Topic 1 — Investigating and comparing data distributions

1.2 Types of data

1.2 Exercise

1 **a** The answers will be numbers, so the data are numerical.
b The answers will be categories, so the data are categorical.
c The results will be numbers, so the data are numerical.

2 The data collected is the different types of cereal on supermarket shelves, so this is categorical data.
When assessing the types of different cereal, the order is not relevant, so this is nominal data.

3 The data collected is the rating of hotels from 'one star' to 'five star', so this is categorical data.
When assessing the ratings of hotels from 'one star' to 'five star', the order is relevant, so this is ordinal data.

4 **a** The data collected is the amount of daily rainfall in Geelong, so this is numerical data.
The data involves measuring the amount of rainfall, so an infinite number of values are possible, with an additional value always possible between any two given values.
The data collected is continuous data.
b The data collected is the heights of players in the National Basketball League, so this is numerical data.
The data involves measuring heights, so an infinite number of values are possible, with an additional value always possible between any two given values.
The data collected is continuous data.
c The data collected is the number of children in families, so this is numerical data.
The data involves counting children, so only whole number values are possible.
The data collected is discrete data.

5 **a** The data collected is the times taken for the place getters in the Olympic 100 m sprinting final, so this is numerical data.
The data involves measuring times, so an infinite number of values are possible, with an additional value always possible between any two given values.
The data collected is continuous data.
b The data collected is the number of gold medals won by countries competing at the Olympic Games, so this is numerical data.
The data involves counting gold medals, so only whole number values are possible.
The data collected is discrete data.
c The data collected is the type of medals won by a country at the Olympic Games, so this is categorical data.
When assessing the type of medals won, the order is relevant, so this is ordinal data.

6 **a** The data collected is wines rated as high, medium or low quality, so this is categorical data.
When assessing the ratings of wines, the order is relevant, so this is ordinal data.
b The data collected is the number of downloads from a website, so this is numerical data.
The data involves counting downloads, so only whole number values are possible.
The data collected is discrete data.
c The data collected is the amount of electricity usage over a three-month period, so this is numerical data.
The data involves measuring the amount of electricity used, so an infinite number of values are possible, with an additional value always possible between any two given values.
The data collected is continuous data.
d The data collected is the volume of petrol sold at a petrol station, so this is numerical data.
The data involves measuring the volume of petrol sold, so an infinite number of values are possible, with an additional value always possible between any two given values.
The data collected is continuous data.

	Data	Type	
a	Wines rated as high, medium or low quality	Categorical	Ordinal
b	The number of downloads from a website	Numerical	Discrete
c	Electricity usage over a three-month period	Numerical	Continuous
d	The daily volume of petrol sold by a petrol station	Numerical	Continuous

7 The data collected is the birthplaces of people, so this is categorical data.
When assessing the birthplaces of people, the order is not relevant, so this is nominal data.

8 **a** **i** The data is measured on a scale, but the scale does not have a true zero, so the data is interval.
ii The data are categories with no order, so the data is nominal.
b The data from **i** are more useful for statistical analysis because interval data is higher ranking than nominal data in levels of measurement.

9 **a** Question 1: The choices are ordered categories so the data will be ordinal.
Question 2: The data will be a number value measured using the scale *hours.* The scale has a meaningful zero, because 0 hours represents no sleep. So, the data will be ratio.
b Question 2 is better for statistical analysis because ratio data is higher ranking than ordinal data in levels of measurement.

10 Weight is continuous data. It is measured on a scale. Continuous data has an infinite set of possible values.

1.2 Exam questions

1 The variables *age* and *preferred travel destination* are both categorical variables. Categories have been given for both. The correct answer is **A**.

2 The second variable also needs to have a small number of categories. The only possibility is *sex* (male, female).

The correct answer is **B**.
VCAA Examination Report **note:**
Students needed to recognise that for a two-way frequency table to be used, both variables had to be categorical variables.

3 Both *blood pressure* and *age*, in this instance, are ordinal variables, as they have an *order*.
The correct answer is **B**.
VCAA Examination Report **note:**
Many students incorrectly identified the variable *age* (under 50 years, 50 years or over) as nominal. The variable *age* (under 50 years, 50 years or over) is ordinal because the process of allocating each of the people to one of these two categories 'under 50 years' or '50 years or over' orders the group of people by age.

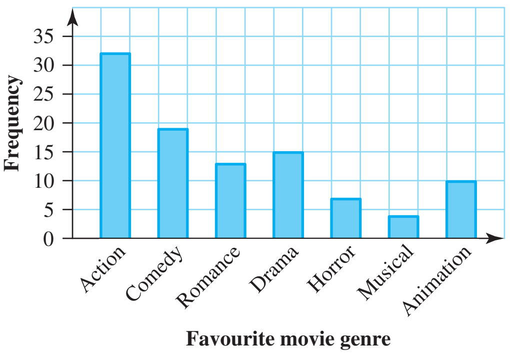
1.3 Categorical data distributions

1.3 Exercise

1 Create a frequency table to capture the data. Then go through the data, filling in the tally column. Finally, sum each tally to complete the frequency column.

Favourite type of music	Tally	Frequency
Pop	~~||||~~ ||||	9
Rock	~~||||~~ ||	7
Classical	||	2
Folk	|||	3
Electronic	~~||||~~ ||||	9

2 a Choose an appropriate scale for the bar chart. As the frequencies go up to 32, we will mark our intervals in 5s. Display the different categories along the horizontal axis.
Draw bars to represent the frequency of each category, making sure there are spaces between the bars.



b The number of people surveyed was stated in the question. It was 100. A description of the bar chart could be:
In a survey, 100 people were asked 'What is your preferred movie genre?' The options were: action, comedy, romance, drama, horror, musical and animation. The most common response was action with 32; the least common was musical with 4.

3 Create a frequency table to capture the data. Then read the frequency of each pizza type off the bar chart to complete the frequency column.

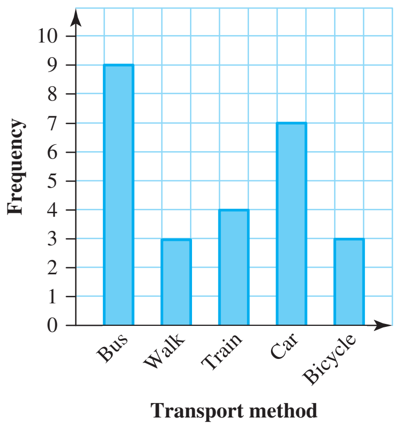
Favourite pizza	Frequency
Margherita	7
Pepperoni	11
Supreme	9
Meat feast	14
Vegetarian	6
Other	13

4 a The data collected is the different methods of travel, so this is categorical data.
When assessing the methods of travel, the order is not relevant, so this is nominal data.

b Create a frequency table to capture the data. Then go through the data, filling in the tally column. Finally, sum each tally to complete the frequency column.

Transport method	Tally	Frequency
Bus	~~||||~~ ||||	9
Walk	|||	3
Train	||||	4
Car	~~||||~~ ||	7
Bicycle	|||	3

c Choose an appropriate scale for the bar chart. As the frequencies only go up to 9, we will mark our intervals in single digits. Display the different categories along the horizontal axis.
Draw bars to represent the frequency of each category, making sure there are spaces between the bars.

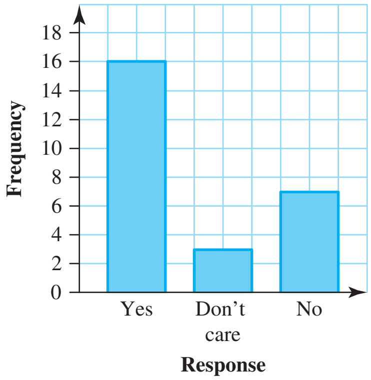


d The number of people surveyed was 26 (from adding bars: $9 + 3 + 4 + 7 + 3 = 26$) (or from counting the number from the data). A description of the bar chart could be:
In a survey, 26 university students were asked 'What is your usual method of travel?' The options were: bus, walk, train, car and bicycle. The most common response was bus with 9; the least common were walk and bicycle, each with 3.

5 a Create a frequency table to capture the data. Then go through the data, filling in the tally column. Finally, sum each tally to complete the frequency column.

Response	Tally	Frequency
Yes	𝍸 𝍸 𝍸 |	16
Don't care	|||	3
No	𝍸 ||	7

b Choose an appropriate scale for the bar chart. As the frequencies go up to 16, we will mark our intervals in 2s. Display the different categories along the horizontal axis.
Draw bars to represent the frequency of each category, making sure there are spaces between the bars.

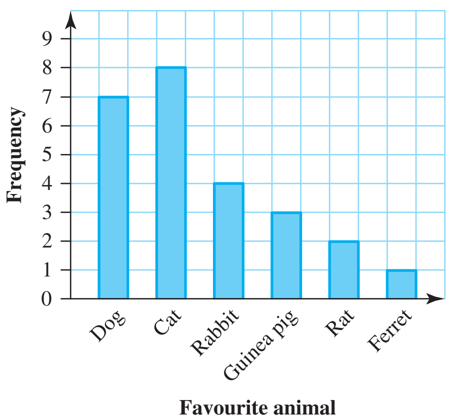


c The data collected is a response of 'Yes', 'No' or 'Don't care', so this is categorical data.
When assessing the responses, the order is relevant (it makes sense to arrange the data in order from 'Yes' to 'No', with 'Don't care' between them), so this is ordinal data.

6 a Create a frequency table to capture the data. Then go through the data, filling in the tally column. Finally, sum each tally to complete the frequency column.

Favourite animal	Tally	Frequency
Dog	𝍸 ||	7
Cat	𝍸 |||	8
Rabbit	||||	4
Guinea pig	|||	3
Rat	||	2
Ferret	|	1

b Choose an appropriate scale for the bar chart. As the frequencies only go up to 8, we will mark our intervals in single digits. Display the different categories along the horizontal axis.
Draw bars to represent the frequency of each category, making sure there are spaces between the bars.



c The cat is the most popular animal among the students.

7 a The mode is the category that has the highest frequency. Therefore, the modal category is flat white.

b To calculate how many coffees were sold in that hour, read off the frequencies of each kind of coffee and add them together.

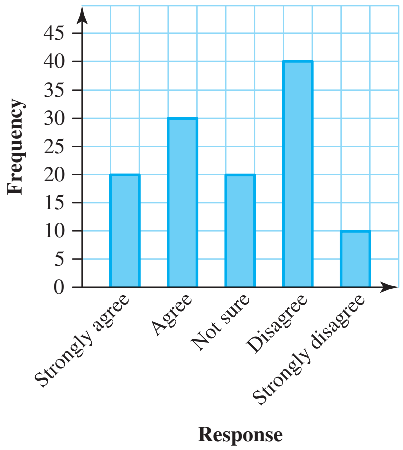
$$\text{Coffees sold} = 8 + 13 + 10 + 5 + 12 + 18 + 4$$
$$= 70$$

70 coffees were sold in that hour.

8 a The data collected is a response ranging from 'Strongly agree' to 'Not sure', so this is categorical data.
When assessing the responses, the order is relevant (it makes sense to arrange the data in the order of 'Strongly agree', 'Agree', 'Not sure', 'Disagree', 'Strongly disagree'), so this is ordinal data.

b Looking at the horizontal axis, the categories should be in a certain order since the data is ordinal.
In the current data display, the data should be in order from 'Strongly agree' through to 'Strongly disagree'.

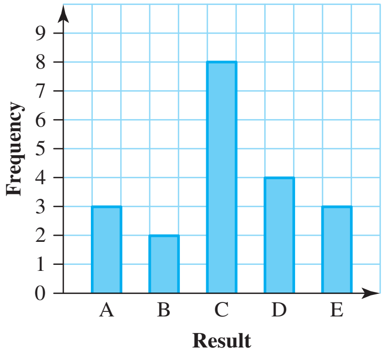
c Redraw the given graph, but reorganise the horizontal axis to the appropriate order. Remember to copy the corresponding frequencies correctly.



9 a Create a frequency table to capture the data. Go through the data, filling in the tally column. Sum each tally to complete the frequency.

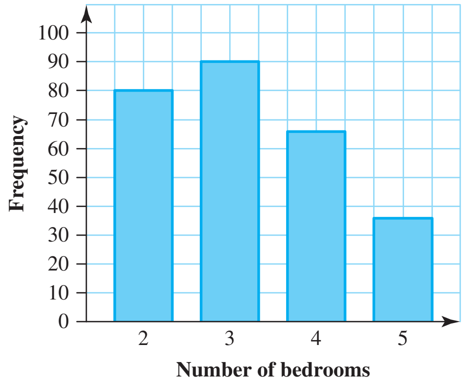
Result	Tally	Frequency
A	|||	3
B	||	2
C	𝍸 |||	8
D	||||	4
E	|||	3

b Choose an appropriate scale for the bar chart. As the frequencies only go up to 8, we will mark our intervals in single digits. Display the different categories along the horizontal axis.
Draw bars to represent the frequency of each category, making sure there are spaces between the bars.



c The data collected is an exam result of ‘A’, ‘B’, etc., so this is categorical data.
When assessing the exam results, the order is relevant (it makes sense to arrange the data in order from ‘A’ to ‘E’), so this is ordinal data.

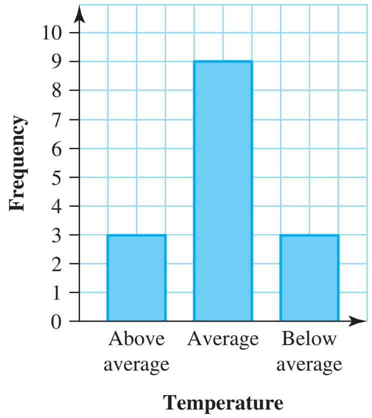
10 Choose an appropriate scale for the bar chart. As the frequencies go up to 90, we will mark our intervals in 10s. Display the different categories along the horizontal axis.
Draw bars to represent the frequency of each category, making sure there are spaces between the bars.
To calculate the frequency for each of the numbers of bedrooms, sum each column:
Number of properties sold with two bedrooms
$= 8 + 15 + 8 + 3 + 16 + 23 + 7$
$= 80$
Number of properties sold with three bedrooms
$= 12 + 11 + 12 + 9 + 18 + 19 + 9$
$= 90$
Number of properties sold with four bedrooms
$= 5 + 8 + 9 + 5 + 12 + 15 + 12$
$= 66$
Number of properties sold with five bedrooms
$= 4 + 6 + 2 + 1 + 11 + 9 + 3$
$= 36$



11 a Create a frequency table to capture the data. Then go through the data, filling in the tally column. Finally, sum each tally to complete the frequency column.
Remember that temperatures greater than or equal to 25 °C are considered above average and those less than 25 °C are considered below average.

<table>
<tr><th>Temperature</th><th>Tally</th><th>Frequency</th></tr>
<tr><td>Above average</td><td>|||</td><td>3</td></tr>
<tr><td>Average</td><td>卌 ||||</td><td>9</td></tr>
<tr><td>Below average</td><td>|||</td><td>3</td></tr>
</table>

b Choose an appropriate scale for the bar chart. As the frequencies only go up to 9, we will mark our intervals in single digits. Display the different categories along the horizontal axis.
Draw bars to represent the frequency of each category, making sure there are spaces between the bars.



c The data collected is temperatures that are either below average, average or above average, so this is categorical data.
When assessing the temperatures, the order is relevant (it makes sense to arrange the data in order from below average to above average, with average between them), so this is ordinal data.

12 a To calculate the total number of items purchased, sum the frequency column:

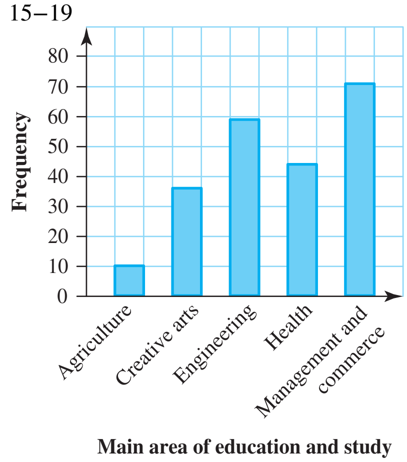
$$\text{Number of items purchased} = 6 + 8 + 5 + 11 + 3 + 7$$
$$= 40$$

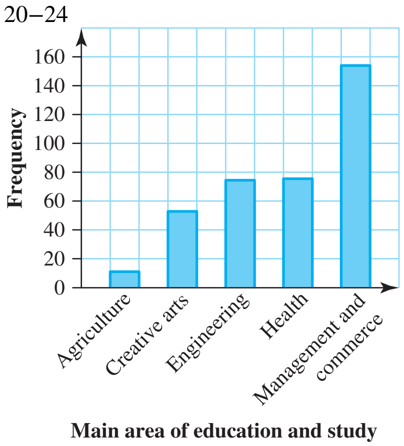
b $\text{Percentage} = \dfrac{6}{40} \times 100\%$
$= 15\%$

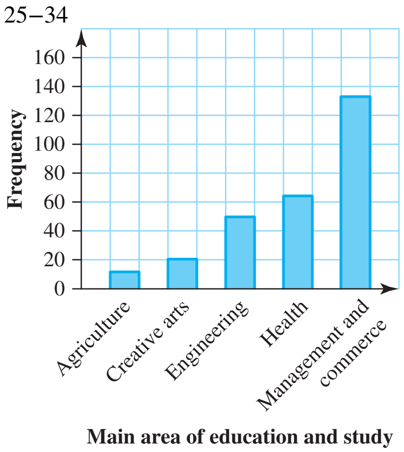
15% of the total purchases were fruit.

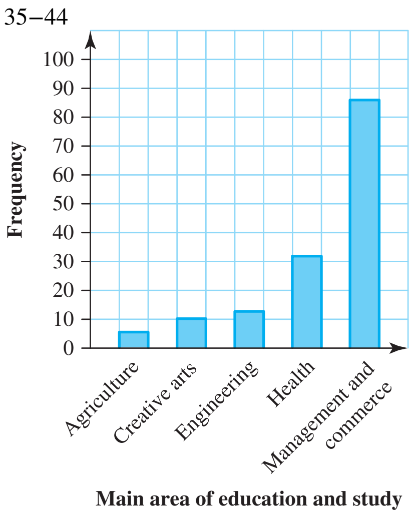
13 a For each age group, choose an appropriate scale for the bar chart. The scale’s highest value and interval size should be selected based on the highest frequency of the five areas of study for each age group. Display the different categories along the horizontal axis.
Draw bars to represent the frequency of each category, making sure there are spaces between the bars.

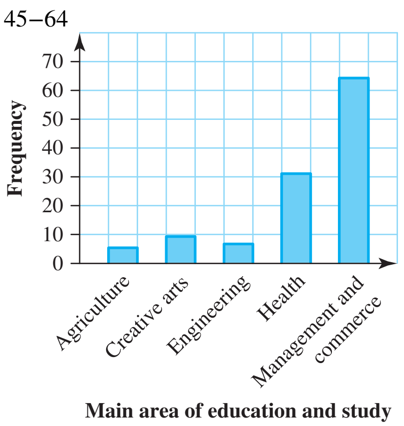
15–19











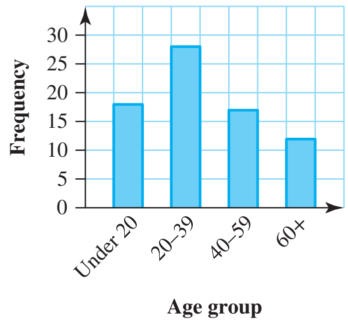
b In health, the highest number of people fall into the category 20–24. The mode is the highest number/most popular category. Therefore, 20–24 is the modal category.

14 a Create a frequency table to capture the data. Then read the frequency of each age group off the bar chart to complete the frequency column.

Age group	Frequency
Under 20	18
20–29	15
30–39	13
40–49	10
50–59	7
60+	12

b The mode is the category that has the highest frequency. Therefore, the modal category is under 20.

c Choose an appropriate scale for the bar chart. As the frequencies go up to 28, we will mark our intervals in 5s. Display the different categories along the horizontal axis.
Draw bars to represent the frequency of each category, making sure there are spaces between the bars.
Calculate the frequencies of each of the new categories by adding the frequencies of 20–29 with 30–39, and 40–49 with 50–59.



d Yes, the group with the highest bar has changed from under 20 on the first graph to 20–39 on the second graph.
The modal category is now 20–39.

1.3 Exam questions

1 The wind direction with the highest frequency (41) was north-west.
The correct answer is **E**.

2 The total number of days on which the wind direction was east or south-east was $10 + 25 = 35$.
The percentage is $\dfrac{35}{214} \times 100\% = 16.36\%$; the closest is 16%.
The correct answer is **B**.

3 There are 5 families with 5 children; therefore, B is false.
The correct answer is **B**.

1.4 Numerical data distributions — frequency tables and histograms

1.4 Exercise

1 Smallest value = 37.5
Largest value = 72.3
We will have class intervals of 10, starting with 30–<40.

Time (seconds)	Frequency
30–<40	3
40–<50	5
50–<60	7
60–<70	4
70–<80	1

2 a Smallest value = 87

Largest value = 124

We will have class intervals of 10, as specified, starting with 80–<90.

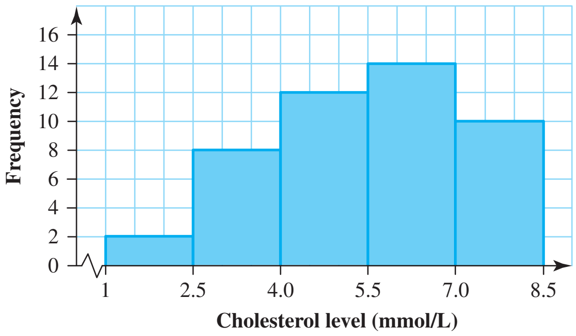
Time (seconds)	Frequency
80–<90	2
90–<100	6
100–<110	5
110–<120	5
120–<130	2

b We will have class intervals of 5, as specified, starting with 85–<90.

Time (seconds)	Frequency
85–<90	2
90–<95	1
95–<100	5
100–<105	2
105–<110	3
110–<115	3
115–<120	2
120–<125	2

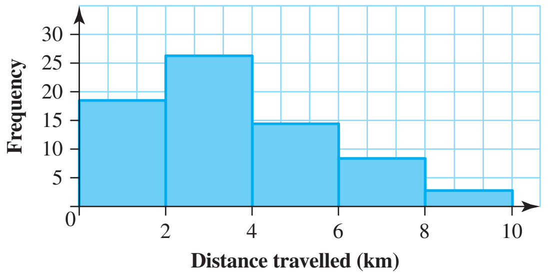
3 a The cholesterol level data in the table has intervals starting from 1 mmol/L and increasing by 1.5 mmol/L.

Draw rectangles for each interval to the height of the frequency indicated by the data in the table.



b The distance data in the table has intervals starting from 0 km and increasing by 2 km.

Draw rectangles for each interval to the height of the frequency indicated by the data in the table.



4 a Smallest value = 5

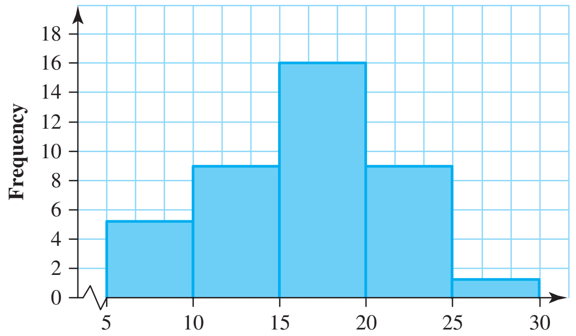
Largest value = 26

We will have class intervals of 5, as specified, starting with 5–<10.

Class interval	Frequency
5–<10	5
10–<15	9
15–<20	16
20–<25	9
25–<30	1

The data in the table has intervals starting from 5 and increasing by 5.

Draw rectangles for each interval to the height of the frequency indicated by the data in the table.



b Smallest value = 11

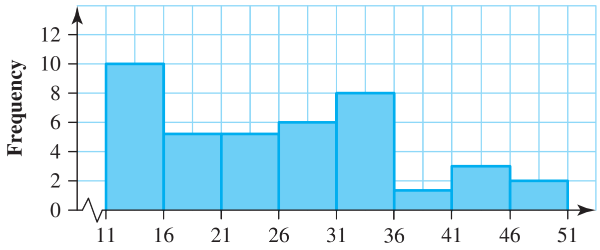
Largest value = 50

We will have class intervals of 5, as specified, starting with 11–<16.

Class interval	Frequency
11–<16	10
16–<21	5
21–<26	5
26–<31	6
31–<36	8
36–<41	1
41–<46	3
46–<51	2

The data in the table has intervals starting from 11 and increasing by 5.

Draw rectangles for each interval to the height of the frequency indicated by the data in the table.



5 **a** Smallest value = 22

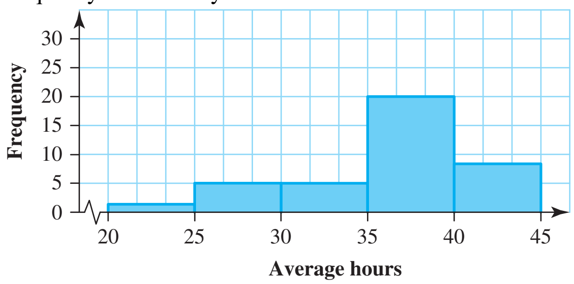
Largest value = 43

We will have class intervals of 5, as specified, starting with 20–<25.

Average hours	Frequency
20–<25	1
25–<30	5
30–<35	5
35–<40	21
40–<45	8

b The data in the table has intervals starting from 20 and increasing by 5.

Draw rectangles for each interval to the height of the frequency indicated by the data in the table.



6 The only option that is not true is C because one person over 90 years old visited the ice cream shop — one person in the 90–<100 category.

The correct answer is **C**.

7 **a** Most common is the tallest column. The most common is working 0–<5 hours.

b The frequency values come from the height of each column.

The frequency table is:

Number of hours worked	Frequency
0–<5	25
5–<10	14
10–<15	8
15–<20	3
20–<25	1

c Adding up the frequency values: 25 + 14 + 8 + 3 + 1 = 51

The total number of students surveyed is 51.

8 Data sets 1 and 3. Data set 2 is not included because the 4 is included in the second column (not the first column).

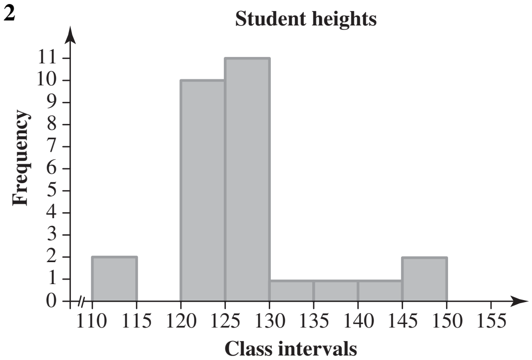
1.4 Exam questions

1

Class interval (5 cm)	Frequency
110–114	2
115–119	0
120–124	10
125–129	11
130–134	1
135–139	1
140–144	1
145–149	2

[1 mark]

2

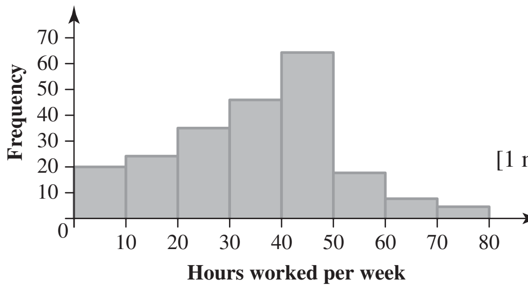


[1 mark]

3 **a** The total number of workers surveyed is the total of the frequencies in the table.

20 + 24 + 35 + 46 + 64 + 18 + 7 + 5 = 220 [1 mark]

b



[1 mark]

c The mode is the midpoint of the modal class (40 to less than 50 hours per week).

45 hours per week [1 mark]

1.5 Numerical data distributions — dot plots and stem plots

1.5 Exercise

1 **a** The money data has values in the 10s and 20s. Since there are 4 or less different place values, we will split each into two.

Stem	Leaf
1*	
2	
2*	

Write the units for each stem place value. Remember to add a key.

Key: 1*| 9 = $19

Stem	Leaf
1*	9
2	1 1 2 2 2 2 2 2 3 3 4 4
2*	5 6

b The time data has values in the units, 10s and 20s. Since there are 4 or less different place values, we will split each into two.

Stem	Leaf
0*	
1	
1*	
2	
2*	

Write the units for each stem place value. Remember to add a key.

Key: $0^{*}|6 = 6$ hours

Stem	Leaf
0*	6 7 9
1	3 4 4
1*	7 7
2	0 0 1 1 1 3 3 4 4
2*	5 5 6

2 a The passengers per day data has values in the 10s, 20s, 30s and 40s. Since there are four or fewer different place values, we will split each into two.

Leaf	Stem	Leaf
	1	
	1*	
	2	
	2*	
	3	
	3*	
	4	
	4*	

Write the units for each stem place value. Remember to add a key.

Key: $1|2 = 12$ passengers

March Leaf	Stem	April Leaf
3	1	2 4
7 5	1*	5 5 7
4 3 3 2	2	0 0 2 3 3
8 7 7 5	2*	7 7 7 8
4 4 3	3	0
7 6 5	3*	5 6 6
4 2	4	3
7	4*	

b The patients per day data has values in the 10s, 20s, 30s, 40s, 50s and 60s.

Leaf	Stem	Leaf
	1	
	2	
	3	
	4	
	5	
	6	

Write the units for each stem place value. Remember to add a key.

Key: $1|7 = 17$ patients

Dr. Hammond Leaf	Stem	Dr. Valenski Leaf
	1*	7 7
4 3 3	2	1 2 3 4 4
6 5	2*	5 5 6 6 8
4 2 1	3	0 0 3 4
8 8 7	3*	5
3 1	4	1
6 5 5 4	4*	
1	5	1
6 5	5*	
	6	0

3 The data has values in the units, 10s and 20s. Since there are 4 or less different place values, we will split each into two.

Stem	Leaf
0*	
1	
1*	
2	
2*	

Write the units for each stem place value. Remember to add a key.

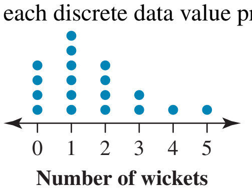
Key: $0^{*}|8 = 8$ people

Stem	Leaf
0*	8 8
1	3 3 4
1*	6 6 7 7 9
2	1 1
2*	5

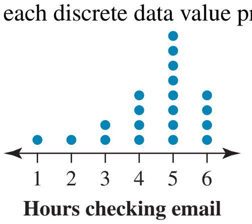
4 Looking at both stem plots, write the units for each stem place value. Remember to add a key.
Key: 0|1 = 1 game played

Squad 1 Leaf	Stem	Squad 2 Leaf
1	0	
4	1	
7	1*	
4 4	2	4
8	2*	
3 3	3	1 2
6 5	3*	6
3 2 1	4	3 4
	4*	5
1 1	5	2
5	6*	
	7	
	8	2
	8*	5 7
1	9	3

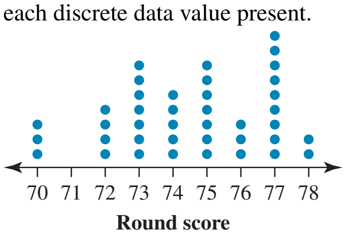
5 **a** The discrete data values are given by the number of wickets.
Draw a horizontal scale from 0 to 5.
Place one dot directly above the number on the scale for each discrete data value present.



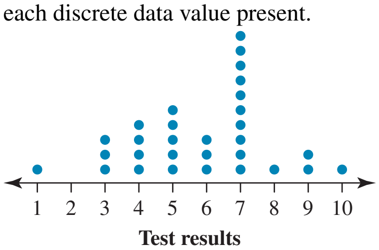
b The discrete data values are given by the number of hours checking emails.
Draw a horizontal scale from 1 to 6.
Place one dot directly above the number on the scale for each discrete data value present.



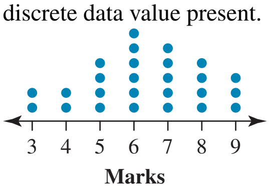
6 **a** The discrete data values are given by the scores per round of a golfer.
Draw a horizontal scale from 70 to 78.
Place one dot directly above the number on the scale for each discrete data value present.



b The discrete data values are given by the scores in a test for a group of students.
Draw a horizontal scale from 1 to 10.
Place one dot directly above the number on the scale for each discrete data value present.



7 The discrete data values are given by the marks on a test.
Draw a horizontal scale from 3 to 9.
Place one dot directly above the number on the scale for each discrete data value present.



8 **a** The data is numerical, continuous, grouped. So, the best option is histogram.
b The data is categorical, ordinal. So, the options are bar chart or dot plot.

9 The data is numerical, discrete. So, the options are histogram, stem plot or dot plot. A stem plot will not work because the values will be low and will not have enough stems.

10 **a** Split each place value into two as specified.

Stem	Leaf
0	
0*	
1	
1*	
2	
2*	

Write the units for each stem place value. Remember to add a key.
Key: 0|1 = 1

Stem	Leaf
0	1
0*	
1	1 1 1 4 4
1*	6 6 7 8
2	3 3 4 4
2*	7 7 9

b Splitting the stem for this data gives a clearer picture of the spread and shape of the distribution of the data set.

1.5 Exam questions

1 The mode is the most frequent value, which is 38 cm. [1 mark]

2 **a** Day number [1 mark]

***VCAA Examination Report* note:**
Most responses given to this question were correct. A small number of students answered 'neither'.

b Key: $4|1 = 4.1$ $n = 15$

minimum temperature (°C)

4	1 8
5	
6	0 7
7	0 5 7
8	0 6
9	0 2 8
10	7
11	8
12	7

Award 1 mark for all five numbers in correct positions.

***VCAA Examination Report* note:**

Some students entered only the values for day 11 and day 15 rather than the five days from day 11 to day 15.

3 Mode means the most frequent score.
Therefore, 2.8 °C is the modal temperature.
The correct answer is **A**.

1.6 Characteristics of numerical data distributions

1.6 Exercise

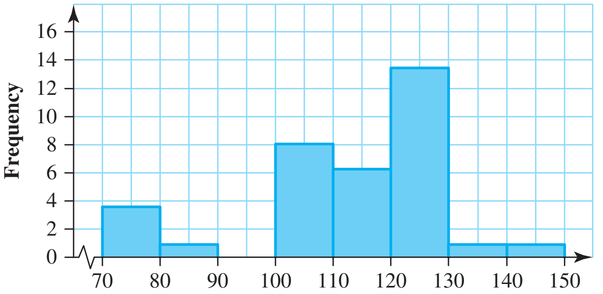
1 a Look for the mode and comment on its value.
Then identify the presence of any potential outliers.
Finally, describe the shape in terms of symmetry or skewness.
The distribution has one mode with data values that are most frequent in the 35–<40 interval. There are no obvious outliers, and there is a negative skew to the distribution.

b Look for the mode and comment on its value.
Then identify the presence of any potential outliers.
Finally, describe the shape in terms of symmetry or skewness.
The distribution has one mode with data values that are most frequent in the 45–<60 interval. There are potential outliers in the 120–<135 interval, and the distribution is either symmetrical (excluding the outliers) or has a slight positive skew (including the outliers).

2 a Smallest value = 70
Largest value = 140
We will have class intervals of 10, as specified, starting with 70–<80.

Class interval	Frequency
70–<80	4
80–<90	1
90–<100	0
100–<110	9
110–<120	7
120–<130	15
130–<140	1
140–<150	1

The data in the table has intervals starting from 70 and increasing by 10.
Draw rectangles for each interval to the height of the frequency indicated by the data in the table.

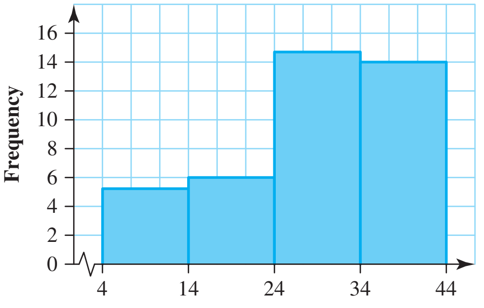


Look for the mode and comment on its value.
Then identify the presence of any potential outliers.
Finally, describe the shape in terms of symmetry or skewness.
The distribution has one mode with data values that are most frequent in the 120–<130 interval. There are potential outliers in the 70–<80 interval, and there is a negative skew to the distribution.

b Smallest value = 4
Largest value = 43
We will have class intervals of 10, as specified, starting with 4–<14.

Class interval	Frequency
4–<14	5
14–<24	6
24–<34	15
34–<44	14

The data in the table has intervals starting from 4 and increasing by 10.
Draw rectangles for each interval to the height of the frequency indicated by the data in the table.



Look for the mode and comment on its value.
Then identify the presence of any potential outliers.
Finally, describe the shape in terms of symmetry or skewness.
The distribution has one mode with data values that are most frequent in the 24–<34 interval. There are no obvious outliers, and there is a negative skew to the distribution.

3 a This information is found in the 'More than 5' row.

$$2 = \text{total} \times \frac{5}{100}$$

$$\text{Total} = 2 \times \frac{100}{5} = 40$$

Therefore, the frequency column should sum to 40.

For the 2–3 row, let a be the missing frequency:

$$\text{Percentage} = \frac{\text{frequency}}{\text{total}} \times 100$$

$$30 = \frac{a}{40} \times 100$$

$$\frac{30 \times 40}{100} = a$$

$$a = 12$$

For the 4–5 row, let b be the missing percentage:

$$\text{Percentage} = \frac{\text{frequency}}{\text{total}} \times 100$$

$$b = \frac{8}{40} \times 100$$

$$b = 20$$

For the 0–1 row, let c be the missing frequency:

$$c + 12 + 8 + 2 = 40$$

$$c = 18$$

For the 0–1 row, let d be the missing percentage:

$$d + 30 + 20 + 5 = 100$$

$$d = 45$$

Number of pets	Frequency	Percentage
0–1	18	45%
2–3	12	30%
4–5	8	20%
More than 5	2	5%
Total	40	100%

b The data collected is the number of pets owned, so this is numerical data.

The data involves counting pets, so only whole number values are possible.

The data collected is discrete data.

c The number of people surveyed corresponds to the total of the frequency column.

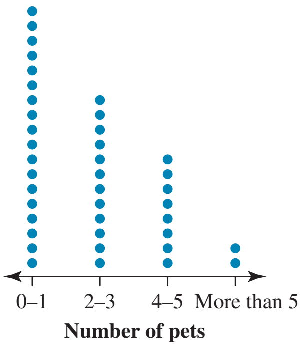
Therefore, 40 people were surveyed.

d Since the data is grouped data, we will not use a stem plot. Because the data is discrete, it is most appropriate to use a dot plot.

The discrete data values are given by the number of pets.

Draw a horizontal scale labelled with divisions for 0–1, 2–3, 4–5 and more than 5.

Place one dot directly above the number on the scale for each discrete data value present.



e Look for the mode and comment on its value.

Then identify the presence of any potential outliers.

Finally, describe the shape in terms of symmetry or skewness.

The distribution has one mode with data values that are most frequent in the 0–1 interval. There are no obvious outliers, and the distribution has a positive skew.

4 a First read off the graph the percentage value for the 90–<120 interval: 40%

Next read off the graph the percentage value for the 120–<150 interval: 25%

45% + 25% = 65%

65% of patients had to wait more than 90 minutes.

b First read off the graph the percentage value for the 0–<30 interval: 5%

Next read off the graph the percentage value for the 30–<60 interval: 10%

5% + 10% = 15%

15% of patients received treatment in less than one hour.

c Look for the mode and comment on its value.

Then identify the presence of any potential outliers.

Finally, describe the shape in terms of symmetry or skewness.

The distribution has one mode with data values that are most frequent in the 90–<120 interval. There are no obvious outliers, and the distribution has a negative skew.

5 a Smallest value = 117

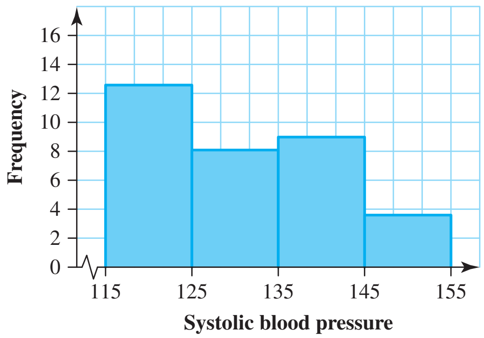
Largest value = 148

We will have class intervals of 10, as specified, starting with 115–<125.

Class interval	Frequency
115–<125	14
125–<135	9
135–<145	10
145–<155	4

The data in the table has intervals starting from 115 and increasing by 10.

Draw rectangles for each interval to the height of the frequency indicated by the data in the table.



b Look for the mode and comment on its value.

Then identify the presence of any potential outliers.

Finally, describe the shape in terms of symmetry or skewness.

The distribution has one mode with data values that are most frequent in the 115–<125 interval. There are no obvious outliers, and the distribution has a positive skew.

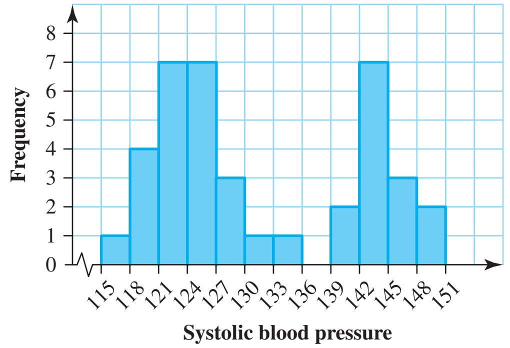
c Smallest value = 117

Largest value = 148

We will have class intervals of 3, as specified, starting with 115–<118.

Class interval	Frequency
115–<118	1
118–<121	4
121–<124	7
124–<127	7
127–<130	3
130–<133	1
133–<136	1
136–<139	0
139–<142	2
142–<145	7
145–<148	3
148–<151	2

The data in the table has intervals starting from 115 and increasing by 3.
Draw rectangles for each interval to the height of the frequency indicated by the data in the table.



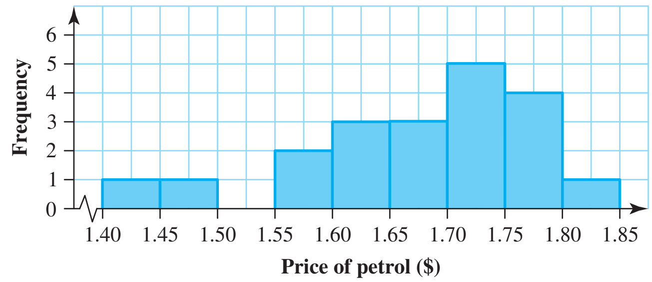
d Look for the mode and comment on its value.
Then identify the presence of any potential outliers.
Finally, describe the shape in terms of symmetry or skewness.
The second histogram is split into two distinct groups with three modes. The lower group is symmetrical around the interval 121–<128. The upper group has a slight positive skew.

e The second histogram might demonstrate that there are two distinct groups present in the data, for example a younger age group and an older age group.

6 a Smallest value = 1.40
Largest value = 1.82
We will have class intervals of 0.05 (5 cents), as specified, starting with 1.40–<1.45.

Class interval	Frequency
1.40–<1.45	1
1.45–<1.50	1
1.50–<1.55	0
1.55–<1.60	2
1.60–<1.65	3
1.65–<1.70	3
1.70–<1.75	5
1.75–<1.80	4
1.80–<1.85	1

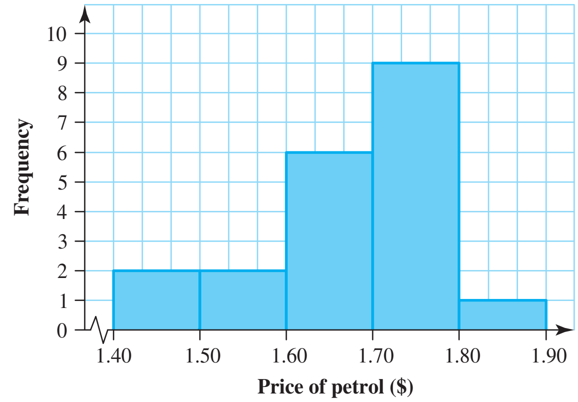
The data in the table has intervals starting from 1.40 and increasing by 0.05. Draw rectangles for each interval to the height of the frequency indicated by the data in the table.



b Smallest value = 1.40
Largest value = 1.82
We will have class intervals of 0.10 (i.e. 10 cents), as specified, starting with 1.40–<1.50.

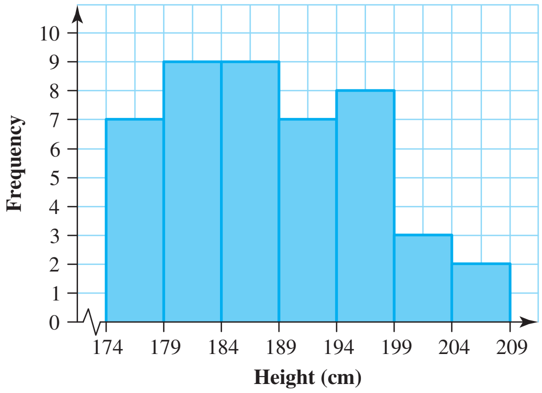
Class interval	Frequency
1.40–<1.50	2
1.50–<1.60	2
1.60–<1.70	6
1.70–<1.80	9
1.80–<1.90	1

The data in the table has intervals starting from 1.40 and increasing by 0.10.
Draw rectangles for each interval to the height of the frequency indicated by the data in the table.

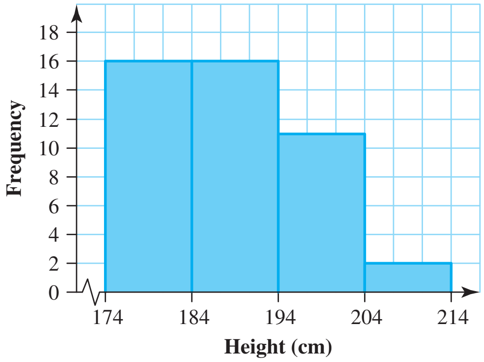


c Look for the mode and comment on its value.
Then identify the presence of any potential outliers.
Finally, describe the shape in terms of symmetry or skewness.
Both histograms have one mode with a negative skew to the distribution. The first histogram gives the impression that there may be outliers at the start of the data set, but this is not as evident in the second histogram.

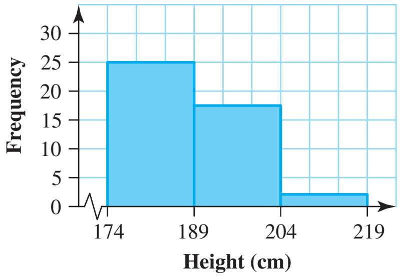
7 a i



ii



iii



b Look for the mode and comment on its value.
Then identify the presence of any potential outliers.
Finally, describe the shape in terms of symmetry or skewness.

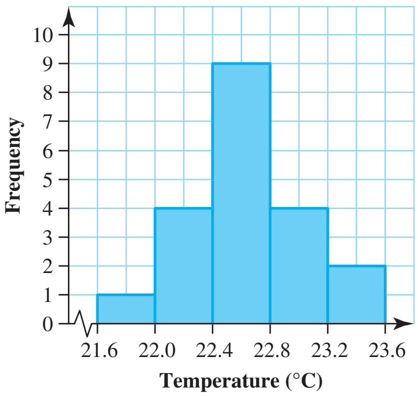
i This distribution has two modes (181.5 and 186.5), with a positive skew and no obvious outliers.

ii This distribution also has two modes (179 and 189), with a positive skew and no obvious outliers.

iii This distribution has one mode (181.5), with a positive skew and no obvious outliers.

The histogram with more intervals allows you to see the shape and details of the distribution better, although the overall shape is fairly similar in all three histograms.

8



Look for the mode and comment on its value.
Then identify the presence of any potential outliers.
Finally, describe the shape in terms of symmetry or skewness.
The distribution for the interval 1993–2012 is approximately symmetrical, with a slight negative skew.
The distribution has one mode.

1.6 Exam questions

1 The data is bunched down the lower end of the histogram with a tail going to the right; therefore, it is positively skewed. Given the large range of the data, it is likely that the data value at the upper end is an outlier.
The correct answer is **B**.

2 There is an outlier present and the data trails on the lower end, so the distribution is negatively skewed with an outlier.
The correct answer is **E**.

3 The data trails off on the positive end so the distribution is positively skewed.
The correct answer is **B**.

1.7 Summarising numerical data — mean and median

1.7 Exercise

1 $108 + 135 + 120 + 132 + 113 + 138 + 125 + 138 + 107 + 131$
$+113 + 136 + 119 + 152 + 134 + 158 + 136 + 132 + 113 + 128$
$= 2568$

$\bar{x} = \dfrac{2568}{20}$
$= 128.4$

The mean of the data set is 128.4.

2 a $25 + 23 + 24 + 25 + 27 + 26 + 23 + 28 + 24 + 20 + 25 + 20 + 29 + 28 + 23 + 27 + 24 = 421.$

$\bar{x} = \dfrac{421}{17}$
$= 24.76$

The mean of the data set is 24.76.

b Replacing the highest value in the data set, 29, with the new value of 79:

$25 + 23 + 24 + 25 + 27 + 26 + 23 + 28 + 24 + 20 + 25 + 20 + 79 + 28 + 23 + 27 + 24 = 471$

$\bar{x} = \dfrac{471}{17}$
$= 27.71$

The new mean of the data set is 27.71.

c Changing the highest value will affect the mean, increasing it if the highest value is increased.

3 $1 + 1 + 5 + 7 + 12 + 16 + 23 + 24 + 24 + 25 + 31 + 33 + 40 + 40 + 43 + 55 + 65 = 445$

$\bar{x} = \dfrac{445}{17}$
$= 26.18$

The mean of the data set is 26.18.

4 a Put the data set in order from lowest to highest.
3, 11, 15, 15, 16, 27, 41, 42, 49, 53, 53, 54, 56, 62, 72, 75, 81, 85
There are 18 data values, so the median will be in position $\left(\dfrac{18+1}{2}\right) = 9.5$, or halfway between position 9 and position 10

3, 11, 15, 15, 16, 27, 41, 42, $\boxed{49, 53,}$ 53, 54, 56, 62, 72, 75, 81, 85

Median $= \dfrac{49 + 53}{2} = 51$

The median of the data set is 51.

b Put the data set in order from lowest to highest.

85, 109, 126, 149, 165, 170, 179, 180, 198, 206, 223, 267, 301, 335, 422, 567, 602

There are 17 data values, so the median will be in position $\left(\dfrac{17+1}{2}\right) = 9$.

85, 109, 126, 149, 165, 170, 179, 180, $\boxed{198}$, 206, 223, 267, 301, 335, 422, 567, 602

Median = 198

The median of the data set is 198.

5 a Read the data values off the table.

$1276 + 654 + 1194 + 563 + 745 + 576 + 1847 + 630 + 326 = 7811$

$$\bar{x} = \frac{7811}{9} = 867.89$$

The mean of the data set is 867.89 mm.

Put the data set in order from lowest to highest.

326, 563, 576, 630, 654, 745, 1194, 1276, 1847

There are 9 data values, so the median will be in position $\left(\dfrac{9+1}{2}\right) = 5$.

326, 563, 576, 630, $\boxed{654,}$ 745, 1194, 1276, 1847

Median = 654

The median of the data set is 654 mm.

b The data set has an outlier at 1847 mm for Darwin. If there are outliers, the median will be significantly less affected by these and would be a better choice to represent the data.

Therefore, in the given case, the best measure of centre is the median, as it is not affected by the extreme values present in the data set.

6 a Put the data set in order from lowest to highest.

20, 21, 21, 22, 22, 23, 23, 23, 24, 24, 24, 25, 26, 27, 27, 28, 31

There are 17 data values, so the median will be in position $\left(\dfrac{17+1}{2}\right) = 9$.

20, 21, 21, 22, 22, 23, 23, 23, $\boxed{24,}$ 24, 24, 25, 26, 27, 27, 28, 31

Median = 24

The median of the data set is 24.

b Put the data set in order from lowest to highest.

20, 21, 21, 22, 22, 23, 23, 23, 24, 24, 24, 25, 26, 27, 27, 28, 96

There are 17 data values, so the median will be in $\left(\dfrac{17+1}{2}\right) = 9$.

20, 21, 21, 22, 22, 23, 23, 23, $\boxed{24,}$ 24, 24, 25, 26, 27, 27, 28, 96

Median = 24

The median of the data set is 24.

c From above we can see that the median is the same in both instances.

The median is unchanged.

7 a Read the data off the dot plot, writing it in order from lowest to highest.

76, 76, 76, 76, 76, 80, 80, 84, 84, 84, 84, 84, 88, 96, 96, 104, 104, 104, 104, 104, 104, 108, 108, 108, 108, 112, 112, 112, 112, 112

There are 30 data values, so the median will be in position $\left(\dfrac{30+1}{2}\right) = 15.5$, or halfway between position 15 and position 16.

76, 76, 76, 76, 76, 80, 80, 84, 84, 84, 84, 84, 88, 96, $\boxed{96, 104}$, 104, 104, 104, 104, 104, 108, 108, 108, 108, 112, 112, 112, 112, 112

Median $= \left(\dfrac{96 + 104}{2}\right)$

Median = 100

The median of the data set is 100.

b Put the data set in order from lowest to highest.

1.02, 1.17, 1.38, 1.38, 1.47, 1.49, 1.70, 1.91, 2.01, 3.21, 3.45, 4.63, 5.02, 8.54

There are 14 data values, so the median will be in position $\left(\dfrac{14+1}{2}\right) = 7.5$, or halfway between position 7 and position 8.

1.02, 1.17, 1.38, 1.38, 1.47, 1.49, $\boxed{1.70, 1.91}$, 2.01, 3.21, 3.45, 4.63, 5.02, 8.54

Median $= \dfrac{1.70 + 1.91}{2} = 1.805$

The median of the data set is 1.805.

8 The data set has two outliers. If there are outliers, the median will be significantly less affected by these and would be a better choice to represent the data.

Therefore, in the given case, the best measure of centre is the median, as the data set has two clear outliers.

9 a $45 + 50 + 55 + 55 + 55 + 60 + 65 + 65 + 70 + 70 + 75 + 80 + 220 = 965$

$\bar{x} = \dfrac{965}{13}$.

$\bar{x} = 74.231$

The mean of the data set, correct to 3 decimal places, is 74.231.

The money is in \$1000s, so we must multiply the result by 1000.

Therefore, the mean of the salaries is \$74 231.

b Note that the given data is already ordered from lowest to highest.

45, 50, 55, 55, 55, 60, 65, 65, 70, 70, 75, 80, 220

There are 13 data values, so the median will be in position $\left(\dfrac{13+1}{2}\right) = 7$.

45, 50, 55, 55, 55, 60, $\boxed{65,}$ 65, 70, 70, 75, 80, 220

Median = 65

The median of the data set is 65.

The money is in \$1000s, so we must multiply the result by 1000.

Therefore, the median of the salaries is \$65 000.

c Note that using the median gives a lower salary, while using the mean gives a higher salary (caused, in part, by the outlier at \$220 000).

It would be in the workers' interest to use a higher figure when negotiating salaries, whereas it would be in the management's interest to use a lower figure.

10 **a** For simplicity, deal with the data values in the \$1000s.

$$\begin{aligned}&4700 + 3160 + 2725 + 2616 + 2560 + 241 + 265\\&\quad +266 + 310 + 320 + 3010 + 2580 + 2450\\&\quad +2300 + 2275 + 286 + 325 + 330 + 435.5\\&\quad +456 + 1350 + 1020 + 900 + 735 + 733\\&\quad +305 + 330 + 347 + 357 + 408\\&= 38\,095.5\end{aligned}$$

$$\bar{x} = \frac{38\,095.5}{30}$$

$$\bar{x} = 1269.85$$

To find the final value, multiply by 1000:

$1269.85 \times 1000 = 1\,269\,850$

The mean of the data set is \$1 269 850.

Put the data set in order from lowest to highest.

241, 265, 266, 286, 305, 310, 320, 325, 330,
330, 347, 357, 408, 435.5, 456, 733, 735,
900, 1020, 1350, 2275, 2300, 2450, 2560
2580, 2616, 2725, 3010, 3160, 4700

There are 30 data values, so the median will be in position $\left(\frac{30+1}{2}\right) = 15.5$, or halfway between position 15 and position 16.

241, 265, 266, 286, 305, 310, 320, 325, 330,
330, 347, 357, 408, 435.5, $\boxed{456,\ 733}$, 735,
900, 1020, 1350, 2275, 2300, 2450, 2560
2580, 2616, 2725, 3010, 3160, 4700

$$\text{Median} = \frac{456 + 733}{2} = 594.5$$

To find the final value, multiply by 1000:

$594.5 \times 1000 = 594\,500$

The median of the data set is \$594 500.

b, c Still dealing with the data values in the \$1000s,

Smallest value = 241

Largest value = 4700

We will have class intervals of 250 (i.e. \$250 000), as specified, starting with 241–<491.

Class interval	Frequency
241–<491	15
491–<741	2
741–<991	1
991–<1241	1
1241–<1491	1
1491–<1741	0
1741–<1991	0
1991–<2241	0
2241–<2491	3
2491–<2741	4
2741–<2991	0
2991–<3241	2
3241–<3491	0
3491–<3741	0
3741–<3991	0
3991–<4241	0
4241–<4491	0
4491–<4741	1

The data in the table has intervals starting from 241 and increasing by 250.

Draw rectangles for each interval to the height of the frequency indicated by the data in the table.

See the image at the bottom of the page.*

d The data set has some high values, which may be considered to be outliers. If there are outliers, the median will be significantly less affected by these and would be a better choice to represent the data.

Therefore, in the given case, the best measure of centre is the median, as the mean is affected by a few very high values.

11 Adding minutes:

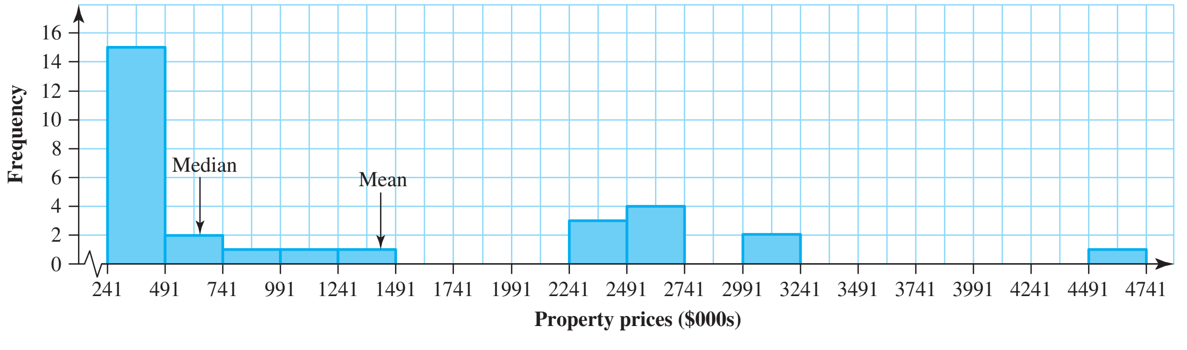
$19+17+14+16+12+11+13+16+11+12+14+17+13+14+15+15+18+11+12+13+13+13+13+14 = \boxed{336}$

Adding seconds:

$28+35+21+22+18+9+15+21+45+26+16+12+42+51+26+13+2+22+26+10+18+41+23+6 = 548$

$$\begin{aligned}\text{Total seconds} &= 336 \times 60 + 548\\&= 20\,160 + 548\\&= 20\,708\end{aligned}$$

***10 b, c**



$\bar{x} = \dfrac{20\,708}{24}$
$= 862.83...$
$= 863$ (to the nearest second)
863 seconds = 14 minutes, 23 seconds.
The mean of the data set is 14:23.

12 **a** $2 \times 4 + 4 \times 12 + 6 \times 8 + 8 \times 5 + 10 \times 4 + 12 \times 4 + 16 + 20 + 34$
$= 302$
$\bar{x} = \dfrac{302}{40}$
$= 7.55$
The mean of the data set is 7.55.
Put the data set in order from lowest to highest.
2, 2, 2, 2, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 6,
6, 6, 6, 6, 6, 6, 6, 8, 8, 8, 8, 8, 10, 10, 10,
10, 12, 12, 12, 12, 16, 20, 34
There are 40 data values, so the median will be in position $\left(\dfrac{40+1}{2}\right) = 20.5$, or halfway between position 20 and position 21.
2, 2, 2, 2, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4,
6, 6, 6, $\boxed{6, 6,}$ 6, 6, 6, 8, 8, 8, 8, 8, 10, 10,
10, 10, 12, 12, 12, 12, 16, 20, 34
Median = 6
The median of the data set is 6.

b The data set has an outlier at 34. If there are outliers, the median will be significantly less affected by these and would be a better choice to represent the data.
Therefore, in the given case, the median would be the preferred choice due to the extreme value of 34.

1.7 Exam questions

1 **a** The median age is 24 years. [1 mark]
b $\dfrac{\sum \text{body density}}{12} = 1.065$ kg/L [1 mark]

2 The median BMI will be in the $\left(\dfrac{32+1}{2}\right) = 16.5$th position.
Therefore, the median is 24.55 kg/m^2. [1 mark]

3 The middle value of any data set is always the median.
The mean is dependent on extreme values, and we do not have the information to know if there are any, so we cannot assume that the middle value will also be the mean.
The correct answer is **B**.

1.8 Summarising numerical data — range, interquartile range and standard deviation

1.8 Exercise

1 Looking at the data, the largest value is 512 and the lowest value is 105. To calculate the range:
$512 - 105 = 407$
The range is 407.

2 Put the data in order.
1.13, 1.21, 1.56, 1.98, 2.12, 2.22, 3.11, 3.16, 3.19,
3.21, 3.34, 4.43, 4.78, 4.89, 5.67, 8.88
There are 16 data values, so the median will be in position $\left(\dfrac{16+1}{2}\right) = 8.5$, or halfway between position 8 and position 9.
1.13, 1.21, 1.56, 1.98, 2.12, 2.22, 3.11, 3.16, 3.19,
3.21, 3.34, 4.43, 4.78, 4.89, 5.67, 8.88
Median $= \dfrac{3.16 + 3.19}{2} = 3.175$
The median is 3.175.
There are 8 values in the lower half of the data, so Q_1 will be halfway between the 4th and 5th of these values.
Q_1
1.13, 1.21, 1.56, 1.98, 2.12, 2.22, 3.11, 3.16
$Q_1 = \dfrac{1.98 + 2.12}{2} = 2.05$
There are 8 values in the upper half of the data, so Q_3 will be halfway between the 4th and 5th of these values.
Q_3
3.19, 3.21, 3.34, 4.43, 4.78, 4.89, 5.67, 8.88
$Q_3 = \dfrac{4.43 + 4.78}{2} = 4.605$
$\text{IQR} = Q_3 - Q_1$
$= 4.605 - 2.05$
$= 2.555$
The interquartile range is 2.555.

3 **a** First look at Class A:.
Put the data in order.
7, 8, 11, 12, 12, 12, 13, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 15, 16, 18
$\text{Range}_A = 18 - 7$
$= 11$
Now look at Class B:
Put the data in order.
9, 11, 11, 12, 12, 12, 13, 13, 13, 14, 14, 14, 15, 17, 17, 17, 17, 18, 18, 19
$\text{Range}_B = 19 - 9$
$= 10$

b First look at Class A:
There are 20 data values, so the median will be in position $\left(\dfrac{20+1}{2}\right) = 10.5$, or halfway between position 10 and position 11.
7, 8, 11, 12, 12, 12, 13, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 15, 16, 18
Median=13
The median is 13.
There are 10 values in the lower half of the data, so Q_1 will be halfway between the 5th and 6th of these values.
Q_1
7, 8, 11, 12, 12, 12, 13, 13, 13, 13
$Q_1 = 12$
There are 10 values in the upper half of the data, so Q_3 will be halfway between the 5th and 6th of these values.
Q_3
13, 14, 14, 14, 14, 14, 14, 15, 16, 18
$Q_3 = 14$

$\text{IQR} = Q_3 - Q_1$
$= 14 - 12$
$= 2$

The interquartile range is 2.

Next look at Class B:

There are 20 data values, so the median will be in position $\left(\frac{20+1}{2}\right) = 10.5$, or halfway between position 10 and position 11.

9, 11, 11, 12, 12, 12, 13, 13, 13, 14, 14, 14, 15, 17, 17, 17, 17, 18, 18, 19

Median=14

The median is 14.

There are 10 values in the lower half of the data, so Q_1 will be halfway between the 5th and 6th of these values.

Q_1

9, 11, 11, 12, 12, 12, 13, 13, 13, 14

$Q_1 = 12$

There are 10 values in the upper half of the data, so Q_3 will be halfway between the 5th and 6th of these values.

Q_3

14, 14, 15, 17, 17, 17, 17, 18, 18, 19

$Q_3 = 17$

$\text{IQR} = Q_3 - Q_1$
$= 17 - 12$
$= 5$

The interquartile range is 5.

4 a Looking at the 'Goals for' column:

Put the data in order

652, 688, 692, 699, 700, 715, 736, 749, 793, 812

$\text{Range} = 812 - 652$
$= 160$

b There are 10 data values, so the median will be in position $\left(\frac{10+1}{2}\right) = 5.5$, or halfway between position 5 and position 6.

652, 688, 692, 699, 700, 715, 736, 749, 793, 812

$\text{Median} = \dfrac{700 + 715}{2} = 707.5$

The median is 707.5.

There are 5 values in the lower half of the data, so Q_1 will be the 3rd of these values.

Q_1

652, 688, 692, 699, 700

$Q_1 = 692$

There are 5 values in the upper half of the data, so Q_3 will be the 3rd of these values.

Q_3

715, 736, 749, 793, 812

$Q_3 = 749$

$\text{IQR} = Q_3 - Q_1$
$= 749 - 692$
$= 57$

The interquartile range is 57.

c Looking at the 'Goals against' column:

Put the data in order

589, 620, 650, 672, 691, 706, 757, 790, 879, 882

$\text{Range} = 882 - 589$
$= 293$

There are 10 data values, so the median will be in position $\left(\frac{10+1}{2}\right) = 5.5$, or halfway between position 5 and position 6.

589, 620, 650, 672, 691, 706, 757, 790, 879, 882.

$\text{Median} = \dfrac{691 + 706}{2} = 698.5.$

The median is 698.5.

There are 5 values in the lower half of the data, so Q_1 will be the 3rd of these values.

Q_1

589, 620, 650, 672, 691

$Q_1 = 650$

There are 5 values in the upper half of the data, so Q_3 will be the 3rd of these values.

Q_3

706, 757, 790, 879, 882

$Q_3 = 790$

$\text{IQR} = Q_3 - Q_1$
$= 790 - 650$
$= 140$

The interquartile range is 140.

The 'Goals against' column is significantly more spread out than the 'Goals for' column.

5 First calculate the mean: $(10 + 11 + 19 + 21) \div 4 = 15.25$

The mean is 15.25.

Next, calculate the sum of the difference between each point and the mean:

$(10 - 15.25)^2 + (11 - 15.25)^2 + (19 - 15.25)^2$
$+ (21 - 15.25)^2 = 92.75$

Substitute everything into the standard deviation formula:

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

$$\sqrt{\frac{92.75}{4 - 1}}$$

$= 5.560\,275...$

The standard deviation is 5.56.

6 a Using CAS:

$s = 7.37$

b Using CAS:

$Q_1 = 82$

$Q_3 = 89$

$\text{IQR} = Q_3 - Q_1$
$= 89 - 82$
$= 7$

c Using CAS:

$s = 11.49$

Using CAS:

$Q_1 = 82$

$Q_3 = 89$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 89 - 82 \\ &= 7\end{aligned}$$

d The standard deviation increased by 4.12, while the interquartile range was unchanged.

7 a Lowest value = 8.0, highest value = 10.9.

$$\begin{aligned}\text{Range} &= \text{highest value} - \text{lowest value} \\ &= 10.9 - 8.0 \\ &= 2.9\end{aligned}$$

b There are 36 values, so the lower half of the data set consists of 18 values and the upper half of the data set consists of 18 values.

So Q_1 will be halfway between the 9th and 10th values, and Q_3 will be halfway between the 27th and 28th values.

Order the data:

8.0, 8.3, 8.4, 8.4, 8.5, 8.5, 8.5, 8.7, 8.8, 8.9, 9.0, 9.0, 9.0, 9.1, 9.3, 9.3, 9.4, 9.5, 9.5, 9.6, 9.8, 9.8, 10.0, 10.0, 10.0, 10.1, 10.1, 10.1, 10.3, 10.3, 10.5, 10.6, 10.6, 10.6, 10.8, 10.9

$$Q_1 = \frac{8.8 + 8.9}{2} = 8.85$$

$$Q_3 = \frac{10.1 + 10.1}{2} = 10.1$$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 10.1 - 8.85 \\ &= 1.25\end{aligned}$$

c The range is less than double the value of the interquartile range. This indicates that the data is quite tightly bunched with no outliers.

8 a Year 1

Put the data in order: 73 302, 191 763, 271 365, 915 059, 1 205 266, 2 138 364, 2 997 856, 3 395 905

There are 8 data values, so the median will be in position $\left(\frac{8+1}{2}\right) = 4.5$, or halfway between position 4 and position 5.

73 302, 191 763, 271 365, 915 059, 1 205 266, 2 138 364, 2 997 856, 3 395 905

$$\text{Median} = \frac{915\,059 + 1\,205\,266}{2} = 1\,060\,162.5.$$

The median is 1 060 162.5.

There are 4 values in the lower half of the data, so Q_1 will be halfway between the 2nd and 3rd of these values.

Q_1

73 302, 191 763, 271 365, 915 059.

$$Q_1 = \frac{191\,763 + 271\,365}{2} = 231\,564$$

There are 4 values in the upper half of the data, so Q_3 will be halfway between the 2nd and 3rd of these values.

Q_3

1 205 266, 2 138 364, 2 997 856, 3 395 905

$$Q_3 = \frac{2\,138\,364 + 2\,997\,856}{2} = 2\,568\,110$$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 2\,568\,110 - 231\,564 \\ &= 2\,336\,546\end{aligned}$$

The interquartile range is 2 336 546.

$$\bar{x} = \frac{3\,395\,905 + 2\,997\,856 + 2\,138\,364 + 915\,059 + 1\,205\,266 + 271\,365 + 73\,302 + 191\,763}{8}$$

$$\bar{x} = 1\,398\,610$$

$$s = \sqrt{\frac{\begin{array}{c}(3\,395\,905 - 1\,398\,610)^2 + (2\,997\,856 - 1\,398\,610)^2 + (2\,138\,364 - 1\,398\,610)^2 + (915\,059 - 1\,398\,610)^2 \\ +(1\,205\,266 - 1\,398\,610)^2 + (271\,365 - 1\,398\,610)^2 + (73\,302 - 1\,398\,610)^2 + (191\,763 - 1\,398\,610)^2\end{array}}{8 - 1}}$$

$$s = 1\,301\,033.5$$

The standard deviation is 1 301 033.5.

Year 2

Put the data in order.

91 071, 229 060, 305 913, 1 016 590, 1 476 743, 2 556 581, 3 446 548, 3 877 515

There are 8 data values, so the median will be in position $\left(\frac{8+1}{2}\right) = 4.5$, or halfway between position 4 and position 5.

91 071, 229 060, 305 913, 1 016 590, 1 476 743, 2 556 581, 3 446 548, 3 877 515

$\text{Median} = \dfrac{1\,016\,590 + 1\,476\,743}{2} = 1\,246\,666.5$

The median is 1 246 666.5.

There are 4 values in the lower half of the data, so Q_1 will be halfway between the 2nd and 3rd of these values.

$$91\,071, \underbrace{229\,060, 305\,913,}_{Q_1} 1\,016\,590$$

$Q_1 = \dfrac{229\,060 + 305\,913}{2} = 267\,486.5$

There are 4 values in the upper half of the data, so Q_3 will be halfway between the 2nd and 3rd of these values.

$$1\,476\,743, \underbrace{2\,556\,581, 3\,446\,548,}_{Q_3} 3\,877\,515$$

$Q_3 = \dfrac{2\,556\,581 + 3\,446\,548}{2} = 3\,001\,564.5$

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 3\,001\,564.5 - 267\,486.5 \\ &= 2\,734\,078 \end{aligned}$$

The interquartile range is 2 734 078.

$$\bar{x} = \frac{3\,877\,515 + 3\,446\,548 + 2\,556\,581 + 1\,016\,590 + 1\,476\,743 + 305\,913 + 91\,071 + 229\,060}{8}$$

$\bar{x} = 1\,625\,002.6$

$$s = \sqrt{\frac{\begin{array}{c}(3\,877\,515 - 1\,625\,002.6)^2 + (34\,46548 - 1\,625\,002.6)^2 + (2\,556\,581 - 1\,625\,002.6)^2 + (1\,016\,591 - 1\,625\,002.6)^2 \\ +(1\,476\,743 - 1\,625\,002.6)^2 + (305\,913 - 1\,625\,002.6)^2 + (91\,071 - 1\,625\,002.6)^2 + (229\,060 - 1\,625\,002.6)^2\end{array}}{8-1}}$$

$s = 1\,497\,303.5$

The standard deviation is 1 497 303.5.

b Year 1

Put the data in order.

915 059, 1 205 266, 2 138 364, 2 997 856, 3 395 905

There are 5 data values, so the median will be in position $\left(\dfrac{5+1}{2}\right) = 3$.

915 059, 1 205 266, 2 138 364, 2 997 856, 3 395 905

The median is 2 138 364.

There are 2 values in the lower half of the data, so Q_1 will be halfway between these values.

$$\underbrace{915\,059, 1\,205\,266}_{Q_1}$$

$Q_1 = \dfrac{915\,059 + 1\,205\,266}{2} = 1\,060\,162.5$

There are 2 values in the upper half of the data, so Q_3 will be halfway between these values.

$$\underbrace{2\,997\,856, 3\,395\,905}_{Q_3}$$

$Q_3 = \dfrac{2\,997\,856 + 3\,395\,905}{2} = 3\,196\,880.5$

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 3\,196\,880.5 - 1\,060\,162.5 \\ &= 2\,136\,718 \end{aligned}$$

The interquartile range is 2 136 718.

$$\bar{x} = \frac{3\,395\,905 + 2\,997\,856 + 2\,138\,364 + 915\,059 + 1\,205\,266}{5}$$

$\bar{x} = 2\,130\,490$

$$s = \sqrt{\frac{\begin{array}{c}(3\,395\,905 - 2\,130\,490)^2 + (2\,997\,856 - 2\,130\,490)^2 + (2\,138\,364 - 2\,130\,490)^2 + (915\,059 - 2\,130\,490)^2 \\ +(1\,205\,266 - 2\,130\,490)^2\end{array}}{5-1}}$$

$s = 1\,082\,470.9$

The standard deviation is 1 082 470.9.

Year 2

Put the data in order.

1 016 590, 1 476 743, 2 556 581, 3 446 548, 3 877 515

There are 5 data values, so the median will be in position $\left(\dfrac{5+1}{2}\right) = 3$.

1 016 590, 1 476 743, 2 556 581, 3 446 548, 3 877 515

The median is 2 556 581.

There are 2 values in the lower half of the data, so Q_1 will be halfway between these values.

$$\overbrace{1\,016\,590,\ 1\,476\,743}^{Q_1}$$

$$Q_1 = \frac{1\,016\,590 + 1\,476\,743}{2} = 1\,246\,666.5$$

There are 2 values in the upper half of the data, so Q_3 will be halfway between these values.

$$\overbrace{3\,446\,548,\ 3\,877\,515}^{Q_3}$$

$$Q_3 = \frac{3\,446\,548 + 3\,877\,515}{2} = 3\,662\,031.5$$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1\\ &= 3\,662\,031.5 - 1\,246\,666.5\\ &= 2\,415\,365\end{aligned}$$

The interquartile range is 2 415 365.

$$\bar{x} = \frac{3\,877\,515 + 3\,446\,548 + 2\,556\,581 + 1\,016\,590 + 1\,476\,743}{5}$$

$\bar{x} = 2\,474\,795.4$

$$s = \sqrt{\frac{(3\,877\,515 - 2\,474\,795.4)^2 + (3\,446\,548 - 2\,474\,795.4)^2 + (2\,556\,581 - 2\,474\,795.4)^2 + (1\,016\,590 - 2\,474\,795.4)^2 + (1\,476\,743 - 2\,474\,795.4)^2}{5-1}}$$

$s = 1\,228\,931$

The standard deviation is 1 228 931.

c Both values are reduced, but there is a bigger impact on the interquartile range than the standard deviation.

9 a $\sum x = 1\,326\,105$

$$\begin{aligned}\bar{x} &= \frac{1\,326\,105}{32}\\ &= 41\,440.78 \text{ (correct to 2 decimal places)}\end{aligned}$$

$$\begin{aligned}s &= \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}\\ &= 2248.92 \text{ (correct to 2 decimal places)}\end{aligned}$$

b There are 32 values, so the median is between the 16th and 17th values.

Q_1 will be halfway between the 8th and 9th value, and Q_3 will be halfway between the 24th and 25th values.

Order the data:

38 273, 38 586, 38 595, 38 823, 38 833, 38 914, 39 095, 39 347, 39 362, 39 456, 39 756,
39 773, 41 038, 41 123, 41 209, 41 301, 41 365, 41 402, 41 583, 41 675, 42 587, 42 673, 42 845,
42 946, 42 981, 43 689, 43 987, 44 212, 44 567, 44 835, 44 892, 46 382

$$\begin{aligned}\text{Median} &= \frac{41\,301 + 41\,365}{2}\\ &= 41\,333\end{aligned}$$

$$Q_1 = \frac{39\,347 + 39\,362}{2} = 39\,354.5$$

$$Q_3 = \frac{42\,946 + 42\,981}{2} = 42\,963.5$$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1\\ &= 42\,963.5 - 39\,354.5\\ &= 3609\end{aligned}$$

c $$\begin{aligned}\bar{x} - s &= 41\,440.78 - 2248.92\\ &= 39\,191.86\end{aligned}$$

$$\begin{aligned}\bar{x} + s &= 41\,440.78 + 2248.92\\ &= 43\,689.7\end{aligned}$$

19 values lie between $\bar{x} - s$ and $\bar{x} + s$.

$$\begin{aligned}\frac{19}{32} &= 0.59375\\ &= 59.38\% \text{ (correct to 2 decimal places)}\end{aligned}$$

d $Q_1 = 39\,354.5$
$Q_3 = 42\,963.5$
16 values lie between Q_1 and Q_3.
$\frac{16}{32} = 0.5$
$= 50\%$

e There is a greater percentage of the sample within one standard deviation of the mean than between the first and third quartiles.

10 a Using CAS:
$s = 0.46$
$\text{IQR} = 0.7$

b Using CAS:
See the table at the bottom of the page.*

c Comparing the answers to parts **a** and **b**, Sydney bears the closest similarity to the entire data set.

d Comparing the answers to parts **a** and **b**, Darwin bears the least similarity to the entire data set.

1.8 Exam questions

1 IQR for average neck size = 26.0 − 23.4 = 2.6 [1 mark]

2 $\text{IQR} = Q_3 - Q_1$
$= 75 - 57$
$= 18$
The correct answer is **C**.

3 Using a CAS calculator, the standard deviation is calculated to be 3.145. The nearest whole number is therefore 3. Students would not be expected to calculate this by hand.
The correct answer is **E**.

1.9 Symmetrical and asymmetrical distributions

1.9 Exercise

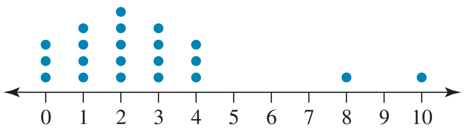
1 a i Asymmetrical because the displayed data shows different shape and different amount of data to the left and right of the centre.

ii Median and interquartile range because they are best for asymmetrical distributions.

b i Symmetrical because the displayed data shows approximately the same shape and same amount of data to the left and right of the centre.

ii Mean and standard deviation because they are best for symmetrical distributions.

2 a



b Asymmetrical because the displayed data shows different shape and different amount of data to the left and right of the centre.

c Median and interquartile range because they are best for asymmetrical distributions.

3 a Lower boundary $= \bar{x} - 2 \times s$
$= 12 - 2 \times 1.45$
$= 9.1$

b Upper boundary $= \bar{x} + 2 \times s$
$= 12 + 2 \times 1.45$
$= 14.9$

c Percentage data $= \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$
$= \frac{19}{21} \times 100$
$= 90.48\%$

4 a Lower boundary $= \bar{x} - s$
$= 6 - 2.88$
$= 3.12$
Upper boundary $= \bar{x} + s$
$= 6 + 2.88$
$= 8.88$
Percentage data $= \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$
$= \frac{13}{20} \times 100$
$= 65\%$

b Lower boundary $= \bar{x} - 2 \times \text{s}$
$= 6 - 2 \times 2.88$
$= 0.24$
Upper boundary $= \bar{x} + 2 \times \text{s}$
$= 6 + 2 \times 2.88$
$= 11.76$
Percentage data $= \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$
$= \frac{18}{20} \times 100$
$= 90\%$

c Lower boundary $= \bar{x} - 3 \times \text{s}$
$= 6 - 3 \times 2.88$
$= -2.64$
Upper boundary $= \bar{x} + 3 \times \text{s}$
$= 6 + 3 \times 2.88$
$= 14.64$
Percentage data $= \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$
$= \frac{20}{20} \times 100$
$= 100\%$

d 100% because all the data is within 3 standard deviations of the mean (found in part **c**); therefore, all data must also be within 4 standard deviations of the mean.

5 a Lower boundary $= \bar{x} - s$
$= 160.07 - 20.5$
$= 139.57$
Upper boundary $= \bar{x} + s$
$= 160.07 + 20.5$
$= 180.57$

***10 b**

	Sydney	Melbourne	Brisbane	Adelaide	Perth	Hobart	Darwin	Canberra
Std dev	0.46	0.40	0.51	0.45	0.44	0.38	0.67	0.41
IQR	0.7	0.7	0.9	0.8	0.6	0.6	1.2	0.6

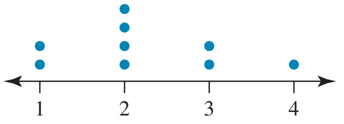
$$\text{Percentage data} = \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$$
$$= \frac{9}{14} \times 100$$
$$= 64.29\%$$

b Lower boundary $= \bar{x} - 2 \times s$
$$= 160.07 - 2 \times 20.5$$
$$= 119.07$$

Upper boundary $= \bar{x} + 2 \times s$
$$= 160.07 + 2 \times 2.81$$
$$= 201.07$$

$$\text{Percentage data} = \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$$
$$= \frac{14}{14} \times 100$$
$$= 100\%$$

6 a



b $(1 + 1 + 2 + 2 + 2 + 2 + 3 + 3 + 4) \div 9 = 2.22$

c Lower boundary $= \bar{x} - s$
$$= 2.22 - 0.97$$
$$= 1.25$$

Upper boundary $= \bar{x} - s$
$$= 2.22 + 0.97$$
$$= 3.19$$

$$\text{Percentage data} = \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$$
$$= \frac{6}{9} \times 100$$
$$= 66.67\%$$

7 a Asymmetrical — the data is positively skewed.

b The median is preferred because it is a better measure of where the centre of the graph lies. Looking at the graph, the centre should be around 5–6, so the median of 6 is much better than the mean of 7.82.

c i Lower boundary $= \bar{x} - 2 \times s$
$$= 7.82 - 2 \times 4.66$$
$$= -1.5$$

ii It is negative.

iii It does not make sense in this context, because you can't play negative hours of video games.

d The best measure of spread is interquartile range because it is best for asymmetrical distributions.

8 a Lower boundary $= \bar{x} - s$
$$= 3.5 - 1.04$$
$$= 2.46$$

Upper boundary $= \bar{x} + s$
$$= 3.5 + 1.04$$
$$= 4.54$$

$$\text{Percentage data} = \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$$
$$= \frac{34}{50} \times 100$$
$$= 68\%$$

b Lower boundary $= \bar{x} - 2 \times s$
$$= 3.5 - 2 \times 1.04$$
$$= 1.42$$

Upper boundary $= \bar{x} + 2 \times s$
$$= 3.5 + 2 \times 1.04$$
$$= 5.58$$

$$\text{Percentage data} = \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$$
$$= \frac{48}{50} \times 100$$
$$= 96\%$$

c Lower boundary $= \bar{x} - 3 \times s$
$$= 3.5 - 3 \times 1.04$$
$$= 0.38$$

Upper boundary $= \bar{x} + 3 \times s$
$$= 3.5 + 3 \times 1.04$$
$$= 6.62$$

$$\text{Percentage data} = \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$$
$$= \frac{50}{50} \times 100$$
$$= 100\%$$

d Yes, it is approximately a normal distribution because our results — 68%, 96% and 100% — are very similar to those of a normal distribution: 68%, 95% and 99.7%.

9 a i Using CAS, the mean is 79.8.

ii Using CAS, the standard deviation is 22.7.

iii Lower boundary $= \bar{x} - s$
$$= 79.8 - 22.7$$
$$= 57.1$$

Upper boundary $= \bar{x} + s$
$$= 79.8 + 22.7$$
$$= 102.5$$

$$\text{Percentage data} = \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$$
$$= \frac{9}{11} \times 100$$
$$= 81.82\%$$

b i Using CAS, the mean is 73.9.

ii Using CAS, the standard deviation is 12.0.

iii Lower boundary $= \bar{x} - s$
$$= 73.9 - 61.9$$
$$= 85.9$$

Upper boundary $= \bar{x} + s$
$$= 73.9 + 12.0$$
$$= 85.9$$

$$\text{Percentage data} = \frac{\text{number of data within boundaries}}{\text{total number of data}} \times 100$$
$$= \frac{7}{10} \times 100$$
$$= 70\%$$

c Removing the outlier decreased the mean (from 79.82 to 73.9) and decreased the standard deviation (from 22.69 to 12).

d 73.9 — the mean without the outlier included, because this number is more representative of what happened for most tomatoes — the outlier was skewing the data.

10 a i 86 and 94 is the same as saying within one standard deviation of the mean, so 68%.

ii 82 and 98 is the same as saying within two standard deviations of the mean, so 95%.

b i 120 and 160 is the same as saying within one standard deviation of the mean, so 68%.

ii 100 and 180 is the same as saying within two standard deviations of the mean, so 95%.

1.9 Exam question

1 Greater than 16% is at 1 standard deviation above the mean:
$\bar{x} + s = 160$
Less than 2.5% is at 2 standard deviations below the mean:
$\bar{x} - 2s = 115$
Solve using CAS: $\bar{x} = 145$ and $s = 15$
The correct answer is **C**.

2 The question asks for the percentage between the mean minus 2 standard deviations and the mean plus 1 standard deviation.
That means the answer will be between 68% and 95%.
The correct answer is **D**.

3 There are 31 countries, so the median is the $\dfrac{31+1}{2} = 16\text{th}$ term. The 16th term is 1.5.
The range is the maximum minus the minimum:
$4.7 - 0.2 = 4.5$.
Finally, the stem plot is positively skewed.
The correct answer is **E**.

1.10 The five-number summary and boxplots

1.10 Exercise

1 a The vertical line inside the box represents the median.
Median = 25
The median is 25 °C.

b Range $= 39 - 18$
$= 21$
The range is 21°C.

c IQR $= 30 - 23$
$= 7$
The IQR is 7 °C.

2 There are 21 values, so the median is in the
$\left(\dfrac{21+1}{2}\right) = 11\text{th}$ position.

29, 31, 33, 36, 38, 39, 42, 44, 45, 45, 46,
47, 47, 48, 50, 50, 53, 55, 58, 61, 62

$Q_2 = 46$

$\left(\dfrac{10+1}{2}\right) = 5.5$

There are 10 values in the lower half of the data, so Q_1 will be between the 5th and 6th values.

29, 31, 33, 36, $\overset{Q_1}{38,}$ 39, 42, 44, 45, 45

$Q_1 = \dfrac{38+39}{2}$
$= 38.5$

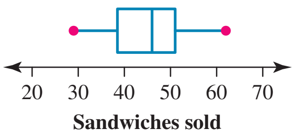
$\left(\dfrac{10+1}{2}\right) = 5.5$

There are 10 values in the upper half of the data, so Q_3 will be between the 5th and 6th values.

47, 47, 48, 50, $\overset{Q_3}{50,}$ 53, 55, 58, 61, 62

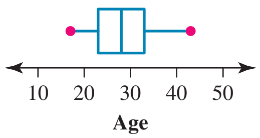
$Q_3 = \dfrac{50+53}{2}$
$= 51.5$

$X_{min} = 29$
$Q_1 = 38.5$
$Q_2 = 46$
$Q_3 = 51.5$
$X_{max} = 62$



3 a $X_{min} = 17$
$X_{max} = 43$
There are 25 pieces of data, so the median is the
$\left(\dfrac{25+1}{2}\right) = 13\text{th}$ piece of data.
$Q_2 = 28$
There are 12 pieces of data in the lower half, so Q_1 is halfway between the 6th and 7th values.
$Q_1 = \dfrac{23+23}{2}$
$= 23$
There are 12 pieces of data in the upper half, so Q_3 is halfway between the 6th and 7th values.
$Q_3 = \dfrac{32+34}{2}$
$= 33$
Five-number summary: 17, 23, 28, 33, 43

b



c Identify the presence of any potential outliers and describe the shape in terms of symmetry or skewness.
The data is fairly symmetrical with no obvious outliers.

4 Given the five-number summary: 45, 56, 70, 83, 92
The median is 70. Therefore, half of the data lies below 70 and half above.
$X_{min} = 45$
Therefore (assuming no outliers), half of the data lies between 45 and 70, so option B must not be true.
The correct answer is **B**.

5

1. Range	e. $X_{max} - X_{min}$
2. Q_2	a. Median
3. Interquartile range	c. $Q_3 - Q_1$
4. Lower fence	b. $Q_1 - 1.5 \times \text{IQR}$
5. Upper fence	d. $Q_3 + 1.5 \times \text{IQR}$

6 a The median is represented by the vertical line inside the main rectangle. It exists on every box plot. Therefore, the statement is true.

b A stem plot involves individual values, not intervals. Therefore, the statement is true.

c The shape of the boxplot will mirror the distribution of the data set. Boxplots display information about the general distribution of the data sets they cover; they lack the detail about this distribution that a histogram or stem plot gives. Therefore, the statement is false.

7 a There are 25 values, so the median is in the
$\left(\dfrac{25+1}{2}\right) = 13\text{th}$ position.

4.4, 6.2, 6.6, 6.9, 7.0, 7.4, 7.7, 7.7, 7.8, 8.0, 8.3, 8.3, 8.5,
8.6, 8.8, 8.9, 9.1, 9.2, 9.4, 9.6, 9.7, 10.2, 10.4, 10.4, 11.5

$Q_2 = 8.5$

$\left(\dfrac{12+1}{2}\right) = 6.5$

There are 12 values in the lower half of the data, so Q_1 will be between the 6th and 7th values.

4.4, 6.2, 6.6, 6.9, 7.0, 7.4, 7.7, 7.7, 7.8, 8.0, 8.3, 8.3

$$Q_1 = \frac{7.4 + 7.7}{2} = 7.55$$

$$\left(\frac{12+1}{2}\right) = 6.5$$

There are 12 values in the upper half of the data, so Q_3 will be between the 6th and 7th values.

8.6, 8.8, 8.9, 9.1, 9.2, 9.4, 9.6, 9.7, 10.2, 10.4, 10.4, 11.5

$$Q_3 = \frac{9.4 + 9.6}{2} = 9.5$$

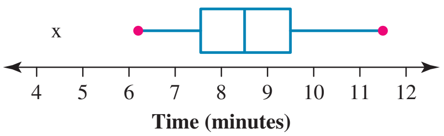
$$\begin{aligned}\text{IQR} &= Q_3 - Q_1\\ &= 9.5 - 7.55\\ &= 1.95\end{aligned}$$

$$\begin{aligned}\text{Lower fence} &= Q_1 - 1.5 \times \text{IQR}\\ &= 7.55 - 1.5 \times 1.95\\ &= 7.55 - 2.925\\ &= 4.625\end{aligned}$$

$$\begin{aligned}\text{Upper fence} &= Q_3 + 1.5 \times \text{IQR}\\ &= 9.5 + 1.5 \times 1.95\\ &= 9.5 + 2.925\\ &= 12.425\end{aligned}$$

b Values below the lower fence (4.625): 4.4.
Values above the upper fence (12.425): none.
There is one outlier: 4.4.

c $X_{\min} = 6.2$
$Q_1 = 7.55$
$Q_2 = 8.5$
$Q_3 = 9.5$
$X_{\max} = 11.5$



8 a $X_{\max} = 104$
The highest amount of points scored 104.

b Lowest value is an outlier at 43.
The lowest amount of points scored was 43.

c $\text{Range} = 104 - 43 = 61$
The range of points scored was 61.

d $\text{IQR} = Q_3 - Q_1 = 89 - 72 = 17$
The interquartile range of points scored was 17.

9 $\text{Range} = X_{\max} - X_{\min}$
$10 = 17 - X_{\min}$
$X_{\min} = 17 - 10$ (add $X_{\min}$ to both sides)
$X_{\min} = 7$
$\text{IQR} = Q_3 - Q_1$
$4 = Q_3 - 9$
$4 + 9 = Q_3$ (add 9 to both sides)
$Q_3 = 13$

10 a $\text{Range} = X_{\max} - X_{\min} = 35 - 9 = 26$

b $\text{IQR} = Q_3 - Q_1 = 29 - 13 = 16$

c The median is Q_2, so the median is 20.

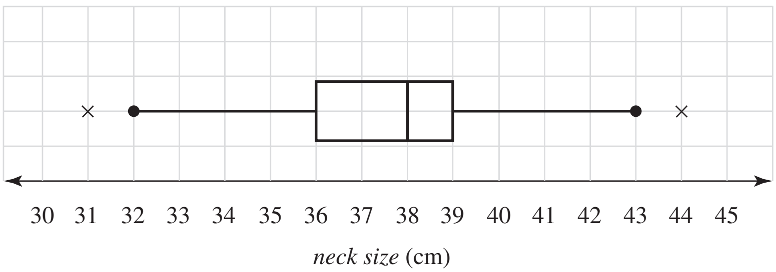
d $\text{Lower fence} = Q_1 - 1.5 \times \text{IQR}.$
$= 13 - 1.5 \times 16$
$= -11$
$\text{Upper fence} = Q_3 + 1.5 \times \text{IQR}$
$= 29 + 1.5 \times 16$
$= 53$

e A commute would be an outlier if it was over 53 minutes.

1.10 Exam questions

1 The value of the mean (220) being greater than the median (150) suggests a positively skewed data set. To determine if there are outliers, we need to calculate the fences:
$LF = Q_1 - 1.5 \times IQR = 10 - 1.5 \times 230 = -335$ $\therefore$ no outliers
$UF = Q_3 + 1.5 \times IQR = 300 + 1.5 \times 230 = 750$
This suggests that *at least* the maximum value (1380) is an outlier, but there may be more.
The correct answer is **C**.

2 First, check for outliers. $LF = 36 - 1.5 \times 3 = 31.5$ (one outlier at the lower end), $UF = 39 + 1.5 \times 3 = 43.5$ (one outlier)



Award 1 mark for outliers.
Award 1 mark for the boxplot.

3 The five-number summary consists of the smallest measurement of 21 cm, lower quartile, $Q_1 = 27.4$ cm, median of 28.7 cm, upper quartile, $Q_3 = 30$ cm and the largest measurement of 35.9 cm.
The correct answer is **B**.
VCAA Examination Report **note:**
Students were required to interpret the provided boxplot and choose the correct five-number summary for the data. While many students did this correctly, a large number of students ignored the outlier points when determining the maximum and minimum values for the five-number summary, leading to the choice of option C, which was incorrect. Even though the points are identified as outliers, they are still valid data points within the data set and must be used as maximum and minimum values if appropriate.

1.11 Comparing the distribution of a numerical variable across two or more groups

1.11 Exercise

1 a First data set (store 1)
$X_{\min} = 44$
$X_{\max} = 90$

There are 15 pieces of data, so the median is the $\left(\dfrac{15+1}{2}\right)$ = 8th piece of data.

$Q_2 = 68$

There are 7 pieces of data in the lower half, so Q_1 is the 4th value.

$Q_1 = 52$

There are 7 pieces of data in the upper half, so Q_3 is the 4th value.

$Q_3 = 75$

Five-number summary: 44, 52, 68, 75, 90

Second data set (store 2)

$X_{min} = 34$

$X_{max} = 67$

There are 15 pieces of data, so the median is the $\left(\dfrac{15+1}{2}\right)$ = 8th piece of data.

$Q_2 = 51$

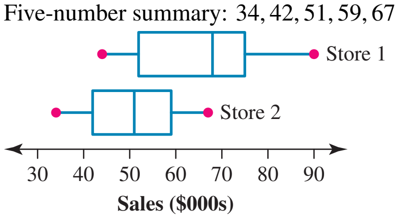
There are 7 pieces of data in the lower half, so Q_1 is the 4th value.

$Q_1 = 42$

There are 7 pieces of data in the upper half, so Q_3 is the 4th value.

$Q_3 = 59$

Five-number summary: 34, 42, 51, 59, 67



b On the whole, store 2 has less sales than store 1; however, the sales of store 2 are much more consistent than store 1's sales.

The sales of store 1 have a negative skew, while the sales of store 2 are symmetrical. There are no obvious outliers in either data set.

2 a The highest overall grade is from the second boxplot (School B).

Therefore, School B has the highest grade.

b The lowest overall grade is the outlier from the second boxplot (School B).

Therefore, School B has the lowest grade.

c
$$\begin{aligned}\text{IQR}_A &= Q_3 - Q_1\\ &= 62 - 52\\ &= 10\end{aligned}$$
$$\begin{aligned}\text{IQR}_B &= Q_3 - Q_1\\ &= 82 - 69\\ &= 13\end{aligned}$$
$$\begin{aligned}\text{IQR}_B - \text{IQR}_A &= 13 - 10\\ &= 3\end{aligned}$$

3 a The vertical line inside the box represents the median. By inspection, the higher median is from the boxplot of Brand X.

Brand X had the better median performance.

b The most consistent performance corresponds to the lesser spread.

Brand Y's boxplot has a smaller spread than Brand X.

Brand Y gave the most consistent performance.

c The lowest value on either of the boxplots belongs to Brand X.

Brand X had the worst performing battery.

d The highest value on either of the boxplots belongs to Brand X.

Brand X had the best performing battery.

4 a First data set (Restaurant A)

There are 7 pieces of data in the lower half, so Q_1 is the 4th value.

$Q_1 = 28$

There are 7 pieces of data in the upper half, so Q_3 is the 4th value.

$Q_3 = 40$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1\\ &= 40 - 28\\ &= 12\end{aligned}$$
$$\begin{aligned}\text{Lower fence} &= Q_1 - 1.5 \times \text{IQR}\\ &= 28 - 1.5 \times 12\\ &= 28 - 18\\ &= 10\end{aligned}$$
$$\begin{aligned}\text{Upper fence} &= Q_3 + 1.5 \times \text{IQR}\\ &= 40 + 1.5 \times 12\\ &= 40 + 18\\ &= 58\end{aligned}$$

$69 in Restaurant A is an outlier.

Second data set (Restaurant B)

There are 7 pieces of data in the lower half, so Q_1 is the 4th value.

$Q_1 = 25$

There are 7 pieces of data in the upper half, so Q_3 is the 4th value.

$Q_3 = 38$

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1\\ &= 38 - 25\\ &= 13\end{aligned}$$
$$\begin{aligned}\text{Lower fence} &= Q_1 - 1.5 \times \text{IQR}\\ &= 25 - 1.5 \times 13\\ &= 25 - 19.5\\ &= 5.5\end{aligned}$$
$$\begin{aligned}\text{Upper fence} &= Q_3 + 1.5 \times \text{IQR}\\ &= 38 + 1.5 \times 13\\ &= 38 + 19.5\\ &= 57.5\end{aligned}$$

There are no outliers for Restaurant B.

b First data set (Restaurant A)

$X_{min} = 24$

$X_{max} = 42$

(Note the value of X_{max} due to the fact that the highest value is an outlier.)

There are 14 pieces of data (because the outlier is excluded), so the median is the $\left(\dfrac{14+1}{2}\right)$ = 7.5th piece of data.

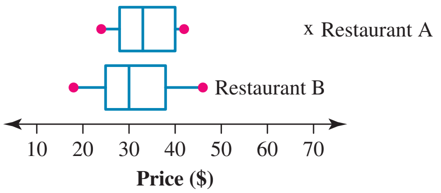
$Q_2 = 33.5$

$Q_1 = 28$

$Q_3 = 38$

Five-number summary: 24, 28, 33.5, 38, 42
Second data set (Restaurant B)
$X_{min} = 18$
$X_{max} = 46$
There are 15 pieces of data, so the median is the
$\left(\frac{15+1}{2}\right) = 8\text{th piece of data.}$
$Q_2 = 30$
$Q_1 = 25$
$Q_3 = 38$
Five-number summary: 18, 25, 30, 38, 46

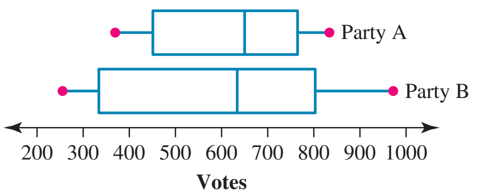
c



d Compare the relative medians of each restaurant and the sizes of the interquartile ranges. Identify the presence of any potential outliers and describe the shape in terms of symmetry or skewness.
The meals in Restaurant A are more consistently priced but are also in general higher priced. The distribution of prices at Restaurant A has a positive skew, while the distribution of prices at Restaurant B is nearly symmetrical.

5 a First data set (Party A)
Put the data in order.
370, 425, 484, 497, 515, 541, 630, 651, 662, 745, 746, 769, 813, 833, 838
$X_{min} = 370$
$X_{max} = 838$
There are 15 pieces of data, so the median is the
$\left(\frac{15+1}{2}\right) = 8\text{th piece of data.}$
$Q_2 = 651$
There are 7 pieces of data in the lower half, so Q_1 is the 4th value.
$Q_1 = 497$
There are 7 pieces of data in the upper half, so Q_3 is the 4th value.
$Q_3 = 769$
Five-number summary: 370, 497, 651, 769, 838
Second data set (Party B)
Put the data in order.
255, 290, 318, 335, 430, 514, 545, 632, 677, 748, 789, 801, 924, 956, 971
$X_{min} = 255$
$X_{max} = 971$
There are 15 pieces of data, so the median is the
$\left(\frac{15+1}{2}\right) = 8\text{th piece of data.}$
$Q_2 = 632$
There are 7 pieces of data in the lower half, so Q_1 is the 4th value.
$Q_1 = 335$
There are 7 pieces of data in the upper half, so Q_3 is the 4th value.
$Q_3 = 801$
Five-number summary: 255, 335, 632, 801, 971

b



c Compare the relative medians of each party and the sizes of the interquartile ranges. Identify the presence of any potential outliers and describe the shape in terms of symmetry or skewness.
The spread of votes for Party B is far larger than it is for Party A. Party A polled more consistently and had a higher median number of votes. Party A had a nearly symmetrical distribution of votes, while Party B's votes had a slight negative skew.

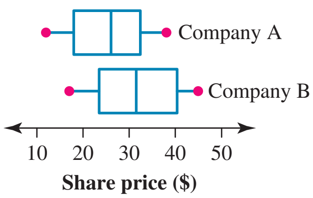
6 a First data set (Company A)
Smallest value = 12
Largest value = 38
We will have class intervals of 5, as specified, starting with 10−<15.

Share price($)	Frequency
10−<15	2
15−<20	3
20−<25	3
25−<30	4
30−<35	4
35−<40	2

Second data set (Company B)
Small value = 17
Largest value = 46
We will have class intervals of 5, as specified, starting with 15−<20.

Share price ($)	Frequency
15−<20	2
20−<25	3
25−<30	3
30−<35	3
35−<40	2
40−<45	3
45−<50	2

b



c Compare the relative medians of each company and the sizes of the interquartile ranges. Identify the presence of any potential outliers and describe the shape in terms of symmetry or skewness.
On the whole, the share price of Company B is greater than the share price of Company A. However, the share price of

Company A is more consistent than the share price of Company B. The share price of Company A has a slightly negative skew, while the share price of Company B has a nearly symmetrical distribution.

7 **a** Farm A's median harvest is higher than that of Farm B. Therefore, the statement is true.

b Both the interquartile range and the overall spread of Farm B's boxplot is less than those of Farm A. Therefore, Farm B's harvests are more consistent and more reliable. Therefore, the statement is true.

c The highest value overall belongs to Farm A's boxplot, so they have had the highest producing month on record. Therefore, the statement is true.

d The lowest value overall belongs to Farm B's data set as an outlier, so they have had the lowest producing month on record. Therefore, the statement is false.

8 **a** The highest value overall belongs to Phone A's boxplot (the outlier), so they have had the highest weekly sales overall.

b The most consistent sales correlate to the smallest interquartile range. Therefore, Phone A had the most consistent sales.

c The largest range in sales correlates to the largest overall spread. Therefore, Phone B has the largest range in sales.

d The largest interquartile range in sales belongs to Phone C.

e The highest median sales figure belongs to Phone C.

9 **a** First data set (Drink 1)

Put the data in order.

35, 39, 39, 41, 42, 44, 47, 47, 47, 50, 51, 52, 53, 55, 56, 58, 59, 60, 62, 64, 66

$X_{\text{min}} = 35$

$X_{\text{max}} = 66$

There are 21 pieces of data, so the median is the

$\left(\dfrac{21+1}{2}\right) = 11\text{th piece of data.}$

$Q_2 = 51$

There are 10 pieces of data in the lower half, so Q_1 is halfway between the 5th and 6th values.

$$Q_1 = \frac{42+44}{2} = 43$$

There are 10 pieces of data in the upper half, so Q_3 is halfway between the 5th and 6th values.

$$Q_3 = \frac{58+59}{2} = 58.5$$

Five-number summary: 35, 43, 51, 58.5, 66.

Second data set (Drink 2)

Put the data in order.

37, 40, 41, 44, 48, 49, 51, 53, 54, 56, 59, 61, 62, 63, 65, 66, 68, 70, 73, 74, 77

$X_{\text{min}} = 37$

$X_{\text{max}} = 77$

There are 21 pieces of data, so the median is the

$\dfrac{21+1}{2} = 11\text{th piece of data.}$

$Q_2 = 59$

There are 10 pieces of data in the lower half, so Q_1 is halfway between the 5th and 6th values.

$$Q_1 = \frac{48+49}{2} = 48.5$$

There are 10 pieces of data in the upper half, so Q_3 is halfway between the 5th and 6th values.

$$Q_3 = \frac{66+68}{2} = 67$$

Five-number summary: 37, 48.5, 59, 67, 77

Third data set (Drink 3)

Put the data in order.

33, 46, 47, 48, 48, 49, 49, 49, 50, 50, 51, 52, 52, 53, 54, 54, 56, 57, 57, 60, 61

Note the outlier 33.

$X_{\text{min}} = 46$

$X_{\text{max}} = 61$

There are 20 pieces of data, so the median is the

$\left(\dfrac{20+1}{2}\right) = 10.5\text{th piece of data.}$

$Q_2 = 51.5$

There are 10 pieces of data in the lower half, so Q_1 is halfway between the 5th and 6th values.

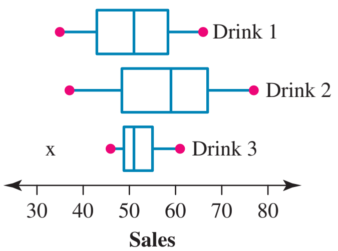
$$Q_1 = \frac{49+49}{2} = 49$$

There are 10 pieces of data in the upper half, so Q_3 is halfway between the 5th and 6th values.

$$Q_3 = \frac{54+56}{2} = 55$$

Five-number summary: 46, 49, 51.5, 55, 61

b



c Compare the relative medians of each drink and the sizes of the interquartile ranges. Identify the presence of any potential outliers and describe the shape in terms of symmetry or skewness.

The sales of Drink 3 are by far the most consistent, although overall Drink 2 has the highest sales. Drink 2's sales are also the most inconsistent of all the drinks. There is one outlier in the data sets (33 in Drink 3).

10 **a** First data set (Suburb A)

$X_{\text{min}} = 275$

$X_{\text{max}} = 315$

Note the outlier 350.

There are 20 pieces of data, so the median will be in position $\left(\dfrac{20+1}{2}\right) = 10.5$, or halfway between position 10 and position 11.

$$Q_2 = \frac{300+300}{2} = 300$$

There are 10 pieces of data in the lower half, so Q_1 is halfway between the 5th and 6th values.

$$Q_1 = \frac{289+289}{2} = 289$$

There are 10 pieces of data in the upper half, so Q_3 is halfway between the 5th and 6th values.

$$Q_3 = \frac{305 + 310}{2}$$
$$= 307.5$$

Five-number summary: 275, 289, 300, 307.5, 315

Second data set (Suburb B)

$X_{min} = 250$

$X_{max} = 340$

There are 20 pieces of data, so the median will be in position $\left(\frac{20+1}{2}\right) = 10.5$, or halfway between position 10 and 11.

$$Q_2 = \frac{290 + 300}{2}$$
$$= 295$$

There are 10 pieces of data in the lower half, so Q_1 is halfway between the 5th and 6th values.

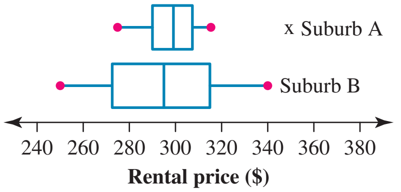
$$Q_1 = \frac{270 + 275}{2}$$
$$= 272.5$$

There are 10 pieces of data in the upper half, so Q_3 is halfway between the 5th and 6th values.

$$Q_3 = \frac{315 + 315}{2}$$
$$= 315$$

Five-number summary: 250, 272.5, 295, 315, 340

b

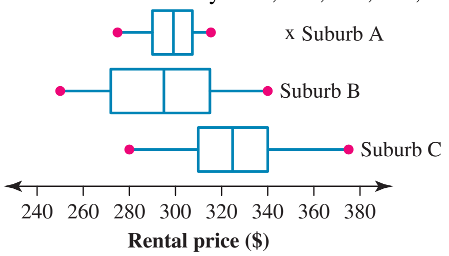


c Compare the relative medians of each suburb and the sizes of the interquartile ranges. Identify the presence of any potential outliers and describe the shape in terms of symmetry or skewness.

The rental prices in Suburb A are far more consistent than the rental prices in Suburb B. There is one outlier in the data sets ($350 in Suburb A). Although Suburb A has a higher median rental price, you could not say that it was definitely more expensive than Suburb B.

d Third data set (Suburb C)

Five-number summary: 280, 310, 325, 340, 375



e Compare the relative medians of each suburb and the sizes of the interquartile ranges. Identify the presence of any potential outliers and describe the shape in terms of symmetry or skewness.

Suburb C has a higher average rental price than either Suburb A or B. The spread of the prices in Suburb C is more similar to those in Suburb B than Suburb A.

1.11 Exam questions

1 There is a numerical variable (*resting pulse rate*) with three categories (*age group*), so **parallel boxplots** are the best wa1y to display the data.
The correct answer is **C**.

2 a i IQR = 17° − 12° = 5° [1 mark]

ii The median for maximum temperature was 25 °C and the median for minimum temperature was 15 °C, so the median for maximum temperature was 10 °C higher. [1 mark]

iii One — the outlier [1 mark]

b If the median is 9.4 °C, then it is expected that 50% will be above this.
There are 30 days in November:
50% × 30 = 15 days [1 mark]

***VCAA Examination Report* note:**
Some gave the percentage of 50% rather than the number of days.

3 Parallel boxplots are used to investigate the association between a numerical variable and a categorical variable. In this question, the variable – monthly rainfall (in mm) is numerical, so the unknown second variable must be categorical. Therefore, Month of the year, option C is the only categorical variable in the given options.
The correct answer is **C**.

1.12 Review

1.12 Exercise

Multiple choice

1 $n = 13$; median $= \frac{13+1}{2} = 7$th score; therefore, quartiles are located between 3rd and 4th scores of each half.

$$Q_1: \frac{15+26}{2} = 20.5$$
$$Q_3: \frac{39+43}{2} = 41$$

IQR: 41 − 20.5 = 20.5
The correct answer is **D**.

2 Consider the lowest 25% of data for group B: the minimum value of group B is higher than Q_1 for group A. Furthermore, Q_1 for group B is lower than the median for group A.
The correct answer is **E**.

3 Number of runs is numerical data; it can only take separate values (which are integers). Therefore, the data is discrete.
The correct answer is **A**.

4 The data arranged in order:
167, 211, 222, 234, 234, 321, 432, 456, 456, 789
$X_{min} = 167$; the median is the average of the 5th and 6th scores.

Median: $\dfrac{234+321}{2} = 277.5$
Q_1 is the 3rd score of the lower half.
$Q_1 = 222$
Q_3 is the 3rd score of the upper half.
$Q_3 = 456$
$X_{\text{max}} = 789$
Therefore, the five-number summary is:
167, 222, 277.5, 456, 789.
The correct answer is **B**.

5 The data is approximately symmetric; there is one outlier.
The correct answer is **B**.

6 Using CAS, mean = 5; standard deviation = 2.725 54 ≈ 2.7.
The correct answer is **D**.

7 $n = 12$; the median is the average of the 6th and 7th scores.
Median: $\dfrac{73+74}{2} = 73.5$
Range: $X_{\text{max}} - X_{\text{min}} = 92 - 51$
$= 41$
The correct answer is **B**.

8 The median value of group A is equal to the second highest value of group B, not the highest value (which is represented by the outlier). Therefore, option C is not true.
The correct answer is **C**.

9 Ordered set: 15, 21, 24, 45, 56, 78, 110, 111, 124, 250
$Q_1 = 24$; $Q_3 = 111$
IQR: $Q_3 - Q_1 = 111 - 24$
$= 87$
$1.5 \times \text{IQR}: 87 \times 1.5 = 130.5$
Lower fence: $24 - 130.5 = -106.5$
Upper fence: $111 + 130.5 = 241.5$
Outliers: 250
1 outlier, $1.5 \times \text{IQR} = 130.5$
The correct answer is **D**.

