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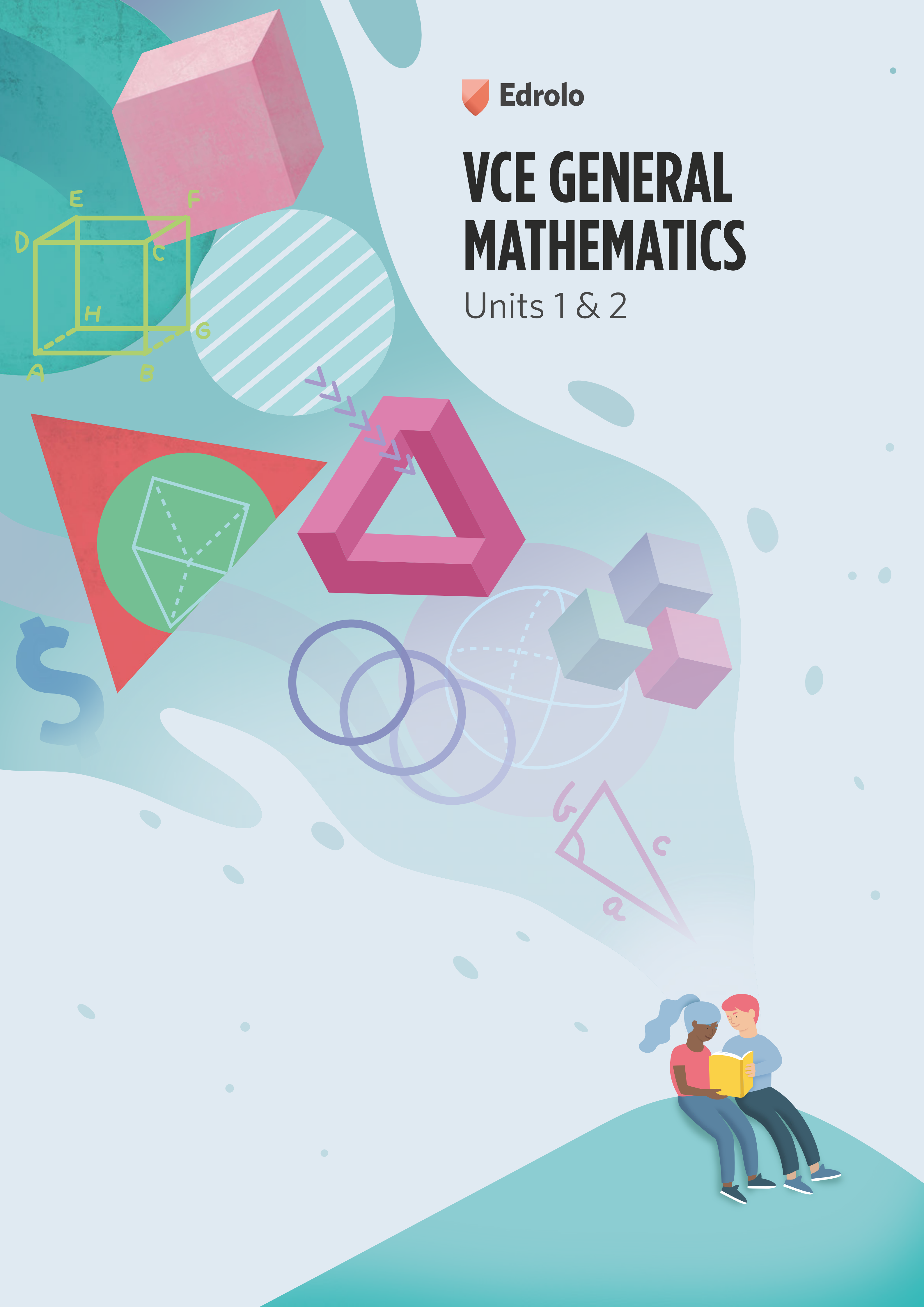
VCE GENERAL MATHEMATICS

Units 1 & 2



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VCE GENERAL MATHEMATICS

Units 1 & 2

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At Edrolo, we're transforming the way the students learn and teachers teach.

Our mission is simple: to improve education.

PUBLISHED IN AUSTRALIA BY Edrolo
321 Exhibition Street Melbourne VIC 3000, Australia

Ref: 1.1.1

National Library of Australia Cataloguing-in-Publication data

TITLE: Edrolo VCE General Mathematics Units 1 & 2
CREATOR: Edrolo et al.
ISBN: 978-1-922901-00-2
TARGET AUDIENCE: For secondary school age.
SUBJECTS: General Mathematics--Study and teaching (Secondary)--Victoria
General Mathematics--Victoria--Textbooks.
General Mathematics--Theory, exercises, etc.
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Printed in Australia by Ligare Printing Pty Ltd

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FEATURES OF THIS PRODUCT

Edrolo's VCE General Maths Units 1 & 2 product has the following features.

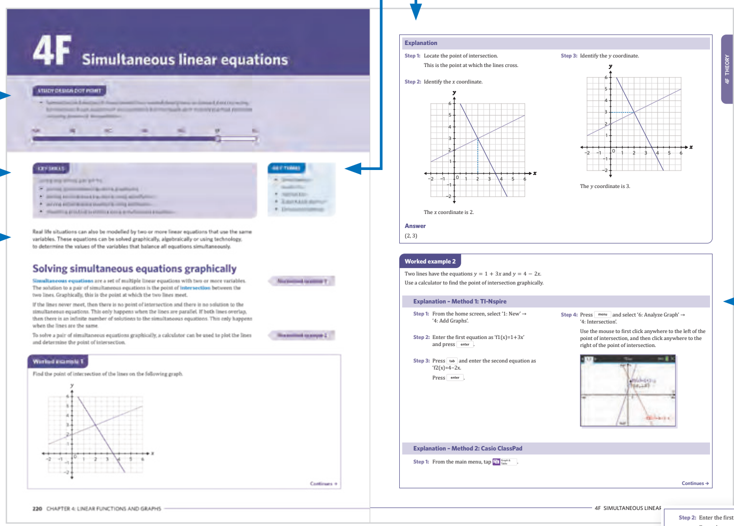
Textbook theory

Study design dot points provide explicit links between the content covered in each lesson and the VCAA curriculum.

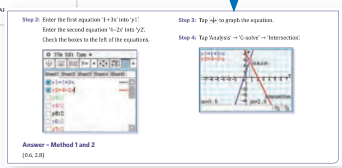
Key terms identify newly defined mathematical terminology and provide a reference for navigating glossary definitions.

Worked examples provide fully stepped out exemplar solutions.

Key skills break the theory down into smaller chunks that focus on only one skill at a time, with key skill headings replicated throughout the theory, questions, and answers for easy navigation.



Calculator methods with screenshots step students through using the 'TI-Nspire' and 'Casio ClassPad' CAS calculators.

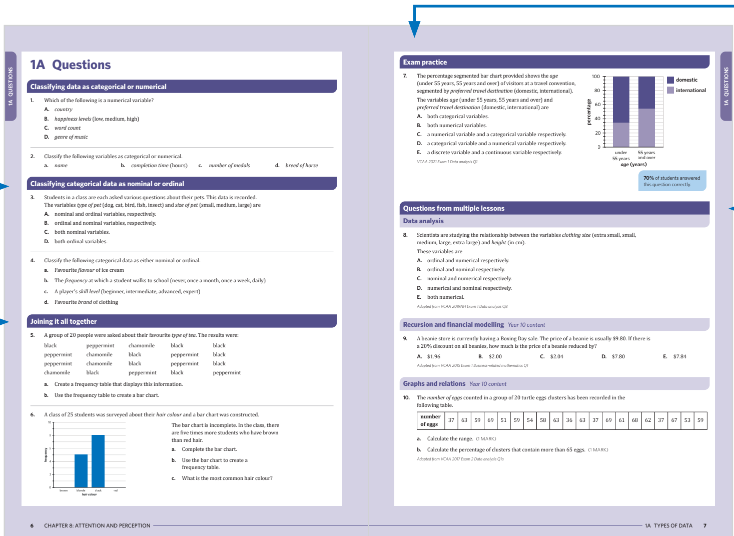


Introductions provide a launchpad for the lesson and serve to give context for the theory.

Textbook questions

Key skills questions link to key skills in the theory and ask students to apply only one skill at a time.

Exam practice questions provide students with past VCAA exam questions to get them ready for exams.



Joining it all together questions scaffold students to link multiple skills from the lesson together.

Questions from multiple lessons provide ongoing revision from a range of topics.

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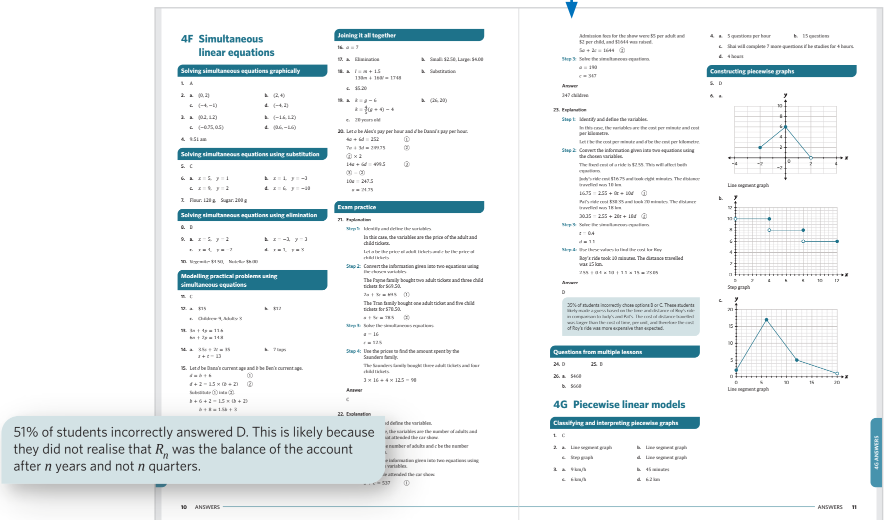
Textbook answers

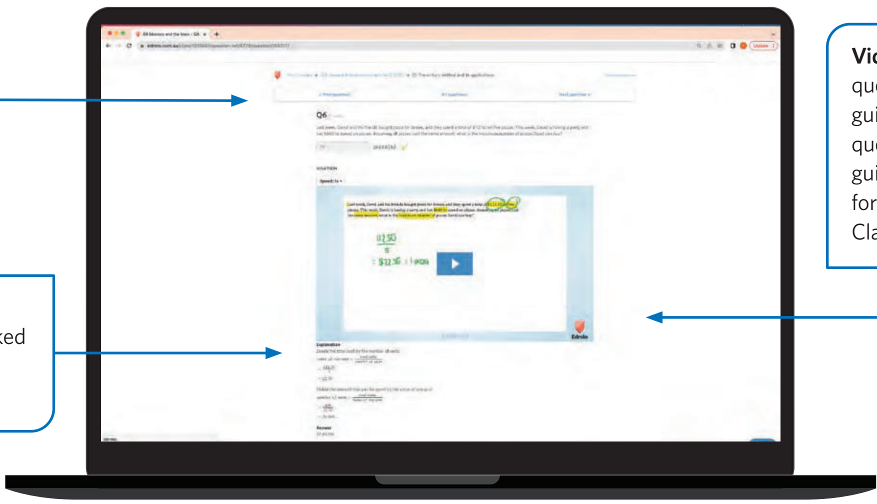
Fully worked solutions are provided for all exam practice questions, complete with commentary on common misconceptions and errors made, based on VCAA statistics (where applicable).



Online – Other resources

Question sets provide the ability to complete all questions online, with instant feedback on student responses.

Video solutions for every question provide extra guidance on how to answer questions, complete with guided calculator solutions for TI-Nspire and Casio ClassPad CAS calculators.



Static solutions provide fully worked solutions for all questions.

Chapter reviews provide visual theory summaries and application questions that scaffold students towards answering questions using multiple skills within the chapter.

CHAPTER 1 CALCULATOR QUICK LOOK-UP GUIDE

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1

UNIT 1 AOS 1
CHAPTER 1

Investigating and comparing data distributions

LESSONS

KEY KNOWLEDGE

- types of data, including categorical (nominal or ordinal) or numerical (discrete or continuous, interval, ratio)
- display and description of categorical data distributions of one or more groups using frequency tables and bar charts, and the mode and its interpretation
- display and description of numerical data distributions using histograms, stem plots and dot plots and choosing between plots according to context and purpose
- summarising numerical data distributions, including use of and calculation of the sample summary statistics, median, range, and interquartile range (IQR) or mean and standard deviation
- the five-number summary and the boxplot as its graphical representation and display, including the use of the lower fence ($Q_1 - 1.5 \times IQR$) and upper fence ($Q_3 + 1.5 \times IQR$) to identify possible outliers
- consideration of a range of distributions (symmetrical, asymmetrical), their summary statistics and the percentage of data lying within several standard deviations of the mean
- use of back-to-back stem plots or parallel boxplots, as appropriate, to compare the distributions of a single numerical variable across two or more groups in terms of centre (median) and spread (IQR and range), and the interpretation of any differences observed in the context of the data.

Image: ChristianChan/Shutterstock.com

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1A Types of data

STUDY DESIGN DOT POINT

- types of data, including categorical (nominal or ordinal) or numerical (discrete or continuous, interval, ratio)

1A 1B 1C 1D 1E 1F 1G 1H

KEY SKILLS

During this lesson, you will be:

- classifying data as categorical or numerical
- classifying categorical data as nominal or ordinal
- classifying numerical data as discrete or continuous
- classifying numerical data as interval or ratio.

KEY TERMS

- Categorical data
- Numerical data
- Nominal data
- Ordinal data
- Discrete data
- Continuous data
- Interval data
- Ratio data

Data is information that has been collected, often for the purposes of analysis or interpretation. Data can be classified as either categorical or numerical, and can then be classified even more specifically. It is important to understand the distinctions between types of data, as this will affect how the data can be used.

Classifying data as categorical or numerical

Categorical data refers to data which is organised into several categories or groups. Examples of categorical data include *city* and *favourite colour*.

Numerical data refers to data that can be counted or measured. Examples of numerical data include *length* and *number of pets*.

Most, but not all, data involving numbers is numerical. For example, a *postcode* such as 3047, while consisting of numbers, is not a numerical variable. This is because a postcode is just a numerical representation of a location. It would not make sense to perform mathematical operations, such as addition or multiplication, on a postcode. Furthermore, it would not make sense to take the average of multiple postcodes.

Worked example 1

Is the variable *hair colour* an example of categorical or numerical data?

Explanation

Step 1: Determine whether the variable is counted/measured or categorised into groups.

Hair colour cannot be counted or measured, but rather sorted into groups e.g. brown, black, red etc.

Step 2: Classify the data as categorical or numerical.

Answer

Categorical

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Classifying categorical data as nominal or ordinal

Categorical data can be further separated into two types.

Nominal data refers to data which has no logical order or ranking of categories. Examples of nominal data include *flavour* (vanilla, chocolate, strawberry etc.) and *instrument* (guitar, piano, violin etc.).

Ordinal data refers to data which can be logically ordered. Examples of ordinal data include *size* (small, medium or large) and *placement* in a race (1^{st}, 2^{nd}, 3^{rd} etc.)

Worked example 2

A poll asked the question 'Ferraris are the greatest cars. Do you agree?' with the possible responses 'strongly agree', 'agree', 'neutral', 'disagree', or 'strongly disagree'.

Are these responses nominal or ordinal?

Explanation

Step 1: Identify whether there is a logical order for the categories.

It makes sense for the categories to start and end with either 'strongly agree' and 'strongly disagree', and for 'neutral' to be in the middle.

Therefore, there is a logical order.

Step 2: Classify the variable as nominal or ordinal.

Answer

Ordinal

Classifying numerical data as discrete or continuous

Numerical data can be further classified as discrete or continuous.

Discrete data refers to data which can be counted using integers. Examples of discrete data include *number of shirts* and *number of people*.

Continuous data refers to data which is measured using a continuous scale. Examples of continuous data include *height* (cm) and *speed* (km/h).

Worked example 3

The *maximum temperature* (°C) on each day in July is recorded in 2018. Is *maximum temperature* (°C) a discrete or continuous variable?

Explanation

Step 1: Identify whether the variable is measured or counted.

The variable *maximum temperature* (°C) is a measured quantity; it can be measured to varying degrees of accuracy.

Step 2: Classify the variable as discrete or continuous.

Answer

Continuous

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Classifying numerical data as interval or ratio

Numerical data can also be classified as either interval or ratio, depending on the scale that the data uses.

Interval data uses a scale where the difference between each adjacent possible value is equal. For example, the difference between *temperatures* of 18 °C and 19 °C is the same as the difference between 19 °C and 20 °C, so *temperature* (°C) is interval data. It is important to note that interval data can contain negative values.

Ratio data uses a scale similar to interval data, with one key difference. The 0 value on a scale for ratio data signifies the lowest possible value ('absolute zero'), or a lack of a measurement. No negative values are possible to obtain using a ratio scale. Examples of ratio data include *length* (cm) and *age* (years).

Worked example 4

Classify the following variables as either interval or ratio.

a. *temperature* (°F)

Explanation

Step 1: Determine whether the scale used has an absolute zero value.

Since *temperature* in Farenheit can have a negative value, its scale does not have an absolute zero value.

Step 2: Classify the data as interval or ratio.

Answer

Interval

b. *length* (m)

Explanation

Step 1: Determine whether the scale used has an absolute zero value.

Since a negative *length* cannot be measured, and 0 m indicates the shortest possible *length*, the scale for *length* has an absolute 0 value.

Step 2: Classify the data as interval or ratio.

Answer

Ratio

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1A Questions

Classifying data as categorical or numerical

1. Which of the following is a numerical variable?

A. *country*

B. *happiness levels* (low, medium, high)

C. *word count*

D. *genre of music*

---

2. Classify the following variables as categorical or numerical.

a. *name*
b. *completion time* (hours)
c. *number of medals*
d. *breed of horse*

Classifying categorical data as nominal or ordinal

3. Students in a class are each asked various questions about their pets. This data is recorded. The variables *type of pet* (dog, cat, bird, fish, insect) and *size of pet* (small, medium, large) are

A. nominal and ordinal variables, respectively.

B. ordinal and nominal variables, respectively.

C. both nominal variables.

D. both ordinal variables.

---

4. Classify the following categorical data as either nominal or ordinal.

a. Favourite *flavour* of ice cream

b. The *frequency* at which a student walks to school (never, once a month, once a week, daily)

c. A player's *skill level* (beginner, intermediate, advanced, expert)

d. Favourite *brand* of clothing

---

5. Data is collected from a local university inquiring about the *age* of their students. The possible answers are 14–17, 18–21, 22–25, and 26+. Classify this data as nominal or ordinal.

Classifying numerical data as discrete or continuous

6. Each member of the audience at a concert is asked the *number of concerts* they have attended in their lifetime, and their *age*, in years. This data is recorded. The variables *number of concerts* and *age* (years) are

A. both discrete variables.

B. both continuous variables.

C. discrete and continuous variables, respectively.

D. continuous and discrete variables, respectively.

---

7. Classify the following numerical data as either discrete or continuous.

a. The *number of people* attending a football match

b. The *distance* (km) from a planet to the sun

c. The *temperature* (°C) of a pool

d. The *number of children* in a classroom

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8. The fastest *time taken* for Usain Bolt to run 100 metres is 9.58 seconds. Is *time taken* a discrete or continuous variable?

Classifying numerical data as interval or ratio

9. Which of the following is not an example of ratio data?
   - **A.** *age* (years)
   - **B.** *elevation* compared to sea level (m)
   - **C.** *width* of a table (cm)
   - **D.** *number of apples*

---

10. The variable *temperature* can be interval or ratio depending on what unit it is measured in. In degrees Celsius (°C), 0 represents the freezing temperature of water. In Kelvin (K), 0 is known as 'absolute zero' and is theoretically the coldest possible temperature. Which of the following statements is true?
    - **A.** *temperature* (°C) and *temperature* (K) are interval and ratio variables respectively.
    - **B.** *temperature* (°C) and *temperature* (K) are ratio and interval variables respectively.
    - **C.** *temperature* (°C) and *temperature* (K) are both interval variables.
    - **D.** *temperature* (°C) and *temperature* (K) are both ratio variables.

Joining it all together

11. Classify the following as either ordinal, nominal, discrete or continuous.
    - **a.** The weight of a bag of apples
    - **b.** The different brands of apples
    - **c.** The classification of apples as small, medium or large
    - **d.** The number of apples in a bag

---

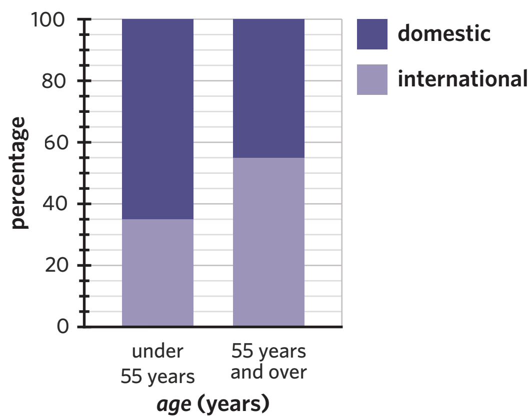
12. Classify the following as either categorical or numerical data.
    - **a.** The number of breeds of cats in a shelter
    - **b.** The ranking of difficulty of maths questions from one to five
    - **c.** A postcode
    - **d.** A car's registration number

Exam practice

13. The percentage segmented bar chart provided shows the *age* (under 55 years, 55 years and over) of visitors at a travel convention, segmented by *preferred travel destination* (domestic, international).

    The variables *age* (under 55 years, 55 years and over) and *preferred travel destination* (domestic, international) are
    - **A.** both categorical variables.
    - **B.** both numerical variables.
    - **C.** a numerical variable and a categorical variable respectively.
    - **D.** a categorical variable and a numerical variable respectively.
    - **E.** a discrete variable and a continuous variable respectively.

    VCAA 2021 Exam 1 Data analysis Q1



70% of students answered this question correctly.

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14. The data in the table shown was collected in a study of the association between the variables *frequency of nightmares* (low, high) and *snores* (no, yes).

frequency of nightmares	*snores*		**total**
	no	**yes**	
low	80	58	138
high	11	12	23
total	91	70	161

Data: adapted from RA Hicks and J Bautista, 'Snoring and nightmares', *Perceptual and Motor Skills*, 1 October 1993, <https://doi.org/10.2466/pms.1993.77.2.433>

The variables in this study, *frequency of nightmares* (low, high) and *snores* (no, yes), are

A. ordinal and nominal respectively.

B. nominal and ordinal respectively.

C. both numerical.

D. both ordinal.

E. both nominal.

VCAA 2020 Exam 1 Data analysis Q10

57% of students answered this question correctly.

Questions from multiple lessons

Data analysis *Year 10 content*

15. The following dot plot displays the age, in years, of the 22 players in a university football team.

The percentage of players that are 20 years of age is closest to

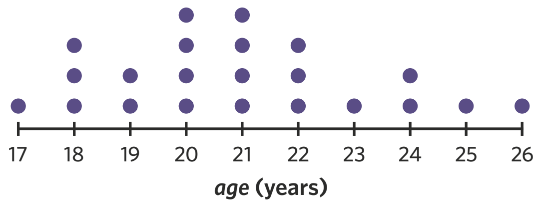
A. 9%

B. 14%

C. 15%

D. 18%

E. 25%



Adapted from VCAA 2018 Exam 1 Data analysis Q1

Data analysis *Year 10 content*

16. The number of minutes spent eating breakfast on a particular day was recorded for a group of 20 high school students. The results are shown in the stem plot provided.

Key: 0 | 6 = 6 minutes

0	6 7 7 8 8 9
1	0 0 0 1 3 7 8
2	2 2 4 5 7
3	1 3

The percentage of these people that spent more than 10 minutes eating breakfast is

A. 30%

B. 40%

C. 45%

D. 55%

E. 70%

Adapted from VCAA 2018NH Exam 1 Data analysis Q2

Data analysis *Year 10 content*

17. The data in the table displays information for 25 different types of fruit.

fruit	*seed location*	*size*	*price* ($ per kg)
apple	internal	medium	$4.99
apricot	internal	medium	$2.30
avocado	internal	medium	$5.20
banana	internal	medium	$3.49
blackcurrant	internal	small	$7.25
blueberry	internal	small	$6.95
boysenberry	internal	small	$6.99
cherry	internal	small	$8.99
cranberry	internal	small	$6.99
fig	internal	medium	$5.49
grape	internal	small	$5.89
grapefruit	internal	large	$9.49
jackfruit	internal	large	$12.49
kiwifruit	internal	medium	$4.99
kumquat	internal	small	$5.99
lime	internal	small	$1.99
lychee	internal	small	$14.99
mango	internal	medium	$6.99
orange	internal	medium	$2.99
peach	internal	medium	$10.49
pineapple	internal	large	$3.49
pomegranate	internal	medium	$7.99
raspberry	internal	small	$4.99
strawberry	external	small	$2.49
watermelon	internal	large	$1.99

The four variables in this data set are:

- *fruit* – type of fruit
- *seed location* – where seeds are found (external, internal)
- *size* – size of the fruit (small, medium large)
- *price* – the average prices for these fruits (dollars per kg)

a. How many variables in this data set are categorical variables? (1 MARK)

b. How many variables in this data set are ordinal variables? (1 MARK)

c. Name the large fruits that cost more than $3 per kg. (1 MARK)

Adapted from VCAA 2018 Exam 2 Data analysis Q1a-c

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1B Displaying and describing categorical data

STUDY DESIGN DOT POINT

- display and description of categorical data distributions of one or more groups using frequency tables and bar charts, and the mode and its interpretation

1A 1B 1C 1D 1E 1F 1G 1H

KEY SKILLS

During this lesson, you will be:

- constructing frequency tables
- constructing bar charts
- describing categorical data.

KEY TERMS

- Frequency table
- Bar chart
- Mode

Categorical data can be displayed visually in order to make it easier to understand and interpret. Two common visual representations of categorical data are frequency tables and bar charts.

Constructing frequency tables

A **frequency table** is a table consisting of two columns that shows how often each category occurs within a data set. One column lists the different categories in the data set, and the other column lists the frequency of the categories. A third 'tally' column can also be included to help find the frequency.

Worked example 1

A class of students were asked about their favourite ice cream flavours. Their answers were:

chocolate	strawberry	chocolate	chocolate	vanilla
chocolate	strawberry	chocolate	chocolate	strawberry
strawberry	vanilla	chocolate	chocolate	chocolate

Display this information in a frequency table.

Explanation

Step 1: Set up a frequency table.

The categories should be displayed in the first column, a tally in the second, and frequency in the third.

favourite flavour	tally	frequency
chocolate		
strawberry		
vanilla		

Step 2: Count the number of occurrences of each category.

Make a mark in the tally column for each student in the row associated with their answer.

favourite flavour	tally	frequency
chocolate	~~IIII~~ IIII	
strawberry	IIII	
vanilla	II	

Continues →

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Step 3: Record the frequency of each category.

Record the totals from the tally column in the frequency column.

Answer

favourite flavour	tally	frequency
chocolate	卌 IIII	9
strawberry	IIII	4
vanilla	II	2

Constructing bar charts

A **bar chart** is a visual representation of categorical data. The categories of the data set are shown on the horizontal axis, and the frequency of the categories on the vertical axis.

Each category is represented by a bar with a height that represents the frequency of the category. Bars are drawn with gaps between them.

Worked example 2

Dr. Barras always works a whole number of hours. Over a month, the number of hours he worked each day was recorded. This information is summarised in the following frequency table.

numbers of hours worked	frequency
0–4	4
5–8	15
9–12	8
13–16	2

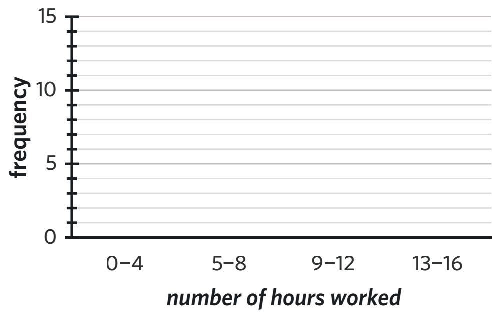
From the frequency table, construct a bar chart.

Explanation

Step 1: Construct the axes with the 'frequency' on the vertical axis and '*hours worked*' on the horizontal axis.

The vertical axis should at least extend to the maximum frequency value.

The horizontal axis should include labels for each of the categories.



Step 2: Draw vertical columns for each category.

The height of each individual bar should match the value in the frequency table.

Remember that each column should be separated by a gap.

Continues →

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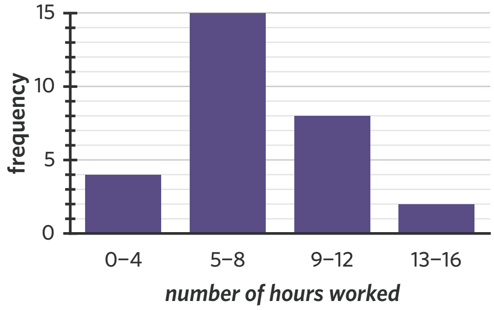
Answer



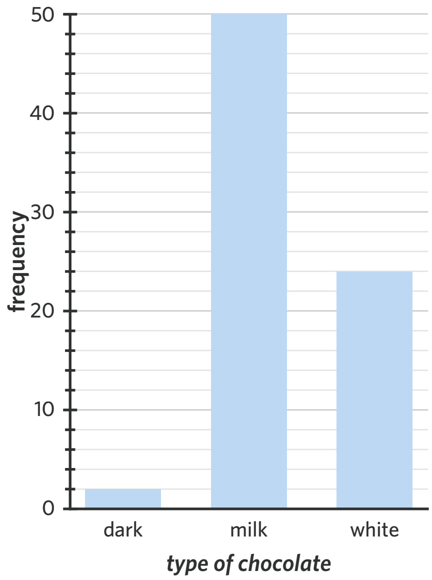
Describing categorical data

Categorical data can be described using the mode. The **mode** (or modal category) is the most frequently occurring category within the data set. There can be multiple modal categories if two or more categories equally occur the most. To determine the modal category in a bar chart, identify the bar with the greatest vertical height.

Worked example 3

Students in a class were surveyed and asked about their favourite type of chocolate.

The results are shown in the following bar chart.



What is the most popular type of chocolate?

Explanation

The most 'popular type' of chocolate is the modal category.

The modal category is the bar with the greatest vertical height.

Answer

Milk chocolate

1B Questions

Note: There are no direct exam questions relevant to this lesson.

Constructing frequency tables

1. A group of people were asked to name their *favourite sport*. The following data was collected.
How many people chose netball as their favourite sport?

favourite sport	frequency
football	11
netball	6
basketball	3

A. 3
B. 6
C. 11
D. 20

2. A group of people were asked about their favourite *pizza topping*. The results of their responses are shown in the following frequency table.

pizza topping	frequency
margherita	9
hawaiian	4
meat lovers	2
vegetarian	4

a. How many people were surveyed?

b. What percentage of people like vegetarian pizza the most? Round to the nearest percent.

3. The *eye colour* of each person in an office was collected. The results are as follows:

blue	blue	brown	brown	brown
green	hazel	brown	blue	brown
green	brown	green	blue	hazel
brown	brown	blue	hazel	brown

Fill in the frequency table.

eye colour	tally	frequency

4. A group of 12 people were asked about their *favourite genre* of music. The results are shown in the following table.

person	*favourite genre*
Demar	pop
Udonis	classical
Stephanie	rock
Tyson	classical
Yuri	pop
Tess	pop
Harris	rock
Emily	pop
Georgia	pop
Omar	rock
Alex	pop
Thomson	rock

Construct a frequency table that displays the information in the table.

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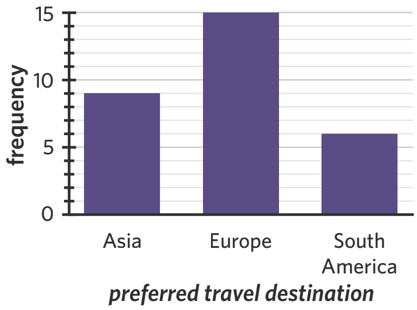
Constructing bar charts

5. A group of 30 people were asked to choose their *preferred travel destination*. The results are shown in the following bar chart.

How many people chose South America as their preferred travel destination?

A. 6
B. 9
C. 14
D. 15



6. The *favourite holiday destination* of 15 people is shown in the following frequency table.

Construct a bar chart to display this information.

favourite holiday destination	frequency
beach	8
snow	3
city	4

7. A cafe recorded how many of each *type of beverage* was sold in one morning. A total of 200 beverages were sold. The data is shown in this frequency table.

a. What is the missing frequency value?

b. Create a bar chart to display the information in the table. Include the missing value found in part **a**.

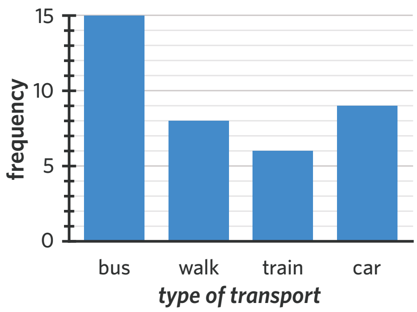
type of beverage	frequency
coffee	38
tea	46
juice	83
milkshake	

Describing categorical data

8. A class of students were asked what *type of transport* they use to get to school. The results are displayed in the bar chart as shown.

Which type of transportation is the most popular?

A. Bus
B. Walk
C. Train
D. Car



9. A group of 1100 people were asked which *city* they were born in. The results are shown in the following frequency table.

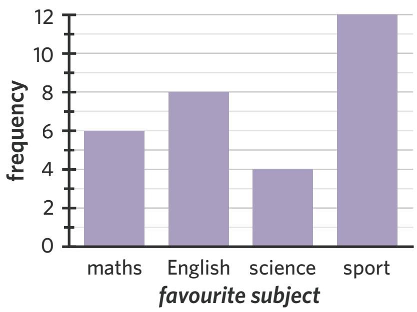
a. What is the missing frequency value?

b. Which city had the greatest number of people born there?

city	frequency
Melbourne	492
Sydney	279
Canberra	

10. Thirty students were asked about their *favourite subject*. The results are shown in the following bar chart.

Which subject is the mode?



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Joining it all together

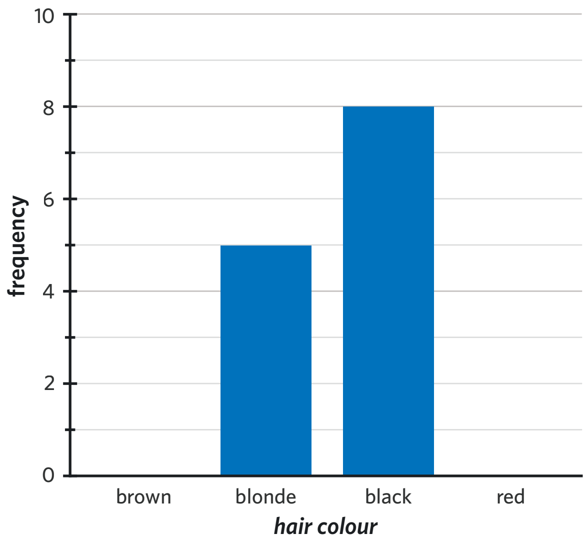
11. A group of 20 people were asked about their favourite *type of tea*. The results were:

black	peppermint	chamomile	black	black
peppermint	chamomile	black	peppermint	black
peppermint	chamomile	black	peppermint	black
chamomile	black	peppermint	black	peppermint

a. Create a frequency table that displays this information.

b. Use the frequency table to create a bar chart.

12. A class of 25 students was surveyed about their *hair colour* and a bar chart was constructed.



The bar chart is incomplete. In the class, there are five times more students who have brown than red hair.

a. Complete the bar chart.

b. Use the bar chart to create a frequency table.

c. What is the most common hair colour?

13. A concert hall was filled with 25 000 people for a concert by the Melbourne Symphony Orchestra. The people who attended the concert were asked to fill out a survey as to their *favourite instrument family*. In the survey 17.5% responded brass, 24.8% responded woodwind, 6.3% responded percussion, and the rest responded strings.

a. Strings is the favourite instrument family for what percentage of people?

b. Use the data to construct a frequency table.

c. From the frequency table, construct a bar chart.

d. What is the most popular instrument family?

Questions from multiple lessons

Data analysis

14. Scientists are studying the relationship between the variables *clothing size* (extra small, small, medium, large, extra large) and *height* (in cm).

These variables are

A. ordinal and numerical respectively.

B. ordinal and nominal respectively.

C. nominal and numerical respectively.

D. numerical and nominal respectively.

E. both numerical.

Adapted from VCAA 2019NH Exam 1 Data analysis Q8

Recursion and financial modelling *Year 10 content*

15. A beanie store is currently having a Boxing Day sale. The price of a beanie is usually $9.80. If there is a 20% discount on all beanies, how much is the price of a beanie reduced by?

A. $1.96 **B.** $2.00 **C.** $2.04 **D.** $7.80 **E.** $7.84

Adapted from VCAA 2015 Exam 1 Business-related mathematics Q1

Data analysis *Year 10 content*

16. The *number of eggs* counted in a group of 20 turtle eggs clusters has been recorded in the following table.

number of eggs	37	63	59	69	51	59	54	58	63	36	63	37	69	61	68	62	37	67	53	59

a. Calculate the range. (1 MARK)

b. Calculate the percentage of clusters that contain more than 65 eggs. (1 MARK)

Adapted from VCAA 2017 Exam 2 Data analysis Q1a

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1C Displaying and describing numerical data

STUDY DESIGN DOT POINT

- display and description of numerical data distributions using histograms, stem plots and dot plots and choosing between plots according to context and purpose

1A 1B **1C** 1D 1E 1F 1G 1H

KEY SKILLS

During this lesson, you will be:

- displaying numerical data using dot plots
- displaying numerical data using stem plots
- displaying numerical data using histograms.

KEY TERMS

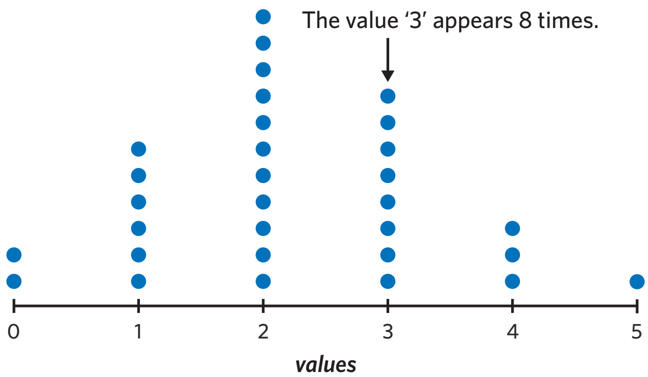
- Dot plot
- Stem plot
- Grouped frequency table
- Histogram
- Modal interval

Organising numerical data and displaying it visually can make it easier to interpret. Dot plots, stem plots, and histograms are three types of visual displays used to represent numerical data. They can be used to identify the frequency or percentage of values occurring within certain ranges of values.

Displaying numerical data using dot plots

A **dot plot** is a visual representation of numerical data that uses stacked dots to convey the frequency of data values. The number of dots above a value on the axis represents the frequency of the value. The mode of the data set (also known as the modal value) is the value with the most dots.

Dot plots are best for displaying discrete numerical data or categorical data. As each dot represents a single data entry, dot plots can only display small/medium sized data sets with a small range of values.



Worked example 1

The following data represents the *number of pets* owned by residents of George St in Fitzroy.

0 1 0 2 2 4 1 0 0 1 2 0 1

1 5 0 2 4 1 0 0 0 2 1 0 1

Continues →

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a. Construct a frequency table using this data.

Explanation

Step 1: Construct a table with *number of pets* in the left column and frequency in the right column.

number of pets	frequency
0	
1	
2	
3	
4	
5	

Step 2: Count the number of occurrences of each data value and fill in the frequency column.

Answer

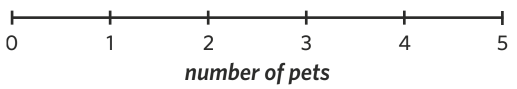
number of pets	frequency
0	10
1	8
2	5
3	0
4	2
5	1

b. Construct a dot plot using this data.

Explanation

Step 1: Draw a number line that covers all relevant data values.

The data set ranges from 0 to 5.



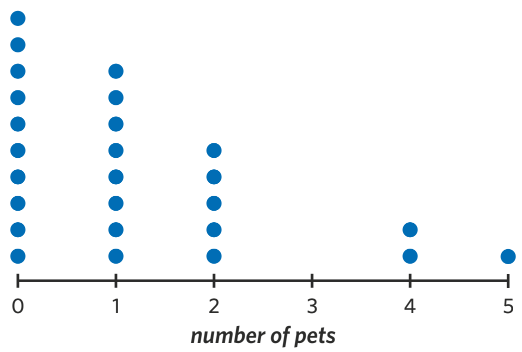
Step 2: Draw dots above each value in accordance with the frequency of the value.

The frequency of 0 pets is 10, so there will be 10 dots above '0' on the dot plot.

1 pet will have 8 dots, 2 will have 5 dots, and so on.

Step 3: Label the plot '*number of pets*'.

Answer



Continues →

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c. What is the modal number of pets?

Explanation

Identify which value has the most dots.

'0' has the most dots.

Answer

0 pets

d. How many residents have one pet?

Explanation

Identify the number of dots above '1'.

There are 8 dots above '1'.

Answer

8 residents

Displaying numerical data using stem plots

A **stem plot** (or stem-and-leaf plot) is a visual representation of numerical data where each data entry is split into a 'stem', composed of the leading digit(s), and a 'leaf', the final digit. Each stem plot has a key that explains how to translate the stem and leaf entries into a number.

A stem plot is constructed by drawing a vertical line. The stems are ordered vertically to the left of the line. The leaves are to the right of the line, aligned horizontally with their corresponding stem, and are ordered numerically (with the smallest number nearest the stem).

Key: 7 | 8 = 78

7	8
8	2 6 9
9	2 5 8 8
10	
11	5 7
12	3 7 8 8 9
13	0 4 8 9

The stem is ordered numerically, with the smallest number at the top.

Empty stems cannot be skipped.

Leaves are ordered numerically, with the smallest number nearest the stem.

A line separates the stems from the leaves.

Stem plots should not be used if the range of data is large or if the data values are large, as it becomes difficult to display the data concisely. On the other hand, if the data has too few values, a stem plot might be unnecessary.

Worked example 2

A local swimming squad recorded their personal best 50-metre freestyle times. The following data set contains the *personal best time* (in seconds) of all the individual members.

43 38 52 41 47 32 39 45 40 28 49 30 43

Construct a stem plot using the data collected.

Explanation

Step 1: Consider an appropriate scale.

The data ranges from 28 to 52.

An appropriate scale will have the 'tens' values as the stem and the 'ones' values as the leaves.

Continues →

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Step 2: Create the stem by listing all the different 'tens' digits.

2	
3	
4	
5	

Note: Each stem within the range of the data needs to be included, even if there are no data values within it.

Step 3: List the 'ones' digits of each data value next to its corresponding stem, ordered smallest to largest.

2	8
3	0 3 8 9
4	0 1 3 3 5 7 9
5	2

Step 4: Add a key.

Answer

Key: 2 | 8 = 28 seconds

2	8
3	0 2 8 9
4	0 1 3 3 5 7 9
5	2

Displaying numerical data using histograms

A **grouped frequency table** is a table that organises numerical data into groups, or intervals, and shows the frequency of each interval.

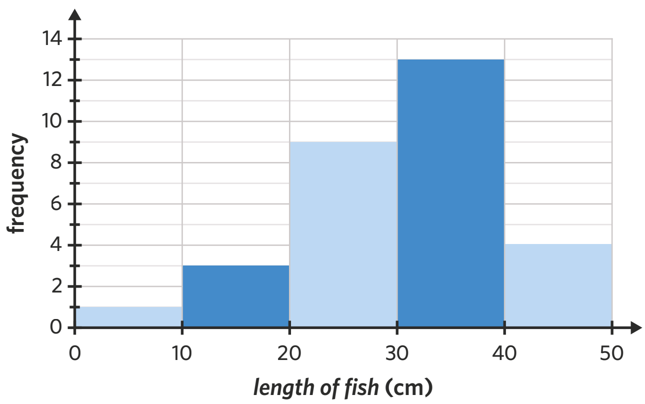
When the data is continuous, the lower bound of each interval is inclusive, whilst the upper bound is not.

For example, the following grouped frequency table was constructed using data on the *length of fish* caught in Mallacoota.

length of fish (cm)	frequency
0–<10	1
10–<20	3
20–<30	9
30–<40	13
40–<50	4

According to the grouped frequency table, there were 13 fish caught that were at least 30 cm but less than 40 cm long, and 4 fish caught that were at least 40 cm long

A **histogram** is a visual representation of a grouped frequency table that uses the height of columns to represent frequency. The **modal interval** is the interval with the tallest column.



The values on the horizontal axis represent the lower bound of each interval. There is no gap between the columns.

Histograms are best at displaying data sets with a large number of numerical values. A calculator can be helpful in creating a histogram directly from the data.

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Worked example 3

A class sat a 30-mark maths quiz. The following data set contains the *scores*, out of 30, of students in the class.

12 18 11 25 10 5 28 27 8 14 3 19 20
11 1 19 28 22 29 15 4 14 18 16 20 21

a. Construct a grouped frequency table to display data.

Explanation

Step 1: Determine how the data should be grouped.

The data ranges from 1 to 28. Splitting the data into intervals of 5 marks is suitable.

Step 2: Construct a frequency table with the groups of values in the left column.

scores	frequency
0–4	
5–9	
10–14	
15–19	
20–24	
25–29	

Note: As the data is discrete, the upper bound of each interval is inclusive.

Step 3: Count the number of data values that fall within each group and fill in the frequency column.

Answer

scores	frequency
0–4	3
5–9	2
10–14	6
15–19	6
20–24	4
25–29	5

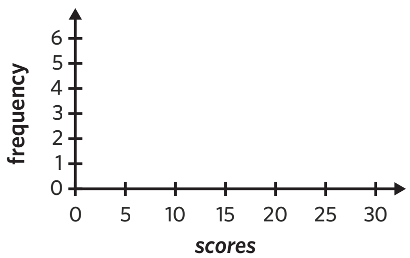
b. Construct a histogram to display the data.

Explanation – Method 1: By hand

Step 1: Construct the axes.

Mark the lower bound of each interval on the horizontal axis (0, 5, 10, etc.).

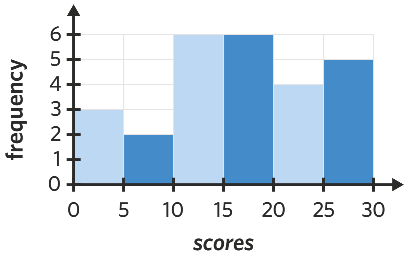
Label the horizontal axis '*scores*' and the vertical axis 'frequency'.



Step 2: Draw a vertical column for each interval with a height equal to its frequency.

Continues →

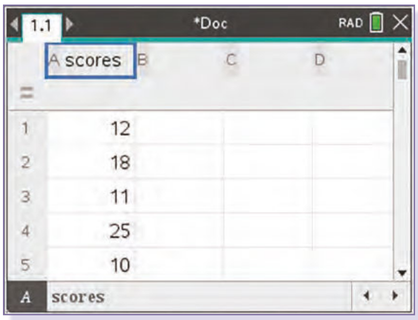
Answer



Explanation – Method 2: TI-Nspire

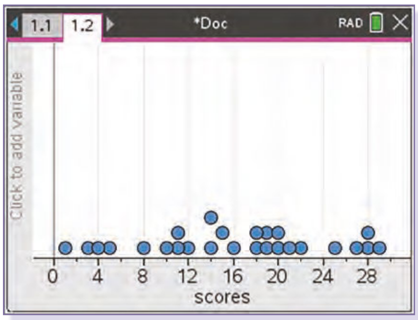
Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name column A 'scores' and enter the data values into column A, starting from row 1.



Step 3: Press ctrl + doc, and select '5: Add Data & Statistics'.

Move the cursor to the horizontal axis and click 'Click to add variable'. Select 'scores'.



Step 4: Press menu, select '1: Plot Type' → '3: Histogram'.

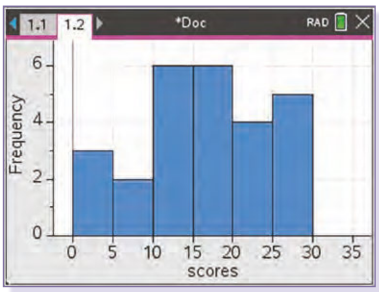
Step 5: To adjust the column width and the starting point, press menu and then select '2: Plot Properties' → '2: Histogram Properties' → '2: Bin Settings' → '1: Equal Bin Width'.

Set the column width by changing 'Width' to '5'.

Set the starting point by changing 'Alignment' to '0'.

Note: To change the view of the histogram press menu and use options within '5: Window/Zoom'.

Answer



Continues →

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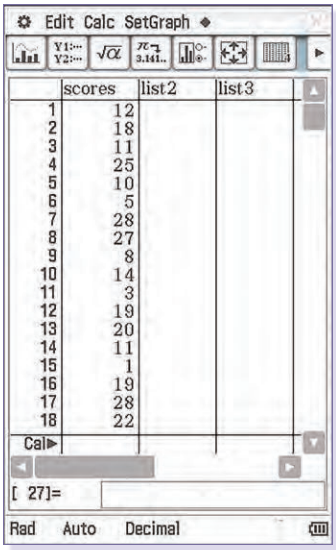
Explanation – Method 3: Casio ClassPad

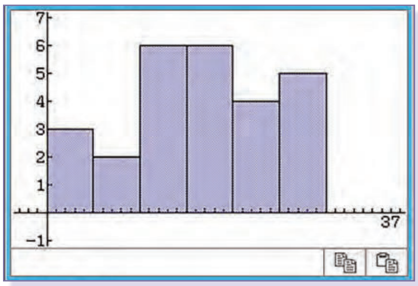
Step 1: From the main menu, tap Statistics.

Step 2: Press keyboard and select abc.

Rename list1 to 'scores' and enter the data values starting from row 1.



Answer



Step 3: Configure the settings of the graph by tapping .

Create a histogram by changing 'Type' to 'Histogram'.

Specify the data set by changing 'XList' to 'main\scores'.

Tap set to confirm.

Step 4: Tap in the icon bar to plot the histogram.

Set the starting point by changing 'HStart' to 0.

Set the column width by changing 'HStep' to 5.

Note: To change the view of the histogram press .

---

c. How many students scored less than 50% on the quiz?

Explanation

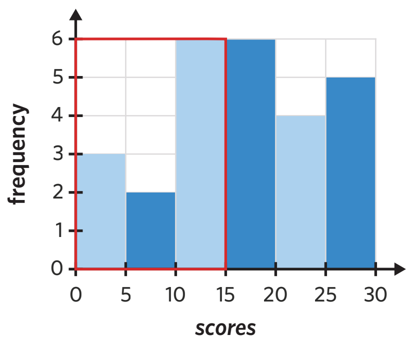
Step 1: Determine the score that is equal to 50% on the quiz.

There are 30 marks.

$0.5 \times 30 = 15$

A score of less than 15 corresponds to a score of less than 50%.

Step 2: Determine the number of scores less than 15.



Sum the heights of the columns.

$3 + 2 + 6 = 11$

Answer

11 students

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1C Questions

Displaying numerical data using dot plots

1. The *number of bedrooms* in 30 houses are shown in the following dot plot.

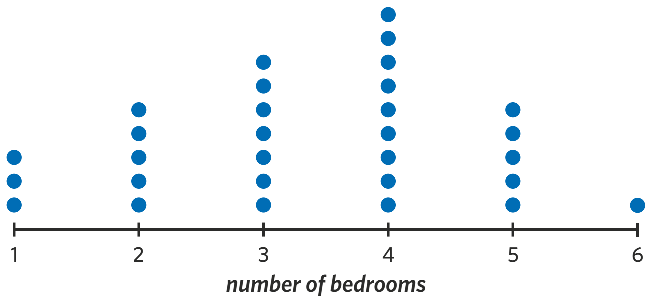
   How many of the houses in the sample have four bedrooms?

   A. 7

   B. 8

   C. 9

   D. 10



2. Consider the following data set.

   5 4 5 4 3 5 2 5 4 3 6 2 4 3 1 1 2 4 2 4 6 4 2 8 5 5

   a. Construct a dot plot.

   b. What is the modal value?

   c. How many data values are equal to or less than 3?

3. Every day, Mr Robinson counts how many students in his General Maths class look at their phones at some point instead of doing questions. The data he collected over 36 days is displayed in the given frequency table.

   a. Mr Robinson decides that a visual display would be useful for understanding the effect of phones on his students. Create a dot plot based on his data.

   b. On what percentage of days do none of the students look at their phones? Round to one decimal place.

students on phones	frequency
0	7
1	5
2	13
3	6
4	3
5	1
6	0
7	1

Displaying numerical data using stem plots

4. The following stem plot displays the *test score*, out of 40, obtained by 20 Year 11 students on a maths test.

   Key: 2 | 5 = 25

0	9
1	5 8 9
2	5 6 8 8 9
3	0 1 1 1 3 3 5 7 8 8
4	0

   What percentage of students failed the test if a mark of 50% was required to pass?

   A. 4% **B.** 5% **C.** 15% **D.** 20%

5. Consider the following data set.

   45 28 73 38 75 27 39 47 76 89 63 75 37
   37 19 37 39 49 57 26 75 55 49 20 70 63

   a. Construct a stem plot using this data.

   b. How many values are greater than 70?

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6. A farmer recently harvested a new crop of watermelons. Before selling them, the farmer recorded the *weight* of all the watermelons in kilograms.

10.8 12.1 11.1 11.4 11.7 12.5 14.6 14.0 10.9 11.1 14.7 12.9 15.3
15.6 14.4 12.2 12.1 11.9 10.9 10.5 10.1 12.5 11.7 14.1 15.0 11.6

a. Construct a stem plot that displays the *weight* of the watermelons within the new crop.

b. What percentage of the watermelon crop weighed more than 13 kg? Round to one decimal place.

Displaying numerical data using histograms

7. The following histogram shows the distribution of *average life span* for 189 countries.

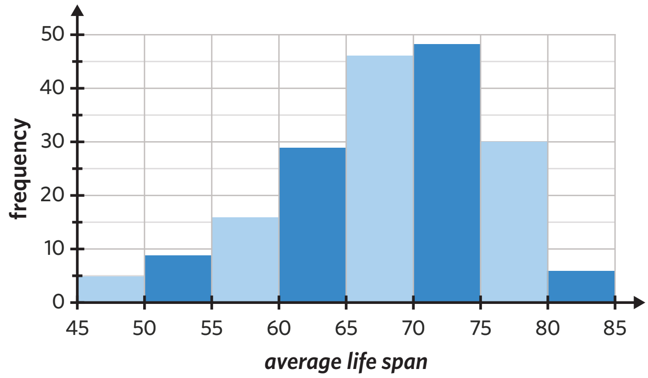
What is the modal interval of the distribution?

A. 45–<50

B. 65–<70

C. 65–<75

D. 70–<75



8. A group of 26 students took part in a survey where they were asked how many *hours of exercise* they complete weekly. The following data set displays the results.

4 8 14 3 7 9 6 19 10 4 6 0 2 7 11 7 8 6 21 15 12 5 3 6 2 5

a. Using intervals of 5 hours, construct a grouped frequency table.

b. How many students exercise less than 5 hours per week?

c. What percentage of students exercise less than 5 hours per week? Round to one decimal place.

d. Find the modal interval and state its frequency.

9. Consider the following data set.

11.4 27.1 29.6 20.8 27.0 10.2 22.1 11.4 9.1 16.4 5.7 24.4 10.5
12.2 3.9 17.4 1.1 25.3 34.6 21.5 16.2 13.7 7.4 23.2 29.5 17.4

a. Construct a histogram with intervals of 5 units, starting with the interval 0–<5.

b. What is the frequency of values within the interval 10–<15?

10. The *circumference*, in centimetres, of trees in a local park are as follows:

15 110 76 22 84 18 94 11 45 72 53 81 118
26 18 104 111 63 38 23 14 65 103 89 31 16

a. Construct a histogram with intervals of 20 cm, starting from 0 cm.

b. The local council are planning to build a treehouse in one of the larger trees. The council estimates that a circumference of more than one metre is necessary for the tree to be suitable for a treehouse. How many trees are suitable?

c. The trees within the park take exactly one year to reach a circumference of 20 cm. What percentage of the trees have been within the park for at least a year? Round to one decimal place.

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11. A group of 100 athletes from different disciplines had their resting *heart rate* measured. The following frequency table details the results.

a. Construct a histogram using the table.

b. Which interval(s) contain more than 25% of the athletes?

c. A resting heart rate of less than 90 BPM is considered healthy. How many athletes are considered unhealthy?

***heart rate* (BPM)**	**frequency**
30–<50	32
50–<70	43
70–<90	15
90–<110	8
110–<130	2

Joining it all together

12. Which visual display would be best suited for displaying the following data set?

3 1 1 4 3 2 4 6 4 2 6 5 5 5 4 5 2 4 5 2 5 4 3 8 2 4

A. Dot plot **B.** Histogram **C.** Stem plot **D.** Grouped frequency table

13. Consider the following set of discrete data.

13 38 32 27 24 22 20 14 11 26 36 31 28

11 15 23 17 34 39 39 37 14 16 20 21 28

a. Construct a grouped frequency table with appropriate intervals.

b. Construct a histogram based on the grouped frequency table.

c. Construct a stem plot.

14. Harry surveyed his whole neighbourhood and collected data on how many children are in each family.

a. Explain why a stem plot is not a suitable display of Harry's data.

b. Which display, out of a dot plot or histogram, would be most suitable for this kind of data? Explain why.

Exam practice

15. The following stem plot displays the *wingspan*, in millimetres, of 19 moths that were captured.

Key: 1 | 2 = 12 ***wingspan* (mm)**

1	8
2	2 2 4 4 4
2	5 5 9
3	0 0 1 2 3 4
3	6 8
4	0 3

Write down the modal *wingspan*, in millimetres, of the moths captured. (1 MARK)

Adapted from VCAA 2017 Exam 2 Data analysis Q2b

87% of students answered this type of question correctly.

16. The following table shows the *day number* and the *minimum temperature*, in degrees Celsius, for 15 consecutive days in May 2017.

day number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
***minimum temperature* (°C)**	12.7	11.8	10.7	9.0	6.0	7.0	4.1	4.8	9.2	6.7	7.5	8.0	8.6	9.8	7.7

Data: Australian Government, Bureau of Meteorology,<www.bom.gov.au/>

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The incomplete ordered stem plot has been constructed using the data values for days 1 to 10.

Key: $4 \mid 1 = 4.1$ $\quad n = 15$

minimum temperature **(°C)**

4	1 8
5	
6	0 7
7	0
8	
9	0 2
10	7
11	8
12	7

Complete the stem plot by adding the data values for days 11 to 15. (1 MARK)

VCAA 2019 Exam 2 Data analysis Q1b

83% of students answered this question correctly.

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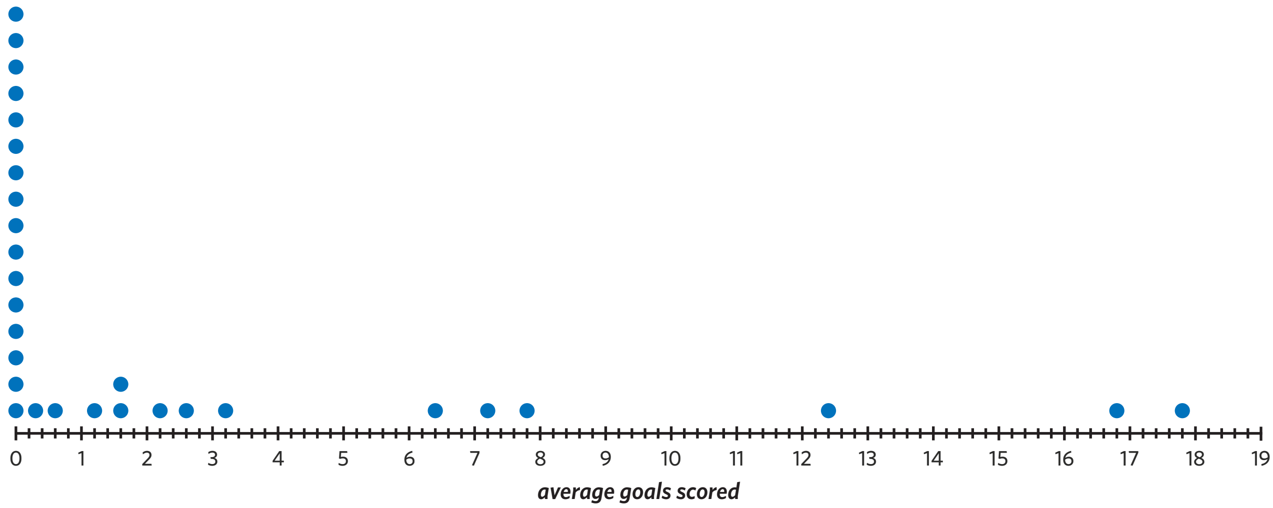
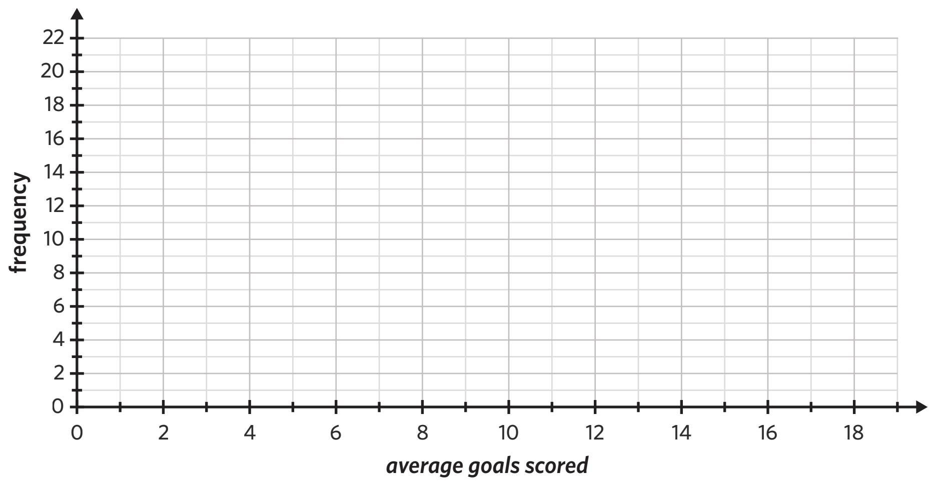
17. The following dot plot shows the average number of goals scored per season by each player within a soccer club.



Use the grid to construct a histogram that displays the distribution of the average number of goals scored per season at the soccer club. Use interval widths of two with the first interval starting at 0. (2 MARKS)



Adapted from VCAA 2016 Exam 2 Data analysis Q1d

The average mark on this type of question was **1.1**.

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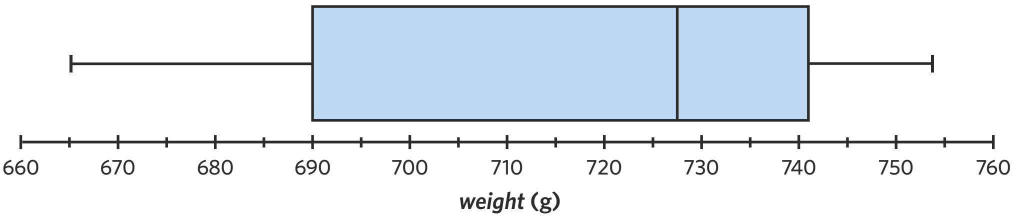
Questions from multiple lessons

Data analysis *Year 10 content*

18. Niall and Louis are holidaying in Victoria and decide to visit the zoo.

The following boxplot displays the *weight*, in grams, of 83 meerkats at Werribee Zoo.



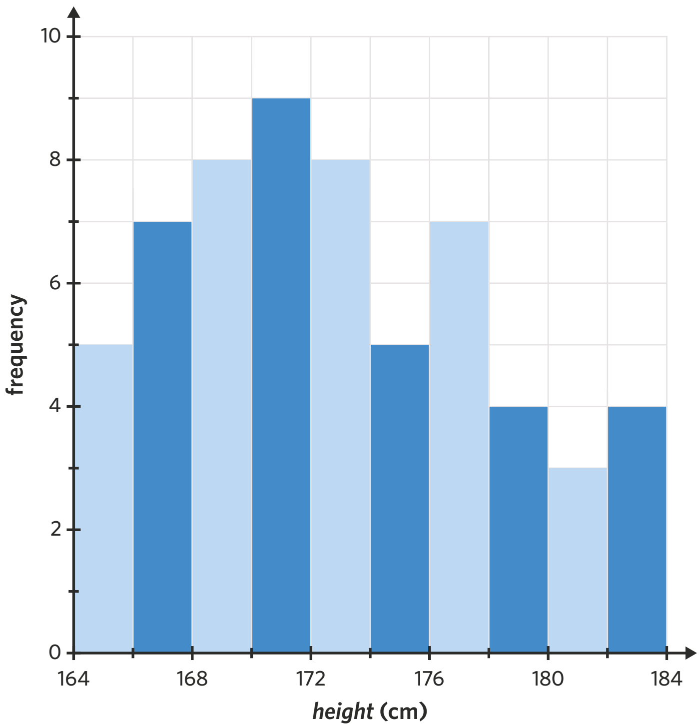
The percentage of these 83 meerkats with a *weight* greater than 690 g is closest to

A. 15% **B.** 25% **C.** 50% **D.** 75% **E.** 85%

Adapted from VCAA 2017 Exam 1 Data analysis Q1

Data analysis

19. The following histogram displays the *height*, in centimetres, of 60 competitors in an E-Sports tournament.



The *height* for this sample of competitors is most frequently

A. greater than or equal to 168 cm and less than 170 cm.

B. greater than or equal to 170 cm and less than 172 cm.

C. greater than or equal to 172 cm and less than 174 cm.

D. greater than or equal to 174 cm and less than 176 cm.

E. greater than or equal to 176 cm and less than 178 cm.

Adapted from VCAA 2018NH Exam 1 Data analysis Q3

Data analysis *Year 10 content*

20. The *number of movies watched* on Netflix within a month, for 13 people, is shown in the following data set.

9 8 4 3 16 5 4 7 3 2 0 4 10

a. Construct a five-number summary for the data set. (1 MARK)

b. Determine if there are any outliers in the data. If there are outliers, what are they? (1 MARK)

c. Use the five-number summary to construct a boxplot. (1 MARK)

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1D Summarising numerical data – median, range and IQR

STUDY DESIGN DOT POINT

- summarising numerical data distributions, including use of and calculation of the sample summary statistics, median, range, and interquartile range (IQR) or mean and standard deviation

1A 1B 1C 1D 1E 1F 1G 1H

KEY SKILLS

During this lesson, you will be:

- determining the median
- calculating the range
- calculating the interquartile range.

KEY TERMS

- Centre
- Median
- Spread
- Range
- Interquartile range
- Quartiles

The median, range, quartiles and interquartile range are useful measures of the centre and spread of numerical data. They help to understand and classify distributions, recognise patterns and identify useful data. They also help in analysing different subsections of data.

Determining the median

The **centre** refers to the middle of a data set, and can be measured in several ways. This lesson will focus on the median as a measure of the centre. The **median** is the middle value of an ordered data set.

In a data set with n values, the median is located in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position.

See worked example 1

If there is an even number of data values, the median is the average of the two middle values.

For example, if there are 20 values in the data set, $\left(\frac{n+1}{2}\right) = 10.5$. On either side of 10.5 are the 10^{th} and 11^{th} values. The median is the average of these two values.

When finding the median from tables and charts, it can be determined either by inspection or by first listing the data in ascending order.

See worked example 2

Worked example 1

For the following data set, determine the median.

43 252 465 859 245 492 453 48 239 4

Explanation

Step 1: Rearrange the data set in ascending order.

4 43 48 239 245 252 453 465 492 859

Step 2: Count the number of values in the data set.

$n = 10$

Step 3: Determine the position of the median.

The median is located in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position.

$\frac{10+1}{2} = 5.5$

Since there are an even number of values, the median is the average of the 5^{th} and 6^{th} values.

Continues →

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Step 4: Calculate the median.

The 5th and 6th values are shown.

4 43 48 239 245 252 453 465 492 859

$\frac{245 + 252}{2} = 248.5$

Answer

248.5

Worked example 2

Determine the median of the following data displays.

a.

number of pets	frequency
0	3
1	4
2	2
3	4
4	1

Explanation

Step 1: List the data in ascending order.

0 0 0 1 1 1 1 2 2 3 3 3 3 4

Step 2: Count the number of values in the data set.

$n = 14$

Step 3: Determine the position of the median.

The median is located in the $\left(\frac{n+1}{2}\right)^{th}$ position.

$\frac{14 + 1}{2} = 7.5$

Since there are an even number of values, the median is the average of the 7th and 8th values.

Step 4: Calculate the median.

The 7th and 8th values are shown.

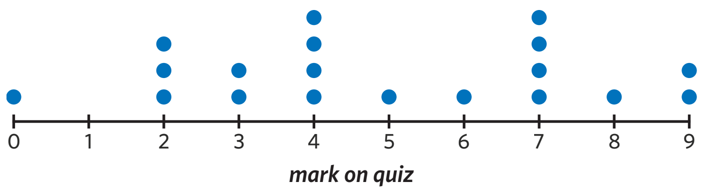
0 0 0 1 1 1 1 2 2 3 3 3 3 4

$\frac{1 + 2}{2} = 1.5$

Answer

1.5 pets

b.



Explanation

Step 1: List the data in ascending order.

0 2 2 2 3 3 4 4 4 4 5 6 7 7 7 7 8 9 9

Step 2: Count the number of values in the data set.

$n = 19$

Step 3: Determine the position of the median.

The median is located in the $\left(\frac{n+1}{2}\right)^{th}$ position.

$\frac{19 + 1}{2} = 10$

Continues →

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Step 4: Determine the median.

The 10^{th} value is shown.

0 2 2 2 3 3 4 4 4 4 5 6 7 7 7 7 8 9 9

Answer

4 marks

---

c. **Key:** 1 | 5 = $15

1	2 4 4 6
2	2 3 4 7 8
3	0 3 9

Explanation

Step 1: List the data in ascending order.

12 14 14 16 22 23 24 27 28 30 33 39

Step 2: Count the number of values in the data set.

$n = 12$

Step 3: Determine the position of the median.

The median is located in the $\left(\frac{n+1}{2}\right)^{th}$ position.

$\frac{12+1}{2} = 6.5$

Since there are an even number of values, the median is the average of the 6^{th} and 7^{th} values.

Step 4: Calculate the median.

The 6^{th} and 7^{th} values are shown.

12 14 14 16 22 23 24 27 28 30 33 39

$\frac{23+24}{2} = 23.5$

Answer

$23.50

Calculating the range

The **spread** refers to how similar or varied a data set is. One useful measure of spread is the range. The **range** of a numerical data set is the difference between the largest number (maximum) and the smallest number (minimum).

$range = maximum - minimum$

Finding the range is the simplest way of measuring the spread of a data set.

Worked example 3

For the following data set, calculate the range.

4 55 6 77 87 34 56 23 11 23 4 6 94 23 123 −3 −6 −2

Explanation

Step 1: Identify the minimum.

The smallest number in the data set is −6.

Step 2: Identify the maximum.

The largest number in the data set is 123.

Step 3: Calculate the range.

$$\begin{aligned} range &= 123 - (-6) \\ &= 129 \end{aligned}$$

Answer

129

Calculating the interquartile range

The **interquartile range** (IQR) is a measure of the spread of the middle 50% of a data set. Since it does not include the upper 25% and lower 25% of values, it is not affected by outliers.

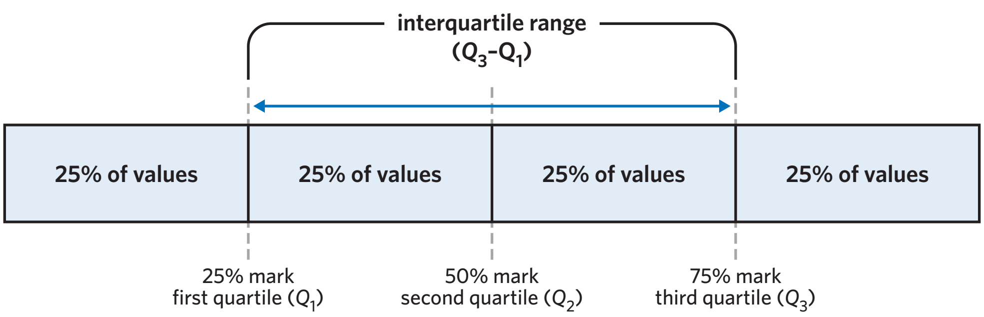
Before the interquartile range can be calculated, the **quartiles** must be found. Quartiles are the values that divide an ordered data set into quarters.

There are three quartiles denoted the first quartile (Q_1), the second quartile (Q_2) and the third quartile (Q_3).

- Q_1, the first quartile, is the median of the lower half of the data set. The median, Q_2, is excluded if there is an odd number of values.
- Q_2, the second quartile, is the median of the data set.
- Q_3, the third quartile, is the median of the upper half of the data set. The median, Q_2, is excluded if there is an odd number of values.

With these values the interquartile range can now be calculated.

$IQR = Q_3 - Q_1$



Worked example 4

Consider the following data set.

3 50 34 2 34 21 4 5 7 4 3 12 4 8 10

a. Determine the quartiles (Q_1, Q_2, Q_3).

Explanation

Step 1: List the data in ascending order.

2 3 3 4 4 4 5 7 8 10 12 21 34 34 50

Step 2: Determine the median (Q_2) of the data set.

$n = 15$

$\frac{n+1}{2} = 8$

The median is located in the 8th position.

$Q_2 = 7$

Step 3: Split the data set into halves.

Since there are an odd number of values, the median will be excluded.

2 3 3 4 4 4 5 | 7 | 8 10 12 21 34 34 50

lower half — median — upper half

Step 4: Determine Q_1.

Q_1 is the median of the lower half of the data set.

$n = 7$

$\frac{n+1}{2} = 4$

Q_1 is located in the 4^{th} position of the lower half.

$Q_1 = 4$

Step 5: Determine Q_3.

Q_3 is the median of the upper half of the data set.

Q_3 is located in the 4^{th} position of the upper half.

$Q_3 = 21$

Answer

$Q_1 = 4, \quad Q_2 = 7, \quad Q_3 = 21$

Continues →

b. Calculate the interquartile range.

Explanation

From part **a**, $Q_1 = 4$ and $Q_3 = 21$.

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 21 - 4 \\ &= 17 \end{aligned}$$

Answer

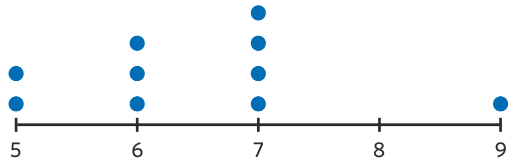
17

1D Questions

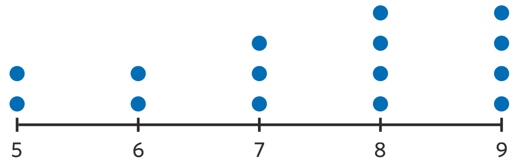
Determining the median

1. Which of the following dot plots has a median of 7?

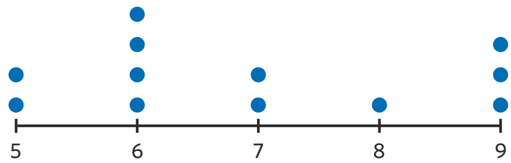
A.



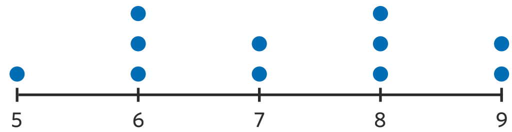
B.



C.



D.



2. Determine the median of the following data.

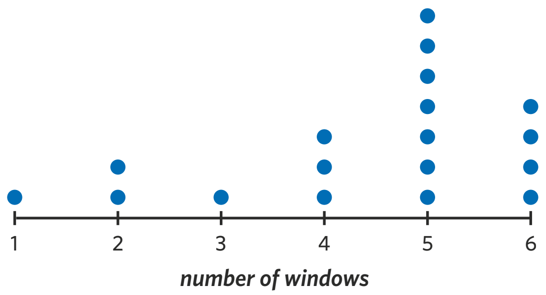
2 41 7 38 −26 25 32 79 −6

3. Josh went to the beach and measured the heights of each of the sandcastles. The *height*, in centimetres, of the sandcastles are shown.

30 17 26 15 88 45 37 44 21 52

Determine the median *height* of the sandcastles.

4. Anastasia counted the *number of windows* on some of the properties in her neighbourhood. She displayed this data using a dot plot. Determine the median *number of windows*.



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5. Maddy constructed a stem plot showing the amount of money each of her friends had in their piggy bank. Determine the median of the stem plot.

Key: 12 | 3 = $123

11	2 4 9
12	2 5 6 9
13	1 5
14	2 5 7 7

Calculating the range

6. What is the range of the following data set?

34 77 93 25 20 46 111 102 62

A. 28 **B.** 62 **C.** 91 **D.** 111

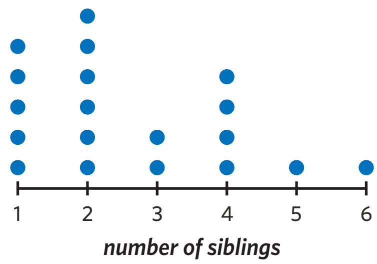
7. Consider the following data set.

224 894 275 560 165 196 575 839 992

a. What are the minimum and maximum values?

b. What is the range?

8. Ms Nalder made a dot plot displaying the *number of siblings* each of her students has. Calculate the range.



Calculating the interquartile range

9. Each of the following data sets contain 11 values.

Which data set has a Q_3 value of 9?

A. 4 3 4 2 7 4 3 9 12 11 11

B. 1 9 12 20 9 10 6 11 7 9 9

C. 14 6 1 10 3 4 3 8 12 1 2

D. 2 1 1 7 8 4 3 9 12 11 2

10. Consider the following data set.

224 894 275 165 196 575 839 992

a. Find Q_1, Q_2 and Q_3.

b. Calculate the interquartile range.

11. What is the interquartile range of the following data set?

12 17 22 25 43 16 29 20 46

12. Calculate the interquartile range of the following stem plot.

Key: 1 | 3 = 13 minutes

1	2 4 5 7 7 7 8 9 9
2	3 4 8
3	0
4	1 1 4
5	1 9

13. Amy asked each of her friends how many holidays they went on last year, and summarised the data in the following frequency table.

Calculate the interquartile range.

number of holidays	frequency
0	2
1	7
2	4
3	3
4	1

Joining it all together

14. Consider the following data set.

22 31 19 46 38 26 15 29 32

a. Determine the median.

b. Calculate the range.

c. Find the quartiles.

15. The *number of students* at 13 different schools is shown.

293 932 536 304 265 482 773 285 582 287 349 223 888

a. What are the minimum and maximum numbers of students?

b. Determine the median number of students.

c. Calculate the interquartile range.

16. Jacob played 16 games of basketball and kept track of the number of *field goals* he scored each game using a frequency table.

field goals	frequency
0	1
1	4
2	6
3	2
4	3

a. Calculate the range.

b. Find Q_3 of the data.

c. Calculate the interquartile range.

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Exam practice

17. In a study of the association between *forearm length* and *neck size*, 250 monkeys were grouped by *neck size* (below average, average and above average) and had their *forearm length* recorded.

The distribution of *forearm length* for each group are summarised in the following table with the group size.

		forearm length (cm)				
neck size	**group size**	**min.**	Q_1	**median**	Q_3	**max.**
below average	50	18.1	20.6	21.6	23.2	26.8
average	124	19.8	23.4	24.6	26.0	33.9
above average	76	23.1	26.3	28.1	29.9	39.1

What is the interquartile range (IQR) of *forearm length* for the monkeys with an average *neck size*? (1 MARK)

Adapted from VCAA 2020 Exam 2 Data analysis Q3b

89% of students answered this type of question correctly.

18. The *number of eggs* counted in a sample of 12 clusters of moth eggs is recorded in the following table.

number of eggs	172	192	159	125	197	135	140	140	138	166	136	131

From the information given, determine the range. (1 MARK)

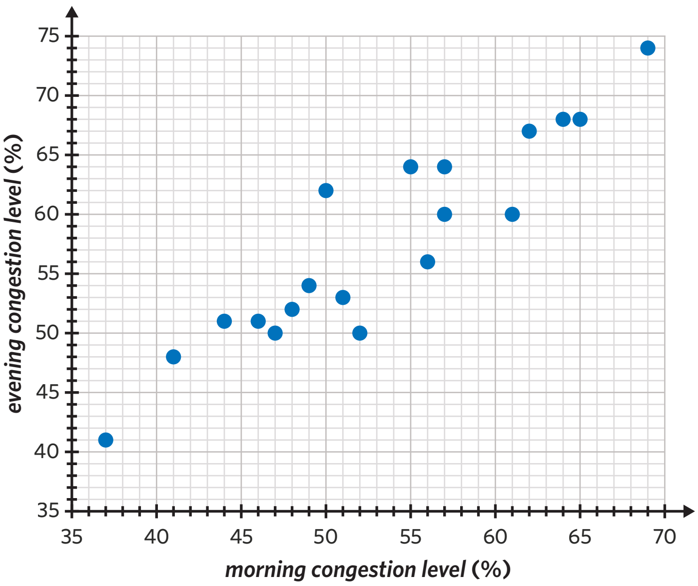
VCAA 2017 Exam 2 Data analysis Q1ai

87% of students answered this question correctly.

19. The congestion level in a city can be recorded as the percentage increase in travel time due to traffic congestion in peak periods (compared to non-peak periods).

This is called the percentage congestion level.

The percentage congestion levels for the morning and evening peak periods for 19 large cities are plotted on the scatterplot.



Determine the median percentage congestion level for the morning peak period and the evening peak period. Write the answers in the appropriate boxes provided. (2 MARKS)

Median percentage congestion level for morning peak period: ____ %

Median percentage congestion level for evening peak period: ____ %

VCAA 2018 Exam 2 Data analysis Q2a

The average mark on this question was **1.1**.

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Questions from multiple lessons

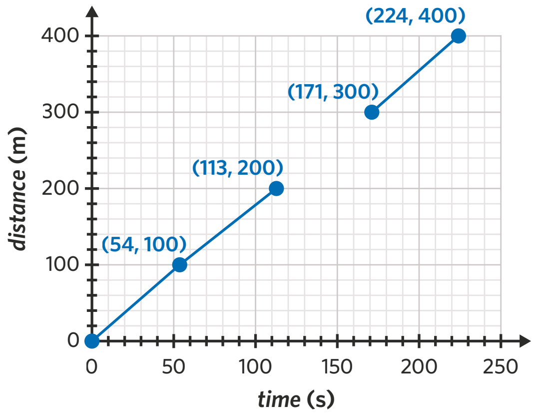
Graphs and relations *Year 10 content*

20. Four swimmers competed in a 4×100 m medley relay in the following order.

order	*name*
first	William
second	Josh
third	Torri
fourth	Gretchen

The following line segment graph displays the progression of the race.

Torri's segment is missing.



Speed is calculated as distance over time, which is represented by the slope of the line. The speed that Torri swam at is closest to

A. 1.5 m/s

B. 1.5 km/h

C. 1.7 m/s

D. 1.7 km/h

E. 1.8 m/s

Adapted from VCAA 2018 Exam 1 Graphs and relations Q4

Recursion and financial modelling *Year 10 content*

21. Due to a faulty zipper, the price of a pair of jeans has been reduced by 60%. If the original price of the jeans was \$45, how much has the price been reduced by?

A. \$18

B. \$19

C. \$22

D. \$23

E. \$27

Adapted from VCAA 2015 Exam 1 Business-related mathematics Q1

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Data analysis *Year 10 content*

22.

type of coffee	*size*	*caffeine level*	*price*
cappuccino	medium	medium	\$3.50
mocha	large	low	\$4.50
white mocha	large	low	\$4.99
dark mocha	large	low	\$4.99
espresso	small	high	\$4.99
latte	medium	medium	\$4.50
cold brew	medium	medium	\$5.50
cold brew latte	medium	medium	\$5.50
iced americano	large	high	\$6.50
iced cappuccino	medium	medium	\$4.50
iced mocha	large	low	\$5.50
iced white mocha	large	low	\$5.99
iced dark mocha	large	low	\$5.99
iced espresso	small	high	\$5.99
iced latte	medium	medium	\$5.50

a. Use the data in the table to complete the following two-way frequency table. (2 MARKS)

		size		
		small	**medium**	**large**
caffeine level	**low**			
	medium			
	high			
total				

b. What percentage of large coffees had a high *caffeine level*? Round to the nearest percent. (1 MARK)

Adapted from VCAA 2018 Exam 2 Data analysis Q1d,e

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1E Summarising numerical data – mean and standard deviation

STUDY DESIGN DOT POINT

- summarising numerical data distributions, including use of and calculation of the sample summary statistics, median, range, and interquartile range (IQR) or mean and standard deviation

1A 1B 1C 1D **1E** 1F 1G 1H

KEY SKILLS

During this lesson, you will be:

- calculating the mean of a data set
- calculating the mean and standard deviation using technology
- using the mean and standard deviation to compare numerical distributions.

KEY TERMS

- Mean
- Standard deviation

Descriptive statistics can aid in the understanding and analysis of a data set. The mean and standard deviation are two types of descriptive statistics that provide inference into the central tendency and dispersion of data.

Calculating the mean of a data set

The **mean** is a measure of centre that averages out all values of a data set into a single, central value. It is calculated by adding all data values in a sample and then dividing the sum by the number of values.

Often the data collected is a sample from a larger population. As such, when calculating the mean of a data set in this course, this is actually a reference to the mean of a sample (or sample mean), which is denoted $\overline{x}$.

The mean is calculated using the formula

$$mean = \frac{\textit{the sum of all elements in the data set}}{\textit{the total number of elements in the data set}}$$

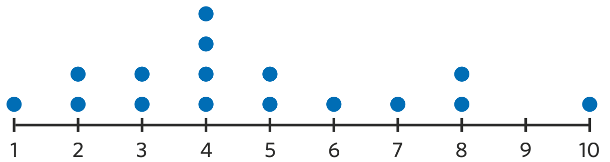
The formula can also be written as:

$\overline{x} = \frac{\Sigma x}{n}$, where

- Σx is the sum of all values
- n is the number of values in the data set.

Worked example 1

Calculate the mean of the data set displayed in the following dot plot.



Explanation

Step 1: Extract the data set from the dot plot.

1 2 2 3 3 4 4 4 4 5 5 6 7 8 8 10

Continues →

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Step 2: Calculate Σx and determine n.

$$\Sigma x = 1 + 2 + 2 + 3 + 3 + 4 + 4 + 4 + 4 + 5 + 5 + 6 + 7 + 8 + 8 + 10$$
$$= 76$$

There are 16 elements in the data set.

$n = 16$

Step 3: Calculate the mean.

$$\overline{x} = \frac{76}{16}$$
$$= 4.75$$

Answer

4.75

Calculating the mean and standard deviation using technology

Once the mean has been calculated, the standard deviation can be used to give an indication of the spread of a data set. The **standard deviation** is a measure of spread that is based on the average deviation (or difference) of each data point compared to the mean. The standard deviation of a data sample is denoted s_x, and has a value that is always greater than or equal to 0.

The standard deviation can be manually calculated using the formula

$s_x = \sqrt{\dfrac{\Sigma(x - \overline{x})^2}{n - 1}}$, where

- $\Sigma(x - \overline{x})^2$ is the sum of the squared differences between each data value and the mean
- n is the number of values in the data set.

Worked example 2

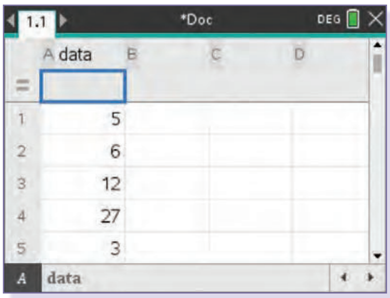
Calculate the mean and standard deviation of the following data set, rounded to two decimal places.

5 6 12 27 3

Explanation – Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name column A 'data' and enter the data starting from row 1.

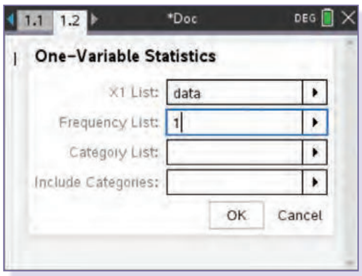


Step 3: Press ctrl + doc, and select '1: Add Calculator'.

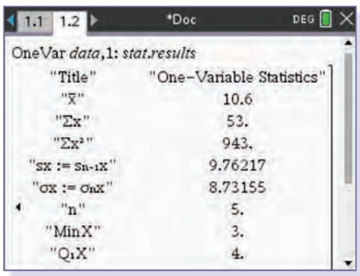
Step 4: Press menu → '6: Statistics' → '1: Stat Calculations' → '1: One-Variable Statistics'.

Set 'Num of Lists:' to '1' and select 'OK'.

Set 'X1 List:' to 'data' and select 'OK'.



Step 5: Identify the mean $(\overline{x})$ and standard deviation (s_x).



Continues →

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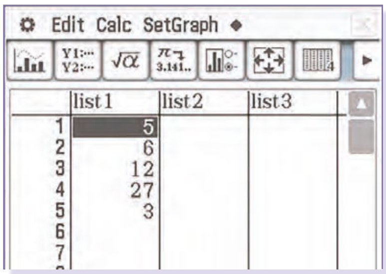
Explanation – Method 2: Casio ClassPad

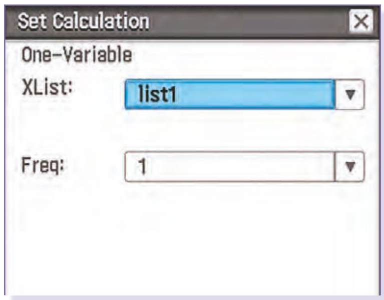
Step 1: From the menu, tap Statistics.

Step 2: Type the data in column 'list1'.

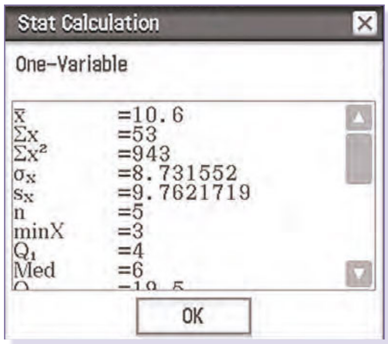


Step 3: Tap 'Calc' → 'One-Variable'.

Set 'XList:' to 'list1' and 'Freq:' to '1'. Tap 'OK'.



Step 4: Identify the mean $(\bar{x})$ and standard deviation (s_x).



Answer – Method 1 and 2

Mean: 10.60

Standard deviation: 9.76

Using the mean and standard deviation to compare numerical distributions

The mean and standard deviation can be used to perform analysis across different data sets to compare the centre and spread of distributions.

A larger mean suggests that the centre of the distribution is a higher value. A smaller mean suggests that the centre of the distribution is a lower value.

A larger standard deviation indicates that the values are more spread out and further from the mean. A smaller standard deviation indicates that the values are closer to the mean, and less spread out.

Worked example 3

The *test score*, in percentages, for students in two Year 11 General Maths classes was recorded. The mean and standard deviation were calculated and summarised in the following table.

	mean	standard deviation
class A	82	3.2
class B	89	6.7

Continues →

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a. Compare the mean *test score* for class A and class B.

Explanation

State the class with the higher mean test score, providing the mean figures for the two classes being compared.

Answer

Class B had a higher mean test score than class A. The mean for class B was 89%, while the mean for class A was 82%.

b. Compare the standard deviation in *test score* for class A and class B.

Explanation

State the class with the smaller standard deviation in test score, providing the standard deviation figures for the two classes being compared.

Answer

Class A had a smaller standard deviation in test score than class B. The standard deviation for class A was 3.2%, while the standard deviation for class B was 6.7%.

1E Questions

Calculating the mean of a data set

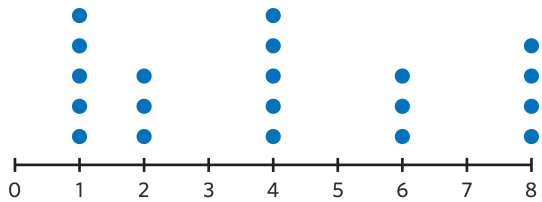
1. Abby recently received her *exam result* for each subject in semester 1. The results are shown in the following table.

	Mathematics	English	Geography	History	Chemistry
***exam result* (%)**	66	95	40	79	10

What is her mean *exam result* for the semester 1 exam period?

A. 10% **B.** 58% **C.** 65% **D.** 95%

2. Calculate the mean of the data set displayed in the following dot plot.



3. It was determined that, in a class of 25 students, the mean score on a chemistry test was 67%. A new student joined the class one week later and sat the same test. He achieved a score of 95%.

a. Would this new score increase or decrease the mean value?

b. Calculate the new mean score, rounded to one decimal place.

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Calculating the mean and standard deviation using technology

4. Consider the following data set.

10.8 12.7 22.4 18.1 32.5 12.6

The standard deviation is closest to

A. −8.2 **B.** −7.5 **C.** 7.5 **D.** 8.2

5. The amount of *money spent* by Mary on online shopping, over one week, is recorded in the following table.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
***money spent* ($)**	20	23	12	5	10	12	30

a. Calculate the standard deviation of this data set, correct to two decimal places.

b. On the following Monday, it was Mary's birthday. Mary decided to celebrate by spending $100 on a gift for herself. What is the standard deviation of *money spent* on her 8-day shopping spree, rounded to two decimal places?

c. Has the standard deviation increased or decreased? Explain why.

6. Nicky is working at Coles. She records the *weight*, in kilograms, of each customer's groceries over the course of 10 minutes.

4.5 6.9 2.4 8.1 2.5 2.6

Nicky then realises that she had recorded one of the values incorrectly. The '8.1' should have been recorded as '10.3' instead.

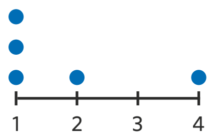
a. What is the implication of this change on the standard deviation?

b. Use a calculator to determine the new standard deviation, rounded to two decimal places.

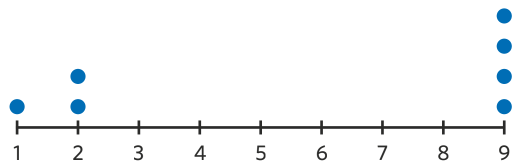
Using the mean and standard deviation to compare numerical distributions

7. Two dot plots are shown. Without doing any calculations, state which dot plot has a larger standard deviation.

A.



B.



8. The *height* (cm) of each player in the senior basketball team and senior netball team was recorded. The following table shows the mean and the standard deviation for each of the two teams.

	mean	standard deviation
basketball team	178.4	3.2
netball team	167.7	8.1

a. Compare the mean *height* of the two teams.

b. Compare the standard deviation in *height* of the two teams.

Joining it all together

9. In the Amazing Chef cooking competition, competitors receive a *rating*, out of 10, on a selection of dishes.

a. The results for one chef, Kaiden, are shown in the following table. Find the mean and standard deviation of his results, rounded to one decimal place.

	jam donuts	spaghetti	caesar salad	cheesecake	pizza	rice
rating	1.4	9.9	4.5	7.8	3.4	10.0

b. Another contestant, Emily, has her results shown in the following table. She is yet to be judged for her last dish. What rating must she receive, rounded to one decimal place, to achieve a mean *rating* of 6.5?

	jam donuts	spaghetti	caesar salad	cheesecake	pizza	rice
rating	5.4	7.8	6.2	5.4	4.9	

c. Suppose Emily received a rating of 9.8 on her final dish. Describe how this would affect the mean and standard deviation of her results.

10. Leon recorded the number of points he scored in his first five basketball games of the season.

game	1	2	3	4	5
points scored	25	26	20	19	28

a. Calculate the mean and standard deviation of *points scored* in Leon's first five games, rounded to two decimal places where necessary.

b. Leon wants to average 25 *points scored* over his first seven games. Given that he scored 27 points in game six, how many points does Leon need to score in game seven?

c. Assuming Leon achieved his goal and averaged exactly 25 *points scored* over his first seven games, calculate the standard deviation in *points scored*, rounded to two decimal places.

d. Compare the standard deviation in *points scored* from part **a** and part **c**.

Exam practice

11. The following table shows the *time taken*, in seconds, for a sample of 10 athletes to complete a 50 m freestyle swim.

time taken (seconds)	23.5	27.1	22.5	23.4	30.0	29.3	26.5	22.8	29.3	22.1

The mean, $\overline{x}$, and the standard deviation, s_x, for this sample of athletes are closest to

A. $\overline{x} = 25.65, \quad s_x = 3.14$

B. $\overline{x} = 3.14, \quad s_x = 25.65$

C. $\overline{x} = 25.65, \quad s_x = 2.98$

D. $\overline{x} = 25.65, \quad s_x = 3.41$

E. $\overline{x} = 2.98, \quad s_x = 25.65$

Adapted from VCAA 2017 Exam 1 Data analysis Q3

88% of students answered this type of question correctly.

12. The *height*, in cm, and *leg length*, in cm, of a sample of 11 men aged 23 to 25 years are shown in the table.

height (cm)	170.0	180.4	192.5	158.6	168.3	175.2	163.9	177.1	192.2	188.3	183.4
leg length (cm)	76.3	80.1	79.3	82.5	87.4	88.5	90.1	86.3	85.2	77.2	76.9

For these 11 men, determine the mean of their *height*, in cm, correct to two decimal places. (1 MARK)

Adapted from VCAA 2020 Exam 2 Data analysis Q4aii

77% of students answered this type of question correctly.

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13. In the sport of heptathlon, athletes compete in seven events.

These events are the 100 m hurdles, high jump, shot-put, javelin, 200 m run, 800 m run and long jump.

Fifteen female athletes competed to qualify for the heptathlon at the Olympic Games.

Their results for two of the heptathlon events – long jump and 100 m run – are shown in Table 1.

Table 1

athlete number	***long jump*** **(metres)**	***100 m run*** **(seconds)**
1	7.99	13.21
2	7.00	15.31
3	6.03	16.28
4	5.53	13.54
5	6.82	12.76
6	7.88	11.56
7	6.92	14.22
8	6.91	12.83
9	7.86	14.37
10	8.05	15.05
11	6.83	16.02
12	5.87	14.40
13	5.95	16.23
14	6.91	12.93
15	6.94	13.63

Complete Table 2 by calculating the standard deviation in the distance jumped in the long jump, in metres, by the 15 athletes. (1 MARK)

Table 2

statistic	***long jump*** **(metres)**	***100 m run*** **(seconds)**
mean	6.90	14.16
standard deviation		1.41

Adapted from VCAA 2021 Exam 2 Data analysis Q1b

87% of students answered this type of question correctly.

Questions from multiple lessons

Data analysis

14. The following two-way frequency table displays the *preferred beverage* (coffee, tea, juice) and *age* (under 40 years, 40 years or over) of 83 people.

		age	
		under 40 years	**40 years or over**
preferred beverage	**coffee**	18	21
	tea	9	16
	juice	14	5
total		41	42

The percentage of people under 40 years old who chose tea as their preferred beverage is closest to

A. 11% **B.** 21% **C.** 22% **D.** 32% **E.** 78%

Adapted from VCAA 2016 Exam 1 Data analysis Q1

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Data analysis

15. A manager is investigating the relationship between the time it takes for employees to complete tasks and the quality of work.

The variables *time taken* and *quality* (below satisfactory, satisfactory, good, excellent) are

A. both numerical.

B. both categorical.

C. a discrete variable and a nominal variable respectively.

D. a continuous variable and an ordinal variable respectively.

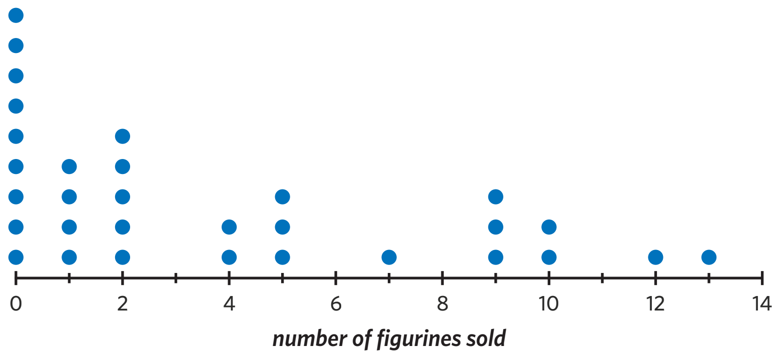
E. a continuous variable and a nominal variable respectively.

Adapted from VCAA 2016 Exam 1 Data analysis Q2

Data analysis

16. Horacio has just opened his start-up business selling custom pet figurines.

The following dot plot shows the number of figurines sold each day over a period of 31 days.



a. Identify the number of days on which Horacio made no sales. (1 MARK)

b. Calculate the percentage of days in which more than 10 figurines were sold. Give the answer correct to one decimal place. (1 MARK)

c. Determine the median value. (1 MARK)

Adapted from VCAA 2016 Exam 2 Data analysis Q1c-d

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1F The five-number summary and boxplots

STUDY DESIGN DOT POINT

- the five-number summary and the boxplot as its graphical representation and display, including the use of the lower fence ($Q_1 - 1.5 \times IQR$) and upper fence ($Q_3 + 1.5 \times IQR$) to identify possible outliers

1A 1B 1C 1D 1E **1F** 1G 1H

KEY SKILLS

During this lesson, you will be:

- calculating the five-number summary
- identifying outliers using fences
- constructing and interpreting boxplots.

KEY TERMS

- Five-number summary
- Minimum
- Maximum
- Outliers
- Upper fence
- Lower fence
- Boxplot

The five-number summary summarises key features of a distribution of numerical data. A boxplot is a visual representation of the distribution and is constructed using the five-number summary. It shows the distribution of the values in a data set within certain intervals. Constructing boxplots from numerical data sets allows conclusions to be drawn about the distribution.

Calculating the five-number summary

When there is a set of numerical data that is arranged in ascending order, a five-number summary can be created. A **five-number summary** summarises the distribution of a data set, as well as providing key information about the spread and centre.

The five figures included in the summary are: minimum, lower quartile (Q_1), median (Q_2), upper quartile (Q_3), and maximum.

The **minimum** is the smallest value in the data set, while the **maximum** value is the largest value in the data set.

For example, the five-number summary for the data set 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 is:

1	2	3	4	5		6	7	8	9	10
↑		↑			↑			↑		↑
minimum		Q_1			median			Q_3		maximum

Minimum: 1

First quartile (Q_1): 3

Median (Q_2): 5.5

Third quartile (Q_3): 8

Maximum: 10

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Worked example 1

Create a five-number summary for the following data set.

18 16 21 4 24 17 34 19 15 10 14

Explanation – Method 1: By hand

Step 1: Rewrite the data in ascending order.

4 10 14 15 16 17 18 19 21 24 34

Step 2: Identify the minimum and the maximum values.

4 10 14 15 16 17 18 19 21 24 34

$minimum = 4$

$maximum = 34$

Step 3: Determine the median (Q_2).

Using the formula $\frac{n+1}{2}$, where n is the number of elements in the data set, determine the position of the median value.

$\frac{11+1}{2} = 6$

4 10 14 15 16 17 18 19 21 24 34

The median is the sixth element of the data set, which is 17.

Step 4: Determine the upper and lower quartiles (Q_1 and Q_3).

Q_1 is the median of the lower half of the data set.

Lower half: 4 10 14 15 16

$Q_1 = 14$

Q_3 is the median of the upper half of the data set.

Upper half: 18 19 21 24 34

$Q_3 = 21$

Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Name column A as 'list' and enter the data values , starting from row 1.

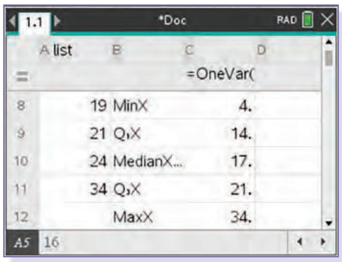
Step 2: Press menu. Select '4: Statistics' → '1: Stat Calculations' → '1: One-Variable Statistics'.

Select 'OK' to confirm one-variable statistics for one data set only.

Step 3: Specify the data set by entering 'list' in 'X1 List:'.

Select 'OK' to exit this window and generate the statistics.

Scroll down to find the five-number summary statistics.



Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap Statistics.

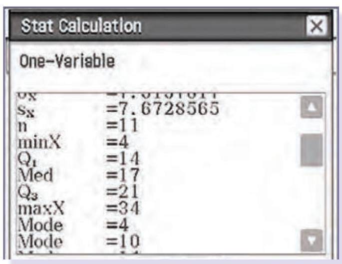
Name list1 as 'list' and enter the data values, starting from row 1.

Step 2: Tap 'Calc' → 'One-Variable'.

Specify the data set by changing 'XList:' to 'main/list'.

Tap 'OK' to confirm.

Step 3: Scroll down to find the five-number summary statistics.



Answer – Method 1, 2 and 3

4, 14, 17, 21, 34

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Identifying outliers using fences

Outliers are values in a data set that are outside of the general spread of data, and can be found by calculating the upper and lower fences.

The **lower fence** defines the boundary of an outlier in the lower half of the data.

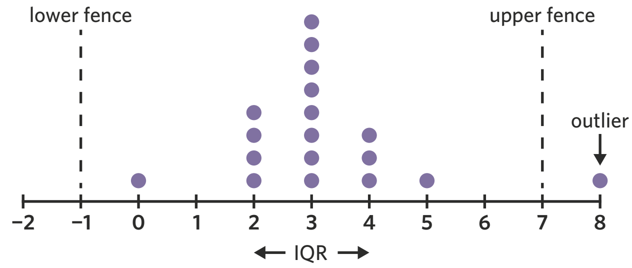
The **upper fence** defines the boundary of an outlier in the upper half of the data.

$\textit{lower fence} = Q_1 - (1.5 \times IQR)$

$\textit{upper fence} = Q_3 + (1.5 \times IQR)$

If a value in the data set is below the lower fence or above the upper fence, it is considered to be an outlier. It is important to note that outliers can still be labelled as the minimum or maximum value, and are included in the five-number summary.

Note: Lower and upper fences are not included in the five-number summary, so it is not necessary to mark them when creating a boxplot unless the question specifies.



Worked example 2

The *number of runs* scored by 13 players in a cricket match was recorded.

75 82 87 76 81 83 82 25 83 79 105 85 78

a. Calculate the lower and upper fences of the data set.

Explanation

Step 1: Arrange the data in ascending order.

25 75 76 78 79 81 82 82 83 83
85 87 105

Step 2: Construct the five-number summary.

Minimum: 25

First quartile (Q_1): 77

Median (Q_2): 82

Third quartile (Q_3): 84

Maximum: 105

Step 3: Calculate the *IQR*.

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 84 - 77 \\ &= 7 \end{aligned}$$

Step 4: Find the lower and upper fence.

$$\begin{aligned} \textit{lower fence} &= Q_1 - (1.5 \times IQR) \\ &= 77 - (1.5 \times 7) \\ &= 66.5 \end{aligned}$$

$$\begin{aligned} \textit{upper fence} &= Q_3 + (1.5 \times IQR) \\ &= 84 + (1.5 \times 7) \\ &= 94.5 \end{aligned}$$

Answer

$\textit{lower fence} = 66.5$

$\textit{upper fence} = 94.5$

b. Identify any outliers.

Explanation

25 | 75 76 78 79 81 82 82 83 83 85 87 | 105

Lower fence: 66.5 Upper fence: 94.5

$25 < 66.5$

25 is under the lower fence of 66.5.

$105 > 95.5$

105 is above the upper fence of 94.5.

Answer

25 and 105

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Constructing and interpreting boxplots

See worked example 3

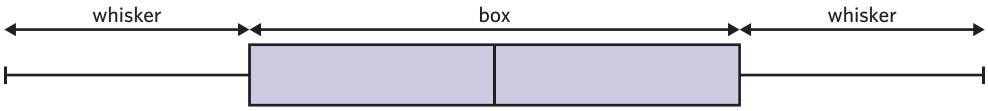
A **boxplot** is a graphical representation of a set of numerical data. It shows the five-number summary and any outliers.

A boxplot consists of a box and whiskers positioned above a scale. If there are no outliers, the leftmost and rightmost ends of the whiskers represent the minimum and maximum values. However, if there are outliers, they are shown as separate dots on the same scale, and the whisker ends represent the most extreme values that aren't outliers.

The central box represents the middle 50% of values, where the left and right borders of the box represent Q_1 and Q_3 respectively. The vertical line dividing the box represents the median.

Note: This line doesn't always divide the box into two equal halves.

The left whisker extends from the minimum non-outlier value to Q_1, while the right whisker extends from Q_3 to the maximum non-outlier value.

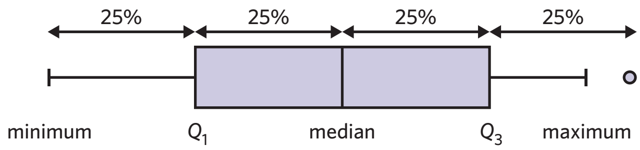


See worked example 4

The boxplot is divided into four quarters where the data found between two consecutive boundaries represents 25% of the total data set.

25% of the data is shown between the following intervals:

- Minimum to Q_1
- Q_1 to median (Q_2)
- Median to Q_3
- Q_3 to maximum.



Worked example 3

Construct a boxplot using the following data.

1 5 5 6 7 7 7 8 8 9

Explanation – Method 1: By hand

Step 1: Construct the five-number summary.

1, 5, 7, 8, 9

Step 2: Check for any outliers.

Calculate the IQR.

$$IQR = Q_3 - Q_1$$
$$= 8 - 5$$
$$= 3$$

Calculate the lower fence.

$$lower\ fence = Q_1 - (1.5 \times IQR)$$
$$= 5 - (1.5 \times 3)$$
$$= 0.5$$

Calculate the upper fence.

$$upper\ fence = Q_3 + (1.5 \times IQR)$$
$$= 8 + (1.5 \times 3)$$
$$= 12.5$$

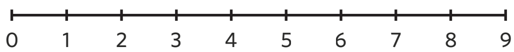
All data values lie between the lower and upper fence. So, there are no outliers.

Step 3: Construct an axis with an appropriate scale.

The scale should have a range that accommodates all data.

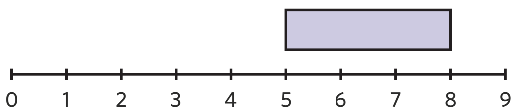
Intervals of the scale should be constant.

Since the data ranges from 1–9, an appropriate scale would run from 0–9.



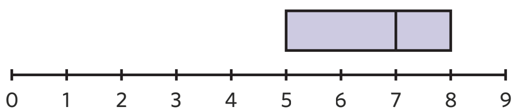
Step 4: Draw the border of the box.

The left border is Q_1, 5, and the right border is Q_3, 8.



Step 5: Mark the median.

The vertical line in the box represents the median, 7.



Continues →

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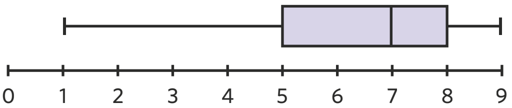
Step 6: Draw the whiskers.

As there is no value below the lower fence, the left whisker extends from Q_1 to the minimum value, 1.

As there is no value above the upper fence, the right whisker extends from Q_3 to the maximum value, 9.

Answer



Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name column A 'list' and enter the data values into column A, starting from row 1.

Step 3: Press ctrl + doc and select '5: Add Data & Statistics'.

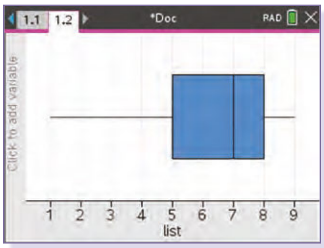
Move the cursor to the horizontal axis and select 'Click to add variable'.

Select 'list'.

Step 4: Press menu. Select '1: Plot Type' → '2: Box Plot'.

Note: To change the view of the histogram press menu and use options within '5: Window/Zoom'.

Answer



Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap Statistics.

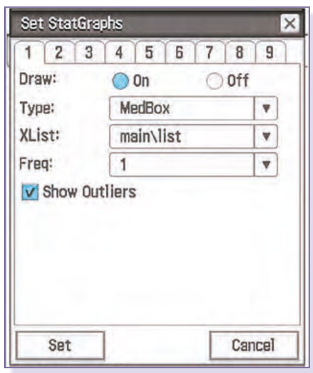
Step 2: Name list1 'list' and input the data values starting from row 1.

Step 3: Configure the settings of the graph by tapping.

Create a histogram by changing 'Type:' to 'MedBox'.

Specify the data set by changing 'XList:' to 'main\list'.

Tick the 'Show Outliers' box.



Tap 'Set' to confirm.

Step 4: Tap in the icon bar to plot the boxplot.

Continues →

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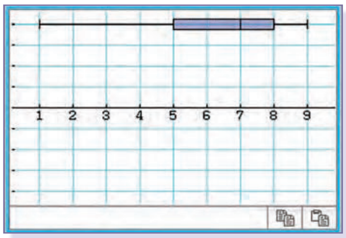
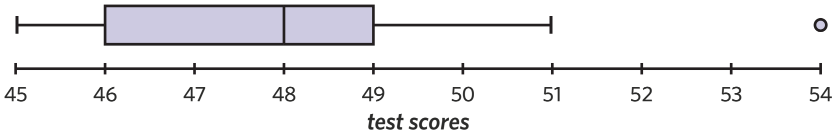
Answer



Worked example 4

The *test scores* of a Year 12 Biology class are represented by the following boxplot.



a. Determine the five-number summary, and identify any outliers.

Explanation

Step 1: Identify the minimum and maximum values.

The leftmost end of the whisker extends to 45. As there are no values found below 45, this represents the minimum.

The rightmost end of the whisker extends to 51. However, since there is a value found above 51, the outlier represents the maximum.

$minimum = 45$

$maximum = 54$

Step 2: Identify Q_1 and Q_3.

The left and right borders of the box represent Q_1 and Q_3 respectively.

$Q_1 = 46$

$Q_3 = 49$

Step 3: Identify the median.

The vertical line in the box represents the median.

$median = 48$

Answer

45, 46, 48, 49, 54

There is an outlier at 54.

b. Between which two values does the middle 50% of the data lie?

Explanation

The middle 50% of the data lies between Q_1 and Q_3.

$Q_1 = 46$

$Q_3 = 49$

Answer

46 and 49

c. What percentage of scores were over 49?

Explanation

$Q_3 = 49$

25% of the data is greater than Q_3.

Continues →

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Answer

25%

1F Questions

Calculating the five-number summary

1. What's the median value of the following data set?

1 2 8 21 23 37 81

A. 14.5 **B.** 21 **C.** 22 **D.** 24.7

2. In an athletics carnival, the *times* (seconds) of the 100 m sprint for the 15–16 year old boys were recorded.

11.33 16.21 10.99 15.89 12.30 15.71 19.20 12.11 12.60 13.01 11.32 14.00

Find Q_3.

3. Employees in an office were asked how much they work a week. The *number of hours* they each logged in one week was recorded.

25.32 23.32 34.56 45.54 12.43 40.01 38.35 34.11 21.34 37.76 38.04 31.89

Construct a five-number summary of the data, rounded to two decimal places.

4. Construct a five-number summary for the following stem plot.

Key: 3 | 9 = 39

0	4
1	0 3 5 8
2	2 6 9
3	6 6
4	3
5	7

5. The *shoe size* of members in a teenage basketball team was recorded.

a. What is the median value of this data set?

b. What are the values of Q_1 and Q_3?

c. Is the value of Q_1 similar to the median? If so, why would this be the case?

d. Construct a five-number summary.

shoe size	frequency
7	3
8	5
9	3
10	2

Identifying outliers using fences

6. A group of 100 Year 12 students were asked the *number of hours* on average they spend looking at a screen per day. The five-number summary for the results is: 1, 4, 6, 7, 11.

The lower fence for this data is:

A. −0.5 **B.** 3 **C.** 4.5 **D.** 11.5

7. Teachers in a high school were asked for their *age*. The following data shows 15 responses. Without calculating the lower and upper fence, determine the potential outlier.

40 41 43 42 39 42 40 38 40 42 19 41 40 39 40

---

8. Calculate the upper and lower fences, and subsequently identify any outliers from the following data sets:

a. 5 5 6 6 6 7 7 8 8 10 14 80

b. 55 59 41 48 48 49 41 41 41 42 50 70

---

9. Customers purchasing items from a pet shop were asked the *number of pets* they own. The following data shows the 13 responses.

0 0 1 1 1 1 2 2 3 4 6 15 21

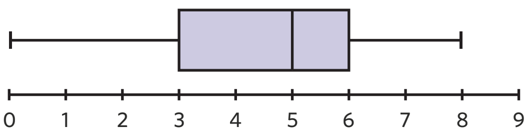
a. Find the lower and upper fences.

b. Identify the outliers.

c. Explain why some of the values are considered outliers.

Constructing and interpreting boxplots

10. What is the five-number summary for the following boxplot?



A. 3, 3, 5, 6, 7

B. 0, 1, 2, 7, 8

C. 0, 3, 5, 6, 8

D. 8, 7, 2, 1, 0

---

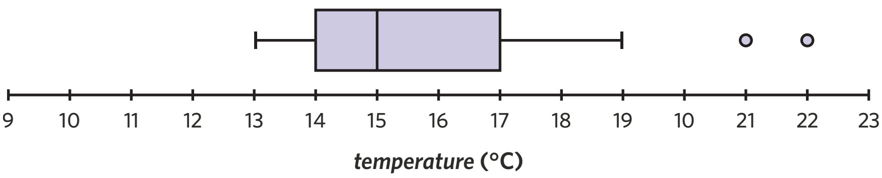
11. A class of 23 students were asked what they thought of rap music. They gave it a *rating* out of 10, and the following results were recorded.

1 4 5 5 6 7 7 7 7 8 8 8 8 9 9 9 9 9 10 10 10 10 10

Construct a boxplot from the data by hand and include any outliers if present.

---

12. The *temperature*, in °C, in Melbourne over 24 days is displayed using a boxplot.



a. The 6 days that recorded the highest *temperature* in Melbourne is between which two data values on the boxplot?

b. What percentage of days were between 14 °C and 17 °C?

Joining it all together

13. The *height*, in centimetres, of a Grade 5 class of 14 girls were recorded. The results are as follows:

130.32 123.56 167.98 123.57 143.16 134.18 119.12

120.46 121.65 110.65 122.87 151.89 112.12 110.56

a. Construct a five-number summary for the data, rounded to two decimal places.

b. Identify any outliers.

c. Show that 110.65 cm is not an outlier.

d. Construct a boxplot by hand to represent the data.

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e. What percentage of girls are between 119.12 cm and 134.18 cm tall?

f. How many girls are between 123.22 cm and 167.98 cm tall?

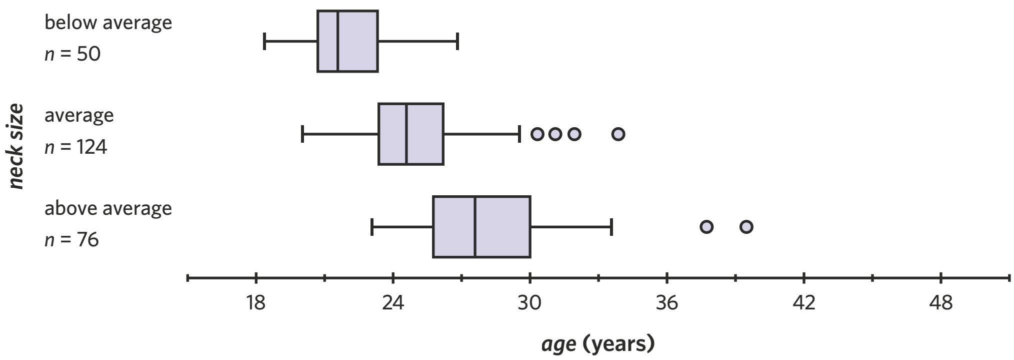
Exam practice

14. In a study of the association between *age* (years) and *neck size*, 250 chimpanzees were grouped by *neck size* (below average, average and above average) and their *age*.

Five-number summaries describing the distribution of age for each group are displayed in the following table along with the group size.

The associated boxplots are also shown.

		age (years)				
neck size	group size	min.	Q_1	median	Q_3	max.
below average	50	18.1	20.6	21.6	23.2	26.8
average	124	19.8	23.4	24.6	26.0	33.9
above average	76	23.1	26.25	28.1	29.95	39.1

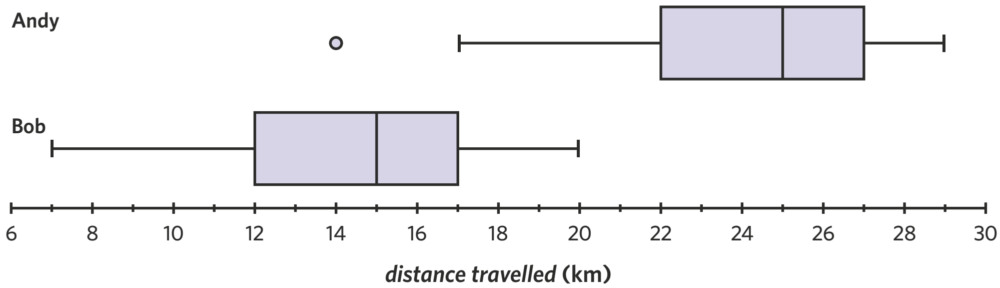


What percentage of these 250 chimpanzees are classified as having a below average neck size? (1 MARK)

Adapted from VCAA 2020 Exam 2 Data analysis Q3a

93% of students answered this type of question correctly.

15. Two friends, Andy and Bob, are training for a triathlon. The following parallel boxplots show the *distance travelled* (km) of 32 bike rides they each did in the lead up to their triathlon.



Use the information in the boxplots to complete the following sentence.

In the lead up to the triathlon, the median value for Bob's *distance travelled* was

[ ] km less than the median value for Andy's *distance travelled*. (1 MARK)

Adapted from VCAA 2019 Exam 2 Data analysis Q2aii

87% of students answered this type of question correctly.

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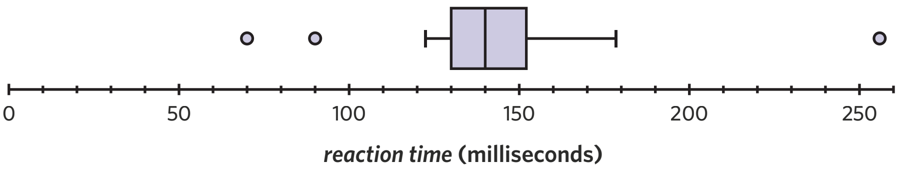
16. The boxplot shows the distribution of the *reaction time* (milliseconds), of 256 firefighters.



The five-number summary for the *reaction time* of these 256 firefighters is closest to

A. 70, 130, 140, 152, 179

B. 70, 130, 140, 152, 255

C. 122, 130, 140, 152, 179

D. 122, 130, 140, 152, 245

E. 122, 130, 140, 152, 265

Adapted from VCAA 2017 Exam 1 Data analysis Q2

58% of students answered this type of question correctly.

Questions from multiple lessons

Data analysis *Year 10 content*

17. Lachlan is working on his fitness so he went for nine runs last week. The total distance he ran was 42.3 km. What is the mean distance that Lachlan ran?

A. 4.23 km **B.** 4.67 km **C.** 4.7 km **D.** 4.8 km **E.** 4.92 km

Adapted from VCAA 2019NH Exam 1 Data analysis Q3

Data analysis

18. Are the variables *exam grade* (high distinction, distinction, pass, fail) and *time spent studying* (less than 5 hours, 5 to 10 hours, more than 10 hours) nominal, ordinal, discrete or continuous?

A. The variables are both nominal.

B. The variables are both ordinal.

C. The variables are nominal and ordinal respectively.

D. The variables are ordinal and discrete respectively.

E. The variables are nominal and discrete respectively.

Adapted from VCAA 2016 Exam 1 Data analysis Q2

Data analysis

19. Class A and B were surveyed on the *number of hours* they spend doing homework each week. Their responses are recorded in the following back-to-back stem plot.

number of hours

Key: 1 | 0 = 10 hours

Class A		Class B
4	0	4
9 8 8 6	0	5 5 8
4 2 2 1 0 0	1	2 3 3 3 4
9	1	8 8
1 0 0	2	0 1 2 2
	2	5

a. Which group has a larger range? (1 MARK)

b. What is the difference in the median of the two groups? (1 MARK)

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1G Investigating data distributions

STUDY DESIGN DOT POINT

- consideration of a range of distributions (symmetrical, asymmetrical), their summary statistics and the percentage of data lying within several standard deviations of the mean

1A 1B 1C 1D 1E 1F **1G** 1H

KEY SKILLS

During this lesson, you will be:

- describing the shape of data distributions
- comparing data distributions
- interpreting the normal distribution.

KEY TERMS

- Perfectly symmetric
- Approximately symmetric
- Positive skew
- Negative skew
- Bimodal
- Normal distribution
- 68–95–99.7% rule

Data can be analysed and described in different ways to better understand trends or to make generalisations. The shape, centre, spread and standard deviation of a data set are all useful when investigating data distributions. The normal distribution is also useful in finding the amount of data within given intervals.

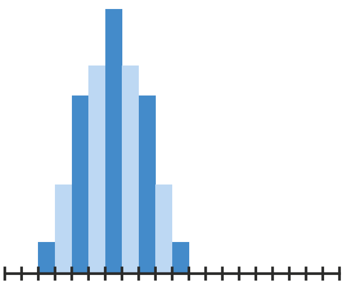
Describing the shape of data distributions

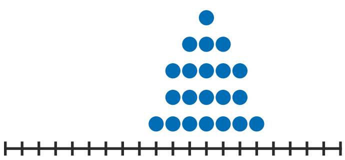
The shape of a distribution refers to how the data is arranged. Depending on the arrangement, data can be described as skewed, symmetric or bimodal. It is important to remember that the location of the data on an axis does not affect the shape of the distribution.

Symmetric data can be described as either perfectly or approximately symmetric.

A **perfectly symmetric** distribution will be the exact same on either side of the centre of data. An **approximately symmetric** distribution will be very similar on either side of the centre, but does not have to be perfect.

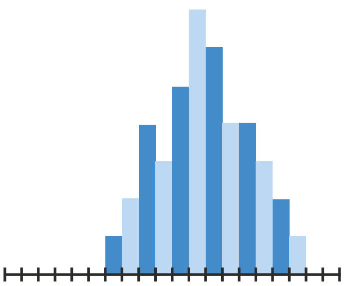
The following examples demonstrate perfectly symmetric distributions.

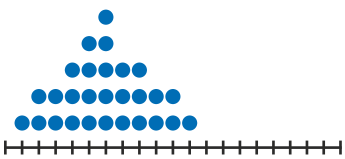




3	9				
4	2	3	8		
5	2	5	6	8	9
6	3	6	9		
7	6				

The following examples demonstrate approximately symmetric distributions.

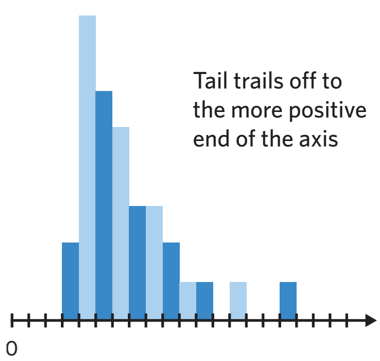




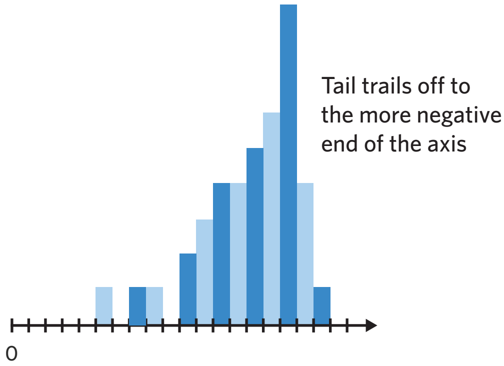
4	1	6			
5	3	5	6	9	
6	2	3	5	8	9
7	2	4	8		
8	5	6			

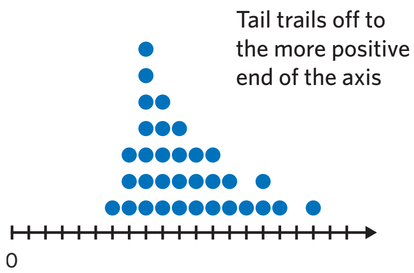
The skew of a distribution describes which direction from the peak the data trails off to.
A **positive skew** occurs when the 'tail' trails off from the peak in a positive direction.
A **negative skew** occurs when the 'tail' trails off from the peak in a negative direction.

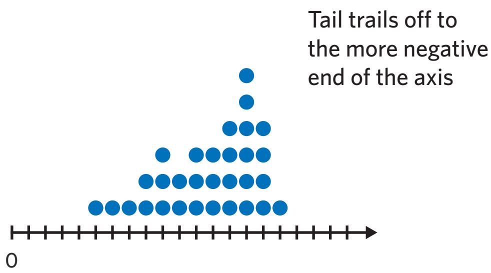
Positive skew



Negative skew







Tail trails off to the more positive end of the stem

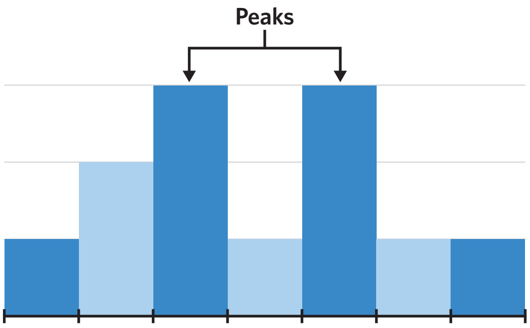
3	5					
4	1	3	4	4	6	9
5	0	3	7	8		
6	2	5	6			
7	1					
8	4	8				
9	1					

Tail trails off to the more negative end of the stem

2	6					
3	8					
4						
5	7					
6	2	3	9			
7	0	2	5	8		
8	1	3	4	6	8	9
9	1					

A distribution is **bimodal** if it has two distinct peaks within the distribution.

Note: The peaks do not have to be the same height for the distribution to be considered bimodal.



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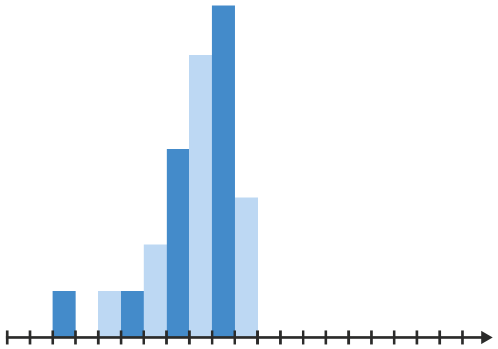
Worked example 1

Describe the shape of the following distributions.

a.



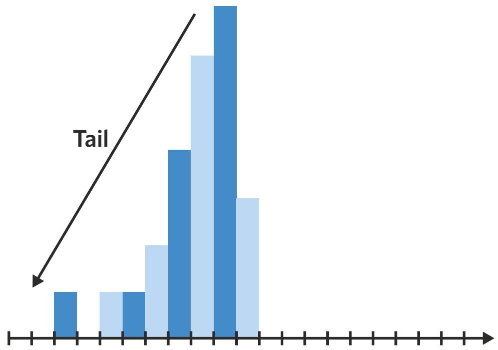
Explanation

Step 1: Determine whether the distribution is approximately or perfectly symmetric, skewed or bimodal.

The distribution is skewed, as there is only one peak and it is not symmetric.

Step 2: Determine whether the distribution is positively or negatively skewed.

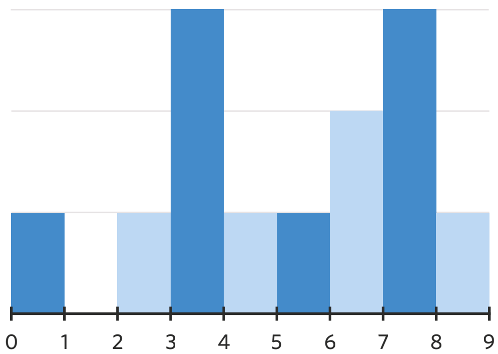
The distribution trails off in a negative direction on the horizontal axis.



Answer

Negatively skewed

b.



Explanation

Determine whether the distribution is approximately or perfectly symmetric, skewed or bimodal.

There are two peaks in the distribution so it is bimodal.

The distribution is neither symmetric nor skewed.

Answer

Bimodal

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Comparing data distributions

Measures of centre and spread can be used when comparing data distributions.

Recall that the measure of centre refers to the middle point of a distribution, and will often be the peak.

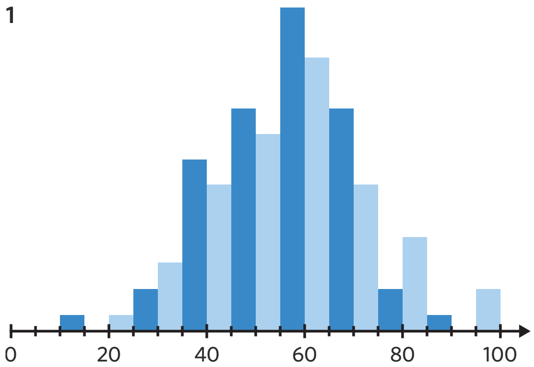
The spread describes how similar or varied the data set is. For numerical data displays, the spread can sometimes be best represented by the range.

Recall that the range can be calculated as *range* = *maximum value* − *minimum value*.

Worked example 2

Compare the centre and spread of the following distributions.

1



2 **Key:** 3 | 2 = 32

2	9
3	2 4
3	5 6 8 8
4	0 1 2 2 4
4	5 6 6 7 8 9 9
5	0 1 3 4 4
5	5 8 9
6	0

Explanation

Step 1: Compare the centre.

Distribution 1 is approximately centred around 55–60. Distribution 2 is centred around 45–49.

Therefore, distribution 1 tends to have larger values.

Step 2: Compare the spread. The range of distribution 1 will be approximate.

$approximate\ range_1 = 100 - 10 = 90$

$range_2 = 60 - 29 = 31$

Therefore distribution 1 has a larger spread.

Answer

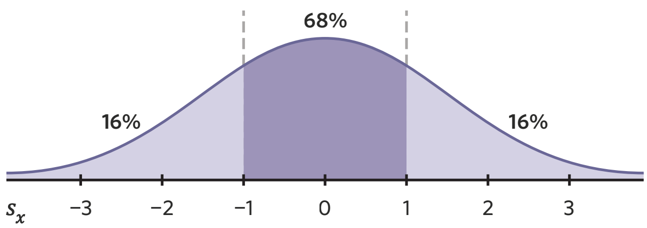
Distribution 1 has a greater centre and spread than distribution 2.

Interpreting the normal distribution

Some symmetric data sets can be further described as having a normal distribution. The **normal distribution** can be used to identify the percentage of data within a number of standard deviations from the centre. In a normal distribution, the mean and the median are equal and both represent the centre of data.

When there is a normally distributed data set, its mean $(\bar{x})$ and standard deviation (s_x) value can be used to compare the data to the standard normal distribution curve, which has a mean of 0, and standard deviation of 1. This comparison helps to visualise intervals in terms of standard deviations.

The **68–95–99.7% rule** determines the percentage of the data that lies within each standard deviation either side of the mean.



68% of the data lies within 1 standard deviation either side of the mean.

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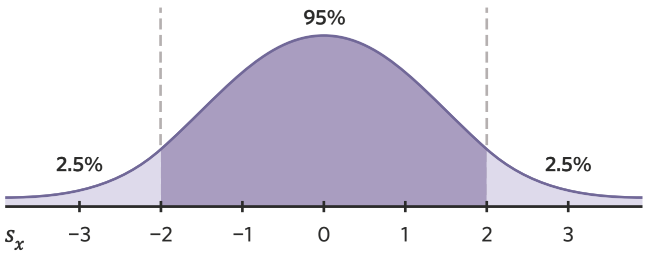
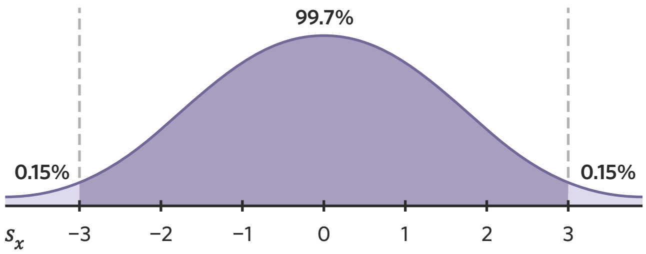
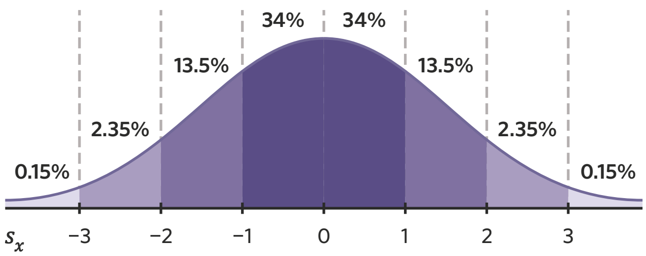


95% of the data lies within 2 standard deviations either side of the mean.



99.7% of the data lies within 3 standard deviations either side of the mean.

The percentage of data lying within one standard deviation to the next is shown in the following image.



Worked example 3

22 members of a football team were asked to record their height. Their height was normally distributed with a mean of 182 cm and a standard deviation of 4 cm.

a. What percentage of players are between 178 cm and 186 cm tall?

Explanation

Step 1: Calculate the number of standard deviations each value is from the mean.

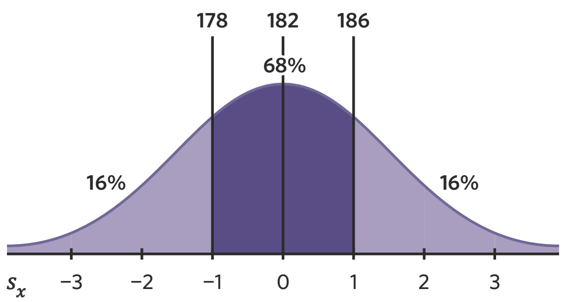
The mean is 182 cm and standard deviation is 4 cm.

$182 - (4 \times 1) = 178$, therefore 178 cm is 1 standard deviation below the mean.

$182 + (4 \times 1) = 186$, therefore 186 cm is 1 standard deviation above the mean.

Step 2: Determine the percentage of data in the defined boundary.

The interval is within 1 standard deviation either side of the mean.



Answer

68%

Continues →

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b. How many players, to the nearest whole number, are taller than 186 cm?

Explanation

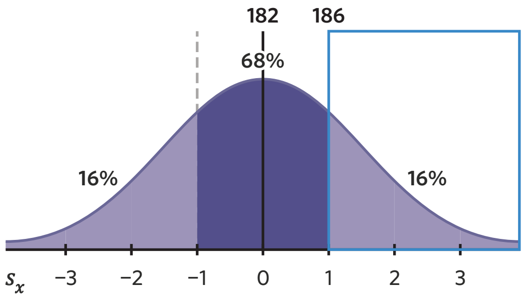
Step 1: Determine the number of standard deviations the value is from the mean.

186 cm is 1 standard deviation above the mean.

Step 2: Determine the percentage of data in the defined boundary.

The interval 'taller than 186 cm' translates to a boundary of everything greater than 1 standard deviation above the mean.

This is equivalent to 16% of players.



Step 3: Calculate the number of players.

There are 22 players in the data set, so find 16% of 22.

$$\frac{16}{100} \times 22 = 3.52$$
$$\approx 4$$

Answer

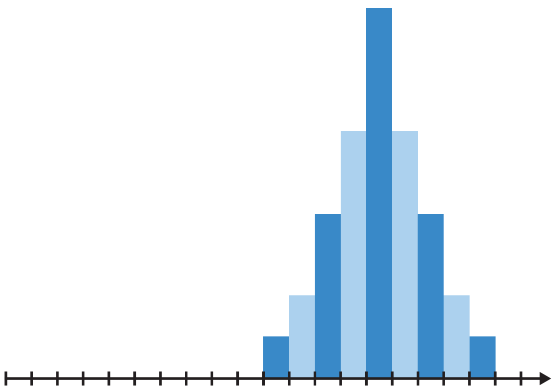
4 players

1G Questions

Describing the shape of data distributions

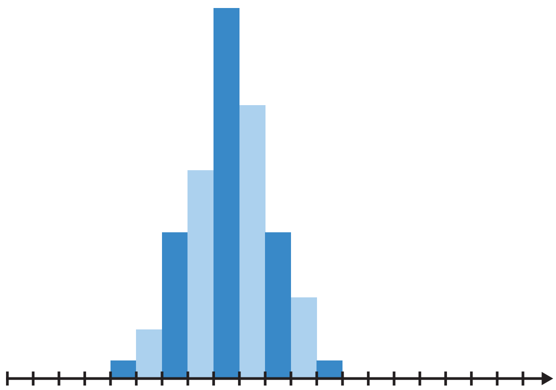
1. Describe the shape of the following distributions.

a.



A. Negatively skewed
B. Perfectly symmetric
C. Approximately symmetric
D. Positively skewed

b.



A. Bimodal
B. Perfectly symmetric
C. Approximately symmetric
D. Positively skewed

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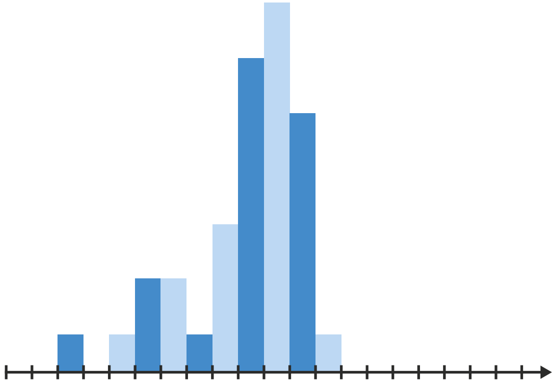
c. **Key:** 3 | 1 = 31

0	0 2 5
1	1 2 5 8 9 9
2	0 0 6 8 2
3	2 3 9
4	0
5	1
6	5

A. Negatively skewed
B. Perfectly symmetric
C. Approximately symmetric
D. Positively skewed

d.



A. Negatively skewed
B. Perfectly symmetric
C. Bimodal
D. Positively skewed

2. The height of 14 basketball players is represented by the stem plot.

Key: 16 | 2 = 162 cm

16	2
16	8
17	1 3
17	5 7 9
18	2 2 3
18	5 6 6 8 9
19	1 1 3 4
19	5 6 6 7 7 7 8
20	2

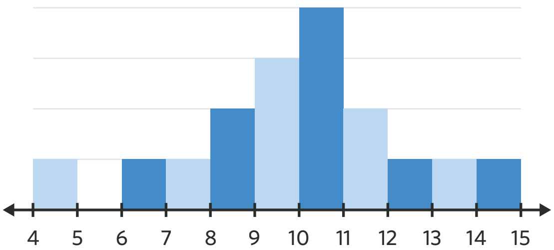
Describe the shape of this data.

3. Explain how a distribution's shape can be described as positively skewed, even if all of its data points are negative.

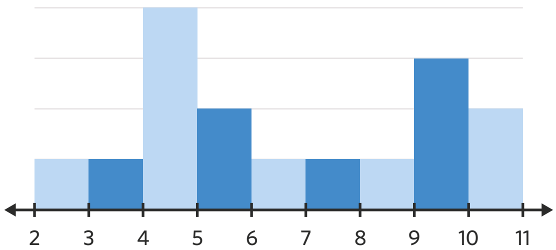
Comparing data distributions

4. Identify the distribution with a centre between 10 and 11 and a spread of approximately 11.

A.



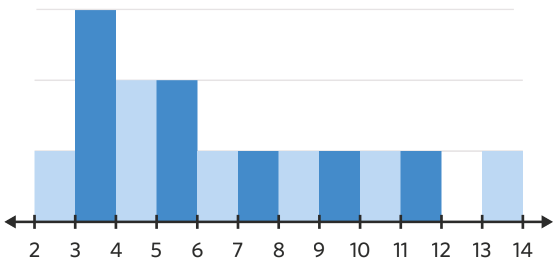
B.



C. **Key:** 1 | 2 = 12

0	2 4 5 5
1	0 1 1 4 6
1	7 8 8 9 9 9
2	0 0 1

D.



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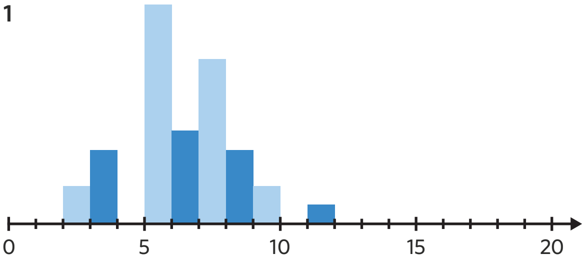
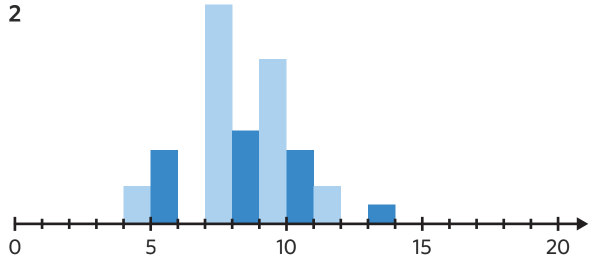
5. Compare the centre of the following pair of histograms.

1

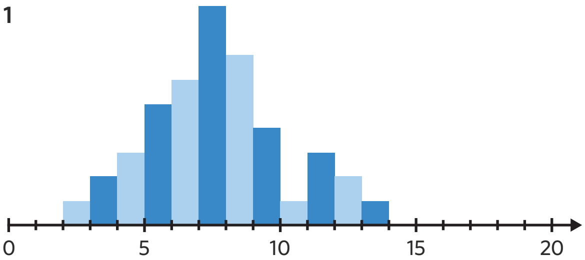


2

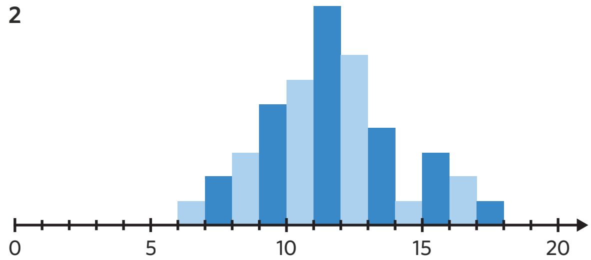


6. Compare the spread of the following pair of histograms.

1

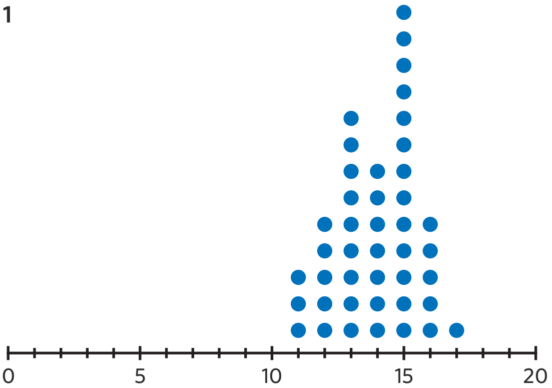


2

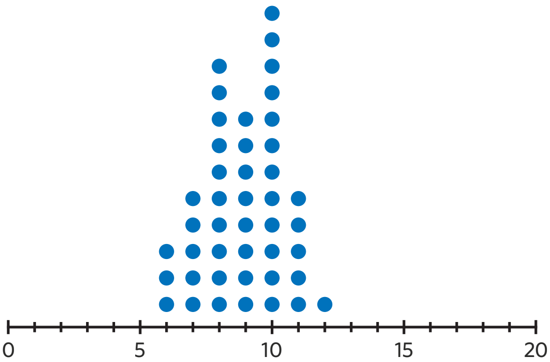


7. Do the following pairs of distributions differ in centre or spread, both or neither?

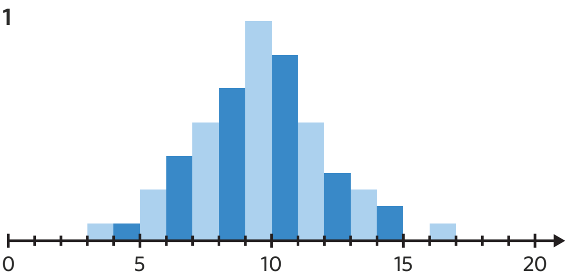
a. 1



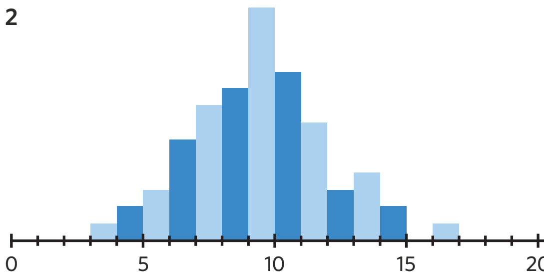
2



b. 1



2



c. 1 **Key:** 17 | 8 = 178

17	8
18	3 6 9
19	
20	6
21	4 4 5 9
22	1 8
23	1 4 6 7 8 9
24	0 6 9
25	2 7

2 **Key:** 30 | 8 = 308

30	8
31	3 6 9
32	
33	6
34	4 4 5 9
35	1 8
36	1 4 6 7 8 9
37	0 6 9
38	2 7

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Interpreting the normal distribution

8. In a normally distributed set of data, what percentage of data is expected to lie between 1 standard deviation either side of the mean?

A. 34% **B.** 68% **C.** 84% **D.** 95%

9. A group of 50 university students were surveyed about how long they study for each week. The resulting data is normally distributed with a mean of 150 minutes and a standard deviation of 25 minutes.

a. What percentage of the students are expected to study between 125 and 150 minutes per week?

b. Which of the following boundaries is expected to contain 17 students?

A. 100 to 125 minutes
B. 125 to 175 minutes
C. 150 to 175 minutes
D. 175 to 200 minutes

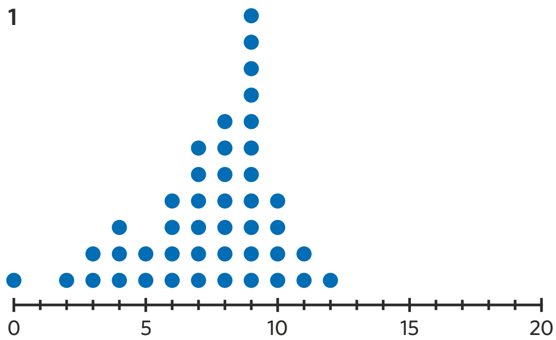
c. How many students are expected to study between 100 and 150 minutes per week? Round to the nearest whole number.

d. How many more students are expected to study 125 minutes or less compared to students who study 200 minutes or more? Round to the nearest whole number.

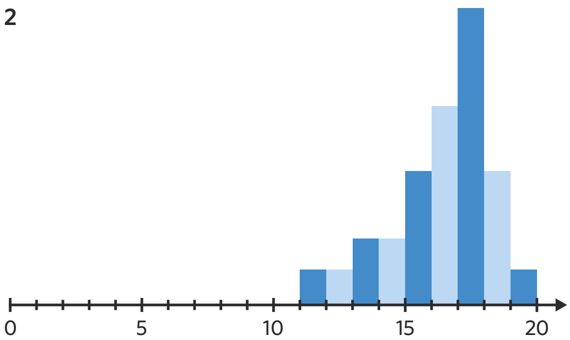
Joining it all together

10. Compare the following pairs of distributions by referencing their shape, centre and spread.

a. **1**



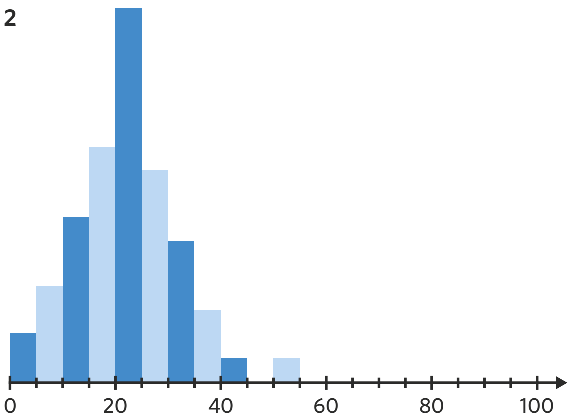
2



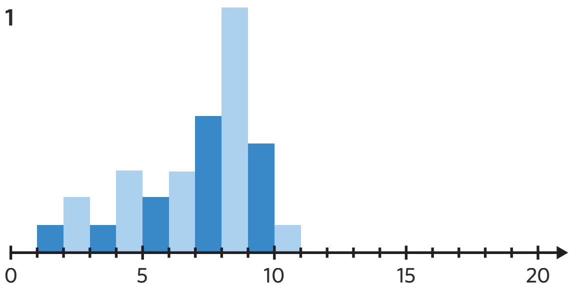
b. **1** **Key:** 1 | 9 = 19

1	9
2	2 3 4 4
2	5 6 6 7 8 8 9
3	0 1 2 2 3 4 4 4
3	5 5 6 6 6 7 7 8 9 9
4	0 0 0 1 1 3 4
4	5 5 6 8 9
5	0 3 3
5	6

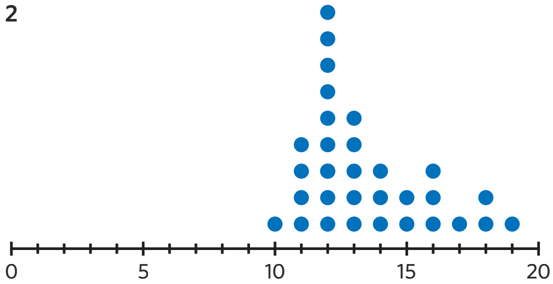
2



c. **1**



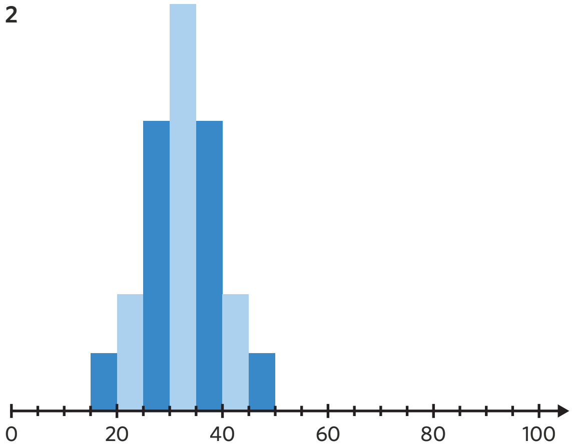
2



d. **1** **Key:** 2 | 4 = 24

2	4
2	5 6 7 9
3	0 0 1 1 2 3 3 3 4 4
3	6 6 6 7 8 8 9 9
4	0 1 1 2 2 3 4
4	6 7 7 8 9 9
5	1 2 2 3 4
5	5 6 8
6	0 4
6	6

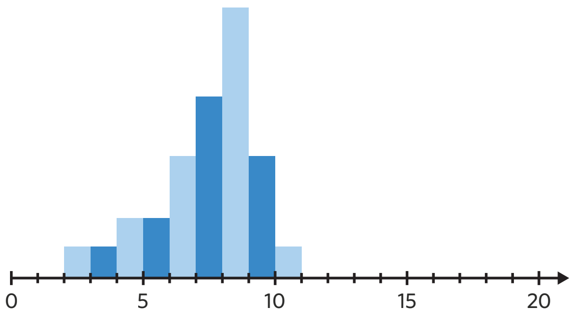
2



11. Fill out the following table referencing distributions I–VI. Note that distributions may be used more than once.

description	distribution
largest spread	
largest centre	
skewed (positively or negatively)	
symmetric (perfectly or approximately)	

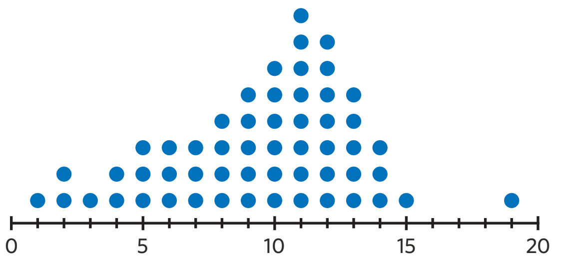
I.



II. **Key:** 5 | 1 = 51

5	1
6	2 6 9
7	4 5 7 8
8	3 7
9	0

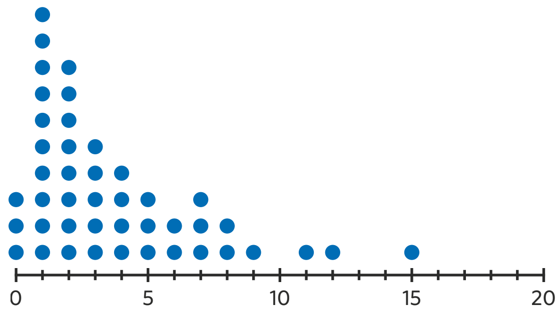
III.



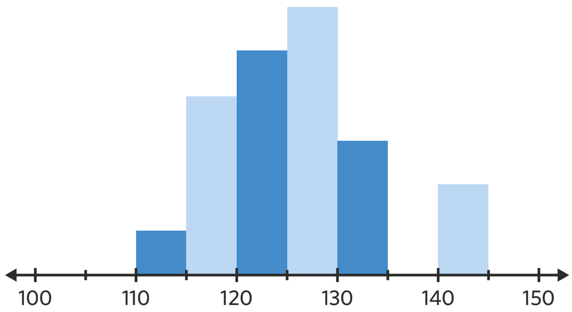
IV. **Key:** 8 | 3 = 83

8	3
8	
9	1
9	6 8
10	0 4
10	6 7 8
11	1 2 2 3 4
11	5 7

V.



VI.



Exam practice

12. The time taken to *travel* between two towns is approximately normally distributed with a mean of 16 minutes and a standard deviation of 4 minutes.

The percentage of *travel* times that are between 8 minutes and 20 minutes is closest to

A. 2.5% **B.** 34% **C.** 68%
D. 81.5% **E.** 95%

Adapted from VCAA 2019 Exam 1 Data analysis Q6

79% of students answered this type of question correctly.

---

13. In a large sample of second hand cars, the average number of kilometres on the odometer is approximately normally distributed with a mean of 210 000 km and a standard deviation of 15 000 km.

Using the 68–95–99.7% rule, determine the percentage of cars expected to have an odometer reading greater than 195 000 kilometres. (1 MARK)

Adapted from VCAA 2017 Exam 2 Data analysis Q1bi

69% of students answered this type of question correctly.

---

14. A sample of 250 bonobos has been drawn at random from a population of bonobos whose *neck size* is normally distributed with a mean of 38 cm and a standard deviation of 2.3 cm.

How many of these 250 bonobos are expected to have a *neck size* that is more than three standard deviations above or below the mean? Round to the nearest whole number. (1 MARK)

Adapted from VCAA 2020 Exam 2 Data analysis Q2bi

40% of students answered this type of question correctly.

Questions from multiple lessons

Data analysis

15. The weights of 22 cavoodles are shown in the following stem plot.

Key: 10 | 5 = 10.5 kg

weight

10	5 8 9
11	0 2 2 3
11	6 6 7 9 9
12	0 1 1 1 3 4
12	5 8 8
13	2

The modal *weight* is

A. 11.0 kg **B.** 11.6 kg **C.** 11.9 kg **D.** 12.1 kg **E.** 12.8 kg

Adapted from VCAA 2016 Exam 1 Data analysis Q3

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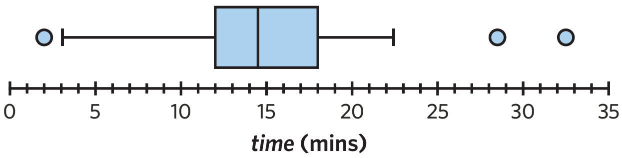
Data analysis

16. The *time*, in minutes, taken for students to get from school to the local fish and chips shop was recorded and is displayed in the following boxplot.



The five-number summary for the *time* taken to get to the fish and chips shop is

A. 2, 11, 14.5, 18, 22.5

B. 2, 12, 14.5, 18, 32.5

C. 3, 11, 14.5, 18, 22.5

D. 3, 12, 14.5, 18, 22.5

E. 3, 12, 15, 18, 22.5

Adapted from VCAA 2017 Exam 1 Data analysis Q2

Data analysis

17. Kevin wants to get feedback on the dinner he made for a group of 16 friends. He asked them all to specify whether they found the meal rancid, unpalatable, mediocre, appetising or divine.

Their responses are recorded as shown.

unpalatable mediocre unpalatable divine rancid rancid mediocre appetising

divine mediocre mediocre mediocre divine appetising divine divine

Use the data to:

a. Complete the following frequency table. (1 MARK)

rating	**frequency**
appetising	
divine	
mediocre	
rancid	
unpalatable	
total	

b. Determine the percentage of Kevin's friends who rated Kevin's dinner as 'mediocre'. (1 MARK)

Adapted from VCAA 2008 Exam 2 Data analysis Q1

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1H Comparing data distributions

STUDY DESIGN DOT POINT

- use of back-to-back stem plots or parallel boxplots, as appropriate, to compare the distributions of a single numerical variable across two or more groups in terms of centre (median) and spread (IQR and range), and the interpretation of any differences observed in the context of the data

1A 1B 1C 1D 1E 1F 1G **1H**

KEY SKILLS

During this lesson, you will be:

- comparing distributions using a back-to-back stem plot
- comparing distributions using parallel boxplots
- interpreting differences between distributions.

KEY TERMS

- Back-to-back stem plot
- Parallel boxplots

Visual displays such as back-to-back stem plots and parallel boxplots show the distribution of a numerical variable for two or more categories. These data displays can be compared in terms of centre and spread in order to identify differences between the distributions.

Comparing distributions using a back-to-back stem plot

A **back-to-back stem plot** is a visual representation that displays numerical data for two categories. The data for both categories share a stem, with the leaves for one category connecting to the left and the leaves for the other category connecting to the right.

Key: 3 | 8 = 38

7 6 5 2	3	8
9 7 7 6 4 1	4	3 6 9
9 7 4 3 0	5	2 5 8 8
8 5	6	
6	7	5 7
4	8	3 7 8 8 9
	9	0 4 8 9

Leaves are still ordered numerically, with the smallest number nearest to the stem

Back-to-back stem plots are used to compare the distributions of two categories. Specifically, the centre and spread can be compared.

To compare the centre of each category, the median is used. When comparing median values, the words ‘higher’ or ‘lower’ are used.

To compare the spread of each category, the interquartile range (IQR) and range are used. However, when the data contains outliers the range will not necessarily give an accurate representation of the data, so only the IQR is used. When comparing the IQR and range, the words ‘larger’ or ‘smaller’ are used.

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Worked example 1

There are two General Maths classes, A and B. The *exam mark* (%) of each student within the two classes was recorded.

Class A:	42	80	62	63	55	57	46	41	56	46	61	75	77
Class B:	76	81	59	99	60	80	97	44	99	98	80	95	78

a. Construct a back-to-back stem plot to display the *exam mark* (%) of each student within each class.

Explanation

Step 1: Consider the most appropriate scale.

The data for both classes ranges from 41 to 99.

An appropriate scale will have the 'tens' values as the stem and the 'ones' values as the leaves.

Step 2: Construct a stem by listing all the different 'tens' values.

4
5
6
7
8
9

Step 3: List the 'ones' values of class A scores to the left of the stem.

Order the leaves numerically from right to left.

6	6	2	1	4
	7	6	5	5
	3	2	1	6
		7	5	7
			0	8
				9

Step 4: List the 'ones' values of class B scores to the right of the stem.

Order the leaves numerically from left to right.

6	6	2	1	4	4				
	7	6	5	5	9				
	3	2	1	6	0				
		7	5	7	6	8			
			0	8	0	0	1		
				9	5	7	8	9	9

Step 5: Label each side of the back-to-back stem plot and construct a key.

Answer

Key: 2 | 7 = 27%

class A					class B				
6	6	2	1	4	4				
	7	6	5	5	9				
	3	2	1	6	0				
		7	5	7	6	8			
			0	8	0	0	1		
				9	5	7	8	9	9

Continues →

b. Compare the median *exam mark* of class A and B.

Explanation

Step 1: Determine the median for class A.

There are 13 values for class A.

$$\left(\frac{n+1}{2}\right) = \frac{13+1}{2} = 7$$

The median is the 7^{th} value: 57.

Step 2: Determine the median for class B.

There are 13 values for class B.

$$\left(\frac{n+1}{2}\right) = \frac{13+1}{2} = 7$$

The median is the 7^{th} value: 80.

Step 3: Compare the median value of the two classes.

Answer

The median for class A is 57% and the median for class B is 80%. Class B has a higher median *exam mark* than class A.

---

c. Compare the *exam mark* interquartile range (IQR) for class A and B.

Explanation

Step 1: Determine the first and third quartiles for class A and calculate the IQR.

$$Q_1 = 46$$
$$Q_3 = 69$$
$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 69 - 46 \\ &= 23 \end{aligned}$$

Step 2: Determine the first and third quartiles for class B and calculate the IQR.

$$Q_1 = 68$$
$$Q_3 = 97.5$$
$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 97.5 - 68 \\ &= 29.5 \end{aligned}$$

Step 3: Compare the IQR of the two classes.

Answer

The IQR for class A is 23% and the IQR for class B is 29.5%. Class B has a larger *exam mark* IQR than class A.

---

d. Compare the *exam mark* range for class A and B.

Explanation

Step 1: Calculate the range for class A.

$$\begin{aligned} range &= maximum - minimum \\ &= 80 - 41 \\ &= 39 \end{aligned}$$

Step 2: Calculate the range for class B.

$$\begin{aligned} range &= maximum - minimum \\ &= 99 - 44 \\ &= 55 \end{aligned}$$

Step 3: Compare the range of the two classes.

Answer

The range for class A is 39% and the range for class B is 55%. Class B has a larger *exam mark* range than class A.

Comparing distributions using parallel boxplots

Parallel boxplots are a sequence of boxplots that display numerical data for two or more categories. Each of the boxplots share the same axis, allowing the distributions of the categories to be compared directly.

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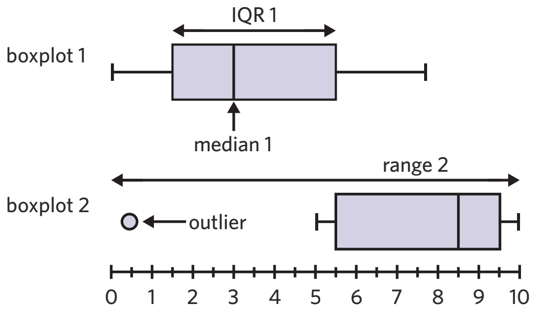
Parallel boxplots can be used to compare data in the same way as back-to-back stem plots. The centre can be compared using the medians, and the spread can be compared using the IQR and range.

Unlike back-to-back stem plots, there is no restriction on the number of categories that can be displayed.



Worked example 2

The *maximum daily temperature*, in degrees Celsius, for each day in May last year was recorded in Melbourne and Sydney.

The five-number summary for each city is:

Melbourne: 14.0, 17.0, 18.0, 19.0, 21.8

Sydney: 16.0, 19.0, 20.0, 22.5, 24.0

a. Construct parallel boxplots to display the *maximum daily temperature* (°C) of Melbourne and Sydney.

Explanation

Step 1: Consider the most appropriate scale.

The data for both cities ranges from 14 to 24.

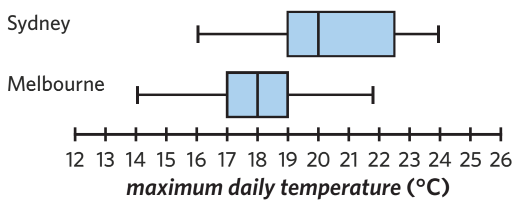
An appropriate scale could range from 12 to 26 with tick marks for each whole number.

Step 2: Construct the parallel boxplots.

Create one axis and use the five-number summary to construct a boxplot for each city, one above the other.

Remember to label each boxplot.

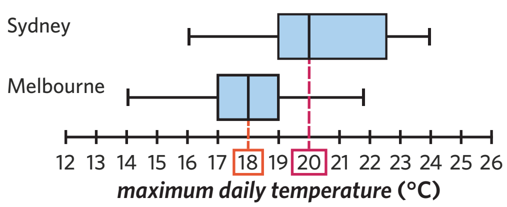
Answer



b. Compare the median *maximum daily temperature* of Melbourne and Sydney.

Explanation

Step 1: Identify the median for both Melbourne and Sydney.



The median for Melbourne is 18 °C.

The median for Sydney is 20 °C.

Step 2: Compare the median value of the two cities.

Continues →

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Answer

The median for Melbourne is 18 °C and the median for Sydney is 20 °C. Sydney has a higher median *maximum daily temperature* than Melbourne.

c. Compare the interquartile range (IQR) of *maximum daily temperature* for Melbourne and Sydney.

Explanation

Step 1: Identify Q_1 and Q_3 from the Melbourne boxplot and calculate the IQR.

$$\begin{aligned} Q_1 &= 17 \\ Q_3 &= 19 \\ IQR &= Q_3 - Q_1 \\ &= 19 - 17 \\ &= 2 \end{aligned}$$

Step 2: Identify Q_1 and Q_3 from the Sydney boxplot and calculate the IQR.

$$\begin{aligned} Q_1 &= 19 \\ Q_3 &= 22.5 \\ IQR &= Q_3 - Q_1 \\ &= 22.5 - 19 \\ &= 3.5 \end{aligned}$$

Step 3: Compare the IQR for the two cities.

Answer

The IQR for Melbourne is 2 °C and the IQR for Sydney is 3.5 °C. Sydney has a larger IQR of *maximum daily temperature* than Melbourne.

d. Compare the range of *maximum daily temperature* for Melbourne and Sydney.

Explanation

Step 1: Calculate the range for Melbourne.

$$\begin{aligned} range &= maximum - minimum \\ &= 21.8 - 14 \\ &= 7.8 \end{aligned}$$

Step 2: Calculate the range for Sydney.

$$\begin{aligned} range &= maximum - minimum \\ &= 24 - 16 \\ &= 8 \end{aligned}$$

Step 3: Compare the range for the two cities.

Answer

The range for Melbourne is 7.8 °C and the range for Sydney is 8 °C. Sydney has a larger range of *maximum daily temperature* than Melbourne.

Interpreting differences between distributions

To interpret the data in a back-to-back stem plot or a parallel boxplot, examine the visual displays for any differences that can be commented on.

The interpretation of any differences needs to be made in the context of the data. The type of data indicates what type of comparison will be made. Data on prices should be compared in terms of which category is cheaper or more expensive. Data on heights should be interpreted in terms of which category is shorter or taller.

The median can be compared and interpreted to determine which category is higher or lower. In addition to this, the IQR and range can be compared to determine which category has a larger spread. The data set with a larger spread is considered to be 'more variable'.

Worked example 3

Interpret the data.

a. Each *exam mark* (%) of students within two classes, A and B, is displayed in the following back-to-back stem plot.

Key: 2 | 7 = 27%

class A		class B
6 6 2 1	4	4
7 6 5	5	9
3 2 1	6	0
7 5	7	6 8
0	8	0 0 1
	9	5 7 8 9 9

Explanation

Step 1: Interpret any difference between the centres.

The median is used to compare the centre.

Class B has a higher median than class A.

In general, class B has higher exam marks than class A.

Step 2: Interpret any difference between the spreads.

The IQR and range are used to compare the spread.

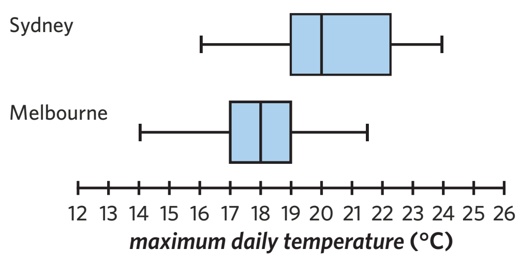
Class B has a larger IQR and range than class A.

Class B has a larger spread than class A, so its exam marks are more variable.

Answer

In general, class B has higher exam marks than class A. Class B exam marks are more variable than class A.

b. The *maximum daily temperature* (°C) for each day in May last year, in Melbourne and Sydney, is displayed in the following parallel boxplots.



Explanation

Step 1: Interpret any difference between the centres.

The median is used to compare the centre.

Sydney had a higher median than Melbourne.

In general, Sydney had higher maximum daily temperatures than Melbourne.

Step 2: Interpret any difference between the spreads.

The IQR and range are used to compare the spread.

Sydney had a larger IQR and range than Melbourne.

Sydney had a larger spread than Melbourne, so its maximum daily temperatures were more variable.

Answer

In general Sydney had higher maximum daily temperatures in May last year compared to Melbourne. Sydney's maximum daily temperatures were more variable than Melbourne's.

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1H Questions

Comparing distributions using a back-to-back stem plot

1. The *sales* figures, over one day, for employees at two small companies are recorded in the following back-to-back stem plot.

Key: 4 | 5 = $4500

company A		company B
5	0	5 7 8
2	1	2 7 8 9
9 1	2	2 5 5 6
9 5 5	3	1
6 6 5 3 1 1	4	
	5	8

Which of the following statements is true?

A. The median *sales* figure of company A is 39.

B. Company B has a higher median *sales* figure than company A.

C. Company A has a larger range of *sales* figures than company B.

D. Company B has a higher maximum *sales* figure than company A.

---

2. A number of men and women were asked how many *litres of milk* they normally drink in a year. The results are shown in the following back-to-back stem plot.

Key: 1 | 5 = 15 litres

women		men
9 9 6 4 1	0	1 4
8 7 4 3 2 1	1	6
2 2	2	8
	3	9
	4	2 2 4 5 6 7
1	5	1 2 5 6

a. What is the minimum value for women?

b. What is the maximum value for men?

c. Which group has the higher median?

d. Which group has a smaller range?

e. Which group has a larger IQR?

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3. Fernando and Lucas are having a competition to see who can score the most points in one season of basketball. The back-to-back stem plot displays the *number of points* that each player scored in each game of the season.

Key: 3 | 2 = 32 points

Fernando		Lucas
4 2	0	2 2
9	0	6 8 9
	1	2 2
8 8 7	1	6
4 4	2	
6	2	8

Compare the values of the following statistics for *number of points* for Fernando and Lucas.

a. Median

b. Range

c. IQR

4. The following data shows the *age* of people from the UK and the USA on a bus tour around Japan.

UK: 29 28 31 18 27 17 22 24 26 23 28 19 25 22 26

USA: 18 21 19 18 22 20 24 18 23 34 24 21 25 19 21

a. Construct a back-to-back stem plot from the data.

b. Compare the centre of the data for the UK and USA.

c. Compare the range of the data for the UK and USA.

Comparing distributions using parallel boxplots

5. The *final score* of two netball teams in every match of a twenty game season is recorded in the parallel boxplots.

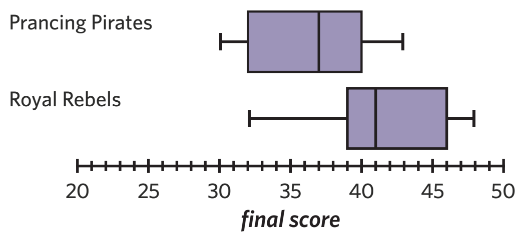
Which of the following statements is **not** true?

A. The median for the Royal Rebels is 41.

B. At least 75% of the Prancing Pirates' scores lie below the Royal Rebels' median.

C. The Royal Rebels scored more than 47 in 25% of their games.

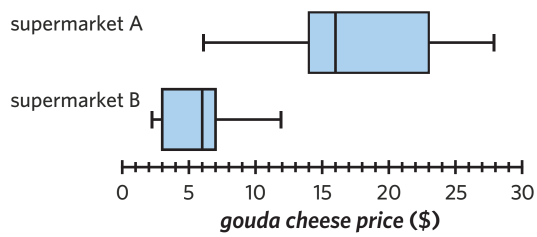
D. 50% of the Prancing Pirates' scores were 37 or less.



6. The parallel boxplots show the *gouda cheese price* ($) at two supermarkets, over a period of five years.

a. Which supermarket has the higher median *gouda cheese price*?

b. Which supermarket has the larger spread of *gouda cheese price*, as indicated by the IQR?

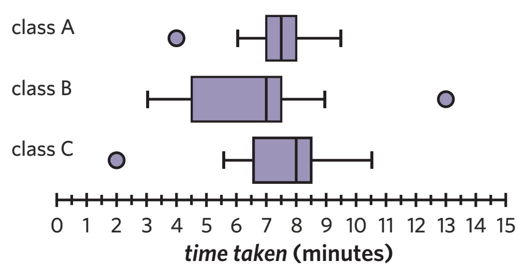


7. Three classes sat the same test on basic multiplication.The *time taken*, in minutes, to sit the test for students in each class is displayed in the following parallel boxplots.

a. Which class had the highest median *time taken*?

b. Which class had the smallest IQR of *time taken*?

c. Which class had the largest range of *time taken*?



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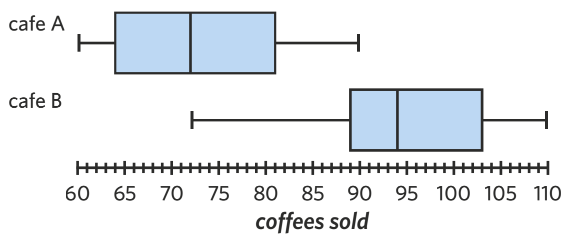
8. The number of *coffees sold* each day in two cafes, over a period of two weeks, is shown.

Compare the values of the following statistics for *coffees sold* for cafe A and B.

a. Median

b. Range

c. IQR



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9. A school records how many days students miss school due to illness over the course of a semester. The following five-number summaries represent the data for three year levels.

Year 10: 0, 4, 6, 9, 14

Year 11: 1, 4, 5, 6, 9

Year 12: 1, 3, 4, 6, 10

a. Construct parallel boxplots using the data provided.

b. Compare the centres.

c. Compare the spreads.

Interpreting differences between distributions

10. The following parallel boxplots display the daily *running distance*, in kilometres, of four friends over a two-week period.

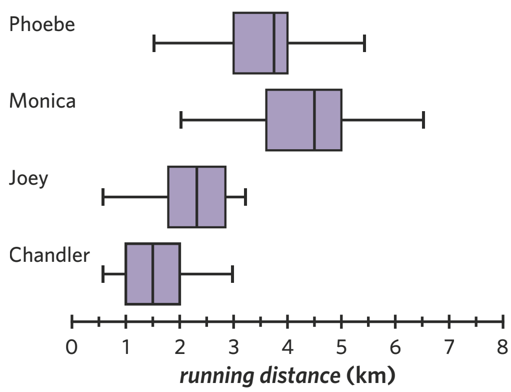
Which of the friends had the most variable running distances over the two weeks?

A. Phoebe

B. Monica

C. Joey

D. Chandler



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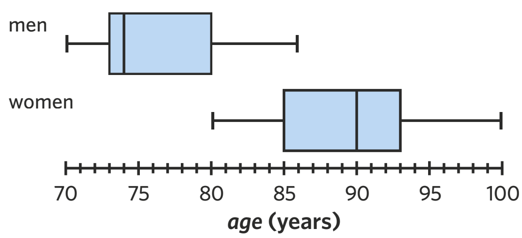
11. The following parallel boxplots display the *age* of residents in a retirement village.

Interpret this data by comparing centre and spread.



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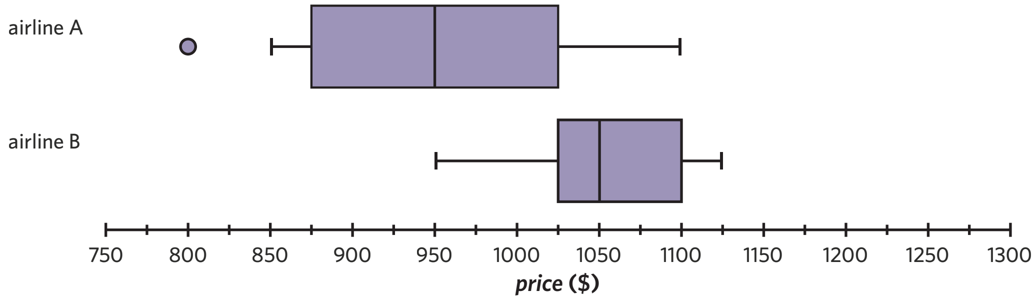
12. The following back-to-back stem plot shows the *hours studied* each week by two students during the twelve-week semester.

Key: 1 | 4 = 14 hours

student A						student B			
				3	0				
9	7	7	5	5	0	6	7		
4	2	1	0	0	1	0	0	4	4
					1	6	7	8	
					2	2	4		
				7	2	8			

Interpret this data by comparing centre and spread.

13. The following parallel boxplots display the *price* ($) of one way economy flight tickets from Melbourne to Los Angeles for two different airlines.



a. Interpret this data by comparing the centres and spreads.

b. Why should the variability be compared by using the IQR?

Joining it all together

14. The following back-to-back stem plot displays the *exam mark* (%) obtained by students in two geography classes.

Key: 3 | 8 = 38%

class A		class B
	4	5
9 5 3 2 1	5	8
5 4 4 3 1 1	6	1 4 7
7 6 4 1	7	1 1 3 4 7 8
5 1	8	2 2 3 4 7 9
9	9	1

For this data,

a. compare the medians.

b. compare the ranges.

c. compare the interquartile ranges.

d. determine which class had the more variable results.

e. interpret the difference between the *exam mark* medians.

15. The following data shows the number of *hours spent studying* per week for 45 children, across three year levels.

Year 10: 12 6 15 25 2 17 8 21 13 19 26 11 9 3 16

Year 11: 10 16 22 17 9 18 21 28 32 19 23 13 21 20 15

Year 12: 25 29 35 21 3 31 28 25 17 28 30 26 34 29 30

a. Construct parallel boxplots using the data.

b. Using the information from the boxplots, explain why *year level* is associated with the number of *hours spent studying* per week. Refer to the centre.

c. Using the information from the boxplots, explain why *year level* is associated with the variance in the number of *hours spent studying* per week. Refer to the spread (using IQR).

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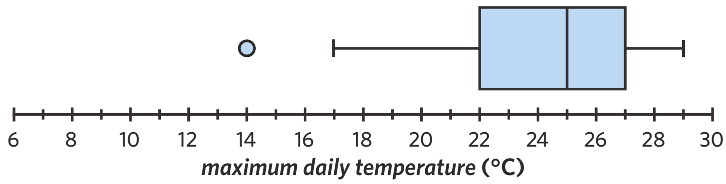
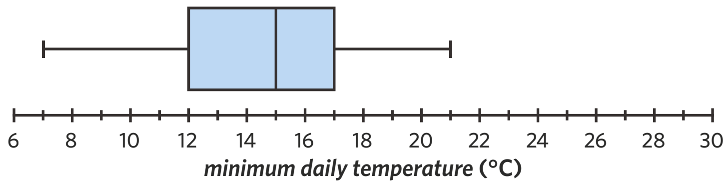
Exam practice

16. The following parallel boxplots show the *maximum daily temperature* and *minimum daily temperature*, in degrees Celsius, for 30 days in November 2017.





Data: Australian Government, Bureau of Meteorology, <www.bom.gov.au/>

Use the information in the boxplots to complete the following sentence.

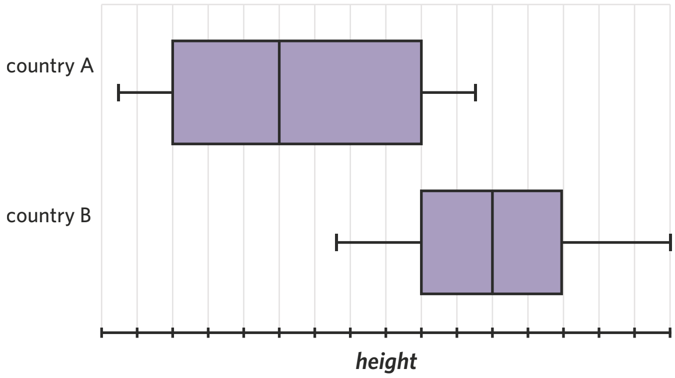
For November 2017, the median value for *maximum daily temperature* was __________ °C higher than the median value for *minimum daily temperature*. (1 MARK)

VCAA 2019 Exam 2 Data analysis Q2aii

87% of students answered this question correctly.

---

17. The following boxplots show the distribution of the *height* of babies born in two different countries, country A and country B.



Based on the boxplots shown, it can be said that

A. 50% of the babies born in country A are the same height as the babies born in country B.

B. 50% of the babies born in country B are taller than all of the babies born in country A.

C. 50% of the babies born in country B are shorter than all of the babies born in country A.

D. 75% of the babies born in country A are shorter than all of the babies born in country B.

E. 75% of the babies born in country B are taller than all of the babies born in country A.

Adapted from VCAA 2021 Exam 1 Data analysis Q4

60% of students answered this type of question correctly.

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Questions from multiple lessons

Data analysis

18. The following histogram shows the distribution of the *height* of Australian Olympians at the 2012 Olympics.

Using this histogram, the percentage of these 399 Australian Olympians that are 190 cm or taller is closest to

A. 17%

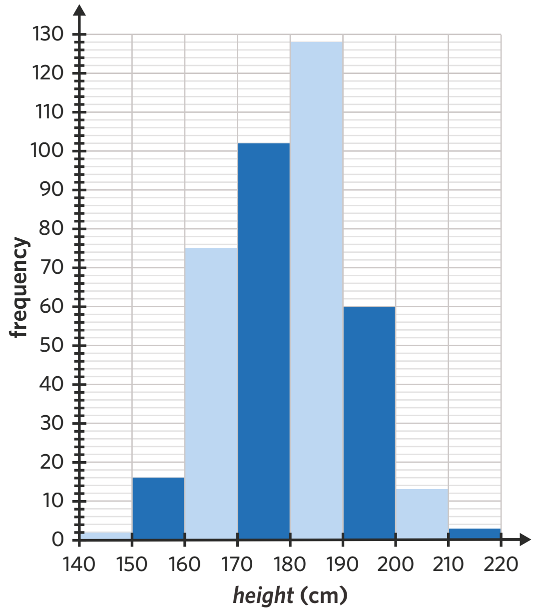
B. 19%

C. 76%

D. 81%

E. 83%

Adapted from VCAA 2016 Exam 1 Data analysis Q6



Recursion and financial modelling *Year 10 content*

19. Kourtney goes to her favourite Japanese restaurant, Nobu, and orders a meal for \$56. Kourtney then tips her waiter \$5. The \$5 tip as a percentage of the price of the meal is closest to

A. 0.09%

B. 0.89%

C. 1.12%

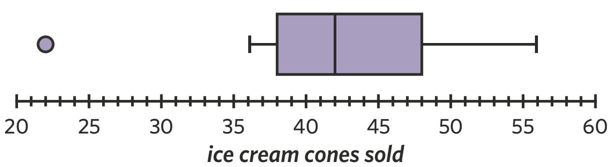
D. 8.93%

E. 11.20%

Adapted from VCAA 2014 Exam 1 Business-related mathematics Q2

Data analysis

20. The boxplot displays the number of *ice cream cones sold* each hour for a new flavour of ice cream, black sesame.



a. Describe the shape of the distribution of the number of *ice cream cones sold* (including any outliers). (1 MARK)

b. Determine the value of the lower fence. (1 MARK)

Adapted from VCAA 2016 Exam 2 Data analysis Q2b

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2

UNIT 1 AOS 2
CHAPTER 2

Financial arithmetic

LESSONS

KEY KNOWLEDGE

- percentage increase and decrease, mark-ups and discounts, and calculating GST in various financial contexts
- determining the impact of inflation on costs and the spending power of money over time
- the unitary method and its use in making comparisons and solving practical problems involving percentages and finance
- comparison of purchase options including cash, credit and debit cards, personal loans, buy now and pay later schemes.

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2A Rates and ratios

STUDY DESIGN DOT POINT

- prerequisite lesson

2A 2B 2C 2D 2E

KEY SKILLS

During this lesson, you will be:

- identifying and simplifying ratios
- identifying and simplifying rates
- performing calculations with rates and ratios.

KEY TERMS

- Ratio
- Rate

Rates and ratios and their use in everyday life are important to understand before exploring their applications to financial arithmetic. While they can be used in a financial context, they are broadly applicable across many aspects of everyday life, and are useful when expressing proportions and making comparisons of different quantities.

Identifying and simplifying ratios

A **ratio** is a set of numbers in the form $a : b$ (read as 'a to b') that expresses the relationship between two or more quantities or sizes. They are an effective way to state the proportions of quantities or amounts, expressed in the same units, without stating specific amounts.

See worked example 1

For example, a car park may contain 4 blue cars, 5 white cars and 2 green cars.

The ratio of blue cars to white cars to green cars is 4 : 5 : 2.

The ratio of white cars to total cars is 5 : 11.

The ratio of green cars to other cars is 2 : 9.

Note: The order of the stated ratio is important. The ratio 2 : 1 is not the same as the ratio 1 : 2.

Ratios can be simplified if all of the values within the ratio have a common factor other than 1.

See worked example 2

For example, the ratio 25 : 50 can be simplified to 1 : 2, and the ratio 9 : 3 : 21 can be simplified to 3 : 1 : 7.

Worked example 1

Bernard loves to eat fruit. His favourite fruits are bananas, strawberries, apples and mangoes. In one week, he calculates that he eats 6 bananas, 22 strawberries, 8 apples and 2 mangoes.

a. What is the ratio of bananas to strawberries to apples to mangoes?

Explanation

Step 1: Write down the number of fruits in the form bananas : strawberries : apples : mangoes.

6 : 22 : 8 : 2

Step 2: Divide all values in the ratio by the highest common factor.

The highest common factor of all four values is 2.

Answer

3 : 11 : 4 : 1

Continues →

b. What is the ratio of strawberries to all other fruits?

Explanation

Step 1: Write down the number of fruits in the form strawberries : bananas + apples + mangoes.

$22 : 6 + 8 + 2$

$22 : 16$

Step 2: Divide all values in the ratio by the highest common factor.

The highest common factor of the values is 2.

Answer

$11 : 8$

Worked example 2

Simplify the following ratios.

a. $2.5 : 10 : 17.5$

Explanation

Step 1: Convert each value to a whole number by multiplying each by the same common multiple.

This can be done by multiplying each of the values by 2.

$2.5 \times 2 = 5$

$10 \times 2 = 20$

$17.5 \times 2 = 35$

$5 : 20 : 35$

Step 2: Divide all values in the ratio by the highest common factor.

The highest common factor of all three values is 5.

Answer

$1 : 4 : 7$

b. $1\frac{1}{3} : 4 : 2 : 6\frac{2}{3}$

Explanation

Step 1: Convert each value to a whole number by multiplying each by the same common multiple.

This can be done by multiplying each of the values by 3.

$1\frac{1}{3} \times 3 = 4$

$4 \times 3 = 12$

$2 \times 3 = 6$

$6\frac{2}{3} \times 3 = 20$

$4 : 12 : 6 : 20$

Step 2: Divide all values in the ratio by the highest common factor.

The highest common factor of all four values is 2.

Answer

$2 : 6 : 3 : 10$

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Identifying and simplifying rates

A **rate** is a comparison of two quantities that are related to each other. Whereas ratios compare quantities of similar type (such as red cars and blue cars), rates are not limited in the same way and can compare any two quantities. The second quantity is often time, so that the rate expresses the rate of change per unit of time. Examples of this include kilometres per hour, metres per second, or orders delivered per day.

As with ratios, rates can be simplified. In the case of rates, they are usually simplified so that the first quantity is expressed per one unit of the second quantity. This means that a highest common factor doesn't need to be found to simplify a rate.

For example, a student wants to analyse how efficient they are at finishing a multiple choice exam. They work for an hour and complete 30 multiple choice questions. Depending on how they want to analyse their efficiency, the student may say that they average 0.5 questions per minute, or 2 minutes per question.

Worked example 3

In 2009, Usain Bolt set the world record 100 m sprint time of 9.58 seconds. On average, how fast did Bolt travel in m/s, correct to two decimal places?

Explanation

Step 1: Determine appropriate division to simplify the rate.

The question asks to express the rate in metres per second. The quantities need to be simplified so that 'metres' is expressed per one second.

Step 2: Divide the metres ran by the number of seconds taken.

$100 \div 9.58 = 10.438...$

Answer

10.44 m/s

Performing calculations with rates and ratios

Multiplication and division can be applied to both rates and ratios to further investigate real-world scenarios.

For example, a ratio of 3 : 5 : 2 : 8 could represent the allocation of tasks between a group of four people. Depending on the total number of tasks, division can be used to determine exactly how many tasks each person completes.

See worked example 4

Ratios can also indicate the scaling of one quantity to another. For example, a ratio of 10 : 1 could represent the scale of a building to a model of the building. Multiplication and division can be used to determine the size of each depending on which value is known.

See worked example 5

Multiplication and division can be used to convert rates into different units.

See worked example 6

For example a speed of 50 km/h is equivalent to 13.89 m/s.

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Worked example 4

Sally, Zhu, Armand and Paul all work at a sales company. Their boss works out that the number of sales they make per week can be expressed in the ratio $2 : 3 : 1 : 6$.

a. If 96 sales were made in one week, how many sales did each person make?

Explanation

Step 1: Determine the value of one unit in the ratio.

The number of units in the ratio sums to $2 + 3 + 1 + 6 = 12$.

As 96 sales were made, one unit in the ratio is equal to $96 \div 12 = 8$.

Step 2: Multiply the value per unit by the number of units in each part of the ratio.

The ratio represents the number of sales in the form Sally : Zhu : Armand : Paul.

Sally: $8 \times 2 = 16$

Zhu: $8 \times 3 = 24$

Armand: $8 \times 1 = 8$

Paul: $8 \times 6 = 48$

Answer

Sally: 16 sales

Zhu: 24 sales

Armand: 8 sales

Paul: 48 sales

b. If Zhu made 12 sales in one week, how many did Sally make?

Explanation

Step 1: Determine the ratio of sales made in the form Zhu : Sally.

Use the values provided in the original ratio and simplify if possible.

$3 : 2$ cannot be simplified any further.

Step 2: Determine the value of one unit in the ratio.

Armand made 12 sales, and is represented by 3 parts in the ratio.

This means one unit is equal to $12 \div 3 = 4$.

Step 3: Calculate the number of sales made by Sally.

Sally is represented by 2 parts in the ratio.

$2 \times 4 = 8$

Answer

8 sales

Worked example 5

A map has a 1 : 10 000 scale ratio. Answer the following questions using the most appropriate unit of measurement.

a. A length of 2.5 cm on the map corresponds to what distance in metres?

Explanation

Step 1: Interpret the scale.

According to the ratio, every cm on the map represents 10 000 cm.

This means a length of 2.5 cm on the map represents $2.5 \times 10\,000 = 25\,000$ cm.

Step 2: Convert this length to metres.

There are 100 cm in 1 m.

25 000 cm corresponds to:

$25\,000 \div 100 = 250$ m

Answer

250 m

b. A distance of 8 km corresponds to what length on the map in centimetres?

Explanation

Step 1: Interpret the scale.

According to the ratio, every length is 10 000 times smaller on the map.

This means a length of 8 km on the map represents $8 \div 10\,000 = 0.0008$ km.

Step 2: Convert this length to centimetres.

There are 1000 m in 1 km and 100 cm in 1 m.

0.0008 km corresponds to:

$0.0008 \times 1000 = 0.8$ m

$0.8 \times 100 = 80$ cm

Answer

80 cm

Worked example 6

Sergio buys a new plant and expects it to grow 1 metre per fortnight. Express the expected growth of Sergio's plant in centimetres per day, correct to two decimal places.

Explanation

Step 1: Convert the current rate to the units specified in the converted rate.

Convert 1 metre to cm: $1 \times 100 = 100$

Convert 1 fortnight to days: $1 \times 14 = 14$

The current rate is 100 cm per 14 days.

Step 2: Simplify to the rate specified in the question.

The rate is centimetres per day. Divide the number of centimetres by the number of days.

$100 \div 14 = 7.142...$

Answer

7.14 cm per day

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2A Questions

Note: There are no direct exam questions relevant to this lesson.

Identifying and simplifying ratios

1. The ratio 1 : 3 : 2 : 6 simplifies to

A. 1 : 1 : 2 : 2

B. 1 : 3 : 1 : 3

C. 2 : 6 : 4 : 12

D. Does not simplify

---

2. Determine the ratios for the following scenarios.

a. Pencils to pens in a shop that has 24 pencils and 40 pens.

b. Rooms to guests in a hotel with 180 rooms and 270 guests.

c. Hawaiian to margherita to meat lovers in a ‘favourite pizza list’ that has hawaiian 14 times, margherita 35 times and meat lovers 7 times.

d. Water consumed in the form A to B to C if person A consumed 4.5 L, person B consumed 2 L and person C consumed 1.5 L.

e. Ten-cent pieces to twenty-cent pieces to fifty-cent pieces to one-dollar pieces in a piggy bank that contains 28 one-dollar pieces, 51 twenty-cent pieces, 39 ten-cent pieces, and 9 fifty-cent pieces.

f. Money spent to money earned if $46.50 was spent and $102 was earned.

g. Sprint time in the form A to B to C to D if person A ran in 16.2 seconds, person B ran in 15.4 seconds, person C ran in 18.8 seconds and person D ran in 14.6 seconds.

h. Flour to brown sugar to white sugar to butter in a recipe that contains $1\frac{1}{4}$ cups of flour, $\frac{3}{4}$ cup of brown sugar, $\frac{1}{2}$ cup of white sugar and $\frac{1}{2}$ cup of butter.

---

3. Judy conducted a survey asking her friends what their favourite sport is. The results are summarised in the following table.

sport	frequency
netball	28
footy	52
tennis	39
soccer	21
other	24

Determine the ratio of:

a. netball to other

b. tennis to soccer

c. other to soccer and tennis

d. netball and footy to other

e. netball to footy to other

f. soccer and tennis to other to footy to netball

Identifying and simplifying rates

4. If a car travels a distance of 102 km in 2 hours, it's speed in km/h is

A. 0.85 **B.** 1.7 **C.** 51 **D.** 102

5. Determine the rates for the following scenarios. Where applicable, round answers to two decimal places.

a. Orders per day for an online store that receives 336 orders in a week (orders/day).

b. Metres per hour for a snail that travels 13 metres in 2.5 hours (m/h).

c. Millimetres per day for a town that receives 135 mm of rain in June (mm/day).

d. Hours per page for an author that writes 225 pages in 60 hours (h/page).

6. Anna and Georgia are knitting scarves for each other for Christmas. Anna works out that the length of the scarf she is knitting increases by an average of 2.8 cm per day. Over the month of November, the scarf Georgia is knitting increased from 13 cm to 85 cm long. Who is knitting the fastest?

Performing calculations with rates and ratios

7. After receiving his weekly income, Bert's spending to saving to investing ratio is 2 : 1 : 1. If he receives $400 in one week, how much does he spend?

A. $100 **B.** $200 **C.** $300 **D.** $600

8. Complete each of the following rate conversions, correct to two decimal places where applicable.

a. 125.4 litres per day to litres per week (L/week)

b. 546 sales per year to sales per month (sales/month)

c. 46 metres per second to kilometres per hour (km/h)

d. 1056 kilometres per year to metres per week (m/week)

9. Three jars contain only red and blue jelly beans. They are filled with the following ratios of red : blue.

If there are 120 jelly beans in each jar,

a. which jar has the most blue jelly beans?

b. how many red jelly beans are in each jar?

10. Samit is an elite runner, training to make the Olympics. He uses a map to plan out a run that he is going on. The map has a ratio of 1 : 150 000, meaning 1 cm on the map represents 150 000 cm.

a. If the distance of the run on the map is 15 cm long, how many kilometres does he plan on running?

b. Samit aims to average a speed of 18 km/h for his whole run. How long, in hours and minutes, will it take him to complete the run?

c. What is Samit's running pace in minutes/km, rounded to two decimal places?

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Joining it all together

11. Anita, Frank, Vincent and Meg all work in a warehouse packing boxes.
The hours that they work per week can be expressed in the ratio $4 : 3 : 7 : 9$.

a. If they collectively work 92 hours per week, determine the number of hours each person works for.

b. Vincent earns \$840 per week while Anita earns \$520 per week. Who has a greater hourly rate?

The following table shows the number of boxes packed in one week by each person.

person	*boxes*
Anita	92
Frank	75
Vincent	126
Meg	144

c. Order the workers from most efficient to least efficient according to the rate at which they pack boxes.

d. How many more boxes will Anita pack than Meg if they both work 144 hours?

---

12. Bronte is training to complete a triathlon. In one of her early training sessions she completes a 600 m swim, 16 km bike ride and 4 km run.

a. Determine the ratio of the distances in the form swim to bike to run.

b. The triathlon that Bronte is training for has distances in the same ratio as her training session. If the triathlon is 51.5 km in total, how far, in km, is each component?

c. Bronte aims to complete each component of the final triathlon in the following times:

- Swim in 50 minutes
- Bike in 2 hours
- Run in 1 hour

Calculate the speed, in km/h, that Bronte is aiming to travel for each component.

d. Convert the speeds in part **c** to minutes per km (minutes/km), to two decimal places.

Questions from multiple lessons

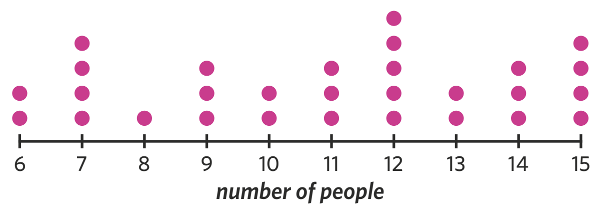
Computation and practical arithmetic *Year 10 content*

13. At a berry picking farm, the price of strawberries is determined by rounding the weight of the fruit to the nearest ten grams, and multiplying by the price per gram. If Dave picks 416 grams worth of strawberries, and the price of strawberries is \$0.011 per gram, how much does Dave pay?

A. \$4.16 **B.** \$4.51 **C.** \$4.57 **D.** \$4.58 **E.** \$4.62

Data analysis *Year 10 content*

14. The following dot plot shows the number of people in 29 different running groups.



The median number of people is

A. 10.5 **B.** 11 **C.** 11.5 **D.** 12 **E.** 12.5

Adapted from VCAA 2019NH Exam 1 Data analysis Q4

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Data analysis

15. The following back-to-back stemplot displays the distribution of the *height* (cm) of 21 jockeys and 21 hockey players, separated by the *type of athlete* they are.

Key: 17 | 4 = 174 cm

height **(cm)**

jockeys		**hockey players**
7	13	
9 9 8 8 8 8 6 5 5 4 2	14	
5 4 4 3 2 1	15	
5 4 2	16	
	17	4 5 5 7 7 7 8 9 9 9
	18	0 1 2 3 3 4 5 7 9
	19	1 3

a. Which variable, *height* or *type of athlete*, is a categorical variable? (1 MARK)

b. Write down the modal *height*(s), in cm, of the hockey players. (1 MARK)

c. Use the information in the back-to-back stem plot to find the values of x and y in the following table. (2 MARKS)

		height **(cm)**				
		minimum	Q_1	median	Q_3	maximum
type of athlete	jockeys	137	x	149	154	165
	hockey players	174	177	180	184.5	y

Adapted from VCAA 2017 Exam 2 Data analysis Q2a-c

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2B Percentages

STUDY DESIGN DOT POINT

- percentage increase and decrease, mark-ups and discounts, and calculating GST in various financial contexts

2A 2B 2C 2D 2E

KEY SKILLS

During this lesson, you will be:

- calculating percentages of numbers
- calculating percentage increase and decrease
- applying percentages to GST calculations.

KEY TERMS

- Percentage
- Mark-up
- Discount
- GST

Percentages are a way of presenting a wide range of different ideas under one scale, 'out of one hundred'. They allow for comparisons and calculations to be made in a variety of application contexts, such as calculating discounts or mark-ups, or calculations involving GST.

Calculating percentages of numbers

Percentage literally translates to 'out of one hundred' and is a standard measure used around the world to compare proportions and perform calculations with them.

For example, any fraction can be converted to a percentage so that it can be compared to other percentages, because they all use the same scale.

See worked example 1

To convert a fraction (or its decimal equivalent) to a percentage, multiply the fraction or decimal by 100.

$percentage\ (\%) = fraction\ or\ decimal \times 100$

This concept can also be used in reverse to calculate the percentage of any value. This is calculated by first converting the percentage to a fraction or decimal, and then multiplying by the value in question.

See worked example 2

$percentage\ of\ a\ value = (percentage \div 100) \times value$

Worked example 1

Express the following proportions as percentages to one decimal place where necessary.

a. 492 out of 1247

Explanation

Step 1: Determine the proportion to be converted.

The proportion must be a fraction or decimal.

Written as a fraction, this proportion is $\frac{492}{1247}$.

Step 2: Convert the proportion to a percentage.

To convert the fraction to a percentage, multiply by 100.

$\frac{492}{1247} \times 100 = 39.454...$

Answer

39.5%

Continues →

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b. 0.31

Explanation

Convert the proportion to a percentage.

To convert the decimal to a percentage, multiply by 100.

$0.31 \times 100 = 31$

Answer

31%

Worked example 2

For the following, calculate:

a. 43.2% of 945.

Explanation

Calculate the percentage of the value.

$$\frac{43.2}{100} \times 945 = 0.432 \times 945$$
$$= 408.24$$

Answer

408.24

b. 157% of 1592.

Explanation

Calculate the percentage of the value.

$$\frac{157}{100} \times 1592 = 1.57 \times 1592$$
$$= 2499.44$$

Answer

2499.44

Calculating percentage increase and decrease

See worked example 3

Percentages also allow for calculations to be performed to increase or decrease a value.

A percentage increase can be calculated using the following formula:

$$\left(1 + \frac{\textit{percentage increase}}{100}\right) \times \textit{original value}$$

When applied to financial mathematics, particularly the sale of a good or service, a percentage increase is commonly referred to as a **mark-up**.

A percentage decrease can be calculated using the following formula:

$$\left(1 - \frac{\textit{percentage decrease}}{100}\right) \times \textit{original value}$$

When applied to financial mathematics, particularly the sale of a good or service, a percentage decrease is commonly referred to as a **discount**.

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See worked example 4

It is also possible to calculate the percentage change of a value if the original value and change in value are known.

To calculate a percentage change, divide the change in value by the original value and multiply by 100.

$\frac{\textit{change in value}}{\textit{original value}} \times 100$, where

$\textit{change in value} = \textit{new value} - \textit{original value}$

Worked example 3

Calculate the following percentage changes.

a. Increase 394 by 12%

Explanation

Step 1: Determine the percentage increase and the original value.

$\textit{percentage increase} = 12$

$\textit{original value} = 394$

Step 2: Substitute these values into the percentage increase formula to calculate the value after the percentage increase.

$$\left(1 + \frac{12}{100}\right) \times 394 = 1.12 \times 394$$
$$= 441.28$$

Answer

441.28

b. Decrease 3457 by 56%

Explanation

Step 1: Determine the percentage decrease and the original value.

$\textit{percentage decrease} = 56$

$\textit{original value} = 3457$

Step 2: Substitute these values into the percentage decrease formula to calculate the value after the percentage decrease.

$$\left(1 - \frac{56}{100}\right) \times 3457 = 0.44 \times 3457$$
$$= 1521.08$$

Answer

1521.08

Worked example 4

An item on sale drops from \$799 to \$647. Calculate the percentage discount, rounded to the nearest percent.

Explanation

Step 1: Calculate the change in value.

$647 - 799 = -152$

Step 2: Calculate the percentage change.

$\textit{change in value} = -152$

$\textit{original value} = 799$

$\frac{\textit{change in value}}{\textit{original value}} \times 100$

$\frac{-152}{799} \times 100 = -19.023...$

The negative indicates that the change is a decrease, or in this case a discount.

Answer

19%

Applying percentages to GST calculations

GST (Goods and Services Tax) is a tax that is applied to most items that people buy or services that people pay for. Currently in Australia, GST is set at 10% of an item's original sale value.

If GST on a good or service has yet to be applied, the price after GST can be calculated by performing a 10% increase calculation on the price before GST. This can be achieved by multiplying the price before GST by 1.1.

See worked example 5

In a similar way, if the cost of a good or service already includes GST, the price before GST can be calculated by dividing the price after GST by 1.1.

$$\begin{tikzcd} \textbf{price before GST} \arrow[r, bend left, "\times 1.1"] & \textbf{price after GST} \arrow[l, bend left, "\div 1.1"] \end{tikzcd}$$

The amount of GST to be added to a good or service can also be determined by calculating 10% of the price before GST.

See worked example 6

$GST\ amount = 10\% \times price\ before\ GST$

Worked example 5

Consider the following scenarios.

a. An item is sold at a price of $42 before GST is applied. Calculate the cost of the item after GST is added.

Explanation

Increase the value of the item by 10%.

$42 \times 1.1 = 46.2$

Answer

$46.20

b. A service is sold at a price of $199, including GST. Determine the cost of the service before GST had been applied, rounded to the nearest cent.

Explanation

Reverse the 10% increase on the value of the service.

$199 \div 1.1 = 180.909...$

Answer

$180.91

Worked example 6

Calculate the amount of GST to be added to an item priced at $427 before GST.

Explanation

Calculate 10% of the price of the item before GST.

$$\begin{aligned}\frac{10}{100} \times 427 &= 0.1 \times 427 \\ &= 42.7\end{aligned}$$

Answer

$42.70

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2B Questions

Calculating percentages of numbers

1. If a block of chocolate has 30 pieces, how many pieces are there in 60% of the chocolate?

   A. 6 **B.** 14 **C.** 18 **D.** 20

2. Express the following proportions as percentages, rounded to two decimal places where necessary.

   a. 4 out of 5 **b.** 0.32 **c.** $\frac{1}{15}$

3. Complete the following calculations.

   a. 35% of 260 **b.** 2% of 15 **c.** 99% of 103

4. Sam has completed six elevenths of her homework. Charlie has completed four sevenths of his homework.

   a. Convert both of these fractions to percentages, rounded to two decimal places.

   b. Who has completed the most homework?

5. Cormac has read 12% of a book with 425 pages. How many pages has Cormac read so far?

6. Out of a 200 gram wheel of cheese, there are two pieces left. One is 24% of the original size, and the other is 11% of the original size. How much cheese, in grams, is left in total?

Calculating percentage increase and decrease

7. What is the percentage change when 20 increases to 32?

   A. 12% increase **B.** 37.5% increase **C.** 60% increase **D.** 62.5% decrease

8. Calculate the new value for each of the following. Round answers to two decimal places where necessary.

   a. 721 increased by 42%

   b. 1415 decreased by 90%

   c. 459 increased by 45.3%

   d. 59 191 decreased by 27.25%

9. Last year at Village Cinemas, the price of a movie ticket for a student was $15. This year the price went up 13.33%. What is the price of a student ticket this year? Round to the nearest cent.

10. Calculate the percentage increase or decrease for the following. Round answers to two decimal places where necessary.

    a. The price of avocados was $8 per kilogram last month but is $12 per kilogram this month.

    b. Monday's maximum temperature was 23 degrees Celsius and Tuesday's maximum temperature was 16 degrees Celsius.

    c. Bianca's height last year was 1.53 m and this year is 1.60 m.

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11. In Round 1, Ken's cricket team scored 124 runs. In Round 2, they scored 36% less than in Round 1. In Round 3, they scored 22% more than in Round 2. Calculate their score in Round 3. At each step round the score to the nearest run.

12. For each of the following, calculate the new price of the item.
    - **a.** A t-shirt bought with an original price of $12 is marked up by 20%.
    - **b.** A giant chocolate bar is usually $25, but it currently has a discount of 45%.
    - **c.** String lights with an original price of $12.50 are marked up 98%.

Applying percentages to GST calculations

13. Fill in the box.

    GST is a ☐ % tax added to goods and services in Australia.

14. Consider the cost of the following goods and services before GST is added. Calculate each cost after GST is applied.
    - **a.** Shoes: $180
    - **b.** Landscape quote: $350
    - **c.** Blu-ray movie: $14.50

15. Consider the cost of the following goods and services after GST has been added. Calculate each cost before GST has been applied. Round answers to the nearest cent.
    - **a.** Bag: $199
    - **b.** PS5 game: $119.95
    - **c.** Financial advisor quote: $249

16. Calculate the amount of GST to be added to the following items. Provide all answers to the nearest cent.
    - **a.** A hat that sells for $45 before GST.
    - **b.** A bicycle that sells for $175 before GST.
    - **c.** An album purchased online for $16.99 (not including GST).

Joining it all together

17. A car is priced at $45 399 after GST. Due to supply shortages and high demand, the car gets a 17% markup.
    - **a.** Calculate the new price of the car, to the nearest cent.
    - **b.** Calculate the amount of GST that has been added to the new cost of the car, rounded to the nearest cent.

18. A vinyl record is purchased for $29.99 after GST has been applied. Over the years since its original purchase, the record has become quite rare and can now be sold online for $147 before GST has been applied.

    What is the percentage markup on the pre-GST price of the record, rounded to the nearest whole percent?

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Exam practice

19. A theme park has four locations, Air World, Food World, Ground World and Water World.

The number of visitors at each of the four locations is counted every hour. By 10 am on Saturday the park had reached its capacity of 2000 visitors and could take no more visitors. The park stayed at capacity until the end of the day.

The following table shows the number of visitors at each location at 10 am on Saturday.

location	*number of visitors*
Air World	600
Food World	600
Ground World	400
Water World	400

What percentage of the park's visitors were at Water World at 10 am on Saturday? (1 MARK)

Adapted from VCAA 2019 Exam 2 Matrices Q2a

91% of students answered this type of question correctly.

20. The number of eggs counted in a sample of 12 clusters of moth eggs is recorded in the following table.

number of eggs	172	192	159	125	197	135	140	140	138	166	136	131

From the information given, determine the percentage of clusters in this sample that contain more than 170 eggs. (1 MARK)

VCAA 2017 Exam 2 Data analysis Q1aii

87% of students answered this question correctly.

21. The following ordered stem plot shows the maximum temperature, in degrees Celsius, for 15 days.

Key: 9 | 2 = 9.2 $n = 15$

maximum temperature **(°C)**

9	2
10	
11	5 6
12	2 5
13	5 5 7
14	9 9
15	0 2 5 6
16	0

Data: Australian Government, Bureau of Meteorology, <www.bom.gov.au>

Use this stem plot to determine the percentage of days with a maximum temperature higher than 15.3°C. (1 MARK)

VCAA 2019 Exam 2 Data analysis Q1cii

85% of students answered this question correctly.

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Questions from multiple lessons

Data analysis *Year 10 content*

22. The following stem plot displays the number of *hours driven* for twenty Year 12 students on their L's.

Key: 2 | 8 = 28 hours

2	8
3	6 6
4	1 4 4 7
5	8 9
6	5 6 6 6 6 9
7	8
8	8 8
9	1
10	6

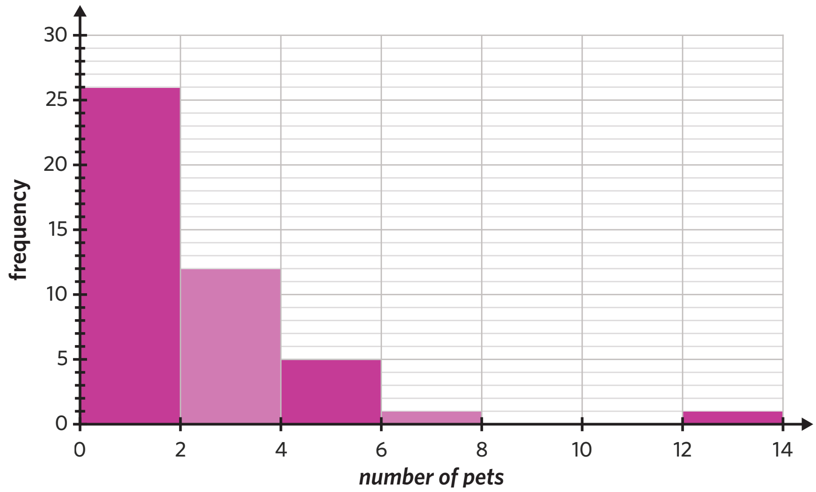
The modal number of *hours driven* is

A. 36 **B.** 44 **C.** 65.5 **D.** 66 **E.** 88

Adapted from VCAA 2016 Exam 1 Data analysis Q3

Data analysis *Year 10 content*

23. The following histogram shows the distribution of the *number of pets* in different households.



Using this histogram, the percentage of households with less than two pets is closest to

A. 26% **B.** 42% **C.** 44% **D.** 57% **E.** 58%

Adapted from VCAA 2016 Exam 1 Data analysis Q6

Computation and practical arithmetic *Year 10 content*

24. A hydrotherapy session costs $800.

Concession card holders are offered a discount of $120 off the full price.

a. Write down the discount for concession card holders as a percentage of the full price. (1 MARK)

Those that do not have concession cards also have to pay an extra $60 per hour that they spend in the hydrotherapy pool after their session.

b. John does not have a concession card and spends an extra five hours in the pool. What is the total amount he has to pay? (1 MARK)

Adapted from VCAA 2015 Exam 1 Business-related mathematics Q1a,b

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2C Inflation

STUDY DESIGN DOT POINT

- determining the impact of inflation on costs and the spending power of money over time

2A 2B 2C 2D 2E

KEY SKILLS

During this lesson, you will be:

- calculating price changes over time
- calculating changes in spending power over time.

KEY TERMS

- Inflation
- Inflation rate
- Spending power

Over time, the cost of living generally increases each year due to the price of everyday goods and services increasing slightly. Inflation rates can be used to find the change in prices of goods and services over time, and to calculate the amount of spending power today's money will have in the future.

Calculating price changes over time

Inflation is a general increase in the prices of goods and services over time. The **inflation rate** is commonly given as an annual figure, which shows the increase in price as a percentage of the previous year's price.

The annual inflation rate can be found using the formula

See worked example 1

$$\textit{inflation rate } (\%) = \frac{\textit{new price} - \textit{original price}}{\textit{original price}} \times 100$$

This formula can be transposed in order to find the new price after one year of inflation.

See worked example 2

$$\textit{new price} = \textit{original price} \times \left(1 + \frac{\textit{inflation rate } (\%)}{100}\right)$$

This can be expressed by the following compounding formula, which also allows the new price to be calculated after several years:

$$A = P \times \left(1 + \frac{r}{100}\right)^n$$

where

- A is the price after inflation
- P is the price before inflation
- r is the annual inflation rate (%)
- n is the number of years

If the formula is used to calculate the new price after one year, $n = 1$.

If the formula is used to calculate the new price after multiple years, r is the average annual inflation rate (%).

Worked example 1

The cost of an item, impacted only by inflation, increases from $474 in one year to $483 in the next year. Calculate that year's inflation rate, rounded to two decimal places.

Explanation

Step 1: Identify the original and new prices of the item.

$$\textit{original price} = 474$$

$$\textit{new price} = 483$$

Step 2: Substitute the values into the inflation rate formula and evaluate.

$$\begin{aligned}\textit{inflation rate } (\%) &= \frac{\textit{new price} - \textit{original price}}{\textit{original price}} \times 100\\ &= \frac{483 - 474}{474} \times 100\\ &= 1.898...\end{aligned}$$

Answer

1.90%

Worked example 2

At the end of 1990, a pizza cost $12. Assume that the price of a pizza increases according to inflation.

a. Inflation occurred at a rate of 3.1% in 1991. What was the price of a pizza at the end of 1991, correct to the nearest cent?

Explanation

Step 1: Identify the original price and the inflation rate.

The original price of a pizza was $12.

The inflation rate in 1991 was 3.1%.

Step 2: Substitute the values into the formula and evaluate.

$$\begin{aligned}\textit{new price} &= \textit{original price} \times \left(1 + \frac{\textit{inflation rate } (\%)}{100}\right)\\ &= 12 \times \left(1 + \frac{3.1}{100}\right)\\ &= 12 \times 1.031\\ &= 12.372\end{aligned}$$

Answer

$12.37

b. In 1992, the annual inflation rate was 3.01%. In 1993, the annual inflation rate was 1.67%. In 1994, the annual inflation rate was 2.56%. Calculate the price of a pizza at the end of 1994, correct to the nearest cent.

Explanation

Step 1: Calculate the price of a pizza at the end of 1992.

From part **a**, a pizza cost $12.372 at the end of 1991.

$$\begin{aligned}\textit{new price} &= \textit{original price} \times \left(1 + \frac{\textit{inflation rate } (\%)}{100}\right)\\ &= 12.372 \times \left(1 + \frac{3.01}{100}\right)\\ &= 12.7443...\end{aligned}$$

Step 2: Calculate the price of a pizza at the end of 1993.

$$\begin{aligned}\textit{new price} &= 12.7443... \times \left(1 + \frac{1.67}{100}\right)\\ &= 12.9572...\end{aligned}$$

Step 3: Calculate the price of a pizza at the end of 1994.

$$\begin{aligned}\textit{new price} &= 12.9572... \times \left(1 + \frac{2.56}{100}\right)\\ &= 13.2889...\end{aligned}$$

Answer

$13.29

Continues →

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c. Between 1994 and 2020, the average annual inflation rate was 2.6%. Calculate the price of a pizza at the end of 2020, correct to the nearest cent.

Explanation

Step 1: Identify P, r and n.

The price of a pizza in 1994 was $13.29.

$P = 13.29$

The average annual inflation rate was 2.6%.

$r = 2.6$

The number of years between 1994 and 2020 is 26.

$n = 26$

Step 2: Substitute the values into the compounding formula and evaluate.

$$A = P \times \left(1 + \frac{r}{100}\right)^n$$

$$= 13.29 \times \left(1 + \frac{2.6}{100}\right)^{26}$$

$$= 13.29 \times 1.026^{26}$$

$$= 25.9033...$$

Answer

$25.90

Calculating changes in spending power over time

The **spending power** of money is the amount that can be purchased with a unit of currency. As inflation continues over time, the spending power of money reduces. For example, $1 in 1900 could buy much more than what $1 in 2000 could. This is because when the general price of goods and services increases, each unit of currency is able to purchase less goods or services.

To determine the future spending power of an amount of money, the original amount of money is 'deflated', as spending power decreases according to the rate of inflation. The future spending power in one year can be calculated using the formula

$$\textit{future spending power} = \textit{original value} \div \left(1 + \frac{\textit{inflation rate } (\%)}{100}\right)$$

This can also be expressed by rearranging the compounding formula as shown, which also allows the future spending power to be calculated after several years:

$$A = P \times \left(1 + \frac{r}{100}\right)^n \rightarrow P = A \div \left(1 + \frac{r}{100}\right)^n$$

where

- A is the original value
- P is the future spending power
- r is the annual inflation rate (%)
- n is the number of years

If the formula is used to calculate the future spending power after one year, $n = 1$.

If the formula is used to calculate the future spending power after multiple years, r is the average annual inflation rate (%).

The 'solve' function on a CAS calculator can be used to calculate the inflation rate when A, P, and n are known.

Worked example 3

Troy has a piggy bank containing $250.

a. If the inflation rate over the coming year is predicted to be 6.5%, what will be the spending power of Troy's piggy bank if he opens it next year? Round to the nearest cent.

Explanation

Step 1: Identify the original value and the inflation rate.

The original value of the piggy bank is $250.

The inflation rate in the coming year is 6.5%.

Continues →

Step 2: Substitute the values into the formula and evaluate.

$$\textit{future spending power} = \textit{original value} \div \left(1 + \frac{\textit{inflation rate}\ (\%)}{100}\right)$$
$$= 250 \div \left(1 + \frac{6.5}{100}\right)$$
$$= 250 \div 1.065$$
$$= 234.7417...$$

Answer

\$234.74

---

b. If the average annual inflation rate over the next 20 years from now is predicted to be 3.3%, what will be the spending power of Troy's piggy bank if he opens it in 20 years' time? Round to the nearest cent.

Explanation

Step 1: Identify A, r and n.

The original value of the piggy bank is \$250.

$A = 250$

The average inflation rate is 3.3%.

$r = 3.3$

The number of years is 20.

$n = 20$

Step 2: Substitute the values into the compound formula and evaluate.

$$P = A \div \left(1 + \frac{r}{100}\right)^n$$
$$= 250 \div \left(1 + \frac{3.3}{100}\right)^{20}$$
$$= 250 \div 1.033^{20}$$
$$= 130.5971...$$

Answer

\$130.60

2C Questions

Note: There are no direct exam questions relevant to this lesson.

Calculating price changes over time

1. Consider that the price of petrol is currently \$1.86 per litre and its price increases according to inflation. If the annual inflation rate is 4.5%, the price per litre of petrol a year from now will be closest to

   A. \$1.78 **B.** \$1.94 **C.** \$2.03 **D.** \$2.31

---

2. In 2023, Sophie has a HECS debt of \$29 445. The balance of a HECS debt increases annually according to inflation.
   - **a.** If the rate of inflation from 2023 to 2024 is 3.5%, how much HECS debt will Sophie have in 2024, correct to the nearest dollar?
   - **b.** If the rate of inflation from 2024 to 2025 is 3.0%, how much HECS debt will Sophie have in 2025, correct to the nearest dollar?
   - **c.** If the rate of inflation for the next 5 years from 2025 has an average of 2.7%, how much HECS debt will Sophie have in 2030, correct to the nearest dollar?
   - **d.** If this annual inflation rate of 2.7% continues, in what year will Sophie's HECS debt increase to over \$40 000?

3. At the start of 2021, a head of lettuce cost $3.20. At the start of 2022, a head of lettuce cost $11.00.
   a. If this price change was due only to inflation, what would the annual inflation rate for 2021 be?
   b. If the price of lettuce continued to increase at this rate of inflation, how much would a head of lettuce cost at the start of 2023, correct to the nearest cent?

4. Calculate the price after inflation for each of the following.
   a. At the start of 2016, a banh mi cost $5.50. Since then, the price of a banh mi has increased annually in accordance with inflation. If the average inflation rate since then was 2.8%, what is the price of a banh mi at the start of 2023, correct to the nearest cent?
   b. In 1998, Oliver bought a house for $145 000 in Fitzroy. If his house's value only increased according to inflation, and the average inflation rate between 1998 and 2018 was 2.6%, calculate the price of Oliver's house in 2018 correct to the nearest dollar.

5. At the start of 2005, a carton of eggs in the USA cost $2.00.
   Shown in the following table are the annual inflation rates in the USA between 2005 and 2014.
   a. Calculate the average annual rate of inflation over the 10 years.
   b. Using the average value from part **a**, calculate the predicted price of a carton of eggs at the start of 2015, correct to the nearest cent.

year	*inflation rate* (%)
2005	3.4
2006	3.2
2007	2.9
2008	3.8
2009	−0.4
2010	1.6
2011	3.2
2012	2.1
2013	1.5
2014	1.6

Calculating changes in spending power over time

6. Patrice has $1000 in his safe. If the inflation rate over the next year is predicted to be 7.7%, the spending power of the money of his safe in one year will be closest to
   A. $923 **B.** $929 **C.** $992 **D.** $1077

7. Freidrich likes to hide his money around his house because he doesn't trust banks. He recently hid a stash containing $1200 in cash but can't remember where he hid it.
   Give answers correct to the nearest cent.
   a. If he finds his cash stash in one year, what is the spending power of his money predicted to be if the inflation rate for the coming year is predicted to be 1.9%?
   b. If he finds his cash stash in 15 years, what is the spending power of his money predicted to be if inflation averages at 2.1% annually?

8. Calculate the future spending power of each of the following, correct to the nearest cent.
   a. Georgie has $3000 in her savings account. What will be the spending power of Georgie's savings next year if the inflation rate in the coming year is 7.2%?
   b. Winston inherited $8000 from his grandparents. What will be the spending power of his inheritance in 10 years if the average inflation rate for the next 10 years is 3.2%?

9. 13 years ago, Heloise lost a wallet containing $145 in cash. Heloise has been told that this money would have a spending power of $100 in today's currency.
   What was the average inflation rate over this period, correct to three decimal places?

10. At the end of 2022, Liesel received an hourly wage of $27.50 at her part-time job and had been receiving this same wage since the end of 2018. Shown in the following table are the annual inflation rates for the years 2019 to 2022.

    Use the inflation rates from each year to calculate the spending power of Liesel's hourly wage at the end of 2022 in comparison to her same wage at the end of 2018, correct to the nearest cent.

year	*inflation rate* (%)
2019	1.7
2020	0.7
2021	3.0
2022	7.3

Joining it all together

11. When Fernando was a child in 1995, he received $10 of pocket money each week from his parents. In 2022, Fernando also gave his daughter Benny $10 of pocket money each week. Between 1995 and 2022, the average annual rate of inflation was 2.45%.
    - **a.** What is the spending power of Benny's pocket money in comparison to Fernando's, to the nearest cent?
    - **b.** If Fernando were to give Benny pocket money with the same spending power as he had, how much would he give her per week, to the nearest cent?

12. At the start of each year, Amelie receives an increase to her salary to match inflation from the previous year. Amelie's salary at the start of 2022 was $64 400 and the inflation rate for 2022 was 8.2%.
    - **a.** By how much should Amelie's salary increase at the start of 2023?

    Assume inflation remains at an annual rate of 3.1% from 2023 onwards.
    - **b.** At the start of which year will Amelie's salary be raised over $80 000?
    - **c.** If Amelie's salary did not increase each year, how much spending power would her 2022 salary of $64 400 have at the start of 2030, correct to the nearest dollar?

Questions from multiple lessons

Recursion and financial modelling

13. The cost of a VPN subscription is $60.00 per year plus GST of 10%.

    The cost of a VPN subscription for three years, including GST, is

    A. $180.00 **B.** $186.00 **C.** $198.00 **D.** $240.00 **E.** $264.00

    Adapted from VCAA 2014 Exam 1 Recursion and financial modelling Q4

Data analysis

14. A new doughnut store just opened up in Melbourne. The store manager wants to investigate the association between the *number of doughnuts sold* and the *day of the week*. These variables are
    - **A.** a numerical variable and an ordinal variable respectively.
    - **B.** a numerical variable and a nominal variable respectively.
    - **C.** an ordinal and numerical variable respectively.
    - **D.** both numerical variables.
    - **E.** both categorical variables.

