1 Investigating and comparing data distributions

Topic	**1**	**Investigating and comparing data distributions**
Subtopic	**1.2**	**Types of data**

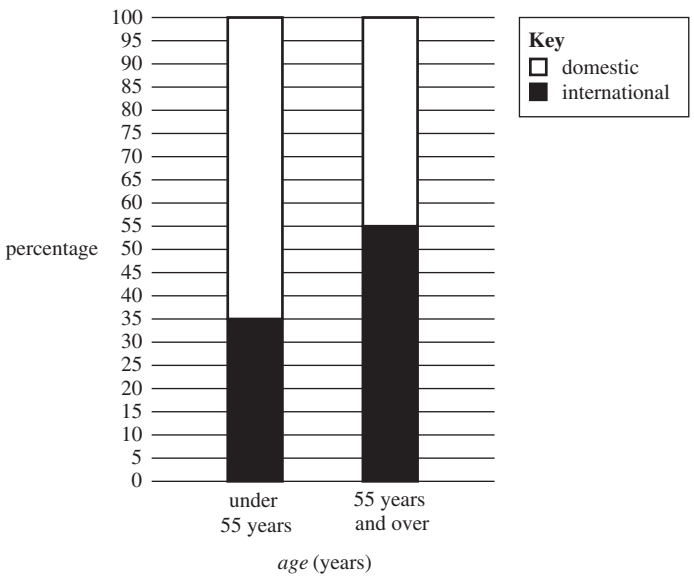
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2021, Further Mathematics Exam 1, Section A, Core, Q.1; © VCAA*

Question 1 (1 mark)

The percentaged segmented bar chart below shows the *age* (under 55 years, 55 years and over) of visitors at a travel convention, segmented by *preferred travel destination* (domestic, international).



The variables *age* (under 55 years, 55 years and over) and *preferred travel destination* (domestic, international) are

A. both categorical variables.
B. both numerical variables.
C. a numerical variable and a categorical variable respectively.
D. a categorical variable and a numerical variable respectively.
E. a discrete variable and a continuous variable respectively.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 1, Section A, Q.8; © VCAA*

Question 2 (1 mark)

Percy conducted a survey of people in his workplace. He constructed a two-way frequency table involving two variables.

One of the variables was *attitude towards shorter working days* (for, against). The other variable could have been

A. *age* (in years).
B. *sex* (male, female).
C. *height* (to the nearest centimetre).
D. *income* (to the nearest thousand dollars).
E. *time* spent travelling to work (in minutes).

Source: *VCE 2016, Further Mathematics Exam 1, Section A, Q.2; © VCAA*

Question 3 (1 mark)

The *blood pressure* (low, normal, high) and the *age* (under 50 years, 50 years or over) of 110 adults were recorded. The results are displayed in the two-way frequency table below.

	Age	
Blood pressure	**Under 50 years**	**50 years or over**
Low	15	5
Normal	32	24
High	11	23
Total	58	52

The variables *blood pressure* (low, normal, high) and *age* (under 50 years, 50 years or over) are

A. both nominal variables.
B. both ordinal variables.
C. a nominal variable and an ordinal variable respectively.
D. an ordinal variable and a nominal variable respectively.
E. a continuous variable and an ordinal variable respectively.

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
The number of children who choose a salad for lunch is counted during term-time. The data in this survey is
A. continuous.
B. nominal.
C. categorical.
D. discrete.
E. ordinal.

Question 5 (1 mark)
The eye colour of all children in a kindergarten is recorded. The data in this survey is
A. categorical.
B. nominal.
C. ordinal.
D. discrete.
E. continuous.

Question 6 (1 mark)
Which of the following is not an example of categorical data?
A. Eye colour
B. Religious denomination
C. Height of 10 students in your class
D. Pizza sizes available at the local pizza shop
E. Types of vehicles available at a second-hand car yard

© John Wiley & Sons Australia, Ltd

Topic	1	Investigating and comparing data distributions
Subtopic	1.3	Categorical data distributions

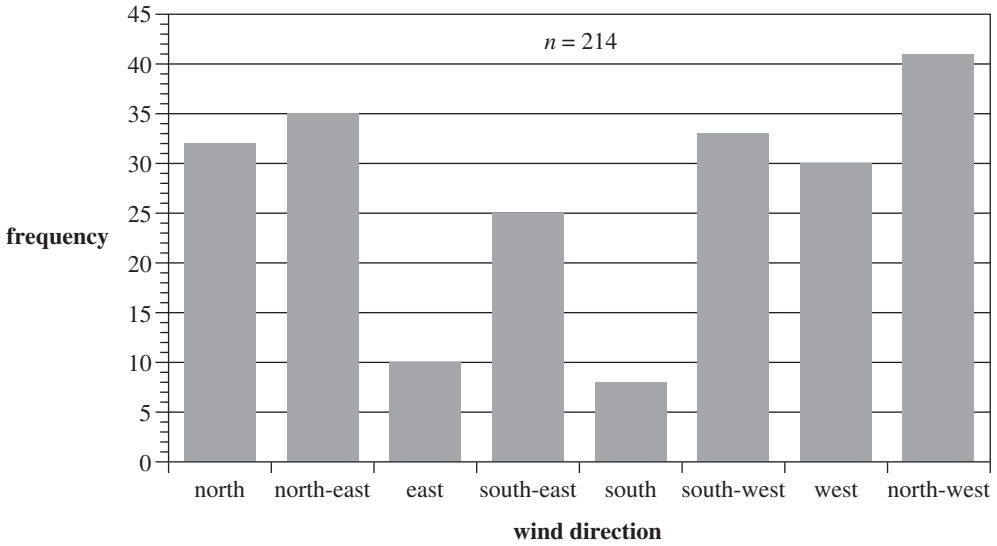
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2012, Further Mathematics Exam 1, Section A, Q.1; © VCAA*

Question 1 (1 mark)

The following bar chart shows the distribution of wind directions recorded at a weather station at 9.00 am on each of 214 days in 2011.



According to the bar chart, the most frequently observed wind direction was

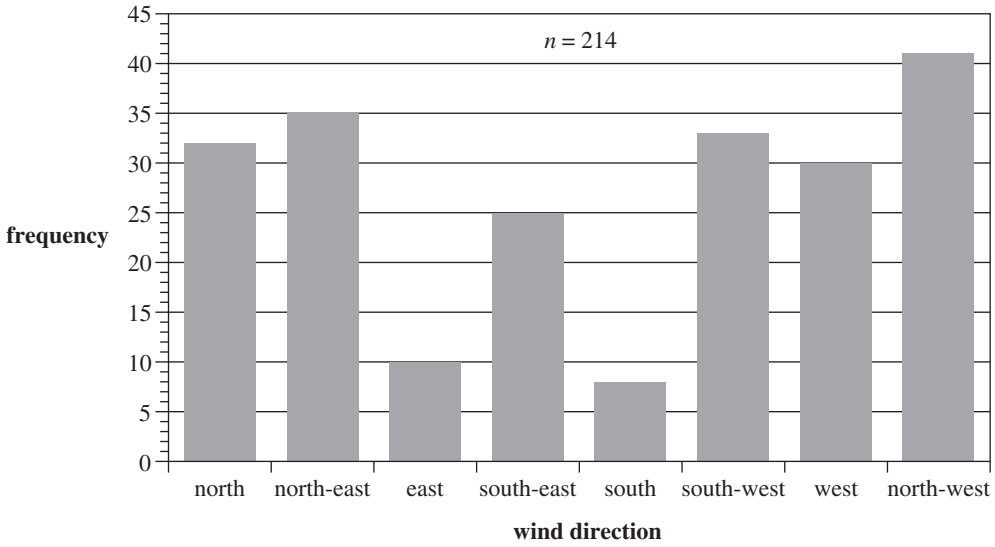
A. south-east.
B. south.
C. south-west.
D. west.
E. north-west.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2012, Further Mathematics Exam 1, Section A, Q.2; © VCAA*

Question 2 (1 mark)

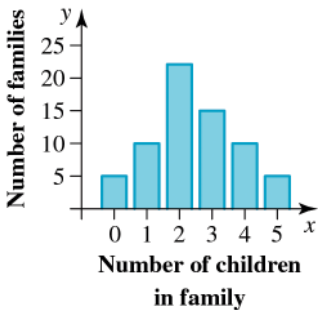
The following bar chart shows the distribution of wind directions recorded at a weather station at 9.00 am on each of 214 days in 2011.



According to the bar chart, the percentage of the 214 days on which the wind direction was observed to be east or south-east is closest to

A. 10%
B. 16%
C. 25%
D. 33%
E. 35%

Question 3 (1 mark)



For the bar chart shown, which of the following statements is false?

A. There are five families with zero children.
B. There are zero families with five children.
C. There are more families with three children than there are families with four children.
D. The modal number of children is two.
E. There are ten families with four children.

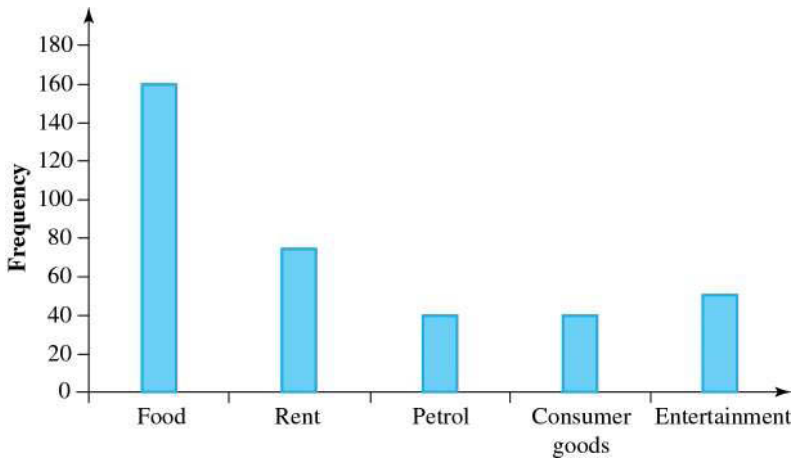
Question 4 (1 mark)

Dawn asks 365 students which of food, rent, petrol, consumer goods and entertainment they spend most of their weekly allowance on. The information is shown in this table.

Area	Frequency
Food	160
Rent	75
Petrol	30
Consumer goods	50
Entertainment	50

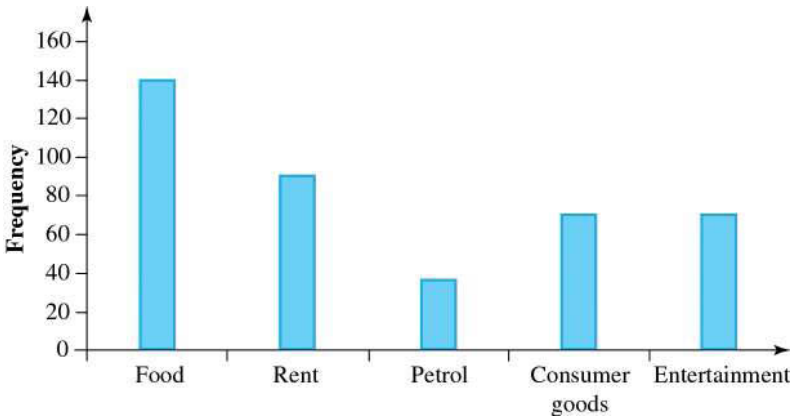
The bar chart from this data is

A.

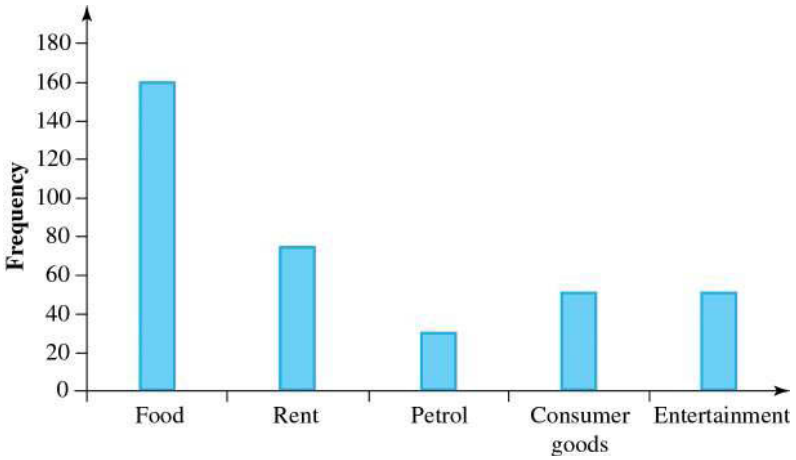


© John Wiley & Sons Australia, Ltd

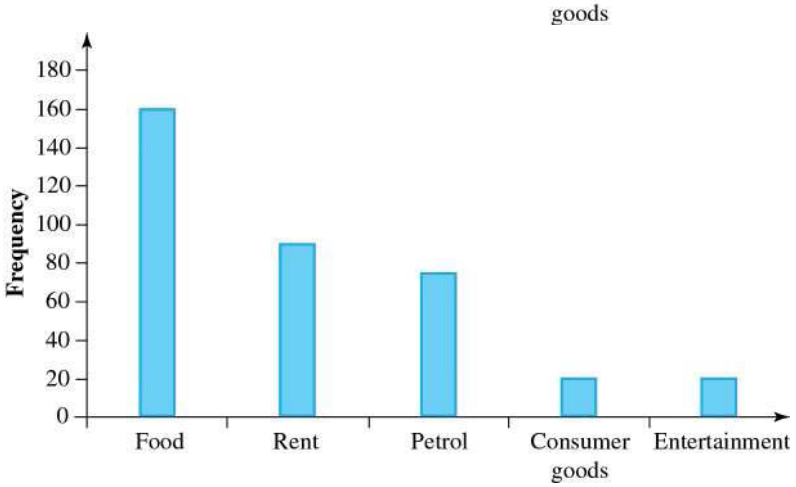
B.



C.

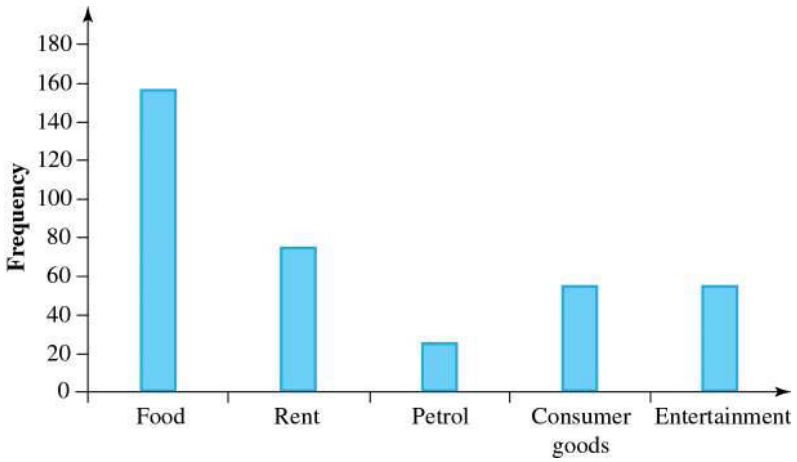


D.



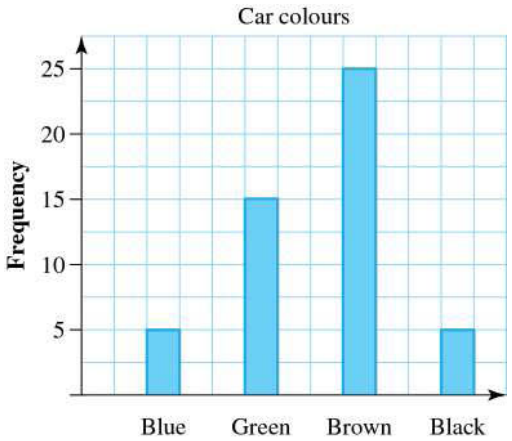
© John Wiley & Sons Australia, Ltd

E.



Question 5 (1 mark)

This bar chart shows the frequency of different car colours recorded in a school car park on a particular day.



The total number of cars counted was:

A. 5
B. 15
C. 4
D. 25
E. 50

© John Wiley & Sons Australia, Ltd

Topic	1	Investigating and comparing data distributions
Subtopic	1.4	Numerical data distributions — frequency tables and histograms

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

A group of 28 students had their heights recorded. See the data set below.

123	125	126	124	111	135	147	125	128	123	126	128	124	124
120	122	125	123	110	130	140	127	128	123	120	149	125	127

Construct a frequency table using a suitable class interval.

Question 2 (1 mark)

A group of 28 students had their heights recorded. See the data set below.

123	125	126	124	111	135	147	125	128	123	126	128	124	124
120	122	125	123	110	130	140	127	128	123	120	149	125	127

Construct a histogram to represent these results.

Question 3 (3 marks)

The average number of hours worked each week was surveyed and a frequency table drawn up.

Hours	Frequency
0–	20
10–	24
20–	35
30–	46
40–	64
50–	18
60–	8
70–	5

a. State the number of workers surveyed. **(1 mark)**

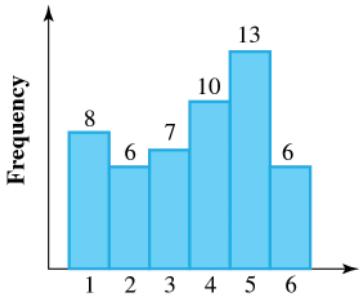
© John Wiley & Sons Australia, Ltd

b. Draw a histogram to show the information. **(1 mark)**

c. Determine the mode of the number of hours worked each week. **(1 mark)**

Question 4 (1 mark)

A die was tossed and the number uppermost noted. The histogram shows the number of times each number was uppermost.



How many times was the die tossed?

A. 50
B. 100
C. 21
D. 13
E. 6

© John Wiley & Sons Australia, Ltd

Topic	1	Investigating and comparing data distributions
Subtopic	1.5	Numerical data distributions — dot plots and stem plots

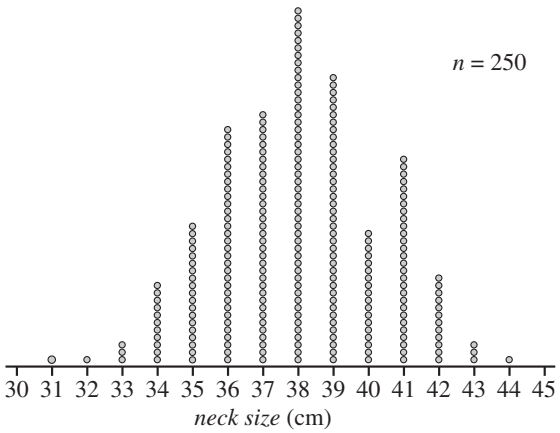
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2020, Further Mathematics Exam 2, Section A, Q.2; © VCAA*

Question 1 (1 mark)

The *neck* size, in centimetres, of 250 men was recorded and displayed in the dot plot below.



Data: RW Johnson, 'Fitting percentage of body fat to simple body measurements', *Journal of Statistics Education*, 4:1, 1996, <https://doi.org/10.1080/10691898.1996.11910505>

Write down the modal neck size, in centimetres, for these 250 men.

Modal neck size = ☐ cm

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 2, Section A, Q.1; © VCAA*

Question 2 (2 marks)

Table 1 shows the *day number* and the *minimum temperature*, in degrees Celsius, for 15 consecutive days in May 2017.

Table 1

Day number	***Minimum temperature*** **(°C)**
1	12.7
2	11.8
3	10.7
4	9.0
5	6.0
6	7.0
7	4.1
8	4.8
9	9.2
10	6.7
11	7.5
12	8.0
13	8.6
14	9.8
15	7.7

Data: Australian Government, Bureau of Meteorology, <www.bom.gov.au/>

a. Which of the two variables in this data set is an ordinal variable? **(1 mark)**

b. The incomplete ordered stem plot below has been constructed using the data values for days 1 to 10.

key: 4|1 = 4.1 $n = 15$

minimum temperature (°C)

4	1 8
5	
6	0 7
7	0
8	
9	0 2
10	7
11	8
12	7

Complete the **stem plot above** by adding the data values for days 11 to 15.
(*Answer on the stem plot above.*) **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics Exam 1, Section A, Q.3; © VCAA*

Question 3 (1 mark)

The stem plot below displays 30 temperatures recorded at a weather station.

key: 2|2 = 2.2 °C

Temperatue

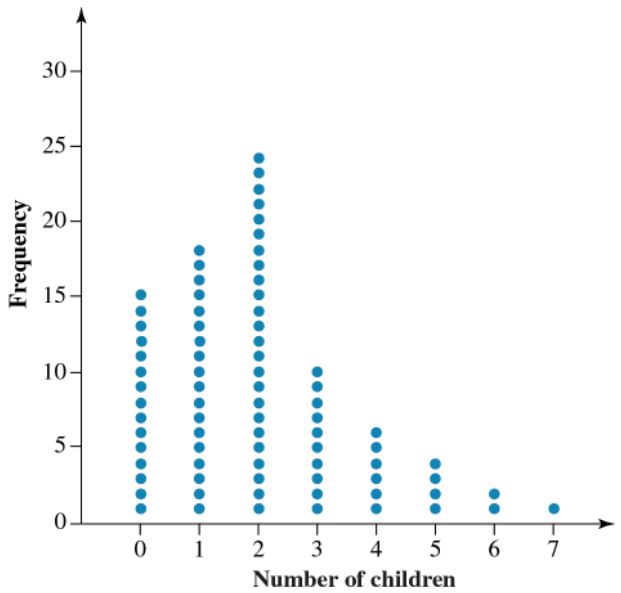
2	2 2 4 4
2	5 7 8 8 8 8 8 8 9 9 9 9
3	1 2 3 3 4 4 4
3	5 6 7 7 7 7
4	1

The modal temperature is

A. 2.8°C
B. 2.9°C
C. 3.7°C
D. 8.0°C
E. 9.0°C

Question 4 (1 mark)

Data on the number of children in each family in a certain town is collected and plotted in a dot plot.



The frequency table for this plot is:

A.

Number	Frequency
0	15
1	18
2	24
3	10
4	6
5	4
6	2
7	1
8	0

B.

Number	Frequency
0	14
1	17
2	23
3	9
4	5
5	3
6	1
7	1
8	0

C.

Number	Frequency
0	15
1	24
2	18
3	10
4	6
5	4
6	2
7	1
8	0

D.

Number	Frequency
0	15
1	18
2	24
3	10
4	6
5	4
6	0
7	0
8	0

© John Wiley & Sons Australia, Ltd

E.

Number	Frequency
0	7
1	9
2	12
3	5
4	3
5	2
6	1
7	1
8	0

Question 5 (1 mark)

The population (in thousands) of 40 randomly selected towns is shown in the frequency table.

Population	Frequency
0–9999	12
10 000–19 999	10
20 000–29 999	7
30 000–39 999	5
40 000–49 999	3
50 000–59 999	1
> 60 000	2

The stemplot for this data could be

A.

Stem	Leaf
0	1 2 3 4 5 7 8 9 9
10	1 2 3 4 5 6 7 9
20	1 2 4 6 8
30	1 3 5 7 9
40	2 5 8
50	3
60	4
70	0
80	1
10|1 = 11,000	

B.

Stem	Leaf
0	1 2 3 3 4 4 5 5 7 8 9 9
10	1 2 3 3 4 4 5 6 7 9
20	1 2 4 4 4 6 8
30	1 3 5 7 9
40	2 5 8
50	3
60	4
70	
80	1
10|1 = 11,000	

© John Wiley & Sons Australia, Ltd

C.

Stem	Leaf
0	1 2 3 3 4 4 5 5 7 8 9 9
10	1 2 3 3 4 4 5 6 7 9
20	1 2 4 4 4 6 8
30	1 3 5 7 9
40	2 5 8
50	3
60	4
70	0
80	1
10|1 = 11,000	

D.

Stem	Leaf
0	1 2 3 3 4 4 5 5 7 8 9 9
10	1 2 3 3 4 4 5 6 7 9 9
20	1 2 4 4 4 6 8
30	1 3 5 7 9
40	2 5 8
50	3
60	4
70	
80	1
10|1 = 11,000	

E.

Stem	Leaf
0	1 2 3 3 4 4 5 5 7 8 9 9
10	11 12 13 13 14 14 15 16 17 19
20	21 22 24 24 24 26 28
30	31 33 35 37 39
40	42 45 48
50	53
60	64
70	
80	81
10|1 = 11,000	

© John Wiley & Sons Australia, Ltd

Question 6 (4 marks)
The marks for a Maths test were collected.

33	50	53	67	83
60	87	40	57	53
70	60	77	90	27
73	93	50	53	20
80	63	83	70	90

a. Complete this frequency table for the data. **(1 mark)**

Marks	Frequency
0–	
10–	
20–	2
30–	1
40–	1
50–	
60–	
70–	
80–	4
90–100	3

b. How many students are in the class? **(n/a mark)**

c. Show the data in a stemplot. **(1 mark)**

© John Wiley & Sons Australia, Ltd

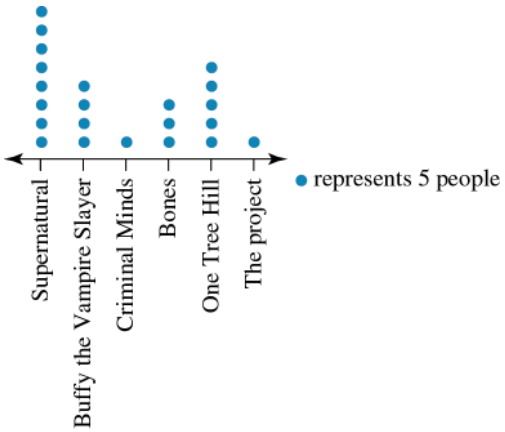
d. The same class' English marks are shown in this stemplot. **(2 marks)**

Leaf	Stem
1 5	2
2 4 5	3
1 2 5 5 7 8	4
1 1 3 4 4 6 9 9	5
2 5 7 8	6
1 3	7
	8
	9
Key 5	2 = 25

Use this to make a back-to-back stemplot and hence state why you might think that, on average, the students performed better in Maths than in English.

Question 7 (1 mark)

Consider the dot plot shown. The percentage, correct to 1 decimal place, of people surveyed who liked *Supernatural or One Tree Hill* is:



A. 59.1%
B. 36.4%
C. 22.7%
D. 65%
E. 13.6%

© John Wiley & Sons Australia, Ltd

Topic	1	Investigating and comparing data distributions
Subtopic	1.6	Characteristics of numerical data distributions

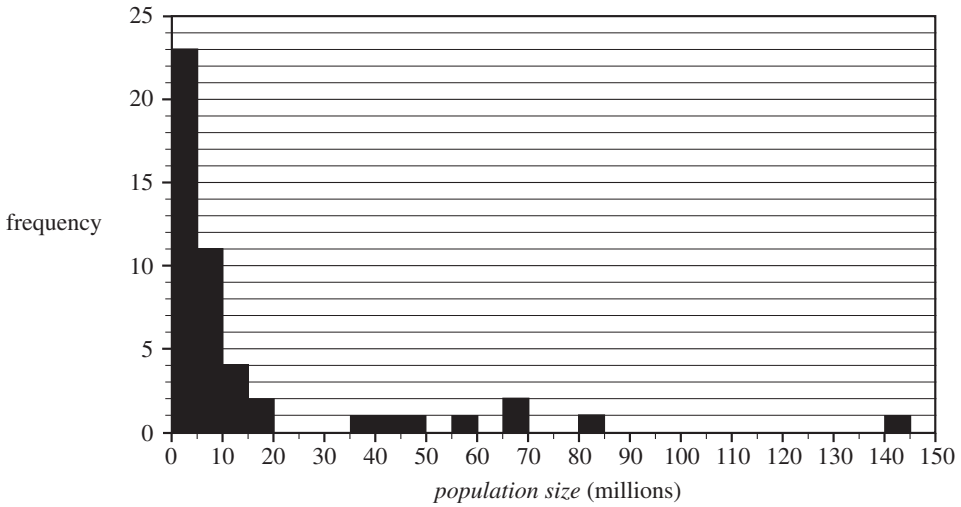
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2019, Further Mathematics Exam 1, Section A, Q.2; © VCAA*

Question 1 (1 mark)

The histogram below shows the distribution of the *population size* of 48 countries in 2018.



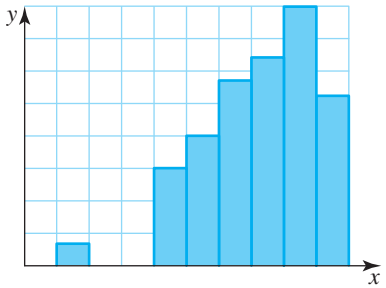
Data: Worldometers, <www.worldometers.info/>

The shape of this histogram is best described as

A. positively skewed with no outliers.
B. positively skewed with outliers.
C. approximately symmetric.
D. negatively skewed with no outliers.
E. negatively skewed with outliers.

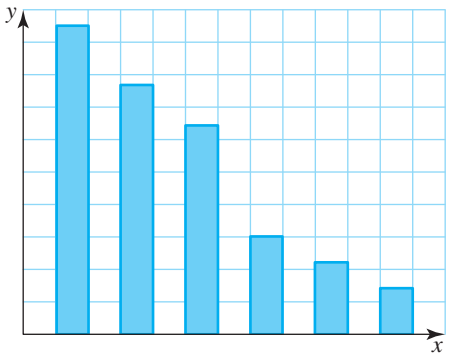
© John Wiley & Sons Australia, Ltd

Question 2 (1 mark)
Describe the shape of the distribution of the data in the graph.



A. Symmetrical
B. Positively skewed
C. Negatively skewed
D. Bimodal
E. Negatively skewed with an outlier

Question 3 (1 mark)
Describe the shape of the distribution of the data in the graph.



A. Symmetrical
B. Positively skewed
C. Negatively skewed
D. Bimodal
E. Positively skewed with an outlier

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
Data was collected on the number of pedestrians in accidents in a city.

Age group (years)	Pedestrians killed
0–	16
10–	40
20–	75
30–	52
40–	72
50–	89
60–	54
70–	38
80–100	30

What type of data is represented by this frequency distribution table?
A. Normal
B. Skewed
C. Clustered
D. Spread
E. Bimodal

© John Wiley & Sons Australia, Ltd

Topic	1	Investigating and comparing data distributions
Subtopic	1.7	Summarising numerical data —mean and median

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2020, Further Mathematics Exam 2, Section A, Q.4; © VCAA*

Question 1 (2 marks)

The *age*, in years, *body density*, in kilograms per litre, and *weight*, in kilograms, of a sample of 12 men aged 23 to 25 years are shown in the table below.

***Age* (years)**	***Body density* (kg/litre)**	***Weight* (kg)**
23	1.07	70.1
23	1.07	90.4
23	1.08	73.2
23	1.08	85.0
24	1.03	84.3
24	1.05	95.6
24	1.07	71.7
24	1.06	95.0
25	1.07	80.2
25	1.09	87.4
25	1.02	94.9
25	1.09	65.3

For these 12 men, determine

a. their median age, in years **(1 mark)**

b. the mean of their body *density*, in kilograms per litre. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 2, Section A, Q.1; © VCAA*

Question 2 (1 mark)

Body mass index (*BMI*), in kilograms per square metre, was recorded for a sample of 32 men and displayed in the ordered stem plot below.

key: 21 | 6 = 21.6 $n = 32$

21	6 9 9
22	1 2 5 6
23	0 1 4 6 6 7 8
24	4 5 6 7 7 9
25	6 8
26	1 7 9
27	3 7
28	2
29	1 8
30	4
31	1

Determine the median *BMI* for this group of men.

Source: *VCE 2015, Further Mathematics Exam 1, Section A, Q.2; © VCAA*

Question 3 (1 mark)

For an ordered set of data containing an odd number of values, the middle value is always

A. the mean.
B. the median.
C. the mode.
D. the mean and the median.
E. the mean, the median and the mode.

Question 4 (1 mark)
Data is collected on the number of cups of coffee drunk by a group of office workers:
0, 0, 0, 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 4, 7
The outlier is removed.
Which of the following is true?
A. The mean will increase.
B. The mean will decrease.
C. The median will change.
D. The mode will increase.
E. There will be no change to the mean, median or mode.

Question 5 (1 mark)
Consider the following data set.
5 4 2 1 3 3 3 7 8 12 1 0 4 9
The median of this data set is:
A. 3
B. 4
C. 5
D. 3.5
E. 0

© John Wiley & Sons Australia, Ltd

Topic	1	Investigating and comparing data distributions
Subtopic	1.8	Summarising numerical data — range, interquartile range and standard deviation

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2020, Further Mathematics Exam 2, Section A, Q.3; © VCAA*

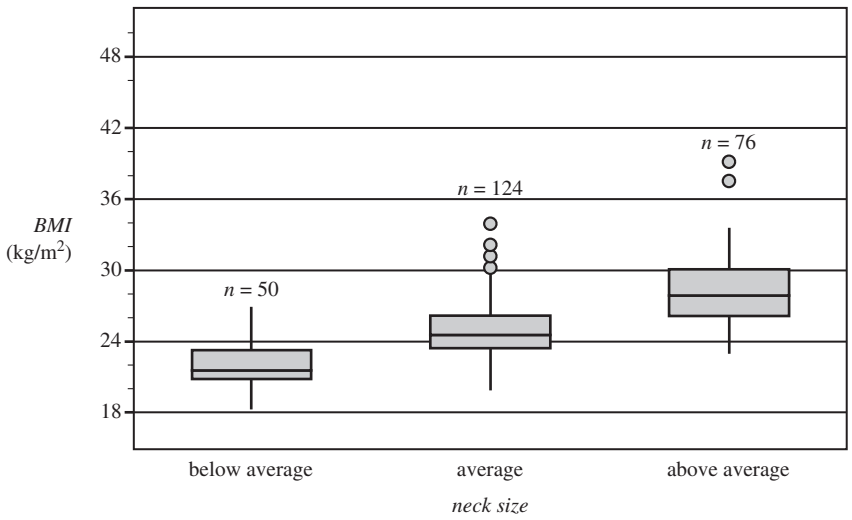
Question 1 (1 mark)

In a study of the association between ***BMI*** and *neck size*, 250 men were grouped by *neck size* (below average, average and above average) and their ***BMI*** recorded.

Five-number summaries describing the distribution of ***BMI*** for each group are displayed in the table below along with the group size.

The associated boxplots are shown below the table.

		BMI (kg/m^2)				
Neck size	**Group size**	**Min.**	**Q_1**	**Median**	**Q_3**	**Max.**
below average	50	18.1	20.6	21.6	23.2	26.8
average	124	19.8	23.4	24.6	26.0	33.9
above average	76	23.1	26.25	28.1	29.95	39.1



Data: RW Johnson, 'Fitting percentage of body fat of simple body measurement', *Journal of Statistics Eduction*, 4:1, 1996, <http://doi.org/10.1080/1069 1898.1996.11910505>

What is the interquartile range (IQR) of *BMI* for the men with an average neck size?

Source: *VCE 2019, Further Mathematics Exam 1, Section A, Q.5; © VCAA*

Question 2 (1 mark)

The stem plot below shows the distribution of mathematics *test scores* for a class of 23 students.

key: 4|2 = 42 $n = 23$

4	0 1 4 4
5	2 7 9 9 9
6	5 6 8 8 9 9
7	0 0 5 6 7 8
8	5 9

For this class, the interquartile range (IQR) of *test scores* is

A. 14.5
B. 17.5
C. 18
D. 24
E. 49

Question 3 (1 mark)

Thirty Year 11 students were asked to measure the number of hours of homework they completed each week. The data are recorded below.

12	11	10	11	9	8	5	13	14	12	12	11	10	11	9
8	5	13	14	12	12	11	10	11	9	8	5	13	3	2

The standard deviation, correct to the nearest whole number, for this data set is

A. 4
B. 10
C. 11
D. 12
E. 3

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
The change in the weights of seven people on diets was recorded at the end of one year. The results were as follows:
−5 kg, −3 kg, 1 kg, 2 kg, −6 kg, 3 kg, 0 kg
The range of the change in weights of the dieters was
A. −9 kg
B. 9 kg
C. 3 kg
D. 8 kg
E. −6 kg

Question 5 (1 mark)
Consider the following data set:
22, 23, 24, 24, 25, 25, 26, 27, 28, 28, 29, 30
The interquartile range is
A. 8
B. 4
C. 24
D. 25.5
E. 3

Question 6 (1 mark)
Calculate the standard deviation of the following data, correct to three decimal places.

Score	Frequency
0	1
1	3
2	4
3	6
4	2

A. 1.138
B. 2.313
C. 1.296
D. 4.000
E. 0.348

© John Wiley & Sons Australia, Ltd

Topic	1	Investigating and comparing data distributions
Subtopic	1.9	Symmetrical and asymmetrical distributions

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2021, Further Mathematics Exam 1, Section A, Core, Q.9; © VCAA*

Question 1 (1 mark)

The heights of females living in a small country town are normally distributed:

- 16% of the females are more than 160 cm tall.
- 2.5% of the females are less than 115 cm tall.

The mean and the standard deviation of this female population, in centimetres, are closest to

A. mean = 135 standard deviation = 15
B. mean = 135 standard deviation = 25
C. mean = 145 standard deviation = 15
D. mean = 145 standard deviation = 20
E. mean = 150 standard deviation = 10

Source: *VCE 2019, Further Mathematics Exam 1, Section A, Q.6; © VCAA*

Question 2 (1 mark)

The time taken to *travel* between two regional cities is approximately normally distributed with a mean of 70 minutes and a standard deviation of 2 minutes.

The percentage of travel times that are between 66 minutes and 72 minutes is closest to

A. 2.5%
B. 34%
C. 68%
D. 81.5%
E. 95%

© John Wiley & Sons Australia, Ltd

Source: *VCE 2015, Further Mathematics Exam 1, Section A, Q.1; © VCAA*

Question 3 (1 mark)

The stem plot below displays the average number of decayed teeth in 12-year-old children from 31 countries.

key: 0|2 = 0.2

0	2
0	5 6 7 7 8 9
1	0 0 0 0 1 4 4 4
1	5 6 7
2	3 3 4
2	7 7 8 9
3	0 4
3	5 6
4	1
4	7

Data: Gapminder

Based on this stem plot, the distribution of the average number of decayed teeth for these countries is best described as

A. negatively skewed with a median of 15 decayed teeth and a range of 45.
B. positively skewed with a median of 15 decayed teeth and a range of 45.
C. approximately symmetric with a median of 1.5 decayed teeth and a range of 4.5.
D. negatively skewed with a median of 1.5 decayed teeth and a range of 4.5.
E. positively skewed with a median of 1.5 decayed teeth and a range of 4.5.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics 1, Section A, Q.5; © VCAA*

Question 4 (1 mark)

The stem plot below shows the distribution of mathematics *test scores* for a class of 23 students.

key: 4|2 = 42 $n = 23$

4	0	1	4	4		
5	2	7	9	9	9	
6	5	6	8	8	9	9
7	0	0	5	6	7	8
8	5	9				

For this class, the interquartile range (IQR) of *test scores* is

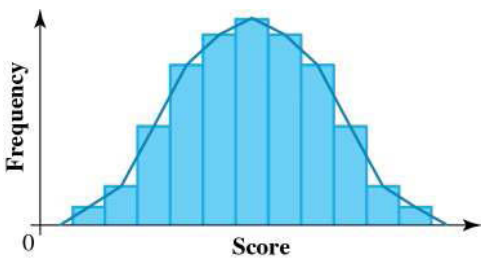
A. 14.5
B. 17.5
C. 18
D. 24
E. 49

© John Wiley & Sons Australia, Ltd

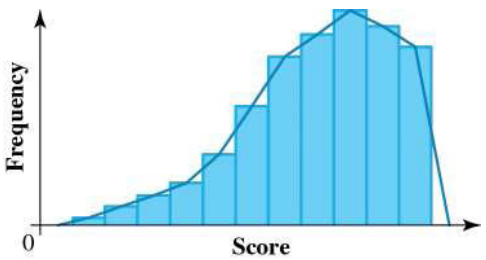
Question 5 (1 mark)

The histogram that represents a set of negatively skewed data is:

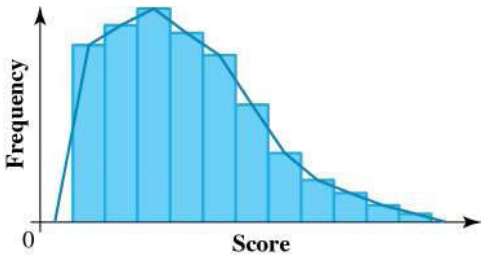
A.



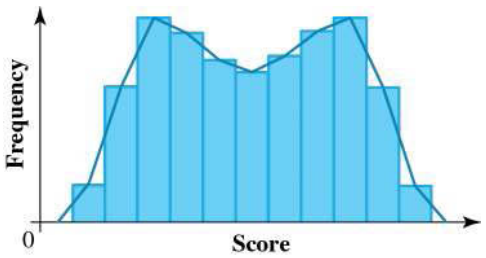
B.



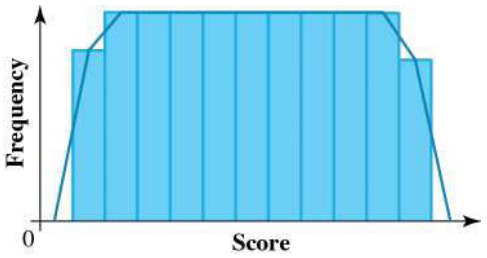
C.



D.

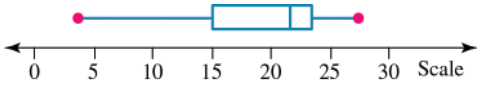


E.



© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)
Consider the boxplot shown.



Which of the following statements is most likely *false*?
A. The IQR for this data could be 8.
B. The data represented here is positively skewed.
C. The minimum value recorded could be 4.
D. The range could be 24.
E. There are no outliers marked on this boxplot.

Question 7 (4 marks)
A group of 28 students had their heights recorded. See the data set below.

123	125	126	124	111	135	147	125	128	123	126	128	124	124
120	122	125	123	110	130	140	127	128	123	120	149	125	127

a. Construct a frequency table using a suitable class interval. **(1 mark)**

b. Construct a histogram to represent these results. **(1 mark)**

c. Comment on the distribution. **(2 marks)**

© John Wiley & Sons Australia, Ltd

Topic	1	Investigating and comparing data distributions
Subtopic	1.10	The five-number summary and boxplots

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2020, Further Mathematics Exam 1, Section A, Q.3; © VCAA*

Question 1 (1 mark)

The times between successive nerve impulses (time), in milliseconds, were recorded.

Table 1 shows the mean and the five-number summary calculated using 800 recorded data values.

Table 1

	***Time* (milliseconds)**
Mean	220
Minimum value	10
First quartile (Q_1)	70
Median	150
Third quartile(Q_3)	300
Maximum value	1380

Data: Adapted from P Fatt and B Katz, 'Spontaneous subthreshold activity at motor nerve endings', *The Journal of Physiology*, 117, 1952, pp. 109–128

The shape of the distribution of these 800 times is best described as

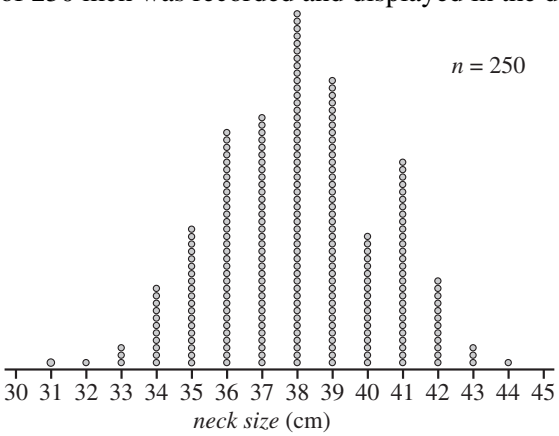
A. approximately symmetric.
B. positively skewed.
C. positively skewed with one or more outliers.
D. negatively skewed.
E. negatively skewed with one or more outliers.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 2, Section A, Q.2c; © VCAA*

Question 2 (2 marks)

The *neck* size, in centimetres, of 250 men was recorded and displayed in the dot plot below.

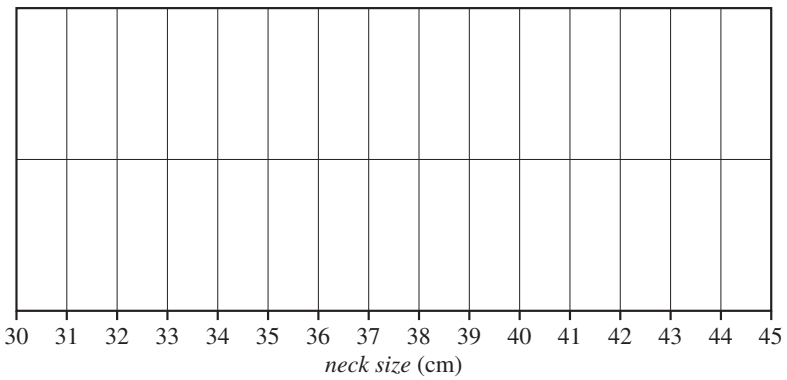


Data: RW Johnson, 'Fitting percentage of body fat to simple body measurements', *Journal of Statistics Education*, 4:1, 1996, <https://doi.org/10.1080/10691898.1996.11910505>

The five-number summary for this sample of neck sizes, in centimetres, is given below.

Minimum	**First quartile** (Q_1)	**Median**	**Third quartile** (Q_3)	**Maximum**
31	36	38	39	44

Use the five-number summary to construct a boxplot (on a grid as shown below) in your workbook, showing any outliers if appropriate.

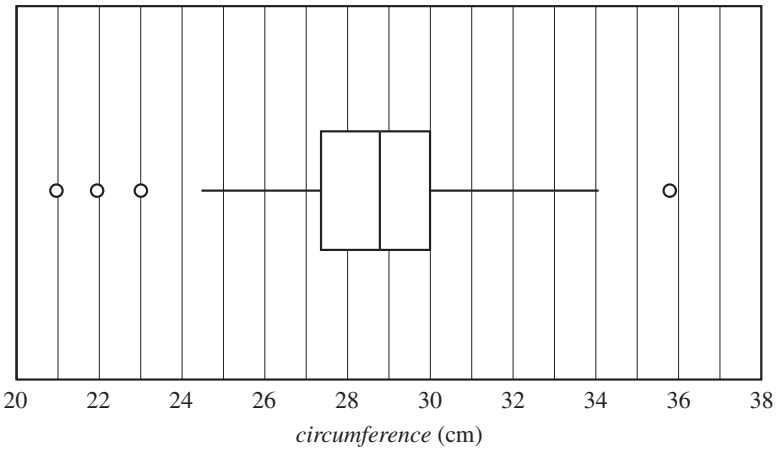


© John Wiley & Sons Australia, Ltd

Source: *VCE 2017, Further Mathematics Exam 1, Section A, Core, Q.2; © VCAA*

Question 3 (1 mark)

The boxplot below shows the distribution of the forearm *circumference*, in centimetres, of 252 people.



The five-number summary for the forearm *circumference* of these 252 people is closest to

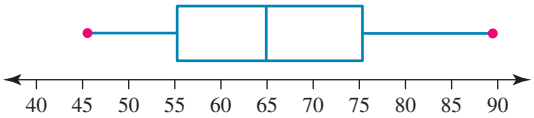
A. 21, 27.4, 28.7, 30, 34
B. 21, 27.4, 28.7, 30, 35.9
C. 24.5, 27.4, 28.7, 30, 34
D. 24.5, 27.4, 28.7, 30, 35.9
E. 24.5, 27.4, 28.7, 30, 36

Question 4 (1 mark)

The five-number summary for a distribution is 7, 9, 12, 15, 17.
The interquartile range (IQR) is

A. 15
B. 10
C. 9
D. 6
E. 3

Question 5 (1 mark)



For this boxplot, the range is

A. 55 to 75
B. 20
C. 45
D. 45 to 90
E. 65

Question 6 (1 mark)

Which of the following is **not** true for a five-number summary?

A. It is not possible to find the shape of the distribution.
B. Half the scores are less than the middle number.
C. Half the scores are between the second and fourth numbers.
D. The range is found from the difference between the first and last numbers.
E. The average score is easy to calculate.

Question 7 (1 mark)

The five-number summary for a distribution is 15 18 20 21 27.

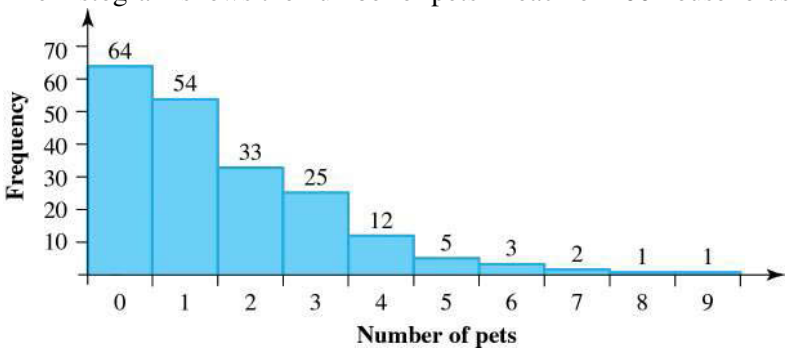
A mild outlier is

A. 26
B. 24
C. 23
D. 22
E. 21

© John Wiley & Sons Australia, Ltd

Question 8 (1 mark)

The histogram shows the number of pets in each of 200 households.



Which of the following is a true statement?

A. Scores of 5 pets are mild outliers.
B. Scores of 8 pets or more are strong outliers.
C. There is not enough information given to find fences and outliers.
D. Using quartiles and the median for fences gives false values when the distribution is skewed.
E. There are no lower outliers.

Question 9 (1 mark)

The five-number summary for a distribution is 52 74 90 100 140.
The upper inner fence is

A. 178
B. 140
C. 139
D. 100
E. 35

© John Wiley & Sons Australia, Ltd

Topic	1	Investigating and comparing data distributions
Subtopic	1.11	Comparing the distribution of a numerical variable across two or more groups

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2021, Further Mathematics Exam 1, Section A, Core, Q.6; © VCAA*

Question 1 (1 mark)

The relationship between *resting pulse rate*, in beats per minute, and *age group* (15–20 years, 21–30 years, 31–50 years, over 50 years) is best displayed using

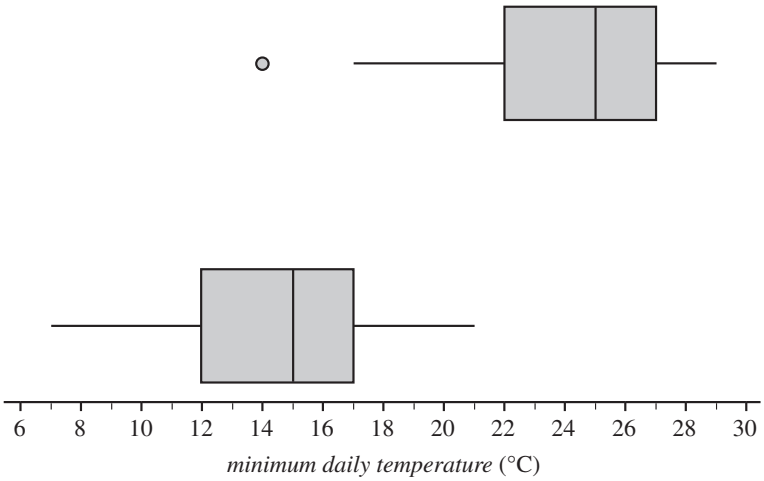
A. a histogram.
B. a scatterplot.
C. parallel boxplots.
D. a time series plot.
E. a back-to-back stem plot.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 2, Section A, Q.2; © VCAA*

Question 2 (4 marks)

The parallel boxplots below show the *maximum daily temperature and minimum daily temperature*, in degrees Celsius, for 30 days in November 2017.



Data: Australian Government, Bureau of Meteorology, <www.bom.gov.au/>

a. Use the information in the boxplots to complete the following sentences. For November 2017

i. the interquartile range for the *minimum daily temperature* was_____ °C **(1 mark)**

ii. the median value for *maximum daily temperature* was _____ °C higher than the median value for *minimum daily temperature* **(1 mark)**

iii. the number of days on which the *maximum daily temperature* was less than the median value for *minimum daily temperature* was _____. **(1 mark)**

b. The *temperature difference* between the *minimum daily temperature* and the *maximum daily temperature* in November 2017 at this location is approximately normally distributed with a mean of 9.4°C and a standard deviation of 3.2°C. **(1 mark)**

Determine the number of days in November 2017 for which this *temperature difference* is expected to be greater than 9.4°C.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics Exam 1, Section A, Q.8; © VCAA*

Question 3 (1 mark)

Parallel boxplots would be an appropriate graphical tool to investigate the association between the monthly median rainfall, in millimetres, and the

A. monthly median wind speed, in kilometres per hour.
B. monthly median temperature, in degrees Celsius.
C. month of the year (January, February, March, etc.).
D. monthly sunshine time, in hours.
E. annual rainfall, in millimetres.

Question 4 (1 mark)

A back-to-back stemplot shows the ages of men and women in a sample when they had their first children.

Men		Women
Leaf	Stem	Leaf
8 7	1	6 7 8 9
9 8 6	2	0 2 2 3 3 4 4 5 7 8
2 3 5 5 6 7 7	3	0 1 2 2 3 4 4 7 8 9
0 3 3 4 5 6 8	4	0 1 1
3 4 6 7 9	5	
0 2 4 5	6	
Key 8|1 = 18 years old		Key 1|8 = 18 years old

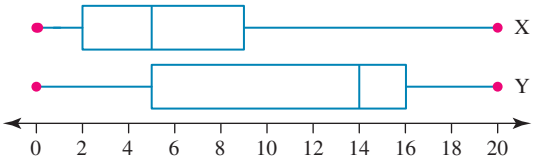
Which of the following statements is **not** true?

A. Men in the sample generally have their first child later than women because the median age of men is higher than that of women.
B. The variability of the ages for men in the sample is greater than for women.
C. The distribution of ages of men is skewed.
D. No women had children when they were over the age of 45 years.
E. There were more women in their 30s when they had their first child than men in their thirties having their first child.

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)

Which of the statements below are true for these boxplots?



1. X has a higher median and more variability than Y.
2. Y has a higher median and more variability than X.
3. Data from X are bimodal.
4. Data from Y are negatively skewed.
5. The third quartile of X is greater than the third quartile of Y.

A. I and III
B. I and V
C. III and IV
D. II and IV
E. II and V

Question 6 (1 mark)

Which of the statements below are **not** true for this back-to-back stemplot?

X		*Y*
Leaf	**Stem**	**Leaf**
2 8	1	0 3 6
3 6	2	7 8 8
4 5 9	3	4 6 6 7 8
3 8 8 8	4	2 3
0 4 7 7	5	0 2 2 6
2 2 4 5 6 8	6	0 0 5 5 8 9 9
0 1 1 5	7	2 2 3 4 4 6 8 8
3 9	8	5 6 7
Key 8|1 = 18 mm		Key 1|0 = 10 mm

1. Y is bimodal.
2. X is positively skewed.
3. X has a larger IQR than Y.
4. X has a larger range than Y.
5. X has a smaller median than Y.

A. I and II
B. II and III
C. III and IV
D. I and V
E. IV and V

© John Wiley & Sons Australia, Ltd

Topic	1	Investigating and comparing data distributions
Subtopic	1.12	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2021, Further Mathematics Exam 1, Section A, Core, Q.5; © VCAA*

Question 1 (1 mark)

The stem plot below shows the *height*, in centimetres, of 20 players in a junior football team.

key: 14|2 = 142 cm $n = 20$

14	2	2	4	7	8	8	9	
15	0	0	1	2	5	5	6	8
16	0	1	1	2				
17	9							

A player with a height of 179 cm is considered an outlier because 179 cm is greater than

A. 162 cm
B. 169 cm
C. 172.5 cm
D. 173 cm
E. 175.5 cm

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 1, Section A, Q.2; © VCAA*

Question 2 (1 mark)

The times between successive nerve impulses (time), in milliseconds, were recorded.

Table 1 shows the mean and the five-number summary calculated using 800 recorded data values.

	***Time* (milliseconds)**
Mean	220
Minimum value	10
First quartile (Q_1)	70
Median	150
Third quartile (Q_3)	300
Maximum value	1380

Data: Adapted from P Fatt and B Katz, 'Spontaneous subthreshold activity at motor nerve endings', *The Journal of Physiology*, 117, 1952, pp. 109 – 128

Of these 800 times, the number of times that are longer than 300 milliseconds is closest to

A. 20
B. 25
C. 75
D. 200
E. 400

Source: *VCE 2019, Further Mathematics Exam 2, Section A, Q.3; © VCAA*

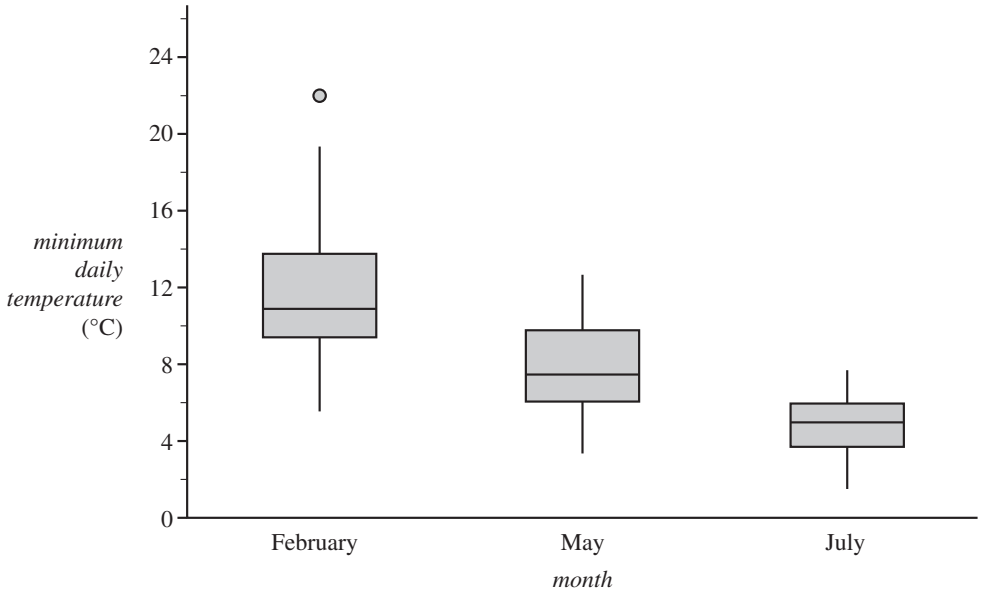
Question 3 (2 marks)

The five-number summary for the distribution of *minimum daily temperature* for the months of February, May and July in 2017 is shown in Table 2.

The associated boxplots are shown below the table.

Table 2. Five-number summary for *minimum daily temperature*

Month	Minimum	Q_2	Median	Q_3	Maximum
February	5.9	9.5	10.9	13.9	22.2
May	3.3	6.0	7.5	9.8	12.7
July	1.6	3.7	5.0	5.9	7.7



Data: Australian Government, Bureau of Meteorology, <www.bom.gov.au/>

Explain why the information given above supports the contention that *minimum daily temperature* is associated with the *month*. Refer to the values of an appropriate statistic in your response.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2017, Further Mathematics Exam 2, Section A, Core, Q.2; © VCAA*

Question 4 (5 marks)

The back-to-back stem plot below displays the wingspan, in millimetres, of 32 moths and their *place of capture* (forest or grassland).

