75 km/h.

Answer

B

✓1 mark for the correct answer

13 ***Data analysis***

Decoding the question

Keywords: least squares regression line, equation

Relevant lesson: 13D

We must use a calculator to find the equation of the least squares regression line.

Information we are given

- The given table

Using the theory

Enter the data within the provided table into your calculator and use its statistics functions to find the least squares regression line.

The calculator will return values of $a = 0.96$ and $b = -1.02$ correct to two decimal places. Substitute these into the equation $y = a + bx$.

$y = 0.96 - 1.02x$

See video solution for a button-by-button demonstration.

Answer

A

✓ *1 mark for the correct answer*

14a ***Graphs and relations***

Decoding the question

Keywords: table, inversely, transformation

Relevant lesson: 11C

We must find the values of $\frac{1}{time\ taken}$ and complete the table.

Information we are given

- The given table

Using the theory

Complete the table by finding $\frac{1}{time\ taken}$ for each value of *time taken*

speed	90.00	85.71	78.26	75.00	72.00
time taken	2.0	2.1	2.3	2.4	2.5
$\frac{1}{time\ taken}$	$\frac{1}{2.0} = 0.50$	$\frac{1}{2.1} = 0.48$	$\frac{1}{2.3} = 0.43$	$\frac{1}{2.4} = 0.42$	$\frac{1}{2.5} = 0.40$

Answer

speed	90.00	85.71	78.26	75.00	72.00
time taken	2.0	2.1	2.3	2.4	2.5
$\frac{1}{time\ taken}$	0.50	0.48	0.43	0.42	0.40

✓ *1 mark for the correct answer*

14b ***Graphs and relations***

Decoding the question

Keywords: inversely, transformation, plot, axes

Relevant lesson: 11C

We must plot each point using the given set of axes.

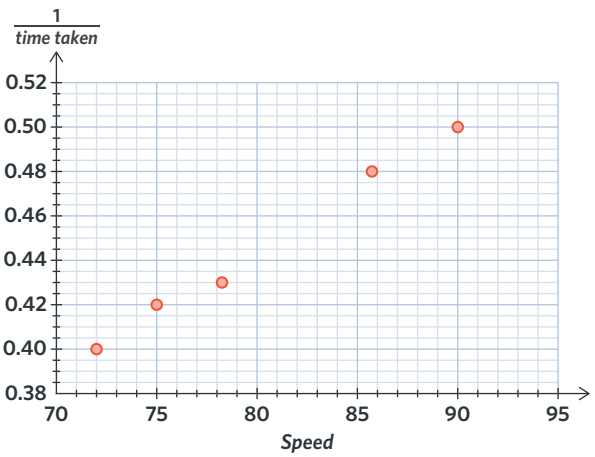
Information we are given

- The given table
- The given axes

Using the theory

Plot each point using the table from part **a** on the given axes.

Answer



✓ *1 mark for the correct answer*

12A

1 Counted

2 Nominal

3 **a** Nominal **b** Ordinal
c Ordinal **d** Nominal

4 Ordinal

5 Discrete

6 **a** Discrete **b** Continuous
c Continuous **d** Discrete

7 Continuous

8 **a** Continuous **b** Nominal
c Ordinal **d** Discrete

9 **a** Numerical **b** Categorical
c Categorical **d** Categorical

10 E

11 D

Questions from multiple lessons

12 ***Data analysis***

Decoding the question

Keywords: dot plot, percentage

Relevant lessons: 2E, 12C

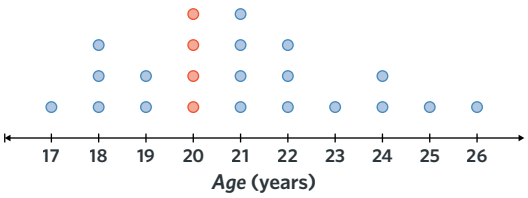
We must determine the number of players that are 20 years old and represent this as a percentage of the total number of players.

Information we are given

- The given dot plot
- There are 22 players in the team.

Using the theory

There are four players who are 20 years old, as highlighted in the following.



Represent this as a percentage of the total number of players using the following formula:

$$percentage\ frequency = \frac{frequency}{total\ frequency} \times 100$$

Substitute in $frequency = 4$ and $total\ frequency = 22$.

$$percentage\ frequency = \frac{4}{22} \times 100$$

$$percentage\ frequency = 18.181...\%$$

$$percentage\ frequency \approx 18\%$$

Hence, 18% of the players are 20 years old.

Answer

D

✓1 mark for the correct answer

13 ***Recursion and financial modelling***

Decoding the question

Keywords: sequence, recurrence relation

Relevant lesson: 6B

We need to use the given recurrence relation to generate a sequence of numbers in order to find V_4

Information we are given

- $V_0 = 1, \quad V_{n+1} = V_n + 6$

Using the theory

We must first find the values of V_1 V_2 and V_3 in order to find V_4

Find the value of V_1:
$V_1 = V_0 + 6$
$V_1 = 1 + 6$
$V_1 = 7$

Find the value of V_2:
$V_2 = V_1 + 6$
$V_2 = 7 + 6$
$V_2 = 13$

Find the value of V_3:
$V_3 = V_2 + 6$
$V_3 = 13 + 6$
$V_3 = 19$

Find the value of V_4:
$V_4 = V_3 + 6$
$V_4 = 19 + 6$
$V_4 = 25$

Answer

E

✓1 mark for the correct answer

14a ***Data analysis***

Decoding the question

Keywords: variables, categorical

Relevant lesson: 12A

We need to determine the number of categorical variables in this data set.

Information we are given

- The given table
- There are four variables in this data set:
  - *Fruit* – type of fruit
  - *Seed location* – where seeds are found (external, internal)
  - *Size* – size of the fruit (small, medium, large)
  - *Price* – the average prices for these fruits (dollars per kg)

Using the theory

Categorical variables are variables that can be organised into a finite number of categories. A numerical variable is represented as a quantity.

The variable *fruit* gives the types of fruit. As different fruits are categorised according to their name, this is a categorical variable.

The variable *seed location* gives the location where seeds are found. Because there are two given categories; external and internal, this is a categorical variable.

The variable *size* describes the sizes of the fruits. Since it has been categorised into three categories; small, medium and large, this is a categorical variable.

The variable *price* determines the costs of these fruits. Since it is measured in dollars and can take any value, this is a numerical variable.

∴ There are three categorical variables in this data set.

Answer

3

✓1 mark for the correct answer

14b ***Data analysis***

Decoding the question

Keywords: variables, ordinal

Relevant lesson: 12A

We need to determine the number of ordinal variables in the data set.

Information we are given

- The given table
- There are four variables in this data set:
  - *Fruit* – type of fruit
  - *Seed location* – where seeds are found (external, internal)
  - *Size* – size of the fruit (small, medium, large)
  - *Price* – the average prices for these fruits (dollars per kg)

Using the theory

Categorical variables can either be nominal or ordinal. Nominal variables do not have a logical order or hierarchy while ordinal variables do.

From part **a**, the three categorical variables are *fruit* *seed location* and *size*.

The variable *fruit* lists the different types of fruit. Since the names of fruits have no logical hierarchy, this is a nominal variable.

The variable *seed location* states where the seeds are found. Since the location of seeds (external, internal) have no logical order, this is a nominal variable.

The variable *size* categorises fruits as either small, medium or large. Since the categories can be logically ordered from small to large, this variable is an ordinal variable.

∴ There is one ordinal variable in this data set.

Answer

1

✓ *1 mark for the correct answer*

14c ***Data analysis***

Decoding the question

Keywords: name, more than

Relevant lesson: 12A

We need to identify the large fruits that cost more than \$3 by referring to the columns *size* and *price*

Information we are given

- The given table

Using the theory

The large fruits are grapefruit, jackfruit, pineapple and watermelon.

Their costs, per kilo, are \$9.49, \$12.49, \$3.49 and \$1.99 respectively.

∴ The only large fruits that cost over \$3 per kilo are grapefruit, jackfruit and pineapple.

Answer

Grapefruit, jackfruit and pineapple

✓ *1 mark for the correct answer*

12B

1 Dogs

2 6

3 **a**

Eye Colour	Tally	Frequency
Blue	𝍸	5
Brown	𝍸 ||||	9
Green	|||	3
Hazel	|||	3

b 5

4

Movie Genre	Frequency
Action	7
Comedy	6
Drama	2
Romance	6
Thriller	4

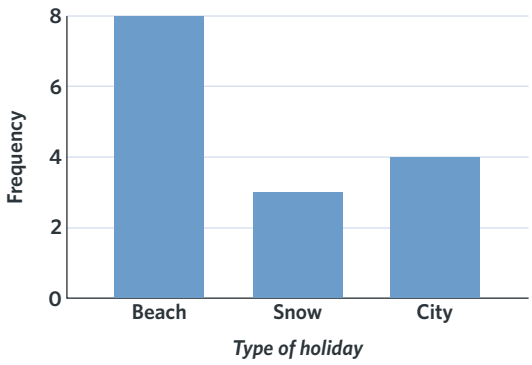
5

Genre Of Music	Frequency
Pop	6
Rock	4
Classical	2

6 **a** 19

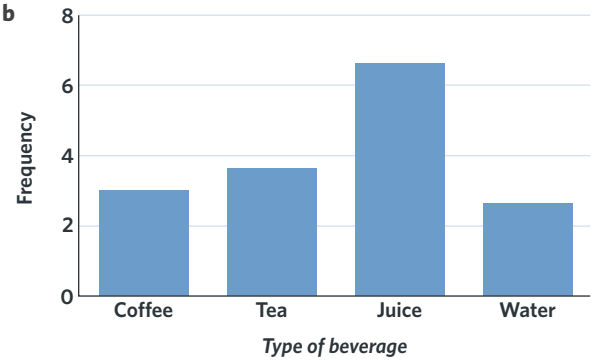
b 21%

7



8 **a** 33

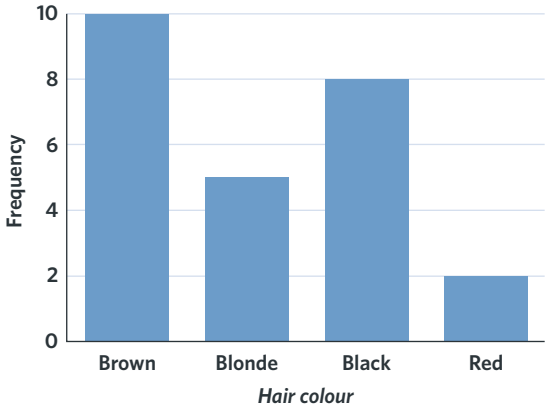
b



9

Age Group	Frequency
20–40	15
41–60	50
61–80	35

10 **a**



b

Hair colour	Frequency
Brown	10
Blonde	5
Black	8
Red	2

c 8%

11 Bus

12 Sport

13 **a** 329

b Melbourne

14 **a**

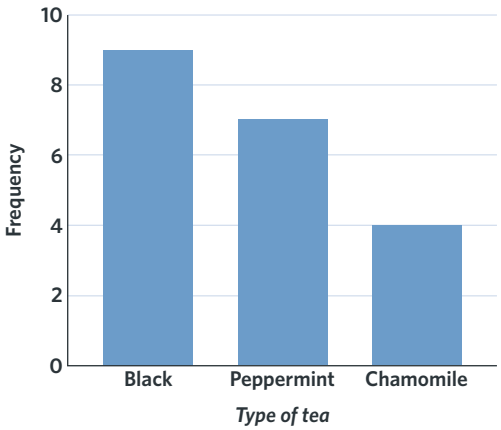
Number of children	Frequency
1	8
2	18
3	18
4	9

b 2 and 3

15 **a**

Type of tea	Frequency
Black	9
Peppermint	7
Chamomile	4

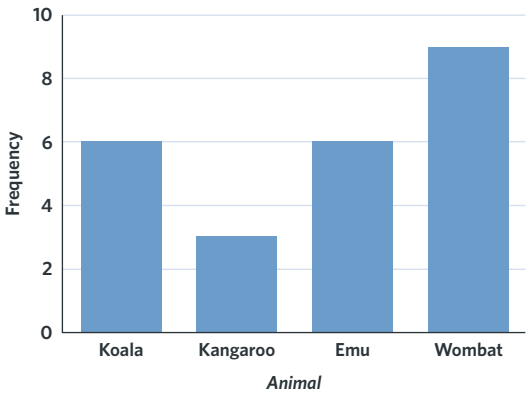
b



16 **a**

Animal	Frequency
Koala	6
Kangaroo	3
Emu	6
Wombat	9

b



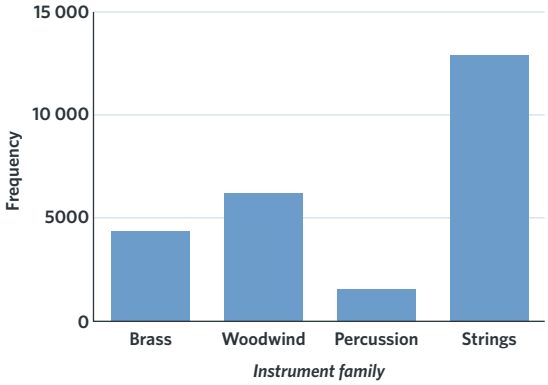
c Wombat

17 **a** 51.4%

b

Instrument family	Frequency
Brass	4375
Woodwind	6200
Percussion	1575
Strings	12850

c



d Strings

Questions from multiple lessons

18 *Data analysis*

Decoding the question

Keywords: boxplot, distribution, not true

Relevant lessons: 12G, 12H

We need to analyse the boxplots to determine which of the statements is false.

Information we are given

- The given boxplots

Using the theory

Option A states that the slowest typer is a Year 10. Looking at the minimum across all three year levels, Year 10's is the lowest of the three. Hence, statement A is true.

Option B states that Year 10s have the least variable typing speed. We need to look at the range of all three Year levels.

For Year 10s, their minimum and maximum are 57 and 66 respectively. This means that the $range = 66 - 57 = 9$.

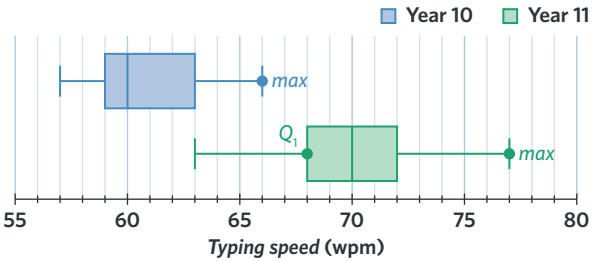
For Year 11s, their minimum and maximum are 63 and 77 respectively. This means that the $range = 77 - 63 = 14$.