    Adapted from VCAA 2017NH Exam 1 Data analysis Q4

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2D The unitary method and its applications

STUDY DESIGN DOT POINT

- the unitary method and its use in making comparisons and solving practical problems involving percentages and finance

2A 2B 2C **2D** 2E

KEY SKILLS

During this lesson, you will be:

- using the unitary method
- using the unitary method to calculate percentages.

KEY TERMS

- Unitary method

The unitary method can be useful in financial mathematics for finding the cost or price of one unit or any given number of units. Additionally, the method can be applied to other real life contexts, such as weight, time or distance. It can also be used as an alternative method of working with percentages and percentage change.

Using the unitary method

The **unitary method** is a technique for finding the total value of a given number of items, by first finding the value of a single item, or unit.

The value of a single unit can be found using the formula

$$\textit{value of one unit} = \frac{\textit{total value}}{\textit{number of units}}$$

The value of any given number of units can then be found using the formula

$$\textit{total value} = \textit{value of one unit} \times \textit{number of units}$$

For financial applications, the unitary method can be used to calculate the cost of one unit so that the cost of the required number of units can then be found.

Worked example 1

Zhao purchased 5 cookies at a bakery for $18.50.

a. Calculate the price of one cookie.

Explanation

Divide the total cost by the number of units.

$$\begin{aligned}\textit{value of one unit} &= \frac{\textit{total value}}{\textit{number of units}} \\ &= \frac{18.5}{5} \\ &= 3.7\end{aligned}$$

Answer

$3.70

Continues →

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b. How much would 12 cookies cost?

Explanation

Multiply the value of one unit by the number of units.

$$\begin{aligned} \textit{total value} &= \textit{value of one unit} \times \textit{number of units} \\ &= 3.70 \times 12 \\ &= 44.4 \end{aligned}$$

Answer

$44.40

Using the unitary method to calculate percentages

The unitary method can also be used as an alternative way of working with percentages and percentage change. To use the unitary method in terms of percentages, divide the initial value by 100. This gives the value of one percent, which can then be multiplied to achieve the required percentage.

Worked example 2

Complete the following calculations.

a. 119% of 500.

Explanation

Step 1: Divide the number by 100 to find one percent of that number.

$$\frac{500}{100} = 5$$

Step 2: Multiply this by the percentage.

$$5 \times 119 = 595$$

Answer

595

b. Decrease 3710 by 28%.

Explanation

Step 1: Divide the number by 100 to find one percent of that number.

$$\frac{3710}{100} = 37.1$$

Step 2: Multiply this by the percentage.

$$37.1 \times 28 = 1038.8$$

Step 3: Subtract this from the original value.

$$3710 - 1038.8 = 2671.2$$

Answer

2671.2

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2D Questions

Note: There are no direct exam questions relevant to this lesson.

Using the unitary method

1. Three cups of flour weigh 360 g. How much do five cups of flour weigh?

   A. 240 g **B.** 480 g **C.** 520 g **D.** 600 g

---

2. Calculate the cost of one unit for each of the following.

   a. Ricardo bought 14 snake lollies for $2.10. How much does one snake lolly cost?

   b. James's yearly audiobook subscription is $420. How much does he pay per month?

   c. An umbrella company sold 18 370 umbrellas in 2022, and had a total revenue of $404 140. How much revenue do they make for each umbrella sold?

   d. Yelian was paid $48 061 last year, and got paid fortnightly. What was the value of each paycheque? Note: Assume that there are 26 fortnights in a year.

---

3. Calculate the cost of each of the following.

   a. At the fruit store, seven oranges cost $2.31. How much would it cost to buy 25 oranges?

   b. Enriques bought 150 grams of brie for $4.20. How much would 715 grams of brie cost?

   c. Damian bought 1.26 kg of pistachios for $28.35. What is the price per 100 g of pistachios?

   d. Leona bought 210 shares for $2839.20. How much would 37 shares cost?

---

4. A stack of 7 Edrolo textbooks weighs 11.55 kg. How much would a stack of 20 Edrolo textbooks weigh?

---

5. Agnes is selling raffle tickets as a fundraiser for the local footy club. She has five books of tickets, and each book has 100 tickets. If she sells all the tickets she will raise $750. How much will she raise if she sells 173 tickets?

---

6. Last week, David and his friends bought pizza for dinner, and they spent a total of $112.50 on five pizzas. This week, David is having a party and has $600 to spend on pizzas. Assuming all pizzas cost the same amount, what is the maximum number of pizzas David can buy?

---

7. Sophie is a savvy shopper and likes to be cost efficient when grocery shopping. She is choosing between two brands of peanut butter. Pic's peanut butter is $4.80 for a 325 g jar. Mayver's peanut butter is $6.10 for a 420 g jar.

   a. Calculate the price per 100 g, correct to the nearest cent, for Pic's peanut butter.

   b. Calculate the price per 100 g, correct to the nearest cent, for Mayver's peanut butter.

   c. Which brand of peanut butter is the most cost efficient?

---

8. Charles cycles at a consistent speed. It takes him 15 minutes to cycle the 6 km distance from his house to his office every day.

   a. Last week, Charles cycled for 4 hours and 35 minutes in total. How far did Charles cycle last week?

   b. How long, in hours, minutes and seconds, would it take Charles to cycle the 153 km distance from Bendigo to Melbourne?

Using the unitary method to calculate percentages

9. Tabit's phone battery lasts 10 hours. If his battery is on 1%, it will last for
   - **A.** 1 minute.
   - **B.** 6 minutes.
   - **C.** 10 minutes.
   - **D.** 1 hour.

10. Use the unitary method to calculate the following.
    - **a.** Matisse is reading Harry Potter and the Chamber of Secrets, which is 352 pages in length. She has read 75% of the book. How many pages has she read?
    - **b.** Miles recently completed an exam that had 240 total possible marks. He received a grade of 85%. How many marks did Miles get on the exam?
    - **c.** Rafael is planning on running 150 km this month. He has already reached 36% of his target. How far has he already run?
    - **d.** Zair is aiming to raise $5000 for Beyond Blue. She is currently at 61% of her goal. How much has she raised so far?

11. Use the unitary method to calculate the following percentage changes.
    - **a.** A $125 pair of shoes has a 30% discount. What is the sale price of the shoes?
    - **b.** Last year, Jerry spent $55 a week on groceries. This year, he spends 12% more. How much does Jerry spend on groceries each week this year?
    - **c.** Last year, Ramid bought $1200 of shares. Since then, the share price has increased by 193%. How much are his shares worth now?
    - **d.** The value of Shonda's car has decreased by 37% since she bought it. If she bought it for $16 000, what is its value now?

12. Justin has $12 350 in his savings account. Kirsty has 39% more than Justin in her savings account. How much does Kirsty have in her savings account?

13. Duncan is travelling around Europe, and started his holiday with $13 500. He has already spent 63% of his money, and wants to go on a Contiki tour that costs $5000. Does he have enough money left for the Contiki tour?

14. A $370 lawnmower has gone on sale for $284.90. By what percentage has the price of the lawnmower been discounted?

Joining it all together

15. Last year, Ella tried a new gym for three weeks. For the three weeks, Ella paid $70.50 in total for her membership.
    - **a.** Calculate the weekly price of a gym membership for this gym.
    - **b.** Calculate the cost of a gym membership for an entire year (52 weeks).
    - **c.** Due to inflation, the gym has raised their membership prices by 10% this year. What is the current price of a weekly gym membership?

    Students receive a discount of 15% on their weekly membership.
    - **d.** What was the weekly membership price for a student last year, correct to the nearest cent?
    - **e.** What is the weekly membership price for a student this year, correct to the nearest cent?

16. A 3862 km flight from Melbourne to Nadi costs \$482.75. Assume that the cost of a plane ticket is based solely on the distance travelled during the flight.
    a. What is the cost, in cents, for each kilometre travelled?
    b. Calculate the cost, in dollars, of a ticket for a 1487 km flight from Paris to Lisbon, correct to the nearest cent.
    c. Calculate the cost, in dollars, of a ticket for a 11 557 km flight from Reykjavik to Singapore, correct to the nearest cent.
    d. Daniel is flying from Melbourne to Berlin, taking three connecting flights. His flight from Melbourne to Doha is 11 950 km, his flight from Doha to Dusseldorf is 4760 km, and his flight from Dusseldorf to Berlin is 477 km. How much will this cost him in total, correct to the nearest dollar?
    e. The flight distance from Sydney to Nadi is 18% shorter than the distance from Melbourne to Nadi. What is the cost of a plane ticket from Sydney to Nadi, correct to the nearest cent?
    f. What is the distance travelled on a flight between Sydney and Nadi, correct to the nearest kilometre?

Questions from multiple lessons

Recursion and financial modelling

17. A rare painting was bought for \$140 000 in 2012.
    In 2019, it sold at auction for \$189 000.
    What is the increase in value, as a percentage of the original purchase price?
    A. 5.0%
    B. 25.9%
    C. 35.0%
    D. 42.1%
    E. 74.1%

    Adapted from VCAA 2015 Exam 1 Business-related mathematics Q2

Data analysis *Year 10 content*

18. The following stem plot displays the *hourly wage*, in dollars, of 35 hospitality workers.

    Key: 13 | 5 = \$13.50 $n = 35$

13	5
14	2 7
15	0 3 6 8 8
16	3 4 5 9
17	0 0 6 7 7 7
18	4 8
19	5 5 8 9
20	0 2 9
21	2 5 5 6
22	0
23	1 4 8

What is the median hourly wage?

A. \$17.60 B. \$17.65 C. \$17.70 D. \$18.05 E. \$18.40

Adapted from VCAA 2018NH Exam 1 Data analysis Q1

Computation and practical arithmetic

19. David Attenborough is producing a documentary on an aardvark population in Namibia.

His team monitored the population over a three-year period.

At the beginning of monitoring, the population contained 65 aardvark cubs and 232 fully-grown aardvarks.

a. What percentage of the aardvarks are cubs? Round to the nearest percent. (1 MARK)

b. After the first year of monitoring, eleven of the aardvarks passed away. Express this change as a percentage of the original population, correct to one decimal place. (1 MARK)

c. From the beginning of monitoring to the end of the three-year period, the aardvark population increased by 24%. What is the current population, correct to the nearest whole number? (1 MARK)

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2E Purchase options

STUDY DESIGN DOT POINT

- comparison of purchase options including cash, credit and debit cards, personal loans, buy now and pay later schemes

2A 2B 2C 2D 2E

KEY SKILLS

During this lesson, you will be:

- evaluating cash and card purchase options
- evaluating personal loans and buy now pay later schemes
- comparing purchase options.

KEY TERMS

- Cash
- Debit cards
- Credit cards
- Interest
- Billing period
- Interest-free period
- Personal loan
- Buy now pay later scheme

In everyday life, various kinds of purchases need to be made, from food, to clothing, to services and expensive items like cars and houses. This means that there are various kinds of purchase options that can be used depending on the circumstance. It is useful to understand the features of these purchase options, and to be able to compare purchase options to decide which would be most suitable in a given situation.

Evaluating cash and card purchase options

See worked example 1

The three most common ways to pay for something are with cash, using a debit card or using a credit card.

Cash is a form of physical currency, such as coins and notes.

Debit cards are cards that allow the cardholder to electronically transfer money they have saved from a linked bank account in order to make a payment.

Credit cards are cards that allow the cardholder to make a payment using the bank's money, which must then be repaid by the cardholder according to the terms of the credit card.

Debit and credit cards are used to make EFTPOS payments, which means Electronic Funds Transfer at Point of Sale. Often, a surcharge will be applied to EFTPOS payments, where the cardholder will be charged a small percentage on top of whatever amount they are required to pay.

See worked example 2

Different credit card providers have different rules and costs associated with usage and typically incurs a fixed annual fee, and a variable fee that represents the cost of borrowing.

Interest refers to the variable fee charged by lenders as a cost for borrowing. For a credit card, interest is typically calculated as a percentage of the balance outstanding and compounds daily.

Credit cards generally have a **billing period**, also known as the statement period, which is the regular period of time in which the bank sends a statement of purchases made, including the amount owing, the minimum payment and the due date. The billing cycle is typically monthly.

Many credit cards have an **interest-free period**, which is the number of days between the purchase date and the date that the monthly balance is due where the bank does not charge interest. The interest-free period is commonly up to 55 days. For example, an item purchased on the 1st day of a billing cycle will receive 55 interest-free days whereas an item purchased on the 30th day of a 30-day billing cycle will receive 26 interest-free days.

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Once the interest-free period ends, the amount of money owed on a credit card, A, is given by the formula

$A = P \times \left(1 + \frac{r}{365 \times 100}\right)^n$, where

- P is the purchase price, or the initial amount owing on the card
- r is the annual interest rate (%)
- n is the number of days since the interest-free period has ended.

Note: this formula assumes that there is no money owing on the card from the previous billing period.

Worked example 1

Cara and her partner Maddie go to the food court to buy lunch.

a. Cara uses a \$20 note to buy her lunch. What purchase option has she used?

Explanation

Physical currency, such as notes and coins, are examples of cash.

Answer

Cash

b. Maddie uses a card to buy her lunch. The purchase price is immediately deducted from her bank account balance. What purchase option has she used?

Explanation

A debit card is linked directly with a bank account, so the individual will pay the purchase price immediately.

Answer

Debit card

c. A 1.5% surcharge applies to all EFTPOS transactions at the restaurant Maddie buys her lunch at. If her lunch costs \$16, how much will she be charged in total?

Explanation

Apply a 1.5% increase to the amount Maddie has to pay.

$$\begin{aligned} \textit{total cost} &= \left(1 + \frac{\textit{percentage increase}}{100}\right) \times \textit{original value} \\ &= \left(1 + \frac{1.5}{100}\right) \times 16 \\ &= 1.015 \times 16 \\ &= 16.24 \end{aligned}$$

Answer

\$16.24

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Worked example 2

Esther's credit card has an interest rate of 15% p.a, with up to 55 interest-free days. She pays $4500 for a 75 inch smart TV with her credit card on the 21st day of her billing cycle and plans to pay the outstanding balance 40 days after the purchase.

a. How many interest-free days does Esther receive for this purchase?

Explanation

Calculate the difference between the day of the billing cycle and 55.

As Esther is making the purchase on the 21st day, 20 days of the current billing cycle have passed.

$55 - 20 = 35$

Answer

35 days

b. How many days does Esther pay interest on her purchase?

Explanation

Calculate the difference between the interest-free days and the number of days before Esther pays the outstanding balance off.

$40 - 35 = 5$

Answer

5 days

c. How much will Esther pay back, correct to the nearest cent?

Explanation

Step 1: Determine the values of P and r.

The purchase price is $4500.

$P = 4500$

The annual interest rate is 15%.

$r = 15$

Step 2: Determine the value of n.

From part **b**, $n = 5$

Step 3: Substitute the values into the formula to solve for A.

$$A = P \times \left(1 + \frac{r}{365 \times 100}\right)^n$$

$$= 4500 \times \left(1 + \frac{15}{365 \times 100}\right)^5$$

$$= 4509.254...$$

Answer

$4509.25

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Evaluating personal loans and buy now pay later schemes

There are several types of loans that are used to make purchases in everyday life.

Some loans are for specific items. For example, a home loan is used for purchasing residential property while a car loan can only be used to purchase vehicles.

Two types of more generic purchase options are personal loans and buy now pay later schemes.

See worked example 3

A **personal loan** is typically between $5000 and $50 000 and usually needs to be paid back within 7 years. This type of loan is commonly used for items such as furniture, appliances, cars and family holidays. Personal loans generally have the following main features:

- Interest is calculated based on the original borrowed amount.
- Regular payments (e.g. weekly, fortnightly, monthly, etc.)
- A set duration of time within which the repayments will reduce the balance of the loan to $0.

See worked example 4

A **buy now pay later scheme** is generally used for purchases under $2000, where the amount is paid over 4 or more equal payments rather than made upfront. Popular buy now pay later scheme providers in Australia include Afterpay and Zip.

Buy now pay later schemes often do not charge interest if the amount is low and the payment is made within a matter of weeks or months. However, providers may charge establishment fees, monthly fees and late fees.

While buy now pay later schemes can be useful for consumers from a budgeting perspective, and to pay for necessary items, it can cause some to overspend as the lower regular payments may seem more affordable than a larger upfront payment.

Worked example 3

Herschelle takes out a loan of $40 000 to make major renovations to his house. The bank charges him monthly payments of $640, and Herschelle will pay the loan back in exactly 6 years.

a. How much does it cost Herschelle to pay back the loan?

Explanation

Step 1: Determine the number of payments made.

Herschelle makes monthly payments, and the loan will be fully paid in 6 years.

This means there are $12 \times 6 = 72$ payments.

Step 2: Multiply the payment amount by the number of payments.

Each payment is $640.

$640 \times 72 = 46\ 080$

Answer

$46 080

b. How much interest does Herschelle pay?

Explanation

The interest paid is equal to the difference between the amount of the loan and the total amount paid.

From part **a**, Herschelle paid $46 080.

$46\ 080 - 40\ 000 = 6080$

Answer

$6080

Continues →

c. What is the annual interest rate charged on Herschelle's loan? Round the answer to two decimal places.

Explanation

Step 1: Calculate the average amount of interest paid per year.

From part **b**, Herschelle pays \$6080 of interest. This occurs over 6 years.

$6080 \div 6 = 1013.333...$

Step 2: Calculate the interest per year as a percentage of the original loan.

$1013.333... \div 40\,000 \times 100 = 2.533...$

Answer

2.53%

Worked example 4

Elena sees a couch online for \$400. She decides to purchase it using a buy now pay later scheme, which will allow her to pay for the couch over 10 weeks. The store charges a fee of \$20 for every payment missed.

a. What are Elena's weekly payments?

Explanation

Divide the total cost by the number of weeks required to pay the total cost.

$400 \div 10 = 40$

Answer

\$40

b. Elena makes the first 6 payments, but fails to make the rest. Once the payment period is completed, how much money does she owe?

Explanation

Step 1: Determine the amount owed on the initial purchase price.

After the first 6 weeks, Elena has made $6 \times 40 = \$240$ in payments.

This means she still owes $400 - 240 = \$160$.

Step 2: Determine the amount owed in additional fees.

There is a fee of \$20 for every payment missed. As Elena misses 4 payments, she owes an additional $4 \times 20 = \$80$.

Step 3: Calculate the total amount owed.

$160 + 80 = 240$

Answer

\$240

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Comparing purchase options

Depending on circumstances, some purchase options may be more suitable in given situations than others.

For example, it may be more cost effective to pay with cash instead of a debit card if there is a surcharge applied to EFTPOS transactions.

Where a purchaser does not have sufficient savings, it may be necessary to use credit cards, personal loans or buy now pay later schemes. Depending on the terms of each payment method, as well as the individual's circumstances, one option is likely more suitable than the others.

In such cases, it is useful to calculate the cost that will be incurred under all possible scenarios to determine the most suitable option.

Worked example 5

For each of the following scenarios, determine the most cost effective purchase option.

a. A store is selling a t-shirt for \$35, with a surcharge of 2% for all EFTPOS transactions. The shirt can be paid for using cash or a debit card.

Explanation

Determine the total cost of each option.

Using cash will cost \$35.

Using a debit card will cost $35 \times 1.02 = \$35.70$ with the surcharge.

Answer

Cash

b. Roberto wants to purchase a secondhand car for \$15 000. He can pay either by using his credit card or by obtaining a personal loan, where the balance is paid off after exactly 12 months.

- His credit card charges 12% interest p.a. with 44 days interest-free. Assume that Roberto makes the purchase on the first day of his billing period, makes no other purchases and pays off the balance in one go.
- His bank offers him a personal loan with 12 monthly payments of \$1387.50.

Explanation

Step 1: Determine the amount paid using the credit card.

$P = 15\,000$

$r = 12$

He will pay back the amount owed one year (365 days) after the initial purchase. Given the 44-day interest-free period, interest will compound for $365 - 44 = 321$ days.

$n = 321$

$$A = P \times \left(1 + \frac{r}{365 \times 100}\right)^n$$

$$= 15\,000 \times \left(1 + \frac{12}{365 \times 100}\right)^{321}$$

$$= \$16\,669.273...$$

Step 2: Determine the amount paid using the personal loan.

Roberto will make 12 payments of \$1387.50.

$12 \times 1387.50 = \$16\,650$

Step 3: Compare the two values to determine which option will be cheaper.

$16\,669.273... > 16\,650$

Answer

Personal loan

2E Questions

Note: There are no direct exam questions relevant to this lesson.

Evaluating cash and card purchase options

1. If a store has an EFTPOS surcharge of 2%, how much will a customer be charged on an item that costs $400?

   A. $400 B. $402 C. $404 D. $408

2. For each of the following, give the answer correct to the nearest cent.
   a. A local bakery offers a 5% discount on purchases made with cash. The regular price of a dozen Nutella doughnuts is $30.
   How much would it cost to buy two dozen nutella doughnuts with cash?
   b. A dumpling restaurant has a 1.2% surcharge on all EFTPOS transactions. A family who ate there received a bill for $62.50.
   How much would it cost the family to pay the bill using a debit card?

3. Surya has a credit card with up to 55 interest-free days. He purchases an Xbox on the 11th day of his billing cycle and pays off his credit card 60 days after the purchase.
   a. How many interest-free days does he receive for the Xbox?
   b. How many days does Surya pay interest on his credit card for the Xbox?

4. Rosa's credit card has a 44-day interest-free period and an interest rate of 15% p.a. Rosa purchases a bicycle for $350 on the 25th day of her billing period, which she pays off 100 days later.
   a. Calculate the number of days that Rosa paid interest on her credit card.
   b. Calculate the total amount owing on Rosa's credit card after 100 days, to the nearest cent.

5. On the first day of her billing period, Evelyn pays $4500 using her credit card for an online course on book editing. The card has an interest-free period of up to 55 days and Evelyn takes 77 days from the day of purchase, to pay the outstanding balance of $4534.71. Calculate the annual interest rate on Evelyn's credit card, correct to two decimal places.

Evaluating personal loans and buy now pay later schemes

6. An item is purchased using a buy now pay later scheme. It is to be repaid in 8 weekly payments of $7.50. How much does the item cost?

   A. $7.50 B. $15.00 C. $30.00 D. $60.00

7. John takes out a personal loan of $14 000. He is required to make monthly repayments of $330 for 5 years in order to repay the loan.
   a. How much will it cost John to fully repay the loan?
   b. How much interest will John pay?

8. Roland obtains a personal loan of $9000 and will repay the loan with weekly payments of $65.25 over a period of 4 years.
   What is the annual interest rate of the loan?

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9. Sanjay is looking to buy a fridge for $1500. He thinks the fridge is a bit expensive, so he elects to pay for it using a buy now pay later scheme. This will allow Sanjay to pay for the fridge over the course of 20 weeks, interest-free. He will be charged an extra $25 for each payment that he misses.

   a. How much does he pay per week for the fridge?

   Sanjay also needs a new couch. After 5 weekly payments for the fridge, he enters a new buy now pay later scheme to purchase a $750 couch that will be paid off in 15 weeks. He will be charged an extra $15 for each payment that he misses.

   b. How much is he now paying per week for both the fridge and the couch?

   12 weeks after the initial payment for the fridge, Sanjay finds that he is struggling to afford the payments and misses 6 of the last 8 payments.

   9 weeks after the initial payment for the couch, he is in a similar situation and misses 3 of the last 6 payments.

   c. How much does Sanjay still owe in total, for both items, 20 weeks after the initial payment for the fridge?

   d. Assuming Sanjay pays back the money he owes instantly, how much more does he pay for the two items, compared to if he had purchased them upfront?

Comparing purchase options

10. Don wants to buy a new cricket bat for $750. He has enough cash, but is considering other purchasing options. Which of the following options will cost the same as the cash option?

    A. Debit card purchase with a 0.5% EFTPOS surcharge.

    B. Credit card purchase with an interest rate of 5% p.a., paid off with a lump sum payment in two years.

    C. Buy now pay later scheme with 10 weekly payments of $75.

    D. Personal loan with an interest rate of 4% p.a.

11. Nathaniel is looking to purchase a new jacket for $120. The store he is at charges a 1.5% surcharge on all debit card transactions.

    The store also offers him a buy now pay later option where he can pay for the jacket in 6 weekly payments of $20. They charge an extra $15 for each payment missed.

    a. Assuming Nathaniel is able to make all the payments, which option will be cheaper?

    b. Why might Nathaniel choose to pay using his debit card instead of the buy now pay later option?

12. Sammy is buying her niece a Lego set for Christmas and is trying to work out whether to use her debit or credit card. The Lego set costs $180.

    - The store has a 1% surcharge on all debit card purchases.
    - Her credit card company charges 15% p.a. and has a 44-day interest-free period. Sammy plans on purchasing the Lego set on the 29^{th} day of the billing period and paying back the outstanding balance 32 days after the purchase.

    a. Which option is more cost effective for Sammy?

    b. How many days from the day of purchase will it take for the amount owing on the credit card to exceed the amount paid using her debit card?

13. Maxwell is looking to purchase a valuable painting, worth $15 000. He can either purchase the painting with his credit card, or by obtaining a personal loan, with plans to pay the loan back in exactly 1 year.

    - His credit card charges 11.99% p.a. and has no interest-free period.
    - His bank offers him a personal loan with interest of 12.5% p.a.

    Which option is cheaper for Maxwell and by how much? Round to the nearest cent.

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Joining it all together

14. Oswald is opening up his own candle business. He calculates that he will require approximately $5000 in order to purchase equipment, materials and host a website and rent a weekly stall at his local farmer's market.

a. Assuming Oswald has $280 in his bank account, which single purchase option is he able to use in order to pay for all the elements required to start his business?

A. Cash

B. Debit card

C. Credit card

D. None of the above

b. Oswald decides instead to obtain a personal loan from his bank and is required to make monthly payments of $115.69 in order to fully repay the loan in exactly 7 years. How much will Oswald pay in total?

c. What annual interest rate is the bank charging on Oswald's loan, to 2 decimal places?

d. In his first month of business, he sells 4 candles for $8 each, 1 candle for $15 and a gift package worth $75. Has he made enough money to make his monthly loan payment from his sales?

As his business grows, Oswald requires more materials. He needs to buy $1200 worth of materials from a wholesaler.

The wholesaler offers Oswald a buy now pay later option where he can pay for the materials with 20 weekly payments of $60.

Alternatively, Oswald could make the purchase using his credit card. His card charges 20.24% p.a. and has a 55-day interest-free period.

Oswald makes the purchase on the 1st day of his billing cycle and will be able to make a lump sum repayment 184 days after the day of purchase.

e. Which option will be more expensive for Oswald and by how much? Provide the answer to the nearest cent.

f. The buy now pay later option charges an extra $50 for each payment missed. Oswald receives most of his income in December when people are buying Christmas gifts. Why might he prefer to pay for materials using his credit card?

Questions from multiple lessons

Data analysis *Year 10 content*

15. The following histogram displays the long jump results of a year level of 86 students.

The interquartile range for this distribution is closest to

A. 0.5 m

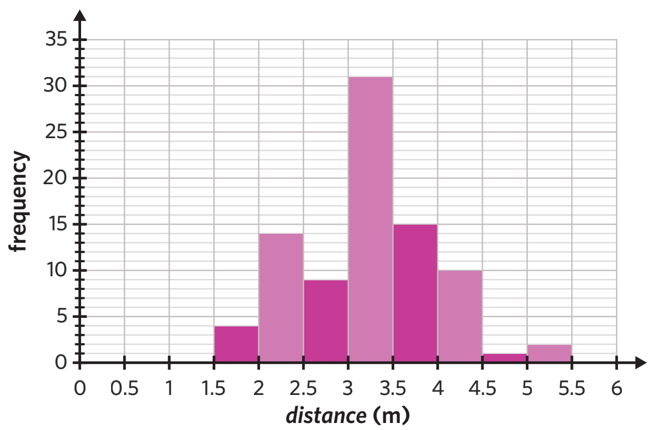
B. 0.75 m

C. 1 m

D. 1.25 m

E. 1.5 m

Adapted from VCAA 2018NH Exam 1 Data analysis Q4



Recursion and financial modelling *Year 10 content*

16. A billboard in New York City costs \$10 320 to hire for promoting concert tickets for a musician.

The total amount earned selling tickets is \$86 000.

The \$10 320 charge as a percentage of the amount earned from selling tickets is

A. 0.10% **B.** 0.12% **C.** 8.33% **D.** 10.0% **E.** 12.0%

Adapted from VCAA 2014 Exam 1 Business-related mathematics Q2

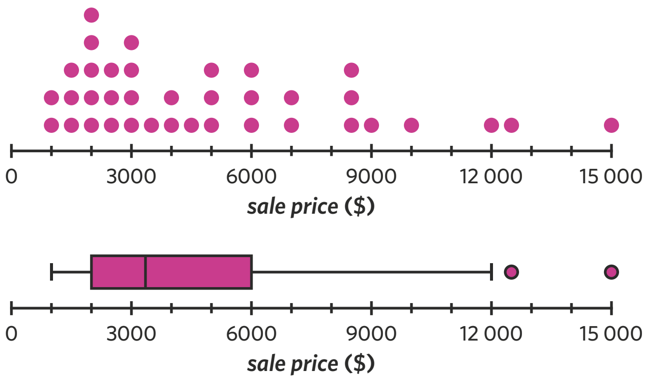
Data analysis

17. Nick and Vickie are artists. Together they sold 34 artworks last financial year. The distribution of the sale prices of these artworks is shown in the dot plot and boxplot.

a. Describe the shape of the distribution of the sale prices of the 34 artworks. (1 MARK)

b. Determine the value of the lower fence. (1 MARK)

Adapted from VCAA 2018 Exam 2 Data analysis Q1f,g



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CHAPTER 3 CALCULATOR QUICK LOOK-UP GUIDE

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3

UNIT 1 AOS 2
CHAPTER 3

Number patterns and recursion

LESSONS

KEY KNOWLEDGE

- the concept of an arithmetic or geometric sequence as a function with the set of non-negative integers as its domain
- tabular and graphical display of sequences, investigation of their behaviour (increasing, decreasing, constant, oscillating, limiting values)
- use of a first-order linear recurrence relation of the form $u_0 = a,\ u_{n+1} = u_n + d$, where a and d are constants, to generate the values of an arithmetic sequence
- use of a first-order linear recurrence relation of the form $u_0 = a,\ u_{n+1} = u_n + d$, where a and d are constants, to model and analyse practical situations involving discrete linear growth or decay such as a simple interest loan or investment, the depreciating value of an asset using the unit cost or flat rate method
- use of a first-order linear recurrence relation of the form $u_0 = a,\ u_{n+1} = Ru_n$, where a and R are constants, to generate the values of a geometric sequence
- use of a first-order linear recurrence relation of the form $u_0 = a,\ u_{n+1} = Ru_n$, where a and R are constants, to model growth and decay and analyse practical situations involving geometric sequences such as the reducing height of a bouncing ball, reducing balance depreciation, compound interest loans or investments
- generation of the explicit rule, u_n, of an arithmetic or geometric sequence, its use and evaluation, including various practical and financial contexts.

Image: ChristianChan/Shutterstock.com

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3A Introduction to sequences and recursion

STUDY DESIGN DOT POINTS

- the concept of an arithmetic or geometric sequence as a function with the set of non-negative integers as its domain
- tabular and graphical display of sequences, investigation of their behaviour (increasing, decreasing, constant, oscillating, limiting values)

3A 3B 3C 3D 3E 3F

KEY SKILLS

During this lesson, you will be:

- identifying arithmetic and geometric sequences
- displaying sequences as tables and graphs
- investigating the behaviour of sequences.

KEY TERMS

- Sequence
- Term
- Arithmetic
- Geometric
- Recursion
- Limiting value

Sequences can be used to model the population of a group of animals over a number of years, the balance of a bank account over a number of months, or the value of an asset over time. Recursion is the process of generating the numbers in a sequence based on a pattern that exists between the numbers. This can be used to predict future values, such as the decreasing value of a laptop.

Identifying arithmetic and geometric sequences

A **sequence** is a set of numbers arranged in a particular order. Each number in a sequence is referred to as a **term**. For example, in the sequence

$-4, 7, 9, -2, -3,$

-4 is the first term, 7 is the second term, and so on.

If a sequence has a pattern, the next term can be predicted. If the pattern is repeated addition or subtraction of a constant value, the sequence is **arithmetic**. For example, in the sequence

$-2, 4, 10, 16,$

each term has 6 added to it to find the next.

$$-2 \xrightarrow{+6} 4 \xrightarrow{+6} 10 \xrightarrow{+6} 16$$

If the pattern is repeated multiplication of a constant value, the sequence is **geometric**. For example, in the sequence

$14, 7, 3.5, 1.75,$

each term is multiplied by $\frac{1}{2}$ to find the next.

$$14 \xrightarrow{\times\frac{1}{2}} 7 \xrightarrow{\times\frac{1}{2}} 3.5 \xrightarrow{\times\frac{1}{2}} 1.75$$

Other types of patterns exist, but a sequence is considered random if there is no recurring pattern.

The pattern can be used to predict the next term, or any term, in the sequence. This is called **recursion**.

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Worked example 1

For the following sequences,

- identify if they are arithmetic, geometric, or have no pattern
- if there is a pattern, determine the next term in the sequence.

a. 2, 5, 8, 11, 14, 17, 20

Explanation

Step 1: Check if the sequence is arithmetic.

Three is added to 2 to give 5.

Three is added to 5 to give 8.

The pattern holds for the remaining terms. The pattern is repeated addition, so the sequence is arithmetic.

Step 2: Determine the next term in the sequence using the pattern.

$20 + 3 = 23$

Answer

Arithmetic

23

b. 6, 12, 24, 48

Explanation

Step 1: Check if the sequence is arithmetic.

Six is added to 6 to give 12.

Twelve is added to 12 to give 24.

The sequence is not arithmetic.

Step 2: Check if the sequence is geometric.

6 is multiplied by two to give 12.

12 is multiplied by two to give 24.

The pattern holds for the remaining terms. The pattern is repeated multiplication, so the sequence is geometric.

Step 3: Determine the next term in the sequence using the pattern.

$48 \times 2 = 96$

Answer

Geometric

96

c. $-3, -1, 5, 2, 7$

Explanation

Step 1: Check if the sequence is arithmetic.

Two is added to -3 to give -1.

Six is added to -1 to give 5.

The sequence is not arithmetic.

Step 2: Check if the sequence is geometric.

-3 is multiplied by $\frac{1}{3}$ to give -1.

-1 is multiplied by -5 to give 5.

The sequence is not geometric.

Answer

No pattern

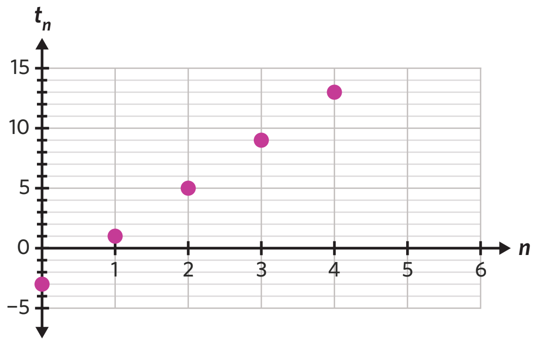
Displaying sequences as tables and graphs

Tables and graphs are common ways to visually represent sequences. They can be used to quickly identify whether a sequence is arithmetic or geometric.

There is a specific notation used for sequences. A term in a sequence is denoted by t_n, where n describes the term's position in the sequence. Note: Any pronumeral can be used in place of t. Usually, the position of the first term in a sequence is $n = 0$. For example, in the sequence $-3, 1, 5, 9, 13$, if $t_0 = -3$, then the third term is $t_2 = 5$.

This sequence is represented in the following table and graph.

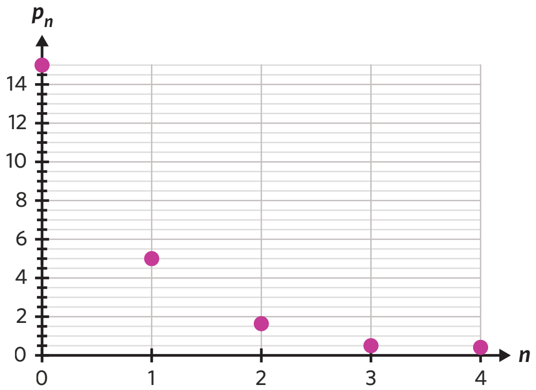
n	0	1	2	3	4
t_n	−3	1	5	9	13



The terms in an arithmetic sequence form a straight line.

The geometric sequence 15, 5, 1.67, 0.56, 0.19 is represented in the following table and graph.

n	0	1	2	3	4
p_n	15	5	1.67	0.56	0.19



The terms in a geometric sequence form a curved line.

Worked example 2

Consider the sequence $6, 3, 0, -3, -6, -9$, where t_0 represents the first term in the sequence.

a. Display the sequence in a table.

Explanation

Step 1: Construct a table with two rows.

Because there are six terms in the sequence, fill in the top row ranging from 0 to 5.

n	0	1	2	3	4	5
t_n						

Step 2: Fill in the rest of the table with the value for each term.

Continues →

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Answer

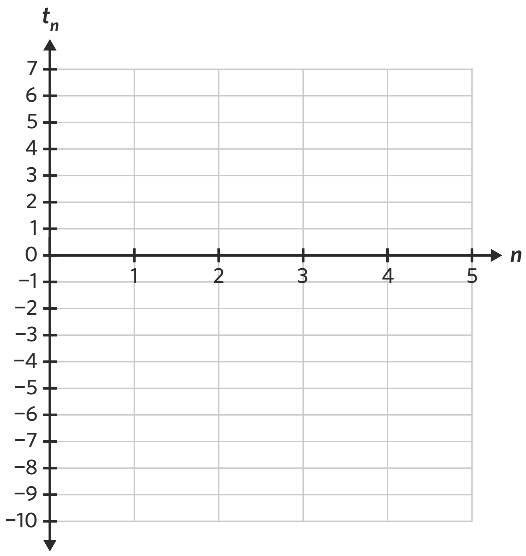
n	0	1	2	3	4	5
t_n	6	3	0	−3	−6	−9

b. Display the sequence in a graph.

Explanation

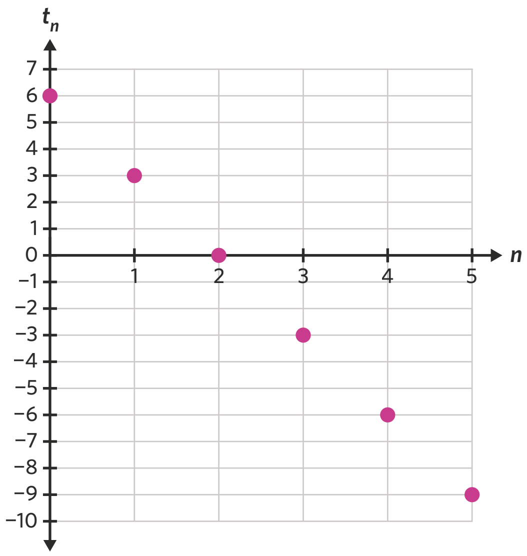
Step 1: Construct a set of axes.

The scale of the vertical axis, t_n, ranges from −10 to 7. The scale of the horizontal axis, n, ranges from 0 to 5.



Step 2: Draw the sequence as a series of points and connect with a line.

Answer



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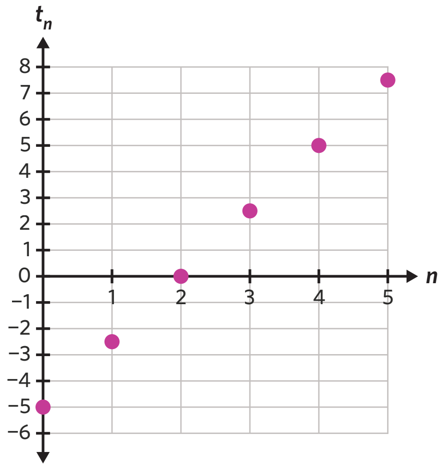
Investigating the behaviour of sequences

The behaviour of sequences can be investigated in two ways: classifying the trend, and identifying any limiting values.

The trend describes the overall change in values for the sequence. It can be classified as increasing, decreasing, constant or oscillating.

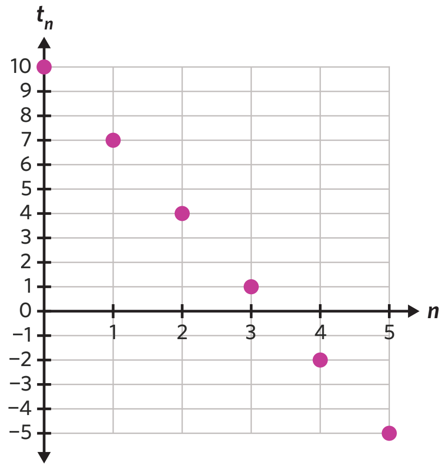
Increasing

−5, −2.5, 0, 2.5, 5, 7.5
The values increase from one to the next.



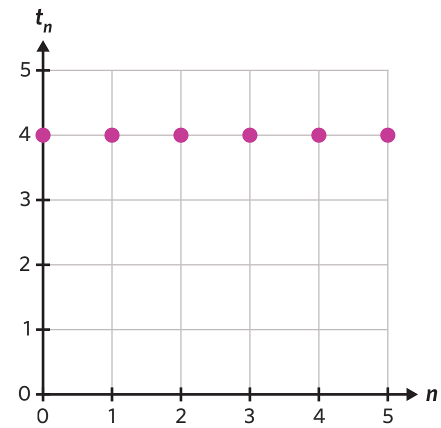
Decreasing

10, 7, 4, 1, −2, −5
The values decrease from one to the next.



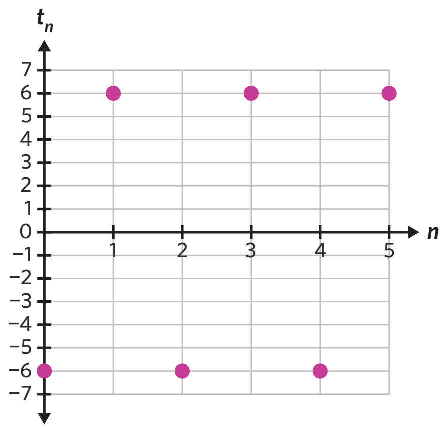
Constant

4, 4, 4, 4, 4, 4
The values do not change from one to the next.



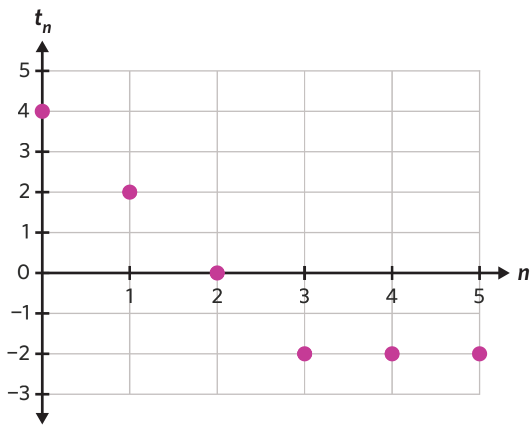
Oscillating

−6, 6, −6, 6, −6, 6
The values increase and decrease in an alternating pattern. This may include between positive and negative values.



If a sequence approaches a certain value and stays constant thereafter, that value is called the **limiting value**.

For example, the sequence 4, 2, 0, −2, −2, −2 is decreasing, approaches the value −2 and then stays at −2. So the limiting value is −2.



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Worked example 3

Classify the behaviour of the following sequences by stating the type of change and identifying any limiting values.

a. 30, 20, 5, 3, 2, 1, 1, 1

Explanation

Step 1: Classify how the values change.

The values increase and decrease in an alternating pattern.

The sequence is decreasing.

Step 2: Identify any limiting values the sequence approaches.

The sequence approaches the value 1 and stays at this value. 1 is the limiting value.

Answer

The sequence is decreasing. There is a limiting value of 1.

b. 3, −4, 5, −6, 7, −8

Explanation

Step 1: Classify how the values change.

The values change between positive and negative.

The sequence is oscillating.

Step 2: Identify any limiting values the sequence approaches.

The sequence does not approach any value.

Answer

The sequence is oscillating. There are no limiting values.

3A Questions

Identifying arithmetic and geometric sequences

1. Consider the sequence 5, 15, 25, 35, 45.

Which of the following statements is true?

A. Each term is equal to the previous term plus 5.

B. Each term is three times the previous term.

C. Each term is 10.

D. Each term is equal to the previous term plus 10.

2. For the following sequences,

- identify if they are arithmetic, geometric, or have no pattern
- if there is a pattern, determine the next term in the sequence.

a. −4, 3, 10, 17, 24, 31 **b.** 13, 9, 8, −4, 0, −1, 8 **c.** 128, 64, 32, 16, 8 **d.** 10, 4, −2, −8, −14

3. Find the missing term(s) in the following sequences.

a. 0.5, 1, 2, 4, __________, 16

b. 11, __________, 5, 2, __________

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Displaying sequences as tables and graphs

4. Which of the following tables correctly displays the sequence 7, 9, 11, 13, given $t_0 = 7$?

A.

n	0	1	2	3
t_n	13	11	9	7

B.

n	1	2	3	4
t_n	7	9	11	13

C.

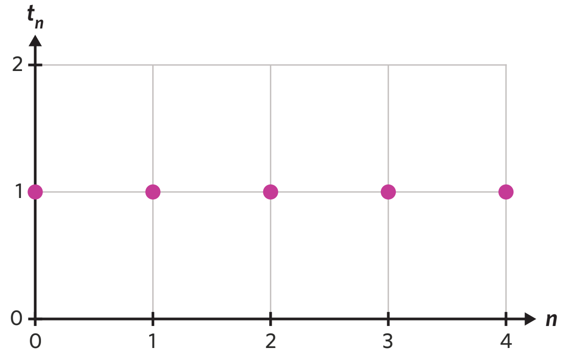
n	1	2	3	4
t_n	7	9	11	13

D.

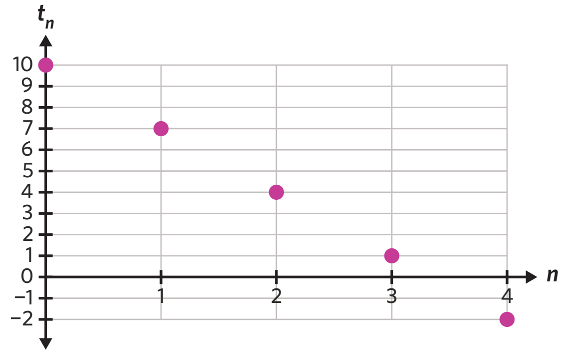
n	0	1	2	3
t_n	7	9	11	13

5. Which of the following graphs correctly displays the sequence 10, 7, 4, 1, −2?

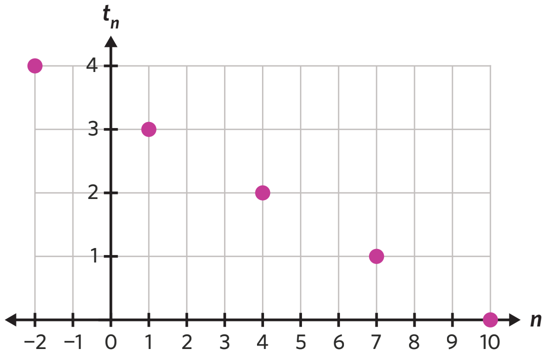
A.



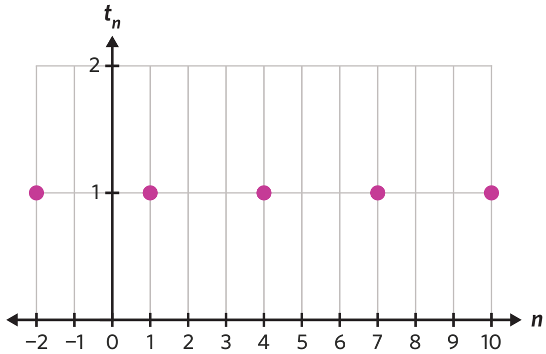
B.



C.



D.



6. Using t_0 to represent the first value, display the following sequences in

- a table and
- a graph.

a. 2, 3, 4, 5, 6

b. −3, 2, 7, 12, 7

c. 10, 5, 2.5, 1.25, 0.625

Investigating the behaviour of sequences

7. For the sequence −0.5, −1, −2, −4, −8, −16, the trend can be best described as

A. increasing. **B.** decreasing. **C.** oscillating. **D.** constant.

8. Classify the behaviour of the following sequences by stating the trend and identifying any limiting values.

a. 0, 4, 8, 12, 16, 20, 20, 20

b. 20, −20, 20, −20, 20

c. 29, 29, 29, 29, 29, 29

d. 52, −44, 36, −28, 20

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Joining it all together

9. A local football club opened a number of years ago. The number of club members, m_n, over a number of years is represented by the sequence 40, 100, 160, 220. Let n be the number of years after the club was founded in 2016, and m_0 be the initial number of members that signed up on opening day.

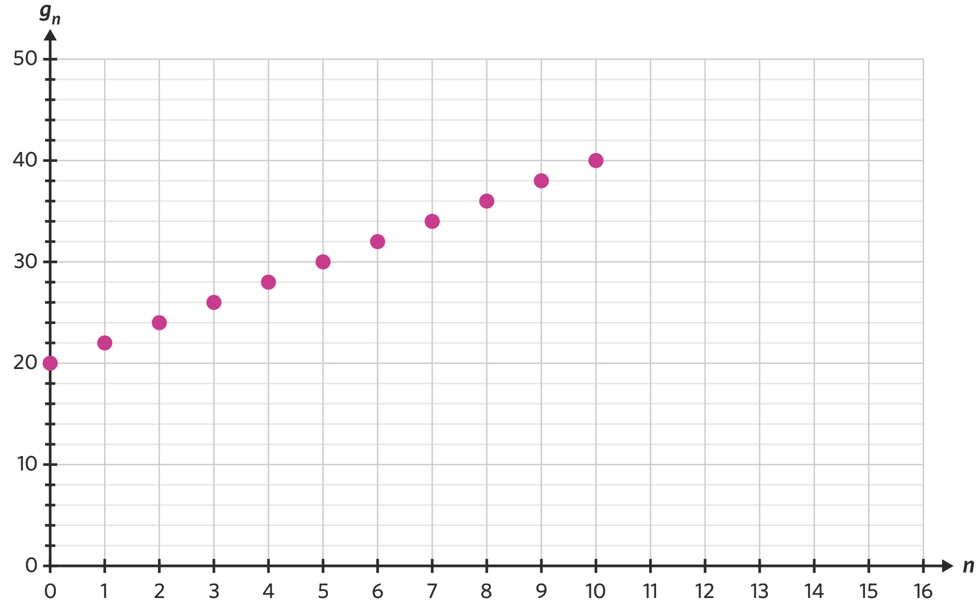
a. Display this information in a graph.

b. Identify the type of sequence.

c. Assuming the sequence continues, predict how many members the club will have in 2022.

---

10. The following graph models the population of a herd of mountain goats between 2005 and 2015.



a. Lucia says that the sequence is geometric because the population in 2015 was double what it was in 2005. Greg says that the sequence is arithmetic. Who is correct? Explain why.

b. The population continues to be counted for the years between 2016 and 2020. The values are 42, 44, 46, 46, 46.

Plot these values on the graph.

c. Describe the behaviour of the entire sequence, including the presence of any limiting values.

---

11. Naid bought a laptop for \$2000. Every year, it decreases in value by half its current value.

a. Using L_n to represent the value of the laptop, and L_0 as the initial purchase price, construct a table to display the value of the laptop over the first 3 years.

b. Display the information from part **a** in a graph.

c. Identify the type of sequence and describe the behaviour of the sequence.

d. After how many years will the laptop be worth less than \$50?

Exam practice

12. The first four terms in a geometric sequence are: 5, 10, 20, 40, ...

The fifth term in this sequence is

A. 45 **B.** 50 **C.** 60
D. 80 **E.** 100

VCAA 2015 Exam 1 Number patterns Q1

97% of students answered this question correctly.

Questions from multiple lessons

Recursion and financial modelling

13. The first five terms in a geometric sequence are: 32, 16, 8, 4, 2, …

What is the next term in this sequence?

A. -2 **B.** -1 **C.** 0 **D.** 1 **E.** 2

Adapted from VCAA 2015 Exam 1 Number patterns Q1

Recursion and financial modelling *Year 10 content*

14. Jacob wants to display a complete collection of ancient Mayan coins in an exhibition in four years.

Each year, he collects 200 more ancient coins than the previous year.

His team of archaeologists predict that there are 4200 coins to collect.

The number of coins that Jacob will need to collect in the first year, if he is to finish collecting the coins in four years, is

A. 700 **B.** 725 **C.** 750 **D.** 775 **E.** 800

Adapted from VCAA 2014 Exam 1 Number patterns Q5

Recursion and financial modelling

15. A car rental company has a special promotion for 2020.

Customers are charged based on the whole number of kilometres they travel each day.

- The first kilometre costs $8.
- Each kilometre after this costs 25 cents less than the previous one.

a. How much does the sixth kilometre cost? (1 MARK)

b. What is the total cost of travel for 6 kilometres? (1 MARK)

Adapted from VCAA 2011 Exam 2 Number patterns Q2a,b

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3B Arithmetic sequences and recurrence relations

STUDY DESIGN DOT POINT

- use of a first-order linear recurrence relation of the form $u_0 = a$, $u_{n+1} = u_n + d$, where a and d are constants, to generate the values of an arithmetic sequence

3A 3B 3C 3D 3E 3F

KEY SKILLS

During this lesson, you will be:

- generating a sequence from an arithmetic recurrence relation
- interpreting an arithmetic recurrence relation
- constructing an arithmetic recurrence relation.

KEY TERMS

- Recurrence relation
- Common difference
- Linear growth
- Linear decay

Arithmetic sequences are used to demonstrate repeated addition or subtraction in a number pattern. This can include the growth of a bank account, or the decreasing value of a population. Recurrence relations can be used to model these sequences, and show how one value in a sequence can be used to calculate the next value.

Generating a sequence from an arithmetic recurrence relation

A **recurrence relation** is a formula that links each term in a pattern-based sequence to the next.

See worked example 1

Recall that an arithmetic sequence is a pattern that has repeated addition or subtraction. This repetition is known as the **common difference**, which is the amount being added or subtracted from one term to the next in a sequence of numbers.

An arithmetic sequence can be modelled by a recurrence relation of the form

$u_0 = a, \quad u_{n+1} = u_n + d$, where

- a is the initial value
- d is the common difference

Here, u_{n+1} represents the next term in the sequence while u_n represents the current term.

A calculator can be used to generate the terms in a sequence from a recurrence relation.

See worked example 2

Worked example 1

Consider the following recurrence relation.

$u_0 = 3, \quad u_{n+1} = u_n + 6$

Calculate the value of u_3.

Explanation

Step 1: Calculate u_1 from u_0.

$$\begin{aligned} u_1 &= u_0 + 6 \\ &= 3 + 6 \\ &= 9 \end{aligned}$$

Step 2: Calculate u_2 from u_1.

$$\begin{aligned} u_2 &= u_1 + 6 \\ &= 9 + 6 \\ &= 15 \end{aligned}$$

Continues →

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Step 3: Calculate u_3 from u_2.

$$u_3 = u_2 + 6$$
$$= 15 + 6$$
$$= 21$$

Answer

21

Worked example 2

Consider the following recurrence relation.

$u_0 = 2000, \quad u_{n+1} = u_n - 200$

Calculate the value of the first five terms in the sequence.

Explanation – Method 1: TI-Nspire

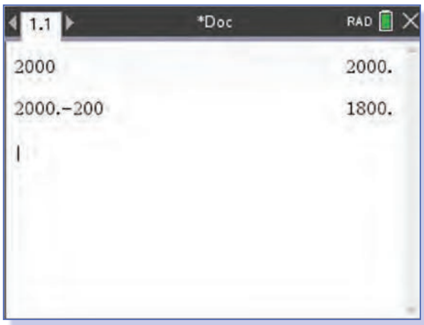
Step 1: From the home screen, select ‘1: New’ → ‘1: Add calculator’.

Step 2: Enter the initial value by typing ‘2000’. Press enter.

Step 3: Press ‘–’.

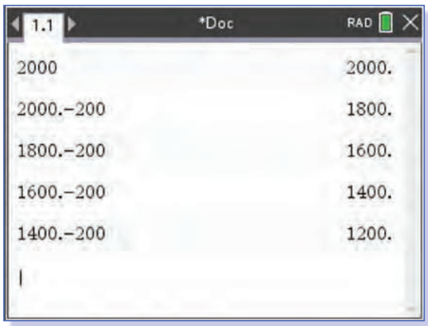
‘Ans–’ will appear to indicate that it is subtracting from the previous answer.

Type ‘200’ and press enter.



Step 4: To find the next value in the sequence, press enter.

Step 5: Repeat step 4 for each of the remaining first five terms.



Explanation – Method 2: Casio ClassPad

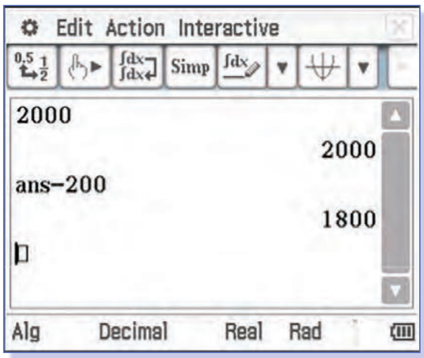
Step 1: From the menu, tap √α Main.

Step 2: Enter the initial value by typing ‘2000’. Press EXE.

Step 3: Type ‘–’.

‘Ans–’ will appear to indicate that it is subtracting from the previous answer.

Type ‘200’ and press EXE.



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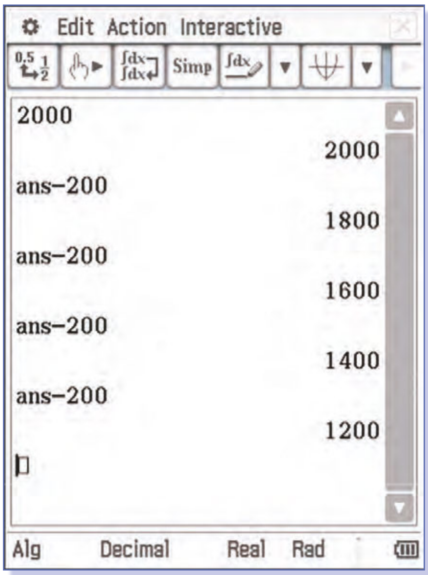
Step 4: To find the next value in the sequence, press **EXE**.

Step 5: Repeat step 4 for each of the remaining first five terms.

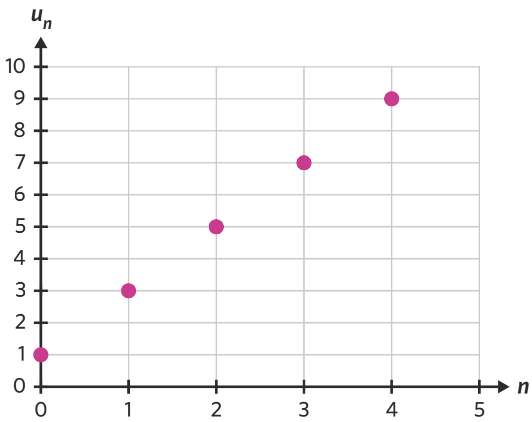


Answer – Method 1 and 2

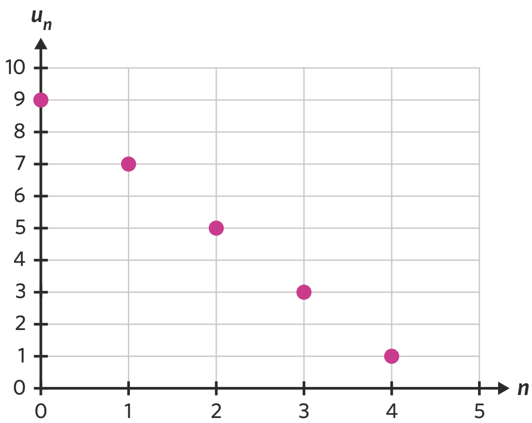
2000, 1800, 1600, 1400, 1200

Interpreting an arithmetic recurrence relation

An arithmetic recurrence relation with a positive common difference $(d > 0)$ will generate a sequence that demonstrates **linear growth**. For example, the first five terms generated by the recurrence relation $u_0 = 1, \quad u_{n+1} = u_n + 2$ have been plotted on the following graph.



On the other hand, a negative common difference $(d < 0)$ will generate a sequence that demonstrates **linear decay**. For example, the first five terms generated by the recurrence relation $u_0 = 9, \quad u_{n+1} = u_n - 2$ have been plotted on the following graph.



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Worked example 3

Let u_n represent the number of umbrellas at a store n days after restocking their shelves.

$u_0 = 120, \quad u_{n+1} = u_n - 50$

a. Determine if the sequence will have linear growth or decay.

Explanation

Step 1: Identify the common difference.

50 is subtracted from the current term, u_n, to find the next term, u_{n+1}, so the common difference, d, is -50.

Step 2: Determine whether the common difference is positive or negative.

The common difference is negative, so the sequence will demonstrate linear decay.

Answer

Linear decay

b. What does u_0 mean in the context of this scenario?

Explanation

The initial value is $u_0 = 120$. This is the number of umbrellas at the store when $n = 0$.

Answer

There were initially 120 umbrellas when the store restocked their shelves.

Constructing an arithmetic recurrence relation

An arithmetic recurrence relation can be determined from a sequence of numbers that demonstrate linear growth or decay, once the initial value and common difference is known.

The initial value, a, is the first value in the sequence.

The common difference, d, can be calculated as the difference between any two consecutive terms in a sequence.

These values can then be substituted into the form for an arithmetic recurrence relation,

$u_0 = a, \quad u_{n+1} = u_n + d.$

Worked example 4

Write down a recurrence relation in terms of u_0, u_{n+1} and u_n to model the following sequences.

a. 0, 5, 10, 15, 20, 25...

Explanation

Step 1: Identify the initial value.

The first term of the sequence is 0, so $a = 0$.

Step 2: Calculate the common difference.

This can be done using any two consecutive terms.

Using $u_1 = 5$ and $u_0 = 0$,

$$d = 5 - 0$$
$$= 5$$

This can be verified by calculating the common difference using other terms in the same sequence.

Using $u_3 = 15$ and $u_2 = 10$,

$$d = 15 - 10$$
$$= 5$$

Continues →

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Step 3: Construct the recurrence relation.

Substitute $a = 0$ and $d = 5$ into
$u_0 = a, \quad u_{n+1} = u_n + d.$

Answer

$u_0 = 0, \quad u_{n+1} = u_n + 5$

b. A tree sapling was initially 5 cm tall when it was planted. Each day, the plant grows by 0.02 cm. Let u_n represent the height of the sapling n days after it was planted, in centimetres.

Explanation

Step 1: Identify the initial value.

The sapling was initially 5 cm tall, so $a = 5$.

Step 2: Calculate the common difference.

The sapling grows by 0.02 cm each day so d must be positive.

$d = 0.02$

Step 3: Construct the recurrence relation.

Substitute $a = 5$ and $d = 0.02$ into
$u_0 = a, \quad u_{n+1} = u_n + d.$

Answer

$u_0 = 5, \quad u_{n+1} = u_n + 0.02$

3B Questions

Generating a sequence from an arithmetic recurrence relation

1. Consider the following recurrence relation.

$t_0 = -3, \quad t_{n+1} = t_n + 1$

What is the value of t_2?

A. -3 **B.** -2 **C.** -1 **D.** 0

2. Find t_3 for the following recurrence relations.

a. $t_0 = 3, \quad t_{n+1} = t_n + 6$

b. $t_0 = 16, \quad t_{n+1} = t_n - 7$

3. Generate the first five terms for the following recurrence relations.

a. $u_0 = -5046, \quad u_{n+1} = u_n + 50$

b. $u_0 = 3216, \quad u_{n+1} = u_n - 78$

4. The population of Australia in 1900 was around 3.76 million people. Over the years 1900–1904, the population can be roughly modelled using the following recurrence relation, where P_0 is the population in 1900.

$P_0 = 3\,760\,000, \quad P_{n+1} = P_n + 65\,000$

a. What was Australia's population in 1903?

b. What was Australia's population in 1904?

Interpreting an arithmetic recurrence relation

5. The following recurrence relation can be used to model the number of user accounts a social media company has.

$S_0 = 2\,000\,000, \quad S_{n+1} = S_n + 1600$

Which of the following statements is true?

A. There were initially 2 000 000 accounts when observations began.

B. There were initially 1600 accounts when observations began.

C. 2 000 000 additional accounts are created every day.

D. 1600 accounts are deleted every day.

---

6. Consider the following recurrence relation, where V_n is the number of varsity jackets owned by a football team after n games.

$V_0 = 12, \quad V_{n+1} = V_n + 1$

a. How many varsity jackets did the football team initially have?

b. Interpret the common difference, d.

---

7. The fish population in a pond after n days of observation, F_n, can be modelled using the following recurrence relation.

$F_0 = 1500, \quad F_{n+1} = F_n - 20$

a. Interpret the initial value, a.

b. Interpret the common difference, d.

---

8. A biotechnologist is attempting to culture a colony of bacteria to test the efficacy of her company's disinfectant. The expected number of bacterial cells in this culture, in thousands, can be represented by the following recurrence relation, where n represents the number of hours after exposure.

$B_0 = 5000, \quad B_{n+1} = B_n - 20$

a. Will this colony demonstrate linear growth or decay?

b. Describe the pattern in words.

Constructing an arithmetic recurrence relation

9. Consider the first five terms of the following sequence.

98, 97, 96, 95, 94, ...

This can be generated using the recurrence relation

A. $u_0 = 98, \quad u_{n+1} = u_n - 1$

B. $u_0 = 98, \quad u_{n+1} = u_n + 1$

C. $u_0 = 99, \quad u_{n+1} = u_n - 1$

D. $u_0 = 99, \quad u_{n+1} = u_n + 1$

---

10. Consider the first five terms of the following sequence.

105, 102, 99, 96, 93, ...

Construct a recurrence relation that can be used to model this sequence, where u_n is the value of the sequence after n iterations.

---

11. Jenna is monitoring the number of books on her bookshelf on a monthly basis. She purchases the same number of books each month.

Two months after her initial count, she has 22 books.

Four months after her initial count, she has 38 books.

Let B_n be the number of books on her bookshelf, after n months. Construct a recurrence relation that can be used to model the value of B_n.

Joining it all together

12. Jackson initially makes 23 cups of lemonade for his lemonade stand.

Each day, he will make 2 more cups than the previous day.

a. How many cups will he make after 5 days?

b. Construct a recurrence relation that can model the number of lemonade cups made after n days, L_n.

c. After how many days will it take Jackson to make 37 lemonade cups?

---

13. Ava's height, in metres, from 12 years of age to 17 years of age is modelled by the following recurrence relation.

$h_0 = 1.25, \quad h_{n+1} = h_n + 0.05$

How tall will she be at age 17?

---

14. Charlotte heard that human hair grows an additional 0.08 cm each day. She measures her hair on the third of April at 23.84 cm long. One week later, she measures it again.

a. Construct a recurrence relation that can be used to model the length of Charlotte's hair from the day of her initial measurement, where h_n is the length of hair in centimetres after n days.

b. Use the recurrence relation from part **a** to work out how long her hair will be one week after April 3rd.

Exam practice

15. Miguelle owns a vinyl collection.

The collection grows as Miguelle's favourite artists continue to release new albums.

The number of vinyls, V_n, after n years of collecting can be modelled by the recurrence relation.

$V_0 = 35, \quad V_{n+1} = V_n + 3$

a. How many vinyls does Miguelle collect each year? (1 MARK)

b. Showing recursive calculations, calculate the number of vinyls Miguelle owns, after two years of collecting. (1 MARK)

Adapted from VCAA 2020 Exam 2 Recursion and financial modelling Q7a,b

Part **a**: **94%** of students answered this type of question correctly.
Part **b**: **72%** of students answered this type of question correctly.

---

16. The following recurrence relation can generate a sequence of numbers.

$S_0 = 7, \quad S_{n+1} = S_n + 4$

The number 11 appears in this sequence as

A. S_1 **B.** S_2 **C.** S_3
D. S_{10} **E.** S_{11}

Adapted from VCAA 2020 Exam 1 Recursion and financial modelling Q21

85% of students answered this type of question correctly.

Questions from multiple lessons

Recursion and financial modelling

17. Lorraine has started a small business selling homemade pottery.

Her business's monthly profit forms an arithmetic sequence.

The following table shows her profit in months 1 to 3.

If this pattern continues, what will Lorraine's profit be in month seven?

month number	*profit* ($)
1	170
2	215
3	260

A. \$380 **B.** \$395 **C.** \$415
D. \$440 **E.** \$485

Adapted from VCAA 2009 Exam 1 Number patterns Q2

Data analysis

18. Lachie is passionate about dogs.

Recently, he made his way to the local dog park and measured the *height*, *weight* and *length* of the first 13 dogs he could find, and recorded them in the following table.

height (cm)	*weight* (kg)	*length* (cm)
58.2	25.8	55.1
20.1	9.7	22.5
38.4	15.9	32.9
23.9	12.1	28.0
61.2	31.9	70.5
34.1	19.1	30.1
42.1	18.1	38.1
50.1	21.1	51.3
29.1	8.4	34.7
32.7	13.8	29.1
38.1	17.1	35.0
35.9	19.2	38.1
40.1	20.1	41.1

The mean, $\overline{x}$, and the standard deviation, s_x, for the *height* of these dogs, in centimetres, are closest to

A. $\overline{x} = 38.7 \quad s_x = 12.06$

B. $\overline{x} = 38.7 \quad s_x = 12.08$

C. $\overline{x} = 38.8 \quad s_x = 12.06$

D. $\overline{x} = 38.8 \quad s_x = 12.08$

E. $\overline{x} = 41.8 \quad s_x = 11.18$

Adapted from VCAA 2018NH Exam 1 Data analysis Q7

Recursion and financial modelling *Year 10 content*

19. A large umbrella factory wants to determine the number of umbrellas they can produce each hour.

In the 1st hour, they produced 30 000 umbrellas.

In the 2nd hour, they produced 32 000 umbrellas.

In the 3rd hour, they produced 34 000 umbrellas.

The number of umbrellas produced in each hour continues in this pattern for 10 hours.

The amount of stock produced in each hour forms the terms of an arithmetic sequence, as shown.

30 000, 32 000, 34 000...

a. Show that the common difference for this sequence is 2000. (1 MARK)

b. How many umbrellas will be produced during the 8th hour? (1 MARK)

c. In total, how many umbrellas can be produced in the first five hours? (1 MARK)

Adapted from VCAA 2015 Exam 2 Number patterns Q1a-c

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3C Arithmetic recursion applications

STUDY DESIGN DOT POINT

- use of a first-order linear recurrence relation of the form $u_0 = a$, $u_{n+1} = u_n + d$, where a and d are constants, to model and analyse practical situations involving discrete linear growth or decay such as a simple interest loan or investment, the depreciating value of an asset using the unit cost or flat rate method

3A 3B 3C 3D 3E 3F

KEY SKILLS

During this lesson, you will be:

- modelling simple interest investments using recurrence relations
- modelling unit cost depreciation using recurrence relations
- modelling flat rate depreciation using recurrence relations.

KEY TERMS

- Simple interest investment
- Depreciation
- Unit cost depreciation
- Flat rate depreciation

There are several financial circumstances that can be modelled using arithmetic recurrence relations. They can be used to model simple interest investments and depreciation (unit cost and flat rate), as well as calculating the value of an item or investment after a certain period of time.

Modelling simple interest investments using recurrence relations

Arithmetic recurrence relations of the form $u_0 = a, \quad u_{n+1} = u_n + d$ can model simple interest investments. For a recurrence relation of this form, u_n is the current term and u_{n+1} is the next term. The recurrence relation can be used to calculate the value of an investment after n periods, u_n.

A **simple interest investment** is where the interest earned is calculated as a fixed percentage of the principal, meaning that the investment increases by the same amount each period.

In the recurrence relation for a simple interest investment

- a is the initial investment, or principal
- d is the interest earned each period, and is always positive.

Worked example 1

Glenys has \$15 000 to invest. She decides to put it in a simple interest savings account, earning 5% p.a. Payments are made monthly.

a. Construct a recurrence relation to model this scenario.

Explanation

Step 1: Determine the value of a.

Glenys invests \$15 000.

$a = 15\ 000$

Continues →

Step 2: Calculate the regular payment, d.

Glenys earns 5% p.a. This means she earns $15\ 000 \times 0.05 = \$750$ in interest per year.

As payments are made monthly.

$$d = 750 \div 12$$
$$= 62.5$$

Step 3: Construct the recurrence relation.

Answer

$u_0 = 15\ 000, \quad u_{n+1} = u_n + 62.5$

---

b. Calculate the value of the investment after 7 months.

Explanation

Use recursion to calculate the value of the investment after 7 months.

$u_0 = 15\ 000$

$u_1 = 15\ 000 + 62.5 = 15\ 062.5$

$u_2 = 15\ 062.5 + 62.5 = 15\ 125$

$u_3 = 15\ 125 + 62.5 = 15\ 187.5$

$u_4 = 15\ 187.5 + 62.5 = 15\ 250$

$u_5 = 15\ 250 + 62.5 = 15\ 312.5$

$u_6 = 15\ 312.5 + 62.5 = 15\ 375$

$u_7 = 15\ 375 + 62.5 = 15\ 437.5$

Answer

\$15 437.50

Modelling unit cost depreciation using recurrence relations

The loss of value of an asset is known as **depreciation**. One type of depreciation that can be modelled using an arithmetic recurrence relation is unit cost depreciation.

Unit cost depreciation is used when an asset loses value after each unit of use. For example, a washing machine might lose 10 cents in value for each wash. The unit of use is often different between cases. The unit of use for a washing machine could be a wash, while the unit of use for a car could be kilometres driven.

Unit cost depreciation can be modelled using a recurrence relation of the form $u_0 = a, \quad u_{n+1} = u_n - d$, where

- a is the initial value of the asset
- d is the depreciation per unit of use
- u_n is the value of the asset after n units of use.

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Worked example 2

Sebastian purchased a camera for \$1500. The value of the camera depreciates by 2 cents for each photo taken.
Construct a recurrence relation to model this scenario.

Explanation

Step 1: Determine the value of a.

The initial value of the camera was \$1500.

$a = 1500$

Step 2: Determine the value of d.

The camera depreciates by 2 cents, or \$0.02 per photo taken.

$d = 0.02$

Step 3: Construct the recurrence relation.

Answer

$u_0 = 1500, \quad u_{n+1} = u_n - 0.02$

Modelling flat rate depreciation using recurrence relations

Another type of depreciation that can be modelled using an arithmetic recurrence relation is flat rate depreciation. **Flat rate depreciation** is used when the value of an asset decreases by a constant amount for each specified period. This amount is a percentage of the initial value of the asset. For example, flat rate depreciation could describe the value of a computer as it gets outdated each year. A computer that had an initial value of \$2200 and depreciates at a flat rate of 10% per year will depreciate by \$220 per year. After 10 years, it will have no value.

Flat rate depreciation can be modelled using a recurrence relation of the form
$u_0 = a, \quad u_{n+1} = u_n - d$, where

- a is the initial value of the asset
- d is the depreciation per period
- u_n is the value of the asset after n periods.

Worked example 3

Lyndell purchased a fridge for \$900. It depreciates at a flat rate of 12.5% each year.

a. Construct a recurrence relation to model this scenario.

Explanation

Step 1: Determine the value of a.

The initial value of the fridge was \$900.

$a = 900$

Step 2: Calculate the value of d.

The rate of depreciation is 12.5% p.a. This means it depreciates by $0.125 \times 900 = \$112.50$ each year.

$d = 112.5$

Step 3: Construct the recurrence relation.

Answer

$u_0 = 900, \quad u_{n+1} = u_n - 112.5$

Continues →

b. After how many years will the fridge have no value?

Explanation

The fridge has an initial value of \$900, and depreciates by \$112.50 each year.

$900 \div 112.5 = 8$

Alternatively, as the asset will have no value once 100% of the value has been depreciated, 100% can be divided by the rate of depreciation.

$100 \div 12.5 = 8$

Answer

8 years

3C Questions

Modelling simple interest investments using recurrence relations

1. The following recurrence relation shows the value of a simple interest investment after n months.

$u_0 = 4000, \quad u_{n+1} = u_n + 50$

The amount of interest earned annually for this investment is

A. \$1.25 **B.** \$15 **C.** \$50 **D.** \$600

2. For each of the following scenarios, construct a recurrence relation and calculate the value after 4 payments.

a. Melanie invests \$80 000 in an account that pays simple interest at a rate of 9.75% p.a. Payments are received weekly.

b. Douglas invests \$1 025 000 in an account that pays 6% simple interest p.a. The account will receive quarterly payments.

c. Stanley invests a sum of money that earns 7% simple interest p.a. He receives annual payments of \$700.

d. Geraldine invests her savings in a simple interest account earning 4.5% p.a. The account receives quarterly payments of \$159.75.

3. Henry inherits \$18 000 and wants to invest it wisely. He decides to invest it in a simple interest account that will provide him with monthly payments of \$150.

His friend Athira invests \$23 920 in a simple interest account. She receives weekly payments of \$41.40.

a. Whose account has a higher interest rate?

b. Construct a recurrence relation to model the value of each investment after n payments.

c. Calculate the value of each investment after 8 payments.

4. Hayden is saving up to buy a new tractor for his farm. He wants to invest enough money to be earning \$350 per month in interest.

His bank offers him an interest rate of 5% p.a.

a. How much would Hayden need to invest to meet his earnings goal?

b. Construct a recurrence relation to model Hayden's investment.

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c. The tractor he wishes to purchase is worth $87 000. How many months will it take for Hayden to be able to afford the tractor?

5. Amir has $20 000 to invest. His bank provides two options that earn simple interest.

Option 1: 7% p.a., with annual payments.

Option 2: 5.98% p.a., with quarterly payments.

He wants to purchase a rare comic book worth $22 000.

Which option will allow him to purchase the comic book sooner?

Modelling unit cost depreciation using recurrence relations

6. Sonoko owns a toastie machine. It cost them $300 and depreciates using the unit cost method by $0.50 per toastie.

For a recurrence relation of the form $u_0 = a, \quad u_{n+1} = u_n - d$, where d is the depreciation per toastie, a and d are given by

A. $a = 0.5, \quad d = 300$ **B.** $a = 300, \quad d = 0.5$ **C.** $a = 300, \quad d = 0.17\%$ **D.** $a = 300, \quad d = 50$

7. For each of the following scenarios, construct a recurrence relation and calculate the value of the asset after 5 units of use.

a. A $20 000 car depreciates by $0.02 for each kilometre driven.

b. A $600 coffee machine depreciates by $1.50 for each coffee made.

c. A $260 pair of running shoes depreciates by $5 for every 100 000 steps.

d. A $799 gaming console depreciates by $10 for every 20 hours of use.

8. A $750 dishwasher has a value of $187.50 after 6 years.

The dishwasher was used an average of 125 times per year every year during those 6 years, and was depreciated using the unit cost method of depreciation.

a. How many washes has the dishwasher completed over the 6 years?

b. What is the difference between the initial value of the dishwasher and its current value?

c. How much does the dishwasher depreciate per wash?

d. Construct a recurrence relation to model the value of the dishwasher after n washes.

9. Percival owns a machine that prints designs on t-shirts. The machine was purchased for $7500 and was worth $7112.80 after 6 months. In this time, Percival printed 352 t-shirts.

Assuming the machine depreciates for each t-shirt printed, construct a recurrence relation to model the value of Percival's machine, and use recursion to show that the value of the machine was worth $7496.70 after 3 t-shirts were printed.

Modelling flat rate depreciation using recurrence relations

10. The following recurrence relation models the depreciating value of an asset after n months using the flat rate method.

$u_0 = 5000, \quad u_{n+1} = u_n - 125$

The depreciation rate of the asset per month is

A. 2.5% **B.** 12.5% **C.** 30% **D.** 125%

11. For each of the following scenarios, construct a recurrence relation and calculate how long it will take, in years (and months where applicable), for the asset to have no value.
    a. An \$800 fridge that depreciates at a flat rate of 12.5% every year.
    b. A \$1100 chandelier that depreciates at a flat rate of 2% every month.
    c. A \$100 set of baseball cards that depreciates by 6.25% quarterly at a flat rate.
    d. A \$650 cricket bat that depreciates by 2% every two months.

12. A new phone was purchased for \$1200. After 2 years, the phone was worth \$1050, and depreciates annually using the flat rate method.
    a. What is the rate of depreciation per period?
    b. Construct a recurrence relation to model the value of the phone after *n* years.
    c. What is the value of the phone after 5 years?

13. Alonzo purchased a luxury watch that depreciates at a flat rate of 5% every 6 months. After 3 years, the watch was worth \$17 500.

    Construct a recurrence relation to model the value of Alonzo's watch after *n* periods, and use recursion to show that the value of the watch after 2 years is \$20 000.

Joining it all together

14. Consider the following recurrence relation.

    $u_0 = 100\,000, \quad u_{n+1} = u_n - 1000$

    This recurrence relation could **not** model

    A. a simple interest investment.
    B. unit cost depreciation.
    C. flat rate depreciation of 1% per month.
    D. flat rate depreciation of 1% per year.

15. Fierro is saving to buy a castle, and wants to invest enough money to earn \$9600 per month in interest. His bank offers to pay 6% p.a. with monthly payments.
    a. Construct a recurrence relation to model the value of his investment after *n* months.
    b. After one year, he decides to take the money in his investment and use all of it to purchase a castle. Unfortunately for Fierro, the castle is in an unpopular location, and its value depreciates by a flat rate of 6.25% every 6 months.

       Construct a recurrence relation to model the value of Fierro's castle after *n* depreciation periods.
    c. The castle stops depreciating after 2 and a half years, and doesn't change in value. At this point, Fierro decides to sell the castle. Does Fierro end up with more or less money than he initially invested, and by how much?

16. Shannon inherits \$39 000 and decides to invest it in a simple interest account.

    The account earns 6.5% p.a., and payments are made quarterly.
    a. Construct a recurrence relation to model Shannon's investment.
    b. Shannon wants to purchase a top quality 3D printer worth \$45 000. How long, in years and months, will it be until she can afford the printer using the money from her investment?
    c. Once she is able to, she purchases the printer. It depreciates by \$1250 for every 120 hours of use. Construct a recurrence relation to model the value of the printer after *n* units of use.
    d. Calculate the value of the printer after 480 hours of use.
    e. The value of the printer can also be modelled using the flat rate method. If the printer is used for an average of 240 hours each year, what is the annual flat rate of depreciation correct to two decimal places?

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Exam practice

17. Xavier owns a set of DJ decks worth \$3200.

The decks are depreciated in value by Xavier using flat rate depreciation.

The value of the decks, in dollars, after n years, u_n, can be modelled by the recurrence relation

$u_0 = 3200, \quad u_{n+1} = u_n - 160$

a. Showing recursive calculations, determine the value of the decks, in dollars, after three years. (1 MARK)

b. What annual flat rate percentage of depreciation is used by Xavier? (1 MARK)

Adapted from VCAA 2020 Exam 2 Recursion and financial modelling Q7b,c

Part **a**: **72%** of students answered this type of question correctly.
Part **b**: **75%** of students answered this type of question correctly.

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18. Sienna owns a coffee shop.

A coffee machine, purchased for \$12 000, is depreciated in value using the unit cost method.

The rate of depreciation is \$0.05 per cup of coffee made.

The recurrence relation that models the year-to-year value, in dollars, of the coffee machine is

$u_0 = 12\ 000, \quad u_{n+1} = u_n - 1440$

Calculate the number of cups of coffee that the machine produces per year. (1 MARK)

VCAA 2021 Exam 2 Recursion and financial modelling Q7a

54% of students answered this question correctly.

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19. Anton is a musician, and he purchases a minivan for \$64 000 to transport equipment.

After four years, the value of the minivan is \$40 000.

The minivan travelled an average of 8000 km in each of the four years since it was purchased.

Assume that the value of the minivan has been depreciated using the unit cost method of depreciation.

By how much is the value of the minivan reduced per kilometre travelled? (1 MARK)

Adapted from VCAA 2016 Exam 2 Recursion and financial modelling Q6c

29% of students answered this type of question correctly.

Questions from multiple lessons

Recursion and financial modelling *Year 10 content*

20. As part of a promotion, a new customer is offered to have his new air conditioner professionally installed in his home for only \$200. However, he must have it installed within the week that he purchases it.

Next week, the charge will increase by 5.2%.

The charge next week will be

A. \$189.60 **B.** \$200.00 **C.** \$210.00 **D.** \$210.40 **E.** \$215.60

Adapted from VCAA 2014 Exam 1 Business-related mathematics Q1

Data analysis *Year 10 content*

21. The variables *car type* (1: Tesla, 2: Mustang, 3: Other) and *number plate type* (custom, auto-generated) are

A. both nominal variables.

B. both ordinal variables.

C. a numerical variable and a categorical variable respectively.

D. a nominal variable and an ordinal variable respectively.

E. an ordinal variable and a nominal variable respectively.

Adapted from VCAA 2017 Exam 1 Data analysis Q7

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Recursion and financial modelling

22. Claire decides to borrow $7000 to purchase a car.

a. If she decides to borrow $7000 from the bank, she will pay interest at a simple interest rate of 4.3% per month. Calculate the interest she will pay each month. (1 MARK)

b. If she borrows from the dealer's finance company, she will pay $8.75 in interest per month. Calculate the annual simple interest rate charged. (1 MARK)

Adapted from VCAA 2009 Exam 2 Business-related mathematics Q2a,b

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3D Geometric sequences and recurrence relations

STUDY DESIGN DOT POINT

- use of a first-order linear recurrence relation of the form $u_0 = a$, $u_{n+1} = Ru_n$, where a and R are constants, to generate the values of a geometric sequence

3A 3B 3C **3D** 3E 3F

KEY SKILLS

During this lesson, you will be:

- generating a sequence from a geometric recurrence relation
- interpreting a geometric recurrence relation
- constructing a geometric recurrence relation.

KEY TERMS

- Common ratio
- Geometric growth
- Geometric decay

Geometric sequences are used to model scenarios where there is repeated multiplication by the same number. These scenarios can include the populations of animals or the balances of savings accounts. Like with arithmetic sequences, recurrence relations can be used to model geometric sequences.

Generating a sequence from a geometric recurrence relation

See worked example 1

Recall that a geometric sequence is a pattern that has repeated multiplication by the same number. The amount each term is multiplied by to give the next term in a sequence of numbers is called the **common ratio**.

The recurrence relation of a geometric sequence has the form

$u_0 = a, \quad u_{n+1} = Ru_n$, where

- a is the initial value
- R is the common ratio

Here, u_{n+1} represents the next term in the sequence while u_n represents the current term.

See worked example 2

Like with arithmetic sequences, a calculator can be used to generate the terms in a geometric sequence from a recurrence relation.

Worked example 1

Consider the following recurrence relation.

$u_0 = 2, \quad u_{n+1} = 3u_n$

Calculate the value of u_3.

Explanation

Step 1: Calculate u_1 from u_0.

$$\begin{aligned} u_1 &= 3u_0 \\ &= 3 \times 2 \\ &= 6 \end{aligned}$$

Step 2: Calculate u_2 from u_1.

$$\begin{aligned} u_2 &= 3u_1 \\ &= 3 \times 6 \\ &= 18 \end{aligned}$$

Continues →

Step 3: Calculate u_3 from u_2.

$$u_3 = 3u_2$$
$$= 3 \times 18$$
$$= 54$$

Answer

54

Worked example 2

Consider the following recurrence relation.

$u_0 = 45, \quad u_{n+1} = 0.6u_n$

Calculate the value of the first five terms in the sequence.

Explanation – Method 1: TI-Nspire

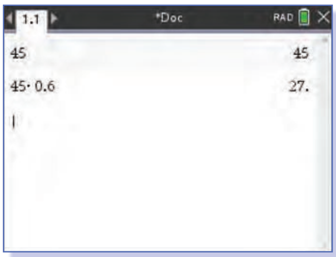
Step 1: From the home screen, select '1: New' → '1: Add calculator'.

Step 2: Enter the initial value by typing '45'. Press enter.

Step 3: Press '×'.

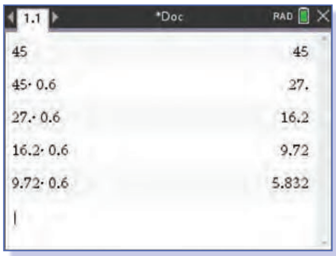
'Ans·' will appear to indicate that it is multiplying by the previous answer.

Type '0.6'. Press enter.



Step 4: To find the next value in the sequence, press enter.

Step 5: Repeat step 4 for each of the remaining first five terms.



Explanation – Method 2: Casio ClassPad

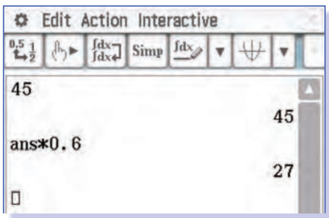
Step 1: From the menu, tap √α Main.

Step 2: Enter the initial value by typing '45'. Press EXE.

Step 3: Type '×'.

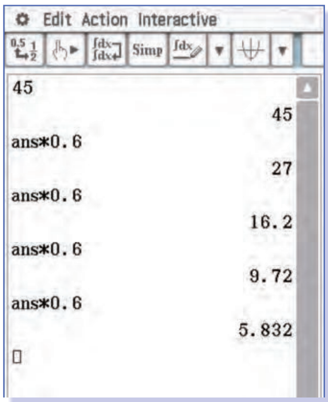
'Ans∗' will appear to indicate that it is multiplying by the previous answer.

Type '0.6' and press EXE.



Step 4: To find the next value in the sequence, press EXE.

Step 5: Repeat step 4 for each of the remaining first five terms.



Answer – Method 1 and 2

45, 27, 16.2, 9.72, 5.832

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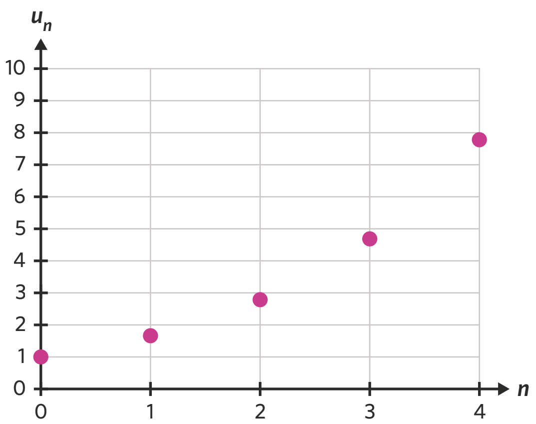
Interpreting a geometric recurrence relation

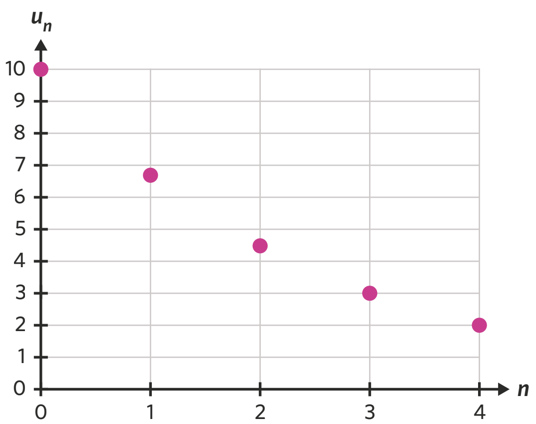
A geometric recurrence relation with a common ratio greater than one ($R > 1$) will generate a sequence that demonstrates **geometric growth**, where the sequence is increasing.

For example, the first five terms generated by the recurrence relation $u_0 = 1, \quad u_{n+1} = 1.67u_n$ have been plotted on the following graph.



On the other hand, a common ratio that is greater than 0 but less than 1 ($0 < R < 1$) will generate a sequence that demonstrates **geometric decay**, where the sequence is decreasing.

For example, the first five terms generated by the recurrence relation $u_0 = 10, \quad u_{n+1} = 0.67u_n$ have been plotted on the following graph.



Worked example 3

Let u_n represent the value, in dollars, of a laptop n months after it was purchased.

$u_0 = 800, \quad u_{n+1} = \frac{7}{8}u_n$

a. Determine if the sequence will have geometric growth or decay.

Explanation

Step 1: Identify the common ratio.

The current term, u_n, is multiplied by $\frac{7}{8}$ to find the next term, u_{n+1}.

$R = \frac{7}{8}$

Step 2: Determine whether the common ratio is greater than or less than 1.

The common ratio is less than 1 $\left(\frac{7}{8} = 0.875\right)$, so the sequence will demonstrate geometric decay.

Answer

Geometric decay

Continues →

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b. What does u_0 mean in the context of this scenario?

Explanation

The initial value is $u_0 = 800$.

This is the value of the laptop when $n = 0$.

Answer

The purchase price of the laptop was $800.

Constructing a geometric recurrence relation

A geometric recurrence relation can be determined from a sequence of numbers that demonstrate geometric growth or decay, once the initial value and common ratio is known.

The initial value, a, is the first value in the sequence.

The common ratio, R, can be calculated as the ratio between any two consecutive terms in a sequence. That is, by dividing one term by the previous term.

These values can then be substituted into the form for a geometric recurrence relation,

$u_0 = a, \quad u_{n+1} = Ru_n$

Worked example 4

Write down a recurrence relation in terms of u_0, u_{n+1} and u_n to model the following sequences.

a. 4, 8, 16, 32, 64

Explanation

Step 1: Identify the initial value.

The first term of the sequence is 4, so $a = 4$.

Step 2: Calculate the common ratio.

This can be done using any two consecutive terms.

Using $u_1 = 8$ and $u_0 = 4$,

$$R = \frac{8}{4} = 2$$

This can be verified by calculating the common ratio using other terms in the same sequence.

Using $u_2 = 16$ and $u_1 = 8$,

$$R = \frac{16}{8} = 2$$

Step 3: Construct the recurrence relation.

Substitute $a = 4$ and $R = 2$ into $u_0 = a, \quad u_{n+1} = Ru_n$.

Answer

$u_0 = 4, \quad u_{n+1} = 2u_n$

Continues →

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b. A substance is weighed and has a mass of 30 g. Every week, the mass halves due to radioactive decay. Let u_n represent the mass of the substance, in grams, n weeks it was first weighed.

Explanation

Step 1: Identify the initial value.

The substance's initial mass is 30 g, so $a = 30$.

Step 2: Calculate the common ratio.

The mass of the substance halves each week.

$R = 0.5$

Step 3: Construct the recurrence relation.

Substitute $a = 30$ and $R = 0.5$ into $u_0 = a, \quad u_{n+1} = Ru_n$.

Answer

$u_0 = 30, \quad u_{n+1} = 0.5u_n$

3D Questions

Generating a sequence from a geometric recurrence relation

1. Consider the following recurrence relation.

$g_0 = 9, \quad g_{n+1} = 2.1g_n$

The value of g_2 is closest to

A. 2.1 **B.** 9.0 **C.** 18.9 **D.** 39.7

2. Find t_3 for the following recurrence relations, rounded to two decimal places where necessary.

a. $t_0 = 2, \quad t_{n+1} = 4t_n$

b. $t_0 = 84, \quad t_{n+1} = \frac{1}{6}t_n$

3. Generate the first five terms for the following recurrence relations, rounded to two decimal places where necessary.

a. $t_0 = -10, \quad t_{n+1} = 5t_n$

b. $t_0 = 25, \quad t_{n+1} = 3.5t_n$

4. The population of rabbits in a forest is given by $p_{n+1} = 3p_n$ where p_n is the population of rabbits after n years. The initial population of rabbits, p_0, in 2020 was 52.

a. What is the population of rabbits in 2022?

b. In which year will the population of rabbits first be greater than 10 000?

Interpreting a geometric recurrence relation

5. The following recurrence relation can be used to model the population of birds in an aviary after n years.

$B_0 = 30, \quad B_{n+1} = 3B_n$

Which of the following statements is true?

A. There were initially 3 birds.

B. The number of birds increases by 30 each year.

C. The number of birds in one year is 3 times the number of birds the previous year.

D. There were initially 90 birds.

6. The following recurrence relation models the average cost of a month's supply of groceries, C_n, n years after the year 2000.

   $C_0 = 300, \quad C_{n+1} = 1.03C_n$

   a. Does the sequence demonstrate geometric growth or decay?

   b. What does C_0 mean in the context of this scenario?

---

7. Consider the following recurrence relation, where P_n represents the number of passengers on a train travelling from Flinders Street Station to Sandringham, and n is the train station number.

   $P_0 = 542, \quad P_{n+1} = \frac{3}{5}P_n$

   a. Interpret the initial value, a.

   b. Interpret the common ratio, R.

Constructing a geometric recurrence relation

8. Consider the following sequence.

   24, 36, 54, 81, ...

   The terms can be generated using the recurrence relation

   A. $u_0 = 16, \quad u_{n+1} = u_n + 8$
   B. $u_0 = 16, \quad u_{n+1} = 1.5u_n$
   C. $u_0 = 24, \quad u_{n+1} = u_n + 12$
   D. $u_0 = 24, \quad u_{n+1} = 1.5u_n$

---

9. Consider the following geometric sequence.

   162, 54, 18, 6, 2, ...

   Construct a recurrence relation that can be used to model this sequence, where u_n is the value of the sequence after n iterations.

---

10. A new company is projecting its profits over a number of weeks. They predict that their profits each week can be modelled by a geometric sequence.

    Three weeks after they started, the company's projected profit is $10 985.00.

    Four weeks after they started, the company's projected profit is $14 280.50.

    Let P_n be the projected profit, in dollars, n weeks after the company started tracking their profits.

    a. What is the common ratio of the sequence?

    b. Calculate the initial value.

    c. Construct a recurrence relation that can be used to model the value of P_n.

Joining it all together

11. Yunku drops a ball from a height of 40 cm. After it first bounces, it reaches a height of 24 cm. The height of the ball, h_n, after n bounces can be modelled by a geometric sequence.

    a. Construct a recurrence relation that models the height of the ball.

    b. Calculate h_2, h_3 and h_4 rounded to the nearest centimetre.

    c. Does the sequence demonstrate geometric growth or geometric decay?

12. Linda is growing a culture of bacteria to observe their growth rate. The following table shows the number of bacteria counted at hourly intervals. The number of bacteria, B_n, after n hours, can be modelled using a geometric sequence.

time (hours)	*number of bacteria*
0	7
1	x
2	28

a. Find the value of x.

b. Construct a recurrence relation to model the number of bacteria.

c. How many bacteria would there be after 6 hours?

d. After how many hours will the number of bacteria first exceed 1000?

13. The population of cats on a farm is given by the recurrence relation

$C_0 = 2, \quad C_{n+1} = 2C_n$

where C_n is the number of cats after n years.

On the same farm, the population of mice is given by the recurrence relation

$M_0 = 64, \quad M_{n+1} = 0.68M_n$

where M_n is the number of mice after n years.

a. What do C_0 and M_0 represent?

b. Which sequence demonstrates geometric growth?

c. How many cats and how many mice are there after the first and second years? Round down to the nearest whole number.

d. After how many years will the population of cats first be greater than the population of mice?

Exam practice

14. Ken has opened a savings account to save money to buy a new caravan.

The amount of money in the savings account after n years, V_n, can be modelled by the following recurrence relation.

$V_0 = 15\ 000, \quad V_{n+1} = 1.04 \times V_n$

How much money did Ken initially deposit into the savings account? (1 MARK)

VCAA 2016 Exam 2 Recursion and financial modelling Q5a

92% of students answered this question correctly.

15. Amahle opens a savings account.

Let B_n be the balance of this savings account, in dollars, n months after it was opened.

The month-to-month value of B_n can be determined using the following recurrence relation.

$B_0 = 6000, \quad B_{n+1} = 1.002B_n$

Write down the value of B_3, the balance of the savings account after three months.

Round to the nearest cent. (1 MARK)

Adapted from VCAA 2020 Exam 2 Recursion and financial modelling Q9a

64% of students answered this type of question correctly.

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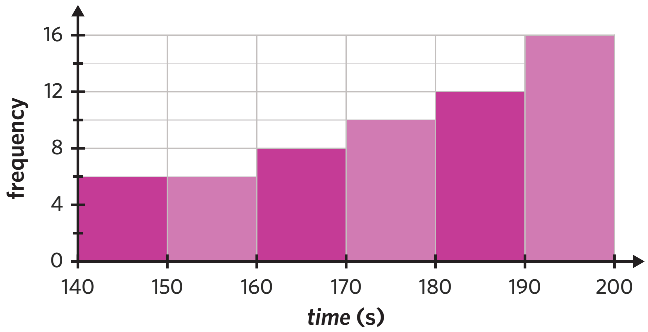
Questions from multiple lessons

Data analysis

16. The following histogram displays the time taken in seconds for a group of primary school students to run 800 metres.



The shape of this distribution can be described as

A. symmetric.

B. approximately symmetric.

C. approximately normal.

D. negatively skewed.

E. positively skewed.

Adapted from VCAA 2017NH Exam 1 Data analysis Q1

Recursion and financial modelling

17. Beatrice has decided to pursue a career as an Instagram influencer.

Before introducing strategies to increase her following, Beatrice has 200 Instagram followers.

Each subsequent day, Beatrice has 20 more followers than she did the day before.

Let F_n be the number of followers Beatrice has n days after the first day.

A recurrence relation that can be used to model this is

A. $F_0 = 200, \quad F_{n+1} = 1.10F_n$

B. $F_0 = 200, \quad F_{n+1} = 1.20F_n$

C. $F_0 = 200, \quad F_{n+1} = F_n + 20$

D. $F_0 = 200, \quad F_{n+1} = F_n + 10$

E. $F_0 = 200, \quad F_{n+1} = F_n + 0.10$

Adapted from VCAA 2014 Exam 1 Number patterns Q4

Recursion and financial modelling

18. An investment of \$1 000 000 was put into a savings account modelled by the following recurrence relation, where V_n is the value of the investment after n years.

$V_0 = 1\,000\,000, \quad V_{n+1} = 1.06 \times V_n$

a. What is the balance of the savings account five years after the investment was made?

Round to the nearest cent. (1 MARK)

b. After how many years will the value of the investment first exceed \$1 500 000? (1 MARK)

Adapted from VCAA 2012 Exam 2 Number patterns Q3a,b

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3E Geometric recursion applications

STUDY DESIGN DOT POINT

- use of a first-order linear recurrence relation of the form $u_0 = a$, $u_{n+1} = Ru_n$, where a and R are constants, to model growth and decay and analyse practical situations involving geometric sequences such as the reducing height of a bouncing ball, reducing balance depreciation, compound interest loans or investments

3A 3B 3C 3D 3E 3F

KEY SKILLS

During this lesson, you will be:

- modelling practical applications using geometric recurrence relations
- modelling financial applications using geometric recurrence relations.

KEY TERMS

- Compound interest
- Compounding period
- Reducing balance depreciation

Geometric recursion relations can be used to model many real-world situations, from the reducing height of a bouncing ball to financial applications like depreciation or investments. In each case geometric recurrence can help to model the past as well as anticipate the future.

Modelling practical applications using geometric recurrence relations

See worked example 1

Geometric recurrence relations can be used to model practical situations that involve increasing values over time, specifically those that have a common ratio greater than 1 (geometric growth). For example, the number of bacterial cells present in a sample every hour can be modelled by the geometric relation

$s_0 = 100, \quad s_{n+1} = 2s_n$, where

- s_0 is the initial number of bacterial cells in the sample
- s_n is the number of bacterial cells in the sample, after n hours

A common ratio of 2 indicates that the number of bacterial cells doubles every hour.

This model can then be used to predict the number of bacterial cells present in the sample after any number of hours.

See worked example 2

Geometric recurrence relations can also be used to model practical situations that involve decreasing values over time, specifically those that have a common ratio between 0 and 1 (geometric decay). For example, the decreasing height, in centimetres, of a bouncing ball after each bounce can be modelled by the geometric relation

$h_0 = 157.4, \quad h_{n+1} = 0.7h_n$, where

- h_0 is the initial height of the ball, in centimetres
- h_n is the height of the height of the ball, after n bounces

A common ratio of 0.7 indicates that the ball bounces to 70% of its previous height with each bounce.

This model can then be used to predict the height of the ball after any number of bounces.

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Worked example 1

In an environment where locusts were destroying farmers' crops, a species of frog was introduced to eliminate the pests. Initially, 200 frogs were let out into the crops. Over the next few years, the farmers realised that the frog population was increasing, with 4 times as many frogs each year than in the previous year.

a. Model the scenario using a recurrence relation, where f_0 represents the number of frogs let out into the crops initially.

Explanation

Step 1: Determine the initial value and common ratio.

$a = 200$

$R = 4$

Step 2: Construct the recurrence relation.

Make sure to use f when referencing terms.

Answer

$f_0 = 200, \quad f_{n+1} = 4f_n$

b. How many frogs will there be in 4 years after their introduction, if nothing is done to stop their growth?

Explanation

Use recursion to calculate the number of frogs there will be each year over the next 4 years.

$f_0 = 200$

$f_1 = 200 \times 4 = 800$

$f_2 = 800 \times 4 = 3200$

$f_3 = 3200 \times 4 = 12\,800$

$f_4 = 12\,800 \times 4 = 51\,200$

Answer

51 200 frogs

Worked example 2

The strength of an elastic band, measured in g/cm^2, decreases by 27% every time it is stretched. A particular elastic band is determined to have a strength of 81.2 g/cm^2 when brand new.

a. Model the scenario using a recurrence relation, where s_0 represents the strength of the elastic band when brand new.

Explanation

Step 1: Determine the initial value and common ratio.

$a = 81.2$

As 27% of the elastic band's strength is lost each time it is stretched, this means it still retains 73% of its strength.

$R = 0.73$

Step 2: Construct the recurrence relation.

Make sure to use s when referencing terms.

Answer

$s_0 = 81.2, \quad s_{n+1} = 0.73s_n$

Continues →

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b. If the elastic band snaps when its strength drops below 10 g/cm^2, how many times can it be stretched before it snaps?

Explanation

Use recursion to calculate the strength of the elastic band each time it is stretched, until a value below 10 is obtained.

$s_0 = 81.2$

$s_1 = 81.2 \times 0.73 = 59.276$

$s_2 = 59.276 \times 0.73 = 43.271...$

$s_3 = 43.271... \times 0.73 = 31.588...$

...

$s_6 = 16.833... \times 0.73 = 12.288...$

$s_7 = 12.288... \times 0.73 = 8.970...$

Since the elastic band snaps the 7$^{\text{th}}$ time it is stretched, it can only be stretched 6 times.

Answer

6 times

Modelling financial applications using geometric recurrence relations

Geometric recurrence relations also have financial applications.

See worked example 3

One such application is compound interest. Different from simple interest (where interest is calculated once, based on the principal value), **compound interest** works from the idea that the amount of interest is recalculated every set period of time from the most recent value. This period of time is called a **compounding period**.

Compound interest can be modelled by the geometric recurrence relation

$V_0 = a, \quad V_{n+1} = RV_n$, where

- a is the initial value
- V_n is the value after n compounding periods
- $R = 1 + \frac{r}{100}$, where r is the interest rate (%) per compounding period

For example, a \$1000 investment returns interest at a compounding rate of 4.5% per month. The value of the investment each month could be modelled by the geometric relation

$V_0 = 1000, \quad V_{n+1} = 1.045V_n$

The total amount of interest earned on a compound interest investment after n months can be calculated using the formula

$I = V_n - V_0$, where

- I is the total amount of interest earned
- V_n is the value of the investment after n months
- V_0 is the principal value

See worked example 4

Another financial application of geometric recurrence relations is reducing balance depreciation, which differs from flat rate and unit cost depreciation. Instead of an item depreciating in value by a set amount, **reducing balance depreciation** reduces the value of an item by a percentage of its current value, and is recalculated every set period of time.

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Reducing balance depreciation can be modelled by the geometric recurrence relation

$V_0 = a, \quad V_{n+1} = RV_n$, where

- a is the initial value
- V_n is the value after n periods
- $R = 1 - \frac{r}{100}$, where r is the depreciation rate (%) per period

For example, a car purchased at a value of \$37 000 depreciates at a rate of 20% per year. The value of the car each year could be modelled by the geometric relation

$V_0 = 37\,000, \quad V_{n+1} = 0.8\,V_n$

Worked example 3

Tully invests \$5000 in a savings account that earns interest at a rate of 4.6% p.a., compounding monthly.

a. Calculate the interest rate, as a percentage, per compounding period. Round to two decimal places.

Explanation

The interest rate is 4.6% over 12 compounding periods (months).

$\frac{4.6}{12} = 0.38...$

Answer

0.38% per compounding period

b. Write a recurrence relation that models the value of the account, V_n, after n months, rounding values to two significant figures where necessary.

Explanation

Step 1: Identify the initial value and interest rate.

$a = 5000$

$r = 0.38\%$ per compounding period, from part **a**.

Step 2: Calculate R.

$R = 1 + \frac{0.38}{100}$

$= 1.0038$

Step 3: Construct the recurrence relation.

Answer

$V_0 = 5000, \quad V_{n+1} = 1.0038V_n$

c. Tully decides to move the money to a different investment opportunity after 6 months. How much money is in the account at this time?

Explanation

Use recursion to calculate the value of the investment after 6 months.

$V_0 = 5000$

$V_1 = 1.0038 \times 5000 = 5019$

$V_2 = 1.0038 \times 5019 = 5038.072...$

$V_3 = 1.0038 \times 5038.072... = 5057.216...$

$V_4 = 1.0038 \times 5057.216... = 5076.434...$

$V_5 = 1.0038 \times 5076.434... = 5095.724...$

$V_6 = 1.0038 \times 5095.724... = 5115.088...$

Continues →

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Answer

$5115.09

d. How much interest was earned on the investment over the 6 months?

Explanation

Subtract the initial value of the investment from the value of the investment after 6 months.

$5115.09 - 5000 = 115.09$

Answer

$115.09

Worked example 4

After many years of investing, Tully is able to purchase a snow plough for $81 000. The vehicle depreciates at a reducing balance rate of 17.2% p.a.

a. Write a recurrence relation that calculates the value of the snow plough, V_n, after n years.

Explanation

Step 1: Identify the initial value and depreciation rate.

$a = 81\,000$

$r = 17.2\%$ per period

Step 2: Calculate R.

$$R = 1 - \frac{17.2}{100}$$
$$= 0.828$$

Step 3: Construct the recurrence relation.

Answer

$V_0 = 81\,000, \quad V_{n+1} = 0.828V_n$

b. How much is the snow plough worth after 2 years?

Explanation

Use recursion to calculate the value of the vehicle after 2 years.

$V_0 = 81\,000$

$V_1 = 0.828 \times 81\,000 = 67\,068$

$V_2 = 0.828 \times 67\,068 = 55\,532.304$

Answer

$55 532.30

Continues →

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c. The snow plough is written off (has no effective value) after its value falls below $20 000. How many years after the purchase will Tully's snow plough be written off?

Explanation

Use recursion to calculate the value of the vehicle after each year until a value below $20 000 is obtained.

$V_0 = 81\,000$

$V_1 = 0.828 \times 81\,000 = 67\,068$

$V_2 = 0.828 \times 67\,068 = 55\,532.30...$

...

$V_7 = 0.828 \times 26\,101.59... = 21\,612.12...$

$V_8 = 0.828 \times 21\,612.12... = 17\,894.83...$

Answer

8 years

3E Questions

Modelling practical applications using geometric recurrence relations

1. Which of the following could represent geometric growth over time?

A. The diameter of a snake increases by 20% every time it sheds its skin.

B. A plant grows at a rate of 2 cm every year.

C. A balloon loses 4% of its volume every day.

D. Danny's weight doubles from when he is born to when he is 6 months old.

---

2. A disease infects a crop over time. Initially, 200 m^2 of the crop is infected, and the area expands by 23% of the previously infected area every month.

a. Model the scenario using a recurrence relation, where a_0 represents the initially infected area.

b. What area of crops will be infected in 6 months if nothing is done to stop the spread? Round to one decimal place.

---

3. A leaky beach ball becomes deflated over time. It initially holds 22 000 cm^3 of air but loses 11.4% of its current volume every hour.

a. Model the scenario using a recurrence relation, where v_0 represents the initial volume of the beach ball.

b. The beach ball needs to be reinflated after it has lost half of its original volume. Use recursion to determine how long it will be after the beach ball is first inflated before it will need to be reinflated.

---

4. For a particular species of flies, females each lay 100 eggs. Of those eggs, 60% will hatch into female flies, but only half of those will survive past one day. All flies of this species live for only 1 year. Initially, there were 300 female flies in a particular population.

a. Model the number of newly hatched female flies per year that survive past one day in this population, representing the initial number of female flies as f_0.

b. Calculate the number of newly hatched female flies in this population after 3 years.

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Modelling financial applications using geometric recurrence relations

5. For an investment with an interest rate of 3.95% per annum, compounding monthly, the monthly interest rate is closest to

A. 0.03%

B. 0.33%

C. 0.40%

D. 0.47%

6. Binh invests $20 000 in an account that pays 5.1% p.a. interest, compounding monthly.

a. What is the monthly interest rate, as a percentage?

b. Write a recurrence relation that models the value of the investment, V_n, after n months.

c. What is the value of the account after 6 months, rounded to the nearest cent?

d. Binh would like to go on a holiday, one year from when she first invested her money. She will need $22 000 for her trip. Will Binh be able to go on her holiday using only the money from her investment? Explain briefly.

7. A new cafe purchases a top-of-the-line coffee machine for $12 499. The machine depreciates at a reducing balance rate of 9.4% every year.

a. Write a recurrence relation that models the value of the coffee machine, M_n, where n is the number of years after its purchase.

b. How many years does it take for the coffee machine to drop below half of its original value?

c. The cafe owner will purchase a new coffee machine after 10 years. What is the value of the old coffee machine at this time? Round to the nearest cent.

Joining it all together

8. Dialo purchased a 1967 Chevy Impala in the year 2000 for $8100. It depreciates by 4% of its value every year.

a. Write a recurrence relation that models the value of the car, V_n, where n is the number of years after 2000.

b. In 2005, there was a sudden surge in popularity of the 1967 Chevy Impala, which saw its value increase by 8.2% of its value every year, starting in 2005. How much was the Impala worth in 2004, right before this sudden surge in popularity? Round to the nearest cent.

c. Write a recurrence relation that models the value of the car, C_n, where n is the number of years after 2004.

d. In what year will the 1967 Chevy Impala first be worth more than double what Dialo paid for it?

9. A cinema chain, 'Town Cinemas', charges $12.50 for all tickets sold in their cinemas. At the end of 2015, Town Cinemas noticed that they had sold 14% fewer tickets than in the previous year, with 21 457 tickets sold in total over the year.

a. Assuming that this alarming trend continues, write a recurrence relation that models the number of tickets sold each year, T_n, where n is the number of years after 2015.

b. Town Cinemas are worried that these numbers would mean that they can't cover their running costs. They attempt to balance this by increasing the cost of a ticket by 14% every year. Write a recurrence relation that models the cost of a ticket, C_n, where n is the number of years after 2015.

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c. Use recursion to fill in the following table. Round tickets to the nearest whole number and the cost per ticket to the nearest 5 cents.

year	*number of tickets sold*	*cost per ticket*	*total revenue*
2015	21 457	\$12.50	\$268 212.50
2016			
2017			
2018			
2019			
2020			

d. Has their strategy worked? Give a reason for this conclusion.

Exam practice

10. Simon works as a clown at children's birthday parties. He has purchased a large set of props that he will use during his shows.

The value of Simon's props is depreciated using the reducing balance method.

The value of the props, in dollars, after n years, V_n, can be modelled by the following recurrence relation.

$V_0 = 20\ 000, \quad V_{n+1} = 0.8V_n$

Use recursion to show that the value of the props after three years, V_3, is \$10 240. (1 MARK)

Adapted from VCAA 2019 Exam 2 Recursion and financial modelling Q7a

74% of students answered this type of question correctly.

11. Tammy is a mobile dog groomer. She sends a bill to her customers after their dog has been groomed.

If a customer does not pay the bill by the due date, interest is charged.

Tammy charges interest after the due date at the rate of 1.7% per month on the amount of an unpaid bill.

The interest on this amount will compound monthly.

a. Tammy sent Robert a bill of \$80 for grooming his Border Collie.
Robert paid the full amount two months after the due date.
How much did Robert pay? Round to the nearest cent. (1 MARK)

b. Tammy sent Summer a bill of \$107 for grooming her Chihuahua.
Summer did not pay the bill by the due date.
Let A_n be the amount of this bill n months after the due date.
Write down a recurrence relation, in terms of A_0, A_{n+1} and A_n, that models the amount of the bill. (2 MARKS)

Adapted from VCAA 2017 Exam 2 Recursion and financial modelling Q6a,b

Part **a**: **61%** of students answered this type of question correctly.
Part **b**: The average mark on this type of question was **1**.

12. Alex is a mobile mechanic. He uses a van to travel to his customers to repair their cars.

The value of Alex's van is depreciated using the reducing balance method of depreciation.

The value of the van, in dollars, after n years, R_n, can be modelled by the following recurrence relation.

$R_0 = 75\ 000, \quad R_{n+1} = 0.943R_n$

At what annual percentage rate is the value of the van depreciated each year? (1 MARK)

VCAA 2017 Exam 2 Recursion and financial modelling Q5c

58% of students answered this question correctly.

Questions from multiple lessons

Graphs and relations *Year 10 content*

13. A group of friends went shopping at Mush for bath products.

- Isobel bought four bath bombs and one face mask for $44.00.
- Jackson bought two bath bombs and two face masks for $43.00.
- Klaudia bought three bath bombs and four face masks.

How much did Klaudia spend in total?

A. $72.00

B. $73.00

C. $78.50

D. $81.00

E. $86.00

Adapted from VCAA 2017 Exam 1 Graphs and relations Q6

Recursion and financial modelling

14. Link wants to manage the amount of money that he spends each week on hairspray. He begins with $150 that he will spend purely on hairspray over a 12-week period. Each week, he plans to spend $12.50.

Let h_n be the amount of hairspray money Link has left after week n.

A recurrence relation that can be used to model this situation is

A. $h_0 = 12.5, \quad h_{n+1} = 150 - 12.5$

B. $h_0 = 12.5, \quad h_{n+1} = h_n - 150$

C. $h_0 = 150, \quad h_{n+1} = h_n = 12.5h_n$

D. $h_0 = 150, \quad h_{n+1} = h_n - 12.5$

E. $h_0 = 150, \quad h_{n+1} = 1 - 0.125h_n$

Adapted from VCAA 2014 Exam 1 Number patterns Q4

Graphs and relations *Year 10 content*

15. Anita sells t-shirts at a local market. The revenue, in dollars, that she makes from selling n t-shirts is given by the following equation.

$revenue = 24n$

The cost, in dollars, of producing these t-shirts is given by the following equation.

$cost = 15.5n + 14$

a. What is the selling price of Anita's t-shirts? (1 MARK)

b. Anita sold 18 t-shirts last Saturday. How much profit did she earn?
Note: Profit is calculated by subtracting cost from revenue. (1 MARK)

Adapted from VCAA 2017NH Exam 2 Graphs and relations Q2

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3F Modelling sequences using a rule

STUDY DESIGN DOT POINT

- generation of the explicit rule, u_n, of an arithmetic or geometric sequence, its use and evaluation, including various practical and financial contexts

3A 3B 3C 3D 3E 3F

KEY SKILLS

During this lesson, you will be:

- modelling an arithmetic sequence using a rule
- modelling a geometric sequence using a rule.

Recurrence relations can only be used to calculate the immediate next term in a sequence. As such, it can be time consuming to calculate u_n for large values of n. A rule can be constructed so that any term in a sequence can be evaluated from one calculation, rather than multiple iterations.

Modelling an arithmetic sequence using a rule

A rule can be constructed that gives the value of any term in a sequence. This can be used to find a specific term far along a sequence, without having to find each of the preceding terms first.

See worked example 1

For arithmetic sequences, this rule is of the form

$u_n = u_0 + d \times n$, where

- u_n is the value after n iterations
- u_0 is the initial value
- d is the common difference

For example, the recurrence relation $u_0 = 2, \quad u_{n+1} = u_n - 8$ can be converted into the rule $u_n = 2 - 8n$.

This general rule can be used to model simple interest loans and investments, flat rate depreciation and unit cost depreciation.

See worked example 2

For simple interest investments and loans, this rule is of the form

$u_n = u_0 + d \times n$, where

- u_n is the value of the investment or loan after n periods
- u_0 is the initial value of the investment or loan
- d is the interest amount per period, calculated as $d = \frac{r}{100} \times u_0$, where r is the interest rate (%) per period.

For flat rate depreciation, this rule is of the form

$u_n = u_0 - d \times n$, where

- u_n is the value of the asset after n periods
- u_0 is the initial value of the asset
- d is the depreciation amount per period, calculated as $d = \frac{r}{100} \times u_0$, where r is the depreciation rate (%) per period.

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For unit cost depreciation, this rule is of the form

$u_n = u_0 - d \times n$, where

- u_n is the value of the asset after n units of use
- u_0 is the initial value of the asset
- d is the depreciation amount per unit of use

Worked example 1

The population of kakapos in New Zealand is being monitored after the species is declared critically endangered.

Initially, there are 100 kakapos. Each month, the population will increase by 2 kakapos due to the introduction of a breeding program.

Let P_n be the kakapos population in New Zealand n months after the introduction of the breeding program.

a. Construct a rule that can be used to calculate P_n.

Explanation

Step 1: Identify the initial value and common difference.

There are initially 100 kakapos, so $P_0 = 100$.

The population of kakapos will increase by 2 each month, so $d = 2$.

Step 2: Construct the rule for P_n.

Substitute these values into the rule $P_n = P_0 + d \times n$.

Answer

$P_n = 100 + 2n$

b. Determine the population of kakapos after 1 year.

Explanation

Step 1: Determine the corresponding n value.

n represents the number of months after the introduction of the breeding program.

There are 12 months in 1 year, so $n = 12$.

Step 2: Calculate P_{12}, the population of kakapos after 12 months.

$$P_{12} = 100 + 2 \times 12$$
$$= 124$$

Answer

124 kakapos

Worked example 2

Consider the following scenarios.

a. Lep borrows \$250 from his older sibling who charges simple interest at a rate of 5% per annum. The value of the loan, V_n, after n years can be modelled by the following recurrence relation.

$V_0 = 250, \quad V_{n+1} = V_n + 12.5$

Construct a rule that can be used to calculate the amount Lep owes after n years.

Explanation

Step 1: Identify the initial value and interest rate.

$V_0 = 250$

$r = 5\%$ per annum

Step 2: Calculate d.

$$d = \frac{5}{100} \times 250$$
$$= 12.5$$

Note: This can also be obtained from the recurrence relation.

Continues →

Step 3: Construct the rule for V_n.

$V_n = 250 + 12.5 \times n$

Answer

$V_n = 250 + 12.5n$

---

b. Clove purchases an asset worth \$2000. Its value depreciates using the flat rate method, at a rate of 2.5% per year.

The value of the asset, C_n, after n years can be modelled using the following rule.

$C_n = 2000 - 50n$

Calculate the value of the asset after 15 years.

Explanation

Step 1: Determine the corresponding n value.

n represents the number of years after the asset was first purchased.

There are 15 years, so $n = 15$.

Step 2: Calculate C_{15}, the value of the asset after 15 years.

$C_{15} = 2000 - 50 \times 15$

$= 1250$

Answer

\$1250

Modelling a geometric sequence using a rule

See worked example 3

A rule can also be constructed to determine u_n for a geometric sequence.

For geometric sequences, this rule is of the form

$u_n = u_0 \times R^n$, where

- u_n is the value after n iterations
- u_0 is the initial value
- R is the common ratio

For example, the recurrence relation $u_0 = 10, \quad u_{n+1} = 0.5u_n$ can be converted into the rule $u_n = 10 \times 0.5^n$.

See worked example 4

This general rule can be used to model compound interest and reducing balance depreciation.

For compound interest loans and investments, this rule is of the form

$u_n = u_0 \times R^n$, where

- u_n is the value of the investment or loan after n compounding periods
- u_0 is the initial value
- R is the common ratio, calculated as $R = 1 + \frac{r}{100}$, where r is the interest rate (%) per compounding period.

For reducing balance depreciation, this rule is of the form

$u_n = u_0 \times R^n$, where

- u_n is the value of the asset after n periods
- u_0 is the initial value
- R is the common ratio, calculated as $R = 1 - \frac{r}{100}$, where r is the depreciation rate (%) per period.

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Worked example 3

The number of stem cells in a live culture is being monitored as part of a drug trial.
Initially, there were 4 cells in the agar plate. Each hour, the number of cells doubles.

a. Construct a rule that can be used to model the number of stem cells, S_n, in the agar plate after n hours.

Explanation

Step 1: Identify the initial value and common ratio.

There were initially 4 cells in the agar plate, so $S_0 = 4$.

The number of cells doubles each hour, so $R = 2$.

Step 2: Construct the rule for S_n.

Substitute these values into the rule $S_n = S_0 \times R^n$.

Answer

$S_n = 4 \times 2^n$

b. How many cells will there be after a full day?

Explanation

Step 1: Determine the corresponding n value.

n represents the number of hours after the cells were cultured.

There are 24 hours in a day so $n = 24$.

Step 2: Calculate S_{24}, the number of stem cells after 24 hours.

$$\begin{aligned} S_{24} &= 4 \times 2^{24} \\ &= 67\,108\,864 \end{aligned}$$

Answer

67 108 864 cells

Worked example 4

Consider the following scenarios.

a. Sam invests \$40 000 in a savings account with her bank. They offer an interest rate of 1.4% per annum, compounding quarterly.

How much will her account be worth in 10 years, correct to the nearest cent?

Explanation

Step 1: Identify the initial value and interest rate.

$u_0 = 40\,000$

$$\begin{aligned} r &= \frac{1.4}{4}\% \\ &= 0.35\% \text{ per compounding period} \end{aligned}$$

Step 2: Calculate R.

$$\begin{aligned} R &= 1 + \frac{0.35}{100} \\ &= 1.0035 \end{aligned}$$

Step 3: Construct the rule for u_n.

$u_n = 40\,000 \times 1.0035^n$

Step 4: Determine the corresponding n value.

n represents the number of compounding periods since the investment was established.

There are 40 quarters in 10 years so $n = 40$.

Step 5: Calculate u_{40}, the value of the account after 40 quarters.

$$\begin{aligned} u_{40} &= 40\,000 \times 1.0035^{40} \\ &= 45\,999.706... \end{aligned}$$

Answer

\$45 999.71

Continues →

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b. Pi purchased a boat that depreciates using the reducing balance method. The value of the boat after n years, P_n, can be modelled using the following rule.

$P_n = 20\,000 \times 0.85^n$

What is the annual rate of depreciation for this boat?

Explanation

Step 1: Identify the common ratio.

$R = 0.85$

Step 2: Substitute R into $R = 1 - \frac{r}{100}$.

$0.85 = 1 - \frac{r}{100}$

Step 3: Solve for r.

$\frac{r}{100} = 0.15$

$r = 0.15 \times 100$

$= 15\%$ per period

The rule models the value of the asset in terms of years, so this is the annual rate of depreciation.

Answer

15% p.a.

3F Questions

Modelling an arithmetic sequence using a rule

1. Which of the following rules can be used to model this arithmetic sequence?

2, −3, −8, −13, −18...

A. $u_n = 2 + 5n$ **B.** $u_n = 2 - 5n$ **C.** $u_n = 5 + 2n$ **D.** $u_n = 5 - 2n$

2. Ian currently has a moth infestation in his house. He counted 45 moths before using a repellent that removes 2 moths per day. Let m_n be the number of moths after n days of using the repellent.

a. Represent this using a rule for m_n.

b. How many moths will remain after 20 days of using the repellent?

3. Poppy invests \$1000 in an account that earns simple interest annually. The value of this investment can be modelled using the following recurrence relation, where p_n is the value of the investment after n years.

$p_0 = 1000, \quad p_{n+1} = p_n + 25$

a. Represent this using a rule for p_n.

b. How long will it take for her account to double in value?

4. Bij purchases an e-reader for \$420. Its value depreciates, using the flat rate method, by \$0.21 per day.

Let b_n be the value of the e-reader after n days.

a. Construct a rule for b_n.

b. Assuming that there were no leap years, how long after the purchase will the e-reader have no value, in years and days?

c. What is the daily rate of depreciation, as a percentage?

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5. A local café decides to upgrade their equipment by purchasing a brand new coffee bean grinder. This machine depreciates by $0.10 in value for every 100 g of coffee beans ground.

   On average, the café goes through 2000 g of coffee beans each day. After 60 days, the grinder is now worth $440.

   a. What was the original purchase price of the coffee grinder?

   b. What will the coffee grinder be worth 90 days after purchasing it?

Modelling a geometric sequence using a rule

6. Which of the following rules can be used to model this geometric sequence?

   0.5, 1, 2, 4, 8, ...

   A. $u_n = 0.5 + 2n$ **B.** $u_n = 2 - 0.5n$ **C.** $u_n = 2 \times 0.5^n$ **D.** $u_n = 0.5 \times 2^n$

7. Each hour, a 500 g sample of radioactive material will reduce its weight by half.

   a. Create a rule that can be used to model the weight, in grams, w_n, of this sample after n hours.

   b. Determine the weight of the sample after 4 hours.

8. John borrows some money from the bank to purchase a refrigerator. His bank charges interest, compounding monthly. The amount that he owes, J_n, after n months can be represented by the following rule.

   $J_n = 2985 \times 1.006^n$

   a. How much does the refrigerator cost?

   b. What annual interest rate does his bank charge?

   c. Assuming he makes no repayments, after how many months will John first owe more than $3450?

9. Michayla purchases a new laptop for $1999.

   Its value depreciates by 42% each year using the reducing balance method.

   Let L_n be the value of the laptop after n years.

   a. Determine a rule for L_n.

   b. How many years after the purchase will the value of the laptop first drop below $200?

Joining it all together

10. Jenny has just purchased her dream apartment and wishes to decorate the living room with a brand new three-seat cloud sofa.

    Edkea has one listed for $10 000. She turns to her bank to establish a loan and they present her with two offers.

    - Loan A: Jenny will pay a 10% deposit, with the loan covering the remaining amount. Simple interest is charged at a rate of 5% per annum.
    - Loan B: Jenny will pay a 7% deposit, with the loan covering the remaining amount. Compound interest is charged at a rate of 6% per annum, compounding monthly.

    After 5 years, she will pay off the loan with one singular payment.

    a. Represent Loan A using a rule for A_n, the value of the loan after n years.

    b. How much would the final payment be if she chooses Loan A?

    c. Represent Loan B using a rule for B_n, the value of the loan after n months.

    d. How much would the final payment be if she chooses Loan B? Round your answer to the nearest cent.

    e. Which loan is the most cost effective?

11. Jaq purchases a luxury, gold-plated stethoscope that comes with an Italian leather case for $15 000.

His accountant, Peppa, can choose to depreciate it using one of three methods.

- The flat rate method, at a rate of 5.9% per year.
- The unit cost method, where the stethoscope depreciates by $0.10 each time it is used. On average, Jaq will use the stethoscope twice a day.
- The reducing balance method, at a rate of 4.4% per year.

a. Represent the value of the asset using flat rate depreciation as a rule for A_n, the value of the asset after n years.

b. Represent the value of the asset using unit cost depreciation as a rule for B_n, the value of the asset after n uses.

c. Represent the value of the asset using reducing balance depreciation as a rule for C_n, the value of the asset after n years.

d. Which depreciation method would give the stethoscope the highest possible value after 2 years?

Exam practice

12. Julie deposits some money into a savings account that will pay compound interest every month.

The balance of Julie's account, in dollars, after n months can be modelled by the recurrence relation shown.

$V_0 = 12\ 000, \quad V_{n+1} = 1.0062 \times V_n$

A rule of the form $V_n = a \times b^n$ can be used to determine the balance of Julie's account after n months.

a. Complete this rule for Julie's investment after n months by writing the appropriate numbers in the boxes provided. (1 MARK)

$$balance = \boxed{} \times \boxed{}^{\,n}$$

b. What would be the value of n if Julie wanted to determine the value of her investment after three years? (1 MARK)

VCAA 2018 Exam 2 Recursion and financial modelling Q4c

Part **a**: **78%** of students answered this question correctly.
Part **b**: **65%** of students answered this question correctly.

13. An asset is purchased for $2480.

The value of this asset after n time periods, V_n, can be determined using the rule

$V_n = 2480 + 45n$

A recurrence relation that also models the value of this asset after n time periods is

A. $V_0 = 2480, \quad V_{n+1} = V_n + 45n$

B. $V_n = 2480, \quad V_{n+1} = V_n + 45n$

C. $V_0 = 2480, \quad V_{n+1} = V_n + 45$

D. $V_1 = 2480, \quad V_{n+1} = V_n + 45$

E. $V_n = 2480, \quad V_{n+1} = V_n + 45$

VCAA 2020 Exam 1 Recursion and financial modelling Q22

55% of students answered this question correctly.

14. Ray deposited $5000 in an investment account earning interest at the rate of 3% per annum, compounding quarterly.

A rule for the balance, R_n, in dollars, after n years is given by

A. $R_n = 5000 \times 0.03^n$ **B.** $R_n = 5000 \times 1.03^n$ **C.** $R_n = 5000 \times 0.03^{4n}$

D. $R_n = 5000 \times 1.0075^n$ **E.** $R_n = 5000 \times 1.0075^{4n}$

VCAA 2020 Exam 1 Recursion and financial modelling Q26

20% of students answered this question correctly.

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Questions from multiple lessons

Recursion and financial modelling

15. What is the sequence generated from the following recurrence relation?

$T_0 = 4, \quad T_{n+1} = -5 \times T_n$

A. 4, −1, −6, −11, −16...

B. 4, 9, 14, 19, 24...

C. 4, 20, 100, 500, 2500...

D. 4, −20, −100, −500, −2500...

E. 4, −20, 100, −500, 2500...

Adapted from VCAA 2017NH Exam 1 Recursion and financial modelling Q17

Data analysis *Year 10 content*

16. The following histogram shows the distribution of *weight*, in kilograms, of 28 cats in an animal shelter.

The most common interval of *weight* for the cats is

A. greater than or equal to 2.5 kg and less than 3.0 kg.

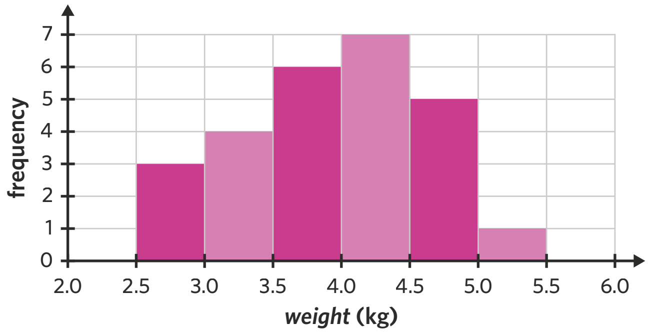
B. greater than or equal to 3.0 kg and less than 3.5 kg.

C. greater than or equal to 3.5 kg and less than 4.0 kg.

D. greater than or equal to 4.0 kg and less than 4.5 kg.

E. greater than or equal to 5.0 kg and less than 5.5 kg.

Adapted from VCAA 2009 Exam 1 Data analysis Q5



Recursion and financial modelling

17. A cosmetics studio needs to predict the number of eyeshadow palettes they are going to sell so they can produce the correct number of units accordingly.

In the first month, they are expected to sell 20 000 units of eyeshadow.

Each subsequent month, they expect to sell 5% more units than the previous month.

a. How many units are they expected to sell after 3 months? Round to the nearest whole number. (1 MARK)

b. Calculate the difference between the units expected to be sold after 2 and 5 months. Round to the nearest whole number. (1 MARK)

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CHAPTER 4 CALCULATOR QUICK LOOK-UP GUIDE

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4

UNIT 1 AOS 3
CHAPTER 4
Linear functions and graphs

LESSONS

KEY KNOWLEDGE

- the linear function $y = a + bx$, its graph, and interpretation of the parameters, a and b in terms of initial value and constant rate of change respectively
- graphing linear relations $Ax + By = C$ and equivalent forms
- formulation and analysis of linear models from worded descriptions or relevant data (including simultaneous linear equations in two variables) and their application to solve practical problems including domain of interpretation
- piecewise linear (line segment, step) graphs and their application to modelling practical situations, including tax scales and charges and payment.

Image: ChristianChan/Shutterstock.com

4A Linear algebra

STUDY DESIGN DOT POINT

- prerequisite lesson

4A 4B 4C 4D 4E 4F 4G

KEY SKILLS

During this lesson, you will be:

- substituting values into linear equations
- solving linear equations.

KEY TERMS

- Substitution
- Subject
- Transpose

Pronumerals are used to represent an unknown value in an equation. When there are multiple pronumerals in an equation, the value of one of the pronumerals can be determined if the rest are known. This can be achieved by transposing the equation, if necessary, and substituting in the known values.

Substituting values into linear equations

Substitution is the replacement of one term with another. In linear equations, substitution is the replacement of a pronumeral with a number. The terms 'let' and 'when' are often used to indicate that a variable is going to be substituted with a particular value in the following calculation. For example, 'let $x = 0$', or 'when $a = 5$'. To substitute a pronumeral with a number, replace the pronumeral with the necessary value.

Worked example 1

For the equation $y = 2x + 1$, find the value of y when $x = 3$.

Explanation

Step 1: Replace the pronumeral with the necessary value.

$$y = 2 \times 3 + 1$$

Step 2: Evaluate the equation.

$$\begin{aligned} y &= 2 \times 3 + 1 \\ &= 6 + 1 \\ &= 7 \end{aligned}$$

Answer

7

Solving linear equations

When a variable is by itself on one side of an '=' sign it is called the **subject** of the equation. For example, in the equation $y = 3x + 1$, the subject is y. When trying to find the value of a variable that is not the subject of an equation, the equation should be transposed before substitution occurs.

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To **transpose** an equation is to rearrange it. This can be thought of as 'moving' everything except the desired variable to the other side of the '=' sign, making the desired variable the subject. Transposing is achieved by 'undoing' the operations that were applied to the desired variable of the equation. To 'undo' an operation, the inverse operation is required. These are shown in the table below.

See worked example 2

operation	$+$	$-$	$\times$	$\div$
inverse operation	$-$	$+$	$\div$	$\times$

Inverse operations can be applied to both numbers and pronumerals, which is useful when transposing linear equations with more than two variables. More complicated equations can be solved using a calculator.

See worked example 3

Worked example 2

Consider the equation $y = 2x + 1$.

Transpose the equation to make x the subject.

Explanation

Step 1: List, in order, the operations that transform the desired variable into its current form in the equation.

In this case, that is x into $2x + 1$.

The first operation is $\times 2$.

The next operation is $+1$.

Step 2: Write down the inverses of the operations, in the opposite order.

The first inverse operation is -1.

The next inverse operation is $\div 2$.

Step 3: Apply these transformations to both sides of the equation.

$$y = 2x + 1$$
$$y - 1 = 2x + 1 - 1$$
$$y - 1 = 2x$$
$$(y - 1) \div 2 = 2x \div 2$$
$$\frac{y - 1}{2} = x$$

Answer

$$x = \frac{y - 1}{2}$$

Worked example 3

Consider the equation $a = \dfrac{5b + 15}{c}$.

a. Transpose the equation to make b the subject.

Explanation – Method 1: By hand

Step 1: List, in order, the operations that transform the desired variable into its current form in the equation.

In this case, that is b into $\dfrac{5b + 15}{c}$.

The first operation is $\times 5$.

The second operation is $+15$.

The third operation is $\div c$.

Step 2: Write down the inverses of the operations, in the opposite order.

The first inverse operation is $\times c$.

The second inverse operation is -15.

The third inverse operation is $\div 5$.

Continues →

Step 3: Apply these transformations to both sides of the equation.

$$a = \frac{5b + 15}{c}$$

$$a \times c = \frac{5b + 15}{c} \times c$$

$$ac = 5b + 15$$

$$ac - 15 = 5b + 15 - 15$$

$$ac - 15 = 5b$$

$$(ac - 15) \div 5 = 5b \div 5$$

$$\frac{ac - 15}{5} = b$$

$$\frac{ac}{5} - 3 = b$$

Explanation – Method 2: TI-Nspire

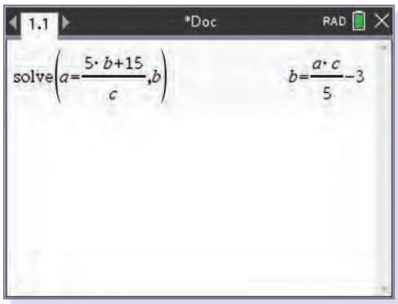
Step 1: From the home screen, select ‘1: New’ → ‘4: Add Calculator’.

Step 2: Press menu and then select ‘3: Algebra’ → ‘1: Solve’

Step 3: Enter ‘$a = \frac{5b+15}{c}, b$’ into the function.

Note: The ‘,b’ tells the calculator to make b the subject.

Press enter.



Explanation – Method 3: Casio ClassPad

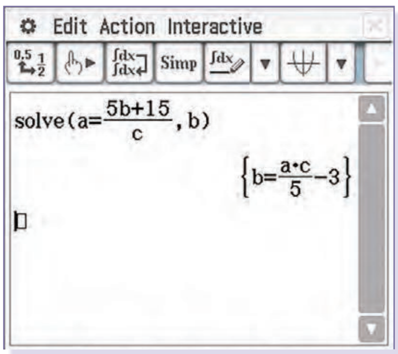
Step 1: From the main menu, tap √α Main.

Step 2: Press keyboard and tap the solve icon Solve.

Step 3: Enter ‘$a = \frac{5b+15}{c}$, b’ into the function.

Note: The ‘,b’ tells the calculator to make b the subject.

Press enter.



Answer – Method 1, 2 and 3

$$b = \frac{ac}{5} - 3$$

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4A Questions

Note: There are no direct exam questions relevant to this lesson.

Substituting values into linear equations

1. Consider the equation $a = 2n + 3$.
   a. Substitute $n = 2$ into the equation by filling in the following box.

   $a = 2 \times \boxed{} + 3$

   b. Find the value of a when $n = 2$.

2. Follow the instructions for the following equations:
   a. $p = q - 4$
      i. Without solving or simplifying, substitute $q = 1$ into the equation.
      ii. Find the value of p.
   b. $f = \frac{1}{3}x + 7$
      i. Without solving or simplifying, substitute $x = 3$ into the equation.
      ii. Find the value of f.
   c. $h = -2.3t + 11$
      i. Without solving or simplifying, substitute $t = 1.7$ into the equation.
      ii. Find the value of h.

3. Follow the instructions for the following equations:
   a. $y = 5x - 4$
   Find the value of y when $x = 3$.
   b. $a = 3b - 2c$
   Find the value of a when $b = 2$ and $c = 8$.
   c. $p = -\frac{1}{4}(q + 3r)$
   Find the value of p when $q = 5$ and $r = -2$.

4. Answer the following questions.
   a. The number of goals, g, Vanessa kicks in an Australian Rules Football match is given by the equation $g = \frac{1}{3}w$, where w is the number of Weet-Bix she ate for breakfast. If she eats 12 Weet-Bix for breakfast, how many goals will she kick?
   b. The equation $T = \frac{2}{3}m$ describes how long in minutes, T, it takes to watch a video that is m minutes long, if it is at 1.5× speed. How long, in minutes, will it take to watch a 5.3-minute-long Edrolo video at this speed? Round to 1 decimal place.
   c. The equation $A = \frac{1}{2}bh$ relates the area of a triangle, A, to the length of its base, b, and its height, h. Find the area, in cm^2, of a triangle that has a base length of 12 cm and a height of 9 cm.

5. Beppe's doona is 210 cm long. This means that the area, A, covered by his doona, in cm^2, is given by the linear equation $A = 210w$, where w is the width, in cm, of his current doona.
   Beppe's dog, Barnesworth, always gets on the bed in the morning and takes up a lot of the doona. To solve this issue, Beppe decides to buy a new doona that is the same length, but 30 cm wider than his current doona. If Beppe's current doona is 170 cm wide, find the area, A, in cm^2, of his new doona.

Solving linear equations

6. Consider the equation $y = \frac{1}{3}x + 5$.

a. What operations, in order, are required to transform x into $\frac{1}{3}x + 5$?

A. $\times 3, -5$ **B.** $\div 3, +5$ **C.** $-5, \times 3$ **D.** $+5, \div 3$

b. What operations, in order, are required to transform $\frac{1}{3}x + 5$ into x?

A. $\times 3, -5$ **B.** $\div 3, +5$ **C.** $-5, \times 3$ **D.** $+5, \div 3$

c. Transpose the equation $y = \frac{1}{3}x + 5$ to make x the subject.

---

7. Transpose each of the following linear relations to make b the subject.

a. $a = 2b + 4$

b. $a = 43 - 3b$

c. $\frac{a}{20} + b = 5$

d. $\frac{3a + 6b}{15} = 14$

---

8. The equation to convert temperatures in Fahrenheit, F, to Celsius, C, is $C = \frac{5}{9}(F - 32)$.

Show that the equation to convert temperatures from Celsius to Fahrenheit is $\frac{9}{5}C + 32 = F$.

Joining it all together

9. Consider the equation $p = \frac{1}{q}(12r + s)$.

a. Find the value of p when $q = 2$, $r = 4$ and $s = 16$.

b. Find the value of r when $p = 4$, $q = 7$ and $s = 10$.

---

10. The equation that converts a distance in miles, M, to kilometres, K, is $K = \frac{8}{5}M$.

a. Find the equation that converts a distance in kilometres to miles.

b. Using the equation from part **a**, convert 8 kilometres into miles.

---

11. The length of time, t, in minutes, that Jane and her dog Bowie spend at the local park is given by $t = 30 + 5d$, where d is the number of other dogs at the park. Her partner, Des, wants to deduce the number of dogs Jane and Bowie encounter from the amount of time they spend at the park.

a. Rearrange the equation to make d the subject.

b. Using the equation from part **a**, find the number of other dogs that were at the park if Jane and Bowie spent an hour and a half at the local park.

---

12. The cost, C, in dollars, of hiring an UberX is $C = 2 + 1.15d + 0.35t$, where d is the distance travelled in km, and t is the time taken in minutes.

a. Determine the cost of hiring an UberX to travel 15 km if the trip took 12 minutes.

b. Angelo hired an UberX to travel 10 km from the Night Cat in Fitzroy to his home in Reservoir. If the total cost was $20.85, how long did the trip take?

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13. In most branches of science, temperature is measured in Kelvin, K. The equation to convert between Kelvin and Celsius is $K = C + 273$.

The equation to convert temperatures in Fahrenheit, F, to Celsius, C, is $C = \frac{5}{9}(F - 32)$.

Note: Temperatures in Kelvin are written without the degree symbol.

a. If the temperature is 18 degrees Fahrenheit, what is the temperature in Kelvin? Give your answer correct to the nearest whole number.

b. If the temperature is 298 Kelvin, what is the temperature in Fahrenheit?

---

14. The winning margin, m, in an AFL game can be calculated using the equation $m = 6g_f + b_f - (6g_a + b_a)$, where g_f is goals for the winning team, b_f is behinds for the winning team, g_a is goals against the winning team and b_a is behinds against the winning team.

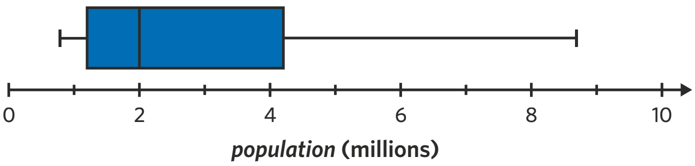
a. Gold Coast defeated Richmond. They scored 14 goals and 10 behinds, whilst Richmond scored 13 goals and 14 behinds. What was the winning margin?

b. Adelaide defeated Carlton by 29 points. They scored 12 goals and 12 behinds. Carlton scored 7 behinds. How many goals did Carlton score?

Questions from multiple lessons

Data analysis *Year 10 content*

15. The following boxplot displays the *population*, in millions of people, of the 65 most populated African cities.



The percentage of these 65 cities with a population greater than 2 000 000 is closest to

A. 15% **B.** 25% **C.** 50% **D.** 75% **E.** 85%

Adapted from VCAA 2017 Exam 1 Data analysis Q1

Recursion and financial modelling

16. A sequence of numbers can be generated using the following recurrence relation.

$V_0 = 1, \quad V_{n+1} = V_n + 6$

What is the value of V_4?

A. 1 **B.** 7 **C.** 13 **D.** 19 **E.** 25

Adapted from VCAA 2019NH Exam 1 Recursion and financial modelling Q17

Data analysis

17. A group of 20 tourists visiting the Sydney Opera House were asked which *country* they were from. The results are displayed in the following frequency table.

a. What is the value of x in the frequency table? (1 MARK)

b. Create a bar chart using the data from the frequency table. (2 MARKS)

country	frequency
New Zealand	3
China	8
USA	5
UK	2
Japan	x

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4B Linear functions

STUDY DESIGN DOT POINT

- the linear function $y = a + bx$, its graph, and interpretation of the parameters, a and b in terms of initial value and constant rate of change respectively

4A 4B 4C 4D 4E 4F 4G

KEY SKILLS

During this lesson, you will be:

- identifying linear functions and their graphs
- identifying features of linear functions and graphs.

KEY TERMS

- Linear function
- Gradient
- x-intercept
- y-intercept

Linear algebra can be applied in various different contexts. A common use is linear functions, where the principles of linear algebra are applied to model different real-life scenarios. Before modelling however, it is important to be able to identify linear functions and their associated graphs, as well as defining features.

Identifying linear functions and their graphs

A **linear function** is a relationship between two variables where one value changes by a constant amount in response to the other. Linear functions are generally represented by equations expressed in the form

$y = a + bx$, where

- a is the initial value
- b is the constant rate of change
- x and y are variables that can change in value

An equation is regarded as linear when each variable has a power of either zero or one. Common forms of linear equations are:

See worked example 1

- $y = a + bx$
- $ax + by = c$
- $y = a$
- $x = c$

An equation is regarded as non-linear when either of the variables have a power other than zero or one. Common examples are:

- $y = x^2$
- $y = \sqrt{x}$
- $y = a^x$
- $y = \log_{10}(x)$
- $a = xy$

Note: $a = xy$ is not linear, even if it may appear to be. This is because x is raised to the power of -1 when the equation is rearranged to $y = \frac{a}{x}$.

Determining whether an equation is linear or not involves examining the equation to make sure that all of the variables are linear.

Linear functions can also be displayed graphically. The graph of a linear function will be represented by a straight line, whereas a non-linear function will be curved.

See worked example 2

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Worked example 1

Determine if the following functions are linear or non-linear.

a. $y = 1 + 2x$

Explanation

Both variables, x and y, have been raised to the power of 1.

Answer

Linear

b. $y = 5x^3 - 4$

Explanation

In the equation $y = 5x^3 - 4$, x has been raised to the power of 3.

Answer

Non-linear

c. $y = 5$

Explanation

The variable y has been raised to the power of 1.

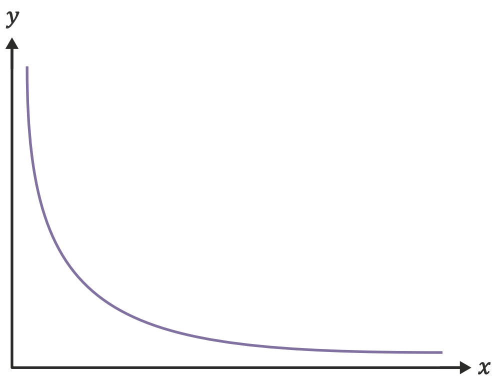
Answer

Linear

Worked example 2

Determine if the following graphs are linear or non-linear.

a.



Explanation

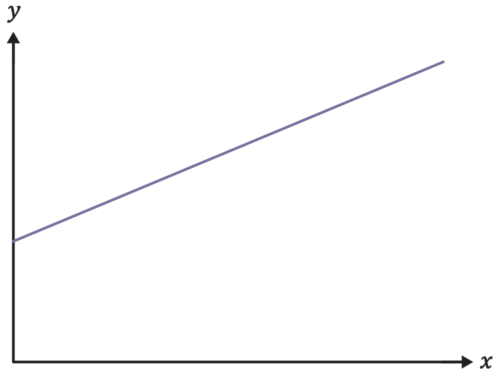
The line on this graph is curved.

Answer

Non-linear

Continues →

b.



Explanation

The line on this graph is straight.

Answer

Linear

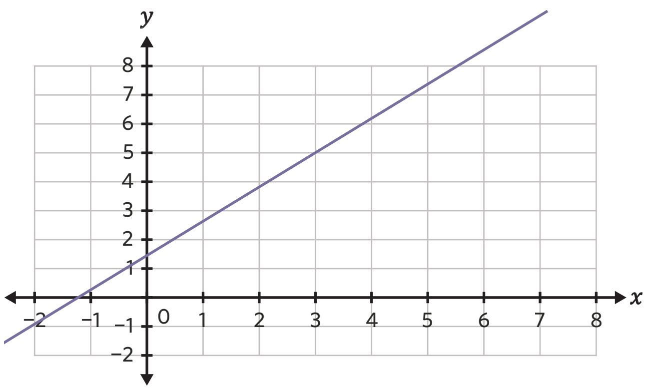
Identifying features of linear functions and graphs

There are many features of linear functions and graphs that can help when interpreting them. This lesson will only focus on identifying these features.

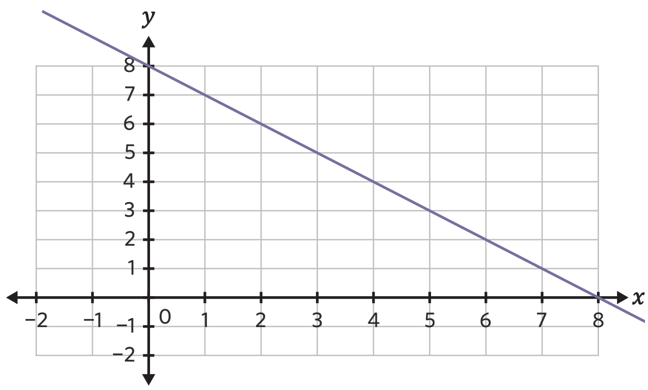
The **gradient** (often called the 'slope') of a line represents its steepness, and is equal to the change in y for every one unit increase in x. There are four types of gradients.

See worked example 3

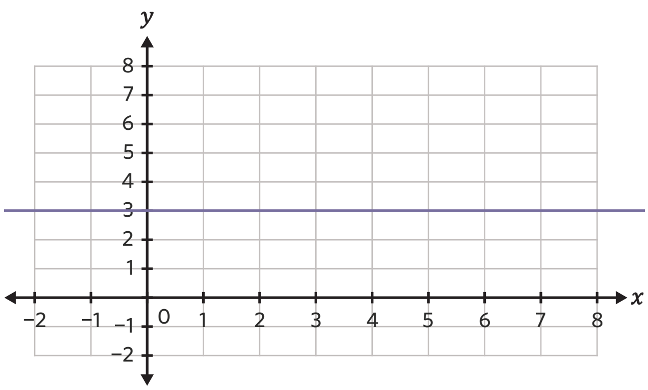
Positive gradient



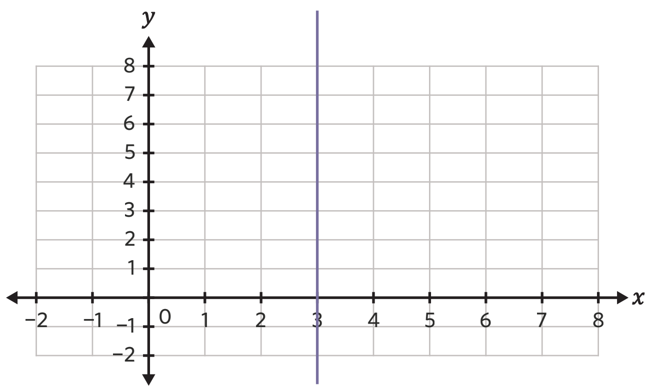
Negative gradient



Zero gradient



Undefined gradient



The ***x*-intercept** refers to the point on the line where it crosses the x-axis, when $y = 0$.

The ***y*-intercept** refers to the point on the line where it crosses the y-axis, when $x = 0$.

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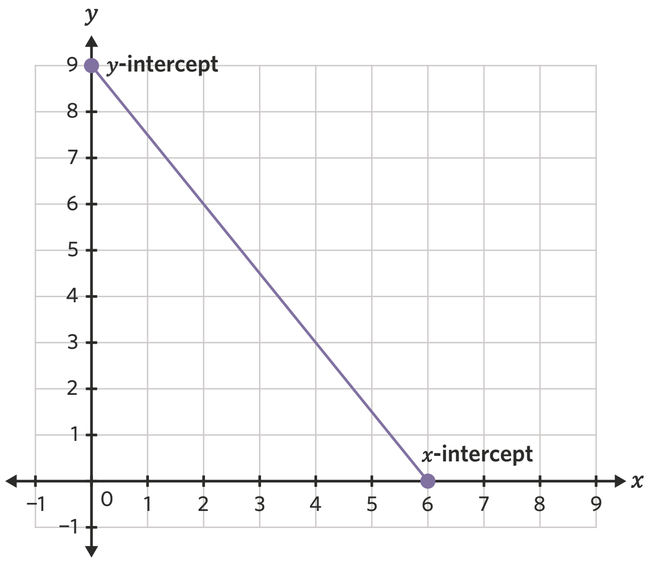
The following graph has an x-intercept of 6 and a y-intercept of 9.



The gradient and y-intercept can also be determined from the equation of the line.

See worked example 4

When a linear function is expressed in the form $y = a + bx$,

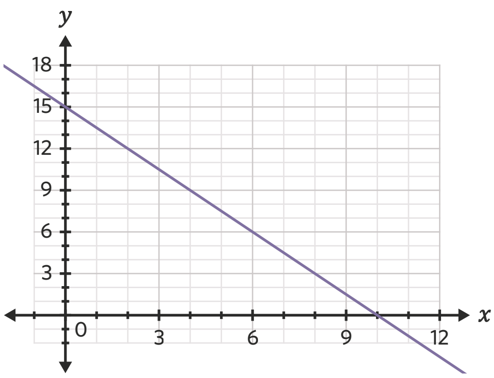
- a is the y-intercept
- b is the gradient

Worked example 3

For the following graphs:

- identify the gradient type (positive, negative, zero or undefined)
- determine the x and y-intercepts (if possible)

a.



Explanation

Step 1: Identify the gradient type.

The line is downward-sloping. This is a negative gradient.

Step 2: Determine the x-intercept.

The line crosses the x-axis at $x = 10$.

Step 3: Determine the y-intercept.

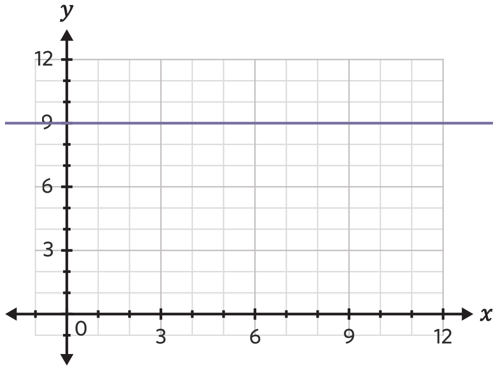
The line crosses the y-axis at $y = 15$.

Answer

Negative gradient, x-intercept $= 10$, y-intercept $= 15$

Continues →

b.



Explanation

Step 1: Identify the gradient type.

The line is horizontal. This means the gradient is zero.

Step 2: Determine the x-intercept.

The line is horizontal and doesn't have a $y = 0$ value. This means it will never cross the x-axis.

Step 3: Determine the y-intercept.

The line crosses the y-axis at $y = 9$.

Answer

Zero gradient, no x-intercept, y-intercept $= 9$

Worked example 4

For the following equations,

- Identify the gradient type (positive, negative, zero or undefined)
- Determine the y-intercept

a. $y = 9x + 8$

Explanation

Step 1: Identify the gradient type.

The gradient is represented by the coefficient of x. The gradient is 9, which is a positive value.

Step 2: Determine the y-intercept.

The y-intercept is the constant, 8.

Answer

Positive gradient, y-intercept $= 8$

b. $x = 5$

Explanation

Step 1: Identify the gradient type.

The value of x is constant (5, regardless of the y value). This means the line will be vertical and the gradient is undefined.

Step 2: Determine the y-intercept.

The line is vertical and doesn't have a $x = 0$ value. This means it doesn't cross the y-axis.

Answer

Undefined gradient, no y-intercept

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4B Questions

Note: There are no direct exam questions relevant to this lesson.

Identifying linear functions and their graphs

1. Which of the following is false for a linear function of the form $y = a + bx$?

A. a is the initial value
B. b is the constant rate of change
C. y is a changing variable
D. x is a constant variable

2. Consider the equation $y = 5x - 18$.

a. Do any of the following terms or transformations appear in the equation?

i. Variables raised to a power other than 0 or 1
ii. Roots of variables
iii. Sets of two or more variables multiplied together
iv. Numbers raised to the power of a variable
v. Trigonometric or logarithmic terms

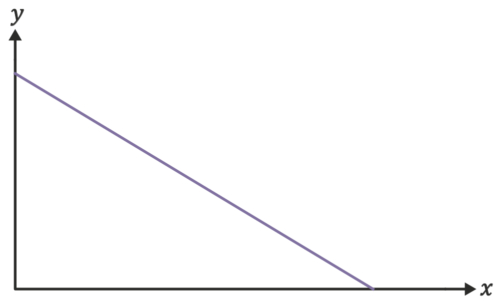
b. Is the equation linear?

3. Determine whether each of the following equations is linear or non-linear.

a. $y^2 = 2x + 4$
b. $y = -9 + 1.8x$
c. $x - 103 = y$
d. $y = \sqrt{x} - 4$
e. $y = \tan(2x)$
f. $y = x$
g. $-19 = y$
h. $y = x^{-4} + 2$

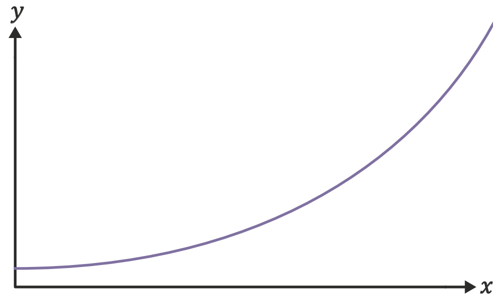
4. Determine whether each of the following graphs is linear or non-linear.

a.

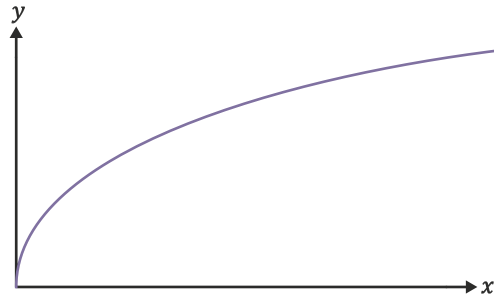


b.

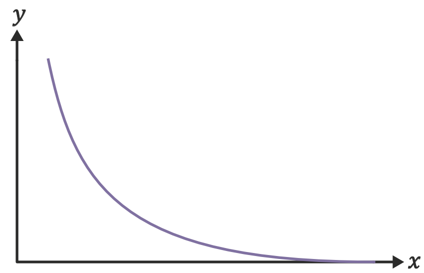
c.



d.



5. Alicia's teacher asked her to graph the equation $y = -1.5x + 12$.
She drew the following graph.
Is it possible that Alicia could be correct? Explain.



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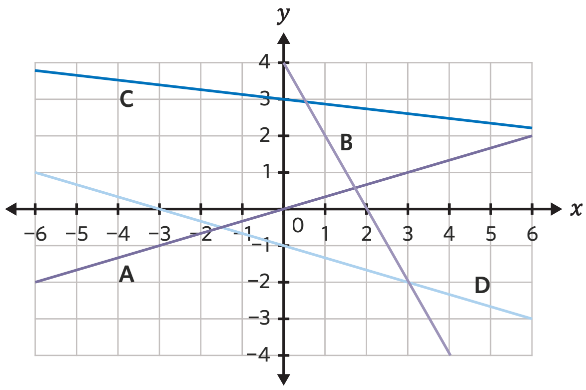
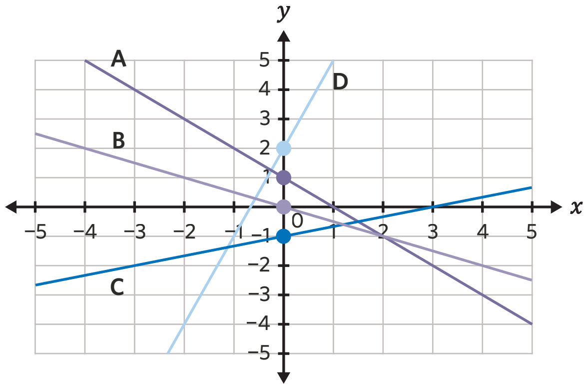
Identifying features of linear functions and graphs

6. Which of the following lines has a positive gradient?

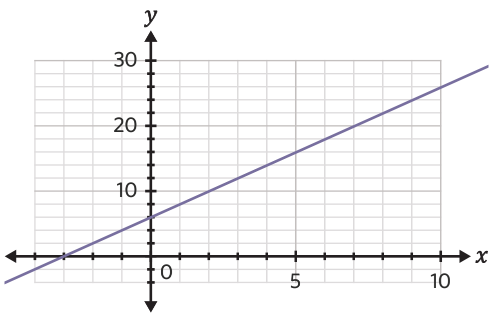


7. Which of the following graphs has a y-intercept of 1?

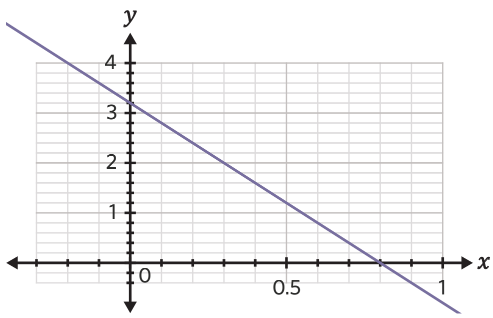


8. For the following graphs, identify the gradient type (positive, negative, zero or undefined) and determine the x and y-intercepts (if possible).

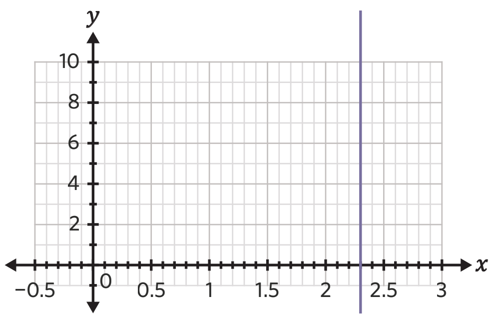
a.



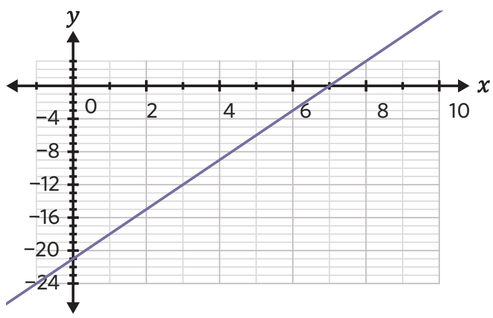
b.



c.



d.



9. For the following equations, identify the gradient type (positive, negative, zero or undefined) and determine the y-intercept.

a. $y = 8x - 4$

b. $y = -1.4x + 8$

c. $y = x$

d. $y = -10 - 9x$

e. $x = 12$

f. $y = -x + 2$

g. $-14 + 0.9x = y$

h. $y = 214$

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10. Which of the following functions is impossible?

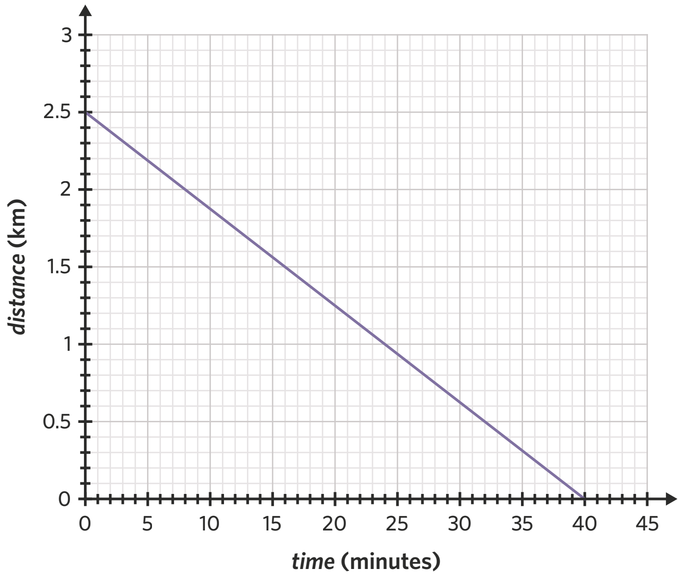
A. Negative gradient, x-intercept $= 5$, y-intercept $= 10$

B. Negative gradient, x-intercept $= 10$, y-intercept $= 1$

C. Zero gradient, x-intercept $= 7$, y-intercept $= 118$

D. Undefined gradient, x-intercept $= 7.8$, no y-intercept

11. Brayden records how long it takes him to walk home from lacrosse training. When he gets home, he produces the following graph.



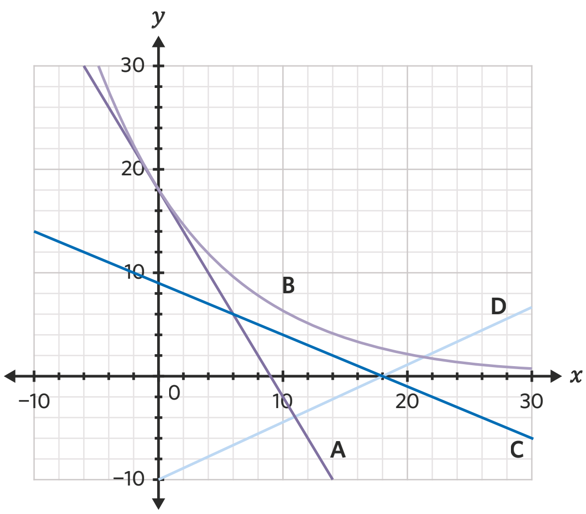
The x-intercept in Brayden's graph represents how long it took him to walk home. The y-intercept represents how far away his house is from lacrosse training.

a. How far away does Brayden live from lacrosse training?

b. How long did it take him to walk home?

Joining it all together

12. Which of the following graphs depicts a linear function with a negative gradient and a y-intercept of 18?



13. Which of the following equations could model a linear function with a positive x-intercept and a negative y-intercept?

A. $y = 2x^2 - 18$ **B.** $y = -0.4 + 21x$ **C.** $y = -1.4x + 10$ **D.** $y = 4^x - 3.8$

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14. For each of the following equations:

- Identify whether it is linear or non-linear
- If linear, identify the gradient type (positive, negative, zero or undefined) and determine the y-intercept.

a. $y = 0.5x^2 - 26$ **b.** $y = 110x + 28$ **c.** $x = 8 + 14$ **d.** $y = -43 - 2^x$

e. $-x + 219.01 = y$ **f.** $x + y = 28$ **g.** $\log_{10}(4x) = y$ **h.** $\frac{y}{x} = 20 + 0.7$

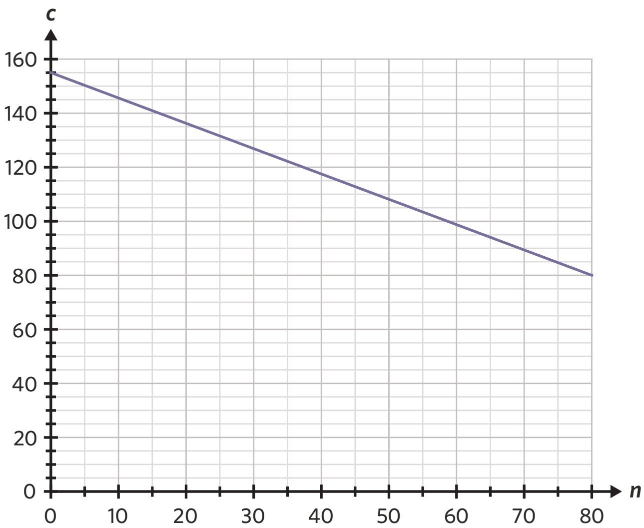
---

15. Freya has a large collection of old footy cards, and wants to replace them with new ones. Each week, she buys a pack of cards and gives away a certain amount. The following graph shows the total number of new and old cards c, she has in her collection after n weeks.

a. The y-intercept represents Freya's initial number of cards. How many cards did Freya begin her collection with?

b. Is Freya's graph a linear or non-linear function?

c. Which is greater? The number of cards Freya purchases each week, or the number of cards she gives away?



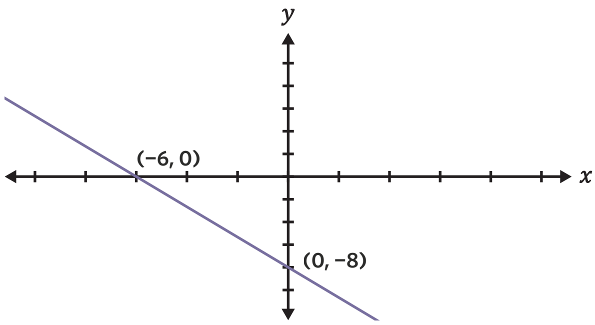
Questions from multiple lessons

Graphs and relations *Year 10 content*

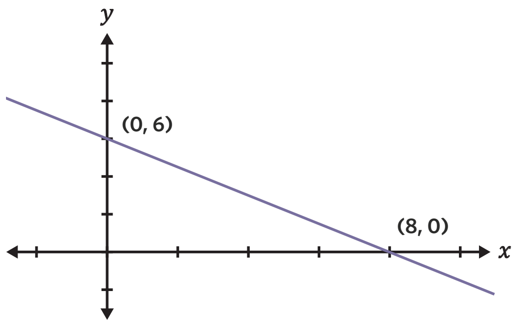
16. A straight line has an x-intercept at $x = 8$ and a y-intercept at $y = 6$.

Which of the following graphs is correct?

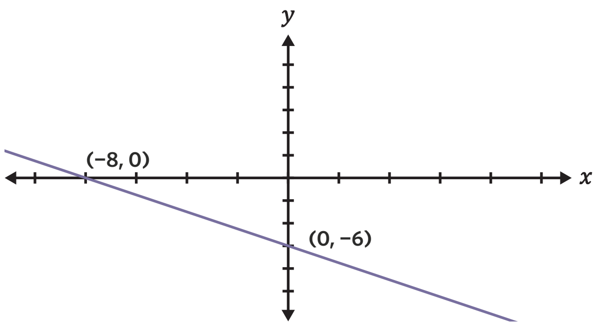
A.



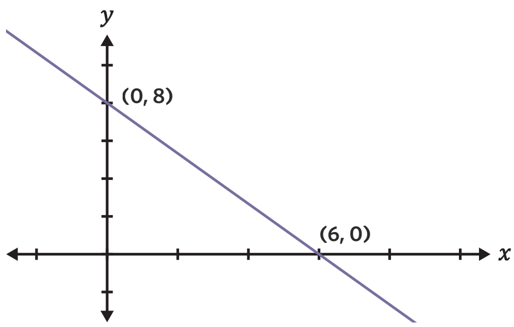
B.



C.



D.



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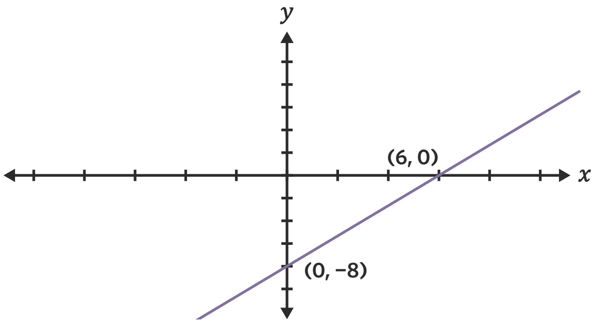
E.



Adapted from VCAA 2018 Exam 1 Graphs and relations Q1

Graphs and relations *Year 10 content*

17. Jonty found a bag of marbles, and wants to start collecting more.

There are 80 marbles in the bag he found. Each week, he plans to collect two more marbles.

This can be represented by the following equation:

$M = 80 + 2w$

Jonty now has 128 marbles.

How many weeks has Jonty been collecting marbles for?

A. 14 **B.** 24 **C.** 48 **D.** 64 **E.** 104

Adapted from VCAA 2017NH Exam 1 Graphs and relations Q2

Graphs and relations *Year 10 content*

18. Within the central district of Paris, there is a linear relationship between the price of a coffee in euros (€), p, and the distance, in km, from the Eiffel Tower, d. This relationship is given by the equation $3d = 26 - 4p$.

a. Transpose this linear relation to make p the subject. (1 MARK)

b. What is the price of a coffee at the Louvre, 4.5 km away from the Eiffel Tower? Round the answer to the nearest cent. (1 MARK)

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4C Graphing linear functions

STUDY DESIGN DOT POINT

- graphing linear relations $Ax + By = C$ and equivalent forms

4A 4B 4C 4D 4E 4F 4G

KEY SKILLS

During this lesson, you will be:

- plotting linear functions from a table
- graphing linear functions from an equation
- graphing horizontal and vertical lines.

KEY TERMS

- Two-points method
- Gradient-intercept method
- Intercept-intercept method

Linear functions can be represented graphically. The graphical depiction of a linear function is, by definition, a straight line. Graphing a linear function is useful to understand the relationship between two variables.

Plotting linear functions from a table

A linear graph is the graphical depiction of a linear function. A linear function can be plotted by using a table of values.

A table of values contains a row of x values, and their corresponding y values.

Filling out a table of values makes it easy to plot a linear graph since each pair of values represents a point on the line.

After plotting, the coordinates of each point are written in brackets, with the x value first. For example, if $y = 5$ when $x = 2$, this is written as $(2, 5)$.

Worked example 1

Consider $y = 2x + 1$.

a. Create a table for $x = -1, 0, 1, 2$ and 3.

Explanation – Method 1: By hand

Step 1: Set up a table over the appropriate values.

Write the x values in the table.

x	-1	0	1	2	3
y					

Step 2: Substitute the x values into the equation.

Substitute the five x values into $y = 2x + 1$.

Let $x = -1$

$$y = 2(-1) + 1$$
$$= -2 + 1$$
$$= -1$$

Repeat for the remaining x values.

Step 3: Enter the y values into the table.

Continues →

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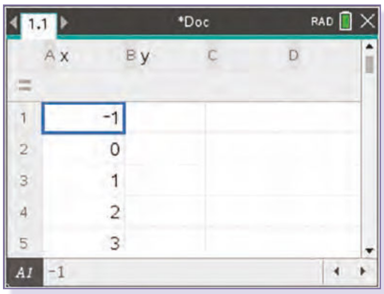
Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

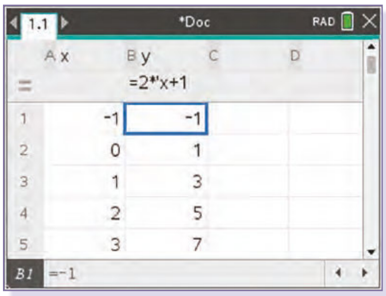
Step 2: Name column A 'x' and column B 'y'.

Enter the x values into column A, starting from row 1.



Step 3: Enter '=2x+1' into the cell below the 'y' heading. Press enter.

Select 'Variable Reference' → 'OK'.

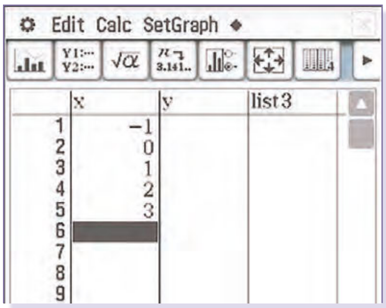


Explanation – Method 3: Casio ClassPad

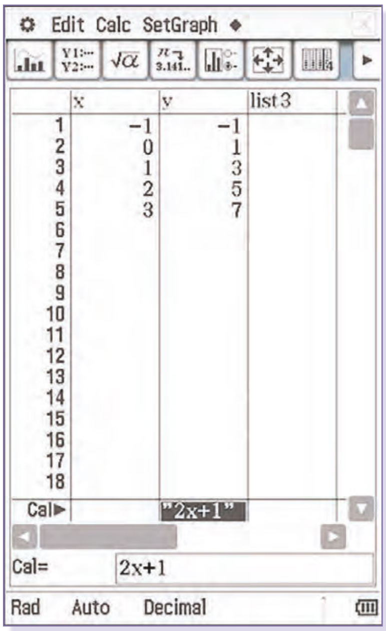
Step 1: From the main menu, tap Statistics.

Step 2: Name the first list 'x' and the second list 'y'.

Enter the x values into list 'x', starting from row 1.



Step 3: In the calculation cell Cal▶, enter '2x+1'. Press **EXE**.



Answer – Method 1, 2 and 3

x	-1	0	1	2	3
y	-1	1	3	5	7

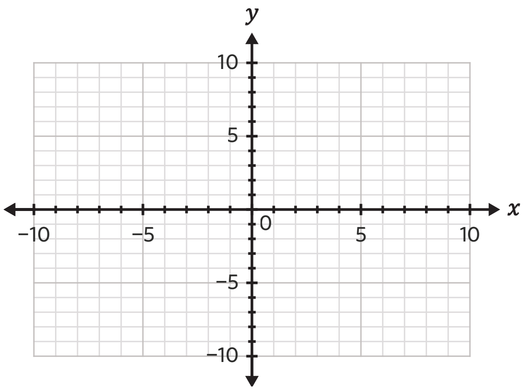
Continues →

b. Plot $y = 2x + 1$ using the table of values from part **a**.

Explanation

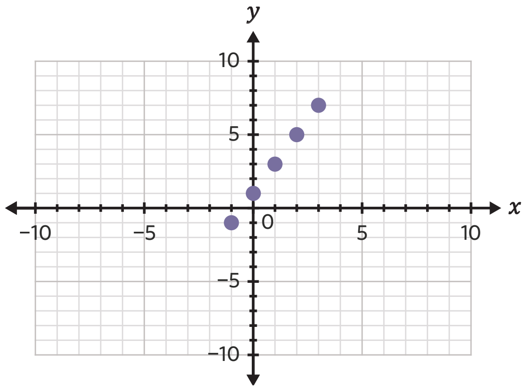
Step 1: Draw and label a set of x and y axes.

Ensure all values in the table are covered.



Step 2: Plot each pair of values from the table as a point.

The set of points here is $(-1, -1)$, $(0, 1)$, $(1, 3)$, $(2, 5)$ and $(3, 7)$.

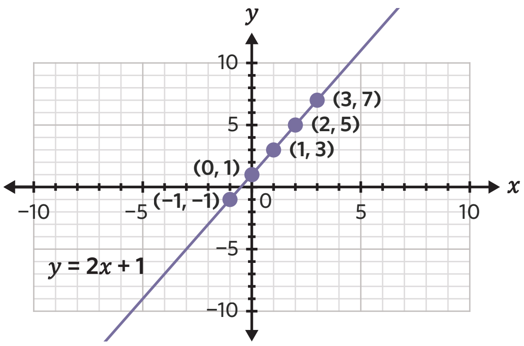


Step 3: Connect the points with a straight line.

Make sure to extend the line.

Label the line $y = 2x + 1$ and each point with its coordinate.

Answer



Graphing linear functions from an equation

See worked example 2

Plotting functions using a table of values can be time consuming. However, it is also possible to graph a linear function by only plotting two points. The **two-points method** is a graphing method that uses any two points to graph a linear function. Two values of x are chosen and substituted into the linear function tofind their corresponding y values. These points are then connected by a straight line, and extended through the points.

Alternatively, the two points needed to graph a linear function can be found using either the gradient-intercept or intercept-intercept method.

The **gradient-intercept method** (or slope-intercept method) is a graphing method that involves identifying and plotting the y-intercept, and then using the gradient to plot another point.

For a linear function in the gradient-intercept form $y = a + bx$,

- a is the y-intercept, which is the point where the line crosses the y-axis.
- b is the gradient, which represents how steep the line is. The gradient can be thought of as: $b = \frac{rise}{run}$.

  For example, if the gradient is $\frac{1}{2}$, this means the line moves up by 1 unit for every 2 units it 'runs' across to the right. If the gradient is negative, for example -2, the line moves down by 2 units for every 1 unit it 'runs' across to the right (since $-2 = \frac{-2}{1}$).

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When using the gradient-intercept method, if the linear function is not already in gradient-intercept form, solve the equation for y to change it to the form $y = a + bx$.

The **intercept-intercept method** is a graphing method that involves finding the x and y-intercepts as the two points required.

- The x-intercept is the point where the line crosses the x-axis and occurs when $y = 0$. To find the x-intercept, substitute $y = 0$ into the equation and solve for x.
- The y-intercept is the point where the line crosses the y-axis and occurs when $x = 0$. To find the y-intercept, substitute $x = 0$ into the equation and solve for y.

When linear functions are given in the form $y = a + bx$, the gradient-intercept method can be applied. However, sometimes linear functions are in the standard form $Ax + By = C$. In this case, the intercept-intercept method is more appropriate.

Linear functions can also be graphed using a calculator instead of by hand.

See worked example 3

Worked example 2

Graph the following linear functions using the specified method.

a. $y = -\frac{1}{2}x - 1$ [two-points method]

Explanation

Step 1: Choose any two values of x.

Suppose $x = -4$ and $x = 4$.

Step 2: Substitute the x values into the equation.

Let $x = -4$

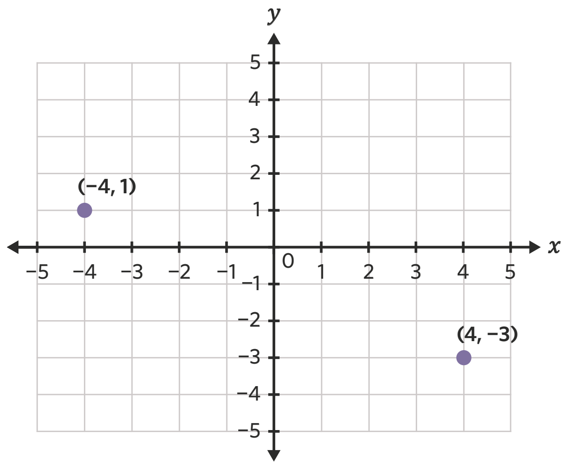
$$y = -\frac{-4}{2} - 1$$
$$= 2 - 1$$
$$= 1$$

Let $x = 4$

$$y = -\frac{4}{2} - 1$$
$$= -2 - 1$$
$$= -3$$

The two points are $(-4, 1)$ and $(4, -3)$.

Step 3: Plot the two points on a set of axes.

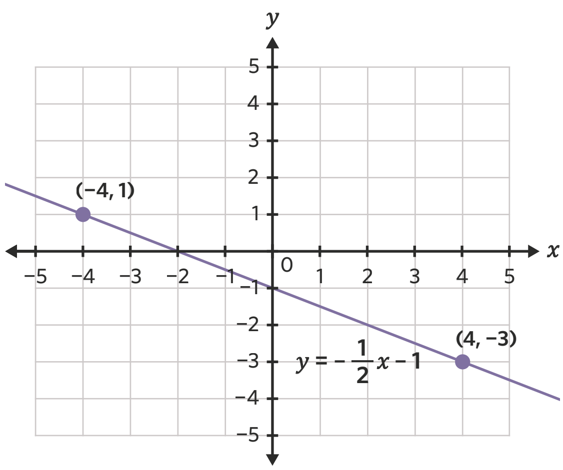


Step 4: Connect the points with a straight line.

Make sure to extend the line.

Label the line $y = -\frac{1}{2}x - 1$.

Answer



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b. $y = -2 + 3x$ [gradient-intercept method]

Explanation

Step 1: Identify the y-intercept.

When a linear function is in the form $y = a + bx$, a is the y-intercept.

$a = -2$

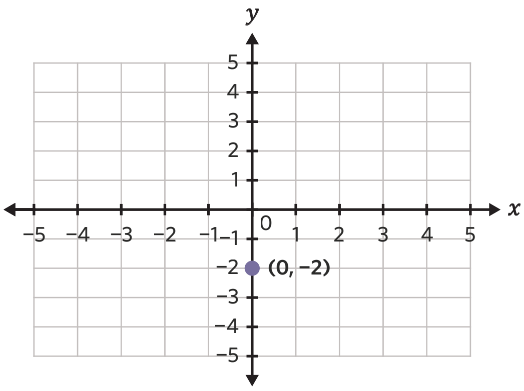
y-intercept: $(0, -2)$

Step 2: Identify the gradient.

When a linear function is in the form $y = a + bx$, b is the gradient.

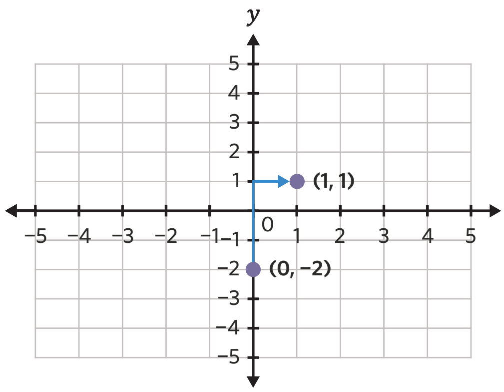
$b = 3 = \frac{3}{1}$

Step 3: Plot the y-intercept.



Step 4: Use the gradient to plot an additional point.

Since $b = \frac{3}{1}$, the line moves up by 3 units for every 1 unit to the right.

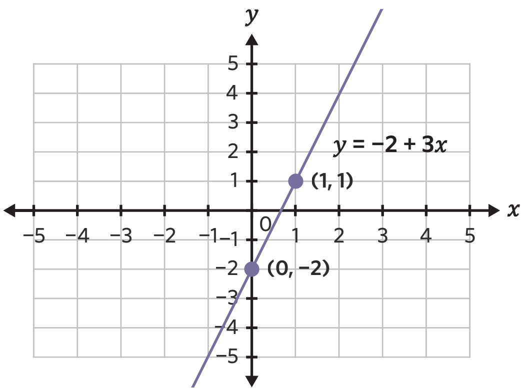


Step 5: Connect the points with a straight line.

Make sure to extend the line.

Label the line $y = -2 + 3x$.

Answer



c. $5x + 2y = 10$ [intercept-intercept method]

Explanation

Step 1: Find the x-intercept.

Substitute $y = 0$ into the equation and solve for x.

$5x + 2(0) = 10$

$5x = 10$

$x = 2$

The x-intercept is $(2, 0)$.

Step 2: Find the y-intercept.

Substitute $x = 0$ into the equation and solve for y.

$5(0) + 2y = 10$

$2y = 10$

$y = 5$

The y-intercept is $(0, 5)$.

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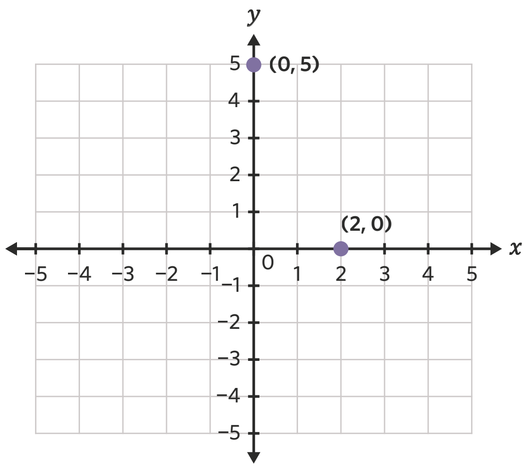
Step 3: Plot the intercepts on a graph.

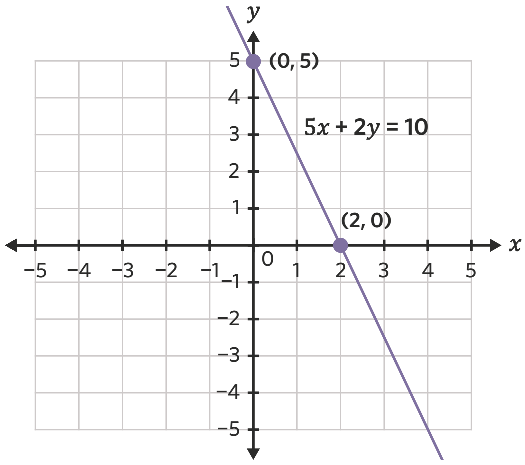


Step 4: Connect the points with a straight line.

Make sure to extend the line.

Label the line $5x + 2y = 10$.

Answer



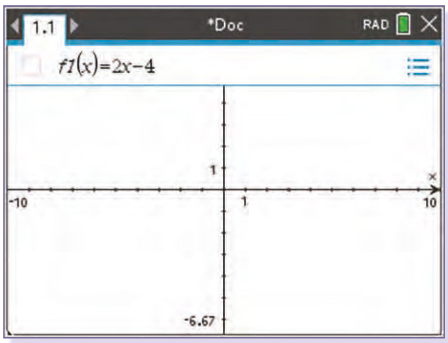
Worked example 3

Use a calculator to graph the linear function $y = 2x - 4$.

Explanation – Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '2: Add Graphs'.

Step 2: On the first line, type '$2x - 4$' for $f1(x)$.



Step 3: Press enter.

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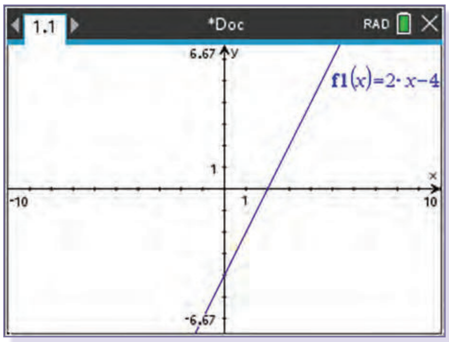
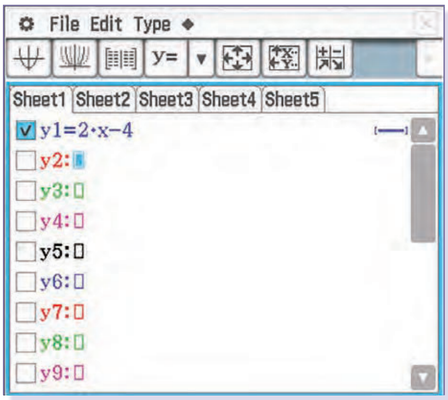
Answer



Explanation – Method 2: Casio ClassPad

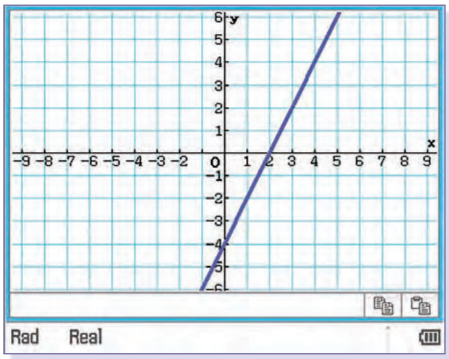
Step 1: From the main menu, tap Graph & Table.

Step 2: On the first line, type '$2x - 4$' for $y1$ and press enter.



Step 3: Tap to draw the graph.

Answer



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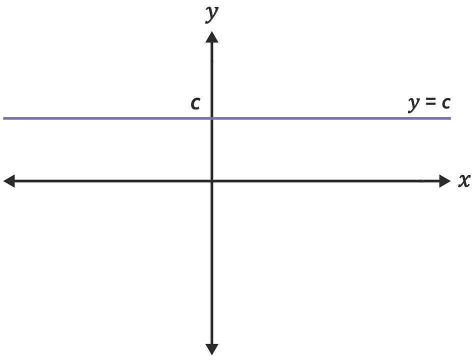
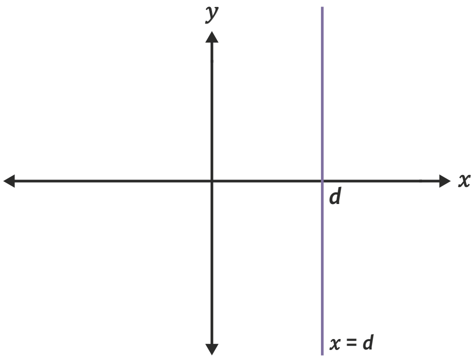
Graphing horizontal and vertical lines

Horizontal and vertical lines are two types of linear graphs.

Horizontal lines share the same y value at every point, so they are written in the form $y = c$.

Vertical lines share the same x value at every point, so they are written in the form $x = d$.





Worked example 4

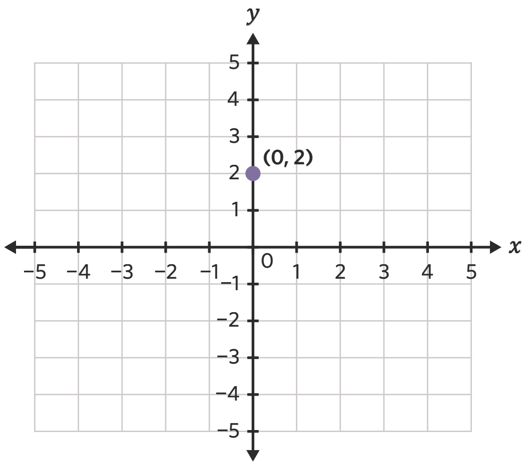
Graph the following lines.

a. $y = 2$

Explanation

Step 1: Plot the y-intercept.

The y-intercept is $(0, 2)$.

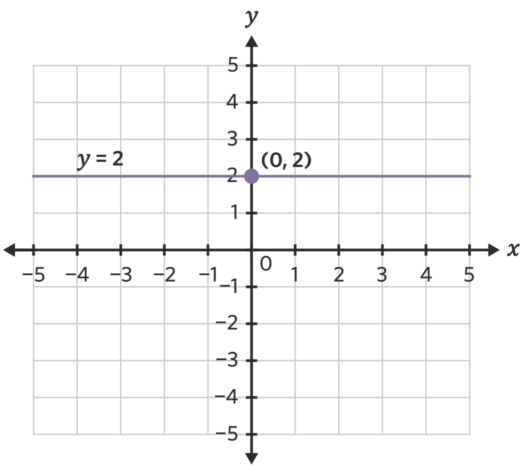


Step 2: Draw the rest of the line.

Every point shares the same y value at $y = 2$.

Label the line $y = 2$.

Answer



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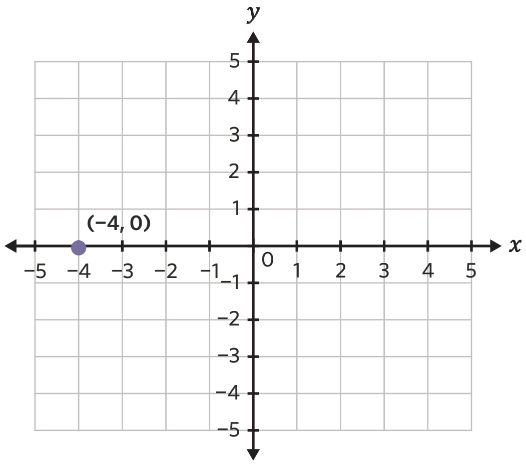
b. $x = -4$

Explanation

Step 1: Plot the x-intercept.

The x-intercept is $(-4, 0)$.

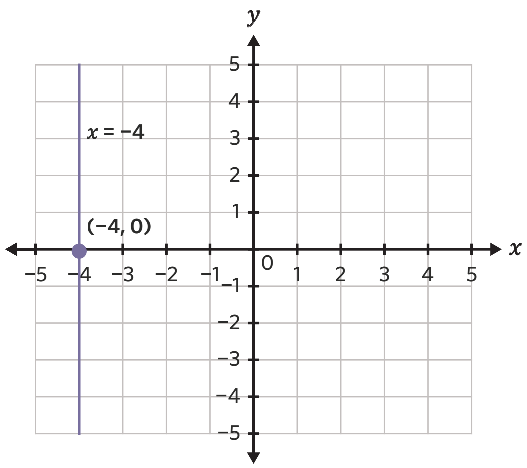


Step 2: Draw the rest of the line.

Every point shares the same x value at $x = -4$.

Label the line $x = -4$.

Answer



4C Questions

Plotting linear functions from a table

1. Which of the following shows a table of values for the line $y = 6x - 1$?

A.

x	−1	0	1
y	−6	0	6

B.

x	−1	0	1
y	−7	−1	4

C.

x	3	6	9
y	17	35	53

D.

x	1	2	3
y	5	11	19

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2. Create a table of values for the following linear functions for the values $x = 0, 1, 2, 3, 4$.

   a. $y = 5x$ b. $y = 2x + 4$ c. $3x + y = 4$

3. Plot $y = 1 - 3x$ by creating a table of values for $x = 0, 1, 2, 3, 4$.

4. Plot $y = -0.5x + 1$ by creating a table of values for $x = -2, -1, 0, 1, 2$.

Graphing linear functions from an equation

5. The coordinates of the x-intercept of the line $y = 3x - 12$ are:

   A. $(0, 4)$ **B.** $(4, 0)$ **C.** $(0, 12)$ **D.** $(12, 0)$

6. Graph the following linear functions using the two-points method.

   a. $y = -x + 2$ b. $y = -6 + 2x$ c. $2x + 2y = 2$

7. Graph the following linear functions using the gradient-intercept method.

   a. $y = 2 - 2x$ b. $y = \frac{1}{3}x + 4$ c. $x + 2y = 6$

8. Graph the following linear functions using the intercept-intercept method.

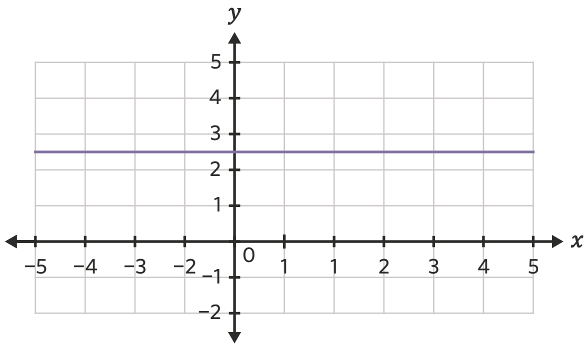
   a. $x + y = 1$ b. $y = 12 - \frac{3}{4}x$ c. $8x - 5y = 2$

9. Use a calculator to graph the following linear functions.

   a. $y = 7x + 2$ b. $y = -20x + 2$

Graphing horizontal and vertical lines

10. The equation of the line shown is closest to



   A. $x = 2.5$ **B.** $y = x + 2.5$ **C.** $y = 2.5x$ **D.** $y = 2.5$

11. Graph the following lines.

   a. $y = 1$ b. $x = 5$ c. $y = -3$

Joining it all together

12. Consider the linear function $3x - y = 1$.

   a. Find the x and y-intercepts.

   b. Create a table of values for $x = 0, 1, 2, 3$.

   c. Graph the linear function $3x - y = 1$ and label the x and y-intercepts.

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13. Alison is comparing ski slopes around her town. She has written the following equations to represent the slopes at two ski resorts,

Sunrise Ridge: $h = 1125 - \frac{9}{4}d$

Cumulus Peak: $h = 760 - \frac{4}{3}d$,

where h is the height of the slope in metres and d is the horizontal distance in metres along the slope.

a. Use a table of values for $d = 0, 100, 200$ to plot the slope at Sunrise Ridge. Put h on the vertical axis and d on the horizontal axis. Scale the d-axis from 0 to 600.

b. Use the intercept-intercept method to graph the slope at Cumulus Peak on the same set of axes.

c. Which slope is the highest?

d. Which slope is the steepest?

e. Which slope covers the most ground horizontally?

Exam practice

14. Kyla's business manufactures car seat covers.

The *monthly revenue*, R, in dollars, from selling n seat covers is given by

$R = 80n$

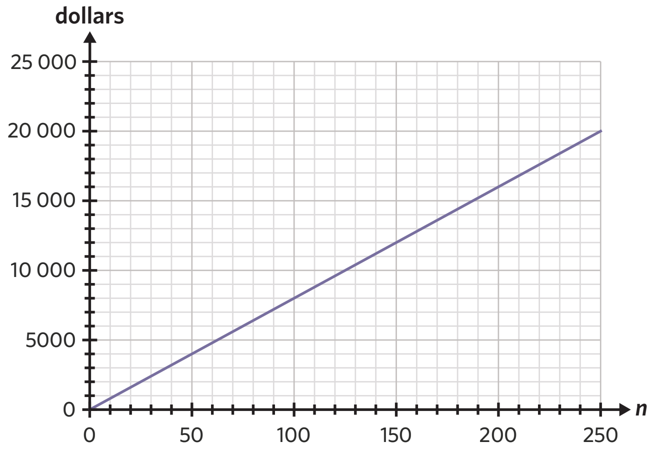
This relationship is shown on the graph.

The *monthly cost*, C, in dollars, of making n seat covers is given by

$C = 36n + 5000$

Sketch the *monthly cost*, C, of making n seat covers on the graph. (1 MARK)

VCAA 2020 Exam 2 Graphs and relations Q3a



62% of students answered this question correctly.

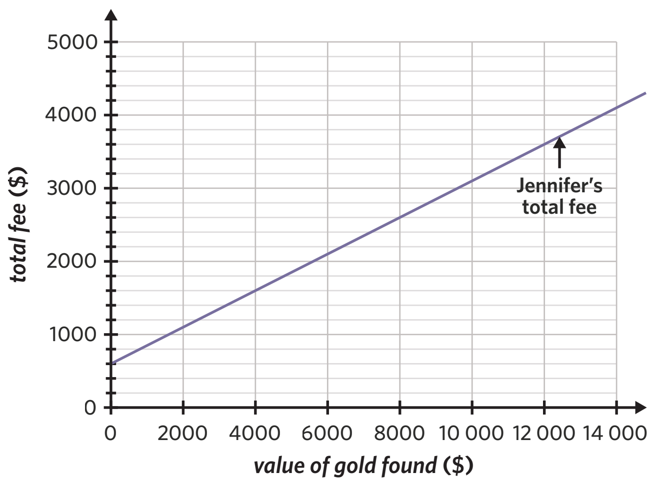
15. Robert wants to hire a geologist to help him find potential gold locations.

One geologist, Jennifer, charges a flat fee of $600 plus 25% commission on the value of gold found. The following graph displays Jennifer's total fee in dollars.

Another geologist, Kevin, charges a total fee of $3400 for the same task.

Draw a graph of the line representing Kevin's fee on the axes. (1 MARK)

VCAA 2018 Exam 2 Graphs and relations Q3a



60% of students answered this question correctly.

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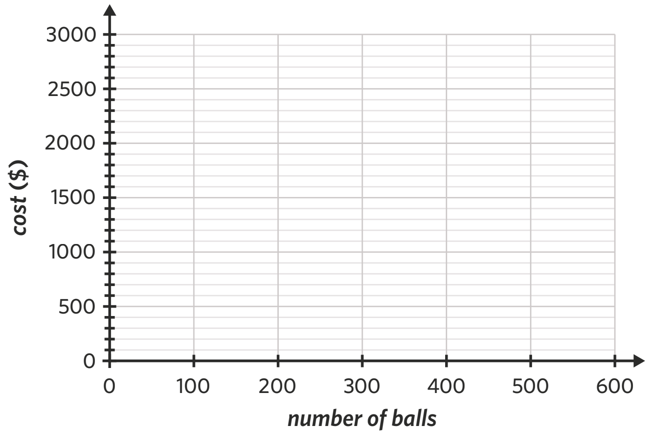
16. A company manufactures and sells hockey balls. The *cost*, in dollars, of manufacturing a certain *number of balls* can be found using the equation

cost = 1200 + 1.5 × *number of balls*

On the grid, sketch the graph of the relationship between the manufacturing *cost* and the *number of balls* manufactured. (1 MARK)

VCAA 2016 Exam 2 Graphs and relations Q2b



58% of students answered this question correctly.

Questions from multiple lessons

Graphs and relations *Year 10 content*

17. Hamish is a carpenter.

He charges a call-out fee of \$140, plus \$90 for each hour of work.

The equation that represents the total cost, \$$C$, Hamish charges, for t hours of carpentry is

A. $C = 230t$ **B.** $C = 90$ **C.** $C = 140 + 90t$ **D.** $C = 140t$ **E.** $C = 90 + 140t$

Adapted from VCAA 2018 Exam 1 Graphs and relations Q2

Data analysis

18. An investigation was carried out to determine the association between the variables *length of a movie* (less than 1 hour, 1–2 hours, over 2 hours) and *popularity* (low, medium, high). These variables are

A. both numerical variables.

B. both nominal variables.

C. both ordinal variables.

D. a numerical and ordinal variable respectively.

E. an ordinal and nominal variable respectively.

Adapted from VCAA 2018NH Exam 1 Data analysis Q5

Recursion and financial modelling

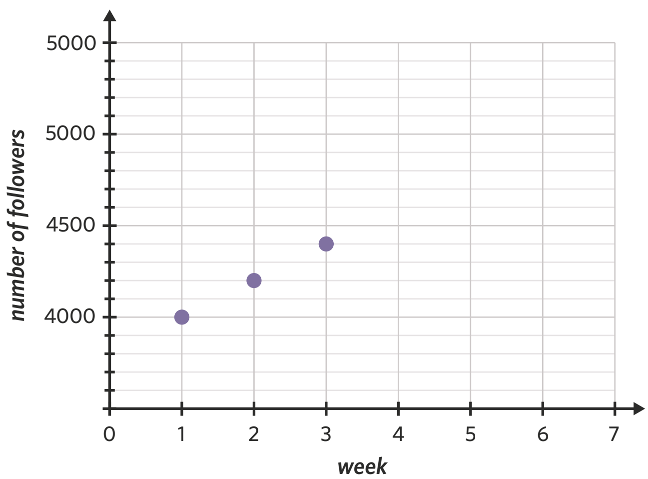
19. Hector is a rising TikTok star and his number of followers increases weekly at a constant rate.

The graph shows his number of followers in the first four weeks of 2020.

a. How many followers does Hector gain each week? (1 MARK)

b. How many followers will Hector have in week 6? (1 MARK)

Adapted from VCAA 2011 Exam 2 Recursion and financial modelling Q1a,b



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4D Finding the equation of a linear function

STUDY DESIGN DOT POINT

- the linear function $y = ax + b$, its graph, and interpretation of the parameters, a and b in terms of initial value and constant rate of change respectively

4A 4B 4C **4D** 4E 4F 4G

KEY SKILLS

During this lesson, you will be:

- finding a linear equation from two known points
- finding a linear equation from a graph.

When working with linear functions, an important skill is to be able to find the equation of a straight line. This is useful when only two points are known or when it is difficult to read specific values from a graph. If the equation can be found, different values of x and y can be substituted into the equation to find the corresponding x and y value. This enables any point of a linear function to be calculated.

Finding a linear equation from two known points

See worked example 1

Recall that the general form of a linear function is $y = a + bx$, where

- a is the y-intercept
- b is the gradient

The gradient can be calculated using the formula

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

where x_1, x_2, y_1 and y_2 represent two known coordinates (x_1, y_1) and (x_2, y_2).

When provided with two or more coordinates, either in a table or graph, the values of a and b can be calculated and substituted into the general form for a linear function to determine the equation. This method can be simplified when one of the coordinates includes the y-intercept.

However, the equation can still be found when the known values don't include the y-intercept.

See worked example 2

An alternative method to find the equation is by using the formula $y - y_1 = b(x - x_1)$.

After calculating the gradient, b, one of the two known points can be substituted into the formula to find the equation of the straight line.

Note: When one value of a or b is positive and the other is negative, the positive value is often placed first in the equation to make it easier to read.

For example, $y = -18 + 5x$ can instead be written as $y = 5x - 18$.

Worked example 1

Determine the equation of the linear function for each of the given sets of values.

a.

x	0	1	2	3	4
y	14	17	20	23	26

Explanation

Step 1: Determine the y-intercept.

This is the y-value when $x = 0$. From the table, the y-intercept is $(0, 14)$.

$a = 14$

Step 2: Determine the gradient.

Using the coordinates $(0, 14)$ and $(1, 17)$:

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{17 - 14}{1 - 0}$$
$$= \frac{3}{1}$$
$$= 3$$

Note: Any two coordinates on the line can be used.

Step 3: Substitute the a and b values into $y = a + bx$.

Answer

$y = 14 + 3x$

b. $(3, 21)$ and $(8, 11)$

Explanation – Method 1: By hand

Step 1: Determine the gradient.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{11 - 21}{8 - 3}$$
$$= \frac{-10}{5}$$
$$= -2$$

Step 2: Determine the y-intercept.

Substitute b and one of the coordinates into $y = a + bx$ and solve for a.

$21 = a - 2 \times 3$

$21 = a - 6$

$27 = a$

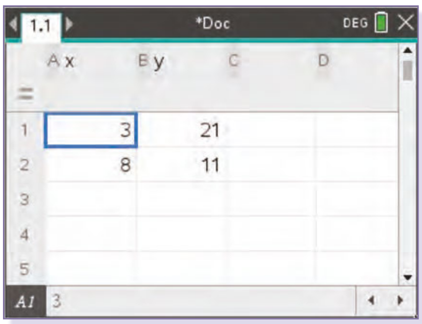
Note: Either of the two coordinates can be used.

Step 3: Substitute the a and b values into $y = a + bx$.

Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select ‘1: New’ → ‘4: Add Lists & Spreadsheet’.

Step 2: Name column A ‘x’ and column B ‘y’ and enter the coordinates.



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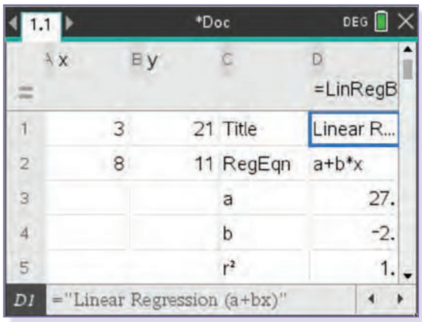
Step 3: Press menu. Select → '4: Statistics' → '1: Stat Calculations' → '4: Linear Regression (a+bx)'.

On the settings window, change 'X List:' to 'x' and 'Y List:' to 'y'. Select 'OK'.

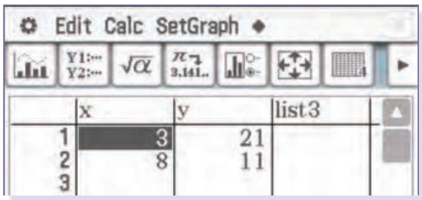


Step 4: Substitute the a and b values shown in the regression analysis into $y = a + bx$.

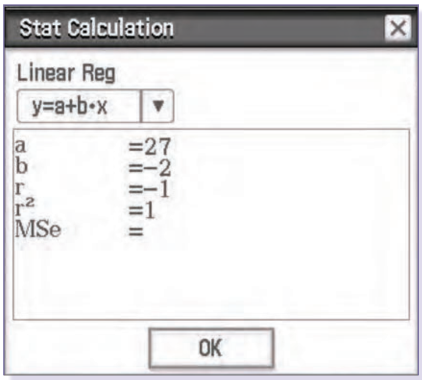
Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap Statistics.

Step 2: Name list1 'x' and list2 'y' and enter the coordinates.



Step 3: Tap 'Calc' → 'Regression' → 'Linear Reg'. Specify the data set by changing 'XList:' to 'main\x' and 'YList:' to 'main\y'. Tap 'OK'.



Step 4: Substitute the a and b values shown in the regression analysis into $y = a + bx$.

Answer – Method 1, 2 and 3

$y = 27 - 2x$

Worked example 2

Using the formula $y - y_1 = b(x - x_1)$, find the equation of the line that passes through the points (6, 21) and (15, 57).

Explanation

Step 1: Determine the gradient.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{57 - 21}{15 - 6}$$
$$= \frac{36}{9}$$
$$= 4$$

Step 2: Substitute b and one of the two coordinates into the formula and solve to find the equation.

$$y - 21 = 4(x - 6)$$
$$y - 21 = 4x - 24$$
$$y = 4x - 3$$

Answer

$y = 4x - 3$

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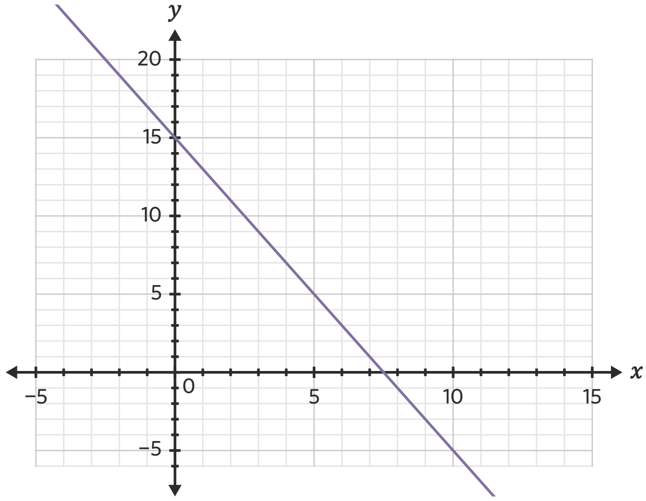
Finding a linear equation from a graph

The equation of a linear function can be determined from a graph if two points can be clearly read. This method can be simplified if one of the points is the y-intercept.

Worked example 3

Find the equation of each of the following lines.

a.



Explanation

Step 1: Determine the y-intercept.

Reading from the graph, the y-intercept is (0, 15).

$a = 15$

Step 2: Determine the gradient.

The point (0, 15) is already known. Another point the line passes through is (5, 5).

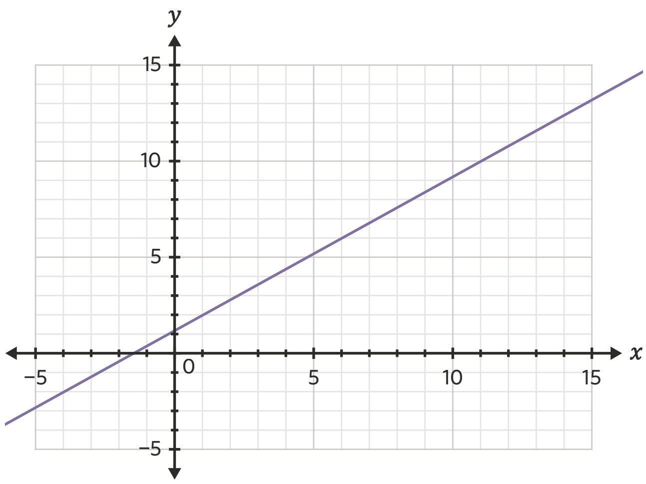
$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 15}{5 - 0}$$
$$= \frac{-10}{5}$$
$$= -2$$

Step 3: Substitute the a and b values into $y = a + bx$.

Answer

$y = 15 - 2x$

b.



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Explanation

Step 1: Determine two points that the line passes through.

Two points are (1, 2) and (6, 6).

Step 2: Determine the gradient.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - 2}{6 - 1}$$

$$= \frac{4}{5}$$

Step 3: Determine the y-intercept.

Substitute b and one point into $y = a + bx$ and solve for a.

$$2 = a + \frac{4}{5} \times 1$$

$$2 = a + \frac{4}{5}$$

$$a = \frac{6}{5}$$

Step 4: Substitute the a and b values into $y = a + bx$.

Answer

$y = \frac{6}{5} + \frac{4}{5}x$ or $y = 1.2 + 0.8x$

Note: The $y - y_1 = b(x - x_1)$ method and calculator can also be used to solve this question.

4D Questions

Finding a linear equation from two known points

1. For the linear function $y = 11 - 4x$, the y-intercept is

A. -11 **B.** -4 **C.** 4 **D.** 11

2. A line goes through $(-1, 3)$ and $(3, 19)$. The gradient of this line is

A. $\frac{1}{4}$ **B.** 4 **C.** 12 **D.** 16

3. Determine the equation of the linear function for each of the following tables. Provide all non-integers as decimals.

a.

x	0	1	2	3
y	-6	1	8	15

b.

x	0	3	5	10
y	107	152	182	257

c.

x	24	26	28	30
y	-1	-5	-9	-13

d.

x	-10	-7	2	13
y	-48	-37.5	-6	32.5

4. Determine the equation of the straight line that passes through the following sets of coordinates. Provide all non-integers as decimals.

a. $(0, -5)$ and $(7, 16)$

b. $\left(-3, \frac{5}{2}\right)$ and $\left(0, -\frac{1}{2}\right)$

c. $(109, 106)$ and $(120, 119.2)$

d. $(-1102, 6)$ and $(-529, -681.6)$

5. Tracy currently has \$156 in her savings account (at 0 weeks), and decides to deposit the same amount each week into the account. After 7 weeks, she will have \$427.25.

- **a.** Determine the linear equation that shows the amount of money, m, that Tracy will have in her savings account after n weeks.
- **b.** How much money will she have in her account after a year?
- **c.** After how many weeks of saving will there be \$1357.25 in the account?

---

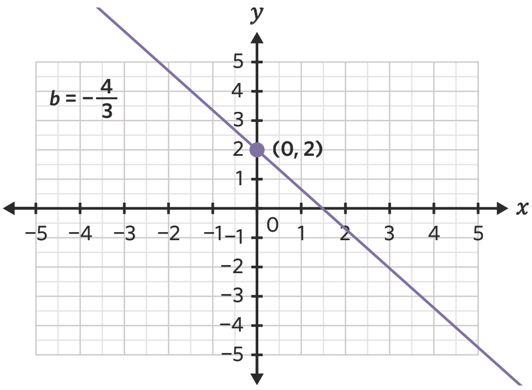
6. Sergio and his wife Sofia have a large library collection and decide to donate a certain number of books each month. After two months, they will have 1746 books left in their library. After one year and three months, there will be 1512 books remaining.

- **a.** Determine the linear equation that shows how many books, b, Sergio and Sofia will have in their library after n months.
- **b.** How many books do they begin with in their library?
- **c.** How long (in years and months) will it be until their library is empty?

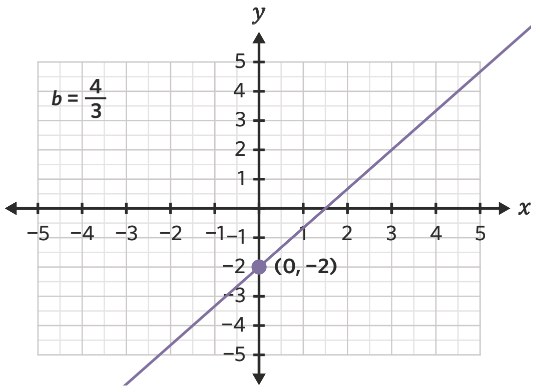
Finding a linear equation from a graph

7. Which of the following lines has the equation $y = 2 + \frac{4}{3}x$?

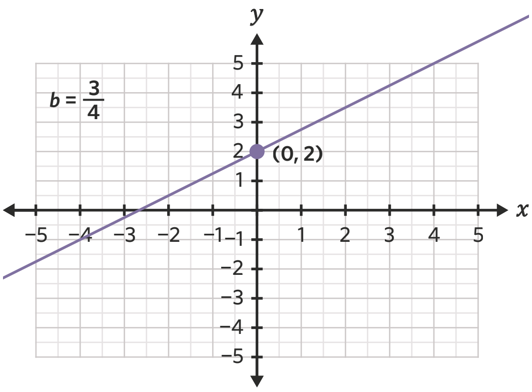
A.



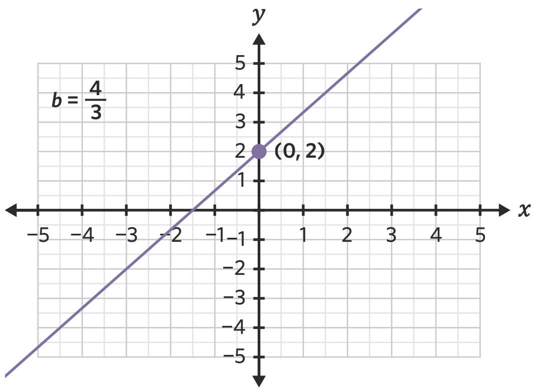
B.



C.

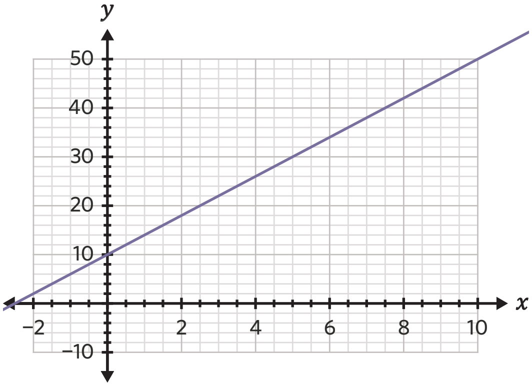


D.

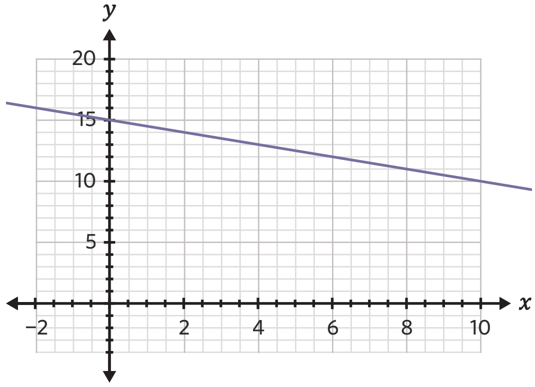


8. Determine the equation for each of the following graphs. Round the coefficients to 3 decimal places where necessary.

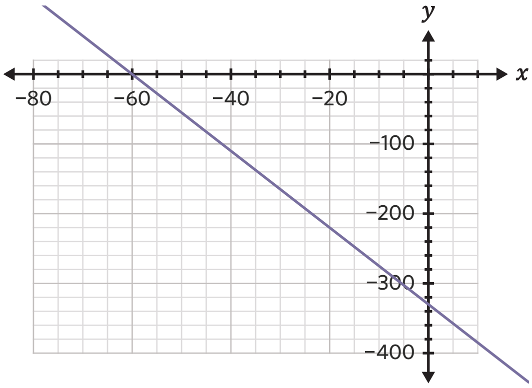
a.



b.



c.



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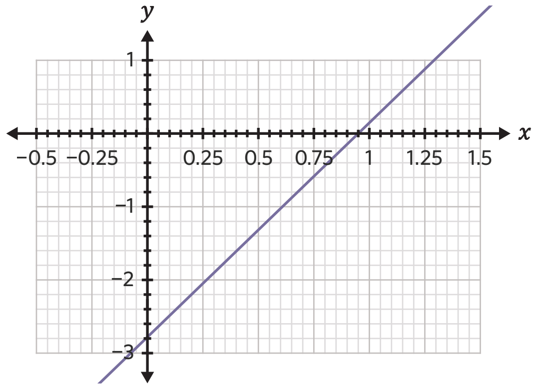


9. Rodney decides to start collecting \$2 coins to add to the ones he already has at home. Each time he gets a new coin, he stacks it on top of all the others. The following graph shows the height of his coin stack after each new coin is added.

a. Determine the linear equation that shows the height, h, in mm, of the coin stack after each new coin, c, is added. Provide all non-integers as decimals.

b. How many coins did Rodney begin with?

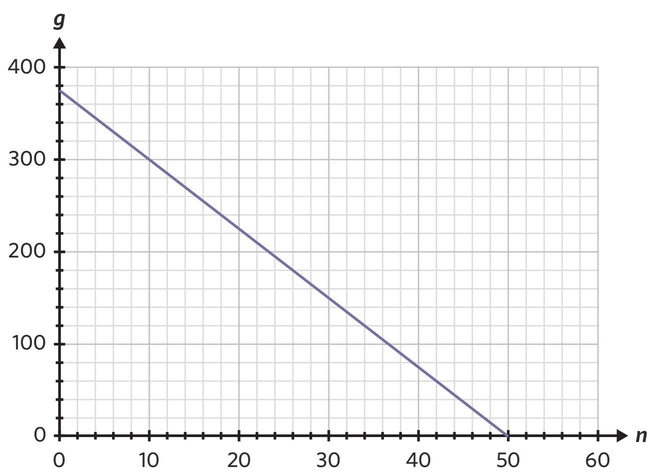
c. What is the height of the coin stack once Rodney adds 3 new coins?

10. Sophie enjoys making hot chocolates. The following graph shows the amount of cocoa powder, in grams, remaining in the tin each day after Sophie purchases a brand new tin.

a. Determine the linear equation that shows the grams, g, remaining in the tin after n days.

b. How many grams of cocoa powder are in the brand new tin?

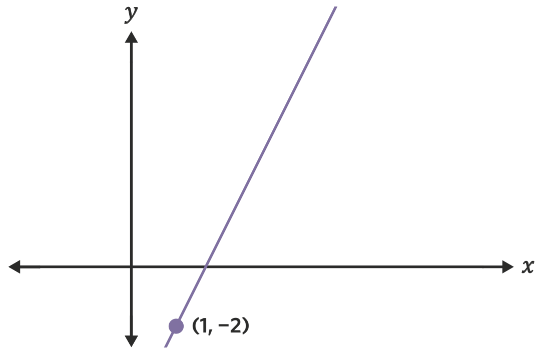
c. How many days does it take Sophie to use 150 grams of cocoa powder?



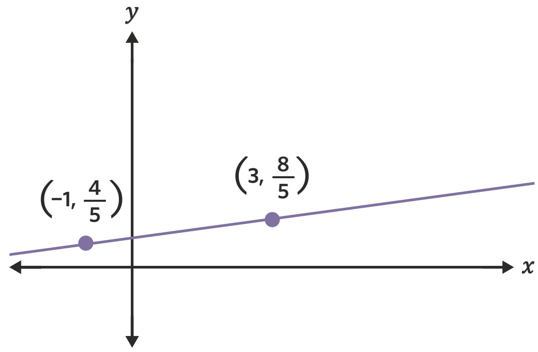
Joining it all together

11. For which of the following scenarios is it possible to determine an equation? **(Select all that apply.)**

I.



II.



III. A line that goes through the point (5.5, 5.5).

IV. A line that has a y-intercept of 2 and a gradient of 10.

---

12. Lawrence is completing a two-and-a-half hour multiple choice exam. He wants to finish early to go over his answers, so he allocates a set amount of time to complete each question.

After completing 6 questions, he has 107 minutes and 15 seconds remaining from his personally allocated total time.