Key: 1|8 = 18 ***wingspan*** **(mm)**

forest (n = 13)		**grassland** (n = 19)
6	1	8
2 1 1 0 0 0 0	2	2 2 4 4
7	2	5 5 9
4 0	3	0 0 1 2 3 4
5	3	6 8
	4	0 3
	4	
2	5	

a. Which variable, *wingspan or place of capture*, is a categorical variable? **(1 mark)**

b. Write down the modal wingspan, in millimetres, of the moths captured in the forest. **(1 mark)**

c. Use the information in the back-to-back stem plot to complete the table below. **(1 mark)**

	Wingspan **(mm)**				
Place of capture	**minimum**	**Q_1**	**median (M)**	**Q_3**	**maximum**
forest		20	21	32	52
grassland	18	24	30		45

d. Show that the moth captured in the forest that had a *wingspan* of 52 mm is an outlier. **(1 mark)**

e. The back-to-back stem plot suggests that *wingspan* is associated with *place of capture*. Explain why, quoting the values of an appropriate statistic. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics Exam 2, Core, Q.2; © VCAA*

Question 5 (6 marks)

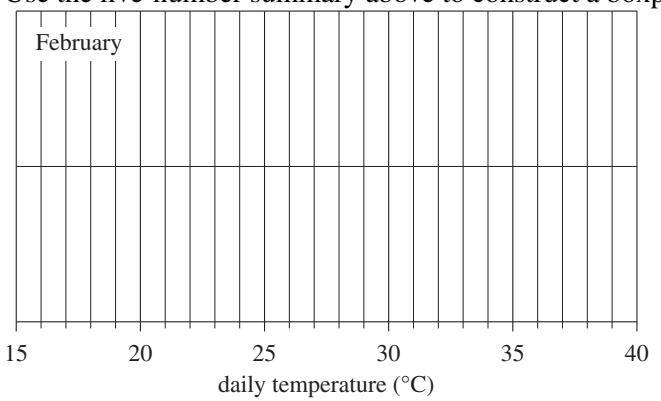
The weather station also records daily maximum temperatures.

a. The five-number summary for the distribution of maximum temperatures for the month of February is displayed in the table below. **(1 mark)**

	Temperature(°C)
Minimum	16
Q_1	21
Median	25
Q_3	31
Maximum	38

There are no outliers in this distribution.

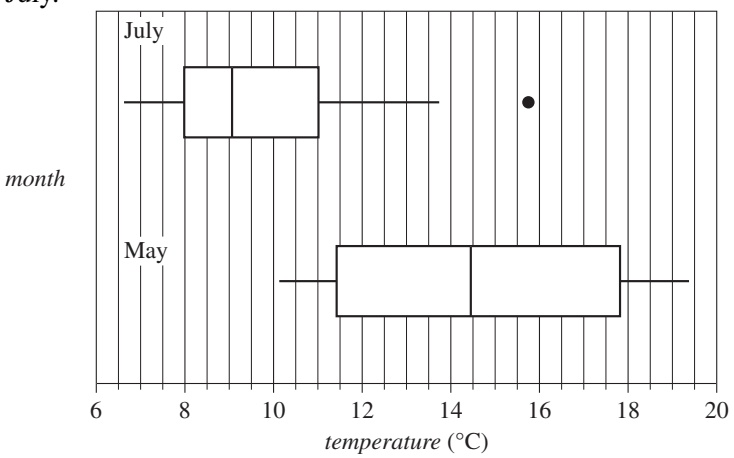
i. Use the five-number summary above to construct a boxplot on the grid below. **(1 mark)**



ii. What percentage of days had a maximum temperature of 21°C, or greater, in this particular February? **(1 mark)**

© John Wiley & Sons Australia, Ltd

b. The boxplots below display the distribution of maximum daily temperature for the months of May and July.



i. Describe the shapes of the distributions of daily *temperature* (including outliers) for July and for May. **(1 mark)**

July _____

May _____

ii. Determine the value of the upper fence for the July boxplot. **(1 mark)**

iii. Using the information from the boxplots, explain why the maximum daily *temperature* is associated with the *month* of the year. Quote the values of appropriate statistics in your response. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 6 (4 marks)

A group of children who play sport on Saturday were surveyed to find the number playing each sport. The results are shown in the table.

Sport	Number playing
Football	49
Soccer	56
Netball	38
Hockey	19
Basketball	14
Cricket	44

a. Create a bar chart showing the frequency distribution of the sports played. **(1 mark)**

b. What is the mode of the distribution? **(1 mark)**

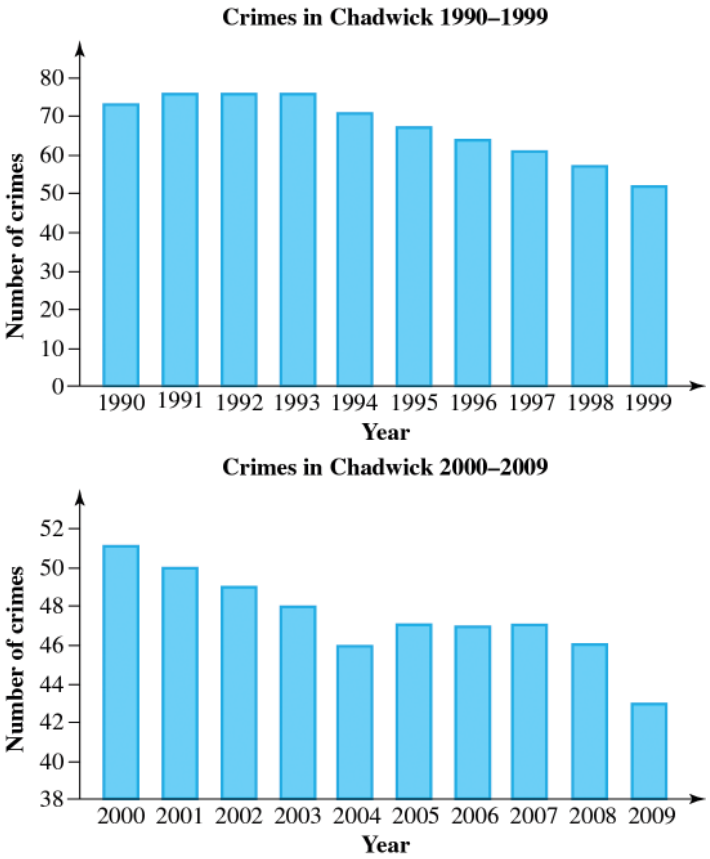
c. How many children were surveyed? (Assume each only plays one sport.) **(1 mark)**

d. What proportion of children played cricket? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 7 (4 marks)

Chadwick Police Department collected data on the number of crimes committed in the city during two decades, 1990–1999 and 2000–2009. The results were shown on bar charts.



a. Which decade had the highest number of crimes committed altogether? **(1 mark)**

b. In which year(s) were the greatest number of crimes committed? **(1 mark)**

c. In which year(s) were the least number of crimes committed? **(1 mark)**

d. How many crimes were committed in 1999? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 8 (1 mark)
The stemplot shows the weights of a random sample of 25 packets of sugar labelled 1 kg.

Stem	Leaf
9	4 5 5 6 7 8 8 8 8 9 9 9
10	0 0 0 1 1 1 1 2 2 2 5 5
11	0
Key 9	4 = 940 g

Which of the following is **not** true?
A. The median is 1 kg.
B. The distribution is bimodal.
C. The mode is 1.1 kg.
D. The median is the 13th score.
E. The score of 1.1 kg is an outlier.

Question 9 (4 marks)
Data was collected on the number of pedestrians in accidents in a city.

Age group (years)	Pedestrians killed
0–	16
10–	40
20–	75
30–	52
40–	72
50–	89
60–	54
70–	38
80–100	30

a. How many pedestrians were killed altogether in the city? **(1 mark)**

b. What is the modal class of the data? **(1 mark)**

c. In which age group is the median number killed? **(1 mark)**

d. What percentage of pedestrians killed were 60 or older? Is that percentage greater or less than the percentage younger than 30? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 10 (4 marks)
The weights of babies at a crèche are as follows.

Weight (kg)	Frequency
7	2
8	4
9	6
10	2
11	1
12	1

Find:

a. the number of babies in the crèche **(1 mark)**

b. the modal weight of the babies **(1 mark)**

c. the median weight of the babies **(1 mark)**

d. the mean weight of the babies, correct to four decimal places. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 11 (5 marks)

The average petrol consumption of 100 randomly selected cars is 6.63 litres per 100 kilometres, and the distribution is shown in the table.

Fuel consumption (litres per 100 km)	Frequency
3	5
4	8
5	10
6	18
7	25
8	25
9	8
10	
11	1

a. What is the range of petrol consumption? **(1 mark)**

b. What percentage of the cars in the sample had a petrol consumption of less than 7 litres per 100 km? **(1 mark)**

c. Find the IQR of petrol consumptions in the sample. **(1 mark)**

d. Use a calculator to find the standard deviation of petrol consumptions in the sample. **(1 mark)**

e. For each of the mean, range, IQR and standard deviation, state what effect the inclusion of the outlier has on the result. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 12 (6 marks)

The radius of discs made by a machine varies with the following distribution.

Radius (mm)	Frequency
22	7
23	6
24	30
25	45
26	10
27	2

The mean radius of discs in the sample is $\bar{x} = 24.51$ mm.

a. What is the range of radii in the sample? **(1 mark)**

b. Use a calculator to find the standard deviation of radii in the sample, correct to four decimal places. **(1 mark)**

c. Use a calculator to find the distribution of the areas of discs. **(1 mark)**

d. Hence, find the mean and standard deviation of the areas of discs. **(1 mark)**

e. Could the mean of the areas have been found from just squaring the mean radius and multiplying by π $\left(\overline{A} = \pi\bar{x}^2\right)$? **(1 mark)**

f. Could the standard deviation of the areas have been found from just squaring the standard deviation of the radius and multiplying by π $\left(sd_A = \pi(sd_\pi)^2\right)$? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 13 (4 marks)
The number of children per family in two different suburbs was surveyed. The data is summarised in five-number summaries.
Suburb A: 0 1 2 3 5
Suburb B: 0 1 3 5 7
For the two different suburbs:

a. find the difference in the median numbers of children per family **(1 mark)**

b. find the difference in the ranges of numbers of children per family **(1 mark)**

c. draw boxplots for the data **(1 mark)**

d. compare the number of children per family in the two suburbs. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 14 (3 marks)

The stemplot of data from a survey is shown below.

Stem	Leaf
3	5 9
4	1 6 8 9
5	3 3 3 5 9 9
6	0 1 3 3 4 6 6 9 9
7	1 1 1 3 5 7
8	0 3 3 5
9	7
Key 3	5 = 35

a. How many scores were collected in the survey? **(1 mark)**

b. Find the five-number summary for the data. **(1 mark)**

c. A boxplot for data from a new survey showed a range of 60 and an IQR of 20. Are these survey results likely to be from the same group as the first survey? Why? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 15 (4 marks)

The cost per week of renting a certain type of flat in the city was surveyed and the data organised in a table.

Rent($)	Frequency
200	2
225	6
250	18
275	31
300	18
325	9
350	9
375	5
400	1
425	1

For the flats surveyed, find:

a. the range of rents **(1 mark)**

b. median rent **(1 mark)**

c. the lower and upper quartile ranges **(1 mark)**

d. if a rent of $450 per week would be considered to be unreasonably high for this type of flat. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 16 (4 marks)
The heights of Year 12 students were measured and the data organised to give the following stemplot.

Stem	Leaf
13	5
14	5
15	5 7 7
16	0 0 0 0 1 3 5 8
17	3 5 5 5 5
18	1 3
key 13|5	= 135

For the sample of measurements:

a. find the range of heights **(1 mark)**

b. find the median height **(1 mark)**

c. find the lower and upper quartile ranges **(1 mark)**

d. draw a boxplot and show the quartiles and fences. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 17 (4 marks)

The average monthly rainfall (in mm) in City A and City B in 12 months is shown in this back-to-back stemplot.

City A		City B
Leaf	Stem	Leaf
8 9 9	4	
0 0 4 8 8 9 9 9	5	
7	6	
	7	0 8
	8	2
	9	
	10	0 6
	11	2
	12	4 6 8 9
Key 9|4 = 49 mm		Key 7|8 = 78 mm

a. Complete the five-number summaries for the two cities. **(1 mark)**

City A:	48	49.5	?	59	?
City B:	?	82	109	?	129

b. Draw parallel boxplots of the data. **(1 mark)**

c. Use these statistics to compare the rainfall in the two cities. **(1 mark)**

d. Which city would be considered to be wetter and why? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 18 (4 marks)

Samples of spaghetti are taken from two machines and the lengths measured (in cm).
The five-number summaries for the two samples are shown below.

A	17	17.6	17.96	18.24	19
B	17	18.02	20	20.3	21

a. Draw parallel boxplots to illustrate the data. **(1 mark)**

b. Comment on the difference between the two median lengths. **(1 mark)**

c. Which machine produces the greatest variability of lengths? **(1 mark)**

d. Machine A is functioning correctly.
Find the fences for the data for machine A and state whether machine B is producing lengths which are outliers. **(1 mark)**

Question 19 (1 mark)

The mean, correct to 2 decimal places, of the data represented in the following frequency distribution table is:

Score (x)	Frequency (f)
1	12
2	13
3	15
4	20
Total	60

A. 2.72
B. 0.15
C. 0.07
D. 2.52
E. 3.26

© John Wiley & Sons Australia, Ltd

Question 20 (1 mark)

Thirty Year 11 students were asked to measure the number of hours of homework they completed each week. The data are recorded below.

12	11	10	11	9	8	5	13	14	12	12	11	10	11	9
8	5	13	14	12	12	11	10	11	9	8	5	13	2	2

Which of the following statements about this data is *incorrect?*

A. The mean number of hours of homework completed was 9.8.

B. The median number of hours of homework completed was 11.

C. The IQR of the data was 4.

D. The range of the data was 30.

E. The five-figure summary for the data set is:

$\text{Min}\, X = 2$

$Q_1 = 8$

$\text{Med} = 11$

$Q_3 = 12$

$\text{Max}\, X = 14$

Question 21 (25 marks)

The following stem-and-leaf plot details the age of 26 offenders who were caught drink driving during a Friday night blitz.

Male offenders (Leaf)	Stem	Female offenders (Leaf)
7 7 7 7 8 8 8 8 8 9	1	8 8 8
1 1 1 1 1 1	2	1 1 2 4
7	3	
	4	2 5
key:	1|8 =	18 years old

Note: Give answers to 1 decimal place where appropriate.

a. Calculate the mean, median, mode and standard deviation for the male offenders. **(4 marks)**

b. Calculate the five-figure summary, and hence the range and IQR, for the male offenders. **(3 marks)**

c. Calculate the mean, median, mode and standard deviation for the female offenders. **(4 marks)**

© John Wiley & Sons Australia, Ltd

d. Calculate the five-figure summary, and hence the range and IQR, for the female offenders. **(3 marks)**

e. Comment on the distribution for the male offenders. **(1 mark)**

f. Comment on the distribution for the female offenders. **(1 mark)**

g. Identify any outliers. **(4 marks)**

h. Using statistics, write a paragraph about the results of the Friday night drink driving blitz. **(5 marks)**

© John Wiley & Sons Australia, Ltd

Answers and marking guide

1.2 Types of data

Question 1
The variables age and *preferred travel destination* are both categorical variables. Categories have been given for both.

Question 2
The second variable also needs to have a small number of categories. The only possibility is sex (male, female).
***VCAA Examination Report* note:**
Students needed to recognise that for a two way-frequency table to be used, both variables had to be categorical variables.

Question 3
Both *blood pressure* and *age*, in this instance, are ordinal variables, as they have an *order*.
***VCAA Assessment Report* note:**
Many students incorrectly identified the variable *age* (under 50 years, 50 years or over) as nominal. The variable *age* (under 50 years, 50 years or over) is ordinal because the process of allocating each of the people to one of these two categories 'under 50 years' or '50 years or over' orders the group of people by age.

Question 4
Discrete data is data that can be counted.

Question 5
Nominal data is categorical data where order is not important.

Question 6
The statement which is not an example of categorical data is 'Height of 10 students in your class'. Height measurements are numerical, continuous data.

1.3 Categorical data distributions

Question 1
The wind direction with the highest frequency (41) was north-west.

Question 2
Total number of days on which the wind direction was east or south-east was $10 + 25 = 35$.
The percentage is $\frac{35}{214} \times 100\% = 16.36\%$; the closest is 16%.

Question 3
There are 5 families with 5 children, therefore B is false.

Question 4
Compare values in the table with the heights of the rectangles in the bar charts.

Question 5
Add the frequency of each car type.
Total cars counted $= 5 + 15 + 25 + 5$
$= 50$

© John Wiley & Sons Australia, Ltd

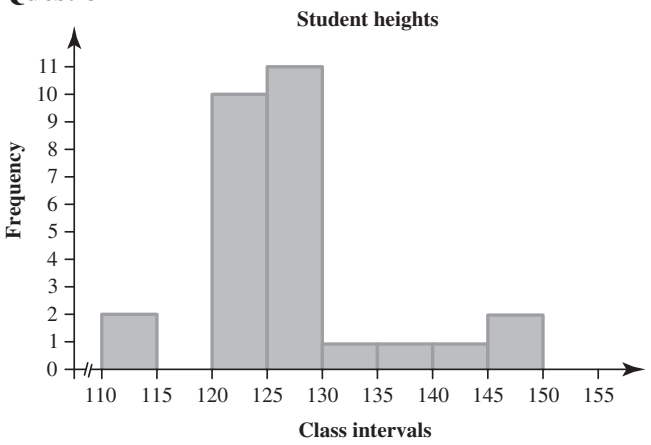
1.4 Numerical data distributions — frequency tables and histograms

Question 1

Class interval (5 cm)	Frequency
110–114	2
115–119	0
120–124	10
125–129	11
130–134	1
135–139	1
140–144	1
145–149	2

[1 mark]

Question 2

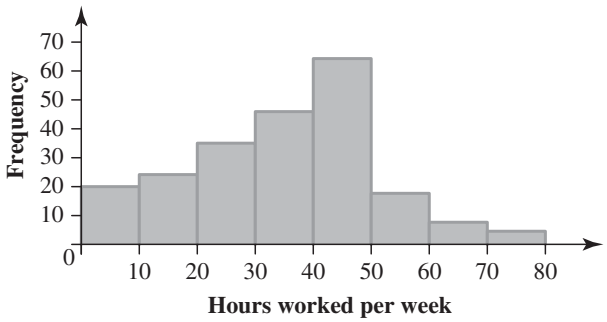


[1 mark]

Question 3

a. The total number of workers surveyed is the total of the frequencies in the table.
$20 + 24 + 35 + 46 + 64 + 18 + 7 + 5 = 220$ **[1 mark]**

b. **[1 mark]**



c. The mode is the midpoint of the modal class (40 to less than 50 hours per week).
45 hours per week **[1 mark]**

Question 4

Total all the frequencies on the histogram.
$8 + 6 + 7 + 10 + 13 + 6 = 50$

© John Wiley & Sons Australia, Ltd

1.5 Numerical data distributions — dot plots and stem plots

Question 1

The mode is the most frequent value, which is 38 cm. **[1 mark]**

Question 2

a. Day number **[1 mark]**

***VCAA Examination Report* note:**

Most responses given to this question were correct. A small number of students answered 'neither'.

b. key: 4|1 = 4.1 $n = 15$

minimum temperature (°C)

4	1 8
5	
6	0 7
7	0 **5 7**
8	**0 6**
9	0 2 **8**
10	7
11	8
12	7

Award **1 mark** for all five numbers in correct positions.

***VCAA Examination Report* note:**

Some students entered only the values for day 11 and day 15 rather than the five days from day 11 to day 15.

Question 3

Mode means the most frequent score.

∴ 2.8°C is the modal temperature.

Question 4

Look at the range of numbers and the most frequent number compared with those in the tables.

Number	Frequency
0	15
1	18
2	24
3	10
4	6
5	4
6	2
7	1
8	0

© John Wiley & Sons Australia, Ltd

Question 5

Stem	Leaf
0	1 2 3 3 4 4 5 5 7 8 9 9
10	1 2 3 3 4 4 5 6 7 9
20	1 2 4 4 4 6 8
30	1 3 5 7 9
40	2 5 8
50	3
60	4
70	
80	1
10|1 = 11,000	

Question 6

a.

Marks	Frequency
0–9	
10–19	
20–29	2
30–39	1
40–49	1
50–59	6
60–69	4
70–79	4
80–89	4
90–100	3

[1 mark]

b. 25 **[1 mark]**

c.

Stem	Leaf
	0 7
3	3
4	0
5	0 0 3 3 3 7
6	0 0 3 7
7	0 0 3 7
8	0 3 3 7
9	0 0 3
Key 2	1 = 21

[1 mark]

d.

English		Maths
Leaf	Stem	Leaf
1 5	2	0 7
2 4 5	3	3
1 2 5 5 7 8	4	0
1 1 3 4 4 6 9 9	5	0 0 3 3 3 7
2 5 7 8	6	0 0 3 7
1 3	7	0 0 3 7
	8	0 3 3 7
	9	0 0 3
Key 5|2 = 25		Key 2|7 = 27

[1 mark]

© John Wiley & Sons Australia, Ltd

On average it can be seen that the students performed better in Maths than in English because the marks in English are generally lower than those in Maths. Also, there are no high marks in English. **[1 mark]**

Question 7

Total people surveyed $= 40 + 20 + 5 + 15 + 25 + 5$

$= 110$

Percentage of people who liked *Supernatural* or *One Tree Hill* $= \frac{40 + 25}{110} \times 100$

$= \frac{65}{110} \times 100$

$= 59.1\%$

1.6 Characteristics of numerical data distributions

Question 1

The data is bunched down the lower end of the histogram with a tail going to the right; therefore, it is positively skewed.

Given the large range of the data, it is likely that the data value at the upper end is an outlier.

Question 2

There is an outlier present and the data trails on the lower end, so the distribution is negatively skewed with an outlier.

Question 3

The data trails off on the positive end so the distribution is positively skewed.

Question 4

This data is bimodal, as the two modal classes are 20– (75 killed) and 50– (89 killed).

1.7 Summarising numerical data — mean and median

Question 1

a. The median age is 24 years. **[1 mark]**

b. $\frac{\sum \text{body} \quad \text{density}}{12} = 1.065\,\text{kg/L}$ **[1 mark]**

Question 2

The median BMI will be in the $\left(\frac{32+1}{2}\right) = 16.5$th position.

Therefore, the median is 24.55 kg/m^2. **[1 mark]**

Question 3

The middle value of any data set is always the median.

The mean is dependent on extreme values, and we do not have the information to know if there are any, so we cannot assume that the middle value will also be the mean.

Question 4

The mean will decrease.

Question 5

Order the data:

0 1 1 2 3 3 3 4 4 5 7 8 9 12

Find the middle two numbers.

© John Wiley & Sons Australia, Ltd

$$\text{Median} = \frac{7\text{th} + 8\text{th numbers}}{2}$$
$$= \frac{3+4}{2}$$
$$= \frac{7}{2}$$
$$= 3.5$$

1.8 Summarising numerical data — range, interquartile range and standard deviation

Question 1

IQR for average neck size $= 26.0 - 23.4 = 2.6$ **[1 mark]**

Question 2

$$\text{IQR} = Q_3 - Q_1$$
$$= 75 - 57$$
$$= 18$$

Question 3

Using a CAS calculator, the standard deviation is calculated to be 3.145. The nearest whole number is therefore 3. Students would not be expected to calculate this by hand.

Question 4

$$x_{\max} - x_{\min} = (+3) - (-6)$$
$$= 9\,\text{kg}$$

Question 5

$$\text{IQR} = 28 - 24$$
$$= 4$$

Question 6

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i f_i}{n}$$

Mean $= 2.313$

$$v = \frac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^2}{n-1}$$

Variance $= 1.296$

$$s = \sqrt{\frac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^2}{n-1}}$$

Standard deviation $= 1.138$

Range $= 4.000$

Range of squared deviations $= 0.348$

1.9 Symmetrical and asymmetrical distributions

Question 1

Greater than 16% is at 1 standard deviation above the mean: $\overline{x} + s = 160$

Less than 2.5% is at 2 standard deviations below the mean: $\overline{x} - 2s = 115$

Solve using CAS: $\overline{x} = 145$ and $s = 15$

© John Wiley & Sons Australia, Ltd

Question 2

The question asks for the percentage between the mean minus 2 standard deviations and the mean plus 1 standard deviation. That means the answer will be between 68% and 95%.

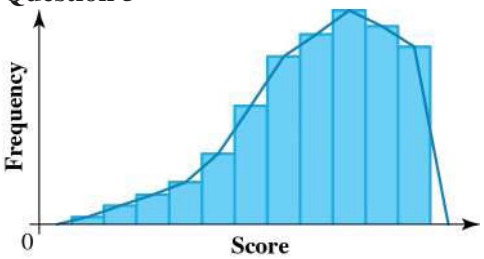
Question 3

There are 31 countries, so the median is the $\frac{31+1}{2} = 16$th term. The 16th term is 1.5.
The range is the maximum minus the minimum: $4.7 - 0.2 = 4.5$.
Finally, the stem plot is positively skewed.

Question 4

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 75 - 57 \\ &= 18\end{aligned}$$

Question 5



This histogram represents negatively distributed data.

Question 6

The incorrect statement is **B**. 'The data represented here is positively skewed.'
The data is negatively skewed.
All other comments could be true.

Question 7

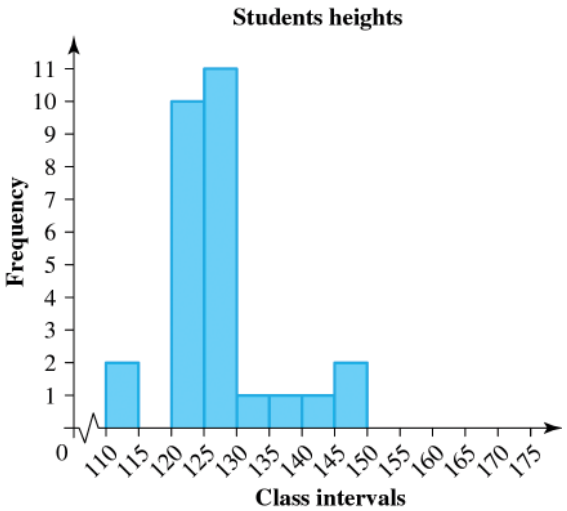
a.

Class interval (5 cm)	Frequency
110–114	2
115–119	0
120–124	10
125–129	11
130–134	1
135–139	1
140–144	1
145–149	2

[1 mark]

© John Wiley & Sons Australia, Ltd

b.



[1 mark]

c. The data is positively skewed. **[1 mark]**

The data is clustered between 120 cm and 130 cm. **[1 mark]**

1.10 The five-number summary and boxplots

Question 1

The value of the mean (220) being greater than the median (150) suggests a positively skewed data set. To determine if there are outliers, we need to calculate the fences:

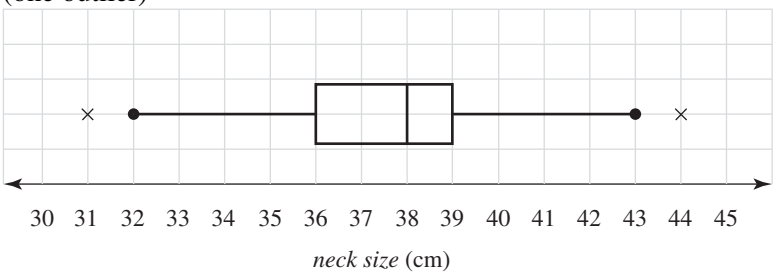
$LF = Q_1 - 1.5 \times IQR = 10 - 1.5 \times 230 = -335$ ∴no outliers

$UF = Q_3 + 1.5 \times IQR = 300 + 1.5 \times 230 = 750$

This suggests that at *least* the maximum value (1380) is an outlier, but there may be more.

Question 2

First, check for outliers. $LF = 36 - 1.5 \times 3 = 31.5$ (one outlier at the lower end), $UF = 39 + 1.5 \times 3 = 43.5$ (one outlier)



Award **1 mark** for outliers.

Award **1 mark** for the boxplot.

Question 3

The five-number summary consists of the smallest measurement of 21cm, lower quartile, $Q_1 = 27.4$ cm, median of 28.7 cm, upper quartile, $Q_3 = 30$ cm and the largest measurement of 35.9 cm.

***VCAA Examination Report* note:**

Students were required to interpret the provided boxplot and choose the correct five-number summary for the data. While many students did this correctly, a large number of students ignored the outlier points when determining the maximum and minimum values for the five-number summary, leading to the choice of option C, which was incorrect. Even though the points are identified as outliers, they are still valid data points within the data set and must be used as maximum and minimum values if appropriate.

Question 4

From the list, the IQR is 9 to 15, or 6.

© John Wiley & Sons Australia, Ltd

Question 5
From the boxplot, the range is 45 to 90, or 45.

Question 6
The average score is easy to calculate.

Question 7

IQR	3	
Lower inner fence:	$Q_1 - 1.5 \times \text{IQR}$	13.5
Upper inner fence:	$Q_3 + 1.5 \times \text{IQR}$	25.5
Lower outer fence:	$Q_1 - 3 \times \text{IQR}$	9
Upper outer fence:	$Q_3 + 3 \times \text{IQR}$	30

Question 8
The histogram is skewed and the calculation of quartiles and fences relies on a distribution that is fairly symmetrical.

Question 9

$$\begin{aligned} \text{IQR} &= 26 \\ 1.5 \times \text{IQR} &= 39 \\ Q_3 + 1.5 \times \text{IQR} &= 100 + 39 \\ &= 139 \end{aligned}$$

1.11 Comparing the distribution of a numerical variable across two or more groups

Question 1
There is a numerical variable (*resting pulse rate*) with three categories (*age group*), so **parallel boxplots** are the best way to display the data.

Question 2
a. i. IQR $= 17° - 12° = 5°$ **[1 mark]**
ii. The median for maximum temperature was 25° C and the median for minimum temperature was 15° C, so the median for maximum temperature was 10° C higher. **[1 mark]**
iii. One the outlier **[1 mark]**
b. If the median is 9.4° C, then it is expected that 50% will be above this.
There are 30 days in November:
$50\% \times 30 = 15$ days **[1 mark]**
***VCAA Examination Report* note:**
Some gave the percentage of 50% rather than the number of days.

Question 3
Parallel boxplots are used to investigate the association between a numerical variable and a categorical variable. In this question, the variable — monthly rainfall (in mm) is numerical, so the unknown second variable must be categorical. Therefore, Month of the year, option C is the only categorical variable in the given options.

Question 4
The distribution of ages of men is skewed.

© John Wiley & Sons Australia, Ltd

Question 5
The median for Y is 14 compared with 5 for X. Y has a bigger range than X. The data from Y is skewed negatively.

Question 6

X		***Y***	
x_{min}	12	x_{min}	10
Q_1	39	Q_1	42
Median	54	Median	60
Q_3	68	Q_3	73
x_{max}		x_{max}	87
IQR	29	IQR	31
Range	77	Range	77

1.12 Review

Question 1
$Q_1 = 148$, $Q_3 = 159$; therefore IQR $= 159 - 148 = 11$.
Upper fence $= 159 + 1.5 \times 11 = 175.5$

Question 2
300 milliseconds is the value at the third quartile (Q_3); therefore, *longer than 300 milliseconds* will be 25% of the times:
$0.25 \times 800 = 200$

Question 3
The *month* is the explanatory variable and the *minimum daily temperature* is the response variable.
The *median* values decrease with the month, which is expected as the year moves from summer into winter months.
Award **1 mark** for identifying an appropriate statistic – e.g. median.
Award **1 mark** for the explanation.
***VCAA Examination Report* note:**
A statement that a **decrease** or **change** in median (or IQR) signals an association was required for the first mark to be awarded. Median (or IQR) values for all three months needed to be quoted correctly for the second mark to be awarded.
Successful responses focused on one statistic only (usually the median) and quoted the values from the table rather than estimating from the boxplots.
Incorrect answers included using the word 'averages' or 'means' rather than medians and quoting only two medians rather than all three.
Some students went on to comment on the minimums and maximums; this additional information compromised an otherwise correct answer. Comments about the shape of the boxplots were also not appropriate.

Question 4
a. Place of capture **[1 mark]**
b. The most frequently occurring value in the forest section of the stem plot is 20, so the modal wingspan is 20 mm. **[1 mark]**
c.

	***Wingspan* (mm)**				
Place of capture	**minimum**	**Q_1**	**median (M)**	**Q_3**	**maximum**
forest	16	20	21	32	52
grassland	18	24	30	36	45

© John Wiley & Sons Australia, Ltd

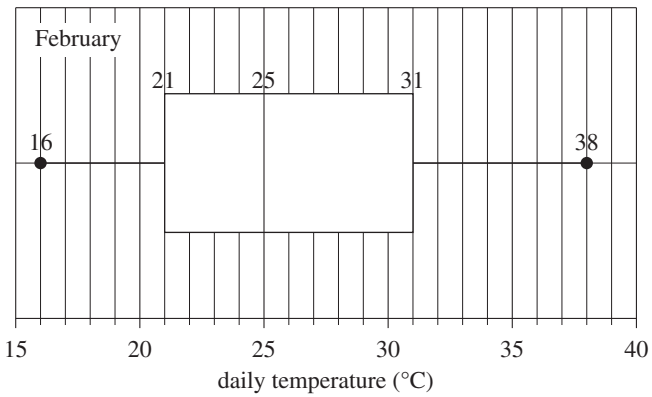
The minimum wingspan in the forest is 16 mm. **[1 mark]**
The upper quartile (Q_3) in the grassland is 36 mm. **[1 mark]**

d. IQR $= Q_3 - Q_1$
$= 32 - 20$
$= 12$
The wingspan of 52 mm is at the upper end of the forest values. Students need to show that this value is greater than the upper fence, that is, greater than $Q_3 + 1.5 \times \text{IQR}$.
$Q_3 + 1.5 \times \text{IQR} = 32 + 1.5 \times 12$
$= 50$ **[1 mark]**
As 52 mm is greater than this upper-fence value of 50 mm, it is an outlier. **[1 mark]**

e. Possible solution:
The wingspan is associated with the place of capture. Those captured in the grassland had a median wingspan of 30 mm, which is greater than the median wingspan of 21 mm of the moths captured in the forest.
Award 1 mark for stating that the median wingspan of those captured in grassland is greater than the median wingspan of those captured in the forest. Award 1 mark for stating the values of the two medians, 30 mm and 21 mm.
***VCAA Examination Report* note:**
The mean is not part of a stemplot five-number summary and was not appropriate to use because of the outlier of 52 in the forest data.
Students who initially gave the required median comparisons and then went further by quoting comparisons of other irrelevant statistics were not awarded full marks.

Question 5

a. i.



Award **1 mark** for the correct plot.

ii. 21 is at the first quartile, so 75% of days have a maximum temperature above 21°C. **[1 mark]**
***VCAA Examination Report* note:**
A common incorrect answer was 25%.

b. i. July: Positively skewed with an outlier at the high end
May: Approximately symmetric with no outliers **[1 mark]**
***VCAA Examination Report* note:**
Common unacceptable answers for July included symmetrically skewed, evenly distributed, bell shaped and normally distributed.

ii. $UF = Q_3 + 1.5 \times IQR$
$= 11 + 1.5 \times 3$
$= 15.5$ **[1 mark]**

iii. The median maximum daily temperature for May is approximately 14.4°C and for July it is approximately 9.1°C. These are very different temperatures, which we would expect as July is in

© John Wiley & Sons Australia, Ltd

winter and May is in autumn. Therefore, the maximum daily temperature is associated with the month of the year. **[1 mark]**

***VCAA Examination Report* note:**

The answer needed to refer to the difference between the two median temperatures. Simply quoting the two median values was not sufficient. Accuracy in reading the scales was an issue for some students.

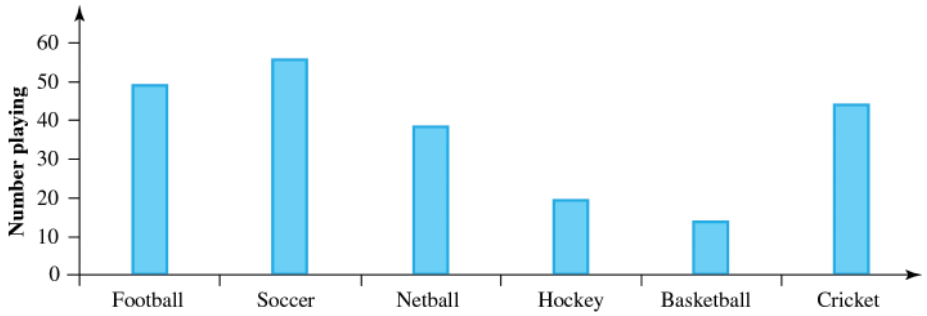
Alternatively, comparing the two interquartile range (IQR) values could have been used as the difference in the IQRs also indicates the presence of an association.

It appeared that some students confused 'maximum daily temperature' with the maximum of the boxplot.

Some students referred to average or mean temperatures; however, this cannot be accurately determined from a boxplot unless the distributions are clearly symmetric.

Question 6

a.



[1 mark]

b. The mode of the distribution is soccer. **[1 mark]**

c. 220 **[1 mark]**

d. 0.2 or one-fifth **[1 mark]**

Question 7

a. 1990 – 1999 **[1 mark]**

b. 1991, 1992 and 1993 **[1 mark]**

c. 2009 **[1 mark]**

d. 52 **[1 mark]**

Question 8

The mode is shown as 10|1, which means 1010 g or 1.01 kg. Therefore, the mode is not 1.1 kg.

Question 9

a. 466 **[1 mark]**

b. 50– **[1 mark]**

c. 40s **[1 mark]**

d. 26.18%; less (younger than 30 is 28.11%) **[1 mark]**

Question 10

a. The number of babies is found from the sum of the frequencies.
$2 + 4 + 6 + 2 + 1 + 1 = 16$ **[1 mark]**

b. The modal weight is the most frequently occurring weight, namely 9 kg. **[1 mark]**

c. The median is the halfway weight when the weights are put in order from smallest to largest: 9 kg. **[1 mark]**

© John Wiley & Sons Australia, Ltd

d. The mean weight is $\dfrac{\sum_{i-1}^{n} W_i f_i}{\sum_{i-1}^{n} f_i} = \dfrac{143}{16} = 8.9375\,\text{kg}$ **[1 mark]**

Question 11

a. The range is from 3 to 11, or 8. **[1 mark]**

b. Less than 7 means 3 to 6 and there are 41 out of the 100 in this range, or 41%. **[1 mark]**

c. The interquartile range is between 25% and 75%, which is consumption from 6 to 8, or 2. **[1 mark]**

d. From the calculator, standard deviation = 1.649. **[1 mark]**

e. The inclusion of the outlier makes the mean a little larger, makes the range a lot larger (increases from 6 to 8), makes very little change to the IQR (because of class groups), makes the standard deviation a little larger (effect is smoothed in calculation). **[1 mark]**

Question 12

a. 5 mm **[1 mark]**

b. 1.0588 mm **[1 mark]**

c.

πr^2 (mm^2)	Frequency
1520.52956	7
1661.90111	6
1809.55584	30
1963.49375	45
2123.71484	10
2290.21911	2

[1 mark]

d. Mean area = 1890.766 mm^2, sd of area = 161.1652 mm^2 **[1 mark]**

e. No; $\pi(\bar{x})^2 = \pi\left(\dfrac{1}{n}\sum_{i-1}^{n} x_i f_i\right)^2 \neq \pi\dfrac{1}{n}\sum_{i-1}^{n}(x_i)^2 f_i$ **[1 mark]**

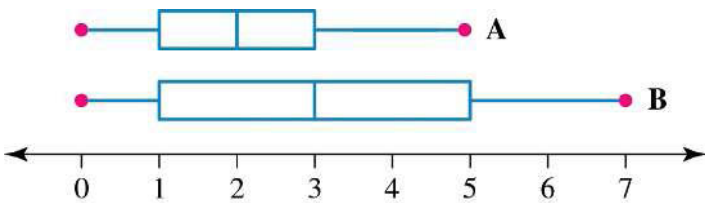
f. No, by similar reasoning to part (e), for standard deviation. **[1 mark]**

Question 13

a. $3 - 2 = 1$ **[1 mark]**

b. $7 - 5 = 2$ **[1 mark]**

c.



[1 mark]

d. Generally, there are more children per family in Suburb B (median). Also there is a bigger range in the number of children per family in Suburb B. The IQR for Suburb B is also bigger. **[1 mark]**

Question 14

a. 32 **[1 mark]**

b. 32 53 63.5 72.5 97 **[1 mark]**

c. Even though the range and IQR match, you cannot be sure it is from the same group since the median is not known nor the values of the first and third quartile. **[1 mark]**

Question 15

a. $x_{\min} = 200$

$x_{\max} = 425$

Range = \$225 **[1 mark]**

© John Wiley & Sons Australia, Ltd

b. \$275 **[1 mark]**

c. $Q_1 = 250$

$Q_3 = 325$

\$200 to \$250 = \$50; \$312.50 to \$425 = \$112.50 **[1 mark]**

d. IQR = 75

Lower inner fence = $Q_1 - 1.5 \times \text{IQR} = 137.5$

Upper inner fence = $Q_3 + 1.5 \times \text{IQR} = 437.5$

Lower outer fence = $Q_1 - 3 \times \text{IQR} = 25$

Upper outer fence = $Q_3 + 3 \times \text{IQR} = 550$

Upper inner fence is \$437.50, so a rent of \$450 would be considered to be a mild outlier. **[1 mark]**

Question 16

a. $x_{\min} = 135$, $x_{\max} = 183$

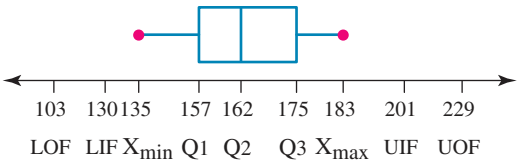
Range: 135 to 183; $183 - 135 = 48$ cm [1 mark]

b. Median is halfway between 161 and 163 cm, therefore it is 162 cm **[1 mark]**

c. $Q_1 = 157$, $Q_3 = 175$

Lower quartile range 135 to 157 cm, so 22 cm; upper quartile range 175 cm to 183 cm, so 8 cm **[1 mark]**

d.



[1 mark]

Lower inner fence: $Q_1 - 1.5 \times IQR = 130$

Upper inner fence: $Q_3 + 1.5 \times IQR = 202$

Lower outer fence: $Q_1 - 3 \times IQR = 103$

Upper outer fence: $Q_3 + 3 \times IQR = 229$

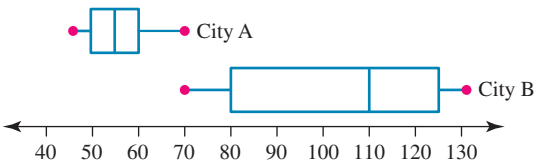
Question 17

a.

City A:	48	49.5	***56***	59	***67***
City B:	***70***	82	109	***126***	129

[1 mark]

b.



[1 mark]

c. City A has lower rainfall median (56) than City B (109), but City B has more variability (IQR 44, range 59) than City A (IQR 9.5, range 19).

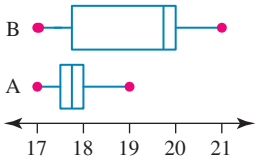
There is no overlap in the range of rainfall averages of City A and City B (highest in City A is 67 and lowest in City B is 70). **[1 mark]**

d. City B would be wetter than City A because all the monthly average rainfalls are higher. **[1 mark]**

© John Wiley & Sons Australia, Ltd

Question 18

a.



[1 mark]

A		B	
x_{min}	17	x_{min}	17
Q_1	17.6	Q_1	18.02
Median	17.96	Median	20
Q_3	18.24	Q_3	20.3
x_{max}	19	x_{max}	21
IQR	0.64	IQR	2.28
Range	2	Range	4

b. The median length from machine A is 17.96 cm and for machine B is 20 cm, so the median of lengths from machine B is much greater than the maximum length of spaghetti from machine A. **[1 mark]**

c. Machine B has greater variability than machine A (from boxplot or IQR). **[1 mark]**

d. Lower inner fence: $Q1 - 1.5 \times IQR = 16.64$
Upper inner fence: $Q3 + 1.5 \times IQR = 19.2$
Lower outer fence: $Q1 - 3 \times IQR = 15.68$
Upper outer fence: $Q3 + 3 \times IQR = 20.16$
The upper inner fence for machine A is 19.2 and the upper outer fence is 20.16 cm. The median length and the upper half of the output of machine B are outside the fences and hence are outliers. It is highly likely that machine B is malfunctioning. **[1 mark]**

Question 19

Score (x)	Frequency (f)	(fx)
1	12	$1 \times 12 = 12$
2	13	$2 \times 13 = 26$
3	15	$3 \times 15 = 45$
4	20	$4 \times 20 = 80$
Total	60	163

$$\text{Mean} = \frac{\sum fx}{\sum x}$$
$$= \frac{163}{60}$$
$$= 2.72 \text{ (correct to 2 decimal places)}$$

Question 20

The incorrect statement is **D**. 'The range of the data was 30.'
The range of the data is $14 - 2 = 12$.

Question 21

a. Using CAS: mean $(\bar{x}) = 20$, median $= 18$, mode $= 21$, standard deviation $= 4.7$
Award 1 mark for each correct value $= 4$ marks

b. Min $X = 17$
$Q_1 = 17.5$
Med $= 18$
$Q_3 = 21$

© John Wiley & Sons Australia, Ltd

Max $X = 37$ **[1 mark]**
Range $= 37 - 17$
$= 20$ **[1 mark]**
IQR $= 21 - 17.5$
$= 3.5$ **[1 mark]**

c. Using CAS: mean $(\bar{x}) = 25.4$, median $= 21$, mode $= 18$, standard deviation $= 10.5$
Award **1 mark** for each correct value = 4 marks

d. Min $X = 18$
$Q_1 = 18$
Med $= 21$
$Q_3 = 33$
Max $X = 45$ **[1 mark]**
Range $= 45 - 18$
$= 27$ **[1 mark]**
IQR $= 33 - 18$
$= 15$ **[1 mark]**

e. The data for male offenders is positively skewed. **[1 mark]**

f. The data for female offenders is almost symmetrical. **[1 mark]**

g. Outliers in male offenders – there are no outliers, see the calculation below.
Lower limit $= Q_1 - 1.5 \times IQR$
$= 17.5 - 1.5 \times 3.5$
$= 12.25$
There are no ages recorded below this limit. **[1 mark]**
Upper limit $= Q_3 + 1.5 \times IQR$
$= 37 + 1.5 \times 3.5$
$= 42.5$
There are no ages recorded above this limit. **[1 mark]**
Outliers in female offenders – there are no outliers, see the calculation below.
Lower limit $= Q_3 + 1.5 \times 15$
$= 18 + 1.5 \times 15$
$= -4.5$
There are no ages recorded below this limit. **[1 mark]**
Upper limit $= Q_3 + 1.5 \times IQR$
$= 45 + 1.5 \times 15$
$= 67.5$
There are no ages recorded above this limit. **[1 mark]**

h. Answers will vary. Students should include at least three of the following.
- There are many more male offenders than female offenders. **[1 mark]**
- The ages of the male offenders include many younger males, even below legal driving age. **[1 mark]**
- The mean age of male offenders is 20, while the mean age for female offenders is 25.4 (much higher). **[1 mark]**
- There is a greater deviation in the female results as compared to the male results, as seen by the different standard deviation values. **[1 mark]**
- Most of the offenders were below 25 (only 3 out of 26 were older). **[1 mark]**

© John Wiley & Sons Australia, Ltd

2 Linear relations and equations

Topic	**2**	**Linear relations and equations**
Subtopic	**2.2**	**Linear relations and solving linear equations**

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
The interest, I, owed on a loan of $1000 borrowed for 1 year depends on the rate of interest $r\%$ p.a., according to the formula $I = 10r$.
For an interest rate of 5%, the interest owed will be
A. $0.50
B. $5
C. $50
D. $500
E. $5000

Question 2 (1 mark)
Solve the equation $2(x + 1) = 5(x - 2)$

Question 3 (1 mark)
Determine which solution satisfies all the following equations.
$3x - \frac{1}{2} = \frac{1}{4}$
$4x + 5 = 6$
$4 - \frac{8x}{3} = \frac{10}{3}$
A. $x = \frac{1}{4}$
B. $x = 4$
C. $x = \frac{1}{12}$
D. $x = -\frac{1}{4}$
E. $x = -4$

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
The table shows the conversion from degrees Celsius, C, to degrees Fahrenheit, F , from 0 °F to 120 °F, using the formula $C = \frac{5}{9}(F - 32)$.

Degrees Fahrenheit	Degrees Celsius
0	−18
10	−12
20	−7
30	−1
40	4
50	10
60	16
70	21
80	27
90	32
100	38
110	43
120	49

The temperature today ranged from 30 °F to 60 °F. The temperature difference in degrees Celsius (°C) was
A. −15
B. 15
C. 16
D. 17
E. 30

Question 5 (1 mark)
The circumference C of a circle of diameter D is given by $C = \pi D$. If a pot has a radius of 3 cm, what is its circumference? Use $\pi = 3.14$.
A. 9.42 cm
B. 18.84 cm
C. 9 cm
D. 18 cm
E. $18\frac{6}{7}$ cm

Question 6 (1 mark)
The Australian dollar can be exchanged for 85 US cents. So the formula $U = 0.85A$ can be used to convert US dollars U to Australian dollars A How many Australian dollars is equivalent to US$20?

© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)

The volume of a cone, v (cm^3), is given by the formula $v = \frac{1}{3}\pi r^2 h$, where r (cm) is the radius of the base and h (cm) is the height. Transpose the formula to make r the subject.

A. $r = \frac{3v}{\pi h}$

B. $r = \sqrt{\frac{v}{3\pi h}}$

C. $r = \sqrt{\frac{3v}{\pi h}}$

D. $r = \frac{v}{3\pi h}$

E. $r = \sqrt{\frac{9v}{\pi h}}$

Question 8 (1 mark)

Transpose the formula $A = \frac{b}{3} - \frac{1}{2}$ to make b the subject.

A. $b = 3A + 12$

B. $b = 3A + \frac{1}{2}$

C. $b = A + \frac{3}{2}$

D. $b = 2A + \frac{3}{2}$

E. $b = 3\left(A + \frac{1}{2}\right)$

© John Wiley & Sons Australia, Ltd

Question 9 (1 mark)

Make n the subject of the formula $r = \dfrac{2Rn}{(n+1)}$

A. $n = \dfrac{2Rn - 1}{r}$

B. $n = \dfrac{r}{2R - r}$

C. $n = \dfrac{r(n+1)}{2R}$

D. $n = \dfrac{r}{2R - 1}$

E. $n = \dfrac{2r}{R - r}$

Question 10 (1 mark)

Which of the following is not a linear equation?

A. $y = 3x + 4$

B. $y = 3$

C. $y = 4 - x$

D. $y = \dfrac{1}{x}$

E. $y = 2x$

Question 11 (1 mark)

The solution of the equation $\dfrac{a}{b}x + c = d$ for x is

A. $x = \dfrac{a(d - c)}{b}$

B. $x = \dfrac{b(d - c)}{a}$

C. $x = \dfrac{b}{a}d - c$

D. $x = \dfrac{db - c}{a}$

E. $x = db - \dfrac{c}{a}$

© John Wiley & Sons Australia, Ltd

Question 12 (1 mark)

Which solution for x satisfies all the following equations?

$ax - b = c$

$ax = b + c$

$-c = b - ax$

A. $x = \frac{c}{a} + b$

B. $x = \frac{c}{a} - b$

C. $x = -\frac{c}{a} - b$

D. $x = \frac{a}{b + c}$

E. $x = \frac{b + c}{a}$

© John Wiley & Sons Australia, Ltd

Topic	2	Linear relations and equations
Subtopic	2.3	Developing linear equations

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
Think of a number, add 3, then multiply the result by 6. The answer is 24. Determine which equation can be solved to find the initial number.
A. $6x + 3 = 24$
B. $6(x - 3) = 24$
C. $6x - 3 = 24$
D. $6(x + 3) = 24$
E. $3x + 6 = 24$

Question 2 (1 mark)
In Year 11 at Orana Secondary College, there are 20% more girls than boys. There are 44 students altogether in Year 11.
A. $b + 0.2b = 44$
B. $b + 20b = 44$
C. $b + 1.2b = 44$
D. $b = 44 - 20b$
E. $b = 44 + 1.2b$

© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)

This table shows the perimeters of paddocks whose lengths are twice their widths.

Length (m)	Width (m)	Perimeter (m)
100	50	300
300	150	900
500	250	1500
700	350	2100
900	450	2700
1100	550	3300
1300	650	3900
1500	750	4500
1700	850	5100
1900	950	5700
2100	1050	6300

Determine the perimeter of the paddock with length 1.1 km.

A. 400 m
B. 550 m
C. 310 m
D. 2200 m
E. 3300 m

Question 4 (1 mark)

To find the average of n numbers, the sum of all the numbers is divided by n.
The sum of a list of n numbers is 60. When the number 4 is added to the list, the average remains the same.
Which equation can be used to find the number of numbers in the original list?

A. $\frac{60}{n} = \frac{64}{n+1}$
B. $60n = 64(n+1)$
C. $60n = \frac{64}{n+1}$
D. $\frac{60}{n+1} = \frac{64}{n}$
E. $n(n+1) = 4$

© John Wiley & Sons Australia, Ltd

Question 5 (3 marks)

At the summer sales, shirts are discounted by 30%. The retailer bought the shirts for $15 each and normally sells them for double that. She wishes to make a total profit of $1920 on the shirts during the sales.

a. Write down the equation that could be used to find the number of shirts, n, she needs to sell at the discounted price in order to make the required profit. **[2 marks]**

b. Hence, find the value of n. **[1 mark]**

Question 6 (3 marks)

Eva and Philip ride from the City to Funston, a distance of 54 km, every Sunday. Last Sunday Eva rode at $\frac{2}{3}$ of Philip's speed and arrived $2\frac{1}{4}$ hours after him.

a. Write an equation that could be used to find Philip's speed. **[1 mark]**

a. Hence, find Philip's and Eva's speeds. **[2 mark]**

© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)

This table shows the perimeters of paddocks whose lengths are three times their widths.

Length (m)	Width (m)	Perimeter (m)
300	100	800
600	200	1600
900	300	2400
1200	400	3200
1500	500	4000
1800	600	4800
2100	700	5600
2400	800	6400
2700	900	7200
3000	1000	8000
3300	1100	8800
3600	1200	9600
3900	1300	10 400

What is the perimeter of a paddock 3 km long?

A. 2400 m
B. 8000 m
C. 1200 m
D. 100 m
E. 900 m

© John Wiley & Sons Australia, Ltd

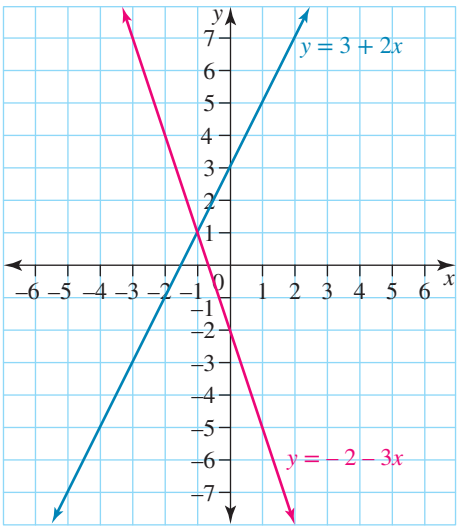
Topic	2	Linear relations and equations
Subtopic	2.4	Simultaneous linear equations

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

The graphs of $y = 3 + 2x$ and $y = -2 - 3x$ are shown.



Their point of intersection is

A. $(1, -1)$
B. $(0, 3)$
C. $(0, -2)$
D. $(0, 0)$
E. $(-1, 1)$

Question 2 (1 mark)

Using the substitution method, the solution to the simultaneous equations $x + 2y = 9$ and $y = -1 + 5x$ is

A. $(1, 4)$
B. $(9, 5)$
C. $(4, 1)$
D. $(-1, 5)$
E. $(2, 9)$

© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)
Using the elimination method, the solution to the simultaneous equations $2x + y = 8$ and $3x - y = 17$ is
A. $(-5,\ 2)$
B. $(2,\ 5)$
C. $(5, -2)$
D. $(5,\ 2)$
E. $(-5, -2)$

Question 4 (1 mark)
The solution to the pair of simultaneous equations $4x - 3y = 9$ and $x = 2y + 1$ is:
A. $(-3, -1)$
B. $(3, 1)$
C. $(1, 3)$
D. $(-1, 3)$
E. $(3, -1)$

Question 5 (1 mark)
The solution to the pair of simultaneous equations $3x + 4y = -2$ and $x - 4y = -6$ is:
A. $(1, -2)$
B. $(2, 1)$
C. $(-2, -1)$
D. $(2, -1)$
E. $(-2, 1)$

Question 6 (1 mark)
The solution to the pair of simultaneous equations $x = 1 - y$ and $y = 3 - 2x$ is:
A. $(-1, 2)$
B. $(2, 1)$
C. $(-2, 1)$
D. $(2, -1)$
E. $(-2, -1)$

© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)
The solution to the pair of simultaneous equations $2x - 5y = -17$ and $x + y = 2$ is:
A. $(1, -3)$
B. $(-1, 3)$
C. $(-1, -3)$
D. $(3, -1)$
E. $(-3, 1)$

Question 8 (1 mark)
The solution to the pair of simultaneous equations $2x - y = 3$ and $8x - 7y = 18$ is:
A. $\left(-\frac{1}{2}, -2\right)$
B. $\left(2, \frac{1}{2}\right)$
C. $\left(\frac{1}{2}, -2\right)$
D. $\left(-2, \frac{1}{2}\right)$
E. $\left(-\frac{1}{2}, 2\right)$

Question 9 (1 mark)
Andrew is solving the pair of simultaneous equations $4x + 3y = -6$ and $7x + 2y = 10$. In order to use the elimination method he could:
A. multiply both equation [1] and equation [2] by the same positive value.
B. multiply equation [1] and equation [2] by the same negative value.
C. multiply equation [1] by 3 and equation [2] by -2, and then add to eliminate y.
D. multiply equation [1] by 2 and equation [2] by -3, and then add to eliminate y
E. multiply equation [1] by 7 and equation [2] by 4, and then add to eliminate y.

© John Wiley & Sons Australia, Ltd

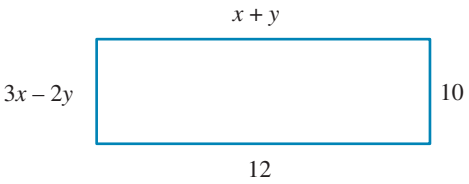
Topic	2	Linear relations and equations
Subtopic	2.5	Problem solving with simultaneous equations

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

The value of x in the following rectangle is:



A. 6.8
B. 5.2
C. 6
D. 4
E. 7

Question 2 (1 mark)

x and y are two consecutive even numbers. The relationship between x and y is:

A. $x + y = 2$
B. $y = x + 1$
C. $x + y = 1$
D. $y = 2 + x$
E. $y = 2x$

© John Wiley & Sons Australia, Ltd

Question 3 (3 marks)
River Car Sales sells sedans and wagons. The profit made on selling six sedans and four wagons is $25 600. The profit made on selling three sedans and one wagon is $10 000.

a. Write simultaneous equations showing the relationship between the profit on one sedan, s, and the profit on one wagon, w. **[1 mark]**

b. Solve the equations for s and w. **[1 mark]**

c. Hence, calculate the profit on selling 10 sedans and 8 wagons. **[1 mark]**

Question 4 (1 mark)
Yesterday, Amy ran for 2 hours and walked for 2 hours, travelling a total distance of 28 km. Today she ran for 1 hour and walked for 3 hours, travelling a total distance of 22 km. To find the speeds she runs (R) and walks (W), she writes this information as two simultaneous equations.

$2R + 2W = 28$

$R + 3W = 22$

The units of measurement of each side of the equations are:

A. km per hour
B. metres
C. hour
D. km
E. None of these

Question 5 (3 marks)
A school tuckshop had 200 muffins for sale. At the end of the day, the boys and girls had eaten them all. If the boys had eaten half as many and the girls twice as many, the tuckshop would have had 10 muffins left.

a. Write simultaneous equations describing the relationship between numbers of muffins eaten by girls, g, and the number of muffins eaten by boys, b. **[1 mark]**

b. How many muffins did the boys eat? **[1 mark]**

c. How many muffins did the girls eat? **[1 mark]**

© John Wiley & Sons Australia, Ltd

Topic	2	Linear relations and equations
Subtopic	2.6	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

Using the formula $F = \frac{9}{5}C + 32$ (where F represents degrees Fahrenheit and C represents degrees Celsius). The number of degrees F when C is 25 is

A. 257
B. 45
C. 57
D. 77
E. 0

Question 2 (1 mark)

The solution to the linear equation $115 - 5p = 45$ is

A. -32
B. 23
C. 14
D. -23
E. -14

Question 3 (1 mark)

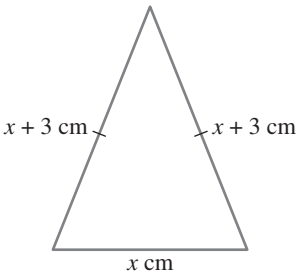
A number is multiplied by 7 and then divided by 3. The result is 2 less than the number multiplied by 4. The equation that best represents this is

A. $\frac{7x}{3} = 2 - 4x$
B. $\frac{7x}{3} = 4x - 2$
C. $\frac{3x}{7} = 4x - 2$
D. $\frac{3x}{7} = 4x + 2$
E. $\frac{7x}{3} = 4x + 2$

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)

In a particular isosceles triangle, the two sides of equal length are each 3 cm longer than the third side.