Short answer

10 a Using CAS, mean = 94.6; median = 93.3 (correct to 1 decimal place)

b $Q_1 = 88.99$; $Q_3 = 97.63$
IQR: $Q_3 - Q_1 = 97.63 - 88.99$
$= 8.64$
Standard deviation = 7.49 (correct to 2 decimal places).

c For this sample, the mean and the standard deviation would be preferred as there are no clear outliers and no apparent skew.

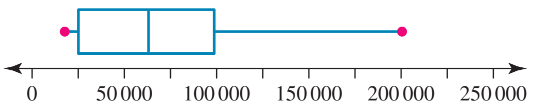
11 a Using CAS:
Year 1: mean = 6432.9; median = 1324
Year 2: mean = 2637.6; median = 1201

b The mean for Year 1 is significantly greater than for Year 2, while the values of the medians are quite similar. In Year 1 the number of incidences of influenza was very large compared to Year 2 (59 090 compared to 13 419). Similarly, the number of incidences of hepatitis C was much larger in Year 1 (11 089 compared to 7286). This resulted in the mean value being much larger for Year 1. Since medians are not affected by extreme values, the medians for both years are similar.

12 a Using CAS: mean = 202 461.4;
standard deviation = 257 819.6

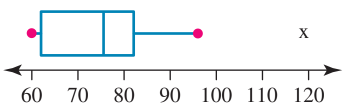
b Median = 7655; it is significantly smaller than the mean. The set of data contains 4 scores whose values are between 5000 and 8000 and 3 very large scores whose values are above 300 000. Since the median is the 4th score in the ordered set, it is not affected by the top 3 extreme values, while the mean is.
IQR: $Q_3 - Q_1 = 480\,625 - 5318$
$= 475\,307$
IQR is nearly double the size of standard deviation.

13 a

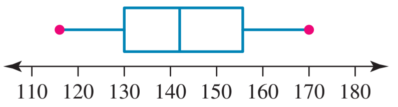


b The distribution is positively skewed.
Five-figure summary:
$X_{\text{min}} = 17\,747$; $Q1 = 25\,025$; median = 63 161;
$Q_3 = 98\,700$; $X_{\text{max}} = 200\,552$
IQR: $Q_3 - Q_1 = 98\,700 - 25\,025$
$= 73\,675$
$1.5 \times \text{IQR}: 1.5 \times 73\,675 = 110\,512.5$
Upper fence: $Q_3 + 1.5 \times \text{IQR} = 98\,700 + 110\,512.5$
$= 209\,212.5$
Lower fence: $Q_1 - 1.5 \times \text{IQR} = 25\,025 - 110\,512.5$
$= -85\,487.5$
Since $X_{\text{min}} > -85\,487.5$ and $X_{\text{max}} < 209\,212.5$, there are no outliers.
Overall, the data is positively skewed with the upper 25% having a much greater spread than the lower 25%. The middle 50% is approximately symmetrical. There are no outliers.

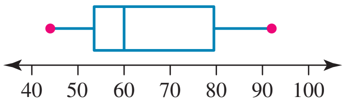
14 a i



ii



iii



b i The data for 'Winning margin' had the following distribution: minimum score, $X_{\text{min}} = 60$;
$Q_1 = 62$; median = 75.5; $Q_3 = 82$; maximum score, $X_{\text{max}} = 119$.
IQR: $82 - 62 = 20$
$1.5 \times \text{IQR}: 1.5 \times 20 = 30$
Lower fence: $Q_1 - 1.5 \times \text{IQR} = 62 - 30$
$= 32$
Upper fence: $Q_3 + 1.5 \times \text{IQR} = 82 + 30$
$= 112$
The score 119 is higher than upper fence; therefore, it is an outlier.
The distribution of winning margins is negatively skewed with an upper outlier.

ii The data for 'Winning score' had the following distribution:
$X_{\text{min}} = 116$; $Q_1 = 130$; median = 142; $Q_3 = 155.5$;
$X_{\text{max}} = 170$

$\text{IQR}: 155.5 - 130 = 25.5$

$1.5 \times \text{IQR}: 1.5 \times 25.5 = 38.25$

$$\text{Lower fence}: Q_1 - 1.5 \times \text{IQR} = 130 - 38.25 = 91.75$$

$$\text{Upper fence}: Q_3 + 1.5 \times \text{IQR} = 155.5 + 38.25 = 193.75$$

All scores are within the fences.

The distribution of winning scores is approximately symmetric with no outliers.

iii The data for 'Losing score' had the following distribution:

$X_{\min} = 44$; $Q_1 = 53.5$; median $= 60$; $Q_3 = 79$; $X_{\max} = 92$

$\text{IQR}: 79 - 53.5 = 25.5$

$1.5 \times \text{IQR}: 1.5 \times 25.5 = 38.25$

$$\text{Lower fence}: Q_1 - 1.5 \times \text{IQR} = 53.5 - 38.25 = 15.25$$

$$\text{Upper fence}: Q_3 + 1.5 \times \text{IQR} = 79 + 38.25 = 117.25$$

All scores are within the fences.

The distribution of losing scores is positively skewed with no outliers.

15 a Five-figure summary for the Americas:

$X_{\min} = 441$; $Q_1 = 515$; median $= 658.5$; $Q_3 = 768$; $X_{\max} = 817$

Five-figure summary for Asia and Oceania:

$X_{\min} = 198$; $Q_1 = 224$; median $= 267.5$; $Q_3 = 348$; $X_{\max} = 382$

Five-figure summary for Europe:

$X_{\min} = 348$; $Q_1 = 374$; median $= 392$; $Q_3 = 419$; $X_{\max} = 428$

Parallel boxplots are shown below.

See the graph at the bottom of the page.*

b Military expenditure was the highest for the Americas. The smallest value of 441 billion US dollars for Americas was higher than the highest values for Asia and Oceania (382 billion US dollars) and for Europe (428 billion). The centre of the distribution, as indicated by the medians was highest for the Americas (658.5 billion), followed by Europe (392 billion) and the lowest for Asia and Oceania (267.5 billion).

The spread of the distribution was again the highest for the Americas, followed by Asia and Oceania and the smallest for Europe. This is evident from both range and IQR of the distributions. The IQR for the Americas was 253 billion US dollars, for Asia and Oceania 124 billion, and for Europe 45 billion.

No distribution had outliers.

Overall it can be concluded that the Americas had the largest and most spread out military expenditure over the period of 1999 to 2012. Asia and Oceania had the smallest military expenditure, while Europe's expenditure was least spread out (most consistent).

Extended response

16 a See the graph at the bottom of the page.*

b Using CAS, mean births per state/territory of Australia for Indigenous groups = 1410.5, while for non-Indigenous groups mean = 35 241.6.

c Median births per state/territory of Australia for Indigenous groups = 1156, while for non-Indigenous groups median = 24 008.

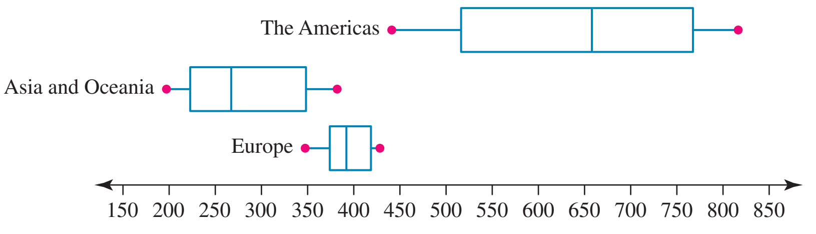
d For Indigenous groups: standard deviation = 1193.8

$$\text{IQR}: Q_3 - Q_1 = 2321 - 445.5 = 1875.5$$

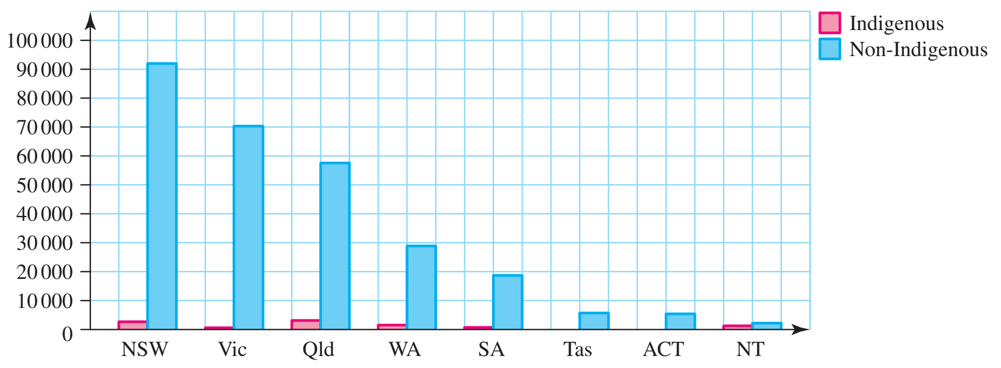
For non-Indigenous groups: standard deviation = 33 949.0

$$\text{IQR}: Q_3 - Q_1 = 63\,996.5 - 5798.5 = 58\,198$$

*** 15 a**



*** 16 a**



e Due to the presence of potential extreme values in the data, the median and IQR are probably more appropriate measures of centre and spread (respectively) than mean and standard deviation.

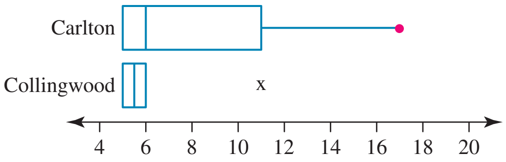
17 a Using CAS, mean temperature for 1980–89 = 19.34 and standard deviation = 0.44 (correct to 2 decimal places).
Mean temperature for 2003–12 = 20.29 and standard deviation = 0.45 (correct to 2 decimal places).

b There is a slight increase in temperature in the 2003–12 decade compared to 1980–89, as mean temperature is about one degree higher in the period 2003–12. The standard deviations have very close values, which indicates that the data have similar spreads over the two 10-year periods.

c Using CAS, mean temperature for the entire data = 19.82 and standard deviation = 0.65 (correct to 2 decimal places).

d The mean temperature of the total data is about half-way between the means of the two separate 10-year periods. The standard deviation is significantly higher for the 20-year period than for either of the 10-year periods, which indicates a much greater variation from the mean for the combined data.

18 a Five-figure summary of number of goals for Carlton:
$X_{\min} = 5$; $Q_1 = 5$; median $= 6$; $Q_3 = 11$; $X_{\max} = 17$
$\text{IQR}: 11 - 5 = 6$
$1.5 \times \text{IQR}: 1.5 \times 6 = 9$
Lower fence: $5 - 9 = -4$
Upper fence: $11 + 9 = 20$
Since all scores are within the fences, there are no outliers.
Five-figure summary of number of goals for Collingwood:
$X_{\min} = 5$; $Q_1 = 5$; median $= 5.5$; $Q_3 = 6$; $X_{\max} = 11$
$\text{IQR}: 6 - 5 = 1$
$1.5 \times \text{IQR}: 1.5 \times 1 = 1.5$
Lower fence: $5 - 1.5 = 3.5$
Upper fence: $6 + 1.5 = 7.5$
The score $11 > 7.5$; therefore, it is an outlier.
Parallel boxplots are shown below.



b Goals scored in grand finals for this sample of players is slightly larger for Carlton, as indicated by the higher median (6 against 5.5). The number of goals for Carlton are more variable compared to Collingwood, as indicated by the larger range and IQR. The number of goals for Collingwood players were very consistent, being either 5 or 6 with the exception of the one high score of 11 being an outlier.

c Five-figure summary of number of kicks:
$X_{\min} = 13$; $Q_1 = 24$; median $= 30$; $Q_3 = 43$; $X_{\max} = 78$
$\text{IQR}: 43 - 24 = 19$
$1.5 \times \text{IQR}: 1.5 \times 19 = 28.5$
Lower fence: $24 - 28.5 = -4.5$
Upper fence: $43 + 28.5 = 71.5$
Since $78 > 71.5$, it is an outlier.
Five-figure summary of number of handballs:
$X_{\min} = 1$; $Q_1 = 7$; median $= 10$; $Q_3 = 16.5$; $X_{\max} = 37$
$\text{IQR}: 16.5 - 7 = 9.5$
$1.5 \times \text{IQR}: 1.5 \times 9.5 = 14.25$
Lower fence: $7 - 14.25 = -7.25$
Upper fence: $16.5 + 14.25 = 30.75$
The score $37 > 30.75$; therefore, it is an outlier.
Parallel boxplots are shown below.
See the graph at the bottom of the page.*

d Both distributions are positively skewed with one upper outlier. The scores for kicks for this sample of players are significantly greater than for handballs, as indicated by the medians (30 compared to 10). The scores for kicks are also more variable than the scores for handballs as indicated by the larger range and IQR.

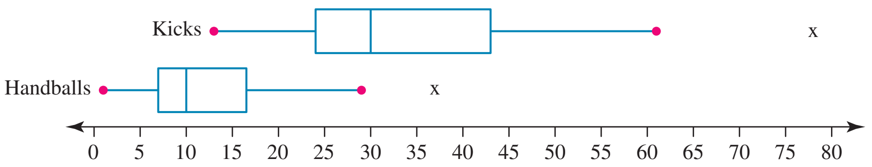
1.12 Exam questions

1 $Q_1 = 148$, $Q_3 = 159$; therefore, $\text{IQR} = 159 - 148 = 11$.
Upper fence $= 159 + 1.5 \times 11 = 175.5$
The correct answer is **E**.

2 300 milliseconds is the value at the third quartile (Q_3); therefore, *longer than 300 milliseconds* will be 25% of the times:
$0.25 \times 800 = 200$
The correct answer is **D**.

3 The *month* is the explanatory variable and the *minimum daily temperature* is the response variable.
The *median* values decrease with the month, which is expected as the year moves from summer into winter months.
Award 1 mark for identifying an appropriate statistic – e.g. median.
Award 1 mark for the explanation.
***VCAA Examination Report* note:**
A statement that a **decrease** or **change** in median (or IQR) signals an association was required for the first mark to be awarded.
Median (or IQR) values for all three months needed to be quoted correctly for the second mark to be awarded.
Successful responses focused on one statistic only (usually the median) and quoted the values from the table rather than estimating from the boxplots.
Incorrect answers included using the word ‘averages’ or ‘means’ rather than medians and quoting only two medians rather than all three.
Some students went on to comment on the minimums and maximums; this additional information compromised an

***18 c**



otherwise correct answer. Comments about the shape of the boxplots were also not appropriate.

4 a Place of capture [1 mark]

b The most frequently occurring value in the forest section of the stem plot is 20, so the modal wingspan is 20 mm. [1 mark]

c See the table at the bottom of the page.*

The minimum wingspan in the forest is 16 mm. [1 mark]

The upper quartile (Q_3) in the grassland is 36 mm. [1 mark]

d $\text{IQR} = Q_3 - Q_1$

$= 32 - 20$

$= 12$

The wingspan of 52 mm is at the upper end of the forest values. Students need to show that this value is greater than the upper fence, that is greater than $Q_3 + 1.5 \times \text{IQR}$.

$Q_3 + 1.5 \times \text{IQR} = 32 + 1.5 \times 12$

$= 50$ [1 mark]

As 52 mm is greater than this upper fence value of 50 mm, it is an outlier. [1 mark]

e Possible solution:

The wingspan is associated with the place of capture. Those captured in the grassland had a median wingspan of 30 mm, which is greater than the median wingspan of 21 mm of the moths captured in the forest.

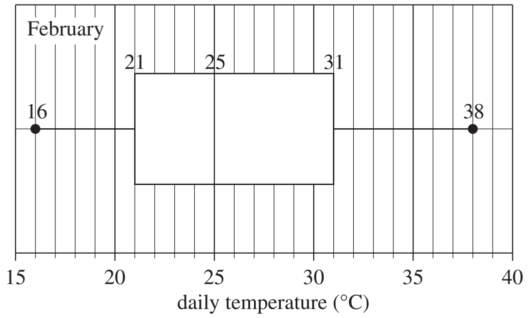
Award 1 mark for stating that the median wingspan of those captured in grassland is greater than the median wingspan of those captured in the forest. Award 1 mark for stating the values of the two medians, 30 mm and 21 mm.

***VCAA Examination Report* note:**

The mean is not part of a stemplot five-number summary and was not appropriate to use because of the outlier of 52 in the forest data.

Students who initially gave the required median comparisons and then went further by quoting comparisons of other irrelevant statistics were not awarded full marks.

5 a i



Award 1 mark for the correct plot.

ii 21 is at the first quartile, so 75% of days have a maximum temperature above 21 °C. [1 mark]

***VCAA Examination Report* note:**

A common incorrect answer was 25%.

b i July: Positively skewed with an outlier at the high end

May: Approximately symmetric with no outliers [1 mark]

***VCAA Examination Report* note:**

Common unacceptable answers for July included symmetrically skewed, evenly distributed, bell shaped and normally distributed.

ii Upper fence $= Q_3 + 1.5 \times \text{IQR}$

$= 11 + 1.5 \times 3$

$= 15.5$ [1 mark]

iii The median maximum daily temperature for May is approximately 14.4 °C and for July it is approximately 9.1 °C. These are very different temperatures, which we would expect as July is in winter and May is in autumn. Therefore, the maximum daily temperature is associated with the month of the year. [1 mark]

***VCAA Examination Report* note:**

The answer needed to refer to the difference between the two median temperatures. Simply quoting the two median values was not sufficient. Accuracy in reading the scales was an issue for some students.

Alternatively, comparing the two interquartile range (IQR) values could have been used as the difference in the IQRs also indicates the presence of an association.

It appeared that some students confused ‘maximum daily temperature’ with the maximum of the boxplot.

Some students referred to average or mean temperatures; however, this cannot be accurately determined from a boxplot unless the distributions are clearly symmetric.

***4 c**

	***Wingspan* (mm)**				
Place of capture	minimum	Q_1	median (M)	Q_3	maximum
forest	**16**	20	21	32	52
grassland	18	24	30	**36**	45

Topic 2 — Linear relations and equations

2.2 Linear relations and solving linear equations

2.2 Exercise

1 a y has a power of 2; therefore, the equation is non-linear.

b x has a power of 3; therefore, the equation is non-linear.

c The powers of the variables x and y are both 1; therefore, the equation is linear.

d The variable x is a power; therefore, the equation is non-linear.

e The powers of the variables x and y are both 1; therefore, the equation is linear.

2 a

Equation	Bethany's response	Correct response
$y = 1 + 4x$	Yes	Yes, the powers of both variables are 1.
$y^2 = 5x - 2$	Yes	No, the power of y is 2.
$y + 6x = 7$	Yes	Yes, the powers of both variables are 1.
$y = x^2 - 5x$	No	No, the highest power of x is 2.
$t = 6d^2 - 9$	No	No, the power of variable d is 2.
$m^3 = n + 8$	Yes	No, the power of variable m is 3.

b Bethany should look at both variables (pronumerals or letters). Both variables need to have a highest power of 1.

3 a The powers of variables y and t are both 1; therefore, the equation is linear.

b The power of x is 2; therefore, the equation is non-linear.

c The powers of variables m and n are both 1; therefore, the equation is linear.

d The powers of variables d and t are both 1; therefore, the equation is linear.

e The power of y is $\frac{1}{2}$ (square root); therefore, the equation is non-linear.

f The power of t is $-1\left(\frac{1}{t}\right)$; therefore, the equation is non-linear.

4 a Yes, as the power of x is 1.

b The power of both variables in a linear relation must be 1.

5 $y = -3 + 6x$

Add 3 to both sides: $y + 3 = -3 + 3 + 6x$

$$y + 3 = 6x$$

Divide both sides by 6: $\frac{6x}{6} = \frac{y+3}{6}$

$$x = \frac{y+3}{6}$$

6 a $y = 5 + 2x$

Subtract 5 from both sides: $y - 5 = 5 - 5 + 2x$

$$y - 5 = 2x$$

Divide both sides by 2: $\frac{y-5}{2} = \frac{2x}{2}$

$$\frac{y-5}{2} = x$$

$$x = \frac{y-5}{2}$$

b $3y = 8 + 6x$

Subtract 8 from both sides: $3y - 8 = 8 - 8 + 6x$

$$3y - 8 = 6x$$

Divide both sides by 6: $\frac{3y}{6} - \frac{8}{6} = \frac{6x}{6}$

$$\frac{3y-8}{6} = x$$

$$x = \frac{3y-8}{6}$$

c $p = -6 + 5x$

Add 6 to both sides: $p + 6 = -6 + 6 + 5x$

$$p + 6 = 5x$$

Divide both sides by 5: $\frac{p+6}{5} = \frac{5x}{5}$

$$\frac{p+6}{5} = x$$

$$x = \frac{p+6}{5}$$

d $6y = 1 + 3x$

Subtract 1 from both sides: $6y - 1 = 1 - 1 + 3x$

$$6y - 1 = 3x$$

Divide both sides by 3: $\frac{6y}{3} - \frac{1}{3} = \frac{3x}{3}$

$$2y - \frac{1}{3} = x$$

$$x = 2y - \frac{1}{3}$$

7 a $2(x + 1) = 8$ (operations on x in order $+1$, $\times 2$)

Opposite operations: $-1, \div 2$

Perform the opposite operations in reverse order to both sides of the equation, one operation at a time:

$\div$ 2 both sides:

$$\frac{2(x+1)}{2} = \frac{8}{2}$$

$$x + 1 = 4$$

-1 from both sides:

$$x + 1 - 1 = 4 - 1$$

$$x = 3$$

b $n - 12 = -2$ (operation on n is -12)

Opposite operation: $+12$

Perform opposite operation to both sides:

$+$ 12 to both sides:

$$n - 12 + 12 = -2 + 12$$

$$n = 10$$

c $4d - 7 = 11$ (operations on d in order: $\times 4$, -7)

Opposite operations: $\div 4$, $+7$

Perform the opposite operations in reverse order to both sides, one operation at a time:

$+\,7$ to both sides:

$4d - 7 + 7 = 11 + 7$

$4d = 18$

$\div\,4$ both sides:

$\frac{4d}{4} = \frac{18}{4}$

$d = 4.5$

d $\frac{x+1}{2} = 9$ (operations performed on x in order: $+\,1, \div\,2$)

Opposite operations: $-\,1,\ \times\,2$

Perform the opposite operations in reverse order to both sides, one operation at a time:

$\times\,2$ both sides:

$\frac{x+1}{2} \times 2 = 9 \times 2$

$x + 1 = 18$

$-\,1$ from both sides:

$x + 1 - 1 = 18 - 1$

$x = 17$

8 a i $\times\,4,\ +\,3$

ii $+\,2,\ \times\,3$

iii $+\,1,\ \div\,2$

iv $\times\,3,\ -\,9,\ \times\,2$

b i $10 = 4a + 3$ (operations on a in order: $\times\,4, +\,3$; opposite operations: $\div\,4, -\,3$)

Perform opposite operations in reverse order, one operation at a time:

$-\,3$ from both sides:

$10 - 3 = 4a + 3 - 3$

$7 = 4a$

$\div\,4$ both sides:

$\frac{7}{4} = \frac{4a}{4}$

$\frac{7}{4} = a$

$a = \frac{7}{4}$

ii $3(x+2) = 12$ (operations on x in order: $+\,2, \times\,3$; opposite operations: $-\,2, \div\,3$)

Perform opposite operation in reverse order, one operation at a time:

$\div\,3$ both sides:

$\frac{3(x+2)}{3} = \frac{12}{3}$

$x + 2 = 4$

$-\,2$ from both sides:

$x + 2 - 2 = 4 - 2$

$x = 2$

iii $\frac{s+1}{2} = 7$ (operations on s in order: $+\,1, \div\,2$; opposite operations: $-\,1, \times 2$)

Perform opposite operation in reverse order, one operation at a time:

$\times\,2$ both sides:

$\frac{s+1}{2} \times 2 = 7 \times 2$

$s + 1 = 14$

$-\,1$ from both sides:

$s + 1 - 1 = 14 - 1$

$s = 13$

iv $16 = 2(3c - 9)$ (operations on c in order: $\times\,3, -\,9, \times\,2$; opposite operations: $\div 3, +9, \div 2$)

Perform opposite operations in reverse order, one operation at a time:

$\div\,2$ both sides:

$\frac{16}{2} = \frac{2(3c-9)}{2}$

$8 = 3c - 9$

$+\,9$ both sides:

$8 + 9 = 3c - 9 + 9$

$17 = 3c$

$\div\,3$ both sides:

$\frac{17}{3} = \frac{3c}{3}$

$\frac{17}{3} = c$

$c = \frac{17}{3}$

9 a $14 = 5 - x$ (operations on x: $\times -1, +5$, opposite operations $\div -1, -5$)

$14 - 5 = 5 - x - 5$

$9 = -x$

$\frac{9}{-1} = \frac{-x}{-1}$

$x = -9$

b $\frac{4(3y-1)}{5} = -2$ (operations on y: $\times\,3, -1, \times 4, \div\,5$; opposite operations: $\div\,3, +\,1, \div\,4, \times\,5$)

$\frac{4(3y-1)}{5} \times 5 = -2 \times 5$

$4(3y - 1) = -10$

$\frac{4(3y-1)}{4} = \frac{-10}{4}$

$3y - 1 = -2.5$

$3y - 1 + 1 = -2.5 + 1$

$3y = -1.5$

$\frac{3y}{3} = \frac{-1.5}{3}$

$y = -0.5$

c $\frac{2(3-x)}{3} = 5$ (operations on $x, \times -1, +3, \times 2, \div 3$; opposite operations: $\div -1, -3, \div 2, \times 3$)

$\frac{2(3-x)}{3} \times 3 = 5 \times 3$

$2(3 - x) = 15$

$\frac{2(3-x)}{2} = \frac{15}{2}$

$3 - x = 7.5$

$3 - x - 3 = 7.5 - 3$

$\frac{-x}{-1} = \frac{4.5}{-1}$

$x = -4.5$

10 a $v = u + at\ (a)$

$v - u = u + at - u$

$v - u = at$

$\frac{v-u}{t} = \frac{at}{t}$

$a = \frac{v-u}{t}$

b $xy - k = m\,(x)$

$$xy - k + k = m + k$$
$$xy = m + k$$
$$\frac{xy}{y} = \frac{m+k}{y}$$
$$x = \frac{m+k}{y}$$

c $\frac{x}{p} - r = s\,(x)$

$$\frac{x}{p} - r + r = s + r$$
$$\frac{x}{p} = s + r$$
$$\frac{x}{p} \times p = p \times (s + r)$$
$$x = p(r + s)$$

11 $y = 5 - 6x, x = 5$

$$= 5 - 6\,(5)$$
$$= 5 - 30$$
$$= -25$$

12 $y = 3 + 3x, x = -3$

$$= 3 + 3\,(-3)$$
$$= 3 - 9$$
$$= -6$$

13 a $w = 120 + 10t$

$$w = 450$$
$$450 = 120 + 10t$$
$$450 - 120 = 120 - 120 + 10t$$
$$330 = 10t$$
$$\frac{330}{10} = \frac{10t}{10}$$
$$t = 33 \text{ minutes}$$

b $w = 1200$

$$1200 = 120 + 10t$$
$$1200 - 120 = 120 - 120 + 10t$$
$$1080 = 10t$$
$$\frac{1080}{10} = \frac{10t}{10}$$
$$t = 108 \text{ minutes}$$

14 $-q + px = r$

$$-q + q + px = r + q$$
$$px = r + q$$
$$\frac{px}{p} = \frac{r+q}{p}$$
$$x = \frac{r+q}{p}$$

15 $C = \pi d$

$$\frac{C}{\pi} = \frac{\pi d}{\pi}$$
$$d = \frac{C}{\pi}$$

16 a Step 1: $\times 5,\ -13$

Step 2: opposite operations $\div 5,\ +13$

Step 3: $5w - 13 = 12$

$+13$ to both sides:

$$5w - 13 + 13 = 12 + 13$$
$$5w = 25$$

Step 4: $\div 5$ both sides

$$\frac{5w}{5} = \frac{25}{5}$$
$$w = 5$$

b Operations need to be performed in reverse order.

17 $F = 1.8(K - 273) + 32$

Operations on K: $-273, \times 1.8, +32$

-32 from both sides:

$$F - 32 = 1.8\,(K - 273) + 32 - 32$$
$$F - 32 = 1.8(K - 273)$$

$\div 1.8$ both sides:

$$\frac{F - 32}{1.8} = \frac{1.8(K - 273)}{1.8}$$
$$\frac{F - 32}{1.8} = K - 273$$

$+273$ to both sides:

$$\frac{F - 32}{1.8} + 273 = K - 273 + 273$$
$$K = \frac{F - 32}{1.8} + 273$$

18 $y = \frac{3x+1}{4}$

a $y = 2$

$2 = \frac{3x+1}{4}$ (operations on x, $\times 3$, $+1$, $\div 4$; opposite operations: $\div 3$, -1, $\times 4$)

$\times 4$ both sides:

$$2 \times 4 = \frac{3x+1}{4} \times 4$$
$$8 = 3x + 1$$

-1 from both sides:

$$8 - 1 = 3x + 1 - 1$$
$$7 = 3x$$

$\div 3$ both sides:

$$\frac{7}{3} = \frac{3x}{3}$$
$$x = \frac{7}{3}$$

b $y = -3$:

$-3 = \frac{3x+1}{4}$ (operations on x, $\times 3$, $+1$, $\div 4$; opposite operations: $\div 3$, -1, $\times 4$)

$\times 4$ both sides:

$$-3 \times 4 = \frac{3x+1}{4} \times 4$$
$$-12 = 3x + 1$$

-1 from both sides:

$$-12 - 1 = 3x + 1 - 1$$
$$-13 = 3x$$

$\div 3$ both sides:

$$\frac{-13}{3} = \frac{3x}{3}$$
$$x = \frac{-13}{3}$$

c $y = \frac{1}{2}$

$\frac{1}{2} = \frac{3x+1}{4}$ (operations on x, $\times 3$, $+1$, $\div 4$; opposite operations: $\div 3, -1, \times 4$)

$\times 4$ both sides:

$\frac{1}{2} \times 4 = \frac{3x+1}{4} \times 4$

$2 = 3x + 1$

-1 from both sides:

$2 - 1 = 3x + 1 - 1$

$1 = 3x$

$\div 3$ both sides:

$\frac{1}{3} = \frac{3x}{3}$

$x = \frac{1}{3}$

d $y = 10$

$10 = \frac{3x+1}{4}$ (operations on x, $\times 3$, $+1$, $\div 4$; opposite operations: $\div 3$, -1, $\times 4$)

$\times 4$ both sides:

$10 \times 4 = \frac{3x+1}{4} \times 4$

$40 = 3x + 1$

-1 from both sides:

$40 - 1 = 3x + 1 - 1$

$39 = 3x$

$\div 3$ both sides:

$\frac{39}{3} = \frac{3x}{3}$

$x = 13$

19 $d = 95t$

a $d = 190$

$190 = 95t$

$\frac{190}{95} = \frac{95t}{95}$

$t = 2$ hours

b $d = 250$

$250 = 95t$

$\frac{250}{95} = \frac{95t}{95}$

$t = 2.632$

$0.632 \times 60 = 37.92$ (to nearest minute, 38)

$t = 2$ hours 38 minutes

c $d = 65$

$65 = 95t$

$\frac{65}{95} = \frac{95t}{95}$

$t = 0.6842$

$0.6842 \times 60 = 41.05$ (to nearest minute, 41)

$t = 41$ minutes

d $d = 356.5$

$356.5 = 95t$

$\frac{356.5}{95} = \frac{95t}{95}$

$t = 3.752\,63$

$0.752\,63 \times 60 = 45.158$ (to nearest minute, 45)

$t = 3$ hours 45 minutes

e $d = 50\,000$ m (50 km)

$50 = 95t$

$\frac{50}{95} = \frac{95t}{95}$

$t = 0.5263$

$0.5263 \times 60 = 31.5789$ (to nearest minute, 32)

$t = 32$ minutes

20 **a** **i** $A = 400 + 150m$

$A = 1750$

$1750 = 400 + 150m$

$1750 - 400 = 400 - 400 + 150m$

$1350 = 150m$

$\frac{1350}{150} = \frac{150m}{150}$

$m = 9$ months

ii $A = 400 + 150m$

$A = 3200$

$3200 = 400 + 150m$

$3200 - 400 = 400 - 400 + 150m$

$2800 = 150m$

$\frac{2800}{150} = \frac{150m}{150}$

$m = 18.67$

To the nearest month,

$m = 19$ months

b $A = 400 + 150m$

$A = 10\,000$

$10\,000 = 400 + 150m$

$10\,000 - 400 = 400 - 400 + 150m$

$9600 = 150m$

$\frac{9600}{150} = \frac{150m}{150}$

$m = 64$ months (5 years, 4 months)

21 **a** -32, $\times 5$, $\div 9$

b $\times 9$, $\div 5$, $+32$

c $C = \frac{5(F-32)}{9}$

$\times 9$ both sides:

$C \times 9 = \frac{5(F-32)}{9} \times 9$

$9C = 5(F - 32)$

$\div 5$ both sides:

$\frac{9C}{5} = \frac{5(F-32)}{5}$

$\frac{9C}{5} = F - 32$

$+32$ to both sides:

$\frac{9C}{5} + 32 = F - 32 + 32$

$\frac{9C}{5} + 32 = F$

$C = 190°$

$F = \frac{9C}{5} + 32$

$F = \frac{9 \times 190}{5} + 32$

$F = 374$

Temperature = 374 °F

22 **a** $A = 3.5\pi(3.5 + h)$

$A = 200$

$200 = 3.5\pi(3.5 + h)$

$$\frac{200}{3.5\pi} = \frac{3.5\pi\,(3.5+h)}{3.5\pi}$$
$$\frac{200}{3.5\pi} = 3.5 + h$$
$$\frac{200}{3.5\pi} - 3.5 = 3.5 + h - 3.5$$
$$\frac{200}{3.5\pi} - 3.5 = h$$

(Use calculator value for $\pi \approx 3.141\,592\,654$.)

$h = 14.689\,136\,35$

Correct to 2 decimal places,

$h = 14.69$ cm.

b $A = 240$

$$240 = 3.5\pi(3.5+h)$$
$$\frac{240}{3.5\pi} = \frac{3.5\pi(3.5+h)}{3.5\pi}$$
$$\frac{240}{3.5\pi} = 3.5 + h$$
$$\frac{240}{3.5\pi} - 3.5 = 3.5 + h - 3.5$$
$$\frac{240}{3.5\pi} - 3.5 = h$$

(Use calculator value for $\pi \approx 3.141\,592\,654$.)

$h = 18.326\,963\,62$

Correct to 2 decimal places, $h = 18.33$ cm.

c $A = 270$

$$270 = 3.5\pi(3.5+h)$$
$$\frac{270}{3.5\pi} = \frac{3.5\pi(3.5+h)}{3.5\pi}$$
$$\frac{270}{3.5\pi} = 3.5 + h$$
$$\frac{270}{3.5\pi} - 3.5 = 3.5 + h - 3.5$$
$$\frac{270}{3.5\pi} - 3.5 = h$$

(Use calculator value for $\pi \approx 3.141\,592\,654$.)

$h = 21.055\,334\,08$

Correct to 2 decimal places, $h = 21.06$ cm.

2.2 Exam questions

1 If $r = 5\%$,

$$\begin{aligned} I &= 10r \\ &= 10 \times 5 \\ &= \$50 \end{aligned}$$

The correct answer is **C**.

2 $2(x+1) = 5(x-2)$

$$\begin{aligned} 2x + 2 &= 5x - 10 \\ 2 &= 3x - 10 \\ 2 + 10 &= 3x - 10 + 10 \\ 12 &= 3x \\ x &= 4 \end{aligned}$$

[1 mark]

3 $x = \frac{1}{4}$

Can either substitute alternatives in each equation ('satisfies') or solve each equation.

The correct answer is **A**.

2.3 Developing linear equations

2.3 Exercise

1 $8P = 17.92$, where P = The price of one art pencil

$$\frac{8P}{8} = \frac{17.92}{8}$$
$$P = 2.24$$

The price of one art pencil is \$2.24.

2 $4V = 9.36$, where V = price for one red velvet cupcake

$$\frac{4V}{4} = \frac{9.36}{4}$$
$$V = 2.34$$

$3C = 7.41$, where C = price for one chocolate delight cupcake

$$\frac{3C}{3} = \frac{7.41}{3}$$
$$C = 2.47$$

$5S = 11.80$, where S = price for one caramel surprise cupcake

$$\frac{5S}{5} = \frac{11.80}{5}$$
$$S = 2.36$$

Red velvet cupcakes price = \$2.34

Chocolate delight cupcake price = \$2.47

Caramel surprise cupcake price = \$2.36

The red velvet cupcakes are the cheapest per cupcake.

3 $\frac{x+3}{4} = 9$, where x is the number.

$$\begin{aligned} \frac{x+3}{4} \times 4 &= 9 \times 4 \\ x + 3 &= 36 \\ x + 3 - 3 &= 36 - 3 \\ x &= 33 \end{aligned}$$

4 Perimeter $= 3l + 3l + l + l$, where l = length of shorter side

$$84 = 8l$$
$$\frac{84}{8} = \frac{8l}{8}$$
$$l = 10.5$$

Other side $= 3 \times 10.5 = 31.5$

Hence, the parallelogram has side lengths 10.5 cm and 31.5 cm.

5 $f + 0.8 \times 2f = 20.54$ where f = price of 1 bag of fruit and nut mix

Note: 20% discount means that he pays 80% of original price.

$$\begin{aligned} 2.6f &= 20.54 \\ \frac{2.6f}{2.6} &= \frac{20.54}{2.6} \\ f &= 7.9 \end{aligned}$$

The original price of a bag of fruit and nut mix is \$7.90.

6 $6(x+4) = 126$, where x is the number.

$$\begin{aligned} \frac{6(x+4)}{6} &= \frac{126}{6} \\ x + 4 &= 21 \\ x + 4 - 4 &= 21 - 4 \\ x &= 17 \end{aligned}$$

The number is 17.

7 a See the table at the foot of the page.*

b Looking at the table of values, 3 km lies between the 7th and 8th minutes into her journey.

8 a See the table at the foot of the page.*

b Looking at the table of values, Tommy needs \$49 to buy the remote control car, so after 10 weeks he will have saved enough money.

9 Let width be x; length $= x + 6$.

Perimeter $= 64 = 2x + 2(x + 6)$

$$64 = 2x + 2x + 12$$
$$64 = 4x + 12$$
$$64 - 12 = 4x + 12 - 12$$
$$52 = 4x$$
$$\frac{52}{4} = \frac{4x}{4}$$
$$x = 13$$

Width $= 13$ metres, length $= 13 + 6 = 19$ metres

The dimensions are 13 metres by 19 metres.

10 a See the table at the foot of the page.*

b See the table at the foot of the page.*

c Subtract the values in the first table from the values in the second table.

d See the table at the foot of the page.*

2.3 Exam questions

1 $6(x + 3) = 24$

Order of operations is important.

The correct answer is **D**.

2 Let b equal the number of boys.

Then $b \times 1.2$ equals the number of girls.

Hence, the total number of students $= b + b \times 1.2$

$b + 1.2b = 44$

Note: 20% more is the same as by multiplying by 1.2.

The correct answer is **C**.

3 3300 m

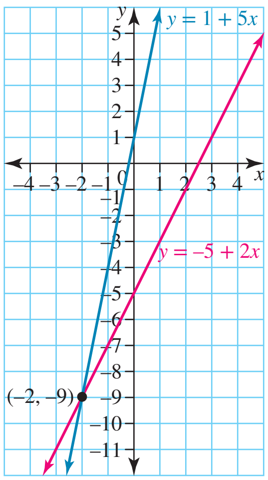
Can use the pattern in the perimeter column.

The correct answer is **E**.

2.4 Simultaneous linear equations

2.4 Exercise

1 Sketch the graphs of $y = 1 + 5x$ and $y = -5 + 2x$ and find the point of intersection by reading off the graph.



$x = -2, y = -9$

***7 a**

Minute	1	2	3	4	5	6	7	8	9	10
Distance (km)	0.4 (0.4×1)	0.8 (0.4×2)	1.2 (0.4×3)	1.6 (0.4×4)	2 (0.4×5)	2.4 (0.4×6)	2.8 (0.4×7)	3.2 (0.4×8)	3.6 (0.4×9)	4 (0.4×10)

***8 a**

Week	0	1	2	3	4	5	6	7	8	9	10	11	12
Money (\$)	20	23	26	29	32	35	38	41	44	47	50	53	56
$M = 20 + 3w$	$M = 20 + 3(0)$ $M = 20$	$M = 20 + 3(1)$ $M = 23$	$M = 20 + 3(2)$ $M = 26$	$M = 20 + 3(3)$ $M = 29$	$M = 20 + 3(4)$ $M = 32$	$M = 20 + 3(5)$ $M = 35$	$M = 20 + 3(6)$ $M = 38$	$M = 20 + 3(7)$ $M = 41$	$M = 20 + 3(8)$ $M = 44$	$M = 20 + 3(9)$ $M = 47$	$M = 20 + 3(10)$ $M = 50$	$M = 20 + 3(11)$ $M = 53$	$M = 20 + 3(12)$ $M = 56$

***10 a**

Number of boards	10	11	12	13	14	15	16	17	18	19	20
Cost (\$)	395	409.50	424	438.50	453	467.50	482	496.50	511	525.50	540

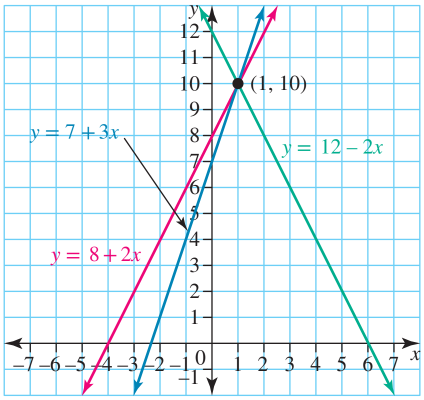
***10 b**

Number of boards	10	11	12	13	14	15	16	17	18	19	20
Revenue (\$)	329.50	362.45	395.40	428.35	461.30	494.25	527.20	560.15	593.01	626.05	659

***10 d**

Number of boards	10	11	12	13	14	15	16	17	18	19	20
Profit (\$)	−65.50	−47.05	−28.60	−10.15	8.30	26.75	45.20	63.65	82.10	100.55	119

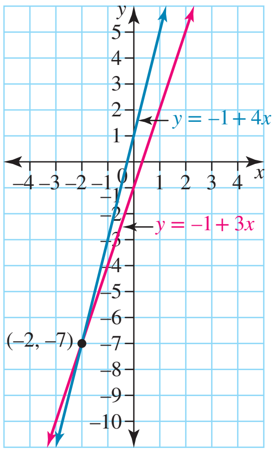
2 Sketch the graphs of $y = 7 + 3x$, $y = 8 + 2x$ and $y = 12 - 2x$ and find the point of intersection by reading off the graph.



$x = 1, y = 10$

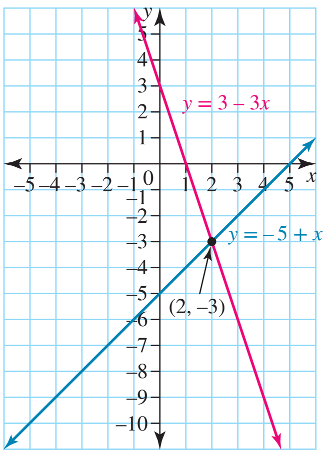
3 Reading off the graph, the point of intersection is at $x = -2, y = 5$.

4 a



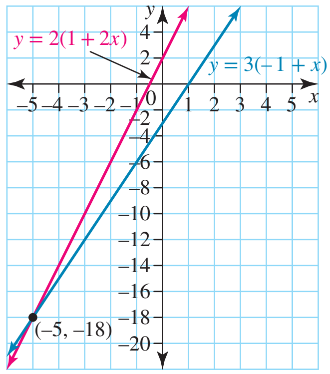
$x = -2, y = -7$

b



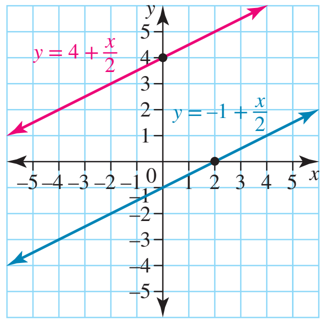
$x = 2, y = -3$

c



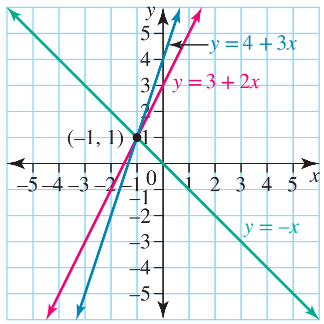
$x = -5, y = -18$

d



No solution

e



$x = -1, y = 1$

5 a $y = 1 + 2x$ and $2y - x = -1$

Substitute $y = 1 + 2x$ into $2y - x = -1$:

$2(1 + 2x) - x = -1$

Expand and simplify:

$2 + 4x - x = -1$

$3x + 2 = -1$

Solve for x:

$3x + 2 - 2 = -1 - 2$

$3x = -3$

$x = -1$

Substitute into either equation:

$y = 1 + 2x$

$y = 1 + 2(-1)$

$y = -1$

$x = -1, y = -1$

b $m = 5 + 2n$ and $m = -1 + 4n$

Substitute $m = 5 + 2n$ into $m = -1 + 4n$:

$5 + 2n = -1 + 4n$

Solve for n:

$5 + 2n - 2n = -1 + 4n - 2n$

$5 = 2n - 1$

$5 + 1 = 2n - 1 + 1$

$6 = 2n$

$\frac{6}{2} = \frac{2n}{2}$

$n = 3$

Substitute into either equation:

$m = 5 + 2n$

$m = 5 + 2(3)$

$m = 11$

$n = 3,\ m = 11$

c $2x - y = 5$ and $y = 1 + x$

Substitute $y = 1 + x$ into $2x - y = 5$:

$2x - (1 + x) = 5$

Expand and simplify:

$2x - 1 - x = 5$

$x - 1 = 5$

Solve for x:

$x - 1 + 1 = 5 + 1$

$x = 6$

Substitute into either equation:

$y = 1 + x$

$y = 1 + 6$

$y = 7$

$x = 6,\ y = 7$

6 a $2(x + 1) + y = 5$ and $y = -6 + x$

Substitute $y = -6 + x$ into

$2(x + 1) + y = 5$:

$2(x + 1) - 6 + x = 5$

Expand and simplify:

$2x + 2 - 6 + x = 5$

$3x - 4 = 5$

Solve for x:

$3x - 4 + 4 = 5 + 4$

$3x = 9$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

Substitute into either equation:

$y = -6 + x$

$y = -6 + 3$

$y = -3$

$x = 3,\ y = -3$

b $\frac{x + 5}{2} + 2y = 11$ and $y = -2 + 6x$

Substitute $y = -2 + 6x$ into $\frac{x + 5}{2} + 2y = 11$:

$\frac{x + 5}{2} + 2(-2 + 6x) = 11$

Expand and simplify:

$\frac{x + 5}{2} - 4 + 12x = 11$

Multiply by both sides by 2.

$\frac{x + 5}{2} \times 2 + 2(-4 + 12x) = 11 \times 2$

$x + 5 - 8 + 24x = 22$

$25x - 3 = 22$

Solve for x:

$25x - 3 + 3 = 22 + 3$

$25x = 25$

$\frac{25x}{25} = \frac{25}{25}$

$x = 1$

Substitute into either equation:

$y = -2 + 6x$

$y = -2 + 6(1)$

$y = 4$

$x = 1,\ y = 4$

7 For the substitution method, one or both equations must be 'pronumeral' =.

The correct answer is **E**.

8 a $y = 5 + 2x$ and $y = -2 + 3x$

Substitute $y = 5 + 2x$ into $y = -2 + 3x$:

$5 + 2x = -2 + 3x$

Solve for x:

$5 + 2x - 2x = -2 + 3x - 2x$

$5 = -2 + x$

$5 + 2 = -2 + 2 + x$

$x = 7$

Substitute into either equation:

$y = 5 + 2x$

$y = 5 + 2(7)$

$y = 19$

$x = 7,\ y = 19$

b $y = -2 + 5x$ and $y = 2 + 7x$

Substitute $y = -2 + 5x$ into $y = 2 + 7x$:

$-2 + 5x = 2 + 7x$

Solve for x:

$-2 + 5x - 5x = 2 + 7x - 5x$

$-2 = 2 + 2x$

$-2 - 2 = 2 + 2x - 2$

$-4 = 2x$

$\frac{-4}{2} = \frac{2x}{2}$

$x = -2$

Substitute into either equation:

$y = -2 + 5x$

$y = -2 + 5(-2)$

$y = -12$

$x = -2,\ y = -12$

c $y = 2(3x + 1)$ and $y = 4(2x - 3)$

Substitute $y = 2(3x + 1)$ into $y = 4(2x - 3)$:

$2(3x + 1) = 4(2x - 3)$

Expand and simplify:

$6x + 2 = 8x - 12$

Solve for x:

$6x + 2 - 6x = 8x - 12 - 6x$

$2 = 2x - 12$

$2 + 12 = 2x - 12 + 12$

$14 = 2x$

$\frac{14}{2} = \frac{2x}{2}$

$x = 7$

Substitute into either equation:
$y = 2(3x + 1)$
$y = 2(3(7) + 1)$
$y = 2(21 + 1)$
$y = 2(22)$
$y = 44$
$x = 7,\ y = 44$

d $y = -9 + 5x$ and $3x - 5y = 1$
Substitute $y = -9 + 5x$ into $3x - 5y = 1$:
$3x - 5(-9 + 5x) = 1$
Expand and simplify:
$3x + 45 - 25x = 1$
$-22x + 45 = 1$
Solve for x:
$-22x + 45 - 45 = 1 - 45$
$-22x = -44$
$\frac{-22x}{-22} = \frac{-44}{-22}$
$x = 2$
Substitute into either equation:
$y = -9 + 5x$
$y = -9 + 5(2)$
$y = 1$
$x = 2,\ y = 1$

e $3(2x + 1) + y = -19$ and $y = -1 + x$
Substitute $y = -1 + x$ into $3(2x + 1) + y = -19$:
$3(2x + 1) - 1 + x = -19$
Expand and simplify:
$6x + 3 - 1 + x = -19$
$7x + 2 = -19$
Solve for x:
$7x + 2 - 2 = -19 - 2$
$7x = -21$
$\frac{7x}{7} = \frac{-21}{7}$
$x = -3$
Substitute into either equation:
$y = -1 + x$
$y = -1 - 3$
$y = -4$
$x = -3, y = -4$

f $\frac{3x + 5}{2} + 2y = 2$ and $y = -2 + x$.
Substitute $y = -2 + x$ into $\frac{3x + 5}{2} + 2y = 2$:
$\frac{3x + 5}{2} + 2(-2 + x) = 2$
Expand and simplify:
$\frac{3x + 5}{2} - 4 + 2x = 2$
Multiply both sides by 2.
$\frac{3x + 5}{2} \times 2 + 2(-4 + 2x) = 2 \times 2$
$3x + 5 - 8 + 4x = 4$
$7x - 3 = 4$
Solve for x:
$7x - 3 = 4$
$7x - 3 + 3 = 4 + 3$
$7x = 7$
$\frac{7x}{7} = \frac{7}{7}$
$x = 1$
Substitute into either equation:
$y = -2 + x$
$y = -1 + 1$
$y = -1$
$x = 1, y = -1$

9 a Both unknowns are on the same side.

b Add the two equations and solve for x. Then substitute x into one of the equations to solve for y.
$3x + y = 8$ [1] and $2x - y = 7$ [2]
[1] + [2]
$3x + 2x + y - y = 8 + 7$
$5x = 15$
Solve for x:
$\frac{5x}{5} = \frac{15}{5}$
$x = 3$
Substitute into either equation:
[2] $2x - y = 7$
$2(3) - y = 7$
$6 - y = 7$
$6 - y - 6 = 7$
$-y = 1$
$y = -1$

10 a $3x + y = 5$ [1] and $4x - y = 2$ [2]
Eliminate y:
$3x + y = 5$ [1]
$4x - y = 2$ [2]
[1] + [2] (coefficients of y are 1 and -1)
$3x + 4x + y - y = 5 + 2$
$7x = 7$
Solve for x:
$\frac{7x}{7} = \frac{7}{7}$
$x = 1$
Substitute into either equation:
$3x + y = 5$[1]
$3(1) + y = 5$
$3 + y - 3 = 5 - 3$
$y = 2$
$x = 1,\ y = 2$

b $2a + b = 7$ and $a + b = 5$
Eliminate b:
$2a + b = 7$ [1]
$a + b = 5$ [2]
[1] − [2] (coefficients of b are both 1)
$2a - a + b - b = 7 - 5$
$a = 2$
Substitute into either equation:
$a + b = 5$ [2]
$2 + b = 5$
$2 + b - 2 = 5 - 2$
$b = 3$
$a = 2, b = 3$

c $3c + 4d = 5$ and $2c + 3d = 4$

Eliminate c.

$3c + 4d = 5 \quad [1]$

$2c + 3d = 4 \quad [2]$

Coefficients of c are 3 and 2 respectively; need to multiply so that coefficients are the same:

$$\begin{array}{lll} 3c + 4d = 5 & [1] \times 2 & \\ 2c + 3d = 4 & [2] \times 3 & \end{array} \longrightarrow \begin{array}{ll} 6c + 8d = 10 & [3] \\ 6c + 9d = 12 & [4] \end{array}$$

$[4] - [3]$

$6c - 6c + 9d - 8d = 12 - 10$

$d = 2$

Substitute into either equation:

$3c + 4d = 5 \quad [1]$

$3c + 4(2) = 5$

$3c + 8 = 5$

$3c + 8 - 8 = 5 - 8$

$3c = -3$

$\frac{3c}{3} = \frac{-3}{3}$

$c = -1$

$c = -1, d = 2$

11 $ax - 3y = -16$ and $3x + y = -2$

$y = 4$; substitute into $3x + y = -2$:

$3x + 4 = -2$

$3x + 4 - 4 = -2 - 4$

$3x = -6$

$\frac{3x}{3} = \frac{-6}{3}$

$x = -2$

Substitute into $ax - 3y = -16$:

$a(-2) - 3(4) = -16$

$-2a - 12 = -16$

$-2a - 12 + 12 = -16 + 12$

$-2a = -4$

$\frac{-2a}{-2} = \frac{-4}{-2}$

$a = 2$

$a = 2$ and $x = -2$

12 **a** $4x + y = 6$ and $x - y = 4$

Eliminate y:

$4x + y = 6 \quad [1]$

$x - y = 4 \quad [2]$

$[1] + [2]$ (coefficients of y are 1 and -1)

$4x + x + y - y = 6 + 4$

$5x = 10$

Solve for x:

$\frac{5x}{5} = \frac{10}{5}$

$x = 2$

Substitute into either equation:

$x - y = 4 \quad [2]$

$2 - y = 4$

$2 - y - 2 = 4 - 2$

$y = -2$

$x = 2, y = -2$

b $x + y = 7$ and $x - 2y = -5$

Eliminate x:

$x + y = 7 \quad [1]$

$x - 2y = -5 \quad [2]$

$[1] - [2]$ (coefficients of x are both 1)

$x - x + y - (-2y) = 7 - (-5)$

$3y = 12$

Solve for y:

$\frac{3y}{3} = \frac{12}{3}$

$y = 4$

Substitute into either equation:

$x + y = 7 \quad [1]$

$x + 4 = 7$

$x + 4 - 4 = 7 - 4$

$x = 3$

$x = 3, y = 4$

c $2x - y = -5$ and $x - 3y = -10$

Eliminate x:

$2x - y = -5 \quad [1]$

$x - 3y = -10 \quad [2]$

Coefficients of x are 2 and 1 respectively; need to multiply so that coefficients are the same:

$$\begin{array}{ll} 2x - y = -5 & [1] \times 1 \\ x - 3y = -10 & [2] \times 2 \end{array} \longrightarrow \begin{array}{ll} 2x - y = -5 & [1] \\ 2x - 6y = -20 & [3] \end{array}$$

$[1] - [3]$

$2x - 2x - y - (-6y) = -5 - (-20)$

$5y = 15$

Solve for y:

$\frac{5y}{5} = \frac{15}{5}$

$y = 3$

Substitute into either equation:

$2x - y = -5 \quad [1]$

$2x - (3) = -5$

Solve for x:

$2x - (3) + 3 = -5 + 3$

$2x = -2$

$\frac{2x}{2} = \frac{-2}{2}$

$x = -1$

$x = -1, y = 3$

d $4x + 3y = 29$ and $2x + y = 13$

Eliminate x:

$4x + 3y = 29 \quad [1]$

$2x + y = 13 \quad [2]$

Coefficients of x are 4 and 2 respectively; need to multiply so that coefficients are the same:

$$\begin{array}{ll} 4x + 3y = 29 & [1] \times 1 \\ 2x + y = 13 & [2] \times 2 \end{array} \longrightarrow \begin{array}{ll} 4x + 3y = 29 & [1] \\ 4x + 2y = 26 & [3] \end{array}$$

$[1] - [3]$

$4x - 4x + 3y - 2y = 29 - 26$

$y = 3$

Substitute into either equation:

$2x + y = 13 \quad [2]$

$2x + 3 = 13 \quad [2]$

Solve for x:

$2x + 3 - 3 = 13 - 3$

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

$x = 5, y = 3$

e $5x - 7y = -33$ and $4x + 3y = 8$

Eliminate x:

$5x - 7y = -33$ [1]

$4x + 3y = 8$ [2]

Coefficients of x are 5 and 4 respectively; need to multiply so that coefficients are the same:

$5x - 7y = -33$ [1] $\times 4$ → $20x - 28y = -132$ [3]

$4x + 3y = 8$ [2] $\times 5$ → $20x + 15y = 40$ [4]

[4] − [3]

$20x - 20x + 15y - (-28y) = 40 - (-132)$

$43y = 172$

Solve for y:

$\frac{43y}{43} = \frac{172}{43}$

$y = 4$

Substitute into either equation:

$4x + 3y = 8$ [2]

$4x + 3(4) = 8$

$4x + 12 = 8$

Solve for x:

$4x + 12 - 12 = 8 - 12$

$4x = -4$

$\frac{4x}{4} = \frac{-4}{4}$

$x = -1$

$x = -1, y = 4$

f $\frac{x}{2} + y = 7$ and $3x + \frac{y}{2} = 20$

The easiest method is to multiply each equation by the denominator:

$2 \times \frac{x}{2} + 2 \times y = 2 \times 7$

$x + 2y = 14$

$2 \times 3x + \frac{y}{2} \times 2 = 2 \times 20$

$6x + y = 40$

Eliminate y:

$x + 2y = 14$ [1]

$6x + y = 40$ [2]

Coefficients of y are 2 and 1 respectively; need to multiply so that coefficients are the same:

$x + 2y = 14$ [1] $\times 1$ → $x + 2y = 14$ [1]

$6x + y = 40$ [2] $\times 2$ → $12x + 2y = 80$ [3]

[3] − [1]

$12x - x + 2y - 2y = 80 - 14$

$11x = 66$

Solve for x:

$\frac{11x}{11} = \frac{66}{11}$

$x = 6$

Substitute into either equation:

$x + 2y = 14$ [1]

$6 + 2y = 14$

Solve for y:

$6 + 2y - 6 = 14 - 6$

$2y = 8$

$\frac{2y}{2} = \frac{8}{2}$

$y = 4$

$x = 6, y = 4$

13 Since the coefficients of y are 1 and -1, adding the two equations together will eliminate y.
The correct answer is **A**.

14 Let r = number of roses, l = number of lillies

$r + l = 19$

$r = 19 - l$ (substitute into equation below)

$6.20r + 4.70l = 98.30$

$6.20(19 - l) + 4.70l = 98.30$

$6.20 \times 19 - 6.20 \times l + 4.70l = 98.30$

$117.80 - 6.20l + 4.70l = 98.30$ (collect like terms)

$117.80 - 1.50l = 98.30$

Solve the equation for l.

$117.80 - 1.50l - 117.80 = 98.30 - 117.80$

$-1.50l = -19.50$

$\frac{-1.50l}{-1.50} = \frac{-19.50}{-1.50}$

$l = 13$

Substitute this value into the equation $r + l = 19$.

$r = 19 - 1$

$r = 19 - 13$

$r = 6$

Therefore, Fredo bought 6 roses and 13 lilies.

15 $s + c = 28$ where s = number of strawberry twists, c = number of chocolate ripple

$s = 28 - c$ (substitute into the equation below)

$5s + 9c = 188$

$5(28 - c) + 9c = 188$

$5 \times 28 - 5 \times c + 9c = 188$

$140 - 5c + 9c = 188$ (collect like terms)

$140 + 4c = 188$

$140 + 4c - 140 = 188 - 140$

$4c = 48$

$\frac{4c}{4} = \frac{48}{4}$

$c = 12$

Substitute $c = 12$ into $s + c = 28$.

$s = 28 - c$

$s = 28 - 12$

$s = 16$

Therefore, Miriam bought 16 strawberry twists and 12 chocolate ripples.

16 Michelle's speed = m

Lydia's speed = $m - 10$

Distance Michelle covered = $2.5m$

Distance Lydia covered = $2.5(m - 10)$

In total they cover 325 km.

$2.5m + 2.5(m - 10) = 325$

$2.5m + 2.5m - 25 = 325$

$$5m = 350$$
$$m = \frac{350}{5}$$
$$m = 70$$

Lydia's speed $= 70 - 10$
$= 60$

Michelle travelled at an average of 70 km/h and Lydia travelled at an average of 60 km/h.

17 $\frac{1290}{86} = 15$, so the shopper bought 15 vegetables in total.

Number of carrots $= c$
Number of potatoes $= 15 - c$
Weight of carrots $= 60c$
Weight of potatoes $= 125(15 - c)$

$$60c + 125(15 - c) = 1290$$
$$60c + 1875 - 125c = 1290$$
$$-65c = -585$$
$$c = \frac{-585}{-65}$$
$$c = 9$$

Number of potatoes $= 15 - 9$
$= 6$

The shopper bought 9 carrots and 6 potatoes.

18 **a** Marcia added the equations together instead of subtracting. The correct result for step 2 is $22y = 11$.

b Step 1: Equation [1] $\times$ 4:
$12x + 16y = 68$
Equation [2] $\times$ 3:
$12x - 6y = 57$
Step 2: [2] − [1]:
$22y = 11$
Step 3: ÷ 22 both sides:
$y = \frac{1}{2}$
Step 4: Substitute $y = \frac{1}{2}$ into [1]:
$3x + 4 \times \frac{1}{2} = 17$ [1]
$3x + 2 = 17$
Step 5: Solve for x:
$3x = 17 - 2$ (− 2 from both sides)
$3x = 15$ (÷ 3 both sides)
$x = 5$
$x = 5, y = \frac{1}{2}$

c Eliminate y:
Step 1:
$3x + 4y = 17$ [1]
$4x - 2y = 19$ [2] $\times 2$
$8x - 4y = 38$ [3]
Step 2: [1] + [3]
$3x + 8x + 4y - 4y = 17 + 38$
$11x = 55$
Step 3: ÷ 11 both sides:
$\frac{11x}{11} = \frac{55}{11}$
$x = 5$
Step 4: Substitute $x = 5$ into [1]:
$3x + 4y = 17$ [1]
$3(5) + 4y = 17$
$15 + 4y = 17$
Step 5: Solve for y:
$4y = 17 - 15$ (− 15 from both sides)
$4y = 2$ (÷ 4 both sides)
$y = \frac{1}{2}$

19 **a** Rockets: $6x + 12y = 54$
Comets: $7x + 5y = 45$
Eliminate y:
$6x + 12y = 54$ [1]
$7x + 5y = 45$ [2]
Coefficients of y are 12 and 5 respectively; need to multiply so that coefficients are the same:

$6x + 12y = 54$ [1] $\times$ 5 → $30x + 60y = 270$ [3]
$7x + 5y = 45$ [2] $\times$ 12 → $84x + 60y = 540$ [4]

[4] − [3]
$84x - 30x + 60y - 60y = 540 - 270$
$54x = 270$
Solve for x:
$\frac{54x}{54} = \frac{270}{54}$
$x = 5$
Substitute into either equation:
$6x + 12y = 54$ [1]
$6(5) + 12y = 54$
$30 + 12y = 54$
Solve for y:
$30 + 12y - 30 = 54 - 30$
$12y = 24$
$\frac{12y}{12} = \frac{24}{12}$
$y = 2$
Goal = 5 points, behind = 2 points

b Jetts: 4 goals, 10 behinds: $4 \times 5 + 10 \times 2 = 40$ points
Meteorites: 6 goals, 9 behinds: $6 \times 5 + 9 \times 2 = 48$ points

20 **a** Equation 2 has unknowns on each side of the equal sign and can be substituted into equation 1.

b $x + y = 15$ and $y = 2x$
Substitute $y = 2x$ into $x + y = 15$:
$x + (2x) = 15$
$3x = 15$
$\frac{3x}{3} = \frac{15}{3}$
$x = 5$
Substitute into $y = 2x$:
$y = 2(5)$
$= 10$
Milan works 5 hours; Yashab works 10 hours.

c $x = \frac{y}{3}$ and $x + y = 15$
Substitute $x = \frac{y}{3}$ into $x + y = 15$:
$\frac{y}{3} + y = 15$
$\frac{4y}{3} = 15$
$\frac{4y}{3} \times 3 = 15 \times 3$
$4y = 45$

$$\frac{4y}{4} = \frac{45}{4}$$
$$y = 11.25$$

Substitute into $x = \frac{y}{3}$:

$$x = \frac{11.25}{3}$$
$$= 3.75$$

Milan works 3 hours 45 minutes (3.75 hours).

2.4 Exam questions

1 Reading the graph, the point of intersection is $(-1, 1)$.
The correct answer is **E**.

2 $x + 2y = 9$ [1]
$y = -1 + 5x$ [2]
Substitute $y = 5x - 1$ into equation [1]:
$$x + 2(-1 + 5x) = 9$$
$$x - 2 + 10x = 9$$
$$11x - 2 = 9$$
$$11x = 9 + 2$$
$$11x = 11$$
$$x = 1$$
Substitute $x = 1$ into equation [1]:
$$x + 2y = 9$$
$$1 + 2y = 9$$
$$2y = 9 - 1$$
$$2y = 8$$
$$y = 4$$
Therefore, the solution is $(1, 4)$.
The correct answer is **A**.

3 $2x + y = 8$ [1]
$3x - y = 17$ [2]
[1] + [2]
$$5x = 25$$
$$x = 5$$
Substitute $x = 5$ into equation [1]
$$2x + y = 8$$
$$2(5) + y = 8$$
$$10 + y = 8$$
$$y = 8 - 10$$
$$y = -2$$
Therefore, the solution is $(5, -2)$.
The correct answer is **C**.

2.5 Problem solving with simultaneous equations

2.5 Exercise

1 $4d + 3c = 10.55$ and $2d + 4c = 9.90$

2 a Equation 1: $a + c = 350$
Equation 2: $25a + 15c = 6650$
Eliminate c:
$a + c = 350$ [1]
$25a + 15c = 6650$ [2]
Coefficients of c are 1 and 15 respectively; need to multiply so that coefficients are the same:

$a + c = 350$ [1] × 15 $\longrightarrow$ $15a + 15c = 5250$ [3]
$25a + 15c = 6650$ [2] × 1

[2] − [3]
$$25a - 15a + 15c - 15c = 6650 - 5250$$
$$10a = 1400$$
Solve for x:
$$\frac{10a}{10} = \frac{1400}{10}$$
$$a = 140$$
Substitute into either equation:
$a + c = 350$ [1]
$$140 + c = 350$$
Solve for c:
$$140 + c - 140 = 350 - 140$$
$$c = 210$$
140 adults and 210 children

b The cost of an adult's ticket is \$25, the cost of a children's ticket is \$15, and the total ticket sales is \$6650.

3 a $C = 80 + 4.50n$ and $R = 12.50n$
$$80 + 4.50n = 12.50n$$
Solve for n:
$$80 + 4.50n - 4.50n = 12.50n - 4.50n$$
$$80 = 8n$$
$$\frac{80}{8} = \frac{8n}{8}$$
$$n = 10$$
Yolanda needs to sell 10 bracelets to cover her costs.

b i $P = R - C$
$$P = 12.50n - (80 + 4.50n)$$
$$P = 12.50n - 80 - 4.50n$$
$$P = 8n - 80$$
$$n = 8$$
$$P = 8(8) - 80$$
$$P = -16$$
\$16 loss

ii $n = 13$
$$P = 8(13) - 80$$
$$P = 24$$
\$24 profit

4 a $a = 18$, \$18 entry fee

b $b = 3$, \$3 per entry cost

c $C = 2550 + 3n$ and $R = 18n$
$$2550 + 3n = 18n$$
Solve for n:
$$2550 + 3n - 3n = 18n - 3n$$
$$2550 = 15n$$
$$\frac{2550}{15} = \frac{15n}{15}$$
$$n = 170$$
170 entries

d $n = 310$
$$C = 2550 + 3n$$
$$= 2550 + 3(310)$$
Cost = \$3480
$$R = 18n$$
$$= 18(310)$$
Revenue = \$5580

Profit $= R - C$
$= 5580 - 3480$
$= \$2100$

e $P = 18n - (2550 + 3n)$
$= 15n - 2250$
Profit = donation to charity = \$5010
$5010 = 15n - 2550$
$5010 + 2550 = 15n$
$7560 = 15n$
$n = 504$
504 entries

5 **a** $s = \$1.50, a = \$3.50$

b Elimination method: Both equations have unknowns on the same side.

c Bus: $3.5a + 1.5s = 42.50$ [1]
Train: $4.75a + 2.25s = 61.75$ [2]
Eliminate s:
Coefficients of s are 1.5 and 2.25 respectively; need to multiply so that coefficients are the same:*
[3] – [4]
$7.875a - 7.125a + 3.375s - 3.375s = 95.625 - 92.625$
$0.75a = 3$
Solve for a:
$\frac{0.75a}{0.75} = \frac{3}{0.75}$
$a = 4$
Substitute into either equation:
$3.5a + 1.5s = 42.50$ [1]
$3.5(4) + 1.5s = 42.50$
$14 + 1.5s = 42.50$
Solve for c:
$14 + 1.5s - 14 = 42.50 - 14$
$1.5s = 28.5$
$\frac{1.5s}{1.5} = \frac{28.5}{1.5}$
$s = 19$
4 adults and 19 students

6 **a** $a = \$19.50, c = \$14.50$

b The total number of tickets sold (both adult and concession)

c Equation 1: $a + c = 544$
Equation 2: $19.50a + 14.50c = 9013$
Eliminate c:
Coefficients of c are 1 and 14.50 respectively; need to multiply so that coefficients are the same:*
[2] – [3]
$19.5a - 14.5a + 14.5c - 14.5c = 9013 - 7888$
$5a = 1125$
Solve for a:
$\frac{5a}{5} = \frac{1125}{5}$
$a = 225$
Substitute into either equation:
$a + c = 544$ [1]
$225 + c = 544$
Solve for c:
$c = 544 - 225$
$= 319$
225 adult tickets and 319 concession tickets

7 **a** $R = 12.50h$

b $C = 45 + 2.50h$ and $R = 12.50h$
$45 + 2.50h = 12.50h$
Solve for h:
$45 + 2.50h - 2.50h = 12.50h - 2.50h$
$45 = 10h$
$\frac{45}{10} = \frac{10h}{10}$
$h = 4.5$
4.5 hours

c **i** See the table at the foot of the page.*
Charlotte made a profit for jobs 1 and 4 and a loss for jobs 2 and 3.

ii Yes, she made \$15 profit.
$(25 + 5 - (10 + 5)) = 30 - 15 = \$15$

d $P = 10h - 45$
$50 = 10h - 45$
$50 + 45 = 10h - 45 + 45$
$95 = 10h$
$h = 9.5$ hours

8 **a** $15t + 12m = 400.50$ and $9t + 13m = 328.75$

b t represents the hourly rate earned by Trudi and m represents the hourly rate earned by Mia.

c $15t + 12m = 400.50$ and $9t + 13m = 328.75$
Eliminate t:

***5 c**
$3.5a + 1.5s = 42.50$ [1] × 2.25 → $7.875a + 3.375s = 95.625$ [3]
$4.75a + 2.25s = 61.75$ [2] × 1.5 → $7.125a + 3.375s = 92.625$ [4]

***6 c**
$a + c = 544$ [1] × 14.5 → $14.5a + 14.5c = 7888$ [3]
$19.50a + 14.50c = 9013$ [2]

***7 c i**

Babysitting job	1	2	3	4
Number of hours (h)	5	3.5	4	7
Profit/loss ($R - C$): $12.5h - 2.5h - 45$ $= 10h - 45$	$10h - 45$ $= 10(5) - 45$ $= 5$ profit	$10h - 45$ $= 10(3.5) - 45$ $= 35 - 45$ $= -10$ loss	$10h - 45$ $= 10(4) - 45$ $= 40 - 5$ $= -5$ loss	$10h - 45$ $= 10(7) - 45$ $= 70 - 45$ $= 25$ profit

Coefficients of t are 15 and 9 respectively; need to multiply so that the coefficients are the same.*

$$135t - 135t + 195m - 108m = 4931.25 - 3604.5$$
$$87m = 1326.75$$

Solve for m.

$$\frac{87m}{87} = \frac{1326.75}{87}$$
$$m = 15.25$$

Substitute into either equation:

$$9t + 13m = 328.75 \qquad [2]$$
$$9t + 13(15.25) = 328.75$$
$$9t + 198.25 = 328.75$$

Solve for t.

$$9t + 198.25 - 198.25 = 328.75 - 198.25$$
$$9t = 130.5$$
$$\frac{9t}{9} = \frac{130.5}{9}$$
$$t = 14.5$$

$t = \$14.50, m = \$15.25$

9 a $5x + 4y = 31.55$ and $4x + 3y = 24.65$

b Eliminate y:

Coefficients of y are 4 and 3 respectively; need to multiply so that the coefficients are the same.

$$\begin{array}{lll} 5x + 4y = 31.55 & [1] \times 3 & \\ 4x + 3y = 24.65 & [2] \times 4 & \end{array} \longrightarrow \begin{array}{ll} 15x + 12y = 94.65 & [3] \\ 16x + 12y = 98.6 & [4] \end{array}$$

[4] − [3]

$$16x - 15x + 12y - 12y = 98.6 - 94.65$$
$$x = 3.95$$

Substitute into either equation:

$$5x + 4y = 31.55 \qquad [1]$$
$$5(3.95) + 4y = 31.55$$
$$19.75 + 4y = 31.55$$

Solve for y.

$$19.75 + 4y - 19.75 = 31.55 - 19.75$$
$$4y = 11.8$$
$$\frac{4y}{4} = \frac{11.8}{4}$$
$$y = 2.95$$

$x = \$3.95, y = \$2.95$

c 2 kg carrots, 1.5 kg apples

$2x + 1.5y$

$2(3.95) + 1.5(2.95) = 12.325$

\$12.35 (to the nearest 5 cents)

10 a $3s + 2g = 1000$ and $4s + 3g = 1430$

b Eliminate g:

Coefficients of g are 2 and 3 respectively; need to multiply so that the coefficients are the same.

$$\begin{array}{ll} 3s + 2g = 1000 & [1] \times 3 \\ 4s + 3g = 1430 & [2] \times 2 \end{array} \longrightarrow \begin{array}{ll} 9s + 6g = 3000 & [3] \\ 8s + 6g = 2860 & [4] \end{array}$$

[3] − [4]

$$9s - 8s + 6g - 6g = 3000 - 2860$$
$$s = 140$$

Substitute into either equation:

$$3s + 2g = 1000 \qquad [1]$$
$$3(140) + 2g = 1000$$
$$420 + 2g = 1000$$

Solve for g.

$$420 + 2g - 420 = 1000 - 420$$
$$2g = 580$$
$$\frac{2g}{2} = \frac{580}{2}$$
$$g = 290$$

One 100 g serve of strawberries has 140 kJ.

11 a $C = 75 + 1.10k$ and $C = 90 + 0.90k$

b $C = 75 + 1.10k$ and $C = 90 + 0.90k$

(Since at least one equation has unknowns on either side of the equals sign, use substitution.)

$$75 + 1.10k = 90 + 0.90k$$
$$75 + 1.10k - 0.9k = 90 + 0.90k - 0.9k$$
$$75 + 0.2k = 90$$
$$75 + 0.2k - 75 = 90 - 75$$
$$0.2k = 15$$
$$k = 75 \text{ km}$$

c They should use GetThere.

$$\begin{aligned} C_{\text{FreeWheels}} &= 75 + 1.10k, \ k = 250 \\ &= 75 + 1.1(250) \\ &= \$350 \end{aligned}$$

$$\begin{aligned} C_{\text{GetThere}} &= 90 + 0.90k, \ k = 250 \\ &= 90 + 0.9\ (250) \\ &= \$315 \end{aligned}$$

12 a The cost is for the three different types of cereal, but the equations only include one type of cereal.

b $2c + 3r + m = 27.45$

$c + 2r + 2m = 24.25$

$3c + 4r + m = 36.35$

c $2c + 3r + m = 27.45 \qquad [1]$

$c + 2r + 2m = 24.25 \qquad [2]$

$3c + 4r + m = 36.35 \qquad [3]$

[1] − [2]:

$c + r - m = 3.2 \qquad [4]$

[3] − [1]:

$c + r = 8.9 \qquad [5]$

[5] − [4]:

$m = 5.7$

Substitute into $2c + 3r + m = 27.45$:

$2c + 3r + 5.7 = 27.45$

$2c + 3r = 21.75 \qquad [6]$

Substitute $m = 5.7$ into [3]:

$3c + 4r + 5.7 = 36.35$

$3c + 4r = 30.65 \qquad [7]$

***8 c**

$$\begin{array}{ll} 15t + 12m = 400.50 & [1] \times 9 \\ 9t + 13m = 328.75 & [2] \times 15 \end{array} \longrightarrow \begin{array}{ll} 135t + 108m = 3604.5 & [3] \\ 135t + 195m = 4931.25 & [4] \end{array}$$

[4] − [3]

[6] × [3]:
$6c + 9r = 65.25$ [8]
[7] × 2:
$6c + 8r = 61.3$ [9]
[8] − [9]:
$r = 3.95$
Substitute into $c + r = 8.9$:
$c + 3.95 = 8.9$
$c = 4.95$
$c = 4.95, r = 3.95, m = 5.7$
Cost of $3c + 2r + 2m = 3(4.95) + 2(3.95) + 2(5.7)$
$= 14.85 + 7.9 + 11.4$
$= \$34.15$

13 a $S = 0.5n$

b $S = 0.5n$ and $C = 0.25n + 2$
$0.5n = 0.25n + 2$
$0.5n - 0.25n = 0.25n + 2 - 0.25n$
$0.25n = 2$
$$\frac{0.25n}{0.25} = \frac{2}{0.25}$$
$n = 8$
8 cups of lemonade

c $P = R - C, n = 20, P = 7$
$7 = a(20) - (0.25(20) + 2)$, where a = new selling price
$7 = 20a - 7$
Solve for a:
$7 + 7 = 20a - 7 + 7$
$14 = 20a$
$$\frac{14}{20} = \frac{20a}{20}$$
$a = 0.7$
70 cents

14 a $C = 7.50$ per T-shirt $+ 810$, where C = cost
$C = 7.5(100) + 810$
$C = \$1560$

b See the table at the foot of the page.*

c $C = 810 + 7.5n$

d $S = 25.50n$

e $C = 810 + 7.5n$ and $S = 25.50n$
Break even when $C = S$:
$810 + 7.5n = 25.5n$
$810 + 7.5n - 7.5n = 25.5n - 7.5n$
$810 = 18n$
$$\frac{810}{18} = \frac{18n}{18}$$
$n = 45$
Therefore, they need to sell 45 T-shirts to break even.

f $P = R - C$
$5000 = 25.5n - (7.5n + 810)$
$5000 = 18n - 810$
$5810 = 18n$
$n = 322.78$
Therefore, they need to sell 323 T-shirts to make at least \$5000 profit.

15 a $5s + 3m + 4p = 19.4$
$4s + 2m + 5p = 17.5$
$3s + 5m + 6p = 24.6$

b $5s + 3m + 4p = 19.4$ [1]
$4s + 2m + 5p = 17.5$ [2]
$3s + 5m + 6p = 24.60$ [3]
[1] − [2]:
$s + m - p = 1.9$ [4]
[2] − [3]:
$s - 3m - p = -7.1$ [5]
[4] − [5]:
$4m = 9$
$m = 2.25$
Substitute into [1]:
$5s + 3(2.25) + 4p = 19.4$
$5s + 6.75 + 4p = 19.4$
$5s + 4p = 12.65$ [6]
Substitute $m = 2.25$ into [2]:
$4s + 2(2.25) + 5p = 17.5$
$4s + 4.5 + 5p = 17.5$
$4s + 5p = 13$ [7]
[6] × 4:
$20s + 16p = 50.6$ [8]
[7] × 5:
$20s + 25p = 65$ [9]
[9] − [8]:
$9p = 14.4$
$p = 1.6$
Substitute $m = 2.25$ and $p = 1.6$ into [4]:
$s + 2.25 - 1.6 = 1.9$
$s + 0.65 = 1.9$
$s = 1.25$
$s = \$1.25, \quad m = \$2.25, \quad p = \$1.60$

c $2s + 4m + 4p = 2(1.25) + 4(2.25) + 4(1.6)$
$= 2.5 + 9 + 6.4$
$= 17.90$
The cost of two starfruit, four mangoes and four papayas is \$17.90.

16 a $24a + 52c + 12s + 15m = 1071$
$35a + 8c + 45s + 27m = 1105.5$
$20a + 55c + 9s + 6m = 961.5$
$35a + 15c + 7s + 13m = 777$

b Using CAS:
Adult ticket = \$13.50, concession = \$10.50,
seniors = \$8.00, members = \$7.00

c i $77 \times 13.5 + 30 \times 10.5 + 15 \times 8 + 45 \times 7$

ii $77 \times 13.5 + 30 \times 10.5 + 15 \times 8 + 45 \times 7 = 1789.5$
$= \$1789.50$

***14 b**

n	0	20	30	40	50	60	80	100	120	140
C	810	960	1035	1110	1185	1260	1410	1560	1710	1860

2.5 Exam questions

1 Since opposite sides of a rectangle are equal, equations can be solved simultaneously.

$x + y = 12 \quad [1]$

$3x - 2y = 10 \quad [2]$

Multiply [1] by 2 and add it to [2].

$2x + 2y = 24 \quad [3]$

$3x - 2y = 10 \quad [2]$

$$5x = 34$$

$$x = \frac{34}{5}$$

$$= 6.8$$

The correct answer is **A**.

2 Even numbers are two apart.
The correct answer is **D**.

3 a $6s + 4w = \$25\,600 \quad [1]$

$3s + w = \$10\,000 \quad [2] \qquad$ [1 mark]

b Multiply [2] by 2:

$6s + 2w = \$20\,000 \quad [3]$

Subtract [3] from [1]:

$$2w = \$5600$$

$$w = \$2800$$

Substitute $w = \$2800$ in [2] :

$$3s + (2800) = \$10\,000$$

$$3s = \$7200$$

$$s = \$2400$$

$\therefore s = \$2400, w = \$2800 \qquad$ [1 mark]

c The profit on selling 10 sedans and 8 wagons is found by substituting the values in P:

$P = 10s + 8w$

$P = 10 \times 2400 + 8 \times 2800$

$= \$46\,400 \qquad$ [1 mark]

2.6 Review

2.6 Exercise

Multiple choice

1
$$y = -6 + 3x$$
$$y + 6 = -6 + 3x + 6$$
$$y + 6 = 3x$$
$$\frac{y+6}{3} = \frac{3x}{3}$$
$$\frac{y+6}{3} = x$$
$$x = \frac{y+6}{3}$$

The correct answer is **B**.

2
$$3(2x + 5) = 12$$
$$\frac{3(2x+5)}{3} = \frac{12}{3}$$
$$2x + 5 = 4$$
$$2x + 5 - 5 = 4 - 5$$
$$2x = -1$$
$$\frac{2x}{2} = \frac{-1}{2}$$
$$x = -0.5$$

The correct answer is **C**.

3
$$v = u + at$$
$$v - u = u + at - u$$
$$v - u = at$$
$$\frac{v-u}{t} = \frac{at}{t}$$
$$\frac{v-u}{t} = a$$
$$a = \frac{v-u}{t}$$

The correct answer is **A**.

4 Alyssa drives an average of 12 km/h faster than Juliana, so if Juliana's average speed is j km/h, Alyssa's average speed is $(j + 12)$ km/h.
The correct answer is **D**.

5 When Juliana and Alyssa pass each other, they have driven a combined 232 km.
In 2 hours, Juliana travels $2j$ km and Alyssa travels $2(j + 12)$ km.

Therefore: $2j + 2(j + 12) = 232$

$$2j + 2j + 24 = 232$$
$$4j + 24 - 24 = 232 - 24$$
$$4j = 208$$
$$\frac{4j}{4} = \frac{208}{4}$$
$$j = 52$$

Therefore, Juliana's average driving speed is 52 km/h.
The correct answer is **B**.

6 $x + 5y = 7 \quad [1]$

$y = 5 - 2x \quad [2]$

Substitute [2] into [1]:

$$x + 5(5 - 2x) = 7$$
$$x + 25 - 10x = 7$$
$$25 - 9x = 7$$
$$25 - 9x - 25 = 7 - 25$$
$$-9x = -18$$
$$\frac{-9x}{-9} = \frac{-18}{-9}$$
$$x = 2$$

Substitute x into [1]:

$$2 + 5y = 7$$
$$2 + 5y - 2 = 7 - 2$$
$$5y = 5$$
$$\frac{5y}{5} = \frac{5}{5}$$
$$y = 1$$

So, the solution to the pair of simultaneous equations is $x = 2, y = 1$.
The correct answer is **C**.

7 a = cost of an adult ticket, c = cost of a child's ticket
Two adults and four children = \$55
Therefore, $2a + 4c = 55$
One adult and three children = \$35
Therefore, $a + 3c = 35$
The correct answer is **E**.

Short answer

8 **a** $7(y+4)=35$

$\frac{7(y+4)}{7}=\frac{35}{7}$

$y+4=5$

$y+4-4=5-4$

$y=1$

b $\frac{5-2m}{3}=-2$

$\frac{5-2m}{3}\times 3=-2\times 3$

$5-2m=-6$

$5-2m-5=-6-5$

$-2m=-11$

$\frac{-2m}{-2}=\frac{-11}{-2}$

$m=5.5$

c $\frac{3s}{4}+6=10$

$\frac{3s}{4}+6-6=10-6$

$\frac{3s}{4}=4$

$\frac{3s}{4}\times 4=4\times 4$

$3s=16$

$\frac{3s}{3}=\frac{16}{3}$

$s=5\frac{1}{3}$

d $\frac{-2(3t+1)}{5}+3=9$

$\frac{-2(3t+1)}{5}+3-3=9-3$

$\frac{-2(3t+1)}{5}=6$

$\frac{-2(3t+1)}{5}\times 5=6\times 5$

$-2(3t+1)=30$

$\frac{-2(3t+1)}{-2}=\frac{30}{-2}$

$3t+1=-15$

$3t+1-1=-15-1$

$3t=-16$

$\frac{3t}{3}=\frac{-16}{3}$

$t=-5\frac{1}{3}$

9 **a** $y=-5+3x$ [1]

$3x+2y=17$ [2]

Substitute [1] into [2]:

$3x+2(-5+3x)=17$

$3x-10+6x=17$

$9x-10=17$

$9x-10+10=17+10$

$9x=27$

$\frac{9x}{9}=\frac{27}{9}$

$x=3$

Substitute $x=3$ into [1]:

$y=-5+3\times 3$

$=9-5$

$=4$

The solution is $x=3, y=4$.

b $2x+y=7$ [1]

$3x-y=3$ [2]

[1] + [2]:

$2x+3x+y-y=7+3$

$5x=10$

$\frac{5x}{5}=\frac{10}{5}$

$x=2$

Substitute $x=2$ into [1]:

$2\times 2+y=7$

$4+y=7$

$4+y-4=7-4$

$y=3$

The solution is $x=2, y=3$.

c $3x-2y=-16$ [1]

$y=2(x+9)$ [2]

Substitute [2] into [1]:

$3x-2(2(x+9))=-16$

$3x-4(x+9)=-16$

$3x-4x-36=-16$

$-x-36=-16$

$-x-36+36=-16+36$

$-x=20$

$\frac{-x}{-1}=\frac{20}{-1}$

$x=-20$

Substitute $x=-20$ into [2]:

$y=2(-20+9)$

$=2\times -11$

$=-22$

The solution is $x=-20, y=-22$.

d $4x+3y=17$ [1]

$3x+2y=13$ [2]

[1] × 2:

$8x+6y=34$ [3]

[2] × 3:

$9x+6y=39$ [4]

[4] − [3]:

$9x-8x+6y-6y=39-34$

$x=5$

Substitute $x = 5$ into [1]:

$$\begin{aligned}4 \times 5 + 3y &= 17\\ 20 + 3y &= 17\\ 20 + 3y - 20 &= 17 - 20\\ 3y &= -3\\ \frac{3y}{3} &= \frac{-3}{3}\\ y &= -1\end{aligned}$$

The solution is $x = 5, y = -1$.

10 The break-even point is where $C = R$.

a
$$\begin{aligned}150 + 2x &= 7.5x\\ 150 + 2x - 2x &= 7.5x - 2x\\ 150 &= 5.5x\\ \frac{150}{5.5} &= \frac{5.5x}{5.5}\\ 27.272... &= x\\ x &= 27.27 \text{ (2 decimal places)}\end{aligned}$$

b
$$\begin{aligned}25 + 13.5x &= 19.7x\\ 25 + 13.5x - 13.5x &= 19.7x - 13.5x\\ 25 &= 6.2x\\ \frac{25}{6.2} &= \frac{6.2x}{6.2}\\ 4.032... &= x\\ x &= 4.03 \text{ (2 decimal places)}\end{aligned}$$

c
$$\begin{aligned}3500 + 22.5x &= 35x\\ 3500 + 22.5x - 22.5x &= 35x - 22.5x\\ 3500 &= 12.5x\\ \frac{3500}{12.5} &= \frac{12.5x}{12.5}\\ 280 &= x\\ x &= 280\end{aligned}$$

11 a
$$\begin{aligned}\frac{2 - 5x}{8} &= \frac{3}{5}\\ \frac{2 - 5x}{8} \times 8 &= \frac{3}{5} \times 8\\ 2 - 5x &= \frac{24}{5}\\ 2 - 5x - 2 &= \frac{24}{5} - 2\\ -5x &= \frac{24}{5} - \frac{10}{5}\\ -5x &= \frac{14}{5}\\ \frac{-5x}{-5} &= \frac{14}{5 \times -5}\\ x &= -\frac{14}{25}\end{aligned}$$

b
$$\begin{aligned}\frac{6(3y - 2)}{11} &= \frac{5}{9}\\ \frac{11 \times 6(3y - 2)}{11} &= \frac{5 \times 11}{9}\\ 6(3y - 2) &= \frac{55}{9}\\ \frac{6(3y - 2)}{6} &= \frac{55}{9 \times 6}\\ 3y - 2 &= \frac{55}{54}\\ 3y - 2 + 2 &= \frac{55}{54} + 2\\ 3y &= \frac{163}{54}\\ \frac{3y}{3} &= \frac{163}{54 \times 3}\\ y &= \frac{163}{162}\end{aligned}$$

c
$$\begin{aligned}\left(\frac{4x}{5} - \frac{3}{7}\right) + 8 &= 2\\ \left(\frac{4x}{5} - \frac{3}{7}\right) + 8 - 8 &= 2 - 8\\ \frac{4x}{5} - \frac{3}{7} &= -6\\ \frac{4x}{5} - \frac{3}{7} + \frac{3}{7} &= -6 + \frac{3}{7}\\ \frac{4x}{5} &= \frac{-39}{7}\\ \frac{4x}{5} \times 5 &= \frac{-39}{7} \times 5\\ 4x &= \frac{-195}{7}\\ \frac{4x}{4} &= \frac{-195}{7 \times 4}\\ x &= \frac{-195}{28}\end{aligned}$$

d
$$\begin{aligned}\frac{7x + 6}{9} + \frac{3x}{10} &= \frac{4}{5}\\ 90 \times \left(\frac{7x + 6}{9} + \frac{3x}{10}\right) &= 90 \times \frac{4}{5}\\ 70x + 60 + 27x &= 72\\ 97x + 60 - 60 &= 72 - 60\\ 97x &= 12\\ \frac{97x}{97} &= \frac{12}{97}\\ x &= \frac{12}{97}\end{aligned}$$

12 a $y = 6 + 5x$ [1]

$3x + 2y = 7$ [2]

Substitute [1] into [2]:

$$3x + 2(6 + 5x) = 7$$
$$3x + 12 + 10x = 7$$
$$13x + 12 - 12 = 7 - 12$$
$$13x = -5$$
$$\frac{13x}{13} = \frac{-5}{13}$$
$$x = \frac{-5}{13}\ (x \approx -0.38)$$

Substitute into $y = 6 + 5x$:

$$y = 6 + 5\left(\frac{-5}{13}\right)$$
$$= 6 + \frac{-25}{13}$$
$$= \frac{53}{13}\ (y \approx 4.08)$$

Rounding answers correct to 2 decimal places, $x = -0.38, y = 4.08$.

b $4(x + 6) = y - 6$ [1]

$2(y + 3) = x - 9$ [2]

Rearranging [1]:

$$4x + 24 = y - 6$$
$$4x + 24 + 6 = y - 6 + 6$$
$$y = 4x + 30 \quad [3]$$

Substitute [3] into [2]:

$$2(4x + 30 + 3) = x - 9$$
$$2(4x + 33) = x - 9$$
$$8x + 66 = x - 9$$
$$8x + 66 + 9 = x - 9 + 9$$
$$8x + 75 = x$$
$$8x - x + 75 = x - x$$
$$7x + 75 = 0$$
$$7x + 75 - 75 = 0 - 75$$
$$7x = -75$$
$$\frac{7x}{7} = \frac{-75}{7}$$
$$x = \frac{-75}{7}\ (x \approx -10.71)$$

Substitute into $y = 4x + 30$:

$$y = 4\left(\frac{-75}{7}\right) + 30$$
$$= \frac{-300}{7} + 30$$
$$= \frac{-90}{7}\ (y \approx 12.86)$$

Rounding answers correct to 2 decimal places, $x = -10.71, y = -12.86$.

c $6x + 5y = 8.95$ [1]

$y = -1.36 + 3x$ [2]

$2x + 3y = 4.17$ [3]

Substitute [2] into [1]:

$$6x + 5(-1.36 + 3x) = 8.95$$
$$6x - 6.8 + 15x = 8.95$$
$$21x - 6.8 = 8.95$$
$$21x - 6.8 + 6.8 = 8.95 + 6.8$$
$$21x = 15.75$$
$$\frac{21x}{21} = \frac{15.75}{21}$$
$$x = 0.75$$

Substitute into $y = -1.36 + 3x$:

$$y = -1.36 + 3\,(0.75)$$
$$= -1.36 + 2.25$$
$$= 0.89$$

Confirm by substituting both solutions into [3]:

$$2(0.75) + 3(0.89) = 4.17$$
$$1.5 + 2.67 = 4.17$$
$$4.17 = 4.17$$

$x = 0.75, y = 0.89$

13 a There are 33 humans and pets, so if there are h humans, there are $33 - h$ pets.

b Total number of human legs $= 2h$

Total number of pet legs $= 4\,(33 - h)$

c Total number of legs $= 2h + 4(33 - h)$

$$= 2h + 132 - 4h$$
$$= 132 - 2h$$

Therefore: $132 - 2h = 94$

$$132 - 2h - 132 = 94 - 132$$
$$-2h = -38$$
$$\frac{-2h}{-2} = \frac{-38}{-2}$$
$$h = 19$$

There are $33 - 19 = 14$ pets.

Therefore, there are 19 humans and 14 pets in Petra's extended family.

Extended response

14 a $h = \dfrac{2(3t + 15)}{3}$

i $20 = \dfrac{2(3t + 15)}{3}$

$$60 = 2(3t + 15)$$
$$60 = 6t + 30$$
$$30 = 6t$$
$$t = 5$$

5 weeks

ii $30 = \dfrac{2(3t + 15)}{3}$

$$90 = 2(3t + 15)$$
$$90 = 6t + 30$$
$$60 = 6t$$
$$t = 10$$

10 weeks

iii $35 = \frac{2(3t+15)}{3}$
$105 = 2(3t+15)$
$105 = 6t + 30$
$75 = 6t$
$t = 12.5$
13 weeks

iv $50 = \frac{2(3t+15)}{3}$
$150 = 2(3t+15)$
$150 = 6t + 30$
$120 = 6t$
$t = 20$
20 weeks

b $t = 0$:
$$\frac{2(3(0)+15)}{3} = \frac{2(15)}{3} = \frac{30}{3} = 10$$
The initial height of the plant is 10 cm.

c $g = 2 + t$
$t = 1$:
$g = 2 + 1 = 3$
Height $= 60 + 3 = 63$ cm
$t = 2$:
$g = 2 + 2 = 4$
Height $= 63 + 4 = 67$ cm
$t = 3$:
$g = 2 + 3 = 5$
Height $= 67 + 5 = 72$ cm
$t = 4$:
$g = 2 + 4 = 6$
Height $= 72 + 6 = 78$ cm
63 cm, 67 cm, 72 cm, 78 cm

15 a i Equate the two values of y:
$-4 + 5x = 8 + 6x$
$-4 + 5x - 5x = 8 + 6x - 5x$
$-4 = 8 + x$
$-4 - 8 = 8 + x - 8$
$x = -12$
Substitute into $y = -4 + 5x$:
$y = -4 + 5(-12)$
$y = -4 - 60$
$y = -64$
$(-12, -64)$

ii Equate the two values of y:
$-5 - 3x = 1 + 3x$
$-5 - 3x + 3x = 1 + 3x + 3x$
$-5 = 1 + 6x$
$-5 - 1 = 1 + 6x - 1$
$-6 = 6x$
$\frac{-6}{6} = \frac{6x}{6}$
$x = -1$
Substitute into $y = 1 + 3x$:
$y = 1 + 3(-1)$
$y = 1 - 3$
$y = -2$
$(-1, -2)$

iii Equate the two values of y:
$6 + 2x = -4 + 2x$
$6 + 2x - 2x = -4 + 2x - 2x$
$6 = -4$
No solution

iv Equate the first two values of y:
$3 - x = 5 + x$
$3 - x + x = 5 + x + x$
$3 = 5 + 2x$
$3 - 5 = 5 + 2x - 5$
$-2 = 2x$
$\frac{-2}{2} = \frac{2x}{2}$
$x = -1$
Substitute into $y = 5 + x$:
$y = 5 - 1$
$y = 4$
Confirm by substituting into $y = 6 + 2x$:
$4 = 6 + 2(-1)$
$4 = 6 - 2$
$4 = 4$
$(-1, 4)$

b No, the graphs in part **iii** are parallel. (They have the same gradient.)

16 a $\frac{62.5}{2.5} = 25$ and $\frac{105}{4.2} = 25$
Therefore, the cooking time is 25 minutes per kilogram of meat.

b t = cooking time in minutes, k = weight of the meat (kg)
Therefore, $t = 15 + 25k$.

c When $m = 0.5$ (500 g):
$t = 15 + 25 \times 0.5$
$= 15 + 12.5$
$= 27.5$
When $m = 0.75$ (750 g):
$t = 15 + 25 \times 0.75$
$= 15 + 18.75$
$= 33.75$
When $m = 1$ (1000 g):
$t = 15 + 25 \times 1$
$= 15 + 25$
$= 40$
When $m = 1.25$ (1250 g):

$t = 15 + 25 \times 1.25$
$= 15 + 31.25$
$= 46.25$
When $m = 1.5$ (1500 g):
$t = 15 + 25 \times 1.5$
$= 15 + 37.5$
$= 52.5$
When $m = 1.75$ (1750 g):
$t = 15 + 25 \times 1.75$
$= 15 + 43.75$
$= 58.75$
When $m = 2$ (2000 g):
$t = 15 + 25 \times 2$
$= 15 + 50$
$= 65$
When $m = 2.25$ (2250 g):
$t = 15 + 25 \times 2.25$
$= 15 + 56.25$
$= 71.25$
When $m = 2.5$ (2500 g):
$t = 15 + 25 \times 2.5$
$= 15 + 62.5$
$= 77.5$
When $m = 2.75$ (2750 g):
$t = 15 + 25 \times 2.75$
$= 15 + 68.75$
$= 83.75$
When $m = 3$ (3000 g):
$t = 15 + 25 \times 3$
$= 15 + 75$
$= 90$
When $m = 3.25$ (3250 g):
$t = 15 + 25 \times 3.25$
$= 15 + 81.25$
$= 96.25$
When $m = 3.5$ (3500 g):
$t = 15 + 25 \times 3.5$
$= 15 + 87.5$
$= 102.5$
When $m = 3.75$ (3750 g):
$t = 15 + 25 \times 3.75$
$= 15 + 93.75$
$= 108.75$

Weight (g)	**Cooking time (minutes)**	**Weight (g)**	**Cooking time (minutes)**
500	27.5	2250	71.25
750	33.75	2500	77.5
1000	40	2750	83.75
1250	46.25	3000	90
1500	52.5	3250	96.25
1750	58.75	3500	102.5
2000	65	3750	108.75

d When $m = 5.5$:
$t = 15 + 25 \times 5.5$
$= 15 + 137.5$
$= 152.5$
152.5 minutes = 2 hours, 33 minutes (to the nearest minute).
Therefore, according to the equation, it will take 2 hours and 33 minutes to cook the turkey.

e This equation will probably vary for different types of meat.

f t = cooking time in minutes, k = weight of the meat (kg)
Therefore, $t = 30 + 45k$.

g When $m = 5.5$:
$t = 30 + 45 \times 5.5$
$= 30 + 247.5$
$= 277.5$
277.5 minutes = 4 hours, 38 minutes (to the nearest minute).
Therefore, it will take 4 hours and 38 minutes to cook the turkey (including resting time).

17 **a** $b = 250$

b $C = 250 + 2.25n$
When $n = 50$:
$C = 250 + 2.25 \times 50$
$= 250 + 112.5$
$= 362.5$
The cost for 50 cupcakes is \$362.50.

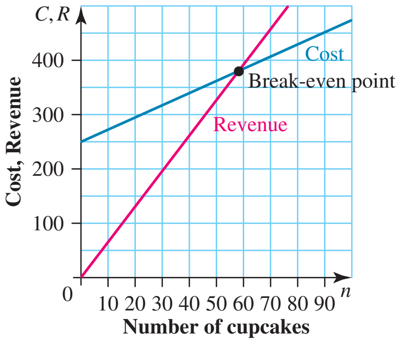
c When $C = 373.75$:
$373.75 = 250 + 2.25n$
$373.75 - 250 = 250 + 2.25n - 250$
$123.75 = 2.25n$
$\frac{123.75}{2.25} = \frac{2.25n}{2.25}$
$n = 55$
Suzanne made 55 cupcakes.

d $R = 6.5n$

e When $n = 150$:
$C = 250 + 2.25 \times 150$
$= 250 + 337.5$
$= 587.5$
When $n = 150$:
$R = 6.5 \times 150$
$= 975$
Profit $= 975 - 587.5$
$= 387.5$
Suzanne made \$387.50 in profit.

f The break-even point occurs when cost = revenue.
$250 + 2.25n = 6.5n$
$250 + 2.25n - 2.25n = 6.5n - 2.25n$
$250 = 4.25n$
$\frac{250}{4.25} = \frac{4.25n}{4.25}$
$58.82... = n$
$n = 59$ (to the nearest whole number)
Therefore, Suzanne will need to sell 59 cupcakes to break even.

g



18 **a** Number of people attending: $f + c = 6473$

Ticket sales: $19.8f + 14.6c = 109\,559.8$

b $f + c = 6473$ [1]

$19.8f + 14.6c = 109\,559.8$ [2]

Transpose [1]:

$f = 6473 - c$ [3]

Substitute [3] into [2]:

$$19.8(6473 - c) + 14.6c = 109\,559.8$$
$$128\,165.4 - 19.8c + 14.6c = 109\,559.8$$
$$128\,165.4 - 5.2c = 109\,559.8$$
$$128\,165.4 - 5.2c - 128\,165.4 = 109\,559.8 - 128\,165.4$$
$$-5.2c = -18\,605.6$$
$$\frac{-5.2c}{-5.2} = \frac{-18\,605.6}{-5.2}$$
$$c = 3578$$

Substitute $c = 3578$ into [3]:

$f = 6473 - 3578$

$f = 2895$

2895 full price tickets and 3578 concession tickets were sold.

c $C = 43\,500 + 5.9x$

d $x = f + c$

e $R = 19.8f + 14.6c$

f When $x = 6473$:

$C = 43\,500 + 5.9 \times 6473$

$= 81\,690.7$

The cost for day one is \$81 690.70.

Profit $= 109\,559.8 - 81\,690.7$

$= 27\,869.1$

The organisers made a profit of \$27 869.10 on day one of the festival.

2.6 Exam questions

1 $F = \frac{9}{5}C + 32$

$= \frac{9}{5}(25) + 32$

$= 77$

25 °C is equivalent to 77 °F.

The correct answer is **D**.

2 $115 - 5p = 45$

$$-5p = 45 - 115$$
$$-5p = 45 - 115$$
$$-5p = -70$$
$$p = \frac{-70}{-5}$$
$$p = 14$$

The correct answer is **C**.

3 $\frac{7x}{3} = 4x - 2$

The correct answer is **B**.

4 $(x + 3) + (x + 3) + x = 24$

$$3x + 6 = 24$$
$$3x = 24 - 16$$
$$3x = 18$$
$$x = \frac{18}{3}$$
$$x = 6\text{ cm}$$

The correct answer is **C**.

5 $2a + 3c = 24.80$ [1]

$2a + c = 16.50$ [2] [1 mark]

Multiply equation [2] by -3 to eliminate the c values by addition.

$2a + 3c = 24.80$ [1]

$\underline{-6a - 3c = -49.50}$ $-3 \times [2]$ [1 mark]

$-4a = -24.70$

$a = \frac{-24.70}{-4}$

$a = 6.175$

$a = \$6.20$ (to the nearest 5 cents) [1 mark]

Substitute $a = 6.20$ into equation [1] to find c.

$2a + 3c = 24.80$ [1]

$2(6.20) + 3c = 24.80$

$12.40 + 3c = 24.80$

$3c = 12.40$

$c = \frac{12.40}{3}$

$c = 4.1333$ [1 mark]

$c = \$4.15$ (to the nearest 5 cents) [1 mark]

Topic 3 — Financial mathematics

3.2 Ratio, rates and percentages

3.2 Exercise

1 a $\frac{62}{80} \times 100 = 77.5\%$

b $\frac{15}{100} \times 20 = 3$

2 a Week 1: $\frac{3.5}{100} \times \$8900 = \$311.50$

Week 2: $\frac{3.5}{100} \times \$10\,000 + \frac{6.5}{100} \times \$1300 = \$350 + \$84.50$
$= \$434.50$

Week 3: $\frac{3.5}{100} \times \$10\,000 + \frac{6.5}{100} \times \$3450 = \$350 + \$224.25$
$= \$574.25$

Week 4: $\frac{3.5}{100} \times \$10\,000 + \frac{6.5}{100} \times \$4200 = \$350 + \$273$
$= \$623$

Total for the month $= \$311.50 + \$434.50 + \$574.25 + \$623$
$= \$1943.25$

b Week 1: $\frac{311.50}{1943.25} \times 100 = 16.03\%$

Week 2: $\frac{434.50}{1943.25} \times 100 = 22.36\%$

Week 3: $\frac{574.25}{1943.25} \times 100 = 29.55\%$

Week 4: $\frac{623}{1943.25} \times 100 = 32.06\%$

3 a $\frac{4.25}{100} \times \$250\,000 = \$10\,625$

b $\frac{4.25}{100} \times \$310\,500 = \$13\,196.25$

c $\frac{4.25}{100} \times \$454\,755 = \$19\,327.087...$
$= \$19\,327.09$ (correct to 2 d.p.)

d $\frac{4.25}{100} \times \$879\,256 = \$37\,368.38$

4 a See the table at the bottom of the page.*

b $16 + 14 + 26 + 36 + 14.5 + 13 + 42 + 26 = 187.5$
$20 + 21 + 34 + 45 + 20 + 39 + 60 + 35 = 274$
$\frac{187.5}{274} \times 100 = 68.430...\%$
$= 68.43\%$ (correct to 2 d.p.)

5 a $280 + 105 + 50 + 85 + 320 = 840$

Food: $\frac{280}{840} \times 100 = 33.333...\%$
$= 33.33\%$ (correct to 2 d.p.)

Electricity: $\frac{105}{840} \times 100 = 12.5\%$

Telephone: $\frac{50}{840} \times 100 = 5.952...\%$
$= 5.95\%$ (correct to 2 d.p.)

Petrol: $\frac{85}{840} \times 100 = 10.119...\%$
$= 10.12\%$ (correct to 2 d.p.)

Rent: $\frac{320}{840} \times 100 = 38.095...\%$
$= 38.10\%$ (correct to 2 d.p.)

b Food: $\frac{280}{1100} \times 100 = 25.454...\%$
$= 25.45\%$ (correct to 2 d.p.)

Electricity: $\frac{105}{1100} \times 100 = 9.545...\%$
$= 9.55\%$ (correct to 2 d.p.)

Telephone: $\frac{50}{1100} \times 100 = 4.545...\%$
$= 4.55\%$ (correct to 2 d.p.)

Petrol: $\frac{85}{1100} \times 100 = 7.727...\%$
$= 7.73\%$ (correct to 2 d.p.)

Rent: $\frac{320}{1100} \times 100 = 29.090...\%$
$= 29.09\%$ (correct to 2 d.p.)

6 a $81 : 27 : 12$
HCF $= 3$
$81 \div 3 = 27$
$27 \div 3 = 9$
$12 \div 3 = 4$
$27 : 9 : 4$

b $4.8 : 9.6 = 48 : 96$
HCF $= 48$
$48 \div 48 = 1$
$96 \div 48 = 2$
$1 : 2$

7 $0.2\text{ kg} = 200\text{ g}$
$195 : 200$
HCF $= 5$
$195 \div 5 = 39$
$200 \div 5 = 40$
$39 : 40$

8 a HCF $= 12$
$36 \div 12 = 3$
$84 \div 12 = 7$
$3 : 7$

b HCF $= 7$
$49 \div 7 = 7$
$77 \div 7 = 11$
$105 \div 7 = 15$
$7 : 11 : 15$

***4 a**

	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8
Mark	$\frac{16}{20}$	$\frac{14}{21}$	$\frac{26}{34}$	$\frac{36}{45}$	$\frac{14.5}{20}$	$\frac{13}{39}$	$\frac{42}{60}$	$\frac{26}{35}$
Percentage	80%	66.67%	76.47%	80%	72.5%	33.33%	70%	74.29%

c 3225 : 1875
HCF = 75
$3225 \div 75 = 43$
$1875 \div 75 = 25$
43 : 25

d 2400 : 960 : 1200
HCF = 240
$2400 \div 240 = 10$
$960 \div 240 = 4$
$1200 \div 240 = 5$
10 : 4 : 5

9 Red: $\frac{5}{16} \times 48 = 15$
Yellow: $\frac{8}{16} \times 48 = 24$
Orange: $\frac{3}{16} \times 48 = 9$

10 a Mark: $\frac{3}{10} \times \$18\,000 = \$5400$
Henry: $\frac{1}{10} \times \$18\,000 = \$1800$
Dale: $\frac{4}{10} \times \$18\,000 = \$7200$
Ben: $\frac{2}{10} \times \$18\,000 = \$3600$

b 750 : 200 : 345 : 615
HCF = 5
$750 \div 5 = 150$
$200 \div 5 = 40$
$345 \div 5 = 69$
$615 \div 5 = 123$
150 : 40 : 69 : 123

11 Melbourne to Ballarat: $\frac{44}{247} \times 473 = 84.25...$ min
$\approx$ 1 hour, 24 minutes
Note: 7 hours and 53 mins is 473 mins
Ballarat to Horsham: $\frac{67}{247} \times 473 = 128.30...$ min
$\approx$ 2 hours, 8 minutes
Horsham to Adelaide: $\frac{136}{247} \times 473 = 260.43...$ min
$\approx$ 4 hours, 20 minutes

12 Convert 100 minutes to hours:
$$100 \text{ minutes} = \frac{100}{60} \text{ hours} = \frac{5}{3} \text{ hours}$$
$$\frac{1770}{\frac{5}{3}} = \frac{1770 \times 3}{5} = \frac{5310}{5} = 1062$$
1062 km/h

13 a Convert 165 minutes to hours:
$$165 \text{ minutes} = \frac{165}{60} \text{ hours} = \frac{11}{4} \text{ hours}$$
$$\frac{1.375}{\frac{11}{4}} = \frac{1.375 \times 4}{11} = \frac{5.5}{11} = 0.5$$
0.5 km/h

b Convert $2\frac{1}{3}$ hours to minutes: $2\frac{1}{3} \times 60 = 140$
$$\frac{1320}{140} = 9.428... = 9.43 \text{ (correct to 2 d.p.)}$$
9.43 mL/min

c $\frac{67.14}{3.6} = 18.65$
\$18.65/m

d $\frac{833}{68} = 12.25$
12.25 points/game

14 a $\frac{\$6.25}{650} \times 100 = \$0.961...$
$= \$0.96$ (correct to 2 d.p.)

b $\frac{\$3.25}{350} \times 100 = \$0.928...$
$= \$0.93$ (correct to 2 d.p.)

c $\frac{\$3.98}{425} \times 100 = \$0.936...$
$= \$0.94$ (correct to 2 d.p.)

d $\frac{\$3.69}{550} \times 100 = \$0.669...$
$= \$0.67$ (correct to 2 d.p.)