After completing 13 questions, he has 88 minutes remaining from his personally allocated total time.

a. Determine the linear equation that shows the time remaining in minutes, t, that Lawrence has to complete the exam from his personally allocated total time after n questions have been completed. Provide all non-integers as decimals.

b. How much time has he allocated himself to complete all the questions in the exam, in minutes and seconds?

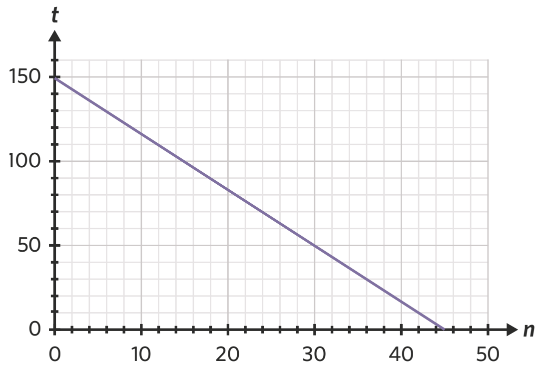
c. How much time does he have remaining to check his answers, in minutes and seconds?

d. How many questions are on the exam?

e. How much of the allocated time for completing questions does Lawrence have remaining after completing 39 questions, in minutes and seconds?

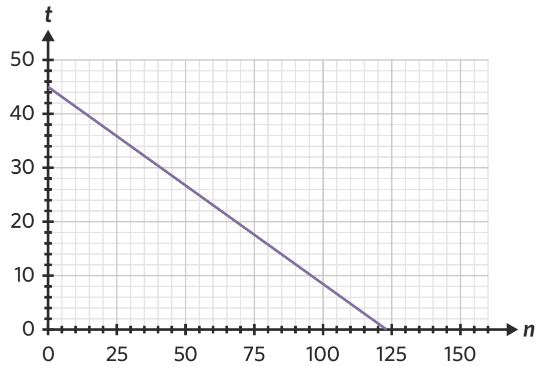
f. Which of the following graphs shows the amount of allocated time remaining, t, for Lawrence to complete the questions in the exam after n questions have been completed?

A.

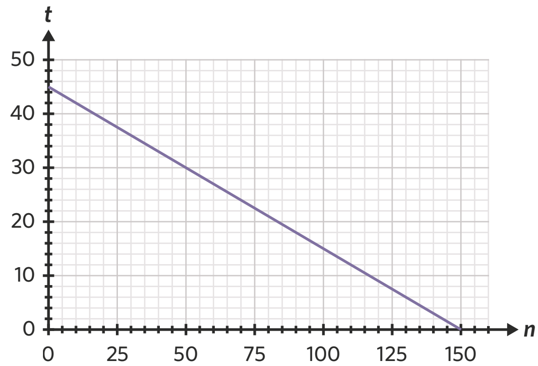


B.

C.



D.



Exam practice

13. A line passes through the points $(-1, 1)$ and $(3, 5)$. Another point that lies on this line is

A. $(0, 1)$ **B.** $(1, 3)$ **C.** $(2, 6)$
D. $(3, 4)$ **E.** $(4, 7)$

VCAA 2014 Exam 1 Graphs and relations Q4

68% of students answered this question correctly.

14. Lam is a builder constructing a community centre at a new housing estate.

The cost, C, in dollars, for Lam to work onsite for n weeks is given by

$C = 10\,000 + k \times n$

The cost of 15 weeks of onsite work is \$92 500.

Show that the value of k is 5500. (1 MARK)

VCAA 2021 Exam 2 Graphs and relations Q3a

29% of students answered this question correctly.

Questions from multiple lessons

Graphs and relations *Year 10 content*

15. Artemis is starting up a t-shirt business. He opens with 100 t-shirts, and each week he will order 65 more. The total number of t-shirts that Artemis will have ordered can be determined from the rule

$N = 100 + 65w$

where w represents the number of weeks the store has been open for.

According to this rule, the total number of t-shirts ordered will be

A. 100 after 1 week.
B. 130 after 2 weeks.
C. 650 after 10 weeks.
D. 975 after 15 weeks.
E. 1075 after 15 weeks.

Adapted from VCAA 2007 Exam 1 Graphs and relations Q2

Recursion and financial modelling

16. A sequence is generated using the following recurrence relation.

$T_0 = 1, \quad T_{n+1} = 5T_n - 3$

The sequence is

A. 1, 2, 3, 4, 5... **B.** 1, 5, 10, 15, 20... **C.** 1, 5, 2, 10, 7... **D.** 1, 2, 7, 32, 157... **E.** 2, 7, 32, 157, 782...

Adapted from VCAA 2017NH Exam 1 Recursion and financial modelling Q17

Graphs and relation *Year 10 content*

17. The extension of a spring is the distance it is stretched, in centimetres, when a force is applied. The following graph shows the relationship between *force*, in Newtons (N) and *extension* (cm) for a linear spring.

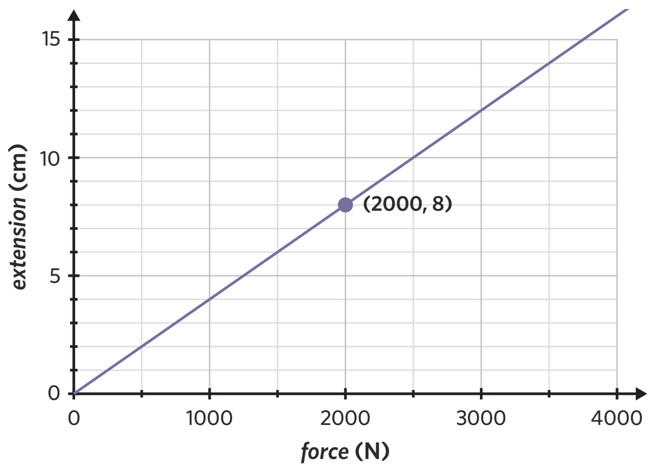
The relationship between *force* and *extension* is given by the equation $extension = \dfrac{force}{k}$.

a. Show that $k = 250$. (1 MARK)

b. Natalia stretched a linear spring 20 cm.

Using the equation, calculate the force applied, in Newtons, to the spring. (1 MARK)

Adapted from VCAA 2018 Exam 2 Graphs and relations Q2a,b



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4E Linear modelling

STUDY DESIGN DOT POINT

- formulation and analysis of linear models from worded descriptions or relevant data (including simultaneous linear equations in two variables) and their application to solve practical problems including domain of interpretation

4A 4B 4C 4D 4E 4F 4G

KEY SKILLS

During this lesson, you will be:

- modelling practical problems using linear equations
- finding the domain of interpretation.

KEY TERMS

- Domain
- Domain of interpretation

Linear equations are often used to represent real life situations where there is a constant change over a period of time. The amount of water in a sink when filling it up, the fare of a taxi ride, and the manufacturing costs of a factory can all be represented by linear equations.

Modelling practical problems using linear equations

A linear equation, written in the form $y = a + bx$, can be used to model how y changes as x changes. In a linear model, a is the initial value and b is the rate of change. Possible rates of change include kilometres per hour, litres per minute and dollars per day. If the rate of change is positive, b is positive, and if the rate of change is negative, b is negative.

See worked example 1

For example, if a ski slope initially had 100 cm of snow and 5 cm melts each day, the linear equation to describe the situation would be $S = 100 - 5n$, where S is the depth (cm) of snow after n days. The rate of change, -5, is negative because the amount of snow decreases each day.

See worked example 2

Worked example 1

Hugo's phone plan provider charges for international calls. The cost of a call can be modelled by the equation $C = 0.55 + 1.06m$, where C is the cost, in dollars, and m is the length of the call, in minutes.

a. How much does the phone plan provider charge for each additional minute of an international call?

Explanation

The charge for each additional minute is the rate of change.

$b = 1.06$

Answer

$1.06

Continues →

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b. What is the phone plan provider's call connection fee?

Explanation

The call connection fee is the initial value.

$a = 0.55$

Answer

$0.55

Worked example 2

A balloon initially contains 200 cm^3 of air and is filled at a rate of 50 cm^3 per second.

a. Construct an equation to describe the amount of air in the balloon, A (cm^3), after t seconds.

Explanation

Step 1: Find the initial amount of air in the balloon.

The balloon initially contains 200 cm^3 of air.

$a = 200$

Step 2: Find the rate of change of the amount of air in the balloon.

The balloon is filled at a rate of 50 cm^3 per second.

$b = 50$

Step 3: Represent the information in a linear equation.

The linear equation will be in the form $A = a + bt$.

Answer

$A = 200 + 50t$

b. Determine the amount of air in the balloon, A (cm^3), after 25 seconds.

Explanation

Step 1: Substitute the pronumeral with the necessary value.

Let $t = 25$.

Step 2: Evaluate the equation.

$A = 200 + 50 \times 25$

$= 1450$

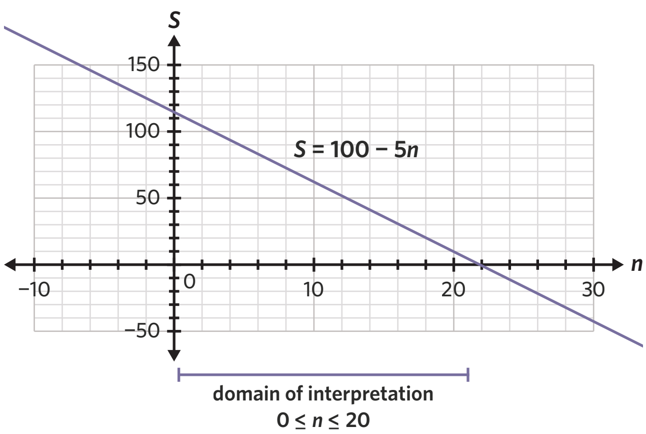
Answer

1450 cm^3

Finding the domain of interpretation

For the linear model $y = a + bx$, the set of possible values of x is known as the **domain**. However, sometimes linear models are only applicable over certain values. The domain of these linear models is known as the **domain of interpretation**, as the model cannot be interpreted appropriately outside of these values.

For example, the equation $S = 100 - 5n$, which describes the depth of snow in centimetres after n days, can only take values of n that are between 0 and 20. The value of n cannot be greater than 20, otherwise the depth of snow would be negative.



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Worked example 3

The manufacturing costs for a fashion label are modelled by the equation $C = 1920 + 18.65g$, where C is the cost, in dollars, and g is the number of garments produced. The label has \$24 300 to spend on manufacturing.

a. What is the domain of interpretation?

Explanation

Step 1: Find the maximum number of garments that can be produced.

The label has \$24 300 to spend on manufacturing.

Let $C = 24\,300$ and solve for g.

$24\,300 = 1920 + 18.65g$

$22\,380 = 18.65g$

$g = 1200$

Hence, $g \leq 1200$.

Step 2: Find the minimum number of garments that can be produced.

The fashion label cannot produce a negative number of garments.

Hence, $g \geq 0$.

Answer

$0 \leq g \leq 1200$

b. Graph the linear model within the domain of interpretation.

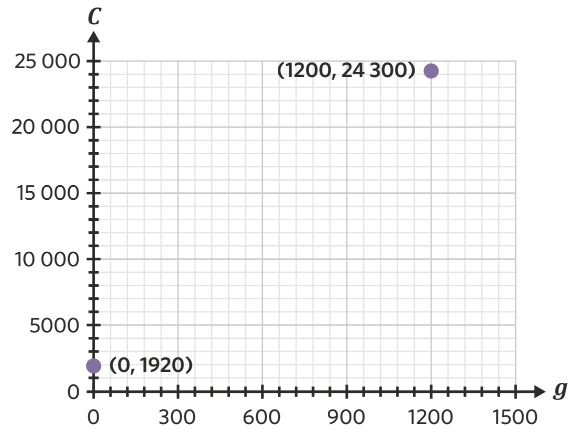
Explanation

Step 1: Find the coordinates of the boundary of the domain of interpretation.

Left boundary point: (0, 1920)

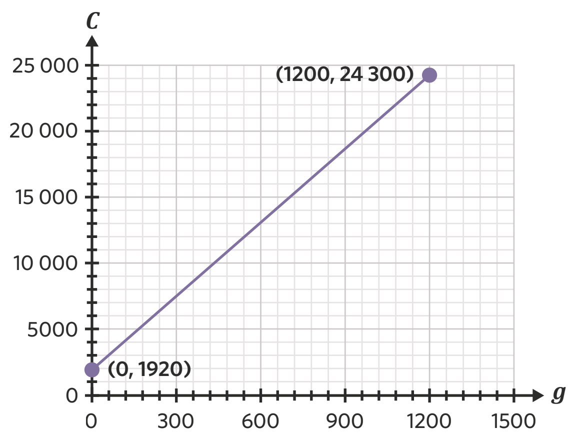
Right boundary point: (1200, 24 300)

Step 2: Plot the two points on a graph.



Step 3: Connect the points with a straight line.

Answer



4E Questions

Modelling practical problems using linear equations

1. Blaise takes his dog, Prince, for a walk every morning.

He walks at a pace of 80 metres per minute.

The following linear equation models the distance he has travelled in metres, D, after t minutes of walking.

Fill in the box to complete the equation.

$D = \boxed{} \times t$

---

2. Rashpal has an extensive collection of funk music on vinyl. He currently owns 350 vinyls and buys 8 new records each month.

- **a.** Write a linear equation that models the number of vinyls, V, that Rashpal will own after M months.
- **b.** How many vinyls will Rashpal own after 24 months?

---

3. A water company charges their customers yearly.

The bill includes a flat fee of \$234.68 per year plus \$0.0047 for each litre of water used.

- **a.** Write a linear equation that models the cost of the bill, C, in dollars, if a customer used L litres of water last year.
- **b.** Determine the water bill for a customer that used 140 000 litres of water last year.

---

4. The number of people on a tram, T, after n stops is given by the equation $T = 30 - 4n$.

- **a.** How many people were initially on the tram?
- **b.** How many less people were on the tram after the first three stops?
- **c.** After how many stops would there be less than eight people on the tram?

---

5. The cost of ordering an Uber is \$3.65 plus an additional fee of \$2.25 per kilometre travelled.

- **a.** Write a linear equation that models the cost of an Uber, C, in dollars, for a trip with a distance of d kilometres.
- **b.** Bailey wants to catch an Uber from his house in Port Melbourne to the Burnley Golf Course. The total distance of the trip is 8.4 km. What will be the total cost of the trip?
- **c.** Max wants to catch an Uber from Revolver Upstairs in Prahran to Middle Park, 6.1 km away. He only has \$18 in his bank account. Can he afford to hire the Uber?
- **d.** The minimum cost of hiring an Uber is \$10. What is the minimum distance, in kilometres, that would have to be travelled in an Uber in order to be charged more than the minimum fare? Round to one decimal place.

---

6. Joe has to write a 1500-word essay. Unfortunately, it is due in four and a half hours and he has not started. A linear equation that models the number of words left to write, W, after h hours of writing is $W = 1500 - 350h$.

- **a.** How many words does Joe write per hour?

It will take Joe half an hour to plan the essay before he starts writing.

- **b.** Joe believes he can still finish the essay in time. Show that Joe is incorrect.
- **c.** What is the minimum number of words Joe must write per hour in order to complete the essay in time?

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7. An inflatable swimming pool was emptied of water at a constant rate of 35 litres per minute. After one minute the amount of water in the swimming pool was 280 L.
   a. How many litres of water were in the swimming pool initially?
   b. Write a linear equation that models the volume of water in the pool, W, in litres, after t minutes.
   c. After how many minutes will the pool be empty?

8. Lola is trying to find a mechanic to fix her car. Toni charges a call-out fee of \$70 and \$55 for every hour of work, whereas John charges a call-out fee of \$85 and \$50 for every hour of work.
   If it will take four hours to fix her car, which mechanic would be cheaper to hire?

9. A car hire company charges an initial fee of \$40 as well as a cost of \$70 per day to hire a car for the first week. After the first week it costs \$50 per day to hire the car.
   a. Construct a linear equation that models the total cost, C (\$), to hire the car for n days during the first week.
   b. How much would it cost to hire the car for one week?
   c. Construct a linear equation that models the total cost, C (\$), to hire the car for m days after the first week.
   d. How much would it cost to hire the car for 15 days?

Finding the domain of interpretation

10. The amount of water in a rainwater tank, W, in litres, after m minutes, is modelled by the equation $W = 1.6m$. The rainwater tank can hold a maximum of 4000 litres of water. The domain of interpretation is
    A. $m \leq 2500$
    B. $0 \leq m \leq 2500$
    C. $0 \leq m \leq 4000$
    D. $0 \leq W \leq 4000$

11. The population of a bird species, B, after n years, is modelled by the equation $B = 10\,000 + 3000 \times n$. The population cannot increase after it reaches 28 000 due to competition for resources with other species.
    a. What is the domain of interpretation?
    b. Graph the linear model within the domain of interpretation.

12. A cup of coffee cools according to the equation $T = 80 - 1.5m$, where T (°C) is the temperature of the coffee and m is the time in minutes.
    a. Find the domain of interpretation, given that the temperature of the coffee does not change after it reaches room temperature, 20 °C.
    b. Graph the linear model within the domain of interpretation.

13. A car has a petrol tank that can hold 42.46 litres of petrol. The volume of petrol in the car, P, in litres, after d kilometres travelled (since last being filled), is modelled by the equation $P = 42.46 - 0.11 \times d$. What is the domain of interpretation?

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Joining it all together

14. A small rural community of 20 people has a water tank with a capacity of 510 000 litres. The tank is currently full.

Each day, each member of the community uses 300 litres of water.

a. Construct a linear equation that models the volume of water in the tank, W, in litres, after d days.

b. Assuming the tank doesn't get refilled at all, how many litres of water will be left in the tank after 30 days?

c. What is the domain of interpretation?

d. What would be the domain of interpretation if there were 50 people in the rural community?

---

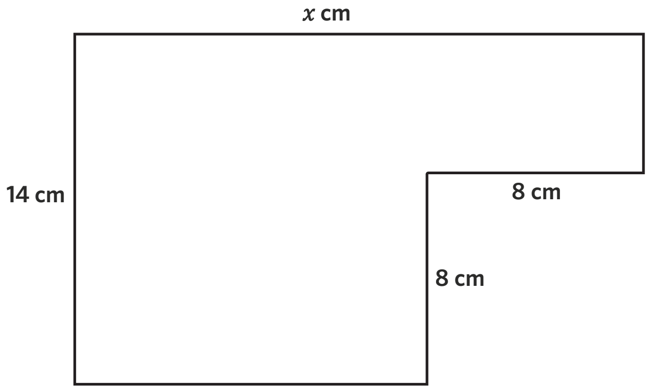
15. The following diagram shows a rectangle with a square cut out of it.

a. Construct a linear equation that models the area of the shape, A, in cm^2, in terms of x.

b. Determine the area of the shape, in cm^2, if the value of x is 54.

c. Determine the value of x if the area of the shape is 202 cm^2.

d. Raf wants to make the shape by cutting an A4 piece of paper. The dimensions of an A4 piece of paper are 21.0 cm × 29.4 cm. What is the domain of interpretation?



Exam practice

16. A phone company charges a fixed, monthly line rental fee of $28 and $0.25 per call.

Let n be the number of calls that are made in a month.

Let C be the monthly phone bill, in dollars.

The equation for the relationship between the monthly phone bill, in dollars, and the number of calls is

A. $C = 28 + 0.25n$ **B.** $C = 28n + 0.25$ **C.** $C = n + 28.25n$
D. $C = 28(n + 0.25)$ **E.** $C = 0.25(n + 28)$

VCAA 2016 Exam 1 Graphs and relations Q2

94% of students answered this question correctly.

---

17. Justin makes and sells electrical circuit boards.

He has one fixed cost of $420 each week.

Each circuit board costs $15 to make.

The selling price of each circuit board is $27.

The weekly profit if Justin makes and sells 200 circuit boards per week is

A. $1980 **B.** $2400 **C.** $2820
D. $4980 **E.** $5400

VCAA 2020 Exam 1 Graphs and relations Q6

74% of students answered this question correctly.

---

18. An energy company charges a monthly service fee of $38.70 plus a supply charge of 2.5 cents per megajoule of energy used.

Caitlin's energy bill for the 30 days of the month of June was $169.90.

On average, the number of megajoules of energy that Caitlin used per day in June is closest to

A. 6 **B.** 52 **C.** 175
D. 227 **E.** 5248

VCAA 2020 Exam 1 Graphs and relations Q7

38% of students answered this question correctly.

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Questions from multiple lessons

Graphs and relations *Year 10 content*

19. The equation of the line that passes through the points (3, 1) and (3, 8) is

A. $x = 3$ **B.** $y = 8$ **C.** $y = 8x$
D. $y = 1 + 3x$ **E.** $y = 3 + 8x$

Adapted from VCAA 2017 Exam 1 Graphs and relations Q1

Data analysis *Year 10 content*

20. The number of minutes spent driving to work on a particular day was recorded for a group of 25 people. The results are shown in the following stem plot.

Key: 1 | 1 = 11 minutes

1	5 6 7 7
2	2 5 6
3	2 7 8 8
4	1 1 2 6 9
5	0 1 3 7 8
6	2 4 5 7

The percentage of these people that spent less than 50 minutes driving to work is

A. 16% **B.** 32% **C.** 34% **D.** 64% **E.** 66%

Adapted from VCAA 2018NH Exam 1 Data analysis Q2

Graphs and relations *Year 10 content*

21. A phone company is keeping records on the cost of manufacturing and selling phone cases.
The *cost*, in dollars, of producing a certain *number of cases* can be found using the following equation.

cost $= 200 + 1.25 \times$ *number of cases*

a. How many cases are produced if the cost is $330? (1 MARK)

b. Sketch the relationship between *cost* and the *number of cases* produced. (1 MARK)

Adapted from VCAA 2016 Exam 2 Graphs and relations Q2a,b

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4F Simultaneous linear equations

STUDY DESIGN DOT POINT

- formulation and analysis of linear models from worded descriptions or relevant data (including simultaneous linear equations in two variables) and their application to solve practical problems including domain of interpretation

4A 4B 4C 4D 4E **4F** 4G

KEY SKILLS

During this lesson, you will be:

- solving simultaneous equations graphically
- solving simultaneous equations using substitution
- solving simultaneous equations using elimination
- modelling practical problems using simultaneous equations.

KEY TERMS

- Simultaneous equations
- Intersection
- Substitution method
- Elimination method

Real life situations can also be modelled by two or more linear equations that use the same variables. These equations can be solved graphically, algebraically or using technology, to determine the values of the variables that balance all equations simultaneously.

Solving simultaneous equations graphically

Simultaneous equations are a set of multiple linear equations with two or more variables. The solution to a pair of simultaneous equations is the point of **intersection** between the two lines. Graphically, this is the point at which the two lines meet.

See worked example 1

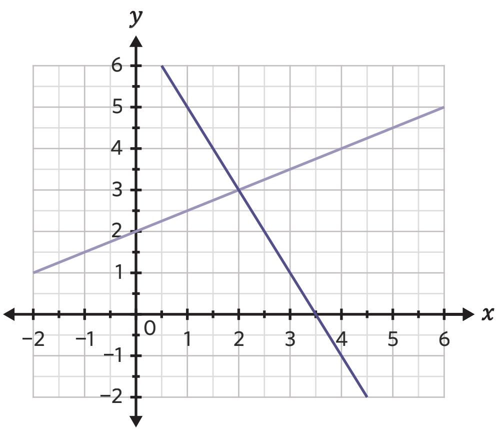
If the lines never meet, then there is no point of intersection and there is no solution to the simultaneous equations. This only happens when the lines are parallel. If both lines overlap, then there is an infinite number of solutions to the simultaneous equations. This only happens when the lines are the same.

To solve a pair of simultaneous equations graphically, a calculator can be used to plot the lines and determine the point of intersection.

See worked example 2

Worked example 1

Find the point of intersection of the lines on the following graph.



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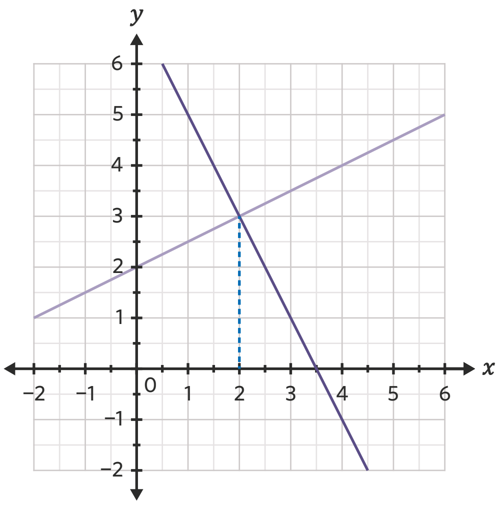
Explanation

Step 1: Locate the point of intersection.

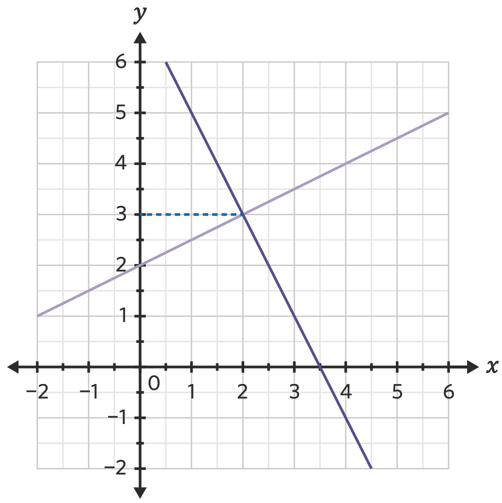
This is the point at which the lines cross.

Step 2: Identify the x coordinate.



The x coordinate is 2.

Step 3: Identify the y coordinate.



The y coordinate is 3.

Answer

(2, 3)

Worked example 2

Two lines have the equations $y = 1 + 3x$ and $y = 4 - 2x$.
Use a calculator to find the point of intersection graphically.

Explanation – Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Graphs'.

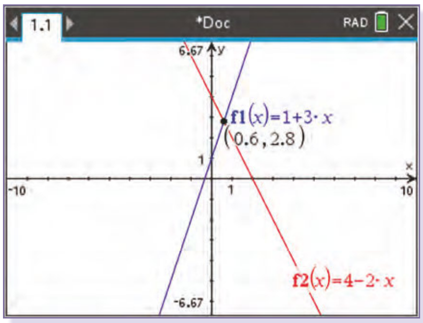
Step 2: Enter the first equation as 'f1(x)=1+3x' and press enter.

Step 3: Press tab and enter the second equation as 'f2(x)=4−2x.

Press enter.

Step 4: Press menu and select '6: Analyze Graph' → '4: Intersection'.

Use the mouse to first click anywhere to the left of the point of intersection, and then click anywhere to the right of the point of intersection.

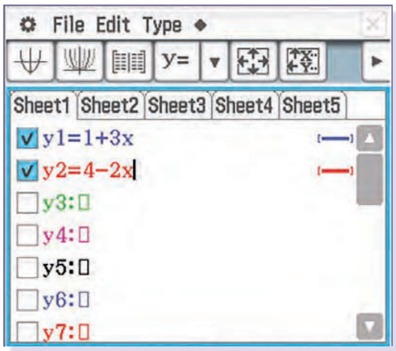


Explanation – Method 2: Casio ClassPad

Step 1: From the main menu, tap Graph & Table.

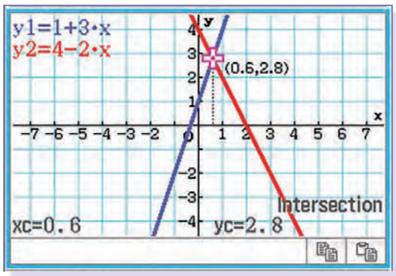
Continues →

Step 2: Enter the first equation '1+3x' into 'y1'.
Enter the second equation '4–2x' into 'y2'.
Check the boxes to the left of the equations.



Step 3: Tap to graph the equation.

Step 4: Tap 'Analysis' → 'G-solve' → 'Intersection'.



Answer – Method 1 and 2

(0.6, 2.8)

Solving simultaneous equations using substitution

Solving simultaneous equations using the **substitution method** involves a variable from one equation being substituted into the other.

Substitution is useful when at least one of the equations has a variable as the subject ($y = a + bx$ or $x = a + by$). This is because substitution can be done directly, without any need for transposing.

The following pairs of equations would be easier to solve using the substitution method.

$y = 2x + 3$ and $x + y = 9$

$x = y - 4$ and $x = -2y + 9$

$y - 2x = 7$ and $x = 3 - 4y$

Worked example 3

Solve the following pairs of simultaneous equations using substitution.

a. $y = 5 - 3x$ and $4x + 3y = 5$

Explanation

Step 1: Label the equations ① and ②.

$y = 5 - 3x$ ①

$4x + 3y = 5$ ②

Step 2: Substitute ① into ②.

It is best to substitute y because it is the subject.

$y = \boxed{5 - 3x}$ ①

$4x + 3y = 5$ ②

$4x + 3(5 - 3x) = 5$

Step 3: Solve for x.

$$
\begin{aligned}
4x + 3(5 - 3x) &= 5 \\
4x + 15 - 9x &= 5 \\
4x - 9x &= -10 \\
-5x &= -10 \\
x &= 2
\end{aligned}
$$

Step 4: Substitute $x = 2$ into one of the equations and solve for y.

Both equations will work, but choosing ① is easier for calculations.

$y = 5 - 3x$

$y = 5 - 3 \times (2)$

$= -1$

Continues →

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Answer

$x = 2, \quad y = -1$

---

b. $x = 4y - 2$ and $x = y + 7$

Explanation

Step 1: Label the equations 1 and 2.

$x = 4y - 2$ ①

$x = y + 7$ ②

Step 2: Since both equations have x as the subject, let equations ① and ② equal each other.

$4y - 2 = y + 7$

Step 3: Solve for y.

$$\begin{aligned} 4y - 2 &= y + 7 \\ 4y &= y + 9 \\ 3y &= 9 \\ y &= 3 \end{aligned}$$

Step 4: Substitute $y = 3$ into one of the equations and solve for x.

Both equations will work, but choosing ② is easier for calculations.

$$\begin{aligned} x &= y + 7 \\ x &= (3) + 7 \\ &= 10 \end{aligned}$$

Answer

$x = 10, \quad y = 3$

Solving simultaneous equations using elimination

Solving simultaneous equations using the **elimination method** involves the equations being added or subtracted in a way that eliminates one of the variables.

Elimination is useful when the equations are in the form $ax + by = c$. That is, both variables are on the same side of the equals sign in both equations.

The following pairs of equations would be easier to solve using the elimination method.

$3x + y = 2$ and $4x - 5y = -1$

$y - 4x = 3$ and $5x + 3y = 11$

$2x + 2y = 16$ and $8y - 2x = -4$

Worked example 4

Solve the following pairs of simultaneous equations using elimination.

---

a. $3x + 2y = 7$ and $-x - 2y = -1$

Explanation

Step 1: Label the equations 1 and 2.

$3x + 2y = 7$ ①

$-x - 2y = -1$ ②

Step 2: Choose a variable to be eliminated.

Since the coefficients for y are equal but opposite, y can be eliminated by simply adding the two equations. On the other hand, the coefficients for x are not equal, and more work must be done to make them equal. This means choosing to eliminate y will be the easier option.

Continues →

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Step 3: Add the equations.

①+②

$$\begin{array}{r} 3x + 2y = 7 \\ + \ (-x - 2y = -1) \\ \hline 2x + 0 = 6 \end{array}$$

Step 4: Solve for x.

$2x = 6$

$x = 3$

Step 5: Substitute $x = 3$ into one of the equations and solve for y.

① is chosen in this solution, however ② works just as well.

$$\begin{aligned} 3x + 2y &= 7 \\ 3 \times (3) + 2y &= 7 \\ 9 + 2y &= 7 \\ 2y &= -2 \\ y &= -1 \end{aligned}$$

Answer

$x = 3, \quad y = -1$

b. $2x + 3y = 2$ and $3x + 4y = 4$

Explanation

Step 1: Label the equations 1 and 2.

$2x + 3y = 2$ ①

$3x + 4y = 4$ ②

Step 2: Choose a variable to be eliminated.

The coefficients for x and y are not equal so neither variable can be eliminated directly.

Choosing x will be easier since the coefficients are smaller than those for y.

Step 3: Multiply each equation such that the coefficients of x will be equal. Label the new equations ③ and ④.

The coefficients of x are 2 and 3. The lowest common multiple is 6.

Remember to multiply both sides of the equation.

①× 3

$6x + 9y = 6$ ③

②× 2

$6x + 8y = 8$ ④

Step 4: Subtract one equation from the other.

③−④

$$\begin{array}{r} 6x + 9y = 6 \\ - \ (6x + 8y = 8) \\ \hline 0 + y = -2 \end{array}$$

Step 5: Substitute $y = -2$ into one of the equations and solve for y.

Equation ① is chosen in this solution, however ② works just as well.

$$\begin{aligned} 2x + 3y &= 2 \\ 2x + 3 \times (-2) &= 2 \\ 2x - 6 &= 2 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

Answer

$x = 4, \quad y = -2$

Modelling practical problems using simultaneous equations

Simultaneous equations can be used to solve real-world problems with multiple unknowns.

To solve a question using simultaneous equations, identify and define the variables, model the equations, and then solve simultaneously.

Simultaneous equations can also be solved algebraically using calculators. One big advantage of using a calculator is it can solve a system of simultaneous equations with more than two equations and unknowns. This would be a lengthy process to do by hand.

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Worked example 5

Tickets for a Justin Bieber concert were $110 for A-reserve seats and $85 for B-reserve seats. Annie and her friends bought 12 tickets in total and spent a total of $1095.

a. Construct two equations to model the situation.

Explanation

Step 1: Identify and define the variables.

In this case, the variables are the number of the A-reserve and B-reserve tickets bought.

Let a be the number of A-reserve tickets and b be the number of B-reserve tickets they bought.

Step 2: Convert the information given into two equations using the chosen variables.

The total number of tickets bought was 12, so the sum of the number of A-reserve tickets and the number of B-reserve tickets will be 12.

$a + b = 12$ ①

$1095 was spent on a $110 tickets and b $85 tickets.

$110a + 85b = 1095$ ②

Answer

$a + b = 12$ ①

$110a + 85b = 1095$ ②

b. How many B-reserve tickets did they buy?

Explanation – Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Calculator'.

Step 2: Press menu and then select '3: Algebra' → '7: Solve System of Equations' → '1: Solve System of Equations'.

Step 3: Set 'Number of equations:' to '2'.

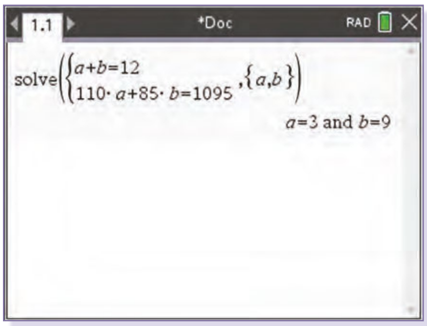
Set 'Variables:' to 'a,b'.

Select 'OK'.

Step 4: Enter equation ① into the top box.

Enter equation ② into the bottom box.

Press enter.



Step 5: Use the solution to the simultaneous equations to answer the question.

The number of B-tickets bought is represented by b.

Explanation – Method 2: Casio ClassPad

Step 1: From the main menu, tap √α Main.

Step 2: Press keyboard and tap the simultaneous equations icon.

Continues →

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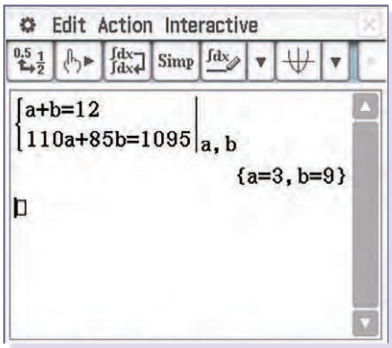
Step 3: Enter equation ① into the top box.

Enter equation ② into the bottom box.

Enter the 'a,b' into the box to the right.

Press **EXE**.



Step 4: Use the solution to the simultaneous equations to answer the question.

The number of B-tickets bought is represented by *b*.

Answer – Method 1 and 2

9 B-reserve tickets

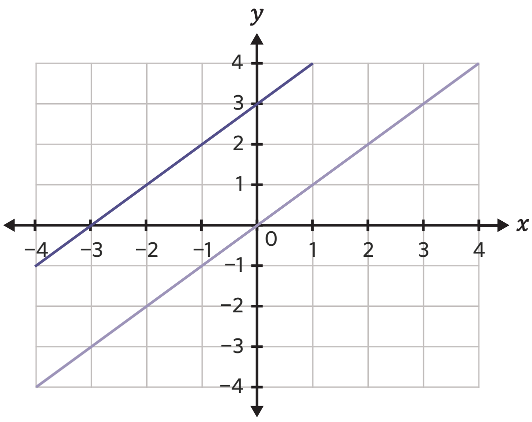
4F Questions

Solving simultaneous equations graphically

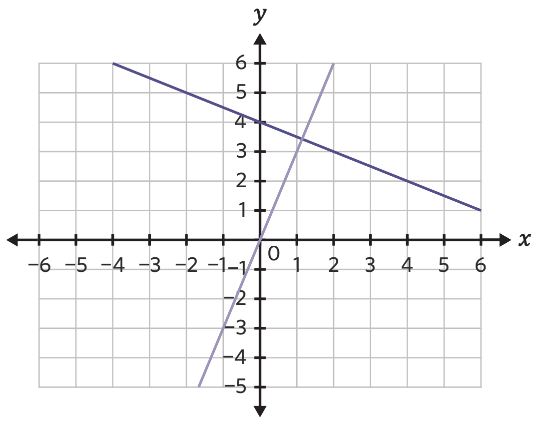
1. Consider the pairs of simultaneous equations represented in the following graphs.

Which pair will have no solution?

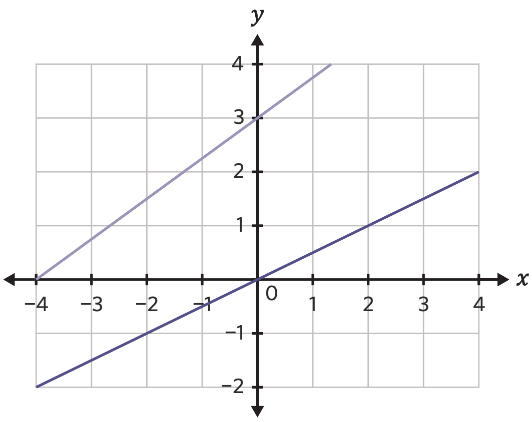
A.



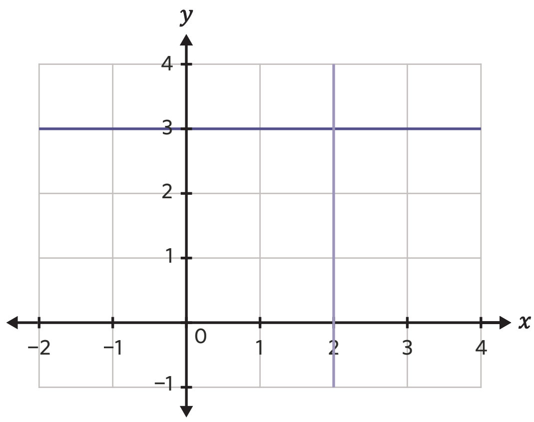
B.



C.



D.



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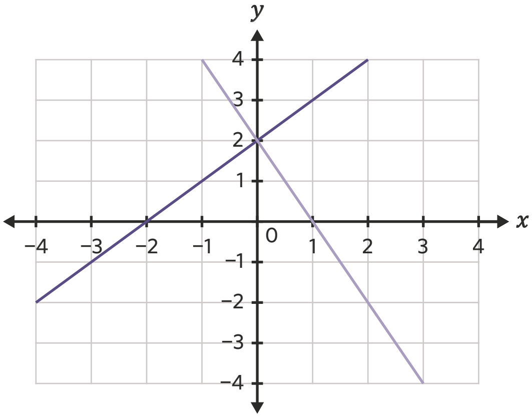
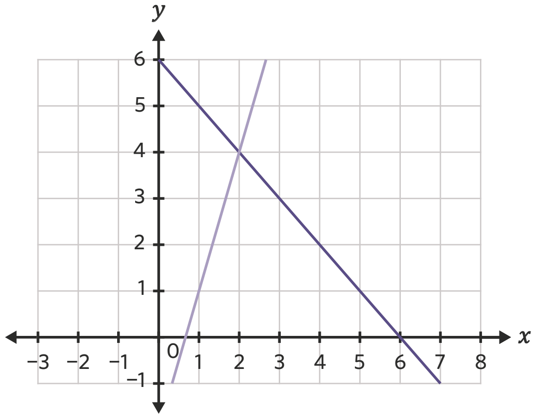
2. State the coordinates of the point of intersection for each of the following graphs.

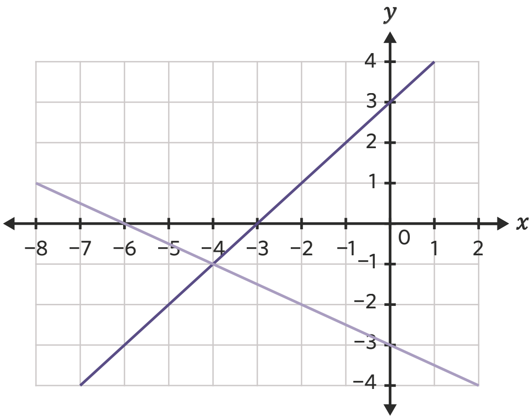
a.



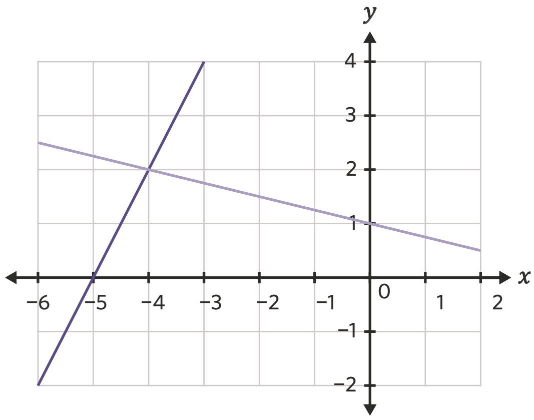
b.



c.



d.



3. Construct a graph on a calculator to find the point of intersection for the following pairs of lines.

a. $y = 2 - 4x$ and $y = 6x$

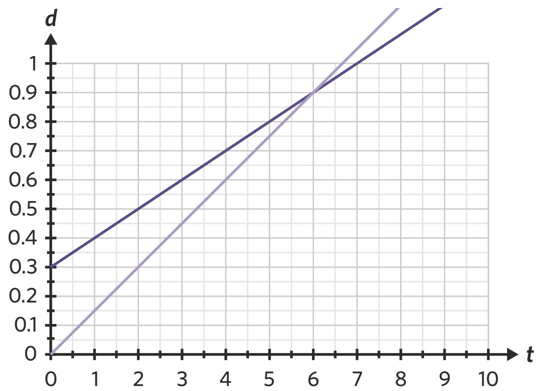
b. $y = 6 + 3x$ and $y = -2 - 2x$

c. $y = -1 - 2x$ and $y = 5 + 6x$

d. $y = -4 + 4x$ and $y = 2 - 6x$

4. Allison and Beatrice are running a race along a straight track. Allison is known to be a better runner, so Beatrice starts ahead of Allison.

The positions of the runners are shown in the following graph where d is the distance, in kilometres, from the start of the track and t is the time, in minutes, after the beginning of the race. If the race starts at 9:45 am, at what time does Allison catch up to Beatrice?



Solving simultaneous equations using substitution

5. Consider the following linear equations.

$y = 3x - 2$

$y - 4x = 2$

Use the substitution method to determine which of the following coordinates is the point of intersection.

A. $(-4, 14)$ **B.** $(14, -4)$ **C.** $(-4, -14)$ **D.** $(-14, -4)$

6. Use the substitution method to solve the following simultaneous equations.
   a. $x = 3 + 2y$ and $2x - 3y = 7$
   b. $4x + 3y = -5$ and $y = x - 4$
   c. $x = 7y - 5$ and $x = 3y + 3$
   d. $y = 14 - 4x$ and $y = x - 16$

7. Zeke is an avid baker and is currently working on perfecting his creme brulée recipe. His scales are broken at the moment so he is trying to figure out the weight of a cup of flour and a cup of sugar using algebra.

   He knows that one cup of sugar weighs 40 grams less than two cups of flour, and that the total weight of two cups of sugar and five cups of flour is one kilogram.

   This information is shown in the following equations, where f is the weight of a cup of flour, in grams, and s is the weight of a cup of sugar, in grams.

   $s = 2f - 40$

   $2s + 5f = 1000$

   Use the substitution method to find the weights of a cup of flour and a cup of sugar, in grams.

Solving simultaneous equations using elimination

8. Consider the following linear equations.

   $2x + y = 8$

   $3x - y = 2$

   Use the elimination method to determine which of the following coordinates is the point of intersection.

   A. $(4, 2)$ **B.** $(2, 4)$ **C.** $(-4, 2)$ **D.** $(2, -4)$

9. Use the elimination method to solve the following simultaneous equations.
   a. $x + 3y = 11$ and $-2x + 7y = 4$
   b. $2x + 5y = 9$ and $4x + 6y = 6$
   c. $3x - 4y = 20$ and $x + 7y = -10$
   d. $7x + 3y = 16$ and $2x + 2y = 8$

10. Monica and Niamh wanted to stock up on spreads, so they went together to the supermarket. Monica bought two jars of Vegemite and four jars of Nutella for \$33, and Niamh bought five jars of Vegemite and three jars of Nutella for \$40.50.

    This is shown in the following equations.

    $2V + 4N = 33$

    $5V + 3N = 40.5$

    Use the elimination method to find the cost of a jar of Vegemite and a jar of Nutella.

Modelling practical problems using simultaneous equations

11. The perimeter of a rectangle is 60 cm. Its length is four times its width.

    This information is shown in the following equations.

    $2l + 2w = 60$

    $l = 4w$

    Use a calculator to solve the simultaneous equations.

    The length of the rectangle is

    A. 6 cm **B.** 10 cm **C.** 24 cm **D.** 30 cm

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12. A group went out for Daniel's 10^{th} birthday party. They first went bowling, and then went to see a movie. The prices, in dollars, of bowling and movie tickets for children and adults are shown in the following equations.

    Let c be the number of children and a be the number of adults.

    Bowling: $12c + 16a = 156$

    Movie: $11c + 15a = 144$

    a. What is the price of an adult movie ticket?

    b. What is the price of bowling for a child?

    c. How many children and adults were in the group?

13. Jason buys 3 notebooks and 4 pens for \$11.60 and Keiran buys 6 notebooks and 2 pens for \$14.80. Let n be the cost of a notebook and p be the cost of a pen. Represent this information in two simultaneous equations.

14. Saskia was at a Vinnies op shop when there was a sale on. All skirts were \$3.50 and all tops were \$2. Saskia spent \$35 and bought 13 pieces of clothing (all of which were either tops or skirts).

    a. Represent this information in two simultaneous equations. Let s be the number of skirts bought and t be the number of tops bought.

    b. How many tops did Saskia buy?

15. Dana is six years older than Ben. In two years' time, Dana will be one and a half times as old as Ben.

    Show that Dana is currently 16 years old.

Joining it all together

16. Two lines intersect at $y = 5$. The equations of the two lines are $y = 1 + 2x$ and $y = a - x$.

    Find the value of a.

17. Mary's Marvellous Milkshakes sells milkshakes in two different sizes: small and large.

    Mary looked at her sales records from Monday and Tuesday this week.

    She found that on Monday, she sold 10 small milkshakes and 12 large milkshakes and received a total of \$73. On Tuesday, she sold 20 small milkshakes and 16 large milkshakes and received a total of \$114.

    This information can be shown in the following simultaneous equations.

    Monday: $10s + 12l = 73$

    Tuesday: $20s + 16l = 114$

    a. Would the method of elimination or substitution be more appropriate in solving the simultaneous equations?

    b. Solve the simultaneous equations using the method chosen in part **a** to find the price of each milkshake size.

18. At a fish and chip shop, large chips are \$1.50 more expensive than medium chips. On Friday, the fish and chip shop sold 130 medium chips and 160 large chips for \$1748.

    a. Represent this information in two simultaneous equations. Let m be the price of medium chips and l be the price of large chips.

    b. Would the method of elimination or substitution be more appropriate in solving the simultaneous equations?

    c. Use a calculator to determine the price of medium chips.

19. Katya is 6 years younger than Ginger. In four years time, Katya's age will be four fifths of Ginger's age.

a. Represent this information in two simultaneous equations. Let k be Katya's current age, and g be Ginger's current age. Transpose the equations such that k is the subject of both.

b. Find the point of intersection of the linear equations graphically, in the form (g, k).

c. How old is Katya?

---

20. Alex and Danni both work part-time as waitresses.

The number of hours they each worked on Monday and Tuesday this week, along with the total amount they were both paid, is shown in the following table.

	Alex (hours)	Danni (hours)	total paid
Monday	4	6	$252.00
Tuesday	7	3	$249.75

Show that Alex gets paid $24.75 per hour.

Exam practice

21. The ticket office at a circus sells adult tickets and child tickets:

- The Payne family bought two adult tickets and three child tickets for $69.50.
- The Tran family bought one adult ticket and five child tickets for $78.50.
- The Saunders family bought three adult tickets and four child tickets.

What is the total amount spent by the Saunders family?

A. $83.40 **B.** $87.50 **C.** $98.00
D. $101.50 **E.** $112.00

VCAA 2017 Exam 1 Graphs and relations Q6

79% of students answered this question correctly.

---

22. Kyla organises fundraiser car shows.

Admission fees for the show are $5 per adult and $2 per child.

$1644 was raised from the 537 people who attended the most recent car show.

How many children attended this car show? (1 MARK)

VCAA 2020 Exam 2 Graphs and relations Q2b

63% of students answered this question correctly.

---

23. A ride-share company has a fee that includes a fixed cost and a cost that depends on both the time spent travelling, in minutes, and the distance travelled, in kilometres.

The fixed cost of a ride is $2.55.

Judy's ride cost $16.75 and took eight minutes. The distance travelled was 10 km.

Pat's ride cost $30.35 and took 20 minutes. The distance travelled was 18 km.

Roy's ride took 10 minutes. The distance travelled was 15 km.

The cost of Roy's ride was

A. $17.00 **B.** $19.55 **C.** $20.50
D. $23.05 **E.** $25.60

VCAA 2018 Exam 1 Graphs and relations Q8

48% of students answered this question correctly.

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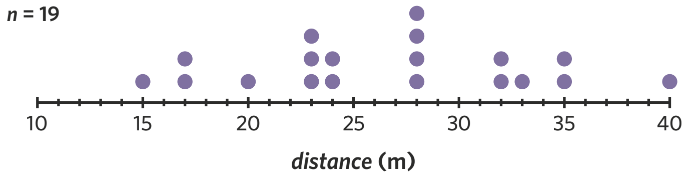
Questions from multiple lessons

Data analysis *Year 10 content*

24. The following dot plot displays the *distance*, in metres, of a football kicked by each student in a Year 7 class.



The median is

A. 23.0 m **B.** 25.5 m **C.** 26.0 m **D.** 28.0 m **E.** 34.0 m

Adapted from VCAA 2018 Exam 1 Data analysis Q2

Recursion and financial modelling

25. A laptop was purchased for $4000 using a loan scheme.

A deposit of $200 was paid.

The balance will be repaid with 20 monthly repayments of $250.

The total amount of interest charged is

A. $1000 **B.** $1200 **C.** $4000 **D.** $5000 **E.** $5200

Adapted from VCAA 2015 Exam 1 Business-related mathematics Q5

Graphs and relations

26. Harry Styles is selling tickets to his last concert before taking up acting full time.

There are two types of tickets, adult and child.

The revenue is then divided into two streams, venue costs and staff wages.

They are expressed as percentages of the ticket prices in the following table.

	adult	**child**
venue costs	40%	30%
staff wages	50%	40%

a. A group of friends decided to buy tickets together. They spent $880 on adult tickets and $360 on child tickets.

Calculate the amount, in dollars, of the group of friends' ticket purchases that went towards the venue costs of the concert. (1 MARK)

b. A different group of friends purchased tickets, and by doing so contributed $402 towards staff wages. They purchased $180 worth of child tickets. How much money did they spend on adult tickets? (2 MARKS)

Adapted from VCAA 2011 Exam 2 Graphs and relations Q1a,b

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4G Piecewise linear models

STUDY DESIGN DOT POINT

- piecewise linear (line segment, step) graphs and their application to modelling practical situations, including tax scales and charges and payment

4A 4B 4C 4D 4E 4F 4G

KEY SKILLS

During this lesson, you will be:

- classifying and interpreting piecewise graphs
- constructing piecewise graphs
- applying piecewise graphs to financial situations.

KEY TERMS

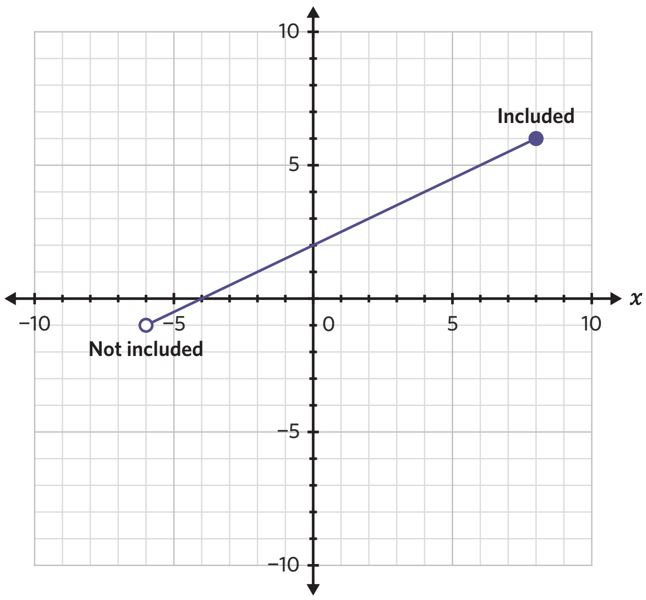
- Piecewise linear graph
- Line segment graph
- Step graph

Linear functions can help to model various situations. They are limited, however, in that the rate of change remains constant as the explanatory variable changes in value. Piecewise linear models allow for multiple linear functions to be combined to model one situation. This has many applications, from distance and time models to financial models.

Classifying and interpreting piecewise graphs

A **piecewise linear graph** is a graph made up of two or more linear equations.

Each segment of a piecewise linear function will have a domain, with any endpoints marked by a circle. An open circle means the point is not included in the line, whereas a closed circle means the point is included. Each point of the function is only included once in the domain.



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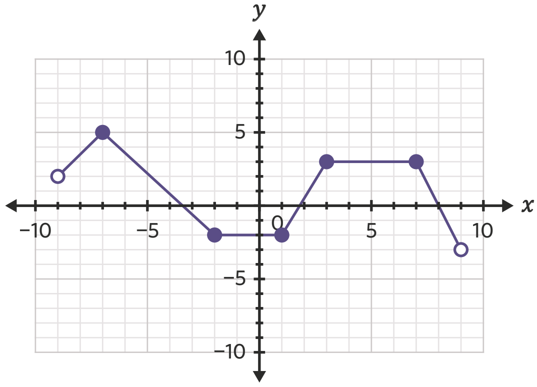
See worked example 1

There are two main types of piecewise linear graphs.

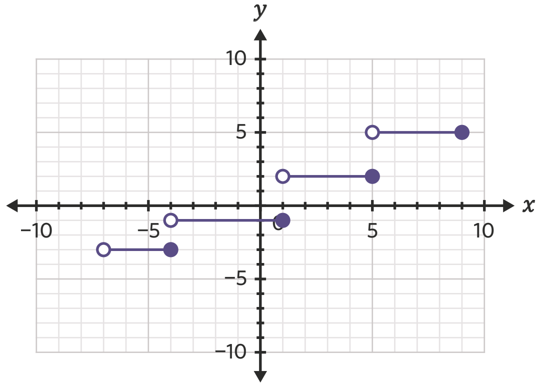
A **line segment graph** connects segments across different domains where the variable on the vertical axis can change in value. If there is an overlap of open and closed circles between the start and end of two line segments, then the closed circle takes precedence and the point is included in the graph.

A **step graph** has segments across different domains where the variable on the vertical axis is constant for each segment, hence, each segment is horizontal. While each segment starts where the previous one finishes, they never connect.

Line segment graph



Step graph



In order to gather information from a piecewise linear graph, it is important to know how to interpret different elements of the graph, as line segment and step graphs can be used to represent a range of different situations.

A line segment can be used to model the change in one variable against another, such as distance and time.

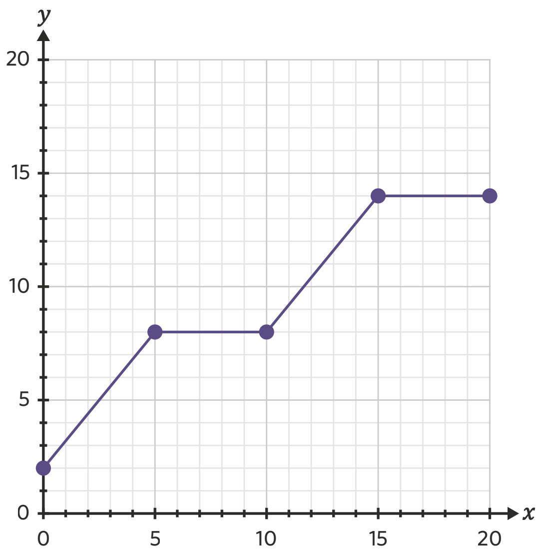
See worked example 2

Step graphs can be used to model applications involving conditional figures.

See worked example 3

Worked example 1

Classify the following graph as either a line segment graph or a step graph.



Explanation

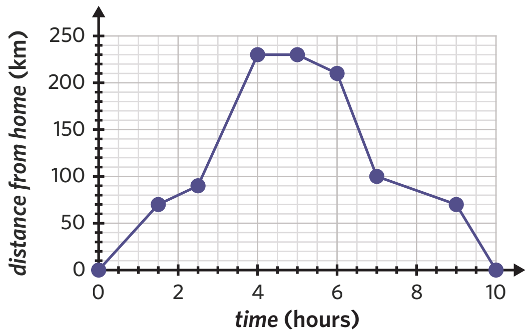
The line segments in the domains $0 \leq x \leq 5$ and $10 \leq x \leq 15$ change in value along the vertical axis.

Answer

Line segment graph

Worked example 2

The following graph shows Ben's *distance from home* (km), over an eight-hour time period, on a day where he went on a road trip.

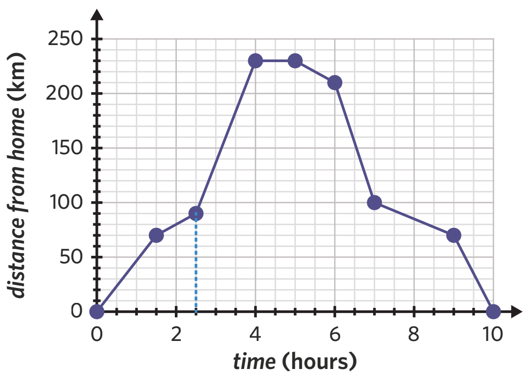


a. How far from home, in km, was Ben after 2 hours and 30 minutes?

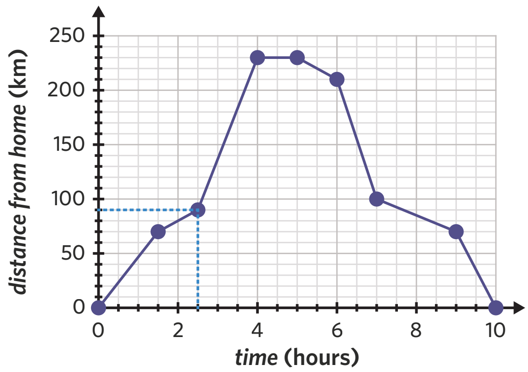
Explanation

Step 1: Locate 2 hours and 30 minutes on the horizontal axis.

This is equivalent to *time* $= 2.5$ hours.



Step 2: Determine the corresponding *distance from home* on the vertical axis.



Answer

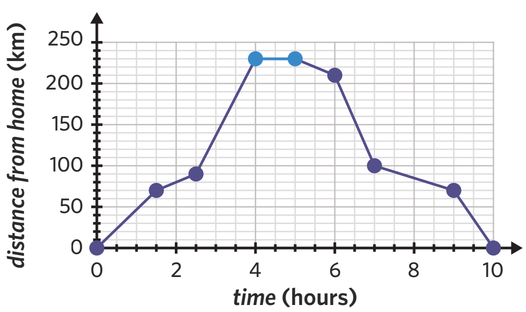
90 km

b. After how many hours did Ben stop to have lunch?

Explanation

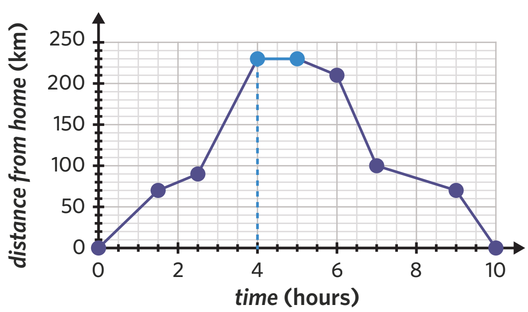
Step 1: Locate any horizontal line segments.

The question states that Ben stopped to have lunch. This will be represented by a line segment where the *distance from home* doesn't change. There is only one horizontal line segment.



Step 2: Determine the *time* value this corresponds to on the vertical axis.

The question asks for when Ben stops working to have lunch, so this will be the time value in which the line segment starts.



Answer

4 hours

Continues →

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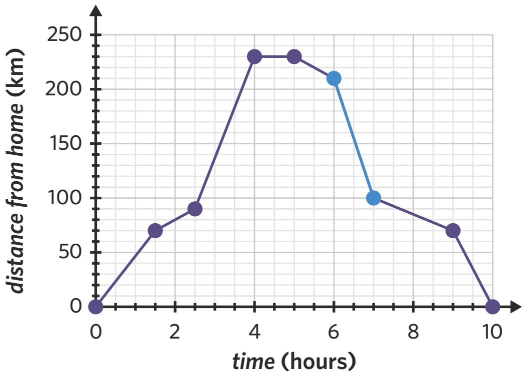
c. On average, how fast, in km/h, was Ben travelling between the 6 and 7-hour mark?

Explanation

Step 1: Locate the line segment that models Ben's *distance from home* from 6 to 7 hours.



Step 2: Calculate how far Ben travelled in this time.

At the 6-hour mark, Ben was 210 km from home.

At the 7-hour mark, Ben was 100 km from home.

This means he travelled $210 - 100 = 110$ km.

Step 3: Determine how fast Ben is travelling.

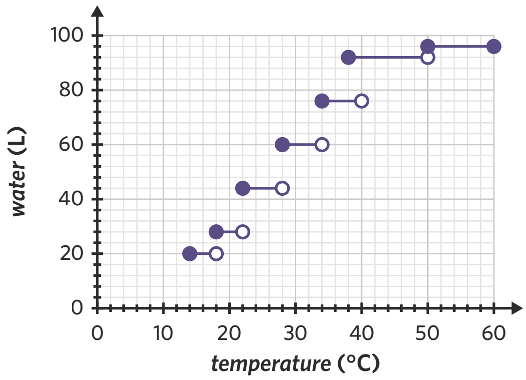
Ben has travelled 110 km in 1 hour.

Answer

110 km/h

Worked example 3

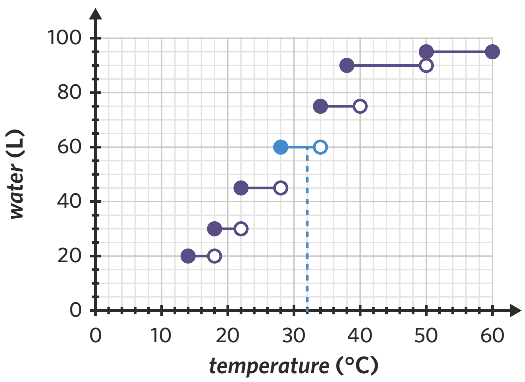
Leslie owns a farm, and spends a lot of time looking after her cows. During the summer, the amount of water she provides to each cow daily is dependent on the temperature. The following step graph shows the daily amount of *water* (L) each cow gets according to the *temperature* (°C).



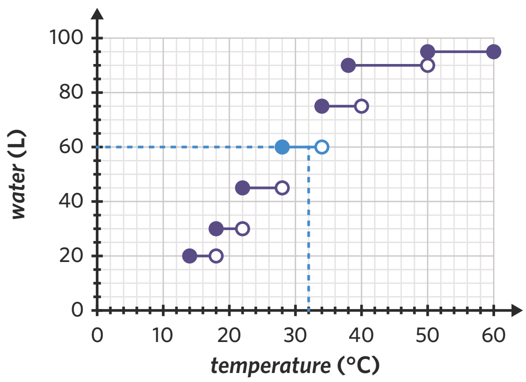
a. How much water does each cow get when it is 32 °C?

Explanation

Step 1: Identify the line segment that corresponds to 32 °C.



Step 2: Determine at the corresponding *water* value for this line segment.



Answer

60 L

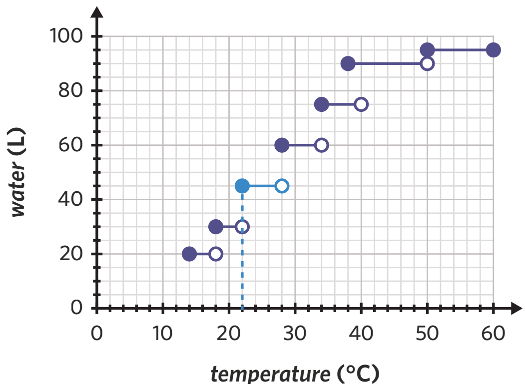
Continues →

b. How much water does each cow get when it is 22 °C?

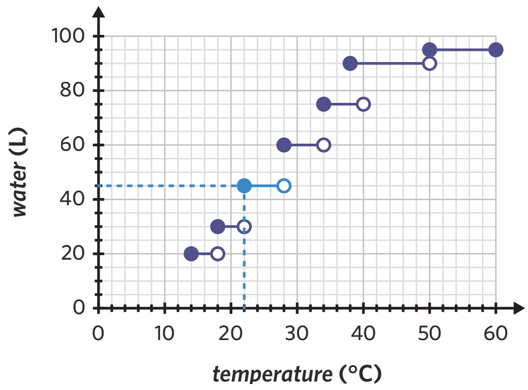
Explanation

Step 1: Identify the line segment that corresponds to 22 °C.

In this case, there are two line segments, so the open and closed circles need to be taken into account. The segment that corresponds to 22 °C will have a closed circle, as this indicates that 22 °C is included in the line segment.



Step 2: Determine the corresponding *water* value for this line segment.



Answer

45 L

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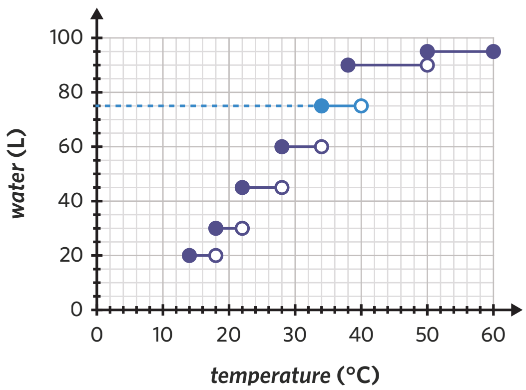
c. What is the lowest temperature possible for each cow to be provided with 75 L of water?

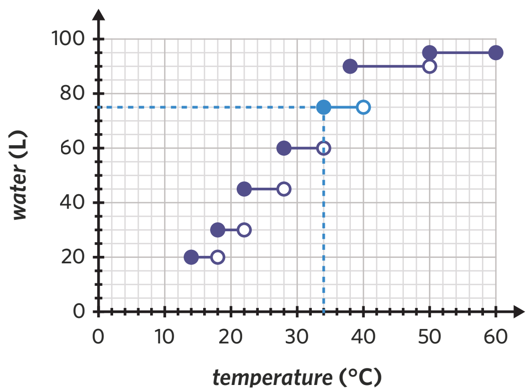
Explanation

Step 1: Identify the line segment that corresponds to 75 L.



Step 2: Determine the minimum *temperature* value for this line segment.

The beginning point of the line segment has a closed circle, meaning it is included and is the lowest horizontal axis value.



Answer

34 °C

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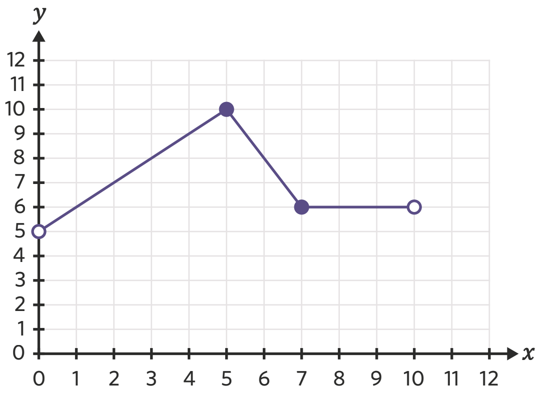
Constructing piecewise graphs

The function for a piecewise linear graph is expressed by listing the function of each line segment, followed by its domain. A piecewise linear function can be graphed by drawing the line of each linear function according to its domain.

The following example shows a piecewise linear function and its associated graph.

$$y = \begin{cases} 5 + x & 0 < x \leq 5 \\ 20 - 2x & 5 < x \leq 7 \\ 6 & 7 < x < 10 \end{cases}$$



Worked example 4

Construct a graph for the following piecewise linear functions.

a. $$y = \begin{cases} -4.5 + 1.5x & 0 < x \leq 3 \\ -6 + 2x & 3 < x \leq 8 \\ 18 - x & 8 < x \leq 12 \end{cases}$$

Explanation

Step 1: Determine the coordinates of the start and end point of the first line segment.

Substitute the start and end domain values of x for the first line segment into the first linear function to solve for y.

Substitute $x = 0$.

$y = -4.5 + 1.5 \times 0$

$= -4.5$

Substitute $x = 3$.

$y = -4.5 + 1.5 \times 3$

$= 0$

The first line segment starts at $(0, -4.5)$ and ends at $(3, 0)$.

Step 2: Determine the coordinates of the end points for the remaining line segments.

The start point will be the same as the end point of the previous line segment, so these do not need to be solved.

Substitute $x = 8$.

$y = -6 + 2 \times 8$

$= 10$

The second line segment starts at $(3, 0)$ and ends at $(8, 10)$.

Substitute $x = 12$.

$y = 18 - 12$

$= 6$

The third line segment starts at $(8, 10)$ and ends at $(12, 6)$.

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Step 3: Construct a set of axes and plot the end points.

Each end point should be either an open or closed circle, depending on the domains of each line segment. Remember that if two end points overlap, a closed circle takes precedence.

For the first line segment:

$0 < x$, so $(0, 4.5)$ requires an open circle.

$x \leq 3$, so $(3, 0)$ requires a closed circle.

For the second line segment:

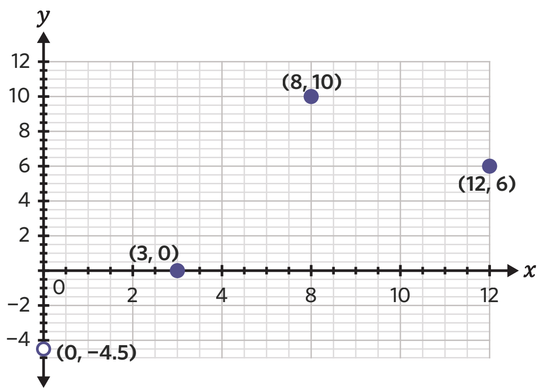
$3 < x$, so $(3, 0)$ would have required an open circle, but the closed circle at the same point in the first line segment takes precedence.

$x \leq 8$, so $(8, 0)$ requires a closed circle.

For the third line segment:

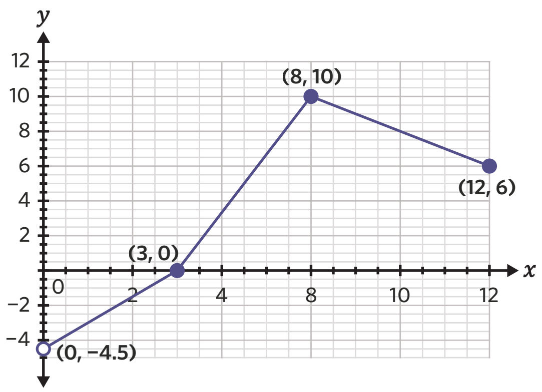
$8 < x$, so $(8, 0)$ would have required an open circle, but the closed circle at the same point in the second line segment takes precedence.

$x \leq 12$, so $(12, 6)$ requires a closed circle.



Step 4: Connect the end points with straight lines.

Answer



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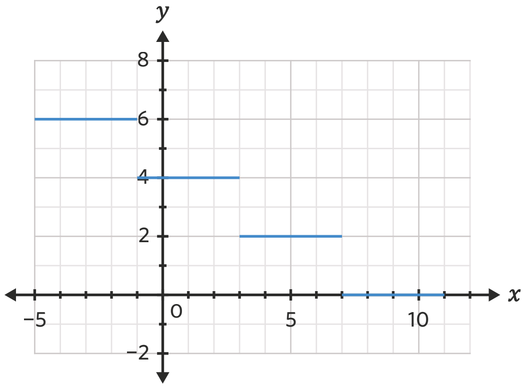
b. $$y = \begin{cases} 6 & -5 \le x < -1 \\ 4 & -1 \le x < 3 \\ 2 & 3 \le x < 7 \\ 0 & 7 \le x < 11 \end{cases}$$

Explanation

Step 1: Construct a set of axes and draw each line segment according to the corresponding domain.

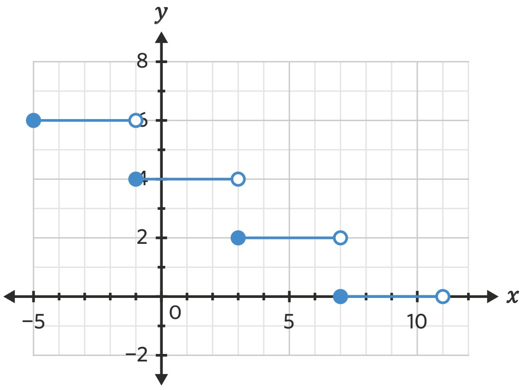
As y is constant in each linear function, all of the line segments will be horizontal.



Step 2: Add an open circle or a closed circle at the start point and end point of each line segment.

A '<' sign indicates an open circle, and a '≤' sign indicates a closed circle.

Answer



Applying piecewise graphs to financial situations

A unique application of piecewise linear graphs is their common use in modelling different financial situations, such as tax scales.

Each country has a different tax structure, where individuals pay a marginal tax rate depending on their level of income. The following table shows the marginal tax rate for Australian residents in the 2022–2023 financial year.

See worked example 5

taxable income	marginal tax rate	tax on this income
0 – \$18 200	0%	Nil
\$18 201 – \$45 000	19%	\$0.19 for each \$1 over \$18 200
\$45 001 – \$120 000	32.5%	\$5092 plus \$0.325 for each \$1 over \$45 000
\$120 001 – \$180 000	37%	\$29 467 plus \$0.37 for each \$1 over \$120 000
\$180 001 and over	45%	\$51 667 plus \$0.45 for each \$1 over \$180 000

Data: Australian Taxation Office <https://www.ato.gov.au/rates/individual-income-tax-rates/>

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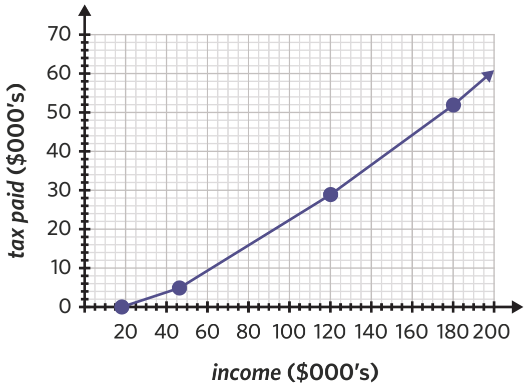
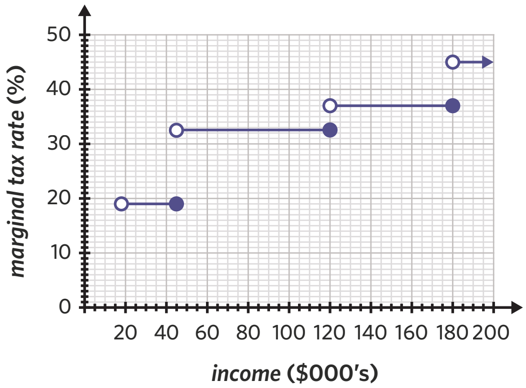
This can be expressed with the following line segment graph, which shows the amount of *tax paid* per level of *income*.



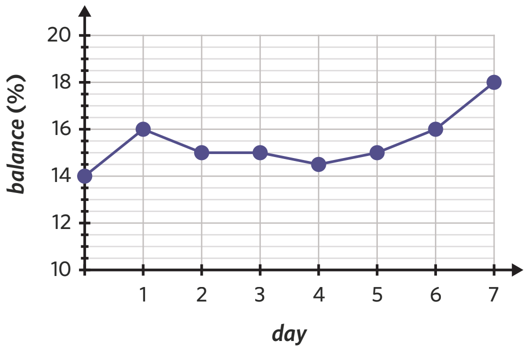
It can also be expressed with the following step graph, which shows the *marginal tax rate* of each *income* bracket.



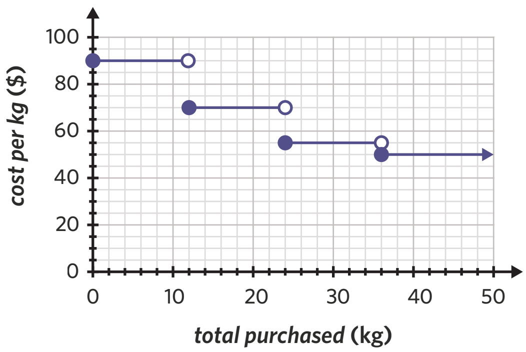
Piecewise graphs can also be used to model various other financial situations.

For example, the following graph shows the account *balance* of a store at the end of each *day* throughout a week.

See worked example 6



Step graphs can also be used to model payment structures or schedules. The following graph shows the *cost per kg* ($), of a certain product, depending on the *total purchased* (kg).



For example, an individual purchasing 12 kg of the product would pay $70/kg, while someone purchasing 30 kg would pay $55/kg.

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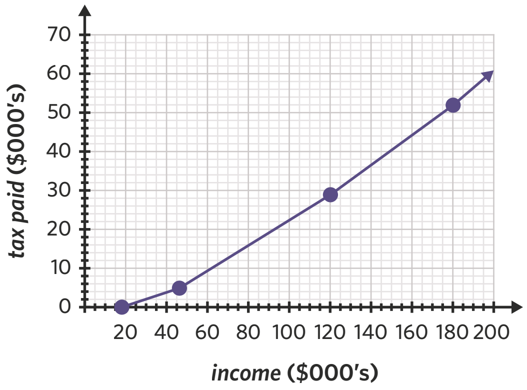
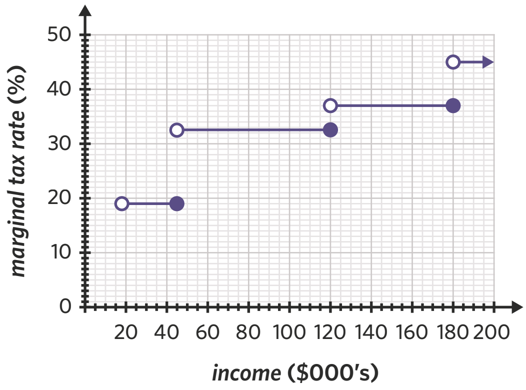
Worked example 5

Consider the following two piecewise linear graphs.





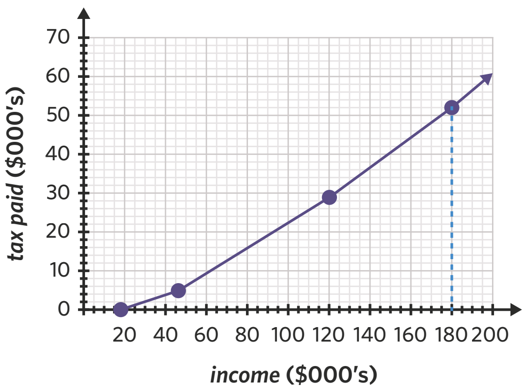
a. Approximately how much tax does an individual earning $180 000 pay?

Explanation

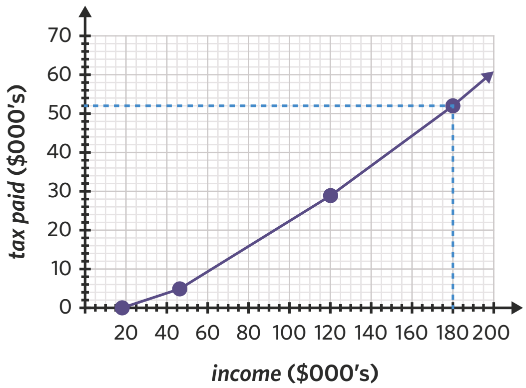
Step 1: Determine which graph is suitable to answer the question.

The question asks for the total amount of tax paid. This can be determined from the graph showing *tax paid* on the vertical axis.

Step 2: Locate $180 000 on the horizontal axis.



Step 3: Find the corresponding *tax paid* on the vertical axis.



Answer

$52 000

b. What is the marginal tax rate for an individual earning $35 000?

Explanation

Step 1: Determine which graph is suitable to answer the question.

The question asks for the marginal tax rate. This can be determined from the graph showing *marginal tax rate* on the vertical axis.

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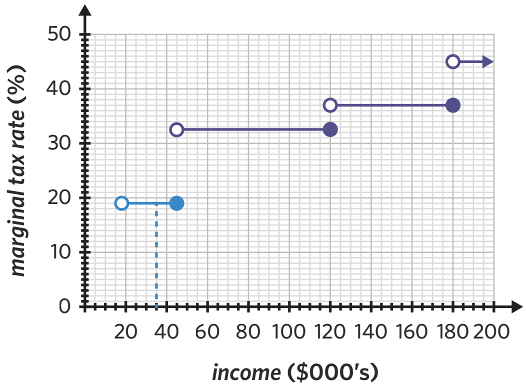
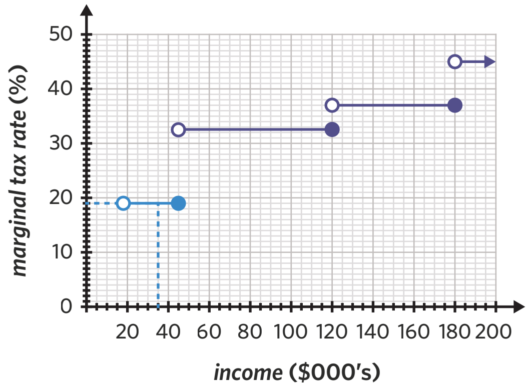
Step 2: Identify the line segment that corresponds to \$35 000 on the horizontal axis.



Step 3: Determine the corresponding *marginal tax rate* for this line segment.

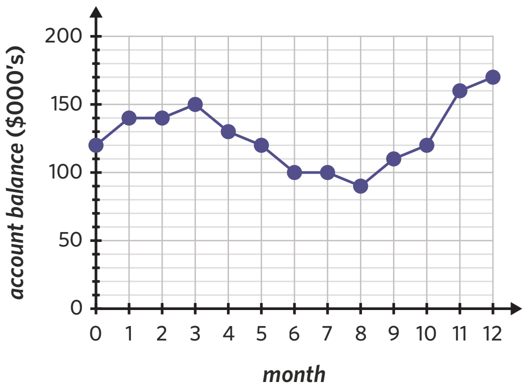


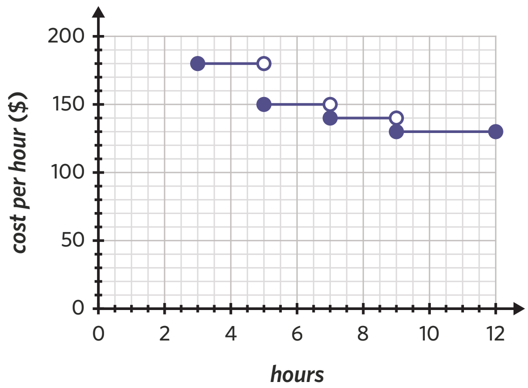
Answer

19%

Worked example 6

The following piecewise linear graphs show the account balance of a jumping castle company at the end of each month of the year (1 = January, etc), as well as the cost per hour of hiring one of their jumping castles.





a. How much money would someone have to pay in total to hire a jumping castle for 8 hours and 30 minutes?

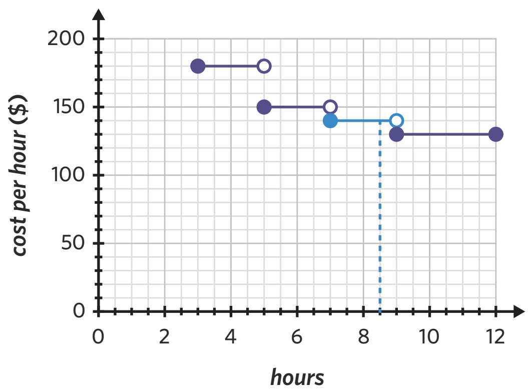
Explanation

Step 1: Determine the appropriate graph.

The question asks for the cost of hiring a jumping castle. This can be determined from the graph showing the *cost per hour* on the vertical axis.

Step 2: Identify the line segment that corresponds to 8 hours and 30 minutes on the horizontal axis.

This is equivalent to *hours* = 8.5.



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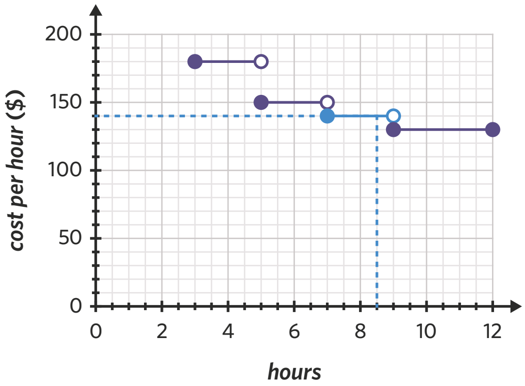
Step 3: Determine the corresponding *cost per hour* for this line segment.



cost per hour = 140

Step 4: Calculate the total amount paid for hiring a jumping castle.

$$\begin{aligned} \textit{total cost} &= \textit{cost per hour} \times \textit{hours} \\ &= 140 \times 8.5 \\ &= 1190 \end{aligned}$$

Answer

$1190

b. By how much has the account balance decreased from March to August?

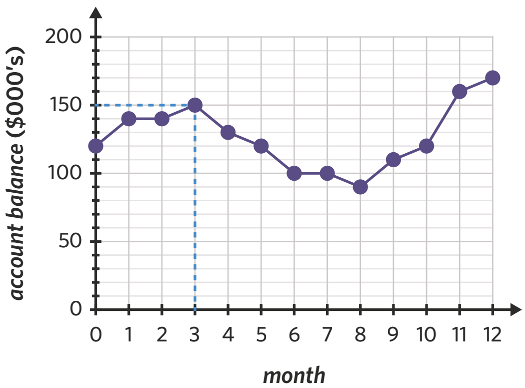
Explanation

Step 1: Determine which graph is suitable to answer the question.

The question asks for the change in account balance. This can be determined from the graph showing the *account balance* on the vertical axis.

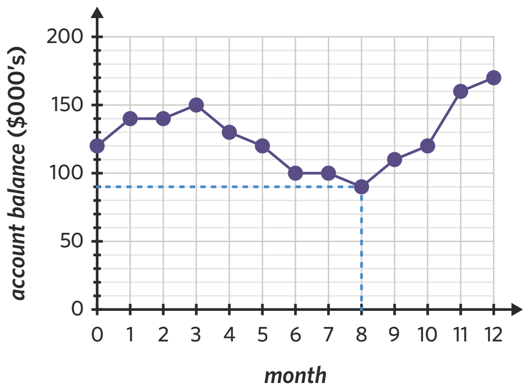
Step 2: Find the *account balance* in March.

This will be where *month* = 3.



account balance = 150

Step 3: Find the *account balance* in August. This will be where *month* = 8.



account balance = 90

Step 4: Calculate the difference between the two values.

$150 - 90 = 60$

Answer

$60 000

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4G Questions

Classifying and interpreting piecewise graphs

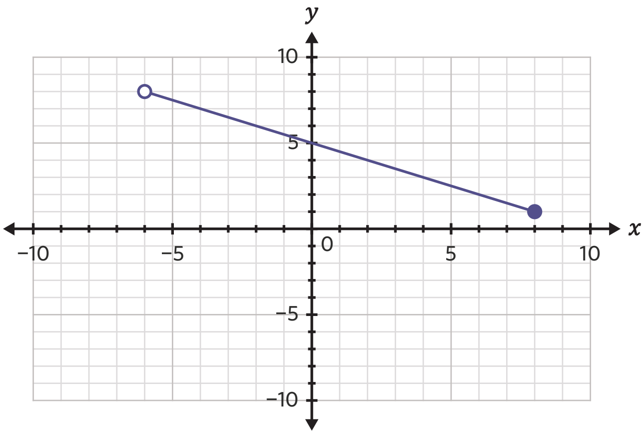
1. What is the domain of the following graph?

A. $-6 < x < 8$

B. $-6 \leq x < 8$

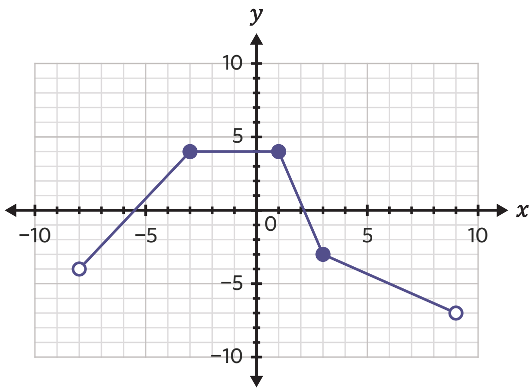
C. $-6 < x \leq 8$

D. $-6 \leq x \leq 8$

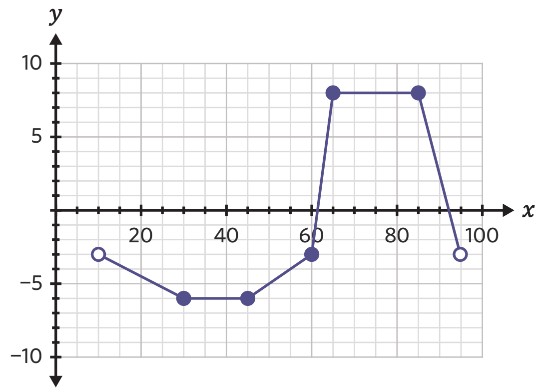


2. Identify whether the following piecewise linear graphs are line segment graphs or step graphs.

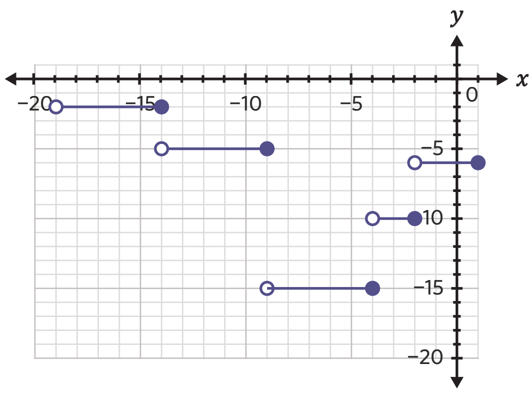
a.



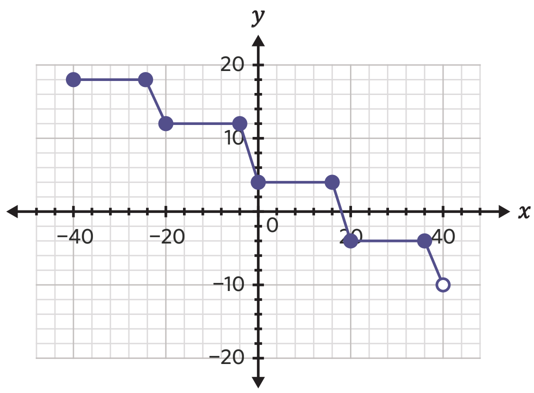
b.



c.



d.



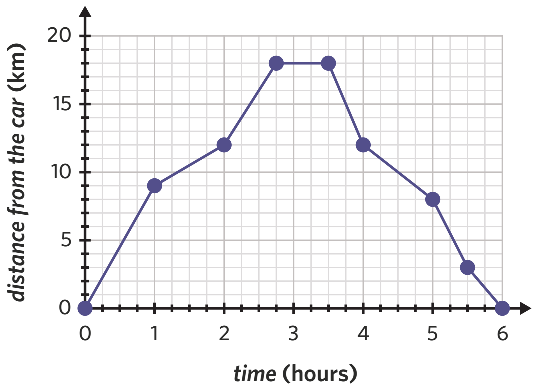
3. Anna is training for a marathon. She decides to drive to the local park and run on the footpath alongside the river. The following graph shows how far away she is, in km, from her car at various intervals throughout her run.

a. How fast, on average, did she run in the first hour?

b. How many minutes did she stop for?

c. How fast, on average, did she run for the entire duration?

d. A marathon is approximately 42.2 km long. How much further would Anna have needed to run in order to reach the same distance as a marathon?



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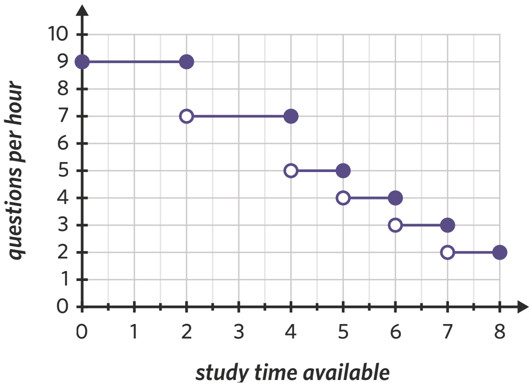
4. Shai is completing some homework over the weekend and finds that he tends to be more productive with his homework if he has less time. The following step graph shows the number of questions he expects to complete per hour depending on the amount of study time he has available, in hours.



   a. How many questions does Shai expect to be able to complete per hour if he studies for 5 hours?

   b. How many questions does Shai expect to be able to complete in total if he studies for 7 hours and 30 minutes?

   c. What is the difference in the amount of questions Shai will complete on a day where he has 7 hours to study compared to a day where he has 4 hours to study? Under which scenario will he complete more questions?

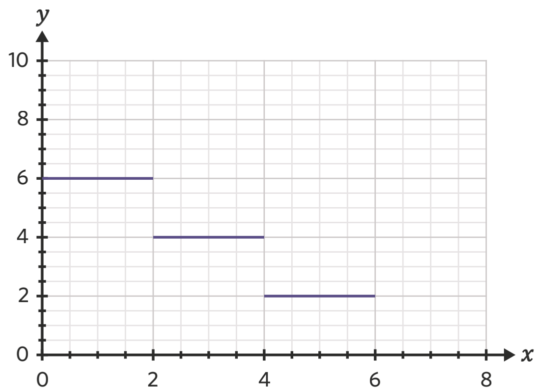
   d. What amount of time will allow Shai to complete the most amount of questions?

Constructing piecewise graphs

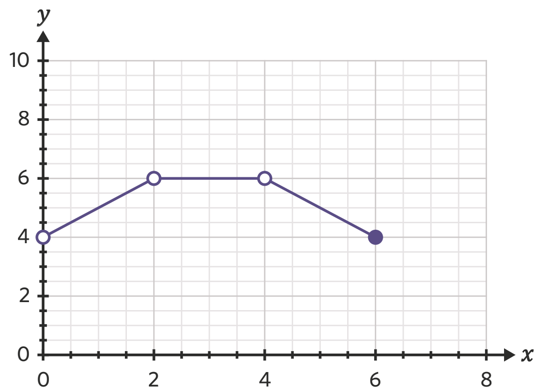
5. Select the graph that models models the piecewise linear function:

$$y = \begin{cases} 4 + x & 0 < x \leq 2 \\ 6 & 2 < x \leq 4 \\ 8 - x & 4 < x \leq 6 \end{cases}$$

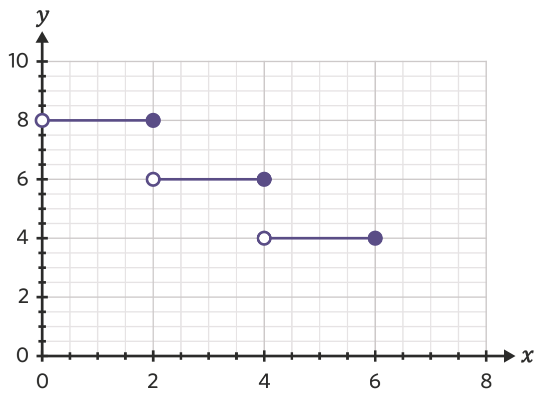
A.



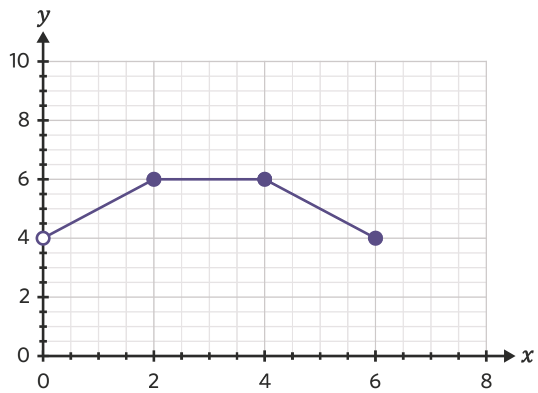
B.



C.



D.



6. For the following piecewise linear functions:
   - Sketch the graph.
   - Identify whether the graph is a line segment or step graph.

   a. $y = \begin{cases} 6 + 2x & -2 \leq x < 0 \\ 6 - 3x & 0 \leq x < 2 \end{cases}$

   b. $y = \begin{cases} 10 & 0 < x \leq 4 \\ 8 & 4 < x \leq 8 \\ 6 & 8 < x \leq 12 \end{cases}$

   c. $y = \begin{cases} 2 + 2.5x & 0 \leq x < 6 \\ 29 - 2x & 6 \leq x < 12 \\ 11 - 0.5x & 12 \leq x < 20 \end{cases}$

   d. $y = \begin{cases} -6 & -10 \leq x < -7 \\ -3.5 & -7 \leq x < -1 \\ 5 & -1 \leq x < 1 \\ 3 & 1 \leq x < 5 \end{cases}$

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Applying piecewise graphs to financial situations

7. The following graph shows the current individual *marginal tax rate* for different *income* brackets in the US.

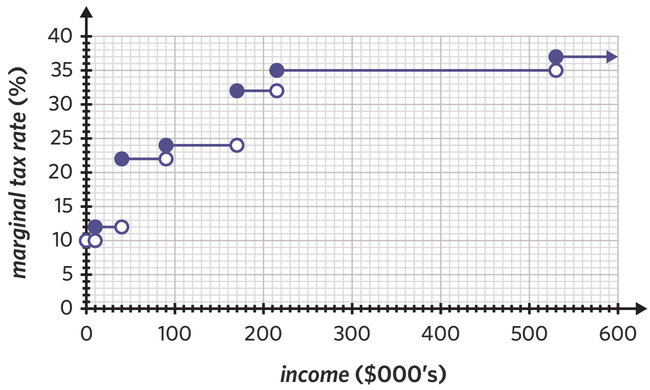
The *marginal tax rate* for an individual earning $200 000 is

A. 24%

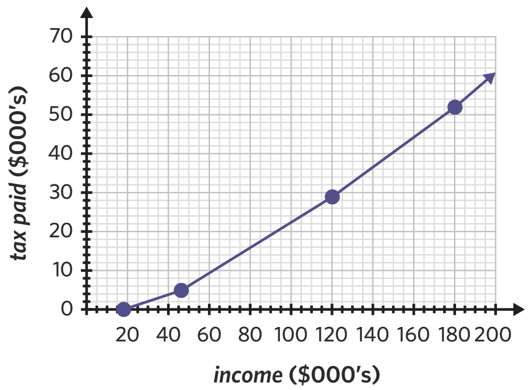
B. 32%

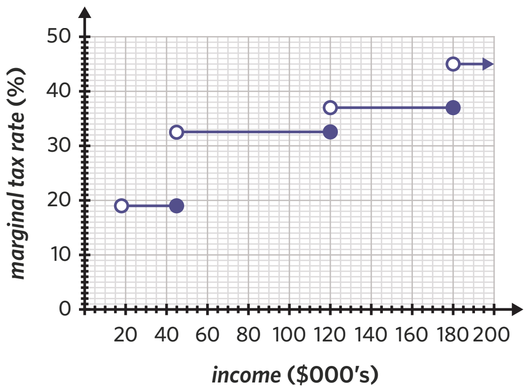
C. 35%

D. 37%



8. Consider the following graphs.





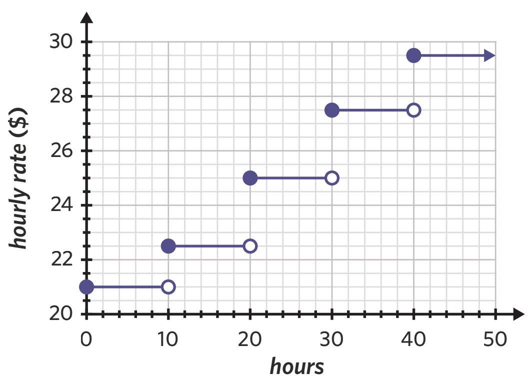
a. Jasmine earns $160 000. What is her *marginal tax rate*?

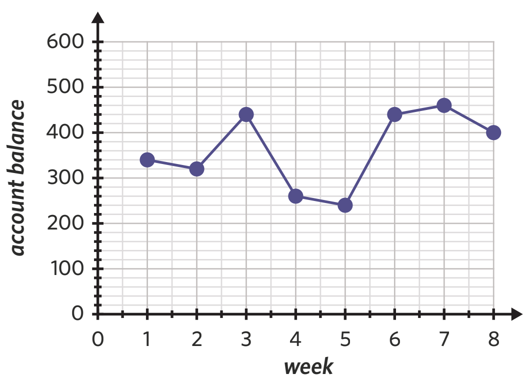
b. What is the maximum amount Jasmine can earn to continue paying the same *marginal tax rate*?

c. Approximately how much tax does Jasmine pay per year? Round to the nearest thousand dollars.

d. Jasmine's brother Jeremy earns $45 000. He pays tax on each dollar he earns over $18 200. Using the answer to part **c**, what is the difference between the amount of tax Jasmine and Jeremy pay per year? Round to the nearest thousand dollars.

9. Shobit is a casual worker, and gets paid an hourly rate. In order to incentivise him to work more, his work increases his hourly rate if he works more hours in a week. Shobit also likes to track the money in his spending account at the end of each week. The following piecewise linear graphs show his *hourly rate* depending on his weekly *hours* of work, as well as his spending *account balance* at the end of each week.





a. If Shobit works 32 hours in a week, how much money will he earn?

b. What is the greatest increase in Shobit's spending account balance? In which week did this occur?

c. In one week, Shobit worked 29 hours and 30 minutes. In another week, he worked 30 hours. What is the difference in the amount of money he earned in these two weeks?

d. If Shobit spent \$105 in week 3, how many hours did he work?

Joining it all together

10. Sonia spends a day in the city and has to pay for parking. The following table shows the *cost per hour* at the carpark according to the number of *hours* a car has been parked.

***cost per hour* (\$)**	8.20	7.80	7.20	6.80	6.20	5.80
hours	<1	1–<2	2–<3	3–<4	4–<5	5+

a. Use the information in the table to construct a piecewise linear graph.

b. Identify whether the piecewise linear graph is a line segment graph or a step graph.

c. How much would Sonia need to pay in total for parking if she parked for 5 hours?

d. Sonia went back to the same carpark the next week. She had been parked for 4 hours and 30 minutes when she went to leave, but realised she had lost her phone. She finally left the carpark 45 minutes later. How much did this additional 45 minutes cost Sonia?

e. Sonia parked for 3 hours. Her friend parked in the same carpark and paid the same amount for parking, but parked for a different period of time. How long did Sonia's friend park for?

11. The following function models the *altitude*, in m, of Horace and his friends on a hike, where *h* represents the number of hours since they began. The altitude was measured in relation to sea level.

$$altitude = \begin{cases} 40 + 53\frac{1}{3} \times h & 0 \le h < 2.25 \\ 2.5 + 70 \times h & 2.25 \le h < 3.25 \\ 230 & 3.25 \le h < 5 \\ 705 - 95 \times h & 5 \le h \le 7 \end{cases}$$

a. Construct the piecewise linear graph from the function.

b. Identify whether the piecewise linear graph is a line segment graph or a step graph.

c. They began hiking at 9 am. Between which times did they stop to have lunch?

d. The hike began at the bottom of a mountain. How far above sea level is the base of the mountain?

e. Horace and his friends hiked to the very top of the mountain. How tall is the mountain?

Exam practice

12. The *delivery fee* for a parcel, in dollars, charged by a courier company is based on the *weight* of the parcel, in kilograms.

This relationship is shown in the following step graph for parcels that weigh up to 20 kg.

Which one of the following statements is not true?

A. The *delivery fee* for a 4 kg parcel is \$20.

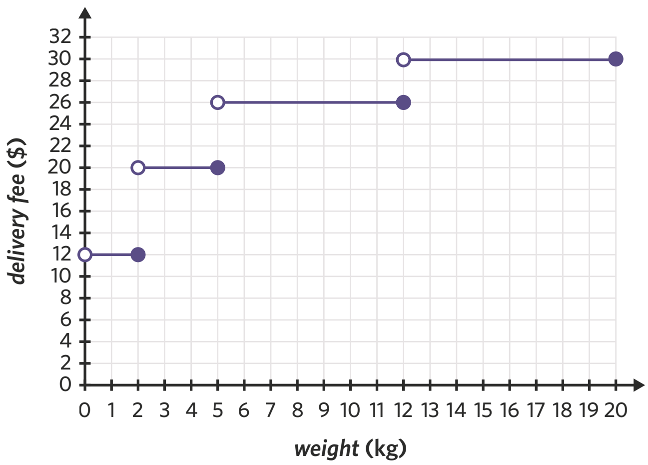
B. The *delivery fee* for a 12 kg parcel is \$26.

C. The *delivery fee* for a 13 kg parcel is the same as the *delivery fee* for a 20 kg parcel.

D. The *delivery fee* for a 10 kg parcel is \$14 more than the *delivery fee* for a 2 kg parcel.

E. The *delivery fee* for a 12 kg parcel is \$18 more than the *delivery fee* for a 2 kg parcel.

VCAA 2020 Exam 1 Graph and relations Q3



91% of students answered this question correctly.

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13. A railway station in the city has two car parks, Eastpark and Northpark.

At Eastpark, cars can be parked for up to 10 hours per day.

The fees for Eastpark are as follows.

$$fee = \begin{cases} \$6, & 0 < hours \le 3 \\ \$10, & 3 < hours \le 6 \\ \$14, & 6 < hours \le 10 \end{cases}$$

Northpark charges fees according to the formula

$fee = \$2.30 \times hours$

Lani wants to park her car for seven hours on Wednesday and four hours on Thursday.

She may choose either car park on each day.

The minimum total fee that Lani will pay for parking for the two days is

A. \$16.00 **B.** \$20.00 **C.** \$23.20
D. \$24.00 **E.** \$25.30

VCAA 2021 Exam 1 Graphs and relations Q4

44% of students answered this question correctly.

---

14. Jenny and Alan's house is 900 m from a supermarket.

Jenny is at the house and Alan is at the supermarket.

At 12 noon Jenny leaves the house and walks towards the supermarket.

At the same time, Alan leaves the supermarket and walks towards the house.

Jenny's planned walk is modelled by the equation

$$j = \begin{cases} 120t & 0 < t \le 2 \\ 100t + 40 & 2 < t \le 6 \\ 65t + 250 & 6 < t \le 10 \end{cases}$$

where j is Jenny's distance, in metres, from the house after t minutes.

Alan's planned walk is modelled by the equation

$a = -80t + 900 \qquad t > 0$

where a is Alan's distance, in metres, from the house after t minutes.

When they meet

A. Jenny will have walked 359 m, to the nearest metre.
B. Alan will have walked 360 m, to the nearest metre.
C. Alan will have walked 382 m, to the nearest metre
D. Alan will have walked 518 m, to the nearest metre.
E. Jenny will have walked 541 m, to the nearest metre.

VCAA 2019 Exam 1 Graphs and relations Q8

32% of students answered this question correctly.

Questions from multiple lessons

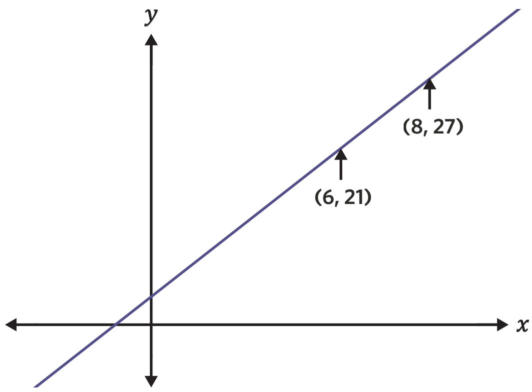
Graphs and relations

15. The following graph shows a straight line that passes through the points (6, 21) and (8, 27).

The point where the line crosses the x-axis

A. $(-2, 0)$
B. $(-1, 0)$
C. $(1, 0)$
D. $(2, 0)$
E. $(0, 3)$

Adapted from VCAA 2017NH Exam 1 Graphs and relations Q4



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Data analysis

16. The following two-way frequency table displays the *favourite soft drink* (Coke, Sprite, Fanta) and *sex* (male, female) of 149 people.

The percentage of females who chose Sprite as their favourite soft drink is closest to

A. 23%

B. 35%

C. 37%

D. 43%

E. 45%

Adapted from VCAA 2016 Exam 1 Data analysis Q1

		sex	
		male	**female**
favourite soft drink	**Coke**	33	26
	Sprite	25	35
	Fanta	13	17
total		71	78

Graphs and relations

17. The graph displays the relationship between *USD* and *AUD* at a currency exchange agency.

The points (100, 52) and (200, 117) are labelled.

The equation for the relationship between the US dollar and the AU dollar is

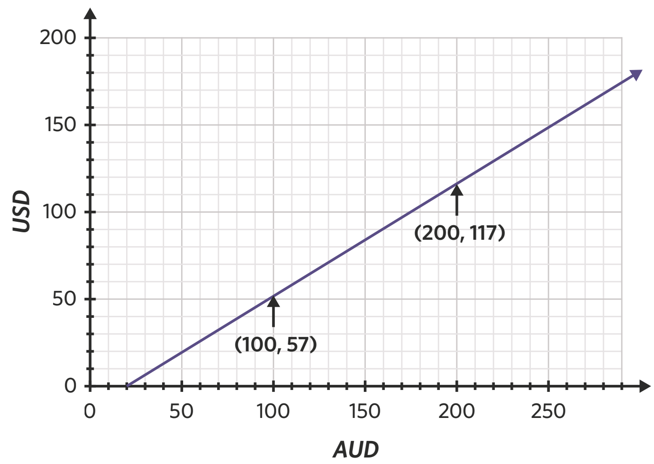
$USD = k + 0.65 \times AUD$

a. Use the point (200, 117) to show that the value of k is -13. (1 MARK)

b. Determine the coordinate for the horizontal axis intercept. (1 MARK)

c. Interpret the slope in terms of the variables *AUD* and *USD*. (1 MARK)

Adapted from VCAA 2015 Exam 2 Graphs and relations Q3



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CHAPTER 5 CALCULATOR QUICK LOOK-UP GUIDE

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5

UNIT 1 AOS 4 CHAPTER 5

Matrices

LESSONS

KEY KNOWLEDGE

- use of matrices to store and display information that can be presented in a rectangular array of rows and columns such as databases and links in social networks and road networks
- types of matrices (row, column, square, zero and identity) and the order of a matrix
- matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using technology as applicable
- use of matrices, including matrix products and powers of matrices, to model and solve problems, for example costing or pricing problems, and squaring a matrix to determine the number of ways pairs of people in a network can communicate with each other via a third person
- inverse matrices and their applications including solving a system of simultaneous linear equations
- introduction to transition matrices (assuming the next state only relies on the current state), working with iterations of simple models linked to, for example, population growth or decay, including informal consideration of long run trends and steady state.

Image: ChristianChan/Shutterstock.com

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5A Introduction to matrices

STUDY DESIGN DOT POINTS

- use of matrices to store and display information that can be presented in a rectangular array of rows and columns such as databases and links in social networks and road networks
- types of matrices (row, column, square, zero and identity) and the order of a matrix

5A 5B 5C 5D 5E 5F 5G

KEY SKILLS

During this lesson, you will be:

- identifying matrix properties
- representing information in a matrix.

KEY TERMS

- Matrix
- Rows
- Columns
- Order
- Element
- Row matrix
- Column matrix
- Square matrix
- Zero matrix
- Identity matrix
- Leading diagonal

Matrices and the different entries in a matrix can be used to model a range of situations. There are various types of matrices and operations that can be performed on these matrices. However, it is important to understand their different properties before these calculations can be completed.

Identifying matrix properties

See worked example 1

A **matrix** is a rectangular array that displays a collection of numerical values. The values can be arranged depending on what the matrix represents. Matrices are typically denoted with a capital letter pronumeral, for example the following matrix, A.

$$A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 9 & 6 & 7 & 8 \end{bmatrix}$$

Rows are horizontal lists of values and are numbered from top to bottom.

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 9 & 6 & 7 & 8 \end{bmatrix}\begin{matrix} \mathbf{1} \\ \mathbf{2} \end{matrix}$$

Columns are vertical lists of values and are numbered from left to right.

$$\begin{array}{c} \begin{matrix} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} \end{matrix} \\ \begin{bmatrix} 4 & 2 & 1 & 3 \\ 9 & 6 & 7 & 8 \end{bmatrix} \end{array}$$

The size of a matrix is known as its **order**. It is written in the form *number of rows* × *number of columns*.

For example, the order of a matrix with two rows and four columns is called a 'two-by-four' matrix, and is written as 2×4.

Every entry in a matrix is called an **element**. The element in row i and column j of matrix A can be written as a_{ij}. For example, in matrix A, the element a_{23} is 7. It is in the second row and third column.

$$A = \begin{array}{c} \begin{matrix} & & \mathbf{3} & \end{matrix} \\ \begin{bmatrix} 4 & 2 & 1 & 3 \\ 9 & 6 & 7 & 8 \end{bmatrix}\begin{matrix} \\ \mathbf{2} \end{matrix} \end{array}$$

See worked example 2

Matrices can be categorised into different types.

A **row matrix** is a matrix with one row and any number of columns.

$\begin{bmatrix} 10 & 5 \end{bmatrix}$ and $\begin{bmatrix} 4 & 4 & 8 & 8 & 3 \end{bmatrix}$

A **column matrix** is a matrix with one column and any number of rows.

$$\begin{bmatrix}2\\14\\6\\9\end{bmatrix} \text{ and } \begin{bmatrix}1\\1\end{bmatrix}$$

A **square matrix** is a matrix with an equal number of rows and columns.

$$\begin{bmatrix}5.2 & 1\\9 & 16\end{bmatrix} \text{ and } \begin{bmatrix}1 & 2 & 3 & 4\\5 & 6 & 7 & 8\\9 & 10 & 11 & 12\\13 & 14 & 15 & 16\end{bmatrix}$$

A **zero matrix** is a matrix of any size where all of the elements are zero.

$$[0] \text{ and } \begin{bmatrix}0 & 0\\0 & 0\\0 & 0\end{bmatrix}$$

An **identity matrix** is a type of square matrix where all of the elements in the leading diagonal are one and the rest of the elements are zero. The **leading diagonal** is the diagonal line from the top left corner of a square matrix to the bottom right corner.

$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} \text{ and } \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Worked example 1

Consider the matrix $B = \begin{bmatrix}9 & 8\\2 & 1\\6 & 3\\4 & 7\end{bmatrix}$.

a. What is the order of matrix B?

Explanation

Step 1: Identify the number of rows in matrix B.

$$\begin{array}{cc}\begin{bmatrix}9 & 8\\2 & 1\\6 & 3\\4 & 7\end{bmatrix} & \begin{array}{c}1\\2\\3\\4\end{array}\end{array}$$

There are 4 rows.

Step 2: Identify the number of columns in matrix B.

$$\begin{array}{c}\begin{array}{cc}1 & 2\end{array}\\ \begin{bmatrix}9 & 8\\2 & 1\\6 & 3\\4 & 7\end{bmatrix}\end{array}$$

There are 2 columns.

Answer

4×2

b. What is element b_{21}?

Explanation

Step 1: Locate the second row.

$$\begin{array}{cc}\begin{bmatrix}9 & 8\\2 & 1\\6 & 3\\4 & 7\end{bmatrix} & \begin{array}{c}\\2\\ \\ \\ \end{array}\end{array}$$

Step 2: Locate the first column.

$$\begin{array}{cc}\begin{array}{cc}1 & \end{array} & \\ \begin{bmatrix}9 & 8\\2 & 1\\6 & 3\\4 & 7\end{bmatrix} & \begin{array}{c}\\2\\ \\ \\ \end{array}\end{array}$$

Answer

$b_{21} = 2$

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Worked example 2

Classify the following matrices as either a row, column, square, zero or identity matrix. There may be more than one answer.

a. $B = [12 \quad 23 \quad 42 \quad 7]$

Explanation

There is one row and four columns in this matrix.
As there is only one row, this is a row matrix.

Answer

Row matrix

b. $C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Explanation

Since all of the elements in the matrix are zero, this is a zero matrix.

Answer

Zero matrix

c. $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Explanation

There are three rows and columns in this matrix.

As the number of rows and columns are equal, this is a square matrix.

Since the elements in the leading diagonal are all one and all other elements are zero, this is also an identity matrix.

Answer

Square and identity matrix

Representing information in a matrix

Matrices can be used to display many types of numerical information, such as the results of a survey, or sales data.

For example, the table shows the number of bagels (B), coffees (C) and doughnuts (D) sold at a cafe on Monday and Tuesday.

	Monday (M)	Tuesday (T)
bagels (B)	17	12
coffee (C)	34	38
doughnuts (D)	19	22

This information can also be displayed in matrix form.

$$\begin{array}{cc} \text{M} & \text{T} \end{array}$$
$$\begin{bmatrix} 17 & 12 \\ 34 & 38 \\ 19 & 22 \end{bmatrix} \begin{array}{l} \text{B} \\ \text{C} \\ \text{D} \end{array}$$

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Worked example 3

The favourite sports of Year 11 and 12 students at Edrolo High are shown in matrix M.

$$M = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{cc} \text{Year 11} & \text{Year 12} \\ \left[\begin{array}{cc} 53 & 29 \\ 16 & 21 \\ 32 & 45 \end{array}\right] \end{array}\begin{array}{l} \\ \text{basketball} \\ \text{netball} \\ \text{soccer} \end{array}$$

a. What information does element m_{32} indicate?

Explanation

Step 1: Locate the third row in matrix M.

$$M = \begin{array}{cc} \text{Year 11} & \text{Year 12} \\ \left[\begin{array}{cc} 53 & 29 \\ 16 & 21 \\ 32 & 45 \end{array}\right] \end{array}\begin{array}{l} \\ \text{basketball} \\ \text{netball} \\ \text{soccer} \end{array}$$

Row 3 is labelled 'soccer'.

Step 2: Locate the second column in matrix M.

$$M = \begin{array}{cc} \text{Year 11} & \text{Year 12} \\ \left[\begin{array}{cc} 53 & 29 \\ 16 & 21 \\ 32 & 45 \end{array}\right] \end{array}\begin{array}{l} \\ \text{basketball} \\ \text{netball} \\ \text{soccer} \end{array}$$

Column 2 is labelled 'Year 12'.

Step 3: Interpret the element.

Answer

The number of Year 12 students who chose soccer as their favourite sport, which is 45.

b. How many Year 12 students are there at Edrolo High?

Explanation

Step 1: Locate the 'Year 12' column.

$$M = \begin{array}{cc} \text{Year 11} & \text{Year 12} \\ \left[\begin{array}{cc} 53 & 29 \\ 16 & 21 \\ 32 & 45 \end{array}\right] \end{array}\begin{array}{l} \\ \text{basketball} \\ \text{netball} \\ \text{soccer} \end{array}$$

Step 2: Find the sum of the elements.

$$\begin{aligned} total &= 29 + 21 + 45 \\ &= 95 \end{aligned}$$

Answer

95 students

c. What is the most popular sport amongst Year 11 students?

Explanation

Step 1: Locate the 'Year 11' column.

$$M = \begin{array}{cc} \text{Year 11} & \text{Year 12} \\ \left[\begin{array}{cc} 53 & 29 \\ 16 & 21 \\ 32 & 45 \end{array}\right] \end{array}\begin{array}{l} \\ \text{basketball} \\ \text{netball} \\ \text{soccer} \end{array}$$

Step 2: Determine the element with the highest value.

The highest value is 53, which corresponds to 'basketball'.

Answer

Basketball

5A Questions

Identifying matrix properties

1. Which of the following is a 3×4 matrix?

A. $\begin{bmatrix} 4 & 3 & 3 & 4 & 3 & 3 \\ 3 & 4 & 3 & 3 & 4 & 3 \end{bmatrix}$ **B.** $\begin{bmatrix} 4 & 7 & 8 \\ 2 & 1 & 3 \\ 6 & 2 & 9 \\ 5 & 4 & 8 \end{bmatrix}$ **C.** $\begin{bmatrix} 7 & 6 & 5 & 4 \\ 0 & 1 & 8 & 1 \\ 9 & 5 & 6 & 3 \end{bmatrix}$ **D.** $[12]$

2. For matrix M, determine each of the following elements.

$$M = \begin{bmatrix} 12 & 3 & 2 & 1 \\ 7 & 9 & 16 & 5 \\ 10 & 11 & 4 & 7 \\ 6 & 8 & 18 & 0 \end{bmatrix}$$

a. m_{14}

b. m_{42}

3. Determine the order for each of the following matrices.

a. $[2 \quad 1 \quad 44 \quad 5 \quad 17]$ **b.** $\begin{bmatrix} 0 & 4 \\ 15 & 2 \\ 6 & 11 \end{bmatrix}$ **c.** $\begin{bmatrix} 6 \\ -4 \end{bmatrix}$ **d.** $\begin{bmatrix} 2 & 33 & 15 & -3 \\ 19 & 6 & 7 & 8 \\ 4 & -55 & 71 & 5 \end{bmatrix}$

4. Consider matrix N.

$$N = \begin{bmatrix} 6 & -9 & 15 \\ 4 & 2 & 11 \\ 3 & 10 & 7 \\ -1 & 8 & -4 \end{bmatrix}$$

a. What is the order of matrix N?

b. Find element n_{32}.

c. Find the sum of column 3.

5. Create the following matrices.

a. A zero row matrix containing 7 elements.

b. An identity matrix with 3 rows.

6. Consider matrices A to G.

$$A = [1 \quad 1 \quad 1 \quad 1] \quad B = [0] \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad F = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad G = [1]$$

List the matrices that are

a. zero matrices.

b. row matrices.

c. column matrices.

d. square matrices.

e. identity matrices.

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Representing information in a matrix

7. The following table displays the highest, lowest and median marks awarded for a maths test in four Year 11 classes.

	highest (%)	lowest (%)	median (%)
Class A	97	34	71
Class B	89	40	68
Class C	100	29	65
Class D	94	42	70

Which of the following matrices does not display information that is in the table?

A. $$\begin{array}{cl} \text{C} & \\ \begin{bmatrix} 100 \\ 29 \\ 65 \end{bmatrix} & \begin{array}{l} \text{highest (\%)} \\ \text{lowest (\%)} \\ \text{median (\%)} \end{array} \end{array}$$

B. $$\begin{array}{cl} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} & \\ \begin{bmatrix} 34 & 40 & 29 & 42 \\ 97 & 89 & 100 & 94 \\ 71 & 68 & 65 & 70 \end{bmatrix} & \begin{array}{l} \text{lowest (\%)} \\ \text{highest (\%)} \\ \text{median (\%)} \end{array} \end{array}$$

C. $$\begin{array}{cl} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} & \\ \begin{bmatrix} 71 & 68 & 65 & 70 \end{bmatrix} & \text{median (\%)} \end{array}$$

D. $$\begin{array}{cl} \begin{array}{cccc} \text{A} & \text{B} & \text{C} & \text{D} \end{array} & \\ \begin{bmatrix} 94 & 100 & 89 & 97 \end{bmatrix} & \text{highest (\%)} \end{array}$$

8. The following table shows the amount of milk (M), flour (F) and sugar (S) needed to make pancakes (P) and cupcakes (C). All measurements are in cups.

	pancakes	cupcakes
milk	2.0	1.5
flour	1.5	2.0
sugar	0.5	1.0

Fill in the matrix to display this information.

$$\begin{array}{cl} \begin{array}{cc} \text{P} & \text{C} \end{array} & \\ \begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix} & \begin{array}{l} \text{M} \\ \text{F} \\ \text{S} \end{array} \end{array}$$

9. Alex has three cats and two dogs. Bella has one cat and one dog. Catherine has two cats and no dogs.

Display this information in a 3×2 matrix.

Joining it all together

10. A group of students were asked to choose their favourite flavour of ice cream out of vanilla (V), strawberry (S) and mint choc chip (M). Their year levels were also recorded. The results have been displayed in a table.

	vanilla	strawberry	mint choc chip
Year 1	9	12	21
Year 2	15	11	17

a. Display this information in a 2×3 matrix.

b. Create a column matrix showing the number of Year 1 and Year 2 students that chose mint choc chip.

c. Create a row matrix showing the ice cream choices for the students in Year 1.

d. Find the sum of the elements in the row matrix from part **c** and interpret the result.

11. Three media classes were asked in a survey what their favourite TV show was out of Friends (F), Grey's Anatomy (G), and RuPaul's Drag Race (R). Matrix X displays the results of the survey separated into classes A, B and C.

$$X = \begin{array}{c} \begin{array}{ccc} F & G & R \end{array} \\ \begin{bmatrix} 7 & 7 & 12 \\ 8 & 9 & 10 \\ 8 & 11 & 6 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \end{array}$$

a. How many students in class A chose Friends?

b. What information does x_{23} represent?

c. Which element represents the number of students in Class C that chose Grey's Anatomy?

d. How many students are there in each of the three classes?

e. What is the most popular TV show?

Exam practice

12. A soccer stadium sells hotdogs (H), party pies (P) and sausage rolls (S).

The number of each item sold over three games is shown in matrix P.

$$P = \begin{array}{c} \begin{array}{ccc} H & P & S \end{array} \\ \begin{bmatrix} 45 & 24 & 49 \\ 29 & 16 & 20 \\ 16 & 31 & 47 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{game 1} \\ \text{game 2} \\ \text{game 3} \end{array}$$

a. In total, how many party pies were sold over the three games? (1 MARK)

b. The element in row i and column j of matrix P is p_{ij}.

What does the element p_{23} indicate? (1 MARK)

Adapted from VCAA 2017 Exam 2 Matrices Q1a,b

Part **a**: **89%** of students answered this type of question correctly.

Part **b**: **75%** of students answered this type of question correctly.

---

13. The element in row i and column j of matrix S is s_{ij}.

S is a 3×3 matrix. It is constructed using the rule $s_{ij} = 2i + j$.

S is

A. $\begin{bmatrix} 3 & 5 & 7 \\ 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix}$

B. $\begin{bmatrix} 5 & 7 & 9 \\ 8 & 11 & 12 \\ 11 & 13 & 15 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$

E. none of these.

Adapted from VCAA 2020 Exam 1 Matrices Q6

73% of students answered this type of question correctly.

---

14. Consider the matrix M, where $M = \begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$.

The element in row i and column j of matrix M is m_{ij}.

The elements in matrix M are determined by the rule

A. $m_{ij} = 2i - j$

B. $m_{ij} = 4 - i$

C. $m_{ij} = 2i + 2j$

D. $m_{ij} = 4 + j$

E. $m_{ij} = 4 - j$

Adapted from VCAA 2019 Exam 1 Matrices Q3

59% of students answered this type of question correctly.

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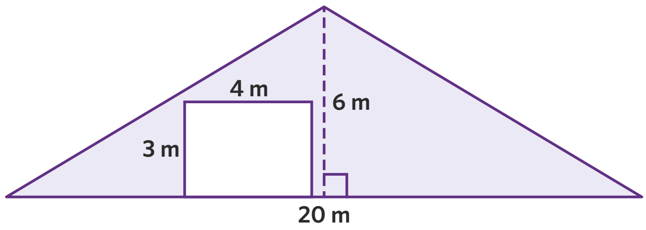
Questions from multiple lessons

Geometry and measurement *Year 10 content*

15. Consider the following diagram.



The shaded area, in square metres, is

A. 12 m^2.

B. 48 m^2.

C. 60 m^2.

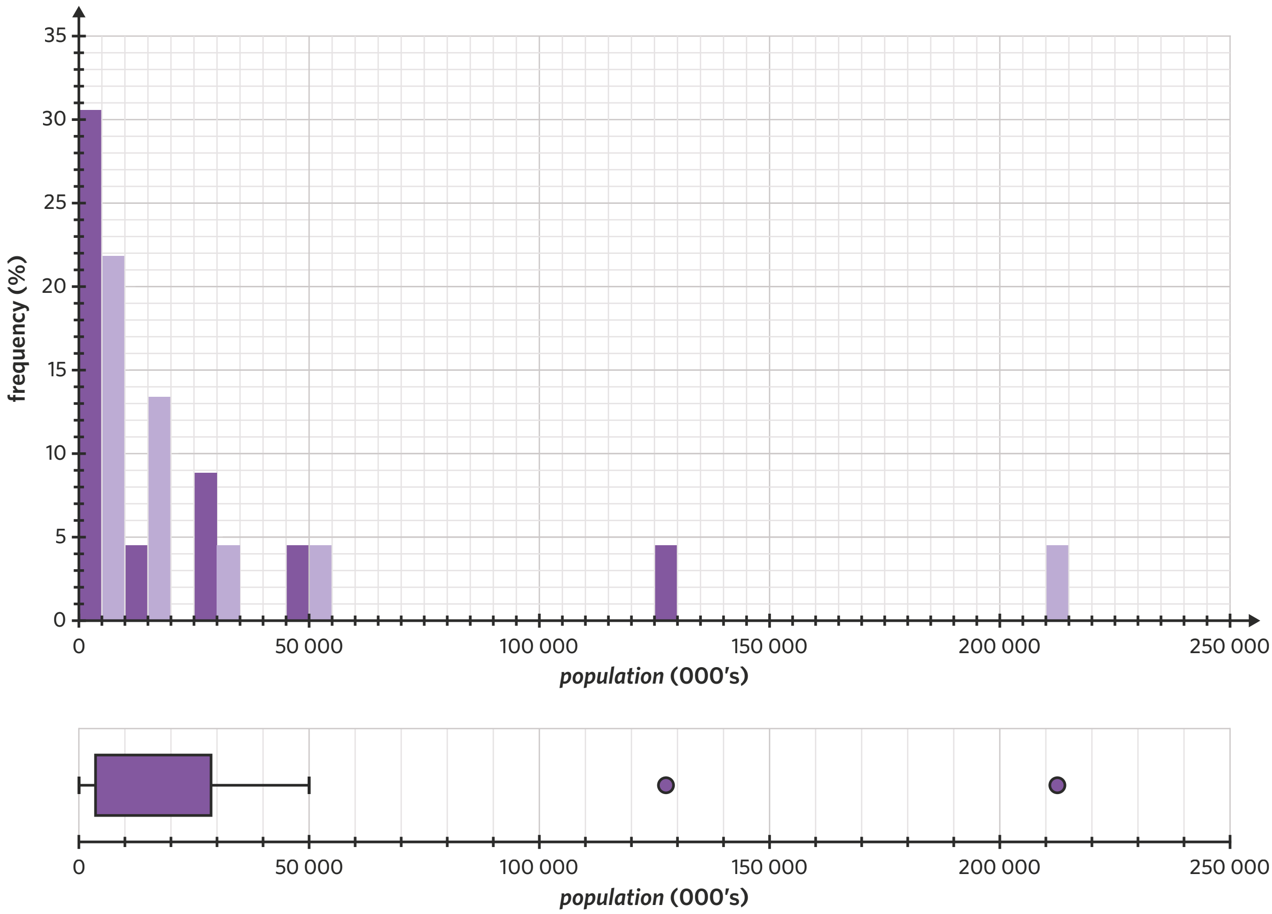
D. 108 m^2.

E. 120 m^2.

Adapted from VCAA 2016 Exam 1 Geometry and measurement Q1

Data analysis *Year 10 content*

16. The histogram and boxplot shown both display the distribution of the *population* of 23 Central and South American countries.



The shape of the distribution for the countries' *population* is best described as

A. approximately symmetric with no outliers.

B. approximately symmetric with outliers.

C. positively skewed with outliers.

D. negatively skewed with outliers.

E. positively skewed with no outliers.

Adapted from VCAA 2019NH Exam 1 Data analysis Q1

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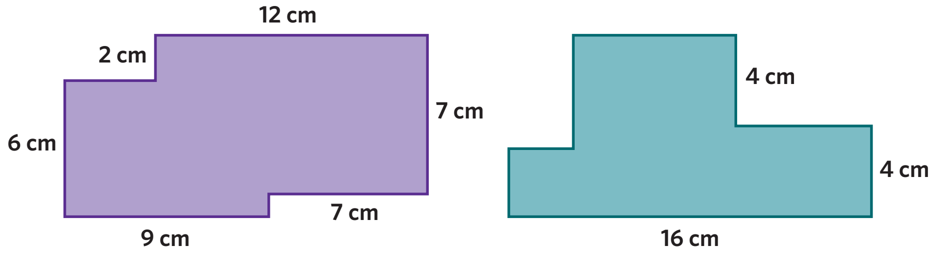
Geometry and measurement *Year 10 content*

17. Consider the following shapes.



Note: All angles within both shapes are right angles.

a. What is the perimeter of the purple shape? (1 MARK)

b. What is the difference between the perimeters of the two shapes? (1 MARK)

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5B Operations with matrices

STUDY DESIGN DOT POINT

- matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using technology as applicable

5A 5B 5C 5D 5E 5F 5G

KEY SKILLS

During this lesson, you will be:

- adding and subtracting matrices
- multiplying matrices by a scalar.

KEY TERMS

- Scalar multiplication

It can be useful to apply operations to matrices in order to represent specific information or scenarios. Addition, subtraction and scalar multiplication are three basic operations applied to matrices that help to demonstrate the changing nature of a matrix and what it represents.

Adding and subtracting matrices

To add or subtract matrices, the expression must first be defined. For a matrix operation involving addition or subtraction to be defined, the matrices involved in the operation must be of the same order.

See worked example 1

When adding or subtracting matrices, elements in the same position are added or subtracted from each other.

See worked example 2

It is also possible to calculate unknown elements in a matrix by forming an equation.

See worked example 3

Worked example 1

$A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$

Determine whether the expression $A + B$ is defined.

Explanation

Determine the order of matrix A and matrix B.

Matrix A: 2×2

Matrix B: 2×2

Both matrices have the same order.

Answer

The expression $A + B$ is defined.

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Worked example 2

If $A = \begin{bmatrix} 2 & 0 \\ 0 & 4 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 3 \\ 2 & 9 \\ 4 & 2 \end{bmatrix}$, find $A + B$.

Explanation – Method 1: By hand

Step 1: Write the calculation.

$$\begin{bmatrix} 2 & 0 \\ 0 & 4 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 3 \\ 2 & 9 \\ 4 & 2 \end{bmatrix}$$

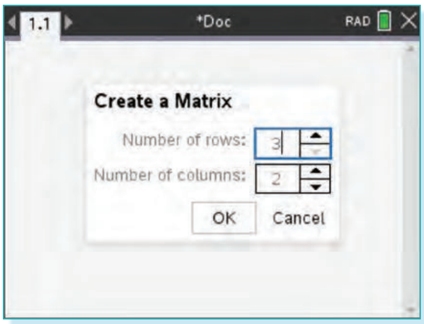
Step 2: Add the elements that are in the same position.

$$\begin{bmatrix} 2+8 & 0+3 \\ 0+2 & 4+9 \\ 6+4 & 7+2 \end{bmatrix}$$

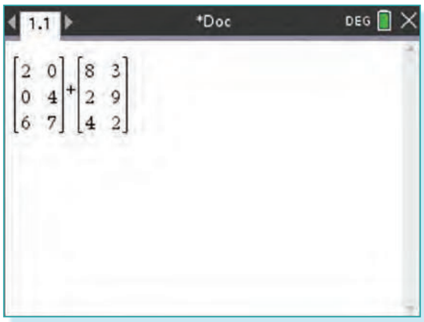
Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select '1: New' → '1: Add Calculator'.

Step 2: Press and select . On the settings window, enter '3' for 'Number of rows' and '2' for 'Number of columns'. Select 'OK'.



Step 3: Enter the values from matrix A. Press the addition sign '+', repeat step 2 and enter the values for matrix B.



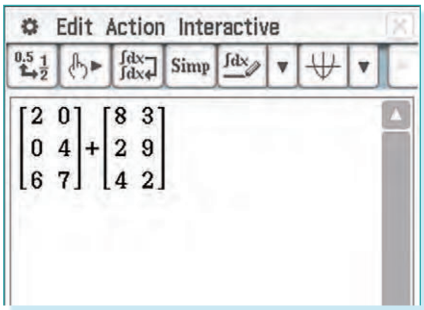
Step 4: Press enter.

Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap √α Main.

Step 2: Press keyboard and tap Math2. Tap to create a matrix, and to add an extra row.

Step 3: Enter the values for matrix A. Press the addition sign '+', repeat step 2 and enter the values for matrix B.



Step 4: Press EXE.

Answer – Method 1, 2 and 3

$$\begin{bmatrix} 10 & 3 \\ 2 & 13 \\ 10 & 9 \end{bmatrix}$$

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Worked example 3

Consider the following matrix equation.

$$\begin{bmatrix} 1 & a \\ -3 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 2 & 15 \end{bmatrix}$$

What is the value of a?

Explanation

Step 1: Determine the position of element a.

Element a is in the first row and second column.

Step 2: Determine the element in the same position in the second and final matrix.

$$\begin{bmatrix} 1 & a \\ -3 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 2 & 15 \end{bmatrix}$$

Step 3: Create an equation and solve to find a.

$$a + 3 = 1$$
$$a = 1 - 3$$

Answer

$a = -2$

Multiplying matrices by a scalar

Scalar multiplication refers to the multiplication of a matrix by a number. The number is known as a scalar as it 'scales' the values in the matrix.

To evaluate an expression that involves scalar multiplication, multiply each value in the matrix by the scalar.

See worked example 4

It is also possible to calculate an unknown scalar by forming an equation.

See worked example 5

Worked example 4

If $A = \begin{bmatrix} 6 & -7 \\ 0 & 3 \end{bmatrix}$, find $3A$.

Explanation – Method 1: By hand

Step 1: Write the expression in matrix form.

$$3A = 3\begin{bmatrix} 6 & -7 \\ 0 & 3 \end{bmatrix}$$

Step 2: Multiply every element inside the matrix by the scalar, 3.

$$3A = \begin{bmatrix} 6 \times 3 & -7 \times 3 \\ 0 \times 3 & 3 \times 3 \end{bmatrix}$$

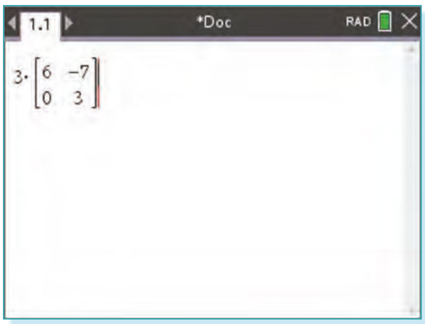
Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select '1: New' → '1: Add Calculator'.

Step 2: Enter the scalar value '3', followed by the multiplication sign.

Step 3: Press and select .

Enter the values from matrix A.



Step 4: Press enter.

Continues →

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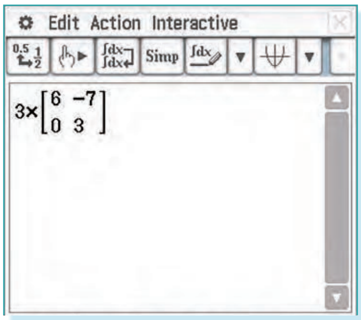
Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap √α Main.

Step 2: Enter the scalar value '3', followed by the multiplication sign.

Step 3: Press keyboard and tap Math2. Tap [▦] and enter the values for matrix A.



Step 4: Press EXE.

Answer – Method 1, 2 and 3

$3A = \begin{bmatrix} 18 & -21 \\ 0 & 9 \end{bmatrix}$

Worked example 5

Consider the following matrix equation.

$a\begin{bmatrix} 2 & 8 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 12 \\ 6 & 9 \end{bmatrix}$

Calculate the value of a.

Explanation

Step 1: Multiply every element inside the matrix by a.

$$\begin{bmatrix} 2 \times a & 8 \times a \\ 4 \times a & 6 \times a \end{bmatrix} = \begin{bmatrix} 3 & 12 \\ 6 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 2a & 8a \\ 4a & 6a \end{bmatrix} = \begin{bmatrix} 3 & 12 \\ 6 & 9 \end{bmatrix}$$

Step 2: Create an equation using a pair of elements in the same position.

Using the first row and first column:

$2a = 3$

Note: Other equations can be created using other pairs of elements in the same position.

Step 3: Solve the equation.

$2a = 3$

$a = \frac{3}{2}$

Answer

$a = \frac{3}{2}$ or 1.5

5B Questions

Adding and subtracting matrices

1. Consider the following matrices.

$$A = \begin{bmatrix} 3 & -7 & 12 \\ 0 & 16 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 7 & -1 \\ -3 & 8 \\ 9 & 2 \end{bmatrix} \quad C = \begin{bmatrix} -14 & 2 \\ 10 & 11 \\ -8 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 6 & -16 \\ 3 & 4 & 7 \\ -8 & 17 & 9 \end{bmatrix} \quad E = \begin{bmatrix} 5 & 10 \\ -6 & 7 \end{bmatrix}$$

$$F = \begin{bmatrix} 28 & 3 & -7 \\ 7 & 15 & 0 \end{bmatrix} \quad G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Which of the following is **not** defined? **(Select all that apply)**

I. $A + B$

II. $B - C$

III. $E + G$

IV. $F + A$

V. $D - C$

---

2. Consider matrices A, B, C and D.

$$A = \begin{bmatrix} -5 & 8 \\ 3 & 10 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 9 \\ 11 & 7 \end{bmatrix} \quad C = \begin{bmatrix} -4 & 5 \\ -6 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 0 & -7 \\ -2 & 12 \end{bmatrix}$$

Calculate the following matrix expressions.

a. $A + B$ **b.** $C + D$ **c.** $B - C$ **d.** $D - B$

---

3. Consider matrices A, B and C

$$A = \begin{bmatrix} 10 & 2 & 9 \\ 5 & 8 & 1 \\ 3 & 10 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -2 & -3 & -9 \\ -5 & -4 & -1 \\ -8 & -11 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 10 & 4 \\ 2 & 1 & -5 \\ -3 & -10 & -2 \end{bmatrix}$$

Calculate the following matrix expressions.

a. $A + B + C$ **b.** $A - B - C$ **c.** $C - A + B$ **d.** $-B - A + C$

---

4. The following matrices show the number of small and large blueberry, banana and chocolate muffins sold at two stores in a day. Construct a matrix that shows the total number of each type of muffin sold that day across the two stores.

store 1

$$\begin{array}{cl} \begin{array}{ccc} \text{blueberry} & \text{banana} & \text{chocolate} \end{array} & \\ \begin{bmatrix} 15 & 11 & 17 \\ 21 & 16 & 19 \end{bmatrix} & \begin{array}{l} \text{small} \\ \text{large} \end{array} \end{array}$$

store 2

$$\begin{array}{cl} \begin{array}{ccc} \text{blueberry} & \text{banana} & \text{chocolate} \end{array} & \\ \begin{bmatrix} 19 & 20 & 22 \\ 25 & 23 & 27 \end{bmatrix} & \begin{array}{l} \text{small} \\ \text{large} \end{array} \end{array}$$

---

5. Find the unknown values in the following matrix equations.

a. $\begin{bmatrix} 0 & 7 \\ 17 & 9 \end{bmatrix} + \begin{bmatrix} 3 & -5 \\ -8 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ \boldsymbol{x} & 13 \end{bmatrix}$

b. $\begin{bmatrix} 6 & \boldsymbol{a} & 1 \\ 9 & 0 & -8 \end{bmatrix} + \begin{bmatrix} 13 & 9 & 11 \\ 2 & 4 & -\boldsymbol{b} \end{bmatrix} = \begin{bmatrix} 19 & 16 & 12 \\ 11 & 4 & 4 \end{bmatrix}$

c. $\begin{bmatrix} 16 & -7 & 3 \\ -5 & 13 & 8 \\ 6 & 10 & \boldsymbol{e} \end{bmatrix} + \begin{bmatrix} 7 & -2 & \boldsymbol{g} \\ \boldsymbol{f} & -3 & 12 \\ 16 & 0 & -7 \end{bmatrix} = \begin{bmatrix} 23 & -9 & 5 \\ -6 & 10 & 20 \\ 22 & 10 & -7 \end{bmatrix}$

d. $\begin{bmatrix} 7 \\ -\boldsymbol{c} \\ 14 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 5 \\ \boldsymbol{d} \\ 11 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \\ 7 \\ -8 \end{bmatrix}$

6. The total number of students who have brown, blue, hazel and green eyes in three Year 11 classes is represented in the following matrix.

$$\begin{bmatrix}37\\18\\11\\3\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$$

Which could **not** have been the number of brown, blue, hazel and green eyes in the three individual classes?

A. Class 11A: $\begin{bmatrix}10\\9\\3\\1\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$ Class 11B: $\begin{bmatrix}13\\5\\3\\2\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$ Class 11C: $\begin{bmatrix}14\\4\\5\\0\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$

B. Class 11A: $\begin{bmatrix}12\\6\\2\\1\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$ Class 11B: $\begin{bmatrix}13\\7\\3\\1\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$ Class 11C: $\begin{bmatrix}12\\5\\6\\1\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$

C. Class 11A: $\begin{bmatrix}13\\7\\3\\1\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$ Class 11B: $\begin{bmatrix}10\\9\\3\\1\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$ Class 11C: $\begin{bmatrix}14\\2\\5\\1\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$

D. Class 11A: $\begin{bmatrix}14\\2\\5\\1\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$ Class 11B: $\begin{bmatrix}11\\10\\5\\1\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$ Class 11C: $\begin{bmatrix}12\\6\\2\\1\end{bmatrix}\begin{matrix}\text{brown}\\\text{blue}\\\text{hazel}\\\text{green}\end{matrix}$

---

7. The AFL Fantasy score and number of handballs of four football players were recorded in round 1 and round 2 of an AFL season.

$$\begin{array}{cc} & \begin{matrix}\text{Steele} & \text{Ridley} & \text{Crisp} & \text{Kelly}\end{matrix}\\ \text{round 1:} & \begin{bmatrix}77 & 56 & 72 & 61\\24 & 22 & 18 & 27\end{bmatrix}\begin{matrix}\text{AFL Fantasy score}\\\text{handballs}\end{matrix}\end{array}$$

$$\begin{array}{cc} & \begin{matrix}\text{Steele} & \text{Ridley} & \text{Crisp} & \text{Kelly}\end{matrix}\\ \text{round 2:} & \begin{bmatrix}76 & 57 & 74 & 59\\19 & 20 & 16 & 24\end{bmatrix}\begin{matrix}\text{AFL Fantasy score}\\\text{handballs}\end{matrix}\end{array}$$

a. Calculate the matrix which shows the change in each player's AFL Fantasy score and number of handballs between round 1 and round 2.

b. Which player's AFL Fantasy score decreased the most?

Multiplying matrices by a scalar

8. Which of the following expressions is equivalent to $\begin{bmatrix}20 & 7.5\\27.5 & 25\end{bmatrix}$?

A. $0.5\begin{bmatrix}40 & 14\\55 & 50\end{bmatrix}$ **B.** $10\begin{bmatrix}4 & -2\\7 & 3\end{bmatrix}$ **C.** $0.25\begin{bmatrix}12 & 4\\16 & 40\end{bmatrix}$ **D.** $2.5\begin{bmatrix}8 & 3\\11 & 10\end{bmatrix}$

---

9. Find the value of x for each of the following scalar multiplications.

a. $x\begin{bmatrix}-2 & 5\\3 & 1\end{bmatrix} = \begin{bmatrix}-18 & 45\\27 & 9\end{bmatrix}$ **b.** $x\begin{bmatrix}2 & 6\\4 & 2\end{bmatrix} = \begin{bmatrix}1 & 3\\2 & 1\end{bmatrix}$

---

10. Due to an increase in demand, the price for Edrolo merchandise will double.

The original prices, in matrix form, were:

$$\begin{bmatrix}\textit{hoodie} & \textit{tracksuit pants}\\\textit{cap} & \textit{tote bag}\end{bmatrix} = \begin{bmatrix}\$70 & \$55\\\$20 & \$10\end{bmatrix}$$

a. Determine the scalar value.

b. Evaluate the scalar product using the value determined from part **a**.

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Joining it all together

11. Which of the following is not defined?

A. $0.27\begin{bmatrix}2.2 & 3.5 & -2.8\\37 & 8.3 & -0.5\end{bmatrix} - \begin{bmatrix}0 & 4 & 3\\6 & 0 & 0\end{bmatrix}$

B. $0.27\begin{bmatrix}-4\\3\\21\end{bmatrix} + 3\begin{bmatrix}2 & -7\\0.4 & 3.3\\-1 & 0.78\end{bmatrix}$

C. $\begin{bmatrix}3 & 4 & 6\\-5 & 0.1 & 3\\4 & -0.6 & 0.33\end{bmatrix} - \begin{bmatrix}-0.1 & 2 & 0.1\\0.2 & 1 & 0\\4 & 3.2 & 43\end{bmatrix}$

D. $45[-3 \quad 1.4] + -5.6[3.1 \quad -2]$

12. Evaluate the following expressions.

a. $8[11 \quad 15 \quad -19] - 0.5[210 \quad 68 \quad 130]$

b. $2\begin{bmatrix}1\\-3\\4\end{bmatrix} + 3\begin{bmatrix}-1\\2\\2\end{bmatrix}$

c. $-3\begin{bmatrix}-2 & 2\\1 & 3\end{bmatrix} + 2\begin{bmatrix}4 & -3\\-1 & 5\end{bmatrix}$

d. $0.5\begin{bmatrix}6 & 4 & -2\\3 & 8 & -5\end{bmatrix} - \begin{bmatrix}-2 & 4 & 1\\1 & 0 & -1\end{bmatrix}$

13. For the following matrix equations, find the unknown values.

a. $3\begin{bmatrix}1 & x & 4\\2 & 3 & 6\end{bmatrix} - 2\begin{bmatrix}-5 & 1 & 2\\3 & y & -2\end{bmatrix} = \begin{bmatrix}13 & 5 & 8\\0 & 9 & 7\end{bmatrix}$

b. $2\begin{bmatrix}1 & 3\\x & 4\end{bmatrix} - 3\begin{bmatrix}y & 2\\3 & 4\end{bmatrix} = \begin{bmatrix}5 & 0\\-3 & -4\end{bmatrix}$

c. $-3\begin{bmatrix}x & y\\2 & 5\end{bmatrix} - 3\begin{bmatrix}-1 & 0\\-3 & 6\end{bmatrix} = \begin{bmatrix}11 & -6\\3 & -33\end{bmatrix}$

14. Gabrielle and Hussain are racing to finish their classwork correctly. Consider their working for the final problem of the lesson.

$$1.5\begin{bmatrix}4 & -3 & 2\\2 & -1 & 0\end{bmatrix} - 2\begin{bmatrix}3 & 1.5 & -2\\-0.5 & 3 & 4.5\end{bmatrix}$$

Gabrielle:

$$= \begin{bmatrix}1.5 \times 4 & 1.5 \times -3 & 1.5 \times 2\\1.5 \times 2 & 1.5 \times -1 & 1.5 \times 0\end{bmatrix} - \begin{bmatrix}2 \times 3 & 2 \times 1.5 & 2 \times -2\\2 \times -0.5 & 2 \times 3 & 2 \times 4.5\end{bmatrix}$$

$$= \begin{bmatrix}6 & -4.5 & 3\\3 & -1.5 & 0\end{bmatrix} - \begin{bmatrix}6 & 3 & -4\\-1 & 6 & 9\end{bmatrix}$$

$$= \begin{bmatrix}0 & -7.5 & 7\\2 & -7.5 & -9\end{bmatrix}$$

Hussain:

$$= \begin{bmatrix}1.5 \times 4 & 1.5 \times -3 & 1.5 \times 2\\1.5 \times 2 & 1.5 \times -1 & 1.5 \times 0\end{bmatrix} - \begin{bmatrix}2 \times 3 & 2 \times 1.5 & 2 \times -2\\2 \times -0.5 & 2 \times 3 & 2 \times 4.5\end{bmatrix}$$

$$= \begin{bmatrix}6 & -4.5 & 3\\3 & -1.5 & 0\end{bmatrix} - \begin{bmatrix}6 & 3 & -4\\-1 & 6 & 9\end{bmatrix}$$

$$= \begin{bmatrix}0 & -7.5 & 7\\4 & -7.5 & -9\end{bmatrix}$$

Despite their rush, who calculated the answer correctly?

Exam practice

15. Elena imports three brands of olive oil: Carmani (C), Linelli (L) and Ohana (O).

The number of 1 litre bottles of these oils sold in January 2021 is shown in matrix J.

$$J = \begin{array}{c} \begin{bmatrix}2800\\1700\\2400\end{bmatrix} \end{array}\begin{array}{l}C\\L\\O\end{array}$$

Elena expected that in February 2021 the sales of all three bands of olive oil would increase by 10%. She multiplied matrix J by a scalar value, k, to determine the expected volume of sales in February.

Write down the value of the scalar k. (1 MARK)

Adapted from VCAA 2021 Exam 2 Matrices Q1b

48% of students answered this type of question correctly.

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Questions from multiple lessons

Matrices

16. Jane recently started working at a grocery store on weekends. The number of hours she works on Saturday and Sunday over three consecutive weeks (W1, W2 and W3) are shown in the following matrix.

$$\begin{array}{cl} \begin{array}{ccc} W1 & W2 & W3 \end{array} & \\ \begin{bmatrix} 4 & 7 & 5 \\ 5 & 7 & 8 \end{bmatrix} & \begin{array}{l} \text{Saturday} \\ \text{Sunday} \end{array} \end{array}$$

How many hours did Jane work on Saturday in week 3?

A. 4 **B.** 5 **C.** 7 **D.** 8 **E.** 13

Adapted from VCAA 2017 Exam 1 Matrices Q1

Data analysis *Year 10 content*

17. The following scatterplot shows the *distance covered* (km) and *price* ($) per ticket for 10 international flights.

A line of good fit has been added to the scatterplot with *distance covered* as the explanatory variable.

The equation of the line of good fit is closest to

A. *distance covered* = 507 + 0.987 × *price*

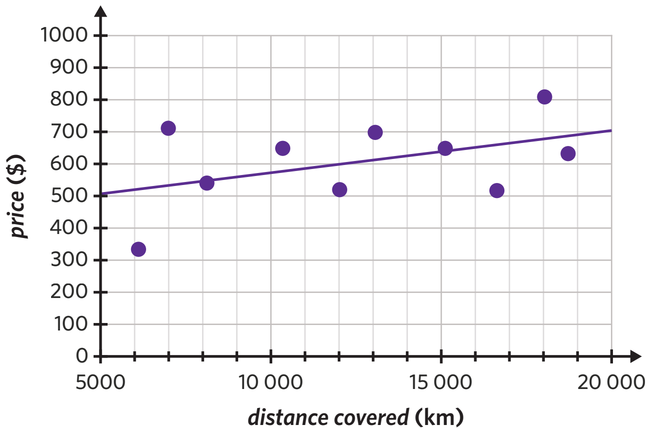
B. *distance covered* = 507 + 0.013 × *price*

C. *distance covered* = 441 + 0.013 × *price*

D. *price* = 507 + 0.013 × *distance covered*

E. *price* = 441 + 0.013 × *distance covered*

Adapted from VCAA 2017 Exam 1 Data analysis Q8



Matrices

18. Cadrolo (C), Edlindt (E), and Ferred Rocholo (F) are three popular brands of chocolate. Woles (W) and Igloo (I) are supermarkets that sell all three brands of chocolate.

The price, in dollars, of one block of each of these brands of chocolate, in each supermarket, is shown in matrix B.

$$B = \begin{array}{cl} \begin{array}{cc} W & I \end{array} & \\ \begin{bmatrix} 2.59 & 2.39 \\ 5.99 & 6.25 \\ 3.89 & 3.99 \end{bmatrix} & \begin{array}{l} C \\ E \\ F \end{array} \end{array}$$

a. What is the price of one block of Edlindt at Woles? (1 MARK)

b. Write down the order of matrix B. (1 MARK)

c. What does the element b_{12} represent? (1 MARK)

Adapted from VCAA 2018 Exam 2 Matrices Q1

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5C Advanced operations with matrices

STUDY DESIGN DOT POINT

- matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using technology as applicable

5A 5B 5C 5D 5E 5F 5G

KEY SKILLS

During this lesson, you will be:

- defining matrix products
- calculating a matrix product
- calculating a matrix power.

KEY TERMS

- Matrix product
- Pre-multiplication
- Post-multiplication
- Matrix power

Multiplying matrices is a key component of matrix arithmetic. It can be used in predicting trends, solving simultaneous equations and mapping transformations. Unlike scalar multiplication, matrix multiplication involves two or more matrices.

Defining matrix products

A **matrix product** is the resulting matrix when two or more matrices are multiplied. However, not all matrix multiplications can be performed.

Whether a matrix product is defined or not is determined by the order of the two (or more) matrices involved. Matrices can only be multiplied together if the number of columns in the first matrix is equal to the number of rows in the second matrix.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad B = \begin{bmatrix} e \\ f \end{bmatrix}$$

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e \\ f \end{bmatrix} \qquad BA = \begin{bmatrix} e \\ f \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Order: $2 \times \underbrace{2 \quad 2}_{\text{equal}} \times 1$ Order: $2 \times \underbrace{1 \quad 2}_{\text{not equal}} \times 1$

The product AB is defined since the number of columns in matrix A equals the number of rows in matrix B. The product BA is not defined since the number of columns in matrix B is not equal to the number of rows in matrix A.

As seen in the previous example, the order of multiplication of two matrices is generally not interchangeable, and must be taken into account when determining if a matrix product is defined. One matrix can either pre-multiply or post-multiply another matrix.

Pre-multiplication is the process of multiplying one matrix before another. For example, the product AB is calculated by pre-multiplying matrix B by matrix A.

Post-multiplication is the process of multiplying one matrix after another. For example, the product BA is calculated by post-multiplying matrix B by matrix A.

If a matrix product is defined, it will have the same number of rows as the first matrix and the same number of columns as the second matrix.

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e \\ f \end{bmatrix}$$

Order: $\underbrace{2 \times 2 \quad 2 \times 1}_{2 \times 1}$

The order of the matrix product AB is 2×1.

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Worked example 1

Determine whether the following matrix products are defined. If defined, determine its order.

$$P = \begin{bmatrix} 3 & -12 \\ 9 & 4 \\ -7 & 6 \end{bmatrix} \quad Q = \begin{bmatrix} 2 & 3 & -11 \\ 4 & 0 & -5 \\ 3 & -2 & 7 \\ 2 & -5 & 0 \\ 1 & 1 & 6 \end{bmatrix}$$

a. PQ

Explanation

Step 1: Determine the order of each matrix.

Matrix P has an order of 3×2.

Matrix Q has an order of 5×3.

Step 2: Determine whether the matrix product is defined.

The number of columns of matrix P must equal the number of rows of matrix Q.

$$PQ = \begin{bmatrix} 3 & -12 \\ 9 & 4 \\ -7 & 6 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & -11 \\ 4 & 0 & -5 \\ 3 & -2 & 7 \\ 2 & -5 & 0 \\ 1 & 1 & 6 \end{bmatrix}$$

Order: $3 \times \underbrace{2 \qquad 5}_{\text{not equal}} \times 3$

The matrix product is not defined.

Answer

PQ is not defined.

b. QP

Explanation

Step 1: Determine the order of each matrix.

Matrix Q has an order of 5×3.

Matrix P has an order of 3×2.

Step 2: Determine whether the matrix product is defined.

The number of columns of matrix Q must equal the number of rows of matrix P.

$$QP = \begin{bmatrix} 2 & 3 & -11 \\ 4 & 0 & -5 \\ 3 & -2 & 7 \\ 2 & -5 & 0 \\ 1 & 1 & 6 \end{bmatrix} \times \begin{bmatrix} 3 & -12 \\ 9 & 4 \\ -7 & 6 \end{bmatrix}$$

Order: $5 \times \underbrace{3 \qquad 3}_{\text{equal}} \times 2$

The matrix product is defined.

Step 3: Determine the order of the matrix product.

QP will have the same number of rows as Q and the same number of columns as P.

$$QP = \begin{bmatrix} 2 & 3 & -11 \\ 4 & 0 & -5 \\ 3 & -2 & 7 \\ 2 & -5 & 0 \\ 1 & 1 & 6 \end{bmatrix} \times \begin{bmatrix} 3 & -12 \\ 9 & 4 \\ -7 & 6 \end{bmatrix}$$

Order: $\underbrace{5 \times 3 \qquad 3 \times 2}_{5 \times 2}$

Answer

QP is defined.

The order of QP is 5×2.

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Calculating a matrix product

To multiply two matrices together, elements in specific rows and columns must be multiplied and summed together. The process of multiplying 2 × 2 matrices is shown in the following matrix equation.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \times e + b \times g & a \times f + b \times h \\ c \times e + d \times g & c \times f + d \times h \end{bmatrix}$$

For example, to find the element in the first row and second column of the product matrix, the elements in the first row of matrix A are multiplied by their corresponding elements in the second column of matrix B, and these values are then added.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a \times e + b \times g & a \times f + b \times h \\ c \times e + d \times g & c \times f + d \times h \end{bmatrix}$$

Worked example 2

Consider the following matrices.

$$X = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \quad Y = \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix}$$

Evaluate the matrix product XY.

Explanation – Method 1: By hand

Step 1: Determine whether the product matrix is defined.

$$XY = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix}$$

Order: $2 \times \underbrace{3 \qquad 3}_{\text{equal}} \times 2$

The matrix product is defined.

Step 2: Determine the order of the product matrix.

XY will have the same number of rows as X and the same number of columns as Y.

The order of XY is 2 × 2.

$$XY = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Step 3: Find the element in the first row and first column of XY.

$$\begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix}$$

$$2 \times (-1) + 3 \times 0 + (-1) \times 4 = -6$$

$$XY = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -6 & \\ & \end{bmatrix}$$

Step 4: Find the element in the first row and second column of XY.

$$\begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix}$$

$$2 \times 2 + 3 \times 3 + (-1) \times 1 = 12$$

$$XY = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -6 & 12 \\ & \end{bmatrix}$$

Step 5: Find the element in the second row and first column of XY.

$$\begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix}$$

$$4 \times (-1) + 0 \times 0 + (-5) \times 4 = -24$$

$$XY = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -6 & 12 \\ -24 & \end{bmatrix}$$

Step 6: Find the element in the second row and second column of XY.

$$\begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix}$$

$$4 \times 2 + 0 \times 3 + (-5) \times 1 = 3$$

$$XY = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & -5 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -6 & 12 \\ -24 & 3 \end{bmatrix}$$

Continues →

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Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select '1: New' → '1: Add calculator'.

Step 2: Press and select .

Enter '2' for 'Number of rows' and '3' for 'Number of columns'.

Select 'OK'.

Enter the values from matrix X.

Step 3: Press the multiplication sign '×'.

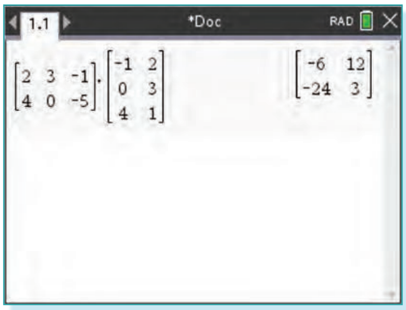
Step 4: Press and select .

Enter '3' for 'Number of rows' and '2' for 'Number of columns'.

Select 'OK'.

Enter the values from matrix Y.

Step 5: Press enter to calculate the matrix product.



Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap √α Main.

Press keyboard and tap Math2.

Step 2: Tap to create a matrix, and to add an extra column.

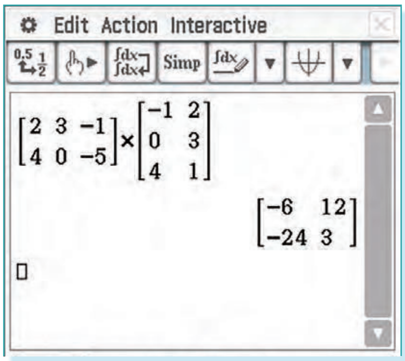
Enter the values from matrix X.

Step 3: Press the multiplication sign '×'.

Step 4: Tap to create a matrix, and to add an extra row.

Enter the values from matrix Y.

Step 5: Press EXE to calculate the matrix product.



Answer – Method 1, 2 and 3

$$XY = \begin{bmatrix} -6 & 12 \\ -24 & 3 \end{bmatrix}$$

Calculating a matrix power

A **matrix power** is the product when a matrix is raised to an index or power. Due to the conditions of matrix multiplication, only square matrices provide a defined matrix product when raised to a given power.

When raised to a given power, matrices behave like any other matrix in a matrix multiplication equation, except that it is the same matrix being multiplied by itself. As such, the order of the resultant matrix product will be the same as the original matrix.

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Worked example 3

Consider the following matrix.

$B = \begin{bmatrix} 7 & 5 \\ 0 & -3 \end{bmatrix}$

Evaluate B^2.

Explanation – Method 1: By hand

Step 1: Express the squared matrix as the product of two matrices.

$$B^2 = \begin{bmatrix} 7 & 5 \\ 0 & -3 \end{bmatrix} \times \begin{bmatrix} 7 & 5 \\ 0 & -3 \end{bmatrix}$$

Step 2: Determine the order of the product matrix.

B^2 will have the same order as B.

The order of B^2 is 2×2.

Step 3: Calculate each element of the matrix power using matrix multiplication.

$$B^2 = \begin{bmatrix} 7 & 5 \\ 0 & -3 \end{bmatrix} \times \begin{bmatrix} 7 & 5 \\ 0 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \times 7 + 5 \times 0 & 7 \times 5 + 5 \times (-3) \\ 0 \times 7 + (-3) \times 0 & 0 \times 5 + (-3) \times (-3) \end{bmatrix}$$

$$= \begin{bmatrix} 49 & 20 \\ 0 & 9 \end{bmatrix}$$

Explanation – Method 2: TI-Nspire

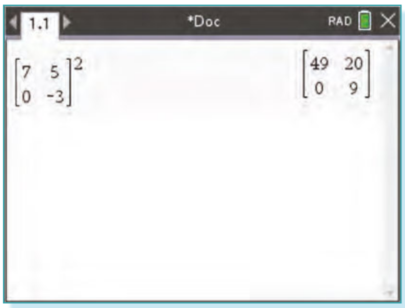
Step 1: From the home screen, select '1: New' → '1: Add calculator'.

Step 2: Press and select .

Enter the values from matrix B.

Step 3: Press '^2'.

Step 4: Press enter to calculate the matrix power.



Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap √α Main.

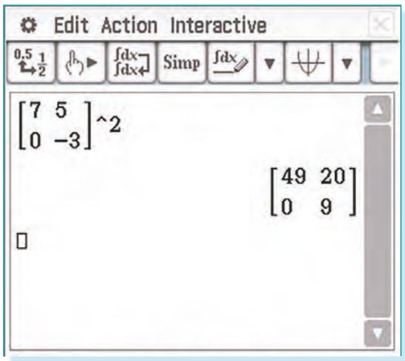
Press keyboard and tap Math2.

Step 2: Tap to create a matrix.

Enter the values from matrix B.

Step 3: Press '^2'.

Step 4: Press EXE to calculate the matrix power.



Answer – Method 1, 2 and 3

$$B^2 = \begin{bmatrix} 49 & 20 \\ 0 & 9 \end{bmatrix}$$

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5C Questions

Defining matrix products

1. The matrix product BA is defined.

If matrix A has two rows, how many columns does matrix B have?

A. 1 **B.** 2 **C.** 3 **D.** 4

---

2. Consider the following matrices.

$A = \begin{bmatrix} 5 & 4 & 7 \\ 8 & 3 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 6 & 4 & 2 & 1 \\ 2 & 5 & 8 & 7 \\ 6 & 9 & 5 & 4 \end{bmatrix}$

a. Is AB defined? Explain briefly.

b. Is BA defined? Explain briefly.

---

3. Is the matrix product AB defined for the following pairs of A and B?

If defined, determine the order of AB.

a. $A = \begin{bmatrix} 2 & 5 \\ -2 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 \\ -2 & 4 \\ 0 & -1 \end{bmatrix}$

b. $A = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ -4 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

---

4. The order of matrix A is 2×3.

The order of matrix B is 3×4.

Is $2A \times 3B$ defined? If so, what is the order of the matrix product?

Calculating a matrix product

5. Which expression will give the correct value for a in the following equation?

$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \times \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} a & 14 \\ 35 & 26 \end{bmatrix}$

A. $4 \times 5 + 4 \times 2$ **B.** $3 \times 4 + 3 \times 3$ **C.** $2 \times 5 + 3 \times 3$ **D.** $2 \times 4 + 3 \times 2$

---

6. Which of the following matrix products does not equal $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$?

A. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ **B.** $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$ **C.** $\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ **D.** $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

---

7. Determine the following matrix products by hand.

a. $A = \begin{bmatrix} 1 & 4 \\ 5 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$

b. $B = \begin{bmatrix} 2 & -1 \\ -4 & 1 \end{bmatrix} \times \begin{bmatrix} 10 & -2 \\ 2 & 6 \end{bmatrix}$

c. $C = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & \frac{1}{2} & 1 \\ 2 & 1 & -\frac{1}{2} \end{bmatrix}$

8. Consider the following matrices.

$$P = \begin{bmatrix} 1 & 4 & -2 & 1 \\ 3 & 0 & -4 & -2 \\ 2 & 5 & -3 & 1 \end{bmatrix} \quad Q = \begin{bmatrix} -3 \\ 4 \\ 2 \\ 0 \end{bmatrix}$$

Use a calculator to evaluate PQ.

---

9. Youssef and Maya's teacher wrote the following question on the board.

$$\begin{bmatrix} a & b \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 5 & 11 \end{bmatrix}$$

Youssef got an answer of $a = 4$ and $b = -2$.

Maya got an answer of $a = -5$ and $b = 1$.

Who is correct?

Calculating a matrix power

10. Which of the following matrices can be raised to any given power?

A. $\begin{bmatrix} 41 & 2 \end{bmatrix}$ **B.** $\begin{bmatrix} 71 & -23 \\ 32 & 4 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ **D.** $\begin{bmatrix} 23 \\ 42 \end{bmatrix}$

---

11. Find the value of a in the following matrix equation.

$$\begin{bmatrix} 4 & -2 \\ 8 & 1 \end{bmatrix}^2 = \begin{bmatrix} a & -10 \\ 40 & -15 \end{bmatrix}$$

A. -13 **B.** 0 **C.** 5 **D.** 12

---

12. Consider the following matrix.

$$A = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$$

Evaluate A^2.

Joining it all together

13. Consider the following matrix product.

$$M = \begin{bmatrix} 3 & 2 & 1 \\ -2 & 3 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ -2 & 3 \\ 6 & -1 \end{bmatrix}$$

Determine if matrix M is defined. If it is defined, state its order and calculate the product.

---

14. Consider the following matrices.

$$A = \begin{bmatrix} 3 & -3 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 4 \\ 0 & 1 \\ -2 & 3 \\ 4 & -1 \end{bmatrix}$$

Calculate BA^2.

---

15. Matrix K has 3 rows and 4 columns.

The matrix product J^2K is defined.

How many rows does matrix J have?

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Exam practice

16. The matrix product $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} \text{T} \\ \text{A} \\ \text{L} \\ \text{E} \\ \text{S} \end{bmatrix}$ is equal to

A. $\begin{bmatrix} \text{S} \\ \text{L} \\ \text{A} \\ \text{T} \\ \text{E} \end{bmatrix}$ **B.** $\begin{bmatrix} \text{S} \\ \text{T} \\ \text{A} \\ \text{L} \\ \text{E} \end{bmatrix}$ **C.** $\begin{bmatrix} \text{S} \\ \text{T} \\ \text{E} \\ \text{A} \\ \text{L} \end{bmatrix}$

D. $\begin{bmatrix} \text{L} \\ \text{E} \\ \text{A} \\ \text{S} \\ \text{T} \end{bmatrix}$ **E.** $\begin{bmatrix} \text{T} \\ \text{E} \\ \text{A} \\ \text{L} \\ \text{S} \end{bmatrix}$

Adapted from VCAA 2016 Exam 1 Matrices Q2

86% of students answered this type of question correctly.

---

17. Matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 1 & 0 \\ 4 & 5 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 2 & 0 & 3 & 1 \\ 4 & 5 & 2 & 0 \end{bmatrix}$.

Matrix $Q = A \times B$.

The element in row i and column j of matrix Q is q_{ij}.

Element q_{41} is determined by the calculation

A. $0 \times 0 + 3 \times 5$ **B.** $1 \times 1 + 2 \times 0$ **C.** $1 \times 2 + 2 \times 4$

D. $4 \times 1 + 5 \times 0$ **E.** $4 \times 2 + 5 \times 4$

VCAA 2020 Exam 1 Matrices Q2

79% of students answered this question correctly.

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18. The matrix product $[2 \quad 1 \quad 4] \times \begin{bmatrix} 8 \\ 0 \\ 11 \end{bmatrix}$ is equal to

A. $[132]$ **B.** $2 \times [1 \quad 1 \quad 2] \times \begin{bmatrix} 4 \\ 0 \\ 11 \end{bmatrix}$ **C.** $4 \times [1 \quad 0 \quad 4] \times \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}$

D. $2 \times [1 \quad 1 \quad 4] \times \begin{bmatrix} 8 \\ 0 \\ 11 \end{bmatrix}$ **E.** $\begin{bmatrix} 16 \\ 44 \\ 60 \end{bmatrix}$

Adapted from VCAA 2018 Exam 1 Matrices Q2

68% of students answered this type of question correctly.

Questions from multiple lessons

Matrices

19. The following table shows information about two matrices, P and Q.

The element in row i and column j of matrix P is p_{ij}.

The element in row i and column j of matrix Q is q_{ij}.

The difference $P - Q$ is

matrix	order	rule
P	3×3	$p_{ij} = 5i - 2j$
Q	3×3	$q_{ij} = 3i - 2j$

A. $\begin{bmatrix} 3 & 1 & -1 \\ 8 & 6 & 4 \\ 13 & 11 & 9 \end{bmatrix}$ **B.** $\begin{bmatrix} 2 & 2 & 2 \\ 4 & 4 & 4 \\ 6 & 6 & 6 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \\ 7 & 5 & 3 \end{bmatrix}$ **D.** $\begin{bmatrix} 2 & 4 & 6 \\ 2 & 4 & 6 \\ 2 & 4 & 6 \end{bmatrix}$ **E.** $\begin{bmatrix} 2 & 4 & 2 \\ 4 & 2 & 6 \\ 2 & 6 & 2 \end{bmatrix}$

Adapted from VCAA 2017 Exam 1 Matrices Q6

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Recursion and financial modelling

20. Sophie wants to open up her own fashion studio. She will take out a loan of $160 000 with interest charged at a rate of 2.6% per annum, compounding monthly.

Each month, Sophie will only pay the interest charged for that month.

After 30 months, the amount Sophie will owe is

A. $84 900 **B.** $122 250 **C.** $149 600 **D.** $160 000 **E.** $170 400

Adapted from VCAA 2017 Exam 1 Recursion and financial modelling Q19

Matrices

21. The number of child (C), student (S) and adult (A) tickets sold for a particular movie is shown in the matrix N.

$$N = \begin{array}{c} \begin{bmatrix} 36 \\ 34 \\ 49 \end{bmatrix} \end{array}\begin{array}{c} C \\ S \\ A \end{array}$$

a. What is the order of matrix N? (1 MARK)

b. What is the element n_{31}? (1 MARK)

c. What is the sum of the elements in the column matrix, and what does it represent? (1 MARK)

Adapted from VCAA 2016 Exam 2 Matrices Q1

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5D Matrix applications

STUDY DESIGN DOT POINT

- use of matrices, including matrix products and powers of matrices, to model and solve problems, for example costing or pricing problems, and squaring a matrix to determine the number of ways pairs of people in a network can communicate with each other via a third person

5A 5B 5C **5D** 5E 5F 5G

KEY SKILLS

During this lesson, you will be:

- using matrix products in financial applications
- using summing matrices to solve application problems
- using communication matrices to model systems.

KEY TERMS

- Summing matrix
- Communication matrix
- One-step communication matrix
- Two-step communication matrix

Matrices have many real-world applications, often being used to model financial and non-financial situations. Matrices can also be used to model the flow of communication in a system. A combination of arithmetic operations, such as matrix multiplication and powers, can be used when solving these problems.

Using matrix products in financial applications

One application of matrix products is their use in calculating business revenue. For these scenarios, a matrix is constructed that allows for the total revenue made from an item, individual or company to be calculated. This matrix will contain the price of each item.

Worked example 1

Jaiden (J) and Nick (N) both sell the same types of fish. They both sell trout (T) for \$40, cod (C) for \$30 and mackerel (M) for \$20. The number of each fish sold by Jaiden and Nick can be found in matrix F.

$$F = \begin{array}{c} \begin{array}{ccc} \text{T} & \text{C} & \text{M} \end{array} \\ \begin{bmatrix} 10 & 19 & 13 \\ 11 & 9 & 16 \end{bmatrix} \end{array} \begin{array}{c} \\ \text{J} \\ \text{N} \end{array}$$

a. Construct P, a matrix which contains the prices of each type of fish, as a column matrix.

Explanation

Step 1: Identify the order of the required matrix.

There are three types of fish being sold, so a 3×1 matrix will be required.

Each element will represent the price for a different type of fish.

Step 2: Construct the matrix.

As matrix F has trout, cod and mackerel in a specific order, this order is maintained in the column matrix.

Trout is sold for \$40, so $p_{11} = 40$.

Cod is sold for \$30, so $p_{21} = 30$.

Mackerel is sold for \$20, so $p_{31} = 20$.

Answer

$$P = \begin{bmatrix} 40 \\ 30 \\ 20 \end{bmatrix} \begin{array}{c} \text{T} \\ \text{C} \\ \text{M} \end{array}$$

Continues →

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b. Using a matrix product, calculate the revenue made from Jaiden and Nick.

Explanation

Step 1: Set up the matrix multiplication.

F and P will be multiplied to calculate the matrix product.

For the matrix product to be defined, matrix P must be post-multiplied.

$$F \times P = \begin{bmatrix} 10 & 19 & 13 \\ 11 & 9 & 16 \end{bmatrix} \times \begin{bmatrix} 40 \\ 30 \\ 20 \end{bmatrix}$$

Step 2: Calculate the matrix product.

The product will be a 2×1 matrix.

$$\begin{array}{cl} \begin{bmatrix} 1230 \\ 1030 \end{bmatrix} & \begin{matrix} \text{J} \\ \text{N} \end{matrix} \end{array}$$

Answer

Jaiden generated \$1230 of revenue whereas Nick generated \$1030 of revenue.

Using summing matrices to solve application problems

A **summing matrix** is a row or column matrix that consists of only the number 1 for each element, and is used to find the sum of either the rows or columns of another matrix.

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

The sum of the elements in each row of a matrix can be found by post-multiplying a summing matrix of one column. The number of rows in the summing matrix must be equal to the number of columns in the matrix to be summed, otherwise the matrix product will be undefined.

For example, to find the sum of the elements in each row in matrix A, the post-multiplication of a 3×1 summing matrix is required, since it has 3 columns.

$$A = \begin{bmatrix} 1 & 4 & 2 \\ -2 & 3 & 0 \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} 1 & 4 & 2 \\ -2 & 3 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} &= \begin{bmatrix} 1 \times 1 + 4 \times 1 + 2 \times 1 \\ -2 \times 1 + 3 \times 1 + 0 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 7 \\ 1 \end{bmatrix} \end{aligned}$$

The sum of the elements in each column of a matrix can be found by pre-multiplying a summing matrix of one row. The number of columns in the summing matrix must be equal to the number of rows in the matrix to be summed, otherwise the matrix product will be undefined.

For example, to find the sum of elements in each column in matrix A, the pre-multiplication of a 1×2 summing matrix is required, since it has 2 rows.

$$A = \begin{bmatrix} 1 & 4 & 2 \\ -2 & 3 & 0 \end{bmatrix}$$

$$\begin{aligned} \begin{bmatrix} 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 & 2 \\ -2 & 3 & 0 \end{bmatrix} &= \begin{bmatrix} 1 \times 1 + 1 \times -2 & 1 \times 4 + 1 \times 3 & 1 \times 2 + 1 \times 0 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 7 & 2 \end{bmatrix} \end{aligned}$$

Worked example 2

Students from three schools, school A, school B and school C, visited a local carnival. Matrix S shows the number of students from each school and the type of snacks they purchased. The carnival had doughnuts (D), hotdogs (H), snowcones (S) and pretzels (P).

$$S = \begin{array}{cc} & \begin{matrix} \text{D} & \text{H} & \text{S} & \text{P} \end{matrix} \\ & \begin{bmatrix} 17 & 18 & 21 & 22 \\ 15 & 21 & 13 & 18 \\ 11 & 14 & 17 & 15 \end{bmatrix} \begin{matrix} \text{A} \\ \text{B} \\ \text{C} \end{matrix} \end{array}$$

Continues →

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a. Using a summing matrix, calculate the number of students from school A that attended the carnival.

Explanation

Step 1: Construct the summing matrix.

The elements of each row need to be added to determine the total number of students at each school. As such, the summing matrix must be a column matrix. As there are four columns in matrix *S*, the summing matrix must have an order of 4×1.

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Step 2: Construct a matrix product that sums each row.

For the matrix product to be defined, the summing matrix must be post-multiplied.

$$\begin{bmatrix} 17 & 18 & 21 & 22 \\ 15 & 21 & 13 & 18 \\ 11 & 14 & 17 & 15 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Step 3: Calculate the matrix product.

The product will be a 3×1 matrix.

$$\begin{array}{cc} \begin{bmatrix} 78 \\ 67 \\ 57 \end{bmatrix} & \begin{array}{c} \mathrm{A} \\ \mathrm{B} \\ \mathrm{C} \end{array} \end{array}$$

Step 4: Interpret the matrix product.

The first row corresponds to students from school A.

Answer

78 students

---

b. Using a summing matrix, determine the most popular snack.

Explanation

Step 1: Construct the summing matrix.

The elements of each column need to be added to determine the total number of each snack sold. As such, the summing matrix must be a row matrix. As there are three rows in matrix *S*, the summing matrix must have an order of 1×3.

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Step 2: Create a matrix product that sums each column.

For the matrix product to be defined, the summing matrix must be pre-multiplied.

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 17 & 18 & 21 & 22 \\ 15 & 21 & 13 & 18 \\ 11 & 14 & 17 & 15 \end{bmatrix}$$

Step 3: Calculate the matrix product.

The product will be a 1×4 matrix.

$$\begin{array}{cccc} \mathrm{D} & \mathrm{H} & \mathrm{S} & \mathrm{P} \end{array}$$
$$\begin{bmatrix} 43 & 53 & 51 & 55 \end{bmatrix}$$

Step 4: Interpret the matrix product.

There were a total of 43 doughnuts, 53 hotdogs, 51 snowcones and 55 pretzels sold.

The most popular snack was pretzels.

Answer

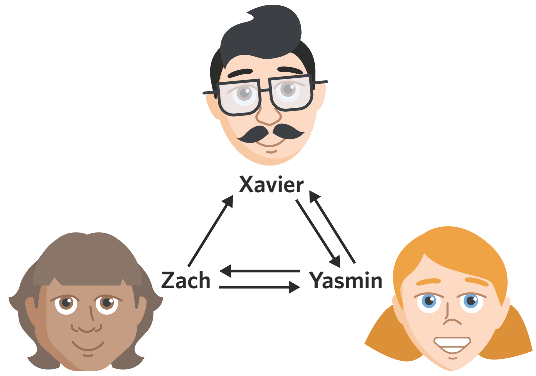
Pretzels

Using communication matrices to model systems

Matrices can also be used to display information about connections in a system. A **communication matrix** is a square matrix where each element indicates the number of communication paths between points. A **one-step communication matrix** models the number of direct connections between two points.

The following is an example of a system that maps who can send messages between Xavier (X), Yasmin (Y) and Zach (Z).

- Xavier can directly send a message to Yasmin but not to Zach.
- Yasmin can directly send a message to both Xavier and Zach.
- Zach can directly send a message to Yasmin and to Xavier.



This can be presented in C, a one-step communication matrix.

$$\begin{array}{cc} & \text{to} \\ & \begin{array}{ccc} X & Y & Z \end{array} \\ C = & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{array}{c} X \\ Y \\ Z \end{array} \text{ from} \end{array}$$

Element c_{11} is '0', because Xavier does not directly communicate with himself.

Element c_{12} is '1', which indicates that Xavier can directly message Yasmin.

Element c_{13} is '1', which indicates that Xavier can directly message Zach.

A two-step link is an indirect path that connects two points through a middle point. A **two-step communication matrix** models the number of two-step connections between points. It can be calculated by squaring the one-step communication matrix.

For example, C^2 is a two-step communication matrix that is used to represent the two-step connections between Xavier, Yasmin and Zach.

$$\begin{array}{cc} & \text{to} \\ & \begin{array}{ccc} X & Y & Z \end{array} \\ C^2 = & \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{array}{c} X \\ Y \\ Z \end{array} \text{ from} \end{array}$$

Element $(C^2)_{13}$ is '1', which indicates that Xavier can indirectly message Zach, even though there is no direct path connecting the two. This can be achieved by Xavier first sending a message to Yasmin, who can then pass the message to Zach. In other words, Xavier can send a message to Zach in two steps.

Element $(C^2)_{22}$ is '2', which indicates that there are two ways that Yasmin can communicate with herself. This can be achieved by her sending a message to either Xavier or Zach, who are both able to send the message back to her.

As a general rule for communication matrices, the rows and columns represent the movement 'from' and 'to', respectively, within a system.

Worked example 3

Lucas (L), Mayoi (M), Omar (O) and Seb (S) are pen pals and are trying to communicate with one another through letters.

- Lucas can send letters directly to Mayoi and Omar.
- Mayoi can send letters directly to Lucas, Omar and Seb.
- Omar can send letters directly to Lucas, Mayoi and Seb.
- Seb can send letters directly to Mayoi and Omar.

a. Construct a communication matrix, C, that represents the direct connections between the four pen pals.

Explanation

Step 1: Set up matrix C.

There are four people in this system. Since communication matrices must be a square matrix, matrix C will be a 4×4 matrix.

$$\begin{array}{cc} & \text{to} \\ & \begin{array}{cccc} L & M & O & S \end{array} \\ C = & \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \begin{array}{c} L \\ M \\ O \\ S \end{array} \text{ from} \end{array}$$

Step 2: Fill out the first row.

The first row corresponds to the direct, or one-step, communication paths from Lucas to everyone else.

Lucas can send letters directly to Mayoi and Omar.

$$\begin{array}{cc} & \text{to} \\ & \begin{array}{cccc} L & M & O & S \end{array} \\ C = & \begin{bmatrix} 0 & 1 & 1 & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \begin{array}{c} L \\ M \\ O \\ S \end{array} \text{ from} \end{array}$$

Continues →

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Step 3: Fill out the second row.

The second row corresponds to the direct, or one-step, communication paths from Mayoi to everyone else.

Mayoi can send letters directly to Lucas, Omar and Seb.

$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} \text{L} & \text{M} & \text{O} & \text{S} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \end{array} \begin{array}{c} \text{L} \\ \text{M} \\ \text{O} \\ \text{S} \end{array} \text{ from}$$

Step 4: Repeat this process to fill out the remaining rows.

Answer

$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} \text{L} & \text{M} & \text{O} & \text{S} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \text{L} \\ \text{M} \\ \text{O} \\ \text{S} \end{array} \text{ from}$$

b. Construct a communication matrix that represents the number of two-step paths between the four pen pals.

Explanation

Calculate the two-step communication matrix.

$$C^2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}^2$$

Answer

$$C^2 = \begin{array}{c} \begin{array}{cccc} \text{L} & \text{M} & \text{O} & \text{S} \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \text{L} \\ \text{M} \\ \text{O} \\ \text{S} \end{array}$$

c. In total, how many ways can Omar send letters to Mayoi? Only include direct letters and letters sent indirectly through one other person.

Explanation

Step 1: Refer to the correct elements in matrix C and matrix C^2.

The element that corresponds to the ways in which Omar can send letters to Mayoi can be found in the third row and second column.

$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} \text{L} & \text{M} & \text{O} & \text{S} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \text{L} \\ \text{M} \\ \text{O} \\ \text{S} \end{array} \text{ from}$$

$$C^2 = \begin{array}{c} \begin{array}{cccc} \text{L} & \text{M} & \text{O} & \text{S} \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \text{L} \\ \text{M} \\ \text{O} \\ \text{S} \end{array}$$

Step 2: Sum the elements.

The sum of the elements is the total number of ways in which Omar can send letters to Mayoi, whether directly or through one other person.

Answer

3

5D Questions

Using matrix products in financial applications

1. Matrix F shows the number of apples (A), bananas (B), oranges (O) and pears (P) sold at Coles (C) and Woolworths (W).

$$F = \begin{array}{c} \begin{array}{cccc} A & B & O & P \end{array} \\ \begin{bmatrix} 17 & 19 & 14 & 12 \\ 14 & 21 & 16 & 13 \end{bmatrix} \end{array} \begin{array}{c} \\ C \\ W \end{array}$$

In both stores, apples are sold for \$3.00, bananas are sold for \$2.50, oranges are sold for \$3.50 and pears are sold for \$4.00. Which of the following matrices can be post-multiplied to find the revenue generated for each store?

A. $\begin{bmatrix} 3.00 & 2.50 & 3.50 & 4.00 \end{bmatrix}$

B. $\begin{bmatrix} 3.00 \\ 2.50 \\ 3.50 \\ 4.00 \end{bmatrix}$

C. $\begin{bmatrix} 51.00 & 47.50 & 49.00 & 48.00 \\ 42.00 & 52.50 & 56.00 & 52.00 \end{bmatrix}$

D. $\begin{bmatrix} 3.00 & 0 & 0 & 0 \\ 0 & 2.50 & 0 & 0 \\ 0 & 0 & 3.50 & 0 \\ 0 & 0 & 0 & 4.00 \end{bmatrix}$

2. A cinema sells child (C), adult (A) and senior (S) tickets. Matrix S shows the number of each type of ticket sold on the weekend. Matrix P, shows the prices of each type of ticket.

$$S = \begin{array}{c} \begin{array}{ccc} C & A & S \end{array} \\ \begin{bmatrix} 248 & 132 & 177 \end{bmatrix} \end{array} \qquad P = \begin{bmatrix} 19.00 \\ 24.00 \\ 17.00 \end{bmatrix} \begin{array}{c} C \\ A \\ S \end{array}$$

a. Write an expression using matrix S and P that calculates the total revenue generated.

b. What was the total revenue generated?

3. Matrix P, shows the costs of hoodies (H), jeans (J) and t-shirts (T) that are sold at an op shop.

$$P = \begin{array}{c} \begin{array}{ccc} H & J & T \end{array} \\ \begin{bmatrix} 18.00 & 14.00 & 12.00 \end{bmatrix} \end{array}$$

Over the weekend, 11 hoodies, 19 jeans and 23 t-shirts were sold.

a. Construct matrix S that shows the number of each clothing item sold as a column matrix.

b. Using matrix multiplication, find the total revenue generated over the weekend.

4. A school canteen sells chips (C), hot dogs (H) and soft drinks (S), and the number of each item sold from Monday to Friday can be found in matrix R.

$$R = \begin{array}{c} \begin{array}{ccc} C & H & S \end{array} \\ \begin{bmatrix} 12 & 17 & 15 \\ 16 & 15 & 13 \\ 13 & 12 & 16 \\ 14 & 14 & 13 \\ 15 & 13 & 17 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Mon} \\ \text{Tue} \\ \text{Wed} \\ \text{Thu} \\ \text{Fri} \end{array}$$

Chips are sold for \$2.50, hot dogs are sold for \$1.50 and soft drinks are sold for \$2.00.

a. Construct P, a matrix which contains the prices of each item sold, as a column matrix.

b. Construct a matrix multiplication expression that could be used to calculate the total amount of money made each day by the canteen.

c. On which day did the canteen earn the most money?

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Using summing matrices to solve application problems

5. Which matrix expression would add the elements of each row?

A. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 6 & 2 \\ 4 & 8 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 8 & 5 & 2 & 3 \\ 4 & 3 & 6 & 7 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 7 \\ 2 & 6 \\ 1 & 5 \\ 6 & 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 8 \\ 9 & 4 \\ 8 & 2 \end{bmatrix}$

---

6. The elements in each column of matrix Q need to be added.

$$Q = \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 5 & 2 \\ 1 & 7 \end{bmatrix}$$

Construct a summing matrix, S, that can be multiplied with Q, to achieve this.

---

7. Matrix G represents the number of cherry (C), pea (P), and tomato (T) seeds planted each day.

$$G = \begin{array}{c} \begin{array}{ccc} C & P & T \end{array} \\ \begin{bmatrix} 2 & 5 & 3 \\ 6 & 4 & 7 \\ 9 & 2 & 8 \\ 8 & 1 & 9 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{day 1} \\ \text{day 2} \\ \text{day 3} \\ \text{day 4} \end{array}$$

a. Construct and evaluate a matrix expression that sums the total number of seeds planted each day.

b. Construct and evaluate a matrix expression that sums the total number of peas, sunflowers and cherries planted.

Using communication matrices to model systems

8. Four towns, A, B, C and D, are connected in various ways by roads.

- Town A has a two-way road connected to each of B and C.
- Town A also has a one-way road connected to Town D.
- Town B has a two-way road connected to each of A and C.
- Town C has a two-way road connected to each of A, B and D.
- Town D has a two-way road connected to Town C.

Matrix C, the one-step communication matrix that represents this information, is

A.
$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \\ C \\ D \end{array} \text{from}$$

B.
$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \\ C \\ D \end{array} \text{from}$$

C.
$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \\ C \\ D \end{array} \text{from}$$

D.
$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \\ C \\ D \end{array} \text{from}$$

9. A, B, C and D are four people who are able to send messages to each other. For the following one-step communication matrices, describe who can send messages and to whom.

a. receiver

$$\begin{array}{c} \\ \text{sender} \end{array}\ \begin{array}{cccc} & \text{A} & \text{B} & \text{C} \\ \text{A} & & & \\ \text{B} & & & \\ \text{C} & & & \end{array}$$

$$\begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 1 \\ B & 1 & 0 & 1 \\ C & 1 & 1 & 0 \end{array}\quad \text{sender}$$

b. receiver

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 1 & 1 \\ B & 1 & 0 & 1 & 1 \\ C & 1 & 1 & 0 & 0 \\ D & 1 & 1 & 0 & 0 \end{array}\quad \text{sender}$$

c. receiver

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 1 \\ B & 1 & 0 & 1 & 0 \\ C & 0 & 1 & 0 & 1 \\ D & 1 & 0 & 1 & 0 \end{array}\quad \text{sender}$$

---

10. C represents the one-step communication matrix showing the connections between W, X, Y and Z.

$$C = \begin{array}{cccc} & \text{to} & & \\ W & X & Y & Z \end{array}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{array}{c} W \\ X \\ Y \\ Z \end{array} \ \text{from}$$

a. Calculate C^2, the two-step communication matrix.

b. How many two-step paths connect X with Z?

c. How many two-step paths connect Y with Z?

Joining it all together

11. A messaging system used by Alex (A), Brynn (B), Cameron (C), Daniel (D) and Ella (E) has malfunctioned and now not everyone is able to send a direct message to each other.

- Alex can send messages to Brynn, Daniel and Ella.
- Brynn can send messages to Alex, Cameron and Daniel.
- Cameron can send messages to Brynn and Daniel.
- Daniel can send messages to Alex, Brynn, Cameron and Ella.
- Ella can send messages to Alex and Daniel.

a. Construct the one-step communication matrix, M, that represents this.

b. Calculate the two-step communication matrix using the answer obtained in part **a**.

c. In total, how many ways can Ella send messages to Daniel? Use the answers obtained in parts **a** and **b**.

---

12. There are three stores, A, B and C that all sell laptops (L), phones (P) and tablets (T). The number of each item sold over the week is contained in matrix S.

$$S = \begin{array}{c} \begin{array}{ccc} L & P & T \end{array} \\ \begin{bmatrix} 13 & 23 & 15 \\ 15 & 31 & 19 \\ 11 & 27 & 18 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \end{array}$$

At all three stores, laptops are sold for \$1119, phones are sold for \$699 and tablets are sold for \$849.

a. Construct S, a summing matrix that can be used to add the total number of each item from the combined sales of the three stores.

b. Construct M, a column matrix showing the selling price of each item.

c. Construct a matrix multiplication expression that can be used to calculate the amount of revenue generated at each store.

d. Which store generated the most revenue and how much was this amount?

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Exam practice

13. The internet in Brent's office has stopped working properly.

The five staff members, Ally (A), Brent (B), Chloe (C), Drevis (D) and Elvin (E), are having problems sending emails to each other.

Matrix H shows the available communication links between the staff workers.

$$H = \begin{array}{c} \text{receiver} \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ A \\ B \\ C \\ D \\ E \end{array} \text{ sender}$$

In this matrix:

- the '1' in row B, column C indicates that Brent can send emails to Chloe
- the '0' in row A, column E indicates that Ally cannot send emails to Elvin.

Which two staff members can send emails directly to each other? (1 MARK)

Adapted from VCAA 2021 Exam 2 Matrices Q2a

92% of students answered this type of question correctly.

---

14. Matrix J is a communication matrix showing the direct paths by which messages can be sent between two people in a group of six people, U to Z.

$$J = \begin{array}{c} \text{receiver} \\ \begin{array}{cccccc} U & V & W & X & Y & Z \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ U \\ V \\ W \\ X \\ Y \\ Z \end{array} \text{ sender}$$

A '1' in the matrix shows that the person named in that row can send a message directly to the person named in that column.

For example, the '1' in row 4, column 3 shows that X can send a message directly to W.

In how many ways can W get a message to U by sending it directly to one other person?

A. 0 **B.** 1 **C.** 2

D. 3 **E.** 4

Adapted from VCAA 2019 Exam 1 Matrices Q7

54% of students answered this type of question correctly.

---

15. A travel company arranges flight (F), hotel (H), performance (P) and tour (T) bookings.

Matrix C contains the number of each type of booking for a month.

$$C = \begin{bmatrix} 85 \\ 38 \\ 24 \\ 43 \end{bmatrix} \begin{array}{c} F \\ H \\ P \\ T \end{array}$$

A booking fee, per person, is collected by the travel company for each type of booking. Matrix G contains the booking fees, in dollars, per booking.

$$G = \begin{array}{c} \begin{array}{cccc} F & H & P & T \end{array} \\ \begin{bmatrix} 40 & 25 & 15 & 30 \end{bmatrix} \end{array}$$

If $J = G \times C$, what does matrix J represent? (1 MARK)

VCAA 2016 Exam 2 Matrices Q1bii

42% of students answered this question correctly.

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16. The following table shows the number of each type of coin saved in a moneybox.

coin	5 cent	10 cent	20 cent	50 cent
number	17	24	53	32

The matrix product that displays the total number of coins and the total value of these coins is

A. $\begin{bmatrix} 5 & 10 & 20 & 50 \end{bmatrix}\begin{bmatrix} 17 \\ 24 \\ 53 \\ 32 \end{bmatrix}$

B. $\begin{bmatrix} 5 & 10 & 20 & 50 \end{bmatrix}\begin{bmatrix} 1 & 17 \\ 1 & 24 \\ 1 & 53 \\ 1 & 32 \end{bmatrix}$

C. $\begin{bmatrix} 17 & 24 & 53 & 32 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 1 & 10 \\ 1 & 20 \\ 1 & 50 \end{bmatrix}$

D. $\begin{bmatrix} 17 & 24 & 53 & 32 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 20 \\ 50 \end{bmatrix}$

E. $\begin{bmatrix} 5 & 10 & 20 & 50 \\ 17 & 24 & 53 & 32 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

Adapted from VCAA 2016 Exam 1 Matrices Q4

34% of students answered this type of question correctly.

Questions from multiple lessons

Matrices

17. Element x_{ij} is the element in row i and column j of matrix X.

The elements in matrix X are determined by the rule $x_{ij} = i + 3j$.

Which of the following matrices **cannot** be matrix X?

A. $\begin{bmatrix} 4 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 7 & 10 & 13 \end{bmatrix}$

C. $\begin{bmatrix} 4 & 7 & 10 \\ 5 & 8 & 11 \\ 6 & 9 & 12 \end{bmatrix}$

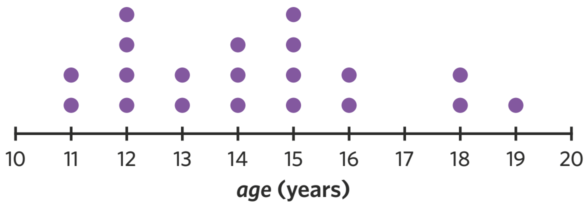
D. $\begin{bmatrix} 4 & 7 \\ 5 & 8 \end{bmatrix}$

E. $\begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix}$

Adapted from VCAA 2017NH Exam 1 Matrices Q4

Data analysis

18. The following dot plot displays the *age*, in years, of 20 members of a Shawn Mendes fan club.



The percentage of members that are 15 years of age is

A. 4% **B.** 5% **C.** 20% **D.** 24% **E.** 25%

Adapted from VCAA 2018 Exam 1 Data analysis Q1

Matrices

19. The cost, in dollars, for some Apple products at a tech shop are shown in matrix M.

$$M = \begin{array}{c} \begin{bmatrix} 1000 \\ 450 \\ 1250 \\ 800 \end{bmatrix} \end{array} \begin{array}{l} \text{iPhone} \\ \text{iPad} \\ \text{MacBook} \\ \text{AppleWatch} \end{array}$$

a. What is the cost of a MacBook? (1 MARK)

b. What is the order of matrix M? (1 MARK)

c. The products have recently been discounted. Let N be the matrix that contains the discounted prices of the apple products. Evaluate N, given the expression $N = 0.75 \times M$. (1 MARK)

Adapted from VCAA 2018 Exam 2 Matrices Q1

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5E Inverse matrices

STUDY DESIGN DOT POINT

- inverse matrices and their applications including solving a system of simultaneous linear equations

5A 5B 5C 5D 5E 5F 5G

KEY SKILLS

During this lesson, you will be:

- calculating the determinant of a matrix
- calculating the inverse of a matrix.

KEY TERMS

- Determinant
- Inverse matrix
- Singular matrix

When two numbers are multiplied together, division can be used to reverse the operation. However, this isn't possible for matrices, so inverse matrices are used instead. Inverse matrices are commonly used to encrypt or decrypt messages, and to solve for unknown matrices.

Calculating the determinant of a matrix

See worked example 1

The **determinant** of a matrix is a value that indicates many properties about the matrix. It can only be calculated for square matrices.

The determinant of matrix A is denoted $\det(A)$.

Consider a 2×2 matrix, such that

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The formula used to calculate the determinant of a 2×2 matrix is

$$\det(A) = ad - bc$$

Note: A calculator will be used to calculate the determinant of 3×3 (or larger) matrices.

See worked example 2

Worked example 1

Calculate the determinant of $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$.

Explanation

Calculate the determinant by substituting each element into the formula.

$$\begin{aligned} \det\left(\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}\right) &= a \times d - b \times c \\ &= 2 \times 3 - 1 \times 4 \\ &= 6 - 4 \end{aligned}$$

Answer

2

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Worked example 2

Using a calculator, calculate the determinant of M.

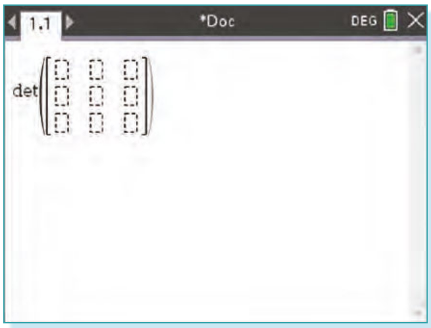
$$M = \begin{bmatrix} 4 & 2 & 13 \\ 6 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Explanation – Method 1: TI-Nspire

Step 1: From the home screen, select ‘1: New’ → ‘1: Add Calculator’.

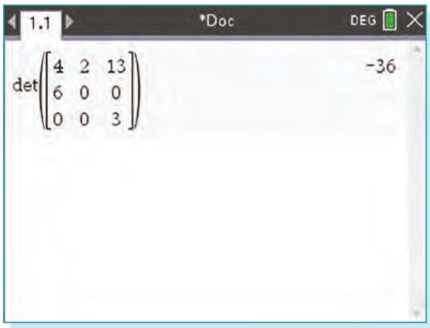
Step 2: Press menu and select ‘7: Matrix & Vector’ → ‘3: Determinant’.

Step 3: Press and select . Insert 3 rows and 3 columns and press OK.



Step 4: Input the values for matrix M.

Press enter.

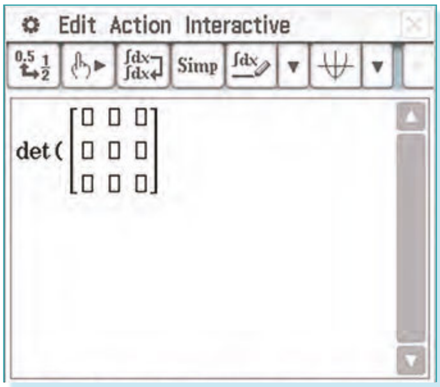


Explanation – Method 2: Casio ClassPad

Step 1: From the main menu, tap √α Main.

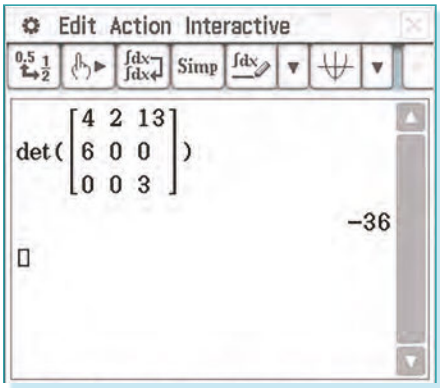
Step 2: Tap ‘Action’ → ‘Matrix’ → ‘Calculation’ → ‘det’.

Step 3: Press keyboard, tap ‘Math2’, and tap twice.



Step 4: Input the values from the given matrix into the appropriate spaces and close the brackets.

Press EXE.



Answer – Method 1 and 2

-36

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Calculating the inverse of a matrix

The **inverse matrix** of A (denoted A^{-1}) is a square matrix such that when it is pre-multiplied or post-multiplied with A, it results in the identity matrix, I. This can be represented by the equation $A \times A^{-1} = A^{-1} \times A = I$. Inverse matrices can only be calculated for a square matrix with a determinant that is not equal to zero. A matrix with no inverse is called a **singular matrix**.

For a 2×2 matrix of the form $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the inverse matrix can be calculated using the formula

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

The matrix A is scalar multiplied by the reciprocal of the determinant, $\det(A)$.

The elements a and d are swapped.

The elements b and c are multiplied by -1.

Note: A calculator will be used to calculate the inverse of 3×3 (or larger) matrices.

Worked example 3

Consider the matrix $X = \begin{bmatrix} 3 & -7 \\ 10 & 1 \end{bmatrix}$.

a. Is X^{-1} defined?

Explanation

Step 1: Calculate the determinant.

$$\begin{aligned} \det(X) &= 3 \times 1 - (-7) \times 10 \\ &= 73 \end{aligned}$$

Step 2: Determine if X can have an inverse.

Since $\det(X) \neq 0$, the inverse of X is defined.

Answer

Yes

b. Calculate X^{-1}, if it is defined.

Explanation – Method 1: By hand

Step 1: Calculate the inverse matrix.

$$\begin{aligned} X^{-1} &= \frac{1}{\det(X)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \\ &= \frac{1}{73} \begin{bmatrix} 1 & 7 \\ -10 & 3 \end{bmatrix} \end{aligned}$$

Step 2: Complete the scalar multiplication.

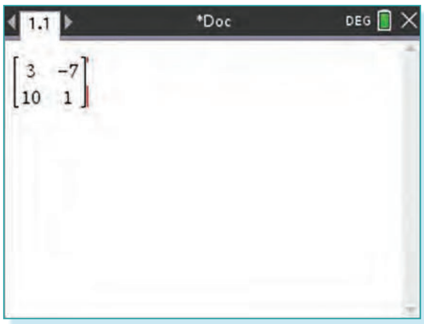
Multiply each matrix element by $\frac{1}{73}$.

Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select ‘1: New’ → ‘1: Add Calculator’.

Step 2: Press and select .

Step 3: Input the values for matrix X.



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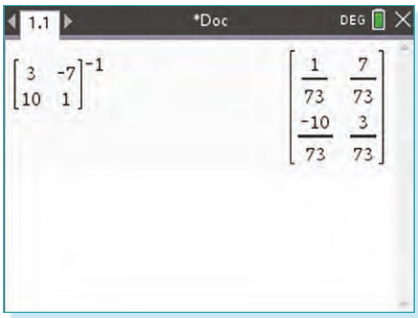
Step 4: Calculate the inverse by raising the matrix to the power of -1.

Press enter.



Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap √α Main.

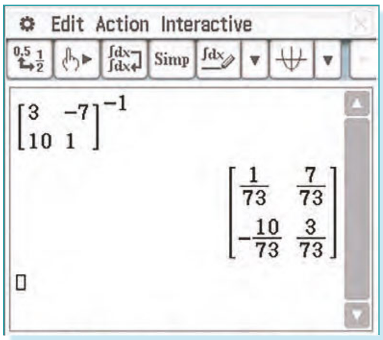
Step 2: Press keyboard, and tap 'Math2'.

Tap [▪▫].

Step 3: Input the values for matrix X.

Step 4: Calculate the inverse by raising the matrix to the power of -1.

Press EXE.



Answer – Method 1, 2 and 3

$$X^{-1} = \begin{bmatrix} \frac{1}{73} & \frac{7}{73} \\ -\frac{10}{73} & \frac{3}{73} \end{bmatrix}$$

5E Questions

Calculating the determinant of a matrix

1. Which of the following is equivalent to the determinant of $\begin{bmatrix} 27 & 39 \\ 33 & 44 \end{bmatrix}$?

A. $27 \times 39 - 33 \times 44$ **B.** $27 \times 44 - 33 \times 39$ **C.** $44 \times 27 + 39 \times 33$ **D.** $33 \times 39 - 44 \times 27$

2. Calculate the determinant of $\begin{bmatrix} 3 & 6 \\ -1 & 2 \end{bmatrix}$.

3. Calculate the determinant of $\begin{bmatrix} 12 & 9 & 4 \\ 8 & 14 & 2 \\ 3 & 1 & 0 \end{bmatrix}$.

4. Calculate the determinant of $\begin{bmatrix} \frac{3}{2} & \frac{4}{3} \\ \frac{3}{4} & \frac{2}{3} \end{bmatrix}$.

Calculating the inverse of a matrix

5. Which of the following matrices has an inverse?

A. $\begin{bmatrix} 2 & 3 \\ 1 & -5 \end{bmatrix}$ **B.** $\begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$ **C.** $\begin{bmatrix} 2 & 1 \\ 8 & 4 \end{bmatrix}$ **D.** $\begin{bmatrix} 0 & 0 \\ 3 & 5 \end{bmatrix}$

6. X is a matrix with order 2×2, and X^{-1} is the inverse of this matrix.

a. What is the product of X and X^{-1}?

b. What is the product of X^{-1} and X?

c. What type of matrix is attained in parts **a** and **b**?

7. Consider the following matrix.

$$E = \begin{bmatrix} 7 & 6 & 2 \\ 8 & 4 & 3 \\ 5 & 1 & 9 \end{bmatrix}$$

Calculate E^{-1}.

8. Explain why the inverse of $\begin{bmatrix} 2 & 8 \\ 4 & 12 \\ 9 & 3 \end{bmatrix}$ does not exist.

9. Calculate the inverse of $\begin{bmatrix} 3 & 2 \\ 5 & 6 \end{bmatrix}$, rounding all elements to two decimal places.

10. Calculate the inverse of $\begin{bmatrix} -\frac{1}{2} & 3 \\ 2 & -\frac{3}{4} \end{bmatrix}$.

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Joining it all together

11. Matrix A is such that its inverse is defined as $A^{-1} = \begin{bmatrix} 2 & -1 \\ -\frac{3}{2} & 1 \end{bmatrix}$. Matrix A is equivalent to which of the following?

A. $\begin{bmatrix} 1 & 1 \\ \frac{3}{2} & 2 \end{bmatrix}$ **B.** $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ **C.** $\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix}$ **D.** $\begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$

---

12. Consider the following matrix.

$R = \begin{bmatrix} 4 & 5 \\ 6 & 4 \end{bmatrix}$

a. Calculate the determinant of R.

b. Does the inverse R^{-1} exist? Why or why not?

c. If it exists, calculate R^{-1}.

---

13. Consider the following matrix.

$X = \begin{bmatrix} \frac{4}{3} & \frac{3}{2} \\ \frac{5}{3} & \frac{3}{2} \end{bmatrix}$

a. Show that matrix X has an inverse.

b. Calculate X^{-1}.

Exam practice

14. Which one of the following matrices has a determinant of zero?

A. $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ **B.** $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$ **C.** $\begin{bmatrix} 5 & 2 \\ -10 & 4 \end{bmatrix}$

D. $\begin{bmatrix} 22 & 0 \\ 11 & 0 \end{bmatrix}$ **E.** $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

Adapted from VCAA 2018 Exam 1 Matrices Q1

78% of students answered this type of question correctly.

---

15. The total number of boys (b) and girls (g) in two music classes can be determined by solving the following equations written in matrix form.

$\begin{bmatrix} 2 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} b \\ g \end{bmatrix} = \begin{bmatrix} 30 \\ 46 \end{bmatrix}$

a. The value of the determinant of the 2×2 matrix is -4.
Use this information to explain why this matrix has an inverse. (1 MARK)

b. Write the three missing values of the inverse of the 2×2 matrix. (1 MARK)

$\begin{bmatrix} & \\ 1 & \end{bmatrix}$

Adapted from VCAA 2020 Exam 2 Matrices Q2a,b

Part **a**: **37%** of students answered this type of question correctly.
Part **b**: **71%** of students answered this type of question correctly.

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Questions from multiple lessons

Matrices

16. The matrix product $[12 \quad 3 \quad 9] \times \begin{bmatrix} 3 \\ 15 \\ 6 \end{bmatrix}$ is equal to

A. $3 \times [4 \quad 3 \quad 9] \times \begin{bmatrix} 1 \\ 15 \\ 6 \end{bmatrix}$

B. $\begin{bmatrix} 36 \\ 45 \\ 54 \end{bmatrix}$

C. $[576]$

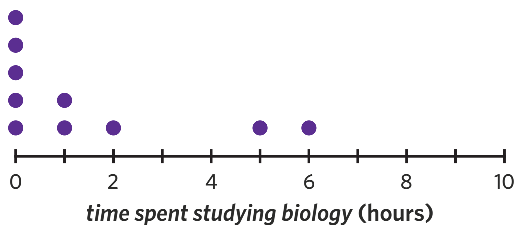
D. $9 \times [4 \quad 1 \quad 3] \times \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$

E. $3 \times [4 \quad 1 \quad 3] \times \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$

Adapted from VCAA 2018 Exam 1 Matrices Q2

Data analysis

17. Sofia recorded how long she spent studying biology each of the last 10 nights. The results are displayed in the following dot plot.



Correct to one decimal place, the mean and standard deviation of the time Sofia spent studying are

A. $\bar{x} = 1.5, \quad s_x = 2.0$

B. $\bar{x} = 1.5, \quad s_x = 2.2$

C. $\bar{x} = 2.2, \quad s_x = 1.5$

D. $\bar{x} = 2.2, \quad s_x = 3.0$

E. $\bar{x} = 3.0, \quad s_x = 2.2$

Adapted from VCAA 2008 Exam 1 Data analysis Q5

Matrices

18. One realm has four kingdoms: the North (N), the Vale (V), the Iron Islands (I) and Dorne (D).

The number of knights in each kingdom is shown in matrix K.

$$K = \begin{array}{c} \begin{bmatrix} 52 \\ 29 \\ 20 \\ 38 \end{bmatrix} \end{array} \begin{array}{l} N \\ V \\ I \\ D \end{array}$$

a. What is the order of matrix K? (1 MARK)

b. How many knights are there in either the Vale or the Iron Islands? (1 MARK)

The following table shows the cost of armour for one knight per kingdom.

kingdom	*cost per knight* ($)
North	15
Vale	29
Iron Islands	11
Dorne	23

c. Write down a matrix that could be multiplied by K to give a total cost to armour all knights. (1 MARK)

Adapted from VCAA 2018NH Exam 2 Matrices Q1

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5F Solving matrix equations

STUDY DESIGN DOT POINTS

- matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using technology as applicable
- inverse matrices and their applications including solving a system of simultaneous linear equations

5A 5B 5C 5D 5E 5F 5G

KEY SKILLS

During this lesson, you will be:

- solving matrix equations
- using matrices to solve sets of simultaneous equations.

KEY TERMS

- Matrix equation
- Coefficient matrix
- Variable matrix
- Answer matrix

Like with numbers, equations can be formed using matrices.. By using matrix operations and inverse matrices, matrix equations can be solved in a similar fashion to regular equations. One application of matrix equations is using them to solve linear simultaneous equations.

Solving matrix equations

See worked example 1

A **matrix equation** is an equation involving matrices. For example, $A + X = B$ is a matrix equation involving three separate matrices. Simple equations may involve adding or subtracting matrices, or multiplying them by a scalar. Often the equation will have to be rearranged using inverse operations in order to solve for the unknown matrix.

See worked example 2

More advanced matrix equations may involve the multiplication of matrices. In order to solve these equations, multiplication by inverse matrices is required.

It is important to consider the order of multiplication by inverse matrices, as this is not generally interchangeable. For example, in the equation

$AX = B$

X is pre-multiplied by A. Therefore, to solve for X, A and B must be pre-multiplied by the inverse of A.

$A^{-1}AX = A^{-1}B$

$X = A^{-1}B$

Worked example 1

If $A = \begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 2 \\ 6 & 10 \end{bmatrix}$, solve the following matrix equations for X.

a. $A + X = B$

Explanation

Step 1: Rearrange the matrix equation to isolate the unknown matrix, X.

$A + X = B$

$X = B - A$

Step 2: Substitute matrices A and B into the equation.

$$X = \begin{bmatrix} 5 & 2 \\ 6 & 10 \end{bmatrix} - \begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix}$$

Continues →

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Step 3: Solve for matrix X.

$$X = \begin{bmatrix} 5-5 & 2-(-3) \\ 6-4 & 10-2 \end{bmatrix}$$

Answer

$$X = \begin{bmatrix} 0 & 5 \\ 2 & 8 \end{bmatrix}$$

b. $3X = A$

Explanation

Step 1: Rearrange the matrix equation to isolate the unknown matrix, X.

To isolate X, a scalar multiplication of $\frac{1}{3}$ needs to be performed to both sides.

$$3X = A$$
$$X = \frac{1}{3}A$$

Step 2: Substitute matrix A into the equation.

$$X = \frac{1}{3}\begin{bmatrix} 5 & -3 \\ 4 & 2 \end{bmatrix}$$

Step 3: Solve for matrix X.

$$X = \begin{bmatrix} \frac{5}{3} & -\frac{3}{3} \\ \frac{4}{3} & \frac{2}{3} \end{bmatrix}$$

Answer

$$X = \begin{bmatrix} \frac{5}{3} & -1 \\ \frac{4}{3} & \frac{2}{3} \end{bmatrix}$$

Worked example 2

Consider the matrix equation $AX = B$.

If $A = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 18 \\ 16 \end{bmatrix}$, solve for matrix X.

Explanation

Step 1: Rearrange the equation to isolate matrix X.

For the equation $AX = B$, pre-multiply both sides by A^{-1}.

$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Step 2: Substitute matrices A^{-1} and B into the equation.

$$X = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 18 \\ 16 \end{bmatrix}$$

Step 3: Solve for matrix X.

Answer

$$X = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Using matrices to solve sets of simultaneous equations

See worked example 3

The pair of simultaneous equations $ax + by = e$ and $cx + dy = f$ can be converted to a matrix equation as shown.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$$

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This is because $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$ can be expanded to $\begin{bmatrix} ax + by \\ cx + dy \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$, which is equivalent to the original equations of $ax + by = e$ and $cx + dy = f$.

Notice that the matrix equation $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$ is of the form $AX = B$. For these types of equations:

- Matrix A, $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, is the **coefficient matrix**, which represents the coefficients of x and y.
- Matrix X, $\begin{bmatrix} x \\ y \end{bmatrix}$, is the **variable matrix**, which represents the unknown variables, x and y.
- Matrix B, $\begin{bmatrix} e \\ f \end{bmatrix}$, is the **answer matrix**, showing what the matrix equation equates to.

Once simultaneous equations have been converted to matrix form, matrix operations can then be used to solve for the matrix containing x and y.

See worked example 4

Worked example 3

Express the following simultaneous equations as matrix equations.

a. $5x + 2y = 9$ and $9x + 3y = 15$

Explanation

Step 1: Set up the matrix equation.

The matrix equation used will be of the form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}.$$

Step 2: Input the values from the first equation into the top row.

$$\begin{bmatrix} 5 & 2 \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ f \end{bmatrix}$$

Step 3: Input the values from the second equation into the bottom row.

Answer

$$\begin{bmatrix} 5 & 2 \\ 9 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \end{bmatrix}$$

b. $y = 5x - 3$ and $2y = -\frac{1}{4}x + 10$

Explanation

Step 1: Rearrange the matrix equations into the form $ax + by = e$ and $cx + dy = f$.

Equation 1: $y = 5x - 3$

$-5x + y = -3$

Equation 2: $2y = -\frac{1}{4}x + 10$

$\frac{1}{4}x + 2y = 10$

Step 2: Set up the matrix equation.

The matrix equation used will be of the form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}.$$

Step 3: Input the values from the first equation into the top row.

$$\begin{bmatrix} -5 & 1 \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ f \end{bmatrix}$$

Step 4: Input the values from the second equation into the bottom row.

Answer

$$\begin{bmatrix} -5 & 1 \\ \frac{1}{4} & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}$$

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Worked example 4

If $\begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -5 \end{bmatrix}$, what are the values of x and y?

Explanation

Step 1: Determine how the matrix equation should be rearranged.

Let:

$A = \begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} 13 \\ -5 \end{bmatrix}$.

The current matrix equation, $AX = B$, needs to be rearranged to be in the form of $X = A^{-1}B$, to solve for x and y.

Step 2: Pre-multiply A and B by the inverse of A.

$$\begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 13 \\ -5 \end{bmatrix}$$

Step 3: Simplify the left side.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 13 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ -1 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 13 \\ -5 \end{bmatrix}$$

Step 4: Solve for $\begin{bmatrix} x \\ y \end{bmatrix}$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{79}{35} \\ -\frac{32}{35} \end{bmatrix}$$

Answer

$x = \frac{79}{35}$ and $y = -\frac{32}{35}$

5F Questions

Solving matrix equations

1. Consider a matrix equation of the form $2A - X = B$. The equation, rearranged to isolate X, is

A. $X = 2A - B$ **B.** $X = 2A + B$ **C.** $X = -2A + B$ **D.** $X = -2A - B$

2. Consider a matrix equation of the form $AX = 2B$. The equation, rearranged to isolate X, is

A. $X = 2B - A$ **B.** $X = \frac{2B}{A}$ **C.** $X = A^{-1}2B$ **D.** $X = 2BA^{-1}$

3. If $A = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$, solve the following matrix equations for X.

a. $A + X = B$ **b.** $A - X = B$ **c.** $2X = B$ **d.** $-2A + X = 2B$ **e.** $3A - 2X = B$

4. If $A = \begin{bmatrix} 10 & -6 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, solve the following matrix equations for X.

a. $AX = B$ **b.** $AX = 4B$ **c.** $AX = -2B$ **d.** $A^2X = B$

5. Joseph collects rare Australian coins, and keeps track of the amount of coins in his collection using matrices. He sorts his coins into gold and silver, and sorts them by which century they were minted (20^{th} or 21^{st}). This information is represented by matrix J, as shown.

$$J = \begin{array}{c} \text{gold} \quad \text{silver} \\ \begin{bmatrix} 40 & 31 \\ 35 & 19 \end{bmatrix} \end{array} \begin{array}{l} \\ 20^{th} \\ 21^{st} \end{array}$$

Martha collects coins in the same way, as shown by matrix M.

$$M = \begin{array}{c} \text{gold} \quad \text{silver} \\ \begin{bmatrix} 28 & 21 \\ 41 & 24 \end{bmatrix} \end{array} \begin{array}{l} \\ 20^{th} \\ 21^{st} \end{array}$$

a. If $X = J + M$, determine matrix X. What does element x_{21} represent?

b. Joseph and Martha have collected many of the same coins, leading to a large overlap in their collections. Matrix U represents all of the unique coins from Joseph and Martha's collections combined.

$$U = \begin{array}{c} \text{gold} \quad \text{silver} \\ \begin{bmatrix} 47 & 39 \\ 50 & 28 \end{bmatrix} \end{array} \begin{array}{l} \\ 20^{th} \\ 21^{st} \end{array}$$

If $U = J + L$, determine matrix L. What does this matrix represent?

Using matrices to solve sets of simultaneous equations

6. Find the pair of simultaneous equations that corresponds to the following matrix equation.

$$\begin{bmatrix} 10 & 4 \\ 3 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 2 \end{bmatrix}$$

A. $10x + 4y = 2$ and $3x + 9y = 16$

B. $10x + 3y = 2$ and $4x + 9y = 16$

C. $10x + 3y = 16$ and $4x + 9y = 2$

D. $10x + 4y = 16$ and $3x + 9y = 2$

---

7. If the pair of simultaneous equations $5x + 10y = 12$ and $-5x + 8y = 15$ corresponds to the matrix equation $\begin{bmatrix} 5 & 10 \\ a & 8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b \\ 15 \end{bmatrix}$, what are the values of a and b?

---

8. Express the following pairs of simultaneous equations as matrix equations.

a. $2x + 3y = 1$ and $3x + 3y = 0$

b. $12x + 6y = 18$ and $4x - 5y = -13$

c. $y = -2x + 1$ and $y = -x - 3$

---

9. Using matrices, solve the following sets of simultaneous equations for x and y.

a. $5x + 4y = 19$ and $10x - 7y = 23$

b. $x + y = 4$ and $x - y = -12$

c. $x - y = 20$ and $4x + 8y = -4$

d. $x + 2y = 20$ and $y = 1 - 3x$

e. $2x + y = 5$ and $x - \frac{3}{2}y = \frac{1}{2}$

f. $y = -4x$ and $y = 3x + 7$

10. Using matrices, solve the following set of simultaneous equations for x, y and z.

$3x + 2y - z = 5$

$4x + 2y + z = 8$

$-x - 2y + 3z = 0$

---

11. Alice and Henry visit a bakery, where they both individually purchase some food.

- Alice buys 2 bagels and 3 muffins for \$10.60.
- Henry buys 3 bagels and 1 muffin for \$8.20.

a. From this information, construct a set of simultaneous equations where x represents the price of a single bagel, and y represents the price of a single muffin.

b. Express the simultaneous equations from part **a** as a matrix equation.

c. Determine the values of x and y.

Joining it all together

12. Henry and George decide to play a game of basketball with a custom scoring system. For one game played, Henry and George's respective performances can be modelled by the pair of simultaneous equations, $11x + 5y = 115$ and $8x + 6y = 112$, where

- x represents the value of a regular score
- y represents the value of scoring from outside the yellow line

a. Express the simultaneous equations in matrix form.

b. Use the matrix equation to determine the point value of scoring regularly, and scoring from behind the yellow line.

---

13. A phone company sells black and silver phones, through stores A, B and C. The number of sales for each type of phone at each store on Monday and Tuesday are displayed in matrix M and T respectively.

$$M = \begin{array}{c c} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 203 & 150 & 382 \\ 392 & 142 & 892 \end{bmatrix} & \begin{array}{l} \text{black} \\ \text{silver} \end{array} \end{array} \qquad T = \begin{array}{c c} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 134 & 120 & 153 \\ 205 & 98 & 390 \end{bmatrix} & \begin{array}{l} \text{black} \\ \text{silver} \end{array} \end{array}$$

It is later found out that matrix T is incorrect due to an error, and only reported half of the actual sales for Tuesday.

a. Write a matrix equation that can be used to find S, the matrix representing the actual sales across Monday and Tuesday.

b. Calculate matrix S, and use it to determine the most popular phone colour at each individual store.

c. It is known that store A made \$135 116 from the combined black and silver phone sales across Monday and Tuesday, and store B made \$79 924.

Using this information and matrix S, construct a set of simultaneous equations, where x represents the price of a black phone, and y represents the price of a silver phone. Assume that a phone sells for the same price regardless of what store it is sold in.

d. Using matrix equations, determine the price of a black and silver phone respectively.