If the perimeter of the triangle is 24 cm, the length of the third side labelled x is

A. 9 cm

B. $\frac{24}{7}$ cm

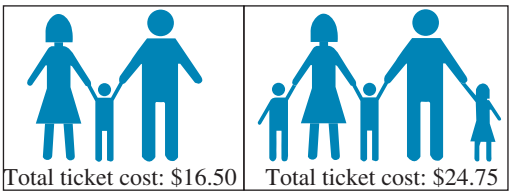
C. 6 cm

D. 8 cm

E. 12 cm

Question 5 (5 marks)

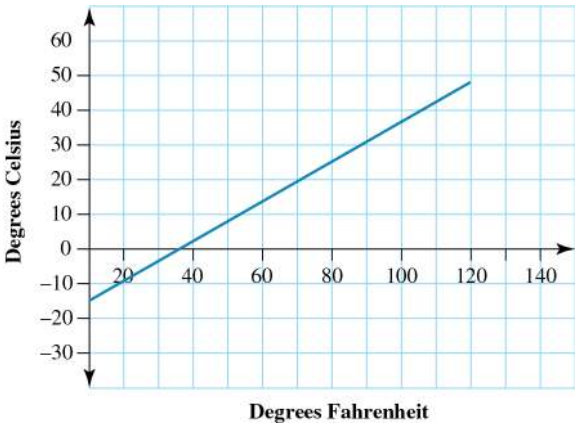
Different combinations are available when purchasing tickets to enter a theme park. The total cost for 2 adults and 3 children is $24.75. The total cost of 2 adults and 1 child is $16.50.



Use simultaneous equations to calculate the cost of an individual child ticket and an adult ticket. Give answers correct to the nearest 5 cents.

© John Wiley & Sons Australia, Ltd

Question 6 (2 marks)
The graph of degrees Celsius against degrees Fahrenheit is a straight line.



Find the approximate temperature change in degrees Celsius when the temperature increases by 60° F.

Question 7 (1 mark)
To solve the equation $3 = \frac{2}{x}$ for x
A. $\times$ both sides by x , then $\div$ by 3
B. $\div$ both sides by 2, then $\times$ by $x\,(x \neq 3)$
C. $\times$ both sides by x, then $\div$ by $3\,(x \neq 0)$
D. $\times$ both sides by x, then $\div$ by $2\,(x \neq 2)$
E. $\times$ both sides by $\frac{2}{3}$

Question 8 (1 mark)
To solve the equation $b = \frac{c}{x}$ for x
A. $\times$ both sides by x, then $\div$ by b
B. $\div$ both sides by c, then $\times$ by $(x \neq b)$
C. $\times$ both sides by x $(x \neq 0)$, then $\div$ by $b\,(b \neq 0)$
D. $\times$both sides by x, then $\div$ by $c\,(x \neq c)$
E. $\times$both sides by $\frac{c}{b}$

© John Wiley & Sons Australia, Ltd

Question 9 (3 marks)
Solve the equation $m(x + 1) = p(x - n)$ for x.

Question 10 (3 marks)
Bridget is testing a spring. She finds that the greater the weight she hangs from its end, the more the spring stretches.
She puts her results in a table.

Weight (kg)	Extension (cm)
1	4
2	8
4	16
6	24
8	32
10	40
12	48
14	56
16	64
18	72
20	80
22	88
24	96
26	104
28	112
30	120
32	128

Bridget assumes that the spring will not break.

a. What extension would she expect if a 20 kg weight were placed at the bottom of the spring? **(1 mark)**

b. What equation relates the weight, W, and the extension, E ? **(1 mark)**

c. What weight would give her an extension of 1 metre? **(1 mark)**

© John Wiley & Sons Australia, Ltd

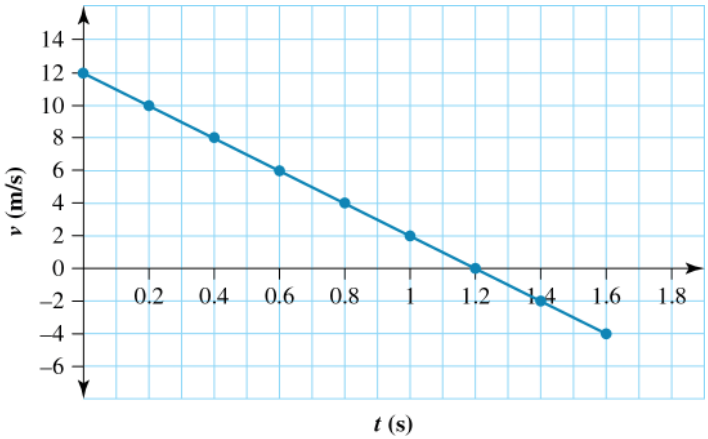
Question 11 (3 marks)

A ball is thrown straight up in the air at a velocity of 12 m/s. It immediately begins to slow down because of gravity. It stops and then begins to fall back to Earth.

The table shows the velocity of the ball v m/s at different times t seconds

t (s)	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6
v (m/s)	12	10	8.1	6.1	4.2	2.2	0.2	−1.7	−3.7

and the graph of these values.



a. What was the velocity of the ball after 1.2 seconds? **(1 mark)**

b. After how many seconds does the ball start to fall? **(1 mark)**

c. What does it mean when the velocity is negative? **(1 mark)**

Question 12 (1 mark)

Using the formula $F = \frac{9}{5}C + 32$ (where F represents degrees Fahrenheit and C represents degrees Celsius), the number of degrees C (correct to two decimal places) when F is 105° is:

A. 131.40
B. 40.56
C. 76.11
D. 58.33
E. 189.00

© John Wiley & Sons Australia, Ltd

Question 13 (1 mark)

In order to solve the equation $\frac{4x}{3} = 12$, the operations which must be performed are:

A. multiply both sides by 3, divide by 4.

B. multiply both sides by $\frac{4}{3}$.

C. multiply both sides by 4, divide by 3.

D. divide both sides by $\frac{3}{4}$.

E. add 3 to both sides and then subtract 4.

Question 14 (1 mark)

When transposed to make t the subject, the formula $v = u + at$ becomes:

A. $t = \frac{v + u}{a}$

B. $t = a(v - u)$

C. $at = v - u$

D. $t = \frac{a}{v - u}$

E. $t = \frac{v - u}{a}$

Question 15 (1 mark)

For the equation $4p - 6 = 14y$:

A. $p = 14y + 2$

B. $p = \frac{14y - 6}{4}$

C. $p = 5$

D. $p = 20$

E. $p = \frac{7y + 3}{2}$

© John Wiley & Sons Australia, Ltd

Question 16 (1 mark)

The missing value in the table below for $y = \frac{2(x-3)}{5}$ is:

x	-3	-2	-1	0	1	2	3
y	$-2\frac{2}{5}$	-2		$-1\frac{1}{5}$	$-\frac{4}{5}$	$-\frac{2}{5}$	0

A. $-1\frac{1}{5}$

B. $-1\frac{3}{5}$

C. $-1\frac{4}{5}$

D. $-1\frac{2}{5}$

E. $\frac{4}{5}$

Question 17 (15 marks)

A small catering company sold four different types of cupcakes. Different packs are available.

- Red velvet with cream cheese frosting (6 for $20.70)
- Chocolate mud with chocolate ganache (4 for $19.00)
- Vanilla with buttercream icing (5 for $17.25)
- Strawberry with fresh cream and jam filling (3 for $19.05)

a. A customer is trying to decide which is the cheapest option per cupcake. Form and solve a linear equation for each type of cupcake and determine the cheapest option. **(5 marks)**

b. The customer decided to purchase 3 red velvet cupcakes, 2 chocolate mud cupcakes and 5 strawberry cupcakes. Form and solve a linear equation to calculate the total cost of the order. **(2 marks)**

© John Wiley & Sons Australia, Ltd

c. Upon calculating the cost of the bundle of cupcakes, the customer decides instead to purchase a two-tiered cylindrical birthday cake instead.

i. The diameter of the bottom tier is 8 inches. Given that 1 inch is the equivalent of 2.54 cm convert the diameter of the bottom tier into centimetres. **(1 mark)**

ii. The diameter of the top tier is 6 inches. Convert the diameter of the top tier into centimetres. **(1 mark)**

d. The height of the bottom tier is 5 cm and the height of the top tier is 4.5 cm. Given that the volume of a cylinder can be calculated using the formula $V = \pi r^2 h$, calculate the total volume of the cake. Give your answer correct to 2 decimal places. **(4 marks)**

e. Given that the average slice of this cake is 48.50 cm^3, how many people will this two-tiered cake serve? **(2 marks)**

Question 18 (1 mark)

Which of the following statements is true when considering the solution to the pair of simultaneous equations: $\frac{x}{2} - \frac{y}{4} = 5$ and $2x + y = 3$.

A. These equations cannot be solved simultaneously.
B. The solution to these equations consists of two positive values.
C. The first equation can be rewritten without the fractions as $2x - y = 20$.
D. When equation [1] is multiplied by 4 and added to equation [2], the x values are eliminated.
E. The solution consists of whole number values.

© John Wiley & Sons Australia, Ltd

Question 19 (8 marks)

The cost of 12 apples and 9 oranges is $14.75. Due to current weather conditions, the price of an apple is twice the price of an orange.

a. Write two equations that can be solved simultaneously to calculate the price of one apple and one orange. **(2 marks)**

b. Calculate the price of an orange. **(2 marks)**

c. Calculate the price of an apple. **(1 mark)**

d. Substitute your values into the first of your simultaneous equations to confirm that your answers are accurate. **(1 mark)**

e. Is there a difference in your answer? Explain why this difference occurs. **(2 marks)**

© John Wiley & Sons Australia, Ltd

Answers and marking guide
2.2 Linear relations and solving linear equations

Question 1

If $r = 5\%$,

$$\begin{aligned} I &= 10r \\ &= 10 \times 5 \\ &= \$50 \end{aligned}$$

Question 2

$2(x + 1) = 5(x - 2)$

$2x + 2 = 5x - 10$	Remove brackets first
$2 = 3x - 10$	Subtract smaller from larger
$12 = 3x - 10 + 10$	Add 10 to both sides
$12 = 3x$	Divide both sides by 3
$x = 4$ **[1 mark]**	

Question 3

$x = \frac{1}{4}$

Can either substitute alternatives in each equation ('satisfies') or solve each equation.

Question 4

From the table, the temperature range from 30 °F to 60 °F is equivalent to the temperature range −1°C to 16°C. Therefore there is a temperature difference of 17°C.

(Notice that the question asks for difference and there is a negative number).

Question 5

$$\begin{aligned} C &= \pi D \\ &= \pi \times 2\,(r) \\ &= 3.14 \times 6 \\ &= 18.84 \end{aligned}$$

The formula includes diameter but it is the radius that is given.

Question 6

$$\begin{aligned} U &= 0.85A \\ 20 &= 0.85A \\ A &= \frac{20}{0.85} \\ &= \$23.53 \textbf{ [1 mark]} \end{aligned}$$

Question 7

Order of operations is: multiply by, divide by πh and take square root.

$$\begin{aligned} v &= \frac{1}{3}\pi r^2 h \\ 3v &= \pi r^2 h \\ \frac{3v}{\pi h} &= r^2 \\ \sqrt{\frac{3v}{\pi h}} &= r \end{aligned}$$

© John Wiley & Sons Australia, Ltd

Question 8

Order of operations is important in 'undoing'; must add $\frac{1}{2}$ then multiply by 3;

$$A = \frac{b}{3} - \frac{1}{2}$$
$$A + \frac{1}{2} = \frac{b}{3}$$
$$3\left(A + \frac{1}{2}\right) = b$$
$$b = 3\left(A + \frac{1}{2}\right)$$

Question 9

Undo division and then collect all the terms in n on one side.

$$r = \frac{2Rn}{(n+1)}$$
$$r(n+1) = 2Rn$$
$$rn + r = 2Rn$$
$$2Rn - rn = r$$
$$n(2R - r) = r$$
$$n = \frac{r}{2R - r}$$

Question 10

$$y = \frac{1}{x}$$

Question 11

$$\frac{a}{b}x = d - c$$
$$ax = b(d - c)$$
$$x = \frac{b(d - c)}{a}$$

Question 12

$$x = \frac{b + c}{a}$$

Can either solve each equation or substitute alternatives in each equation ('satisfies'), which is harder.

2.3 Developing linear equations

Question 1

$6(x + 3) = 24$

Order of operations is important.

Question 2

Let b equal the number of boys.

Then $b \times 1.2$ equals the number of girls.

Hence, the total number of students $= b + b \times 1.2$

$b + 1.2b = 44$

Note: 20% more is the same as by multiplying by 1.2.

© John Wiley & Sons Australia, Ltd

Question 3
3300 m
Can use the pattern in the perimeter column.

Question 4
$\frac{60}{n} = \frac{64}{n+1}$
Definition of average used for both sides of the equation.

Question 5
a. Cost price = \$15
Normal selling price = \$30
Sale selling price = 70% of \$30 = \$21
Profit per shirt = \$21 − \$15 = \$6
Profit for shirts = \$6n **[1 mark]**
Equation: $6n = 1920$ **[1 mark]**
b. $n = \frac{1920}{6} = 320$ **[1 mark]**

Question 6
a. If $v =$ Philip's speed then Eva's speed $= \frac{2}{3}v$
Difference in time to Funston: Philip's time – Eva's time $= 2\frac{1}{4}$
$\frac{54}{\frac{2}{3}v} - \frac{54}{v} = 2.25$ **[1 mark]**
b. $\frac{54}{\frac{2}{3}v} - \frac{54}{v} = 2.25$
$\frac{3 \times 54}{2v} - \frac{54}{v} = 2.25$
$\frac{3 \times 54 - 108}{2v} = 2.25$ **[1 mark]**
$3 \times 54 - 108 = 2.25 \times 2v$
$v = \frac{54}{4.5} = 12$ **[1 mark]**
Philip's speed = 12 km/h and Eva's speed = 8 km/h

Question 7
8000 m
Can use the pattern in the perimeter column.

2.4 Simultaneous linear equations

Question 1
Reading the graph, the point of intersection is $(-1,\ 1)$.

Question 2
$x + 2y = 9$ [1]
$y = -1 + 5x$ [2]
Substitute $y = 5x - 1$ into equation [1];

© John Wiley & Sons Australia, Ltd

$x + 2(-1 + 5x) = 9$
$x - 2 + 10x = 9$
$11x - 2 = 9$
$11x = 9 + 2$
$11x = 11$
$x = 1$
Substitute $x = 1$ into equation [1];
$x + 2y = 9$
$1 + 2y = 9$
$2y = 9 - 1$
$2y = 8$
$y = 4$
Therefore, the solution is $(1, 4)$.

Question 3
$2x + y = 8$ [1]
$3x - y = 17$ [2]
[1] + [2]
$5x = 25$
$x = 5$
Substitute $x = 5$ into equation [1]:
$2x + y = 8$
$2(5) + y = 8$
$10 + y = 8$
$y = 8 - 10$
$y = -2$
Therefore, the solution is $(5, -2)$.

Question 4
$4x - 3y = 9$ [1]
$x = 2y + 1$ [2]
Substitute $x = 2y + 1$ into equation [1].
$4x - 3y = 9$
$4(2y + 1) - 3y = 9$
$8y + 4 - 3y = 9$
$5y + 4 = 9$
$5y = 9 - 4$
$5y = 5$
$y = 1$
Substitute $y = 1$ into equation [2] to find x.
$x = 2y + 1$
$x = 2(1) + 1$
$x = 2 + 1$
$x = 3$
$\therefore$ The solution is $(3, 1)$.

Question 5
$3x + 4y = -2$
$x - 4y = -6$ [1]
[1] = [2]

© John Wiley & Sons Australia, Ltd

$4x = -8$

$x = \frac{-8}{4}$

$x = -2$

Substitute $x = -2$ into equation [1]

$3x + 4y = -2$

$3(-2) + 4y = -2$

$-6 + 4y = -2$

$4y = -2 + 6$

$4y = 4$

$y = 1$

$\therefore$ The solution is $(-2, 1)$.

Question 6

$x = 1 - y$ [1]

$y = 3 - 2x$ [2]

Substitute $y = 3 - 2x$ into equation [1]

$x = 1 - y$

$x = 1 - (3 - 2x)$

$x = 1 - 3 + 2x$

$x = -2 + 2x$

$x - 2x = -2$

$-x = -2$

$x = 2$

Substitute $x = 2$ into equation [2] to find y.

$y = 3 - 2x$

$y = 3 - 2(2)$

$y = 3 - 4$

$y = -1$

$\therefore$ The solution is $(2, -1)$

Question 7

$2x - 5y = -17$ [1]

$x + y = 2$ [2]

Multiply equation [2] by 5 to eliminate y by addition

$2x - 5y = -17$ [1]

$5x = 5y = 10$ $5 \times [2]$

[1] + (5 × [2])

$7x = -7$

$x = -1$

Substitute $x = -1$ into equation [2] to find y.

$x + y = 2$

$(-1) + y = 2$

$y = 2 + 1$

$y = 3$

$\therefore$ The solution is $(-1, 3)$.

Question 8

$2x - y = 3$ [1]

$8x - 7y = 18$ [2]

© John Wiley & Sons Australia, Ltd

Multiply equation [1] by -7 to eliminate y by addition.

$-14x + 7y = -21 \quad ([1] \times -7)$

$$\begin{array}{rl} 8x - 7y = 18 & \\ \hline -6x & = -3 \\ x & = \dfrac{-3}{-6} \\ x & = \dfrac{1}{2} \end{array}$$

Substitute $x = \dfrac{1}{2}$ into equation [1] to find y.

$$\begin{aligned} 2x - y &= 3 \\ 2\left(\frac{1}{2}\right) - y &= 3 \\ 1 - y &= 3 \\ -y &= 3 - 1 \\ -y &= 2 \\ y &= -2 \end{aligned}$$

$\therefore$ The solution is $\left(\dfrac{1}{2}, -2\right)$.

Question 9

$4x + 3y = -6 \quad [1]$

$7x + 2y = 10 \quad [2]$

To solve: multiply equation [1] by 2 and equation [2] by -3, and then add to eliminate y.

$8x + 6y = -12 \quad [1] \times 2$

$-21x - 6y = -30 \quad [2] \times -3$

These equations can now be added to eliminate the y values and solve for x.

2.5 Problem solving with simultaneous equations

Question 1

Since opposite sides of a rectangle are equal, equations can be solved simultaneously.

$x + y = 12 \quad [1]$

$3x - 2y = 10 \quad [2]$

Multiply [1] by 2 and add it to [2].

$2x + 2y = 24 \quad [3]$

$3x - 2y = 10 \quad [2]$

$$\begin{aligned} 5x &= 34 \\ x &= \frac{34}{5} \\ &= 6\frac{4}{5} = 6.8 \end{aligned}$$

Question 2

Even numbers are 2 apart.

Question 3

a. $6s + 4w = \$25\,600 \quad [1]$

$3s + w = \$10\,000 \quad [2]$ **[1 mark]**

b. Multiply [2] by 2:

© John Wiley & Sons Australia, Ltd

$6s + 2w = \$20\,000$ [3]
Subtract [3] from [1]:
$2w = \$5600$
$w = \$2800$
Substitute $w = \$2800$ in [2]:
$3s + (2800) = \$10\,000$
$3s = \$7200$
$s = \$2400$
$\therefore s = \$2400, w = \$2800$ **[1 mark]**

c. The profit on selling 10 sedans and 8 wagons is found by substituting the values in p:
$P = 10s + 8w$
$P = 10 \times 2400 + 8 \times 2800$
$= \$46\,400$ **[1 mark]**

Question 4
The units of measurement of the right hand side is kilometres, so the units of measurement of the left hand side must also be kilometres.

Question 5
a. $b + g = 200$ (1)
$\frac{b}{2} + 2g = 190$ (2) **[1 mark]**

b. Multiply (1) by 2:
$2b + 2g = 400$ (3)
$\frac{b}{2} + 2g = 190$ (2)
Subtract (2) from (3):
$\frac{3b}{2} = 210$
$b = 140$ **[1 mark]**

c. Substitute $b = 140$ into (1)
$(140) + g = 200$
$g = 60$ **[1 mark]**

2.6 Review

Question 1
$F = \frac{9}{5}C + 32$
$= \frac{9}{5}(25) + 32$
$= 77$
25° C is equivalent to 77° F.

© John Wiley & Sons Australia, Ltd

Question 2

$115 - 5p = 45$

$-5p = 45 - 115$

$-5p = 45 - 115$

$-5p = -70$

$p = \dfrac{-70}{-5}$

$p = 14$

Question 3

$\dfrac{7x}{3} = 4x - 2$

Question 4

$(x + 3) + (x + 3) + x = 24$

$3x + 6 = 24$

$3x = 24 - 16$

$3x = 18$

$x = \dfrac{18}{3}$

$x = 6$ cm

Question 5

$2a + 3c = 24.80$ [1]

$2a + c = 16.50$ [2] **[1 mark]**

Multiply equation [2] by -3 to eliminate the c values by addition.

$2a + 3c = 24.80$ [1]

$-6a - 3c = -49.50$ $-3 \times$ [2] **[1 mark]**

$-4a = -24.70$

$a = \dfrac{-24.70}{-4}$

$a = 6.175$

$a = \$6.20$ (to the nearest 5 cents) **[1 mark]**

Substitute $a = 6.20$ into equation [1] to find c.

$2a + 3c = 24.80$ [1]

$2(6.20) + 3c = 24.80$

$12.40 + 3c = 24.80$

$3c = 12.40$

$c = \dfrac{12.40}{3}$

$c = 4.1333$ **[1 mark]**

$c = \$4.15$ (to the nearest 5 cents) **[1 mark]**

Question 6

Reading from the graph: **[1 mark]**

Approximately 13°C **[1 mark]**

Question 7

$\times$ both sides by x, then 3 ($x \neq 0$)

Note that the zero denominator would make the equation undefined.

Question 8

$\times$ both sides by ($x \neq 0$), then $\div$ by b ($b \neq 0$)

© John Wiley & Sons Australia, Ltd

Question 9

$m(x + 1) = p(x - n)$

$mx + m = px - pn$ Remove brackets first **[1 mark]**

$mx - px = -m - pn$ Sort x terms from literal (other) terms

$x(m - p) = -m - pn$ Take x out as common factor **[1 mark]**

$x = \frac{-m - pn}{m - p}$ or $x = -\frac{m + pn}{m - p}$ or $x = \frac{m + pn}{p - m}$ Divide both sides by $(m - p)$ **[1 mark]**

Note: Watch for order of operations and negative numbers.

Question 10

a. a.Reading from the table gives an extension of 80 cm. **[1 mark]**

b. The value of the extension is always four times the value of the weight, so the relationship is $E = 4W$. **[1 mark]**

c. The table has values for 96 cm and 104 cm. The weight must be between 24 kg and 26 kg. Interpolation gives the weight 25 kg. Alternatively the equation gives $4 \times 24 = 100$. **[1 mark]**

Question 11

a. Read from the table 0.2m/s. **[1 mark]**

b. Read from the table or from the graph; estimated 1.22s **[1 mark]**

c. Velocity is negative when the ball is on the way down. **[1 mark]**
Positive velocity for the ball on the way up, zero when it is stopped and negative on the way down.

Question 12

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C$$

$$5(F - 32) = 9C$$

$$\frac{5(F - 32)}{9} = C$$

$$C = \frac{5(F - 32)}{9}$$

$$= \frac{5(105 - 32)}{9}$$

$$= \frac{5(73)}{9}$$

$$= 40.555\,555...$$

$C = 40.56$ (correct to two decimal places)

105° F is equivalent to 40.56° C

Question 13

$\frac{4x}{3} = 12$

$4x = 12 \times 3$

$4x = 36$

$x = \frac{36}{4}$

$x = 9$

© John Wiley & Sons Australia, Ltd

Question 14

$$v = u + at$$
$$v - u = at$$
$$\frac{v-u}{a} = t$$
$$t = \frac{v-u}{a}$$

Question 15

$$4p - 6 = 14y$$
$$4p = 14y + 6$$
$$p = \frac{14y+6}{4}$$
$$p = \frac{2(7y+3)}{4}$$
$$p = \frac{7y+3}{2}$$

Question 16

$$y = \frac{2(x-3)}{5}$$
$$= \frac{2(-1-3)}{5}$$
$$= \frac{2(-4)}{5}$$
$$= \frac{-8}{5}$$
$$= -1\frac{3}{5}$$

Question 17

a. $6c = 20.70$

$$c = \frac{20.70}{6}$$
$$c = 3.45$$

The cost of one red velvet cupcake with cream cheese frosting is $3.45. **[1 mark]**

$$4c = 19.00$$
$$c = \frac{19.00}{4}$$
$$c = 4.75$$

The cost of one chocolate mud cupcake with chocolate ganache is $4.75. **[1 mark]**

$$5c = 17.25$$
$$c = \frac{17.25}{5}$$
$$c = 3.45$$

The cost of one vanilla cupcake with buttercream icing is $3.45. **[1 mark]**

$$3c = 19.05$$
$$c = \frac{19.05}{3}$$
$$c = 6.35$$

© John Wiley & Sons Australia, Ltd

The cost of one strawberry cupcake with fresh cream and jam filling is \$6.35. **[1 mark]**
∴ The cheapest options are red velvet cupcake with cream cheese frosting and vanilla cupcake with buttercream icing. [1 mark]

b. Let r represent the price of red velvet cupcakes, c represent the price of chocolate mud cupcakes and s represent the price of strawberry cupcakes.

$r = \$3.45$
$c = \$4.75$
$s = \$6.35$

Total cost $= 3r + 2c + 5s$ **[1 mark]**
$= 3(3.45) + 2(4.75) + 5(6.35)$
$= 10.35 + 9.50 + 31.75$
$= 51.60$

∴ The total cost of the order is \$51.60. **[1 mark]**

c. i. $8 \times 2.54 = 20.32$ cm **[1 mark]**

ii. $6 \times 2.54 = 15.24$ cm **[1 mark]**

d. $V_{\text{bottom tier}} = \pi r^2 h$
$= \pi \times (10.16)^2 \times 5$
$= 1621.463\,933$ cm^3 **[1 mark]**

$V_{\text{top tier}} = \pi r^2 h$
$= \pi \times (7.62)^2 \times 4.5$
$= 820.866\,116\,1$ cm^3 **[1 mark]**

Total volume $= 1621.463\,933 + 820.866\,116\,1$
$= 2442.330\,0491$ cm^3 **[1 mark]**
$= 2442.33$ cm^3 (correct to 2 decimal places) **[1 mark]**

e. Total volume $= 2442.33$

Numbers of serves $= \dfrac{2442.33}{48.50}$ **[1 mark]**
≈ 50.36
≈ 50 serves **[1 mark]**

Question 18

The first equation can be rewritten without the fractions as $2x - y = 20$

$\dfrac{x}{2} - \dfrac{y}{4} = 5$ [1]

$2x + y = 3$ [2]

Re write the equation [1] without fractions by multipliying each term by 4. This gives:

$2x - y = 20$ [1]

Therefore:

$2x - y = 20$ [1]
$2x + y = 3$
$4x = 23$
$x = \dfrac{23}{4}$
$x = 5\dfrac{3}{4}$

Substitute $x = 5\dfrac{3}{4}$ into equation [2] to find y.

$2x + y = 3$ [2]
$2\left(5\dfrac{3}{4}\right) + y = 3$

© John Wiley & Sons Australia, Ltd

$$11\frac{1}{2} + y = 3$$
$$y = 3 - 11\frac{1}{2}$$
$$y = -8\frac{1}{2}$$

$\therefore$ The solution is $\left(5\frac{3}{4}, \ -8\frac{1}{2}\right)$

Question 19

a. Let x represent the cost of an apple and let y represent the cost of an orange.
$12x + 9y = 14.75$ and $x = 2y$ are the two equations which represent the situation.
Award 1 mark for each correct equation.

b. $12x + 9y = 14.75$ [1]
$x = 2y$ [2]
Substitute equation [2] into equation [1] **[1 mark]**
$12x + 9y = 14.75$ [1]
$12(2y) + 9y = 14.75$
$24y + 9y = 14.75$
$33y = 14.75$
$y = 0.45$ (correct to the nearest cent) **[1 mark]**

c. Substitute the price of an orange into equation [2] to find the price of an apple.
$y = 0.45$ (correct to the nearest cent)
$x = 2y$ [2]
$x = 2(0.45)$
$x = 0.90$ (correct to the nearest cent) **[1 mark]**

d. $12x + 9y = 14.75$
$12(0.90) + 9(0.45) = 14.75$
$10.80 + 4.05 = 14.75$
$14.85 \neq 14.75$ **[1 mark]**

e. There is a difference in the answers. The total is supposed to be \$14.75 but when you check to confirm the answer is \$14.85.
This difference is due to rounding. The price of an orange is actually $0.4\overline{6}$ **[1 mark]**
It is not possible to charge this amount, so the price is rounded to the nearest cent. This is where the difference occurs **[1 mark]**.

© John Wiley & Sons Australia, Ltd

3 Financial mathematics

Topic	**3**	**Financial mathematics**
Subtopic	**3.2**	**Ratio, rates and percentages**

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
Angelique, Jerome and Summer were mentioned as beneficiaries in a will. The total inheritance was $50 000. This money was to be divided in the ratio of 2:3:1.
The amount of money received by Summer is
A. $500
B. $5000
C. $16 666.67
D. $25 000
E. $8333.34

Question 2 (1 mark)
Simon divides his pay into three parts in the ratio 3:2:1, with 3 parts for rent, 2 parts for living and the rest for saving. He earns $900 per week.
Select from the following the amount that he pays in rent.
A. $300
B. $450
C. $600
D. $200
E. $250

Question 3 (5 marks)
A group of Year 11 students was shopping together to gather provisions for their Outward Bound experience. The sizes and costs are: 150 g for $2.09; 250 g for $2.89; 500 g for $5.99.
a. State which of the three jars of sauce was the most economical. Explain your reasoning. **(4 marks)**

© John Wiley & Sons Australia, Ltd

b. State whether the most economical purchase was the one you expected. **(1 mark)**

Question 4 (1 mark)
Which ratio is equivalent to the ratio 15:7?
A. 45:49
B. 7:15
C. 45:14
D. 90:63
E. 60:28

Question 5 (1 mark)
The bread dough rose so that its volume increased in a ratio 2:1. The dough's volume was originally 450 cm^3. What was its final volume?
A. 500 cm^3
B. 625 cm^3
C. 800 cm^3
D. 900 cm^3
E. 1000 cm^3

Question 6 (2 marks)
The area of the paddock was 6 hectares after it had been increased in area in the ratio 3:2. Find the original area.

Question 7 (2 marks)
Julie invested $5000 capital in a bank term deposit for 3 years at an interest rate of 4% per annum. At the end of the term, the money had grown to $5624.32. What proportion of this money was interest? Express the answer as a decimal correct to 3 decimal places.

© John Wiley & Sons Australia, Ltd

Question 8 (1 mark)
15 copies of a paperback novel weigh 1.8 kg. How much would 40 copies of that novel weigh?
A. 120 g
B. 1.2 kg
C. 4.8 kg
D. 480 g
E. 3.6 kg

Question 9 (1 mark)
A restaurant agrees to provide dinner for 17 people for $1224. An extra three people turned up on the night. How much should the restaurant charge now?
A. $61.20
B. $72
C. $1040.40
D. $1440
E. $3590.40

Question 10 (1 mark)
The bank offers a currency exchange of US$820 for A$1100. The travel agent says he can get an exchange rate of US$500 for A$600. Daniel exchanged A$1000. How many whole US dollars did he get if he chose the better of the two offers?
A. $1200
B. $833
C. $745
D. $1341
E. $913

© John Wiley & Sons Australia, Ltd

Topic	3	Financial mathematics
Subtopic	3.3	Percentage applications and GST

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

The price of petrol was $1.38 per litre, but has now risen to $1.59 per litre. The percentage change in the price of petrol is

A. 21%

B. $\frac{21}{138}\%$

C. $\frac{7}{46}\%$

D. 350%

E. $15\frac{5}{23}\%$

Question 2 (1 mark)

During a supermarket sale campaign, the price of bread went 'down, down, down'.

If the original price of the bread was $1.95 and the new price is $1.05, the percentage decrease in price is

A. 90%

B. $46\frac{2}{13}\%$

C. $\frac{6}{13}\%$

D. 600%

E. $\frac{90}{195}\%$

© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)
In one week, Max earns $127 working at a drive-through coffee shop.
If he saved $34, the percentage of his weekly earnings that he spent, correct to the nearest percent, is
A. 127%
B. 34%
C. 73%
D. 93%
E. 161%

Question 4 (1 mark)
A machine fills bottles of orange juice labelled as 1 L. The actual volume of juice in a bottle ranges from 992 mL to 1008 mL. The range of volumes in bottles as a percentage of the labelled volume is:
A. 8%
B. 16%
C. 1.6%
D. 10%
E. 98.4%

Question 5 (1 mark)
The value of Bill's house was $1.2 million. That value increased by 6% in a year. What was the value of the house at the end of that year?
A. $1.272 million
B. $1.92 million
C. $72 000
D. $1 207 200
E. $1 920 000

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)
The shares Maria bought are now worth $55 000. This is 125% of their value 12 months ago. How much did Maria pay for the shares 12 months ago?
A. $13 750
B. $22 000
C. $27 000
D. $44 000
E. $68 750

Question 7 (1 mark)
When football teams are on the same rung of the 'ladder', their position can be decided on the ratio of points for : points against, expressed as a percentage, to the nearest whole number.
After two rounds, the ratio of points for : points against for the Maggies and the Puppies was 224:165. Expressed as a percentage, this is
A. 380%
B. 136%
C. 74%
D. 36%
E. 26%

© John Wiley & Sons Australia, Ltd

Question 8 (1 mark)

When the factory opened in Geeton, the population increased by 3% in 1 year. To find the new population you need to multiply by

A. 1.03

B. 1.3

C. $\frac{100}{97}$

D. $\frac{97}{100}$

E. $\frac{3}{100}$

Question 9 (1 mark)

A text book costs $54.99, including 10% GST. The price excluding GST is

A. $60.49

B. $56.09

C. $53.89

D. $49.99

E. $43.83

© John Wiley & Sons Australia, Ltd

Topic	3	Financial mathematics
Subtopic	3.4	Simple interest applications

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
Ragnar invests $1500 at 15% p.a. simple interest for 18 months. At the end of the investment, the total amount that Ragnar will have in the investment account is
A. $337.50
B. $4050
C. $5550
D. $1837.50
E. $33 750

Question 2 (1 mark)
Alice's bank account pays 0.01% interest per month on the minimum monthly balance. She never has less than $275 in the bank. The least interest she earns is
A. $27.50
B. $2.75
C. 27.5 cents
D. 2.75 cents
E. 0.275 cents

Question 3 (1 mark)
Michelle borrowed $500 for 2 years at a simple interest rate of 8.5%. The amount she had to pay back was
A. $670
B. $585
C. $542.50
D. $85
E. $42.50

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
Philip is saving up for a new bike. He puts $2000 in a term deposit for 6 months at 3.05% per annum interest paid at maturity. At maturity the deposit will be worth
A. $2061
B. $2060
C. $2030.50
D. $3050
E. $30.50

Question 5 (1 mark)
The sum to the nearest dollar that must be invested for 1 year at 3.5% per annum simple interest in order to earn $2100 interest is
A. $600
B. $2029
C. $7350
D. $73 500
E. $60 000

© John Wiley & Sons Australia, Ltd

Topic	3	Financial mathematics
Subtopic	3.5	Compound interest applications

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2020, Further Mathematics Exam 1, Section A, Q.27; © VCAA*

Question 1 (1 mark)

Gen invests $10 000 at an interest rate of 5.5% per annum, compounding annually.
After how many years will her investment first be more than double its original value?

A. 12
B. 13
C. 14
D. 15
E. 16

Question 2 (1 mark)

When considering the compound interest formula $A = P\left(1 + \frac{r}{100}\right)^n$, select which of the following statements is *false*.

A. The formula calculates only the interest earned.
B. P represents principal, r represents rate and n represents the number of compounding periods.
C. The A in the formula represents the final amount in the investment account at the end of the investment.
D. When using the formula, the order of operations must be followed.
E. When considering compounding periods you must calculate the number of times that interest is calculated and added to the account over the life of the loan.

© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)
If $5000 is invested at 8% p.a. compound interest for 3 years, compounding bi-annually, the interest earned is
A. $6326.60
B. $5624.32
C. $1326.60
D. $624.32
E. $5826.60

Question 4 (1 mark)
In 1960, the cost of a loaf of bread was 25 cents. The average inflation rate for the next 50 years was 3% per annum.
The cost of a loaf of bread in 2010 would have been, approximately,
A. $4.98
B. $4.38
C. $1.10
D. $1.24
E. $2.50

Question 5 (1 mark)
My private health insurance premiums have averaged an increase of 6% every year for the last 5 years. The premium 5 years ago was $1240 per annum. What will the premium be after five rises?
A. $1659.40
B. $1565.47
C. $1758.96
D. $1476.86
E. $3318.80

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)
$10 000 is invested for 1 year at 4.4% per annum compound interest. What is the difference between the final amounts if the interest is adjusted quarterly compared with if the interest is adjusted annually?
A. $330
B. $331.31
C. $447.31
D. $7.31
E. $440

Question 7 (1 mark)
$4000 was invested in shares paying a dividend of 4.88% per annum, compounded quarterly. The value of the investment at the end of 3 years was nearest to
A. $4615
B. $4148
C. $4627
D. $4049
E. $11 566

Question 8 (5 marks)
$15 000 is invested for 10 years at an interest rate of 3.05% per annum compound interest.
a. If the interest is adjusted annually, find the value of the investment at the end of 10 years. **(1 mark)**

b. How much interest is earned on this investment? **(1 mark)**

c. If the interest is adjusted quarterly, find the value of the investment at the end of 10 years. **(1 mark)**

© John Wiley & Sons Australia, Ltd

d. How much interest is earned in this case? **(1 mark)**

e. Find the difference in the interest earned on these investments. **(1 mark)**

Question 9 (3 marks)

Mark started work on a salary of $38 000 per annum with an annual increase of 4%.
Marianne started work in the same year on the same salary. For the first 2 years, Marianne had an annual increase of 5% per annum, then she had an increase of 3% per annum for the next 8 years.

a. How much will Mark's salary be after completing 10 years of employment? **(1 mark)**

b. How much will Marianne's salary be after completing 10 years of employment? **(1 mark)**

c. Find the difference in their salaries after 10 years. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Topic	3	Financial mathematics
Subtopic	3.6	Purchasing options

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2016, Further Mathematics Exam 1, Section A, Q.23; © VCAA*

Question 1 (1 mark)

Sarah invests $5000 in a savings account that pays interest at the rate of 3.9% per annum compounding quarterly. At the end of each quarter, immediately after the interest has been paid, she adds $200 to her investment.

After two years, the value of her investment will be closest to

A. $5805
B. $6600
C. $7004
D. $7059
E. $9285

Source: *VCE 2015, Further Mathematics Exam 1, Section B, Module 4, Q.1; © VCAA*

Question 2 (1 mark)

Fong's gas bill is $368.40. If he pays this bill on time, it will be reduced by 5%.

In this case, the bill would be reduced by

A. $1.84
B. $5.00
C. $18.42
D. $184.20
E. $349.98

© John Wiley & Sons Australia, Ltd

Question 3 (12 marks)

A Year 11 student wishes to invest some money in order to help him save for a holiday more quickly. He is looking at two different investment options.

Option 1: Simple interest at 5% per annum for 4 years

Option 2: Compound interest at 3.5% per annum for 2 years

a. Calculate the interest earned if he invests $3500 in the account offering option 1. **(1 mark)**

b. Calculate the interest earned if he invests $3500 in the account offering option 2. **(2 marks)**

He decides that he really doesn't want to lock his money away for 4 years, but he definitely wants more interest than option 2 offers. He does some negotiating with the bank and they agree to change option 2 slightly so that it compounds biannually.

c. Calculate the interest earned if he selects the new and improved option 2. **(3 marks)**

The interest is better! But it is not quite enough. The bank agrees to extend the lending period on option 2.

d. Determine for how many compounding periods he needs to invest the money to have a total of $5000 (investment and interest). **(5 marks)**

Before he decides, he needs to calculate how long this investment will be for.

e. Determine for how long he needs to invest his money in this investment for him to achieve his goals, and state whether you think he will select this option. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
Sue is buying a new phone. If she uses her Byer credit card the phone will cost her $850 plus interest. The interest rate on the card is 18% per annum and there is a $25 penalty for late payment. Calculate the accumulated debt on the phone if she doesn't pay the full amount on the due date this month.
A. $887.75
B. $859.75
C. $875
D. $850
E. $1003

Question 5 (1 mark)
Denis is offered 7% discount if he buys a new smart TV with cash. The TV is advertised for $499. How much will Denis pay if he pays cash?
A. $34.93
B. $533.93
C. $499
D. $464.07
E. $848.30

Question 6 (3 marks)
Paul is purchasing his $80 000 delivery truck on hire purchase. He traded in his old truck for $25 000 and agreed to repay the rest by paying $2770 per month for 2 years.
a. How much interest is he being charged? **(1 mark)**

b. What is the real rate of interest? **(1 mark)**

c. What is the effective rate of interest? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Topic	3	Financial mathematics
Subtopic	3.7	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2015, Further Mathematics Exam 1, Section B, Module 4, Q.2; © VCAA*

Question 1 (1 mark)

An investment property was purchased for $600 000. Over a 10-year period, its value increased to $850 000.

The increase in value, as a percentage of the purchase price, is closest to

A. 4.2%
B. 25.0%
C. 29.4%
D. 41.7%
E. 70.6%

Source: *VCE 2015, Further Mathematics Exam 2, Module 4, Q.1; © VCAA*

Question 2 (3 marks)

Jane and Michael have started a business that provides music at parties.

The business charges customers $88 per hour.

The $88 per hour includes a 10% goods and services tax (GST).

a. Calculate the amount of GST included in the $88 hourly rate. **(1 mark)**

b. Jane and Michael's first booking was a party where they provided music for four hours.
Calculate the total amount they were paid for this booking. **(1 mark)**

c. After six months of regular work, Jane and Michael decided to increase the hourly rate they charge by 12.5%.
Calculate the new hourly rate (including GST). **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)
A newspaper reported that the price of petrol had increased by 35% in the last five years. If the current price is $1.59, the original price of petrol mentioned in the article was
A. $0.56
B. $2.14
C. $1.18
D. $2.94
E. $1.35

Question 4 (1 mark)
A television was advertised as being marked down by $33\frac{1}{3}\%$. If the original price of the television is $2999 , the discounted price will be
A. $2666.33
B. $999.67
C. $2000
D. $1999.33
E. $1500

Question 5 (7 marks)
On occasion items can be purchased by paying a deposit and then paying monthly instalments. Leonard was planning to purchase a surround sound entertainment system worth $5495 and was investigating different ways in which this purchase could be made. If he paid a 20% deposit, he would be able to pay the system off over the next year.
a. Calculate a 20% deposit on this system. **(1 mark)**

b. Calculate the balance owing. **(1 mark)**

© John Wiley & Sons Australia, Ltd

c. Leonard is able to make monthly payments for 12 months in order to pay the remainder of the cost. **(1 mark)**
Calculate each monthly instalment, correct to the nearest dollar.

d. The shop assistant offers Leonard a discount to try to entice him to buy the system now. The shop assistant offers a 32% discount on the total cost.

i. **Calculate** the amount of the discount. **(1 mark)**

ii. **Calculate the cost of** the entertainment system with the discount applied. **(1 mark)**

e. Leonard decides to go with the payment plan, but he decides to pay slightly more each month. He decides to pay $425 each month.

i. Determine how quickly Leonard will pay off the balance remaining. **(1 mark)**

ii. Calculate the final payment. **(1 mark)**

Question 6 (5 marks)
A sheep eats about 3% of its body weight in grass each day.
An average sheep weighs 68 kg.

a. What weight of grass will a sheep eat each day? **(1 mark)**

© John Wiley & Sons Australia, Ltd

b. For 1 hectare (ha) of land, every centimetre of growth in grass height weighs 175 kg.
Bob has a 1 ha paddock with grass 6 cm $\frac{1}{3}$ of that height for forage.
What height of grass will he leave for forage? **(1 mark)**

c. What weight of grass will be available for the sheep to eat? **(1 mark)**

d. In fact, Bob has found that 25% of the feed is wasted.
What weight of grass is actually available for the sheep to eat? **(1 mark)**

e. How many sheep can Bob feed on his 1-ha paddock to be sure that he can supply enough feed for 1 day? **(1 mark)**

Question 7 (3 marks)
33 adults earned $160 380 for 27 days' work. How much will 17 teenagers earn for 12 days' work if a teenager earns 70% of the adults' wage?

© John Wiley & Sons Australia, Ltd

Question 8 (3 marks)
A workforce of 35 painters is able to finish painting all the apartments in a high-rise building in 45 days. However, the painting firm loses 7 of the painters after 30 days.

a. What fraction of the job is left after they lose the 7 painters? **(1 mark)**

b. How long would it take 28 painters to do the rest of the job? **(1 mark)**

c. How long will it take to paint all the apartments now? **(1 mark)**

Question 9 (3 marks)
Harrison's Car Sales imported a sports car valued at $62 000. Customs required 40% tax to be paid.

a. How much tax did Harrison pay? **(1 mark)**

b. How much did the car cost him altogether? **(1 mark)**

c. He put a mark-up of 60% on the price of the car. How much was the car priced to the customer? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 10 (5 marks)

Aggie's Boutique was set up with Frank and Nick making contributions of $24 000 and $36 000, respectively.

a. What is the ratio of the contributions by Frank and Nick? **(1 mark)**

b. The shop made a profit of $15 000 in the first year. The men received a share of the profits in the same ratio as their original contributions.
What was the profit share for:

i. Frank **(1 mark)**

ii. Nick? **(1 mark)**

c. In the second year, the profit decreased by $750.
What was the profit share for:

i. Frank **(1 mark)**

ii. Nick? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 11 (2 marks)
Dean's bank statement is:

Date	Comment	Deposit	Withdrawal	Balance
1-Mar				$ 510.00
1-Mar	Fee		$ 5.00	$ 505.00
2-Mar	Allowance	$1450		$ 1955.00
2-Mar	EFTPOS fee		$ 0.65	$ 1954.35
9-Mar	Supermarket		$ 120.00	$ 1834.35
9-Mar	EFTPOS fee		$ 0.65	$ 1833.70
15-Mar	Rent		$ 640	$ 1193.70
16-Mar	Supermarket		$ 120.00	$ 1073.70
16-Mar	EFTPOS fee		$ 0.65	$ 1073.05
17-Mar	Phone		$ 65.00	$ 1008.05
24-Mar	Supermarket		$ 120.00	$ 888.05
24-Mar	EFTPOS fee		$ 0.65	$ 887.40
31-Mar	Cash out		$ 125.00	$ 762.40
31-Mar	Bank fee		$ 2.00	$ 760.40

a. The interest rate is 2.5% per annum on the minimum monthly balance. How much interest did Dean earn? **(1 mark)**

b. Because he was concerned that he would not have enough money to live on, he decided to pay the rent by credit card and pay the money back after 2 months and only be charged 1 month's interest. The interest rate is 19.49% per annum simple interest. How much will it cost Dean in monthly credit card interest to borrow the money for paying the rent? **(1 mark)**

Question 12 (4 marks)
Sue has three alternatives for investing her $3500 for 6 months. She knows that she will not need access to her money in that time. Calculate the amount returned on her investment in each case and hence find which is the best option.

a. 2.5% per annum simple interest **(1 mark)**

© John Wiley & Sons Australia, Ltd

b. 3.05% per annum simple interest, payable after 6 months, and a $5 per month fee paid at the end **(1 mark)**

c. An upfront fee of $2 and 2.8% per annum on the remainder, payable after 6 months **(2 mark)**

Question 13 (1 mark)

In order to calculate the principal on an investment, the simple interest formula $I = \frac{Prn}{100}$ may be transposed into

A. $P = \frac{rn}{100I}$

B. $P = \frac{100r}{In}$

C. $P = \frac{100I}{rn}$

D. $P = \frac{100n}{rI}$

E. $P = \frac{Irn}{100}$

Question 14 (1 mark)

A small business owner purchases a car for $22 900. The car depreciates at a flat rate of 8% per annum. The book value of the car after 4 years will be:

A. $7328

B. $15 572

C. $17 404

D. $30 228

E. $5496

© John Wiley & Sons Australia, Ltd

Question 15 (3 marks)
A secondary school buys a new photocopier for $92 000. It depreciates at 11% per annum, reducing balance. How much will the photocopier be worth after 3 years?

Question 16 (5 marks)
Answer the following

a. Find the cost of each of these purchase options for a cruise to Fiji:

i. Pay cash price of $6000 with a discount of 5% for immediate payment **(1 mark)**

ii. Take a personal loan of $6000 from the bank with repayments of $120 per week for 52 weeks **(1 mark)**

iii. Borrow $6000 from a finance company with the full amount of $7200 to be repaid at the end of 1 year **(1 mark)**

iv. Pay $6000 by credit card with a flat rate of interest of 18% per annum and repay principal and interest at the end of 6 months. **(1 mark)**

b. Hence, find the most expensive method of payment. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Answers and marking guide

3.2 Ratio, rates and percentages

Question 1

Angelique : Jerome : Summer

2 : 3 : 1 (6 parts)

$\frac{2}{6} \times 50\,000 : \frac{3}{6} \times 50\,000 : \frac{1}{6} \times 50\,000$

$\frac{1}{3} \times 50\,000 : \frac{1}{2} \times 50\,000 : \frac{1}{6} \times 50\,000$

\$16 666.67 : \$25 000 : \$8333.34

Summer receives a one-sixth share or \$8333.34

Question 2

Pay needs to be divided into $3 + 2 + 1 = 6$ parts.

Rent:

$$\frac{3}{6} = \frac{1}{2}$$

$\frac{1}{2} \times \$900 = \$450$

Therefore, \$450 is paid in rent.

Question 3

a. Jar 1

$\$2.09 \div 150 = 0.013\,93\dot{3}$ per gram

$= 0.013933\dot{3} \times 100$

= \$1.39 per 100 grams **[1 mark]**

Jar 2

$\$2.89 \div 250 = 0.011\,56$ per gram

$= 0.01156 \times 100$

= \$1.16 per 100 grams **[1 mark]**

Jar 3

$\$5.99 \div 500 = 0.011\,98$ per gram

$= 0.01198 \times 100$

= \$1.20 per 100 grams **[1 mark]**

The medium-sized jar is the cheapest per 100 grams of sauce. The students should purchase this one. **[1 mark]**

b. It should be expected that the largest jar would be the cheapest item; however, this is not always the case. **[1 mark]**

Question 4

$\frac{15}{7} = \frac{60}{28}$ multiplying numerator and denominator by 4.

Question 5

$\frac{2}{1} \times 450\ \text{cm}^3 = 900\ \text{cm}^3$

© John Wiley & Sons Australia, Ltd

Question 6

The new area is $\frac{3}{2}$ times the original area so the original area is $\frac{2}{3}$ of the new area. **[1 mark]**

The original area $= \frac{2}{3} \times 6 = 4$ hectares

Question 7

Proportion is ratio or fraction:

624.32 :5000 **[1 mark]**

$= 0.124864$

≈ 0.125 **[1 mark]**

Question 8

1 copy weighs:

$1.8\,\text{kg} \div 15 = 0.12\,\text{kg}$

40 copies weigh:

$0.12\,\text{kg} \times 40 = 4.8\,\text{kg}$

Question 9

1 person costs $= 1224 \div 17$

20 people costs $= 20 \times 1224 \div 17$

$= \$1440$

Question 10

Bank exchange rate: A\$1 = US\$0.745 45

Travel agent exchange rate: A\$1 = US\$0.833 33

$\therefore$ The travel agent provides the better exchange rate.

$\$1000 \times 0.83333 = \$833.33$

$\therefore$ Daniel would have received \$833.

3.3 Percentage applications and GST

Question 1

$$\text{Increase} = \$1.59 - \$1.38$$
$$= \$0.21$$
$$\frac{0.21}{1.38} = \frac{21}{138}$$
$$= \frac{7}{46}$$
$$\text{Percentage increase} = \frac{7}{46} \times 100$$
$$= \frac{7}{46} \times \frac{100}{1}$$
$$= \frac{7}{23} \times \frac{50}{1}$$
$$= \frac{350}{23}$$
$$= 15\frac{5}{23}\%$$

© John Wiley & Sons Australia, Ltd

Question 2

$$\text{Decrease} = \$1.95 - \$1.05$$
$$= \$0.90$$
$$\frac{0.90}{1.95} = \frac{90}{195}$$
$$= \frac{6}{13}$$
$$\text{Percentage increase} = \frac{6}{13} \times 100$$
$$= \frac{6}{13} \times \frac{100}{1}$$
$$= \frac{600}{13}$$
$$= 46\frac{2}{13}\%$$

Question 3

$$\text{Dollars spent} = \$127 - \$34$$
$$= \$93$$
$$\text{Percentage spent} = \frac{93}{127} \times 100$$
$$= 73.23$$
$$= 73\% \text{ (correct to the nearest percent)}$$

Max spent 73% of his weekly earnings.

Question 4

Range is 16 mL in 1000 L, which is 1.6 in 100 or 1.6%

Question 5

Take 6% of $1.2 million and add it or multiply $1.2 million by 1.06.

$1.06 \times 1.2 = 1.272$

∴The value of the house was $1.272 million.

It is easier to work in millions rather than having to deal with all the zeros in the full numbers.

Question 6

Dividing by 1.25 is the easiest method. An increase of 125% means the shares' value has multiplied by 1.25.

$55\,000 \div 1.25 = 44\,000$

∴ The original value of the shares was $44 000.

Question 7

$$\frac{224}{165} \times \frac{100}{1} = 136\%$$

Question 8

1.03

Question 9

$$\$54.99 \times \frac{10}{11} = \$49.99$$

© John Wiley & Sons Australia, Ltd

3.4 Simple interest applications

Question 1

18 months = 1.5 years

Simple interest: $I = \frac{Prn}{100}$

$$= \frac{1500 \times 15 \times 1.5}{100}$$
$$= \$337.50$$

Total investment $= \$1500 + 337.50$
$= 1837.50$

Question 2

$0.01\% \times \$275 = \$0.0275$
$= 2.75$ cents

Question 3

Simple interest: $I = \frac{Prn}{100}$

$$= \frac{500 \times 8.5 \times 2}{100}$$
$$= \$85$$

The amount she had to pay the bank was $\$500 + 85 = \$585$.

Question 4

$$\left(\frac{1}{2} \times 3.05\right)\% \times \$2000 = \$30.50$$

Deposit $= \$2000 + \$30.50$
$= \$2030.50$

Question 5

$P \times 0.035 = \$2100$

So, $P = \frac{2100}{0.035}$
$= \$60\,000$

3.5 Compound interest applications

Question 1

Solve on CAS:

$10\,000 \times 1.055^n = 20\,000$
$n = 12.946$
$= 13$

Question 2

The false statement is 'The formula calculates only the interest earned.' The value calculated using the formula $A = P\left(1 + \frac{r}{100}\right)^n$ is the total amount of the investment; that is, the principal and the interest.

© John Wiley & Sons Australia, Ltd

Question 3

$$A = P\left(1 + \frac{r}{c \times 100}\right)^{n \times c}$$

$$A = 5000\left(1 + \frac{8}{2 \times 100}\right)^{3 \times 2}$$

$$A = 5000(1.04)^6$$

$$A = \$6326.60$$

$$I = A - P$$

$$I = \$6326.60 - \$5000$$

Interest earned $= \$1326.60$

Question 4

$$0.25 \times (1.03)^{50} \approx 0.25 \times 4.3839$$

$$\approx \$1.10$$

Question 5

$$\$1240\left(1 + \frac{6}{100}\right)^5 = \$1240 \times 1.06^5$$

$$\approx \$1659.40$$

Question 6

$$\$10\,000\left(1 + \frac{4.4}{400}\right)^4 - \$10\,000\left(1 + \frac{4.4}{100}\right) \approx \$10\,447.31 - \$10\,440$$

$$= \$7.31$$

Question 7

$$\text{Amount} = \$4000\left(1 + \frac{4.88}{400}\right)^{12}$$

$$= \$4000 \times (1.0122)^{12}$$

$$\approx \$4000 \times 1.1566$$

$$\approx \$4627$$

Question 8

a. $\$15\,000\left(1 + \frac{3.05}{100}\right)^{10} \approx \$20\,256.82$ **[1 mark]**

b. $\$15\,000\left(1 + \frac{3.05}{100}\right)^{10} - \$15\,000 \approx \$5256.82$ **[1 mark]**

c. $\$15\,000\left(1 + \frac{3.05}{100}\right)^{40} \approx \$20\,325.8$ **[1 mark]**

d. $\$15\,000\left(1 + \frac{3.05}{100}\right)^{40} - \$15\,000 \approx \$5325.85$ **[1 mark]**

e. $\$5\,325.85 - \$5\,256.82 = \$69.03$ **[1 mark]**

Question 9

a. $\$38\,000\left(1 + \frac{4}{100}\right)^{10} \approx \$56\,249.28$ **[1 mark]**

b. $\$38\,000\left(1 + \frac{5}{100}\right)^{2}\left(1 + \frac{3}{100}\right)^{8} \approx \$53\,071.33$ **[1 mark]**

c. $\$56\,249.28 - \$53\,071.33 = \$3177.95$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

3.6 Purchasing options

Question 1

Use Finance Solver on the CAS:
N: 8
I: 3.9
PV: −5000
PMT: −200
FV: 7059.249 ...
PPY: 4
CPY: 4
Thus, the answer is closest to D ($7059.25).

Question 2

Reduction = 5%

$$368.40 \times 5\% = 384.40 \times \frac{5}{100}$$

Reduction = 18.42
The reduction in price is $18.42.

Question 3

a. Simple interest: $I = \dfrac{Prn}{100}$

$$= \frac{3500 \times 5 \times 4}{100}$$

$$= \$700 \textbf{ [1 mark]}$$

b.
$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$= 3500\left(1 + \frac{3.5}{100}\right)^2$$

$$= 3500 \times (1.035)^2$$

$$= 3749.29 \textbf{ [1 mark]}$$

Interest earned = $3749.29 − $3500

$$= \$249.29 \textbf{ [1 mark]}$$

c.
$$n = 2 \times 2$$

$$= 4 \textbf{ [1 mark]}$$

$$r = \frac{3.5}{2} = 1.75$$

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$= 3500\left(1 + \frac{1.75}{100}\right)^4$$

$$= 3500 \times (1.0175)^4$$

$$= 3751.51 \textbf{ [1 mark]}$$

Interest earned = $3751.51 − $3500

$$= \$251.51 \textbf{ [1 mark]}$$

© John Wiley & Sons Australia, Ltd

d. $n = ?$ **[1 mark]**

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$5000 = 3500\left(1 + \frac{3.5}{100}\right)^n \textbf{ [1 mark]}$$

$$5000 = 3500 \times (1.035)^n$$

$$\frac{5000}{3500} = (1.035)^n$$

$$1.43 = (1.035)^n$$

Use CAS to solve this and find n.