15 a Batsman A: $\frac{48}{66} \times 100 = 72.727...\%$
Batsman B: $\frac{34}{42} \times 100 = 80.952...\%$
Batsman B is scoring at the fastest rate.

b $\frac{48 + 34}{66 + 42} = \frac{82}{110}$
$= 75.925...$
$= 75.93$ runs per 100 deliveries

16 a 1.5 km = 0.0015 m
3600 seconds in an hour
$0.0015 \times 3600 = 5.4$
5.4 km/h

b 60 km = 60 000 m
3600 seconds in an hour
$60\,000 \div 3600 = 16.666...$
$= 16.67$ (correct to 2 d.p.)
16.67 m/s

c 65 cents = \$0.65
1000 grams in a kilogram
$0.65 \times 1000 = 650$
\$650 per kilo

d \$5.65 = 565 cents
1000 grams in a kilogram
$565 \div 1000 = 0.565$
$= 0.57$ (correct to 2 d.p.)
0.57 cents per gram

17 **a** $\frac{\$75\,000}{38 \times 52} = \$37.955...$

$= \$37.96$ (correct to 2 d.p.)

b $\frac{\$90\,000}{40 \times 52} = \$43.269...$

$= \$43.27$ (correct to 2 d.p.)

c $\frac{\$64\,000}{35 \times 52} = \$35.164...$

$= \$35.16$ (correct to 2 d.p.)

d $\frac{\$48\,000}{30 \times 52} = \$30.769...$

$= \$30.77$ (correct to 2 d.p.)

18 **a** $\frac{\$8000}{2054} = \$3.894...$

$= \$3.89$ (correct to 2 d.p.)

b $\$3.89 \times \frac{115}{100} = \$4.473...$

$= \$4.47$ (correct to 2 d.p.)

c $\frac{\$7000}{1770} = \$3.954...$

$= \$3.95$ (correct to 2 d.p.)

$\$3.95 \times \frac{115}{100} = \$4.5425$

$= \$4.54$ (correct to 2 d.p.)

Therefore the seller needs to charge \$4.54 to maintain a 15% profit.

19 **a** BBQ lamb chop: $\frac{\$15.50}{12} = \$1.291...$

$= \$1.29$ (correct to 2 d.p.)

Steak: $\frac{\$13.80}{5} = \$2.76$

Chicken drumstick: $\frac{\$11.33}{11} = \$1.03$

b Cost: $2 \times (\$15.50 + \$13.80 + \$11.33) = \$81.26$

Package A weight:

$2.535\text{ kg} + 1.045\text{ kg} + 1.441\text{ kg} = 5.021\text{ kg}$

$\frac{\$81.26}{5.021} = \$16.184...$

$= \$16.18$ (correct to 2 d.p.)

Package B weight:

$2.602\text{ kg} + 1.068\text{ kg} + 1.453\text{ kg} = 5.123\text{ kg}$

$\frac{\$81.26}{5.123} = \$15.861...$

$= \$15.86$ (correct to 2 d.p.)

20 **a** All teams have played 27 games.

West Sydney Wanderers:

Win: $\frac{18}{27} \times 100 = 66.666...\%$

$= 66.67\%$ (correct to 2 d.p.)

Loss: $\frac{6}{27} \times 100 = 22.222...\%$

$= 22.22\%$ (correct to 2 d.p.)

Draw: $\frac{3}{27} \times 100 = 11.111...\%$

$= 11.11\%$ (correct to 2 d.p.)

Central Coast Mariners:

Win: $\frac{16}{27} \times 100 = 59.259...\%$

$= 59.26\%$ (correct to 2 d.p.)

Loss: $\frac{5}{27} \times 100 = 18.518...\%$

$= 18.52\%$ (correct to 2 d.p.)

Draw: $\frac{6}{27} \times 100 = 22.22...\%$

$= 22.22\%$ (correct to 2 d.p.)

Melbourne Victory:

Win: $\frac{13}{27} \times 100 = 48.148...\%$

$= 48.15\%$ (correct to 2 d.p.)

Loss: $\frac{9}{27} \times 100 = 33.333...\%$

$= 33.33\%$ (correct to 2 d.p.)

Draw: $\frac{5}{27} \times 100 = 18.518...\%$

$= 18.52\%$ (correct to 2 d.p.)

Adelaide United:

Win: $\frac{12}{27} \times 100 = 44.444...\%$

$= 44.44\%$ (correct to 2 d.p.)

Loss: $\frac{10}{27} \times 100 = 37.037...\%$

$= 37.04\%$ (correct to 2 d.p.)

Draw: $\frac{5}{27} \times 100 = 18.518...\%$

$= 18.52\%$ (correct to 2 d.p.)

Team	Win	Loss	Draw
1. West Sydney Wanderers	66.67%	22.22%	11.11%
2. Central Coast Mariners	59.26%	18.52%	22.22%
3. Melbourne Victory	48.15%	33.33%	18.52%
4. Adelaide United	44.44%	37.04%	18.52%

b Western Sydney Wanderers: $\frac{41}{21} \times 100 = 195.238...\%$

$= 195.24\%$ (correct to 2 d.p.)

Central Coast Mariners: $\frac{48}{22} \times 100 = 218.181...\%$

$= 218.18\%$ (correct to 2 d.p.)

Melbourne Victory: $\frac{48}{45} \times 100 = 106.666...\%$

$= 106.67\%$ (correct to 2 d.p.)

Adelaide United: $\frac{38}{37} \times 100 = 102.702...\%$

$= 102.70\%$ (correct to 2 d.p.)

Team	Goal percentage
1. West Sydney Wanderers	195.24%
2. Central Coast Mariners	218.18%
3. Melbourne Victory	106.67%
4. Adelaide United	102.70%

3.2 Exam questions

1

Angelique	:	Jerome	:	Summer
2	:	3	:	1 (6 parts)
$\frac{2}{6} \times 50\,000$	:	$\frac{3}{6} \times 50\,000$	:	$\frac{1}{6} \times 50\,000$
$\frac{1}{3} \times 50\,000$	:	$\frac{1}{2} \times 50\,000$	:	$\frac{1}{6} \times 50\,000$
\$16 666.67	:	\$25 000	:	\$8333.34

Summer receives a one-sixth share or \$8333.34.
The correct answer is **E**.

2 Pay needs to be divided into $3 + 2 + 1 = 6$ parts.
Rent:

$$\frac{3}{6} = \frac{1}{2}$$

$$\frac{1}{2} \times \$900 = \$450$$

Therefore, \$450 is paid in rent.
The correct answer is **B**.

3 a Jar 1

$$\begin{aligned}\$2.09 \div 150 &= 0.013\,933 \text{ per gram}\\ &= 0.013\,933 \times 100\\ &= \$1.39 \text{ per } 100 \text{ grams}\end{aligned}$$ [1 mark]

Jar 2

$$\begin{aligned}\$2.89 \div 250 &= 0.011\,56 \text{ per gram}\\ &= 0.011\,56 \times 100\\ &= \$1.16 \text{ per } 100 \text{ grams}\end{aligned}$$ [1 mark]

Jar 3

$$\begin{aligned}\$5.99 \div 500 &= 0.011\,98 \text{ per gram}\\ &= 0.011\,98 \times 100\\ &= \$1.20 \text{ per } 100 \text{ grams}\end{aligned}$$ [1 mark]

The medium-sized jar is the cheapest per 100 grams of sauce. The students should purchase this one. [1 mark]

b It should be expected that the largest jar would be the cheapest item; however, this is not always the case. [1 mark]

3.3 Percentage applications and GST

3.3 Exercise

1 a $\frac{(3.25 - 2.65)}{2.65} \times 100 = 22.64 \rightarrow$ A 22.64% increase

b $\frac{(4.15 - 2.65)}{2.65} \times 100 = 56.60 \rightarrow$ A 56.6% increase

c $\frac{(1.95 - 2.65)}{2.65} \times 100 = -26.42 \rightarrow$ A 26.42% decrease

d $\frac{(2.28 - 2.65)}{2.65} \times 100 = -13.96 \rightarrow$ A 13.96% decrease

2 a $\frac{(1.44 - 1.48)}{1.48} \times 100 = -2.70 \rightarrow$ A 2.7% decrease

b $\frac{(23\,300 - 28\,500)}{28\,500} \times 100 = -18.25 \rightarrow$ An 18.25% decrease

c $\$6.50/\text{kg} = \frac{6.50}{1000} \times 50$

$= 32.5$ cents/50 g

$\frac{(75 - 32.5)}{32.5} \times 100 = 130.77 \rightarrow$ A 130.77% increase

d $\frac{(147 - 168)}{168} \times 100 = -12.5 \rightarrow$ A 12.5% decrease

3 a i $\frac{1610 - 1620}{1620} \times 100 = -0.617 \rightarrow$ A reduction of 0.62%

ii $\frac{1640 - 1600}{1600} \times 100 = 2.5 \rightarrow$ An increase of 2.5%

b $\frac{1640 - 1620}{1620} \times 100 = 1.23 \rightarrow$ An increase of 1.23%

4 Car 1: $\frac{14\,991 - 18\,750}{18\,750} \times 100 = -20.05$

Car 2: $\frac{9999 - 12\,250}{12\,250} \times 100 = -18.38$

Car 3: $\frac{19\,888 - 23\,990}{23\,990} \times 100 = -17.10$

The first car has the largest percentage reduction at 20.05%.

5 a $\frac{108}{100} \times 35 = 37.8 \rightarrow \$37.80$

b $\frac{112.5}{100} \times 96 = 108 \rightarrow \$108$

c $\frac{122.15}{100} \times 142.85 = 174.49 \rightarrow \$174.49$

d $\frac{100.285}{100} \times 42\,184 = 42\,304.22 \rightarrow \$42\,304.22$

6 a $\frac{84}{100} \times 54 = 45.36 \rightarrow \$45.36$

b $\frac{96.8}{100} \times 7.65 = 7.405 \rightarrow \$7.41$

c $\frac{67.85}{100} \times 102.15 = 69.308 \rightarrow \$69.31$

d $\frac{99.9545}{100} \times 12\,043 = 12\,037.52 \rightarrow \$12\,037.52$

7 a $\frac{94 - 88}{88} \times 100 = 6.82\%$

b $\frac{106 - 92}{92} \times 100 = 15.22\%$

8 Cost at \$1.45 : $50 + 0.25 \times 25 = \$56.25$

Cost at \$1.52 : $50 + 0.32 \times 25 = \$58.00$

Percentage change: $\frac{58 - 56.25}{56.25} \times 100 = 3.11\%$

9 a $\frac{34.98}{11} = \$3.18$

b $\frac{586.85}{11} = \$53.35$

c $\frac{56\,367.85}{11} = \$5124.35$

d $\frac{2.31}{11} = \$0.21$

10 Company A = \$5575

$$\begin{aligned}\text{Company B} &= \$5800 + \frac{10}{100} \times 5800\\ &= \$6380\end{aligned}$$

$$\begin{aligned}\text{Less 10\% for cash payment} &= \$6380 - \frac{10}{100} \times 6380\\ &= \$5742\end{aligned}$$

$\therefore$ Company A is cheaper by $5742 - 5575 = \$167$.

11 a $250 + 4 \times 74.50 = 548$

$$\begin{aligned}\text{Quote} &= 548 + \frac{10}{100} \times 548\\ &= \$602.80\end{aligned}$$

b Actual charge: $250 + 3 \times 74.50 = 473.50$

$$\begin{aligned}\text{with GST} &= 473.50 + \frac{10}{100} \times 473.50\\ &= \$520.85\end{aligned}$$

Percentage change: $\frac{520.85-602.80}{602.80}\times 100=-13.59\%$

$\therefore$ A 13.59% discount

12 a Price after receiving the award: $\frac{112.95}{100}\times 19.95=\$22.39$

Final price: $\frac{84.5}{100}\times 22.39=\$18.92$

b $\frac{18.92-19.95}{19.95}\times 100=-5.16\rightarrow$ The final price is a 5.16% decrease from the original price.

13 a $\frac{385-440}{440}\times 100=-12.5\rightarrow$ The price is reduced by 12.5%.

b Sale price at second store $=\frac{95}{100}\times 385$

$=\$365.75$

$\frac{365.75-440}{440}\times 100=-16.875\rightarrow$ A reduction of 16.875%

14 $\frac{85}{100}\times(\text{original price})=\$127.50$

$$\begin{aligned}\text{original price}&=\frac{127.50\times 100}{85}\\&=\$150\end{aligned}$$

15 Increase price by 15%:

$\frac{115}{100}(\text{original price})=1.15(\text{original price})$

Then decrease by 10%:

$\frac{90}{100}\times 1.15(\text{original price})=1.035(\text{original price})$

Then increase by 5%:

$\frac{105}{100}\times 1.035(\text{original price})=1.086\,75(\text{original price})$

which is an overall increase of 9%.

16 Normally pay \$250 per month.

$$\begin{aligned}\text{Saving \$1800 over two years}&=\frac{1800}{24}\\&=\$75\text{ per month}\end{aligned}$$

$$\begin{aligned}\text{New monthly bill}&=250-75\\&=\$175\end{aligned}$$

$\frac{175-250}{250}\times 100=-30$

Therefore, the bills will be reduced by 30%.

17 a $\frac{377\,600-320\,000}{320\,000}\times 100=18\rightarrow$ An 18% increase

b Deposit: $\frac{15}{100}\times 377\,600=\$56\,640$

Amount borrowed: $377\,600-56\,640=\$320\,960$

Pay bank: $\frac{5}{100}\times 320\,960=\$16\,048$

Monthly rent: $\frac{16\,048}{12}=\$1337.33$

18 a GST $=\frac{10}{100}(5.80+6.90+2.90)=\$1.56$

b Additional GST $=\frac{10}{100}(3.30+5.50+5.00)=\$1.38$

c Total paid:

$$\frac{90}{100}\times\left[(3.30+5.50+5.80+6.90+5.00+2.90)+\frac{10}{100}(5.80+6.90+2.90)\right]=\$27.86$$

19 a $\frac{87.5}{100}\times(\text{original price})=32\,250$

$$\begin{aligned}\text{original price}&=\frac{32\,250\times 100}{87.5}\\&=\$36\,857.15\end{aligned}$$

b Percentage discount: $\frac{35\,000-36\,857.15}{36\,857.15}\times 100=-5.04\%$

$\therefore$ There is a 5.04% discount compared to buying direct from the carpet company.

20 Total at 10%: GST $1.1\times(2.80+5.30+6.15+7.60+8.35+3.50)=\$37.07$

Total at 12.5%: GST $1.125\times(2.80+5.30+6.15+7.60+8.35+3.50)=\$37.91$

$37.91-37.07=\$0.84\rightarrow$ The bill would increase by 84 cents.

21 a $\frac{2153-2170}{2170}\times 100=-0.78\rightarrow$ A 0.78% decrease

b $\frac{6181-4283}{4283}\times 100=44.31\rightarrow$ A 44.31% increase

c $\frac{8334-6453}{6453}\times 100=29.15\rightarrow$ A 29.15% increase

22 a 2014: $\frac{35\,750-34\,000}{34\,000}\times 100=5.147\rightarrow$ An increase of 5.15%

2015: $\frac{38\,545-35\,750}{35\,750}\times 100=7.818\rightarrow$ An increase of 7.82%

2016: $\frac{42\,280-38\,545}{38\,545}\times 100=9.6\,899\rightarrow$ An increase of 9.69%

2017: $\frac{46\,000-42\,280}{42\,280}\times 100=8.798\rightarrow$ An increase of 8.8%

b The individual received the biggest percentage increase in salary in 2016 (9.69%).

3.3 Exam questions

1

$$\begin{aligned}\text{Increase}&=\$1.59-\$1.38\\&=\$0.21\\\frac{0.21}{1.38}&=\frac{21}{138}\\&=\frac{7}{46}\\\text{Percentage increase}&=\frac{7}{46}\times 100\\&=\frac{7}{46}\times\frac{100}{1}\\&=\frac{7}{23}\times\frac{50}{1}\\&=\frac{350}{23}\\&=15\frac{5}{23}\%\end{aligned}$$

The correct answer is **E**.

2 $\text{Decrease} = \$1.95 - \$1.05$

$= \$0.90$

$\frac{0.90}{1.95} = \frac{90}{195}$

$= \frac{6}{13}$

$\text{Percentage increase} = \frac{6}{13} \times 100$

$= \frac{6}{13} \times \frac{100}{1}$

$= \frac{600}{13}$

$= 46\frac{2}{13}\%$

The correct answer is **B**.

3 $\text{Dollars spent} = \$127 - \$34$

$= \$93$

$\text{Percentage increase} = \frac{93}{127} \times 100$

$= 73.23$

$= 73\%$ (correct to the nearest percent)

Max spent 73% of his weekly earnings.

The correct answer is **C**.

3.4 Simple interest applications

3.4 Exercise

1 a Simple interest: $I = \frac{Prn}{100}$

$= \frac{2575 \times 8.25 \times 4}{100}$

$= \$849.75$

b Simple interest: $I = \frac{Prn}{100}$

$= \frac{12\,250 \times 5.15 \times 6.5}{100}$

$= \$4100.69$

c Simple interest: $I = \frac{Prn}{100}$

$= \frac{43\,500 \times 12.325 \times 8.25}{100}$

$= \$44\,231.34$

d Simple interest: $I = \frac{Prn}{100}$

$= \frac{103\,995 \times 2.015 \times 8.75}{100}$

$= \$18\,335.62$

2 a Simple interest: $I = \frac{Prn}{100}$

$= \frac{500 \times 3.55 \times 3}{100}$

$= \$53.25$

$\text{Investment} = 500 + 53.25$

$= \$553.25$

b Simple interest: $I = \frac{Prn}{100}$

$= \frac{2054 \times 4.22 \times 7.75}{100}$

$= \$671.76$

$\text{Investment} = 2054 + 671.76$

$= \$2725.76$

c Simple interest: $I = \frac{Prn}{100}$

$= \frac{3500 \times 11.025 \times 9.25}{100}$

$= \$3569.34$

$\text{Investment} = 3500 + 3569.34$

$= \$7069.34$

d Simple interest: $I = \frac{Prn}{100}$

$= \frac{10\,201 \times 1.008 \times 5.25}{100}$

$= \$539.84$

$\text{Investment} = 10\,201 + 539.84$

$= \$10\,740.84$

3 a $n = \frac{100I}{P \times r} = \frac{100 \times 216}{675 \times 3.2} = 10\text{ years}$

b $n = \frac{100I}{P \times r} = \frac{100 \times 850}{1000 \times 4.25} = 20\text{ years}$

c $n = \frac{100I}{P \times r} = \frac{100 \times 2100}{5000 \times 5.25} = 8\text{ years}$

d $n = \frac{100I}{P \times r} = \frac{100 \times 775}{2500 \times 7.75} = 4\text{ years}$

4 a $r = \frac{100I}{P \times n} = \frac{100 \times 590}{2000 \times 5} = 5.9\%$

b $r = \frac{100I}{P \times n} = \frac{100 \times 648}{1800 \times 3} = 12\%$

c $P = \frac{100I}{r \times n} = \frac{100 \times 408}{4.25 \times 6} = \$1600$

d $P = \frac{100I}{r \times n} = \frac{100 \times 3750}{3.125 \times 12} = \$10\,000$

5 a $I = \frac{8000 \times 12.25 \times 3}{100} = 2940$

$\text{Monthly repayments} = \frac{8000 + 2940}{36} = \$303.89$

b $I = \frac{23\,000 \times 15.35 \times 6}{100} = 21\,183$

$\text{Monthly repayments} = \frac{23\,000 + 21\,183}{72} = \$613.65$

c $I = \frac{21\,050 \times 11.734 \times 6.25}{100} = 15\,437.54$

$\text{Monthly repayments} = \frac{21\,050 + 15\,437.54}{75} = \$486.50$

d $I = \frac{33\,224 \times 23.105 \times 4.5}{100} = 34\,543.82$

$\text{Monthly repayments} = \frac{33\,224 + 34\,543.82}{54} = \$1254.96$

6 a $130\text{ weeks} = \frac{130}{52} = 2.5\text{ years}$

$I = \frac{6225 \times 7.025 \times 2.5}{100} = 1093.27$

$\text{Monthly repayments} = \frac{6225 + 1093.27}{30} = \$243.94$

b 1095 days $= \dfrac{1095}{365} = 3$ years

$I = \dfrac{13\,328 \times 9.135 \times 3}{100} = 3652.54$

Monthly repayments $= \dfrac{13\,328 + 3652.54}{36} = \$471.68$

7 $\dfrac{800 \times \frac{r}{12}}{100} = 3.60$

$r = \left(\dfrac{3.6 \times 100}{800}\right) \times 12$

$= 5.4\%$

8 a Interest per year: $\dfrac{6.36}{100} \times 25\,000 = \$1590$

b Value of investment after 5 years:
$25\,000 + (5 \times 1590) = \$32\,950$

c Interest needed over two years: $35\,000 - 32\,950 = 2050$

$r = \dfrac{100I}{Pn} = \dfrac{100 \times 2050}{32\,950 \times 2} = 3.11\%$

9 a $P = \dfrac{100I}{rn} = \dfrac{100 \times 3744}{7.8 \times 6} = \$8000$

b Total amount to be repaid: $8000 + 3744 = 11\,744$

Fortnightly repayments for 6 years:

$\dfrac{11\,744}{(6 \times 26)} = 75.282 \rightarrow \$75.28$

10 a $I = \dfrac{19\,245 \times 7.8 \times 3.5}{100} = 5253.885 \rightarrow \$5253.89$

b Second investment rate: $0.625 \times 12 = 7.5\%$ for the first 2.5 years and $0.665 \times 12 = 7.98\%$

Total interest on the second investment:

$I = \left(\dfrac{19\,245 \times 7.5 \times 2.5}{100}\right) + \left(\dfrac{19\,245 \times 7.98 \times 1}{100}\right)$

$= \$5144.19$

Therefore, the first investment is the best (7.8% p.a.).

11 a Total repaid: $545 \times 12 \times 15 = \$98\,100$

Total interest paid: $98\,100 - 35\,000 = \$63\,100$

$r = \dfrac{100I}{Pn} = \dfrac{100 \times 63\,100}{35\,000 \times 15} = 12.019 \rightarrow 12.02\%$

b Total repaid: $(545 \times 12 \times 5) + (650 \times 12 \times 7) = \$87\,300$

Total interest paid: $87\,300 - 35\,000 = \$52\,300$

c $r = \dfrac{100I}{Pn} = \dfrac{100 \times 52\,300}{35\,000 \times 12} = 12.452 \rightarrow 12.45\%$

12 a $r = \dfrac{100I}{Pn} = \dfrac{100 \times 28}{100 \times \frac{2}{3}} = 42\%$ p.a

b $I = \dfrac{100 \times 42.75 \times \frac{2}{3}}{100} = \$28.50$

3.4 Exam questions

1 18 months = 1.5 years

Simple interest: $I = \dfrac{Prn}{100}$

$= \dfrac{1500 \times 15 \times 1.5}{100}$

$= \$337.50$

Total investment $= \$1500 + \$337.50$

$= \$1837.50$

The correct answer is **D**.

2 $0.01\% \times \$275 = \$0.0275$

$= 2.75$ cents

The correct answer is **D**.

3 Simple interest: $I = \dfrac{Prn}{100}$

$= \dfrac{500 \times 8.5 \times 2}{100}$

$= \$85$

The amount she had to pay the bank was $\$500 + 85 = \$585$.
The correct answer is **B**.

3.5 Compound interest applications

3.5 Exercise

1 a $A = 358\left(1 + \dfrac{1.22}{100}\right)^6 = \$385.02$

b $A = 1276\left(1 + \dfrac{2.41}{100}\right)^4 = \$1403.52$

c $A = 4362\left(1 + \dfrac{4.204}{100}\right)^3 = \$4935.59$

d $A = 275\,950\left(1 + \dfrac{6.18}{100}\right)^{16} = \$720\,300.86$

2 a $A = P\left(1 + \dfrac{r}{100}\right)^n = 4655\left(1 + \dfrac{4.55}{100}\right)^3 = \$5319.76$

$A - P = \$664.76$

b $A = P\left(1 + \dfrac{r}{100}\right)^n = 12\,344\left(1 + \dfrac{6.35}{100}\right)^6 = \$17\,859.98$

$A - P = \$5515.98$

c $A = P\left(1 + \dfrac{r}{100}\right)^n = 3465\left(1 + \dfrac{2.015}{100}\right)^8 = \$4064.58$

$A - P = \$599.58$

d $A = P\left(1 + \dfrac{r}{100}\right)^n = 365\,000\left(1 + \dfrac{7.65}{100}\right)^{20}$

$= 1\,594\,312.85$

$A - P = \$1\,229\,312.85$

3 a $P = \dfrac{A}{\left(1 + \frac{r}{100}\right)^n} = \dfrac{15\,000}{\left(1 + \frac{5.25}{100}\right)^8} = \$9961.26$

b $P = \dfrac{A}{\left(1 + \frac{r}{100}\right)^n} = \dfrac{22\,500}{\left(1 + \frac{7.15}{100}\right)^{10}} = \$11\,278.74$

c $P = \dfrac{A}{\left(1 + \frac{r}{100}\right)^n} = \dfrac{1000}{\left(1 + \frac{1.25}{100}\right)^2} = \$975.46$

d $P = \dfrac{A}{\left(1 + \frac{r}{100}\right)^n} = \dfrac{80\,000}{\left(1 + \frac{6.18}{100}\right)^{15}} = \$32\,542.37$

4 a $r = 100\left(\left(\dfrac{A}{P}\right)^{\frac{1}{n}} - 1\right) = 100\left(\left(\dfrac{1000}{500}\right)^{\frac{1}{2}} - 1\right) = 41.42\%$

b $r = 100\left(\left(\dfrac{A}{P}\right)^{\frac{1}{n}} - 1\right) = 100\left(\left(\dfrac{2500}{850}\right)^{\frac{1}{3}} - 1\right) = 43.28\%$

c $r = 100\left(\left(\dfrac{A}{P}\right)^{\frac{1}{n}} - 1\right) = 100\left(\left(\dfrac{2900}{1600}\right)^{\frac{1}{4}} - 1\right) = 16.03\%$

d $r = 100\left(\left(\dfrac{A}{P}\right)^{\frac{1}{n}} - 1\right) = 100\left(\left(\dfrac{9000}{3490}\right)^{\frac{1}{3}} - 1\right) = 37.13\%$

5 a $A = P\left(1 + \dfrac{r}{1200}\right)^n = 2876\left(1 + \dfrac{3.12}{1200}\right)^{24} = \$3060.93$

$A - P = \$184.93$

b $A = P\left(1+\frac{r}{1200}\right)^n = 23\,560\left(1+\frac{6.17}{1200}\right)^{36} =$
$\$28\,337.22$
$A - P = \$4777.22$

c $A = P\left(1+\frac{r}{1200}\right)^n = 85.50\left(1+\frac{2.108}{1200}\right)^{24} = \$89.18$
$A - P = \$3.68$

d $A = P\left(1+\frac{r}{1200}\right)^n = 12\,345\left(1+\frac{5.218}{1200}\right)^{72} =$
$\$16\,871.95$
$A - P = \$4526.95$

6 a $A = P\left(1+\frac{r}{5200}\right)^n = 675\left(1+\frac{2.42}{5200}\right)^{104} = \$708.47$

b $A = P\left(1+\frac{r}{400}\right)^n = 4235\left(1+\frac{6.43}{400}\right)^{12} = \$5128.17$

c $A = P\left(1+\frac{r}{2600}\right)^n = 85\,276\left(1+\frac{8.14}{2600}\right)^{104} =$
$\$118\,035.38$

d $A = P\left(1+\frac{r}{36\,500}\right)^n = 53\,412\left(1+\frac{4.329}{36\,500}\right)^{365} =$
$\$55\,774.84$

7 a $A = 8000\left(1+\frac{7.8}{100}\right)^{3} = \$10\,021.81$
$A - P = \$2021.81$

b $A = 8000\left(1+\frac{7.8}{1200}\right)^{36} = \$10\,101.50$
$A - P = \$2101.50$

c $A = 8000\left(1+\frac{7.8}{5200}\right)^{156} = \$10\,107.38$
$A - P = \$2107.38$

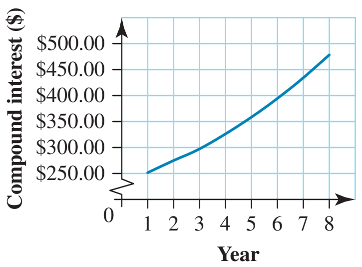
d $A = 8000\left(1+\frac{7.8}{36\,500}\right)^{1095} = \$10\,108.90$
$A - P = \$2108.90$

8 $A = 325\,000\left(1+\frac{2.73}{100}\right)^{5} = \$371\,851.73$
The inflated value is $371 851.73, so it was not profitable.

9 $A = 180\,000\left(1+\frac{1.8}{100}\right)^{2} = \$186\,538.32$
The inflated value is $186 538.32, so it is profitable.

10 a See the table at the bottom of the page.*

b



11 $P + I = 65 + 35 = 100$

Daily interest rate: $r = 100\left(\left(\frac{100}{65}\right)^{\frac{1}{14}} - 1\right) = 3.12\,485\,043$

Annual interest rate: $365 \times 3.125 = 1140.57\%$ p.a.

12 a Monthly rate: $r = 100\left(\left(\frac{1450}{1000}\right)^{\frac{1}{36}} - 1\right) = 1.0374$

Annual rate: $12 \times 1.0374 = 12.45\%$ p.a.

b New amount required: $1450\left(1+\frac{2}{100}\right)^{3} = 1538.75$

New principal needed: $\frac{1538.75}{\left(1+\frac{12.45}{1200}\right)^{36}} = \$1061.19$

13 a Year 1: $200 - \left(\frac{10}{100} \times 200\right) = \$180$

Year 2: $180 - \left(\frac{10}{100} \times 180\right) = \$162$

Year 3: $162 - \left(\frac{10}{100} \times 162\right) = \$145.80$

b Year 1:
Inflated price: $200\left(1+\frac{3}{100}\right) = \$206$

Decreased price: $206 - \left(\frac{10}{100} \times 206\right) = \$185.40$

Year 2:
Inflated price: $185.40\left(1+\frac{3}{100}\right) = \$190.96$

Decreased price: $190.96 - \left(\frac{10}{100} \times 190.96\right) = \$171.86$

Year 3:
Inflated price: $171.86\left(1+\frac{3}{100}\right) = \$177.02$

Decreased price: $177.02 - \left(\frac{10}{100} \times 177.02\right) = \$159.32$

14 The equivalent value of 2006 income in 2016:
$45\,000\left(1+\frac{3}{100}\right)^{10} = \$60\,476.24$
Therefore, he was earning comparatively more in 2016.

15 a The equivalent value of the sculpture in 2007:
$12\,000\left(1+\frac{3.3}{100}\right)^{9} = \$16\,072.53$
Therefore, the series of prints cost more in real terms.

b Sculpture: $12\,000\left(1+\frac{7.5}{100}\right)^{17} = \$41\,032$

Prints: $17\,000\left(1+\frac{6.8}{100}\right)^{8} = \$28\,775$

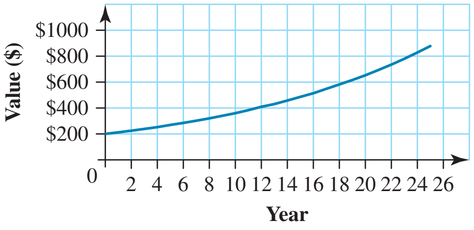
16

Principal ($)	**Final amount ($)**	**Interest earned ($)**	**Interest rate (p.a.)**	**Number of years**
11 000	12 012.28	**1012.28**	**4.5**	2
14 000	**15 409.84**	**1409.84**	3.25	3
22 050	25 561.99	3511.99	**3**	5
108 000	**110 070**	2700	2.5	1

***10 a**

Year	1	2	3	4	5	6	7	8
$A_{\text{Year}} = 2600\left(1+\frac{9.65}{100}\right)^{\text{Year}}$	2850.9	3126.01	3427.67	3758.44	4121.13	4518.82	4954.89	5433.03
Interest accrued ($)$I = A_{\text{Year}} - A_{\text{Year}-1}$	250.90	275.11	301.66	330.77	362.69	397.69	436.07	478.15

17 **a** See the table at the bottom of the page.*



b From the table we can see that it takes:

i 12 years for the investment to double:

$$200\left(1+\frac{6.1}{100}\right)^{11} = \$383.62 \text{ and}$$

$$200\left(1+\frac{6.1}{100}\right)^{12} = \$407.02$$

ii 19 years for the investment to triple:

$$200\left(1+\frac{6.1}{100}\right)^{18} = \$580.64 \text{ and}$$

$$200\left(1+\frac{6.1}{100}\right)^{19} = \$616.06$$

iii 24 years for the investment to quadruple:

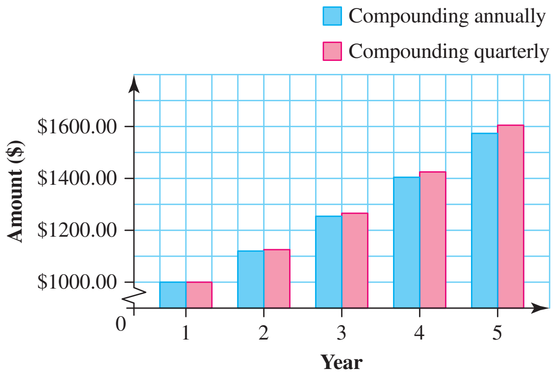
$$200\left(1+\frac{6.1}{100}\right)^{23} = \$780.70 \text{ and}$$

$$200\left(1+\frac{6.1}{100}\right)^{24} = \$828.32$$

18 **a**

Compound annually	
Year	**Amount**
1	$1000.00
2	$1120.00
3	$1254.40
4	$1404.93
5	$1573.52

Compound annually	
Year	**Amount**
1	$1000.00
2	$1030.00
3	$1060.90
4	$1092.73
5	$1125.51
6	$1159.27
7	$1194.05
8	$1229.87
9	$1266.77
10	$1304.77
11	$1343.92
12	$1384.23
13	$1425.76
14	$1468.53
15	$1512.59
16	$1557.97
17	$1604.71
18	$1652.85
19	$1702.43
20	$1753.51



Note: The graph shows the amounts at the beginning of each year.

b Compounding at regular intervals during the year accumulates more interest than compounding only once a year.

3.5 Exam questions

1 Solve on CAS:

$$\begin{aligned} 10\,000 \times 1.055^{n} &= 20\,000 \\ n &= 12.946 \\ &= 13 \end{aligned}$$

The correct answer is **B**.

2 The false statement is 'The formula calculates only the interest earned.' The value calculated using the formula $A = P\left(1+\frac{r}{100}\right)^{n}$ is the total amount of the investment — that is, the principal and the interest.

The correct answer is **A**.

3 $A = P\left(1+\frac{r}{c \times 100}\right)^{n \times c}$

$$\begin{aligned} A &= 5000\left(1+\frac{8}{2 \times 100}\right)^{3 \times 2} \\ A &= 5000\,(1.04)^{6} \\ A &= 6326.60 \\ I &= A - P \\ I &= 6326.60 - 5000 \end{aligned}$$

Interest earned = $1326.60

The correct answer is **C**.

***17 a**

Year	0	1	2	3	4	5	6
Value ($)	200.00	212.20	225.14	238.88	253.45	268.91	285.31

Year	7	8	9	10	11	12
Value ($)	302.72	321.18	340.78	361.56	383.62	407.02

Year	13	14	15	16	17	18	19
Value ($)	431.85	458.19	486.14	515.79	547.26	580.64	616.06

Year	20	21	22	23	24	25
Value ($)	653.64	693.51	735.81	780.70	828.32	878.85

3.6 Purchasing options

3.6 Exercise

1 **a** $\frac{92.5}{100} \times 200 = \$185$

b $\frac{92.5}{100} \times 312 = \$288.60$

c $\frac{92.5}{100} \times 126 = \$116.55$

2 Normal fee for minding two cats for 4 hours:
$(2 \times 20) + (4 \times 9) = \$76$
Discount fee for cash: $\frac{94}{100} \times 76 = \$71.44$
As payment is in cash, round to the nearest 5 cents = \$71.45.
The correct answer is **C**.

3 1st month: $\frac{311.55 \times 22.75 \times \frac{1}{12}}{100} = 5.906 \to \$5.91$

2nd month: $\frac{494.44 \times 22.75 \times \frac{1}{12}}{100} = 9.373 \to \$9.37$

23rd month: $\frac{639.70 \times 22.75 \times \frac{1}{12}}{100} = 12.127 \to \$12.13$

Total interest: $5.91 + 9.37 + 12.13 = \$27.41$

4 **a** Balance on card: $2365.24 - 70.96 = \$2294.28$

Interest: $\frac{2294.28 \times 24.28 \times \frac{1}{12}}{100} = 46.421 \to \$46.42$

b Balance on card: $(2365.24 - 500) - 70.96 = \$1794.28$

Interest: $\frac{1794.28 \times 24.28 \times \frac{1}{12}}{100} = 36.304 \to \$36.30$

$\therefore$ The interest paid would be reduced by:
$46.42 - 36.10 = \$10.12$

5 1st payment calculation: $I = \frac{5500 \times 6.85 \times \frac{1}{12}}{100} = 31.40$

Balance after 1st payment: $5500 + 31.40 - 425 = \$5106.40$

2nd payment calculation: $I = \frac{5106.4 \times 6.85 \times \frac{1}{12}}{100} = 29.15$

Balance after 2nd payment:
$5106.40 + 29.15 - 425 = \$4710.55$

3rd payment calculation: $I = \frac{4710.55 \times 6.85 \times \frac{1}{12}}{100} = 26.89$

Balance after 3rd payment:
$4710.55 + 26.89 - 425 = \$4312.44$

6 1st payment calculation: $I = \frac{2500 \times 5.5 \times \frac{1}{12}}{100} = 11.46$

Balance after 1st payment: $2500 + 11.46 - 450 = \$2061.46$

2nd payment calculation: $I = \frac{2061.46 \times 5.5 \times \frac{1}{12}}{100} = 9.45$

Balance after 2nd payment:
$2061.46 + 9.45 - 450 = \$1620.91$

3rd payment calculation: $I = \frac{1620.91 \times 5.5 \times \frac{1}{12}}{100} = 7.43$

Balance after 3rd payment: $1620.91 + 7.43 - 450 = \$1178.34$

4th payment calculation: $I = \frac{1178.34 \times 5.5 \times \frac{1}{12}}{100} = 5.40$

Balance after 4th payment: $1620.91 + 7.43 - 450 = \$733.74$

5th payment calculation: $I = \frac{733.74 \times 5.5 \times \frac{1}{12}}{100} = 3.36$

Balance after 5th payment: $733.74 + 3.36 - 450 = \$287.10$

6th payment calculation: $I = \frac{287.10 \times 5.5 \times \frac{1}{12}}{100} = 1.32$

Total interest:
$11.46 + 9.45 + 7.43 + 5.40 + 3.36 + 1.32 = \$38.42$

7 Plumber A: $(100 + (80 \times 1.5)) \times 0.95 = \$209$
Plumber B: $= \$200$
Plumber C: $(130 \times 1.5) \times 0.90 = \$175.50$
Plumber D: $(70 + (90 \times 1.5)) \times 0.92 = \$188.60$
Plumber E: $120 \times 1.5 = \$180.00$
Therefore, Plumber C is the cheapest one to use.

8 **a** Deposit: $\frac{12.5}{100} \times 150 = \$18.75$

Balance: $131.25\left(1 + \frac{\left(\frac{7.5}{12}\right)}{100}\right)^9 = \$138.82$

Total paid: $18.75 + 138.82 = \$157.57$

b Deposit: $\frac{12.5}{100} \times 550 = \$68.75$

Balance: $481.25\left(1 + \frac{\left(\frac{7.5}{12}\right)}{100}\right)^9 = \$509.01$

Total paid: $68.75 + 509.01 = \$577.76$

c Deposit: $\frac{12.5}{100} \times 285 = \$35.63$

Balance: $249.37\left(1 + \frac{\left(\frac{7.5}{12}\right)}{100}\right)^9 = \$263.75$

Total paid: $35.63 + 263.75 = \$299.38$

d Deposit: $\frac{12.5}{100} \times 675 = \$84.38$

Balance: $590.62\left(1 + \frac{\left(\frac{7.5}{12}\right)}{100}\right)^9 = \$624.69$

Total paid: $84.38 + 624.69 = \$709.07$

9 Sound system: $499\left(1 + \frac{13.55}{36\,500}\right)^7 = \$500.30$

Interest: $500.30 - 499 = \$1.30$

Blue-Ray films: $39\left(1 + \frac{13.55}{36\,500}\right)^{12} = \$39.17$

Interest: $39.17 - 39 = \$0.17$

Food shopping: $56\left(1 + \frac{13.55}{36\,500}\right)^2 = \$56.04$

Interest: $56.04 - 56 = \$0.04$

Coffee machine: $85\left(1 + \frac{13.55}{36\,500}\right)^{18} = \$85.57$

Interest: $85.57 - 85 = \$0.57$

Total interest: $1.30 + 0.17 + 0.04 + 0.57 = \$2.08$

10 **a** Balance after deposit: $1265 - 126.50 = \$1138.50$

Interest: $\frac{1138.50 \times \frac{7.64}{2}}{100} = 43.49$

Total amount paid: $126.50 + 1138.50 + 43.49 = \$1308.49$

b Balance after deposit: $1450 - 145 = \$1305$

Interest: $\frac{1305 \times \frac{7.64}{2}}{100} = 49.85$

Total amount paid: $145 + 1305 + 49.85 = \$1499.85$

c Balance after deposit: $2018 - 201.80 = \$1816.20$

Interest: $\frac{1816.20 \times \frac{7.64}{2}}{100} = 69.38$

Total amount paid: $201.80 + 1816.20 + 69.38 = \$2087.38$

d Balance after deposit: $3124 - 312.40 = \$2811.60$

Interest: $\frac{2811.60 \times \frac{7.64}{2}}{100} = 107.40$

Total amount paid: $312.40 + 2811.60 + 107.40 = \$3231.40$

11 Total payments: $100 + 27\,500 + 163 = \$27\,763$
Gift card value received: $27\,500 \times 0.008 = \$220$
Total value received (purchases and gift card):

$27\,500 + 220 = 27\,720$
Net value is −\$43, so the credit card was not a profitable investment as Elise loses \$43.

12 **a** $12\,000 + (24 \times 750) = \$30\,000$

b $\dfrac{\left(\frac{23.75}{12}\right)}{100} \times 215 \times 24 = \$102.13$

c Total saving = interest saved on car + interest saved on credit card
$= (30\,000 - 25\,000) + 102.13$
$= \$5102.13$

3.6 Exam questions

1 Using Finance Solver on the CAS:
N: 8
I: 3.9
PV: −5000
PMT: −200
FV: 7059.249 …
PPY: 4
CPY: 4
Thus, the answer is closest to D (\$7059).
The correct answer is **D**.

2 Reduction = 5%

$368.40 \times 5\% = 384.40 \times \dfrac{5}{100}$

Reduction = 18.42
The reduction in price is \$18.42.
The correct answer is **C**.

3 **a** Simple interest: $I = \dfrac{Prn}{100}$

$= \dfrac{3500 \times 5 \times 4}{100}$

$= \$700$ [1 mark]

b $A = p\left(1 + \dfrac{r}{100}\right)^n$

$= 3500\left(1 + \dfrac{3.5}{100}\right)^2$

$= 3500 \times (1.035)^2$

$= 3749.29$ [1 mark]

Interest earned $= \$3749.29 - \$3500$

$= \$249.29$ [1 mark]

c $n = 2 \times 2$

$= 4$ [1 mark]

$r = \dfrac{3.5}{2} = 1.75$

$A = P\left(1 + \dfrac{r}{100}\right)^n$

$= 3500\left(1 + \dfrac{1.75}{100}\right)^4$

$= 3500 \times (1.0175)^4$

$= 3751.51$ [1 mark]

Interest earned $= \$3751.51 - \$3500$

$= \$251.51$ [1 mark]

d $n = ?$ [1 mark]

$A = P\left(1 + \dfrac{r}{100}\right)^n$

$5000 = 3500\left(1 + \dfrac{3.5}{100}\right)^n$ [1 mark]

$5000 = 3500 \times (1.035)^n$

$\dfrac{5000}{3500} = (1.035)^n$

$1.43 = (1.035)^n$

Use CAS to solve this and calculate n.

$\log_{10} 1.43 = n \log_{10} 1.035$ [1 mark]

$n = \dfrac{\log_{10} 1.43}{\log_{10} 1.035}$

$n = 10.4$ [1 mark]

The number of compounding periods required for this option will be 11. [1 mark]

e The account has interest compounded biannually, so in order to achieve this goal he will need to invest his money for 11 ÷ 2 = 5.5 years. He will not select this option as he did not wish to tie his money up for this long. [1 mark]

3.7 Review

3.7 Exercise

Multiple choice

1 $\dfrac{3.7}{100} \times \$14\,990 = \$554.63$
The correct answer is **C**.

2 Convert to improper fractions:
$\dfrac{9}{5} : \dfrac{21}{5}$
× 5 both sides:
9 : 21
÷ 3 (common factor):
3 : 7
The correct answer is **E**.

3 $\dfrac{3.70}{120} = 0.03$
The correct answer is **B**.

4 $\dfrac{130.9 - 118.4}{118.4} \times 100 = 10.557$
≈ 10.6
The correct answer is **A**.

5 $(100 - 24)\,\% \times \text{original price} = 28.50$

$\text{original price} = \dfrac{28.50}{0.76}$

$= 37.5$

The correct answer is **D**.

6 Simple interest: $I = \dfrac{Prn}{100}$

$100I = Prn$

$= \dfrac{100I}{Pn}$

The correct answer is **D**.

7 $0.932 \times 244 = 227.408 \approx \$227.41$
The correct answer is **C**.

8 Normal charge: $(3 \times 14) + (6 \times 2) = 54$
Cash price: $0.95 \times 54 = \$51.30$
The correct answer is **A**.

9 Simple interest: $I = \dfrac{Prn}{100}$

$P = 12\,000,\ r = 4.5,\ n = \dfrac{30}{12} = 2.5$

$I = \dfrac{12\,000 \times 4.5 \times 2.5}{100}$

$I = 1350$

Total to repay: $12\,000 + 1350 = \$13\,350$

The correct answer is **E**.

10 $P = 29\,000,\ r = 6.2,\ c = 12$

$45\,000 = 29\,000 \times \left(1 + \dfrac{6.2}{12 \times 100}\right)^n$

Solving using CAS gives $n = 85.25\ldots$ months.

Sotiris will have to wait 86 months to have \$45 000 as he does not have enough at the end of 85 months.

The correct answer is **D**.

Short answer

11 a $1 + 24 = 25$

$\dfrac{600}{25} = 24$

$1 \times 24 = 24$ g (powdered milk)

$24 \times 24 = 576$ mL (water)

b $\dfrac{1200}{25} = 48$

$1 \times 48 = 48$ g (powdered milk)

$24 \times 48 = 1152$ mL (water)

12 a 7.5×100 g in 750 g of Weet-Bix

$\dfrac{4.99}{7.5} = 0.665$

$= 0.67$

\$0.67 per 100 g

b 9×100 g in 900 g of jelly beans

$\dfrac{16.80}{9} = 1.8667$

$= 1.87$

\$1.87 per 100 g

c \$4.50 for 1.5 L of milk (per 100 mL)

1.5 L = 1500 mL (15×100 mL)

$\dfrac{4.50}{15} = 0.30$

\$0.30 per 100 mL

d \$126.95 for 15 L of paint (per L)

$\dfrac{126.95}{15} = 8.463$

$= 8.46$

\$8.46 per 1 L

13 a $\dfrac{112.6}{100} \times 65.85 = \$74.15$

b $\dfrac{76.6}{100} \times 14.56 = \$11.15$

c $\dfrac{102.83}{100} \times 150.50 = \$154.76$

d $\dfrac{99.35}{100} \times 453.25 = \$450.30$

14 a $45.50 \div 11 = \$4.14$

b $109.00 \div 10 = \$10.90$

c $448.75 \div 11 = \$40.80$

d $13.25 \div 10 = \$1.33$

15 a Simple interest: $I = \dfrac{Prn}{100}$

$= \dfrac{4500 \times 6.87 \times 5.5}{100}$

$= \$1700.33$

b $n = \dfrac{100I}{Pr}$

$= \dfrac{100 \times 360}{1260 \times 4.08}$

$= 7$ years

c $r = \dfrac{100I}{Pn}$

$= \dfrac{100 \times 645}{5300 \times 3}$

$= 4.06\%$

d Simple interest: $I = \dfrac{Prn}{100}$

$= \dfrac{6250 \times 9.32 \times 7.25}{100}$

$= 4223.13$

Monthly repayments $= \dfrac{P + I}{7.25 \times 12}$

$= \dfrac{6250 + 4223.13}{87}$

$= \dfrac{10\,473.13}{87}$

$= \$120.38$

16 a $A = P\left(1 + \dfrac{r}{100}\right)^n$

$= 3655\left(1 + \dfrac{6.54}{100}\right)^{2.5}$

$= 4282.22$

$I = A - P$

$= 4282.22 - 3655$

$= \$627.22$

b $A = P\left(1 + \dfrac{r}{100}\right)^n$

$= 478\left(1 + \dfrac{2.27}{100}\right)^{10}$

$= \$598.29$

c $r = 100\left(\left(\dfrac{A}{P}\right)^{\frac{1}{n}} - 1\right)$

$= 100\left(\left(\dfrac{3550}{1640}\right)^{\frac{1}{3.25}} - 1\right)$

$= 26.82\%$

d $P = \dfrac{A}{\left(1 + \frac{r}{100}\right)^n}$

$= \dfrac{22\,000}{\left(1 + \frac{11.2}{100}\right)^{15}}$

$= \$4475.60$

17 $P = 250\,000$

$r = 2.82$

$n = 4$

$$A = P\left(1 + \frac{r}{100}\right)^n$$
$$= 250\,000\left(1 + \frac{2.82}{100}\right)^4$$
$$= \$279\,415.44$$

Inflated amount = \$279 415.44
Selling price = \$275 000
Therefore, in real terms she made a loss of \$4415.44 when inflation is taken into account.

Extended response

18 a $\frac{1230}{250\,000} \times 100 = 0.492$
0.492%

b $0.492 \times 12 = 5.904\%$

c Total repayments $= 25 \times 12 \times \$1230$
$= \$369\,000$
Total repayments – original borrowed amount
$= \$369\,000 - \$250\,000$
$= \$119\,000$

d Yearly percentage = 5.904
Over 25 years: $5.904 \times 25 = 147.6\%$
$147.6\% - 100\% = 47.6\%$
or
$\frac{119\,000}{250\,000} \times 100 = 47.6\%$

19 a Calculate price per unit (100 g):
Brand A: $\frac{7.25}{2.50} = 2.9$
Brand B: $\frac{22.50}{12} = 1.875$
Brand B is best value for money. Its price per unit (\$/100g) = \$1.88, which is lower than that of Brand A (\$2.90 per 100 g).

b $1.88 \times 2.5 = \$4.69$ per 250 g
$\approx \$4.70$ per 250 g

20 a 72 : 56 : 48
Divide by highest common factor (8):
9 : 7 : 6

b $9 + 7 + 6 = 22$
$\frac{250}{22} = 11.364$
Non-vegetarian dishes: $(9 + 7) \times 11.364 = 181.82$
Approximately 182 people would be expected to order non-vegetarian dishes.

c i $5 + 3 = 8$
$\frac{\$268}{8} = \$33.50$

ii $\frac{5}{8} \times 100 = 62.5\%$

21 a $\$185\,000 - \$130\,000 = \$55\,000$
$\frac{55\,000}{130\,000} = 42.31\%$

b $A = P\left(1 + \frac{r}{100}\right)^n$
$= 55\,000\left(1 + \frac{12.75}{100}\right)^5$
$= \$100\,217.93$

$I = A - P$
$= 100\,217.93 - 55\,000$
$= \$45\,217.93$

22 a $P_1 = 290 \times 3 + 20 \times 120$
$= \$3270$
$P_2 = 4000 \times \frac{90}{100}$
$= \$3600$
$P_3 = (80 \times 3 \times 7 + 25 \times 120) \times \frac{95}{100}$
$= \$4446$

b Painter 1

c

Date	Details	Amount	Balance
1st	Withdrawal	\$4000	6000 – 4000 = \$2000
10th	Deposit	\$151	2000 + 151 = \$2151
15th	Withdrawal	\$220	2151 – 220 = \$1931
22nd	Deposit	\$1500	1931 + 1500 = \$3431
29th	Withdrawal	\$50	3431 – 50 = \$3381
30th	Deposit	\$250	3381 + 250 = \$3631

Simple interest: $I = \frac{Prn}{100}$
$= \frac{1931 \times \left(12.4 \times \frac{1}{12}\right) \times 1}{100}$
$= \$19.95$

d $6000 - 3631 = 2369$
$\frac{2369}{6000} \times 100 = 39.48\%$

3.7 Exam questions

1 Increase in value $= 850\,000 - 600\,000 = 250\,000$
As a percentage of the purchase price $= \frac{250\,000}{600\,000} \times 100\%$
As a percentage of the purchase price $= 41.666\ldots\%$
Option D is the most correct with 41.7%.
The correct answer is **D**.

2 a Price without GST = (price with GST included) ÷ 1.1
Price without GST = 88 ÷ 1.1
= 80
GST = 88 − 80 = \$8 [1 mark]

b Total paid = 88×4
Total paid = \$352 [1 mark]

c To increase something by 12.5%, multiply by (100 + 12.5) %.
New hourly rate = $88 \times 112.5\%$
New hourly rate = $88 \times \frac{112.5}{100}$
New hourly rate = \$99 [1 mark]

3 $135\% \times$ original price = \$1.59
$1.35 \times$ original price = \$1.59
original price $= \frac{\$1.59}{1.35}$
original price = \$1.1777 …
original price = \$1.18
The correct answer is **C**.

4 $33\frac{1}{3}\% \times \$2999 = \$999.67$

$$\begin{aligned}\text{New price} &= \$2999 - \$999.67 \\ &= \$1999.33\end{aligned}$$

The correct answer is **D**.

5 **a** $20\% \times \$5495 = 0.20 \times \$5495$

$= \$1099$ [1 mark]

b $\$5495 - \$1099 = \$4396$ [1 mark]

c $\$4396 \div 12 = \$366$ [1 mark]

d **i** $32\% \times \$5495 = 0.32 \times \$5495$

$= \$1758.40$ [1 mark]

ii $\text{New price} = \$5495 - \$1758.40$

$= \$3736.60$ [1 mark]

e **i** Balance owing after 20% deposit = \$4396

After 1 month = \$3971

After 2 months = \$3546

After 3 months = \$3121

After 4 months = \$2696

After 5 months = \$2271

After 6 months = \$1846

After 7 months = \$1421

After 8 months = \$996

After 9 months = \$571

After 10 months = \$146

After 11 months, the debt is cleared. [1 mark]

ii The final payment is \$146. [1 mark]

Topic 4 — Matrices

4.2 Types of matrices

4.2 Exercise

1 $\begin{bmatrix} 45 \\ 30 \end{bmatrix}$ in order shower water savers and energy-saving globes, order 2×1 (2 rows and 1 column)

2 a 2 rows, 2 columns
2×2
b 2 rows, 1 column
2×1
c 1 row, 4 columns
1×4
d 2 rows, 3 columns
2×3
e 3 rows, 3 columns
3×3
f 4 rows, 2 columns
4×2

3 a y_{13} is in row 1, column 3.
$y_{13} = 5$
b y_{24} is in row 2, column 4.
$y_{24} = 8$
c y_{31} is in row 3, column 1.
$y_{31} = 1$
d y_{43} is in row 4, column 3.
$y_{43} = 0$
e y_{41} is in row 4, column 1.
$y_{41} = 2$
f y_{12} is in row 1, column 2.
$y_{12} = 7$

4 Rewriting the data into a matrix:
$$\begin{bmatrix} 18 & 12 & 8 \\ 13 & 10 & 11 \end{bmatrix}$$

5 The scores are 6 – 2, 4 – 6, 7 – 6, 3 – 6, 6 – 4.
Placing Newton's scores into row 1 and Isaac's scores into row 2 results in the following matrix. Alternatively, the rows can be reversed (e.g. Isaac's scores can be placed in row 1 and Newton's scores in row 2).
$$\begin{bmatrix} 6 & 4 & 7 & 3 & 6 \\ 2 & 6 & 6 & 6 & 4 \end{bmatrix}$$

6 a b_{12} is in row 1, column 2 of matrix B.
$b_{12} = 5$
b c_{11} is in row 1, column 1 of matrix C.
$c_{11} = 6$
c a_{21} is in row 2, column 1 of matrix A.
$a_{21} = 6$
d a_{11} is in row 1, column 1 of matrix A.
$a_{11} = 8$
e c_{12} is in row 1, column 2 of matrix C.
$c_{12} = 3$
f b_{22} is in row 2, column 2 of matrix B.
$b_{22} = 6$

7 a e_{24} element in 2nd row, 4th column; however there is no 4th column, so element does not exist.
b −3 is in the 2nd row and 3rd column so it will be element e_{23}.
c Nadia thought that e_{12} was read as 1st column, 2nd row. The correct value is 0.

8 a 3 rows, 2 columns
The order of matrix K is 3×2.
b Element k_{22} is in row 2 (Friday) and column 2 (Section B). Element k_{22} shows that 800 Section B tickets were sold on Friday.
c Saturday is row 3 and Section A tickets are column 1. The number of Section A tickets sold on Saturday is given by element k_{31}. 400 Section A tickets were sold on Saturday.
d Section B tickets are in column 2. To find the total Section B tickets sold, sum all values in column 1.
$600 + 800 + 700 = 2100$
2100 Section B tickets were sold in total over Thursday, Friday and Saturday.

9 a Adding the participants in the column for Friday:
$19 + 12 + 25 = 56$ participants
b Adding all entries in the table:
$19 + 12 + 25 + 23 + 17 + 33 + 30 + 18 + 36$
$= 213$ entries
c Looking only at the beginner participants (Friday 12, Saturday 17 and Sunday 18) construct the row matrix as shown: $\begin{bmatrix} 12 & 17 & 18 \end{bmatrix}$

10 a Reading the subscripts for each listed element, there are 3 rows and 2 columns so the order is 3×2.
b $h_{12} = 3$ (element in 1st row, 2nd column)
$h_{11} = 4$ (element in 1st row, 1st column)
$h_{21} = -1$ (element in 2nd row, 1st column)
$h_{31} = -4$ (element in 3rd row, 1st column)
$h_{32} = 6$ (element in 3rd row, 2nd column)
$h_{22} = 7$ (element in 2nd row, 2nd column)
$$H = \begin{bmatrix} 4 & 3 \\ -1 & 7 \\ -4 & 6 \end{bmatrix}$$

11 a Place values as listed from the table into a column matrix as shown:
$$\begin{bmatrix} 0.4 \\ 4.2 \\ 6.8 \\ 0.2 \\ 1.6 \\ 2.1 \\ 0.5 \\ 5.2 \end{bmatrix}$$
b Order the land sizes in ascending order (smallest to largest) and place into a row matrix.
$[2358 \quad 68\,330 \quad 227\,600 \quad 801\,428 \quad 984\,000 \quad 1\,346\,200 \quad 1\,727\,200 \quad 2\,529\,875]$
c i Three states would mean three rows with two columns (land size and population), so order is 3×2.

ii Enter values from the table into matrix in order: New South Wales, Victoria and Queensland. Matrix is as shown.

$$\begin{bmatrix} 801\,428 & 6.8 \\ 227\,600 & 5.2 \\ 1\,727\,200 & 4.2 \end{bmatrix}$$

12 a 8×2 (8 rows and 2 columns) entering in the values in order as they appear in the table, the matrix is as follows:

$$\begin{bmatrix} 148\,178 & 2.2 \\ 30\,839 & 0.6 \\ 146\,429 & 3.6 \\ 26\,044 & 1.7 \\ 77\,928 & 3.8 \\ 16\,900 & 3.4 \\ 66\,582 & 31.6 \\ 4043 & 1.2 \end{bmatrix}$$

b. i Reading from the table, there are 66 582 Aboriginal and Torres Strait Islander persons living in the Northern Territory.

ii 16 900 Aboriginal and Torres Strait Islander persons live in Tasmania.

iii Adding the number of Aboriginal and Torres Strait Islander persons in Queensland, NSW and Victoria $= 146\,429 + 148\,178 + 30\,839 = 325\,446$

c Adding 1st column: estimated number of Aboriginal and Torres Strait Islander persons living in Australia $= 516\,943$

4.2 Exam questions

1 ‘Row then column’: the number of rows by the number of columns. Therefore, this is a 3×2 matrix.
The correct answer is **C**.

2 ‘Row then column’:
$a_{21} = 1$ and $a_{32} = -2$ [1 mark]
$a_{21} + a_{32} = 1 + -2 = -1$ [1 mark]

3 A row matrix is a $1 \times m$ order matrix — that is, a matrix consisting of a single row of m elements.
The correct answer is **D**.

4.3 Adding and subtracting matrices

4.3 Exercise

1 a $\begin{bmatrix} 2 & -3 \\ -1 & -8 \end{bmatrix} + \begin{bmatrix} -1 & 9 \\ 0 & 11 \end{bmatrix}$

$= \begin{bmatrix} 2 + -1 & -3 + 9 \\ -1 + 0 & -8 + 11 \end{bmatrix}$

$= \begin{bmatrix} 1 & 6 \\ -1 & 3 \end{bmatrix}$

b $\begin{bmatrix} 0.5 \\ 0.1 \\ 1.2 \end{bmatrix} + \begin{bmatrix} -0.5 \\ 2.2 \\ 0.9 \end{bmatrix} + \begin{bmatrix} -0.1 \\ -0.8 \\ 2.1 \end{bmatrix}$

$= \begin{bmatrix} 0.5 + -0.5 + -0.1 \\ 0.1 + 2.2 + -0.8 \\ 1.2 + 0.9 + 2.1 \end{bmatrix}$

$= \begin{bmatrix} -0.1 \\ 1.5 \\ 4.2 \end{bmatrix}$

2 $-3 + a = 1$, solving for $a = 1 + 3$
$b + 2 = -4$, solving for $b = -4 - 2$
$a = 4, b = -6$

3 $\begin{bmatrix} 1 & 1 \\ -3 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -2 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1 - 2 & 1 - 3 \\ -3 - (-2) & -1 - 4 \end{bmatrix}$

$= \begin{bmatrix} -1 & -2 \\ -1 & -5 \end{bmatrix}$

4 a $\begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} + \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 0 \end{bmatrix}$

b $\begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix} + \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 6 \end{bmatrix}$

c $\begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 - (-1) \\ 4 - 0 \\ -2 - 4 \end{bmatrix}$

$= \begin{bmatrix} 6 \\ 4 \\ -6 \end{bmatrix}$

d $\begin{bmatrix} 5 \\ 4 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix} - \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 5 + (-1) - (-4) \\ 4 + 0 - 3 \\ -2 + 4 - 2 \end{bmatrix}$

$= \begin{bmatrix} 8 \\ 1 \\ 0 \end{bmatrix}$

5 a Both matrices must be of the same order for it to be possible to add and subtract them.

b Let $B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$B - A = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$a - 0 = 4$, solve for $a = 4$
$b - 1 = 0, b = 1$
$c - 3 = 2, c = 5$

Hence, $B = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$

Alternatively, solve:
$a + 0 = 4$
$b + 1 = 2$
$c + 3 = 8$

6 a $W + Y = \begin{bmatrix} -2 & 5 \\ 3 & 8 \end{bmatrix} + \begin{bmatrix} 5 & -6 \\ 1 & 9 \end{bmatrix}$

$= \begin{bmatrix} -2+5 & 5+-6 \\ 3+1 & 8+9 \end{bmatrix}$

$= \begin{bmatrix} 3 & -1 \\ 4 & 17 \end{bmatrix}$

b Matrix X has an order of 3×1 and matrix Y has an order of 2×2. Because the orders don't match, subtraction cannot be performed, and the answer is not defined.

c Matrix Z has an order of 1×2 and matrix W has an order of 2×2. Because the orders don't match, subtraction cannot be performed, and the answer is not defined.

d Matrix Y has an order of 2×2 and matrix Z has an order of 1×2. Because the orders don't match, addition cannot be performed, and the answer is not defined.

7 a $[0.5 \;\; 0.25 \;\; 1.2] - [0.75 \;\; 1.2 \;\; 0.9]$

$= [0.5 - 0.75 \;\; 0.25 - 1.2 \;\; 1.2 - 0.9]$

$= [-0.25 \;\; -0.95 \;\; 0.3]$

b $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 1+2 & 0+(-1) \\ 3+6 & 1+0 \end{bmatrix}$

$= \begin{bmatrix} 3 & -1 \\ 9 & 1 \end{bmatrix}$

c $\begin{bmatrix} 12 & 17 & 10 \\ 35 & 20 & 25 \\ 28 & 32 & 29 \end{bmatrix} - \begin{bmatrix} 13 & 12 & 9 \\ 31 & 22 & 22 \\ 25 & 35 & 31 \end{bmatrix}$

$= \begin{bmatrix} 12-13 & 17-12 & 10-9 \\ 35-31 & 20-22 & 25-22 \\ 28-25 & 32-35 & 29-31 \end{bmatrix}$

$= \begin{bmatrix} -1 & 5 & 1 \\ 4 & -2 & 3 \\ 3 & -3 & -2 \end{bmatrix}$

d $\begin{bmatrix} 11 & 6 & 9 \\ 7 & 12 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 8 & 8 \\ 6 & 7 & 6 \end{bmatrix} - \begin{bmatrix} -2 & -1 & 10 \\ 4 & 9 & -3 \end{bmatrix}$

$= \begin{bmatrix} 11+2-(-2) & 6+8-(-1) & 9+8-10 \\ 7+6-4 & 12+7-9 & -1+6-(-3) \end{bmatrix}$

$= \begin{bmatrix} 15 & 15 & 7 \\ 9 & 10 & 8 \end{bmatrix}$

8 $\begin{bmatrix} 3 & 0 \\ 5 & a \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ -b & 1 \end{bmatrix} = \begin{bmatrix} c & 2 \\ 3 & 4 \end{bmatrix}$

$3 + 2 = c$, therefore $c = 5$

$a + 1 = -4$, solving for $a = -5$

$5 + (-b) = 3$, solving for $b = 2$

$a = -5, b = 2, c = 5$

9 $\begin{bmatrix} 12 & 10 \\ 25 & 13 \\ 20 & a \end{bmatrix} - \begin{bmatrix} 9 & 11 \\ 26 & c \\ b & 9 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 8 \\ 21 & -3 \end{bmatrix}$

$13 - c = 8$, solving for $c = 5$

$a - 9 = -3$, solving for $a = 6$

$20 - b = 21$, solving for $b = -1$

$a = 6,\ b = -1,\ c = 5$

10 To add or subtract matrices, the matrices must be of the same order.

A is of the order 1×2
B is of the order 2×1
C is of the order 2×1
D is of the order 1×1
E is of the order 1×2
A and E have the same order of 1×2, and B and C have the same order of 2×1, so they can be added or subtracted.

11 a Using the information as shown in the table, the matrix (2 rows and 3 columns) is:

$\begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 3 \end{bmatrix}$

b i Barntown order = total order − Appleton order

A matrix sum is: $\begin{bmatrix} 3 & 4 & 8 \\ 6 & 8 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 3 \end{bmatrix}$

ii $\begin{bmatrix} 3 & 4 & 8 \\ 6 & 8 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3-2 & 4-3 & 8-5 \\ 6-4 & 8-6 & 5-3 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 3 \\ 2 & 2 & 2 \end{bmatrix}$

Eggs	Small	Medium	Large
Free-range	1	1	3
Barn-laid	2	2	2

12 a Both matrices are of the order 2×3, so the answer matrix must also be of the order 2×3. The student's answer matrix is of the order 2×1, which is incorrect.

b A possible response is:
Step 1:
Check that all matrices are the same order.
Step 2:
Add or subtract the corresponding elements.

13 a From the table for week 1, the column matrix (four rows and one column) is:

$\begin{bmatrix} 150 \\ 165 \\ 155 \\ 80 \end{bmatrix}$

b i $\begin{bmatrix} 150 \\ 165 \\ 155 \\ 80 \end{bmatrix} + \begin{bmatrix} 145 \\ 152 \\ 135 \\ 95 \end{bmatrix} + \begin{bmatrix} 166 \\ 155 \\ 156 \\ 110 \end{bmatrix}$

ii $\begin{bmatrix} 150 \\ 165 \\ 155 \\ 80 \end{bmatrix} + \begin{bmatrix} 145 \\ 155 \\ 135 \\ 95 \end{bmatrix} + \begin{bmatrix} 166 \\ 155 \\ 156 \\ 110 \end{bmatrix}$

$= \begin{bmatrix} 150+145+166 \\ 165+152+155 \\ 155+135+156 \\ 80+95+110 \end{bmatrix}$

$= \begin{bmatrix} 461 \\ 472 \\ 446 \\ 285 \end{bmatrix}$

c $\begin{bmatrix} 35 \\ 41 \\ 38 \\ 25 \end{bmatrix} + \begin{bmatrix} 32 \\ 36 \\ 35 \\ 30 \end{bmatrix} + \begin{bmatrix} 38 \\ 35 \\ 35 \\ 32 \end{bmatrix} = \begin{bmatrix} 105 \\ 112 \\ 108 \\ 87 \end{bmatrix}$

14 a Using the order of speckles, googly eyes and fantails, the possible matrix sums are shown:

$\begin{bmatrix} 12 \\ 9 \\ 8 \end{bmatrix} + \begin{bmatrix} -3 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$ or $\begin{bmatrix} 12 \\ 9 \\ 8 \end{bmatrix} - \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$

b $\begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \\ 10 \end{bmatrix}$ or $\begin{bmatrix} 12 \\ 4 \\ 10 \end{bmatrix} - \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$

15 a To add or subtract matrices, all matrices must be of the same order. Since the resultant matrix D is of the order 3×2, all other matrices must also be of the order 3×2.

b $\begin{bmatrix} x & 7 \\ 20 & y \\ 3(5) & -8 \end{bmatrix} - \begin{bmatrix} 12 & 9 \\ \frac{1}{2}(20) & 2y \\ 5 & 2x \end{bmatrix} + \begin{bmatrix} x & y \\ 2x & 5 \\ 3x & 6 \end{bmatrix}$

$= \begin{bmatrix} x - 12 + x & 7 - 9 + y \\ 20 - 10 + 2x & y - 2y + 5 \\ 15 - 5 + 3x & -8 - 2x + 6 \end{bmatrix}$

$= \begin{bmatrix} 2x - 12 & y - 2 \\ 2x + 10 & 5 - y \\ 3x + 10 & -2 - 2x \end{bmatrix}$

c $\begin{bmatrix} 2x - 12 & y - 2 \\ 10 + 2x & 5 - y \\ 10 + 3x & -2 - 2x \end{bmatrix} = \begin{bmatrix} -8 & 1 \\ 14 & 2 \\ 16 & -8 \end{bmatrix}$

Solving any of the following equations

$2x - 12 = -8;\ 10 + 2x = 14;\ 10 + 3x = 16;\ -2 - 2x = -8:$

$2x = 4$

$x = 2$

$y - 2 = 1;\ 5 - y = 2:$

$y = 3$

16 $\begin{bmatrix} \frac{5}{8} & \frac{5}{8} & 0 \\ \frac{17}{30} & \frac{5}{42} & \frac{1}{36} \\ \frac{11}{8} & \frac{1}{8} & -\frac{2}{9} \end{bmatrix}$

4.3 Exam questions

1 $\begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 + 1 \\ -4 + 2 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

The correct answer is **D**.

2 $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 5 & 2 \end{bmatrix}$

$= \begin{bmatrix} -4 & 5 \\ 8 & 3 \end{bmatrix}$

$\therefore\ a = -4$

The correct answer is **B**.

3 Working from left to right:

$A + B - C + O = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} -4 & 2 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2 + 1 & 3 + 3 \\ -4 + -3 & 5 + 5 \end{bmatrix} - \begin{bmatrix} -4 & 2 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ [1 mark]

$= \begin{bmatrix} 3 & 6 \\ -7 & 10 \end{bmatrix} - \begin{bmatrix} -4 & 2 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 3 - -4 & 6 - 2 \\ -7 - 4 & 10 - 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 7 & 4 \\ -11 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 7 & 4 \\ -11 & 5 \end{bmatrix}$ [1 mark]

4.4 Multiplying matrices

4.4 Exercise

1 a $4 \times \begin{bmatrix} 2 & 3 & 7 \\ 1 & 4 & 6 \end{bmatrix}$

$= \begin{bmatrix} 4 \times 2 & 4 \times 3 & 4 \times 7 \\ 4 \times 1 & 4 \times 4 & 4 \times 6 \end{bmatrix}$

$= \begin{bmatrix} 8 & 12 & 28 \\ 4 & 16 & 24 \end{bmatrix}$

b $\frac{1}{5} \times \begin{bmatrix} 2 & 3 & 7 \\ 1 & 4 & 6 \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{5} \times 2 & \frac{1}{5} \times 3 & \frac{1}{5} \times 7 \\ \frac{1}{5} \times 1 & \frac{1}{5} \times 4 & \frac{1}{5} \times 6 \end{bmatrix}$

$= \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & \frac{7}{5} \\ \frac{1}{5} & \frac{4}{5} & \frac{6}{5} \end{bmatrix}$

c $0.3 \times \begin{bmatrix} 2 & 3 & 7 \\ 1 & 4 & 6 \end{bmatrix}$

$= \begin{bmatrix} 0.3 \times 2 & 0.3 \times 3 & 0.3 \times 7 \\ 0.3 \times 1 & 0.3 \times 4 & 0.3 \times 6 \end{bmatrix}$

$= \begin{bmatrix} 0.6 & 0.9 & 2.1 \\ 0.3 & 1.2 & 1.8 \end{bmatrix}$

2 $3D = \begin{bmatrix} 15 & 0 \\ 21 & 12 \\ 33 & 9 \end{bmatrix}$, each element of D has been multiplied by 3, so:

$D = \begin{bmatrix} 5 & 0 \\ 7 & 4 \\ 11 & 3 \end{bmatrix}$

$xD = \begin{bmatrix} 12.5 & 0 \\ 17.5 & 10 \\ 27.5 & 7.5 \end{bmatrix}$,

$x \times 5 = 12.5$, so $x = \dfrac{12.5}{5}$

Hence, $x = 2.5$.

3 Take out a common factor of 3 and divide each element by 3. The result is C.

4 **a** $\boldsymbol{X}$ has order 1×2. $\boldsymbol{Y}$ has order 2×1.

$1 \times 2 \quad 2 \times 1$

Number of columns = number of rows, so XY exists and is of order 1×1.

b D is of the order 3×2

C is of the order 2×2

E is of the order 2×3

DC: $3 \times 2 \;\; 2 \times 2$; product matrix exists; order 3×2

DE: $3 \times 2 \;\; 2 \times 3$; product matrix exists; order 3×3

CD: $2 \times 2 \;\; 3 \times 2$; product matrix does not exist.

CE: $2 \times 2 \;\; 2 \times 3$; product matrix exists; order 2×3

ED: $2 \times 3 \;\; 3 \times 2$; product matrix exists; order 2×2

EC: $2 \times 3 \;\; 2 \times 2$; product matrix does not exist.

DE: 3×3, DC: 3×2, ED: 2×2, CE: 2×3

5 Let S be of order $s \times 2$ (given it has 2 columns).

Let T be of order $m \times n$.

ST: $s \times 2 \;\; m \times n$

Since ST exists, $m = 2$

Order of ST is 3×4, which results from the rows in S and columns in T.

Therefore $s = 3$ and $n = 4$

Hence the orders are: S: 3×2, T: 2×4

6

Matrix	Rows	Columns	Order
D	3	1	3×1
E	3	2	3×2
F	2	3	2×3
G	1	2	1×2

For a product matrix to exist, the columns in the first matrix must equal the rows in the second matrix, so the product matrices that exist are, with their respective orders:

DG: 3×2, FD: 2×1, FE: 2×2, EF: 3×3, GF: 1×3.

7 $MN = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \times \begin{bmatrix} 7 & 12 \end{bmatrix}$

$= \begin{bmatrix} 4 \times 7 & 4 \times 12 \\ 3 \times 7 & 3 \times 12 \end{bmatrix}$

$= \begin{bmatrix} 28 & 48 \\ 21 & 36 \end{bmatrix}$

8 $ST = \begin{bmatrix} 1 & 4 & 3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 3 \\ t \end{bmatrix}$

$= [1 \times 2 + 4 \times 3 + 3 \times t]$

$= [14 + 3t]$

$= [5]$

Solve $14 + 3t = 5$

$3t = -9$

$t = -3$

9 $PQ = \begin{bmatrix} 3 & 7 \\ 8 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 2 + 7 \times 5 & 3 \times 1 + 7 \times 6 \\ 8 \times 2 + 4 \times 5 & 8 \times 1 + 4 \times 6 \end{bmatrix}$

$= \begin{bmatrix} 41 & 45 \\ 36 & 32 \end{bmatrix}$

10 **a** In order as stated in the question, the column matrix is $\begin{bmatrix} 12.50 \\ 8.50 \\ 6.00 \end{bmatrix}$.

b Total number of tickets requires an order of 1×1, and the order of the ticket price is 3×1. The number of people must be of order 1×3 to result in a product matrix of order 1×1. Therefore the answer must be a row matrix.

c $\begin{bmatrix} 65 & 40 & 85 \end{bmatrix} \begin{bmatrix} 12.50 \\ 8.50 \\ 6.00 \end{bmatrix}$

$= [65 \times 12.50 + 40 \times 8.50 + 85 \times 6.00]$

$= [1662.5]$

Therefore, total amount of ticket sales is \$1662.50.

11 **a** $\begin{bmatrix} 6 & 9 \end{bmatrix} \times \begin{bmatrix} 5 \\ 4 \end{bmatrix} = [6 \times 5 + 9 \times 4]$

$= [66]$

b $\begin{bmatrix} 5 \\ 4 \end{bmatrix} \times \begin{bmatrix} 6 & 9 \end{bmatrix} = \begin{bmatrix} 5 \times 6 & 5 \times 9 \\ 4 \times 6 & 4 \times 9 \end{bmatrix}$

$= \begin{bmatrix} 30 & 45 \\ 24 & 36 \end{bmatrix}$

c $\begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix} \times \begin{bmatrix} 10 & 15 \end{bmatrix} = \begin{bmatrix} 7 \times 10 & 7 \times 15 \\ 2 \times 10 & 2 \times 15 \\ 9 \times 10 & 9 \times 15 \end{bmatrix}$

$= \begin{bmatrix} 70 & 105 \\ 20 & 30 \\ 90 & 135 \end{bmatrix}$

d $\begin{bmatrix} 6 & 5 \\ 8 & 3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 6 \times 2 + 5 \times 9 \\ 8 \times 2 + 3 \times 9 \end{bmatrix}$

$= \begin{bmatrix} 57 \\ 43 \end{bmatrix}$

e $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 6 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 3 \times 6 + 5 \times 4 & 3 \times 3 + 5 \times 2 \\ 1 \times 6 + 2 \times 4 & 1 \times 3 + 2 \times 2 \end{bmatrix}$

$= \begin{bmatrix} 38 & 19 \\ 14 & 7 \end{bmatrix}$

12 **a** $\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 \times 1 + 6 \times 0 & 4 \times 0 + 6 \times 1 \\ 2 \times 1 + 3 \times 0 & 2 \times 0 + 3 \times 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$

b $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 \times 4 + 0 \times 2 & 1 \times 6 + 0 \times 3 \\ 0 \times 4 + 1 \times 2 & 0 \times 6 + 1 \times 3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$

13 $P = \begin{bmatrix} 8 & 2 \\ 4 & 7 \end{bmatrix}$

$P^2 = \begin{bmatrix} 8 & 2 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 8 & 2 \\ 4 & 7 \end{bmatrix}$

$P^2 = \begin{bmatrix} 8 \times 8 + 2 \times 4 & 8 \times 2 + 2 \times 7 \\ 4 \times 8 + 7 \times 4 & 4 \times 2 + 7 \times 7 \end{bmatrix}$

$P^2 = \begin{bmatrix} 72 & 30 \\ 60 & 57 \end{bmatrix}$

14 $T = \begin{bmatrix} 3 & 5 \\ 0 & 6 \end{bmatrix}$

$T^3 = \begin{bmatrix} 3 & 5 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 6 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 6 \end{bmatrix}$

$T^2T = \begin{bmatrix} 3 \times 3 + 5 \times 0 & 3 \times 5 + 5 \times 6 \\ 0 \times 3 + 6 \times 0 & 0 \times 5 + 6 \times 6 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 6 \end{bmatrix}$

$T^2T = \begin{bmatrix} 9 & 45 \\ 0 & 36 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 6 \end{bmatrix}$

$T^3 = \begin{bmatrix} 9 \times 3 + 45 \times 0 & 9 \times 5 + 45 \times 6 \\ 0 \times 3 + 36 \times 0 & 0 \times 5 + 36 \times 6 \end{bmatrix}$

$T^3 = \begin{bmatrix} 27 & 315 \\ 0 & 216 \end{bmatrix}$

15 **a** $I_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b $I_3^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c $I_3^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d Whatever power you raise I_3 to, the matrix stays the same.

16 **a** Place the number of students in order as they are stated into a column matrix, $\begin{bmatrix} 250 \\ 185 \end{bmatrix}$.

b The values shown are in percentages.

Convert the percentages into decimal form by dividing each by 100; for example: $5\% = \dfrac{5}{100} = 0.05$

Place each value in order as shown in the table, into a row matrix:

$[0.05 \;\; 0.18 \;\; 0.45 \;\; 0.25 \;\; 0.07]$

c The number of expected grades (A–E) for students studying Maths and Physics

d $SA = \begin{bmatrix} 250 \\ 185 \end{bmatrix} [0.05 \;\; 0.18 \;\; 0.45 \;\; 0.25 \;\; 0.07]$

$= \begin{bmatrix} 250 \times 0.05 & 250 \times 0.18 & 250 \times 0.45 & 250 \times 0.25 & 250 \times 0.07 \\ 185 \times 0.05 & 185 \times 0.18 & 185 \times 0.45 & 185 \times 0.25 & 185 \times 0.07 \end{bmatrix}$

$= \begin{bmatrix} 13 & 45 & 113 & 63 & 18 \\ 9 & 33 & 83 & 46 & 13 \end{bmatrix}$

e SA_{12} (element in 1st row and 2nd column) = 45. Therefore 45 students studying Maths are expected to be awarded a B grade.

17 $N = MPR$

$m \times \; n \;\; 1 \; \times \; q \;\; 2 \; \times s$

Since the product matrix exists, the columns in matrix M must equal the rows in matrix P, therefore $n = 1$; similarly, $q = 2$. The order of matrix N is 3×4 which results from the rows in matrix M and columns in matrix R; therefore, $m = 3$ and $s = 4$.

$n = 1, m = 3, s = 4, q = 2$

18 **a** Matrix G is of order 3×2. Matrix H is of order 2×1. Therefore, GH is of order 3×1. The student's matrix has an order of 3×2.

b $GH = \begin{bmatrix} 6 & 5 \\ 3 & 8 \\ 5 & 9 \end{bmatrix} \begin{bmatrix} 10 \\ 13 \end{bmatrix}$

$= \begin{bmatrix} 6 \times 10 + 5 \times 13 \\ 3 \times 10 + 8 \times 13 \\ 5 \times 10 + 9 \times 13 \end{bmatrix}$

$= \begin{bmatrix} 125 \\ 134 \\ 167 \end{bmatrix}$

c A possible answer: The student multiplied the first column with the first row, and then the second column with the second row.

d A possible answer:

Step 1: Determine the order of the product matrix.

Step 2: Multiply the elements in the first row by the elements in the first column.

19 **a** Represent the number of vehicles in a row matrix and the cost for each vehicle in a column matrix, then multiply the two matrices together. The product matrix will have an order of 1×1.

b $\begin{bmatrix} 5 & 8 & 4 \end{bmatrix} \begin{bmatrix} 4\,000 \\ 12\,500 \\ 8\,500 \end{bmatrix}$

Alternatively, the product matrix can be found by number of sales × monthly sales, but the product matrix must have order 1×1.

$= [5 \times 4000 + 8 \times 12\,500 + 4 \times 8500]$

$= [154\,000]$ or $\$154\,000$

c A possible answer:

In this multiplication the number of each type of vehicle is multiplied by the price of each type of vehicle, which is incorrect. For example, the ute is valued at \$12 500, but in this multiplication the eight utes sold are multiplied by \$4000, \$12 500 and \$8500 respectively.

20 a

	Adults	Children	Seniors
Friday	125	245	89
Saturday	350	456	128
Sunday	421	523	102

$$\begin{bmatrix} 125 & 245 & 89 \\ 350 & 456 & 128 \\ 421 & 523 & 102 \end{bmatrix}\begin{bmatrix} 35 \\ 25 \\ 20 \end{bmatrix} = \begin{bmatrix} 12\,280 \\ 26\,210 \\ 29\,850 \end{bmatrix}$$

Friday \$12 280, Saturday \$26 210, Sunday \$29 850

b $350 \times 35 + 456 \times 25 + 128 \times 20$

c No, because you cannot multiply the matrix of the entry price (3×1) by the matrix number of people (3×3).

4.4 Exam questions

1
$$2A - 3B = 2 \times \begin{bmatrix} -1 & 2 \\ -3 & 2 \end{bmatrix} - 3 \times \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} -1 \times 2 & 2 \times 2 \\ -3 \times 2 & 2 \times 2 \end{bmatrix} - \begin{bmatrix} -1 \times 3 & 2 \times 3 \\ 1 \times 3 & -2 \times 3 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 4 \\ -6 & 4 \end{bmatrix} - \begin{bmatrix} -3 & 6 \\ 3 & -6 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -2 \\ -9 & 10 \end{bmatrix}$$

Handling subtraction of negative numbers may be more of a problem than the scalar multiplication.
The correct answer is **D**.

2 $AB = A \times B$
$$= \begin{bmatrix} 3 & 4 \\ -5 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & -1 & 5 \\ 0 & 2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 3 \times 4 + 4 \times 0 & 3 \times -1 + 4 \times 2 & 3 \times 5 + 4 \times 0 \\ -5 \times 4 + 0 \times 0 & -5 \times -1 + 0 \times 2 & -5 \times 5 + 0 \times 0 \end{bmatrix}$$
$$= \begin{bmatrix} 12 & 5 & 15 \\ -20 & 5 & -25 \end{bmatrix}$$

A is 2×2 and B is 2×3, so the correct answer should have order 2×3. Be careful when multiplying negative numbers.
The correct answer is **D**.

3
$$X = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$
$$X^2 = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix}$$ [1 mark]
$$2X = \begin{bmatrix} 2 \times 2 & 2 \times -1 \\ 2 \times 0 & 2 \times 4 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -2 \\ 0 & 8 \end{bmatrix}$$ [1 mark]
$$X^2 + 2X = \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 0 & 8 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & -8 \\ 0 & 24 \end{bmatrix}$$ [1 mark]

4.5 Inverse matrices and problem-solving with matrices

4.5 Exercise

1
$$\begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1.5 & -2.5 \\ -1 & 2 \end{bmatrix}$$
$$AB = \begin{bmatrix} 4 \times 1.5 + 5 \times -1 & 4 \times (-2.5) + 5 \times 2 \\ 2 \times 1.5 + 3 \times (-1) & 2 \times (-2.5) + 3 \times 2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, A and B are multiplicative inverses of each other.

2 a determinant $= ad - bc$
$= 3 \times 2 - 5 \times 8$
$= -34$
This matrix has an inverse.

b determinant $= ad - bc$
$= -2 \times 5 - -6 \times 3$
$= 8$
This matrix has an inverse.

c determinant $= ad - bc$
$= 3 \times 8 - 6 \times 4$
$= 0$
This matrix does not have an inverse.

d determinant $= ad - bc$
$= -9 \times 3 + 2 \times 4$
$= -35$
This matrix has an inverse.

3 a
$$\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = \frac{1}{5 \times 1 - 2 \times 2}\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$
$$= \frac{1}{1}\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

b
$$\begin{bmatrix} 7 & 4 \\ 3 & 2 \end{bmatrix} = \frac{1}{7 \times 2 - 4 \times 3}\begin{bmatrix} 2 & -4 \\ -3 & 7 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 2 & -4 \\ -3 & 7 \end{bmatrix}$$ (multiply $\frac{1}{2}$ into the *sim matrix*)
$$= \begin{bmatrix} 1 & -2 \\ -1.5 & 3.5 \end{bmatrix}$$

c
$$\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix} = \frac{1}{2 \times (-2) - 1 \times (-3)}\begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= -1\begin{bmatrix} -2 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}$$

4 det $B = 3 \times 6 - 2 \times 9 = 0$; we cannot divide by zero therefore B^{-1} does not exist.

5 **a** $\det(A) = 2 \times 3 - 1 \times 5 = 1$

$\det(B) = 3 \times 6 - 9 \times 2 = 0$

$\det(C) = -3 \times (-8) - 6 \times 4 = 0$

$\det(D) = 0 \times (-5) - 1 \times 3 = -3$

Therefore, matrices A and D have inverses.

b $A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$D^{-1} = \frac{1}{-3}\begin{bmatrix} -5 & -1 \\ -3 & 0 \end{bmatrix}$$

$$D^{-1} = \begin{bmatrix} \frac{5}{3} & \frac{1}{3} \\ 1 & 0 \end{bmatrix}$$

6 $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -8 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3 \times 4 - 1 \times 2}\begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 \times 3 + (-1) \times (-8) \\ (-2) \times 3 + 3 \times (-8) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -30 \end{bmatrix}$$

Multiply $\frac{1}{10}$ into the matrix

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Therefore, $x = 2$, $y = -3$

7 **a** Matrix X is found by the matrix equation $X = BA^{-1}$.

Matrix B order is 1×2, matrix A^{-1} 2×2.

The order of matrix X is found by the number of rows of matrix B and number of columns in matrix A^{-1}: 1×2 2×2, so matrix X has an order of 1×2.

$$X = \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1.5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 1 + 2 \times (-1.5) & 1 \times (-1) + 2 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 3 \end{bmatrix}$$

b A and A^{-1} are multiplicative inverses of each other, therefore the inverse of A^{-1} is A.

$$(A^{-1})^{-1} = \frac{1}{1 \times 2 - (-1) \times (-1.5)}\begin{bmatrix} 2 & 1 \\ 1.5 & 1 \end{bmatrix}$$

$$(A^{-1})^{-1} = \frac{1}{0.5}\begin{bmatrix} 2 & 1 \\ 1.5 & 1 \end{bmatrix}$$

$$(A^{-1})^{-1} = 2\begin{bmatrix} 2 & 1 \\ 1.5 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$$

8 **a** Find the inverse of $\begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$ and multiply by $\begin{bmatrix} 9 \\ 7 \end{bmatrix}$, using $AX = B$ and $X = A^{-1}B$.

b $\frac{1}{4 \times 1 - 1 \times 3}\begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} = 1 \times \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$$

c $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 9 \\ 7 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \times 9 + -1 \times 7 \\ -3 \times 9 + 4 \times 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$x = 2$, $y = 1$

9 $\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} d \\ c \end{bmatrix} = \begin{bmatrix} 14.00 \\ 12.25 \end{bmatrix}$

$$\begin{bmatrix} d \\ c \end{bmatrix} = \frac{1}{2 \times 2 - 3 \times 3}\begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 14.00 \\ 12.25 \end{bmatrix}$$

$$\begin{bmatrix} d \\ c \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} 2 \times 14 + -3 \times 12.25 \\ -3 \times 14 + 2 \times 12.25 \end{bmatrix}$$

$$\begin{bmatrix} d \\ c \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} -8.75 \\ -17.5 \end{bmatrix}$$

$$\begin{bmatrix} d \\ c \end{bmatrix} = \begin{bmatrix} \frac{-8.75}{-5} \\ \frac{-17.5}{-5} \end{bmatrix}$$

$$\begin{bmatrix} d \\ c \end{bmatrix} = \begin{bmatrix} 1.75 \\ 3.5 \end{bmatrix}$$

4 donuts and 3 cupcakes

$4 \times 1.75 + 3 \times 3.5 = \$17.50$

10 $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1 \times (-5) - 2 \times 3}\begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -5 \times 4 + (-2) \times 1 \\ (-3) \times 4 + 1 \times 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} -22 \\ -11 \end{bmatrix}$$

Multiply $\frac{1}{-11}$ into matrix:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-22}{-11} \\ \frac{-11}{-11} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Therefore, $x = 2$, $y = 1$

11 $\begin{bmatrix} 3 & 5 \\ 4.5 & 7.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

To solve the matrix equation, the inverse of the matrix $\begin{bmatrix} 3 & 5 \\ 4.5 & 7.5 \end{bmatrix}$ must exist.

Determinant $= 3 \times 7.5 - 5 \times 4.5 = 0$. Since the determinant $= 0$, the inverse does not exist.

This means that there is no solution to the simultaneous equations (graphically, this means that the lines are parallel).

12 Let c be the number of children and a be the number of adults attending the party.

Setting up the matrix equation (note the simultaneous equations are

$4.5\,c + 6.5\,a = 51$, $3.25\,c + 4.95\,a = 37.60$)

$$\begin{bmatrix} 4.5 & 6.5 \\ 3.25 & 4.95 \end{bmatrix} \begin{bmatrix} c \\ a \end{bmatrix} = \begin{bmatrix} 51 \\ 37.60 \end{bmatrix}$$

Solving the matrix equation:

$$\begin{bmatrix} c \\ a \end{bmatrix} = \frac{1}{4.5 \times 4.95 - 6.5 \times 3.25} \begin{bmatrix} 4.95 & -6.5 \\ -3.25 & 4.5 \end{bmatrix} \begin{bmatrix} 51 \\ 37.6 \end{bmatrix}$$

$$\begin{bmatrix} c \\ a \end{bmatrix} = \frac{1}{1.15} \begin{bmatrix} 4.95 & -6.5 \\ -3.25 & 4.5 \end{bmatrix} \begin{bmatrix} 51 \\ 37.6 \end{bmatrix}$$

$$\begin{bmatrix} c \\ a \end{bmatrix} = \frac{1}{1.15} \begin{bmatrix} 4.95 \times 51 + -6.5 \times 37.6 \\ -3.25 \times 51 + 4.5 \times 37.6 \end{bmatrix}$$

$$\begin{bmatrix} c \\ a \end{bmatrix} = \frac{1}{1.15} \begin{bmatrix} 8.05 \\ 3.45 \end{bmatrix}$$

Multiply $\frac{1}{1.15}$ into the matrix

$$\begin{bmatrix} c \\ a \end{bmatrix} = \begin{bmatrix} \frac{8.05}{1.15} \\ \frac{3.45}{1.15} \end{bmatrix}$$

$$\begin{bmatrix} c \\ a \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

Therefore, 7 children (including Ben) and 3 adults attended Ben's party.

13 Let d = cost of one drink, p = cost of one bag of popcorn

Setting up the matrix equation (note the simultaneous equations are $5d + 4p = 14$, $4d + 3p = 10.8$):

$$\begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} d \\ p \end{bmatrix} = \begin{bmatrix} 14 \\ 10.8 \end{bmatrix}$$

Solving the matrix equation:

$$\begin{bmatrix} d \\ p \end{bmatrix} = \frac{1}{5 \times 3 - 4 \times 4} \begin{bmatrix} 3 & -4 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} 14 \\ 10.8 \end{bmatrix}$$

$$\begin{bmatrix} d \\ p \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 3 & -4 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} 14 \\ 10.8 \end{bmatrix}$$

$$\begin{bmatrix} d \\ p \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 3 \times 14 + (-4) \times 10.8 \\ -4 \times 14 + 5 \times 10.8 \end{bmatrix}$$

$$\begin{bmatrix} d \\ p \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} -1.2 \\ -2 \end{bmatrix}$$

Multiply $\frac{1}{-1} = -1$ into the matrix:

$$\begin{bmatrix} d \\ p \end{bmatrix} = \begin{bmatrix} 1.2 \\ 2 \end{bmatrix}$$

1 drink = \$1.20. 1 bag of popcorn = \$2.00

Cost of 2 drinks and 2 bags of popcorn:

$2 \times 1.2 + 2 \times 2 = 6.4 = \$6.40$

14 **a** $3b + 2n = 31.80$, $5b + 3n = 49.80$ where b = cost of a bracelet and n = cost of a necklace

$$\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} b \\ n \end{bmatrix} = \begin{bmatrix} 31.80 \\ 49.80 \end{bmatrix}$$

$$\begin{bmatrix} b \\ n \end{bmatrix} = \frac{1}{3 \times 3 - 2 \times 5} \begin{bmatrix} 3 & -2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 31.80 \\ 49.80 \end{bmatrix}$$

$$\begin{bmatrix} b \\ n \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 3 & -2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 31.80 \\ 49.80 \end{bmatrix}$$

$$\begin{bmatrix} b \\ n \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 3 \times 31.80 + -2 \times 49.80 \\ -5 \times 31.80 + 3 \times 49.80 \end{bmatrix}$$

$$\begin{bmatrix} b \\ n \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} -4.2 \\ -9.6 \end{bmatrix}$$

$$\begin{bmatrix} b \\ n \end{bmatrix} = \begin{bmatrix} \frac{-4.2}{-1} \\ \frac{-9.6}{-1} \end{bmatrix}$$

Each bracelet costs \$4.20 and each necklace costs \$9.60.

b $7 \times 4.20 + 4 \times 9.60 = \$67.80$

15 **a** Any one of: did not swap the elements on the diagonal, or multiply the other elements by -1, or multiply the matrix by $\frac{1}{\text{det}}$

b $$\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}^{-1} = \frac{1}{2 \times 3 - 0 \times 1} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$$

c The respective order of matrices is 2×2 and 1×2. The number of columns in the first matrix does not equal the number of rows in the second matrix.

d Possible answers:

Step 1: Find the correct inverse.

Step 2:

Multiply $\begin{bmatrix} 5 & 9 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$.

e $\begin{bmatrix}5 & 9\end{bmatrix}\frac{1}{6}\begin{bmatrix}3 & 0\\ -1 & 2\end{bmatrix} = \begin{bmatrix}5 & 9\end{bmatrix}\begin{bmatrix}\frac{3}{6} & 0\\ \frac{-1}{6} & \frac{2}{6}\end{bmatrix}$

$= \begin{bmatrix}5\times\frac{1}{2}+9\times\frac{-1}{6} & 5\times 0+9\times\frac{1}{3}\end{bmatrix}$

$x = 1, y = 3$

16 a $\begin{bmatrix}5 & 7\\ 6 & 6\end{bmatrix}\begin{bmatrix}s\\ p\end{bmatrix} = \begin{bmatrix}156.80\\ 155.40\end{bmatrix}$

b $\begin{bmatrix}s\\ p\end{bmatrix} = \begin{bmatrix}5 & 7\\ 6 & 6\end{bmatrix}^{-1}\begin{bmatrix}156.80\\ 155.40\end{bmatrix}$

$\begin{bmatrix}s\\ p\end{bmatrix} = \begin{bmatrix}12.25\\ 13.65\end{bmatrix}$

\$12.25

c $K = \begin{bmatrix}30 & 30\end{bmatrix}$, $(6\times 5,\ 6\times 5)$

d Order of product matrix KS: (total selling price) has order 1×1.

Matrix multiplication KS: order K is 1×2 therefore S must have order 2×1.

Note: Product matrix SK will have order 2×2, which will not give the total selling price.

e $KS = [30\ 30]\begin{bmatrix}3.49\\ 4.50\end{bmatrix}$

$KS = [30\times 3.49 + 30\times 4.50]$

$KS = \$239.70$

f Profit = selling price − cost price

$= 239.70 - 155.40$

$= \$84.30$

17 a a = the number of adult tickets sold

c = the number of concession tickets sold

Therefore, $a + c = 400$ and $15a + 9.5c = 5422.5$

b $\begin{bmatrix}1 & 1\\ 15.00 & 9.50\end{bmatrix}\begin{bmatrix}a\\ c\end{bmatrix} = \begin{bmatrix}400\\ 5422.50\end{bmatrix}$, where

a = the number of adult tickets sold and

c = the number of concession tickets sold

c $\begin{bmatrix}a\\ c\end{bmatrix} = \begin{bmatrix}1 & 1\\ 15.00 & 9.50\end{bmatrix}^{-1}\begin{bmatrix}400\\ 5422.50\end{bmatrix}$

$\begin{bmatrix}a\\ c\end{bmatrix} = \begin{bmatrix}295\\ 105\end{bmatrix}$

295 adult tickets were sold.

18 a $\begin{bmatrix}1 & -1 & 2\\ 3 & 4 & 5\\ 2 & 0 & 1\end{bmatrix}^{-1} = \begin{bmatrix}-\frac{4}{19} & -\frac{1}{19} & \frac{13}{19}\\ -\frac{7}{19} & \frac{3}{19} & -\frac{1}{19}\\ \frac{8}{19} & \frac{2}{19} & -\frac{7}{19}\end{bmatrix}$

b $\begin{bmatrix}2 & 1 & -1 & -2\\ -1 & 2 & 0 & 2\\ 0 & 3 & 5 & 3\\ 1 & 1 & 4 & 1\end{bmatrix}^{-1} = \frac{1}{9}\begin{bmatrix}0 & 21 & -24 & 30\\ 3 & -7 & 11 & -13\\ 0 & -9 & 9 & -9\\ -3 & 22 & -23 & 28\end{bmatrix}$

$= \begin{bmatrix}0 & \frac{7}{3} & -\frac{8}{3} & \frac{10}{3}\\ \frac{1}{3} & -\frac{7}{9} & \frac{11}{9} & -\frac{13}{9}\\ 0 & -1 & 1 & -1\\ -\frac{1}{3} & \frac{22}{9} & -\frac{23}{9} & \frac{28}{9}\end{bmatrix}$

19 $\begin{bmatrix}1 & 2 & 3 & 1\\ 2 & -1 & -1 & 3\\ 3 & 1 & 1 & -2\\ -1 & 2 & -4 & -1\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix} = \begin{bmatrix}7\\ -4\\ 12\\ -14\end{bmatrix}$

$\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix} = \begin{bmatrix}1 & 2 & 3 & 1\\ 2 & -1 & -1 & 3\\ 3 & 1 & 1 & -2\\ -1 & 2 & -4 & -1\end{bmatrix}^{-1}\begin{bmatrix}7\\ -4\\ 12\\ -14\end{bmatrix}$

$a = 2,\ b = -1,\ c = 3,\ d = -2$

4.5 Exam questions

1 $\det(A) = (1\times 0) - (-3\times 2)$

$= 0 + 6$

$= 6$

Be careful when dealing with the negative numbers.

The correct answer is **D**.

2

$$\begin{bmatrix}2 & 2\\ 2 & -5\end{bmatrix}X = \begin{bmatrix}8 & 6\\ -13 & -1\end{bmatrix}$$

$$\frac{1}{-14}\begin{bmatrix}-5 & -2\\ -2 & 2\end{bmatrix}\begin{bmatrix}2 & 2\\ 2 & -5\end{bmatrix}X = \frac{1}{-14}\begin{bmatrix}-5 & -2\\ -2 & 2\end{bmatrix}\begin{bmatrix}8 & 6\\ -13 & -1\end{bmatrix}$$

$$LA = \frac{1}{-14}\begin{bmatrix}-14 & -28\\ -42 & -14\end{bmatrix}$$

$$A = \begin{bmatrix}1 & 2\\ 3 & 1\end{bmatrix}$$

The correct answer is **B**.

3 Notice that the second equation must be written in the same order as the first; namely, $-6x + 3y = 7$.

$3x - y = 0 \quad (1)$

$3y - 6x = 7 \quad (2)$

$-6x + 3y = 7 \quad (2')$

$\begin{bmatrix}3 & -1\\ -6 & 3\end{bmatrix}$

The correct answer is **E**.

4.6 Communications and connections

4.6 Exercise

1 a

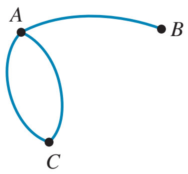
$$\begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array}\\ \begin{array}{c} A\\ B\\ C \end{array} & \begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}\end{array}$$

b
$$\begin{array}{c} \\ D \\ E \\ F \\ G \end{array}\begin{array}{c} \begin{array}{cccc} D & E & F & G \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

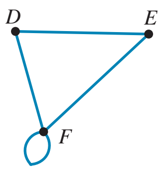
c
$$\begin{array}{c} \\ H \\ I \\ J \\ K \end{array}\begin{array}{c} \begin{array}{cccc} H & I & J & K \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

d
$$\begin{array}{c} \\ L \\ M \\ N \\ O \end{array}\begin{array}{c} \begin{array}{cccc} L & M & N & O \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

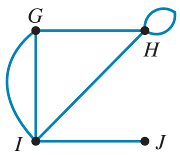
2 a



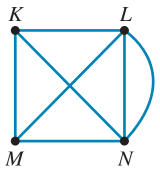
b



c



d



3
$$\begin{array}{c} \\ K \\ W \\ N \\ G \end{array}\begin{array}{c} \begin{array}{cccc} K & W & N & G \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}^2 = \begin{array}{c} \\ K \\ W \\ N \\ G \end{array}\begin{array}{c} \begin{array}{cccc} K & W & N & G \end{array} \\ \begin{bmatrix} 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix} \end{array}$$

There are 2 two-step communications between Karina and Winter.

$K - G - W$

$K - N - W$

4
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Square the matrix to find the number of two-step connections.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}^2 \end{array} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 3 & 1 & 1 & 2 & 2 \\ 1 & 3 & 3 & 0 & 1 \\ 1 & 3 & 3 & 0 & 1 \\ 2 & 0 & 0 & 2 & 2 \\ 2 & 1 & 1 & 2 & 3 \end{bmatrix} \end{array}$$

The number of two-step connections between Amira and Dev can be obtained from the element in row 1, column 4 or row 4, column 1.

There are 2 two-step connections between Amira and Dev.

There are 2 ways Amira can get a message to Dev, via one other person.

5 There are five computers; construct a 5×5 adjacency matrix using the pronumerals provided. Count the number of cables that join between each pair of computers. For example, there is one cable joining computers A and B. Record this in the adjacency matrix as shown.

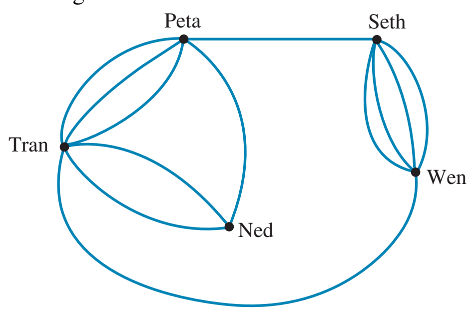
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

6 a Locating Peta (P) on the top and Tran (T) on the side of the matrix (or vice versa), there are 3 communications between the two.

b No, because there is a '0' entered against Seth (S) and Ned (N).

c They did not communicate with themselves.

d Set up a diagram with five points and label them using the pronumerals provided. Use the matrix to determine the number of communications and represent these as lines on the diagram as shown.



7 a
$$\begin{array}{c} \\ W \\ C \\ H \\ K \\ A \end{array}\begin{array}{c} \begin{array}{ccccc} W & C & H & K & A \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

b
$$\begin{bmatrix} 4 & 0 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 2 & 1 & 3 & 1 & 1 \\ 1 & \cancel{1} & \cancel{1} & \cancel{2} & \cancel{2} \\ 1 & 1 & 1 & 2 & 2 \end{bmatrix}$$

1 way between Williamton and Kokialah.

c Yes. The matrix raised to the power of 3 will provide the number of ways possible.

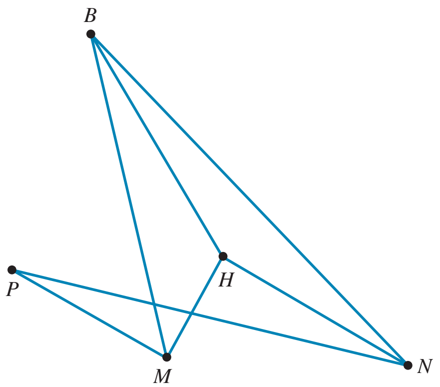
8 Glenorchy to St Arnaud via Campbell's Bridge would be a two-step matrix. Using a spreadsheet or calculator, find:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}^2$$

$$= \begin{array}{c} \\ G \\ S \\ C \end{array} \begin{array}{c} \begin{array}{ccc} G & S & C \end{array} \\ \begin{bmatrix} 2 & 2 & 2 \\ 2 & 5 & 1 \\ 2 & 1 & 5 \end{bmatrix} \end{array}$$

There are 2 ways to travel from Glenorchy to St Arnaud via Campbell's Bridge.

9 a



b $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}^2$

$$= \begin{array}{c} \\ B \\ H \\ M \\ N \\ P \end{array} \begin{array}{c} \begin{array}{ccccc} B & H & M & N & P \end{array} \\ \begin{bmatrix} 3 & 2 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 & 2 \\ 2 & 2 & 4 & 3 & 1 \\ 2 & 2 & 3 & 4 & 1 \\ 2 & 2 & 1 & 1 & 2 \end{bmatrix} \end{array}$$

c Raise the matrix to the power of 2.

d Yes. Raise the matrix to a power of 4, as there are five cities in total.

10 a

$$\begin{array}{c} \\ S \\ R \\ T \end{array} \begin{array}{c} \begin{array}{ccc} S & R & T \end{array} \\ \begin{bmatrix} 0 & 3 & 2 \\ 3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} \end{array}$$

There are 3 text messages sent between Stacey and Ruth.

b $3 + 2 + 1 = 6$ messages in total were sent between all three friends.

4.6 Exam questions

1 Y cannot send directly to U or V, only directly to X or Z. However, X and Z do not communicate directly with W. Therefore the answer is 0 — Y cannot send a message to W by sending it via one other person.
The correct answer is **A**.

2 Options D and E are not correct, as some points are connected twice.
Points A, B and C are connected to each other only once.
However, in options A and B, some are not connected at all.
The correct answer is **C**.

3

$$\text{From} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \text{To} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix} \end{array}$$

Notice that A, B, C and D are connected to each other once, except for B and D, which are connected twice. [1 mark]

4.7 Transition matrices

4.7 Exercise

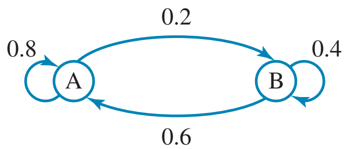
1

		From	
		Rain	No rain
To	Rain	0.3	0.2
	No rain	0.7	0.8

$$\begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \end{bmatrix}$$

2 40% of trains that start the week in West depot will end up in East depot the next week.
75% of trains that start the week in East depot will end up in West depot the next week.

3



4 20% of Melburnians move to Canberra each year.
65% of Canberra residents move to Melbourne each year.

5 It helps to make a transition table first.

		From	
		Cooktown	Dieterviller
To	Cooktown	0.3	0.9
	Dieterviller	0.7	0.1

$$\begin{bmatrix} 0.3 & 0.9 \\ 0.7 & 0.1 \end{bmatrix}$$

6 Transition table:

		From	
		J	K
To	J	0.75	0.2
	K	0.25	0.8

$$T = \begin{bmatrix} 0.75 & 0.2 \\ 0.25 & 0.8 \end{bmatrix}$$

Town J started with 2000. Town K started with 5000.

$$S_0 = \begin{bmatrix} 2000 \\ 5000 \end{bmatrix}$$

$S_n = T^n S_0$

$S_2 = T^2 S_0$

$$S_2 = \begin{bmatrix} 0.613 & 0.31 \\ 0.388 & 0.69 \end{bmatrix}^2 \begin{bmatrix} 2000 \\ 5000 \end{bmatrix}$$

$$= \begin{bmatrix} 2775 \\ 4225 \end{bmatrix}$$

After two years, there would be 2775 people in Town J and 4225 people in Town K.

7 $S_n = T^n S_0$

$$S_5 = \begin{bmatrix} 0.6 & 0.55 \\ 0.4 & 0.45 \end{bmatrix}^5 \begin{bmatrix} 400 \\ 300 \end{bmatrix}$$

$$= \begin{bmatrix} 405 \\ 295 \end{bmatrix}$$

8

		From	
		X	Y
To	X	0.85	0.55
	Y	0.15	0.45

$$T = \begin{bmatrix} 0.85 & 0.55 \\ 0.15 & 0.45 \end{bmatrix}$$

$$S_0 = \begin{bmatrix} 16 \\ 28 \end{bmatrix}$$

$S_{50} = T^{50} S_0$

$$= \begin{bmatrix} 0.85 & 0.55 \\ 0.15 & 0.45 \end{bmatrix}^{50} \begin{bmatrix} 16 \\ 28 \end{bmatrix}$$

$$= \begin{bmatrix} 35 \\ 9 \end{bmatrix}$$

$S_{51} = T^{51} S_0$

$$= \begin{bmatrix} 0.85 & 0.55 \\ 0.15 & 0.45 \end{bmatrix}^{51} \begin{bmatrix} 16 \\ 28 \end{bmatrix}$$

$$= \begin{bmatrix} 35 \\ 9 \end{bmatrix}$$

9 $S_0 = \begin{bmatrix} 800 \\ 300 \end{bmatrix}$

$$T = \begin{bmatrix} 0.35 & 0.2 \\ 0.65 & 0.8 \end{bmatrix}$$

$$S_{50} = \begin{bmatrix} 0.35 & 0.2 \\ 0.65 & 0.8 \end{bmatrix}^{50} \begin{bmatrix} 800 \\ 300 \end{bmatrix}$$

$$= \begin{bmatrix} 259 \\ 841 \end{bmatrix}$$

$$S_{51} = \begin{bmatrix} 0.35 & 0.2 \\ 0.65 & 0.8 \end{bmatrix}^{51} \begin{bmatrix} 800 \\ 300 \end{bmatrix}$$

$$= \begin{bmatrix} 259 \\ 841 \end{bmatrix}$$

There are 259 birds in Australia and 841 birds in Antarctica at the steady state.