---

14. A merchandise stand outside of a theatre sells hats, shirts and jumpers for a theatre company performing multiple shows.

- On Monday, they sell 13 hats, 20 shirts and 20 jumpers and make \$2090.
- On Wednesday, they sell 20 hats, 22 shirts and 16 jumpers and make \$2200.
- On Saturday, they sell 30 hats, 48 shirts and 26 jumpers and make \$3990.

Using matrix equations, determine how much it would cost to buy 2 shirts from this merchandise stand.

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Exam practice

15. Consider the matrix equation

$$\begin{bmatrix} 35 & 24 & 60 \\ 28 & 32 & 43 \\ 32 & 30 & 56 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 491.55 \\ 428.00 \\ 487.60 \end{bmatrix}$$

where $a =$ cost of one pie, $b =$ cost of one roll and $c =$ cost of one sandwich.

What is the cost of one sandwich? (1 MARK)

VCAA 2017 Exam 2 Matrices Q1ci

56% of students answered this question correctly.

16. The preferred number of cafes (x) and sandwich bars (y) in Grandmall's food court can be determined by solving the following equations written in matrix form.

$$\begin{bmatrix} 5 & -9 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

Determine the preferred number of sandwich bars for Grandmall's food court. (1 MARK)

VCAA 2020 Exam 2 Matrices Q2c

51% of students answered this question correctly.

Questions from multiple lessons

Matrices

17. Sandra loves running and is training for her first marathon in one month's time. The distances of her three favourite tracks, Albert Park Lake, The Tan, and Princes Park, are shown in the following table.

Albert Park Lake	4.70 km
The Tan	3.83 km
Princes Park	3.21 km

In her training, Sandra will run the Albert Park Lake 7 times, The Tan 19 times, and Princes Park 13 times.

Which of the following matrix products will result in a matrix that contains the total *distance* Sandra will run in her training, in kilometres?

A. $\begin{bmatrix} 4.70 \\ 3.83 \\ 3.21 \end{bmatrix} \begin{bmatrix} 7 & 19 & 13 \end{bmatrix}$

B. $\begin{bmatrix} 4.70 \\ 3.83 \\ 3.21 \end{bmatrix} \begin{bmatrix} 7 \\ 19 \\ 13 \end{bmatrix}$

C. $\begin{bmatrix} 4.70 & 3.83 & 3.21 \end{bmatrix} \begin{bmatrix} 7 \\ 19 \\ 13 \end{bmatrix}$

D. $\begin{bmatrix} 4.70 & 3.83 & 3.21 \end{bmatrix} \begin{bmatrix} 7 & 19 & 13 \end{bmatrix}$

E. $\begin{bmatrix} 4.70 & 3.83 & 3.21 \end{bmatrix} \begin{bmatrix} 7 & 0 & 0 \\ 0 & 19 & 0 \\ 0 & 0 & 13 \end{bmatrix}$

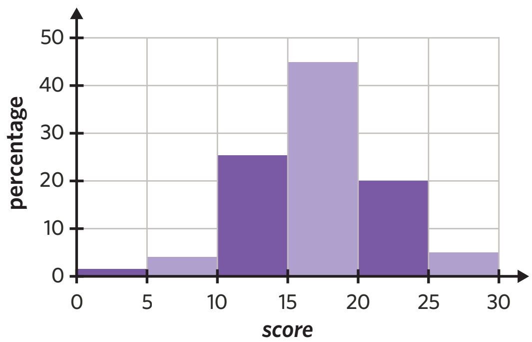
Adapted from VCAA 2017NH Exam 1 Matrices Q2

Data analysis *Year 10 content*

18. The histogram provided displays the distribution of *scores*, out of 30, on a spelling test for 80 primary school students.

The number of students with a *score* between 10 and 15 is closest to

A. 15
B. 16
C. 20
D. 25
E. 36



Adapted from VCAA 2019NH Exam 1 Data analysis Q2

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Matrices

19. At the cinema, there are three sizes of popcorn: small, medium and large.

The number of each size sold last month and their individual prices are shown in the following table.

size	*number sold*	*cost*
small	120	$4.50
medium	380	$8.00
large	710	$11.50

a. The column matrix Q displays the cost of each popcorn size.

$$Q = \begin{array}{c} \begin{bmatrix} 4.50 \\ 8.00 \\ 11.50 \end{bmatrix} \end{array} \begin{array}{l} \text{small} \\ \text{medium} \\ \text{large} \end{array}$$

What is the order of matrix Q? (1 MARK)

b. Matrix R contains the number of each size sold in the cinema.

$$R = \begin{array}{ccc} \text{small} & \text{medium} & \text{large} \\ \end{array}$$
$$R = \begin{bmatrix} 120 & 380 & 710 \end{bmatrix}$$

i. Determine the matrix product RQ. (1 MARK)

ii. Explain what the matrix product RQ represents. (1 MARK)

Adapted from VCAA 2017NH Exam 2 Matrices Q1

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5G Transition matrices

STUDY DESIGN DOT POINT

- introduction to transition matrices (assuming the next state only relies on the current state), working with iterations of simple models linked to, for example, population growth or decay, including informal consideration of long run trends and steady state

5A 5B 5C 5D 5E 5F **5G**

KEY SKILLS

During this lesson, you will be:

- identifying transition matrix properties
- constructing transition and initial state matrices
- modelling applied problems using transition matrices.

KEY TERMS

- Transition matrix
- State matrix
- Initial state matrix
- Steady state matrix
- Equilibrium matrix

Transition matrices can be used in many real-life situations to find the projection of data over time using matrix recursion. It takes into account a system and manipulates it to find the change or the lack of change at a given point in time. Along with initial and state matrices, matrix equations can be applied to model the movement from one state to another.

Identifying transition matrix properties

The unidirectional movement from one state to another is represented by a **transition matrix**. It follows the understanding that the next state is dependent on the previous state and is represented by T. It gives information about how much of the data from the system remains the same, or changes. This helps determine future and past states based on the values of a given state.

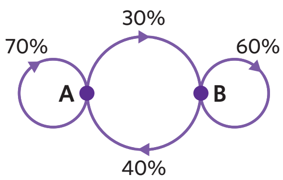
All transition matrices are square matrices, and each column sums to 1. Transition matrices are labelled to indicate that the columns represent the current state and the rows represent the next state.

For example, every week a group of trucks visit either warehouse A or B.

The movement of the group of trucks from one warehouse to the other every week is represented as a transition matrix, T.

transition from

$$T = \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \end{array} \text{ transition to}$$



This matrix shows that:

- 70% of the trucks that visit warehouse A will visit warehouse A next week.
- 30% of the trucks that visit warehouse A will visit warehouse B next week.
- 40% of the trucks that visit warehouse B will visit warehouse A next week.
- 60% of the trucks that visit warehouse B will visit warehouse B next week.

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Worked example 1

A group of car buyers purchase either a Hyundai (H) or a Toyota (T). The transition of this group purchasing either brand from one car to the next is represented in a transition matrix.

$$T = \begin{array}{c} \text{this car} \\ \begin{array}{cc} \text{H} & \text{T} \end{array} \\ \begin{bmatrix} 0.3 & 0.1 \\ 0.7 & x \end{bmatrix} \end{array} \begin{array}{l} \text{H} \\ \text{T} \end{array} \text{ next car}$$

a. Find the value of x, and explain what it represents.

Explanation

Step 1: Identify the necessary column.

The unknown value, x, is located in column T.
All columns need to sum to 1.

$$T = \begin{array}{c} \text{this car} \\ \begin{array}{cc} \text{H} & \text{T} \end{array} \\ \begin{bmatrix} 0.3 & 0.1 \\ 0.7 & x \end{bmatrix} \end{array} \begin{array}{l} \text{H} \\ \text{T} \end{array} \text{ next car}$$

Step 2: Calculate x.

$0.1 + x = 1$

$x = 1 - 0.1$

$x = 0.9$

Answer

$x = 0.9$ which shows that 90% of car buyers that purchased a Toyota will purchase a Toyota as their next car.

b. What proportion of car buyers who purchased Hyundai will purchase a Toyota as their next car? Express your answer as a percentage.

Explanation

Step 1: Read the 'Hyundai' column for this car.

$$T = \begin{array}{c} \text{this car} \\ \begin{array}{cc} \text{H} & \text{T} \end{array} \\ \begin{bmatrix} 0.3 & 0.1 \\ 0.7 & x \end{bmatrix} \end{array} \begin{array}{l} \text{H} \\ \text{T} \end{array} \text{ next car}$$

Step 2: Read the 'Toyota' row for next car.

$$T = \begin{array}{c} \text{this car} \\ \begin{array}{cc} \text{H} & \text{T} \end{array} \\ \begin{bmatrix} 0.3 & 0.1 \\ 0.7 & x \end{bmatrix} \end{array} \begin{array}{l} \text{H} \\ \text{T} \end{array} \text{ next car}$$

Step 3: Convert the decimal to a percentage.

$0.7 \times 100 = 70\%$

Answer

70%

c. If there are 30 car buyers who bought a Hyundai this year, how many will purchase a Hyundai again as their next car?

Explanation

Step 1: Determine the proportion of car buyers purchasing a Hyundai as both this car and the next.

$t_{11} = 0.3$

Step 2: Multiply this proportion by the number of car buyers who purchased a Hyundai as their current car.

$0.3 \times 30 = 9$

Answer

9 car buyers

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Constructing transition and initial state matrices

The state of a system at a given time that is separated by regular time intervals can be represented by a **state matrix**. This is a column matrix represented as S_n, where n refers to the state number.

An **initial state matrix** is a column matrix that presents the first state of the system and is denoted as S_0.

For example, the following initial state matrix, S_0, outlines the number of phones (P) and tablets (T) sold in a store.

$$S_0 = \begin{array}{c c} \begin{bmatrix} 240 \\ 127 \end{bmatrix} & \begin{array}{c} \text{P} \\ \text{T} \end{array} \end{array}$$

It can be seen that

- Row 1 (P) represents the number of phones that were sold, 240.
- Row 2 (T) represents the number of tablets that were sold, 127.

Worked example 2

The town of Bendigo has two main restaurants, Coconut Lime and The Moon.

Initially, there were 300 customers at Coconut Lime, and 200 customers at The Moon.

Reviews on the internet have shown that 80% of customers who eat at Coconut Lime one day will eat there the next day, and 60% of customers who eat at The Moon on one day will eat there the following day. The remaining customers swap restaurants from one day to the next.

a. Construct the initial state matrix, S_0.

Explanation

Step 1: Set up a blank column matrix and label it with the names of the restaurants.

Since there are 2 restaurants (Coconut Lime and The Moon), a 2 × 1 matrix is required.

Row 1 will be labelled Coconut Lime (C).

Row 2 will be labelled The Moon (M).

$$S_0 = \begin{array}{c c} \begin{bmatrix} \quad \\ \quad \end{bmatrix} & \begin{array}{c} \text{C} \\ \text{M} \end{array} \end{array}$$

Step 2: Fill in the elements.

Coconut Lime had 300 customers.

The Moon had 200 customers.

Answer

$$S_0 = \begin{array}{c c} \begin{bmatrix} 300 \\ 200 \end{bmatrix} & \begin{array}{c} \text{C} \\ \text{M} \end{array} \end{array}$$

b. Construct a transition matrix that describes the change in restaurant visits.

Explanation

Step 1: Set up a blank transition matrix and label it with the names of the restaurants.

Since there are 2 restaurants, a 2 × 2 matrix is required.

The columns represent the restaurant people eat from on one day, and the rows represent the restaurant people eat from on the following day.

$$\begin{array}{c c c} & \text{today} & \\ & \begin{array}{c c} \text{C} & \text{M} \end{array} & \\ T = & \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} & \begin{array}{c} \text{C} \\ \text{M} \end{array} \ \text{tomorrow} \end{array}$$

Step 2: Fill in the matrix elements.

Since 80% of customers who eat from Coconut Lime one day will eat there the next day, $t_{11} = 0.8$.

Since 60% of customers who eat from The Moon will eat there the next day, $t_{22} = 0.6$.

$$\begin{array}{c c c} & \text{today} & \\ & \begin{array}{c c} \text{C} & \text{M} \end{array} & \\ T = & \begin{bmatrix} 0.8 & \\ & 0.6 \end{bmatrix} & \begin{array}{c} \text{C} \\ \text{M} \end{array} \ \text{tomorrow} \end{array}$$

Continues →

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Step 3: Calculate the missing elements.

$$T = \begin{array}{c} \text{today} \\ \begin{array}{cc} \text{C} & \text{M} \end{array} \\ \begin{bmatrix} 0.8 & y \\ x & 0.6 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \text{C} \\ \text{M} \end{array} \text{ tomorrow}$$

$0.8 + x = 1$

$x = 1 - 0.8$

$x = 0.2$

$y + 0.6 = 1$

$y = 1 - 0.6$

$y = 0.4$

Answer

$$T = \begin{array}{c} \text{today} \\ \begin{array}{cc} \text{C} & \text{M} \end{array} \\ \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \text{C} \\ \text{M} \end{array} \text{ tomorrow}$$

Modelling applied problems using transition matrices

The next state matrix, denoted S_{n+1}, can be calculated recursively by pre-multiplying the current state matrix, S_n, by the transition matrix, T.

This can be modelled using a recurrence relation of the form

$S_0 = \textit{initial state matrix}, \quad S_{n+1} = T \times S_n$, where

- n refers to the state number
- S_n is the current state matrix
- S_{n+1} is the next state matrix
- T is the transition matrix

For example, there are two kinds of bird seeds bought by customers at Chantelle's pet shop: Valley Grains (V) and Hissan Grains (H).

The following transition matrix describes how customers transition between the two products.

$$T = \begin{array}{c} \text{this week} \\ \begin{array}{cc} \text{V} & \text{H} \end{array} \\ \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \text{V} \\ \text{H} \end{array} \text{ next week}$$

The following initial state matrix describes how many customers bought Valley Grains and Hissan Grains initially.

$$S_0 = \begin{bmatrix} 400 \\ 200 \end{bmatrix} \begin{array}{c} \text{V} \\ \text{H} \end{array}$$

The number of customers who bought each type of grain after 1 week, S_1, can be calculated by multiplying the transition matrix by the initial state matrix.

$S_1 = T \times S_0$

$$S_1 = \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \times \begin{bmatrix} 400 \\ 200 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 300 \\ 300 \end{bmatrix}$$

This shows that after 1 week 300 customers bought Valley Grains, and 300 customers bought Hissan Grains from Chantelle's bird shop.

After a certain number of iterations, a system will achieve a steady state. The **steady state matrix**, also known as the **equilibrium matrix**, represents a point in time in which the transition matrix no longer affects the overall state of the system. A steady state matrix is the same as the directly preceding and following state matrix.

This is represented as $S_n = S_{n+1} = S_{n-1}$.

State matrices generally reach a steady state, or equilibrium, after a large number of iterations. The number of iterations required to reach a steady state depends on the data in the system.

Worked example 3

Every night, each seagull settles on Island A or Island B. On Sunday, there were 52 seagulls on Island A and 98 seagulls on Island B.

The initial state and transition matrices have been provided.

$$S_0 = \begin{array}{c} \\ \begin{bmatrix} 52 \\ 98 \end{bmatrix} \end{array}\begin{array}{l} \\ \text{A} \\ \text{B} \end{array} \qquad T = \begin{array}{c} \text{tonight} \\ \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \end{array}\begin{array}{l} \\ \\ \text{A} \\ \text{B} \end{array} \text{ next night}$$

a. Construct a recurrence relation to model this scenario.

Explanation

Construct the recurrence relation.

$S_0 = \textit{initial state matrix}, \quad S_{n+1} = T \times S_n$

Answer

$S_0 = \begin{bmatrix} 52 \\ 98 \end{bmatrix}, \quad S_{n+1} = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \times S_n$

b. Find the number of seagulls on Island A and B on Tuesday, rounded to the nearest whole number.

Explanation

Step 1: Calculate S_1, the state matrix for Monday.

$S_1 = T \times S_0$

$S_1 = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \times \begin{bmatrix} 52 \\ 98 \end{bmatrix}$

$S_1 = \begin{bmatrix} 56 \\ 94 \end{bmatrix}$

Step 2: Calculate S_2, the state matrix for Tuesday.

$S_2 = T \times S_1$

$S_2 = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \times \begin{bmatrix} 56 \\ 94 \end{bmatrix}$

$S_2 = \begin{bmatrix} 58 \\ 92 \end{bmatrix}$

Answer

On Tuesday, there will be 58 seagulls on Island A, and 92 on Island B.

c. The steady state matrix for the seagulls is represented by the following matrix.

$$\begin{bmatrix} 60 \\ 90 \end{bmatrix}\begin{array}{l} \text{A} \\ \text{B} \end{array}$$

If this represents the number of seagulls at each island on Sunday, 13 weeks after the initial Sunday, determine the number of seagulls at each island on the following Monday.

Explanation

The steady state matrix represents a state that is not affected by the transition matrix. This means that each following period in time will be exactly the same.

Answer

Island A: 60 seagulls

Island B: 90 seagulls

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5G Questions

Identifying transition matrix properties

1. Identify the proportion of customers purchasing a KitKat this month that will purchase a KitKat again in the next month.

$$T = \begin{array}{c} \text{this month} \\ \begin{array}{cc} \text{Mars} & \text{KitKat} \end{array} \\ \begin{bmatrix} 0.5 & 0.7 \\ 0.5 & 0.3 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \text{Mars} \\ \text{KitKat} \end{array} \ \text{next month}$$

A. 0.1% **B.** 0.3% **C.** 30% **D.** 50%

---

2. A group of students participating in an exchange program travel monthly between two countries: A and B. A transition matrix can be used to determine which country they will travel to next month based on their choice this month, with all students starting in January and 50 students starting in country A, 50 in country B.

$$\begin{array}{c} \text{this month} \\ \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 0.1 & 0.8 \\ 0.9 & 0.2 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \text{A} \\ \text{B} \end{array} \ \text{next month}$$

a. Of the students who travelled to country A in January, what percentage remained there in February?

b. Of the students who travel to country B in one month, what percentage travel to country A in the following month?

c. How many students travelled from country A in January to country B in February?

d. How many students travelled first to country B and remained there the next month?

Constructing transition and initial state matrices

3. During the COVID lockdowns, Mitchell started a hairdressing side hustle out of his house. He is open one day per week, and customers can either get their hair cut (C) or dyed (D).

In his first year, he had 270 customers. 200 of these had their hair cut, and the remaining 70 had their hair dyed.

Which of the following initial state matrices best describes this?

A. $\begin{bmatrix} 200 \\ 70 \end{bmatrix} \begin{array}{l} \text{cut} \\ \text{dyed} \end{array}$ **B.** $\begin{bmatrix} 70 \\ 200 \end{bmatrix} \begin{array}{l} \text{cut} \\ \text{dyed} \end{array}$ **C.** $\begin{bmatrix} 270 \\ 200 \end{bmatrix} \begin{array}{l} \text{cut} \\ \text{dyed} \end{array}$ **D.** $\begin{bmatrix} 270 \\ 270 \end{bmatrix} \begin{array}{l} \text{cut} \\ \text{dyed} \end{array}$

---

4. Employees in a Melbourne office have the option of working at home (H) or in the office (O).

- 70% of the employees who work from home one week, work from home the following week.
- 30% of the employees who work from home one week, work at the office the following week.
- 20% of the employees who work at the office one week, work from home the following week.
- 80% of the employees who work at the office one week, work at the office the following week.

Construct a transition matrix to represent this information.

---

5. Around Australia, the weather is either sunny (S) or rainy (R).

On the first day of the year there were 250 towns that had sunny weather, and 120 towns that had rainy weather.

The Bureau of Meteorology shows that 70% of the Australian towns that have sunny weather on one day, have sunny weather the following day, and 40% of the Australian towns that have rainy weather on one day have sunny weather on the following day.

a. Construct the initial state matrix.

b. Construct the transition matrix.

Modelling applied problems using transition matrices

6. In a kindergarten, there are two types of crayons: black and blue. Of the children who use these crayons daily, 30% of those who use the black today will use the black tomorrow, and 50% of the children who use the blue crayon today, will use the black crayon tomorrow. On Monday, 25 children used the black crayon and 15 used the blue crayon.

Which calculation will show the state matrix for Tuesday?

A. $\begin{bmatrix} 0.3 & 0.5 \\ 0.7 & 0.5 \end{bmatrix} \times \begin{bmatrix} 15 \\ 25 \end{bmatrix}$ **B.** $\begin{bmatrix} 0.3 & 0.5 \\ 0.7 & 0.5 \end{bmatrix} \times \begin{bmatrix} 25 \\ 15 \end{bmatrix}$ **C.** $\begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.5 \end{bmatrix} \times \begin{bmatrix} 15 \\ 25 \end{bmatrix}$ **D.** $\begin{bmatrix} 0.3 & 0.7 \\ 0.5 & 0.5 \end{bmatrix} \times \begin{bmatrix} 25 \\ 15 \end{bmatrix}$

7. Which of the following pairs of matrices shows a steady state?

A. $M_{41} = \begin{bmatrix} 12 \\ 4 \\ 84 \end{bmatrix}, \ M_{42} = \begin{bmatrix} 84 \\ 4 \\ 12 \end{bmatrix}$

B. $M_{41} = \begin{bmatrix} 6.05 \\ 8 \\ 9.95 \end{bmatrix}, \ M_{42} = \begin{bmatrix} 6.10 \\ 8.05 \\ 9.85 \end{bmatrix}$

C. $M_{41} = \begin{bmatrix} 900 \\ 900 \\ 900 \end{bmatrix}, \ M_{42} = \begin{bmatrix} 800 \\ 800 \\ 800 \end{bmatrix}$

D. $M_{41} = \begin{bmatrix} 72 \\ 103 \\ 45 \end{bmatrix}, \ M_{42} = \begin{bmatrix} 72 \\ 103 \\ 45 \end{bmatrix}$

8. The population of fish at four different underwater locations changes each year. The system reaches equilibrium after 65 years, with the populations at each location shown in the following matrix.

$$P_{65} = \begin{array}{c} \begin{bmatrix} 128 \\ 529 \\ 311 \\ 485 \end{bmatrix} \end{array} \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array}$$

Construct a matrix to show the population at each location after 83 years.

9. A cinema has a promotion deal where customers can sign up to watch one movie per month, chosen by the cinema, at a discounted price.

The cinema uses three screening rooms: (A), (B) and (C) for the customers involved in the monthly promotion deal.

The following transition matrix describes the movement of customers between the screening rooms from one month to the next.

$$T = \begin{array}{c} \text{this month} \\ \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{bmatrix} 0.30 & 0.50 & 0.20 \\ 0.60 & 0.25 & 0.10 \\ 0.10 & 0.25 & 0.70 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \text{A} \\ \text{B} \\ \text{C} \end{array} \ \text{next month}$$

The cinema opened in March, and in this month 180 customers watched a movie in screening room A, 80 watched a movie in screening room B and 160 watched a movie in screening room C. The initial state matrix is given.

$$S_0 = \begin{bmatrix} 180 \\ 80 \\ 160 \end{bmatrix} \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \end{array}$$

a. Calculate S_1.

b. Use recursion to show the number of customers in screening rooms A, B and C for May, June and July. Round the elements to two decimal places where necessary.

c. Using the answer from part **b**, how many customers will watch a movie in screening room A two months after opening, rounded to the nearest whole number?

d. Using the answer from part **b**, in which month will there be approximately 130 customers watching a movie in screening room B?

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Joining it all together

10. Zebras either live in savannah A or B, but migrate between them every week. From the zebras that participate in this migration, 10% of those who live in savannah A in one week, live there the following week. Additionally, 20% of those who live in savannah B in one week, live there the following week.

a. What percentage of zebras in savannah A will move to savannah B from one week to the next?

b. Construct the transition matrix, T.

In the first week of the year, there were 130 zebras in savannah A and 200 in savannah B.

c. Construct the initial state matrix, S_0.

d. Construct the matrix recurrence relation.

e. Use recursion to show the number of zebras living in savannah A and B for the second and third weeks of the year, rounded to the nearest whole number.

f. How many zebras will be living in savannah A in the sixth week, rounded to the nearest whole number?

Exam practice

11. A theme park has four locations, Air World (A), Food World (F), Ground World (G) and Water World (W).

The number of visitors at each of the four locations is counted every hour.

a. Let S_n be the state matrix that shows the number of visitors expected at each location n hours after 10 am on Saturday.

The number of visitors expected at each location n hours after 10 am on Saturday can be determined by the following matrix recurrence relation.

$$S_0 = \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix}, \quad S_{n+1} = T \times S_n \quad \text{where} \quad T = \begin{array}{c} \text{this hour} \\ \begin{array}{cccc} A & F & G & W \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.1 & 0.2 \\ 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ A \\ F \\ G \\ W \end{array} \text{ next hour}$$

Complete the state matrix, S_1, to show the number of visitors expected at each location at 11 am on Saturday. (1 MARK)

$$S_1 = \begin{bmatrix} \\ \\ 300 \\ \\ \end{bmatrix} \begin{array}{c} A \\ F \\ G \\ W \end{array}$$

b. On Sunday, matrix V is used when calculating the expected number of visitors at each location every hour after 10 am. It is assumed that the park will be at its capacity of 2000 for all of Sunday. Let L_0 be the state matrix that shows the number of visitors at each location at 10 am on Sunday. The number of visitors expected at each location at 11am on Sunday can be determined by the matrix product:

$$V \times L_0 \quad \text{where} \quad L_0 = \begin{bmatrix} 500 \\ 600 \\ 500 \\ 400 \end{bmatrix} \begin{array}{c} A \\ F \\ G \\ W \end{array} \quad \text{and} \quad V = \begin{array}{c} \text{this hour} \\ \begin{array}{cccc} A & F & G & W \end{array} \\ \begin{bmatrix} 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ A \\ F \\ G \\ W \end{array} \text{ next hour}$$

Safety restrictions require that all four locations have a maximum of 600 visitors.

Which location is expected to have more than 600 visitors at 11 am on Sunday? (1 MARK)

VCAA 2019 Exam 2 Matrices Q2b,3a

Part **a**: **84%** of students answered this question correctly.
Part **b**: **75%** of students answered this question correctly.

12. A market research study of shoppers showed that the buying preferences for the three olive oils, Carmani (C), Linelli (L) and Ohana (O), change from month to month according to the following transition matrix T.

$$T = \begin{array}{c} \text{this month} \\ \begin{array}{ccc} C & L & O \end{array} \\ \begin{bmatrix} 0.85 & 0.10 & 0.05 \\ 0.05 & 0.80 & 0.05 \\ 0.10 & 0.10 & 0.90 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ C \\ L \quad \text{next month} \\ O \end{array}$$

The initial state matrix S_0 shows the number of shoppers who bought each brand of olive oil in July 2021.

$$S_0 = \begin{bmatrix} 3200 \\ 2000 \\ 2800 \end{bmatrix} \begin{array}{l} C \\ L \\ O \end{array}$$

Let S_n represent the state matrix describing the number of shoppers buying each brand n months after July 2021.

How many of these 8000 shoppers bought a different brand of olive oil in August 2021 from the brand bought in July 2021? (1 MARK)

VCAA 2021 Exam 2 Matrices Q3a

28% of students answered this question correctly.

Questions from multiple lessons

Matrices

13. The number of music festivals (F), concerts (C) and gigs (G) that Heidi (H), Ingrid (I), Joey (J), Kathy (K) and Leo (L) attended last summer is given in matrix M.

$$M = \begin{array}{c} \begin{array}{ccccc} H & I & J & K & L \end{array} \\ \begin{bmatrix} 3 & 2 & 0 & 0 & 6 \\ 1 & 5 & 4 & 7 & 0 \\ 8 & 3 & 2 & 1 & 8 \end{bmatrix} \end{array} \begin{array}{l} \\ F \\ C \\ G \end{array}$$

The element in row i and column j in matrix M is m_{ij}.

The element m_{23} is the number of

A. gigs Joey attended last summer.

B. gigs Ingrid attended last summer.

C. concerts Kathy attended last summer.

D. concerts Joey attended last summer.

E. concerts Ingrid attended last summer.

Adapted from VCAA 2015 Exam 1 Matrices Q1

Recursion and financial modelling

14. Find the first five terms of the following recurrence relation.

$B_0 = -1, \quad B_{n+1} = -3B_n + 4$

A. $-1, 3, 7, 11, 15$

B. $-1, 7, -17, 55, -161$

C. $-1, 1, 7, 25, 79$

D. $7, -17, 55, -161, 487$

E. $-1, 3, -9, 27, -81$

Adapted from VCAA 2016 Exam 1 Recursion and financial modelling Q17

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Matrices

15. The four most popular plants sold at a plant shop in Brunswick are the succulent (S), cactus (C), fern (F), and monstera (M).

Matrix P_{2019} shows the sale price of each of the plants, in dollars.

$$P_{2019} = \begin{array}{c} \begin{bmatrix} 15 \\ 20 \\ 30 \\ 40 \end{bmatrix} \begin{array}{l} S \\ C \\ F \\ M \end{array} \end{array}$$

a. What is the order of matrix P_{2019}? (1 MARK)

b. Nick buys a cactus and a monstera plant. How much does Nick have to pay? (1 MARK)

c. The following table shows the number of each plant sold in the past week.

plant	number sold
succulent	22
cactus	24
fern	17
monstera	31

Write down a matrix that, when multiplied by matrix P_{2019}, would give the total revenue from these plant sales for the past week. (1 MARK)

d. The shop decides to implement a 20% price increase at the start of 2020 due to increased demand. Fill in the following box with a scalar so the product is the matrix of 2020 plant prices. (1 MARK)

$P_{2020} = \boxed{} \times P_{2019}$

Adapted from VCAA 2018NH Exam 2 Matrices Q1

CHAPTER 6 CALCULATOR QUICK LOOK-UP GUIDE

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6

UNIT 2 AOS 1
CHAPTER 6

Investigating relationships between two numerical variables

LESSONS

KEY KNOWLEDGE

- response and explanatory variables
- scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction, form and strength
- informal interpretation of association and causation
- use of a line of good fit by eye to make predictions, including the issues of interpolation and extrapolation
- interpretation of a line of good fit, its intercept and slope in the context of the data.

Image: ChristianChan/Shutterstock.com

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6A Introduction to scatterplots

STUDY DESIGN DOT POINTS

- response and explanatory variables
- scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction, form and strength

6A 6B 6C 6D

KEY SKILLS

During this lesson, you will be:

- identifying the response and explanatory variables
- constructing scatterplots.

KEY TERMS

- Response variable
- Explanatory variable
- Scatterplot

When analysing data sets, it is important to understand how to effectively investigate the relationship between two numerical variables to answer statistical questions. In such circumstances, it is useful to be able to identify response and explanatory variables, as well as present the data using scatterplots. This enables the data to be further analysed in order to make more definitive statements about the relationship between the variables.

Identifying the response and explanatory variables

Investigations are performed on bivariate data to examine whether there is an association between two variables. In these situations, the data is used to determine whether the change in one variable can be explained by the change in another variable. These are called the response and explanatory variables.

The **response variable**, *RV*, may be explained or predicted by changes in the explanatory variable. It can also be called the dependent variable.

The **explanatory variable**, *EV*, is used to explain or predict the changes observed in the response variable. It can also be called the independent variable.

For example, consider data collected on the *age* of a group of people from 5-20 years old and their *height*.

The *height* of the group is the response variable, as increases in height may be predicted from increases in *age*.

Worked example 1

Data is collected to investigate the relationship between the *combined income* per household and the *number of adults* per household.

Determine the response variable and explanatory variable.

Explanation

Step 1: Determine which variable is predicted from the other.

It is most reasonable to expect that the combined income of a household will change according to the number of adults in the house.

Step 2: Classify each variable as either response or explanatory.

The *RV* is predicted from changes in the EV.

Answer

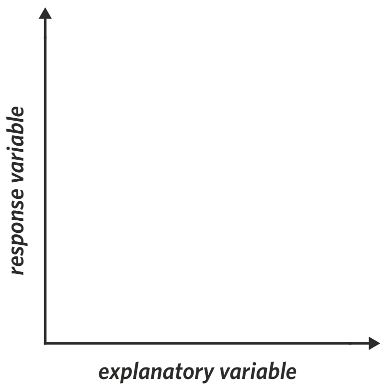
RV: *combined income*

EV: *number of adults*

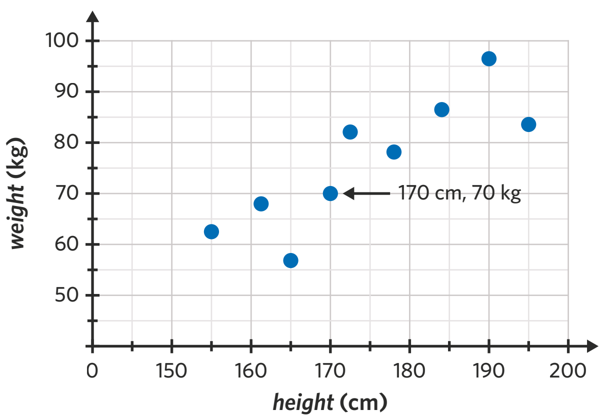
Constructing scatterplots

A **scatterplot** is a display used to represent data relating to two numerical variables. Each point represents an individual data entry with the axes providing the numerical measurements.

When comparing the relationship between a response variable and its explanatory variable, the response variable is positioned on the vertical axis, and the explanatory variable on the horizontal axis.



For example, the point indicated on the following scatterplot shows an individual who has a *height* of 170 cm and a *weight* of 70 kg.



When provided with a table of data, calculators can be used to construct scatterplots.

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Worked example 2

A study was conducted to determine whether the amount of time a family spends on holiday could be predicted from their household income. The following data shows the *income* ($000's) and *days spent on holiday*, per year, of 10 families.

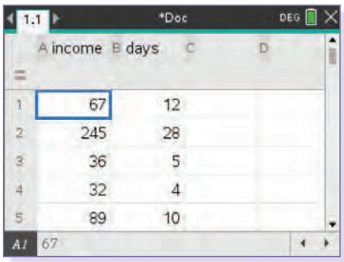
***income* ($000's)**	67	245	36	32	89	122	96	45	75	103
days spent on holiday	12	28	5	4	10	23	18	13	16	20

With a calculator, use the data to construct a scatterplot.

Explanation – Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name a list 'income' and another list 'days' and enter the data from the table.



Step 3: Identify the response and explanatory variables.

The days spent on holiday is being predicted from the income.

RV: *days spent on holiday*

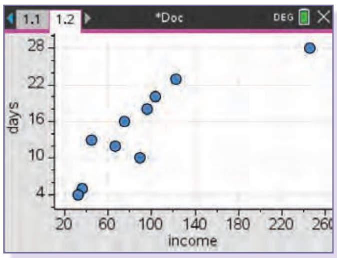
EV: *income*

Step 4: Press ctrl + doc, and select 'Add Data & Statistics'.

Step 5: Add the variables on each axis using the 'Click to add variable' function.

The *RV* will be positioned on the vertical axis and the *EV* will be positioned on the horizontal axis.

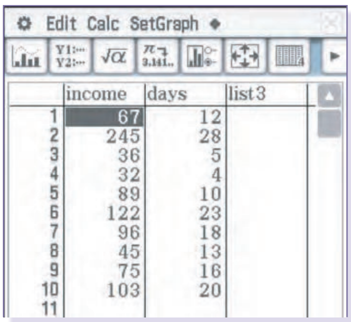
Answer



Explanation – Method 2: Casio ClassPad

Step 1: From the main menu, tap Statistics

Step 2: Name a list 'income' and another list 'days' and enter the data from the table.



Step 3: Identify the response and explanatory variables.

The *days spent on holiday* is being predicted from the *income*.

RV: *days spent on holiday*

EV: *income*

Step 4: Configure the settings of the graph by tapping in the icon bar.

Create a scatterplot by keeping 'Type' as 'Scatter'.

Specify the data set by changing 'XList:' to 'main\income' and 'YList:' to 'main\days'.
Tap 'Set' to confirm.

Step 5: Tap in the icon bar to plot the graph.

To analyse the graph, tap in the toolbar. Use the left and right arrow keys to navigate along the data points.

Continues →

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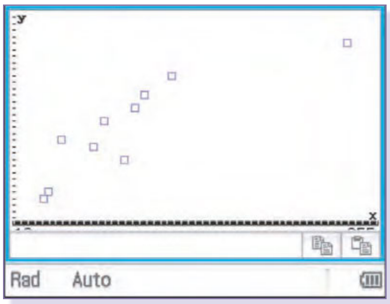
Answer



6A Questions

Identifying the response and explanatory variables

1. A research study investigates how *sleep* (hours) impacts the *number of disposals*, *distance run* (km) and *disposal efficiency* (%) of a group of football midfielders. Which of these variables is the explanatory variable?
   - **A.** *sleep*
   - **B.** *number of disposals*
   - **C.** *distance run*
   - **D.** *disposal efficiency*

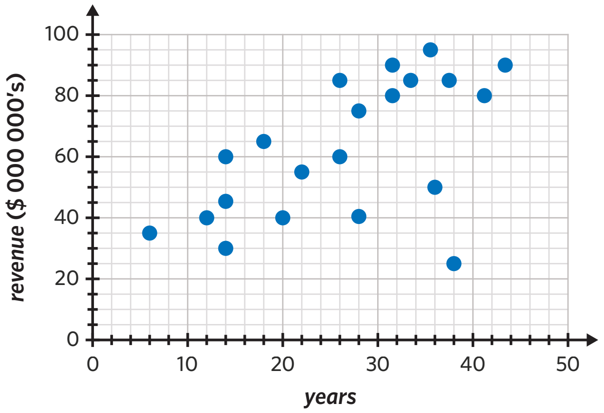
2. Identify the response variable, *RV*, and the explanatory variable, *EV*, in each of the following studies.
   - **a.** Data collected on *jackets sold* and *temperature* to determine if the weather can predict the jacket sales for a store.
   - **b.** Collecting data on the *number of bedrooms* and *selling price* of houses to determine whether the number of bedrooms can predict how much a house sells for.
   - **c.** Data collected on *cost of golf clubs* and *handicap* to determine whether a golfer's handicap can predict how much they spend on clubs.
   - **d.** Collecting data on *age* and *screen time* to determine whether someone's age can predict how long they spend in front of screens.

3. Identify what the response variable, *RV*, and the explanatory variable, *EV*, would most likely be in each of the following pairs of variables.
   - **a.** *distance* from school and *time taken* to get to school
   - **b.** *money earned* and *hours worked*
   - **c.** *years of education* and *age*
   - **d.** *maximum deadlift* and *weight*

Constructing scatterplots

4. Which of the following pairs of variables would not be used to construct a scatterplot?
   - **A.** *time spent exercising* (minutes) and *amount of sleep* (hours)
   - **B.** *number of beach visitors* and *season* (Summer, Autumn, Winter, Spring)
   - **C.** *distance* (km) and *time taken* (minutes)
   - **D.** *high score* and *number of attempts*

5. From the following graph, identify the response variable and explanatory variable.

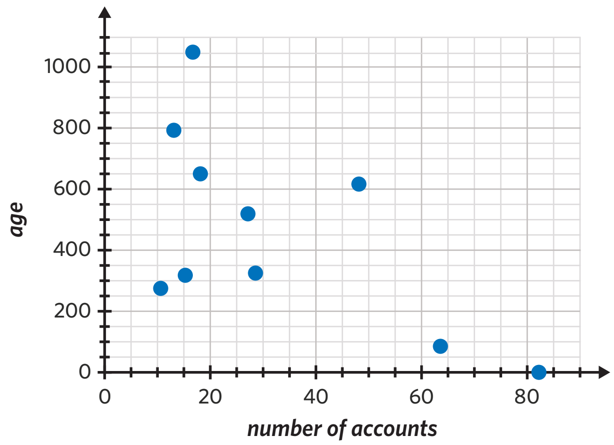


6. Kane collects the following data on the *age* of his family members and the *number of accounts* they each follow on instagram.

age	13	64	27	28	48	82	10	16	18	15
number of accounts	798	89	522	326	615	0	278	1056	650	318

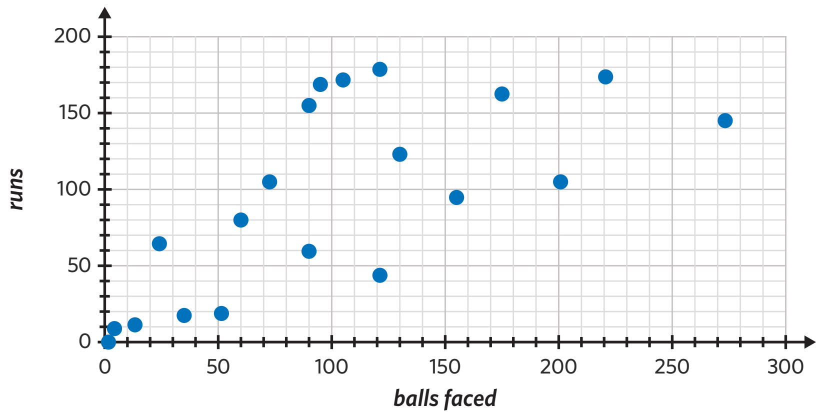
Kane uses the *number of accounts* as the *RV* and the *age* as the *EV*.

He uses the data to construct the following scatterplot.



Identify the mistake Kane has made in constructing the scatterplot, and explain how the mistake could be fixed.

7. The following scatterplot shows the *runs* made in one innings by 20 different cricketers and the number of *balls faced* by each player.



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a. Identify the response and explanatory variables.

b. How many runs did the player who faced 60 balls make?

c. One player made 123 runs. How many balls did they face?

d. Annabel made 155 runs and faced 95 balls. Is her score included on the scatterplot?

8. Construct a scatterplot for each of the following tables of data. In each case, the variable in the first row is used to predict the value of the variable in the second row.

a.

age	12	15	9	38	18	31	71	13	4	39	65	10
age of oldest sibling	13	14	15	32	31	32	69	13	6	40	60	23

b.

critic rating	3.4	9.8	7.6	5.7	8.0	4.1	4.8	3.1	6.8
movie attendees	28	164	48	83	119	32	51	17	102

9. Students at a local secondary school must wear ties regardless of the temperature. The following table shows the number of *students* who disobeyed this policy over the last two weeks of the school year, alongside the *temperature* (°C) on each of the days.

students	12	17	24	118	17	10	8	15	33	28
temperature (°C)	23	24	26	33	25	21	20	23	27	26

The data is used to predict the number of *students* disobeying the policy from the *temperature*. Construct a scatterplot using this data.

Joining it all together

10. Henderson is conducting a research study on *age* and its impact on *salary* ($000's), *years of education*, and *weekly time on social media* (hours). If he wanted to construct a scatterplot for any of his collections of data, the variable on the horizontal axis would be

A. *age*

B. *salary*

C. *years of education*

D. *weekly time on social media*

11. Students were surveyed on how long they spent working on their history projects. Ms Riosa compared this data with the number of words contained in each project. The data for eight students is displayed in the following table.

time (hours)	15	12	2	7	10	13	22	3
words	1230	1106	709	1058	976	1362	2489	542

a. Identify the response and explanatory variables.

b. Construct a scatterplot using this data.

12. Josh is frustrated that the Wi-Fi in his room is very weak. He decides to record the internet *speed* in the centre of every room of his house, and also the *distance* from the room to the router. The data he collected is displayed in the following table.

room	living room	balcony	kitchen	bedroom	office	bathroom
speed (mbps)	14	9	11	6	23	17
distance (m)	10	18	13	22	3	7

Construct a scatterplot using this data.

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13. The owners of a pub receive constant noise complaints from neighbours. They decide to record the number of *patrons* at their pub and the resulting *noise level*, in decibels, every night, in order to understand how to restrict the noise. They collected data over an entire month, and the following table shows the average results for each day of the week

night	Mon	Tue	Wed	Thu	Fri	Sat	Sun
***noise level* (dB)**	74	85	62	71	104	112	54
patrons	34	46	24	35	95	113	32

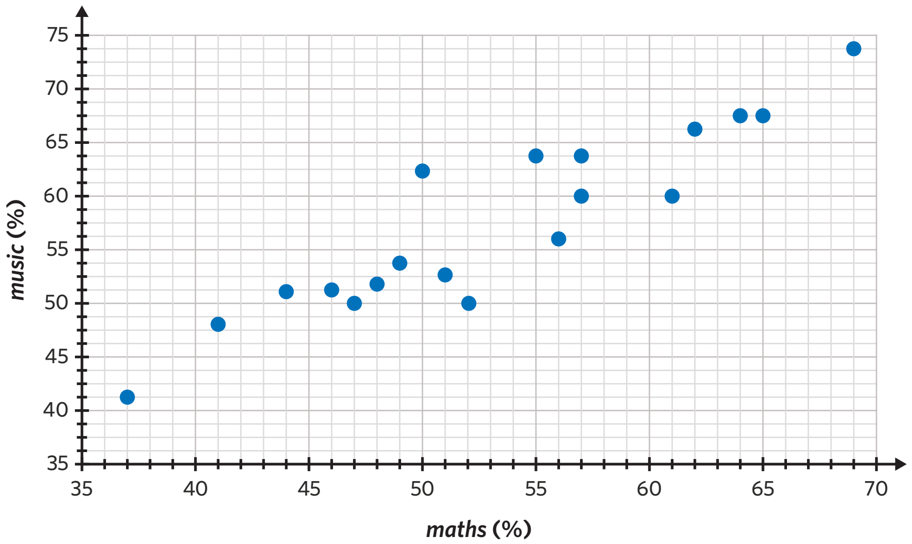
a. Construct a scatterplot using the table.

b. The council sets a maximum noise level of 98 dB for the pub. On which nights is this regulation currently broken?

c. The council set a limit of 100 patrons in the pub at once. Based on the data, are the owners going to continue to break council regulations? Explain your answer.

Exam practice

14. A music teacher wants to determine whether maths ability is a good predictor of musical ability. The teacher collects data on the *maths* and *music* exam results (%) of 19 of their students. The results are displayed in the following scatterplot.



Name the response variable in this scatterplot. (1 MARK)

Adapted from VCAA 2018 Exam 2 Data analysis Q2b

90% of students answered this type of question correctly.

15. The relative humidity (%) at 9 am and 3 pm on 14 days in November 2017 is shown in the following table.

***relative humidity* (%)**	**9 am**	100	99	95	63	81	94	96	81	73	53	57	77	51	41
	3 pm	87	75	67	57	57	74	71	62	53	54	36	39	30	32

Data: Australian Government Bureau of Meteorology

The data is analysed with the aim of predicting the relative humidity at 3 pm (*humidity 3 pm*) from the relative humidity at 9 am (*humidity 9 am*).

Name the explanatory variable. (1 MARK)

Adapted from VCAA 2019 Exam 2 Data analysis Q4a

84% of students answered this type of question correctly.

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16. The *age*, in years, *total number of jobs* held across each respective career, and *current salary*, in $000's, of a sample of 12 individuals is shown in the following table.

age	25	28	72	42	58	29	51	64	31	22	49
total number of jobs	6	9	15	3	12	3	10	16	2	4	11
***current salary* ($000's)**	65	72	612	163	89	71	258	312	491	56	750

The data is analysed with the aim of predicting *total number of jobs* from *age*.

Name the explanatory variable. (1 MARK)

Adapted from VCAA 2020 Exam 2 Data analysis Q4bi

78% of students answered this type of question correctly.

Questions from multiple lessons

Recursion and financial modelling *Year 10 content*

17. A house was purchased for $800 000 with a deposit of $50 000.

The balance will be completely repaid with 85 monthly repayments of $10 000.

The total amount of interest charged is

A. $10 000

B. $50 000

C. $100 000

D. $850 000

E. $900 000

Adapted from VCAA 2015 Exam 1 Business-related mathematics Q5

Recursion and financial modelling

18. Gonzalo is saving up for a trip to Machu Picchu. He invests $4000 in a savings account with an interest rate of 4.4% per annum, compounding quarterly. Correct to the nearest cent, how much interest will Gonzalo earn in three years?

A. $528.00

B. $551.57

C. $557.91

D. $561.14

E. $563.33

Adapted from VCAA 2015 Exam 1 Business-related mathematics Q4

Matrices

19. A local game shop sells Jenga (J), Pictionary (P), Monopoly (M) and Scrabble (S). Matrix N contains the number of sales of each game in the past month.

$$N = \begin{bmatrix} 58 \\ 14 \\ 167 \\ 115 \end{bmatrix}\begin{array}{l} J \\ P \\ M \\ S \end{array}$$

a. Write down the order of matrix N. (1 MARK)

The price of each game is contained within Matrix P.

$$P = \begin{array}{c} \begin{array}{cccc} J & P & M & S \end{array} \\ \begin{bmatrix} 29 & 13 & 25 & 34 \end{bmatrix} \end{array}$$

b. Calculate the matrix product $R = P \times N$. (1 MARK)

Adapted from VCAA 2016 Exam 2 Matrices Q1

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6B Interpreting scatterplots

STUDY DESIGN DOT POINTS

- scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction, form and strength
- informal interpretation of association and causation

6A 6B 6C 6D

KEY SKILLS

During this lesson, you will be:

- describing the relationship between numerical variables
- understanding the limitations of correlation with respect to causation.

KEY TERMS

- Strength
- Direction
- Form
- Correlation
- Causation
- Common response
- Confounding variables
- Coincidence

Once data has been displayed as a scatterplot, it is useful to determine whether any relationship exists between the numerical variables. This association can be described in terms of strength, direction and form. However, further analysis is needed before it can be determined whether causation exists.

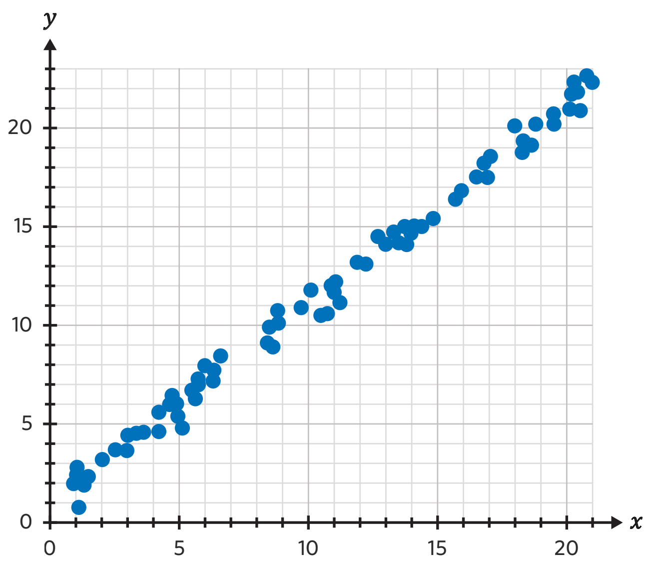
Describing the relationship between numerical variables

Scatterplots can be used to describe the relationship between two numerical variables in terms of strength, direction and form.

Strength refers to how close the data points are to the general trend of the scatterplot.

A relationship can be described as weak, moderate or strong.

There is a strong relationship if the points follow a distinct trend closely.



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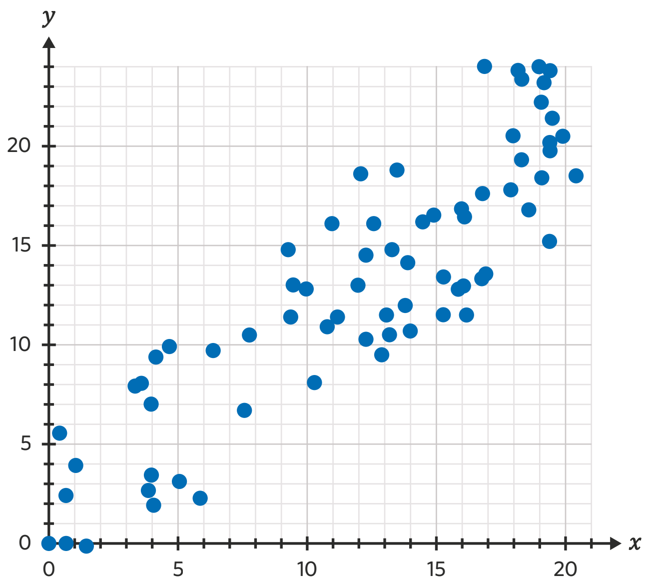
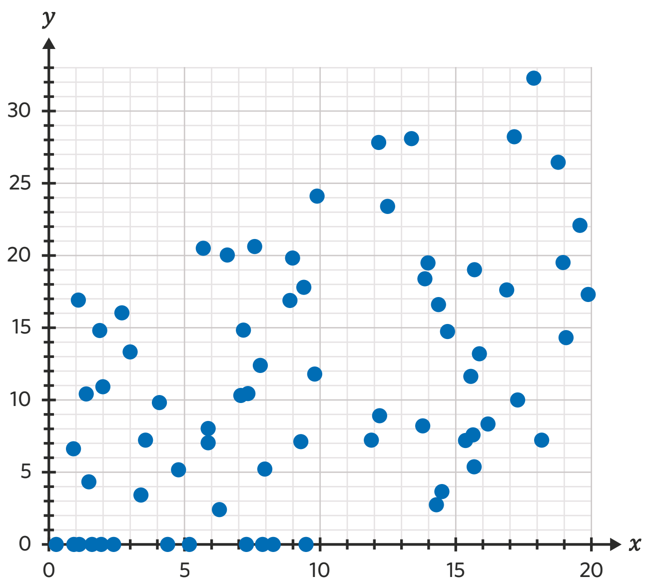
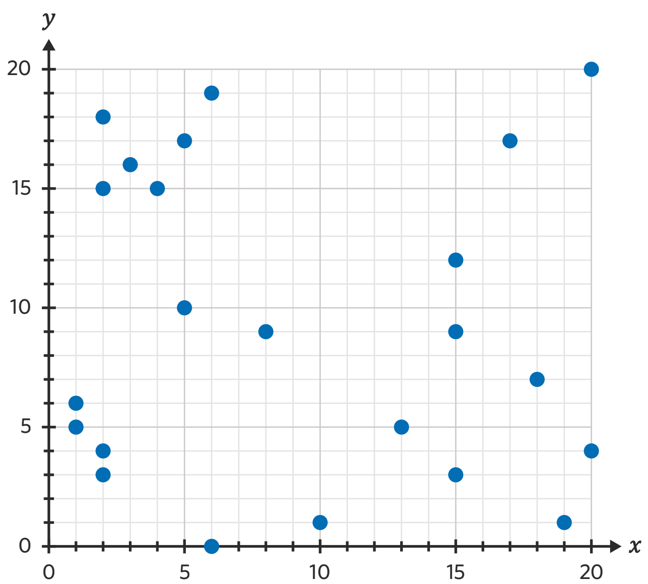
There is a moderate relationship if there is a distinct trend visible, but the points are more spread out.



There is a weak relationship if a trend is barely visible.



There is no relationship if the points do not follow any visible trend.



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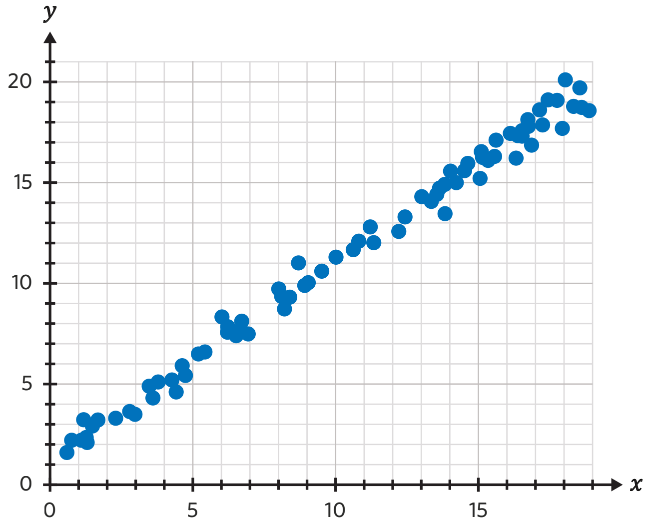
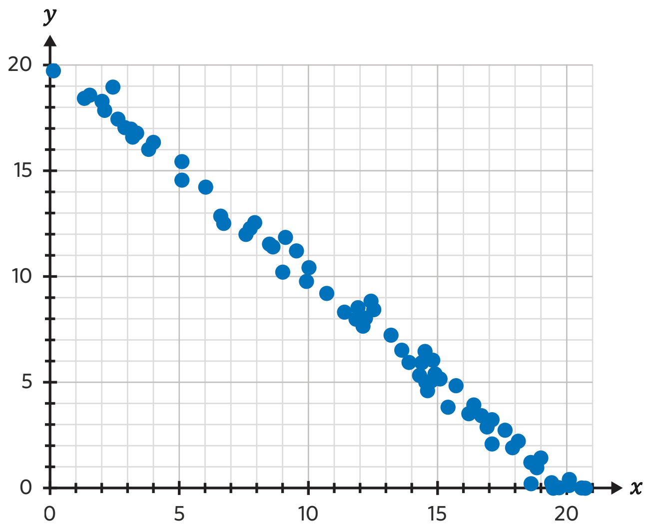
Direction refers to how the response variable changes as the explanatory variable increases. A relationship must exist for the direction to be defined.

There is a positive relationship if the response variable increases as the explanatory variable increases.

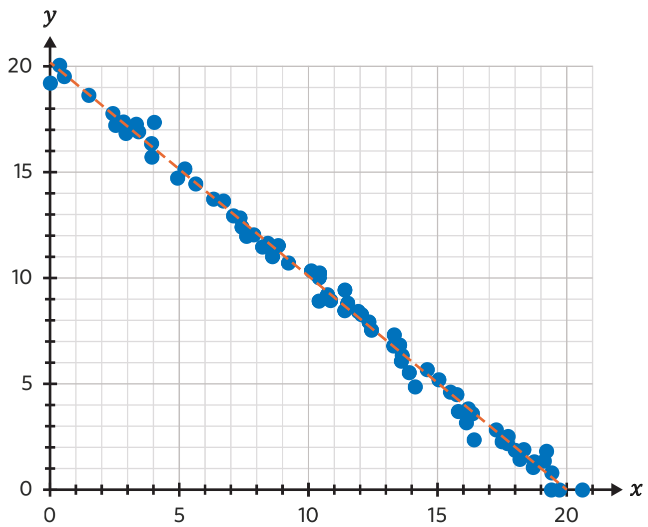


There is a negative relationship if the response variable decreases as the explanatory variable increases.

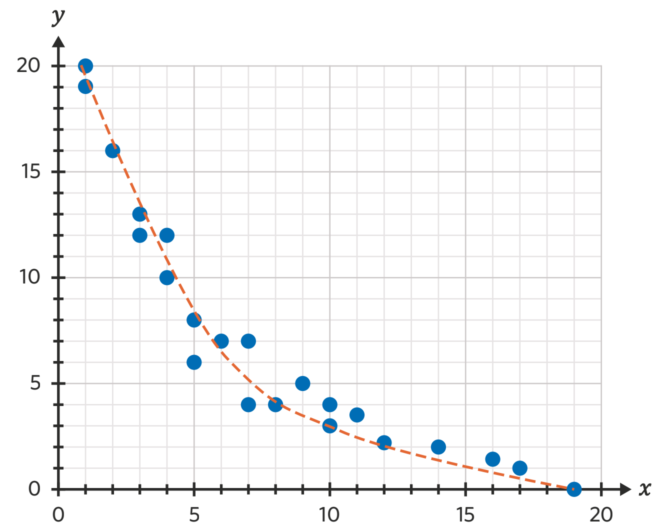


Form refers to whether the relationship is linear or non-linear. A relationship must exist for the form to be defined.

The relationship is linear if the trend resembles a straight line.

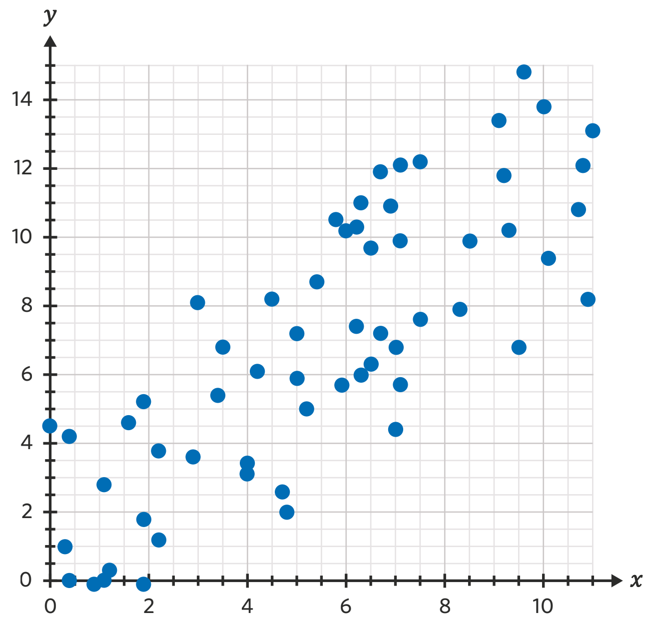


The relationship is non-linear if the trend does not resemble a straight line.



Worked example 1

Consider the following scatterplot.



Describe the relationship in terms of strength, direction and form.

Explanation

Step 1: Identify the strength of the relationship.

There is a distinct trend visible, but the points are more spread out.

There is a moderate relationship.

Step 2: Identify the direction of the relationship.

The value of y tends to increase as the value of x increases.

There is a positive relationship.

Step 3: Identify the form of the relationship.

The trend resembles a straight line.

There is a linear relationship.

Answer

Moderate, positive, linear relationship

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Understanding the limitations of correlation with respect to causation

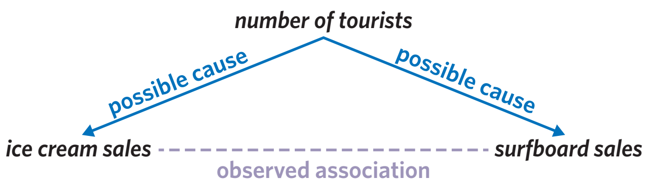
Correlation is a measure of the strength and direction of a relationship between the explanatory and response variables. **Causation** indicates that a change in the explanatory variable definitively causes the change in the response variable.

While two variables may have a strong correlation, this does not necessarily mean that the association implies causation.

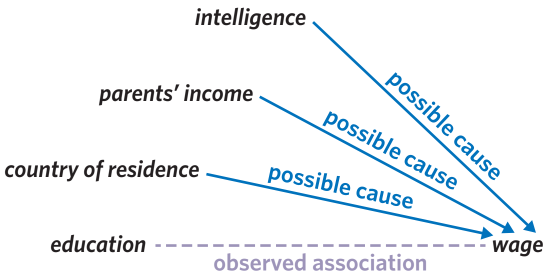
A correlation between two variables that have a strong relationship but do not have a causal relationship can occur due to three different circumstances.

A **common response** to a third variable may be present in the two variables with the observed association. For example, there is a positive correlation between *ice cream sales* and *surfboard sales*. However, it is unlikely that an increase in *ice cream sales* causes an increase in *surfboard sales*. A common response to a third variable, *number of tourists*, is the likely cause of this correlation.

See worked example 2



There could be **confounding variables** that also produce a change in the response variable. For example, there is likely a positive association between the level of *education* and a person's *wage*. Although *wage* could be affected by *education*, other related factors such as *intelligence*, *parents' income* and *country of residence* could also have an effect.



Lastly, a correlation may exist by pure **coincidence**. For example, a positive association between *average number of cats owned* and *number of annual lightning strikes* is likely purely coincidental.

The presence of correlation in a scatterplot is not evidence of causation unless definitively stated. Therefore, without further exploration, the most informed statement that can be made based on correlation seen in a scatterplot is that a change in the explanatory variable is associated with either a positive or negative change in the response variable.

See worked example 3

Worked example 2

It is observed that the more time a sprinter spends warming up, the worse they perform in a 100-metre race.

a. Identify a potential third variable that could cause a common response.

Explanation

Identify a variable that could both increase the amount of time a sprinter spends warming up and negatively influence their performance in the race.

If a sprinter is returning from a recent injury, they will likely spend longer warming up and perform worse in the race.

Answer

recent injury

Note: There are multiple third variables that may cause a common response.

Continues →

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b. Identify a potential confounding variable.

Explanation

Identify a variable that could increase the time it takes for a sprinter to complete the 100-metre sprint, but not affect the amount of time they spend warming up.

The amount of wind during the race is likely to affect the performance of sprinters without changing the amount of time spent warming up.

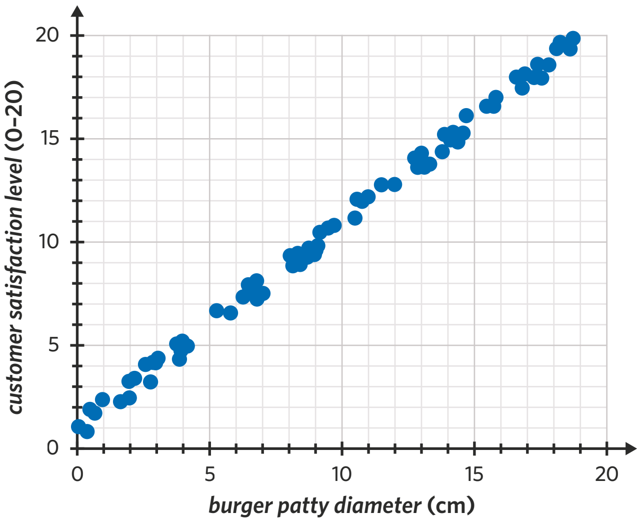
Answer

wind

Note: There are multiple confounding variables.

Worked example 3

The relationship between customer satisfaction and burger patty size at a fast food restaurant is shown in the following scatterplot.



Interpret the correlation between *burger patty diameter* and *customer satisfaction level*.

Explanation

Identify the direction of the relationship.

The values follow an upward trend from left to right.

There is a positive relationship.

Answer

An increase in *burger patty diameter* is associated with an increase in *customer satisfaction level*.

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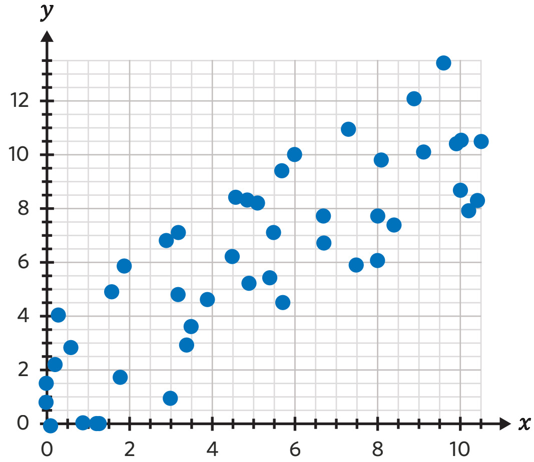
6B Questions

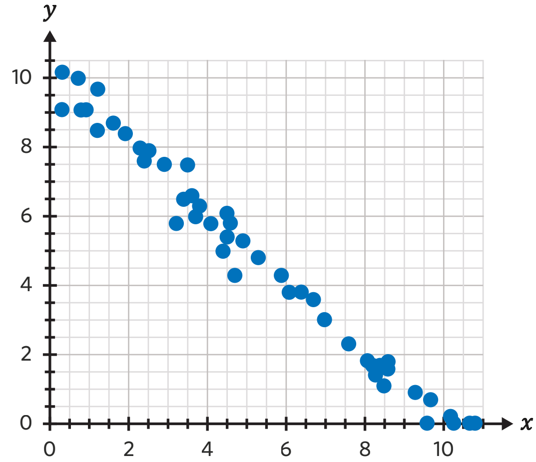
Describing the relationship between numerical variables

1. Which of the following graphs has the strongest relationship between x and y?

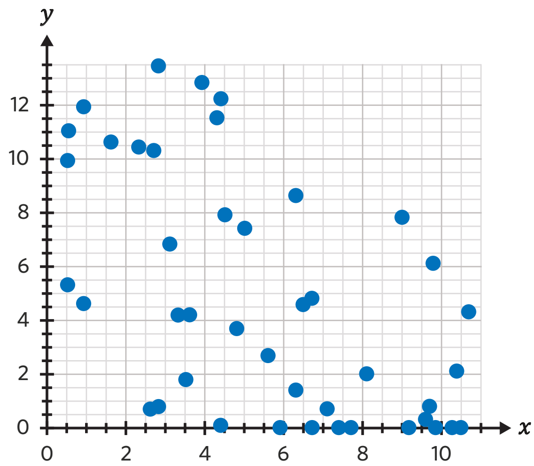
A.



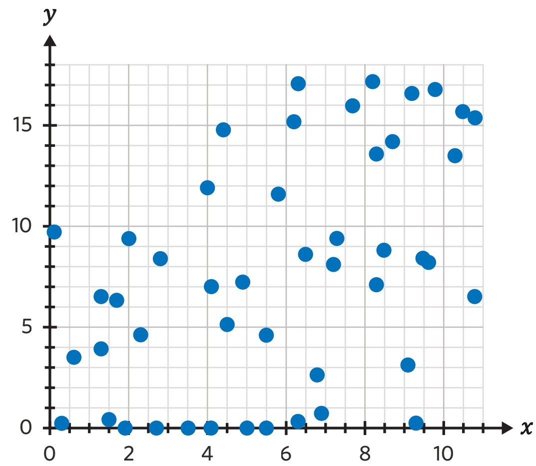
B.



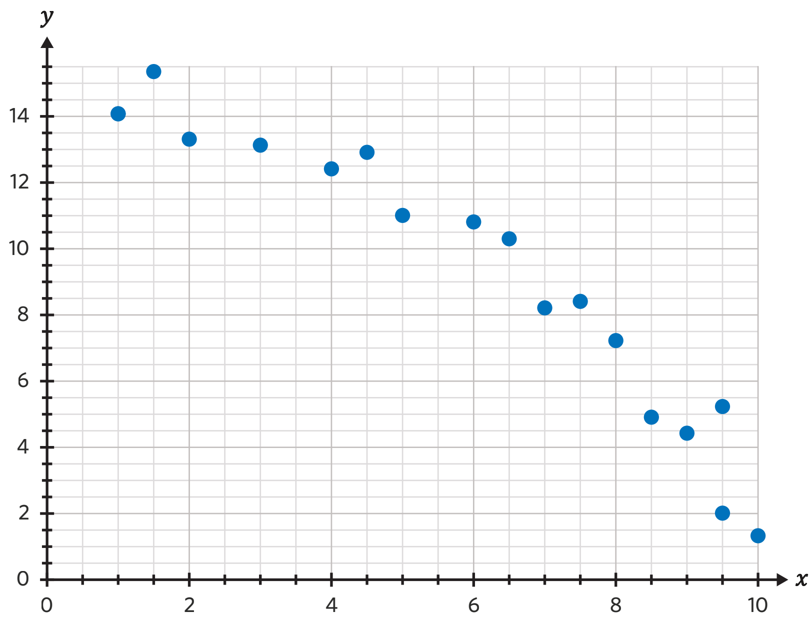
C.



D.



2. Determine the form of the following scatterplot.



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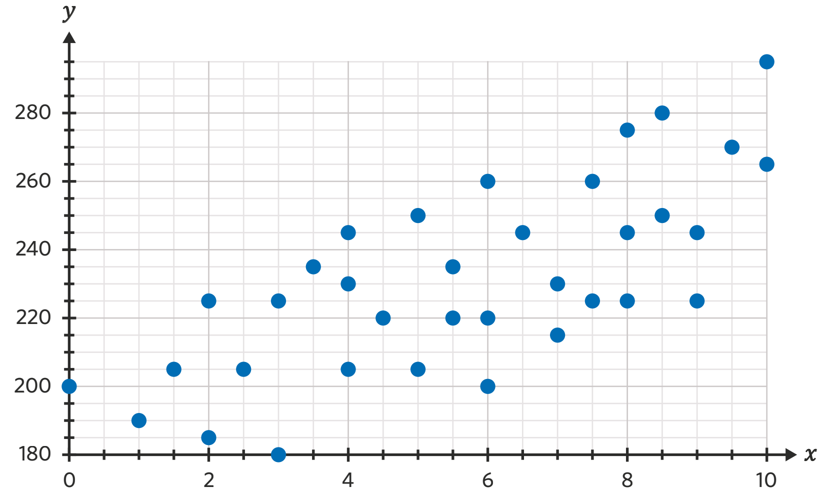
3. Consider the following scatterplot.



Describe the relationship in terms of

a. strength.

b. direction.

c. form.

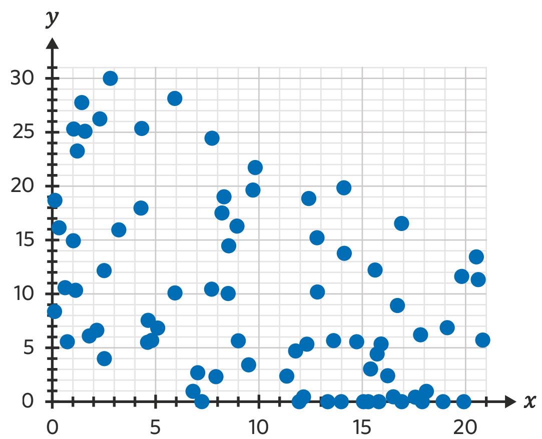
---

4. Jon is organising a party in the North. He finds that the more attendees there are that are over 18, the more cars there are in the parking lot. State the direction of the correlation.

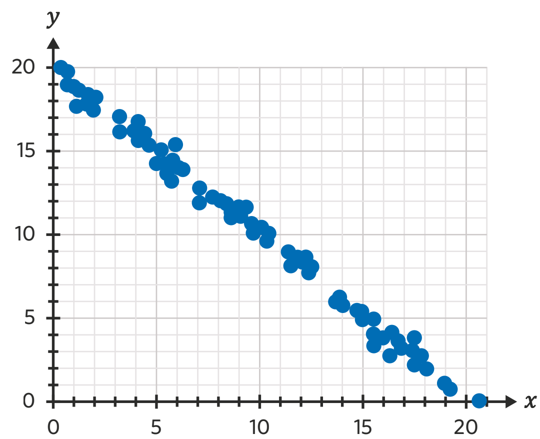
---

5. For each of the following scatterplots, describe the relationship in terms of strength, direction and form.

a.

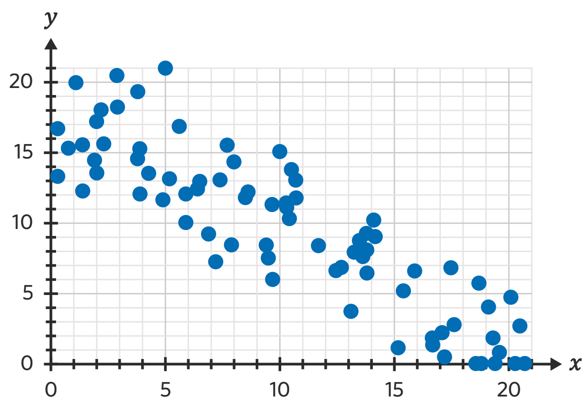


b.



---

6. After studying the following scatterplot for some time, Toby concluded that the correlation is weak, negative and linear. Is Toby's conclusion correct? Explain.



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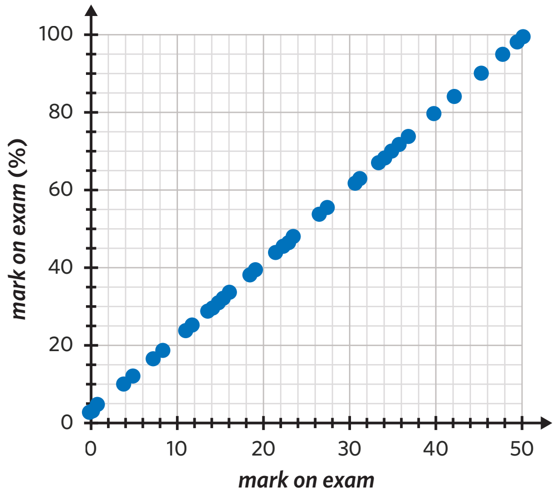
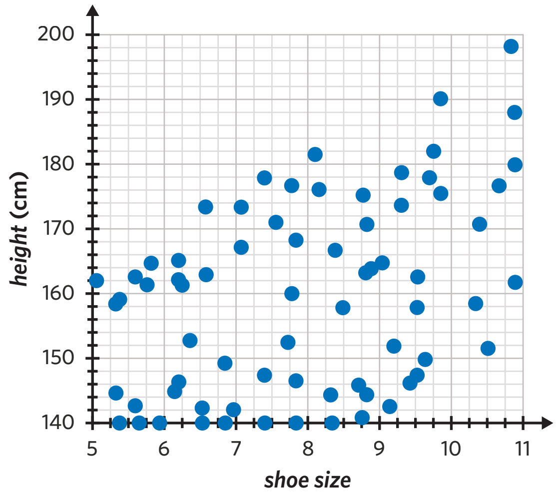
7. Describe the relationship between the explanatory and response variables in each of the following scatterplots in terms of strength, direction and form.

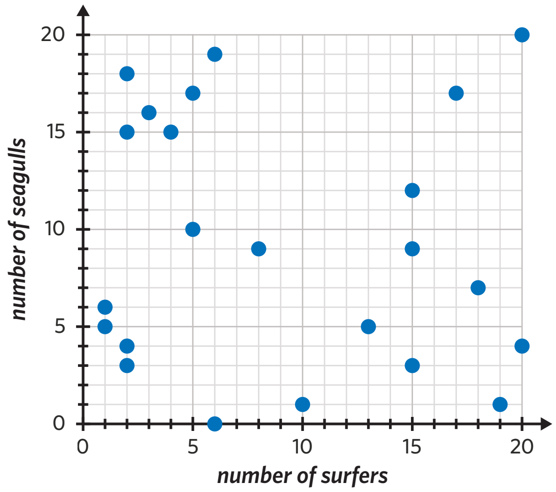
a.



b.



c.



Understanding the limitations of correlation with respect to causation

8. Which of the following statements is not always true?
   A. There can be more than one variable that affects the response variable.
   B. It is possible for no relationship to exist between explanatory and response variables.
   C. Strong, negative correlation means that an increase in the explanatory variable is associated with a decrease in the response variable.
   D. Strong, negative correlation means that an increase in the explanatory variable causes a decrease in the response variable.

9. It is known that there is a strong positive correlation between household income and the number of pets owned. Which of the following statements is always true?
   A. An increase in household income is associated with an increase in the number of pets owned.
   B. A decrease in household income is associated with an increase in the number of pets owned.
   C. Increasing the income for a household leads to the household adopting more pets.
   D. Increasing the income for a household leads to the household adopting less pets.

10. Bailey spins a roulette wheel once a day for a month. He finds that the number that the ball lands on is associated with the maximum temperature of that day (°C).
    Why might this correlation exist?
    A. A third variable **B.** A confounding variable **C.** Coincidence **D.** Causation

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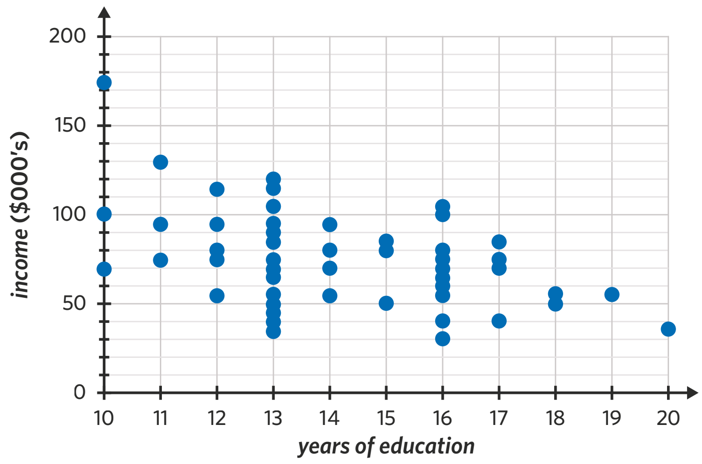
11. The relationship between number of *years of education* and annual *income* for a sample of 25 year olds is shown in the following scatterplot.



Interpret the correlation between *years of education* and *income*.

---

12. There is a strong negative correlation between the number of supernatural scenes in a movie and its popularity with young children. It can be concluded that

A. young children dislike supernatural scenes.

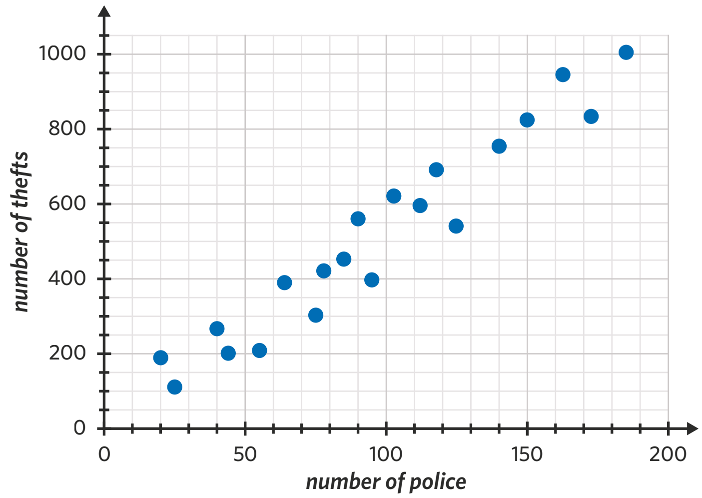
B. increasing the number of supernatural scenes in a movie will decrease its popularity with young children.

C. movies with a lesser number of supernatural scenes tend to be more popular with young children.

D. this association is a chance occurrence.

Joining it all together

13. The following scatterplot shows the relationship between the *number of police* and *number of thefts* in different Perth suburbs within a year.



a. Describe the relationship between *number of police* and *number of thefts* in terms of strength, direction and form.

b. Identify a potential third variable that could cause a common response.

c. It can be concluded that

A. suburbs with more police tend to have less thefts.

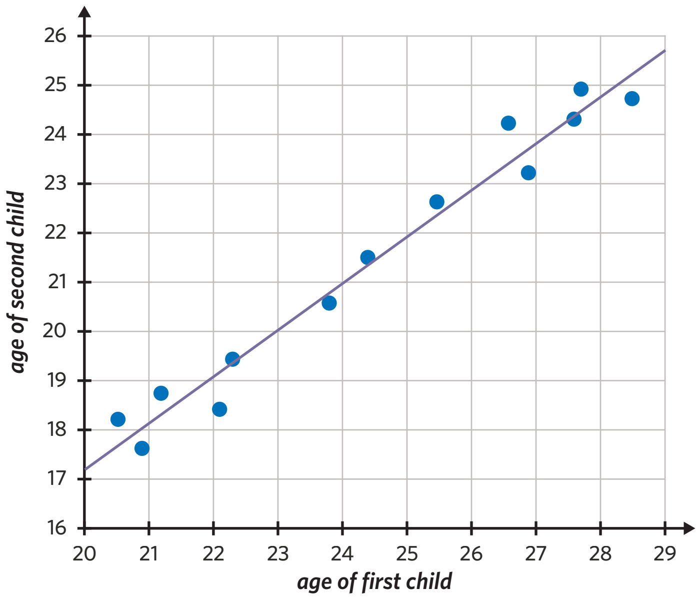
B. suburbs with more police tend to have more thefts.

C. an increased police presence causes a decrease in theft.

D. an increased police presence causes an increase in theft.

Exam practice

14. The data in the following scatterplot shows the *age of first child* and *age of second child* for 13 families.



Use the scatterplot to describe the association between *age of first child* and *age of second child* in terms of strength, direction and form. (1 MARK)

Adapted from VCAA 2016 Exam 2 Data analysis Q3a

78% of students answered this type of question correctly.

---

15. There is a strong positive association between a country's Gross Domestic Product (GDP) and the percentage of citizens who drive to work. From this information, it can be concluded that

A. increasing the percentage of citizens that drive to work will increase the GDP of the country.

B. decreasing the percentage of citizens that drive to work will increase the GDP of the country.

C. this association must be a chance occurrence and can be safely ignored.

D. countries that have a larger GDP tend to have a larger percentage of citizens who drive to work.

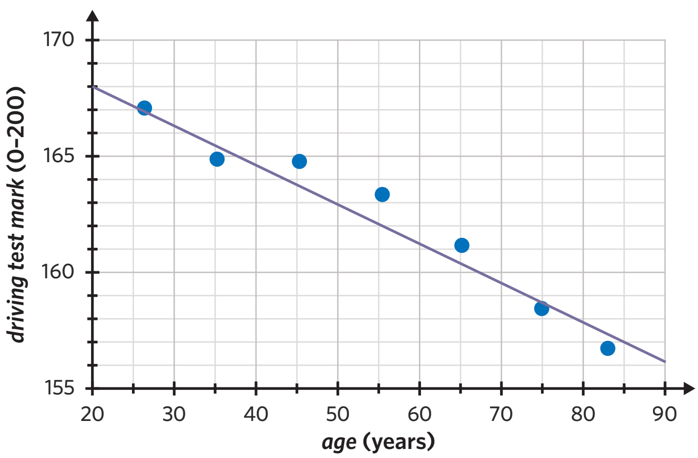
E. countries that have a larger GDP tend to have a smaller percentage of citizens who drive to work.

Adapted from VCAA 2016 Exam 1 Data analysis Q12

67% of students answered this type of question correctly.

---

16. The following scatterplot shows an association between *age* (years) and *driving test mark* (0–200) of a group of people.



Describe this association in terms of strength and direction. (1 MARK)

Adapted from VCAA 2020 Exam 2 Data analysis Q6b

51% of students answered this type of question correctly.

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Questions from multiple lessons

Data analysis

17. The ATARs of a sample of eight students were recorded and are shown in the following table.

ATAR	78.05	93.65	88.20	61.40	82.85	49.95	98.10	79.00

The mean, $\bar{x}$, and standard deviation, s_x, of the ATARs for this sample are closest to

A. $\bar{x} = 15.1 \quad s_x = 78.9$

B. $\bar{x} = 15.7 \quad s_x = 78.9$

C. $\bar{x} = 78.9 \quad s_x = 16.2$

D. $\bar{x} = 78.9 \quad s_x = 15.1$

E. $\bar{x} = 78.9 \quad s_x = 15.7$

Adapted from VCAA 2017 Exam 1 Data analysis Q3

Recursion and financial modelling

18. The first five terms of a sequence are

$-17, -8, 1, 10, 19$

Which of the following could be the recurrence relation that generates this sequence?

A. $T_0 = -17, \quad T_{n+1} = T_n + 9$

B. $T_0 = -17, \quad T_{n+1} = T_n - 9$

C. $T_0 = -17, \quad T_{n+1} = -2T_n$

D. $T_0 = -26, \quad T_{n+1} = \frac{1}{2}T_n$

E. $T_0 = -26, \quad T_{n+1} = 10T_n$

Adapted from VCAA 2017 Exam 1 Recursion and financial modelling Q19

Data analysis

19. Sonya skips each morning to get ready for the day. She records the *time spent skipping*, in minutes, per day over 6 days, as well as the number of *total skips* performed.

time spent skipping **(mins)**	5	7	3	8	12	4
total skips	510	720	300	780	1220	390

Construct a scatterplot using *time spent skipping* as the explanatory variable. (2 MARKS)

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6C Lines of good fit by eye

STUDY DESIGN DOT POINT

- use of a line of good fit by eye to make predictions, including the issues of interpolation and extrapolation

6A 6B 6C 6D

KEY SKILLS

During this lesson, you will be:

- constructing a line of good fit by eye
- using a line of good fit to make predictions.

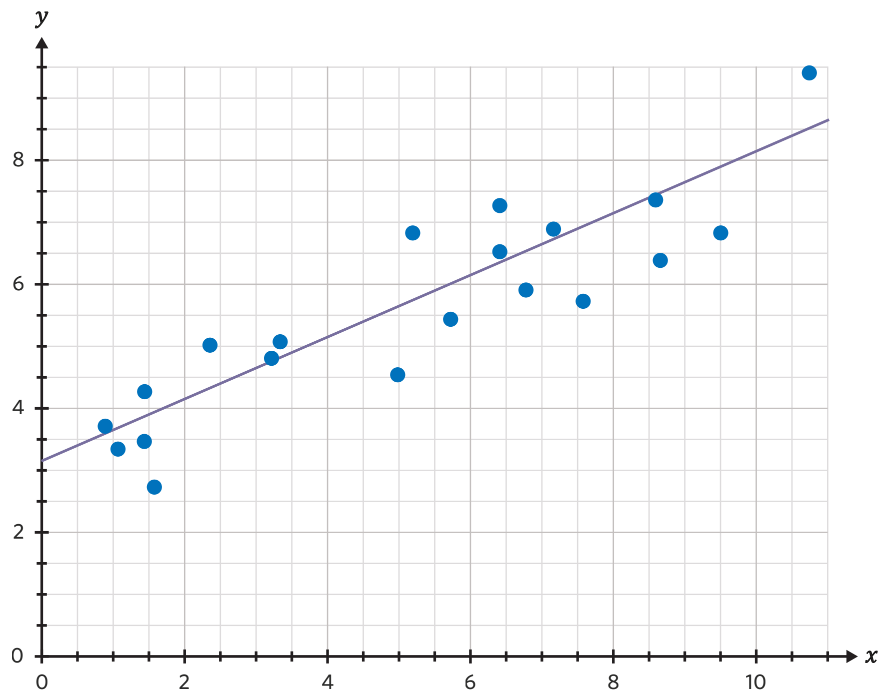
KEY TERMS

- Line of good fit
- Interpolation
- Extrapolation

When the form of a scatterplot is linear, a straight line can be drawn to approximate the general trend of the data. This allows for predictions to be made, within some assumptions and limitations. Many real life relationships are approximately linear, and can be effectively modelled and analysed using this method.

Constructing a line of good fit by eye

A **line of good fit** is a straight line which approximates the general trend of a scatter plot. It is drawn such that approximately half of the data points lie below the line, and half above.



A line of good fit does not have to be perfect. It should be drawn approximately through the middle of the scatterplot, and should follow the direction that the data appears to be moving in.

A line of good fit with a positive gradient indicates a positive relationship between the variables, while a line of good fit with a negative gradient indicates a negative relationship.

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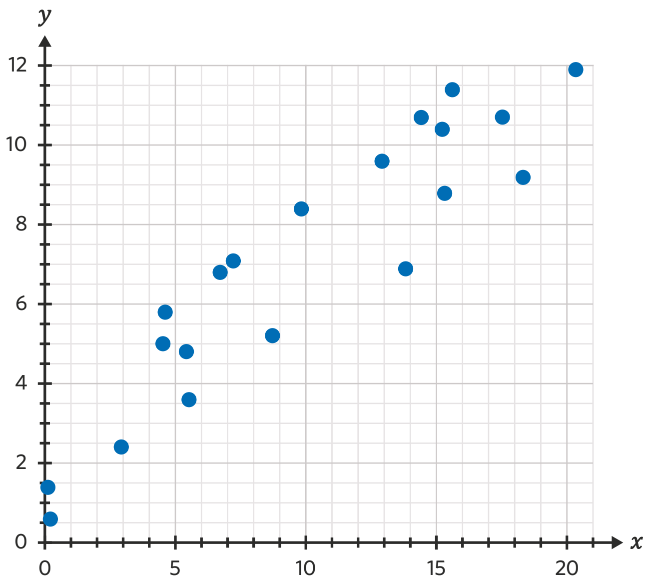
Worked example 1

Draw a line of good fit for the following scatterplot.



Explanation

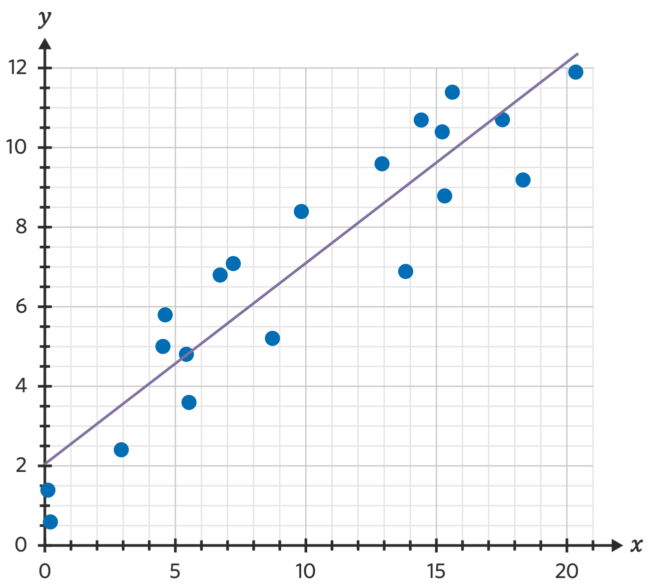
Step 1: Determine the general trend of the data.

As the x values increase, the y values increase, therefore the line of good fit must have a positive gradient.

Step 2: Draw a line of good fit.

Ensure that approximately half the data points lie below the line and half lie above the line.

Answer



Note: The line of good fit does not have to pass through exactly the same coordinates. Anything similar is acceptable.

Using a line of good fit to make predictions

A line of good fit can be used to predict the value of one of the variables, given the value of the other. Predictions can be performed visually by reading off the line of good fit.

For example, the line of good fit in the following scatterplot can be used to estimate the value of y when $x = 4$. In this case, the value of y is estimated to be 3.

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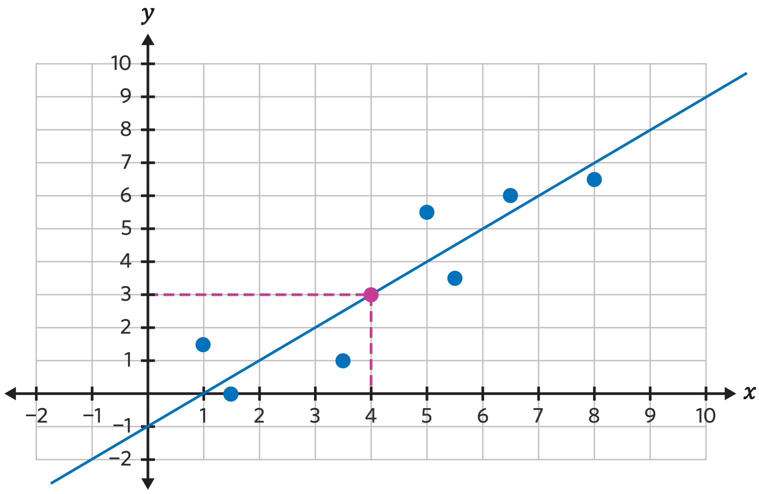
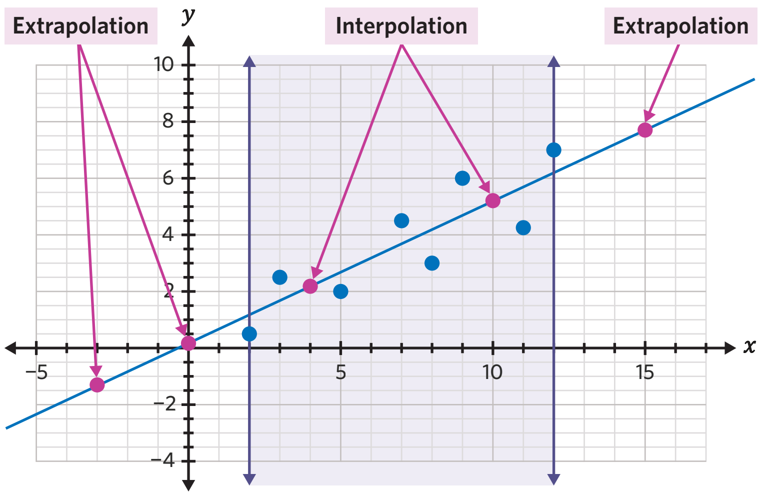


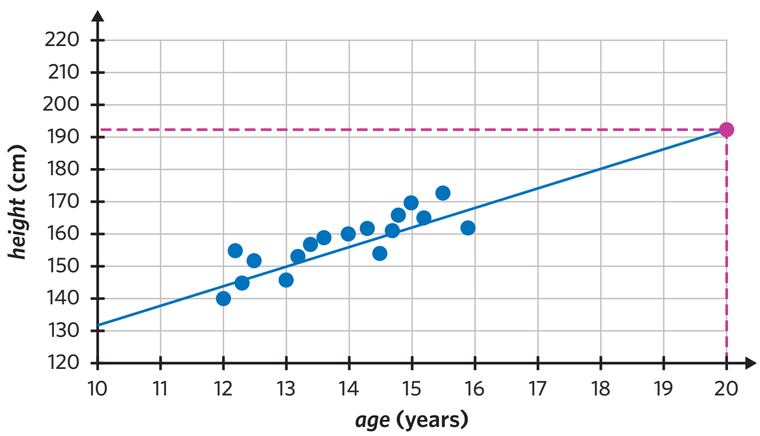
There are two types of predictions: interpolation and extrapolation. An **interpolation** is a prediction that uses the existing data points to make a prediction within the domain of the data set. An **extrapolation** is a prediction that uses the existing data points to make a prediction outside the domain of the data set.

In the following scatterplot the original data points are shown in blue, while predictions are shown in pink. An interpolation is any prediction in which the value of x is between 2 and 12 (inclusive). An extrapolation is any prediction in which the value of x is less than 2 or greater than 12.



Extrapolation has limited reliability, and should be treated carefully. This is because a line of good fit is estimated using data within the domain. Extending the line of good fit beyond the domain assumes that the general trend will continue beyond what is observed. This assumption cannot be tested, as there is no data with values outside the domain.

For example, the following line of good fit is estimated using data on the *age* and *height* of a group of students between 12 and 16 years old. When extrapolating the data to age 20, the average height is estimated to be 192 cm. This value is very inaccurate and will only continue to become more inaccurate as age increases.



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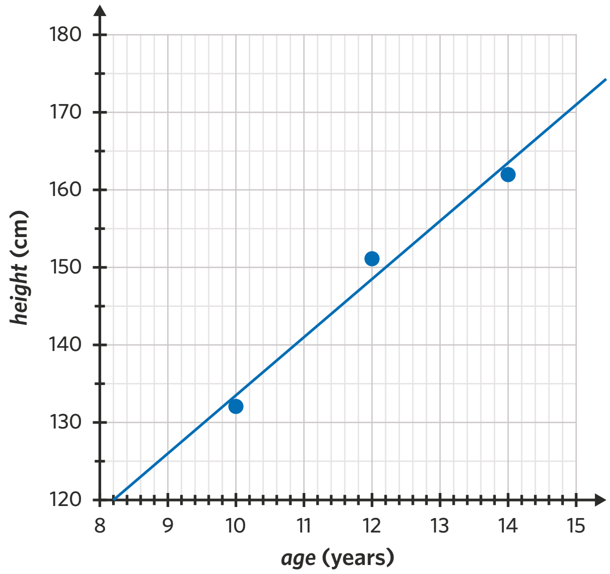
Worked example 2

The following scatterplot shows Clara's height measured on her 10^{th}, 12^{th}, and 14^{th} birthdays.

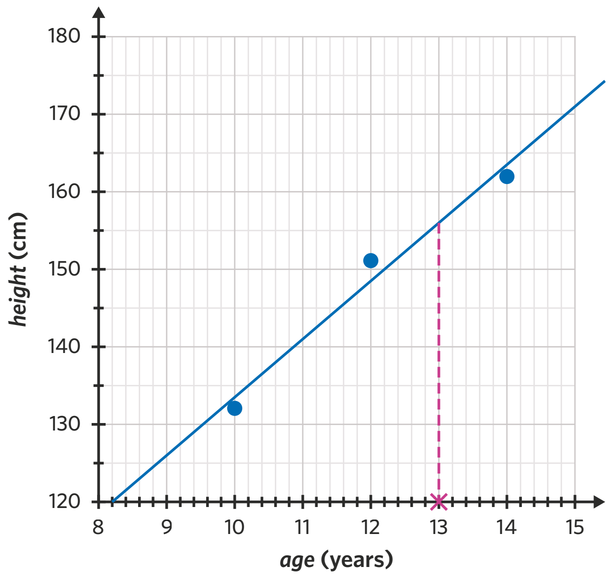


a. Estimate Clara's *height* on her 13^{th} birthday.

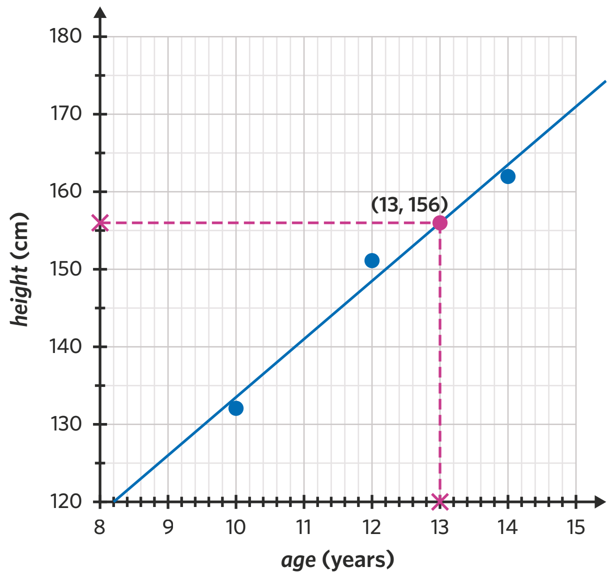
Explanation

Step 1: Draw a vertical line from the known value on the horizontal axis to the line of good fit.

The known value is *age* = 13.



Step 2: Draw a line horizontally from the line of good fit to the vertical axis.



The value at which this horizontal line intersects the vertical axis is the predicted value of *height* when *age* = 13.

Answer

156 cm

Continues →

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b. Is this prediction an interpolation or an extrapolation?

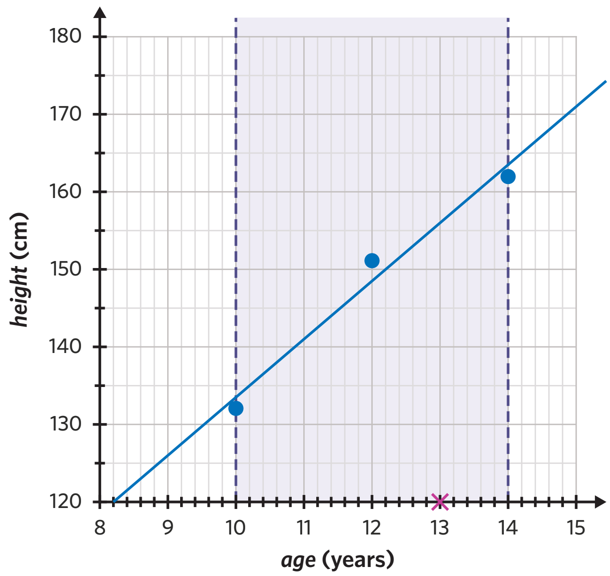
Explanation

Step 1: Determine the domain of the data set.

The minimum *age* value is 10.

The maximum *age* value is 14.

The domain of *age* values is 10 to 14.



Step 2: Determine whether the prediction is an interpolation or an extrapolation.

The domain of *age* values is 10 to 14.

The known value is *age* $= 13$.

13 is between 10 and 14.

Answer

Interpolation

c. Explain, with reference to the context in the question, why estimating Clara's *height* on her 20th birthday is of limited reliability.

Explanation

Step 1: Determine whether the prediction is an interpolation or an extrapolation.

The known value is *age* $= 20$.

20 is not between 10 and 14.

The prediction is an extrapolation.

Step 2: Explain the assumption made when extrapolating in terms of the context in the question.

Answer

The prediction is an extrapolation. The line of good fit was estimated using ages from 10 to 14. When extending the line of good fit to an age of 20, an assumption is being made that the general trend will continue beyond the observed data. This is not reliable, as height does not increase at a constant rate beyond the typical ages of puberty.

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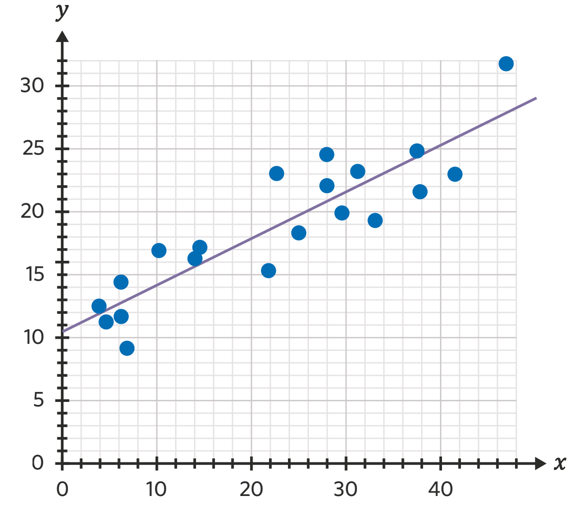
6C Questions

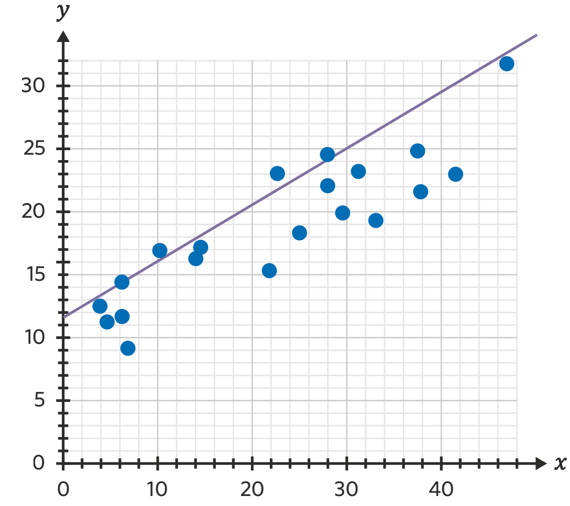
Constructing a line of good fit by eye

1. Which of the following scatterplots has the most appropriate line of good fit?

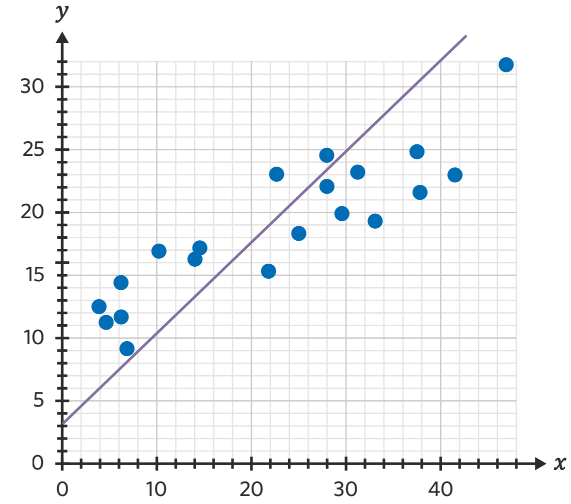
A.



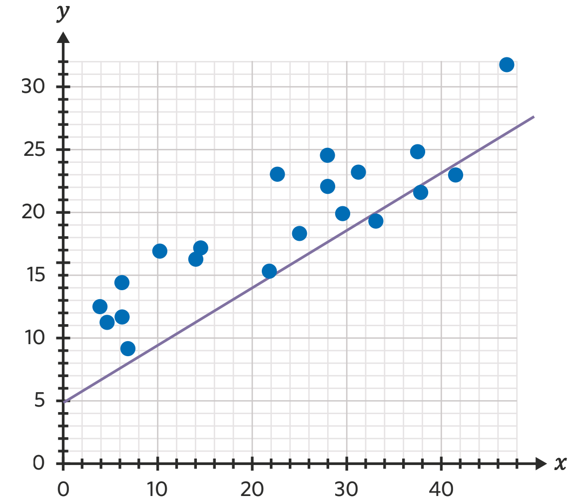
B.



C.

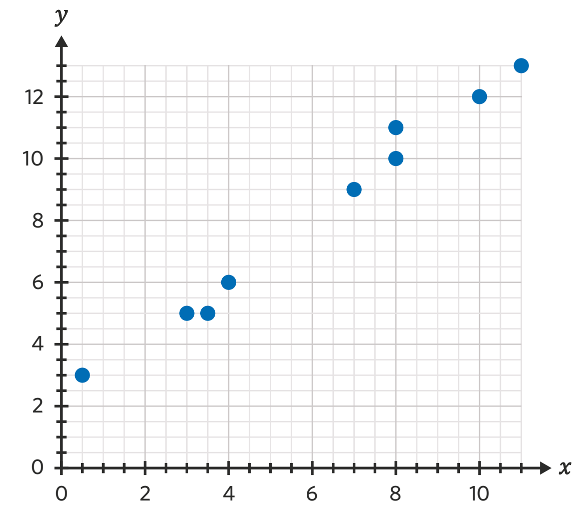


D.

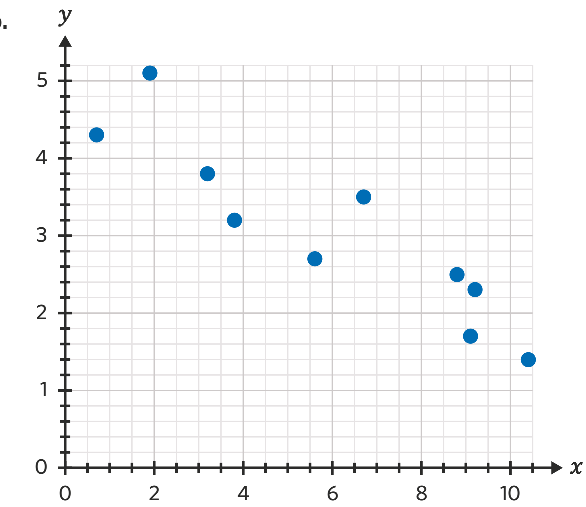


2. Draw a line of good fit for each of the following scatterplots.

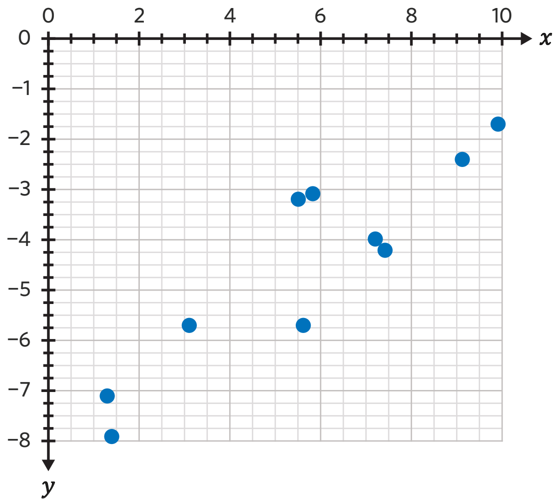
a.



b.



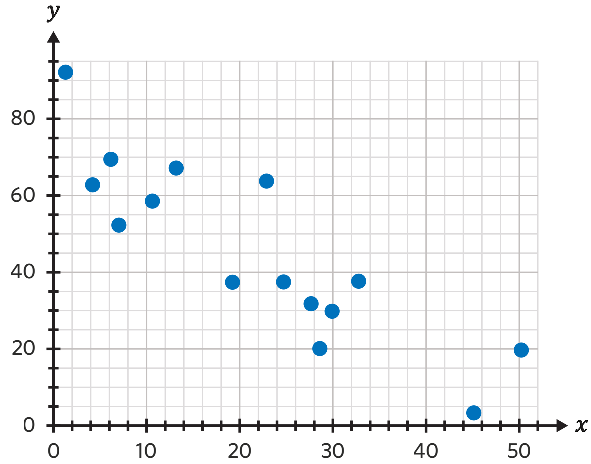
c.



3. Two students are asked to draw a line of good fit for the following scatterplot.

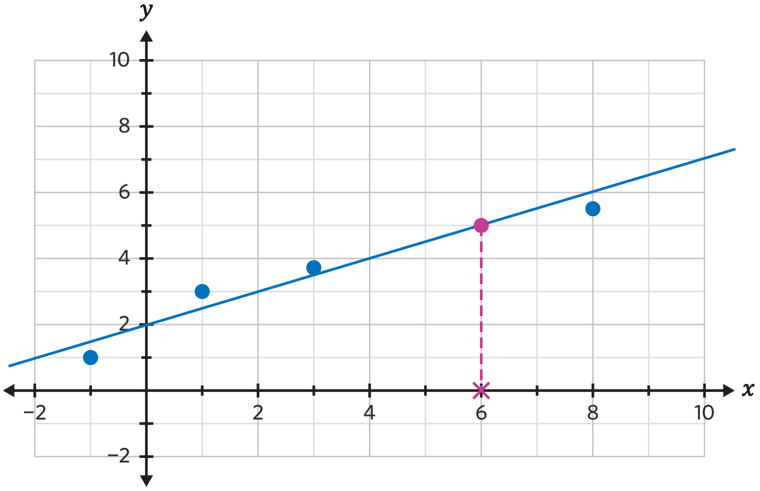
Hamish draws the line through the coordinates (0, 80) and (50, 20), whereas Andy draws the line through the coordinates (10, 60) and (40, 20).

Who drew the more appropriate line of good fit?



Using a line of good fit to make predictions

4. The following scatterplot shows four data points and a line of good fit.



What is the estimated value of y when $x = 6$?

A. 4 **B.** 4.5 **C.** 5 **D.** 6

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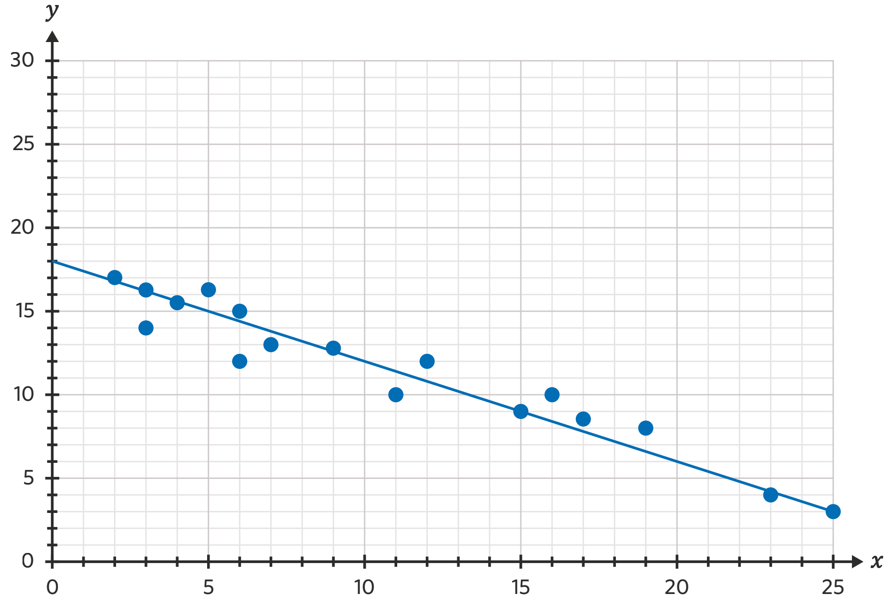
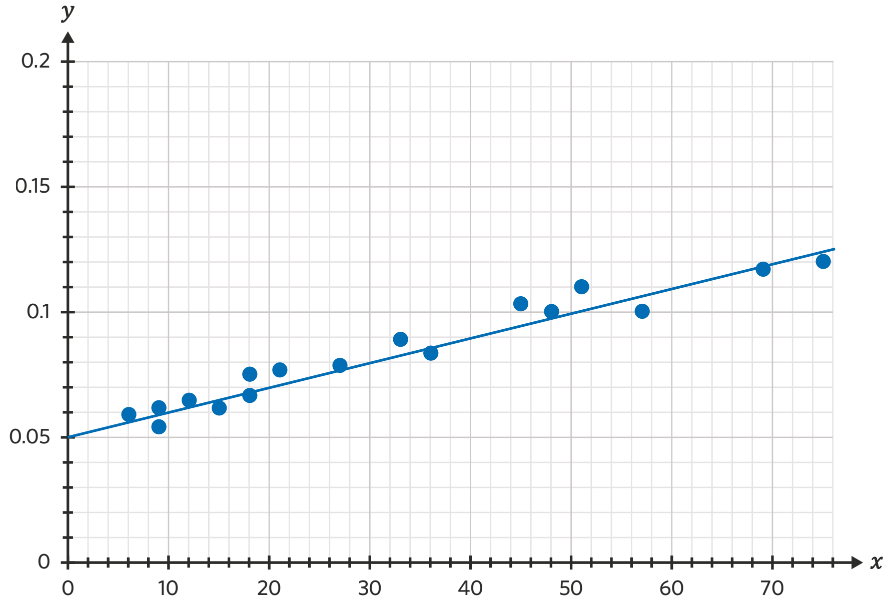
5. For each of the following scatterplots, estimate the value of y when $x = 10$, and determine if the prediction is an interpolation or an extrapolation.

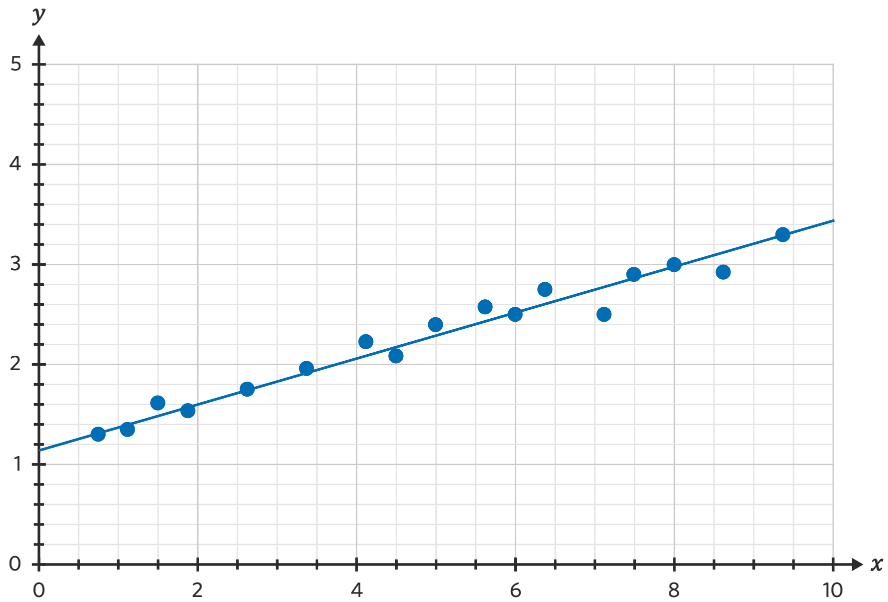
a.



b.

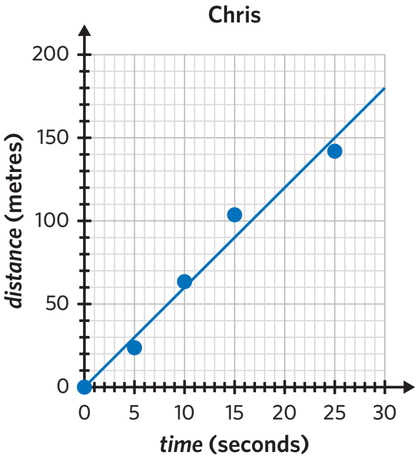


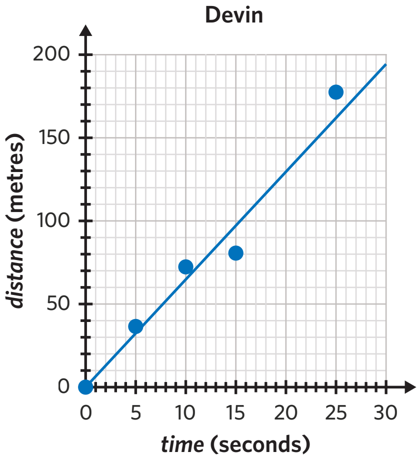
c.

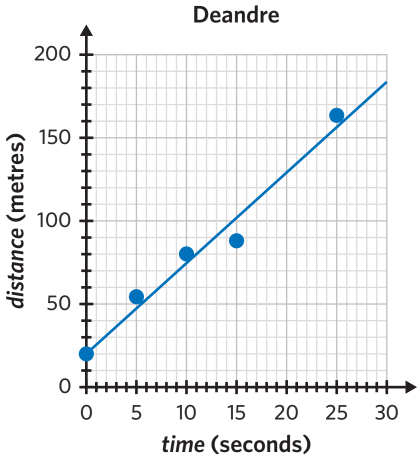


6. Three students, Chris, Devin and Deandre, participated in a 30-second running race. The *distance* that each of the students were from the starting line is known after 0, 5, 10, 15 and 25 seconds.

The following scatterplots show the relationship between *distance* and *time* for each of the students. A line of good fit is also shown. Deandre was given a head-start because he is younger than the others.







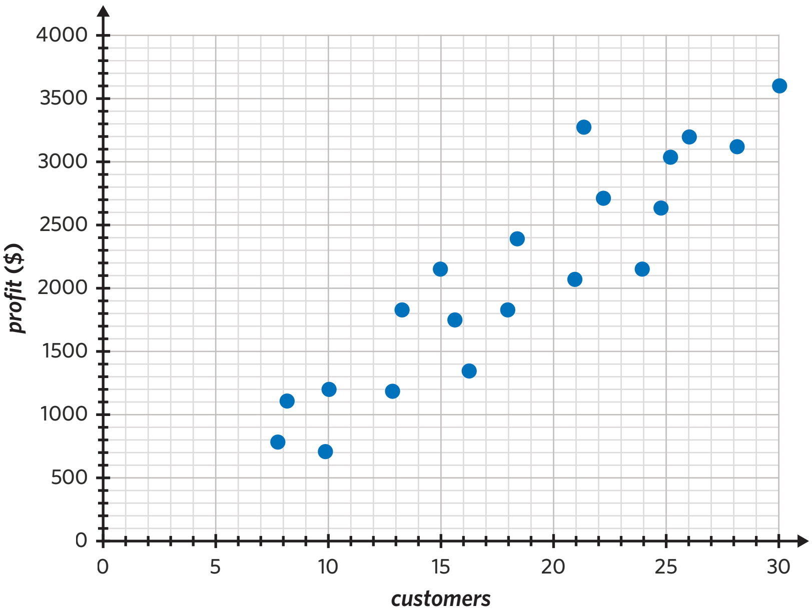
a. Estimate how far each student was from the start line after 20 seconds.

b. Use the lines of good fit to predict which student was the furthest from the starting line at the completion of the race.

c. Explain, with reference to the context in the question, why the prediction in part **b** is of limited reliability.

Joining it all together

7. A store manager recorded the *profit* ($) along with the number of *customers* that entered her store for each business day in February. This information is shown on the following scatterplot.



a. Draw a line of good fit for the scatterplot.

b. Estimate how many customers need to enter the store in a day to make a $3000 profit.

c. Is this prediction reliable? If not, provide a reason.

d. Estimate the profit when there are 6 customers.

e. Is this prediction reliable? If not, provide a reason.

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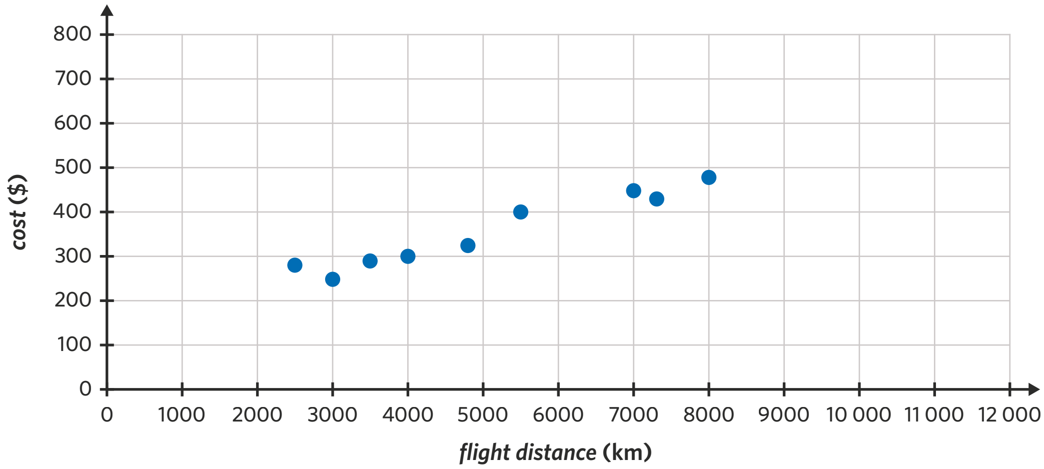
8. The following scatterplot shows the relationship between *flight distance* and *cost* for various flights.



a. Draw a line of good fit for the scatterplot.

Vincent wants to use this data to estimate the *cost* of a 6000 km flight, a 9000 km flight, and a 12 000 km flight.

b. Which of these flight costs can be found by interpolation and which can be found by extrapolation?

c. Complete all interpolations. Give your answer as a sentence, in terms of the context in the question.

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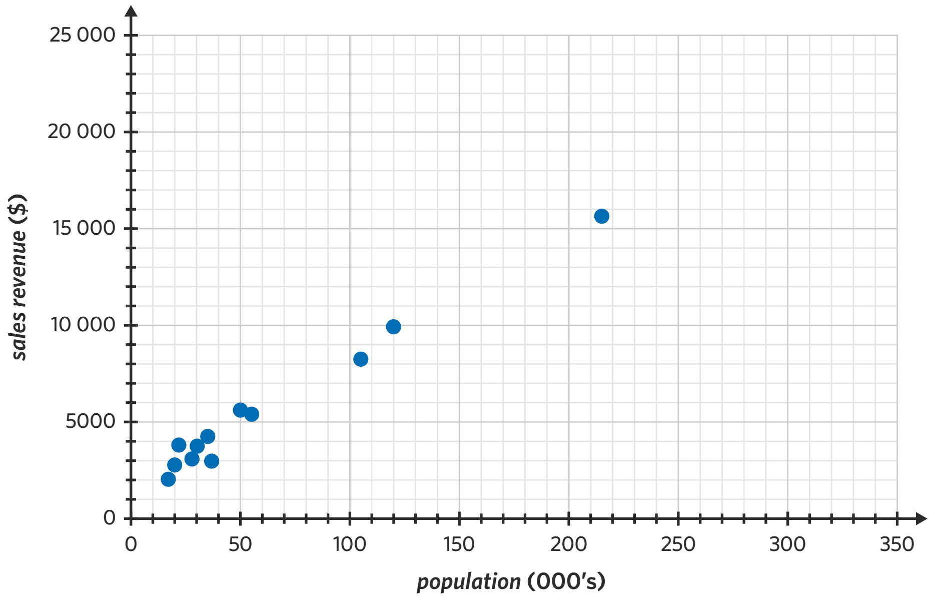
9. Julian is a regional salesman for an art supplier. He records the *population* (000's) of each town he visits as well as the *sales revenue* ($) he makes. The information is shown in the following scatterplot.



a. Draw a line of good fit for the scatterplot.

b. The *population* of Bathurst is 38 000. Estimate the *sales revenue*, to the nearest thousand dollars, that Julian is expected to make if he visits Bathurst. Determine if this prediction is an interpolation or an extrapolation.

c. The *population* of Wollongong is 230 000. Estimate *sales revenue*, to the nearest thousand dollars, that Julian is expected to make if he visits Wollongong. Determine if this prediction is an interpolation or an extrapolation.

d. Estimate the *population* of a town, to the nearest 10 000 people, in which Julian's *sales revenue* was $15 000. Determine whether this prediction is an interpolation or an extrapolation.

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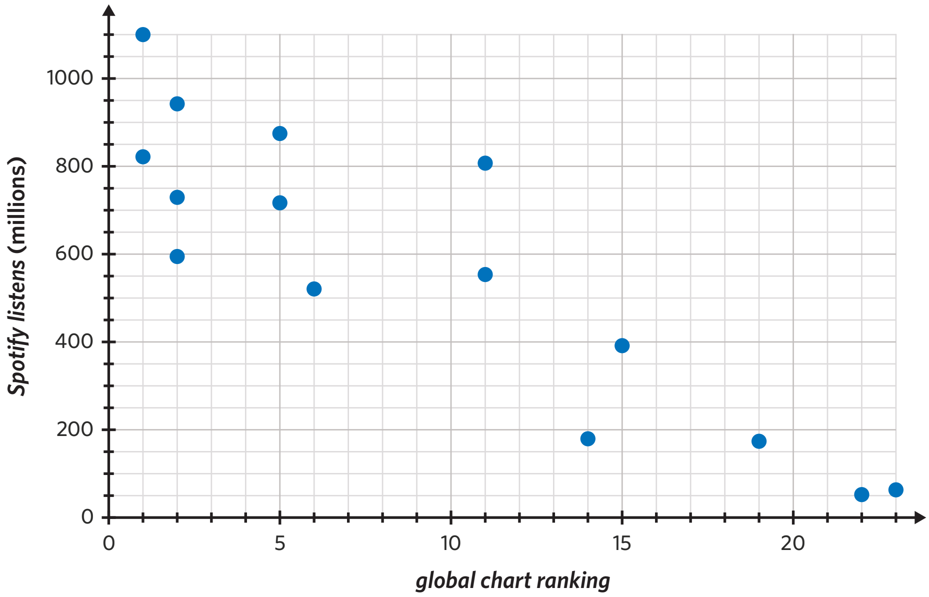
10. Sophie is a diehard Belieber. She has constructed the following scatterplot to post on her Twitter account, @L0veTheBiebz. It shows the global chart ranking and number of Spotify listens for some of Justin Bieber's top hits.



a. Draw a line of good fit for the scatterplot.

b. Justin Bieber's 'Where Are Ü Now' reached a global chart ranking of 3. Estimate how many Spotify listens it has, to the nearest 50 million.

c. Justin Bieber's 'Yummy' has 445 million Spotify listens. Estimate its global chart ranking, to the nearest whole number.

d. Justin Bieber's newest single has a global chart ranking of 35. Explain, with reference to the context in the question, why this prediction is of limited reliability.

Exam practice

11. A large colony of ants grows in size each year. The association between the *population* of the colony and *year* is approximately linear. The data is shown in the following table.

year	2014	2015	2016	2017	2018	2019	2020	2021	2022
***population* (mil)**	18.2	19.0	19.9	20.8	22.0	22.8	23.2	23.8	25.3

The line of good fit predicts that the *population* of the ant colony by the year 2026 will be approximately 28.5 million.

Give a reason why this prediction may have limited reliability. (1 MARK)

Adapted from VCAA 2017 Exam 2 Data analysis Q4cii

53% of students answered this type of question correctly.

12. Data has been collected on the *winning time*, in seconds, for the women's 100 m freestyle swim for each year that the Olympic Games were held during the period 1956 to 2016.

What assumption is being made when this data is used to predict the *winning time* for the women's 100 m freestyle in 2032? (1 MARK)

Adapted from VCAA 2021 Exam 2 Data analysis Q3eii

18% of students answered this type of question correctly.

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Questions from multiple lessons

Data analysis

13. Olajide plays FIFA Ultimate Team, a video game based on the sport of soccer. He records the *team value* (in terms of an in-game currency) of his last 10 opponents' teams, along with the number of *goals scored* by his opponents in the FIFA Ultimate Team games.

Olajide finds that there is a strong positive relationship between *team value* and *goals scored*.

From this information, it can be concluded that

A. an increase in the value of an opponents' team will decrease the number of goals they score in the FIFA game.

B. an increase in the value of an opponents' team will increase the number of goals they score in the FIFA game.

C. this association must be a chance occurrence and can be safely ignored.

D. an increase in the value of an opponents' team is associated with a decrease in the number of goals they score in the FIFA game.

E. an increase in the value of an opponents' team is associated with an increase in the number of goals they score in the FIFA game.

Adapted from VCAA 2016 Exam 1 Data analysis Q12

Recursion and financial modelling

14. Each year, the value of a rare Italian violin increases by 5.5%.

In 2019, the violin has a value of $170 500.

In 2017, the value of the violin was closest to

A. $151 790 **B.** $152 261 **C.** $153 186 **D.** $153 649 **E.** $189 771

Adapted from VCAA 2013 Exam 1 Business-related mathematics Q6

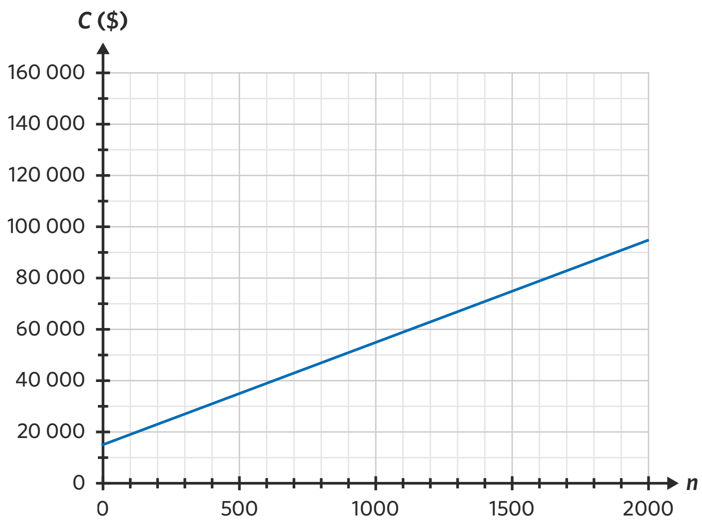
Graphs and relations

15. Melbourne label Crumprolo makes fashionable, yet practical, laptop bags for high school students.

The cost, C, in dollars, of producing n laptop bags is given by $C = 40n + 15\ 000$.

The revenue, R, in dollars, from selling n laptop bags is given by $R = 80n$.

The cost, C, for the production of n laptop bags is shown in the following graph.



a. Draw the revenue equation line, $R = 80n$, on the same graph as the given cost equation. (1 MARK)

b. What profit will Crumprolo make if they sell 1500 laptop bags? (2 MARKS)

Adapted from VCAA 2014 Exam 2 Graphs and relations Q2a,b

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6D Lines of good fit – applications

STUDY DESIGN DOT POINT

- interpretation of a line of good fit, its intercept and slope in the context of the data

6A 6B 6C **6D**

KEY SKILLS

During this lesson, you will be:

- determining the equation of a line of good fit
- applying equations of lines of good fit.

Once a line of good fit has been drawn, its equation can be estimated. It is useful to be able to interpret the intercept and slope of the equation of a line of good fit as it provides information about the relationship between the two variables. The equation can also be used to make predictions using the observed data.

Determining the equation of a line of good fit

The equation of a line of good fit can be determined from a graph if at least two points can be clearly read. This method can be simplified if one of the known points is the y-intercept.

Recall that the general form of a linear function is $y = a + bx$, where

- a is the y-intercept
- b is the slope

The slope, also known as the gradient, can be calculated using the formula

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

where x_1, x_2, y_1 and y_2 represent the x and y values of two known coordinates (x_1, y_1) and (x_2, y_2).

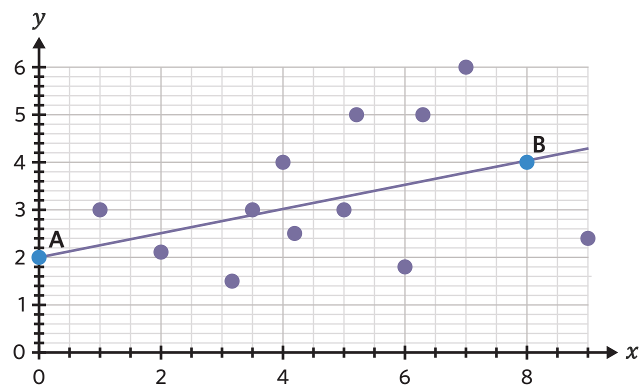
If the y-intercept can be read off the graph, it will be the value of a.

If not, after calculating the slope, b, one of the two known points can be substituted into the linear function to find the equation of the straight line.

See worked example 1
See worked example 2

Worked example 1

Find the equation of the line of good fit using the points A and B.



Continues →

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Explanation

Step 1: Determine the coordinates of points A and B.

A: (0, 2)

B: (8, 4)

Step 2: Calculate the slope.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{4 - 2}{8 - 0}$$
$$= \frac{2}{8}$$
$$= 0.25$$

Step 3: Determine the y-intercept.

$a = 2$

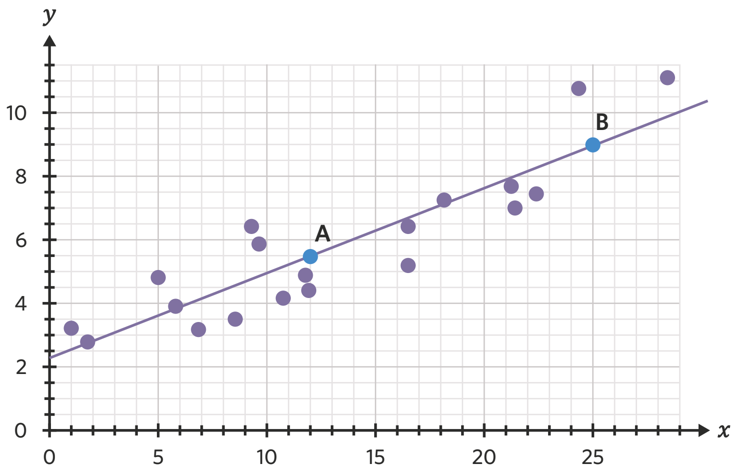
Step 4: Substitute the values of a and b into $y = a + bx$.

Answer

$y = 2 + 0.25x$

Worked example 2

Find the equation of the line of good fit using the points A and B, rounding to 2 decimal places.



Explanation

Step 1: Determine the coordinates of points A and B.

A: (12, 5.5)

B: (25, 9)

Step 2: Calculate the slope.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{9 - 5.5}{25 - 12}$$
$$= \frac{3.5}{13}$$
$$= 0.269...$$
$$\approx 0.27$$

Step 3: Determine the y-intercept.

Substitute $b = 0.269...$ and the point (25, 9) into $y = a + bx$.

$$9 = a + 0.269... \times 25$$
$$9 = a + 6.730...$$
$$a = 9 - 6.730...$$
$$= 2.269...$$
$$\approx 2.27$$

Step 4: Substitute the values of a and b into $y = a + bx$.

Answer

$y = 0.27 + 2.27x$

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Applying equations of lines of good fit

Once the equation of a line of good fit has been determined, the intercept, a, and the slope, b, can be used to provide further analysis about the relationship between the variables.

The intercept of the line of good fit is the expected value of the response variable when the explanatory variable is zero.

The slope of the line of good fit is the average change in the response variable for every one-unit increase in the explanatory variable.

For example, the equation of the line of good fit between the *price* ($) of a property and its *distance to CBD* (km) in Melbourne is

$\textit{price} = 1\ 050\ 000 - 17\ 000 \times \textit{distance to CBD}$

From this it can be interpreted that:

- When the *distance to CBD* is 0, the expected price of a property is $1 050 000.
- The price of a property decreases by $17 000, on average, for every additional kilometre away from the city.

The equation of a line of good fit can also be used to make predictions. This is achieved by substituting the known value into the equation and solving for the unknown value. As with making predictions using a graph, extrapolation has limited reliability as the trend seen in the data may not actually continue outside the domain of the data.

Worked example 3

Tallie, Emilie and Tara have Korean BBQ after every group study session. The equation of the line of good fit between the number of *hours spent studying* and the *Korean BBQ bill* ($) is

$\textit{Korean BBQ bill} = 35.6 + 8.4 \times \textit{hours spent studying}$

a. Interpret the slope of the line of good fit in terms of the variables *hours spent studying* and *Korean BBQ bill*.

Explanation

Step 1: Determine the value of the slope.

$b = 8.4$

Step 2: Interpret the slope in terms of the variables.

The slope is the average change in the response variable for every one-unit increase in the explanatory variable.

Answer

Tallie, Emilie and Tara's Korean BBQ bill increases by $8.40, on average, for each additional hour of study.

b. Interpret the intercept of the line of good fit in terms of the variables *hours spent studying* and *Korean BBQ bill*.

Explanation

Step 1: Determine the value of the intercept.

$a = 35.6$

Step 2: Interpret the intercept in terms of the variables.

The intercept is the expected value of the response variable when the explanatory variable is zero.

Answer

Tallie, Emilie and Tara's Korean BBQ bill is expected to be $35.60 when they do not study beforehand.

Continues →

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c. Tallie, Emilie and Tara begin studying at 10 am and finish at 4 pm before heading to the Korean BBQ restaurant. What is their expected Korean BBQ bill?

Explanation

Step 1: Determine the known value.

Tallie, Emilie and Tara begin studying at 10 am and finish at 4 pm.

$\textit{hours spent studying} = 6$

Step 2: Substitute the known value into the equation of the line of good fit and evaluate.

$\textit{Korean BBQ bill} = 35.6 + 8.4 \times 6$

$= 86$

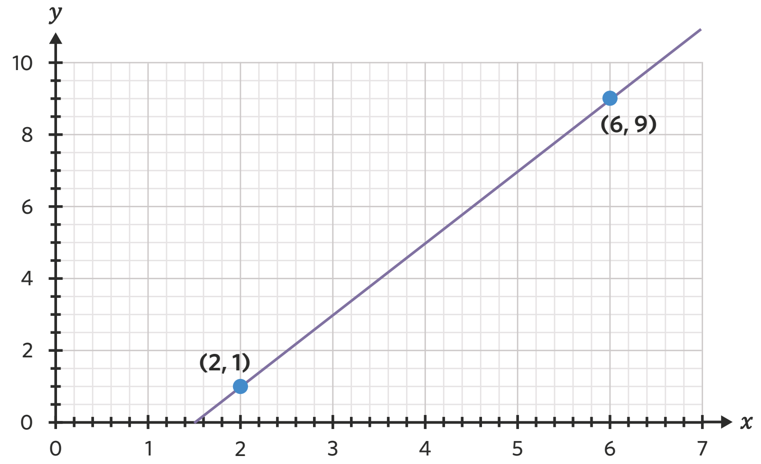
Answer

\$86

6D Questions

Determining the equation of a line of good fit

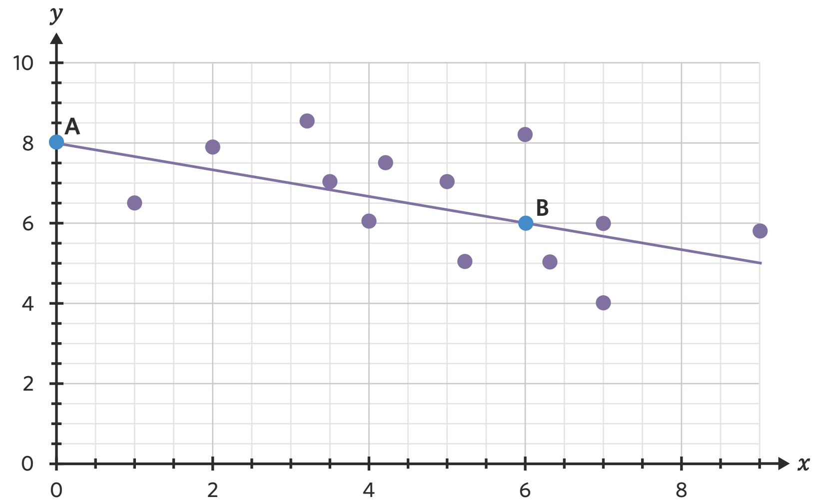
1. What is the equation of the following line?



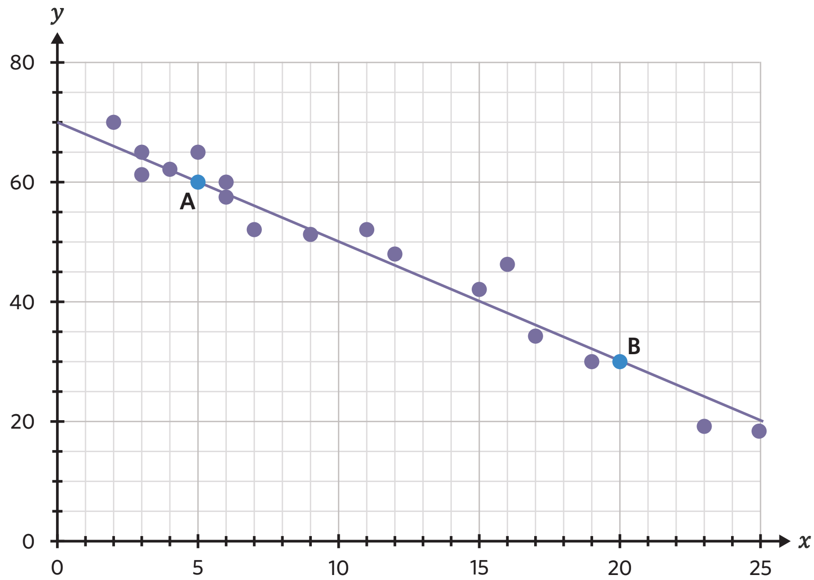
A. $y = 2 - 3x$ **B.** $y = 3 + 2x$ **C.** $y = -3 + 2x$ **D.** $y = 2 + 3x$

2. Find the equation of the line of good fit for the following scatterplots, using points A and B. Round to two decimal places where relevant.

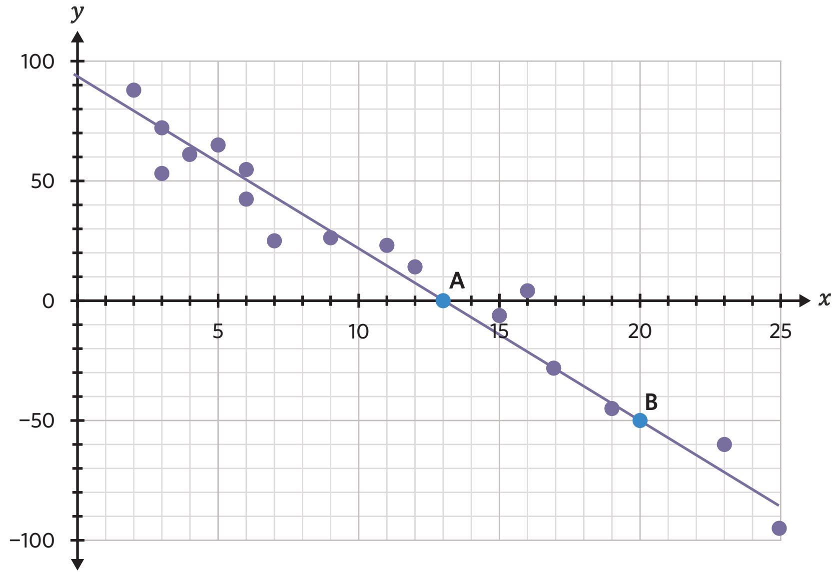
a.



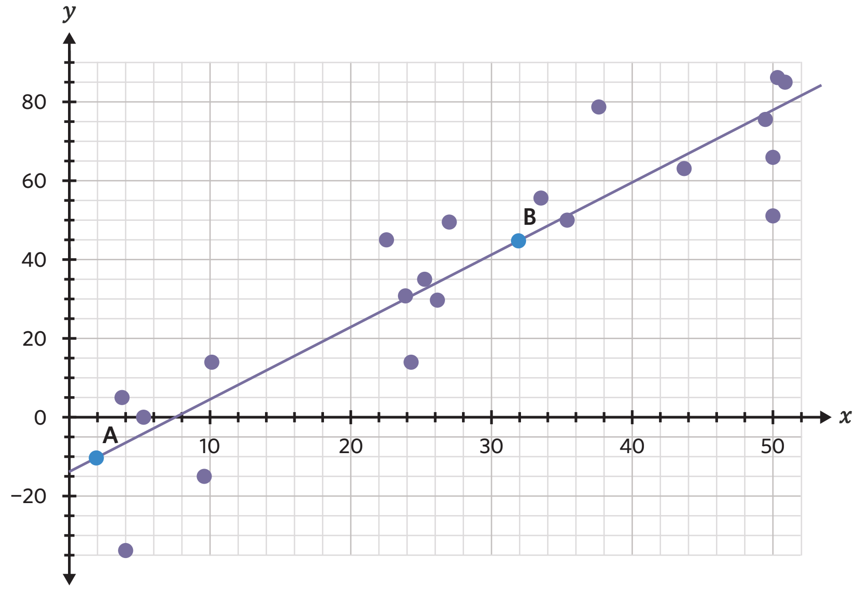
b.


y
80
60
40
20
0
A
B
0
5
10
15
20
25
x

c.


y
100
50
0
−50
−100
A
B
5
10
15
20
25
x

d.


y
80
60
40
20
0
−20
A
B
10
20
30
40
50
x

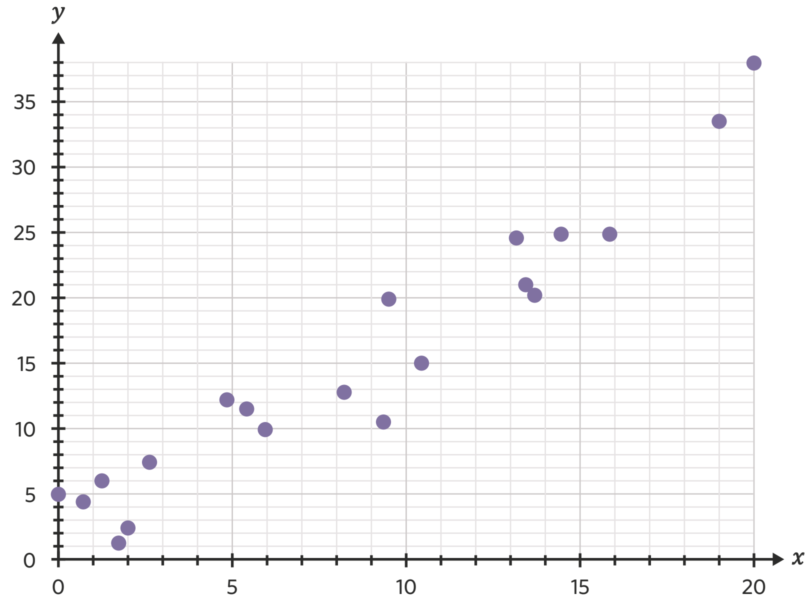
3. The speed, S (km/hr), of a car after t seconds can be modelled using a line of good fit with the equation

$S = 7 + 6.2t$

Which of the following pairs of coordinates do not lie along the line of good fit?

A. (2, 19.4) and (0.5, 10.1)
B. (0.7, 11.34) and (3.9, 31.18)
C. (8.5, 59.7) and (2.1, 20.02)
D. (3.3, 20.46) and (6.3, 39.06)

4. Which of these equations is the most appropriate line of good fit for the following scatterplot?



A. $y = 3x$
B. $y = 2 + x$
C. $y = 2 + \frac{3}{2}x$
D. $y = 5 - \frac{1}{2}x$

5. Charlie made a mistake while working out the equation of the line of good fit for some data. He chose two points on the line, but mistakenly used the coordinate (3, 7) instead of (3, 9).

The equation that Charlie found was $y = 1 + 2x$.

If the x-coordinate of the other point he used was -1, what is the correct equation of the line of good fit?

Applying equations of lines of good fit

6. Harry planted a bamboo plant. The equation of a line of good fit, where h is the height in centimetres, and d is the number of days since the bamboo was planted, is

$h = 10d + 5$

By how much does Harry's bamboo plant grow each day, on average?

A. 5 cm
B. 10 cm
C. 20 cm
D. 50 cm

7. Caitlin completed ten practice exam papers the week before her maths exam, and got 100%. Hannah did not complete any practice exam papers the week before the exam, and got 30%.

The equation of the line of good fit using these data points is shown.

$score = 30 + 7 \times practice\ exams$

a. Interpret the slope of the line of good fit.

b. Interpret the intercept of the line of good fit.

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c. Assuming the relationship between practice exam papers and maths exam score holds true for the rest of the class, what is the expected score of a student that completed six practice exam papers in the week before the exam?

d. A score of 50% is a pass. What is the minimum whole number of practice exam papers that need to be completed in the week before to pass the exam?

8. Toni has been studying pets and their owners and has estimated the equation of the line of good fit that best models her data. It relates the number of pets a student has, P, with the number of hours that a student spends outside per day, T.

$T = 1.5 + 0.75 \times P$

a. Interpret the slope of the line of good fit. Use minutes as the unit of time.

b. Interpret the intercept of the line of good fit. Use minutes as the unit of time.

c. Estimate the amount of time, in hours and minutes, that Toni's friend Karl spends outside per day given that he has two guinea pigs and a blue tongued lizard.

9. The relationship between the number of All Star players on an NBA team and the number of games that team wins in the regular season is captured in the following equation of the line of good fit.

$\textit{wins} = 27 + 11 \times \textit{all stars}$

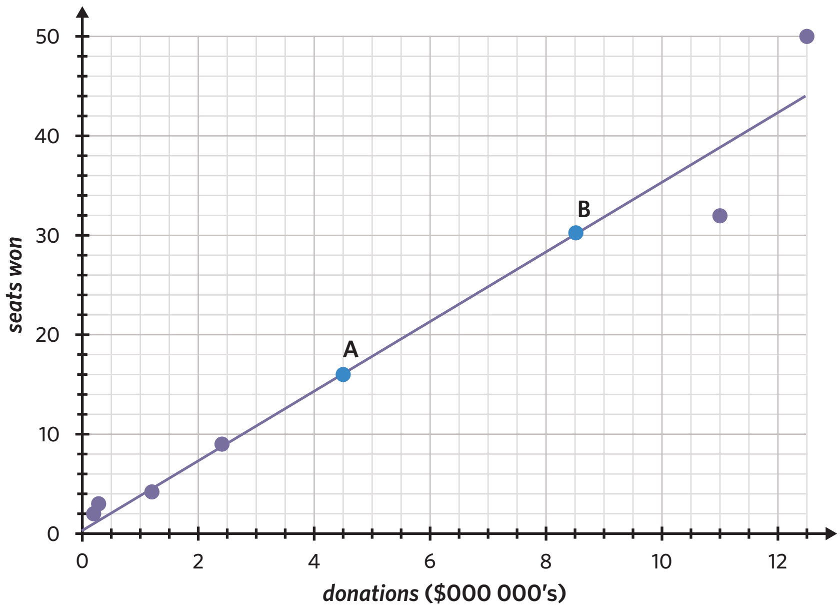
a. How many games is a team without an All Star expected to win?

b. How many games is a team with 3 All Stars expected to win?

c. The Minnesota Timberwolves gained an extra player, Rudy Gobert, who is an All Star. How many additional games are they expected to win due to the trade?

Joining it all together

10. A study was conducted to find the relationship between political *donations* ($000 000's) and *seats won* at an election. The results are shown in the following scatterplot, along with a line of good fit.



a. Find the equation of the line of good fit using points A and B.

b. Interpret the slope of the line of good fit.

c. Interpret the intercept of the line of good fit.

d. A new party wants to criminalise skinny jeans. They have received $7 500 000 in donations. How many seats are they expected to win?

e. It takes 51 seats to win a majority government in this election. Estimate the amount of donations required to win a majority government.

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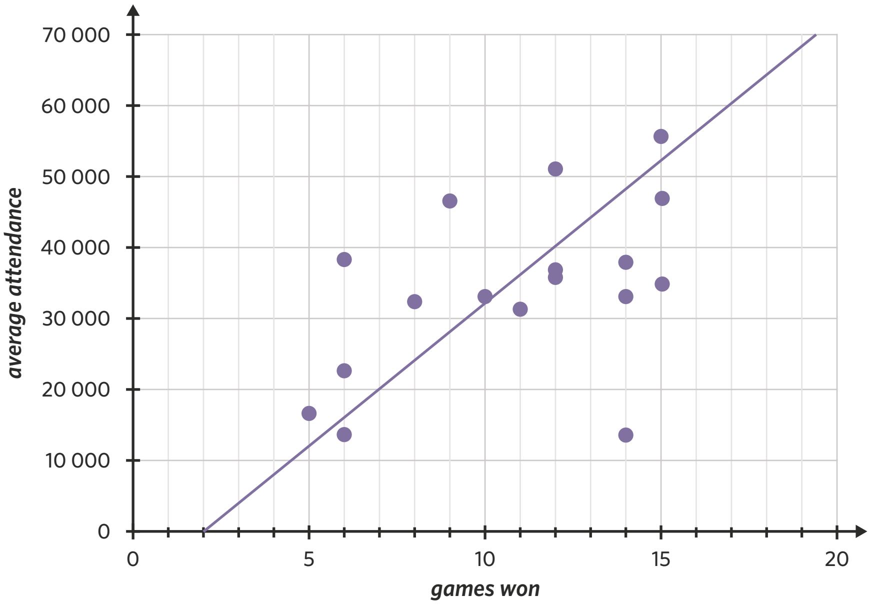
11. The following scatterplot shows the *average attendance* figures at home games, plotted against the number of *games won* in the 2017 AFL season. The line of good fit is also shown.



a. Determine the equation of the line of good fit using the points at 2 games won and 17 games won.

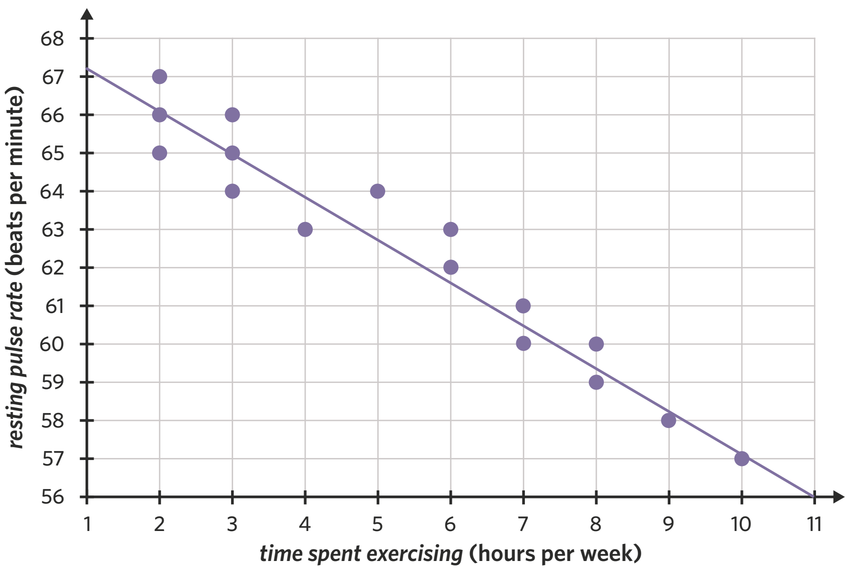
b. Interpret the slope of the line of good fit.

c. A new Tasmanian AFL team is expected to have an average attendance of 28 000 people. Estimate the number of games they are expected to win.

d. Discuss the limitation of extrapolation with reference to the value of the intercept.

Exam practice

12. The following scatterplot displays the *resting pulse rate*, in beats per minute, and the *time spent exercising*, in hours per week, of 16 students. A line of good fit has been fitted to the data.



The equation of this line of good fit is closest to

A. *resting pulse rate* $= 67.2 - 0.91 \times$ *time spent exercising*

B. *resting pulse rate* $= 67.2 - 1.10 \times$ *time spent exercising*

C. *resting pulse rate* $= 68.3 - 0.91 \times$ *time spent exercising*

D. *resting pulse rate* $= 68.3 - 1.10 \times$ *time spent exercising*

E. *resting pulse rate* $= 67.2 + 1.10 \times$ *time spent exercising*

Adapted from VCAA 2018 Exam 1 Data analysis Q8

46% of students answered this type of question correctly.

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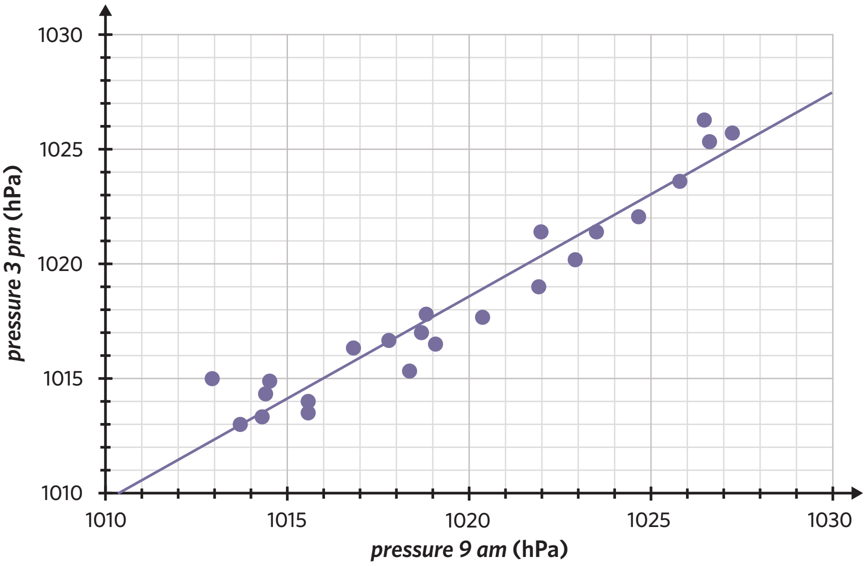
13. The following scatterplot shows the atmospheric pressure, in hectopascals (hPa), at 3 pm (*pressure 3 pm*) plotted against the atmospheric pressure, in hectopascals, at 9 am (*pressure 9 am*) for 23 days in November 2017 at a particular weather station.



Data: Australian Government, Bureau of Meteorology, <www.bom.gov.au/>

A line of good fit has been fitted to the scatterplot as shown. The equation of this line is

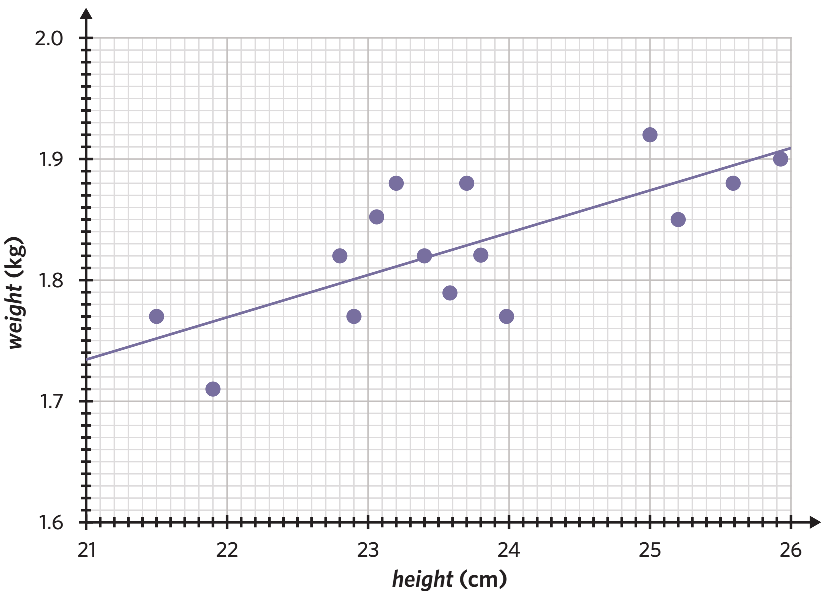
$pressure\ 3\ pm = 111.4 + 0.8894 \times pressure\ 9\ am$

Interpret the slope of this line of good fit in terms of the atmospheric pressure at this weather station at 9 am and at 3 pm. (1 MARK)

Adapted from VCAA 2019 Exam 2 Data analysis Q5a

44% of students answered this type of question correctly.

14. The following scatterplot shows the *height* and *weight*, both in centimetres, of 13 chihuahuas. A line of good fit has been fitted to the scatterplot with *height* as the explanatory variable.



The equation of the line of good fit is closest to

A. $height = 1.02 + 0.0342 \times weight$

B. $weight = 1.02 + 0.0342 \times height$

C. $height = 1.74 + 0.0342 \times weight$

D. $weight = 1.74 + 0.0342 \times height$

E. $weight = 1.74 + 0.0731 \times height$

Adapted from VCAA 2017 Exam 1 Data analysis Q8

43% of students answered this type of question correctly.

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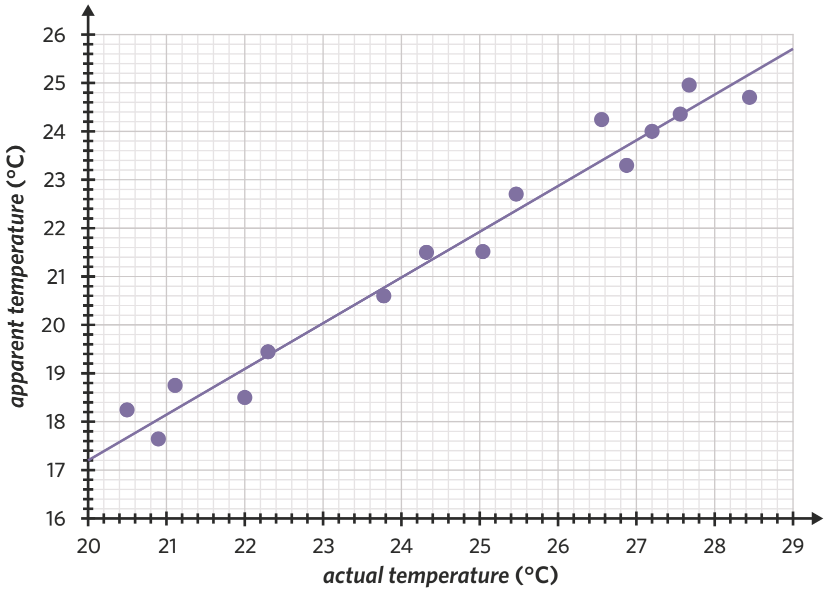
15. The data in the following scatterplot shows a sample of actual temperatures and apparent temperatures recorded at a weather station.



A line of good fit has the equation

apparent temperature $= -1.7 + 0.94 \times$ *actual temperature*

Interpret the intercept of the line of good fit in terms of the variables *apparent temperature* and *actual temperature*. (1 MARK)

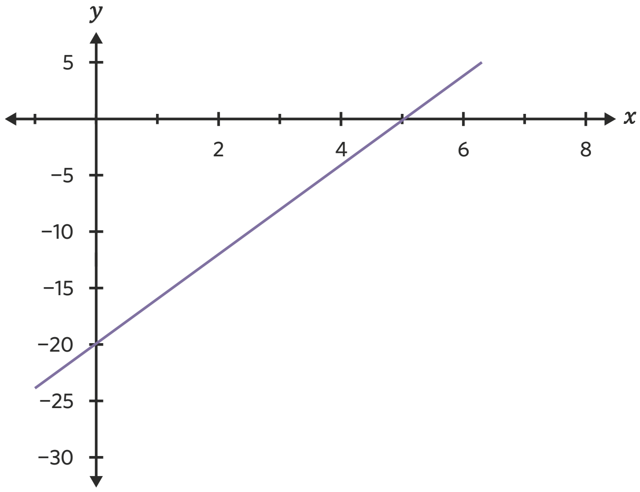
Adapted from VCAA 2016 Exam 2 Data analysis Q3bii

28% of students answered this type of question correctly.

Questions from multiple lessons

Graphs and relations

16. A straight line is shown.



The equation for this line is

A. $y = 4x - 20$

B. $y = 5x - 20$

C. $y = 20x + 5$

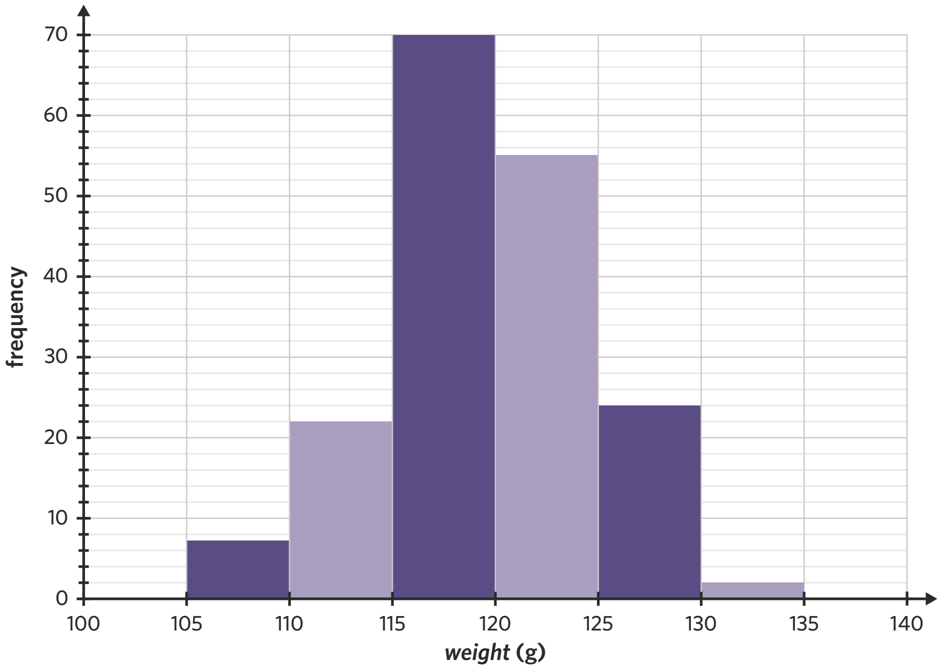
D. $x = 5y - 20$

E. $x = 20y + 5$

Adapted from VCAA 2015 Exam 1 Graphs and relations Q4

Data analysis

17. The following histogram shows the distribution of the *weight* of 180 ducks, in grams.



Ducks weighing 125 grams or higher are adopted as pets.

The percentage of ducks adopted as pets is closest to

A. 13% **B.** 14% **C.** 17% **D.** 24% **E.** 26%

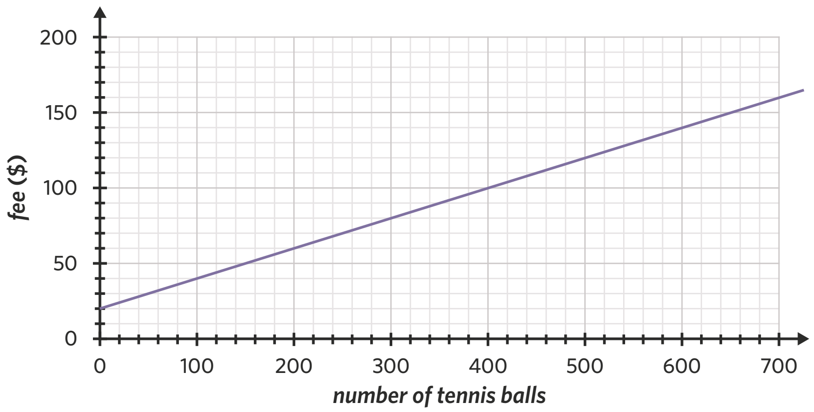
Adapted from VCAA 2017NH Exam 1 Data analysis Q2

Graphs and relations

18. A company wants to sell tennis balls.

Hopkins and Hoppers are two factories that produce tennis balls and have given the company two different offers that consist of a fixed amount and a charge per tennis ball made.

The following graph shows the offer made by Hopkins.



a. Complete the following sentence.

Hopkins charges a fixed amount of $ ______ and an additional $ ______ per tennis ball.
(1 MARK)

b. Hopkins and Hoppers both charge a fee of $100 for 400 tennis balls.
Hoppers' charge per tennis ball is $0.15.
Draw a linear relationship representing the fee charged by Hoppers for the number of tennis balls on the same graph as the linear relation that represents the fee charged by Hopkins. (1 MARK)

Adapted from VCAA 2019NH Exam 2 Graphs and relations Q3a,b

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7

UNIT 2 AOS 2
CHAPTER 7
Graphs and networks

LESSONS

KEY KNOWLEDGE

- introduction to the notations, conventions and representations of types and properties of graphs, including edge, loop, vertex, the degree of a vertex, isomorphic and connected graphs and the adjacency matrix
- description of graphs in terms of faces (regions), vertices and edges and the application of Euler's formula for planar graphs
- connected graphs: walks, trails, paths, cycles and circuits with practical applications
- weighted graphs and networks, and an introduction to the shortest path problem (solution by inspection only) and its practical application
- trees and minimum spanning trees, greedy algorithms and their use to solve practical problems.

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7A Introduction to graphs and networks

STUDY DESIGN DOT POINT

- introduction to the notations, conventions and representations of types and properties of graphs, including edge, loop, vertex, the degree of a vertex, isomorphic and connected graphs and the adjacency matrix

7A 7B 7C 7D 7E 7F

KEY SKILLS

During this lesson, you will be:

- identifying properties of graphs
- identifying and constructing graphs.

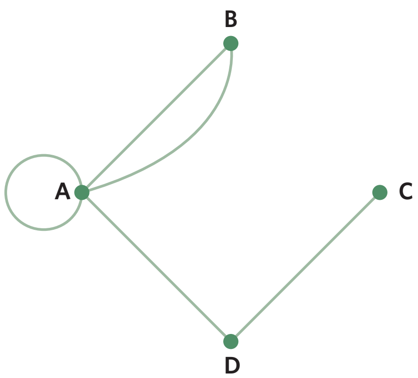
KEY TERMS

- Graph
- Vertex
- Edge
- Degree
- Loop
- Duplicate edges
- Connected graph
- Disconnected graph
- Bridge
- Simple graph
- Complete graph
- Isomorphic graphs

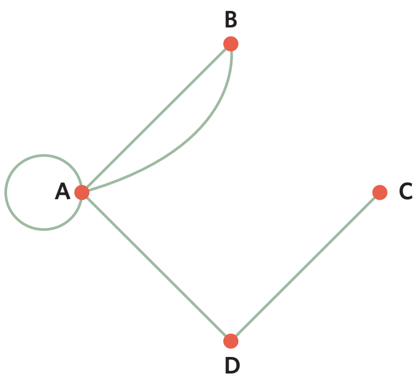
Many situations in everyday life involve connections. Graphs and networks are a way of visualising these connections in order to understand them better. They can be used to solve many types of problems.

Identifying properties of graphs

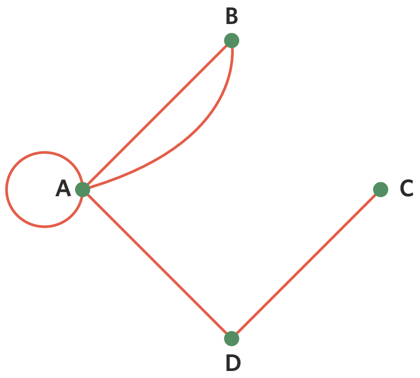
A **graph**, also known as a network, is a diagram that is used to show the connections between a group of common elements, such as objects, locations, people, or activities. Graphs are made up of vertices and edges.



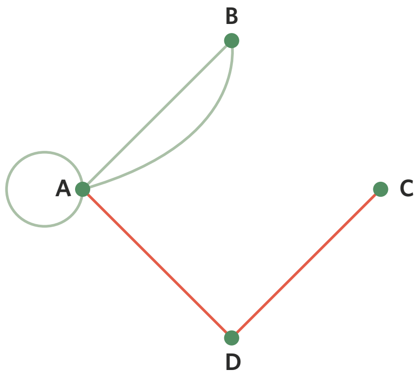
A **vertex** is a point on a graph. Points A, B, C and D are all vertices.



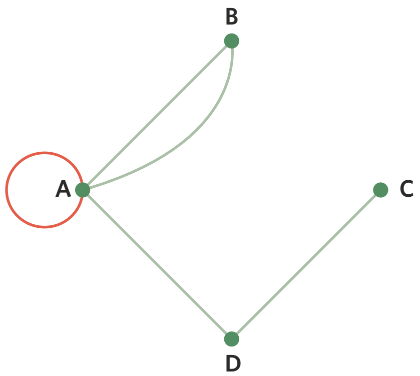
An **edge** is a line connecting one vertex to either another vertex or itself. They represent connections between vertices, and can overlap each other. The graph shown has 5 edges.



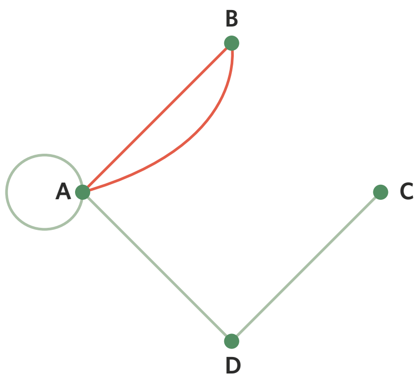
The **degree** of a vertex is equal to the number of edges that connect to it.
Vertex D has a degree of 2.



An edge that connects a vertex back to itself is called a **loop**. A loop adds two degrees to a vertex as it connects to the vertex twice. Vertex A has a loop, so vertex A has a degree of 5.

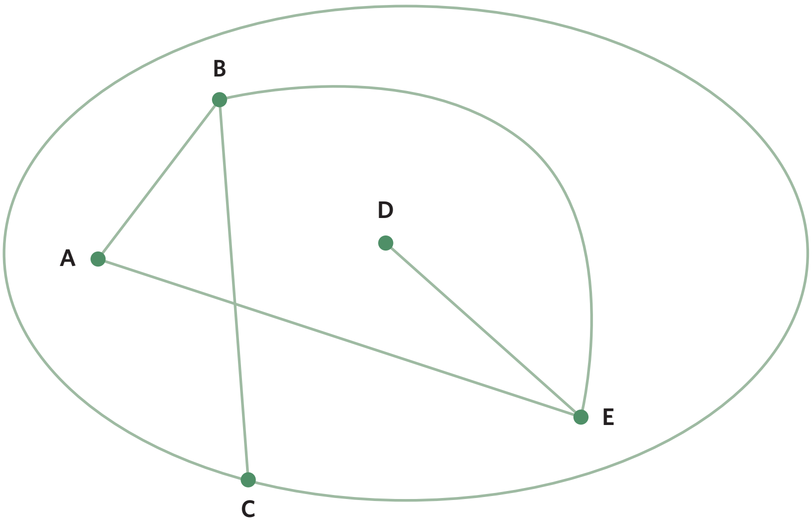


Duplicate edges are multiple edges that connect the same two vertices. There are duplicate edges connecting vertices A and B.



Worked example 1

Consider the following graph.



a. Determine the number of vertices and edges.

Explanation

Step 1: Count the vertices.

Vertices are the points on the graph. There are 5 vertices.

Step 2: Count the edges.

Edges are the lines connecting vertices. There are 6 edges.

Answer

5 vertices and 6 edges

b. Determine the degree of vertex E.

Explanation

The degree of a vertex is equal to the number of edges that connect to it.

Vertex E has 3 edges connected to it.

Answer

3

c. Which vertex has a loop?

Explanation

A loop is an edge that connects a vertex back to itself.

There is an edge connecting vertex C back to itself.

Answer

Vertex C

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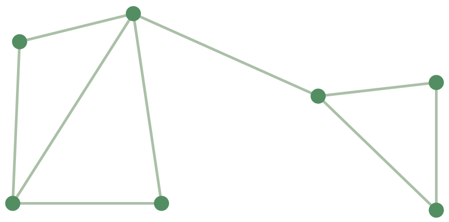
Identifying and constructing graphs

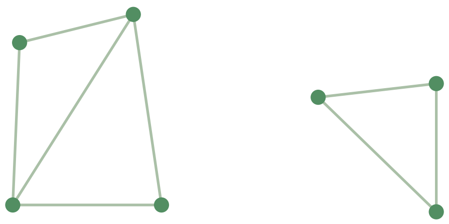
There are many types of graphs. These types include connected, disconnected, simple, complete, and isomorphic graphs.

A **connected graph** is a graph in which all vertices are connected to each other, either directly or indirectly. Every vertex is accessible from any other vertex by travelling along the edges.

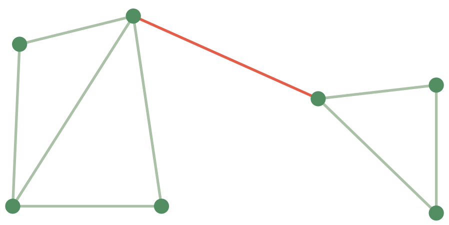
See worked example 2



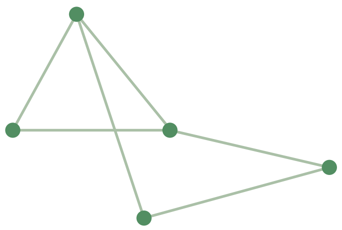
A **disconnected graph** is a graph in which it is not possible to reach every vertex from all other vertices.



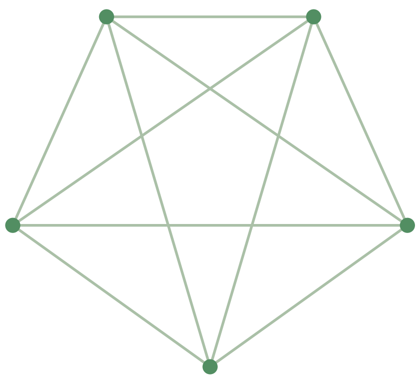
A **bridge** is an edge that is keeping a graph connected. If a bridge were removed, the graph would become disconnected.



A **simple graph** does not contain any loops or duplicate edges. Each pair of vertices is connected by a maximum of one edge.



A **complete graph** is a graph in which every vertex is directly connected to every other vertex exactly once.



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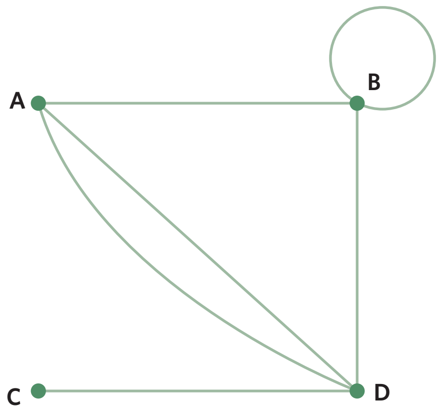
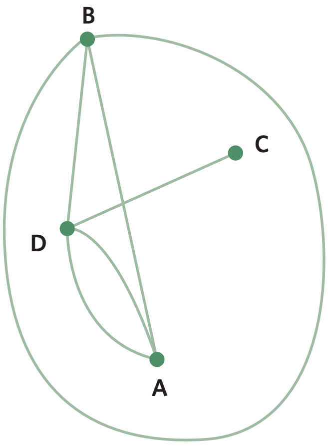
See worked example 3

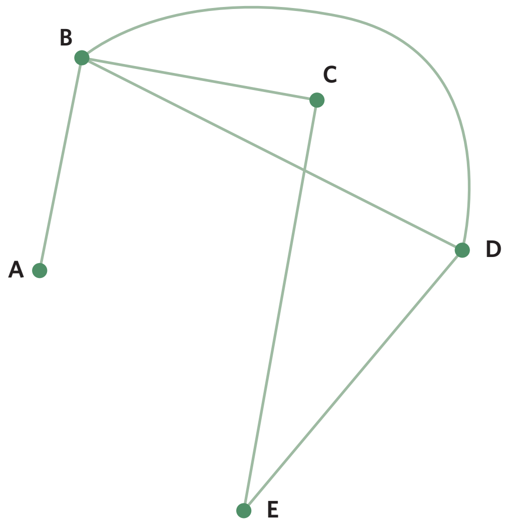
Isomorphic graphs are graphs that display the same information as each other. They have the same vertices and connections, but they are drawn differently. The following graphs are isomorphic.





Worked example 2

Consider the following graph.



a. Classify the graph using one or more of the following terms: connected, disconnected, simple, complete.

Explanation

Step 1: Determine whether the graph is connected or disconnected.

Every vertex is able to be reached by any other vertex by travelling along the edges. Hence, the graph is connected.

Step 2: Determine if the graph is simple.

A simple graph does not contain any loops or duplicate edges. There are two edges connecting vertices B and D. Hence, the graph is not simple.

Step 3: Determine if the graph is complete.

A complete graph is a graph in which every vertex is directly connected to every other vertex once. This is not the case here. For example, vertex A is not directly connected to vertex E. Hence, the graph is not complete.

Answer

Connected

Continues →

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b. Is there a bridge in the graph? If so, which vertices does it connect?

Explanation

If a bridge were removed, the graph would become disconnected.

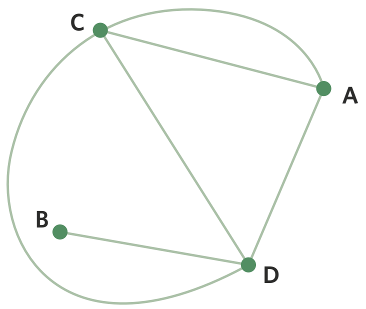
In this graph, if the edge between vertices A and B were removed, vertex A would become isolated. The graph would no longer be connected.

Answer

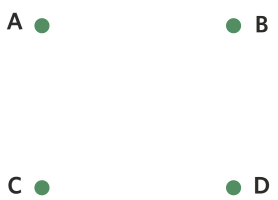
Yes, A and B

Worked example 3

Consider the following graph.



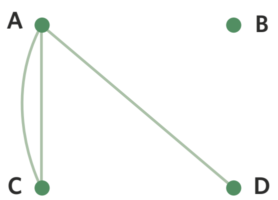
An unfinished graph containing the same vertices is shown. Draw the vertices so that the two graphs are isomorphic.



Explanation

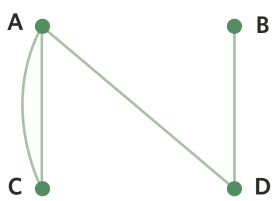
Step 1: Draw the edges connected to vertex A.

In the original graph, there are 2 edges connecting A and C and 1 edge connecting A and D.



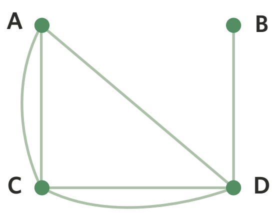
Step 2: Draw the edges connected to vertex B.

In the original graph, there is 1 edge connecting B and D.



Step 3: Continue for the remaining vertices.

Answer



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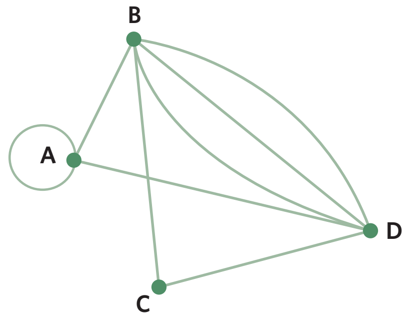
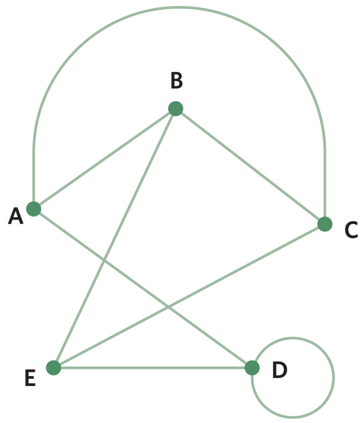
7A Questions

Identifying properties of graphs

1. Which of the vertices in the following graph has a degree of 4?
   A. Vertex A
   B. Vertex B
   C. Vertex C
   D. Vertex D

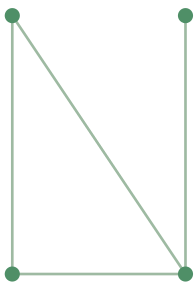


2. Consider the following graph.
   a. State the number of vertices and edges.
   b. Determine the degree of vertex C.
   c. Which vertex has a loop?

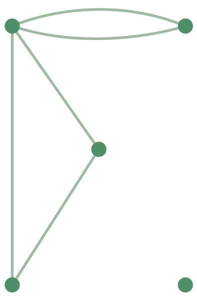


3. For each of the following graphs, state the number of vertices and edges.

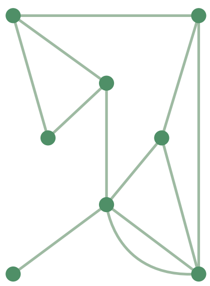
a.



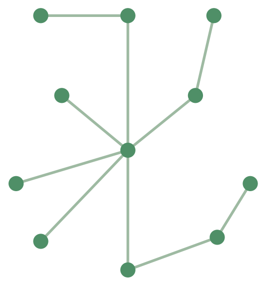
b.



c.

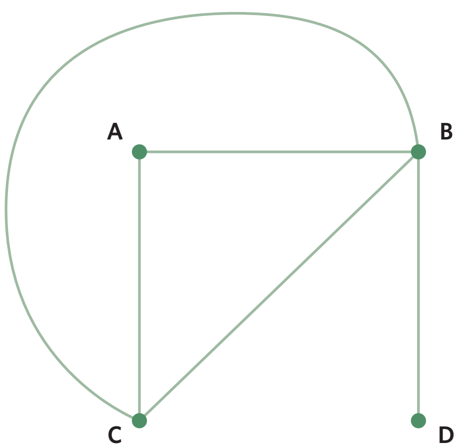


d.

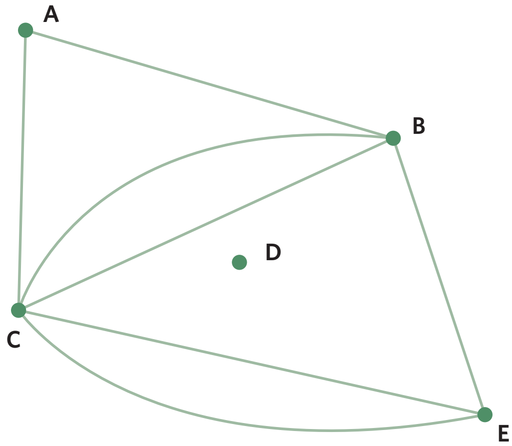


4. Determine the degree of vertex D in each of the following graphs.

a.



b.



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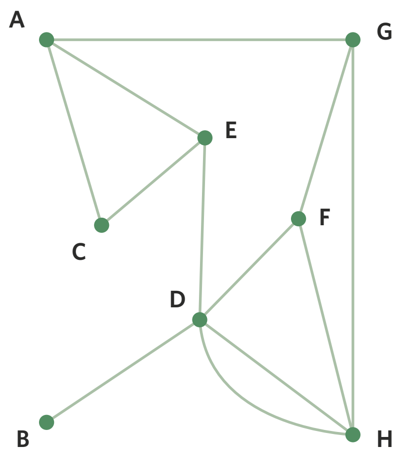
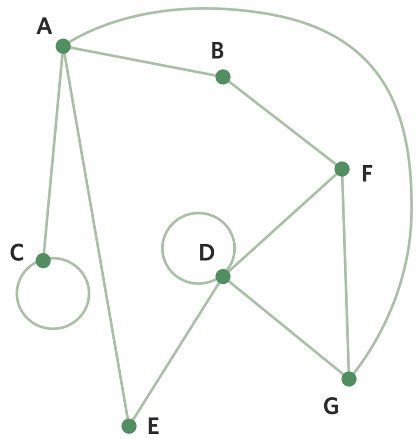
c.



d.



5. The following graph shows the roads between several towns in New Zealand's south island.

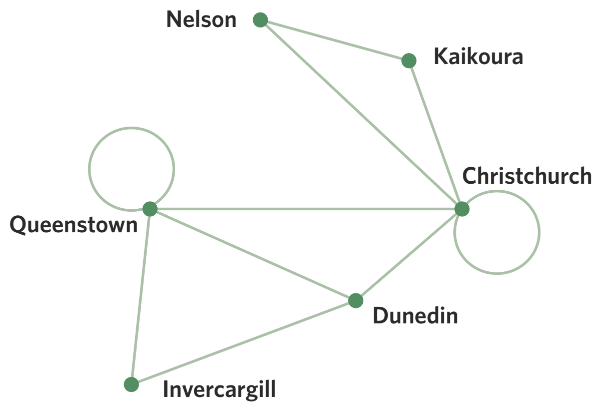
a. How many towns are shown?

b. How many roads are there in total?

c. How many roads lead into Dunedin?

d. Which town has the most roads leading to it?

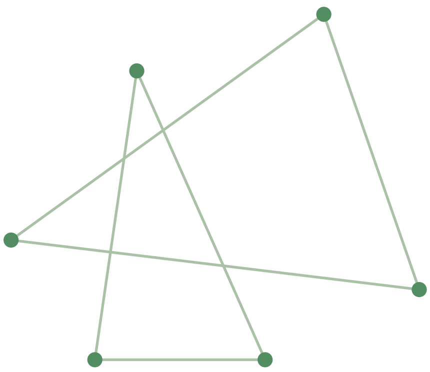
e. Which towns can be travelled to, starting from Queenstown and using only one road?



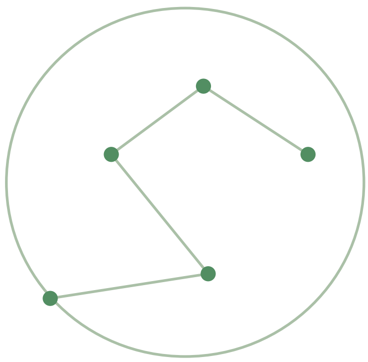
Identifying and constructing graphs

6. Which of the following graphs is not simple?

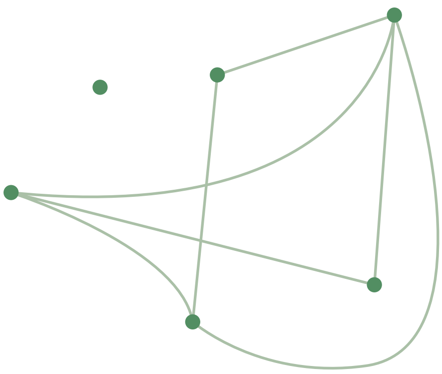
A.



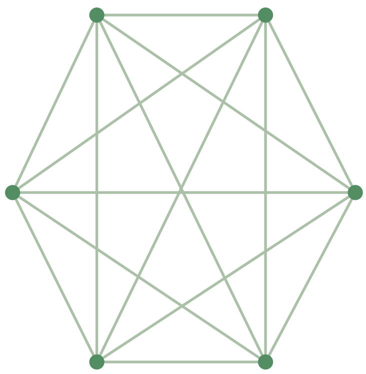
B.



C.



D.



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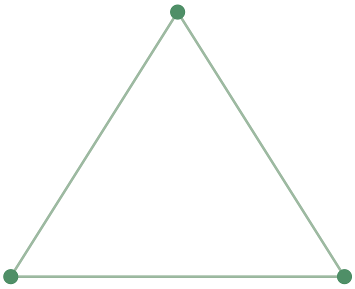
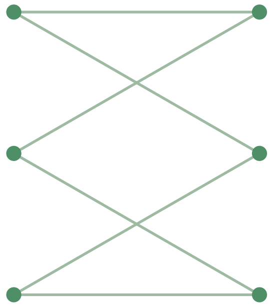
7. Classify each graph using one or more of the following terms: connected, disconnected, simple, complete.

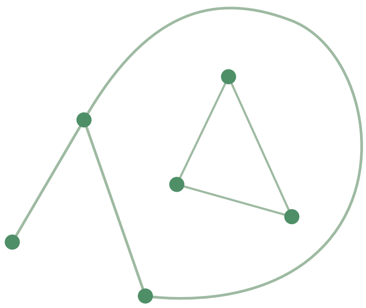
a.



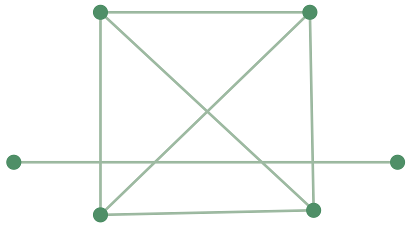
b.



c.

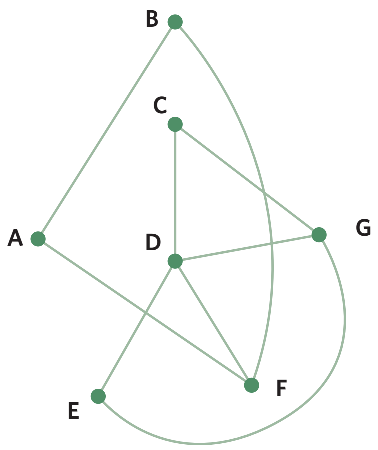


d.

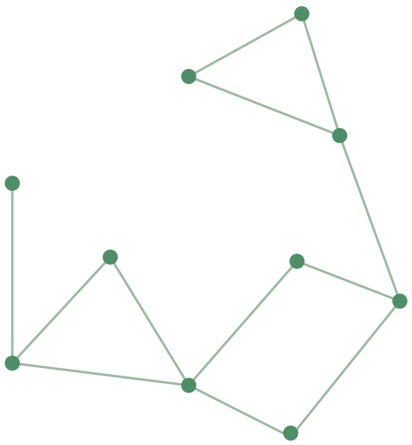


8. Consider the following graphs.

a. Between which two vertices is there a bridge?



b. How many bridges are in the following graph?



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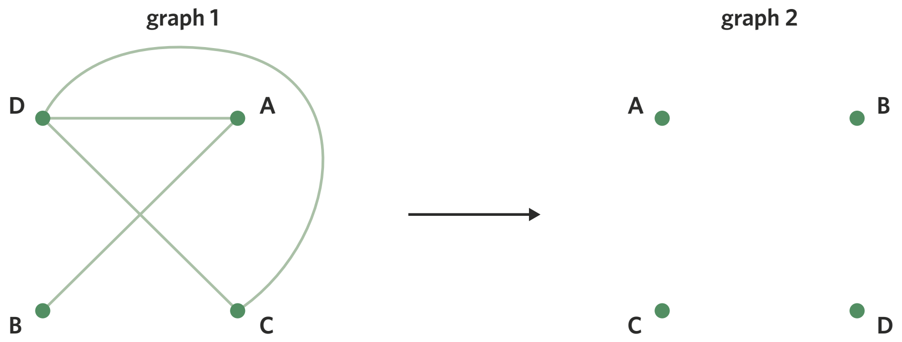
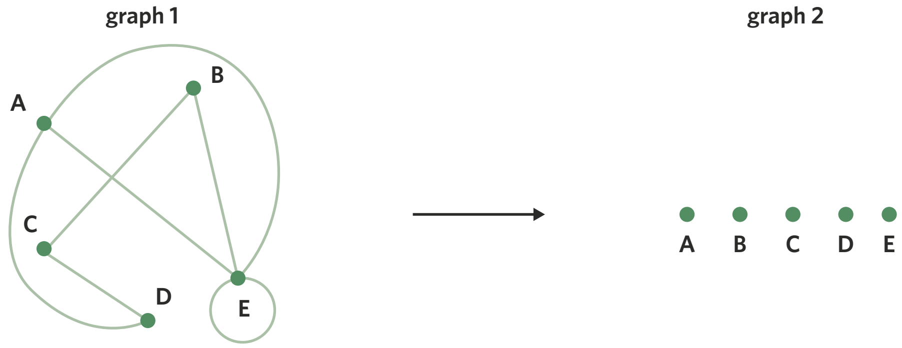
9. For each of the following, draw edges for graph 2 so that graphs 1 and 2 are isomorphic.

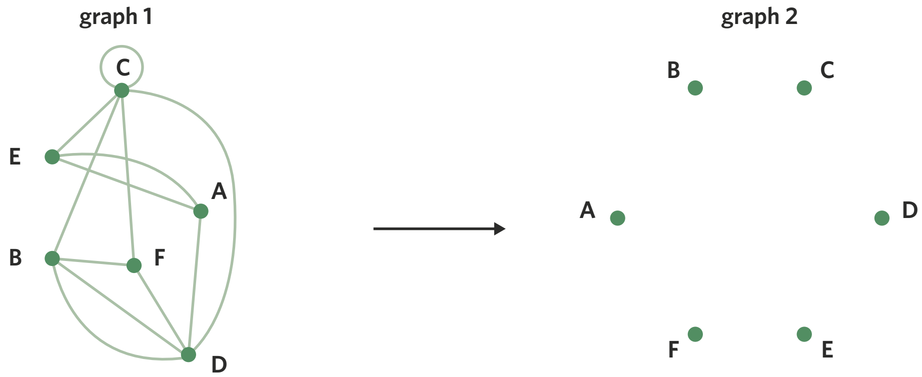
a.



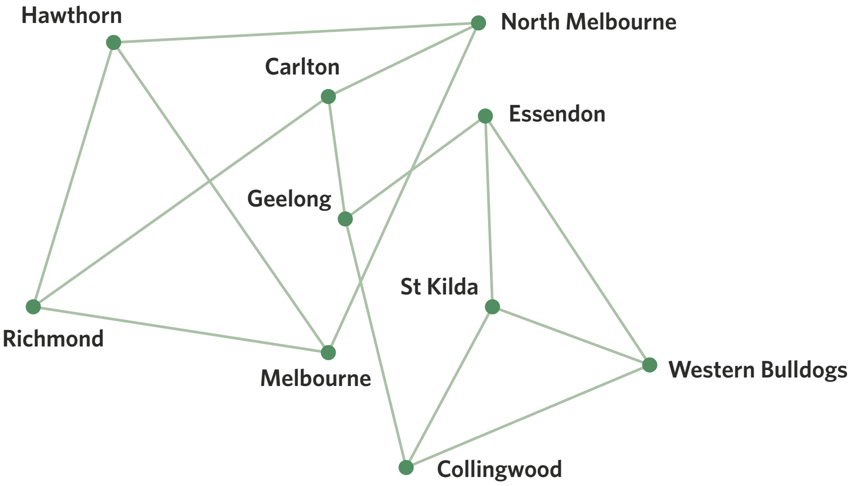
b.



c.



10. After the first three rounds of the AFL season, Zephyr constructed a graph to show which teams had already played each other. An edge on the graph indicates that the two teams have played each other.

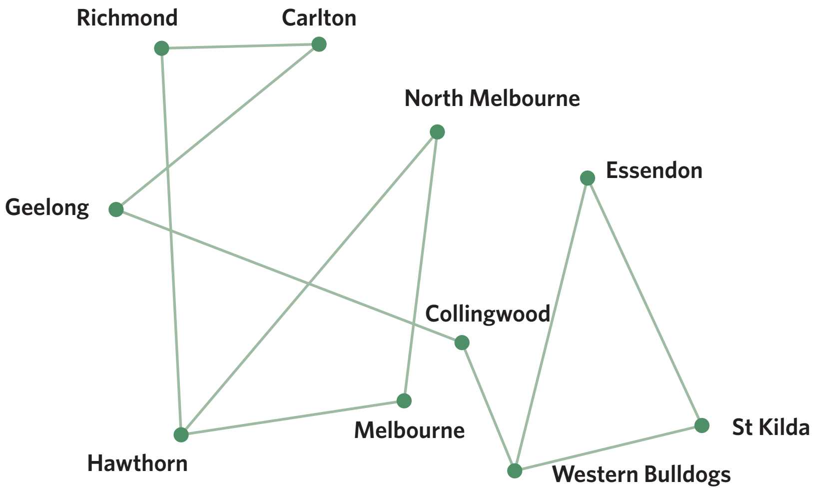


a. Which teams have the Western Bulldogs already played?

b. Classify the graph using one or more of the following terms: connected, disconnected, simple, complete.

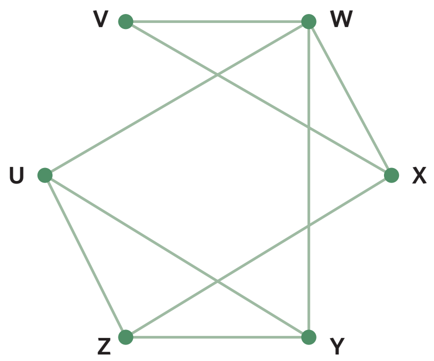
c. Is there a bridge on the graph? If so, which teams does it connect?

d. Zephyr's friend James constructed an isomorphic graph to show the same information, but left it unfinished. Finish the graph by adding the last four edges.



Joining it all together

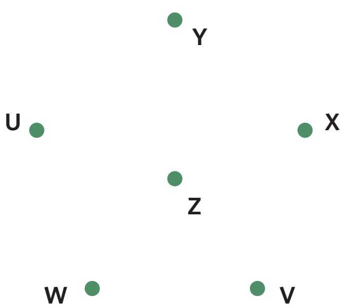
11. Wilhelmina made a graph to show the Facebook connections between her and 5 of her family members; Urma, Venus, Xanthe, Yosef, and Zeke. An edge on the graph indicates that the two people are Facebook friends.



a. Who has the most friends out of the family?

b. Who has the least friends out of the family?

c. Zeke also began to make a graph showing the family's Facebook connections but ran out of time to add the edges. Draw the edges to connect the following vertices.



d. Recently, Wilhelmina decided to block her parents, Urma and Yosef, on Facebook. Xanthe and Venus also finally accepted Yosef's Facebook friend request. Redraw Wilhelmina's original graph to reflect this information.

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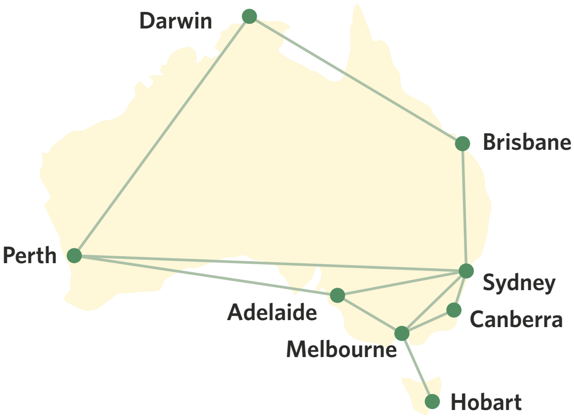
12. The following graph shows the flight paths between capital cities in Australia.



a. What is the total number of flight paths represented in the graph?

b. How many flights are available from Sydney?

c. Classify the graph using one or more of the following terms: connected, disconnected, simple, complete.

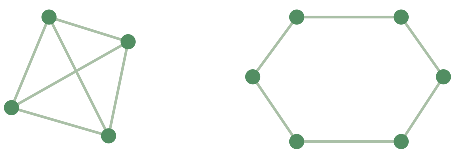
d. Determine the sum of the degrees of the vertices on the graph.

e. Is there a bridge in this graph? If so, which two cities does it connect?

f. In one million years, the capital cities will have moved around due to tectonic plate movements, to the positions shown in the following diagram. Draw the same flight paths onto the following vertices.

Exam practice

13. Two graphs, labelled graph 1 and graph 2, are shown.



graph 1 graph 2

The sum of the degrees of the vertices of graph 1 is

A. two less than the sum of the degrees of the vertices of graph 2.

B. one less than the sum of the degrees of the vertices of graph 2.

C. equal to the sum of the degrees of the vertices of graph 2.

D. one more than the sum of the degrees of the vertices of graph 2.

E. two more than the sum of the degrees of the vertices of graph 2.

VCAA 2017 Exam 1 Networks and decision mathematics Q2

83% of students answered this question correctly.

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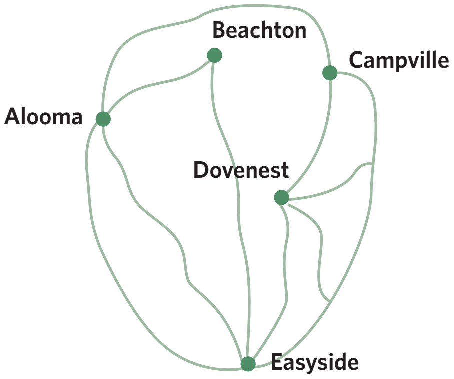
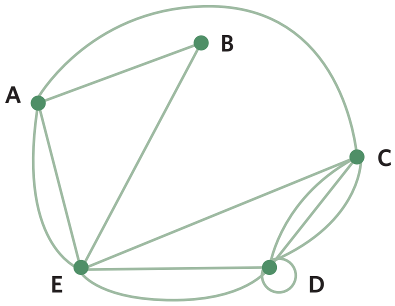
14. A map of the roads connecting five suburbs of a city, Alooma (A), Beachton (B), Campville (C), Dovenest (D) and Easyside (E), is shown.



a. Starting at Beachton, which two suburbs can be driven to using only one road? (1 MARK)

b. A graph that represents the map of the roads is shown.



One of the edges that connects to vertex E is missing from the graph.

Add the missing edge to the graph. (1 MARK)

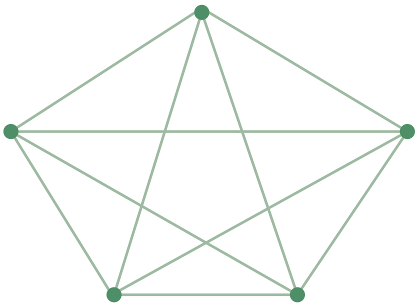
VCAA 2016 Exam 2 Networks and decision mathematics Q1a,bi

Part **a**: **83%** of students answered this question correctly.

Part **b**: **53%** of students answered this question correctly.

---

15. The following graph with five vertices is a complete graph.



Edges are removed so that the graph will have the minimum number of edges to remain connected.
The number of edges that are removed is

A. 4

B. 5

C. 6

D. 9

E. 10

VCAA 2016 Exam 1 Networks and decision mathematics Q3

35% of students answered this question correctly.

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Questions from multiple lessons

Recursion and financial modelling

16. Pauline deposited $3500 into a savings account with an interest rate of 2.8% per annum, compounding annually. Which one of the following recurrence relations can be used to determine the amount in the savings account, S_n, after n years?

A. $S_0 = 3500, \quad S_{n+1} = S_n + 98$

B. $S_0 = 3500, \quad S_{n+1} = 9.8 \times S_n$

C. $S_0 = 3500, \quad S_{n+1} = 2.8 \times S_n$

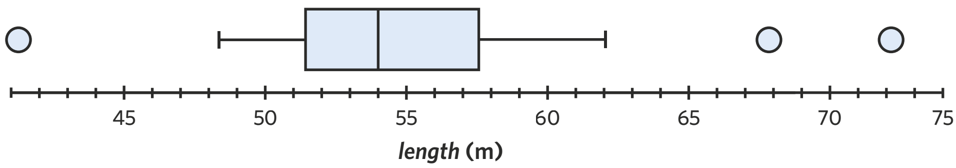
D. $S_0 = 3500, \quad S_{n+1} = 1.028 \times S_n$

E. $S_0 = 3500, \quad S_{n+1} = S_n + 2.8$

Adapted from VCAA 2017NH Exam 1 Recursion and financial modelling Q18

Data analysis

17. The lengths of 189 dragons, in metres, were recorded in the following boxplot.



The five-number summary for the length of these 189 dragons is closest to

A. 41.2, 51.4, 54, 57.4, 72

B. 41.2, 51.4, 54, 62, 67.8

C. 48.2, 51.4, 54, 57.4, 67.8

D. 48.2, 51.4, 54, 57.4, 62

E. 51.4, 54, 56.4, 62, 72

Adapted from VCAA 2017 Exam 1 Data analysis Q2

Recursion and financial modelling

18. A koala population is represented by the recurrence relation

$K_{2020} = 85\,000, \quad K_{n+1} = 0.8K_n,$

K_n is the projected population at the beginning of year n.

a. What is the projected population of koalas at the beginning of 2020? (1 MARK)

b. What is the projected percentage decrease of the koala population from year to year? (1 MARK)

c. What is the projected change in the population of koalas during the year 2021? (1 MARK)

Adapted from VCAA 2014 Exam 2 Number patterns Q1a,b,d

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7B The adjacency matrix and its applications

STUDY DESIGN DOT POINT

- introduction to the notations, conventions and representations of types and properties of graphs, including edge, loop, vertex, the degree of a vertex, isomorphic and connected graphs and the adjacency matrix

7A 7B 7C 7D 7E 7F

KEY SKILLS

During this lesson, you will be:

- constructing adjacency matrices and graphs
- constructing and interpreting directed graphs
- applying adjacency matrices to communication problems.

KEY TERMS

- Adjacency matrix
- Directed graph

Adjacency matrices can be used to display information shown in networks. They are useful for summarising connections or pathways between different vertices, and can be constructed from both directed and undirected networks. Communication matrices are a unique application of adjacency matrices.

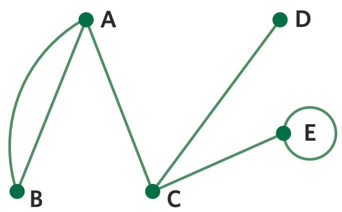
Constructing adjacency matrices and graphs

An **adjacency matrix** represents the number of direct connections between vertices in a graph. It is a square matrix that has a row and column for each vertex. Each element represents the number of edges connecting two vertices.

For example, the following graph can be used to construct the accompanying adjacency matrix.

See worked example 1

Note: A loop counts as one edge.



$$\begin{array}{c c} \begin{array}{ccccc} A & B & C & D & E \end{array} & \\ \begin{bmatrix} 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} \end{array}$$

The ‘2’ in row 1, column 2, indicates that there are two edges connecting vertices A and B. This is also shown in row 2, column 1.

The ‘1’ in row 5, column 5, indicates that there is a loop at vertex E.

The ‘0’ in row 2, column 4, indicates that there are no edges connecting vertices B and D.

The columns and rows of the adjacency matrix should add up to the degree of that vertex. However, if a loop is involved, the columns and rows will add up to one less than the degree of the vertex for each loop involved, as a loop is counted as two degrees but only one edge.

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An adjacency matrix for an undirected graph will be symmetric about the leading diagonal, as the number of edges between vertices A and B is the same as the number of edges between vertices B and A. This is shown in the following matrix.

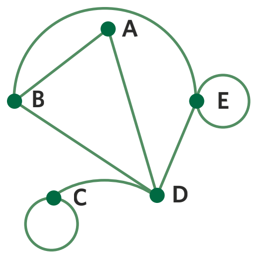
$$\begin{array}{cc} \begin{matrix} A & B & C & D & E \end{matrix} & \\ \begin{bmatrix} 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} & \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \end{array}$$

An adjacency matrix can also be used to construct a graph, by drawing the connections between each of the vertices as described by the matrix.

See worked example 2

Worked example 1

Consider the following graph.



a. Construct an adjacency matrix, *M*, from the graph.

Explanation

Step 1: Set up the adjacency matrix.

There are five vertices, A to E.

The matrix will be a 5 × 5 matrix with the rows and columns labelled A to E.

$$\begin{array}{cc} \begin{matrix} A & B & C & D & E \end{matrix} & \\ \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} & \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \end{array}$$

Step 2: Fill in the first row and column.

There are 0 edges between A and A.

There is 1 edge between A and B.

There are 0 edges between A and C.

There is 1 edge between A and D.

There are 0 edges between A and E.

$$\begin{array}{cc} \begin{matrix} A & B & C & D & E \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & & & & \\ 0 & & & & \\ 1 & & & & \\ 0 & & & & \end{bmatrix} & \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \end{array}$$

Step 3: Fill in the second row and column.

There are 0 edges between B and B.

There are 0 edges between B and C.

There is 1 edge between B and D.

There is 1 edge between B and E.

$$\begin{array}{cc} \begin{matrix} A & B & C & D & E \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & & & \\ 1 & 1 & & & \\ 0 & 1 & & & \end{bmatrix} & \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \end{array}$$

Step 4: Complete this process for the remaining rows and columns.

Note: The loops at vertices C and E count as 1 edge.

Continues →

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Answer

$$\begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} \end{array}$$

b. What does the element m_{33} represent?

Explanation

Step 1: Identify the element.

The element in row 3 (vertex C), column 3 (vertex C) is 1.

Step 2: Interpret the element.

Each element represents an edge connecting the vertices represented by the row and column.

The row and column are the same, so the edge is a loop.

Answer

There is a loop at vertex C.

Worked example 2

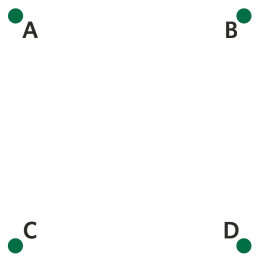
Use the following adjacency matrix to construct a graph.

$$\begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array}$$

Explanation

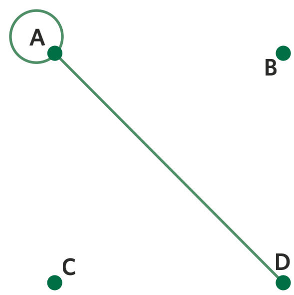
Step 1: Set up the vertices.

There should be 4 vertices, labelled A to D.



Step 2: Add the edges for vertex A.

Vertex A is connected to vertices A (a loop) and D.



Step 3: Repeat this process for the remaining vertices.

Vertex B is connected to vertices C and D.

Vertex C is connected to vertices B and D, with 2 connections to vertex D.

Vertex D is connected to vertices A, B and C, with 2 connections to vertex C.

Continues →

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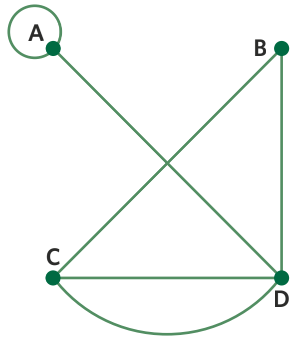
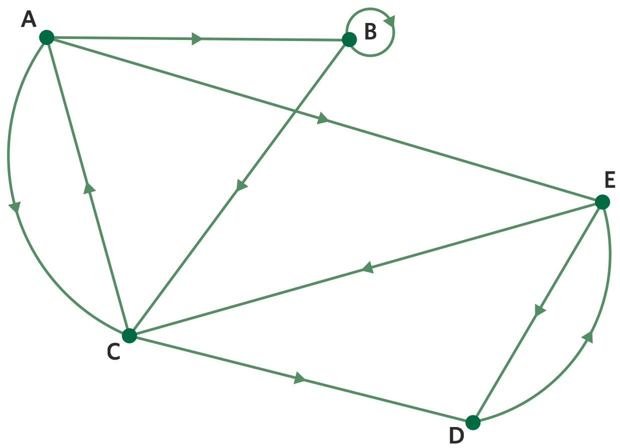
Answer



Constructing and interpreting directed graphs

A **directed graph** is a network containing arrows on each edge that show the direction in which one can travel between two vertices.

For example, the following directed graph shows the roads connecting five stores in a town.



The edge between vertices B and C shows a one-way road from store B to store C.

The edges between vertices E and D show that traffic can go in both directions between these stores.

The directed graph can be represented by the following adjacency matrix.

See worked example 3

$$\begin{array}{c} \text{to} \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \\ C \;\; \text{from} \\ D \\ E \end{array}$$

A directed graph can be represented by an adjacency matrix by labelling the rows ‘from’ and the columns ‘to’ in order to show the direction of the connection between vertices. Each element indicates the number of edges going from the row vertex to the column vertex. Adjacency matrices for directed graphs are usually not symmetric.

As with undirected graphs, directed graphs can be used to construct adjacency matrices and vice versa.

See worked example 4

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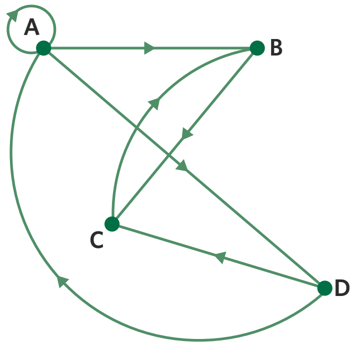
Worked example 3

Use the following directed graph to construct an adjacency matrix.



Explanation

Step 1: Set up the adjacency matrix.

There are four vertices, A to D.

The matrix will be a 4 × 4 matrix with the rows and columns labelled A to D.

The rows will be labelled 'from' and the columns will be labelled 'to'.

$$\begin{array}{cc} \begin{array}{cccc} & \text{to} & & \\ A & B & C & D \end{array} & \\ \begin{bmatrix} \ & \ & \ & \ \\ \ & \ & \ & \ \\ \ & \ & \ & \ \\ \ & \ & \ & \ \end{bmatrix} & \begin{array}{l} A \\ B \\ C \\ D \end{array} \ \text{from} \end{array}$$

Step 2: Fill in the first row.

There is an edge from vertex A to each of vertex A, B and D.

$$\begin{array}{cc} \begin{array}{cccc} & \text{to} & & \\ A & B & C & D \end{array} & \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ \ & \ & \ & \ \\ \ & \ & \ & \ \\ \ & \ & \ & \ \end{bmatrix} & \begin{array}{l} A \\ B \\ C \\ D \end{array} \ \text{from} \end{array}$$

Step 3: Repeat this process for the remaining rows.

There is an edge from vertex B to vertex C.

There is an edge from vertex C to vertex B.

There is an edge from vertex D to each of vertex A and C.

Answer

$$\begin{array}{cc} \begin{array}{cccc} & \text{to} & & \\ A & B & C & D \end{array} & \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{l} A \\ B \\ C \\ D \end{array} \ \text{from} \end{array}$$

Worked example 4

Use the following adjacency matrix to construct a directed graph.

$$\begin{array}{cc} \begin{array}{cccc} & \text{to} & & \\ A & B & C & D \end{array} & \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{l} A \\ B \\ C \\ D \end{array} \ \text{from} \end{array}$$

Continues →

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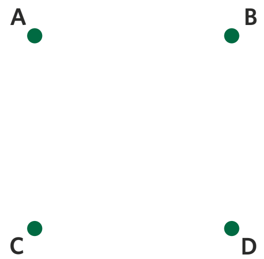
Explanation

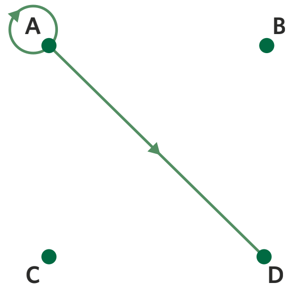
Step 1: Set up the vertices.

There are 4 vertices labelled A to D.



Step 2: Add the edges from A to the other vertices, represented by the first row of the adjacency matrix.

There is an edge from vertex A to each of vertex A (a loop) and vertex D.



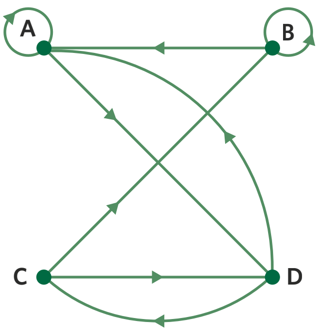
Step 3: Repeat this process for the remaining rows.

There is an edge from vertex B to each of vertex A and vertex B (a loop).

There is an edge from vertex C to each of vertex B and D.

There is an edge from vertex D to each of vertex A and C.

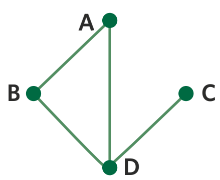
Answer



Applying adjacency matrices to communication problems

Adjacency matrices can be applied to various situations. One of these is communication problems, where the matrix is used to demonstrate the communication links between different vertices. These usually represent different types of connections between people or groups of people, such as social network connections, or communication paths between locations.

A one-step communication matrix models the number of direct connections between vertices. The following one-step communication matrix, denoted C, shows the one-step communication links in the accompanying network.



$$C = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}$$

A two-step link is an indirect path that connects two points through a middle point. A two-step communication matrix models the number of two-step links between points. It can be calculated by squaring the one-step communication matrix. The following two-step communication matrix, denoted C^2, shows the two-step communication links in the previous network.

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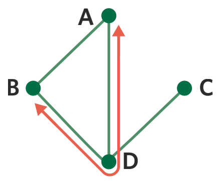
7B THEORY

$$C^2 = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 3 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}$$

The '1' in row 2, column 1, indicates that there is 1 way that A and B can communicate with each other via another vertex. This communication path is highlighted in the following network.

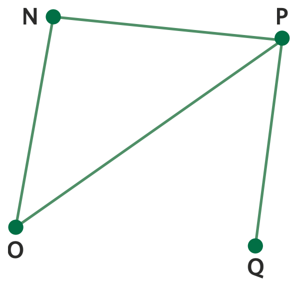


As with regular adjacency matrices, a communication matrix can also be directed. In this case, a directed communication matrix is constructed in the same way as a directed adjacency matrix.

Worked example 5

The following networks demonstrate the communication links between vertices. For each network, construct the communication matrix specified.

a. One-step:



Explanation

Step 1: Set up the communication matrix.

There are four vertices, N to Q.

The matrix will be a 4 × 4 matrix with the rows and columns labelled N to Q.

A one-step communication matrix is denoted by C.

$$C = \begin{array}{c} \begin{array}{cccc} N & O & P & Q \end{array} \\ \begin{bmatrix} \ & \ & \ & \ \\ \ & \ & \ & \ \\ \ & \ & \ & \ \\ \ & \ & \ & \ \end{bmatrix} \end{array} \begin{array}{c} \\ N \\ O \\ P \\ Q \end{array}$$

Step 2: Fill in the first row and column.

There is a connection from vertex N to each of vertex O and P.

$$C = \begin{array}{c} \begin{array}{cccc} N & O & P & Q \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & & & \\ 1 & & & \\ 0 & & & \end{bmatrix} \end{array} \begin{array}{c} \\ N \\ O \\ P \\ Q \end{array}$$

Step 3: Repeat this process for the remaining rows and columns.

There is a connection from vertex O to each of vertex N and P.

There is a connection from vertex P to each of vertex N, O and Q.

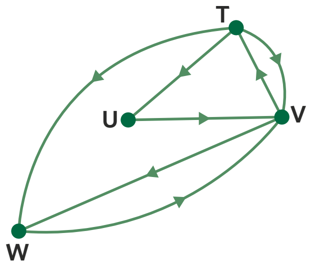
There is a connection from vertex Q to vertex P.

Continues →

Answer

$$C = \begin{array}{c} \begin{array}{cccc} \text{N} & \text{O} & \text{P} & \text{Q} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \text{N} \\ \text{O} \\ \text{P} \\ \text{Q} \end{array}$$

b. Two-step:



Explanation

Step 1: Set up the one-step communication matrix.

There are four vertices, T to W.

The matrix will be a 4 × 4 matrix with the rows and columns labelled T to W.

A one-step communication matrix is denoted by C.

As the graph is directed, the rows will be labelled 'from' and the columns will be labelled 'to'.

$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} \text{T} & \text{U} & \text{V} & \text{W} \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \text{T} \\ \text{U} \\ \text{V} \\ \text{W} \end{array} \text{from}$$

Step 2: Fill in the first row.

There is a connection from vertex T to each of vertex U, V and W.

$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} \text{T} & \text{U} & \text{V} & \text{W} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \text{T} \\ \text{U} \\ \text{V} \\ \text{W} \end{array} \text{from}$$

Step 3: Repeat this process for the remaining rows.

There is a connection from vertex U to vertex V.

There is a connection from vertex V to each of vertex T and W.

There is a connection from vertex W to vertex V.

$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} \text{T} & \text{U} & \text{V} & \text{W} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \text{T} \\ \text{U} \\ \text{V} \\ \text{W} \end{array} \text{from}$$

Step 4: Square the matrix to show the two-step communications.

A two-step communication matrix is denoted C^2.

Answer

$$C^2 = \begin{array}{c} \text{to} \\ \begin{array}{cccc} \text{T} & \text{U} & \text{V} & \text{W} \end{array} \\ \begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ \text{T} \\ \text{U} \\ \text{V} \\ \text{W} \end{array} \text{from}$$

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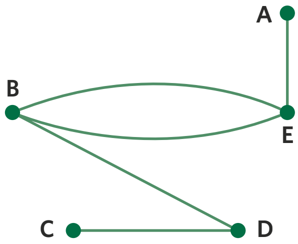
7B Questions

Constructing adjacency matrices and graphs

1. Consider the following graph and its adjacency matrix.



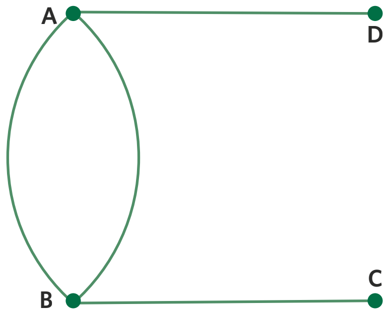
$$\begin{array}{cc} \begin{array}{ccccc} A & B & C & D & E \end{array} & \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & & 0 & 0 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} \end{array}$$

The missing element is

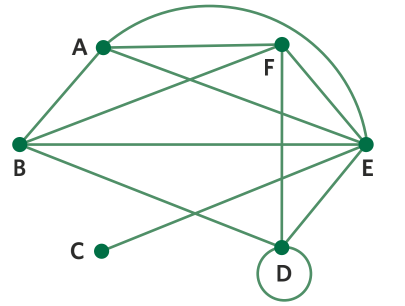
A. 0 **B.** 1 **C.** 2 **D.** 3

2. Construct an adjacency matrix for each of the following graphs.

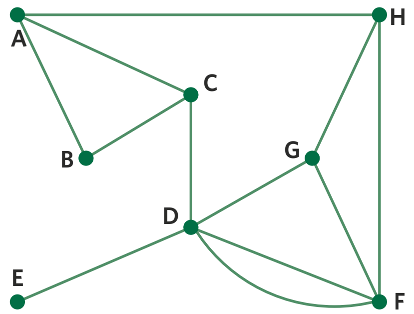
a.



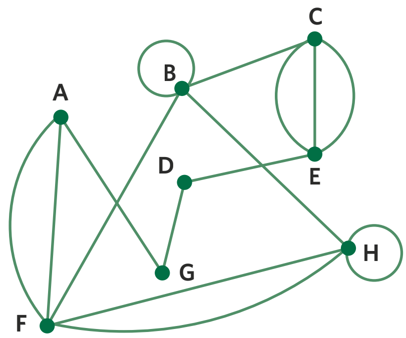
b.



c.



d.



3. Construct a graph for each of the following adjacency matrices.

a.
$$\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

b.
$$\begin{array}{cc} \begin{array}{cccc} A & B & C & D \end{array} & \\ \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} & \begin{array}{c} A \\ B \\ C \\ D \end{array} \end{array}$$

c.
$$\begin{array}{cc} \begin{array}{ccccc} H & I & J & K & L \end{array} & \\ \begin{bmatrix} 0 & 0 & 2 & 1 & 0 \\ 0 & 1 & 0 & 2 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} & \begin{array}{c} H \\ I \\ J \\ K \\ L \end{array} \end{array}$$

d.
$$\begin{array}{cc} \begin{array}{cccccc} Q & R & S & T & U & V \end{array} & \\ \begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 1 \\ 1 & 0 & 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 & 0 & 2 \\ 1 & 0 & 0 & 1 & 2 & 0 \end{bmatrix} & \begin{array}{c} Q \\ R \\ S \\ T \\ U \\ V \end{array} \end{array}$$

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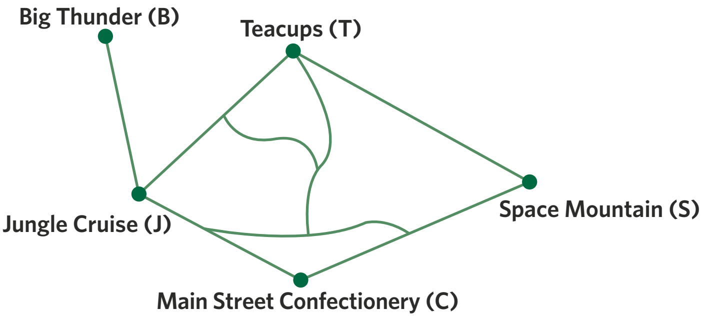
4. The following adjacency matrix represents the routes between six popular stores in a shopping centre.

$$\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \left[\begin{array}{cccccc} 1 & 2 & 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 1 & 2 & 0 & 1 \\ 1 & 2 & 2 & 0 & 2 & 1 \\ 1 & 0 & 0 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 & 2 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}$$

Use the matrix to construct a graph.