For Year 12s, their minimum and maximum are 73 and 86 respectively. This means that the $range = 86 - 73 = 13$.

The Year 10s have the smallest range out of the three, which means that they have the least variable data. Hence, statement B is true.

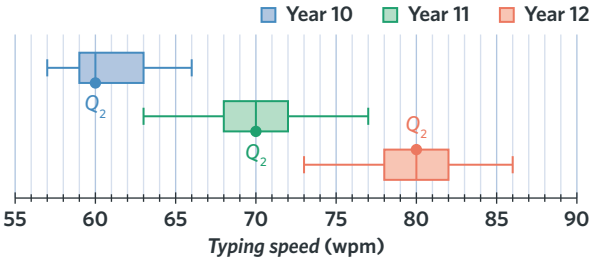
Option C states that at least 75% of Year 11s type faster than any Year 10. We know that for any given five-number summary, 75% of the data will always be greater or equal to Q_1

Comparing the two boxplots, we see that Q_1 for Year 11s is greater than the maximum for Year 10s. This means that at least 75% of Year 11s type faster than any Year 10, which is illustrated in the following diagram.



Hence, statement C is true.

Option D states that in terms of the median, Year 12s are the fastest typers. We need to compare the medians of all three year levels.



We can see that the median for Year 12s is the largest out of all three. Hence statement D is true.

Option E states that the fastest typer is a Year 11. We can see that the maximum value across all three year levels is 86, which is a Year 12. Hence, statement E is false.

Answer

E

✓1 mark for the correct answer

19 ***Recursion and financial modelling***

Decoding the question

Keywords: sequence, recurrence relation

Relevant lessons: 6B, 6D

We must identify which of the recurrence relations could model the given sequence.

Information we are given

- 3, 6, 12, 24...

Using the theory

We must generate the first four terms of each sequence and see if they match the sequence given. Starting from T_0, the given sequence has the values $T_0 = 3$, $T_1 = 6$, $T_2 = 12$, and $T_3 = 24$.

A $T_0 = 2, \quad T_{n+1} = 3T_n$
$T_0 = 2 \neq 3$
A is incorrect.

B $T_0 = 3, \quad T_{n+1} = 2T_n$
$T_0 = 3$
$T_1 = 2 \times 3 = 6$
$T_2 = 2 \times 6 = 12$
$T_3 = 2 \times 12 = 24$
B is correct.

C $T_0 = 3, \quad T_{n+1} = 3T_n$
$T_0 = 3$
$T_1 = 3 \times 3 = 9 \neq 6$
C is incorrect.

D $T_0 = 3, \quad T_{n+1} = T_n + 2$
$T_0 = 3$
$T_1 = 3 + 2 = 5 \neq 6$
D is incorrect.

E $T_0 = 3, \quad T_{n+1} = T_n + 3$
$T_0 = 3$
$T_1 = 3 + 3 = 6$
$T_2 = 6 + 3 = 9 \neq 12$
E is incorrect.

Answer

B

✓1 mark for the correct answer

20a ***Data analysis***

Decoding the question

Keywords: data, frequency table

Relevant lesson: 12B

We must count the number of responses for each rating and complete the frequency table.

Information we are given

- The given responses
- The given table

Using the theory

Count the number of each response provided in the question. The number of each response is as follows:

Rancid: 2
Unpalatable: 2
Mediocre: 5
Appetising: 2
Divine: 5

Write each of these values in the corresponding row of the frequency table.

NOTE: As there were 16 people at Kevin's dinner, the total frequency must add up to 16.

Answer

Rating	Frequency
Rancid	2
Unpalatable	2
Mediocre	5
Appetising	2
Divine	5
Total	16

✓1 mark for the correct answer

20b *Data analysis*

Decoding the question

Keywords: data, percentage, 'mediocre'

Relevant lesson: 2E

We need to express the number of 'mediocre' responses as a percentage out of the total number of responses.

Information we are given

- The given responses

Using the theory

Referring to the completed frequency table from part **a**, the number of 'mediocre' responses is five. Convert this to a percentage of the total number of responses.

$$percentage = \frac{frequency}{total\ frequency} \times 100$$

$$percentage = \frac{5}{16} \times 100 = 31.25$$

∴ 31.25% of Kevin's friends rated the dinner as 'mediocre'.

Answer

31.25%

✓*1 mark for the correct answer*

12C

1

Group	Values
0–4	0, 2
5–9	5, 6, 7, 7, 9
10–14	10, 11, 13
15–19	15, 16, 18

2 **a** 1

b

Group	Frequency
10–19	1
20–29	7
30–39	5
40–49	6
50–59	4
60–69	3

c Group: 20–29, Frequency: 7

3 **a**

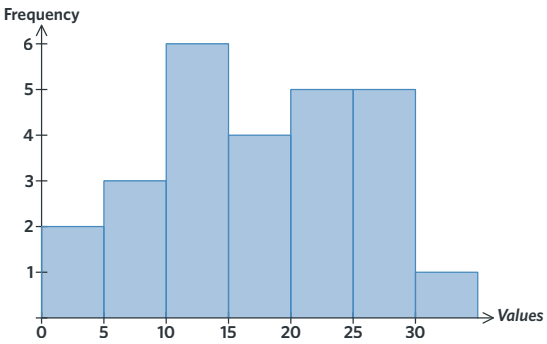
Hours	Frequency
0–4	7
5–9	12
10–14	4
15–19	2
20–24	1

b 7 **c** 26.9%

4 **a** 201

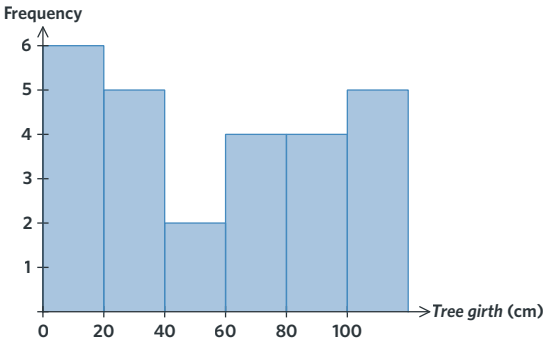
b 12 (must exclude Grant's apartment)

5 **a**



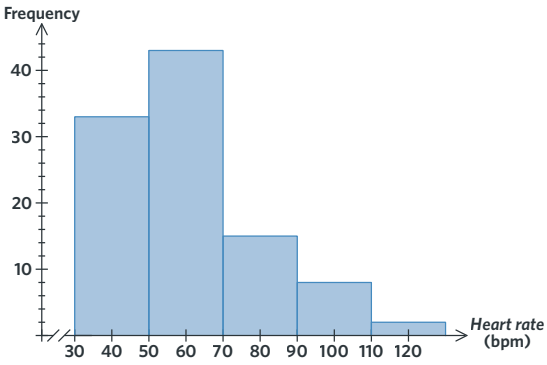
b 6

6 **a**



b 5 **c** 76.9%

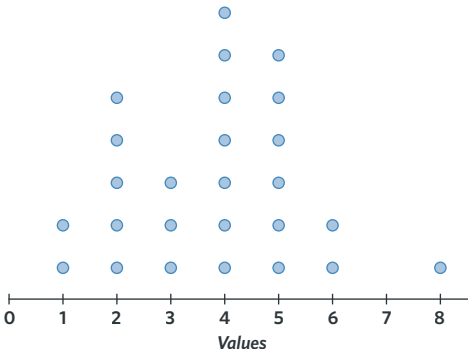
7 **a**



b 30−<50 and 50−<70

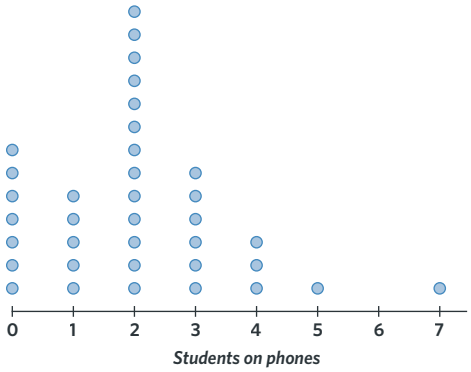
c 10

8 **a**



b 4 **c** 10

9 **a**



b 19.4%

10 **a** Key: 1|9 = 19

1	9
2	0 6 7 8
3	7 7 7 8 9 9
4	5 7 9 9
5	5 7
6	3 3
7	0 3 5 5 5 6
8	9

b 6

11 **a** Key: 18|2 = 182 cm

8 7 7 7 7 5 5 5 3 2	17	
5 3 0	18	2 4 5 5 8 8
	19	0 0 1 2 5 7

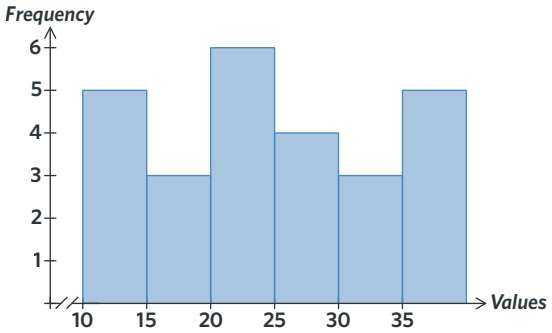
b 33.3% **c** 2 **d** 66.7%

12 **a** 3 **b** 9

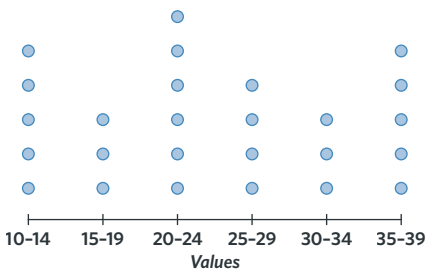
13 **a**

Group	Frequency
10–14	5
15–19	3
20–24	6
25–29	4
30–34	3
35–39	5

b



c



d Key: 1|1 = 11

1	1 1 3 4 4 5 6 7
2	0 0 1 2 3 4 6 7 8 8
3	1 2 4 6 7 8 9 9

14 **a** As there aren't many different values that the data can take, a stem plot is not necessary.

b It is unnecessary to use a histogram as the data is discrete and does not need to be grouped. A dot plot is more suited as it can display ungrouped data.

15 **a** Histogram. There is a very large data set and a large range of values that the data will span. Therefore, the data should be grouped. Histograms are most suited to display large amounts of grouped data.

b Back to back stem plot. The statistician wants to compare the data sets, and a back to back stem plot is best at comparisons.

16 **a** 70–<75 **b** 14 **c** 16.4%

17 C

Questions from multiple lessons

18 ***Data analysis***

Decoding the question

Keywords: boxplot, percentage

Relevant lesson: 12G

We must use our knowledge of boxplots and quartiles to deduce the percentage of cities with a population greater than 2 000 000.

Information we are given

- The given boxplot

Using the theory

First, find the value of 2 000 000 on the boxplot.

The value '2 000 000' is represented by '2000' on the boxplot as the population scale is in thousands of people.

2 000 000 lies on the median line.

We know that the median represents the middle value of an ordered set of data, hence 50% of the data lies above the median and 50% of the data lies below the median.

Hence, the percentage of cities with a population greater than 2 000 000 is closest to 50%.

Answer

C

✓1 mark for the correct answer

19 ***Recursion and financial modelling***

Decoding the question

Keywords: sequence, recurrence relation

Relevant lesson: 6D

We must generate a sequence using the given recurrence relation.

Information we are given

- $T_0 = 4, \quad T_{n+1} = -5 \times T_n$

Using the theory

Use the recurrence relation $T_0 = 4, \quad T_{n+1} = -5 \times T_n$ to find T_1 T_2 T_3 and T_4

Find the value of T_1:

$T_1 = -5 \times T_0$

$T_1 = -5 \times 4$

$T_1 = -20$

Find the value of T_2:

$T_2 = -5 \times T_1$

$T_2 = -5 \times (-20)$

$T_2 = 100$

Find the value of T_3:

$T_3 = -5 \times T_2$

$T_3 = -5 \times 100$

$T_3 = -500$

Find the value of T_4:

$T_4 = -5 \times T_3$

$T_4 = -5 \times (-500)$

$T_4 = 2500$

The sequence generated from the recurrence relation is $4, -20, 100, -500, 2500...$

Answer

E

✓1 mark for the correct answer

20a ***Data analysis***

Decoding the question

Keyword: two-way frequency table

Relevant lesson: 12B

This question requires us to present the given information in a two-way frequency table.

Information we are given

- The given tables

Using the theory

The two-way frequency table only considers the variables *coffee size* and *caffeine level*

In regards to *coffee size*, there are three categories; 'small', 'medium' and 'large'. We will fill out the table one column at a time, starting at the top left.

Count the number of small coffees with low, medium and high levels of caffeine. The numbers are 0, 0 and 2 respectively.

Count the number of medium coffees with low, medium and high levels of caffeine. The numbers are 0, 6, and 0 respectively.

Count the number of large coffees with low, medium and high caffeine levels. The numbers are 6, 0, and 1 respectively.

Calculate the sum of each column and fill out the 'total' row at the bottom.

Answer

		Coffee size		
		Small	**Medium**	**Large**
Caffeine level	**Low**	0	0	6
	Medium	0	6	0
	High	2	0	1
Total		2	6	7

✓2 marks for the correct answer

20b ***Data analysis***

Decoding the question

Keyword: percentage

Relevant lessons: 2E, 12B

We need to find the percentage of large coffees that have a high caffeine level.

Information we are given

- The given table

Using the theory

Using the completed two-way frequency table from part **a**, we can see that the number of large coffees with high caffeine levels is 1. There are a total of 7 large coffees.

The percentage can be found using the following formula:

$$percentage = \frac{frequency}{total\ frequency} \times 100$$

Substitute $frequency = 1$ and $total\ frequency = 7$ into the formula.

$percentage = \frac{1}{7} \times 100$

$percentage = 14.2857...\%$

$\therefore$ 14% of the large coffees have a high caffeine level.

Answer

14%

✓1 mark for the correct answer

12D

1 **a** Symmetrical **b** Asymmetrical

2 **a** Perfectly symmetrical

b Approximately symmetrical

3 **a** Positively skewed

b Negatively skewed

4 The shape of a distribution is independent from its location.

5 **a** Negatively skewed **b** Perfectly symmetrical

c Approximately symmetrical

d Positively skewed

6 **a** Histogram 2 has a more positive centre than histogram 1.

b The histograms share a similar location.

7 **a** Histogram 2 has a greater spread than histogram 1.

b The histograms share a similar spread.