10 a

		From	
		Coldes	Sheepies
To	Coldes	0.7	0.35
	Sheepies	0.3	0.65

$$T = \begin{bmatrix} 0.7 & 0.35 \\ 0.3 & 0.65 \end{bmatrix}$$

b $S_0 = \begin{bmatrix} 1000 \\ 3000 \end{bmatrix}$

$S_n = T^n S_0$

$S_3 = T^3 S_0$

$$= \begin{bmatrix} 0.7 & 0.35 \\ 0.3 & 0.65 \end{bmatrix}^3 \begin{bmatrix} 1000 \\ 3000 \end{bmatrix}$$

$$= \begin{bmatrix} 2104 \\ 1896 \end{bmatrix}$$

Sheepies would have 1896 customers after three weeks.

c $S_{50} = T^{50} S_0$

$$= \begin{bmatrix} 0.7 & 0.35 \\ 0.3 & 0.65 \end{bmatrix}^{50} \begin{bmatrix} 1000 \\ 3000 \end{bmatrix}$$

$$= \begin{bmatrix} 2154 \\ 1846 \end{bmatrix}$$

We can see this is the steady state because it has the same value as the third state.

At the steady state, 2154 customers shop at Coldes and 1846 customers shop at Sheepies.

4.7 Exam questions

1 $m = 0.8m + 0.48s$ and $0.48s = 12$ (for Tuesday night)

Solve simultaneously on your CAS calculator to give $m = 60$ and $s = 25$

Therefore, the airline has 85 planes.

The correct answer is **E**.

2 $\dfrac{400}{2000} \times 100\% = 20\%$ [1 mark]

3 $S_1 = \begin{bmatrix} 0.1 & 0.2 & 0.1 & 0.2 \\ 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix} \times \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 300 \\ 780 \\ 300 \\ 620 \end{bmatrix}$ [1 mark]

4.8 Review

4.8 Exercise

Multiple choice

1 $a - (-1) = 2$

$a + 1 = 2$

$a = 2 - 1$

$a = 1$

$2 - b = 1$

$2 - 1 = b$

$b = 1$

The correct answer is **A**.

2 There are 4 rows and 3 columns in the matrix, so the order is 4×3.
The correct answer is **C**.

3 Use a CAS and enter:
$$\det\left(\begin{bmatrix} 5 & 4 \\ 2 & 7 \end{bmatrix}\right)$$
The answer is shown on screen.
$$\det\left(\begin{bmatrix} 5 & 4 \\ 2 & 7 \end{bmatrix}\right) = 27$$
The correct answer is **C**.

4 $$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 3 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 3 \end{bmatrix}$$
A can communicate to D via 2 other people.
The correct answer is **C**.

5 Let the number of apples purchased $= x$ and the number of bananas purchased $= y$.
The two following simultaneous equations can then be formed:
$2x + 3y = 3.8$ and $4x + 4y = 6.2$
These can be set up as the matrix equation
$$\begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.8 \\ 6.2 \end{bmatrix}.$$
The correct answer is **A**.

6 $$\begin{bmatrix} 5 & 1 & 2 \\ 3 & 7 & 8 \end{bmatrix} \begin{bmatrix} 12 \\ 10 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \times 12 + 1 \times 10 + 2 \times 9 \\ 3 \times 12 + 7 \times 10 + 8 \times 9 \end{bmatrix}$$
$$= \begin{bmatrix} 60 + 10 + 18 \\ 36 + 70 + 72 \end{bmatrix}$$
$$= \begin{bmatrix} 88 \\ 178 \end{bmatrix}$$
The correct answer is **A**.

7 The percentage of population across the three age groups for Indonesia is represented by the bottom row of the table.
The matrix in E represents this information.
The correct answer is **E**.

8 A: 3×2 by 2×1; product matrix exists.
B: 1×3 by 3×2; product matrix exists.
C: 1×3 by 2×1; product matrix does not exist.
D: 2×1 by 1×3; product matrix exists.
E: 3×2 by 2×1 by 1×3; product matrix exists.
The correct answer is **C**.

9 $$\frac{1}{16 \times 9 - 15 \times 10} \begin{bmatrix} 9 & -15 \\ -10 & 16 \end{bmatrix} = \frac{1}{-6} \begin{bmatrix} 9 & -15 \\ -10 & 16 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{9}{-6} & \frac{-15}{-6} \\ \frac{-10}{-6} & \frac{16}{-6} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{-3}{2} & \frac{5}{2} \\ \frac{5}{3} & \frac{-8}{3} \end{bmatrix}$$
The correct answer is **E**.

10 $S_n = TS_0$
$$= \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 200 \\ 400 \end{bmatrix}$$
$$= \begin{bmatrix} 160 \\ 440 \end{bmatrix}$$
The correct answer is **B**.

Short answer

11 a $$\begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 + 1 \\ 2 + 0 \end{bmatrix}$$
$$= \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

b $$\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 2\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$
$$= \begin{bmatrix} 1 - 6 \\ 0 - 4 \end{bmatrix}$$
$$= \begin{bmatrix} -5 \\ -4 \end{bmatrix}$$

c $$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + 3 \times 0 \\ 1 \times 1 + 4 \times 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 2 + 0 \\ 1 + 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

d $$1.5\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 \times 1.5 & 3 \times 1.5 \\ 1 \times 1.5 & 4 \times 1.5 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 4.5 \\ 1.5 & 6 \end{bmatrix}$$

12 a $1 \times p$ by 3×2
For the product matrix ED to exist $p = 3$.

b For the product matrix H to exist, $p = 3$.
1×3 by 3×2 by $2 \times 2 = 1 \times 2$ by 2×2
Therefore H has order 1×2.

13 a $$\begin{bmatrix} 12.5 \\ 6 \\ 10 \end{bmatrix}$$

b $$\begin{bmatrix} 140 & 225 & 90 \end{bmatrix} \begin{bmatrix} 12.5 \\ 6 \\ 10 \end{bmatrix} = \begin{bmatrix} 140 \times 12.5 + 225 \times 6 + 90 \times 10 \end{bmatrix}$$
$$= [1750 + 1350 + 900]$$
$$= \begin{bmatrix} 4000 \end{bmatrix}$$
The total amount of ticket sales is \$4000.

14 a i $\begin{bmatrix}3 & 5\end{bmatrix}$ is a 1×2 matrix, $\begin{bmatrix}2\\6\end{bmatrix}$ is a 2×1 matrix. The product matrix has an order of 1×1.

ii $\begin{bmatrix}4 & 6\end{bmatrix}$ is a 1×2 matrix, $\begin{bmatrix}2 & 3\\4 & 7\end{bmatrix}$ is a 2×2 matrix. The product matrix has an order of 1×2.

iii $\begin{bmatrix}-1 & 9\\10 & 5\end{bmatrix}$ is a 2×2 matrix, $\begin{bmatrix}3 & -2\\5 & 11\end{bmatrix}$ is a 2×2 matrix. The product matrix has an order of 2×2.

iv $\begin{bmatrix}2\\7\\8\end{bmatrix}$ is a 3×1 matrix, $\begin{bmatrix}5 & 3 & 4\end{bmatrix}$ is a 1×3 matrix. The product matrix has an order of 3×3.

b i
$$\begin{bmatrix}3 & 5\end{bmatrix}\begin{bmatrix}2\\6\end{bmatrix} = \begin{bmatrix}3\times 2+5\times 6\end{bmatrix} = \begin{bmatrix}6+30\end{bmatrix} = \begin{bmatrix}36\end{bmatrix}$$

ii
$$\begin{bmatrix}4 & 6\end{bmatrix}\begin{bmatrix}2 & 3\\4 & 7\end{bmatrix} = \begin{bmatrix}4\times 2+6\times 4 & 4\times 3+6\times 7\end{bmatrix} = \begin{bmatrix}8+24 & 12+42\end{bmatrix} = \begin{bmatrix}32 & 54\end{bmatrix}$$

iii
$$\begin{bmatrix}-1 & 9\\10 & 5\end{bmatrix}\begin{bmatrix}3 & -2\\5 & 11\end{bmatrix} = \begin{bmatrix}-1\times 3+9\times 5 & -1\times -2+9\times 11\\10\times 3+5\times 5 & 10\times -2+5\times 11\end{bmatrix}$$
$$= \begin{bmatrix}-3+45 & 2+99\\30+25 & -20+55\end{bmatrix} = \begin{bmatrix}42 & 101\\55 & 35\end{bmatrix}$$

iv
$$\begin{bmatrix}2\\7\\8\end{bmatrix}\begin{bmatrix}5 & 3 & 4\end{bmatrix} = \begin{bmatrix}2\times 5 & 2\times 3 & 2\times 4\\7\times 5 & 7\times 3 & 7\times 4\\8\times 5 & 8\times 3 & 8\times 4\end{bmatrix} = \begin{bmatrix}10 & 6 & 8\\35 & 21 & 28\\40 & 24 & 32\end{bmatrix}$$

15 a i
$$\det A = 6\times 3-5\times 3 = 18-15 = 3$$

ii
$$\det B = -2\times 3-2\times -1 = -6+2 = -4$$

b. i
$$A^{-1} = \frac{1}{3}\begin{bmatrix}3 & -3\\-5 & 6\end{bmatrix} = \begin{bmatrix}\frac{3}{3} & \frac{-3}{3}\\\frac{-5}{3} & \frac{6}{3}\end{bmatrix} = \begin{bmatrix}1 & -1\\-\frac{5}{3} & 2\end{bmatrix}$$

ii
$$B^{-1} = \frac{1}{-4}\begin{bmatrix}3 & -2\\1 & -2\end{bmatrix} = \begin{bmatrix}\frac{3}{-4} & \frac{-2}{-4}\\\frac{1}{-4} & \frac{-2}{-4}\end{bmatrix} = \begin{bmatrix}-\frac{3}{4} & \frac{1}{2}\\-\frac{1}{4} & \frac{1}{2}\end{bmatrix}$$

c Matrix M is not a square matrix; therefore, it does not have an inverse.

16 a
$$\begin{bmatrix}2 & 3\\4 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1\\9\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{2\times -1-3\times 4}\begin{bmatrix}-1 & -3\\-4 & 2\end{bmatrix}\begin{bmatrix}1\\9\end{bmatrix} = \frac{1}{-2-12}\begin{bmatrix}-1\times 1+-3\times 9\\-4\times 1+2\times 9\end{bmatrix} = \frac{1}{-14}\begin{bmatrix}-1-27\\-4+18\end{bmatrix} = \frac{1}{-14}\begin{bmatrix}-28\\14\end{bmatrix} = \begin{bmatrix}\frac{-28}{-14}\\\frac{14}{-14}\end{bmatrix} = \begin{bmatrix}2\\-1\end{bmatrix}$$

Therefore $x = 2$ and $y = -1$.

b
$$\begin{bmatrix}3 & -2\\1 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}12\\19\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix} = \frac{1}{3\times 2-(-2)\times 1}\begin{bmatrix}2 & 2\\-1 & 3\end{bmatrix}\begin{bmatrix}12\\19\end{bmatrix} = \frac{1}{6+2}\begin{bmatrix}2\times 12+2\times 19\\-1\times 12+3\times 19\end{bmatrix} = \frac{1}{8}\begin{bmatrix}24+38\\-12+57\end{bmatrix} = \frac{1}{8}\begin{bmatrix}62\\45\end{bmatrix} = \begin{bmatrix}\frac{62}{8}\\\frac{45}{8}\end{bmatrix} = \begin{bmatrix}7.75\\5.625\end{bmatrix}$$

Therefore $x = 7.75$ and $y = 5.625$.

Extended response

17 a Select any element from the matrix and divide by the corresponding element from the table.
For example, fat content of the bread in lunch and fat content in one slice of bread: $\frac{7.6}{0.95} = 8$. Checking using other elements:
fat content of the cheese in lunch and fat content of one slice of cheese: $\frac{44}{5.5} = 8$
So Pedro used 8 slices of bread.

b $\frac{8}{2} = 4$
Pedro made 4 cheese sandwiches.

c $\frac{44}{5.5} = 8$, so 8 slices of cheese are used.
$\frac{8}{4} = 2$, so each sandwich has 2 slices of cheese.

d $x = 8 \times 1.6$
$= 12.8$

e $\frac{13.2}{3.3} = 4$
$\frac{4}{4} = 1$, so each sandwich has 1 tsp of margarine.
Each sandwich has 2 slices of bread, 2 slices of cheese and 1 tsp of margarine.
This can be represented by the row matrix $\begin{bmatrix} 2 & 2 & 1 \end{bmatrix}$.

18 a $C = \begin{bmatrix} 3890 \\ 2178 \\ 868 \end{bmatrix}$
C has an order of 3×1.

b $P = \begin{bmatrix} 62 & 125 & 270 \end{bmatrix}$

c 1×3 by 3×1. The product matrix PC has an order of 1×1.

d $100\% - 5\% = 95\%$
$95\% = 0.95$
$d = 0.95$

e $E = 0.95 \begin{bmatrix} 3890 \\ 2178 \\ 868 \end{bmatrix}$
$= \begin{bmatrix} 3890 \times 0.95 \\ 2178 \times 0.95 \\ 868 \times 0.95 \end{bmatrix}$
$= \begin{bmatrix} 3695.50 \\ 2069.10 \\ 824.60 \end{bmatrix}$

19 a $\begin{bmatrix} 10 & 9 \\ 8 & 12 \\ 9 & 11 \end{bmatrix}$

b $\begin{bmatrix} 45.95 & 25.50 & 18.60 \end{bmatrix}$

c i $\begin{bmatrix} 45.95 & 25.50 & 18.60 \end{bmatrix} \begin{bmatrix} 10 & 9 \\ 8 & 12 \\ 9 & 11 \end{bmatrix}$

ii $\begin{bmatrix} 45.95 & 25.50 & 18.60 \end{bmatrix} \begin{bmatrix} 10 & 9 \\ 8 & 12 \\ 9 & 11 \end{bmatrix}$

$= \begin{bmatrix} 45.95 \times 10 + 25.50 \times 8 + 18.60 \times 9 & 45.95 \times 9 + 25.50 \times 12 + 18.60 \times 11 \end{bmatrix}$

$= \begin{bmatrix} 459.5 + 204 + 167.4 & 413.55 + 306 + 204.6 \end{bmatrix}$

$= \begin{bmatrix} 830.9 & 924.15 \end{bmatrix}$

LeisureLand: \$830.90, SportLand: \$924.15

20 a

		From	
		H	O
To	H	0.4	0.2
	O	0.6	0.8

b $\begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}$

c $S_0 = \begin{bmatrix} 5000 \\ 1000 \end{bmatrix}$

$S_3 = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}^3 \begin{bmatrix} 5000 \\ 1000 \end{bmatrix}$

$= \begin{bmatrix} 1528 \\ 4472 \end{bmatrix}$

After three weeks, 1528 people are predicted to be buying the *Herald Moon*.

d Steady state = S

$S = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.8 \end{bmatrix}^{50} \begin{bmatrix} 5000 \\ 1000 \end{bmatrix}$

$= \begin{bmatrix} 1500 \\ 4500 \end{bmatrix}$

At the steady state, it is predicted that there will be 1500 people buying the *Herald Moon* and 4500 people buying *The Old Times*.

4.8 Exam questions

1 Check an element that is different in all matrices, for example row 2, column 3:
$m_{23} = 3 \times 2 + 2 \times 3 = 12$
The correct answer is **B**.

2 Use the diagram to set up a communication matrix.

$$\textit{sender}\ \begin{array}{c} \\ S \\ T \\ U \\ V \end{array}\!\!\begin{array}{c} \textit{receiver} \\ \begin{array}{cccc} S & T & U & V \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

The correct answer is **D**.

3 To add matrices together, all matrices must be of the same order. Therefore, only $\begin{bmatrix} 8 \\ 12 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 8 & 0 \\ 12 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ are possible.
The correct answer is **C**.

4 a 3×2 [1 mark]

b Add up column 2: $50 + 20 + 40 = 110$ [1 mark]

c $L = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$ [1 mark]

5 The matrix multiplication

$$\begin{bmatrix} 4 & 2 & 0 \end{bmatrix} \times \begin{bmatrix} 4 \\ 12 \\ 8 \end{bmatrix} = [4 \times 4 + 2 \times 12 + 0 \times 8] = [40].$$

A $[144] \neq [40]$

B $\begin{bmatrix} 16 \\ 24 \\ 0 \end{bmatrix} \neq [40]$

C $4 \times \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 12 \\ 8 \end{bmatrix} =$

$[4 \times 1 \times 1 + 4 \times 2 \times 12 + 4 \times 0 \times 8] = [100] \neq [40]$

D $2 \times \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} =$

$[2 \times 2 \times 2 + 2 \times 1 \times 6 + 2 \times 0 \times 4] = [20] \neq [40]$

E $4 \times \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} =$

$[4 \times 2 \times 2 + 4 \times 1 \times 6 + 4 \times 0 \times 4] = [40]$

The correct answer is **E**.

Topic 5 — Linear functions, graphs and models

5.2 Linear functions and their features

5.2 Exercise

1 a $y = 1 + 2x$

Equation is in the form $y = a + bx$,
coefficient of $b = 2$, a-value $= 1$;
gradient $= 2$, y-intercept $= 1$

b $y = 3 - x$

Equation is in the form $y = a + bx$,
coefficient of $b = -1$, a-value $= 3$;
gradient $= -1$, y-intercept $= 3$

c $y = 4 + \frac{1}{2}x$

Equation is in the form $y = a + bx$,
coefficient of $b = \frac{1}{2}$, a-value $= 4$;
gradient $= \frac{1}{2}$, y-intercept $= 4$

d $-4x + 4y = 1$

Equation is not in form $y = a + bx$;
rearrange the equation:

$$-\frac{4x}{4} + \frac{4y}{4} = \frac{1}{4}$$
$$y = \frac{1}{4} + x$$

Coefficient of $b = 1$, a-value $= \frac{1}{4}$;
gradient $= 1$, y-intercept $= \frac{1}{4}$

e $3x + 2y = 6$

Equation is not in form $y = a + bx$;
rearrange the equation:

$$3x + 2y - 3x = 6 - 3x$$
$$2y = 6 - 3x$$
$$\frac{2y}{2} = \frac{6}{2} + \frac{-3x}{2}$$
$$y = 3 - \frac{3}{2}x$$

coefficient of $b = -\frac{3}{2}$, a-value $= 3$;
Gradient $= -\frac{3}{2}$, y-intercept $= 3$

2 a $y = \frac{3x - 1}{5}$

Rearrange equation into form $y = a + bx$:
$y = -\frac{1}{5} + \frac{3}{5}x$
Coefficient of $b = \frac{3}{5}$, a-value $= -\frac{1}{5}$;
gradient $= \frac{3}{5}$, y-intercept $= -\frac{1}{5}$

b $y = 5(2x - 1)$

Expand:
$y = 10x - 5 = -5 + 10x$
Equation is in form $y = a + bx$;
coefficient of $b = 10$, a-value $= -5$;
gradient $= 10$, y-intercept $= -5$

c $y = \frac{3 - x}{2}$

Rearrange equation into form $y = a + bx$:
$y = \frac{3}{2} - \frac{x}{2}$
Coefficient of $b = -\frac{1}{2}$, (*note:* $-\frac{x}{2} = -\frac{1}{2}x$), a-value $= \frac{3}{2}$;
gradient $= -\frac{1}{2}$, y-intercept $= \frac{3}{2}$

3 a Select two points on the line: $(-4, 0)$ and $(0, 4)$

Determine the rise (change in y-values): $4 - 0 = 4$
Determine the run (change in x-value): $0 - (-4) = 4$
Gradient $= \frac{\text{rise}}{\text{run}} = \frac{4}{4} = 1$

b Select two points on the line: $(3, 0)$ and $(0, 6)$

Determine the rise (change in y-values): $6 - 0 = 6$
Determine the run (change in x-value): $0 - 3 = -3$
Gradient $= \frac{\text{rise}}{\text{run}} = \frac{6}{-3} = -2$

4 a Taking two points $(2, 0)$ and $(4, 6)$

Gradient: rise $= 6 - 0 = 6$
run $= 4 - 2 = 2$

$$\text{Gradient} = \frac{6}{2}$$
$$= 3$$

y-intercept $(0, a)$ using point $(2, 0)$

$$3 = \frac{a - 0}{0 - 2}$$
$$3 = \frac{a}{-2}$$
$$a = -6$$

y-intercept $= -6$
Gradient $= 3$, y-intercept $= -6$

b Taking two points $(-2, 0)$ and $(3, -3)$

Gradient: rise $= -3 - 0 = -3$
run $= 3 - (-2) = 5$

$$\text{Gradient} = \frac{-3}{5}$$

y-intercept $(0, a)$ using point $(-2, 0)$

$$-\frac{3}{5} = \frac{a - 0}{0 - (-2)}$$
$$-\frac{3}{5} = \frac{a}{2}$$
$$a = -\frac{6}{5}$$

Gradient $= -\frac{3}{5}$, y-intercept $= -\frac{6}{5}$

5 A Points $(4, 0), (0, 8)$

Rise: $8 - 0 = 8$
Run: $0 - 4 = -4$
Gradient $= \frac{8}{-4} \neq -\frac{1}{4}$

B Points $(-2, 0), (1, 4)$

Rise: $4 - 0 = 4$
Run: $1 - (-2) = 3$
Gradient $= \frac{4}{3} \neq -\frac{1}{4}$

C Points $(-1, 0), (0, -4)$

Rise: $-4 - 0 = -4$

Run: $0 - (-1) = 1$

$\text{Gradient} = \frac{-4}{1} \neq -\frac{1}{4}$

D Points $(-3, 2), (13, -2)$

Rise: $-2 - 2 = -4$

Run: $13 - (-3) = 16$

$\text{Gradient} = \frac{-4}{16} = -\frac{1}{4}$

E Points $(1, -4), (-3, 0)$

Rise: $0 - (-4) = 4$

Run: $(-3) - 1 = -4$

$\text{Gradient} = \frac{4}{-4} \neq -\frac{1}{4}$

The correct answer is **D**.

6 a $(2, 3)$ and $(5, 12)$

Difference in y-values: $12 - 3 = 9$

Write as numerator: $\underline{9}$

Difference in x-values: $5 - 2 = 3$

Write as denominator: $\frac{9}{3}$

Simplify: $\frac{9}{3}$

Gradient $= 3$

b $(-1, 3)$ and $(2, 7)$

Difference in y-values: $7 - 3 = 4$

Write as numerator: $\underline{4}$

Difference in x-values: $2 - (-1) = 3$

Write as denominator: $\frac{4}{3}$

Gradient $= \frac{4}{3}$

c $(-0.2, 0.7)$ and $(0.5, 0.9)$

Difference in y-values: $0.9 - 0.7 = 0.2$

Write as numerator: $\underline{0.2}$

Difference in x-values: $0.5 - (-0.2) = 0.7$

Write as denominator: $\frac{0.2}{0.7}$

Simplify: $\frac{0.2}{0.7}$

Gradient $= \frac{2}{7}$

7 $(1, 4)$ and $(a, 8)$

Difference in y-values: $8 - 4 = 4$

Write as numerator: $\underline{4}$

Difference in x-values: $a - 1$; write as denominator $\frac{4}{a-1}$

$\frac{4}{a-1} = -2$

Solve for a:

$$\frac{4}{a-1} \times (a-1) = -2(a-1)$$

$$\begin{aligned} \frac{4}{-2} &= a - 1 \\ -2 &= a - 1 \\ -2 + 1 &= a \\ a &= -1 \end{aligned}$$

8 Substitute the a- and b-values into $y = a + bx$

a $y = 7 + 2x$

b $y = -3 + 4x$

c $y = 2 - x$

d $y = -1 + 3x$

9 a $(3, 6)$ and $(2, 9)$

Rise: $9 - 6 = 3$

Run: $2 - 3 = -1$

$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{3}{-1}$

Gradient $= -3$

b $(-4, 5)$ and $(1, 8)$

Rise: $8 - 5 = 3$

Run: $1 - (-4) = 5$

$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-2}{2}$

$\text{Gradient} = \frac{3}{5}$

c $(-0.9, 0.5)$ and $(0.2, -0.7)$

Rise: $-0.7 - 0.5 = -1.2$

Run: $02 - (-0.9) = 1.1$

$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-1.2}{1.1}$

$\text{Gradient} = -\frac{12}{11}$

d $(1.4, 7.8)$ and $(3.2, 9.5)$

Rise: $9.5 - 7.8 = 1.7$

Run: $3.2 - 1.4 = 1.8$

$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{1.7}{1.8}$

$\text{Gradient} = \frac{17}{18}$

e $\left(\frac{4}{5}, \frac{2}{5}\right)$ and $\left(\frac{1}{5}, \frac{-6}{5}\right)$

Rise: $\frac{-6}{5} - \frac{2}{5} = -\frac{8}{5}$

Run: $\frac{1}{5} - \frac{4}{5} = \frac{3}{5}$

$$\begin{aligned} \text{Gradient} = \frac{\text{rise}}{\text{run}} &= \frac{-\frac{8}{5}}{-\frac{3}{5}} \\ &-\frac{8}{5} \div -\frac{3}{5} \\ &= -\frac{8}{5} \times -\frac{5}{3} \\ &= \frac{8}{3} \end{aligned}$$

$\text{Gradient} = \frac{8}{3}$

f $\left(\frac{2}{3}, \frac{1}{4}\right)$ and $\left(\frac{3}{4}, \frac{-2}{3}\right)$

Rise: $\frac{-2}{3}$

$$\begin{aligned} -\frac{1}{4} &= \frac{-2(4) - 1(3)}{12} \\ &= \frac{-8 - 3}{12} \\ &= -\frac{11}{12} \end{aligned}$$

Run:

$$\frac{3}{2}-\frac{2}{3}=\frac{3(3)-2(4)}{12}$$
$$=\frac{9-8}{12}$$
$$=\frac{1}{12}$$

$$\text{Gradient}=\frac{\text{rise}}{\text{run}}=\frac{-\frac{11}{12}}{\frac{1}{12}}$$
$$=-\frac{11}{12}\div\frac{1}{12}$$
$$=-\frac{11}{12}\times\frac{12}{1}$$
$$=-11$$

Gradient $= -11$

10 **a** $a = 4$

At $y = 0, x = 3$

$y = a + bx$

$0 = 4 + b(3)$

$b = -\frac{4}{3}$

The equation is $y = 4 - \frac{4}{3}x$

b $a = 9$

At $y = 0,\ x = 3$

$y = a + bx$

$0 = 9 + b(3)$

$b = -\frac{9}{3} = -3$

The equation is $y = 9 - 3x$

c $a = 2$

At $y = 0,\ x = -2$

$y = a + bx$

$0 = 2 + b(-2)$

$b = \frac{-2}{-2} = 1$

The equation is $y = 2 + x$

d $a = -3$

At $y = 0,\ \ x = -5$

$y = a + bx$

$0 = -3 + b(-5)$

$b = -\frac{3}{5}$

The equation is $y = -3 - \frac{3}{5}x$

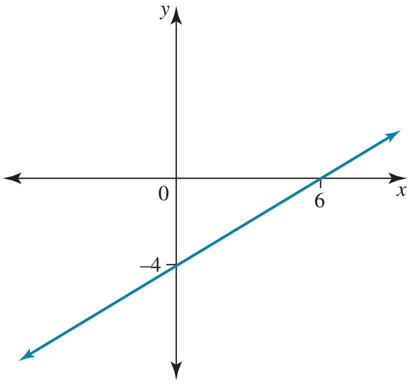
5.2 Exam questions

1 x-intercept, $y = 0$

$$2x - 3y = 12$$
$$2x - 3(0) = 12$$
$$2x = 12$$
$$x = 6 \text{ or } (6, 0)$$

y-intercept, $x = 0$

$$2x - 3y = 12$$
$$2(0) - 3y = 12$$
$$-3y = 12$$
$$y = -4 \text{ or } (0, -4)$$



The correct answer is **E**.

2 $3x - 4y + 12 = 0$

$4y = 12 + 3x$

$y = 3 + \frac{3}{4}x$

$\therefore b = \frac{3}{4}$ and $a = 3$

The correct answer is **E**.

3
$$b=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{10-2}{2-(-6)}$$
$$=\frac{8}{8}$$
$$=1$$

The correct answer is **A**.

5.3 Sketching linear graphs

5.3 Exercise

1 When $x = 0$

$y = 6 - 3(0)$

$y = 6$

When $x = 1$

$y = 6 - 3(1)$

$y = 3$

When $x = 2$

$y = 6 - 3(2)$

$y = 0$

When $x = 3$

$y = 6 - 3(3)$

$y = -3$

When $x = 4$

$y = 6 - 3(4)$

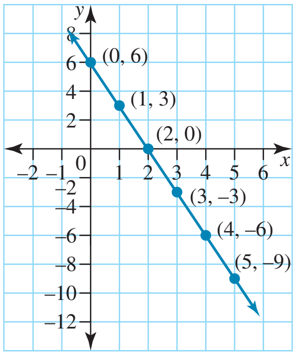
$y = -6$

When $x = 5$

$y = 6 - 3(5)$

$y = -9$

x	0	1	2	3	4	5
y	6	3	0	–3	–6	–9



2 When $x = 0$

$4x + 2y = 12$

$2x + y = 6$

$2(0) + y = 6$

$y = 6$

When $x = 1$

$4x + 2y = 12$

$2x + y = 6$

$2(1) + y = 6$

$y = 6 - 2$

$y = 4$

When $x = 2$

$4x + 2y = 12$

$2x + y = 6$

$2(2) + y = 6$

$y = 6 - 4$

$y = 2$

When $x = 3$

$4x + 2y = 12$

$2x + y = 6$

$2(3) + y = 6$

$y = 0$

When $x = 4$

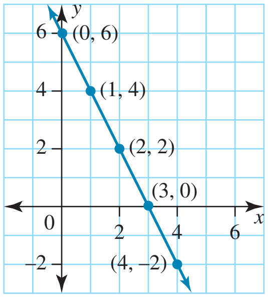
$4x + 2y = 12$

$2x + y = 6$

$2(4) + y = 6$

$y = -2$

x	0	1	2	3	4
y	6	4	2	0	–2



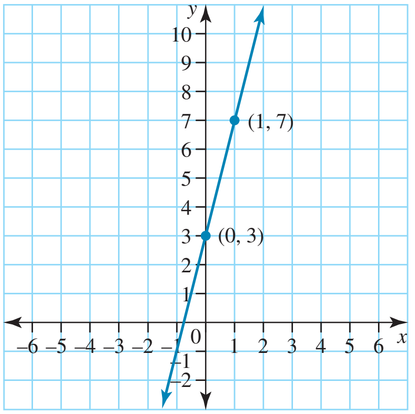
3 a Gradient = 4, y-intercept = 3

Gradient = 4 (for every increase of 1 in the x-value there is an increase of 4 in the y-value)

y-intercept = 3, x-value = 0, y-value = 3

New point: x-point = 0 + 1 = 1, y-point = 3 + 4 = 7

Point = (1, 7)



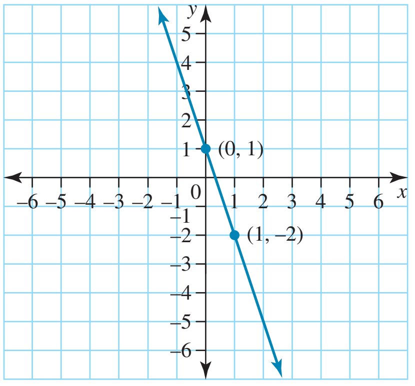
b Gradient = −3, y-intercept = 1

Gradient = −3 (for every increase of 1 in the x-value there is a decrease of 3 in the y-value)

y-intercept = 1, x-value = 0, y-value = 1

New point: x-point = 0 + 1 = 1, y-point = 1 − 3 = −2

Point = (1, −2)



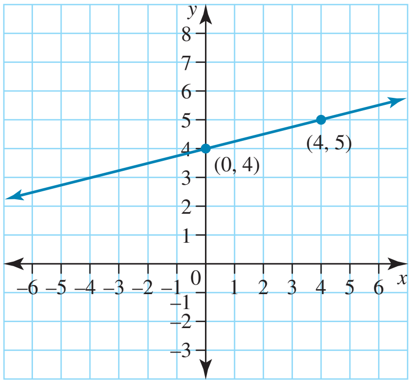
4 a Gradient = $\frac{1}{4}$, y-intercept = 4

Gradient = $\frac{1}{4}$ (for every increase of 4 in the x-value there is an increase of 1 in the y-value)

y-intercept = 4, x-value = 0, y-value = 4

New point: x-point = 0 + 4 = 4, y-point = 4 + 1 = 5

Point = (4, 5)



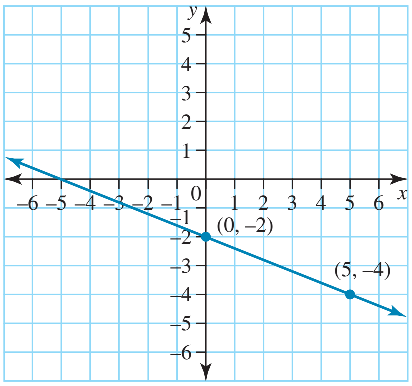
b Gradient $= \dfrac{-2}{5}$, y-intercept $= -2$

Gradient $= \dfrac{-2}{5}$ (for every increase of 5 in the x-value there is a decrease of 2 in the y-value)

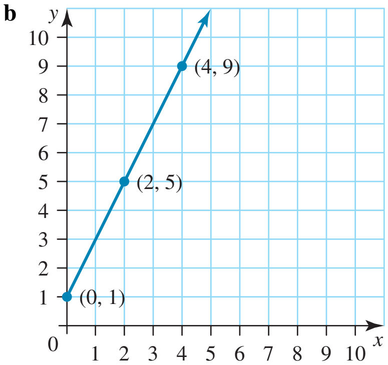
y-intercept $= -2$, x-value $= 0$, y-value $= -2$

New point: x point $= 0 + 5 = 5$, y point $= -2 - 2 = -4$

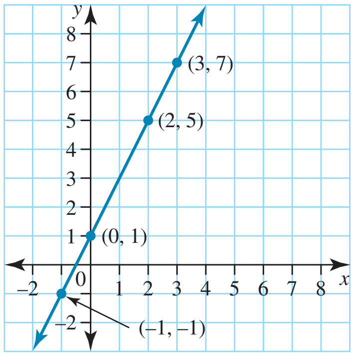
Point $= (5, -4)$



5 a, b



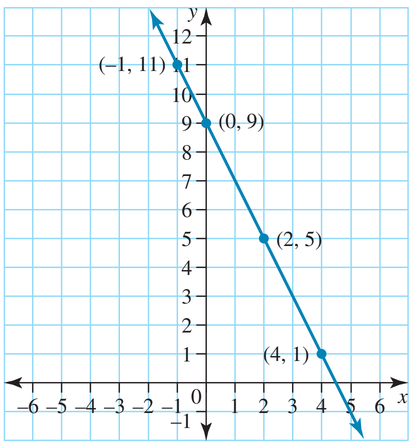
6



Since the point $(0, a)$ lies on the line, reading off the graph:

$a = 1$

7



Since the point $(4, a)$ lies on the same line, the gradient is the same between any two points $(2, 5)$, $(0, 9)$.

Rise $= 9 - 5 = 4$

Run $= 0 - 2 = -2$

Gradient $= \dfrac{4}{-2} = -2$

$(-1, 11)$, $(4, a)$

Rise $= a - 11$

Run $= 4 - (-1) = 5$

Gradient $= \dfrac{a - 11}{5} = -2$

Solve for a:

$$\frac{a - 11}{5} \times 5 = -2 \times 5$$

$$a - 11 = -10$$

$$a = 1$$

8 Rise $= 7 - b$

Run $= -1 - (-2) = 1$

Gradient $= \dfrac{7 - b}{1} = 5$

Solve for b:

$7 - b = 5$

$-b = 5 - 7$

$-b = -2$

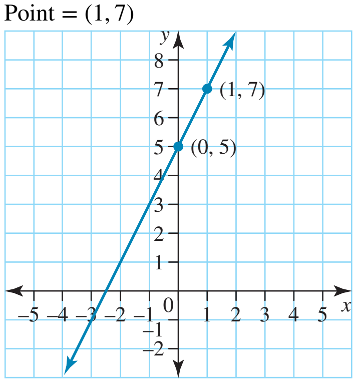
$b = 2$

9 a Gradient $= 2$ (for every increase of 1 in the x-value there is an increase of 2 in the y-value)

y-intercept $= 5$, x-value $= 0$, y-value $= 5$

New point: x point $= 0 + 1 = 1$, y point $= 5 + 2 = 7$

Point $= (1, 7)$

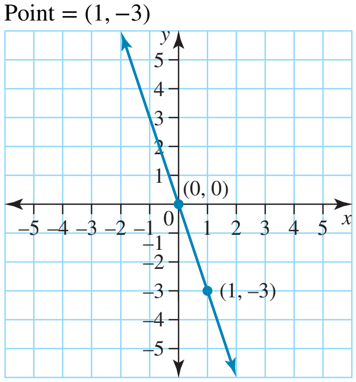


b Gradient $= -3$ (for every increase of 1 in the x-value there is a decrease of 3 in the y-value)

y-intercept $= 0$, x-value $= 0$, y-value $= 0$

New point: x point $= 0 + 1 = 1$, y point $= 0 - 3 = -3$

Point $= (1, -3)$

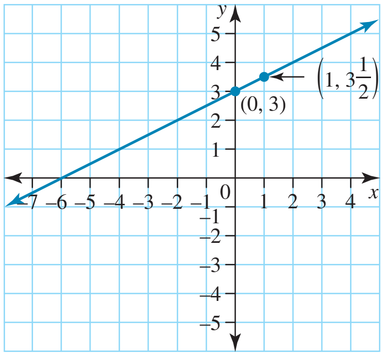


c Gradient $= \frac{1}{2}$ (for every increase of 2 in the x-value there is an increase of 1 in the y-value)

y-intercept $= 3$, x-value $= 0$, y-value $= 3$

New point: x point $= 0 + 2 = 2$, y point $= 3 + 1 = 4$

Point $= (1, 3.5)$ or $\left(1, 3\frac{1}{2}\right)$



10 a $2x + 5y = 20$

x-intercept, $y = 0$

$2x + 5(0) = 20$

$2x = 20$

$x = 10$

x-intercept: $(10, 0)$

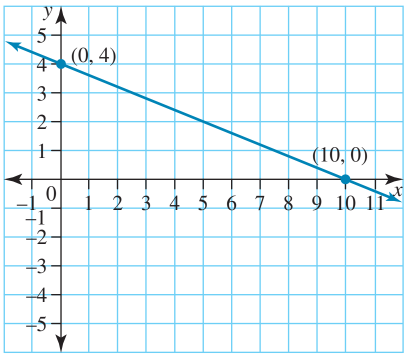
y-intercept, $x = 0$

$2(0) + 5y = 20$

$5y = 20$

$y = 4$

y-intercept: $(0, 4)$



b $y = 4 + 2x$

In form $y = a + bx$

y-intercept $= 4\,(0, 4)$

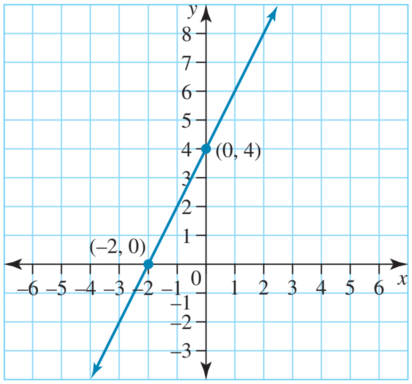
x-intercept, $y = 0$

$0 = 4 + 2x$

$-4 = 2x$

$x = -2$

x-intercept: $(-2, 0)$



c $4y = 5 + 3x$

x-intercept, $y = 0$

$0 = 5 + 3x$

$-5 = 3x$

$x = -\frac{5}{3}$

x-intercept: $\left(-\frac{5}{3}, 0\right)$

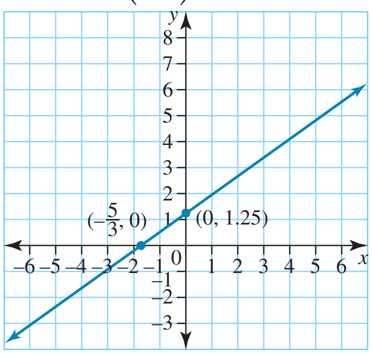
y-intercept, $x = 0$

$4y = 5 + 3\,(0)$

$4y = 5$

$y = \frac{5}{4}$

y-intercept: $\left(0, \frac{5}{4}\right)$



11 a $2x + y = 6$

x-intercept, $y = 0$

$2x + 0 = 6$

$2x = 6$

$x = 3$

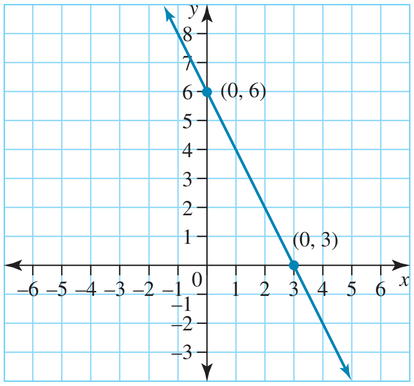
x-intercept: $(3, 0)$

y-intercept, $x = 0$

$2\,(0) + y = 6$

$y = 6$

y-intercept: $(0, 6)$



b $y = 9 + 3x$

In form $y = a + bx$

y-intercept $= 9 (0, 9)$

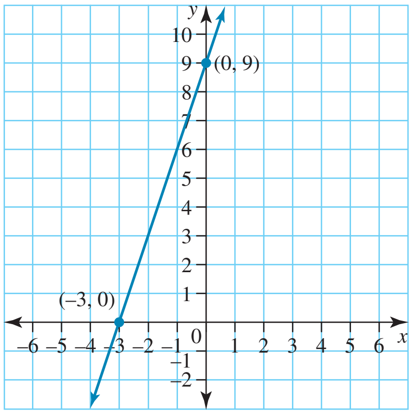
x-intercept, $y = 0$

$0 = 9 + 3x$

$-9 = 3x$

$x = -3$

x-intercept: $(-3, 0)$



c $2y = 4 + 3x$

x – intercept, $y = 0$

$0 = 4 + 3x$

$-4 = 3x$

$x = -\frac{4}{3}$

x-intercept: $\left(-\frac{4}{3}, 0\right)$

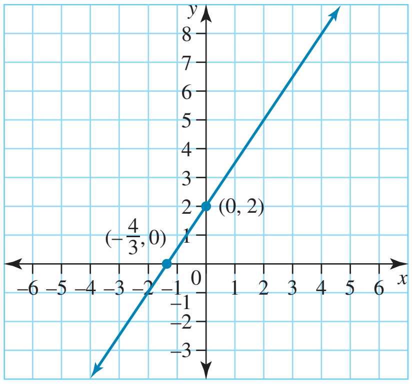
y-intercept, $x = 0$

$2y = 4 + 3(0)$

$2y = 4$

$y = \frac{4}{2} = 2$

y-intercept: $(0, 2)$



d $3y - 4 = 5x$

x-intercept, $y = 0$

$-4 = 5x$

$x = -\frac{4}{5}$

x-intercept: $\left(-\frac{4}{5}, 0\right)$

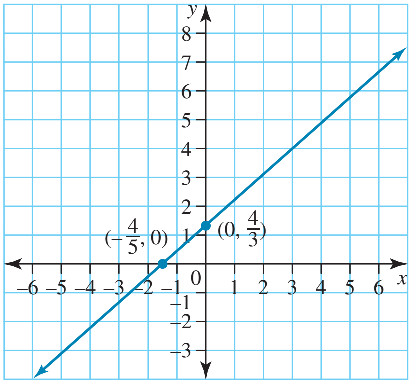
y-intercept, $x = 0$

$3y - 4 = 0$

$3y = 4$

$y = \frac{4}{3}$

y-intercept: $\left(0, \frac{4}{3}\right)$



12 a The y-intercept is the number separate from the x (the constant) and the x-intercept is equal to $\frac{-y\text{-intercept}}{\text{gradient}}$.

This method only works when the equation is in the form $y = a + bx$.

b y-intercept $= a$, x-intercept $= \frac{-a}{b}$

c x-intercept $= \frac{-y\text{-intercept}}{\text{gradient}}$ $\frac{-4}{5}$, therefore, y-intercept (a value) $= 4$. Gradient (b) $= 5$

Therefore, $y = 4 + 5x$

5.3 Exam questions

1 The equation of a straight line is $y = mx + c$, where in this case the y-intercept is $c = q$.

$m = \frac{0 - 8}{-10 - -6} = \frac{-8}{-4} = 2$

$y = 2x + q$

Substitute in the coordinate $(-10, 0)$ to calculate the value of q.

$0 = 2(-10) + q$

$q = 20$

Or using only gradients:

$$\frac{q-8}{0-(-6)} = \frac{8-0}{-6-(-10)}$$

$$\frac{q-8}{6} = \frac{8}{4}$$

$q - 8 = 12$

$q = 20$

The correct answer is **D**.

2 First, determine the equation of the straight line.

The gradient is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-6}{8-6} = \frac{3}{2}$.

So $y = a + \frac{3}{2}x$.

Substitute in one of the points to find the value of a, for example (6, 6).

$6 = a + \frac{3}{2} \times 6$

$6 = a + 9$

$a = 6 - 9$

$a = -3$

$\therefore y = -3 + \frac{3}{2}x$

The x-intercept is when $y = 0$:

$0 = -3 + \frac{3}{2}x$

$x = 2$

The correct option is **D**.

3 **a** As $y = 4 + \frac{1}{2}x$ is in the form $y = a + bx$:

$b = \frac{1}{2}$

$a = 4$ [1 mark]

b x – intercept ($y = 0$)

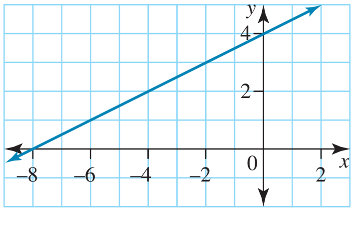
$y = \frac{1}{2}x + 4$

$0 = \frac{1}{2}x + 4$

$\frac{1}{2}x = -4$

$x = -8$ [1 mark]

c



[1 mark]

5.4 Linear modelling

5.4 Exercise

1 Constant change = 65, starting point = 90, $C = 90 + 65t$.

2 Constant change = −250 (leaking), starting point = 125 000
$A = 125\,000 - 250t$.

3 **a** Both variables in the equation have a power of 1.

b y-intercept = 5; this represents the amount of water initially in the pool.

c y-intercept = 5, (0, 5) using y-intercept-gradient method to sketch

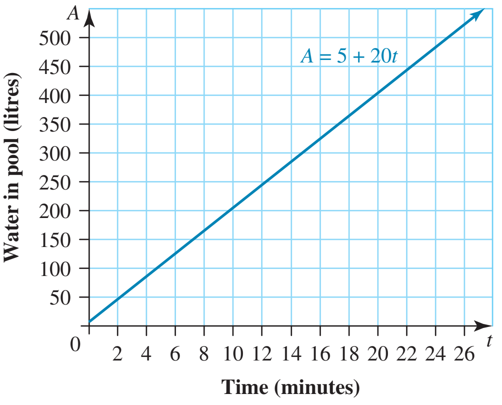
Gradient = 20

Identify another point:

x-value = 0 + 1 = 1

y-value = 5 + 20 = 25

New point (1, 25)



d $A = 5 + 20t, A = 500$

$500 = 5 + 20t$

Solve for t.

$495 = 20t$

$t = 24.75$

Correct to the nearest minute = 25 minutes.

4 **a** Constant change = 40, starting point = 100

$A = 100 + 40t$

b How much air was initially in the ball

c $t = 120$ (2 mins), $A = 100 + 40(120)$

$A = 4900\text{ cm}^3$

d $A = 100\,000$

$100\,000 = 100 + 40t$

Solve for t.

$100\,000 - 100 = 40t$

$99\,900 = 40t$

$$\frac{99\,900}{40} = \frac{40t}{40}$$

$t = 2497.7s$

$$t = \frac{2497.7}{60}$$

41 minutes, 38 seconds

5 **a** a = constant change = 12 km/h Therefore, $a = 12$

b y-intercept = −0.5. This means that the runner starts 0.5 km before the starting point of the race.

c $d = 0.5 + 12t$

$t = 0.5$ (30 minutes = 0.5 hour)

$d = -0.5 + 12(0.5)$

$d = 5.5$ km

d $21 = -0.5 + 12t$

Solve for t.

$21.5 = 12t$

$$\frac{21.5}{12} = \frac{12t}{12}$$

$t = 1.792h$

$0.792 \times 60 = 47.5$

Correct to the nearest minute = 1 hour, 48 minutes.

6 a (1, 2) (4, 6) calculating the change in water height

Rise = 6 − 2 = 4

Run = 4 − 1 = 3

Gradient $= \frac{4}{3}$

b The increase in the height of the water each minute

c Take the point (1, 2); y-intercept occurs at $(0, a)$

Rise = $2 - a$

Run = 1 − 0

$$\frac{2-a}{1} = \frac{4}{3}$$

$$-a = \frac{4}{3} - 2$$

$$-a = -\frac{2}{3}$$

$$a = \frac{2}{3}$$

$$a = 0.667$$

Correct to 2 decimal places, y-intercept = 0.67

d No, the y-intercept calculated in part **c** is not 0, so there was water in the tank to start with.

7 a Constant change = 40, $b = 40$

Starting point = 120, $a = 120$

b The amount of money in the family account at the start of the year

c $A = 120 + 40t$

$A = 120 + 40t$

$A = 3000$

$3000 = 120 + 40t$

Solve for t.

$2880 = 40t$

$t = 72$ weeks

8 a Constant change = 19.20, starting point = 0

P = Julie's weekly pay, t = hours worked

$P = 19.2t$

Domain: she works minimum 10 hours ($t \geq 10$), maximum 20 hours ($t \leq 20$)

$10 \leq t \leq 20$

b Constant change = −4 (4 less marks for each incorrect question), starting point = 100

Let R = results, e = number of errors (incorrect questions)

$R = 100 - 4e$

Domain: minimum number of errors = 0 $e \geq 0$, maximum number of errors = 15, $e \leq 15$ (lowest possible score = 40)

Therefore −60 marks, 60 ÷ 4 = 15 incorrect questions.

$0 \leq e \leq 15$

9 a Constant change = 15, $P = 15t$

b The additional amount of petrol in the tank each minute

c $75 = 15t$

Solve for t.

$t = 5$ minutes

d Starting point = 15, same constant change, $P = 15 + 15t$

e Tank is full after 5 minutes; therefore, minimum time to fill = 0, maximum time = 15, $0 \leq t \leq 5$

10 a When $t = 0$ Gert has yet to leave for work, so the distance he needs to travel to work is 37 km (37 − 22(0)).

b The distance to Gert's home is reducing as time passes.

c $d = 37 - 22t$, $d = 0$ (he has arrived at his destination)

$0 = 37 - 22t$

Solve for t.

$-37 = -22t$

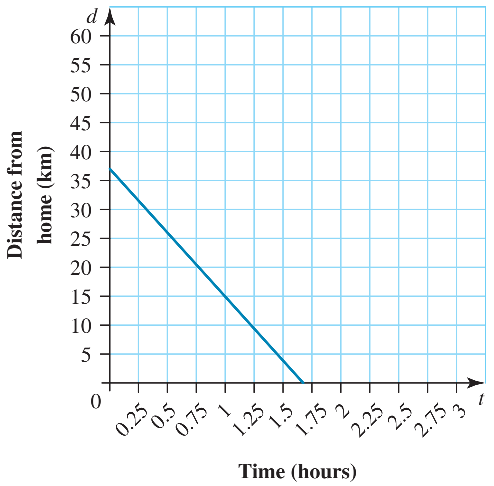
$t = 1.682$ hours

$(0.682 \times 60 = 40.9$ minutes)

Correct to the nearest minute 1 hour, 41 minutes

d $0 \leq t \leq 1.682$

e



11 a Commission (constant change) = 1.5% = 0.015, $b = 0.015$, Base salary (starting point) = 800, $a = 800$.

b No, there is no limit to how much the real estate agent can earn in a month.

c $W = 800 + 0.015x$

$x = 452\,000$

$W = 800 + 0.015(452\,000)$

$W = \$7580$

d $10\,582.10 = 800 + 0.015x$

Solve for x.

$9782.10 = 0.015x$

$x = \$652\,140$

12 a Constant change = 3.5, starting point = 0

Let C = cost of t T-shirts

$C = 3.5t$

Domain: she needs a minimum of 100 T-shirts $t \geq 100$, maximum number of T-shirts 1000, $t \leq 1000$

Domain = $100 \leq t \leq 1000$

b The domain represents the number of T-shirts Monique can buy.

c There is an upper limit as the deal is valid only up to 1000 T-shirts.

13 a $C = 15 + 0.13t$, where C is the call cost, in dollars, for any time t minutes spent on the phone.

b The gradient represents the cost per minute and the y-intercept represents the flat fee.

Time	$C = 15 + 0.13t$	Cost($)	Time	$C = 15 + 0.13t$	Cost($)
5	$15 + 0.13(5)$	15.65	35	$15 + 0.13(35)$	19.55
10	$15 + 0.13(10)$	16.30	40	$15 + 0.13(40)$	20.20
15	$15 + 0.13(15)$	16.95	45	$15 + 0.13(45)$	20.85
20	$15 + 0.13(20)$	17.60	50	$15 + 0.13(50)$	21.50
25	$15 + 0.13(25)$	18.25	55	$15 + 0.13(55)$	22.15
30	$15 + 0.13(30)$	18.90	60	$15 + 0.13(60)$	22.80

5.4 Exam questions

1 Steve charges his clients a fixed fee = \$500 + \$250 per hour

$$C = 500 + 250t$$

The correct answer is **D**.

2

$h = 70.2 + 6.5a$
$161.2 = 70.2 + 6.5a$
$91 = 6.5a$
$a = 14$ years

The correct answer is **C**.

3 Looking at the alternatives:

$d = 50t$ distance travelled, not how far away
$d + 15 = 50t$ has the distance away increasing
$d + 50t + 15 = 0$ is not transposed correctly
$d = 50t - 15$ gives negative distance

The correct answer is **D**.

5.5 Determining equations of straight lines

5.5 Exercise

1 a $a = -3$
$b = 2$
$y = -3 + 2x$

b $a = 4$
$b = 1$
$y = 4 + x$

c $a = 1$
$b = -3$
$y = 1 - 3x$

d $a = -1$
$b = -2$
$y = -1 - 2x$

2 a $b = 1$
$y = a + x$
At $(3, 7)$,
$7 = a + 3$
$a = 4$
$y = 4 + x$

b $b = 3$
$y = a + 3x$
At $(3, 3)$
$3 = a + 3(3)$
$= 3 - 9 = -6$
$y = -6 + 3x$

c $b = -2$
$y = a - 2x$
At $(-4, 10)$
$y = a - 2x$
$10 = a - 2(-4)$
$a = 10 - 8 = 2$
$y = 2 - 2x$

d $b = \frac{1}{2}$
$y = a + \frac{1}{2}x$
At $(8, 0)$
$0 = a + 8 \times \frac{1}{2}$
$a = -\frac{8}{2} = -4$
$y = -4 + \frac{1}{2}x$

3 a $b = \frac{6 - 4}{1 - -3} = \frac{2}{4} = \frac{1}{2}$
$y = a + \frac{1}{2}x$
$6 = a + \frac{1}{2}(1)$
$a = \frac{11}{2}$
$y = \frac{11}{2} + \frac{1}{2}x$

b $b = \frac{7 - 7}{0 - 3} = 0$
$y = a + 0$
$a = 7$
$y = 7$

c $b = \frac{14 - 8}{5 - 2} = \frac{6}{3} = 2$
$y = a + 2x$
$8 = a + 2(2)$
$a = 4$
$y = 4 + 2x$

d $b = \frac{10 - -6}{-3 - 1} = \frac{16}{-4} = -4$
$y = a - 4x$
$-6 = a - 4(6)$
$a = 3(6) = 18$
$y = 18 - 4x$

4 Calculate the gradient using points (3, 8) and (12, 35).

$b = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{35 - 8}{12 - 3}$

$= \dfrac{27}{9}$

$= 3$

Substitute into $y = a + bx$, $y = a + 3x$.

Substitute either point into $y = a + 3x$.

$(3, 8), 8 = a + 3(3)$

$a = -9 + 8$

$a = -1$

$y = -1 + 3x$

The correct option is **C**.

5 **a** The increase in price of 4 cents for every additional person the venue holds.

b The price of a ticket if a venue has no capacity.

c No, as the smallest venues would still have some capacity.

6 (5, 3) (9, 5)

$b = \dfrac{5-3}{9-5} = \dfrac{2}{4} = \dfrac{1}{2}$

$y = a + \dfrac{1}{2}x$

$3 = a + \dfrac{1}{2}(5)$

$a = \dfrac{1}{2}$

$y = \dfrac{1}{2} + \dfrac{1}{2}x$

$= 0.5 + 0.5x$

7 $a = 50\,000$

$b = 500$

$V = 50\,000 + 500t$

8 **a** $a = 110$

$b = 15$

$C = 110 + 15n$

b $C = 110 + 15(12)$

$C = 290$

It would cost \$290 for Tommy to invite 12 friends.

9 **a** (1, 1.5) (3, 4)

$b = \dfrac{4 - 1.5}{3 - 1} = \dfrac{2.5}{2} = 1.25$

$H = a + 1.25t$

$4 = a + 1.25(3)$

$a = 0.25$

$H = 0.25 + 1.25t$

b $a = 0.25$

The tree was 25 cm tall when it was first planted.

c $H = 0.25 + 1.25(5)$

$H = 6.5$

The tree is predicted to be 6.5 m tall at the start of the fifth year.

10 **a** $a = 9.5$

$b = 0.5$

$C = 9.5 + 0.5x$

b $200 = 9.5 + 0.5x$

$190.5 = 0.5x$

$x = 190.5 \times 2$

$x = 381$

The cab had travelled 381 km before Snoozy woke up.

5.5 Exam questions

1 $b = \dfrac{0-2}{4-0} = \dfrac{-2}{4} = -\dfrac{1}{2}$

The correct answer is **C**.

2 $a = 2$

$b = \dfrac{\text{rise}}{\text{run}} = \dfrac{2}{2} = 1$

$y = a + bx$

$y = 2 + x$

The correct answer is **B**.

3 (10, 40)

(15, 55)

$b = \dfrac{55 - 40}{15 - 10} = \dfrac{15}{5} = 3$

The correct answer is **C**.

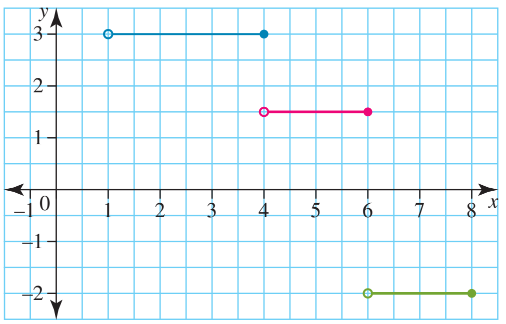
5.6 Piecewise linear graphs and their application

5.6 Exercise

1 $y = 3$, $1 < x \le 4$ open end at $x = 1$ and closed end point at $x = 4$

$y = 1.5$, $4 < x \le 6$ open end at $x = 4$ and closed end point at $x = 6$

$y = -2$, $6 < x \le 8$ open end at $x = 6$ and closed end point at $x = 8$



2 $y = 1$, $1 \le x \le 1$ both end points are closed therefore, inequality needs to include the ‘equal’ sign, $y = 2.5$, $1 < x < 2$; both end points are open therefore, inequality does not include ‘equal’, $y = 3$, $2 \le x \le 4$, both end points are closed therefore, inequality needs to include the ‘equal’ sign.

3 **a** $y = -3 - 3x$, $x \le a$

$y = 1 + x$, $x \ge a$

$-3 - 3x = 1 + x$

$-3 = 1 + 4x$

$-4 = 4x$

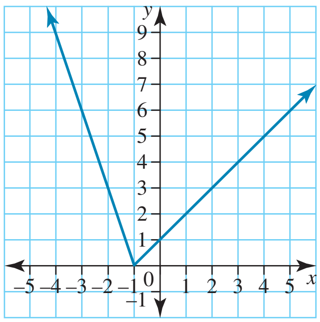
$x = -1$; substitute into either equation:

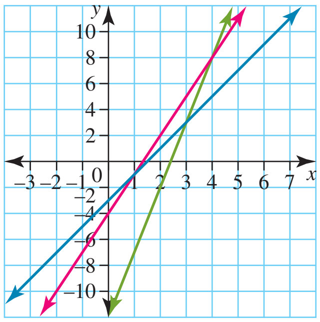
$y = 1 + x$

$y = 1 - 1$

$y = 0$

Point of intersection $= (-1, 0)$; therefore, $a = -1$.

b 

4 a 

b $y = -3 + 2x,\ x \le a,$

$y = -4 + 3x,\ a \le x \le b$

$y = -12 + 5x,\ x \ge b$

Determine the point of intersection between pairs of equations:

$y = -3 + 2x,\ x \le a$

$y = -4 + 3x,\ a \le x \le b$

$-3 + 2x = -4 + 3x$

$-3 = -4 + x$

$1 = x \Rightarrow a = 1$

Substitute into $y = -3 + 2x$, $y = -3 + 2(1) = -1$

$y = -4 + 3x,\ a \le x \le b$

$y = -12 + 5x,\ x \ge b$

$-4 + 3x = -12 + 5x$

$-4 = -12 + 2x$

$8 = 2x$

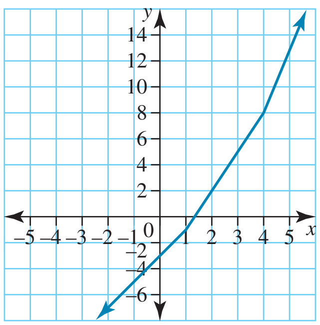
$x = 4 \Rightarrow b = 4$

Substitute into either equation, $y = -4 + 3x$:

$y = -4 + 3(4) = 8$

Points of intersection: $(1, -1)$ and $(4, 8)$

c Refer to part *b* for solutions $a = 1$ and $b = 4$

d 

5 a $t = 5$; can use either equation.

Equation 1: $w = 25t,\ 0 \le t \le 5$

$w = 25(5)$

$w = 125$

Equation 2: $w = -25 + 30t,\ 5 \le t \le 15$

$w = -25 + 30(5)$

$w = 125$

$w = 125\,\text{L}$

b i After $t = 5$, water levels follow equation 2:

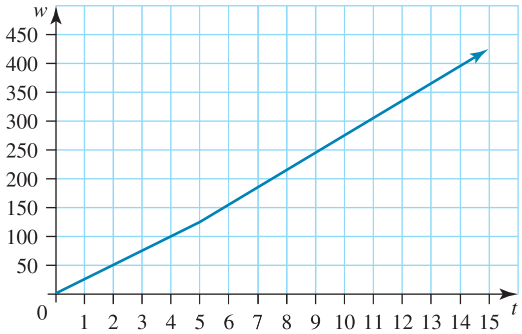
$w = -25 + 30t,\ 5 \le t \le 15$

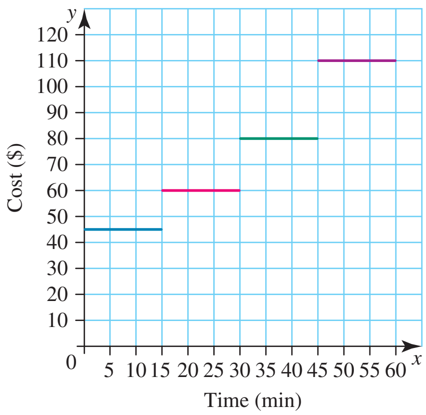
Rate = gradient = 30

Rate = 30 L/h

ii Looking at the domain $5 \le t \le 15$: $15 - 5 = 10$ hours

c Point of intersection occurs at $t = 5$ and $w = 125$ (from part **a**)



6 a 

b 23 minutes lies within the time interval 15–30; therefore, the charge is \$60.

7 a Reading off the graph, the cost for 31 kg is \$65.

b Reading off the vertical axis (\$40), this corresponds to the luggage weights 20–30; therefore, the maximum excess is 10 kg.

c 32 kg charge = \$65, 25 kg charge = \$40, total = \$105

Place 3 kg from the 32 kg bag in the 25 kg bag.

$32 - 3 = 29$ kg $25 + 3 = 28$ kg; charge for each is \$40;

total = \$80

8 a $C = 50 + ak, 0 \le k \le b,$

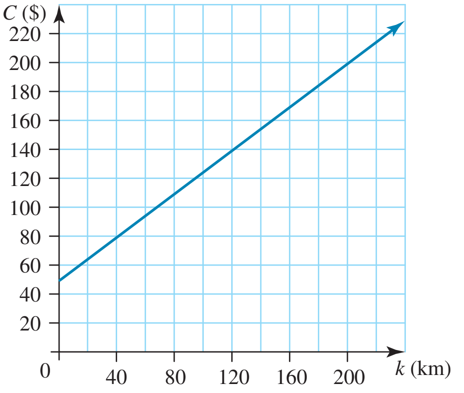
75 c/km is the gradient (constant change); therefore, $a = 0.75$.

This only applies per kilometre up to and including 150; therefore, $b = 150$.

b Starting at $C = 50$ (y-intercept) add the gradient to determine the next point to sketch.

x-value $= 0 + 1 = 1$

y-value $= 50 + 0.75 = 50.75$; new point $(1, 50.75)$



c Reading gradient from equation: $0.5 = 50$ cents/km

d $50 + 0.75k = 87.50 + 0.5k$

$50 + 0.25k = 87.50$

$0.25k = 37.50$

$k = 150$

Substitute into either equation: $C = 87.50 + 0.5k$

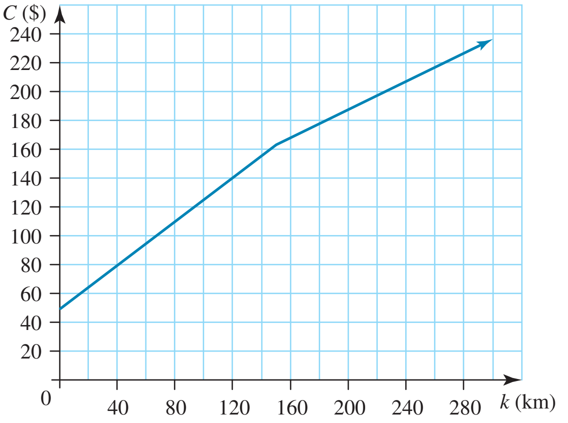
$C = 87.50 + 0.5(150)$

$C = 162.5$

$k = 150,\ C = 162.50$

This means that the point of intersection (150, 162.5) is the point where the charges change, and at this point both equations will have the same value and therefore the graph will be continuous.

e



9 Graph from $0 \leq x < 3$

Identify two points: (0, 0) and (3, 3)

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{3-0}$$

$= 1$

Goes through origin, so $a = 0$

$y = x$

Graph from $3 \leq x < 9$

Identify two points: (3, 3) and (9, 0)

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{9-3}$$

$= -\frac{3}{6}$

$= -\frac{1}{2}$

$b = -0.5$

$y = a + bx$

$y = a - 0.5x$

at $(3, 3)$

$3 = a - 0.5(3)$

$a = 3 + 1.5$

$a = 4.5$

$y = 4.5 + 0.5\,x$

10 a At $t = 1$, we use $V = 90\,t$

$V = 90(1)$

$V = 90$

At minute 1, the train was moving at 90 km/h

b At $t = 4$, we use $V = 270$

$V = 270$

At minute 4, the train was moving at 270 km/h

c At $t = 8$, we use $V = 270$

$V = 270$

At minute 8, the train was moving at 270 km/h

d At $t = 12$, we use $V = 1260 - 90\,t$

$V = 1260 - 90(12)$

$V = 180$

At minute 12, the train was moving at 180 km/h

11 Read off the graph.

At $t = 1$, $v = 2.5$ m/s

At $t = 2$, $v = 5$ m/s

At $t = 5$, $v = 10$ m/s

At $t = 7$, $v = 5$ m/s

12 The growth of the plant is the gradient of the graph.

The first equation is $H = 4\,t$

The growth of the plant starts changing at the intersection of the two graphs.

$H = 4t$

$H = 5 + 3t$

$4t = 5 + 3t$

$t = 5$

The growth of the plant will slow down at the 5-month mark.

13 a Starting point $T = 18$ (0, 18), $T = 200$ in $t = 10$ (10, 200)

$x_1 = 0 \quad x_2 = 10$

$y_1 = 18 \quad y_2 = 200$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{200 - 18}{10 - 0}$$

$$= \frac{182}{10}$$

$= 18.2$

Substitute into $y = a + bx$, $T = a + 18.2t$.

Substitute either point into $T = a + 18.2t$.

$(0, 18)$ $18 = a + 18.2(0)$

$a = 18$

$T = 18 + 18.2t, 0 \leq t \leq 10$

b **i** Bread is put into the oven at $t = 10$ and cooks for 20 minutes; $t = 20 + 10$; therefore, $a = 10$, $b = 30$.

ii a is the time the oven first reaches 200 °C and b is the time at which the bread stops being cooked.

c (30, 200) (60, 60)

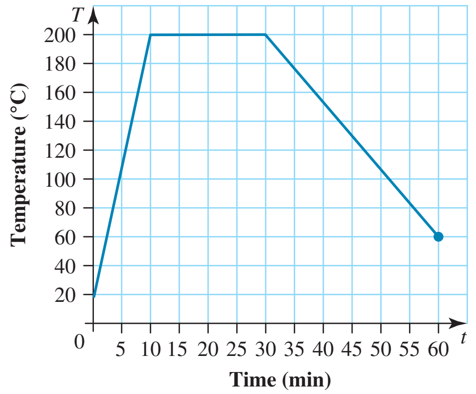
$x_1 = 30\, x_2 = 60$ (after another 30 minutes the oven reaches 60 °C)

$y_1 = 200\, y_2 = 60$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{60 - 200}{60 - 30}$$
$$= -\frac{140}{30}$$
$$= -\frac{14}{3}$$
$$= \frac{-14}{3}, d = 30, e = 60$$

d The change in temperature for each minute in the oven

e



14 **a** Reading from the graph, the first section of the piecewise graph is between 0 and 4; therefore $a = 4$.

b $A = 2000 - 150t$ and $A = b - 50t$; intersect at (4, 1400)

$2000 - 150(4) = b - 50(4)$

$1400 = b - 200$

$1600 = b$

Check: substitute $t = 4$ into $A = 1600 - 50t$

$A = 1600 - 50(4)$

$A = 1400$ (correct)

c $A = 4100 - 300t$ and $A = 1600 - 50t$

$4100 - 300t = 1600 - 50t$

$4100 = 1600 + 250t$

$2500 = 250t$

$t = 10$

Since over 12 months, the time interval is $10 \le t \le 12$.

d $t = 12$, $A = 4100 - 300t$

$A = 4100 - 300(12)$

$A = 500$

$A = \$500$

15 **a** Equation 1: $d = 20t$, $0 \le t \le 0.75$

Equation 2: $d = 3.75 + 15t$, $0.75 \le t \le 1.25$

Equation 3: $d = 37.5 - 12t$, $1.25 \le t \le b$

Solve equations 1 and 2 simultaneously.

$20t = 3.75 + 15t$

$5t = 3.75$

$t = 0.75$

Substitute into $d = 20t$:

$d = 20(0.75)$

$d = 15$

Solve equations 2 and 3 simultaneously.

$3.75 + 15t = 37.5 - 12t$

$3.75 + 27t = 37.5$

$27t = 33.75$

$t = 1.25$

Substitute into $d = 3.75 + 15t$:

$d = 3.75 + 15(1.25)$

$d = 22.5$

The points of intersection are (0.75, 15) and (1.25, 22.5).

b The yacht is returning to the yacht club during this time period.

c The point of intersection before gradient is negative, 22.5 km.

d $0 = 37.5 - 12t$, $d = 0$ since yachts have returned to the club

$-37.5 = -12t$

$t = 3.125$

$(0.125 \times 60 = 7.5)$

Correct to the nearest minute, the time is 3 hours, 8 minutes; therefore $b = 3.13$.

16 **a** Read off the graph or determine the gradient of the line using points (0, 0) and (25, 300).

Gradient $= \frac{300}{25} = 12$; this gives the water flow rate (12 litres/ min) in 45 minutes: $45 \times 12 = 540$ L.

b Gradient $= \frac{300}{25} = 12$

The flow rate is 12 L/min.

c $A = -359 + 20t$

$1500 = -359 + 20t$

$1859 = 20t$

$t = 92.95$

Correct to the nearest whole minute = 93 minutes.

17 **a** 16 000 is between 0 and 18 200, so we refer to the first equation.

$T = 0$

For an income of \$16 000, no income tax needs to be paid.

b 40 000 is between 18 200 and 45 000, so we refer to the second equation.

$T = 0.19(x - 18\,200)$

$= 0.19(40\,000 - 18\,200)$

$= \$4142$

For an income of \$40 000, \$4142 of income tax needs to be paid.

c 82 000 is between 45 000 and 120 000, so we refer to the third equation.

$T = 5092 + 0.325(x - 45\,000)$

$= 5092 + 0.325(82\,000 - 45\,000)$

$= 5092 + 0.325(37\,000)$

$= \$17\,117$

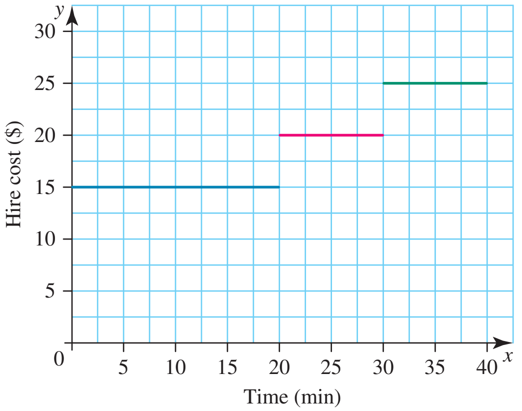
For an income of \$82 000, \$17 117 of income tax needs to be paid.

d 65 000 is between 45 000 and 120 000, so we refer to the third equation.

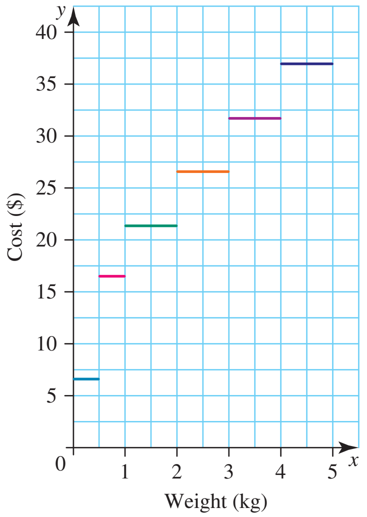
$$\begin{aligned}T &= 5092 + 0.325(x - 45\,000)\\ &= 5092 + 0.325(65\,000 - 45\,000)\\ &= 5092 + 0.325(20\,000)\\ &= \$11\,592\end{aligned}$$

For an income of \$65 000, \$11 592 of income tax needs to be paid.

18



19 **a**



b Individually: 450 g costs \$6.60, 525 g costs \$16.15; total cost = \$22.75

Together total weight = 450 + 525 = 975; costs \$16.15 to send.

It is cheaper to post them together (\$16.15 together vs. \$22.75 individually).

5.6 Exam questions

1 When $n = 200$, $n + 150 = 0.6n + p$ Therefore, $p = 230$
The correct answer is **C**.

2 Ensure that each part of the graph matches the correct part of the equation.
The correct answer is **D**.

3 Notice that the gradient of the first part of the line is negative and the second part is positive. The change in gradient occurs at $x = 2.5$.
The correct answer is **B**.

5.7 Review

5.7 Exercise

Multiple choice

1 $$\begin{aligned}b &= \frac{\text{rise}}{\text{run}}\\ &= \frac{(-6) - 6}{(-2) - 4}\\ &= \frac{-12}{-6}\\ &= 2\end{aligned}$$
The correct answer is **E**.

2 x-intercept ($y = 0$):
$3x = 6$
$x = 2$
$(2,\ 0)$
y-intercept ($x = 0$):
$-y = 6$
$y = -6$
$(0,\ -6)$
The correct answer is **D**.

3 Gradient $= \dfrac{\text{rise}}{\text{run}}$
Rise $= -1$ (descending)
Run $= 2$
Gradient $= -\dfrac{1}{2}$
The correct answer is **C**.

4 y-intercept ($x = 0$), $y = 2$ $(0,\ 2)$; therefore D is incorrect.
Gradient is 4.
Beginning at the y-intercept add the gradient (rise = 4, run = 1)
$x = 0$, new point: $x = 1$
$y = 2$, new point: $y = 2 + 4 = 6$
Look for the line that passes through $(1,\ 6)$. A is correct.
Alternatively, determine the x-intercept, $y = 0$:
$0 = 2 + 4x$
Solve for x:
$-2 = 4x$
$x = -\dfrac{1}{2}$
B has a negative gradient.
C and D have the incorrect x-intercept.
The correct answer is **A**.

5 Read off the graph.
The correct answer is **D**.

6 Read off the graph.
The correct answer is **C**.

7 Solving pairs of equations simultaneously:
$y = -5 + 3x, x \le a$
$y = -4 + 4x, a \le x \le b$
$-5 + 3x = -4 + 4x$
Solve for x:
$-3x$ both sides
$-5 = x - 4$
+4 both sides
$-1 = x$, so $a = -1$

Solving for b:
$y = -4 + 4x,\ a \le x \le b$
$y = -5 + 6x, x \ge b$
$-4 + 4x = -5 + 6x$
$-4x$ both sides
$-4 = 2x - 5$
$+5$ both sides
$1 = 2x$
$\div 2$ both sides
$x = \frac{1}{2}$; therefore $b = \frac{1}{2}$
The correct answer is **A**.

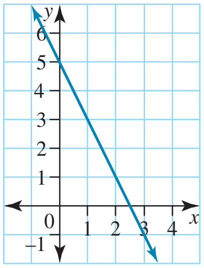
8 V = initial volume − decrease per minute
$= 1000 - 5t$
The correct answer is **D**.

9 Using equation from question **8**, note that the balloon has lost $650\,\text{cm}^3$, so there is $350\,\text{cm}^3$ in the balloon.
$350 = 1000 - 5t$
Solve for t:
-350 both sides
$0 = 650 - 5t$
$+5t$ both sides
$5t = 650$
$\div 5$ both sides
$t = 130$ minutes
The correct answer is **B**.

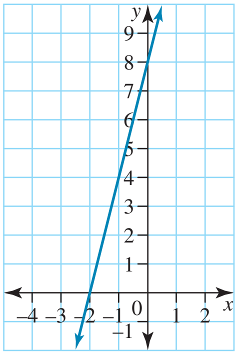
10 $b = \frac{\text{rise}}{\text{run}}$
$= \frac{1 - 3}{5 - (-2)}$
$= \frac{-2}{7}$
Substitute either point and b into $y = a + bx$
$1 = a - \frac{2}{7} \times 5$
$1 + \frac{10}{7} = a$
$a = \frac{17}{7}$
$a = 2\frac{3}{7}$
The correct answer is **A**.

Short answer

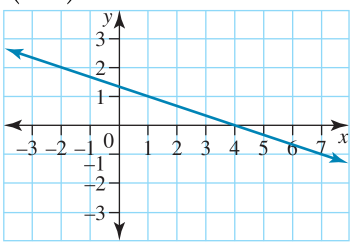
11 a $2x + y = 5$
x-intercept ($y = 0$):
$2x + 0 = 5$
$2x = 5$
$x = 2.5$
$(2.5,\ 0)$
y-intercept ($x = 0$):
$2(0) + y = 5$
$0 + y = 5$
$y = 5$
$(0,\ 5)$



b $y - 4x = 8$
x-intercept ($y = 0$):
$0 + -4x = 8$
$-4x = 8$
$x = -2$
$(-2,\ 0)$,
y-intercept ($x = 0$):
$y - 4(0) = 8$
$y - 0 = 8$
$y = 8$
$(0,\ 8)$



c $4(x + 3y) = 16$
x-intercept ($y = 0$):
$4(x + 3(0)) = 16$
$4x = 16$
$x = 4$
$(4,\ 0)$
y-intercept ($x = 0$):
$4(0 + 3y) = 16$
$4(3y) = 16$
$12y = 16$
$y = \frac{16}{12}$
$y = \frac{4}{3}$
$\left(0,\ \frac{4}{3}\right)$



d $3x + 4y - 10 = 0$
x-intercept ($y = 0$):

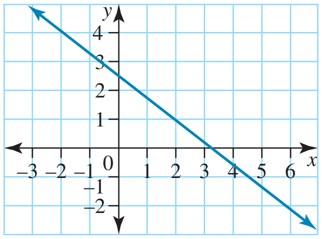
$$3x + 4(0) - 10 = 0$$
$$3x = 10$$
$$x = \frac{10}{3}$$

$\left(\frac{10}{3}, 0\right)$

y-intercept ($x = 0$):

$$3(0) + 4y - 10 = 0$$
$$4y = 10$$
$$y = 2.5$$

(0, 2.5)



12 a (3, −2) and (0, 4)

$$b = \frac{\text{rise}}{\text{run}} = \frac{4 - (-2)}{0 - 3} = \frac{6}{-3} = -2$$

b (5, 11) and (−2, 18)

$$b = \frac{\text{rise}}{\text{run}} = \frac{18 - 11}{(-2) - 5} = \frac{7}{-7} = -1$$

c (0.3, 4.1) and (1.2, 5.3)

$$b = \frac{\text{rise}}{\text{run}} = \frac{5.3 - 4.1}{1.2 - 0.3} = \frac{1.2}{0.9} = \frac{12}{9} = \frac{4}{3}$$

d $\left(\frac{2}{5}, \frac{1}{4}\right)$ and $\left(-\frac{1}{4}, \frac{3}{5}\right)$

$$b = \frac{\text{rise}}{\text{run}} = \frac{\frac{3}{5} - \frac{1}{4}}{-\frac{1}{4} - \frac{2}{5}} = \frac{\frac{12-5}{20}}{\frac{-5-8}{20}} = -\frac{7}{13}$$

13 $b = \frac{\text{rise}}{\text{run}}$

$$-\frac{3}{4} = \frac{6 - 3}{(-2) - (-a)}$$
$$-\frac{3}{4} = \frac{3}{-2 + a}$$
$$-3(-2 + a) = 4 \times 3$$
$$-2 + a = -4$$
$$a = -4 + 2$$
$$a = -2$$

14 See table bottom of the page*

15 **a** $1 - 2x = 2 - 3x$

$$\begin{aligned} 1 + x &= 2 \\ x &= 1 \end{aligned}$$

$x = 1$; therefore, $a = 1$

b Substitute $x = 1$ into $y = 1 - 2x$:

$$\begin{aligned} y &= 1 - 2(1) \\ &= 1 - 2 \\ &= -1 \end{aligned}$$

$(1, -1)$

Identify another point on the line $y = 1 - 2x$:

y-intercept = 1:

$(0, 1)$

Draw a line that passes through the points $(1, -1)$ and $(0, 1)$ and ends at the point $(1, -1)$.

Both lines pass through the point $(1, -1)$ so identify another point that lies on the line $y = 2 - 3x$.

Since line only exists for x-values ≥ 1 choose an x-value ≥ 1,

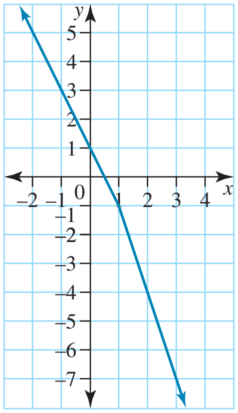
$$\begin{aligned} x = 3 : y &= 2 - 3(3) \\ &= 2 - 9 \\ &= -7 \end{aligned}$$

$(3, -7)$

14*

	Equation	**Gradient**	***y*-intercept**	***x*-intercept**
a	$y = -3 + 5x$	5	$x = 0:$ $y = -3 + 5(0)$ $= -3$	$y = 0:$ $0 = -3 + 5x$ $3 = 5x$ $x = 0.6$
b	$y = 1 + 3x$	3	$x = 0:$ $y = 1 + 3(0)$ $= 1$	$y = 0:$ $0 = 1 + 3x$ $-3x = 1$ $x = -\dfrac{1}{3}$
c	$6x - 3y = 9$	Rearrange to form $y = a + bx$ $3y = -9 + 6x$ $y = \dfrac{-9 + 6x}{3}$ $y = -3 + 2x$ Gradient = 2	$x = 0:$ $3y = -9 + 6(0)$ $y = \dfrac{-9}{3}$ $y = -3$	$y = 0:$ $0 = 6x - 9$ $9 = 6x$ $x = \dfrac{9}{6}$ $x = 1.5$
d	$2y + 4x = 8$	Rearrange to form $y = a + bx$ $2y + 4x = 8$ $2y = 8 - 4x$ $y = 4 - 2x$ Gradient = −2	$x = 0:$ $2y + 4(0) = 8$ $2y = 8$ $y = \dfrac{8}{2}$ $y = 4$	$y = 0:$ $0 + 4x = 8$ $x = \dfrac{8}{4}$ $x = 2$
e	$y = 5 + x$	$\text{Gradient} = \dfrac{y\text{-intercept}}{-x\text{-intercept}}$ $= \dfrac{5}{-(-5)}$ Gradient = 1	5	−5
f	$y = -4 + 2x$	2	$\text{Gradient} = \dfrac{y\text{-intercept}}{-x\text{-intercept}}$ $2 = \dfrac{y\text{-intercept}}{-2}$ $y\text{-intercept} = 2 \times -2$ $y\text{-intercept} = -4$	2

Draw a line that begins at the point $(1, -1)$ and passes through the point $(3, -7)$.



Extended response

16 a The power of both variables in the equation (H and A) is 1.

b $H = 0.85\,(220 - A)$

$A = 25$

$H = 0.85(220 - 25)$

$= 165.75$

$= 166$ (nearest whole number)

c Rearrange formula to the form $y = a + bx$:

$H = 0.85(220 - A)$

$= 0.85 \times 220 - 0.85 \times A$

$= 187 - 0.85A$

Gradient $= -0.85$, y-intercept $= 187$

d $A = 20$

$H = 187 - 0.85A$

$= 187 - 0.85 \times 20$

$= 170$

$(20, 170)$

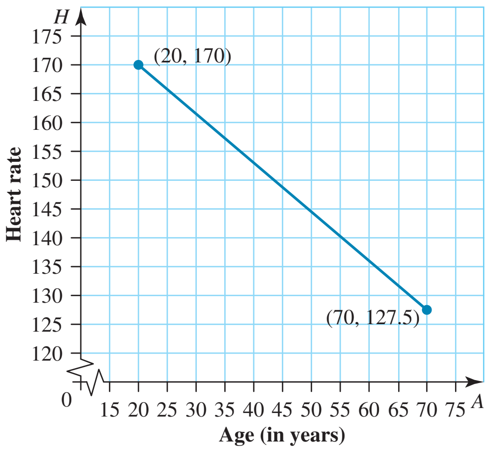
$A = 70$

$H = 187 - 0.85A$

$= 187 - 0.85 \times 70$

$= 127.5$

$(70, 127.5)$



e $H = 187 - 0.85A$

$162 = 187 - 0.85A$

$-25 = -0.85A$

$A = \dfrac{-25}{-0.85}$

$A = 29.4118$

Charlie is approximately 29 years of age.

f At the x-intercept, heart rate $= 0$, so the person would no longer be living.

17 a $d = -0.1 + 6t$

$10 = -0.1 + 6t$

$10.1 = 6t$

$t = \dfrac{10.1}{6}$

$t = 1.6833$

$t = 1$ hour, 41 minutes

b Jerri started 0.1 km (100 metres) behind the starting line.

c $T = 0.5$ (time is measured in hours)

$d = 4\,(0.5)$

$d = 2$ km

d The gradient of the equation = speed, so Samantha was travelling at 4 km/h.

e After 30 minutes, Samantha increased her speed from 4 km/h to 8 km/h.

f i $d = 10$

$10 = -2 + 8t$

$12 = 8t$

$t = 1.5$ hours; therefore, $b = 1.5$

ii Samantha took 1 hour, 30 minutes to run 10 km; Jerri took 1 hour, 41 minutes. Difference:

$41 - 30$ minutes $= 11$ minutes

g By solving a pair of simultaneous equations:

i Samantha:

$d = 4t,\ 0 \le t \le \dfrac{1}{2}$

$d = -2 + 8t,\ \dfrac{1}{2} \le t \le 1.5$

Jerri: $d = -0.1 + 6t$

After 30 minutes, Jerri is $d = -0.1 + 6\,(0.5) = 2.9$ km from the starting line; Samantha was 2 km (from part **c**), so Samantha passes Jerri between 30 and 90 minutes into the run.

Solving equations: $d = -2 + 8t$ and $d = -0.1 + 6t$

$-0.1 + 6t = -2 + 8t$

$-0.1 = 2t - 2$

$1.9 = 2t$

$t = 0.95$ hours (57 minutes)

ii $d = -2 + 8t,\ t = 0.95$

$d = -2 + 8\,(0.95)$

$d = 5.6$ km

h Jerri $d = -0.1 + 6t$; determine two points that the line passes through from previous parts.

$t = 0,\ d = -0.1\ (0, -0.1)$

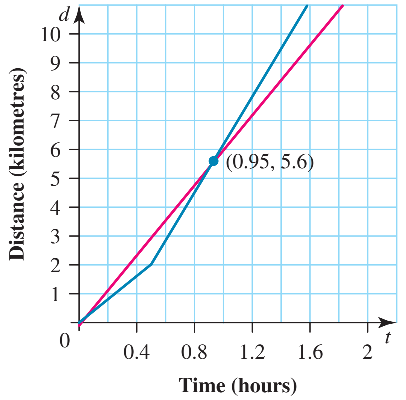
$t = 1.6833, d = 10\ (1.6833, 10)$

Draw a line that connects these two points.

Samantha:

$d = 4t,\ 0 \le t \le \dfrac{1}{2}$

Determine two points:
$t = 0, d = 0$ (0, 0)
$t = 0.5, d = 2$ (0.5, 2)
Draw a line that connects these two points.
$d = -2 + 8t, \frac{1}{2} \le t \le 1.5$
$t = 0.5,\ d = 2$ (0.5, 2)
$t = 1.5,\ d = 10$ (1.5, 10)
Draw a line that connects these two points.



18 a Leaking rate = gradient
Initial petrol = y-intercept

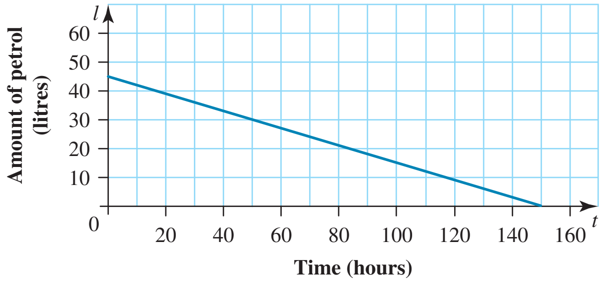
b 5 mL/ min
Convert to litres/hour:
$$\frac{\frac{5}{1000}}{60} = 0.3 \text{ L/h}$$
It is assumed that the petrol is leaking at a constant rate.

c $0.3 \times 4 = 1.2$ litres

d Initially there are 45 litres.
Lost = 45 − 39.75
Lost = 5.25 litres
$$\frac{5.25}{0.3} = 17.5 \text{ hours}$$

e i Petrol is leaking at a constant rate (gradient).
ii Petrol is leaking, so the amount of petrol is decreasing.
iii $l = 45 - 0.3t$

f



g
$$l = 45 - 0.3t$$
$$l = 0$$
$$0 = 45 - 0.3t$$
$$-45 = -0.3t$$
$$t = \frac{-45}{-0.3}$$
$$t = 150 \text{ hours}$$

5.7 Exam questions

1 Number of rooms (r) = explanatory variable
Cost (C) = response variable
(r, C) or $C = a + br$
Two points are given: (1, 25) and (6, 75).
$$b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{75 - 25}{6 - 1} = \frac{50}{5} = 10$$
Determine the value of a. Substitute in one of the two given points.
$$C = a + br$$
$$C = a + 10r$$
$$25 = a + 10(1)$$
$$25 = 10 + a$$
$$25 - 10 = a$$
$$a = 15$$
The linear relationship is $C = 15 + 10r$.
The correct answer is **C**.

2 The gradient of a vertical line is 0.
The correct answer is **E**.

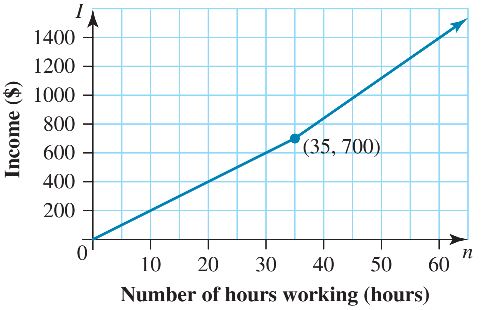
3 y-intercept is \$4.50
Gradient is determined to be \$0.50.
Distance (k) is the explanatory variable and cost (C) is the response variable. (Cost depends on the distance travelled.)
Linear model is $C = 4.50 + 0.50k$
If travelling 25 km, cost will be: $C = 4.50 + 0.50\,(25)$
$= \$17.00$
The correct answer is **A**.

4 a \$700
$n = 35$ in first equation: $35 \times \$20$ [1 mark]

b \$730
$n = 36$ in second equation or calculate $\$700 + 1 \times 30$ [1 mark]

c $I = \begin{cases} 20n & \text{if } 0 \le n \le 35 \\ -350 + 30n & \text{if } n > 35 \end{cases}$ [1 mark]

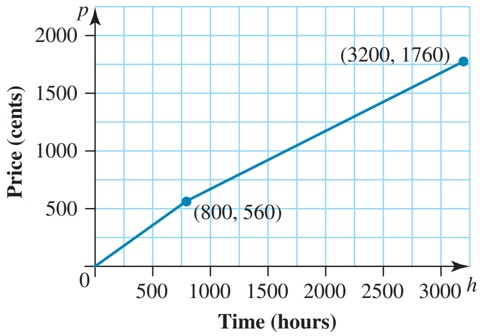
d



[1 mark]

e 45 hours
$I = 1000$ in second equation [1 mark]

5 a



[1 mark]

b 560 cents = \$5.60 [1 mark]

c 1760 cents = \$17.60

$h = 3200$ in new globes equation [1 mark]

d 1680 hours

$P = 1000$ in new globes equation [1 mark]

Topic 6 — Sequences and first-order linear recurrence relations

6.2 Arithmetic sequences

6.2 Exercise

1 **a** The 2nd term, u_1, is 8.
b The 4th term, u_3, is 12.
c The 6th term, u_5, is 16.

2 **a** The 1st term, u_0, is 2.
b The 3rd term, u_2, is 13.
c The 6th term, u_5, is 13.
d The 7th term, u_6, is 6.

3 **a** The 2nd term, u_1, is 12.
The 3rd term, u_2, is 6.
The 5th term, u_4, is 6.
b The 2nd term, u_1, is 6.
The 3rd term, u_2, is 9.
The 5th term, u_4, is 15.
c The 2nd term, u_1, is 35.
The 3rd term, u_2, is 30.
The 5th term, u_4, is 20.
d The 2nd term, u_1, is 2.0.
The 3rd term, u_2, is 2.5.
The 5th term, u_4, is 3.5.
e The 2nd term, u_1, is 32.
The 3rd term, u_2, is 16.
The 5th term, u_4, is 4.
f The 2nd term, u_1, is 2.
The 3rd term, u_2, is 4.
The 5th term, u_4, is 10.

4 The 1st term, u_0, is 5.
The 3rd term, u_2, is 9.
The 4th term, u_3, is 11.
The correct answer is **B**.

5 **a**
$$\begin{aligned}u_1 - u_0 &= 68 - 23\\ &= 45\\ u_2 - u_1 &= 113 - 68\\ &= 45\\ u_3 - u_2 &= 158 - 113\\ &= 45\\ u_4 - u_3 &= 203 - 158\\ &= 45\end{aligned}$$
The common differences are constant, so the sequence is arithmetic.
$a = 23$ and $d = 45$

b
$$\begin{aligned}u_1 - u_0 &= 8 - 3\\ &= 5\\ u_2 - u_1 &= 23 - 8\\ &= 15\\ u_3 - u_2 &= 68 - 23\\ &= 45\\ u_4 - u_3 &= 203 - 68\\ &= 135\end{aligned}$$
The common differences are not constant, so the sequence is not arithmetic.

c
$$\begin{aligned}u_1 - u_0 &= \frac{3}{4} - \frac{1}{2}\\ &= \frac{1}{4}\\ u_2 - u_1 &= 1 - \frac{3}{4}\\ &= \frac{1}{4}\\ u_3 - u_2 &= \frac{5}{4} - 1\\ &= \frac{1}{4}\\ u_4 - u_3 &= \frac{3}{2} - \frac{5}{4}\\ &= \frac{1}{4}\\ u_5 - u_4 &= \frac{7}{4} - \frac{3}{2}\\ &= \frac{1}{4}\end{aligned}$$
The common differences are constant, so the sequence is arithmetic.
$a = \frac{1}{2}$ and $d = \frac{1}{4}$

6 **a**
$$\begin{aligned}u_1 - u_0 &= -12 - 13\\ &= -25\end{aligned}$$
Therefore, $d = -25$.
The missing value f is the fourth term in the sequence.
$$\begin{aligned}u_3 - u_2 &= -25\\ f - -37 &= -25\\ f &= -62\end{aligned}$$
The missing value is -62.

b
$$\begin{aligned}u_3 - u_2 &= 12.1 - 8.9\\ &= 3.2\end{aligned}$$
Therefore, $d = 3.2$.
The missing value j is the second term in the sequence.
$$\begin{aligned}u_1 - u_0 &= 3.2\\ j - 2.5 &= 3.2\\ j &= 5.7\end{aligned}$$
The missing value k is the fifth term in the sequence.
$$\begin{aligned}u_4 - u_3 &= 3.2\\ k - 12.1 &= 3.2\\ k &= 15.3\end{aligned}$$
The missing values are 5.7 and 15.3.

c
$$\begin{aligned}u_4 - u_3 &= \frac{25}{4} - \frac{9}{2}\\ &= \frac{7}{4}\end{aligned}$$
Therefore, $d = \frac{7}{4}$.

The missing value r is the third term in the sequence.

$$u_3 - u_2 = \frac{7}{4}$$
$$\frac{9}{2} - r = \frac{7}{4}$$
$$\frac{9}{2} - \frac{7}{4} = r$$
$$r = \frac{11}{4}$$

The missing value q is the second term in the sequence.

$$u_2 - u_1 = \frac{7}{4}$$
$$\frac{11}{4} - q = \frac{7}{4}$$
$$\frac{11}{4} - \frac{7}{4} = q$$
$$q = 1$$

The missing value p is the first term in the sequence.

$$u_1 - u_0 = \frac{7}{4}$$
$$1 - p = \frac{7}{4}$$
$$1 - \frac{7}{4} = p$$
$$p = -\frac{3}{4}$$

The missing values are $p = -\frac{3}{4},\ \ q = 1$ and $r = \frac{11}{4}$.

7 **A** $u_1 - u_0 = 6 - 4$
$= 2$
$u_2 - u_1 = 8 - 6$
$= 2$
$u_3 - u_2 = 10 - 8$
$= 2$
There is a common difference, so this is arithmetic.

B $u_1 - u_0 = 3 - 1$
$= 2$
$u_2 - u_1 = 5 - 3$
$= 2$
$u_3 - u_2 = 7 - 5$
$= 2$
There is a common difference, so this is arithmetic.

C $u_1 - u_0 = 7 - 4$
$= 3$
$u_2 - u_1 = 10 - 7$
$= 3$
$u_3 - u_2 = 13 - 10$
$= 3$
There is a common difference, so this is arithmetic.

D $u_1 - u_0 = 6 - 3$
$= 3$
$u_2 - u_1 = 9 - 6$
$= 3$
$u_3 - u_2 = 12 - 9$
$= 3$
There is a common difference, so this is arithmetic.

E $u_1 - u_0 = 4 - 2$
$= 2$
$u_2 - u_1 = 8 - 4$
$= 4$
$u_3 - u_2 = 16 - 8$
$= 8$
The common difference of sequence E is not constant.
Hence, option E is not an arithmetic sequence.
The correct answer is **E.**

8 **a** $u_0 = a$
$= 2$
$d = u_1 - u_0$
$= 4 - 2$
$= 2$

b $u_0 = a$
$= 3$
$d = u_1 - u_0$
$= 7 - 3$
$= 4$

c $u_0 = a$
$= 23$
$d = u_1 - u_0$
$= 20 - 23$
$= -3$

d $u_0 = a$
$= 12$
$d = u_1 - u_0$
$= 11 - 12$
$= -1$

9 $u_1 - u_0 = 10 - 4$
$= 6$
$d = 6$
$u_4 = 22 + 6$
$u_4 = 28$
$u_5 = 28 + 6$
$u_5 = 34$
$u_6 = 34 + 6$
$u_6 = 40$
$u_7 = 40 + 6$
$u_7 = 46$
The next four terms are 28, 34, 40 and 46.

10 $a = 12, d = 5$
$u_0 = 12$
$u_1 = 12 + 5$
$= 17$
$u_2 = 17 + 5$
$= 22$
$u_3 = 22 + 5$
$= 27$
$u_4 = 27 + 5$
$= 32$
The first five terms are12, 17, 22, 27 and 32 .

11 $u_0 = a = 3$
So, we eliminate options C and E.
$d = 5$

So, there must be a common difference of 5.

A $u_0 = a = 3$
$d = u_1 - u_0$
$= 6 - 3$
$= 3$

B $u_0 = a = 3$
$d = u_1 - u_0$
$= 5 - 3$
$= 2$

C $u_0 = a = 5$
$d = u_1 - t$
$= 10 - 5$
$= 5$

D $u_0 = a = 3$
$d = u_1 - u_0$
$= 8 - 3$
$= 5$

E $u_0 = a = 5$
$d = u_1 - u_0$
$= 8 - 5$
$= 3$

The correct answer is **D.**

12 Answers will vary.
The difference between each consecutive term must be identical.
Sample response:
For the sequence 1, 10, 19, 28, 37
$u_1 - u_0 = 10 - 1$
$= 9$
$u_2 - u_1 = 19 - 10$
$= 9$
$u_3 - u_2 = 28 - 19$
$= 9$
$u_4 - u_3 = 37 - 28$
$= 9$
The common differences are constant at 9, so this is an arithmetic sequence with a common difference of 9.

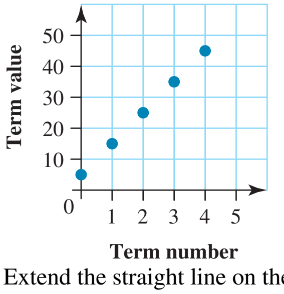
13 1 square required 4 matchsticks, so:
$a = 4$
$d = u_1 - u_0$
$= 7 - 4$
$= 3$

14 $u_1 = u_2 - d$
$= 10 - 3$
$= 7$
$u_0 = u_1 - d$
$= 7 - 3$
$= 4$

15 **a** Given $u_n = 5 + 10n$,
$u_0 = 5 + 10(0)$
$= 5$
$u_1 = 5 + 10(1)$
$= 15$
$u_2 = 5 + 10(2)$
$= 25$
$u_3 = 5 + 10(3)$
$= 35$
$u_4 = 5 + 10(4)$
$= 45$

Term number	0	1	2	3	4
Term value	5	15	25	35	45

b The points to be plotted are (0, 5), (1, 15), (2, 25), (3, 35) and (4, 45).

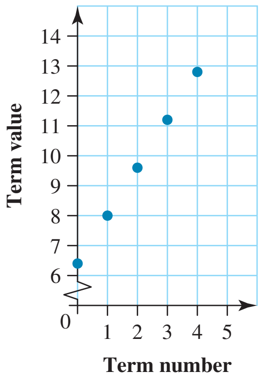


c Extend the straight line on the graph to cover future values of the sequence. Then read the value of when $n = 8$ from the graph. The 9th term of the sequence is 85.

16 **a** Given $u_n = 6.4 + 1.6n$,
$u_0 = 6.4 + 1.6(0)$
$= 6.4$
$u_1 = 6.4 + 1.6(1)$
$= 8$
$u_2 = 6.4 + 1.6(2)$
$= 9.6$
$u_3 = 6.4 + 1.6(3)$
$= 11.2$
$u_4 = 6.4 + 1.6(4)$
$= 12.8$

Term number	0	1	2	3	4
Term value	6.4	8	9.6	11.2	12.8

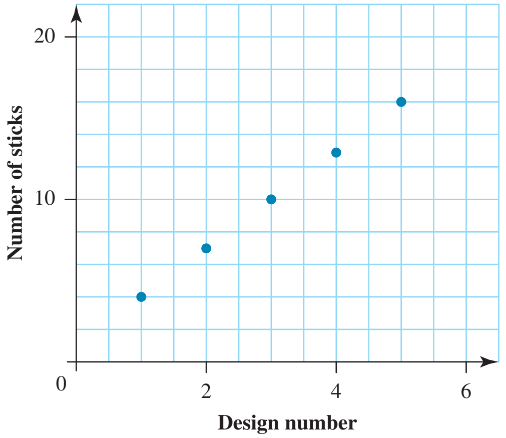
b The points to be plotted are (0, 6.4), (1, 8), (2, 9.6), (3, 11.2) and (4, 12.8).

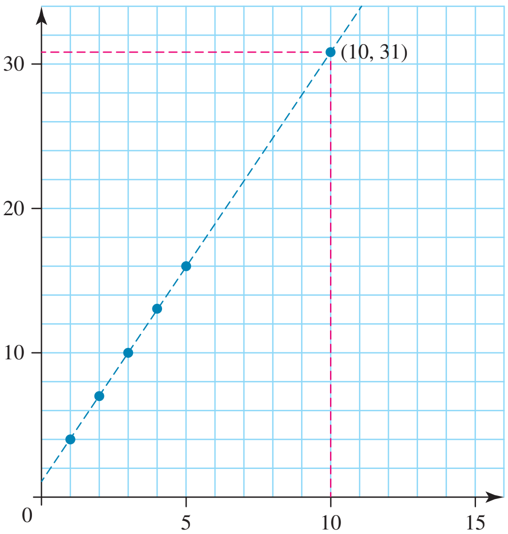


c Extend the straight line on the graph to cover future values of the sequence. Then read the value of when $n = 12$ from the graph. The13th term of the sequence is 25.6.

17

Design number	1	2	3	4	5
Number of sticks	4	7	10	13	16



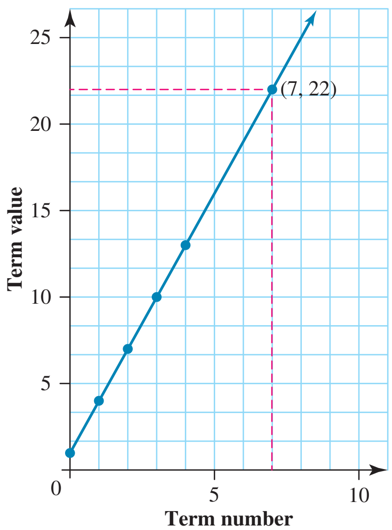


Namjoon will use 31 sticks to create the 10th design.

18 a

Term number	0	1	2	3	4
Term value	1	4	7	10	13

b

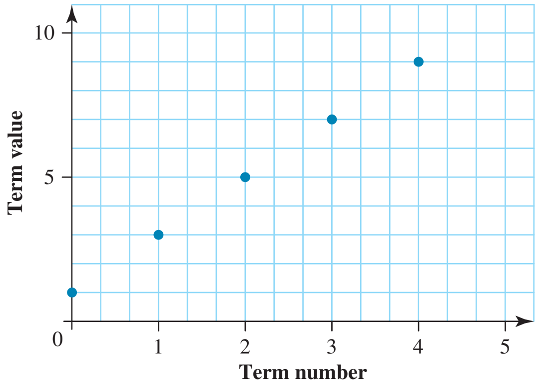


c Interpreting from the graph, the value of the 8th term is 22.

d
- It can be tedious and time consuming.
- Could lack accuracy if drawn incorrectly.
- Difficult to find the values of very high term numbers.

19 a

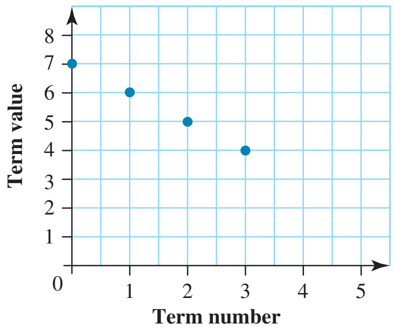
Term number	0	1	2	3	4
Term value	1	3	5	7	9



The graph is increasing in a linear pattern.

b

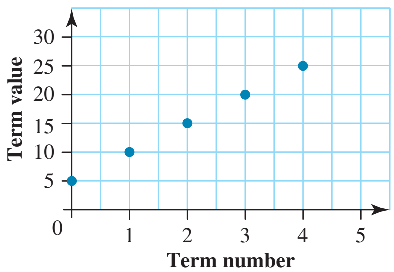
Term number	0	1	2	3
Term value	7	6	5	4



The graph is decreasing in a linear pattern.

c

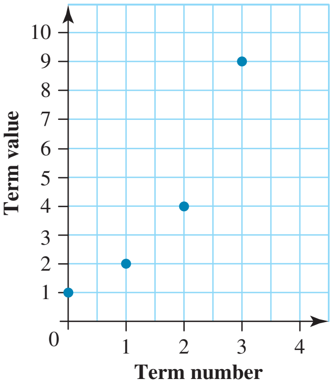
Term number	0	1	2	3	4
Term value	5	10	15	20	25



The graph is increasing in a linear pattern.

d

Term number	0	1	2	3
Term value	1	2	4	9



The graph is not in a straight line, so it is non-linear. The graph is increasing in a non-linear pattern.

20 **a** The values are increasing but not linearly. The sequence is increasing in a non-linear pattern.

b The values are decreasing linearly. The sequence is decreasing in a linear pattern.

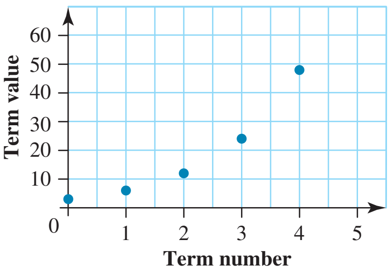
c The values do not change and stay the same throughout. The sequence has a constant value of 2.

d The values are increasing at a constant rate. The sequence is increasing in a linear pattern.

21 The graph is decreasing in a linear pattern.
The correct answer is **B**.

22

Term number	0	1	2	3	4
Term value	3	6	12	24	48



As term numbers increase, term values increase too, so the pattern of the sequence is increasing. The graph is not in a straight line, so it is non-linear.
The graph is increasing in a non-linear pattern.

6.2 Exam questions

1 As u_1 is the 2nd term of the sequence, u_1 for the given series is $-\frac{1}{4}$. [1 mark]

2 -20
$-44, -40, -36, -32, -28, -24, -20, \ldots$
The correct answer is **B**.

3 $-2, -6, -10$
The difference between the second term and the first term is -4. That is, $u_1 - u_0 = 10 - 6 = -4$.
The difference between the third term and the second term is -4. That is, $u_2 - u_1 = 2 - 6 = -4$.
According to this pattern, the difference between each term is -4.
So, the pattern is to add -4 to each term to get the next.
$10, 6, 2, -2, -6, -10$
Therefore, the next three terms are $-2, -6, -10$.
The correct answer is **E**.

6.3 Arithmetic sequence applications

6.3 Exercise

1 **a** $a = -1$

$$\begin{aligned} d &= u_1 - u_0 \\ &= 3 - (-1) \\ &= 4 \end{aligned}$$

$u_n = -1 + 4n$

b $a = 1.5$

$$\begin{aligned} d &= u_1 - u_0 \\ &= -2 - 1.5 \\ &= -3.5 \end{aligned}$$

$$\begin{aligned} u_n &= a + nd \\ &= 1.5 + n \times -3.5 \\ &= 1.5 - 3.5n \end{aligned}$$

c $a = \frac{7}{2}$

$$\begin{aligned} d &= u_1 - u_0 \\ &= \frac{11}{2} - \frac{7}{2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} u_n &= a + nd \\ &= \frac{7}{2} + n \times 2 \\ &= \frac{7}{2} + 2n \end{aligned}$$

2 **a** Given $u_n = 5 + 3n$,

$$\begin{aligned} u_0 &= 5 + 3(0) \\ &= 5 \\ u_1 &= 5 + 3(1) \\ &= 8 \\ u_2 &= 5 + 3(2) \\ &= 11 \\ u_3 &= 5 + 3(3) \\ &= 14 \\ u_4 &= 5 + 3(4) \\ &= 17 \end{aligned}$$

The first five terms of the sequence are 5, 8, 11, 14 and 17.

b Given $u_n = -1 - 7n$,

$$\begin{aligned} u_0 &= -1 - 7(0) \\ &= -1 \\ u_1 &= -1 - 7(1) \\ &= -8 \\ u_2 &= -1 - 7(2) \\ &= -15 \\ u_3 &= -1 - 7(3) \\ &= -22 \\ u_4 &= -1 - 7(4) \\ &= -29 \end{aligned}$$

The first five terms of the sequence are $-1, -8, -15, -22$ and -29.

c Given $u_n = \frac{1}{3} + \frac{2}{3}n$,

$$\begin{aligned} u_0 &= \frac{1}{3} + \frac{2}{3}(0) \\ &= \frac{1}{3} \\ u_1 &= \frac{1}{3} + \frac{2}{3}(1) \\ &= 1 \\ u_2 &= \frac{1}{3} + \frac{2}{3}(2) \\ &= \frac{5}{3} \\ u_3 &= \frac{1}{3} + \frac{2}{3}(3) \\ &= \frac{7}{3} \\ u_4 &= \frac{1}{3} + \frac{2}{3}(4) \\ &= 3 \end{aligned}$$

The first five terms of the sequence are $\frac{1}{3}, 1, \frac{5}{3}, \frac{7}{3}$ and 3.

3 a As the sequence has a common difference of -13, it is an arithmetic sequence.

$a = 85, d = -13, n = 19$

$$\begin{aligned} u_n &= a + nd \\ u_{19} &= 85 + (19)(-13) \\ &= 85 - 247 \\ &= -162 \end{aligned}$$

The 20th term of the sequence is -162.

b $d = -43, n = 69, u_{69} = 500$

$$\begin{aligned} a &= u_n - nd \\ &= 500 - (69)(-43) \\ &= 500 - -2967 \\ &= 500 + 2967 \\ &= 3467 \end{aligned}$$

The first term of the sequence is 3467.