$\log_{10} 1.43 = n \log_{10} 1.035$ **[1 mark]**

$$n = \frac{\log_{10} 1.43}{\log_{10} 1.035}$$

$n = 10.4$ **[1 mark]**

The number of compounding periods required for this option will be 11. **[1 mark]**

e. The account has interest compounded biannually, so in order to achieve his goal he will need to invest his money for $11 \div 2 = 5.5$ years. He will not select this option as he did not wish to tie his money up for this long. **[1 mark]**

Question 4

Interest per month on \$850 at 18% per annum $= \frac{1}{12} \times \frac{18}{100} \times \$850 = \$12.75$

Penalty means total is $\$12.75 + \$25 = \$37.75$ so the total cost is $\$850 + \$37.75 = \$887.75$

Question 5

7% of \$499 is the discount. The price paid is 93% of \$499, which equals \$464.07.

Question 6

a. Paul owes: $\$80\,000 - \$25\,000 = \$55\,000$

Paul paid: $\$2770 \times 24 = \$66\,480$

Interest charged: $\$66\,480 - \$55\,000 = \$11\,480$ **[1 mark]**

b. Real rate is the interest expressed as the percentage of the amount owed.

$$\frac{x}{y} \times \frac{100}{1}\% = \frac{\$11\,480}{\$55\,000} \times \frac{100}{1}\%$$

$= 20.87\%$ for 2 years

$= 10.435\%$ for 1 year

$\approx 10.44\%$ for 1 year **[1 mark]**

c. The effective rate is double the real rate: $2 \times 10.435 = 20.87\%$ **[1 mark]**

There is no point in using the larger effective rate formula.

3.7 Review

Question 1

Increase in value $= \$850\,000 - \$600\,000 = \$250\,000$

As a percentage of the purchase price $= \frac{250\,000}{600\,000} \times 100\%$

As a percentage of the purchase price $= 41.666 \ldots \%$

Option D is the most correct with 41.7%.

© John Wiley & Sons Australia, Ltd

Question 2

a. Price without GST = (price with GST included) ÷ 1.1

Price without GST = 88 ÷ 1.1

= 80

GST = 88 − 80 **[1 mark]**

b. Total paid = 88 × 4

Total paid = \$352 **[1 mark]**

c. To increase something by 12.5%, multiply by (100 + 12.5) %

New hourly rate = 88 × 112.5%

$$\text{New hourly rate} = 88 \times \frac{112.5}{100}$$

New hourly rate = \$99 **[1 mark]**

Question 3

135% × original price = \$1.59

1.35 × original price = \$1.59

$$\text{Original price} = \frac{\$1.59}{1.35}$$

Original price = \$1.1777...

Original price = \$1.18

Question 4

$33\frac{1}{3}\% \times \$2999 = \$999.67$

New price = \$2999 − \$999.67

= \$1999.33

Question 5

a. 20% × \$5495 = 0.20 × \$5495

= \$1099 **[1 mark]**

b. \$5495 − \$1099 = \$4396 **[1 mark]**

c. \$4396 ÷ 12 = \$366 **[1 mark]**

d. i. 32% × \$5495 = 0.32 × \$5495

= \$1758.40 **[1 mark]**

ii. New price = \$5495 − \$1758.40

= \$3736.60 **[1 mark]**

e. i. Balance owing after 20% deposit = \$4396

After 1 month = \$3971

After 2 months = \$3546

After 3 months = \$3121

After 4 months = \$2696

After 5 months = \$2271

After 6 months = \$1846

After 7 months = \$1421

After 8 months = \$996

After 9 months = \$571

After 10 months = \$146

After 11 months, the debt is cleared. **[1 mark]**

ii. The final payment is \$146. **[1 mark]**

© John Wiley & Sons Australia, Ltd

Question 6

a. Weight of grass $= 3\%$ of 68 kg
$= 0.03 \times 68$ kg
$= 2.04$ kg **[1 mark]**

b. $\frac{1}{3} \times 6$ cm $= 2$ cm **[1 mark]**

c. Since there is 4 cm available and each centimetre weighs 175 kg,
4 cm $\times$ 175 kg $= 700$ kg **[1 mark]**

d. 25% wastage means there is 75% available.
75% of 700 kg $= 525$ kg **[1 mark]**

e. Each sheep eats 2.04 kg in a day and there are 525 kg available.
$525 \div 2.04 = 257$ sheep **[1 mark]**

Question 7

1 adult earns: $\$160\,380 \div 33 = \$4860$ in 27 days
1 adult earns: $\$4860 \div 27 = \$180$ in 1 day **[1 mark]**
1 teenager earns: $\$180 \times 0.7 = \$126$ in 1 day **[1 mark]**
17 teenagers earn: $\$126 \times 17 \times 12 = \$25\,704$ in 12 days **[1 mark]**

Question 8

a. 35 painters can finish the whole job in 45 days.
so 35 painters can finish $\frac{30}{45}$ of the job in 30 days and there is $\frac{15}{45} = \frac{1}{3}$ of the job left to do by 28 painters. **[1 mark]**
There are 35 painters for 30 days and 28 painters for the rest of the time.

b. 35 painters can finish the whole job in 45 days so 1 painter can do the whole job in 35×45 days.
28 painters can do the whole job in $\frac{35 \times 45}{28}$ days and 28 painters can do $\frac{1}{3}$ of the job in $\frac{1}{3} \times \frac{35 \times 45}{28} = 18.75$ days. **[1 mark]**
There are 35 painters for 30 days and 28 painters for the rest of the time.

c. The job will take $30 + 18.75 = 48.75$ days. **[1 mark]**
There are 35 painters for 30 days and 28 painters for the rest of the time.

Question 9

a. 40% of $\$62\,000 = \frac{40}{100} \times 62\,000$
$= 24\,800$
$= \$24\,800$ **[1 mark]**

b. Total cost $= 62\,000 + 24\,800$
$= 86\,800$
$= \$86\,800$ **[1 mark]**

c. 160% of $\$86\,800 = \frac{160}{100} \times \$86\,800$
$= \$138\,880$
$= \$138\,880$ **[1 mark]**

Question 10

a. 24 000:36 000 simplifies to 2:3 **[1 mark]**

b. **i.** $\frac{2}{5} \times \$15\,000 = \$6000$ **[1 mark]**

ii. $\frac{3}{5} \times \$15\,000 = \$9000$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

c. i. Profit $= \$15\,000 - \$750$

$= \$14\,250$

$\frac{2}{5} \times \$14\,250 = \$5700$ **[1 mark]**

ii. Profit $= \$15\,000 - \$750$

$= \$14\,250$

$\frac{3}{5} \times \$14\,250 = \$8550$ **[1 mark]**

Question 11

a. $\left(\frac{1}{12} \times 2.5\right)\% \times \$120.40 = \$0.25$ **[1 mark]**

b. $\left(\frac{1}{12} \times 19.49\right)\% \times \$640 \approx \$10.39$ **[1 mark]**

Question 12

1. $\left(\frac{1}{2} \times 2.5\right)\% \times \$3500 = \$43.75$ **[1 mark]**

2. $\left(\frac{1}{2} \times 3.05\right)\% \times \$3500 - \$5 \times 6 \approx \$53.38 - \$30$

$= \$23.38$ **[1 mark]**

3. $\left(\frac{1}{2} \times 2.8\right)\% \times (\$3500 - \$2) \approx \$48.97$ **[1 mark]**

The best return would be alternative c. **[1 mark]**

Question 13

$$I = \frac{Prn}{100}$$

$$100I = Prn$$

$$P = \frac{100I}{rn}$$

Question 14

$$D = \frac{Prn}{100}$$

$$= \frac{22\,900 \times 8 \times 4}{100}$$

$$= 7328$$

The value of the car will depreciate by \$7328 in 4 years.

$V = 22\,900 - 7328$

$= 15\,572$

The value of the car after 4 years will be \$15 572.

Question 15

$V = P\left(1 - \frac{r}{100}\right)^n$ **[1 mark]**

$= 92\,000 \times \left(1 - \frac{11}{100}\right)^3$

$= 92\,000 \times (0.89)^3$ **[1 mark]**

$= 64\,857.15$

After 3 years, the photocopier will be worth \$68 857.15. **[1 mark]**

© John Wiley & Sons Australia, Ltd

Question 16

a. i. $\$6000 - \left(\$6000 \times \frac{5}{100}\right) = \$6000 - \left(\$6000 \times \frac{5}{100}\right)$
$= \$6000 - \$300$
$= \$5700$ **[1 mark]**

ii. $P_{52} = 120 \times 52$
$= \$6240$ **[1 mark]**

iii. $\$7200$ **[1 mark]**

iv. $P = \$6000$
$R = 18\%$p.a
$T = 6$ months
$= \frac{6}{12}$ or 0.5 year
(or 18% interest for 12 months is 9% for 6 months)
$A = \$6000 + \left(\$6000 \times \frac{18}{100} \times 0.5\right)$
$= \$6000 + \left(\$6000 \times \frac{9}{100}\right)$
$= \$6000 + \$540$
$= \$6540$ **[1 mark]**

b. The finance company is the most expensive at $\$7200$. **[1 mark]**

© John Wiley & Sons Australia, Ltd

4 Matrices

Topic	4	Matrices
Subtopic	4.2	Types of matrices

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

Select the order of the following matrix.

$$\begin{bmatrix} 1 & 2 \\ 5 & 4 \\ 3 & 7 \end{bmatrix}$$

A. 6
B. 2×3
C. 3×2
D. 9×13
E. 22

Question 2 (2 marks)

Determine the sum of the elements a_{21} and a_{32} in matrix

$$\begin{bmatrix} 5 & 2 \\ 1 & 0 \\ 3 & -2 \end{bmatrix}.$$

© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)

Select the row matrix from the following matrices.

A. $\begin{bmatrix} 10 & -4 \\ 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & -10 \end{bmatrix}$

C. $\begin{bmatrix} 6 \\ -9 \\ 3 \end{bmatrix}$

D. $\begin{bmatrix} -4 & 6 & 5 & 2 \end{bmatrix}$

E. [24]

Question 4 (1 mark)

The element a_{21} of the matrix $\begin{bmatrix} 4 \\ -2 \\ \frac{1}{4} \\ 7 \end{bmatrix}$ is

A. 4

B. -2

C. $\frac{1}{4}$

D. 7

E. does not exist

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)

Which element does not exist in the matrix $\begin{bmatrix} 5 & 10 & 9 \\ -5 & \frac{2}{3} & 4 \\ 0 & 3 & -1 \end{bmatrix}$?

A. a_{22}
B. a_{23}
C. a_{31}
D. a_{13}
E. a_{42}

Question 6 (1 mark)

Which of these matrices is a column matrix?

A. $\begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 0 \end{bmatrix}$

C. $\begin{bmatrix} -1 \\ 4 \\ 7 \end{bmatrix}$

D. $\begin{bmatrix} 3 & -1 & 2 & 5 \end{bmatrix}$

E. [12]

© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)
A matrix is
A. a list of numbers.
B. a linear array of numbers arranged into rows and columns.
C. a rectangular array of numbers arranged into rows and columns.
D. a square box containing numbers.
E. an array of numbers contained in three rows and four columns.

Question 8 (1 mark)
The order of this matrix is

$$\begin{bmatrix} 2 & 3 & 1 & 5 & 8 \\ 4 & 2 & 3 & 3 & 7 \\ 5 & 7 & 4 & 1 & 6 \\ 1 & 8 & 2 & 0 & 9 \end{bmatrix}$$

A. 5×4
B. 4×5
C. 4×4
D. 5×5
E. 2×5

© John Wiley & Sons Australia, Ltd

Topic	4	Matrices
Subtopic	4.3	Adding and subtracting matrices

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

Select the matrix that can be added to the matrix $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$.

A. $\begin{bmatrix} 1 & 5 & 3 \\ 2 & -4 & 6 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 \\ 5 & -4 \\ 3 & 6 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 5 \\ 2 & -4 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

E. [12]

Question 2 (1 mark)

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 5 & 2 \end{bmatrix}$, determine the value of a.

A. $a = 0$

B. $a = -4$

C. $a = 4$

D. $a = 2$

E. $\begin{bmatrix} -4 & 5 \\ 8 & 3 \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Question 3 (2 marks)

If $A = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -3 & 5 \end{bmatrix}$, $C = \begin{bmatrix} -4 & 2 \\ 4 & 5 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, calculate $A + B - C + O$.

Question 4 (1 mark)

Which of the following is *not* equal to $\begin{bmatrix} 1 & 3 \\ -4 & 1 \end{bmatrix}$?

A. $\begin{bmatrix} 2 & 6 \\ -8 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -4 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 0.5 & 1.5 \\ -2 & 0.5 \end{bmatrix} + \begin{bmatrix} 0.5 & 1.5 \\ -2 & 0.5 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 1 \\ 1 & -4 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 3 \\ -4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -4 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 2 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ -4 & 9 \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)

Which of these sets of matrices can be added?

A. $\begin{bmatrix} 3 & 4 & 5 \\ 2 & 6 & 4 \\ 9 & 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 4 & 2 & 7 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 3 & 4 \\ 5 & 1 & 2 \\ 9 & 1 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & 2 & 1 & 9 \\ 8 & 6 & 3 & 0 \\ 7 & 8 & 7 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 6 \end{bmatrix}$

D. $\begin{bmatrix} 5 & 8 \\ 4 & 1 \end{bmatrix}$ and $\begin{bmatrix} 9 & 4 & 2 \\ 8 & 5 & 1 \\ 4 & 2 & 5 \end{bmatrix}$

E. $\begin{bmatrix} 3 & 4 \\ 6 & 2 \end{bmatrix}$ and $\begin{bmatrix} 4 & 1 \\ 5 & 3 \end{bmatrix}$

Question 6 (1 mark)

$\begin{bmatrix} 5 & 1 & 2 \\ 1 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 8 & 1 \\ 2 & 1 & 0 \\ 6 & 3 & 6 \end{bmatrix} =$

A. $\begin{bmatrix} 4 & 7 & 1 \\ 1 & 3 & 3 \\ 3 & 2 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 5 & 1 & 2 \\ 1 & 4 & 3 \\ 3 & 5 & 7 \end{bmatrix}$

C. $\begin{bmatrix} 6 & 9 & 3 \\ 3 & 5 & 3 \\ 9 & 8 & 13 \end{bmatrix}$

D. $\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 8 & 1 \\ 2 & 1 & 0 \\ 6 & 3 & 6 \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)

$\begin{bmatrix} 4 & 2 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} =$

A. $\begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 6 & 3 \\ 8 & 10 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

D. $\begin{bmatrix} -2 & -1 \\ -2 & -2 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Topic	4	Matrices
Subtopic	4.4	Multiplying matrices

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

If $A = \begin{bmatrix} -1 & 2 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$, determine $2A - 3B$.

A. $\begin{bmatrix} -2 & 4 \\ -6 & 2 \end{bmatrix}$

B. $\begin{bmatrix} -3 & 6 \\ 3 & -6 \end{bmatrix}$

C. $\begin{bmatrix} -5 & 10 \\ -3 & -2 \end{bmatrix}$

D. $\begin{bmatrix} 1 & -2 \\ -9 & 10 \end{bmatrix}$

E. $\begin{bmatrix} -5 & -2 \\ -9 & 2 \end{bmatrix}$

Question 2 (1 mark)

If $A = \begin{bmatrix} 3 & 4 \\ -5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -1 & 5 \\ 0 & 2 & 0 \end{bmatrix}$ then $AB = A \times B$ equals

A. $\begin{bmatrix} 12 & 5 \\ -20 & 5 \end{bmatrix}$

B. $\begin{bmatrix} 12 & 5 & 15 \\ -20 & 5 & -25 \end{bmatrix}$

C. $\begin{bmatrix} 32 \\ -40 \end{bmatrix}$

D. $\begin{bmatrix} 16 & 5 & 15 \\ 20 & 5 & -25 \end{bmatrix}$

E. $\begin{bmatrix} 72 \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Question 3 (3 marks)

Given $X = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$, determine the matrix $X^2 + 2X$.

Question 4 (1 mark)

If $X = \begin{bmatrix} 2 & -1 \\ -5 & 0 \end{bmatrix}$, what is $7X$?

A. $\begin{bmatrix} 9 & 6 \\ 2 & 7 \end{bmatrix}$

B. $\begin{bmatrix} 14 & -7 \\ -35 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 14 & -7 \\ -35 & 7 \end{bmatrix}$

D. $\begin{bmatrix} 14 & 7 \\ 35 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 14 & -7 \\ -42 & 0 \end{bmatrix}$

Question 5 (1 mark)

If $P = \begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix}$ then $4(P + Q) =$

A. $\begin{bmatrix} 0 & 12 \\ 28 & 12 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 3 \\ 7 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 4 & -8 \\ 16 & 4 \end{bmatrix}$

D. $\begin{bmatrix} -4 & 20 \\ 12 & 8 \end{bmatrix}$

E. $\begin{bmatrix} 2 & -7 \\ 1 & -1 \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Question 6 (3 marks)

If $C = \begin{bmatrix} 1 & 4 & -3 \\ 0 & 2 & 5 \\ -1 & 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} -1 & 1 & 3 \\ 5 & 4 & 2 \\ 0 & 6 & 0 \end{bmatrix}$, find $2C + D - E$.

Question 7 (2 marks)

If $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 8 \\ 9 & 0 & 0 \end{bmatrix}$, find the element a_{23} in the matrix $-A$.

Question 8 (1 marks)

For $A = \begin{bmatrix} 1 & 6 & -2 \end{bmatrix}$, $L = \begin{bmatrix} 4 & 2 \\ -5 & 9 \\ 0 & 2 \end{bmatrix}$, $M = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}$, $N = \begin{bmatrix} 3 & 5 \\ -1 & 0 \end{bmatrix}$, $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $P = \begin{bmatrix} 2 & 5 & 6 \\ -1 & 0 & 4 \\ 3 & -7 & 1 \end{bmatrix}$, which matrices cannot be multiplied?

A. $A \times L$
B. $A \times M$
C. $A \times N$
D. $A \times 0$
E. $A \times P$

Question 9 (1 mark)

If $A = \begin{bmatrix} -2 & 4 \\ 1 & 0 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, which pair of equations is correct?

A. $AI = I$ and $A + I = I$
B. $A + I = A$ and $AO = O$
C. $AI = A$ and $A + I = I$
D. $A + O = O$ and $AI = A$
E. $AO = O$ and $AI = A$

© John Wiley & Sons Australia, Ltd

Topic	4	Matrices
Subtopic	4.5	Inverse matrices and problem-solving with matrices

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

The determinant of $A = \begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix}$ is

A. -7
B. -6
C. 5
D. 6
E. -5

Question 2 (1 mark)

Solve this matrix equation for X.

$$\begin{bmatrix} 2 & 2 \\ 2 & -5 \end{bmatrix} X = \begin{bmatrix} 8 & 6 \\ -13 & -1 \end{bmatrix}$$

A. $\begin{bmatrix} -1 & -2 \\ -3 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

C. $\frac{1}{-14}\begin{bmatrix} -14 & -28 \\ -42 & -14 \end{bmatrix}$

D. $\frac{1}{-14}\begin{bmatrix} -14 & -28 \\ -42 & -14 \end{bmatrix}$

E. $\frac{1}{-14}\begin{bmatrix} -52 & -4 \\ 68 & 24 \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)

To solve the simultaneous equations $3x - y = 0$ and $3y - 6x = 7$ using the matrix equation $AX = B$, the coefficient matrix A is

A. $\begin{bmatrix} 3 & -1 \\ 3 & -6 \end{bmatrix}$

B. $\begin{bmatrix} 3 & 1 \\ -6 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 1 \\ 3 & 6 \end{bmatrix}$

D. $\begin{bmatrix} 0 \\ 7 \end{bmatrix}$

E. $\begin{bmatrix} 3 & -1 \\ -6 & 3 \end{bmatrix}$

Question 4 (1 mark)

If $A = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$, choose the correct statement.

A. $AB = I$, so $A^{-1} = B$

B. $AB = 4I$, so $A^{-1} = 5B$

C. $AB = 5I$, so $A^{-1} = \frac{1}{5}B$

D. $AB = I$, so $A = B$

E. $AB = I$, so $A = \frac{I}{B}$

Question 5 (2 marks)

If the multiplicative inverse of C is $C^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$, find the matrix X in the equation $XC = \begin{bmatrix} 2 & 9 \\ 8 & 8 \end{bmatrix}$.

© John Wiley & Sons Australia, Ltd

Question 6 (3 marks)

Answer the following.

a. If $B = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & -4 \\ 2 & 2 \end{bmatrix}$, find the matrix BC. **(1 mark)**

b. Hence, solve the equation $BX = C$ for X. **(2 marks)**

Question 7 (1 mark)

The term 'multiplicative inverse' means:

A. a matrix multiplied by itself will result in the identity matrix.
B. a matrix multiplied by its multiplicative inverse will result in a matrix identical to the original.
C. a matrix multiplied by its multiplicative inverse will result in a zero matrix.
D. a matrix multiplied by its multiplicative inverse will result in a multiple of the identity matrix.
E. a matrix multiplied by its multiplicative inverse will result in a multiple of the original matrix.

Question 8 (1 mark)

The inverse of the matrix $\begin{bmatrix} 4 & 7 \\ 2 & 9 \end{bmatrix}$ is

A. $\begin{bmatrix} 9 & -7 \\ -1 & 2 \end{bmatrix}$
B. $\begin{bmatrix} \frac{9}{22} & \frac{-7}{22} \\ \frac{-1}{11} & \frac{2}{11} \end{bmatrix}$
C. 22
D. $\frac{1}{22}$
E. $\begin{bmatrix} \frac{4}{22} & \frac{7}{22} \\ \frac{2}{22} & \frac{9}{22} \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Question 9 (3 marks)

Find the multiplicative inverse of the matrix $\begin{bmatrix} 6 & 2 \\ -3 & 2 \end{bmatrix}$.

© John Wiley & Sons Australia, Ltd

Topic	4	Matrices
Subtopic	4.6	Communications and connections

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2019, Further Mathematics Exam 1, Section B, Module 1, Q.7; © VCAA*

Question 1 (1 mark)

The communication matrix below shows the direct paths by which messages can be sent between two people in a group of six people, U to Z.

$$\begin{array}{cc} & \begin{matrix} \multicolumn{6}{c}{\textit{receiver}} \\ U & V & W & X & Y & Z \end{matrix} \\ \textit{sender} & \begin{matrix} U \\ V \\ W \\ X \\ Y \\ Z \end{matrix} \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

A '1' in the matrix shows that the person named in that row can send a message directly to the person named in that column.

For example, the '1' in row 4, column 2 shows that X can send a message directly to V.

In how many ways can Y get a message to W by sending it directly to one other person?

A. 0
B. 1
C. 2
D. 3
E. 4

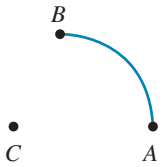
© John Wiley & Sons Australia, Ltd

Question 2 (1 mark)

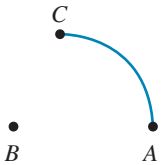
Select the network from the following that can be described by the adjacency matrix.

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

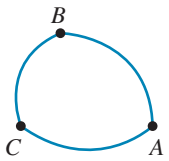
A.



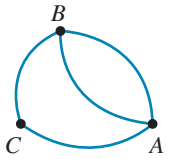
B.



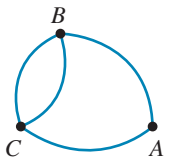
C.



D.



E.



© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)
Fill in the matrix below, which describes the numbers of roads out of each of towns A, B, C and D , as represented by the network.

From \ To	A	B	C	D
A				
B				
C				
D				

Question 4 (1 mark)
Five competitors, Andy (A), Brie (B), Cleo (C), Della (D)and Eddie (E), participate in a darts tournament. Each competitor plays each of the other competitors once only, and each match results in a winner and a loser.
The matrix below shows the results of this darts tournament.
There are still two matches that need to be played.

winner \ *loser*	A	B	C	D	E
A	0	...	0	1	0
B	...	0	1	0	1
C	1	0	0	...	1
D	0	1	...	0	0
E	1	0	0	1	0

A '1' in the matrix shows that the competitor named in that row defeated the competitor named in that column.
For example, the '1' in row 2, column 3 shows that Brie defeated Cleo.
A '...' in the matrix shows that the competitor named in that row has not yet played the competitor named in that column.
The winner of this darts tournament is the competitor with the highest sum of their one-step and two-step dominances.
Which player, by winning their remaining match, will ensure that they are ranked first by the sum of their one-step and two-step dominances?
A. Andy
B. Brie
C. Cleo
D. Della
E. Eddie

© John Wiley & Sons Australia, Ltd

Question 5 (1 marks)

Four teams, A, B, C and D, competed in a round-robin competition where each team played each of the other teams once. There were no draws.

The results are shown in the matrix below.

$$\begin{array}{cc} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} \\ winner \\ \\ \\ \end{matrix} & \begin{matrix} A \\ B \\ C \\ D \end{matrix} \begin{bmatrix} 0 & 0 & f & 1 \\ 1 & 0 & 0 & 0 \\ 1 & g & 0 & 1 \\ 0 & 1 & 0 & h \end{bmatrix} \end{array} \quad loser$$

A '1' in the matrix shows that the team named in that row defeated the team named in that column.

For example, the '1' in row 2 shows that team B defeated team A.

In this matrix, the values of f, g and h are

A. $f = 0, \quad g = 1, \quad h = 0$

B. $f = 0, \quad g = 1, \quad h = 1$

C. $f = 1, \quad g = 0, \quad h = 0$

D. $f = 1, \quad g = 1, \quad h = 0$

E. $f = 1, \quad g = 1, \quad h = 1$

© John Wiley & Sons Australia, Ltd

Topic	4	Matrices
Subtopic	4.7	Transition matrices

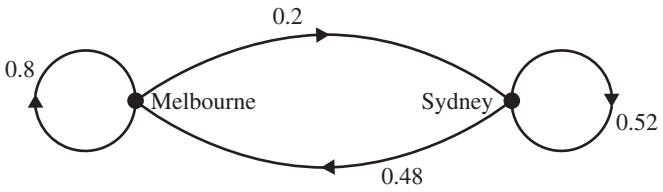
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2019, Further Mathematics Exam 1, Section B, Module 1, Q.8; © VCAA*

Question 1 (1 mark)

An airline parks all of its planes at Sydney airport or Melbourne airport overnight.
The transition diagram below shows the change in the location of the planes from night to night.



There are always m planes parked at Melbourne airport.
There are always s planes parked at Sydney airport.
Of the planes parked at Melbourne airport on Tuesday night, 12 had been parked at Sydney airport on Monday night.
How many planes does the airline have?

A. 25
B. 37
C. 62
D. 65
E. 85

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 2, Section B, Module 1, Q.2a; © VCAA*

Question 2 (1 mark)

The theme park has four locations, Air World (A), Food World (F), Ground World (G)and Water World (W).
The number of visitors at each of the four locations is counted every hour.
By 10 am on Saturday the park had reached its capacity of 2000 visitors and could take no more visitors.
The park stayed at capacity until the end of the day.
The state matrix, S_0, below, shows the number of visitors at each location at 10 am on Saturday.

$$S_0 = \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix}$$

What percentage of the park's visitors were at Water World (W) at 10 am on Saturday?

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 2, Section B, Module 1, Q.2b; © VCAA*

Question 3 (1 mark)

The theme park has four locations, Air World (A), Food World (F), Ground World (G)and Water World (W). The number of visitors at each of the four locations is counted every hour.

By 10 am on Saturday the park had reached its capacity of 2000 visitors and could take no more visitors. The park stayed at capacity until the end of the day.

The state matrix, S_0, below, shows the number of visitors at each location at 10 am on Saturday.

$$S_0 = \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix}$$

Let S_n be the state matrix that shows the number of visitors expected at each location n hours after 10 am on Saturday.

The number of visitors expected at each location n hours after 10 am on Saturday can be determined by the matrix recurrence relation below.

$$S_1 = \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix}, \quad S_{n+1} + T \times S_n$$

$$\text{where } T = \begin{array}{c} \\ A \\ F \\ G \\ W \end{array} \begin{array}{c} \textit{this hour} \\ \begin{matrix} A & F & G & W \end{matrix} \\ \begin{bmatrix} 0.1 & 0.2 & 0.1 & 0.2 \\ 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix} \\ \textit{next hour} \end{array}$$

Complete the state matrix, S_1, below to show the number of visitors expected at each location at 11 am on Saturday.

$$S_1 = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \\ 300 \\ \_\_\_\_\_ \end{bmatrix} \begin{matrix} A \\ F \\ G \\ W \end{matrix}$$

© John Wiley & Sons Australia, Ltd

Source: *VCE 2014, Further Mathematics Exam 1, Section B, Module 6, Q.8; © VCAA*

Question 4 (1 mark)

Wendy will have lunch with one of her friends each day of this week.
Her friends are Angela (A), Betty (B), Craig (C), Daniel (D) and Edgar (E).
On Monday, Wendy will have lunch with Craig.
Wendy will use the transition matrix below to choose a friend to have lunch with for the next four days of the week.

$$T = \begin{array}{c} \\ \\ \\ \\ \\ \text{tomorrow} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{today} & & & & \\ A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} A \\ B \\ C \\ D \\ E \end{array}$$

The order in which Wendy has lunch with her friends for the next four days is

A. Angela, Betty, Craig, Daniel
B. Daniel, Betty, Angela, Craig
C. Daniel, Betty, Angela, Edgar
D. Edgar, Angela, Daniel, Betty
E. Edgar, Daniel, Betty, Angela

© John Wiley & Sons Australia, Ltd

Source: *VCE 2014, Further Mathematics Exam 1, Section B, Module 6, Q.7; © VCAA*

Question 5 (1 mark)

A transition matrix, T, and a state matrix, S_2, are defined as follows.

$$T = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \quad S_2 = \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$$

If $S_2 = TS_1$, the state matrix S_1 is

A. $\begin{bmatrix} 200 \\ 250 \\ 150 \end{bmatrix}$

B. $\begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix}$

C. $\begin{bmatrix} 300 \\ 0 \\ 300 \end{bmatrix}$

D. $\begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$

E. undefined

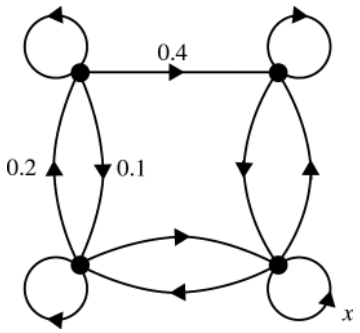
© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)

A transition matrix, V, is shown below.

$$V = \begin{array}{c} \\ L \\ T \\ F \\ M \end{array} \begin{array}{c} \begin{array}{cccc} L & T & F & M \end{array} \\ \begin{bmatrix} 0.6 & 0.6 & 0.2 & 0.0 \\ 0.1 & 0.2 & 0.0 & 0.1 \\ 0.3 & 0.0 & 0.8 & 0.4 \\ 0.0 & 0.20 & 0.0 & 0.5 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \textit{this month} \\ \\ \\ \\ \uparrow \textit{next month} \end{array}$$

The transition diagram below has been constructed from the transition matrix V.
The labelling in the transition diagram is not yet complete.



The proportion for one of the transitions is labeled x.
The value of x is

A. 0.2
B. 0.5
C. 0.6
D. 0.7
E. 0.8

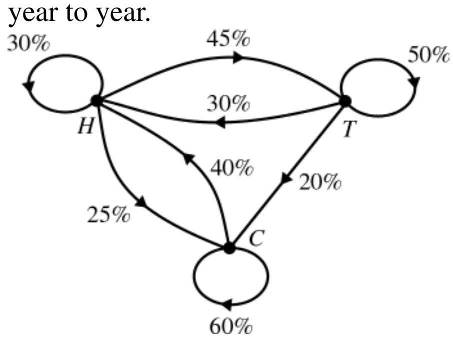
© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)

Families in a country town were asked about their annual holidays.

Every year, these families choose between staying at home (*H*), travelling (*T*) and camping (*C*).

The transition diagram below shows the way families in the town change their holiday preferences from year to year.



A transition matrix that provides the same information as the transition diagram is

A. $$\begin{array}{c} \textit{from} \\ \begin{array}{ccc} H & T & C \end{array} \\ \begin{bmatrix} 0.30 & 0.75 & 0.65 \\ 0.75 & 0.50 & 0.20 \\ 0.65 & 0.20 & 0.60 \end{bmatrix} \begin{array}{l} H \\ T\ \textit{to} \\ C \end{array} \end{array}$$

B. $$\begin{array}{c} \textit{from} \\ \begin{array}{ccc} H & T & C \end{array} \\ \begin{bmatrix} 0.30 & 0.30 & 0.40 \\ 0.45 & 0.50 & 0 \\ 0.25 & 0.20 & 0.60 \end{bmatrix} \begin{array}{l} H \\ T\ \textit{to} \\ C \end{array} \end{array}$$

C. $$\begin{array}{c} \textit{from} \\ \begin{array}{ccc} H & T & C \end{array} \\ \begin{bmatrix} 0.30 & 0.30 & 0.40 \\ 0.45 & 0.50 & 0.20 \\ 0.25 & 0.20 & 0.60 \end{bmatrix} \begin{array}{l} H \\ T\ \textit{to} \\ C \end{array} \end{array}$$

D. $$\begin{array}{c} \textit{from} \\ \begin{array}{ccc} H & T & C \end{array} \\ \begin{bmatrix} 0.30 & 0.30 & 0.40 \\ 0.45 & 0.50 & 0.20 \\ 0.25 & 0.20 & 0.40 \end{bmatrix} \begin{array}{l} H \\ T\ \textit{to} \\ C \end{array} \end{array}$$

E. $$\begin{array}{c} \textit{from} \\ \begin{array}{ccc} H & T & C \end{array} \\ \begin{bmatrix} 0.30 & 0.45 & 0.25 \\ 0.30 & 0.50 & 0.20 \\ 0.40 & 0 & 0.60 \end{bmatrix} \begin{array}{l} H \\ T\ \textit{to} \\ C \end{array} \end{array}$$

© John Wiley & Sons Australia, Ltd

Question 8 (1 mark)

There are 3 brands of instant coffee that dominate the market: Brand *A*, Brand *B*, and Brand *C*. Consumers change from one brand of coffee to another all the time.

If consumers use Brand *A* this week, next week the probability that they will change to Brand *B* is 0.1, the probability that they will change to brand *C* is 0.3 and the probability that they will continue to use Brand *A* is 0.6.

If consumers use Brand *B* this week, next week the probability that they will change to Brand *A* is 0.5, the probability that they will change to brand *C* is 0.1 and the probability that they will continue to use Brand *B* is 0.4.

If consumers use Brand *C* this week, next week the probability that they will change to Brand *A* is 0.2, the probability that they will change to brand *B* is 0.3 and the probability that they will continue to use Brand *C* is 0.5.

Which of the following transition matrices represents this situation?

A. $T = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0.1 & 0.3 & 0.6 \\ 0.5 & 0.1 & 0.4 \\ 0.2 & 0.3 & 0.5 \end{bmatrix} \end{matrix}$

B. $T = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0.6 & 0.5 & 0.2 \\ 0.1 & 0.4 & 0.3 \\ 0.3 & 0.1 & 0.5 \end{bmatrix} \end{matrix}$

C. $T = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0.1 & 0.5 & 0.2 \\ 0.3 & 0.1 & 0.3 \\ 0.6 & 0.4 & 0.5 \end{bmatrix} \end{matrix}$

D. $T = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0.5 & 0.6 & 0.2 \\ 0.4 & 0.1 & 0.3 \\ 0.1 & 0.3 & 0.5 \end{bmatrix} \end{matrix}$

© John Wiley & Sons Australia, Ltd

Topic	4	Matrices
Subtopic	4.8	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2020, Further Mathematics Exam 1, Section B, Module 1, Q.6; © VCAA*

Question 1 (1 mark)

The element in row i and column j of matrix M is m_{ij}.
M is a 3×3 matrix. It is constructed using the rule $m_{ij} = 3i + 2j$.
M is

A. $\begin{bmatrix} 5 & 7 & 9 \\ 7 & 9 & 11 \\ 11 & 13 & 15 \end{bmatrix}$

B. $\begin{bmatrix} 5 & 7 & 9 \\ 8 & 10 & 12 \\ 11 & 13 & 15 \end{bmatrix}$

C. $\begin{bmatrix} 5 & 7 & 10 \\ 8 & 10 & 13 \\ 11 & 13 & 16 \end{bmatrix}$

D. $\begin{bmatrix} 5 & 8 & 11 \\ 7 & 10 & 13 \\ 9 & 12 & 15 \end{bmatrix}$

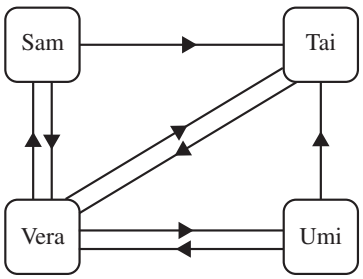
E. $\begin{bmatrix} 5 & 8 & 11 \\ 8 & 11 & 14 \\ 11 & 14 & 17 \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 1, Section B, Module 1, Q.5; © VCAA*

Question 2 (1 mark)

The diagram below shows the direct communication links that exist between Sam (S), Tai (T), Umi (U)and Vera (V). For example, the arrow from Umi to Vera indicates that Umi can communicate directly with Vera.



A communication matrix can be used to convey the same information.

In this matrix:

- a '1' indicates that a direct communication link exists between a sender and a receiver
- a '0' indicates that a direct communication link does not exist between a sender and a receiver.

The communication matrix could be

A.

sender \ *receiver*	S	T	U	V
S	0	1	0	1
T	0	0	0	1
U	0	1	0	1
V	1	0	1	0

B.

sender \ *receiver*	S	T	U	V
S	0	1	0	1
T	1	0	0	1
U	0	1	0	1
V	1	1	1	0

C.

sender \ *receiver*	S	T	U	V
S	0	1	0	1
T	0	0	0	1
U	0	1	0	0
V	1	1	1	0

D.

sender \ *receiver*	S	T	U	V
S	0	1	0	1
T	0	0	0	1
U	0	1	0	1
V	1	1	1	0

© John Wiley & Sons Australia, Ltd

E.

$$\begin{array}{c} \\ sender \end{array} \begin{array}{c} \begin{array}{cccc} S & T & U & V \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 2 \\ 2 & 2 & 2 & 0 \end{bmatrix} \end{array} \begin{array}{l} S \\ T \\ U \\ V \end{array} \quad receiver$$

Source: *VCE 2019, Further Mathematics Exam 1, Section B, Module 1, Q.1; © VCAA*

Question 3 (1 mark)

Consider the following four matrix expressions.

$\begin{bmatrix} 8 \\ 12 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 8 \\ 12 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 8 & 0 \\ 12 & 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 8 & 0 \\ 12 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

How many of these four matrix expressions are defined?

A. 0
B. 1
C. 2
D. 3
E. 4

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 2, Section B, Module 1, Q.1; © VCAA*

Question 4 (3 marks)

The car park at a theme park has three areas, A, B and C.

The number of empty (E) and full (F) parking spaces in each of the three areas at 1 pm on Friday are shown in matrix Q below.

$$Q = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} E & F \\ \end{matrix} \begin{bmatrix} 70 & 50 \\ 30 & 20 \\ 40 & 40 \end{bmatrix} \begin{matrix} \\ \\ \\ area \end{matrix}$$

a. What is the order of matrix Q? **(1 mark)**

b. Write down a calculation to show that 110 parking spaces are full at 1 pm. **(1 mark)**

c. Drivers must pay a parking fee for each hour of parking.

Matrix P, below, shows the hourly fee, in dollars, for a car parked in each of the three areas.

$$P = \begin{matrix} \\ area \end{matrix} \begin{matrix} A & B & C \\ \end{matrix} \begin{bmatrix} 1.30 & 3.50 & 1.80 \end{bmatrix}$$

The total parking fee, in dollars, collected from these 110 parked cars if they were parked for one hour is calculated as follows.

$P \times L = [207.00]$

where matrix L is a 3×1 matrix.

Write down matrix L. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2018, Further Mathematics Exam 1, Section B, Module 1, Q.2; © VCAA*

Question 5 (1 mark)

The matrix product $\begin{bmatrix}4 & 2 & 0\end{bmatrix} \times \begin{bmatrix}4\\12\\8\end{bmatrix}$ is equal to

A. [144]

B. $\begin{bmatrix}16\\24\\0\end{bmatrix}$

C. $4 \times \begin{bmatrix}1 & 2 & 0\end{bmatrix} \times \begin{bmatrix}1\\12\\8\end{bmatrix}$

D. $2 \times \begin{bmatrix}2 & 1 & 0\end{bmatrix} \times \begin{bmatrix}2\\6\\4\end{bmatrix}$

E. $4 \times \begin{bmatrix}2 & 1 & 0\end{bmatrix} \times \begin{bmatrix}2\\6\\4\end{bmatrix}$

Question 6 (2 marks)

Give the order for each of the following matrices:

a. $A = \begin{bmatrix}5\\6\\4\end{bmatrix}$ **(1 mark)**

b. $B = \begin{bmatrix}-4 & 0 & 6 & 7\\2 & 3 & -5 & 8\end{bmatrix}$ **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 7 (2 marks)
There are two Maths classes at Keystone Secondary College. Class 11W contains 10 boys and 8 girls. Class 11M contains 7 boys and 12 girls. Complete the matrix equation and use it to find the total number of boys and girls in Year 11 Maths.

$$\begin{matrix} & 11W \\ \text{Boys} \\ \text{Girls} \end{matrix}\begin{bmatrix} & \\ & \end{bmatrix} + \begin{matrix} & 11M \\ \text{Boys} \\ \text{Girls} \end{matrix}\begin{bmatrix} & \\ & \end{bmatrix} = \begin{matrix} & \text{Year 11} \\ \text{Boys} \\ \text{Girls} \end{matrix}\begin{bmatrix} \\ \end{bmatrix}$$

Question 8 (3 marks)
Movie tickets at Lakes Theatre cost $20 for adults and $12 for children. On Friday afternoon, 220 adults and 80 children attended the session. On Friday night, attendance was 250 adults and 40 children.

a. Fill in the attendance matrix: **(1 mark)**

$$\begin{matrix} & \text{Adult} & \text{Child} \\ \text{Afternoon} \\ \text{Evening} \end{matrix}\begin{bmatrix} & \\ & \end{bmatrix}$$

b. Fill in the cost matrix: **(1 mark)**

$$\begin{matrix} & \$ \\ \text{Adult} \\ \text{Child} \end{matrix}\begin{bmatrix} \\ \end{bmatrix}$$

c. Use matrix multiplication to find the amounts of money taken by the theatre on Friday afternoon and evening. **(1 mark)**

$$\begin{matrix} & \text{Adult} & \text{Child} \\ \text{Afternoon} \\ \text{Evening} \end{matrix}\begin{bmatrix} & \\ & \end{bmatrix} \times \begin{matrix} & \$ \\ \text{Afternoon} \\ \text{Evening} \end{matrix}\begin{bmatrix} \\ \end{bmatrix} = \begin{matrix} & \$ \\ \text{Adult} \\ \text{Child} \end{matrix}\begin{bmatrix} \\ \end{bmatrix}$$

© John Wiley & Sons Australia, Ltd

Question 9 (1 mark)

$\begin{bmatrix}1 & 4\\6 & 3\end{bmatrix} \times \begin{bmatrix}8 & 1 & 3\\0 & 9 & 4\end{bmatrix}$ equals to

A. $\begin{bmatrix}8 & 19\\48 & 37\end{bmatrix}$

B. $\begin{bmatrix}8 & 37 & 19\\48 & 33 & 30\end{bmatrix}$

C. $\begin{bmatrix}48 & 37\\30 & 33\end{bmatrix}$

D. $\begin{bmatrix}8 & 48\\37 & 33\\19 & 30\end{bmatrix}$

E. $\begin{bmatrix}48 & 33 & 30\\8 & 37 & 19\end{bmatrix}$

Question 10 (1 mark)

The numbers of tyres and toolkits sold at two shops are recorded for the first 3 months of the year. These are shown in two tables.

Shop A

	January	**February**	**March**
Tyres	240	180	140
Toolkits	150	125	95

Shop B

	January	**February**	**March**
Tyres	240	180	140
Toolkits	150	125	95

If the data were put into two matrices to determine total sales, what matrix operation would be used?

A. Addition
B. Subtraction
C. Multiplication
D. Division
E. Scalar multiplication

© John Wiley & Sons Australia, Ltd

Question 11 (1 marks)

The weekly sales of cereals at two grocery stores is shown in the matrix:

$$\begin{bmatrix} 80 & 75 \\ 40 & 60 \\ 25 & 30 \\ 60 & 100 \end{bmatrix}$$

The sales at both stores doubled last week. What is the matrix showing the sales last week at the two stores?

A. $\begin{bmatrix} 310 \\ 200 \\ 110 \\ 320 \end{bmatrix}$

B. [940]

C. $\begin{bmatrix} 80 & 75 \\ 40 & 60 \\ 25 & 30 \\ 60 & 100 \end{bmatrix}$

D. $\begin{bmatrix} 160 & 150 \\ 80 & 120 \\ 50 & 60 \\ 120 & 200 \end{bmatrix}$

E. 410 530

Question 12 (1 marks)

Albert, Brian and Clara each bought shares in two companies. The numbers of shares they purchased are shown in the matrices:

$$\begin{matrix} A \\ B \\ C \end{bmatrix} \begin{bmatrix} 1000 & 500 \\ 1500 & 750 \\ 2000 & 1200 \end{bmatrix}$$

$$\text{Share price} = \begin{bmatrix} \$1.50 \\ \$3.00 \end{bmatrix}$$

How much did Clara spend on the two parcels of shares?

A. $\begin{bmatrix} \$3000 \\ \$4500 \\ \$6600 \end{bmatrix}$

B. $4400

C. $6600

D. $3000

E. $14 100

© John Wiley & Sons Australia, Ltd

Question 13 (2 marks)

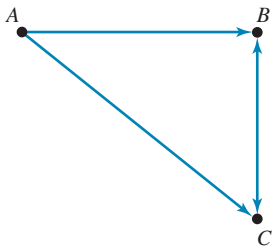
Four menswear shops sell T-shirts. *A* buys the T-shirts for $9.00 and sells them for $27.95, *B* buys them for $10 and sells them for $30.00, *C* buys them for $12.50 and sells them for $38.50, and *D* buys them for $20.00 and sells them for $62.00. Complete the matrix equation and use it to find the profit that each shop makes on the sale of a T-shirt.

$$\begin{matrix} & \text{Selling price \$} \\ A \\ B \\ C \\ D \end{matrix}\begin{bmatrix} \\ \\ \\ \end{bmatrix} - \begin{matrix} & \text{Cost price \$} \\ A \\ B \\ C \\ D \end{matrix}\begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{matrix} & \text{Profit \$} \\ A \\ B \\ C \\ D \end{matrix}\begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

© John Wiley & Sons Australia, Ltd

Question 14 (3 marks)

Three children are asked who their friends are and this information is shown on a friendship network diagram. The network shows that Andy says he is friends with Barbie and Con, Barbie says she is friends with Con only and Con says he is friends with Barbie only.



a. Fill in the matrix M to show this friendship network. **(1 mark)**

$$M = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array}$$

b. Which child has nobody calling them a friend and how is that shown in the matrix? **(1 mark)**

c. Find M^2 to show the number of connections between two children through a third child. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 15 (9 marks)

Kylie bought 3 Mars Bars and 1 Snickers for $9. Jarrad bought 2 Mars Bars and 3 Snickers for $13.

a. Express these situations in two simultaneous equations. **(1 mark)**

b. Write the simultaneous equations in matrix form. **(1 mark)**

c. Calculate the determinant of the coefficient matrix. **(2 marks)**

d. Use the determinant to find the inverse of the coefficient matrix. **(2 marks)**

e. Use the inverse of the coefficient matrix to solve for both variables. **(3 marks)**

Question 16 (1 mark)

$2 \times \begin{bmatrix} 3 & 2 \\ 9 & 6 \end{bmatrix}$ equals to

A. $\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}$

B. $\begin{bmatrix} 3 & 2 \\ 18 & 12 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 6 & 4 \\ 18 & 12 \end{bmatrix}$

E. $\begin{bmatrix} 3 & 3 & 3 & 2 \\ 9 & 6 & 9 & 6 \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Question 17 (1 mark)

The set of matrices which can be multiplied is:

A. $\begin{bmatrix}4\\1\\4\end{bmatrix}$ and $\begin{bmatrix}5\\1\\3\end{bmatrix}$

B. $\begin{bmatrix}4 & 2\end{bmatrix}$ and $\begin{bmatrix}5 & 3\end{bmatrix}$

C. $\begin{bmatrix}3 & 1 & 2\end{bmatrix}$ and $\begin{bmatrix}5\\3\\1\end{bmatrix}$

D. $\begin{bmatrix}4 & 2\\7 & 3\end{bmatrix}$ and $\begin{bmatrix}9 & 1 & 5\\3 & 2 & 4\\8 & 1 & 3\end{bmatrix}$

E. $\begin{bmatrix}4 & 1 & 5\\9 & 2 & 3\end{bmatrix}$ and $\begin{bmatrix}9 & 1 & 8\end{bmatrix}$

Question 18 (1 mark)

$\begin{bmatrix}1 & 4\\6 & 3\end{bmatrix} \times \begin{bmatrix}8 & 1 & 3\\0 & 9 & 4\end{bmatrix}$ equals to

A. $\begin{bmatrix}8 & 19\\48 & 37\end{bmatrix}$

B. $\begin{bmatrix}8 & 37 & 19\\48 & 33 & 30\end{bmatrix}$

C. $\begin{bmatrix}48 & 37\\30 & 33\end{bmatrix}$

D. $\begin{bmatrix}8 & 48\\37 & 33\\19 & 30\end{bmatrix}$

E. $\begin{bmatrix}48 & 33 & 30\\8 & 37 & 19\end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Question 19 (1 marks)

Fill in the following matrix, which describes the numbers of boys and girls with different-coloured eyes, as shown in the table.

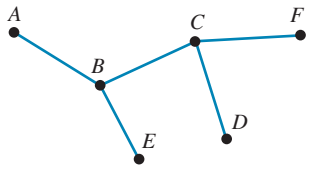
Gender	Blue eyes	Green eyes	Brown eyes	Black eyes
Boy	32	15	24	12
Girl	18	10	14	8

$$\begin{array}{c} \\ \text{Blue} \\ \text{Green} \\ \text{Brown} \\ \text{Black} \end{array} \begin{array}{c} \begin{array}{cc} \text{B} & \text{G} \end{array} \\ \begin{bmatrix} & \\ & \\ & \\ & \end{bmatrix} \end{array}$$

© John Wiley & Sons Australia, Ltd

Question 20 (1 mark)

A communication network is represented by the diagram shown.



The matrix that best represents this communication matrix is

A.

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	1	0	1	0	1	0
C	0	1	0	1	0	1
D	0	0	1	0	0	0
E	0	1	0	0	0	0
F	0	0	1	0	0	0

B.

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	1	0	1	0	1
D	1	0	1	0	0	0
E	0	1	0	0	0	0
F	0	0	1	0	0	0

C.

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	1	0	1	0	1	0
C	0	1	0	1	0	1
D	0	0	1	0	1	1
E	0	1	0	1	0	0
F	0	0	1	1	0	0

D.

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	1	0	1	0	1
D	1	0	1	0	1	0
E	0	1	0	1	0	1
F	0	0	1	0	1	0

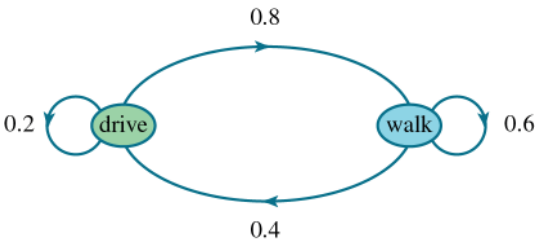
E.

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	1	0	1	0	1	0
C	0	1	0	1	0	1
D	0	0	1	0	0	0
E	0	1	0	0	0	1
F	0	0	1	0	1	0

© John Wiley & Sons Australia, Ltd

Question 21 (1 mark)

To get to school in the morning, Tessa and her sister either walk (w) or are driven by their father (d). The diagram below shows the probability that the girls will walk or their father will drive them to school tomorrow based on how they got to school today.



Which of the following transition matrices represents the diagram above?

A. $T = \begin{matrix} \\ d \\ w \end{matrix} \begin{matrix} \begin{matrix} d & w \end{matrix} \\ \begin{bmatrix} 0.2 & 0.4 \\ 0.8 & 0.6 \end{bmatrix} \end{matrix}$

B. $T = \begin{matrix} \\ d \\ w \end{matrix} \begin{matrix} \begin{matrix} d & w \end{matrix} \\ \begin{bmatrix} 0.2 & 0.8 \\ 0.4 & 0.6 \end{bmatrix} \end{matrix}$

C. $T = \begin{matrix} \\ d \\ w \end{matrix} \begin{matrix} \begin{matrix} d & w \end{matrix} \\ \begin{bmatrix} 0.2 & 0.6 \\ 0.8 & 0.4 \end{bmatrix} \end{matrix}$

D. $T = \begin{matrix} \\ d \\ w \end{matrix} \begin{matrix} \begin{matrix} d & w \end{matrix} \\ \begin{bmatrix} 0.8 & 0.4 \\ 0.2 & 0.6 \end{bmatrix} \end{matrix}$

E. $T = \begin{matrix} \\ d \\ w \end{matrix} \begin{matrix} \begin{matrix} d & w \end{matrix} \\ \begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{bmatrix} \end{matrix}$

© John Wiley & Sons Australia, Ltd

Question 22 (1 mark)

The transition matrix below is missing values 'Assume the population of the initial state remains the same throughout the process'.

$$T = \begin{bmatrix} 0.2 & 0.1 & \\ 0.5 & & 0.3 \\ & 0.6 & 0.4 \end{bmatrix}$$

Which of the following matrices is the completed transition matrix?

A. $T = \begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.5 & 0.2 & 0.3 \\ 0 & 0.6 & 0.4 \end{bmatrix}$

B. $T = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.5 & 0.2 & 0.3 \\ 0.7 & 0.6 & 0.4 \end{bmatrix}$

C. $T = \begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.5 & 0 & 0.3 \\ 0.2 & 0.6 & 0.4 \end{bmatrix}$

D. $T = \begin{bmatrix} 0.2 & 0.1 & 0.3 \\ 0.5 & 0.3 & 0.3 \\ 0.2 & 0.6 & 0.4 \end{bmatrix}$

E. $T = \begin{bmatrix} 0.2 & 0.1 & 0.3 \\ 0.5 & 0.3 & 0.3 \\ 0.3 & 0.6 & 0.4 \end{bmatrix}$

© John Wiley & Sons Australia, Ltd

Question 23 (1 mark)

When buying fruit in winter, people alternate between oranges (O) and apples (A). 75% of those who buy apples will buy oranges next time and 55% of the people who buy oranges will buy apples next time. On the first day of winter, 85 apples and 120 oranges are sold.

The initial state matrix for this situation is

A. $\begin{matrix} & A & O \\ A & 0.75 & 0.55 \\ O & 0.25 & 0.45 \end{matrix}$

B. $\begin{matrix} & A & O \\ A & 0.75 & 0.25 \\ O & 0.55 & 0.45 \end{matrix}$

C. $\begin{matrix} A & O \\ 85 & 120 \end{matrix}$

D. $\begin{matrix} A \\ O \end{matrix} \begin{bmatrix} 85 \\ 120 \end{bmatrix}$

E. $\begin{matrix} & A & O \\ A & 75 & 25 \\ O & 55 & 45 \end{matrix}$

Question 24 (1 mark)

When buying fruit in winter, people alternate between oranges (O) and apples (A). 75% of those who buy apples will buy oranges next time and 55% of the people who buy oranges will buy apples next time.
On the first day of winter, 85 apples and 120 oranges are sold.

The transition matrix for this situation is

A. $\begin{matrix} & A & O \\ A & 0.25 & 0.55 \\ O & 0.75 & 0.45 \end{matrix}$

B. $\begin{matrix} & A & O \\ A & 0.75 & 0.25 \\ O & 0.55 & 0.45 \end{matrix}$

C. $\begin{matrix} A & O \\ 85 & 120 \end{matrix}$

D. $\begin{matrix} A \\ O \end{matrix} \begin{bmatrix} 85 \\ 120 \end{bmatrix}$

E. $\begin{matrix} & A & O \\ A & 75 & 25 \\ O & 55 & 45 \end{matrix}$

© John Wiley & Sons Australia, Ltd

Answers and marking guide

4.2 Types of matrices

Question 1
'Row then column': the number of rows by the number of columns. Therefore, this is a 3×2 matrix.

Question 2
'Row then column':
$a_{21} = 1$ and $a_{32} = -2$ **[1 mark]**
$a_{21} + a_{32} = 1 + -2 = -1$ **[1 mark]**

Question 3
A row matrix is an $1 \times m$ order matrix; that is, a matrix consisting of a single row of m elements.

Question 4
'Row then column': the number of rows then the number of columns. The element is in the second row and the first column.

Question 5
a_{42}
'Row then column': the number of rows then the number of columns. As this is a 3×3 matrix, there is no fourth row.

Question 6
A column matrix is an $m \times 1$ order matrix; that is, a matrix consisting of a single column of m elements.

Question 7
A matrix is a rectangular array of numbers arranged into rows and columns.

Question 8
This matrix has four rows and five columns, and is therefore of the order 4×5

4.3 Adding and subtracting matrices

Question 1

$$\begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3+1 \\ -4+2 \end{bmatrix}$$
$$= \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

Question 2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} -2 & 1 \\ 5 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} -4 & 5 \\ 8 & 3 \end{bmatrix}$$
$\therefore \quad a = -4$

Question 3
Work from left to right:

© John Wiley & Sons Australia, Ltd

$$A+B-C+O=\begin{bmatrix}2&3\\-4&5\end{bmatrix}+\begin{bmatrix}1&3\\-3&5\end{bmatrix}-\begin{bmatrix}-4&2\\4&5\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}$$
$$=\begin{bmatrix}2+1&3+3\\-4+-3&5+5\end{bmatrix}-\begin{bmatrix}-4&2\\4&5\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}$$ **[1 mark]**
$$=\begin{bmatrix}3&6\\-7&10\end{bmatrix}-\begin{bmatrix}-4&2\\4&5\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}$$
$$=\begin{bmatrix}3--4&6-2\\-7-4&10-5\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}$$
$$=\begin{bmatrix}7&4\\-11&5\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}$$
$$=\begin{bmatrix}7&4\\-11&5\end{bmatrix}$$ **[1 mark]**

Question 4

$$\begin{bmatrix}3&1\\1&-4\end{bmatrix}-\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

Notice the word *not*. Could evaluate each alternative, but can immediately tell the correct alternative is option C because in it, the zero matrix has been subtracted from a matrix that is not the matrix in the question.

Question 5

To be added, matrices must be of the same order. Option E contains two matrices and therefore 2×2 can be added.

Question 6

$$\begin{bmatrix}5&1&2\\1&4&3\\3&5&7\end{bmatrix}+\begin{bmatrix}1&8&1\\2&1&0\\6&3&6\end{bmatrix}=\begin{bmatrix}5+1&1+8&2+1\\1+2&4+1&3+0\\3+6&5+3&7+6\end{bmatrix}$$
$$=\begin{bmatrix}6&9&3\\3&5&3\\9&8&13\end{bmatrix}$$

Question 7

$$\begin{bmatrix}4&2\\5&6\end{bmatrix}-\begin{bmatrix}2&1\\3&4\end{bmatrix}=\begin{bmatrix}4-2&2-1\\5-3&6-4\end{bmatrix}$$
$$=\begin{bmatrix}2&1\\2&2\end{bmatrix}$$

4.4 Multiplying matrices

Question 1

$$2A-3B=2\times\begin{bmatrix}-1&2\\-3&2\end{bmatrix}-3\times\begin{bmatrix}-1&2\\1&-2\end{bmatrix}$$
$$=\begin{bmatrix}-1\times 2&2\times 2\\-3\times 2&2\times 2\end{bmatrix}-\begin{bmatrix}-1\times 3&2\times 3\\1\times 3&-2\times 3\end{bmatrix}$$
$$=\begin{bmatrix}-2&4\\-6&4\end{bmatrix}-\begin{bmatrix}-3&6\\3&-6\end{bmatrix}$$
$$=\begin{bmatrix}1&-2\\-9&10\end{bmatrix}$$

Handling subtraction of negative numbers may be more of a problem than the scalar multiplication.