8 **a** Location **b** Neither

c Both **d** Location

9 **a** Both distributions have a negative skew. Distribution 1 has a larger spread and a less positive centre than distribution 2.

b Both distributions are approximately symmetrical. Distribution 1 has a smaller spread and a more positive centre than distribution 2.

c Distribution 1 is negatively skewed, while distribution 2 is positively skewed. The distributions share a similar spread, but differ in centre, with distribution 2 having a more positive centre.

d Distribution 1 is positively skewed, while distribution 2 is perfectly symmetrical. Distribution 1 has a larger spread and a more positive centre than distribution 2.

10

Description	Distribution
Largest spread	B
Largest centre value	F
Skewed (positively or negatively)	A,D,E
Symmetrical (perfectly or approximately)	B,C,F

Questions from multiple lessons

11 ***Data analysis***

Decoding the question

Keywords: boxplot, five-number summary

Relevant lesson: 12G

We can read features of the boxplot to deduce the five-number summary of the data.

Information we are given

- The given boxplot

Using the theory

A five-number summary is listed as: minimum, Q_1 median, Q_3, maximum.

The minimum is the leftmost data point. This is the outlier at 2 represented by a dot.

Q_1 is the left border of the box. This is located at 12.

The median is the vertical line in the middle of the box. This is located at 14.5.

Q_3 is the right border of the box. This is located at 18.

The maximum is the rightmost data point. This is the outlier at 32.5 represented by a dot.

∴ The five-number summary is 2, 12, 14.5, 18, 32.5.

NOTE: Outliers are included when identifying a five-number summary.

Answer

B

✓*1 mark for the correct answer*

12 ***Recursion and financial modelling***

Decoding the question

Keywords: compounding quarterly, interest

Relevant lesson: 3D

We can use the compound interest formula to find the amount of interest earned during the three-year period.

Information we are given

- Gonzalo invests $4000 for three years.
- The interest rate is 4.4% p.a, compounding quarterly.

Using the theory

The value of a compound-interest investment is given by the following formula.

$A = P\left(1 + \frac{r}{100}\right)^n$, where

- A is the total value of the investment.
- P is the principal value.
- r is the interest rate per compounding period, as a percentage.
- n is the number of compounding periods.

In this case, $P = 4000$, $r = \frac{4.4\%}{4} = 1.1\%$ and $n = 4 \times 3 = 12$.

Substitute these values into the formula.

$A = 4000\left(1 + \frac{1.1}{100}\right)^{12}$

$A = 4561.144...$

Find the interest earned using the formula: $I = A - P$

Substitute $A = 4561.144...$ and $P = 4000$ into the formula.

$I = 4561.144... - 4000$

$I = 561.144...$

∴ The interest Gonzalo will earn to the nearest cent is $561.14.

Answer

D

✓*1 mark for the correct answer*

13a *Data analysis*

Decoding the question

Keywords: frequency table, value

Relevant lesson: 12B

We must find the value of x using the data in the frequency table and the information given in the question.

Information we are given

- The given table
- 20 tourists were surveyed.

Using the theory

We know that 20 tourists were surveyed. Thus, the frequencies in the table must add up to 20.

$3 + 8 + 5 + 2 + x = 20$

Solve for x.

$x = 20 - 3 - 8 - 5 - 2$

$x = 2$

Answer

2

✓1 mark for the correct answer

13b *Data analysis*

Decoding the question

Keywords: frequency table, bar chart

Relevant lesson: 12B

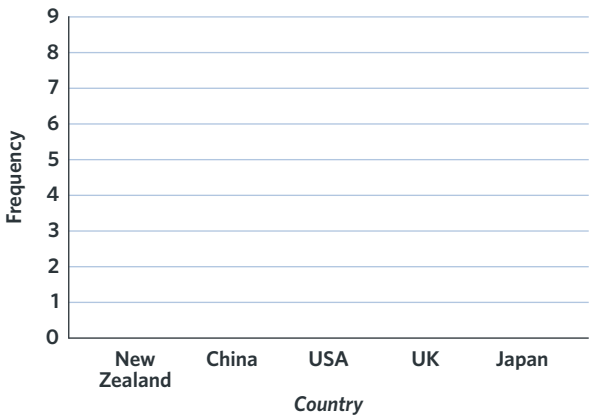
We must create a bar chart using the frequency table given in the question and the information found in part **a**.

Information we are given

- The given table

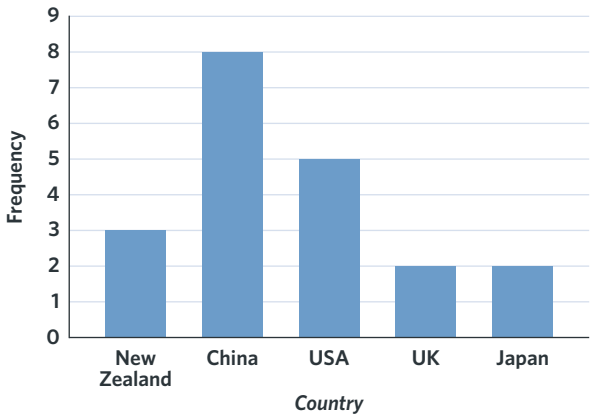
Using the theory

Draw a set of axes with *country* on the horizontal axis and frequency on the vertical axis.



Add in a bar for each country. The height of each country's bar should correspond with its value in the frequency table.

Answer



✓1 mark for the correct axes
✓1 mark for the correct bar heights

12E

1 10

2 343.5

3 90

4 **a** 58% **b** 51%

c Mean score: 69%, Median score: 86%

5 $590

6 Mean weight: 6.7 kg

Median weight: 6.5 kg

Mean weight is greater than median weight.

7

x	f
4	1
5	4
6	0
7	1
8	2
9	1
10	1

Make an xf column by multiplying x and f together. Sum up frequency column and xf column. Divide the xf total by the frequency total to find the mean.

8 4

9 11.63

10 Mean: 4.95, Median: 4.50

11 Table 2

12

Interval	Midpoint x	Frequency f	Cumulative frequency cf
0–<2	1	6	6
2–<4	3	3	9
4–<6	5	8	17
6–<8	7	2	19
8–<10	9	1	20

As $n = 20$, the median is the average of the $\frac{n}{2}^{th}$ and $\frac{n}{2} + 1^{th}$ values.

The median is the average of the 10^{th} and 11^{th} values.

Interval 4–<6 is the interval that holds the 10^{th} and 11^{th} values. Thus, the 10^{th} and 11^{th} values are both 5.

The median is 5 chocolate bars.

13 C

14 **a** The median would represent this data set best because there is an extreme outlier at 75–<80 which would increase the overall mean significantly, rendering it an inaccurate representation of the data set.

b Both measures of centre are acceptable because the data set is perfectly symmetrical. The value of the mean and the value of median will be equal.

c The mean would be affected the most, because 170 is an extremely large outlier.

15 Median is the most appropriate measure of centre since there is a large outlier present (169).

16 There is no inappropriate choice for the data set. This is because both the mean and median are equal.

17 **a** Mean: 6.8, Median: 7

The median is greater than the mean.

b 13

18 **a**

Interval	Midpoint	Frequency	xf
150–<155	152.5	1	152.5
155–<160	157.5	1	157.5
160–<165	162.5	2	325
165–<170	167.5	3	502.5
170–<175	172.5	4	690
175–<180	177.5	5	887.5
180–<185	182.5	6	1095
185–<190	187.5	7	1312.5
190–<195	192.5	3	577.5
195–<200	197.5	1	197.5
		$\sum f = 33$	$\sum xf = 5897.5$

Mean: 178.71 cm

b The median is the more appropriate measure of centre as the data is skewed.

c For a distribution to be symmetrical, the mean must equal the median. As the mean is 178.71 cm, the median would have to be 178.71 cm.

19 B

Questions from multiple lessons

20 ***Data analysis***

Decoding the question

Keywords: two-way frequency table, percentage

Relevant lessons: 2E, 12B

We must calculate the number of people under 40 years old who chose tea as their preferred beverage as a percentage of the total people under 40 years old surveyed.

Information we are given

- The given table

Using the theory

The percentage can be found using the following formula:

$$percentage = \frac{frequency}{total\ frequency} \times 100$$

The number of people under 40 years old who chose tea as their preferred beverage is 9.

The total number of people under 40 years old surveyed is 41.

Substitute $frequency = 9$ and $total\ frequency = 41$ into the formula.

$$percentage = \frac{9}{41} \times 100$$

$$percentage = 21.95\ldots$$

∴ The percentage of people under 40 years old who chose tea as their preferred beverage is closest to 22%.

Answer

D

✓*1 mark for the correct answer*

21 ***Recursion and financial modelling***

Decoding the question

Keywords: sequence, recurrence relation, positive

Relevant lesson: 6D

We must use the recurrence relation to generate a sequence and identify the position of the first term greater than 10.

Information we are given

- $T_{n+1} = -3T_n \quad T_0 = -\frac{1}{3}$

Using the theory

Since we have been given the value of T_0, we can use the recurrence relation to generate the sequence term by term.

Find T_1

$T_1 = -3T_0$

$T_1 = -3 \times \left(-\frac{1}{3}\right) = 1$

Find T_2

$T_2 = -3T_1$

$T_2 = -3 \times 1 = -3$

Find T_3

$T_3 = -3T_2$

$T_3 = -3 \times (-3) = 9$

Find T_4

$T_4 = -3T_3$

$T_4 = -3 \times 9 = -27$

Find T_5

$T_5 = -3T_4$

$T_5 = -3 \times (-27) = 81$

$81 > 10$

$\therefore T_5$ is the first term greater than 10.

Answer

E

✓1 mark for the correct answer

22a ***Data analysis***

Decoding the question

Keywords: describe, shape, histogram

Relevant lesson: 12D

We need to determine the shape of the given histogram.

Information we are given

- The given histogram

Using the theory

We can see that the majority of the data is clustered around the right-hand side of the histogram and that the distribution trails off to the left.

$\therefore$ This distribution has a negative skew.

Answer

Negatively skewed

✓1 mark for the correct answer

22b ***Data analysis***

Decoding the question

Keywords: determine, number

Relevant lesson: 12C

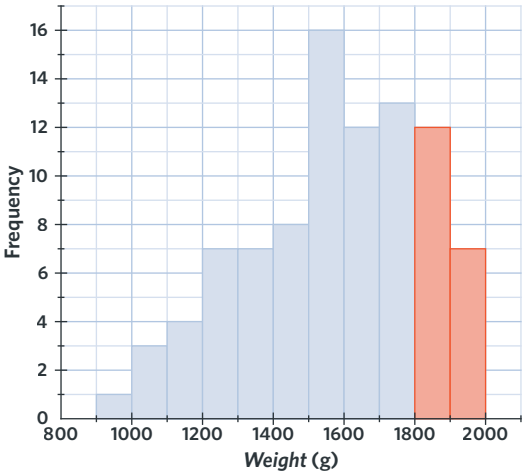
We can use the histogram to find the number of customers who purchased 1800 g or more of potatoes.

Information we are given

- The given graph

Using the theory

The graph below shows the intervals that are 1800 and greater.



Count the frequency of each bar and calculate the sum.

number of customers $= 12 + 7$

number of customers $= 19$

$\therefore$ 19 customers purchased 1800 g or more of potatoes.

Answer

19 customers

✓1 mark for the correct answer

22c ***Data analysis***

Decoding the question

Keywords: percentage, between

Relevant lessons: 2E, 12C

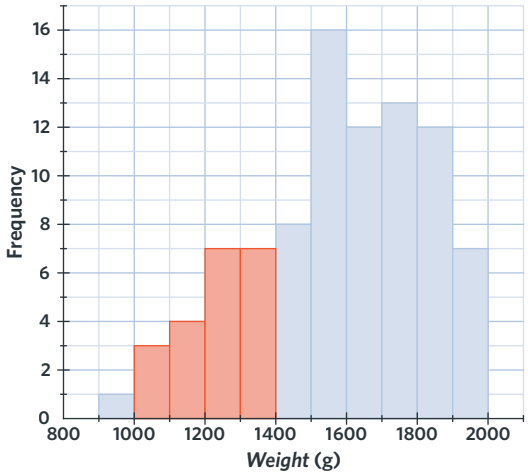
We need to determine the number of customers that purchased between 1000 g and 1400 g of potatoes, then represent this as a percentage of the total number of customers.

Information we are given

- The given graph
- There were 90 customers in total.

Using the theory

The graph below shows the intervals between 1000 and 1400.



Count the frequency of each bar and calculate the sum.

number of customers $= 3 + 4 + 7 + 7$

number of customers $= 21$

Calculate the percentage using the formula

$$percentage = \frac{frequency}{total\ frequency} \times 100.$$

$$percentage = \frac{21}{90} \times 100$$

$$percentage = 23.33...$$

$\therefore$ 23.3% of customers purchased between 1000 g and 1400 g of potatoes.

Answer

23.3%

✓1 mark for the correct answer

12F

1 D

2 997

3 24 309

4 5.1

5 9

6 Most number of times fined is 12

Least number of times fined is 1

$12 - 1 = 11$

The range is 11.

7 Jasper's height at the end of 10 years is 1.55 metres.

The range of Murphy's heights is: $2.01 - 1.23 = 0.78$

$0.78 > 0.24$, therefore Murphy's range of height is greater.

8 **a** 9 **b** 6 **c** 14

9 Q_1: 2 Q_2: 5 Q_3: 9

10 Q_1: 13, Q_2: 16.5, Q_3: 18

11 **a** {12, 14, 25, 29, 33, 41}

b 27

c {12, 14, 25}, Q_1: 14

d {29, 33, 41}, Q_3: 33

12 B

13 **a** 2.4 **b** 2.6

14 **a** 6.5 **b** 7.6

15 7

16 **a** 75 **b** 8.66 **c** 30.76

17 B

Standard deviation measures of how much the data is spread out from the mean. The data points of B are more spread out than the data points of A.

18 Variance: 6.15

Standard deviation: 2.48

Nicky got the variance wrong.

19 8

20 **a** 484 seconds

b Daily vlog IQR: 6.65

China vlog IQR: 1.55

Daily vlog IQR is higher.

Amy could adjust every daily vlog except the one that runs for 12.4 minutes to ensure she has a higher Q_3 or lower Q_1 for an overall larger IQR.

c 3.433 minutes

21 2.6 mm

Questions from multiple lessons

22 ***Data analysis***

Decoding the question

Keyword: variables

Relevant lesson: 12A

This question asks us to classify the variables *time taken* and *quality*

Information we are given

- *Time taken*
- *Quality* (below satisfactory, satisfactory, good, excellent)

Using the theory

We must first decide if the variables *time taken* and *quality* are categorical or numerical variables. If they are categorical variables, determine if they are nominal or ordinal. If they are numerical variables, determine if they are discrete or continuous.