4 a $a = -32, n = 7, u_7 = 304$

$$\begin{aligned} d &= \frac{u_n - a}{n} \\ &= \frac{304 - (-32)}{7} \\ &= \frac{336}{7} \\ &= 48 \end{aligned}$$

The common difference is 48.

b $a = 5, d = 40, u_n = 85$

$$\begin{aligned} n &= \frac{u_n - a}{d} \\ &= \frac{85 - 5}{40} \\ &= 2 \end{aligned}$$

The 3rd term in the sequence has a value of 85.

c $a = 40, d = 12, u_n = 196$

$$\begin{aligned} n &= \frac{u_n - a}{d} \\ &= \frac{196 - 40}{12} \\ &= 13 \end{aligned}$$

The 14th term in the sequence has a value of 196.

5 a $a = 6$

$$\begin{aligned} d &= u_1 - u_0 \\ &= 13 - 6 \\ &= 7 \\ u_n &= a + nd \\ &= 6 + 7n \end{aligned}$$

Given $n = 14$,

$$\begin{aligned} u_{14} &= 6 + 7(14) \\ &= 6 + 98 \\ &= 104 \end{aligned}$$

b $a = 9$

$$\begin{aligned} d &= u_1 - u_0 \\ &= 23 - 9 \\ &= 14 \\ u_n &= a + nd \\ &= 9 + 14n \end{aligned}$$

Given $n = 19$,

$$\begin{aligned} u_{19} &= 9 + 14(19) \\ &= 9 + 266 \\ &= 275 \end{aligned}$$

c $a = 56$

$$\begin{aligned} d &= u_1 - u_0 \\ &= 48 - 56 \\ &= -8 \\ u_n &= a + nd \\ &= 56 + (n - 1) \times (-8) \\ &= 56 - 8n \end{aligned}$$

Given $n = 29$,

$$\begin{aligned} u_{29} &= 56 - 8(29) \\ &= 56 - 232 \\ &= -176 \end{aligned}$$

d $a = \frac{72}{5}$

$$\begin{aligned} d &= u_1 - u_0 \\ &= \frac{551}{40} - \frac{72}{5} \\ &= -\frac{5}{8} \\ u_n &= a + nd \\ &= \frac{72}{5} + n \times \left(-\frac{5}{8}\right) \\ &= \frac{72}{5} - \frac{5}{8}n \end{aligned}$$

Given $n = 54$,

$$\begin{aligned} u_{55} &= \frac{72}{5} - \frac{5}{8}(54) \\ &= \frac{72}{5} - \frac{135}{4} \\ &= -\frac{387}{20} \end{aligned}$$

6 a $d = 6, n = 30, u_{30} = 904$

$$\begin{aligned} a &= u_n - nd \\ &= 904 - (30) \times 6 \\ &= 904 - 180 \\ &= 724 \end{aligned}$$

The first term of the sequence is 724.

b $d = \frac{2}{5}, n = 39, u_{39} = -37.2$

$$\begin{aligned} a &= u_n - nd \\ &= -37.2 - (39) \times \frac{2}{5} \\ &= -37.2 - \frac{78}{5} \\ &= -37.2 - 15.6 \\ &= -52.8 \end{aligned}$$

The first term of the sequence is -52.8.

c $a = 564, n = 50, u_{50} = 54$

$$\begin{aligned} d &= \frac{u_n - a}{n} \\ &= \frac{54 - 564}{50} \\ &= \frac{-510}{50} \\ &= -10.2 \end{aligned}$$

The common difference is -10.2.

d $a = -87, n = 60, u_{60} = 43$

$$\begin{aligned} d &= \frac{u_n - a}{n} \\ &= \frac{43 - (-87)}{60} \\ &= \frac{130}{60} \\ &= \frac{13}{6} \end{aligned}$$

The common difference is $\frac{13}{6}$.

7 **a** $a = 120, d = 16, u_n = 712$

$$\begin{aligned} n &= \frac{u_n - a}{d} \\ &= \frac{712 - 120}{16} \\ &= 37 \end{aligned}$$

The 37th term in the sequence has a value of 712.

b $a = 320, d = 4, u_n = 1160$

$$\begin{aligned} n &= \frac{u_n - a}{d} \\ &= \frac{1160 - 320}{4} \\ &= 210 \end{aligned}$$

The 210th term in the sequence has a value of 1160.

8 **a** Using the points $(2, 12)$ and $(4, 9)$,

$$\begin{aligned} d &= \frac{9 - 12}{4 - 2} \\ &= \frac{-3}{2} \\ &= -1.5 \end{aligned}$$

b Using the point $(1, 12)$,

$d = -1.5, n = 1, u_1 = 12$

$$\begin{aligned} a &= u_n - nd \\ &= 12 - (1)(-1.5) \\ &= 12 - -1.5 \\ &= 12 + 1.5 \\ &= 13.5 \end{aligned}$$

The first term of the sequence is 13.5.

c $a = 13.5$ and $d = -1.5$.

$$\begin{aligned} u_n &= a + nd \\ &= 13.5 + n(-1.5) \\ &= 13.5 - 1.5n \\ u_{11} &= 13.5 - 1.5(11) \\ &= 13.5 - 16.5 \\ &= -3 \end{aligned}$$

The 12th term in the sequence is -3.

9 $u_0 = 55$

$d = 40$

$u_n = 55 + 40n$

$u_7 = 55 + 40(7)$

$u_7 = 335$

The account would have \$335 in it after 7 weeks.

10 $u_0 = 7$

$d = 3$

$u_n = 7 + 3n$

$u_6 = 7 + 3(6)$

$u_6 = 25$

There will be 25 elephants in the herd after 6 years.

6.3 Exam questions

1 Be careful about numbering the production. If \$250 is a production of 50 and is counted as $n = 0$, then we require $n = 50$ for a production of 100. $u_{50} = ?$, but $a = u_0 = \$250$ and $d = -\$1.50$.

$$u_n = a + n \times d$$

where $u_{50} = a = 250,\ n = 50$ and $d = -\$1.50$

$$\begin{aligned} u_{50} &= 250 + (50) \times -\$1.50 \\ u_{50} &= \$175 \end{aligned}$$

The correct choice is **A**.

2 The equation (rule) of the function is of the form $y = a + bx$, where $a = -5$ and $b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{3 - 1} = 2$.

$\therefore u_n = 5 + 2n$

The correct choice is **D**.

3 $u_n = a + n \times d$

where $a = 20,\ d = 4,$ and $n = 59$

$$\begin{aligned} u_{59} &= 20 + (59)(4) \\ &= 256 \end{aligned}$$

The correct choice is **C**.

6.4 Generate and analyse an arithmetic sequence using a recurrence relation

6.4 Exercise

1
$$\begin{aligned} d &= u_1 - u_0 \\ &= -3 - 2 \\ &= -5 \\ u_0 &= 2 \end{aligned}$$

$u_{n+1} = u_n - 5, u_0 = 2$

2 Given $u_{n+1} = u_n + 3.5, u_0 = -2.2$

$$\begin{aligned} u_1 &= u_0 + 3.5 \\ &= -2.2 + 3.5 \\ &= 1.3 \end{aligned}$$

$u_2 = u_1 + 3.5$
$= 1.3 + 3.5$
$= 4.8$
$u_3 = u_2 + 3.5$
$= 4.8 + 3.5$
$= 8.3$
$u_4 = u_3 + 3.5$
$= 8.3 + 3.5$
$= 11.8$
The first five terms of the sequence are $-2.2, 1.3, 4.8, 8.3$ and 11.8.

3 $d = u_3 - u_2$
$= -11.5 - (-7)$
$= -4.5$
$u_1 = u_2 - d$
$= -7 - (-4.5)$
$= -2.5$
$u_0 = u_1 - d$
$= -2.5 - (-4.5)$
$= 2$
$u_{n+1} = u_n - 4.5, u_0 = 2$

4 a $u_0 = 1$
$d = u_1 - u_0$
$= 4 - 1$
$= 3$
$u_0 = 1,\ u_{n+1} = u_n + 3$

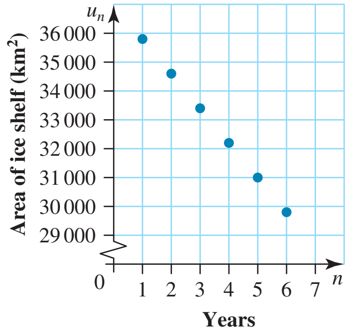
b $u_n = a + nd$
$= 1 + 3n$

c $u_{14} = 1 + 3 \times 14$
$= 43$
Aryan would have 43 chairs in his 15th stack.

5 a $d = -1200$
$a = 37\,000$
$u_0 = 37\,000,\ u_{n+1} = u_n - 1200$

b $u_0 = 37\,000,\ u_{n+1} = u_n - 1200$
$u_1 = u_0 - 1200$
$= 37\,000 - 1200$
$= 35\,800$
$u_2 = u_1 - 1200$
$= 35\,800 - 1200$
$= 34\,600$
$u_3 = u_2 - 1200$
$= 33\,400$
$u_4 = u_3 - 1200$
$= 33\,400 - 1200$
$= 32\,200$
$u_5 = u_4 - 1200$
$= 32\,200 - 1200$
$= 31\,000$
$u_6 = 31\,000 - 1200$
$u_6 = 29\,800$
The area of the ice shelf after each of the first 6 years is $35\,800$ km^2, $34\,600$ km^2, $33\,400$ km^2, $32\,200$ km^2 and $31\,000$ km^2.

c See table at the bottom of the page*
The points to be plotted are $(1, 35\,800)$, $(2, 34\,600)$, $(3, 33\,400)$, $(4, 32\,200)$, $(5, 31\,000)$ and $(6, 29\,800)$.



6.4 Exam questions

1 $u_0 = 13$, not $u_0 = -3$
$u_0 - u_0 = 9 - 13$
$= -4$ not 4
$u_{n+1} = u_n - 4,\ u_0 = 13$
The correct answer is **E**.

2 a $u_{n+1} = u_n + 4;\ u_0 = 3$ [1 mark]

b From the table, $u_0 = 3$. Since n increases by 2, to calculate the difference we need to go halfway between:
$n = 1, u_n = \frac{1}{2}(11 - 3)$
$= 4$ [1 mark]
$u_n = 4n + 3$

3 a $u_{n+1} - u_n = 9;\ u_0 = 30$ [1 mark]

b $u_n = 30 + 9n$ [1 mark]

c Since 8th strut will be u_7
$u_7 = 30 + 9 \times 7 = 93$ cm [1 mark]

6.5 Geometric sequences

6.5 Exercise

1 a $\frac{u_1}{u_0} = \frac{6}{3}$
$= 2$
$\frac{u_2}{u_1} = \frac{12}{6}$
$= 2$
$\frac{u_3}{u_2} = \frac{24}{12}$
$= 2$
$\frac{u_4}{u_3} = \frac{48}{24}$
$= 2$
The ratios between consecutive terms are constant, so this is a geometric sequence; $a = 3, R = 2$.

*5c

Year	1	2	3	4	5	6
Area (km^2)	35 800	34 600	33 400	32 200	31 000	29 800

b $\frac{u_1}{u_0}=\frac{\left(\frac{5}{4}\right)}{\left(\frac{1}{2}\right)}$
$=\frac{5}{4}\times\frac{2}{1}$
$=\frac{5}{2}$

$\frac{u_2}{u_1}=\frac{\left(\frac{25}{8}\right)}{\left(\frac{5}{4}\right)}$
$=\frac{25}{8}\times\frac{4}{5}$
$=\frac{5}{2}$

$\frac{u_3}{u_2}=\frac{\left(\frac{125}{16}\right)}{\left(\frac{25}{8}\right)}$
$=\frac{125}{16}\times\frac{8}{25}$
$=\frac{5}{2}$

The ratios between consecutive terms are constant, so this is a geometric sequence; $a=\frac{1}{2}, R=\frac{5}{2}=2\frac{1}{2}$.

c $\frac{u_1}{u_0}=\frac{6}{9}$
$=\frac{2}{3}$

$\frac{u_2}{u_1}=\frac{3}{6}$
$=\frac{1}{2}$

$\frac{u_3}{u_2}=\frac{0}{3}$
$=0$

$\frac{u_4}{u_3}=\frac{-3}{0}$
= undefined

The ratios between consecutive terms are not constant, so this is not a geometric sequence.

d $\frac{u_1}{u_0}=\frac{\left(\frac{1}{5}\right)}{\left(\frac{1}{2}\right)}$
$=\frac{1}{5}\times\frac{2}{1}$
$=\frac{2}{5}$

$\frac{u_2}{u_1}=\frac{\left(\frac{2}{25}\right)}{\left(\frac{1}{5}\right)}$
$=\frac{2}{25}\times\frac{5}{1}$
$=\frac{2}{5}$

$\frac{u_3}{u_2}=\frac{\left(\frac{4}{125}\right)}{\left(\frac{2}{25}\right)}$
$=\frac{4}{125}\times\frac{25}{2}$
$=\frac{2}{5}$

The ratios between consecutive terms are constant, so this is a geometric sequence; $a=\frac{1}{2}, R=\frac{2}{5}$.

2 a $R=\frac{u_1}{u_0}$
$=\frac{6}{1}$
$=6$

Since the ratios between consecutive terms are constant, to find the missing value c,

$\frac{u_2}{u_1}=R$
$\frac{c}{6}=6$
$c=36$

b $R=\frac{u_4}{u_3}$
$=\frac{48}{-24}$
$=-2$

Since the ratios between consecutive terms are constant, to find the missing value g,

$\frac{u_1}{u_0}=R$
$\frac{g}{3}=-2$
$g=-2\times 3$
$=-6$

To determine the missing value h,

$R=\frac{u_3}{u_2}$
$-2=\frac{-24}{h}$
$-2h=-24$
$h=\frac{-24}{-2}$
$h=12$

c $R=\frac{u_4}{u_3}$
$=\frac{1500}{300}$
$=5$

Since the ratios between consecutive terms are constant, to find the missing value s,

$s=\frac{u_3}{u_2}$
$5=\frac{300}{s}$
$5s=300$
$s=\frac{300}{5}$
$s=60$

To find the missing value q,

$$R = \frac{u_3}{u_2}$$
$$5 = \frac{60}{q}$$
$$5q = 60$$
$$q = \frac{60}{5}$$
$$q = 12$$

To find the missing value p,

$$R = \frac{u_3}{u_2}$$
$$5 = \frac{12}{p}$$
$$5p = 12$$
$$p = \frac{12}{5}$$
$$p = 2.4$$

3 a $\frac{u_1}{u_0} = \frac{15}{3}$
$= 5$

$\frac{u_2}{u_1} = \frac{75}{15}$
$= 5$

$\frac{u_3}{u_2} = \frac{375}{75}$
$= 5$

$\frac{u_4}{u_3} = \frac{1875}{375}$
$= 5$

The ratios between consecutive terms are constant, so this is a geometric sequence; first term = 3, common ratio = 5.

b $\frac{u_1}{u_0} = \frac{13}{7}$

$\frac{u_2}{u_1} = \frac{25}{13}$

$\frac{u_3}{u_2} = \frac{49}{25}$

$\frac{u_4}{u_3} = \frac{97}{49}$

The ratios between consecutive terms are not constant, so this is not a geometric sequence.

c $\frac{u_1}{u_0} = \frac{24}{-8}$
$= -3$

$\frac{u_2}{u_1} = \frac{-72}{24}$
$= -3$

$\frac{u_3}{u_2} = \frac{216}{-72}$
$= -3$

$\frac{u_4}{u_3} = \frac{-648}{216}$
$= -3$

The ratios between consecutive terms are constant, so this is a geometric sequence; first term = −8, common ratio = −3.

d $\frac{u_1}{u_0} = \frac{32}{128}$
$= \frac{1}{4}$

$\frac{u_2}{u_1} = \frac{8}{32}$
$= \frac{1}{4}$

$\frac{u_3}{u_2} = \frac{2}{8}$
$= \frac{1}{4}$

$\frac{u_4}{u_3} = \frac{\left(\frac{1}{2}\right)}{2}$
$= \frac{1}{4}$

The ratios between consecutive terms are constant, so this is a geometric sequence; first term = 128, common ratio = $\frac{1}{4}$.

e $\frac{u_1}{u_0} = \frac{6}{2}$
$= 3$

$\frac{u_2}{u_1} = \frac{12}{6}$
$= 2$

$\frac{u_3}{u_2} = \frac{20}{12}$
$= \frac{5}{3}$

$\frac{u_4}{u_3} = \frac{30}{20}$
$= \frac{3}{2}$

The ratios between consecutive terms are not constant, so this is not a geometric sequence.

f $\frac{u_1}{u_0} = \frac{3\sqrt{3}}{3}$
$= \sqrt{3}$

$\frac{u_2}{u_1} = \frac{9}{3\sqrt{3}}$
$= \frac{3}{\sqrt{3}}$
$= \sqrt{3}$

$\frac{u_3}{u_2} = \frac{9\sqrt{3}}{9}$
$= \sqrt{3}$

$\frac{u_4}{u_3} = \frac{27}{9\sqrt{3}}$
$= \frac{3}{\sqrt{3}}$
$= \sqrt{3}$

The ratios between consecutive terms are constant, so this is a geometric sequence; first term = 3, common ratio = $\sqrt{3}$.

4 $R = \frac{u_1}{u_0} = \frac{4}{2}$

$R = 2$
$u_5 = 16 \times 2$
$u_5 = 32$
$u_6 = 32 \times 2$
$u_6 = 64$

$u_7 = 64 \times 2$
$u_7 = 128$
$u_8 = 128 \times 2$
$u_8 = 256$
The next four terms are 32, 64, 128 and 256.

5 **a** Given $u_n = 64 \times \left(\frac{1}{2}\right)^n$,

$u_0 = 64 \times \left(\frac{1}{2}\right)^0$
$= 64 \times 1$
$= 64$

$u_1 = 64 \times \left(\frac{1}{2}\right)^1$
$= 64 \times \frac{1}{2}$
$= 32$

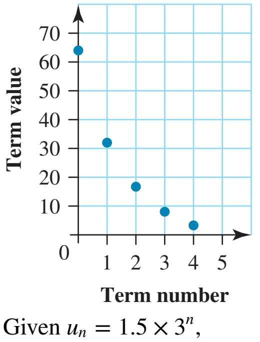
$u_2 = 64 \times \left(\frac{1}{2}\right)^2$
$= 64 \times \frac{1}{4}$
$= 16$

$u_3 = 64 \times \left(\frac{1}{2}\right)^3$
$= 64 \times \frac{1}{8}$
$= 8$

$u_4 = 64 \times \left(\frac{1}{2}\right)^4$
$= 64 \times \frac{1}{16}$
$= 4$

Term number	0	1	2	3	4
Term value	64	32	16	8	4

b The points to be plotted are (0, 64), (1, 32), (2, 16), (3, 8) and (4, 4).



6 **a** Given $u_n = 1.5 \times 3^n$,

$u_0 = 1.5 \times 3^0$
$= 1.5 \times 1$
$= 1.5$

$u_1 = 1.5 \times 3^1$
$= 1.5 \times 3$
$= 4.5$

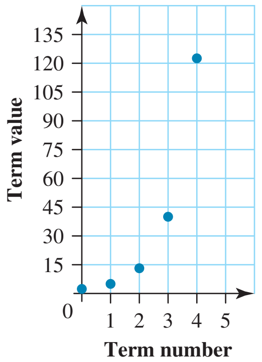
$u_2 = 1.5 \times 3^2$
$= 1.5 \times 9$
$= 13.5$

$u_3 = 1.5 \times 3^3$
$= 1.5 \times 27$
$= 40.5$

$u_4 = 1.5 \times 3^4$
$= 1.5 \times 81$
$= 121.5$

Term number	0	1	2	3	4
Term value	1.5	4.5	13.5	40.5	121.5

b The points to be plotted are (0, 1.5), (1, 4.5), (2, 13.5), (3, 40.5) and (4, 121.5).



7 $u_3 = 9, u_4 = 4.5$

$R = \frac{u_4}{u_3}$
$= \frac{4.5}{9}$
$= \frac{1}{2}$
$a = 72$

$u_n = 72 \times \left(\frac{1}{2}\right)^n$

$u_1 = 72 \times \left(\frac{1}{2}\right)^1$
$= 72 \times \frac{1}{2}$
$= 36$

$u_2 = 72 \times \left(\frac{1}{2}\right)^2$
$= 72 \times \frac{1}{4}$
$= 18$

The value of the 2nd term is 36 and the value of the 3rd term is 18.

6.5 Exam questions

1 Option A is an arithmetic sequence.
Option B is an arithmetic sequence.
Option C is an arithmetic sequence.
Option D is a geometric sequence where numbers are multiplied by 3.
Option E is a random sequence.
The correct answer is **D**.

2 $u_n = aR^n$
$u_n = 5 \times 3^n$
where $u_0 = 5$
The correct answer is **B**.

3 The sequence shows numbers being multiplied by 1.5.
$9 \times 1.5 = 13.5$
The correct answer is **B**.

6.6 Geometric sequence applications

6.6 Exercise

1 a $a = 4, R = 3, n = 14$
$$u(n) = aR^n$$
$$u_{14} = 4 \times 3^{14}$$
$$= 19\,131\,876$$

b $u_{11} = 97\,656\,250, a = 2, n = 11$
$$R = \left(\frac{u_n}{a}\right)^{\frac{1}{n}}$$
$$= \left(\frac{97\,656\,250}{2}\right)^{\frac{1}{11}}$$
$$= 48\,828\,125^{\frac{1}{11}}$$
$$= 5$$

c $u_5 = 13.125, R = -\frac{1}{2}, n = 5$
$$a = \frac{u_n}{R^n}$$
$$= \frac{13.125}{\left(-\frac{1}{2}\right)^5}$$
$$= \frac{13.125}{\left(-\frac{1}{2}\right)^5}$$
$$= \frac{13.125}{-\frac{1}{32}}$$
$$= 13.125 \times -32$$
$$= -420$$

2 a $a = 1.2, R = 4, n = 10$
$$u_n = aR^n$$
$$u_{10} = 1.2 \times 4^{10}$$
$$= 1\,258\,291.2$$

b $u_9 = 768, a = -1.5, n = 9$
$$R = \left(\frac{u_n}{a}\right)^{\frac{1}{n}}$$
$$= \left(\frac{768}{-1.5}\right)^{\frac{1}{9}}$$
$$= (-512)^{\frac{1}{9}}$$
$$= -2$$

c $u_5 = 6.5536, r = 0.4, n = 6$
$$a = \frac{u_n}{R^n}$$
$$= \frac{6.5536}{0.4^5}$$
$$= \frac{6.5536}{0.010\,24}$$
$$= 640$$

3 a $u_5 = 243, n = 5$
$$a = \frac{u_n}{R^n}$$
$$= \frac{243}{R^5} \quad \ldots [1]$$
$u_7 = 2187, n = 7$
$$a = \frac{u_n}{r^n}$$
$$= \frac{2187}{R^7} \quad \ldots [2]$$
Equate the two equations:
$$\frac{243}{R^5} = \frac{2187}{R^7}$$
$$\frac{R^7}{R^5} = \frac{2187}{243}$$
$$R^2 = 9$$
$$R = 3$$
Substitute $R = 3$ into equation [1]:
$$a = \frac{243}{3^5}$$
$$= \frac{243}{243}$$
$$= 1$$
$$u_n = 1 \times 3^n$$
$$u_0 = 1 \times 3^0$$
$$= 1 \times 1$$
$$= 1$$
$$u_1 = 1 \times 3^1$$
$$= 1 \times 3$$
$$= 3$$
$$u_2 = 1 \times 3^2$$
$$= 1 \times 9$$
$$= 9$$
$$u_3 = 1 \times 3^3$$
$$= 1 \times 27$$
$$= 27$$
The first four terms of the sequence are 1, 3, 9, and 27.

b $u_2 = 331, n = 2$
$$a = \frac{u_n}{R^n}$$
$$= \frac{331}{R^2} \quad \ldots [1]$$
$u_4 = 8275, n = 4$
$$a = \frac{u_n}{R^n}$$
$$= \frac{8275}{R^4} \quad \ldots [2]$$

Equate the two equations:

$$\frac{331}{R^3} = \frac{8275}{R^5}$$

$$\frac{R^5}{R^3} = \frac{8275}{331}$$

$$R^2 = 25$$

$$R = 5$$

Substitute $R = 5$ into equation [2]:

$$a = \frac{331}{5^2} = \frac{331}{25} = 13.24$$

$$u_n = 13.24 \times 5^n$$

$$u_0 = 13.24 \times 5^0 = 13.24 \times 1 = 13.24$$

$$u_1 = 13.24 \times 5^1 = 13.24 \times 5 = 66.2$$

$$u_3 = 13.24 \times 5^3 = 13.24 \times 125 = 1655$$

The first four terms of the sequence are 13.24, 66.2, 331 and 1655.

4 $a = 200, u_5 = 2.048, n = 5$

$$R = \left(\frac{u_n}{a}\right)^{\frac{1}{n}} = \left(\frac{2.048}{200}\right)^{\frac{1}{5}} = 0.01\,024^{\frac{1}{5}} = 0.4$$

$$u_n = a \times R^n = 200 \times 0.4^n$$

$$u_1 = 200 \times 0.4^1 = 200 \times 0.4 = 80$$

$$u_2 = 200 \times 0.4^2 = 32$$

$$u_3 = 200 \times 0.4^3 = 12.8$$

$$u_4 = 200 \times 0.4^4 = 5.12$$

The values of the 2nd, 3rd, 4th and 5th terms are 80, 32, 12.8 and 5.12 respectively.

5 **a** $a = 200, P = 4$

$$R = 1 + \frac{4}{100}$$

$R = 1.04$ or 104%

The first four terms are 200, 208, 216.32 and 224.97.

b $a = 600, P = 3$

$$R = 1 - \frac{3}{100}$$

$R = 0.97$ or 97%

The first four terms are 600, 582, 564.54 and 547.60.

6 $a = 670\,000, P = 8.0$

$$R = 1 + \frac{8.0}{100}$$

$R = 1.08$ or 108%

Substitute $n = 9$ into the formula for u_n.

$$u_n = 670\,000 \times 1.08^9 = 1\,339\,333 = \$1.34 \text{ million}$$

The value of Sanka's house after 10 years would be \$1.34 million.

7 **a** $a = 400\,600$

$$R = 1 + \frac{15}{100} = 1.15$$

b $u_n = aR^n$

$u_n = 400\,600 \times 1.15^n$

c $u_n = aR^n$

$$u_n = 400\,600 \times 1.15^{15} = 3\,259\,706.90 = \$3.26 \text{ million}$$

8 **a** $a = 5.0$ and $R = 0.5$

b In 24 days, the number of 8-day intervals is $\frac{24}{8}$ or 3.

Substituting values in the formula:

$u_n = aR^n$

$$u_3 = 5.0 \times 0.5^3 = 0.625$$

Thus, the amount of iodine-131 remaining after 24 days will be 0.625 mg.

6.6 Exam questions

1 $a = -4$ and $R = -3$

7th term is $u_6 = aR^6$

$$u_6 = (-4)(-3)^6 = -2916 \text{ years}$$

Be careful of the negatives.

The correct answer is **B**.

2 Be careful about numbering the months. April is month 0. After 1 month Henry's mark will be 52.2%, after 2 months his mark will be 55.125% and so on. $u_n > 70\%$ but $a = u_0 = 50\%$ and $R = 1.05$.

Mark = 50%, 52.5%, 55.13%, 57.88%, 60.78%, 63.81%, 67%, 70.36%

which is greater than 70%. This corresponds to November.

The correct answer is **D**.

3 **a** This information would be expressed as a geometric sequence, because a percentage increase in the total amount will be represented by multiplication, not addition. [1 mark]

b $u_{n+1} = R \times u_n$ [1 mark]

$u_{n+1} = 1.07 \times u_n$ and $u_0 = 1000$ [1 mark]

c After 1 year, $u_1 = 1.07 \times 1000 = \$1070$ [2 marks]

d After 18 years, $u_{18} = 1.07^{18} \times 1000 = \$3379.93$ [2 marks]

e If $u_0 = 1500$, [1 mark]

$u_{18} = 1.07^{18} \times 1500$ [1 mark]

$= \$5069.90$ [1 mark]

6.7 Generate and analyse a geometric sequence using a recurrence relation

6.7 Exercise

1
$$R = \frac{u_1}{u_0} = \frac{-7.5}{2.5} = -3$$
$$u_0 = 2.5$$
$$u_{n+1} = -3u_n, u_0 = 2.5$$

2 Given $u_{n+1} = -3.5u_n, u_0 = -4$

$$u_1 = -3.5u_0 = -3.5 \times (-4) = 14$$
$$u_2 = -3.5u_1 = -3.5 \times 14 = -49$$
$$u_3 = -3.5u_2 = -3.5 \times (-49) = 171.5$$
$$u_4 = -3.5u_3 = -3.5 \times 171.5 = -600.25$$

The first five terms of the sequence are −4, 14, −49, 171.5 and −600.25.

3
$$R = \frac{u_4}{u_3} = \frac{-1}{-4} = \frac{1}{4}$$
$$a = \frac{u_n}{R^n} = \frac{-4}{\left(\frac{1}{4}\right)^3} = -4 \times \frac{4^3}{1} = -256$$
$$u_{n+1} = \frac{1}{4}u_n,\ u_0 = -256$$

4 a 1st bounce: 165 cm

2nd bounce: 90.75 cm

3rd bounce: 49.91 cm

$$\frac{u_1}{u_0} = \frac{90.75}{165} = 0.55$$
$$\frac{u_2}{u_1} = \frac{49.91}{90.75} = 0.549... \approx 0.55$$

There is a common ratio between consecutive terms of 0.55.

$$a = 165, R = 0.55$$
$$u_{n+1} = Ru_n,\ u_0 = a$$
$$u_{n+1} = 0.55u_n,\ u_0 = 165$$

b $u_2 = 49.91$

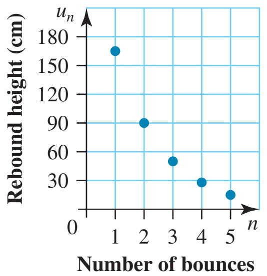
$$u_{n+1} = 0.55u_n$$
$$u_3 = 0.55 \times 49.91 = 27.4505 = 27.45 \text{ (correct to 2 d.p.)}$$
$$u_{n+1} = 0.55u_n$$
$$u_4 = 0.55u_3 = 0.55 \times 27.4505 = 15.097\,775 = 15.10 \text{ (correct to 2 d.p.)}$$

The estimated height of the 4th rebound is 27.45 cm, and the estimated height of the 5th rebound is 15.10 cm.

c

Bounce number	1	2	3	4	5
Rebound height (cm)	165	90.75	49.91	27.45	15.10

The points to be plotted are (1, 165), (2, 90.75), (3, 49.91), (4, 27.45) and (5, 15.10).



5 a
$$R = \frac{2}{5}$$
$$a = \frac{2}{5} \times 500 = 200,$$
$$u_{n+1} = Ru_n,\ u_0 = a$$
$$u_{n+1} = \frac{2}{5}u_n,\ u_0 = 200$$

b $u_0 = 200$

$$u_{n+1} = \frac{2}{5}u_n$$
$$u_1 = \frac{2}{5}u_0 = \frac{2}{5} \times 200 = 80$$
$$u_{n+1} = \frac{2}{5}u_n$$
$$u_2 = \frac{2}{5}u_1 = \frac{2}{5} \times 80 = 32$$
$$u_{n+1} = \frac{2}{5}u_n$$
$$u_3 = \frac{2}{5}u_2 = \frac{2}{5} \times 32 = 12.8$$

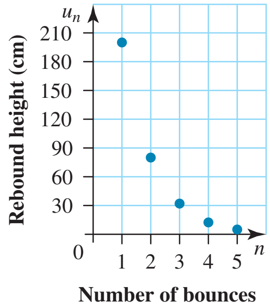
$$u_{n+1} = \frac{2}{5}u_n$$
$$u_4 = \frac{2}{5}u_3$$
$$= \frac{2}{5} \times 12.8$$
$$= 5.12$$

The estimated heights of the first 5 rebounds are 200 cm, 80 cm, 32 cm, 12.8 cm and 5.12 cm.

c

Bounce number	1	2	3	4	5
Rebound height (cm)	200	80	32	12.8	5.12

The points to be plotted are (1, 200), (2, 80), (3, 32), (4, 12.8) and (5, 5.12).



6 a $R = 70\%$

$$= \frac{70}{100}$$
$$= \frac{7}{10}$$

$u_{10} = 50, n = 10$

$$a = \frac{u_n}{R^n}$$
$$= \frac{50}{\left(\frac{7}{10}\right)^9}$$
$$= 50 \times \frac{10^9}{7^9}$$
$$= 1239.05$$

The first bounce reaches 1239.05 cm.

$a = \text{drop height} \times R$

$$\text{drop height} = \frac{a}{R}$$
$$= \frac{1239.05}{\left(\frac{7}{10}\right)}$$
$$= 1239.05 \times \frac{10}{7}$$
$$= 1770.07$$

The ball would have to be dropped from a height of 1770.07 cm, or 17.7 m.

b $a = 12.39, R = \frac{7}{10} = 0.7$ where a is in metres.

$u_{n+1} = Ru_n, \ u_0 = a$

$u_{n+1} = 0.7u_n, \ u_0 = 12.39$

6.7 Exam questions

1 The sequence shows numbers being multiplied by 2.
Therefore, $u_{n+1} = 2u_n$ where $u_0 = -8$.
The correct answer is **E**.

2 $u_0 = 100$ (1st term) – 0 minutes
$u_1 = 200$ (2nd term) – 5 minutes
$u_2 = 400$ (3rd term) – 10 minutes
$u_3 = 800$ (4th term) – 15 minutes
$u_4 = 1600$ (5th term) – 20 minutes
$u_5 = 3200$ (6th term) – 25 minutes
$u_6 = 6400$ (7th term) – 30 minutes
100, 200, 400, 800, 1600, 3200, 6400 ....
$\frac{1}{2}$ hour is 6 lots of 5 min or u_6: 6400
The correct answer is **D**.

3 The amount of water is reducing by 10% each month. That is, the amount of water in the tank in a particular month is 90% of that in the previous month.
$u_0 = 5000$ is the initial amount.
After 1 month, $u_1 = 4500$.
So, $R = \frac{4500}{5000} = 0.9$ and we want after 6 months, so
$u_6 = 0.9^6 \times 5000 = 2657$ litres
The correct answer is **A**.

6.8 Modelling growth and decay using recurrence relations

6.8 Exercise

1 a $u_0 = 10\,000$
3% of \$10 000 = $0.03 \times 10\,000$
The interest added to the investment each year is \$300.
$d = I$
$u_0 = 10\,000, \ u_{n+1} = u_n + 300$

b $u_n = a + nd$
$u_n = 10\,000 + 300n$

c $u_5 = 10\,000 + 300 \times 5$
$u_5 = 11\,500$
After 5 years, Sean's investment would have a value of \$11 500.

2 $u_0 = 6000$
8% of \$6000 = 0.08×6000
The interest added to the investment each year is \$480.
$I = d$
$u_n = a + nd$
$u_3 = 6000 + 3 \times 480$
$u_3 = 7440$
Oceania would have to pay back \$7440 to pay off the loan.

3 a Given that the rate is 4.8% per year,
$R = \frac{4.8\%}{12 \text{ months}} = 0.4\%$ per month
0.4% of \$1500 = 0.004×1500
The interest added to the investment each year is \$6.
$u_0 = 1500$
$d = 6$
$u_n = 1500 + 6n$

b $u_1 = 1500 + 6(1)$
$= 1506$
$u_2 = 1500 + 6(2)$
$= 1512$
$u_3 = 1500 + 6(3)$
$= 1518$

$u_4 = 1500 + 6(4)$
$= 1524$
$u_5 = 1500 + 6(5)$
$= 1530$
$u_6 = 1500 + 6(6)$
$= 1536$

The amounts in Grigor's account at the end of each of the first 6 months are \$1506, \$1512, \$1518, \$1524, \$1530 and \$1536.

c $u_{18} = 1500 + 6(18)$
$u_{18} = 1608$
Grigor's account will have \$1608 after 18 months.

4 a $u_0 = 8000$
Justine invested \$8000 in the account.

b The interest added to the investment each year is 50.
Principal value $= u_0 = 8000$
$I = 50$
$R = \dfrac{5000}{8000}$
$= 0.625$
Annual rate $= 0.625 \times 12 = 7.5\%$

5 a $u_0 = 24\,000$
$d = -0.25$
$u_n = a + nd$
$u_n = 24\,000 - 0.25n$

b Given $n = 12\,000$
$u_{12\,000} = 24\,000 - 0.25(12\,000)$
$= 24\,000 - 3000$
$= 21\,000$
After 12 000 km the car will be worth \$21 000.

6 a $u_n = 5400$
The photocopier cost \$5400.

b $d = -0.001$
Rate of depreciation = \$0.001
= 0.1 cents per copy

7 a $u_0 = 60\,000$
$d = 2500$
$u_n = a + nd$
$u_n = 60\,000 + 2500n$
$For\ n = 6,$
$u_6 = 60\,000 + 2500(5)$
$= 60\,000 + 12\,500$
$= 72\,500$
In the 6th year her salary will be \$72 500.

b $u_0 = 60\,000,\ d = 2500,\ u_n = 85\,000$
$n = \dfrac{u_n - u_0}{d}$
$= \dfrac{85\,000 - 60\,000}{2500}$
$= 10$
Her salary will reach \$85 000 after 10 years.

8 a Given that the rate is 6% per year,
6% of \$90 000 = $0.06 \times 90\,000$
The interest added to the investment each year is \$5400.
Nadia will receive \$5400 interest after 1 year.

b $n_0 = 90\,000$
$d = 5400$
$u_n = a + nd$
$u_n = 90\,000 + 5400n$

c $u_0 = 90\,000,\ d = 5400,\ u_n = 154\,800$
$n = \dfrac{u_n - u_0}{d}$
$= \dfrac{154\,800 - 90\,000}{5400}$
$= 12$
Nadia should keep her money invested for 12 years.

9 a $u_0 = 23\,000$
$d = -210$
$u_n = a + nd$
$u_n = 23\,000 - 210n$
For $n = 18$,
$u_{18} = 23\,000 - 210(18)$
$= 23\,000 - 3780$
$= 19\,220$
After 18 months the car will be worth \$19 220.

b $n = 3 \times 12$
$= 36$
$u_{36} = 23\,000 - 210(36)$
$= 23\,000 - 7560$
$= 15\,440$
Amount depreciated $= 23\,000 - 15\,440 = 7560$
In 3 years the value of the car depreciates by \$7560.

c $u_0 = 23\,000,\ d = -210,\ u_n = 6200$
$n = \dfrac{u_n - u_0}{d}$
$= \dfrac{6200 - 23\,000}{-210}$
$= 80$
It will take 80 months.

10 a $a = 50\,000$
$d = 50\,000 \times \dfrac{30}{100}$
$= 15\,000$
$u_n = a + nd$
$= 50\,000 + n \times 15\,000$
$= 50\,000 + 15\,000n$
For $n = 19$,
$u_{19} = 50\,000 + 15\,000n$
$= 50\,000 + 15\,000 \times 19$
$= 50\,000 + 285\,000$
$= 335\,000$
335 000 packets are manufactured in the 20th week.

b $a = 50\,000, d = 15\,000, u_n = 5\,540\,000$
$n = \dfrac{u_n - a}{d}$
$= \dfrac{5\,540\,000 - 50\,000}{15\,000}$
$= 366$
In the 366th week

11 a Unit depreciation $= -\dfrac{250\,000}{500\,000\,000}$
$= -0.0005$
The machine depreciates by \$0.0005 or 0.05 cents per can.

b Given the machine makes 40 200 000 cans per year,
$d = -0.0005 \times 40\,200\,000$
$= -20\,100$
So the machine depreciates in value by \$20 100 each year.
$a = \text{original price} + d$
$= 250\,000 - 20\,100$
$= 229\,900$
The write-off value is the point at which the machine has a value of \$0.
For $u_n = 0$
$n = \dfrac{u_n - a}{d}$
$= \dfrac{0 - 229\,900}{-20\,100}$
$= 11.44$
It will be written off in the 12th year.

c For $u_n = 89\,200$
$n = \dfrac{u_n - a}{d}$
$= \dfrac{89\,200 - 229\,900}{-20\,100}$
$= 7$
After 7 years

12 a $a = 5000, d = 1200$
$u_n = 5000 + 1200n$
For $n = 14$,
$u_{14} = 5000 + 1200(14)$
$= 5000 + 16\,800$
$= 21\,800$
In 15 years' time they will have 21 800 members.

b $a = 200, n = 5, u_5 = 320$
$d = \dfrac{u_n - a}{n}$
$= \dfrac{320 - 200}{5}$
$= \dfrac{120}{5}$
$= 24$
$u_n = 200 + 24n$
For $n = 14$,
$u_{14} = 200 + 24(14)$
$= 200 + 336$
$= 536$
In 15 years' time the tickets would cost \$536 each.

c For the first year,
total membership income $= \$200 \times 5000$
$= \$1\,000\,000$
For the 15th year,
total membership income $= \$536 \times 21\,800$
$= \$11\,684\,800$

13 a Determine the common ratio by substituting the values in the equation $R = 1 + \dfrac{P}{100}$.
$R = 1 + \dfrac{0.3}{100}$
$R = 1.003$
$u_0 = 2500, u_{n+1} = 1.003 \times u_n$
For the first month,
$u_1 = 2500 \times 1.003$
$= 2507.5$
Then calculate the common ratio:
$R = \dfrac{u_1}{u_0}$
$= \dfrac{2507.5}{2500}$
$= 1.003$
$a = 2500, R = 1.003$
$u_n = 2500 \times 1.003^n$

b $u_2 = 2500 \times 1.003^2$
$= 2515.0225$
≈ 2515.02
$u_3 = 2500 \times 1.003^3$
$= 2522.5675675$
≈ 2522.57
$u_4 = 2500 \times 1.003^4$
$= 2530.1352702$
≈ 2530.14
$u_5 = 2500 \times 1.003^5$
$= 2537.72567601$
≈ 2537.73
The amounts in Hussein's account at the end of each of the first 6 months are \$2507.50, \$2515.02, \$2522.57, \$2530.14, \$2537.73.

c $u_{15} = 2500 \times 1.003^{15}$
$= 2614.893...$
≈ 2614.89
After 15 months Hussein has \$2614.89 in his account.

14 a Given $u_n = 4500 \times 1.0035^n$,
$a = 4500$ and $R = 1.0035$ by inspection.
$P = 4500$
Tim invested \$4500 in the account.

b The common ratio in the geometric sequence equation is equal to $1 + \dfrac{R}{100}$ from the compound interest formula.
$1 + \dfrac{R}{100} = 1.0035$
$\dfrac{R}{100} = 0.0035$
$R = 0.35$
Therefore,
annual interest rate $= 0.35 \times 12$
$= 4.2\%$

15 a $a = 55\,000$
Jonas' salary each year is 3% more than the previous year.
$R = 1 + \dfrac{3}{100}$
$= 1.03$
$u_n = a \times R^n$
$u_n = 55\,000 \times 1.03^n$

b $u_4 = 55\,000 \times 1.03^4$
$= 61\,902.984...$
$\approx 61\,902.98$
In his 5th year Jonas will earn \$61 902.98.

16 a $100\% - 7\% = 93\%$

Each year the value of the item is 93% of the previous value.

$$93\% = \frac{93}{100} = 0.93$$
$$R = 0.93$$
$$a = 1470 \times 0.93 = 1367.1$$
$$u_n = 1367.1 \times 0.93^n$$

b For $n = 7$,

$$u_7 = 1367.1 \times 0.93^7 = 822.585... = 822.59$$

After 8 years the book value of the refrigerator is \$822.59.

17 a Given $u_n = 1665 \times 0.925^n$,

$$a = 1665$$
$$R = 0.925 = \frac{92.5}{100} = 92.5\%$$

Each year the value of the item is 92.5% of the previous value.

Let c be the original cost of the oven.

$$a = c \times R$$
$$1665 = c \times 0.925$$
$$\frac{1665}{0.925} = c$$
$$c = 1800$$

The oven cost \$1800.

b $R = 92.5\%$

$100\% - 92.5\% = 7.5\%$

The annual rate of depreciation for the oven is 7.5%.

18 Given $r = 5.5\%$

Determine the common ratio by substituting the values in the equation $R = 1 + \frac{P}{100}$.

$$R = 1 + \frac{5.5}{100}$$
$$R = 1.055$$

$u_0 = 5000, u_{n+1} = 1.055 \times u_n$

For the first year,

$$u_1 = 5000 \times 1.055 = 5275$$

Then calculate the common ratio:

$$R = \frac{u_1}{u_0} = \frac{5275}{5000} = 1.055$$
$$a = 5275, R = 1.055$$
$$u_n = 5275 \times 1.055^n$$

When Julio turns 18,

$n = 13$

$$u_{12} = 5275 \times 1.055^{12} = 10\,028.869... \approx 10\,028.87$$

When Julio turns 18, the fund will be worth \$10 028.87.

6.8 Exam questions

1 a 4% per annum is $\frac{4}{12}\%$ per month. Therefore, the common ratio is $1 + \frac{4}{1200}$.

After 6 months the amount is:

$$500 \times \left(1 + \frac{4}{1200}\right)^6 = 500 \times 1.00333^6 = \$510.08$$ [1 mark]

b A calculator, spreadsheet or CAS calculator can be used to determine how many months are needed to reach \$520.

12 (\$ 520.37) [1 mark]

c No; it would take 16 months. [1 mark]

0.25% per month means the common ratio is $1 + \frac{0.25}{100}$.

See the table at the bottom of the page*

etc.

Compounded: $A = 500\left(1 + \frac{4}{1200}\right)^n$ Simple: $A = 500\left(1 + \frac{0.25}{100}\right)^n$

2 a $u_{n+1} = \frac{3}{4}u_n$ and $u_0 = 64$ [1 mark]

b The first bounce corresponds to u_0, so the nth bounce is given by: $u_n = 64\left(\frac{3}{4}\right)^n$

c 5th bounce is:

$$u_5 = 64\left(\frac{3}{4}\right)^5 = 15.19\,\text{m}$$ [1 mark]

3 a $u_0 = \$\,18\,200$, $R = 20\%$, $n = 3$

Calculate the common ratio by identifying the value of the item in any given year as a percentage of the value in the previous year.

$100\% - 20\% = 80\%$

$$R = 0.8$$
$$u_0 = 18\,200$$
$$u_n = 18\,200 \times 0.8^n$$

After 3 years:

$$u_3 = \$18\,200(0.8)^3 = \$9318.40$$ [1 mark]

b Accumulated depreciation $= u_0 - u_3 = \$18\,200 - \$9318.40 = \$8881.60$ [1 mark]

c 9 years [1 mark]

The number of years after which the book value falls below \$2500 can be found by trial and error or with a spreadsheet. Choose the first year when value is less than \$2500.

*1c

n	0	1	2	3	4	5	
A	\$500.00	\$501.25	\$502.50	\$503.76	\$505.02	\$506.28	etc.

6.9 Review

6.9 Exercise

Multiple choice

1 **A** $\frac{16}{12} = 1.33, \frac{20}{16} = 1.25$

There is no common ratio, so this is not a geometric sequence.

B $\frac{4.8}{12} = 0.4, \frac{1.92}{4.8} = 0.4, \frac{0.77}{1.92} = 0.40, \frac{0.31}{0.77} = 0.40.$

This is a geometric sequence with a common ratio of 0.40.

C $\frac{15}{12} = 1.25, \frac{21}{15} = 1.4$

There is no common ratio, so this is not a geometric sequence.

D $\frac{6.2}{12} = 0.52, \frac{3.3}{6.2} = 0.53, \frac{1.88}{3.3} = 0.57$

There is no common ratio, so this is not a geometric sequence.

E $\frac{14.8}{12} = 1.23, \frac{17.6}{4.8} = 1.19$

There is no common ratio, so this is not a geometric sequence.

The correct answer is **B**.

2 $a = -16$ (the first term)

$d = -11.2 - (-16)$

$= 4.8$

The correct answer is **C**.

3 $u_1 = 2u_0 - 10$

$= 2 \times 65 - 10$

$= 130 - 10$

$= 120$

$u_2 = 2u_1 - 10$

$= 2 \times 120 - 10$

$= 240 - 10$

$= 230$

The correct answer is **B**.

4 $54.5 - 58 = -3.5$; therefore, $d = -3.5$

$65 - 3.5 = 61.5$

The correct answer is **D**.

5 $\frac{14.4}{4.8} = 3, \frac{43.2}{14.4} = 3, \frac{129.6}{43.2} = 3, \frac{388.8}{129.6} = 3$

The common ratio for the sequence is 3.

The correct answer is **A**.

6 $u_3 = 0.6u_2 + 4.5$

$= 0.6 \times 18 + 4.5$

$= 10.8 + 4.5$

$= 15.3$

The correct answer is **E**.

7 $\frac{21}{7} = 3$, so the common ratio for the sequence is 3.

Third term: $21 \times 3 = 63$

Fourth term: $63 \times 3 = 189$

The correct answer is **B**.

8 $\frac{11.25}{45} = 0.25, \frac{2.81}{11.25} = 0.25, \frac{0.70}{2.81} = 0.25, \frac{0.18}{0.70} = 0.26$

This is a geometric sequence with a common ratio of approximately 0.25.

The first term is 45.

This can be represented by the recurrence relation $u_{n+1} = 0.25u_n,\ u_0 = 45$.

The correct answer is **D**.

9 Arithmetic sequences have linear (straight line) graphs

The correct answer is **A**.

10 **A** $u_n = a + dn$ represents an arithmetic sequence.

B $u_{n+1} = Ru_n + d, u_0 = a$ represents a mixed sequence.

C $u_{n+1} = u_n + d, u_0 = a$ represents an arithmetic sequence.

D $u_{n+1} = u_n, u_0 = a$ represents a steady sequence.

E $u_n = aR^n$ represents a geometric sequence.

The correct answer is **E**.

Short answer

11 **a** $u_0 = 2 + 5(0)$

$= 2$

$u_1 = 2 + 5(1)$

$= 7$

$u_2 = 2 + 5(2)$

$= 12$

$u_3 = 2 + 5(3)$

$= 17$

$u_4 = 2 + 5(4)$

$= 22$

The first five terms of the sequence are 2, 7, 12, 17 and 22.

b $u_0 = 17 \times 2.2^0$

$= 17 \times 1$

$= 17$

$u_1 = 17 \times 2.2^1$

$= 17 \times 2.2$

$= 37.4$

$u_2 = 17 \times 2.2^2$

$= 82.28$

$u_3 = 17 \times 2.2^3$

$= 181.02$

$u_4 = 17 \times 2.2^4$

$= 398.24$

The first five terms of the sequence are 17, 37.4, 82.28, 181.02 and 398.24.

c $u_0 = 15$

$u_1 = u_0 - 6$

$= 15 - 6$

$= 9$

$u_2 = u_1 - 6$

$= 9 - 6$

$= 3$

$u_3 = u_2 - 6$

$= 3 - 6$

$= -3$

$u_4 = u_3 - 6$

$= -3 - 6$

$= -9$

The first five terms of the sequence are 15, 9, 3, -3 and -9.

12 **a** $0.5 - 3 = -2.5$, so $d = -2.5$

$a = 3$ (as given)

Therefore, $u_n = 3 - 2.5n$.

b $\frac{-8}{-2} = 4$; therefore, $R = 4$

$a = -2$ (as given)

Therefore, $u_n = -2 \times 4^n$.

c $\frac{143}{22} = 6.5, \frac{929.5}{143} = 6.5, \frac{929.5}{143} = 6.5$

This is a geometric sequence with $a = 22$ and $u_0 = 22$.

Therefore, $u_{n+1} = 6.5u_n, u_0 = 22$.

13 a $18.75 - 14 = 4.75$; therefore, $d = 4.75$

$a = 14$ (given)

Therefore, $u_n = 14 + 4.75n$.

$$\begin{aligned} u_7 &= 14 + 4.75(7) \\ &= 14 + 33.25 \\ &= 47.25 \end{aligned}$$

b $\frac{\frac{33}{50}}{\frac{11}{25}} = 1.5$; therefore, $R = 1.5$

$a = \frac{11}{25}$ (given)

Therefore, $u_n = \frac{11}{25} \times 1.5^n$

$$\begin{aligned} u_7 &= \frac{11}{25} \times 1.5^7 \\ &= 7.52 \text{ (2 decimal places)} \end{aligned}$$

c $28 - 45 = -17, 11 - 28 = -17, -6 - 11 = -17,$
$-23 - -6 = -17$

Therefore, this is an arithmetic sequence with a common difference of -17.

$u_{n+1} = u_n - 17, \quad u_0 = 45$

$$\begin{aligned} u_5 &= u_4 - 17 \\ &= -23 - 17 \\ &= -40 \end{aligned}$$

$$\begin{aligned} u_6 &= u_5 - 17 \\ &= -40 - 17 \\ &= -57 \end{aligned}$$

$$\begin{aligned} u_7 &= u_6 - 17 \\ &= -57 - 17 \\ &= -74 \end{aligned}$$

14 a $29.2 - 27 = 2.2$; therefore, the common difference is 2.2.

b $a = 27$

c $u_n = 27 + 2.2n$

d
$$\begin{aligned} u_8 &= 27 + 2.2(8) \\ &= 27 + 17.6 \\ &= 44.6 \end{aligned}$$

15 a
$$\begin{aligned} u_0 &= 48 \times \left(\frac{1}{2}\right)^0 \\ &= 48 \times 1 \\ &= 48 \end{aligned}$$

$$\begin{aligned} u_1 &= 48 \times \left(\frac{1}{2}\right)^1 \\ &= 48 \times \frac{1}{2} \\ &= 24 \end{aligned}$$

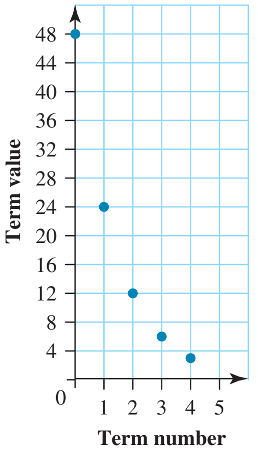
$$\begin{aligned} u_2 &= 48 \times \left(\frac{1}{2}\right)^2 \\ &= 48 \times \frac{1}{4} \\ &= 12 \end{aligned}$$

$$\begin{aligned} u_3 &= 48 \times \left(\frac{1}{2}\right)^3 \\ &= 48 \times \frac{1}{8} \\ &= 6 \end{aligned}$$

$$\begin{aligned} u_4 &= 48 \times \left(\frac{1}{2}\right)^4 \\ &= 48 \times \frac{1}{16} \\ &= 3 \end{aligned}$$

Term number	0	1	2	3	4
Term value	48	24	12	6	3

b



16 $u_0 = -4$

$u_1 = 2$

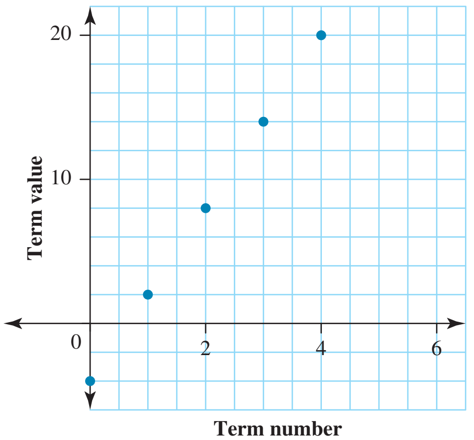
$$\begin{aligned} d &= 2 - -4 \\ &= 6 \end{aligned}$$

$$\begin{aligned} u_2 &= 2 + 6 \\ &= 8 \end{aligned}$$

$$\begin{aligned} u_3 &= 8 + 6 \\ &= 14 \end{aligned}$$

$$\begin{aligned} u_4 &= 14 + 6 \\ &= 20 \end{aligned}$$

Team number	0	1	2	3	4
Term value	–4	2	8	14	20



Extended Response

17 a $a = u_0 = 900$

8.2% of \$900 = 0.082×900

The interest added each year is \$73.8. Therefore, interest each month would be \$6.15 $\left(\frac{73.8}{12}\right)$

$d = 6.15$

$u_n = a + nd$

$u_n = 900 + 6.15n$

b $u_0 = 900$

$u_1 = 900 + 6.15(1)$
$= 906.15$

$u_2 = u_1 + 6.15$
$= 912.30$

$u_3 = u_2 + 6.15$
$= 918.45$

$u_4 = u_3 + 6.15$
$= 924.60$

$u_5 = u_4 + 6.15$
$= 930.75$

The amount in Chris' account after each of the first 5 months is \$906.15, \$912.30, \$918.45, \$924.60 and \$930.75.

c $u_{20} = 900 + 6.15(20)$
$= 1023$

After 20 months there will be \$1023 in the savings account.

d $1200 = 900 + 6.15n$

$1200 - 900 = 6.15n$

$300 = 6.15n$

$n = \frac{300}{6.15}$

$n = 48.8$

$n \approx 49$

It will take Chris 49 months to save \$1200.

18 a $R = \frac{2}{3}$

After 1 bounce:

$u_0 = 600 \times \frac{2}{3}$
$= 400$

Therefore, $u_{n+1} = \frac{2}{3}u_n, \quad u_0 = 400$

b $u_1 = \frac{2}{3} \times u_0$
$= \frac{2}{3} \times 400$
$= 266.67$

$u_2 = \frac{2}{3} \times u_1$
$= \frac{2}{3} \times 266.67$
$= 177.78$

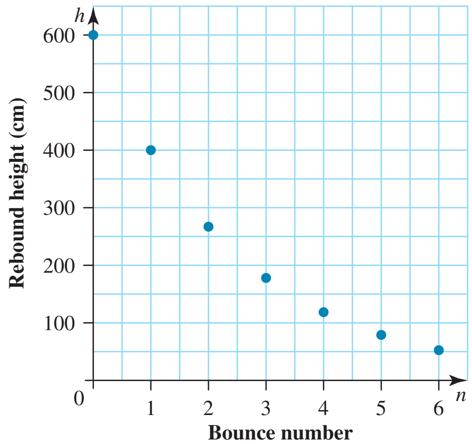
$u_3 = u_4$
$= \frac{2}{3} \times 177.78$
$= 118.52$

$u_4 = \frac{2}{3} \times u_3$
$= \frac{2}{3} \times 118.52$
$= 79.01$

$u_5 = \frac{2}{3} \times u_4$
$= \frac{2}{3} \times 79.01$
$= 52.67$

The fifth bounce reaches 79.01 cm and the sixth bounce reaches 52.67 cm.

c



19 a $a = 68\,000,\ R = 1.025$

$u_n = 68\,000 \times 1.025^n$

b $u_3 = 68\,000 \times 1.025^3$
$= \$73\,228.56$

c By inspection of the formula, we can see Kane originally put \$1500 into the account at an interest rate of 3.4%.

d $u_5 = 1500 \times 1.034^5$
$= 1772.94$

$\$1772.94 - \$1500 = \$272.94$

20 a $R = 3,\ u_6 = 10\,935$

$u_{n+1} = 3 \times u_n$

$u_7 = 3 \times u_6$
$= 3 \times 10\,935$
$= 32\,805$

$u_8 = 3 \times u_7$
$= 3 \times 32\ 805$
$= 98\,415$

There will be 98 415 bacteria on the 9th day.

b $u_{n+1} = 3 \times u_n$

$\frac{u_{n+1}}{3} = u_n$

$u_n = \frac{u_{n+1}}{3}$

$u_5 = \frac{u_{6a}}{3}$
$= \frac{10\,935}{3}$
$= 3645$

$$u_4 = \frac{u_5}{3} = \frac{3645}{3} = 1215$$

$$u_3 = \frac{u_4}{3} = \frac{1215}{3} = 405$$

$$u_2 = \frac{u_3}{3} = \frac{405}{3} = 135$$

$$u_1 = \frac{u_2}{3} = \frac{135}{3} = 45$$

$$u_0 = \frac{u_1}{3} = \frac{45}{3} = 15$$

Originally there were 15 bacteria.

21 a $u_0 = 3000, d = -250$

$u_n = u_0 + nd$

$u_n = 3000 - 250n$

b $u_5 = 3000 - 250(5)$

$u_5 = 1750$

$$\text{Depreciation} = u_0 - u_5 = 3000 - 1750 = 1250$$

The computer will be worth \$1750 in 5 years. It will have depreciated by \$1250.

c $u_n = 0, u_0 = 3000,\ d = -250$

$$n = \frac{u_n - u_0}{d}$$

$$n = \frac{0 - 3000}{-250} = 12$$

It will take 12 years for the computer to reach its write-off value.

22 a $u_0 = 170\,000, u_{n+1} = u_n + 13\,000$

b $u_n = a + nd$

$u_n = 170\,000 + 13\,000n$

$u_5 = 170\,000 + 13\,000 \times 5 = 235\,000$

235 000 people are expected to migrate to Australia in 5 years' time.

6.9 Exam questions

1 Option A shows numbers increasing by 2.
Option B shows numbers increasing by 6.
There is no recognisable pattern in option C.
Option D shows numbers being divided by 2.
Option E shows numbers decreasing by 7.
The correct answer is **C**.

2 The sequence shows numbers decreasing by 9.
Therefore, $u_{n+1} = u_n - 9$, where $u_0 = 39$.
The correct answer is **A**.

3 $a = 4$

$d = 11$

$u_n = a + nd$

$u_n = 4 + 11n$ [1 mark]

where $u_0 = 4$ and $n = 3$

$u_3 = 4 + 11 \times 3$

$u_3 = 37$ [1 mark]

Therefore, in the 4th year, the apple tree grew 37 apples.

4 $u_n = a + nd$

$u_n = 67 + n \times -6$

$u_n = 67 - 6n$

where $u_0 = 67$

The correct answer is **C**.

5 $u_0 = 1850$

$$\text{The common ratio, } R = 1 - \frac{R}{100} = 1 - \frac{25}{100}$$

$R = 0.75$

$$u_n = u_0 \times R^n = 1850 \times 0.75^3 = \$780.47$$

The correct answer is **C**.

Topic 7 — Financial mathematics extension

7.2 Reducing balance loans modelled using recurrence relations

7.2 Exercise

1 a $V_0 = 4200 \quad d = 380 \quad r = \dfrac{10.5}{12} = 0.875$

$R = 1 + \dfrac{0.875}{100} = 1.008\,75$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.008\,75\,V_n - 380, \; V_0 = 4200$

b i $V_1 = 1.008\,75 \times 4200 - 380$

$= 3856.75$

$V_2 = 1.008\,75 \times 3856.75 - 380$

$= 3510.50$

The outstanding balance after 2 payments is $3510.50

ii Using a calculator:

4200	4200
4200 × 1.008 75 − 380	3856.75
3856.75 × 1.008 75 − 380	3510.50
3510.50 × 1.008 75 − 380	3161.21
3161.21 × 1.008 75 − 380	2808.87
2808.87 × 1.008 75 − 380	2453.45

The outstanding balance after 5 payments is $2453.45.

c Using a calculator:

4200	4200
4200 × 1.008 75 − 380	3856.75
3856.75 × 1.008 75 − 380	3510.50
3510.50 × 1.008 75 − 380	3161.21
3161.21 × 1.008 75 − 380	2808.87
2808.87 × 1.008 75 − 380	2453.45
2453.45 × 1.008 75 − 380	2094.92
2094.92 × 1.008 75 − 380	1733.25
1733.25 × 1.008 75 − 380	1368.42
1368.42 × 1.008 75 − 380	1000.39
1000.39 × 1.008 75 − 380	629.143
629.143 × 1.008 75 − 380	254.648
254.648 × 1.008 75 − 380	−123.124

Stephen would need to make 12 payments to repay the loan. (*Note:* The last payment would be reduced by $123.12 to give a zero balance.)

2 a $V_0 = 2600 \quad d = 345 \quad r = \dfrac{15}{12} = 1.25$

$R = 1 + \dfrac{1.25}{100} = 1.0125$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.0125\,V_n - 345, \; V_0 = 2600$

b i $V_1 = 1.0125 \times 2600 - 345$

$= 2287.5$

$V_2 = 1.0125 \times 2287.5 - 345$

$= 1971.09$

$V_3 = 1.0125 \times 1971.09 - 345$

$= 1650.73$

The outstanding balance after 3 payments is $1650.73.

ii Using a calculator:

2600	2600
2600 × 1.0125 − 345	2287.5
2287.5 × 1.0125 − 345	1971.09
1971.09 × 1.0125 − 345	1650.73
1650.73 × 1.0125 − 345	1326.37
1326.37 × 1.0125 − 345	997.946
997.946 × 1.0125 − 345	665.42
665.42 × 1.0125 − 345	328.738

Outstanding balance after 7 payments is $328.74.

c Using a calculator:

2600	2600
2600 × 1.0125 − 345	2287.5
2287.5 × 1.0125 − 345	1971.09
1971.09 × 1.0125 − 345	1650.73
1650.73 × 1.0125 − 345	1326.37
1326.37 × 1.0125 − 345	997.946
997.946 × 1.0125 − 345	665.42
665.42 × 1.0125 − 345	328.738
328.738 × 1.0125 − 345	−12.1525

Sallyanne would need to make 8 payments to repay the loan.
(*Note:* The last payment would be reduced by $12.15 to give a zero balance.)

3 a Amount borrowed = $ (12500 − 7200) = $5300
Hazel borrowed $5300 to purchase her first car.

b $V_0 = 5300 \; d = 575 \; r = \dfrac{12.6}{12} = 1.05$

$R = 1 + \dfrac{1.05}{100} = 1.0105$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.0105\,V_n - 575, \; V_0 = 5300$

c Using a calculator

5300	5300
5300 × 1.0105 − 575	4780.65
4780.65 × 1.0105 − 575	4255.85
4255.85 × 1.0105 − 575	3725.53
3725.53 × 1.0105 − 575	3189.65
3189.65 × 1.0105 − 575	2648.14
2648.14 × 1.0105 − 575	2100.95
2100.95 × 1.0105 − 575	1548.01
1548.01 × 1.0105 − 575	989.262
989.262 × 1.0105 − 575	424.649
424.649 × 1.0105 − 575	−145.892

Hazel would need to make 10 payments to repay the loan. (*Note:* The last payment would be reduced by $145.89 to give a zero balance.)

4 a Amount borrowed = \$ (7500 − 5000) = \$2500
Leo borrowed \$2 500 to purchase his first car.

b $V_0 = 2500 \quad d = 300 \quad r = \dfrac{8.4}{12} = 0.7$

$R = 1 + \dfrac{0.7}{100} = 1.007$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.007\,V_n - 300, \quad V_0 = 2500$

c Using a calculator

2500	2500
2500 × 1.007 − 300	2217.5
2217.5 × 1.007 − 300	1933.02
1933.02 × 1.007 − 300	1646.55
1646.55 × 1.007 − 300	1358.08
1358.08 × 1.007 − 300	1067.59
1067.59 × 1.007 − 300	775.059
775.059 × 1.007 − 300	480.485
480.485 × 1.007 − 300	183.848
183.848 × 1.007 − 300	−114.865

Leo would need to make 9 payments to repay the loan.
(*Note:* The last payment would be reduced by \$114.87 to give a zero balance.)

5

Payment number, *n*	**Payment (\$)**	**Interest (\$)**	**Principal reduction (\$)**	**Balance of loan (\$)**
0				4500.00
1	1200.00	132.75	1067.25	3432.75
2	1200.00	101.27	1098.73	2334.02
3	1200.00	68.85	1131.15	1202.87
4	1200.00	35.48	1164.52	38.35

a 11.8% per year
$= \dfrac{11.8}{4}\%$ per quarter
= 2.95% per quarter
The interest rate is 2.95% per quarter.

b 2.95% × 4500
$= \dfrac{2.95}{100} \times 4500$
= 132.75
Interest charged in first quarter is \$132.75

c Previous balance − principal reduction
= 3432.75 − 1098.73
= 2334.02
Balance owing after 2 payments is \$2334.02

d Payment − interest charged
= 1200 − 68.85
= 1131.15
The principal is reduced by \$1131.15 after 3 payments.

e Outstanding balance = \$38.35
Last payment = \$ (1200 + 38.35)
The final payment would be \$1238.35.

f Total interest = \$ (132.75 + 101.27 + 68.85 + 35.48)
Total interest paid on loan is \$338.35.

6 Interest rate per month $= \dfrac{16.5}{12} = 1.375\%$

$A_0 = 20\,000,\ A_{n+1} = 1.013\,75\ A_n - 421.02$

Using the iterative process on your calculator,
$A_8 = \$18\,774.05$.
The correct answer is **A**.

7

Payment number, *n*	**Payment (\$)**	**Interest (\$)**	**Principal reduction (\$)**	**Balance of loan (\$)**
0				7600.00
1	1375.00	182.40	1192.60	6407.40
2	1375.00	153.78	1221.22	5186.18
3	1375.00	124.47	1250.53	3935.65
4	1375.00	94.46	1280.54	2655.11
5	1375.00			
6	1375.00			

a 9.6% per year
$= \dfrac{9.6}{4}\%$ per quarter
= 2.4% per quarter
The interest rate is 2.4% per quarter

b 2.4% × 7600
$= \dfrac{2.4}{100} \times 7600$
= 182.40
Interest charged in first quarter is \$182.40

c Previous balance − principal reduction
= 6407.40 − 1221.22
= 5186.18
Balance owing after 2 payments is \$5186.18

d Payment − interest charged
= 1375 − 124.47
= 1250.53
The principal is reduced by \$1250.53 after 3 payments.

e Continue filling in the blanks.
See the table bottom of the page*

f Final payment = \$ (1375 + 1.08)
Final payment to give a zero balance is \$1376.08.

8 a 15% per year
$= \dfrac{15}{4}\%$ per quarter
= 3.75% per quarter
The interest rate is 3.75% per quarter

*7e

5	1375.00	63.72	1311.28	1343.83
6	1375.00	32.25	1342.75	1.08

b $V_0 = 25000 \quad d = 3250 \quad r = 3.75\% \quad R = 1 + \frac{3.75}{100} = 1.0375$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.0375\,V_n - 3250, \quad V_0 = 25\,000$

c Using a calculator

25 000	25 000
25 000 × 1.0375 − 3250	22 687.5
22 687.5 × 1.0375 − 3250	20 288.3
20 288.3 × 1.0375 − 3250	17 799.1
17 799.1 × 1.0375 − 3250	15 216.6
15 216.6 × 1.0375 − 3250	12 537.2
12 537.2 × 1.0375 − 3250	9757.32

Outstanding balance after 3 payments is $17 799.10

d Outstanding balance after 18 months = outstanding balance after 6 quarters or payments
Outstanding balance after 18 months is $9757.32.

9 Interest rate per quarter = 3.75% = 0.0375
See the table bottom of the page*

10 **a** 9% per year

$= \frac{9}{12}\%$ per month

= 0.75% per month

The interest rate is 0.75% per month

b $V_0 = 3750 \quad d = 390 \quad r = 0.75\% \quad R = 1 + \frac{0.75}{100} = 1.0075$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.0075\,V_n - 390, \quad V_0 = 3750$

c Using a calculator

3750	3750
3750 × 1.0075 − 390	3388.13
3388.13 × 1.0075 − 390	3023.54
3023.54 × 1.0075 − 390	2656.21
2656.21 × 1.0075 − 390	2286.13
2286.13 × 1.0075 − 390	1913.28
1913.28 × 1.0075 − 390	1537.63
1537.63 × 1.0075 − 390	1159.16
1159.16 × 1.0075 − 390	777.86
777.86 × 1.0075 − 390	393.69
393.69 × 1.0075 − 390	6.64

The balance owing after 2 payments is $3 023.54

d The balance owing after 10 payments is $6.64, so last payment is increased by $6.64.
Final payment = $ (390 + 6.64)
Jamie's final payment will be $396.64.

7.2 Exam questions

1 Use the initial value of $300 000 to calculate the interest of $900.

$\frac{3.6}{100 \times n} \times 300\,000 = 900$

$\frac{3.6}{100 \times n} = 0.003$

$3.6 = 0.3\,n$

$n = 12$

The correct answer is **B**.

2 $V_1 = 1.003 \times (26\,000) - 400$
$= 25\,678$
$V_2 = 1.003 \times (25\,678) - 400$
$= 25\,355.034$
$V_3 = 1.003 \times (25\,355.034) - 400$
$= 25\,031.0991$
$V_4 = 1.003 \times (25\,031.0991) - 400$
$= 24\,706.1924$
$V_5 = 1.003 \times (24\,706.1924) - 400$
$= 24\,380.3110$

The correct answer is **A**.

3

2	1500	$I = 249\,500 \times \frac{4.8}{100 \times 12}$ = $998	1500 − 998 = $502	$248 998

The correct answer is **B**.

7.3 Solving reducing balance loan problems with technology

7.3 Exercise

1

N	48
I%	8.25
PV	28 500
PMT	−503
FV	
P/Y	12
C/Y	12

FV = −11 108.0545
Amount owing after 4 years = $11 108.05

*9

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance of loan ($)
0				25 000.00
1	3250	0.0375 × 25 000 = 937.50	3250 − 937.50 = 2312.5	25 000 − 2312.5 = 22 687.50
2	3250	0.0375 × 22 687.5 = 850.78	3250 − 850.78 = 2399.22	22 687.5 − 2399.22 = 20 288.28
3	3250	0.0375×20 288.28 = 760.81	3250 − 760.81 = 2489.19	20 288.28 − 2489.19 = 17 799.09
4	3250	0.0375×17 700.09 = 667.47	3250 − 667.47 = 2582.53	17 799.09 − 2582.53 = 15 216.56

2

N	18
I%	12.45
PV	3750
PMT	−126
FV	
P/Y	12
C/Y	12

FV = −2036.1057
Amount owing after 18 months = $2036.11

3 a

N	360
I%	4.28
PV	475 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −2345.0643
Charlotte's monthly payment = $2345.06

b Total payments
= $2345.06 × 360
= $844 221.60
Charlotte paid in total $844 221.60.

c interest = total repayments − loan
= 844 221.60 − 475 000
= 369 221.60
Charlotte paid $369 221.60 in interest over the 30 years repaying the loan.

4 a

N	48
I%	9.8
PV	75 800
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −1915.2115
Thomas's monthly payment = $1915.21

b Total payments
= $1915.21 × 48
= $91 930.08
Thomas paid in total $91 930.08.

c interest = total repayments − loan
= 91 930.08 − 75 800
= 16 130.08
Thomas paid $16 130.08 in interest over the 4 years of repaying the loan.

5

N	300
I%	6
PV	
PMT	−4580
FV	0
P/Y	12
C/Y	12

PV = 710 847.437
Henry could borrow $710 847 for his apartment.

6

N	18
I%	5.2
PV	
PMT	−350
FV	0
P/Y	12
C/Y	12

PV = 6047.975
The amount that could borrowed, to the nearest $100, is $6000.

7 a

N	
I%	6.35
PV	32 500
PMT	−1910
FV	0
P/Y	4
C/Y	4

N = 19.992 05
The loan would be repaid in 20 quarterly payments, a total of 5 years.

b

N	20
I%	6.35
PV	32 500
PMT	−1910
FV	
P/Y	4
C/Y	4

FV = 15.05307, the loan is overpaid by $15.05, so the last payment needs to be reduced.
The last payment will be $1894.95.

8 a

N	
I%	7.75
PV	17 500
PMT	−1535.75
FV	0
P/Y	4
C/Y	4

N = 12.9997
The loan would be repaid in 13 quarterly payments, a total of 3 years and 3 months.

b

N	13
I%	7.75
PV	17 500
PMT	−1535.75
FV	
P/Y	4
C/Y	4

FV = 0.3558, the loan is overpaid by \$0.36, so the last payment needs to be reduced.
The last payment will be \$1535.39.

9 Liam:

N	36
I%	
PV	12500
PMT	−388.55
FV	0
P/Y	12
C/Y	12

I = 7.4516
Amelia:

N	12
I%	
PV	12 500
PMT	−1177.50
FV	0
P/Y	4
C/Y	4

I = 7.752
Liam has the lower interest rate of 7.45%, lower by 0.3%.

10 A-bank

N	48
I%	
PV	34 000
PMT	−850
FV	0
P/Y	12
C/Y	12

I = 9.241 77
B-bank

N	16
I%	
PV	34 000
PMT	−2551
FV	0
P/Y	4
C/Y	4

I = 8.9408
B-bank has the lower interest rate of 8.94%, lower by 0.3%.

11 a

N	300
I%	4.9
PV	315 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −1823.1526
Grace's monthly payment = \$1823.15

b Total payments
= \$1823.15 × 300
= \$546 945
Grace paid in total \$546 945.

c interest = total repayments − loan
= 546 945 − 315 000
= 231 945
Grace paid \$231 945 in interest over the 25 years repaying the loan.

12 a

N	32
I%	9.8
PV	35 650
PMT	
FV	0
P/Y	4
C/Y	4

PMT = −1620.187
James would need to pay \$1620.19 per quarter to repay the loan in 8 years.

b

N	20
I%	9.8
PV	35 650
PMT	−1620.19
FV	
P/Y	4
C/Y	4

FV = −16 669.860 9
After 5 years, James would need to pay \$16 669.86 to pay out the loan.

13 a

N	360
I%	4.25
PV	475 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −2336.714
Their monthly payment would be \$2336.71

b

N	180
I%	4.25
PV	475 000
PMT	−2336.71
FV	
P/Y	12
C/Y	12

FV = −310 619.435
Amount owing after 15 years is $310 619.44

c

N	60
I%	4.25
PV	310619.44
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −5755.6402
Their new monthly payment would be $5755.64

14 a i Quarterly

N	20
I%	6.2
PV	38 500
PMT	
FV	0
P/Y	4
C/Y	4

PMT = −2253.529
Their quarterly payment would be $2253.53

ii Monthly

N	60
I%	6.2
PV	38 500
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −747.8985
Their monthly payment would be $747.90

iii Fortnightly

N	130
I%	6.2
PV	38 500
PMT	
FV	0
P/Y	26
C/Y	26

PMT = −344.775 75
Their fortnightly payment would be $344.78

b Total payments:
Quarterly
= $2253.53 × 20
= $45 070.60
Monthly
= $747.90 × 60
= $44 874
Fortnightly
= $344.78 × 130
= $44 821.40
The best deal is paying fortnightly (by a minimum of $52.60)

15 a

N	300
I%	3.95
PV	395 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −2074.0659
Their monthly payment would be $2074.07

b After 10 years:

N	120
I%	3.95
PV	395 000
PMT	−2074.07
FV	
P/Y	12
C/Y	12

FV = −281 348.9254
Balance of loan after 10 years is $281 348.93
Interest rate increased for remaining 15 years:

N	180
I%	5.2
PV	281 348.93
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −2254.3115
Their new monthly payment would be $2254.31

7.3 Exam questions

1 Calculate with financial solver on a CAS calculator using N = 60, I = 3.72, PV = 175 260.56, PMT = −3200 and PpY = 12. The final value will be −368.116 …
Since the final value is negative, Adam must still pay this amount.
Therefore, Adam's final payment will be
3200 + 368.116 ≈ $3568.12

***VCAA Examination Report* note:**
Students who chose option D did not add the interest that was required to be paid with the final payment.
The correct answer is **E**.

2 a Use Finance Solver on CAS:
N: 12
I(%): 6.9
PV: 70 000
PMT: −800
PpY/CpY: 12
Therefore, Ken will owe $65 076.22 after 12 months. [1 mark]

***VCAA Assessment Report* note:**
Rather than use a financial solver to answer the question above, a number of students adopted a formulaic approach, almost always unsuccessfully, based on the compound interest formula.

b Total interest after 12 payments = 800 × 12 − (70 000 −65 076.22) = $4676.22 [1 mark]

***VCAA Assessment Report* note:**
A common incorrect answer was $4923.78, which is the reduction in the principal over the year. This failed to take into account the $9600 total of repayments made in the year.

3 Via TVM solver:
N = 240
I = 6.95
PV = 90 000
PMT = ?
FV = 0
PpY = 12
CpY = 12
Therefore, monthly payment = $695.09 ≈ $695.
The correct answer is **C**.

7.4 Annuities

7.4 Exercise

1 a $V_0 = 8500 \quad d = 2145 \quad r = \dfrac{4.8}{12} = 0.4\%$

$R = 1 + \dfrac{0.4}{100} = 1.004$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.004\,V_n - 2145, \quad V_0 = 8500$

b $V_1 = 1.004\,V_0 - 2145$
$= 1.004 \times 8500 - 2145$
$= 6389$

$V_2 = 1.004\,V_1 - 2145$
$= 1.004 \times 6389 - 2145$
$= 4269.56$

$V_3 = 1.004\,V_2 - 2145$
$= 1.004 \times 4269.56 - 2145$
$= 2141.63$

Balance in the annuity at the end of 3 months is $2141.63.

c $V_4 = 1.004\,V_3 - 2145$
$= 1.004 \times 2141.63 - 2145$
$= 5.20$

The annuity still has a balance of $5.20, so last payment will have to be increased by $5.20 to $2150.20.

2 a $V_0 = 6500 \quad d = 2180 \quad r = \dfrac{3.24}{12} = 0.27\%$

$R = 1 + \dfrac{0.27}{100} = 1.0027$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.0027\,V_n - 2180, \quad V_0 = 6500$

b $V_1 = 1.0027\,V_0 - 2180$
$= 1.0027 \times 6500 - 2180$
$= 4337.55$

$V_2 = 1.0027\,V_1 - 2180$
$= 1.0027 \times 4337.55 - 2180$
$= 2169.26$

Balance in the annuity at the end of 2 months is $2169.26.

c $V_3 = 1.0027\,V_2 - 2180$
$= 1.0027 \times 2169.26 - 2180$
$= -4.8816$

The annuity has been overdrawn by $4.88, so the last payment will have to be decreased by $4.88 to $2175.12 to give a zero balance.

d See the table bottom of the page*

3 a $V_0 = 12\,500 \quad d = 1580 \quad r = \dfrac{3.6}{12} = 0.3\%$

$R = 1 + \dfrac{0.3}{100} = 1.003$

$V_{n+1} = RV_n - d, \quad V_0 = a$

$V_{n+1} = 1.003\,V_n - 1580, \quad V_0 = 12\,500$

b i After 4 months:

12 500	12 500
12 500 × 1.003 − 1580	10957.5
10 957.5 × 1.003 − 1580	9410.37
9410.37 × 1.003 − 1580	7858.6
7858.6 × 1.003 − 1580	6302.18

The balance of the annuity after 4 months is $6302.18

ii After 8 months:

12 500	12500
12 500 × 1.003 − 1580	10 957.5
10 957.5 × 1.003 − 1580	9410.37

***2d**

Payment number, *n*	**Payment ($)**	**Interest ($)**	**Annuity reduction ($)**	**Balance of annuity ($)**
0				6 500.00
1	2180.00	0.0027 × 6500 = 17.55	2180 − 17.55 = 2162.45	6500 − 2162.45 = 4337.55
2	2180.00	0.0027 × 4337.55 = 11.71	2180 − 11.71 = 2168.29	4337.55 − 2168.29 = 2169.26
3	2180.00	0.0027 × 2169.26 = 5.86	2180 − 5.86 = 2174.14	2169.26 − 2174.14 = −4.88

9410.37 × 1.003 − 1580	7858.6
7858.6 × 1.003 − 1580	6302.18
6302.18 × 1.003 − 1580	4741.09
4741.09 × 1.003 − 1580	3175.31
3175.31 × 1.003 − 1580	1604.84
1604.84 × 1.003 − 1580	29.6497

The balance of the annuity after 8 months is $29.65

c The annuity still has a balance of $29.65, so Terry's last payment will have to be increased by $29.65 to $1609.65.

4 a $V_0 = 15\,250 \quad d = 1725 \quad r = \frac{4.5}{12} = 0.375\%$

$R = 1 + \frac{0.375}{100} = 1.003\,75$

$V_{n+1} = RV_n - d, \quad V_0 = a$

$V_{n+1} = 1.003\,75\,V_n - 1725, \quad V_0 = 15\,250$

b

15 250	15 250
15 250 × 1.003 75 − 1725	13 582.2
13 852.2 × 1.003 75 − 1752	11 908.1
11 908.1 × 1.003 75 − 1725	10 227.8
10 227.8 × 1.003 75 − 1725	8541.13
8541.13 × 1.003 75 − 1725	6848.16
6848.16 × 1.003 75 − 1725	5148.84

Balance of the annuity after 6 months is $5148.84

c

15 250	15 250
15 250 × 1.00 375 − 1725	13 582.2
13 582.2 × 1.00 375 − 1725	11 908.1
11 908.1 × 1.00 375 − 1725	10 227.8
10 227.8 × 1.00 375 − 1725	8541.13
8541.13 × 1.00 375 − 1725	6848.16
6848.16 × 1.00 375 − 1725	5148.84
5148.84 × 1.00 375 − 1725	3443.15
3443.15 × 1.00 375 − 1725	1731.06
1731.06 × 1.00 375 − 1725	12.5516

The annuity still has a balance of $12.55, so Melanie's last payment will have to be increased by $12.55 to $1737.55.

5 a $V_0 = 16\,000 \quad d = 4150 \quad r = \frac{6.4}{4} = 1.6\%$

$R = 1 + \frac{1.6}{100} = 1.016$

$V_{n+1} = R\;V_n - d, \quad V_0 = a$

$V_{n+1} = 1.016\,V_n - 4150, \quad V_0 = 16\,000$

b

16 000	16 000
11 600 × 1.016 − 4150	12 106
12 106 × 1.016 − 4150	8149.7
8149.7 × 1.016 − 4150	4130.09
4130.09 × 1.016 − 4150	46.1726

Balance in the annuity at the end of the year is $46.17.

c The annuity still has a balance of $46.17, so the last payment will have to be increased by $46.17 to $4196.17.

d See the table bottom of the page*

6 a

N	
I%	5.25
PV	−725 000
PMT	5500
FV	0
P/Y	12
C/Y	12

N = 196.9289 months

Sam's funds will last 16 years and 5 months (to the nearest month)

b

N	300
I%	5.25
PV	−725 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = 4344.5459

To last 25 years, Sam's payment should be $4344.55 (to the nearest cent).

7 a

N	360
I%	5.4
PV	−850 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = 4773.0117

Georgia's payment should be $4773.01 to last for 30 years.

*5d

Payment number, n	**Payment** ($)	**Interest** ($)	**Annuity reduction** ($)	**Balance of annuity** ($)
0				16 000.00
1	4150	0.016 × 16 000 = 256	4150 − 256 = 3894	16 000 − 3894 = 12 106
2	4150	0.016 × 12 106 = 193.70	4150 − 193.70 = 3956.30	12 106 − 3956.3 = 8149.70
3	4150	0.016 × 8149.7 = 130.39	4150 − 130.39 = 4019.61	8149.70 − 4019.61 = 4130.09
4	4150	0.016 × 4130.09 = 66.08	4150 − 66.08 = 4083.92	4130.09 − 4083.02 = 46.17

b

N	
I%	5.4
PV	−850 000
PMT	5500
FV	0
P/Y	12
C/Y	12

N = 264.801
Georgia's funds will last 22 years and 1 month (to the nearest month).

8

N	60
I%	7.3
PV	−236 000
PMT	
FV	0
P/Y	4
C/Y	4

PMT = 6504.652
Patricia's payment should be $6504.65 per quarter for the annuity to last for 15 years.

9

N	300
I%	6.2
PV	
PMT	6200
FV	0
P/Y	12
C/Y	12

PV = −944 284.048
To reach his goals, Stephen would need a superannuation fund of $944 284 (to the nearest dollar).

10

N	80
I%	7.32
PV	
PMT	13 200
FV	0
P/Y	4
C/Y	4

PV = −552 242.726
To reach her goals, Marion would need a superannuation fund of $552 243 (to the nearest dollar).

11 a

N	
I%	8.45
PV	−560 000
PMT	5000
FV	0
P/Y	12
C/Y	12

N = 221.507 88
James's annuity will last 18 years and 6 months (to the nearest month).

b

N	60
I%	8.45
PV	−560 000
PMT	5000
FV	
P/Y	12
C/Y	12

FV = 481 441.9489
After 5 years, James's annuity has a balance of $481 442 (to the nearest dollar).

c

N	
I%	8.45
PV	−481 442
PMT	6 500
FV	0
P/Y	12
C/Y	12

N = 105.0634
James's annuity will last a further 8 years and 9 months (to the nearest month).

12 a i Quarterly:

N	8
I%	5.6
PV	−120 000
PMT	
FV	0
P/Y	4
C/Y	4

PMT = 15 960.3247
Quarterly payments would be $15 960.32

ii Monthly:

N	24
I%	5.6
PV	−120 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = 5296.871 03
Monthly payments would be $5296.87

iii Weekly:

N	104
I%	5.6
PV	−120 000
PMT	
FV	0
P/Y	52
C/Y	52

PMT = 1220.287 98
Weekly payments would be $1220.29

b Quarterly: $\$15\,960.32 \times 8 = \$127\,682.56$
Monthly: $\$5296.87 \times 24 = \$127\,124.88$
Weekly: $\$1220.29 \times 104 = \$126\,910.16$
The quarterly payment gives Samantha the largest amount in total.

13 a

N	240
I%	6.48
PV	−468 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = 3483.7739
Billy's monthly payment should be \$3483.77 to last for 20 years

b After 6 years:

N	72
I%	6.48
PV	−468 000
PMT	3483.77
FV	
P/Y	12
C/Y	12

FV = 384 093.360 038
After 6 years, the annuity has a balance of \$384 093.36.
His inheritance is added to give a new PV.
For another 14 years with inheritance added to annuity:

N	168
I%	6.48
PV	−459 093.36
PMT	
FV	0
P/Y	12
C/Y	12

PMT = 4164.0369
Billy's new monthly payment should be \$4164.04 to last for the remaining 14 years.

14 After 5 years with interest rate of 6.73%

N	60
I%	6.73
PV	−256 000
PMT	4000
FV	
P/Y	12
C/Y	12

FV = 736 93.9786
The annuity has a balance of \$73 693.98 after 5 years.
Interest rate changed to 5.95%

N	
I%	5.95
PV	−73 693.98
PMT	4000
FV	0
P/Y	12
C/Y	12

N = 19.3679
After the change, it will last another 19 months, or 1 year and 7 months.
Matilda's annuity would last 6 years and 7 months under these conditions.

15 After the first 10 years:

N	120
I%	7.24
PV	−1 645 250
PMT	10 500
FV	
P/Y	12
C/Y	12

FV = 1 544 636.162
After 10 years, the balance of the annuity is \$1 544 636.16
Increased interest rate for next 15 years:

N	180
I%	8.49
PV	−1 544 636.16
PMT	
FV	0
P/Y	12
C/Y	12

PMT = 15 201.5904
Their monthly payment for the remaining 15 years would be \$15 201.59.

7.4 Exam questions

1 The value of A is not changing, so the recurrence relation is subtracting the same value as the interest earned.
$0.025 \times 200\,000 = 5000$
The correct answer is **B**.

2 $V_1 = 1.0025(100\,000) - 555 = 99\,695$
$V_2 = 1.0025(99\,695) - 555 = 99\,389.2375$
$V_3 = 1.0025(99\,389.2375) - 555 = 99\,082.710\,59$
$V_4 = 1.0025(99\,082.710\,59) - 555 = 98\,775.417\,37$
$V_5 = 1.0025(98\,775.417\,37) - 555 = 98\,467.3559$
The correct answer is **C**.

3 Use Finance Solver on CAS:
N: 5 years × 12 months = 60
I(%): 5.2 p.a.
PV: −130 784.93
FV: −66 992.27
PpY/CpY: 12
Therefore, the payment (PMT) each month is \$3460.15.
***VCAA Assessment Report* note:**
1A common error was to choose option E, which corresponded to both the PV and FV being incorrectly allocated the same sign.
The correct answer is **E**.

7.5 Perpetuities

7.5 Exercise

1 $d = \frac{V_0 r}{100}$

$V_0 = 75\,000$ and $r = 6.25$

$d = \frac{75\,000 \times 6.25}{100}$

$= 4687.5$

The annual grant to the language centre will be $4687.50.

2 **a** $d = \frac{V_0 r}{100}$

$V_0 = 200\,000$ and $r = 5.82$

$d = \frac{200\,000 \times 5.82}{100}$

$= 11\,640$

The annual grant to the sporting club will be $11 640.

b $11 640 × 10 = $116 400

In 10 years, the sporting club would receive $116 400 in grants.

3 **a** Annually

$V_0 = \frac{100\,d}{r}$

$d = 15\,000$ $r = 5$

$V_0 = \frac{100 \times 15\,000}{5}$

$= 300\,000$

The amount invested in the perpetuity would be $300 000.

b Quarterly – d and r in per quarter

$V_0 = \frac{100\,d}{r}$

$d = \frac{15\,000}{4} = 3750 \qquad r = \frac{5}{4} = 1.25$

$V_0 = \frac{100 \times 3750}{1.25}$

$= 300\,000$

The amount invested in the perpetuity would be $300 000.

4 **a** $23 400 annually = $5850 per quarter

Quarterly payments would be $5850.

b $V_0 = \frac{100\,d}{r}$

$d = 5850 \quad r = \frac{6}{4} = 1.5$

$V_0 = \frac{100 \times 5850}{1.5}$

$= 390\,000$

The amount invested in the perpetuity would be $390 000.

5 One annual payment, but with interest compounded:

a yearly

N	1
I%	4.95
PV	−1 500 000
PMT	
FV	1 500 000
P/Y	1
C/Y	1

PMT = 74 250

The annual grant is $74 250.

b monthly

N	1
I%	4.95
PV	−1 500 000
PMT	
FV	1 500 000
P/Y	1
C/Y	12

PMT = 75 957.9257

The annual grant is $75 957.93

c daily

N	1
I%	4.95
PV	−1 500 000
PMT	
FV	1 500 000
P/Y	1
C/Y	365

PMT = 76 113.098 55

The annual grant is $76 113.10

6 One yearly payment, but with interest compounded:

a quarterly

N	1
I%	7.85
PV	−350 000
PMT	
FV	350 000
P/Y	1
C/Y	4

PMT = 28 294.4289

The yearly amount available is $28 294

b monthly

N	1
I%	7.85
PV	−350 000
PMT	
FV	350 000
P/Y	1
C/Y	12

PMT = 28 485.4036

The yearly amount available is $28 485

c daily

N	1
I%	7.85
PV	−350 000
PMT	
FV	350 000
P/Y	1
C/Y	365

PMT = 28 578.979

The yearly amount available is $28 579

7

N	1
I%	
PV	−625 000
PMT	12 500
FV	625 000
P/Y	2
C/Y	365

I = 3.9607

The bond interest rate would need to be 3.96% p.a. (correct to 2 decimal places).

8 a

N	1
I%	5.8
PV	−85 000
PMT	
FV	85 000
P/Y	1
C/Y	12

PMT = 5063.1904

The annual grant would be $5063.19

b

N	1
I%	
PV	−85 000
PMT	6750
FV	85 000
P/Y	1
C/Y	365

I = 7.642

The interest rate, compounded daily, would be 7.6% p.a. (correct to one decimal place).

9 a

N	1
I%	6.75
PV	−185 000
PMT	
FV	185 000
P/Y	1
C/Y	12

PMT = 12 881.1682

The annual scholarship amount would be $12 881 (to the nearest dollar).

b

N	1
I%	
PV	−185 000
PMT	13 881
FV	185 000
P/Y	1
C/Y	365

I = 7.2358

The new interest rate would be 7.24% p.a. (to 2 decimal places).

10 a Quarterly − d and r in per quarter

$$V_0 = \frac{100\,d}{r}$$

$$d = \frac{48\,000}{4} = 12\,000 \quad r = \frac{9.6}{4} = 2.4$$

$$V_0 = \frac{100 \times 12\ 000}{2.4}$$

$$= 500\,000$$

The amount invested in the perpetuity would be $500 000.

b

N	1
I%	9.6
PV	−500 000
PMT	
FV	500 000
P/Y	1
C/Y	365

PMT = 12 143.5428

The new quarterly payment would be $12 143.54.

11 a Annually

$$V_0 = \frac{100\,d}{r}$$

$d = 4600 \quad r = 6.25$

$$V_0 = \frac{100 \times 4\,600}{6.25}$$

$$= 73\,600$$

The amount invested in the perpetuity would be $73 600.

b With interest rate of 6.22%:

N	1
I%	6.22
PV	−73 600
PMT	
FV	73 600
P/Y	1
C/Y	365

PMT = 4722.876 615

The payment with the new fund would be $4722.88, an increase of $122.88 from the original fund.

12 a

N	1
I%	5.8
PV	−500 000
PMT	
FV	500 000
P/Y	1
C/Y	365

PMT = 29 855.0564

The Rugby Club could give $29 855 annually to encourage participation at the local level.

b $29 855 × 15 = $447 825

The local clubs would have received $447 825 over the 15 years.

13 Plan A: quarterly payments
interest compounded monthly

N	1
I%	6.2
PV	−950 000
PMT	
FV	950 000
P/Y	4
C/Y	12

PMT = 14 801.210 19
The quarterly payment would be \$14 801.21
Over a two year period:
\$14 801.21 × 8 = \$118 409.68
Plan B: monthly payments
interest compounded daily

N	1
I%	6.2
PV	−950 000
PMT	
FV	950 000
P/Y	12
C/Y	365

PMT = 4920.616 076
The monthly payment would be \$4920.62
Over a two year period:
\$4920.62 × 24 = \$118 094.88
Over a two-year period, Plan A would give more funds for grants by \$314.80.

14 **a** Monthly

$$V_0 = \frac{100\,d}{r}$$

$$d = 2750 \qquad r = \frac{6.6}{12} = 0.55$$

$$V_0 = \frac{100 \times 2750}{0.55} = 500\,000$$

The amount invested in the perpetuity would be \$500 000.

b

N	1
I%	6.6
PV	−500 000
PMT	
FV	500 000
P/Y	12
C/Y	365

PMT = 2757.3264
The new monthly payment would be
\$2757.33, an increase of \$7.33 per month.

15 Trust A
Biannual payment
Interest compounded monthly

N	1
I%	
PV	−800 000
PMT	12 500
FV	800 000
P/Y	2
C/Y	12

I = 3.104 847
Interest rate is 3.10% p.a.
Trust B
Monthly payments
Interest compounded monthly

N	1
I%	
PV	−800 000
PMT	2100
FV	800 000
P/Y	12
C/Y	12

I = 3.15
Interest rate is 3.15% p.a.
Trust C
Fortnightly payments
Interest compounded daily

N	1
I%	
PV	−800 000
PMT	980
FV	800 000
P/Y	26
C/Y	365

I = 3.1831
Interest rate is 3.18% p.a.
Trust C is giving the higher rate of interest on the invested perpetuity.

7.5 Exam questions

1 **a** \$1890 × 12 = \$22 680 [1 mark]

b \$420 000 – the final value of a perpetuity remains the same as the principal value [1 mark]

c Interest rate per month is $\frac{5.4\%}{12} = 0.45\%$

$S_0 = 420\,000 \; S_{n+1} = 1.0045 \times S_n - 1890$ [1 mark]

2 **a** 5.2% p.a. is equivalent to $\frac{5.2}{12}\%$ per month. [1 mark]

$$\frac{\frac{5.2}{12}}{100} \times 360\,000 = \$1560 \text{ per month.}$$

b Use Finance Solver on CAS: [2 marks]

N: 4 years 12 months = 48
I (%): 3.8 p.a.
PV: −360 000
PMT: 500
FV : 444 872.9444992

PpY/CpY: 12
After 4 years, Alex's investment grows to $444 872.94 [1 mark]
N: 2 years 12 months = −24
I3.8
PV: −444 872.94
PMT: −805.65070094875
FV : 500 000
PpY/CpY: 12
To grow to $500 000 in a further two years, Alex's new monthly payment will be $805.65. [1 mark]

3 a $Q = \dfrac{P \times r}{100}$

$460 = \dfrac{P \times 3.68}{100}$ [1 mark]

$P = \$12\,500$

b A perpetuity is an investment that provides regular payments that continue forever. Therefore, they will be able to provide the scholarship for an infinite number of years. [1 mark]
scholarship for an infinite number of years. [1 mark]

***VCAA Assessment Report* note:**
Many students did not know that perpetuities pay out only the interest earned, while the principal remains unchanged.
The most common incorrect answer was $\dfrac{12\,500}{460} \approx 27$ years

7.6 Review

7.6 Exercise

Multiple choice

1 N: 104
I%: 9.75
PV: 14 000
PMT: unknown
FV: 0
PY: 26
CY: 26
The repayment is $162.82.
The correct answer is **C**.

2 $r = \dfrac{8.2}{4} = 2.05\%$
Interest = 0.0205 × 22 000 = 451
Interest = $451
The correct answer is **B**.

3 N: unknown
I%: 10.5
PV: 41 000
PMT: − 588.39
FV: 0
PY: 12
CY: 12
N = 108
N is closest to 110
The correct answer is **E**.

4 N: unknown
I%: 5
PV: 56 000
PMT: − 1500
FV: 0
PY: 4
CY: 4
N = 50.6 ≈ 51
≈ 12.75 years
The correct answer is **C**.

5 N: 130
I%: 6.5
PV: 24 000
PMT: unknown
FV: 0
PY: 26
CY: 26
d = $216.47
The correct answer is **E**.

6 Enter the values into Finance solver and solve for FV.
N: 20
I%: 6.5
PV: −24 000
PMT: ?
FV: 0
PpY: 4
CpY: 4
The payment (FV) = $1415.18
The correct answer is **D**.

7 N: 20
I%: 5.5
PV: 24 000
PMT: unknown
FV: 0
PY: 4
CY: 4
$d = \$1380.73$
The correct answer is **B**.

8 $V_0 = a,\ V_{n+1} = RV_n + d$
$V_0 = 2000$
$r = \dfrac{6}{12} = 0.5$
$R = 1 + \dfrac{0.5}{100} = 1.005$
$d = 360$
$V_0 = 2000,\ V_{n+1} = 1.005V_n + 360$
The correct answer is **D**.

9 Interest only loans:PV = −FV
PpY = 12
N = any number because loan is not paid off
Using Finance Solver
PMT = $1644.458
The correct answer is **D**.

10 Using Finance Solver,
N: 204
I%: 9.6
PV: − 92 200
PMT: unknown
FV: 800 000
PY: 12
CY: 12
PMT : \$649.91. Her employee is contributing \$500 per month, so Claire needs to contribute \$149.91.
The correct answer is **D**.

11 a $V_0 = 16\,000 \quad d = 340 \quad r = \dfrac{9.6}{12} = 0.8\%$

$R = 1 + \dfrac{0.8}{100} = 1.008$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.008\,V_n - 340, \quad V_0 = 16\,000$

b $V_1 = 1.008\,V_0 - 340$
$= 1.008 \times 16\,000 - 340$
$= 15\,788$

$V_2 = 1.008\,V_1 - 340$
$= 1.008 \times 15\,788 - 340$
$= 15\,574.304$

$V_3 = 1.008\,V_2 - 340$
$= 1.008 \times 15\,574.304 - 340$
$= 15\,358.8984$

Claire owes \$15 358.90 after 3 payments.

12 a $V_0 = 6500 \quad d = 750 \quad r = \dfrac{12.24}{12} = 1.02\%$

$R = 1 + \dfrac{1.02}{100} = 1.0102$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.0102\,V_n - 750, \quad V_0 = 6500$

b $V_1 = 1.0102\,V_0 - 750$
$= 1.0102 \times 6500 - 750$
$= 5816.3$

$V_2 = 1.0102\,V_1 - 750$
$= 1.0102 \times 5816.3 - 750$
$= 5125.63$

$V_3 = 1.0102\,V_2 - 750$
$= 1.0102 \times 5125.63 - 750$
$= 4427.91$

$V_4 = 1.0102\,V_3 - 750$
$= 1.0102 \times 4427.91 - 750$
$= 3723.07$

Tom owes \$3723.07 after 4 payments.

c

6500	6500
6500 × 1.0102 − 750	5816.3
5816.3 × 1.0102 − 750	5125.63
5125.63 × 1.0102 − 750	4427.91
4427.91 × 1.0102 − 750	3723.07
3723.07 × 1.0102 − 750	3011.05
3011.05 × 1.0102 − 750	2291.76
2291.76 × 1.0102 − 750	1565.14
1565.14 × 1.0102 − 750	831.10
831.10 × 1.0102 − 750	89.5779

Nine monthly payments will reduce the loan to under \$100.

d For a zero balance the last payment needs to be increased by \$89.58.
The last payment will be \$839.58

13 a 9.2% per year = 2.3% per quarter = 0.023 per quarter
See the table bottom of the page*

b The loan has been overpaid by \$9.55, so the last payment needs to be decreased by this amount. The last payment would be \$1315.45.

14 a

N	60
I%	7.35
PV	76 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −1517.4726
Zoe's monthly payments would be \$1517.47.

b Total repaid in 5 years
= \$1517.47 × 60
= \$91 048.20
Total interest paid in 5 years
= \$91 048.20 − 76 000
= \$15 048.20
Zoe paid \$15 048.20 in interest over the 5 years.

15 a $V_0 = 85\,000 \; d = 7475 \; r = \dfrac{10.08}{12} = 0.84\%$

$R = 1 + \dfrac{0.84}{100} = 1.0084$

$V_{n+1} = R\,V_n - d, \quad V_0 = a$

$V_{n+1} = 1.0084\,V_n - 7475, \quad V_0 = 85\,000$

b $V_1 = 1.0084\,V_0 - 7475$
$= 1.0084 \times 85\,000 - 7475$
$= 78\,239$

*13a

Payment number, *n*	Payment (\$)	Interest (\$)	Principal reduction (\$)	Balance of loan (\$)
0				5000.00
1	1325.00	0.023 × 5000 = 115	1325 − 115 = 1210	5000 − 1210 = 3790
2	1325.00	0.023 × 3790 = 87.17	1325 − 87.17 = 1237.83	3790 − 1237.83 = 2552.17
3	1325.00	0.023 × 2552.17 = 58.70	1325 − 58.70 = 1266.30	2552.17 − 1266.30 = 1285.87
4	1325.00	0.023 × 1285.87 = 29.58	1325 − 29.58 = 1295.42	1285.87 − 1295.42 = −9.55

$V_2 = 1.0084\ V_1 - 7475$
$= 1.0084 \times 78239 - 7475$
$= 71\,421.2$

$V_3 = 1.0084\ V_2 - 7475$
$= 1.0084 \times 71421.2 - 7475$
$= 64546.1$

Penelope's annuity has a balance of \$64 546 after 3 months.

16 perpetuity at 3.52% p.a. compounded annually.

$d = \dfrac{V_0\,r}{100}$

$V_0 = 550\,000$ and $r = 3.52$

$d = \dfrac{550\,000 \times 3.52}{100}$

$= 19\,360$

The annual payment to the college would be \$19 360.

17 perpetuity: $d = 8250$ and $r = \dfrac{10}{12} = \dfrac{5}{6}\%$ per month

$V_0 = \dfrac{100\,d}{r}$

$V_0 = \dfrac{100 \times 8250}{\frac{5}{6}}$

$= 990\,000$

The company would need to invest \$990 000 in a perpetuity fund.

18

N	300
I%	3.95
PV	480 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −2520.3839

Their monthly payment would be \$2520.38 (to the nearest cent)

19

N	240
I%	4.2
PV	
PMT	−3500
FV	0
P/Y	12
C/Y	12

PV = 567 655.874 49

Anita could borrow \$567 656 for the home loan (to the nearest dollar)

20 a

N	180
I%	4.65
PV	125 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −965.852 07

Paul's monthly instalments will be \$965.85, to the nearest cent.

b

N	120
I%	4.65
PV	125 000
PMT	−965.85
FV	
P/Y	12
C/Y	12

FV = −51 618.95 801

After 10 years, Paul owes \$51 618.96 on his loan.

c

N	24
I%	4.65
PV	51 618.96
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −2256.5131

To repay the remaining loan in two years, Paul would need to make monthly payments of \$2256.51.

21 a interest compounded monthly

N	1
I%	9.25
PV	−250 000
PMT	
FV	250 000
P/Y	1
C/Y	12

PMT = 24 131.0369

If the interest is compounded monthly, the annual grant is \$24 131, to the nearest dollar.

b interest compounded daily

N	1
I%	9.25
PV	−250 000
PMT	
FV	250 000
P/Y	1
C/Y	365

PMT = 24 225.0717

If the interest is compounded daily, the annual grant is \$24 225, to the nearest dollar.

Extended response

22 a

N	
I%	5.8
PV	−657 500
PMT	5250

FV	0
P/Y	12
C/Y	12

N = 192.8107

The annuity would last 192.8 months.

Mabel's funds would last 16 years and 1 month, to the nearest month.

b

N	240
I%	5.8
PV	−657 500
PMT	
FV	0
P/Y	12
C/Y	12

PMT = 4634.9868

To last 20 years, Mabel's monthly payment would be \$4635 (to the nearest dollar)

23 a

N	104
I%	
PV	23 400
PMT	−275
FV	0
P/Y	26
C/Y	26

I% = 10.307 199

Interest rate is 10.307% p.a. (correct to 3 decimal places)

b

N	48
I%	10.307
PV	23 400
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −596.9403

Monthly payment is \$596.94

c Fortnightly: $\$275 \times 26 \times 4 = \$28\,600$

Monthly: $\$596.94 \times 12 \times 4 = \$28\,653.12$

The fortnightly loan would be the better choice as you would pay \$54.12 less over the 4 years.

24 Plan A:

Interest: 7.8% p.a. compounded quarterly

N	1
I%	7.8
PV	−750 000
PMT	
FV	750 000
P/Y	12
C/Y	4

PMT = 4843.6513

Plan B:

Interest: 7.8% p.a. compounded daily

N	1
I%	7.8
PV	−750 000
PMT	
FV	750 000
P/Y	12
C/Y	365

PMT = 4890.3539

Plan B is the better plan by an extra \$46.70 per month.

25 a i annuity lasting 25 years

N	300
I%	5.32
PV	−525 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = 3167.769

Jacinta's monthly annuity payment is \$3167.77 (to the nearest cent)

ii perpetuity

N	1
I%	5.32
PV	−525 000
PMT	
FV	525 000
P/Y	12
C/Y	365

PMT = 2332.4965

Jacinta's monthly perpetuity payment is \$2332.50 (to the nearest cent)

b i For the annuity after 15 years:

N	180
I%	5.32
PV	−525 000
PMT	3167.77
FV	
P/Y	12
C/Y	12

FV = 294 302.211

After 15 years, the annuity investment has a balance of \$294 302.21

ii After 15 years, the perpetuity investment has a balance of \$525 000.

26 a

N	360
I%	3.85
PV	540 000
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −2531.56445
Sally's monthly payment is $2531.56.

b Find the balance after 5 years

N	60
I%	3.85
PV	540 000
PMT	−2531.56
FV	
P/Y	12
C/Y	12

FV = −487 223.932 55
Balance after 5 years: $487 223.93
For next 25 years, with increased interest rate

N	300
I%	4.35
PV	487 223.93
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −2666.8339
New payment would be $2666.83
Sally's new monthly payment after the 5 years would be $2666.83.

c After next 20 years at increased interest rate and new payment:

N	120
I%	4.35
PV	487 223.93
PMT	−2666.83
FV	
P/Y	12
C/Y	12

FV = −352 127.8479
After paying inheritance of $150 000, outstanding balance is $202 127.85, with another 5 years of payments.

N	60
I%	4.35
PV	202 147.85
PMT	
FV	0
P/Y	12
C/Y	12

PMT = −3754.502 507
After paying her inheritance into the loan, Sally's new payment would be $3755 per month, to the nearest dollar.

7.6 Exam questions

1 $449 060.08 − $422 051.93 = $27 008.15
The correct answer is **C**.

2 Use Finance Solver on CAS:
N: ??
I(%): 4
PV: −500 000
PMT: 44 970.55
FV: 0
PpY: 1
CpY: 1
Therefore, N = 15 years.
The correct answer is **B**.

3 a i Principal reduction = $318 718.08 − $318 074.23
= $643.85 [1 mark]

ii Monthly interest = $\frac{3.6}{12}\% \times \$318\,074.23 = \$954.22$
Therefore, the balance after payment 4 will be
$318 074.23 + $954.22 − $1600 = $317 428.45[1 mark]

b Note that the monthly interest rate will be $\frac{3.6}{12} = 3\%$
$S_0 = 320\,000,\ S_{n+1} = 1.003 \times S_n - 1600$ [1 mark]

4 Change in rate:
$\frac{I_{28}}{V_{27}} - \frac{I_{27}}{V_{26}} = \frac{1002.26}{227\,785.76} - \frac{961.90}{229\,023.86} \approx +0.02\%$ per month
$+0.02 \times 12 = +0.24\%$ per annum.
The interest rate increased by 0.24% per annum.
The correct answer is **A**.

5 The smallest effective interest rate for the loans will cost Daniel the least amount of money.
Since Loans I, II and III are all compounding weekly, the lowest rate of the three will be Loan I with 12.6%.
Loans IV and V are both compounding quarterly, so the lowest rate out of them will be Loan IV with 12.7%.
Effective interest rate of Loan I:
$$r = \left(1 + \frac{i}{n}\right)^n - 1 = \left(1 + \frac{0.126}{52}\right)^{52} - 1 \approx 13.41\% \text{ annually}$$
Effective interest rate of Loan IV:
$$r = \left(1 + \frac{i}{n}\right)^n - 1 = \left(1 + \frac{0.127}{4}\right)^{4} - 1 \approx 13.32\% \text{ annually}$$
Since Loan IV has a lower effective interest rate over the year, it will cost Daniel the least amount of money.
The correct answer is **D**.
VCCA Examiner report **note:**
This question required an understanding that both the interest rate and compounding period in combination determine the overall cost of the loan. Students who selected option A seemed to simply select the lowest interest rate without considering the compounding period.

Topic 8 — Investigating relationships between two numerical variables

8.2 Response and explanatory variables

8.2 Exercise

1 **a** An exam result will depend on the amount of time studying, which means that percentage will be the response variable.
b As a person's age increases, their income will tend to increase. This means that income will be the response variable.

2 **a** In general, those with more money will have better access to health services and thus are likely to live longer. Thus, age will be the response variable as it will respond to changes in income.
b Those who spend more time exercising each week will have better resting heart rate. As the heart rate is dependent on the amount of exercise, BPM is the response variable.

3 **a** If a person puts on weight, it tends to be stored as fat around the body. Therefore, waist size will respond to changes in body weight. So, waist is the response variable.
b Since weight is being predicted from length, weight is the response variable.