---

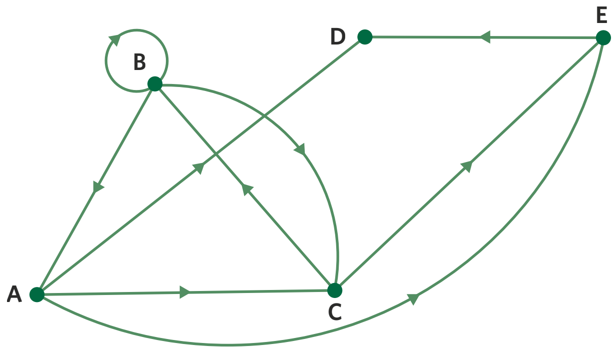
5. Robin is going to Disney World. He constructs the following network that shows the paths between his four favourite rides and his favourite food destination.



Use the network to construct an adjacency matrix that represents the paths between the five destinations. Label the columns and rows in alphabetical order.

Constructing and interpreting directed graphs

6. Consider the following directed graph.



How many vertices can be reached directly from vertex B?

A. 1 **B.** 2 **C.** 3 **D.** 4

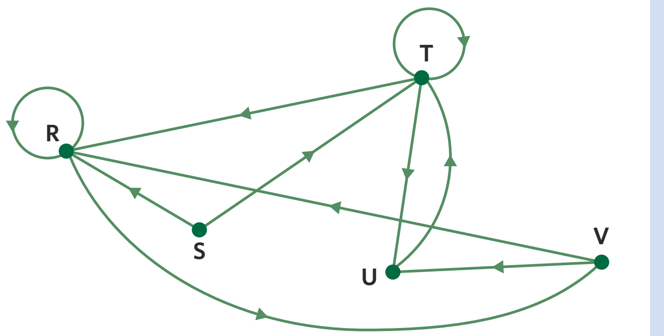
---

7. Consider the following directed graph.

a. Use the graph to construct an adjacency matrix.

b. Which vertices can be used to travel directly to vertex R?

c. Which vertex is it impossible to reach?



8. Construct a directed graph from each of the following adjacency matrices.

a.

to

$$\begin{array}{c}\begin{array}{ccc}A & B & C\end{array}\\ \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 0\\ 0 & 1 & 1\end{bmatrix}\end{array}\begin{array}{l}A\\B\\C\end{array}\text{ from}$$

b.

to

$$\begin{array}{c}\begin{array}{cccc}A & B & C & D\end{array}\\ \begin{bmatrix}1 & 0 & 0 & 1\\ 1 & 1 & 1 & 1\\ 0 & 0 & 1 & 1\\ 0 & 1 & 1 & 0\end{bmatrix}\end{array}\begin{array}{l}A\\B\\C\\D\end{array}\text{ from}$$

c.

to

$$\begin{array}{c}\begin{array}{ccccc}E & F & G & H & I\end{array}\\ \begin{bmatrix}0 & 1 & 1 & 1 & 0\\ 0 & 0 & 0 & 1 & 0\\ 1 & 1 & 0 & 0 & 0\\ 1 & 0 & 0 & 1 & 1\\ 1 & 0 & 1 & 0 & 1\end{bmatrix}\end{array}\begin{array}{l}E\\F\\G\\H\\I\end{array}\text{ from}$$

d.

to

$$\begin{array}{c}\begin{array}{cccccc}H & I & J & K & L & M\end{array}\\ \begin{bmatrix}1 & 0 & 1 & 1 & 0 & 1\\ 0 & 1 & 1 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 1\\ 1 & 0 & 0 & 0 & 1 & 0\\ 1 & 1 & 1 & 1 & 0 & 1\\ 0 & 0 & 1 & 0 & 0 & 0\end{bmatrix}\end{array}\begin{array}{l}H\\I\\J\\K\\L\\M\end{array}\text{ from}$$

---

9. The following information details the direct flights available on a particular day between five different countries.

- From Australia (A) to New Zealand (N) and Indonesia (I)
- From New Zealand to Australia, Vietnam (V) and Thailand (T)
- From Indonesia to Vietnam and Thailand
- From Vietnam to Indonesia and Thailand
- From Thailand to Australia, Vietnam and Indonesia

a. Use this information to construct a directed graph.

b. Use this information to construct an adjacency matrix. Label the columns and rows in alphabetical order.

Applying adjacency matrices to communication problems

10. Consider the following one-step communication matrix.

$$C = \begin{array}{c}\begin{array}{ccc}A & B & C\end{array}\\ \begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}\end{array}\begin{array}{l}A\\B\\C\end{array}$$

Which of the following statements is false?

A. B cannot communicate with A.

B. A can communicate with C.

C. C can communicate with B.

D. If a communication network was constructed from this matrix, there would be 0 loops.

---

11. The following communication matrix shows the one-step communication links between Adam (A), Beau (B), Cassie (C) and Deandra (D).

to

$$C = \begin{array}{c}\begin{array}{cccc}A & B & C & D\end{array}\\ \begin{bmatrix}0 & 0 & 1 & 1\\ 1 & 0 & 1 & 1\\ 1 & 0 & 0 & 0\\ 1 & 1 & 1 & 0\end{bmatrix}\end{array}\begin{array}{l}A\\B\\C\\D\end{array}\text{ from}$$

a. How many people can Deandra send a message to?

b. How many people can Beau send a message to via another person?

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12. The following one-step communication matrix shows the communication links between six different security patrol stations.

$$C = \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}$$

Use the matrix to construct a communication network.

13. Alison (A) likes playing computer games with her friends Baxter (B), Candace (C) and Dinesh (D).

Recently, there was a glitch in one of the games they were playing and each player could only send messages to certain players.

- Alison could send messages to Baxter and Dinesh.
- Baxter could send messages to Candace.
- Candace could send messages to Alison and Baxter.
- Dinesh could send messages to Baxter and Candace.

a. Use this information to construct a directed communication network.

b. Construct a one-step communication matrix.

c. Construct a two-step communication matrix.

Joining it all together

14. Consider the following undirected adjacency matrix.

$$\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 2 & & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}$$

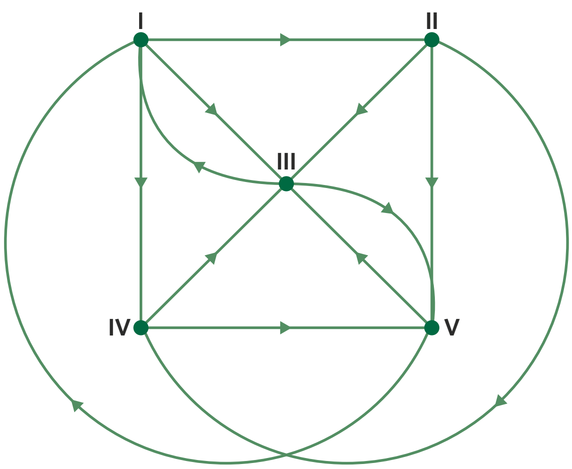
a. The missing element is

A. 0 **B.** 1 **C.** 2 **D.** 3

b. If the adjacency matrix was directed, would it be possible to determine the missing element? Why or why not?

c. If a network was constructed from this matrix, how many loops would there be?

15. The following directed graph shows the shipping routes between five main ports.



a. Use the directed graph to construct an adjacency matrix.

b. Which ports can ship directly to port V?

c. Which ports receive shipped goods from port III?

The matrix from part **a** also represents the one-step communication links between each of the ports.

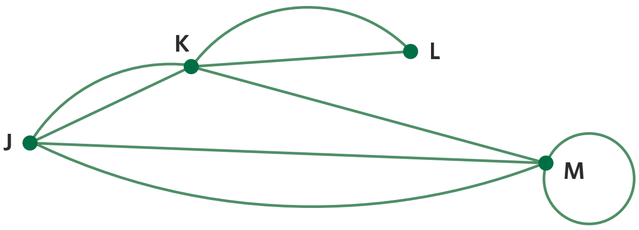
d. Construct a two-step communication matrix.

e. How many ways can port IV send a message to port I via a third port?

f. Which two ports have the most two-step communication links to each other?

Exam practice

16. Consider the following graph.



The adjacency matrix for this graph, with some elements missing, is shown.

$$\begin{array}{cc} \begin{matrix} J & K & L & M \end{matrix} & \\ \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 1 \end{bmatrix} & \begin{matrix} J \\ K \\ L \\ M \end{matrix} \end{array}$$

This adjacency matrix contains 16 elements when complete.

Of the 12 missing elements

A. six are ‘0’ and six are ‘1’.

B. four are ‘1’ and eight are ‘2’.

C. eight are ‘1’ and four are ‘2’.

D. two are ‘0’, four are ‘1’ and six are ‘2’.

E. four are ‘0’, two are ‘1’ and six are ‘2’.

Adapted from VCAA 2017 Exam 1 Networks and decision mathematics Q3

92% of students answered this type of question correctly.

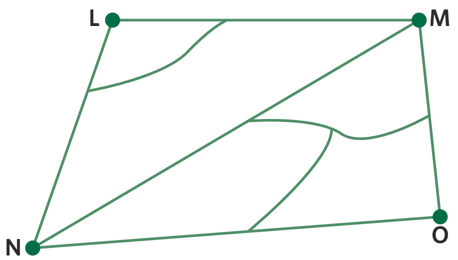
---

17. The following adjacency matrix shows the number of pathway connections between four bus stops: L, M, N and O.

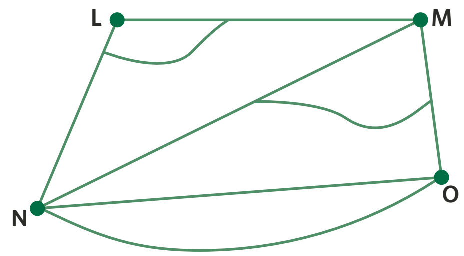
$$\begin{array}{cc} \begin{matrix} L & M & N & O \end{matrix} & \\ \begin{bmatrix} 1 & 2 & 2 & 0 \\ 2 & 1 & 5 & 4 \\ 2 & 5 & 1 & 4 \\ 0 & 4 & 4 & 1 \end{bmatrix} & \begin{matrix} L \\ M \\ N \\ O \end{matrix} \end{array}$$

A network of pathways that would be represented by the adjacency matrix is

A.



B.



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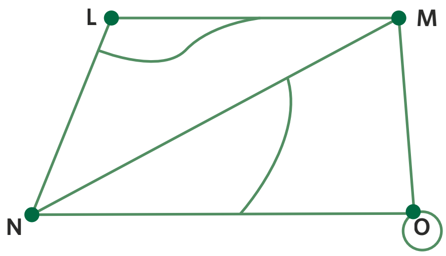
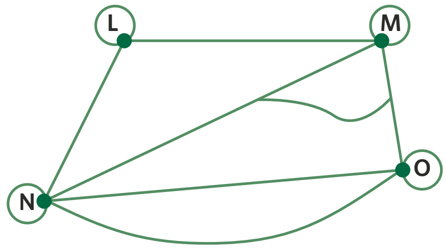
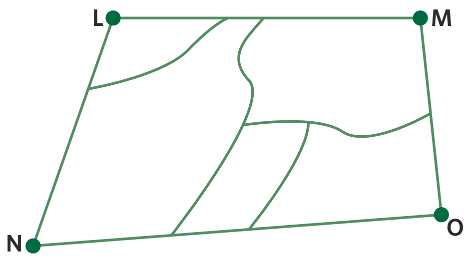
C.



D.



E.

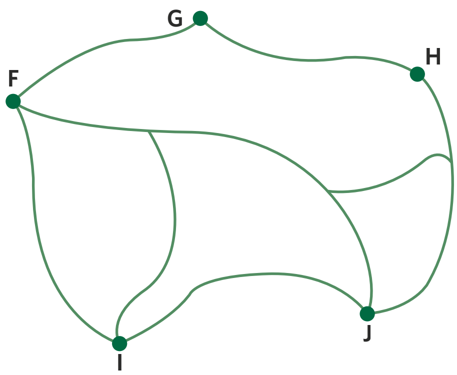


Adapted from VCAA 2020 Exam 1 Networks and decision mathematics Q8

75% of students answered this type of question correctly.

---

18. The following map shows all the river connections between five lookout points, F, G, H, I, and J.



The river connections could be represented by the adjacency matrix

A.
$$\begin{array}{cc} \begin{matrix} F & G & H & I & J \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 0 & 2 & 2 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 2 \\ 2 & 0 & 2 & 2 & 1 \end{bmatrix} & \begin{matrix} F \\ G \\ H \\ I \\ J \end{matrix} \end{array}$$

B.
$$\begin{array}{cc} \begin{matrix} F & G & H & I & J \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 2 \\ 1 & 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 2 & 1 \end{bmatrix} & \begin{matrix} F \\ G \\ H \\ I \\ J \end{matrix} \end{array}$$

C.
$$\begin{array}{cc} \begin{matrix} F & G & H & I & J \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 1 & 2 & 2 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 2 \\ 2 & 0 & 1 & 0 & 3 \\ 2 & 0 & 2 & 3 & 1 \end{bmatrix} & \begin{matrix} F \\ G \\ H \\ I \\ J \end{matrix} \end{array}$$

D.
$$\begin{array}{cc} \begin{matrix} F & G & H & I & J \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 0 & 2 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 2 \\ 2 & 0 & 1 & 0 & 3 \\ 1 & 0 & 2 & 3 & 1 \end{bmatrix} & \begin{matrix} F \\ G \\ H \\ I \\ J \end{matrix} \end{array}$$

E.
$$\begin{array}{cc} \begin{matrix} F & G & H & I & J \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 1 & 2 & 3 \\ 1 & 0 & 2 & 0 & 0 \\ 1 & 2 & 0 & 1 & 2 \\ 2 & 0 & 1 & 0 & 3 \\ 3 & 0 & 2 & 3 & 0 \end{bmatrix} & \begin{matrix} F \\ G \\ H \\ I \\ J \end{matrix} \end{array}$$

Adapted from VCAA 2019 Exam 1 Networks and decision mathematics Q6

39% of students answered this type of question correctly.

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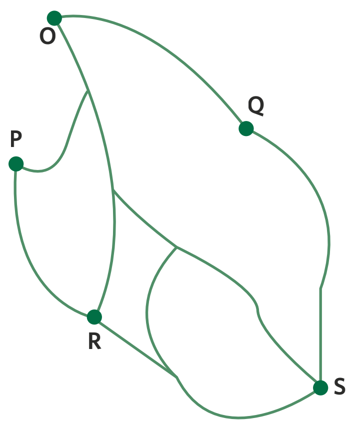
19. The following network shows the pathways between five shipping locations.



An adjacency matrix for this network is formed.

The number of zeros in this matrix is

A. 6

B. 7

C. 8

D. 9

E. 10

Adapted from VCAA 2021 Exam 1 Networks and decision mathematics Q7

33% of students answered this type of question correctly.

Questions from multiple lessons

Graphs and relations

20. Harry is a very successful musician. His yearly salary has two components. He receives a yearly fixed base wage of \$100 000 and an extra \$5000 for each concert he plays.

Let n be the number of concerts he plays in a year.

Let S be his total yearly salary.

What is the equation that models the relationship between Harry's yearly salary and the number of concerts he plays?

A. $S = 100\,000n + 5000$

B. $S = 5000n + 100\,000$

C. $S = 5000(n + 100\,000)$

D. $S = 100\,000(n + 5000)$

E. $S = n + 105\,000$

Adapted from VCAA 2016 Exam 1 Graphs and relations Q2

Data analysis

21. Research is being conducted into trees in Victorian forests. The focus is on two variables:

- *height* (less than 20 m, 20 m–40 m, more than 40 m)
- *type of tree* (Eucalypt, Mountain Ash, Redwood)

These variables are

A. a numerical variable and a categorical variable respectively.

B. an ordinal variable and nominal variable respectively.

C. both ordinal variables.

D. a nominal variable and ordinal variable respectively.

E. both nominal variables.

Adapted from VCAA 2017 Exam 1 Data analysis Q7

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Graphs and relations

22. A local cinema charges $20 per ticket.

They pay $60 a day to rent the venue and spend an average of $4 per ticket sold to pay their employees.

Let n be the number of tickets sold per day at the cinema.

a. Write an expression for the daily profit, p, that the cinema will make in terms of n. (1 MARK)

b. The cinema wants to make at least $373 of profit per day.
Determine the minimum number of tickets they need to sell per day. (1 MARK)

Adapted from VCAA 2013 Exam 2 Graphs and relations Q3a,b

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7C Planar graphs

STUDY DESIGN DOT POINT

- description of graphs in terms of faces (regions), vertices and edges and the application of Euler's formula for planar graphs

7A 7B 7C 7D 7E 7F

KEY SKILLS

During this lesson, you will be:

- identifying planar and non-planar graphs
- applying Euler's rule.

KEY TERMS

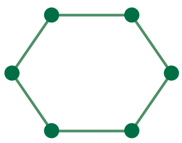
- Planar graph
- Non-planar graph
- Euler's rule
- Face

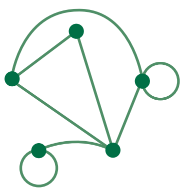
Planar graphs are useful for modelling applications where it is essential that the lines between points on the graph must not cross. For example, when designing a train network, it might be important to design it without any intersections.

Identifying planar and non-planar graphs

A **planar graph** is a graph that can be drawn with no overlapping edges. If a graph cannot be drawn this way, it is a **non-planar graph**.

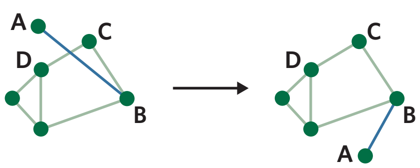
The following graphs are planar as none of their edges intersect.





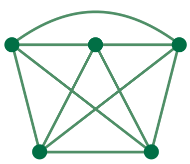
Consider the following graphs. Note that there is no vertex at the point where the edges overlap in graph 1.

graph 1 graph 2



Initially it appears that the network is non-planar as the edge connecting A and B in graph 1 overlaps with the edge connecting C and D. However, the graph can be redrawn in planar form by moving vertex A to create graph 2, which is an isomorph of graph 1.

In the following graph, however, it is not possible to redraw the graph with no overlapping edges. Therefore, it is non-planar.



A complete graph, in which every vertex is directly connected to every other vertex once, with five or more vertices is non-planar.

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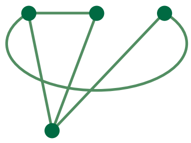
Worked example 1

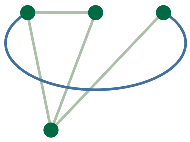
For the following graphs, determine if they are planar or non-planar by inspection.

a.

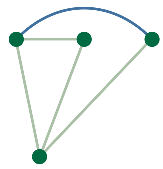


Explanation

Step 1: Identify any overlapping edges.



Step 2: Redraw the graph by moving edges or vertices so that no edges overlap.

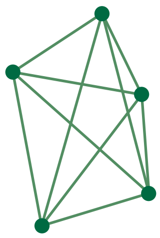


Step 3: State whether the graph is planar or non-planar.

Answer

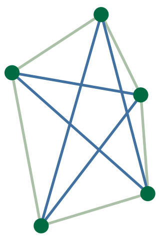
Planar

b.

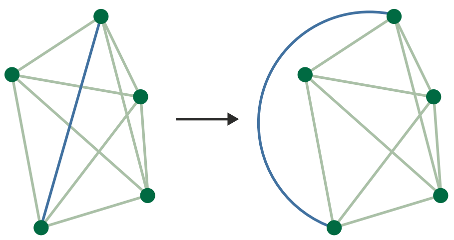


Explanation

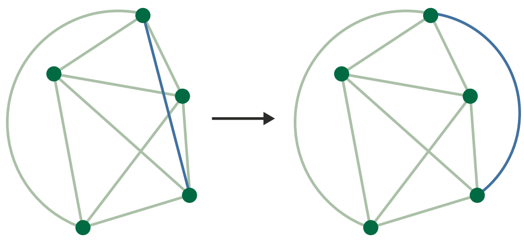
Step 1: Identify any overlapping edges.



Step 2: Redraw the graph by moving edges or vertices so that no edges overlap.



Step 3: Repeat this for all other overlapping edges, where possible. State whether the graph is planar or non-planar.



Answer

Non-planar

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Applying Euler's rule

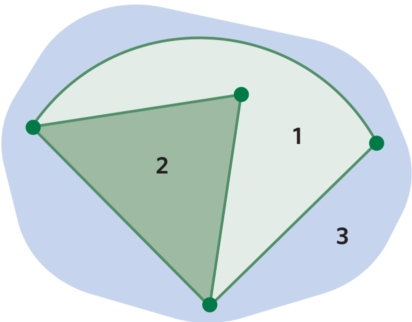
Euler's rule, also known as Euler's formula, describes the relationship between the number of vertices, edges and faces for all connected planar graphs. This relationship can be written as

$v - e + f = 2$, where

- v is the number of vertices
- e is the number of edges
- f is the number of faces

A **face**, also known as a region, is an area on a graph that is bordered by edges. For the number of faces to be counted directly from a graph, there must be no overlapping edges (that is, the graph must be drawn in planar form).

Note: The space outside the graph is counted as an additional face. For example, in the following graph, there are 3 faces where the blue region (3) represents the area outside the graph.



Given the number of vertices, edges and faces of a graph, Euler's rule can be used to determine whether any graph is planar or non-planar. For example, the planar graph shown has 4 vertices, 5 edges and 3 faces.

See worked example 2

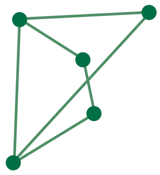
$4 - 5 + 3 = 2$

If a graph is known to be planar, Euler's rule can also be used to calculate an unknown number of vertices, edges or faces if two of these values are known.

See worked example 3

Worked example 2

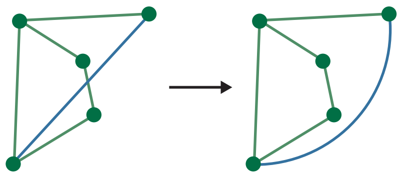
Consider the following planar graph.



a. State the number of vertices, edges and faces.

Explanation

Step 1: Redraw the graph in planar form.



Step 2: Count the number of vertices, edges and faces.

Answer

Vertices: 5

Edges: 6

Faces: 3

Continues →

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b. Show that the graph is planar using Euler's rule.

Explanation

Substitute the number of vertices, edges and faces into Euler's rule.

Answer

$$v - e + f = 2$$
$$5 - 6 + 3 = 2$$
$$2 = 2$$

Worked example 3

A planar graph has 8 edges and 4 faces. Use Euler's rule to calculate the number of vertices.

Explanation

Step 1: Identify the known values.

$e = 8$

$f = 4$

Step 2: Substitute into Euler's rule.

$$v - e + f = 2$$
$$v - 8 + 4 = 2$$
$$v - 4 = 2$$

Step 3: Solve for v.

$$v - 4 = 2$$
$$v = 6$$

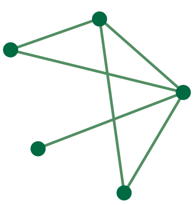
Answer

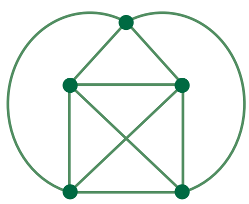
6 vertices

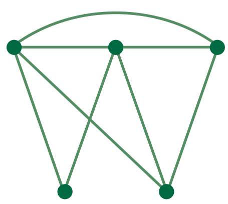
7C Questions

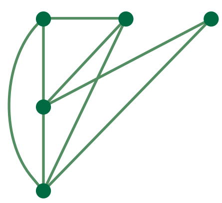
Identifying planar and non-planar graphs

1. Which of the following graphs is non-planar?

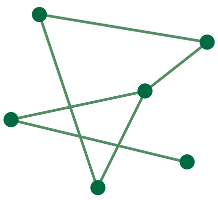
A.


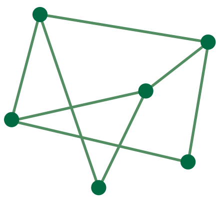
B.


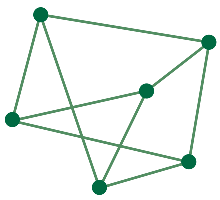
C.


D.


2. For the following graphs, determine if they are planar or non-planar by inspection.

a.


b.


c.


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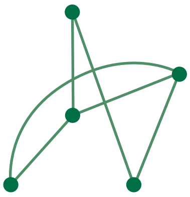
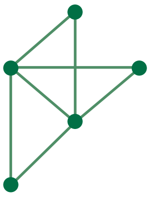
3. Redraw the following graphs in planar form.

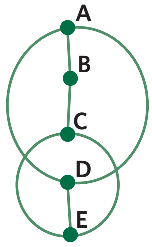
a.



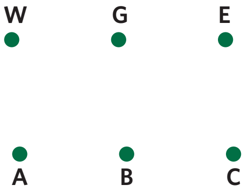
b.



c.



4. Sariah is an engineer and is planning the utility connections (water, gas and electricity) to three newly built houses in a street. She decides to model this situation using a connected graph. The following graph shows the vertices only, representing the utility connections (W, G and E) and the houses (A, B and C).



The following conditions must be satisfied:

- each edge represents a connection to a utility
- every house must be connected to each utility

Using this information, determine whether it is possible to complete the graph with the missing edges without having overlapping utility connections.

Applying Euler's rule

5. Which combination of vertices, edges and faces cannot represent a connected planar graph?

A. Three vertices, four edges and three faces

B. Three vertices, four edges and six faces

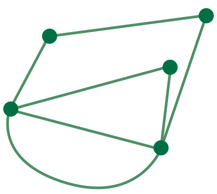
C. Four vertices, four edges and two faces

D. Five vertices, five edges and two faces

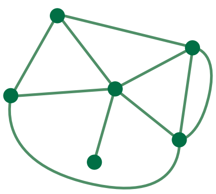
6. For each of the following connected planar graphs:

- State the number of vertices, edges and faces.
- Show that the graph is planar using Euler's rule.

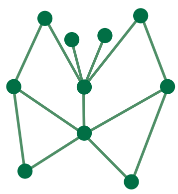
a.



b.



c.



7. A planar graph has six vertices and three faces. Use Euler's rule to calculate the number of edges in the graph.

8. A network of 5 telecommunications towers and their connections with each other can be modelled using a planar graph. If there are 10 connections in total, represented using edges, how many regions will be covered by this network?

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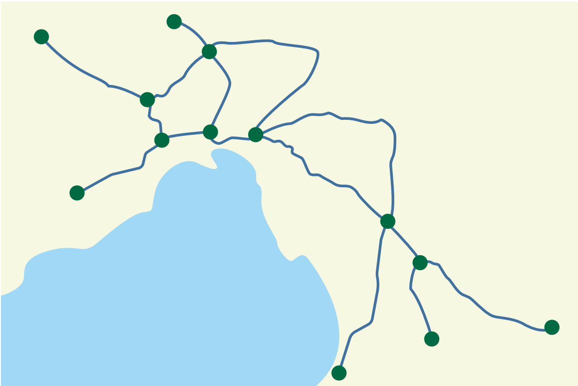
Joining it all together

9. Angel drew the following graph based on Melbourne's freeway system.



a. Explain why this graph is planar.

b. Show that Euler's rule holds for this graph.

---

10. An urban designer, Emma, is contracted to design the walking trails in a proposed city park. They will use graphs to model the routes. There will be twelve drinking fountains, represented by vertices. Each edge represents a walking trail which starts and ends at a drinking fountain. There can be no overlap between walking trails.

a. Each area enclosed by the drinking fountains and walking trails will form a forest area. If there will be four forest areas, how many walking trails will there be?

b. Draw a possible graph to represent the city park.

Exam practice

11. A planar graph has five faces.

This graph could have

A. eight vertices and eight edges.

B. six vertices and eight edges.

C. eight vertices and five edges.

D. eight vertices and six edges.

E. five vertices and eight edges.

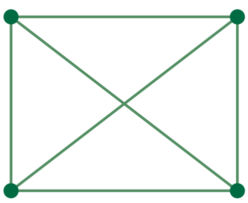
VCAA 2018 Exam 1 Networks and decision mathematics Q3

76% of students answered this question correctly.

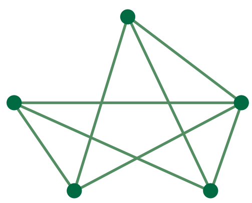
---

12. Which one of the following graphs is not a planar graph?

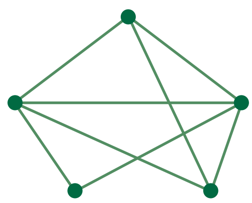
A.



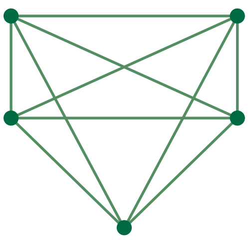
B.



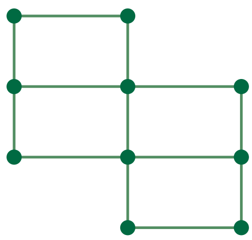
C.



D.



E.



VCAA 2018 Exam 1 Networks and decision mathematics Q6

41% of students answered this question correctly.

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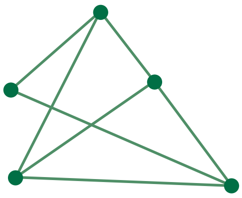
7C QUESTIONS

13. Consider the following graph.



The number of faces is

A. 2 **B.** 3 **C.** 4

D. 5 **E.** 6

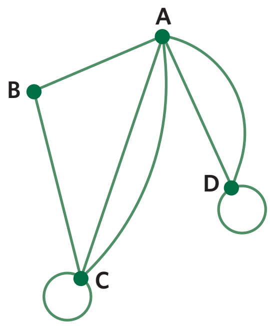
Adapted from VCAA 2021 Exam 1 Networks and decision mathematics Q3

34% of students answered this type of question correctly.

Questions from multiple lessons

Networks and decision mathematics

14. A graph and its corresponding adjacency matrix, with some elements missing, is shown.



$$\begin{array}{cc} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} & & & 2 \\ & 0 & 1 & \\ & 1 & & 0 \\ 2 & & 0 & \end{bmatrix} \end{array}$$

Of the nine missing elements, there are

A. four ‘0s’, three ‘1s’ and two ‘2s’

B. three ‘0s’, four ‘1s’ and two ‘2s’

C. three ‘0s’, five ‘1s’ and one ‘2’

D. four ‘0s’, four ‘1s’ and one ‘2’

E. three ‘0s’, three ‘1s’ and two ‘2s’

Adapted from VCAA 2017 Exam 1 Networks and decision mathematics Q3

Recursion and financial modelling

15. Consider the following recurrence relation.

$W_0 = 12\,000, \quad W_{n+1} = W_n + 180$

This recurrence relation could be used to model a

A. simple interest investment of \$12 000 with an interest rate of 0.015% per period.

B. simple interest investment of \$12 000 with an interest rate of 1.5% per period.

C. simple interest investment of \$12 000 with an interest rate of 15% per period.

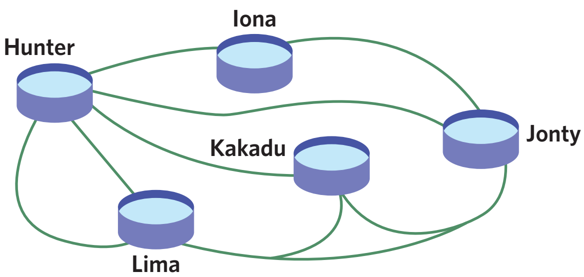
D. compound interest investment of \$12 000 with an interest rate of 1.5% per period.

E. compound interest loan of \$12 000 with an interest rate of 1.5% period.

Adapted from VCAA 2017NH Exam 1 Recursion and financial modelling Q19

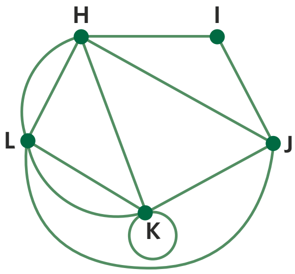
Networks and decision mathematics

16. A map of the roads between five public pools is shown in the following diagram.



a. Alejandro is currently at Iona pool. Which two other pools can he visit travelling only along one road? (1 MARK)

The map has been converted into the following graph, with each vertex representing a pool; Hunter (H), Iona (I), Jonty (J), Kakadu (K), and Lima (L). However, the graph is missing an edge.



b. Which two vertices is the missing edge between? (1 MARK)

c. What does the loop at K represent in the context of travel from Kakadu pool? (1 MARK)

Adapted from VCAA 2016 Exam 2 Networks and decision mathematics Q1

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7D Connected graphs

STUDY DESIGN DOT POINT

- connected graphs: walks, trails, paths, cycles and circuits with practical applications

7A 7B 7C 7D 7E 7F

KEY SKILLS

During this lesson, you will be:

- identifying walks, trails and paths
- identifying circuits and cycles.

KEY TERMS

- Route
- Walk
- Trail
- Path
- Circuit
- Cycle

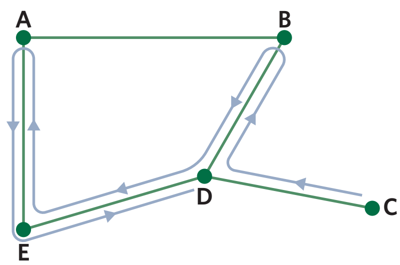
Graphs have several practical applications that include navigating and travelling. This is because graphs can be used similarly to maps, where vertices are locations and edges are paths or roads. For example, public transport network maps are graphs. In these situations, it can be useful to be able to classify specific ways of travelling through graphs. These concepts can be extended further to help solve navigation problems.

Identifying walks, trails and paths

A **route** is a written list of the vertices travelled through, in order, when moving from one vertex to another. It shows the pathway travelled in a graph. A route can be named based on the type of movement that has occurred.

A **walk** is the most general type of movement through a network. It is any continuous sequence of edges that passes through any number of vertices, in any order, starting and finishing at any vertex.

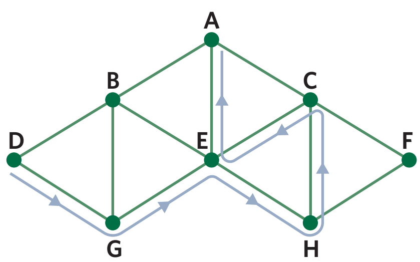
The following graph shows the walk C–D–B–D–E–A–E–D. Note that for a walk, not all vertices need to be passed through.



A **trail** is a walk that does not repeat any edges. However, it may pass through the same vertices multiple times.

A **path** is a walk in which no edges or vertices are repeated.

The following graph shows the walk D–G–E–H–C–E–A. This is a trail but it is not a path, because the vertex E is visited more than once.



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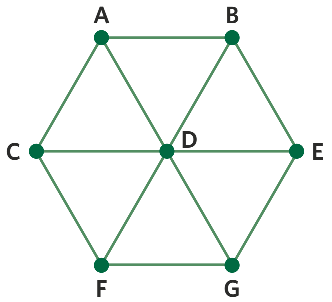
If the walk went straight from vertex C to A instead of going via vertex E, this walk would be a path.

When naming the route through a network, it is important to note that only the most specific name for the route is used. For example, a path is also a trail and a walk, but only the word path is used to name this route.

Worked example 1

Consider the following graph.

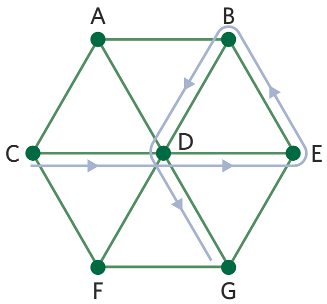


Determine whether a walk, trail or path most accurately describes the following routes.

a. C–D–E–B–D–G

Explanation

Step 1: Draw the route onto the graph.



Step 2: Check whether any edges are repeated.

No edges are repeated. This means that the route is a trail.

Step 3: Check whether any vertices are repeated.

The vertex D is repeated. This means that the trail is not a path.

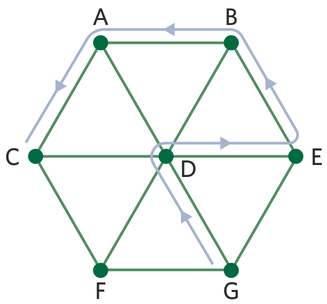
Answer

Trail

b. G–D–E–B–A–C

Explanation

Step 1: Draw the route onto the graph.



Step 2: Check whether any edges are repeated.

No edges are repeated. This means that the route is a trail.

Step 3: Check whether any vertices are repeated.

No vertices are repeated. This means that the trail is a path.

Answer

Path

Continues →

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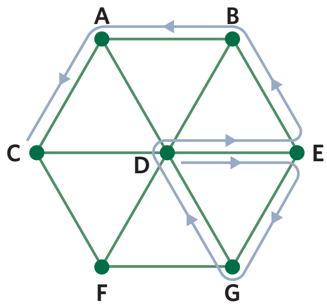
c. D–E–G–D–E–B–A–C

Explanation

Step 1: Draw the route on the graph.



Step 2: Check whether any edges are repeated.

The edge between vertices D and E is repeated.
This means that the route is not a trail or a path.

Answer

Walk

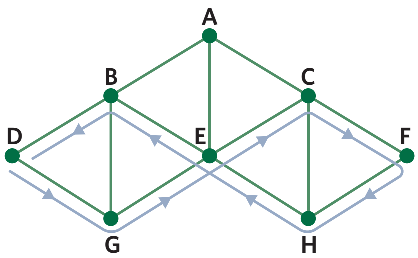
Identifying circuits and cycles

Circuits and cycles are types of trails and paths that start and finish at the same vertex.

A **circuit** is a trail that starts and finishes at the same vertex. Since it is a type of trail, no edges are repeated, but vertices may be.

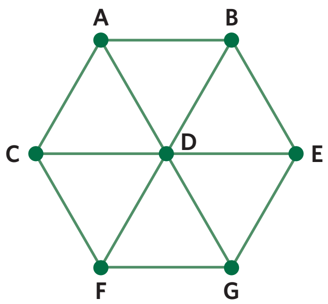
A **cycle** is a path that starts and ends at the same vertex. Since it is a type of path, no edges or vertices are repeated, except the starting and ending vertex.

For example, the walk D–G–E–C–F–H–E–B–D shown in the following graph is a circuit but not a cycle, because it passes through vertex E twice.



Worked example 2

Consider the following graph.



Determine whether a walk, circuit or cycle most accurately describes the following routes.

Continues →

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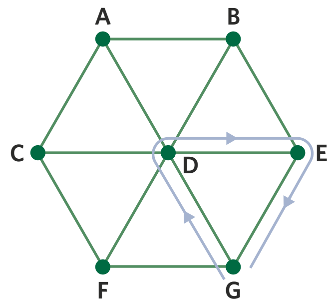
a. G–D–E–G

Explanation

Step 1: Draw the route onto the graph.



Step 2: Check whether the route starts and ends at the same vertex.

The walk starts and ends at vertex G.

Since it starts and ends at the same vertex, consider whether it is a circuit or a cycle.

Step 3: Check whether any edges or vertices are repeated.

There are no repeated edges or vertices, except the starting and ending vertex.

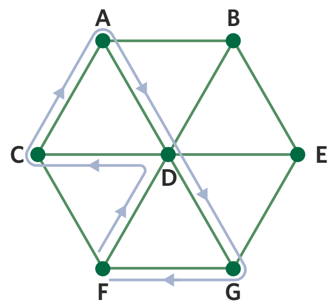
Answer

Cycle

b. F–D–C–A–D–G–F

Explanation

Step 1: Draw the route on the graph.



Step 2: Check whether the route starts and ends at the same vertex.

The walk starts and ends at vertex F.

Since it starts and ends at the same vertex, consider whether it is a circuit or a cycle.

Step 3: Check whether any edges or vertices are repeated.

There are no repeated edges, but vertex D is repeated.

Answer

Circuit

7D Questions

Identifying walks, trails and paths

1. Which of the following statements is false?

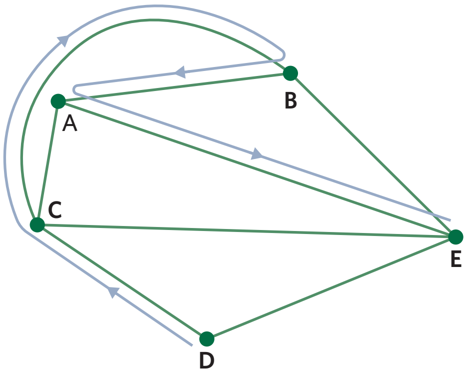
A. Trails and paths are also walks.

B. A trail has no repeated edges.

C. Paths are also trails.

D. Paths can pass through the same vertex twice.

2. Which term best describes the route shown in the following graph?



A. Route **B.** Walk **C.** Trail **D.** Path

3. Consider the following graph.

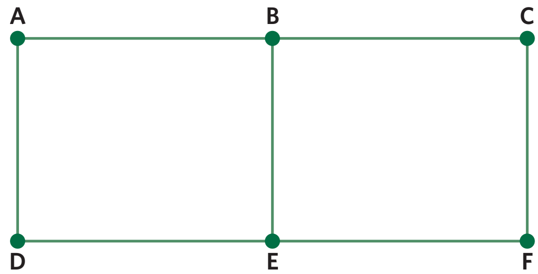
Determine whether a walk, trail or path most accurately describes the following routes.

a. E–D–A–B–E–F–C–B

b. D–A–B–E–F–C–B–E–D

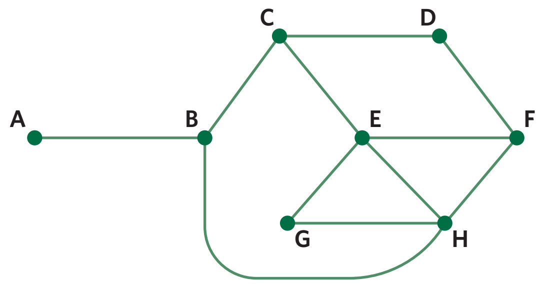
c. A–B–C–F–E–B

d. C–F–E–B–A–D



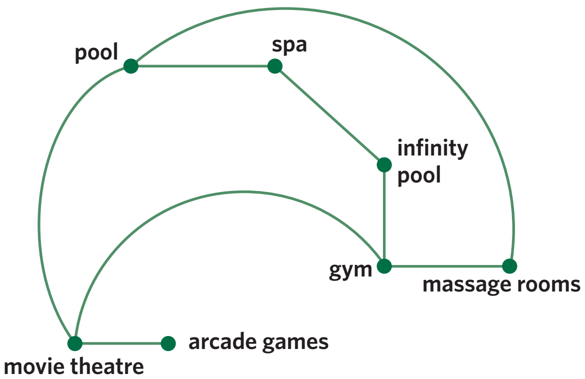
4. Eddie wants to deliver Christmas cards to all his friends in his neighbourhood. He constructs the following graph that shows his house, at vertex E, and all the friends he wants to deliver Christmas cards to shown as the remaining vertices.

Eddie wants to have dinner at his friend Alexa's house, at vertex A. Find a path that starts at Eddie's house and ends at Alexa's house, while passing all the other houses.



5. Lucia is on holiday at a resort with several amenities that she wants to use.

The following graph shows each available amenity, and the paths between them.



a. If Lucia begins at the pool, is it possible for her to visit each location via a trail?

b. If Lucia begins at the pool, is it possible for her to visit each location via a path?

c. If Lucia begins at the gym and wants to visit as many locations as possible while walking a path, which location(s) will she miss out on visiting?

d. Draw a path beginning at the arcade games and finishing at the infinity pool, passing through every location.

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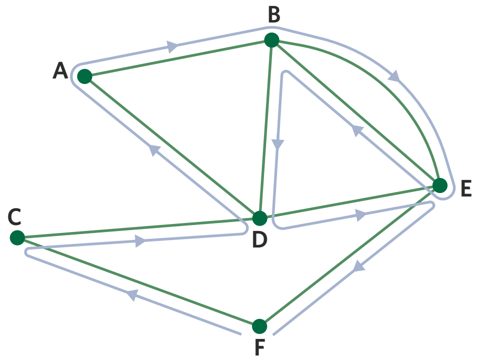
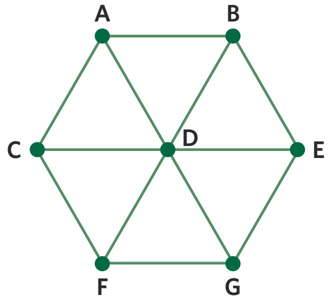
Identifying circuits and cycles

6. What type of route is shown in the following graph?



A. Route **B.** Walk **C.** Circuit **D.** Cycle

7. Consider the following graph.



Determine whether a walk, circuit or cycle most accurately describes the following routes.

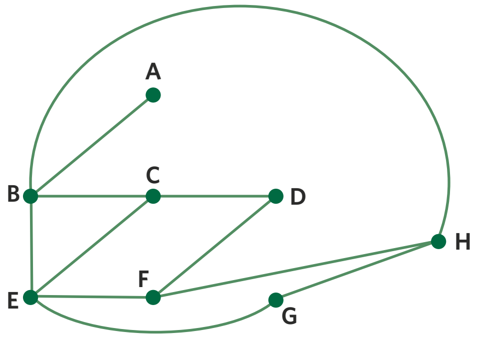
a. A–D–F–C–D–A

b. D–E–G–D–F–C–D–A–B–D

c. B–E–D–F–C–A–B

d. D–G–F–C–A–B–E–D

8. The following graph shows the destinations that the Davis family want to visit on a particular day while on holiday, and the roads between them.



From their hotel, they will take a taxi to any of the destinations and plan a route from there. The taxi driver has agreed to pick them up later at the same destination they were dropped off at.

a. Find a cycle that visits the greatest number of destinations.

The Davis family was dropped off at destination E.

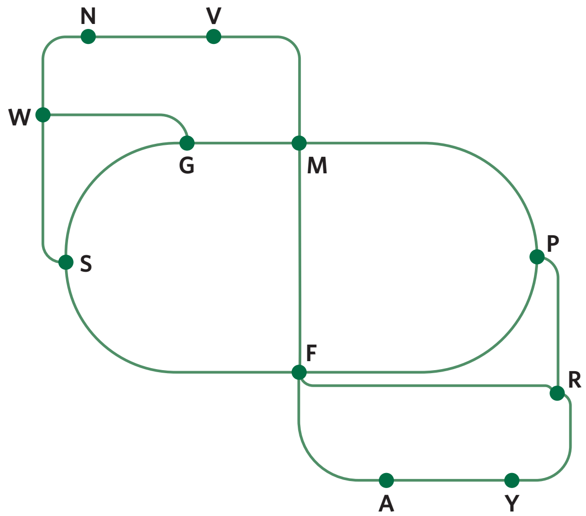
b. They want to travel on as many of the roads as possible in case there are other destinations they didn't know about. What is the maximum number of roads on which they can travel via a circuit?

c. If they only plan a route that is a cycle or a circuit, which destination(s) will they be unable to visit?

Joining it all together

9. The Victorian State Government is planning the final upgrade to the City Loop and the surrounding train lines. The following graph shows the new plan.

The stations are North Melbourne (N), Parkville (V), West Melbourne (W), Flagstaff (G), Melbourne Central (M), Southern Cross (S), Parliament (P), Flinders Street (F), Richmond (R), Anzac (A) and South Yarra (Y).



a. What is the term that most accurately describes the route G–M–P–R–F–A?

b. What is the term that most accurately describes the route F–M–G–W–N–V–M–P–R–F?

c. Find a cycle that starts at Flagstaff and visits every station.

d. A child boarded a train without their parents. Two stations later they realised their mistake and got off the train. They are now at Flagstaff Station, G. List all possible stations at which they may have boarded the train.

e. The train that the child boarded follows a cycle around all of the stations shown in the graph. Use your answer from part **d** to identify the two stations at which the child may have boarded the train.

---

10. Graeme, Bill, and Tim are going to Germany. Their friend Hans recommends that they take a walk in the Black Forest, in the country's south west. He gives them the following map of the region with his must-see locations marked with the letters O, P, Q, R, S, T, U, V, W, Y, X, and Z.

a. Graeme wants to visit each location only once.

i. List the five locations from which they can leave in order to achieve this.

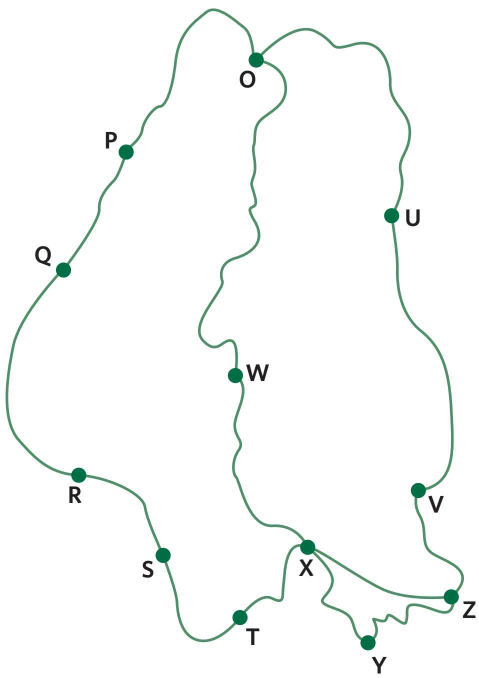
ii. Which term (walk, trail, path, circuit, or cycle) best describes these routes.

b. Bill wants to travel each road exactly once.

Which term (walk, trail, path, circuit, or cycle) best describes this route.

c. Tim wants to finish where they started.

The local government of the Black Forest is planning to construct another road that will connect W to either P, T, U, or Z.
Which road would not allow Tim, Graeme, and Bill to plan a cycle that visits every location? W–Z, W–T, W–U, or W–P?



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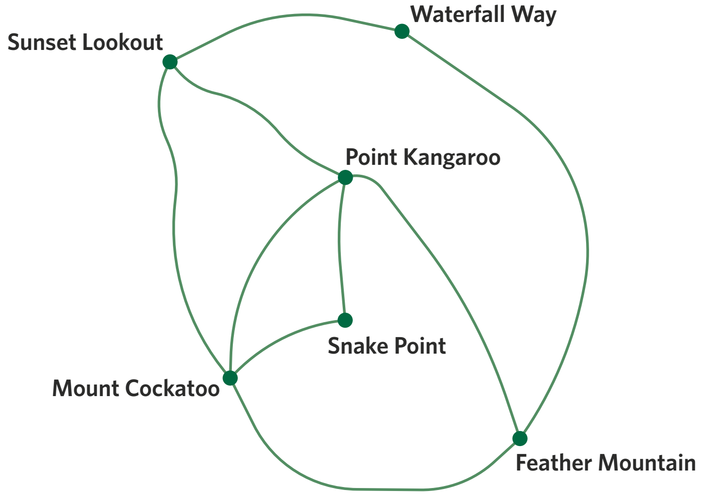
Exam practice

11. A hiking trail has six lookout points. The following network shows these lookout points represented by vertices. The edges of the network represent the paths between the lookout points.



A hiking tour starts and finishes at Mount Cockatoo, visiting each lookout only once.

Draw a possible route for this tour. (1 MARK).

Adapted from VCAA 2019 Exam 2 Networks and decision mathematics Q1bii

93% of students answered this type of question correctly.

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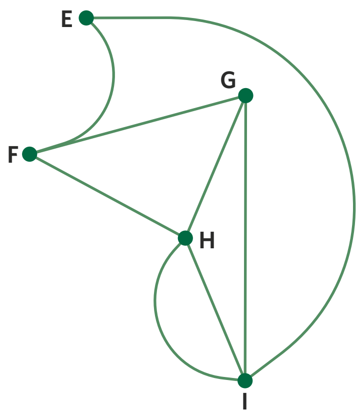
12. Consider the following graph.



Which of the following is **not** a path for this graph?

A. I–E–F–G–H **B.** H–G–F–E–I **C.** F–H–I–G–E
D. E–F–H–I–G **E.** G–F–E–I–H

Adapted from VCAA 2018 Exam 1 Networks and decision mathematics Q4

88% of students answered this type of question correctly.

Questions from multiple lessons

Graphs and relations

13. Elizabeth and Ash are buying souvenirs for their friends and family in Australia. A particular store sells small and large fridge magnets. The following equations describe the amount of money Elizabeth and Ash each spent on fridge magnets, where s is the price of small fridge magnets and l is the price of large fridge magnets.

Elizabeth: $3s + 4l = 21.80$

Ash: $6s + 3l = 24.60$

What is the price of a small fridge magnet?

A. \$2.20 **B.** \$2.40 **C.** \$2.60 **D.** \$3.60 **E.** \$3.80

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Data analysis

14. The following table shows the *height* in centimetres of a group of 14 students selected from a cohort of 130 students.

height **(cm)**	122.6	162.3	183.7	152.1	142.5	156.6	162.5	172.2	125.2	123.2	162.5	152.9	172.5	159.6

The mean, $\overline{x}$, and the standard deviation, s_x, of the *height* for this sample of students are closest to

A. $\overline{x} = 19.03, \quad s_x = 152.59$

B. $\overline{x} = 19.03, \quad s_x = 153.60$

C. $\overline{x} = 153.59, \quad s_x = 18.33$

D. $\overline{x} = 153.60, \quad s_x = 18.33$

E. $\overline{x} = 153.60, \quad s_x = 19.03$

Adapted from VCAA 2017 Exam 1 Data analysis Q3

Recursion and financial modelling

15. A small emergency fund of $14 000 was invested in a compound interest account for eight years.

After eight years of earning interest, the value of this investment was $15 783.77.

a. How much interest was earned during the eight years of this investment? (1 MARK)

b. Interest on the account had been calculated and paid monthly.

What was the annual rate of interest for this investment? Round to two decimal places. (1 MARK)

Adapted from VCAA 2014 Exam 2 Business-related mathematics Q3a,b

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7E Weighted graphs

STUDY DESIGN DOT POINT

- weighted graphs and networks, and an introduction to the shortest path problem (solution by inspection only) and its practical application

7A 7B 7C 7D **7E** 7F

KEY SKILLS

During this lesson, you will be:

- interpreting weighted graphs
- identifying the shortest path through a network.

KEY TERMS

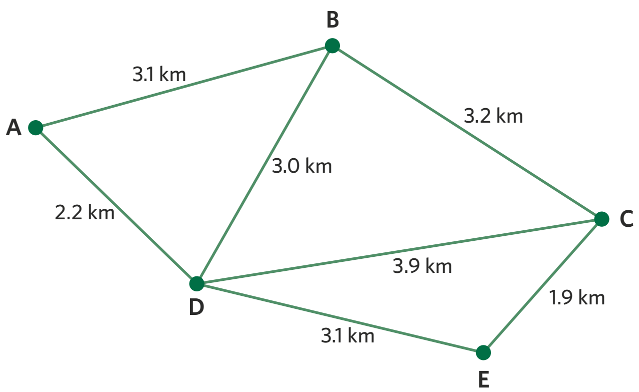
- Weighted graph
- Shortest path

From the distance of roads, to the time taken to travel, to the cost of each task in a larger activity, daily life contains numerous examples of 'weighting' being an important factor in decision making. A plumber, for example, is not only asked whether they can do a job; they are asked how long it will take or how much it will cost. Weighted graphs map out multiple pathways and the 'weights' associated with each one, so that more informed decisions can be made.

Interpreting weighted graphs

A **weighted graph** is a graph that has numeric values assigned to each edge. These values represent the 'weight' of each edge and are usually associated with distance, time or cost.

For example, in a weighted graph representing a road network, the weight of an edge could represent the distance between two cities. The following weighted graph shows the distance in kilometres between five locations, A, B, C, D, and E.



By incorporating these weights, weighted graphs provide a more detailed and informative representation of the connections between vertices. These weights make it possible to investigate the total distance, time or cost of a particular journey or task.

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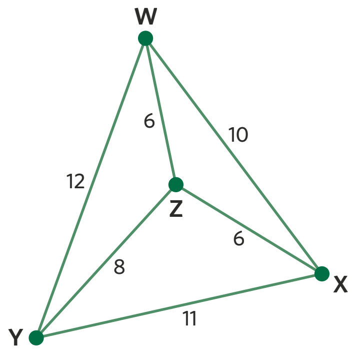
Worked example 1

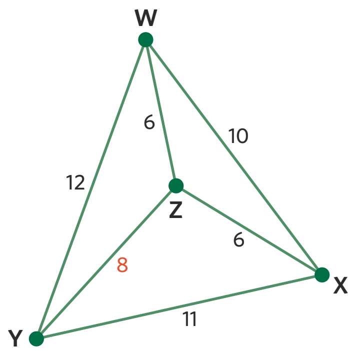
The distances between W, X, Y and Z in kilometres are shown in the following graph.



What is the total length of the path Y–Z–W–X?

Explanation

Step 1: Determine the distance between Y and Z.



Y to Z: 8 km

Step 2: Repeat step 1 for the remaining distances.

Z to W: 6 km

W to X: 10 km

Step 3: Add the distances together.

$8 + 6 + 10 = 24$

Answer

24 km

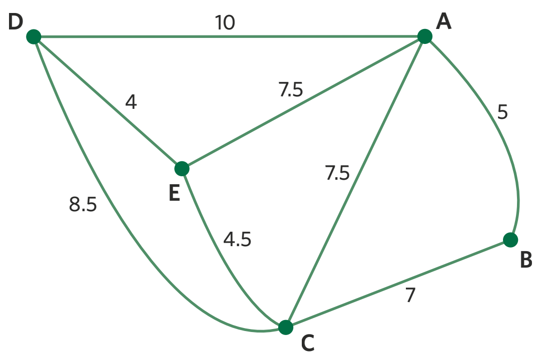
Identifying the shortest path through a network

Weighted graphs can be used to determine the shortest distance or the least amount of time needed to travel between two locations, or the lowest cost of performing a task.

The **shortest path** is the minimum total weighted value between two vertices, found by identifying all of the possible paths from one vertex to another, and comparing their lengths. This is usually done through visual inspection.

Worked example 2

Consider the following network.



Continues →

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What is the shortest route from B to D?

Explanation

Step 1: Through inspection, write out all of the possible shortest paths from B to D.

B–C–D

B–A–D

B–C–E–D

B–A–E–D

Step 2: Determine the weight of all the routes.

B–C–D = 7 + 8.5 = 15.5

B–A–D = 5 + 10 = 15

B–C–E–D = 7 + 4.5 + 4 = 15.5

B–A–E–D = 5 + 7.5 + 4 = 16.5

Step 3: Identify the shortest route.

The shortest route is B–A–D with a total weight of 15.

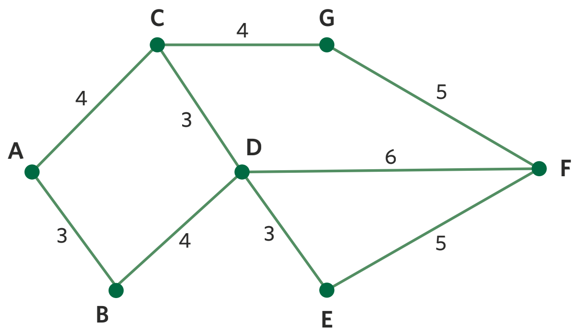
Answer

B–A–D

7E Questions

Interpreting weighted graphs

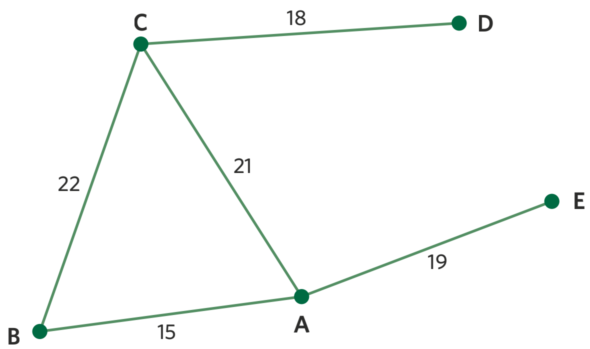
1. Consider the following weighted graph.



A route through the graph has a total weight of 20. The route could be

A. A–B–D–C–G–F **B.** A–C–D–E–F–G **C.** B–D–F–E–D **D.** G–F–D–E–F

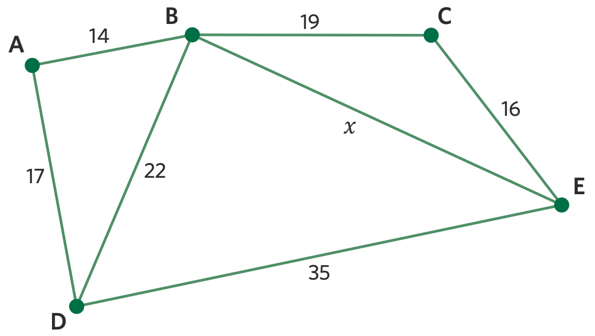
2. The following weighted graph shows the distance between five country towns A, B, C, D and E in kilometres.



Determine the distances of the paths between the following towns.

a. C to D **b.** E to A to C **c.** B to C to A to E

3. The following weighted graph shows the time taken to walk between five locations, in minutes. If it takes 59 minutes to walk the path D–A–B–E, what is the value of x?



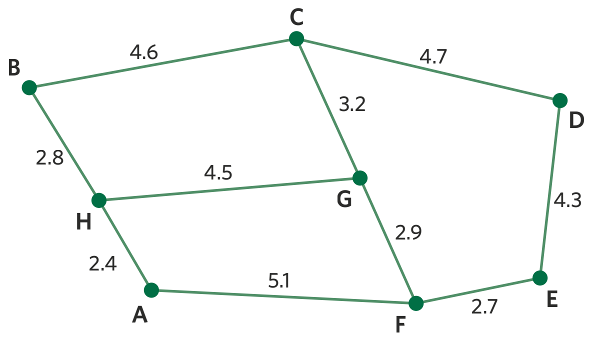
4. Sabrina (S), Mary (M), Jessica (J) and Victoria (V) are four friends who all live in the same suburb. Sabrina lives three minutes away from Mary and 5 minutes away from Jessica.

Mary and Jessica are four minutes away from each other. Victoria lives two minutes away from Mary and three minutes away from Sabrina.

Draw a weighted graph to represent the situation.

Identifying the shortest path through a network

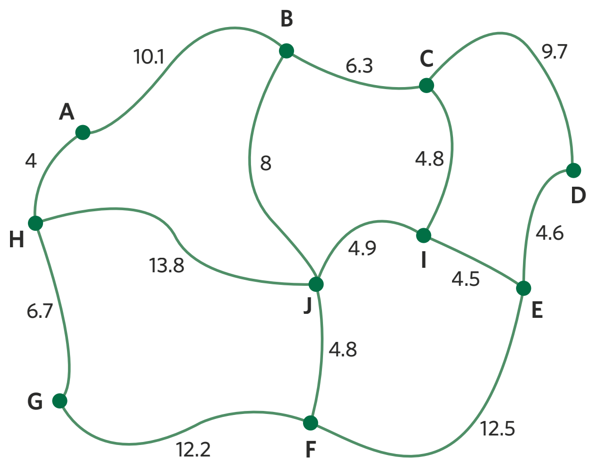
5. The following weighted graph shows the time taken in minutes to walk between seven locations.



In order to walk from B to E in the shortest possible time, which of the following routes should be taken?

A. B–C–G–F–E **B.** B–C–D–E **C.** B–H–G–F–E **D.** B–H–A–F–E

6. The following weighted graph shows the distance, in kilometres, of cycling trails in a park.



What is the distance of the shortest path between the following locations?

a. H and E **b.** A and F **c.** G and D

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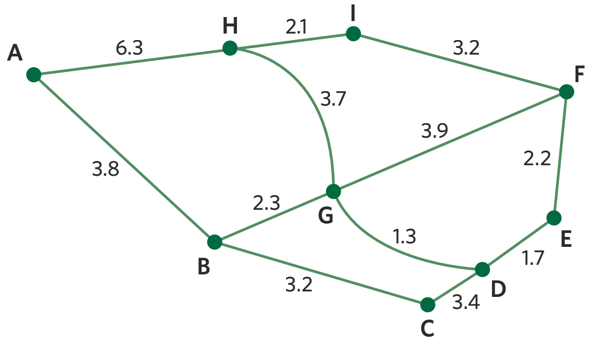
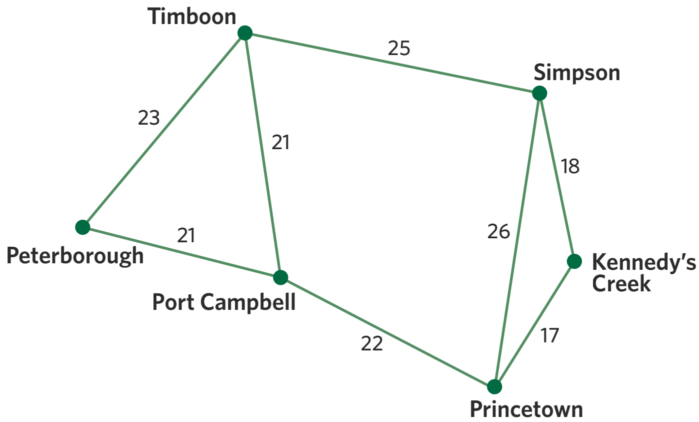
7. The numbers in the following graph represent distances in metres.



What is the shortest path from A to E, while avoiding vertex G? What is the distance of this path?

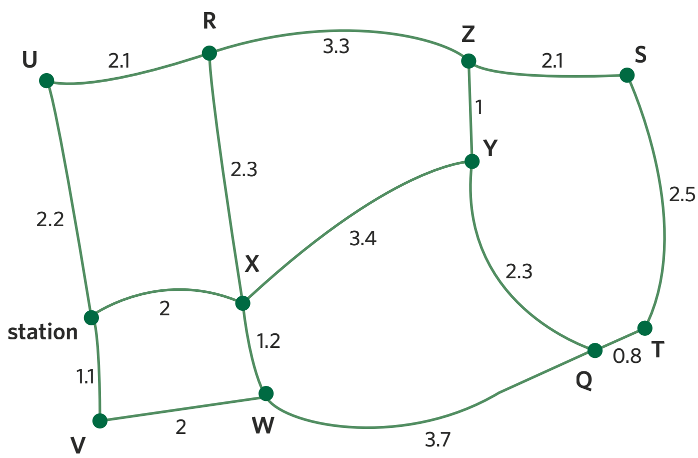
Joining it all together

8. Bailey is on holiday in Timboon and wants to visit a few nearby towns, as shown in the following graph.



If the numbers represent the distance between towns in kilometres, what is the distance of the path that starts at Timboon, travels through each town exactly once and ends at Kennedy's Creek?

9. The following weighted graph shows a series of roads surrounding Angus' home.



Angus is travelling from Glen Waverley railway station to his home located at Y. He usually goes from the station to X to Y but the intersection at X is blocked off due to roadworks.

The numbers represent distances in kilometres.

How much further does Angus have to travel compared to his usual route due to the roadworks, assuming that he takes the shortest path?

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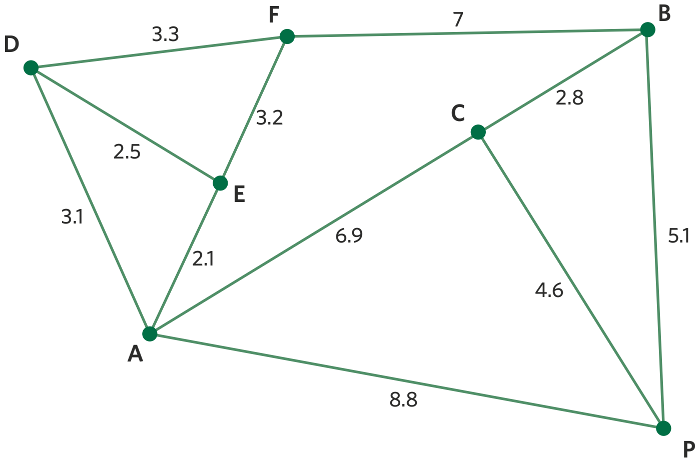
10. The following weighted graph represents a series of roads connected to a nearby post office (P).



A postman leaves the post office and makes deliveries to a number of locations (represented by vertices A to F) before returning to the post office. The numbers represent distances in kilometres. What is the shortest route he can take if he is to visit each of the locations?

11. The capital cities of Australia are shown on the following map.

The distances between these cities, in kilometres, are represented in the following table. Entries with a dash indicate no direct path between the cities.

	Adelaide	Brisbane	Darwin	Hobart	Melbourne	Perth	Sydney
Adelaide	–	–	2620	–	650	2130	–
Brisbane	–	–	2850	–	–	–	730
Darwin	2620	2850	–	–	–	2660	3150
Hobart	–	–	–	–	600	3010	1060
Melbourne	650	–	–	600	–	–	710
Perth	2130	–	2660	3010	–	–	–
Sydney	–	730	3150	1060	710	–	–

a. Draw a weighted graph to represent this situation.

b. What is the shortest route beginning and ending at Melbourne that passes through all the cities exactly once?

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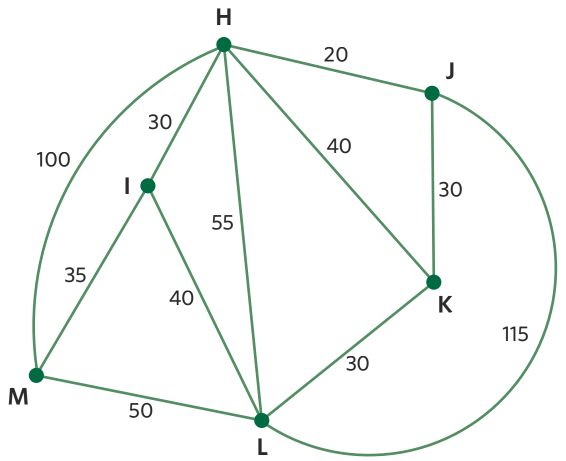
Exam practice

12. A local taxi driver works within six towns.

The towns are Hampshire (H), Idina (I), Jaspine (J), Kranskie (K), Louisberg (L) and Manifold (M).

The following graph gives the cost, in dollars, for the taxi driver to transport people between these towns.



Mai lives in Hampshire (H) and she will take the taxi to visit her grandmother in Manifold (M).

a. Mai considers asking the taxi driver to travel along the route Hampshire (H)–Louisberg (L)–Manifold (M). How much would Mai have to pay? (1 MARK)

b. If Mai takes the cheapest route from Hampshire (H) to Manifold (M), which other town(s) will she pass through? (1 MARK)

Adapted from VCAA 2017 Exam 2 Networks and decision mathematics Q1a,b

Part **a**: **94%** of students answered this type of question correctly.

Part **b**: **79%** of students answered this type of question correctly.

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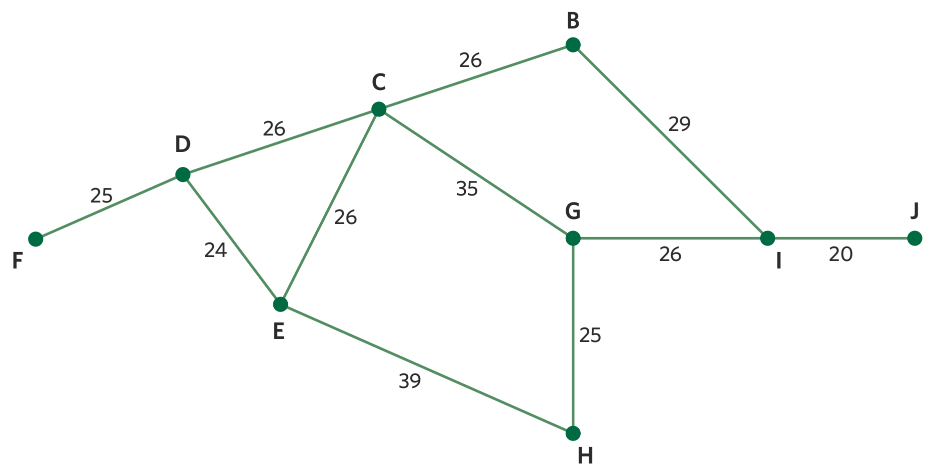
13. Lewis owns a flower shop in town F. He regularly picks the flowers he sells from his greenhouse in town J.

The following diagram shows the network of main roads between town F and town J.

The vertices B, C, D, E, F, G, H, I and J represent towns.

The edges represent the main roads. The numbers on the edges indicate the distances, in kilometres, between adjacent towns.



What is the shortest distance, in kilometres, between town F and town J? (1 MARK)

Adapted from VCAA 2021 Exam 2 Networks and decision mathematics Q2a

85% of students answered this type of question correctly.

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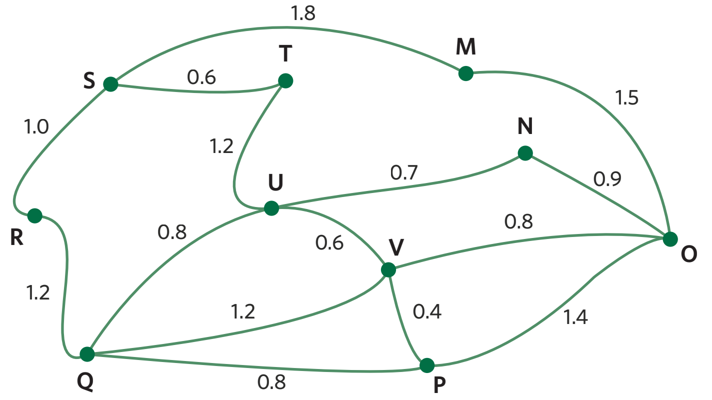
14. A local fitness park has 10 exercise stations: M to V.

The edges on the following graph represent the paths between the exercise stations.

The number on each edge represents the length, in kilometres, of each track.



The Sunny Coast cricket coach designs three different training programs, **all starting at exercise station S**.

training program number	training details
1	The team must run to exercise station O.
2	The team must run along all tracks just once.
3	The team must visit each exercise station and return to exercise station S.

What is the shortest distance, in kilometres, covered in training program 1? (1 MARK)

VCAA 2020 Exam 2 Networks and decision mathematics Q3a

58% of students answered this question correctly.

Questions from multiple lessons

Graphs and relations *Year 10 content*

15. The following table of values is formed using the equation $y = -2x - 18$.

x	-2	3	b	5	1
y	-14	a	-12	-28	c

What are the values of a, b and c?

A. $a = -24, \quad b = -12, \quad c = -3$

B. $a = -24, \quad b = -3, \quad c = -20$

C. $a = -20, \quad b = -12, \quad c = -3$

D. $a = -12, \quad b = -24, \quad c = -20$

E. $a = -12, \quad b = -3, \quad c = -24$

Computation and practical arithmetic *Year 10 content*

16. Which of the following expressions has a value of 1?

A. $\dfrac{(-10) \times (-4)}{8} + (-2)^2$

B. $\dfrac{6(4 - 3)^2}{(8 - 6)(7 + (-2) \times 5)}$

C. $3((9 - 2) + (-6)) + (-7) \times 2$

D. $\dfrac{9 - (-3)}{4} - \dfrac{1}{2}(11 + (-7))$

E. $\dfrac{(-3) \times (5 - (-1))}{(4 - 1)^2}$

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Graphs and relations *Year 10 content*

17. Boston and Alana are renting games and movies from RentalMania. Boston rents two games and seven movies for \$46.40 while Alana rents four games and five movies for \$49.60. Let g represent the cost of renting a game at RentalMania and m represent the cost of renting a movie at RentalMania.

a. Write down two simultaneous equations that represent this information. (2 MARKS)

b. Find the cost of a game rental at RentalMania. (1 MARK)

c. Lauren goes to RentalMania and rents six games and three movies. How much money did she spend? (1 MARK)

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7F Trees and their applications

STUDY DESIGN DOT POINT

- trees and minimum spanning trees, greedy algorithms and their use to solve practical problems

7A 7B 7C 7D 7E 7F

KEY SKILLS

During this lesson, you will be:

- identifying trees in a network
- identifying a minimum spanning tree using greedy algorithms.

KEY TERMS

- Tree
- Spanning tree
- Minimum spanning tree
- Greedy algorithm
- Prim's algorithm
- Kruskal's algorithm

Finding the best possible solution to a real world network problem can be difficult without the proper tools. Trees are an important feature of networks that allow people to define network problems mathematically, whether minimising the amount of NBN cabling required to service a city and its surrounding suburbs, or deciding the optimal way to connect the passageways of the state library. These trees allow for the use of greedy algorithms that help solve these real world problems efficiently.

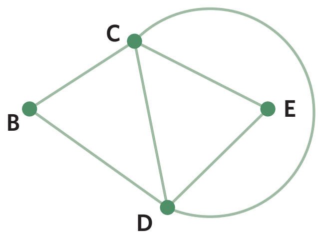
Identifying trees in a network

A **tree** is a simple connected graph that contains no loops, circuits or duplicate edges. A tree is often part of a larger graph. The number of edges in a tree is always one less than the number of vertices.

$$\textit{number of edges} = \textit{number of vertices} - 1$$

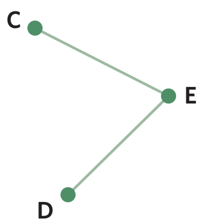
A **spanning tree** is a tree that contains all of the vertices in a larger graph, hence 'spanning' the entire graph.

For example, consider the following graph.



The graph is not a tree as it has four vertices and six edges.

One tree that can be identified within this graph is



This tree does not include all vertices of the larger graph, so it is not a spanning tree.

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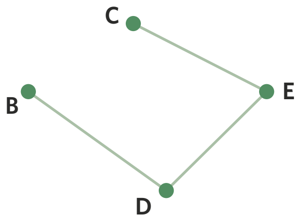
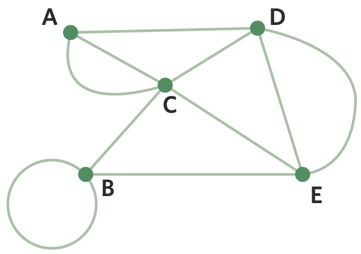
A spanning tree that can be identified within this graph is



Not only does this spanning tree include all four vertices from the larger graph, but also contains only three edges. In other words, all loops, cycles and multiple edges have been removed whilst still keeping the graph connected.

Worked example 1

Consider the following graph.

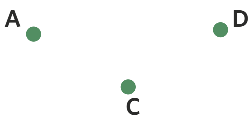


a. Draw a tree that can be identified from the graph.

Explanation

Step 1: Draw any number of included vertices.

As this is not a spanning tree, any number of vertices greater than one is acceptable.

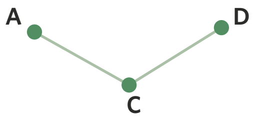


Step 2: Draw included edges.

The number of edges to include will be one less than the number of vertices.

As 3 vertices have been drawn in this example, the tree must have 2 edges from the original graph whilst still keeping the graph connected.

Answer



Note: There are many other possible trees that can be identified.

Continues →

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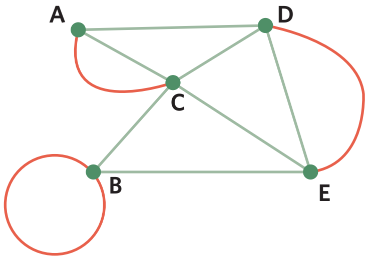
b. Draw a spanning tree that can be identified from the graph.

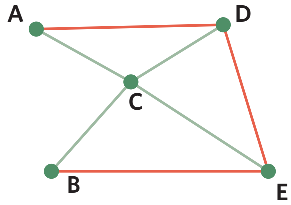
Explanation

Step 1: Remove any loops or duplicate edges.

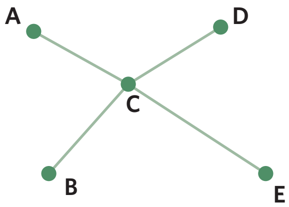


Step 2: Continue to remove edges until there is one less than the number of vertices.

As there are 5 vertices, the spanning tree must have 4 edges from the original graph whilst still keeping the graph connected.



Answer



Note: There are many other possible spanning trees that can be identified.

Identifying a minimum spanning tree using greedy algorithms

When identifying a spanning tree for a weighted graph, the objective is often to identify the minimum spanning tree. A **minimum spanning tree** is the spanning tree of a weighted graph that has the lowest possible total weight.

Note: Some weighted graphs have more than one minimum spanning tree.

Although it is possible to identify a minimum spanning tree of a weighted graph by eye, a greedy algorithm can be applied to reduce the effort required. A **greedy algorithm** is any procedure in which the optimal (best) solution is found for every step of a multi-step problem. Two such greedy algorithms are **Prim's algorithm** and **Kruskal's algorithm**, both of which help to identify the minimum spanning tree of a weighted graph.

Prim's algorithm contains the following steps:

See worked example 2

1. Locate and draw any vertex from the weighted graph.
2. Look at the edges that connect to this vertex and choose the edge with the lowest weight. If there are two edges of the same weight, pick either. Draw the edge and the new vertex it connects to.
3. Look at the edges that connect to either of the vertices and choose the one with the lowest weight that connects to a new vertex. Draw this edge and the new vertex it connects to.
4. Repeat this process until all vertices are connected.

Kruskal's algorithm contains the following steps:

See worked example 3

1. Identify the edge with the smallest weight. Draw it and the two vertices it connects to.
2. Identify the edge with the next smallest weight. If it does not create a loop, cycle, or multiple edge with the spanning tree so far, draw it and the vertices it connects to (if not already drawn).
3. Repeat this process until all vertices are connected.

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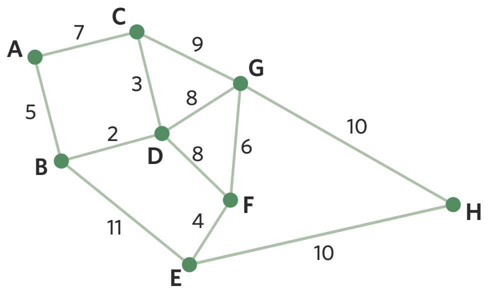
Worked example 2

Use Prim's algorithm to draw the minimum spanning tree for the following graph.



Explanation

Step 1: Choose a starting vertex and identify the connecting edge with the lowest weight.

In this case, vertex A will be chosen, but any vertex can be used.

There are two edges that connect to vertex A, with weights of 7 and 5.

5 is the lowest weight, so this edge is chosen.

The chosen edge connects vertex B to vertex A.

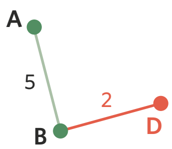
Step 2: Identify the edge with the lowest weight that connects vertex A or B to a new vertex.

From vertex A, there is one more edge with a weight of 7.

From vertex B, there are two more edges with weights of 2 and 11.

2 is the lowest weight, so this edge is chosen.

The chosen edge connects vertex D to vertex B.



Step 3: Identify the edge with the lowest weight that connects vertex A, B or D to a new vertex.

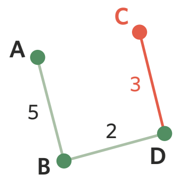
From vertex A, there is one more edge with a weight of 7.

From vertex B, there is one more edge with a weight of 11.

From vertex D, there are three more edges with weights of 3, 8 and 8.

3 is the lowest weight, so this edge is chosen.

The chosen edge connects vertex C to vertex D.



Step 4: Identify the edge with the lowest weight that connects vertex A, B, C or D to a new vertex.

The edge connecting vertices A and C would create a cycle and will no longer be considered.

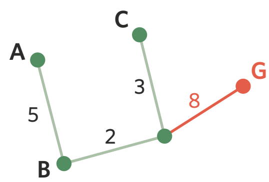
From vertex B, there is one more edge with a weight of 11.

From vertex D, there are two more edges, both with weights of 8.

From vertex C, there is one more edge with a weight of 9.

8 is the lowest weight, so either of the two edges can be chosen.

In this case, the edge connecting vertices D and G is chosen.



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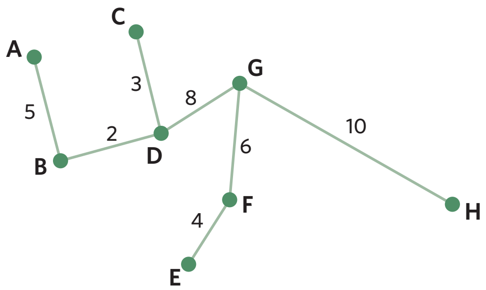
Step 5: Continue the process until all vertices are connected.

The minimum spanning tree has a total weight of 38.

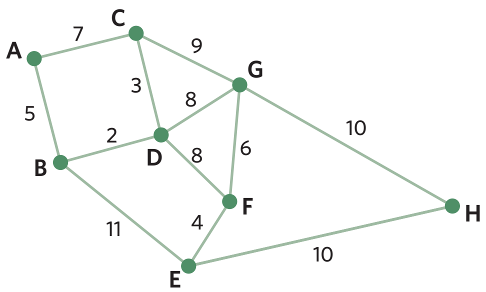
Answer



Note: There are other possible minimum spanning trees for this graph.

Worked example 3

Use Kruskal's algorithm to draw the minimum spanning tree for the following graph.



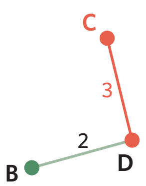
Explanation

Step 1: Identify the edge with the lowest weight.

2 is the lowest weight, so the edge connecting vertices B and D is chosen.

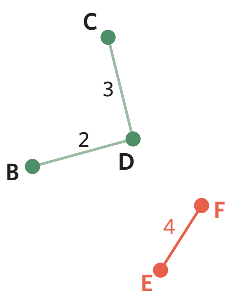
Step 2: Identify the edge with the next lowest weight.

3 is the lowest weight remaining, so the edge connecting vertices C and D is chosen.



Step 3: Identify the edge with the next lowest weight.

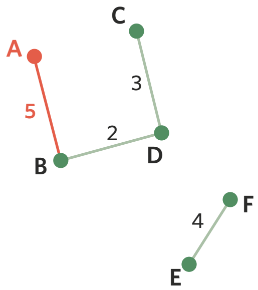
4 is the lowest weight remaining, so the edge connecting vertices E and F is chosen.



Continues →

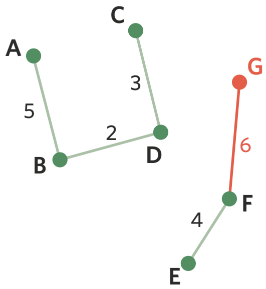
Step 4: Identify the edge with the next lowest weight.

5 is the lowest weight remaining, so the edge connecting vertices A and B is chosen.



Step 5: Identify the edge with the next lowest weight.

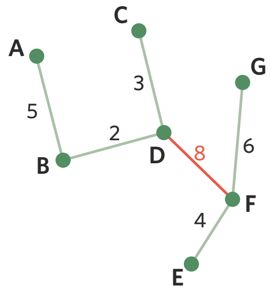
6 is the lowest weight remaining, so the edge connecting vertices F and G is chosen.



Step 6: Identify the edge with the next lowest weight.

Although 7 is the lowest weight remaining, an edge connecting vertices A and C would create a cycle. Therefore, it is disregarded.

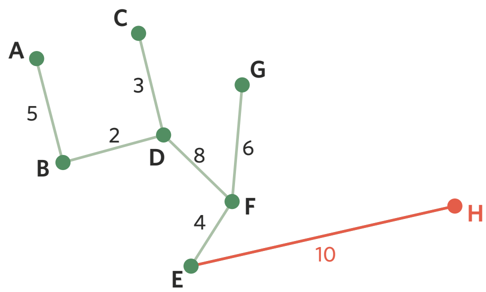
8 is the next lowest weight remaining, and there are two edges with this weight. One connects vertices D and F, and the other connects vertices D and G. Either of these edges can be added.



Step 7: Identify the edge with the next lowest weight.

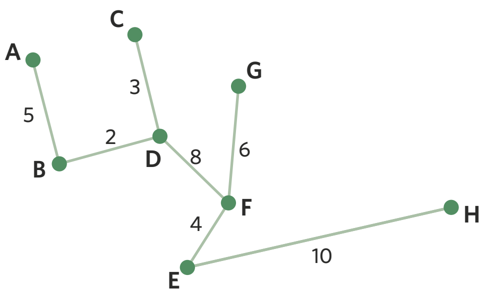
Although there are edges with weights of 8 (connecting vertices D and G) and 9 (connecting vertices C and G), these would create cycles and are therefore disregarded.

10 is the next lowest weight remaining, and there are two edges with this weight. One connects vertices G and H, and the other connects vertices E and H. Either of these edges can be added.



The graph is now connected. There are 6 edges connecting 7 vertices, so the minimum spanning tree has been identified. It has a total weight of 38.

Answer



Note: There are other possible minimum spanning trees for this graph.

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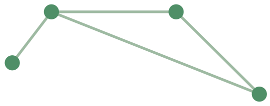
7F Questions

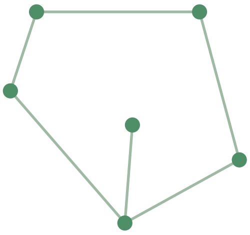
Identifying trees in a network

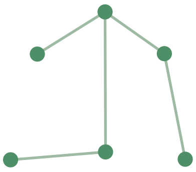
1. A tree has 7 edges. How many vertices does it have?

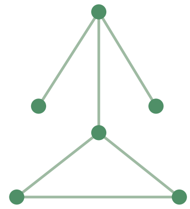
A. 5 **B.** 6 **C.** 7 **D.** 8

2. Which of the following is a tree?

A.


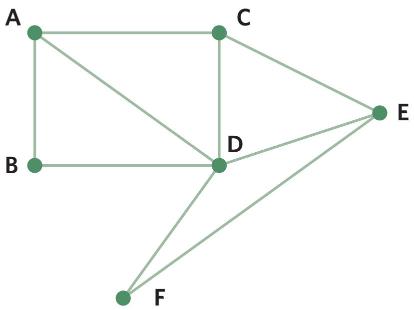
B.


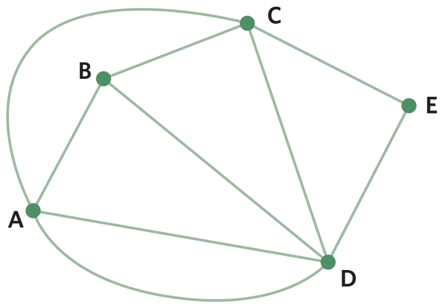
C.


D.


3. Draw a tree with 6 vertices.

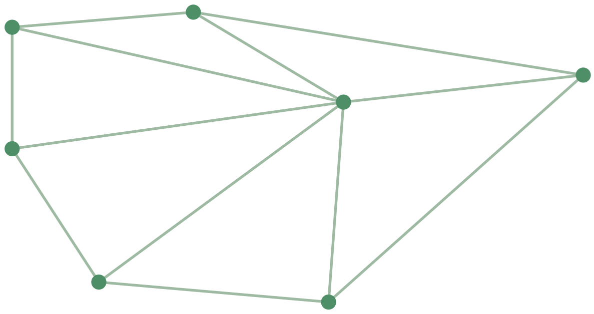
4. For each of the following graphs, draw three different spanning trees.

a.


b.


5. Sally is a software engineer. She is setting up a computer network and is trying to connect all of the computers using the least number of cables possible.

The following diagram shows a graph in which each vertex represents a computer and each edge is a possible connection path where Sally can place a cable.



a. What is the least number of cables Sally can use?

b. Draw two possible layouts of the network.

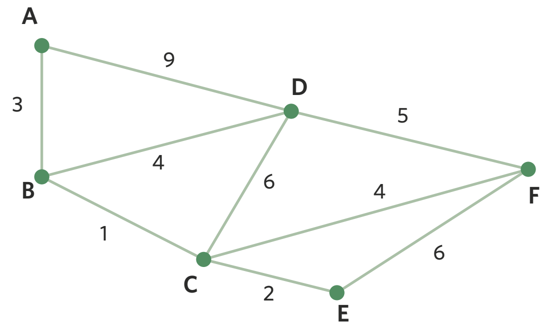
Identifying a minimum spanning tree using greedy algorithms

6. Fill in the blanks using the following words.

algorithm | graph | greedy | Kruskal's | minimum spanning tree

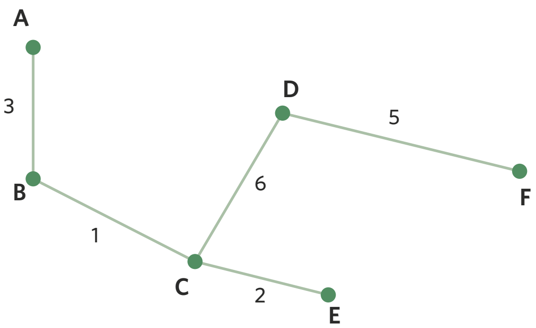
A ______ algorithm is a set of steps used to find the best solution at each step of a multi-step problem. Prim's ______ and ______ algorithm are examples of greedy algorithms. They are both used to find the ______ of a ______.

7. Consider the following graph.

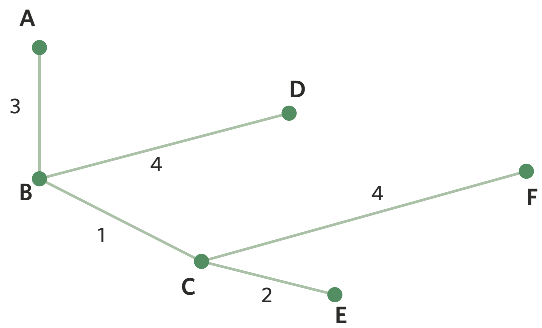


Which of the following is the minimum spanning tree?

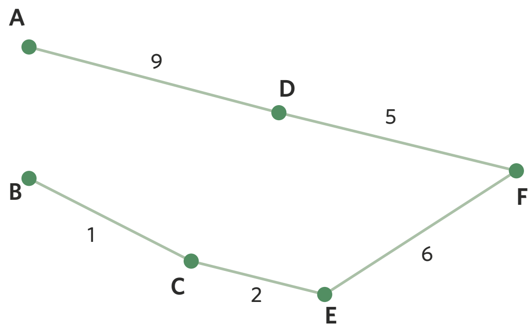
A.



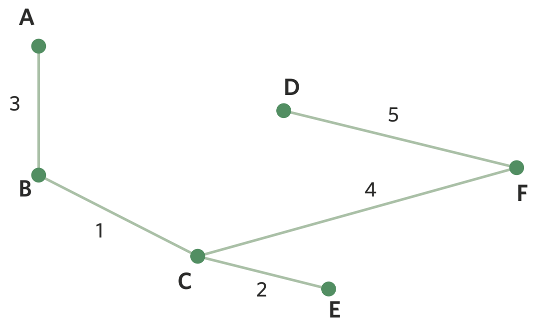
B.



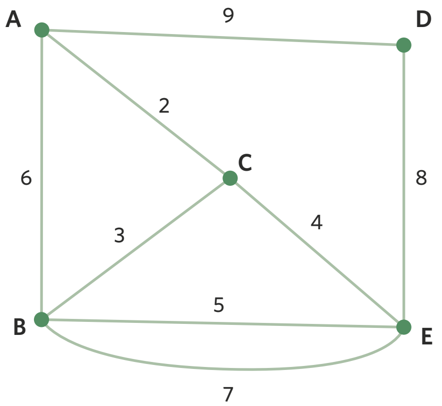
C.



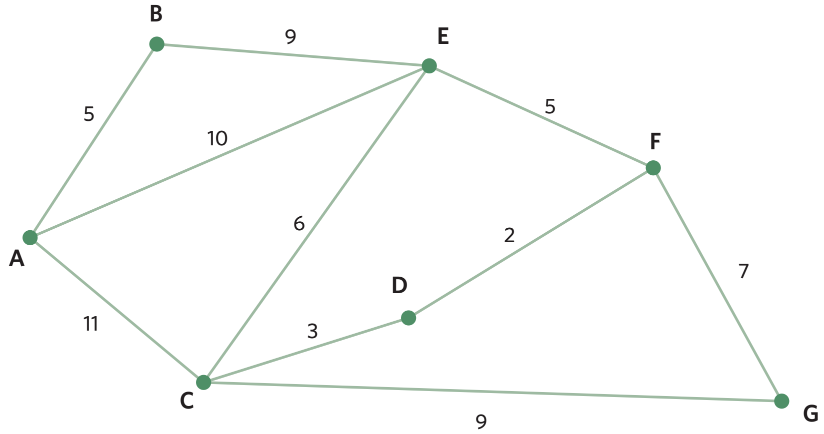
D.



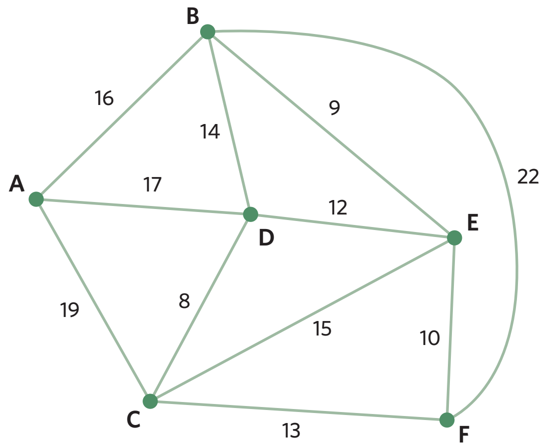
8. For the following graph, use Prim's algorithm to draw the minimum spanning tree and determine its total weight.



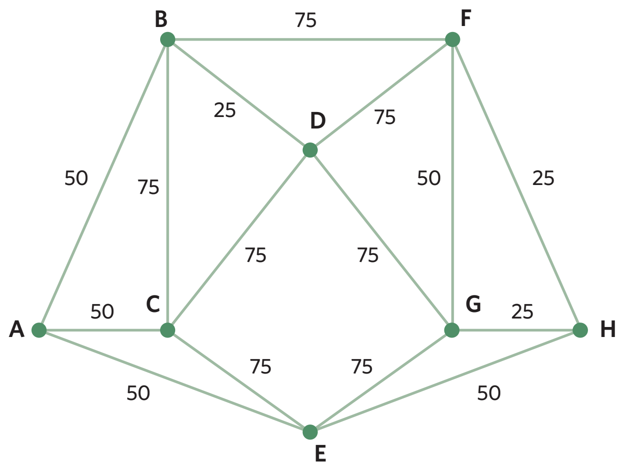
9. For the following graph, use Kruskal's algorithm to draw the minimum spanning tree and determine its total weight.



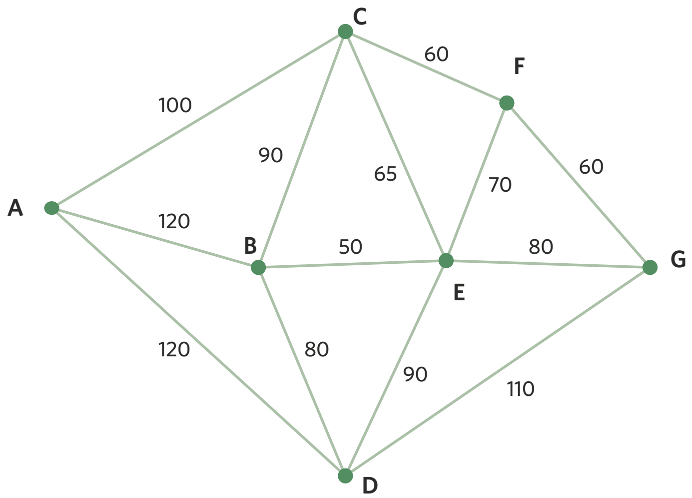
10. For the following graph, use Prim's algorithm to draw the minimum spanning tree and determine its total weight.



11. For the following graph, use Kruskal's algorithm to draw the minimum spanning tree and determine its total weight.



12. Underground tunnels are being built between seven different gold mines (A, B, C, D, E, F and G) so that they are all accessible directly or indirectly from every other mine. The possible tunnel routes between each mine and their length in, metres, are shown in the following graph.



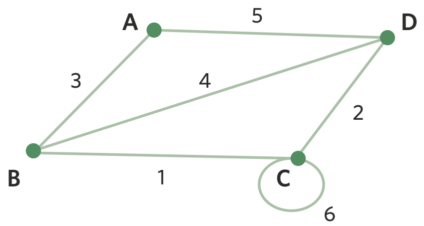
a. Draw the tunnel layout that would allow for the shortest total distance in tunnels.

b. What is the total distance of all tunnels in this layout?

c. In this layout, which mine is directly accessible from the most other mines?

Joining it all together

13. Consider the following graph.

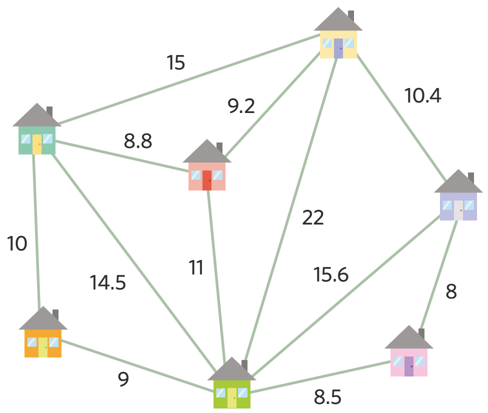


a. How many different spanning trees are there for this graph?

b. Draw the minimum spanning tree and determine its total weight.

c. Draw the spanning tree with the maximum total weight and determine its weight.

14. An electricity company is trying to connect seven bungalows at a resort in Bora Bora. The amount of electric cable, in metres, needed to connect the bungalows is shown in the following graph.



What is the minimum amount of cable needed?

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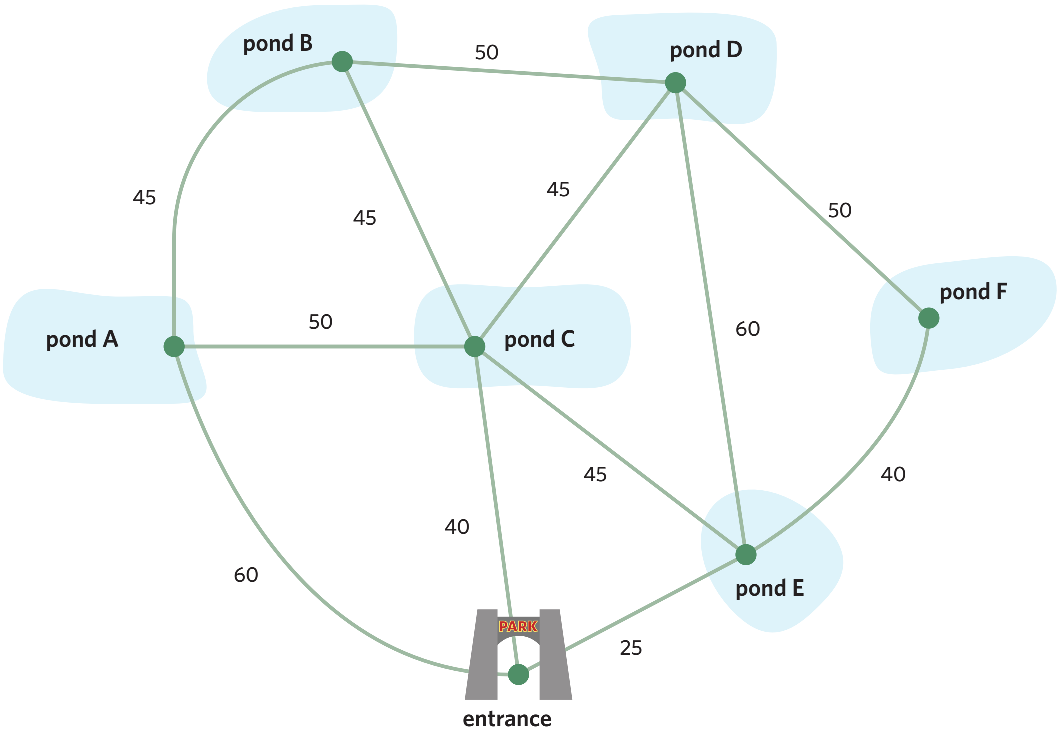
15. A Japanese garden is being built in a park. The council wants to pave paths so that the six small ponds are accessible by path from the entrance. The distances, in metres, of the possible paths are shown in the following graph.

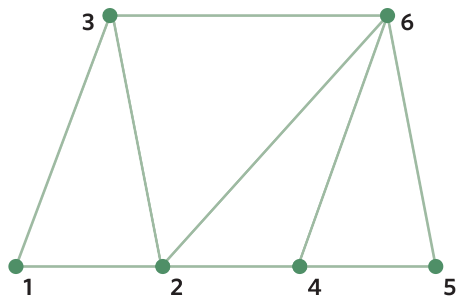


a. The council wants to minimise the number of the paths to minimise cost. Draw the path layout with the minimum total length that reaches every pond.

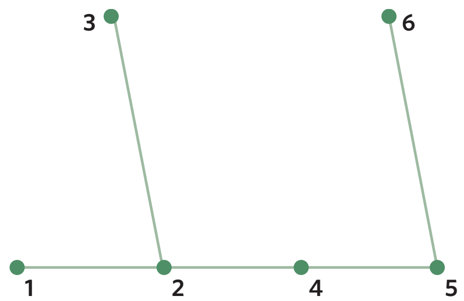
b. The cost to pave the paths is $120 per metre. How much will the council have to pay?

Exam practice

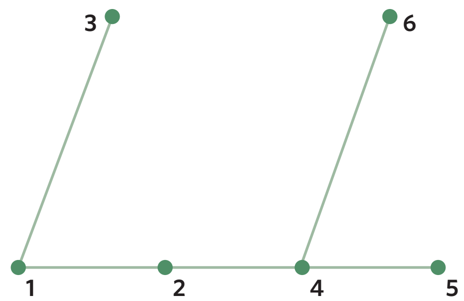
16. Which of the following is **not** a spanning tree for the given network?



A.



B.



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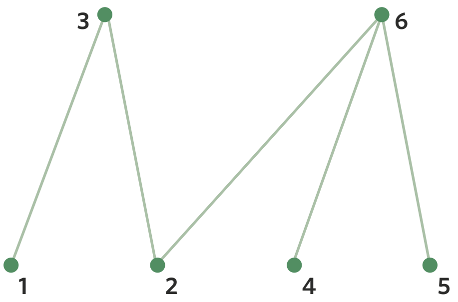
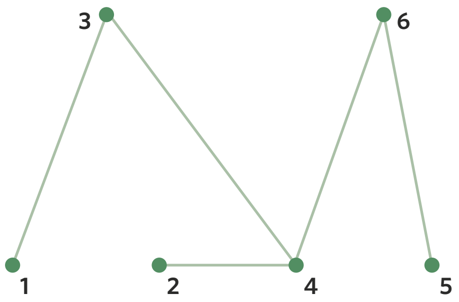
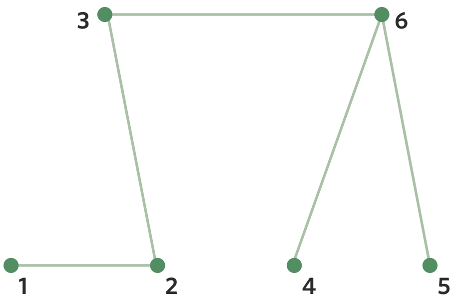
C.



D.



E.



Adapted from VCAA 2020 Exam 1 Networks and decision mathematics Q3

81% of students answered this type of question correctly.

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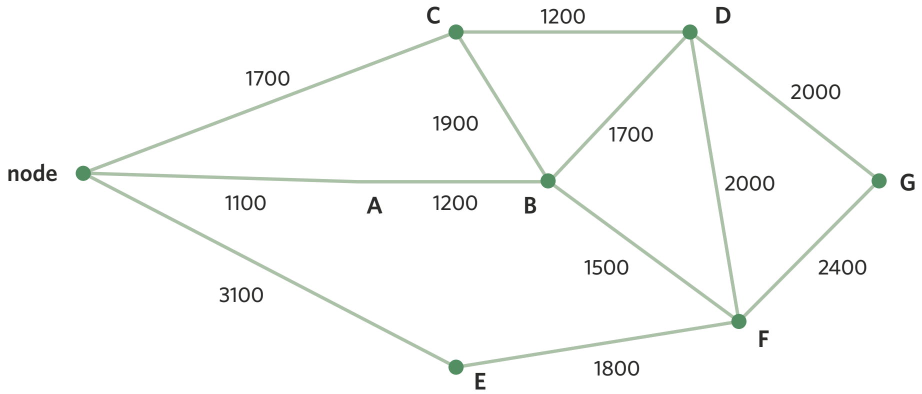
17. The following diagram shows the cost, in dollars, to lay NBN cabling to connect a series of local suburbs (A, B, C, D, E, F, G) to the node.



The minimum cost, in dollars, required to ensure that each suburb is connected to the node directly or via another suburb is

A. $10 200

B. $10 500

C. $10 800

D. $11 000

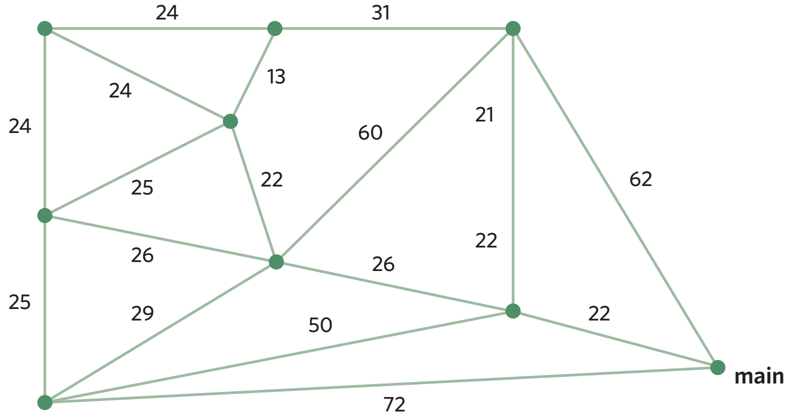
E. $11 500

Adapted from VCAA 2019 Exam 1 Networks and decision mathematics Q5

75% of students answered this type of question correctly.

18. Stacey and Ishmael are building a house. There will be 9 powerpoints throughout the house. In the following graph, the positions of the powerpoints and the main power source are indicated by the vertices.

The numbers on the edges represent the distances, in centimetres, between the powerpoints.



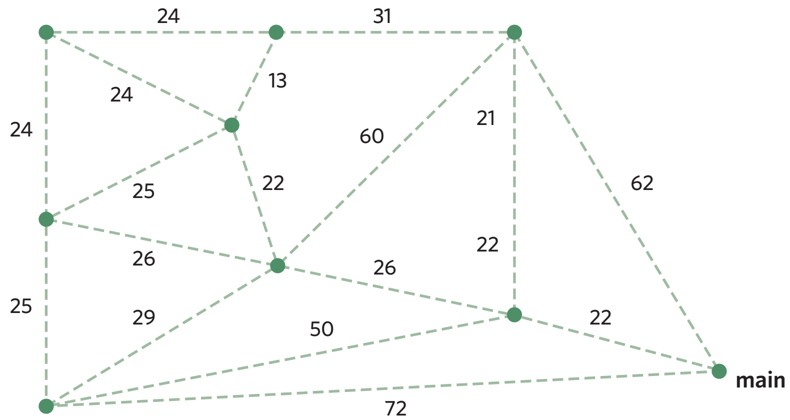
Electrical wires are required to connect the powerpoints to the main power source.

These wires will form a connected graph.

The shortest total length of cable will be used.

a. Give a mathematical term to describe a graph that represents these wires. (1 MARK)

b. Draw in the graph that represents these wires on the following diagram. (1 MARK)



Adapted from VCAA 2017 Exam 2 Networks and decision mathematics Q3a

Part **a**: **63%** of students answered this type of question correctly.

Part **b**: **53%** of students answered this type of question correctly.

Questions from multiple lessons

Computation and practical arithmetic *Year 10 content*

19. Clem needs 0.00076 kg of aluminium hydroxide for a science experiment. Written in scientific notation, this value can be expressed as

A. 76×10^{5} kg

B. 76×10^{-5} kg

C. 7.6×10^{4} kg

D. 7.6×10^{-4} kg

E. $-4 \times 10^{7.6}$ kg

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Computation and practical arithmetic

20. At Sinead's favourite vegan cafe, a small smoothie is 260 mL and a regular smoothie is 350 mL.

The increase in volume if Sinead sizes up from a small to a regular smoothie, as a percentage of the small smoothie's volume, is closest to

A. 25.7%

B. 31.4%

C. 34.6%

D. 38.5%

E. 42.3%

Recursion and financial modelling

21. Riley is travelling around Central America. She spends exactly \$170 per day. At the start of her trip she had \$7100 in her bank account.

a. Construct a recurrence relation, using B_n, B_{n+1} and B_0, that represents the daily balance of Riley's bank account, where n is the number of days she has spent travelling. (1 MARK)

b. Use recursion to show that Riley has \$5910 in her bank account after one week. (1 MARK)

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CHAPTER 8 CALCULATOR QUICK LOOK-UP GUIDE

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8

UNIT 2 AOS 3 CHAPTER 8

Variation and transformations

LESSONS

KEY KNOWLEDGE

- numerical, graphical and algebraic approaches to direct and inverse variation
- transformation of data to linearity to establish relationships between variables, for example y and x^2, y and $1/x$, and y and $\log_{10}(x)$
- modelling of given non-linear data using the relationships $y = kx^2 + c$, $y = k/x$, where $k > 0$, and $y = k\log_{10}(x) + c$, where $k > 0$.

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8A Variation

STUDY DESIGN DOT POINT

- numerical, graphical and algebraic approaches to direct and inverse variation

8A 8B 8C 8D

KEY SKILLS

During this lesson, you will be:

- classifying variation as direct or inverse
- constructing rules for variation
- graphing variation.

KEY TERMS

- Variation
- Constant of proportionality
- Direct variation
- Inverse variation

It is often important to understand how values and patterns are related to each other. When cooking rice, the amount of water needed is approximately double the amount of rice, so as the amount of rice increases, the amount of water must increase proportionally. Similarly, the faster a car travels, the more distance it covers, and the less time it takes to arrive at a particular destination. These are examples of the relationship between two or more variables.

Classifying variation as direct or inverse

Variation is the change in one variable as a result of the change in another variable.

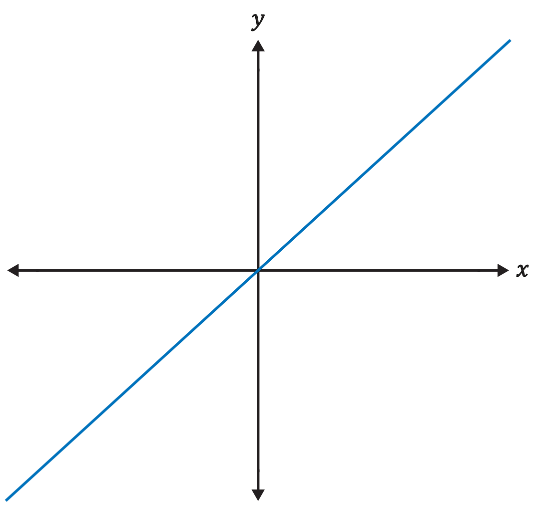
If two variables vary with each other, they are said to be proportional. The degree of proportionality is defined by k, the **constant of proportionality**. There are two distinct types of variation, direct and inverse.

Direct variation is when an increase in one variable results in a scalar change in the other variable.

The graph of direct variation between x and y is a straight line with the equation

$y = kx$

A graph of direct variation is no different to other linear equations. However, graphs of direct variation always have a y-intercept equal to 0.



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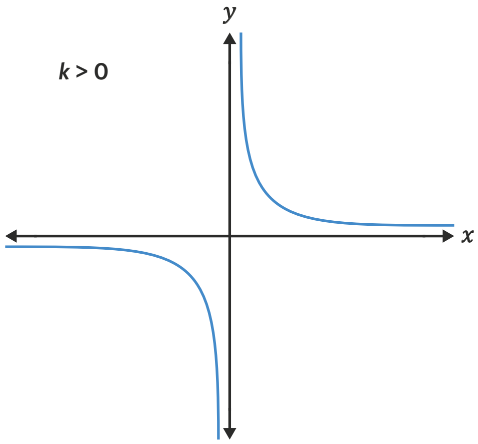
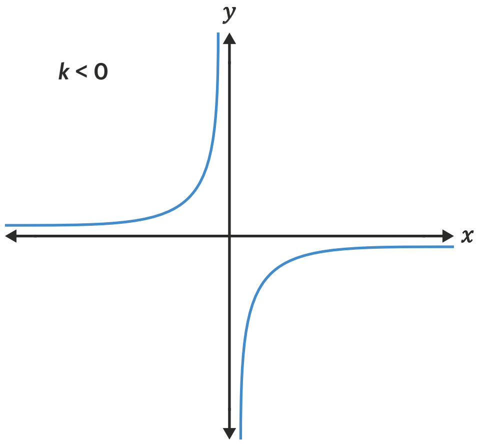
Inverse variation is when an increase in one variable results in a scalar reciprocal change in the other variable.

The graph of inverse variation between x and y is a hyperbola with the equation

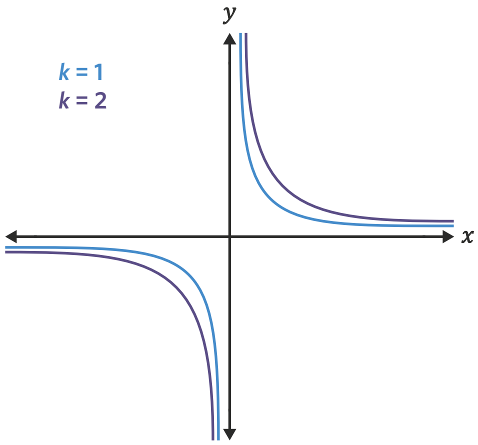
$y = \frac{k}{x}$

A hyperbola is a graph shape in which a curved line approaches a vertical and horizontal limiting value from opposite sides. The vertical and horizontal limiting values for a graph of inverse variation are the two axes. That is, $x = 0$ and $y = 0$. The hyperbola will be in either of the following two forms, depending on whether k is positive or negative.





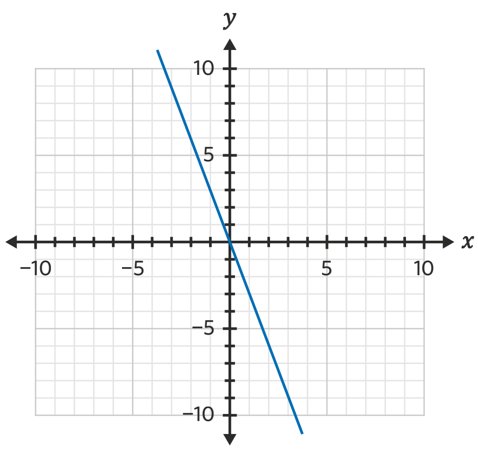
Larger values of k will shift the hyperbola away from the axes as shown.



Worked example 1

Determine whether the following graphs show direct variation, inverse variation or neither.

a.



Continues →

Explanation

Step 1: Determine whether the shape resembles direct or inverse variation.

The graph is a straight line, so it resembles direct variation.

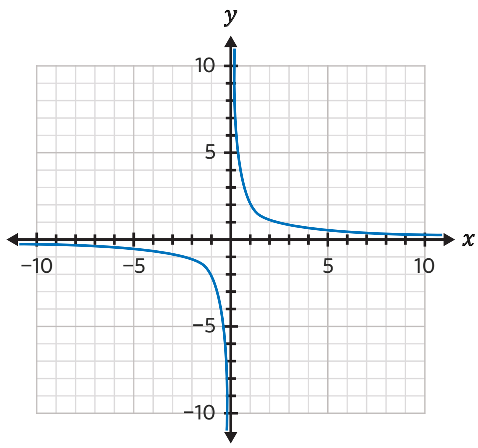
Step 2: Confirm whether it is direct variation.

The line passes through the origin, so it is direct variation.

Answer

Direct variation

b.



Explanation

Step 1: Determine whether the shape resembles direct or inverse variation.

The graph is a hyperbola, so it resembles inverse variation.

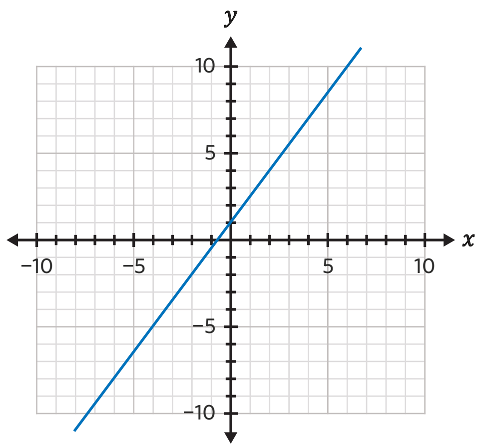
Step 2: Confirm whether it is inverse variation.

The hyperbola approaches each of the axes, so it is inverse variation.

Answer

Inverse variation

c.



Explanation

Step 1: Determine whether the shape resembles direct or inverse variation.

The graph is a straight line, so it resembles direct variation.

Step 2: Confirm whether it is direct variation.

The line does not pass through the origin, so it is not direct variation.

Answer

Neither

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Constructing rules for variation

There are multiple ways to describe the relationship between two or more variables. The following table shows a few examples of how relationships can be described, alongside their corresponding rules.

rule	description
$y = kx$	y is directly proportional to x. y varies directly with x.
$y = \frac{k}{x}$	y is inversely proportional to x. y varies inversely with x. y varies directly with $\frac{1}{x}$.
$y = kx^2$	y is directly proportional to the square of x. y varies directly with x^2.
$y = \frac{k}{\sqrt{x}}$	y is inversely proportional to the square root of x. y varies inversely with $\sqrt{x}$. y varies directly with $\frac{1}{\sqrt{x}}$.

The value of k can be determined if the values of the variables are known at a certain point. This can be given in the form of coordinates. Once the value of k is determined, the rule can be used to find y from any given x value, or vice versa.

Worked example 2

y is directly proportional to x cubed. It is known that $y = 32$ when $x = 4$.

a. Construct the rule for the variation between y and x.

Explanation

Step 1: Determine the variation between y and x.

y is directly proportional to x^3:

$y = kx^3$

Step 2: Substitute the known values of x and y and solve for k.

$32 = k \times 4^3$

$32 = k \times 64$

$k = \frac{32}{64}$

$= \frac{1}{2}$

Step 3: Substitute the value of k into the general rule.

Answer

$y = \frac{x^3}{2}$

b. Determine the value of y when $x = 8$.

Explanation

Substitute $x = 8$ into the rule.

$y = \frac{8^3}{2}$

$= \frac{512}{2}$

$= 256$

Answer

256

Graphing variation

Once a rule has been determined and the constant of proportionality, k, is known, the graph representing the variation between the variables can be constructed.

Worked example 3

Draw a graph to represent the following relationships.

a. y varies directly with x, and $k = 2$.

Explanation

Step 1: Determine the variation between y and x.

y is directly proportional to x:

$y = kx$

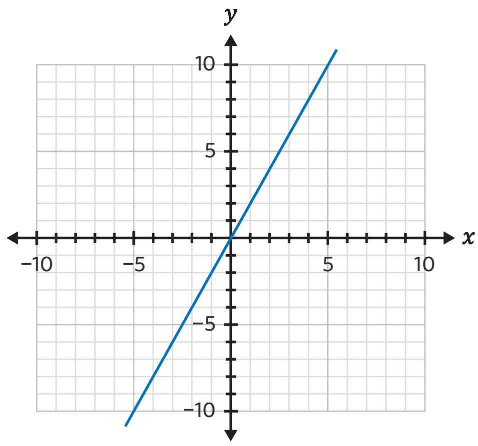
Step 2: Substitute the value of k into the rule.

$y = 2x$

Step 3: Graph the rule.

$y = 2x$ is represented by a line that passes through the origin with a gradient of 2.

Answer



b. y varies inversely with x, and $k = -3$.

Explanation

Step 1: Determine the variation between y and x.

y is inversely proportional to x:

$y = \frac{k}{x}$

Step 2: Substitute the value of k into the rule.

$y = -\frac{3}{x}$

Step 3: Find a point on the graph.

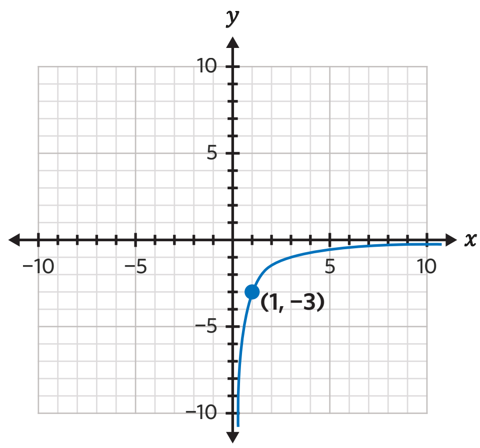
Pick an x value to substitute into the rule.

$x = 1$ is a simple choice.

$y = -\frac{3}{1}$

$= -3$

Step 4: Graph a hyperbola that goes through the point $(1, -3)$.



Step 5: Graph a mirrored hyperbola on the opposite side of both the x and y-axis.

Continues →

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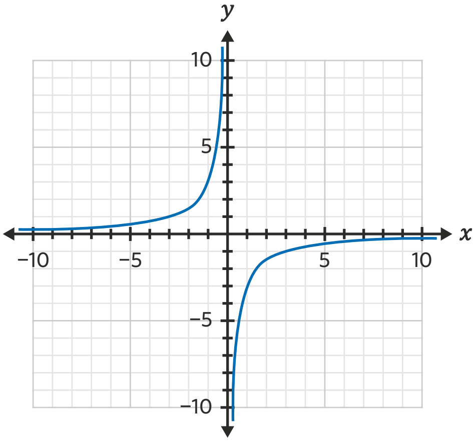
Answer



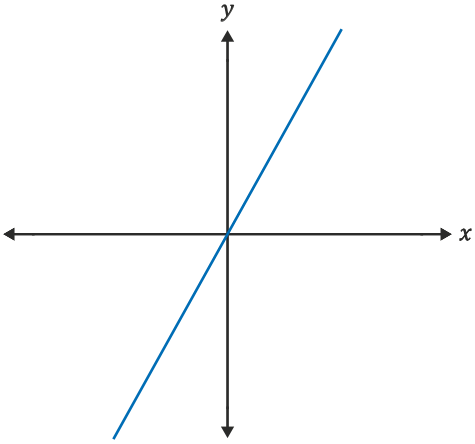
8A Questions

Note: There are no direct exam questions relevant to this lesson.

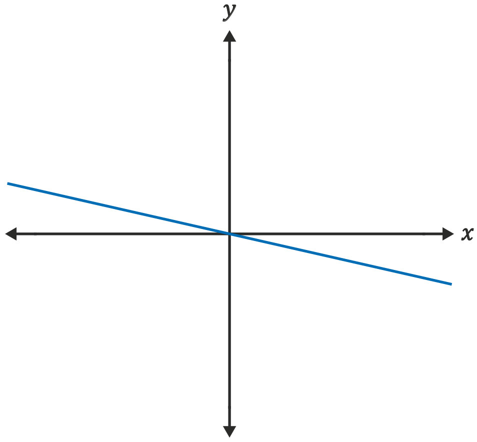
Classifying variation as direct or inverse

1. If y varies directly with x and $k < 0$, which graph could represent the relationship between x and y?

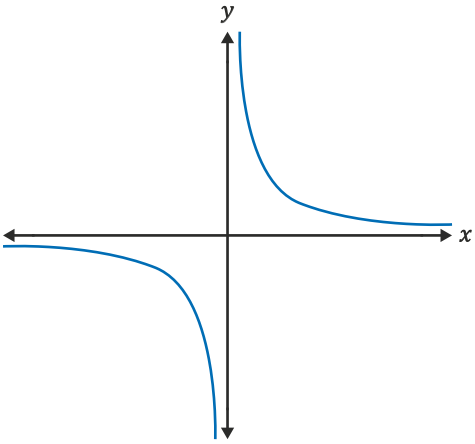
A.



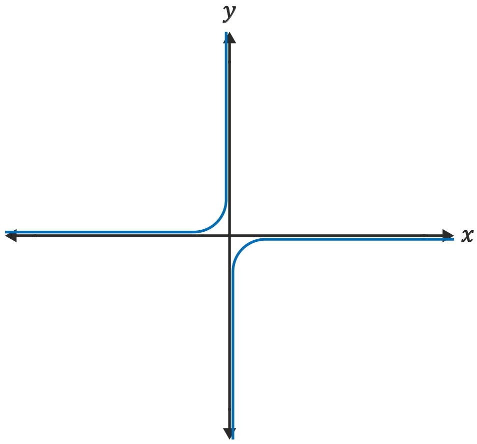
B.



C.

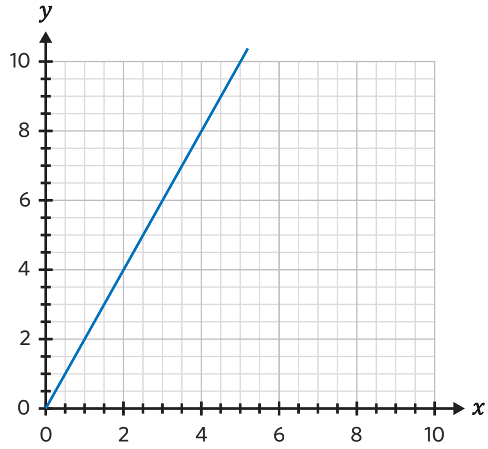


D.

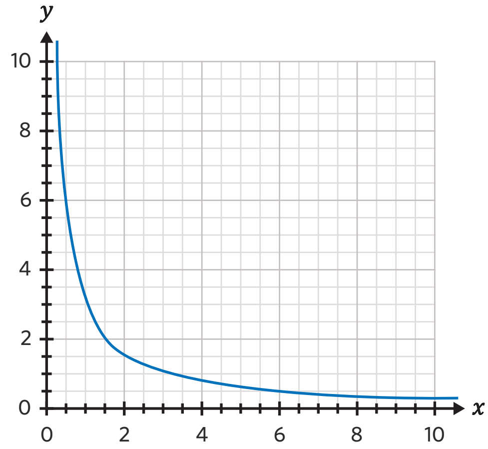


2. Determine whether the following graphs show direct or inverse variation.

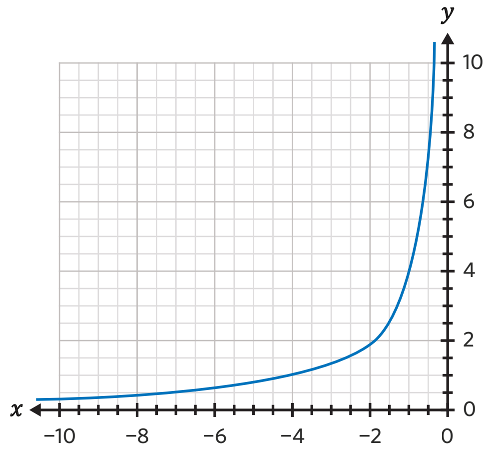
a.



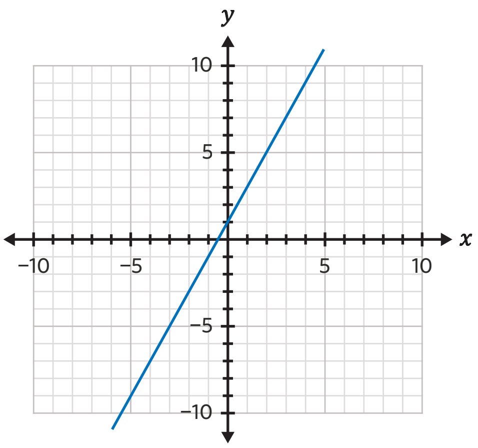
b.



c.



3. In the following graph, does y vary directly with x?



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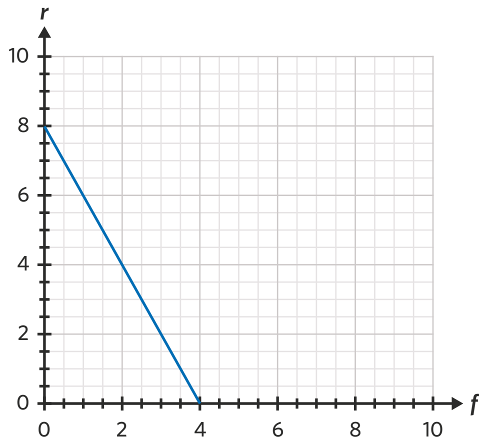
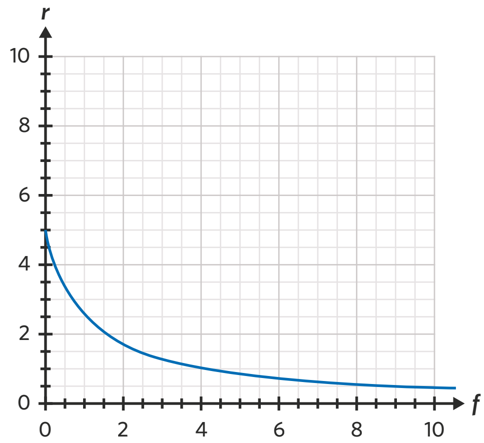
4. The population of rabbits in a town, r, is inversely proportional to the population of foxes, f. Which graph could represent the relationship between foxes and rabbits?

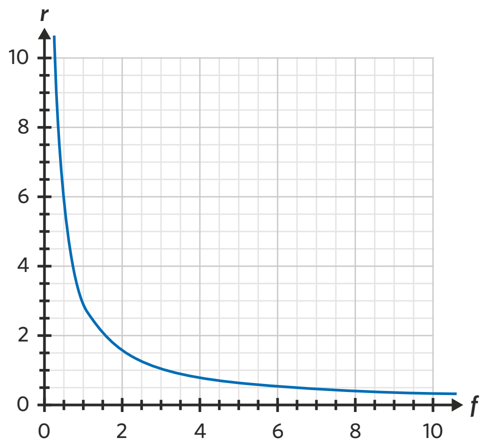
A.



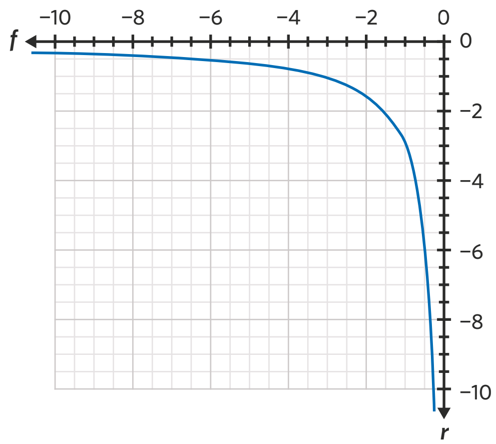
B.



C.



D.



Constructing rules for variation

5. In which of the following rules does y vary directly with x?

A. $y = 3x + 1$ **B.** $2x = -\frac{1}{y}$ **C.** $y = -\frac{2}{x}$ **D.** $y = -x$

---

6. Write rules for the following statements, in terms of k.

a. y is inversely proportional to x.

b. y varies directly with x^2.

c. y varies directly with $\frac{1}{x}$.

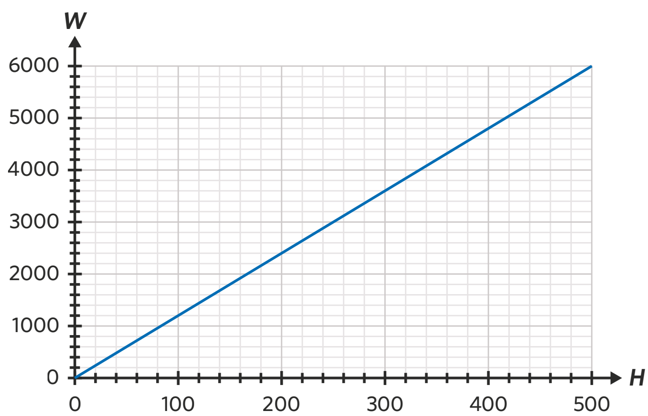
d. y varies inversely with $\frac{1}{\sqrt{x}}$.

---

7. The weight of an elephant, in kilograms, W, varies directly with its height, in centimetres, H, as shown in the following graph.

a. Construct a rule to describe the relationship between W and H.

b. Determine the height of an elephant that weighs 3000 kg.



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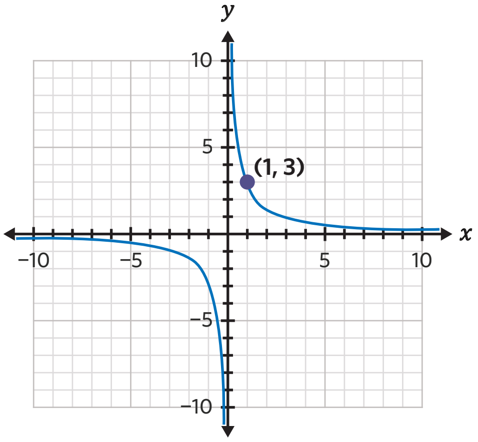
8. Consider the following graph.



a. What is the constant of proportionality?

b. Determine the value of y when $x = -15$.

---

9. Fill in the following boxes.

a. y varies directly with x^2.

$k = 4$

$y =$ ☐ when $x = 1$.

b. y varies inversely with x.

$k =$ ☐

$y = 5$ when $x = -2$.

---

10. If $k = 6$, determine the value of y when $x = -3$ for each of the following relationships.

a. $y = kx$

b. $y = kx^2$

c. $y = \frac{k}{x^3}$

---

11. Which of the following rules is not the same as the others?

A. y varies directly with $\frac{1}{x^2}$.

B. x^2 varies inversely with y.

C. y varies inversely with x^2.

D. x^2 varies directly with y.

---

12. Mary is working on a maths problem in class, but she is shortsighted and cannot see the board clearly.

She can see that y varies inversely with x raised to a certain power, but she cannot see what the power is.

If $k = -192$, and $x = 4$ when $y = -\frac{3}{4}$, what power is x raised to?

---

13. The energy, E, of an object is directly proportional to its mass, m, multiplied by the square of the speed of light, c.

If the constant of proportionality is 1, find a rule that relates E with m and c.

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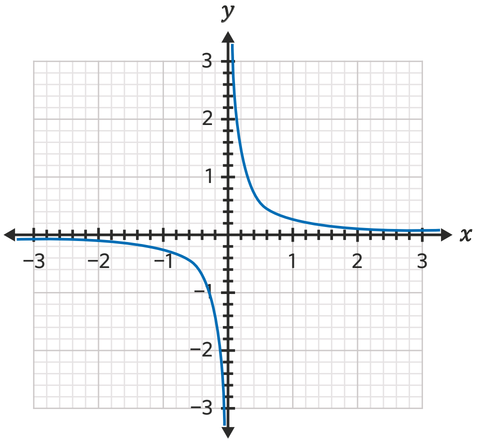
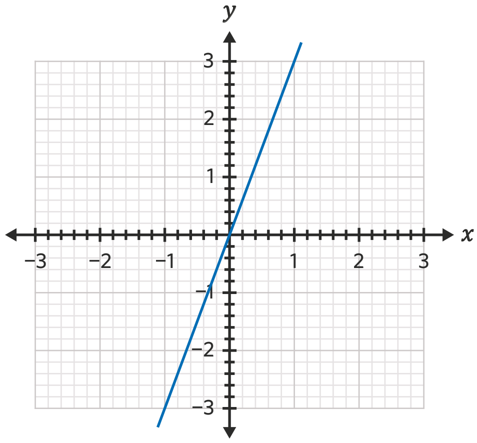
Graphing variation

14. If y is directly proportional to x and $k = 0.3$, which of the following graphs represent the relationship between y and x?

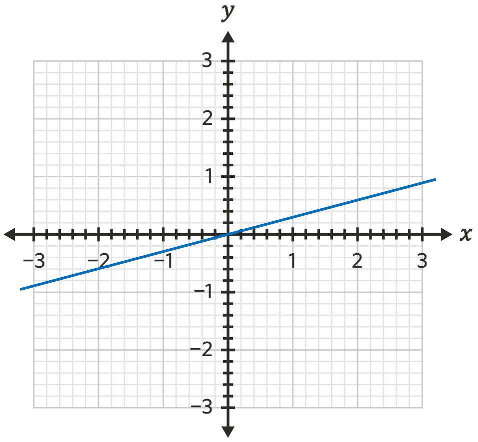
A.



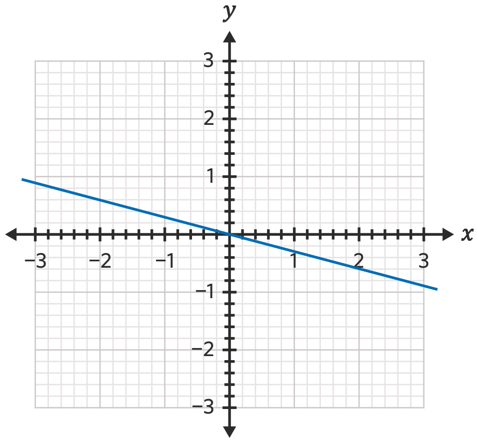
B.



C.



D.



15. Draw a graph to represent the following relationships.

a. y varies directly with x, and $k = -0.5$.

b. y varies inversely with x, and $k = 4$.

c. y varies directly with x, and $k = 2$.

16. Consider the following relationship.

$V = \frac{kT}{P}$

where V is volume in litres, T is temperature in Kelvin and P is pressure in kilopascals.

Draw a graph for when

a. $k = 10$ and $P = 100$

b. $k = 2$ and $T = 300$

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Joining it all together

17. The weight of a pole vaulting pole varies with its length. The rule connecting weight, W (kg) and length, L (m) is

$W = 0.7L$

a. Does weight vary directly or inversely with length?

b. What is the constant of proportionality?

c. Draw a graph of the relationship between W and L.

d. How much would a 4.5 m long pole weigh in kilograms?

e. How many centimetres long is a pole weighing 2100 grams?

---

18. The cost of a bunch of grapes varies directly with its weight.

A bunch of grapes weighing 0.5 kg costs $5.50.

a. Determine the rule relating the cost, C ($), with weight, w (kg), with C as the subject.

b. How much would a bunch of grapes weighing 700 grams cost?

---

19. Consider the following table.

x	1	2	3	4
y	-4	-2	$-\frac{4}{3}$	-1

a. What type of variation is shown in the table?

b. Determine the rule relating y with x.

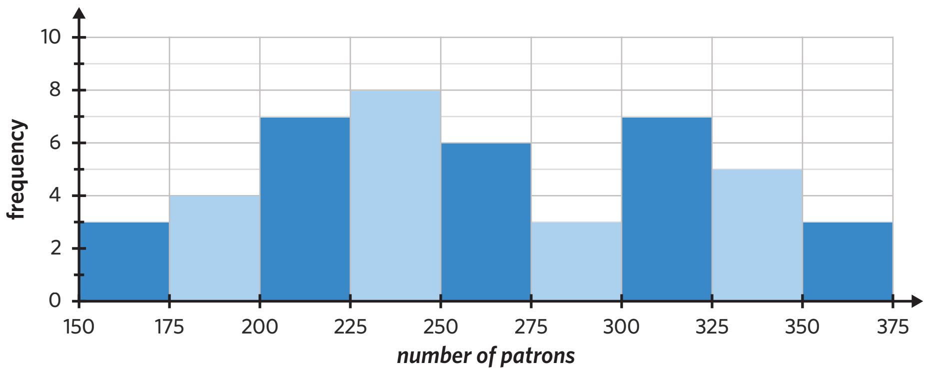
c. What is the value of y when $x = 24$?

Questions from multiple lessons

Data analysis

20. Angelo's favourite hangout spot is Revolver Upstairs in Prahran. Each night, he counts the number of patrons inside the establishment at a particular time.

The following histogram displays his results over a period of 46 days.



The median *number of patrons* is

A. greater than 200 but less than 225.

B. greater than 225 but less than 250.

C. greater than 250 but less than 275.

D. greater than 275 but less than 300.

E. greater than 300 but less than 325.

Adapted from VCAA 2017NH Exam 1 Data analysis Q3

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Recursion and financial modelling

21. Consider the following recurrence relation.

$M_0 = 50\,000, \quad M_{n+1} = M_n + 3250$

Which of the following scenarios could this recurrence relation be used to model?

A. A simple interest investment of \$50 000 with an annual interest rate of 3.25%

B. A simple interest investment of \$50 000 with an annual interest rate of 6.5%

C. A simple interest investment of \$50 000 with an annual interest rate of 65%

D. A compound interest investment of \$50 000 with an annual interest rate of 3.25%

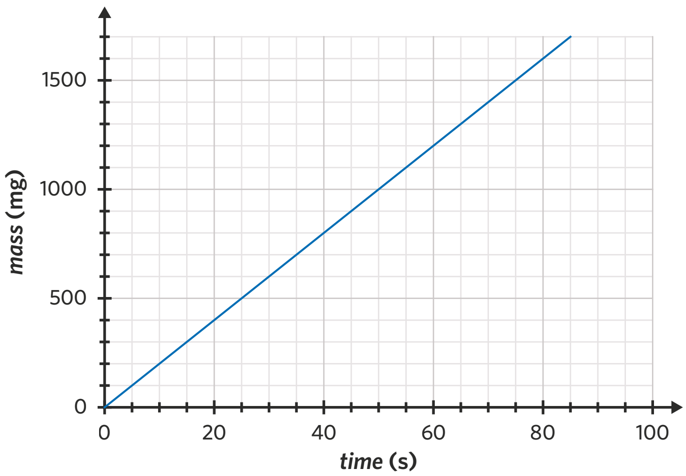
E. A compound interest investment of \$50 000 with an interest rate of 6.5%

Adapted from VCAA 2017NH Exam 1 Recursion and financial modelling Q19

Graphs and relations *Year 10 content*

22. A researcher is trying to determine the time taken for different masses of an unknown compound to completely break down and dissolve in a strong acid.

The following graph displays the relationship between *time* in seconds and *mass* in milligrams.



a. Lara has an unknown mass of the compound. She does not have a scale so she decides to test and see how long it takes to completely dissolve. It dissolves in 60 seconds.

How much of the compound does she have? (1 MARK)

b. The slope of this graph is the rate at which the compound dissolves per second.

How much can she dissolve in one second? (1 MARK)

Adapted from VCAA 2015 Exam 2 Graphs and relations Q2

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8B Transformations – kx^2

STUDY DESIGN DOT POINTS

- transformation of data to linearity to establish relationships between variables, for example y and x^2, y and $\frac{1}{x}$, and y and $\log_{10}(x)$
- modelling of given non-linear data using the relationships $y = kx^2 + c$, $y = \frac{k}{x}$, where $k > 0$, and $y = k\log_{10}(x) + c$, where $k > 0$

8A 8B 8C 8D

KEY SKILLS

During this lesson, you will be:

- applying a square transformation to numerical data
- modelling non-linear data using squared variation.

KEY TERMS

- x-squared transformation

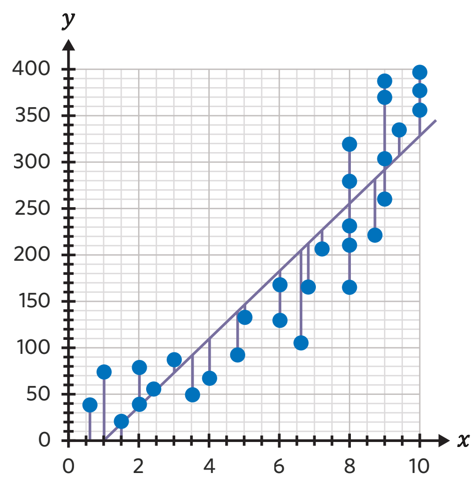
In order to have accurate predictions of a data set using a straight line of good fit, the data needs to be linear. However, in some cases the data presented is not linear and requires transformations to be applied to it. For example, bacteria populations increase more and more rapidly as time goes on. Applying a squared transformation may help to linearise the data and allow for conclusions to be drawn.

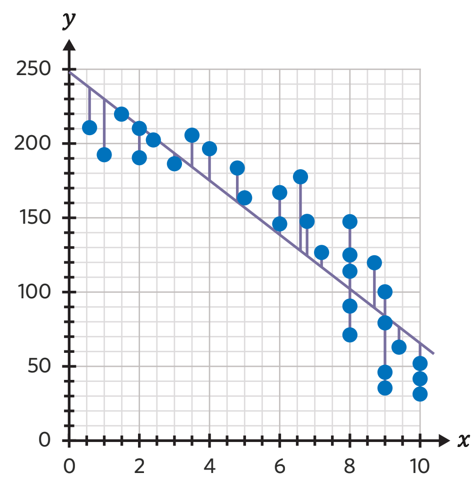
Applying a square transformation to numerical data

A numerical data set may not always follow a linear trend. As such, predictions using a line of good fit will be unreliable, since it will not appropriately model the behaviour of the data set.

A squared transformation is one type of transformation that can be applied to a set of data to linearise it. In this lesson, squared transformations will involve the explanatory variable only.

A data set may require the explanatory variable to undergo a squared transformation when its scatterplot looks similar to one of the following graphs.





A data set can be transformed by hand or using a calculator.

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Worked example 1

Consider the following table.

x	1	3	5	9	11
x^2					
y	6	3	5	12	21

Perform a squared transformation on x by filling in the table.

Explanation – Method 1: By hand

Step 1: Calculate the square of each x value.

$$1^2 = 1 \times 1$$
$$= 1$$
$$3^2 = 3 \times 3$$
$$= 9$$
$$5^2 = 5 \times 5$$
$$= 25$$

Repeat this for the remaining x values.

Step 2: Fill in the table with the squared x values.

Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name column A 'x' and column B 'y'.

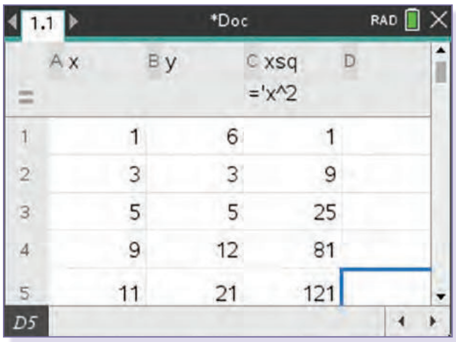
Enter the x values into column A, starting from row 1.

Enter the y values into column B, starting from row 1.

Step 3: Name column C 'xsq' (short for x squared).

Enter '=x^2' into the cell below the 'xsq' heading.

Select 'Variable Reference' → 'OK'.



Step 4: Fill in the table with the squared x values.

Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap Statistics.

Step 2: Name the first list 'x' and the second list 'y'.

Enter the x values into list 'x', starting from row 1.

Enter the y values into list 'y', starting from row 1.

Continues →

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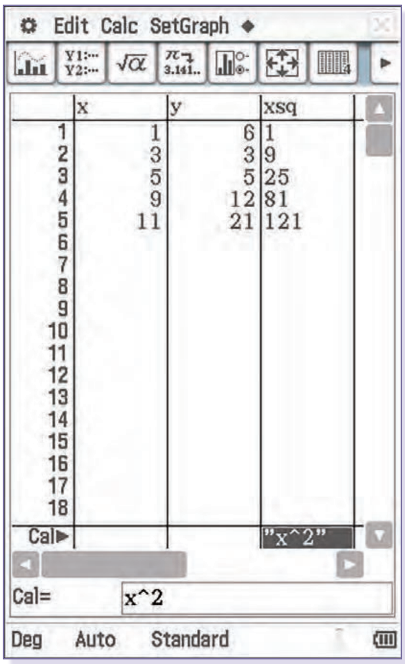
Step 3: Name the third list 'xsq' (short for x-squared).

In the third list, go down to the calculator cell Cal▶ and enter 'x^2'.



Step 4: Fill in the table with the squared x values.

Answer – Method 1, 2 and 3

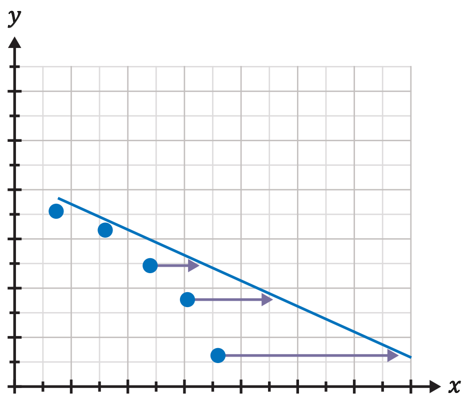
x	1	3	5	9	11
x^2	1	9	25	81	121
y	6	3	5	12	21

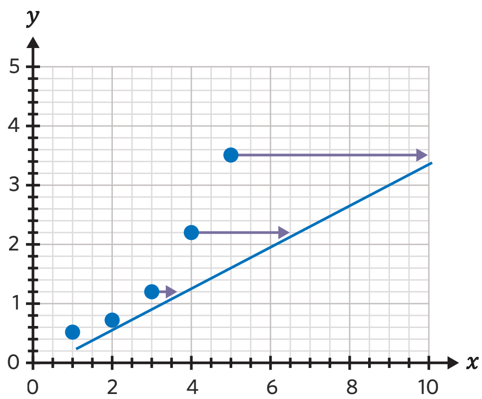
Modelling non-linear data using squared variation

On a graph, an **x-squared transformation** involves 'stretching' the larger explanatory variable (x) values more than the smaller values. The response variable values remain the same.

See worked example 2

The following graphs demonstrate how an x-squared transformation can linearise the data by stretching the larger values of the explanatory variable.





For transformed data, the process of determining the equation for a line of good fit is very similar to when determining one for a linear data set. However, one of the original variables will be replaced by the transformed variable in the equation. In the case of x-squared transformations, the variable 'x' is replaced with 'x^2'. This transformed equation for the line of good fit can then be used to draw conclusions about the association between the two variables and make predictions.

See worked example 3

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Worked example 2

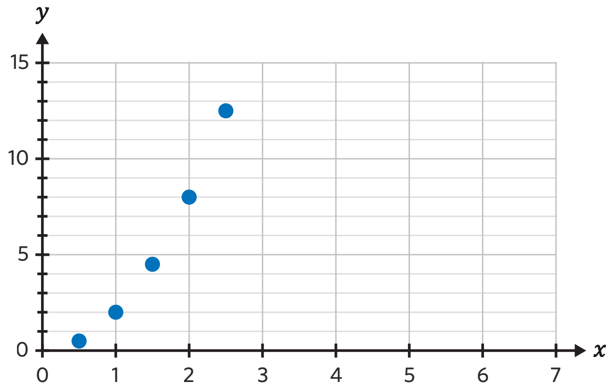
Consider the following data.

x	0.5	1	1.5	2	2.5
y	0.5	2	4.5	8	12.5

a. Why might an x-squared transformation linearise the data?

Explanation

Plot the data and observe its form.



Answer

Since the graph is curved, with y values increasing at an increasing rate, an x-squared transformation could linearise the data by stretching the larger x values.

b. Plot the transformed data.

Explanation – Method 1: By hand

Step 1: Apply an x-squared transformation.

x	0.5	1	1.5	2	2.5
x^2	0.25	1	2.25	4	6.25
y	0.5	2	4.5	8	12.5

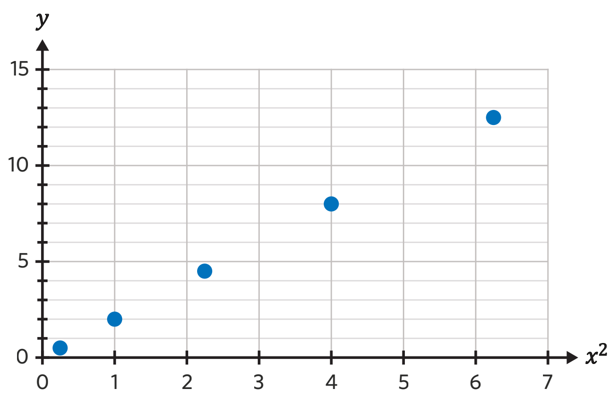
Step 2: Construct a set of axes.

A scale from 0 to 7 is appropriate for the horizontal axis, while the vertical axis should extend from 0 to 15.

The horizontal axis should be labelled x^2.

Step 3: Plot the data points using the x^2 and y values.

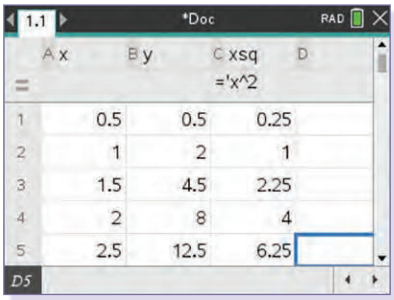
Answer



Continues →

Explanation – Method 2: TI-Nspire

Step 1: Apply an x-squared transformation.



Step 2: Press ctrl + doc, and select '5: Add Data & Statistics'.

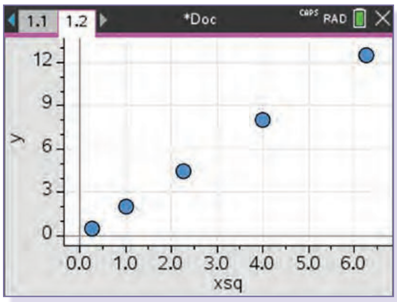
Move the cursor to the horizontal axis and select 'Click to add variable'.

Select 'xsq'.

Move the cursor to the vertical axis and select 'Click to add variable'.

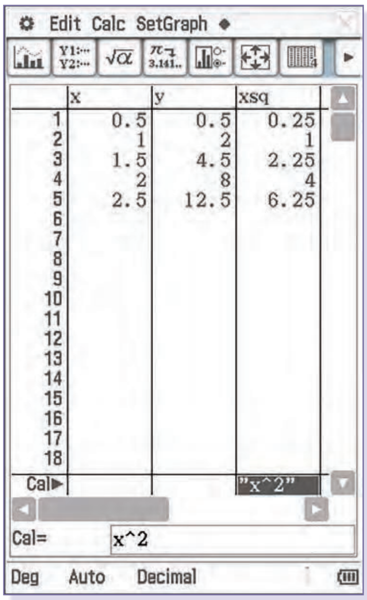
Select 'y'.

Answer



Explanation – Method 3: Casio ClassPad

Step 1: Apply an x-squared transformation.



Step 2: Configure the settings of the graph by tapping .

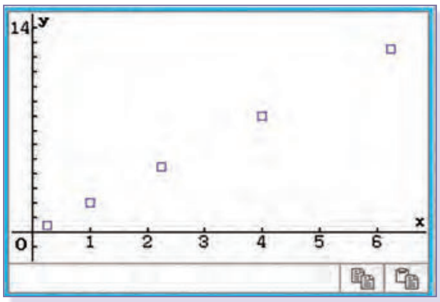
Create a scatterplot by changing 'Type' to 'Scatter'.

Specify the data set by changing 'XList:' to 'main\xsq' and 'YList:' to 'main\y'.

Tap 'Set' to confirm.

Step 3: Tap to plot the graph.

Answer



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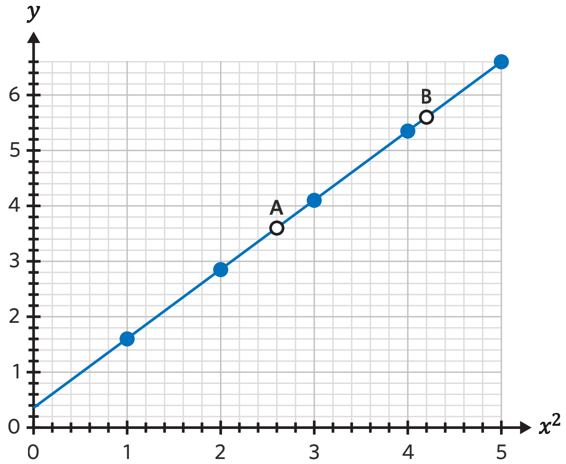
Worked example 3

An x-squared transformation was applied to a non-linear data set. A line of good fit was then added to show the relationship between x^2 and y. Points A and B are (2.6, 3.6) and (4.2, 5.6) respectively.



Find the equation of this line.

Explanation

Step 1: Calculate b using the gradient formula.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5.6 - 3.6}{4.2 - 2.6}$$

$$= \frac{2}{1.6}$$

$$= 1.25$$

Step 2: Substitute a coordinate and the gradient into the equation for a straight line to calculate the y-intercept.

$y = a + bx$

$5.6 = a + 1.25 \times 4.2$

$a = 0.35$

Step 3: Write the equation for the line of good fit.

Make sure to replace x with x^2.

Answer

$y = 0.35 + 1.25x^2$

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8B Questions

Note: There are no direct exam questions relevant to this lesson.

Applying a square transformation to numerical data

1. What value is missing from the table?

x	1	2	3	4
x^2		4	9	16
y	15	12	9	6

A. 0 **B.** 1 **C.** 3 **D.** 5

---

2. Apply an x-squared transformation to the following tables.

a.

x	-3	-1	1	3
x^2				
y	36	24	19	17

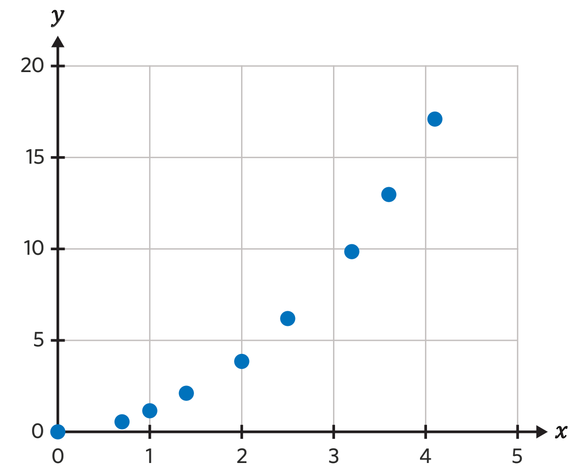
b.

x	-8	-7	-6	-5
x^2				
y	52	41	36	32

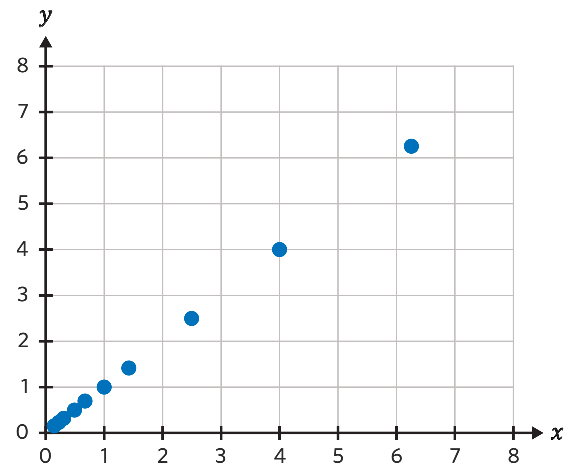
Modelling non-linear data using squared variation

3. Which graph requires an x-squared transformation to linearise the data?

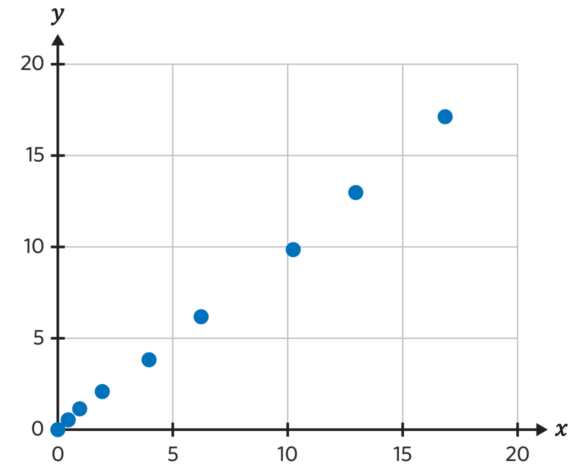
A.



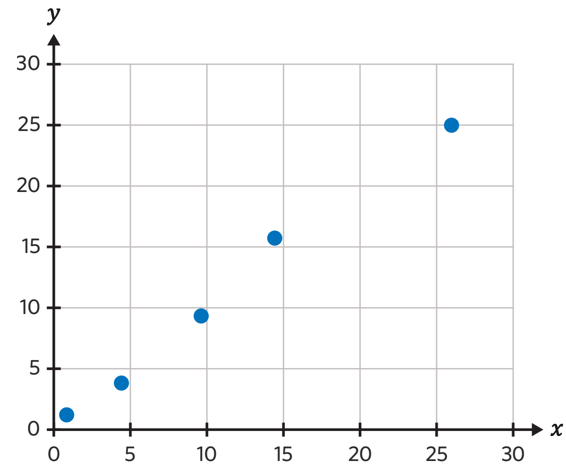
B.



C.



D.



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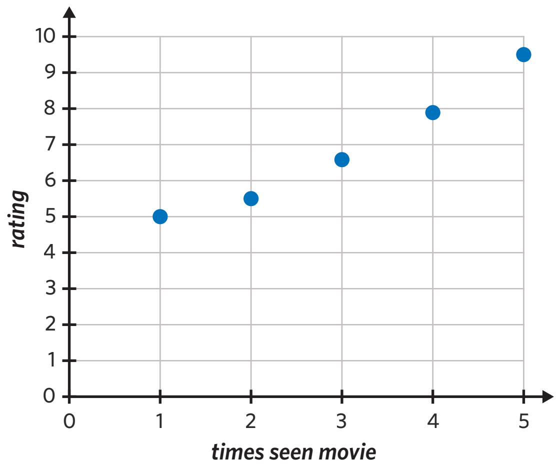
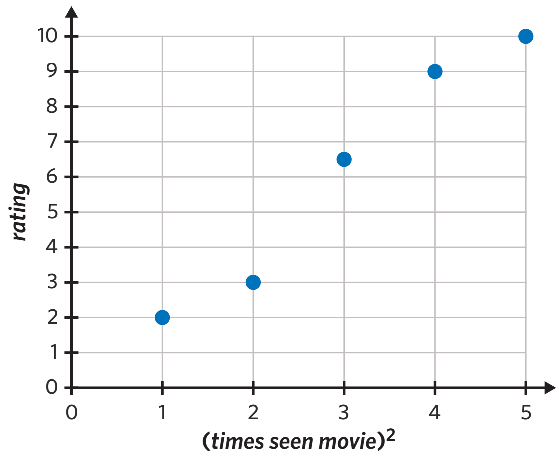
4. A teacher asked their students how many times they had seen a particular movie, and how they rate it out of 10. The results are shown in the following graph.

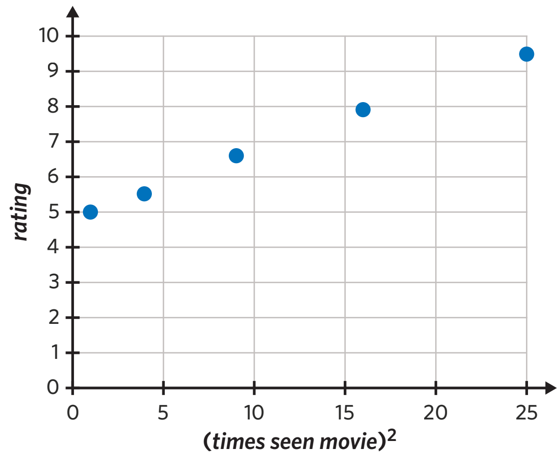


The data set was determined to be non-linear and an x-squared transformation was applied. Which graph shows the transformed data?

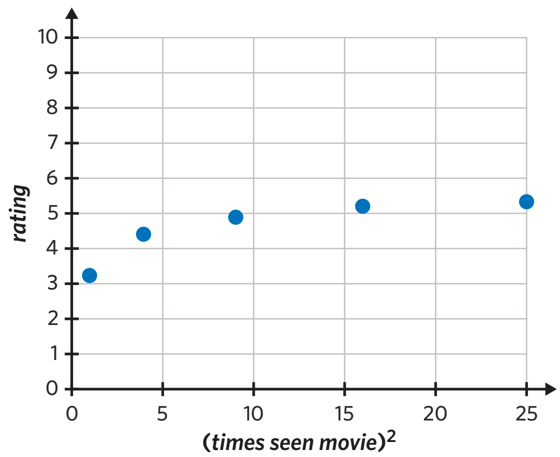
A.



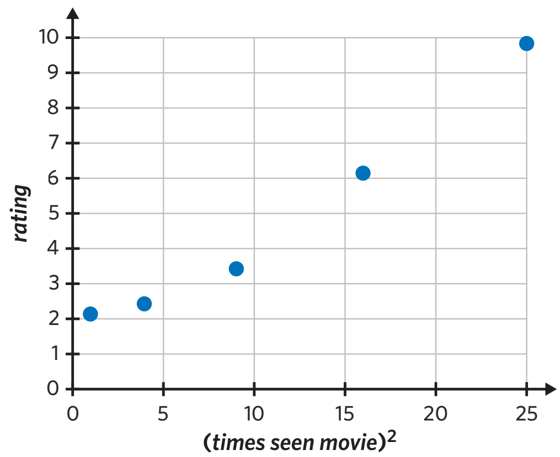
B.



C.



D.



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5. For the following tables, apply an x-squared transformation and plot the transformed data.

a.

x	0.9	2.1	3.1	3.8	5.1
y	1.2	3.8	9.3	15.7	24.9

b.

x	1	2	3	4	5	6
y	1.2	7.8	27.8	66	119	220

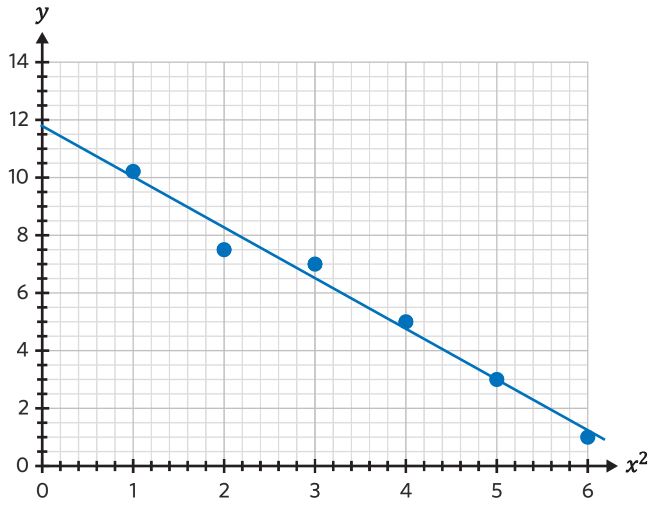
c.

x	1	2	3	4	5	6
y	0.9	4.3	9.5	15.3	24.3	34.8

d.

x	1	1.5	2	2.5	3	3.5
y	2.2	5.6	11.7	22.5	37	54

6. An x-squared transformation was applied to a non-linear data set in an attempt to linearise it. Find the equation of the line of good fit, rounding values to two decimal places.



Joining it all together

7. The average number of ice creams bought through heat waves of differing durations is shown in the following table.

***duration of heat wave* (days)**	1	2	3	4	5
average number of ice creams	0.9	1.4	2.0	3.5	5.4

a. Construct a scatterplot displaying the relationship between the two variables.

b. If an x-squared transformation is to be applied, which variable will this affect?

c. Apply an x-squared transformation and plot the transformed data.

d. A line of good fit for the transformed data passes through the points (0, 0.6) and (15, 3.2). Find the equation of this line, rounding values to two decimal places.

8. Anna's test scores have increased over the course of the year.

test number	1	2	3	4	5	6	7
test score	40	42	46	52	60	74	87

a. Construct a scatterplot displaying the relationship between the two variables.

b. If an x-squared transformation is to be applied, which variable will this affect?

c. Apply an x-squared transformation and plot the transformed data.

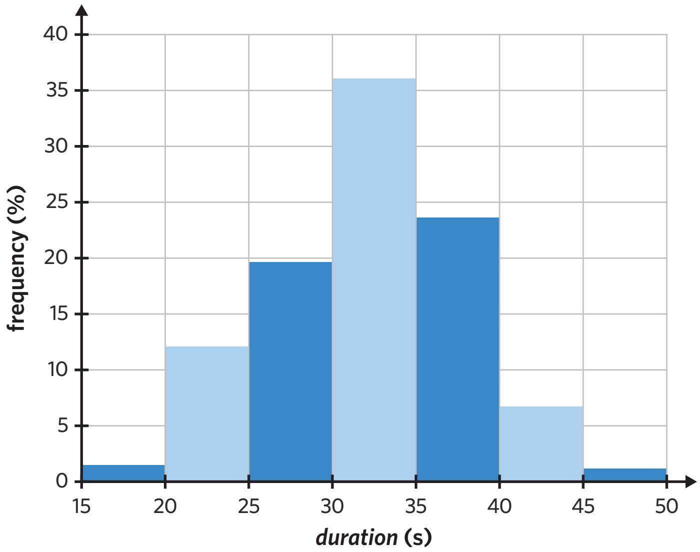
d. Draw a line of good fit for the transformed data.

e. Find the equation of this line, rounding values to one decimal place.

Questions from multiple lessons

Data analysis

9. The following histogram displays the distribution of the *duration*, in seconds, of 150 television advertisements.



The shape of the distribution is best described as

A. approximately symmetric with outliers.

B. perfectly symmetric with no outliers.

C. positively skewed with outliers.

D. approximately symmetric with no outliers.

E. negatively skewed with no outliers.

Adapted from VCAA 2019NH Exam 1 Data analysis Q1

Recursion and financial modelling

10. Calliope is a Year 12 student studying for her VCE exams.

Calliope studied for 150 minutes in the first week and will successively increase the amount of time she spends studying by 15 minutes each week. Hence, in the second week she will study for 165 minutes, and in the third week she will study for 180 minutes.

For how long will Calliope study in the 13th week of study?

A. 315 minutes **B.** 330 minutes **C.** 345 minutes **D.** 350 minutes **E.** 365 minutes

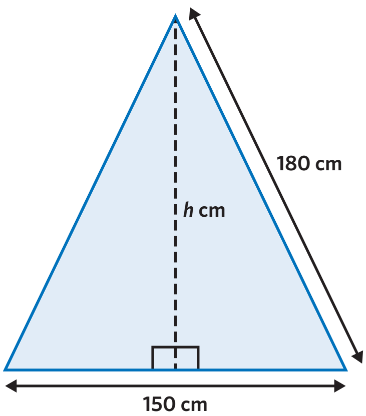
Adapted from VCAA 2012 Exam 1 Number patterns Q3

Geometry and measurement *Year 10 content*

11. The front view of a tent is shown.

a. Show that the height, h, of the tent is 163.6 cm, correct to the nearest millimetre. (1 MARK)

b. Convert 163.6 centimetres to metres. (1 MARK)



8C Transformations – k/x

STUDY DESIGN DOT POINTS

- transformation of data to linearity to establish relationships between variables, for example y and x^2, y and $\frac{1}{x}$, and y and $\log_{10}(x)$
- modelling of given non-linear data using the relationships $y = kx^2 + c$, $y = \frac{k}{x}$, where $k > 0$, and $y = k\log_{10}(x) + c$, where $k > 0$

8A 8B 8C 8D

KEY SKILLS

During this lesson, you will be:

- applying a reciprocal transformation to numerical data
- modelling non-linear data using reciprocal variation.

KEY TERMS

- Reciprocal
- x–reciprocal transformation

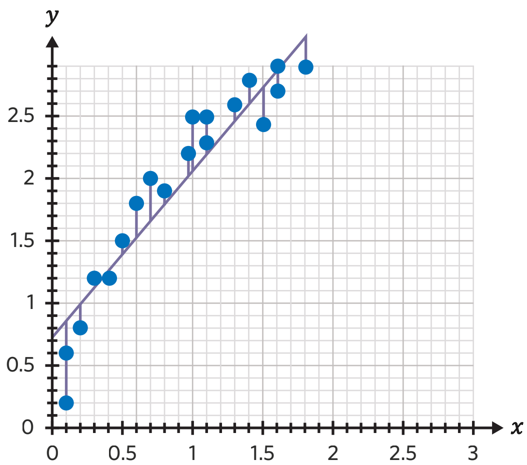
Another type of data transformation that can be applied is a reciprocal transformation. For example, if the rate of reaction of an experiment increases rapidly at first but plateaus as time goes on, applying a reciprocal transformation may help to linearise the data and allow conclusions to be drawn.

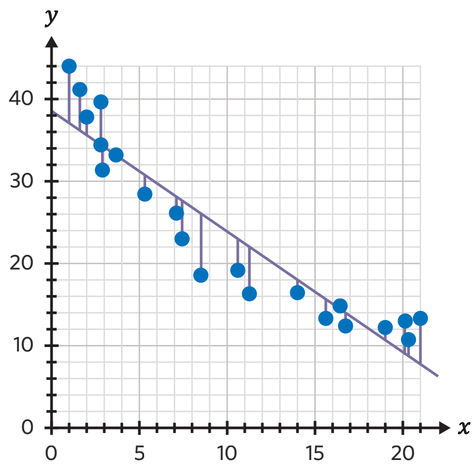
Applying a reciprocal transformation to numerical data

A reciprocal transformation is another type of transformation that can be applied to a set of data to linearise it. In this lesson, reciprocal transformations will involve the explanatory variable only.

A **reciprocal** is the inverse of a number and can be calculated by raising the number to the power of -1, or dividing 1 by the number. For example, the reciprocal of 7 can be written in two ways: $\frac{1}{7}$ or 7^{-1}.

A data set may require the explanatory variable to undergo a reciprocal transformation when its scatterplot looks similar to one of the following graphs.





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Worked example 1

Consider the following table.

x	1	3	5	7	9
$\frac{1}{x}$					
y	28	16	9	5	3

Perform a reciprocal transformation on x by filling in the table. Round values to two decimal places where necessary.

Explanation – Method 1: By hand

Step 1: Calculate the reciprocal of each x value.

$$1^{-1} = \frac{1}{1}$$
$$= 1$$
$$3^{-1} = \frac{1}{3}$$
$$= 0.333...$$
$$\approx 0.33$$
$$5^{-1} = \frac{1}{5}$$
$$= 0.2$$

Repeat this for the remaining x values.

Step 2: Fill in the table with the rounded reciprocal x values.

Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name column A 'x' and column B 'y'.

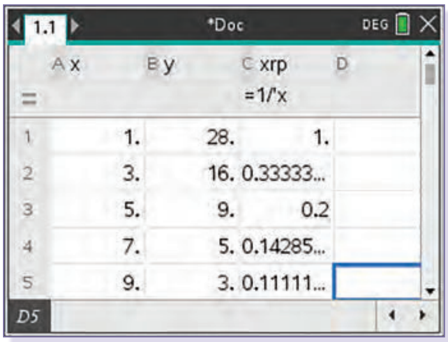
Enter the x values into column A, starting from row 1.

Enter the y values into column B, starting from row 1.

Step 3: Name column C 'xrp' (short for x reciprocal).

Enter '=1/x' into the cell below the 'xrp' heading.

Select 'Variable Reference' → 'OK'.



Step 4: Fill in the table with the rounded reciprocal x values.

Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap Statistics.

Step 2: Name the first list 'x' and the second list 'y'.

Enter the x values into list 'x', starting from row 1.

Enter the y values into list 'y', starting from row 1.

Continues →

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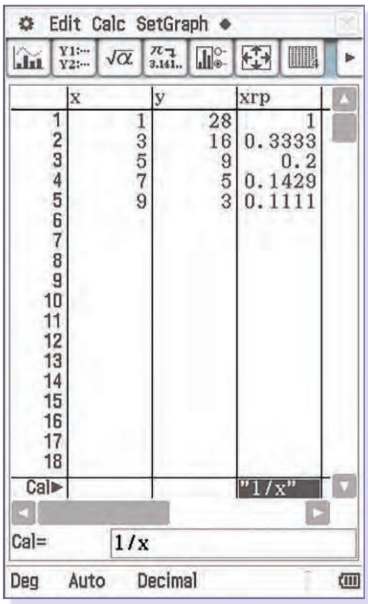
Step 3: Name the third list 'xrp' (short for x-reciprocal).

In the third column, go down to the calculator cell Cal▶ and enter '1/x'.



Step 4: Fill in the table with the rounded reciprocal x values.

Answer – Method 1, 2 and 3

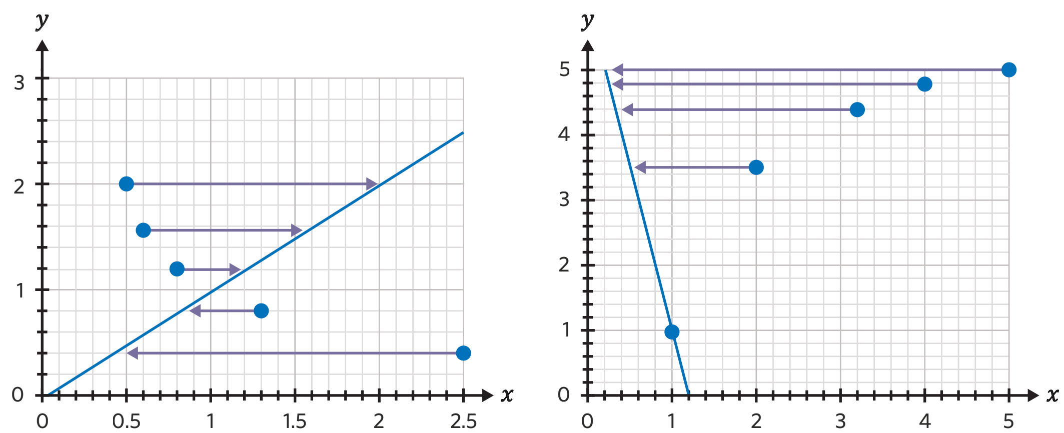
x	1	3	5	7	9
$\frac{1}{x}$	1	0.33	0.2	0.14	0.11
y	28	16	9	5	3

Modelling non-linear data using reciprocal variation

See worked example 2

On a graph, an ***x*-reciprocal transformation** involves reducing the explanatory variable (x) values that are greater than one, whilst increasing the values that are less than one. Values closer to one are altered less in comparison to numbers that are further away. The response variable values remain the same.

The following graphs demonstrate how an x-reciprocal transformation can linearise the data by changing the values of the explanatory variable.



See worked example 3

As with other transformations, it is possible to determine the equation for a line of good fit for data transformed using an x-reciprocal transformation. After the equation for the line of good fit has been determined for the transformed data using $y = a + bx$, the variable 'x' is replaced with '$\frac{1}{x}$'.

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Worked example 2

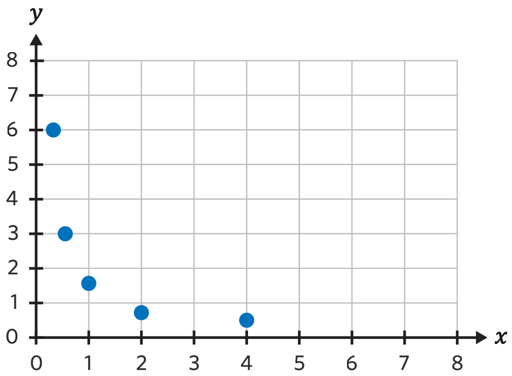
Consider the following data.

x	0.25	0.5	1	2	4
y	6	3	1.5	0.75	0.375

a. Why might an x-reciprocal transformation linearise the data?

Explanation

Plot the data and observe its form.



Answer

Since the graph is curved, with y values decreasing at a decreasing rate, an x-reciprocal transformation could linearise the data by reducing the larger x values while increasing the smaller ones.

b. Apply an x-reciprocal transformation and plot the transformed data.

Explanation – Method 1: By hand

Step 1: Apply an x-reciprocal transformation.

x	0.25	0.5	1	2	4
$\frac{1}{x}$	4	2	1	0.5	0.25
y	6	3	1.5	0.75	0.375

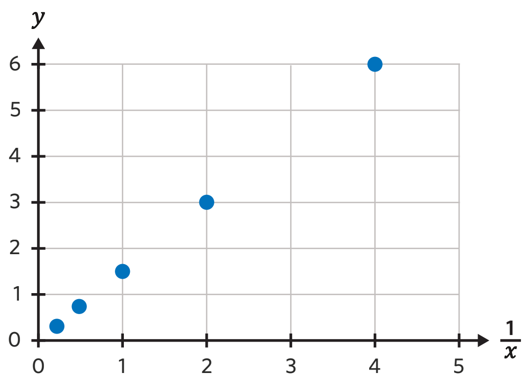
Step 2: Construct a set of axes.

A scale from 0 to 5 is appropriate for the horizontal axis, while the vertical axis needs to extend from 0 to 6.

The horizontal axis should be labelled $\frac{1}{x}$.

Step 3: Plot the data points using the $\frac{1}{x}$ and y values.

Answer



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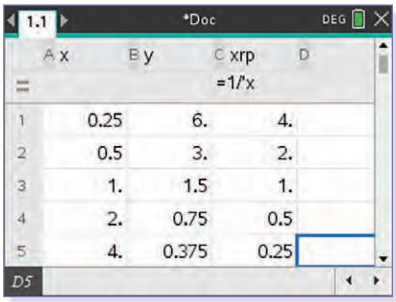
Explanation – Method 2: TI-Nspire

Step 1: Apply an x-reciprocal transformation.



Step 2: Press ctrl + doc, and select '5: Add Data & Statistics'.

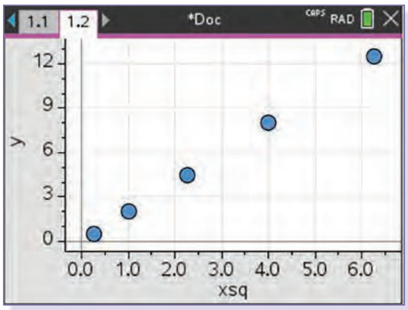
Move the cursor to the horizontal axis and select 'Click to add variable'.

Select 'xrp'.

Move the cursor to the vertical axis and select 'Click to add variable'.

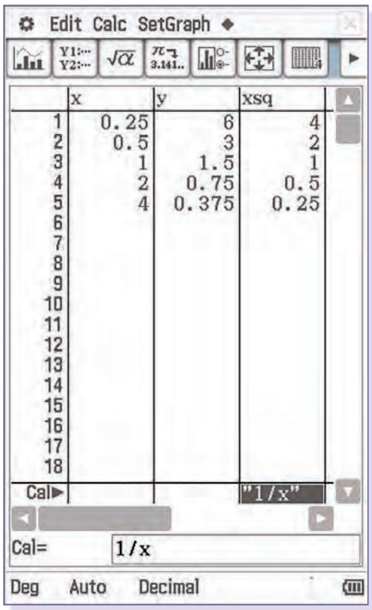
Select 'y'.

Answer



Explanation – Method 3: Casio ClassPad

Step 1: Apply an x-reciprocal transformation.



Step 2: Configure the settings of the graph by tapping .

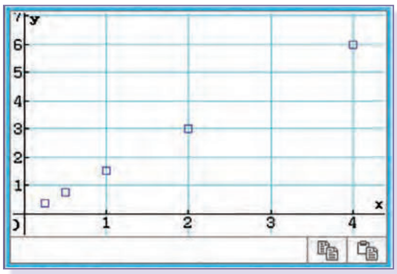
Create a scatterplot by changing 'Type' to 'Scatter'.

Specify the data set by changing 'XList:' to 'main\xrp' and 'YList:' to 'main\y'.

Tap 'Set' to confirm.

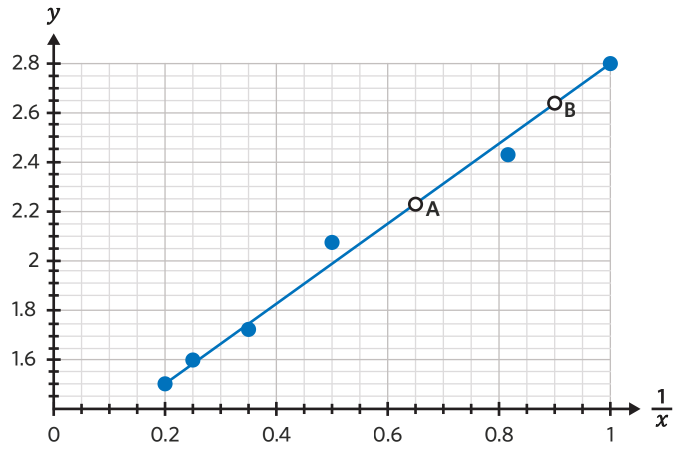
Step 3: Tap to plot the graph.

Answer



Worked example 3

An x-reciprocal transformation was applied to a non-linear data set. A line of good fit was then added to show the relationship between $\frac{1}{x}$ and y. Points A and B are (0.65, 2.25) and (0.9, 2.65) respectively.



Find the equation of this line.

Explanation

Step 1: Calculate b using the gradient formula.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2.65 - 2.25}{0.9 - 0.65}$$

$$= \frac{0.4}{0.25}$$

$$= 1.6$$

Step 2: Substitute a pair of coordinates and the gradient into the equation for a straight line to calculate the y-intercept.

$$y = a + bx$$

$$2.25 = a + 1.6 \times 0.65$$

$$a = 1.21$$

Step 3: Write the equation for the line of good fit.

Make sure to replace x with $\frac{1}{x}$.

Answer

$$y = 1.21 + 1.6 \times \frac{1}{x}$$

8C Questions

Note: There are no direct exam questions relevant to this lesson.

Applying a reciprocal transformation to numerical data

1. What value is missing from the table?

x	1	2	3	4
$\frac{1}{x}$	1		0.33	0.25
y	3	10	15	17

A. -0.5
B. 0
C. 0.5
D. 2

2. Apply an x-reciprocal transformation to the following tables, rounding values to two decimal places where necessary.

a.

x	2	4	6	8
$\frac{1}{x}$				
y	1	8	13	15

b.

x	1	3	5	7
$\frac{1}{x}$				
y	-3	2	4	5

c.

x	-3	-2	-1	1
$\frac{1}{x}$				
y	25	19	15	12

3. Alvin, Simon and Theodore are students in a maths class. They are applying an x-reciprocal transformation to the following set of data.

x	1	5	9	13	17
y	1	15	24	30	32

a. Alvin constructs the following table.

x	1	5	9	13	17
$\frac{1}{x}$	-1	-5	-9	-13	-17
y	1	15	24	30	32

What has Alvin done wrong?

b. Simon constructs the following table.

x	1	5	9	13	17
$\frac{1}{x}$	1	0.07	0.04	0.03	0.03
y	1	15	24	30	32

What has Simon done wrong?

c. Theodore constructs a table with the x-reciprocal transformation correctly applied. Construct Theodore's table, rounding values to two decimal places where necessary.

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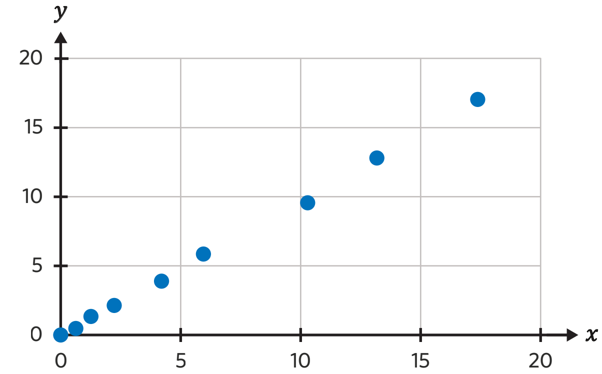
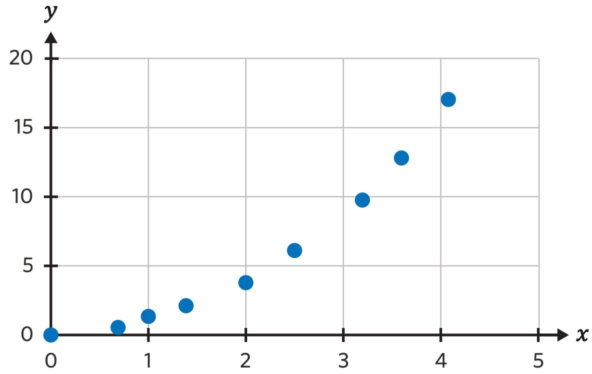
Modelling non-linear data using reciprocal variation

4. Which graph requires an x-reciprocal transformation to linearise the data?

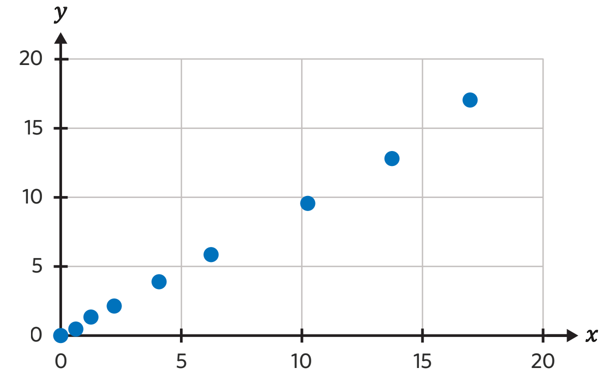
A.



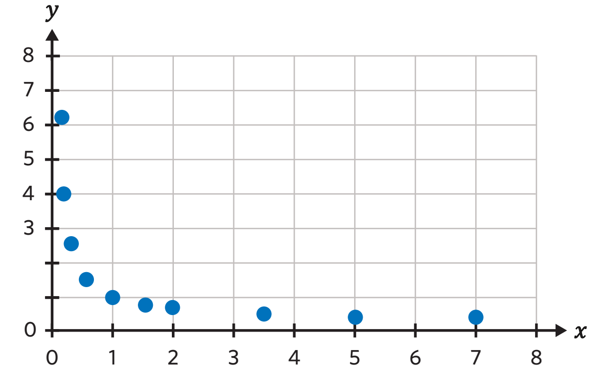
B.



C.



D.



5. For the following tables, apply an x-reciprocal transformation and plot the transformed data.

a.

x	0.25	0.5	1	2	3	4
y	38	19	9	5.2	3.4	2.4

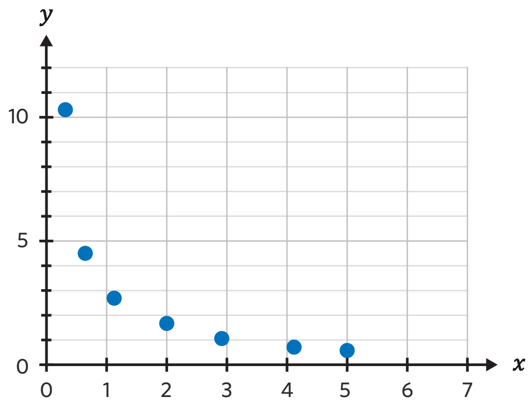
b.

x	0.25	0.5	1	2	3	4
y	1.5	3.2	4.7	5.4	5.6	6.0

c.

x	1	2	3	4	5
y	0.5	7.0	9.2	10.5	12.0

6. The following scatterplot displays a data set that is non-linear.

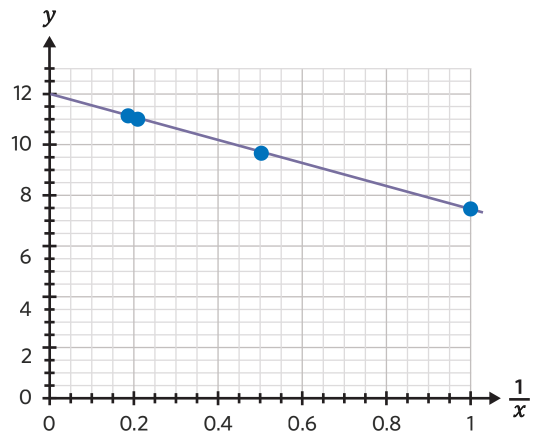


The data used in the scatterplot is provided in the following table. Apply an x-reciprocal transformation to this data set and plot the transformed data.

x	0.3	0.7	1.1	2.0	2.9	4.1	5.0
y	10.2	4.5	2.7	1.6	1.0	0.8	0.6

7. An x-reciprocal transformation was applied to a non-linear data set in an attempt to linearise it. The resulting scatterplot is shown.

Find the equation of the line of good fit, rounding values to one decimal place.



Joining it all together

8. Mr Nadan wanted to see if there was a correlation between students playing cricket and football throughout the year. He asked a group of students to record the number of times a month they play each sport over the course of a year. Five data points representing the averages for different months are shown in the table.

times playing cricket	1	1.7	2.2	2.9	4.5
times playing football	4.8	3.1	2.1	1.8	1.1

Assume that *times playing cricket* is the explanatory variable.

a. Construct a scatterplot displaying the relationship between the two variables.

b. If an x-reciprocal transformation is to be applied, which variable will this affect?

c. Apply an x-reciprocal transformation and plot the transformed data.

d. A line of good fit was drawn for the transformed data, passing through the points (0.05, 0.33) and (0.95, 4.62). Find the equation of this line, rounding values to two decimal places.

---

9. Anika is working towards getting enough hours to do her driving test. The total number of hours she has completed has increased over the course of the year.

month	1	2	3	4	5	6
number of hours	0	39	62	63	70	73

a. Construct a scatterplot displaying the relationship between the two variables.

b. If an x-reciprocal transformation is to be applied, which variable will this affect?

c. Apply an x-reciprocal transformation and plot the transformed data.

d. Draw a line of good fit for the transformed data.

e. Find the equation of this line, rounding values to two decimal places.

Questions from multiple lessons

Data analysis *Year 10 content*

10. The variables *typing speed* (words per minute) and *language* (English, French, German) are

A. both numerical.

B. both categorical.

C. an ordinal variable and nominal variable respectively.

D. a numerical variable and nominal variable respectively.

E. a numerical variable and ordinal variable respectively.

Adapted from VCAA 2019NH Exam 1 Data analysis Q8

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Data analysis *Year 10 content*

11. The following stem plot displays the *number of teabags* consumed each day in the Edrolo office over a period of 31 days.

Key: 0 | 1 = 1 $n = 31$

0	1	2				
0	5	6	8	8	9	
1	0	1	1	3	4	4
1	6	7	8	9	9	9
2	2	2	3	4	4	
2	5	5	6	7		
3	0	1				
3	6					

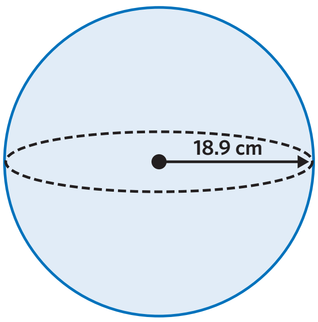
What is the median *number of teabags* consumed?

A. 17.5 **B.** 18 **C.** 18.5 **D.** 19 **E.** 22

Adapted from VCAA 2018NH Exam 1 Data analysis Q1

Geometry and measurement *Year 10 content*

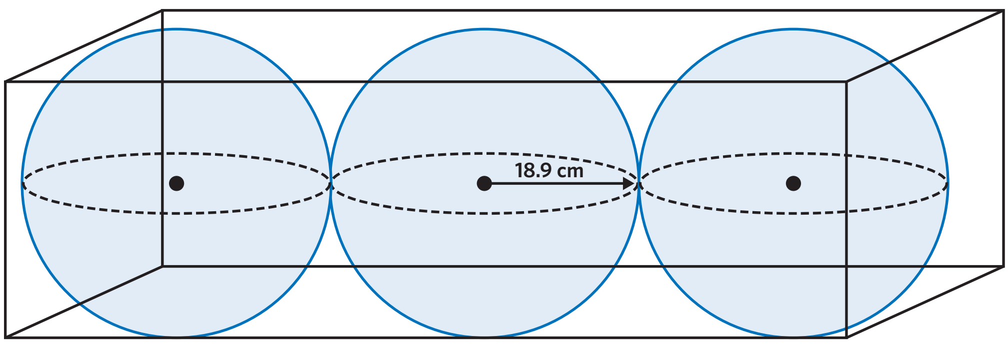
12. A beach ball is spherical in shape and has a radius of 18.9 cm.



Assume that the surface of the beach ball is smooth.

a. The surface area of a sphere is calculated using the formula
$A = 4\pi r^2$
What is the surface area of the beach ball? Round to the nearest square centimetre. (1 MARK)

b. Beach balls are sold in a rectangular box that contains three identical beach balls, as shown.



What is the minimum length, in centimetres, of the box? (1 MARK)

Adapted from VCAA 2016 Exam 2 Geometry and measurement Q1a,b

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8D Transformations – $k\log_{10}(x)$

STUDY DESIGN DOT POINTS

- transformation of data to linearity to establish relationships between variables, for example y and x^2, y and $\frac{1}{x}$, and y and $\log_{10}(x)$
- modelling of given non-linear data using the relationships $y = kx^2 + c$, $y = \frac{k}{x}$, where $k > 0$, and $y = k\log_{10}(x) + c$, where $k > 0$

8A 8B 8C 8D

KEY SKILLS

During this lesson, you will be:

- performing calculations using logarithms
- applying a logarithmic transformation to numerical data
- modelling non-linear data using logarithmic variation.

KEY TERMS

- Logarithm
- $\log(x)$ transformation

Another data transformation that can be useful is the log transformation. For example, when measuring sounds, due to the large range between the quietest and loudest noises, not only are the measurements not manageable, but they form a non-linear shape when plotted. Applying a log transformation may help to linearise the data and allow for conclusions to be drawn.

Performing calculations using logarithms

A **logarithm**, commonly referred to as a log, is a function that gives the power to which a fixed number (the base) must be raised to produce a given number. They allow for all numbers to be represented as the power of a predetermined base.

argument

If $\log_b(x) = y$ then $b^y = x$

base exponent

For example, $\log_{10}(1000) = 3$ since $10^3 = 1000$.

Since the most common logarithmic base is 10, and it is the only one used in this course, it is not necessary to snow the base value in calculations.

$\log(1000) = 3$

Although some logarithms involve integer values, many include decimals. Therefore, calculating logarithms is primarily, and most efficiently, performed using a calculator.

See worked example 1

Some scientific scales, such as the Richter scale and pH scale, use log values (with a base of 10) due to the large range of values that are measured. To convert from these log values to actual values, raise 10 to the power of the log value.

See worked example 2

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Worked example 1

Calculate the log of 542, rounded to two decimal places.

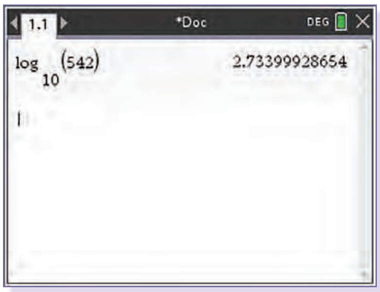
Explanation – Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '1: Add Calculator'.

Step 2: Press ctrl + 10^x.

Step 3: Type '10' as the base and '542' inside the brackets.

Press enter.



Explanation – Method 2: Casio ClassPad

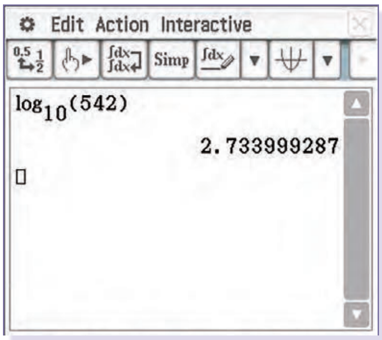
Step 1: From the main menu, tap $\sqrt{\alpha}$ Main.

Step 2: Press keyboard and tap 'Math1'.

Select $\log_{\square}\square$.

Step 3: Type '10' as the base and '542' inside the brackets.

Press EXE.



Answer – Method 1 and 2

2.73

Worked example 2

Earthquakes are measured on the Richter scale, a base-10 logarithmic scale.

In 2022, Melbourne experienced an earthquake measuring 2.4 on the Richter scale. The largest recorded earthquake in New Zealand occurred in 1855, measuring 8.2 on the Richter scale.

How many times greater was the New Zealand earthquake compared to the Melbourne earthquake, rounded to the nearest whole number?

Explanation

Step 1: Determine the actual magnitude of each earthquake.

Raise 10 to the power of the Richter scale measurement for each earthquake.

New Zealand: $10^{8.2}$

Melbourne: $10^{2.4}$

Step 2: Divide the New Zealand measurement by the Melbourne measurement.

$\frac{10^{8.2}}{10^{2.4}} = 630\ 957.344...$

Note: The same value is found when raising 10 to the difference between the two measurements (5.8).

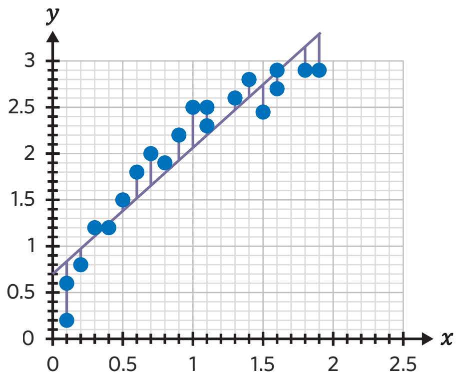
Answer

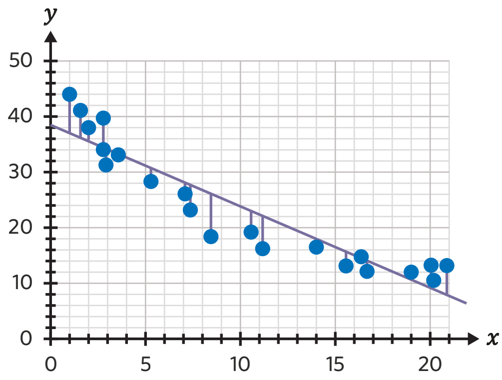
630 957 times greater

Applying a logarithmic transformation to numerical data

Understanding how to calculate the log of a number allows for a logarithmic transformation to be applied to a set of data to linearise it. In this lesson, log transformations will involve the explanatory variable only.

A data set may require the explanatory variable to undergo a log transformation when its scatterplot looks similar to one of the following graphs.





Worked example 3

Consider the following table.

Perform a log transformation on x by filling in the table. Round values to two decimal places where necessary.

x	1	3	5	7	9
log(x)					
y	32	14	10	7	5

Explanation – Method 1: By hand

Step 1: Calculate the log of each x value.

$\log_{10}(1) = 0$

$\log_{10}(3) = 0.477...$

≈ 0.48

$\log_{10}(5) = 0.698...$

≈ 0.70

Repeat this process for the remaining x values.

Step 2: Fill in the table with the rounded log(x) values.

Explanation – Method 2: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name column A 'x' and column B 'y'.

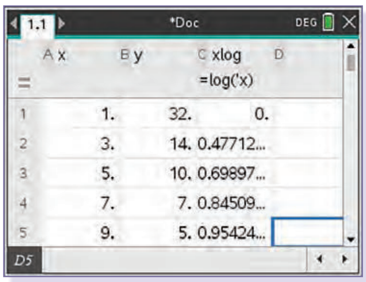
Enter the x values into column A, starting from row 1.

Enter the y values into column B, starting from row 1.

Step 3: Name column C 'xlog'.

Enter '=log(x)' into the cell below the 'xlog' heading.

Select 'Variable Reference' → 'OK'.



Step 4: Fill in the table with the rounded log(x) values.

Continues →

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Explanation – Method 3: Casio ClassPad

Step 1: From the main menu, tap Statistics.

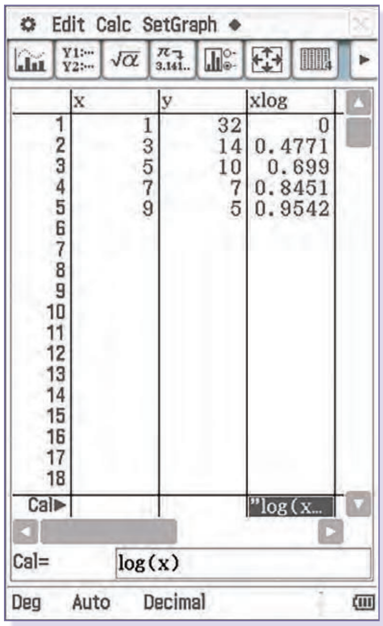
Step 2: Name the first list 'x' and the second list 'y'.

Enter the x values into list 'x', starting from row 1.

Enter the y values into list 'y', starting from row 1.

Step 3: Name the third list 'xlog'.

In the third list, go down to the calculator cell Cal▶ and enter 'log(x)'



Step 4: Fill in the table with the rounded $\log(x)$ values.

Answer – Method 1,2 and 3

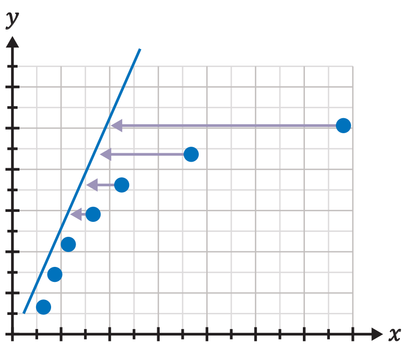
x	1	3	5	7	9
$\log(x)$	0	0.48	0.70	0.85	0.95
y	32	14	10	7	5

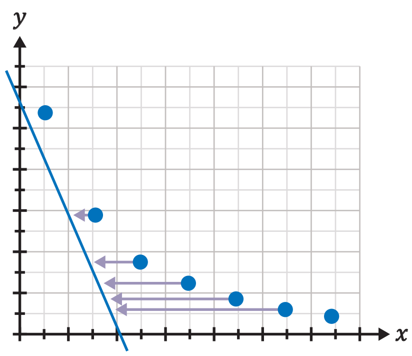
Modelling non-linear data using logarithmic variation

On a graph, a **$\log(x)$ transformation** involves 'compressing' the larger explanatory variable (x) values more than the smaller values. The response variable values remain the same.

See worked example 4

The following graphs demonstrate how a $\log(x)$ transformation can linearise the data by compressing the larger values of the explanatory variable.





Determining the equation of a line of good fit for data that has undergone a $\log(x)$ transformation is very similar to other transformations. After the equation for the line of good fit has been determined for the transformed data using $y = a + bx$, the variable 'x' is replaced with '$\log(x)$'.

See worked example 5

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Worked example 4

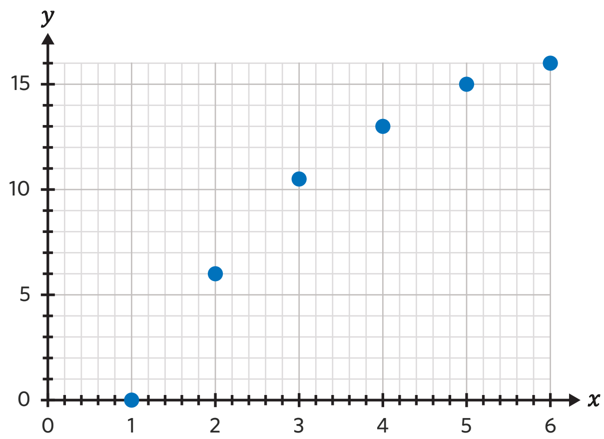
Consider the following data.

x	1	2	3	4	5	6
y	0	6	10.5	13	15	16

a. Why might a $\log(x)$ transformation linearise the data?

Explanation

Plot the data and observe its form.



Answer

Since the graph is curved, with the y values increasing at a decreasing rate, a $\log(x)$ transformation could linearise the data by compressing the larger x values more than the smaller x values.

b. Plot the transformed data.

Explanation – Method 1: By hand

Step 1: Apply a $\log(x)$ transformation.

x	1	2	3	4	5	6
$\log(x)$	0	0.30	0.48	0.60	0.70	0.78
y	0	6	10.5	13	15	16

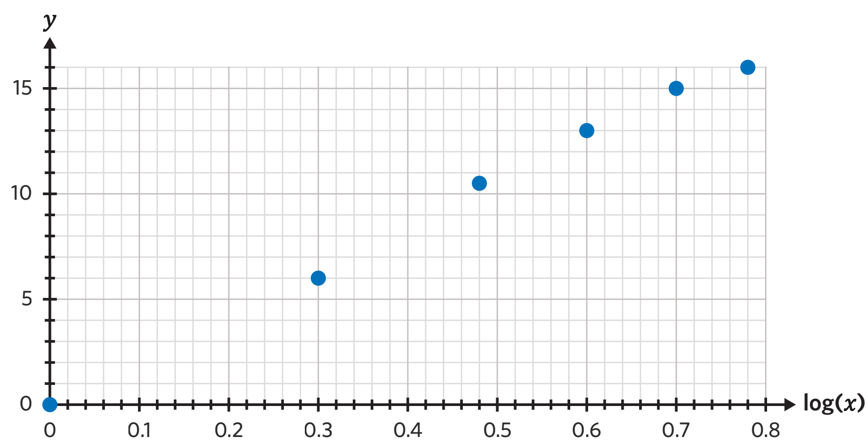
Step 2: Construct a set of axes.

A scale from 0 to 0.8 is appropriate for the horizontal axis, while the vertical axis needs to extend from 0 to at least 16.

The horizontal axis should be labelled $\log(x)$.

Step 3: Plot the data points using the $\log(x)$ and y values.

Answer



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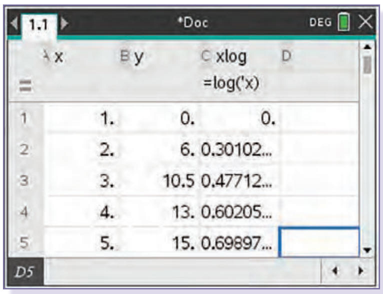
Explanation – Method 2: TI-Nspire

Step 1: Apply a log(x) transformation.



Step 2: Press ctrl + doc▾, and select '5: Add Data & Statistics'.

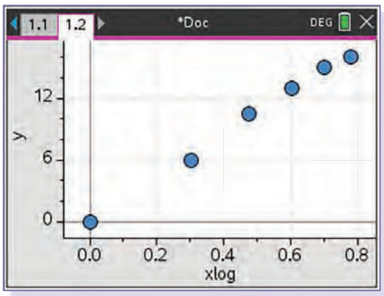
Move the cursor to the horizontal axis and select 'Click to add variable'.

Select 'xlog'.

Move the cursor to the vertical axis and select 'Click to add variable'.

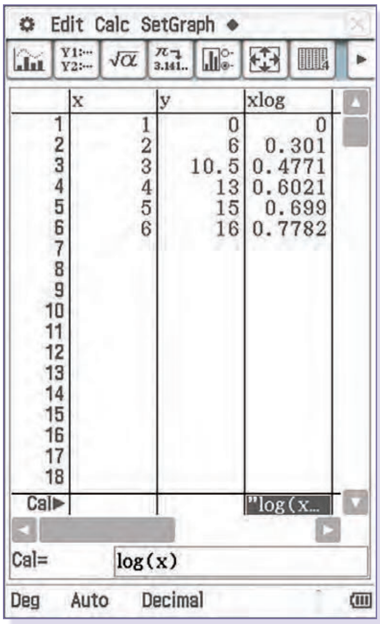
Select 'y'.

Answer



Explanation – Method 3: Casio ClassPad

Step 1: Apply a log(x) transformation.



Step 2: Configure the settings of the graph by tapping .

Create a scatterplot by changing 'Type' to 'Scatter'.

Specify the data set by changing 'XList:' to 'main\xlog' and 'YList:' to 'main\y'.

Tap 'Set' to confirm.

Step 3: Tap to plot the graph.

Answer

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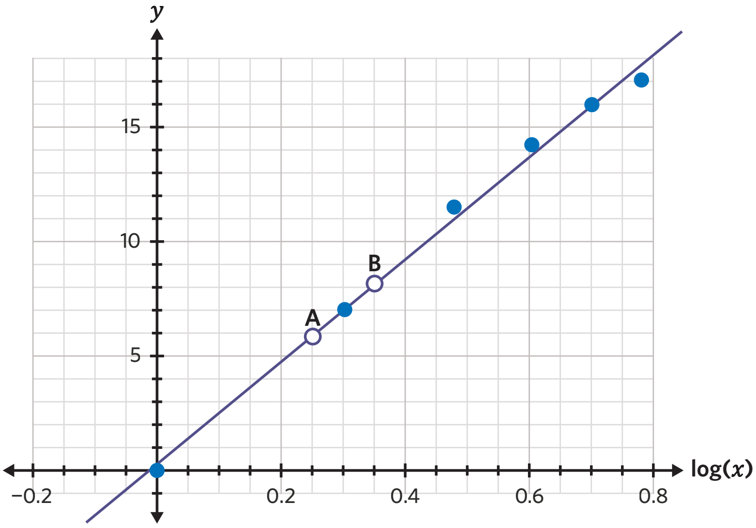
Worked example 5

A log(x) transformation was applied to a non-linear data set. A line of good fit was then added to show the relationship between log(x) and y. Points A and B are (0.25, 5.8875) and (0.35, 8.1345) respectively.



Find the equation of this line.

Explanation

Step 1: Calculate b using the gradient formula.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{8.1345 - 5.8875}{0.35 - 0.25}$$
$$= \frac{2.247}{0.1}$$
$$= 22.47$$

Step 2: Substitute a coordinate and the gradient into the equation for a straight line to calculate the y-intercept.

$y = a + bx$

$5.8875 = a + 22.47 \times 0.25$

$a = 0.27$

Step 3: Write the equation for the line of good fit.

Make sure to replace x with log(x).

Answer

$y = 0.27 + 22.47 \times \log(x)$

8D Questions

Performing calculations using logarithms

1. The log of 10 000 is

   A. 3 **B.** 4 **C.** 5 **D.** 10

2. Calculate the log of the following numbers, rounded to two decimal places.

   a. 34 **b.** 1045 **c.** 0.0078 **d.** 6 495 913

3. The pH scale is a base-10 logarithmic scale that measures the acidity of a chemical solution. A lower pH value indicates a stronger acid. How many times more acidic is orange juice than water, if water has a pH of 7 and orange juice has a pH of 3?

4. The table shows the log(*weight*), in kilograms, of a number of different animals.

animal	log(*weight*) (kg)
blue whale	5.1
elephant	3.7
moose	2.6
crocodile	3
dolphin	2
orangutan	1.8
gorilla	2.2
hummingbird	−2.7
pygmy rabbit	−0.4
cat	0.6

What are the actual weights, in kilograms, of the following animals? Round to one decimal place where necessary.

a. Dolphin **b.** Pygmy rabbit **c.** Blue whale

5. An unknown solution is 1000 times less acidic than coffee, which has a pH of 5. Which of the following solutions is it most likely to be?

A. Tomato juice (pH of 4)
B. Seawater (pH of 8)
C. Lemon juice (pH of 2)
D. Soapy water (pH of 12)

6. Earthquakes are measured on the Richter scale, a base-10 logarithmic scale.

An earthquake in Japan was 2000 times stronger than an earthquake the same day in Australia.

If the one in Australia had a magnitude of 4.3 on the Richter scale, what was the magnitude of the one in Japan, rounded to one decimal place?

Applying a logarithmic transformation to numerical data

7. What value is missing from the table?

x	1	2	3	4
$\log(x)$	0		0.48	0.60
y	1	7	11	12

A. −0.30 **B.** 0.30 **C.** 0.85 **D.** 2

8. Apply a $\log(x)$ transformation to the values in the following tables. Round to two decimal places where necessary.

a.

x	1	2	3	4
$\log(x)$				
y	20	26	29	30

b.

x	5	10	15	20
$\log(x)$				
y	20	15	13	12

c.

x	10	12	14	16
$\log(x)$				
y	400	300	260	221

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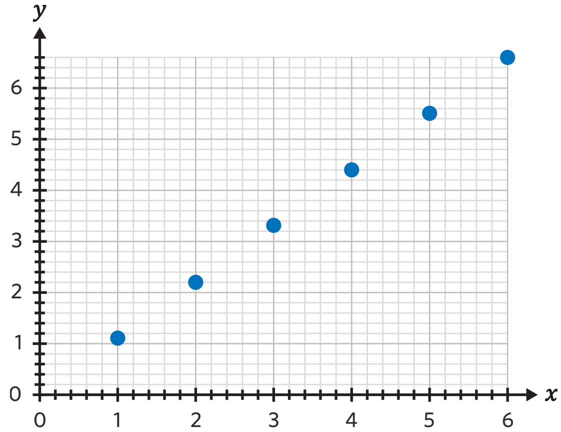
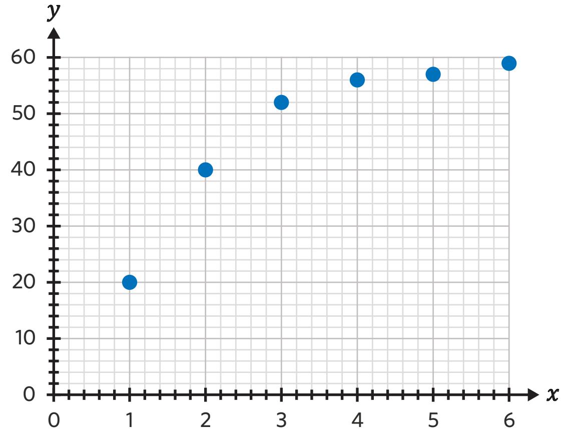
Modelling non-linear data using logarithmic variation

9. Which of the following graphs could require a $\log(x)$ transformation to linearise the data?

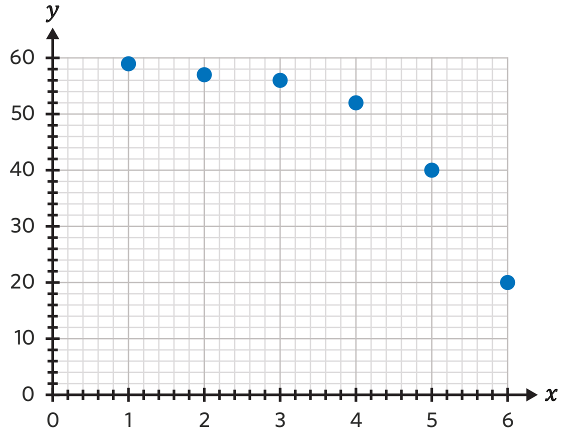
A.



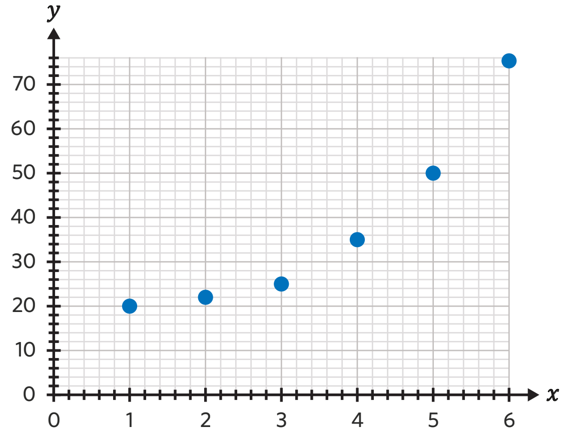
B.



C.



D.



10. For the following tables, apply a $\log(x)$ transformation and plot the transformed data.

a.

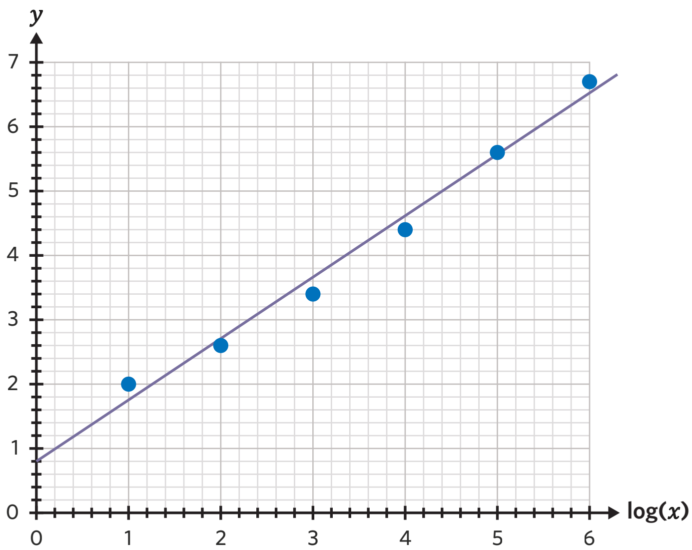
x	0.25	0.5	0.75	1	1.25
y	40	52	60	65	68

b.

x	10	20	30	40	50
y	68	58	50	44	40

11. A $\log(x)$ transformation was applied to a non-linear data set in an attempt to linearise it.

Find the equation of the line of good fit if it passes through the points (1.50, 2.21) and (4.34, 4.92). Round the value of the intercept and slope to two decimal places.



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Joining it all together

12. The value of Vineth's action figure model from a science film increases over the years.

year	1	2	3	4	5
value ($)	30	52	65	70	73

a. Construct a scatterplot displaying the relationship between the two variables.

b. If a $\log(x)$ transformation is to be applied, which variable will this affect?

c. Apply a $\log(x)$ transformation and plot the transformed data.

d. A line of good fit was drawn for the transformed data, passing through the points (0, 31.7) and (0.6, 69.6). Find the equation of this line, rounding values to the nearest whole number.

13. The number of books sold at a bookstore from Monday to Friday has been recorded in the following table.

day	1	2	3	4	5
books sold	42	61	70	79	84

a. Construct a scatterplot displaying the relationship between the two variables.

b. If a $\log(x)$ transformation is to be applied, which variable will this affect?

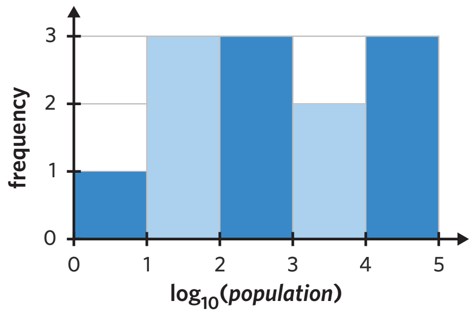
c. Apply a $\log(x)$ transformation and plot the transformed data.

d. Draw a line of good fit for the transformed data.

e. Find the equation of this line, rounding values to one decimal place.

Exam practice

14. The following histogram shows the distribution of $\log_{10}(population)$ of 12 towns.



The median population of these towns is between

A. 2 and 3 **B.** 3 and 4 **C.** 100 and 1000
D. 1000 and 10 000 **E.** 10 000 and 100 000

Adapted from VCAA 2017 Exam 1 Data analysis Q4

62% of students answered this type of question correctly.

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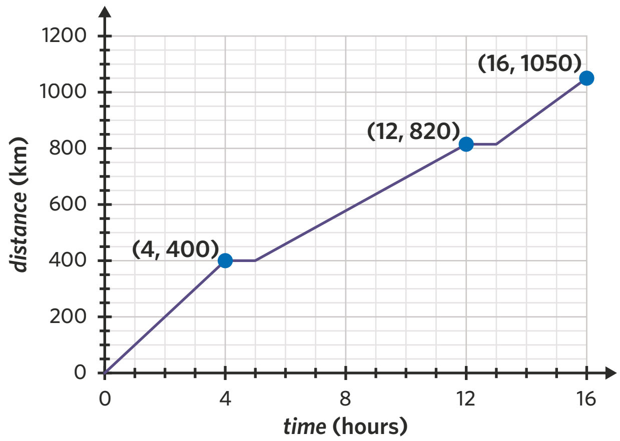
Questions from multiple lessons

Graphs and relations

15. Johnny's job requires him to transport race horses during the spring carnival. The following graph shows the distance Johnny travelled throughout an entire work day.



The average speed that Johnny was travelling over the entire work day, including the time he spent resting is closest to

A. 50.5 km/h **B.** 65.6 km/h **C.** 68.4 km/h **D.** 75.0 km/h **E.** 85.2 km/h

Adapted from VCAA 2014 Exam 1 Graphs and relations Q5

Recursion and financial modelling

16. Hugo purchased a new yacht. To do so he established a loan from the bank for \$1 200 000 with interest charged at a rate of 4.9% per annum, compounding monthly.

Each month, Hugo will only pay for the interest charged that month.

After 7 months, the amount that he still owes is closest to

A. \$844 199 **B.** \$1 166 117 **C.** \$1 200 000 **D.** \$1 234 723 **E.** \$1 677 295

Adapted from VCAA 2017 Exam 1 Recursion and financial modelling Q19

Graphs and relations *Year 10 content*

17. Markus owns a local supermarket that sells exotic fruits.

He sells lychees and dragon fruits.

All lychees are sold for the same price and all dragon fruits are sold for the same price.

Let L represent the selling price of each lychee and D represent the selling price of each dragon fruit.

One customer, Alan, purchased 267 lychees and 41 dragon fruits and paid \$484.40.

A linear relation representing Alan's purchase is $267L + 41D = 484.40$.

A second customer, Mary, purchased 129 lychees and 26 dragon fruits and paid \$258.80.

a. Write down a linear relation representing Mary's purchase. (1 MARK)

b. The selling price of each dragon fruit in Markus' shop is \$4.00.

What is the selling price of each lychee? (1 MARK)

Adapted from VCAA 2019NH Exam 2 Graphs and relations Q1a,b

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9

UNIT 2 AOS 4
CHAPTER 9
Measurement

LESSONS

KEY KNOWLEDGE

- units of measurement of length, angle, area, volume and capacity
- exact and approximate answers, scientific notation, significant figures and rounding
- similar shapes including the conditions for similarity
- perimeter and areas of triangles, quadrilaterals, circles including arcs and sectors and composite shapes, and practical applications
- volumes and surface areas of solids (spheres, cylinders, pyramids and prisms and composite objects) and practical applications, including simple applications of Pythagoras' theorem in three dimensions
- the use of trigonometric ratios and Pythagoras' theorem to solve practical problems involving a right-angled triangle in two dimensions, including the use of angles of elevation and depression
- similar objects and the application of linear scale factor $k > 0$ to scale lengths, surface areas and volumes with practical applications.

Image: ChristianChan/Shutterstock.com

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9A Units of measurement

STUDY DESIGN DOT POINT

- units of measurement of length, angle, area, volume and capacity

9A 9B 9C 9D 9E 9F 9G 9H 9I

KEY SKILLS

During this lesson, you will be:

- identifying appropriate units of measurement
- converting units of length
- converting units of area
- converting units of volume and capacity.

KEY TERMS

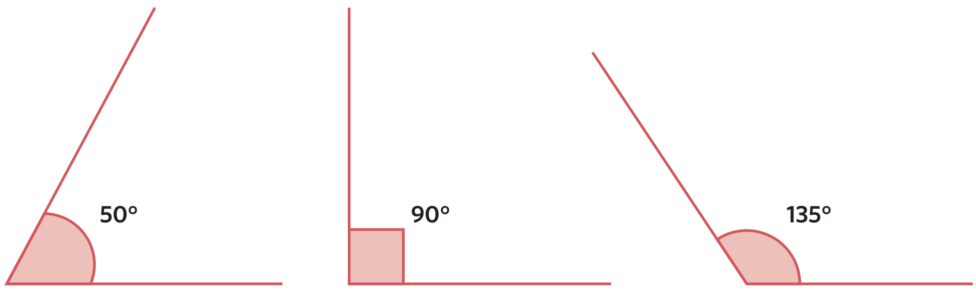
- Angle
- Length
- Area
- Volume
- Capacity

Units of measurement are important concepts to understand as they allow for the size of objects and spaces to be quantified and compared. These concepts are used in a wide range of real-world applications, including construction, engineering and trade.