Time taken is a numerical variable as time is measured, and in this case, isn't broken down into categories. Time exists on a continuous scale and isn't represented only as separate numerical values. Hence, *time taken* is a continuous variable.

Quality is a categorical variable as it has four categories: below satisfactory, satisfactory, good and excellent. As these categories can be logically ordered from the lowest quality to highest quality, *quality* is an ordinal variable.

∴ The variables *time taken* and *quality* are a continuous variable and an ordinal variable respectively.

Answer

D

✓1 mark for the correct answer

23 ***Recursion and financial modelling***

Decoding the question

Keywords: price, reduced

Relevant lesson: 3A

We must find 60% of \$45 to determine the reduction in the price of the jeans.

Information we are given

- The original price of the jeans was \$45.
- The price of the jeans has been reduced by 60%.

Using the theory

Find the price change using the following formula:

$$price\ change = original\ price \times \frac{percentage\ change}{100}$$

Substitute in $original\ price = 45$ and $percentage\ change = 60$.

$$price\ change = 45 \times \frac{60}{100}$$

$price\ change = 27$

∴ The price of the jeans was reduced by $27.

Answer

E

✓1 mark for the correct answer

24a *Data analysis*

Decoding the question

Keywords: dot plot, identify, number of days

Relevant lesson: 12C

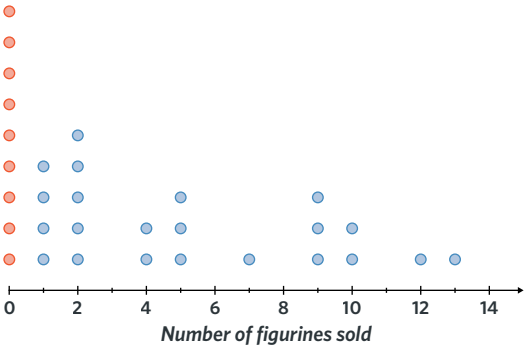
We can identify by eye the number of days on which no sales were made from looking at the dot plot.

Information we are given

- The given dot plot

Using the theory

A value of 0 on the dot plot indicates no sales were made on a particular day. Count the number of dots there are over 0.



There are nine dots at 0 on the dot plot.

∴ Horacio made no sales on nine days during the 31 day period.

Answer

9 days

✓1 mark for the correct answer

24b *Data analysis*

Decoding the question

Keywords: percentage, 'more than 10'

Relevant lesson: 12C

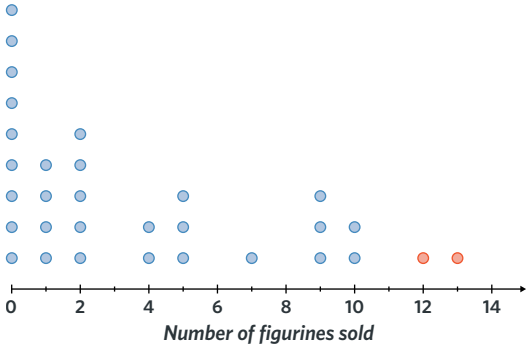
We must find the number of days on which more than 10 sales were made and then convert this to a percentage.

Information we are given

- The given dot plot
- Data was collected over a period of 31 days.

Using the theory

There are two values greater than 10 in the dot plot. These are highlighted.



NOTE: The two values at 10 are not included because we are looking for the days on which **more than** 10 are sold.

More than 10 figurines were sold on two out of the 31 days.

Convert this to a percentage using the formula $percentage = \frac{frequency}{total\ frequency} \times 100$.

$percentage = \frac{2}{31} \times 100$

$percentage = 6.45...$

∴ More than 10 figurines were sold on 6.5% of the days during the 31-day period.

Answer

6.5%

✓1 mark for the correct answer

24c *Data analysis*

Decoding the question

Keywords: histogram, distribution, intervals

Relevant lesson: 12C

We must use the information provided in the dot plot to construct a histogram using the provided axis and specifications.

Information we are given

- The given dot plot
- The given axis
- Intervals should start at 0 and have a width of two.

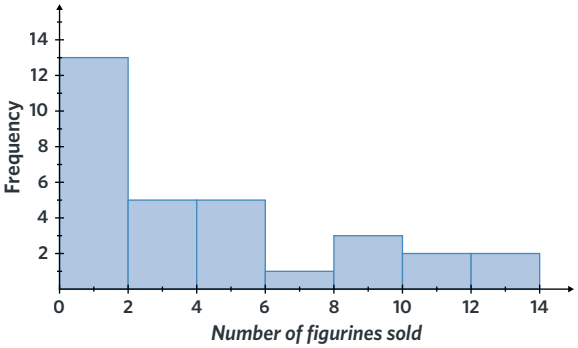
Using the theory

First, group the values from the dot plot into intervals of two. The intervals must start at 0 and end after the highest data value (13). Count the number of data values there are within each interval, and display this data in a frequency chart.

Number of figurines sold	Frequency
0–<2	13
2–<4	5
4–<6	5
6–<8	1
8–<10	3
10–<12	2
12–<14	2

We can now use the frequency chart to draw the histogram. Draw vertical columns at the border of each interval with a height equal to the group's frequency.

Answer



✓1 mark for the correct interval width
✓1 mark for the correct interval frequencies

12G

1 191

2
Minimum: 3
Q_1: 5
Median: 6
Q_3: 6
Maximum: 8

3
Minimum: 4
Q_1: 14
Median: 24
Q_3: 36
Maximum: 57

4
Minimum: 7
Q_1: 7.5
Median: 8
Q_3: 9
Maximum: 10

5 {6, 6, 6, 7, 8, 9}

6 19

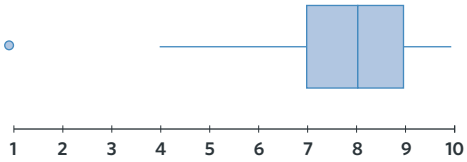
7 **a** 14 **b** 41, 43, 59

8 4, 6

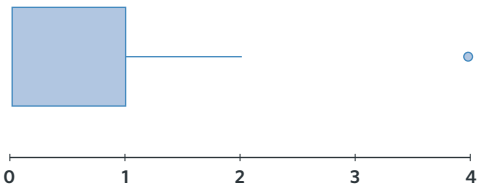
9 0.9, 3.1, 4.0

10
Minimum: 0
Q_1: 3
Median: 5
Q_3: 6
Maximum: 8

11



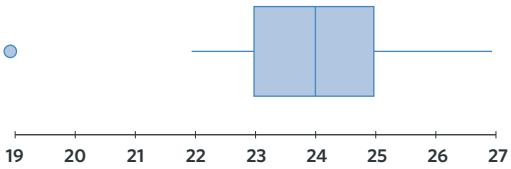
12 **a**



b 1

13 D

14 **a**



b 19 cm

c
Minimum: 19
Q_1: 23
Median: 24
Q_3: 25
Maximum: 27

15 **a**
Minimum: 47
Q_1: 65.5
Median: 70
Q_3: 71.5
Maximum: 91

b 6 **c** Increases IQR to 6.5

16 C

17 A

Questions from multiple lessons

18 ***Data analysis***

Decoding the question

Keywords: stem plot, modal weight

Relevant lessons: 12B, 12C

We must identify the most frequently occurring value in the data set by eye.

Information we are given

- The given stem plot

Using the theory

The modal weight is the weight that occurs most frequently in the stem plot.

Key: 10 | 5 = 10.5 kg

Weight

10	5	8	9		
11	0	2	2	3	
11	6	6	7	9	9
12	0	1	1	1	3	4
12	5	8	8		
13	2				

The weight 12.1 kg occurs three times in the stem plot. All other weights occur two times or less. 12.1 kg is the most common value and is, therefore, the modal weight.

Answer

D

✓*1 mark for the correct answer*

19 ***Recursion and financial modelling***

Decoding the question

Keywords: loan, interest rate, compounding period, lump sum

Relevant lesson: 3D

We must determine which loan will cost Sonya the least amount of money taking into account the different compounding periods.

Information we are given

- Sonya borrows $10 000.
- Sonya will fully repay the loan in a lump sum payment after eighteen months.
- Loan 1 – 11.30% per annum, compounding monthly
- Loan 2 – 11.35% per annum, compounding monthly
- Loan 3 – 11.39% per annum, compounding monthly
- Loan 4 – 11.40% per annum, compounding quarterly
- Loan 5 – 11.45% per annum, compounding quarterly

Using the theory

We can only directly compare the loans that have the same compounding periods.

Loans 1, 2 and 3 all compound monthly and have annual interest rates of 11.30%, 11.35% and 11.39% respectively. Loan 1 has the lowest annual interest rate of 11.30%, so out of the three options, loan 1 will cost Sonya the least amount of money.

Loans 4 and 5 both compound quarterly and have annual interest rates of 11.40% and 11.45% respectively. Loan 4 has the lower annual interest rate of 11.40%, so out of the two options, loan 4 will cost Sonya the least amount of money.

Now we must compare loans 1 and 4. We can use the compound interest formula to find the total value of the loans after 18 months. The compound interest formula states: $A = P\left(1 + \frac{r}{100}\right)^n$, where

- A is the total value of the loan
- P is the principal
- r is the interest rate per compounding period
- n is the number of compounding periods

Loan 1:

Since interest compounds monthly, $r = \frac{11.30}{12}$ and $n = 18$.

Substitute $P = 10\ 000$, $r = \frac{11.30}{12}$ and $n = 18$ into the compound interest formula.

$$A = 10\ 000\left(1 + \frac{\left(\frac{11.30}{12}\right)}{100}\right)^{18}$$

$$A = 11\ 837.73\ldots$$

Loan 4:

Since interest compounds quarterly, $r = \frac{11.40}{4}$ and $n = \frac{18}{3} = 6$.

Substitute $P = 10\ 000$, $r = \frac{11.40}{4}$ and $n = 6$ into the compound interest formula.

$$A = 10\ 000\left(1 + \frac{\left(\frac{11.40}{4}\right)}{100}\right)^{6}$$

$$A = 11\ 836.56\ldots$$

Since $11\ 836.56\ldots < 11\ 837.73\ldots$, loan 4 will cost Sonya the least amount of money.

Answer

D

✓*1 mark for the correct answer*

20a ***Data analysis***

Decoding the question

Keywords: variable, categorical

Relevant lesson: 12A

We need to identify the categorical variable out of *height* and *type of athlete*

Information we are given

- The given stem plot
- *height* (cm) and *type of athlete* (jockeys, hockey players)

Using the theory

Categorical variables are variables that have categories. Numerical variables are counted or measured variables.

The variable *height* is measured and can take on any value.

Hence, it is a numerical variable.

The variable *type of athlete* has two categories, 'jockeys' and 'hockey players'.

Hence, it is a categorical variable.

Answer

type of athlete

✓*1 mark for the correct answer*

20b *Data analysis*

Decoding the question

Keyword: modal height

Relevant lessons: 12B, 12C

We need to find the modal *height* (s) of the set of data for hockey players.

Information we are given

- The given stem plot

Using the theory

The modal value is the most frequently occurring value in the set of data. The most frequently occurring height of hockey players is the most frequently occurring height on the right-hand side of the stem plot.

These are shown in the following stem plot.

Key: 17 | 4 = 174 cm

Jockeys	*Height* (cm)	Hockey players
7	13	
9 9 8 8 8 8 6 5 5 4 2	14	
5 4 4 3 2 1	15	
5 4 2	16	
	17	4 5 5 7 7 7 8 9 9 9
	18	0 1 2 3 3 4 5 7 9
	19	1 3

∴ The modal heights of hockey players are 177 cm and 179 cm.

Answer

177 cm and 179 cm

✓1 mark for the correct answer

20c *Data analysis*

Decoding the question

Keywords: values, table

Relevant lesson: 12E

This question requires us to find components of a five-figure summary when given a stem plot.

Information we are given

- The given table

Decoding the question

The value of x is Q_1 of the jockeys' heights.

The median is in the $\left(\frac{21+1}{2}\right) = 11^{\text{th}}$ position of the data set. It is circled in the following.

Key: 17 | 4 = 174 cm

Jockeys	*Height* (cm)	Hockey players
7	13	
9 (9) 8 8 8 8 6 5 5 4 2	14	
5 4 4 3 2 1	15	
5 4 2	16	
	17	4 5 5 7 7 7 8 9 9 9
	18	0 1 2 3 3 4 5 7 9
	19	1 3

Q_1 is the median of the lower half of the data. Hence, Q_1 will be in the $\left(\frac{10+1}{2}\right) = 5.5^{\text{th}}$ position. This means Q_1 is the average of the 5^{th} and 6^{th} values as shown in the following.

Key: 17 | 4 = 174 cm

Jockeys	*Height* (cm)	Hockey players
7	13	
9 9 8 8 8 8 (6 5) 5 4 2	14	
5 4 4 3 2 1	15	
5 4 2	16	
	17	4 5 5 7 7 7 8 9 9 9
	18	0 1 2 3 3 4 5 7 9
	19	1 3

The average of these two values is $\frac{145+146}{2} = 145.5$.

$\therefore x = 145.5$

The value of y is the maximum height of the hockey players.

It is circled in the following.

Key: 17 | 4 = 174 cm

Jockeys	*Height* (cm)	Hockey players
7	13	
9 9 8 8 8 8 6 5 5 4 2	14	
5 4 4 3 2 1	15	
5 4 2	16	
	17	4 5 5 7 7 7 8 9 9 9
	18	0 1 2 3 3 4 5 7 9
	19	1 (3)

$\therefore y = 193$

Answer

$x = 145.5$ and $y = 193$

✓1 mark for the correct value of x
✓1 mark for the correct value of y

12H

1 Cordelia

2 **a** 1 litre **b** 56 litres
c Men **d** Men

3 **a** Fernando: 18, Lucas: 9
Fernando had a higher median score.
b Fernando: 17.5, Lucas: 10
Fernando had a larger IQR of points scored.
c Fernando: 24, Lucas: 26
Fernando had a smaller range of points scored.

4 **a** Company A: 3900, Company B: 1900
Company A had higher median sales figure.
b Company A: 1900, Company B: 1550
Company A had a larger IQR of sales figures.

c Company A: 4100, Company B: 5300

Company A had a smaller range of sales figures.