© John Wiley & Sons Australia, Ltd

Question 2

$$AB = A \times B$$
$$= \begin{bmatrix} 3 & 4 \\ -5 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & -1 & 5 \\ 0 & 2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 3\times 4 + 4\times 0 & 3\times -1 + 4\times 2 & 3\times 5 + 4\times 0 \\ -5\times 4 + 0\times 0 & -5\times -1 + 0\times 2 & -5\times 5 + 0\times 0 \end{bmatrix}$$
$$= \begin{bmatrix} 12 & 5 & 15 \\ -20 & 5 & -25 \end{bmatrix}$$

A is 2×2 and B is 2×3, so the correct answer should have order 2×3. Be careful when multiplying negative numbers.

Question 3

$$X = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$
$$X^2 = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix} \textbf{ [1 mark]}$$
$$2X = \begin{bmatrix} 2\times 2 & 2\times -1 \\ 2\times 0 & 2\times 4 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -2 \\ 0 & 8 \end{bmatrix} \textbf{ [1 mark]}$$
$$X^2 + 2X = \begin{bmatrix} 4 & -6 \\ 0 & 16 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 0 & 8 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & -8 \\ 0 & 24 \end{bmatrix} \textbf{ [1 mark]}$$

Question 4

if $X = \begin{bmatrix} 2 & -1 \\ -5 & 0 \end{bmatrix}$

then $7X = \begin{bmatrix} 2\times 7 & -1\times 7 \\ -5\times 7 & 0\times 7 \end{bmatrix}$

$$= \begin{bmatrix} 14 & -7 \\ -35 & 0 \end{bmatrix}$$

Question 5

$$4(P+Q) = 4\left(\begin{bmatrix} 1 & -2 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ 3 & 2 \end{bmatrix}\right)$$
$$= 4\left(\begin{bmatrix} 1+-1 & -2+5 \\ 4+3 & 1+2 \end{bmatrix}\right)$$
$$= 4 \times \begin{bmatrix} 0 & 3 \\ 7 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 0\times 4 & 3\times 4 \\ 7\times 4 & 3\times 4 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 12 \\ 28 & 12 \end{bmatrix}$$

© John Wiley & Sons Australia, Ltd

Question 6

$$2C + D - E = 2\begin{bmatrix} 1 & 4 & -3 \\ 0 & 2 & 5 \\ -1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 0 \\ -1 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 & 3 \\ 5 & 4 & 2 \\ 0 & 6 & 0 \end{bmatrix} \textbf{ [1 mark]}$$

$$= \begin{bmatrix} 1\times 2 & 4\times 2 & -3\times 2 \\ 0\times 2 & 2\times 2 & 5\times 2 \\ -1\times 2 & 2\times 2 & 1\times 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 5 \\ 0 & 4 & 0 \\ -1 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 & 3 \\ 5 & 4 & 2 \\ 0 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 8+3 & -6+5 \\ 0+0 & 4+4 & 10+0 \\ -2+-1 & 4+0 & 2+1 \end{bmatrix} - \begin{bmatrix} -1 & 1 & 3 \\ 5 & 4 & 2 \\ 0 & 6 & 0 \end{bmatrix} \textbf{ [1 mark]}$$

$$= \begin{bmatrix} 4 & 11 & -1 \\ 0 & 8 & 10 \\ -3 & 4 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 1 & 3 \\ 5 & 4 & 2 \\ 0 & 6 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 & -4 \\ -5 & 4 & 8 \\ -3 & -2 & 3 \end{bmatrix} \textbf{ [1 mark]}$$

Question 7

$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 8 \\ 9 & 0 & 0 \end{bmatrix}$, so $-A = \begin{bmatrix} -2 & 1 & -3 \\ -4 & -6 & -8 \\ -9 & 0 & 0 \end{bmatrix}$ **[1 mark]**

$a_{23} = -8$ is the element in the second row and third column. **[1 mark]**

If the answer given is zero then the student has probably selected the element in the second column and third row.

Question 8

A is 1×3, so matrices that will 'fit' for multiplication must be $3\times$ something. N is 2×2.

Question 9

Multiplying the matrices, $AO = O$ and $AI = A$

4.5 Inverse matrices and problem-solving with matrices

Question 1

$$\det(A) = (1 \times 0) - (-3 \times 2)$$
$$= 0 + 6$$
$$= 6$$

Be careful when dealing with the negative numbers.

Question 2

$$\begin{bmatrix} 2 & 2 \\ 2 & -5 \end{bmatrix} X = \begin{bmatrix} 8 & 6 \\ -13 & 1 \end{bmatrix}$$

$$\frac{1}{-14}\begin{bmatrix} -5 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & -5 \end{bmatrix} X = \frac{1}{-14}\begin{bmatrix} -5 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 8 & 6 \\ -13 & -1 \end{bmatrix}$$

$$LA = \frac{1}{-14}\begin{bmatrix} -14 & -28 \\ -42 & -14 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

Question 3

Notice that the second equation must be written in the same order as the first; namely,

© John Wiley & Sons Australia, Ltd

$-6x + 3y = 7$
$3x - y = 0$ (1)
$3y - 6x = 7$ (2)
$-6x + 3y = 7$ (2′)

$$\begin{bmatrix} 3 & -1 \\ -6 & 3 \end{bmatrix}$$

Question 4

$$AB = 5L$$
$$A \times \left(\frac{1}{5}B\right) = L$$
$$A^{-1} = \frac{1}{5}B$$

Question 5

$$XC = \begin{bmatrix} 2 & 9 \\ 8 & 8 \end{bmatrix}$$

Postmultiply to 'undo' multiplying by C.

$$XCC^{-1} = \begin{bmatrix} 2 & 9 \\ 8 & 8 \end{bmatrix} C^{-1}$$
$$X = \begin{bmatrix} 2 & 9 \\ 8 & 8 \end{bmatrix} \times \frac{1}{4}\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$$ **[1 mark]**
$$= \frac{1}{4}\begin{bmatrix} 4 & 16 \\ 16 & 8 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 4 \\ 4 & 2 \end{bmatrix}$$ **[1 mark]**

Question 6

a. $$BC = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 4 & -4 \\ 2 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$ **[1 mark]**

b. $B\left(\frac{1}{8}C\right) = I$, so $B^{-1} = \frac{1}{8}\begin{bmatrix} 4 & -4 \\ 2 & 2 \end{bmatrix}$ **[1 mark]**

$BX = C$, so $X = B^{-1}C$

$$X = \frac{1}{8}\begin{bmatrix} 4 & -4 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 4 & -4 \\ 2 & 2 \end{bmatrix}$$
$$= \frac{1}{8}\begin{bmatrix} 8 & -24 \\ 12 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 3 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$ **[1 mark]**

Question 7

A matrix multiplied by its multiplicative inverse will result in a multiple of the identity matrix.

© John Wiley & Sons Australia, Ltd

Question 8

$$\begin{bmatrix} 4 & 7 \\ 2 & 9 \end{bmatrix}^{-1} = \frac{1}{(4 \times 9 - 7 \times 2)} \times \begin{bmatrix} 9 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{22} \times \begin{bmatrix} 9 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{22} \times 9 & \frac{1}{22} \times -7 \\ \frac{1}{22} \times -2 & \frac{1}{22} \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{22} & \frac{-7}{22} \\ \frac{-1}{11} & \frac{2}{11} \end{bmatrix}$$

Question 9

$$\begin{bmatrix} 6 & 2 \\ -3 & 2 \end{bmatrix}^{-1} = \frac{1}{6 \times 2 - 2 \times -3} \times \begin{bmatrix} 2 & -2 \\ 3 & 6 \end{bmatrix}$$ **[1 mark]**

$$= \frac{1}{18} \times \begin{bmatrix} 2 & -2 \\ 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{18} & \frac{-2}{18} \\ \frac{3}{18} & \frac{6}{18} \end{bmatrix}$$ **[1 mark]**

$$= \begin{bmatrix} \frac{1}{9} & \frac{-1}{9} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix}$$ **[1 mark]**

4.6 Communications and connections

Question 1

Y cannot send directly to U or V, only directly to X or Z. However, X and Z do not communicate directly with W. Therefore the answer is 0 - – – Y cannot send a message to W by sending it via one other person.

Question 2

Options D and E are not correct, as some points are connected twice.
Points A, B and C are connected to each other only once. However, in options A and B, some are not connected at all.

Question 3

$$\text{From} \begin{matrix} \\ A \\ B \\ C \\ D \end{matrix} \begin{matrix} \text{To} \\ \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix} \end{matrix}$$

Notice that A, B, C and D are connected to each other once, except for B and D, which are connected twice.

© John Wiley & Sons Australia, Ltd

Question 4
There are four scenarios that need to be considered: Brie losing to Andy or Andy losing to Brie, and Cleo losing to Della or Della losing to Cleo.
When the sum of one-step and two-step dominances is calculated (use CAS to do), we find that Brie will win with the highest dominance of 9.

Question 5
f : Represents the result of the $A - C$ match. We know that C defeated (1 in row 3, column 1), so $f = 0$.
g : Represents the result of the $B - C$ match. We know that B lost to C (0 in row 2, column 3), so $g = 1$
h : The position of the h on the diagonal shows that D does not play itself, so $h = 0$.

4.7 Transition matrices

Question 1
$m = 0.8m + 0.48s$ and $0.48s = 12$ (for Tuesday night)
Solve simultaneously on your CAS calculator to give $m = 60$ and $s = 25$
Therefore, the airline has 85 planes.

Question 2
$\frac{400}{2000} \times 100\% = 20\%$ **[1 mark]**

Question 3
$$S_1 = \begin{bmatrix} 0.1 & 0.2 & 0.1 & 0.2 \\ 0.3 & 0.4 & 0.6 & 0.3 \\ 0.1 & 0.2 & 0.2 & 0.1 \\ 0.5 & 0.2 & 0.1 & 0.4 \end{bmatrix} \times \begin{bmatrix} 600 \\ 600 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 300 \\ 780 \\ 300 \\ 620 \end{bmatrix}$$ **[1 mark]**

Question 4
Looking at the matrix, the 1 in the columns signifies the next person Wendy has lunch with.
Starting with C (Craig): E – D – B – A – C
Therefore, the order after Craig is Edgar, Daniel, Betty then Angela.

Question 5
$S_2 = TS_1$
$T^{-1}S_2 = T^{-1}TS_1$
$S_1 = T^{-1}S_2$
$$= \begin{bmatrix} 400 \\ 0 \\ 200 \end{bmatrix}$$

Question 6
The only transition of 0.4 is from M to F. Thus, M is top left and F is top right of the diagram.
From M there is a transition of 0.1 down. The matrix shows a transition of 0.1 from $M$to T; thus, T is the bottom left vertex.
This leaves L as the bottom right vertex.
x is the transition from L to L. Reading from the matrix, the transition x equals 0.6.

Question 7
Note that there is no transition from C to T, so that element will be 0. Otherwise, set up the matrix and follow the transitions.

© John Wiley & Sons Australia, Ltd

Question 8

Each of the columns in a transition matrix must sum to 1.

Disregard answers A and B.

The information in the question must be read very carefully and a transition matrix should be created where

this week

$$T = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{matrix} \text{ next week}$$

$$T = \begin{matrix} \\ A \\ B \\ C \end{matrix} \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0.6 & 0.5 & 0.2 \\ 0.1 & 0.4 & 0.3 \\ 0.3 & 0.1 & 0.5 \end{bmatrix} \end{matrix}$$

4.8 Review

Question 1

Check an element that is different is all matrices, for example row 2, column 3:

$m_{23} = 3 \times 2 + 2 \times 3 = 12$

Question 2

Use the diagram to set up a communication matrix.

$$\begin{matrix} \\ \textit{sender} \\ \\ \\ \end{matrix} \begin{matrix} \begin{matrix} & \textit{receiver} & & \end{matrix} \\ \begin{matrix} S & T & U & V \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{matrix} \begin{matrix} S \\ T \\ U \\ V \end{matrix}$$

Question 3

To add matrices together, all matrices must be of the same order. Therefore, only

$\begin{bmatrix} 8 \\ 12 \end{bmatrix} + \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 8 & 0 \\ 12 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ are possible.

Question 4

a. 3×2 **[1 mark]**

b. Add up column 2: $50 + 20 + 40 = 110$ **[1 mark]**

c. $L = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$ **[1 mark]**

Question 5

The matrix multiplication $\begin{bmatrix} 4 & 2 & 0 \end{bmatrix} \times \begin{bmatrix} 4 \\ 12 \\ 8 \end{bmatrix} = [4 \times 4 + 2 \times 12 + 0 \times 8] = [40]$.

A. $[144] \neq [40]$

B. $\begin{bmatrix} 16 \\ 24 \\ 0 \end{bmatrix} \neq [40]$

© John Wiley & Sons Australia, Ltd

C. $4 \times \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 12 \\ 8 \end{bmatrix} = [4 \times 1 \times 1 + 4 \times 2 \times 12 + 4 \times 0 \times 8] = [100] \neq [40]$

D. $2 \times \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} = [2 \times 2 \times 2 + 2 \times 1 \times 6 + 2 \times 0 \times 4] = [20] \neq [40]$

E. $4 \times \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} = [4 \times 2 \times 2 + 4 \times 1 \times 6 + 4 \times 0 \times 4] = [40]$

Question 6

a. A is 3×1; A has 3 rows and 1 column. **[1 mark]**

b. B is 2×4; B has 2 rows and 4 columns. **[1 mark]**

Question 7

$$\begin{matrix} & 11W \\ \text{Boys} \\ \text{Girls} \end{matrix} \begin{bmatrix} 10 \\ 8 \end{bmatrix} + \begin{matrix} & 11M \\ \text{Boys} \\ \text{Girls} \end{matrix} \begin{bmatrix} 7 \\ 12 \end{bmatrix} = \begin{matrix} & \text{Year 11} \\ \text{Boys} \\ \text{Girls} \end{matrix} \begin{bmatrix} 10+7 \\ 8+12 \end{bmatrix} \textbf{ [1 mark]}$$

$$= \begin{matrix} \text{Boys} \\ \text{Girls} \end{matrix} \begin{bmatrix} 17 \\ 20 \end{bmatrix} \textbf{ [1 mark]}$$

Question 8

a.
$$\begin{matrix} & \text{Adult} & \text{Child} \\ \text{Afternoon} \\ \text{Evening} \end{matrix} \begin{bmatrix} 220 & 80 \\ 250 & 40 \end{bmatrix} \textbf{ [1 mark]}$$

b.
$$\begin{matrix} & \$ \\ \text{Adult} \\ \text{Child} \end{matrix} \begin{bmatrix} 20 \\ 12 \end{bmatrix} \textbf{ [1 mark]}$$

c.
$$\begin{matrix} & \$ \\ \text{Afternoon} \\ \text{Evening} \end{matrix} \begin{bmatrix} 5360 \\ 5480 \end{bmatrix} \textbf{ [1 mark]}$$

Question 9

$$\begin{bmatrix} 1 & 4 \\ 6 & 3 \end{bmatrix} \times \begin{bmatrix} 8 & 1 & 3 \\ 0 & 9 & 4 \end{bmatrix} = \begin{bmatrix} 1 \times 8 + 4 \times 0 & 1 \times 1 + 4 \times 9 & 1 \times 3 + 4 \times 4 \\ 6 \times 8 + 3 \times 0 & 6 \times 1 + 3 \times 0 & 6 \times 3 + 3 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 37 & 19 \\ 48 & 33 & 30 \end{bmatrix}$$

Question 10

Addition, as the total sales is the sum of the sales in the two shops.

Question 11

Scalar multiplication by 2 because all the sales have doubled.

$$\begin{bmatrix} 80 \times 2 & 75 \times 2 \\ 40 \times 2 & 60 \times 2 \\ 25 \times 2 & 30 \times 2 \\ 60 \times 2 & 100 \times 2 \end{bmatrix} = \begin{bmatrix} 160 & 150 \\ 80 & 120 \\ 50 & 60 \\ 120 & 200 \end{bmatrix}$$

Question 12

\$6600

Amount spent = number of shares × share price

© John Wiley & Sons Australia, Ltd

$$\begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} 1000 & 500 \\ 1500 & 750 \\ 2000 & 1200 \end{bmatrix}\begin{bmatrix} \$1.50 \\ \$3.00 \end{bmatrix} = \begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} \$3000 \\ \$4500 \\ \$6600 \end{bmatrix}$$

Question 13

Selling price $ Cost price $ Profit $

$$\begin{matrix} A \\ B \\ C \\ D \end{matrix}\begin{bmatrix} 27.95 \\ 30.00 \\ 38.50 \\ 62.00 \end{bmatrix} - \begin{matrix} A \\ B \\ C \\ D \end{matrix}\begin{bmatrix} 9.00 \\ 10.00 \\ 12.50 \\ 20.00 \end{bmatrix} = \begin{matrix} A \\ B \\ C \\ D \end{matrix}\begin{bmatrix} 18.95 \\ 20.00 \\ 26.00 \\ 42.00 \end{bmatrix}$$ **[2 mark]**

Question 14

a.
$$M = \begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{matrix}$$ **[1 mark]**

b. Andy; the column under A has all zeros. **[1 mark]**

c.
$$\begin{matrix} \\ M = \\ \\ \\ \end{matrix}\begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{matrix} \times \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{matrix} = \begin{matrix} \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{matrix}$$
[1 mark]

Question 15

a. $3m + s = 9$

$2m + 3s = 13$ **[1 mark]**

b. $\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} m \\ s \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \end{bmatrix}$ **[1 mark]**

c. $\det \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} = 3 \times 3 - 1 \times 2$ **[1 mark]**

$= 9 - 2$

$= 7$ **[1 mark]**

d. $\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}^{-1} = \frac{1}{7} \times \begin{bmatrix} 3 & -1 \\ -2 & 3 \end{bmatrix}$ **[1 mark]**

$= \begin{bmatrix} \frac{3}{7} & \frac{-1}{7} \\ \frac{-2}{7} & \frac{3}{7} \end{bmatrix}$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

e. $\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} m \\ s \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \end{bmatrix}$

$$\begin{bmatrix} m \\ s \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} 9 \\ 13 \end{bmatrix}$$ **[1 mark]**

$$= \begin{bmatrix} \frac{3}{7} & \frac{-1}{7} \\ \frac{-2}{7} & \frac{3}{7} \end{bmatrix} \times \begin{bmatrix} 9 \\ 13 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{7} \times 9 + \frac{-1}{7} \times 13 \\ \frac{-2}{7} \times 9 + \frac{3}{7} \times 13 \end{bmatrix}$$ **[1 mark]**

$$= \begin{bmatrix} \frac{27}{7} - \frac{13}{7} \\ \frac{39}{7} - \frac{18}{7} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{14}{7} \\ \frac{21}{7} \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Therefore, Mars Bars cost \$2 each and Snickers cost \$3. [1 mark]

Question 16

$$2 \times \begin{bmatrix} 3 & 2 \\ 9 & 6 \end{bmatrix} = \begin{bmatrix} 2 \times 3 & 2 \times 2 \\ 2 \times 9 & 2 \times 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 \\ 18 & 12 \end{bmatrix}$$

Question 17

To multiply matrices, the number of columns of the first matrix much be the same as the number of rows of the second matrix. Option C contains a 1×3 matrix followed by a 3×1 matrix. The number of columns of the first matrix, 3, is the same as the number of rows of the second matrix, 3, and therefore the matrices can be multiplied.

Question 18

$$\begin{bmatrix} 1 & 4 \\ 6 & 3 \end{bmatrix} \times \begin{bmatrix} 8 & 1 & 3 \\ 0 & 9 & 4 \end{bmatrix} = \begin{bmatrix} 1 \times 8 + 4 \times 0 & 1 \times 1 + 4 \times 9 & 1 \times 3 + 4 \times 4 \\ 6 \times 8 + 3 \times 0 & 6 \times 1 + 3 \times 9 & 6 \times 3 + 3 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 37 & 19 \\ 48 & 33 & 30 \end{bmatrix}$$

Question 19

$$\begin{array}{c} \\ \text{Blue} \\ \text{Green} \\ \text{Brown} \\ \text{Black} \end{array} \begin{array}{c} \begin{array}{cc} \text{B} & \text{G} \end{array} \\ \begin{bmatrix} 32 & 18 \\ 15 & 10 \\ 24 & 14 \\ 12 & 8 \end{bmatrix} \end{array}$$ **[1 mark]**

Notice that the values in the matrix are the other way round compared with the values in the table.

© John Wiley & Sons Australia, Ltd

Question 20

From the communication network it can be seen there are 5 single connections. As these connections represent communication in both directions of the nodes, the required matrix must be symmetrical about the main diagonal. Option A is the matrix that correctly displays the communications between nodes.
A is only connected to B, so there is only a single 1 in both its row and column matching with B.
B is connected to A, C, E so there are 3 single 1 s in both its row and column.
C is connected to B, D, F, so there are 3 single 1 s in both its row and column.
D is only connected to C, so there is only a single 1 in both its row and column matching with C.
E is only connected to B, so there is only a single 1 in both its row and column matching with B.
F is only connected to C, so there is only a single 1 in both its row and column matching with C.

Question 21

The transition matrix is then

$$T = \begin{array}{c} \text{today} \\ \begin{array}{cc} d & w \end{array} \\ \begin{bmatrix} & \\ & \end{bmatrix} \begin{array}{l} d \\ w \end{array} \end{array} \text{ tomorrow}$$

$$T = \begin{array}{cc} d & w \\ \begin{bmatrix} 0.2 & 0.4 \\ 0.8 & 0.6 \end{bmatrix} & \begin{array}{l} d \\ w \end{array} \end{array}$$

Question 22

Each of the columns in a transition matrix add to 1.

$$T = \begin{bmatrix} 0.2 & 0.1 & 0.3 \\ 0.5 & 0.3 & 0.3 \\ 0.3 & 0.6 & 0.4 \\ 1 & 1 & 1 \end{bmatrix}$$

Question 23

The initial state matrix should contain information about the starting state of the number of apples and oranges. Disregard options A, B and E.
The matrix should also be able to be continuously multiplied by the transition matrix which is a (2×2) matrix. Therefore, the transition matrix must have 2 rows.

Question 24

The transition matrix must show the proportions and not the percentages. Disregard option E.
The transition matrix is set up as

$$\begin{array}{c} \text{from} \\ \begin{array}{cc} A & O \end{array} \\ \text{to} \begin{array}{l} A \\ O \end{array} \begin{bmatrix} & \\ & \end{bmatrix} \end{array}$$

$$\begin{array}{cc} A & O \\ \begin{array}{l} A \\ O \end{array} \begin{bmatrix} 0.25 & 0.55 \\ 0.75 & 0.45 \end{bmatrix} \end{array}$$

75% of apple purchasers purchase oranges next time. There should be 0.75 from A to O.

© John Wiley & Sons Australia, Ltd

5 Linear functions, graphs and models

Topic	5	Linear functions, graphs and models
Subtopic	5.2	Linear functions and their features

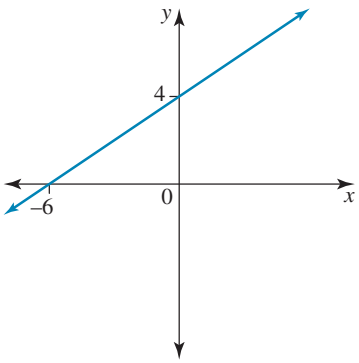
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

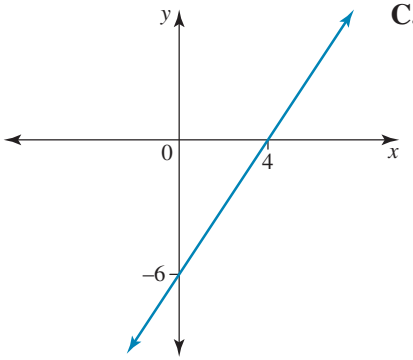
Question 1 (1 mark)

Which of the following is an accurate sketch of $2x - 3y = 12$?

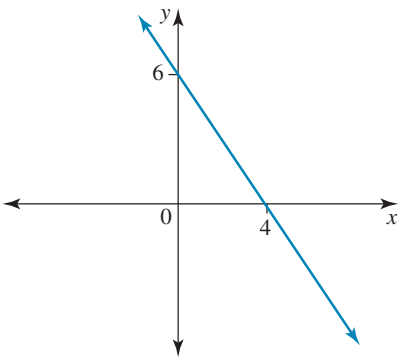
A.



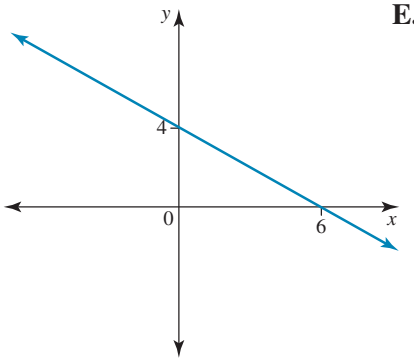
B.



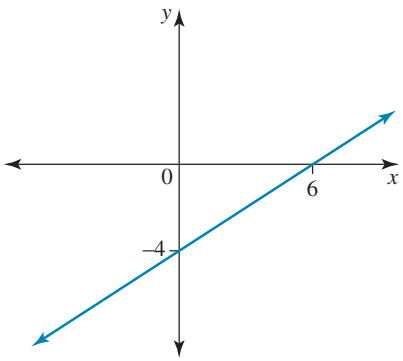
C.



D.



E.



© John Wiley & Sons Australia, Ltd

Question 2 (1 mark)
The gradient and y-intercept of the graph with equation $3x - 4y + 12 = 0$ are
A. $b = \frac{3}{4}$ and $a = 4$
B. $b = \frac{4}{3}$ and $a = 3$
C. $b = \frac{4}{3}$ and $a = 4$
D. $b = 3$ and $a = 12$
E. $b = \frac{3}{4}$ and $a = 3$

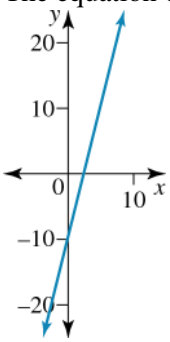
Question 3 (1 mark)
The gradient of the line connecting the two points $(-6, 2)$ and $(2, 10)$ is:
A. 1
B. -1
C. $\frac{4}{3}$
D. $-\frac{4}{3}$
E. 3

Question 4 (1 mark)
$x = -3$ is the solution to the equation
A. $7x - 5 = 16$
B. $2(x - 4) = 5x + 1$
C. $4 - \frac{4x - 1}{2} = 1$
D. $\frac{3x - 4}{2} = 6$
E. $2x + 6 = 3x - 5$

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)

The equation of the graph is



A. $4x - 3y = 12$
B. $4x + y = 12$
C. $x - 3y = 12$
D. $4x - y = 12$
E. $4x + 3y = 12$

© John Wiley & Sons Australia, Ltd

Topic	5	Linear functions, graphs and models
Subtopic	5.3	Sketching linear graphs

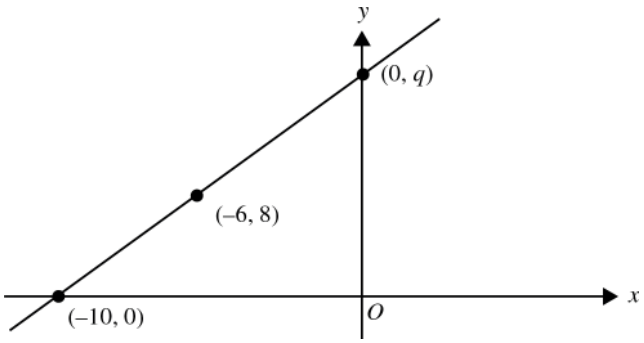
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: VCE 2019, Further Mathematics Exam 1, Section B, Module 4, Q.4; © VCAA

Question 1 (1 mark)

The graph below shows a straight line that crosses the x-axis at $(-10,\ 0)$, passes through the point $(-6,\ 8)$ and crosses the y-axis at $(0,\ q)$.



What is the value of q ?

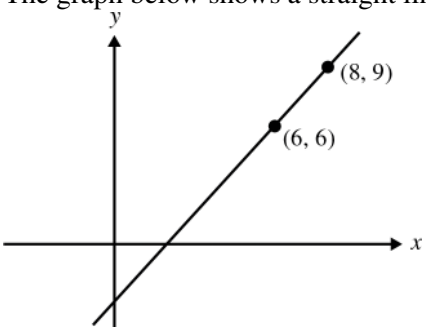
A. 14
B. 16
C. 18
D. 20
E. 22

© John Wiley & Sons Australia, Ltd

Source: VCE 2016, Further Mathematics Exam 1, Section B, Module 4, Q.3; © VCAA

Question 2 (1 mark)

The graph below shows a straight line that passes through the points (6, 6) and (8, 9).



The coordinates of the point where the line crosses the x-axis are

A. $(-3, 0)$
B. $(1, 0)$
C. $(1.5, 0)$
D. $(2, 0)$
E. $(4, 0)$

Question 3 (4 marks)

The equation of a straight line is $y = 4 + \frac{1}{2}x$.

a. State the gradient and the y-intercept. **[2 marks]**

b. Determine the x-intercept. **[1 mark]**

c. Draw the graph. **[1 mark]**

Question 4 (1 mark)
The table of values shows the linear relationship between q and p.

p	0	1	2	3	4	5
q	4.2	5.7	7.2	8.7	10.2	11.7

The equation of the straight line joining these points is
A. $q = 4.2 + 1.5p$
B. $q + 1.5p = 4.2$
C. $p = 1.5 + 4.2q$
D. $q = 1.5 + 4.2p$
E. $p = 4.2 + 1.5q$

Question 5 (7 marks)
Taxi charges comprise a fixed fee (called the 'flag fall') of $3.20, a $2.00 booking fee and a travel charge of $1.62 per kilometre.

a. If you book and get into a taxi, how much does it cost before you travel anywhere? **[1 mark]**

b. What is the travel charge if you travel x kilometres? **[1 mark]**

c. Write down a linear equation relating the total cost of a trip of x km, C, in a taxi that has been booked. **[1 mark]**

d. Draw graphs of the total cost, C, against the length of the trip, x, with and without the booking fee. **[2 marks]**

© John Wiley & Sons Australia, Ltd

e. Con paid $51.80 for a taxi ride. If he did **not** book the taxi ahead of time, how far did he travel? **[1 mark]**

f. What are the limits on the values of the length of the trip, x? **[1 mark]**

© John Wiley & Sons Australia, Ltd

Topic	5	Linear functions, graphs and models
Subtopic	5.4	Linear modelling

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: VCE 2018, Further Mathematics Exam 1, Section B, Module 4, Q.2; © VCAA

Question 1 (1 mark)

Steven is a wedding photographer.
He charges his clients a fixed fee of $500, plus $250 per hour of photography.
The equation that represents the total amount, $\$C$, Steven charges, for t hours of photography is

A. $C = 250t$
B. $C = 500t$
C. $C = 750t$
D. $C = 500 + 250t$
E. $C = 250 + 500t$

Question 2 (1 mark)

The height of a boy, h cm, is related to his age, a years, according to the equation $h = 70.2 + 6.5a$.
How old was the boy when he was 161.2 cm tall?

A. 9.8
B. 13
C. 14
D. -14
E. 15.2

Question 3 (1 mark)

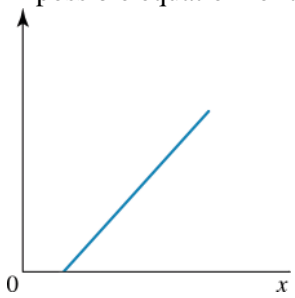
According to my smart phone, a hire car was 15 km away when I called. Its driver undertook that she could average 50 km/h when she drove to pick me up. What equation can be used to model the distance the car is away from me, d km $(0 \leq d \leq 15)$, after time t hours?

A. $d = 50t$
B. $d + 15 = 50t$
C. $d + 50t + 15 = 0$
D. $15 - d = 50t$
E. $d = 50t - 15$

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)

A possible equation for the graph shown is



A. $y = 3x - 6$
B. $y = 3x + 6$
C. $y = -3x - 6$
D. $y = -3x + 6$
E. $y = 3x$

Question 5 (1 mark)

A cleaning company charges their clients using the equation $C\,(\$) = 15 + 20n$, where C represents the total cost of the cleaning and n represents the number of half-hour increments taken for the job.
If the cleaner took two-and-a-half hours to complete one job, the total cost would be

A. $95.00
B. $40.00
C. $115.00
D. $65.00
E. $100.00

Question 6 (1 mark)

A cleaning company charges their clients using the equation $C\,(\$) = 15 + 20n$, where C represents the total cost of the cleaning and n represents the number of half-hour increments taken for the job. If the total cost was $615, how many hours did the clean take?

A. 30 hours
B. 75 hours
C. 37.5 hours
D. 15 hours
E. 20 hours

© John Wiley & Sons Australia, Ltd

Topic	5	Linear functions, graphs and models
Subtopic	5.5	Determining equations of straight lines

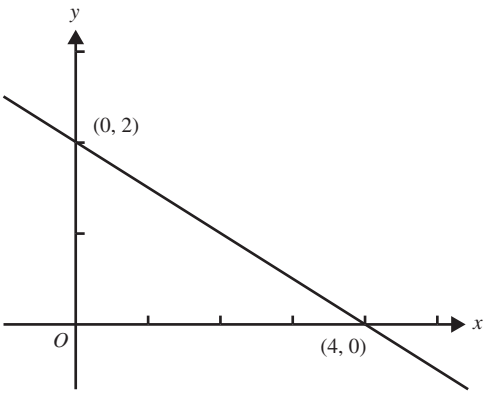
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: VCE 2018, Further Mathematics Exam 1, Section B, Module 4, Q.1; © VCAA

Question 1 (1 mark)

The graph below shows a line intersecting the x-axis at $(4,\ 0)$ and the y-axis at $(0, 2)$.



The gradient of this line is

A. -4

B. -2

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

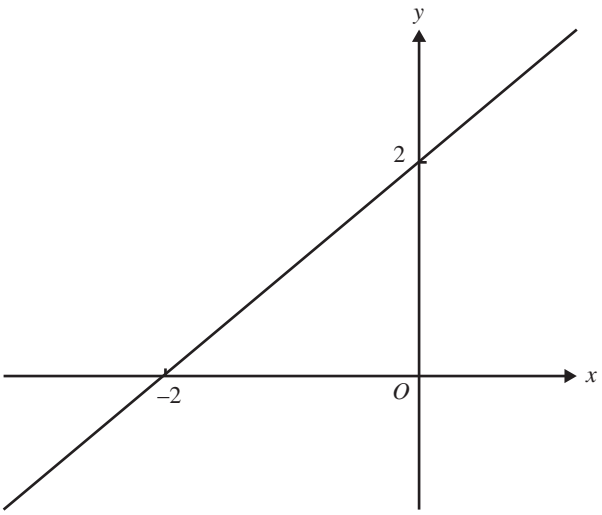
E. 4

© John Wiley & Sons Australia, Ltd

Source: VCE 2018, NH Further Mathematics Exam 1, Module 4, Q.1; © VCAA

Question 2 (1 mark)

A straight line passes through the point $(-2, 0)$ and $(0, 2)$, as shown in the diagram below.



The equation of this straight line is

A. $y = 2x$
B. $y = x + 2$
C. $y = 2x + 2$
D. $y = x - 2$
E. $y = 2x - 2$

Question 3 (1 mark)

The amount needed to pay a taxi fare, $\$C$, to travel a distance of x kilometres is given by the rule:

$C = a + bx$

To travel a distance for 10 kilometres, the taxi fare is \$40.

To travel a distance of 15 kilometres, the taxi fare is \$55.

The cost per kilometre, b, is

A. 2
B. 4
C. 3
D. 2.5
E. 3.5

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)

A line with gradient of $-\frac{5}{4}$ and a y-intercept of 3 could have the equation

A. $y = \frac{5}{4}x + 3$

B. $4y + 5x = 12$

C. $y = -\frac{5}{4}x - 3$

D. $4y - 5x = 12$

E. $4y + 5x = 3$

Question 5 (1 mark)

The equation of the line with a gradient of -3 and passing through the point $\left(4, -11\frac{1}{2}\right)$ is

A. $y = -23\frac{1}{2} - 3x$

B. $y = 3x - \frac{1}{2}$

C. $y = 3 - \frac{1}{2}x$

D. $y = \frac{1}{2} - 3x$

E. $y = 23\frac{1}{2} + 3x$

© John Wiley & Sons Australia, Ltd

Topic	5	Linear functions, graphs and models
Subtopic	5.6	Piecewise linear graphs and their application

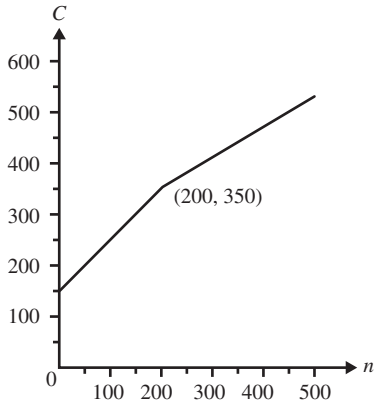
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: VCE 2015, Further Mathematics Exam 1, Section B, Module 3, Q.7; © VCAA

Question 1 (1 mark)

The graph below shows the cost, C, of printing n wedding invitations.



A function that can be used to model this is

$$C = \begin{cases} n + 150 & 0 \le n < 200 \\ 0.6n + p & 200 \le n \le 500 \end{cases}$$

The value of p is

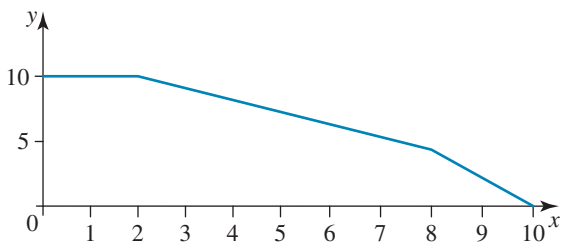
A. 30
B. 150
C. 230
D. 380
E. 470

Question 2 (1 mark)

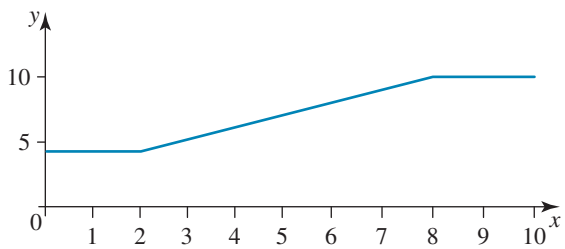
The graph for the relationship described below is

$$y = \begin{cases} 2x & \text{if } 0 \leq x \leq 2 \\ x + 2 & \text{if } 2 \leq x \leq 8 \\ 10 & \text{if } 8 \leq x \leq 10 \end{cases}$$

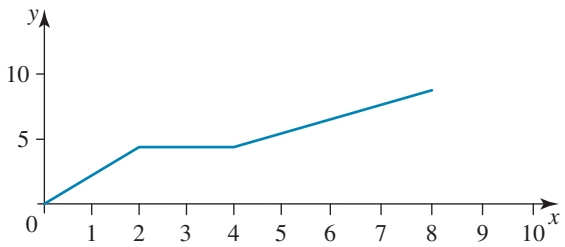
A.



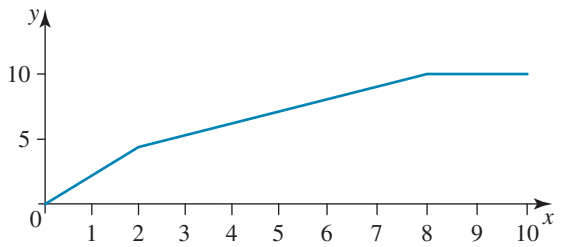
B.



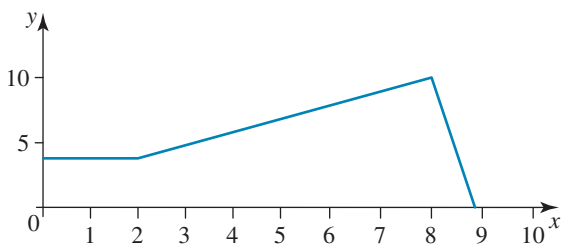
C.



D.



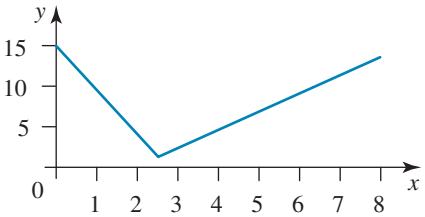
E.



© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)

The relationship for this graph is



A. $y = \begin{cases} 15 - 5x & \text{if } 0 \le x \le 2.5 \\ -2.5 + 3x & \text{if } 2.5 \le x \le 8 \end{cases}$

B. $y = \begin{cases} 15 - 5x & \text{if } 0 \le x \le 2.5 \\ -2.5 + 2x & \text{if } 2.5 \le x \le 8 \end{cases}$

C. $y = \begin{cases} 15 - 5x & \text{if } 0 \le x \le 3 \\ -2.5 + 2x & \text{if } 3 \le x \le 8 \end{cases}$

D. $y = \begin{cases} 15 - 5x & \text{if } 0 \le x \le 2 \\ -2.5 + 2x & \text{if } 2 \le x \le 8 \end{cases}$

E. $y = \begin{cases} 2x & \text{if } 0 \le x \le 2 \\ x + 2 & \text{if } 8 \le x \le 10 \\ 10 & \text{if } 8 \le x \le 10 \end{cases}$

Question 4 (1 mark)

A doctor has the following scale of fees.

Type of appointment	Consultation time	Fee
Standard	0 – 15 minutes	$70
Long	15 – 30 minutes	$120
Prolonged	30 – 45 minutes	$180
1 hour	45 – 60 minutes	$215

The equation for the relationship between the fee, F , and the consultation time, t, is given by

A. $F = \begin{cases} 70t & \text{if } 0 \le t \le 15 \\ 120t & \text{if } 15 \le t \le 30 \\ 180t & \text{if } 30 \le t \le 45 \\ 215t & \text{if } t = 60 \end{cases}$

B. $t = \begin{cases} 70 & \text{if } 0 \le F \le 15 \\ 120t & \text{if } 15 \le F \le 30 \\ 180 & \text{if } 30 \le F \le 45 \\ 215 & \text{if } F = 60 \end{cases}$

C. $F = \begin{cases} 70 & \text{if } 0 < t \le 15 \\ 120 & \text{if } 15 < t \le 30 \\ 180 & \text{if } 30 < t \le 45 \\ 215 & \text{if } 45 < t \le 60 \end{cases}$

D. $F = \begin{cases} 70 & \text{if } 0 \le t \le 15 \\ 120 & \text{if } 15 \le t \le 30 \\ 215 & \text{if } 30 \le t \le 45 \\ 215 & \text{if } 45 < t \le 60 \end{cases}$

E. $t = \begin{cases} 70F & \text{if } 0 \le t \le 15 \\ 120F & \text{if } 15 \le t \le 30 \\ 180F & \text{if } 30 \le t \le 45 \\ 215F & \text{if } F = 60 \end{cases}$

© John Wiley & Sons Australia, Ltd

Topic	5	Linear functions, graphs and models
Subtopic	5.7	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

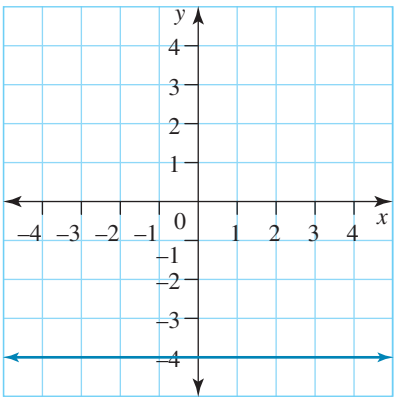
Question 1 (1 mark)

A carpet steam-cleaning company charges $25 to clean one room and $75 to clean six rooms. Which of the given equations is a possible linear model for this steam-cleaning charge (C) if r = number of rooms?

A. $C = 75 + 25r$
B. $C = 25 + 75r$
C. $C = 15 + 10r$
D. $C = 10 + r$
E. $C = 10 + 15r$

Question 2 (1 mark)

The gradient of the line shown is



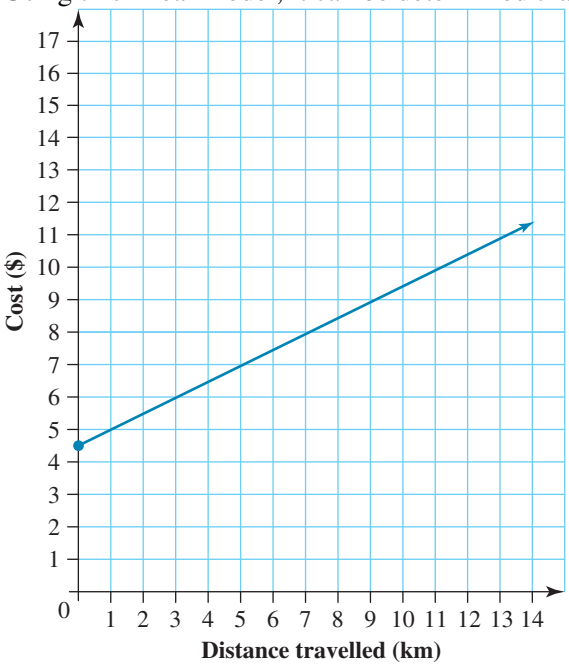
A. undefined
B. 1
C. -1
D. -4
E. 0

© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)

A local taxi company uses the following graph to calculate the cost of a taxi trip. Determine the linear model which represents this scenario.

Using this linear model, it can be determined that the cost of a 25-km trip will be



A. $17.00
B. $4.50
C. $25.00
D. $12.50
E. $112.50

Question 4 (5 marks)

In her new job, Josie is paid $20 per hour up to 35 hours in a week and then $30 per hour for any extra hours she works in that week.

a. How much will Josie be paid if she works 35 hours a week? **[1 mark]**

b. How much will Josie be paid if she works 36 hours a week? **[1 mark]**

© John Wiley & Sons Australia, Ltd

c. Write an equation relating Josie's income in a week, I, and the number of hours she works, n. **[1 mark]**

d. Draw a graph of I against n. **[1 mark]**

e. How many hours did Josie work if she got paid $1000? **[1 mark]**

Question 5 (4 marks)

When his light globes stopped working, Alex changed to halogen globes.
His records showed that it cost him 0.7 cents per hour for the 800 hours his original light globes had lasted.
Then, the halogen globes cost him 0.5 cents per hour and they lasted 2400 hours.
The relationship showing the total amount he paid per globe, P cents, after h hours from the beginning, is:

$$P = \begin{cases} 0.7h & \text{if } 0 \leq h \leq 800 \\ 160 + 0.5h & \text{if } 800 < h \leq 3200 \end{cases}$$

a. Plot the graph of P against h. **[1 mark]**

b. How much did running the original globes cost altogether? **[1 mark]**

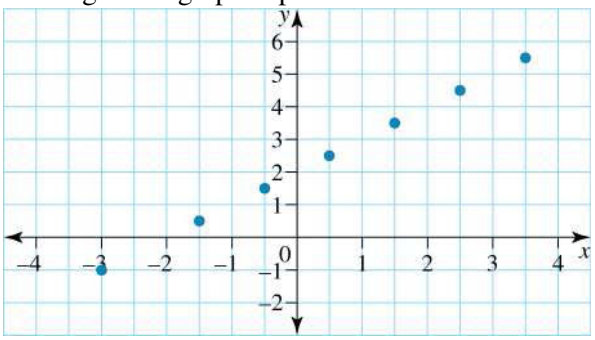
c. How much did running both globes cost altogether? **[1 mark]**

d. After how many hours had he spent $10 on running the globes? **[1 mark]**

© John Wiley & Sons Australia, Ltd

Question 6 (5 marks)

A straight line graph is plotted from a table of values on an Excel spreadsheet:



a. Draw a straight line through the points. **[1 mark]**

b. State the gradient and y-intercept of the graph. **[2 marks]**

c. Hence, or using another method, state the equation of the straight line. **[1 mark]**

d. What is the value of y when $x = 5$? **[1 mark]**

Question 7 (7 marks)

In 1898, A.E. Dolbear described an equation relating the rate crickets chirp and the air temperature:

$T = 50 + \dfrac{N - 40}{4}$

where T = temperature in degrees Fahrenheit and N = number of chirps per minute.

a. Write Dolbear's equation in the form $Ax + By + C = 0$ and show that the relationship is linear. **[2 mark]**

b. Sketch the graph of N against T. **[1 mark]**

© John Wiley & Sons Australia, Ltd

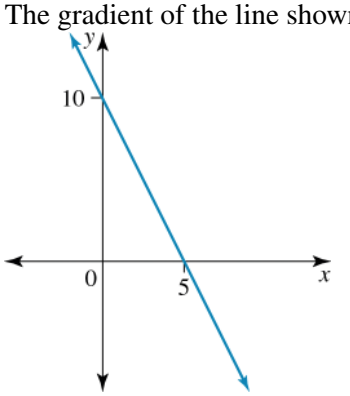
c. What are the limits on the values of N? **[1 mark]**

d. Find the number of chirps of the cricket each minute when the temperature is 60°F. **[1 mark]**

e. At what temperature does the cricket stop chirping? **[2 marks]**

Question 8 (1 mark)

The gradient of the line shown is



A. 2
B. −2
C. −5
D. 5
E. 15

© John Wiley & Sons Australia, Ltd

Question 9 (1 mark)

The gradient of the line connecting the two points $(-6, 2)$ and $(2, 10)$ is

A. 1

B. -1

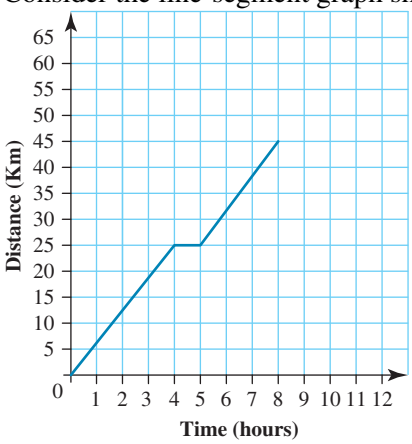
C. $\frac{4}{3}$

D. $-\frac{4}{3}$

E. 3

Question 10 (1 mark)

Consider the line-segment graph shown. Which of the following statements is *incorrect*?



A. The speed for the first part of the journey was approximately 5.6 km/h.

B. A rest break was had after $4\frac{1}{2}$ hours of travelling.

C. The rest break was approximately an hour long.

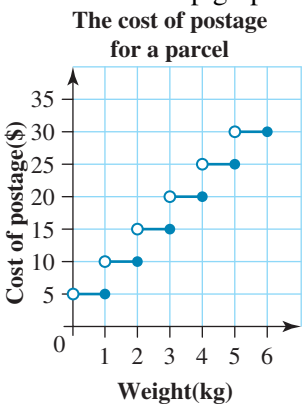
D. The speed for the second part of the journey was approximately 16 km/h.

E. The total distance travelled was 40 km.

© John Wiley & Sons Australia, Ltd

Question 11 (1 mark)

Consider the step graph for postage below. Which of the following statements is *incorrect*?



A. A parcel weighing 3.75 kg will cost $20 to post.
B. A parcel weighing less than 1 kg will cost $5 to post
C. A parcel weighing exactly 4 kg will cost $20 to post.
D. A parcel weighing just over 5 kg will cost $25 to post.
E. A parcel weighing just over 2 kg will cost $15 to post.

© John Wiley & Sons Australia, Ltd

Answers and marking guide

5.2 Linear functions and their features

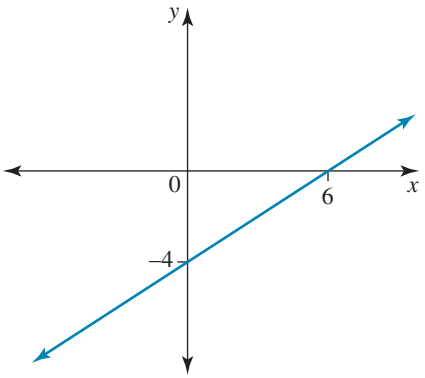
Question 1

$x-$ intercept, $y=0$

$$2x-3y=12$$
$$2x-3(0)=12$$
$$2x=12$$
$$x=6 \quad \text{or} \quad (6,0)$$

$y-$ intercept, $x=0$

$$2x-3y=12$$
$$2(0)-3y=12$$
$$-3y=12$$
$$y=-4 \quad \text{or} \quad (0,-4)$$



Question 2

$$3x-4y+12=0$$
$$4y=12+3x$$
$$y=3+\frac{3}{4}x$$
$$\therefore b=\frac{3}{4} \text{ and } a=3$$

Question 3

$$b=\frac{y_2-y_1}{x_2-x_1}$$
$$=\frac{10-2}{2-(-6)}$$
$$=\frac{8}{8}$$
$$=1$$

Question 4

Solve each equation to find the correct solution:

$$2(x-4)=5x+1$$
$$2x-8=5x+1$$
$$-3x=9$$
$$x=-3$$

Alternatively, substitute $x=-3$ into the equations:

$$2(-3-4)=-14$$
$$5(-3)+1=-14$$
$$\therefore 2(x-4)=5x+1$$

© John Wiley & Sons Australia, Ltd

Question 5

From the graph: the gradient is positive and y-intercept is -12.

$4x - y = 12$

$y = -12 + 4x$

$\therefore b = 4, a = -12$

5.3 Sketching linear graphs

Question 1

The equation of a straight line is $y = mx + c$, where in this case the y-intercept is $c = q$.

$$m = \frac{0 - 8}{-10 - -6} = \frac{-8}{-4} = 2$$

$y = 2x + q$

Substitute in the coordinate $(-10,\ 0)$ to find the value of q.

$0 = 2(-10) + q$

$q = 20$

OR using only gradients:

$$\frac{q - 8}{0 - (-6)} = \frac{8 - 0}{-6 - (-10)}$$

$$\frac{q - 8}{6} = \frac{8}{4}$$

$q - 8 = 12$

$q = 20$

Question 2

First, find the equation of the straight line.

The gradient is $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 6}{8 - 6} = \frac{3}{2}$.

so $y = a + \frac{3}{2}x$.

Substitute in one of the points to find the value of a, e.g. $(6,\ 6)$:

$6 = a + \frac{3}{2} \times 6$

$6 = a + 9$

$a = 6 - 9$

$a = -3$

$\therefore y = -3 + \frac{3}{2}x$

The x-intercept is when $y = 0$:

$0 = -3 + \frac{3}{2}x$

$x = 2$

Question 3

a. As $y = 4 + \frac{1}{2}x$ is in the form $y = a + bx$:

$b = \frac{1}{2}$ **[1 mark]**

$a = 4$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

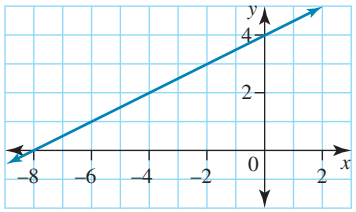
b. x-intercept $(y = 0)$

$$y = \frac{1}{2}x + 4$$

$$0 = \frac{1}{2}x + 4$$

$$\frac{1}{2}x = -4$$

$$x = -8$$ **[1 mark]**

c. 

[1 mark]

Question 4

From the table:

- the gradient is positive, and can be found using table values:

$$\text{Gradient } (b) = \frac{\text{rise}}{\text{run}}$$

$$= \frac{5.7 - 4.2}{1 - 0}$$

$$\therefore b = 1.5$$

- the q-intercept (when $p = 0$) is 4.2.

The equation should be of the form:

$q = a + bp$

$q = 4.2 + 1.5p$

Question 5

a. Cost = flag fall + booking fee

$$= 3.20 + 2.00$$

$$= \$5.20$$ **[1 mark]**

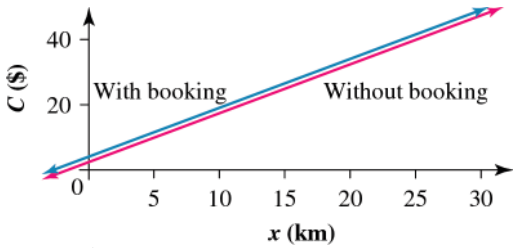
b. Travel charge $= 1.62 \times x$

$$= 1.62x$$ **[1 mark]**

c. Total cost (C) = flag fall + booking fee + $1.62 \times x$

$$\therefore C = 5.20 + 1.62x$$

[1 mark]

d. 

Award **1 mark** for the correct graph with booking fee.

Award **1 mark** for the correct graph without booking fee.

© John Wiley & Sons Australia, Ltd

e. Cost for Con's trip:

$$C = 3.20 + 1.62 \times x$$
$$51.80 = 3.20 + 1.62x$$
$$1.62x = 48.60$$
$$x = 30\,\text{km}$$ **[1 mark]**

f. The trip distance x must be a positive number. **[1 mark]**

5.4 Linear modelling

Question 1

The fixed fee of \$500 is a in the equation $y = a + bx$. The price increases by \$250 per hour (t), so the gradient, b, is 250.
The equation is $C = 500 + 250t$.

Question 2

$$h = 70.2 + 6.5a$$
$$161.2 = 70.2 + 6.5a$$
$$91 = 6.5a$$
$$a = 14\,\text{years}$$ **[1 mark]**

Question 3

Looking at the alternatives:
$d = 50t$ is distance travelled, not how far away
$d + 15 = 50t$ has the distance away increasing
$d + 50t + 15 = 0$ is not transposed correctly
$d = 50t - 15$ gives negative distance

Question 4

From the graph, we can see that the gradient is positive, the x-intercept is positive and the y-intercept will be negative. The only equation that fits these characteristics is $y = 3x - 6$.
Alternatively, to find the equation the x-intercept can be calculated. $y = 3x - 6$ has an x-intercept of $(2,\ 0)$. Therefore, this could be the equation for this graph, as the x-intercept is shown to be quite close to the origin.

Question 5

$2.5\,\text{hours} = 5\,\text{half} - \text{hourly increments}$

$$C\,(\$) = 15 + 20n$$
$$C\,(\$) = 15 + 20\,(5)$$
$$C\,(\$) = 15 + 100$$
$$C\,(\$) = 115$$

The total cost of the clean is \$115.00.

Question 6

$$C\,(\$) = 15 + 20n$$
$$75 = 15 + 20n$$
$$75 - 15 = 20n$$
$$20n = 600$$
$$n = \frac{600}{20}$$
$$n = 30\,\text{half-hour increments}$$

The number of hours is therefore $30 \div 2 = 15\,\text{hours}$.

© John Wiley & Sons Australia, Ltd

5.5 Determining equations of straight lines

Question 1

The gradient can be found by using $\frac{rise}{run}$.

$b = \frac{0-2}{4-0} = \frac{-2}{4} = -\frac{1}{2}$

The gradient of the graph is $= -\frac{1}{2}$.

Question 2

$a = 2$

$b = \frac{rise}{run} = \frac{2}{2} = 1$

$y = a + bx$

$y = 2 + x$

Question 3

$(10, 40)$

$(15, 55)$

$b = \frac{55-40}{15-10} = \frac{15}{5} = 3$

Question 4

$4y + 5x = 12$

$4y = -5x + 12$

$y = -\frac{5x}{4} + \frac{12}{4}$

$y = -\frac{5x}{4} + 3$

Question 5

$y = a + bx$

$y = a - 3x$

Substitute in $\left(4, -11\frac{1}{2}\right)$ to calculate a.

$-11\frac{1}{2} = a - 3(4)$

$-11\frac{1}{2} = a - 12$

$-11\frac{1}{2} + 12 = a$

$a = \frac{1}{2}$

The equation is $y = \frac{1}{2} - 3x$.

5.6 Piecewise linear graphs and their application

Question 1

When $n = 200$, $n + 150 = 0.6n + p$.

Therefore, $p = 230$.

Question 2

Ensure that each part of the graph matches the correct part of the equation.

© John Wiley & Sons Australia, Ltd

Question 3
Notice that the gradient of the first part of the line is negative and the second part is positive.
The change in gradient occurs at $x = 2.5$.

Question 4
Ensure that each part of the graph matches the correct part of the table.