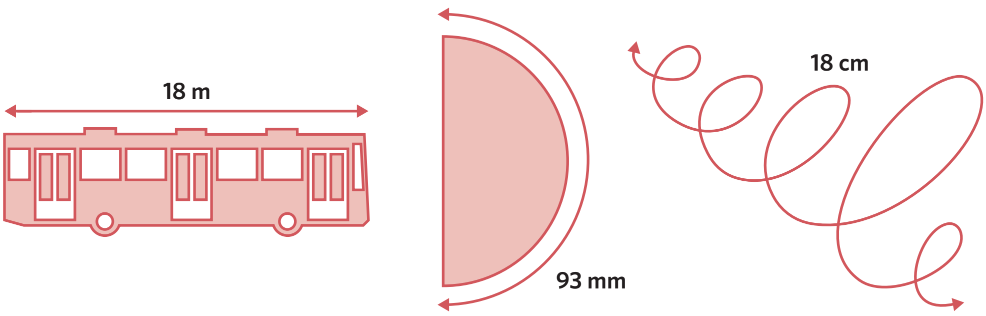
Identifying appropriate units of measurement

Five of the most important characteristics when studying shapes are angle, length, area, volume, and capacity.

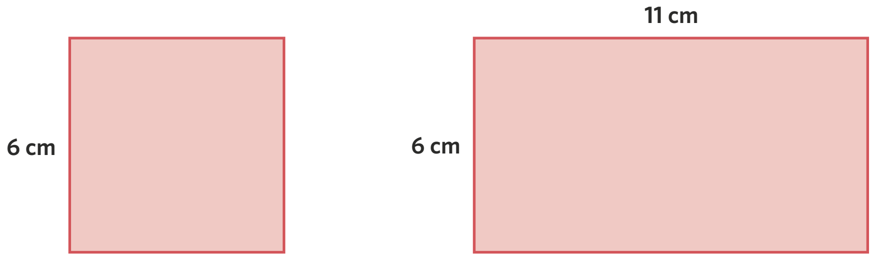
An **angle** is a measure of the space between two intersecting lines, measured close to the point of intersection. Angles are often measured in degrees (°), and labelled on diagrams using a curved line at the point of intersection. An angle of 90° is labelled using a square at the point of intersection.



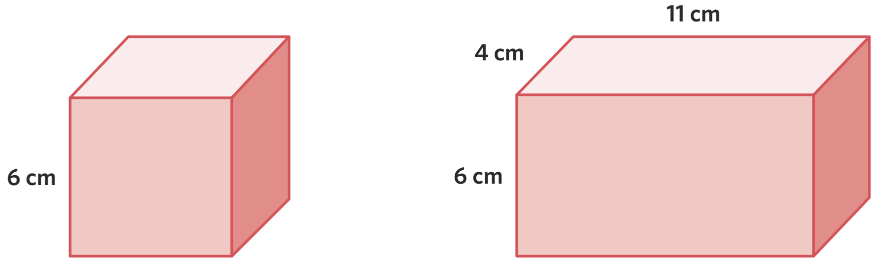
Length is a measure of the straight-line distance from one point to another. Length has one dimension, and is measured in millimetres (mm), centimetres (cm), metres (m) or kilometres (km) in the metric system.



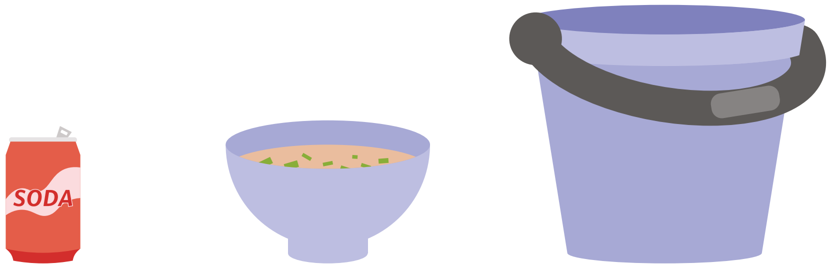
Area is a measure of the space taken up by a two-dimensional shape. Area has two dimensions, and is measured in square units, such as mm^2, cm^2, m^2 and km^2.



Volume is a measure of the space taken up by a three-dimensional object. Volume has three dimensions, and is measured in cubic units, such as mm^3, cm^3, m^3 and km^3.



Capacity is similar to volume, but instead is a measure of how much can be held inside a three-dimensional object. Capacity is often measured in millilitres (mL) and litres (L).



Worked example 1

Determine which unit of measurement (angle, length, area, volume or capacity) is appropriate for each of the following scenarios.

a. Jerry is interested in measuring how long a piece of paper is.

Explanation

As Jerry is measuring the distance from one end of the paper to the other, he is measuring length.

Answer

Length

b. Hannah wants to know how much each of her suitcases can fit so she can pack for her upcoming holiday.

Explanation

As Hannah is determining the total amount that can be held inside her suitcases, she is measuring capacity.

Answer

Capacity

Continues →

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c. Jason would like to know how much space his basketball will take up in his bag.

Explanation

As Jason wants to know how much space his basketball will occupy in his bag, he is measuring volume.

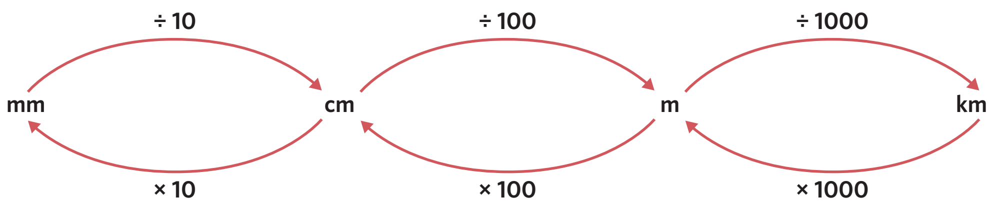
Answer

Volume

Converting units of length

To convert between different units of length, a conversion factor must be used.

To convert lengths from smaller units to larger ones, divide the smaller unit of length by the conversion factor. To convert lengths from larger units to smaller ones, multiply the larger unit of length by the conversion factor.



For example, there are 10 millimetres in 1 centimetre. This means that to convert from millimetres to centimetres, the measurement must be divided by 10. To convert from centimetres to millimetres, the measurement must be multiplied by 10.

Worked example 2

Convert the following.

a. 245 cm to m

Explanation

Step 1: Determine the type of conversion.

Centimetres are smaller units than metres. Therefore, division is required.

Step 2: Divide the measurement by the conversion factor.

Converting centimetres to metres requires a conversion factor of 100.

$245 \div 100 = 2.45$

Answer

2.45 m

b. 3.65 km to cm

Explanation

Step 1: Determine the type of conversion.

Kilometres are larger units than centimetres. Therefore, multiplication is required.

Step 2: Multiply the measurement by the conversion factor.

Converting kilometres to metres requires a conversion factor of 1000.

$3.65 \times 1000 = 3650$

Converting metres to centimetres requires a conversion factor of 100.

$3650 \times 100 = 365\ 000$

Answer

365 000 cm

Continues →

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c. 219 mm to m

Explanation

Step 1: Determine the type of conversion.

Millimetres are smaller units than metres. Therefore, division is required.

Step 2: Divide the measurement by the conversion factor.

Converting millimetres to centimetres requires a conversion factor of 10.

$219 \div 10 = 21.9$

Converting centimetres to metres requires a conversion factor of 100.

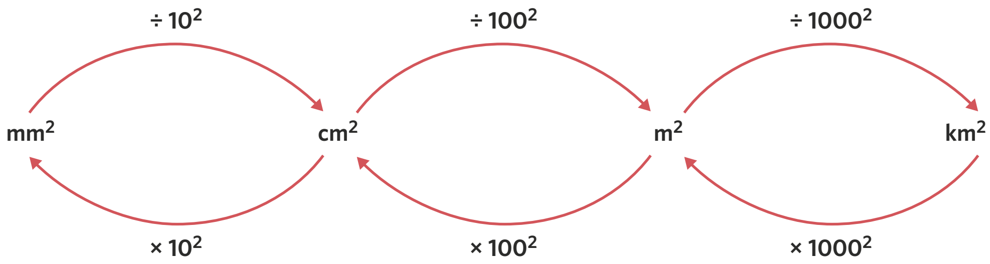
$21.9 \div 100 = 0.219$

Answer

0.219 m

Converting units of area

To convert between different units of area, the same method is applied as converting between different units of length. However, as area is measured in square units, the conversion factor used to convert between different units of area must also be squared.



For example, to convert from square millimetres to square centimetres, the measurement must be divided by 10^2. To convert from square centimetres to square millimetres, the measurement must be multiplied by 10^2.

Worked example 3

Convert the following.

a. 3 mm^2 to cm^2

Explanation

Step 1: Determine the type of conversion.

Square millimetres are smaller units than square centimetres. Therefore, division is required.

Step 2: Divide the measurement by the conversion factor.

Converting square millimetres to square centimetres requires a conversion factor of 10^2.

$3 \div 10^2 = 0.03$

Answer

0.03 cm^2

Continues →

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b. 0.456 m^2 to mm^2

Explanation

Step 1: Determine the type of conversion.

Square metres are larger units than square millimetres. Therefore, multiplication is required.

Step 2: Multiply the measurement by the conversion factor.

Converting square metres to square centimetres requires a conversion factor of 100^2.

$0.456 \times 100^2 = 4560$

Converting square centimetres to square millimetres requires a conversion factor of 10^2.

$4560 \times 10^2 = 456\,000$

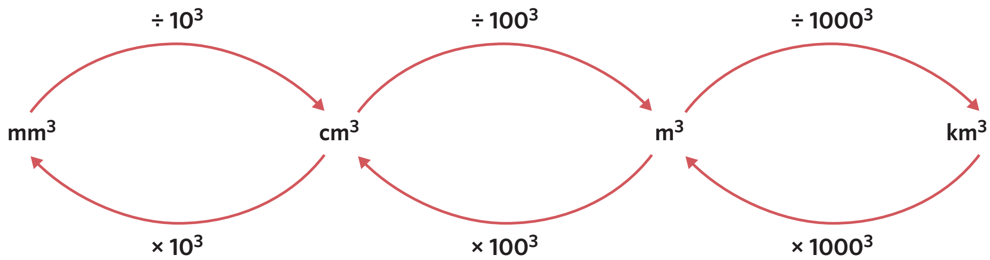
Answer

$456\,000\text{ mm}^2$

Converting units of volume and capacity

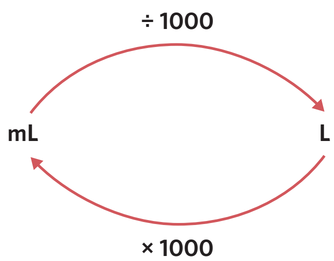
See worked example 4

To convert between different units of volume, the same method is applied as converting between different units of length or area. However, as volume is measured in cubic units, the conversion factor used to convert between different units of volume must also be cubed.



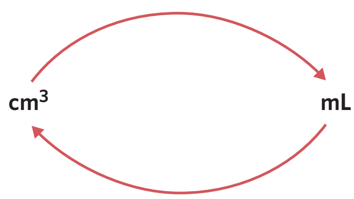
For example, to convert from cubic millimetres to cubic centimetres, the measurement must be divided by 10^3. To convert from cubic centimetres to cubic millimetres, the measurement must be multiplied by 10^3.

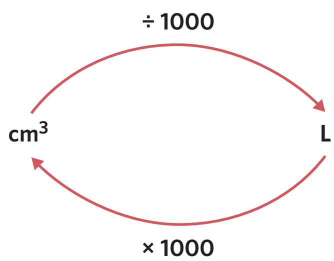
To convert between different units of capacity, a similar method is applied. As there are 1000 millilitres in 1 litre, division or multiplication by 1000 will convert between the units.



See worked example 5

The capacity of an object can be calculated by performing a conversion on its volume, as one cubic centimetre is equivalent to one millilitre and 1000 cubic centimetres is equivalent to one litre.





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Worked example 4

Convert the following.

a. 56 320 000 m^3 to km^3

Explanation

Step 1: Determine the type of conversion

Cubic metres are smaller units than cubic kilometres. Therefore, division is required.

Step 2: Divide the measurement by the conversion factor.

Converting cubic metres to cubic kilometres requires a conversion factor of 1000^3.

$56\,320\,000 \div 1000^3 = 0.05632$

Answer

0.05632 km^3

b. 5.656 L to mL

Explanation

Step 1: Determine the type of conversion.

Litres are larger units than millilitres. Therefore, multiplication is required.

Step 2: Multiply the measurement by the conversion factor.

Converting litres to millilitres requires a conversion factor of 1000.

$5.656 \times 1000 = 5656$

Answer

5656 mL

Worked example 5

Convert the following.

a. 35 890 cm^3 to L

Explanation

Divide the measurement by the conversion factor.

Converting cubic centimetres to litres requires a conversion factor of 1000.

$35\,890 \div 1000 = 35.89$

Answer

35.89 L

b. 2437.5 L to m^3

Explanation

Step 1: Convert litres to cubic centimetres.

Converting litres to cubic centimetres requires a conversion factor of 1000.

$2437.5 \times 1000 = 2\,437\,500$ cm^3

Step 2: Convert cubic centimetres to cubic metres.

Converting cubic centimetres to cubic metres requires a conversion factor of 100^3.

As a smaller unit is being converted to a larger unit, division is required.

$2\,437\,500 \div 100^3 = 2.4375$

Answer

2.4375 m^3

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9A Questions

Note: There are no direct exam questions relevant to this lesson.

Identifying appropriate units of measurement

1. Which of the following is a unit of area?

A. mL **B.** cm^3 **C.** m **D.** mm^2

2. Which of the following is a unit of capacity?

A. mm^2 **B.** m^3 **C.** L **D.** km

3. What unit of measurement (angle, length, area, volume or capacity) do the markings on a ruler represent?

4. Determine which unit of measurement (angle, length, area, volume or capacity) is appropriate for each of the following scenarios.

a. Jack wants to measure the size of his farm.

b. Blaise wants to know how much water can fit in his esky.

c. Juliet is interested in knowing the distance of her tram ride from home to work.

d. Ella wants to know how the direction of her flight path from Melbourne to Barcelona is related to the equator.

e. Johnny wants to find out how much space his towel takes up in his backpack.

Converting units of length

5. Which of the following lengths is equivalent to 730 cm?

A. 0.73 km **B.** 7.3 m **C.** 73 m **D.** 73 000 mm

6. Convert the following lengths to kilometres.

a. 100 m **b.** 18 cm **c.** 38.6 mm **d.** 6.5 cm

7. Convert the following lengths to millimetres.

a. 50 cm **b.** 165 km **c.** 80.5 m **d.** 23.72 cm

8. Jordy just completed a marathon and wants to show off to her Instagram followers. However, the 42.125 kilometre length of a marathon isn't impressive enough, so Jordy wants to convert this distance to millimetres. Help Jordy find the length of a marathon in millimetres.

Converting units of area

9. Which of the following areas is equivalent to 2900 m^2?

A. 2.9 km^2 **B.** 290 000 cm^2 **C.** 2 900 000 mm^2 **D.** 29 000 000 cm^2

10. Convert the following areas to square metres.

a. 150 cm^2 **b.** 163 mm^2 **c.** 18.95 cm^2 **d.** 185.2 mm^2

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11. Convert the following areas to square millimetres.

a. $26\ cm^2$ b. $0.0000183\ km^2$ c. $160\ m^2$ d. $0.286\ m^2$

12. Henry and Lily have bought a property in Clifton Hill and are planning on building a house together. The property has a land area of $535\ m^2$. Henry and Lily paid 30 cents for every cm^2 of land area. How much, in dollars, did Henry and Lily pay for their property?

Converting units of volume and capacity

13. Which of the following volumes is equivalent to $37\,000\,000\ cm^3$?

A. $0.000000037\ km^3$
B. $0.00037\ m^3$
C. $37\,000\ m^3$
D. $370\,000\,000\ mm^3$

14. Convert the following volumes.

a. $16\ cm^3$ to mm^3
b. $13.6\ mm^3$ to cm^3
c. $0.06\ m^3$ to cm^3
d. $750\,000\ m^3$ to km^3

15. Convert the following capacities.

a. 245 mL to L
b. 1.231 L to mL
c. 381262 ml to L
d. 0.045 L to mL

16. Convert the following volumes to capacities.

a. $123.56\ cm^3$ to mL

b. $154.6\ cm^3$ to L

c. $14.2\ m^3$ to mL

17. Convert the following capacities to volumes.

a. 2.563 L to cm^3

b. 31.32 L to m^3

c. 7.67 L to mm^3

18. Georgie and Sofia are having a water balloon fight. They fill up 30 balloons. Each balloon has a capacity of 0.8 L, however, they are only filled up to 80% of their capacity. What volume of water, in cubic metres, did Georgie and Sofia use during their water balloon fight?

Joining it all together

19. The HTC DROID DNA was the first mobile phone to have a full HD screen.

a. What unit of measurement (angle, length, area, volume or capacity) is commonly used to represent the size of a phone screen?

b. The area of the screen is $68.97\ cm^2$. Determine the area of the screen, expressed in mm^2.

c. A customer is interested in buying the phone, however, he is concerned with how large the phone is. He will not buy the phone if it takes up more than $135\ cm^3$ of space in his pocket. The phone has a volume of $134\,430\ mm^3$. Should the customer buy the phone?

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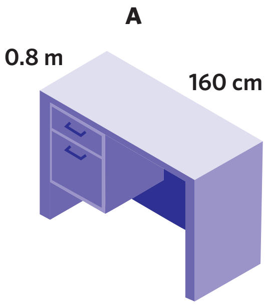
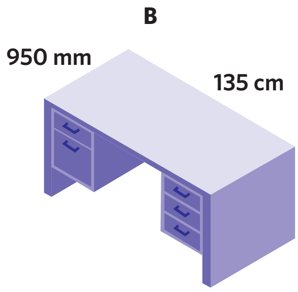
20. Consider the following diagrams showing desk A and desk B.





a. James is interested in determining how much space the table takes in his room. Which unit of measurement would be most appropriate?

b. He realises that the length and width of the table are given in different units. He would like the lengths to be expressed in the same units. What are the dimensions of each of the desks, expressed in centimetres?

c. James calculates the tabletop area of desk A to be 12 800 cm^2. What is this area expressed in mm^2?

---

21. A model Olympic swimming pool is a rectangle of dimensions 1.5 m by 0.8 m. The pool is 0.3 metres deep.

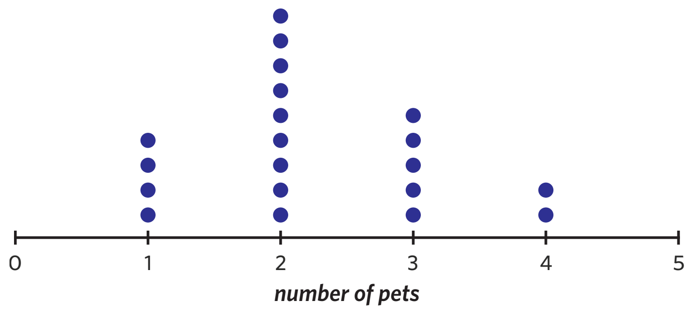
a. The pool was determined to have a volume of 0.36 m^3. What is the volume of the model pool, expressed in cm^3?

b. What is the capacity of the pool, expressed in L?

Questions from multiple lessons

Data analysis *Year 10 content*

22. The following dot plot displays the *number of pets* owned by 20 zookeepers.



What is the median *number of pets* for this group of zookeepers?

A. 1 **B.** 2 **C.** 2.5 **D.** 3 **E.** 4

Adapted from VCAA 2019NH Exam 1 Data analysis Q4

Recursion and financial modelling

23. Kelly wants to buy a show dog and deposits \$2000 into a savings account with an interest rate of 1.8% per annum, compounding monthly. Which of the following recurrence relations can be used to determine the amount in the savings account, S_n, after n months?

A. $S_0 = 2000, \quad S_{n+1} = S_n + 36$

B. $S_0 = 2000, \quad S_{n+1} = 1.018 \times S_n$

C. $S_0 = 2000, \quad S_{n+1} = 1.0015 \times S_n$

D. $S_0 = 2000, \quad S_{n+1} = 1.8 \times S_n$

E. $S_0 = 2000, \quad S_{n+1} = S_n + 3$

Adapted from VCAA 2017NH Exam 1 Recursion and financial modelling Q18

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Graphs and relations *Year 10 content*

24. A carpet store recently received an order of new carpets which need to be measured before they can be sold. The following table shows the relationship between the width of a particular style of carpet, and the area it takes up.

***width* (m)**	2	3	5	6
***area* (m^2)**	5	7.5	12.5	15

a. The relationship between *width* and *area* is linear. Find the equation of the line in the form $area = m \times width + c$ (1 MARK)

b. Draw the graph that shows the relationship between *width* and *area*. (1 MARK)

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9B Exact answers, rounding and scientific notation

STUDY DESIGN DOT POINT

- exact and approximate answers, scientific notation, significant figures and rounding

9A 9B 9C 9D 9E 9F 9G 9H 9I

KEY SKILLS

During this lesson, you will be:

- approximating values using rounding
- expressing values using scientific notation.

KEY TERMS

- Rounding
- Significant figure
- Scientific notation
- Order of magnitude

Whether it is buying a cup of coffee for $5.60 or asking for 200 g of oranges, in many real-world situations, the numbers are easy to manage. However, there are also instances where values have a large number of digits. For example, the Avogadro constant, the approximate number of nucleons in one gram of ordinary matter, has over 20 decimal places, making it difficult to write simply. As a result, there are processes, such as rounding or using scientific notation, to condense the number into a form that is easier to read and perform calculations with.

Approximating values using rounding

Rounding is the process of condensing a number so that it is more convenient to write and use in calculations. This can include rounding to a whole number, a specified number of decimal places, or the nearest 5, 10 or 100 (or some other place value). The drawback of rounding is a loss of accuracy.

See worked example 1

When rounding, the following rules are applied:

- The digit directly following the required rounded digits is used to determine rounding.
- If this digit is between 0 and 4, the number is rounded down.
- If this digit is between 5 and 9, the number is rounded up.

Another way of rounding is to a certain number of significant figures. A **significant figure** is a digit in a number that contributes to the value of the number with certainty.

See worked example 2

There are several ways to identify if a digit is a significant figure.

- From left to right, the first significant figure is always the first non-zero digit.
- All non-zero digits are significant.
- All zeros in between significant figures are significant.
- In a number with a decimal point, all zeros after any non-zero digit are significant.
- All other zeros are not significant.

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In the following examples, the **green** digits are significant, and the **red** digits are not significant.

number	explanation
153	Each digit is non-zero, so all three are significant figures.
100	The two 0's in 100 are not significant. Therefore, only the 1 is significant.
1070	The 1 and 7 are significant. The first 0 is in between the 1 and 7, and is therefore significant.
0.00470	The 4 and the 7 are significant. The 0 after the non-zero digit is significant as the number has a decimal point. The other 0's are not significant.
2000.00	The 2 is significant as it is a non-zero digit. All 0's after the non-zero digit are significant as the number has a decimal point.

Rounding significant figures is performed using the same method as rounding to the nearest whole number or decimal place.

See worked example 3

Worked example 1

Round the following numbers.

a. 62 to the nearest ten.

Explanation

Step 1: Identify the nearest multiples of ten.

In this case, they are 60 and 70.

Step 2: Round the number based on the digit directly following the required rounded digits.

2 is between 0 and 4.

Therefore, 62 should be rounded down to 60.

Answer

60

b. 4.6 to the nearest whole number.

Explanation

Step 1: Identify the nearest whole numbers.

In this case, they are 4 and 5.

Step 2: Round the number based on the digit directly following the required rounded digits.

6 is between 5 and 9.

Therefore, 4.6 should be rounded up to 5.

Answer

5

c. 3.74 to one decimal place.

Explanation

Step 1: Identify the nearest values with one decimal place.

In this case, they are 3.7 and 3.8.

Step 2: Round the number based on the digit directly following the required rounded digits.

4 is between 0 and 4.

Therefore, 3.74 should be rounded down to 3.7.

Answer

3.7

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Worked example 2

Determine the number of significant figures for the following values.

a. 1030

Explanation

Step 1: Find the non-zero digits.

The non-zero digits, 1 and 3, are significant figures.

Step 2: Determine which zeros are significant.

The 0 between the two non-zero digits is significant.

However, the last 0 is not significant.

1030

Answer

3 significant figures

b. 16.040

Explanation

Step 1: Find the non-zero digits.

The non-zero digits, 1, 6 and 4, are significant figures.

Step 2: Determine which zeros are significant.

The 0 between the non-zero digits is significant.

The 0 at the end of the number is also significant as the number has a decimal point.

16.040

Answer

5 significant figures

Worked example 3

Round the following to the indicated number of significant figures.

a. 84.823 to three significant figures.

Explanation

Step 1: Determine the first three significant figures.

All digits are non-zero.

84.823

Step 2: Round the number based on the digit directly following the third significant figure.

2 is between 0 and 4.

Therefore, 84.823 should be rounded down to 84.8.

Answer

84.8

b. 0.5182 to two significant figures.

Explanation

Step 1: Determine the first two significant figures.

All non-zero digits in this case are significant.

0.5182

Step 2: Round the number based on the digit directly following the second significant figure.

8 is between 5 and 9.

Therefore, 0.5182 should be rounded up to 0.52.

Answer

0.52

Expressing values using scientific notation

Scientific notation is a way to express values with a condensed number of significant figures. When a number is expressed using scientific notation, it is written in the form

$x \times 10^y$, where

- x is a value between 1 and 10 and is commonly expressed with a maximum of 3 significant figures
- y is any positive or negative whole number.

Here, ‘10’ is known as the base, and ‘y’ is known as the exponent, or the **order of magnitude** of the number, which allows for a quick comparison between numbers.

For example, the following shows 600 expressed in scientific notation. The order of magnitude is 2.

exponent
(or index, or power)

$$6 \times 10^2$$

base

Numbers can be converted to and from scientific notation.

See worked example 4

To convert a number to scientific notation, the following must be done:

1. Place a decimal point after the first significant figure.
2. To determine the value of the exponent, y, count the number of places the decimal point has moved from its original position. If the decimal point has been moved to the left of its original position, the exponent is positive. If the decimal point has been moved to the right, the exponent is negative.
3. To determine the value of x, round the changed number to three significant figures, if necessary.
4. Express in the form $x \times 10^y$.

To convert a number from scientific notation, a multiplication or division must be completed on the number value, x.

See worked example 5

When the exponent is positive, the number, x, is multiplied by 10 y times.

This can be achieved by moving the decimal point to the right. As a result, a 0 is added as a placeholder to the end if needed.

scientific notation	expression	number
7×10^1	7×10	70
7×10^2	$7 \times 10 \times 10$	700
7×10^3	$7 \times 10 \times 10 \times 10$	7000
7×10^4	$7 \times 10 \times 10 \times 10 \times 10$	70000

When the exponent is negative, the number, x, is divided by 10 y times.

This can be achieved by moving the decimal point to the left. As a result, a 0 is added as a placeholder to the left if needed.

scientific notation	expression	number
7×10^{-1}	$7 \div 10$	0.7
7×10^{-2}	$7 \div 10 \div 10$	0.07
7×10^{-3}	$7 \div 10 \div 10 \div 10$	0.007
7×10^{-4}	$7 \div 10 \div 10 \div 10 \div 10$	0.0007

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Worked example 4

Convert the following to scientific notation.

a. 946 000

Explanation

Step 1: Rewrite the number with a decimal point after the first significant figure.

9.46000

Step 2: Determine the value of the exponent.

Count the number of places the decimal point has been moved and in which direction.

9 . 4 6 0 0 0

The decimal point has been moved five places to the left.

$y = 5$

Step 3: Determine the value of x.

Round the rewritten number to three significant figures.

$x = 9.46$

Step 4: Write the number using scientific notation.

Answer

9.46×10^5

b. 0.45186412

Explanation

Step 1: Rewrite the number with a decimal point after the first significant figure.

4.5186412

Step 2: Determine the value of the exponent.

Count the number of places the decimal point has been moved and in which direction.

0 4 . 5 1 8 6 4 1 2

The decimal point has been moved one place to the right.

$y = -1$

Step 3: Determine the value of x.

Round the rewritten number to three significant figures.

$x = 4.52$

Step 4: Write the number using scientific notation.

Answer

4.52×10^{-1}

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Worked example 5

Convert the following from scientific notation.

a. 6.2×10^2

Explanation

Multiply 6.2 by 10^2 by moving the decimal point two places to the right.

6 2 0 .

Answer

620

b. 3.58×10^{-4}

Explanation

Multiply 3.58 by 10^{-4} by moving the decimal point 4 places to the left.

0 . 0 0 0 3 5 8

Answer

0.000358

9B Questions

Approximating values using rounding

1. What is 3.7 rounded to the nearest whole number?

A. 3 **B.** 4 **C.** 7 **D.** 8

2. Round the following to the nearest whole number.

a. 2.1 **b.** 12.5 **c.** 89.92 **d.** 1000.01

3. Natalia lives 2.7 kilometres from school, and walks to school and back every day. How far does Natalia walk along this route every day, rounded to the nearest kilometre?

4. Round the following to the nearest ten.

a. 81 **b.** 33 **c.** 85 **d.** 186.5

5. Round the following to:

a. one decimal place.

i. 3.12

ii. 5.24

iii. 4.76

iv. 8.18

b. two decimal places.

i. 13.231

ii. 234.114

iii. 34.785

iv. 67.879

---

6. A butcher rounds the weight of meat to the nearest ten grams before calculating the price.
If the scale reads 0.665 kg and the meat is \$8 per kg, how much will the customer have to pay?

---

7. Which of the following does not have exactly three significant figures?

A. 0.0236 **B.** 20.50 **C.** 37.0 **D.** 10 500

---

8. How many significant figures are in the following numbers?

a. 25 **b.** 103.2 **c.** 2580 **d.** 0.0105
e. 65.0 **f.** 12 120

---

9. Ms. Tao asked four of her students to write one number each that had the same amount of significant figures as the number of letters in their name.

- Jake: 4030
- Emily: 2.0155
- An: 65 000
- Annabelle: 4.0010013

Which student(s) got it right?

---

10. Round the following numbers to three significant figures.

a. 237.01 **b.** 153.1 **c.** 6.607 **d.** 9.6234
e. 92.763 **f.** 0.00327876

Expressing values using scientific notation

11. 3×10^{-5} is equivalent to

A. 0.000003 **B.** 0.00003 **C.** 30 000 **D.** 300 000

---

12. Which number is the smallest?

A. 1.6×10^{-5} **B.** 2.31×10^{3} **C.** 4.234×10^{-7} **D.** 5.834×10^{0}

---

13. Convert the following to scientific notation with three significant figures.

a. 32 815 **b.** 8 234 400 **c.** 0.000782

---

14. How many orders of magnitude do the following values differ by?

a. 780 and 78 000 000 **b.** 0.1 and 0.001 **c.** 0.0024 and 24 000

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15. Which of these is not written in correct scientific notation?

A. 1.003×10^{-8} **B.** 5.8×10^{-6} **C.** 3.7823×10^{2} **D.** 0.9×10^{3}

16. The distance around an athletics track is 40 000 cm.

What is the distance, in cm, of half the track written in scientific notation?

17. Convert the following from scientific notation.

a. 3.54×10^{5} **b.** 4.92×10^{-3} **c.** 8.7408×10^{-5}

Joining it all together

18. The radius of a tennis ball is 0.033 metres.

What is the radius in kilometres written in scientific notation?

19. A student makes a mistake when converting 0.0000000027 to scientific notation and gets an answer of 2.7×10^{-8}.

What is the correct answer?

20. Lipika, Anaya and Holly are trying to estimate the area of their school. The school grounds have a rectangular shape with dimensions 207.85 m by 322.37 m.

Their estimates are shown in the following table.

name	Lipika	Anaya	Holly
estimate (m^2)	67 200	60 000	66 976

Before calculating the estimated area, one student rounded the dimensions to the nearest metre, another student rounded to the nearest 10 metres and the other student rounded to the nearest 100 metres.

a. Which method did each student use?

b. Which method(s) estimated a larger area than the actual area, and which method(s) estimated a smaller area?

c. Which student's estimate was closest to the actual area?

Exam practice

21. A line of good fit is used to model the relationship between the monthly *average temperature* and *latitude* recorded at seven different weather stations. The equation of the line of good fit is found to be

average temperature = 42.9842 − 0.877447 × *latitude*

When the numbers in this equation are correctly rounded to three significant figures, the equation will be

A. *average temperature* = 42.984 − 0.877 × *latitude*

B. *average temperature* = 42.984 − 0.878 × *latitude*

C. *average temperature* = 43.0 − 0.878 × *latitude*

D. *average temperature* = 42.9 − 0.878 × *latitude*

E. *average temperature* = 43.0 − 0.877 × *latitude*

Adapted from VCAA 2019 Exam 1 Data analysis Q9

67% of students answered this type of question correctly.

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22. The two running events in the heptathlon are the 200 m run and the 800 m run. The times taken by the athletes in these two events, *time200* and *time800*, are linearly related.

When a line of good fit is fitted to the data, the equation of this line is found to be

$time800 = 0.03921 + 5.2756 \times time200$

Round the values for the intercept and the slope to three significant figures.
Write your answers in the boxes provided. (1 MARK)

$time800 = \boxed{} + \boxed{} \times time200$

Adapted from VCAA 2021 Exam 2 Data analysis Q2a

48% of students answered this type of question correctly.

Questions from multiple lessons

Recursion and financial modelling

23. Ash borrows \$6500 which she has to repay fully in a lump sum payment after two years.

She has a choice of one of the following five loans, with the given interest rates and compounding periods:

- Loan 1 – 9.70% per annum, compounding monthly
- Loan 2 – 9.72% per annum, compounding monthly
- Loan 3 – 9.74% per annum, compounding monthly
- Loan 4 – 9.76% per annum, compounding quarterly
- Loan 5 – 9.78% per annum, compounding quarterly

The loan that will cost Ash the least amount of money is

A. Loan 1. **B.** Loan 2. **C.** Loan 3. **D.** Loan 4. **E.** Loan 5.

Adapted from VCAA 2018 Exam 1 Recursion and financial modelling Q19

Matrices

24. Consider the following matrix equation.

$$2 \times \begin{bmatrix} 6 & -4 \\ 3 & 0 \end{bmatrix} - M = \begin{bmatrix} 5 & 1 \\ 4 & -1 \end{bmatrix}$$

Which of the following matrices is M?

A. $\begin{bmatrix} 1 & -5 \\ -1 & 1 \end{bmatrix}$ **B.** $\begin{bmatrix} 3 & -3 \\ 1 & 3 \end{bmatrix}$ **C.** $\begin{bmatrix} -3 & 3 \\ -1 & -3 \end{bmatrix}$ **D.** $\begin{bmatrix} 7 & -9 \\ 2 & 1 \end{bmatrix}$ **E.** $\begin{bmatrix} -7 & 9 \\ -2 & -1 \end{bmatrix}$

Adapted from VCAA 2018NH Exam 1 Matrices Q2

Computation and practical arithmetic *Year 10 content*

25. While Dylan is on holiday in Japan, two earthquakes occur. The first measures 3.4 on the Richter scale and the second measures 2.7 on the Richter scale. Note that the Richter scale is a logarithmic scale.

How many times stronger was the first earthquake than the second, rounded to the nearest whole number? (1 MARK)

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9C Similarity

STUDY DESIGN DOT POINT

- similar shapes including the conditions for similarity

9A 9B 9C 9D 9E 9F 9G 9H 9I

KEY SKILLS

During this lesson, you will be:

- identifying similar shapes by eye
- identifying conditions for similar triangles
- identifying conditions for other similar shapes.

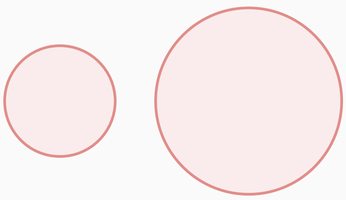
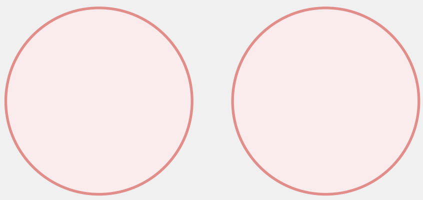
KEY TERMS

- Similarity
- Congruent
- Linear scale factor
- Image

Different two dimensional shapes can be analysed to determine whether they are proportionally the same. This is a practical skill that can be used to scale diagrams and their real-life sizes up or down. This allows different components to be accurately represented on a scale that is easier to visualise.

Identifying similar shapes by eye

Similarity refers to objects that are the same shape, but different sizes. Objects are considered similar even if one has been reflected or rotated. If objects have exactly the same shape and size, they are considered **congruent**. For example, all circles are either similar or congruent as they are all the same shape. Similarly, all regular polygons (shapes where all sides have the same length, such as a hexagon) with the same number of sides are either similar or congruent.

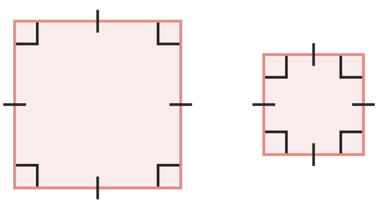
Similar	Congruent
Same shape, different sizes.	Same shape, same size.
	

There are certain conditions that need to be met in order to determine whether shapes are similar or not. This can be determined by eye.

Worked example 1

Determine whether the following pairs of shapes are similar or not.

a.



Continues →

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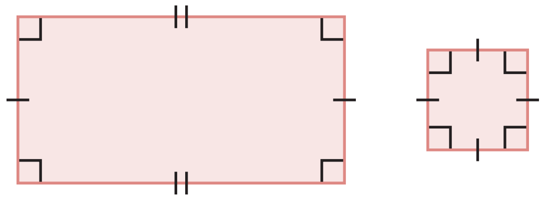
Explanation

In each shape, there are four side lengths that are equal, all at 90° to each other. These objects have the same shape but different size, meaning they are considered similar.

Answer

Similar

b.



Explanation

The first shape contains two pairs of side lengths that are equal.

The second shape has four side lengths that are all equal.

These objects do not have the same shape, meaning they are not considered similar.

Answer

Not similar

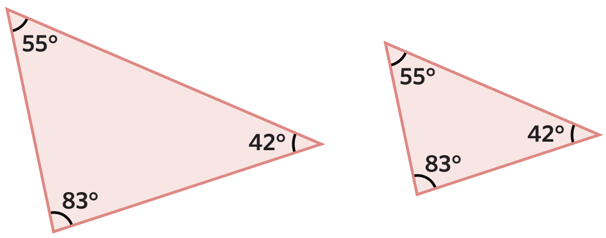
Identifying conditions for similar triangles

Identifying similarity in triangles is often simpler than for other 2D shapes, as not all side lengths and angles are needed to determine similarity.

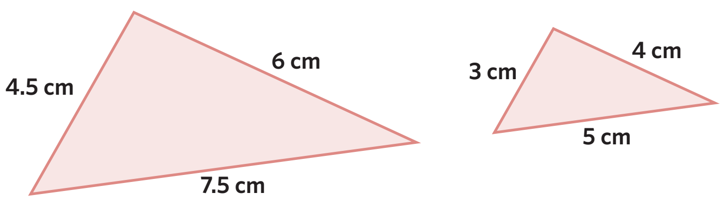
Two triangles are similar if one of the following conditions is met:

1. AAA (angle-angle-angle): all angles are equal.

   Recall that all internal angles in a triangle add up to 180°.



2. SSS (side-side-side): all corresponding side lengths are proportional.



   In this case, the triangles are similar because dividing each side length of one of the triangles by its corresponding side length in the other triangle results in the same ratio of 1.5.

   $4.5 \div 3 = 1.5$

   $6 \div 4 = 1.5$

   $7.5 \div 5 = 1.5$

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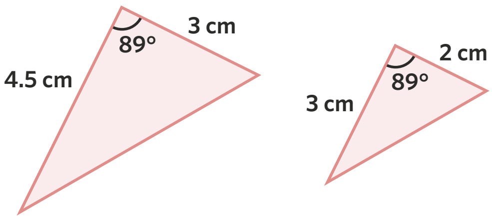
3. SAS (side-angle-side): two corresponding side lengths are proportional and the angle between them (the included angle) is equal.



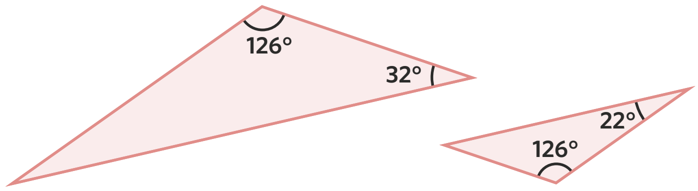
$4.5 \div 3 = 1.5$

$3 \div 2 = 1.5$

Worked example 2

Determine whether the following pairs of triangles are similar or not.

a.



Explanation

Step 1: Determine which condition should be used to test for similarity.

Two angles are known for each triangle. This means the third angle can be calculated, and the AAA condition can be tested.

Step 2: Calculate the missing angle in each triangle.

Triangle 1: $180 - 126 - 32 = 22°$

Triangle 2: $180 - 126 - 22 = 32°$

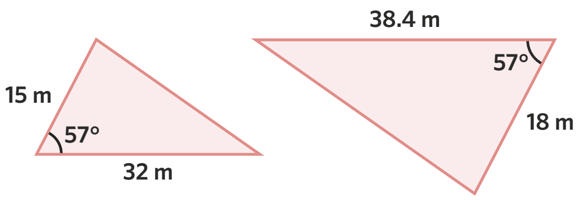
Step 3: Determine whether the condition has been met.

Both triangles have angles of 126°, 32° and 22°. All the angles in the two triangles are equal.

Answer

Similar

b.



Explanation

Step 1: Determine which condition should be used to test for similarity.

Two corresponding side lengths have been provided, as well as the included angle. The SAS condition can be tested.

Step 2: Calculate the proportions of the corresponding side lengths.

$38.4 \div 32 = 1.2$

$18 \div 15 = 1.2$

Step 3: Determine whether the condition has been met.

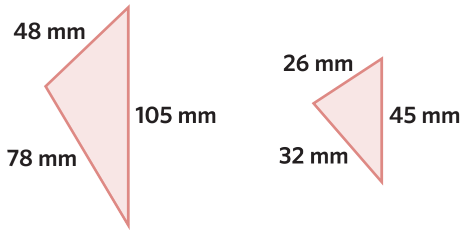
The proportions of the corresponding side lengths are equal, as is the angle between them.

Answer

Similar

Continues →

c.



Explanation

Step 1: Determine which condition should be used to test for similarity.

All side lengths have been provided, so the SSS condition can be tested.

Step 2: Calculate the proportions of all the corresponding side lengths.

$45 \div 105 = 0.428...$

$26 \div 48 = 0.541...$

$32 \div 78 = 0.410...$

Step 3: Determine whether the condition has been met.

The proportions of all the corresponding side lengths are not equal.

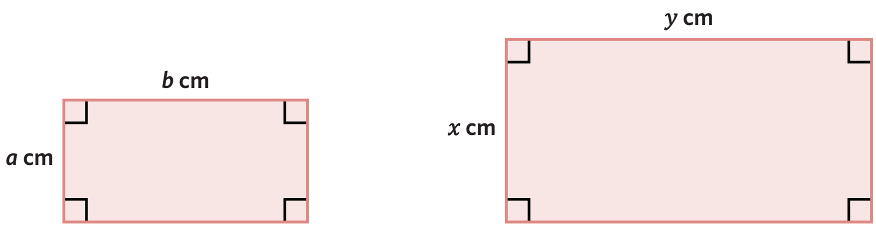
Answer

Not similar

Identifying conditions for other similar shapes

Two dimensional shapes are similar when all their corresponding angles are equal, and their corresponding side lengths are proportional.

The following objects have four internal angles equal to 90°. These objects are similar if $\frac{x}{a} = \frac{y}{b}$ or $\frac{a}{x} = \frac{b}{y}$.



This ratio is known as the **linear scale factor**, denoted k. The scale factor is always calculated as the side length of the image divided by the side length of the original. In geometry, an **image** is a copy of a similar object after it has been rotated, reflected, enlarged, or reduced.

$$k = \frac{\textit{length of image}}{\textit{length of original}}$$

$k > 1$	$k < 1$	$k = 1$
Image is larger than the original.	Image is smaller than the original.	Image is the same size as the original.
		

Congruent objects always have a linear scale factor of 1.

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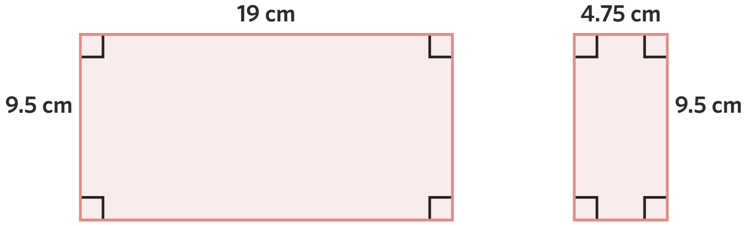
Worked example 3

Determine whether the following shapes are similar or not.

a.



Explanation

Step 1: Check if the corresponding angles are the same.

Both objects have four internal angles equal to 90°.

Step 2: Calculate the linear scale factor of the corresponding sides.

Make sure to match the longer side of one rectangle with the longer side of the other.

Shorter side: $k = 4.75 \div 9.5 = 0.5$

Longer side: $k = 9.5 \div 19 = 0.5$

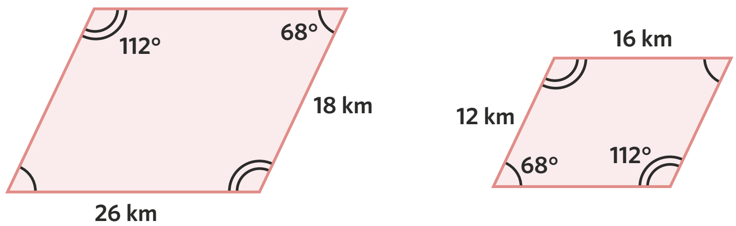
Step 3: Compare the linear scale factors.

Both linear scale factors are equal.

Answer

Similar

b.



Explanation

Step 1: Check if the corresponding angles are the same.

Both shapes have two angles of 112° and two angles of 68°.

Step 2: Calculate the linear scale factor of the corresponding sides.

Shorter side: $k = 12 \div 18 = 0.666...$

Longer side: $k = 16 \div 26 = 0.615$

Step 3: Compare the linear scale factors.

The linear scale factors aren't equal.

Answer

Not similar

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9C Questions

Identifying similar shapes by eye

1. Which of the following two shapes will **always** be similar or congruent?

A. Two equilateral triangles

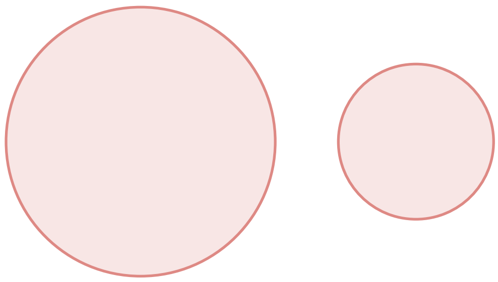
B. Two rectangles

C. Two trapeziums

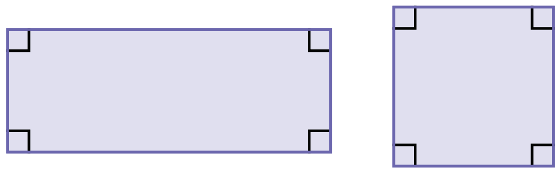
D. Two rhombuses

2. Determine whether the following pairs of shapes are considered similar, congruent or neither.

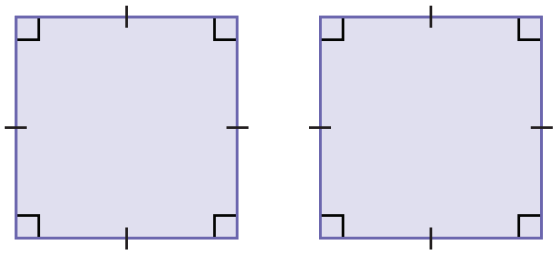
a.



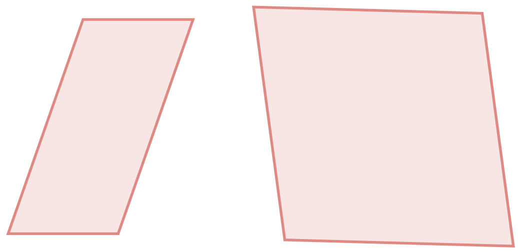
b.



c.



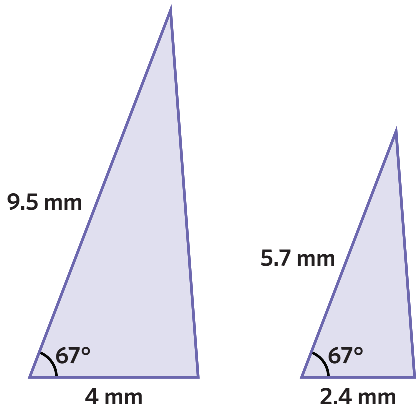
d.



3. Anton has drawn a square with side lengths of 14 cm. He wants to draw a similar shape with a side length of 8 cm. What will be the difference in perimeter between the two shapes?

Identifying conditions for similar triangles

4. Which condition would be used to determine whether the following triangles are similar?



A. AAA

B. SSS

C. SAS

D. Not enough information to test

5. Which of the following conditions would **not** be sufficient to determine if two triangles are similar?

A. Two corresponding sides and one included angle labelled in both triangles.

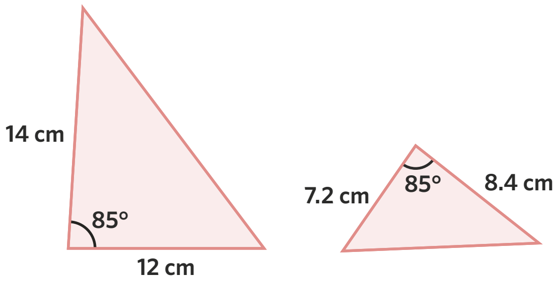
B. Three sides labelled in both triangles.

C. Two corresponding sides and one non-included angle labelled in both triangles.

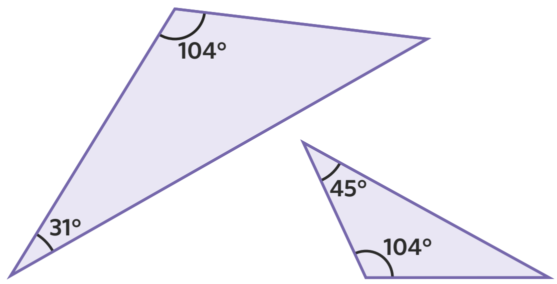
D. Two angles labelled in both triangles.

6. Determine whether the following pairs of triangles are similar or not.

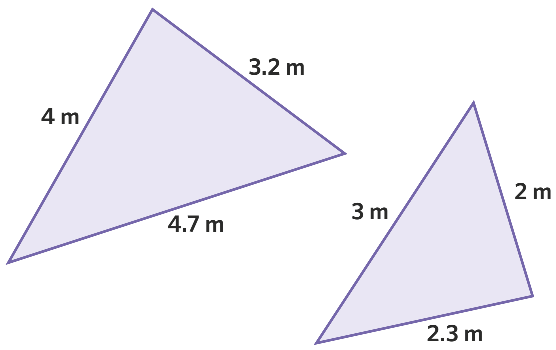
a.



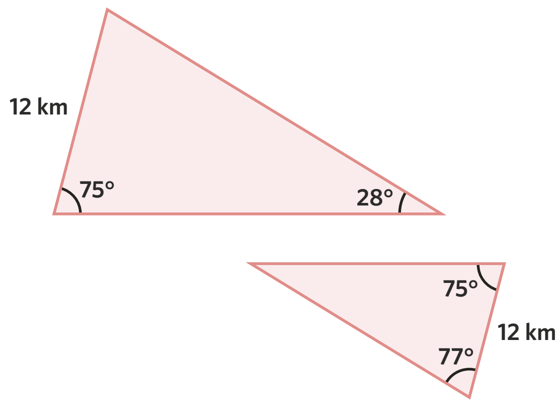
b.



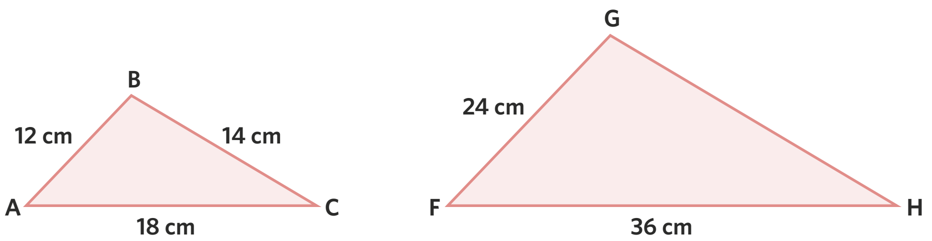
c.



d.

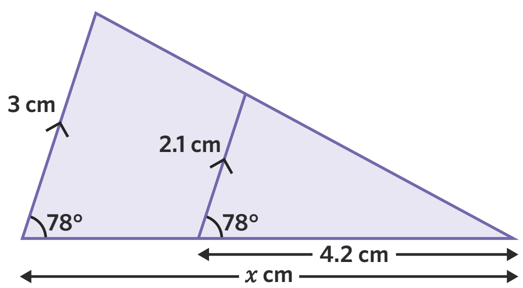


7. The following triangles are similar.



What is the length of GH?

8. Lawrence has a triangular piece of land that he needs to divide.
The following diagram is a scaled down drawing of his piece of land.
What is the value of x in Lawrence's diagram?



Identifying conditions for other similar shapes

9. Consider the following shapes and the incomplete working out.

$\frac{3}{4} = 0.75$

$\frac{a}{8} = b$

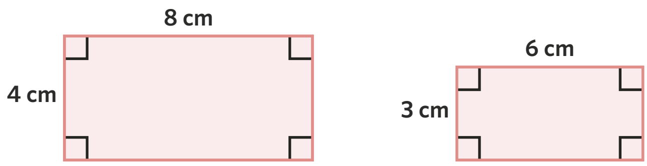
What are the values of a and b?

A. $a = 3, \quad b = 0.75$

B. $a = 0.75, \quad b = 3$

C. $a = 6, \quad b = 0.75$

D. $a = 0.75, \quad b = 6$



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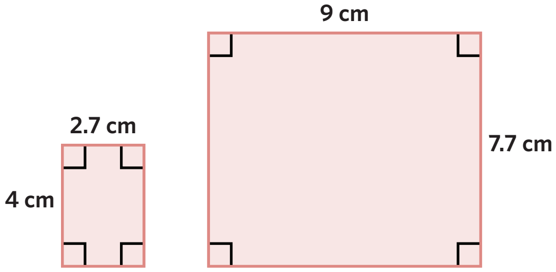
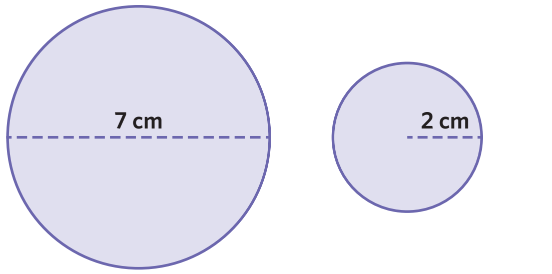
10. Determine whether the following pairs of shapes are similar or not.

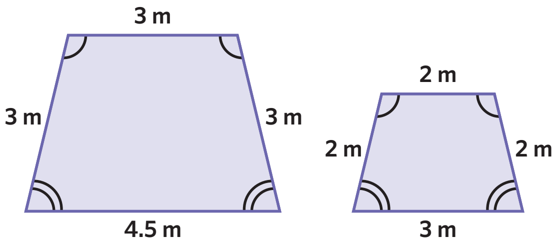
a.



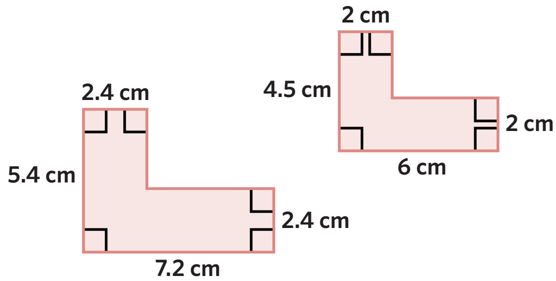
b.



c.



d.



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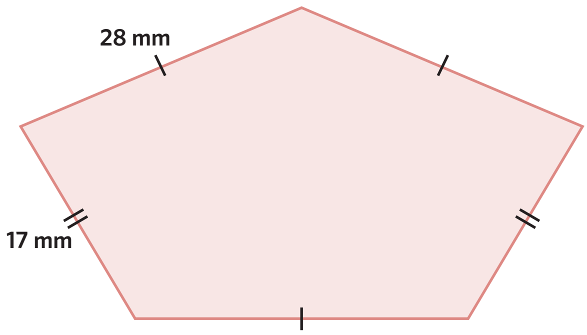
11. Consider the following shape.



If a similar shape was drawn with the longer side length equal to 42 mm, what would be the length of the shorter side?

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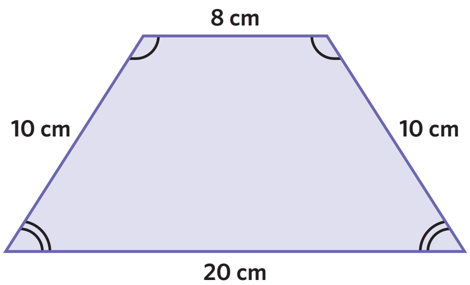
12. Gabby wants to make a two dimensional outline of the following trapezium but she only has 36 cm of string.



Draw a similar trapezium using all of the string that Gabby has.

Joining it all together

13. Which of the following statements is false?

A. All circles are either similar or congruent.

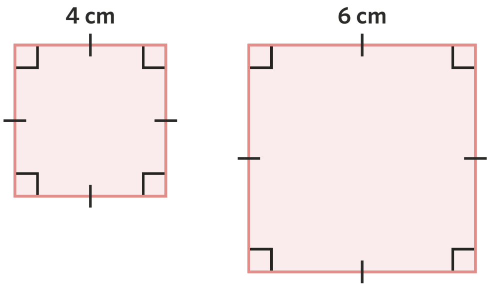
B. Two triangles can be tested for similarity if two internal angles are known.

C. Two triangles can be tested for similarity if two side lengths are known.

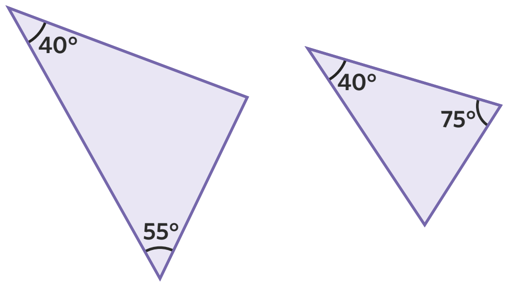
D. Two rectangles can be tested for similarity if the length and width of each are known.

14. Which of the following pairs of shapes are not similar?

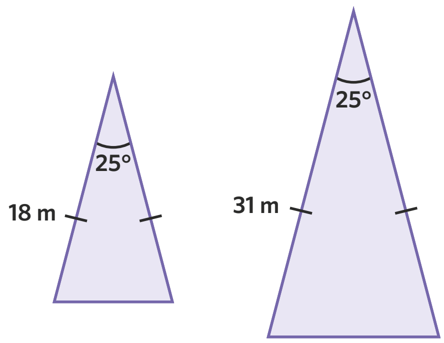
A.



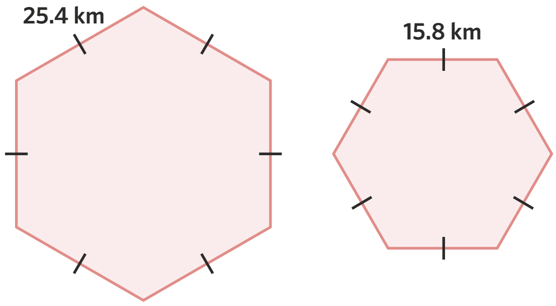
B.



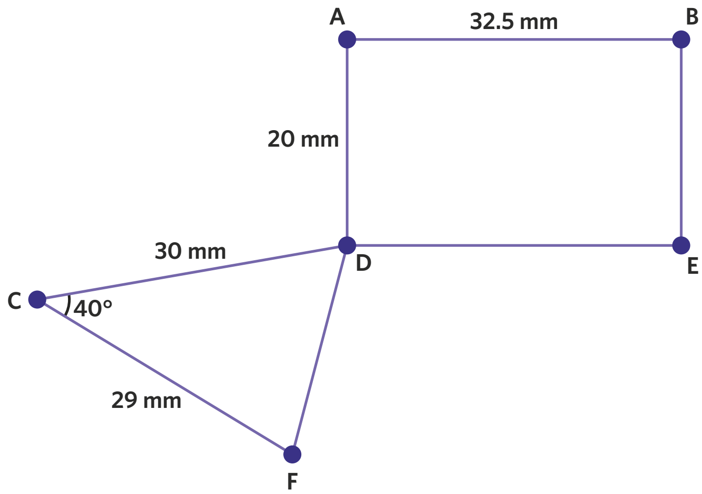
C.



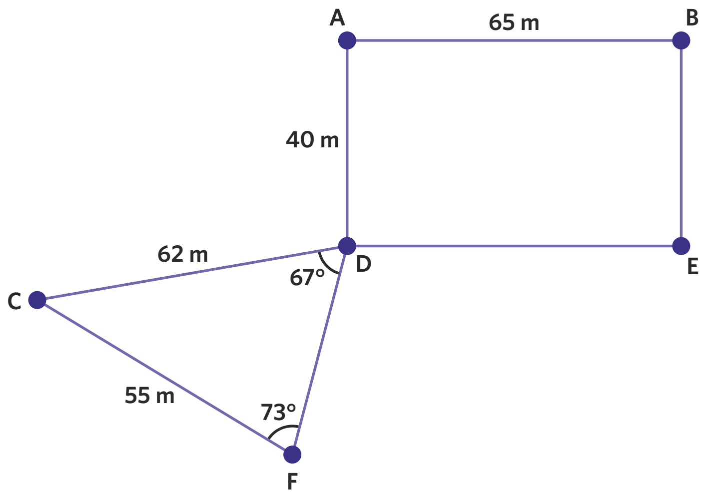
D.



15. The following map was drawn to show the roads between 6 locations.



The following map shows the actual distances of each road.



Has the original map been drawn to the correct proportions?

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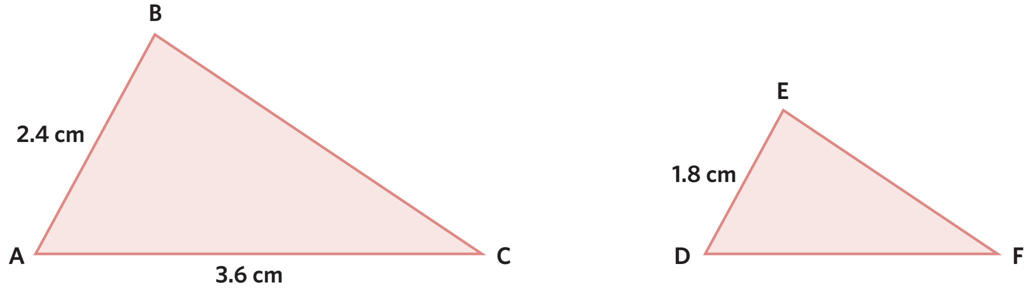
Exam practice

16. Triangle ABC is similar to triangle DEF.



The length of DF, in centimetres, is

A. 0.9 **B.** 1.2 **C.** 1.8
D. 2.7 **E.** 3.6

VCAA 2016 Exam 1 Geometry and measurement Q2

92% of students answered this question correctly.

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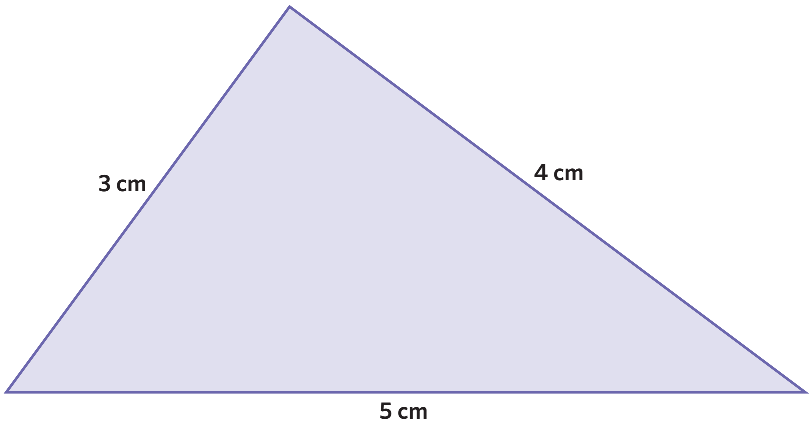
17. The following triangle, *M*, has side lengths of 3 cm, 4 cm and 5 cm.



Four other triangles have the following side lengths:

- Triangle *N* has side lengths of 3 cm, 6 cm and 8 cm.
- Triangle *O* has side lengths of 4 cm, 8 cm and 12 cm.
- Triangle *P* has side lengths of 6 cm, 8 cm and 10 cm.
- Triangle *Q* has side lengths of 9 cm, 12 cm and 15 cm.

The triangles that are similar to triangle *M* are

A. triangle *N* and triangle *O*.
B. triangle *N*, triangle *O* and triangle *P*.
C. triangle *O* and triangle *P*.
D. triangle *O* and triangle *Q*.
E. triangle *P* and triangle *Q*.

VCAA 2019 Exam 1 Geometry and measurement Q4

87% of students answered this question correctly.

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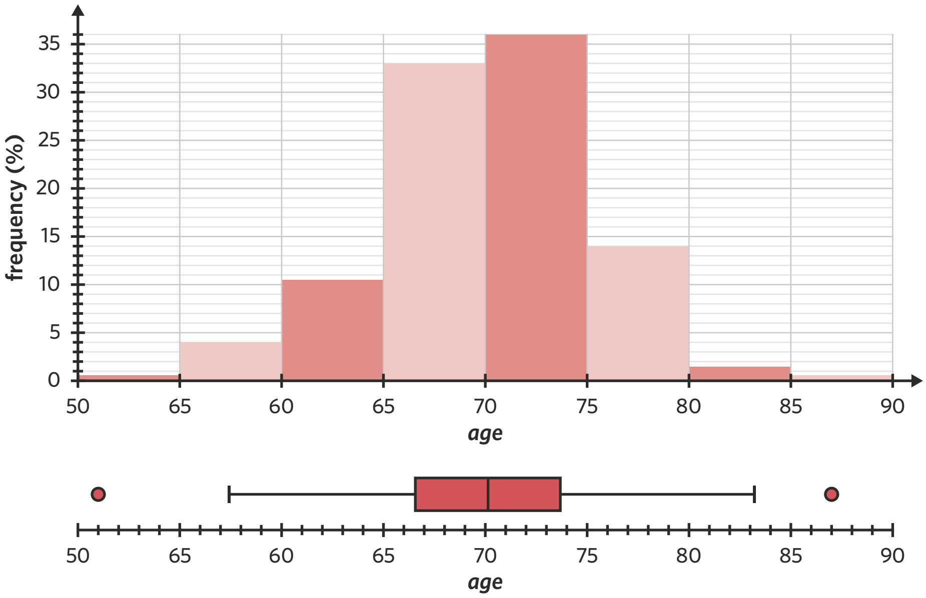
Questions from multiple lessons

Data analysis

18. The following histogram and boxplot display the distribution of the *age* of 200 residents at a retirement village in Nepal.



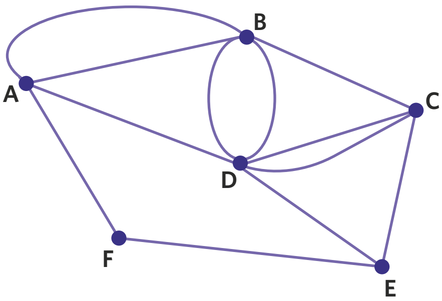
The number of people between the ages of 65 and 70 years is closest to

A. 21 **B.** 33 **C.** 35 **D.** 66 **E.** 70

Adapted from VCAA 2019NH Exam 1 Data analysis Q2

Networks and decision mathematics

19. Consider the following graph.



Which of the following routes is **not** a path?

A. D–E–F–A–B **B.** D–C–B–A–F **C.** D–B–A–F–E **D.** D–A–B–C–E **E.** D–A–C–E–F

Adapted from VCAA 2018 Exam 1 Networks and decision mathematics Q4

Recursion and financial modelling

20. A house was purchased in Eltham for \$750 000. A 20% deposit was paid.

a. Calculate the deposit. (1 MARK)

b. Determine the amount that the homeowners still owe after the deposit is paid. (1 MARK)

The price of \$750 000 included 10% GST.

c. Calculate the price of the house before GST was added. Round to the nearest dollar. (1 MARK)

Adapted from VCAA 2012 Exam 2 Business-related mathematics Q1a,bi,c

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9D Pythagoras' theorem

STUDY DESIGN DOT POINT

- the use of trigonometric ratios and Pythagoras' theorem to solve practical problems involving a right-angled triangle in two dimensions, including the use of angles of elevation and depression

9A 9B 9C **9D** 9E 9F 9G 9H 9I

KEY SKILLS

During this lesson, you will be:

- using Pythagoras' theorem in two dimensions
- using Pythagoras' theorem in three dimensions.

KEY TERMS

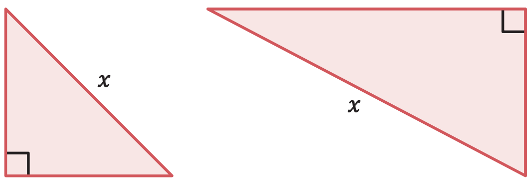
- Right-angled triangle
- Hypotenuse
- Pythagoras' theorem

Pythagoras was an ancient Greek philosopher and mathematician, credited with discovering the mathematical relationship between the side lengths of right-angled triangles. Pythagoras' theorem can be applied in many situations across two and three dimensions.

Using Pythagoras' theorem in two dimensions

A **right-angled triangle** is a triangle in which there is one angle of 90° (a right angle). This angle is shown geometrically using a small square inside the angle.

The longest side in a right-angled triangle is known as the **hypotenuse**, and is always opposite the right angle. The hypotenuse is marked by an x in the following triangles.

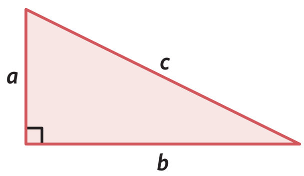


Pythagoras' theorem is a rule that states that for every right-angled triangle:

See worked example 1

$a^2 + b^2 = c^2$, where

- c is the length of the hypotenuse
- a and b are the other two side lengths



Note: Sides a and b are interchangeable.

This formula only applies to right-angled triangles.

For example, consider a right-angled triangle with side lengths 3 and 4, and a hypotenuse of length 5.

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

These side lengths satisfy Pythagoras' theorem.

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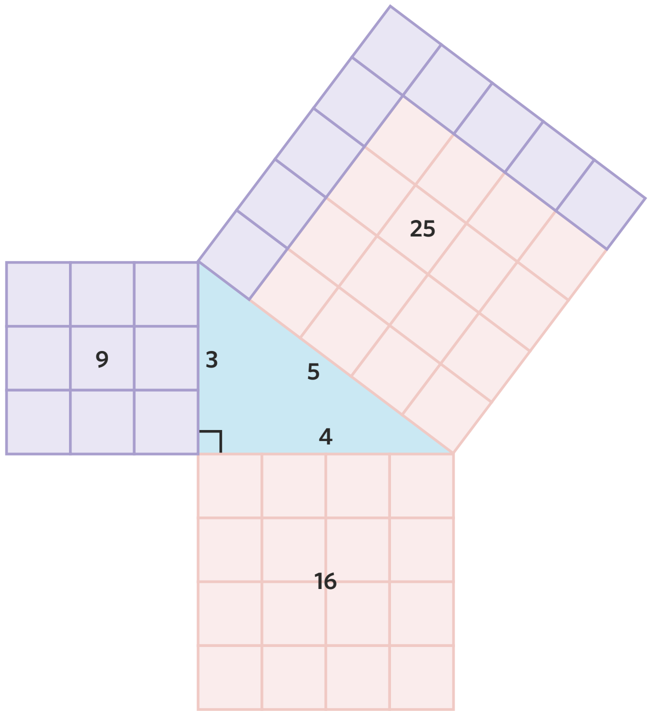
This can be shown visually:



If the values of a and b are known, Pythagoras' theorem can be used to calculate c, the length of the hypotenuse.

See worked example 2

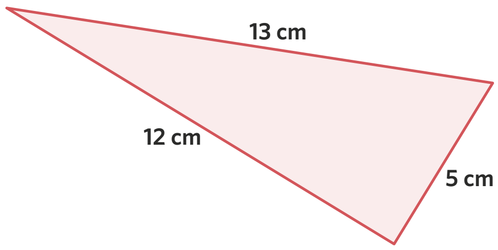
More generally, if the lengths of any two sides of a right-angled triangle are known, Pythagoras' theorem can be used to find the remaining side length.

See worked example 3

Worked example 1

Use Pythagoras' theorem to determine whether the following triangles have a right angle.

a.



Explanation

Step 1: Identify the condition under which the triangle will have a right angle.

If Pythagoras' theorem $a^2 + b^2 = c^2$ holds for the triangle, it contains a right angle.

Step 2: Identify the side lengths.

The hypotenuse is the longest side.

$c = 13$

a and b are interchangeable.

$a = 5$

$b = 12$

Step 3: Evaluate the LHS.

$$\begin{aligned} a^2 + b^2 &= 5^2 + 12^2 \\ &= 25 + 144 \\ &= 169 \end{aligned}$$

Step 4: Evaluate the RHS and determine if it is equal to the LHS.

$$\begin{aligned} c^2 &= 13^2 \\ &= 169 \end{aligned}$$

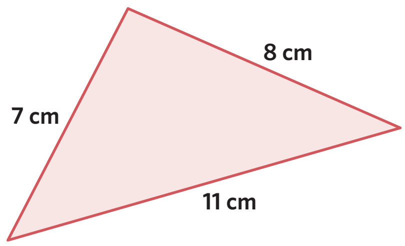
$169 = 169$ ✓

Answer

The triangle has a right angle.

Continues →

b.



Explanation

Step 1: Identify the condition under which the triangle will have a right angle.

If Pythagoras' theorem $a^2 + b^2 = c^2$ holds for the triangle, it contains a right angle.

Step 2: Identify the side lengths.

The hypotenuse is the longest side.

$c = 11$

a and b are interchangeable.

$a = 7$

$b = 8$

Step 3: Evaluate the LHS.

$a^2 + b^2 = 7^2 + 8^2$

$= 49 + 64$

$= 113$

Step 4: Evaluate the RHS and determine if it is equal to the LHS.

$c^2 = 11^2$

$= 121$

$113 \neq 121$ ✗

Answer

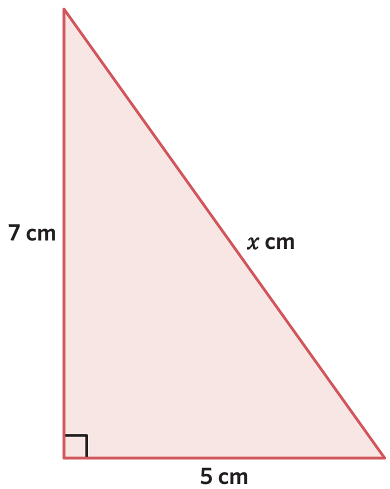
The triangle does not have a right angle.

Worked example 2

For a right-angled triangle with a height of 7 cm and a width of 5 cm, find the length of the hypotenuse, rounded to two decimal places.

Explanation

Step 1: Draw the triangle.



Step 2: Substitute values for a, b and c into Pythagoras' theorem.

$a^2 + b^2 = c^2$

$5^2 + 7^2 = x^2$

Step 3: Solve for x.

$x^2 = 25 + 49$

$= 74$

$x = \sqrt{74}$

$= 8.6023...$

Answer

8.60 cm

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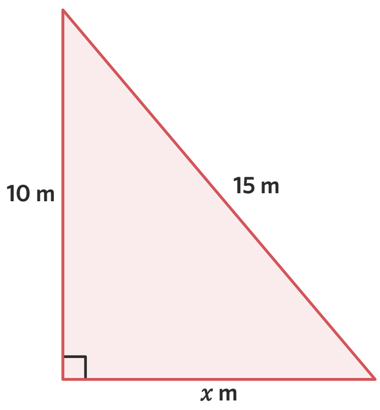
Worked example 3

Find the value of x in the following triangle, rounded to two decimal places.



Explanation

Step 1: Substitute values for a, b and c into Pythagoras' theorem.

$a^2 + b^2 = c^2$

$x^2 + 10^2 = 15^2$

Step 2: Solve for x.

$$x^2 = 15^2 - 10^2$$
$$= 225 - 100$$
$$= 125$$
$$x = \sqrt{125}$$
$$= 11.1803...$$

Answer

$x = 11.18$

Using Pythagoras' theorem in three dimensions

In many cases, a missing side length or diagonal in a three-dimensional object can be found using Pythagoras' theorem. To do so, split the problem up into separate steps, and draw a diagram for each one.

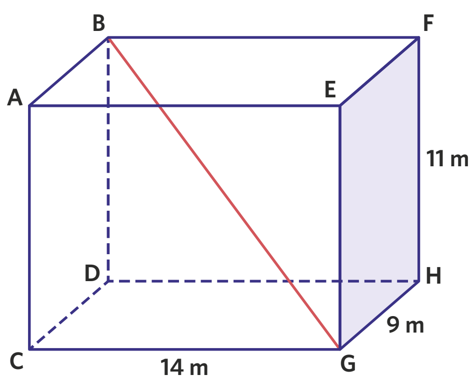
See worked example 4

Pythagoras' theorem can also be applied to real-world scenarios in three dimensions.

See worked example 5

Worked example 4

A rectangular prism has a length of 14 metres, a width of 9 metres, and a height of 11 metres. Find the length of the diagonal BG, rounded to two decimal places.



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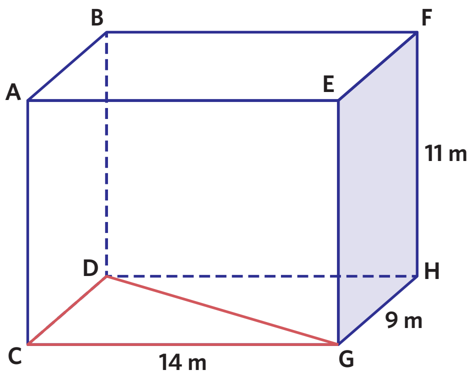
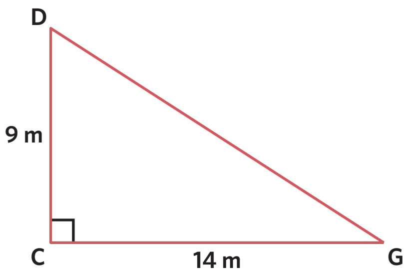
Explanation

Step 1: Draw the triangle CDG and find the length of DG.



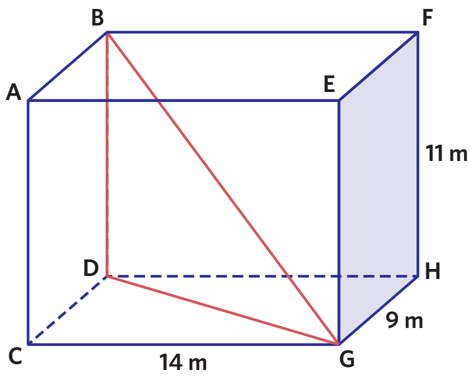


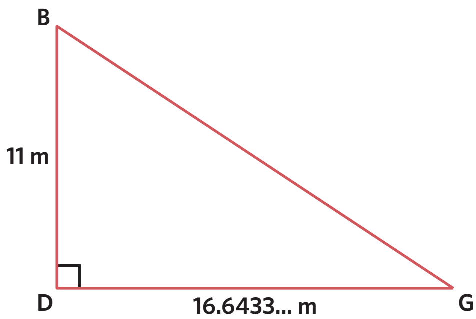
$a^2 + b^2 = c^2$

$DG = \sqrt{9^2 + 14^2}$

$= 16.6433...$

Step 2: Draw the triangle BDG and find the length of BG.





$BG = \sqrt{11^2 + 16.6433...^2}$

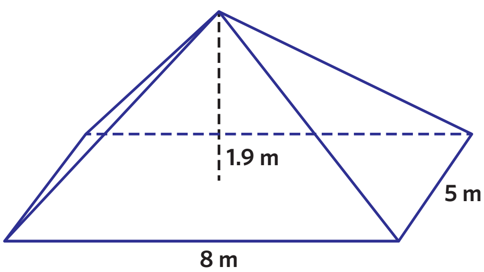
$= 19.9499...$

Answer

19.95 m

Worked example 5

The roof of a house is in the shape of a rectangular-based pyramid with a width of 5 metres, a length of 8 metres and a height of 1.9 metres. What is the distance from the top of the roof to each corner, rounded to two decimal places?



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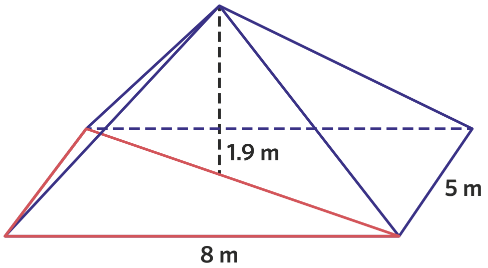
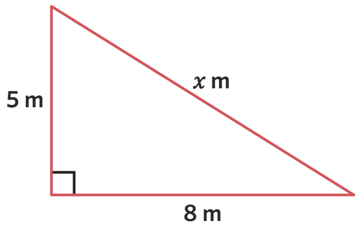
Explanation

Step 1: Find the length of the base diagonal.



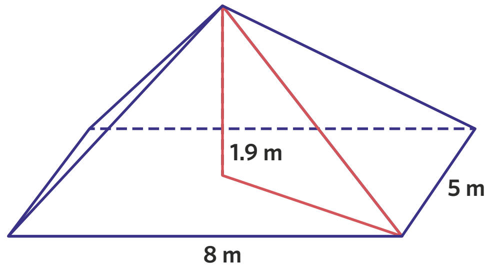


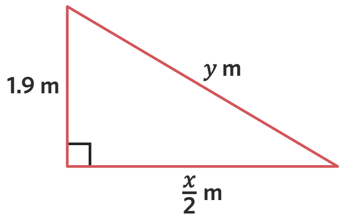
$a^2 + b^2 = c^2$

$x = \sqrt{8^2 + 5^2}$

$= 9.4339...$

Step 2: Find the length from the top to the corner.





The base of this triangle is half the length of the hypotenuse that was found in step 1.

$\frac{x}{2} = \frac{9.4339...}{2} = 4.7169...$

$a^2 + b^2 = c^2$

$y = \sqrt{1.9^2 + 4.7169...^2}$

$= 5.0852...$

Answer

5.09 m

9D Questions

Using Pythagoras' theorem in two dimensions

1. Which of the following is an accurate statement about Pythagoras' theorem, $a^2 + b^2 = c^2$?

A. a is the hypotenuse length, while b and c are the two shorter side lengths of the triangle.

B. b is the hypotenuse length, while a and c are the two shorter side lengths of the triangle.

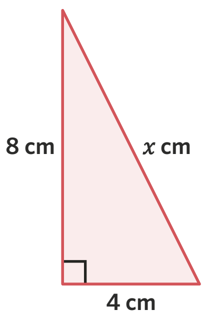
C. c is the hypotenuse length, while a and b are the two shorter side lengths of the triangle.

D. It does not matter which pronumeral represents each side length.

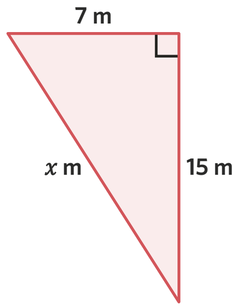
---

2. Determine the length of the hypotenuse for the following triangles. Round to two decimal places.

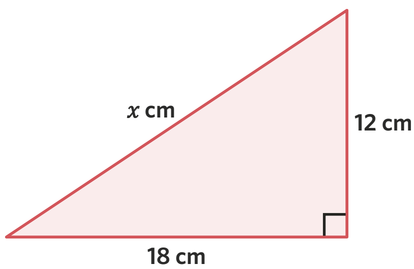
a.



b.

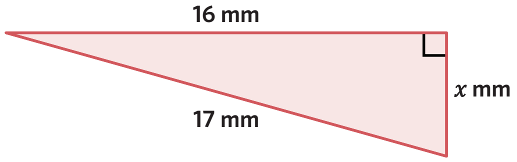


c.

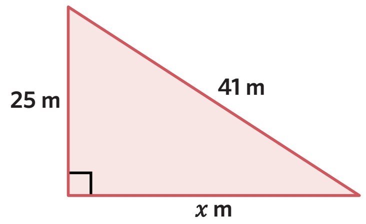


3. Determine the length of the unknown side for the following triangles. Round to two decimal places.

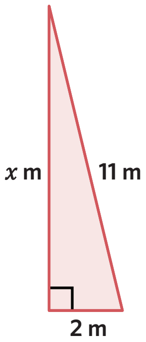
a.



b.



c.

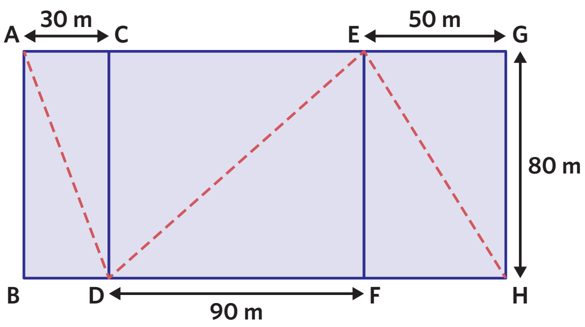


4. Determine the unknown side length for the following.

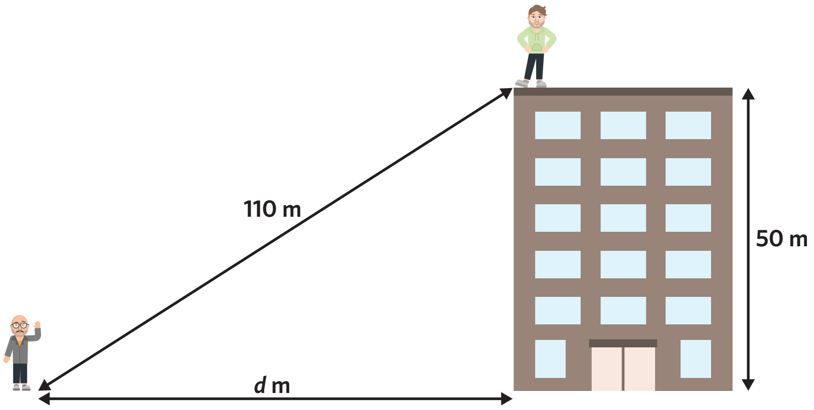
a. A right-angled triangle has a height of 7 cm and a width of 10 cm. What is the length of its hypotenuse in centimetres, rounded to two decimal places?

b. A right-angled triangle has a height of 9 millimetres and a hypotenuse with a length of 17 millimetres. What is the width of the triangle in millimetres, rounded to two decimal places?

5. Lucy started at point A and walked to point D, then to point E and finished at point H, as shown in the diagram. How far did Lucy walk in total, rounded to the nearest metre?

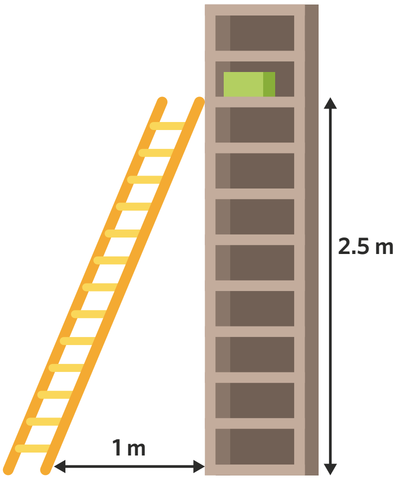


6. Adam is on top of a 50-metre-tall building and can see his friend, James, on the ground. He is at a distance of 110 metres from James. What is the distance between James and the base of the building, d, rounded to two decimal places?



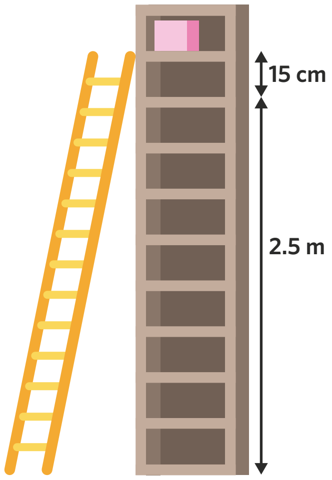
7. Jim is doing an orienteering course. He walks three kilometres north from the starting point. He then turns right and walks six kilometres east. How many kilometres will he have to walk to get directly back to the start, rounded to one decimal place?

8. Allen needs to get something from the second highest shelf in his garage, so he gets out his ladder. The shelf is 2.5 m high and he positions the ladder 1 m from the base of the shelf.



a. If the ladder reaches exactly to the second highest shelf, calculate the length of the ladder in metres, rounded to two decimal places.

b. Allen now needs to put something back on the very top shelf, so he moves the base of the ladder closer to the shelf. The top shelf is 15 centimetres higher than the second highest shelf. Using the rounded answer from part **a** for the length of the ladder, calculate the new distance between the bottom of the ladder and the base of the shelf, in metres, rounded to two decimal places.

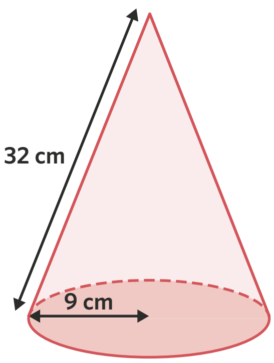


Using Pythagoras' theorem in three dimensions

9. Consider the following cone.

The vertical height of the cone is closest to

A. 29 cm
B. 30 cm
C. 31 cm
D. 32 cm



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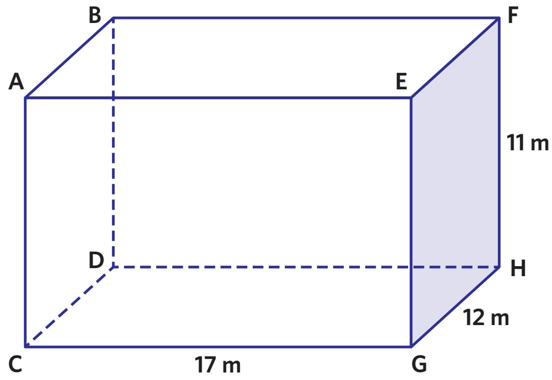
9D QUESTIONS

10. In the rectangular prism shown, determine the following, rounded to two decimal places:

a. the length of the diagonal from C to H.

b. the length of the diagonal from C to F.

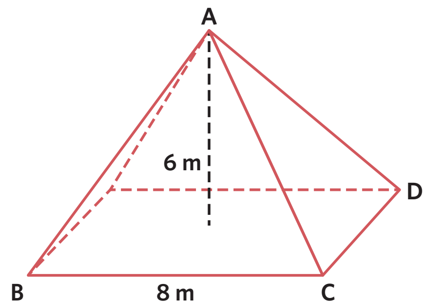


11. A square-based pyramid has a width of 8 metres and a height of 6 metres.

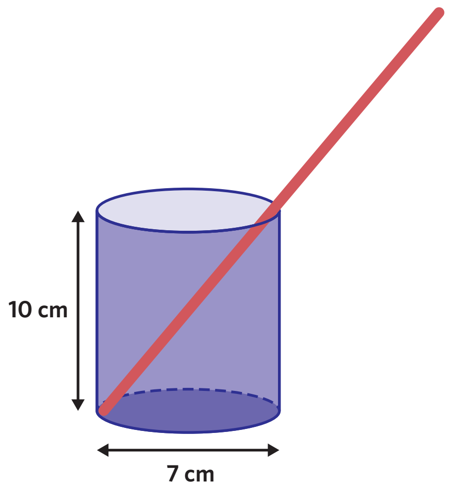
Determine:

a. the length of the diagonal from B to D, rounded to two decimal places.

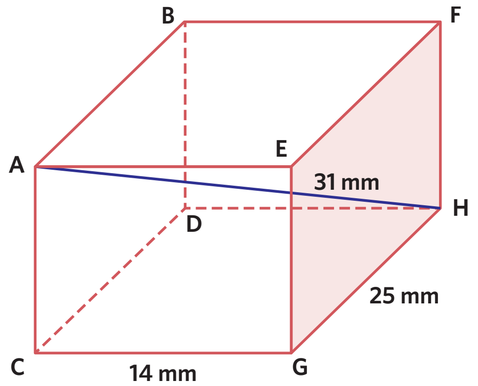
b. the length of the edge AB, rounded to two decimal places.



12. A straw is placed diagonally in a cylindrical cup so that exactly half of the straw is inside the cup. The cup has a height of 10 centimetres and a width of 7 centimetres. What is the length of the straw, rounded to one decimal place?



13. The length of the diagonal between A and H in the following rectangular prism is 31 millimetres. What is the height of the prism, rounded to two decimal places?



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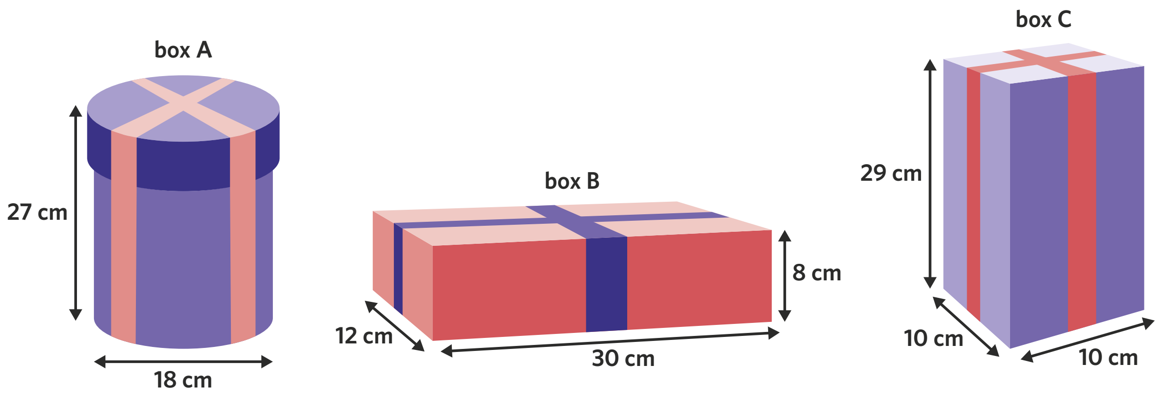
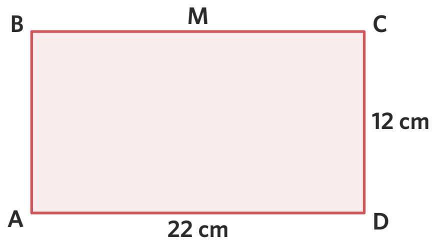
14. Alex knows that his friend Sammy loves Harry Potter, so he gets Sammy a replica wand for his birthday. The wand is 33 centimetres long. Alex is looking for a gift box that the wand will fit inside. Which of the following boxes is big enough?



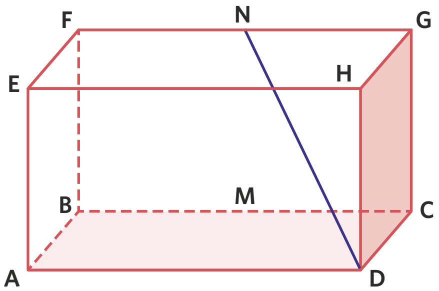
Joining it all together

15. Consider the following rectangle.

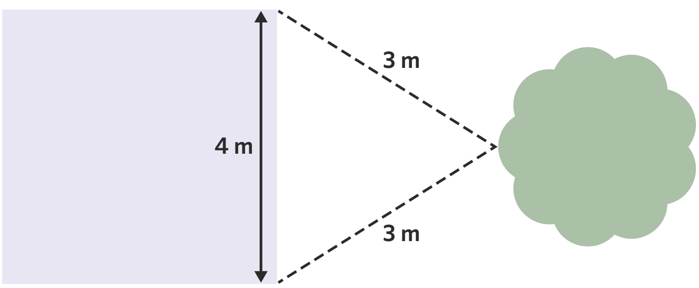


a. M is the midpoint between points B and C. Determine the distance between D and M, rounded to two decimal places.

b. The rectangular prism shown features the rectangle from part **a** as the base, and has a height of 10 cm. N is the midpoint between points F and G. What is the length of the line from D to N, rounded to one decimal place?



16. A tree stands next to a garden shed, as shown in the following bird's-eye view diagram. The base of the tree is three metres away from two corners of the shed, and the shed is four metres wide.



a. Calculate the shortest distance between the tree and the shed in metres, rounded to two decimal places.

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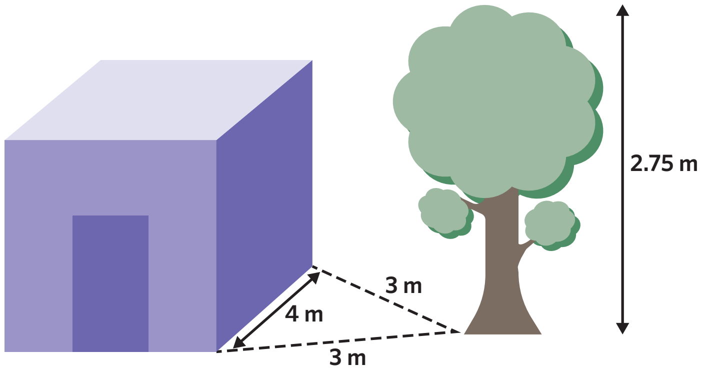
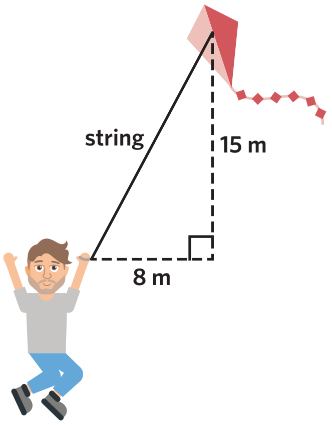
b. A side view of the tree and shed is provided. The tree fell down due to strong wind, hitting the wall of the shed. If the tree is 2.75 m tall, what is the maximum height above the ground at which the tree could have hit the wall? Round to one decimal place.



Exam practice

17. Henry flies a kite attached to a long string, as shown in the diagram.



The horizontal distance of the kite to Henry's hand is 8 m.

The vertical distance of the kite above Henry's hand is 15 m.

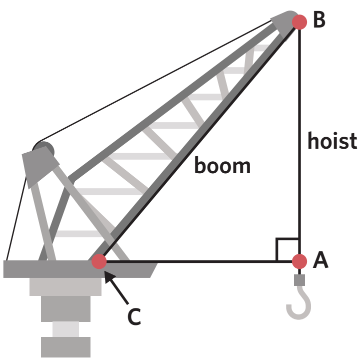
The length of the string, in metres, is

A. 13 **B.** 17 **C.** 23
D. 161 **E.** 289

VCAA 2018 Exam 1 Geometry and measurement Q1

92% of students answered this question correctly.

18. The following diagram shows a crane that is used to transfer shipping containers between the port and the cargo ship.



The length of the boom, BC, is 25 m. The length of the hoist, AB, is 15 m.

Write a calculation to show that the distance AC is 20 m. (1 MARK)

VCAA 2019 Exam 2 Geometry and measurement Q3ai

60% of students answered this question correctly.

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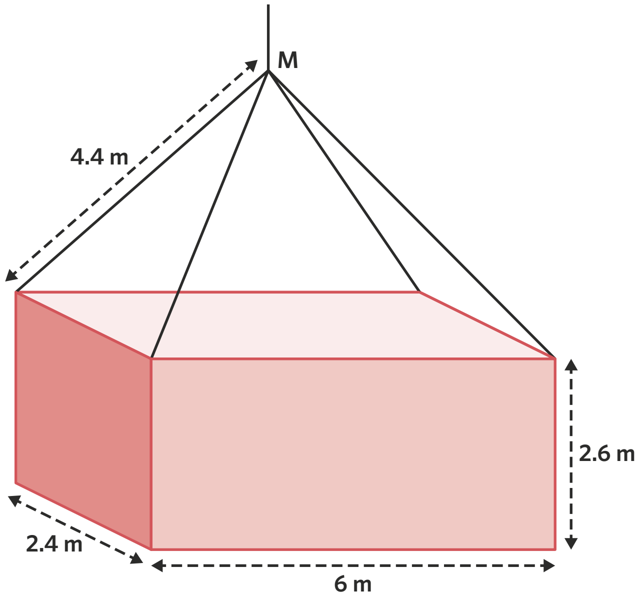
19. A shipping container is a rectangular prism.

Four chains connect a shipping container to a hoist at point M, as shown in the following diagram.



The shipping container has a height of 2.6 m, a width of 2.4 m and a length of 6 m.
Each chain on the hoist is 4.4 m in length.

What is the vertical distance, in metres, between point M and the top of the shipping container? Round to the nearest metre. (2 MARKS)

VCAA 2019 Exam 2 Geometry and measurement Q3c

The average mark on this question was **1**.

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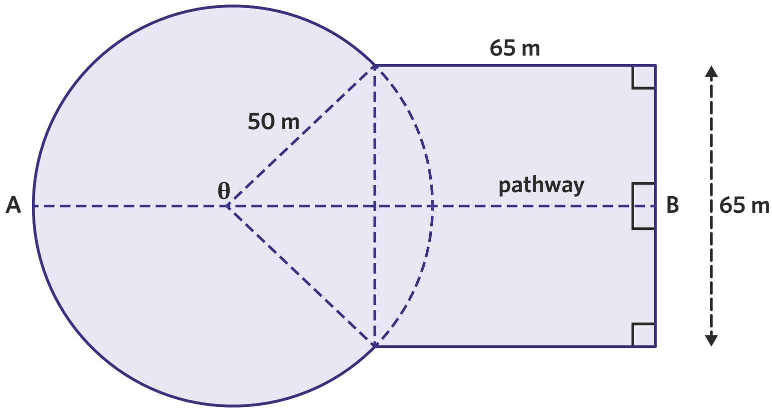
20. A hostel's management is planning to build a pathway from point A to point B, as shown on the diagram.



Calculate the length, in metres, of the planned pathway.

Round to the nearest metre. (2 MARKS)

VCAA 2017 Exam 2 Geometry and measurement Q3c

The average mark on this question was **0.7**.

Questions from multiple lessons

Geometry and measurement *Year 10 content*

21. A cartographer is sketching a map. The scale used for the map is 1 : 100 000.

On the map, a distance of 10 km will be represented by

A. 0.1 cm **B.** 1 cm **C.** 10 cm **D.** 100 cm **E.** 1000 cm

Adapted from VCAA 2013 Exam 1 Geometry and trigonometry Q5

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Data analysis *Year 10 content*

22. The sum of the ages of a family of twelve people is 312 years.

The mean age of members in this family is

A. 15.5 **B.** 16.5 **C.** 25 **D.** 26 **E.** 27

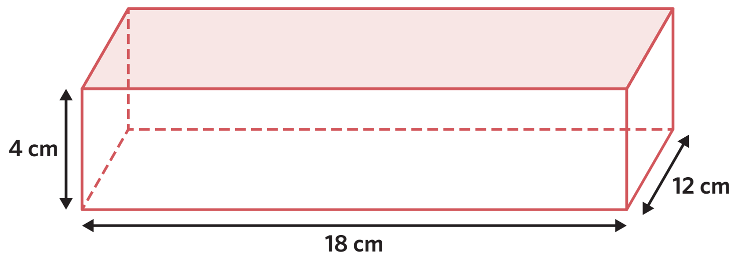
Adapted from VCAA 2019NH Exam 1 Data analysis Q3

Geometry and measurement *Year 10 content*

23. A new eyeshadow palette is currently in production.

The palette is in the shape of a rectangular prism.

The following diagram shows the dimensions of the palette.



The lid of the palette is shaded.

a. What is the area of the top of the lid? (1 MARK)

b. What is the total surface area of the palette? (1 MARK)

Adapted from VCAA 2017 Exam 2 Geometry and measurement Q1a

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9E Perimeter

STUDY DESIGN DOT POINT

- perimeter and areas of triangles, quadrilaterals, circles including arcs and sectors and composite shapes, and practical applications

9A 9B 9C 9D 9E 9F 9G 9H 9I

KEY SKILLS

During this lesson, you will be:

- calculating the perimeter of polygons
- calculating the perimeter of circles
- calculating the perimeter of composite shapes.

KEY TERMS

- Perimeter
- Polygon
- Circumference
- Radius
- Diameter
- Arc
- Composite shape

Sometimes it is useful to measure the total length around one or more shapes. This could be used when deciding how many metres of Christmas lights should be purchased, how much fencing is needed to put up around the yard, or how much electrical wiring is needed in a building.

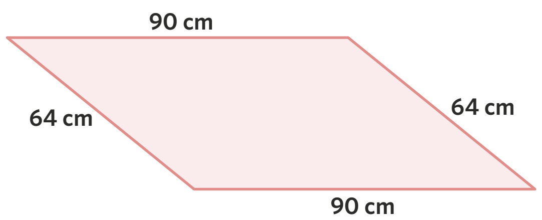
Calculating the perimeter of polygons

The **perimeter** is the total distance around a two dimensional shape. For any **polygon**, a shape consisting of three or more edges, the perimeter is calculated by summing the lengths of all of its sides.

Worked example 1

Calculate the perimeter of the following shapes, rounded to two decimal places where necessary.

a.



Explanation

Sum the side lengths.

$P = 90 + 90 + 64 + 64$

$= 308$ cm

Answer

308 cm

Continues →

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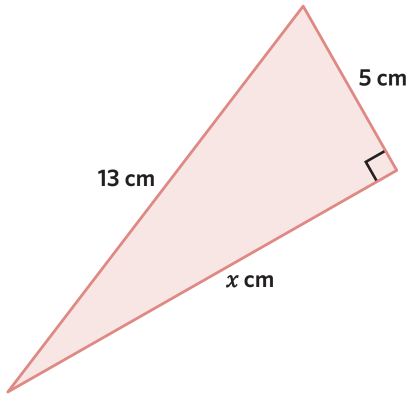
b.



Explanation

Step 1: Use Pythagoras' theorem to calculate the unknown side, x.

$$x^2 + 5^2 = 13^2$$
$$x^2 = 13^2 - 5^2$$
$$= 144$$
$$x = \sqrt{144}$$
$$= 12 \text{ m}$$

Step 2: Calculate the perimeter.

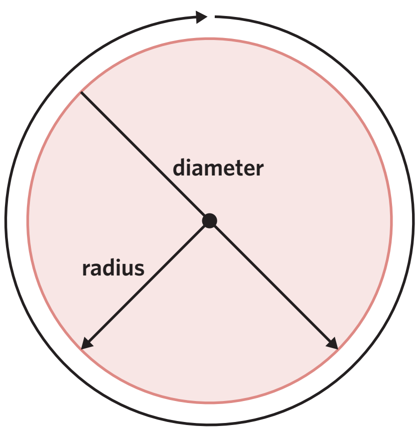
$$P = 5 + 13 + 12$$
$$= 30 \text{ m}$$

Answer

30 m

Calculating the perimeter of circles

The perimeter of a circle is known as the **circumference**, and can be calculated using the radius or diameter of the circle.



The **radius** of a circle, denoted r, is the distance from the edge of a circle to its centre.

The **diameter** of a circle, denoted d, the distance from one side of a circle to the other, passing through its centre. The length of the diameter is twice that of the radius.

$d = 2 \times r$

If the radius is known, the circumference can be calculated as

See worked example 2

$C = 2\pi r$

where r is the radius of the circle.

If the diameter is known, the circumference can be calculated as

$C = \pi d$

where d is the diameter of the circle.

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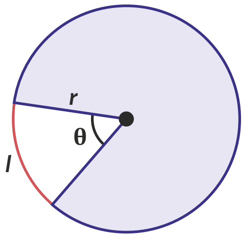
The **arc** of a circle is a portion of its circumference. The length of an arc is calculated as

See worked example 3

$l = 2\pi r \times \dfrac{\theta}{360°}$

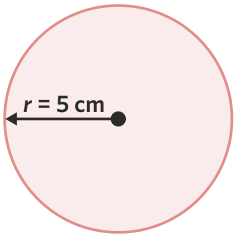
where r is the radius and θ is the central angle of the arc, in degrees.



Worked example 2

Calculate the circumference of the following circles, rounded to two decimal places.

a.



Explanation

Step 1: Identify the radius or diameter.

$r = 5$ cm

Step 2: Calculate the circumference.

Since the radius is known, the formula $P = 2\pi r$ is most appropriate.

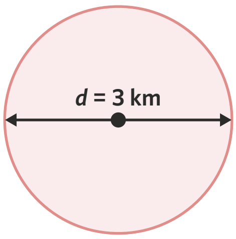
$P = 2 \times \pi \times 5$

$= 31.4159...$ cm

Answer

31.42 cm

b.



Explanation

Step 1: Identify the radius or diameter.

$d = 3$ km

Step 2: Calculate the circumference.

Since the diameter is known, the formula $P = \pi d$ is most appropriate.

$P = \pi \times 3$

$= 9.4247...$ km

Answer

9.42 km

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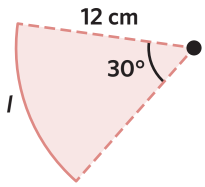
Worked example 3

Calculate the length of the following arcs, rounded to four significant figures.

a.



Explanation

Step 1: Determine the radius and central angle.

$r = 12$ cm

$\theta = 30°$

Step 2: Calculate the arc length.

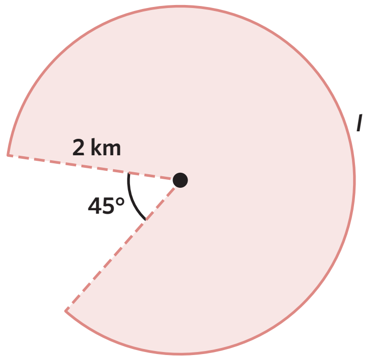
$l = 2 \times \pi \times 12 \times \frac{30°}{360°}$

$= 6.2831...$ cm

Answer

6.283 cm

b.



Explanation

Step 1: Determine the radius and central angle.

$r = 2$ km

$\theta = 360° - 45°$

$= 315°$

Step 2: Calculate the arc length.

$l = 2 \times \pi \times 2 \times \frac{315°}{360°}$

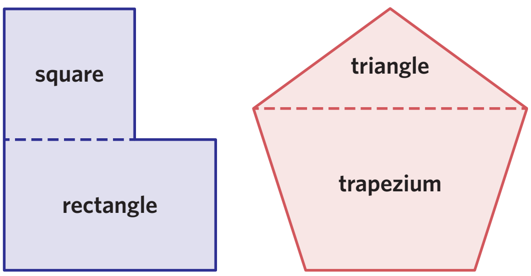
$= 10.9955...$ km

Answer

11.00 km

Calculating the perimeter of composite shapes

A **composite shape** consists of two or more basic shapes.



The perimeter of a composite shape can be found by summing the lengths of all its outside edges.

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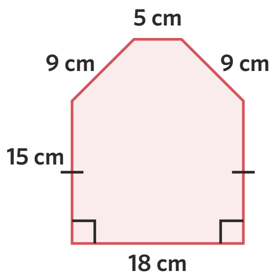
Worked example 4

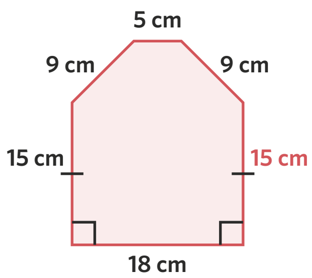
Calculate the perimeter of the following shape.



Explanation

Step 1: Determine the length of all sides.

There is one side length missing.



Step 2: Calculate the perimeter.

$P = 15 + 9 + 5 + 9 + 15 + 18$

$= 71$ cm

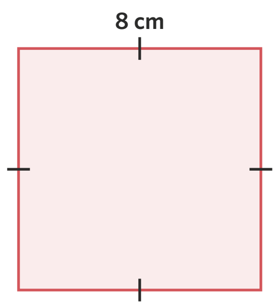
Answer

71 cm

9E Questions

Calculating the perimeter of polygons

1. Consider the following square.



Its perimeter is

A. 8 cm

B. 16 cm

C. 32 cm

D. 64 cm

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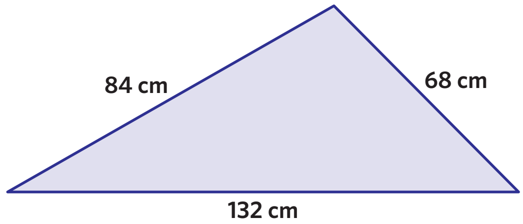
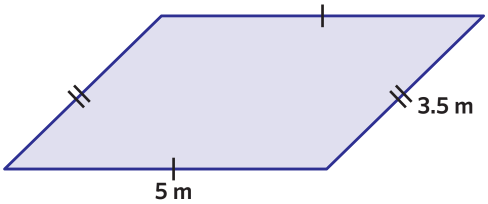
2. Calculate the perimeter of the following shapes. Round to the nearest whole number where necessary.

a.

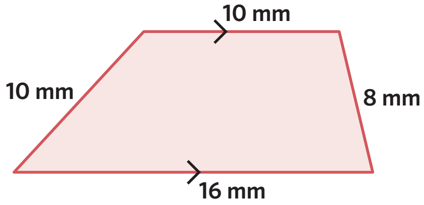
b.



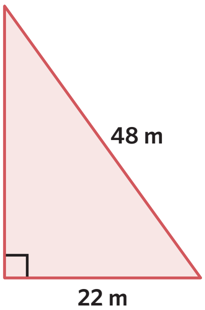
c.



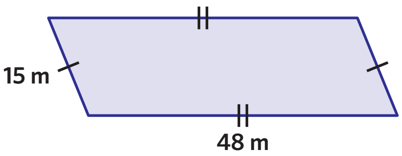
d.



e.

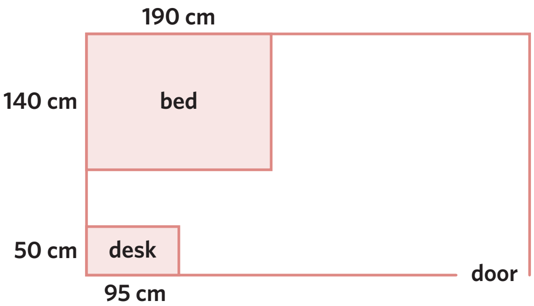


3. Strider wishes to construct a fence around the outside of his property to keep the foxes away from his chickens. The following diagram shows the shape and measurements of his property.



Each metre of fencing costs $70. How much will the fencing cost?

4. Monica is decorating her room. She wants to put up some string lights around her desk and bed. The following is a floor plan of Monica's room.



Monica wants enough string lights to wrap around all sides of her desk and the two sides of the bed that are away from the wall. The string lights cost $12.50 per metre. How much will Monica spend on the string lights?

5. Resh is training for his own version of the Tour de France. He cycles around the local park, which is in the shape of a rectangle. The distance is 7.6 km to go around the park four times, and the horizontal length of the park is 500 m. Calculate the vertical length of the park, in metres.

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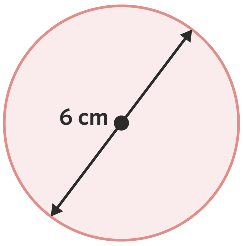
Calculating the perimeter of circles

6. Fill in the blanks. Round to two decimal places where necessary.

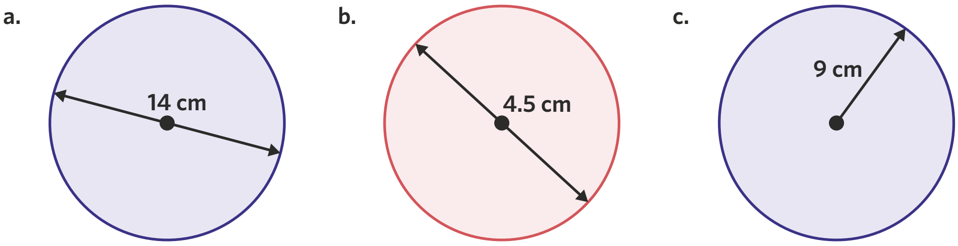


$P = 2 \times \pi \times r$

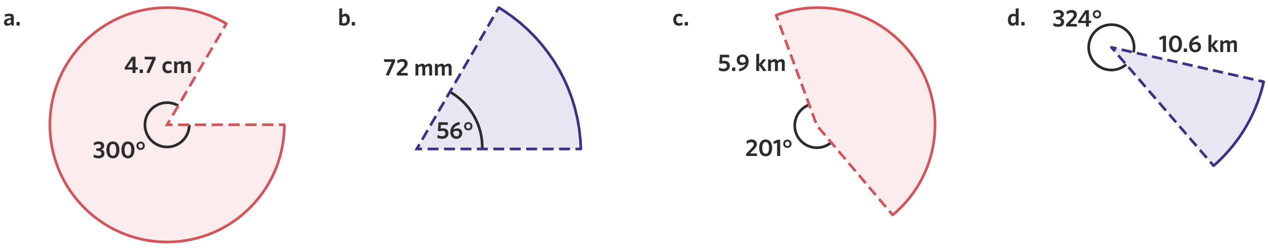
$= 2 \times \pi \times$ ___

$=$ ___ cm

7. Calculate the circumference of the following circles, rounded to three decimal places.



8. Calculate the length of the following arcs, rounded to four significant figures.

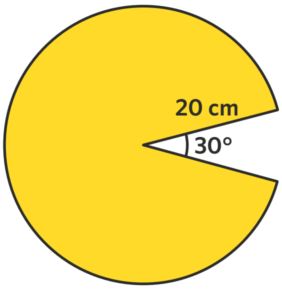


9. A 2016 Mustang has wheels with a diameter of 48.26 cm.

a. What is the radius, in cm, of the wheel?

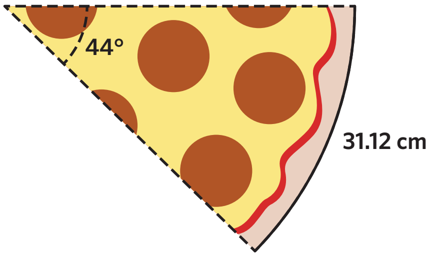
b. For each complete turn of the wheel, how far does the Mustang move forward? Round to two decimal places.

10. For her school art project, Nora is required to make a simple Pac-Man model. She wants to line the entire border with black pipe cleaner.



How much pipe cleaner does she need? Round to two decimal places.

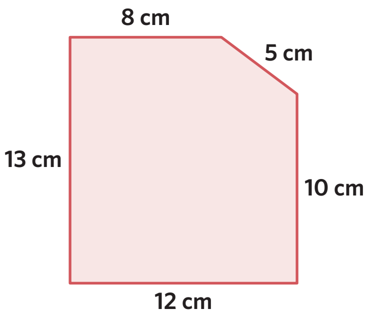
11. Judy is craving some New York pizza. She orders a single slice. The length of the crust is 31.12 cm.



What is the diameter, in cm, of their full pizza? Round to two decimal places.

Calculating the perimeter of composite shapes

12. Fill in the missing numbers.



$P = 8 + \boxed{} + 10 + 12 + 13$

$= \boxed{}$ cm

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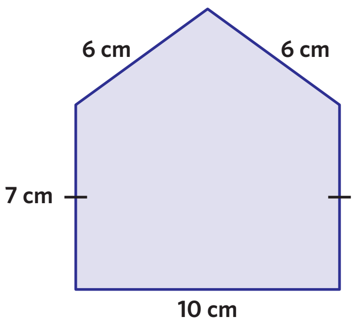
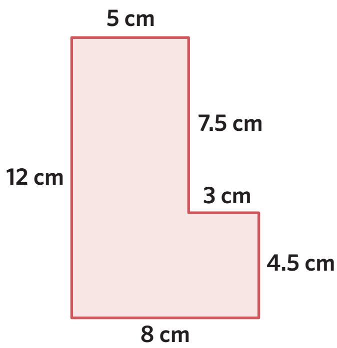
13. Calculate the perimeter of the following composite shapes. Round to four significant figures where necessary.

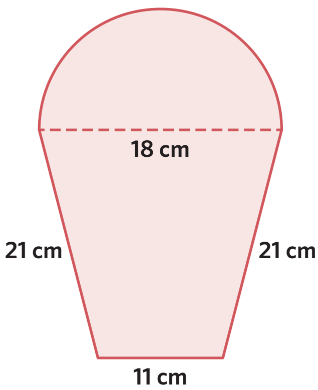
a.



b.



c.



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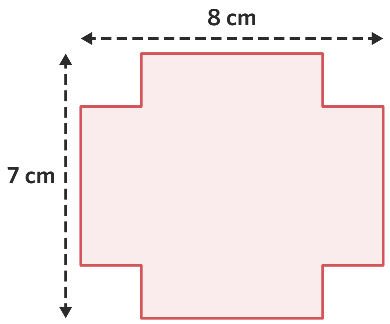
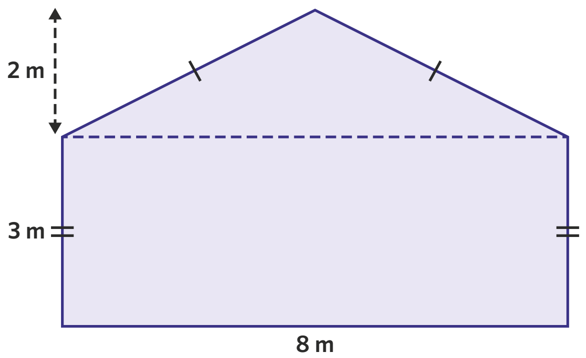
14. Calculate the perimeter of the following shape.

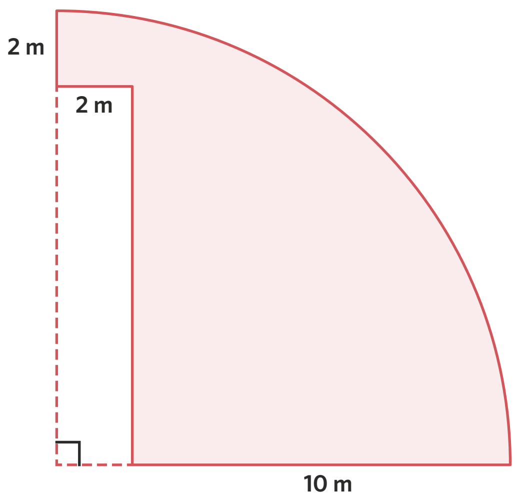


15. The shape of the front of a house is shown. Find the perimeter, rounded to two decimal places.



Joining it all together

16. A swimming pool has opened up a new jacuzzi section. The shape of the jacuzzi is a quadrant, a quarter of a circle, with a rectangle carved out.



a. What is the radius of the quadrant?

b. What is the total perimeter of the jacuzzi? Round to three decimal places.

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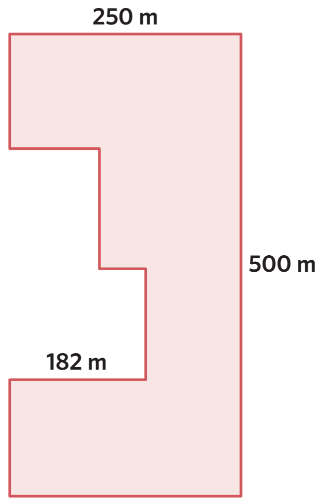
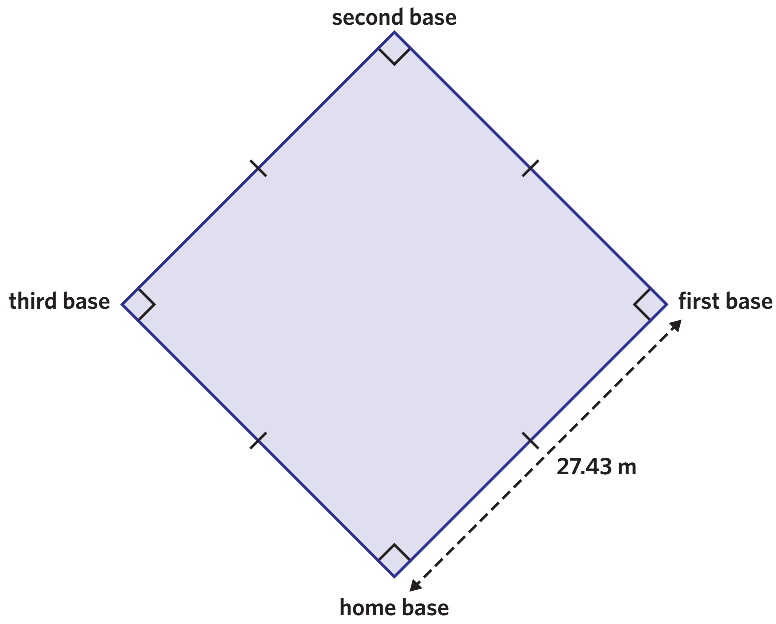
17. A truck driver delivers Amazon packages to the same suburb every day. In the week leading up to, and including, Black Friday, he repeats the same route 3 times a day. The following diagram shows the truck driver's route. Assume that all roads intersect at a right angle.



Assuming that he only delivers to this suburb on the 5 business days, what is the total distance, in kilometres, that he travels in this week?

Exam practice

18. The four bases of a baseball field form four corners of a square with side length 27.43 m, as shown in the following diagram.



A player ran from home base to first base, then to second base, then to third base and finally back to home base.

The minimum distance, in metres, that the player ran is

A. 27.43

B. 54.86

C. 82.29

D. 109.72

E. 164.58

VCAA 2019 Exam 1 Geometry and measurement Q1

91% of students answered this question correctly.

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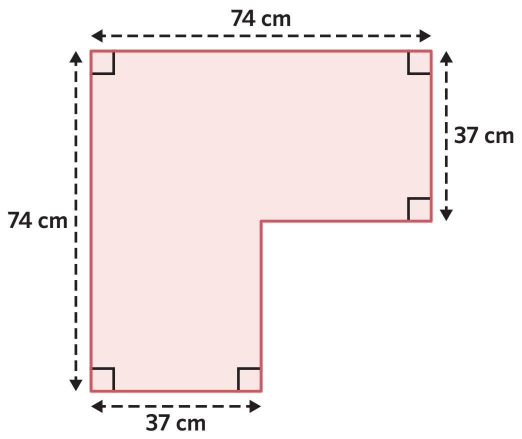
19. The following diagram shows the dimensions of a shelf that will display containers.



What is the perimeter of the shelf, in centimetres? (1 MARK)

VCAA 2020 Exam 2 Geometry and measurement Q1d

76% of students answered this question correctly.

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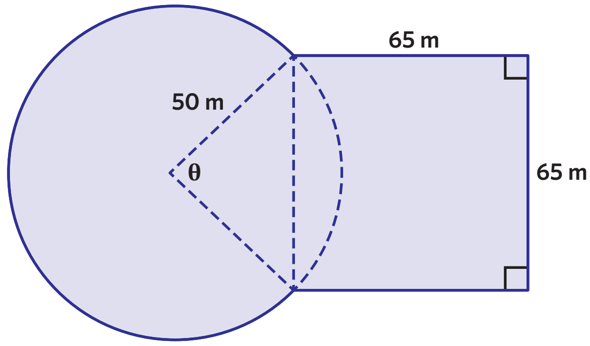
20. Miki will travel by train from Tokyo to Nemuro and she will stay in a hostel when she arrives.

The hostel buildings are arranged around a grassed area.

The grassed area is shown in the following diagram.



The grassed area is made up of a square overlapping a circle.

The square has side lengths of 65 m.

The circle has a radius of 50 m.

The angle, θ, is 81°.

What is the perimeter, in metres, of the entire grassed area?
Round to the nearest metre. (1 MARK)

VCAA 2017 Exam 2 Geometry and measurement Q3b

34% of students answered this question correctly.

Questions from multiple lessons

Geometry and measurement

21. The following graphic shows a juice box that is 6 cm long, 4 cm wide and 10 cm high.

A straw sits on the inside of the box. One end of the straw sits at A and the other sits at K.

The point A lies on the line FG at a distance of 2 cm from G.

The length of the straw, in centimetres, is closest to

A. 10.77 cm

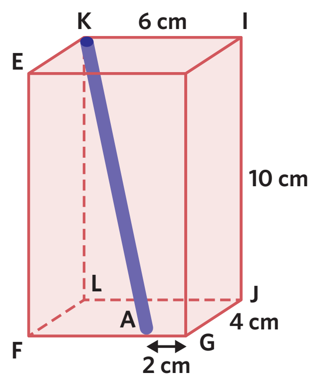
B. 10.95 cm

C. 11.49 cm

D. 12.33 cm

E. 13.42 cm

Adapted from VCAA 2014 Exam 1 Geometry and trigonometry Q5



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Data analysis

22. Zoe measured the *height*, in centimetres, and *weight*, in kilograms, of nine koalas she saved during the 2019–20 bushfires. They are displayed in the following table.

height **(cm)**	***weight*** **(kg)**
78	4.2
92	9.9
80	7.2
81	5.4
72	4.8
75	5.5
78	5.0
84	8.2
74	7.6

The mean, $\overline{x}$, and the standard deviation, s_x, of the *weight* of these koalas, in kilograms, are closest to

A. $\overline{x} = 6.28 \quad s_x = 1.79$

B. $\overline{x} = 6.28 \quad s_x = 1.90$

C. $\overline{x} = 6.42 \quad s_x = 1.90$

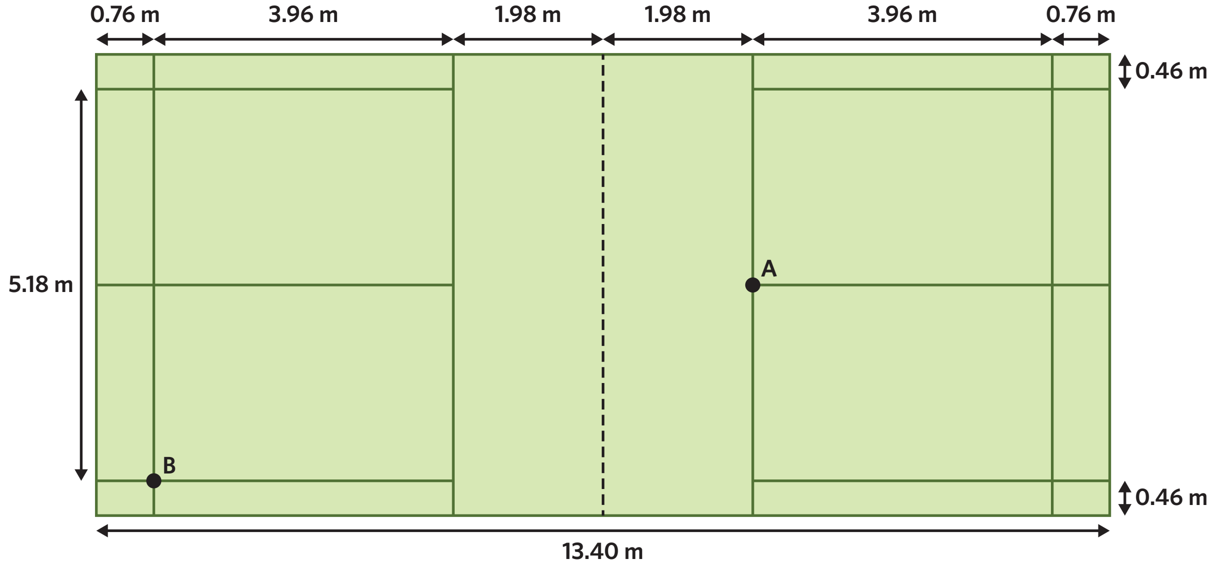
D. $\overline{x} = 6.42 \quad s_x = 1.79$

E. $\overline{x} = 6.53 \quad s_x = 1.79$

Adapted from VCAA 2018NH Exam 1 Data analysis Q7

Geometry and measurement

23. Boris has a badminton court in his backyard. A diagram of the badminton court is shown. It can be assumed that all intersecting lines meet at right angles.



Boris stands at point A and serves to point B.

a. What is the straight-line distance, in metres, between points A and B? Round to two decimal places. (1 MARK)

b. Boris serves at a height of 2 m directly above point A. The ball travels in a straight line to the ground at point B. What is the straight-line distance, in metres, that the ball travels? Round to two decimal places. (1 MARK)

Adapted from VCAA 2018 Exam 2 Geometry and measurement Q3a,b

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9F Area

STUDY DESIGN DOT POINT

- perimeter and areas of triangles, quadrilaterals, circles including arcs and sectors and composite shapes, and practical applications

9A 9B 9C 9D 9E 9F 9G 9H 9I

KEY SKILLS

During this lesson, you will be:

- calculating the area of quadrilaterals
- calculating the area of circles and sectors
- calculating the area of triangles
- calculating the area of composite shapes.

KEY TERMS

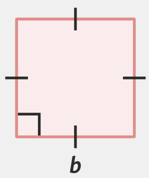
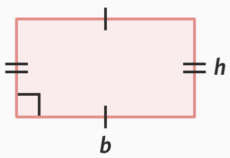
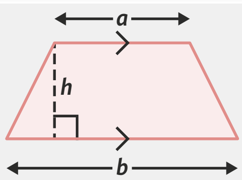
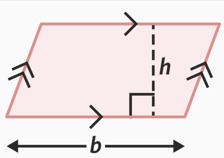
- Sector
- Heron's formula

When working with two-dimensional shapes, it can be useful to calculate the amount of space they take up. There are many real-world applications of this skill, such as determining the amount of land within a property or the amount of carpet required to cover the space within a house.

Calculating the area of quadrilaterals

Recall that area is a measurement of the space taken up by a two-dimensional shape and is measured in square units.

The area of quadrilaterals can be found using the given formulas.

quadrilateral	features	example	area
square	two pairs of parallel sides, all sides of equal length, all at right angles		$A = b^2$
rectangle	two pairs of parallel sides of equal length, all at right angles		$A = b \times h$
trapezium	one pair of parallel sides		$A = \frac{(a + b)}{2} \times h$
parallelogram	two pairs of parallel sides of equal length		$A = b \times h$

Continues →

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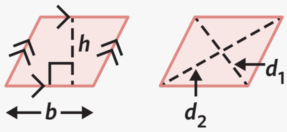
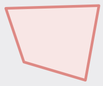
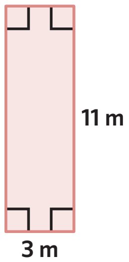
quadrilateral	features	example	area
rhombus	two pairs of parallel sides, all sides of equal length		$A = b \times h$ or $A = \dfrac{d_1 \times d_2}{2}$
irregular	no parallel sides		none

Worked example 1

Calculate the area of the following quadrilaterals.

a.



Explanation

Step 1: Identify the appropriate formula.

The given shape has two pairs of parallel sides, all at right angles. It is a rectangle.

$A = b \times h$

Step 2: Identify the required dimensions.

$b = 3$ m

$h = 11$ m

Step 3: Substitute the known values into the formula.

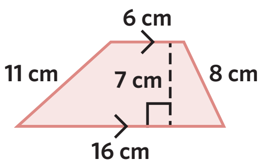
$A = 3 \times 11$

$= 33 \text{ m}^2$

Answer

33 m^2

b.



Explanation

Step 1: Identify the appropriate formula.

The given shape has one pair of parallel sides. It is a trapezium.

$$A = \frac{(a + b)}{2} \times h$$

Step 2: Identify the required dimensions.

$a = 6$ cm

$b = 16$ cm

$h = 7$ cm

Step 3: Substitute the known values into the formula.

$$A = \frac{(6 + 16)}{2} \times 7$$

$= 11 \times 7$

$= 77 \text{ cm}^2$

Answer

77 cm^2

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Calculating the area of circles and sectors

The area of a circle can be found using the formula

See worked example 2

$A = \pi \times r^2$

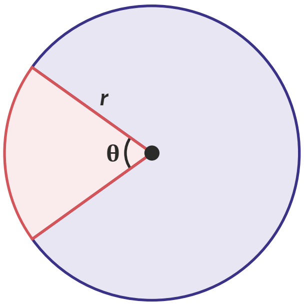
where r is the radius of the circle.

A **sector** is a portion of a circle, and its area can be found using the formula

See worked example 3

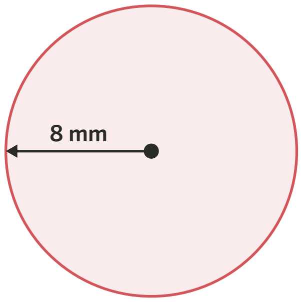
$A = \frac{\theta}{360°} \times \pi \times r^2$

where r is the radius of the circle and θ is the central angle of the sector, in degrees.



Worked example 2

Calculate the area of the following circle, rounded to two decimal places.



Explanation

Step 1: Identify the radius.

$r = 8$ mm

Step 2: Substitute the known value into the formula.

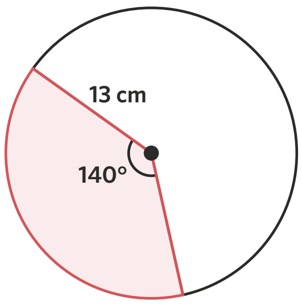
$$
\begin{aligned}
A &= \pi \times r^2 \\
&= \pi \times 8^2 \\
&= 201.061\ldots \text{ mm}^2
\end{aligned}
$$

Answer

201.06 mm^2

Worked example 3

Calculate the area of the shaded sector, rounded to two decimal places.



Continues →

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Explanation

Step 1: Identify the radius and central angle.

$r = 13$ cm

$\theta = 140°$

Step 2: Substitute the known values into the formula.

$$A = \frac{\theta}{360°} \times \pi \times r^2$$
$$= \frac{140°}{360°} \times \pi \times 13^2$$
$$= 206.472... \text{ cm}^2$$

Answer

206.47 cm^2

Calculating the area of triangles

The area of a triangle can be found in two ways.

If the height of the triangle and the perpendicular side length (known as the base) are known, the area can be found using the formula

See worked example 4

$$A = \frac{1}{2} \times b \times h$$

where b is the base and h is the height.

Alternatively, if all three side lengths are known, the area of a triangle can be found using **Heron's formula**

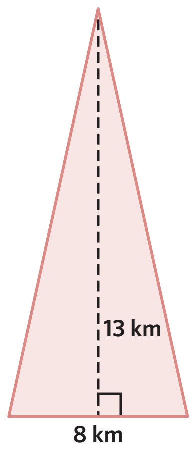
See worked example 5

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$

where $S = \frac{a+b+c}{2}$ and a, b and c are the three side lengths.

Worked example 4

Calculate the area of the following triangle.



Explanation

Step 1: Identify the appropriate formula.

The base and height are given.

$A = \frac{1}{2} \times b \times h$

Step 2: Identify the required dimensions.

$b = 8$ km

$h = 13$ km

Step 3: Substitute the known values into the formula.

$$A = \frac{1}{2} \times 8 \times 13$$
$$= 52 \text{ km}^2$$

Answer

52 km^2

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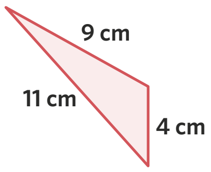
Worked example 5

Calculate the area of the following triangle, rounded to two decimal places.



Explanation

Step 1: Identify the appropriate formula.

The three side lengths are given.

$A = \sqrt{S(S - a)(S - b)(S - c)}$

Step 2: Identify the required dimensions.

$a = 4$ cm

$b = 9$ cm

$c = 11$ cm

Step 3: Calculate S.

$$S = \frac{a + b + c}{2}$$
$$= \frac{4 + 9 + 11}{2}$$
$$= 12$$

Step 4: Substitute the known values into the formula.

$$A = \sqrt{12(12 - 4)(12 - 9)(12 - 11)}$$
$$= \sqrt{288}$$
$$= 16.970\ldots \text{ cm}^2$$

Answer

16.97 cm^2

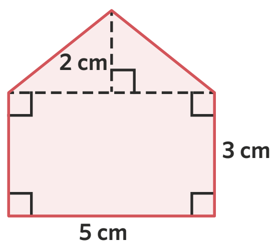
Calculating the area of composite shapes

The area of a composite shape can be found by calculating the area of the individual shapes and adding or subtracting them where necessary.

Worked example 6

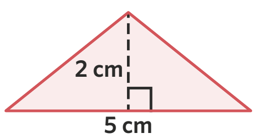
Calculate the area of the following shapes, rounded to two decimal places where necessary.

a.



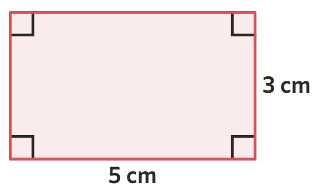
Explanation

Step 1: Calculate the area of the triangle.



$$A = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times 5 \times 2$$
$$= 5 \text{ cm}^2$$

Step 2: Calculate the area of the rectangle.



$$A = b \times h$$
$$= 5 \times 3$$
$$= 15 \text{ cm}^2$$

Continues →

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Step 3: Calculate the total area.

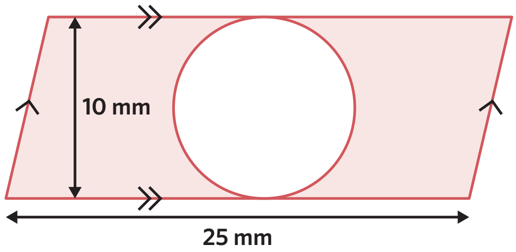
$A = 5 + 15$

$= 20\ \text{cm}^2$

Answer

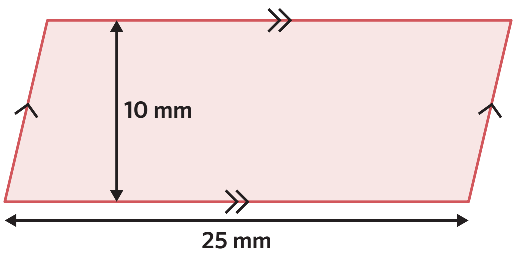
$20\ \text{cm}^2$

b.



Explanation

Step 1: Calculate the area of the parallelogram.

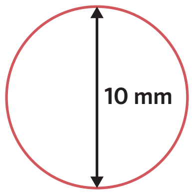


$A = b \times h$

$= 25 \times 10$

$= 250\ \text{mm}^2$

Step 2: Calculate the area of the circle.



$r = \frac{10}{2} = 5\ \text{mm}$

$A = \pi \times r^2$

$= \pi \times 5^2$

$= 78.539...\ \text{mm}^2$

Step 3: Subtract the area of the circle from the area of the parallelogram.

$A = 250 - 78.539...$

$= 171.460...\ \text{mm}^2$

Answer

$171.46\ \text{mm}^2$

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9F Questions

Calculating the area of quadrilaterals

1. A square has an area of 36 cm^2.

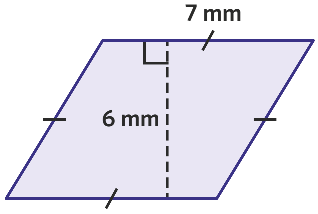
It has side lengths of

A. 3 cm **B.** 6 cm **C.** 8 cm **D.** 12 cm

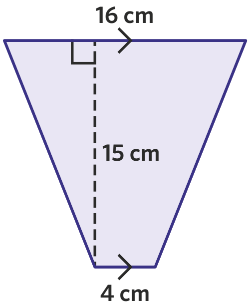
2. Calculate the area of the following quadrilaterals.

a.

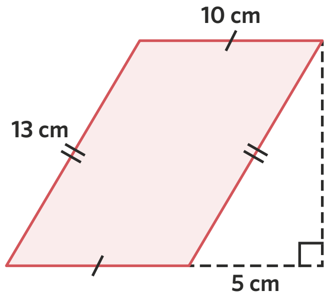
b.



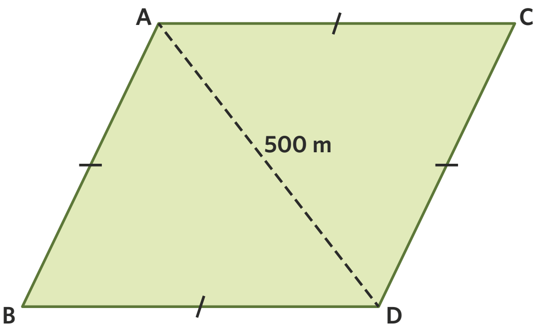
c.



d.



3. One of the paddocks in Alistair's farm is in the shape of a rhombus, and is shown in the following diagram.

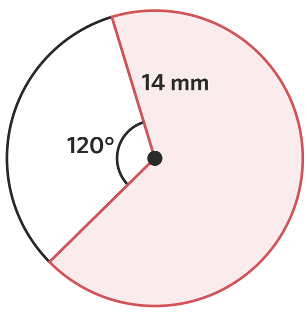


There is a 500 m fence between corners A and D of the paddock. Alistair also knows that the paddock has an area of 157 000 m^2.

Alistair wants to put up a second fence from corner B to corner C. How long will this fence be?

Calculating the area of circles and sectors

4. Consider the following diagram.



The area, in mm^2, of the shaded region can be found using the expression

A. $A = \frac{120°}{360°} \times \pi \times 14^2$

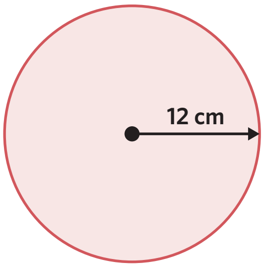
B. $A = \frac{240°}{360°} \times \pi \times 14^2$

C. $A = 120° \times \pi \times 14^2$

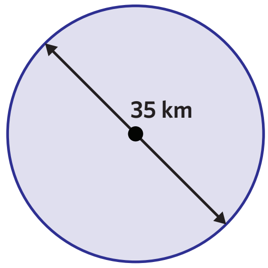
D. $A = 240° \times \pi \times 14^2$

5. Calculate the area of the following circles and sectors, rounded to two decimal places.

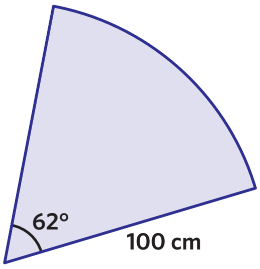
a.



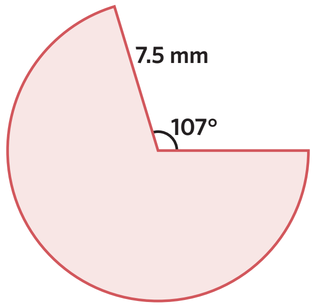
b.



c.



d.



6. A circular pizza is 36 cm wide and is cut into 8 equal slices.

a. What is the area of the entire pizza, rounded to two decimal places?

b. What is the central angle of each pizza slice?

c. What is the area of one pizza slice, rounded to two decimal places?

Calculating the area of triangles

7. The area of the following triangle can be found using Heron's formula, $A = \sqrt{S(S-a)(S-b)(S-c)}$.

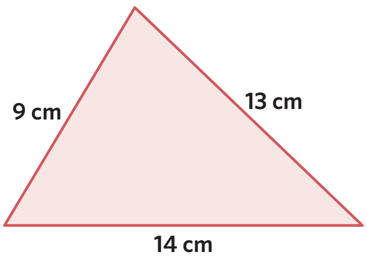
For this triangle, the value of S is

A. 13

B. 14

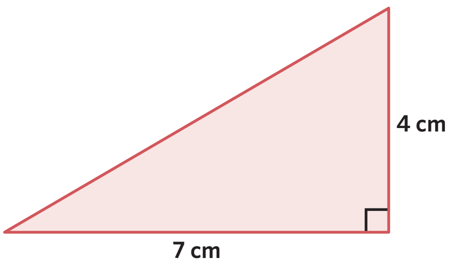
C. 18

D. 22

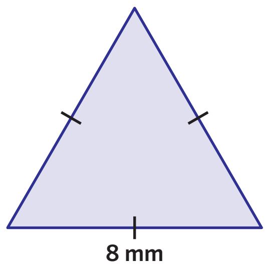


8. Calculate the area of the following triangles, using the most appropriate formula. Round to two decimal places where necessary.

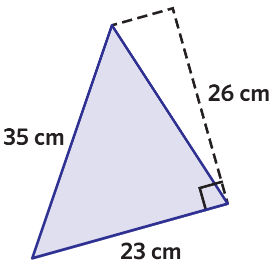
a.



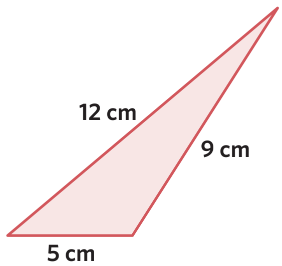
b.



c.



d.



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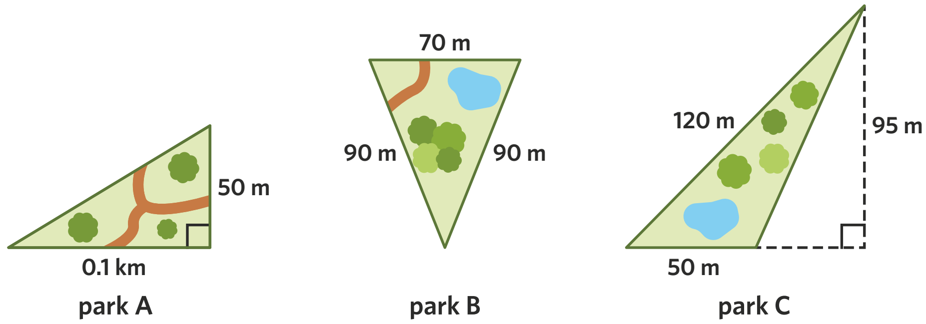
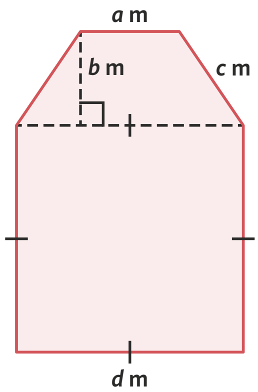
9. Which of the following parks has the largest area?



Calculating the area of composite shapes

10. Consider the following composite shape.



The area of this shape, in m^2, is

A. $ac + bd$

B. $abc + d^2$

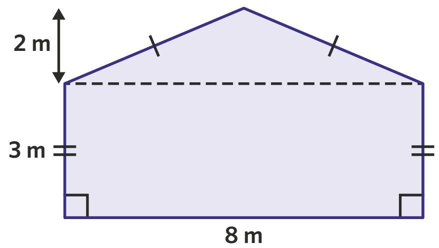
C. $\frac{(a + b)}{2} \times c + d^2$

D. $\frac{(a + d)}{2} \times b + d^2$

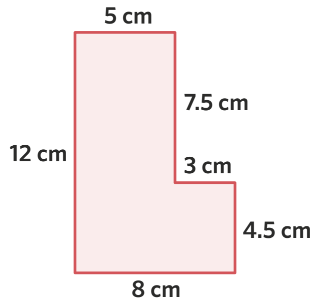
---

11. Calculate the area of the shaded region in each of the following shapes. Round to two decimal places where necessary.

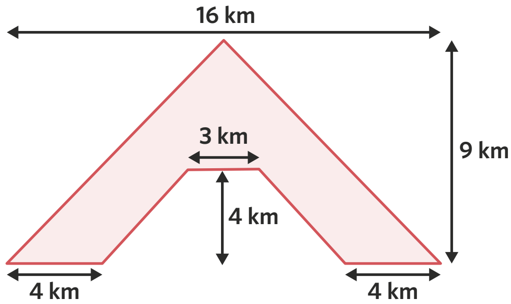
a.



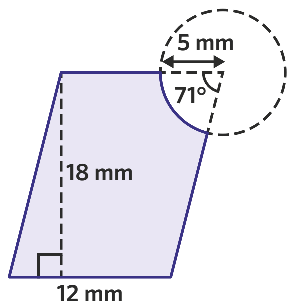
b.



c.



d.



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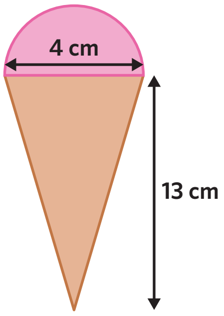
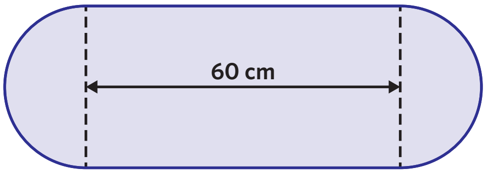
12. Find the area of the following drawing of an ice cream, rounded to the nearest square centimetre, given that the cone is in the shape of an isosceles triangle and the ice cream on top is in the shape of a semicircle.



13. A rectangular piece of paper with side lengths 21 cm and 25 cm has a semicircle of radius 20 mm cut out of it. What is the remaining area of the piece of paper in cm^2, rounded to one decimal place?

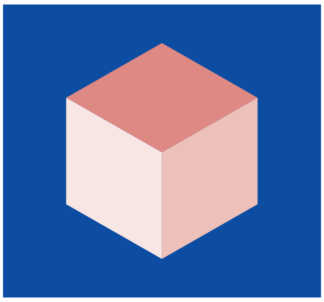
14. Jack recently bought a new skateboard. It consists of a rectangle and two semi-circles. The width of the rectangle is 60 cm and the perimeter of the entire skateboard is 182 cm.



Calculate the area of the skateboard, rounded to the nearest square centimetre.

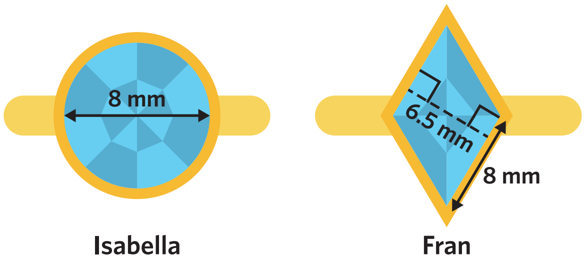
Joining it all together

15. Ellie is opening her psychology practice ‘Think Outside the Box’, and wants to design a logo. She comes up with the following design using quadrilaterals.



The cube in the logo is made up of three rhombuses with a base length of 20 mm and a height of 17.32 mm. If the blue rectangle has a height of 60 mm and a width of 65 mm, what area of the logo is blue?

16. Isabella and Fran recently got engaged. Their engagement rings are shown.



By comparing the area, whose ring has a larger stone, and by how much? Round to two decimal places.

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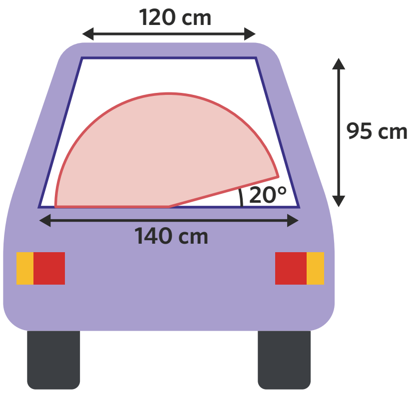
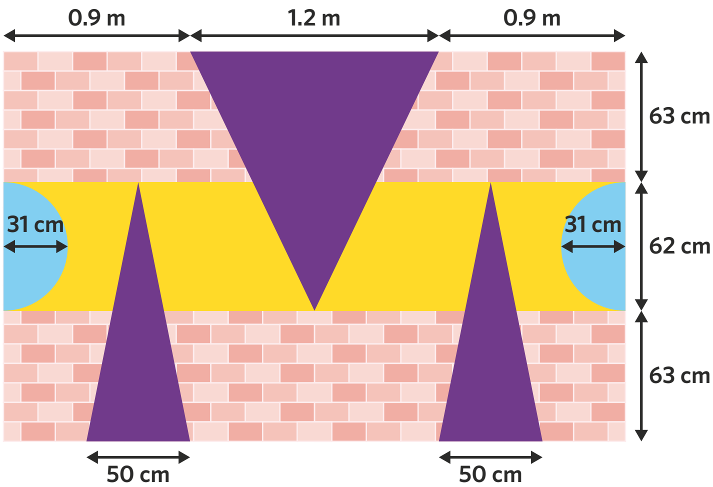
17. Karl-Anthony is frustrated that his car's back windscreen wiper doesn't wipe enough of the windscreen. His windscreen wiper is 60 cm long, and wipes the shaded area in the following diagram.



What percentage of the area of the back windscreen is not wiped by the windscreen wiper? Round to the nearest percent.

18. Otis is painting an abstract mural on the wall of his local community centre.



a. What is the area of the wall, in cm^2, that will be covered in blue paint? Round to two decimal places.

b. What is the area of the wall, in cm^2, that will be covered in purple paint?

c. What is the area of the wall, in m^2, that will be covered in yellow paint? Round to two decimal places.

d. Calculate the total area of the wall, in m^2, that will have paint on it. Round to two decimal places.

Exam practice

19. A flag consists of three different coloured sections: red, white and blue.

The flag is 3 m long and 2 m wide, as shown in the diagram.

The blue section is an isosceles triangle that extends to half the length of the flag.

The area of the blue section, in square metres, is

A. 0.75

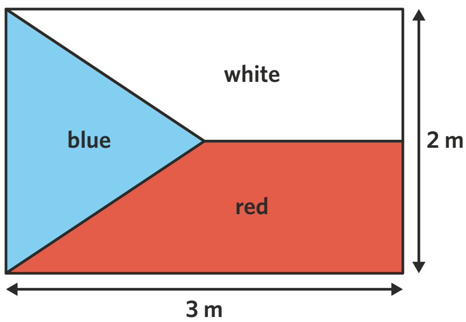
B. 1.5

C. 2

D. 3

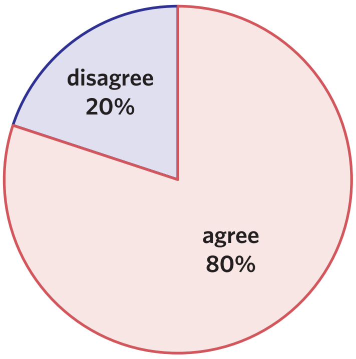
E. 6

VCAA 2020 Exam 1 Geometry and measurement Q2



82% of students answered this question correctly.

20. The following pie chart displays the results of a survey.



Eighty per cent of the people surveyed selected 'agree'.

Twenty per cent of the people surveyed selected 'disagree'.

The radius of the pie chart is 16 mm.

The area of the sector representing 'agree', in square millimetres, is closest to

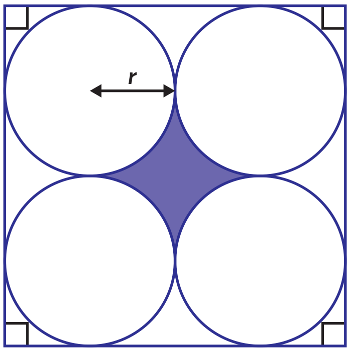
A. 80 **B.** 161 **C.** 483
D. 643 **E.** 804

VCAA 2020 Exam 1 Geometry and measurement Q5

65% of students answered this question correctly.

---

21. Four identical circles of radius r are drawn inside a square, as shown in the following diagram.
The region enclosed by the circles has been shaded in the diagram.



The shaded area can be found using

A. $4r^2 - 2\pi r$ **B.** $4r^2 - \pi r^2$ **C.** $4r - \pi r^2$
D. $2r^2 - \pi r^2$ **E.** $2r - 2\pi r$

VCAA 2019 Exam 1 Geometry and measurement Q8

51% of students answered this question correctly.

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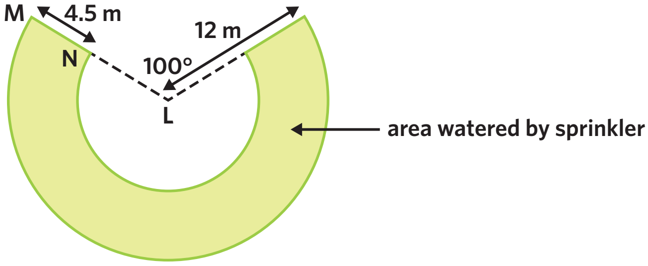
22. A sprinkler can water the shaded section of grass shown in the following diagram.



The section of grass that is watered is 4.5 m wide at all points.

Water can reach a maximum of 12 m from the sprinkler at L.

What is the area of grass that this sprinkler will water?

Round to the nearest square metre. (2 MARKS)

VCAA 2016 Exam 2 Geometry and measurement Q5b

The average mark on this question was **0.5**.

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Questions from multiple lessons

Geometry and measurement

23. In triangle TVW, point X lies on side TW and point U lies on side TV.

The lines XU and WV are parallel.

The length of TU is 8 cm, the length of UV is 4 cm and the length of XU is 10 cm.

The length of WV is

A. 10 cm

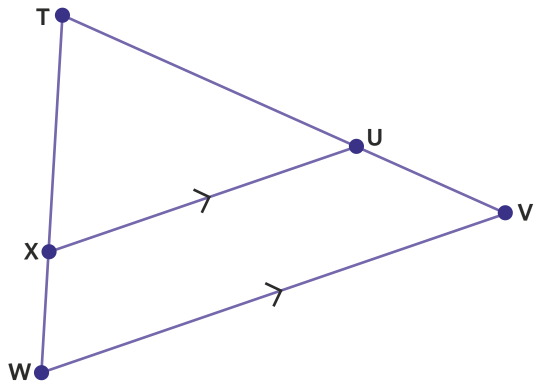
B. 12 cm

C. 15 cm

D. 18 cm

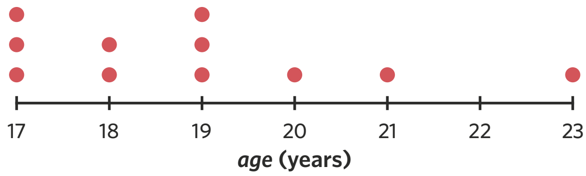
E. 20 cm

Adapted from VCAA 2013 Exam 1 Geometry and trigonometry Q6



Data analysis

24. The following dot plot displays the *age*, in years, of eleven students.



The mean and standard deviation of the *age* of the students are closest to

A. $\bar{x} = 1.4 \quad s_x = 18.5$

B. $\bar{x} = 1.9 \quad s_x = 18.9$

C. $\bar{x} = 18.5 \quad s_x = 1.4$

D. $\bar{x} = 18.9 \quad s_x = 1.8$

E. $\bar{x} = 18.9 \quad s_x = 1.9$

Adapted from VCAA 2008 Exam 1 Data analysis Q5

Graphs and relations

25. A patient is being administered an IV drip.

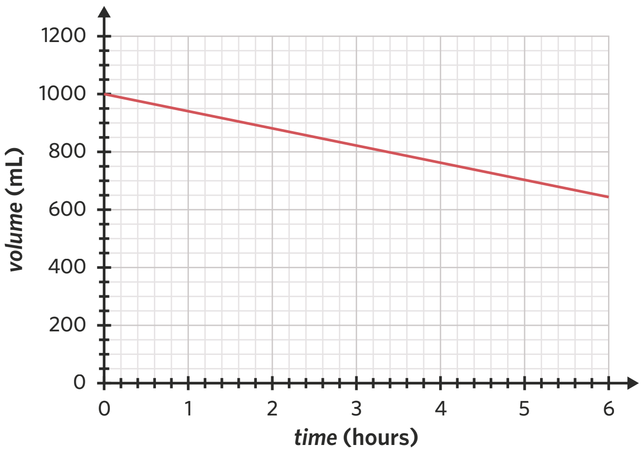
When the bag was first administered, it contained 1000 mL of saline solution.

After six hours, there is 640 mL of saline solution left.

The volume of saline solution remaining in the bag followed a linear trend as shown in the following graph.

a. Determine the equation of the line shown in the graph. (2 MARKS)

b. Assume this linear trend continues. How much longer, in hours and minutes, will it take for the IV drip to be completely administered? (1 MARK)



Adapted from VCAA 2006 Exam 2 Graphs and relations Q2a,b

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9G Volume

STUDY DESIGN DOT POINT

- volumes and surface areas of solids (spheres, cylinders, pyramids and prisms and composite objects) and practical applications, including simple applications of Pythagoras' theorem in three dimensions

9A 9B 9C 9D 9E 9F 9G 9H 9I

KEY SKILLS

During this lesson, you will be:

- calculating the volume of prisms and cylinders
- calculating the volume of tapered solids
- calculating the volume of spheres
- calculating the volume of composite solids.

KEY TERMS

- Prism
- Cylinder
- Tapered solid
- Cone
- Sphere
- Hemisphere
- Composite solid

One of the most important aspects of a three-dimensional object is its potential to store, carry or hold things, which relates to its volume. Sometimes the volume of an object must be measured accurately to ensure that requirements are met, such as the volume of cargo transported from one place to another using a delivery truck.

Calculating the volume of prisms and cylinders

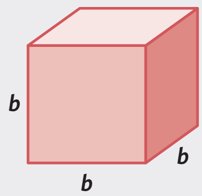
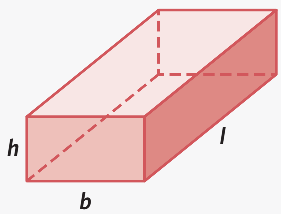
Recall that volume is the amount of three-dimensional space taken up by an object. It is measured in cubic units of length, commonly cubic centimetres (cm^3) or cubic metres (m^3), and is calculated differently depending on the object that is being considered.

One such set of objects are called prisms. A **prism** is a three-dimensional object that has the same cross-section across its entire length. In other words, if the front face of a prism is a triangle, the back face of the prism will be the exact same triangle, connected by straight lines.

A **cylinder** is similar in principle to a prism, but with a curved side. Its front and back face are circles connected by straight lines.

In all instances, the volume of a prism or cylinder can be calculated by first calculating the area of the cross-section and then multiplying by the length of the prism.

Some common formulas are given in the following table.

solid	example	area of cross-section	volume
cube		$A = b^2$	$V = b^3$
rectangular prism		$A = bh$	$V = bhl$

Continues →

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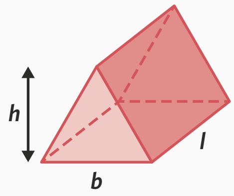
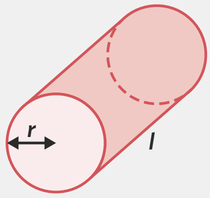
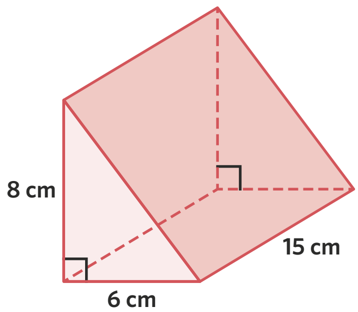
solid	example	area of cross-section	volume
triangular prism		$A = \frac{1}{2}bh$	$V = \frac{1}{2}bhl$
cylinder		$A = \pi r^2$	$V = \pi r^2 l$

Recall that capacity is similar to volume, but instead is the amount of three-dimensional space an object can hold. It is commonly measured in millilitres (mL) and litres (L). 1 cm^3 is equal to 1 mL.

Worked example 1

Consider the following triangular prism.



a. Calculate the volume of the object.

Explanation

Step 1: Identify the required dimensions.

The volume of a triangular prism is calculated using the base and height of the triangular cross-section and the length of the prism.

$b = 6$ cm

$h = 8$ cm

$l = 15$ cm

Step 2: Substitute the values into the appropriate formula and evaluate.

$$V = \frac{1}{2}bhl$$
$$= \frac{1}{2} \times 6 \times 8 \times 15$$
$$= 360 \text{ cm}^3$$

Answer

360 cm^3

b. Calculate the capacity of the object in mL.

Explanation

1 cm^3 is equivalent to 1 mL.

Therefore, 360 cm^3 is equivalent to 360 mL.

Answer

360 mL

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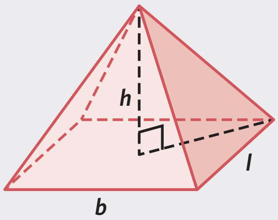
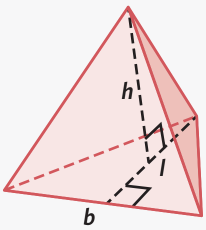
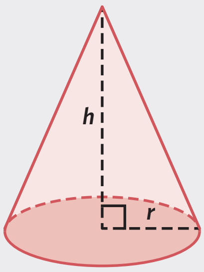
Calculating the volume of tapered solids

A second set of objects to consider are tapered solids. A **tapered solid** is a three-dimensional object that starts at a base shape but gradually reduces to a point, its apex.

Tapered solids are generally called pyramids and are named from the shape of the base, such as rectangular pyramid or triangular pyramid. A pyramid with a circular-shaped base is called a **cone**.

In all instances, the volume of a tapered solid can be calculated by first calculating the area of the base and then multiplying this by one third of its height.

Some common formulas are given in the following table.

solid	example	area of base	volume
rectangular pyramid		$A = bl$	$V = \frac{1}{3}blh$
triangular pyramid		$A = \frac{1}{2}bl$	$V = \frac{1}{6}blh$
cone		$A = \pi r^2$	$V = \frac{1}{3}\pi r^2 h$

Worked example 2

Calculate the volume of a cone with a radius of 1.2 metres and a height of 4.1 metres, rounded to two decimal places.

Explanation

Step 1: Identify the required dimensions.

The volume of a cone is calculated using the radius of the base circle and the height of the cone.

$r = 1.2$ m

$h = 4.1$ m

Step 2: Substitute the values into the appropriate formula and evaluate.

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 1.2^2 \times 4.1$$

$$= 6.182... \text{ m}^3$$

Answer

6.18 m^3

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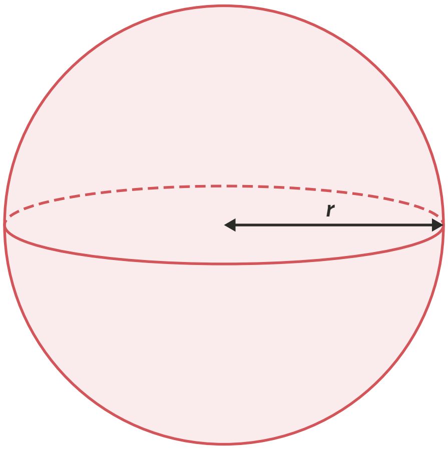
Calculating the volume of spheres

A third object to consider is the sphere. A **sphere** is a perfectly round object where every point on its surface is the same distance from its centre.

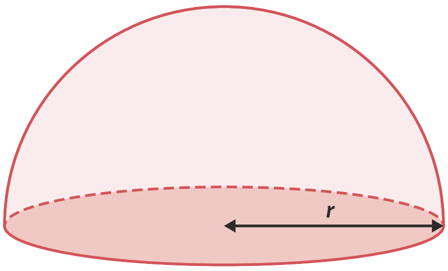


The volume of a sphere can be calculated using the formula

$V = \frac{4}{3}\pi r^3$

where r is the radius of the sphere.

Half of a sphere is called a **hemisphere**.



The volume of a hemisphere can be calculated using the formula

$V = \frac{2}{3}\pi r^3$

where r is the radius of the hemisphere.

Worked example 3

A bouncy ball has a diameter of four centimetres. What is the volume of the ball, rounded to two decimal places?

Explanation

Step 1: Identify the required dimensions.

A bouncy ball is spherical in shape.

The volume of a sphere is calculated using the radius of the sphere.

A diameter of 4 cm is given, which is double the length of the radius.

$r = 2$ cm

Step 2: Substitute the radius into the appropriate formula and evaluate.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 2^3$

$= 33.510...\text{ cm}^3$

Answer

33.51 cm^3

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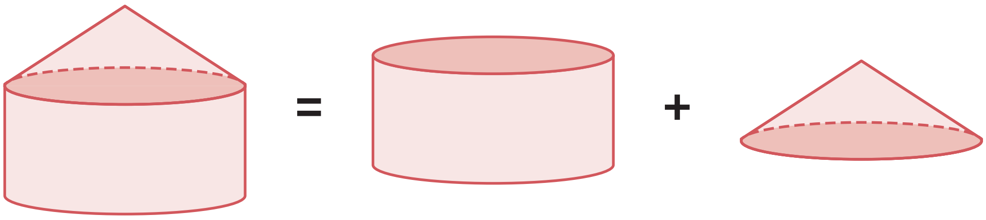
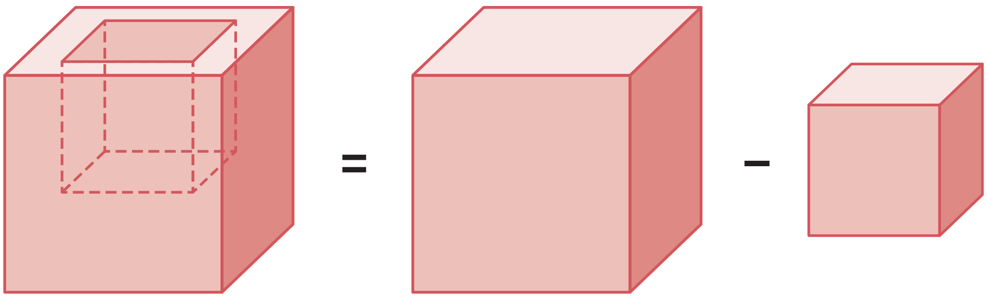
Calculating the volume of composite solids

A **composite solid** is a solid that is made up of a combination of three-dimensional objects.



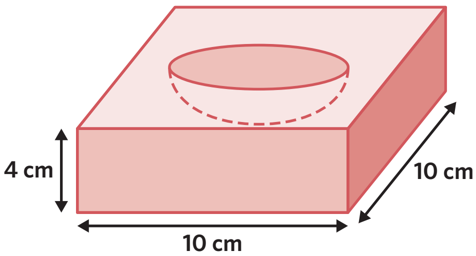
Sometimes a composite solid is formed when a smaller three-dimensional object is removed from a larger one.



Composite solids don't have specific formulas that can be used to calculate their volumes. Instead, volumes can be calculated through the addition and subtraction of each basic three-dimensional object.

Worked example 4

A block of wood has had a hemisphere with a radius of three centimetres carved out of it, as shown in the following diagram. What is the volume of the remaining block of wood, rounded to two decimal places?



Explanation

Step 1: Calculate the volume of the rectangular prism.

$$V = bhl$$
$$= 10 \times 4 \times 10$$
$$= 400 \text{ cm}^3$$

Step 2: Calculate the volume of the hemisphere.

$$V = \frac{2}{3}\pi r^3$$
$$= \frac{2}{3} \times \pi \times 3^3$$
$$= 56.5486... \text{ cm}^3$$

Step 3: Subtract the volume of the hemisphere from the volume of the rectangular prism.

$$400 - 56.5486... = 343.4513... \text{ cm}^3$$

Answer

343.45 cm^3

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9G Questions

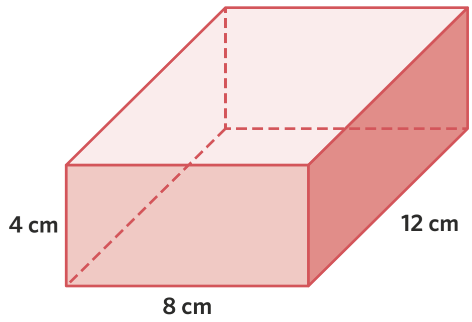
Calculating the volume of prisms and cylinders

1. Which of the following is not the name of a prism?

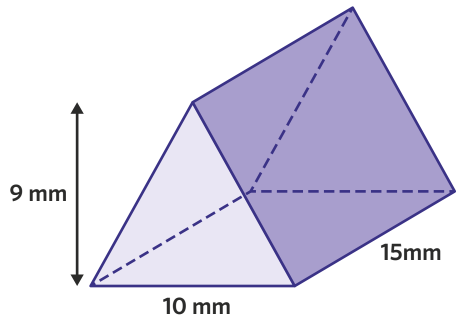
   A. Rectangular prism **B.** Octagonal prism **C.** Spherical prism **D.** Cube

2. Calculate the volume of each of the following prisms, rounded to the nearest whole number where necessary.

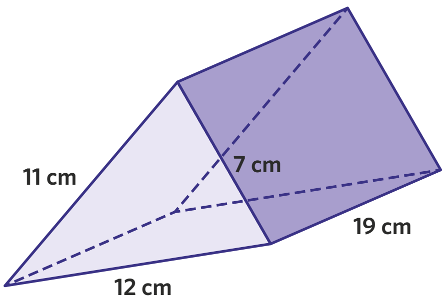
   a.



   b.



   c.



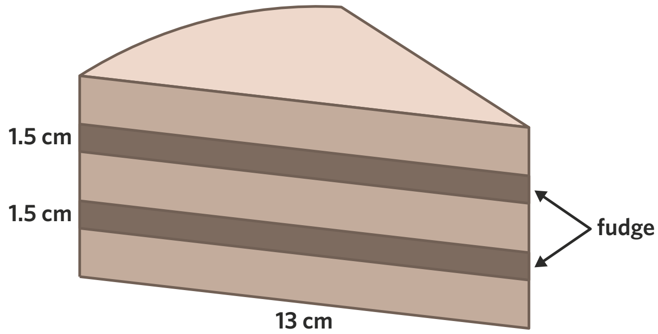
3. A cube has a side length of nine millimetres. What is its volume?

4. A cylinder has a height of 12 centimetres and a radius of 13 centimetres. What is its volume, rounded to two decimal places?

5. For Eloise's birthday, she buys a cylindrical chocolate fudge cake. It has a radius of 13 centimetres and a height of 11 centimetres.

   a. If the cake is split evenly between eight people, what volume of cake does each person receive, rounded to two decimal places?

   b. Eloise notices that the cake has two layers of fudge inside it, each 1.5 centimetres thick, as shown in the following diagram.



   How much fudge does each person receive in their serving, rounded to the nearest mL?

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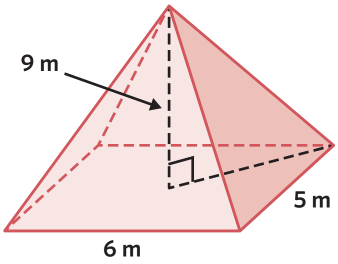
Calculating the volume of tapered solids

6. Which of the following is not a tapered solid?

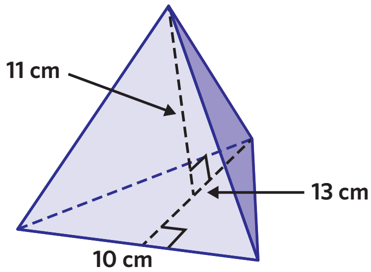
A. Square pyramid **B.** Triangular pyramid **C.** Triangular prism **D.** Cone

7. Calculate the volume of each of the following pyramids, rounded to two decimal places where necessary.

a.

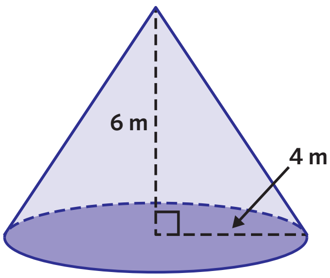


b.

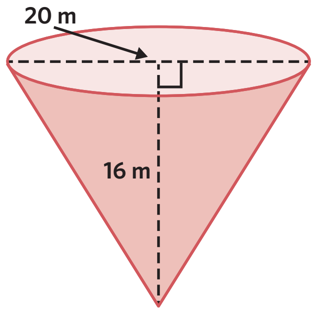


8. Calculate the volume of each of the following cones, rounded to two decimal places.

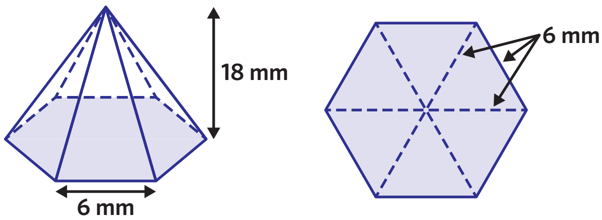
a.



b.



9. A hexagonal pyramid has a height of 18 millimetres and a side length of 6 millimetres. The hexagonal base can be split up into six identical equilateral triangles, as shown. What is the volume of the pyramid, rounded to one decimal place?



10. The Pyramid of Khafre in Egypt is a square-based pyramid with a volume of 2 115 072 m^3. The square base of the pyramid has side lengths of 216 metres. What is the height of the pyramid?

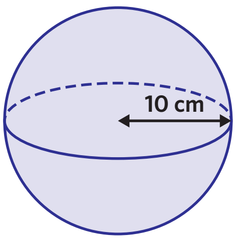
Calculating the volume of spheres

11. A hemisphere has a radius of 7.2 cm. Its volume, rounded to two decimal places is

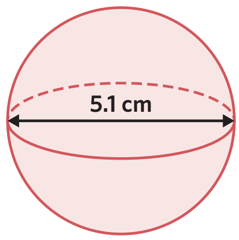
A. 162.86 cm^3 **B.** 651.44 cm^3 **C.** 781.73 cm^3 **D.** 1563.46 cm^3

12. Calculate the volume of the following spheres rounded to two decimal places.

a.



b.



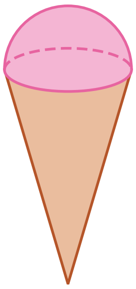
13. A sphere has a volume of 177 cm^3. What is its diameter, rounded to two decimal places?

14. Mary made scale models of the Earth and Venus for her science class. Her model of Earth had a radius of 4 cm, and her model of Venus had a volume of 230 cm^3. Which planet model did Mary make larger, Earth or Venus?

15. The radius measured from the inside of a particular basketball is 11.5 centimetres.
   a. What volume of air does the basketball contain when pumped up, in millilitres, rounded to two decimal places?
   b. The basketball is made of rubber 5 millimetres thick so the ball at its widest point is 24 centimetres wide. What volume of rubber was used to make the ball, in cm^3, rounded to one decimal place?

Calculating the volume of composite solids

16. Consider the following ice cream.

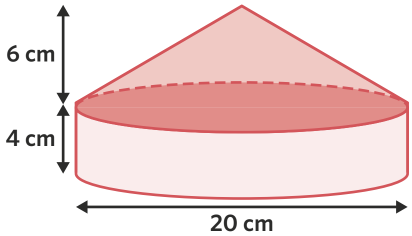


Which two solids could be used to model the ice cream?

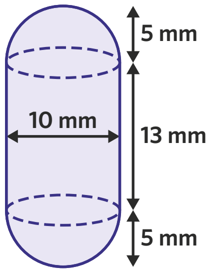
A. Cube and cone
B. Hemisphere and cone
C. Sphere and cone
D. Hemisphere and triangular prism

17. Calculate the volume of each of the following objects, rounded to two decimal places.

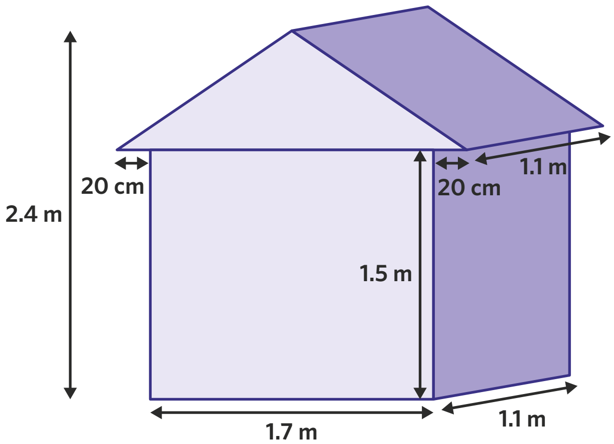
a.



b.



18. A treehouse consists of a rectangular prism and a triangular prism, as shown in the following diagram. What is the volume of the treehouse, in cubic metres, rounded to three decimal places?



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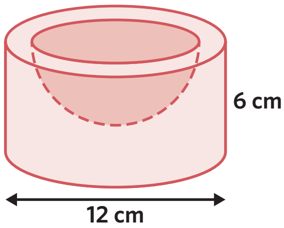
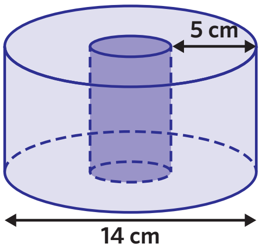
19. A glass bowl is in the shape of a cylinder with a hemispherical hole in the top. The cylinder is six centimetres in height and 12 centimetres at its widest point. The radius of the hemisphere is one centimetre less than the radius of the cylinder. What is the volume of glass that makes up the bowl, rounded to two decimal places?



20. The volume of the roll of toilet paper shown is 1700 cm^3.

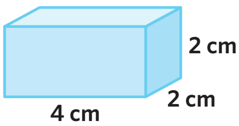


What is its height, rounded to two decimal places?

Joining it all together

21. A rectangular box containing 12 chocolate truffles is four centimetres in height. Each truffle in the box is spherical and has a radius of 1.5 centimetres. The box fits exactly four truffles along its length and exactly three truffles along its width.
    a. What is the total volume of chocolate in the box, rounded to the nearest mL?
    b. What is the volume of empty space in the box, rounded to the nearest cm^3?

22. 250 mL of water is poured into a cylindrical drinking glass, which fills 65% of its capacity. The glass has a diameter of 6 centimetres.
    a. What is the height of the glass, rounded to one decimal place?
    b. 3 ice cubes are added to the glass. The ice cubes are in the shape of rectangular prisms, with measurements as shown in the following diagram.



To what percent of its capacity is the glass now full? Round to one decimal place.

Exam practice

23. The game of squash is played with a spherical ball that has a radius of 2 cm.
Show that the volume of one squash ball, rounded to two decimal places, is 33.51 cm^3. (1 MARK)
VCAA 2021 Exam 2 Geometry and measurement Q1a

87% of students answered this question correctly.

24. A cone and a cylinder both have a radius of r centimetres.
The height of the cone is 12 cm.
If the cylinder and the cone have the same volume, then the height of the cylinder, in centimetres, is

A. 4 **B.** 6 **C.** 8
D. 12 **E.** 36

VCAA 2021 Exam 1 Geometry and measurement Q5

46% of students answered this question correctly.

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Questions from multiple lessons

Recursion and financial modelling

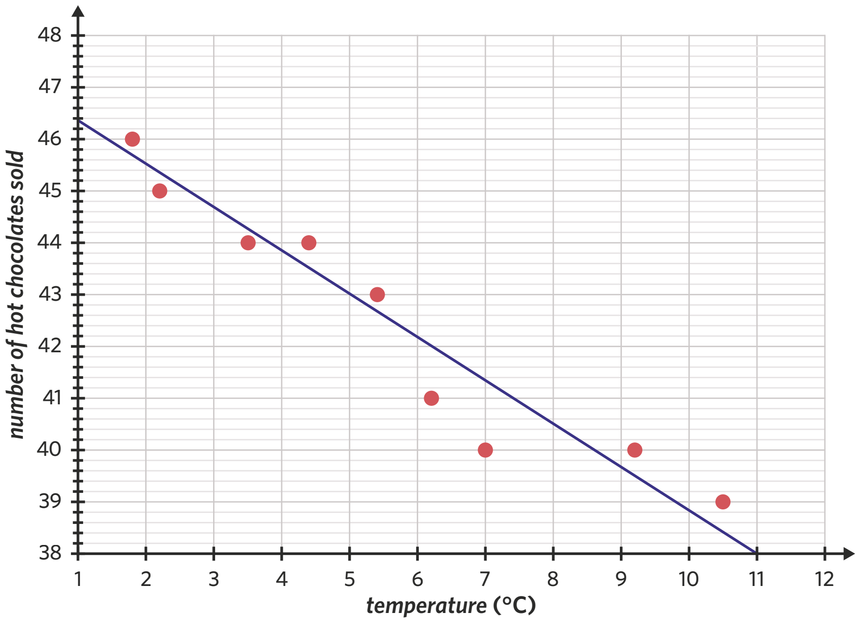
25. Jackson invests \$12 000 for 200 days. If his interest compounds daily at a rate of 4.5% per annum, the value of Jackson's investment after 200 days is closest to

A. \$12 296 **B.** \$12 300 **C.** \$25 368 **D.** \$79 880 235 **E.** \$80 821 398

Adapted from VCAA 2014 Exam 1 Business-related mathematics Q3

Data analysis

26. The following scatterplot displays the daily *number of hot chocolates sold*, and the *temperature*, in degrees Celsius, for a cafe in Toronto. A line of good fit has been fitted to the data.



The equation of this line of good fit is closest to

A. *number of hot chocolates* = 46.4 − 0.84 × *temperature*

B. *number of hot chocolates* = 47.2 − 0.91 × *temperature*

C. *number of hot chocolates* = 46.4 + 0.84 × *temperature*

D. *number of hot chocolates* = 46.4 − 0.91 × *temperature*

E. *number of hot chocolates* = 47.2 − 0.84 × *temperature*

Adapted from VCAA 2018 Exam 1 Data analysis Q8

Computation and practical arithmetic *Year 10 content*

27. Rafael is asked to evaluate the expression $\dfrac{-\left(\frac{3}{4}+12\right)\times 4}{21-38}$.

a. Consider Rafael's working out as shown.

Line 1: $\dfrac{-\left(\frac{3}{4}+12\right)\times 4}{21-38} = \dfrac{-\left(\frac{3}{4}+\frac{48}{4}\right)\times 4}{-17}$

Line 2: $= \dfrac{-\left(\frac{51}{4}\right)\times 4}{-17}$

Line 3: $= \dfrac{51}{-17}$

Line 4: $= -3$

Which line does the first mistake appear on? (1 MARK)

b. Evaluate the expression correctly. (1 MARK)

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9H Surface area

STUDY DESIGN DOT POINT

- volumes and surface areas of solids (spheres, cylinders, pyramids and prisms and composite objects) and practical applications, including simple applications of Pythagoras' theorem in three dimensions

9A 9B 9C 9D 9E 9F 9G **9H** 9I

KEY SKILLS

During this lesson, you will be:

- calculating the surface area of solids with planar faces
- calculating the surface area of solids with curved faces
- calculating the surface area of composite solids.

KEY TERMS

- Surface area
- Planar solid
- Net

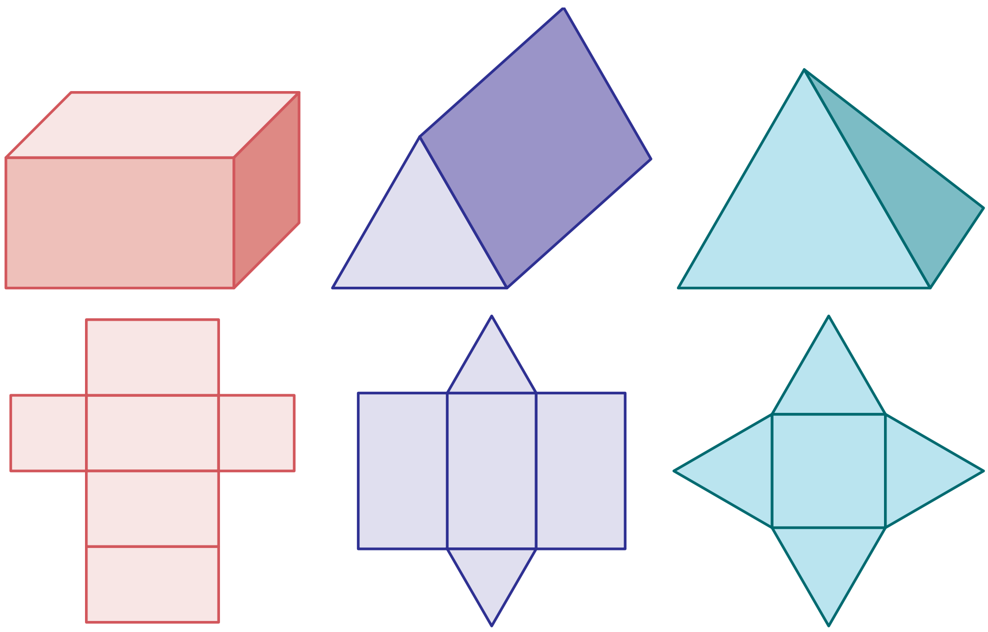
When analysing three-dimensional objects, the total area of the external surface of the object can be calculated. This can have a number of practical applications, such as knowing how much material is required to cover an object, or how many objects can be constructed given a certain amount of material.

Calculating the surface area of solids with planar faces

The **surface area** of a three-dimensional solid is the total area of all the surfaces of the object.

The surface area of a **planar solid**, an object which only has flat faces, can be found by calculating the area of each face, and adding these areas together. Planar solids include prisms and pyramids.

Another way to find the surface area of a planar solid is by finding the area of the net of the solid. The **net** of a solid is a two-dimensional representation of what a three-dimensional planar object would look like if unfolded, or deconstructed. The following examples show the nets of a rectangular prism, a triangular prism and a square pyramid.



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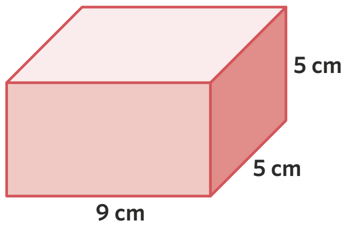
Worked example 1

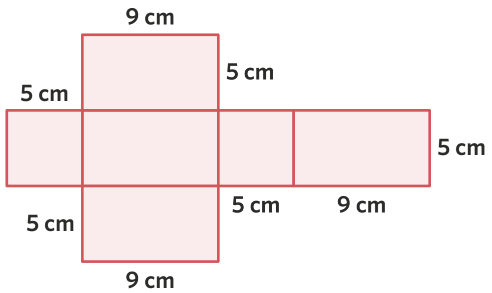
Calculate the surface area of the following solids.

a.



Explanation

Step 1: Identify all the faces.



There are two 5×5 cm square faces and four 5×9 cm rectangular faces.

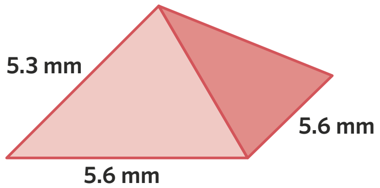
Step 2: Sum the areas of the faces.

$$SA = 2(5 \times 5) + 4(5 \times 9)$$
$$= 50 + 180$$
$$= 230 \text{ cm}^2$$

Answer

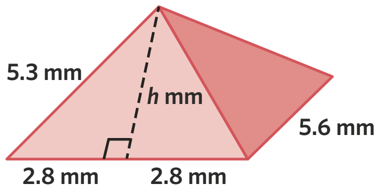
230 cm^2

b.



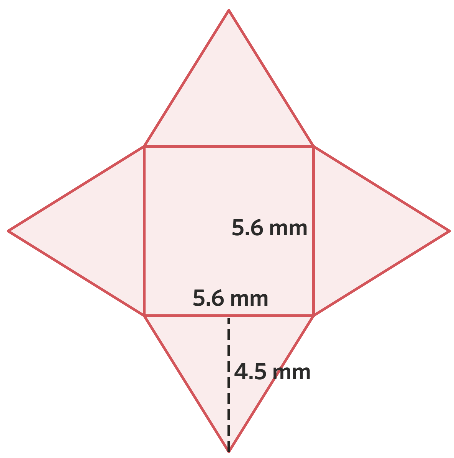
Explanation

Step 1: Calculate the height of the triangular planar face using Pythagoras' theorem.



$$2.8^2 + h^2 = 5.3^2$$
$$7.84 + h^2 = 28.09$$
$$h = 4.5 \text{ mm}$$

Step 2: Identify all the faces.



There are four triangular faces with a base of 5.6 mm and a height of 4.5 mm.

There is one 5.6×5.6 mm face.

Continues →

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Step 3: Sum the areas of the faces.

$$SA = 4\left(\frac{1}{2} \times 5.6 \times 4.5\right) + (5.6 \times 5.6)$$
$$= 50.4 + 31.36$$
$$= 81.76 \text{ mm}^2$$

Note: Heron's formula can also be used to calculate the area of each triangular face.

Answer

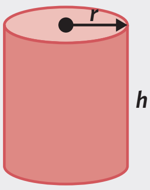
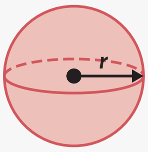
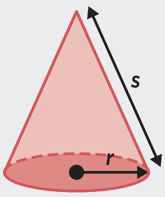
81.76 mm^2

Calculating the surface area of solids with curved faces

Some three-dimensional solids have curved faces, such as cylinders, spheres and cones.

The surface area of these solids can be calculated by adding the area of any planar faces with the area of the curved face.

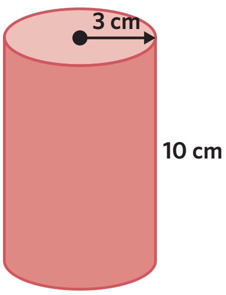
The formulas for the surface area of these solids are given in the following table.

solid	example	surface area
cylinder		$SA = \textit{area of two circles} + \textit{area of curved side}$ $SA = 2\pi r^2 + 2\pi rh$
sphere		$SA = \textit{area of curved side}$ $SA = 4\pi r^2$
cone		$SA = \textit{area of circle} + \textit{area of curved side}$ $SA = \pi r^2 + \pi rs$

Worked example 2

Calculate the surface area of the following solids, rounded to two decimal places.

a.



Continues →

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Explanation

Step 1: Identify the appropriate formula.

The given solid is a cylinder.

$SA = 2\pi r^2 + 2\pi rh$

Step 2: Identify the required dimensions.

$r = 3$ cm

$h = 10$ cm

Step 3: Substitute the known values into the formula.

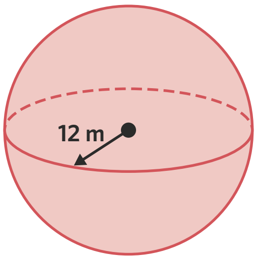
$SA = (2 \times \pi \times 3^2) + (2 \times \pi \times 3 \times 10)$

$= 245.044...\ \text{cm}^2$

Answer

$245.04\ \text{cm}^2$

b.



Explanation

Step 1: Identify the appropriate formula.

The given solid is a sphere.

$SA = 4\pi r^2$

Step 2: Identify the required dimensions.

$r = 12$ m

Step 3: Substitute the known value into the formula.

$SA = 4 \times \pi \times 12^2$

$= 1809.557...\ \text{m}^2$

Answer

$1809.56\ \text{m}^2$

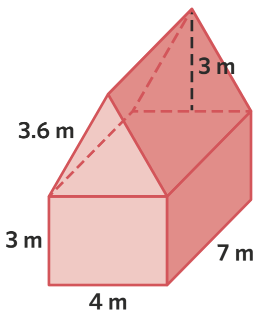
Calculating the surface area of composite solids

The surface area of a composite solid can be found by adding the area of the faces on the outer surface of the solid together. It is important not to include the area of any internal surfaces in the total surface area. In such situations, it may be necessary to adjust the formulas to remove certain faces.

Worked example 3

Calculate the surface area of the following solids. Round to two decimal places where necessary.

a.



Continues →

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Explanation

Step 1: Identify all the external faces.

The rectangular prism has two 3 m × 4 m faces, two 3 m × 7 m faces and one 4 m × 7 m face exposed.

The triangular prism has two 3.6 m × 7 m faces exposed, as well as two triangular faces with a base of 4 m and a height of 3 m.

Step 2: Sum the areas of the faces.

$$SA = 2(3 \times 4) + 2(3 \times 7) + (4 \times 7) + 2(3.6 \times 7) + 2\left(\frac{1}{2} \times 4 \times 3\right)$$

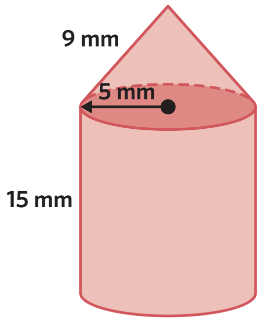
$$= 156.4 \text{ m}^2$$

Answer

156.4 m^2

---

b.



Explanation

Step 1: Identify all the external faces.

The cylinder has a curved face and a circular face exposed.

The cone has a curved face exposed.

Step 2: Adjust any formulas used.

The formula for a cylinder is $SA = 2\pi r^2 + 2\pi rh$. As only one circular face is exposed, the area of one circle can be removed.

$SA = \pi r^2 + 2\pi rh$

The formula for a cone is $SA = \pi r^2 + \pi rs$. As the circular face is not exposed, the area of the circle can be removed.

$SA = \pi rs$

The formula for the total surface area of this composite solid is $SA = \pi r^2 + 2\pi rh + \pi rs$.

Step 3: Identify the required dimensions.

$r = 5$ mm

$h = 15$ mm

$s = 9$ mm

Step 4: Substitute the known values into the formula.

$$SA = (\pi \times 5^2) + (2 \times \pi \times 5) \times (15 + \pi \times 5 \times 9)$$

$$= 691.150... \text{ mm}^2$$

Answer

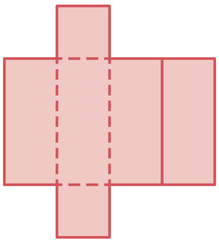
691.15 mm^2

9H Questions

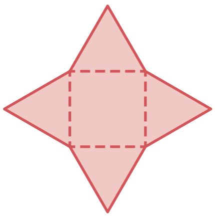
Calculating the surface area of solids with planar faces

1. Which of the following nets forms a triangular prism?

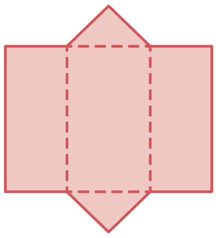
A.



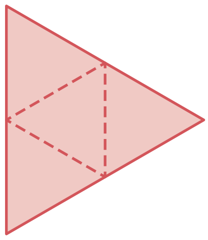
B.



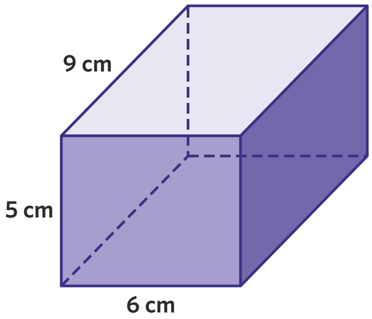
C.



D.



2. Consider the following solid.

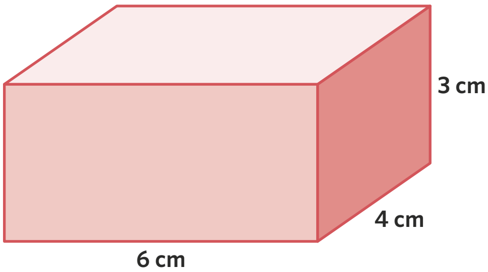


What is the value of x in the following equation?

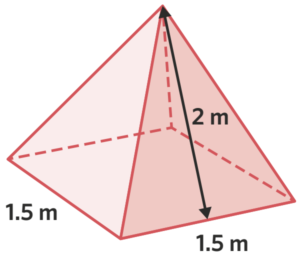
$SA = 2(5 \times 6) + 2(5 \times x) + 2(6 \times 9)$

3. Calculate the surface area of the following solids. Round to two decimal places where necessary.

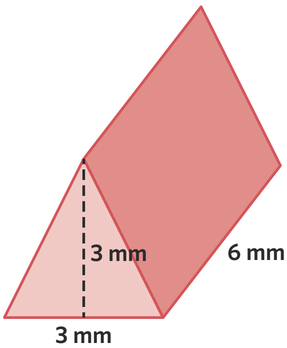
a.



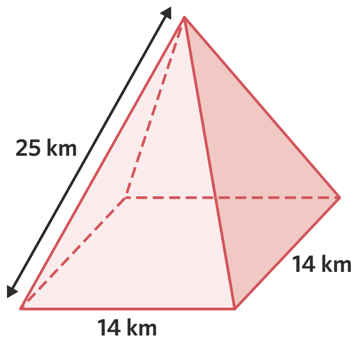
b.



c.

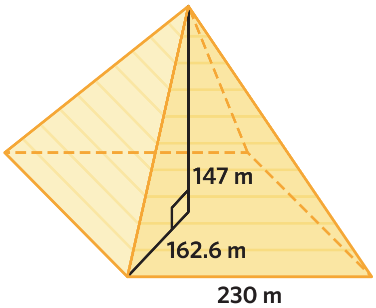


d.



4. The surface area of a triangular pyramid is 87.6 cm^2. If the base of the pyramid is an equilateral triangle with side lengths of 6 cm, what is the height of the other three triangles that make up the pyramid? Round to the nearest centimetre.

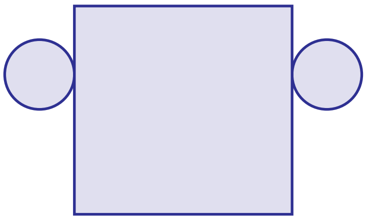
5. The Great Pyramid of Giza is approximately in the shape of a square-based pyramid, as shown in the following diagram.



Considering the base of the Pyramid is not exposed, calculate the exposed surface area of the Great Pyramid of Giza, rounded to the nearest square metre.

Calculating the surface area of solids with curved faces

6. The following net forms a regular solid.



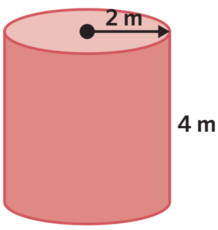
The regular solid is a

A. cylinder. **B.** sphere. **C.** cone. **D.** hemisphere.

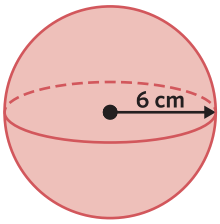
---

7. Calculate the surface area of the following solids, rounded to two decimal places.

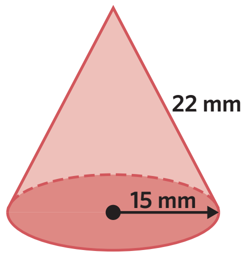
a.



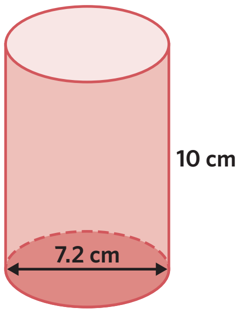
b.



c.



d.



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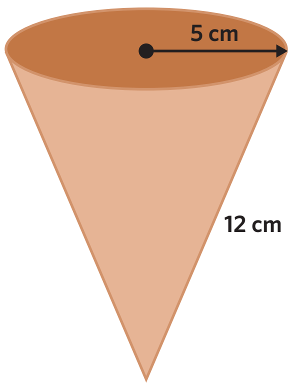
8. Six basketballs need to be made for a basketball tournament. If the diameter of a basketball is 24 cm, what is the total area of material needed to cover the basketballs? Round to the nearest square centimetre.

---

9. Usman runs an ice-cream store, and sells waffle cones. The following diagram shows the dimensions of the waffle cones he makes.

He currently has enough ingredients to make 3800 cm^2 worth of the mixture used to shape the waffle cones. How many waffle cones can Usman make?



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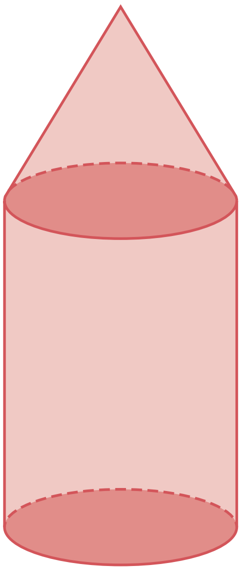
Calculating the surface area of composite solids

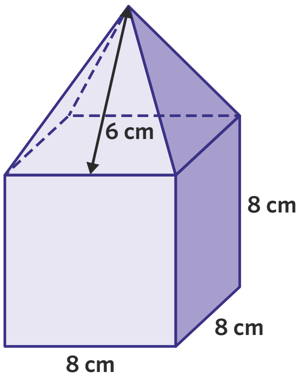
10. Which two regular solids make up the following composite object?



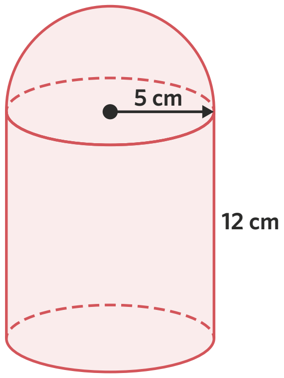
A. Cylinder and cone
B. Cylinder and pyramid
C. Rectangular prism and cone
D. Rectangular prism and pyramid

11. Calculate the surface area of the following solids. Round to two decimal places where necessary.

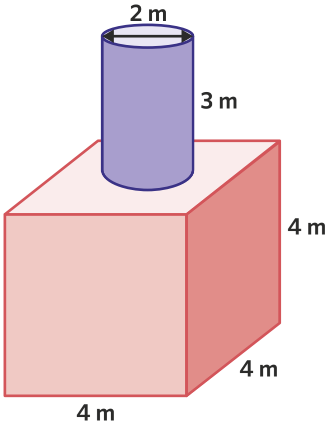
a.



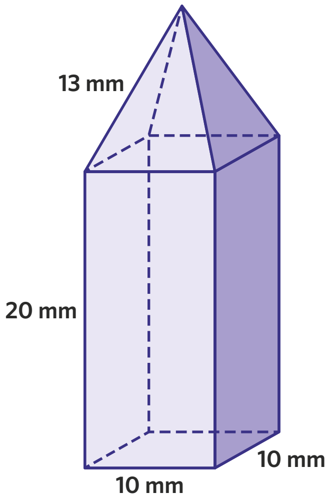
b.



c.



d.



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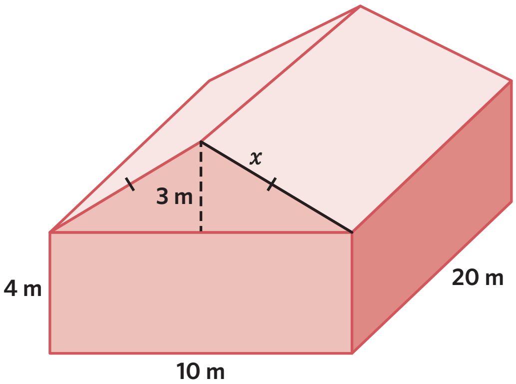
12. A new house is made up of a rectangular prism and triangular prism as shown. The roof has a vertical height of 3 m.



a. What is the length of side x, rounded to two decimal places?

b. If the entire exterior of the house needs to be painted, including the roof but not including the base, what is the total surface area that needs to be painted? Round to the nearest square metre.

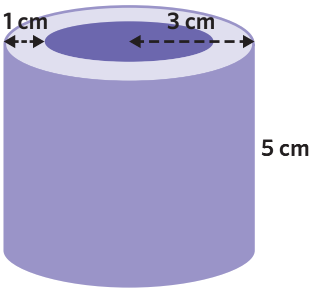
---

13. A perfume bottle is in the shape of a cube with side lengths of 5 cm, and a cylindrical pump on top with a height of 2 cm and radius of 0.6 cm.

What is the total surface area of the perfume bottle, rounded to two decimal places?

Joining it all together

14. Consider the hollow tube shown.



Which expression will calculate the total surface area of the tube?

A. $2\pi \times 3^2 + 2\pi \times 3 \times 5$

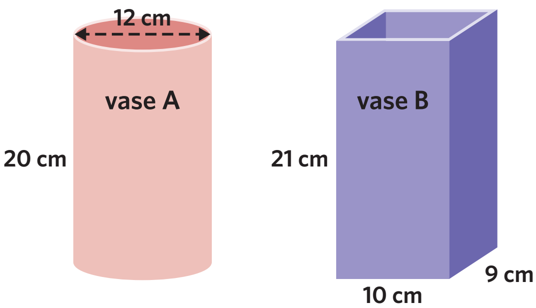
B. $(2\pi \times 3^2 + 2\pi \times 3 \times 5) - (2\pi \times 2^2 + 2\pi \times 2 \times 5)$

C. $(2\pi \times 3^2 + 2\pi \times 3 \times 5) + (2\pi \times 2^2 + 2\pi \times 2 \times 5)$

D. $(2\pi \times 3^2 + 2\pi \times 3 \times 5) - (2\pi \times 2^2) + (2\pi \times 2 \times 5)$

---

15. The following diagram shows two vases.



Which vase has a larger external surface area (only including the sides and base) and by how much? Round to two decimal places.

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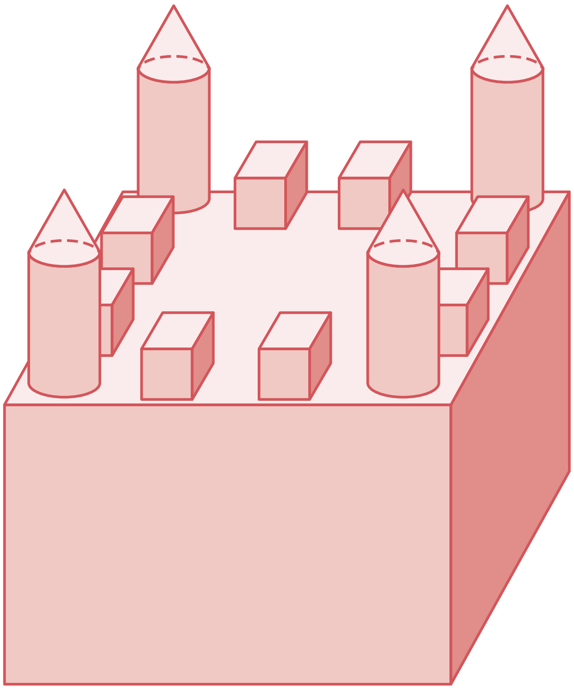
16. The following diagram shows a model castle that Charles constructed for a school project.



The model is constructed from several solids.

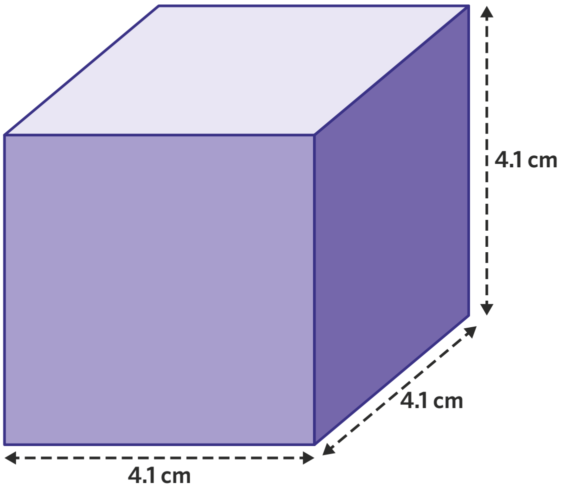
- The base has a length and width of 12 cm, and a height of 8 cm.
- Each corner has a cone on a cylinder. The cylinder has a diameter of 2 cm and a height of 6 cm. The cone has a slant height of 3 cm.
- Each side has 2 cubes with side lengths of 2 cm.

Calculate the total exposed surface area of Charles' model, including the base. Round to two decimal places.

Exam practice

17. Squash balls are sold in cube-shaped boxes.

Each box contains one ball and has a side length of 4.1 cm, as shown in the following diagram.



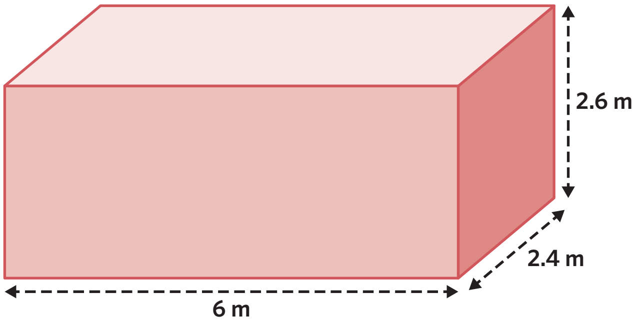
Calculate the total surface area, in square centimetres, of one box. (1 MARK)

VCAA 2021 Exam 2 Geometry and measurement Q1c

81% of students answered this question correctly.

18. A cargo ship has shipping containers in the shape of a rectangular prism.

Each shipping container has a height of 2.6 m, a width of 2.4 m and a length of 6 m, as shown in the following diagram.



What is the total surface area, in square metres, of the outside of one shipping container? (1 MARK)

VCAA 2019 Exam 2 Geometry and measurement Q1c

70% of students answered this question correctly.

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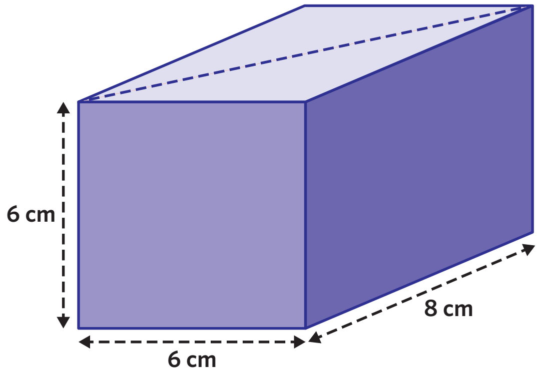
19. A cake is in the shape of a rectangular prism, as shown in the following diagram.



The cake is cut in half to create two equal portions. The cut is made along the diagonal, as represented by the dotted line. The total surface area, in square centimetres, of one portion of the cake is

A. 132 **B.** 180 **C.** 192
D. 212 **E.** 264

VCAA 2020 Exam 1 Geometry and measurement Q8

57% of students answered this question correctly.

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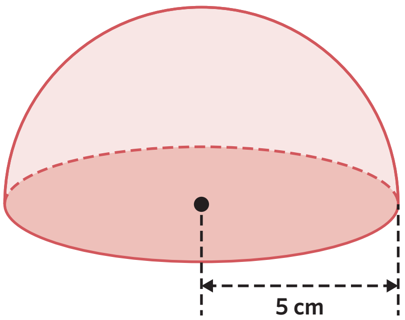
20. An ice cream dessert is in the shape of a hemisphere. The dessert has a radius of 5 cm.



The top and the base of the dessert are covered in chocolate. The total surface area, in square centimetres, that is covered in chocolate is closest to

A. 52 **B.** 157 **C.** 236
D. 314 **E.** 942

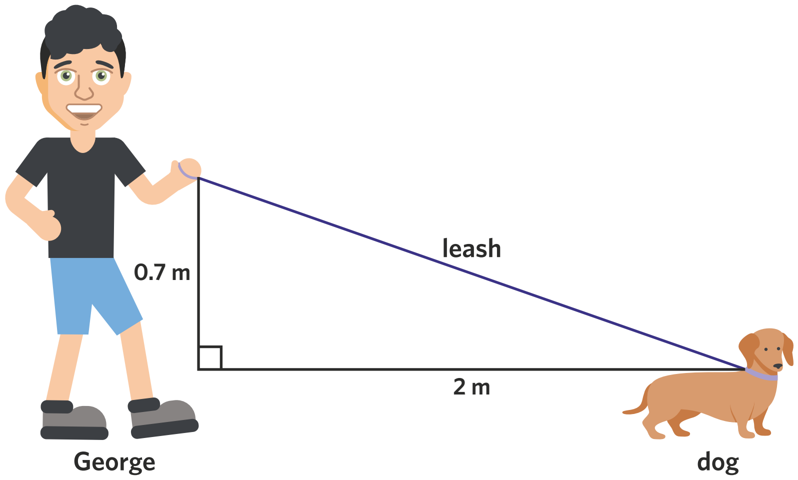
VCAA 2019 Exam 1 Geometry and measurement Q3

48% of students answered this question correctly.

Questions from multiple lessons

Geometry and measurement *Year 10 content*

21. George decides to walk his dog on a leash as shown in the following diagram.



The length of the leash, in metres, is closest to

A. 2.1 **B.** 2.2 **C.** 2.7 **D.** 3.5 **E.** 4.5

Adapted from VCAA 2018 Exam 1 Geometry and measurement Q1

Recursion and financial modelling

22. A sequence of numbers can be generated by the following recurrence relation.

$V_0 = 23 \quad V_{n+1} = 4 \times V_n$

What is the value of V_7?

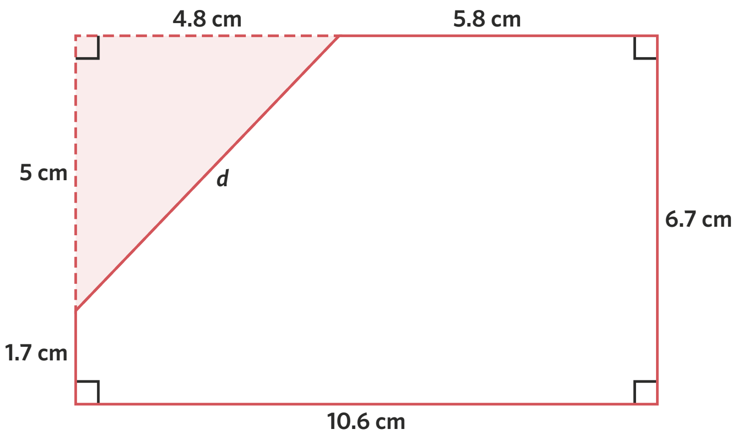
A. 5888 **B.** 23 552 **C.** 94 208 **D.** 376 832 **E.** 1 507 328

Adapted from VCAA 2019NH Exam 1 Recursion and financial modelling Q17

Geometry and measurement

23. Kyle intends to build a toy house for his daughter using a piece of cardboard as the base.

The unshaded area in the following diagram represents the cardboard piece.



a. Show that the length d is 6.9 cm, rounded to one decimal place. (1 MARK)

b. Calculate the perimeter, in centimetres, of this cardboard piece. (1 MARK)

c. Calculate the area of this piece of cardboard. Round to the nearest square centimetre. (1 MARK)

Adapted from VCAA 2017NH Exam 2 Geometry and measurement Q1a-c

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9I Scale factor

STUDY DESIGN DOT POINT

- similar objects and the application of linear scale factor $k > 0$ to scale lengths, surface areas and volumes with practical applications

9A 9B 9C 9D 9E 9F 9G 9H 9I

KEY SKILLS

During this lesson, you will be:

- using scale factors to scale area
- using scale factors to scale volume.

KEY TERMS

- Area scale factor
- Volume scale factor

Sometimes there is a need to change the size of an object while still keeping its dimensions to scale. Enlarging or reducing an object or measurement has important real-world applications, such as constructing a model of a building or reducing the volume of an object to fit it inside a smaller shape.

Using scale factors to scale area

In geometry, an image is a copy of an object after it has been rotated, reflected, enlarged, or reduced.

Recall that the linear scale factor, k, of similar objects can be calculated using the formula

$$k = \frac{\textit{length of image}}{\textit{length of original}}$$

The **area scale factor** compares the area of a shape with its image. If two similar shapes have a linear scale factor of k, the area scale factor will be k^2.

$$k^2 = \frac{\textit{area of image}}{\textit{area of original}}$$

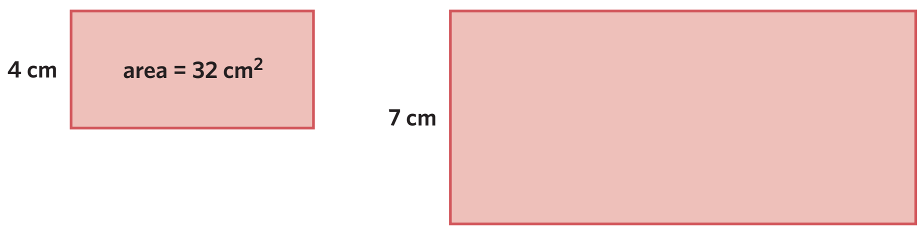
To calculate the area of the image, the formula can be rearranged as
$\textit{area of image} = k^2 \times \textit{area of original}$

The surface area of a three-dimensional object is calculated in the same way. To calculate the surface area of an image, the formula is

$\textit{surface area of image} = k^2 \times \textit{surface area of original}$

Worked example 1

Consider the following similar rectangles.



Continues →

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a. Calculate the linear scale factor, k.

Explanation

Substitute an original side length and its equivalent image side length into the linear scale factor formula and evaluate.

$$k = \frac{\textit{length of image}}{\textit{length of original}}$$
$$= \frac{7}{4}$$
$$= 1.75$$

Answer

1.75

b. Calculate the area of the larger rectangle.

Explanation

Substitute the original area and the linear scale factor, k, into the area formula and evaluate.

$$\textit{area of image} = k^2 \times \textit{area of original}$$
$$= 1.75^2 \times 32$$
$$= 98$$

Answer

98 cm^2

Using scale factors to scale volume

The **volume scale factor** compares the volume of an object with its image. If two similar objects have a length scale factor of k, the volume scale factor will be k^3.

$$k^3 = \frac{\textit{volume of image}}{\textit{volume of original}}$$

To calculate the volume of the image, the formula can be rearranged as

$$\textit{volume of image} = k^3 \times \textit{volume of original}$$

Worked example 2

In Alex's chemistry class, there are two similarly-shaped beakers. Alex knows that the larger beaker has a capacity of 500 mL, but she has forgotten the capacity of the smaller beaker. The large beaker is 15 centimetres tall and the small beaker is 12 centimetres tall.

a. Calculate the linear scale factor, k.

Explanation

Step 1: Identify the required values.

As the smaller beaker has an unknown capacity, it will be referred to as the image. Therefore, the larger beaker will be the original.

As height is related to length, the scaling calculation can be performed in the same way.

$\textit{height of original} = 15$

$\textit{height of image} = 12$

Step 2: Substitute the heights into the linear scale factor formula and evaluate.

$$k = \frac{\textit{height of image}}{\textit{height of original}}$$
$$= \frac{12}{15}$$
$$= 0.8$$

Continues →

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Answer

0.8

---

b. Calculate the capacity of the smaller beaker.

Explanation

Substitute the original capacity and linear scale factor into the volume formula and evaluate.

As capacity is related to volume, the scaling calculation can be performed in the same way.

$\textit{capacity of original} = 500$

$k = 0.8$

$$\begin{aligned}\textit{capacity of image} &= k^3 \times \textit{capacity of original}\\ &= 0.8^3 \times 500\\ &= 256\end{aligned}$$

Answer

256 mL

---

c. Alex has a third beaker with a capacity of 1.5 L. Considering the 500 mL beaker as the original and the 1.5 L beaker as the image, calculate the linear scale factor, k. Round to two decimal places.

Explanation

Step 1: Substitute the capacities into the formula for volume scale factor.

Make sure that both capacities are in the same units.

$$\begin{aligned}k^3 &= \frac{\textit{capacity of image}}{\textit{capacity of original}}\\ &= \frac{1.5}{0.5}\\ &= 3\end{aligned}$$

Step 2: Calculate the linear scale factor.

$$\begin{aligned}k &= \sqrt[3]{3}\\ &= 1.442...\end{aligned}$$

Answer

1.44

9I Questions

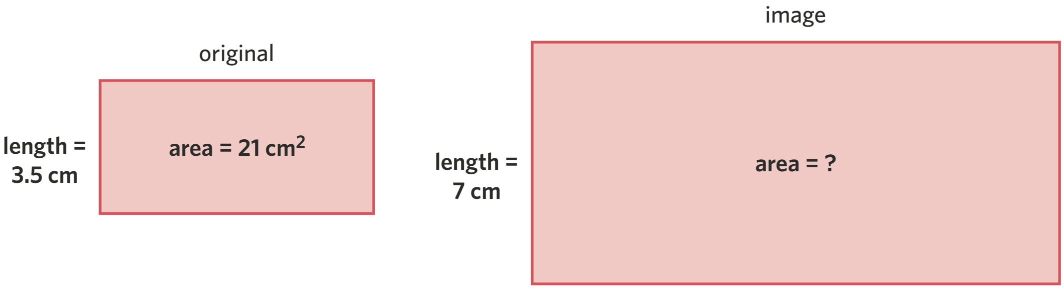
Using scale factors to scale area

1. Which of the following is the scale factor for area?

A. k **B.** k^2 **C.** k^3 **D.** $\frac{1}{k}$

---

2. Consider the following rectangles.



a. Calculate the linear scale factor, k.

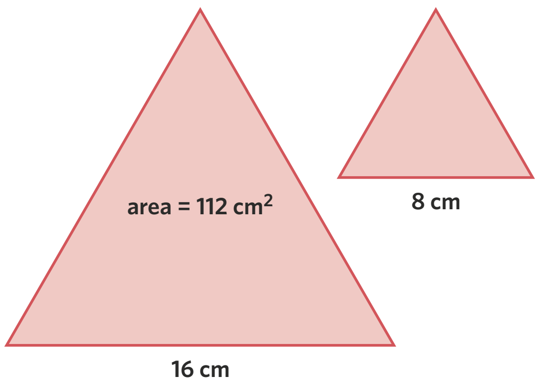
b. Calculate the area scale factor, k^2.

c. Calculate the area of the larger rectangle.

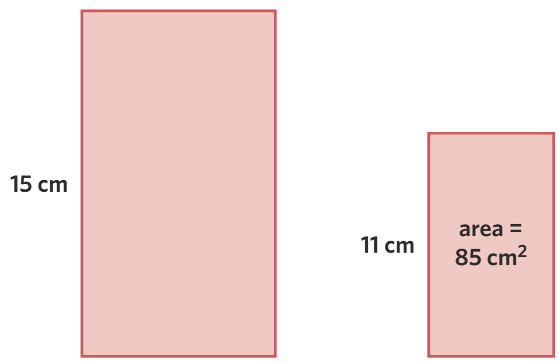
---

3. For the following pairs of similar shapes, calculate the unknown area, rounded to two decimal places where necessary.

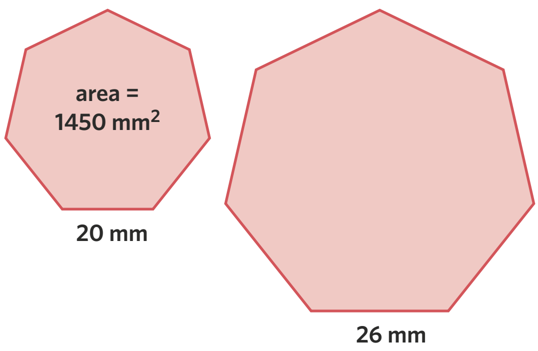
a.



b.



c.



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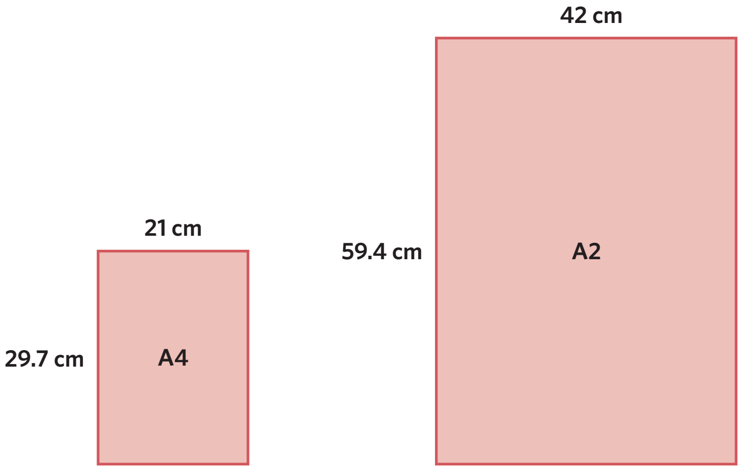
4. The dimensions of an A4 and A2 piece of paper are shown in the following diagram.