5 a

Key: 1|5 = 15 years old

UK				USA							
		9	8	7	1	8	8	8	9	9	
		4	3	2	2	0	1	1	1	2	3	4	4
9	8	8	7	6	6	5	2	5					
					1	3	4						

b UK: 14 years, USA: 16 years

c People from the USA had the most variable ages.

6 a Supermarket A **b** Supermarket A

7 a Before: 7, After: 25

The median number of push-ups before was lower than after.

b Before: 5, After: 10

The IQR before was smaller than after.

c Before: 14, After: 27

The range before was smaller than after.

8 a Cafe A: 72, Cafe B: 94

Cafe A had a lower median number of coffees sold.

b Cafe A: 17, Cafe B: 14

Cafe A had a greater IQR of coffees sold than Cafe B.

c Cafe A: 30, Cafe B: 38

Cafe A had a smaller range of coffees sold than Cafe B.

9 a Class C **b** Class A

c Class A because it had the smallest IQR and range.

10 C

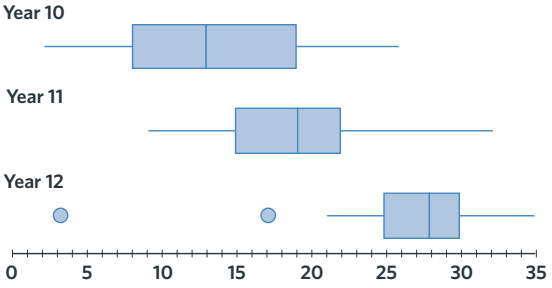
11 Women in this retirement village are generally older than men.

12 The median price for Airline A is $950. The median price for Airline B is $1050, $100 more expensive. The spread in terms of IQR for Airline A is $150. The spread in terms of IQR for Airline B is $75, $75 less. Airline A has an outlier at $800. Airline A generally has a lower price than Airline B, but is more variable.

13 a 29

b This data shows that in this group of people men drink more milk than women, but the sample is too small to make a general statement about all men and women.

14 a



b The median number of hours of study per week in Year 10 is 13, in Year 11 is 19, and in Year 12 is 28. The number of hours studying a week generally increases as the Year level increases. The spread in terms of IQR for Year 10 is 11, for Year 11 it is 7, and for Year 12 it is 5. The variability decreases as the year increases.

15 a Class A: 64, Class B: 75.5

Class A had a lower median score than Class B.

b Class A: 17, Class B: 16

Class A had a larger IQR of scores than Class B.

c Class A: 48, Class B: 46

Class A had a larger range of scores than Class B.

d Class A had the more variable results because both the IQR and range of scores were larger than Class B.

e In general Class B got higher exam marks than Class A.

16 The medians for the two months differ. In May, the median maximum temperature is about 14.5°C, while in July, the median maximum temperature is about 9°C.

or

Comparing the two IQR values can be used as the difference is the IQRs indicates the presence of an association.

Questions from multiple lessons

17 ***Data analysis***

Decoding the question

Keywords: histogram, percentage

Relevant lessons: 2E, 12C

We need to analyse the histogram to find the percentage of Australian Olympians that are 190 cm or taller.

Information we are given

- The given histogram
- The sample size is 399.

Using the theory

The number of Australian Olympians that are 190 cm or taller is the sum of the frequencies of the three rightmost intervals in the histogram. The frequency of an interval in a histogram is given by the height of the column.

Hence, $60 + 13 + 3 = 76$ Australian Olympians are 190 cm or taller.

Represent this as a percentage of the total number of Australian Olympians using the formula $percentage = \frac{frequency}{total\ frequency} \times 100$.

Substitute $frequency = 76$ and $total\ frequency = 399$ into the formula.

$percentage = \frac{76}{399} \times 100$

$percentage = 19.04...\%$

∴ 19% of Australian Olympians at the 2012 Olympics were 190 cm or taller.

Answer

B

✓1 mark for the correct answer

18 ***Recursion and financial modelling***

Decoding the question

Keyword: percentage

Relevant lesson: 3A

We must represent the $5 tip as a percentage of the $56 meal.

Information we are given

- The meal cost $56.
- Kourtney tipped the waiter $5.

Using the theory

We can find V_2 as a percentage of V_1 using the following formula:

$percentage = \frac{V_2}{V_1} \times 100$

We are finding $5 as a percentage of $56, so substitute $V_1 = 56$ and $V_2 = 5$.

$percentage = \frac{5}{56} \times 100$

$percentage = 8.928...\%$

∴ The $5 tip as a percentage of the price of the meal is closest to 8.93%.

Answer

D

✓1 mark for the correct answer

19a ***Data analysis***

Decoding the question

Keywords: describe, shape, distribution

Relevant lessons: 12D, 12G

We need to describe the shape of each boxplot.

Information we are given

- The given boxplot

Using the theory

The boxplot for green tea flavoured ice creams shows that the whisker on the right-hand side is longer than the left. The median is also on the left side of the box. This means that the bulk of the data lies in the left side of the distribution, and trails off to the right side, indicating a positive skew. There are no outliers present. Hence, the distribution is positively skewed with no outliers.

The boxplot for black sesame flavoured ice creams also shows the whisker on the right-hand side longer than the left. The median is again further to the left of the data. This means that the data trails off on the right side of the distribution, indicating a positive skew. There is one outlier on the left-hand side. Hence, the distribution is positively skewed with an outlier.

Answer

Green tea: positively skewed with no outliers.

Black sesame: positively skewed with an outlier.

✓1 mark for the correct answer

19b ***Data analysis***

Decoding the question

Keywords: lower fence, boxplot

Relevant lesson: 12G

We need to use the formula $lower\ fence = Q_1 - 1.5 \times IQR$ to determine the lower fence for the black sesame boxplot.

Information we are given

- The given boxplot

Using the theory

The lower fence can be calculated using the formula $lower\ fence = Q_1 - 1.5 \times IQR$, where $IQR = Q_3 - Q_1$

By looking at the boxplot, we can see that Q_1 is 38 and Q_3 is 48.

Substitute $Q_1 = 38$ and $Q_3 = 48$ into the equation.

$lower\ fence = 38 - 1.5 \times (48 - 38)$

$lower\ fence = 38 - 1.5 \times 10$

$lower\ fence = 23$

Answer

23

✓1 mark for the correct answer

13A

1 **a** Yes **b** No

2 **a** EV: Temperature

RV: Number of people at the pool

This is because it is temperature that dictates the attendance rates.

b EV: Age of participants

RV: Amount of time spent exercising daily

Age cannot be impacted by anything other than time. Therefore the age of the participants cannot be the response variable, and must be the explanatory variable.

c EV: Workers' pay

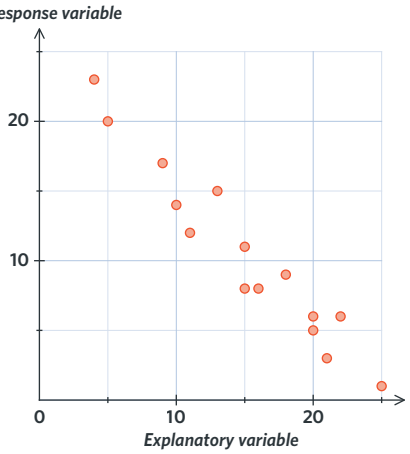
RV: Happiness

A workers' pay can influence their happiness, but their happiness cannot change their pay.

3 a The amount of sleep that residents get can be changed by the amount of dogs in the neighbourhood, but the amount of dogs in the neighbourhood cannot be changed by the amount of sleep the residents get. Therefore the amount of sleep is the response variable and the amount of dogs is the explanatory variable.

b The two variables are unrelated so there is no explanatory or response variable that makes sense.

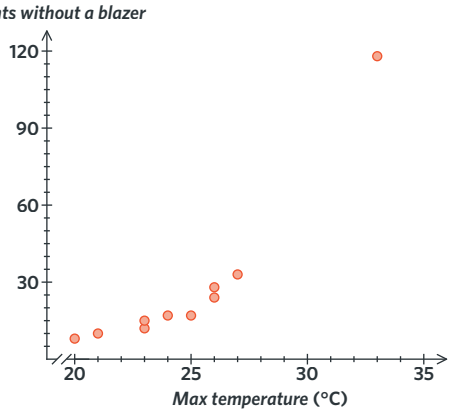
4 a



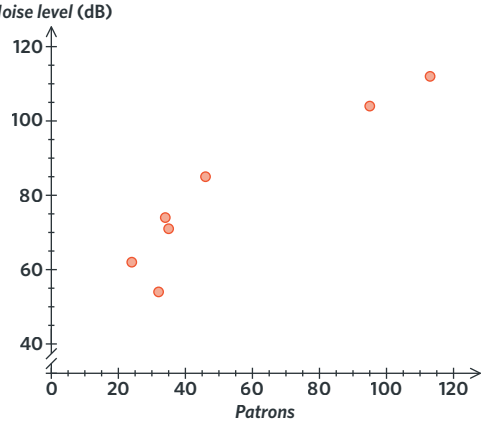
b 8

5 B

6



7 a

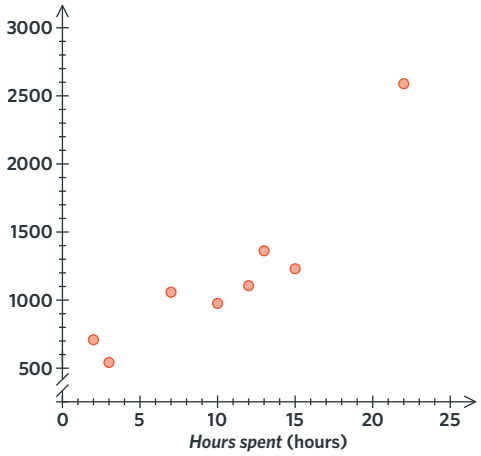


b Friday and Saturday

c Yes. On Friday nights the pub averages 95 patrons, which is under the 100 patron limit, yet exceeds the noise limit of 98 decibels.

8 a The explanatory variable is the time spent on the project and the response variable is the number of words written.

b



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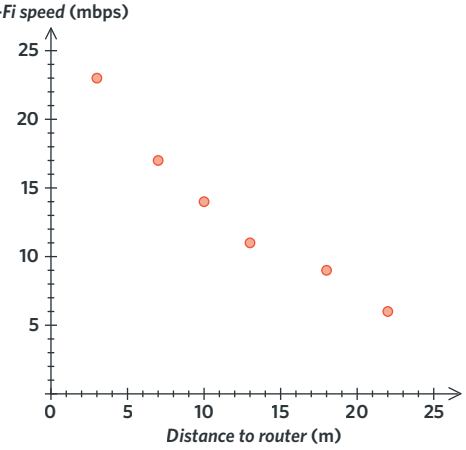
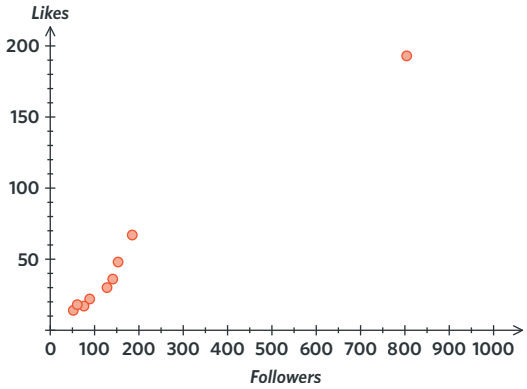
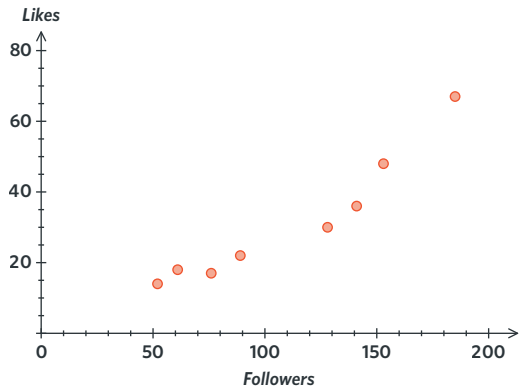


10 a



b



Questions from multiple lessons

11 *Data analysis*

Decoding the question

Keywords: histogram, most frequently

Relevant lesson: 12B

We must identify by eye the most frequently occurring interval of the histogram.

Information we are given

- The given histogram

Using the theory

The question asks us to find the 'most frequently' occurring interval. This is the modal interval. In a histogram, the modal interval is the interval with the tallest column. The interval 170–<172 cm is the tallest with a frequency of nine. All other intervals have a frequency of eight or less.

∴ The *height* for this sample of competitors is most frequently greater than or equal to 170 cm and less than 172 cm.

Answer

B

✓1 mark for the correct answer

12 *Recursion and financial modelling*

Decoding the question

Keywords: deposit, monthly repayments, interest

Relevant lesson: 3E

This question requires us to determine the total amount of money paid in order to find the interest charged.

Information we are given

- A house was purchased for $800 000.
- A deposit of $50 000 was made.
- The balance will be completely repaid with 85 monthly repayments of $10 000.

Using the theory

Calculate the total amount of money paid, A

$A = deposit + (payment \times number\ of\ payments)$

$A = 50\ 000 + (10\ 000 \times 85)$

$A = 900\ 000$

Calculate the total amount of interest paid by subtracting the original price of the house, P_0, from the total amount of money paid, A

$I = A - P_0$

$I = 900\ 000 - 800\ 000$

$I = 100\ 000$

Answer

C

✓1 mark for the correct answer

13a *Data analysis*

Decoding the question

Keyword: five-number summary

Relevant lesson: 12G

We must find the five-number summary of the given data.

Information we are given

- The given data
- There are 13 people in the sample.

Using the theory

The five-number summary is as follows: minimum value, Q_1, median, Q_3, maximum value.

Order the data in ascending order.

0 2 3 3 4 4 4 5 7 8 9 10 16

Identify the minimum and maximum values.

$minimum\ value = 0$

$maximum\ value = 16$

Find the position of the median using the formula $\frac{n+1}{2}$ where n is the number of elements in the data set.

$\frac{13+1}{2} = \frac{14}{2} = 7$

The median is the 7th element of the data set, which is 4.