5.7 Review

Question 1
Number of rooms (r) = explanatory variable
Cost (C) = response variable
(r, C) or $C = a + br$
Two points are given: $(1, 25)$ and $(6, 75)$.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{75 - 25}{6 - 1}$$
$$= \frac{50}{5}$$
$$= 10$$

Determine the value for a. Substitute in one of the two given points.

$$C = a + br$$
$$C = a + 10r$$
$$25 = a + 10(1)$$
$$25 = 10 + a$$
$$25 - 10 = a$$
$$a = 15$$

The linear relationship is $C = 15 + 10r$.

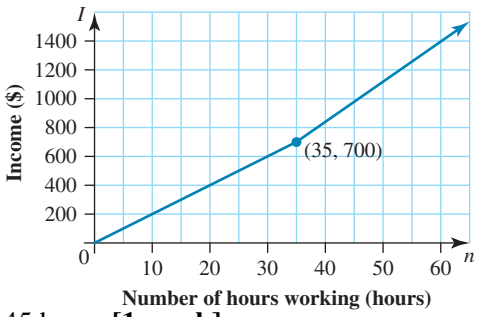
Question 2
The gradient of a vertical line is 0.

Question 3
y-intercept is \$4.50
Gradient is determined to be \$0.50.
Distance (k) is the explanatory variable and cost (C) is the response variable. (Cost depends on the distance travelled).
Linear model is $C = 4.50 + 0.50k$
If travelling 25 km, cost will be: $C = 4.50 + 0.50(25)$

$$= \$17.00$$

Question 4
a. \$700 **[1 mark]**
$n = 35$ in first equation: $35 \times \$20$
b. \$730 **[1 mark]**
$n = 36$ in second equation or calculate $\$700 + 1 \times 30$
c. $I = \begin{cases} 20n & \text{if } 0 \leq n \leq 35 \\ -350 + 30n & \text{if } n > 35 \end{cases}$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

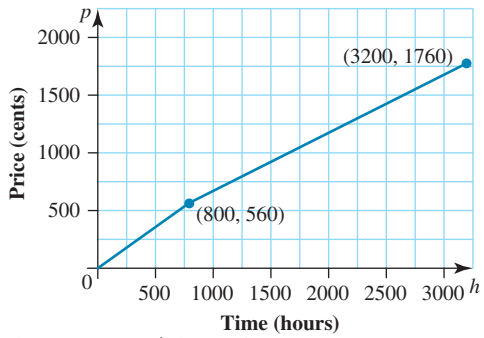
d. 

[1 mark]

e. 45 hours **[1 mark]**

$I = 1000$ in second equation

Question 5

a. 

[1 mark]

b. 560 cents = $5.60 **[1 mark]**

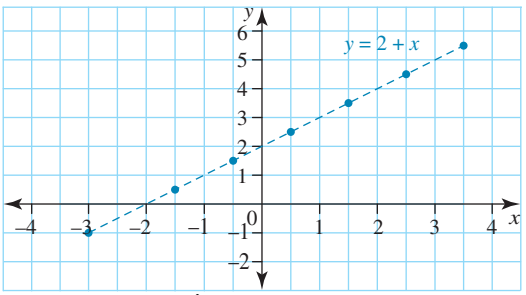
c. 1760 cents = $17.60 **[1 mark]**

$h = 3200$ in new globes equation

d. 1680 hours **[1 mark]**

$P = 1000$ in new globes equation

Question 6

a. 

[1 mark]

b. Gradient $(b) = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{2}{2}$

$\therefore b = 1$ **[1 mark]**

From the graph:

$a = 2$ **[1 mark]**

c. Substitude $b = 1$ and $a = 2$

$y = a + bx$

$= (2) + (1)\,x$

$\therefore y = 2 + x$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

d. When $x = 5$,

$$y = 5 + 2$$
$$\therefore (5, 7)$$ **[1 mark]**

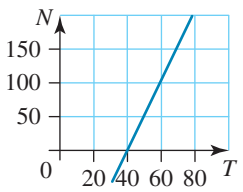
Question 7

a.
$$T = 50 + \frac{N - 40}{4}$$
$$4T = 200 + N - 40$$
$$N - 4T + 160 = 0$$

Or

$$4T - N - 160 = 0$$ **[1 mark]**

b. **[1 mark]**



c. N must be positive since there can't be a negative number of chirps. **[1 mark]**

d. $N - 4T + 160 = 0$

$$N = -160 + 4T$$
$$= -160 + 4 \times 60$$
$$= 80 \text{ chirps per minute}$$ **[1 mark]**

e. When $N = 0$:

$$0 - 4T + 160 = 0$$
$$4T = 160$$
$$T = 40$$
$$\therefore \text{At } 40°\text{F the cricket stops chirping.}$$ **[1 mark]**

Question 8

$$b = \frac{\text{rise}}{\text{run}}$$
$$= -\frac{10}{5}$$
$$= -2$$

Question 9

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{10 - 2}{2 - -6}$$
$$= \frac{8}{8}$$
$$= 1$$

Question 10

The speed for the second part of the journey is calculated by:

$$\text{speed} = \frac{16}{13.5}$$
$$\approx 46 \text{ km/h}$$

which is not 16 km/h.

Therefore, 'The speed for the second part of the journey was approximately 16 km/h.' is the incorrect statement, making it the correct choice in the question.

© John Wiley & Sons Australia, Ltd

Question 11

Any parcel weighing over 5 kg will cost $30 to post.

Therefore, 'A parcel weighing just over 5 kg will cost $25 to post.' is the incorrect statement, making it the correct choice in the question.

© John Wiley & Sons Australia, Ltd

6 Sequences and first-order linear recurrence relations

Topic	6	Sequences and first-order linear recurrence relations
Subtopic	6.2	Arithmetic sequences

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
Determine the value of u_1 in the following sequence.
$1, -\frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \frac{1}{256}$
$u_1 = \square$

Question 2 (1 mark)
For the sequence $-44, -40, -36, ...$, the seventh term is
A. -24
B. -20
C. -18
D. -16
E. -12

Question 3 (1 mark)
Select the three numbers that complete the pattern of the sequence with terms $u_1 = 10, u_2 = 6$ and $u_3 = 2$
A. -2
B. $0, -4, -8$
C. $1, 0, -1$
D. $4, 5, 6$
E. $-2, -6, -10$

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
State which number completes the pattern of the sequence
3, 5, 8, 12, ..., ..., ...
A. 14
B. 15
C. 16
D. 17
E. 18

Question 5 (1 mark)
A sequence is described by the rule $u_n = n^2 - 1$.
Which of the numbers 625 and 168 is a term of the sequence?

Question 6 (1 mark)
Which sequence is **not** an arithmetic sequence?
A. 3, 8, 13, ...
B. 5, 4.2, 3.4, ...
C. $6\frac{1}{2}, 7, 7\frac{1}{2}, \ldots$
D. $-4, -1, 2, \ldots$
E. $-6, -3.5, 6, \ldots$

© John Wiley & Sons Australia, Ltd

Topic	6	Sequences and first-order linear recurrence relations
Subtopic	6.3	Arithmetic sequence applications

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
The cost of a television depends on the number that are made. If 50 are made, the cost per television is $250. If 51 are made, the cost is $1.50 less. If 52 are made then the cost is $33 less. Determine the cost if 100 are made.
A. $175
B. $173.50
C. $176.50
D. $178
E. $179.50

Question 2 (1 mark)
For the information shown in the table, the function has the rule

Term number	0	1	2	3	4
Term value	−5	−3	−1	1	3

A. $u_n = u\,(n-1) + 2$
B. $u_n = 2n - 6$
C. $u_n = n - 2$
D. $u_n = -5 + 2n$
E. $u_n - u_n - 1 = 2$

Question 3 (1 mark)
The seating in an arena is arranged in 60 rows. There are 20 seats in the front row, 24 in the second row, 28 in the third row and so on.
The number of seats in the back row is
A. 260
B. 220
C. 256
D. 20
E. 32

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
Which rule describes the pattern for the sequence shown in the table?

Number	n	1	2	3
Term	u_n	4	8	12

A. $u_n = n + 3$

B. $u_n = 4_n$

C. $u_n = 4 + n$

D. $u_n = 2n + 2$

E. $u_n = 2n$

Question 5 (3 marks)
Which of the following sequences is generated by the formula $u_n = -3n + 2$

A. $2, -1, -4, \ldots$

B. $-3, -1, 1, \ldots$

C. $-1, -4, -7, \ldots$

D. $1, -2, -5, \ldots$

E. $-5, -3, -1, \ldots$

© John Wiley & Sons Australia, Ltd

Topic	6	Sequences and first-order linear recurrence relations
Subtopic	6.4	Generate and analyse an arithmetic sequence using a recurrence relation

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

For the sequence $13, 9, 5, 1, -3, \ldots$, the recurrence relation is

A. $u_{n+1} = u_n - 4, u_0 = -3$

B. $u_{n+1} = u_n - 3, u_0 = 13$

C. $u_{n+1} = u_n + 4, u_0 = 13$

D. $u_{n+1} = u_n + 3, u_0 = 13$

E. $u_{n+1} = u_n - 4, u_0 = 13$

Question 2 (1 mark)

From the information in the table

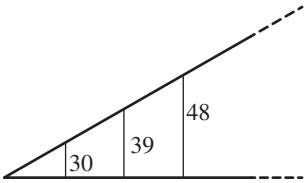
Term number	0	2	4	6	8
Term value	3	11	19	27	35

a. write down the recurrence relation. **(1 mark)**

b. write down the rule for the function relating term value and term number. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 3 (3 mark)
A ramp is supported by equally spaced vertical struts, as shown. The struts are, in sequence, 30 cm, 39 cm, 48 cm, ...



a. Write down the recurrence relation for the sequence of strut lengths. **(1 mark)**

b. Write down the rule for the length of the nth strut. **(1 mark)**

c. Calculate the height of the ramp if it finishes at the eighth strut. **(1 mark)**
Height of the ramp = ▢

Question 4 (1 mark)
The fourth term generated by the recurrence relation $u_{n+1} = u_n + 3, u_0 = -5$ is:
A. 12
B. 10
C. 7
D. 4
E. 1

Question 5 (1 mark)
Sara's starting salary for her new job is $50 000 per year. She has been promised that her salary will rise by $5500 per year every year.
If she doesn't change jobs, she can expect to have a salary of $83 000 after
A. 3 years
B. 4 years
C. 5 years
D. 6 years
E. 7 years

© John Wiley & Sons Australia, Ltd

Topic	6	Sequences and first-order linear recurrence relations
Subtopic	6.5	Geometric sequences

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
Select the geometric sequence from the following.

A. $3, 6, 9, 12, \ldots$
B. $2, 5, 8, \ldots$
C. $8, 11, 14, 17, \ldots$
D. $3, 9, 27, \ldots$
E. $3, 45, 36, 42, \ldots$

Question 2 (1 mark)
The rule for the sequence $5, 15, 45, \ldots$ is
A. $u_n = 3 \times 5^{n-1}$
B. $u_n = 5 \times 3^n$
C. $u_n = 3 \times 5^n$
D. $u_n = 5 \times 3^{n-1}$
E. $u_n = 5 + 3^{n-1}$

Question 3 (3 marks)
The next number in the geometric sequence $4, 6, 9, \ldots$ would be
A. 13
B. 13.5
C. 14
D. 14.5
E. 12.5

Question 4 (1 mark)
Which sequence is not a geometric sequence?
A. $3, 15, 75, \ldots$
B. $2, 1, \frac{1}{2}, \ldots$
C. $1.1, 2.2, 4.4, \ldots$
D. $-4, 2, -1, \ldots$
E. $-6, -12, 24, \ldots$

© John Wiley & Sons Australia, Ltd

Topic	6	Sequences and first-order linear recurrence relations
Subtopic	6.6	Geometric sequence applications

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
For the sequence −4, 12, −36 … the seventh term is

A. 2916
B. −2916
C. 972
D. −972
E. −324

Question 2 (1 mark)
Henry's Maths mark in April is 50%. He promises his mother that he will work so that his mark rises by 5% every month.
If he is able to keep his promise, he can expect to a have a mark of more than 70% in maths in

A. August
B. September
C. October
D. November
E. December

Question 3 (10 marks)
When Jenny was born, her parents decided to invest $1000 into a term deposit account that she could access once she turned 18.
The account received 7% interest per year.

a. State whether this information, when expressed as a sequence, would be represented by an arithmetic sequence or a geometric sequence. Explain your answer. **(1 mark)**

b. Write the rule for this sequence in terms of the number of years of Jenny's life. **(2 marks)**

© John Wiley & Sons Australia, Ltd

c. Determine how much money will be in the account after 1 year.
Money in the account = $ ▢ **(2 marks)**

d. Determine how much money Jenny will receive from her term deposit when she turns 18, correct to 2 decimal places.
Amount of money Jenny will receive = $ ▢ **(2 mark)**

e. Determine how much Jenny would have received if her parents had invested $1500 instead, correct to 2 decimal places.
Amount of money Jenny would have received = $ ▢ **(3 marks)**

Question 4 (1 mark)

Which of the following sequences is generated by the formula $u_n = \frac{1}{2}(2)^{n-1}$?

A. 1, 2, 4, ...

B. $\frac{1}{2}$, 1, $1\frac{1}{2}$, ...

C. $\frac{1}{2}$, 1, 2, ...

D. $\frac{1}{2}$, −1, 2, ...

E. $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, ...

© John Wiley & Sons Australia, Ltd

Topic	6	Sequences and first-order linear recurrence relations
Subtopic	6.7	Generate and analyse a geometric sequence using a recurrence relation

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
The recurrence relation to represent the geometric sequence 8, 16, 32 is

A. $u_{n+1} = u_n + 2$
B. $u_{n+1} = u_n - 2$
C. $u_{n+1} = \frac{1}{2}u_n$
D. $u_{n+1} = 2u_{n+1}$
E. $u_{n+1} = 2u_n$

Question 2 (1 mark)
A colony of bacteria grows by each organism splitting into two every 5 minutes. If there are 100 bacteria in a dish initially, the number in the dish after half an hour is

A. 800
B. 1600
C. 3200
D. 6400
E. 400

Question 3 (1 mark)
The 5000 litres of water in a tank is evaporating, being reduced to 4500 litres and 4050 litres in successive months.
Determine approximately how much water will be in the tank after 6 months.

A. 2657 litres
B. 2657 millilitres
C. 2391 litres
D. 2952 litres
E. 3281 litres

© John Wiley & Sons Australia, Ltd

Topic	6	Sequences and first-order linear recurrence relations
Subtopic	6.8	Modelling growth and decay using recurrence relations

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (3 marks)

Jacqui's mother invests $500 for her daughter in a bank account that pays 4% per annum compound interest, adjusted monthly.

Jacqui wishes to use the money to buy furniture for $520.

She draws up a table to help her work out when she will have enough.

n	0	1	2	3	4	5
A	$500.00	$501.67	$503.34	$505.62	$506.70	$508.39

a. Calculate how much will she have after 6 months, correct to 2 decimal places.
Amount of money saved = $ ▢ **(1 mark)**

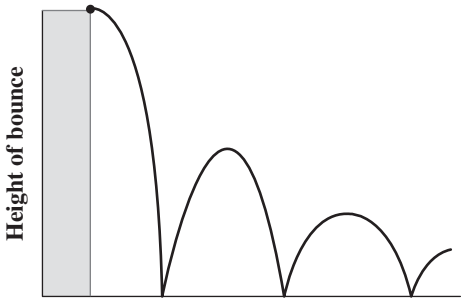
b. Determine how many months it will take for her to have enough to buy the furniture.
Number of months = ▢ months **(1 mark)**

c. Determine whether it would be quicker if she invested with simple interest of 0.25% per month.
Calculate how long the simple interest investment would take to reach her goal. **(1 mark)**
xs

© John Wiley & Sons Australia, Ltd

Question 2 (3 marks)

A ball dropped from the top of a 64-metre-high building rebounds to a height of 48 metres on the first bounce, 36 metres on the second bounce and so on. The height of each bounce is $\frac{3}{4}$ of the height of the previous bounce.



a. Write the recurrence relation for the sequence of bounces. **(1 mark)**

b. Write down the rule for the height of the *n*th bounce. **(1 mark)**

c. Determine the height of the fifth bounce, correct to 2 decimal places.
Height of the fifth bounce = ▢ m **(1 mark)**

Question 3 (3 marks)

A swimming pool pump is purchased for $18 200 with reducing-balance depreciation at 20% per annum.

a. Calculate the future value of the pump after 3 years, correct to 2 decimal places.
Future value = $ ▢ **(1 mark)**

b. Calculate the accumulated depreciation over the period, correct to 2 decimal places.
Accumulated depreciation = $ ▢ **(1 mark)**

c. Determine after how many full years the book value of the pump will be less than $2500.
Number of years = ▢ years
Height of the fifth bounce = ▢ m **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
A coffee machine was purchased for $3500. It has an expected lifetime of 8 years and zero residual (scrap) value. If flat rate depreciation is assumed, the amount of depreciation per year is
A. $400
B. $12.50
C. $3062.25
D. $412.50
E. $437.50

Question 5 (1 mark)
New refrigeration equipment, costing 52 000, depreciates at a constant rate of 15% per annum. If the equipment is valued at less than 25 000, it is considered to be a health risk needing major servicing. After how many whole number years should the equipment be serviced?
A. 2
B. 3
C. 4
D. 5
E. 6

© John Wiley & Sons Australia, Ltd

Topic	6	Sequences and first-order linear recurrence relations
Subtopic	6.9	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
Select the sequence that has no recognisable pattern.
A. $2, 4, 6, 8, \ldots$
B. $2, 8, 14, 20, \ldots$
C. $1, 34, 35, 67, \ldots$
D. $32, 16, 8, 4, \ldots$
E. $89, 82, 75, 68, \ldots$

Question 2 (1 mark)
The recursive equation for the sequence $39, 30, 21, 12, \ldots$ is
A. $u_{n+1} = u_n - 9$
B. $u_n = u_{n+1} - 9$
C. $u_n = 9u_{n+1}$
D. $u_n = u_{n+1} + 9$
E. $u_{n+1} = 9$

Question 3 (1 mark)
The number of apples on a tree is seen to increase by 11 each year.
If in the first year after the tree was planted it grew 4 apples, determine how many apples it grew in the 4th year.
Number of apples = ▢

Question 4 (1 mark)
The rule for the sequence $67, 61, 55, 49, \ldots$ is
A. $u_n = 67 + 6(n - 1)$
B. $u_n = 67 - 6(n + 1)$
C. $u_n = 67 - 6n$
D. $n = 67 - 6(n + 1)$
E. $u_n = 67 + 6n$

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)
The reducing balance depreciation on sporting equipment is 25%.
After 3 years, the book value of sporting equipment originally costing $1850 will be
A. $462.50
B. $1387.50
C. $780.47
D. $1040.63
E. $616.67

Question 6 (1 mark)
The next number in the arithmetic sequence 3, 11, 19, 27, ... would be.
A. 36
B. 32
C. 37
D. 34
E. 35

Question 7 (4 marks)
The depreciation on $1200 tools is $90 per year.
a. Complete the table showing the depreciated value $V(n)$ in each of the first 5 years. **(1 mark)**

n	0	1	2	3	4	5
***V*(*n*)**	1200					

b. Write down the recurrence relation. **(1 mark)**

© John Wiley & Sons Australia, Ltd

c. Write down the rule for the function relating $V(n)$ and n. **(1 mark)**

d. Find when the depreciated value is $660.

Question 8 (3 marks)

Answer the following.

a. Complete the table of an arithmetic sequence **(1 mark)**

n	1	2	3	4	5
$u(n)$	−3	−1	1	3	5
$u(n+1)$	−1				
$u(n+1)-u(n)$	2				

b. Use the table to write the recurrence relation for the sequence. **(1 mark)**

c. Write the rule for $u(n)$ for the sequence. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 9 (3 marks)
George has decided to invest $5000 in a bank account that pays 4% per annum simple interest. He will use the interest to repay a loan from his parents of $10 000.
He draws up a table to help him work out how long he will need to repay his parents:

Year	Interest *4*%	Repaid $
0	0	0
1	200	200
2	200	400
3	200	600
4	200	800
5	200	1000

a. How much has he repaid after 10 years? **(1 mark)**

b. After how many years will he have paid off half the money? **(1 mark)**

c. After how many years will his parents be repaid? **(1 mark)**

Question 10 (3 marks)
Answer the following
a. What is the unit cost of a $15 000 machine with a scrap value $3000 after 5 years? **(1 mark**

b. By how much has the value depreciated after 3 years? **(1 mark**

c. What is the book value of the machine after 3 years? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 11 (3 marks)

Answer the following.

a. Complete the table of a geometric sequence. **(1 mark)**

n	1	2	3	4	5
$u(n)$	3	6	12	24	48
$u(n+1)$	6				
$u(n+1) \div u(n)$	2				

b. Use the table to write the recurrence relation for the sequence. **(1 mark)**

c. Write the rule for $u(n)$ for the sequence. **(1 mark)**

Question 12 (3 marks)

A shopkeeper's equipment has been valued at $245 000. It depreciates at 12.5% per annum reducing balance. Calculate

a. the book value of the equipment after 5 years **(1 mark)**

b. the total depreciation over the first 5 years **(1 mark)**

c. the number of years until the equipment has a value of less than $100 000. Give the answer to the nearest month. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 13 (3 mark)
A taxi company buys 10 new cars, each at a cost of $18 500. The useful life of a taxi is 150 000 km. The estimated trade-in value of a taxi is $2000. Calculate:

a. the unit cost depreciation **(2 marks)**

b. the book value of the cars after they all have travelled 100 000 km. **(1 mark)**

c. the number of years until the equipment has a value of less than $100 000. Give the answer to the nearest month.

Question 14 (1 mark)
The next number in the arithmetic sequence 4, 5.5, 7, 8.5, ... would be

A. 10
B. 10.5
C. 9.5
D. 11
E. 9

Question 15 (1 mark)
The rule for the sequence 67, 61, 55, 49 ... is

A. $u_n = 67 + 6n$
B. $u_n = 67 - 6n$
C. $u_n = -67 + 6n$
D. $u_n = -67 - 6n$
E. $u_n = -6n$

© John Wiley & Sons Australia, Ltd

Answers and marking guide

6.2 Arithmetic sequences

Question 1

As u_1 is the 2nd term of the sequence, u_1 for the given series is $-\frac{1}{4}$. **[1 mark]**

Question 2

$-44, -40, -36, -32, -28, -24, -20, ...$

The seventh term is -20.

Question 3

$-2, -6, -10$

The difference between the second term and the first term is -4. That is,

$u_1 - u_0 = 10 - 6 = -4$.

The difference between the third term and the second term is -4. That is,

$u_2 - u_1 = 2 - 6 = -4$

According to this pattern, the difference between each term is -4.

So, the pattern is to add -4 to each term to get the next.

$10, 6, 2, -2, -6, -10$

Therefore, the next three terms are $-2, -6, -10$

Question 4

The difference between the second term and the first term is 2. That is,

$u_1 - u_0 = 5 - 3 = 2$

The difference between the third term and the second term is 3. That is,

$u_2 - u_1 = 8 - 5 = 3$

The difference between the fourth term and the third term is 4. That is,

$u_3 - u_2 = 12 - 8 = 4$

According to this pattern, the difference between the fifth term and the fourth term is 5.That is $u_4 = u_3 + 5 = 12 + 5 = 17$.

Therefore, the fifth term is 17.

Question 5

The rule $u_n = n^2 - 1$ states that the number is one less than a square number.

$13^2 - 1 = 169 - 1 = 168$

$625 = 25^2$

625 is not a square number, but 169 is a square number; that is, $169 = 13^2$. 168 is one less than 169 and therefore is a term of the given sequence. **[1 mark]**

Question 6

$-6, -3.5, 6, ...$

$a = -6$, d is not common between terms -6 and -3.5; -3.5 and 6

For other alternatives the difference between terms is common.

6.3 Arithmetic sequence applications

Question 1

Be careful about numbering the production. If \$250 is a production of 50 and is counted as $n = 0$, then we require $n = 50$ for a production of 100. $u_{50} = ?$, but

$a = u_0 = \$250$ and $d = -\$1.50$.

© John Wiley & Sons Australia, Ltd

$u_n = a + n \times d$

where $u_{50} = a = 250,\ n = 50$ and $d = -\$1.50$

$u_{50} = 250 + (50) \times -\$1.50$

$u_{50} = \$175$

Question 2

The equation (rule) of the function is of the form $y = a + bx$, where $a = -5$ and $b = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 3}{3 - 1} = 2$.

$\therefore u_n = 5 + 2n$

Question 3

$u_n = a + n \times d$

where $a = 20,\ d = 4,$ and $n = 59$

$u_{59} = 20 + (59)\,(4)$

$= 256$

Question 4

$u_n = 4_n$

There are a number of ways to find the answer to this question. One way is to look for the pattern between the value of n and the value of u: each u is 4 times u.

Another way is to check each option.

Option A

$u_n = n + 3$

$u_1 = 1 + 3 = 4$

$u_2 = 2 + 3 = 5$

$u_3 = 3 + 3 = 6$

This option is not correct.

Option B

$u_n = 4n$

$u_1 = 4 \times 1 = 4$

$u_2 = 4 \times 2 = 8$

$u_3 = 4 \times 3 = 12$

This option is correct.

There is no need to check the other options.

Question 5

Substitute $n = 1, 2, 3,$ ... into the formula $u_n = -3n + 2$

$u_1 = -3(1) + 2$

$= -3 + 2$

$= -1$

$u_2 = -3(2) + 2$

$= -6 + 2$

$= -4$

$u_3 = -3(3) + 2$

$= -9 + 2$

$= -7$

The correct answer is $-1, -4, -7, \ldots$

© John Wiley & Sons Australia, Ltd

6.4 Generate and analyse an arithmetic sequence using a recurrence relation

Question 1

$u_0 = 13$, not $u_0 = -3$

$u_0 - u_0 = 9 - 13$

$= -4$ not 4

$u_{n+1} = u_n - 4, u_0 = 13$

Question 2

a. $u_{n+1} = u_n + 4; u_0 = 3$ **[1 mark]**

b. From the table, $u_0 = 3$. Since n increases by 2, to calculate the difference we need to go halfway between:

$n = 1, u_n = \frac{1}{2}(11 - 3)$ **[1 mark]**

$= 4$

$u_n = 4n + 3$

Question 3

a. $u_{n+1} - u_n = 9;\ u_0 = 30$ **[1 mark]**

b. $u_n = 30 + 9n$ **[1 mark]**

c. Since 8th strut will be u_7

$u_7 = 30 + 9 \times 7 = 93$ cm **[1 mark]**

Question 4

$u_0 = -5$

$u_1 = -5 + 3 = -2$

$u_2 = -2 + 3 = 1$

$u_3 = 1 + 3 = 4$

$u_4 = 4 + 3 = 7$

Question 5

Be careful about numbering the years. \$50 000 is year 0. After 1 year, the salary will be \$55 500 and after 2 years, the salary will be \$61 000, etc.

$u(n) = a + (n) \times d$

Where $u(n) = 83\,000$, $a = 55\,500$ and $d = 5500$

$83\,000 = 55\,500 + (n) \times 5500$

$27\,500 = n \times 5500$

$n = 5$

Sara can expect to have a salary of \$83000 after 6 years.

6.5 Geometric sequences

Question 1

3, 6, 9, 12, ... is an arithmetic sequence.

2, 5, 8, ... is an arithmetic sequence.

8, 11, 14, 17, ... is an arithmetic sequence.

3, 9, 27, ... is a geometric sequence where numbers are multiplied by 3.

3, 45, 36, 42, ... is a random sequence.

The geometric sequence is 3, 9, 27, ...

© John Wiley & Sons Australia, Ltd

Question 2
$u_n = aR^n$
$u_n = 5 \times 3^n$
where $u_0 = 5$

Question 3
The sequence shows numbers being multiplied by 1.5.
$9 \times 1.5 = 13.5$

Question 4
$a = -6$, r is not common $\frac{-12}{-6} \neq \frac{24}{-12}$
For other alternatives the difference between terms is not common.

6.6 Geometric sequence applications

Question 1
$a = -4$ and $R = -3$
7th term is $u_6 = aR^6$
$u_6 = (-4)(-3)^6$
$= -2916$ years
Be careful of the negatives.
The 7th term is -2916 years.

Question 2
Be careful about numbering the months. April is month 0. After 1 month Henry's mark will be 52.2%, after 2 months his mark will be 55.125% and so on. $u_n > 70\%$ but $a = u_0 = 50\%$ and $R = 1.05$.
Mark = 50%, 52.5%, 55.13%, 57.88%, 60.78%, 63.81%, 67%, 70.36%
which is greater than 70%.
This corresponds to November.

Question 3
a. This information would be expressed as a geometric sequence, because a percentage increase in the total amount will be represented by multiplication, not addition.
b. $u_{n+1} = R \times u_n$
$u_{n+1} = 1.07 \times u_n$ and $u_0 = 1000$
c. After 1 year, $u_1 = 1.07 \times 1000 = \$1070$
d. After 18 years, $u_{18} = 1.07^{18} \times 1000 = \$3379.93$
e. If $u_0 = 1500$,
$u_{18} = 1.07^{18} \times 1500$
$= \$5069.90$

Question 4
Substitute $n = 1, 2, 3, \ldots$ into the formula $u_n = \frac{1}{2}(2)^{n-1}$

© John Wiley & Sons Australia, Ltd

$n = 1,\ u_1 = \frac{1}{2}(2)^{1-1}$

$= \frac{1}{2}(2)^0$

$= \frac{1}{2};$

$n = 2,\ u_2 = \frac{1}{2}(2)^{2-1}$

$= \frac{1}{2}(2)^1$

$= 1;$

$n = 3,\ u_3 = \frac{1}{2}(2)^{3-1}$

$= \frac{1}{2}(2)^2$

$= 2;$

$\therefore \frac{1}{2}, 1, 2, \ldots$

6.7 Generate and analyse a geometric sequence using a recurrence relation

Question 1

The sequence shows numbers being multiplied by 2.
Therefore, $u_{n+1} = 2u_n$ where $u_0 = -8$.

Question 2

$u_0 = 100$ (1st term) – 0 minutes
$u_1 = 200$ (2nd term) – 5 minutes
$u_2 = 400$ (3rd term) – 10 minutes
$u_3 = 800$ (4th term) – 15 minutes
$u_4 = 1600$ (5th term) – 20 minutes
$u_5 = 3200$ (6th term) – 25 minutes
$u_6 = 6400$ (7th term) – 30 minutes
100, 200, 400, 800, 1600, 3200, 6400....
$\frac{1}{2}$ hour is 6 lots of 5 min or u_6: 6400

Question 3

The amount of water is reducing by 10% each month. That is, the amount of water in the tank in a particular month is 90% of that in the previous month.
$u_0 = 5000$ is the initial amount.
After 1 month, $u_1 = 4500$
So, $R = \frac{4500}{5000} = 0.9$ and we want after 6 months, so
$u_6 = 0.9^6 \times 5000 = 2657$ litres

© John Wiley & Sons Australia, Ltd

6.8 Modelling growth and decay using recurrence relations

Question 1

a. 4% per annum is $\frac{4}{12}$% per month. Therefore, the common ratio is $1 + \frac{4}{1200}$.
After 6 months the amount is:

$$500 \times \left(1 + \frac{4}{1200}\right)^6 = 500 \times 1.00333^6$$
$$= \$510.08$$

b. A calculator, spreadsheet or CAS calculator can be used to determine how many months are needed to reach $520
12 ($520.37)

c. No, it would take 16 months.
0.25% per month means the common ratio is $1 + \frac{0.25}{100}$.

n	0	1	2	3	4	5	
A	$500.00	$501.25	$502.50	$503.76	$505.02	$506.28	etc.

Compounded: $A = 500\left(1 + \frac{4}{200}\right)^n$

Simple: $A = 500\left(1 + \frac{4}{200}\right)^n$

Question 2

a. $u_{n+1} = \frac{3}{4}u_n$ and $u_0 = 64$

b. The first bounce corresponds to u_0, so the nth bounce is given by: $u_n = 64\left(\frac{3}{4}\right)^n$

c. 5th bounce is:

$$u_5 = 64\left(\frac{3}{4}\right)^5 = 15.19 \text{ m}$$

Question 3

a. $u_0 = \$18\,200, R = 20\%, n = 3$
Calculate the common ratio by identifying the value of the item in any given year as a percentage of the value in the previous year.

$100\% - 20\% = 80\%$

$R = 0.8$

$u_0 = 18\,200$

$u_n = 18\,200 \times 0.8^n$

After 3 years:

$u_3 = \$18\,200(0.8)^3$

$= \$9318.40$

b. Accumulated depreciation $= u_0 - u_3$
$= \$18\,200 - \$9318.40$
$= \$8881.60$

c. 9 years
The number of years after which the book value falls below $2500 can be found by trial and error or with a spreadsheet. Choose the first year when value is less than $2500.

© John Wiley & Sons Australia, Ltd

Question 4

The value falls by equal amounts every year for 8 years: $\frac{\$3500}{8} = \$437.50$.

Question 5

The constant depreciation per year:

$15\% \times \$52\,000 = \$7800$

To be serviced the value must be below \$25 000.

$\$52\,000 - \$25\,000 = \$27\,000$

The number of years for this to happen is found using:

$\frac{\$27\,000}{\$7800} = 3.46$ years

$\therefore$The equipment should be serviced after 3 years.

(4 years will be too long to wait: the value will be below \$25 000.)

6.9 Review

Question 1

Option A shows numbers increasing by 2.

Option B shows numbers increasing by 6.

There is no recognisable pattern in option C.

Option D shows numbers being divided by 2.

Option E shows numbers decreasing by 7.

The sequence is $1, 34, 35, 67, \ldots$

Question 2

The sequence shows numbers decreasing by 9.

Therefore, $u_{n+1} = u_n - 9$, where $u_0 = 39$.

Question 3

$a = 4$

$d = 11$

$u_n = a + nd$

$u_n = 4 + 11n$ **[1 mark]**

where $u_0 = 4$ and $n = 3$

$u_3 = 4 + 11 \times 3$

$u_3 = 37$ **[1 mark]**

Therefore, in the 4th year, the apple tree grew 37 apples.

Question 4

$u_n = a + nd$

$u_n = 67 + n \times -6$

$u_n = 67 - 6n$

where $u_0 = 67$

Question 5

$u_0 = 1850$

The common ratio, $R = 1 - \frac{R}{100}$

$= 1 - \frac{25}{100}$

© John Wiley & Sons Australia, Ltd

$R = 0.75$

$u_n = u_0 \times R^n$
$= 1850 \times 0.75^3$
$= \$780.47$

Question 6

The sequence shows numbers increasing by 8.
$27 + 8 = 35$

Question 7

a. The value falls by \$90 each year and the initial value is \$1200. **[1 mark]**

n	0	1	2	3	4	5
$V(n)$	1200	1110	1020	930	840	750

b. $V(n) = V(n-1) - 90;\ V(0) = 1200$ **[1 mark]**

c. $V(n) = 1200 - 90n$ **[1 mark]**

Note the negative gradient of the line with equation of the form $y = a + bx$, where $a = 1200$ and $b = -90$

d. 6 years **[1 mark]**

The time when the depreciated value is \$660 can be found by looking at the next term in the table or by substituting $V = 660$ in the rule in part iii.

Question 8

a.

n	1	2	3	4	5
$u(n)$	-3	-1	1	3	5
$u(n+1)$	-1	1	3	5	7
$u(n+1) - u(n)$	2	2	2	2	2

[1 mark]

b. $u(n+1) - u(n) = 2; u(1) = -3$ **[1 mark]**

c. For an arithmetic sequence, the formula is $a + nd$.

From the table, $a = -3$ and $d = 2$.

$u(n) = -3 + 2n$ **[1 mark]**

Question 9

a. $u_n = 200 + 200n$

$u_9 = 200 + 200 \times 9$
$= \$2000$ **[1 mark]**

b. $u_n = 200 + 200n$

$5000 = 200 + 200n$
$4800 = 200n$
$n = 24$ **[1 mark]**

He would have paid off half the money in 25 years.

c. $u_n = 200 + 200n$

$10\,000 = 200 + 200n$
$9800 = 200n$
$n = 49$ **[1 mark]**

His parents would be repaid after 50 years.

© John Wiley & Sons Australia, Ltd

Question 10

a. Unit cost $= \dfrac{\text{purchase price} - \text{scrap value}}{\text{number of repetitions expected}}$

$= \dfrac{15\,000 - 3000}{5}$

$= \$2400$ **[1 mark]**

b. \$7200 **[1 mark]**

c. Be careful about the book value. \$15 000 is in year 0. After 1 year, the book value will be \$12 600 and after 2 years the book value will be \$10 200, which is an arithmetic sequence with $a = 12\,600$ and $d = -2400$.

Book value after 3 years is t when $n = 3$.

$\$15\,000 - \$7200 = \$7800$ **[1 mark]**

Question 11

a.

n	1	2	3	4	5
$u(n)$	3	6	12	24	48
$u(n+1)$	6	12	24	48	96
$u(n+1) \div u(n)$	2	2	2	2	2

[1 mark]

b. $u(n+1) \div u(n) = 2;\ u(1) = 3$ **[1 mark]**

c. For a geometric sequence, the formula is

$u_n = a(R)^n$

$u_n = 3(2)^n$ **[1 mark]**

Question 12

a. Depreciation table

years	book value
0	\$245 000.00
1	\$214 375.00
2	\$187 578.13
3	\$164 130.86
4	\$143 614.50
5	\$125 662.69
6	\$109 954.85
7	\$96 210.50
8	\$84 184.18

\$125 662.69 **[1 mark]**

© John Wiley & Sons Australia, Ltd

b. Depreciation table

years	book value
0	$245 000.00
1	$214 375.00
2	$187 578.13
3	$164 130.86
4	$143 614.50
5	$125 662.69
6	$109 954.85
7	$96 210.50
8	$84 184.18

Total depreciation = $245 000 − $125 662.69 = $119 337.31 **[1 mark]**

c. After 6 years the book value is $109 954.85 and after 7 years is $96 210.50. So the answer is 6 years and some months. To find the number of months, use 12.5% per year = $\frac{12.5}{12}$% per month.

Depreciation table

Months	book value
0	$109 954.85
1	$108 809.49
2	$107 676.05
3	$106 554.43
4	$105 444.49
5	$104 346.11
6	$103 259.17
7	$102 183.55
8	$101 119.14
9	$100 065.82
10	$99 023.46

6 years and 9 months **[1 mark]**

Question 13

a. Unit cost depreciation $= \frac{185\,000 - 20\,000}{150\,000}$ **[1 mark]**

$= 1.1$

$\therefore$ \$1.1 per kilometre **[1 mark]**

b. Depreciation = Unit cost × number of times used

$= 1.1 \times 100\,000$

= \$110 000

Total book value = \$185 000 − \$110 000

= \$75 000 **[1 mark]**

Question 14

The sequence shows numbers increasing by 1.5.

8.5 + 1.5 = 10

Question 15

$u_n = a + nd$

$u_n = 67 + n \times (-6)$

$u_n = 67 - 6n$

where $u_0 = 67$

© John Wiley & Sons Australia, Ltd

7 Financial mathematics extension

Topic	**7**	**Financial mathematics extension**
Subtopic	**7.2**	**Reducing balance loans modelled using recurrence relations**

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2019, Further Mathematics Exam 1, Section A, Q.20; © VCAA*

Question 1 (1 mark)

Consider the following amortisation table for a reducing balance loan.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	300 000.00
1	1050.00	900.00	150.00	299 850.00
2	1050.00	899.55	150.45	299 699.55
3	1050.00	899.10	150.90	299 548.65

The annual interest rate for this loan is 3.6%.
Interest is calculated immediately before each payment.
For this loan, the repayments are made

A. weekly.
B. fortnightly.
C. monthly.
D. quarterly.
E. yearly.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2017, Further Mathematics Exam 1, Section A, Core, Q.17; © VCAA*

Question 2 (1 mark)

The value of a reducing balance loan, in dollars, after n months, V_n, can be modelled by the recurrence relation shown below.

$V_0 = 26\,000, \quad V_{n+1} = 1.003V_n - 400$

What is the value of this loan after five months?

A. $24 380.31
B. $24 706.19
C. $25 031.10
D. $25 355.03
E. $25 678.00

Source: *VCE 2016, Further Mathematics Exam 1, Section A, Q.22; © VCAA*

Question 3 (1 mark)

The first three lines of an amortisation table for a reducing balance home loan are shown below.
The interest rate for this home loan is 4.8% per annum compounding monthly.
The loan is to be repaid with monthly payments of $1500

Payment number, *n*	**Payment** ($)	**Interest** ($)	**Principal reduction** ($)	**Balance of loan** ($)
0	0	0	0	250 000.00
1	1500	1000.00	500.00	249 500.00
2	1500			

The amount of payment number 2 that goes towards reducing the principal of the loan is

A. $486
B. $502
C. $504
D. $996
E. $998

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 1, Section A, Q.23; © VCAA*

Question 4 (1 mark)
Joseph borrowed $50 000 to buy a new car.
Interest on this loan is charged at the rate of 7.5% per annum, compounding monthly.
Joseph will fully repay this loan with 60 monthly repayments over five years.
Immediately after the 59th repayment is made, Joseph still owes $995.49
The value of his final repayment, to the nearest cent, will be
A. $995.49
B. $998.36
C. $1001.71
D. $1001.90
E. $1070.15

Source: *VCE 2014, Further Mathematics 2, Module 4, Q.4; © VCAA*

Question 5 (2 mark)
The cricket club borrowed $400 000 to build a clubhouse.
Interest is calculated at the rate of 4.5% per annum, compounding monthly.
The cricket club will make monthly repayments of $2500.
After a number of monthly repayments, the balance of the loan will be reduced to $143 585.33.
What percentage of the next monthly repayment will reduce the balance of the loan?
Write your answer, correct to the nearest percentage.

Question 6 (1 mark)
Jane and Hashim have been paying $2500 in monthly instalments towards their home loan. At the start of the month, there is $70 000 still outstanding on their 6.3% p.a. loan.
How much of the principal will still be outstanding at the end of the first month?
A. $2132.50
B. $65 590.00
C. $67 500.00
D. $67 867.50
E. $69 632.50

© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)
Interest is charged on a loan of $25 000 at a rate of 0.8% per month. The monthly repayments are $225. How much of the principal is still outstanding after 2 months?
A. $24 949.80
B. $24 959.60
C. $24 965.00
D. $24 975.00
E. $24 987.50

Question 8 (1 mark)
Interest is charged on a loan of $75 000 at a rate of 0.7% per month. The monthly repayments are $725. How much of the principal is still outstanding after 2 months?
A. $74 949.80
B. $74 859.00
C. $74 965.00
D. $74 800.00
E. $74 598.60

© John Wiley & Sons Australia, Ltd

Topic	7	Financial mathematics extension
Subtopic	7.3	Solving reducing balance loan problems with technology

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2018, Further Mathematics Exam 1, Section A, Q.22; © VCAA*

Question 1 (1 mark)

Adam has a home loan with a present value of $175 260.56
The interest rate for Adam's loan is 3.72% per annum, compounding monthly.
His monthly repayment is $3200.
The loan is to be fully repaid after five years.
Adam knows that the loan cannot be exactly repaid with 60 repayments of $3200.
To solve this problem, Adam will make 59 repayments of $3200. He will then adjust the value of the final repayment so that the loan is fully repaid with the 60th repayment.
The value of the 60th repayment will be closest to

A. $368.12
B. $2831.88
C. $3200.56
D. $3557.09
E. $3568.12

Source: *VCE 2016, Further Mathematics Exam 2, Core, Q.7a; © VCAA*

Question 2 (2 marks)

Ken has borrowed $70 000 to buy a new caravan.
He will be charged interest at the rate of 6.9% per annum, compounding monthly.
For the first year (12 months), Ken will make monthly repayments of $800.

a. Find the amount that Ken will owe on his loan after he has made 12 repayments. **(1 mark)**

b. What is the total interest that Ken will have paid after 12 repayments? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2014, Further Mathematics Exam 1, Section B, Module 4, Q.5; © VCAA*

Question 3 (1 mark)

A bank approves a $90 000 loan for a customer.
The loan is to be repaid fully over 20 years in equal monthly payments.
Interest is charged at a rate of 6.95% per annum on the reducing monthly balance.
To the nearest dollar, the monthly payment will be

A. $478
B. $692
C. $695
D. $1409
E. $1579

Source: *VCE 2017, Further Mathematics Exam 1, Section A, Core, Q.24; © VCAA*

Question 4 (1 mark)

Xavier borrowed $245 000 to pay for a house.
For the first 10 years of the loan, the interest rate was 4.35% per annum, compounding monthly.
Xavier made monthly repayments of $1800.
After 10 years, the interest rate changed.
If Xavier now makes monthly repayments of $2000, he could repay the loan in a further five years.
The new annual interest rate for Xavier's loan is closest to

A. 0.35%
B. 4.1%
C. 4.5%
D. 4.8%
E. 18.7%

© John Wiley & Sons Australia, Ltd

Source: *VCE 2015, Further Mathematics Exam 1, Section B, Module 4, Q.9; © VCAA*

Question 5 (1 mark)

Ravi borrowed $160 000 at an interest rate of 6.18% per annum.
Interest is calculated monthly on the reducing balance of the loan.
The loan will be fully repaid with monthly payments of $1950.
Which one of the following statements is **not** true?

A. His first payment reduces the loan by less than $1950.
B. His second payment reduces the loan by more than the first payment.
C. Repaying more than $1950 per month will reduce the term of the loan.
D. His final payment will be less than $1760.
E. His final payment includes interest.

Source: *VCE 2014, Further Mathematics Exam 1, Section B, Module 4, Q.9; © VCAA*

Question 6 (1 mark)

Leslie borrowed $35 000 from a bank.
Interest is charged at the rate of 4.75% on the reducing monthly balance.
The loan is to be repaid with 47 monthly payments of $802.00 and a final payment that is to be adjusted so that the loan will be fully repaid after exactly 48 monthly payments.
Correct to the nearest cent, the amount of the final payment will be Responses

A. $0.39
B. $3.57
C. $802.00
D. $802.39
E. $805.57

© John Wiley & Sons Australia, Ltd

Source: *VCE 2013, Further Mathematics Exam 1, Section B, Module 4, Q.9; © VCAA*

Question 7 (1 mark)

The following information relates to the repayment of a home loan of $300 000.

- The loan is to be repaid fully with monthly payments of $2500.
- Interest compounds monthly.
- After the first monthly payment has been made, the amount owing on the loan is $299 000.

Which one of the following statements is true?

A. After two months, $297 995 is still owing on the loan.
B. $1000 of interest has been paid in the first month.
C. The loan will be fully repaid in less than 15 years.
D. Halfway through the term of the loan, the amount still owing will be $150 000.
E. Payments of $2750 rather than $2500 per month will reduce the time to repay the loan fully by more than three years.

Source: *VCE 2013, Further Mathematics Exam 2, Module 4, Q.4; © VCAA*

Question 8 (1 mark)

Hugo took out a reducing balance loan of $25 000 to compete in road races overseas.
Interest was charged at a rate of 12% per annum compounding quarterly.
His loan is to be repaid fully in four years with equal quarterly payments.
After two years, how much of the $25 000 will Hugo have repaid?
Write your answer, correct to the nearest dollar.

© John Wiley & Sons Australia, Ltd

Topic	7	Financial mathematics extension
Subtopic	7.4	Annuities

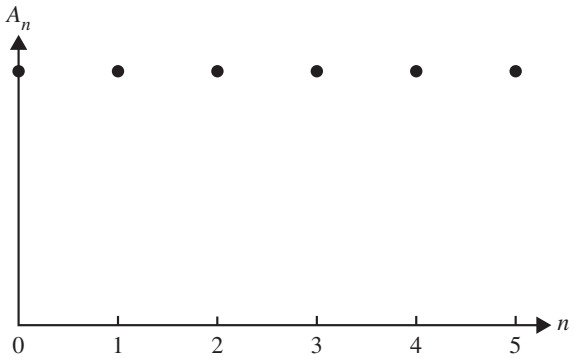
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2020, Further Mathematics Exam 1, Section A, Q.25; © VCAA*

Question 1 (1 mark)

The graph below represents the value of an annuity investment, A_n, in dollars, after n time periods.



A recurrence relation that could match this graphical representation is

A. $A_0 = 200\,000, A_{n+1} = 1.015A_n - 2500$
B. $A_0 = 200\,000, A_{n+1} = 1.025A_n - 5000$
C. $A_0 = 200\,000, A_{n+1} = 1.03A_n - 5500$
D. $A_0 = 200\,000, A_{n+1} = 1.04A_n - 6000$
E. $A_0 = 200\,000, A_{n+1} = 1.05A_n - 8000$

Source: *VCE 2016, Further Mathematics Exam 1, Section A, Q.18; © VCAA*

Question 2 (1 mark)

The value of an annuity, V_n, after n monthly payments of \$555 have been made, can be determined using the recurrence relation

$V_0 = 100\,000, \qquad V_n + 1 = 1.0025\,V_n - 555$

The value of the annuity after five payments have been made is closest to

A. \$97 225
B. \$98 158
C. \$98 467
D. \$98 775
E. \$110 224

© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics Exam 1, Section A, Q.24; © VCAA*

Question 3 (1 mark)

Mai invests in an annuity that earns interest at the rate of 5.2% per annum compounding monthly.
Monthly payments are received from the annuity.
The balance of the annuity will be $130 784.93 after five years.
The balance of the annuity will be $66 992.27 after 10 years.
The monthly payment that Mai receives from the annuity is closest to

A. $1270
B. $1400
C. $1500
D. $2480
E. $3460

Source: *VCE 2018, Further Mathematics Exam 1, Section A, Q.17; © VCAA*

Question 4 (1 mark)

The value of an annuity investment, in dollars, after n years, V_n, can be modelled by the recurrence relation shown below.

$V_0 = 46\,000, \qquad V_{n+1} = 1.0034V_n + 500$

What is the value of the regular payment added to the principal of this annuity investment?

A. $34.00
B. $156.40
C. $466.00
D. $500.00
E. $656.40

Source: *VCE 2018, Further Mathematics Exam 1, Section A, Q.18; © VCAA*

Question 5 (1 mark)

The value of an annuity investment, in dollars, after n years, V_n, can be modelled by the recurrence relation shown below.

$V_0 = 46\,000, \qquad V_{n+1} = 1.0034V_n + 500$

Between the second and third years, the increase in the value of this investment is closest to

A. $656
B. $658
C. $661
D. $1315
E. $1975

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)

Using the annuities recurrence relation of $V_{n+1} = V_n\left(1 + \frac{r}{100}\right) - d$, V_0 = Principal and the information below:

Principal (V_0) = \$25 000, Interest = 6.3% per annum, d = \$450, Compounding periods = monthly, the value of V_3 is

A. \$23 714.92
B. \$24 038.72
C. \$24 360.83
D. \$24 681.00
E. \$25 000.00

Question 7 (1 mark)

Using the annuities recurrence relation of $V_{n+1} = V_n\left(1 + \frac{r}{100}\right) - d$, V_0 = Principal and the information below:

A_3 = \$150 000, Interest = 6.2% per annum, d = \$1100, Compounding periods = monthly.

The value of V_2 is

A. \$13 977.50
B. \$15 000.00
C. \$150 323.33
D. \$150 645.00
E. \$14 528.00

Question 8 (1 mark)

Interest is paid monthly into an account at a rate of 3.5% per annum. Each month, immediately after the interest is paid, the account is debited \$10 in fees. No other money is taken from the account. The initial amount of money in the account is \$15 500.

After all the interest has been paid and fees debited, the balance in the account at the end of **two** months is

A. \$15 555.20
B. \$15 610.58
C. \$15 500.00
D. \$15 535.21
E. \$15 570.52

© John Wiley & Sons Australia, Ltd

Topic	7	Financial mathematics extension
Subtopic	7.5	Perpetuities

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2021, Further Mathematics Exam 2, Section A, Core, Q.6; © VCAA*

Question 1 (3 marks)

Sienna invests $420 000 in a perpetuity from which she will receive a regular monthly payment of $1890. The perpetuity earns interest at the rate of 5.4% per annum.

a. Determine the total amount, in dollars, that Sienna will receive after one year of monthly payments. **(1 mark)**

b. Write down the value of the perpetuity after Sienna has received one year of monthly payments. **(1 mark)**

c. Let S_n be the value of Sienna's perpetuity after n months.
Complete the recurrence relation, in terms of S_0, S_{n+1} and S_n, that would model the value of this perpetuity over time. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2017, Further Mathematics Exam 2, Section A, Core, Q.7; © VCAA*

Question 2 (3 marks)

Alex sold his mechanics' business for $360 000 and invested this amount in a perpetuity.
The perpetuity earns interest at the rate of 5.2% per annum.
Interest is calculated and paid monthly.

a. What monthly payment will Alex receive from this investment? **(1 mark)**

b. Later, Alex converts the perpetuity to an annuity investment.
This annuity investment earns interest at the rate of 3.8% per annum, compounding monthly.
For the first four years Alex makes a further payment each month of $500to his investment.
This monthly payment is made immediately after the interest is added.
After four years of these regular monthly payments, Alex increases the monthly payment.
This new monthly payment gives Alex a balance of $500 000 in his annuity after a further two years.
What is the value of Alex's new monthly payment?
Round your answer to the nearest cent. **(2 marks)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2015, Further Mathematics Exam 2, Module 4, Q.3; © VCAA*

Question 3 (2 marks)

Jane and Michael decide to set up an annual music scholarship.
To fund the scholarship, they invest in a perpetuity that pays interest at a rate of 3.68% per annum.
The interest from this perpetuity is used to provide an annual $460 scholarship.

a. Determine the minimum amount they must invest in the perpetuity to fund the scholarship. **(1 mark)**

b. For how many years will they be able to provide the scholarship? **(1 mark)**

Source: *VCE 2018, Further Mathematics Exam 1, Section A, Q.21; © VCAA*

Question 4 (1 mark)

Which one of the following recurrence relations could be used to model the value of a perpetuity investment, P_n, after n months?

A. $P_0 = 120\,000, \quad P_{n+1} = 1.0029 \times P_n - 356$
B. $P_0 = 180\,000, \quad P_{n+1} = 1.0047 \times P_n - 846$
C. $P_0 = 210\,000, \quad P_{n+1} = 1.0071 \times P_n - 1534$
D. $P_0 = 240\,000, \quad P_{n+1} = 0.0047 \times P_n - 2232$
E. $P_0 = 250\,000, \quad P_{n+1} = 0.0085 \times P_n - 2125$

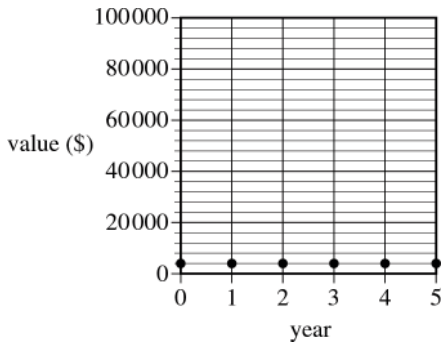
© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics Exam 1, Section A, Q.21; © VCAA*

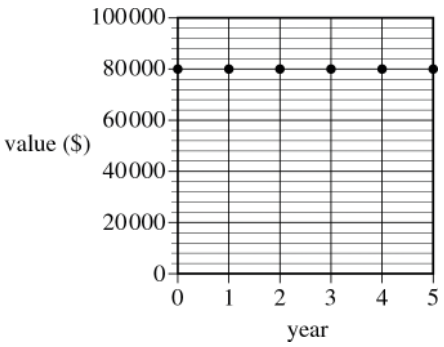
Question 5 (1 mark)

Juanita invests $80 000 in a perpetuity that will provide $4000 per year to fund a scholarship at a university. The graph that shows the value of this perpetuity over a period of five years is

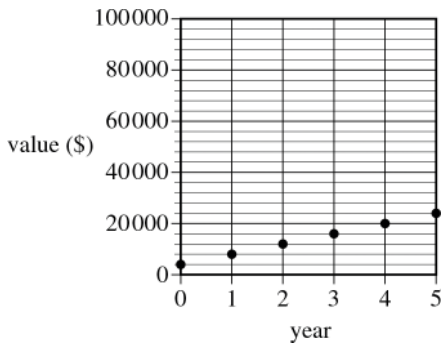
A.



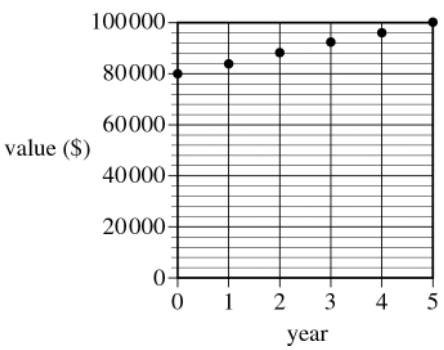
B.



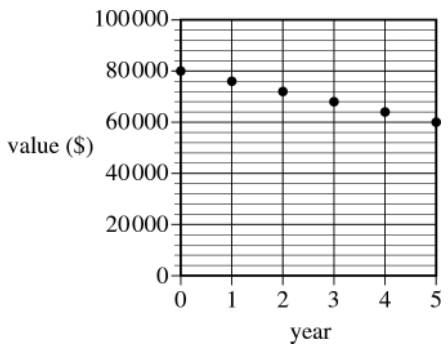
C.



D.



E.



© John Wiley & Sons Australia, Ltd

Source: *VCE 2014, Further Mathematics Exam 2, Module 4, Q.2; © VCAA*

Question 6 (4 marks)

A sponsor of the cricket club has invested $20 000 in a perpetuity.
The annual interest from this perpetuity is $750.
The interest from the perpetuity is given to the best player in the club every year, for a period of 10 years.

a. What is the annual rate of interest for this perpetuity investment? **(1 mark)**

b. After 10 years, how much money is still invested in the perpetuity? **(1 mark)**

c. The average rate of inflation over the next 10 years is expected to be 3% per annum.

i. Michael was the best player in 2014 and he considered purchasing cricket equipment that was valued at $750.
What is the expected price of this cricket equipment in 250? **(1 mark)**

ii. What is the 2014 value of cricket equipment that could be bought for $750 in 2024? **(1 mark)**

Source: *VCE 2013, Further Mathematics Exam 1, Section B, Module 4, Q.5; © VCAA*

Question 7 (1 mark)

$100 000 is invested in a perpetuity at an interest rate of 6% per annum.
After 10 quarterly payments have been made, the amount of money that remains invested in the perpetuity is

A. $15 000
B. $40 000
C. $85 000
D. $94 000
E. $100 000

© John Wiley & Sons Australia, Ltd

Question 8 (1 mark)
An ex-student would like to provide an annual trophy to their former school by setting up an account which will pay for the trophy each year.
If the annual interest is 3%, what is the minimum amount that the ex-student should invest so that a $100 trophy can be bought in perpetuity?
A. $103.00
B. $1000.00
C. $1300.00
D. $2222.22
E. $3333.33

Question 9 (1 mark)
A grandmother sets up an account for her first granddaughter so that she will receive a $200 birthday present each year. If the grandmother invests $4000, what is the minimum interest required so that the granddaughter's present continues for perpetuity?
A. 5%
B. 7.5%
C. 10%
D. 12.5%
E. 15%

Question 10 (1 mark)
A benefactor left an amount of $115 000 to the local bowls club. The club's committee decides to invest the money at a rate of 6% p.a.
What is the maximum amount the bowls club could remove annually if the total is to remain at $115 000 for perpetuity?
A. $6000.00
B. $6509.43
C. $6900.00
D. $7124.37
E. $8315.07

© John Wiley & Sons Australia, Ltd

Topic	7	Financial mathematics extension
Subtopic	7.6	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2021, Further Mathematics Exam 1, Section A, Core, Q.18; © VCAA*

Question 1 (1 mark)

Deepa invests $500 000 in an annuity that provides an annual payment of $44 970.55
Interest is calculated annually.
The first five lines of the amortisation table are shown below.