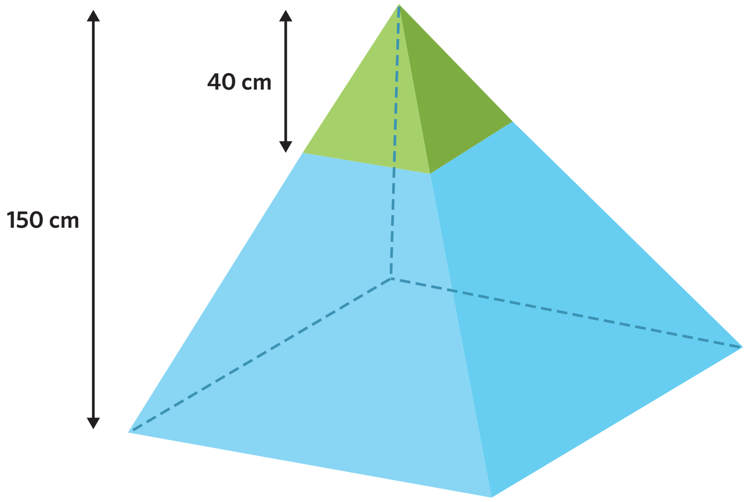


The A4 and A2 pieces of paper are similar. A piece of A4 paper has an area of 623.7 cm^2.

a. Considering the A2 piece of paper as the image, calculate the linear scale factor, *k*.

b. Use the area scale formula to show that the area of an A2 piece of paper is 2494.8 cm^2.

5. Bella is an avid hiker and her favourite tent is in the shape of a pyramid.



The surface area of the green portion of the tent is 3000 cm^2.

Calculate the surface area of the blue portion of the tent, excluding the base.

6. Angelica is an architecture student and has to make a model of a building she designed for an assignment. She makes a model with a height of 18 centimetres and a surface area of 980 cm^2. The actual building will have a surface area of 310 000 cm^2. What will the actual height of the building be, rounded to the nearest centimetre?

Using scale factors to scale volume

7. The image of an object has a linear scale factor of 1.75. Its volume scale factor is closest to

A. 1.75

B. 3.06

C. 5.25

D. 5.36

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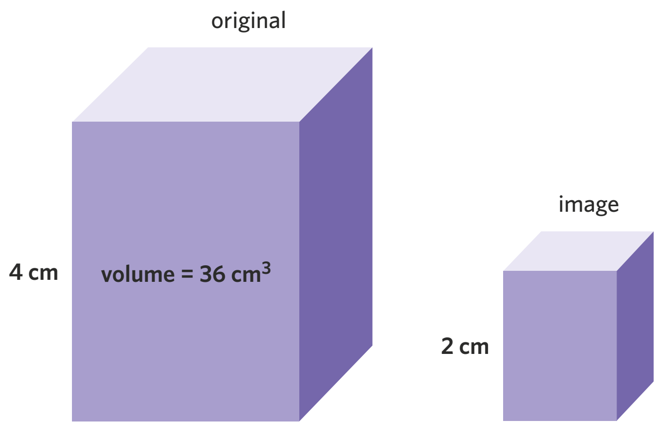
8. Consider the following rectangular prisms.



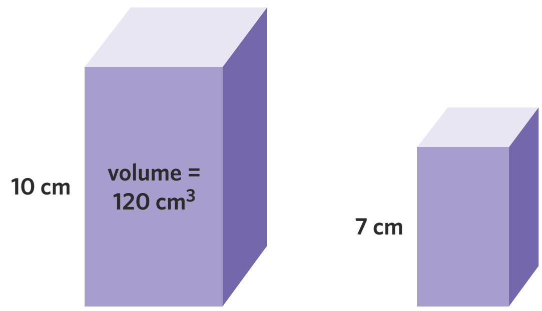
a. Calculate the linear scale factor, k.

b. Calculate the volume scale factor, k^3.

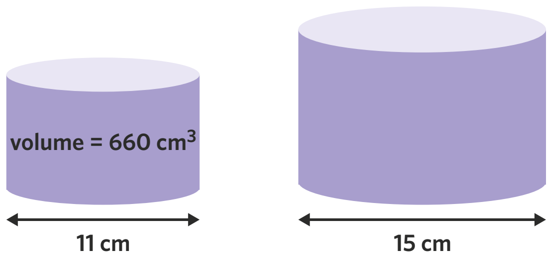
c. Calculate the volume of the smaller prism.

9. For the following pairs of similar objects, calculate the unknown volume, rounded to two decimal places.

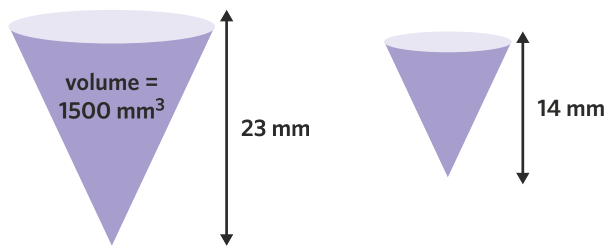
a.



b.



c.



10. A soccer ball has a diameter of 22 cm and a volume of 5575.28 cm^3. A tennis ball has a volume of 143.79 cm^3.

a. Considering the tennis ball as the image, calculate the linear scale factor, k, rounded to three decimal places.

b. Using the value of k from part **a**, calculate the diameter of the tennis ball, rounded to one decimal place.

11. Caleb's Cardboard Container Company has three different sizes of similarly shaped cardboard boxes; small, medium and large.

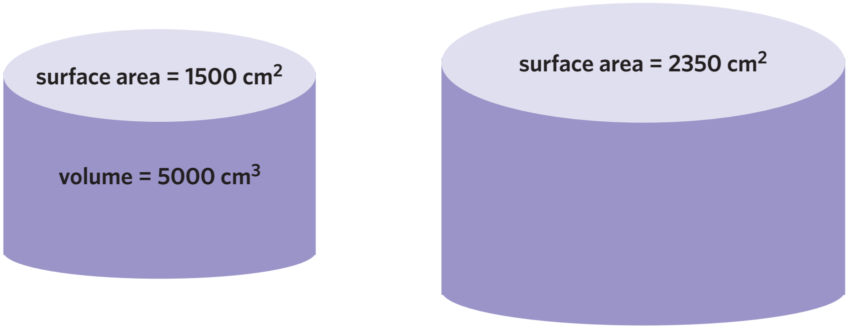
The width of the large box is four times the width of the small box.

The volume of the medium box is eight times the volume of the small box.

If the medium box has a volume of 64 000 cm^3, what is the volume of the large box?

Joining it all together

12. Consider the following similar cylinders.



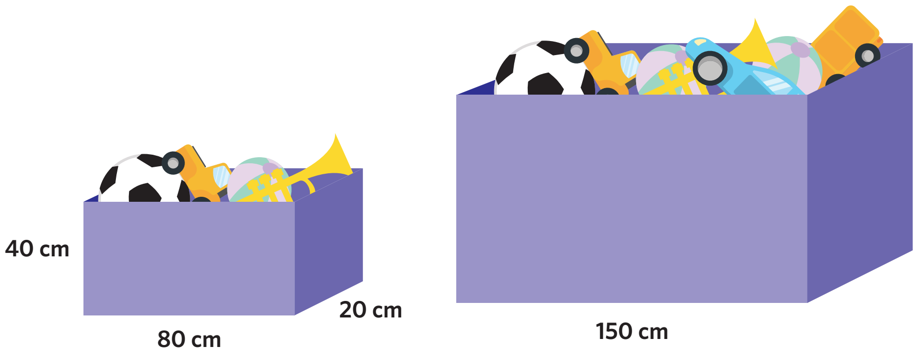
Calculate the volume of the larger cylinder, rounded to one decimal place.

13. India is going on holiday, so she buys a travel-size bottle of her favourite perfume. She has the full-size bottle of the same perfume and notices that the two bottles are similarly shaped, and the height of the travel-size bottle is half the height of the full-size bottle.

a. If the full-size perfume bottle can hold 100 mL, what is the capacity of travel-size bottle?

b. If the surface area of the travel-size bottle is 15.7 cm^2, what is the surface area of the full-size bottle?

14. Derek's old toy box is too small to store all of his toys. His parents buy him a new toy box for his 10th birthday. The old and new toy boxes are similar rectangular prisms as shown in the following diagram.



a. Considering the new toy box as the image, calculate the linear scale factor, k.

b. What is the volume of his new toy box?

c. Derek would like to cover the sides and bottom of his new toy box in his favourite unicorn wrapping paper. What is the total surface area that needs to be covered on his new toy box?

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Exam practice

15. A photograph was enlarged by an area scale factor of 9.

The length of the original photograph was 12 cm.

The original photograph and the enlarged photograph are similar in shape.

The length of the enlarged photograph, in centimetres, is

A. 4 **B.** 9 **C.** 27
D. 36 **E.** 108

VCAA 2021 Exam 1 Geometry and measurement Q3

38% of students answered this question correctly.

---

16. Miki is planning a gap year in Japan.
She will store some of her belongings in a small storage box while she is away.
This small storage box is in the shape of a rectangular prism.

Miki has a large storage box that is also a rectangular prism.
The large storage box and the small storage box are similar in shape.
The volume of the large storage box is eight times the volume of the small storage box.

The length of the small storage box is 40 cm.
What is the length of the large storage box, in centimetres? (1 MARK)

VCAA 2017 Exam 2 Geometry and measurement Q1b

32% of students answered this question correctly.

---

17. A string of seven flags consisting of equilateral triangles in twosizes is hanging at the end of a racetrack, as shown in the following diagram.

The edge length of each black flag is twice the edge length of each white flag.
For this string of seven flags, the total area of the black flags would be

A. two times the total area of the white flags.

B. four times the total area of the white flags.

C. $\frac{4}{3}$ times the total area of the white flags.

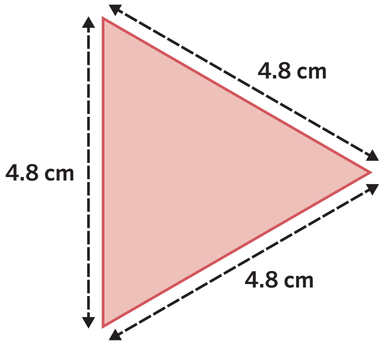
D. $\frac{16}{3}$ times the total area of the white flags.

E. $\frac{16}{9}$ times the total area of the white flags.

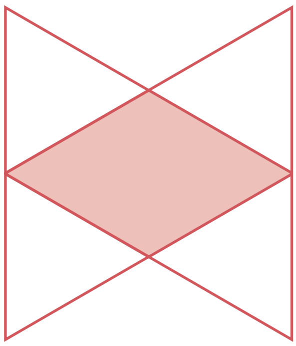
VCAA 2016 Exam 1 Geometry and measurement Q8

30% of students answered this question correctly.

18. Khaleda has designed a logo for her business.
The logo contains two identical equilateral triangles.
The side length of each triangle is 4.8 cm, as shown in the following diagram.



In the logo, the two triangles overlap. Part of the logo is shaded and part of the logo is not shaded, as shown in the following diagram.



The logo is enlarged and printed on boxes for shipping.
The enlarged logo and the original logo are similar in shape.
The area of the enlarged logo is four times the area of the original logo.
What is the height, in centimetres, of the enlarged logo? (1 MARK)

VCAA 2020 Exam 2 Geometry and measurement Q2d

17% of students answered this question correctly.

Questions from multiple lessons

Recursion and financial modelling

19. Consider the following recurrence relation.

$t_0 = -10, \quad t_{n+1} = 2t_n + 11$

Which term of the sequence generated by this relation is the first to be positive?

A. t_1
B. t_2
C. t_3
D. t_4
E. t_5

Adapted from VCAA 2018NH Exam 1 Recursion and financial modelling Q18

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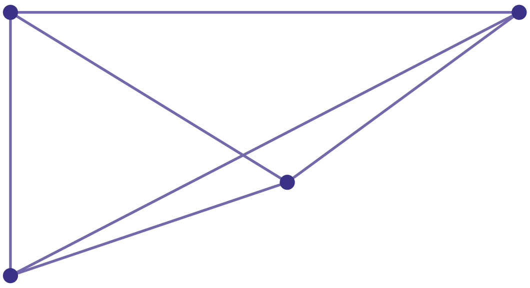
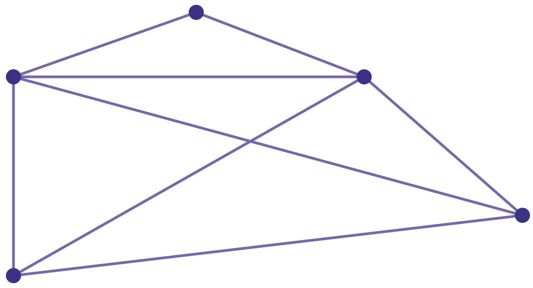
Networks and decision mathematics

20. Which one of the following graphs is **not** a planar graph?

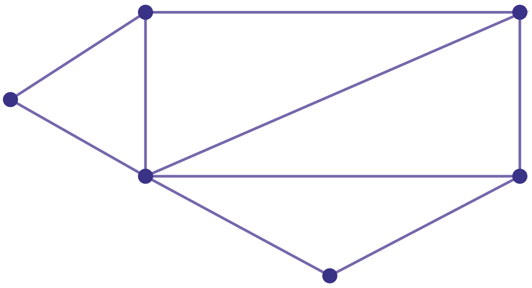
A.



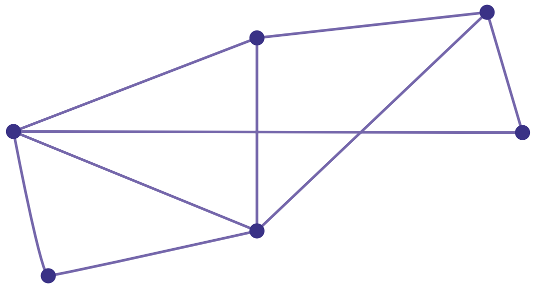
B.



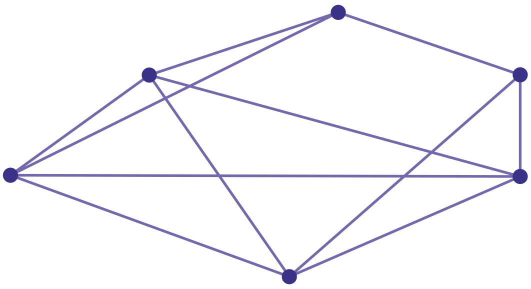
C.



D.



E.



Adapted from VCAA 2018 Exam 1 Networks and decision mathematics Q6

Computation and practical arithmetic *Year 10 content*

21. A water bottle contains 0.00327 litres of liquid.

a. What is 0.00327 converted to scientific notation? (1 MARK)

b. If 1.893×10^2 millilitres of water is added to the bottle, how much liquid, in mL, does the bottle now contain? (1 MARK)

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CHAPTER 10 CALCULATOR QUICK LOOK-UP GUIDE

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10

UNIT 2 AOS 4
CHAPTER 10
Trigonometry

LESSONS

KEY KNOWLEDGE

- the use of trigonometric ratios and Pythagoras' theorem to solve practical problems involving a right-angled triangle in two dimensions, including the use of angles of elevation and depression
- the use of the sine rule, including the ambiguous case, the cosine rule, as a generalisation of Pythagoras' theorem, and their application to solving practical problems involving non-right-angled triangles, including three-figure (true) bearings in navigation.

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10A Using trigonometry

STUDY DESIGN DOT POINT

- the use of trigonometric ratios and Pythagoras' theorem to solve practical problems involving a right-angled triangle in two dimensions, including the use of angles of elevation and depression

10A 10B 10C 10D 10E

KEY SKILLS

During this lesson, you will be:

- calculating side lengths of right-angled triangles using trigonometry
- calculating angles in right-angled triangles using trigonometry.

KEY TERMS

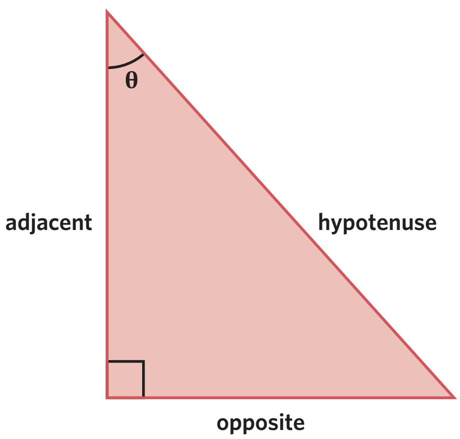
- Trigonometry
- Opposite side
- Adjacent side
- Sine
- Cosine
- Tangent

Triangles are everywhere. As most shapes can be split into multiple triangles, they are a powerful tool when performing calculations involving measurements. Triangles with a right angle are particularly unique, and their special properties allow a range of trigonometric calculations to be performed.

Calculating side lengths of right-angled triangles using trigonometry

Trigonometry is the study of the sides and angles that make up triangles, and the relationships between them. In trigonometry, the three sides of a right-angled triangle are called the hypotenuse, the opposite side and the adjacent side. Recall that the hypotenuse is the longest side of a right-angled triangle, and is always located directly opposite the right angle. The location of the opposite side and the adjacent side depend on the location of the reference angle, commonly labelled θ (theta). As the names suggest, the **opposite side** is located opposite to θ, and the **adjacent side** is located adjacent to (next to) θ.

Consider the following two triangles.



The hypotenuse, opposite side and adjacent side have been labelled with reference to the angle θ.

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The trigonometric ratios, **sine**, **cosine** and **tangent** (commonly referred to as sin, cos and tan) can be used to find the length of an unknown side or the size of an unknown angle in a right-angled triangle. The ratios can be written as the following equations

$$\sin(\theta) = \frac{opposite}{hypotenuse}$$

$$\cos(\theta) = \frac{adjacent}{hypotenuse}$$

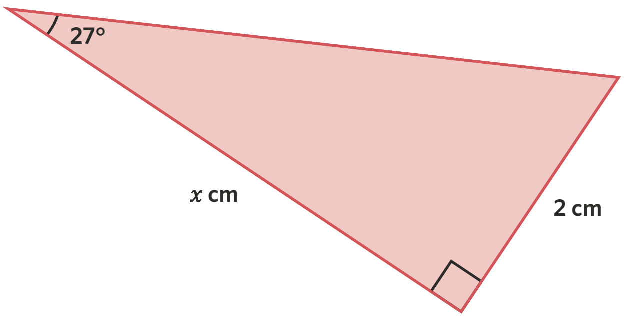
$$\tan(\theta) = \frac{opposite}{adjacent}$$

A helpful phrase often used to remember the equations is SOH CAH TOA. This phrase is a combination of the first letter of each component of these equations. For example, SOH is a reference to the equation $\mathbf{s}\text{in}(\theta) = \frac{\boldsymbol{o}pposite}{\boldsymbol{h}ypotenuse}$.

When calculating the length of an unknown side, it is important to first determine which trigonometric ratio should be applied. The values of the known side and angle are then substituted into the equation.

Worked example 1

Consider the following triangle.



a. Write the equation that can be used to calculate the value of x.

Explanation

Step 1: Identify the trigonometric ratio to be used.

The 2 cm side length is opposite the reference angle.

The unknown side length is adjacent to the reference angle.

The trigonometric ratio that uses the opposite and adjacent side lengths is tan.

Step 2: Substitute the known values into the trigonometric equation.

$$\tan(\theta) = \frac{opposite}{adjacent}$$

Answer

$\tan(27) = \frac{2}{x}$

b. Calculate the value of x, rounded to two decimal places.

Explanation – Method 1: TI-Nspire

Note: It is important to make sure that the CAS calculator is set to degrees mode and approximate (or decimal) mode when performing trigonometric calculations.

Step 1: From the home screen, select '5: Settings' → '2: Document Settings'.

Step 2: Make sure that 'Angle:' is set to 'Degree' and 'Calculation Mode:' is set to 'Approximate'.

Step 3: Select 'OK' to save the settings.

Step 4: From the home screen, select '1: New' → '1: Add Calculator'.

Continues →

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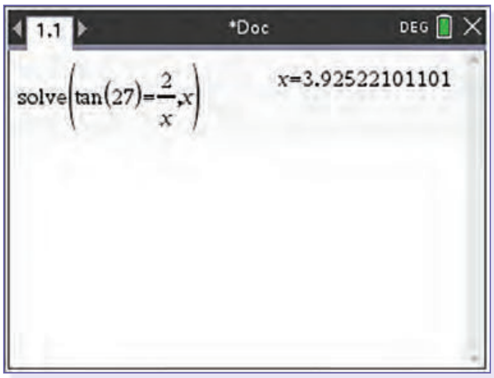
Step 5: Press menu. Select '3: Algebra' → '1: Solve'.

Step 6: Enter the equation and press enter.

sin, cos and tan can be found by pressing the trig button.

Make sure to end the equation with ',x'.



Explanation – Method 2: Casio ClassPad

Note: It is important to make sure that the CAS calculator is set to degrees mode and approximate (or decimal) mode when performing trigonometric calculations.

Step 1: From the main menu, tap √α Main.

Step 2: Along the bottom of the touch screen is a status bar.

The second status word cycles between 'Standard' and 'Decimal' when tapped.

Tap the status until 'Decimal' is displayed.

The last status word cycles between 'Rad', 'Deg' and 'Gra' when tapped.

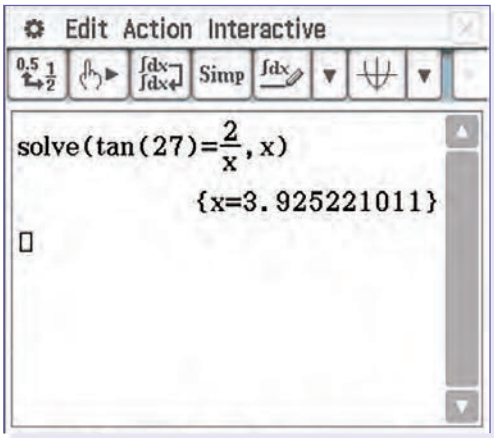
Tap the status until 'Deg' is displayed.

Step 3: Tap 'Action' in the menu bar. Select 'Advanced'→ 'solve'.

Step 4: Enter the equation and press EXE.

sin, cos and tan can be found by pressing keyboard and selecting 'Trig'.

Make sure to end the equation with ',x'.



Answer – Method 1 and 2

3.93 cm

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Calculating angles in right-angled triangles using trigonometry

The trigonometric ratios can also be used to find the size of an unknown angle in a right-angled triangle. Rearranged versions of the equations using the inverse functions of sin, cos and tan are used.

$$\theta = \sin^{-1}\left(\frac{opposite}{hypotenuse}\right)$$

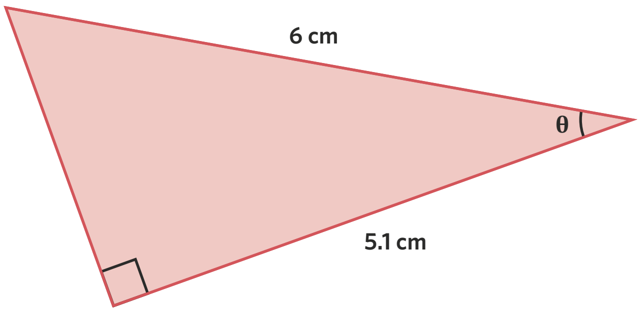
$$\theta = \cos^{-1}\left(\frac{adjacent}{hypotenuse}\right)$$

$$\theta = \tan^{-1}\left(\frac{opposite}{adjacent}\right)$$

When calculating the size of an unknown angle, it is important to first determine which trigonometric ratio should be applied. The values of the two known sides are then substituted into the equation.

Worked example 2

Consider the following triangle.



a. Write the equation that can be used to calculate the value of θ.

Explanation

Step 1: Identify the trigonometric ratio to be used.

The 5.1 cm side length is adjacent to the reference angle.

The 6 cm side length is the hypotenuse.

The trigonometric ratio that uses the adjacent and hypotenuse side lengths is cos.

Step 2: Substitute the known values into the trigonometric equation.

Since an angle is being calculated, use the inverse function.

$$\theta = \cos^{-1}\left(\frac{adjacent}{hypotenuse}\right)$$

Answer

$$\theta = \cos^{-1}\left(\frac{5.1}{6}\right)$$

Continues →

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b. Calculate the value of θ, rounded to two decimal places.

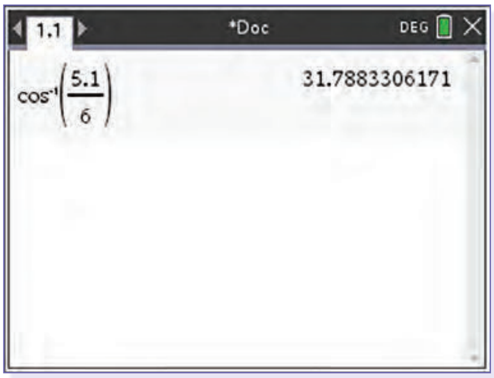
Explanation – Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '1: Add Calculator'.

Step 2: Enter the expression and press enter.

$\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ can be found by pressing the trig button.

There is no need to input 'θ = '.

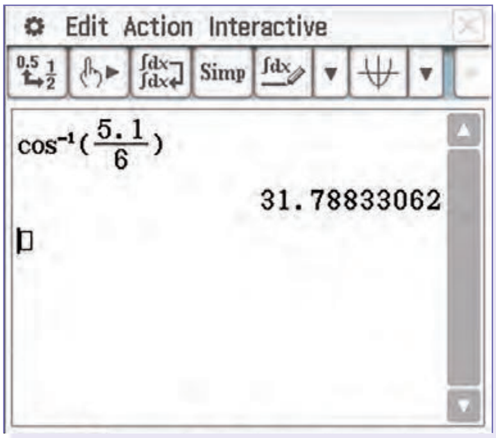


Explanation – Method 2: Casio ClassPad

Step 1: From the main menu, tap √α Main.

Step 2: Enter the equation and press EXE.

$\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ can be found by pressing keyboard and selecting 'Trig'.



Answer – Method 1 and 2

31.79°

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10A Questions

Calculating side lengths of right-angled triangles using trigonometry

1. Fill in the blanks.

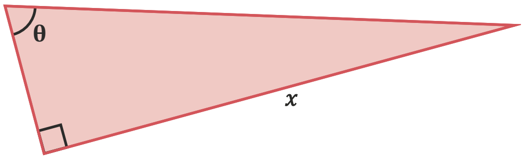
$\sin(\theta) = \dfrac{\boxed{}}{\textit{hypotenuse}}$

$\cos(\theta) = \dfrac{\boxed{}}{\textit{hypotenuse}}$

$\tan(\theta) = \dfrac{\boxed{}}{\boxed{}}$

---

2. Consider the following diagram.

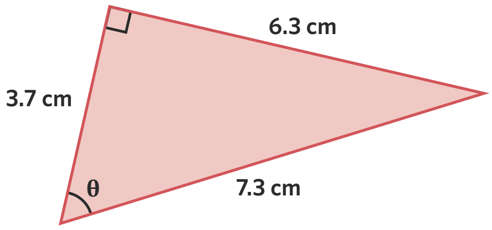


The side length labelled x is known as the

A. hypotenuse. **B.** opposite side. **C.** reference side. **D.** adjacent side.

---

3. What is the length of the adjacent side of the following triangle, with reference to the angle θ?

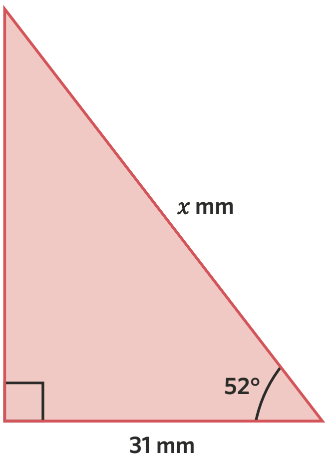


---

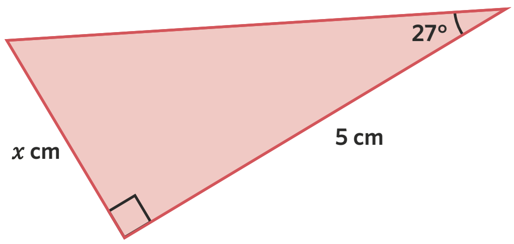
4. For the following triangles:

- write the equation that can be used to calculate the value of x.
- calculate the value of x, rounded to two decimal places.

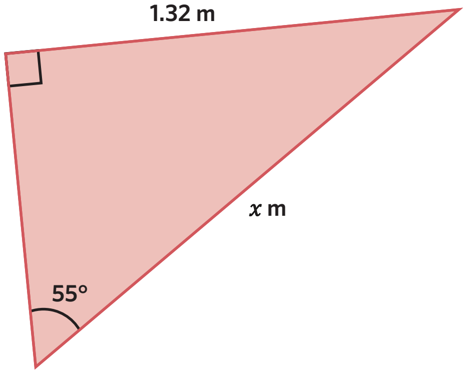
a.



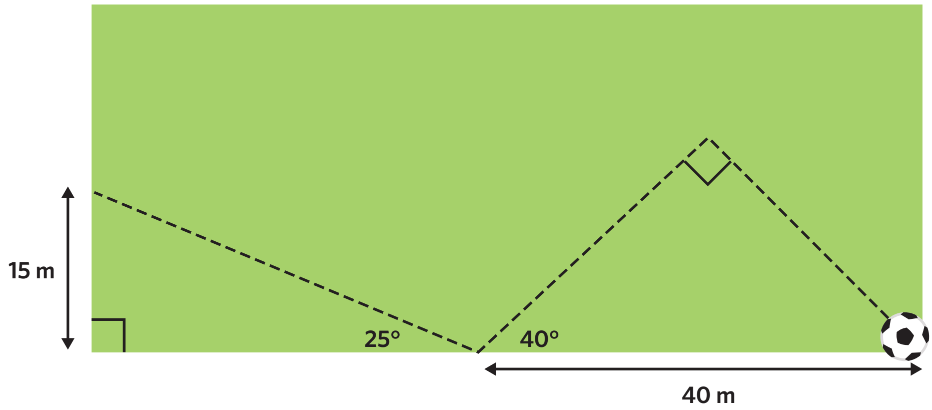
b.



c.



5. A soccer ball travels along the dotted path shown in the following diagram.

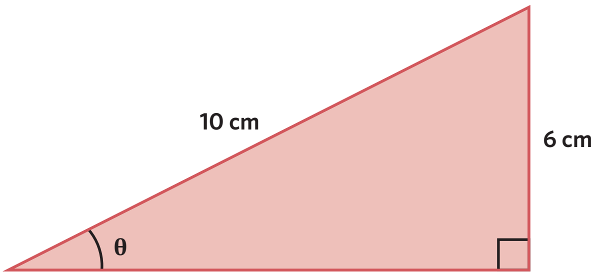


What is the total distance travelled by the soccer ball, rounded to the nearest metre?

6. A bird swoops, in a straight path, from a tree with a height of four metres, to a worm on the ground. If the path the bird takes makes an angle of 30° with the ground, what is the distance travelled by the bird?

Calculating angles in right-angled triangles using trigonometry

7. Consider the following triangle.



Which trigonometric ratio can be used to find the value of θ?

A. adjacent

B. sin

C. cos

D. tan

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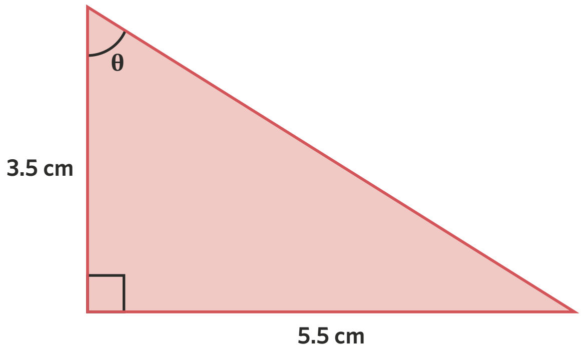
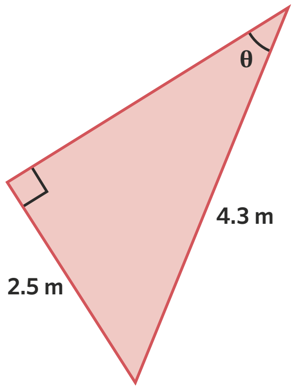
8. For the following triangles:
   - write the equation that can be used to calculate the value of θ.
   - calculate the value of θ, rounded to two decimal places.

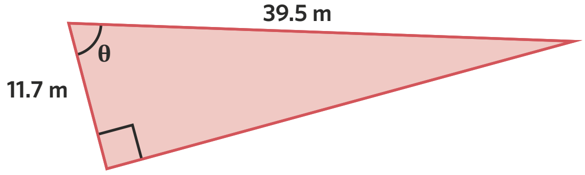
a.



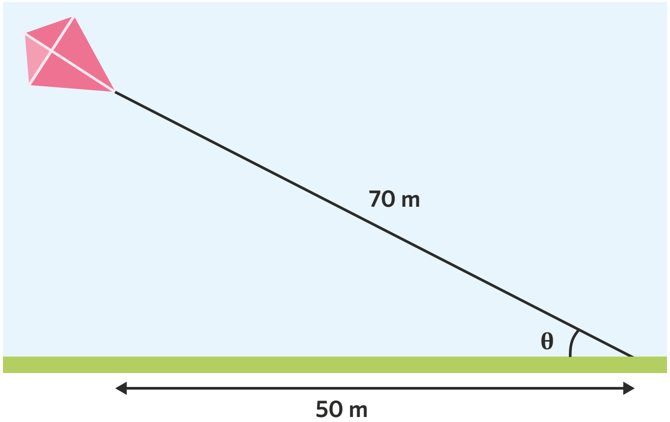
b.



c.



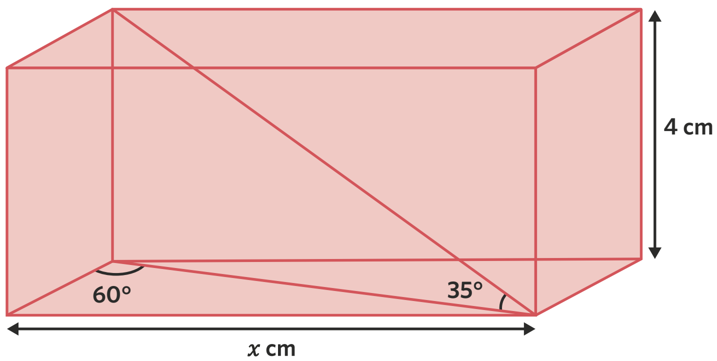
9. A student is flying a kite on a windy day. The string of the kite is 70 m long and the kite flies at a horizontal distance of 50 m away from the student.



What angle does the string of the kite form with the ground, rounded to the nearest degree?

Joining it all together

10. Consider the following rectangular prism.



Calculate the value of x, rounded to two decimal places.

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11. A student worked out that an angle in a right-angled triangle was 47°. However, instead of using $\tan(\theta) = \frac{opposite}{adjacent}$, they mistakenly used $\tan(\theta) = \frac{adjacent}{opposite}$. If the length of the opposite side was 3.7 cm, what should the angle have actually been, rounded to the nearest degree?

12. A three metre ladder leans against the wall such that the angle between the ladder and the wall is 30°.

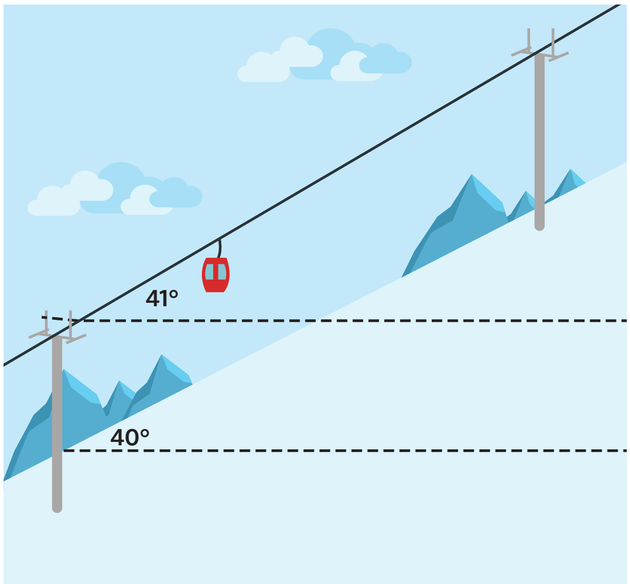
a. How far away is the base of the ladder from the wall?

b. The ladder slips 20 cm down the wall. What is the new angle between the ladder and the wall, rounded to the nearest degree?

13. A ski slope has an incline of 40°, whereas the ski lift has an incline of 41°.

a. Over a horizontal distance of 100 metres, how many more metres are travelled by making a descent in the ski lift compared to skiing directly down the slope? Round to two decimal places.

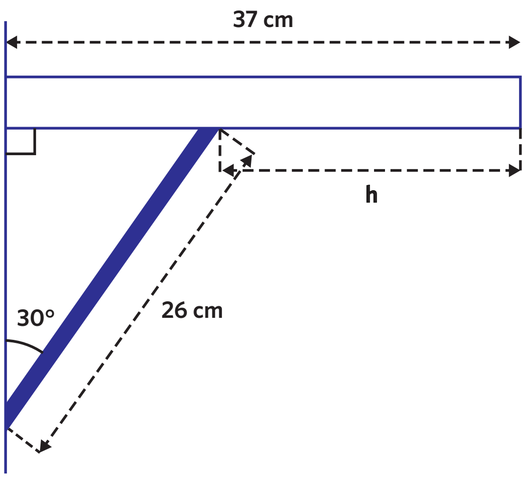
b. Over what minimum horizontal distance will the ski lift rise two metres more than the slope? Round to two decimal places.



Exam practice

14. A shelf sits against a wall at a 90° angle.

The shelf is supported by a 26 cm bracket that forms a 30° angle with the wall, as shown in the following diagram.



Find the value of h, the distance between the edge of the shelf and the bracket, in centimetres. (1 MARK)

VCAA 2020 Exam 2 Geometry and measurement Q1f

67% of students answered this question correctly.

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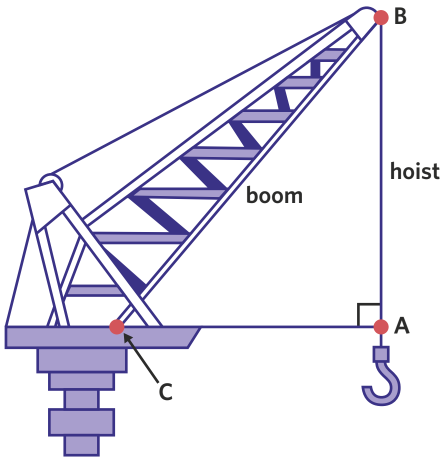
15. The following diagram shows a crane that is used to transfer shipping containers between a port and cargo ship.



The length of the boom, BC, is 25 m. The length of the hoist, AB, is 15 m.

Find the angle ACB.

Round to the nearest degree. (1 MARK)

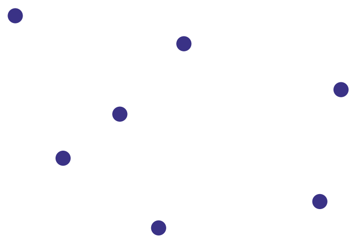
VCAA 2019 Exam 2 Geometry and measurement Q3aii

62% of students answered this question correctly.

Questions from multiple lessons

Networks and decision mathematics

16. The following graph is made up of seven isolated vertices.



The minimum number of edges that must be added to the graph to form a tree is

A. 1

B. 5

C. 6

D. 7

E. 8

Adapted from VCAA 2018 Exam 1 Networks and decision mathematics Q1

Recursion and financial modelling

17. A sequence can be generated using the following recurrence relation.

$a_0 = 3, \quad a_{n+1} = 5 \times a_n$

An expression for the value in the sequence after n periods is given by

A. $a_n = 3 + 5n$ **B.** $a_n = 5 + 3n$ **C.** $a_n = 3 \times 5^n$ **D.** $a_n = 5 \times n^3$ **E.** $a_n = 5 \times 3^n$

Adapted from VCAA 2013 Exam 1 Number patterns Q5

Recursion and financial modelling

18. Carl has just opened his business selling backpacks made out of recycled plastic.

The business charges customers \$140 per backpack.

This price includes GST.

a. Calculate the amount of GST included in the price of the backpack, rounded to the nearest cent. (1 MARK)

b. A customer purchases 8 backpacks. If the GST amount is to be paid to the government in taxes, calculate the total amount of money that Carl makes from this purchase, rounded to the nearest dollar. (1 MARK)

Adapted from VCAA 2015 Exam 2 Business-related mathematics Q1a,b

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10B Applications of trigonometry

STUDY DESIGN DOT POINT

- the use of trigonometric ratios and Pythagoras' theorem to solve practical problems involving a right-angled triangle in two dimensions, including the use of angles of elevation and depression

10A 10B 10C 10D 10E

KEY SKILLS

During this lesson, you will be:

- identifying and using angles of elevation
- identifying and using angles of depression.

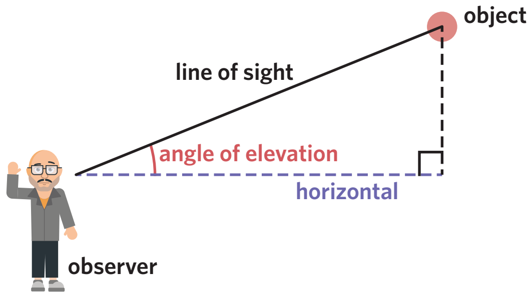
KEY TERMS

- Angle of elevation
- Angle of depression

Trigonometry is useful in a wide range of real-world applications, like calculating the lift-off angle of a plane, or the distance of a ship being observed from a lookout point. Angles of elevation and depression are useful ways of defining angles for use in these real-world problems.

Identifying and using angles of elevation

An **angle of elevation** is an angle measured up from a horizontal. The term is often used to describe the line of sight from an observer looking up to an object being observed.



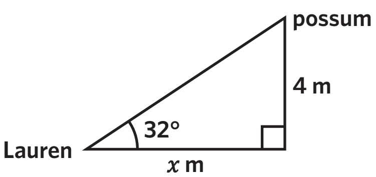
Worked example 1

Lauren is standing in a park and sees a ringtail possum four metres up a tree. The angle of elevation from Lauren to the possum is 32°. What is the horizontal distance between Lauren and the tree, rounded to the nearest centimetre?

Explanation

Step 1: Draw a diagram.

Make sure to label the angle of elevation as the angle measured up from the horizontal.



Step 2: Identify the trigonometric ratio that can be used to calculate the unknown distance, x.

The 4 m side length is opposite the angle of elevation.

The unknown side length is adjacent to the angle of elevation.

Therefore, the tan ratio will be used.

Continues →

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Step 3: Substitute the known side lengths into the equation and solve for x.

$$\tan(\theta) = \frac{opposite}{adjacent}$$

$$\tan(32°) = \frac{4}{x}$$

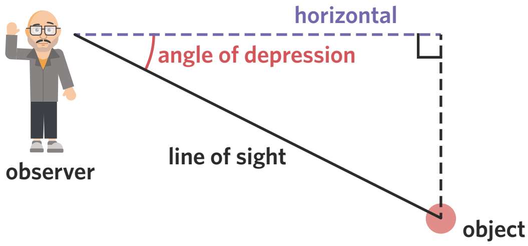
$$x = 6.4013...$$

Answer

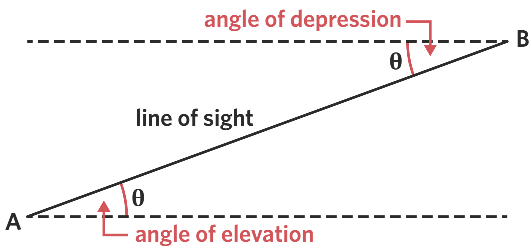
6.40 m

Identifying and using angles of depression

An **angle of depression** is an angle measured down from a horizontal. The term is often used to describe the line of sight from an observer looking down to an object being observed.



Angles of elevation and depression are alternate angles. Hence, the angle of elevation of point B from A will always be equal to the angle of depression of point A from B.



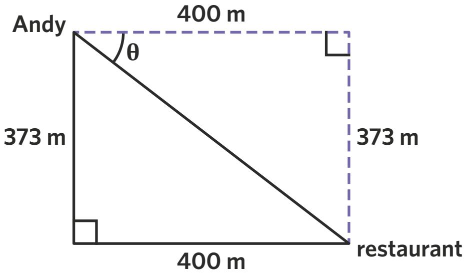
Worked example 2

Andy is standing on the observation deck of the Empire State Building, which is 373 metres above ground level. Andy can see his favourite restaurant on the street below, which he knows is 400 metres from the base of the building. What is the angle of depression, rounded to the nearest degree, from Andy to the restaurant?

Explanation

Step 1: Draw a diagram.

Make sure to label the angle of depression as the angle measured down from the horizontal.



Step 2: Identify the trigonometric ratio that can be used to calculate the unknown angle, θ.

The 373 m side length is opposite the unknown angle of depression.

The 400 m side length is adjacent to the unknown angle of depression.

Therefore, the tan ratio will be used.

Continues →

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Step 3: Substitute the known side lengths into the equation and evaluate.

$$\tan(\theta) = \frac{opposite}{adjacent}$$

$$\theta = \tan^{-1}\left(\frac{opposite}{adjacent}\right)$$

$$= \tan^{-1}\left(\frac{373}{400}\right)$$

$$= 42.999...^\circ$$

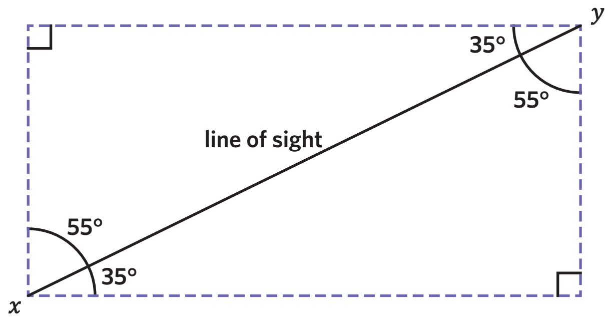
Answer

43°

10B Questions

Identifying and using angles of elevation

1. Consider the following diagram.



What is the angle of elevation of y from x?

A. 35°

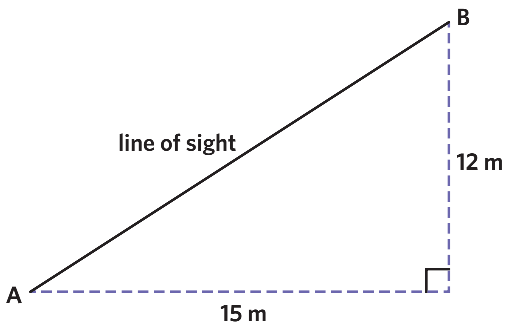
B. 55°

C. 90°

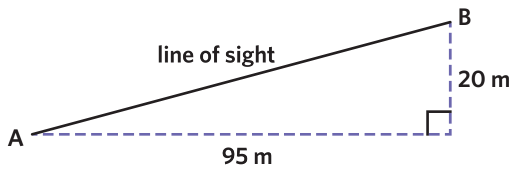
D. 180°

2. For each of the following diagrams, calculate the angle of elevation of point B from point A, rounded to two decimal places.

a.



b.



3. Sam is standing in a field when he spots a UFO in the sky. The UFO is 120 metres away from him horizontally, and is 45 metres above the ground. What is the angle of elevation of the UFO from Sam, rounded to two decimal places?

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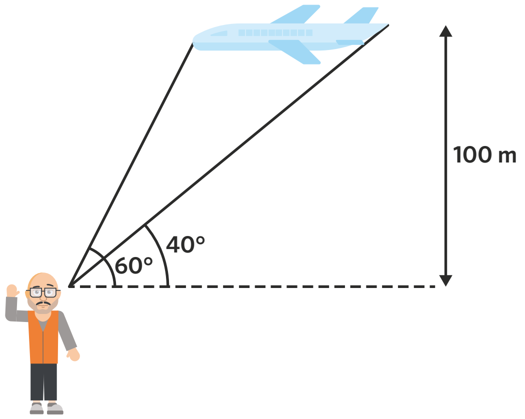
4. Mason is on holiday in Dubai and decides to visit the tallest structure in the world, the Burj Khalifa. He stands on the ground, 270 metres from the base of the tower and the angle of elevation to the top of the tower from where he is standing is 72°. What is the vertical height of the Burj Khalifa, in metres, rounded to two decimal places?

5. An airport control officer is standing on the runway and sees an aeroplane in the sky facing in his direction. From where he is standing, the angle of elevation to the front of the plane is 60°, and the angle of elevation to the back of the plane is 40°.



Given that the plane is flying parallel to the ground at an altitude of 100 metres, what is the length of the aeroplane, in metres, rounded to one decimal place?

Identifying and using angles of depression

6. Consider the following diagram.

What is the angle of depression, in degrees, of point x from point y?

A. 20°

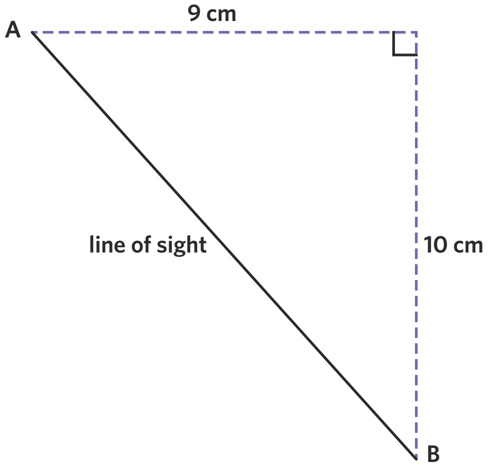
B. 70°

C. 90°

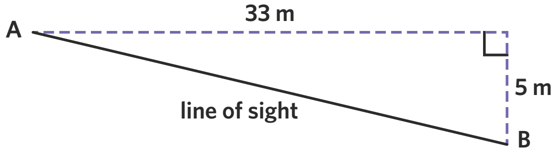
D. 180°

7. For each of the following diagrams, calculate the angle of depression of point B from point A, rounded to two decimal places.

a.

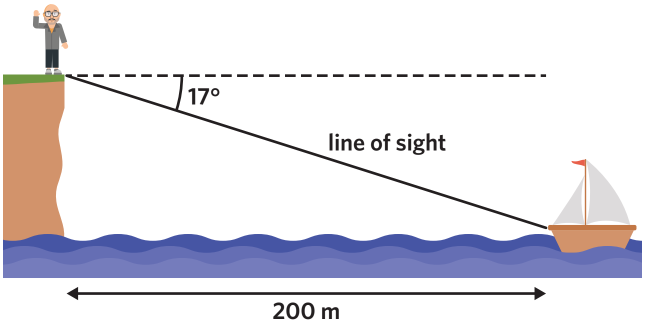


b.



8. A person standing on the edge of a cliff can see a boat at an angle of depression of 17° from where they are standing.

   Calculate the height of the cliff, in metres, rounded to two decimal places.



9. Angie is attending a Beyoncé concert but could only afford tickets at the very back of the arena. Her seat is 200 metres horizontally from the centre of the stage, and is 20 metres higher than the stage. If Beyoncé stands in the centre of the stage, what is the angle of depression, in degrees, from Angie's seat to where Beyoncé stands? Round to one decimal place.

10. Gordon is sitting in his treehouse 3.2 metres above the ground when he sees a kookaburra's nest in a nearby tree. The nest is 2.55 metres above the ground and the angle of depression of the nest from Gordon is 10°. What is the horizontal distance, in metres, between the two trees? Round to the nearest centimetre.

11. Lauren is in a hot air balloon, flying at a height of 300 metres, and has spotted both her house and her school. The angle of depression to her house is 65°, and the angle of depression to her school is 22°. Calculate the shortest possible distance between Lauren's house and school, in metres, rounded to one decimal place.

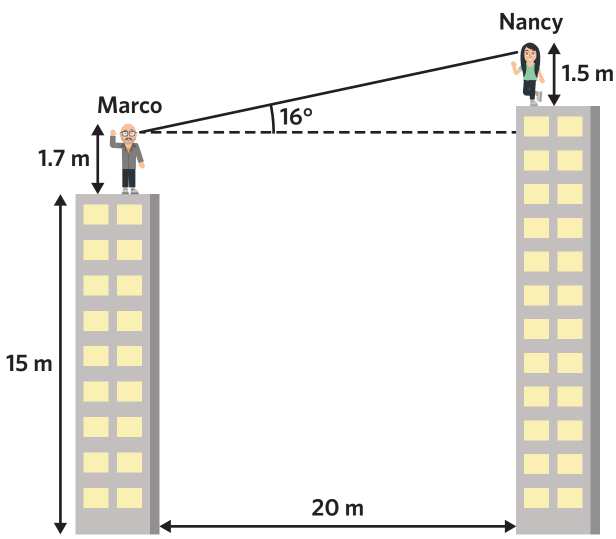
Joining it all together

12. A zipline is being constructed through a jungle in Thailand. The zipline will start from an eight-metre platform and finish on the ground. The length of rope needed for the zipline is 170 metres.
    a. When standing on the starting platform, what will be the angle of depression to the bottom of the zipline, in degrees? Rounded to two decimal places.
    b. What will be the horizontal distance spanned by the zipline, rounded to the nearest centimetre?
    c. The company constructing the zipline decides to construct an even higher platform that people can zipline from. This platform is three metres higher than the original platform.

       If the angle of elevation from the bottom of the second zipline to its starting platform is 4°, calculate the length of rope needed for this zipline, rounded to the nearest metre.

13. Marco is standing on the roof of a 15-metre-tall building. Marco is 1.7 metres tall. He can see his friend Nancy, who is 1.5 metres tall, on top of a nearby building, at an angle of elevation of 16°. The buildings are 20 metres apart.

    How tall is the second building, in metres, rounded to one decimal place?



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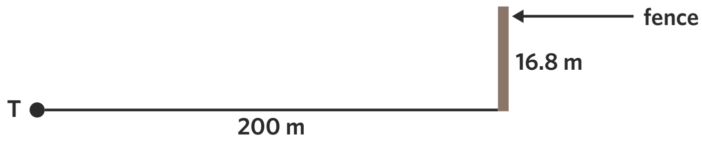
Exam practice

14. A fence is positioned at the end of a golf driving range.

This fence is 16.8 m high and 200 m from the point T.



What is the angle of elevation from T to the top of the fence?

Round to the nearest degree. (1 MARK)

VCAA 2016 Exam 2 Geometry and measurement Q2b

66% of students answered this question correctly.

Questions from multiple lessons

Data analysis

15. The following scatterplot displays the *length* and *weight* of a group of newborn babies. A line of good fit has been fitted to the data, as shown.

The equation of the line is closest to

A. $length = 2.60 + 0.12 \times weight$

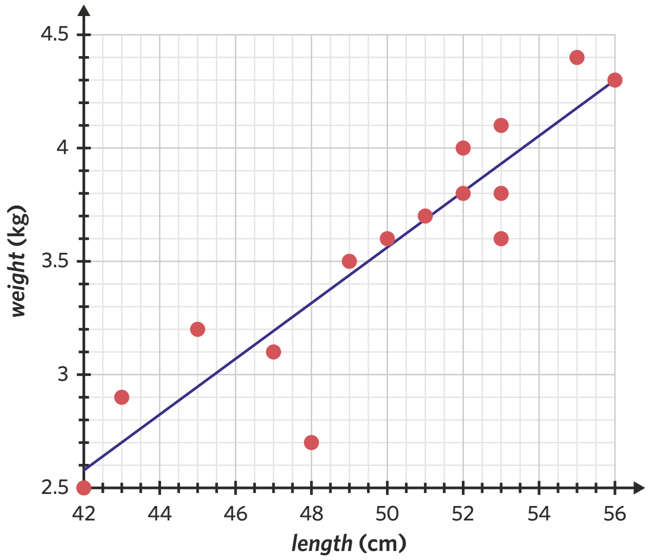
B. $weight = 2.60 + 0.12 \times length$

C. $length = 0.12 + 2.60 \times weight$

D. $length = -2.60 + 0.12 \times weight$

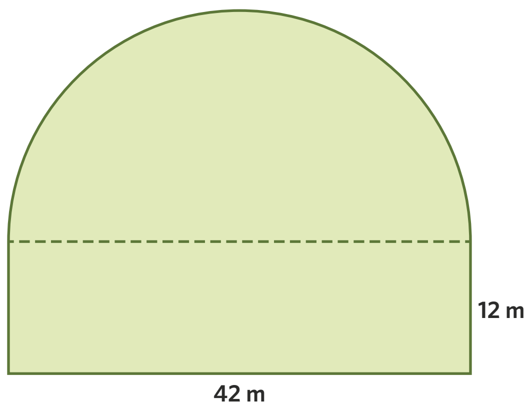
E. $weight = -2.60 + 0.12 \times length$

Adapted from VCAA 2017NH Exam 1 Data analysis Q10



Geometry and measurement

16. A dairy farmer is planning on expanding his farm. There is a plot of land for sale and he plans to purchase it. It consists of a rectangle and a semicircle, as shown in the following diagram.



The farmer wants to construct a fence around the perimeter. The distance of fence that he needs is closest to

A. 42 m **B.** 66 m **C.** 104 m **D.** 132 m **E.** 198 m

Adapted from VCAA 2015 Exam 1 Geometry and trigonometry Q2

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Recursion and financial modelling

17. Corey is a footballer and decides to set himself a challenge in regards to the number of goals he can kick every day after school.

On the 1st day, he kicked 42 goals.

On the 2nd day, he kicked 60 goals.

On the 3rd day, he kicked 78 goals.

The number of goals he kicked each day forms the terms of an arithmetic sequence:

42, 60, 78…

This sequence continues for each night for a week.

a. Write down a calculation which shows that the common difference for this sequence is 18. (1 MARK)

b. How many goals will Corey kick on the 6th day? (1 MARK)

Adapted from VCAA 2015 Exam 2 Number patterns Q1a,b

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10C The sine rule

STUDY DESIGN DOT POINT

- the use of the sine rule, including the ambiguous case, the cosine rule, as a generalisation of Pythagoras' theorem, and their application to solving practical problems involving non-right-angled triangles, including three-figure (true) bearings in navigation

10A 10B 10C 10D 10E

KEY SKILLS

During this lesson, you will be:

- calculating side lengths of a triangle using the sine rule
- calculating angles in a triangle using the sine rule
- solving the ambiguous case of the sine rule
- calculating the area of a triangle using the sine rule.

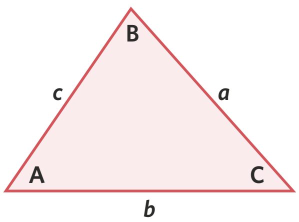
KEY TERMS

- Sine rule

When working with triangles that don't contain a right angle, trigonometry is no longer useful. The sine rule provides one way to perform calculations on triangles to find unknown side lengths or angles, and has powerful applications, such as calculating the area of a triangle.

Calculating side lengths of a triangle using the sine rule

When considering non-right-angled triangles, a standard labelling system is used to make referencing side lengths and angles easier. Consider the following diagram.



The angles of the triangle are labelled with uppercase letters. The sides are labelled with the lowercase letter of the angle directly opposite them.

The **sine rule** states that in the triangle ABC:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

In other words, the ratio between a side and the sine value of its opposite angle will be consistent for all three sides of the triangle.

To calculate an unknown side length of a non-right-angled triangle, two ratios from the sine rule are isolated, such as

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

Note: Any two ratios can be used.

Therefore, to calculate the unknown side length, the following information must be known:

- the value of its opposite angle
- one other side length and its opposite angle

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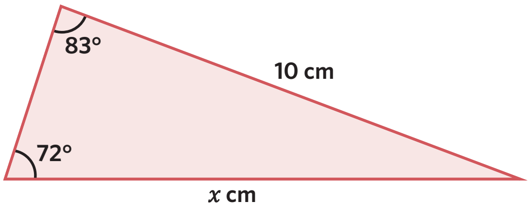
Worked example 1

Consider the following diagram.



Use the sine rule to calculate the value of x, rounded to two decimal places.

Explanation

Step 1: Identify two angle/side length pairs.

Make sure that there is exactly one unknown value only.

The 83° angle pairs with the unknown x side length.

$A = 83°$

$a = x$ cm

The 72° angle pairs with the 10 cm side length.

$B = 72°$

$b = 10$ cm

Step 2: Substitute the values into the unknown side length application of the sine rule.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

$$\frac{x}{\sin(83°)} = \frac{10}{\sin(72°)}$$

$$x = \frac{10 \times \sin(83°)}{\sin(72°)}$$

Step 3: Use a calculator to evaluate x.

$x = 10.436...$ cm

Answer

10.44 cm

Calculating angles in a triangle using the sine rule

The sine rule can also be used to calculate unknown angles in a non-right-angled triangle.

When the side length application is rearranged, the sine rule becomes

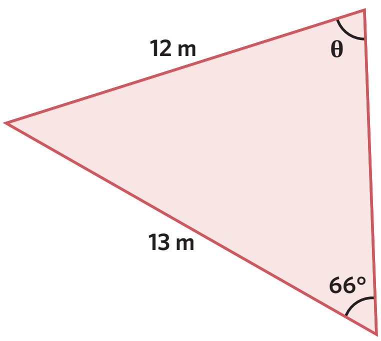
$A = \sin^{-1}\left(\frac{a \times \sin(B)}{b}\right)$, where A is an unknown angle.

Therefore, to calculate the unknown angle, the following information must be known:

- the length of its opposite side
- one other side length and its opposite angle

Worked example 2

Consider the following diagram.



Use the sine rule to calculate the value of θ, rounded to two decimal places.

Continues →

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Explanation

Step 1: Identify two angle/side length pairs.

Make sure that there is exactly one unknown value only.

The unknown angle θ pairs with the 13 m side length.

$A = \theta$

$a = 13$ m

The 66° angle pairs with the 12 m side length.

$B = 66°$

$b = 12$ m

Step 2: Substitute the values into the unknown angle application of the sine rule.

$$A = \sin^{-1}\left(\frac{a \times \sin(B)}{b}\right)$$

$$\theta = \sin^{-1}\left(\frac{13 \times \sin(66°)}{12}\right)$$

Step 3: Use a calculator to evaluate θ.

$\theta = 81.759...°$

Answer

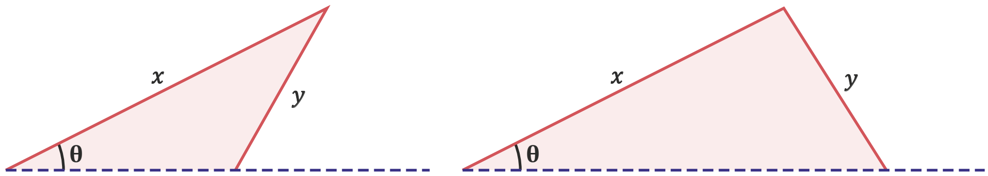
81.76°

Solving the ambiguous case of the sine rule

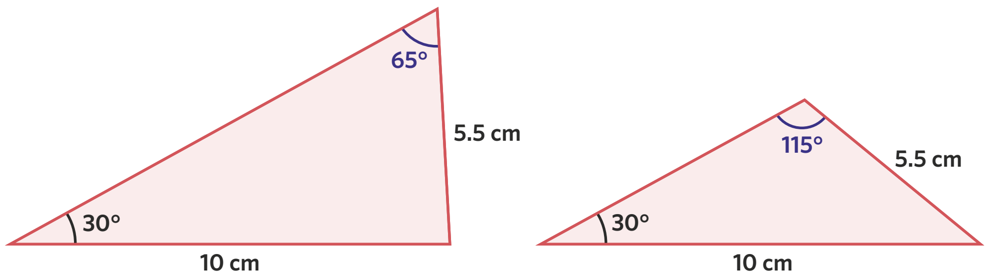
In some cases, the information given for a triangle can lead to an ambiguous application of the sine rule. The following leads to the ambiguous case:

- two sides and one non-included angle are known (that is, an angle not made where the two known sides meet)
- the known angle is acute
- the shorter of the two known sides is opposite the known angle

Two possible triangles can be drawn with the given information, as shown in the following diagram.



For example, consider the following triangles.



Both triangles have

- known side lengths 10 cm and 5.5 cm
- a known non-included angle of 30°
- the shorter known side opposite the known angle.

However, one of the triangles has an acute angle (65°) and the other an obtuse angle (115°) between the short side and the unknown side. These two angles will sum to 180° when both triangles exist (the ambiguous case).

When performing the sine rule calculations for such triangles, the values entered are the same for both triangles, so only one possible solution is obtained. The other possible solution must be calculated separately by subtracting the angle obtained from the sine rule calculation from 180°.

Worked example 3

Consider the following information about triangle ABC:

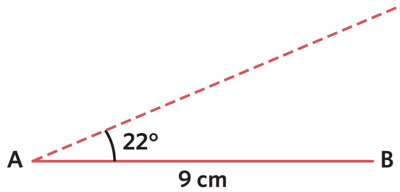
- side AB has a length of 9 centimetres
- side BC has a length of 4 centimetres
- angle BAC has a magnitude of 22°

a. Draw all possible triangles that satisfy this information.

Explanation

Step 1: For both triangles, draw the longer side and the known angle.

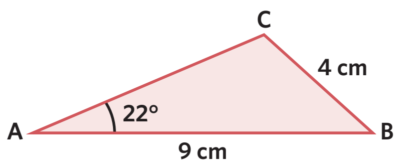
In this case, draw side AB and angle BAC.



Point C will lie somewhere along the dotted line.

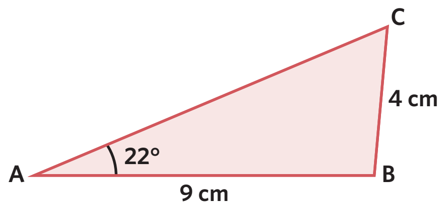
Step 2: For the first triangle, draw the shorter side so that the angle between the shorter side and the unknown side is obtuse.

In this case, draw side BC so that angle ACB is obtuse.

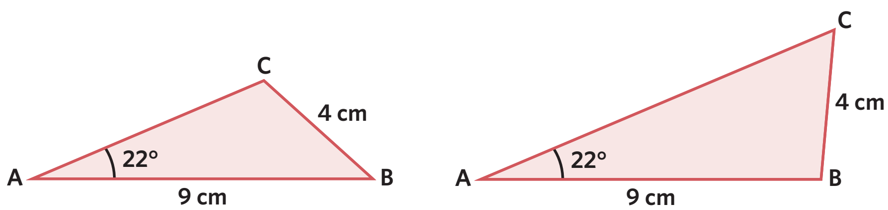


Step 3: For the second triangle, draw the shorter side so that the angle between the shorter side and the unknown side is acute.

In this case, draw side BC so that angle ACB is acute.



Answer



b. Find all possible values of the angle ACB, rounded to two decimal places.

Explanation

Step 1: Identify two angle/side length pairs.

The unknown angle ACB pairs with the 9 cm side length.

$A = ACB$

$a = 9$ cm

The 22° angle pairs with the 4 cm side length.

$B = 22°$

$b = 4$ cm

Step 2: Substitute the values into the unknown angle application of the sine rule.

$$A = \sin^{-1}\left(\frac{a \times \sin(B)}{b}\right)$$

$$ACB = \sin^{-1}\left(\frac{9 \times \sin(22°)}{4}\right)$$

$$= 57.44...°$$

The first possible value for angle ACB is 57.44°.

Continues →

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Step 3: Determine the second possible value for angle ACB.

Subtract the angle obtained from the sine rule calculation from 180°.

$ACB = 180 - 57.44...$

$= 122.56...°$

The second possible value for angle ACB is 122.56°.

Answer

57.44° and 122.56°

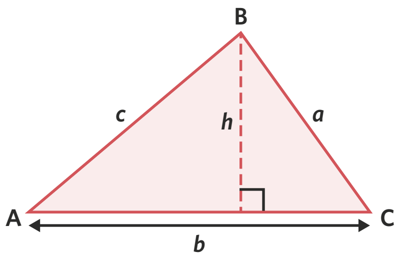
Calculating the area of a triangle using the sine rule

Recall that the basic formula for the area of a triangle is

$A = \frac{1}{2}bh$

In some cases, the perpendicular height is not known. An application of the sine rule allows for the area of a triangle to still be calculated.

Consider the following triangle.



Using an application of the sine rule, the basic area formula can be rewritten as

$A = \frac{1}{2}ab \times \sin(C)$

This formula can be used to calculate the area of a triangle when any two sides and their included angle are known.

Worked example 4

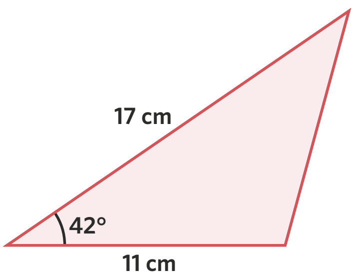
Consider a triangle with the following properties:

- one side has a length of 17 centimetres
- another side has a length of 11 centimetres
- the included angle has a magnitude of 42°

Calculate the area of the triangle, rounded to two decimal places.

Explanation

Step 1: Draw the diagram.



Step 2: Determine the values of each variable.

a and b are side lengths and C is their included angle.

$a = 17$ cm

$b = 11$ cm

$C = 42°$

Continues →

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Step 3: Substitute the values into the area formula and evaluate.

$$A = \frac{1}{2}ab \times \sin(C)$$
$$= \frac{1}{2} \times 17 \times 11 \times \sin(42°)$$
$$= 62.563...\ \text{cm}^2$$

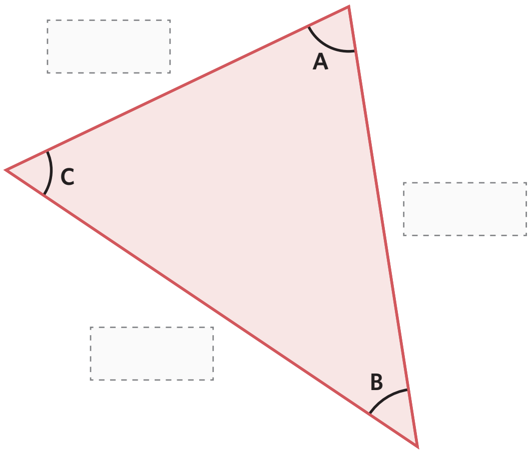
Answer

62.56 cm^2

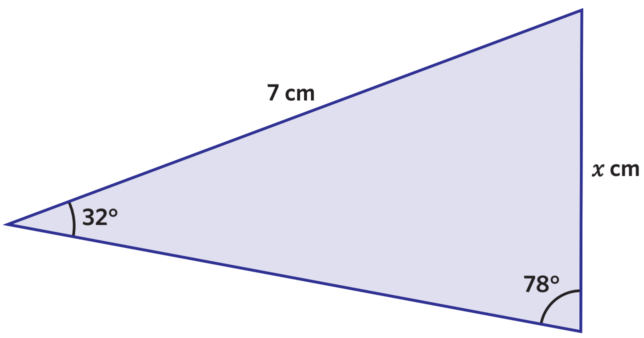
10C Questions

Calculating side lengths of a triangle using the sine rule

1. Finish the following diagram by labelling the side lengths.



2. Consider the following triangle.



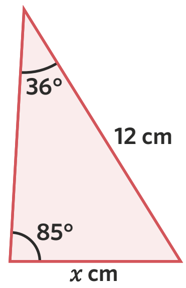
a. Fill the blank spaces in the sine rule shown.

$$\frac{x}{\sin(\square°)} = \frac{\square}{\sin(\square°)}$$

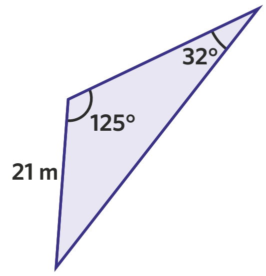
b. Calculate the value of x, rounded to two decimal places.

3. Use the sine rule to find the value of x in each of the following triangles. Round to two decimal places.

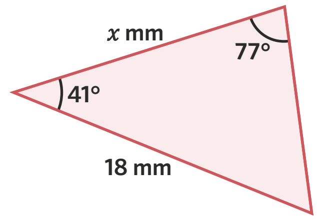
a.



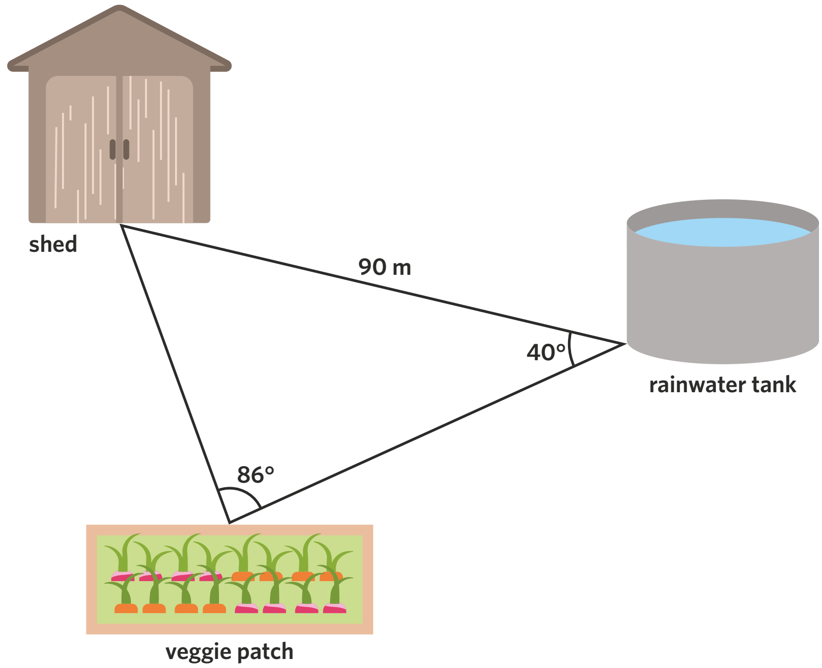
b.



c.



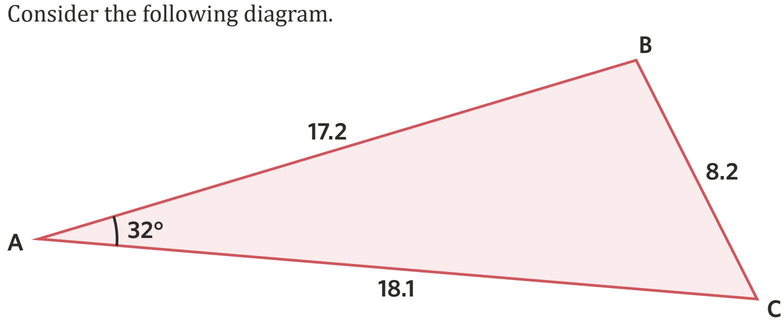
4. Charlie took some measurements of his garden and drew them on a map.



Show that the distance, rounded to the nearest metre, between the shed and the veggie patch is 58 metres.

Calculating angles in a triangle using the sine rule

5. Consider the following diagram.



Which of the following equations can be used to calculate angle ACB?

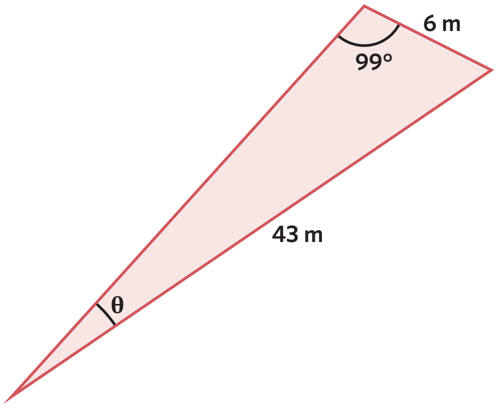
A. $ACB = \sin^{-1}\left(\frac{18.1 \times \sin(32°)}{8.2}\right)$

B. $ACB = \sin^{-1}\left(\frac{17.2 \times \sin(32°)}{8.2}\right)$

C. $ACB = \sin^{-1}\left(\frac{8.2 \times \sin(32°)}{17.2}\right)$

D. $ACB = \sin^{-1}\left(\frac{32 \times \sin(17.2°)}{8.2}\right)$

6. Consider the following triangle.



a. Fill in the blank spaces in the following working.

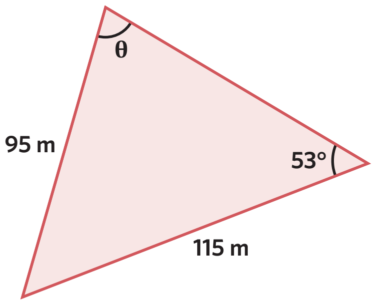
$$\theta = \sin^{-1}\left(\boxed{} \times \frac{\sin\left(\boxed{}^{\circ}\right)}{\boxed{}}\right)$$

b. Find the value of θ, rounded to one decimal place.

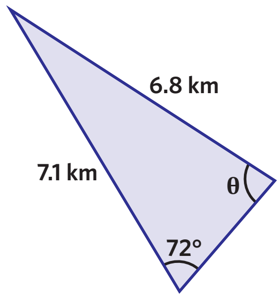
---

7. Use the sine rule to find the value of the unknown angle θ in each of the following triangles, rounded to two decimal places.

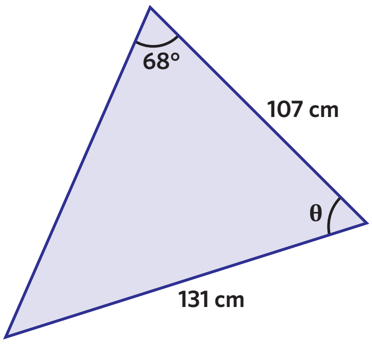
a.



b.



c.



Solving the ambiguous case of the sine rule

8. Which of the following is not a condition for the ambiguous case of the sine rule?

A. Two sides and one non-included angle are known.

B. The known angle is acute.

C. The triangle contains a right angle.

D. The shorter of the two known sides is opposite the known angle.

---

9. In an ambiguous case of the sine rule, the magnitude of the unknown angle is calculated as 61°. What is the other possible magnitude of the angle?

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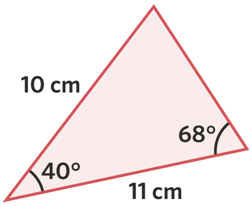
10. In the triangle ABC, side AC has a length of 9 cm, side BC has a length of 12 cm, and angle ABC has a magnitude of 32°.
    a. Draw the two possible triangles that satisfy this information.
    b. Find the two possible values of the angle BAC, each rounded to two decimal places.
    c. Find the two possible values of the angle ACB, each rounded to two decimal places.

11. Triangle ABC satisfies the following properties:
    - side AB has a length of 20 cm
    - side AC has a length of 30 cm
    - angle ACB has a magnitude of 40°

    Calculate all possible lengths of the side BC, rounded to one decimal place.

Calculating the area of a triangle using the sine rule

12. Consider the following triangle.

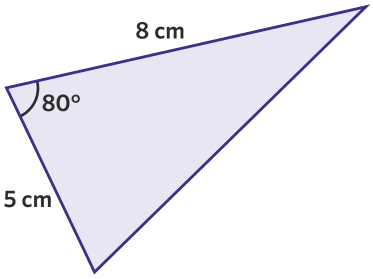


Fill in the blank spaces in the following working.

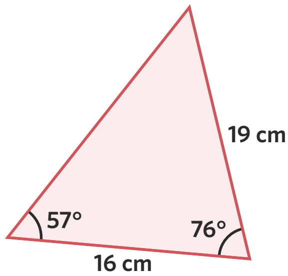
$$A = \frac{1}{2} \times 11 \times \boxed{} \times \sin\left(\boxed{}^{\circ}\right)$$

13. Calculate the area of each of the following triangles, rounded to two decimal places.

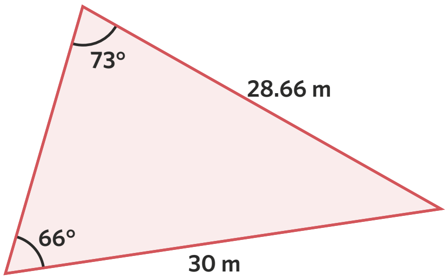
a.



b.



c.



14. Triangle XYZ has the following properties:
    - side XY has a length of 21 cm
    - side YZ has a length of 27.4 cm
    - angle XYZ has a magnitude of 38°
    - angle YXZ has a magnitude of 92°

    Draw the triangle and find the area of the triangle XYZ, rounded to two decimal places.

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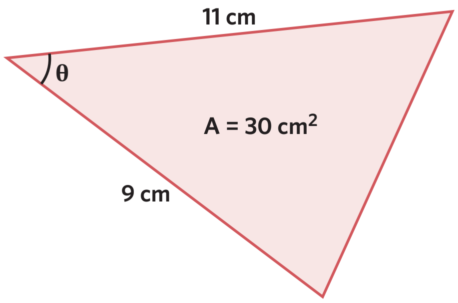
15. Consider the following triangle and the rearranged formula for finding angle.

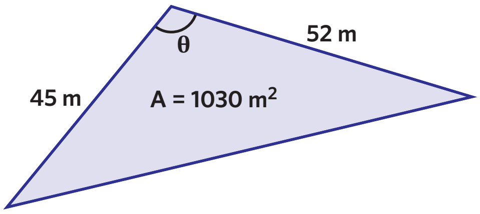
$$C = \sin^{-1}\left(\frac{A}{\frac{1}{2}ab}\right)$$



Calculate θ, rounded to two decimal places.

Joining it all together

16. Find the value of θ in the following triangle, rounded to one decimal place, given that the angle θ is obtuse.



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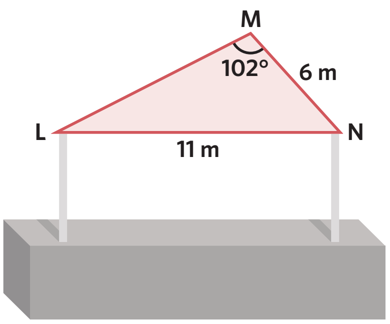
17. The design for a new shade cloth for the local skate park is shown, but some information is missing.



a. From the given information, which angle should be calculated first using the sine rule?

b. Calculate the magnitude of the angle chosen in part **a**, rounded to two decimal places.

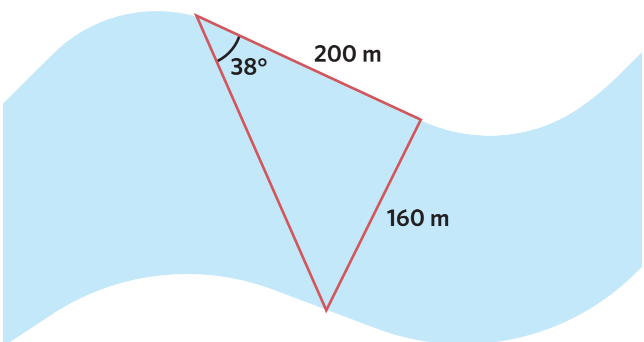
c. Calculate the magnitude of the last unknown angle, rounded to two decimal places.

d. Calculate the length of the remaining unknown side, rounded to two decimal places.

---

18. A triangular section of the Yarra River with an area of 15 993 m^2 has been fenced off to allow for a waterskiing showcase at Melbourne's annual Moomba Festival.

What was the total length of fence used, rounded to one decimal place?



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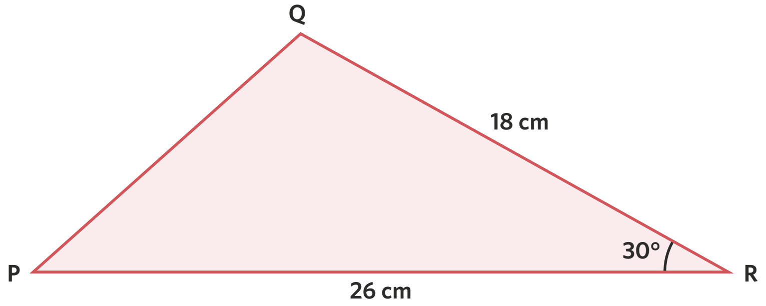
Exam practice

19. A triangle PQR is shown in the following diagram.



The length of the side QR is 18 cm.

The length of the side PR is 26 cm.

The angle QRP is 30°.

The area of triangle PQR, in square centimetres, is closest to

A. 117 **B.** 162 **C.** 171
D. 234 **E.** 468

VCAA 2018 Exam 1 Geometry and measurement Q2

80% of students answered this question correctly.

---

20. A triangle ABC has:

- one side, AB, of length 4 cm
- one side, BC, of length 7 cm
- one angle, ACB, of 26°.

Which one of the following angles, correct to the nearest degree, could **not** be another angle in triangle ABC?

A. 24° **B.** 50° **C.** 104°
D. 130° **E.** 144°

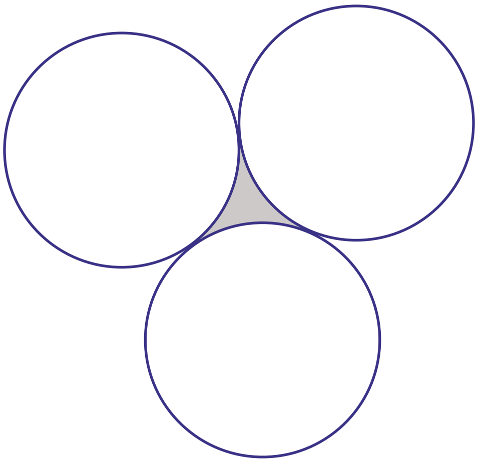
VCAA 2017 Exam 1 Geometry and measurement Q7

57% of students answered this question correctly.

---

21. Three circles of radius 50 mm are placed so that they just touch each other.

The region enclosed by the circles is shaded in the following diagram.



The area of the shaded region, in square millimetres, is closest to

A. 403 **B.** 436 **C.** 1309
D. 2844 **E.** 4330

VCAA 2017 Exam 1 Geometry and measurement Q8

35% of students answered this question correctly.

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22. Frank hits two balls from point A on a tennis court.

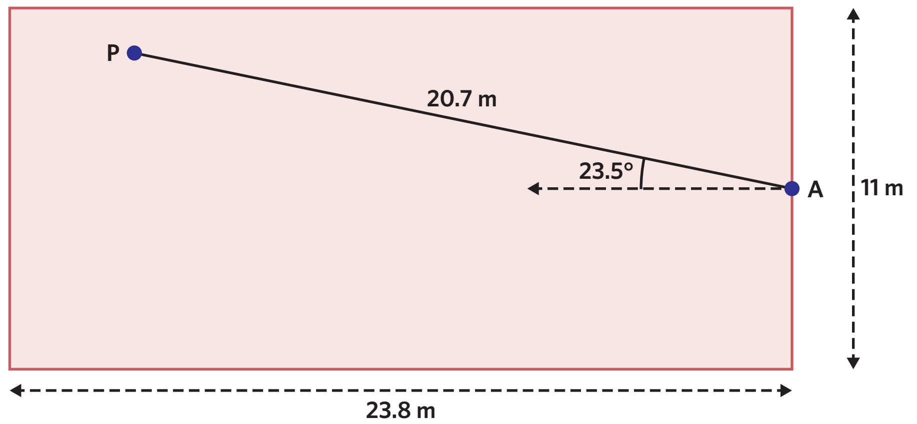
For Frank's first hit, the ball strikes the ground at point P, 20.7 m from point A.

For Frank's second hit, the ball strikes the ground at point Q.

Point Q is x metres from point A.

Point Q is 10.4 m from point P.

The angle, PAQ, formed is 23.5°.



a. Determine two possible values for angle AQP.
Round to one decimal place. (1 MARK)

b. If point Q is within the boundary of the court, what is the value of x?
Round to the nearest metre. (1 MARK)

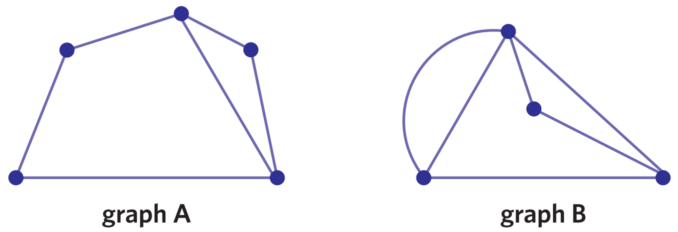
VCAA 2018 Exam 2 Geometry and measurement Q3c

Part **a**: **25%** of students answered this question correctly.
Part **b**: **18%** of students answered this question correctly.

Questions from multiple lessons

Networks and decision mathematics

23. Consider graphs A and B.



The sum of the degrees of the vertices of graph B is

A. two less than the sum of the degrees of the vertices of graph A.
B. one less than the sum of the degrees of the vertices of graph A.
C. equal to the sum of the degrees of the vertices of graph A.
D. one more than the sum of the degrees of the vertices of graph A.
E. two more than the sum of the degrees of the vertices of graph A.

Adapted from VCAA 2017 Exam 1 Networks and decision mathematics Q2

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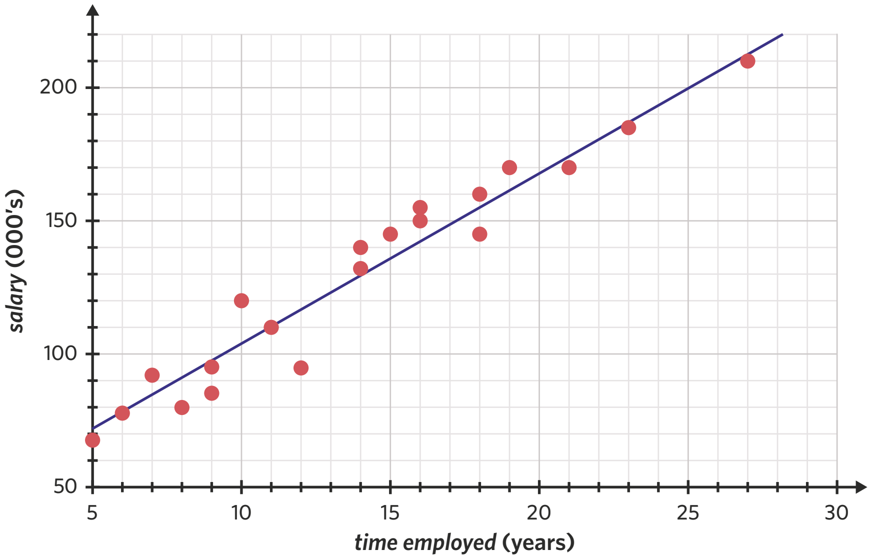
Data analysis

24. The following scatterplot shows the *salary* and the *time employed*, in years, of 20 employees at a company. A line of good fit has been fitted to the data.



The equation of this line is closest to

A. *time employed* = 40 + 4.5 × *salary*

B. *time employed* = 72 + 6.4 × *salary*

C. *salary* = 40 + 4.5 × *time employed*

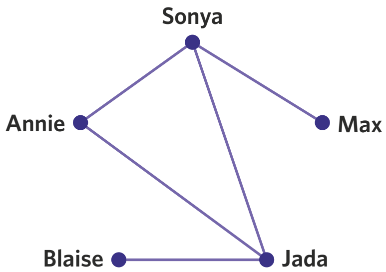
D. *salary* = 72 + 4.5 × *time employed*

E. *salary* = 40 + 6.4 × *time employed*

Adapted from VCAA 2018 Exam 1 Data analysis Q8

Networks and decision mathematics

25. Five colleagues are connected by different friendships, represented by edges in the following graph.



The following adjacency matrix also shows the different friendships between the five colleagues.

$$\begin{array}{c} \begin{matrix} S & M & J & B & A \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & x & 0 \\ 1 & 0 & 0 & y & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} \\ S \\ M \\ J \\ B \\ A \end{matrix}$$

a. Explain what a zero in the adjacency matrix represents in the context of friendships. (1 MARK)

b. What are the values of x and y in the adjacency matrix? (1 MARK)

Adapted from VCAA 2010 Exam 2 Networks and decision mathematics Q1

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10D The cosine rule

STUDY DESIGN DOT POINT

- the use of the sine rule, including the ambiguous case, the cosine rule, as a generalisation of Pythagoras' theorem, and their application to solving practical problems involving non-right-angled triangles, including three-figure (true) bearings in navigation

10A 10B 10C **10D** 10E

KEY SKILLS

During this lesson, you will be:

- calculating side lengths of a triangle using the cosine rule
- calculating angles in a triangle using the cosine rule.

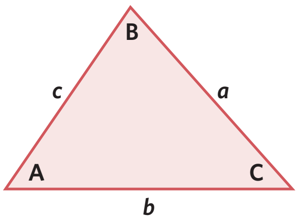
KEY TERMS

- Cosine rule

Another useful tool when performing calculations on triangles without a right angle is the cosine rule. This rule can also be used to calculate unknown side lengths or angles and gives a helpful alternative to right-angled trigonometry or the sine rule.

Calculating side lengths of a triangle using the cosine rule

Recall that in trigonometry, side lengths and angles of non-right-angled triangles are labelled in the following way.



The **cosine rule** is a generalisation of Pythagoras' theorem that can be used to calculate unknown values in any triangle.

The cosine rule states that in triangle ABC:

$c^2 = a^2 + b^2 - 2ab \times \cos(C)$

It can be used to find the missing side length in a triangle when two sides and the included angle between them are known.

When solving for an unknown side length (c), the cosine rule can be rearranged to

$c = \sqrt{a^2 + b^2 - 2ab \times \cos(C)}$, where

- C is the angle opposite the unknown side
- a and b are the two known side lengths.

Note: Side lengths a and b are interchangeable.

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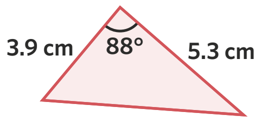
Worked example 1

Consider the following triangle.



Calculate the length of the unknown side, rounded to one decimal place.

Explanation

Step 1: Identify the known values.

$a = 3.9$ cm

$b = 5.3$ cm

$C = 88°$

Step 2: Substitute the values into the cosine rule and evaluate.

$c = \sqrt{a^2 + b^2 - 2ab \times \cos(C)}$

$= \sqrt{3.9^2 + 5.3^2 - 2 \times 3.9 \times 5.3 \times \cos(88°)}$

$= 6.469...$ cm

Answer

6.5 cm

Calculating angles in a triangle using the cosine rule

The cosine rule can also be used to calculate an unknown angle in any triangle when all three side lengths are known.

When solving for an unknown angle (C), the cosine rule can be rearranged to

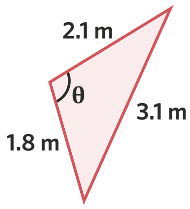
$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$, where

- c is the side length opposite the unknown angle
- a and b are the two other known side lengths.

Note: Sides lengths a and b are interchangeable.

Worked example 2

Consider the following triangle.



Calculate the value of θ, rounded to the nearest degree.

Explanation

Step 1: Identify the known values.

c is the side length opposite θ.

$a = 1.8$ m

$b = 2.1$ m

$c = 3.1$ m

$C = \theta$

Step 2: Substitute the values into the cosine rule and evaluate.

$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$

$\theta = \cos^{-1}\left(\frac{1.8^2 + 2.1^2 - 3.1^2}{2 \times 1.8 \times 2.1}\right)$

$= 105.026...°$

Answer

105°

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10D Questions

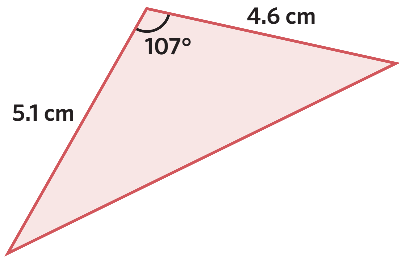
Calculating side lengths of a triangle using the cosine rule

1. A triangle has two known side lengths of 3 cm and 5 cm and an angle of 80° between the two sides. Fill in the blanks for the following equation, where c is the unknown side length.

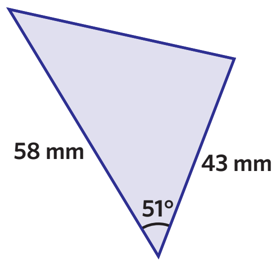
$$c^2 = 3^2 + 5^2 - 2 \times \boxed{} \times 5 \times \cos\left(\boxed{}^\circ\right)$$

2. For the following triangles, calculate the length of the unknown side, rounded to two decimal places.

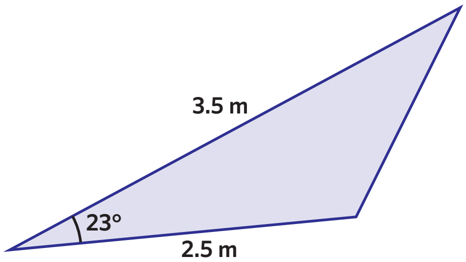
a.



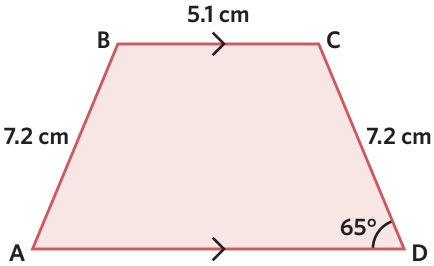
b.



c.



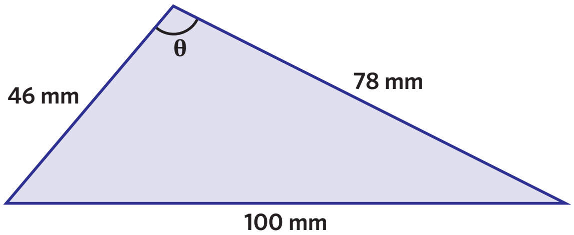
3. Consider the following trapezium.



What is the length of the diagonal AC, rounded to one decimal place?

Calculating angles in a triangle using the cosine rule

4. Consider the following triangle.



To calculate θ using $C = \cos^{-1}\left(\dfrac{a^2 + b^2 - c^2}{2ab}\right)$, the value of c is

A. θ°
B. 46 mm
C. 78 mm
D. 100 mm

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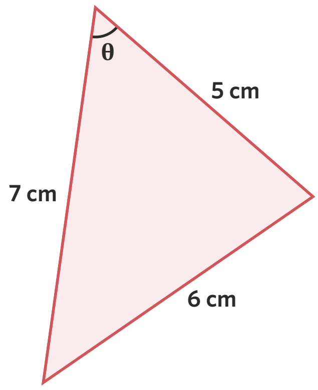
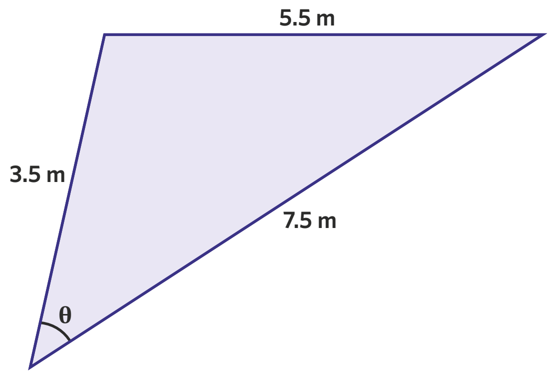
5. For the following triangles, calculate the value of θ, rounded to two decimal places.

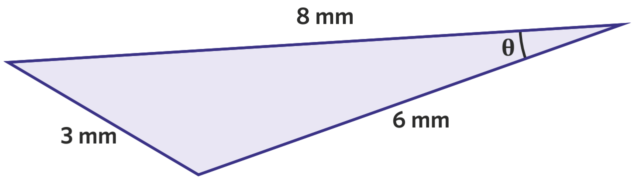
a.



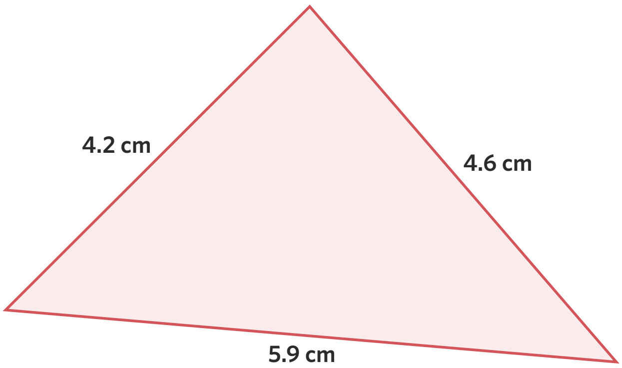
b.



c.



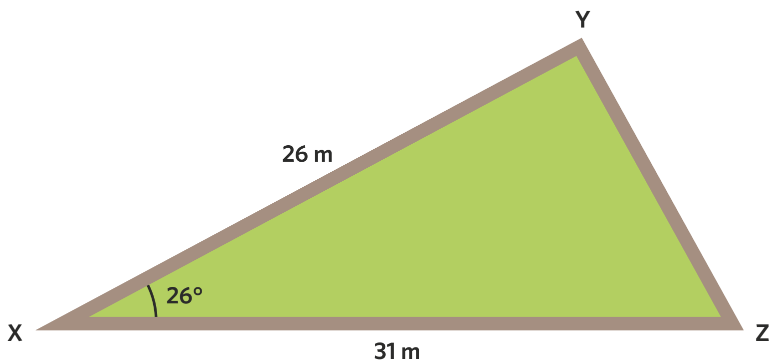
6. Consider the following triangle.



What is the value of the smallest angle in the triangle, rounded to two decimal places?

Joining it all together

7. As part of their studies of the cosine rule, a maths class is designing a garden bed for the school yard.



a. What is the distance, rounded to one decimal place, between points Y and Z?

b. Using the rounded answer from part **a**, calculate the value of the angle XZY, rounded to the nearest degree.

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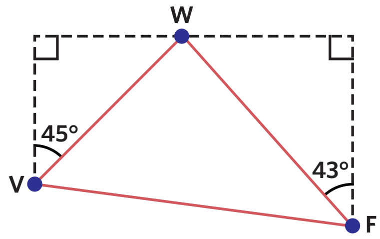
8. Two friends, Victoria (V) and Felicity (F), are trying to find each other in the bush. The following diagram shows their positions relative to a waterfall (W).



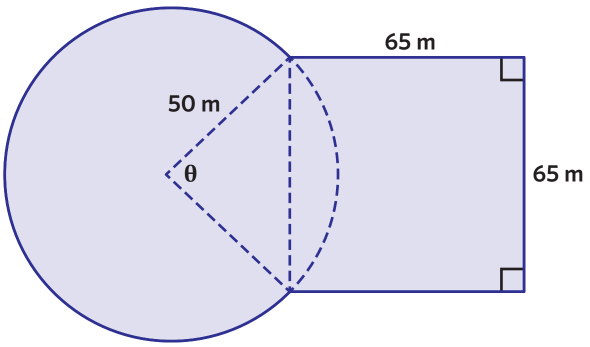
If Victoria is 600 m away from the waterfall and Felicity is 0.75 km away from the waterfall, how far away are they from each other, rounded to the nearest metre?

9. A student finds one of the angles in a non-right-angled triangle to be 105°. The length of the side opposite the angle is 5.3 cm and the lengths of the other two sides are x cm and $2x$ cm. What is the length of the shortest side of the triangle, rounded to two decimal places?

Exam practice

10. The buildings of a hostel are arranged around a grassed area.

The grassed area is shown shaded in the following diagram.



The grassed area is made up of a square overlapping a circle.

The square has side lengths of 65 m.

The circle has a radius of 50 m.

An angle, θ, is also shown on the diagram.

Use the cosine rule to show that the angle θ, correct to the nearest degree, is equal to 81°. (1 MARK)

VCAA 2017 Exam 2 Geometry and measurement Q3a

56% of students answered this question correctly.

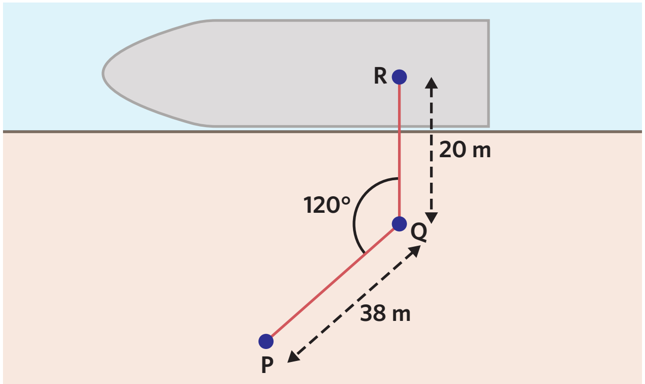
11. The following diagram shows a cargo ship next to a port. The base of a crane is shown at point Q.

The base of the crane (Q) is 20 m from a shipping container at point R. The shipping container will be moved to point P, 38 m from Q. The crane rotates 120° as it moves the shipping container anticlockwise from R to P.

What is the distance RP, in metres?

Round to the nearest metre. (1 MARK)

VCAA 2019 Exam 2 Geometry and measurement Q3b



56% of students answered this question correctly.

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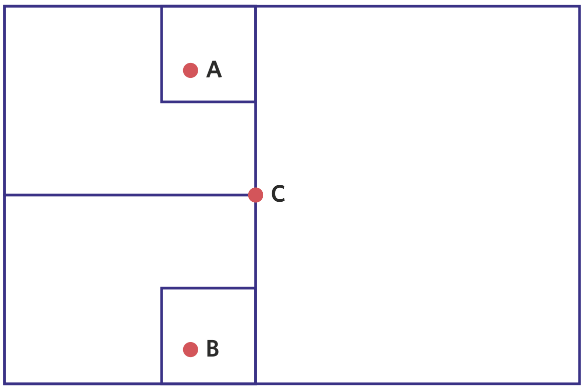
12. Two squash players, Wei-Yi and Bao, are playing a match.
The following diagram shows the lines on the court floor.



Wei-Yi is serving from point A and Bao is standing at point B.
Point A is 2.7 m from point C.
Point B is 3.1 m from point C.
The angle ACB is 119°.
Show that the distance between Wei-Yi and Bao is 5 m, rounded to the nearest metre. (1 MARK).

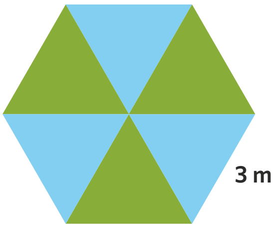
VCAA 2021 Exam 2 Geometry and measurement Q2c

42% of students answered this question correctly.

Questions from multiple lessons

Geometry and measurement

13. A helicopter landing pad is hexagonal and has a side length of three metres.
The landing pad can be divided into six equilateral triangles.
The area of the green section of the landing pad is closest to

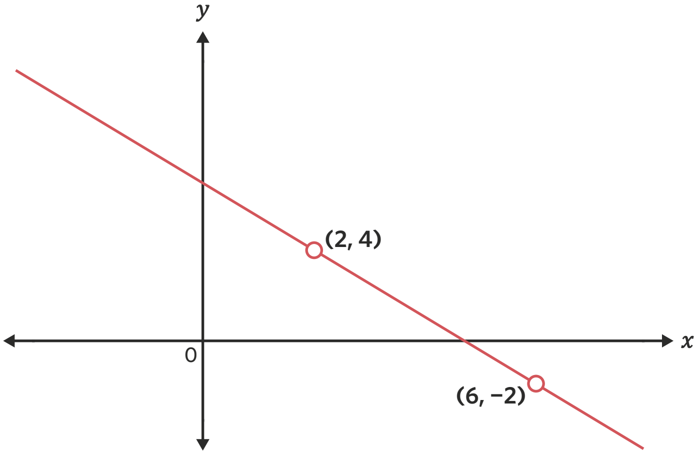


A. 3.9 m^2 **B.** 11.7 m^2 **C.** 14.7 m^2
D. 15.2 m^2 **E.** 15.6 m^2

Adapted from VCAA 2015 Exam 1 Geometry and trigonometry Q5

Graphs and relations

14. A line passes through the points (2, 4) and (6, −2), as shown.



The coordinates of the line's x-intercept are

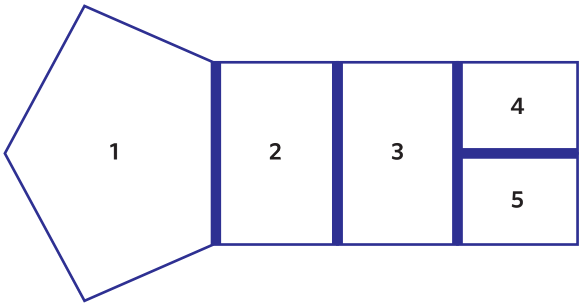
A. (0, 7) **B.** (4.5, 0) **C.** (0, 2) **D.** $\left(\frac{11}{3}, 0\right)$ **E.** $\left(\frac{14}{3}, 0\right)$

Adapted from VCAA 2016 Exam 1 Graphs and relations Q3

Networks and decision mathematics

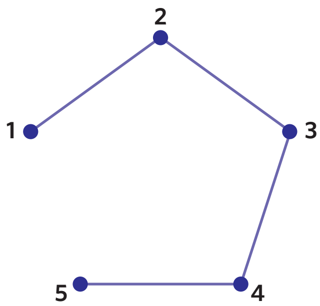
15. Paulo wants to design a new, unique house. He has decided to build it according to the following diagram, where each of the rooms is labelled 1 to 5.

The bold lines represent the walls between two rooms.



In the following graph, the five rooms are represented as vertices.

The edges of the graph represent the walls between two rooms.



One of the edges is missing from this graph.

a. Draw the complete graph. (1 MARK)

b. With the missing edge included, what is the sum of the degrees of the vertices of the graph? (1 MARK)

Adapted from VCAA 2018NH Exam 2 Networks and decision mathematics Q1

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10E Bearings

STUDY DESIGN DOT POINT

- the use of the sine rule, including the ambiguous case, the cosine rule, as a generalisation of Pythagoras' theorem, and their application to solving practical problems involving non-right-angled triangles, including three-figure (true) bearings in navigation

10A 10B 10C 10D 10E

KEY SKILLS

During this lesson, you will be:

- identifying and drawing true bearings
- solving problems using true bearings.

KEY TERMS

- True bearing
- Three-figure bearing

Bearings are vital for exploration and location. From organising the flight paths of multiple planes, to plotting a course by sea, to planning a trip across the country, bearings are the global tool for communicating location and navigation.

Identifying and drawing true bearings

Although there are different ways to define bearings, the most universal method is using true bearings. A **true bearing** measures the position of one object relative to another, with reference to north. A true bearing is measured clockwise from north and is always written using three digits, even if the first digit is a zero. For example, a true bearing of 37° will be written as 037°. A true bearing is sometimes referred to as a **three-figure bearing**.

When determining the true bearing of an object, it is important to identify a starting point ('from' is frequently used to denote this) and an ending point ('to' is frequently used to denote this). For example, 'the true bearing from the ship to the lighthouse'.

See worked example 1

If the true bearing in the opposite direction is required ('the true bearing from the lighthouse to the ship'), an addition or subtraction of 180° is required. It is important when completing this calculation that the value remains between 0° and 360°.

Sometimes the angle provided in a problem is not the bearing, but can be used to calculate it. In cases like this, it is important to remember that a full revolution is 360°.

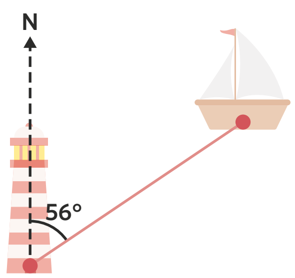
See worked example 2

It is also important to know how to draw bearings using bearing diagrams. These help to visualise applied problems, which is often necessary to find a solution. On bearing diagrams, the direction of north is shown as a ray (arrow) and labelled 'N' or 'north'.

See worked example 3

Worked example 1

A ship has sailed past a lighthouse as shown in the following diagram.



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a. What is the true bearing from the lighthouse to the ship?

Explanation

Identify the angle that indicates the bearing from the lighthouse to the ship.

Because the lighthouse is the starting point, the angle required is formed clockwise between north and the ship's location, in relation to the lighthouse.

In this case, the angle is already labelled as 56°.

Answer

056°

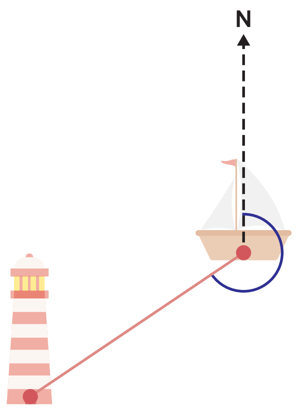
---

b. What is the true bearing from the ship to the lighthouse?

Explanation

Step 1: Identify the angle that indicates the bearing from the ship to the lighthouse.

In this case, the angle is formed clockwise between north and the lighthouse's location, both from the perspective of the ship.



Step 2: Calculate the angle.

As this bearing is in the reverse direction to part **a**, the angle represents a 180° turn.

The bearing can be calculated by adding or subtracting 180° (whichever one returns a value between 0° and 360°).

$056° + 180° = 236°$

$056° - 180° = -124°$

Answer

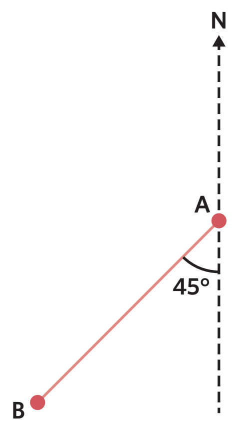
236°

Worked example 2

Calculate the true bearing from point A to point B in each of the following bearing diagrams.

---

a.



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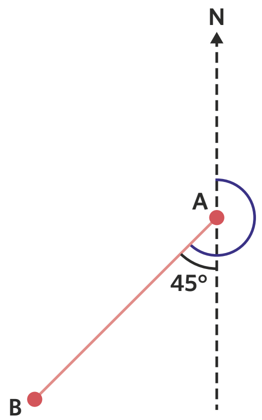
Explanation

Step 1: Identify the angle that indicates the true bearing from point A to point B.

The angle is formed clockwise between north and point B, both from the perspective of point A.



Step 2: Calculate the angle.

From north to south is a straight line, measuring 180°.

From south to point B is an additional 45°.

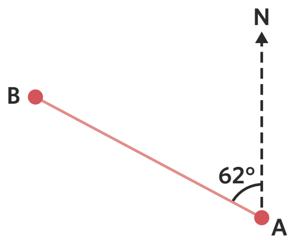
$180° + 45° = 225°$

Answer

225°

---

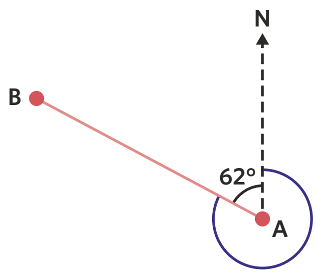
b.



Explanation

Step 1: Identify the angle that indicates the true bearing from point A to point B.

The angle is formed clockwise between north and point B, both from the perspective of point A.



Step 2: Calculate the angle.

A full revolution, starting and finishing at north, measures 360°.

The required true bearing angle is 62° less than 360°.

$360° - 62° = 298°$

Answer

298°

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Worked example 3

A small fishing boat has stopped 25 km off the coast of an island, on a true bearing of 260° from the island.
Draw a diagram of the scenario.

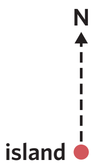
Explanation

Step 1: Mark the starting point.

The scenario states the bearing is 'from the island', so the island is the starting point.

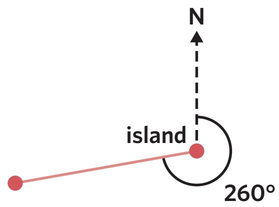
Step 2: Draw a ray (arrow) marking north from the starting point.

Label north 'N'.



Step 3: Using this ray, measure the required angle clockwise and draw a line segment to mark the angle.

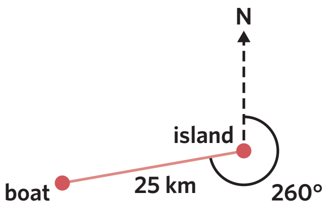
This scenario requires a measured angle of 260°.



Step 4: Label any known points and distances.

The boat can be labelled, as well as the distance between the boat and the island.

Answer



Solving problems using true bearings

Bearings can be one component of multi-part problems. These problems usually involve elements of Pythagoras' theorem and trigonometry, including the sine and cosine rules.

Worked example 4

The true bearing of Sydney from Melbourne is 053.92°, and the true bearing of Adelaide from Melbourne is 297.48°.

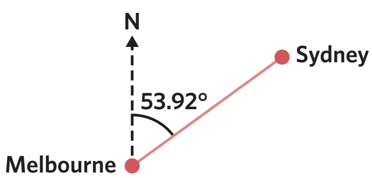
a. If Sydney is 420 km north of Melbourne, what is the direct distance between Sydney and Melbourne, to the nearest kilometre?

Explanation

Step 1: Draw a bearing diagram.

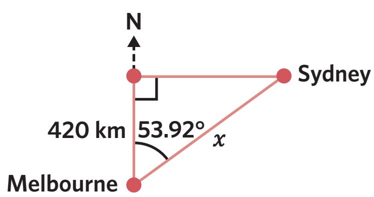
It is only necessary to draw Melbourne and Sydney.

Bearings are given from Melbourne, so Melbourne will be the starting point.



Step 2: Form a right-angled triangle using the north ray and the line connecting Melbourne and Sydney.

Label the side lengths. In this case, x has been used to represent the direct distance between Sydney and Melbourne.



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Step 3: Use trigonometry to calculate the unknown side length.

$\cos(53.92°) = \frac{420}{x}$

$x = 713.17...$ km

Answer

713 km

---

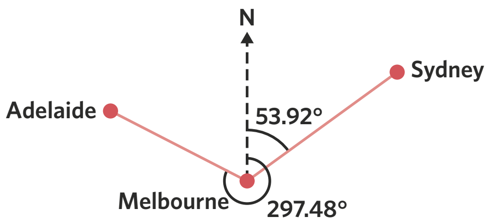
b. Calculate the angle Adelaide–Melbourne–Sydney.

Explanation

Step 1: Draw a bearing diagram.

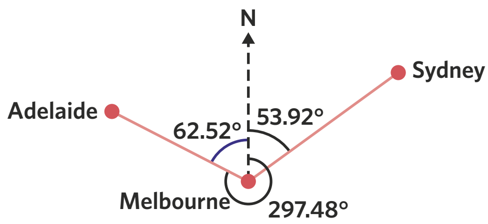
As well as marking north, the diagram will also include Adelaide, Melbourne and Sydney, and all known angles.

Bearings are given from Melbourne, so Melbourne will be the starting point.



Step 2: Calculate the angle between north and Adelaide.

$360° - 297.48° = 62.52°$



Step 3: Add the angle Adelaide–Melbourne–north to the angle north–Melbourne–Sydney.

$62.52° + 53.92° = 116.44°$

Answer

116.44°

---

c. If the direct distance between Adelaide and Melbourne is 654 km, calculate the direct distance between Adelaide and Sydney, rounded to the nearest kilometre.

Use the rounded direct distance between Sydney and Melbourne found in part **a**.

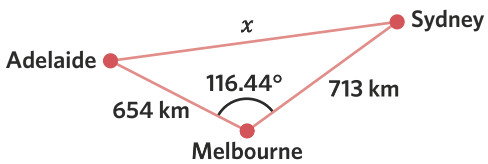
Explanation

Step 1: Draw a diagram.

Make sure to only include information that might be helpful.

The angle Adelaide–Melbourne–Sydney is known from part **b**.

Label the side lengths. In this case, x has been used to represent the direct distance between Adelaide and Sydney.



Step 2: Calculate the unknown side length.

As two side lengths and their included angle are known, the third side of the triangle can be calculated using the cosine rule.

$x = \sqrt{654^2 + 713^2 - 2 \times 654 \times 713 \times \cos(116.44°)}$

$= 1162.47...$ km

Answer

1162 km

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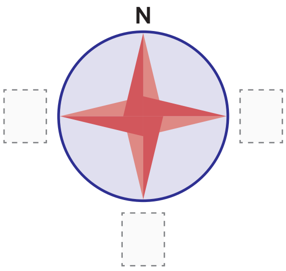
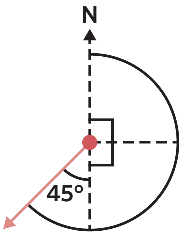
10E Questions

Identifying and drawing true bearings

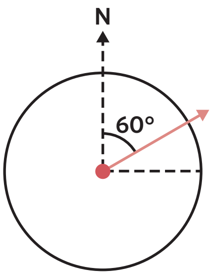
1. Label the directions east (E), south (S) and west (W) on the compass.

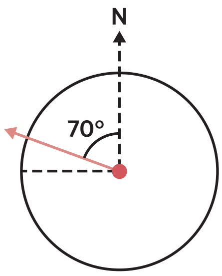


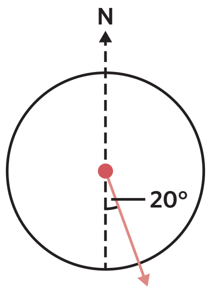
2. Determine the value of the following bearing.

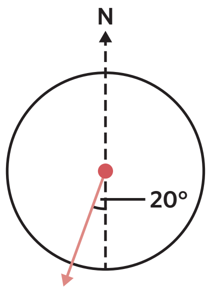


3. Which of the following shows a bearing of 200°?

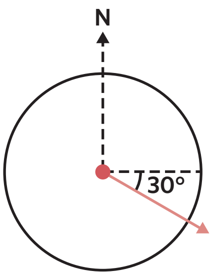
A.


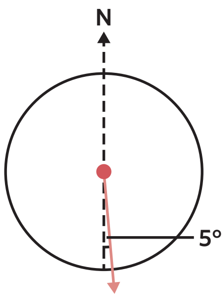
B.


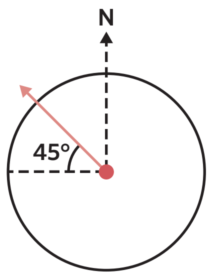
C.


D.


4. Determine the value of the following bearings.

a.


b.


c.


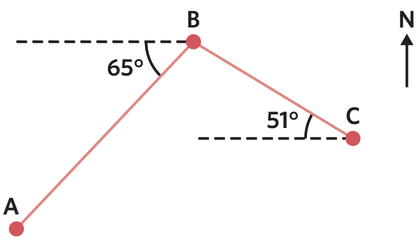
5. Three points, A, B and C, are different campsites in a national park.
What is the true bearing of

a. campsite B from campsite C?

b. campsite C from campsite B?

c. campsite A from campsite B?

d. campsite B from campsite A?



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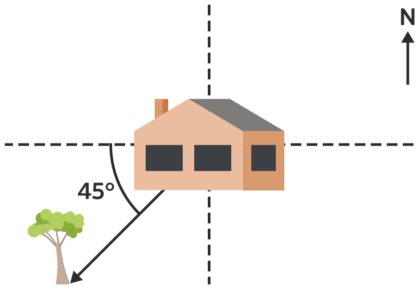
6. Plot the following.
   a. A bearing of 140°
   b. A bearing of 300°
   c. The position of Tim, who is on a true bearing of 190° from Celine

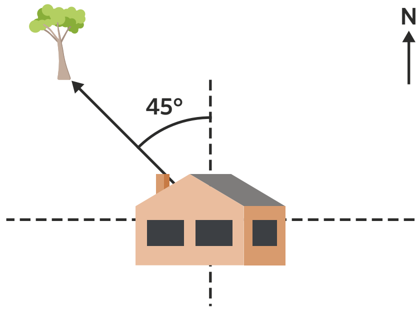
Solving problems using true bearings

7. A gum tree is planted at a true bearing of 315° from a house. Which of the following diagrams shows the tree in the correct position?

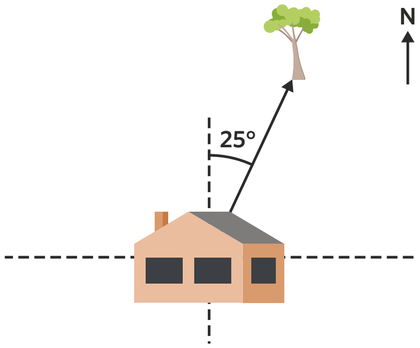
A.



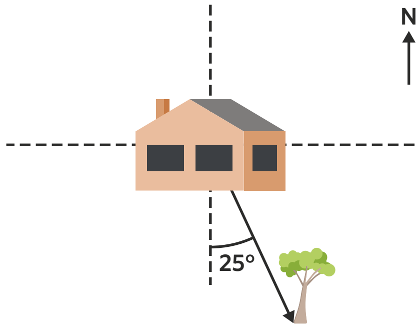
B.



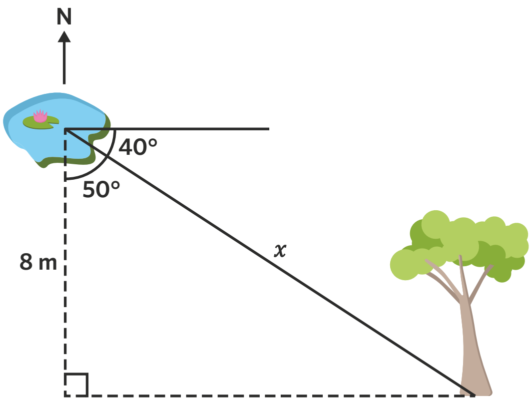
C.



D.



8. Which of the following equations can be used to determine the distance between the tree and the pond?



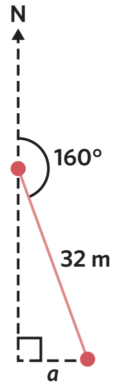
A. $\sin(50°) = \frac{x}{8}$

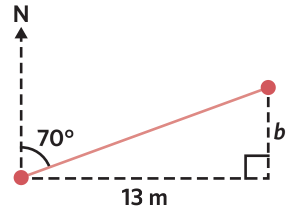
B. $\cos(40°) = \frac{8}{x}$

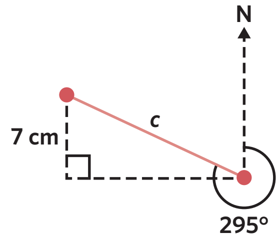
C. $\sin(45°) = \frac{8}{x}$

D. $\cos(50°) = \frac{8}{x}$

9. Calculate the distance of a, b, and c in the following diagrams, rounded to two decimal places.

a. 

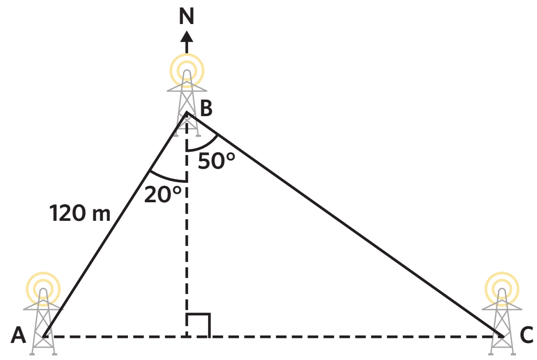
b. 

c. 

10. A group of cyclists travelled from point A to point B on a true bearing of 180° for 13 kilometres, then turned and travelled to point C on a true bearing of 270° for 6 kilometres. Calculate the true bearing of point C from point A, rounded to two decimal places.

11. Three transmission towers, A, B and C, are situated as shown in the following diagram.

Tower C is directly east of tower A. What is the distance between towers A and C, rounded to the nearest metre?



Joining it all together

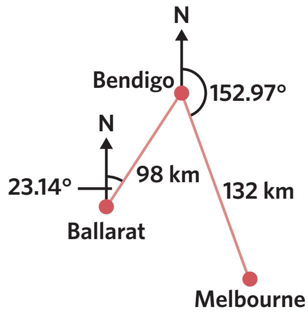
12. The distances between Ballarat and Bendigo, and Bendigo and Melbourne, are shown in the following diagram.

   a. What is the angle Ballarat–Bendigo–Melbourne?

   b. What is the true bearing of Bendigo from Melbourne?

   c. How many kilometres north of Melbourne is Bendigo, rounded to two decimal places?

   d. Wangaratta is at a bearing of 040° from Melbourne. Calculate the angle Bendigo–Melbourne–Wangaratta.



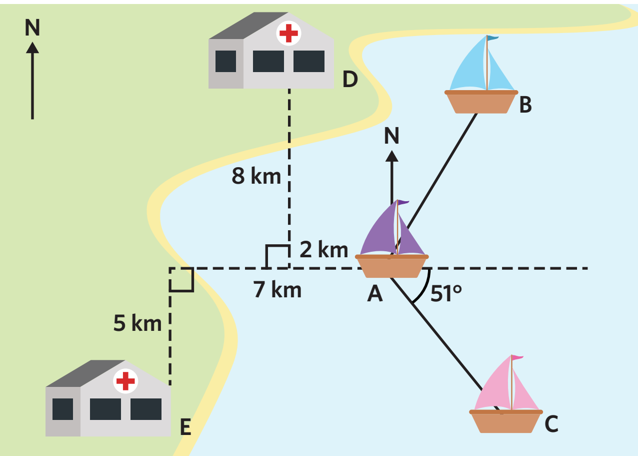
13. The positions of three ships are shown in the following diagram.

   a. Ship B is 2.5 km east and 4 km north of ship A. Show that angle BAC, rounded to the nearest degree, is 109°.

   b. Ship A hits an iceberg. If ship C is 3.5 km south of ship A, how far away is the closest ship to ship A? Round to two decimal places.

   c. Calculate the distance between ships B and C, rounded to two decimal places.

   d. Calculate the true bearing of the closest hospital from ship A, and the distance between them, both rounded to two decimal places.



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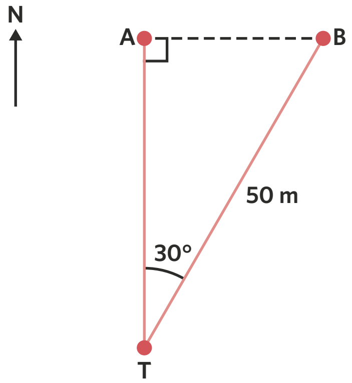
Exam practice

14. Salena practises golf at a driving range by hitting golf balls from point T.

The first ball that Salena hits travels directly north, landing at point A.

The second ball that Salena hits travels 50 m on a bearing of 030°, landing at point B.

The following diagram shows the positions of the two balls after they have landed.



How far apart, in metres, are the two golf balls? (1 MARK)

VCAA 2016 Exam 2 Geometry and measurement Q2a

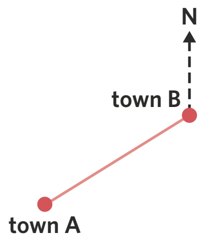
78% of students answered this question correctly.

---

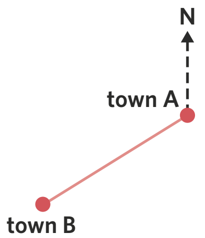
15. Town B is located on a bearing of 060° from town A.

The diagram that could illustrate this is

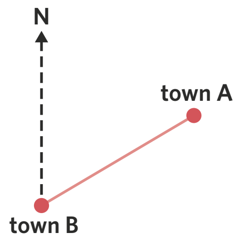
A.



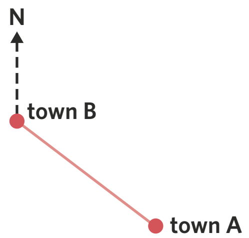
B.



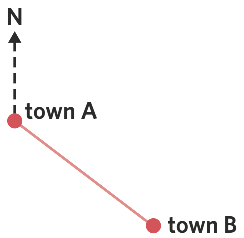
C.



D.



E.



VCAA 2019 Exam 1 Geometry and measurement Q2

68% of students answered this question correctly.

---

16. Marcus is on the opposite side of a large lake from a horse and its stable. The stable is 150 m directly east of the horse. Marcus is on a bearing of 170° from the horse and on a bearing of 205° from the stable.

The straight-line distance, in metres, between Marcus and the horse is closest to

A. 45

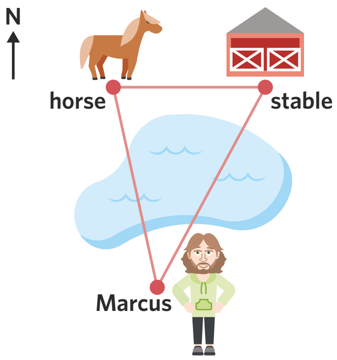
B. 61

C. 95

D. 192

E. 237

VCAA 2016 Exam 1 Geometry and measurement Q6



51% of students answered this question correctly.

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17. Miki will travel by train from Tokyo to Nemuro and she will stay in a hostel when she arrives.

The hostel is located 186 m north and 50 m west of the Nemuro railway station.

a. What distance will Miki have to walk if she were to walk in a straight line from the Nemuro railway station to the hostel?

Round to the nearest metre. (1 MARK)

b. What is the three-figure bearing of the hostel from the Nemuro railway station?

Round to the nearest degree. (1 MARK)

VCAA 2017 Exam 2 Geometry and measurement Q2b

Part **a**: **61%** of students answered this question correctly.

Part **b**: **27%** of students answered this question correctly.

Questions from multiple lessons

Data analysis

18. The variables *weight* (less than 1 kg, 1–2 kg, over 2 kg) and *size* (small, medium, large) are

A. both numerical variables.

B. a numerical and an ordinal variable respectively.

C. both nominal variables.

D. an ordinal and a nominal variable respectively.

E. both ordinal variables.

Adapted from VCAA 2018NH Exam 1 Data analysis Q5

Recursion and financial modelling

19. A sequence can be generated using the following recurrence relation.

$T_{n+1} = T_n + 3, \quad T_0 = -1$

What are the first four terms of the sequence?

A. $-1, -4, -7, -10$ **B.** $-1, 2, 5, 8$ **C.** $-1, -3, -9, -27$ **D.** $1, 4, 7, 10$ **E.** $2, 5, 8, 11$

Adapted from VCAA 2016 Exam 1 Recursion and financial modelling Q17

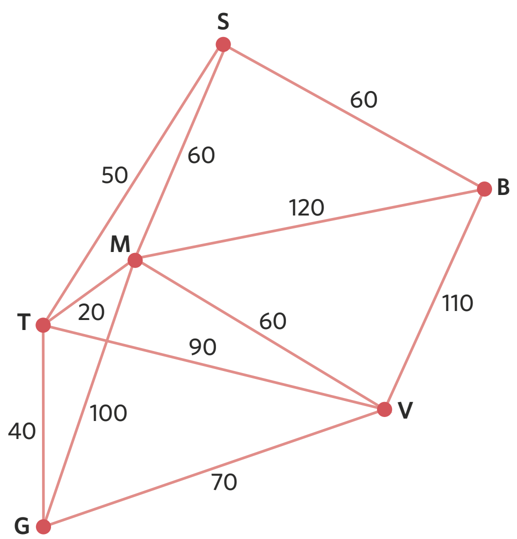
Networks and decision mathematics

20. Train routes connect different cities in Spain.

Some of the cities are Madrid (M), Barcelona (B), Toledo (T), San Sebastian (S), Valencia (V), and Granada (G).

The following graph gives the cost, in dollars, of train travel along these routes.

Ella is currently staying in Barcelona (B) and she wants to travel to Granada (G).



a. Ella considers travelling by train along the route B–M–G. How much will she have to pay? (1 MARK)

b. If Ella takes the cheapest route from Barcelona to Granada, which other town(s) will she pass through? (1 MARK)

c. Euler's formula, $v + f = e + 2$ holds for this graph.

Complete Euler's formula for this graph by replacing v, f, and e with their respective values. (1 MARK)

Adapted from VCAA 2017 Exam 2 Networks and decision mathematics Q1

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ANSWERS

CONTENTS

Image: ChristianChan/Shutterstock.com

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1A Types of data

Classifying data as categorical or numerical

1. C

2. **a.** Categorical **b.** Numerical **c.** Numerical **d.** Categorical

Classifying categorical data as nominal or ordinal

3. A

4. **a.** Nominal **b.** Ordinal **c.** Ordinal **d.** Nominal

5. Ordinal

Classifying numerical data as discrete or continuous

6. C

7. **a.** Discrete **b.** Continuous **c.** Continuous **d.** Discrete

8. Continuous

Classifying numerical data as interval or ratio

9. B

10. A

Joining it all together

11. **a.** Continuous **b.** Nominal **c.** Ordinal **d.** Discrete

12. **a.** Numerical **b.** Categorical **c.** Categorical **d.** Categorical

Exam practice

13. **Explanation**

Step 1: Determine whether the variables are counted/measured or categorised into groups.

The variable *age* (under 55 years, 55 years and over) is categorised, and the variable *preferred travel destination* (domestic, international) is also categorised.

Step 2: Classify each variable as categorical or numerical.

The variable *age* (under 55 years, 55 years and over) is categorical.

The variable *preferred travel destination* (domestic, international) is also categorical.

Answer

A

21% of students chose option C, presumably because age appears to be numerical as it is based on numbers. This answer is incorrect however, because the question specifies that age is broken into two categories (under 55 years, 55 years and over), making it a categorical variable.

14. **Explanation**

Step 1: Classify each variable as categorical or numerical.

The variable *frequency of nightmares* (low, high) is categorical as it specifies two categories, low and high.

The variable *snores* (no, yes) is categorical as it specifies two categories, no and yes.

Step 2: Identify whether there is a logical order for the categories of each variable.

Even though there are only two categories, *frequency of nightmares* (low, high) does have a logical order. The ascending order of frequency would be low, high, and the descending order of frequency would be high, low.

The variable *snores* (no, yes) does not have an implicit ascending or descending order.

Step 3: Classify the variables as nominal or ordinal.

The variable *frequency of nightmares* (low, high) is ordinal.

The variable *snores* (no, yes) is nominal.

Answer

A

24% of students chose option D due to the misconception that the variable *snores* (no, yes) is ordinal.

Questions from multiple lessons

15. D **16.** D

17. **a.** 3

b. 1

c. Grapefruit, jackfruit, pineapple

1B Displaying and describing categorical data

Constructing frequency tables

1. B

2. **a.** 19 people **b.** 21%

3.

eye colour	tally	frequency
blue	卌	5
brown	卌 IIII	9
green	III	3
hazel	III	3

4.

favourite genre	frequency
pop	6
rock	4
classical	2

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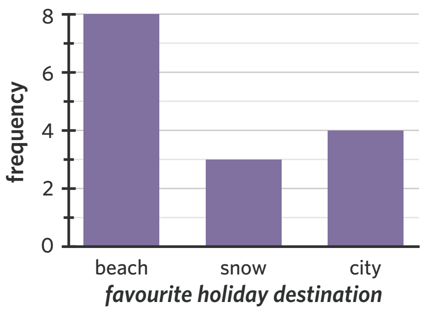
Constructing bar charts

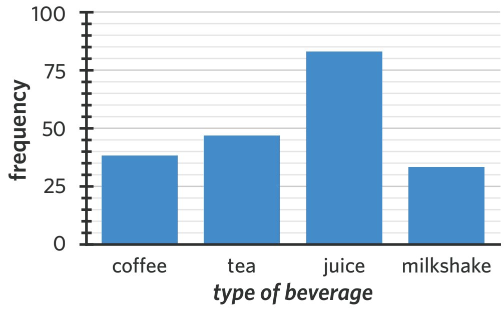
5. A

6.



7. a. 33

b.



Describing categorical data

8. A

9. a. 329 **b.** Melbourne

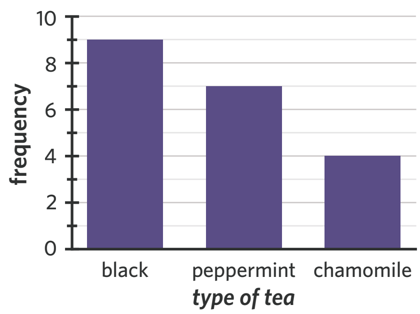
10. Sport

Joining it all together

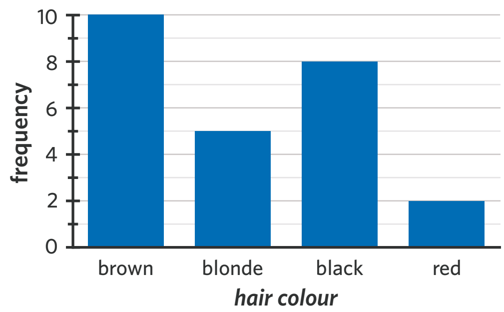
11. a.

type of tea	frequency
black	9
peppermint	7
chamomile	4

b.



12. a.



b.

hair colour	frequency
brown	10
blonde	5
black	8
red	2

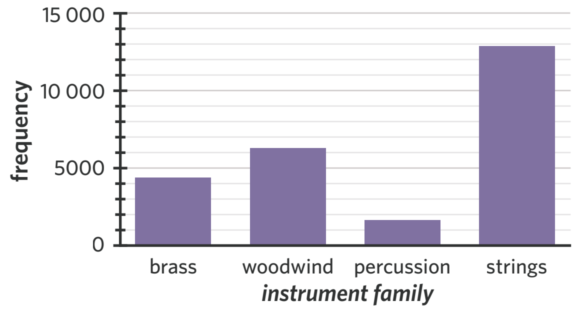
c. Brown

13. a. 51.4%

b.

instrument family	frequency
brass	4375
woodwind	6200
percussion	1575
strings	12 850

c.



d. Strings

Questions from multiple lessons

14. A **15.** A

16. a. 33 eggs

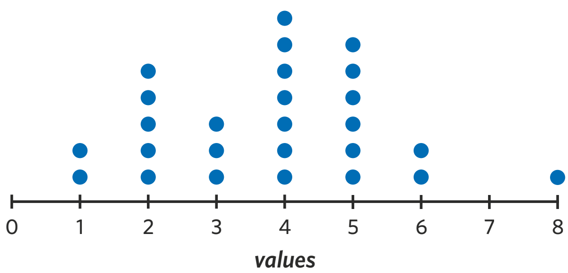
b. 20%

1C Displaying and describing numerical data

Displaying numerical data using dot plots

1. C

2. a.



b. 4

c. 10 values

1C ANSWERS

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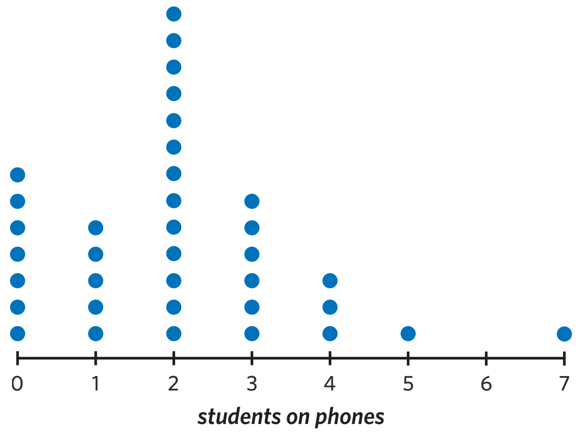
3. a.



b. 19.4%

Displaying numerical data using stem plots

4. D

5. a. **Key:** 1 | 9 = 19

1	9
2	0 6 7 8
3	7 7 7 8 9 9
4	5 7 9 9
5	5 7
6	3 3
7	0 3 5 5 5 6
8	9

b. 6 values

6. a. **Key:** 10 | 1 = 10.1 kg

10	1 5 8 9 9
11	1 1 4 6 7 7 9
12	1 1 2 5 5 9
13	
14	0 1 4 6 7
15	0 3 6

b. 30.8%

Displaying numerical data using histograms

7. D

8. a.

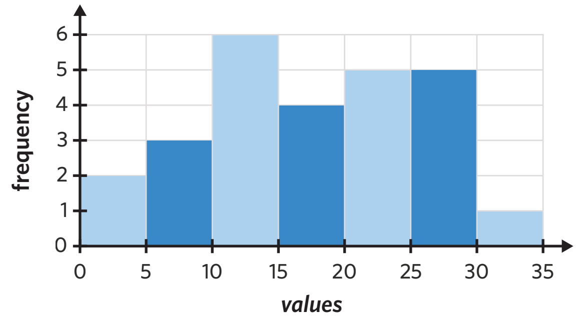
hours	frequency
0–4	7
5–9	12
10–14	4
15–19	2
20–24	1

b. 7 students

c. 26.9%

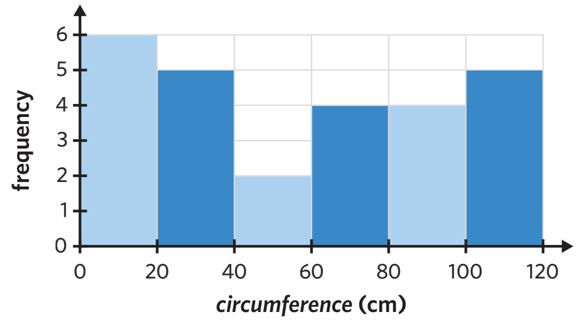
d. 5–9; 12 students

9. a.



b. 6 values

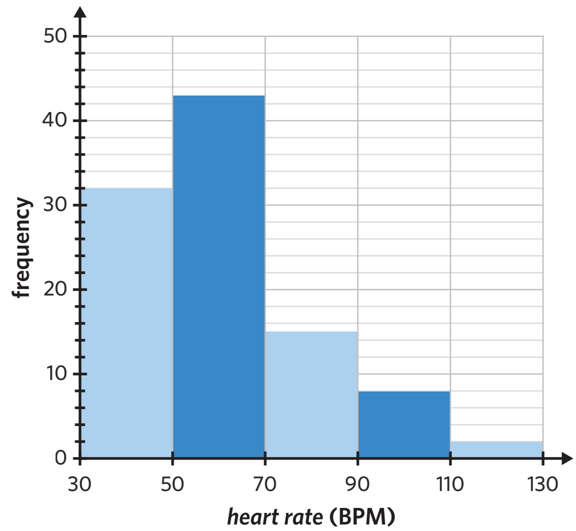
10. a.



b. 5 trees

c. 76.9%

11. a.



b. 30–<50 and 50–<70

c. 10 athletes

Joining it all together

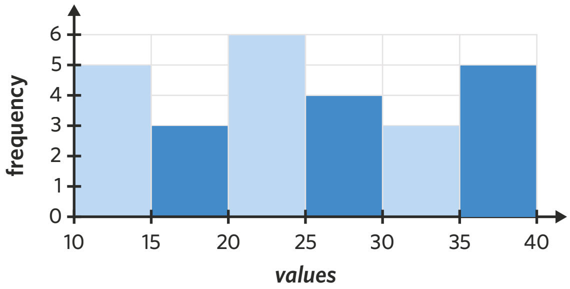
12. A

13. a.

values	frequency
10–14	5
15–19	3
20–24	6
25–29	4
30–34	3
35–39	5

1C ANSWERS

b.



c. **Key:** 1 | 1 = 11

1	1 1 3 4 4 5 6 7
2	0 0 1 2 3 4 6 7 8 8
3	1 2 4 6 7 8 9 9

14. a. The range of possible values is not large enough for a stem plot to be suitable.

b. A dot plot is most suitable as it can display discrete data with a small range of possible values. It is unnecessary to use a histogram as the data does not need to be grouped.

Exam practice

15. Explanation

Determine the modal *wingspan* of the moths captured.

The modal *wingspan* is the wingspan with the highest frequency.

Answer

24 mm

16. Explanation

Step 1: Identify the values that need to be added to the stem plot.

The values for days 11 to 15 need to be added:

Day 11: 7.5 °C

Day 12: 8.0 °C

Day 13: 8.6 °C

Day 14: 9.8 °C

Day 15: 7.7 °C

Step 2: Use the key to determine where the values will be plotted.

Key: 4 | 1 = 4.1 °C

Day 11: 5 will be added to the 7 stem.

Day 12: 0 will be added to the 8 stem.

Day 13: 6 will be added to the 8 stem.

Day 14: 8 will be added to the 9 stem.

Day 15: 7 will be added to the 7 stem.

Answer

Key: 4 | 1 = 4.1 $n = 15$

***minimum temperature* (°C)**

4	1 8
5	
6	0 7
7	0 5 7
8	0 6
9	0 2 8
10	7
11	8
12	7

A number of students did not fully answer the question. They only added the values for days 11 and 15 rather than all the days from day 11 to 15.

17. Explanation

Step 1: Construct a grouped frequency table using interval widths of two.

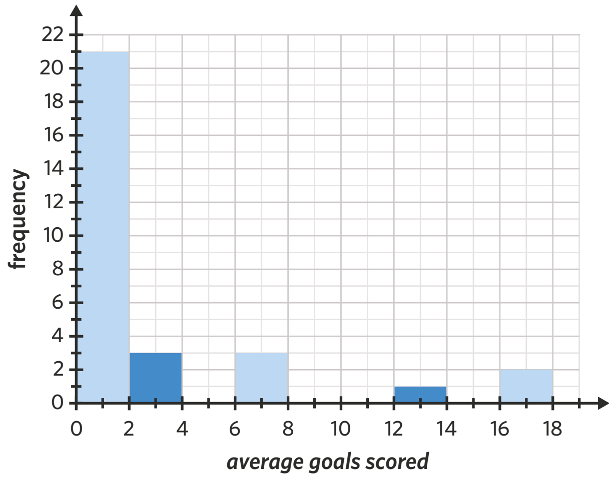
Count the number of dots falling within each interval.

average goals scored	frequency
0–<2	21
2–<4	3
4–<6	0
6–<8	3
8–<10	0
10–<12	0
12–<14	1
14–<16	0
16–<18	2

Step 2: Draw a column for each interval on the grid provided.

The height of each column must correspond to the frequency in the table.

Answer



Many students incorrectly drew columns with interval widths of only one, even though the question had specified intervals of two.

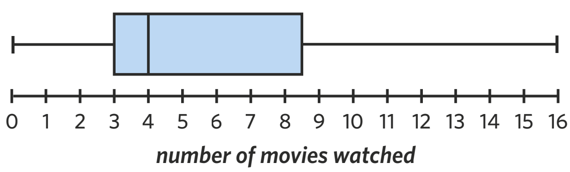
Questions from multiple lessons

18. D **19.** B

20. a. 0, 3, 4, 8.5, 16

b. No outliers

c.



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1D Summarising numerical data – median, range and IQR

Determining the median

1. D **2.** 25 **3.** 33.5 cm

4. 5 windows **5.** $129

Calculating the range

6. C

7. a. minimum: 165
maximum: 992

b. 827

8. 5 siblings

Calculating the interquartile range

9. D

10. a. $Q_1 = 210$, $Q_2 = 425$, $Q_3 = 866.5$

b. 656.5

11. 19.5 **12.** 24 minutes **13.** 1.5 holidays

Joining it all together

14. a. 29 **b.** 31

c. $Q_1 = 20.5$, $Q_2 = 29$, $Q_3 = 35$

15. a. minimum: 223 students
maximum: 932 students

b. 349 students

c. 391.5 students

16. a. 4 field goals **b.** 3 field goals **c.** 2 field goals

Exam practice

17. Explanation

From the table, Q_1 and Q_3 for the 'average' row are 23.4 and 26 respectively.

$$IQR = Q_3 - Q_1$$
$$= 26 - 23.4$$
$$= 2.6$$

Answer

2.6

18. Explanation

Step 1: Identify the minimum.

The smallest number in the data set is 125.

Step 2: Identify the maximum.

The largest number in the data set is 197.

Step 3: Calculate the range.

$$range = 197 - 125$$
$$= 72$$

Answer

72

Some students incorrectly wrote the range as the interval [125, 197], which is incorrect. The range must be a single value showing the difference between the maximum and minimum.

19. Explanation

Step 1: Determine the position of the median.

The median is located in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position.

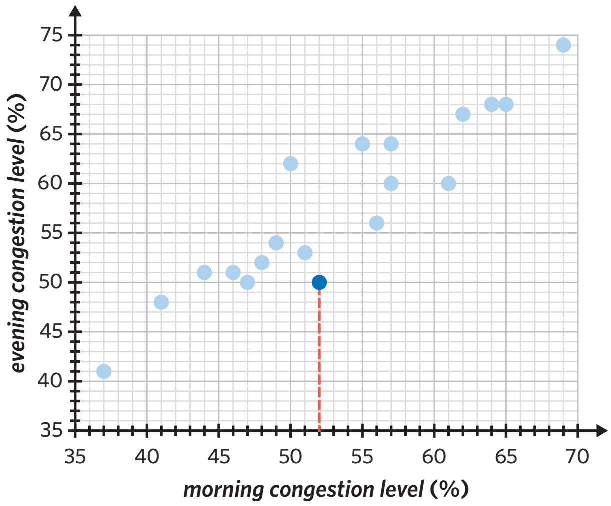
There are 19 cities in the sample, so $n = 19$.

$$\frac{19+1}{2} = 10$$

Step 2: Determine the median percentage congestion level for the morning peak period.

Count from left to right to the 10$^{\text{th}}$ data point.

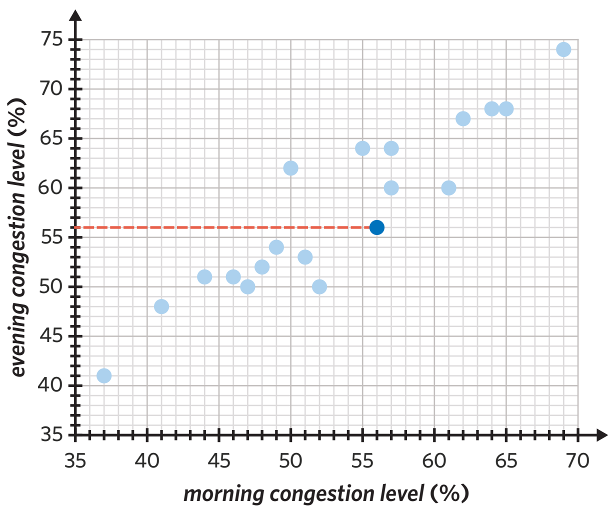
The 10$^{\text{th}}$ data point across corresponds to the horizontal axis value of 52%.



Step 3: Determine the median percentage congestion level for the evening peak period.

Count from bottom to top to the 10$^{\text{th}}$ data point.

The 10$^{\text{th}}$ data point up corresponds to the vertical axis value of 56%.



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Answer

Median percentage congestion level for morning peak period: 52%

Median percentage congestion level for evening peak period: 56%

> A number of students found the correct median for the morning peak period from the horizontal axis value, yet incorrectly chose to use the same point for finding the median of the evening period.

Questions from multiple lessons

20. C

21. E

22. **a.**

		size		
		small	**medium**	**large**
caffeine level	**low**	0	0	6
	medium	0	6	0
	high	2	0	1
total		2	6	7

b. 14%

1E Summarising numerical data – mean and standard deviation

Calculating the mean of a data set

1. B

2. 4.05

3. **a.** Increase **b.** 68.1%

Calculating the mean and standard deviation using technology

4. D

5. **a.** \$8.66

b. \$30.76

c. Increased.
The new value is significantly larger than the original values, so the average deviation from the mean has increased.

6. **a.** The standard deviation will increase.

b. 3.18 kg

Using the mean and standard deviation to compare numerical distributions

7. B

8. **a.** The basketball team has a higher mean height than the netball team. The mean for the basketball team is 178.4 cm, while the mean for the netball team is 167.7 cm.

b. The basketball team has a smaller standard deviation in height than the netball team. The standard deviation for the basketball team is 3.2 cm, while the standard deviation for the netball team is 8.1 cm.

Joining it all together

9. **a.** Mean: 6.2
Standard deviation: 3.6

b. 9.3

c. The mean and standard deviation would both increase.

10. **a.** Mean: 23.6 points
Standard deviation: 3.91 points

b. 30 points

c. 4.08 points

d. The standard deviation in points scored was smaller in the first five games than in all seven games. The standard deviation for the first five games was 3.91 points, while the standard deviation for all seven games was 4.08 points.

Exam practice

11. **Explanation**

Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name column A 'time' and enter the data starting from row 1.

Step 3: Press ctrl + doc▾, and select '1: Add Calculator'.

Step 4: Press menu → '6: Statistics' → '1: Stat Calculations' → '1: One-Variable Statistics'.

Set 'Num of Lists:' to '1' and select 'OK'.

Set 'X1 List:' to 'time' and select 'OK'.

Step 5: Identify the mean ($\bar{x}$) and standard deviation (s_x).

Mean: 25.65

Standard deviation: 3.14

Method 2: Casio ClassPad

Step 1: From the menu, tap Statistics.

Step 2: Type the data in column 'list1'.

Step 3: Tap 'Calc' → 'One-Variable'.

Set 'XList:' to 'list1' and 'Freq:' to '1'.

Tap 'OK'.

Step 4: Identify the mean ($\bar{x}$) and standard deviation (s_x).

Mean: 25.65

Standard deviation: 3.14

Answer – Method 1 and 2

A

12. **Explanation**

Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name column A 'height' and enter the data for *height* starting from row 1.

Step 3: Press ctrl + doc▾, and select '1: Add Calculator'.

Step 4: Press menu → '6: Statistics' → '1: Stat Calculations' → '1: One-Variable Statistics'.

Set 'Num of Lists:' to '1' and select 'OK'.

Set 'X1 List:' to 'height' and select 'OK'.

Step 5: Identify the mean ($\bar{x}$).

Method 2: Casio ClassPad

Step 1: From the menu, tap Statistics.

Step 2: Type the data for *height* in column 'list1'.

Step 3: Tap 'Calc' → 'One-Variable'.

Set 'XList:' to 'list1' and 'Freq:' to '1'.

Tap 'OK'.

Step 4: Identify the mean ($\bar{x}$).

Answer – Method 1 and 2

177.26 cm

13. **Explanation**

Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name column A 'long' and enter the data for *long jump* starting from row 1.

Step 3: Press ctrl + doc, and select '1: Add Calculator'.

Step 4: Press menu → '6: Statistics' → '1: Stat Calculations' → '1: One-Variable Statistics'.

Set 'Num of Lists:' to '1' and select 'OK'.

Set 'X1 List:' to 'long' and select 'OK'.

Step 5: Identify the standard deviation (s_x).

Method 2: Casio ClassPad

Step 1: From the menu, tap Statistics.

Step 2: Type the data for *long jump* in column 'list 1'.

Step 3: Tap 'Calc' → 'One-Variable'.

Set 'XList:' to 'list1' and 'Freq:' to '1'.

Tap 'OK'.

Step 4: Identify the standard deviation (s_x).

Answer – Method 1 and 2

statistic	*long jump* (metres)	*100 m run* (seconds)
mean	6.90	14.16
standard deviation	0.80	1.41

Questions from multiple lessons

14. C

15. D

16. **a.** 9 days **b.** 6.5% **c.** 2 days

1F The five-number summary and boxplots

Calculating the five-number summary

1. B

2. 15.8

3. 12.43, 24.32, 34.34, 38.20, 45.54

4. 4, 14, 24, 36, 57

5. **a.** 8

b. $Q_1 = 7.5$
$Q_3 = 9$

c. Yes, $Q_1 = 7.5$ and the *median* $= 8$ which is nearly equal. This is the case because the lower 25% of data is between the 7–8 interval.

d. 7, 7.5, 8, 9, 10

Identifying outliers using fences

6. A

7. 19

8. **a.** *lower fence* $= 1.5$ and *upper fence* $= 13.5$.
2 outliers at 14 and 80.

b. *lower fence* $= 23.75$ and *upper fence* $= 69.75$.
1 outlier at 70.

9. **a.** *lower fence* $= -5$ and *upper fence* $= 11$.

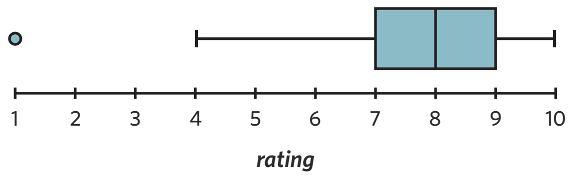
b. 2 outliers at 15 and 21.

c. 15 and 21 are both greater than the upper fence value of 11, meaning they are outliers.

Constructing and interpreting boxplots

10. C

11.



12. **a.** 17 and 22 °C **b.** 50%

Joining it all together

13. **a.** 110.56, 119.12, 123.22, 134.18, 167.98

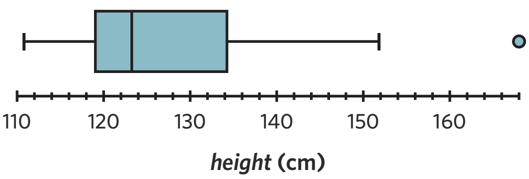
b. One outlier at 167.98 cm.

c. *lower fence* $= 119.12 - (15.06 \times 1.5)$
$= 96.53$

upper fence $= 134.18 + (15.06 \times 1.5)$
$= 156.77$

110.65 is within the lower and upper fences so is not an outlier.

d.



e. 50%

f. 7 girls

Exam practice

14. **Explanation**

Step 1: Identify the total number of chimpanzees with below average neck sizes.

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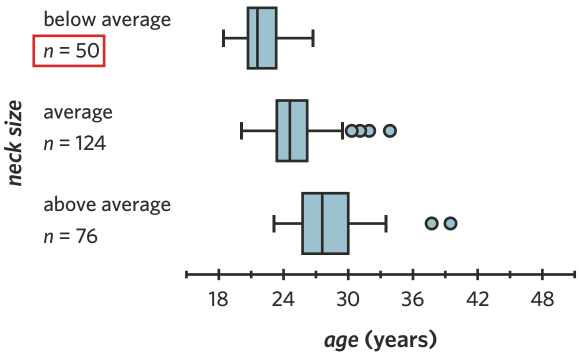


50 chimpanzees have below average neck sizes.

Step 2: Represent this as a percentage of the total.

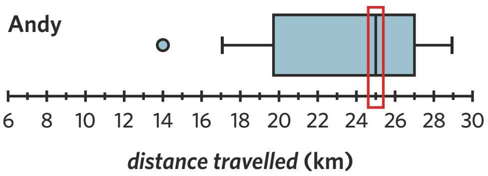
$\frac{50}{250} \times 100$

Answer

20%

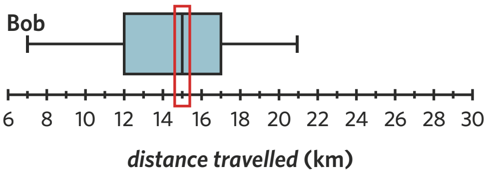
15. Explanation

Step 1: Identify the median for Andy's *distance travelled*.



median = 25 km

Step 2: Identify the median for Bob's *distance travelled*.



median = 15 km

Step 3: Find the difference between the medians of Andy's *distance travelled* and Bob's *distance travelled*.

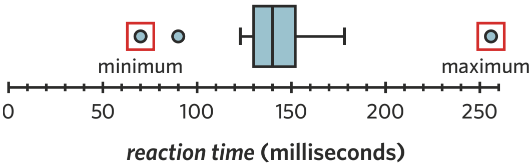
25 − 15 = 10

Answer

In the lead up to the triathlon, the median value for Bob's *distance travelled* was 10 km less than the median value for Andy's *distance travelled*.

16. Explanation

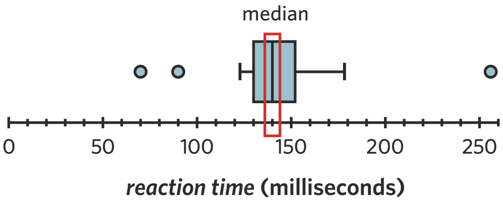
Step 1: Identify the minimum and maximum.



minimum = 70 milliseconds

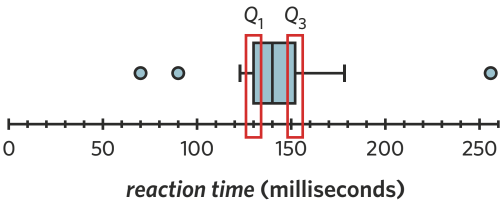
maximum = 255 milliseconds

Step 2: Identify the median.



median = 140 milliseconds

Step 3: Identify Q_1 and Q_3.



Q_1 = 130 milliseconds

Q_3 = 152 milliseconds

Step 4: Construct the five-number summary.

70, 130, 140, 152, 255

Answer

B

> Many students ignored the outlier points for this type of question when determining the maximum and minimum values for the five-number summary, leading to an incorrect response of option C. Even though the points are identified as outliers, they are still valid data points within the data set and must be used as the maximum and minimum values if appropriate.

Questions from multiple lessons

17. C **18.** B

19. **a.** Class B **b.** 2.5 hours

1G Investigating data distributions

Describing the shape of data distributions

1. **a.** B **b.** C **c.** D **d.** A

2. Negatively skewed

3. A positive skew suggests that the data trails off from a peak towards the more positive end of the distribution. The shape of the distribution is independent of its location on an axis.

Comparing data distributions

4. A

5. Histogram 2 has a greater centre than histogram 1.

6. Both histograms have a similar spread.

7. **a.** Centre **b.** Neither **c.** Centre

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Interpreting the normal distribution

8. B

9. **a.** 34% **b.** C

c. 24 students **d.** 7 students

Joining it all together

10. **a.** Both distributions have a negative skew. Distribution 1 has a lower centre and a greater spread than distribution 2. Answers may vary.

b. Both distributions are approximately symmetric. Distribution 1 has a greater centre and a smaller spread than distribution 2. Answers may vary.

c. Distribution 1 is negatively skewed, while distribution 2 is positively skewed. Distribution 1 has a lower centre and a larger spread than distribution 2.
Answers may vary.

d. Distribution 1 is positively skewed, while distribution 2 is perfectly symmetric. Both distributions have a similar centre, and distribution 1 has a larger spread than distribution 2.
Answers may vary.

11.

description	distribution
largest spread	II
largest centre	VI
skewed (positively or negatively)	I, III, IV, V, VI
symmetric (perfectly or approximately)	II

Exam practice

12. Explanation

Step 1: Determine the number of standard deviations each value is from the mean.

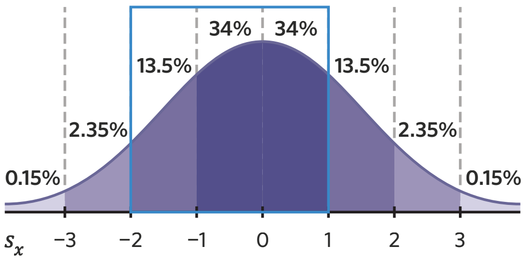
The mean is 16 and the standard deviation is 4.

$16 - (4 \times 2) = 8$, therefore 8 minutes is 2 standard deviations below the mean.

$16 + (4 \times 1) = 20$, therefore 20 minutes is 1 standard deviation above the mean.

Step 2: Calculate the percentage of data in the defined boundary.

The interval is between two standard deviations below and one standard deviation above the mean.



$13.5\% + 34\% + 34\% = 81.5\%$

Answer

D

13. Explanation

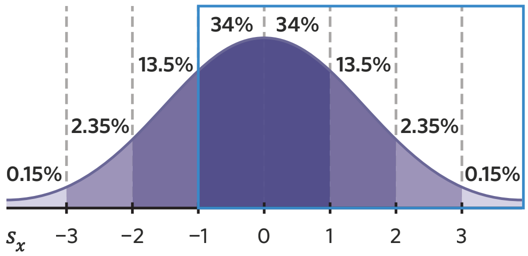
Step 1: Determine the number of standard deviations the value is from the mean.

The mean is 210 000 km and the standard deviation is 15 000 km.

$210\,000 - (15\,000 \times 1) = 195\,000$, therefore 195 000 km is 1 standard deviation below the mean.

Step 2: Calculate the sum of the percentages in this boundary.

The interval is everything greater than 1 standard deviation below the mean.



$34\% + 34\% + 13.5\% + 2.35\% + 0.15\% = 84\%$

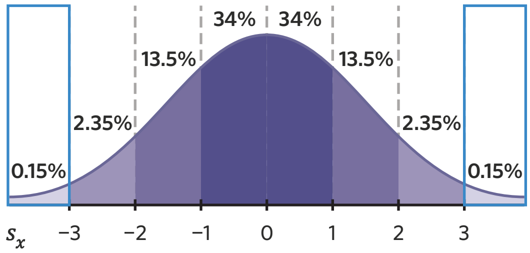
Answer

84%

14. Explanation

Step 1: Calculate the percentage of data that lies in the specified boundaries.

The interval is everything above and below three standard deviations.



$0.15\% + 0.15\% = 0.3\%$

Step 2: Calculate the number of bonobos.

$\frac{0.3}{100} \times 250 = 0.75$

≈ 1 bonobo

Answer

1

Students commonly round incorrectly when answering questions similar to this. They would either have rounded down to 0 or left the answer unrounded as 0.75, rather than rounding up to 1.

Questions from multiple lessons

15. D **16.** B

17. **a.**

rating	frequency
appetising	2
divine	5
mediocre	5
rancid	2
unpalatable	2
total	**16**

b. 31.25%

1H Comparing data distributions

Comparing distributions using a back-to-back stem plot

1. D

2. **a.** 1 litre **b.** 56 litres **c.** Men
d. Women **e.** Men

3. **a.** Fernando: 18, Lucas: 9
Fernando had a higher median *number of points* than Lucas.

b. Fernando: 24, Lucas: 26
Fernando had a smaller range of *number of points* than Lucas.

c. Fernando: 17.5, Lucas: 10
Fernando had a larger IQR of *number of points* than Lucas.

4. **a.** **Key:** 1 | 5 = 15 years old

UK		USA
9 8 7	1	8 8 8 9 9
4 3 2 2	2	0 1 1 1 2 3 4 4
9 8 8 7 6 6 5	2	5
1	3	4

b. The median for UK passengers was 25 and the median for USA passengers was 21. Passengers from the UK had a higher median *age* than passengers from the USA.

c. The range for UK passengers was 14 and the range for USA passengers was 16. Passengers from the UK had a smaller range of *age* than passengers from the USA.

Comparing distributions using parallel boxplots

5. C

6. **a.** Supermarket A
b. Supermarket A

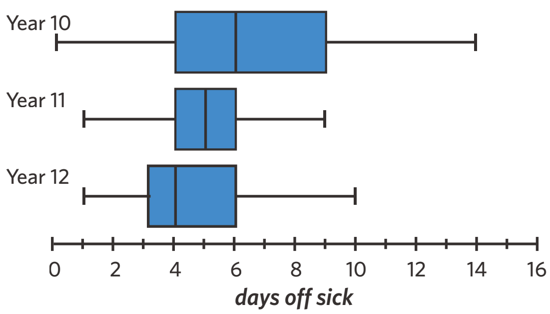
7. **a.** Class C **b.** Class A **c.** Class B

8. **a.** Cafe A: 72, Cafe B: 94
Cafe A had a lower median *coffees sold* than cafe B.

b. Cafe A: 30, Cafe B: 38
Cafe A had a smaller range of *coffees sold* than cafe B.

c. Cafe A: 17, Cafe B: 14
Cafe A had a larger IQR of *coffees sold* than cafe B.

9. **a.**



b. Year 10 median: 6, Year 11 median: 5, Year 12 median: 4
Year 10 had the highest centre of *days off sick*, followed by Year 11 and then Year 12.

c. Year 10 IQR: 5, Year 11 IQR: 2, Year 12 IQR: 3
Year 10 range: 14, Year 11 range: 8, Year 12 range: 9
Year 10 had the largest spread of *days off sick*, followed by Year 12 and then Year 11.

Interpreting differences between distributions

10. B

11. The median age of men is 74 and the median age of women is 90. In general, the men in the retirement village are younger than the women.
The IQR of the ages of men is 7 and the range is 16. The IQR of the ages of women is 8 and the range is 20. The ages of men in the retirement village are less variable than the ages of women.

12. The median number of hours student A spent studying was 9.5 and the median number of hours student B spent studying was 15. In general, student A spent less hours studying each week than student B.
The IQR for student A was 5.5 hours and the IQR for student B was 10. The number of hours student A spent studying was less variable than student B.

13. **a.** The median price for airline A is \$950 and the median price for airline B is \$1050. In general, one-way economy flights from airline A are less expensive than airline B.
The IQR for airline A is \$150 and the IQR for airline B is \$75. The prices of one-way economy flights for airline A are more variable than airline B.

b. The data for airline A contains an outlier.

Joining it all together

14. **a.** Class A: 64%, Class B: 75.5%
Class A had a lower median *exam mark* than Class B.

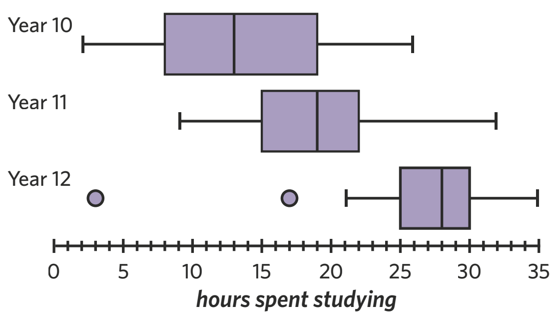
b. Class A: 48%, Class B: 46%
Class A had a larger *exam mark* range than Class B.

c. Class A: 17%, Class B: 16%
Class A had a larger *exam mark* IQR than Class B.

d. Class A

e. In general, Class A achieved lower exam marks than Class B.

15. **a.**



b. The median number of *hours spent studying* is 13 for Year 10, 19 for Year 11, and 28 for Year 12. In general, *hours spent studying* increases as *year level* increases.

c. The IQR of *hours spent studying* is 11 for Year 10, 7 for Year 11, and 5 for Year 12. The variability in *hours spent studying* decreases as *year level* increases.

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Exam practice

16. Explanation

Step 1: Use the parallel boxplots to identify the median *maximum daily temperature* and *minimum daily temperature*.

The median *maximum daily temperature* is 25 °C.

The median *minimum daily temperature* is 15 °C.

Step 2: Calculate the difference.

$25 - 15 = 10$

Answer

For November 2017, the median value for *maximum daily temperature* was 10 °C higher than the median value for *minimum daily temperature*.

17. Explanation

To solve this question, check whether each option is true or false.

A: This is false. Boxplots show the distribution of several data points. Not all the babies in country B have the same height. ✗

B: This is true. The median height of the babies in country B is greater than the maximum height of the babies in country A. ✓

C: This is false. The minimum height of the babies in country A is less than the minimum height in country B, so 50% of the babies born in country B cannot be shorter than all the babies in country A. ✗

D: This is false. Q_3 for the babies in country A is greater than the minimum height of the babies in country B. ✗

E: This is false. Q_1 for the babies in country B is less than the maximum height of the babies in country A. ✗

Answer

B

Many students incorrectly chose option E. Students may not have recognised that in order for 75% of the babies in country B to be taller than all of the babies in country A, the line representing the first quartile needed to be greater than the maximum value in country A. When a multiple choice option has the word 'all' in it, this implies there are no exceptions.

Questions from multiple lessons

18. B

19. D

20. a. The distribution of *ice cream cones sold* is positively skewed with an outlier.

b. 23

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2A Rates and ratios

Identifying and simplifying ratios

1. D
2. a. 3 : 5 b. 2 : 3
   c. 2 : 5 : 1 d. 9 : 4 : 3
   e. 39 : 51 : 9 : 28 f. 31 : 68
   g. 81 : 77 : 94 : 73 h. 5 : 3 : 2 : 2
3. a. 7 : 6 b. 13 : 7
   c. 2 : 5 d. 10 : 3
   e. 7 : 13 : 6 f. 15 : 6 : 13 : 7

Identifying and simplifying rates

4. C
5. a. 48 orders/day b. 5.2 m/h
   c. 4.5 mm/day d. 0.27 h/page
6. Anna

Performing calculations with rates and ratios

7. B
8. a. 877.8 L/week b. 45.5 sales/month
   c. 165.6 km/h d. 20 307.69 m/week
9. a. Jar 2 b. Jar 1: 84
   Jar 2: 64
   Jar 3: 94
10. a. 22.5 km b. 1 hour and 15 minutes
    c. 3.33 minutes/km

Joining it all together

11. a. Anita: 16 hours
    Frank: 12 hours
    Vincent: 28 hours
    Meg: 36 hours
    b. Anita
    c. Frank, Anita, Vincent, Meg d. 252 boxes
12. a. 3 : 80 : 20 b. Swim: 1.5 km
    Bike: 40 km
    Run: 10 km
    c. Swim: 1.8 km/h
    Bike: 20 km/h
    Run: 10 km/h
    d. Swim: 33.33 minutes/km
    Bike: 3 minutes/km
    Run: 6 minutes/km

Questions from multiple lessons

13. E 14. B
15. a. *type of athlete*
    b. 177 cm and 179 cm
    c. $x = 145.5$
    $y = 193$

2B Percentages

Calculating percentages of numbers

1. C
2. a. 80% b. 32% c. 6.67%
3. a. 91 b. 0.3 c. 101.97
4. a. Sam: 54.55%
   Charlie: 57.14%
   b. Charlie
5. 51 pages
6. 70 grams

Calculating percentage increase and decrease

7. C
8. a. 1023.82 b. 141.5
   c. 666.93 d. 43 061.45
9. $17.00
10. a. 50% increase b. 30.43% decrease
    c. 4.58% increase
11. 96 runs
12. a. $14.40 b. $13.75 c. $24.75

Applying percentages to GST calculations

13. GST is a 10% tax added to goods and services in Australia.
14. a. $198 b. $385 c. $15.95
15. a. $180.91 b. $109.05 c. $226.36
16. a. $4.50 b. $17.50 c. $1.70

Joining it all together

17. a. $53 116.83 b. $4828.80
18. 439%

Exam practice

19. **Explanation**

Calculate the Water World visitors as a percentage of the total.

There were 400 visitors at Water World and the park was at its capacity of 2000 visitors.

$\frac{400}{2000} \times 100 = 20$

Answer

20%

20. **Explanation**

Calculate the number of clusters that contain more than 170 eggs as a percentage of the total number of clusters recorded.

There are 3 clusters containing more than 170 eggs and 12 clusters in total.

$\frac{3}{12} \times 100 = 25$

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Answer

25%

21. **Explanation**

Calculate the number of days with a maximum temperature higher than 15.3 °C as a percentage of the total number of days measured.

There are 3 days with a maximum temperature higher than 15.3 °C and 15 days in total.

$\frac{3}{15} \times 100 = 20$

Answer

20%

Questions from multiple lessons

22. D
23. E

24. a. 15%
 b. $1100

2C Inflation

Calculating price changes over time

1. B
2. a. $30 476 b. $31 390 c. $35 863 d. 2035
3. a. 243.75% b. $37.81
4. a. $6.67 b. $242 279
5. a. 2.29% b. $2.51

Calculating changes in spending power over time

6. B
7. a. $1177.63 b. $878.61
8. a. $2798.51 b. $5838.39
9. 2.899%
10. $24.30

Joining it all together

11. a. $5.20 b. $19.22
12. a. $5280.80 b. 2028 c. $48 067

Questions from multiple lessons

13. C
14. B
15. a. 7100 cm b. 10.41 cm

2D The unitary method and its applications

Using the unitary method

1. D
2. a. $0.15 b. $35 c. $22 d. $1848.50
3. a. $8.25 b. $20.02 c. $2.25 d. $500.24
4. 33 kg
5. $259.50
6. 26 pizzas
7. a. $1.48 b. $1.45 c. Mayver's
8. a. 110 km b. 6 hours, 22 minutes and 30 seconds

Using the unitary method to calculate percentages

9. B
10. a. 264 pages b. 204 marks c. 54 km d. $3050
11. a. $87.50 b. $61.60 c. $3516 d. $10 080
12. $17 166.50
13. No
14. 23%

Joining it all together

15. a. $23.50 b. $1222 c. $25.85 d. $19.98 e. $21.97
16. a. 12.5 cents b. $185.88 c. $1444.63 d. $2148 e. $395.86 f. 3167 km

Questions from multiple lessons

17. C
18. C
19. a. 22% b. 3.7% decrease c. 368 aardvarks

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2E Purchase options

Evaluating cash and card purchase options

1. D
2. a. $57.00 b. $63.25
3. a. 45 days b. 15 days
4. a. 80 days b. $361.70
5. 12.75% p.a.

Evaluating personal loans and buy now pay later schemes

6. D
7. a. $19 800 b. $5800
8. 12.7%
9. a. $75 b. $125 c. $795 d. $195

Comparing purchase options

10. C
11. a. Buy now pay later
    b. The buy now pay later option could end up being far more expensive if he misses any payments.
12. a. Credit card b. 41 days
13. The personal loan is cheaper by $35.43.

Joining it all together

14. a. C b. $9717.96
    c. 13.48% d. Yes
    e. The credit card option will be more expensive by $88.96.
    f. Given Oswald receives most of his income in December, he might fail to meet the regular payments, and the buy now pay later option could end up being more expensive.

Questions from multiple lessons

15. C 16. E
17. a. Positively skewed
    b. −$4000

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3A Introduction to sequences and recursion

Identifying arithmetic and geometric sequences

1. D

2. **a.** Arithmetic
38

b. No pattern

c. Geometric
4

d. Arithmetic
−20

3. **a.** 0.5, 1, 2, 4, 8, 16

b. 11, 8, 5, 2, −1

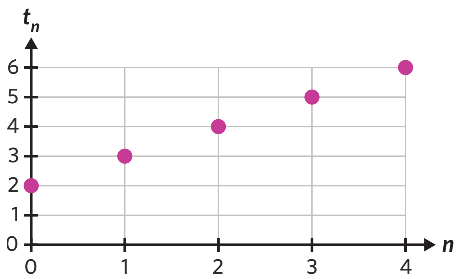
Displaying sequences as tables and graphs

4. D

5. B

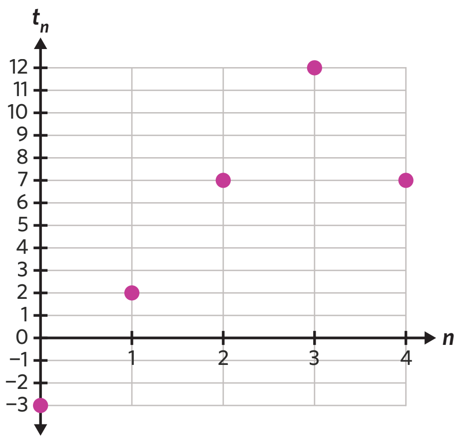
6. **a.**

n	0	1	2	3	4
t_n	2	3	4	5	6



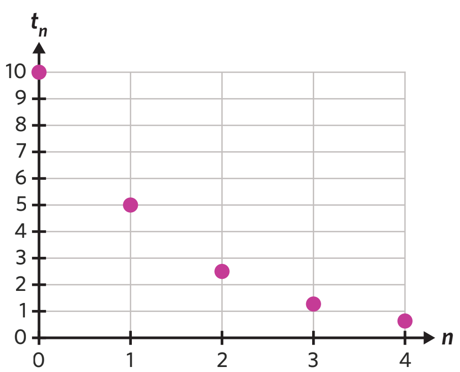
b.

n	0	1	2	3	4
t_n	−3	2	7	12	7



c.

n	0	1	2	3	4
t_n	10	5	2.5	1.25	0.625



Investigating the behaviour of sequences

7. B

8. **a.** Increasing
20 is a limiting value.

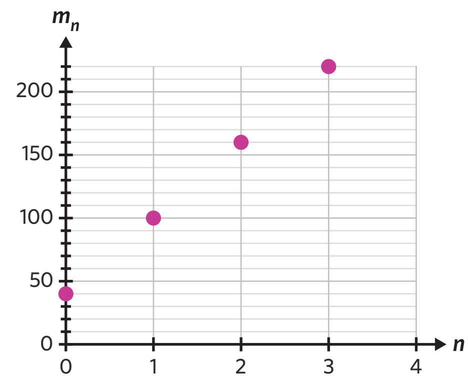
b. Oscillating

c. Constant

d. Oscillating

Joining it all together

9. **a.**

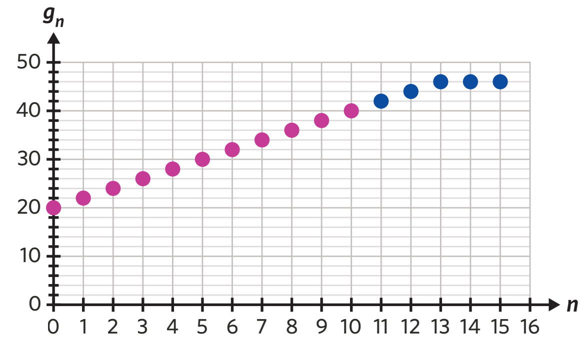


b. Arithmetic

c. 400 members

10. **a.** Greg is correct. The type of sequence is dependent on the pattern between individual terms. Each term is 2 more than the previous term, so the sequence is arithmetic.
Answers may vary.

b.

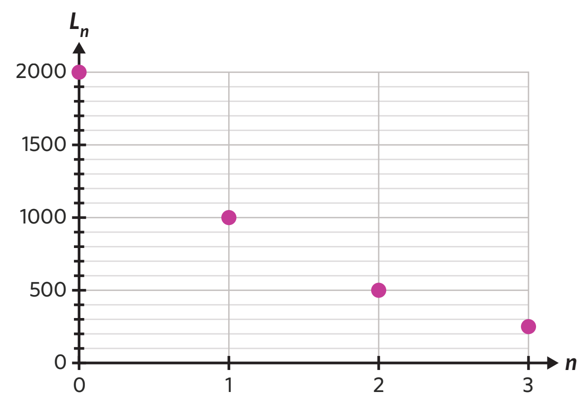


c. The trend is increasing, and there is a limiting value of 46.

11. **a.**

n	0	1	2	3
L_n	2000	1000	500	250

b.



c. Geometric
The sequence is decreasing with no limiting values.

d. 6 years

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Exam practice

12. Explanation

Step 1: Identify the pattern in the sequence.

The second term, 10, is double the first term.

The third term, 20, is double the second term.

Each term is double the value of the previous term.

Step 2: Calculate the value of the fifth term using the fourth term and the pattern.

$40 \times 2 = 80$

Answer

D

Questions from multiple lessons

13. D **14.** C

15. a. $6.75

b. $44.25

3B Arithmetic sequences and recurrence relations

Generating a sequence from an arithmetic recurrence relation

1. C

2. a. 21 **b.** -5

3. a. $-5046, -4996, -4946, -4896, -4846$

b. 3216, 3138, 3060, 2982, 2904

4. a. 3 955 000 people **b.** 4 020 000 people

Interpreting an arithmetic recurrence relation

5. A

6. a. 12 jackets

b. The team obtains another jacket after each game.

7. a. There were initially 1500 fish in the pond.

b. The pond loses 20 fish each day.

8. a. Linear decay

b. There are initially 5 000 000 bacterial cells in the culture. It is expected that the culture will lose 20 000 cells each hour.

Constructing an arithmetic recurrence relation

9. A

10. $u_0 = 105, \quad u_{n+1} = u_n - 3$

11. $B_0 = 6, \quad B_{n+1} = B_n + 8$

Joining it all together

12. a. 33 cups

b. $L_0 = 23, \quad L_{n+1} = L_n + 2$

c. 7 days

13. 1.50 m

14. a. $h_0 = 23.84, \quad h_{n+1} = h_n + 0.08$

b. 24.40 cm

Exam practice

15. a. Explanation

Identify the common difference.

3 is added to the current term, V_n, to find the next term, V_{n+1}, so the common difference, d, is 3.

This means that the collection increases by 3 vinyls each year.

Answer

3 vinyls

b. Explanation

Step 1: Calculate V_1, the number of vinyls in the collection after 1 year.

$$\begin{aligned} V_1 &= V_0 + 3 \\ &= 35 + 3 \\ &= 38 \end{aligned}$$

Step 2: Calculate V_2, the number of vinyls in the collection after 2 years.

$$\begin{aligned} V_2 &= V_1 + 3 \\ &= 38 + 3 \\ &= 41 \end{aligned}$$

Answer

$V_1 = 35 + 3 = 38$

$V_2 = 38 + 3 = 41$

> A number of students did not show recursive calculations as requested and instead simply stated the final answer.

16. Explanation

Calculate S_1 from S_0.

$$\begin{aligned} S_1 &= S_0 + 4 \\ &= 7 + 4 \\ &= 11 \end{aligned}$$

This means that 11 is the value of S_1.

Answer

A

Questions from multiple lessons

17. D **18.** D

19. a. $d = 32\,000 - 30\,000 = 2000$
$d = 34\,000 - 32\,000 = 2000$

b. 44 000 umbrellas

c. 170 000 umbrellas

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3C Arithmetic recursion applications

Modelling simple interest investments using recurrence relations

1. D

2. a. $u_0 = 80\,000, \quad u_{n+1} = u_n + 150$
   $u_4 = \$80\,600$

   b. $u_0 = 1\,025\,000, \quad u_{n+1} = u_n + 15\,375$
   $u_4 = \$1\,086\,500$

   c. $u_0 = 10\,000, \quad u_{n+1} = u_n + 700$
   $u_4 = \$12\,800$

   d. $u_0 = 14\,200, \quad u_{n+1} = u_n + 159.75$
   $u_4 = \$14\,839$

3. a. Henry

   b. Henry: $u_0 = 18\,000, \quad u_{n+1} = u_n + 150$
   Athira: $u_0 = 23\,920, \quad u_{n+1} = u_n + 41.4$

   c. Henry: $u_8 = \$19\,200$
   Athira: $u_8 = \$24\,251.20$

4. a. \$84 000

   b. $u_0 = 84\,000, \quad u_{n+1} = u_n + 350$

   c. 9 months

5. Option 2

Modelling unit cost depreciation using recurrence relations

6. B

7. a. $u_0 = 20\,000, \quad u_{n+1} = u_n - 0.02$
   $u_5 = \$19\,999.90$

   b. $u_0 = 600, \quad u_{n+1} = u_n - 1.5$
   $u_5 = \$592.50$

   c. $u_0 = 260, \quad u_{n+1} = u_n - 5$
   $u_5 = \$235$

   d. $u_0 = 799, \quad u_{n+1} = u_n - 10$
   $u_5 = \$749$

8. a. 750 washes

   b. \$562.50

   c. \$0.75

   d. $u_0 = 750, \quad u_{n+1} = u_n - 0.75$

9. $u_0 = 7500, \quad u_{n+1} = u_n - 1.1$

$$\begin{aligned} u_1 &= 7500 - 1.1 \\ &= 7498.9 \\ u_2 &= 7498.9 - 1.1 \\ &= 7497.8 \\ u_3 &= 7497.8 - 1.1 \\ &= 7496.7 \end{aligned}$$

Modelling flat rate depreciation using recurrence relations

10. A

11. a. $u_0 = 800, \quad u_{n+1} = u_n - 100$
   8 years

   b. $u_0 = 1100, \quad u_{n+1} = u_n - 22$
   4 years and 2 months

   c. $u_0 = 100, \quad u_{n+1} = u_n - 6.25$
   4 years

   d. $u_0 = 650, \quad u_{n+1} = u_n - 13$
   8 years and 4 months

12. a. 6.25%

   b. $u_0 = 1200, \quad u_{n+1} = u_n - 75$

   c. \$825

13. $u_0 = 25\,000, \quad u_{n+1} = u_n - 1250$

$$\begin{aligned} u_1 &= 25\,000 - 1250 \\ &= 23\,750 \\ u_2 &= 23\,750 - 1250 \\ &= 22\,500 \\ u_3 &= 22\,500 - 1250 \\ &= 21\,250 \\ u_4 &= 21\,250 - 1250 \\ &= 20\,000 \end{aligned}$$

Joining it all together

14. A

15. a. $u_0 = 1\,920\,000, \quad u_{n+1} = u_n + 9600$

   b. $u_0 = 2\,035\,200, \quad u_{n+1} = u_n - 127\,200$

   c. Fierro ends up with \$520 800 less than he initially invested.

16. a. $u_0 = 39\,000, \quad u_{n+1} = u_n + 633.75$

   b. 2 years and 6 months

   c. $u_0 = 45\,000, \quad u_{n+1} = u_n - 1250$

   d. \$40 000

   e. 5.56%

Exam practice

17. a. **Explanation**

Step 1: Use the recurrence relation to calculate the value of the decks after one year.

$$\begin{aligned} u_1 &= u_0 - 160 \\ &= 3200 - 160 \\ &= 3040 \end{aligned}$$

Step 2: Continue this process until the value after 3 years, u_3, is calculated.

Note: The question requires the recursive calculations as part of the solution.

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Answer

$u_1 = 3200 - 160$
$= 3040$
$u_2 = 3040 - 160$
$= 2880$
$u_3 = 2880 - 160$
$= 2720$

Many students who received no marks on this type of question did not include recursive calculations as asked, and instead just provided the final answer.

b. Explanation

Step 1: Determine the depreciation amount per year.

According to the recurrence relation, the decks depreciate by $160 per year.

Step 2: Calculate the annual depreciation as a proportion of the initial value.

$160 \div 3200 = 0.05$

Answer

5%

18. Explanation

Step 1: Determine the depreciation amount per year.

According to the recurrence relation, the machine depreciates by $1440 per year.

Step 2: Convert this amount to the number of coffees made per year.

The machine depreciates by $0.05 per coffee made.

$1440 \div 0.05 = 28\,800$

Answer

28 800 coffees

19. Explanation

Step 1: Calculate how much the van has depreciated each year.

The total depreciation across the four years is $64\,000 - 40\,000 = \$24\,000$.

$24\,000 \div 4 = 6000$

Step 2: Calculate the depreciation per kilometre travelled.

The minivan travelled an average of 8000 km each year, and depreciates by $6000 each year.

$6000 \div 8000 = 0.75$

Answer

$0.75

A common error with this type of question was ignoring the time in which the depreciation occurred. This would result in an answer of $3.00, found by dividing $24 000 by 8000 km.

Questions from multiple lessons

20. D **21.** A

22. a. $301
b. 1.5%

3D Geometric sequences and recurrence relations

Generating a sequence from a geometric recurrence relation

1. D

2. a. 128 **b.** 0.39

3. a. −10, −50, −250, −1250, −6250
b. 25, 87.5, 306.25, 1071.88, 3751.56

4. a. 468 **b.** 2025

Interpreting a geometric recurrence relation

5. C

6. a. Geometric growth
b. The average cost of a month's supply of groceries in the year 2000 was $300.

7. a. There were 542 passengers on the train at Flinders Street Station.
b. The number of passengers who remain on the train after each train station is $\frac{3}{5}$ of the current number of passengers on the train.

Constructing a geometric recurrence relation

8. D

9. $u_0 = 162, \quad u_{n+1} = \frac{1}{3}u_n$

10. a. 1.3 **b.** 5000
c. $P_0 = 5000, \quad P_{n+1} = 1.3P_n$

Joining it all together

11. a. $h_0 = 40, \quad h_{n+1} = 0.6h_n$ **b.** $h_2 = 14$, $h_3 = 9$, $h_4 = 5$
c. Geometric decay

12. a. 14 **b.** $B_0 = 7, \quad B_{n+1} = 2B_n$
c. 448 bacteria **d.** 8 hours

13. a. C_0 represents the initial population of cats, which is 2.
M_0 represents the initial population of mice, which is 64.
b. The population of cats
c. First year: 4 cats; 43 mice
Second year: 8 cats; 29 mice
d. 4 years

Exam practice

14. Explanation

The amount of money Ken initially deposited into the savings account is the initial value of the sequence, V_0.

Answer

$15 000

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15. **Explanation**

Step 1: Calculate B_1, the balance of the savings account after one month, from B_0.

$$B_1 = 1.002B_0$$
$$= 1.002 \times 6000$$
$$= 6012$$

Step 2: Repeat to find B_2 and B_3.

$$B_2 = 1.002B_1$$
$$= 1.002 \times 6012$$
$$= 6024.024$$
$$B_3 = 1.002B_2$$
$$= 1.002 \times 6024.024$$
$$= 6036.072...$$

Answer

$6036.07

Students need to ensure they round to the nearest cent, and not the nearest 10 cents.

Questions from multiple lessons

16. D   17. C

18. a. $1 338 225.58

b. 7 years

3E Geometric recursion applications

Modelling practical applications using geometric recurrence relations

1. A

2. a. $a_0 = 200, \quad a_{n+1} = 1.23a_n$

b. 692.6 m^2

3. a. $v_0 = 22\,000, \quad v_{n+1} = 0.886v_n$

b. 6 hours

4. a. $f_0 = 300, \quad f_{n+1} = 30f_n$

b. 8 100 000 flies

Modelling financial applications using geometric recurrence relations

5. B

6. a. 0.425%

b. $V_0 = 20\,000, \quad V_{n+1} = 1.00425V_n$

c. $20 515.45

d. No. The investment will only be worth $21 044.18 after one year.

7. a. $M_0 = 12\,499, \quad M_{n+1} = 0.906M_n$

b. 8 years

c. $4657.55

Joining it all together

8. a. $V_0 = 8100, \quad V_{n+1} = 0.96V_n$

b. $6879.71

c. $C_0 = 6879.71, \quad C_{n+1} = 1.082C_n$

d. 2015

9. a. $T_0 = 21\,457, \quad T_{n+1} = 0.86T_n$

b. $C_0 = 12.5, \quad C_{n+1} = 1.14C_n$

c.

year	*number of tickets sold*	*cost per ticket*	*total revenue*
2015	21 457	$12.50	$268 212.50
2016	18 453	$14.25	$262 955.25
2017	15 870	$16.25	$257 887.50
2018	13 648	$18.50	$252 488.00
2019	11 737	$21.10	$247 650.70
2020	10 094	$24.05	$242 760.70

d. No. Their total revenue has decreased over the 5 years. Answers may vary.

Exam practice

10. **Explanation**

Use recursion to calculate the value of the props after each year for 3 years.

Answer

$$V_1 = 0.8 \times 20\,000 = 16\,000$$
$$V_2 = 0.8 \times 16\,000 = 12\,800$$
$$V_3 = 0.8 \times 12\,800 = 10\,240$$

Students needed to make sure that the full recursive sequence is shown to obtain the mark.

11. a. **Explanation**

Step 1: Calculate R.

$$R = 1 + \frac{1.7}{100}$$
$$= 1.017$$

Step 2: Determine the cost of the bill after 1 month.

$$\textit{cost after one month} = 1.017 \times 80 = 81.36$$

Step 3: Determine the cost of the bill after 2 months.

$$\textit{cost after two months} = 1.017 \times 81.36 = 82.74...$$

Answer

$82.74

A number of students did not realise that the interest rate was already a monthly value.

b. **Explanation**

Step 1: Determine the initial value and common ratio.

$$a = 107$$
$$R = 1.017 \text{ from part } \mathbf{a}.$$

Step 2: Construct the recurrence relation.

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Answer

$A_0 = 107, \quad A_{n+1} = 1.017A_n$

There were a number of things students did incorrectly in this question. First, students needed to write the recurrence relation in the right form, with the initial value written first. Second, students needed to ensure that the variable was consistent throughout the recurrence relation. Finally, some students misunderstood the question to be asking for a rule, or equation, instead of the recurrence relation.

12. Explanation

Step 1: Identify the common ratio.

$R = 0.943$

Step 2: Substitute R into $R = 1 - \frac{r}{100}$.

$0.943 = 1 - \frac{r}{100}$

Step 3: Solve for r.

$\frac{r}{100} = 0.057$

$r = 0.057 \times 100$

$= 5.7\%$ per period

The recurrence relation models the value of the asset in terms of years, so this is the annual rate of depreciation.

Answer

5.7% p.a.

A number of students converted 0.943 to a percentage to get 94.3%, and did not realise that this was the percentage of the value remaining after depreciation.

Questions from multiple lessons

13. C **14.** D

15. a. \$24

b. \$139

3F Modelling sequences using a rule

Modelling an arithmetic sequence using a rule

1. B

2. a. $m_n = 45 - 2n$ **b.** 5 moths

3. a. $p_n = 1000 + 25n$ **b.** 40 years

4. a. $b_n = 420 - 0.21n$ **b.** 5 years and 175 days

c. 0.05%

5. a. \$560 **b.** \$380

Modelling a geometric sequence using a rule

6. D

7. a. $w_n = 500 \times 0.5^n$ **b.** 31.25 g

8. a. \$2985 **b.** 7.2% p.a.

c. 25 months

9. a. $L_n = 1999 \times 0.58^n$ **b.** 5 years

Joining it all together

10. a. $A_n = 9000 + 450n$ **b.** \$11 250

c. $B_n = 9300 \times 1.005^n$ **d.** \$12 544.31

e. Loan A

11. a. $A_n = 15\,000 - 885n$ **b.** $B_n = 15\,000 - 0.10n$

c. $C_n = 15\,000 \times 0.956^n$ **d.** Unit cost method

Exam practice

12. a. Explanation

Step 1: Identify the initial value and common ratio.

$V_0 = 12\,000$

$R = 1.0062$

Step 2: Construct the rule for V_n.

$V_n = 12\,000 \times 1.0062^n$

Answer

balance $= 12\,000 \times 1.0062^n$

A number of students put the values into the incorrect boxes.

b. Explanation

Determine the corresponding n value.

n represents the number of months after the account was opened.

There are 36 months in 3 years, so $n = 36$.

Answer

36

13. Explanation

Step 1: Identify the initial value and common difference.

$V_0 = 2480$

$d = 45$

Step 2: Construct the recurrence relation.

$V_0 = 2480, \quad V_{n+1} = V_n + 45$

Answer

C

35% of students incorrectly answered A. This is likely due to its similarity to the rule for V_n.

14. Explanation

Step 1: Determine the initial value and interest rate.

$R_0 = 5000$

$r = \frac{3}{4}\%$

$= 0.75\%$ per compounding period

Step 2: Calculate the common ratio, R.

$R = 1 + \frac{0.75}{100}$

$= 1.0075$

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Step 3: Construct the rule for R_n.

R_n is the balance of the account after n years but the interest compounds quarterly so this must be reflected in the recurrence relation.

The number of compounding periods after n years is equivalent to $4 \times n$.

$R_n = 5000 \times 1.0075^{4n}$

Answer

E

51% of students incorrectly answered D. This is likely because they did not realise that R_n was the balance of the account after n years and not n quarters.

Questions from multiple lessons

15. E **16.** D

17. **a.** 23 153 units

b. 3476 units

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4A Linear algebra

Substituting values into linear equations

1. a. $a = 2 \times 2 + 3$ b. 7

2. a. i. $p = 1 - 4$
   ii. -3
   b. i. $f = \frac{1}{3} \times 3 + 7$
   ii. 8
   c. i. $h = -2.3 \times 1.7 + 11$
   ii. 7.09

3. a. 11 b. -10 c. $\frac{1}{4}$

4. a. 4 goals b. 3.5 minutes
   c. $54\ \text{cm}^2$

5. $42\,000\ \text{cm}^2$

Solving linear equations

6. a. B b. C
   c. $x = 3y - 15$

7. a. $b = \frac{a}{2} - 2$
   b. $b = \frac{43}{3} - \frac{a}{3}$ or $b = \frac{-(a - 43)}{3}$
   c. $b = 5 - \frac{a}{20}$ or $b = \frac{-(a - 100)}{20}$
   d. $b = 35 - \frac{a}{2}$ or $b = \frac{-(a - 70)}{2}$

8. $C = \frac{5}{9}(F - 32)$
   $9C = 5(F - 32)$
   $\frac{9}{5}C = F - 32$
   $F = \frac{9}{5}C + 32$

Joining it all together

9. a. 32 b. $\frac{3}{2}$ or 1.5

10. a. $M = \frac{5}{8}K$ b. 5 miles

11. a. $d = \frac{t}{5} - 6$ b. 12 dogs

12. a. \$23.45 b. 21 minutes

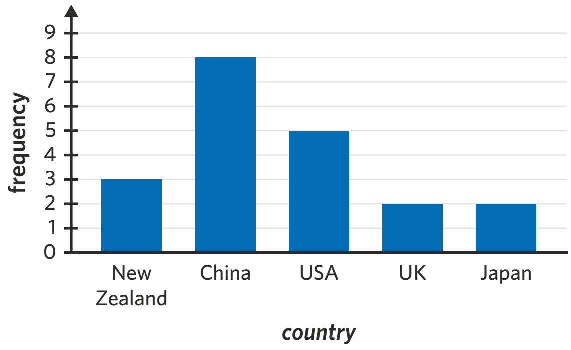
13. a. 265 K b. 77 °F

14. a. 2 points b. 8 goals

Questions from multiple lessons

15. C 16. E

17. a. 2
   b.



4B Linear functions

Identifying linear functions and their graphs

1. D

2. a. No b. Yes

3. a. Non-linear b. Linear
   c. Linear d. Non-linear
   e. Non-linear f. Linear
   g. Linear h. Non-linear

4. a. Linear b. Linear
   c. Non-linear d. Non-linear

5. No. The equation given is linear whereas the graph is non-linear.

Identifying features of linear functions and graphs

6. A 7. A

8. a. Positive gradient, x-intercept at -3, y-intercept at 6
   b. Negative gradient, x-intercept at 0.8, y-intercept at 3.2
   c. Undefined gradient, x-intercept at 2.3, no y-intercept
   d. Positive gradient, x-intercept at 7, y-intercept at -21

9. a. Positive gradient, y-intercept at -4
   b. Negative gradient, y-intercept at 8
   c. Positive gradient, y-intercept at 0
   d. Negative gradient, y-intercept at -10
   e. Undefined gradient, no y-intercept
   f. Negative gradient, y-intercept at 2
   g. Positive gradient, y-intercept at -14
   h. Zero gradient, y-intercept at 214

10. C

11. a. 2.5 km b. 40 minutes

Joining it all together

12. A 13. B

14. a. Non-linear
   b. Linear
   Positive gradient, y-intercept at 28

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c. Linear
Undefined gradient, no y-intercept

d. Non-linear

e. Linear
Negative gradient, y-intercept at 219.01

f. Linear
Negative gradient, y-intercept at 28

g. Non-linear

h. Linear
Positive gradient, y-intercept at 0

15. **a.** 155 cards **b.** Linear

c. The number of cards she gives away

Questions from multiple lessons

16. B **17.** B

18. **a.** $p = \frac{-3d}{4} + 6.5$ or $p = \frac{-3d + 26}{4}$

b. 3.13 €

4C Graphing linear functions

Plotting linear functions from a table

1. C

2. **a.**

x	0	1	2	3	4
y	0	5	10	15	20

b.

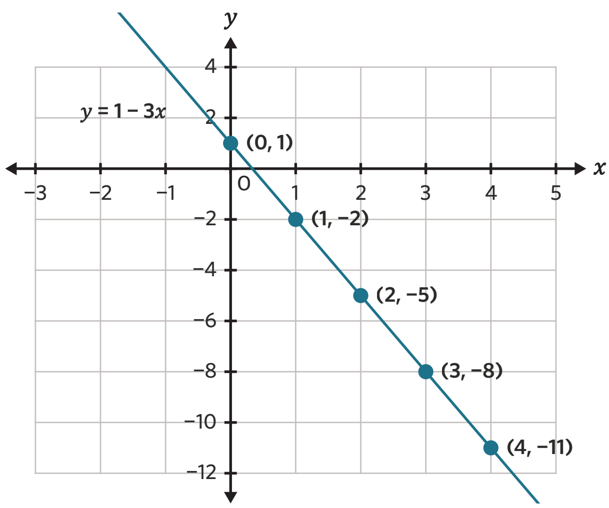
x	0	1	2	3	4
y	4	6	8	10	12

c.

x	0	1	2	3	4
y	4	1	−2	−5	−8

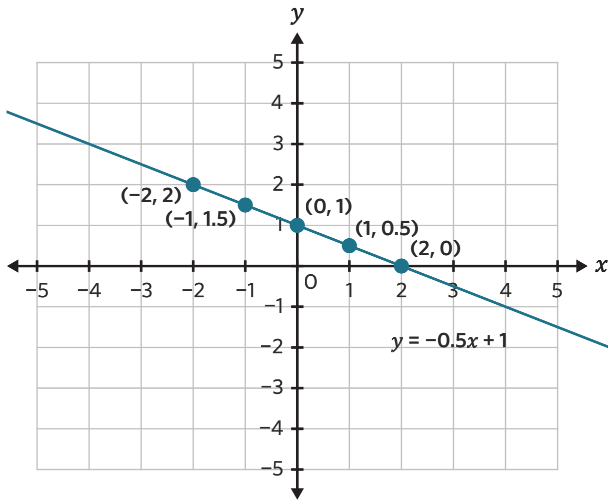
3.

x	0	1	2	3	4
y	1	−2	−5	−8	−11



4.

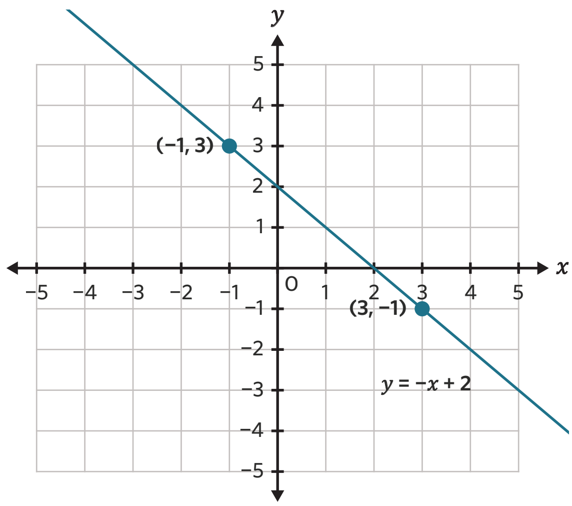
x	−2	−1	0	1	2
y	2	1.5	1	0.5	0



Graphing linear functions from an equation

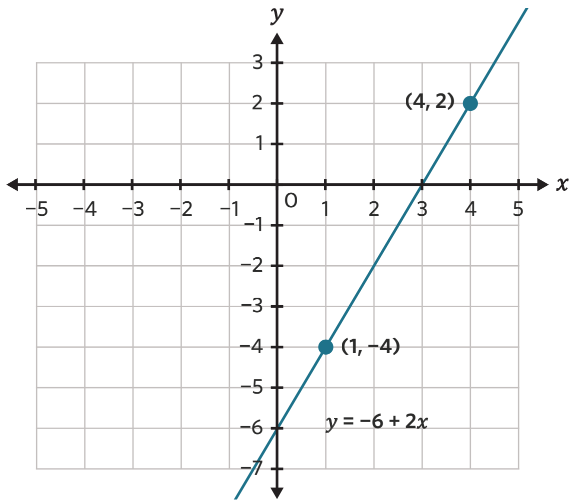
5. B

6. **a.**



Note: any two points along the line can be used.

b.



Note: any two points along the line can be used.

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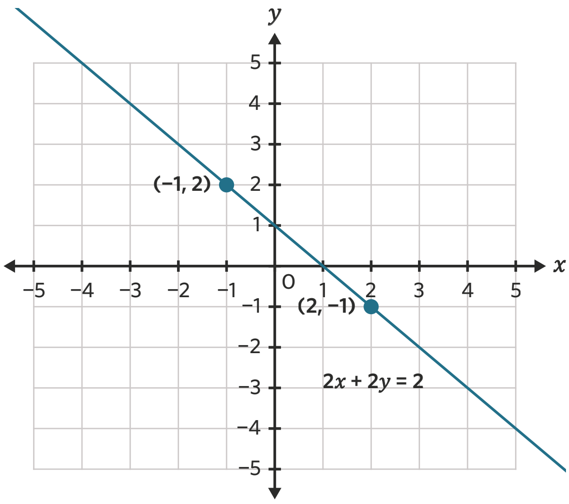
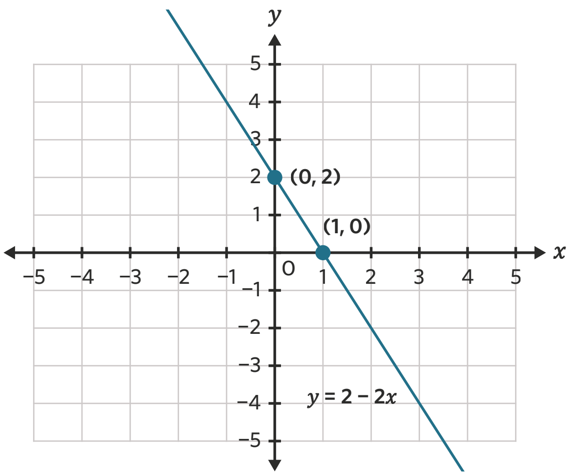
c.

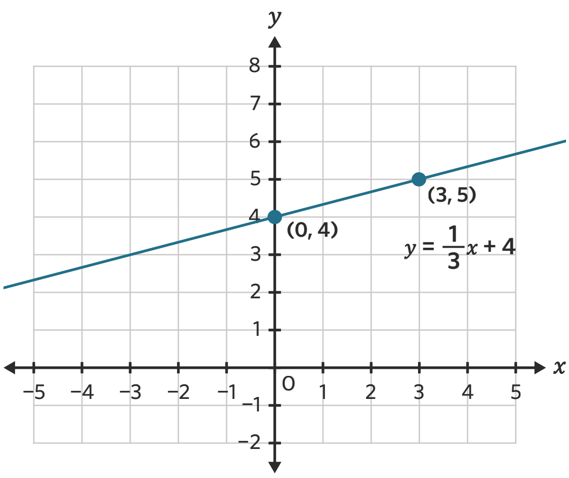


Note: any two points along the line can be used.

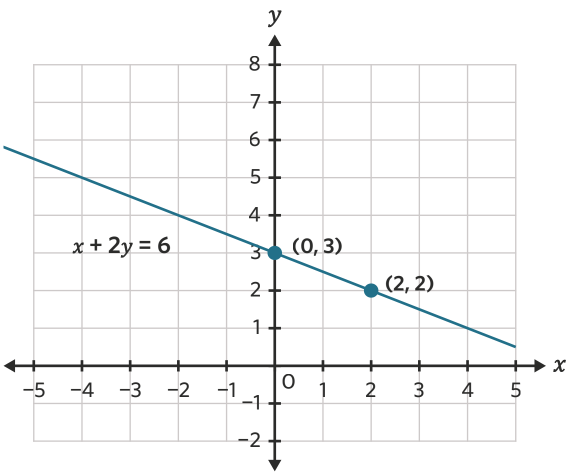
7. a.



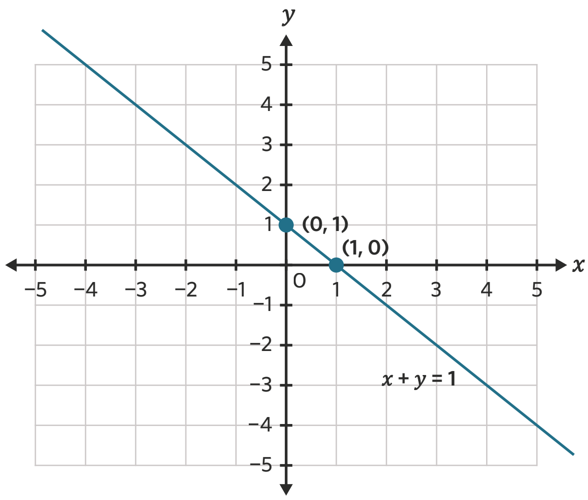
b.



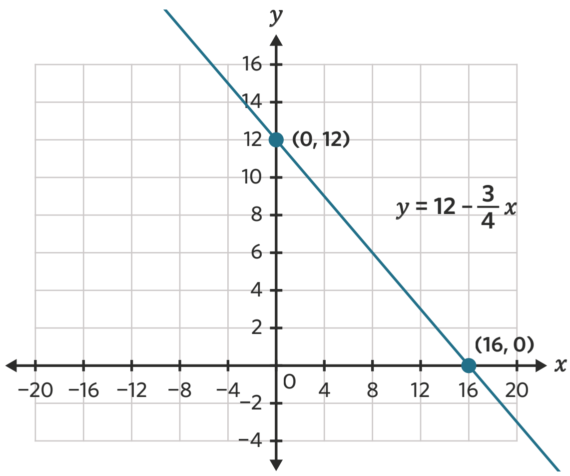
c.



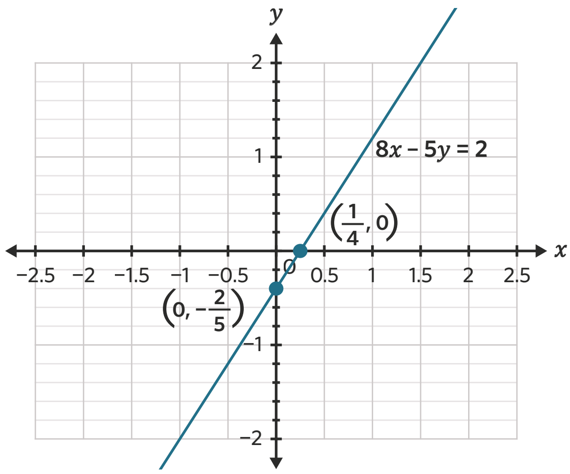
8. a.



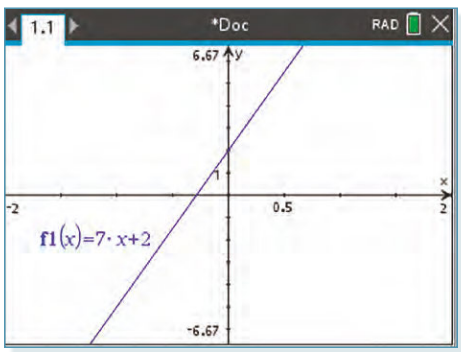
b.



c.

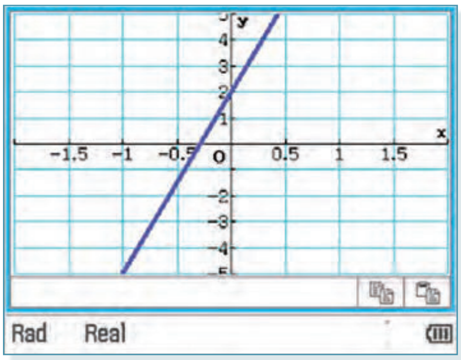


9. a. **TI-Nspire**



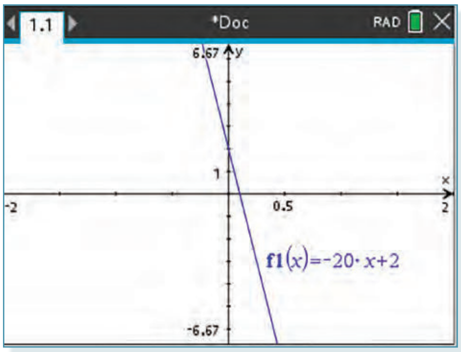
Note: Window settings have been adjusted.

Casio ClassPad



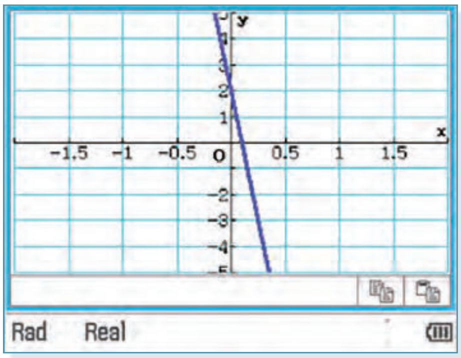
Note: Window settings have been adjusted.

b. **TI-Nspire**



Note: Window settings have been adjusted.

Casio ClassPad

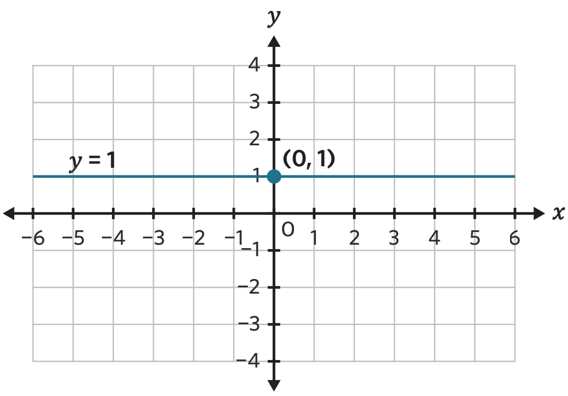


Note: Window settings have been adjusted.

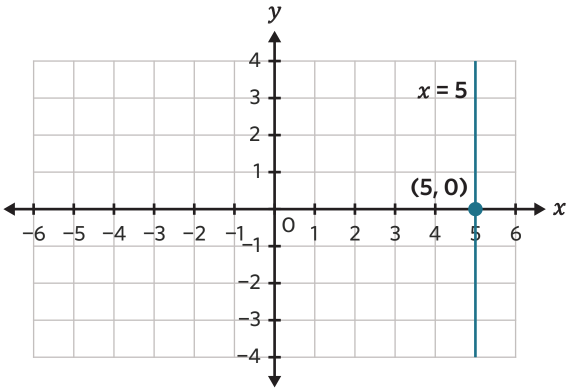
Graphing horizontal and vertical lines

10. D

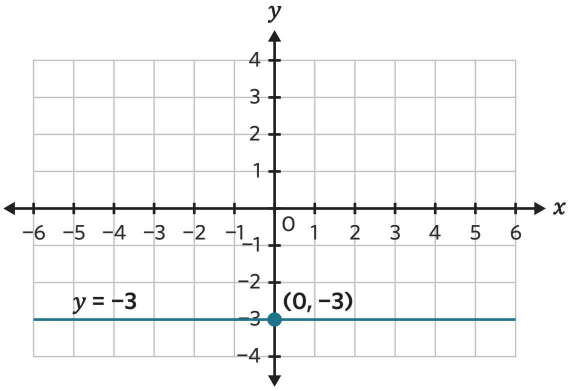
11. **a.**



b.



c.



Joining it all together

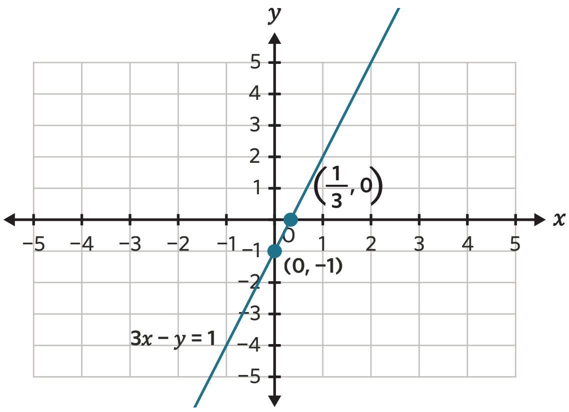
12. **a.** x-intercept: $\left(\frac{1}{3}, 0\right)$

y-intercept: $(0, -1)$

b.

x	0	1	2	3
y	-1	2	5	8

c.



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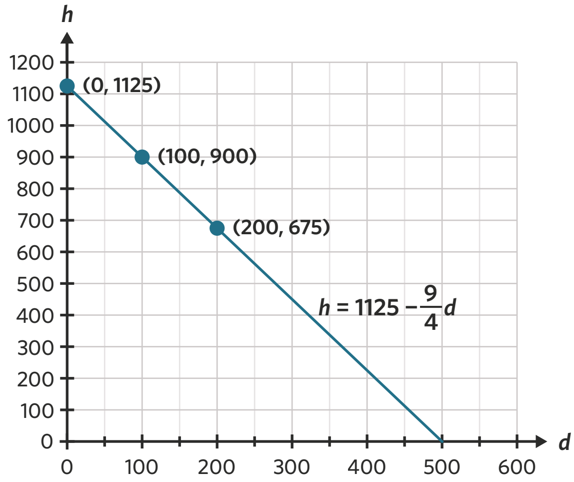
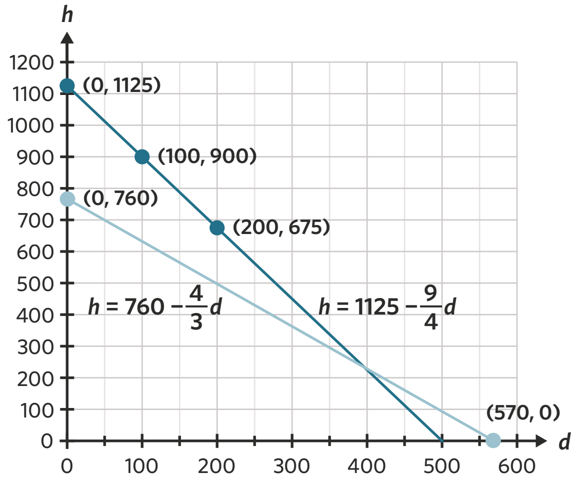
13. a.

d	0	100	200
h	1125	900	675



b. d-intercept: (570, 0)

h-intercept: (0, 760)



c. Sunrise Ridge

d. Sunrise Ridge

e. Cumulus Peak

Exam practice

14. Explanation

Step 1: Determine the most appropriate method to graph the function.

Since the axes are provided, the two-points method is most appropriate as the two endpoints can be used.

Step 2: Choose endpoint values of n.

$n = 0$ and $n = 250$.

Step 3: Substitute the n values into the equation.

Let $n = 0$

$C = 36(0) + 5000$

$= 5000$

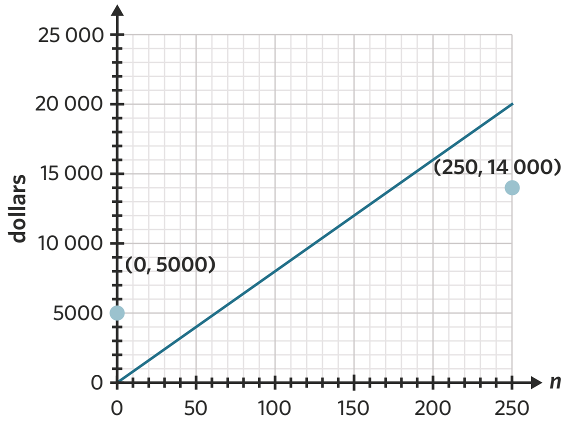
Let $n = 250$

$C = 36(250) + 5000$

$= 14\ 000$

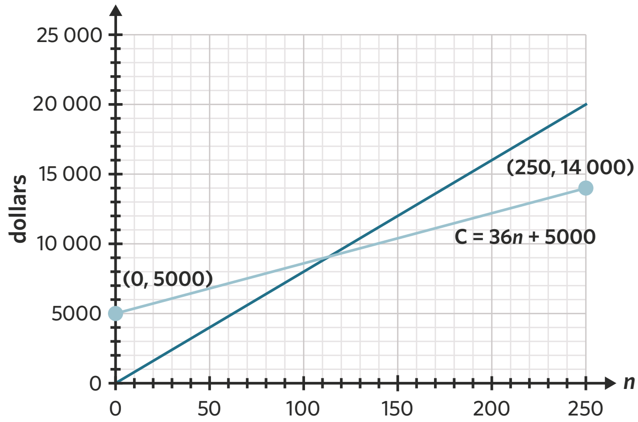
The two points are (0, 5000) and (250, 14 000).

Step 4: Plot the two points on the set of axes provided.



Step 5: Connect the points with a straight line.

Answer



A tip for students when answering this question is to ensure that the tip of the pen, and not the ruler, is aligned with the correct point. Otherwise, the line will be moved and will not be in the correct position.

15. Explanation

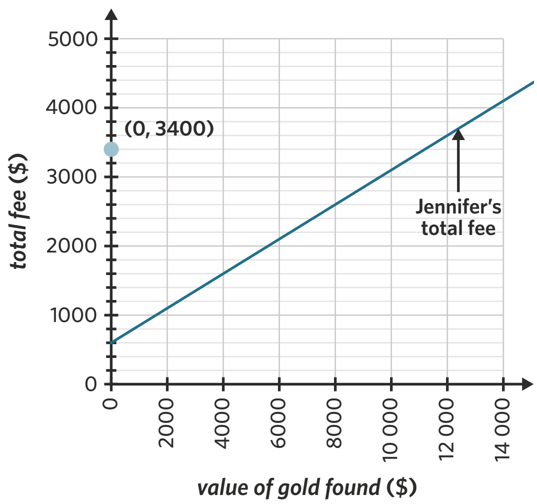
Step 1: Determine the type of linear function being graphed.

Kevin charges a total fee of \$3400. This remains constant over all values of the horizontal axis variable (*value of gold found*).

Therefore this is a horizontal line.

Step 2: Plot the vertical axis intercept.

The vertical axis intercept is (0, 3400).



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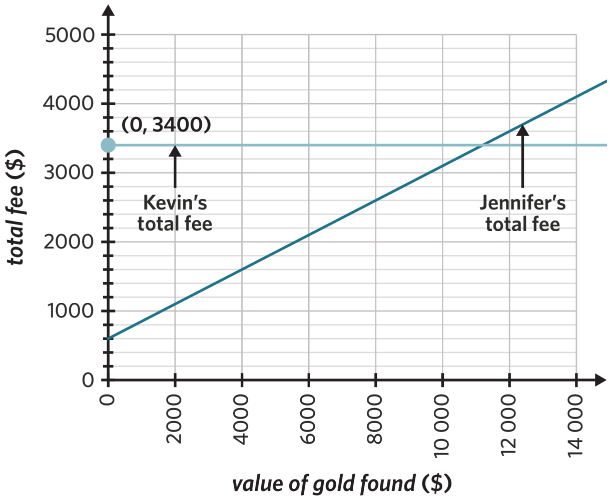
Step 3: Draw the rest of the line.

Every point shares the same vertical axis value at 3400.

Answer



Students who drew an incorrect line often began at the origin. Kevin charges a total fee that remains constant for all values of *value of gold found* and is therefore a horizontal line.

16. Explanation

Step 1: Determine the most appropriate method to graph the function.

Since the axes are provided, the two-points method is most appropriate as the two endpoints can be used.

Step 2: Choose endpoint values of *number of balls*.

number of balls = 0 and 600.

Step 3: Substitute the values into the equation.

Let *number of balls* = 0

cost = 1200 + 1.5 × 0

= 1200

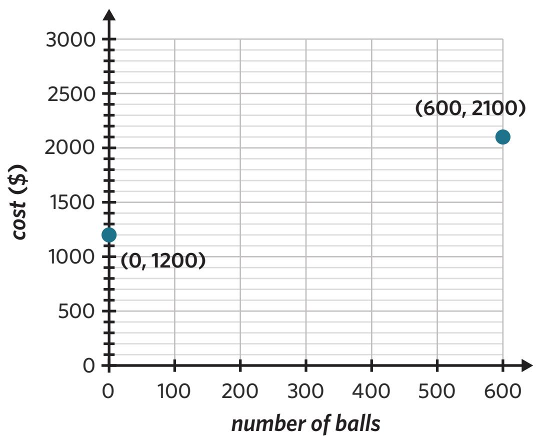
Let *number of balls* = 600

cost = 1200 + 1.5 × 600

= 2100

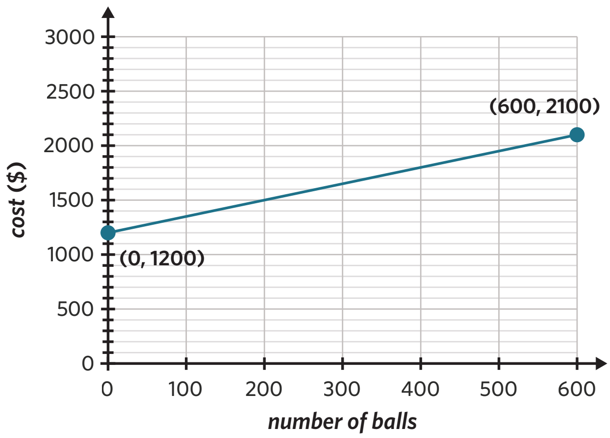
The two points are (0, 1200) and (600, 2100).

Step 4: Plot the two points on a set of axes.



Step 5: Connect the points with a straight line.