0 2 3 3 4 4 4 5 7 8 9 10 16

$median = 4$

Q_1 is the median of the lower half of the data.
The lower half of the data is: 0 2 3 3 4 4
Find the position of Q_1 using the formula $\frac{n+1}{2}$, where n is the number of elements in the lower half of the data set.
$\frac{6+1}{2} = \frac{7}{2} = 3.5$
Q_1 is the mean of the 3rd and 4th elements of the data set, which is $\frac{3+3}{2} = \frac{6}{2} = 3$

$Q_1 = 3$

Q_3 is the median of the upper half of the data.
The upper half of the data is: 5 7 8 9 10 16
Q_3 is the mean of the 3rd and 4th elements of the data set, which is $\frac{8+9}{2} = \frac{17}{2} = 8.5$

$Q_3 = 8.5$

Hence, the five-number summary is:

0, 3, 4, 8.5, 16

Answer

0, 3, 4, 8.5, 16

✓1 mark for the correct answer

13b ***Data analysis***

Decoding the question

Keyword: outliers

Relevant lesson: 12G

We must find the lower and upper fences of the data to determine if there are any outliers.

Information we are given

- The given data
- The five-number summary from part **a**

Using the theory

Any points outside the lower or upper fences are classified as outliers.

$lower\ fence = Q_1 - (1.5 \times IQR)$

$upper\ fence = Q_3 + (1.5 \times IQR)$

To find the lower and upper fences we must find the IQR.

$IQR = Q_3 - Q_1$

Substitute $Q_3 = 8.5$ and $Q_1 = 3$ into the equation.

$IQR = 8.5 - 3 = 5.5$

Substitute $IQR = 5.5$, $Q_1 = 3$, and $Q_3 = 8.5$ into the formulas for the lower and upper fences.

$lower\ fence = 3 - (1.5 \times 5.5) = 3 - 8.25 = -5.25$

$upper\ fence = 8.5 + (1.5 \times 5.5) = 8.5 + 8.25 = 16.75$

Nothing lies beyond the lower or upper fences so there are no outliers.

Answer

No outliers

✓1 mark for the correct answer

13c ***Data analysis***

Decoding the question

Keywords: boxplot

Relevant lesson: 12G

We must use our answers from part **a** and b to draw a boxplot for the data.

Information we are given

- The given data
- The five-number summary from part **a**
- The distribution contains no outliers, as found in part **b**.

Using the theory

A boxplot is a graphical representation of the five-number summary and any outliers. It consists of a central box and whiskers. If outliers exist in the data set, they are shown as separate dots on the number line. The left and right borders of the box represent Q_1 and Q_3, and the vertical line in the centre of the box represents the median. The whiskers extend to the most extreme value that is not an outlier.

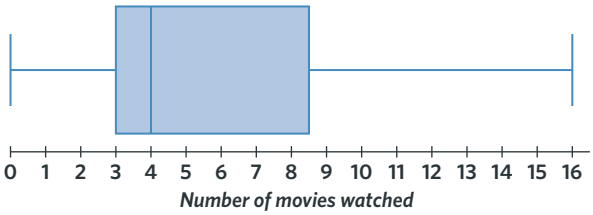
The five-number summary of the data is:

0, 3, 4, 8.5, 16

There are no outliers in the data set.

Use this information to draw the boxplot.

Answer



✓1 mark for the correct answer

13B

1 **a** No **b** Yes **c** Yes

2 A

3 **a** Weak, linear, negative relationship

b Strong, linear, negative relationship

4 x-axis – Length of hair (cm)

y-axis – Driving speed (kilometres per hour)

The values of the scatterplot should be distributed randomly and follow no pattern.

5 No, the relationship between the variables is linear, moderate, and negative.

The relationship is moderate because the variables are packed relatively closely together.

The relationship is linear because the values vaguely follow a straight line instead of a curved one.

The relationship is negative because the values trend downwards, from top left to bottom right.

6 B

7 **a** There is a strong, linear, positive relationship between the marks out of 50 on the exam and the percentage scored on the exam.

b There is a weak, linear, positive relationship between shoe size and height.

c There is no relationship between the number of surfers at a beach and the number of seagulls at a beach.

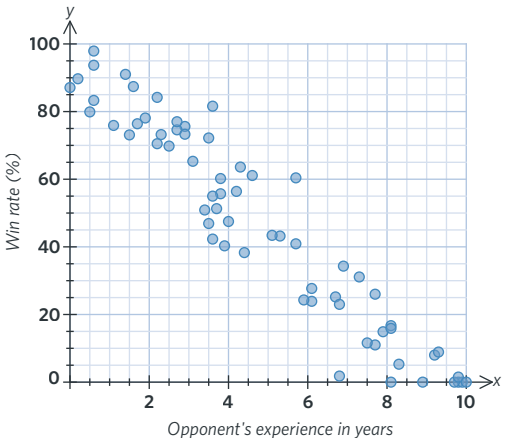
8 Explanatory variable: the number of attendees over 18 years old

Response variable: the number of cars in the parking lot

Positive correlation.

9 B

10 **a** **Relationship between opponent experience and win rate**



The scatterplot should show moderate to strong correlation strength, linear form, and have a negative direction. A possible scatterplot is shown above.

b No, unfavourable. The scatterplot shows a strong/moderate negative correlation. Therefore, the more experience an opponent has, the lower Quinn's win rate.

Against an opponent with 9 years of experience, Quinn has a predicted win rate of 0–10%.

NOTE: answer may vary depending on your graph from part **a**.

11 There is a strong, linear, positive correlation between actual temperature and apparent temperature.

Questions from multiple lessons

12 *Data analysis*

Decoding the question

Keywords: histogram, interquartile range

Relevant lesson: 12F

We must identify the values of Q_1 and Q_3 from the histogram to calculate the interquartile range.

Information we are given

- The given histogram
- The sample contains 55 students.

Using the theory

First, we must find the values of Q_1 and Q_3

There are 55 data values in the sample. The median will be in the $\frac{55+1}{2} = 28^{\text{th}}$ position of the data set, meaning there are 27 values above and below the median.

The first quartile, Q_1, is the median of the first half of the data. It is the value in the $\frac{27+1}{2} = 14^{\text{th}}$ position of the first half of the data. The 14^{th} value lies in the interval from 0 to 2 books.

The midpoint of this interval is 1 book. Hence, $Q_1 = 1$.

The third quartile, Q_3, is the median of the second half of the data. It is the value in the $\frac{27+1}{2} = 14^{\text{th}}$ position of the second half of the data, which is the data point in the 42^{nd} position overall (there are 28 data values before the second half of the data begins). The 42^{nd} value lies in the interval from 6 to 8 books.

The midpoint of this interval is 7 books. Hence, $Q_3 = 7$.

The interquartile range is given by the formula $IQR = Q_3 - Q_1$

Substitute $Q_3 = 7$ and $Q_1 = 1$ into the equation.

$IQR = 7 - 1$

$IQR = 6$

∴ The interquartile range is 6 books.

Answer

D

✓1 mark for the correct answer

13 *Recursion and financial modelling*

Decoding the question

Keywords: sequence, recurrence relation

Relevant lessons: 6B, 6D

We need to generate a recurrence relation for the given sequence.

Information we are given

- The given sequence

Using the theory

First, determine whether the sequence is arithmetic or geometric.

Check if the sequence has a common difference using the formula $common\ difference = T_n - T_{n-1}$

Substitute $T_n = -8$ and $T_{n-1} = -17$ into the equation as -8 follows -17 in the sequence.

$common\ difference = -8 - (-17) = 9$

Check using $T_n = 1$ and $T_{n-1} = -8$ as 1 follows -8 in the sequence.

$common\ difference = 1 - (-8) = 9$

Hence, this sequence is an arithmetic sequence with a common difference of 9.

Arithmetic sequences are of the form $T_0 = a$ $T_{n+1} = T_n + d$ where a is the first element in the sequence and d is the common difference.

$\therefore T_0 = -17, \quad T_{n+1} = T_n + 9$

Answer

E

✓1 mark for the correct answer

14a ***Data analysis***

Decoding the question

Keywords: back-to-back stem plot, range

Relevant lessons: 12F, 12H

We must find the range for men and women and determine which one is larger.

Information we are given

- The given back-to-back stem plot

Using the theory

The range of a data set is the difference between the maximum and minimum value.

Find the range for both men and women using the formula $range = maximum - minimum$

Men: $range = 25 - 5 = 20$

Women: $range = 21 - 4 = 17$

$20 > 17$

$\therefore$ The men surveyed have a larger range than the women.

Answer

Men

✓1 mark for the correct answer

14b ***Data analysis***

Decoding the question

Keywords: back-to-back stem plot, median

Relevant lessons: 12E, 12H

We must find the median response of the men and women surveyed, and then find the difference between the two.

Information we are given

- The given back-to-back stem plot

Using the theory

The median is located in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position of an ordered data set, where n is the number of elements in the data set.

For men:

$n = 18$

$\frac{18+1}{2} = \frac{19}{2} = 9.5$

The median is the mean of the elements located in the 9th and 10th positions.

Key: 1 | 0 = 10

Number of hours

Men						Women					
				0	4						
		8	5	5	0	6	8	8	9		
4	3	3	3	2	1	0	0	1	2	2	4
8	8	7	6	5	1	7	7	9			
	2	2	1	0	2	0	0	1			
				5	2						

15 is located in the 9th position and 16 is located in the 10th position.

$\frac{15+16}{2} = 15.5$

Hence, the median for men is 15.5 hours.

For women:

$n = 17$

$\frac{17+1}{2} = \frac{18}{2} = 9$

The median is the element located in the 9th position.

Key: 1 | 0 = 10

Number of hours

Men						Women					
				0	4						
		8	5	5	0	6	8	8	9		
4	3	3	3	2	1	0	0	1	2	2	4
8	8	7	6	5	1	7	7	9			
	2	2	1	0	2	0	0	1			
				5	2						

12 is located in the 9th position.

Hence, the median for women is 12 hours.

Find the difference in the median for men and women.

$difference = 15.5 - 12 = 3.5$

$\therefore$ The difference in the median of the two groups is 3.5 hours.

Answer

3.5 hours

✓1 mark for the correct answer

13C

1 C

2 No

3 C

4 D

5 Jeremy's level of activity beforehand is a confounding variable. If he was doing strenuous activity before eating the candy, he would be tired.

6 C

7 **a** Moderate, linear, positive.

b Moderate, linear, positive.

c No relationship.

8 Yes he is correct. The scatter plot values follow a positive trend, packed closely together – strong positive relationship can be inferred.

A strong positive relationship has r between 0.75 and 1. His guess, 0.8, is within the range 0.75 to 1.

9 **a** The first scatterplot, A.

b A: within 0.4 to 0.6

B: within -0.8 to -0.9

10 B

11 **a** 0.60 **b** 0.65 **c** 0.91

12 **a** -0.96

Strong, negative correlation.

A negative correlation implies an inverse relationship.

This implies that when the immune system decreases, Whitescale severity increases.
This verifies the observation.

b The second trial will have a stronger value of r than the first.

c Jora has immune system strength of 0 and a severity level of 10. This is an extreme example of the relationship we were expecting – the weaker the host's immune system, the more severe the symptoms of Whitescale. This is strong evidence for the relationship between the two variables, and therefore strengthens the relationship.

13 **a** Data points are approximately linear.
The upwards trend of the data indicates a positive r value.

The points being packed closely together implies a strong relationship. The value of r is between 0.75 and 1.

b 0.98

c Correlation. It isn't causal because confounding variables exist, such as good customer service.

Good customer service while serving larger patties would also increase customer satisfaction.

Therefore, larger patties may not necessarily be the cause of customer satisfaction.

14 E

Questions from multiple lessons

15 ***Data analysis***

Decoding the question

Keywords: explanatory, response, variables, correlation

Relevant lesson: 13B

We must determine which is the explanatory and response variable and if the correlation is negative or positive.

Information we are given

- The higher the temperature on a particular day, the more drinks are sold.

Using the theory

The explanatory variable can affect, or explain, the results of the response variable.

The response variable is expected to change as a result of a change in the explanatory variable.

Since it is possible that the *temperature* could affect the number of *drinks sold*, *temperature* is the explanatory variable and *drinks sold* is the response variable. It is not logical to assume that *temperature* could be affected by the number of *drinks sold*

We are told that as the *temperature* increases, the number of *drinks sold* also increases. Hence, the correlation between *temperature* and *drinks sold* is positive.

Answer

A

✓1 mark for the correct answer

16 ***Recursion and financial modelling***

Decoding the question

Keywords: increase in value, percentage

Relevant lesson: 3A

We must first find the difference between the two values and express this as a percentage of the original value.

Information we are given

- The coin was bought for \$4500.
- The coin then sold at auction for \$6000.

Using the theory

The percentage change can be found using the following formula:

$$percentage\ change = \frac{change}{original\ value} \times 100$$

In this case, the original value of the coin was \$4500 and the change in value was $\$6000 - \$4500 = \$1500$.

Substitute these values into the formula.

$$percentage\ change = \frac{change}{original\ value} \times 100$$

$$percentage\ change = \frac{1500}{4500} \times 100$$

$$percentage\ change = 33.333...\%$$

∴ There was a 33.3% increase in value, correct to one decimal place.

Answer

C

✓1 mark for the correct answer

17a *Data analysis*

Decoding the question

Keywords: table, scatterplot

Relevant lesson: 13A

We must create a scatterplot using the data in the table.

Information we are given

- The given table

Using the theory

First, determine which variable is the explanatory variable and which is the response variable.

The explanatory variable can affect, or explain, the values of the response variable.

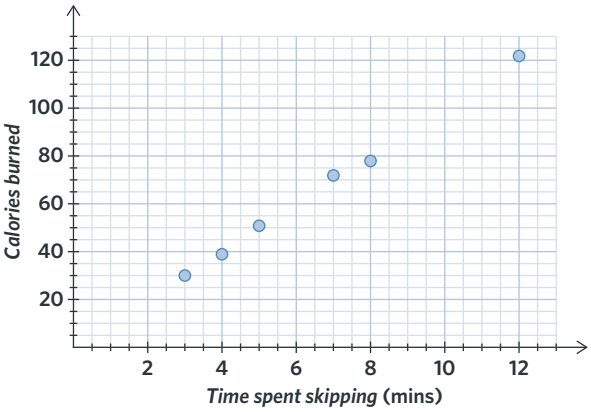
The response variable is expected to change as a result of the explanatory variable.

Since the time Sonya spends skipping could affect the number of calories she burns, *time spent skipping* is the explanatory variable and *calories burned* is the response variable. Hence, *time spent skipping* is located on the x-axis and *calories burned* is located on the y-axis.

Appropriately scale the axes, making sure it covers all of the given data values.

Plot the data points.