Payment number	Payment ($)	Interest ($) P	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	500 000.00
1	44 970.55	20 000.00	24 970.55	475 029.45
2	44 970.55	19 001.18	25 969.37	449 060.08
3	44 970.55	17 962.40		422 051.93
4	44 970.55	16 882.08	28 088.47	393 963.46

The principal reduction associated with payment number 3 is

A. $17 962.40
B. $25 969.37
C. $27 008.15
D. $28 088.47
E. $44 970.55

© John Wiley & Sons Australia, Ltd

Source: *VCE 2021, Further Mathematics Exam 1, Section A, Core, Q.19; © VCAA*

Question 2 (1 mark)

Deepa invests $500 000 in an annuity that provides an annual payment of $44 970.55
Interest is calculated annually.
The first five lines of the amortisation table are shown below.

Payment number	Payment ($)	Interest ($) P	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	500 000.00
1	44 970.55	20 000.00	24 970.55	475 029.45
2	44 970.55	19 001.18	25 969.37	449 060.08
3	44 970.55	17 962.40		422 051.93
4	44 970.55	16 882.08	28 088.47	393 963.46

The number of years, in total, for which Deepa will receive the regular payment of $44 970.55 is closest to

A. 12
B. 15
C. 16
D. 18
E. 20

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 2, Section A, Q.8; © VCAA*

Question 3 (3 marks)

Samuel has a reducing balance loan.

The first five lines of the amortisation table for Samuel's loan are shown below.

Payment number	**Payment** ($)	**Interest** ($)	**Principal reduction** ($)	**Balance** ($)
0	0.00	0.00	0.00	320 000.00
1	1600.00	960.00	640.00	319 360.00
2	1600.00	958.08	641.92	318 718.08
3	1600.00	956.15		318 074.23
4	1600.00			

Interest is calculated monthly and Samuel makes monthly payments of $1600.

Interest is charged on this loan at the rate of 3.6% per annum.

a. Using the values in the amortisation table

i. calculate the principal reduction associated with payment number 3 **(1 mark)**

ii. calculate the balance of the loan after payment number 4 is made.
Round your answer to the nearest cent. **(1 mark)**

b. Let S_n be the balance of Samuel's loan after n months.
Write down a recurrence relation, in terms of S_0, S_{n+1} and S_n, that could be used to model the month-to-month balance of the loan. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2018, Further Mathematics Exam 1, Section A, Q.23; © VCAA*

Question 4 (1 mark)

Five lines of an amortisation table for a reducing balance loan with monthly repayments are shown below.

Repayment number	Repayment	Interest	Principal reduction	Balance of loan
25	$2200.00	$972.24	$1227.76	$230 256.78
26	$2200.00	$967.08	$1232.92	$229 023.86
27	$2200.00	$961.90	$1238.10	$227 785.76
28	$2200.00	$1002.26	$1197.74	$226 588.02
29	$2200.00	$996.99	$1203.01	$225 385.01

The interest rate for this loan changed immediately before repayment number 28
This change in interest rate is best described as

A. an increase of 0.24% per annum.
B. a decrease of 0.024% per annum.
C. an increase of 0.024% per annum.
D. a decrease of 0.0024% per annum.
E. an increase of 0.00024% per annum.

Source: *VCE 2018, Further Mathematics Exam 1, Section A, Q.19; © VCAA*

Question 5 (1 mark)

Daniel borrows $5000, which he intends to repay fully in a lump sum after one year.
The annual interest rate and compounding period for five different compound interest loans are given below:

- Loan I – 12.6% per annum, compounding weekly
- Loan II – 12.8% per annum, compounding weekly
- Loan III – 12.9% per annum, compounding weekly
- Loan IV – 12.7% per annum, compounding quarterly
- Loan V – 13.2% per annum, compounding quarterly

When fully repaid, the loan that will cost Daniel the least amount of money is

A. Loan I.
B. Loan II.
C. Loan III.
D. Loan IV.
E. Loan V.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 2, Section A, Q.10; © VCAA*

Question 6 (3 marks)

Samuel now invests $500 000 in an annuity from which he receives a regular monthly payment. The balance of the annuity, in dollars, after n months, A_n, can be modelled by a recurrence relation of the form

$A_0 = 500\,000, \qquad A_{n+1} = kA_n - 2000$

a. Calculate the balance of this annuity after two months if $k = 1.0024$ **(1 mark)**

b. Calculate the annual compound interest rate percentage for this annuity if $k = 1.0024$ **(1 mark)**

c. For what value of k would this investment act as a simple perpetuity? **(1 mark)**

Source: *VCE 2020, Further Mathematics Exam 2, Section A, Q.11; © VCAA*

Question 7 (2 marks)

Later, Samuel took out a new reducing balance loan.
The interest rate for this loan was 4.1% per annum, compounding monthly.
The balance of the loan after four years of monthly repayments was $329 587.25.
The balance of the loan after seven years of monthly repayments was $280 875.15.
Samuel will continue to make the same monthly repayment.
To ensure the loan is fully repaid, to the nearest cent, the required final repayment will be lower.
In the first seven years, Samuel made 84 monthly repayments.
From this point on, how many more monthly repayments will Samuel make to fully repay the loan?

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 2, Section A, Q.8; © VCAA*

Question 8 (4 marks)

Phil invests $200 000 in an annuity from which he receives a regular monthly payment.
The balance of the annuity, in dollars, after n months, A_n, can be modelled by the recurrence relation
$A_0 = 200\,000, \quad A_{n+1} = 1.0035A_n - 3700$

a. What monthly payment does Phil receive? **(1 mark)**

b. Show that the annual percentage compound interest rate for this annuity is 4.2%. **(1 mark)**

c. At some point in the future, the annuity will have a balance that is lower than the monthly payment amount.
What is the balance of the annuity when it first falls below the monthly payment amount?
Round your answer to the nearest cent. **(1 mark)**

d. If the payment received each month by Phil had been a different amount, the investment would act as a simple perpetuity.
What monthly payment could Phil have received from this perpetuity? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 2, Section A, Q.9; © VCAA*

Question 9 (4 marks)

Phil would like to purchase a block of land.
He will borrow $350 000 to make this purchase.
Interest on this loan will be charged at the rate of 4.9% per annum, compounding fortnightly.
After three years of equal fortnightly repayments, the balance of Phil's loan will be $262 332.33

a. What is the value of each fortnightly repayment Phil will make?
Round your answer to the nearest cent. **(1 mark)**

b. What is the total interest Phil will have paid after three years?
Round your answer to the nearest cent. **(1 mark)**

c. Over the next four years of his loan, Phil will make monthly repayments of $3517.28 and will be charged interest at the rate of 4.8% per annum, compounding monthly.
Let B_n be the balance of the loan n months after these changes apply.
Write down a recurrence relation, in terms of B_0, B_{n+1} and B_n, that could be used to model the balance of the loan over these four years. **(2 marks)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2018, Further Mathematics Exam 2, Section A, Q.4; © VCAA*

Question 10 (5 marks)

Julie deposits some money into a savings account that will pay compound interest every month.
The balance of Julie's account, in dollars, after n months, V_n, can be modelled by the recurrence relation shown below.

$V_0 = 12\,000, \quad V_{n+1} = 1.0062\,V_n$

a. How many dollars does Julie initially invest? **(1 mark)**

b. Recursion can be used to calculate the balance of the account after one month.

i. Write down a calculation to show that the balance in the account after one month, V_1, is \$12 074.40 **(1 mark)**

ii. After how many months will the balance of Julie's account first exceed \$12 300? **(1 mark)**

c. A rule of the form $V_n = a \times b^n$ can be used to determine the balance of Julie's account after n months.

i. Complete this rule for Julie's investment after n months by writing the appropriate numbers in the spaces provided below.
balance = _____ × _____^{n} **(1 mark)**

ii. What would be the value of n if Julie wanted to determine the value of her investment after three years? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2018, Further Mathematics Exam 2, Section A, Q.6; © VCAA*

Question 11 (4 marks)

Julie has retired from work and has received a superannuation payment of $492 800.
She has two options for investing her money.

Option 1

Julie could invest the $492 800 in a perpetuity. She would then receive $887.04 each fortnight for the rest of her life.

a. At what annual percentage rate is interest earned by this perpetuity? **(1 mark)**

b. Option 2

Julie could invest the $492 800 in an annuity, instead of a perpetuity.
The annuity earns interest at the rate of 4.32% per annum, compounding monthly.
The balance of Julie's annuity at the end of the first year of investment would be $480 242.25

i. What monthly payment, in dollars, would Julie receive? **(1 mark)**

ii. How much interest would Julie's annuity earn in the second year of investment?
Round your answer to the nearest cent. **(2 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2017, Further Mathematics Exam 1, Section A, Core, Q.23; © VCAA*

Question 12 (1 mark)

Four lines of an amortisation table for an annuity investment are shown below.
The interest rate for this investment remains constant, but the payment value may vary.

Payment number	Payment	Interest	Principal addition	Balance of investment
17	100.00	27.40	127.40	6977.50
18	100.00	27.91	127.91	7105.41
19	100.00	28.42	128.42	7233.83
20	100.00			7500.00

The balance of the investment after payment number 20 is $7500.
The value of payment number 20 is closest to

A. $29
B. $100
C. $135
D. $237
E. $295

Source: *VCE 2016, Further Mathematics Exam 2, Core, Q.6; © VCAA*

Question 13 (3 marks)

Ken's first caravan had a purchase price of $38 000.
After eight years, the value of the caravan was $16 000.

a. Show that the average depreciation in the value of the caravan per year was $2750. **(1 mark)**

b. Let C_n be the value of the caravan n years after it was purchased.
Assume that the value of the caravan has been depreciated using the flat rate method of depreciation.
Write down a recurrence relation, in terms of C_{n+1} and C_n, that models the value of the caravan. **(1 mark)**

c. The caravan has travelled an average of 5000 km in each of the eight years since it was purchased.
Assume that the value of the caravan has been depreciated using the unit cost method of depreciation.
By how much is the value of the caravan reduced per kilometre travelled? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2015, Further Mathematics Exam 1, Section B, Module 4, Q.5; © VCAA*

Question 14 (1 mark)

The purchase price of a car is $20 000.
A deposit of $5000 is paid.
The balance will be repaid with 60 monthly repayments of $400.
The total amount of interest charged is

A. $1000
B. $4000
C. $9000
D. $19 000
E. $24 000

Source: *VCE 2015, Further Mathematics Exam 2, Module 4, Q.4; © VCAA*

Question 15 (3 marks)

As their business grows, Jane and Michael decide to invest some of their earnings.
They each choose a different investment strategy.
Jane opens an account with Red Bank, with an initial deposit of $4000.
Interest is calculated at a rate of 3.6% per annum, compounding monthly.

a. Determine the amount in Jane's account at the end of six months.
Write your answer correct to the nearest cent. **(1 mark)**

b. Michael decides to open an account with Blue Bank, with an initial deposit of $2000.
At the end of each quarter, he adds an additional $200 to his account.
Interest is compounded at the end of each quarter.
The equation below can be used to determine the balance of Michael's account at the end of the first quarter.
account balance $= 2000 \times (1 + 0.008) + 200$
Show that the annual compounding rate of interest is 3.2%. **(1 mark)**

c. Determine the amount in Michael's account, after the $200 has been added, at the end of five years.
Write your answer correct to the nearest cent. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 16 (1 mark)

Using the annuities investment recurrence relation of $V_{n+1} = V_n\left(1 + \frac{r}{100}\right) + d, \quad V_0 = \text{Initial balance},$ and the following information:

$V_3 = \$28\,000$, Interest $= 7.5\%$ per annum, $d = \$1100$, Compounding periods $=$ monthly, the value of V_2 is

A. \$26 732.92
B. \$29 275.00
C. \$25 023.26
D. \$31 200.00
E. \$28 740.14

Question 17 (1 mark)

Using the annuities investment recurrence relation of $V_{n+1} = V_n\left(1 + \frac{r}{100}\right) + d, \quad V_0 = P$ and the following information:

Initial balance $(P) = \$220\,000$, Interest $= 5.2\%$ per annum, $d = \$450$, Compounding periods $=$ weekly, the value of V_3 is

A. \$22 0670.00
B. \$221 340.67
C. \$222 000.00
D. \$222 012.01
E. \$222 684.02

Question 18 (1 mark)

Peter has an investment account of \$80 000 that includes monthly contributions of \$350, earning an interest rate of 7.7% p.a., debited quarterly.

How much will be in Peter's account in 10 years' time?

A. \$229 128.57
B. \$231 274.23
C. \$233 826.43
D. \$235 327.34
E. \$234 321.38

© John Wiley & Sons Australia, Ltd

Question 19 (1 mark)
Sienna has a loan of $125 000 that includes monthly repayments of $1264.41, with an interest rate of 3.2% p.a., debited monthly.
How much of the loan will still be outstanding in 15 years' time?
A. $35 234.74
B. $36 420.78
C. $37 412.09
D. $38 675.63
E. $39 744.32

Question 20 (1 mark)
Voula has an investment of $8000 that includes monthly contributions of $250 and interest payments.
If the amount in the account in 9 years' time is $50 593.31, what was the per annum interest rate closest to?
A. 5.1%
B. 5.4%
C. 5.8%
D. 6.4%
E. 7.3%

© John Wiley & Sons Australia, Ltd

Answers and marking guide

7.2 Reducing balance loans modelled using recurrence relations

Question 1

Use the initial value of $300 000 to calculate the interest of $900.

$$\frac{3.6}{100 \times n} = 300\,000 = 900$$

$$\frac{3.6}{100 \times n} = 0.003$$

$$3.6 = 0.3n$$

$$n = 12$$

Question 2

$$V_1 = 1.003 \times (26\,000) - 400$$
$$= 25\,678$$
$$V_2 = 1.003 \times (25\,678) - 400$$
$$= 25\,355.034$$
$$V_3 = 1.003 \times (25\,355.034) - 400$$
$$= 25\,031.0991$$
$$V_4 = 1.003 \times (25\,031.0991) - 400$$
$$= 24\,706.1924$$
$$V_5 = 1.003 \times (24\,706.1924) - 400$$
$$= 24\,380.3110$$

Question 3

Payment number,*n*	**Payment**($)	**Interest**($)	**Principal reduction**($)	**Balance of loan**($)
2	1500	$I = 249\,500 \times \frac{4.8}{100 \times 12}$ $= \$998$	$1500 - 998 = \$502$	$248 998

Question 4

Joseph's final payment will be the $995.49 plus any interest accumulated for the month.

$$\text{interest} = \frac{7.5}{12 \times 100} \times 995.49 = 6.22$$

$$\$995.49 + \$6.22 = \$1001.71$$

Question 5

Balance = $143 585.33

After next month:

$$V_n = V_0 R^n - \text{repayment}$$
$$= 143\,585.33\left(1 + \frac{I}{100}\right)^1 - 2500$$
$$= \$141\,623.77$$

Amount loan has been reduced by $= 143\,585.33 - 141\,623.77$

$= \$1961.56$ [1 mark]

© John Wiley & Sons Australia, Ltd

Percentage of next monthly repayment $= \frac{1961.56}{2500} \times 100$
$= 78.46\%$
$= 78\%$ **[1 mark]**

***VCAA Assessment Report* note:**
Many students made poor attempts at this question, gave a single number as the answer or did not attempt it at all. A common incorrect answer was 22%, which was the interest component of the next repayment. A method mark was available for an incorrect answer only if the working out of a significant step towards a solution could be followed, but this was rare.

Question 6
Interest rate per month: $6.3\% \div 12 = 0.525\%$
$70\,000 \times 0.525\% = 70\,000 \times 0.005\,25 = \$367.50$
Repayment interest $= 2500 - 367.50 = \$2132.50$
Principal outstanding $= 70\,000 - 2132.50 = \$67\,867.50$

Question 7

End of month	Interest	Repayment	Balance
1	200	225	24 975
2	199.80	225	24 949.80

Question 8

End of month	Interest	Repayment	Balance
1	525	725	74 800
2	523.6	725	74 598.60

7.3 Solving reducing balance loan problems with technology

Question 1
Calculate with financial solver on a CAS calculator using
$N = 60,\ I = 3.72,\ PV = 175\,260.56$, $PMT = -3200$ and $Ppy = 12$. The final value will be $-368.116\ldots$
Since the final value is negative, Adam must still pay this amount.
Therefore, Adam's final payment will be $3200 + 368.116 \approx \$3568.12$

Question 2
a. Use Finance Solver on CAS:
N: 12
I(%):6.9
PV: 70 000
Pmt: −800
PpY/Cpy: 12
Therefore, Ken will owe $65 076.22 after 12 months. **[1 mark]**
***VCAA Assessment Report* note:**
Rather than use a financial solver to answer the question above, a number of students adopted a formulaic approach, almost always unsuccessfully, based on the compound interest formula.

b. Total interest after 12 payments $= 800 \times 12 - (70\,000 - 65\,076.22) = \$4676.22$ **[1 mark]**
***VCAA Assessment Report* note:**
A common incorrect answer was $4923.78, which is the reduction in the principal over the year. This failed to take into account the $9600 total of repayments made in the year.

© John Wiley & Sons Australia, Ltd

Question 3
Via TVM solver:
N = 240
I = 6.95
PV = 90 000
Pmt = ?
FV = 0
PpY = 12
CpY = 12
Therefore, monthly payment = \$695.09 ≈ \$695.

Question 4
Use Finance Solver on CAS:
N: 10 years 12 months = 120
I (%) :4.35 p.a.
PV: 245 000
Pmt: −1800
FV: *−108219.1611158*
PpY/CpY: 12
After 10 years, Xavier still owes \$108 219.16
N: 5 years 12 months = 60
I* (%) :*4.142 758 985 ... p.a.
PV: 108 219.16
Pmt: −2000
FV: 0
PpY/CpY: 12
To be paid off in 5 more years, the annual interest rate is closest to 4.1%.
***VCAA Assessment Report* note:**
Understanding of the sign convention for TVM use is very important, as is the careful tracking of values used in subsequent calculations.

Question 5
Using the finance solver on the CAS:
N = unknown
I (%) = 6.18
PV = 160 000
Pmt = −1950
FV = 0
PpY = 12
CpY = 12
Therefore N = 106.906 93
The final payment will therefore be $1950 \times 0.906\,93$ = \$1768.51. So option D is incorrect.

Question 6
Using TVM solver:
N = 48
I = 4.75
PV = 35 000
Pmt = −802
FV = ?

© John Wiley & Sons Australia, Ltd

PpY = 12
CpY = 12
Therefore FV = −3.57.
If 48 payments of $802 were made, the balance would still be $3.57 owing.
Therefore the final payment must be 802 + 3.57 = $805.57
***VCAA Assessment Report* note:**
This question asked students to calculate the amount of the final payment for a reducing balance loan. this was a two-step problem.
Step 1 involved calculating the future value of the loan after 47 payments.
Using a financial solver, this amount is found to be $802.39...
Step 2 required adding a month's interest to this amount to find the final payment.

$$\text{final payment} = 802.3911... + 802.3911... \times \left(\frac{4.75}{12 \times 100}\right) = \$805.57 \text{ to the nearest cent (option E)}$$

While most students could correctly use their financial solver to find the amount still owed after the second last payment had been made ($802.39), they apparently failed to realise that this amount would attract interest during the last month of the loan.

Question 7
Using Finance solver in CAS, first find the interest rate per annum.
Enter N = 1, PV = 300 000, Pmt = 2500, FV = −299 000, PpY = 12, CpY = 12.
Solving for I gives I = 6%.
Check whether option A is correct:
Enter N = 2, PV = 300 000, Pmt = 2500, PpY = 12, CpY = 12
Solving for FV gives FV = −297 995.
Therefore, option A ('After 2 months, $297 995 is still owing on the loan') is correct.
***VCAA Assessment Report* note:**
In this question, students needed to test the truth of five statements relating to the repayment of a reducing balance loan. Many students struggled with this question. The key to answering this question was to use a TVM to work out the interest rate applying to the loan. This knowledge, with the aid of a TVM solver, could then be used to test the truth of each of the five statements.

Question 8
First, calculate monthly repayments using Finance solver on CAS.
N = 4 × 4 = 16; I = 12; PV = 25 000; FV = 0; P/Y = 4; C/Y = 4
Solving for Pmt gives −1990.27. Therefore, the quarterly payment is .27. [1 mark]
Next, calculate the amount owed after 2 years.
N = 2 × 4 = 8; I = 12; PV = 25 000; Pmt = −1990.27; P/Y = 4 and C/Y = 4.
Solving for FV gives −13 971.09.
Therefore, after 2 years the amount still owed is $13 971.09.
Hence, the amount repaid = $25 000 − $13 971.09
= $11 028.91
= $11 029 (to the nearest dollar) [1 mark]
***VCAA Assessment Report* note:**
Students needed to read this question carefully to identify the three steps needed for the solution. Writing and labelling such steps can be very helpful in organising thoughts, but very few students showed any TVM input or other working.

© John Wiley & Sons Australia, Ltd

7.4 Annuities

Question 1

The value of A is not changing, so the recurrence relation is subtracting the same value as the interest earned.

$0.025 \times 200\,000 = 5000$

Question 2

$V_1 = 1.0025\,(100\,000) - 555 = 99\,695$

$V_2 = 1.0025\,(99\,695) - 555 = 99\,389.2375$

$V_3 = 1.0025\,(99\,389.2375) - 555 = 99\,082.71\,059$

$V_4 = 1.0025\,(99\,082.710\,59) - 555 = 98\,775.417\,37$

$V_5 = 1.0025\,(98\,775.417\,37) - 555 = 98\,467.3559$

Question 3

Use Finance Solver on CAS:

N: 5 years $\times$12 months = 60

I(%):5.2 p.a.

PV: $-130\,784.93$

FV: 66 992.27

PpY/CpY: 12

Therefore, the payment (Pmt) each month is \$1500.

***VCAA Assessment Report* note:**

A common error was to choose option E, which corresponded to both the PV and FV being incorrectly allocated the same sign.

Question 4

The regular payment is d in $V_{n+1} = V_n\left(1 + \frac{r}{100}\right) + d$, so the payment in

$V_{n+1} = 1.0034V_n + 500$

Question 5

The difference in the value of the annuity investment between $n = 2$ and $n = 3$ is equal to

$V_3 - V_2$

$V_0 = 46\,000$

$V_1 = 1.0034V_0 + 500 = 1.0034 \times 46\,000 + 500 = \$46\,656.4$

$V_2 = 1.0034V_1 + 500 = 1.0034 \times 46\,656.4 + 500 = \$47\,315.03$

$V_3 = 1.0034V_2 + 500 = 1.0034 \times 47\,315.03 + 500 = \$47\,975.902$

Difference between $n = 2$ and $n = 3$: $V_3 - V_2 = 47\,975.902 - 47\,315.03 \approx \$661$

Question 6

Substituting the relevant values:

$$V_1 = V_0\left(1 + \frac{\frac{6.3}{12}}{100}\right) - 450$$

$$= 25\,000 \times \left(1 + \frac{\frac{6.3}{12}}{100}\right) - 450$$

$$= \$24\,681.25$$

© John Wiley & Sons Australia, Ltd

$$V_2 = V_1\left(1 + \frac{\frac{6.3}{12}}{100}\right) - 450$$

$$= 24\,681.25 \times \left(1 + \frac{\frac{6.3}{12}}{100}\right) - 450$$

$$= \$24\,360.83$$

$$V_3 = V_2\left(1 + \frac{\frac{6.3}{12}}{100}\right) - 450$$

$$= 24\,360.83 \times \left(1 + \frac{\frac{6.3}{12}}{100}\right) - 450$$

$$= \$24\,038.72$$

Question 7

Substituting the relevant values:

$$V_3 = V_2\left(1 + \frac{r}{100}\right) - d \quad , \quad r = \frac{6.2}{12} \approx 0.5167$$

$$150\,000 = V_2\left(1 + \frac{\frac{6.2}{100}}{100}\right) - 1100$$

Then using a CAS to solve for V_2:

$V_2 = \$150\,323.33$

Question 8

$$V_1 = V_0\left(1 + \frac{r}{100}\right) - d \quad , \quad r = \frac{3.5}{12} \approx 0.2917 \quad , \quad V_0 = 15\,500 \quad , \quad d = 10$$

$$V_1 = 15\,500\left(1 + \frac{\frac{3.5}{12}}{100}\right) - 10 = 15\,535.21$$

$$V_2 = 15\,535.21\left(1 + \frac{\frac{3.5}{12}}{100}\right) - 10 = 15\,570.52$$

7.5 Perpetuities

Question 1

a. $\$1890 \times 12 = \$22\quad 680$

b. \$420 000 – the final value of a perpetuity remains the same as the principal value **[1 mark]**

c. Interest rate per month is $\frac{5.4\%}{12} = 0.45\%$

$S_0 = 420\quad 000 \qquad S_{n+1} = 1.0045 \times S_n - 1890$ **[1 mark]**

Question 2

a. 5.2% p.a. is equivalent to $\frac{5.2}{12}\%$ per month.

$\frac{\frac{5.2}{12}}{100} \times 360\,000 = \$1560$ per month. **[1 mark]**

© John Wiley & Sons Australia, Ltd

***VCAA Assessment Report* note:**
Some students gave the annual payment of \$18 720, while others knew the correct method but rounded $\frac{5.2}{1000}$ to two decimal places before multiplying by \$360 000, giving \$1 548.

b. Use Finance Solver on CAS:
N: 4 years 12 months = 48
I(%):3.8 p.a.
PV: −360 000
Pmt: −500
FV:***444872.9444992***
PpY/CpY: 12
After 4 years, Alex's investment grows to \$444 872.94 [1 mark]
N: 2 years 12 months = 24
I3.8
PV: −444 872.94
Pmt: ***−805.65070094875***
FV: 500 000
PpY/CpY: 12
To grow to \$500 000 in a further two years, Alex's new monthly payment will be \$805.65 **[1 mark]**
***VCAA Assessment Report* note:**
Some students entered the \$500 payment as a positive into their finance solver, giving \$393 121.15. **[1 mark]**

Question 3

a. $Q = \frac{P \times r}{100}$

$460 = \frac{P \times 3.68}{100}$ [1 mark]

$P = \$12\,500$

b. A perpetuity is an investment that provides regular payments that continue forever. Therefore, they will be able to provide the scholarship for an infinite number of years. **[1 mark]**
***VCAA Assessment Report* note:**
Many students did not know that perpetuities pay out only the interest earned, while the principal remains unchanged.
The most common incorrect answer was $\frac{12\,500}{460} \approx 27$ years

Question 4

Since it is a perpetuity account, the balance must remain the same after each iteration. Therefore, $d = RV_0$
Options D and E are not perpetuity accounts as they are reduced before each payment (both have a negative interest rate).
Option A: $d = 356$, $RV_0 = 0.0029 \times 120\,000 = 348 \neq 356$
Option B: $d = 846$, $RV_0 = 0.0047 \times 180\,000 = 846 = 846$
Option C: $d = 1534$, $RV_0 = 0.0071 \times 210\,000 = 1491 \neq 1534$
Option B shows the increase in value is equal to the payment out; therefore, it is a perpetuity account.

Question 5

For a perpetuity, funds last for an indefinite period of time as long as the amount paid out is the same as the interest earned on the initial amount. Therefore, the amount in the account does not change each year from the initial \$80 000

© John Wiley & Sons Australia, Ltd

Question 6

a. $d = \frac{V_0 r}{100}$ **[1 mark]**

$750 = \frac{20\,000 \times r}{100}$

$r = 3.75\%$

***VCAA Assessment Report* note:**

A common, unacceptable answer was 0.0375%

b. As only the interest is given away each year, the principal remains unchanged. Therefore the amount still invested = \$20 000. **[1 mark]**

***VCAA Assessment Report* note:**

Many students did not understand what a perpetuity is. A perpetuity balance remains constant since only the interest earned is withdrawn in each compounding period.

c.

i. Cost $= 750 + \frac{3}{100} \times 750$ **[1 mark]**

$= 750\,(1 + 0.03)$

$= 772.50$

***VCAA Assessment Report* note:**

The answer must have been written correct to the nearest cent and so the zero at the second decimal place was required in this question.

ii. Cost in 2014 $= x$

Cost in 2024 $= 750$

$750 = x(1 + 0.03)^{10}$

$x = 558.07$

Therefore the value in 2014 is \$558. **[1 mark]**

Question 7

The amount of money invested in perpetuity always remains the same. In this case, it is \$100 000.

Question 8

$100 = 0.03 \times x$

$x = 3333.33$

Question 9

$200 = \left(1 + \frac{r}{100}\right) \times 4000$

$r = 5$

Question 10

$0.6 \times 115\,000 = \$6900$

7.6 Review

Question 1

\$449 060.08 − \$422 051.93 = \$27 008.15

Question 2

Use Finance Solver on CAS:

N: ??

I(%): 4

PV: −500 000

© John Wiley & Sons Australia, Ltd

Pmt: 44 970.55
FV: 0
PpY: 1
CpY: 1
Therefore, N = 15 years.

Question 3

a. i. Principal reduction \$318 718.08 − \$318 074.23 = \$643.85 **[1 mark]**

ii. Monthly interest $\frac{3.6}{12}\% \times \$318\,074.23 = \$954.22$
Therefore, the balance after payment 4 will be
\$318 074.23 + \$954.22 − \$1600 = \$317 428.45 **[1 mark]**

b. Note that the monthly interest rate will be $\frac{3.6}{12} = 3\%$
$S_0 = 320\,000, \quad S_{n+1} = 1.003 \times S_n - 1600$ [1 mark]

Question 4

Change in rate: $\frac{I_{28}}{V_{27}} - \frac{I_{27}}{V_{26}} = \frac{1002.26}{227\,785.76} - \frac{961.90}{229\,023.86} \approx +0.02\%$ per month
$+0.02 \times 12 = +0.24\%$ per annum.
The interest rate increased by 0.24% per annum.

Question 5

The smallest effective interest rate for the loans will cost Daniel the least amount of money.
Since Loans I, II and III are all compounding weekly, the lowest rate of the three will be Loan I with 12.6%
Loans IV and V are both compounding quarterly, so the lowest rate out of them will be Loan IV with 12.7%

Effective interest rate of Loan I: $r = \left(1 + \frac{i}{n}\right)^n - 1 = \left(1 + \frac{0.126}{52}\right)^{52} - 1 \approx 13.41\%$ annually

Effective interest rate of Loan IV: $r = \left(1 + \frac{i}{n}\right)^n - 1 = \left(1 + \frac{0.127}{4}\right)^4 - 1 \approx 13.32\%$ annually

Since Loan IV has a lower effective interest rate over the year, it will cost Daniel the least amount of money.
***VCAA Assessment Report* note:**
This question required an understanding that both the interest rate and compounding period in combination determine the overall cost of the loan. Students who selected option A seemed to simply select the lowest interest rate without considering the compounding period.

Question 6

a. After one month: $A_1 = 1.0024 \times 500\,000 - 2000 = \$499\,200$
After two months: $A_2 = 1.0024 \times 499\,200 - 2000 = \$498\,398.08$ **[1 mark]**

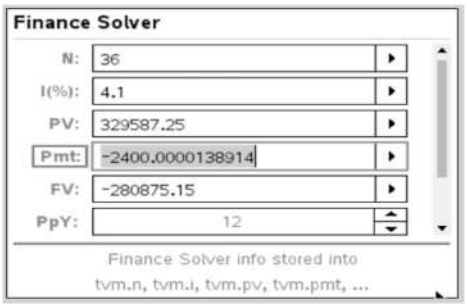
b. $1.0024 = 1 + \frac{r}{1200}$, so $r = 2.88\%$. **[1 mark]**

c. Perpetuity means that the balance will always remain the same, i.e. $A_{n+1} = A_n$
$500\,000 = k \times 500\,000 - 2000$
$k = 1.004$ **[1 mark]**

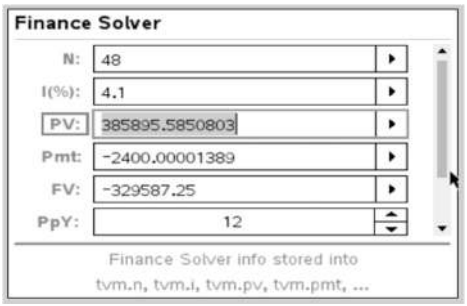
Question 7

From the information given, we know that there are 36 payments (3 years) between the two balances.
Using Finance Solver on the CAS, calculate the monthly payment to be \$2400.

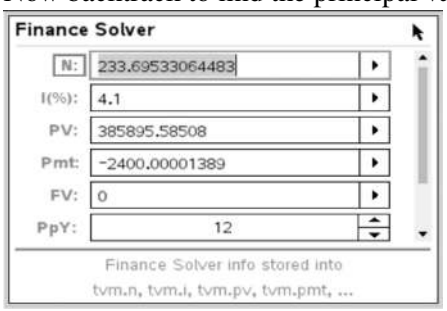
© John Wiley & Sons Australia, Ltd



Now backtrack to find the principal value of the loan to be $385 895.59



Now backtrack to find the principal value of the loan to be $385 895.59



But the question says that the final payment will be lower, so we need 234 payments.
We are told that 84 payments have already been made, so we will need $234 - 84 = 150$ more payments.
Award 1 mark for the number of payments and 1 mark for the correct calculation of the additional payments required.

Question 8

a. $3700 **[1 mark]**
***VCAA Assessment Report* note:**
The question was straightforward but often misunderstood. Some gave the answer to part d. while others gave $3000 after calculating A_1

b. $0.0035 \times 12 = 0.042$, which is 4.2% **[1 mark]**
***VCAA Assessment Report* note:**
Responses needed to show the calculation using basic arithmetic that resulted in an answer of 4.2%, not use the 4.2% and attempt to verify it.

c. Use the finance solver and allow I = 4.2, PV = 200 000, Pmt = −3700, FV = 0 and PpY/CpY = 12 to give N = 60.02495
Now set N = 60 to find the FV = $92.15
After 60 months, the value of the annuity is $92.15. **[1 mark]**

d. $0.35\% \times 200\,000 = \$700$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

Question 9

a. Use the finance solver to allow N = 78, I = 4.9, PV = −350 000, FV = 262 332.33 and PpY/CpY = 26 to find Pmt = $1704.03
Each fortnightly repayment is $1704.03 **[1 mark]**
***VCAA Assessment Report* note:**
$7954.54 was a common incorrect answer obtained by entering the FV in technology as a positive rather than as −262 332.33.
This very large fortnightly repayment should have been a signal that something had been entered incorrectly.

b. $1704.03 \times 78 - (350000 - 262332.33) = \$45\ 246.67$ **[1 mark]**
***VCAA Assessment Report* note:**
Some students followed through with the incorrect response they obtained for part a. but did not show the working required for the consequential mark.

c. $1 + \frac{4.8}{1200} = 1.004$
$B_{n+1} = 1.004\ B_n - 3517.28$ where $B_0 = 262\ 332.33$
Award 1 mark for correct B_0.
Award 1 mark for the correct equation.
The question was done reasonably well, but some responses gave B_0 as 350 000, or R as 1.048, and added rather than subtracted the 3517.28.

Question 10

a. Julie initially invests V_0, which is $12 000. **[1 mark]**

b. i. $V_1 = 1.0062 \times V_0 = 1.0062 \times 12\ 000 = \$12\ 074.40$ **[1 mark]**
***VCAA Examination Report* note:**
This question was answered well, although some students used methods other than simple recursion, which were not acceptab

ii. $V_1 = 1.0062 \times V_0 = 1.0062 \times 12\,000 = \$12\,074.40$
$V_2 = 1.0062 \times V_1 = 1.0062 \times 12\,074.4 \approx \$12\,149.26$
$V_3 = 1.0062 \times V_2 = 1.0062 \times 12\,149.26 \approx \$12\,224.59$
$V_4 = 1.0062 \times V_3 = 1.0062 \times 12\,224.59 \approx \$12\,300.38$
The answer is 4 months. **[1 mark]**

c. i. a is equal to the principle value of $12 000 and b is equal to 1 plus the percentage increase.
The equation is $balance = 12\ 000 \times 1.0062^n$. **[1 mark]**
***VCAA Assessment Report* note:**
A common error was for students to write the two numbers in the incorrect boxes.

ii. Since n is in months, there are $3 \times 12 = 36$ months in 3 years. Therefore, $n = 36$. **[1 mark]**

Question 11

a. Since the account is a perpetuity, the interest earned on the initial amount is equal to the payout. The payout is fortnightly, so there are 26 payments.
$492\ 800 \times R = 887.04 \times 26$

$$R = 0.0468$$

$R = 4.68\%$ per annum **[1 mark]**
***VCAA Examination Report* note:**
A common error was 0.18%, which did not account for the fortnightly payments. Perpetuities was an area that requires improvement for many students.

© John Wiley & Sons Australia, Ltd

b. i. Use the financial solver on your CAS calculator with the inputs $n = 12$, $I = 4.32$, $PV = -492\,800$, $FV = 480\,242.25$, $Ppy = 12$, and solve for Pmt. The payment is equal to \$2800.00. **[1 mark]**

ii. The second year of investment is from $n = 12$ to $n = 24$.
Using the financial solver with inputs
$n = 12$, $I = 4.32$, $PV = -480\,242.25$, $Ppy = 12$ and $Pmt = 2800$, we can solve for FV.
$FV = 467\,131.13$ **[1 mark]**
Interest earned = *payout* − *change in balance* = $2800 \times 12 - (480\,242.25 - 467\,131.13) =$ \$20 488.88**[1 mark]**
***VCAA Examination Report* note:**
Many students did not attempt this question. Many found the future value after two years of \$467 131 but could not proceed further.
A few students tried unsuccessfully to use the annuities formula.

Question 12
Use the first balance and second interest amount to calculate the interest rate per period (x).
x% of 6977.50 = 27.91 Solving for x,
$x = 0.4\%$ per period
The interest earned for payment number 20 is 0.4% of \$7233.83 = \$28.94
The principal addition for payment number 20 is \$7500 − \$7233.83 = \$266.17
So, interest + payment = principal addition

$$\begin{aligned} \text{Payment} + 28.94 &= 266.17 \\ \text{Payment} &= 266.17 - 28.94 \\ &= \$237.23 \end{aligned}$$

***VCAA Assessment Report* note:**
This question required the interpretation of an amortisation table and the calculation of a missing payment value. Many students seemed to assume that the payment value would be constant at \$100 (option B), despite the question stating that the payment may vary.

Question 13

a. Average depreciation per year $= \dfrac{38\,000 - 16\,000}{8} = \$2750$ **[1 mark]**
***VCAA Assessment Report* note:**
Show that' questions give the answer to a basic and appropriate calculation, and students must write that calculation. The given number in a 'show that' question is sometimes needed in a following question. With this number, students can attempt the following question, even if they cannot complete the 'show that' question.
The given number in a 'show that' question is not to be used in a calculation; it must be the result of a calculation. The following calculations use the 2750 to show a different result than what is required and were not acceptable for this question:
- $38\,000 - 8 \times 2750 = 16\,000-$ shows how to find the depreciated value after eight years
- $16\,000 + 8 \times 2750 = 38\,000-$ shows how to find the initial value

b. $C_{n+1} = C_n - 2750$, $C_0 = 38\,000$ **[1 mark]**
***VCAA Assessment Report* note:**
Common errors included:
- failure to include the initial value, C_0
- writing the initial value as C_n, not C_0

© John Wiley & Sons Australia, Ltd

- using different symbols for different parts of the recurrence relation, e.g. $V_0 = 38\,000,\ C_{n+1} = C_n - 2750$
- using a rule for the nth term $C_n = 38\,000 - 2750n$ instead of the recurrence rule.

c. In 8 years, the caravan depreciates by $38\,000 - 16\,000 = 22\,000$.

$\therefore$ depreciation $= \dfrac{22\,000}{5000 \times 8} = \$0.55$ per km travelled. **[1 mark]**

***VCAA Assessment Report* note:**

A very common incorrect answer of $4.40 was found by ignoring the eight years over which the depreciation occurred.

Question 14

Total repaid $= 60 \times \$400 = \$24\,000$

Interest paid $= \$24\,000 - \$15\,000 = \$9000$

Question 15

a. Jane's investment uses compound interest:

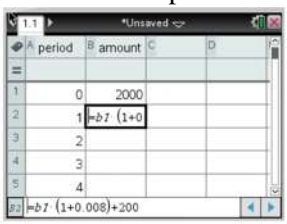
$$V_n = V_0\left(1 + \frac{r}{100}\right)^n$$
$$= 4000\left(1 + \frac{3.6/12}{100}\right)^6 \qquad [1 \text{ mark}]$$
$$= \$4072.54$$

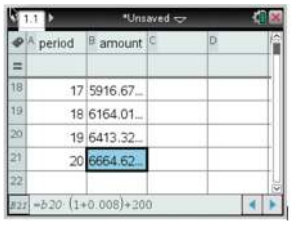
***VCAA Assessment Report* note:**

Answers rounded to $4072.55 or $4072.50 were not accepted.

b. From the equation, we know that $\dfrac{r}{100} = 0.008$ which will be 0.8% per quarter (as the investment compounds quarterly). So the percentage rate per year is $0.8 \times 4 = 3.2\%$. **[1 mark]**

c. Use CAS – a spreadsheet is best.





Therefore, the amount that Michael has in his account at the end of 5 years is $6664.63. **[1 mark]**

***VCAA Assessment Report* note:**

Answers of $6664.60 or $6664.65 were not accepted.

Question 16

Substituting the relevant values:

$$V_3 = V_2\left(1 + \frac{r}{100}\right) + d,\ \ r = \frac{7.5}{12} = 0.625$$
$$28\,000 = V_2\left(1 + \frac{\frac{7.5}{12}}{100}\right) + 1100$$

Using a CAS to solve for V_2,

$V_2 = \$26\,732.92$

Question 17

$V_1 = V_0\left(1 + \frac{r}{100}\right) + d, \quad r = \frac{5.2}{52} = 0.1, \; V_0 = 220\,000, \quad d = 450$

$V_1 = 220\,000\left(1 + \frac{0.1}{100}\right) + 450 = \$220\,670$

$V_2 = 220\,670\left(1 + \frac{0.1}{100}\right) + 450 = \$221\,340.67$

$V_3 = 221\,340.67\left(1 + \frac{0.1}{100}\right) + 450 = \$222\,012.01$

Question 18

$N = 120$
$I\% = 7.7$
$PV = 80000$
$PMT = 350$
$FV = ?$
$P/Y = 12$
$C/Y = 4$
Therefore, $FV = \$234\,321.38$

Question 19

$N = 180$
$I\% = 3.2$
$PV = 125\,000$
$PMT = 1264.41$
$FV = ?$
$P/Y = 12$
$C/Y = 12$
Therefore, $FV = \$39\,744.32$

Question 20

$N = 108$
$1\% = ?$
$PV = 8\,000$
$PMT = 250$
$FV = 50\,593.31$
$P/Y = 12$
$C/Y = 12$
Therefore the per annum interest rate 6.4%

© John Wiley & Sons Australia, Ltd

8 Investigating relationships between two numerical variables

Topic	**8**	**Investigating relationships between two numerical variables**
Subtopic	**8.2**	**Response and explanatory variables**

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2019, Further Mathematics Exam 2, Section A, Q.4; © VCAA*

Question 1 (1 mark)

The relative humidity (%) at 9 am and 3 pm on 14 days in November is shown in the table below.

Relative humidity (%)	
9 am	3 pm
100	87
99	75
95	67
63	57
81	57
94	74
96	71
81	62
73	53
53	54
57	36
77	39
51	30
41	32

Data: Australian Government, Bureau of Meteorology, www.bom.gov.au

A least squares line is to be fitted to the data with the aim of predicting the relative humidity at 3 pm (*humidity* 3 *pm*) from the relative humidity at 9 am (*humidity* 9 *am*).

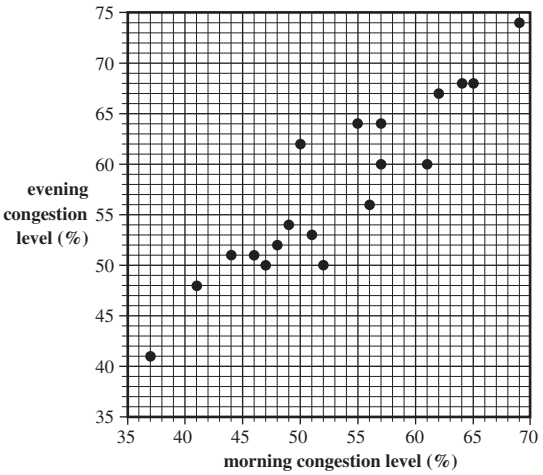
Name the explanatory variable.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2018, Further Mathematics Exam 2, Section A, Q.2b; © VCAA*

Question 2 (1 mark)

The congestion level in a city can also be recorded as the percentage increase in travel time due to traffic congestion in peak periods (compared to non-peak periods). This is called the percentage congestion level. The percentage congestion levels for the morning and evening peak periods for 19 large cities are plotted on the scatterplot below.



A least squares line is to be fitted to the data with the aim of predicting evening congestion level from morning congestion level.

The equation of this line is

$$\textit{evening congestion level} = 8.48 + 0.922 \times \textit{morning congestion level}$$

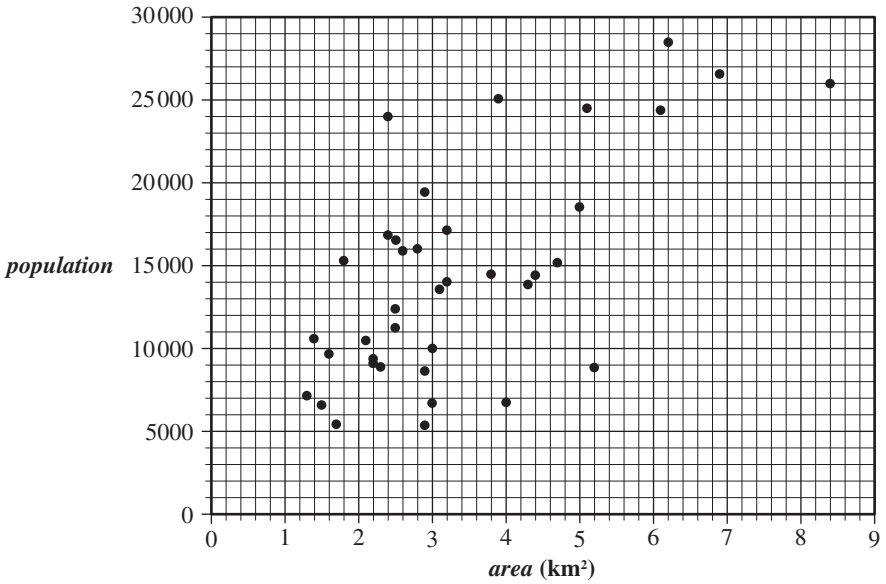
Name the response variable in this equation.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2014, Further Mathematics Exam 2, Core, Q.2a; © VCAA*

Question 3 (1 mark)

The scatterplot below shows the *population* and *area* (in square kilometres) of a sample of inner suburbs of a large city.



The equation of the least squares regression line for the data in the scatterplot is

$population = 5330 + 2680 \times area$

Write down the response (dependent) variable.

Question 4 (1 mark)

A chef is testing a bread recipe by changing the temperature of the oven and then measuring the amount the baked bread has risen.

He plots the data on a scatterplot. The chef should plot

A. The temperature on the *x*-axis because it is the explanatory variable and the amount risen on the *y*-axis because it is the response variable.

B. The temperature on the *y* -axis because it is the explanatory variable and the amount risen on the *x*-axis because it is the response variable.

C. The temperature on the *x*-axis because it is the response variable and the amount risen on the *y* -axis because it is the explanatory variable.

D. The temperature on the *y*-axis because it is the response variable and the amount risen on the *x*-axis because it is the explanatory variable.

E. The temperature and the amount risen on either axis because the variables are related.

© John Wiley & Sons Australia, Ltd

Topic	8	Investigating relationships between two numerical variables
Subtopic	8.3	Scatterplots and basic correlation

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2016, Further Mathematics Exam 1, Section A, Q.12; © VCAA*

Question 1 (1 mark)

There is a strong positive association between a country's Human Development Index and its carbon dioxide emissions.

From this information, it can be concluded that

A. increasing a country's carbon dioxide emissions will increase the Human Development Index of the country.

B. decreasing a country's carbon dioxide emissions will increase the Human Development Index of the country.

C. this association must be a chance occurrence and can be safely ignored.

D. countries that have higher human development indices tend to have higher levels of carbon dioxide emissions.

E. countries that have higher human development indices tend to have lower levels of carbon dioxide emissions.

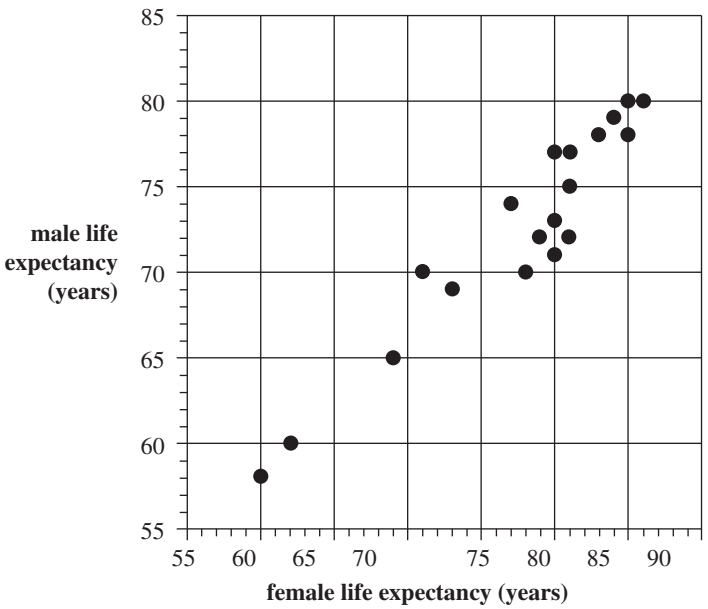
© John Wiley & Sons Australia, Ltd

Source: *VCE 2015, Further Mathematics Exam 2, Core, Q.4a; © VCAA*

Question 2 (1 mark)

The table below shows male life expectancy (*male*) and female life expectancy (*female*) for a number of countries in 2013. A scatterplot has been constructed from this data.

Life expectancy (in years) in2013	
male	***female***
80	85
60	62
73	80
70	71
70	78
78	83
77	80
65	69
74	77
70	78
75	81
58	60
80	86
69	73
79	84
72	81
78	85
72	79
77	81
71	80



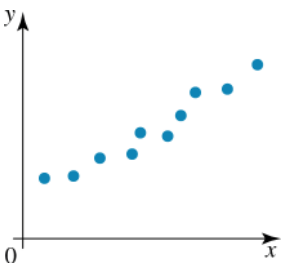
The correlation coefficient, r, is equal to 0.9496.

Use the scatterplot to describe the association between *male* life expectancy and *female* life expectancy in terms of strength, direction and form.

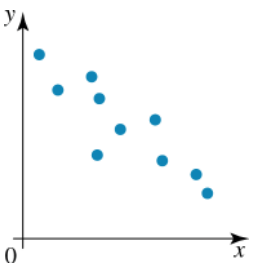
Question 3 (1 mark)

Which of the following scatterplots shows weak negative correlation?

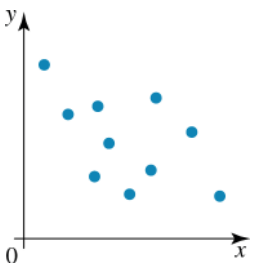
A.



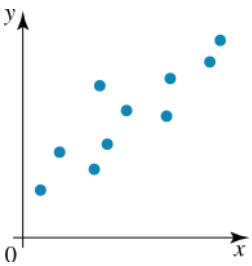
B.



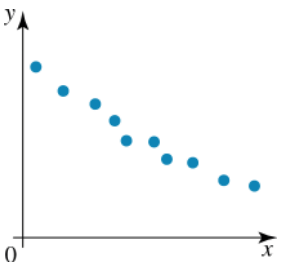
C.



D.

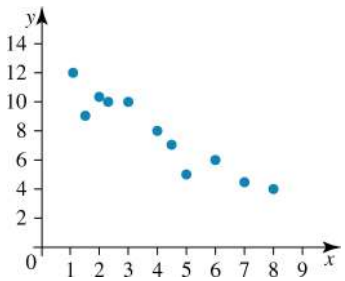


E.



Question 4 (1 mark)

What type of correlation is shown on the graph?



A. Non-linear, strong, negative
B. Linear, strong, positive
C. Linear, strong, negative
D. Non-linear, weak, positive
E. No correlation

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)

Adfair suggested that the more a company spent on advertising, the greater the profit the company would make. To prove this, they collected data on profits and advertising costs and calculated a correlation coefficient of 0.85.

Adfair can conclude that

A. there is a moderate positive correlation between profit and advertising costs, so the lower the amount spent on advertising, the greater the profit.

B. there is a strong negative correlation between profit and advertising costs, so the lower the amount spent on advertising, the greater the profit.

C. there is a strong positive correlation between profit and advertising costs, so the greater the amount spent on advertising, the greater the profit.

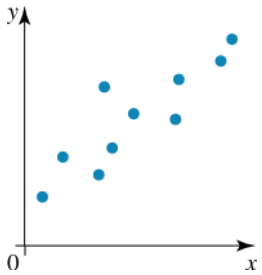
D. there is a strong positive correlation between profit and advertising costs, so the greater the amount spent on advertising, the lower the profit.

E. high advertising costs cause high profits.

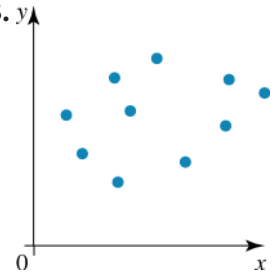
Question 6 (1 mark)

Which of the following scatterplots shows moderate positive correlation?

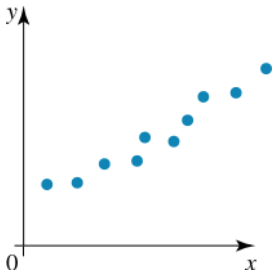
A.



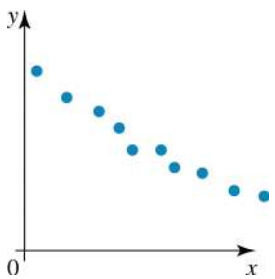
B.



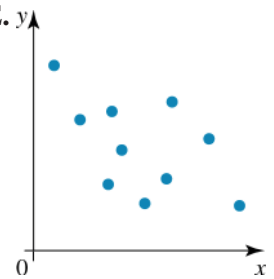
C.



D.

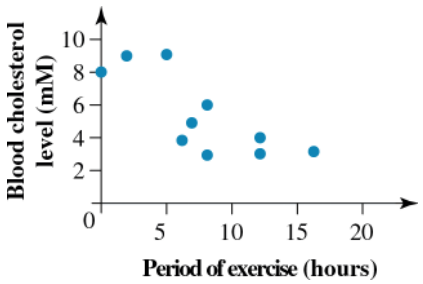


E.



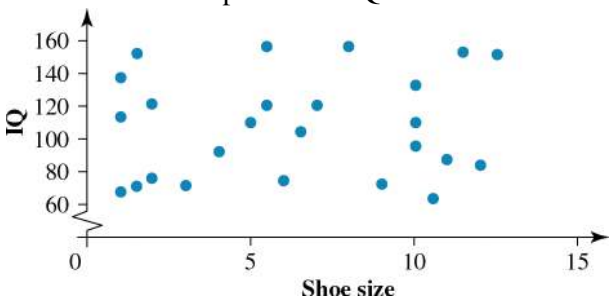
© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)
The following scatterplot shows the relationship between the period of exercise (hours) and blood cholesterol level (mM).



This correlation could be interpreted as
A. moderate positive correlation.
B. strong negative correlation.
C. no correlation.
D. strong positive correlation.
E. weak negative correlation.

Question 8 (1 mark)
The following scatterplot shows the relationship between IQ and shoe size.



This correlation could be interpreted as:
A. strong positive correlation.
B. moderate negative correlation.
C. weak positive correlation.
D. no correlation.
E. strong negative correlation.

© John Wiley & Sons Australia, Ltd

Topic	8	Investigating relationships between two numerical variables
Subtopic	8.4	Informal interpretation of association and causation

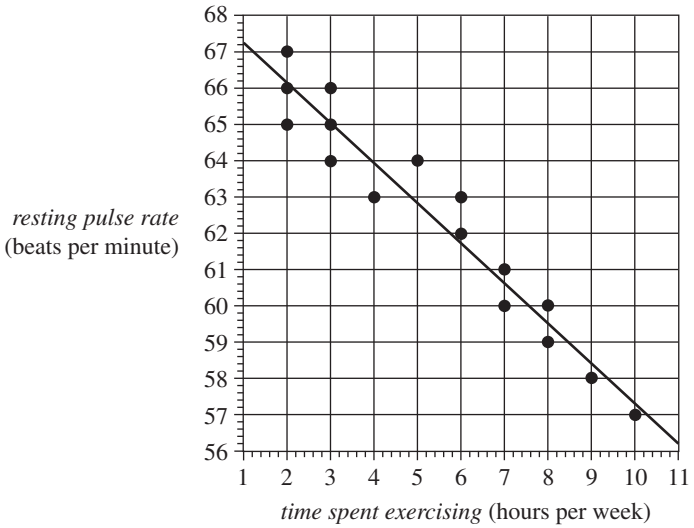
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2018, Further Mathematics Exam 1, Section A, Q.9; © VCAA*

Question 1 (1 mark)

The scatterplot below displays the *resting pulse rate*, in beats per minute, and the *time spent exercising*, in hours per week, of 16 students. A least squares line has been fitted to the data.



The coefficient of determination is 0.8339

The correlation coefficient r is closest to

A. −0.913

B. −0.834

C. −0.695

D. 0.834

E. 0.913

Source: *VCE 2017, Further Mathematics Exam 2, Section A, Core, Q.3biv; © VCAA*

Question 2 (1 mark)

The *number of male moths* caught in a trap set in a forest and the *egg density* (eggs per square metre) in the forest are shown in the table below.

Number of male moths	35	37	45	49	65	74	77	86	95
Egg density (eggs per square metre)	471	635	664	997	1350	1100	2010	1640	1350

The correlation coefficient is $r = 0.862$.
Determine the percentage of the variation in *egg density* in the forest explained by the variation in the *number of female moths* caught in the trap.
Round your answer to 1 decimal place.

Source: *VCE 2017, Further Mathematics Exam 1, Section A, Core, Q.12; © VCAA*

Question 3 (1 mark)

Data collected over a period of 10 years indicated a strong, positive association between the number of stray cats and the number of stray dogs reported each year ($r = 0.87$) in a large, regional city.
A positive association was also found between the population of the city and both the number of stray cats ($r = 0.61$) and the number of stray dogs ($r = 0.72$).
During the time that the data was collected, the population of the city grew from 34 564 to 51 055.From this information, we can conclude that

A. if cat owners paid more attention to keeping dogs off their property, the number of stray cats reported would decrease.
B. the association between the number of stray cats and stray dogs reported cannot be causal because only a correlation of $+1$ or -1 shows causal relationships.
C. there is no logical explanation for the association between the number of stray cats and stray dogs reported in the city, so it must be a chance occurrence.
D. because larger populations tend to have both a larger number of stray cats and stray dogs, the association between the number of stray cats and the number of stray dogs can be explained by a common response to a third variable, which is the increasing population size of the city.
E. more stray cats were reported because people are no longer as careful about keeping their cats properly contained on their property as they were in the past.

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)

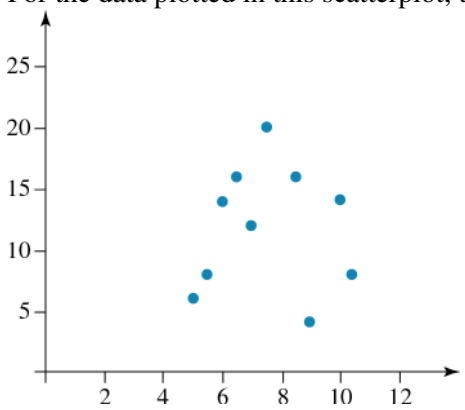
A coffee shop owner is concerned that the number of sick days taken by staff is affecting his income. To prove this he collected data on his income and the number of sick days of staff, and calculated a correlation coefficient of −1.2.