Answer



Questions from multiple lessons

17. C **18.** C

19. a. 200 followers

b. 5000 followers

4D Finding the equation of a linear function

Finding a linear equation from two known points

1. D **2.** B

3. a. $y = -6 + 7x$ **b.** $y = 107 + 15x$

c. $y = 47 - 2x$ **d.** $y = -13 + 3.5x$

4. a. $y = -5 + 3x$ **b.** $y = -0.5 - x$

c. $y = -24.8 + 1.2x$ **d.** $y = -1316.4 - 1.2x$

5. a. $m = 156 + 38.75n$ **b.** $2171

c. 31 weeks

6. a. $b = 1782 - 18n$ **b.** 1782 books

c. 8 years and 3 months

Finding a linear equation from a graph

7. D

8. a. $y = 10 + 4x$ **b.** $y = 15 - 0.5x$

c. $y = -330 - 5.5x$ **d.** $y = -2.8 + 2.947x$

9. a. $h = 14 + 2.8c$ **b.** 5 coins

c. 22.4 mm

10. a. $g = 375 - 7.5n$ **b.** 375 grams

c. 20 days

Joining it all together

11. II and IV

12. a. $t = 123.75 - 2.75n$

b. 123 minutes and 45 seconds

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c. 26 minutes and 15 seconds

d. 45 questions

e. 16 minutes and 30 seconds

f. B

Exam practice

13. Explanation

Method 1: By hand

Step 1: Determine the gradient.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{5 - 1}{3 - (-1)}$$
$$= 1$$

Step 2: Determine the y-intercept.

Substitute b and one set of coordinates into $y = a + bx$ and solve for a.

$1 = a + 1 \times -1$

$a = 2$

Step 3: Substitute the a and b values into $y = a + bx$.

$y = 2 + x$

Step 4: Substitute each set of coordinates into the equation and determine whether the result is true.

Method 2: TI-Nspire

Step 1: From the home screen, select '1: New' → '4: Add Lists & Spreadsheet'.

Step 2: Name column A 'x' and column B 'y' and enter the coordinates.

Step 3: Press menu. Select → '4: Statistics' → '1: Stat Calculations' → ' 4: Linear Regression (a+bx)'.

On the settings window, change 'X List:' to 'x' and 'Y List:' to 'y'. Select 'OK'.

Step 4: Substitute the a and b values shown in the regression analysis into $y = a + bx$.

$y = 2 + x$

Step 5: Substitute each set of coordinates into the equation and determine whether the result is true.

Method 3: Casio ClassPad

Step 1: From the main menu, tap Statistics.

Step 2: Name list1 'x' and list2 'y' and enter the coordinates.

Step 3: Tap 'Calc' → 'Regression' → 'Linear Reg'. Specify the data set by changing 'XList:' to 'main\x' and 'YList:' to 'main\y'. Tap 'OK'.

Step 4: Substitute the a and b values shown in the regression analysis into $y = a + bx$.

$y = 2 + x$

Step 5: Substitute each set of coordinates into the equation and determine whether the result is true.

Answer – Method 1, 2 and 3

B

14. Explanation

Step 1: Substitute known values into the given equation.

The question states that the cost is $92 500, so $C = 92\ 500$.

This is the cost after 15 weeks, so $n = 15$.

$92\ 500 = 10\ 000 + 15k$

Step 2: Solve to find k, ensuring to show the steps by hand.

Answer

$92\ 500 = 10\ 000 + 15k$

$\frac{92\ 500 - 10\ 000}{15} = k$

$k = 5500$

> 71% of students got no marks on this question. Many of these students were likely able to solve for k, but failed to provide the appropriate working to 'show that' $k = 5500$.

Questions from multiple lessons

15. E **16.** D

17. a. $8 = \frac{2000}{k}$

$k = \frac{2000}{8}$

$= 250$

b. 5000 N

4E Linear modelling

Modelling practical problems using linear equations

1. $D = 80 \times t$

2. a. $V = 350 + 8M$ b. 542 vinyls

3. a. $C = 234.68 + 0.0047L$ b. $892.68

4. a. 30 people b. 12 people
c. 6 stops

5. a. $C = 3.65 + 2.25d$ b. $22.55
c. Yes d. 2.9 km

6. a. 350 words per hour
b. Let $h = 4$.
$W = 1500 - 350 \times 4 = 100$
$W > 0$, hence Joe will not finish the essay in time.
c. 375 words

7. a. 315 L b. $W = 315 - 35t$
c. 9 minutes

8. John

9. a. $C = 40 + 70n$ b. $530
c. $C = 530 + 50m$ d. $930

Finding the domain of interpretation

10. B

11. a. $0 \le n \le 6$

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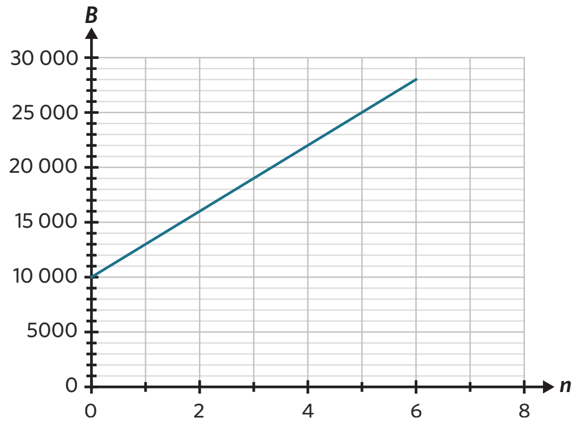
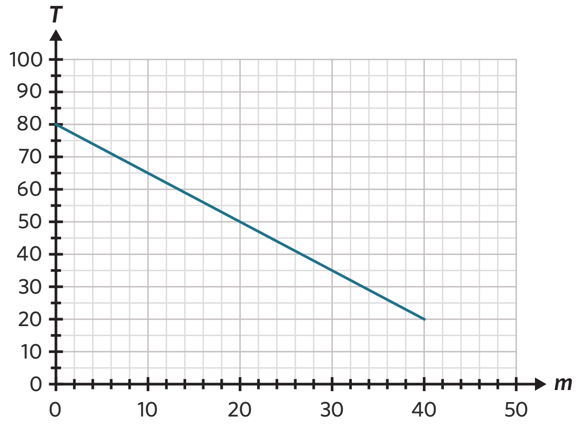
b.



12. a. $0 \leq m \leq 40$

b.



13. $0 \leq d \leq 386$

Joining it all together

14. a. $W = 510\,000 - 6000d$ **b.** 330 000 L

c. $0 \leq d \leq 85$ **d.** $0 \leq d \leq 34$

15. a. $A = -64 + 14x$ **b.** 692 cm^2

c. 19 cm **d.** $8.0 < x \leq 29.4$

Exam practice

16. Explanation

Step 1: Find the initial value.

The phone company charges a fixed, monthly line rental fee of $28.

$a = 28$

Step 2: Find the rate of change.

The phone company charges $0.25 per call.

$b = 0.25$

Step 3: Represent the information in a linear equation.

The linear equation will be in the form $C = a + bn$.

$C = 28 + 0.25n$

Answer

A

17. Explanation

Step 1: Find the initial value.

Justin has one fixed cost of $420 each week. As it is a cost, it is a negative number.

$a = -420$

Step 2: Find the rate of change.

Each circuit board costs $15 to make.

The selling price of each circuit board is $27.

The profit for each sale will be the difference between the selling price and the cost.

$b = 27 - 15$

$= 12$

Step 3: Represent the information in a linear equation.

The linear equation will be in the form $P = a + bn$, where P is the profit, in dollars, and n is the number of circuit boards sold.

$P = -420 + 12n$

Step 4: Substitute the pronumeral with the necessary value and evaluate the equation.

Let $n = 200$.

$P = -420 + 12 \times 200$

$= 1980$

Answer

A

12% of students incorrectly chose option C. These students counted the initial cost of $420 as a positive value in the profit equation.

18. Explanation

Step 1: Find the initial value.

The energy company charges a monthly service fee of $38.70.

$a = 38.70$

Step 2: Find the rate of change.

The energy company charges a supply charge of 2.5 cents per megajoule of energy used.

2.5 cents is $0.025.

$b = 0.025$

Step 3: Represent the information in a linear equation.

The linear equation will be in the form $C = a + bm$, where C is the monthly cost of the energy, in dollars, and m is the number of megajoules used in the month.

$C = 38.70 + 0.025m$

Step 4: Substitute the pronumeral with the necessary value and solve the equation.

Let $C = 169.90$.

$169.90 = 38.70 + 0.025m$

$m = 5248$

Step 5: Calculate the daily number of megajoules used.

There were 30 days in the month.

$5248 \div 30 = 174.933...$

Answer

C

30% of students incorrectly chose option B. These students found the monthly number of megajoules used with a supply charge $2.5 instead of $0.025. 23% of students incorrectly chose option E. These students found the monthly number of megajoules used instead of the daily number of megajoules used.

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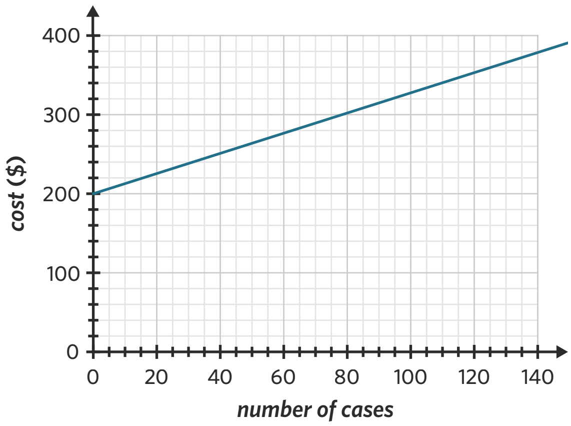
Questions from multiple lessons

19. A **20.** D

21. a. 104 cases

b.



4F Simultaneous linear equations

Solving simultaneous equations graphically

1. A

2. a. (0, 2) **b.** (2, 4)

c. $(-4, -1)$ **d.** $(-4, 2)$

3. a. (0.2, 1.2) **b.** $(-1.6, 1.2)$

c. $(-0.75, 0.5)$ **d.** $(0.6, -1.6)$

4. 9:51 am

Solving simultaneous equations using substitution

5. C

6. a. $x = 5, \quad y = 1$ **b.** $x = 1, \quad y = -3$

c. $x = 9, \quad y = 2$ **d.** $x = 6, \quad y = -10$

7. Flour: 120 g, Sugar: 200 g

Solving simultaneous equations using elimination

8. B

9. a. $x = 5, \quad y = 2$ **b.** $x = -3, \quad y = 3$

c. $x = 4, \quad y = -2$ **d.** $x = 1, \quad y = 3$

10. Vegemite: $4.50, Nutella: $6.00

Modelling practical problems using simultaneous equations

11. C

12. a. $15 **b.** $12

c. Children: 9, Adults: 3

13. $3n + 4p = 11.6$
$6n + 2p = 14.8$

14. a. $3.5s + 2t = 35$
$s + t = 13$ **b.** 7 tops

15. Let d be Dana's current age and b be Ben's current age.

$d = b + 6$ (1)

$d + 2 = 1.5 \times (b + 2)$ (2)

Substitute (1) into (2).

$b + 6 + 2 = 1.5 \times (b + 2)$

$b + 8 = 1.5b + 3$

$5 = 0.5b$

$10 = b$

Substitute $b = 10$ into (1).

$d = 10 + 6$

$d = 16$

Joining it all together

16. $a = 7$

17. a. Elimination **b.** Small: $2.50, Large: $4.00

18. a. $l = m + 1.5$
$130m + 160l = 1748$ **b.** Substitution

c. $5.20

19. a. $k = g - 6$
$k = \frac{4}{5}(g + 4) - 4$ **b.** (26, 20)

c. 20 years old

20. Let a be Alex's pay per hour and d be Danni's pay per hour.

$4a + 6d = 252$ (1)

$7a + 3d = 249.75$ (2)

(2) $\times$ 2

$14a + 6d = 499.5$ (3)

(3) $-$ (2)

$10a = 247.5$

$a = 24.75$

Exam practice

21. Explanation

Step 1: Identify and define the variables.

In this case, the variables are the price of the adult and child tickets.

Let a be the price of adult tickets and c be the price of child tickets.

Step 2: Convert the information given into two equations using the chosen variables.

The Payne family bought two adult tickets and three child tickets for $69.50.

$2a + 3c = 69.5$ (1)

The Tran family bought one adult ticket and five child tickets for $78.50.

$a + 5c = 78.5$ (2)

Step 3: Solve the simultaneous equations.

$a = 16$

$c = 12.5$

4F ANSWERS

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Step 4: Use the prices to find the amount spent by the Saunders family.

The Saunders family bought three adult tickets and four child tickets.

$3 \times 16 + 4 \times 12.5 = 98$

Answer

C

22. Explanation

Step 1: Identify and define the variables.

In this case, the variables are the number of adults and children that attended the car show.

Let a be the number of adults and c be the number of children.

Step 2: Convert the information given into two equations using the chosen variables.

537 people attended the car show.

$a + c = 537$ (1)

Admission fees for the show were \$5 per adult and \$2 per child, and \$1644 was raised.

$5a + 2c = 1644$ (2)

Step 3: Solve the simultaneous equations.

$a = 190$

$c = 347$

Answer

347 children

23. Explanation

Step 1: Identify and define the variables.

In this case, the variables are the cost per minute and cost per kilometre.

Let t be the cost per minute and d be the cost per kilometre.

Step 2: Convert the information given into two equations using the chosen variables.

The fixed cost of a ride is \$2.55. This will affect both equations.

Judy's ride cost \$16.75 and took eight minutes. The distance travelled was 10 km.

$16.75 = 2.55 + 8t + 10d$ (1)

Pat's ride cost \$30.35 and took 20 minutes. The distance travelled was 18 km.

$30.35 = 2.55 + 20t + 18d$ (2)

Step 3: Solve the simultaneous equations.

$t = 0.4$

$d = 1.1$

Step 4: Use these values to find the cost for Roy.

Roy's ride took 10 minutes. The distance travelled was 15 km.

$2.55 + 0.4 \times 10 + 1.1 \times 15 = 23.05$

Answer

D

35% of students incorrectly chose options B or C. These students likely made a guess based on the time and distance of Roy's ride in comparison to Judy's and Pat's. The cost of distance travelled was larger than the cost of time, per unit, and therefore the cost of Roy's ride was more expensive than expected.

Questions from multiple lessons

24. D **25.** B

26. a. \$460

b. \$660

4G Piecewise linear models

Classifying and interpreting piecewise graphs

1. C

2. a. Line segment graph **b.** Line segment graph

c. Step graph **d.** Line segment graph

3. a. 9 km/h **b.** 45 minutes

c. 6 km/h **d.** 6.2 km

4. a. 5 questions per hour **b.** 15 questions

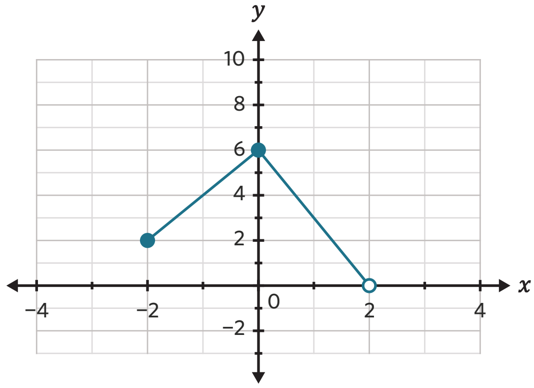
c. Shai will complete 7 more questions if he studies for 4 hours.

d. 4 hours

Constructing piecewise graphs

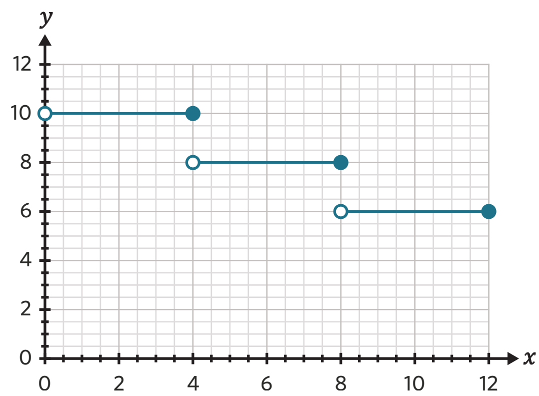
5. D

6. a.

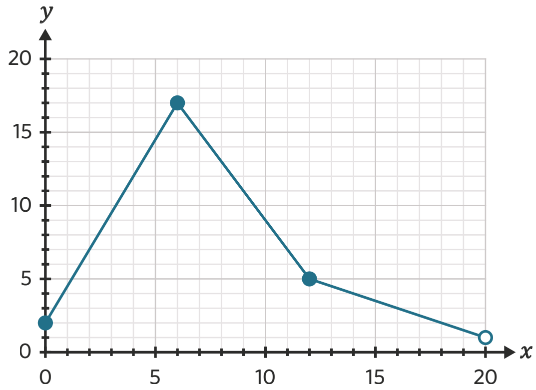


Line segment graph

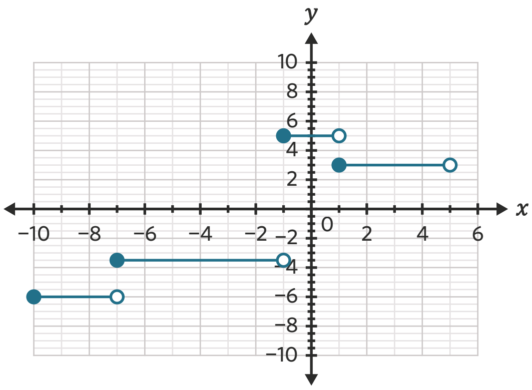
b.



Step graph

c. 

Line segment graph

d. 

Step graph

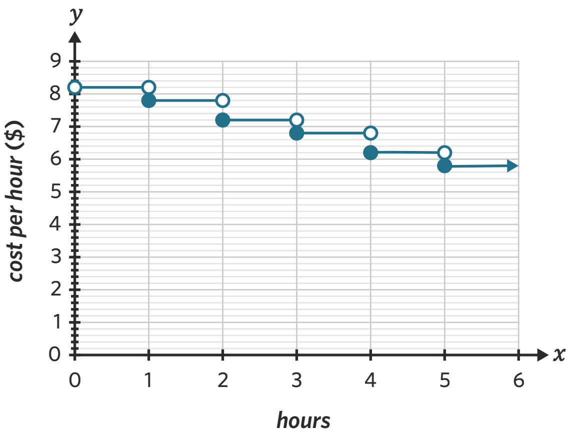
Applying piecewise graphs to financial situations

7. B

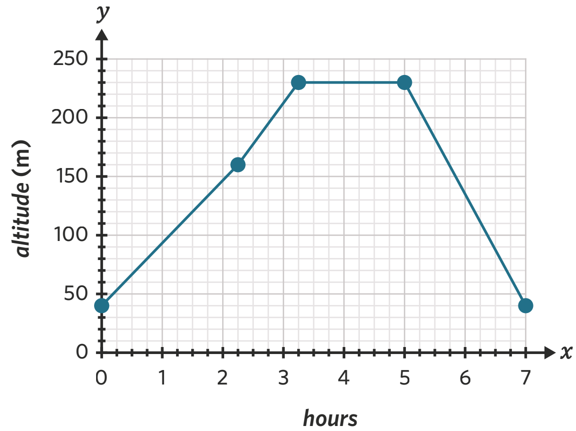
8. **a.** 37% **b.** $180 000
c. $44 000 **d.** $39 000

9. **a.** $880 **b.** $200 in week 6
c. $87.50 **d.** 10 hours

Joining it all together

10. **a.** 

b. Step graph **c.** $29
d. $2.55 **e.** 2 hours and 50 minutes

11. **a.** 

b. Line segment graph **c.** 12:15 pm to 2:00 pm
d. 40 m **e.** 190 m

Exam practice

12. **Explanation**

To solve this question, check whether each option is true or false.

A: The line segment for a 4 kg parcel corresponds to a $20 *delivery fee*. This statement is true. ✗

B: The line segment for a 12 kg parcel corresponds to a $26 *delivery fee*. This statement is true. ✗

C: The *delivery fee* for a 13 kg parcel and a 20 kg parcel are represented by the same line segment. This statement is true. ✗

D: The *delivery fee* for a 10 kg parcel is $26 and the *delivery fee* for a 2 kg parcel is $12.

$26 - 12 = 14$. This statement is true. ✗

E: The *delivery fee* for a 12 kg parcel is $26 and the *delivery fee* for a 2 kg parcel is $12.

$26 - 12 = 14$. This statement is false. ✓

Answer

E

13. **Explanation**

Step 1: Determine how much Lani would pay on Wednesday.

Seven hours is between 6 and 10, so she will pay $14 at Eastpark.

At Northpark, she will pay $2.30 \times 7 = \$16.10$.

$14 < 16.10$

Step 2: Determine how much Lani would pay on Thursday.

Four hours is between 3 and 6, so she will pay $10 at Eastpark.

At Northpark, she will pay $2.30 \times 4 = 9.20$.

$10 > 9.20$

Step 3: Add the two smallest values.

$14 + 9.20 = 23.30$

Answer

C

31% of students incorrectly answered option D. The question stated that ‘she may choose either car park on each day’. These students likely calculated the minimum total fee if she parked at the same carpark on both days.

4G ANSWERS

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14. Explanation

Step 1: Determine the coordinates of the start and end point of the first line segment.

Substitute the start and end domain values of t for the first line segment into the first linear function to solve for j.

Substitute $t = 0$.

$j = 120 \times 0$

$= 0$

Substitute $t = 2$.

$j = 120 \times 2$

$= 240$

The first line segment starts at $(0, 0)$ and ends at $(2, 240)$.

Step 2: Determine the coordinates of the end points for the remaining line segments.

Substitute the end domain value of t for the remaining line segments.

Substitute $t = 6$.

$j = 100 \times 6 + 40$

$= 640$

The second line segment starts at $(2, 240)$ and ends at $(6, 640)$.

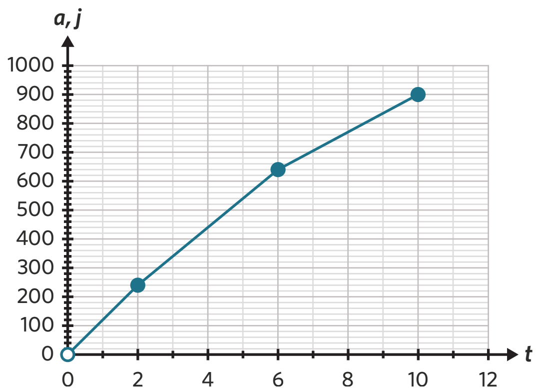
Substitute $t = 10$.

$j = 65 \times 10 + 250$

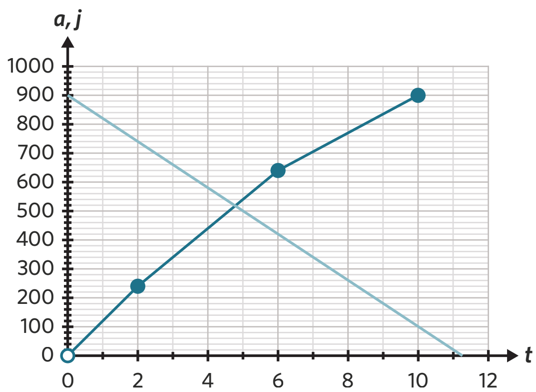
$= 900$

The third line segment starts at $(6, 640)$ and ends at $(10, 900)$.

Step 3: Construct a set of axes and plot each of Jenny's coordinates connected with straight lines.



Step 4: Graph Alan's planned walk on the same graph.



Step 5: Determine the function for j where the lines intersect.

The lines intersect where $2 < t \le 6$.

The corresponding function is $j = 100t + 40$.

Step 6: Solve for the value of t where the lines intersect using simultaneous equations.

$100t + 40 = -80t + 900$

$180t + 40 = 900$

$180t = 860$

$t = \frac{43}{9}$

Step 7: Find the distance each person has walked.

$100 \times \frac{43}{9} + 40 = 517.778$

Jenny and Alan are 518 metres from their house.

Jenny walked from her house, so she has walked 518 m.

Alan walked from the supermarket, so he has walked $900 - 518 = 382$ m.

Answer

C

30% of students incorrectly answered option D. These students likely worked out that Alan and Jenny intersect at 518 metres, however failed to recognise that this was how far Alan was from his house and not how far he had walked.

Questions from multiple lessons

15. B

16. E

17. **a.** $117 = k + 0.65 \times 200$

$k = 117 - 130$

$= -13$

b. $(20, 0)$

c. For every Australian dollar, the currency exchange will provide 65 US cents.

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5A Introduction to matrices

Identifying matrix properties

1. C

2. a. 1 b. 8

3. a. 1×5 b. 3×2 c. 2×1 d. 3×4

4. a. 4×3 b. 10 c. 29

5. a. $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. a. B, F b. A, B, G
   c. B, E, F, G d. B, C, D, G
   e. C, G

Representing information in a matrix

7. D

8. $\begin{array}{cc} \begin{array}{cc} \text{P} & \text{C} \end{array} & \\ \begin{bmatrix} 2.0 & 1.5 \\ 1.5 & 2.0 \\ 0.5 & 1.0 \end{bmatrix} & \begin{array}{c} \text{M} \\ \text{F} \\ \text{S} \end{array} \end{array}$

9. $\begin{array}{cc} \begin{array}{cc} \text{cats} & \text{dogs} \end{array} & \\ \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 2 & 0 \end{bmatrix} & \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \end{array} \end{array}$

Joining it all together

10. a. $\begin{array}{cc} \begin{array}{ccc} \text{V} & \text{S} & \text{M} \end{array} & \\ \begin{bmatrix} 9 & 12 & 21 \\ 15 & 11 & 17 \end{bmatrix} & \begin{array}{c} \text{Year 1} \\ \text{Year 2} \end{array} \end{array}$ b. $\begin{array}{cc} \text{M} & \\ \begin{bmatrix} 21 \\ 17 \end{bmatrix} & \begin{array}{c} \text{Year 1} \\ \text{Year 2} \end{array} \end{array}$

    c. $\begin{array}{cc} \begin{array}{ccc} \text{V} & \text{S} & \text{M} \end{array} & \\ \begin{bmatrix} 9 & 12 & 21 \end{bmatrix} & \text{Year 1} \end{array}$

    d. 42. A total of 42 Year 1 students were surveyed.

11. a. 7 students
    b. The number of people in Class B who chose RuPaul's Drag Race, which was 10.
    c. x_{32}
    d. Class A: 26 students, Class B: 27 students, Class C: 25 students
    e. RuPaul's Drag Race

Exam practice

12. a. **Explanation**

Step 1: Locate the 'party pies' column.

$$P = \begin{array}{cc} \begin{array}{ccc} \text{H} & \text{P} & \text{S} \end{array} & \\ \begin{bmatrix} 45 & 24 & 49 \\ 29 & 16 & 20 \\ 16 & 31 & 47 \end{bmatrix} & \begin{array}{c} \text{game 1} \\ \text{game 2} \\ \text{game 3} \end{array} \end{array}$$

Step 2: Find the sum of the elements.

$$\begin{aligned} total &= 24 + 16 + 31 \\ &= 71 \end{aligned}$$

Answer

71 party pies

A common error is misinterpreting this type of question. Some students may answer the number of party pies sold in the third game, rather than the total number of party pies sold across all three games.

b. **Explanation**

Step 1: Locate row 2.

$$P = \begin{array}{cc} \begin{array}{ccc} \text{H} & \text{P} & \text{S} \end{array} & \\ \begin{bmatrix} 45 & 24 & 49 \\ 29 & 16 & 20 \\ 16 & 31 & 47 \end{bmatrix} & \begin{array}{c} \text{game 1} \\ \text{game 2} \\ \text{game 3} \end{array} \end{array}$$

This corresponds to the sales in game 2.

Step 2: Locate column 3.

$$P = \begin{array}{cc} \begin{array}{ccc} \text{H} & \text{P} & \text{S} \end{array} & \\ \begin{bmatrix} 45 & 24 & 49 \\ 29 & 16 & 20 \\ 16 & 31 & 47 \end{bmatrix} & \begin{array}{c} \text{game 1} \\ \text{game 2} \\ \text{game 3} \end{array} \end{array}$$

This corresponds to the number of sausage rolls sold at the soccer stadium.

Step 3: Interpret the element.

Answer

The number of sausage rolls sold in game 2, which was 20.

A common error is misinterpreting this type of question. Some students may answer '20' without explaining what the element represents.

13. **Explanation**

Step 1: Set up the matrix.

$$S = \begin{array}{cc} \begin{array}{ccc} 1 & 2 & 3 \end{array} & \\ \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

Step 2: Calculate the elements in the first column, s_{11}, s_{21} and s_{31}, using $s_{ij} = 2i + j$.

$$\begin{aligned} s_{11} &= 2 \times 1 + 1 \\ &= 3 \\ s_{21} &= 2 \times 2 + 1 \\ &= 5 \\ s_{31} &= 2 \times 3 + 1 \\ &= 7 \end{aligned}$$

Step 3: Repeat step 2 to calculate the rest of the elements.

Elements in the second column.

$s_{12} = 2 \times 1 + 2 = 4$

$s_{22} = 2 \times 2 + 2 = 6$

$s_{32} = 2 \times 3 + 2 = 8$

Elements in the third column.

$s_{13} = 2 \times 1 + 3 = 5$

$s_{23} = 2 \times 2 + 3 = 7$

$s_{33} = 2 \times 3 + 3 = 9$

$$S = \begin{bmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \\ 7 & 8 & 9 \end{bmatrix}$$

Answer

C

14. Explanation

Step 1: Set up the matrix.

$$M = \begin{array}{cc} \begin{array}{ccc} \ \ 1 & \ \ 2 & \ \ 3 \end{array} & \\ \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \end{bmatrix} & \begin{array}{c} 1 \\ 2 \end{array} \end{array}$$

Step 2: Calculate the matrix for each option and compare it M.

A: This is incorrect. ✗

$$M_A = \begin{bmatrix} 2\times1-1 & 2\times1-2 & 2\times1-3 \\ 2\times2-1 & 2\times2-2 & 2\times2-3 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$
$M_A \neq M$

B: This is incorrect. ✗

$$M_B = \begin{bmatrix} 4-1 & 4-1 & 4-1 \\ 4-2 & 4-2 & 4-3 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix}$$
$M_B \neq M$

C: This is incorrect. ✗

$$M_C = \begin{bmatrix} 2\times1+2\times1 & 2\times1+2\times2 & 2\times1+2\times3 \\ 2\times2+2\times1 & 2\times2+2\times2 & 2\times2+2\times3 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & 6 & 8 \\ 6 & 8 & 10 \end{bmatrix}$$
$M_C \neq M$

D: This is incorrect. ✗

$$M_D = \begin{bmatrix} 4+1 & 4+2 & 4+3 \\ 4+1 & 4+2 & 4+3 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 7 \\ 5 & 6 & 7 \end{bmatrix}$$
$M_D \neq M$

E: This is correct. ✓

$$M_E = \begin{bmatrix} 4-1 & 4-2 & 4-3 \\ 4-1 & 4-2 & 4-3 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$
$M_E = M$

Answer

E

A high proportion of students incorrectly answered this type of question. This is likely because they had only worked through a select number of elements in matrix *M*. In such questions, it is important to spend the time to calculate each matrix element individually.

Questions from multiple lessons

15. B

16. C

17. a. 48 cm

b. 0 cm

5B Operations with matrices

Adding and subtracting matrices

1. I and V

2. a. $\begin{bmatrix} -2 & 17 \\ 14 & 17 \end{bmatrix}$

b. $\begin{bmatrix} -4 & -2 \\ -8 & 15 \end{bmatrix}$

c. $\begin{bmatrix} 7 & 4 \\ 17 & 4 \end{bmatrix}$

d. $\begin{bmatrix} -3 & -16 \\ -13 & 5 \end{bmatrix}$

3. a. $\begin{bmatrix} 11 & 9 & 4 \\ 2 & 5 & -5 \\ -8 & -11 & -2 \end{bmatrix}$

b. $\begin{bmatrix} 9 & -5 & 14 \\ 8 & 11 & 7 \\ 14 & 31 & 2 \end{bmatrix}$

c. $\begin{bmatrix} -9 & 5 & -14 \\ -8 & -11 & -7 \\ -14 & -31 & -2 \end{bmatrix}$

d. $\begin{bmatrix} -5 & 11 & 4 \\ 2 & -3 & -5 \\ 2 & -9 & -2 \end{bmatrix}$

4.

$$\begin{array}{cc} \begin{array}{ccc} \text{blueberry} & \text{banana} & \text{chocolate} \end{array} & \\ \begin{bmatrix} 34 & 31 & 39 \\ 46 & 39 & 46 \end{bmatrix} & \begin{array}{l} \text{small} \\ \text{large} \end{array} \end{array}$$

5. a. $x = 9$

b. $a = 7, \quad b = -12$

c. $e = 0, \quad f = -1, \quad g = 2$

d. $c = 3, \quad d = 7$

6. D

7. a.

$$\begin{array}{cc} \begin{array}{cccc} \text{Steele} & \text{Ridley} & \text{Crisp} & \text{Kelly} \end{array} & \\ \begin{bmatrix} -1 & 1 & 2 & -2 \\ -5 & -2 & -2 & -3 \end{bmatrix} & \begin{array}{l} \text{AFL Fantasy score} \\ \text{handballs} \end{array} \end{array}$$

b. Kelly

Multiplying matrices by a scalar

8. D

9. a. $x = 9$

b. $x = \frac{1}{2}$

10. a. 2

b. $\begin{bmatrix} \$140 & \$110 \\ \$40 & \$20 \end{bmatrix}$

Joining it all together

11. B

12. a. $\begin{bmatrix} -17 & 86 & -217 \end{bmatrix}$

b. $\begin{bmatrix} -1 \\ 0 \\ 14 \end{bmatrix}$

c. $\begin{bmatrix} 14 & -12 \\ -5 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 5 & -2 & -2 \\ 0.5 & 4 & -1.5 \end{bmatrix}$

13. a. $x = \frac{7}{3}, \quad y = 0$

b. $x = 3, \quad y = -1$

c. $x = \frac{-8}{3}, \quad y = 2$

14. Hussain

Exam practice

15. Explanation

Step 1: Convert the percentage increase to a decimal.

$$\frac{10}{100} = 0.1$$

Step 2: As the sales are increasing, add the decimal to 1 to find the scalar value.

$$k = 1 + 0.1$$
$$= 1.1$$

Answer

1.1

A common answer was 0.1. These students likely did not realise that the decimal needed to be added to 1 for an increase in value.

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Questions from multiple lessons

16. B **17.** E

18. **a.** $5.99

b. 3×2

c. The price of one block of Cadrolo at Igloo.

5C Advanced operations with matrices

Defining matrix products

1. B

2. **a.** Defined; The number of columns in matrix A is equivalent to the number of rows in matrix B.

b. Not defined; The number of columns in matrix B is not equivalent to the number of rows in matrix A.

3. **a.** Not defined **b.** Defined; 3×1

4. Defined; 2×4

Calculating a matrix product

5. C **6.** B

7. **a.** $A = \begin{bmatrix} 10 & 13 \\ 16 & 14 \end{bmatrix}$ **b.** $B = \begin{bmatrix} 18 & -10 \\ -38 & 14 \end{bmatrix}$

c. $C = \begin{bmatrix} 6 & 3 & 1 \\ -1 & -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$

8. $PQ = \begin{bmatrix} 9 \\ -17 \\ 8 \end{bmatrix}$ **9.** Youssef

Calculating a matrix power

10. B **11.** B **12.** $A^2 = \begin{bmatrix} 1 & -8 \\ 16 & -7 \end{bmatrix}$

Joining it all together

13. Matrix M is defined; 2×2;

$M = \begin{bmatrix} 11 & 11 \\ 18 & 0 \end{bmatrix}$

14. $BA^2 = \begin{bmatrix} 50 & 82 \\ 14 & 10 \\ 36 & 72 \\ -2 & -94 \end{bmatrix}$

15. 3 rows

Exam practice

16. **Explanation**

Calculate the matrix product.

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} T \\ A \\ L \\ E \\ S \end{bmatrix} = \begin{bmatrix} L \\ E \\ A \\ S \\ T \end{bmatrix}$$

Answer

D

17. **Explanation**

Step 1: Determine the location of the element.

Element q_{41} is in row 4 and column 1 of the matrix product.

Step 2: Determine an expression for q_{41}.

The expression will be the sum of the products of the values in row 4 of matrix A and column 1 of matrix B.

$q_{41} = 4 \times 2 + 5 \times 4$

Answer

E

18. **Explanation**

Step 1: Determine the order of the matrix product.

The matrix product is 1×1, as the number of rows in the first matrix is 1 and the number of columns in the second matrix is 1.

Step 2: Calculate the value of the matrix product.

$[2 \times 8 + 1 \times 0 + 4 \times 11] = [60]$

Step 3: Check whether each option is correct.

A: This is incorrect, as $[132] \neq [60]$. ✗

B: This is incorrect, as $2 \times \begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 4 \\ 0 \\ 11 \end{bmatrix} = [52]$. ✗

C: This is correct, as $4 \times \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \times \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix} = [60]$. ✓

D: This is incorrect, as $2 \times \begin{bmatrix} 1 & 1 & 4 \end{bmatrix} \times \begin{bmatrix} 8 \\ 0 \\ 11 \end{bmatrix} = [104]$. ✗

E: This is incorrect, as $\begin{bmatrix} 16 \\ 44 \\ 60 \end{bmatrix} \neq [60]$. ✗

Answer

C

Questions from multiple lessons

19. B **20.** D

21. **a.** 3×1

b. 49

c. 119; the total number of tickets sold for the movie.

5D Matrix applications

Using matrix products in financial applications

1. B

2. **a.** $S \times P$ or $\begin{bmatrix} 248 & 132 & 177 \end{bmatrix} \times \begin{bmatrix} 19.00 \\ 14.00 \\ 17.00 \end{bmatrix}$

b. $10 889

3. **a.** $S = \begin{array}{c|l} \begin{bmatrix} 11 \\ 19 \\ 23 \end{bmatrix} & \begin{array}{l} H \\ J \\ T \end{array} \end{array}$ **b.** $740

4. **a.** $P = \begin{array}{c|l} \begin{bmatrix} 2.50 \\ 1.50 \\ 2.00 \end{bmatrix} & \begin{array}{l} C \\ H \\ S \end{array} \end{array}$ **b.** $\begin{bmatrix} 12 & 17 & 15 \\ 16 & 15 & 13 \\ 13 & 12 & 16 \\ 14 & 14 & 13 \\ 15 & 13 & 17 \end{bmatrix} \times \begin{bmatrix} 2.50 \\ 1.50 \\ 2.00 \end{bmatrix}$

c. Friday

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Using summing matrices to solve application problems

5. B

6. $S = [1 \quad 1 \quad 1 \quad 1]$

7. a. $\begin{bmatrix} 2 & 5 & 3 \\ 6 & 4 & 7 \\ 9 & 2 & 8 \\ 8 & 1 & 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 17 \\ 19 \\ 18 \end{bmatrix}$

b. $[1 \quad 1 \quad 1 \quad 1] \times \begin{bmatrix} 2 & 5 & 3 \\ 6 & 4 & 7 \\ 9 & 2 & 8 \\ 8 & 1 & 9 \end{bmatrix} = [25 \quad 12 \quad 27]$

Using communication matrices to model systems

8. B

9. a. A can send messages to B and C.
B can send messages to A and C.
C can send messages to A and B.

b. A can send messages to B, C and D.
B can send messages to A, C and D.
C can send messages to A and B.
D can send messages to A and B.

c. A can send messages to B and D.
B can send messages to A and C.
C can send messages to B and D.
D can send messages to A and C.

10. a. $C^2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$

b. 0

c. 0

Joining it all together

11. a.

$$M = \begin{array}{c} \text{receiver} \\ \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \ \text{sender}$$

b.

$$M^2 = \begin{array}{c} \text{receiver} \\ \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{bmatrix} 3 & 1 & 2 & 2 & 1 \\ 1 & 3 & 1 & 2 & 2 \\ 2 & 1 & 2 & 1 & 1 \\ 2 & 2 & 1 & 4 & 1 \\ 1 & 2 & 1 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \ \text{sender}$$

c. 2

12. a. $S = [1 \quad 1 \quad 1]$

b. $M = \begin{bmatrix} 1119 \\ 699 \\ 849 \end{bmatrix} \begin{array}{c} \text{L} \\ \text{P} \\ \text{T} \end{array}$

c. $\begin{bmatrix} 13 & 23 & 15 \\ 15 & 31 & 19 \\ 11 & 27 & 18 \end{bmatrix} \times \begin{bmatrix} 1119 \\ 699 \\ 849 \end{bmatrix}$

d. Store B – \$54 585

Exam practice

13. Explanation

Step 1: Identify the communication links for each individual.

Ally can send emails to Brent and Drevis.

Brent can send emails to Chloe.

Chloe can send emails to Ally and Elvin.

Drevis can send emails to Elvin.

Elvin can send emails to Ally and Chloe.

Step 2: Determine the overlap of communication links.

Chloe and Elvin can both email each other.

Answer

Chloe and Elvin

14. Explanation

Step 1: Calculate the two-step communication matrix.

$$J^2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}^2$$

$$= \begin{bmatrix} 1 & 2 & 0 & 3 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 & 2 \\ 3 & 1 & 1 & 2 & 2 & 1 \\ 0 & 1 & 1 & 2 & 2 & 3 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 2 & 0 & 2 & 0 & 3 & 1 \end{bmatrix}$$

Step 2: Interpret the two-step communication matrix.

The number of two-step communication paths from W to U is located in row 3, column 1.

There are three possible ways for W to send a message to U via a third individual.

Answer

D

20% of incorrect answers referred to the entry in the correct position but in the one-step communication matrix. It is important to ensure that when a question asks for the number of two-steps paths from one point to another, that a two-step communication matrix is constructed.

15. Explanation

Interpret the expression.

Matrix J is a matrix product expression. Matrix G, a 1×4 matrix containing the price of each booking, is post-multiplied by matrix C, a 4×1 matrix containing the number of each type of booking for the month. The result will be a 1×1 matrix.

Answer

Matrix J represents the total booking fees collected for the month.

16. Explanation

To answer this question, check whether each option is correct or incorrect.

A: This is incorrect. It only calculates the total value of the coins.

$$[5 \quad 10 \quad 20 \quad 50] \begin{bmatrix} 17 \\ 24 \\ 53 \\ 32 \end{bmatrix} = [2985] \times$$

5D ANSWERS

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B: This is incorrect. Despite the matrix product calculating the total value of the coins, it sums the value of each type of coin, rather than the total number of coins.

$$\begin{bmatrix} 5 & 10 & 20 & 50 \end{bmatrix}\begin{bmatrix} 1 & 17 \\ 1 & 24 \\ 1 & 53 \\ 1 & 32 \end{bmatrix} = \begin{bmatrix} 85 & 2985 \end{bmatrix} \times$$

C: This is correct. The matrix product calculates the total value of the coins, as well as the total number of coins.

$$\begin{bmatrix} 17 & 24 & 53 & 32 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 1 & 10 \\ 1 & 20 \\ 1 & 50 \end{bmatrix} = \begin{bmatrix} 126 & 2985 \end{bmatrix} \checkmark$$

D: This is incorrect. It only calculates the total value of the coins.

$$\begin{bmatrix} 17 & 24 & 53 & 32 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 20 \\ 50 \end{bmatrix} = \begin{bmatrix} 2985 \end{bmatrix} \times$$

E: This is incorrect. Despite the matrix product summing the total number of coins, it sums the value of each type of coin and not the total value of all the coins.

$$\begin{bmatrix} 5 & 10 & 20 & 50 \\ 17 & 24 & 53 & 32 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 85 \\ 126 \end{bmatrix} \times$$

Answer

C

The main problem that the students found with this question was understanding what it was asking for. 31% of students incorrectly answered option A which only calculated the total amount of money, rather than both the total value and number of coins.

Questions from multiple lessons

17. E **18.** C

19. a. $1250

b. 4×1

c. $N = \begin{bmatrix} 750.00 \\ 337.50 \\ 937.50 \\ 600.00 \end{bmatrix}$

5E Inverse matrices

Calculating the determinant of a matrix

1. B **2.** 12 **3.** -106 **4.** 0

Calculating the inverse of a matrix

5. A

6. a. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c. An identity matrix

7. $E^{-1} = \begin{bmatrix} -\frac{11}{45} & \frac{52}{135} & -\frac{2}{27} \\ \frac{19}{45} & -\frac{53}{135} & \frac{1}{27} \\ \frac{4}{45} & -\frac{23}{135} & \frac{4}{27} \end{bmatrix}$

8. This is not a square matrix.

9. $\begin{bmatrix} 0.75 & -0.25 \\ -0.63 & 0.38 \end{bmatrix}$

10. $\begin{bmatrix} \frac{2}{15} & \frac{8}{15} \\ \frac{16}{45} & \frac{4}{45} \end{bmatrix}$

Joining it all together

11. D

12. a. -14

b. The inverse exists as R is a square matrix and $\det(R) \neq 0$.

c. $R^{-1} = \begin{bmatrix} -\frac{2}{7} & \frac{5}{14} \\ \frac{3}{7} & -\frac{2}{7} \end{bmatrix}$

13. a. The inverse exists as X is a square matrix and $\det(X) \neq 0$.

b. $X^{-1} = \begin{bmatrix} -3 & 3 \\ \frac{10}{3} & -\frac{8}{3} \end{bmatrix}$

Exam practice

14. Explanation

To answer this question, calculate the determinant for each matrix and check which one gives a value of 0.

A: This matrix does not have a determinant of 0.

$ad - bc = 2 \times 2 - 1 \times 1 = 3$ ✗

B: This matrix does not have a determinant of 0.

$ad - bc = 0 \times 0 - 4 \times 4 = -16$ ✗

C: This matrix does not have a determinant of 0.

$ad - bc = 5 \times 4 - (-10) \times 2 = 40$ ✗

D: This matrix has a determinant of 0.

$ad - bc = 22 \times 0 - 0 \times 11 = 0$ ✓

E: This matrix does not have a determinant of 0.

$ad - bc = 2 \times 1 - 0 \times 0 = 2$ ✗

Answer

D

15. a. Explanation

Recall the condition under which an inverse matrix is defined.

A square matrix is defined if the determinant is not equal to 0.

The determinant for this matrix is -4.

Answer

The determinant of the square matrix is not equal to 0.

A number of students struggled with recalling the requirements for a matrix to have an inverse. A square matrix can only have an inverse when the determinant is not equal to 0. Some students stated that it had to be greater than 0 instead.

b. Explanation

Method 1: TI-Nspire

Step 1: From the home screen, select '1: New' → '1: Add Calculator'.

Step 2: Press and select .

Step 3: Input the values of the 2×2 matrix.

Step 4: Calculate the inverse by raising the matrix to the power of -1.

Press enter.

5E ANSWERS

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Method 2: Casio ClassPad

Step 1: From the main menu, tap √α Main.

Step 2: Press keyboard, and tap 'Math2'.

Tap [icon].

Step 3: Input the values of the 2×2 matrix.

Step 4: Calculate the inverse by raising the matrix to the power of -1.

Press EXE.

Answer - Method 1 and 2

$$\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ 1 & -\frac{1}{2} \end{bmatrix}$$

There were a number of students who incorrectly calculated the inverse matrix. Some students did not correctly swap the positions of the a and d elements or multiply the b and c elements by -1. Students could have used a calculator instead, as there was no requirement for the question to be done manually.

Questions from multiple lessons

16. D

17. B

18. a. 4×1

b. 49

c. $[15 \quad 29 \quad 11 \quad 23]$

5F Solving matrix equations

Solving matrix equations

1. A

2. C

3. a. $X = \begin{bmatrix} 2 \\ -7 \end{bmatrix}$

b. $X = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$

c. $X = \begin{bmatrix} 2 \\ -\frac{3}{2} \end{bmatrix}$

d. $X = \begin{bmatrix} 12 \\ 2 \end{bmatrix}$

e. $X = \begin{bmatrix} 1 \\ \frac{15}{2} \end{bmatrix}$

4. a. $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

b. $X = \begin{bmatrix} 8 \\ 12 \end{bmatrix}$

c. $X = \begin{bmatrix} -4 \\ -6 \end{bmatrix}$

d. $X = \begin{bmatrix} -10 \\ -17 \end{bmatrix}$

5. a.

$$X = \begin{array}{c} \begin{array}{cc} \text{gold} & \text{silver} \end{array} \\ \begin{bmatrix} 68 & 52 \\ 76 & 43 \end{bmatrix} \end{array} \begin{array}{l} \\ 20^{th} \\ 21^{st} \end{array}$$

Element x_{21} represents the total amount of 21st century gold coins across Joseph and Martha's collections combined.

b.

$$L = \begin{array}{c} \begin{array}{cc} \text{gold} & \text{silver} \end{array} \\ \begin{bmatrix} 7 & 8 \\ 15 & 9 \end{bmatrix} \end{array} \begin{array}{l} \\ 20^{th} \\ 21^{st} \end{array}$$

This matrix represents the amount of Martha's coins that are different (unique) to the types that Joseph has collected.

Using matrices to solve sets of simultaneous equations

6. D

7. $a = -5$ and $b = 12$.

8. a. $\begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

b. $\begin{bmatrix} 12 & 6 \\ 4 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ -13 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

9. a. $x = 3, \quad y = 1$

b. $x = -4, \quad y = 8$

c. $x = 13, \quad y = -7$

d. $x = -\frac{18}{5}, \quad y = \frac{59}{5}$

e. $x = 2, \quad y = 1$

f. $x = -1, \quad y = 4$

10. $x = 2, \quad y = -\frac{1}{4}, \quad z = \frac{1}{2}$

11. a. $2x + 3y = 10.6$
$3x + y = 8.2$

b. $\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10.6 \\ 8.2 \end{bmatrix}$

c. $x = 2$ and $y = 2.2$

Joining it all together

12. a. $\begin{bmatrix} 11 & 5 \\ 8 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 115 \\ 112 \end{bmatrix}$

b. Scoring regularly: 5 points
Scoring from outside the yellow line: 12 points

13. a. $S = M + 2T$

b.

$$S = \begin{array}{c} \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{bmatrix} 471 & 390 & 688 \\ 802 & 338 & 1672 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{black} \\ \text{silver} \end{array}$$

The silver phone is the most popular at stores A and C, whereas the black phone is the most popular at store B.

c. $471x + 802y = 135\,116$ and $390x + 338y = 79\,924$

d. A black phone costs \$120 and a silver phone costs \$98.

14. \$80

Exam practice

15. Explanation

Step 1: Rearrange the matrix equation to isolate $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 35 & 24 & 60 \\ 28 & 32 & 43 \\ 32 & 30 & 56 \end{bmatrix}^{-1} \begin{bmatrix} 491.55 \\ 428.00 \\ 487.60 \end{bmatrix}$$

Step 2: Solve for $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4.65 \\ 4.20 \\ 3.80 \end{bmatrix}$$

Step 3: Interpret the elements in the matrix.

The cost of one sandwich is represented by c.

$c = 3.80$

Answer

\$3.80

Many students who attempted this question struggled to solve the matrix equation, and likely didn't apply the proper use of the inverse matrix.

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16. Explanation

Step 1: Rearrange the matrix equation to isolate $\begin{bmatrix} x \\ y \end{bmatrix}$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & -9 \\ 4 & -7 \end{bmatrix}^{-1} \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

Step 2: Solve for $\begin{bmatrix} x \\ y \end{bmatrix}$.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Step 3: Interpret the elements in the matrix.

The preferred number of sandwich bars for Grandmall's food court is represented by y.

$y = 2$

Answer

2

Students who failed to answer this question correctly likely did not realise that the inverse matrix was required to solve the question.

Questions from multiple lessons

17. C **18.** C

19. a. 3×1

b. i. $RQ = [11\,745]$

ii. The total revenue from all popcorn sales in the last month.

5G Transition matrices

Identifying transition matrix properties

1. C

2. a. 10% **b.** 80%

c. 45 students **d.** 10 students

Constructing transition and initial state matrices

3. A

4.

$$T = \begin{array}{c} \text{this week} \\ \begin{array}{cc} \text{H} & \text{O} \end{array} \\ \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix} \end{array} \begin{array}{l} \text{H} \\ \text{O} \end{array} \text{ next week}$$

5. a. $S_0 = \begin{bmatrix} 250 \\ 120 \end{bmatrix} \begin{array}{l} \text{S} \\ \text{R} \end{array}$

b.

$$T = \begin{array}{c} \text{today} \\ \begin{array}{cc} \text{S} & \text{R} \end{array} \\ \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix} \end{array} \begin{array}{l} \text{S} \\ \text{R} \end{array} \text{ tomorrow}$$

Modelling applied problems using transition matrices

6. B **7.** D **8.** $P_{83} = \begin{bmatrix} 128 \\ 529 \\ 311 \\ 485 \end{bmatrix} \begin{array}{l} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \end{array}$

9. a. $S_1 = \begin{bmatrix} 126 \\ 144 \\ 150 \end{bmatrix} \begin{array}{l} \text{A} \\ \text{B} \\ \text{C} \end{array}$

b. $S_1 = \begin{bmatrix} 126 \\ 144 \\ 150 \end{bmatrix}$

$$S_2 = \begin{bmatrix} 0.30 & 0.50 & 0.20 \\ 0.60 & 0.25 & 0.10 \\ 0.10 & 0.25 & 0.70 \end{bmatrix} \times \begin{bmatrix} 126 \\ 144 \\ 150 \end{bmatrix} = \begin{bmatrix} 139.8 \\ 126.6 \\ 153.6 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0.30 & 0.50 & 0.20 \\ 0.60 & 0.25 & 0.10 \\ 0.10 & 0.25 & 0.70 \end{bmatrix} \times \begin{bmatrix} 139.8 \\ 126.6 \\ 153.6 \end{bmatrix} = \begin{bmatrix} 135.96 \\ 130.89 \\ 153.15 \end{bmatrix}$$

$$S_4 = \begin{bmatrix} 0.30 & 0.50 & 0.20 \\ 0.60 & 0.25 & 0.10 \\ 0.10 & 0.25 & 0.70 \end{bmatrix} \times \begin{bmatrix} 135.96 \\ 130.89 \\ 153.15 \end{bmatrix} = \begin{bmatrix} 136.86 \\ 129.61 \\ 153.52 \end{bmatrix}$$

Note: Due to rounding, the sum of the column won't add up exactly to 420.

c. 140 customers

d. July

Joining it all together

10. a. 90%

b.

$$T = \begin{array}{c} \text{this week} \\ \begin{array}{cc} \text{A} & \text{B} \end{array} \\ \begin{bmatrix} 0.1 & 0.8 \\ 0.9 & 0.2 \end{bmatrix} \end{array} \begin{array}{l} \text{A} \\ \text{B} \end{array} \text{ next week}$$

c. $S_0 = \begin{bmatrix} 130 \\ 200 \end{bmatrix} \begin{array}{l} \text{A} \\ \text{B} \end{array}$

d. $S_0 = \begin{bmatrix} 130 \\ 200 \end{bmatrix}$, $S_{n+1} = \begin{bmatrix} 0.1 & 0.8 \\ 0.9 & 0.2 \end{bmatrix} \times S_n$

e. $S_1 = \begin{bmatrix} 0.1 & 0.8 \\ 0.9 & 0.2 \end{bmatrix} \times \begin{bmatrix} 130 \\ 200 \end{bmatrix} = \begin{bmatrix} 173 \\ 157 \end{bmatrix}$

$S_2 = \begin{bmatrix} 0.1 & 0.8 \\ 0.9 & 0.2 \end{bmatrix} \times \begin{bmatrix} 173 \\ 157 \end{bmatrix} = \begin{bmatrix} 143 \\ 187 \end{bmatrix}$

f. 160 zebras

Exam practice

11. a. Explanation

Step 1: Calculate S_1 using recursion.

$S_1 = T \times S_0$

$$\begin{bmatrix} 0.1 & 0.2 & 0.1 & 0.2 \\ 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix} \times \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 300 \\ 780 \\ 300 \\ 620 \end{bmatrix}$$

Step 2: Fill the matrix with the missing elements.

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Answer

$$\begin{array}{c}\begin{bmatrix}300\\780\\300\\620\end{bmatrix}\end{array}\begin{array}{l}A\\F\\G\\W\end{array}$$

b. **Explanation**

Step 1: Calculate L_1 using recursion.

$$L_1 = \begin{bmatrix}0.3 & 0.4 & 0.6 & 0.3\\0.1 & 0.2 & 0.1 & 0.2\\0.1 & 0.2 & 0.2 & 0.1\\0.5 & 0.2 & 0.1 & 0.4\end{bmatrix} \times \begin{bmatrix}500\\600\\500\\400\end{bmatrix}$$

$$L_1 = \begin{bmatrix}810\\300\\310\\580\end{bmatrix}$$

Step 2: Identify the row that has an expected number of visitors greater than 600.

$$L_1 = \begin{bmatrix}810\\300\\310\\580\end{bmatrix}$$

The only row with more than 600 expected number of visitors is Row 1, which is location A.

Answer

Air World

12. **Explanation**

Step 1: Calculate the proportion of shoppers buying a different brand of olive oil.

C: $0.05 + 0.10 = 0.15$

L: $0.10 + 0.10 = 0.20$

O: $0.05 + 0.05 = 0.10$

Step 2: Multiply this proportion of shoppers by the number of shoppers who bought each respective brand of olive oil in July 2021.

C: $0.15 \times 3200 = 480$

L: $0.20 \times 2000 = 400$

O: $0.10 \times 2800 = 280$

Step 3: Add the number of shoppers of each olive oil brand to find the total.

$total = 480 + 400 + 280 = 1160$

Answer

1160 shoppers

A significant number of students were unable to completely answer this question. Students were required to look beyond the transition matrix as a whole, and interpret its individual elements in the given context to solve this question.

Questions from multiple lessons

13. D **14.** B

15. **a.** 4×1

b. \$60

c. $\begin{bmatrix}22 & 24 & 17 & 31\end{bmatrix}$

d. $P_{2020} = 1.2 \times P_{2019}$

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6A Introduction to scatterplots

Identifying the response and explanatory variables

1. A

2. a. *RV*: *jackets sold*
   EV: *temperature*

   b. *RV*: *selling price*
   EV: *number of bedrooms*

   c. *RV*: *cost of golf clubs*
   EV: *handicap*

   d. *RV*: *screen time*
   EV: *age*

3. a. *RV*: *time taken*
   EV: *distance*

   b. *RV*: *money earned*
   EV: *hours worked*

   c. *RV*: *years of education*
   EV: *age*

   d. *RV*: *maximum deadlift*
   EV: *weight*

Constructing scatterplots

4. B

5. *RV*: *revenue*
   EV: *years*

6. Kane has labelled the axes incorrectly. The vertical axis should be labelled *number of accounts* and the horizontal axis should be labelled *age*.

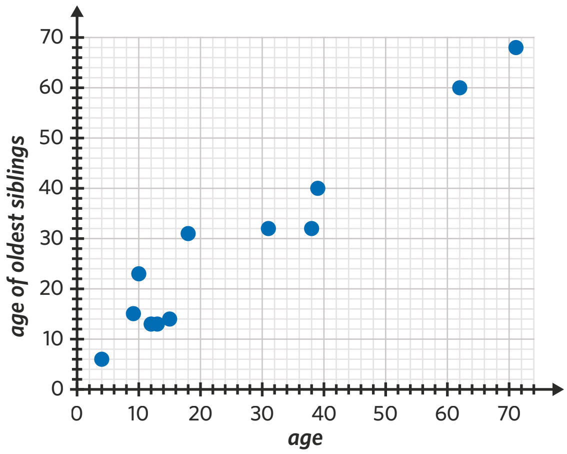
7. a. *RV*: *runs*
   EV: *balls faced*

   b. 80 runs

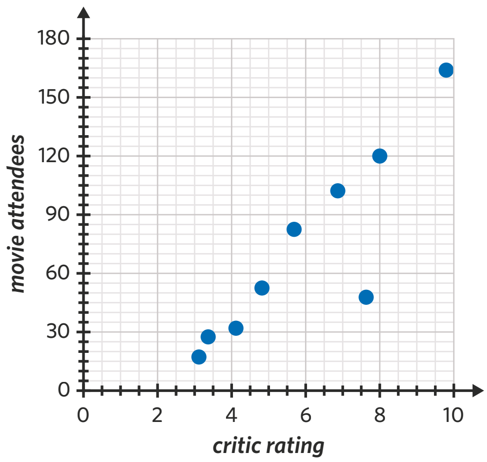
   c. 130 balls

   d. No

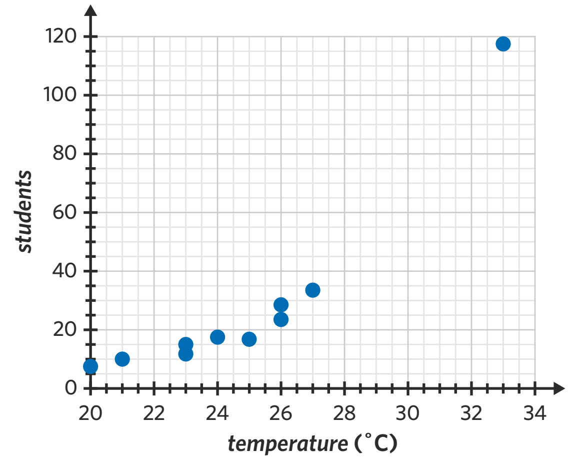
8. a.



b.



9.

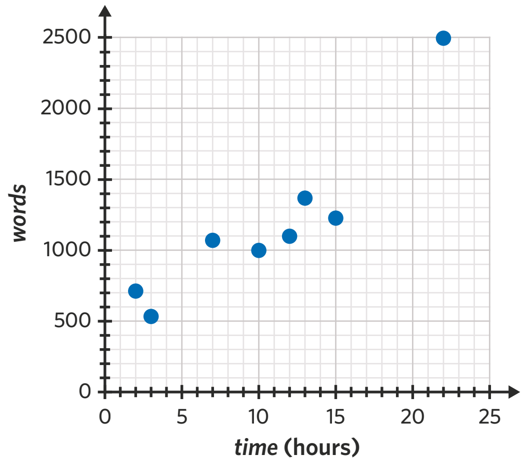


Joining it all together

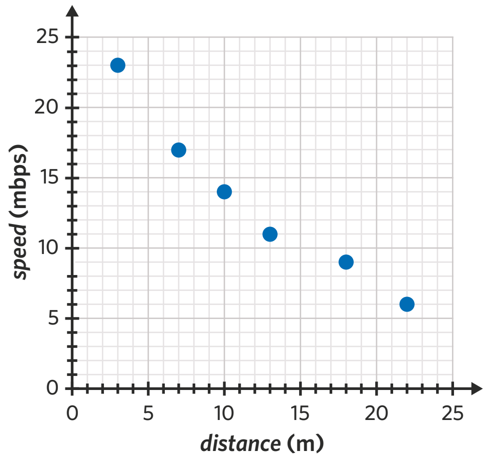
10. A

11. a. *RV*: *words*
    EV: *time*

    b.



12.



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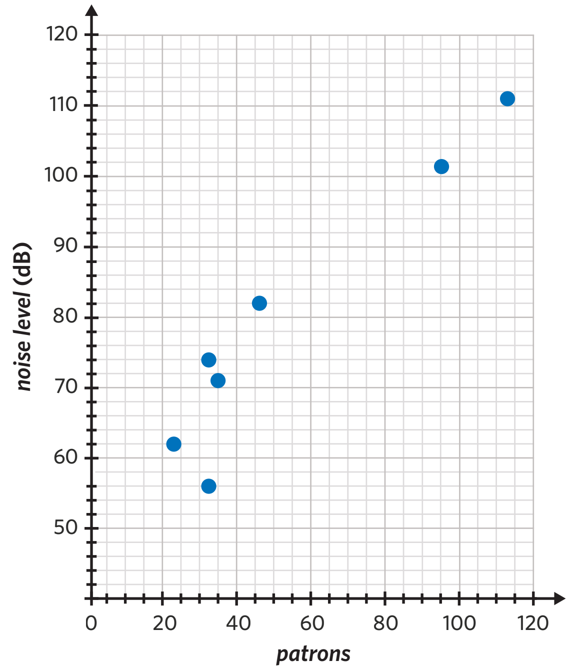
13. a.



b. Friday and Saturday

c. Yes, because they have broken the noise regulations with less than 100 patrons.
Answers may vary.

Exam practice

14. Explanation

On a scatterplot, the response variable is placed on the vertical axis.

Additionally, the question states that maths ability is used to predict musical ability.

Answer

music

15. Explanation

The question states that *humidity* 3 *pm* is predicted from *humidity* 9 *am*.

The change in the response variable is predicted from the change in the explanatory variable.

Answer

humidity 9 *am*

16. Explanation

The question states that *total number of jobs* is predicted from *age*.

The change in the response variable is predicted from the change in the explanatory variable.

Answer

age

Some students who answered this question incorrectly selected a variable that was not a part of the analysis mentioned. This would have resulted in an incorrect answer of current salary.

Questions from multiple lessons

17. C 18. D

19. a. 4×1

b. [9949]

6B Interpreting scatterplots

Describing the relationship between numerical variables

1. B

2. Non-linear

3. a. Moderate b. Positive c. Linear

4. Positive

5. a. Weak, negative, linear relationship

b. Strong, negative, linear relationship

6. No. The strength of the relationship is moderate.

7. a. There is a strong, positive, linear relationship between *marks on exam* and *marks on exam* (%).

b. There is a weak, positive, linear relationship between *shoe size* and *height* (cm).

c. There is no relationship between *number of surfers* and *number of seagulls*.

Understanding the limitations of correlation with respect to causation

8. D 9. A 10. C

11. For 25 years olds, an increase in *years of education* is associated with a decrease in *income*.

12. C

Joining it all together

13. a. Strong, positive, linear relationship

b. *population*
Answers may vary.

c. B

Exam practice

14. Explanation

Step 1: Identify the strength of the relationship.

There is a distinct trend visible and the points follow it closely.

There is a strong relationship.

Step 2: Identify the direction of the relationship.

The *age of second child* tends to increase as the *age of first child* increases.

There is a positive relationship.

Step 3: Identify the form of the relationship.

The trend is a straight line.

There is a linear relationship.

Answer

Strong, positive, linear relationship

Some students incorrectly answered 'Strong, linear and positively skewed'. The direction of a relationship is not related to skew, as this is related to the shape of a distribution.

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15. Explanation

To solve this question, check whether each option is true or false.

A: This is false. This statement suggests a positive causal relationship, which cannot be assumed from strong positive correlation. ✗

B: This is false. This statement suggests a negative causal relationship, while the direction of the relationship is known to be positive. ✗

C: This is false. It cannot be assumed that the association is a coincidence. ✗

D: This is true. This statement suggests a positive association, which can be assumed from the strong positive correlation. ✓

E: This is false. This statement suggests a negative association, while the direction of the relationship is known to be positive. ✗

Answer

D

A common mistake was to assume the relationship was causal. This resulted in an incorrect answer of option A.

16. Explanation

Step 1: Identify the strength of the relationship.

There is a distinct trend visible and the points follow it closely.

There is a strong relationship.

Step 2: Identify the direction of the relationship.

The *driving test mark* tends to decrease as *age* increases.

There is a negative relationship.

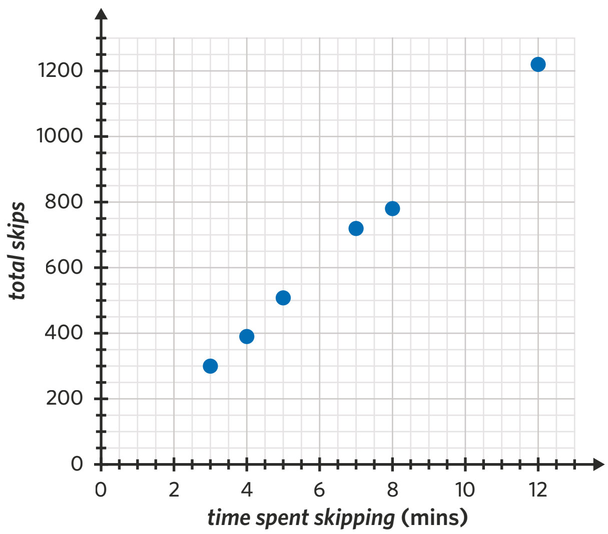
Answer

Strong, negative relationship

Questions from multiple lessons

17. C **18.** A

19.

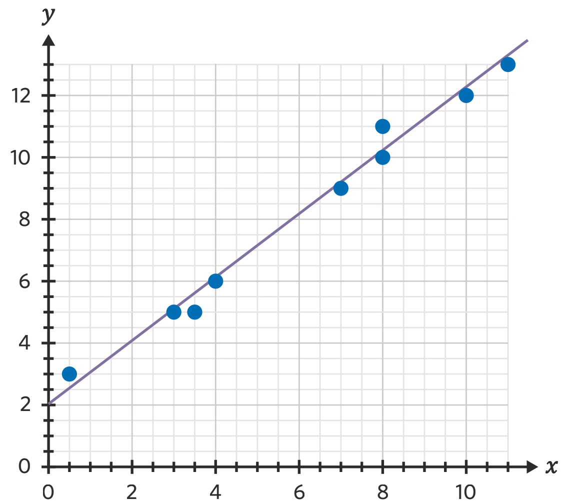


6C Lines of good fit by eye

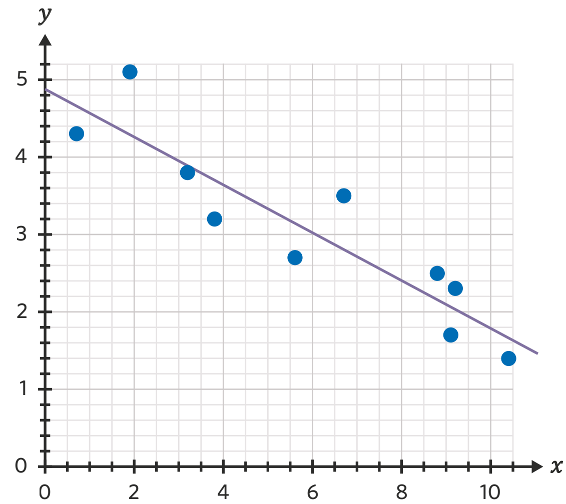
Constructing a line of good fit by eye

1. A

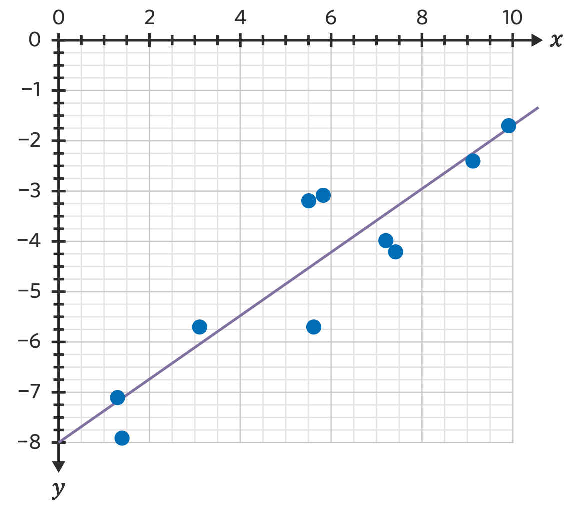
2. a.



b.



c.



3. Andy

Using a line of good fit to make predictions

4. C

5. a. $y = 12$; interpolation

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b. $y = 0.06$; interpolation

c. $y = 3.4$; extrapolation

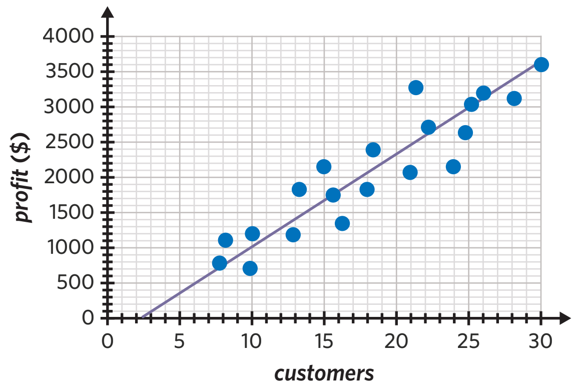
6. a. Chris: 120 m
Devin: 130 m
Deandre: 130 m

b. Devin

c. The prediction is an extrapolation. The lines of good fit were estimated using distance values at times from 0 to 25 seconds. When extending the lines of good fit to a time value of 30 seconds, an assumption is being made that the trends will continue beyond the observed data. This is not reliable as some students may slow down as they tire, or some may have saved energy for the last few seconds.

Joining it all together

7. a.



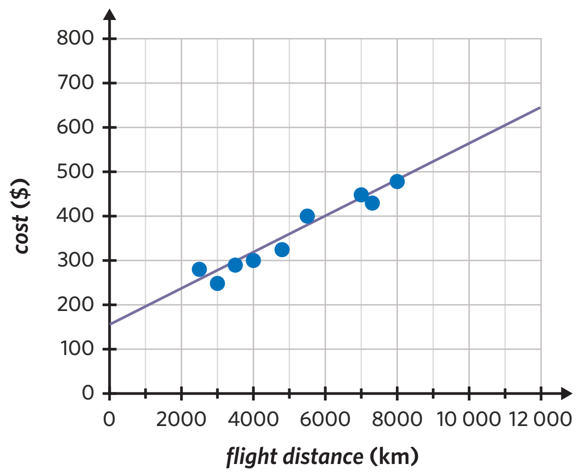
b. 25 customers

c. Yes, the prediction is reliable.

d. $500

e. No, the prediction has limited reliability because it extrapolates beyond the domain of the data.

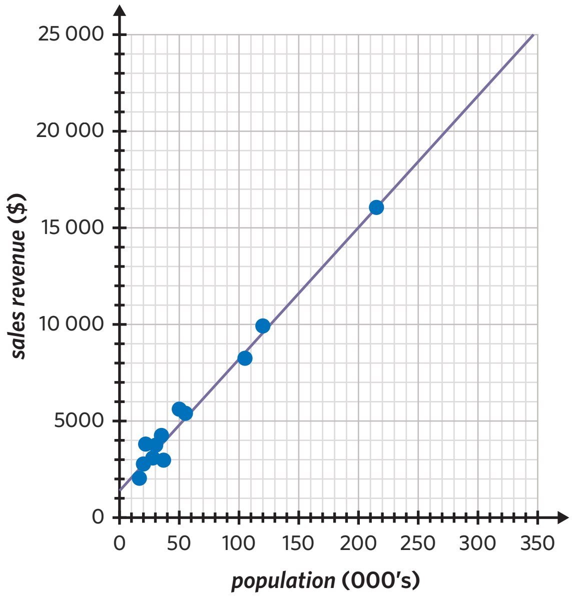
8. a.



b. Interpolation: 6000 km
Extrapolation: 9000 km, 12 000 km

c. The cost of a flight with a distance of 6000 km is expected to be $400.

9. a.

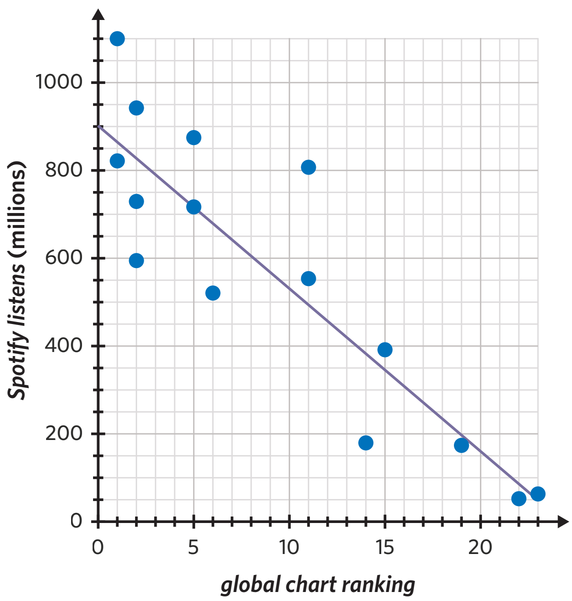


b. $4000; interpolation

c. $17 000; extrapolation

d. 200 000; interpolation

10. a.



b. 800 000 000 Spotify listens

c. 12

d. The prediction is an extrapolation. The line of good fit was estimated using Spotify listen values for songs with global chart rankings from 1 to 23. When extending the line of good fit to a global chart ranking of 35, an assumption is being made that the general trend will continue beyond the observed data. This is not reliable, as the Spotify listens value, given the line of good fit, will be negative for a global chart ranking of 35.

Exam practice

11. **Explanation**

Step 1: Determine the domain of the data set.

The minimum *year* is 2014.

The maximum *year* is 2022.

The domain of *year* is 2014 to 2022.

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Step 2: Determine whether the prediction is an interpolation or an extrapolation.

The known value is $year = 2026$.

2026 is not between 2014 and 2022.

The prediction is an extrapolation.

Answer

The prediction has limited reliability because it extrapolates beyond the domain of the data.

12. Explanation

Step 1: Determine the domain of the data set.

The minimum *year* is 1956.

The maximum *year* is 2016.

The domain of *year* is 1956 to 2016.

Step 2: Determine whether the prediction is an interpolation or an extrapolation.

The known value is $year = 2032$.

2032 is not between 1956 and 2016.

The prediction is an extrapolation.

Step 3: Recall the assumption made when extrapolating.

The assumption is that the general trend will continue beyond what is observed.

Answer

The general trend will continue into the future.

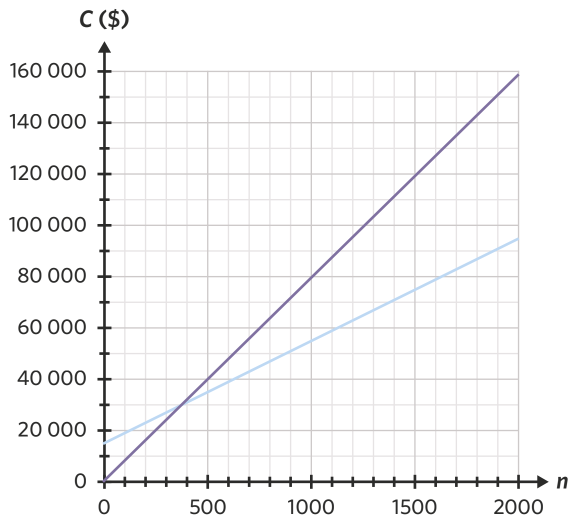
> Students struggled to convey the idea of the trend continuing with this type of question. Many answers referred to extrapolation or linearity without linking it to the predicted trend.

Questions from multiple lessons

13. E

14. C

15. a.



b. $45 000

6D Lines of good fit – applications

Determining the equation of a line of good fit

1. C

2. a. $y = 8 - 0.33x$

b. $y = 70 - 2x$

c. $y = 92.86 - 7.14x$

d. $y = -13.67 + 1.83x$

3. D

4. C

5. $y = 1.5 + 2.5x$

Applying equations of lines of good fit

6. B

7. a. Maths exam scores are expected to increase by 7% for each additional practice exam paper completed.

b. The score on the maths exam is expected to be 30% when no practice exam papers are completed.

c. 72%

d. 3 practice exam papers

8. a. The amount of time spent outside is expected to increase by 45 minutes for each additional pet.

b. The amount of time spent outside is expected to be 90 minutes for students who do not own a pet.

c. 3 hours and 45 minutes

9. a. 27 games **b.** 60 games **c.** 11 games

Joining it all together

10. a. $seats\ won = 0.25 + 3.5 \times donations$

b. The number of seats won is expected to increase by 3.5 seats for each additional $1 000 000 donation.

c. A party that receives no donations is expected to win 0.25 seats.

d. 26.5 seats

e. $14 500 000

11. a. $average\ attendance = 4000 \times games\ won - 8000$

b. The average attendance figure is expected to increase by 4000 for each additional game won.

c. 9 games

d. Extrapolation has limited reliability as it relies on the assumption that the trend seen in the data continues outside the domain of the data.
For example, the intercept of the line of good fit is −8000. This would mean that an AFL team that wins no games is expected to have an attendance of -8000 people. This is impossible, as an attendance figure must be positive.

Exam practice

12. Explanation

Step 1: Find the intercept.

When *time spent exercising* = 1, *resting pulse rate* = 67.2.

As the line of good fit has a negative slope, the intercept will be greater than 67.2.

The only options with an intercept greater than 67.2 are options C and D.

$a = 68.3$

Step 2: Find the slope.

The point (3, 65) is approximately on the line of good fit.

Substitute the point into the equation *resting pulse rate* = 68.3 + b × *time spent exercising*.

$$65 = 68.3 + 3b$$
$$3b = -3.3$$
$$b = -1.1$$

Answer

D

41% of students incorrectly chose either option A or option B. These students correctly determined that the slope was negative, but found the intercept to be 67.2. This is the intercept that is found visually when looking at the left edge of the graph, however the value of the horizontal axis is 1 at this point, not 0.

13. Explanation

Step 1: Determine the value of the slope.

$b = 0.8894$

Step 2: Interpret the slope in terms of the variables.

The slope is the average change in the response variable for every one-unit increase in the explanatory variable.

Answer

Atmospheric pressure at 3 pm is expected to increase by 0.8894 hPa for every one-unit increase in atmospheric pressure at 9 am.

Some students gave a response that failed to reference the one-unit increase in pressure at 9 am. This is incorrect as the increase of pressure at 3 pm is in relation to a one-unit increase at 9 am.

14. Explanation

Step 1: Find the intercept.

When *height* = 21, *weight* = 1.74.

As the line of good fit has a positive slope, the intercept will be less than 1.74.

The only options with an intercept less than 1.74 are options A and B.

Step 2: Find the closest line of good fit.

height is the explanatory variable, so the line of good fit should be in the form:

weight = a + b × *height*

Out of options A and B, only option B is in the correct form.

Answer

B

39% of students incorrectly chose option D. These students found the intercept to be 1.74. This is the intercept that is found visually when looking at the left edge of the graph, however the value of the horizontal axis is 21 at this point, not 0.

15. Explanation

Step 1: Determine the value of the intercept.

$a = -1.7$

Step 2: Interpret the intercept in terms of the variables.

The intercept is the expected value of the response variable when the explanatory variable is zero.

Answer

The apparent temperature is expected to be −1.7 °C, on average, when the actual temperature is 0 °C.

Some students gave a response that failed to reference the one-unit increase in actual temperature. This is incorrect as the increase in apparent temperature is in relation to a one-unit increase of the actual temperature.

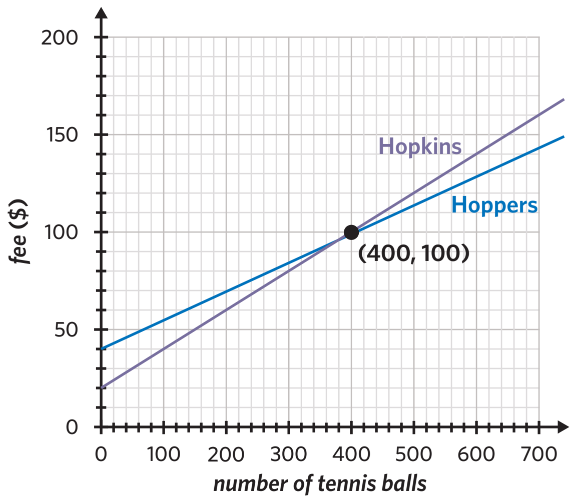
Questions from multiple lessons

16. A

17. B

18. a. Hopkins charges a fixed amount of \$20 and an additional \$0.20 per tennis ball.

b.



6D ANSWERS

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7A Introduction to graphs and networks

Identifying properties of graphs

1. A

2. a. 5 vertices, 8 edges b. 3
   c. D

3. a. 4 vertices, 4 edges b. 5 vertices, 5 edges
   c. 8 vertices, 12 edges d. 11 vertices, 10 edges

4. a. 1 b. 0
   c. 5 d. 5

5. a. 6 b. 10
   c. 3 d. Christchurch
   e. Christchurch, Dunedin, Invercargill, Queenstown

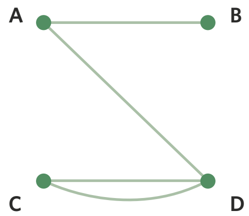
Identifying and constructing graphs

6. B

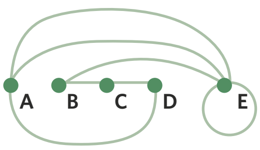
7. a. Connected, simple, complete
   b. Connected, simple
   c. Disconnected
   d. Disconnected, simple

8. a. D and F b. 2

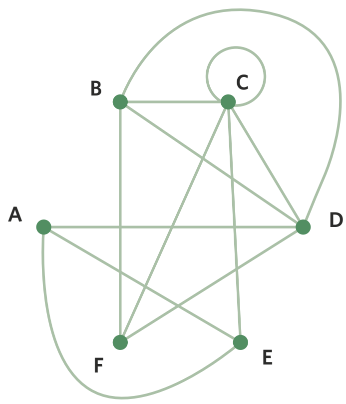
9. a.



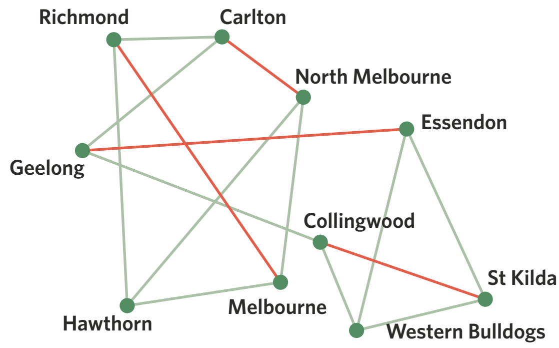
b.



c.

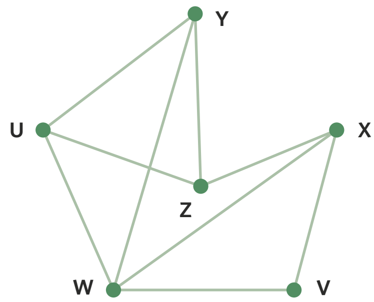


10. a. Collingwood, Essendon and St Kilda
    b. Connected, simple
    c. Yes, Carlton and Geelong
    d.

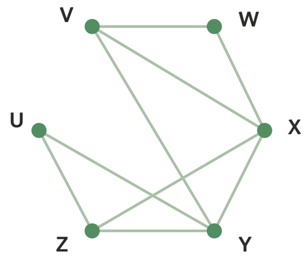


Joining it all together

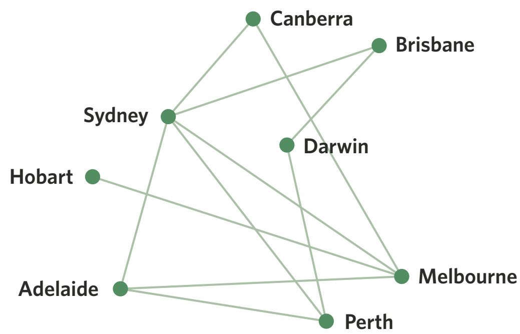
11. a. Wilhelmina b. Venus
    c.



d.



12. a. 11 b. 5
    c. Connected, simple d. 22
    e. Yes, Melbourne and Hobart
    f.



Exam practice

13. **Explanation**

Step 1: Sum the degrees of the vertices in graph 1.
Each vertex in graph 1 has a degree of 3.
There are 4 vertices.
$3 \times 4 = 12$

7A ANSWERS

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Step 2: Sum the degrees of the vertices in graph 2.

Each vertex in graph 2 has a degree of 2.

There are 6 vertices.

$2 \times 6 = 12$

Step 3: Compare the sums.

$12 = 12$

Answer

C

> 9% of students chose option A. This may be because they incorrectly understood the question to be asking the difference in the number of vertices between the graphs, rather than the difference in degrees.

14. a. Explanation

There are two roads leading from Beachton. These roads each go to Alooma and Easyside.

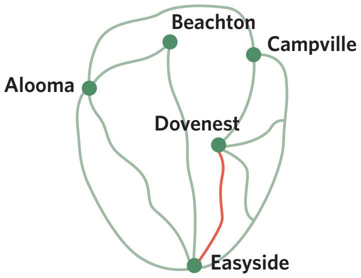
Answer

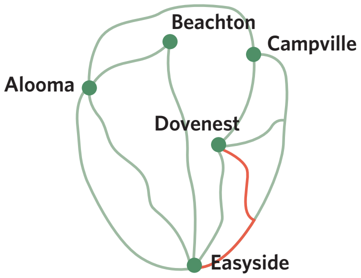
Alooma and Easyside

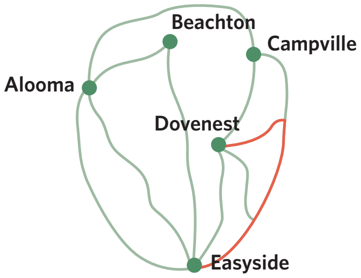
b. Explanation

In the graph, there are only two edges connecting vertices D and E.

On the map, there are three different ways to travel between Dovenest and Easyside without passing any other towns.

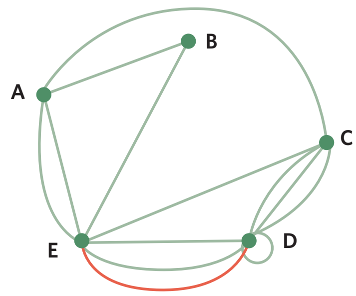






Hence, there should be one more edge connecting vertices D and E.

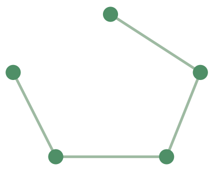
Answer



15. Explanation

The graph has 10 edges and 5 vertices.

In order for a graph with 5 vertices to remain connected, it would need to have at least 4 edges.



This means that $10 - 4 = 6$ edges are removed.

Answer

C

> 49% of students chose option B. This is likely because they incorrectly assumed that 5 edges would be needed to keep 5 vertices connected.

Questions from multiple lessons

16. D **17.** A

18. a. 85 000 koalas

b. 20%

c. Decrease of 13 600 koalas

7B The adjacency matrix and its applications

Constructing adjacency matrices and graphs

1. C

2. a.

$$\begin{array}{cc} \begin{matrix} A & B & C & D \end{matrix} & \\ \begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} & \begin{matrix} A \\ B \\ C \\ D \end{matrix} \end{array}$$

b.

$$\begin{array}{cc} \begin{matrix} A & B & C & D & E & F \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 2 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} & \begin{matrix} A \\ B \\ C \\ D \\ E \\ F \end{matrix} \end{array}$$

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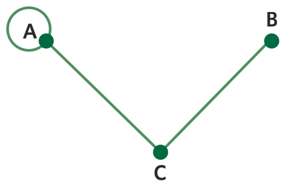
c.

$$\begin{array}{c}\begin{matrix} A & B & C & D & E & F & G & H \end{matrix}\\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}\begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{matrix}\end{array}$$

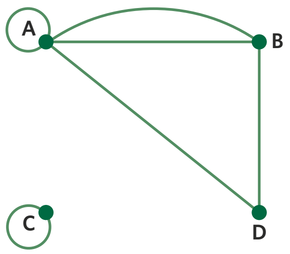
d.

$$\begin{array}{c}\begin{matrix} A & B & C & D & E & F & G & H \end{matrix}\\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & 0 & 1 \end{bmatrix}\begin{matrix} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{matrix}\end{array}$$

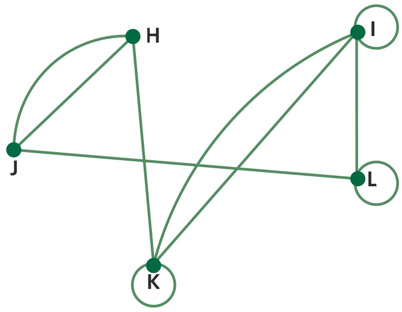
3. a.



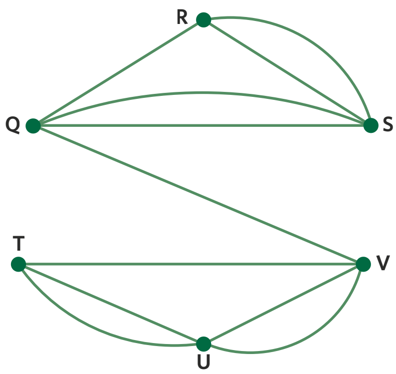
b.



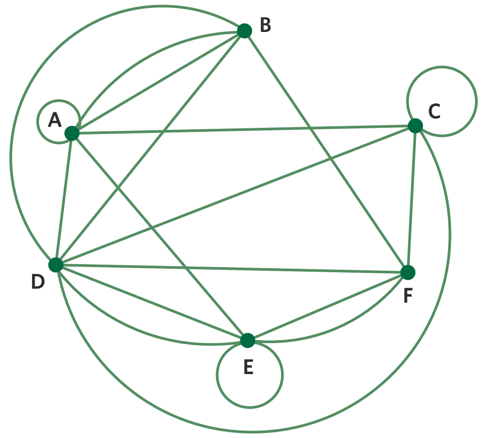
c.



d.



4.



5.

$$\begin{array}{c}\begin{matrix} B & C & J & S & T \end{matrix}\\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 4 & 2 & 4 \\ 1 & 4 & 1 & 2 & 4 \\ 0 & 2 & 2 & 0 & 3 \\ 0 & 4 & 4 & 3 & 1 \end{bmatrix}\begin{matrix} B \\ C \\ J \\ S \\ T \end{matrix}\end{array}$$

Constructing and interpreting directed graphs

6. C

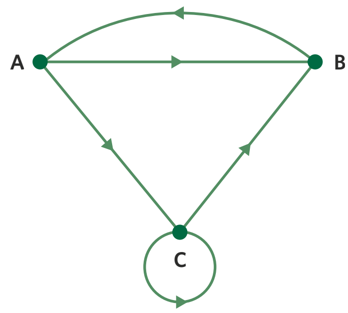
7. a.

$$\begin{array}{c}\text{to}\\ \begin{matrix} R & S & T & U & V \end{matrix}\\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}\begin{matrix} R \\ S \\ T \\ U \\ V \end{matrix}\ \text{from}\end{array}$$

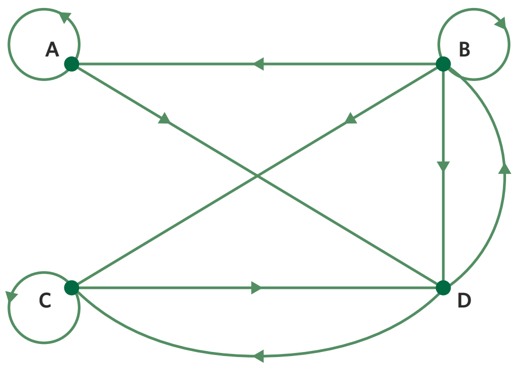
b. R, S, T, V

c. S

8. a.



b.



7B ANSWERS

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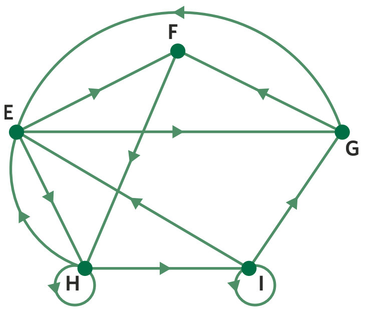
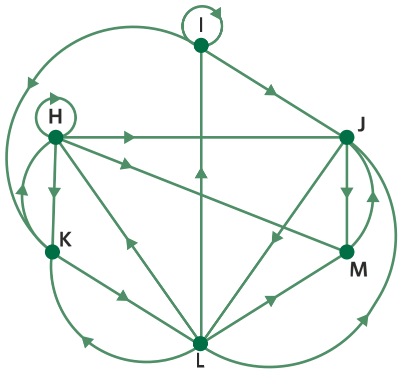
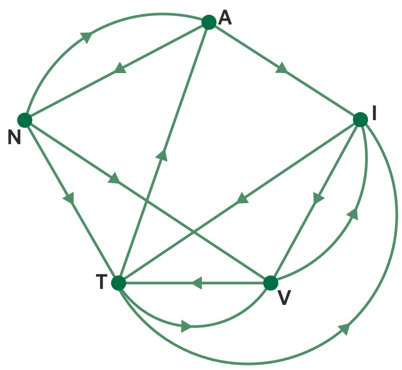
c.



d.



9. a.



b.

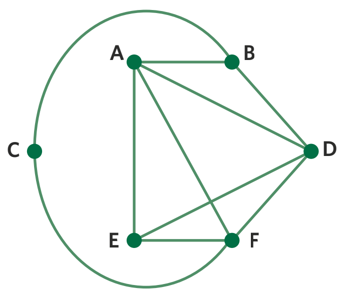
$$\begin{array}{c} \text{to} \\ \begin{array}{ccccc} A & I & N & T & V \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ I \\ N \\ T \\ V \end{array} \text{ from}$$

Applying adjacency matrices to communication problems

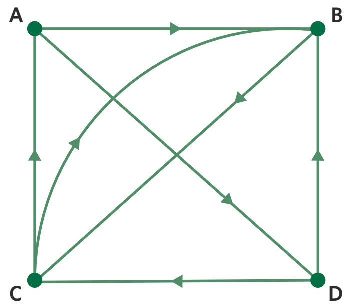
10. A

11. a. 3 **b.** 4

12.



13. a.



b.

$$C = \begin{array}{c} \text{to} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \\ C \\ D \end{array} \text{ from}$$

c.

$$C^2 = \begin{array}{c} \text{to} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ A \\ B \\ C \\ D \end{array} \text{ from}$$

Joining it all together

14. a. D

b. No, because directed adjacency matrices are not always symmetrical.

c. 2

15. a.

$$\begin{array}{c} \text{to} \\ \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ I \\ II \\ III \\ IV \\ V \end{array} \text{ from}$$

b. II, III and IV

c. I and V

d.

$$C^2 = \begin{array}{c} \text{to} \\ \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \begin{bmatrix} 1 & 0 & 2 & 1 & 3 \\ 2 & 0 & 2 & 0 & 2 \\ 1 & 1 & 2 & 1 & 0 \\ 2 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ I \\ II \\ III \\ IV \\ V \end{array} \text{ from}$$

e. 2

f. I and V

Exam practice

16. Explanation

Step 1: Fill in the first row and column of the matrix.

There are 2 paths between J and K.

There are 0 paths between J and L.

There are 2 paths between J and M.

$$\begin{array}{cc} \begin{matrix} J & K & L & M \end{matrix} & \\ \begin{bmatrix} 0 & 2 & 0 & 2 \\ 2 & 0 & & \\ 0 & & 0 & \\ 2 & & & 1 \end{bmatrix} & \begin{matrix} J \\ K \\ L \\ M \end{matrix} \end{array}$$

Step 2: Complete this process for the remaining rows and columns.

$$\begin{array}{cc} \begin{matrix} J & K & L & M \end{matrix} & \\ \begin{bmatrix} 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 1 \\ 0 & 2 & 0 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} & \begin{matrix} J \\ K \\ L \\ M \end{matrix} \end{array}$$

Step 3: Count the number of 0's, 1's and 2's that have been added.

Answer

E

17. Explanation

To solve this question, check whether the adjacency matrix matches each network.

A: All the pathways in the matrix are shown in this network. ✓

B: The first incorrect component of this network is the number of pathways between M and N. This network has only 3 pathways between these vertices, while the matrix states that there are 5. ✗

C: The first incorrect component of this network is the loop at M. The matrix states there is a loop at M but it is not present in this network. ✗

D: The first incorrect component of this network is the number of pathways between L and M. There is only 1 pathway between L and M, while the matrix states that there are 2. ✗

E: The first incorrect component of this network is the number of pathways between L and M. There are 6 pathways between L and M, while the matrix states that there are 2. ✗

Answer

A

18. Explanation

Step 1: Set up the adjacency matrix.

There are five vertices, F to J.

The matrix will be a 5 × 5 matrix with the rows and columns labelled F to J.

$$\begin{array}{cc} \begin{matrix} F & G & H & I & J \end{matrix} & \\ \begin{bmatrix} \; & \; & \; & \; & \; \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} & \begin{matrix} F \\ G \\ H \\ I \\ J \end{matrix} \end{array}$$

Step 2: Fill in the first row and column. Take note of all the possible routes that can be followed without passing through another vertex.

There are 0 connections between F and F.

There is 1 connection between F and G.

There is 1 connection between F and H.

There are 2 connections between F and I.

There are 2 connections between F and J.

$$\begin{array}{cc} \begin{matrix} F & G & H & I & J \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 1 & 2 & 2 \\ 1 & & & & \\ 1 & & & & \\ 2 & & & & \\ 2 & & & & \end{bmatrix} & \begin{matrix} F \\ G \\ H \\ I \\ J \end{matrix} \end{array}$$

Step 3: Complete this process for the remaining rows and columns.

Remember that loops only count as one edge.

$$\begin{array}{cc} \begin{matrix} F & G & H & I & J \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 1 & 2 & 2 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 2 \\ 2 & 0 & 1 & 0 & 3 \\ 2 & 0 & 2 & 3 & 1 \end{bmatrix} & \begin{matrix} F \\ G \\ H \\ I \\ J \end{matrix} \end{array}$$

Answer

C

> Many students were unable to recognise loops with this type of question. Failing to recognise the loop at J could have led to an incorrect answer of E.

19. Explanation

Step 1: Set up the adjacency matrix.

There are five vertices, O to S.

The matrix will be a 5 × 5 matrix with the rows and columns labelled O to S.

$$\begin{array}{cc} \begin{matrix} O & P & Q & R & S \end{matrix} & \\ \begin{bmatrix} \; & \; & \; & \; & \; \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} & \begin{matrix} O \\ P \\ Q \\ R \\ S \end{matrix} \end{array}$$

Step 2: Fill in the first row and column. Take note of all the possible routes that can be followed without passing through another vertex.

There are 0 direct routes between O and O.

There is 1 direct route between O and P.

There is 1 direct route between O and Q.

There are 2 direct routes between O and R.

There are 2 direct routes between O and S.

$$\begin{array}{cc} \begin{matrix} O & P & Q & R & S \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 1 & 2 & 2 \\ 1 & & & & \\ 1 & & & & \\ 2 & & & & \\ 2 & & & & \end{bmatrix} & \begin{matrix} O \\ P \\ Q \\ R \\ S \end{matrix} \end{array}$$

Step 3: Complete this process for the remaining rows and columns.

Remember that loops only count as one edge.

$$\begin{array}{cc} \begin{matrix} O & P & Q & R & S \end{matrix} & \\ \begin{bmatrix} 0 & 1 & 1 & 2 & 2 \\ 1 & 0 & 0 & 3 & 2 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & 3 & 0 & 1 & 4 \\ 2 & 2 & 1 & 4 & 1 \end{bmatrix} & \begin{matrix} O \\ P \\ Q \\ R \\ S \end{matrix} \end{array}$$

Step 4: Count the number of zeros in the matrix.

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Answer

B

Questions from multiple lessons

20. B

21. B

22. **a.** $p = 16n - 60$

b. 28

7C Planar graphs

Identifying planar and non-planar graphs

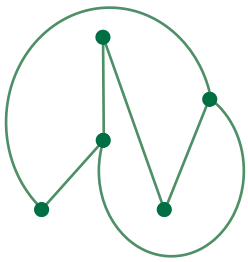
1. B

2. **a.** Planar

b. Planar

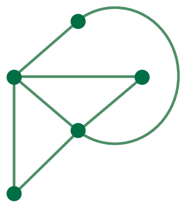
c. Non-planar

3. **a.**



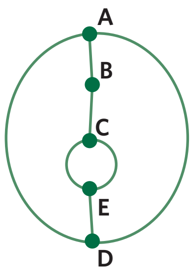
Answers may vary.

b.



Answers may vary.

c.



Answers may vary.

4. It is not possible.

Applying Euler's rule

5. B

6. **a.** Vertices: 5

Edges: 7

Faces: 4

$$v - e + f = 2$$
$$5 - 7 + 4 = 2$$
$$2 = 2$$

b. Vertices: 6

Edges: 10

Faces: 6

$$v - e + f = 2$$
$$6 - 10 + 6 = 2$$
$$2 = 2$$

c. Vertices: 10

Edges: 13

Faces: 5

$$v - e + f = 2$$
$$10 - 13 + 5 = 2$$
$$2 = 2$$

7. 7 edges

8. 7 regions

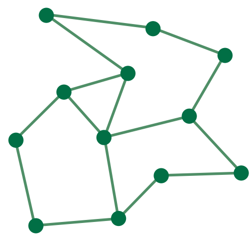
Joining it all together

9. **a.** There are no overlapping edges.

b.
$$v - e + f = 2$$
$$13 - 15 + 4 = 2$$
$$2 = 2$$

10. **a.** 15 walking trails

b.



Answers may vary.

Exam practice

11. **Explanation**

For each option, substitute the number of vertices, edges and faces into Euler's rule, $v - e + f = 2$, and determine if both sides of the rule are equal.

A: This is incorrect.

$8 - 8 + 5 = 5$

$5 \neq 2$ ✗

B: This is incorrect.

$6 - 8 + 5 = 3$

$3 \neq 2$ ✗

C: This is incorrect.

$8 - 5 + 5 = 8$

$8 \neq 2$ ✗

D: This is incorrect.

$8 - 6 + 5 = 7$

$7 \neq 2$ ✗

E: This is correct.

$5 - 8 + 5 = 2$

$2 = 2$ ✓

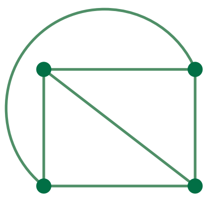
Answer

E

12. **Explanation**

For each option, redraw the graph in planar form, if possible.

A: This graph can be redrawn in planar form with no overlapping edges. ✗



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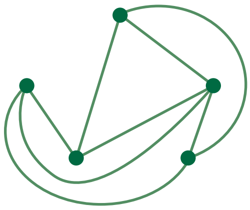
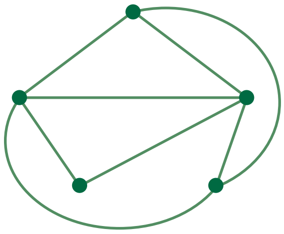
B: This graph can be redrawn in planar form with no overlapping edges. ✗



C: This graph can be redrawn in planar form with no overlapping edges. ✗



D: This graph is a complete graph with five or more vertices, so it is non-planar. ✓

E: This graph has no overlapping edges, so it is planar. ✗

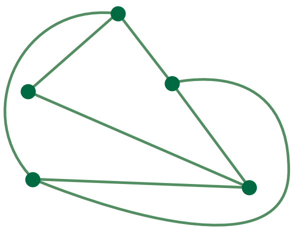
Answer

D

Students may need to try different rearrangements of edges and vertices to draw graphs in planar form. Students can also recognise that a complete graph with five or more vertices is non-planar.

13. **Explanation**

Step 1: Redraw the graph in planar form.



Step 2: Count the number of faces, including the region outside the graph.

The graph has 4 faces.

Answer

C

Students need to ensure they redraw the graph in planar form before counting the faces.

Questions from multiple lessons

14. B **15.** B

16. **a.** Jonty and Hunter

b. J and K

c. People at Kakadu can travel from the pool and return without passing any of the other pools.

7D Connected graphs

Identifying walks, trails and paths

1. D

2. D

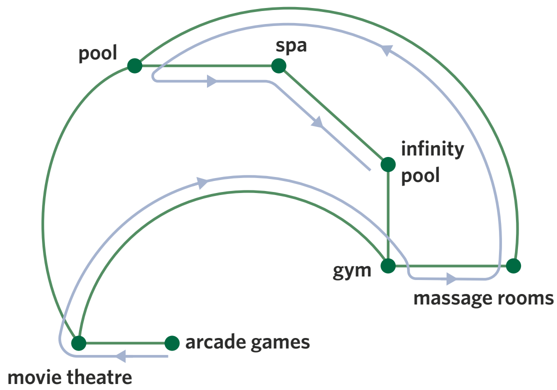
3. **a.** Trail **b.** Walk **c.** Trail **d.** Path

4. E–G–H–F–D–C–B–A

5. **a.** Yes **b.** No

c. Massage rooms

d.



Answers may vary.

Identifying circuits and cycles

6. C

7. **a.** Walk **b.** Circuit **c.** Cycle **d.** Cycle

8. **a.** E–B–C–D–F–H–G–E
Answers may vary.

b. 8 **c.** A

Joining it all together

9. **a.** Path **b.** Circuit

c. G–S–F–A–Y–R–P–M–V–N–W–G or G–W–N–V–M–P–R–Y–A–F–S–G

d. North Melbourne, Parkville, Southern Cross, Parliament, Flinders Street, West Melbourne

e. North Melbourne, Flinders Street

10. **a.** **i.** P, T, U, W, Y **ii.** Path

b. Trail **c.** W–Z

Exam practice

11. **Explanation**

Step 1: Begin at Mount Cockatoo.

Note that Point Kangaroo cannot be visited first because in order to get back to Mount Cockatoo, Snake Point must be reached via Point Kangaroo.

Step 2: Move to either Sunset Lookout, Snake Point, or Feather Mountain.

From here, follow the vertices until all have been passed once and the route reaches back to Mount Cockatoo.

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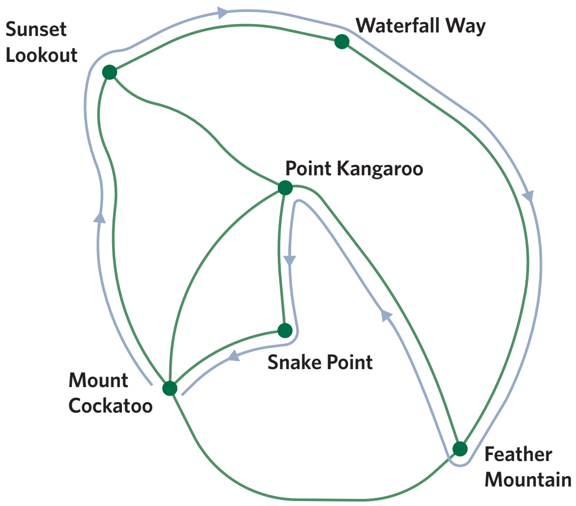
Answer



Answers may vary.

12. **Explanation**

For this question, work through each of the options to determine which is correct.

A path travels to vertices via edges, and no vertices or edges are repeated.

Option C is the correct answer because vertex E cannot be reached from vertex G.

Answer

C

Questions from multiple lessons

13. A **14.** E

15. **a.** \$1783.77

b. 1.50% p.a.

7E Weighted graphs

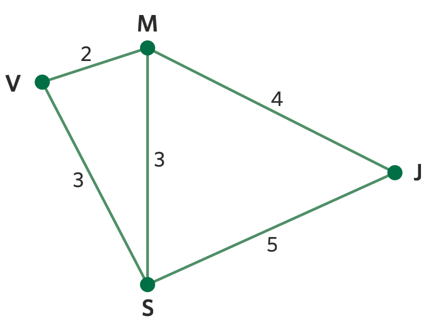
Interpreting weighted graphs

1. B

2. **a.** 18 km **b.** 40 km **c.** 62 km

3. 28

4.



Answers may vary.

Identifying the shortest path through a network

5. C

6. **a.** 23.2 km **b.** 22.6 km **c.** 29.3 km

7. A–B–C–D–E; 12.1 m

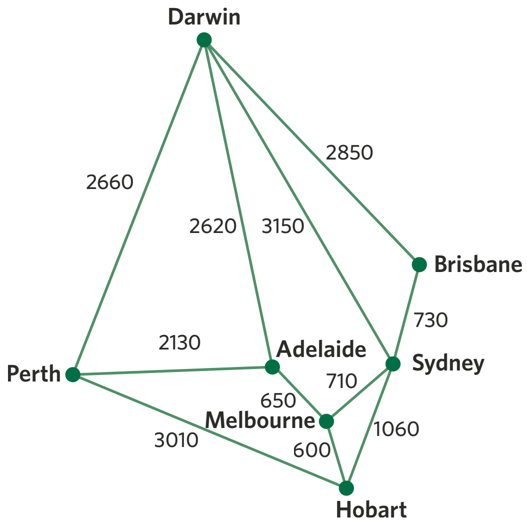
Joining it all together

8. 110 km

9. 3.2 km

10. P–C–B–F–E–D–A–P or P–A–D–E–F–B–C–P

11. **a.**



Answers may vary.

b. Melbourne, Hobart, Sydney, Brisbane, Darwin, Perth, Adelaide, Melbourne

or

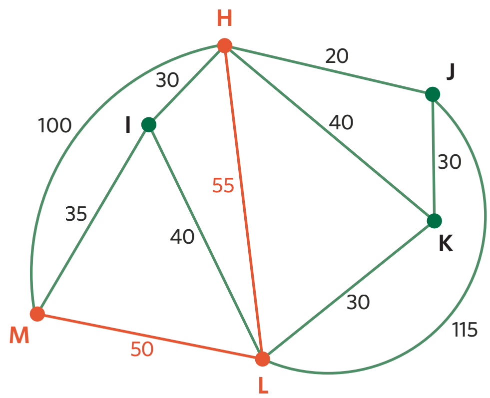
Melbourne, Adelaide, Perth, Darwin, Brisbane, Sydney, Hobart, Melbourne

Exam practice

12. **a.** **Explanation**

Step 1: Identify the route considered by Mai in the diagram.

Mai considers travelling from Hampshire (H) to Louisberg (L) to Manifold (M).



Step 2: Calculate the total amount by adding the weight of each edge along the route.

$55 + 50 = 105$

Answer

\$105

Students who answered this question incorrectly may have given the actual route as the answer rather than the cost of the route.

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b. Explanation

Step 1: Identify all possible cheapest routes between Hampshire (H) and Manifold (M).

H–M

H–I–M

H–L–M

Step 2: Calculate the total amount for each possible route and identify the lowest value.

H–M = 100

H–I–M = 30 + 35 = 65

H–L–M = 55 + 50 = 105

Therefore, H–I–M is the cheapest route.

Step 3: Identify which town Mai passes through using this route.

Answer

Idina

Students who answered this question incorrectly may have given the cost of the route as the answer, rather than identifying which town is passed through along this route.

13. Explanation

Step 1: Identify all possible shortest routes between town F and town J.

F–D–C–B–I–J

F–D–C–G–I–J

F–D–E–H–G–I–J

F–D–E–C–G–I–J

Step 2: Calculate the total distance for each possible route and identify the lowest value.

F–D–C–B–I–J = 25 + 26 + 26 + 29 + 20 = 126

F–D–C–G–I–J = 25 + 26 + 35 + 26 + 20 = 132

F–D–E–H–G–I–J = 25 + 24 + 39 + 25 + 26 + 20 = 159

F–D–E–C–G–I–J = 25 + 24 + 26 + 35 + 26 + 20 = 156

Therefore, F–D–C–B–I–J is the shortest route.

Answer

126 km

Students who answered this question incorrectly may have given the actual route as the answer rather than the distance of the route.

14. Explanation

Step 1: Through inspection, write out all possible shortest paths from S to O.

S–M–O

S–T–U–N–O

S–T–U–V–O

S–T–U–V–P–O

S–R–Q–V–O

S–R–Q–P–O

S–R–Q–P–V–O

Step 2: Determine the weight of all the routes.

S–M–O = 1.8 + 1.5 = 3.3

S–T–U–N–O = 0.6 + 1.2 + 0.7 + 0.9 = 3.4

S–T–U–V–O = 0.6 + 1.2 + 0.6 + 0.8 = 3.2

S–T–U–V–P–O = 0.6 + 1.2 + 0.6 + 0.4 + 1.4 = 4.2

S–R–Q–V–O = 1.0 + 1.2 + 1.2 + 0.8 = 4.2

S–R–Q–P–O = 1.0 + 1.2 + 0.8 + 1.4 = 4.4

S–R–Q–P–V–O = 1.0 + 1.2 + 0.8 + 0.4 + 0.8 = 4.2

Step 3: Identify the shortest route.

The shortest route is S–T–U–V–O with a total weight of 3.2.

Answer

3.2 km

Students commonly gave an answer of 3.3 km as they mistakenly assumed the most direct route S–M–O would yield the shortest path, and therefore did not explore other possible paths.

Questions from multiple lessons

15. B **16.** D

17. a. $2g + 7m = 46.40$

$4g + 5m = 49.60$

b. \$6.40

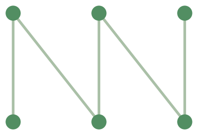
c. \$52.80

7F Trees and their applications

Identifying trees in a network

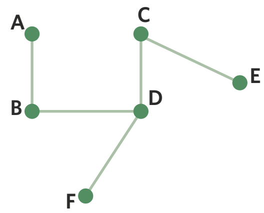
1. D **2.** C

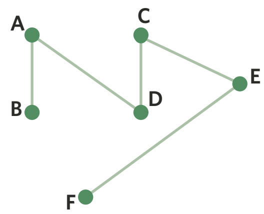
3.

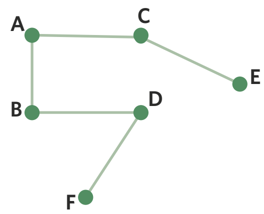


Answers may vary.

4. a.

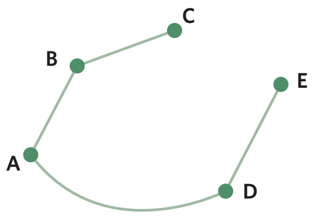


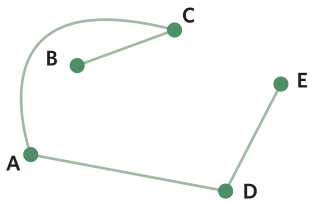


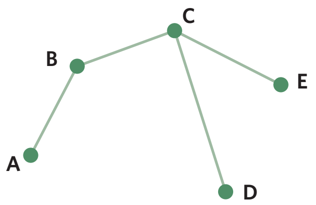


Answers may vary.

b.



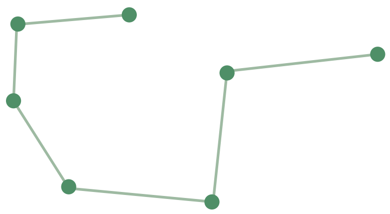


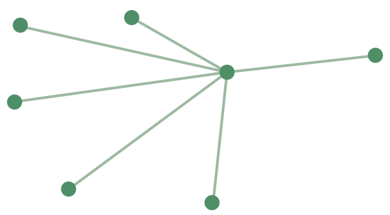


Answers may vary.

5. a. 6

b.





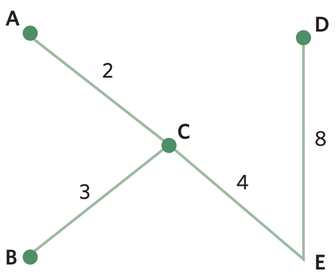
Answers may vary.

Identifying a minimum spanning tree using greedy algorithms

6. A greedy algorithm is a set of steps used to find the best solution at each step of a multi-step problem. Prim's algorithm and Kruskal's algorithm are examples of greedy algorithms. They are both used to find the minimum spanning tree of a graph.

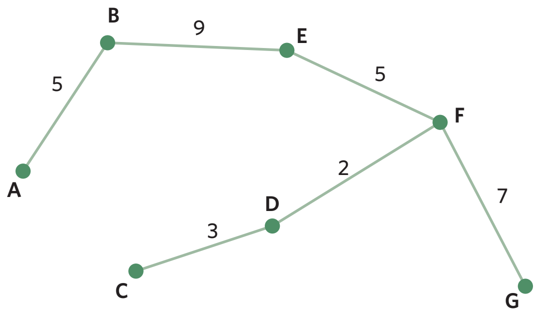
7. B

8.



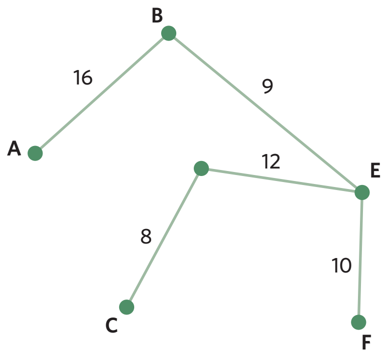
Total weight: 17

9.



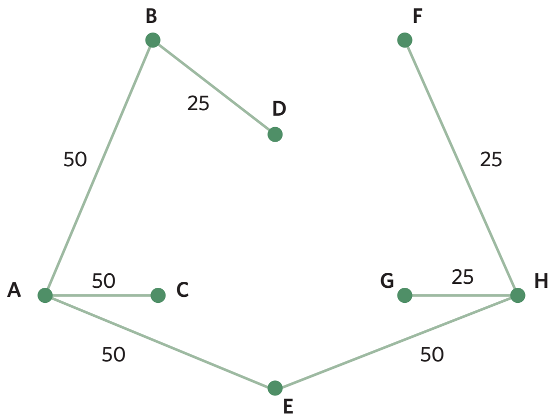
Total weight: 31

10.



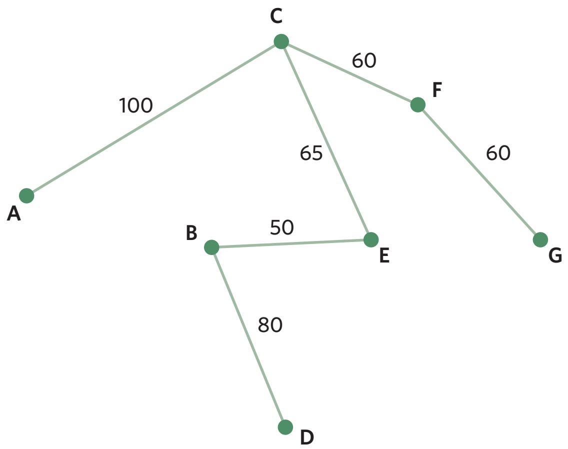
Total weight: 55

11.



Total weight: 275

12. a.



b. 415 m

c. Mine C

Joining it all together

13. a. 8

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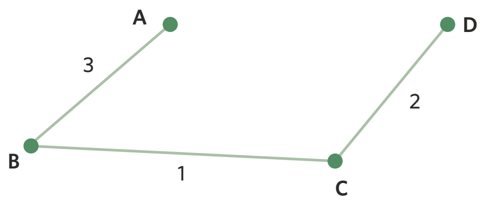
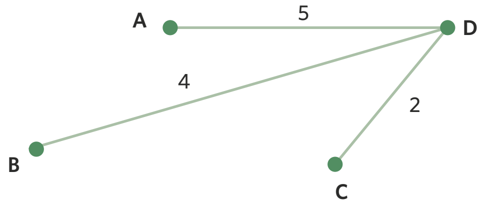
b.



Total weight: 6

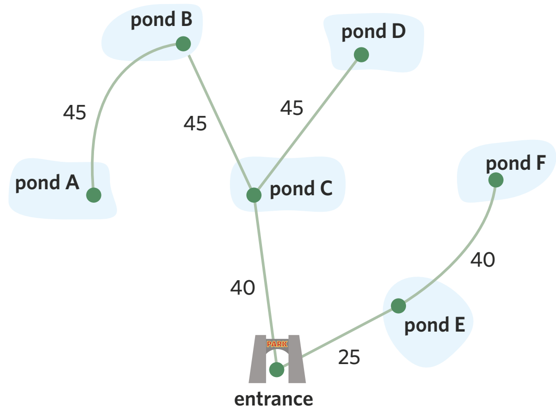
c.



Total weight: 11

14. 53.5 m

15. a.



b. \$28 800

Exam practice

16. Explanation

To answer this question, check if each option is a spanning tree for the network.

A: This is a spanning tree for the network. ✗

B: This is a spanning tree for the network. ✗

C: This is a spanning tree for the network. ✗

D: This is **not** a spanning tree for the network. Although it has all of the features of a spanning tree, the edge connecting vertices 3 and 4 does not exist in the original graph. ✓

E: This is a spanning tree for the network. ✗

Answer

D

17. Explanation

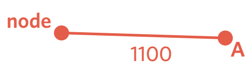
Step 1: Using Prim's algorithm, choose a starting vertex and identify the connecting edge with the lowest weight.

In this case, the node will be chosen, but any vertex can be used.

There are three edges that connect to the node, with weights of 1700, 1100 and 3100.

1100 is the lowest weight, so this edge is chosen.

The chosen edge connects vertex A to the node.



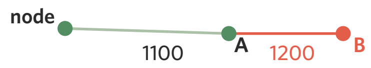
Step 2: Identify the edge with the lowest weight that connects the node or vertex A to a new vertex.

From the node, there are two more edges with weights of 1700 and 3100.

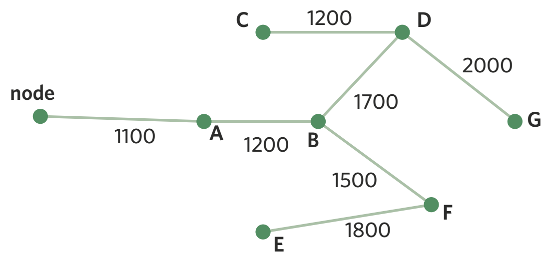
From vertex A, there is one more edge with a weight of 1200.

1200 is the lowest weight, so this edge is chosen.

The chosen edge connects vertex B to vertex A.



Step 3: Continue the process until all vertices are connected.



Step 4: Sum the weights of the edges.

$1100 + 1200 + 1200 + 1700 + 2000 + 1500 + 1800 = 10500$

Answer

B

Kruskal's algorithm could also have been applied to obtain the same answer.

18. a. Explanation

The graph with the lowest weight required to connect all vertices with no loops, cycles or repeated edges is a minimum spanning tree.

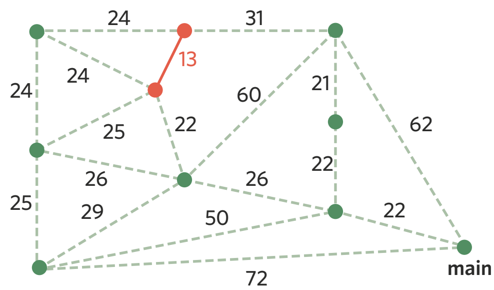
Answer

Minimum spanning tree

b. Explanation

Step 1: Using Kruskal's algorithm, identify the edge with the lowest weight.

13 is the lowest weight, so it is drawn on the graph.



Step 2: Identify the edge with the next lowest weight.

21 is the next lowest weight, so it is drawn on the graph.

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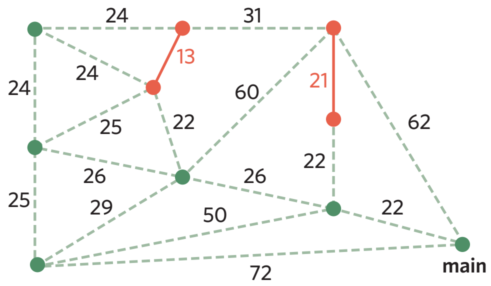
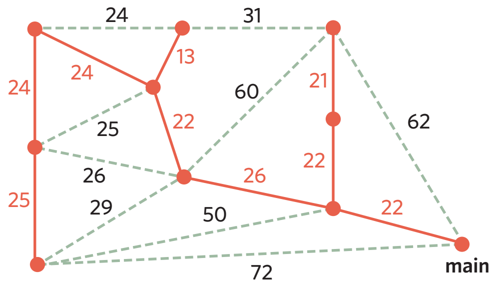


Step 3: Continue the process until all vertices are connected.

Answer



Prim's algorithm could also have been applied to obtain the same answer. There is more than one possible answer.

Questions from multiple lessons

19. D **20.** C

21. **a.** $B_0 = 7100, \quad B_{n+1} = B_n - 170$

b. $B_0 = 7100$

$B_1 = 7100 - 170 = 6930$

$B_2 = 6930 - 170 = 6760$

$B_3 = 6760 - 170 = 6590$

$B_4 = 6590 - 170 = 6420$

$B_5 = 6420 - 170 = 6250$

$B_6 = 6250 - 170 = 6080$

$B_7 = 6080 - 170 = 5910$

7F ANSWERS

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8A Variation

Classifying variation as direct or inverse

1. B

2. **a.** Direct variation **b.** Inverse variation

c. Inverse variation

3. No **4.** C

Constructing rules for variation

5. D

6. **a.** $y = \frac{k}{x}$ **b.** $y = kx^2$

c. $y = \frac{k}{x}$ **d.** $y = k\sqrt{x}$

7. **a.** $W = 12H$ **b.** 250 cm

8. **a.** 3 **b.** $\frac{-1}{5}$

9. **a.** $y = 4$ when $x = 1$. **b.** $k = -10$

10. **a.** $y = -18$ **b.** $y = 54$

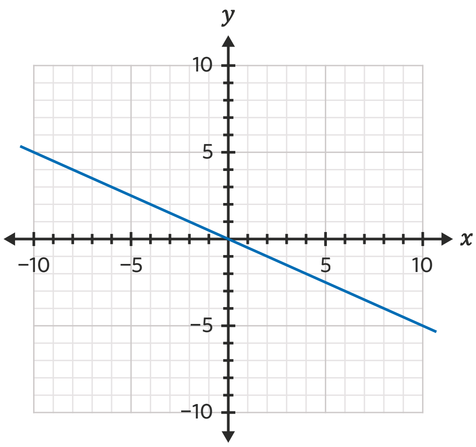
c. $y = \frac{-2}{9}$

11. D **12.** 4 **13.** $E = mc^2$

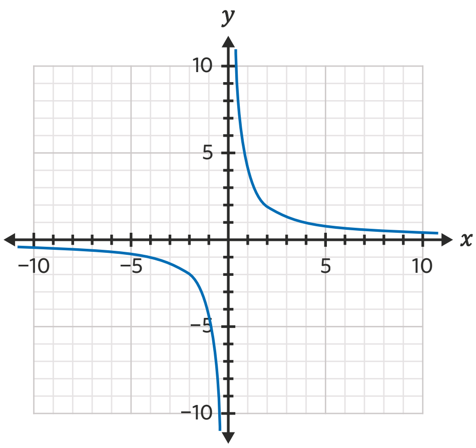
Graphing variation

14. C

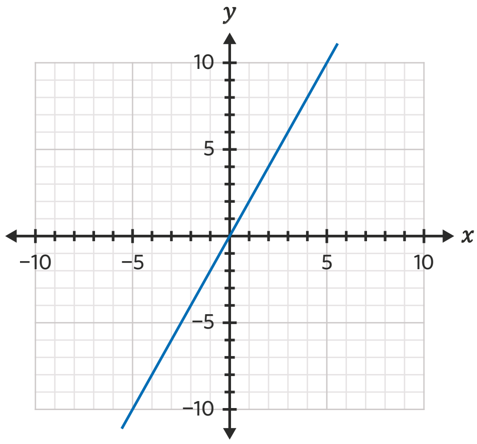
15. **a.**



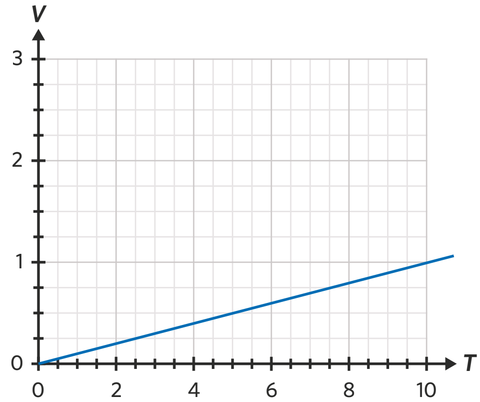
b.



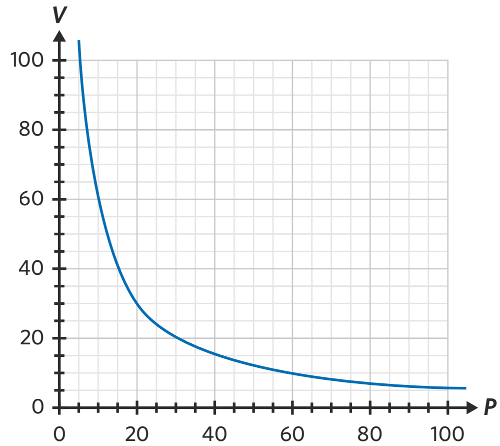
c.



16. **a.**



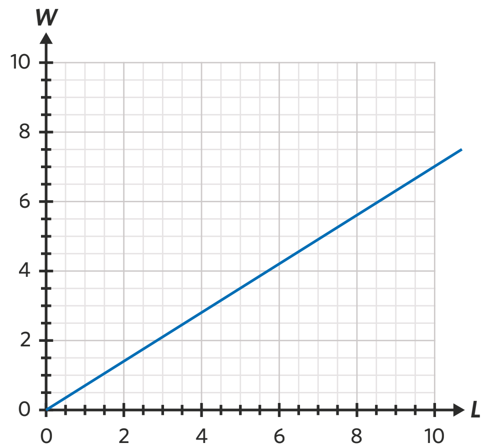
b.



Joining it all together

17. **a.** Directly **b.** $k = 0.7$

c.



d. 3.15 kg **e.** 300 cm

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18. **a.** $C = 11w$ **b.** \$7.70

19. **a.** Inverse variation **b.** $y = -\frac{4}{x}$

c. $-\frac{1}{6}$

Questions from multiple lessons

20. C **21.** B

22. **a.** 1200 mg

b. 20 mg

8B Transformations – kx^2

Applying a square transformation to numerical data

1. B

2. **a.**

x	-3	-1	1	3
x^2	9	1	1	9
y	36	24	19	17

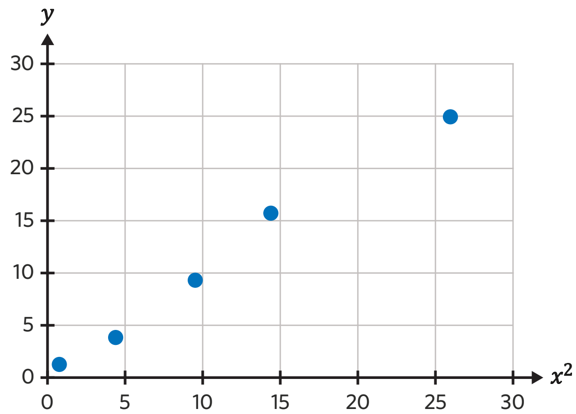
b.

x	-8	-7	-6	-5
x^2	64	49	36	25
y	52	41	36	32

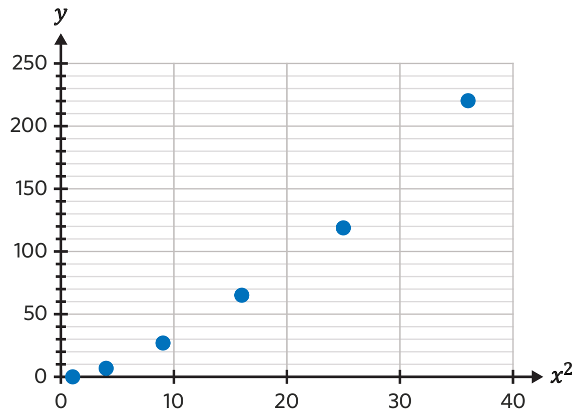
Modelling non-linear data using squared variation

3. A **4.** B

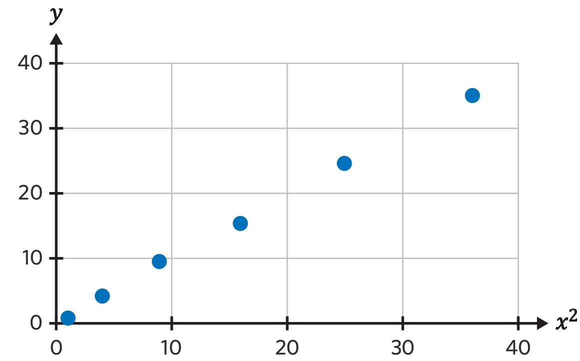
5. **a.**



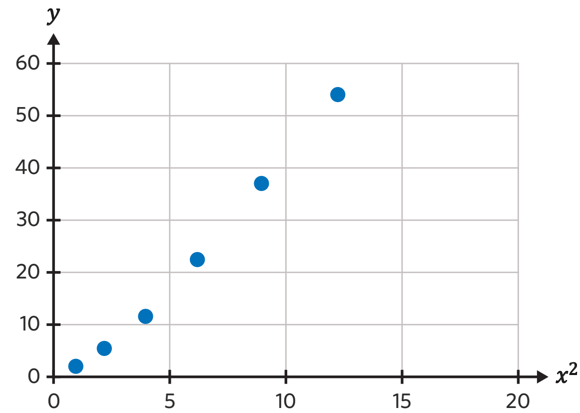
b.



c.



d.

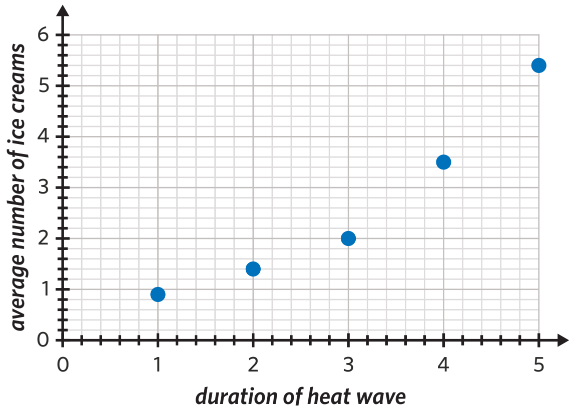


6. $y = 11.75 - 1.75x^2$

Answers may vary.

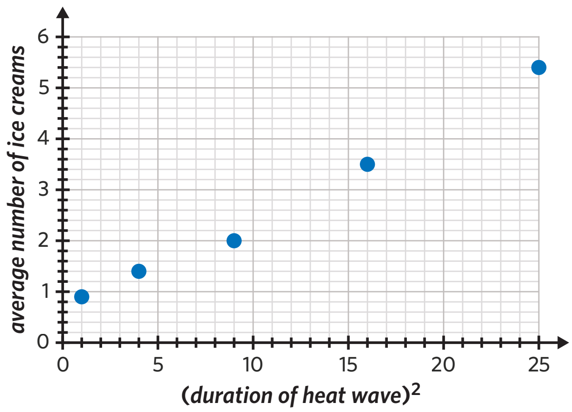
Joining it all together

7. **a.**



b. *duration of heat wave*

c.



d. *average number of ice creams* $= 0.60 + 0.17 \times$ (*duration of heat wave*)^2

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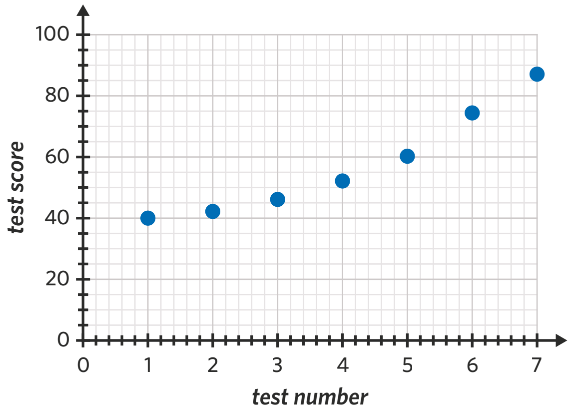
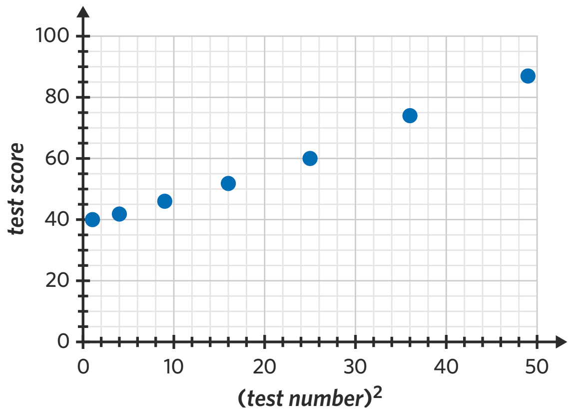
8. a.

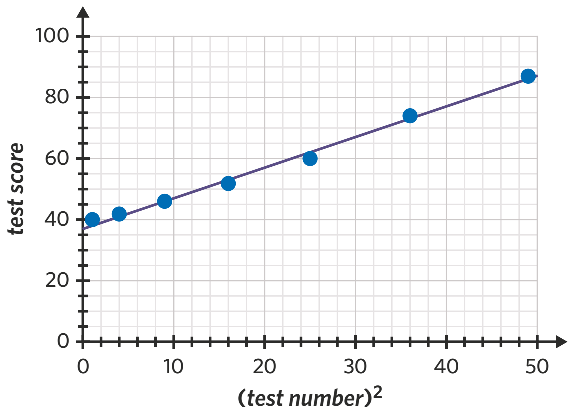


b. *test number*

c.



d.



Answers may vary.

e. *test score* $= 37 + ($*test number*$)^2$

Answers may vary.

Questions from multiple lessons

9. D 10. B

11. a. $h = \sqrt{180^2 - 75^2}$

$= 163.630...$

$= 163.6$ cm

b. 1.636 m

8C Transformations – k/x

Applying a reciprocal transformation to numerical data

1. C

2. a.

x	2	4	6	8
$\frac{1}{x}$	0.5	0.25	0.17	0.13
y	1	8	13	15

b.

x	1	3	5	7
$\frac{1}{x}$	1	0.33	0.2	0.14
y	−3	2	4	5

c.

x	−3	−2	−1	1
$\frac{1}{x}$	−0.33	−0.5	−1	1
y	25	19	15	12

3. a. Alvin has calculated the negative of each x value, rather than the reciprocal.

b. Simon has calculated the reciprocal of each y value, rather than the x values.

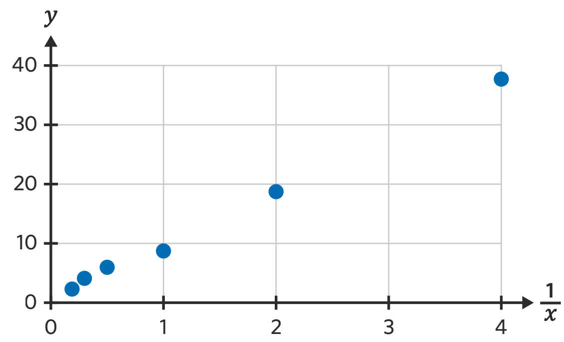
c.

x	1	5	9	13	17
$\frac{1}{x}$	1	0.2	0.11	0.08	0.06
y	1	15	24	30	32

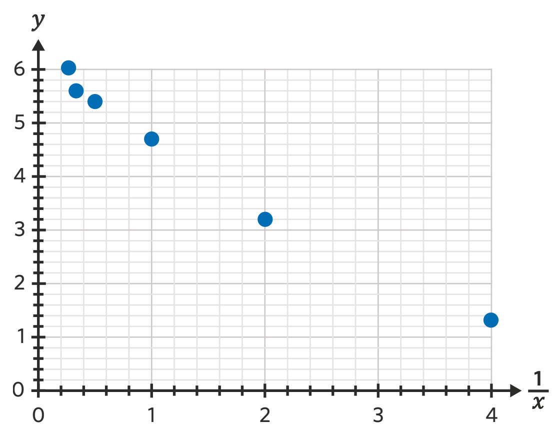
Modelling non-linear data using reciprocal variation

4. D

5. a.



b.



8C ANSWERS

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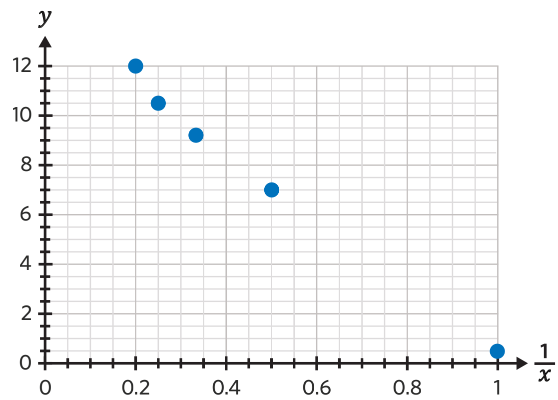
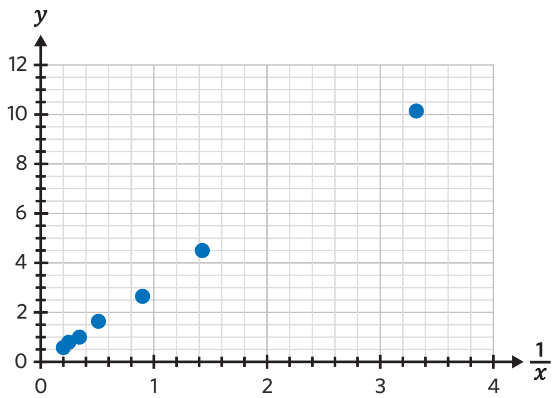
c.



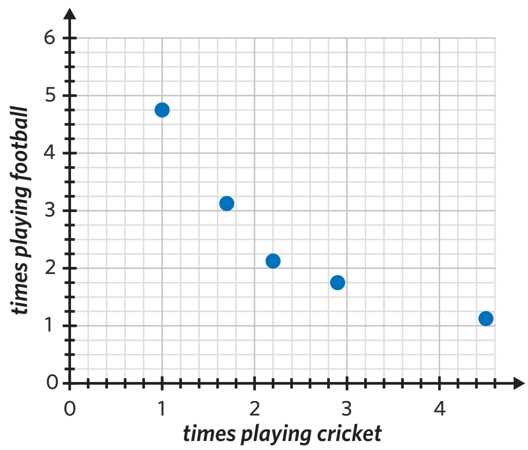
6.



7. $y = 12.0 - 4.5 \times \frac{1}{x}$

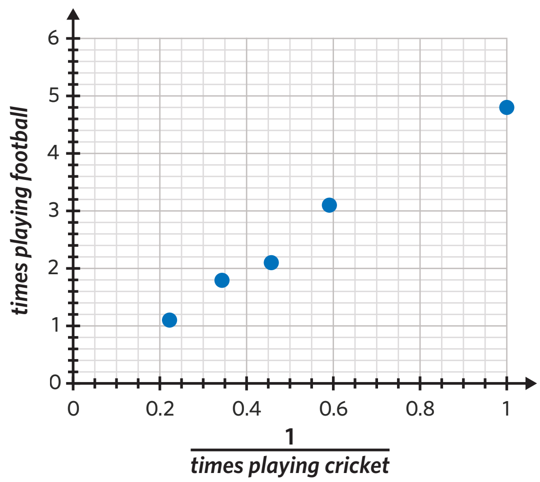
Joining it all together

8. a.



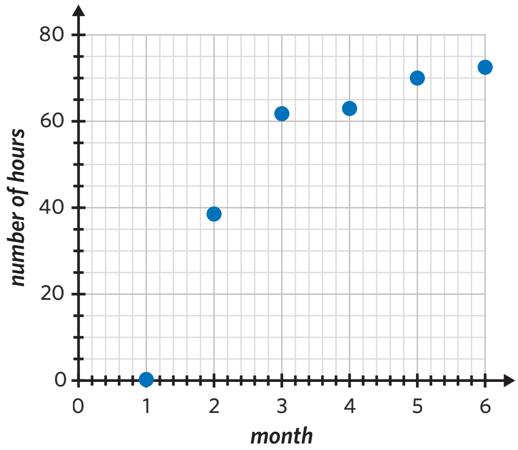
b. *times playing cricket*

c.



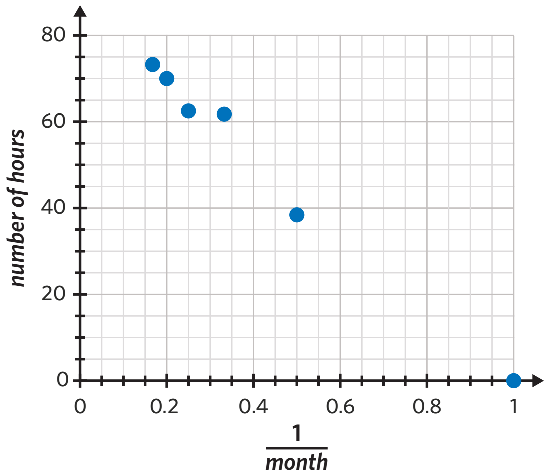
d. $\textit{times playing football} = 0.09 + 4.77 \times \frac{1}{\textit{times playing cricket}}$

9. a.

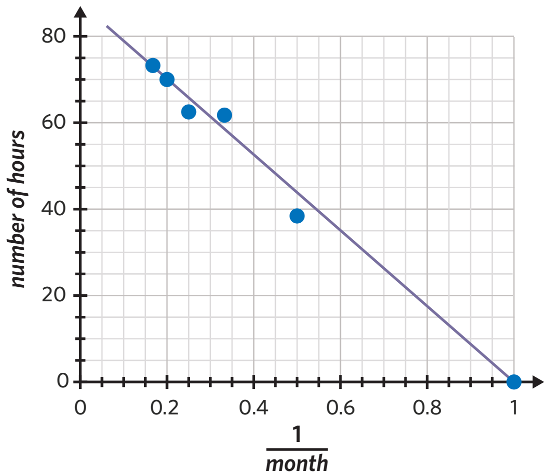


b. *month*

c.



d.



Answers may vary.

e. $\textit{number of hours} = 87.5 - 87.5 \times \frac{1}{\textit{month}}$

Answers may vary.

Questions from multiple lessons

10. D

11. B

12. a. 4489 cm^2

b. 113.4 cm

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8D Transformations – $k\log_{10}(x)$

Performing calculations using logarithms

1. B

2. **a.** 1.53 **b.** 3.02
c. −2.11 **d.** 6.81

3. 10 000

4. **a.** 100 kg **b.** 0.4 kg
c. 125 892.5 kg

5. B

6. 7.6

Applying a logarithmic transformation to numerical data

7. B

8. **a.**

x	1	2	3	4
$\log(x)$	0	0.30	0.48	0.60
y	20	26	29	30

b.

x	5	10	15	20
$\log(x)$	0.70	1	1.18	1.30
y	20	15	13	12

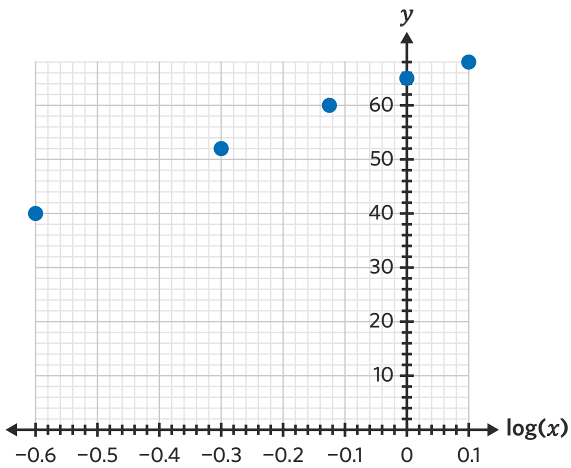
c.

x	10	12	14	16
$\log(x)$	1	1.08	1.15	1.20
y	400	300	260	221

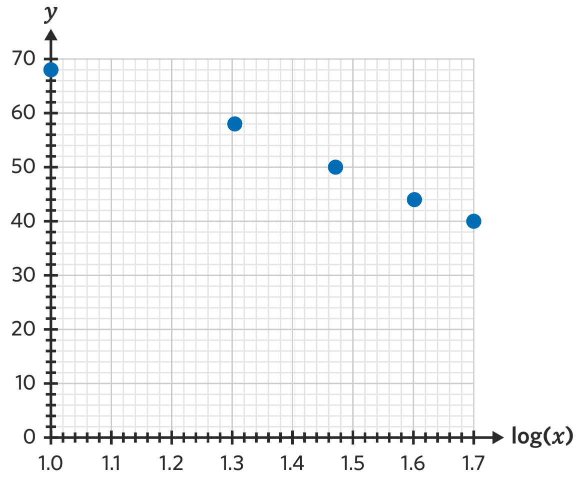
Modelling non-linear data using logarithmic variation

9. B

10. **a.**



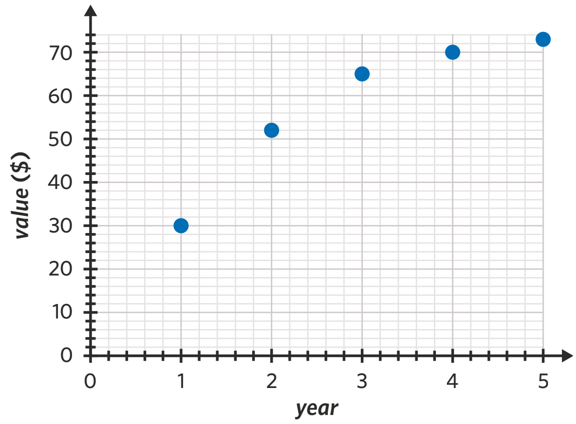
b.



11. $y = 0.78 + 0.95 \times \log(x)$

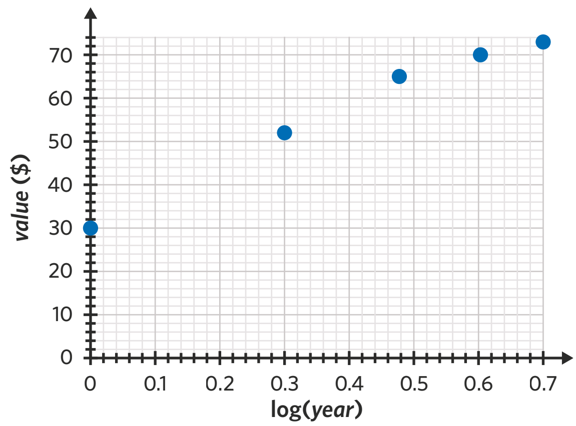
Joining it all together

12. **a.**



b. *year*

c.



d. $value = 32 + 63 \times \log(year)$

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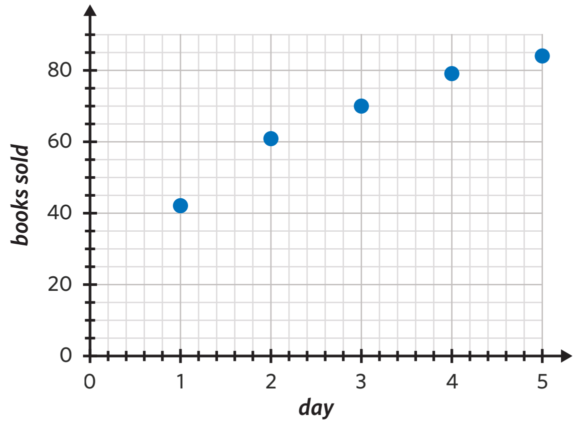
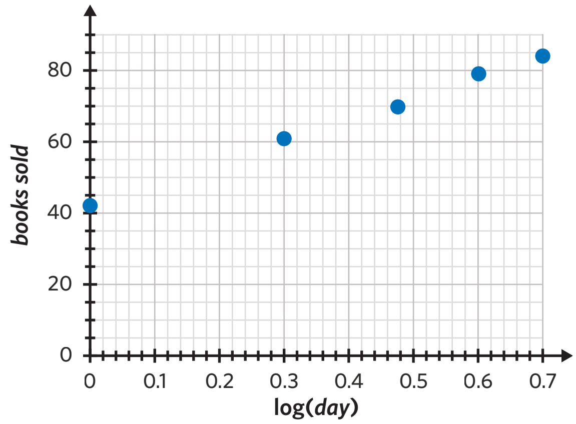
13. a.

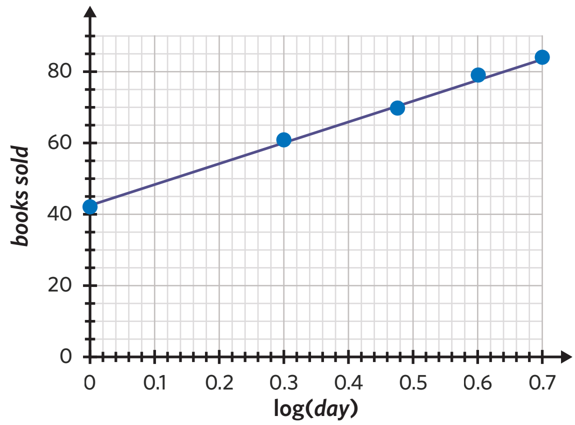


b. *day*

c.



d.



Answers may vary.

e. $books\ sold = 43 + 58 \times \log(day)$

Answers may vary.

Exam practice

14. Explanation

Step 1: Determine the position of the median.

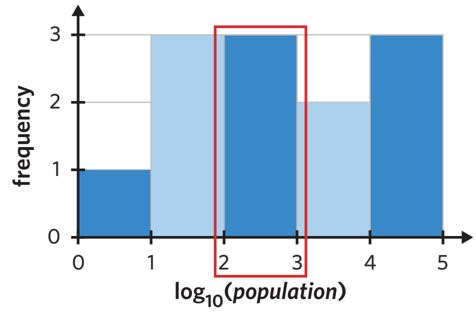
There are 12 data points.

The median is located in the $\left(\frac{n+1}{2}\right)^{\text{th}}$ position.

$\frac{12+1}{2} = 6.5$

Since there are an even number of values, the median is the average of the 6th and 7th values.

Step 2: Identify which interval the median is in.



The median of $\log_{10}(population)$ is between 2 and 3.

Step 3: Raise 10 to the power of 2 and 3.

$10^2 = 100$

$10^3 = 1000$

The median of *population* is between 100 and 1000.

Answer

C

Many students incorrectly chose options A or B because they forgot to convert the log values to actual values.

Questions from multiple lessons

15. B

16. C

17. a. $129L + 26D = 258.80$

b. $1.20

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9A Units of measurement

Identifying appropriate units of measurement

1. D **2.** C **3.** Length

4. **a.** Area **b.** Capacity
c. Length **d.** Angle
e. Volume

Converting units of length

5. B

6. **a.** 0.1 km **b.** 0.00018 km
c. 0.0000386 km **d.** 0.000065 km

7. **a.** 500 mm **b.** 165 000 000 mm
c. 80 500 mm **d.** 237.2 mm

8. 42 125 000 mm

Converting units of area

9. D

10. **a.** 0.015 m^2 **b.** 0.000163 m^2
c. 0.001895 m^2 **d.** 0.0001852 m^2

11. **a.** 2600 mm^2 **b.** 18 300 000 mm^2
c. 160 000 000 mm^2 **d.** 286 000 mm^2

12. $1 605 000

Converting units of volume and capacity

13. A

14. **a.** 16 000 mm^3 **b.** 0.0136 cm^3
c. 60 000 cm^3 **d.** 0.00075 km^3

15. **a.** 0.245 L **b.** 1231 mL
c. 381.262 L **d.** 45 mL

16. **a.** 123.56 mL **b.** 0.1546 L
c. 14 200 000 mL

17. **a.** 2563 cm^3 **b.** 0.03132 m^3
c. 7 670 000 mm^3

18. 0.0192 m^3

Joining it all together

19. **a.** Length **b.** 6897 mm^2
c. Yes

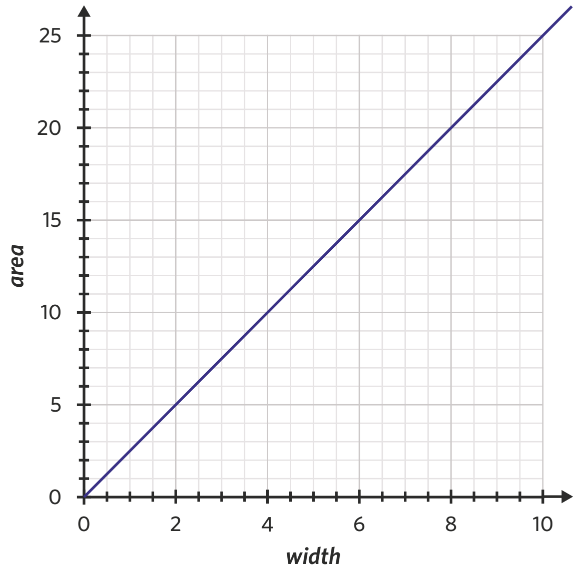
20. **a.** Volume **b.** Desk A: 160 × 80 cm
Desk B: 135 × 95 cm
c. 1 280 000 mm^2

21. **a.** 360 000 cm^3 **b.** 360 L

Questions from multiple lessons

22. B **23.** C

24. **a.** $area = 2.5 \times width$
b.



9B Exact answers, rounding and scientific notation

Approximating values using rounding

1. B

2. **a.** 2 **b.** 13 **c.** 90 **d.** 1000

3. 5 km

4. **a.** 80 **b.** 30 **c.** 90 **d.** 190

5. **a.** **i.** 3.1 **ii.** 5.2
iii. 4.8 **iv.** 8.2
b. **i.** 13.23 **ii.** 234.11
iii. 34.79 **iv.** 67.88

6. $5.36

7. B

8. **a.** 2 **b.** 4 **c.** 3 **d.** 3
e. 3 **f.** 4

9. Emily and An

10. **a.** 237 **b.** 153
c. 6.61 **d.** 9.62
e. 92.8 **f.** 0.00328

Expressing values using scientific notation

11. B **12.** C

13. **a.** 3.28×10^4 **b.** 8.23×10^6
c. 7.82×10^{-4}

9B ANSWERS

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14. a. 5 b. 2 c. 7

15. D

16. 2×10^4 cm

17. a. 354 000 b. 0.00492

c. 0.000087408

Joining it all together

18. 3.3×10^{-5} km

19. 2.7×10^{-9}

20. a. Lipika: nearest 10 metres
Anaya: nearest 100 metres
Holly: nearest metre

b. Larger: nearest 10 metres
Smaller: nearest 100 metres and nearest metre

c. Holly

Exam practice

21. **Explanation**

Step 1: Determine the first three significant figures of the intercept and slope.

Intercept: 42.9842

Slope: 0.877447

Step 2: Round the intercept based on the digit directly following the third significant figure.

8 is between 5 and 9.

Therefore, 42.9842 should be rounded up to 43.0.

Step 3: Round the slope based on the digit directly following the third significant figure.

4 is between 0 and 4.

Therefore, 0.877447 should be rounded down to 0.877.

Answer

E

Some students rounded to three decimal places instead of three significant figures. Additionally, some students did not round 42.9 up to 43.0.

22. **Explanation**

Step 1: Determine the first three significant figures of the intercept and slope.

Intercept: 0.03921

Slope: 5.2756

Step 2: Round the intercept based on the digit directly following the third significant figure.

1 is between 0 and 4.

Therefore, 0.03921 should be rounded down to 0.0392.

Step 3: Round the slope based on the digit directly following the third significant figure.

5 is between 5 and 9.

Therefore, 5.2756 should be rounded up to 5.28.

Answer

$time800 = 0.0392 + 5.28 \times time200$

Common mistakes for this question included students either writing the answer to two significant figures instead of three, or rounding to three decimal places instead of three significant figures. Some students wrote the values in the wrong order.

Questions from multiple lessons

23. D

24. D

25. 5 times stronger

9C Similarity

Identifying similar shapes by eye

1. A

2. a. Similar b. Neither

c. Congruent d. Neither

3. 24 cm

Identifying conditions for similar triangles

4. C

5. C

6. a. Similar b. Similar

c. Not similar d. Similar

7. 28 cm

8. 6 cm

Identifying conditions for other similar shapes

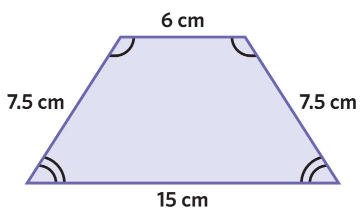
9. C

10. a. Not similar b. Similar

c. Similar d. Similar

11. 25.5 mm

12.



Joining it all together

13. C

14. B

15. No

Exam practice

16. **Explanation**

Step 1: Calculate the linear scale factor using side lengths DE and AB.

$1.8 \div 2.4 = 0.75$

Step 2: Use the linear scale factor to calculate the side length DF from its corresponding side length AC.

$3.6 \times 0.75 = 2.7$

Answer

D

17. **Explanation**

Calculate the linear scale factor of all the corresponding side lengths for each triangle in relation to triangle *M*. The triangles similar to triangle *M* will have three equal linear scale factors.

9C ANSWERS

Triangle N: $3 \div 3 = 1$, $6 \div 4 = 1\frac{1}{2}$, $8 \div 5 = 1\frac{3}{5}$

Triangle O: $4 \div 3 = 1\frac{1}{3}$, $8 \div 4 = 2$, $12 \div 5 = 2\frac{2}{5}$

Triangle P: $6 \div 3 = 2$, $8 \div 4 = 2$, $10 \div 5 = 2$

Triangle Q: $9 \div 3 = 3$, $12 \div 4 = 3$, $15 \div 5 = 3$

Triangles P and Q have three equal linear scale factors.

Answer

E

Questions from multiple lessons

18. D **19.** E

20. **a.** \$150 000

b. \$600 000

c. \$681 818

9D Pythagoras' theorem

Using Pythagoras' theorem in two dimensions

1. C

2. **a.** 8.94 cm **b.** 16.55 m **c.** 21.63 cm

3. **a.** 5.74 mm **b.** 32.50 m **c.** 10.82 m

4. **a.** 12.21 cm **b.** 14.42 mm

5. 300 m **6.** 97.98 m **7.** 6.7 km

8. **a.** 2.69 m **b.** 0.46 m

Using Pythagoras' theorem in three dimensions

9. C

10. **a.** 20.81 m **b.** 23.54 m

11. **a.** 11.31 m **b.** 8.25 m

12. 24.4 cm **13.** 11.83 mm **14.** Box B

Joining it all together

15. **a.** 16.28 cm **b.** 19.1 cm

16. **a.** 2.24 m **b.** 1.6 m

Exam practice

17. **Explanation**

Step 1: Rearrange Pythagoras' theorem to give the value of c.

$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

Step 2: Substitute the values of a and b into the formula.

c is the hypotenuse (the side opposite the right angle) and a and b are the other sides, so let $a = 8$ and $b = 15$.

$c = \sqrt{8^2 + 15^2}$

$= 17$ m

Answer

B

18. **Explanation**

Step 1: Identify the relationship between each side using Pythagoras' theorem.

$AB^2 + AC^2 = BC^2$

Step 2: Rearrange the formula to give the value of AC.

$AC^2 = BC^2 - AB^2$

$AC = \sqrt{BC^2 - AB^2}$

Step 3: Substitute the lengths of BC and AB into the formula.

$AC = \sqrt{25^2 - 15^2}$

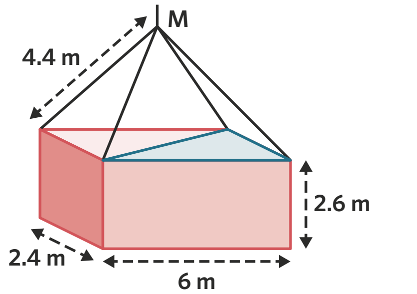
Answer

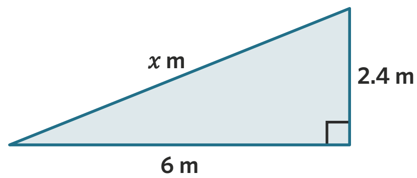
$\sqrt{25^2 - 15^2} = 20$

Some students wrote relevant, but incorrect calculations, such as $25^2 - 15^2 = \sqrt{400} = 20$. Additionally, writing that the equation $AC^2 = 25^2 - 15^2$ is to be solved using technology was not sufficient, as a CAS calculator will provide a positive and negative solution for AC. Using this method, only the positive root is correct and must therefore be specified.

19. **Explanation**

Step 1: Find the length of the base diagonal of the rectangular-based pyramid.



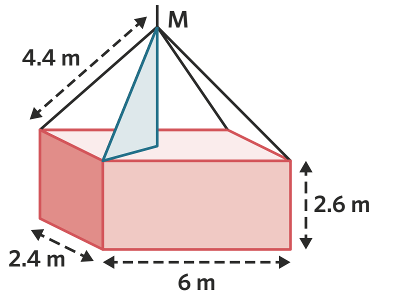


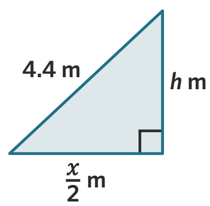
$a^2 + b^2 = c^2$

$x = \sqrt{6^2 + 2.4^2}$

$= 6.4621...$

Step 2: Find the length from M to the top of the container.





The base of this triangle is half the length of the hypotenuse that was found in step 1.

9D ANSWERS

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$$\frac{x}{2} = \frac{6.4621...}{2} = 3.231...$$

$$a^2 = c^2 - b^2$$

$$h = \sqrt{4.4^2 - 3.231...^2}$$

$$= 2.9866...$$

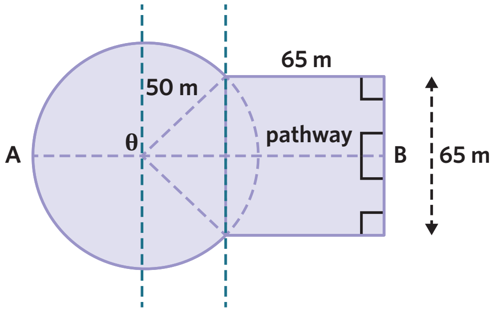
Answer

3 m

20. Explanation

Step 1: Separate the pathway into its components.

The pathway from A to B can be separated into three components.



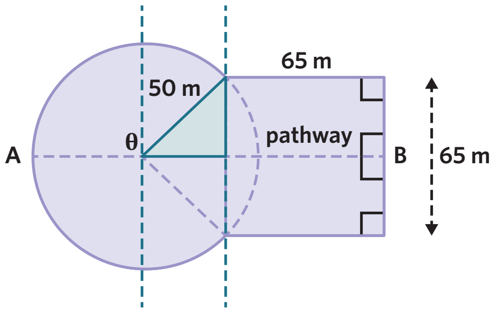
Step 2: Determine the length of the left and right components.

The length of the left component is equal to the radius of the circle, which is 50 m.

The length of the right component is equal to the width of the square, which is 65 m.

Step 3: Determine the length of the middle component.

The middle component is the base of a right-angled triangle, and can be found using Pythagoras' theorem.



$$b = \sqrt{c^2 - a^2}$$

The triangle's hypotenuse is 50 m long and its height is half the height of the square.

$$c = 50$$

$$a = \frac{65}{2} = 32.5$$

$$b = \sqrt{50^2 - 32.5^2}$$

$$= 37.9967...$$

Step 4: Sum the components of the pathway.

$$AB = 50 + 65 + 37.9967...$$

$$= 152.9967...$$

Answer

153 m

Many students struggled with this question, likely due to the overlap between the circle and square. It is important to break down the overall pathway into its solvable parts before making calculations.

Questions from multiple lessons

21. C **22.** D

23. **a.** 216 cm^2

b. 672 cm^2

9E Perimeter

Calculating the perimeter of polygons

1. C

2. **a.** 172 km **b.** 284 cm **c.** 17 m **d.** 44 mm

e. 113 m

3. $8820 **4.** $77.50 **5.** 450 m

Calculating the perimeter of circles

6. $P = 2 \times \pi \times r$

$= 2 \times \pi \times 3$

$= 18.85$ cm

7. **a.** 43.982 cm **b.** 14.137 cm

c. 56.549 cm

8. **a.** 24.61 cm **b.** 70.37 mm

c. 16.37 km **d.** 6.660 km

9. **a.** 24.13 cm **b.** 151.61 cm

10. 155.19 cm **11.** 81.05 cm

Calculating the perimeter of composite shapes

12. $P = 8 + 5 + 10 + 12 + 13$

$= 48$ cm

13. **a.** 36 cm **b.** 40 cm

c. 81.27 cm

14. 30 cm **15.** 22.94 m

Joining it all together

16. **a.** 12 m **b.** 42.850 m

17. 27.96 km

Exam practice

18. Explanation

Step 1: Determine the length of all sides.

$l = 24.43$ cm

Step 2: Calculate the perimeter.

$P = 27.43 + 27.43 + 27.43 + 27.43$

$= 109.72$ cm

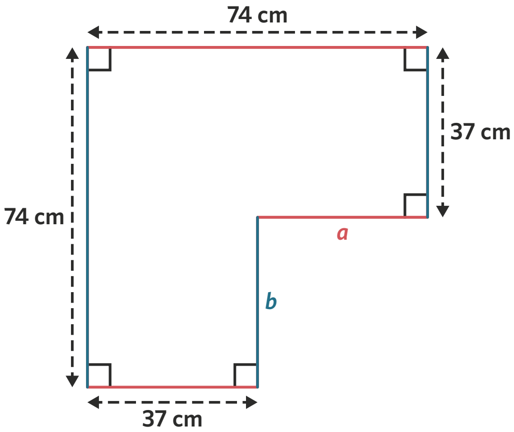
Answer

D

19. Explanation

Step 1: Determine the length of all sides.

Let a and b be the unknown lengths.



$74 = a + 37$

$a = 37$ cm

$74 = b + 37$

$b = 37$ cm

Step 2: Calculate the perimeter.

$P = 74 + 37 + 37 + 37 + 37 + 74$

$= 296$ cm

Answer

296 cm

20. Explanation

Step 1: Calculate the perimeter of the square section.

The perimeter is the sum of the 3 sides of the square.

$65 + 65 + 65 = 195$ m

Step 2: Calculate the perimeter of the circle.

The perimeter is the length of the arc.

$r = 50$ m

$\theta = 360° - 81°$

$= 279°$

$l = 2 \times \pi \times 50 \times \frac{279°}{360°}$

$= 243.473...$ m

Step 3: Calculate the total perimeter.

$P = 195 + 243.473...$

$= 438.473...$ m

Answer

438 m

A number of students struggled to calculate the arc length. This is likely because the question used θ to define the angle as 81°. In this case, the angle created by the arc is $360° - 81°$.

Questions from multiple lessons

21. C **22.** C

23. a. 8.33 m

b. 8.57 m

9F Area

Calculating the area of quadrilaterals

1. B

2. a. 48 km^2 **b.** 42 mm^2

c. 150 cm^2 **d.** 120 cm^2

3. 628 m

Calculating the area of circles and sectors

4. B

5. a. 452.39 cm^2 **b.** 962.11 km^2

c. 5410.52 cm^2 **d.** 124.19 mm^2

6. a. 1017.88 cm^2 **b.** 45°

c. 127.23 cm^2

Calculating the area of triangles

7. C

8. a. 14 cm^2 **b.** 27.71 mm^2

c. 299 cm^2 **d.** 20.40 cm^2

9. Park B

Calculating the area of composite shapes

10. D

11. a. 32 m^2 **b.** 73.5 cm^2

c. 50 km^2 **d.** 200.51 mm^2

12. 32 cm^2 **13.** 518.7 cm^2

14. 1490 cm^2

Joining it all together

15. 2860.8 mm^2 **16.** Fran; 1.73 mm^2

17. 59%

18. a. 3019.07 cm^2 **b.** 13 750 cm^2

c. 1.22 m^2 **d.** 2.90 m^2

Exam practice

19. Explanation

Step 1: Determine the base and height of the triangle.

The base of the triangle is the left side length of the flag.

$b = 2$ m

The height is perpendicular to the base. The height is half the length of the flag.

$h = 3 \div 2$

$= 1.5$ m

Step 2: Substitute the known values into the formula for the area of a triangle.

$$A = \frac{1}{2} \times b \times h$$
$$= \frac{1}{2} \times 2 \times 1.5$$
$$= 1.5 \text{ m}^2$$

Answer

B

20. Explanation

Step 1: Determine the area of the circle.

The pie chart has a radius of 16 mm.

$$A = \pi \times r^2$$
$$= \pi \times 16^2$$
$$= 804.24... \text{ mm}^2$$

Step 2: Determine the area of the sector representing 'agree'.

80% of people selected 'agree'.

$$\frac{80}{100} \times 804.24... = 643.39... \text{ mm}^2$$

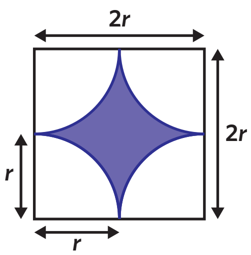
Answer

D

A number of students did not read the question carefully enough. 22% of students incorrectly chose option B, which is the area of the sector representing 'disagree' rather than the sector representing 'agree'.

21. Explanation

Step 1: Construct a square with its four corners at the centre of each circle.



Step 2: Determine the area of the square.

$$A = b \times h$$
$$= 2r \times 2r$$
$$= 4r^2$$

Step 3: Determine the area of the four quarter-circles.

$$A = 4 \times \frac{1}{4} \times \pi r^2$$
$$= \pi r^2$$

Step 4: Write an expression for the area of the shaded region.

$$A = 4r^2 - \pi r^2$$

Answer

B

22. Explanation

Step 1: Calculate the area of the entire sector (from L to M).

$$\theta = 360° - 100° = 260°$$
$$r = 12 \text{ m}$$
$$A = \frac{\theta}{360°} \times \pi \times r^2$$
$$= \frac{260°}{360°} \times \pi \times 12^2$$
$$= 326.725... \text{ m}^2$$

Step 2: Calculate the area of the smaller sector (from L to N).

$$\theta = 260°$$
$$r = 12 - 4.5$$
$$= 7.5 \text{ m}$$
$$A = \frac{\theta}{360°} \times \pi \times r^2$$
$$= \frac{260°}{360°} \times \pi \times 7.5^2$$
$$= 127.627... \text{ m}^2$$

Step 3: Calculate the shaded area.

$$A = 326.725... - 127.627...$$
$$= 199.098... \text{ m}^2$$

Answer

199 m^2

Many students used a central angle of 100° rather than 260°, or found the areas of entire circles, rather than sectors.

Questions from multiple lessons

23. C **24.** E

25. a. *volume* = 1000 − 60 × *time*

b. 10 hours and 40 minutes

9G Volume

Calculating the volume of prisms and cylinders

1. C

2. a. 384 cm^3 **b.** 675 mm^3

c. 721 cm^3

3. 729 mm^3

4. 6371.15 cm^3

5. a. 730.03 cm^3 **b.** 199 mL

Calculating the volume of tapered solids

6. C

7. a. 90 m^3 **b.** 238.33 cm^3

8. a. 100.53 m^3 **b.** 1675.52 m^3

9. 561.2 mm^3 **10.** 136 m

Calculating the volume of spheres

11. C

12. a. 4188.79 cm^3 **b.** 69.46 cm^3

13. 6.97 cm **14.** Earth

15. a. 6370.63 mL **b.** 867.6 cm^3

Calculating the volume of composite solids

16. B

17. a. 1884.96 cm^3 **b.** 1544.62 mm^3

18. 3.845 m^3 **19.** 416.78 cm^3 **20.** 12.03 cm

Joining it all together

21. a. 170 mL **b.** 262 cm^3

22. a. 13.6 cm **b.** 77.5%

Exam practice

23. Explanation

Step 1: Identify the appropriate formula.

The squash ball is spherical.

$V = \frac{4}{3}\pi r^3$

Step 2: Substitute known values into the formula and evaluate.

Answer

$r = 2 \text{ cm}$

$V = \frac{4}{3} \times \pi \times 2^3$

$= 33.5103...$

$\approx 33.51 \text{ cm}^3$

'Show that...' questions require students to show their full calculations in order to receive the mark. The value that is being shown cannot be included in the calculations until the final line.

24. Explanation

Step 1: Compare the equations for the volume of a cone and a cylinder.

Cone: $V = \frac{1}{3}\pi r^2 h$

Cylinder: $V = \pi r^2 h$

The volume of a cone is a third of the volume of a cylinder with the same radius and height.

Step 2: Determine the relationship between the heights of a cone and cylinder with an equal volume and radius.

If a cone and cylinder have the same volume and radius, the height of the cone must be three times the height of the cylinder.

Step 3: Determine the height of the cylinder.

Cone: $h = 12 \text{ cm}$

Cylinder: $h = 12 \div 3 = 4 \text{ cm}$

Answer

A

Questions from multiple lessons

25. B **26.** E

27. a. Line 3

b. 3

9H Surface area

Calculating the surface area of solids with planar faces

1. C **2.** 9

3. a. 108 cm^2 **b.** 8.25 m^2

c. 67.25 mm^2 **d.** 868 km^2

4. 8 cm **5.** $85\,840 \text{ m}^2$

Calculating the surface area of solids with curved faces

6. A

7. a. 75.40 m^2 **b.** 452.39 cm^2

c. 1743.58 mm^2 **d.** 307.62 cm^2

8. $10\,857 \text{ cm}^2$ **9.** 20 waffle cones

Calculating the surface area of composite solids

10. A

11. a. 416 cm^2 **b.** 612.61 cm^2

c. 114.85 m^2 **d.** 1140 mm^2

12. a. 5.83 m **b.** 503 m^2

13. 157.54 cm^2

Joining it all together

14. D **15.** Vase B; 20.92 cm^2

16. 975.93 cm^2

Exam practice

17. Explanation

Step 1: Identify all the faces.

The solid is a cube, so there are six $4.1 \text{ cm} \times 4.1 \text{ cm}$ square faces.

Step 2: Sum the areas of the faces.

$SA = 6(4.1 \times 4.1)$

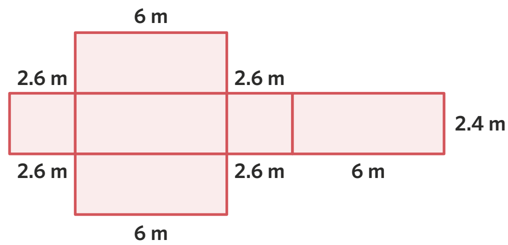
$= 100.86 \text{ cm}^2$

Answer

100.86 cm^2

18. Explanation

Step 1: Identify all the faces.



There are two $6 \text{ m} \times 2.6 \text{ m}$ faces, two $2.6 \text{ m} \times 2.4 \text{ m}$ faces and two $6 \text{ m} \times 2.4 \text{ m}$ faces.

Step 2: Sum the areas of the faces.

$SA = 2(6 \times 2.6) + 2(2.6 \times 2.4) + 2(6 \times 2.4)$

$= 31.2 + 12.48 + 28.8$

$= 72.48 \text{ m}^2$

Answer

72.48 m^2

Many students who answered this question incorrectly calculated four 6×2.6 faces instead of two 6×2.6 faces and two 6×2.4 faces. This would have produced an incorrect answer of 74.88 m^2.

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19. Explanation

Step 1: Calculate the diagonal length using Pythagoras' theorem.

$6^2 + 8^2 = c^2$

$c^2 = 36 + 64$

$c = 10$ cm

Step 2: Identify all the faces.

There are two triangular faces with a base of 6 cm and a height of 8 cm. There is also one 8 cm × 6 cm rectangular face, one 10 cm × 6 cm rectangular face and one 6 cm × 6 cm square face.

Step 3: Sum the areas of the faces.

$$SA = 2\left(\frac{6 \times 8}{2}\right) + (8 \times 6) + (10 \times 6) + (6 \times 6)$$
$$= 48 + 48 + 60 + 36$$
$$= 192 \text{ cm}^2$$

Answer

C

24% of students incorrectly chose option A, likely due to the misconception that cutting an object in half reduces its surface area by half.

20. Explanation

Step 1: Identify all the faces.

There is one dome and one circular face, both with a radius of 5 cm.

Step 2: Sum the areas of the faces.

The surface area of the dome can be found by halving the formula for the surface area of the sphere.

$$SA = 2\pi \times 5^2 + \pi \times 5^2$$
$$= 235.619... \text{ cm}^2$$

Answer

C

41% of students incorrectly chose option B. The question states that both the top and the base of the dessert are covered in chocolate. These students didn't calculate the area of the circular base.

Questions from multiple lessons

21. A **22.** D

23. a. $d = \sqrt{5^2 + 4.8^2} = 6.931... \approx 6.9$

b. 31.7 cm

c. 59 cm^2

91 Scale factor

Using scale factors to scale area

1. B

2. a. 2 **b.** 4 **c.** 84 cm^2

3. a. 28 cm^2 **b.** 158.06 cm^2 **c.** 2450.5 mm^2

4. a. 2

b.
$$\textit{area of image} = k^2 \times \textit{area of original}$$
$$= 2^2 \times 623.7$$
$$= 2494.8 \text{ cm}^2$$

5. 39 187.5 cm^2 **6.** 320 cm

Using scale factors to scale volume

7. D

8. a. 0.5 **b.** 0.125 **c.** 4.5 cm^3

9. a. 41.16 cm^3 **b.** 1673.55 cm^3 **c.** 338.29 mm^3

10. a. 0.295 **b.** 6.5 cm

11. 512 000 cm^3

Joining it all together

12. 9804.7 cm^3

13. a. 12.5 mL **b.** 62.8 cm^2

14. a. 1.875 **b.** 421 875 cm^3 **c.** 33 750 cm^2

Exam practice

15. Explanation

Step 1: Determine the linear scale factor, k.

The area scale factor, k^2, is 9.

$k^2 = 9$

$k = \sqrt{9}$

$= 3$

Step 2: Multiply the original length by the linear scale factor.

$$\textit{length of image} = k \times \textit{length of original}$$
$$= 3 \times 12$$
$$= 36$$

Answer

D

46% of students multiplied the length by the area scale factor, rather than first finding the linear scale factor. This resulted in an answer of 108 cm (E). Students also needed to ensure that they were multiplying by the scale factor, not dividing.

16. Explanation

Step 1: Determine the linear scale factor, k.

The volume scale factor, k^3, is 8.

$k^3 = 8$

$k = \sqrt[3]{8}$

$= 2$

Step 2: Multiply the length of the small storage box by the linear scale factor.

$$\textit{length of image} = k \times \textit{length of original}$$
$$= 2 \times 40$$
$$= 80$$

Answer

80 cm

Many students did not find the linear scale factor and instead multiplied the length of the small storage box by 8 to give an answer of 320 cm.

17. Explanation

Step 1: Determine the linear scale factor, k.

The edge length of each black flag is twice the edge length of each white flag.

$k = 2$

Step 2: Calculate the area scale factor, k^2.

$k^2 = 4$

Step 3: Determine the total area of the black flags with respect to the area of a white flag using the area scale factor.

There are 4 black flags.

The area of 4 black flags is equal to the area of $4 \times 4 = 16$ white flags.

Step 4: Express the total area of the black flags in relation to the area of the white flags.

The black flags are equal to 16 white flags.

There are 3 white flags.

$$\frac{\text{total area}_{black}}{\text{total area}_{white}} = \frac{16}{3}$$

Answer

D

30% of students incorrectly answered B. Many of these students did not account for the fact that there are 4 black flags and only 3 white flags in the diagram.

18. Explanation

Step 1: Determine the linear scale factor, k.

The area scale factor, k^2, is 4.

$$k^2 = 4$$
$$k = \sqrt{4}$$
$$= 2$$

Step 2: Multiply the height of the original logo by the linear scale factor.

The height of the logo is given in the side length of the triangles that make it up.

The height of the original logo is 4.8 cm.

$$\begin{aligned} \textit{length of image} &= k \times \textit{length of original} \\ &= 2 \times 4.8 \\ &= 9.6 \end{aligned}$$

Answer

9.6 cm

Many students did not convert the area scale factor to a linear scale factor before multiplying the side length, leading to an answer of 19.2 cm.

Questions from multiple lessons

19. D **20.** E

21. a. 3.27×10^{-3}

b. 192.57 mL

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10A Using trigonometry

Calculating side lengths of right-angled triangles using trigonometry

1. $\sin(\theta) = \dfrac{opposite}{hypotenuse}$

$\cos(\theta) = \dfrac{adjacent}{hypotenuse}$

$\tan(\theta) = \dfrac{opposite}{adjacent}$

2. B

3. 3.7 cm

4. **a.** $\cos(52) = \dfrac{31}{x}$

50.35 mm

b. $\tan(27) = \dfrac{x}{5}$

2.55 cm

c. $\sin(55) = \dfrac{1.32}{x}$

1.61 m

5. 92 m

6. 8 m

Calculating angles in right-angled triangles using trigonometry

7. B

8. **a.** $\theta = \tan^{-1}\left(\dfrac{5.5}{3.5}\right)$

57.53°

b. $\theta = \sin^{-1}\left(\dfrac{2.5}{4.3}\right)$

35.55°

c. $\theta = \cos^{-1}\left(\dfrac{11.7}{39.5}\right)$

72.77°

9. 44°

Joining it all together

10. 4.95 cm

11. 43°

12. **a.** 1.5 m **b.** 37°

13. **a.** 1.96 m **b.** 66.25 m

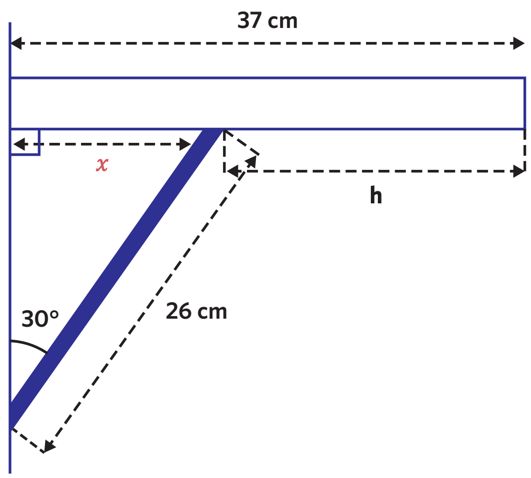
Exam practice

14. Explanation

Step 1: Identify what needs to be calculated.

To calculate h, the remaining depth of the shelf will need to be calculated.

Label this distance 'x'.



Step 2: Identify the trigonometric ratio to be used.

The 26 cm side length is the hypotenuse.

The unknown x side length is opposite to the reference angle.

The trigonometric ratio that uses the opposite and hypotenuse side lengths is sin.

Step 3: Substitute the known values into the trigonometric equation and solve using a calculator.

$\sin(\theta) = \dfrac{opposite}{hypotenuse}$

$\sin(30) = \dfrac{x}{26}$

$x = 13$

Step 4: Subtract x from the total depth of the shelf.

$$\begin{aligned} h &= 37 - x \\ &= 37 - 13 \\ &= 24 \end{aligned}$$

Answer

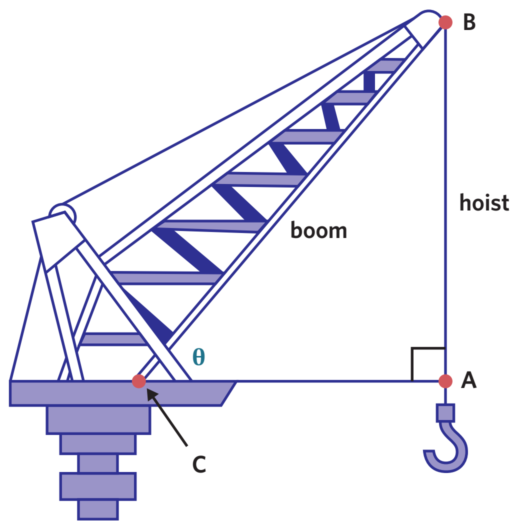
24 cm

15. Explanation

Step 1: Identify what needs to be calculated.

When labelling an angle using its vertices, the middle vertex is where the angle is located.

Therefore, angle ACB is located at vertex C.
Label this angle θ.



Step 2: Identify the trigonometric ratio to be used.

The 25 m side length (the boom) is the hypotenuse.

The 15 m side length (the hoist) is opposite to the unknown reference angle.

The trigonometric ratio that uses the opposite and hypotenuse side lengths is sin.

Step 3: Substitute the known values into the trigonometric equation and evaluate using a calculator.

Since an angle is being calculated, use the inverse function.

$$\begin{aligned} \theta &= \sin^{-1}\left(\dfrac{opposite}{hypotenuse}\right) \\ &= \sin^{-1}\left(\dfrac{15}{25}\right) \\ &= 36.869... \end{aligned}$$

Answer

37°

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Questions from multiple lessons

16. C **17.** C

18. a. $12.73

b. $1018

10B Applications of trigonometry

Identifying and using angles of elevation

1. A

2. a. 38.66° **b.** 11.89°

3. 20.56° **4.** 830.97 m **5.** 61.4 m

Identifying and using angles of depression

6. A

7. a. 48.01° **b.** 8.62°

8. 61.15 m **9.** 5.7° **10.** 3.69 m **11.** 602.6 m

Joining it all together

12. a. 2.70° **b.** 169.81 m

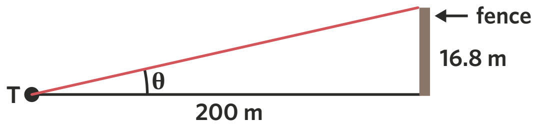
c. 158 m

13. 20.9 m

Exam practice

14. Explanation

Step 1: Complete the diagram and identify the angle of elevation.



Step 2: Identify the trigonometric ratio that can be used to calculate the angle of elevation.

The 16.8 m side length is opposite the unknown angle of elevation.

The 200 m side length is adjacent to the unknown angle of elevation.

Therefore, the tan ratio will be used.

Step 3: Substitute the known side lengths into the equation and evaluate the angle of elevation.

$$\theta = \tan^{-1}\left(\frac{opposite}{adjacent}\right)$$
$$= \tan^{-1}\left(\frac{16.8}{200}\right)$$
$$= 4.801...°$$

Answer

5°

Students needed to ensure that they rounded their answer to the nearest degree, as asked.

Questions from multiple lessons

15. E **16.** D

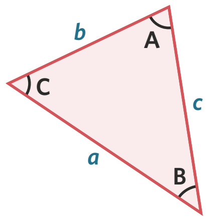
17. a. $60 - 42 = 18$

b. 132 goals

10C The sine rule

Calculating side lengths of a triangle using the sine rule

1.



2. a. $\frac{x}{\sin(32°)} = \frac{7}{\sin(78°)}$ **b.** 3.79 cm

3. a. 7.08 cm **b.** 32.46 m

c. 16.31 mm

4.
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$
$$\frac{x}{\sin(40°)} = \frac{90}{\sin(86°)}$$
$$x = 57.99...$$
$$\approx 58 \text{ m}$$

Calculating angles in a triangle using the sine rule

5. B

6. a. $\theta = \sin^{-1}\left(6 \times \frac{\sin(99°)}{43}\right)$ **b.** 7.9°

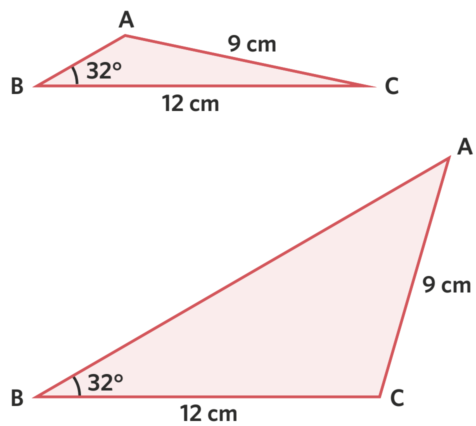
7. a. 75.19° **b.** 83.22°

c. 62.77°

Solving the ambiguous case of the sine rule

8. C **9.** 119°

10. a.



b. 44.96° and 135.04° **c.** 12.96° and 103.04°

11. 17.7 cm and 28.3 cm

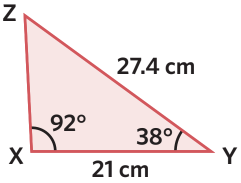
Calculating the area of a triangle using the sine rule

12. $A = \frac{1}{2} \times 11 \times 10 \times \sin(40°)$

13. **a.** 19.70 cm^2 **b.** 147.48 cm^2

c. 282.04 m^2

14.



177.13 cm^2

15. 37.31°

Joining it all together

16. 118.3°

17. **a.** MLN or NLM **b.** 32.24°

c. 45.76° **d.** 8.06 m

18. 619.8 m

Exam practice

19. Explanation

Step 1: Determine the values of each variable.

a and b are side lengths and C is their included angle.

$a = 18$ cm

$b = 26$ cm

$C = 30°$

Step 2: Substitute the values into the area formula and evaluate.

$$A = \frac{1}{2}ab \times \sin(C)$$
$$= \frac{1}{2} \times 18 \times 26 \times \sin(30°)$$
$$= 117 \text{ cm}^2$$

Answer

A

8% of students incorrectly chose option D because they forgot to multiply by $\frac{1}{2}$ when using the formula.

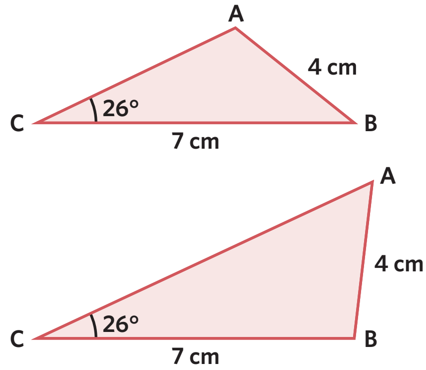
20. Explanation

Step 1: Interpret the information provided.

- Two side lengths are known.
- One non-included acute angle is known.
- The shorter of the two known sides is opposite the known angle.

This information means that the ambiguous case of the sine rule can be applied.

Step 2: Draw the possible triangles.



Step 3: Calculate the value of angle CAB for both triangles.

Triangle 1:

$$A = \sin^{-1}\left(\frac{a \times \sin(B)}{b}\right)$$
$$CAB = \sin^{-1}\left(\frac{7 \times \sin(26°)}{4}\right)$$
$$= 50.098...°$$

Triangle 2:

$$CAB = 180 - 50.098...$$
$$= 129.901...°$$

Step 4: Calculate the value of angle ABC for both triangles.

Triangle 1:

$$ABC = 180 - 26 - 50.098...$$
$$= 103.901...°$$

Triangle 2:

$$ABC = 180 - 26 - 129.901...$$
$$= 24.098...°$$

Step 5: Identify all possible angles in triangle ABC, rounded to the nearest degree.

50°, 130°, 104°, 24°

Answer

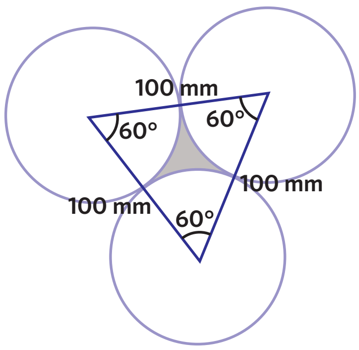
E

The key to this question was to understand that the only way two remaining unknown angles could account for four values in a multiple choice question was if the triangle followed the ambiguous case of the sine rule.

21. Explanation

Step 1: Draw a triangle connecting the centre of each circle.

The triangle will have 100 mm side lengths (double the radius of the circles) and angles of 60° (equilateral).



10C ANSWERS

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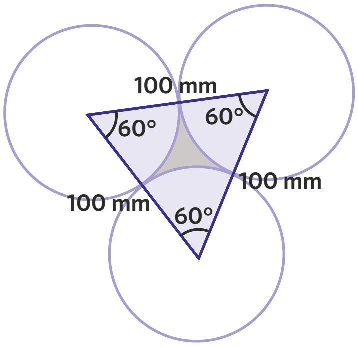
Step 2: Calculate the area of the triangle using the sine rule area formula.

Any side length or angle values can be used.

$$A = \frac{1}{2}ab \times \sin(C)$$
$$= \frac{1}{2} \times 100 \times 100 \times \sin(60°)$$
$$= 4330.127...\text{ mm}^2$$

Step 3: Calculate the area of the circle sectors within the triangle.



1 sector:

$$A = \frac{60°}{360°} \times \pi \times 50^2$$
$$= 1308.996...\text{ mm}^2$$

3 sectors:

$$A = 3 \times 1308.996...$$
$$= 3926.990...\text{ mm}^2$$

Step 4: Subtract the area of the sectors from the area of the triangle.

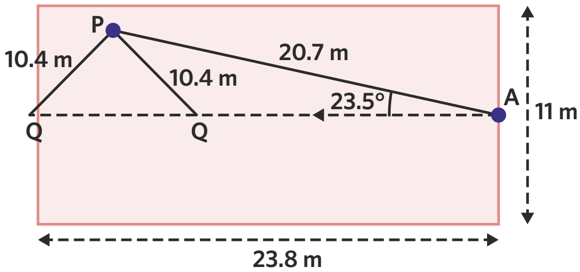
$$4330.127... - 3926.990... = 403.136...\text{ mm}^2$$

Answer

A

22. a. Explanation

Step 1: Complete the triangle drawing.



Step 2: Identify two angle/side length pairs.

The unknown angle AQP pairs with the 20.7 m side length.

$A = AQP$

$a = 20.7$ m

The 23.5° angle pairs with the 10.4 m side length.

$B = 23.5°$

$b = 10.4$ m

Step 3: Substitute the values into the unknown angle application of the sine rule.

$$A = \sin^{-1}\left(\frac{a \times \sin(B)}{b}\right)$$
$$AQP = \sin^{-1}\left(\frac{20.7 \times \sin(23.5°)}{10.4}\right)$$
$$= 52.529...°$$

The first possible value for angle AQP is 52.5°.

Step 4: Determine the second possible value for angle AQP.

Subtract the angle obtained from the sine rule calculation from 180°.

$$AQP = 180 - 52.529...$$
$$= 127.470...°$$

The second possible value for angle AQP is 127.5°.

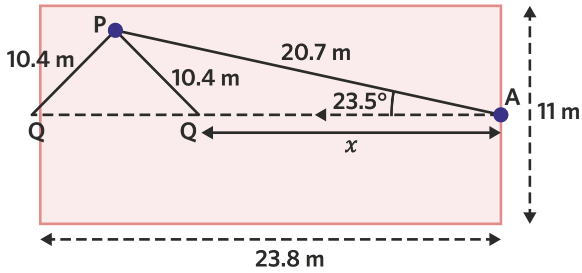
Answer

52.5° and 127.5°

Most students only provided the 52.5° solution, without considering the ambiguous case of the sine rule.

b. Explanation

Step 1: Decide which possible value of angle AQP fits the conditions of the question.



If point Q is within the boundary of the court, angle AQP is the obtuse option.

$AQP = 127.5°$ (from part **a**)

Step 2: Calculate the value of angle APQ.

$$AQP = 180 - 23.5 - 127.5$$
$$= 29°$$

Step 3: Calculate the length of side AQ.

$$\frac{AQ}{\sin(29°)} = \frac{10.4}{\sin(23.5°)}$$
$$AQ = \frac{10.4 \times \sin(29°)}{\sin(23.5°)}$$
$$= 12.644...\text{ m}$$

Answer

13 m

Many incorrect answers came from students choosing the 52.5° angle, leading to an answer of 25 m. This is outside the boundary of the court.

Questions from multiple lessons

23. C **24.** E

25. a. A zero represents no friendship between two people.

b. $x = 0$
$y = 1$

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10D The cosine rule

Calculating side lengths of a triangle using the cosine rule

1. $c^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos(80°)$

2. a. 7.80 cm b. 45.54 mm
   c. 1.55 m

3. 10.4 cm

Calculating angles in a triangle using the cosine rule

4. D

5. a. 57.12° b. 43.23° c. 18.57°

6. 45.08°

Joining it all together

7. a. 13.7 m b. 56°

8. 944 m

9. 2.16 cm

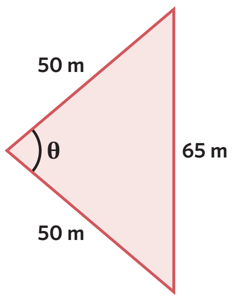
Exam practice

10. **Explanation**

Step 1: Draw and label the triangle.

The two shorter sides of 50 m are equal as they both represent the radius of the circle.

The longer side is 65 m, the side length of the square.



Step 2: Identify the known values.

$a = 50$ m

$b = 50$ m

$c = 65$ m

$C = \theta$

Step 3: Substitute the values into the cosine rule and evaluate.

Answer

$$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

$$\theta = \cos^{-1}\left(\frac{50^2 + 50^2 - 65^2}{2 \times 50 \times 50}\right)$$

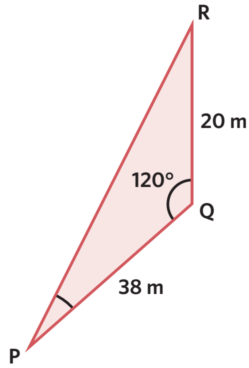
$= 81.083...°$

$\approx 81°$

In order to receive the mark, students needed to write the steps required to calculate the angle as 81°. Many students who calculated an incorrect answer did not assign the correct values to a, b and c.

11. **Explanation**

Step 1: Draw and label the triangle.



Step 2: Identify the known values.

$a = 20$ m

$b = 38$ m

$c = RP$

$C = 120°$

Step 3: Substitute the values into the cosine rule and evaluate.

$$c = \sqrt{a^2 + b^2 - 2ab \times \cos(C)}$$

$$RP = \sqrt{20^2 + 38^2 - 2 \times 20 \times 38 \times \cos(120°)}$$

$= 51.029...$ m

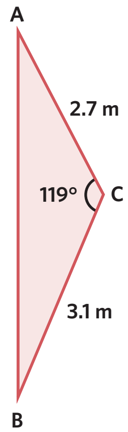
Answer

51 m

12. **Explanation**

Step 1: Draw and label the triangle.

The distance between Wei-Yi and Bao is labelled as AB.



Step 2: Identify the known values.

$a = 2.7$ m

$b = 3.1$ m

$c = AB$

$C = 119°$

Step 3: Substitute the values into the cosine rule and evaluate.

Answer

$$c = \sqrt{a^2 + b^2 - 2ab \times \cos(C)}$$

$$AB = \sqrt{2.7^2 + 3.1^2 - 2 \times 2.7 \times 3.1 \times \cos(119°)}$$

$= 5.001...$ m

≈ 5 m

As for all 'show that' questions, students needed to write the steps required to calculate the side length as 5 m to get the mark.

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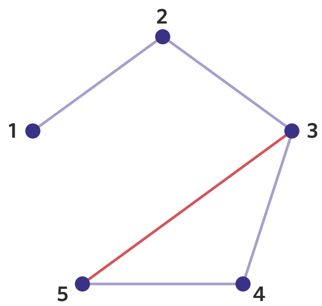
Questions from multiple lessons

13. B 14. E

15. a.

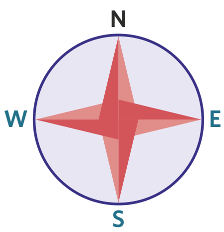


b. 10

10E Bearings

Identifying and drawing true bearings

1.



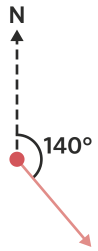
2. 225°

3. D

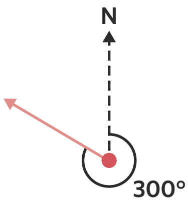
4. a. 120° b. 175° c. 315°

5. a. 321° b. 141° c. 205° d. 25°

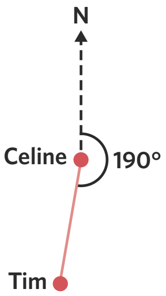
6. a.



b.



c.



Solving problems using true bearings

7. B 8. D

9. a. 10.94 m b. 4.73 m

c. 16.56 cm

10. 204.78° 11. 175 m

Joining it all together

12. a. 50.17° b. 332.97°

c. 117.58 km d. 67.03°

13. a. $BAC = \tan^{-1}\left(\frac{4}{2.5}\right) + 51$

$= 57.99... + 51$

$= 108.99...$

$\approx 109°$

b. 4.50 km c. 7.51 km

d. 8.25 km at a true bearing of 345.96°

Exam practice

14. **Explanation**

Step 1: Identify the appropriate formula.

The triangle is right-angled.

One angle is known: 30°

The hypotenuse is known: 50 m

The side opposite the reference angle requires calculation: AB

The sine trigonometric equation can be used to calculate AB.

Step 2: Substitute the values into the sine formula and solve for the unknown side length.

$\sin(\theta) = \frac{opposite}{hypotenuse}$

$\sin(30°) = \frac{AB}{50}$

$AB = \sin(30°) \times 50$

$= 25$ m

Answer

25 m

15. **Explanation**

Step 1: Mark the starting point.

The scenario states the bearing is 'from town A', so town A is the starting point.

Step 2: Draw a ray (arrow) marking north from the starting point.

10E ANSWERS

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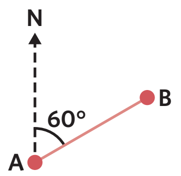
Step 3: Using this ray, measure the required angle clockwise and draw a line segment to mark the angle.

The scenario requires a measured angle of 60°.



Note: Only the relative position of the towns is important. The position of the north ray does not change the true bearing.

Answer

A

Some students incorrectly eliminated options A, C and D due to the position of the north ray.

16. Explanation

Step 1: Label any known measurements relating to the triangle.

The bearing from the horse to Marcus is 170°.

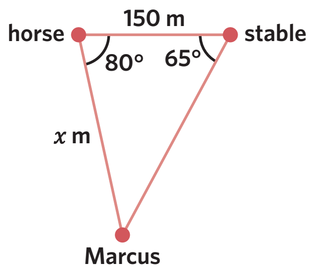
The internal angle at the horse is $170° - 90° = 80°$.

The bearing from the stable to Marcus is 205°.

The internal angle at the stable is $270° - 205° = 65°$.

The straight-line distance between the horse and its stable is 150 m.

The straight-line distance between the Marcus and the horse is unknown (x).



Step 2: Identify the appropriate formula.

The triangle is not right-angled.

x requires calculation and its opposite angle is known.

The 150 m side length is known, but not its opposite angle.

If the opposite angle were known, the sine rule could be used to calculate x.

Step 3: Calculate the unknown internal angle.

$180° - 80° - 65° = 35°$

Step 4: Substitute the known values into the sine rule and solve for the unknown side length.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

$$\frac{x}{\sin(65°)} = \frac{150}{\sin(35°)}$$

$$x = \frac{150 \times \sin(65°)}{\sin(35°)}$$

$$= 237.01... \text{ m}$$

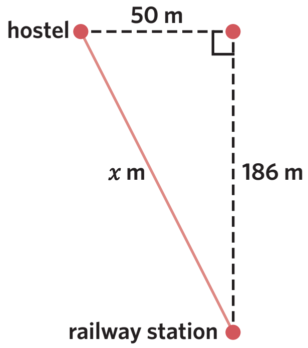
Answer

E

17. a. Explanation

Step 1: Draw a diagram, labelling all known measurements.

Label the distance from Nemuro railway station to the hostel as x.



Step 2: Identify the appropriate formula.

The two shorter sides of a right-angled triangle are known.

Pythagoras' theorem can be used to calculate the hypotenuse.

Step 3: Substitute the values into Pythaogoras' theorem and evaluate.

$$c = \sqrt{a^2 + b^2}$$

$$x = \sqrt{50^2 + 186^2}$$

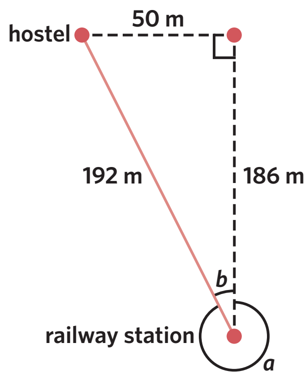
$$= 192.60... \text{ m}$$

Answer

193 m

b. Explanation

Step 1: Identify the required angle.



The bearing angle is measured clockwise from the starting point (railway station) to the end point (hostel).

This angle, a, and the internal angle of the triangle, b, will add to 360°.

Therefore, angle b will be calculated first.

Step 2: Identify the appropriate formula.

One angle in a right-angled triangle requires calculation.

The side lengths opposite and adjacent to the angle are known.

The tangent trigonometric equation can be used to calculate b.

Step 3: Calculate angle b.

$$\theta = \tan^{-1}\left(\frac{opposite}{adjacent}\right)$$

$$b = \tan^{-1}\left(\frac{50}{186}\right)$$

$$= 15.046...°$$

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Step 4: Calculate angle a.

$$\begin{aligned} a &= 360° - b \\ &= 360° - 15.046...° \\ &= 344.953...° \end{aligned}$$

Answer

345°

Some students did not factor in that the internal angle of the triangle was not the bearing and did not subtract this from 360°, giving an incorrect answer of 15°. Some students also gave an answer of N15°W, which is not a three-figure bearing and was not an acceptable response.

Questions from multiple lessons

18. E

19. B

20. a. \$220

b. San Sebastian and Toledo

c. $6 + 7 = 11 + 2$

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GLOSSARY

#

68–95–99.7% rule A rule that approximates the proportion of data within a certain number of standard deviations from the mean in a normally distributed data set. p. 60

A

adjacency matrix A square binary matrix that represents the number of direct connections between vertices in a graph. p. 374

adjacent side The side adjacent to (next to) the reference angle, θ. p. 578

angle A measure of the space between two intersecting lines, measured close to the point of intersection. p. 476

angle of depression An angle measured down from a horizontal. pp.589

angle of elevation An angle measured up from a horizontal. p.588

answer matrix The matrix representing the values each simultaneous equation evaluates to. p.298

approximately symmetric A distribution that is very similar on either side of the centre, but does not have to be perfect. p.57

arc A portion of the circumference of a circle. p.521

area A measure of the space taken up by a two-dimensional shape. p.478

area scale factor The ratio between the areas of similar shapes. It is commonly denoted as k^2. p.566

arithmetic A type of sequence where the pattern is repeated addition or subtraction of a constant value. p.124

B

back-to-back stem plot A stem plot that displays and compares the distribution of two categories. p.69

bar chart A visual representation of categorical data. The categories of the data set are shown on the horizontal axis, and the frequency of the categories on the vertical axis. p.10

billing period The regular period of time in which the bank sends a statement of purchases made, the amount owing and when interest will start being charged. It is also known as the statement period. p.112

bimodal When a data set has two distinct peaks. p.58

boxplot A graphical representation of a five-number summary that also shows s any outliers. p.50

bridge An edge that is keeping a graph connected. If a bridge were removed, the graph would become disconnected. p.363

buy now pay later scheme A purchase option that involves the consumer splitting up the total cost of an item evenly over several interest-free payments. p.115

C

capacity A measure of how much can be held inside a three-dimensional object. p.478

cash A form of physical currency, such as coins and notes. p.112

categorical data Data that is organised into categories or groups. p.2

causation An indication that a change in the explanatory variable definitively causes the change in the response variable. p.328

centre The middle of a numerical distribution. p.29

circuit A trail that starts and ends at the same vertex. No edges are repeated, though vertices may be repeated. p.400

circumference The perimeter of a circle. p.520

coefficient matrix The matrix representing the coefficients in a set of simultaneous equations. p.298

coincidence An association observed between two unrelated variables. p.328

column matrix A matrix with one column and any number of rows. p.253

columns Vertical lists of values and are numbered from left to right. p.252

common difference The amount being added or subtracted from one term to the next in an arithmetic sequence. p.133

common ratio The amount each term in a geometric sequence is multiplied by to give the next term. p.149

common response When an observed association is caused by a third variable that produces the same response in the initial two variables. p.328

communication matrix A square matrix that models the number of connections between points. p.280

complete graph A graph in which every vertex is directly connected to every other vertex once. p.363

composite shape A shape consisting of two or more basic shapes. p.522

composite solid A solid that is made up of a combination of three-dimensional objects. p.548

compound interest Interest accrued each period that is calculated as a percentage of the previous value. p.159

compounding period The set period of time at which compound interest is calculated. p.159

cone A tapered solid that has a circular-shaped base. p.546

confounding variables External variables that also contribute to an observed association. p.328

congruent Objects that have exactly the same shape and size. p.495

connected graph A graph in which all vertices are connected to each other, either directly or indirectly. Every vertex is accessible from any other vertex by travelling along the edges. p.363

constant of proportionality The degree to which two variables are proportional to each other, defined by k. There are two distinct types of variation, direct and inverse. p.432

continuous data Numerical data that can be measured using a continuous scale. p.3

correlation A measure of the strength and direction of a relationship between the explanatory and response variables. p.328

cosine The ratio of the adjacent side to the hypotenuse. p.579

cosine rule A generalisation of Pythagoras' theorem that can be used to calculate unknown values in non-right-angled triangles. p.608

credit cards Cards that allow the owner to make a payment using the bank's money, which then must be repaid by the cardholder according to the terms of the credit card. p.112

cycle A path that starts and ends at the same vertex. No edges or vertices are repeated. p.400

cylinder A curved three-dimensional object that has the same circular cross-section across its entire length. p.544

D

debit cards Cards that allow the owner to electronically transfer money from a linked bank account in order to make a payment. p.112

degree The number of edges connected to a vertex. p.361

depreciation The loss in value of an asset. p.142

determinant A value that indicates many properties about a square matrix, in particular, if its inverse exists. p.288

diameter The distance from one side of a circle to the other, passing through its centre. It is commonly denoted as d. p.520

direct variation Variation in which an increase in one variable results in a linear change in the other variable. p.432

directed graph A network containing arrows on each edge that show directional information between vertices. p.377

direction An indication of whether the response variable increases or decreases as the explanatory variable changes. It can be positive or negative. p.326

disconnected graph A graph in which it is not possible to reach every vertex from all other vertices. p.363

discount A percentage decrease when applied to financial mathematics, particularly the sale of a good or service. p.92

discrete data Numerical data that can be counted using integers. p.3

domain The set of possible values of x for the linear model $y = a + bx$. p.214

domain of interpretation The domain in which a linear model is applicable. p.214

dot plot A visual representation of numerical data that uses stacked dots to convey the frequency of data values. p.16

duplicate edges Multiple edges that connect the same two vertices. p.361

E

edge A line connecting one vertex to either another vertex or itself. p.361

element An entry in a matrix. p.252

elimination method A method of solving simultaneous equations that involves the equations being added or subtracted in a way that eliminates one of the variables. p.223

equilibrium matrix A state matrix that represents a point in time in which the transition matrix no longer affects the state of the system. p.307

Euler's rule A rule that describes the relationship between the number of vertices, edges and faces of connected planar graphs, $v - e + f = 2$. It is also known as Euler's formula. p.392

explanatory variable The variable that is used to explain or predict changes observed in the response variable. It is also known as the independent variable. p.316

extrapolation A prediction made outside of the range of the data set. These predictions have limited reliability. p.338

F

face An area on a graph that is bordered by edges, including the space outside the graph. It is also known as a region. p.392

five-number summary A summary that provides key information about a set of data and its distribution, including centre and spread. p.47

flat rate depreciation When the value of an asset is depreciated by a constant amount for each specified time period, calculated as a percentage of the principal. p.143

form An indication of whether a relationship is linear or non-linear. p.326

frequency table A table consisting of two columns that displays the number of times each category occurs in a data set. p.9

G

geometric A type of sequence where the pattern is repeated multiplication of a constant value. p.124

geometric decay A type of geometric sequence where the common ratio is greater than 0 and less than 1 and the terms are decreasing. p.151

geometric growth A type of geometric sequence where the common ratio is greater than 1 and the terms are increasing. p.151

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gradient The steepness of a line, equal to the change in y for every one unit increase in x. It is also known as the slope. p.184

gradient-intercept method A graphing method that involves identifying and plotting the y-intercept, and then using the gradient to plot more points and drawing a line connecting them. p.194

graph A diagram that is used to show the connections between a group of common elements, such as objects, locations, people or activities. It is also known as a network. p.360

greedy algorithm A procedure in which the optimal (or best) solution to be found for every step of a multi-step problem. p.418

grouped frequency table A table that organises numerical data into groups, or intervals, and shows the frequency of data points that fall within each group or interval. p.19

GST In Australia, this is a 10% tax that is applied to most items that people buy or services that people pay for. It stands for the Goods and Services Tax. p.94

H

hemisphere Half of a sphere. p.547

Heron's formula A formula for the area of a triangle when all side lengths are known. p.534

histogram A visual representation of a grouped frequency table that shows the frequency of data values within different intervals. p.19

hypotenuse The longest side in a right-angled triangle. It is always opposite the right angle. p.506

I

identity matrix A square matrix where all of the elements in the leading diagonal are one and the rest of the elements are zero. p.253

image A copy of a similar object after it has been rotated, reflected, enlarged or reduced. p.498

inflation A general increase in the prices of goods and services over time. p.99

inflation rate The increase in price as a percentage of the previous year's price, commonly given as an annual figure. p.99

initial state matrix A column matrix that presents the first, or initial, state of a system. It is commonly denoted as S_0. p.306

intercept-intercept method A graphing method that involves finding the x- and y-intercepts as the two points required, drawing a line connecting them, and then extending that line. p.195

interest An additional fee charged by lenders as a cost for borrowing. p.112

interest-free period The amount of time after the billing period where repayments can be made before interest is charged on the amount owing. p.112

interpolation A prediction made within the range of the data set. p.338

interquartile range (IQR) A measure of the spread of the middle 50% of a data set. p.32

intersection The solution to a pair of simultaneous equations. Graphically, this is the point at which the two lines meet. p.220

interval data Numerical data that has equal spacing or significance between values on its scale, but with no absolute 0 value. p.4

inverse matrix A matrix that has the property such that when it is pre-multiplied or post-multiplied with the original matrix, the result is the identity matrix, I. p.290

inverse variation Variation in which an increase in one variable results in a reciprocal change in the other variable. p.433

isomorphic graphs Graphs that display the same information as each other. They have the same vertices and edges, but are drawn differently. p.364

K

Kruskal's algorithm A greedy algorithm that helps to identify the minimum spanning tree of a graph. p.418

L

leading diagonal The diagonal line from the top left corner of a square matrix matrix to the bottom right corner. p.253

length A measure of the straight-line distance from one point to another. p.476

limiting value A value that a sequence approaches and stays constant at. p.128

line of good fit A straight line which approximates the general trend of a scatterplot. p.336

line segment graph A piecewise linear model that connects segments across different domains where the variable on the vertical axis can change in value. p.233

linear decay A type of arithmetic sequence where the common difference is less than 0 and the terms are decreasing. p.135

linear function A relationship between two variables where one value changes by a constant amount in response to the other. p.182

linear growth A type of arithmetic sequence where the common difference is greater than 0 and the terms are increasing. p.135

linear scale factor The ratio between the corresponding side lengths in two similar objects. it is commonly denoted as k. p.498

logarithm A function that returns a value representing the power to which a fixed number (the base) must be raised to produce a given number. p.464

log(x) transformation A type of transformation that can be used to linearise non-linear data, where the logarithm of each x-value is taken. It 'compresses' larger x-values more than smaller x-values. p.467

loop An edge that connects a vertex back to itself. p.361

lower fence Defines the boundary of an outlier in the lower half of the data. p.49

M

mark-up A percentage increase when applied to financial mathematics, particularly the sale of a good or service. p.92

matrix A rectangular array that displays a collection of numerical values. p.252

matrix equation An equation involving matrices. p.296

matrix power The product when a matrix is raised to an index or power. p.272

matrix product The resulting matrix when two or more matrices are multiplied. p.269

maximum The largest value in the data set. p.47

mean A measure of centre that averages out all values of a data set into a single value. p.39

median The middle value in an ordered set of data. p.29

minimum The smallest value in the data set. p.47

minimum spanning tree The spanning tree of a weighted graph that has the lowest possible total weight. p.418

modal interval The interval or group with the highest frequency. p.19

mode The most frequently occurring category within a data set. p.11

N

negative skew A distribution that trails off in the negative direction. p.58

net A two-dimensional representation of what a three-dimensional planar object would look like if unfolded, or deconstructed. p.554

nominal data Categorical data that has no logical order or ranking of categories. p.3

non-planar graph A graph that cannot be drawn with no overlapping edges, even after redrawing. p.390

normal distribution A symmetrical (or approximately symmetrical) numerical data set that is centred around the mean, with a width determined by the standard deviation. p.60

numerical data Data that can be counted or measured. p.2

O

one-step communication matrix A square matrix that models the number of direct, one-step connections between points. It is commonly denoted as C. p.280

opposite side The side opposite to the reference angle, θ. p.578

order The size or dimensions of a matrix, expressed in the form *number of rows* × *number of columns*. p.252

order of magnitude The exponent of a number expressed in scientific notation. p.489

ordinal data Categorical data that can be logically ordered. p.3

outliers A value in a data set that falls far outside of the general spread of data, and can be found by calculating the upper and lower fences. p.49

P

parallel boxplots A sequence of boxplots that display data for two or more categories. p.71

path A walk in which no edges or vertices are repeated. p.398

percentage Translates to 'out of one hundred' and is a standard measure used to compare proportions and perform calculations with them. p.91

perfectly symmetric A distribution that is evenly distributed on both sides of the centre of the data. p.57

perimeter The total distance around a two dimensional shape. p.519

personal loan A purchase option generally used for large expenses that involves regular payments, a set duration and an interest rate charged to the borrower. p.115

piecewise linear graph A graph made up of two or more linear equations. p.232

planar graph A graph that can be drawn with no overlapping edges. p.390

planar solid An object which only has flat faces. p.554

polygon A shape consisting of three or more straight lines. p.519

positive skew A distribution that trails off in the positive direction. p.58

post-multiplication The process of multiplying one matrix after another. p.269

pre-multiplication The process of multiplying one matrix before another. p.269

Prim's algorithm A greedy algorithm that helps to identify the minimum spanning tree of a graph. p.418

prism A three-dimensional object that has the same cross-section across its entire length. p.544

Pythagoras' theorem A rule that states that for every right-angled triangle, the square of the hypotenuse is equal to the sum of the two squared sides. This is expressed as $a^2 + b^2 = c^2$. p.506

Q

quartiles Values that divide an ordered data set into quarters. p.32

R

radius The distance from the edge of a circle to its centre. It is commonly denoted as *r*. p.520

range A measure of the spread of the entire data set, found by calculating the difference between the maximum and minimum values. p.31

rate A ratio of two quantities that are related to each other, expressed as the rate of change in one quantity per unit of the other quantity. p.84

ratio A set of numbers in the form $a : b$ that expresses the relationship between two or more quantities or sizes. p.82

ratio data Numerical data that uses the same scale as interval data, but with an absolute 0 value. p.4

reciprocal The inverse of a number, calculated by raising that number to the power of -1, or dividing 1 by the number. p.454

recurrence relation A formula that links each term in a pattern-based sequence to the next. p.133

recursion The process where terms in a sequence are generated using a pattern. p.124

reducing balance depreciation When the value of an asset is depreciated by a constant ratio for each specified time period, calculated as a percentage of the previous value. p.159

response variable The variable that is explained or predicted by changes in the explanatory variable. It is also known as the dependent variable. p.316

right-angled triangle A triangle in which there is one angle of 90° (a right angle). p.506

rounding The process of condensing a number so that it is more convenient for use in calculations at the cost of accuracy. p.486

route A list of the vertices travelled through when moving from one vertex to another, showing the pathway travelled in a graph. p.398

row matrix A matrix with one row and any number of columns. p.252

rows Horizontal lists of values and are numbered from top to bottom. p.252

S

scalar multiplication An operation where each element in a matrix is multiplied by a single number (the scalar). p.263

scatterplot A display used to represent data relating to two numerical variables. Each point is determined by the value of one variable and the corresponding variable of the second variable, creating a coordinate. p.317

scientific notation A way to express values with a condensed number of significant figures by multiplying the value by varying powers of 10. p.489

sector A portion of a circle. p.533

sequence A set of numbers arranged in a particular order. p.124

shortest path The minimum total weighted value between two vertices, found by identifying all of the possible paths from one vertex to another, and comparing their lengths. p.408

significant figure A digit in a number that contributes to the value of the number with certainty, and not as a result of rounding. p.486

similarity Objects that have the same shape, but different sizes. p.495

simple graph A graph which does not contain any loops or duplicate edges. p.363

simple interest investment A type of investment where the interest earned each period is calculated as a percentage of the principal, meaning that the investment increases by the same amount each period. p.141

simultaneous equations A set of multiple linear equations with two or more variables. p.220

sine The ratio of the opposite side to the hypotenuse. p.579

sine rule A rule that states that the ratio between a side length of a triangle and the sine value of its opposite angle will be consistent for all three sides of a triangle. p.595

singular matrix A square matrix that does not have an inverse, where its determinant is equal to zero. p.290

spanning tree A tree that contains all of the vertices in a larger graph, hence 'spanning' the entire graph. p.416

spending power The amount that can be purchased with a unit of currency. The spending power of money reduces as inflation continues. p.101

sphere A perfectly round object where every point on its surface is the same distance from its centre. p.547

spread A measure that refers to how similar or varied a set of data is. p.31

square matrix A matrix with an equal number of rows and columns. p.253

standard deviation A measure of spread that represents the average deviation, or difference, of each data point compared to the mean. p.40

state matrix A column matrix representing the state of a system at a given time, separated by regular time intervals. It is commonly denoted as S_n, where n refers to the state number. p.306

steady state matrix See equilibrium matrix. p.307

stem plot A visual representation of numerical data where each data entry is split into a 'stem' (the largest unit) and a 'leaf' (the smallest unit). p.18

step graph A piecewise linear model that has segments across different domains where the variable on the vertical axis is constant for each segment. Hence, each segment is horizontal. p.233

strength An indication of how close the data points are to the general trend of the scatterplot. It can be weak, moderate or strong. p.324

subject A variable that is by itself on one side of an '=' sign. p.176

substitution The replacement of one term with another. In linear equations, substitution is the replacement of a pronumeral with a number. p.176

substitution method A method of solving simultaneous equations that involves a variable from one equation being substituted into the other. p.222

summing matrix A row or column matrix that is used to sum the rows or columns of another matrix. p.279

surface area The total area of all the surfaces of a three-dimensional object. p.554

T

tangent The ratio of the opposite side to the adjacent side. p.579

tapered solid A three-dimensional object that starts at a base shape but reduces to a point at its top, or apex. p.546

term In the context of number patterns, a term refers to any individual number within a sequence. p.124

three-figure bearing A navigational bearing measured as the clockwise angle from true north. p.615

trail A walk in which no edges are repeated, though vertices may be repeated. p.398

transition matrix A square matrix that represents the unidirectional movement from one state to another. It is commonly denoted as T. p.304

transpose The rearranging of an equation to make a variable the subject. p.177

tree A simple connected graph that contains no loops, circuits or duplicate edges, and is often part of a larger graph. p.416

trigonometry The study of the sides and angles that make up triangles, and the relationships between them. p.578

true bearing See three-figure bearing. p.615

two-points method A graphing method that uses any two points to graph a linear function. p.194

two-step communication matrix A square matrix that models the number of two-step connections between points. It is commonly denoted as C^2. p.281

U

unit cost depreciation When the value of an asset is depreciated by a constant amount after each unit of use. p.142

unitary method A method for calculating the total value of a given number of items, by first finding the value of a single item. p.106

upper fence Defines the boundary of an outlier in the upper half of the data. p.49

V

variable matrix The matrix representing the variables (often x and y) in a set of simultaneous equations. p.298

variation The change in one variable as a result of the change in another variable. p.432

vertex A point on a graph. p.360

volume A measure of the space taken up by a three-dimensional object. p.478

volume scale factor The ratio between the volumes of similar objects. It is commonly denoted as k^3. p.567

W

walk A route that passes through any number of vertices, in any order, starting and finishing at any vertex. p.398

weighted graph A graph that has numeric values assigned to each edge. These values represent the 'weight' of each edge and are usually associated with distance, time or cost. p.407

X

x-intercept The point on a line where it crosses the x-axis (when $y = 0$). p.184

x-reciprocal transformation A type of transformation that can be used to linearise non-linear data, where 1 is divided by each x-value. It 'compresses' x-values that are greater than one, and 'stretches' x-values that are less than one. p.456

x-squared transformation A type of transformation that can be used to linearise non-linear data, where each x-value is squared. It 'stretches' larger x-values more than smaller x-values. p.446

Y

y-intercept The point on a line where it crosses the y-axis (when $x = 0$). p.184

Z

zero matrix A matrix of any size where all of the elements are zero. p.253