4 In this case, the research is being conducted on sleep and the impact of screen time. This means that the researcher is treating screen time as the explanatory variable and investigating how it affects sleep duration.

5 The office manager wants to predict the amount of coffee being consumed in the office, so coffee is the response variable in this scenario.

8.2 Exam questions

1 Since humidity at 3 pm is the value being predicted, it is the response variable, making humidity at 9 am the explanatory variable. [1 mark]

2 Evening congestion level is placed on the vertical axis making it the response variable. [1 mark]

3 Population is placed on the vertical axis making it the response variable. [1 mark]

8.3 Scatterplots and basic correlation

8.3 Exercise

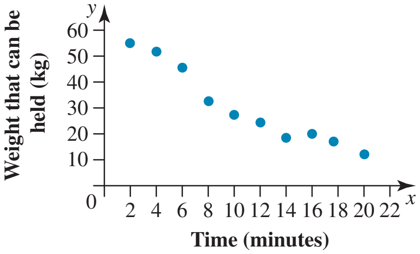
1 Explanatory variable = time of day
Response variable = time taken to eat a pizza

2 Explanatory variable = time spent using phone apps
Response variable = amount of data required

3 **a** Explanatory variable = age
Response variable = number of star jumps
b Explanatory variable = quantities of chocolate
Response variable = cost
c Explanatory variable = number of songs
Response variable = memory used
d Explanatory variable = food supplied
Response variable = growth rate

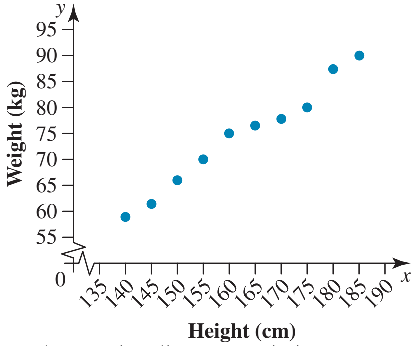
4 **a** File size
b Strong positive correlation

5 **a** Explanatory variable = time
Response variable = weight that can be held (kg)
b



c Strong negative correlation

6 **a** Explanatory variable = height
Response variable = weight
b



7 **a** Weak, negative, linear association
b No association
c Moderate, positive, linear association

8 Various possible answers, for example:
a Loss of money over time
b Temperature and number of shoes owned.

9 As the EV increases, so does the RV showing a positive relationship. The data is clearly curved, so the form is non-linear. Given that r is above 0.75, the strength is high.

10 As the EV increases, the RV decreases showing a negative relationship. The data is straight, so the form is linear. Since r falls between −0.50 and −0.75, the relationship is moderate in strength.

8.3 Exam questions

1 Since the association is positive, as the EV increases, we expect to see the RV increase. Avoid language like 'will increase' as we are exploring the association and so cannot make definite statements.
Thus, the correct answer is **D**.

2 Since $r = 0.9496$ the association must be positive and strong. The data clearly forms a straight line, so the form is linear. [1 mark]

3 The scatterplot that shows weak negative correlation is **C**. The correct answer is **C**.

8.4 Informal interpretation of association and causation

8.4 Exercise

1 a $r = 0.8947$, which indicates a strong positive, linear association.

b $r^2 = 0.8005$, which suggests that 80% of the variation of the subscription costs can be explained by the variation in the number of issues per year.

Other factors in the remaining 20% might include number of coloured pages, weight of postage, amount of advertising in each issue, etc.

2 a It indicates a positive linear association between the x and y-variables, when $r = 0.7211$, and a negative linear association when $r = -0.7211$.

b It could be suggested that 52% of the variation in y can be explained by the variation in x.

c As the coefficient of determination provides information about the strength of the data rather than the causation.

3 a Indicates a moderate, positive, linear association

b Indicates a strong, negative, linear association

c Indicates no linear association

d Indicates a weak, positive, linear association

4 a $r^2 = 0.7962$. Therefore, 79.62% of the variation in a child's health can be explained by the variation in the child's diet.

b $r^2 = 0.9994$. Therefore, 99.94% of the variation in the amount of water in the ocean can be explained by the variation in global warming.

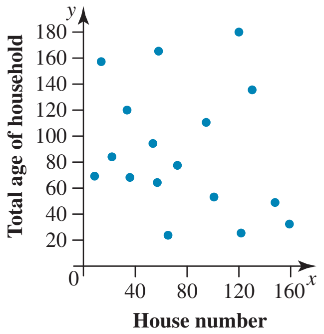
5 $r = 0.989$

$r^2 = 0.978$

6 42.19%

7 A strong positive relationship between the amount of time spent warming up and the number of matches won.

8 a



b The data points appear random, which indicates no correlation.

c $r = -0.2135$

$r^2 = 0.0456$

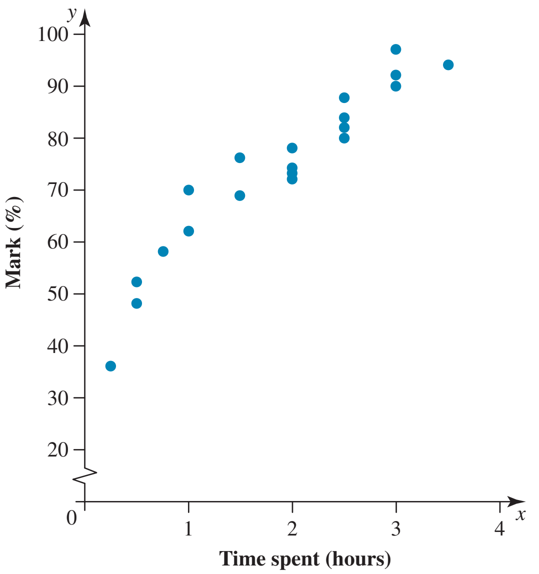
d There is no relationship between the house number and the age of the household.

e 95.44%

9 a Explanatory variable = time spent (hours)

Response variable = mark (%)

b



c Strong, positive, linear correlation

d Each person's understanding of the topic is different and their study habits are unique. Therefore, 1 hour spent on the assignment does not guarantee a consistent result. Individual factors will also influence the resulting assignment mark.

e $r = 0.952$

$r^2 = 0.906$

f There is a strong relationship between the time spent on an assignment and the resulting grade. As the time spent increased, so did the mark.

10 Various answers are possible.

An example data set would be:

x	y
1	30
2	25
2	18
3	11
4	18
5	17
5	10
6	11
7	16
7	8
8	6
9	4
10	12
12	10
14	7

8.4 Exam questions

1 $r = \pm\sqrt{r^2} = \pm\sqrt{0.8339} = \pm 0.913$

Since the graph shows a negative slope, r must be a negative value.

The correct answer is **A**.

***VCAA Examination Report* note:**

The value of the correlation coefficient r was required, given that the coefficient of determination was $r^2 = 0.8339$. Many students incorrectly took the positive square root value and chose option E.

The scatterplot on the examination showed that the direction of the association was negative and therefore –0.913 was required.

2 The percentage of variation is found from $r^2\%$

$r^2 = 0.862^2 = 0.743044$

The percentage of variation is 74.3% to 1 decimal place. [1 mark]

3 The answer is D as both variables (number of stray dogs and number of stray cats) is tied to a common third variable, which is population.

Options A and E are not correct as they try to offer an explanation rather than relating the variables to causation, common response, coincidence or confounding.

Option B is simply an incorrect statement about correlation.

Option C states the association is coincidence, which is less correct than a common response to population size.

The correct answer is **D**.

8.5 The line of good fit and predictions

8.5 Exercise

1 **a** The increase in price of 4 cents for every additional person the venue holds

b The price of a ticket if a venue has no capacity

c No, as the smallest venues would still have some capacity.

2 Pick two points the line passes through: (155, 160) and (175, 185).

Calculate the gradient using the points (155, 160) and (175, 185).

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{185 - 160}{175 - 155}$$
$$= \frac{25}{20}$$
$$= 1.25$$

Substitute into $y = a + bx$, $y = a + 1.25x$

Substitute either point into $y = a + 1.25x$

Let's substitute the point (155, 160):

$160 = a + 1.25(155)$

$a = 160 - 193.75$

$a = -33.75$

$y = -33.75 + 1.25x$

3 **a** $y = 86.86 + 0.57x$

b Calculate the gradient using the points given (170, 184) and (177, 188).

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{188 - 184}{177 - 170}$$
$$= \frac{4}{7}$$
$$= 0.57142$$

Substitute into $y = a + bx$, $y = a + 0.57\,142\,x$

Substitute either point into $y = a + 0.57\,142x$

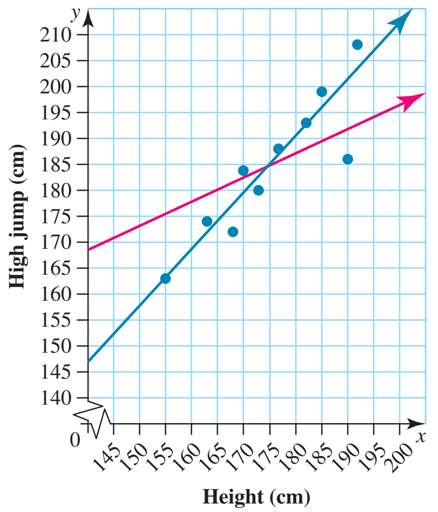
Let's substitute the point (170, 184):

$184 = a + 0.57\,142(170)$

$a = 184 - 97.1414$

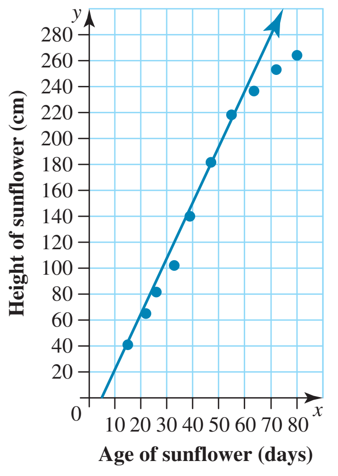
$a = 86.8586$

Rounding correct to 2 decimal places (Note: If you round the gradient, your answer to **c** will be different to the one given.)



c Nidya's line of good fit is not a good representation of the data. In this instance having only two points of data to create the line of best fit was not sufficient.

4 **a**



Xavier's line is closer to the values above the line than those below it, and there are more values below the line than above it, so this is not a great line of good fit.

b Calculate gradient using the points (10, 16) and (70, 280).

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{280 - 16}{70 - 10}$$
$$= \frac{264}{60}$$
$$= 4.4$$

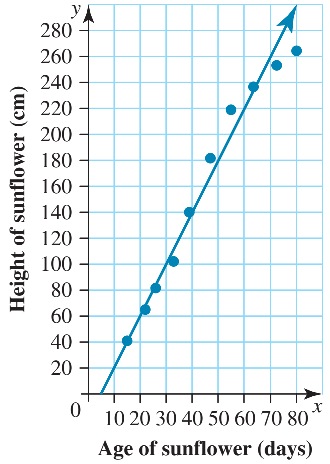
Substitute into $y = a + bx, y = a + 4.4x$

Substitute either point into $y = a + 4.4x$

Let's substitute the point (10, 16):

$$16 = a + 4.4(10)$$
$$a = 16 - 44$$
$$a = -28$$
$$y = -28 + 4.4x$$

c



Patricia's line is more appropriate as the data points lie on either side of the line and the total distance of the points from the line appears to be minimal.

d Calculate the gradient using the points (10, 18) and (70, 258).

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{258 - 18}{70 - 10}$$
$$= \frac{240}{60}$$
$$= 4$$

Substitute into $y = a + bx,\ y = a + 4x$

Substitute either point into $y = a + 4x$

Let's substitute the point (10, 18):

$$18 = a + 4(10)$$
$$a = 18 - 40$$
$$a = -22$$
$$y = -22 + 4x$$

e The line of good fit does not approximate the height for values that appear outside the parameters of the data set, and the y-intercept lies well outside these parameters.

5 a Substitute $b = 2.5$ into $y = a + bx,\ y = a + 2.5x$

Substitute (16, 41.5) into $y = a + 2.5x$

$$41.5 = a + 2.5(16)$$
$$a = 1.5$$
$$y = 1.5 + 2.5x$$

b i $y = 1.5 + 2.5(25)$

$y = \$64.00$

ii $65 = 1.5 + 2.5x$

$63.5 = 2.5x$

$x = \$25.40$

iii $y = 1.5 + 2.5(11)$

$y = \$29.00$

iv $28 = 1.5 + 2.5x$

$26.5 = 2.5x$

$x = \$10.60$

6 a The student did not assign the x- and y-values for each point before calculating the gradient, and mixed up the values.

b (−2, 5) (3, 1)

$x_1 = -2 \quad x_2 = 3$

$y_1 = 5 \quad y_2 = 1$

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$b = \frac{1 - 5}{3 - (-2)}$$
$$b = -\frac{4}{5}$$

Substitute into $y = a + bx,\ y = c - \frac{4}{5}x$

Substitute either point into $y = c - \frac{4}{5}x,\ y = -\frac{4}{5}x + c$

Substitute (3, 1)

$$1 = -\frac{4}{5} \times 3 + c$$
$$1 = -\frac{12}{5} + c$$
$$\frac{17}{5} = c$$
$$y = \frac{17}{5} - \frac{4}{5}x$$

or

$$y = -\frac{4}{5}x + \frac{17}{5}$$

7 a Lines of good fit will vary, but should split the data points on either side of the line and minimise the total distance from the points to the line.

b Answers will vary. Students can take any two points on the scatterplot and determine the equation of line.

c The amount of crime in a suburb with 0 people.

d No, if there are 0 people in a suburb, there should be no crime.

8 a (0, −5) and (100, 127)

$x_1 = 0,\ x_2 = 100$

$y_1 = -5,\ y_2 = 127$

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{127 - (-5)}{100 - 0}$$
$$= \frac{132}{100}$$
$$= 1.32$$

Substitute into $y = a + bx$, $y = a + 1.32x$

Substitute either point into $y = a + 1.32x$

Substitute $(0, -5)$:

$-5 = a + 1.32(0)$

$a = -5$

$y = -5 + 1.32x$

b For each additional nest the number of surviving turtles increases by 1.32.

c The y-intercept represents the number of surviving turtles from 0 nests. This value is not realistic as you can't have a negative amount of turtles.

d **i** $y = -5 + 1.32(135)$

$y = 173.2$

Estimate: 173

ii $12 = -5 + 1.32x$

$17 = 1.32x$

$x = 12.8787$

Estimate: 13

e The answer to **d i** was made using extrapolation, so is not as reliable as the answer to part **d ii**, which was made using interpolation. However, due to the nature of the data in question, we would expect this relationship to continue and for both answers to be quite reliable.

9 **a** $(-2, 2)$ and $(-2, 6)$

$x_1 = -2, \; x_2 = -2$

$y_1 = 2, \; y_2 = 6$

$$b = \frac{6-2}{(-2)-(-2)}$$

$$b = \frac{4}{0}$$

Cannot divide by zero; therefore, the gradient is undefined.

b Line is a vertical line passing through $x = -2$; therefore, equation is $x = -2$.

10 **a** $y = -77 + 9x$

$y = -77 + 9(27.9)$

$y = 174.1$

Therefore 174 ice-creams (cannot have 0.1 of an ice-cream).

b $y = -77 + 9x$

$y = -77 + 9(15.2)$

$y = 59.8$

Therefore 60 ice-creams.

c The estimate in part **a** is reliable as it was made using interpolation. It is located within the parameters of the original data set and appears consistent with the given data.

The estimate in part **b** is unreliable as it was made using extrapolation and is located well outside the parameters of the original data set.

11 **a** $y = 55 + 0.08x$

$y = 55 + 0.08(713)$

$y = 112.04$

Estimate: \$112

b $y = 55 + 0.08x$

$y = 55 + 0.08(3121)$

$y = 304.68$

Estimate: \$305

c All estimates outside the parameters of Georgio's original data set (400 km to 2000 km) will be unreliable, with estimates further away from the data set more unreliable than those closer to the data set.

Other factors that might affect the cost of flights include air taxes, fluctuating exchange rates and the choice of airlines for various flight paths.

12 **a** Points $(3.5, 55.8)$ and $(4.8, 33.3)$

$x_1 = 3.5, \; x_2 = 4.8$

$y_1 = 55.8, \; y_2 = 33.3$

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{33.3 - 55.8}{4.8 - 3.5}$$

$$= -\frac{22.5}{1.3}$$

$$= -17.30\,769$$

Substitute into $y = a + bx$, $y = a - 17.30\,769x$

Substitute either point into $y = a - 17.30\,769x$

Substitute $(4.8, 33.3)$:

$33.3 = a - 17.30\,769(4.8)$

$33.3 + 83.07\,692 = a$

$a = 116.3769$

$y = 116.38 - 17.31x$

b For each increase in 1L of lung capacity, the swimmer will take less time to swim 25 metres.

c **i** $y = 116.38 - 17.31(3.2)$

$y = 60.988$

Estimate: 61.0 seconds

ii $y = 116.38 - 17.31(4.4)$

$y = 40.216$

Estimate: 40.2 seconds

iii $y = 116.38 - 17.31(5.3)$

$y = 24.637$

Estimate: 24.6 seconds

d As Mariana has only two data points and we have no idea of how typical these are of the data set, the equation for the line of good fit and the estimates established from it are all very unreliable.

13 **a** Points $(66, 79)$ and $(84, 90)$

$x_1 = 66, \; x_2 = 84$

$y_1 = 79, \; y_2 = 90$

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

$$b = \frac{90 - 79}{84 - 66}$$

$$b = \frac{11}{18}$$

$$b = 0.61\,111$$

Substitute into $y = a + bx$, $y = a + 0.611x$

Substitute either point into $y = a + 0.611x$

Substitute $(66, 79)$:

$79 = a + 0.611(66)$

$79 - 40.333 = a$

$a = 38.6667$

$y = 38.67 + 0.61x$

b $y = 38.67 + 0.61(75.3) = 84.603$

$y = 38.67 + 0.61(65.6) = 78.686$

$y = 38.67 + 0.61(83.1) = 89.361$

$y = 38.67 + 0.61(73.9) = 83.749$

$y = 38.67 + 0.61(79.0) = 86.86$

$y = 38.67 + 0.61(84.7) = 90.337$

$y = 38.67 + 0.61(64.4) = 77.954$

$y = 38.67 + 0.61(72.4) = 82.834$

$y = 38.67 + 0.61(68.7) = 80.577$

$y = 38.67 + 0.61(80.2) = 87.592$

See table at bottom of the page*

c The predicted and actual handball efficiencies are very similar in values. A couple of results are identical, and only a couple of them are significantly different.

14 a i (400, 80) and (3400, 97)

$x_1 = 400,\ \ x_2 = 3400$

$y_1 = 80,\ \ y_2 = 97$

$$\begin{aligned} b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{97 - 80}{3400 - 400} \\ &= \frac{17}{3000} \\ &= 0.0057 \end{aligned}$$

Substitute into $y = a + bx$, $y = a + 0.0057x$

Substitute either point into $y = a + 0.0057x$

Substitute $(400, 80)$:

$$\begin{aligned} 80 &= a + 0.0057\,(400) \\ 80 - 2.26667 &= a \\ a &= 77.733 \\ y &= 77.7333 + 0.0057x \end{aligned}$$

ii (300, 78) and (3200, 95)

$x_1 = 300,\ \ x_2 = 3200$

$y_1 = 78,\ \ y_2 = 95$

$$\begin{aligned} b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{95 - 78}{3200 - 300} \\ &= \frac{17}{2900} \\ &= 0.005\,86 \end{aligned}$$

Substitute into $y = a + bx$, $y = a + 0.005\,86x$

Substitute either point into $y = a + 0.005\,86x$

Substitute $(300, 78)$:

$$\begin{aligned} 78 &= a + 0.00\,586(300) \\ 78 - 1.758 &= a \\ a &= 76.242 \\ y &= 76.242 + 0.0059x \end{aligned}$$

iii (400, 75) and (2400, 95)

$x_1 = 400,\ \ x_2 = 2400$

$y_1 = 75,\ \ y_2 = 95$

$$\begin{aligned} b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{95 - 75}{2400 - 400} \\ &= \frac{20}{2000} \\ &= 0.01 \end{aligned}$$

Substitute into $y = a + bx$, $y = a + 0.01x$

Substitute either point into $y = a + 0.01x$

Substitute $(400, 75)$:

$$\begin{aligned} 75 &= a + 0.01(400) \\ 75 - 4 &= a \\ a &= 71 \\ y &= 71 + 0.01x \end{aligned}$$

iv (430, 67) and (1850, 95)

$x_1 = 430,\ x_2 = 1850$

$y_1 = 67,\ \ y_2 = 95$

$$\begin{aligned} b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{95 - 67}{1850 - 430} \\ &= \frac{28}{1420} \\ &= 0.019\,72 \end{aligned}$$

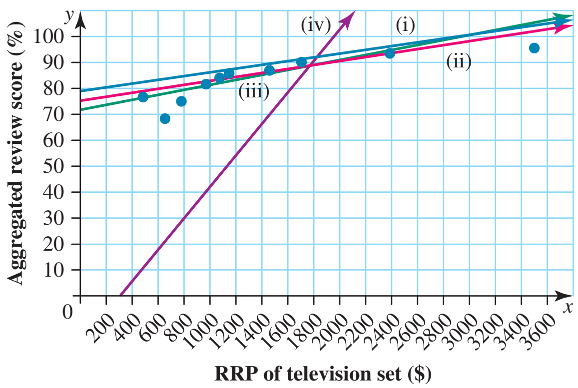
Substitute into $y = a + bx$, $y = a + 0.019\,72x$

Substitute either point into $y = a + 0.019\,72x$

Substitute $(430, 67)$:

$$\begin{aligned} 67 &= a + 0.019\,72(430) \\ 67 - 8.47\,887 &= a \\ a &= 58.5211 \\ y &= 58.5211 + 0.0197x \end{aligned}$$

b



c Line **iii** is the most appropriate line of good fit for this data.

15 a This point of data is clearly an outlier in terms of the data set.

b Lines of good fit will vary but should split the data points on either side of the line and minimise the total distance from the points to the line.

c Answers will vary. Students can take any two points on the scatterplot and determine the equation of line.

d The increase in box office taking per $1m increase in the leading actor/actress salary

*13 b

Kicking efficiency (%)	75.3	65.6	83.1	73.9	79.0	84.7	64.4	72.4	68.7	80.2
Predicted handball efficiency (%)	84.6	78.7	89.4	83.7	86.9	90.3	78.0	82.8	80.6	87.6

e Answers will vary. Sample responses are given below:

i. 18
ii. 355
iii. 5
iv. Hard to predict as the graph could either go up or down and the data after \$14 million is not shown on the scatterplot.

f The answers to parts **iii** and **iv** are going to be considerably less reliable than the answers to parts **i** and **ii**, as they're created using extrapolation instead of interpolation, which is less reliable.

16 a (0, 267) and (2000, 627)

$x_1 = 0, \quad x_2 = 2000$

$y_1 = 267, \quad y_2 = 627$

$$b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{627 - 267}{2000 - 0} = \frac{360}{2000} = 0.18$$

Substitute into $y = a + bx$, $y = a + 0.18x$

Substitute either point into $y = a + 0.18x$

Substitute (0, 267):

$$267 = a + 0.18(0)$$
$$a = 267$$
$$y = 267 + 0.18x$$

b $\dfrac{450 + 1120 + 330 + 1750 + 200 + 1400 + 630 + 800 + 2050}{10}$

= 971 years

c $\dfrac{345 + 485 + 305 + 560 + 240 + 525 + 390 + 430 + 465 + 590}{10}$

= 433.5 cm in diameter

d $\dfrac{433.5}{971}$

0.446

e Differ by a factor of 3. They are very similar.

17 a −1.837

b 1.701

c Positive trend

18 a 105.9

b −1.476

c A negative trend

19 a 60

b −5

c Negative

d $y = 60 - 5x$

$y = 60 - 5 \times 40$

$y = -140$

20 a Age in months

b $M = 0.157 + 0.312A$

$= 0.157 + 0.312 \times 6$

$= 2.029$ mL

c $M = 0.157 + 0.312A$

$2.5 = 0.157 + 0.312x$

$x = \dfrac{2.5 - 0.157}{0.312}$

$x = 7.5$

$x = 7.5$ months old

21 a Cost per night

b $C = 281.92 - 50.471d$

$= 281.92 - 50.471 \times 2.5$

$= \$155.74$

c $C = 281.92 - 50.471d$

$115 = 281.92 - 50.471d$

$d = \dfrac{115 - 281.92}{-50.471}$

$= 3.31$ km

22 a Number of insects caught

b 1.1

c Positive

d $I = 0.43 + 1.1\,s$

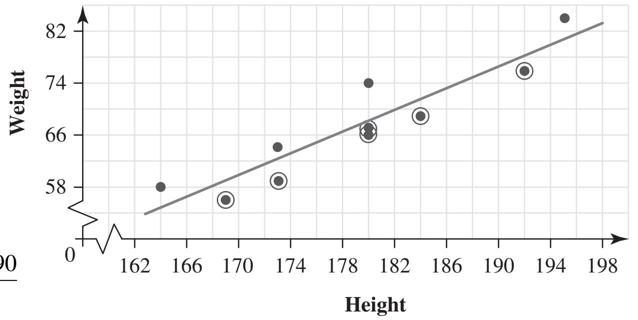
$= 0.43 + 1.1 \times 60$

$= 66.43$

≈ 66 (correct to the nearest whole number)

8.5 Exam questions

1 On your CAS calculator, enter the data, generate the scatterplot and add the LSR line. 6 values lie below the regression line.



The correct answer is **D**.

2 b Body density = $1.195 - 0.001512 \times 65 = 1.09672 =$ 1.10 kg / litre [1 mark]

c Extrapolation since the lowest value of waist measurement on the scatterplot is approximately 69 cm, so 65 cm is outside the range of data. [1 mark]

d The slope is −0.001 512. This indicates that, on average, body density decreases by 0.001 512 kg/litre for each 1 cm increase in waist measurement. [1 mark]

3 a On average, for each 1 hPa increase in pressure at 9 am, the pressure at 3 pm increases by 0.8894 hPa. [1 mark]

***VCAA Examination Report* note:**

Students had to be careful when answering this interpretative question. Many gave a response that was almost correct but failed to reference the one-unit increase in pressure 9 am.

Interpreting the slope in terms of the given variables required identifying the correct constant and then describing it.

Describing both constants was to ignore the first step; specific knowledge was required, not various statements provided in the hope of including something relevant.

b Pressure at 3 pm = $111.4 + 0.8894 \times 1025 = 1023.035 =$ 1023 hPa [1 mark]

c Interpolation [1 mark]

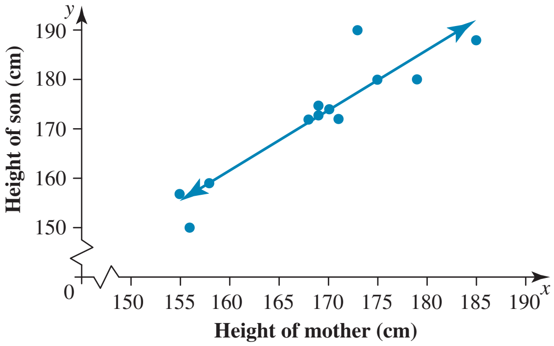
The value 1025 hPa is within the data range for pressure at 9 am

8.6 Introduction to the least squares line of best fit (extending)

8.6 Exercise

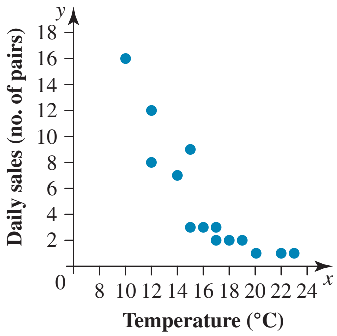
1 a Response variable = height of son

b



c $S = -33.49 + 1.21M$

2 a



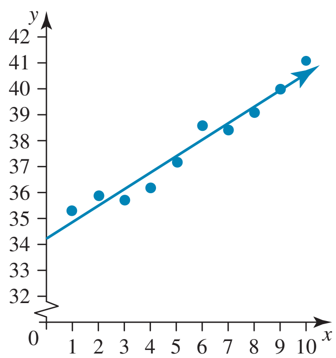
b $D = 22.50 - 1.071T$

c $r = -0.8621$

$r^2 = 0.7432$

d There is a strong negative relationship between the number of gumboots sold and the temperature. The data indicates that 74% of the sales can be explained by the temperature; therefore, 26% are due to other factors.

3 a

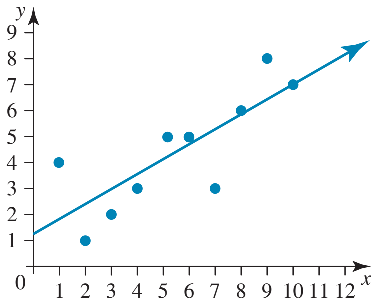


b $y = 34.23 + 0.6412x$

$= 34.23 + 0.6412 \times 15$

$= 43.85$

4 a



b $y = 1.2 + 0.58x$

c $y = 1.2 + 0.58x$

$= 1.2 + 0.58 \times 20$

$= 12.8$

d $y = 1.2 + 0.58x$

$9 = 1.2 + 0.58x$

$x = \dfrac{9 - 1.2}{0.58}$

$x = 13.45$

5 a Number of mosquitos around a camp fire

$= 10.2 + 0.5 \times$ temperature of the fire

$= 10.2 + 0.5x$

$= 10.2 + 0.5 \times 240$

$= 130.2$

$= 130$ (nearest whole number)

b Number of mosquitos around a camp fire

$= 10.2 + 0.5 \times$ temperature of the fire

$12 = 10.2 + 0.5t$

$t = \dfrac{12 - 10.2}{0.5}$

$= 3.6$

$t = 3.6\,°\text{C}$

c The location of the fire, air temperature, proximity to water, etc.

6 a Using the line, 76.

Using the equation:

$y = 13.33 + 2.097x$

$= 13.33 + 2.097 \times 30$

$= 76.24$

$= 76$ (nearest whole number)

b $y = 13.33 + 2.097x$

$150 = 13.33 + 2.097x$

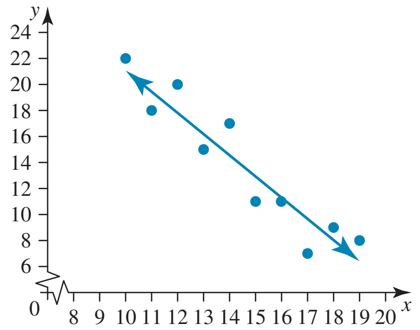
$x = \dfrac{150 - 13.33}{2.097}$

$x = 65.17$

$x = 65$ (nearest whole number)

c Part **a** looks at data within the original data set range, while part **b** predicts data outside of the original data set range of 0–125 number of new customers each hour.

7 a



b $y = 37.70 - 1.648x$

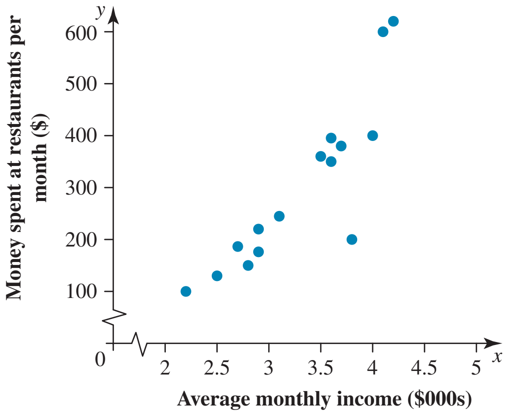
c $y = 37.70 - 1.648x$

$= 37.70 - 1.648 \times 23$

$= -0.204$

d It is assumed that the data will continue to behave in the same manner as the data originally supplied.

8 a



b $R = -459.8 + 229.5I$

c $R = -459.8 + 229.5I$

$I = -459.8 + 229.5 \times 5$

$I = 687.7$

$I = \$687.70$

d Part **c** asks you to predict data outside of the original data set range.

e $R = -459.8 + 229.5I$

$265 = -459.8 + 229.5R$

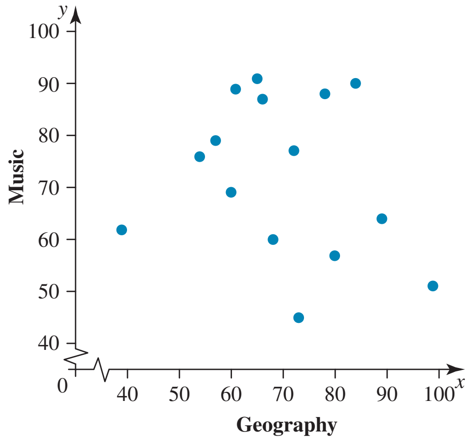
$R = \dfrac{265 + 459.8}{229.5}$

$R = 3.158$

Correct to the nearest \$10, the estimated monthly income is \$3160

9 a No obvious explanatory variable

b



c $M = 87.63 - 0.2195G$

d $M = 87.63 - 0.2195G$

$M = 87.63 - 0.2195 \times 85$

$= 68.97$

$= 69$ (correct to the nearest whole number)

e Not very confident. The graph does not indicate a strong correlation between the two variables.

f $r = 0.2172$. This indicates a very weak correlation between the data. Therefore supporting the view that conclusions cannot be drawn from this data.

10 a Calories burned

b Calories burned = 14 301 + 115.02 × distance walked

c Calories burned $= 14\,301 + 115.02x$

$= 14\,301 + 115.02 \times 50$

$= 20\,052$

This is an example of interpolating as the explanatory variable provided is within the original data range.

d Calories burned $= 14\,301 + 115.02x$

$= 14\,301 + 115.02 \times 10$

$= 15\,451.2$

This is an example of extrapolating as the explanatory variable provided is outside of the original data range.

e An r value of 0.9678 indicates a very strong positive linear relationship, indicating the relationship between the two variables is very strong and can be used to draw conclusions.

f Examples: Speed of walking, difficulty of walking surface, foods eaten

8.6 Exam questions

1 The only possible correct options are C, D or E since time is the RV. Use regression on your CAS to find the correct solution. The correct answer is **E**.

2 Option **C** is not true as the value stated (43) is for the vertical intercept, yet it has been interpreted as if that number was the slope.
The correct answer is **C**.

3 b Use regression on your CAS to find the correct solutions.

i Apparent temperature $= -1.7 + 0.94 \times$ actual temperature [1 mark]

ii On average, when the actual temperature is 0°C, the apparent temperature is −1.7 °C [1 mark]

c Since $r^2 = 0.97$, then 97% of the variation in apparent temperature can be explained by the variation in actual temperature. [1 mark]

8.7 Review

8.7 Exercise

Multiple choice

1 **a** The y-intercept is approximately 3.8 and the gradient is positive.
The correct answer is **A**.
b The correlation shown in the graph is moderate and positive.
The correct answer is **B**.

2 An r value between 0.5 and 0.75 indicates a moderate, positive correlation.
The correct answer is **C**.

3 In this instance the amount of fertiliser is the explanatory variable and the growth rate of the treess is the response variable. An r value of 0.79 indicates a strong, positive correlation. This means that the growth rate of the trees is influenced by the amount of fertiliser used.
The correct answer is **C**.

4 The line of good fit should have approximately an even number of points on either side of it, and should be in the direction of the trend of the data.
The correct answer is **B**.

5 $y = 0.54 + 15.87x$
$= 0.54 + 15.87 \times 2.5$
$= 0.54 + 39.675$
$= 40.215$
The correct answer is **B**.

6 The values of x range from 1 to 25 and the values of y range from 10 to 22. Interpolating means predicting within the given range of data.
An example of interpolating data for this data set is calculating the value of y when $x = 17$.
The correct answer is **B**.

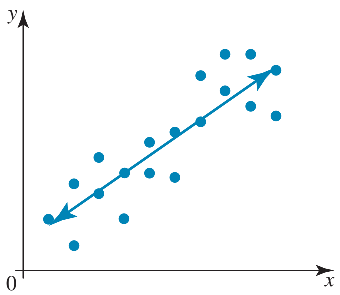
7 From the equation we know that the y-intercept is 85 and the gradient is negative.
The correct answer is **C**.

8 $r^2 = 0.82$
$r = \sqrt{0.82}$
$r = 0.91$ (2 decimal places)
The correct answer is **D**.

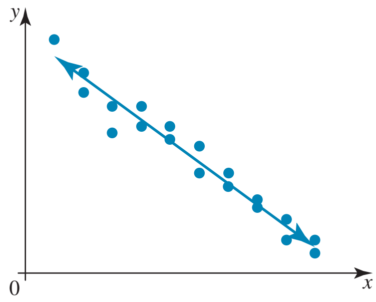
9 Using a CAS, the equation of the regression line is:
$y = 11.456 + 0.435x$.
The correct answer is **D**.

Short answer

10 **i** **a** Moderate positive correlation
b No correlation
c Strong negative correlation
ii **a**

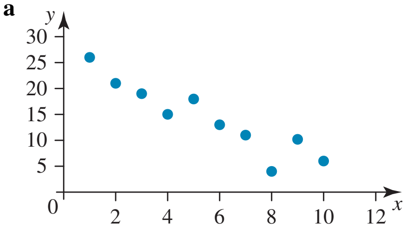


b No line of good fit possible.
c



11 **a** Explanatory variable = quantity of water
Response variable = time
b Explanatory variable = number of students
Response variable = number of buses required

12 **a**



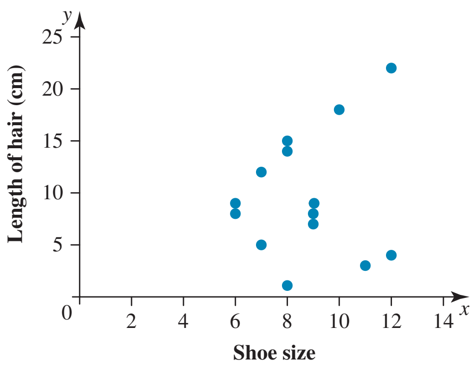
b Strong, negative correlation
c Using a CAS, $r = -0.9329$
$r^2 = (-0.9329)^2$
$= 0.8703$
d Pearson's product-moment correlation coefficient and the coefficient of determination confirm a strong relationship between the two variables.

13 **a** $r^2 = (-0.7564)^2$
$= 0.5721$
b Pearson's product-moment coefficient indicates a strong negative relationship between the variables. The coefficient of determination suggests that 57% of the variation in the y-variable is due to changes in the x-variable, and 43% is due to other factors.

14 While there appears to be a link between the laziness of people and the increase in the sales of healthy foods, there are also many other factors. Based on this observation alone, the cause of an increase in sales of healthy foods cannot be concluded to be due to laziness.

Extended response

15 **a**



b Using a CAS, $y = 4.0386 + 0.6157x$

c Using a CAS, $r = 0.2055$

$$r^2 = (0.2055)^2 = 0.04211$$

d Based on the graph as well as the Pearson's product-moment correlation coefficient and the coefficient of determination, there is no association between the two variables. Therefore, no solid conclusions can be made to suggest that a change in a person's shoe size will affect the length of their hair.

16 **a** Average fuel consumption

b Average fuel consumption = 0.6968 + 0.1119 × fuel tank capacity

c $y = 0.6968 + 0.1119x$

$= 0.6968 + 0.1119 \times 40$

$= 5.17$

d Extrapolation, as the x-value is outside the original data range

e $y = 0.6968 + 0.1119x$

$10.2 = 0.6968 + 0.1119x$

$$x = \frac{10.2 - 0.6968}{0.1119}$$

$x = 84.9$

$x \approx 85$ L

f This value indicates a moderate relationship between the variables. Therefore, the data can be used; however, other factors should be considered.

g Various possible answers; for example, the manner in which a person drives the vehicle, weather conditions, road conditions.

17 **a** Weight (grams)

b No correlation

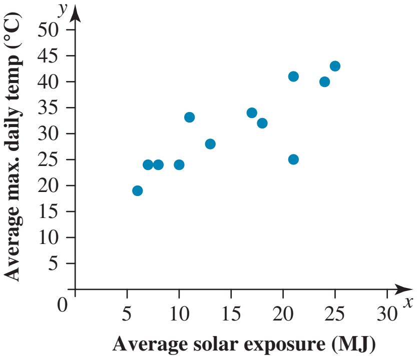
c This supports the view that there is no correlation between the variables. Based on this value, no conclusions can be made from the data.

d Various possible answers; for example, popularity of the shoe, desired profits

18 **a** Explanatory variable = average solar exposure

Response variable = max. daily temperature

b



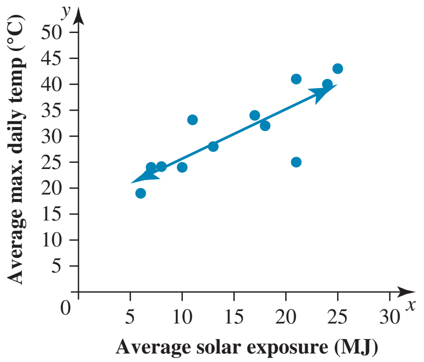
c Strong, positive correlation

d Using a CAS, $r = 0.8242$

$$r^2 = 0.8242^2 = 0.6793$$

These values indicate a strong relationship between the two variables. The coefficient of determination suggests that nearly 70% of the maximum daily temperature can be explained by the amount of solar exposure.

e



Max. daily temperature = 16.232 + 0.9515 × average solar exposure

f $y = 16.232 + 0.9515x$

$37 = 16.232 + 0.9515x$

$$x = \frac{37 - 16.232}{0.9515}$$

$x = 21.8266$

$x \approx 22$ MJ

g $y = 16.232 + 0.9515x$

$= 16.232 + 0.9515 \times 3$

$= 19.0865$

$= 19\,°\mathrm{C}$

h An x-value of 3 MJ is outside the original data set.

8.7 Exam questions

1 On your CAS, enter the data and find the LSR equation:

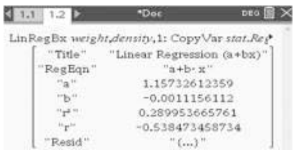
time = 44 − *day*

When *day* = 10, *time* = 44 − 10 = 34, which is the same as day 4.

The correct answer is **B**.

2 **b** **i** Body density is being predicted *from* weight; therefore, *weight* is the explanatory variable. [1 mark]

ii Using your CAS calculator, perform a least-squares regression analysis.



Slope = −0.00112 correct to 3 significant figures. [1 mark]

c This question is referring to the coefficient of determination (r^2), which is found on the least-squares regression analysis screen on CAS.

$r^2 = 0.289\,95...$

Therefore, 29% of the variation in body density may be explained by the variation in weight. [1 mark]

3 Note that the scales do not start at 0, so the vertical axis intercept is not 67.2.

Choose any two points along the line, say (1, 67.2) and (4, 64).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{64 - 67.2}{4 - 1} = \frac{-3.3}{3} = 1.10$$

Equation:
resting pulse rate $= a + b \times$ *time spent exercising*
resting pulse rate $= a - 1.10 \times$ *time spent exercising*
Passes through (4, 64):
$64 = a - 1.1 \times 4$
$68.4 = a$
So the closest equation is
resting pulse rate $= 68.3 - 1.10 \times$ *time spent exercising*
The correct answer is **D**.
VCAA Examination Report **note:**
Students were asked to identify the equation of the least squares line drawn on a graph that contained 16 points. While it was not possible to determine exact slope and intercept values from the graph, students should have been able to approximate these values. Many students incorrectly assumed that the intercept value of the line was 67.2, read directly from the graph; however, this is only possible if the horizontal axis begins at value zero. Students are encouraged to look carefully at graphs before choosing what might seem to be the obvious answer.

4 The association *body surface area* $= -1.1 + 0.019 \times$ *height* has a y-intercept $a = -1.1$ and a gradient $b = 0.019$.
The slope $b = 0.019$ indicates that the *body surface area* increased by $0.019\,\text{m}^2$ for every 1 cm added to a person's height.
The correct answer is **B**.

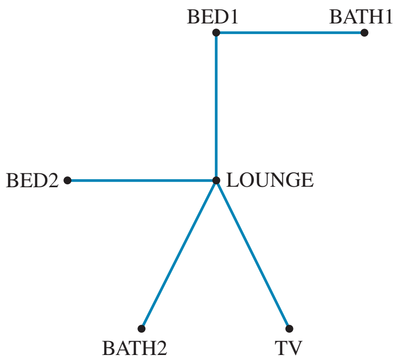
5 Given the equation $y = a + bx$, if y and x were switched and then the equation was solved for y, the equation would become $y = -\frac{a}{b} + \frac{1}{b}x$.
Looking at the two equations, the slopes and intercepts are different. Since the slopes are different, the predicted values and thus the residual values will also be different.
The only statistic that will not change is the correlation coefficient r, as s_x and s_y are not affected by the least squares line.
The correct answer is **C**.
VCAA Examination Report **note:**
Students needed to be aware that reversing the two variables will give a different equation.
The slope and intercept will therefore both change in value, the predictions the line gives will change and hence the residual values will also change.
The correlation coefficient will not change in value as the degree of scattering of the points remains unchanged – i.e. the scattering of 'y' values relative to 'x' is the same as the scattering of 'x' values relative to 'y'.

Topic 9 — Graphs and networks

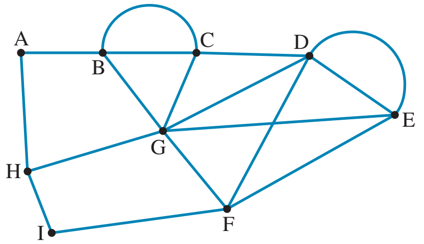
9.2 Definitions and terms

9.2 Exercise

1



2



3 **a** Edges = 7; degree sum = 14

b Edges = 10; degree sum = 20

4 **a** Edges = 9; degree sum = 18

b Edges = 9; degree sum = 18

5 **a** $\deg(A) = 5;\ \deg(B) = 3;\ \deg(C) = 4;$
$\deg(D) = 1; \deg(E) = 1$

b $\deg(A) = 0;\ \deg(B) = 2; \deg(C) = 2;$
$\deg(D) = 3; \deg(E) = 3$

c $\deg(A) = 4;\ \deg(B) = 2;\ \deg(C) = 2;$
$\deg(D) = 2; \deg(E) = 4$

d $\deg(A) = 1;\ \deg(B) = 2;\ \deg(C) = 1;$
$\deg(D) = 1; \deg(E) = 3$

6 **a** The graphs are isomorphic as:

i they both have five edges

ii their vertices have the same degree:
$\deg(A) = 1;\ \deg(B) = 3;\ \deg(C) = 2;$
$\deg(D) = 1;\ \deg(E) = 3$

iii and they have the same connections:

Vertex	Connections
A	E
B	C, D, E
C	B, E
D	B
E	A, B, C

b The graphs are isomorphic as:

i they both have 10 edges

ii their vertices have the same degree:
$\deg(A) = 3;\ \deg(B) = 4;\ \deg(C) = 3;\ \deg(D) = 3;$
$\deg(E) = 3;\ \deg(F) = 2;\ \deg(G) = 2$

iii and they have the same connections:

Vertex	Connections
A	C, E, G
B	C, D, E, F
C	A, B, E
D	B, F, G
E	A, B, C
F	B, D
G	A, D

c The graphs are not isomorphic as they don't have the same number of edges:
Graph 1 has nine edges and Graph 2 has eight.

d The graphs are isomorphic as:

i they both have 12 edges

ii their vertices have the same degree:
$\deg(A) = 5;\ \deg(B) = 1;\ \deg(C) = 3;\ \deg(D) = 3;$
$\deg(E) = 3;\ \deg(F) = 2;\ \deg(G) = 2;\ \deg(H) = 2;$
$\deg(I) = 1;\ \deg(J) = 2$

iii and they have the same connections:

Vertex	Connections
A	C, D, E, G, J
B	F
C	A, E, J
D	A, G, H
E	A, C, I
F	B, H
G	A, D
H	D, F
I	E
J	A, C

7 **a** Different degrees and connections

b Different connections

8 Isomorphic pairs are: Graphs 2 and 4; Graphs 5 and 6; as each pair has the same number of vertices with the same degrees and connections.

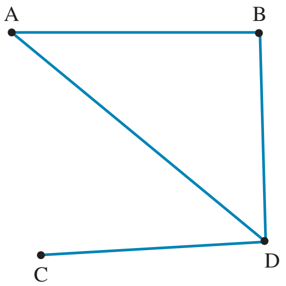
9 **a** $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$

b $\begin{bmatrix} 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$

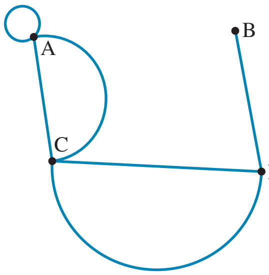
c $\begin{bmatrix} 0 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 3 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 & 0 & 0 \end{bmatrix}$

d $\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 & 4 & 0 \end{bmatrix}$

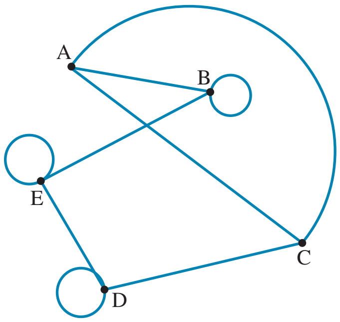
10 a



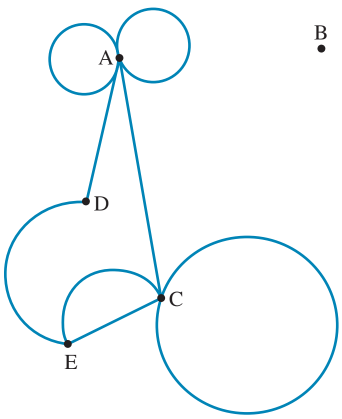
b



c



d



11 Graph 1

$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Graph 2

$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

Graph 3

$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$

Graph 4

$\begin{bmatrix} 0 & 2 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Graph 5

$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

12 a $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 2 \\ 1 & 2 & 0 \end{bmatrix}$

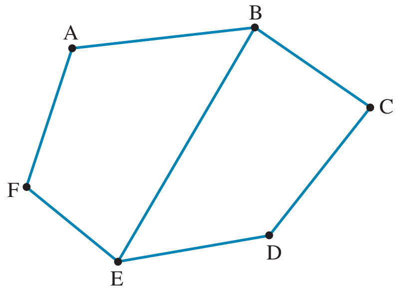
b $\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$

c $\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 \end{bmatrix}$

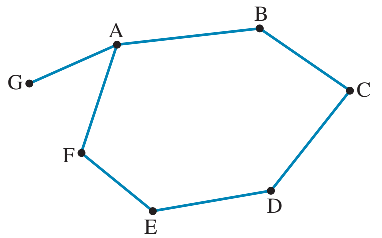
d $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$

13 Answers will vary. Possible answers are shown.

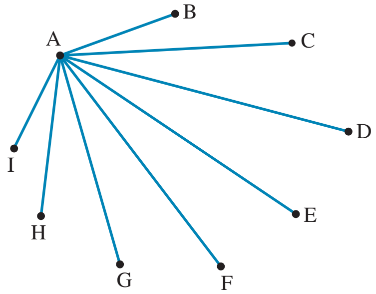
a



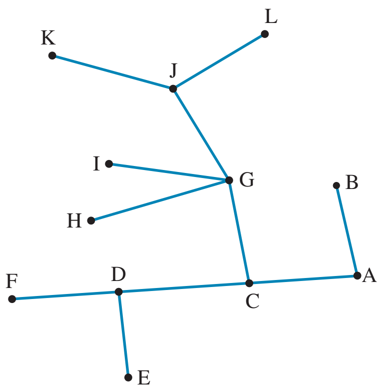
b



c



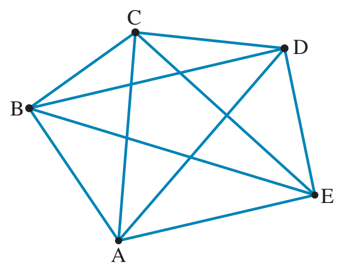
14



15

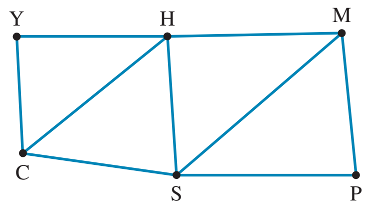
Graph	Simple	Connected
Graph 1	Yes	Yes
Graph 2	Yes	Yes
Graph 3	Yes	Yes
Graph 4	No	Yes
Graph 5	No	Yes

16 a



b The total number of edges represents the total games played, as each edge represents one team playing another.

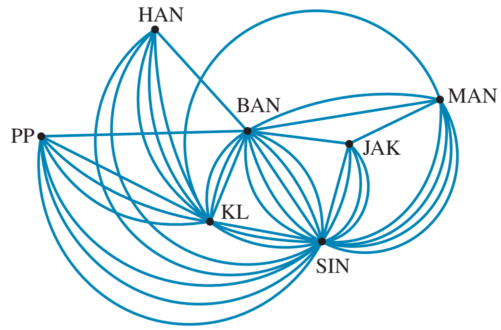
17 a



b deg(Huairou) = deg(Shunyi) = 4

c This is a simple, connected graph as there are no loops or multiple edges, and all vertices are reachable.

18 a



b Directed as it would be important to know the direction of the flight.

c i 10 ways of travelling:

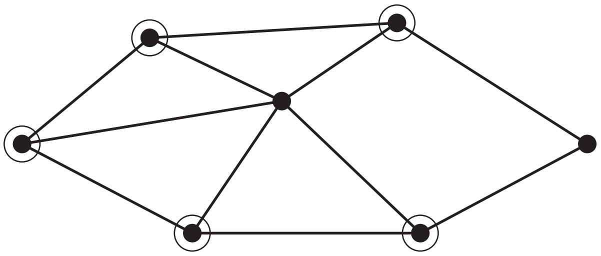
Phnom Penh → Bangkok → Manila
Phnom Penh → Bangkok → Jakarta → Manila
Phnom Penh → Bangkok → Singapore → Manila
Phnom Penh → Bangkok → Singapore → Jakarta → Manila
Phnom Penh → Bangkok → Kuala Lumpur → Singapore → Manila
Phnom Penh → Bangkok → Kuala Lumpur → Singapore → Jakarta → Manila
Phnom Penh → Bangkok → Hanoi → Kuala Lumpur → Singapore → Jakarta → Manila
Phnom Penh → Bangkok → Hanoi → Kuala Lumpur → Singapore → Manila
Phnom Penh → Bangkok → Kuala Lumpur → Hanoi → Singapore → Jakarta → Manila
Phnom Penh → Bangkok → Kuala Lumpur → Hanoi → Singapore → Manila

ii 7 ways of travelling:

Hanoi → Bangkok
Hanoi → Kuala Lumpur → Bangkok
Hanoi → Kuala Lumpur → Singapore → Bangkok
Hanoi → Singapore → Bangkok
Hanoi → Singapore → Jakarta → Bangkok
Hanoi → Singapore → Hanoi → Bangkok
Hanoi → Singapore → Kuala Lumpur → Bangkok

9.2 Exam questions

1 There are 5 vertices with a degree of 3:



The correct answer is **E**.

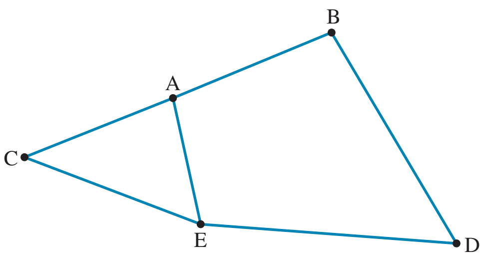
2 The correct answer is **E**.

3 A loop joins a vertex to itself, so option B has the only loop. The correct answer is **B**.

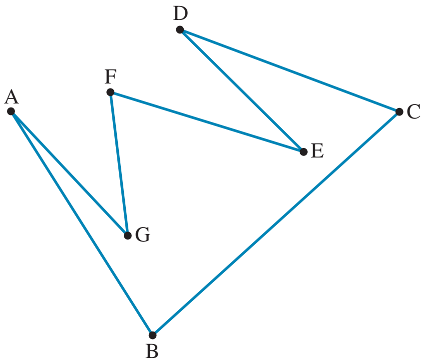
9.3 Planar graphs

9.3 Exercise

1 a



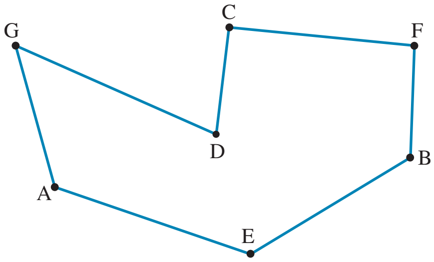
b



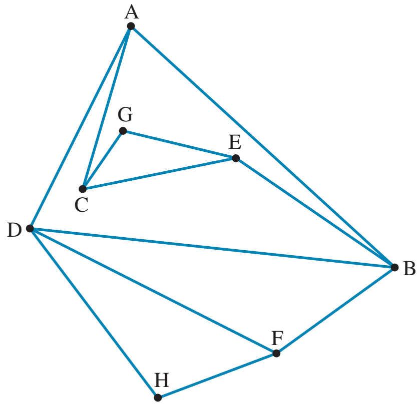
2 a All of them

b All of them

3 a



b



4 Graph 3 is not planar as it cannot be redrawn without intersecting edges.

5 a $v - e + f = 2$

$\rightarrow 8 - 10 + f = 2$

$\rightarrow f = 2 - 8 + 10$

$\rightarrow f = 4$

b $v - e + f = 2$

$\rightarrow 11 - 14 + f = 2$

$\rightarrow f = 2 - 11 + 14$

$\rightarrow f = 5$

6 a $v - e + f = 2$

$\rightarrow 5 - e + 3 = 2$

$\rightarrow e = 5 - 2 + 3$

$\rightarrow e = 6$

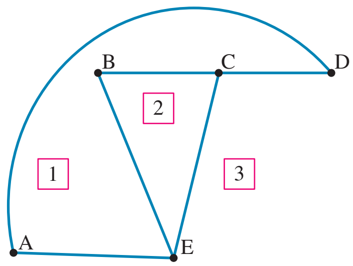
b $v - e + f = 2$

$\rightarrow v - 8 + 5 = 2$

$\rightarrow v = 2 + 8 - 5$

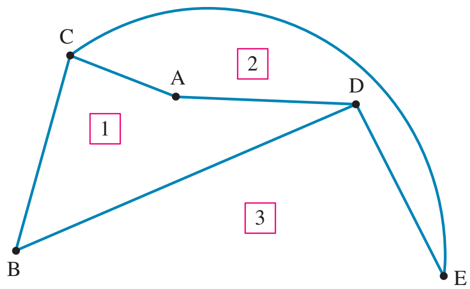
$\rightarrow v = 5$

7 a



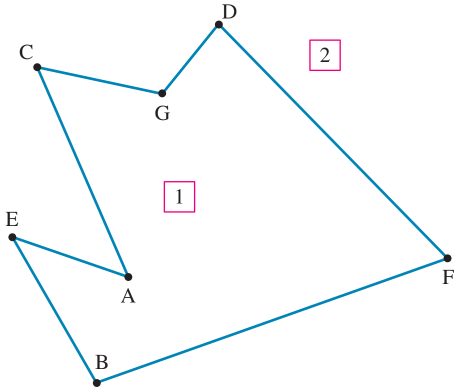
3 faces

b



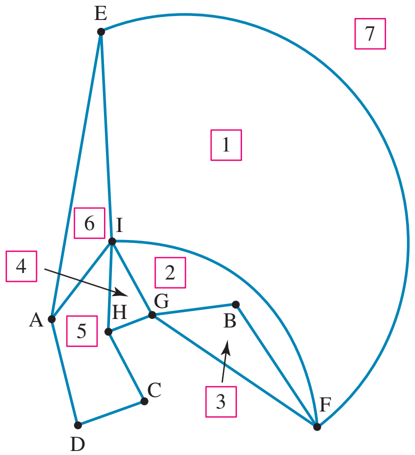
3 faces

c



2 faces

d



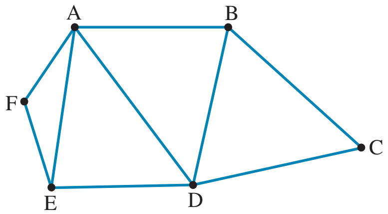
7 faces

8 a $v - e + f = 2$

$\rightarrow 6 - e + 5 = 2$

$\rightarrow e = 6 - 2 + 5$

$\rightarrow e = 9$

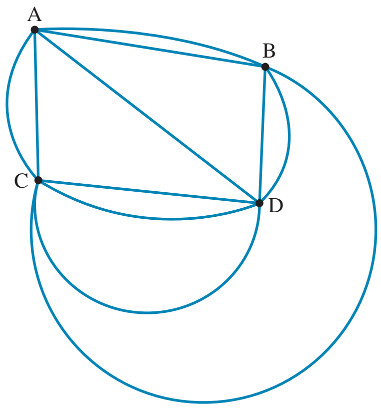


b $v - e + f = 2$

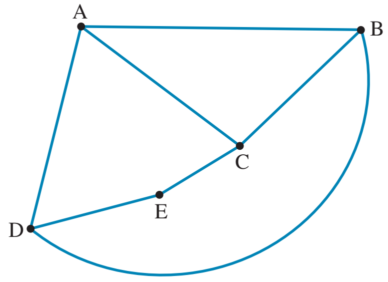
$\rightarrow v - 11 + 9 = 2$

$\rightarrow v = 2 + 11 - 9$

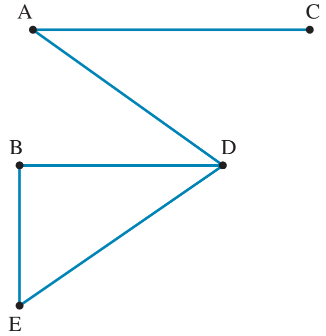
$\rightarrow v = 4$



9 a



b



10 a i 3 enclosed faces

ii 2 additional edges

b i 1 enclosed face

ii 4 additional edges

11 a

Graph	Total edges	Total degrees
Graph 1	3	6
Graph 2	5	10
Graph 3	8	16
Graph 4	14	28

b Total degrees $= 2 \times$ total edges

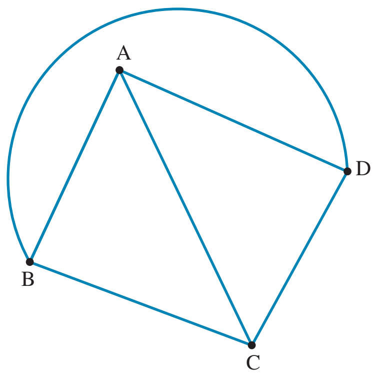
12 a

Graph	Total vertices of even degree	Total vertices of odd degree
Graph 1	3	2
Graph 2	4	2
Graph 3	4	4
Graph 4	6	6

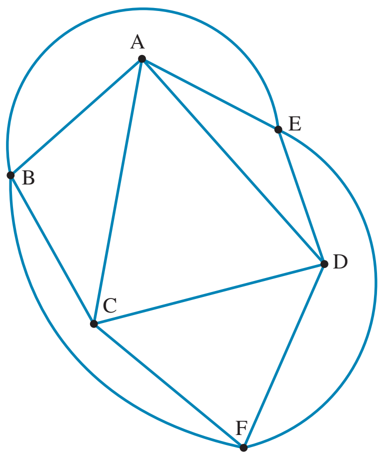
b No clear pattern evident

13

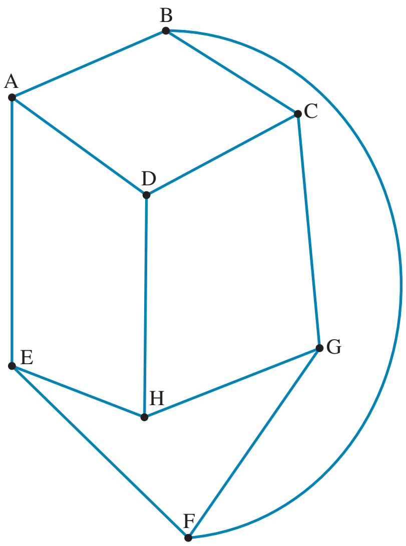
Tetrahedron



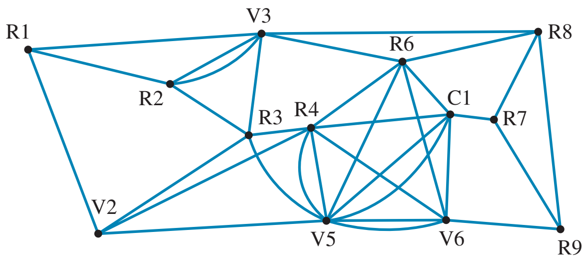
Octahedron



Cube

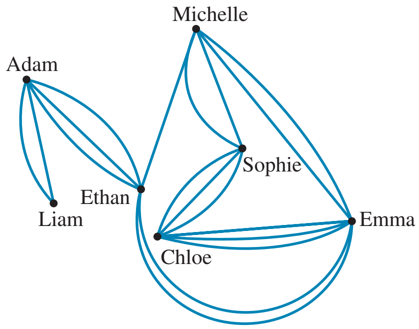


14 a



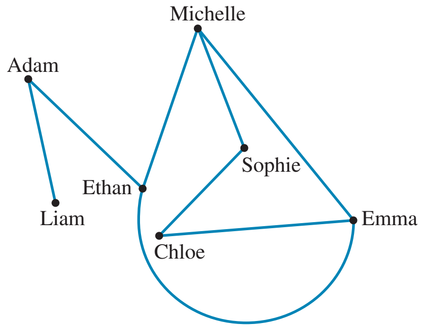
b No, as there will always be some intersecting edges.

15 a

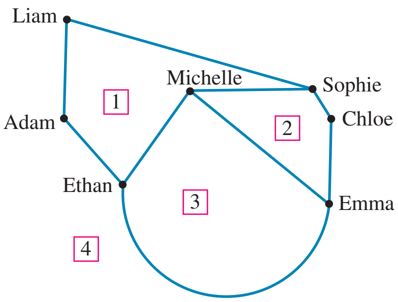


b Sophie or Emma, as they are in contact via more methods.

c



d



4 faces

9.3 Exam questions

1 $v + f - e = 2$

$7 + f - 9 = 2$

$f = 4$

The correct answer is **D**.

2 The planar graph has 8 edges, so the original must have 8 edges too.

The correct answer is **A**.

3 Using Euler's formula, the number of faces for this graph is $f = 2 - v + e = 2 - 5 + 4 = 1$.

Because there is only 1 face, the graph can always be redrawn so that no edges meet. Therefore the graph is always planar, so statement (1) is true.

As there is only 1 face, statement (2) is not true.

Because the number of edges is 1 less than the number of vertices, at least 2 vertices will not be directly connected by the edge and will hence have degree 1 (odd degree).

Therefore, statement (3) is not true.

The sum of degrees of vertices $=$ number of edges $\times 2 = 4 \times 2 = 8$. Therefore, statement (4) is true.

For the graph with v vertices to be connected, the minimum number of edges is $e = v - 1$, which is the case for this graph: $4 = 5 - 1$.

Because the graph has the minimum number of edges, a loop is not possible. (If an edge is used to form a loop, there will not be enough edges left to connect all vertices.) Therefore, statement (5) is true.

Statements (1), (4) and (5) are true. Therefore, the number of statements that are always true for this graph is 3.

The correct answer is **C**.

9.4 Connected graphs

9.4 Exercise

1 Cycle: ABECA (others exist)

Circuit: BECDB (others exist)

2 Path: ABGFHDC (others exist)
Cycle: DCGFHD (others exist)
Circuit: AEBGFHDCA (others exist)

3 **a** Walk, trail and circuit
b Walk, trail and path
c Walk, trail, path, cycle and circuit
d Walk and trail

4 **a** MCHIJGFAED
b AEDBLKMC
c MDEAFGJIHCM
d FMCHIJGF

5 **a** Euler: AFEDBECAB; Hamiltonian: BDECAF
b Euler: GFBECGDAC; Hamiltonian: BECADGF

6 **a** Euler: AIBAHGFCJBCDEGA;
Hamiltonian: none exist
b Euler: ABCDEFGHA (others exist);
Hamiltonian: HABCDEFGH (others exist)

7 **a** Graphs **i**, **ii** and **iv** as each have exactly two vertices with odd degree.
b Graph **i**: ACDABDECB (others exist)
Graph **ii**: CFBCEDBADCA (others exist)
Graph **iv**: CFBCEDCADBAH (others exist)
c Graphs **i** and **ii**
d Graph **i**: CEDABC
Graph **ii**: CEDABFC
e Graph **i**:

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Graph **ii**:

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Graph **iii**:

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Graph **iv**:

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

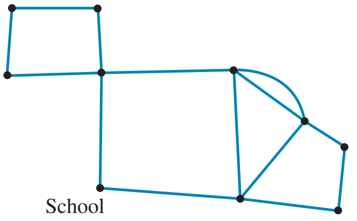
f The presence of Euler trails and circuits can be identified by using the adjacency matrix to check the degree of the vertices. The presence of Hamiltonian paths and cycles can be identified by using the adjacency matrix to check the connections between vertices.

8 E, as Vertex A and E are of odd degree.

9 **a** The possible Hamiltonian paths are BEADC or BECDA, so it finishes at either A or C.
b The possible Hamiltonian paths are EABCD or EADCB, so it finishes at either B or D.

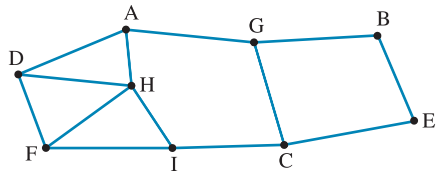
10 **a** Adding an edge from G to C creates the Hamiltonian path: GCDAEBF.
b Adding an edge from F to E creates the Hamiltonian path: FEABCDG.

11 **a**



b Yes, because the degree of each intersection or corner point is an even number.
c Yes, because the degree of each remaining intersection or corner point is still an even number.

12 **a**



b deg(H) = 4
c **i** ADHFICEBGA
ii AHDFICEBGA
d **i** Yes, because two of the checkpoints have odd degree.
ii H and C

13 **a**

	Hamiltonian cycle
1.	ABCDA
2.	ABDCA
3.	ACBDA
4.	ACDBA
5.	ADBCA
6.	ADCBA

b Yes, commencing on vertices other than A. For example: BCDAB.

14 **a** B : BCA
C : CBA
D : DBA or DCA
F : FCA
G : GBA
Therefore, B, C, D, F or G.
b B or C, as once A is reached, the next option is returning to the start.
c None possible
d D or E, as both have odd degree.
e D to E, in order to make all vertices of even degree.

9.4 Exam questions

1 There is no Eulerian trail, because it is not possible to get to and from the vertex on the right without passing over the edge twice. The other 4 statements are true.
The correct answer is **D**.

2 A Hamiltonian cycle passes through each vertex only once and starts and finishes at the same vertex. There is no edge connecting E and D.
The correct answer is **D**.

3 For a Eulerian circuit to be possible, all vertices must be of even degree. There are four vertices with odd degree vertices, so two more edges are needed.
The correct answer is **C**.

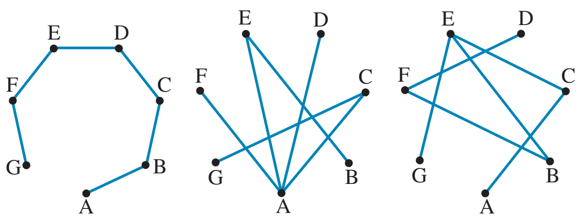
9.5 Weighted graphs and networks, and trees

9.5 Exercise

1 Travelling: AEGBCHFD = 2 + 5 + 4 + 2 + 3 + 2 + 3 = 21

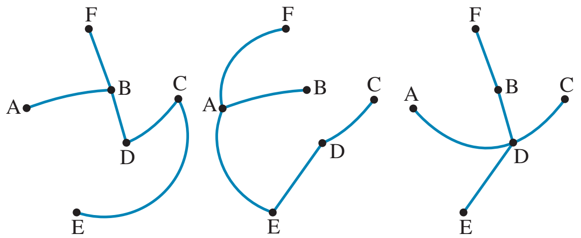
2 Travelling:
ABCGDFEHI = 2.68 + 2.42 + 2.5 + 2.92 + 2.28 + 2.97 + 2.36 + 2.65 = 20.78

3 a



Other possibilities exist.

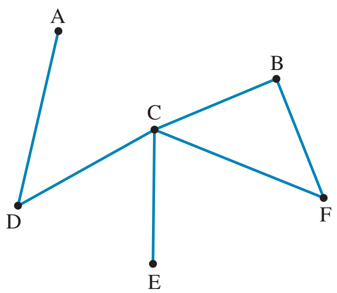
b



Other possibilities exist.

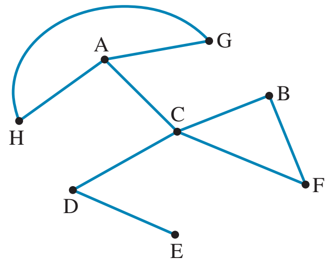
4 Travelling: ACDEGBA = 14 + 18 + 8 + 7 + 8 + 11 = 66
Or travelling: ABGEDCA = 11 + 8 + 7 + 8 + 18 + 14 = 66

5 a i



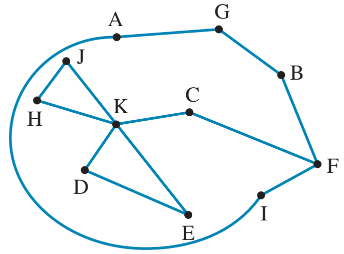
ii ADCBFCE or ADCFBCE

b i Add two edges (GH and BF) to create only two vertices of odd degree.



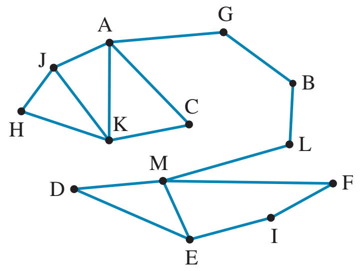
ii AHGACBFCDE or similar

c i Add three edges (AI, DE and JH) to create only two vertices of odd degree.



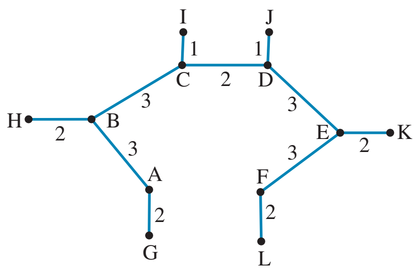
ii KDEKHJKCFIAGBF or similar

d i Add five edges (HK, CK, AK, DM and FI) to create only two vertices of odd degree.



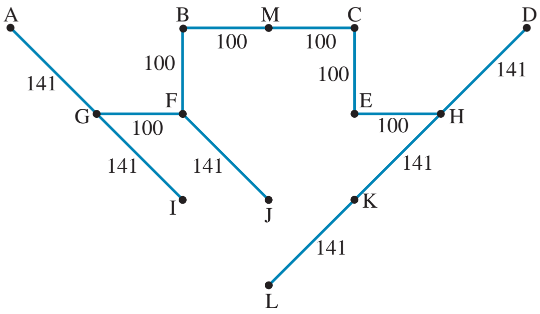
ii EDMEIFMLBGACKHJKA or similar

6 a

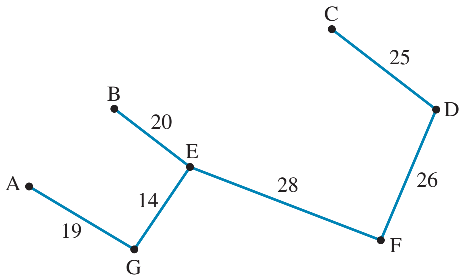


b length = 2 + 3 + 2 + 3 + 1 + 2 + 1 + 3 + 2 + 3 + 2 = 24

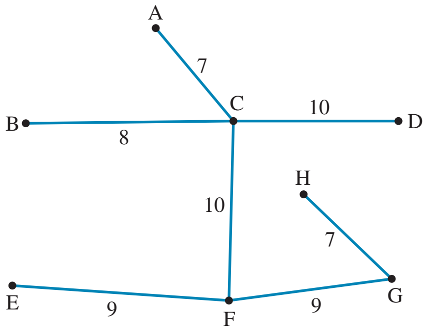
7



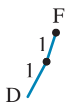
8 a



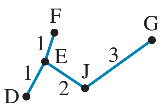
b



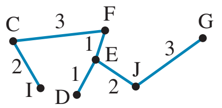
9 Step 1



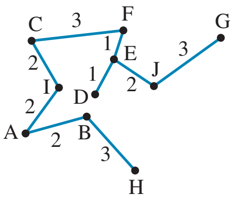
Step 2



Step 3



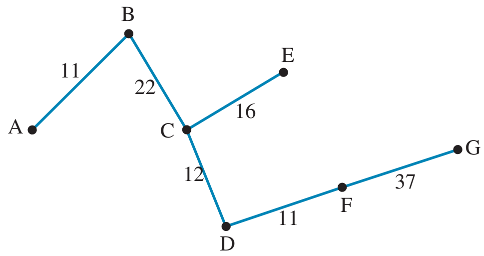
Step 4



10 a i The number of vertices in the graph is 7.
The minimum spanning tree should have 6 edges (7 – 1).
Order the edges by weight, from lowest to highest.

Weight	Edge
11	AB
11	DF
12	CD
16	CE
22	BC
25	AD
25	EF
35	BE
37	FG
48	EG

Starting at the top of the list and working your way down, progressively add edges into the minimum spanning tree. If including an edge would create a cycle, disregard that edge and keep moving through the list until 6 edges have been included.

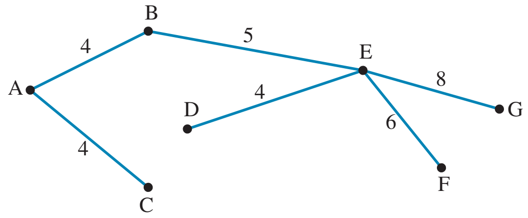


ii Length of minimum spanning tree = 11 + 11 + 12 + 16 + 22 + 37 = 109

b i The number of vertices in the graph is 7.
The minimum spanning tree should have 6 edges (7 – 1).
Order the edges by weight, from lowest to highest.

Weight	Edge
4	AB
4	AC
4	DE
5	BE
6	EF
7	AD
7	CD
8	EG
9	CF
9	FG

Starting at the top of the list and working your way down, progressively add edges into the minimum spanning tree. If including an edge would create a cycle, disregard that edge and keep moving through the list until 6 edges have been included.

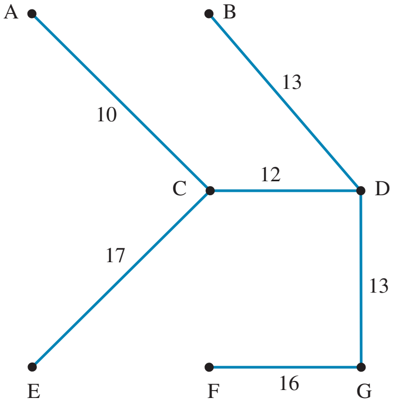


ii Length of minimum spanning tree = 4 + 4 + 4 + 5 + 6 + 8 = 31

c i The number of vertices in the graph is 7.
The minimum spanning tree should have 6 edges (7 – 1).
Order the edges by weight, from lowest to highest.

Weight	Edge
10	AC
12	CD
13	BD
13	DG
15	AB
16	FG
17	CE
19	DF
21	EF

Starting at the top of the list and working your way down, progressively add edges into the minimum spanning tree. If including an edge would create a cycle, disregard that edge and keep moving through the list until 6 edges have been included.

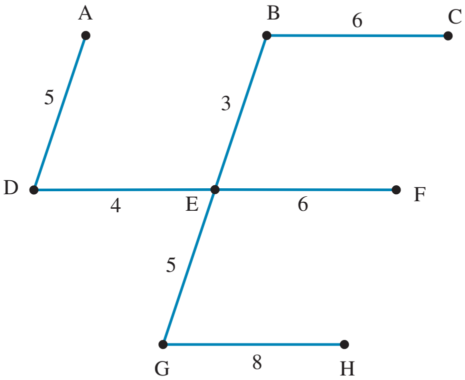


ii Length of minimum spanning tree $= 10 + 12 + 13 + 13 + 16 + 17 = 81$

d i The number of vertices in the graph is 8.
The minimum spanning tree should have 7 edges $(8 - 1)$.
Order the edges by weight, from lowest to highest.

Weight	Edge
3	BE
4	DE
5	AD
5	EG
6	BC
6	EF
7	AB
8	CF
8	GH
9	FH

Starting at the top of the list and working your way down, progressively add edges into the minimum spanning tree. If including an edge would create a cycle, disregard that edge and keep moving through the list until 7 edges have been included.



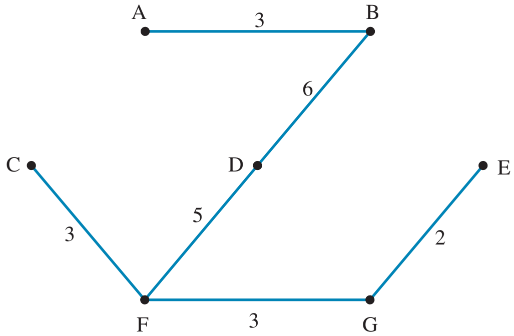
ii Length of minimum spanning tree $= 3 + 4 + 5 + 5 + 6 + 6 + 8 = 37$

e i The number of vertices in the graph is 7.
The minimum spanning tree should have 6 edges $(7 - 1)$.

Order the edges by weight, from lowest to highest.

Weight	Edge
2	EG
3	AB
3	CF
3	FG
5	DF
6	BD
6	DG
9	AD
10	AC
11	BE

Starting at the top of the list and working your way down, progressively add edges into the minimum spanning tree. If including an edge would create a cycle, disregard that edge and keep moving through the list until 6 edges have been included.



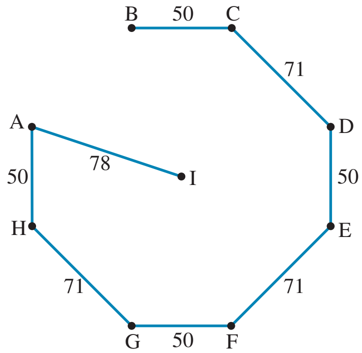
ii Length of minimum spanning tree $= 2 + 3 + 3 + 3 + 5 + 6 = 22$

11 a FDCGBAE (other solutions exist)

b FDCBAEG (other solutions exist)

12 a Longest: IFEDCBAHG: $80 + 71 + 50 + 71 + 50 + 71 + 50 + 71 = 514$ (or similar variation of the same values);
Shortest: IAHGFEDCB: $78 + 50 + 71 + 50 + 71 + 50 + 71 + 50 = 491$ (or similar variation of the same values)

b

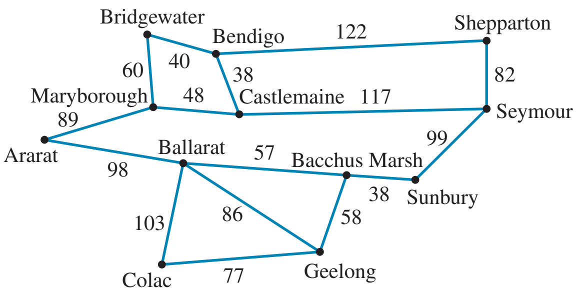


13 a ADEG: $170 + 120 + 100 = 390$

b BHG: $320 + 140 = 460$

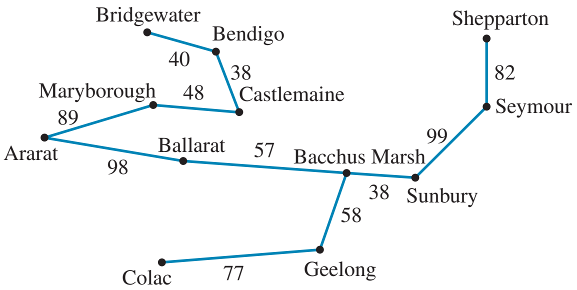
c EGFCDABHE : $100 + 120 + 310 + 110 + 170 + 180 + 320 + 130 = 1440$

14 a



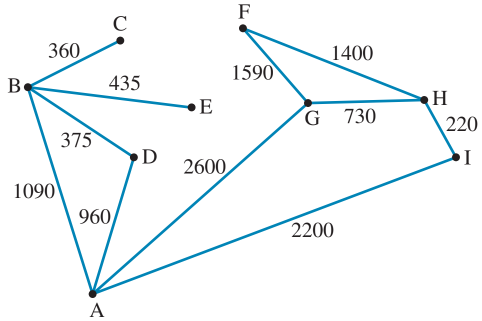
b Geelong → Ballarat → Ararat → Maryborough → Bridgewater → Bendigo → Castlemaine → Seymour → Sunbury → Bacchus Marsh → Geelong = 723 km

c



d 82 + 99 + 38 + 58 + 77 + 77 + 58 + 57 + 98 + 89 + 48 + 38 + 40 = 859 km

15 a

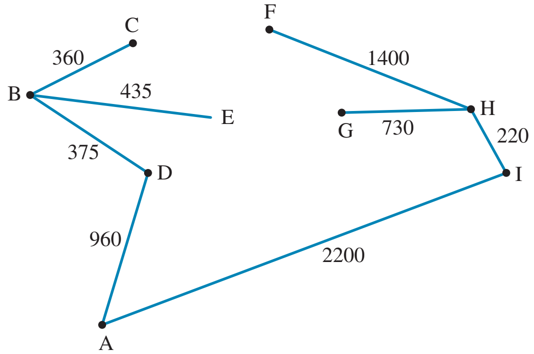


b No; C and E are both only reachable from B.

c i ABCBEBDAIHFGA = 12 025

ii GFHIABCBEBDAG = 12 025

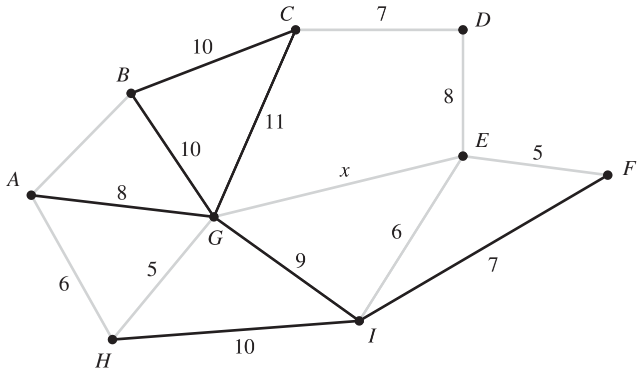
d



9.5 Exam questions

1. A spanning tree includes all the vertices and some of the edges of the original network, and no loops, multiple edges or cycles. Option B has an edge connecting vertices 3 and 5 that is not present in the original network.
The correct answer is **B**.

2 Draw a minimum spanning tree that has a minimum length of 53 m.



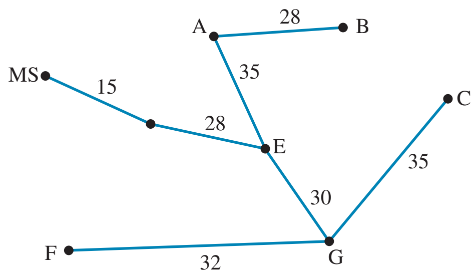
$x = 53 - (7 + 6 + 5 + 6 + 5 + 8 + 7)$

$= 53 - 44$

$= 9$

The correct answer is **D**.

3 A minimum spanning tree is required.



Add up the edges:

15 + 28 + 30 + 32 + 35 + 35 + 28 = 203

The correct answer is **B**.

9.6 Review

9.6 Exercise

Multiple choice

1 Edges = vertices − 1

8 − 1 = 7

The correct answer is **C**.

2 Option D is correct as it has four vertices of odd degree, so it will not have an Euler path.

The correct answer is **D**.

3 $v - e + f = 2$

$v - e + f = 2$

$9 - e + 10 = 2$

$e = 9 - 2 + 10$

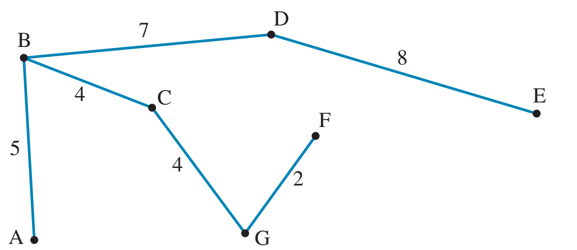
$e = 17$

The correct answer is **C**.

4 Option B is correct as it is the only graph that is a connected graph with no loops.

The correct answer is **B**.

5

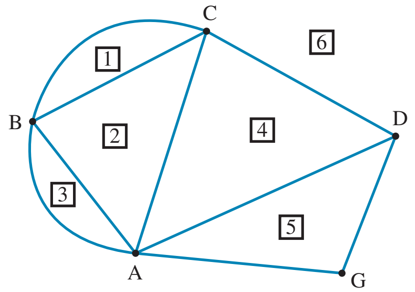


Minimum spanning tree:
$5 + 4 + 4 + 2 + 7 + 8 = 30$
The correct answer is **E**.

6 Adding an edge from A to D means all vertices are of even degree, so an Euler circuit will exist.
The correct answer is **A**.

7 Option E is the only matrix that lists the correct connections between the vertices.
The correct answer is **E**.

8



There are six faces.
The correct answer is **A**.

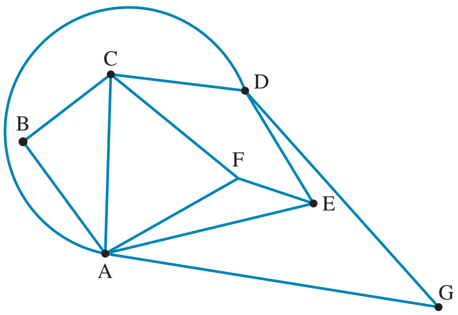
9 Option A is correct as it is the only one that visits all vertices only once before returning to the start.
The correct answer is **A**.

10 Recall that a bridge is an edge that when removed disconnects the graph. Removing any edge would disconnect the graph, hence, all 4 edges are bridges.
The correct answer is **E**.

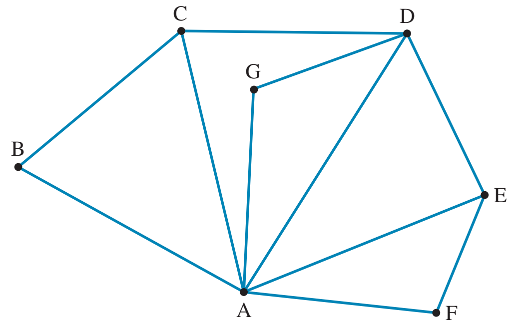
Short answer

11 a i Planar
ii Planar
iii Planar
iv Planar

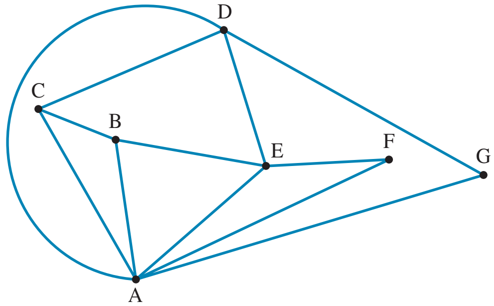
b i



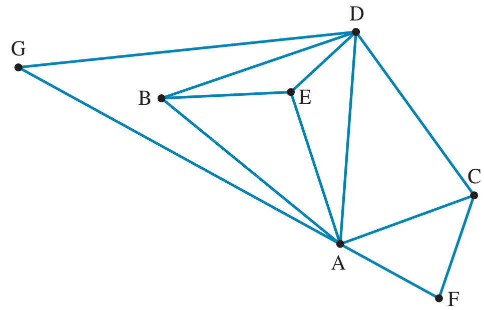
ii



iii



iv



12 a $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

b $\begin{bmatrix} 0 & 1 & 2 & 1 & 1 \\ 1 & 0 & 2 & 0 & 1 \\ 2 & 2 & 0 & 2 & 3 \\ 1 & 0 & 2 & 2 & 2 \\ 1 & 1 & 3 & 2 & 0 \end{bmatrix}$

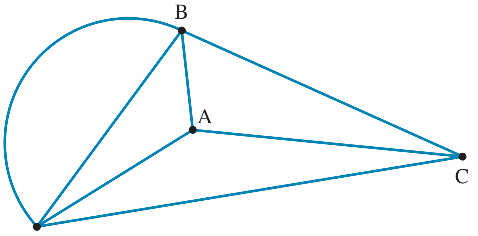
c $\begin{bmatrix} 0 & 2 & 1 & 3 & 1 & 2 \\ 2 & 0 & 3 & 1 & 1 & 0 \\ 1 & 3 & 0 & 2 & 3 & 1 \\ 3 & 1 & 2 & 2 & 2 & 1 \\ 1 & 1 & 3 & 2 & 3 & 1 \\ 2 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$

d $\begin{bmatrix} 0 & 2 & 1 & 3 & 0 & 1 & 0 \\ 2 & 0 & 2 & 1 & 2 & 1 & 0 \\ 1 & 2 & 0 & 1 & 1 & 0 & 1 \\ 3 & 1 & 1 & 0 & 0 & 2 & 1 \\ 0 & 2 & 1 & 0 & 0 & 0 & 3 \\ 1 & 1 & 0 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 & 3 & 2 & 0 \end{bmatrix}$

13 **a** Simple; planar
b Simple; planar
c Simple; planar
d Simple; planar
e Simple
f Simple; planar
g Simple; planar
h Simple; planar

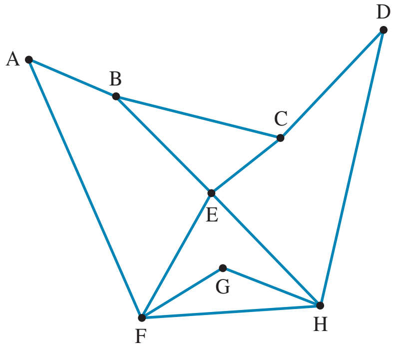
14 Graphs **a** and **d** are isomorphic as they have the same number of vertices with the same connections.

15 **a** **i**



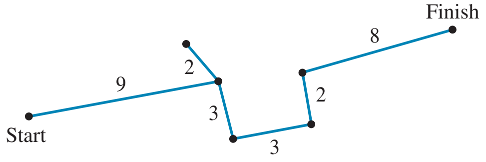
ii ABDBCADC

b **i**



ii BAFEHGFHDCEBC

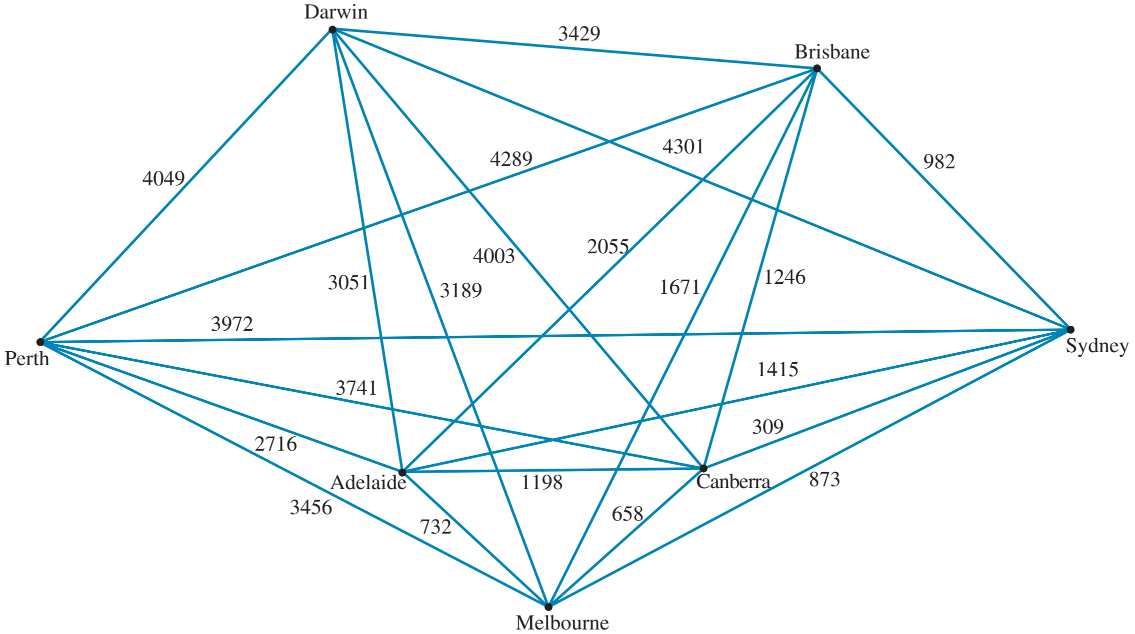
16 **a** $9 + 4 + 2 + 8 = 23$
b $9 + 2 + 5 + 4 + 3 + 11 = 34$
c



Extended response

17 **a** See the figure at foot of the page.*
b Via Canberra: $658 + 4003 = 4661$ km
c Via Sydney: $982 + 3972 = 4954$ km

***17 a**



d See the figure at foot of the page.*
Total distance: $2716 + 3051 + 732 + 658 + 873 + 982 = 9012\text{ km}$

18 a i $815 + 565 + 325 + 365 + 350 + 805 + 580 = 3805\text{ m}$

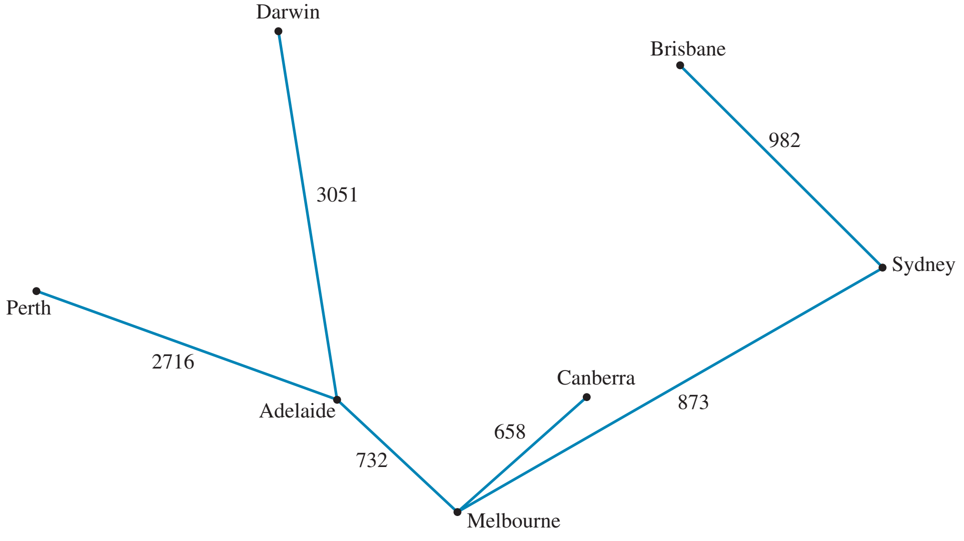
ii See the figure at foot of the page.*

b HEDA: $565 + 480 + 815 = 1860\text{ m}$

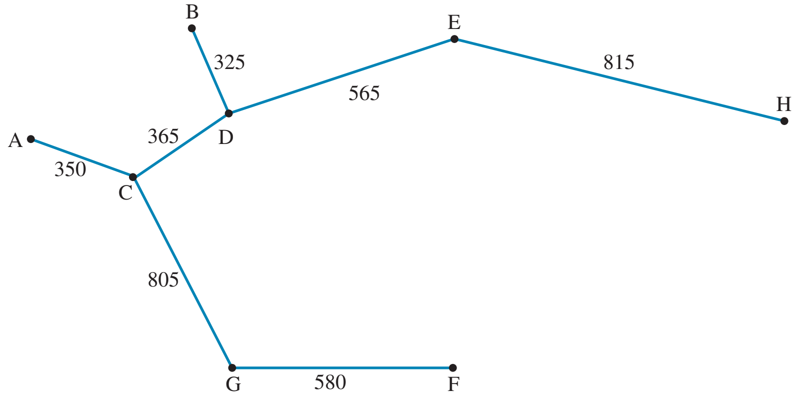
c DFGCABEH:
$1035 + 580 + 805 + 350 + 625 + 695 + 815 = 4905\text{ m}$

d DEHFGCABD:
$565 + 815 + 1195 + 580 + 805 + 350 + 625 + 325 = 5260\text{ m}$

***17 d**



***18 a ii**



19 a See the figure at foot of the page.*

b $\begin{bmatrix} 0 & 2 & 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 & 1 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

c No, as there are more than two vertices of odd degree.

d AFEDCBA: $7 + 10 + 4 + 3 + 9 + 6 = 39$

20 a See the figure at foot of the page.*

b Hobart–Bruny–Robbins: $715 + 65 = 780$ km

c Hobart–Bruny–Robbins–King–Devonport–Flinders–Maria: $65 + 715 + 120 + 395 + 330 + 450 = 2075$ km

d King–Devonport–Flinders–Maria–Hobart–Bruny–Robbins–King: $395 + 330 + 450 + 145 + 65 + 715 + 120 = 2220$ km

9.6 Exam questions

1 a Shortest distance is $G - O - N - M$ for 86 km [1 mark]

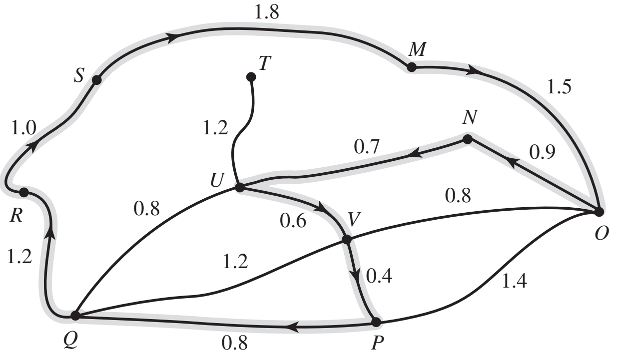
b George will pass through K twice
($G - H - I - K - L - K - J - O - N - M$) [1 mark]

2 a Shortest distance $= 0.6 + 1.2 + 0.6 + 0.8 = 3.2$ kilometers [1 mark]

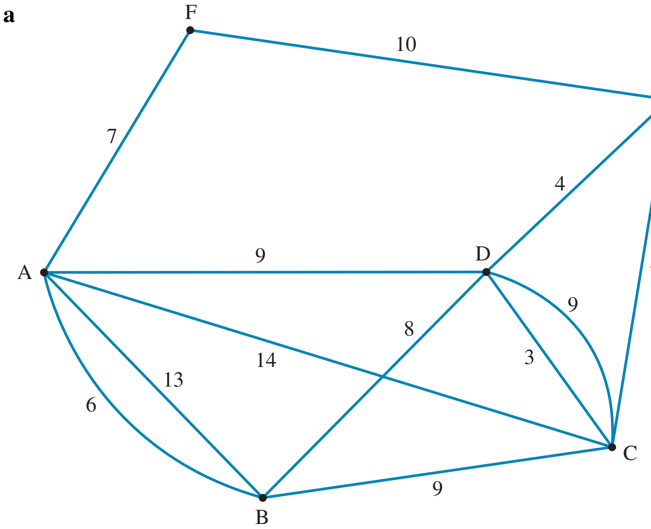
b i Eulerian trail [1 mark]

ii Eulerian trails start and finish at vertices with an odd degree. The training program starts at S, with a degree of 3, and will finish at P, also with a degree of 3. [1 mark]

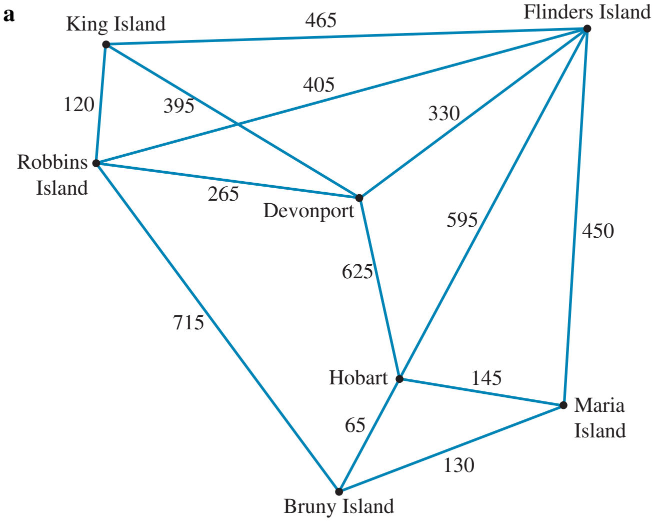
c This track is between exercise station S and exercise station T. [1 mark]



***19 a**



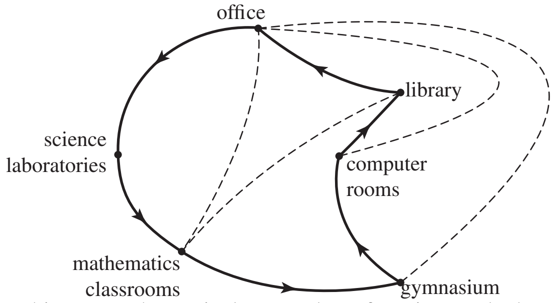
***20 a**



3 **a** The office [1 mark]

b **i** Hamiltonian cycle [1 mark]

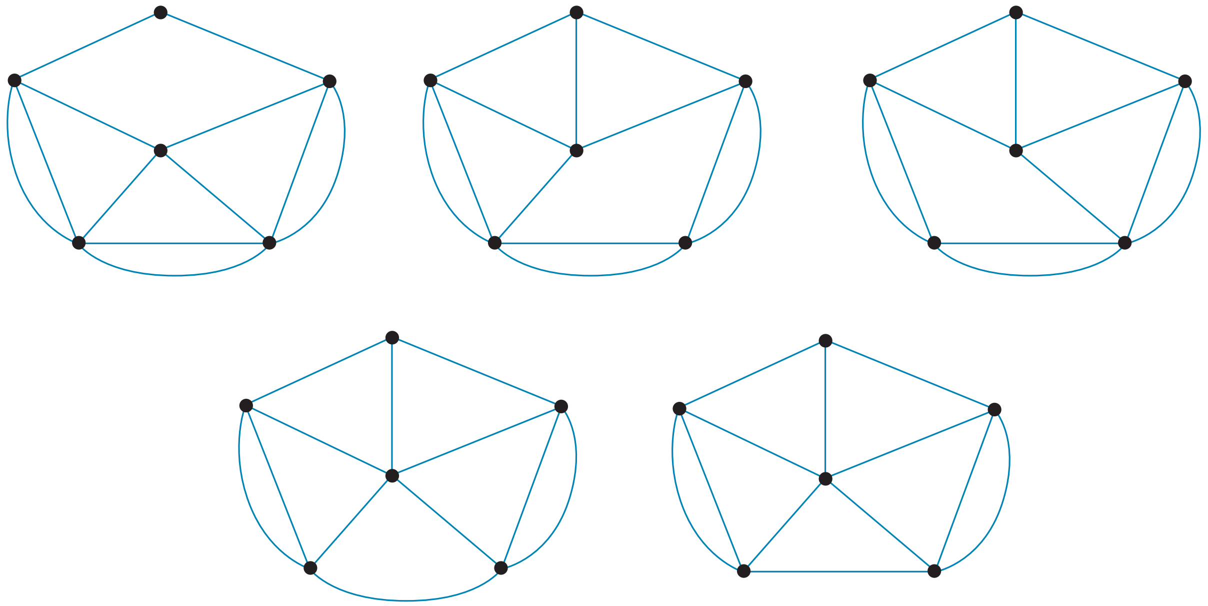
ii One possibility:



[1 mark]

4 Isomorphic means the equivalent number of vertices and edges.
Graph 1 has four vertices and five edges, while Graph 2 has five vertices and six edges. Therefore, Graphs 1 and 2 are not isomorphic.
The correct answer is **A**.

5 An Eulerian trail exists if the graph has two vertices with an odd degree and the degree of the other vertices are even. The given graph has six vertices of which four are of an odd degree and two are of an even degree.
Removing any edge between two vertices that are of an odd degree will change the network to an Eulerian trail. There are five different ways in which this can be done.



The correct answer is **E**.

Topic 10 — Variation

10.2 Direct and inverse variation

10.2 Exercise

1 $k = \frac{3000}{50}$

$k = 60$

$C = kN$

$C = 60N$

2 $d \propto t$

$\rightarrow d = kt$

$t = 1,\ d = 90$

$\rightarrow 90 = 1 \times k$

$\rightarrow k = 90$

$\rightarrow d = 90t$

3 a If $y \propto x,\ \ y = kx.$

$2.8 = k(0.7) \rightarrow k = 4$

$4.8 = k(1.2) \rightarrow k = 4$

$12 = k(4) \rightarrow k = 3$

$16.4 = k(4.1) \rightarrow k = 4$

No, as $y \neq kx$ for all values.

b If $y \propto x,\ \ y = kx.$

$0.84 = k(0.4) \rightarrow k = 2.1$

$3.15 = k(1.5) \rightarrow k = 2.1$

$4.62 = k(2.2) \rightarrow k = 2.1$

$7.14 = k(3.4) \rightarrow k = 2.1$

Yes, as $y = 2.1x$ for all values.

4 a $I \propto A$

b $I = kA$

c $I = kA$

$\rightarrow 15\,000 = 30\,000k$

$\rightarrow k = \frac{1}{2}$

d $I = \frac{1}{2}A$

e $I = \frac{1}{2}A$

$= \frac{1}{2} \times 55\,000$

$= 27\,500$

$I = \$27\,500$

f $I = \frac{1}{2}A$

$75\,000 = \frac{1}{2}A$

$75\,000 \times 2 = A$

$150\,000 = A$

$A = \$150\,000$

5 a $K \propto m,\ K = km.$

$27.9 = k(6.2) \rightarrow k = \frac{27.9}{6.2} = 4.5$

b $K = 4.5m$

c $K = 4.5m$

$= 4.5 \times 72$

$= 324$

d $K = 4.5m$

$450 = 4.5 \times m$

$m = \frac{450}{4.5}$

$= 100$

6 $T \propto \frac{1}{s}$

$\rightarrow T = \frac{k}{s}$

$s = 100,\ T = 1$

$\rightarrow 1 = \frac{k}{100}$

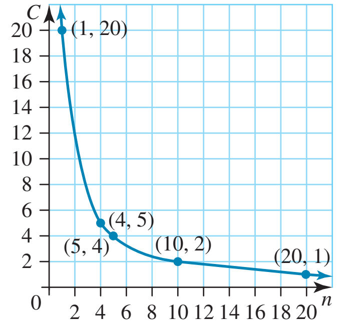
$\rightarrow k = 100$

$\rightarrow T = \frac{100}{s}$

7 a

Family members	20	10	5	4	1
Calculation	$\frac{20}{20} = 1$	$\frac{20}{10} = 2$	$\frac{20}{5} = 4$	$\frac{20}{4} = 5$	$\frac{20}{1} = 20$
Number of chocolates	1	2	4	5	20

b



c $C \propto \frac{1}{n}$

$\rightarrow C = \frac{k}{n}$

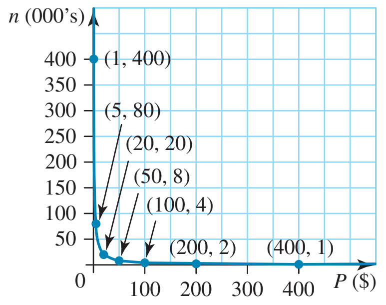
$n = 20,\ C = 1$

$\rightarrow 20 = \frac{k}{1}$

$\rightarrow k = 20$

$\rightarrow C = \frac{20}{n}$

8 a



b $n \propto \frac{1}{P}$

$\rightarrow n = \frac{k}{P}$

$P = 1, n = 400$

$\rightarrow 400 = \frac{k}{1}$

$\rightarrow k = 400$

$\rightarrow n = \frac{400}{P}$

c $n = \frac{400}{P}$

$n = \frac{400}{25}$

$n = 16$

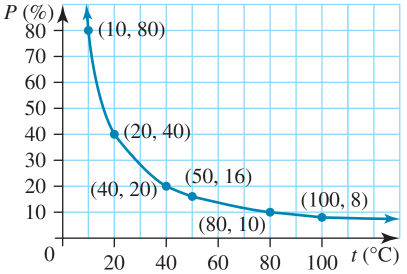
d $n = \frac{400}{P}$

$250\,000 = \frac{400}{P}$

$P = \frac{400}{250\,000}$

$P = \$0.0016$

9 a



b $P \propto \frac{1}{t}$

$\rightarrow P = \frac{k}{t}$

$t = 10, P = 80$

$\rightarrow 80 = \frac{k}{10}$

$\rightarrow k = 800$

$P = \frac{800}{t}$

c $P = \frac{800}{t}$

$= \frac{800}{25}$

$= 32$

$P = 32\%$

d $P = \frac{800}{t}$

$3.2 = \frac{800}{t}$

$t = \frac{800}{3.2}$

$t = 250\,°\text{C}$

10.2 Exam questions

1 As the number of hours worked increases, the pay received will also increase.
When hours worked is zero, pay received will also be zero.
As the hourly rate is consistent, the graph connecting these two variables will be linear.
Therefore, these variables are directly proportional.
The correct answer is **D**.

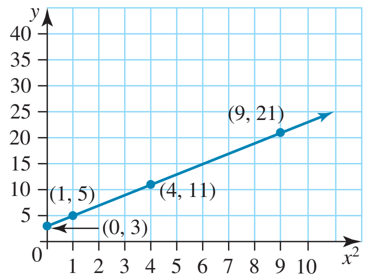
2 As one variable increases, the other must decrease.
The graph relating the two variables must be a hyperbola.
The correct answer is **E**.

3 As x increases, y also increases. [1 mark]
The graph relating y and x goes through the origin. [1 mark]
The graph relating y and x is linear. [1 mark]
As these variables satisfy all three conditions, y and x are directly proportional.