Answer



✓1 mark for the correct answer

17b *Data analysis*

Decoding the question

Keywords: correlation, strength, direction, form

Relevant lesson: 13B

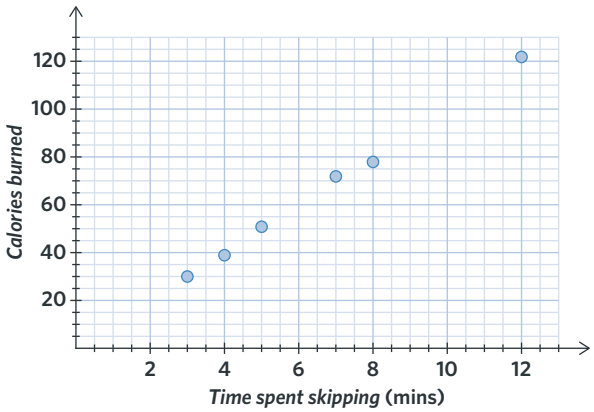
We must use the scatterplot form part **a** to describe the correlation in terms of strength, direction and form.

Information we are given

- The given table

Using the theory

Recall the scatterplot from part **a**.



Find the strength of the correlation.

The points follow the same line and are packed closely together.

Hence, the strength of the correlation is strong.

Identify the direction of the correlation.

The values are following an increasing trend from left to right.

Hence, the direction of the correlation is positive.

Identify the form of the correlation.

The values roughly follow a straight line.

Hence, the form of the correlation is linear.

∴ The correlation between the two variables is strong, positive and linear.

Answer

Strong, positive and linear

✓1 mark for the correct answer

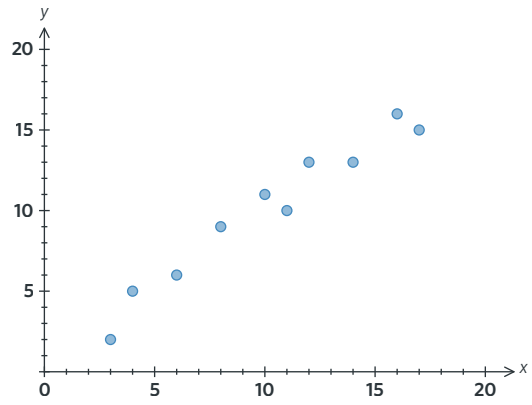
13D

1 A: II

B: III

C: I

2 **a**

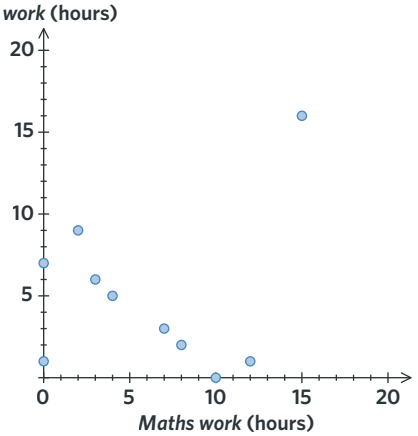


b $y = x$ or similar **c** $y = 0.174 + 0.963x$

3 **a** $y = -165.915 + 7.621x$

b $sales = -165.915 + 7.621 \times max\ temperature$

4 **a**



b *hours of art work completed* = $3.979 + 0.167 \times$ *hours of maths work completed*

c *hours of art work completed* = $8.136 - 0.698 \times$ *hours of maths work completed*

5 **a** a will be negative, b will be positive

b a will be positive, b will be negative

c a and b will be positive

6 **a** *number of children* =

$1 + 0.00002 \times$ *household income*

b 1 child

7 **a** 4 wins **b** 0.5 wins

8 **a** *pollution index* = $64.435 - 1.185 \times$ *cyclists' share of transportation*

b 64.435

c -1.185

9 **a** *average daily max temperature* =

$38.652 - 0.491 \times$ *degrees latitude south*

b -0.491°C

c 38.652°C

10 **a** On average, when the minimum temperature is 0°C, the maximum temperature will be 13°C.

b On average, the maximum temperature increases by 0.67°C for each 1°C increase in the minimum temperature.

11 On average, population increases by 2680 people for each additional square kilometre of area.

Questions from multiple lessons

12 ***Data analysis***

Decoding the question

Keyword: correlation coefficient

Relevant lesson: 13C

We need to estimate the strength of the correlation using the given least squares regression line and scatterplot.

Information we are given

- The given graph

Using the theory

Estimate the strength of correlation between the data points.

The data points follow an upwards trend and are situated relatively closely around the regression line.

Hence, there is a moderate, positive, linear relationship, so $0.5 \leq r < 0.75$.

The only option with $0.5 \leq r < 0.75$ is option B.

Answer

B

✓*1 mark for the correct answer*

13 ***Recursion and financial modelling***

Decoding the question

Keywords: compounding monthly, value, fifth month

Relevant lesson: 3D

We need to apply the compound interest formula to determine the value of Kingsley's investment after five months.

Information we are given

- He invested \$25 000.
- The interest rate is 1.5% p.a. and interest compounds monthly.

Using the theory

The total value of a compound interest investment is given by the following formula.

$A = P\left(1 + \frac{r}{100}\right)^n$ where

- A is the total value of the investment,
- P is the principal,
- r is the interest rate per compounding period as a percentage, and
- n is the number of compounding periods.

First, we will work out r. As interest compounds monthly, this is given as $r = \frac{1.5}{12} = 0.125$.

As Kingsley invested \$25 000, $P = 25\ 000$. As we are looking for the value after the fifth month, $n = 5$. Substitute these values into the compound interest formula to find the value of A

$A = 25\ 000\left(1 + \frac{0.125}{100}\right)^5$

$A = 25\ 156.64\ldots$

This is closest to \$25 157.

Answer

A

✓*1 mark for the correct answer*

14a *Data analysis*

Decoding the question

Keywords: shape of the distribution, dot plot, boxplot

Relevant lesson: 12D

This question requires us to determine the shape of the data by looking at the dot plot and/or boxplot.

Information we are given

- The given dot plot
- The given boxplot

Using the theory

In this solution, we are going to focus on the dot plot.

We can see that the majority of the data is clustered around the left-hand side of the dot plot and that the distribution trails off to the right.

∴ This distribution has a positive skew.

Answer

Positively skewed

✓1 mark for the correct answer

14b *Data analysis*

Decoding the question

Keyword: lower fence

Relevant lesson: 12G

We need to read values off the boxplot to calculate the lower fence.

Information we are given

- The given boxplot

Using the theory

The formula to calculate the lower fence is $lower\ fence = Q_1 - 1.5 \times IQR$, where $IQR = Q_3 - Q_1$

We need to find Q_1 and Q_3. These can be read off the boxplot.

$Q_1 = 2000$ and $Q_3 = 6000$.

Substitute $Q_1 = 2000$ and $Q_3 = 6000$ into the formula for the lower fence.

$lower\ fence = Q_1 - 1.5 \times IQR$

$lower\ fence = Q_1 - 1.5 \times (Q_3 - Q_1)$

$lower\ fence = 2000 - 1.5 \times (6000 - 2000)$

$lower\ fence = -4000$

Answer

-4000

✓1 mark for the correct answer

13E

1 160 cm to 185 cm

2 6

3 11.813

4 **a** \$5791; interpolation **b** Over; extrapolation

5 **a** 78%; interpolation **b** 26.4%; extrapolation

c 4 hours and 30 minutes; interpolation

6 B

7 **a** This prediction uses extrapolation.

b The sample size is too small for a prediction to be reliable.

c There is not enough correlation for a prediction to be reliable.

8 This would not be a reliable prediction. As the value of r is -0.299, there is a weak relationship between the variables. Estimations based on this regression line have no validity.

9 This would not be a reliable prediction. As the domain of the data is between 10 and 200 dollars, the response variable takes values between 63 and 633 days. Two years = 730 days. Therefore, this prediction is an extrapolation and unreliable.

10 **a** 17 m/s; interpolation

b 131 surfers at Surfers Paradise

161 surfers at Bondi Beach

This is a difference of 30 surfers.

c In a hurricane, you'd expect there to be no surfers, yet extrapolating both of these regression lines to 32 m/s gives 131 and 161 surfers at each beach. When extrapolating data, a presumption is made that the trend that is within the range of the data set will continue outside the domain of the data. As seen in this case, this presumption does not always hold true, making extrapolation unreliable.

d As the sample size is so small (a week of data), the sample may have lacked diversity, such as different weather conditions. Making predictions based off one week of data at a random time during the year could lead to unreliable predictions.

11 **a** \$643 million; extrapolation

b \$111 538 462; interpolation

c The value of r is 0.43, which indicates a weak positive correlation. If there is a weak correlation between the variables, the regression line becomes a weak predictor of values. Interpolating values is therefore unreliable.

12 **a** 3 years

b The regression equation was used to make predictions outside the available range of data.

Questions from multiple lessons

13 *Data analysis*

Decoding the question

Keywords: explanatory variable, least squares regression line

Relevant lesson: 13D

In this question, we must evaluate which of the given equations would best represent the regression line on the scatterplot.

Information we are given

- The given scatterplot
- The *value of opponent's team* is the explanatory variable.

Using the theory

A least squares regression equation is of the form $y = a + bx$, where a is the y-intercept, and b is the gradient. As stated in the question, the *value of opponent's team* is the explanatory variable, x, so *goal difference* must be the response variable, y. After substituting in the variables, the equation becomes:
$goal\ difference = a + b \times value\ of\ opponent's\ team$

In options C, D and E, *goal difference* is the explanatory variable, which is incorrect. Hence, the correct option must be either A or B.

The only difference between options A and B is the value of a, the y-intercept.

A common error would be to assume that the y-intercept of the regression line is 1.81 by reading directly from the graph. However, the horizontal axis begins at 50 000, not 0, so a cannot be 1.81. Option B is eliminated.

The regression equation must be closest to
$goal\ difference = 2.11 - 0.0000063 \times value\ of\ opponent's\ team$

Answer

A

✓ *1 mark for the correct answer*

14 *Recursion and financial modelling*

Decoding the question

Keywords: increases by 5.5%, value

Relevant lesson: 3D

We must use the compound interest formula to calculate the value of the violin in 2017.

Information we are given

- The value increases by 5.5% each year.
- The violin has a value of \$170 500 in 2019.

Using the theory

Since the value of the violin increases by 5.5% each year, the value is increasing under compounding conditions.

The value of a compound-interest investment is given by the following formula.

$A = P\left(1 + \frac{r}{100}\right)^n$, where

- A is the total value of the investment
- P is the principal value
- r is the interest rate per compounding period, as a percentage
- n is the number of compounding periods.

In this case, $A = 170\ 500$, P is the value of the violin in 2017, $r = 5.5\%$, and $n = 2$.

Substitute these values into the formula.

$170\ 500 = P\left(1 + \frac{5.5}{100}\right)^2$

Use the solve function on your calculator to solve for P

$P = 153\ 186.136\ldots$

See video solution for button-by-button demonstration.

∴ The value of the violin in 2017 was closest to \$153 186.

Answer

C

✓ *1 mark for the correct answer*

15a *Data analysis*

Decoding the question

Keywords: 'determine the equation', least squares regression line

Relevant lesson: 13D

We need to enter the data provided in the table into our calculators to find the equation of the least squares regression line.

Information we are given

- The given table
- The equation should be in the form $cricket\ players\ at\ training = a + b \times temperature$

Using the theory

Enter the data within the provided table into your calculator and use its statistics functions to find the least squares regression line.

The calculator will return values of $a = 60.90$ and $b = -1.23$ correct to two decimal places. Substitute these into the equation
$cricket\ players\ at\ training = a + b \times temperature$

$cricket\ players\ at\ training = 60.90 - 1.23 \times temperature$

See video solution for a button-by-button demonstration.

Answer

$cricket\ players\ at\ training = 60.90 - 1.23 \times temperature$

✓ *1 mark for finding the correct equation*

15b ***Data analysis***

Decoding the question

Keywords: relationship, interpret, slope, 'in terms of the variables'

Relevant lesson: 13D

We need to explain what the slope of the least squares regression line represents in terms of the variables *patrons at the cricketers' bar* and *temperature*

Information we are given

- $patrons\ at\ the\ cricketers'\ bar = -33.46 + 3.27 \times temperature$

Using the theory

When interpreting a regression line $y = a + bx$, the value of b, or the slope, predicts the change in the response variable for a one-unit change in the explanatory variable.

The response variable is the *patrons at the cricketers' bar* and the explanatory variable is the *temperature*

$\therefore$ For every one-degree Celsius increase in *temperature* there are 3.27 extra *patrons at the cricketers' bar*

Answer

On average, for every one-degree increase in *temperature*, there are an estimated 3.27 extra *patrons at the cricketers' bar*

✓*1 mark for the correct answer*

Glossary

A

Adjacency matrices Adjacency matrices are symmetric, square matrices which have a row and column for each vertex. p. 174

Adjacent (adj) Adjacent is used to refer to the side that is directly adjacent to the angle of interest in a right-angled triangle. p. 330

Ambiguous case The ambiguous case of the sine rule occurs when two different triangles can be created using the given information. p. 355

Angle of depression The angle of depression is the angle below the horizontal. p. 338

Angle of elevation The angle of elevation is the angle above the horizontal. p. 337

Area The area is the amount of two-dimensional space a shape takes up. p. 252

Area scale factor Area scale factors relate the difference in area between two similar figures.

$Area\ scale\ factor = \frac{are\ of\ image}{area\ of\ original} = k^2$ p. 322

Arithmetic recurrence relation Arithmetic recurrence relations model arithmetic growth or decay, where each iteration increases or decreases by a constant amount. p. 222

Arithmetic sequence Arithmetic sequences are sequences with repeated addition or subtraction. p. 213

B

Back-to-back stem plot A back-to-back stem plot is a type of stem plot that can be used to compare a numerical variable with a categorical variable that has exactly two categories. p. 544, 593

Bar chart A bar chart is a graph that is commonly used to display categorical data shown in a frequency table. p. 532

Base-height formula The base-height formula is a formula used for finding the area of a triangle. The base-height formula is: $area = \frac{1}{2} \times base \times height$ p. 281

Boxplot A boxplot is a graphical representation of a five-number summary as well as any outliers. p. 585

Bridge A bridge is an edge that would turn a connected graph into a disconnected graph if it were removed. p. 173

C

Capacity The capacity is similar to the volume, however it is the amount that the object can hold. p. 253

Categorical data Categorical data represents a quality or attribute. Categorical data has a group of categories. p. 526

Circuit A circuit is a trail beginning and ending at the same vertex such that no edges are repeated. Vertices may be repeated. p. 192

Column matrix A column matrix is a matrix with only one column and any number of rows. It is sometimes called a column vector. p. 121

Complete graph A complete graph is a graph in which every vertex is directly connected to every other vertex by an edge. p. 172

Composite shapes A composite shape is a shape made up of two or more simple shapes. p. 290

Compound Interest Compound interest is interest that is added to the principal amount after each time period, causing future interest earnings to increase more each period.