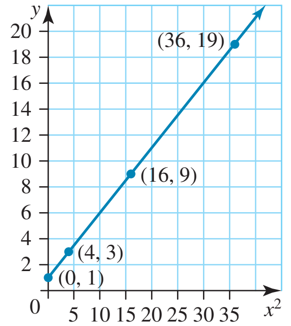
10.3 Data transformations

10.3 Exercise

1 a

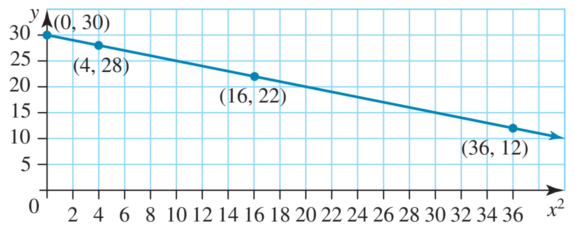


b

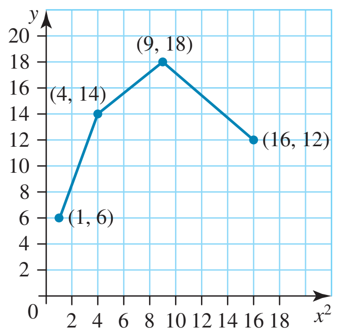


c See the graph at the bottom of the page.*

***1 c**

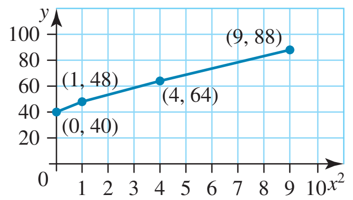


2 a i



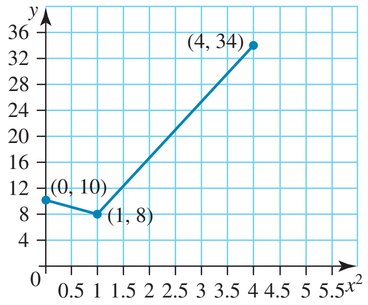
ii The graph is not linearised.

b i



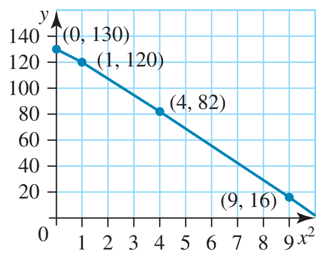
ii The graph is linearised, but not perfectly.

c i



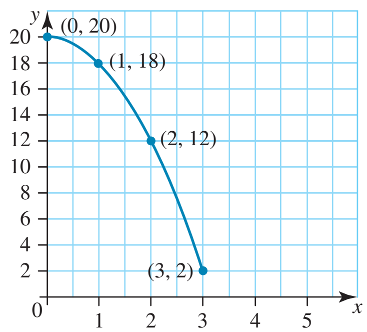
ii The graph is not linearised.

d i



ii The graph is linearised, but not perfectly.

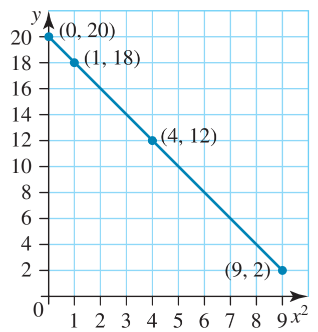
3 a i



ii

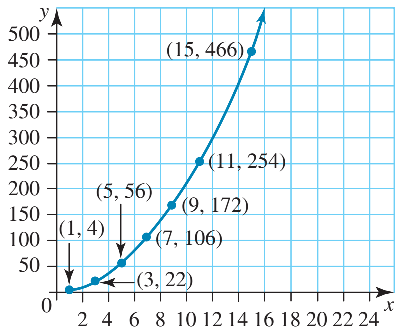
x^2	0	1	4	9
y	20	18	12	2

iii



iv The transformed data has been linearised.

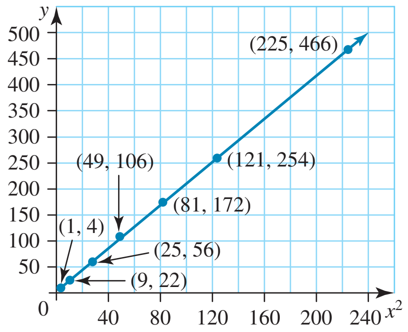
b i



ii

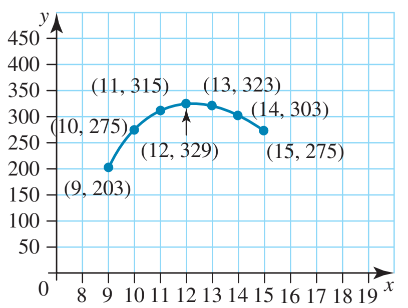
x^2	1	9	25	49	81	121	225
y	4	22	56	106	172	254	466

iii



iv The transformed data has been linearised.

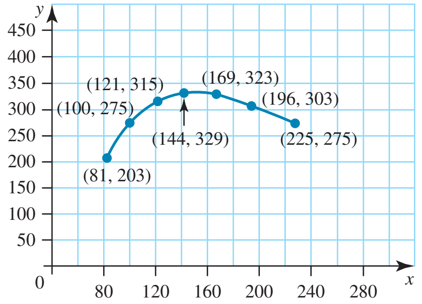
c i



ii

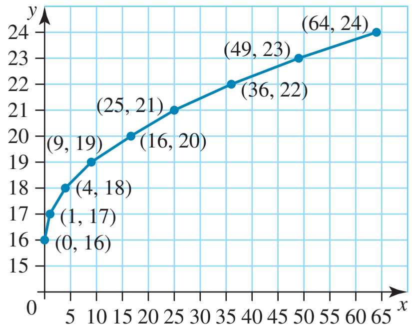
x^2	81	100	121	144	169	196	225
y	203	275	315	329	323	303	275

iii



iv The transformed data has not been linearised.

d i



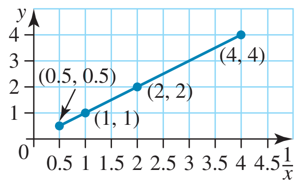
ii

x^2	0	1	16	81	256	625	1296	2401	4096
y	16	17	18	19	20	21	22	23	24

iii See the graph at the bottom of the page.*

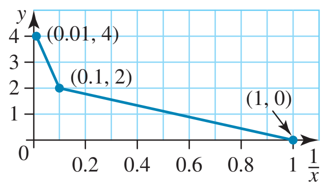
iv The transformed data has not been linearised.

4 a



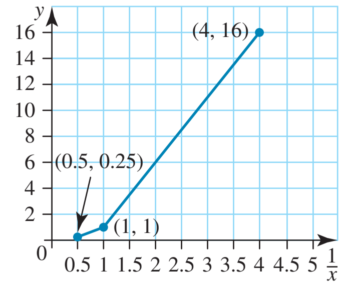
The graph is linearised.

b



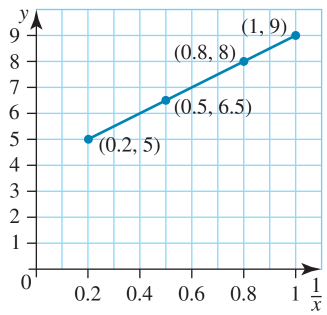
The graph is not linearised.

5 a i



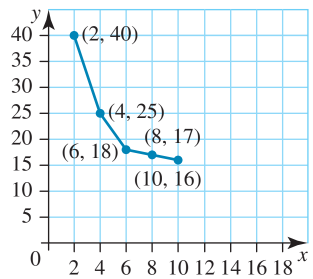
ii The graph is not linearised, but appears more linear than original one.

b i



ii The graph is linearised.

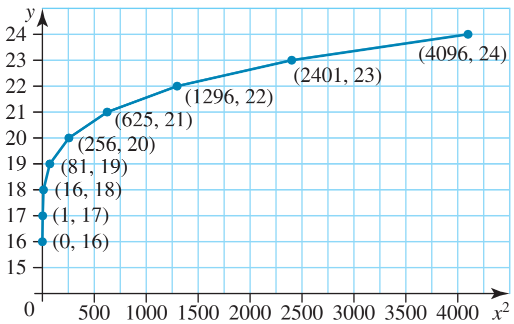
6 a i



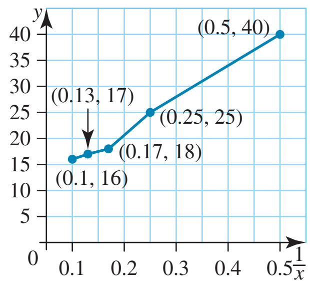
ii

$\frac{1}{x}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$
y	40	25	18	17	16

***3 d iii**

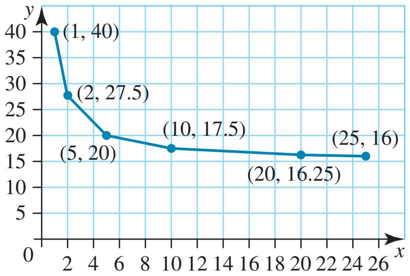


iii



iv The transformed data has been made more linear.

b i



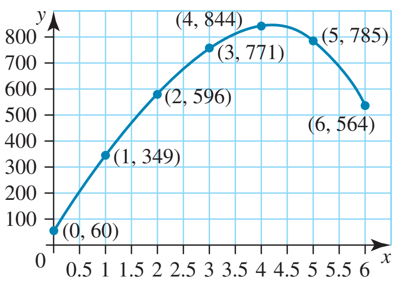
ii

$\frac{1}{x}$	1	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{25}$
y	40	27.5	20	17.5	16.25	16

iii See the graph at the bottom of the page.*

iv The transformed data has been linearised.

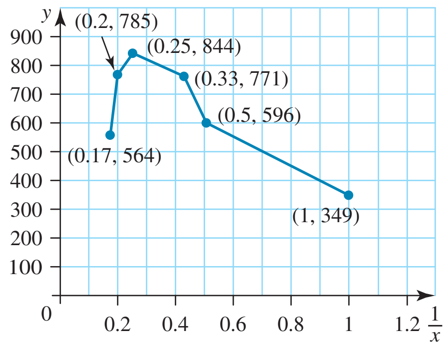
c i



ii

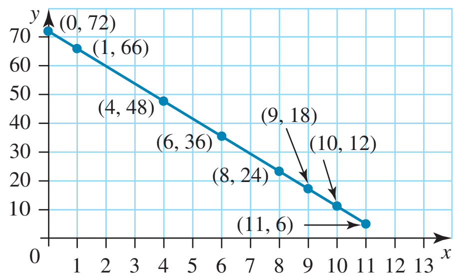
$\frac{1}{x}$	undefined	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
y	60	349	596	771	844	785	564

iii



iv The transformed data has not been linearised.

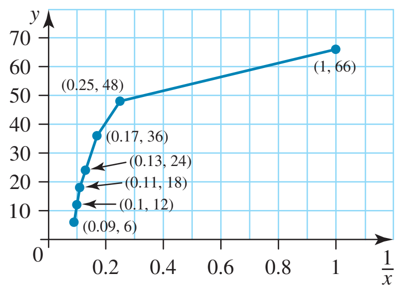
d i



ii

$\frac{1}{x}$	Undefined	1	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$
y	72	66	48	36	24	18	12	6

iii

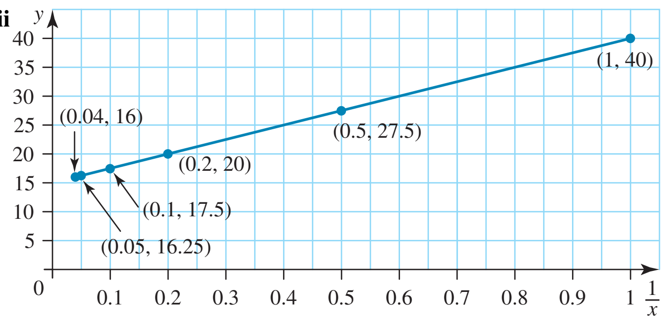


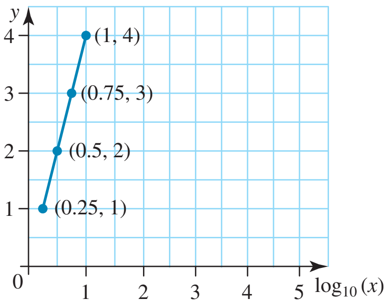
iv The original data was linear but the transformed data is not.

7 a

x	1.8	3.2	5.6	10
$\log_{10}(x)$	0.25	0.5	0.75	1
y	1	2	3	4

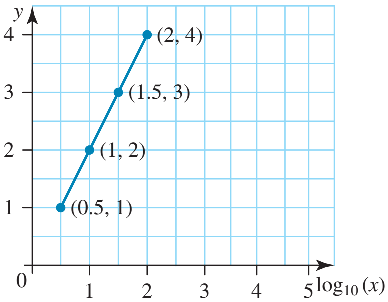
*6 b iii





b

x	3.2	10	31.6	100
$\log_{10}(x)$	0.5	1	1.5	2
y	1	2	3	4



8 a

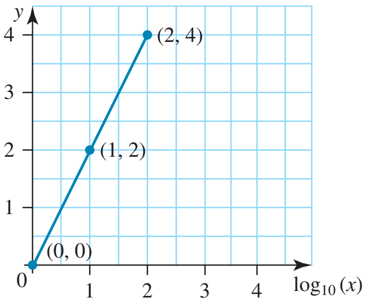
x	0.25	0.5	1	2
$\log_{10}(x)$	−0.6	−0.30	0	0.30
y	4	2	1	0.5

See the graph at the bottom of the page.*

This graph is not linearised by the $\log_{10}(x)$ transformation.

b

x	1	10	100
$\log_{10}(x)$	0	1	2
y	0	2	4

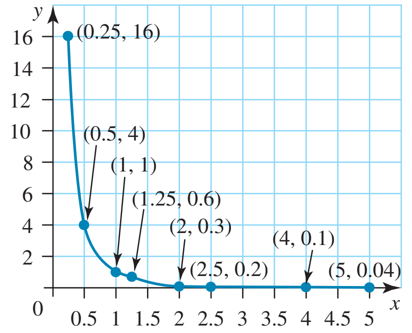


This graph is linearised by the $\log_{10}(x)$ transformation.

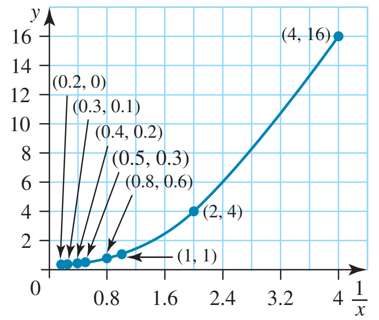
9 a

x	0.25	0.5	1	1.25	2	2.5	4	5
$\frac{1}{x}$	4	2	1	0.8	0.5	0.4	0.3	0.2
y	16	4	1	0.64	0.25	0.16	0.0625	0.04

b Original data:

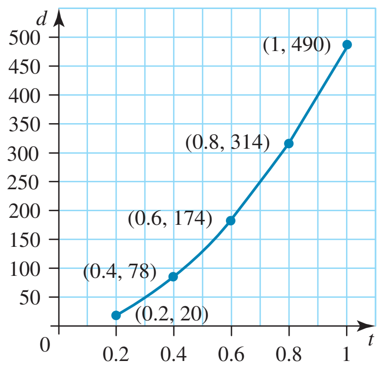


Transformed data:

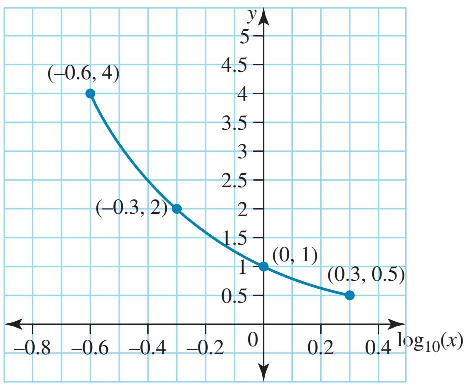


c This transformation is not very effective to linearise this data.

10 a

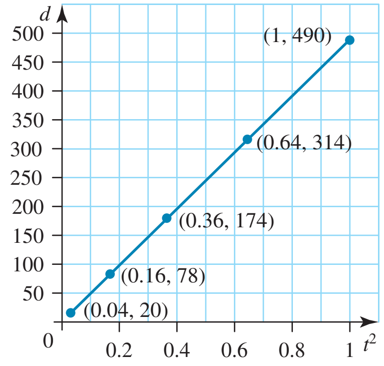


***8 a**

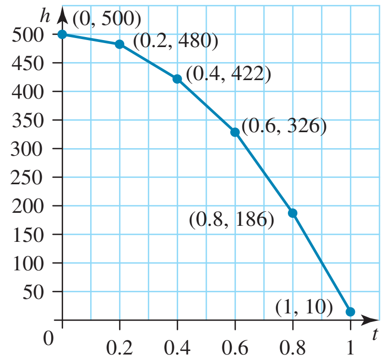


b

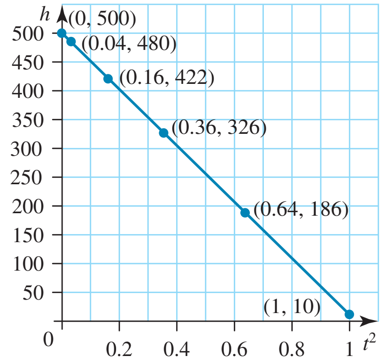
Time^2 (s)^2	0.04	0.16	0.36	0.64	1.0
Distance (cm)	20	78	174	314	490



c

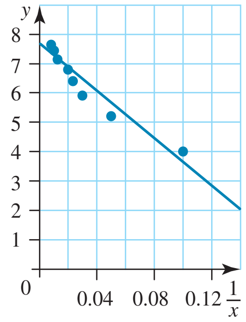


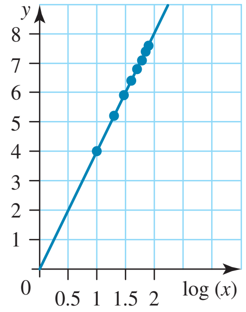
d The same data transformation will work in this case as the original data is parabolic in its shape, so the transformed data will look like this.



11 a See the table at the bottom of the page.*

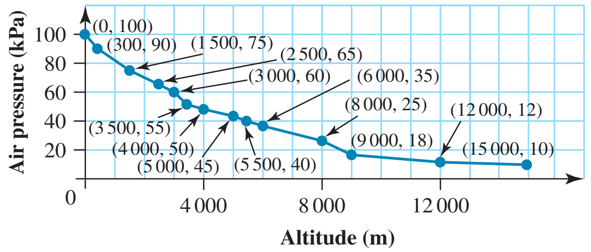
b



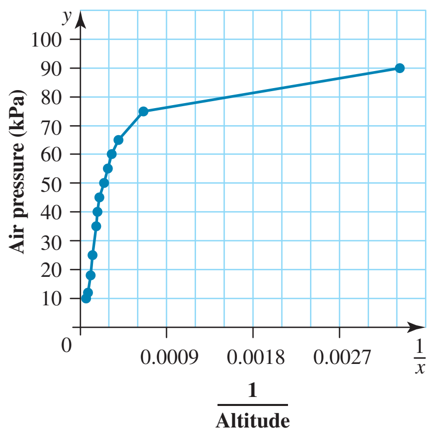


c When using the calculator, the log (x) transformation gives the best linearisation of the data.

12 a



b The general shape of the graph of the original data indicates that a reciprocal $\left(\frac{1}{x}\right)$ transformation might be appropriate.



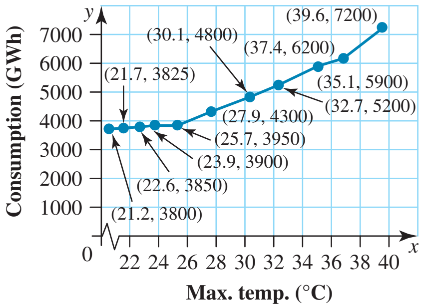
*11 a								
	x	10	20	30	40	50	60	70	80
	y	4	5.2	5.9	6.4	6.8	7.1	7.4	7.6
	$\frac{1}{x}$	0.10	0.05	0.033	0.025	0.02	0.017	0.014	0.013
	$\log(x)$	1.00	1.30	1.48	1.60	1.70	1.78	1.85	1.90

Altitude (m)	Air pressure (kPa)	$\frac{1}{\text{Altitude (m)}}$
0	100	Undefined
300	90	0.003 33
1500	75	0.000 66
2500	65	0.000 40
3000	60	0.000 33
3500	55	0.000 29
4000	50	0.000 25
5000	45	0.000 20
5500	40	0.000 18
6000	35	0.000 17
8000	25	0.000 13
9000	18	0.000 11
12 000	12	0.000 08
15 000	10	0.000 06

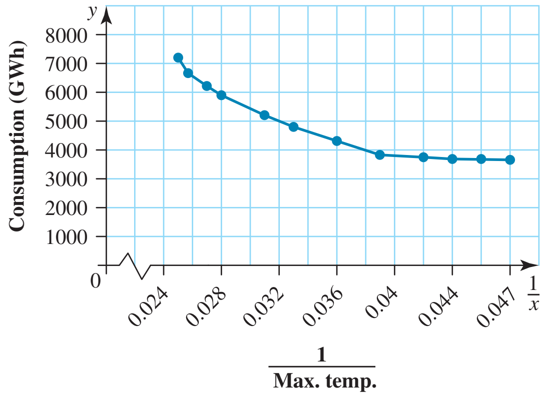
c The $\frac{1}{x}$ transformation doesn't put all the data in a straight line. Most of the values seem to be more linear, except for the lower altitude values. This would seem to indicate that an alternate transformation (other than $\frac{1}{x}$ or x^2) would be needed.

13 a As it is likely that the consumption of electricity would be influenced by the maximum temperature, the temperature should be used for the x-axis.

b i

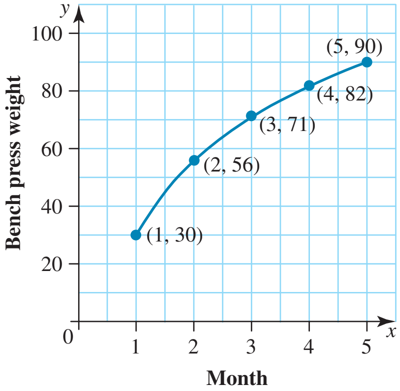


ii See the graph at the bottom of the page.*



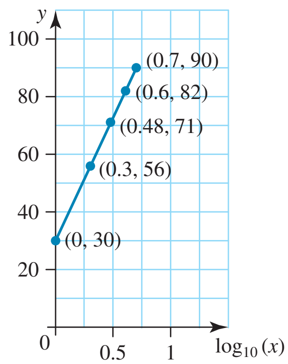
iii Both transformations have a linearising effect, but neither appear to be substantially better than the original data.

14 a

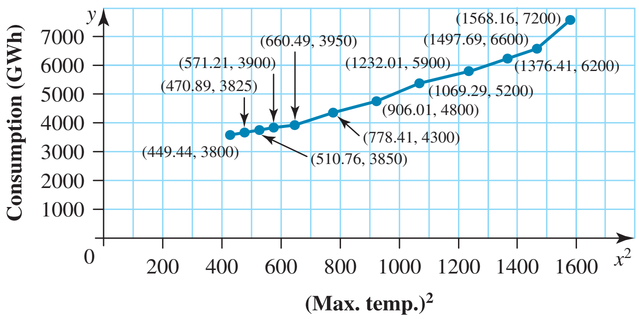


b

x (months)	1	2	3	4	5
$\log_{10}(x)$	0	0.30	0.48	0.60	0.70
y (weight)	30	56	71	82	90



***13 b ii**



10.3 Exam questions

1 Option C would benefit most from an $x \to x^2$ transformation, based on the shape of the $y \leftrightarrow x$ graph.
The correct answer is **C**.

2 $y = a + \frac{b}{x}$

$6 = a + \frac{b}{1}$

$6 = a + b \qquad (1)$

$4 = a + \frac{b}{2} \qquad (2)$

Solving simultaneous equations

$8 = 2a + b \qquad (3)$

Subtract (1) from (3):

$a = 2$

Substitute $a = 2$ into (1):

$6 = 2 + b$

$b = 4$

$\therefore a = 2,\ b = 4$

The correct answer is **B**.

3 **a** $C = F + kl^2$

$45 = F + k10^2 \quad (1)$

$90 = F + k21^2 \quad (2)$

Solve the equations simultaneously.

Subtract (1) from (2):

$45 = 341k$

$k = \frac{45}{341}$

≈ 0.132

$\therefore F \approx 31.80$ cents [1 mark]

b $C = F + kl^2$

$= 31.80 + 0.132l^2$ [1 mark]

c If $l = 14.8$,

$C = 31.80 + 0.132 \times 14.8^2$

$= 31.80 + 28.91328$

$= 60.71328$

$1000C = 1000 \times 60.713\,28$ cents

$= 60713.28$ cents

$= \$607.13$ [1 mark]

Don't round off before completing calculations, since you will be multiplying by 1000.

10.4 Orders of magnitude

10.4 Exercise

1 $\frac{3000}{0.3} = 10\,000$

$= 10^4$

The order of magnitude that expresses the difference in distance is 4.

2 $\frac{4000}{4} = 1\,000$

$= 10^3$

The order of magnitude that expresses the difference in mass is 3.

3 Both numbers in scientific notation have the same coefficient (3.2) so we can compare the powers of 10:

$2 - -1 = 2 + 1$

$= 3$

The weights of the bears differ by 3 orders of magnitude.

4 $7.2 \times 10^3 = 7.2 \times 1000$

$= 7200$

It will take the snail 7200 seconds to cover 100 m.

5 $10\,000\,000\,000 = 10^{10}$, so to be 10 000 000 000 smaller, multiply by 10^{-10}:

$1 \times 10^{-20} \times 10^{-10}$kg $= 1 \times 10^{-30}$ kg

The correct answer is **B**.

6 $5 - 3 = 2$

$10^2 = 100$

An earthquake of magnitude 5 is 100 times stronger than an earthquake of magnitude 3.

7 **a** $5 - 2 = 3$

The difference in the order of magnitude between the acidity of the soft drink and lemon juice is 3.

b $10^3 = 1000$

The lemon juice is 1000 times more acidic than the soft drink.

8 $10\,000 = 10^4$, so to be 10 000 times more acidic or alkaline, the pH level must be 4 more or less than 7:

$7 - 4 = 3$ or $7 + 4 = 11$

The correct answer is **C**.

9 **a** $9.5 - 9 = 0.5$

There is a 0.5 difference in magnitude between the two earthquakes.

b $10^{0.5} = 3.162...$

The earthquake in Chile had an amplitute 3.16 times larger than the earthquake in Japan.

10 $1000 = 10^3$; therefore, reducing the acidity by 1000 means adding 3 to the pH level.

$1 + 3 = 4$

It would register a pH level of 4.

11 **a** Compare powers of 10:

$3 - 1 = 2$

The masses of the balls differ by 2 orders of magnitude.

b $5 \times 10^1 = 5 \times 10$

$= 50$

$5 \times 10^3 = 5 \times 1000$

$= 5000$

Tennis ball: 50 g

Medicine ball: 5000 g

12 **a** Compare powers of 10:

$(-10) - (-15) = (-10) + 15$

$= 5$

The sizes of the hydrogen atom and the nucleus differ by 5 orders of magnitude.

b $10^5 = 100\,000$

The atom is100 000 times larger than the nucleus.

13 **a** $5 \times 10^3 \times 10^{-1}$

b $5 \times 10^3 \times 10^{-1} = 5 \times 10^2$

$= 500$

The new volume of the container is 500 mL.

14 **a** $2.5 \times 10^1 \times 10^{-2} = 2.5 \times 10^{-1}$ km

b 2.5×10^{-1} km $= 2.5 \times 0.1$ km

$= 0.25$ km

The distance from Zoe's house to school is 0.25 km (or 250 m).

10.4 Exam questions

1 1 day = 24 hours

$= 24 \times 60$ minutes

$= 24 \times 60 \times 60$ seconds

$= 86\,400$ seconds

$= 8.64 \times 10^4 \approx 10^5$ seconds

The correct answer is **B**.

2 $10^6 = 1000 \times 10^3$

The correct answer is **A**.

3 $100\,000\,000\,000\,000 = 10^{14}$

The correct answer is **C**.

10.5 Non-linear data modelling

10.5 Exercise

1 $y = kx^2 + c$

$(9, 30)\ \&\ (4, 15) \rightarrow k = \dfrac{30-15}{9-4} = \dfrac{15}{5} = 3$

$(0, 3) \rightarrow c = 3$

$\therefore y = 3x^2 + 3$

2 a $y = kx^2 + c$

$(1, 23)$ and $(0, 25) \rightarrow k = \dfrac{23-25}{1-0} = -2$

$(0, 25) \rightarrow c = 25$

$\therefore y = -2x^2 + 25$

b $y = kx^2 + c$

$(1, 10.4)$ and $(0, 10) \rightarrow k = \dfrac{10.4-10}{1-0} = 0.4$

$(0, 10) \rightarrow c = 10$

$\therefore y = 0.4x^2 + 10$

3 a i $d \propto m \rightarrow d = km$

$d = 40,\ m = 6$

$\rightarrow 40 = 6k$

$\therefore k = \dfrac{40}{6}$

$\rightarrow d = \dfrac{40}{6}m$

ii $m = 15,$

$d = \dfrac{40}{6} \times 15 = 100$

$d = \dfrac{40}{6}m$ and 100 cm

b i $L \propto \dfrac{1}{f} \rightarrow L = \dfrac{k}{f}$

$L = 15, f = 600$

$\rightarrow 15 = \dfrac{k}{600}$

$k = 15 \times 600 = 9000$

$\rightarrow L = \dfrac{9000}{f}$

ii $f = 1600$

$\rightarrow L = \dfrac{9000}{1600} = 5.625$

$L = \dfrac{9000}{f}$ and 5.625 m

c i $d \propto s^2 \rightarrow d = ks^2$

$d = 40, s = 100$

$\rightarrow 40 = k \times 100^2$

$k = \dfrac{40}{10\,000} = 0.004$

$\rightarrow d = 0.004\,s^2$

ii $d = 25$

$\rightarrow 25 = 0.004\,s^2$

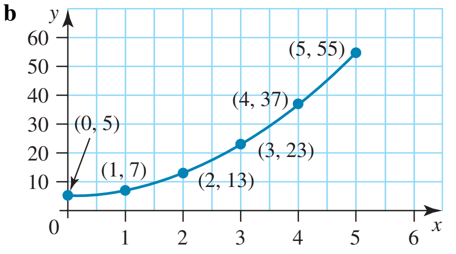
$s = \sqrt{\dfrac{25}{0.004}} = 79$

$s = 79$ km/h

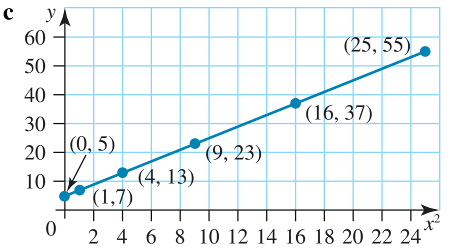
4 a

x	0	1	2	3	4	5
y	5	7	13	23	37	55

b

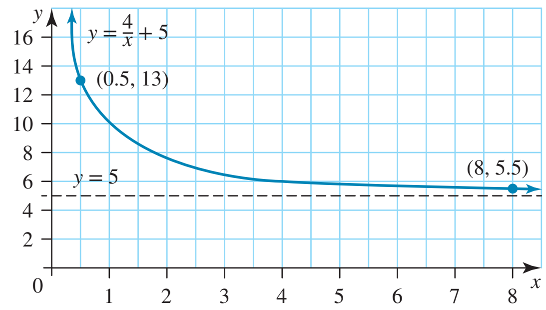


c



d Gradient $= \dfrac{7-5}{1-0} = 2$

5



6 a $y = \dfrac{k}{x} + c$

$(1, 18) \rightarrow 18 = \dfrac{k}{1} + c$ (Equation 1)

$(2, 12) \rightarrow 12 = \dfrac{k}{2} + c$ (Equation 2)

$\rightarrow 6 = \dfrac{k}{2}$ (Equation 1 subtract equation 2)

$\therefore k = 12$

$\rightarrow 18 = \dfrac{12}{1} + c$ (Equation 1)

$\therefore c = 6$

$\therefore y = \dfrac{12}{x} + 6$

b $y = \frac{k}{x} + c$

$(1, 16) \rightarrow 16 = \frac{k}{1} + c$ (Equation 1)

$(2, 6) \rightarrow 6 = \frac{k}{2} + c$ (Equation 2)

$\rightarrow 10 = \frac{k}{2}$ (Equation 1 subtract equation 2)

$\therefore k = 20$

$\rightarrow 16 = \frac{20}{1} + c$ (Equation 1)

$\therefore c = -4$

$y = \frac{20}{x} - 4$

7 a $y = \frac{k}{x} + c$

$(1, 14) \rightarrow 14 = \frac{k}{1} + c$ (Equation 1)

$(3, 6) \rightarrow 6 = \frac{k}{3} + c$ (Equation 2)

$\rightarrow 8 = \frac{2k}{3}$ (Equation 1 subtract equation 2)

$\therefore k = \frac{8 \times 3}{2} = 12$

$\rightarrow 14 = \frac{12}{1} + c$ (Equation 1)

$\therefore c = 2$

$k = 12$ and $c = 2$

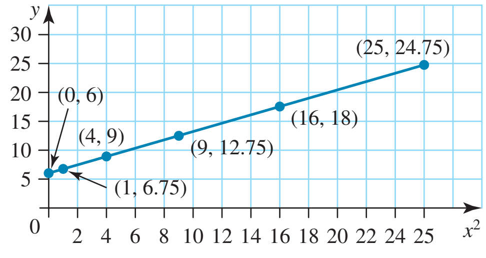
b

x	0.25	0.5	0.75	1.5	2	12
y	50	26	18	10	8	3
$\frac{1}{x}$	4	2	1.33	0.67	0.5	0.08

c See the graph at the bottom of the page.*

d Gradient $= \frac{50 - 26}{4 - 2} = 12$

8 a



b $y = kx^2 + c$

$(0, 6) \rightarrow 6 = k \times 0 + c$

$\therefore c = 6$

From $(4, 9)$ and $(0, 6)$:

$k = \frac{9 - 6}{4 - 0} = \frac{3}{4}$

$\therefore k = 0.75$

$y = 0.75x^2 + 6$

9 See the graph at the bottom of the page.*

10 a $y = k \log_{10}(x) + c$

$(1, 1) \rightarrow 1 = k \log_{10}(1) + c$

$\rightarrow 1 = k \times 0 + c$

$\therefore c = 1$

$(10, 3) \rightarrow 3 = k \log_{10}(10) + 10$

$\rightarrow 3 = k \times 1 + 1$

$\therefore k = 2$

$y = 2 \log_{10}(x) + 1$

b $y = k \log_{10}(x) + c$

$(10, 1) \rightarrow 1 = k \log_{10}(10) + c$

$\rightarrow 1 = k + c$ (Equation 1)

$(100, 7) \rightarrow 7 = k \log_{10}(100) + c$

$\rightarrow 7 = 2k + c$ (Equation 2)

$7 - 1 = 2k - k + c - c$ (Equation 2 − 1)

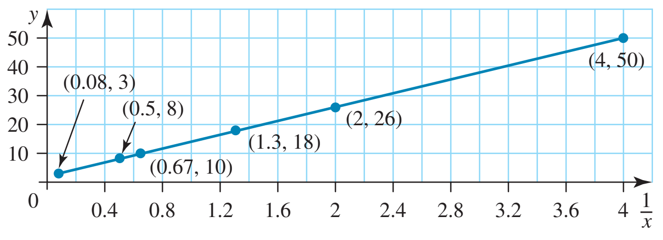
$\therefore k = 6$

$\rightarrow 1 = 6 + c$ (Equation 1)

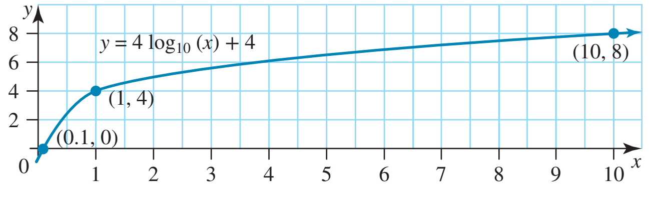
$\therefore c = -5$

$y = 6 \log_{10}(x) - 5$

***7 c**



***9**



11 a

t	1	10	100
P	600	610	620

b See the graph at the bottom of the page.*

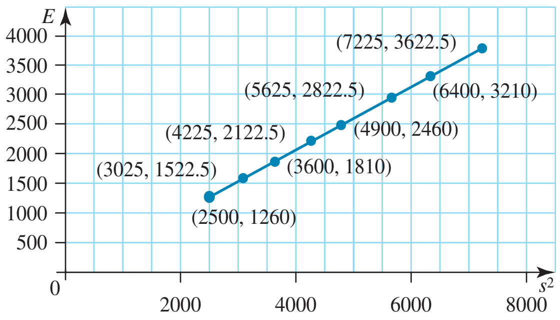
c $P = 10\log_{10}(t) + 600$

$= 10\log_{10}(1000) + 600$

$= 10 \times 3 + 600$

$= 630$

12 a



b $E \propto s^2 \rightarrow E = ks^2 + c$

From (s^2, E) : (3600, 1810) and (2500, 1260)

$$k = \frac{1810 - 1260}{3600 - 2500} = 0.5$$

$\rightarrow E = 0.5s^2 + c$

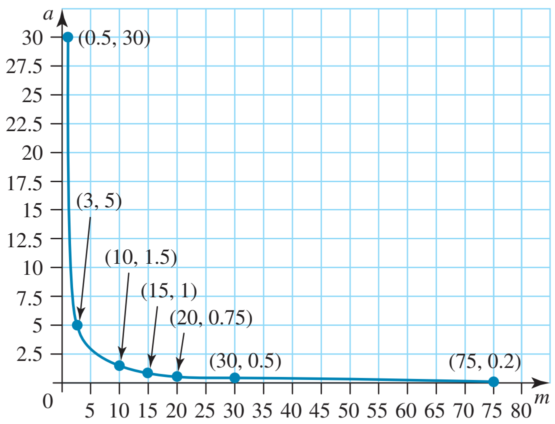
$1260 = 0.5(2500) + c$

$c = 1260 - 1250 = 10$

Therefore, $E = 0.5s^2 + 10$

13 a Inverse variation

b



c $a \propto \frac{1}{m} \rightarrow a = \frac{k}{m}$

$a = 1, m = 15$

$\rightarrow 1 = \frac{k}{15}$

$\therefore k = 15$

$a = \frac{15}{m}$

14 a $f \propto \frac{1}{L} \rightarrow f = \frac{k}{L}$

$f = 5.25,\ L = 62$

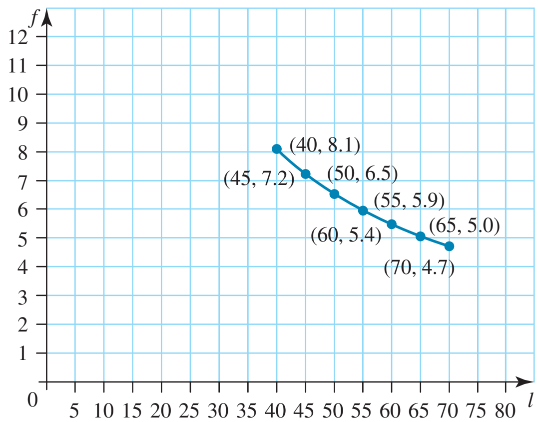
$\rightarrow 5.25 = \frac{k}{62}$

$\therefore k = 325.5$

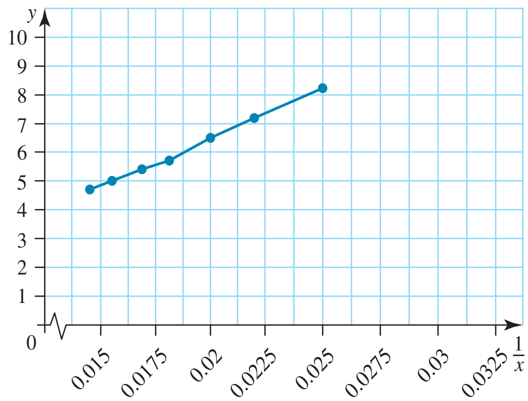
$\rightarrow f = \frac{325.5}{L}$

String length (cm)	70	65	60	55	50	45	40
Vibrations per second	4.7	5.0	5.4	5.9	6.5	7.2	8.1

b



c



d The transformed data is linearised.

15 a $y = \log_{10}(x) + c$

$(1, 3) \rightarrow 3 = k\log_{10}(1) + c$

$\rightarrow 3 = k \times 0 + c$

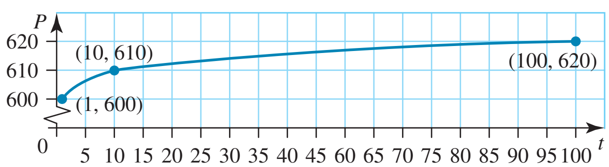
$\therefore c = 3$

$(10, 5) \rightarrow 5 = k\log_{10}(10) + 3$

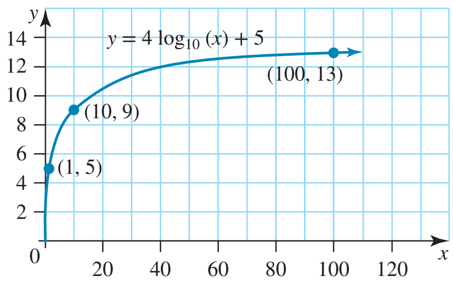
$\rightarrow 5 = k \times 1 + 3$

$\therefore k = 2$

***11 b**



b



16 a i $N = 100\log_{10}(t) + 20$

$t = 1$:

$N = 100\log_{10}(1) + 20$

$N = 100 \times 0 + 20$

$N = 20$

ii $N = 100\log_{10}(t) + 20$

$t = 2$:

$N = 100\log_{10}(2) + 20$

$N = 100 \times 0.3 + 20$

$N = 50$

iii $N = 100\log_{10}(t) + 20$

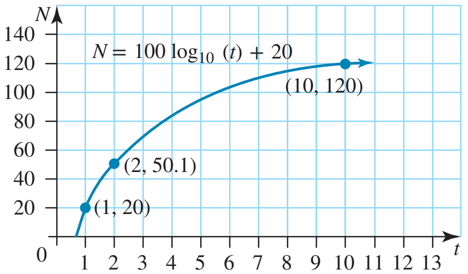
$t = 10$:

$N = 100\log_{10}(10) + 20$

$N = 100 \times 1 + 20$

$N = 120$

b



c $N = 100\log_{10}(t) + 20$

$N = 240$:

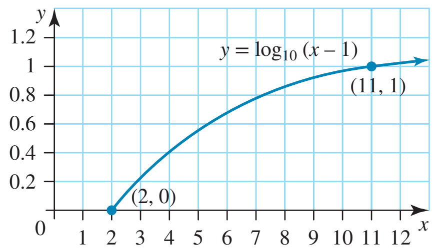
$240 = 100\log_{10}(t) + 20$

$\rightarrow \dfrac{220}{100} = \log_{10}(t)$

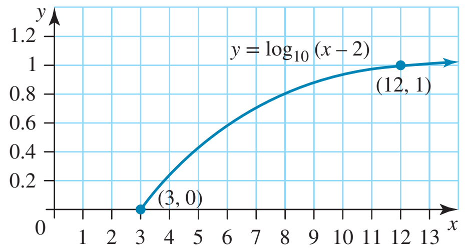
$t = 10^{2.2}$

$t = 158$ days

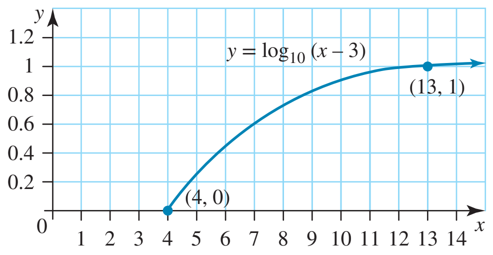
17 a i



ii



iii

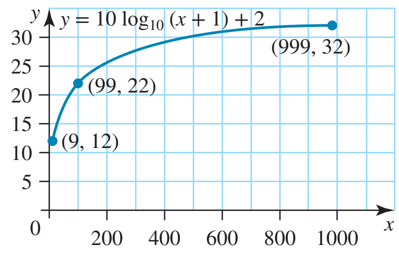


b As the value of b increases, the graph moves an equivalent distance from the y-axis. The x-axis intercept is 1 more than the value of b.

18 a

x	9	99	999
y	12	22	32

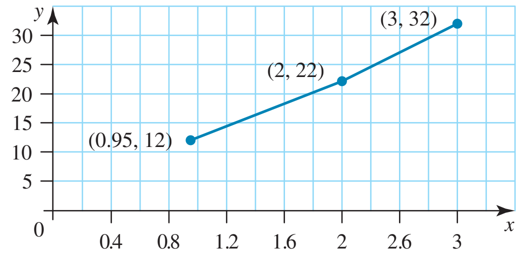
b



c

x	9	99	999
$\log_{10}(x)$	0.9542	1.9956	2.9995
y	12	22	32

d



e The transformed data has been linearised.

10.5 Exam questions

1 $y = kx^2$

$y = 300$ when $x = 25$

$300 = k \times 25^2$

$300 = k \times 625$

$k = \dfrac{300}{625}$

$k = 0.48$

Therefore, $y = 0.48x^2$

The correct answer is **C**.

2 $y = \dfrac{k}{x}$

$y = 50$ when $x = 2$

$50 = \dfrac{k}{2}$

$k = 50 \times 2$

$k = 100$

Therefore, $y = \dfrac{100}{x}$

The correct answer is **D**.

3 $y = kx^2$
$y = 90$ when $x = 5$
$90 = k \times 5^2$
$90 = k \times 25$
$k = \frac{90}{25}$
$k = 3.6$
Therefore, $y = 3.6x^2$
The correct answer is **A**.

10.6 Review

10.6 Exercise

Multiple choice

1 Use the gradient formula to find k:
$k = \frac{\text{rise}}{\text{run}}$
$= \frac{2200}{1000}$
$= 2.2$
The correct answer is **A**.

2 $b \propto c^2 \rightarrow b = kc^2$
$(c, b) \rightarrow b = kc^2$
$(12, 72) \rightarrow 72 = k \times 12^2$
$k = \frac{72}{144}$
$= 0.5$
The correct answer is **A**.

3 $y \propto \frac{1}{x} \rightarrow y = \frac{k}{x}$
$(x, y) \rightarrow y = \frac{k}{x}$
$(50, 12) \rightarrow 12 = \frac{k}{50}$
$k = 12 \times 50$
$= 600$
Therefore, $y = \frac{600}{x}$
When $x = 25$,
$y = \frac{600}{25}$
$= 24$
The correct answer is **D**.

4 $P \propto \frac{1}{V} \rightarrow P = \frac{k}{V}$
$PV = k$
Option **E** states $\frac{P}{V} = k$, which is not true.
The correct answer is **E**.

5 $y \propto x^2 \rightarrow y = kx^2$
$(x, y) \rightarrow y = kx^2$
$(2, 100) \rightarrow 100 = k \times 2^2$
$k = \frac{100}{4}$
$= 25$
The correct answer is **E**.

6 $8.2 - 5.9 = 2.3$
The correct answer is **C**.

7 $9 - 4 = 5$
$10^5 = 100\,000$
The correct answer is **B.**

8 $y = kx^2 + c$
Substitute in the point $(0, 2)$:
$2 = 0 + c$
$c = 2$
Substitute in the point $(1, 2.25)$ and $b = 2$:
$2.25 = k \times 1^2 + 2$
$k = 2.25 - 2$
$= 0.25$
$= \frac{1}{4}$
Therefore, $y = \frac{x^2}{4} + 2$.
The correct answer is **D**.

9 By visual inspection, it can be seen that the graph is likely to follow a logarithmic relationship, $y = k\log_{10}(x) + c$
Substitute in the point $(1, 3)$:
$3 = k\log_{10}(1) + c$
$3 = k \times 0 + c$
$c = 3$
Substitute in the point $(10, 5)$ and $c = 3$:
$5 = k\log_{10}(10) + 3$
$5 = k \times 1 + 3$
$k = 2$
Therefore, $y = 2\log_{10}(x) + 3$.
The correct answer is **D**.

10 Substitute in $x^2 = 16$ to determine the corresponding y-value:
$y = \frac{x^2}{4} + 1$
$= \frac{16}{4} + 1$
$= 5$
Therefore $(16, 5)$ is one point.
Substitute in $x^2 = 0$ to find the corresponding y-value:
$y = \frac{x^2}{4} + 1$
$= \frac{0^2}{4} + 1$
$= 1$
Therefore, $(0, 1)$ is another point.
The correct answer is **B**.

Short answer

11 a As one variable increases, so does the other. Therefore, this is a case of direct variation.

b Graphing the results shows that the variables have a parabolic relationship.
$y = ax^2 + b$
$d = at^2 + b$
Substitute in the point $(0, 0)$:
$0 = a \times 0^2 + b$
$b = 0$
Substitute in the point $(1, 4.9)$ and $b = 0$:
$4.9 = a \times 1^2 + 0$
$a = 4.9$
Therefore, $d = 4.9t^2$.

c The time taken is 12 seconds, so substitute in $t = 12$:

$d = 4.9 \times 12^2$

$= 705.6$

Therefore, in 12 seconds the distance fallen is 705.6 metres.

d We want the time taken to fall a distance of 25 m, so substitute in $d = 12$:

$25 = 4.9t^2$

$t^2 = \dfrac{25}{4.9}$

$t = \sqrt{\dfrac{25}{4.9}}$

$= 2.26$

Therefore, the time taken to fall a distance of 25 m is 2.26 seconds.

12 a $t \propto \dfrac{1}{v}$

b $t = \dfrac{k}{v}$

c Time to complete 100 km at 50 km/hr:

$\text{Speed} = \dfrac{\text{dist.}}{\text{time}}$

$\text{Time} = \dfrac{\text{dist.}}{\text{speed}}$

$= \dfrac{100}{50}$

$= 2$

Therefore, the time to complete is 2 hours.

d Substitute in $v = 50$ and $t = 2$:

$(v, t) \to t = \dfrac{k}{v}$

$(50, 2) \to 2 = \dfrac{k}{50}$

$k = 2 \times 50$

$= 100$

e $t = \dfrac{100}{v}$

13 a $I \propto \dfrac{1}{d^2}$

b $I \propto \dfrac{1}{d^2} \to I = \dfrac{k}{d^2}$

$(d, I) \to I = \dfrac{k}{d^2}$

$(1.0, 5.0) \to 5.0 = \dfrac{k}{1.0^2}$

$k = 5$

Therefore, $I = \dfrac{5}{d^2}$

c Substitute in $d = 3.5$:

$I = \dfrac{5}{3.5^2}$

$= 0.41$

Therefore, the intensity of light is 0.41 lux.

d Substitute in $I = 0.05$:

$0.05 = \dfrac{5}{d^2}$

$0.05d^2 = 5$

$d^2 = \dfrac{5}{0.05}$

$d = \sqrt{100}$

$= 10$

Therefore, the distance is 10 metres.

14 a $w \propto h \to w = kh$

b Substitute in the first pair of values in the table:

$(h, w) \to w = kh$

$(7.0, 119.00) \to w = kh$

$k = \dfrac{119.00}{7.0}$

$= 17$

c See the table at the bottom of the page.*

d The $\dfrac{w}{h}$ value is the same for each day. This indicates that there is direct variation between the two variables: wages and hours worked.

15 a

x	3.5	6.1	9.7	11.2
y	7.35	12.81	21.34	23.52
$\dfrac{y}{x}$	$\dfrac{7.35}{3.5} = 2.1$	$\dfrac{12.81}{6.1} = 2.1$	$\dfrac{21.34}{9.7} = 2.2$	$\dfrac{23.52}{11.2} = 2.1$

y is not directly proportional to x because the $\dfrac{y}{x}$ values are not constant for all values in the table, i.e. $k \neq \dfrac{y}{x}$.

b $y = 2.1x$

$= 2.1 \times 9.7$

$= 20.37$

So change the third value, $y = 21.34$ to $y = 20.37$.

*14 c

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Hours worked (h)	7.0	6.5	9.0	7.0	8.5
Wages (w)	\$119.00	\$110.50	\$153.00	\$119.00	\$144.50
Wages/hour (w/h)	$\dfrac{\$119.00}{7.0} = \$17$	$\dfrac{\$110.50}{6.5} = \$17$	$\dfrac{\$153.00}{9.0} = \$17$	$\dfrac{\$119.00}{7.0} = \$17$	$\dfrac{\$144.50}{8.5} = \$17$

Extended response

16 a See the table at the bottom of the page.*

b The data supports Kepler's Second Law because $\frac{R^3}{T^2}$ is approximately the same value for all the planets in the table.

17 a $16 - 11 = 5$

b $1\text{ m} = 10^{-3}\text{ km}$

$1 \times 10^{11} \times 10^{-3} = 1 \times 10^{8}\text{ km}$

$1 \times 10^{16} \times 10^{-3} = 1 \times 10^{13}\text{ km}$

$13 - 8 = 5$

$10^5 = 100\,000$

c $\frac{12\,742}{1\,391\,684} \times 100 = 0.915\,581$

$= 0.916\%$

18 a By similar triangles,

$\frac{h}{100} = \frac{r}{20}$

$h = 5r$

Substitute in $r = 4$:

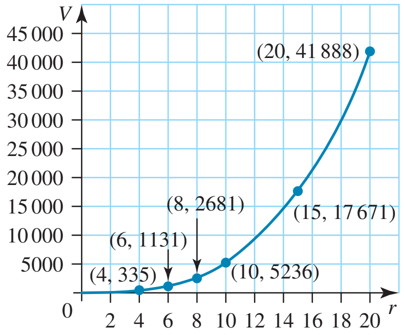
$h = 5 \times 4$

$= 20$

Therefore, the height is 20 cm.

b See the table at the bottom of the page.*

c



d See the table at the bottom of the page.*

$\frac{V}{r}$ is not constant; therefore, V does not vary directly as r varies.

***16 a**

Planet	**Orbital radius, R (m)**	R^3	**Orbital period, T (s)**	T^2	$\frac{R^3}{T^2}$
Mercury	5.79×10^{10}	1.94×10^{32}	7.60×10^{6}	5.78×10^{13}	3.36×10^{18}
Venus	1.08×10^{11}	1.26×10^{33}	1.94×10^{7}	3.76×10^{14}	3.35×10^{18}
Earth	1.49×10^{11}	3.31×10^{33}	3.16×10^{7}	9.99×10^{14}	3.31×10^{18}
Mars	2.28×10^{11}	1.19×10^{34}	5.94×10^{7}	3.53×10^{15}	3.37×10^{18}
Jupiter	7.78×10^{11}	4.71×10^{35}	3.74×10^{8}	1.40×10^{17}	3.36×10^{18}
Saturn	1.43×10^{12}	2.92×10^{36}	9.30×10^{8}	8.65×10^{17}	3.38×10^{18}
Uranus	2.87×10^{12}	2.36×10^{37}	2.66×10^{9}	7.08×10^{18}	3.33×10^{18}
Neptune	4.50×10^{12}	9.11×10^{37}	5.20×10^{9}	2.70×10^{19}	3.37×10^{18}

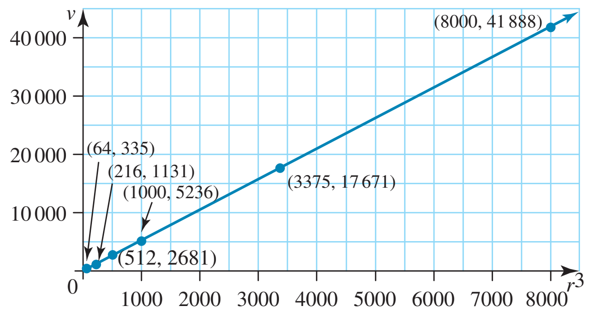
***18 b**

r (cm)	**4**	**6**	**8**
Volume: $V = \frac{\pi r^2 h}{3}$ (cm^3)	$\frac{\pi \times 4^2(5 \times 4)}{3} = 335$	$\frac{\pi \times 6^2(5 \times 6)}{3} = 1131$	$\frac{\pi \times 8^2(5 \times 8)}{3} = 2681$
r (cm)	**10**	**15**	**20**
Volume: $V = \frac{\pi r^2 h}{3}$ (cm^3)	$\frac{\pi \times 10^2(5 \times 10)}{3} = 5236$	$\frac{\pi \times 15^2(5 \times 15)}{3} = 17\,671$	$\frac{\pi \times 20^2(5 \times 20)}{3} = 41\,888$

***18 d**

r (cm)	**4**	**6**	**8**	**10**	**15**	**20**
Volume: $V = \frac{\pi r^2 h}{3}$ (cm^3)	335	1131	2681	5236	17 671	41 888
$\frac{V}{r}$	83.8	188.5	335.1	523.6	1178.1	2094.4

e See the table at the bottom of the page.*



f See the table at the bottom of the page.*

Therefore, $k = 5.24$.

$V = 5.24r^3$

10.6 Exam questions

1 $1\text{ Gb} = 10^9$ bytes

and $1\text{ Mb} = 10^6$ bytes

$\dfrac{10^9}{10^6}$ gives a difference of 3 orders of magnitude.

The correct answer is **C**.

2 $p \propto \dfrac{1}{q}$

$p = \dfrac{25}{q}$

$\therefore p$ varies inversely as q.

The correct answer is **A**.

3 $y = \dfrac{k}{x}$

$y = 75$ when $x = 2$

$75 = \dfrac{k}{2}$

$k = 75 \times 2$

$k = 150$

Therefore, $y = \dfrac{150}{x}$

The correct answer is **D**.

4 As the amount of fuel in the car's tank increases, the distance that can be travelled will also increase.

When the fuel tank is empty, the car will not be able to travel any distance.

As the car is travelling at a constant rate, the graph connecting these two variables will be linear.

Therefore, these variables are directly proportional.

The correct answer is **C**.

5 a See the graph at the bottom of the page.*

b **ii, iv** [1 mark]

c $t \propto \dfrac{1}{v}$ [1 mark]

d t against $\dfrac{1}{v}$ [1 mark]

e $t = \dfrac{880}{v}$ [1 mark]

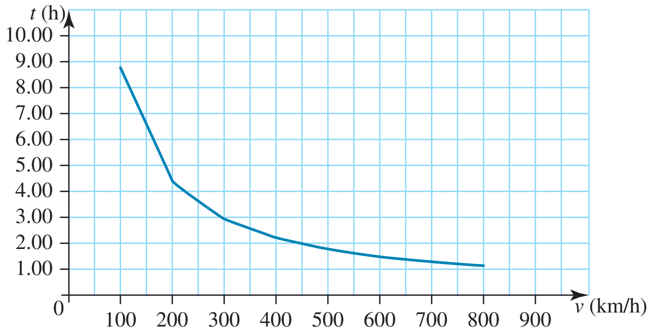
***18 e**

r (cm)	4	6	8	10	15	20
r^3	64	216	512	1000	3375	8000
Volume: $V = \dfrac{\pi r^2 h}{3}$ **(cm³)**	335	1131	2681	5236	17 671	41 888

***18 f**

r (cm)	4	6	8	10	15	20
r^3	64	216	512	1000	3375	8000
Volume: $V = \dfrac{\pi r^2 h}{3}$ **(cm³)**	335	1131	2681	5236	17 671	41 888
$\dfrac{V}{r^3}$	5.24	5.24	5.24	5.24	5.24	5.24

***5 a**



[1 mark]

Topic 11 — Space and measurement

11.2 Scientific notation, significant figures and rounding

11.2 Exercise

1 $2\,600\,000 = 2.6 \times 10^6$
2 $0.000\,000\,55 = 5.5 \times 10^{-7}$
3 a $7319 = 7.319 \times 10^3$
b $0.080\,425 = 8.0425 \times 10^{-2}$
c $13\,000\,438 = 1.300\,043\,8 \times 10^7$
d $0.000\,260 = 2.6 \times 10^{-4}$
e $92\,630\,051 = 9.263\,005\,1 \times 10^7$
f $0.000\,569\,2 = 5.692 \times 10^{-4}$
4 3×10^{-9} m $= 0.000\,000\,003$ m
5 1.38×10^{10} years old $= 13\,800\,000\,000$ years old
6 a $1.64 \times 10^{-4} = 0.000\,164$
b $2.3994 \times 10^{-8} = 0.000\,000\,023\,994$
c $1.4003 \times 10^9 = 1\,400\,300\,000$
d $8.6 \times 10^5 = 860\,000$
7 a 5 significant figures
b 40 080 000 m
8 a 0.0012 cm
b 2 significant figures
c 0.001 cm
9 a i 4
ii 1900
b i 3
ii 0.0015
c i 3
ii 20 000
d i 5
ii 0.0943
e i 7
ii 1.081
f i 4
ii 401
10 a $235.47 + 1952.99 - 489.73 = 1698.73$
$= 1698.7$ (correct to 1 d.p.)
b $200 + 2000 - 500 = 1700$
c Yes, the answers are very close so the approximate answer is a reasonable result.
11 a $100 \times 40 = 4000$ litres
b $102 \times 37 = 3774$ litres
12 a 9
b 240 000 000 km
c $240\,000\,000 = 2.4 \times 10^8$ km
13 a $c = \pi d$
$= \pi \times 110$
$= 345.575...$
$= 345.58$ m
b $c = \pi d$
$= 3.1 \times 110$
$= 341$ m
This changes the answer by approximately 4.5. This is a reasonable approximation.
14 a Rounding to the nearest whole number:
$12.6 \times 7.8 \times 12.7 \approx 13 \times 8 \times 13$
$\approx \$1352$
b $12.6 \times 7.8 \times 12.7 = \$1248.16$
There is a significant difference in the estimated and actual costs (over \$100).
c Answers will vary. The above answer shows rounding to the nearest whole number. Rounding to 2 significant figures will produce a more accurate result.
15 a $A = \pi r^2$
$= \pi \times 1.2^2$
$= \pi \times 1.44$
$= 1.44\pi \text{ cm}^2$
b 4.52 cm^2
c $(76 \times 20) - (162 \times 4.52) = 1520 - 732$
$= 788 \text{ cm}^2$
d $788 \times 10^2 = 78\,800 \text{ mm}^2$
e $7.88 \times 10^4 \text{mm}^2$

11.2 Exam questions

1 0.0250
The first two zeros are not significant but the last zero is.
The correct answer is **D**.
2 $5.14 \times 10^{-3} \times 7.32 \times 10^{-3} \times 2.56 \times 10^{-6} = 96.3 \times 10^{-12}$
$= 9.63 \times 10^{-11} \text{ m}^3$
The correct answer is **E**.
3 a π from the computer [1 mark]
b $509\,497\,627.1 \text{ km}^2$ [1 mark]
c $5.094\,976\,271 \times 10^8 \text{ km}^2$ [1 mark]

11.3 Pythagoras' theorem

11.3 Exercise

1 a $x = \sqrt{9^2 + 6^2}$
$= \sqrt{117}$
≈ 10.8
b $x = \sqrt{25^2 + 14^2}$
$= \sqrt{821}$
≈ 28.7
2 $40^2 + 96^2 = 10\,816 = 104^2$; a right-angled triangle with side lengths 40 cm and 96 cm will have a hypotenuse of length 104 cm.
3 a $x = \sqrt{6^2 + 3^2}$
$= \sqrt{45}$
≈ 6.71
b $x = \sqrt{8.5^2 + 11^2}$
$= \sqrt{\frac{773}{4}}$
≈ 13.90

c $x = \sqrt{5.8^2 + 4.6^2}$
$= \sqrt{\frac{274}{5}}$
≈ 7.40

d $x = \sqrt{11.4^2 + 2.5^2}$
$= \sqrt{\frac{13\,621}{100}}$
≈ 11.67

e $x = \sqrt{31^2 + 22^2}$
$= \sqrt{1445}$
≈ 38.01

f $x = \sqrt{33^2 + 8^2}$
$= \sqrt{1153}$
≈ 33.96

4 a $x = \sqrt{92^2 - 28^2}$
$= \sqrt{7680}$
≈ 87.6

b $x = \sqrt{35^2 - 16^2}$
$= \sqrt{969}$
≈ 31.1

5 $24^2 - 19.2^2 = 207.36$ and $14.4^2 = 207.36$
A right-angled triangle with a hypotenuse of 24 cm and a side length of 19.2 cm will have a third side length of 14.4 cm.

6 a $x = \sqrt{26.73^2 - 22.45^2}$
$= \sqrt{\frac{263\,113}{1250}}$
≈ 14.51

b $x = \sqrt{23.39^2 - 18.35^2}$
$= \sqrt{\frac{131\,481}{625}}$
≈ 14.50

c $x = \sqrt{46.13^2 - 12.15^2}$
$= \sqrt{\frac{2\,475\,443}{1250}}$
≈ 44.50

d $x = \sqrt{11.31^2 - 8^2}$
$= \sqrt{\frac{639\,161}{10000}}$
≈ 7.99

e $x = \sqrt{8.62^2 - 3.2^2}$
$= \sqrt{\frac{160161}{2500}}$
≈ 8.00

f $x = \sqrt{3.62^2 - 3.2^2}$
$= \sqrt{\frac{7161}{2500}}$
≈ 1.69

7 $x = \sqrt{9^2 + 7^2}$
$= \sqrt{130}$
≈ 11.40

8 $y = \sqrt{1.2^2 + 0.55^2}$
$= 1.32$
$x = \sqrt{1.32^2 + 0.83^2}$
$= 1.56$
The maximum length of the metal rod will be 1.56 m.

9 $y = \sqrt{2.3^2 + 1.2^2}$
$= 2.59$
$x = \sqrt{2.59^2 + 0.8^2}$
$= 2.71$
The maximum length of the metal rod will be 2.71 m so a rod of length 2.8 m will not fit.

10 a $x^2 + x^2 = 20^2 \rightarrow 2x^2 = 400$
$x = \sqrt{200}$
$= 14.14$
The length of the shorter sides is 14.14 cm.

b $x^2 + x^2 = 48^2 \rightarrow 2x^2 = 2304$
$x = \sqrt{1152}$
$= 33.94$
The length of the shorter sides is 33.94 cm.

c $x^2 + x^2 = 5.5^2 \rightarrow 2x^2 = \frac{121}{4}$
$x = \sqrt{\frac{121}{8}}$
$= 3.89$
The lengths of the shorter sides is 3.89 cm.

d $x^2 + x^2 = 166^2 \rightarrow 2x^2 = 27\,556$
$x = \sqrt{13\,778}$
$= 117.38$
The length of the shorter sides is 117.38 cm.

11 Diagonal of pyramid base:
$x = \sqrt{25^2 + 25^2}$
$= \sqrt{1250}$
Height of pyramid:
$y^2 = 45^2 - \left(\frac{1}{2}\sqrt{1250}\right)^2$
$= \frac{3425}{2}$
$\rightarrow y = \sqrt{\frac{3425}{2}} \approx 41.38$
The height of the pyramid is 41.4 m.

12 Pyramid 1:
Diagonal of pyramid base:
$x = \sqrt{18^2 + 18^2}$
$= 18\sqrt{2}$
Height of pyramid:
$y^2 = 30^2 - \left(9\sqrt{2}\right)^2$
$= 738$
$\rightarrow y = \sqrt{738} \approx 27.17$
The height of pyramid 1 is 27.17 m.
Pyramid 2:
Diagonal of pyramid base:
$x = \sqrt{22^2 + 22^2}$
$= 22\sqrt{2}$
Height of pyramid:
$y^2 = 28^2 - \left(11\sqrt{2}\right)^2$
$= 542$
$\rightarrow y = \sqrt{542} \approx 23.28$

The height of pyramid 2 is 23.28 m.
Therefore, pyramid 1 has the greatest height.

13 a $y = \sqrt{19.5^2 + 15^2}$
$= 24.6$
$x = \sqrt{24.6^2 + 18^2}$
$= 30.48$
The maximum length of the metal rod will be 30.48 cm.

b $y = \sqrt{1.6^2 + 1.1^2}$
$= 1.94$
$x = \sqrt{1.94^2 + 1.75^2}$
$= 2.61$
The maximum length of the metal rod will be 2.61 m.

c $y = \sqrt{13.04^2 + 42.13^2}$
$= 44.10$
$x = \sqrt{44.10^2 + 17.02^2}$
$= 47.27$
The maximum length of the metal rod will be 47.27 cm.

14 a $y = \sqrt{89^2 + 44^2}$
$= 99.28$
$x = \sqrt{99.28^2 + 21^2}$
$= 101.5\text{ cm}$
$= 1.015\text{ m}$
Yes, the 1-m umbrella will fit in the suitcase.

b The maximum length that is possible is 101.5 cm or 1.015 m.

15 $y = \sqrt{22^2 - 15^2}$
$= 16.09$
$x = 16.09 - 8$
$= 8.09$

16 $x = \sqrt{60^2 + 35^2}$
$= 69.46$
Therefore, the minimum length of cable required is 69.46 m.

17 a $x = \sqrt{14.5^2 + 2.4^2}$
$= 14.70$
Therefore, the longest object that can be placed on the floor is 14.70 m.

b $x = \sqrt{14.7^2 + 2.9^2}$
$= 14.98$
Therefore, the longest object that can be placed in the container with only one end on the floor is 14.98 m.

c $x = \sqrt{13.3^2 + 2.4^2}$
$= 13.51$
$y = \sqrt{13.51^2 + 2.9^2}$
$= 13.82$
Therefore, the longest object that can be placed in the container with one end on the floor adjacent to the box is 13.82 m.

18 a $x = \sqrt{10\,000^2 + 1000^2}$
$= 10\,049.88$
Therefore, it travels 10 050 m (nearest metre).

b $x = \sqrt{6000^2 + 1000^2}$
$= 6082.76$
Therefore, it is now 6083 m (nearest metre) from the landing point.

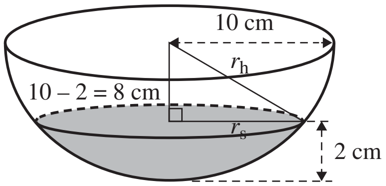
c $x = \sqrt{600^2 + 4500^2}$
$= 4539.82$
$y = \sqrt{400^2 + 5500^2}$
$= 5514.53$
Therefore, the ultralight travels a total distance of
$4539.82 + 5514.53 = 10\,054.35\text{ m}$
$= 10\,054\text{ m}$

11.3 Exam questions

1 Horizontal distance AB $= \sqrt{25^2 - 15^2} = 20$
The correct answer is **C**.

2 $(XZ)^2 = (XY)^2 + (YZ)^2$
$= 38.5^2 + 24.0^2$
$\therefore XZ = \sqrt{38.5^2 + 24.0^2}$
$\simeq 45.37$
The correct answer is **D**.

3 Use Pythagoras' theorem in the hemisphere.



Let r_h = radius of the hemisphere and let
r_s = radius of the surface of the water
$a^2 + b^2 = c^2$
$8^2 + (r_s)^2 = (r_h)^2$
$8^2 + (r_s)^2 = 10^2$
$\therefore r_s = \sqrt{10^2 - 8^2}$
$= 6$
The correct answer is **B**.

11.4 Perimeter and area of polygons and triangles

11.4 Exercise

1 Perimeter $= (2 \times 22) + (2 \times 18) = 80\text{ cm}$
Area $= 20 \times 18 = 360\text{ cm}^2$

2 Circumference $= \pi \times 16 \approx 50.27\text{ cm}$
Area $= \pi \times 8^2 \approx 201.06\text{ cm}^2$

3 a Perimeter $= 22 + 22 + 15 = 59\text{ m}$
Height : $h = \sqrt{22^2 - 7.5^2}$
$= \sqrt{427.75}$
Area $= \frac{1}{2} \times 15 \times \sqrt{427.75}$
$= 155.12\text{ m}^2$

b Perimeter $= 8 + 7 + 13.83 = 28.83\text{ cm}$
Area $= \frac{1}{2} \times 8 \times 5 = 20\text{ cm}^2$

c Perimeter $= 2 \times \pi \times 7 = 43.98\text{ cm}$
Area $= \pi \times 7^2 = 153.94\text{ cm}^2$

d Slant length: $\sqrt{6^2+8^2}=10\,\text{cm}$

Perimeter $=10+8+10+20=48\,\text{cm}$

$$\text{Area}=\frac{1}{2}(20+8)\,8=112\,\text{cm}^2$$

4 a Circumference $=2\times\pi\times5=31.42\,\text{cm}$

$\text{Area}=\pi\times5^2=78.54\,\text{cm}^2$

b Circumference $=18\times\pi=56.55\,\text{cm}$

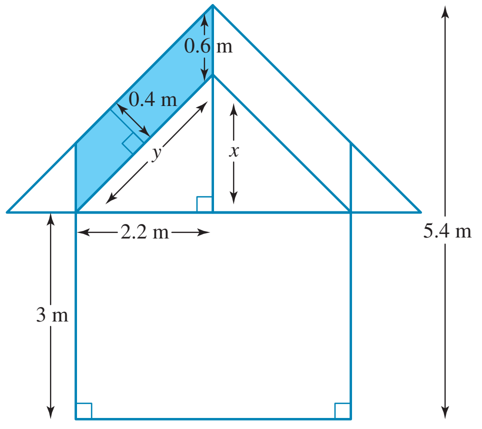
$\text{Area}=\pi\times9^2=254.47\,\text{cm}^2$

5 Perimeter $=(2\times12)+(2\times22)=68\,\text{cm}$

$\text{Area}=12\times16=192\,\text{cm}^2$

6 $\text{Area}=\frac{1}{2}\times11.63\times5.81=33.79\,\text{cm}^2$

7



$x=(5.4-3)-0.6=1.8\,\text{m};\ y=\sqrt{2.2^2+1.8^2}=2.84$

$\text{Area}=2.84\times0.4=1.14\,\text{m}^2$

8 Unknown side on large triangle: $\sqrt{40^2-32^2}=24$

Perimeter: $40+32+24=96$ units

Unknown side on small triangle: $\sqrt{10^2-8^2}=6$

Shaded area: $=\frac{1}{2}(24+6)\,24=360\ \text{units}^2$

9 $3140=\pi r^2\rightarrow r=\sqrt{\frac{3140}{\pi}}\approx31.61\,\text{cm}$

10 $s=\frac{16.8+25.3+9.7}{2}=25.9$

$A=\sqrt{25.9\,(25.9-16.8)\,(25.9-25.3)\,(25.9-9.7)}$

$=47.9\,\text{mm}^2$

11 Triangle 1: $s=\frac{10.6+13.5+16.2}{2}=20.15$

$A=\sqrt{20.15\,(20.15-10.6)\,(20.15-13.5)\,(20.15-16.2)}$

$=71.1\,\text{cm}^2$

Triangle 2: $s=\frac{10.8+14.2+24.6}{2}=24.8$

$A=\sqrt{24.8\,(24.8-10.8)\,(24.8-14.2)\,(24.8-24.6)}$

$=27.1\,\text{cm}^2$

Triangle 3: $s=\frac{12.1+12.6+12.7}{2}=18.7$

$A=\sqrt{18.7\,(18.7-12.1)\,(18.7-12.6)\,(18.7-12.7)}$

$=67.2\,\text{cm}^2$

Therefore, triangle 1 has the largest area.

12 $s=\frac{23+28+32}{2}=41.5$

$A=\sqrt{41.5\,(41.5-23)\,(41.5-28)\,(41.5-32)}$

$=313.8\,\text{m}^2$, therefore the area is not suitable.

13 Side length: $968=2l\times l=2l^2\rightarrow l=\sqrt{\frac{968}{2}}=22$

Diagonal: $x=\sqrt{44^2+22^2}=49.19\,\text{cm}$

14 a i Circumference $=2\times\pi\times32=201.06\,\text{cm}$

ii $\text{Area}=\pi\left(32^2-30^2\right)=389.56\,\text{cm}^2$

b New frame area $=\frac{110}{100}\times389.56=428.51\,\text{cm}^2$

Area of inside frame $=\left(\pi\times32^2\right)-428.51=2788.48\,\text{cm}^2$

Radius $=\sqrt{\frac{2788.48}{\pi}}=29.79\,\text{cm}$

Circumference $=2\times\pi\times29.79=187.19\,\text{cm}$

15 a Inner radius of semi-circle (lane 1):

$100=\pi\times r\rightarrow r=\frac{100}{\pi}=31.83\,\text{m}$

Radius to inside of lane 8 $=31.83+(7\times1.2)=40.23\,\text{m}$

Circumference of inside of lane 8 (semi-circle)

$=\pi\times40.23=126.39\,\text{m}$

Therefore, the runner in lane 8 will run 26.39 metres further.

b Total area of curved section of track = (area of semi-circle to outside of lane 8) − (area of inner semi-circle):

$$A=\left(\frac{\pi\times(40.23+1.2)^2}{2}\right)-\frac{\left(\pi\times31.83^2\right)}{2}=1104.73\,\text{m}^2$$

16 a Height of triangle $=\sqrt{20^2-10^2}=17.32$

Area of triangle $=\frac{1}{2}\times20\times17.32=173.21\,\text{m}^2$

b Each paver has a side length of 5 m so adding two rows adds 10 m to each side of the triangular paved area. Therefore, the perimeter $=30+30+30=90\,\text{m}$

Height of new paved area $=\sqrt{30^2-15^2}=25.98$

Area of triangle $=\frac{1}{2}\times30\times25.98=389.71\,\text{m}^2$

c Each rectangular paver will be $10\,\text{m}\times2.5\,\text{m}$ so adding two rows around the outside creates a perimeter:

$(9\times10)+(6\times5)=120\,\text{m}$

Area = Area of triangle

$+\,(18\times\text{area of one rectangular paver})$

$=389.7+(18\times10\times2.5)=839.7\,\text{m}^2$

11.4 Exam question

1 $2l+2w=2(4.2)+2(8.2)$

$=8.4+16.4$

$=24.8\,\text{cm}$

The correct answer is **D**.

2 Total area = area of rectangle + area of traiangle

$=lw+\frac{1}{2}hb$

$=(5\times7)+\left(\frac{1}{2}\times(18-7)\times5\right)$

$=35\,\text{cm}^2+27.5\,\text{cm}^2$

$=62.5\,\text{cm}^2$

The correct answer is **A**.

3 $\text{Area}=lw-\frac{1}{2}hb$

$=(20\times12)-\left(\frac{1}{2}\times10\times5\right)$

$=240\,\text{cm}^2-25\,\text{cm}^2$

$=215\,\text{cm}^2$

The correct answer is **E**.

11.5 Perimeter and area of composite shapes and sectors

11.5 Exercise

1 Area $= \frac{1}{2} \times 12 \times (4+4+4) = 72\,\text{cm}^2$

2 Angled length $= \sqrt{4^2 + 5.5^2} = 6.8$;
Perimeter $= 7 + 2 \times 12 + 2 \times 2 + 2 \times 6.8 = 48.6\,\text{cm}$
Area $= (7 \times 12) + \left(\frac{1}{2} \times 11 \times 4\right) = 106\,\text{cm}^2$

3 Area $= (20 \times 20) - \left(\pi \times 8^2\right) = 198.94\,\text{cm}^2$

4 **a** Perimeter $= (\pi \times 0.5) + (2 \times 1.5) + 1 = 5.57\,\text{m}$
b Area $= (4 \times 2) - \left((1.5 \times 1) + \left(\frac{1}{2} \times \pi \times 0.5^2\right)\right)$
$= 6.11\,\text{m}^2$

5 **a** Length of inner square $= \sqrt{\frac{108^2}{2}} = 76.37\,\text{cm}$
Area $= (108 \times 108) - (76.37 \times 76.37) = 5831.62\,\text{cm}^2$
b Area $= (10 \times 10) - \left(\left(\pi \times 0.82^2\right) + \left(\pi \times 3.13^2\right)\right.$
$\left. + \left(\pi \times 1.4^2\right) + \left(\pi \times 1.18^2\right)\right)$
$= 56.58\,\text{cm}^2$

6 Area $= \left(\pi \times 1.2^2\right) - \left(\pi \times 0.3^2\right) = 4.24\,\text{m}^2$

7 Perimeter $= (2 \times 16) + \left(\frac{75}{360} \times 2 \times \pi \times 16\right) = 52.94\,\text{cm}$
Area $= \frac{75}{360} \times \pi \times 16^2 = 167.55\,\text{cm}^2$
Arc length $= \pi \times 16 \times \frac{75}{180} = 20.94\,\text{cm}$

8 Perimeter $= (2 \times 30) + \left(\frac{264}{360} \times 2 \times \pi \times 30\right) = 198.23\,\text{cm}$
Area $= \frac{264}{360} \times \pi \times 30^2 = 2073.45\,\text{cm}^2$

9 Perimeter $= (2 \times (45 - 22)) + \left(\frac{85}{360} \times 2 \times \pi \times 45\right)$
$+ \left(\frac{85}{360} \times 2 \times \pi \times 22\right) = 145.40\,\text{cm}$
Area $= \left(\frac{85}{360} \times \pi \times 45^2\right) - \left(\frac{85}{360} \times \pi \times 22^2\right)$
$= 1143.06\,\text{cm}^2$

10 Perimeter $= (2 \times 20) + (\pi \times 35) + (\pi \times 15) = 197\,\text{cm}$
Area $= \left(\frac{1}{2} \times \pi \times 35^2\right) - \left(\frac{1}{2} \times \pi \times 15^2\right) = 1570.79\,\text{cm}^2$
$= 1571\,\text{cm}^2$

11 Area $= \frac{1}{9} \times \left(\left(\pi \times 6^2\right) - \left(\pi \times 3^2\right)\right) = 9.42\,\text{cm}^2$

12 Area $= \left(\pi \times 4.5^2\right) - \left(\pi \times 3.5^2\right) = 25.13\,\text{cm}^2$

13 Area $= \left(\pi \times 34^2\right) + \left(\pi \times 31^2\right) + \left(\pi \times 29^2\right) = 9292.83\,\text{cm}^2$

14 Angle length of triangle $= \sqrt{6^2 + 18^2} = 18.97\,\text{cm}$
Perimeter $= 8 \times 18.97 = 151.76\,\text{cm}$
Area $= (12 \times 12) + 4\left(\frac{1}{2} \times 12 \times 18\right) = 576\,\text{cm}^2$

15 Height of triangle $= \sqrt{2^2 - 1^2} = 1.73$
Area $= \left(\pi \times 1.16^2\right) - \left(\frac{1}{2} \times 2 \times 1.73\right) = 2.50\,\text{m}^2$

16 Height of triangle $= \sqrt{2^2 - 1^2} = 1.73$
Area $= \left(\frac{1}{2} \times 2 \times 1.73\right) - \left(\pi \times 0.58^2\right) = 0.67\,\text{m}^2$

17 Area of annulus $= \left(\pi \times 35^2\right) - \left(\pi \times 20^2\right) = 2591.81\,\text{cm}^2$
Area of remaining $= 2591.81$
$- \left(\left(\frac{38}{360} \times 2591.81\right) + \left(\frac{25}{360} \times 2591.81\right)\right) = 2138.25\,\text{cm}^2$

18 Area $= 2 \times \frac{1}{2} \times 30 \times 30 = 900\,\text{cm}^2$
Triangle side length $= \sqrt{15^2 + 30^2} = 33.54$
Perimeter $= (4 \times 33.54) + (2 \times 30) = 194.16\,\text{cm}$

11.5 Exam questions

1 The area of the rectangle is: $6 \times 21 = 126\,\text{cm}^2$
The radius of each circle is 3 cm.
There are 3.5 circles in the rectangle, so the area of all the circles is: $\pi \times 3^2 \times \frac{7}{2} = \frac{63\pi}{2}\,\text{cm}^2$
Therefore, the area of the shaded region is:
$126 - \frac{63\pi}{2} = 27.0398 = 27\,\text{cm}^2$
The correct answer is **B**.

2 $A = \frac{1}{2} \times 2 \times 1.5 = 1.5\text{m}^2$
The correct answer is **B**.

3 Area of the circle $= \pi \times 16^2 = 804.2477\,\text{mm}^2$
$80\% \times 804.2477 = 643.398 = 643\,\text{mm}^2$
The correct answer is **D**.

11.6 Volume

11.6 Exercise

1 $V = \frac{1}{2} \times 0.6 \times 0.8 \times 2.5$
$= 0.6\,\text{m}^3$

2 **a** $V = 200 \times 102.5 = 20\,500\,\text{cm}^3$
b $V = 25.25 \times 12.65 \times 42 = 13\,415\,\text{cm}^3$
c Triangle side-length $= \sqrt{73^2 - 48^2} = 55$
$V = \frac{1}{2} \times 55 \times 48 \times 96 = 126\,720\,\text{cm}^3$
d $V = \frac{1}{2}(25 + 40) \times 15 \times 105 = 51\,188\,\text{cm}^3$

3 Rectangular pool $= \frac{120 \times 410 \times 225}{1000} = 11\,070$ litres
Square pool $= \frac{120 \times 300 \times 300}{1000} = 10\,800$ litres
Circular pool $= \frac{120 \times \pi \times \left(\frac{330}{2}\right)^2}{1000} = 10\,263.58$ litres

4 **a** Perimeter $= 9 + 3 + 6 + 1 + 1 + 5 + 7 + 8 = 40\,\text{m}$
Volume $= 40 \times 0.6 \times 1.05 = 25.2\,\text{m}^3$
b Volume $= 3 \times 0.6 \times 4 = 7.2\,\text{m}^3$

5 $V = \pi \times 22.5^2 \times 35.4 = 56\,301\,\text{cm}^3$

6 **a** Radius $= \frac{314}{2\pi} \to V = \pi \times \left(\frac{314}{2\pi}\right)^2 \times 62.5 = 490\,376\,\text{cm}^3$
b Radius $= 0.75 \times 4.25 = 3.19$
$V = \pi \times (3.19)^2 \times 4.25 = 136\,\text{m}^3$

7 Volume of cylinder $= \left(\pi \times 30^2 \times 150\right) = 424\,115\text{ cm}^3$
Volume below $= 0.75 \times 42\,4115 = 318\,086\text{ cm}^3$
Volume of hemisphere $= \dfrac{1}{2}\left(\dfrac{4}{3} \times \pi \times 30^3\right) = 56\,549\text{ cm}^3$
Volume above $= 56\,549 + (424\,115 - 318\,086)$
$= 162\,578\text{ cm}^3$

8 **a** $= \dfrac{49 \times \pi \times \left(\frac{30}{2}\right)^2}{1000} = 34.64$ litres
b $V = (0.85 \times 0.595 \times 0.525) - (\pi \times 0.15^2 \times 0.49)$
$= 0.231\text{ m}^3$

9 $V = \dfrac{1}{3} \times \pi \times 30^2 \times 42 = 39\,584.1\text{ cm}^3$

10 **a** Radius $= \dfrac{628}{2\pi} \rightarrow V = \dfrac{1}{3} \times \pi \times \left(\dfrac{628}{2\pi}\right)^2 \times 72 = 753\,218\text{ cm}^3$
b Radius $= \dfrac{2}{3} \times 0.36 = 0.24$
$V = \dfrac{1}{3} \times \pi \times 0.24^2 \times 0.36$
$= 0.022\text{ cm}^3$

11 **a** $V = \dfrac{1}{2} \times (30 + 15) \times 12 \times 55 = 14\,850\text{ cm}^3$
b $V = 65 \times 58 \times 125 = 471\,250\text{ cm}^3$
c $V = \left[\left(\pi \times 18^2\right) - \left(\pi \times 9^2\right)\right] \times 60 = 14\,580\pi\text{ cm}^3$
d $V = \left(\pi \times 12^2 \times 88\right) + \left(\dfrac{1}{3} \times \pi \times 12^2 \times 18\right)$
$= 13\,536\pi\text{ cm}^3$

12 $V = \dfrac{1}{3} \times 1.05 \times 0.0745 \times 2.025 = 0.0528\text{ m}^3$

13 **a** $V = \dfrac{1}{3} \times 366 \times 187.5 = 22\,875\text{ cm}^3$
b $V = \dfrac{1}{3} \times 18.45 \times 26.55 \times 96 = 15\,675.12\text{ cm}^3$
c $V = \dfrac{1}{3} \times \left(\dfrac{1}{2} \times 1.2 \times 0.6\right) \times 3.6 = 0.432\text{ m}^3$

14 $V = \dfrac{1}{8} \times \left(\dfrac{1}{2} \times 5 \times 2.5 \times 8\right) = 6.25\text{ m}^3$

15 $V = \dfrac{4}{3} \times \pi \times 0.27^3 = 0.0824\text{ m}^3$

16 **a** Radius $= \sqrt[3]{\dfrac{3V}{4\pi}}$
$= \sqrt[3]{\dfrac{3 \times 248\,398.88}{4\pi}} = 39\text{ cm}$
b $r = \sqrt[3]{\dfrac{3 \times 4.187}{4\pi}} = 1\text{ m}$

17 Volume of hemisphere $= \dfrac{1}{2}\left(\dfrac{4}{3} \times \pi \times 1.5^3\right) = 7.07\text{ m}^3$
Volume of cylinder $= \pi \times 1.5^2 \times 2.1 = 14.84\text{ m}^3$
Total volume $= 7.07 + 14.84 = 21.91\text{ m}^3$

18 **a** Volume of cube $= 34^3 = 39\,304\text{ cm}^3$
Volume of pyramid $= \dfrac{1}{3} \times 34^2 \times 48 = 18\,496\text{ cm}^3$
Total volume $= 39\,304 + 18\,496 = 57\,800\text{ cm}^3$
b Volume of cone $= \dfrac{1}{3} \times \pi \times 16^2 \times 75 = 20\,106.19\text{ cm}^3$
Volume of cylinder $= \pi \times 16^2 \times 60 = 48\,254.86\text{ cm}^3$
Total volume $= 20\,106.19 + 48\,254.86 = 68\,361.05\text{ cm}^3$

19 Volume of hemispherical ends:
$V = \dfrac{4}{3} \times \pi \times \left(\dfrac{176}{2}\right)^3 = 2\,854\,543.238\text{ cm}^3$
Volume of cylinder:
$V = \pi \times \left(\dfrac{176}{2}\right)^2 \times (870 - 176) = 16\,883\,974.5\text{ cm}^3$
Total volume:
$V = \dfrac{2\,854\,543.238 + 16\,883\,974.5}{1000} = 19\,738.52$ litres

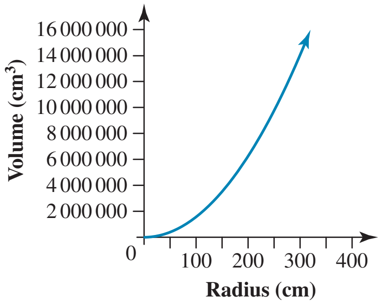
20 **a** $V = \dfrac{1}{3} \times 35^2 \times 22 = 8983.3\text{ m}^3$
b $V = \dfrac{1}{3} \times \left(\dfrac{3500}{3}\right)^2 \times \dfrac{2200}{3} = 332\,716\,049.4\text{ cm}^3$

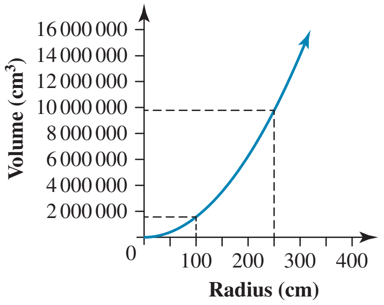
21 Conical-lidded silo:
$V = \left(\pi \times 2.25^2 \times 9.6\right) + \left(\dfrac{1}{3} \times \pi \times 2.25^2 \times 1.5\right)$
$= 160.633\text{ m}^3$
Hemispherical-lidded silo:
$V = \left(\pi \times 2.75^2 \times 6.5\right) + \dfrac{1}{2}\left(\dfrac{4}{3} \times \pi \times 2.75^3\right) = 197.985\text{ m}^3$
Therefore, the hemispherical-lidded silo holds $197.985 - 160.633 = 37.35\text{ m}^3$ more.

22 **a** Volume of tennis ball $= \dfrac{4}{3} \times \pi \times \left(\dfrac{6.7}{2}\right)^3 = 157.5\text{ cm}^3$
Volume of free space:
$\left[\pi \times \left(\dfrac{6.7 + 0.5}{2}\right)^2 \times 26.95\right] - [4 \times 157.5] = 467.27\text{ cm}^3$
b Volume of box $= (8 \times (6.7 + 0.5)) \times (4 \times (6.7 + 0.5)) \times 27$
$= 44\,789.76\text{ cm}^3$
Volume of free space
$= 44\,789.76 - \left[8 \times 4 \times \left(\pi \times \left(\dfrac{6.7 + 0.5}{2}\right)^2 \times 26.95\right)\right]$
$= 9677.11\text{ cm}^3$

23 **a**

Cylinder radius (cm)	Calculation $50\pi r^2$	Volume
10	$50\pi \times 10^2$	$15\,708\text{ cm}^3$
20	$50\pi \times 20^2$	$62\,832\text{ cm}^3$
40	$50\pi \times 40^2$	$251\,327\text{ cm}^3$
80	$50\pi \times 80^2$	$1\,005\,310\text{ cm}^3$
160	$50\pi \times 160^2$	$4\,021\,239\text{ cm}^3$
320	$50\pi \times 320^2$	$16\,084\,954\text{ cm}^3$

b


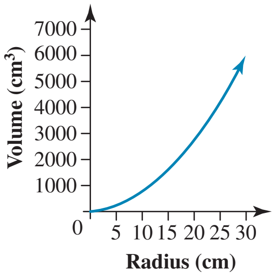
c


i Approx. $1\,600\,000\text{ cm}^3$
ii Approx. $9\,800\,000\text{ cm}^3$

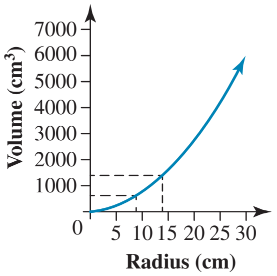
24 a

Base length (cm)	Calculation $\frac{1}{3} \times 20\,(\text{base length})^2$	Volume (cm^3)
5	$\frac{1}{3} \times 20 \times 5^2$	166.7
10	$\frac{1}{3} \times 20 \times 10^2$	666.7
15	$\frac{1}{3} \times 20 \times 15^2$	1500.0
20	$\frac{1}{3} \times 20 \times 20^2$	2666.7
25	$\frac{1}{3} \times 20 \times 25^2$	4166.7
30	$\frac{1}{3} \times 20 \times 30^2$	6000.0

b



c



i Approx. 550 cm^3
ii Approx. 1300 cm^3

11.6 Exam questions

1
$$C = 2\pi r$$
$$r = \frac{18.85}{2\pi} = 3.0\text{ cm}$$
$$V = \pi r^2 h$$
$$311 = \pi \times 3^2 \times h$$
$$h = 10.99937$$
$$= 11\text{ cm}$$
The correct answer is **D**.

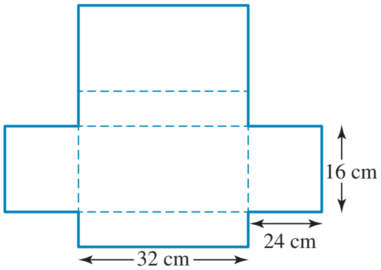
2
$$V = \left(\pi r^2 h\right) + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$
$$= \left(\pi \times 2^2 \times 5\right) + \frac{1}{2}\left(\frac{4}{3} \times \pi \times 2^3\right)$$
$$= \frac{76\pi}{3}$$
$$= 79.587$$
$$= 80\text{ m}^3$$
The correct answer is **A**.

3
$$V_{\text{cylinder}} - V_{\text{hemisphere}} = \pi r_c^2 h - \frac{2}{3}\pi r_h^3$$
$$= \left[\pi \times (6)^2 \times 8\right] - \left[\frac{2}{3} \times \pi \times (5)^3\right]$$
$$= 288\pi - \frac{250\pi}{3}$$
$$= 642.9793$$
$$\approx 643\text{ cm}^3$$
The correct answer is **C**.

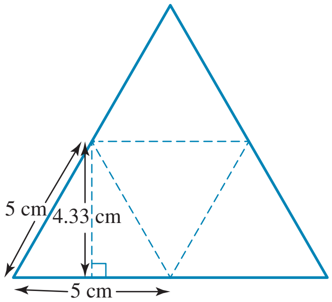
11.7 Surface area

11.7 Exercise

1 $\text{SA} = (2 \times 32 \times 24) + (2 \times 32 \times 16) + (2 \times 16 \times 24)$
$= 3328\text{ cm}^2$



2 Height of triangles $= \sqrt{5^2 - 2.5^2} = 4.33$
$$\text{SA} = \left(\frac{1}{2} \times 5 \times 4.33\right) \times 4 = 43.3\text{ cm}^2$$



3 a $\text{SA} = \left(\frac{1}{2}bh\right) \times 4 + b^2 = \left(\frac{1}{2} \times 25 \times 37\right) \times 4 + 25^2$
$= 2475\text{ m}^2$

b $\text{SA} = (2 \times 32 \times 17) + (2 \times 32 \times 23) + (2 \times 17 \times 23)$
$= 3342\text{ cm}^2$

4 a Height of triangles $= \sqrt{12^2 - 6^2} = \sqrt{108}$
$\text{SA} = \left(\frac{1}{2}bh\right) \times 4 = \left(\frac{1}{2} \times 12 \times \sqrt{108}\right) \times 4 = 249.42\text{ cm}^2$

b $\text{SA} = 4\pi r^2 = 4 \times \pi \times 98^2 = 120\,687.42\text{ cm}^2$

c $\text{SA} = (2\pi r) \times (r + h) = (2 \times \pi \times 15) \times (15 + 22)$
$= 3487.17\text{ cm}^2$

d $\text{SA} = (\pi r) \times (s + r) = (\pi \times 12.5) \times (27.2 + 12.5)$
$= 1559.02\text{ cm}^2$

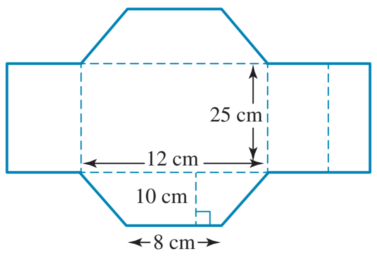
5 a $\text{SA} = (2 \times 8 \times 12) + (2 \times 8 \times 5) + (2 \times 12 \times 5) = 392\text{ cm}^2$

b $\text{SA} = (2 \times \pi \times 9)(9 + 20) = 1639.91\text{ cm}^2$

c Slant height $= \sqrt{18^2 + 7.5^2} = 19.5$
$\text{SA} = 4 \times \left(\frac{1}{2} \times 15 \times 19.5\right) + 15^2 = 810\text{ cm}^2$

d $\text{SA} = 4 \times \pi \times 10^2 = 1256.64\text{ cm}^2$

6 a



b Slant lengths $= \sqrt{10^2 + 2^2} = \sqrt{104}$

$\text{SA} = 2 \times \frac{1}{2}(12 + 8) \times 10 + (12 \times 25) + (8 \times 25)$
$+ \left(2 \times 25 \times \sqrt{104}\right) = 1210\text{ cm}^2$

7 Slant length of roof $= \sqrt{2.5^2 + 2.5^2} = 3.54$

$\text{SA} = (5 \times 18) + (2 \times 5 \times 3.5) + (2 \times 18 \times 3.5)$
$+ \left(2 \times \frac{1}{2} \times 5 \times 2.5\right) + (2 \times 18 \times 3.54) = 390.94\text{ m}^2$

8 Slant height of cone $= \sqrt{12^2 + 18^2} = 21.63$

$\text{SA} = \left(\pi \times 12^2\right) + (2 \times \pi \times 12 \times 88) + (\pi \times 12 \times 21.63)$
$= 7902.86\text{ cm}^2$

9 a $\text{SA} = \pi \times 5^2 + \frac{1}{2} \times 4 \times \pi \times 5^2 = 236\text{ cm}^2$

b $\text{SA} = \pi \times 5^2 + \frac{1}{2} \times 4 \times \pi \times 5^2 + 2 \times \pi \times 5 \times 8 = 487\text{ cm}^2$

10 Cone 1:
Slant height $= \sqrt{6.5^2 + 2.2^2} = 6.86$
$\text{SA} = \pi \times 2.2 \times 6.86 = 47.41\text{ cm}^2$
Cone 2:
Slant height $= \sqrt{7.5^2 + 1.7^2} = 7.69$
$\text{SA} = \pi \times 1.7 \times 7.69 = 41.07\text{ cm}^2$
Cone 1 − cone 2 = 47.41 − 41.07 = 6.34
The cone with height 6.5 cm and radius 2.2 cm has the greater surface area by 6.34 cm^2.

11 Triangle slant height $= \sqrt{6^2 + 2.1^2} = 6.36$

$\text{SA} = \left(4 \times \frac{1}{2} \times 4.2 \times 6.36\right) + (4 \times 4.2 \times 1.1) = 71.90\text{ m}^2$

12 a $\text{SA} = (2 \times \pi \times 12 \times 66) + \left(\pi \times 12^2\right)$
$+ \left[\left(\pi \times 12^2\right) - \left(\pi \times 9^2\right)\right]$
$= 5627\text{ cm}^2$

b $\text{SA} = (2 \times \pi \times 9 \times 48) + \frac{1}{2}\left(4 \times \pi \times 9^2\right) = 3223\text{ cm}^2$

13 $\text{SA} = 2 \times (2 \times \pi \times 4.5 \times 2) + 2 \times (2 \times \pi \times 6 \times 2)$
$+ 2 \times \left[\left(\pi \times 4.5^2\right) - \left(\pi \times 1.5^2\right)\right] + 2 \times \left[\left(\pi \times 6^2\right)\right.$
$\left.- \left(\pi \times 4.5^2\right)\right] + 2 \times \left[\left(\pi \times 6^2\right) - \left(\pi \times 1.5^2\right)\right]$
$= 688\text{ cm}^2$

14 a $\text{SA} = 9 \times (16 \times 80) + 9 \times (25 \times 80) + (65 \times 80)$
$= 34\,720\text{ cm}^2$

b $\text{SA} = 9 \times (16 \times 40) + 9 \times (25 \times 40) = 14\,760\text{ cm}^2$

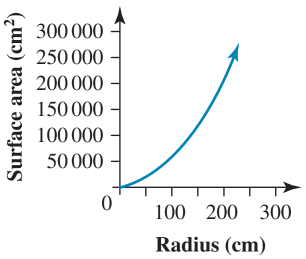
15 $\text{SA} = 2 \times (12.5 \times 1.5) + 2 \times (4.3 \times 1.5) + (12.5 \times 4.3)$
$= 104.15\text{ m}^2$

16 $\text{SA} = \frac{1}{4} \times (2 \times \pi \times 1.5 \times 2.4) + 2 \times \left(1.5^2 - \frac{1}{4} \times \pi \times 1.5^2\right)$
$+ (2.4 \times 1.5) = 10.22\text{ m}^2$

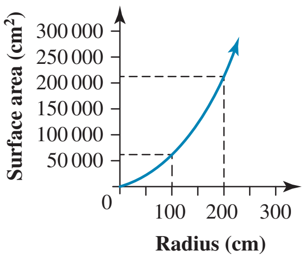
17 a

Cone radius (cm)	Calculation $\pi r(120 + r)$	Surface area (cm^3)
15	$15\pi(120 + 15)$	6361.7
30	$30\pi(120 + 30)$	14 437.2
60	$60\pi(120 + 60)$	33 929.2
120	$120\pi(120 + 120)$	90 477.9
240	$240\pi(120 + 240)$	271 433.6

b



c

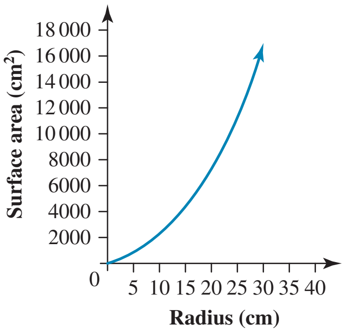


i Approx. 60 000 cm^2
ii Approx. 210 000 cm^2

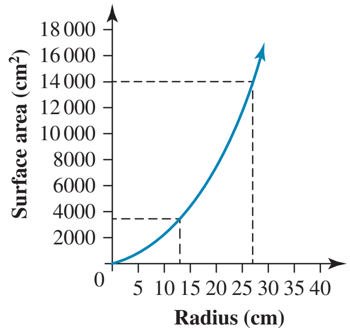
18 a

Cylinder radius (cm)	Calculation $2\pi r(3r)$	Surface area (cm^3)
5	$10\pi(15)$	471.24
10	$20\pi(30)$	1884.96
15	$30\pi(45)$	4241.15
20	$40\pi(60)$	7539.82
25	$50\pi(75)$	11 780.97
30	$60\pi(90)$	16 964.60

b



c



i Approx. $14\,000\text{ cm}^2$
ii Approx. 3200 cm^2

11.7 Exam questions

1 The length of the diagonal is $\sqrt{6^2+8^2}=10$.
So, the total surface area will be
$(6\times 6)+(6\times 8)+2\times\left(\frac{1}{2}\times 6\times 8\right)+(10\times 6)=192\text{ cm}^2$.
The correct answer is **C**.

2 $$\begin{aligned}SA&=\pi r^2+\frac{1}{2}\left(4\pi r^2\right)\\&=\pi(5)^2+2\pi(5)^2\\&=235.6194\\&=236\text{ cm}^2\end{aligned}$$
The correct answer is **C**.

3 Determine the missing side length:
$$\begin{aligned}c^2&=a^2+b^2\\c^2&=10^2+24^2\\c&=26\end{aligned}$$
$$\begin{aligned}\text{SA}&=(24\times 30)+\left(\frac{1}{2}\times 24\times 10\right)+\left(\frac{1}{2}\times 24\times 10\right)\\&\quad+(10\times 30)+(26\times 30)\\&=720+120+120+300+780\\&=2040\text{ m}^2\end{aligned}$$
The correct answer is **D**.

11.8 Review

11.8 Exercise

Multiple choice

1 $\sqrt{24^2+7^2}=25$
Correct answer is **C**.

2 $$\begin{aligned}x^2+x^2&=32^2\\2x^2&=1024\\x^2&=512\\x&=\sqrt{512}\\x&=22.63\ (2\text{ decimal places})\end{aligned}$$
The correct answer is **E**.

3 $$\begin{aligned}x&=\sqrt{4^2-2^2}\\&=3.46\ (2\text{ decimal places})\end{aligned}$$
Correct answer is **A**.

4 $$\begin{aligned}\text{BA}&=\frac{1}{2}(a+b)h\\148&=\frac{1}{2}(a+b)8\\37&=a+b\end{aligned}$$
From the available options: $37=12+15$
The correct answer is **D**.

5 $$\begin{aligned}75.4&=2\times\pi\times r\\r&=\frac{75.4}{2\pi}\\\text{Area}&=\pi\times\left(\frac{75.4}{2\pi}\right)^2\\&\approx 452\end{aligned}$$
Correct answer is **E**.

6 $$\begin{aligned}V&=\pi r^2h\\1570&=\pi r^2\times 20\\r&=\sqrt{\frac{1570}{20\pi}}\\\text{Diameter}&=2\times\sqrt{\frac{1570}{20\pi}}\\&\approx 10\end{aligned}$$
The correct answer is **D**.

7 $$\begin{aligned}A&=\pi r(r+s)\\2713&=12\pi(12+s)\\s&=\frac{2713}{12\pi}-12\\&\approx 60\end{aligned}$$
The correct answer is **C**.

8 $$\begin{aligned}V&=\frac{1}{2}\times\frac{4}{3}\pi r^3\\&=\frac{2}{3}\pi(22.5)^3\\&\approx 23\,856\end{aligned}$$
$$\begin{aligned}\text{Surface area}&=\frac{1}{2}\times 4\pi r^2+\pi r^2\\&=2\pi(22.5)^2+\pi(22.5)^2\\&\approx 4771\end{aligned}$$
The correct answer is **D**.

9 $$\begin{aligned}V&=\frac{1}{3}\times l^2\times h\\500&=\frac{1}{3}\times l^2\times 15\\l&=\sqrt{100}\\&=10\end{aligned}$$
Triangle height: $\sqrt{15^2+5^2}=15.8$
$$\begin{aligned}\text{Surface area}&=10^2+4\times\frac{1}{2}\times 10\times 15.8\\&=416\end{aligned}$$
The correct answer is **A**.

10 Slant height: $\sqrt{1.5^2+\left(\frac{1}{2}(1.8-1.2)\right)^2}=1.53$
$$\begin{aligned}\text{Surface area}&=2\times\frac{1}{2}(1.2+1.8)1.5+2\times 1.53\times 3.6\\&\quad+1.2\times 3.6\\&=19.8\end{aligned}$$
$$\begin{aligned}V&=\text{base area}\times\text{length}\\&=\frac{1}{2}(1.2+1.8)1.5\times 3.6\\&=8.1\end{aligned}$$
The correct answer is **B**.

Short answer

11 $\sqrt{10.8^2+3.6^2}=11.38\text{ m}$

12 $x^2+x^2=1.2^2$

$2x^2=1.44$

$x^2=0.72$

$x=\sqrt{0.72}$

$x\approx 0.849$

The ends are 0.849 m from the top of the pole.

13 $\sqrt{3600^2+4800^2}=6000\text{ mm}$

14 a Surface area $=25\times 2.5-(2\times 2.5\times 2+3\times 0.9\times 3.025)$

$=44.33$

Therefore, 44.33 m^2 remains.

b $l=2\times(2+2.5+2)+3\times(0.9+0.9+3.025+3.025)$

$=36.55$

Therefore, 36.55 m is required.

15 Column width: $\dfrac{7.6-5.8}{2}=0.9\text{ m}$

Arch width: $\dfrac{3}{4}x=0.9$

$x=\dfrac{0.9\times 4}{3}$

$=1.2\text{ m}$

Overhang: $\dfrac{1.2}{8}=0.15\text{ m}$

Inner radius: $\dfrac{5.8}{2}-0.15=2.75\text{ m}$

Outer radius: $\dfrac{7.6}{2}+0.15=3.95\text{ m}$

Tiled area: $\dfrac{1}{2}\left(\pi\times 3.95^2-\pi\times 2.75^2\right)=12.63\text{ m}^2$

16 a Diagonal length: $\sqrt{15^2+8^2}=17\text{ m}$

Pond radius: $\dfrac{1}{4}\times\dfrac{17}{2}=2.125\text{ m}$

Pond circumference: $2\times\pi\times 2.125=13.352\text{ m}$

b $(15\times 8)-\left(\pi\times 2.125^2\right)=105.8\text{ m}^2$

c $\pi\times 212.5^2\times 85=12\,058\,316\text{ cm}^3$

$\dfrac{12\,058\,316}{1000}=12\,058.32\text{ litres}$

The volume of water is 12 058 L.

Extended response

17 a $(2\times 1.5\times 10)+\left(\pi\times 3.5^2-\pi\times 2^2\right)=55.92\text{ m}^2$

b $V=(10\times 4\times 1.5)+\left(\pi\times 2^2\times 0.9\right)=71.3097\text{ m}^3$

$V=71.3097\times 1000$

$=71\,309.7\text{ litres}$

The volume of water is 71 310 L.

18 a $A=(1000\times 800)-4\left((200\times 200)-\dfrac{1}{4}\times\pi\times 200^2\right)$

$=765\,663.7\text{ mm}^2$

b $P=(2\times 600)+(2\times 400)+4\left(\dfrac{1}{4}\times 2\times\pi\times 200\right)$

$=3256.6\text{ mm}$

19 a $V=(0.4\times 0.4\times 1.8)-2\left(\dfrac{1}{2}\times 0.09\times 0.09\times 1.8\right)$

$=0.273\text{ m}^3$

b Base: $400\times 1800=720\,000\text{ mm}^2$

Sides: $2\times 310\times 1800=1\,116\,000\text{ mm}^2$

Ends: $2\times((400\times 400)-(90\times 90))=303\,800\text{ mm}^2$

Top: $220\times 1800=396\,000\text{ mm}^2$

Slanted edges: $2\times\left(\sqrt{90^2+90^2}\right)\times 1800=458\,205\text{ mm}^2$

Total surface area:

$720\,000+1\,116\,000+303\,800+396\,000+458\,205$

$=2\,994\,005\text{ mm}^2$

c Volume:

$V=2\left(\dfrac{1}{2}\times 90\times 90\times 1800\right)$

$=14\,580\,000\text{ mm}^3$

Surface area:

$=4\left(\dfrac{1}{2}\times 90\times 90\right)+4\,(90\times 1800)+2\left(1800\times\sqrt{90^2+90^2}\right)$

$=1\,122\,405\text{ mm}^2$

20 a $\text{DE}=\sqrt{10^2+10^2}$

$=14.14\text{ km}$

$\text{AC}=\sqrt{15^2+15^2}$

$=21.21\text{ km}$

b Area ADEC $=\left(\dfrac{1}{2}\times 15\times 15\right)-\left(\dfrac{1}{2}\times 10\times 10\right)$

$=62.5$

Also area ADEC $=\dfrac{1}{2}(\text{DE}+\text{AC})\times\text{GF}$

$=62.5$

$62.5=\dfrac{1}{2}(14.14+21.21)\times\text{GF}$

$\text{GF}=\dfrac{62.5}{\frac{1}{2}(14.14+21.21)}$

≈ 3.536

Therefore, GF is 3.54 km.

c $V=\left(\pi\times 20^2\times 21\,210\right)+\left(\pi\times 20^2\times 14\,140\right)$

$=44\,422\,120\text{ m}^3$

d $\dfrac{44\,422\,120}{85}=522\,613$

e $V=\left(\pi\times 20^2\times 21\,210\right)-\left(\pi\times 16.5^2\times 21\,210\right)$

$+\left(\pi\times 20^2\times 14\,140\right)-\left(\pi\times 16.5^2\times 14\,140\right)$

$=14\,187\,314.6$

Therefore, the volume of concrete is $14\,187\,314.6\text{ m}^3$.

f Area $=2\times\pi\times 16.5(14\,140+21\,210)$

$=3\,664\,824.9\text{ m}^2$

11.8 Exam questions

1 The cylinder and the cone have the same volume and the same radius:

$\pi r^2 h_{\text{cylinder}}=\dfrac{1}{3}\pi r^2 h_{\text{cone}}$

$h_{\text{cylinder}}=\dfrac{1}{3}h_{\text{cone}}$

The height of the cone is 12 cm:

$h_{\text{cylindier}}=\dfrac{1}{3}\times 12$

$=4\text{ cm}$

The correct answer is **A**.

2 **a** $V = \frac{4}{3} \times \pi \times r^3$

$= \frac{4}{3} \times \pi \times 2^3$ [1 mark – this is a SHOW THAT question. Must see the substitution]

$= 33.51$

b $V_{\text{box}} = 4.1^3 = 68.92\,\text{cm}^3$

$V_{\text{empty}} = 68.91 - 33.51 = 35.41\,\text{cm}^3$ [1 mark – rounding to 2 decimal places applies]

c $\text{TSA} = 6 \times 4.1^2 = 100.86\,\text{cm}^2$ [1 mark]

d length: $\frac{17}{4.1} = 4.146 \rightarrow$ so 4 boxes will fit.

width: $\frac{12.5}{4.1} = 3.049 \rightarrow$ so 3 boxes will fit.

height: $\frac{8.5}{4.1} = 2.073 \rightarrow$ so 2 boxes will fit.

Therefore, the maximum number of boxes that can fit in the display unit is $4 \times 3 \times 2 = 24$ boxes [1 mark]

3 **a** Volume $= 43\,\text{cm}^2 \times 7\,\text{cm} = 301\,\text{cm}^3$ [1 mark]

b Area $= \pi r^2$

$43 = \pi r^2$

$r = \sqrt{\frac{43}{\pi}}$ [1 mark]

$r = 3.6996\ldots$

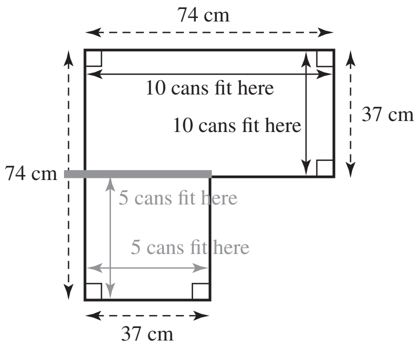
$= 3.7$

c $\text{TSA} = 2 \times 43 + 2\pi \times 3.7 \times 7 = 248.734\ldots = 249\,\text{cm}^2$ [1 mark]

d $74\,\text{cm} \times 4 = 296\,\text{cm}$ [1 mark]

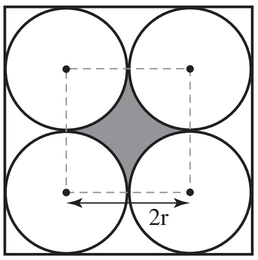
e We know that the radius of each can is 3.7 cm, which means the diameter will be 7.4 cm.

Break the shape into two sections:



So $10 \times 5 + 5 \times 5 = 75$ cans can fit on the shelf. [1 mark]

4 Form a square with the vertices at the centre of each circle and side length of $2r$.



The four quarter circles inside the smaller square make up one whole circle. Take this circle away from the square. i.e. Area of the small square – area of a circle (or 4 quarter circles)

$= (2r)^2 - \pi r^2$

$= 4r^2 - \pi r^2$

$= (2r \times 2r) - \left(4 \times \frac{1}{4}\pi r^2\right) = 4r^2 - \pi r^2$

The correct answer is **B**.

5 The volume of a cone is $\frac{1}{3}\pi r^2 h$. Given the volume equation, $36 = \frac{1}{3}\pi \times 2.5^2 \times h$, h can be found.

$h = \frac{3 \times 36}{\pi \times 2.5^2} = 5.5\,\text{cm}$

x can then be found by using Pythagoras' theorem.

$x = \sqrt{r^2 + h^2} = \sqrt{2.5^2 + 5.5^2} = 6.04\,\text{cm}$

The surface area of a cone $= \pi r(r + x)$.

Substituting in $x = 6.04$, the surface area is

$\pi \times 2.5 \times (2.5 + 6.04) \approx 67\,\text{cm}^2$

The correct answer is **D**.

Topic 12 — Applications of trigonometry

12.2 Trigonometric ratios

12.2 Exercise

1 $\frac{x}{2.7} = \sin(53°)$

$x = 2.7\sin(53°)$
$= 2.156\ldots$
$= 2.16$ cm (correct to 2 decimal places)

2 $\frac{5.3}{x} = \sin(30°)$

$x = \frac{5.3}{\sin(30°)}$
$= 10.6$ mm

3 $\sin(\theta) = \frac{4.6}{6.3}$

$\theta = \sin^{-1}\left(\frac{4.6}{6.3}\right)$
$= 46.899\ldots°$
$= 46.90°$ (correct to 2 decimal places)

4 $\sin(\theta) = \frac{8.2}{11.5}$

$\theta = \sin^{-1}\left(\frac{8.2}{11.5}\right)$
$= 45.483\ldots°$
$= 45.48°$ (correct to 2 decimal places)

5 $\frac{1.8}{y} = \cos(25.2°)$

$y = \frac{1.8}{\cos(25.2°)}$
$= 1.989\ldots$
$= 1.99$ cm (correct to 2 decimal places)

6 $\frac{y}{3.19} = \cos(34.5°)$

$y = 3.19\cos(34.5°)$
$= 2.628\ldots$
$= 2.63$ cm (correct to 2 decimal places)

7 $\cos(\theta) = \frac{4.5}{16.3}$

$\theta = \cos^{-1}\left(\frac{4.5}{16.3}\right)$
$= 73.973\ldots$
$= 73.97°$ (correct to 2 decimal places)

8 $\cos(\theta) = \frac{6.1}{7.5}$

$\theta = \cos^{-1}\left(\frac{6.1}{7.5}\right)$
$= 35.577\ldots°$
$= 35.58°$ (correct to 2 decimal places)

9 $\frac{x}{25.8} = \tan(27°)$

$x = 25.8\tan(27°)$
$= 13.145\ldots$
$= 13.15$ cm (correct to 2 decimal places)

10 $\frac{1.3}{y} = \tan(60°)$

$y = \frac{1.3}{\tan(60°)}$
$= 0.750\ldots$
$= 0.75$ cm (correct to 2 decimal places)

11 $\tan(\theta) = \frac{12.7}{9.5}$

$\theta = \tan^{-1}\left(\frac{12.7}{9.5}\right)$
$= 53.202\ldots°$
$= 53.20°$ (correct to 2 decimal places)

12 $\tan(\theta) = \frac{2.35}{3.19}$

$\theta = \tan^{-1}\left(\frac{2.35}{3.19}\right)$
$= 36.378°$
$= 36.38°$ (correct to 2 decimal places)

13 **a** $180° - 125° = 55°$

$\sin(55°) = 0.819\ldots$
$= 0.82$ (correct to 2 decimal places)
$\sin(125°) = \sin(55°)$
$= 0.82$ (correct to 2 decimal places)

b $180° - 152° = 28°$

$\cos(28°) = 0.882\ldots$
$= 0.88$ (correct to 2 decimal places)
$\cos(152°) = -\cos(28°)$
$= -0.88$ (correct to 2 decimal places)

14 **a** $180° - 99.2° = 80.8°$

$\sin(80.8°) = 0.987\ldots$
$= 0.99$ (correct to 2 decimal places)
$\sin(99.2°) = \sin(80.2°)$
$= 0.99$ (correct to 2 decimal places)

b $180° - 146.7° = 33.3°$

$\cos(33.3°) = 0.835\ldots$
$= 0.84$ (correct to 2 decimal places)
$\cos(146.7°) = -\cos(33.3°)$
$= -0.84$ (correct to 2 decimal places)

15 $y^2 + y^2 = 9$

$2y^2 = 9$
$y = 2.12$ (correct to 2 decimal places)

$\sin(24.25°) = \frac{2.12}{x}$

$x = \frac{2.12}{\sin(24.25°)}$
$= 5.161\ldots$
$= 5.16$ (correct to 2 decimal places)

16 $11.3 - 6.4 = 4.9$

$\tan(\theta) = \frac{5.7}{4.9}$

$\theta = \tan^{-1}\left(\frac{5.7}{4.9}\right)$
$= 49.316\ldots°$
$= 49.32°$ (correct to 2 decimal places)

17 $\sin(\theta) = \dfrac{1.25}{7}$

$$\begin{aligned}\theta &= \sin^{-1}\left(\frac{1.25}{7}\right)\\ &= 10.286\ldots^\circ\\ &= 10.29^\circ \text{ (correct to 2 decimal places)}\end{aligned}$$

$\cos(\alpha) = \dfrac{1.25}{7}$

$$\begin{aligned}\alpha &= \cos^{-1}\left(\frac{1.25}{7}\right)\\ &= 79.713^\circ\\ &= 79.71^\circ \text{ (correct to 2 decimal places)}\end{aligned}$$

18 $\dfrac{20}{x} = \tan(64.3^\circ)$

$$\begin{aligned}x &= \frac{20}{\tan(64.3^\circ)}\\ &= 9.625\ldots\\ &= 9.63 \text{ m (correct to 2 decimal places)}\end{aligned}$$

19 a $\dfrac{8.5}{y} = \tan(49^\circ)$

$$\begin{aligned}y &= \frac{8.5}{\tan(49^\circ)}\\ &= 7.38\ldots\\ &= 7.4 \text{ km (correct to 1 decimal place)}\end{aligned}$$

b $AB^2 = 7.4^2 + 8.5^2$

$$\begin{aligned}&= 127.01\\ AB &= 11.26\ldots\\ &= 11.3 \text{ km (correct to 1 decimal place)}\end{aligned}$$

$$\begin{aligned}\text{Total distance} &= 7.4 + 8.5 + 11.3\\ &= 27.2 \text{ km}\end{aligned}$$

20 $\dfrac{300}{x} = \sin(6^\circ)$

$$\begin{aligned}x &= \frac{300}{\sin(6^\circ)}\\ &= 2870.0\ldots\\ &= 2870 \text{ m (correct to 1 decimal place)}\end{aligned}$$

21 $\tan(\theta) = \dfrac{1.5}{4}$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{1.5}{4}\right)\\ &= 20.556^\circ\\ &= 20.56^\circ \text{ (correct to 2 decimal places)}\end{aligned}$$

22 $\dfrac{x}{3.5} = \sin(35.2^\circ)$

$$\begin{aligned}x &= 3.5\sin(35.2^\circ)\\ &= 2.017\ldots\\ &= 2.02 \text{ m (correct to 2 decimal places)}\end{aligned}$$

23 a $\cos(\theta) = \dfrac{1.7}{2.5}$

$$\begin{aligned}\theta &= \cos^{-1}\left(\frac{1.7}{2.5}\right)\\ &= 47.156\ldots^\circ\\ &= 47.16^\circ \text{ (correct to 2 decimal places)}\end{aligned}$$

b $2.5^2 = y^2 + 1.7^2$

$$\begin{aligned}y^2 &= 2.5^2 - 1.7^2\\ &= 3.36\\ y &= 1.833\ldots\\ &= 1.83 \text{ m (correct to 2 decimal places)}\end{aligned}$$

24 a $\dfrac{x}{4.2} = \cos(10.5^\circ)$

$$\begin{aligned}x &= 4.2\cos(10.5^\circ)\\ &= 4.12\ldots\\ &= 4.1 \text{ m (correct to 1 decimal place)}\end{aligned}$$

b $\dfrac{0.5}{x} = \sin(5.7^\circ)$

$$\begin{aligned}x &= \frac{0.5}{\sin(5.7^\circ)}\\ &= 5.034\ldots\\ &= 5.03 \text{ m (correct to 2 decimal places)}\end{aligned}$$

25 $2.5 - 1.6 = 0.9$

$\dfrac{0.9}{x} = \tan(11.87^\circ)$

$$\begin{aligned}x &= \frac{0.9}{\tan(11.87^\circ)}\\ &= 4.281\ldots\\ &= 4.28 \text{ m (correct to 2 decimal places)}\end{aligned}$$

26 AB distance:

$\dfrac{22}{x} = \cos(12.23^\circ)$

$$\begin{aligned}x &= \frac{22}{\cos(12.23^\circ)}\\ &= 22.510\ldots\\ &= 22.51 \text{ m (correct to 2 decimal places)}\end{aligned}$$

CD distance:

$\dfrac{20.5}{x} = \sin(54.8^\circ)$

$$\begin{aligned}x &= \frac{20.5}{\sin(54.8^\circ)}\\ &= 25.087\ldots\\ &= 25.09 \text{ m (correct to 2 decimal places)}\end{aligned}$$

BC distance:

$\dfrac{y}{25.09} = \cos(54.8^\circ)$

$$\begin{aligned}y &= 25.09\cos(54.8^\circ)\\ &= 14.462\ldots\\ &= 14.46 \text{ m (correct to 2 decimal places)}\end{aligned}$$

DE distance:

$\dfrac{15.7}{x} = \cos(48.48^\circ)$

$$\begin{aligned}x &= \frac{15.7}{\cos(48.48^\circ)}\\ &= 23.684\ldots\\ &= 23.68 \text{ m (correct to 2 decimal places)}\end{aligned}$$

AE distance:

$\dfrac{5.2}{x} = \sin(20^\circ)$

$$\begin{aligned}x &= \frac{5.2}{\sin(20^\circ)}\\ &= 15.203\ldots\\ &= 15.20 \text{ m (correct to 2 decimal places)}\end{aligned}$$

$$\begin{aligned}\text{Total distance} &= \overline{AB} + \overline{BC} + \overline{CD} + \overline{DE} + \overline{AE}\\ &= 22.51 + 14.46 + 25.09 + 23.68 + 15.20\\ &= 100.94 \text{ m}\end{aligned}$$

12.2 Exam questions

1 $\cos(\theta) = \frac{3.25}{4.5}$

$$\begin{aligned}\theta &= \cos^{-1}\left(\frac{3.25}{4.5}\right)\\ &= 43.8°\\ &\approx 44°\end{aligned}$$

The correct answer is **B**.

2 $\tan(\theta) = \frac{\text{Opposite side}}{\text{Adjacent side}}$

$\tan(35°) = \frac{x}{14}$

$$\begin{aligned}x &= \tan(35°) \times 14\\ &= 9.8\text{ cm}\end{aligned}$$

The correct answer is **D**.

3 $\sin(\theta) = \frac{\text{Opposite side}}{\text{Hypotenuse}}$

$\sin(z) = \frac{30}{45}$

$$\begin{aligned}z &= \sin^{-1}\left(\frac{30}{45}\right)\\ &= 41.8°\end{aligned}$$

The correct answer is **E**.

12.3 Angles of elevation and depression, and bearings

12.3 Exercise

1 $\frac{32}{x} = \tan(41°)$

$$\begin{aligned}x &= \frac{32}{\tan(41°)}\\ &= 36.811\ldots\\ &= 36.81\text{ m (correct to 2 decimal places)}\end{aligned}$$

2 $\frac{33}{x} = \tan(35°)$

$$\begin{aligned}x &= \frac{33}{\tan(35°)}\\ &= 47.128\ldots\\ &= 47.13\text{ m (correct to 2 decimal places)}\end{aligned}$$

3 $\cos(\theta) = \frac{16.2}{34}$

$$\begin{aligned}\theta &= \cos^{-1}\left(\frac{16.2}{34}\right)\\ &= 61.544\ldots°\\ &= 61.54° \text{ (correct to 2 decimal places)}\end{aligned}$$

4 $\tan(\theta) = \frac{22}{30}$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{22}{30}\right)\\ &= 36.253\ldots°\\ &= 36.25° \text{ (correct to 2 decimal places)}\end{aligned}$$

5 $\frac{x}{2} = \sin(60°)$

$$\begin{aligned}x &= 2\sin(60°)\\ &= 1.732\ldots\\ &= 1.73\text{ m (correct to 2 decimal places)}\end{aligned}$$

$$\begin{aligned}\text{Total distance} &= 1.73 + 1\\ &= 2.73\text{ m (correct to 2 decimal places)}\end{aligned}$$

6 $\frac{707}{x} = \sin(45°)$

$$\begin{aligned}x &= \frac{707}{\sin(45°)}\\ &= 999.848\ldots\\ &= 999.85\text{ m (correct to 2 decimal places)}\end{aligned}$$

7 **a** 049°
b $360° - 18° = 342°$
c $270° - 39° = 231°$
d $90° + 22° = 112°$

8 **a** $180° + 5° = 185°$
b $360° - 63° = 297°$
c $90° - 11° = 079°$
d $180° - 56° = 124°$

9 **a** The angle measured from north is 110°.
The true bearing from Town A to Town B is 110°T.
b The angle measured from north is $360° - 70° = 290°$.
The true bearing from Town B to Town A is 290°T.

10 **a** The angle measured from north is $360° - 123° = 237°$.
The true bearing from Town A to Town B is 237°T.
b The angle measured from north is $180° - 123° = 57°$.
The true bearing from Town A to Town B is 057°T.

11 **a** The angle measured from north is 50°.
The true bearing of B from A is 050°T.
b $103° - 50° = 53°$
The angle measured from north is $180° - 53° = 127°$.
The true bearing of C from B is 127°T.
c $180° - 48° - 53° = 79°$
The angle measured from north is $180° + 79° = 259°$.
The true bearing of C from B is 259°T.

12 **a** Reference angle = 65°
Hypotenuse = 36

$\sin(65°) = \frac{\text{O}}{36}$

$$\begin{aligned}\text{O} &= 36\sin(65°)\\ &= 32.627\ldots\\ &= 32.63\text{ km (correct to 2 decimal places)}\end{aligned}$$

b $\cos(65°) = \frac{\text{A}}{36}$

$$\begin{aligned}\text{A} &= 36\cos(65°)\\ &= 15.214\ldots\\ &= 15.21\text{ km (correct to 2 decimal places)}\end{aligned}$$

13 Total journey = 4 km north, 3 km west

$$\begin{aligned}\tan(\theta) &= \frac{\text{O}}{\text{A}}\\ &= \frac{4}{3}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{4}{3}\right)\\ &= 53.13\ldots°\\ &= 53.1° \text{ (correct to 2 decimal places)}\end{aligned}$$

$$\begin{aligned}\text{True bearing} &= 270° + 53.1°\\ &= 323.1°\text{T}\end{aligned}$$

14 $\frac{x}{724} = \tan(50°)$

$x = 724\tan(50°)$
$= 862.829\ldots$
$= 862.83$ m (correct to 2 decimal places)

15 $\frac{x}{11} = \tan(88°)$

$x = 11\tan(88°)$
$= 314.9\ldots$
$= 315$ m (correct to the nearest metre)

16 $\frac{x}{7} = \tan(54°)$

$x = 7\tan(54°)$
$= 9.634\ldots$
$= 9.63$ m (correct to 2 decimal places)

Total height $= 1.5 + 9.63$
$= 11.13$ m (correct to 2 decimal places)

17 $\tan(\theta) = \frac{63.28}{50}$

$\theta = \tan^{-1}\left(\frac{63.28}{50}\right)$
$= 51.6\ldots°$
$= 52°$ (correct to the nearest degree)

18 $\tan(\theta) = \frac{5000}{150}$

$\theta = \tan^{-1}\left(\frac{5000}{150}\right)$
$= 88.281\ldots°$
$= 88.28°$ (correct to 2 decimal places)

19 $500 - 87 = 413$

$\tan(7°) = \frac{413}{x}$

$x = \frac{413}{\tan(7°)}$
$= 3363.6\ldots$
$= 3364$ m (correct to the nearest metre)

$\tan(5°) = \frac{413}{y}$

$y = \frac{413}{\tan(5°)}$
$= 4720.6\ldots$
$= 4721$ m (correct to the nearest metre)

Distance $= 4721 - 3364$
$= 1357$ m (correct to the nearest metre)

20 a $10.5 - 1.6 = 8.9$

$\tan(\theta) = \frac{8.9}{1}$

$\theta = \tan^{-1}(8.9)$
$= 83.589\ldots°$
$= 83.59°$ (correct to 2 decimal places)

b $16.5 - 1.6 = 14.9$

$\tan(\theta) = \frac{14.9}{2}$

$\theta = \tan^{-1}\left(\frac{14.9}{2}\right)$
$= 82.354\ldots°$
$= 82.35°$ (correct to 2 decimal places)

c $18.5 - 1.6 = 16.9$

$\tan(\theta) = \frac{16.9}{3}$

$\theta = \tan^{-1}\left(\frac{16.9}{3}\right)$
$= 79.934\ldots°$
$= 79.93°$ (correct to 2 decimal places)

d $16.5 - 1.6 = 14.9$

$\tan(\theta) = \frac{14.9}{4}$

$\theta = \tan^{-1}\left(\frac{14.9}{4}\right)$
$= 74.972\ldots°$
$= 74.97°$ (correct to 2 decimal places)

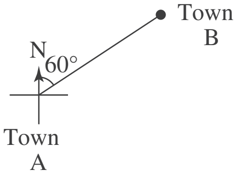
e $10.5 - 1.6 = 8.9$

$\tan(\theta) = \frac{8.9}{5}$

$\theta = \tan^{-1}\left(\frac{8.9}{5}\right)$
$= 60.672\ldots°$
$= 60.67°$ (correct to 2 decimal places)

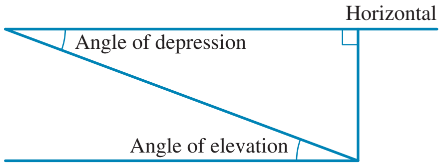
12.3 Exam questions

1 The true bearing of 60° would start from North and move in a clockwise direction. Start by drawing Town A and a compass rose, then add the bearing to Town B.



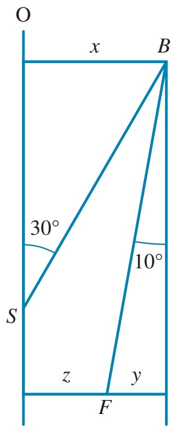
The correct answer is **A**.

2 Note the equality of angles of elevation and depression.



The correct answer is **D**.

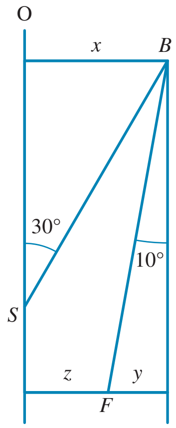
3 a



$\sin(30°) = \frac{x}{6}$

$x = 6\sin(30°)$
$x = 3$ km [1 mark]

b

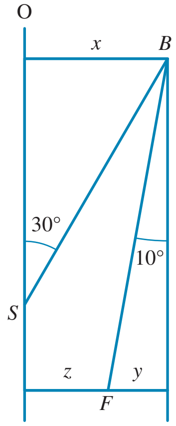


$\sin(10°) = \dfrac{y}{10}$

$y = 10\sin(10°)$

$y = 1.74$ km [1 mark]

c



$z = x - y$

$= 3 - 1.74$

$= 1.26$ km [1 mark]

12.4 The sine rule

12.4 Exercise

1 $\dfrac{x}{\sin(45°)} = \dfrac{12}{\sin(55°)}$

$x = \dfrac{12\sin(45°)}{\sin(55°)}$

$= 10.358\ldots$

$= 10.36$ cm (correct to 2 decimal places)

2 $\dfrac{x}{\sin(39°)} = \dfrac{16}{\sin(76°)}$

$x = \dfrac{16\sin(39°)}{\sin(76°)}$

$= 10.377\ldots$

$= 10.38$ cm (correct to 2 decimal places)

3 $\dfrac{x}{\sin(53°)} = \dfrac{8.45}{\sin(48.2°)}$

$x = \dfrac{8.45\sin(53°)}{\sin(48.2°)}$

$= 9.052\ldots$

$= 9.05$ cm (correct to 2 decimal places)

4 $\dfrac{x}{\sin(84°)} = \dfrac{13.1}{\sin(50°)}$

$x = \dfrac{13.1\ \sin(84°)}{\sin(50°)}$

$= 17.0\ldots$

$= 17$ cm (correct to the nearest cm)

5 $\dfrac{10.5}{\sin(B)} = \dfrac{8.4}{\sin(22.3°)}$

$\sin(B) = \dfrac{10.5\ \sin(22.3°)}{8.4}$

$B = \sin^{-1}\left(\dfrac{10.5\sin(22.3°)}{8.4}\right)$

$= 28.31\ldots$

$= 28.3°$ (correct to 1 decimal place)

6 $\dfrac{7.63}{\sin(B)} = \dfrac{4.56}{\sin(15.8°)}$

$\sin(B) = \dfrac{7.63\ \sin(15.8°)}{4.56}$

$B = \sin^{-1}\left(\dfrac{7.63\sin(15.8°)}{4.56}\right)$

$= 27.10\ldots°$

$= 27.1°$ (correct to 1 decimal place)

7 $\dfrac{22}{\sin(\theta)} = \dfrac{15}{\sin(30°)}$

$\sin(\theta) = \dfrac{22\sin(30°)}{15}$

$\theta = \sin^{-1}\left(\dfrac{22\sin(30°)}{15}\right)$

$= 47.16\ldots°$

$= 47.2°$ (correct to 2 decimal places)

8 $\dfrac{8}{\sin(A)} = \dfrac{6}{\sin(43°)}$

$\sin(A) = \dfrac{8\sin(43°)}{6}$

$A = \sin^{-1}\left(\dfrac{8\sin(43°)}{6}\right)$

$= 65.413\ldots°$

$= 65.41°$ (correct to 2 decimal places)

$A' = 180° - A$

$= 180° - 65.41°$

$= 114.59°$ (correct to 2 decimal places)

9 $\dfrac{7.5}{\sin(A)} = \dfrac{5}{\sin(32°)}$

$\sin(A) = \dfrac{7.5\sin(32°)}{5}$

$A = \sin^{-1}\left(\dfrac{7.5\sin(32°)}{5}\right)$

$= 52.643\ldots$

$= 52.64°$ (correct to 2 decimal places)

$A' = 180° - A$

$= 180° - 52.64°$

$= 127.36°$ (correct to 2 decimal places)

10 Given:

- A is acute.
- length $a <$ length b
- $h = 19.5\sin(25.3)$

  $= 8.33\ldots$

$\therefore$ length $a >$ length h

We have the ambiguous case of the sine rule.

$$\frac{19.5}{\sin(B)} = \frac{11.4}{\sin(25.3°)}$$

$$\sin(B) = \frac{19.5\sin(25.3°)}{11.4}$$

$$B = \sin^{-1}\left(\frac{19.5\sin(25.3°)}{11.4}\right)$$

$= 46.970\ldots°$

$= 46.97°$ (correct to 2 decimal places)

$B' = 180° - B$

$= 180° - 46.97°$

$= 133.03°$ (correct to 2 decimal places)

11 $A = 180° - (60° + 72°)$

$= 48°$

$$\frac{b}{\sin(60°)} = \frac{10.5}{\sin(48°)}$$

$$b = \frac{10.5\sin(60°)}{\sin(48°)}$$

$= 12.236\ldots$

$= 12.24$ (correct to 2 decimal places)

$$\frac{c}{\sin(72°)} = \frac{10.5}{\sin(48°)}$$

$$c = \frac{10.5\sin(72°)}{\sin(48°)}$$

$= 13.437\ldots$

$= 13.43$ (correct to 2 decimal places)

12 $\sin(\theta) = 0.57358$

$\theta = \sin^{-1}(0.57358)$

$\theta = 35°$

$\sin(35°) = \sin(180 - 35)°$

$= \sin(145°)$

The two angles are 35° and 145°

13 $A = 75°$

$C = 180 - 75° - 75°$

$= 30°$

$$\frac{40}{\sin(75°)} = \frac{x}{\sin(30°)}$$

$$x = \frac{40\sin(30°)}{\sin(75°)}$$

$= 20.705\ldots$

$= 20.71$ m (correct to 2 decimal places)

14 a $180° - 54° = 126°$

$180° - 126° - 31° = 23°$

$$\frac{x}{\sin(31°)} = \frac{10}{\sin(23°)}$$

$$x = \frac{10\sin(31°)}{\sin(23°)}$$

$= 13.181\ldots$

$= 13.18$ m (correct to 2 decimal places)

Total length $= 13.18 + 10$

$= 23.18$ m (correct to 2 decimal places)

b $\dfrac{PX}{13.18} = \sin(54°)$

$PX = 13.18\sin(54°)$

$= 10.662\ldots$

$= 10.66$ m (correct to 2 decimal places)

15 $$\frac{10.5}{\sin(y)} = \frac{7}{\sin(24°)}$$

$$\sin(y) = \frac{10.5\sin(24°)}{7}$$

$$y = \sin^{-1}\left(\frac{10.5\sin(24°)}{7}\right)$$

$= 37.597\ldots°$

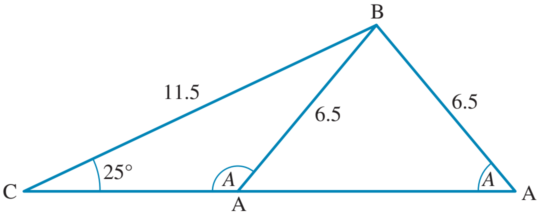
$= 37.6°$ (correct to 1 decimal place)

$x = 180° - y$

$= 180° - 37.6°$

$= 142.4°$ (correct to 1 decimal place)

16 a



b $$\frac{11.5}{\sin(A)} = \frac{6.5}{\sin(25°)}$$

$$\sin(A) = \frac{11.5\sin(25°)}{6.5}$$

$$A = \sin^{-1}\left(\frac{11.5\sin(25°)}{6.5}\right)$$

$= 48.392\ldots°$

$= 48.39°$ (correct to 2 decimal places)

$A' = 180 - A$

$= 180 - 48.39°$

$= 131.61°$ (correct to 2 decimal places)

$B = 180° - (25° + 48.39°)$

$= 106.61°$ (correct to 2 decimal places)

$B' = 180° - (25° + 131.61°)$

$= 23.39°$ (correct to 2 decimal places)

17 We have an isosceles triangle with base angles

$$= \frac{(180° - 122°)}{2}$$

$= 29°$

$$\frac{x}{\sin(122°)} = \frac{5.6}{\sin(29°)}$$

$$x = \frac{5.6\ \sin(122°)}{\sin(29°)}$$

$= 9.795\ldots$

$= 9.80$ m (correct to 2 decimal places)

18 $$\frac{6}{\sin(y)} = \frac{5.5}{\sin(55°)}$$

$$\sin(y) = \frac{6\sin(55°)}{5.5}$$

$$y = \sin^{-1}\left(\frac{6\sin(55°)}{5.5}\right)$$

$= 63.331\ldots°$

$= 63.33°$ (correct to 2 decimal places)

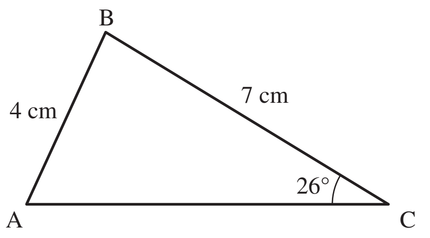
$x = 180° - 55° - 63.33°$

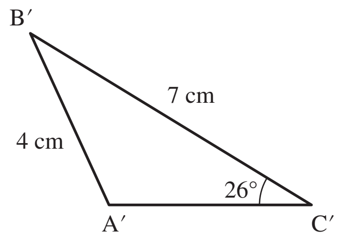
$= 61.67°$ (correct to 2 decimal places)

$$\frac{x}{\sin(61.67°)} = \frac{5.5}{\sin(55°)}$$
$$x = \frac{5.5\sin(61.67°)}{\sin(55°)}$$
$$= 5.910\ldots$$
$$= 5.91 \text{ km (correct to 2 decimal places)}$$

12.4 Exam questions

1



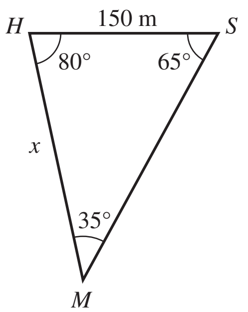


From the first triangle, we can find angle A using the sine rule.

$$\frac{\sin(26)°}{4} = \frac{\sin(A)°}{7}$$

Solving for A, $A = 50°$ (to the nearest degree)
Using the angle sum of a triangle, $A + B + C = 180°$
So, $50° + B + 26° = 180°$,
$B = 104°$
From the ambiguous case, $A' = 180° - 50° = 130°$ (to the nearest degree)
Again, using the angle sum of a triangle, $A' + B' + C' = 180°$
So, $130° + B' + 26° = 180°$,
$B' = 24°$
So the angle that could not be an angle in the triangle ABC is 144°.
The correct answer is **E**.

2



$$\frac{x}{\sin 65°} = \frac{150}{\sin 35°}$$
$$x = 237.0149$$
$$= 237 \text{ m}$$

The correct answer is **E**.

3
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$
$$\frac{5}{\sin(B)} = \frac{4}{\sin(30)}$$
$$\sin(B) = \frac{5\sin(30)}{4}$$
$$B = \sin^{-1}\left(\frac{5\sin(30)}{4}\right)$$
$$= 38.68° \text{ and}$$
$$180 - 38.68° = 141.32°$$
$$\therefore B \text{ is } 38.68° \text{ and } 141.32°$$

The correct answer is **E**.

12.5 The cosine rule

12.5 Exercise

1 $x^2 = 4^2 + 6^2 - 2 \times 4 \times 6\cos(22°)$
$$x = \sqrt{4^2 + 6^2 - 2 \times 4 \times 6\cos(22°)}$$
$$= 2.737\ldots$$
$$= 2.74 \text{ km (correct to 2 decimal places)}$$

2 $x^2 = 16.5^2 + 14.3^2 - 2 \times 16.5 \times 14.3\cos(39°)$
$$x = \sqrt{16.5^2 + 14.3^2 - 2 \times 16.5 \times 14.3\cos(39°)}$$
$$= 10.488\ldots$$
$$= 10.49 \text{ m (correct to 2 decimal places)}$$

3 $x^2 = 9^2 + 15^2 - 2 \times 9 \times 15\cos(30°)$
$$x = \sqrt{9^2 + 15^2 - 2 \times 9 \times 15\cos(30°)}$$
$$= 8.49\ldots$$
$$= 8.5 \text{ km (correct to 1 decimal place)}$$

4 $x^2 = 4.2^2 + 6.7^2 - 2 \times 4.2 \times 6.7\cos(41°)$
$$x = \sqrt{4.2^2 + 6.7^2 - 2 \times 4.2 \times 6.7\cos(41°)}$$
$$= 4.478\ldots$$
$$= 4.48 \text{ m (correct to 2 decimal places)}$$

5 $\cos(A) = \dfrac{13^2 + 17^2 - 8^2}{2 \times 13 \times 17}$
$$A = \cos^{-1}\left(\frac{13^2 + 17^2 - 8^2}{2 \times 13 \times 17}\right)$$
$$= 26.949\ldots°$$
$$= 26.95° \text{ (correct to 2 decimal places)}$$

6 $\cos A = \dfrac{9^2 + 5^2 - 11^2}{2 \times 5 \times 9}$
$$A = \cos^{-1}\left(\frac{9^2 + 5^2 - 11^2}{2 \times 5 \times 9}\right)$$
$$= 99.594\ldots°$$
$$= 99.59° \text{ (correct to 2 decimal places)}$$

7 $\cos A = \left(\dfrac{4^2 + 7^2 - 5^2}{2 \times 4 \times 7}\right)$
$$A = \cos^{-1}\left(\frac{4^2 + 7^2 - 5^2}{2 \times 4 \times 7}\right)$$
$$= 44.415\ldots$$
$$= 44.42° \text{ (correct to 2 decimal places)}$$

8 $b^2 = 12^2 + 8^2 - 2 \times 8 \times 12\cos(57°)$
$$b = \sqrt{12^2 + 8^2 - 2 \times 8 \times 12\cos(57°)}$$
$$= 10.170\ldots$$
$$= 10.17 \text{ (correct to 2 decimal places)}$$

9 The largest angle lies opposite the largest side length.

$$\cos A = \left(\frac{15^2 + 13^2 - 18^2}{2 \times 13 \times 15}\right)$$

$$A = \cos^{-1}\left(\frac{15^2 + 13^2 - 18^2}{2 \times 13 \times 15}\right)$$

$$= 79.660\ldots°$$

$$= 79.66° \text{ (correct to 2 decimal places)}$$

10 $$\cos A = \frac{40^2 + 50^2 - 60^2}{2 \times 40 \times 50}$$

$$A = \cos^{-1}\left(\frac{40^2 + 50^2 - 60^2}{2 \times 40 \times 50}\right)$$

$$= 82.819\ldots°$$

$$= 82.82° \text{ (correct to 2 decimal places)}$$

11 $$\cos A = \frac{7^2 + 9^2 - 5^2}{2 \times 7 \times 9}$$

$$A = \cos^{-1}\left(\frac{7^2 + 9^2 - 5^2}{2 \times 7 \times 9}\right)$$

$$= 33.557\ldots°$$

$$= 33.56° \text{ (correct to 2 decimal places)}$$

12 In a parallelogram the diagonals bisect the angles and opposite angles are equal.

$180° - 43° = 137°$

$$AC^2 = 6.2^2 + 8^2 - 2 \times 6.2 \times 8\cos(137°)$$

$$AC = \sqrt{6.2^2 + 8^2 - 2 \times 6.2 \times 8\cos(137°)}$$

$$= 13.228\ldots$$

$$= 13.23 \text{ cm (correct to 2 decimal places)}$$

13 $$x^2 = 14.5^2 + 8^2 - 2 \times 14.5 \times 8\cos(35°)$$

$$x = \sqrt{14.5^2 + 8^2 - 2 \times 14.5 \times 8\cos(35°)}$$

$$= 9.176\ldots$$

$$= 9.18 \text{ km (correct to 2 decimal places)}$$

Total distance $= 14.5 + 8 + 9.18$

$= 31.68$ km (correct to 2 decimal places)

14 $180° - 105° = 75°$

$$x^2 = 2.1^2 + 3.3^2 - 2 \times 2.1 \times 3.3\cos(75°)$$

$$x = \sqrt{2.1^2 + 3.3^2 - 2 \times 2.1 \times 3.3\cos(75°)}$$

$$= 3.4223\ldots$$

$$= 3.422 \text{ km (correct to 3 decimal places)}$$

Total distance $= 2.1 + 3.3 + 3.422$

$= 8.822$ km (correct to 3 decimal places)

$= 8822$ m (correct to the nearest metre)

15 $4.5 \times 48 = 216$ km

$180° - 98° = 82°$

$6 \times 54 = 324$ km

$$x^2 = 216^2 + 324^2 - 2 \times 216 \times 324\cos(82°)$$

$$x = \sqrt{216^2 + 324^2 - 2 \times 216 \times 324\cos(82°)}$$

$$= 363.527\ldots$$

$$= 363.53 \text{ km (correct to 2 decimal places)}$$

Travelling at 50 km/h:

$$\frac{363.53}{50} = 7.2706 \text{ hours}$$

$= 7$ hours, (0.276×60) minutes

$= 7$ hours, 16.236 minutes

$= 7$ hours, 16 minutes (correct to the nearest minute)

16 $$x^2 = 100^2 + 180^2 - 2 \times 100 \times 180\cos(25°)$$

$$x = \sqrt{100^2 + 180^2 - 2 \times 100 \times 180\cos(25°)}$$

$$= 98.858\ldots$$

$$= 98.86 \text{ km (correct to 2 decimal places)}$$

12.5 Exam questions

1 We are given two side lengths and an included angle. To find the third side, use the cosine rule.

$$AB = \sqrt{12.6^2 + 19.2^2 - 2 \times 12.6 \times 19.2 \times \cos 63°}$$

The correct answer is **D**.

2 $\angle GAB = 90° - 50°$

$= 40°$

$$a^2 = b^2 + c^2 - 2bc\ \cos(A)$$

$$x^2 = 60^2 + 80^2 - 2 \times 60 \times 80\cos(40°)$$

$$= 10\,000 - 9600\cos(40°)$$

$$x = \sqrt{(10\,000 - 9600\cos(40°))}$$

$$= 51.44 \text{ km}$$

The correct answer is **D**.

3 $$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$5^2 = 8^2 + 10^2 - 2 \times 8 \times 10\cos(\theta)$$

$$-\cos(\theta) = \frac{5^2 - 8^2 - 10^2}{2 \times 8 \times 10}$$

$$\cos(\theta) = \frac{8^2 + 10^2 - 5^2}{2 \times 8 \times 10}$$

The correct answer is **B**.

12.6 Area of triangles

12.6 Exercise

1 $$\text{Area} = \frac{1}{2}bc\sin(A)$$

$$= \frac{1}{2} \times 11.9 \times 14.4\sin(38°)$$

$$= 52.749\ldots$$

$$= 52.75 \text{ mm}^2 \text{ (correct to 2 decimal places)}$$

2 $$\text{Area} = \frac{1}{2}bc\sin(A)$$

$$= \frac{1}{2} \times 14.3 \times 6.5\sin(32°)$$

$$= 24.627\ldots$$

$$= 24.63 \text{ mm}^2 \text{ (correct to 2 decimal places)}$$

3 $$\text{Area} = \frac{1}{2}bc\sin(A)$$

$$18.54 = \frac{1}{2} \times x \times 8\sin(45.5°)$$

$$x = \frac{2 \times 18.54}{8\sin(45.5°)}$$

$$= 6.498\ldots$$

$$= 6.50 \text{ cm (correct to 2 decimal places)}$$

4 The largest angle in a triangle is opposite the largest side, so this angle is enclosed by the two smallest sides.

$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin(A)\\ &= \frac{1}{2}\times 10.2\times 16.2\sin(104.5°)\\ &= 79.988\ldots\\ &= 79.99\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

5 $$\begin{aligned}s &= \frac{11+12+13}{2}\\ &= \frac{36}{2}\\ &= 18\end{aligned}$$

$$\begin{aligned}\text{Area} &= \sqrt{18(18-11)(18-12)(18-13)}\\ &= \sqrt{18\times 7\times 6\times 5}\\ &= \sqrt{3780}\\ &= 61.481\ldots\\ &= 61.48\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

6 $$\begin{aligned}s &= \frac{22.2+13.5+10.1}{2}\\ &= \frac{45.8}{2}\\ &= 22.9\end{aligned}$$

$$\begin{aligned}\text{Area} &= \sqrt{22.9(22.9-22.2)(22.9-13.5)(22.9-10.1)}\\ &= \sqrt{22.9\times 0.7\times 9.4\times 12.8}\\ &= \sqrt{1928.7296}\\ &= 43.917\ldots\\ &= 43.92\text{ mm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

7 a $$\begin{aligned}\text{Area} &= \frac{1}{2}bh\\ &= \frac{1}{2}\times 19.4\times 11.7\\ &= 113.49\text{ cm}^2\end{aligned}$$

b $$\begin{aligned}s &= \frac{9.1+10.7+12.4}{2}\\ &= \frac{32.2}{2}\\ &= 16.1\end{aligned}$$

$$\begin{aligned}\text{Area} &= \sqrt{16.1(16.1-9.1)(16.1-10.7)(16.1-12.4)}\\ &= \sqrt{16.1\times 7\times 5.4\times 3.7}\\ &= \sqrt{2251.746}\\ &= 47.452\ldots\\ &= 47.45\text{ mm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

c $$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin(A)\\ &= \frac{1}{2}\times 31.2\times 22.5\sin(38°)\\ &= 216.097\ldots\\ &= 216.10\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

d $$\begin{aligned}\text{Missing angle} &= 180° - 65° - 41°\\ &= 74°\end{aligned}$$

$$\begin{aligned}\frac{x}{\sin(65°)} &= \frac{19.9}{\sin(74°)}\\ x &= \frac{19.9\sin(65°)}{\sin(74°)}\\ &= 18.762\ldots\end{aligned}$$

$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin(A)\\ &= \frac{1}{2}\times 19.9\times 18.76\ldots\ \sin(41°)\\ &= 122.476\ldots\\ &= 122.48\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

8 a $$\begin{aligned}s &= \frac{12+15+20}{2}\\ &= \frac{47}{2}\\ &= 23.5\end{aligned}$$

$$\begin{aligned}\text{Area} &= \sqrt{23.5(23.5-12)(23.5-15)(23.5-20)}\\ &= \sqrt{23.5\times 11.5\times 8.5\times 3.5}\\ &= \sqrt{8039.9375}\\ &= 89.665\ldots\\ &= 89.67\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

b $$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin(A)\\ &= \frac{1}{2}\times 10.5\times 11.2\sin(40°)\\ &= 37.795\ldots\\ &= 37.80\text{ mm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

c $$\begin{aligned}\frac{25.6}{\sin(x)} &= \frac{19.8}{\sin(33°)}\\ \sin(x) &= \frac{25.6\sin(33°)}{19.8}\\ x &= \sin^{-1}\left(\frac{25.6\sin(33°)}{19.8}\right)\\ &= 44.763\ldots°\end{aligned}$$

$180° - 33° - 44.763\ldots° = 102.236\ldots°$

$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin(A)\\ &= \frac{1}{2}\times 19.8\times 25.6\sin(102.236\ldots°)\\ &= 247.68\ldots\\ &= 247.68\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

d $$\begin{aligned}P &= 180° - 45.5° - 67.2°\\ &= 67.3°\end{aligned}$$

$$\begin{aligned}\frac{q}{\sin(45.5°)} &= \frac{45.9}{\sin(67.3°)}\\ q &= \frac{45.9\sin(45.5°)}{\sin(67.3°)}\\ &= 35.487\ldots\end{aligned}$$

$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin(A)\\ &= \frac{1}{2}\times 45.9\times 35.487\ldots\sin(67.2°)\\ &= 750.791\ldots\\ &= 750.79\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

9 $$\begin{aligned}s &= \frac{1.9+2.3+2.5}{2}\\ &= \frac{6.7}{2}\\ &= 3.35\end{aligned}$$

$$\begin{aligned}\text{Area} &= \sqrt{3.35(3.35-1.9)(3.35-2.3)(3.35-2.5)}\\ &= \sqrt{3.35\times 1.45\times 1.05\times 0.85}\\ &= \sqrt{4.33531875}\\ &= 2.0821\ldots\\ &= 2.082\text{ km}^2 \text{ (correct to 3 decimal places)}\end{aligned}$$

10 a $\dfrac{14}{\sin(B)} = \dfrac{11}{\sin(31.3°)}$

$$\begin{aligned}\sin(B) &= \frac{14\sin(31.3°)}{11}\\ B &= \sin^{-1}\left(\frac{14\sin(31.3°)}{11}\right)\\ &= 41.391\ldots\\ &= 41.39° \text{ (correct to 2 decimal places)}\\ C &= 180° - 31.3° - 41.39°\\ &= 107.31° \text{ (correct to 2 decimal places)}\end{aligned}$$

b $\dfrac{c}{\sin(107.31°)} = \dfrac{11}{\sin(31.3°)}$

$$\begin{aligned}c &= \frac{14\sin(107.31°)}{\sin(31.3°)}\\ &= 20.214\ldots\\ &= 20.21\text{ cm (correct to 2 decimal places)}\end{aligned}$$

c
$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin(A)\\ &= \frac{1}{2}\times 11\times 14\sin(107.31°)\\ &= 73.512\ldots\\ &= 73.51\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

11 $3x$, $4x$ and $5x$ form a Pythagorean triad, so $3x$ and $4x$ are the base length and height respectively.

$$\begin{aligned}\text{Area} &= \frac{1}{2}bh\\ 121.5 &= \frac{1}{2}\times 3x\times 4x\\ &= 6x^2\\ x^2 &= \frac{121.5}{6}\\ &= 20.25\\ x &= \sqrt{20.25}\\ &= 4.5\text{ cm}\end{aligned}$$

Note: Other methods can be used to reach the same answer, but this method is the easiest and quickest.

12 Using trial and error:

If the 3rd side length = 10 cm:

$$\begin{aligned}s &= \frac{8+8+10}{2}\\ &= \frac{26}{2}\\ &= 13\end{aligned}$$

$$\begin{aligned}\text{Area} &= \sqrt{13(13-10)(13-8)(13-8)}\\ &= \sqrt{13\times 3\times 5\times 5}\\ &= \sqrt{975}\\ &= 31.224\ldots\\ &= 31.22\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

If 3rd side length = 11 cm:

$$\begin{aligned}s &= \frac{8+8+11}{2}\\ &= \frac{27}{2}\\ &= 13.5\end{aligned}$$

$$\begin{aligned}\text{Area} &= \sqrt{13.5(13.5-11)(13.5-8)(13.5-8)}\\ &= \sqrt{13.5\times 2.5\times 5.5\times 5.5}\\ &= \sqrt{1020.9375}\\ &= 31.952\ldots\\ &= 31.95\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

If 3rd side length = 11.2 cm:

$$\begin{aligned}s &= \frac{8+8+11.2}{2}\\ &= \frac{27.2}{2}\\ &= 13.6\end{aligned}$$

$$\begin{aligned}\text{Area} &= \sqrt{13.6(13.6-11.2)(13.6-8)(13.6-8)}\\ &= \sqrt{13.6\times 2.4\times 5.6\times 5.6}\\ &= \sqrt{1023.5904}\\ &= 31.993\ldots\\ &= 31.99\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

If 3rd side length = 11.1 cm:

$$\begin{aligned}s &= \frac{8+8+11.1}{2}\\ &= \frac{27.1}{2}\\ &= 13.55\end{aligned}$$

$$\begin{aligned}\text{Area} &= \sqrt{13.55(13.55-11.1)(13.55-8)(13.55-8)}\\ &= \sqrt{13.55\times 2.45\times 5.55\times 5.55}\\ &= \sqrt{1022.565994}\\ &= 31.977\ldots\\ &= 31.98\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

Therefore, correct to 1 decimal place, the length of the third side is 11.1 cm.

13 If the 40.2° angle is between the two given sides:

$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin(A)\\ &= \frac{1}{2}\times 9.5\times 13.5\sin(40.2°)\\ &= 41.389\ldots\\ &= 41.39\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

If the 40.2° angle is opposite the 9.5 cm side:

$$\begin{aligned}\frac{13.5}{\sin(x)} &= \frac{9.5}{\sin(40.2°)}\\ \sin(x) &= \frac{13.5\sin(40.2°)}{9.5}\\ x &= \sin^{-1}\left(\frac{13.5\sin(40.2°)}{9.5}\right)\\ &= 66.524\ldots°\end{aligned}$$

$180° - 40.2° - 66.524\ldots° = 73.275\ldots°$

$$\begin{aligned}\text{Area} &= \frac{1}{2}bc\sin(A)\\ &= \frac{1}{2}\times 9.5\times 13.5\sin(73.275\ldots°)\\ &= 61.412\ldots\\ &= 61.41\text{ cm}^2 \text{ (correct to 2 decimal places)}\end{aligned}$$

If the 40.2° angle is opposite the 13.5 cm side:

$$\frac{9.5}{\sin(x)} = \frac{13.5}{\sin(40.2°)}$$

$$\sin(x) = \frac{9.5\sin(40.2°)}{13.5}$$

$$x = \sin^{-1}\left(\frac{9.5\sin(40.2°)}{13.5}\right)$$

$$= 27.014\ldots°$$

$180° - 40.2° - 27.014\ldots° = 112.785\ldots°$

$$\text{Area} = \frac{1}{2}bc\sin(A)$$

$$= \frac{1}{2} \times 9.5 \times 13.5\sin(112.785\ldots°)$$

$$= 59.120\ldots$$

$$= 59.12\text{ cm}^2 \text{ (correct to 2 decimal places)}$$

14 For the left-hand section of the track:

Missing angle $= 180° - 93° - 35°$

$= 52°$

$$\frac{x}{\sin(93°)} = \frac{330}{\sin(35°)}$$

$$c = \frac{330\sin(93°)}{\sin(35°)}$$

$$= 574.548\ldots$$

$$\text{Area} = \frac{1}{2}bc\sin(A)$$

$$= \frac{1}{2} \times 330 \times 574.548\ldots\sin(52°)$$

$$= 74\,703.8\ldots$$

$$= 74\,704\text{ m}^2 \text{ (correct to the nearest m}^2\text{)}$$

For the right-hand section of the track:

$$s = \frac{445 + 425 + 550}{2}$$

$$= \frac{1420}{2}$$

$$= 710$$

$$\text{Area} = \sqrt{710(710-445)(710-425)(710-550)}$$

$$= \sqrt{710 \times 265 \times 285 \times 160}$$

$$= \sqrt{8\,579\,640\,000}$$

$$= 92\,626.3\ldots$$

$$= 92\,626\text{ m}^2 \text{ (nearest m}^2\text{)}$$

Total area $= 74\,704 + 92\,626$

$= 167\,330\text{ m}^2$ (correct to the nearest m^2)

15 A diagonal of a parallelogram splits the parallelogram into two equal triangles.

Split the given parallelogram into two triangles by bisecting the 111° (180° − 69°) angles.

$$A_{\text{triangle}} = \frac{1}{2}bc\sin(A)$$

$$= \frac{1}{2} \times 22 \times 10\sin(69°)$$

$$= 102.693\ldots\text{ cm}^2$$

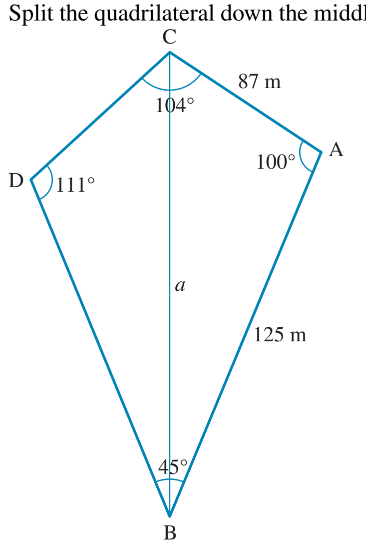
$$A_{\text{parallelogram}} = 2 \times A_{\text{triangle}}$$

$$= 2 \times 102.693\ldots$$

$$= 205.387\ldots$$

$$= 205.39\text{ cm}^2$$

16 Split the quadrilateral down the middle and label this length a.



$$a^2 = 87^2 + 125^2 - 2 \times 87 \times 125\cos(100°)$$

$$= 23\,194 - (-3776.847\ldots)$$

$$= 26\,970.847\ldots$$

$$a = \sqrt{26\,970.847\ldots}$$

$$= 164.228\ldots$$

$$\frac{87}{\sin(B)} = \frac{164.228\ldots}{\sin(100°)}$$

$$\sin(B) = \frac{87\sin(100°)}{164.228\ldots}$$

$$B = \sin^{-1}\left(\frac{87\sin(100°)}{164.228\ldots}\right)$$

$$= 31.446\ldots°$$

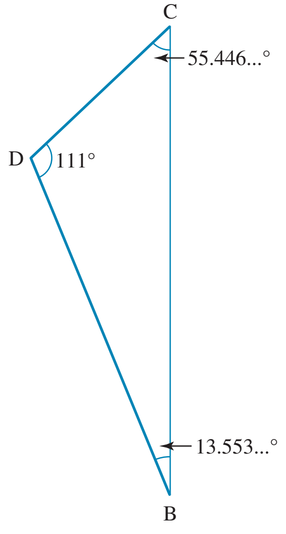
For ΔBCD:

∠DBC $= 45° - 31.446\ldots°$

$= 13.553\ldots°$

∠DCB $= 180° - 111° - 13.553\ldots°$

$= 55.446\ldots°$



$$\frac{x}{\sin(55.446\ldots°)} = \frac{164.228\ldots}{\sin(111°)}$$

$$x = \frac{164.228\ldots \sin(55.446\ldots°)}{\sin(111°)}$$

$$= 144.880\ldots$$

$$\text{Area of } \Delta\text{BCD} = \frac{1}{2} \times 144.880\ldots \times 164.228\ldots \times \sin(13.553\ldots°)$$

$$= 2788.236\ldots$$

$$= 2788.24 \text{ m}^2 \text{ (correct to 2 decimal places)}$$

$$\text{Area of } \Delta\text{ABC} = \frac{1}{2} \times 87 \times 125 \times \sin(100°)$$

$$= 5354.892\ldots$$

$$= 5354.89 \text{ m}^2 \text{ (correct to 2 decimal places)}$$

$$\text{Total area} = 2788.24 + 5354.89$$

$$= 8143.13 \text{ m}^2 \text{ (correct to 2 decimal places)}$$

$$\text{Volume} = A \times H$$

$$= 8143.13 \times 0.001$$

$$= 8.14313$$

$$= 8.14 \text{ m}^3 \text{ (correct to 2 decimal places)}$$

12.6 Exam questions

1 Using Heron's formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

$$= \frac{1}{2}(20+24+26)$$

$$= 35$$

$$\text{Area} = \sqrt{35(35-20)(35-24)(35-26)}$$

$$= \sqrt{35 \times 15 \times 11 \times 9}$$

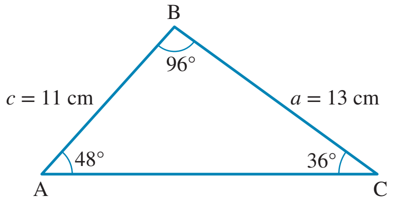
$$= \sqrt{51975}$$

$$= 227.98$$

$$\approx 228 \text{ cm}^2$$

The correct answer is **D**.

2



$$B = 180° - (48° + 36°)$$

$$= 96°$$

$$\text{Area} = \frac{1}{2}ac\sin(B)$$

$$= \frac{1}{2} \times 13 \times 11 \times \sin(96°)$$

$$= 71.11 \text{ cm}^2$$

The correct answer is **A**.

3 $\text{Area} = \frac{1}{2}bc\sin(A)$

$$= \frac{1}{2} \times 8 \times 5.22\sin(100°)$$

$$= 20.88\sin(100°)$$

The correct answer is **C**.

12.7 Review

12.7 Exercise

Multiple choice

1 x is the adjacent side to the angle; therefore, cosine is used.

$$\cos(25°) = \frac{x}{37}$$

$$x = 37\cos(25°)$$

The correct answer is **A**.

2 Use the sine rule because triangle has no right angle and two angles and one opposite side length are given.

Find the value of angle opposite side length x:

$180 - (35 + 42) = 103°$

Sine rule:

$$\frac{x}{\sin(103°)} = \frac{16}{\sin(35°)}$$

$$x = \frac{16 \times \sin(103°)}{\sin(35°)}$$

$$x = 27.180$$

Closest to 27.

The correct answer is **E**.

3 Use sine rule:

$$\frac{15}{\sin(A)} = \frac{10}{\sin(34°)}$$

$$\sin(A) = \frac{\sin(34°) \times 15}{10}$$

$$\sin(A) = 0.8388$$

$$A = \sin^{-1}(0.8388)$$

$$A = 57.012°$$

Nearest degree = 57°

The correct answer is **E**.

4 Largest value is opposite longest side length, 8.

Using cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$8^2 = 3^2 + 5^2 - 2 \times 3 \times 5\cos(A)$$

$$\cos(A) = \frac{8^2 - 3^2 - 5^2}{-2 \times 3 \times 5}$$

$$A = \cos^{-1}\left(\frac{8^2 - 3^2 - 5^2}{-2 \times 3 \times 5}\right)$$

Multiplying numerator and denominator by −1:

$$A = \cos^{-1}\left(\frac{5^2 + 3^2 - 8^2}{2 \times 3 \times 5}\right)$$

The correct answer is **C**.

5 Acute angle $0 < x < 90°$: $\sin^{-1}(0.52992) = 32°$

Obtuse angle $90° < x < 180°$: $(180 - 32) = 148°$

The correct answer is **D**.

6 $s = \frac{a+b+c}{2}$

$$= \frac{4.2 + 5.1 + 9}{2}$$

$$= 9.15$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{9.15(9.15-4.2)(9.15-5.1)(9.15-9)}$$

$$= \sqrt{27.52}$$

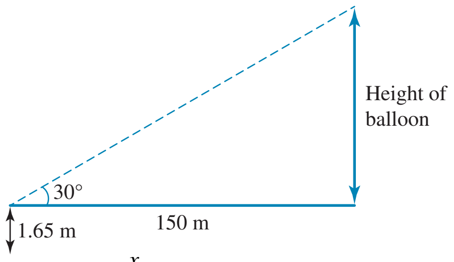
$$= 5.25$$

$$\text{Area} = 5.3 \text{ cm}^2$$

The correct answer is **A**.

7 Since both Jo's and Nelson's houses are located relative to Sammy's house, Sammy's house is the reference point, *N*.
Jo's home is 060°T.
Nelson's home is 090°T.
The correct answer is **B**.

8



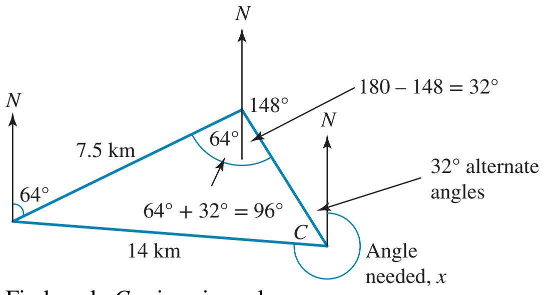
$$\tan(30°) = \frac{x}{150}$$
$$x = 150 \times \tan(30°)$$
$$x = 86.6025$$

Height balloon from ground: $86.6025 + 1.65 = 88.2525$ m
Height balloon is above building: $88.2525 - 20 = 68.2525$ m
$= 68$ m

The correct answer is **E**.

9 $A = \frac{1}{2}bc\sin A$
$= \frac{1}{2} \times 3.2 \times 2.1 \times \sin(41°)$
$= 2.20$
The correct answer is **A**.

10



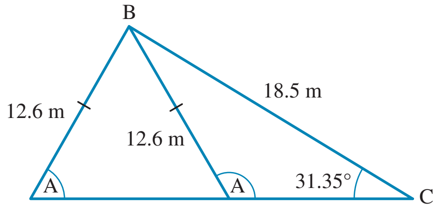
Find angle *C* using sine rule:
$$\frac{14}{\sin(96)} = \frac{7.5}{\sin(C)}$$
$$\sin(C) = \frac{\sin(96°) \times 7.5}{14}$$
$$\sin(A) = 0.53278$$
$$A = \sin^{-1}(0.532\,78)$$
$$A = 32.193°$$
Angle needed, $x = (360 - 32.193 - 32)°$
$x = 295.807°$
True bearing 296°T
The correct answer is **D**.

Short answer

11 a Use cosine rule:
$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$x^2 = 5^2 + 8^2 - 2 \times 5 \times 8\cos(112°)$$
$$x = \sqrt{118.9685}$$
$$x = 10.91$$
$$x = 10.91 \text{ cm}$$

b
$$\frac{40}{\sin(x)} = \frac{57}{\sin(99°)}$$
$$\sin(x) = \frac{\sin(99°) \times 40}{57}$$
$$\sin(x) = 0.6931$$
$$x = \sin^{-1}(0.6931)$$
$$x = 43.88°$$
Nearest degree, $x = 44°$

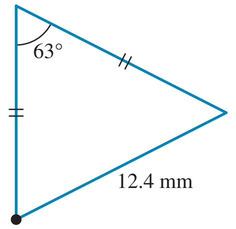
12 a



b Find acute *A* value, using sine rule:
$$\frac{18.5}{\sin(A)} = \frac{12.6}{\sin(31.35°)}$$
$$\sin(A) = \frac{\sin(31.35°) \times 18.5}{12.6}$$
$$\sin(A) = 0.7639$$
$$A = \sin^{-1}(0.7639)$$
$$A = 49.81°$$
$B = 180 - (49.81 + 31.35)$
$= 98.84°$
Obtuse angle *A* : $180 - 49.81° = 130.19°$
$B = 180 - (130.19 + 31.35)$
$= 18.46°$

13 a $A = \frac{1}{2}bc\sin A$
$= \frac{1}{2} \times 4.8 \times 5.2 \times \sin(103°)$
$= 12.16$
$A = 12.16\text{ cm}^2$

b Calculate the area of the triangle shown correct to 2 decimal places.



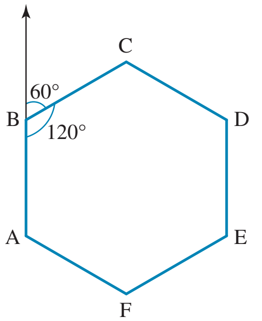
Isosceles triangle, therefore unknown angles are equal:
$$\text{Angle} = \frac{180 - 63}{2}$$
$= 58.5$
Use sine rule to find unknown side length:
$$\frac{x}{\sin(58.5°)} = \frac{12.4}{\sin(63°)}$$
$$x = \frac{\sin(58.5°) \times 12.4}{\sin(63°)}$$
$$x = 11.87$$

$$\text{Area} = \frac{1}{2}bc\sin A$$
$$= \frac{1}{2} \times 11.87 \times 11.87 \times \sin(63°)$$
$$= 62.73$$
$$\text{Area} = 62.73\text{ mm}^2$$

14 a Interior angles for a regular hexagon:

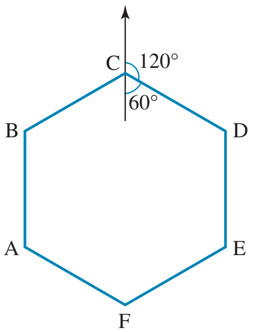
$$\frac{(n-2)\times 180°}{n} = \frac{4 \times 180°}{6}$$
$$= 120°$$

Straight line = 180°, therefore exterior angle at B = 60°



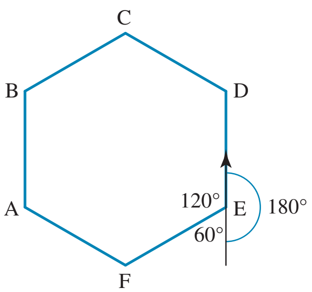
Bearing of C from B: 060°T

b



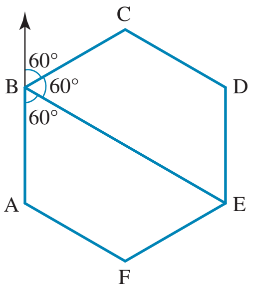
Bearing of D from C: 120°T

c



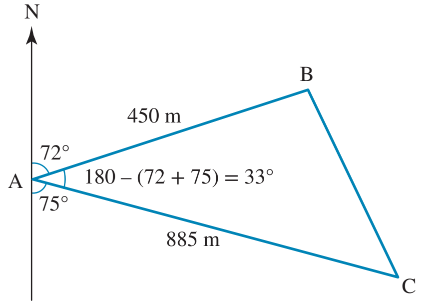
Bearing of F from E: 180 + 60 = 240°T

d



Bearing of E from B: 60 + 60 = 120°T

15 a



Angle A: $180 - (75 + 72) = 33°$

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$BC^2 = 450^2 + 885^2 - 2 \times 450 \times 885\cos(33°)$$
$$x = \sqrt{317\,723.8926}$$
$$x = 563.67\text{ m}$$

b
$$\text{Area} = \frac{1}{2}bc\sin A$$
$$= \frac{1}{2} \times 450 \times 885 \times \sin(33°)$$
$$= 108\,451$$
$$\text{Area} = 108\,451\text{ m}^2$$

16 a Consider smaller triangle with angle 30°:

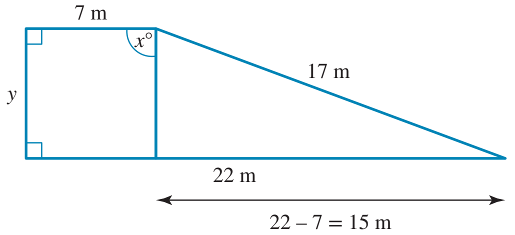
Let vertical height = h

$$\tan(30°) = \frac{h}{30}$$
$$h = 17.3205$$

Consider the larger triangle, angle: 30 + 19 = 49°

$$\tan(49°) = \frac{x+h}{30}$$
$$x + h = 30 \times \tan(49°)$$
$$x + h = 34.511\,05$$
$$x = 34.511\,05 - 17.3205$$
$$x = 17.19\text{ cm}$$

b



Using triangle, find the unknown angle:

$$\sin(b) = \frac{15}{17}$$
$$b = \sin^{-1}\left(\frac{15}{17}\right)$$
$$b = 61.9275°$$
$$b = 61.93°$$

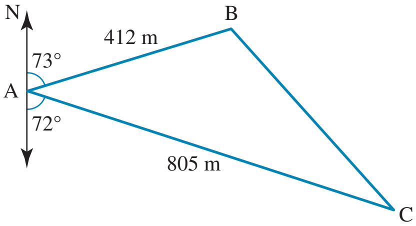
$$x = 90 + b$$
$$= 90 + 61.93$$
$$= 151.93°$$

Using Pythagoras' theorem:

$$y^2 = 17^2 - 15^2$$
$$y^2 = 64$$
$$y = 8\text{ m}$$

Extended response

17 a



b $A = 180 - (73 + 72)$

$= 35°$

c $a^2 = b^2 + c^2 - 2bc\cos(A)$

$BC^2 = 412^2 + 805^2 - 2 \times 412 \times 805\cos(35°)$

$x = \sqrt{274\,409.066}$

$x = 523.84$

Correct to 1 decimal place, 523.8 m

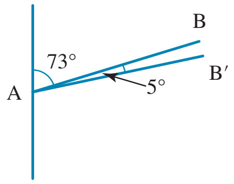
d Area $= \frac{1}{2}bc\sin A$

$= \frac{1}{2} \times 412 \times 805 \times \sin(35°)$

$= 95116.18$

Area $= 95\,116.2\text{ m}^2$

e i



ii $a^2 = b^2 + c^2 - 2bc\cos(A)$

$BB'^2 = 412^2 + 412^2 - 2 \times 412 \times 412\cos(5°)$

$BB' = \sqrt{1291.854}$

$BB' = 35.942$

Distance between the actual treasure at B and the incorrect location

B′ is 35.9 m.

iii Using sine rule to find angle at B′:

$\frac{35.94}{\sin(5°)} = \frac{412}{\sin(B')}$

$\sin(B') = \frac{\sin(5°) \times 412}{35.94}$

$\sin(B') = 0.99911$

$B' = \sin^{-1}(0.999111)$

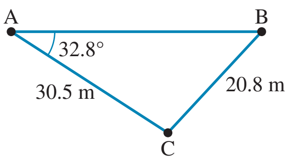
$B' = 87.588°$

Bearing: $360 - (87.588 + 78 + 180) = 14.412°$

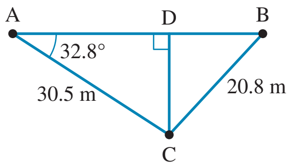
Bearing is $360 - 14.412 = 345.588$

345.6°T

18 a



b i



$\sin(32.8°) = \frac{CD}{30.5}$

$CD = 30.5 \times \sin(32.8°)$

$CD = 16.522$

$CD = 16.5$

ii Use sine rule to find angle B:

$\frac{30.5}{\sin(B°)} = \frac{20.8}{\sin(32.8°)}$

$\sin(B°) = \frac{\sin(32.8°) \times 30.5}{20.8}$

$\sin(B°) = 0.7943$

$B = \sin^{-1}(0.7943)$

$B = 52.592°$

Angle C: $180 - (52.592 + 32.8) = 94.608°$

Use cosine rule to find length AB:

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$AB^2 = 30.5^2 + 20.8^2 - 2 \times 30.5 \times 20.8\cos(94.608°)$

$AB = \sqrt{1464.823}$

$AB = 38.27$

$AB = 38.3$ m correct to 1 decimal place

c $s = \frac{a+b+c}{2}$

$= \frac{30.5 + 20.8 + 38.3}{2}$

$= 44.787$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{44.8(44.8 - 30.5)(44.8 - 20.8)(44.8 - 38.3)}$

$= \sqrt{99\,939.84}$

$= 316.13$

Correct to 1 decimal place, 316.1 m^2

19 a Angle at O = 360°

Since in a regular pentagon all angles are equal,

$\frac{360}{5} = 72°.$

b ΔAOB is isosceles, therefore $\angle ABO = \frac{180 - 72}{2}$

$= 54°$

c $\frac{OB}{\sin(54°)} = \frac{30}{\sin(72°)}$

$OB = \frac{\sin(54°) \times 30}{\sin(72°)}$

$OB = 25.51952$

OB = 25.5 cm

d $25.5 \times 5 + 5 \times 30 = 277.5$ cm

e $A = \frac{1}{2} \times 25.5 \times 25.5\sin(72°)$

$= 309.21$

Total area of coloured glass $= 3 \times 309.21$

Area $= 927.64$

Nearest cm^2 = 928 cm^2

20 a Using similar triangles:

$\frac{h}{1} = \frac{1.5 + h}{1.5}$

$1.5h = 1.5 + h$

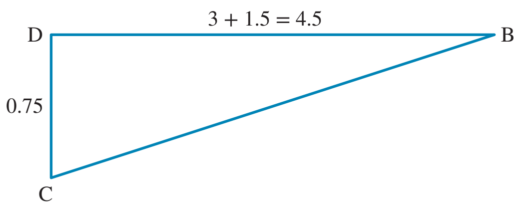
$0.5h = 1.5$

$h = \frac{1.5}{0.5}$

$h = 3$

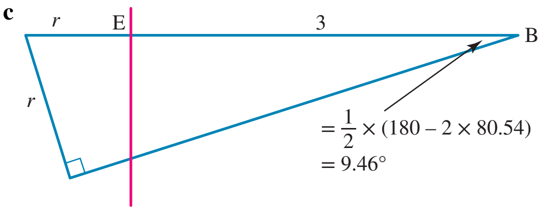
Therefore $h = 3$ m

b



$$\tan(C) = \frac{4.5}{0.75}$$
$$C = \tan^{-1}(6)$$
$$C = 80.537°$$
$$C = 80.54°$$

c



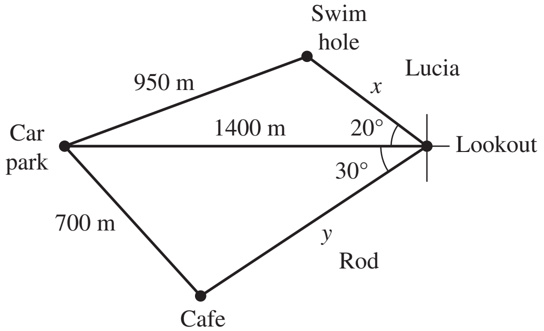
$$\sin(9.46) = \frac{r}{3+r}$$
$$0.16436 = \frac{r}{3+r}$$
$$0.16436(3+r) = r$$
$$0.4938 + 0.16436r = r$$
$$0.4938 = 0.83504r$$
$$r = \frac{0.4938}{0.83504}$$
$$r = 0.59$$
Radius = 0.59 m

d Area $= \pi r^2$
$= \pi(0.59)^2$
$= 1.0935$
Area of circle $= 1.09\text{ m}^2$

e Area $= \frac{1}{2}bh$
$= \frac{1}{2} \times 1.5 \times 4.5$
$= 3.38$
Area of flag $= 3.38\text{ m}^2$

12.7 Exam questions

1 The trick to this question is to be able to draw the diagram as close to scale as possible:



To find how far Lucia walked:
$950^2 = x^2 + 1400^2 - 2 \times x \times 1400 \times \cos 20°$
$x \simeq 495\text{ m}$
Therefore, Lucia walked 1400 + 495 + 950 = 2845 m
To find how far Rod walked:
$700^2 = y^2 1400^2 - 2 \times y \times 1400 \times \cos 30°$
$y = 1212\text{ m}$
Therefore, Rod walked 1400 + 1212 + 700 = 2845 m
So, Rod and Lucia walked the same distance.
The correct answer is **A**.

2 **a** distance $PR = \sqrt{80^2 + 100^2 - 2(80)(100)\cos 104°}$ [1 mark]
$= 142.375$
$= 142\text{ m}$ [1 mark]

b $\frac{\sin(\angle RPQ)}{100} = \frac{\sin 104°}{142}$
$\angle RPQ = \sin^{-1}\left(\frac{100 \times \sin 104°}{142}\right)$
$= 43.1025955$
$= 43°$
$\therefore$ bearing of R from P $= 130° - 43° = 087°$ [1 mark]

***VCAA Assessment Report* note:**
Some students found the angle *PRQ*, which did not readily help to give the required bearing.

3 **a** $\sin 30° = \frac{\text{distance}}{50}$
distance $= 50 \times \sin 30°$
$= 25\text{ m}$ [1 mark]

b $\tan\theta = \frac{16.8}{200}$
$\theta = \tan^{-1}\frac{16.8}{200}$
$= 4.80157°$
$= 5°$ [1 mark]

4 $s = \frac{1}{2}(2250 + 1900 + 2050)$
$= 3100$
$A = \sqrt{3100(3100-1900)(3100-2050)(3100-2250)}$
$= \sqrt{3100 \times 1200 \times 1050 \times 850}$
The correct answer is **A**.

5 **a.** Angles in a triangle add to 180°.
$\theta = 180 - (45 + 60)$
$= 75°$
[1 mark for first line, not answer]

***VCAA Assessment Report* note:**
A suitable calculation that resulted in 75 was required.

b $\frac{d}{\sin D} = \frac{a}{\sin A}$
$\frac{AX}{\sin(45°)} = \frac{3.16}{\sin(75°)}$ [1 mark]

c $\frac{AX}{\sin(45°)} = \frac{3.16}{\sin(75°)}$
$AX = \frac{3.16}{\sin(75°)} \times \sin(45°)$
$= 2.31\text{ m}$ [1 mark]

d $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 3.16 \times 2$

$= 3.2\,\text{m}^2$ [1 mark]

e $CX = 5 - 3.16$

$= 1.84\,\text{m}$

$A = Atop + AsideAB + AsideCB + AsideCX$ [1 mark]

$= \frac{1}{2}(3 + 1.84) \times 2 + 3 \times 1.8 + 2 \times 1.8 + 1.84 \times 1.8$

$= 17.15\,\text{m}^2$

$= 17\,\text{m}^2$ [1 mark]

VCAA Assessment Report **note:**

Many students misread the question and covered the nesting and eating spaces. Some others who covered only the eating space as required, then incorrectly included the wall *AX*.

A method mark was available in this two-mark question if the final answer was incorrect. Many calculations were set out poorly and a method mark could not be allocated. Students are encouraged to clearly label each step in an extended calculation and to draw supporting diagrams where applicable.

Topic 13 — Similarity and scale

13.2 Similar objects

13.2 Exercise

1 a The ratio of the corresponding sides is the same:
$\frac{15}{8} = \frac{11.25}{6} = \frac{7.5}{4} = 1.875$, and all angles are equal, so they are similar.

b The ratio of the corresponding sides is the same:
$\frac{41.04}{11.4} = \frac{26.1}{7.25} = \frac{9}{2.5} = \frac{13.68}{3.8} = 3.6$, and all angles are equal, so they are similar.

2 The ratio of the corresponding sides is the same:
$\frac{106.43}{18.35} = \frac{24.65}{4.25} = 5.8$, and all angles are equal, so they are similar.

3 a All side lengths are not in proportion: $\frac{4.8}{3.2} = 1.5$,
$\frac{2.24}{1.6} = 1.4$, therefore, they are not similar.

b All side lengths are in proportion: $\frac{10.224}{2.84} = 3.6$,
$\frac{2.34}{0.65} = 3.6$, therefore, they are similar.

c All side lengths are in proportion: $\frac{4.4352}{2.112} = 2.1$,
$\frac{2.184}{1.04} = 2.1$, therefore, they are similar.

d All side lengths are not in proportion: $\frac{12.535}{11.5} = 1.09$,
$\frac{10.152}{9.4} = 1.08$, therefore, they are not similar.

4 a All side lengths are in proportion: $\frac{35}{25} = 1.4$, $\frac{22.4}{16} = 1.4$,
$\frac{12.6}{9} = 1.4$, and all angles are equal, therefore, they are similar.

b All side lengths are in proportion: $\frac{4.94}{2.6} = 1.9$, $\frac{6.65}{3.5} = 1.9$,
$\frac{2.09}{1.1} = 1.9$, and all angles are equal, therefore, they are similar.

c All side lengths are in proportion:
$\frac{13}{8} = 1.625$, $\frac{6.5}{4} = 1.625$, $\frac{3.25}{2} = 1.625$, and all angles are equal, therefore, they are similar.

d All side lengths are in proportion:
$\frac{3.24}{2.7} = 1.2$, $\frac{19.44}{16.2} = 1.2$, $\frac{2.16}{1.8} = 1.2$, and all angles are equal, therefore, they are similar.

5 a Two sides are in proportion $\frac{44.275}{7.7} = \frac{78.2}{13.6} = 5.75$, and the included angles are equal, so they are similar (SAS).

b All sides are in proportion $\frac{38.72}{17.6} = \frac{16.28}{7.4} = \frac{14.85}{6.75} = 2.2$, so they are similar (SSS).

6 a The ratio of the corresponding side lengths is not equal:
$\frac{32.4}{18} = \frac{21.6}{12} = 1.8$, $\frac{23.8}{14} = 1.7$, therefore, they are not similar.

b Two side lengths are in proportion: $\frac{54.32}{19.4} = \frac{22.96}{8.2} = 2.8$, and all angles are equal, therefore, they are similar.

7 a A and C are similar as the ratio of their corresponding side lengths is equal: $\frac{A}{C} = \frac{5.44}{3.4} = \frac{2.56}{1.6} = \frac{4.64}{2.9} = 1.6$.

b B and C are similar as the ratio of their corresponding side lengths is equal: $\frac{B}{C} = \frac{7}{5} = \frac{6.3}{4.5} = \frac{4.2}{3} = 1.4$.

c A and C are similar as the ratio of two corresponding side lengths is equal: $\frac{A}{C} = \frac{12.6}{14} = \frac{8.1}{9} = 0.9$ and the included angle is the same: 82°.

d The unmarked side length in B is: $\sqrt{21^2 - 12.6^2} = 16.8$.
Then A and B are similar as the ratio of their corresponding side lengths is equal: $\frac{B}{A} = \frac{21}{5} = \frac{16.8}{4} = \frac{12.6}{3} = 4.2$.

8 a The objects are similar because all angles are equal and side lengths are in proportion.

b The objects are similar because all measurements (radius and circumference) are in proportion.

c The objects are similar because all angles are equal and side lengths are in proportion.

9 a The ratio of the corresponding sides is:
$\frac{5.44}{3.4} = \frac{22.16}{13.85} = 1.6$. Therefore, the ratio is 1.6 : 1.

b The ratio of the corresponding sides is:
$\frac{79.1}{35} = 2.26$. Therefore, the ratio is 2.26 : 1.

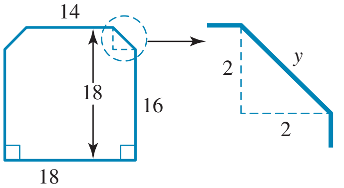
c The ratio of the corresponding sides is: $\frac{18}{5} = 3.6$.
Therefore, the ratio is 3.6 : 1.

10 a $\sqrt{26^2 - 24^2} = 10$
The ratio of the corresponding sides is: $\frac{10}{7.5} = \frac{4}{3}$.
Therefore, the ratio is 4 : 3.

b The ratio of the corresponding sides is: $\frac{69.3}{12.6} = \frac{11}{2} = 5.5$.
Therefore, the ratio is 5.5 : 1.

11 a Using Pythagoras' theorem the corresponding side length in the larger object is: $\sqrt{52^2 - 20^2} = 48$. As the objects are similar: $\frac{x}{1.2} = \frac{48}{12} \rightarrow x = \frac{48}{12} \times 1.2 = 4.8$.

b Since the objects are similar, the ratio of corresponding side lengths must be equal: $\frac{\text{small}}{\text{large}} = \frac{18}{45} = 0.4$.
The unmarked side length on the smaller object is:
$40 \times 0.4 = 16$.



The angled length y would then be given by:
$y = \sqrt{2^2 + 2^2} = 2.828$.
The length of x would then be given by:
$\frac{2.828}{x} = 0.4 \rightarrow x = \frac{2.828}{0.4} = 7.07$

12 a The ratio of the corresponding side lengths is given by:
$\frac{50.84}{8.2} = 6.2$, therefore, the squares are similar as all sides are in proportion and all angles equal.

b The ratio of the corresponding side lengths is given by:
$\frac{14.34}{12.6} = 1.138$, therefore, the triangles are similar as all sides are in proportion and all angles are equal.

13 **a** For similarity: $\frac{x}{10} = \frac{52}{16.25} \rightarrow x = \frac{52}{16.25} \times 10 = 32$

b For similarity: $\frac{x}{30} = \frac{28}{42} \rightarrow x = \frac{28}{42} \times 30 = 20$

14 As the objects are similar, the ratio of corresponding side lengths is: $\frac{13.2}{8} = 1.65$. Therefore, the radius of the larger object must be: $1.65 \times 10 = 16.5$

This gives: $x = \frac{1}{2} \times 2\pi r = \frac{1}{2} \times 2\pi(16.5) = 51.8$

(correct to 1 decimal place)

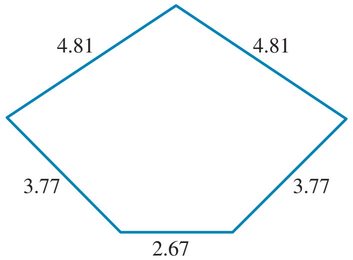
15 $\frac{x}{4.2} = \frac{(3.6 + 1.2)}{3.6} \rightarrow x = \frac{4.8}{3.6} \times 4.2 = 5.6$

$\frac{y}{4.2} = \frac{(3.6 + 1.2 + 1.2)}{3.6} \rightarrow y = \frac{6}{3.6} \times 4.2 = 7$

16 **a** $\frac{3.61}{x} = \frac{3}{4} \rightarrow x = \frac{3.61 \times 4}{3} = 4.81$

$\frac{2.83}{y} = \frac{3}{4} \rightarrow y = \frac{2.83 \times 4}{3} = 3.77$

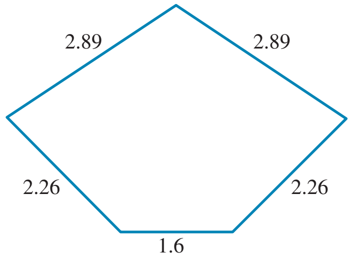
$\frac{2}{z} = \frac{3}{4} \rightarrow z = \frac{2 \times 4}{3} = 2.67$



b $\frac{3.61}{x} = \frac{5}{4} \rightarrow x = \frac{3.61 \times 4}{5} = 2.89$

$\frac{2.83}{y} = \frac{5}{4} \rightarrow y = \frac{2.83 \times 4}{5} = 2.26$

$\frac{2}{z} = \frac{5}{4} \rightarrow z = \frac{2 \times 4}{5} = 1.6$



13.2 Exam question

1 Similar triangles have corresponding side lengths in the same ratio.

Triangle *P* is produced by multiplying the side lengths of triangle *M* by 2.

Triangle *Q* is produced by multiplying the side lengths of triangle *M* by 3.

The correct answer is **E**.

2 $\frac{DF}{3.6} = \frac{1.8}{2.4}$

$DF = \frac{1.8 \times 3.6}{2.4}$

$= 2.7\,\text{cm}$

The correct answer is **D**.

3 Triangles *RPQ* and *RMN* are similar (using the AAA test).

Therefore, corresponding side lengths are in the same ratio.

$\frac{RQ}{RN} = \frac{PQ}{MN}$

$\frac{4}{4+6} = \frac{5}{MN}$

$\frac{4}{10} = \frac{5}{MN}$

$MN = 12.5$

The correct answer is **D**.

13.3 Linear scale factors

13.3 Exercise

1 **a** $\frac{22}{10} = \frac{13.86}{6.3} = \frac{9.68}{4.4} = 2.2$

b $\frac{34.374}{10.11} = \frac{24.514}{7.21} = \frac{23.12}{6.8} = 3.4$

c $\frac{8.4}{6.72} = \frac{5.375}{4.3} = \frac{4.55}{3.64} = 1.25$

d $\frac{4}{2.5} = \frac{(4.2 + 2.52)}{4.2} = \frac{6.72}{4.2} = 1.6$

2 **a** $\frac{9.36}{1.8} = \frac{10.4}{2} = \frac{15.6}{3} = 5.2$

b $\frac{8.64}{3.2} = \frac{18.9}{7} = \frac{7.56}{2.8} = \frac{5.4}{2} = 2.7$

c $\frac{16.72}{4.4} = \frac{13.68}{3.6} = \frac{11.4}{3} = \frac{7.6}{2} = \frac{5.32}{1.4} = 3.8$

d $\frac{16.17}{5.39} = \frac{21.36}{7.12} = \frac{10.83}{3.61} = \frac{8.52}{2.84} = 3$

3 **a** $3:2 \rightarrow \frac{3}{2} = 1.5$

b $12:5 \rightarrow \frac{12}{5} = 2.4$

c $3:4 \rightarrow \frac{3}{4} = 0.75$

d $85:68 \rightarrow \frac{85}{68} = 1.25$

4 **a** $\frac{3}{x} = 6 \rightarrow x = \frac{3}{6} = 0.5$

$\frac{y}{12} = 6 \rightarrow y = 6 \times 12 = 72$

$\rightarrow \frac{3}{\boxed{0.5}} = \frac{\boxed{72}}{12} = 6$

b $\frac{44}{11} = 4$

$\therefore \frac{5}{x} = 4 \rightarrow x = \frac{5}{4} = 1.25$

$\rightarrow \frac{5}{\boxed{1.25}} = \frac{44}{11} = \boxed{4}$

c $\frac{81}{9} = 9$

$\therefore \frac{x}{7} = 9 \rightarrow x = 9 \times 7 = 63$

$\rightarrow \frac{\boxed{63}}{7} = \frac{81}{9} = \boxed{9}$

d $\frac{2}{x} = 0.625 \rightarrow x = \frac{2}{0.625} = 3.2$

$\frac{y}{2} = 0.625 \rightarrow y = 0.625 \times 12 = 1.25$

$\rightarrow \frac{2}{\boxed{3.2}} = \frac{\boxed{1.25}}{2} = 0.625$

5 **a** Scale factor is: $\frac{12.6}{3} = \frac{16.8}{4} = \frac{8.4}{2} = 4.2$

$\therefore \frac{4.2}{x} = 4.2 \rightarrow x = 1$

$\frac{y}{2.8} = 4.2 \rightarrow y = 4.2 \times 2.8 = 11.76$

b Scale factor is: $\frac{12.24}{5.1} = 2.4$

$\therefore \frac{3.384}{x} = 2.4 \rightarrow x = \frac{3.384}{2.4} = 1.41$

c Scale factor is: $\frac{31}{5} = 6.2$

$\therefore \frac{17.36}{x} = 6.2 \rightarrow x = \frac{17.36}{6.2} = 2.8$

d Scale factor is: $\frac{3.1}{2.48} = \frac{1.4}{1.12} = \frac{2.8}{2.24} = 1.25$

$\therefore \frac{x}{1.6} = 1.25 \rightarrow x = 1.25 \times 1.6 = 2$

$\frac{2}{y} = 1.25 \rightarrow y = \frac{2}{1.25} = 1.6$

$\frac{4.47}{z} = 1.25 \rightarrow z = \frac{4.47}{1.25} = 3.576$

6 **a** Scale factor is: $\frac{(4.48 + 2.8)}{2.8} = 2.6$

$\therefore \frac{x + 2.2}{2.2} = 2.6 \rightarrow x = (2.6 \times 2.2) - 2.2 = 3.52$

$\frac{7.8}{y} = 2.6 \rightarrow y = \frac{7.8}{2.6} = 3$

b Scale factor is: $\frac{5}{2.25} = \frac{20}{9}$

$\frac{7.8}{(7.8 - y)} = \frac{20}{9}$

$\rightarrow 20\,(7.8 - y) = 9 \times 7.8$

$\rightarrow y = 7.8 - \frac{9 \times 7.8}{20} = 4.29$

7 $\frac{18}{BC} = \frac{36}{32} \rightarrow BC = \frac{18 \times 32}{36} = 16$

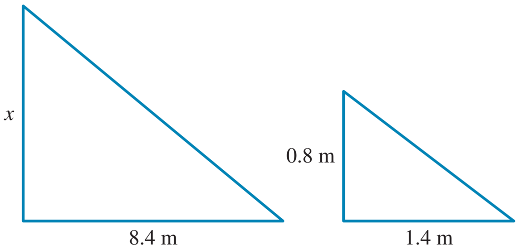
8 $\frac{10.4}{4} = \frac{x + 4}{x} \rightarrow 10.4x = 4\,(x + 4)$

$\rightarrow 10.4x = 4x + 16$

$\rightarrow 6.4x = 16$

$\rightarrow x = \frac{16}{6.4} = 2.5$

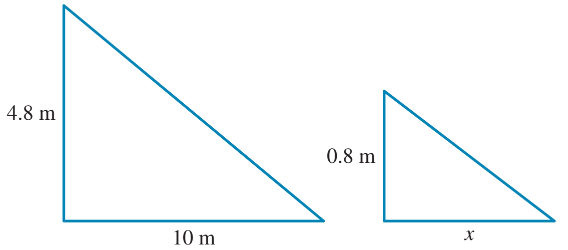
9 **a**



$\frac{8.4}{1.4} = \frac{x}{0.8}$

$\rightarrow x = \frac{8.4}{1.4} \times 0.8 = 4.8$ m

b



$\frac{10}{x} = \frac{4.8}{0.8}$

$\rightarrow x = \frac{10 \times 0.8}{4.8} = 1.67$ m

10 $\frac{16}{10} = \frac{x + (x - 2)}{x}$

$= \frac{2x - 2}{x}$

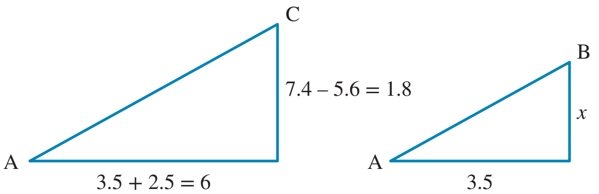
$\rightarrow 16x = 10\,(2x - 2)$

$\rightarrow 16x = 20x - 20$

$\rightarrow 4x = 20$

$\rightarrow x = 5$

11



$\frac{6}{3.5} = \frac{1.8}{x}$

$\rightarrow 6x = 1.8 \times 3.5$

$\rightarrow x = \frac{1.8 \times 3.5}{6}$

$\rightarrow x = 1.05$

$\therefore$Height of B $= 5.6 + 1.05 = 6.65$ m

12 $\frac{18}{4} = \frac{12 + x}{8}$

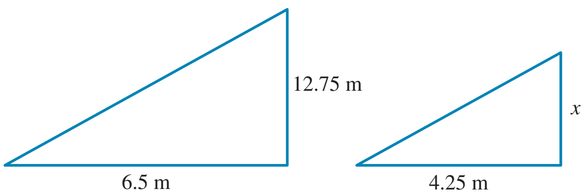
$\rightarrow 18 \times 8 = 4\,(12 + x)$

$\rightarrow 18 \times 8 = 48 + 4x$

$\rightarrow x = \frac{(18 \times 8) - 48}{4} = 24$

$\therefore$ Distance across ravine $= 24$ m

13



$\frac{12.75}{x} = \frac{6.5}{4.25}$

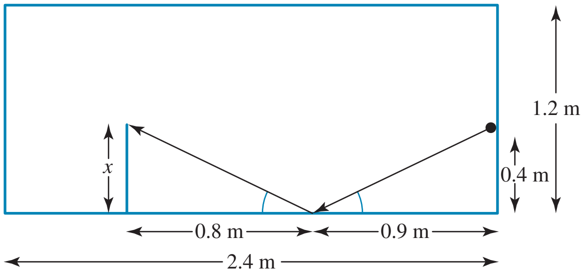
$\rightarrow 6.5x = 12.75 \times 4.25$

$\rightarrow x = \frac{12.75 \times 4.25}{6.5}$

$\rightarrow x = 8.34$

$\therefore$Vertical distance $= 8.34$ m

14 a

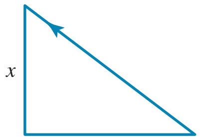


$\frac{x}{0.4} = \frac{0.8}{0.9}$

$\rightarrow x = \frac{0.8}{0.9} \times 0.4$

$\rightarrow x = 0.36$ m

b



$\frac{x}{0.4} = \frac{1.5}{0.9}$

$x = \frac{1.5}{0.9} \times 0.4$

$x = 0.67$ m (2 decimal places)

0.67 m $\left(\frac{2}{3}\text{ m}\right)$ from the bottom of the left side

13.3 Exam questions

1 When enlarged by a factor of 2, all side lengths are doubled. Remember that the side lengths must correspond.
The correct answer is **A**.

2 $x = 36$ cm, $y = 40$ cm
The correct answer is **C**.

3 The statement in option C is incorrect.
The angles in a triangle enlarged by a factor of 2 will be the same size as in the original. All other comments are correct.
The correct answer is **C**.

13.4 Area and volume scale factors

13.4 Exercise

1 a Linear scale factor: $\frac{8.75}{5} = \frac{5.25}{3} = \frac{7}{4}$

Area scale factor: $\left(\frac{7^2}{4^2}\right) = \frac{49}{16} = 3.0625$

b Linear scale factor: $\frac{12.32}{5.5} = \frac{9.408}{4.2} = \frac{5.6}{2.5} = \frac{56}{25}$

Area scale factor: $\left(\frac{56^2}{25^2}\right) = \frac{3136}{625} = 5.0176$

c Linear scale factor: $\frac{16.45}{4.7} = \frac{15.05}{4.3} = \frac{12.95}{3.7} = \frac{7}{2}$

Area scale factor: $\left(\frac{7^2}{2^2}\right) = \frac{49}{4} = 12.25$

2 a Linear scale factor: $\frac{2.64}{1.65} = \frac{2.16}{1.35} = \frac{1.92}{1.2} = \frac{8}{5}$

Area scale factor: $\left(\frac{8^2}{5^2}\right) = \frac{64}{25} = 2.56$

b Linear scale factor: $\frac{47.6}{34} = \frac{30.8}{22} = \frac{7}{5}$

Area scale factor: $\left(\frac{7^2}{5^2}\right) = \frac{49}{25} = 1.96$

3 a Area of smaller triangle: $\frac{1}{2} \times 8 \times 12 = 48$ square units

Area of larger triangle: $\frac{1}{2} \times 16 \times 24 = 192$ square units

b $\frac{192}{48} = 4$; The larger triangle is four times the area of the smaller triangle.

c Linear scale factor: $\frac{24}{12} = \frac{16}{8} = 2$

d Area scale factor: $2^2 = 4$

4 Area of hexagon = 6 × area of triangle:

$6 \times \frac{1}{2} \times 2 \times \sqrt{3} = 6\sqrt{3}\text{ cm}^2$

Area scale factor: $\frac{24\sqrt{3}}{6\sqrt{3}} = 4$

Linear scale factor: $\sqrt{4} = 2$

5 a Area scale factor: $250^2 = 62\,500$

b Surface area of the pool:

$(6 \times 2.5) \times 62\,500 = 937\,500\text{ cm}^2$

$= 93.75\text{ m}^2$

6 Linear scale factor: $\frac{2}{1} = 2$

Area scale factor: $2^2 = 4$

Area of triangle ABC $= \frac{100}{4} = 25\text{ cm}^2$

7 Linear scale factor: $\frac{250}{1} = 250$; Area scale factor:

$250^2 = 62\,500$

Area the room takes up in the diagram: $\frac{120\,000}{62\,500} = 1.92\text{ cm}^2$

8 Linear scale factor: $\frac{6}{1.5} = 4$

Volume scale factor: $4^3 = 64$

9 a Linear scale factor: $\frac{7.5}{3} = \frac{5}{2}$

Volume scale factor: $\left(\frac{5}{2}\right)^3 = \frac{125}{8} = 15.625$

b Linear scale factor: $\frac{25.6}{8} = \frac{6.4}{2} = \frac{4.8}{1.5} = \frac{16}{5}$

Volume scale factor: $\left(\frac{16}{5}\right)^3 = \frac{4096}{125} = 32.768$

c Linear scale factor: $\frac{8.4}{7} = \frac{3.6}{3} = \frac{4.8}{4} = \frac{6}{5}$

Volume scale factor: $\left(\frac{6}{5}\right)^3 = \frac{216}{125} = 1.728$

10 a Linear scale factor: $\frac{2625}{15} = 175$, therefore, the scale of the model is 1 : 175.

b Volume scale factor: $(175)^3 = 5\,359\,375$, therefore, the ratio of the volume of the building to the volume of the model is 5 359 375 : 1.

11 **a** Volume scale factor: $\frac{400}{50} = 8$, therefore, the linear scale factor is $\sqrt[3]{8} = 2$.

b The length of the smaller cylinder is $\frac{8}{2} = 4$ cm.

12 $25 \times (2.5)^3 = 390.625\text{ cm}^3$

13 Volume scale factor: $\frac{1250}{600} = \frac{25}{12}$ therefore, the linear scale factor is $\sqrt[3]{\frac{25}{12}} = 1.28$.

14 Area scale factor: $\frac{7120\,000}{44.5} = 160\,000$ therefore, the linear scale factor is $\sqrt{160\,000} = 400$.
Actual length: $400 \times 5.3 = 2120\text{ cm} = 21.2\text{ m}$

15 **a** Area scale factor: $12^2 = 144$, therefore, the equivalent painted area on the model car is $\frac{43\,200}{144} = 300\text{ cm}^2$.

b Volume scale factor: $12^3 = 1728$, therefore, the equivalent volume for the model car is $\frac{1780\,000}{1728} = 1030\text{ cm}^3$.

16 **a**
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi\left(\frac{D}{2}\right)^2 \times \frac{4D}{5} \\ &= \pi\frac{D^2}{4} \times \frac{4D}{5} \\ &= \frac{\pi D^3}{5} \end{aligned}$$

b
$$\begin{aligned} V_2 &= \pi r^2 h \\ &= \pi\left(\frac{1.5D}{2}\right)^2 \times \frac{4\,(1.5D)}{5} \\ &= \pi \times \frac{9D^2}{16} \times \frac{6D}{5} \\ &= \pi\frac{54D^3}{80} \\ &= \frac{27\pi D^3}{40} \end{aligned}$$

13.4 Exam questions

1 Area scale factor $= 9:1$
Length scale factor $= \sqrt{9}:\sqrt{1} = 3:1$
Length of enlarged photograph: $3 \times 12\text{ cm} = 36\text{ cm}$
The correct answer is **D**.

2 If we let the area of each white flag be 1 unit squared, then the area of each black flag is 4 units squared.
$$\frac{\text{Total area of black flags}}{\text{Total area of white flags}} = \frac{4 \times 4}{1 \times 3} = \frac{16}{3}$$
VCAA Assessment Report **note:**
Many students, in choosing option D, ignored the fact that there were different numbers of black and white flags in the string of flags.
The correct answer is **D**.

3 The side ration is $6:4 = 3:2$; the volume ratio is $3^3:2^3 = 27:8$.
As the prices are proportional to volumes,
$27:8$
$5.40:x$
$27x = 43.2$
$x = 1.60$
Therefore, the small cupcake will cost $1.60.
The correct answer is **A**.

13.5 Review

13.5 Exercise

Multiple choice

1 $\frac{12.4}{4} = \frac{9.3}{3} = \frac{6.2}{2} = 3.1$
The correct answer is **D**.

2 $1\text{ cm}:50\,000\text{ cm}$
$1\text{ cm} = 0.5\text{ km}$
$\frac{11\text{ km}}{0.5} = 22\text{ cm}$
The correct answer is **B**.

3 Area scale factor: $\frac{192}{12} = 16$
Linear scale factor: $\sqrt{16} = 4$
The correct answer is **B**.

4 Volume scale factor: $\frac{128}{16} = 8$
Linear scale factor: $\sqrt[3]{8} = 2$
Area scale factor: $2^2 = 4$
The correct answer is **C**.

5 $\frac{x}{5} = \frac{5.6}{3.5}$
$$\begin{aligned} x &= \frac{5.6 \times 5}{3.5} \\ &= 8.0 \end{aligned}$$
The correct answer is **C**.

6 $\frac{x}{136} = \frac{45}{153}$
$$\begin{aligned} x &= \frac{45 \times 136}{153} \\ &= 40 \end{aligned}$$
The correct answer is **E**.

7 Volume of smaller cube: $\frac{1728}{64} = 27$
Side length of smaller cube: $\sqrt[3]{27} = 3$
Surface area of smaller cube: $3^2 \times 6 = 54$
The correct answer is **C**.

8 $\frac{162}{18^2} = 0.5$
The correct answer is **A**.

9 12.5 cm : 15 m
12.5 cm : 1500 cm
1 cm : $\frac{1500}{12.5}$ cm
1 : 120
The correct answer is **D**.

10 $1\text{ cm}:2.25\text{ m}$
$2.25 \times 12.5 = 28.125$
$2.25 \times 8.4 = 18.9$
$2.25 \times 0.25 = 0.5625$
$$\begin{aligned} \text{Actual volume} &= 28.125 \times 18.9 \times 0.5625 \\ &= 299.003 \end{aligned}$$
The correct answer is **D**.

Short answer

11 $\frac{x}{8} = \frac{15}{6}$

$x = \frac{15 \times 8}{6}$

$= 20$

12 Linear scale factor: $4:5 = 1:\frac{5}{4}$

Volume scale factor $= \left(\frac{5}{4}\right)^3$

Volume $= 1600 \times \left(\frac{5}{4}\right)^3$

$= 3125$

The volume is $3125\ \text{cm}^3$.

13 a $\frac{x}{8.4} = \frac{7.6}{4.4}$

$x = \frac{7.6 \times 8.4}{4.4}$

$= 14.509$

≈ 14.51

b $\frac{x}{84} = \frac{15}{63}$

$x = \frac{15 \times 84}{63}$

$= 20$

$\frac{y}{8} = \frac{63}{15}$

$y = \frac{63 \times 8}{15}$

$= 33.6$

$x = 20;\ y = 33.6$

c $\frac{35}{28} = \frac{2x - 6}{x}$

$35x = 28(2x - 6)$

$35x = 56x - 168$

$21x = 168$

$x = \frac{168}{21}$

$= 8$

14 a $2:3 \rightarrow$ linear scale factor $= 1.5$

$1.5 \times 32 = 48$

$1.5 \times 45 = 67.5$

$1.5 \times 58 = 87$

The side lengths are 48 mm, 67.5 mm and 87 mm.

b Drawing : Triangle $= 5:2 \rightarrow$ Triangle : Drawing $= 2:5$

$2:5 \rightarrow$ linear scale factor $= 2.5$

$2.5 \times 48 = 120$

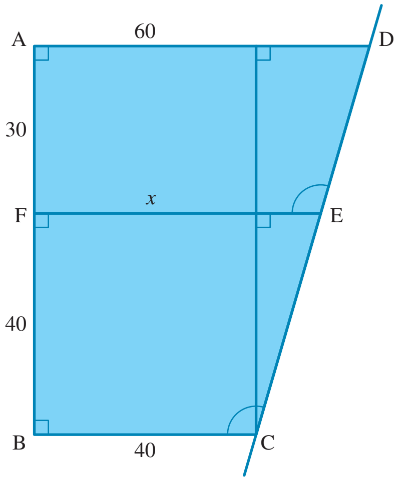
$2.5 \times 67.5 = 168.75$

$2.5 \times 87 = 217.5$

The side lengths in the drawing would be 120 mm, 168.75 mm and 217.5 mm.

15 a $70:40 \rightarrow 7:4$

b Use the two similar triangles created by taking a perpendicular line from AD to the vertex C:



$\frac{x - 40}{20} = \frac{40}{70}$

$x - 40 = \frac{40 \times 20}{70}$

$x = \frac{40 \times 20}{70} + 40$

$= 51.429\ \text{m}$

5143 cm of fencing is required.

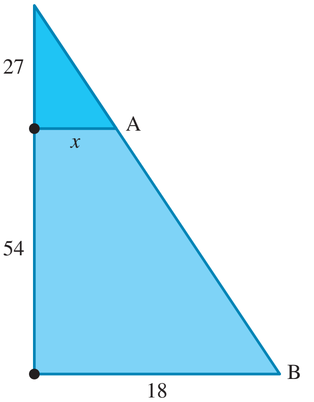
16 a $\frac{2}{3}$total height $= 54$

$\frac{1}{3}$total height $= \frac{54}{2}$

$= 27$

27 cm is removed.

b



$\frac{x}{18} = \frac{27}{81}$

$x = \frac{27 \times 18}{81}$

$x = 6$

$\text{AB} = \left(\sqrt{81^2 + 18^2}\right) - \left(\sqrt{27^2 + 6^2}\right)$

$= \sqrt{6885} - \sqrt{765}$

$= 55.32$

The distance from A to B is 55.32 cm.

Extended response

17 a 225 000 cm = 2.25 km

$88 \times 2.25\text{ km} = 198\text{ km}$

b see the figure bottom of the page*

$$\frac{x+100}{100} = \frac{198}{90}$$

$$x + 100 = \frac{198 \times 100}{90}$$

$$x = \frac{198 \times 100}{90} - 100$$

$$= 120\text{ km}$$

c $x = \sqrt{(120+100)^2 - 198^2}$

$= 95.90\text{ km}$

d $\dfrac{95.9}{2.25} = 42.62\text{ cm}$

18 a $4:7 \rightarrow$ linear scale factor $= 1.75$

$1.75 \times 94 = 164.5$

$1.75 \times 31 = 54.25$

$164.5\text{ cm} \times 54.25\text{ cm}$

b $x = \sqrt{94^2 - 52^2}$

$= 78.31\text{ cm}$

c $\dfrac{x}{78.31} = \dfrac{164.5}{94}$

$$x = \frac{164.5}{94} \times 78.31$$

$$x = 137.04\text{ cm}$$

19 a $\dfrac{x}{2} = \dfrac{5}{1.8}$

$$x = \frac{5 \times 2}{1.8}$$

$$x = 5.6\text{ m}$$

b $\dfrac{x}{5} = \dfrac{2}{8.05}$

$$x = \frac{5 \times 2}{8.05}$$

$$x = 1.242$$

$$\approx 1.24\text{ m}$$

20 a $200 \times$ volume scale factor $= 300$

$$\text{Volume scale factor} = \frac{300}{200}$$

$$\text{Linear scale factor} = \sqrt[3]{\frac{300}{200}}$$

$$= 1.14$$

b $4:5 \rightarrow$ linear scale factor $= \dfrac{5}{4}$

$$\text{Capacity of large cup} = 200 \times \left(\frac{5}{4}\right)^3$$

$$= 390.6\text{ mL}$$

c Small cups: $\dfrac{300}{200} = 1.5$ cents/mL

Medium cups: $\dfrac{350}{300} = 1.17$ cents/mL

Large cups: $\dfrac{400}{390.6} = 1.02$ cents/mL

Therefore, the large cups are the better value.

13.5 Exam questions

1 Length: $\dfrac{w}{m} = \dfrac{1}{1.25} = \dfrac{4}{5}$

Area: $\left(\dfrac{4}{5}\right)^2 = \dfrac{16}{25}$

The correction answer is **E**.

2 a Each angle in an equilateral triangles is 60°.

$$\text{Area} = \frac{1}{2} \times 4.8 \times 4.8 \times \sin 60^\circ$$

$$= 9.9766\ldots = 10\text{ cm}$$ [1 mark]

b Each triangle can be divided into 4 smaller triangles. The two triangles together will consist of 8 smaller triangles, but 2 of them overlap in the logo, so the logo has only 6 of the small triangles, or $\dfrac{6}{8} = \dfrac{3}{4}$ of the area of the two larger triangles.

Therefore, $\dfrac{3}{4} \times \left(10\text{ cm}^2 + 10\text{ cm}^2\right) = 15\text{ cm}^2$.

c Two shaded small triangles: 4 non-shaded small triangles, give a ratio of 1 : 2. [1 mark]

d original area : enlarged area

4 : 1

Therefore, the length will be in the ratio $\sqrt{4}:\sqrt{1} = 2:1$.

If the original height of the logo is 4.8 cm, then the enlarged logo will be $2 \times 4.8 = 9.6$ cm. [1 mark]

3 a i Surface area $= l \times w$

$= 40 \times 19$

$= 760\text{ cm}^2$ [1 mark]

ii Surface area $= 2(l \times w + l \times h + w \times h)$

$= 2(40 \times 19 + 40 \times 32 + 19 \times 32)$

$= 2(760 + 1280 + 608)$

$= 2 \times 2648$

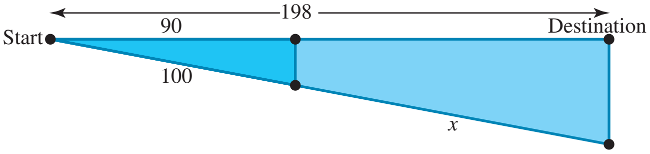
$= 5296\text{ cm}^2$ [1 mark]

b The scale factor of the volume is 8, or 2^3. Therefore, the scale factor of the lengths is 2.The length of the small storage box is 40 cm so the length of the large storage box is $2 \times 40 = 80$ cm. [1 mark]

***VCAA Examination Report* note:**

This question was not answered well. Many students did not calculate the linear scale factor of 2 and gave an answer of 320 cm.

***17b**



4 a $SA = \frac{1}{2} \times 4\pi r^2$

$= \frac{1}{2} \times 4\pi \times 5^2$

$= 157.08$

$= 157\,\text{cm}^2$ [1 mark]

***VCAA Assessment Report* note:**

Some students calculated the surface area of a full sphere, while others used an incorrect radius or formula.

b $V = V_{\text{cylinder}} + V_{\text{hemisphere}}$

$= \pi r^2 h + \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \pi \times 5^2 \times 15 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$

$= 1439.9\,\text{cm}^3$

$= 1440\,\text{cm}^3$

[1 mark is awarded for correct formula and correct radius]

[1 mark for correct answer]

***VCAA Assessment Report* note:**

A method mark was available in this two-mark question if the final answer was incorrect. Many calculations were poorly set out and a method mark could not be allocated. Some students used the wrong formula for the volume of a sphere, while others did not halve this for a hemisphere.

c Volume ratio:

$V_{\text{feed}} : V_{\text{water}}$

$1 : \frac{3}{4}$

Length ratio:

$\sqrt[3]{1} : \sqrt[3]{\frac{3}{4}}$

$= 1 : 0.90856$

SA ratio:

$1^2 : 0.9085^2$

$= 1 : 0.825482$

[1 mark is awarded for the surface area ratio]

If $SA_{water} = 628$

$\therefore SA_{\text{feed}} = 628 \times \frac{1}{0.825482}$

$= 760.77$

$= 761\,\text{cm}^2$ [1 mark]

***VCAA Assessment Report* note:**

This question was not answered well. Many students only converted the volume factor $\frac{3}{4}$ of or $\frac{4}{3}$ into a linear factor, while others calculated the area factor as $\sqrt{\text{volume factor}}$, which was inappropriate. Many students did not attempt conversion of the volume factor.

Some students rounded too early within the question, such as rounding the linear factor to 1.1 before squaring it again to produce the incorrect surface area of $759.88\,\text{cm}^2$.

5 The ratio of the areas is $720 : 500 = 36 : 25$.

The ratio of the radii is $\sqrt{36} : \sqrt{25} = 6 : 5$.

The ratio of the volumes is $6^3 : 5^3 = 216 : 125$. [1 mark]

Therefore, to give the volume of the senior discus, the volume of the intermediate discus must be multiplied by $\frac{216}{125}$ or 1.728

[1 mark]

***VCAA Assessment Report* note:**

The ratio of dimensions of the larger discus to the smaller discus was required. The value of $k > 1$ since a larger volume is scaled up from the smaller volume.

A common error was to work with the reciprocal of the area ratio. This would give a value less than one for the linear and the volume ratios. If this is multiplied with the smaller discus dimensions, the result would be a smaller discus, not a larger one. This question was very poorly answered. Many students did not attempt this question.