$A = P\left(1 + \frac{r}{100}\right)^n$ p. 102

Compounding periods The compounding period is the length of time between the points when interest is calculated for an investment or loan. p. 102

Confounding variables Confounding variables are related external factors that can contribute to an association. They sometimes cause researchers to erroneously believe a relationship exists. p. 617

Connected graph A connected graph is one in which every vertex is connected, directly or indirectly. Every vertex is able to be reached by any other vertex by travelling along the edges. p. 172

Continuous data Continuous data is numerical data that can be measured and exist on a continuous scale. p. 527

Corner-point principle For an objective function in a feasible region, the maximum and minimum points are always corners of the feasible region. p. 465

Correlation Correlation is the measure of the strength of the relationship between two variables. p. 609

Cosine rule The cosine rule is a formula that relates one angle in a triangle to the three side lengths. The cosine rule states that for triangle ABC: $c^2 = a^2 + b^2 - 2ab\cos(C)$ p. 362

Cycle A cycle is a path beginning and ending at the same vertex such that no edges or vertices are repeated (other than the starting/ending vertex). p. 192

D

Degree (of a vertex) The degree of a vertex is the number of connections it has to edges. p. 171

Determinant The determinant of a matrix is a number that tells us whether or not the matrix has an inverse. p. 163

Direct variation Direct variation is where an increase in one variable results in an increase in the other. It can be seen as a linear equation that goes through the origin with the rule $y = kx$. p. 476

Dividends Dividends are a share of the profit made by a company that is distributed to shareholders. p. 86

Discrete data Discrete data is numerical data that is counted, not measured. It only exists as whole units and not as part of a continuous scale. p. 527

Domain The domain is the set of possible values of the explanatory variable. p. 403

Dot plot A dot plot is a simple way to display numerical data. p. 542

E

Edge An edge is a line that joins vertices on a graph. p. 171

Element The numbers contained within a matrix are known as elements. The element in row i and column j of matrix X is denoted x_{ij}. p. 120

Elimination (simultaneous equations) A method of solving simultaneous equations that involves the equations being added or subtracted in a way that eliminates one of the variables. p. 27

Euler's formula For a connected planar graph, Euler's formula is: $V - E + F = 2$ p. 185

Explanatory (independent) variables The explanatory variable, EV, can be useful in explaining or predicting the changes observed in the response variable. p. 603

Extrapolation Extrapolation is the method of making predictions that are outside of the scope of the data set. p. 423, 635

F

Face Faces are separate areas that are bordered by the edges in a planar graph. p. 183

Feasible regions A feasible region is a region that satisfies a set of inequalities. It is normally shaded for clarity. p. 456

Fibonacci-related sequences Fibonacci-related sequences are sequences where each term is found by adding the previous two terms. p. 245

Five-number summary The five-number summary provides key information about a set of data and its distribution, including spread and centre. A five-number summary is written as minimum, Q_1, median, Q_3, maximum. p. 583

Frequency tables A frequency table is a table that tallies how often each value or category in a data set occurs. p. 531

G

Geometric recurrence relation Geometric recurrence relations model geometric growth or decay, where each iteration is multiplied by a constant factor. p. 235

Geometric sequence A geometric sequence is a sequence with repeated multiplication or division. p. 213

Goods and Services Tax (GST) GST is a 10% tax applied to most sales in Australia. p. 85

Gradient The gradient (slope) is the steepness of a line, which represents the change in y for each one-unit increase in x. It can be found using the formula $gradient = \frac{rise}{run}$ or $gradient = \frac{y_2 - y_1}{x_2 - x_1}$. p. 390, 628

Graphs A graph is a diagram that is used to show the connections between a group of things, people or activities. p. 171

Grouped data Grouped data is data that has been organised into set groups or intervals, making large data sets easier to work with. p. 539

Grouped frequency distribution table A grouped frequency distribution table groups data in regular intervals. p. 539

H

Heron's formula Heron's formula is a formula used to find the area of a triangle when all three side lengths are known. Heron's formula is $A = \sqrt{S(S-a)(S-b)(S-c)}$ where $S = \frac{a+b+c}{2}$ p. 282

Hire-purchase agreement A hire-purchase agreement involves a customer paying a deposit on an item and then making small regular payments until the item is paid off. p. 111

Histogram A histogram is a visual representation of a grouped frequency distribution table. p. 540

Horizontal lines Horizontal lines are represented by the equation $y = c$, where c is the y-intercept. p. 384

Hyperbolic relationships Hyperbolic relationships are graphs of relations of the form $y = \frac{k}{x} + c$ where $k \neq 0$ and $x \neq 0$. p. 510

Hypotenuse (hyp) The hypotenuse is the longest side of a right-angled triangle. p. 260, 330

I

Identity matrix The identity matrix (I) is a square matrix in which each element in the main diagonal is one. All other elements are zero. p. 121, 162

Included angle formula The included angle formula is a formula used for finding the area of a triangle. The included angle formula is: $area = \frac{1}{2}ab\sin(C)$ p. 375

Inequalities An inequality is an equation that, instead of an $=$ sign, uses a $<$, $>$, $\leq$ or $\geq$ sign. p. 439

Interpolation Interpolation is the method of making predictions that are within the scope of a data set. p. 415, 635

Interquartile range The interquartile range, IQR, is the spread of the middle 50% of data values. $IQR = Q_3 - Q_1$ p. 575

Inverse matrix Some square matrices have an inverse matrix. The inverse matrix has the special property that when it is multiplied with the original matrix from the left or right, the answer is the identity matrix. The inverse matrix of A is denoted by A^{-1}. p. 163

Inverse variation Inverse variation is where an increase in one variable results in a decrease in the other. The graph of the relationship between x and y is a hyperbola with the rule $y = \frac{k}{x}$. p. 476

J

Joint variation Joint variation is where one variable is affected by two or more variables. p. 477

L

Least squares regression Least squares regression is the process of finding the line of best fit with the minimum sum of squared residuals for the data. p. 626

Line of best fit The line of best fit is a linear equation that shows the general trend of a data set. p. 408

Linear graphs Linear graphs possess a constant gradient, and result in a straight line when graphed. p. 382

Linear relationship A linear relationship is a relationship between two quantities in which the quantities change in response to each other by a constant amount. p. 2

Logarithmic scale Logarithmic scale are scales which use logarithms of base 10 to plot data that has a large range. p. 56

Logarithms A mathematical operation with a base, argument, and exponent. p. 55, 519

Logarithms (logs) to the base 10 The log to base 10 of a number is the power to which 10 must be raised to create this number. $\log_{10}(x) = y$ implies that $10^y = x$. p. 56, 520

Loop A loop is an edge that connects a vertex back to itself. p. 171

M

Matrix A matrix is a tool for displaying a collection of numerical values. The values are arranged into rows and columns depending on what the matrix represents. p. 120

Mean The mean of a set of data is one measure of its centre. It is calculated by adding all the data values together and then dividing the sum by the number of values. p. 560

Median The median is the middle value in an ordered set of data. p. 560

Minimum spanning tree The minimum spanning tree is the spanning tree for a graph with the lowest weight. p. 205

Modal interval The modal interval is the interval with highest frequency, similar to mode. p. 539

Mode The mode is the most frequently occurring category or data value. p. 533

N

Negatively skewed distribution A negatively skewed distribution is when a distribution of data trails off as the values decrease. p. 551

Nominal data Nominal data is categorical data that can be grouped but not ordered. p. 526

Numerical data Numerical data represents a quantity (a numeric value). p. 527

O

Objective functions An objective function represents a quantity to be minimised or maximised in a linear programming problem, such as distance, cost or time. p. 465

Opposite (opp) Opposite is used to refer to the side that is directly opposite the angle of interest in a right-angled triangle. p. 330

Order (of matrices) The order of a matrix describes its dimensions. The order is expressed as $number\ of\ rows \times number\ of\ columns$. p. 120

Order of magnitude Orders of magnitude relate to powers of 10, and are used to describe the approximate size of a number. p. 55

Ordinal data Ordinal data is categorical data that can be grouped and ordered. p. 526

Outliers Outliers are values which fall far outside the rest of the data set. p. 565, 585

P

Parallel boxplots Parallel boxplots are multiple boxplots drawn one above the other. They allow for easy comparison between data sets. p. 594

Path A path is a walk in which no edges or vertices are repeated. p. 191

Pearson's correlation coefficient (r) Pearson's correlation coefficient (r) is a measure of the strength and direction of a linear relationship between two variables. p. 618

Percentage change The percentage change is an expression of the difference between two values as a percentage of the initial value. It is found using the formula

$$Percentage\ change = \frac{Difference}{Original\ value} \times 100.$$ p. 62

Perimeter The perimeter is the distance around the outside of a shape. p. 251

Personal loans A personal loan is a compound interest loan, often used to pay for large personal expenses. Mathematically, loans function in the same way as investments: both start with a principal amount and involve the payment of interest. p. 112

Piecewise linear model A piecewise linear model is a graph composed of different straight line segments. p. 430

Planar graph A planar graph is a graph that can be drawn such that no edges overlap. p. 183

Point of intersection The point of intersection is the point at which two lines meet. p. 20

Positively skewed distribution A positively skewed distribution is a distribution that trails off as the values increase. p. 551

Prim's algorithm Prim's algorithm is an algorithm that finds the minimal spanning tree. p. 205

Prism A prism is a three-dimensional object with a uniform cross-section. p. 296

Pyramid A pyramid is a three-dimensional object with a polygon base and triangular sides that meet at the top. p. 297

Pythagoras' theorem Pythagoras' theorem states that for a right-angled triangle, $a^2 + b^2 = c^2$. p. 260

Q

Quadratic relationships Quadratic relationships are graphs of relations of the form $y = kx^2 + c$. p. 502

Quartiles Quartiles divide a distribution of data in quarters. The symbols used to refer to the quartiles are Q_1, Q_2 and Q_3. p. 573

R

Range Range is a measure of a data set's spread. $Range = maximum\ value - minimum\ value$ p. 572

Recurrence relations A recurrence relation is a formula that relates the current term in a sequence to the next term in the sequence. p. 222

Recursion Recursion is a method of defining a number sequence that follows a pattern. p. 9

Response (dependent) variables The response variable may be explained or predicted by changes in the explanatory variable. p. 603

Row matrix A row matrix is a matrix with only one row, and any number of columns. It is sometimes called a row vector. p. 121

S

Scalar multiplication Scalar multiplication is the multiplication of a matrix by a number. Each element in the matrix is multiplied by the number (scalar). p. 133

Scale factor (k) The scale factor indicates the change in size between the original figure, and the image. All sides scale proportionately. p. 316

Scatterplot A scatterplot is a bivariate graph where each data point is represented by a dot. p. 603

Scientific notation A number written in scientific notation is of the form $x = 10^y$, where $1 \leq x < 10$, and y is an integer. This is useful for very large and very small numbers. p. 45

Sequence A sequence is a list of numbers separated by commas. They often follow a pattern, but can also be random. p. 213

Shares Shares are units of ownership in a company. p. 86

Significant figures Significant figures are digits within a number that contribute to the value of that number with certainty. p. 50

Similar figures Similar figures refer to two figures that have the same shape but different size. p. 314

Simple graph A simple graph does not contain any loops or duplicate edges. p. 172

Simple interest Simple interest is interest that is a fixed portion of the initial investment. $I = \frac{Prn}{100}$ p. 92

Simultaneous equations Simultaneous equations are a set of equations which are used to solve for unknown values. The solution is the point where each equation is true. p. 20

Sine rule The sine rule is a formula that relates the angles in a triangle to the sides opposite them. The sine rule states that for triangle ABC: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$ p. 353

Sliding line method The sliding line method is a way of determining the maximum and minimum solutions of an objective function. It involves transposing the objective function so it is in terms of y and sliding the line up and down the y-axis, to intersect the corners of the feasible region. p. 469

Spanning trees A spanning tree is a tree which connects all vertices in the original graph. p. 204

Spread The spread refers to the range of values included in the data set. p. 553

Square matrix A square matrix has an equal number of rows and columns. p. 121

Standard deviation Standard deviation is a measure of the spread of a distribution around the mean. It is equal to the square root of the variance. p. 576

Stem plot A stem plot is a plot that splits each number into two parts, and groups points according to their left-most digits. p. 543

Substitution (simultaneous equations) A method of solving simultaneous equations that involves a variable from one equation being substituted into the other. p. 26

Surface area The surface area (S) of a three-dimensional object is the sum of the areas of all the faces. p. 307

Symmetric distribution A symmetric distribution is a distribution where the data is evenly distributed around the mean, and does not trail off in either direction more than the other. p. 551

T

Term A term is a number in a sequence. p. 213

Trail A trail is a walk in which no edges are repeated. p. 191

Transforming data Transforming data is the process of taking non-linear relationships between two variables and making them linear. p. 489

Trees Trees are a special type of connected graph which use as few edges as possible to connect a group of vertices. They contain no loops or cycles. p. 204

True bearings A true bearing is a measurement of the direction of one point from another point. It is measured clockwise from north and presented as a three-digit number. p. 343

U

Unit-cost depreciation Unit-cost depreciation is the depreciation of an asset by a set amount for each unit of use. p. 229

Unitary method The unitary method is used to find the value for a large number of identical items by first calculating the value for one item. p. 72

V

Vertex A vertex is a dot on a graph that represent entities such as objects, people or activities. p. 171

Vertical lines Vertical lines are represented by the equation $x = b$, where b is the x-intercept. p. 384

Volume The volume is the amount of three-dimensional space an object takes up. p. 252, 296

Volume scale factor Volume scale factors relate the difference in volume between two similar figures. $Volume\ scale\ factor = \frac{volume\ of\ image}{volume\ of\ original} = k^3$ p. 322

W

Walk A walk is a series of edges that pass through any number of vertices, in any order, starting and finishing at any vertex. p. 191

Weighted graphs Weighted graphs are graphs where each edge is assigned a 'weight', often representing time, cost, or distance. p. 198

X

x-reciprocal transformation $\left(\frac{1}{x}\right)$ The x-reciprocal transformation is a type of transformation used to linearise non-linear data. It works by 'compressing' x-values that are greater than one, whilst 'stretching' x-values that are less than one. p. 491

x-squared transformation The x-squared transformation is a type of transformation used to linearise non-linear data. It works by 'stretching' the larger x-values more than the smaller x-values. p. 489

Z

Zero matrix The zero matrix is a matrix in which every element is 0. p. 121