The owner can conclude that

A. there is a strong positive correlation between income and number of sick days.
B. there is a strong negative correlation between income and number of sick days.
C. there is no correlation between income and number of sick days.
D. there is an error in the data or in his calculations.
E. staff illness is affecting his income.

Question 5 (1 mark)

For the data plotted in this scatterplot, the Pearson correlation coefficient is most likely to be



A. 0.98
B. −0.98
C. 0.42
D. −0.42
E. 0.02

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)

When comparing the number of hours spent on private study and the results achieved in exams, which of the following statements is *incorrect*?

A. When plotting the scatterplot for these variables, number of hours would be placed on the horizontal axis.

B. It is reasonable to expect that there is no correlation between the variables.

C. The number of hours spent studying is the explanatory variable. The results achieved in the exams would be the response variable.

D. When plotting the scatterplot for these variables, results achieved in exams would be placed on the vertical axis.

E. It is reasonable to expect that there is some positive correlation between the variables.

© John Wiley & Sons Australia, Ltd

Topic	8	Investigating relationships between two numerical variables
Subtopic	8.5	The line of good fit and predictions

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2021, Further Mathematics Exam 1, Section A, Core, Q.11; © VCAA*

Question 1 (1 mark)

The table below shows the *weight*, in kilograms, and the *height*, in centimetres, of 10 adults.

Weight **(kg)**	***Height*** **(cm)**
59	173
67	180
69	184
84	195
64	173
74	180
76	192
56	169
58	164
66	180

A least squares line is fitted to the data.

The least squares line enables an adult's weight to be predicted from their *height*.

The number of times that the predicted value of an adult's *weight* is greater than the actual value of their *weight* is

A. 3

B. 4

C. 5

D. 6

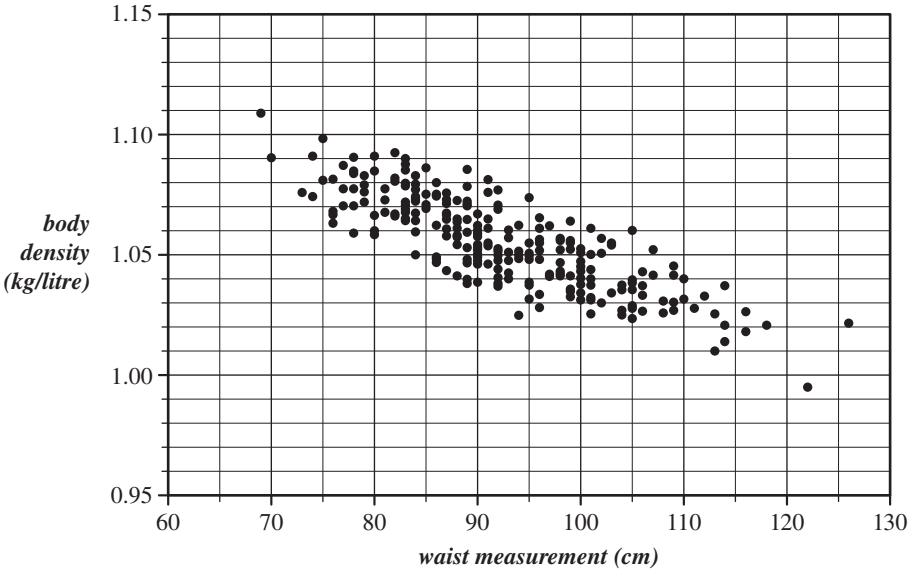
E. 7

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 2, Section A, Q.5b–d; © VCAA*

Question 2 (1 mark)

The scatterplot below shows *body density*, in kilograms per litre, plotted against *waist measurement*, in centimetres, for 250 men.



Data: RW Johnson, 'Fitting percentage of body fat to simple body measurements', *Journal of Statistics Education*, 4:1, 1996, <https://doi.org/10.1080/10691898.1996.11910505>

When a least squares line is fitted to the scatterplot, the equation of this line is

$body\ density = 1.195 - 0.001512 \times waist\ measurement$

b. Use the equation of this least squares line to predict the *body density* of a man whose *waist measurement* is 65 cm.

Round your answer to two decimal places. (1 mark)

c. When using the equation of this least squares line to make the prediction in part **b**, are you extrapolating or interpolating? (1 mark)

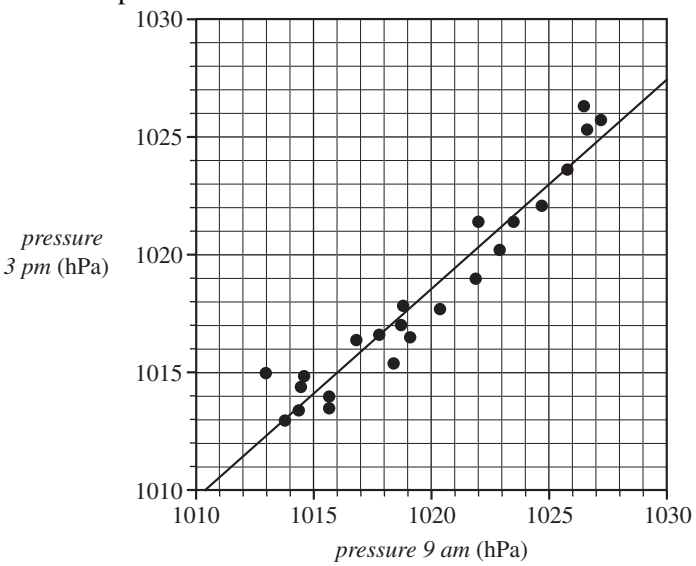
d. Interpret the slope of this least squares line in terms of a man's *body density* and *waist measurement*. (1 mark)

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 2, Section A, Q.5a–c; © VCAA*

Question 3 (1 mark)

The scatterplot below shows the atmospheric pressure, in hectopascals (hPa), at 3 pm (*pressure 3 pm*) plotted against the atmospheric pressure, in hectopascals, at 9 am (*pressure 9 am*) for 23 days in November 2017 at a particular weather station.



Data: Australian Government, Bureau of Meterology, <www.bom.gov.au/>

A least squares line has been fitted to the scatterplot as shown.

The equation of this line is

$pressure\ 3\ pm = 111.4 + 0.8894 \times pressure\ 9\ am$

a. Interpret the slope of this least squares line in terms of the atmospheric pressure at this weather station at 9 am and at 3 pm.

b. Use the equation of the least squares line to predict the atmospheric pressure at 3 pm when the atmospheric pressure at 9 am is 1025 hPa.
Round your answer to the nearest whole number.

c. Is the prediction made in part **b** an example of extrapolation or interpolation?

© John Wiley & Sons Australia, Ltd

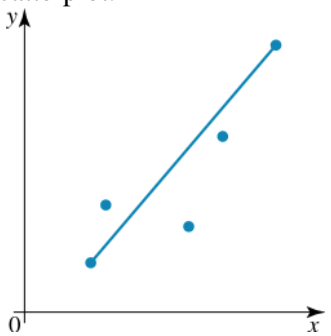
Question 4 (1 mark)

A linear relationship between two variables, x and y, is given by the equation $y = 3200 - 12.2x$.
Predict the value of x when $y = 2585.12$.

A. -50.4
B. 50.4
C. 262.3
D. $-28\,338.5$
E. 34 738.5

Question 5 (1 mark)

A trend line has been drawn through a scatterplot.



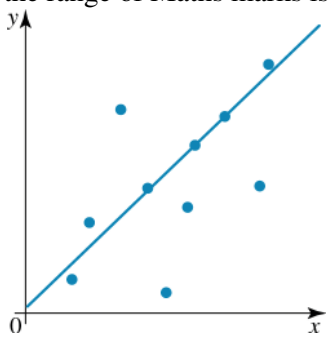
Any results predicted from this line will be

A. reliable, because the correlation between the variables is strong.
B. reliable, because the line is accurately drawn.
C. unreliable, because there are not enough data points.
D. unreliable, because the correlation between the variables is weak.
E. unreliable for extrapolation but not interpolation.

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)

The scatterplot shows the relationship between height of students, h, and the mark in Maths, M. The range of heights is from 160 cmto 180 cm and the range of Maths marks is from 48% to 98%.



The trend line $M = mh + c$ is to be extrapolated to find more data.
In which case can there be extrapolation that is reliable?

A. Find the Maths mark for a student with height
B. Find the Maths mark for a student with height
C. Find the Maths mark for a student with height
D. Find the Maths mark for children aged 5 years.
E. No extrapolation will be reliable.

Question 7 (1 mark)

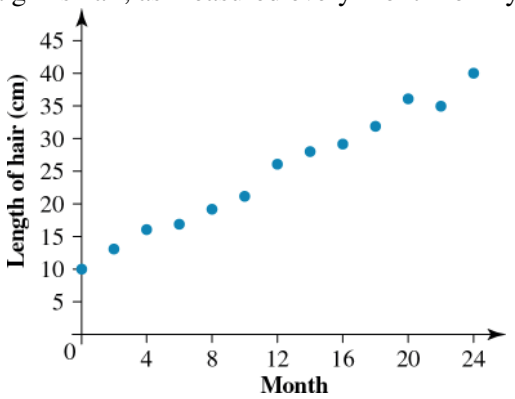
For girls under the age of 12 years, height, h cm, at different ages, a years, was modelled by a linear equation $h = 76 + 6.6a$. Which of the following statements is true?

A. The height of a person aged 25 years can be predicted using this model.
B. This model accurately reflects the rate of growth of all females.
C. This model can predict the approximate height of a 10-year-old girl.
D. Under this model, girls grow at 76 cm per year.
E. From the equation, 195 cm is a reasonable value for the approximate height of an 18-year-old female.

© John Wiley & Sons Australia, Ltd

Question 8 (1 mark)

This graph shows the length of a girl's hair, as measured every month for 2 years.



This graph could be used to

A. interpolate the length of hair after 10 months.
B. interpolate the thickness of hair after 3 months.
C. extrapolate the length of hair after 10 years.
D. extrapolate the thickness of hair after 30 months.
E. interpolate the length of hair after 10 years.

Question 9 (1 mark)

Which of the following statements is/are true?

I. Extrapolation is unreliable because it assumes that the relationship is true for untested values.
II. A straight line graph can be fitted to model any data so that interpolation can be used.
III. Interpolation is the use of the line of best fit to predict values smaller than the smallest value and larger than the largest value in the data set.
IV. Extrapolation is the use of the line of best fit to predict values smaller than the smallest value and larger than the largest value in the data set.
V. For any model, interpolation is reliable but extrapolation is not.

A. I only
B. III only
C. IV only
D. I and IV
E. II and V

© John Wiley & Sons Australia, Ltd

Topic	8	Investigating relationships between two numerical variables
Subtopic	8.6	Introduction to the least squares line of best fit (extending)

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

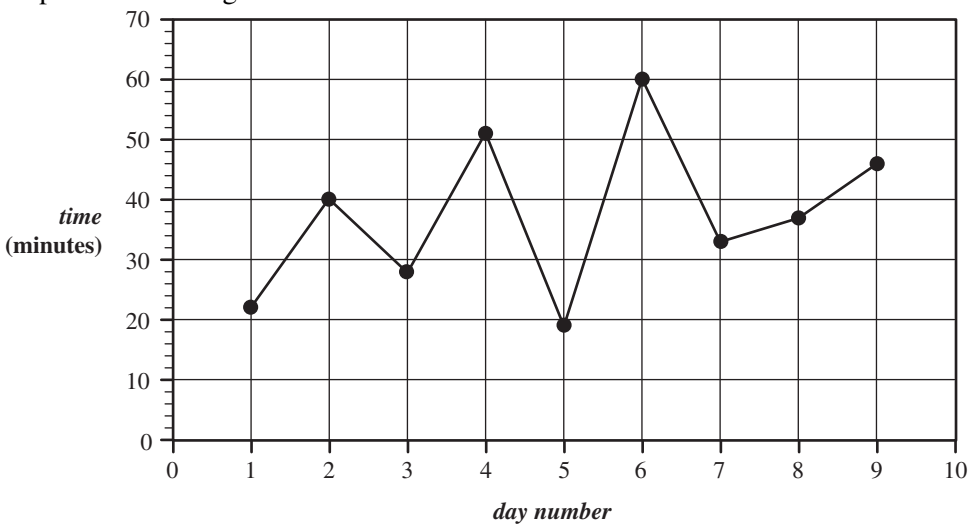
Source: *VCE 2019, Further Mathematics Exam 1, Section A, Q.14; © VCAA*

Question 1 (1 mark)

The *time*, in minutes, that Liv ran each day was recorded for nine days.
These times are shown in the table below.

Day number	1	2	3	4	5	6	7	8	9
***Time* (minutes)**	22	40	28	51	19	60	33	37	46

The time series plot below was generated from this data.



A least squares line is to be fitted to the time series plot shown above.
The equation of this least squares line, *with day number* as the explanatory variable, is closest to

A. $day\ number = 23.8 + 2.29 \times time$
B. $day\ number = 28.5 + 1.77 \times time$
C. $time = 23.8 + 1.77 \times day\ number$
D. $time = 23.8 + 2.29 \times day\ number$
E. $time = 28.5 + 1.77 \times day\ number$

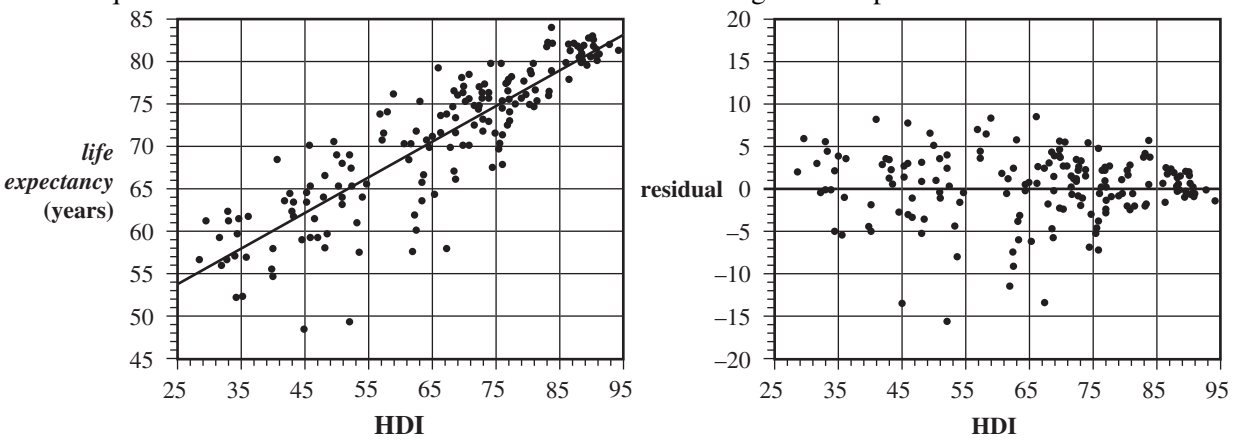
© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics Exam 1, Section A, Q.9; © VCAA*

Question 2 (1 mark)

The scatterplot below shows life expectancy in years (*life expectancy*) plotted against the Human Development Index (*HDI*) for a large number of countries in 2011.

A least squares line has been fitted to the data aaand the resulting residual plot is also shown.



Data: Gapminder

The equation of this least squares line is

$life\ \ expectancy = 43.0 + 0.422 \times HDI$

The coefficient of determination is $r^2 = 0.875$

Given the information above, which one of the following statements is **not** true?

A. The value of the correlation coefficient is close to 0.94.

B. 12.5% of the variation in life expectancy is not explained by the variation in the Human Development Index.

C. On average, life expectancy increases by 43.0 years for each 10-point increase in the Human Development Index.

D. Ignoring any outliers, the association between life expectancy and the Human Development Index can be described as strong, positive and linear.

E. Using the least squares line to predict the life expectancy in a country with a Human Development Index of 75 is an example of interpolation.

© John Wiley & Sons Australia, Ltd

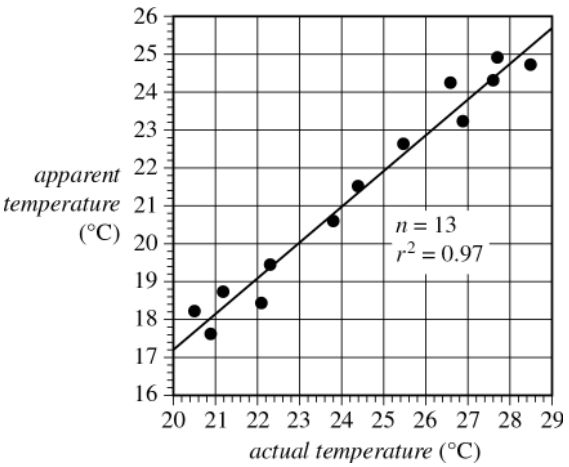
Source: *VCE 2016, Further Mathematics Exam 2, Core, Q.3bc; © VCAA*

Question 3 (1 mark)

The data in the table below shows a sample of actual temperatures and apparent temperatures recorded at the weather station. A scatterplot of the data is also shown.

The data will be used to investigate the association between the variables *apparent temperature* and *actual temperature*.

Apparent temperature (° C)	*Actual temperature* (°C)
24.7	28.5
24.3	27.6
24.9	27.7
23.2	26.9
24.2	26.6
22.6	25.5
21.5	24.4
20.6	23.8
19.4	22.3
18.4	22.1
17.6	20.9
18.7	21.2
18.2	20.5



b. Answer the following.

i. Determine the equation of the least squares line that can be used to predict the *apparent temperature* from the *actual temperature*.

Write the values of the intercept and slope of this least squares line in the appropriate spaces provided below.

Round your answers to two significant figures. (1 mark)

apparent temperature = _____ + _____ × *actual temperature*

ii. Interpret the intercept of the least squares line in terms of the variables *apparent temperature* and *actual temperature*. (1 mark)

c. The coefficient of determination for the association between the variables *apparent temperature* and *actual temperature* is 0.97.

Interpret the coefficient of determination in terms of these variables. (1 mark)

© John Wiley & Sons Australia, Ltd

Question 4 (3 marks)

Data is collected on the time taken for a drug to take effect, y days, when the amount of the dose of the drug, x mg, is changed. Analysis of this data yielded the following results.

	x	y
Mean	$\bar{x} = 30$	$\bar{y} = 3.1$
Standard deviation	$s_x = 8.8$	$s_y = 1.2$
Pearson coefficient	$r = 0.6$	

If the line of good fit has equation $y = a + bx$, calculate (correct to two decimal places):

a. the gradient (1 mark)

b. the y-intercept (1 mark)

c. the equation of the line. (1 mark)

Question 5 (9 marks)

The cost of caring for cats includes food, litter and vet bills. A cat refuge collected data on the total cost and the number of cats in their care.

Number of cats, n	5	12	16	22	28	35
Total cost, $C	2455	5544	5200	6820	8680	7945

a. Plot a scatterplot for the data. (2 marks)

b. If the line of good fit has equation $C = a + bn$, calculate:

i. the gradient (1 mark)

ii. the y-intercept (1 mark)

© John Wiley & Sons Australia, Ltd

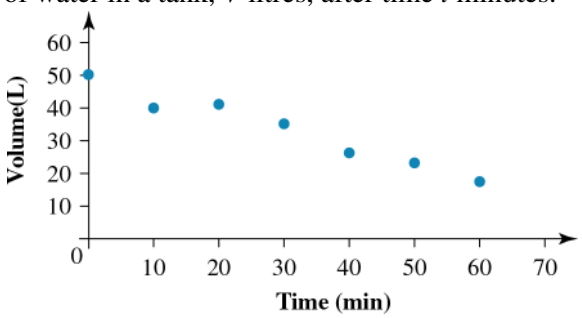
iii. the Pearson correlation coefficient (1 mark)

iv. the equation of the line. (1 mark)

c. Draw in the line of good fit on the scatterplot. (3 marks)

Question 6 (1 mark)

The graph shows the amount of water in a tank, V litres, after time t minutes.



If a line of good fit (V against t) was drawn by eye, it would have

A. no intercept with the t-axis.

B. a t-intercept of 70.

C. a V-intercept of 0.

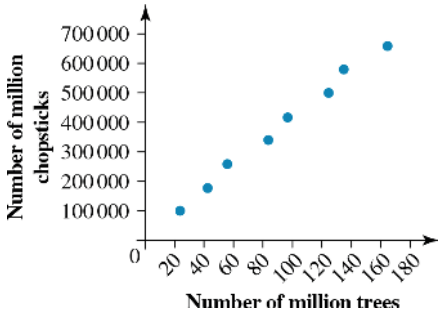
D. a positive gradient.

E. a negative gradient.

© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)

According to the National People's Council of China, it takes 20 million mature trees each year to make the 80 billion pairs of disposable chopsticks needed annually by the Chinese population of 1.4 billion people. To confirm this, information was collected and plotted on a scatter diagram of number of chopsticks, C million, against the number of trees, n million.

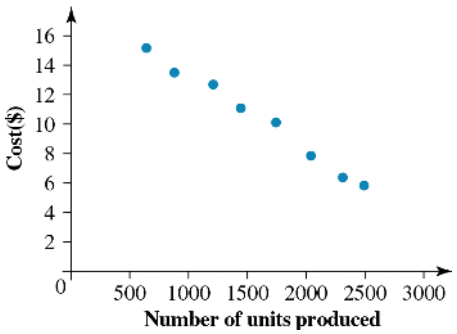


A line of good fit would show that

A. there were already over 15 000 million chopsticks when they began counting trees.
B. there were no chopsticks when they began counting trees.
C. 40 trees are needed to make 200 chopsticks.
D. 500 billion chopsticks can be made with 100 million trees.
E. the 1.4 billion population need 35 million trees chopped down.

Question 8 (1 mark)

The cost of manufacture of a toy, $\$C$, depends on the number produced, n.
This is shown on a scatterplot.



If a line of good fit was drawn by eye, it would have

A. a gradient of 0.005.
B. an n-intercept of 3000.
C. a C-intercept at 28.
D. four points above the line and four points below.
E. a positive gradient.

© John Wiley & Sons Australia, Ltd

Topic	8	Investigating relationships between two numerical variables
Subtopic	8.7	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2021, Further Mathematics Exam 1, Section A, Core, Q.10; © VCAA*

Question 1 (1 mark)

Oscar walked for nine consecutive *days*. The *time*, in minutes, that Oscar spent walking on each day is shown in the table below.

Day	1	2	3	4	5	6	7	8	9
Time	46	40	45	34	36	38	39	40	33

A least squares line is fitted to the data.

The equation of this line predicts that on day 10 the time Oscar spends walking will be the same as the time he spent walking on

A. day 3
B. day 4
C. day 6
D. day 8
E. day 9

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 2, Section A, Q.4bc; © VCAA*

Question 2 (1 mark)

The age, in years, body density, in kilograms per litre, and weight, in kilograms, of a sample of 12 men aged 23 to 23 years are shown in the table below.

Age **(years)**	***Body density*** **(kg/litre)**	***Weight*** **(kg)**
23	1.07	70.1
23	1.07	90.4
23	1.08	73.2
23	1.08	85.0
24	1.03	84.3
24	1.05	95.6
24	1.07	71.7
24	1.06	95.0
25	1.07	80.2
25	1.09	87.4
25	1.02	94.9
25	1.09	65.3

b. A least squares line is to be fitted to the data with the aim of predicting *body density* from *weight*.

i. Name the explanatory variable for this least squares line. (1 mark)

ii. Determine the slope of this least squares line.
Round your answer to three significant figures. (1 mark)
Round your answer to three significant figures. (1 mark)

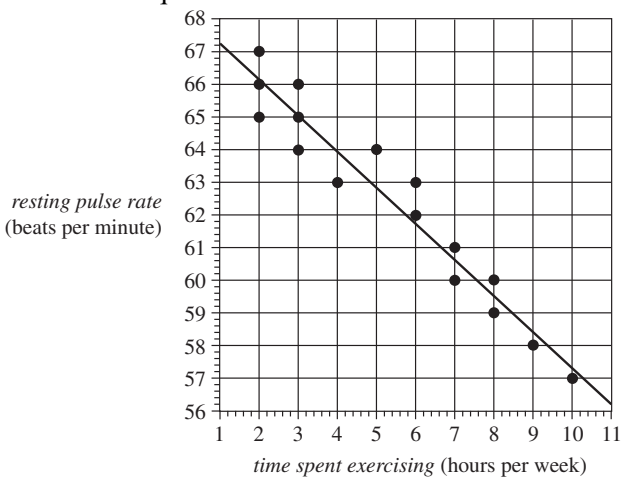
c. What percentage of the variation in *body density* can be explained by the variation in *weight*?
Round your answer to the nearest percentage.(1 mark)

© John Wiley & Sons Australia, Ltd

Source: *VCE 2018, Further Mathematics Exam 1, Section A, Q.8; © VCAA*

Question 3 (1 mark)

The scatterplot below displays the resting pulse rate, in beats per minute, and the time spent exercising, in hours per week, of 16 students. A least squares line has been fitted to the data.



The equation of this least squares line is closest to

A. *resting pulse rate* $= 67.2 - 0.91 \times$ *time spent exercising*

B. *resting pulse rate* $= 67.2 - 1.10 \times$ *time spent exercising*

C. *resting pulse rate* $= 68.3 - 0.91 \times$ *time spent exercising*

D. *resting pulse rate* $= 68.3 - 1.10 \times$ *time spent exercising*

E. *resting pulse rate* $= 67.2 + 1.10 \times$ *time spent exercising*

Source: *VCE 2018, Further Mathematics Exam 1, Section A, Q.10; © VCAA*

Question 4 (1 mark)

In a study of the association between a person's *height*, in centimetres, and *body surface area*, in square metres, the following least squares line was obtained.

body surface area $= -1.1 + 0.019 \times$ *height*

Which one of the following is a conclusion that can be made from this least squares line?

A. An increase of 1 m^2 in *body surface area* is associated with an increase of 0.019 cm in *height*.

B. An increase of 1 cm in *height* is associated with an increase of 0.019 m^2 in *body surface area*.

C. The correlation coefficient is 0.019.

D. A person's *body surface area*, in square metres, can be determined by adding 1.1 cm to their *height*.

E. A person's *height*, in centimetres, can be determined by subtracting 1.1 from their *body surface area*, in square metres.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2018, Further Mathematics Exam 1, Section A, Q.14; © VCAA*

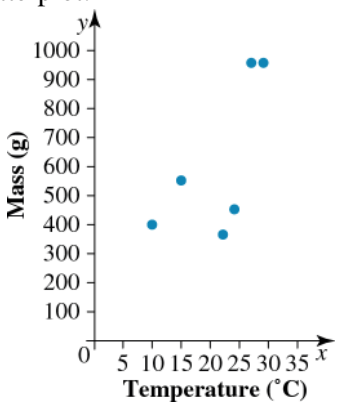
Question 5 (1 mark)

A least squares line is fitted to a set of bivariate data.
Another least squares line is fitted with response and explanatory variables reversed.
Which one of the following statistics will **not** change in value?

A. the residual values
B. the predicted values
C. the correlation coefficient r
D. the slope of the least squares line
E. the intercept of the least squares line

Question 6 (1 mark)

In Penny's Science project she measured the mass of tomato plants, grown at different temperatures, after 2 months. She plotted the results on a scatterplot.



From these results Penny can conclude that:

A. the higher temperature caused the plants to grow better.
B. there is not enough evidence to show that the growth of tomato plants depends on temperature.
C. there is evidence to show that the growth of tomato plants depends on temperature.
D. better growth of tomato plants is caused by the higher temperature.
E. tomato plants should never be grown at temperatures between 20°Cand 25°C.

© John Wiley & Sons Australia, Ltd

Question 7 (4 marks)

The number of murders recorded from 1996 to 2012 is shown in the table.

Year	Number
1996	354
1997	364
1998	334
1999	385
2000	362
2001	347
2002	366
2003	341
2004	302
2005	301
2006	321
2007	283
2008	293
2009	293
2010	261
2011	276
2012	297

a. State which is the response variable and which is the explanatory variable. **(1 mark)**

b. Plot a scatterplot of the data. **(1 mark)**

c. State the form, strength and direction of the relationship between number of murders and time. **(1 mark)**

d. Can it be said that the crime rate is coming down as the years pass? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 8 (4 marks)
Angus thought that the price of sausages was likely to depend on the percentage of meat used. He chose eight different brands to measure and tabled his results.

Percentage of meat	Price/kg
29	$4.30
27	$7.00
27	$7.00
26	$5.30
25	$6.40
25	$9.20
25	$8.10
23	$9.40

a. State which is the response variable and which is the explanatory variable. **(1 mark)**

b. Plot a scatterplot of the data. **(1 mark)**

c. State the form, strength and direction of the relationship between price per kilogram and percentage of meat. **(1 mark)**

d. Can it be said from this data that adding more meat causes the price of sausages to increase? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 9 (3 marks)
The number of accidents per week on a particular road is thought to be related to the number of cars per hour using that road. Data is collected and the results are shown in the table.

Number of cars per hour	Average number of accidents per week
700	1
800	1.4
900	1.6
1000	1.7
1100	1.7
1200	2
1300	2.3
1400	2.5

a. Plot a scatterplot for the data. **(1 mark)**

b. It is found that there is a strong positive correlation. Which of these is the most likely value of the Pearson correlation coefficient: **(1 mark)**
$0.65, 0.98, -0.72$ or -0.98?

c. Give reasons why the relationship is likely to be proven. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 10 (1 mark)

Ms Rogers insists that there is a correlation between the students' marks in French and their marks in Mathematics. To prove her point she collects the students' marks in the final exams and organises them in a table:

French mark, x	Maths mark, y
65	69
92	89
30	12
50	45
58	64
58	61
85	77
70	52
57	37
35	40

a. Which mark has she chosen to be the explanatory variable and which the response variable? Does it matter? **(1 mark)**

b. Plot a scatterplot of the data. **(1 mark)**

c. Do you expect the correlation coefficient to be positive or negative? **(1 mark)**

d. The average marks and their standard deviations are shown here. **(1 mark)**

Mean	60	54.6
Standard deviation	18.4824	21.1149

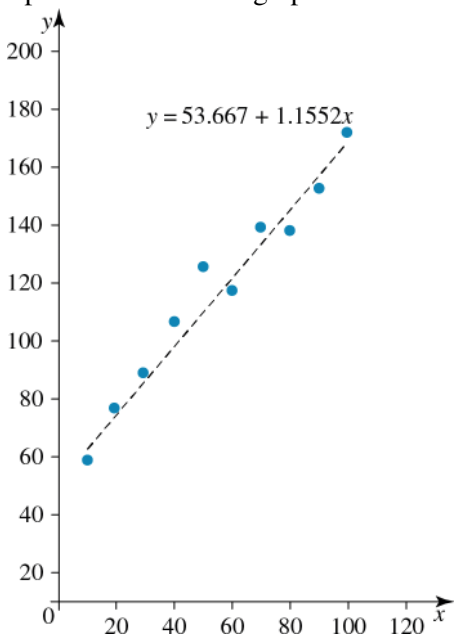
Describe the difference between the distribution of French marks and the distribution of Maths marks.

e. Calculate the Pearson correlation coefficient and comment on the result. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 11 (8 marks)

Data connecting variables x and y is plotted on a scatter graph.



a. Use the graph to predict:

i. the value of y when $x = 40$ **(1 mark)**

ii. the value of y when $x = 110$ **(1 mark)**

iii. the value of x when $y = 150$ **(1 mark)**

iv. the value of x when $y = 200$. **(1 mark)**

b. Find the equation of the trend line. **(3 marks)**

© John Wiley & Sons Australia, Ltd

c. Use the equation to calculate the value of y when $x = 1200$. **(1 mark)**

Question 12 (1 mark)

According to the OECD, the total fertility rates of women in Australia has changed over time as follows.

Year	1960	1970	1980	1990	1997
Children per female	3.45	2.86	1.9	1.91	1.78

a. Plot a scatterplot of the data. **(1 mark)**

b. Draw a line of good fit for the data. **(2 marks)**

c. Using 1950 as $n = 0$, 1960 as $n = 10$ etc., determine an equation relating fertility F and n. **(1 mark)**

d. Estimate the fertility rate in **(1 mark)**

i. 1975

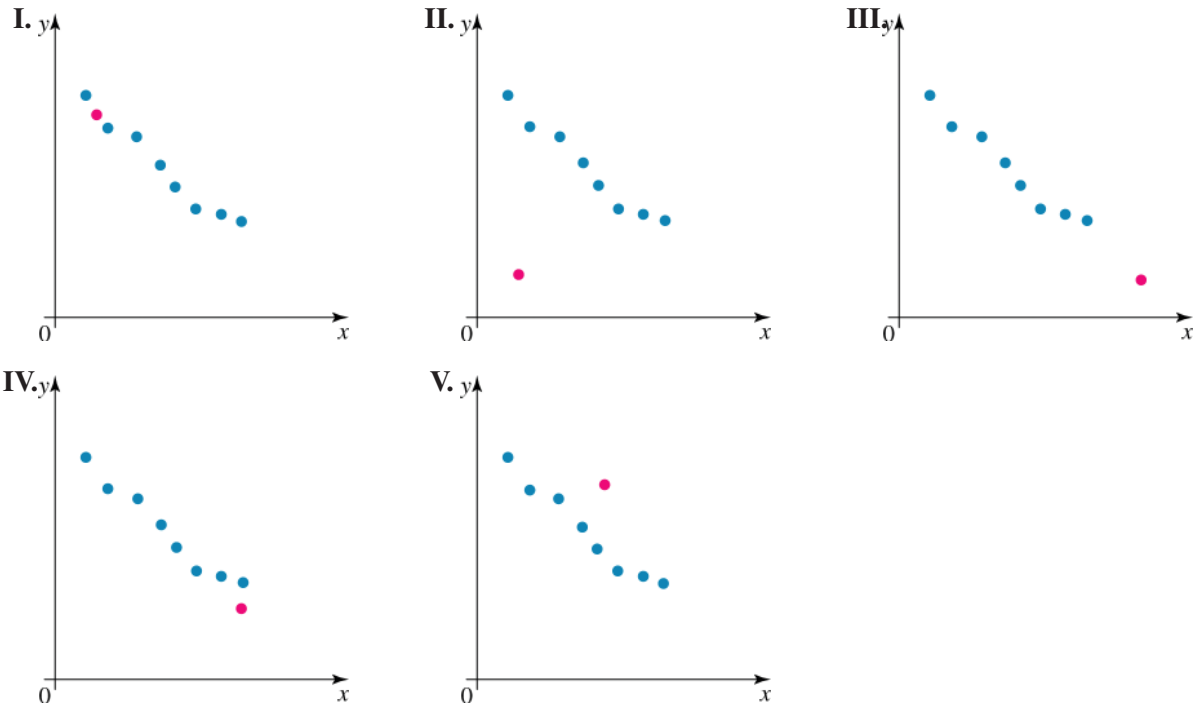
ii. 2000.

e. Comment on the reliability of the estimates. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 13 (1 mark)

An extra piece of data (shown in a lighter shade) is added to these scatterplots.

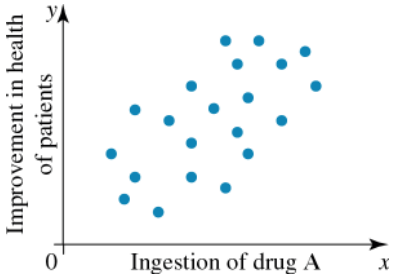


Which of the following is true for the correlation coefficient?

A. Decreased in V only
B. Decreased II and V
C. Increased in I and III
D. Increased in I only
E. Decreased in II only

© John Wiley & Sons Australia, Ltd

Question 14 (1 mark)
For the scatterplot shown, it would be reasonable to state that:



A. the correlation coefficient has a value of 1.
B. the correlation coefficient has a value of −0.75.
C. the correlation coefficient has a value of 0.
D. the correlation coefficient has a value of −0.5.
E. the correlation coefficient has a value of 0.25.

Question 15 (1 mark)
If a scatterplot is drawn and the correlation coefficient is calculated to be −0.42, it would be reasonable to say that:
A. there is strong positive correlation between the variables.
B. there is weak negative correlation between the variables.
C. there is strong negative correlation between the variables.
D. there is no correlation between the variables.
E. there is weak positive correlation between the variables.

Question 16 (1 mark)
A cleaning company charges their clients using the equation $C(\$) = 15 + 20n$, where Crepresents the total cost of the cleaning and n represents the number of half-hour increments taken for the job.
If the cleaner took two-and-a-half hours to complete one job, the total cost would be
A. $95.00
B. $40.00
C. $115.00
D. $65.00
E. $100.00

© John Wiley & Sons Australia, Ltd

Question 17 (1 mark)

A group of senior students collect the following data, comparing their English and Maths results.

English (%)	49	54	60	55	32	75	83	90	20	45	73	85
Maths (%)	72	61	72	68	51	82	91	93	50	51	83	91

Note: Give answers correct to 2 decimal places where appropriate.

a. Present this data as a scatterplot. **(1 mark)**

b. Determine the equation of the line of good fit placed by eye. **(3 marks)**

c. Write your equation in terms of the relationship between English and Maths results. **(1 mark)**

d. Based on your scatterplot, what type of correlation exists between these variables? **(1 mark)**

Question 18 (1 mark)

An AFL analyst collects the following data as he is exploring the relationship between the number of times players go inside the 50 m line and the number of goals kicked per game.

Inside 50 m	9	12	15	21	17	5	11	13	22	14	17	20
Goals kicked	7	10	10	15	11	2	9	9	18	12	12	16

a. Present this data as a scatterplot. **(1 mark)**

b. Determine the equation of the line of good fit placed by eye. **(3 marks)**

© John Wiley & Sons Australia, Ltd

c. Write your equation in terms of the relationship between the number of times the players go inside 50 m and the number of goals kicked. **(1 mark)**

d. Using a CAS calculator, determine the equation for the least squares line. **(1 mark)**

e. What do you notice about your equation found using two points and the equation of the least squares regression line. **(1 mark)**

f. Based on your scatterplot, what type of correlation exists between these variables? **(1 mark)**

g. Using your CAS, calculate Pearson's correlation coefficient. Does this match your prediction in part **f**? **(1 mark)**

h. Could you use these findings to predict the number of goals that will be kicked in a game? Explain. **(1 mark)**

Question 19 (1 mark)

The monthly profit of a dress shop, $\$P$, depends on the number of designers employed each month, n.

No. of designers (*n*)	2	3	3	4	4	4	3	2	2	3	4	5
Profit (\$)	4400	5000	6400	7600	8000	7800	7000	4800	3600	5200	7400	9000

a. Plot a scatterplot and draw a line of good fit by eye. **(1 mark)**

© John Wiley & Sons Australia, Ltd

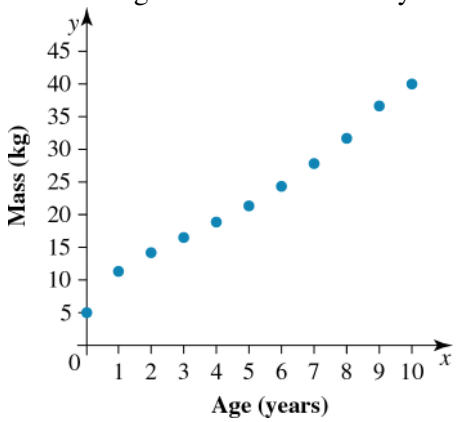
b. Determine the equation of the line of good fit. **(1 mark)**

c. Estimate the profit if there is only one designer available. **(1 mark)**

d. Estimate how many designers would be required in order to make a profit of $18 000. **(1 mark)**

Question 20 (4 mark)

A mother recorded the mass of her son in kilograms until he was 10 years old and produced this graph.



She worked out that the equation of the straight line of best fit was $42a - 13m + 81 = 0$, where a is the age in years and m is the mass in kg.

a. Draw the line of good fit that she drew. **(1 mark)**

b. How much did her son weigh when he was 10 years old, according to the model? **(1 mark)**

c. What is the difference between the weight according to the model and the actual weight at 10 years? **(1 mark)**

© John Wiley & Sons Australia, Ltd

d. How old will her son be when he weighs 48 kg, according to the model? **(1 mark)**

Question 21 (7 marks)

A student measures the speed of a ball, v cm/s, at time t seconds. The table shows the results.

t(s)	v (cm/s)
0	10
1	14.9
2	21.3
3	24.1
4	30.5
5	36
6	40

a. Use a CAS calculator or other technology to create a scatterplot of the data. **(1 mark)**

b. Does the scatterplot appear to be linear over the range of the data? **(1 mark)**

c. Use the CAS calculator or another method to determine the equation of the line of good fit. **(1 mark)**

d. Use the equation to predict:

i. the speed after 3.5 seconds **(1 mark)**

ii. the speed after 10 seconds (1 mark)

© John Wiley & Sons Australia, Ltd

iii. the time when the speed is 20 cm/s. **(1 mark)**

e. Which predictions are most likely to be reliable? **(1 mark)**

Question 22 (8 marks)

A group of 12 to 15 year olds was asked how many hours per day they spent on a computer. The table shows the percentage of the group using a computer for different numbers of hours per day.

No. hours	0	0.5	1	2	3	4	5	5.5
Percentage	8.9	26.5	21.2	18.2	13.6	6.5	4.8	0.3

a. Use a CAS calculator or other technology to create a scatterplot of the data. **(1 mark)**

b. Does the scatterplot appear to be linear over the range of the data? **(1 mark)**

c. Use the CAS calculator to determine the equation of the line of good fit over all the data and draw it on the same axes as the scatterplot. **(1 mark)**

d. Determine the equation of the line of good fit over the linear range of the data by omitting values outside the range. **(1 mark)**

e. Use the equation from part **d**. to predict:

i. the percentage of 12 to 15 year olds who use a computer for 3.5 hours per day **(1 mark)**

© John Wiley & Sons Australia, Ltd

ii. the percentage of 12 to 15 year olds who use a computer for 6 hours per day **(1 mark)**

iii. the number of hours 25% of 12 to 15 year olds use a computer. **(1 mark)**

f. Which prediction in part **e.** is likely to be most reliable? **(1 mark)**

Question 23 (3 marks)

The results (in percentages) obtained by 8 students studying both Maths and Science are shown in the table below.

Maths	25	35	45	50	65	75	85	95
Science	15	45	60	55	55	75	90	100

a. Plot a scatterplot and draw in the line of good fit by eye. **(1 mark)**

b. Estimate the result for Maths, if the student achieves 75% for Science. **(1 mark)**

c. If a student achieves 80% for Maths, what is their estimated result for Science? **(1 mark)**

Question 24 (1 mark)

A group of students sat a practice test in the lead-up to their exam period. Their practice exam marks and their actual exam marks are recorded in the table below.

Practice exam mark(%)	48	32	76	91	87	49	32	89
Actual exam mark(%)	55	47	83	95	92	58	68	95

a. Draw a scatterplot of the results. **(2 marks)**

© John Wiley & Sons Australia, Ltd

b. Draw in a line of good fit by eye. **(1 mark)**

c. Are there any results which seem out of place within this data set? **(1 mark)**

d. Calculate the equation of the line of good fit. **(4 marks)**

e. What does the line of good fit show about the correlation between the practice exam result and the actual exam result? **(2 marks)**

© John Wiley & Sons Australia, Ltd

Answers and marking guide

8.2 Response and explanatory variables

Question 1
Since humidity at 3 pm is the value being predicted, it is the response variable, making humidity at 9 am the explanatory variable. **[1 mark]**

Question 2
Evening congestion level is placed on the vertical axis, making it the response variable. **[1 mark]**

Question 3
Population is placed on the vertical axis, making it the response variable. **[1 mark]**

Question 4
The temperature on the x-axis because it is the explanatory variable and the amount risen on the y-axis because it is the response variable.

8.3 Scatterplots and basic correlation

Question 1
Since the association is positive, as the EV increases, we expect to see the RV increase. Avoid language like 'will increase' as we are exploring the association and so cannot make definite statements.

Question 2
Since $r = 0.9496$ the association must be positive and strong. The data clearly forms a straight line, so the form is linear. **[1 mark]**

Question 3
The scatterplot that shows weak negative correlation is C.

Question 4
Linear, strong, negative

Question 5
There is a strong positive correlation between profit and advertising costs, so the greater the amount spent on advertising, the greater the profit.

Question 6
The scatterplot that shows moderate positive correlation is A.

Question 7
The scatterplot shows there to be weak negative correlation between the variables.

Question 8
The scatterplot shows there to be no correlation between the variables.

8.4 Informal interpretation of association and causation

Question 1
$r = \pm\sqrt{r^2} = \pm\sqrt{0.8339} = \pm 0.913$
Since the graph shows a negative slope, r must be a negative value.
VCAA Examination Report **note:**
The value of the correlation coefficient r was required, given that the coefficient of determination was $r^2 = 0.8339$. Many students incorrectly took the positive square root value and chose option E.

© John Wiley & Sons Australia, Ltd

The scatterplot on the examination showed that the direction of the association was negative and therefore −0.913 was required.

Question 2

The percentage of variation is found from r^2%.

$r = 0.862^2 = 0.743\,044$

The percentage of variation is 74.3% to 1 decimal place. **[1 mark]**

Question 3

The answer is D as both variables (number of stray dogs and number of stray cats) is tied to a common third variable, which is population.

Options A and E are not correct as they try to offer an explanation rather than relating the variables to causation, common response, coincidence or confounding. Option B is simply an incorrect statement about correlation. Option C states the association is coincidence, which is less correct than a common response to population size.

Question 4

The correlation coefficient must lie between −1 and +1. Hence, there is an error in the data or in his calculations.

Question 5

There appears to be little or no correlation so the coefficient will be close to zero, or given the options, 0.02.

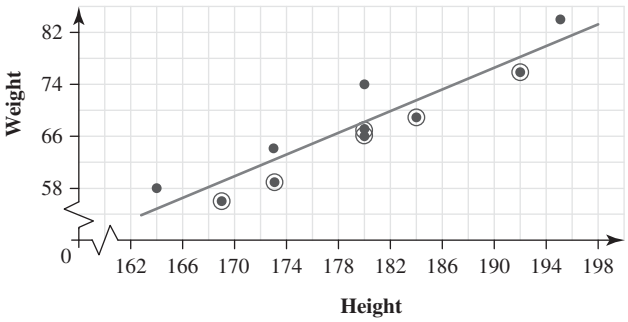
Question 6

It would be reasonable to expect some correlation between the variables.

8.5 The line of good fit and predictions

Question 1

On your CAS calculator, enter the data, generate the scatterplot and add the LSR line. 6 values lie below the regression line.



Question 2

b. Body density = $1.195 - 0.001512 \times 65 = 1.09672 = 1.10$ kg/litre **[1 mark]**

c. Extrapolation since the lowest value of waist measurement on the scatterplot is approximately 69 cm, so 65 cm is outside the range of data. **[1 mark]**

d. The slope is −0.001512. This indicates that, on average, body density decreases by 0.001512 kg/litre for each 1 cm increase in waist measurement. **[1 mark]**

© John Wiley & Sons Australia, Ltd

Question 3

a. On average, for each 1 hPa increase in pressure at 9 am, the pressure at 3 pm increases by 0.8894 hPa. **[1 mark]**

***VCAA Examination Report* note:**

Students had to be careful when answering this interpretative question. Many gave a response that was almost correct but failed to reference the one-unit increase in *pressure* 9 *am*.

Interpreting the slope in terms of the given variables required identifying the correct constant and then describing it.

Describing both constants was to ignore the first step; specific knowledge was required, not various statements provided in the hope of including something relevant.

b. Pressure at 3 pm $= 111.4 + 0.8894 \times 1025 = 1023.035 = 1023$ hPa **[1 mark]**

c. Interpolation **[1 mark]**

The value 1025 hPa is within the data range for pressure at 9 am.

Question 4

$2585.12 = 3200 - 12.2x$

$12.2x = 3200 - 2585.12$

$$x = \frac{614.88}{12.2}$$

$$= 50.4$$

Question 5

Any results predicted from this line will be unreliable, because there are not enough data points.

Question 6

No extrapolation will be reliable because the model does not describe the situation. There is no qualitative reason for a relationship between Maths marks and heights.

Question 7

This model can predict the approximate height of a 10-year-old girl.

Question 8

This graph could be used to interpolate the length of hair after 10 months.

Question 9

I and IV

8.6 Introduction to the least squares line of best fit (extending)

Question 1

The only possible correct options are C, D or E since time is the RV. Use regression on your CAS to find the correct solution.

Question 2

Option C is not true as the value stated (43) is for the vertical intercept, yet it has been interpreted as if that number was the slope.

Question 3

b. i. Use regression on your CAS to find the correct solutions.

Apparent temperature $= -1.7 + 0.94 \times$ actual temperature **[1 mark]**

ii. On average, when the actual temperature is 0 °C, the apparent temperature is −1.7 °C. **[1 mark]**

c. Since $r^2 = 0.97$, then 97% of the variation in apparent temperature can be explained by the variation in the actual temperature. **[1 mark]**

© John Wiley & Sons Australia, Ltd

Question 4

a. The gradient of the line of best fit:

$b = r\frac{s_y}{s_x}$

$= 0.08$ **[1 mark]**

b. The y-intercept of the line of good fit:

$a = \overline{y} - b\,\overline{x}$

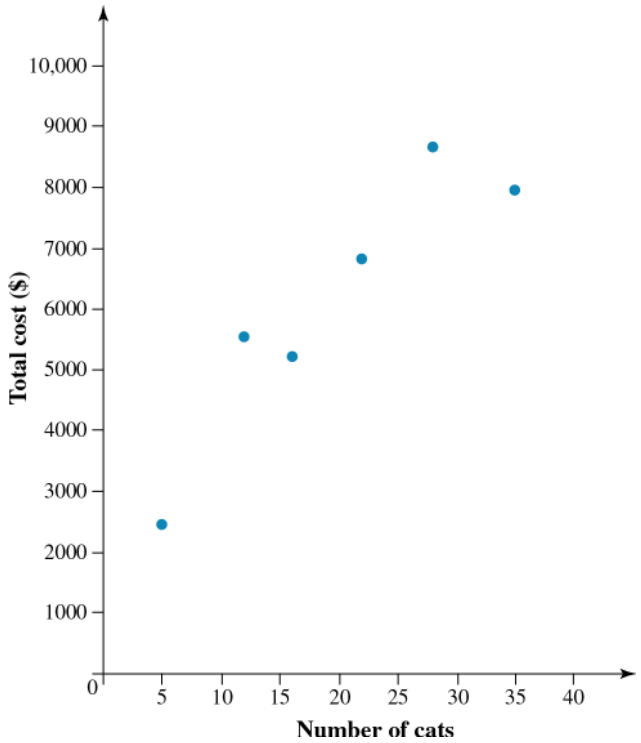
$= 0.65$ **[1 mark]**

c. $y = a + bx$

$y = 0.65 + 0.08x$ **[1 mark]**

Question 5

a. **[2 marks]**



b. **i.** $b = 188.32$ **[1 mark]**

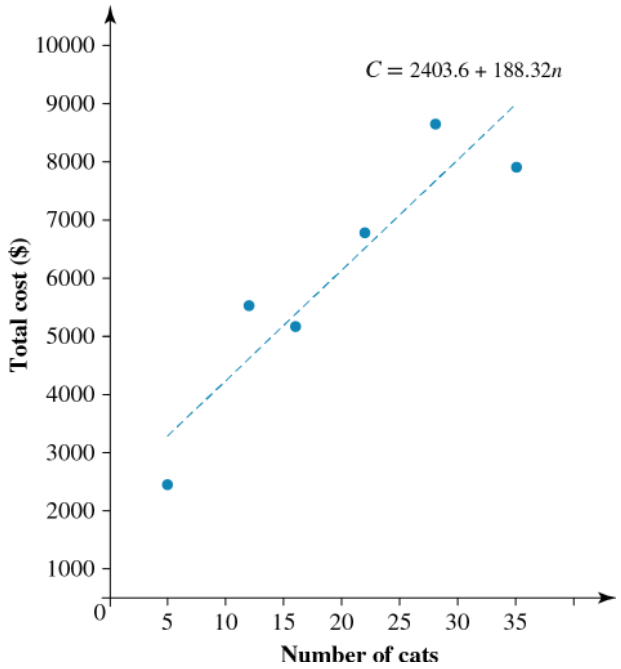
ii. $a = 2403.6$ **[1 mark]**

iii. 0.92 **[1 mark]**

iv. $C = 2403.6 + 188.32n$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

c. **[3 marks]**

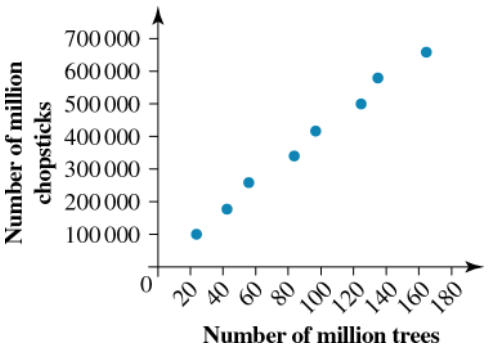


Line should pass through $(0,\ 2403)$ and $(30,\ 8053)$.

Question 6

The amount of water in the tank is reducing with time so the graph will have a negative gradient.

Question 7



$C = 15\,520 + 4000n$

Question 8

Actually, gradient $b \approx -0.0051$ and the line of good fit is $C = 18.542 - 0.0051n$

8.7 Review

Question 1

On your CAS, enter the data and find the LSR equation: $time = 44 - day$
When $day = 10$, $time = 44 - 10 = 34$, which is the same as day 4.

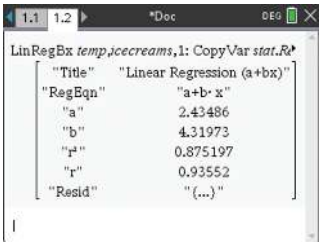
Question 2

b.

i. Body density is being predicted from weight; therefore, weight is the explanatory variable. **[1 mark]**

ii. Using your CAS calculator, perform a least-squares regression analysis.

© John Wiley & Sons Australia, Ltd



Slope $= -0.00112$ correct to 3 significant figures. **[1 mark]**

c. This question is referring to the coefficient of determination $\left(r^2\right)$, which is found on the least-squares regression analysis screen on CAS.

$r^2 = 0.289\,95\ldots$

Therefore, 29% of the variation in body density may be explained by the variation in weight. **[1 mark]**

Question 3

Note that the scales do not start at 0, so the vertical axis intercept is not 67.2.

Choose any two points along the line, say $(1, 67.2)$ and $(4, 64)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{64 - 67.2}{4 - 1} = \frac{-3.3}{3} = -1.10$$

Equation:

$resting\ pulse\ rate = a + b \times time\ spent\ exercising$

$resting\ pulse\ rate = a - 1.10 \times time\ spent\ exercising$

Passes through $(4, 64)$:

$64 = a - 1.1 \times 4$

$68.4 = a$

So the closest equation is:

$resting\ pulse\ rate = 68.3 - 1.10 \times time\ spent\ exercising$

***VCAA Examination Report* note:**

Students were asked to identify the equation of the least squares line drawn on a graph that contained 16 points. While it was not possible to determine exact slope and intercept values from the graph, students should have been able to approximate these values. Many students incorrectly assumed that the intercept value of the line was 67.2, read directly from the graph; however, this is only possible if the horizontal axis begins at value zero. Students are encouraged to look carefully at graphs before choosing what might seem to be the obvious answer.

Question 4

The association $body\ surface\ area = -1.1 + 0.019 \times height$ has a y-intercept $a = -1.1$ and a gradient $b = 0.019$.

The slope $b = 0.019$ indicates that the body surface area increased by 0.019 m^2 for every 1 cm added to a person's height.

Question 5

Given the equation $y = a + bx$, if y and x were switched and then the equation was solved for y, the equation would become $y = -\frac{a}{b} + \frac{1}{b}x$.

Looking at the two equations, the slopes and intercepts are different. Since the slopes are different, the predicted values and thus the residual values will also be different.

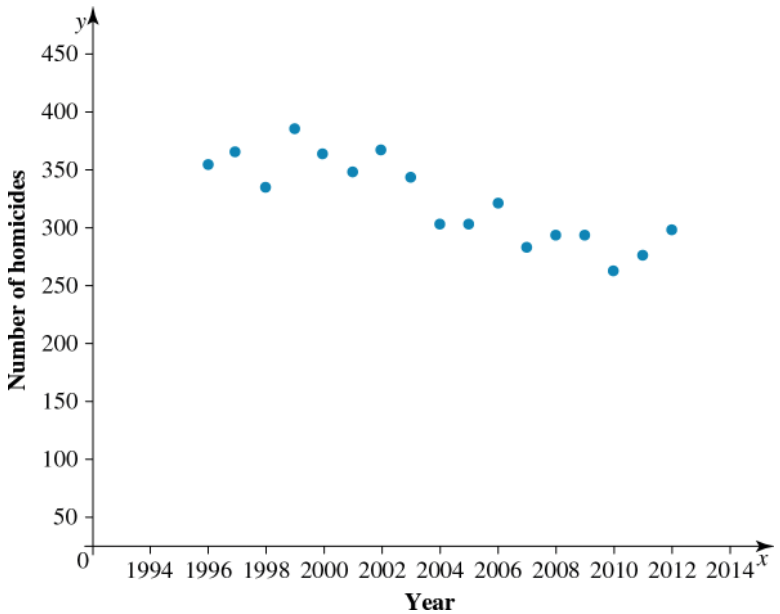
The only statistic that will not change is the correlation coefficient r, as s_x and s_y are not affected by the least squares line.

© John Wiley & Sons Australia, Ltd

***VCAA Examination Report* note:**
Students needed to be aware that reversing the two variables will give a different equation.
The slope and intercept will therefore both change in value, the predictions the line gives will change and hence the residual values will also change.
The correlation coefficient will not change in value as the degree of scattering of the points remains unchanged – i.e. the scattering of '*y*' values relative to '*x*' s the same as the scattering of '*x*' values relative to '*y*'.

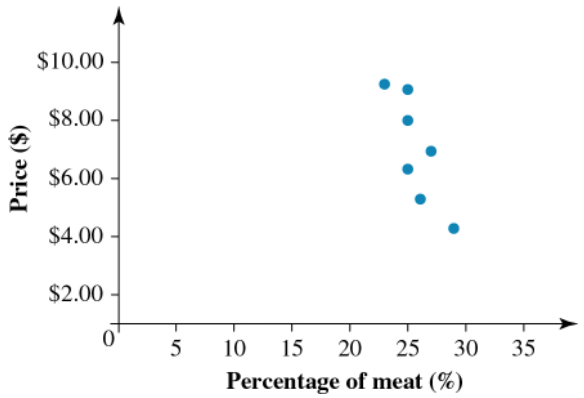
Question 6
From these results Penny can conclude that there is not enough evidence to show that the growth of tomato plants depends on temperature.

Question 7
a. Number of homicides is the response variable and year is the explanatory variable **[1 mark]**
b. **[1 mark]**



c. Linear, moderate and negative **[1 mark]**
d. The number of homicides is trending downwards as the years pass but there is no information given about other crimes. **[1 mark]**

Question 8
a. Price per kilogram is the response variable and percentage of meat is the explanatory variable. **[1 mark]**
b. **[1 mark]**

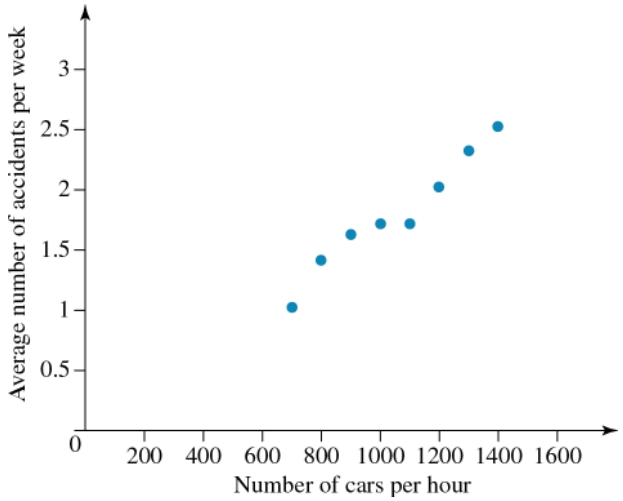


c. Linear, moderate and negative **[1 mark]**
d. No, association does not mean causality. **[1 mark]**

© John Wiley & Sons Australia, Ltd

Question 9

a. **[1 mark]**



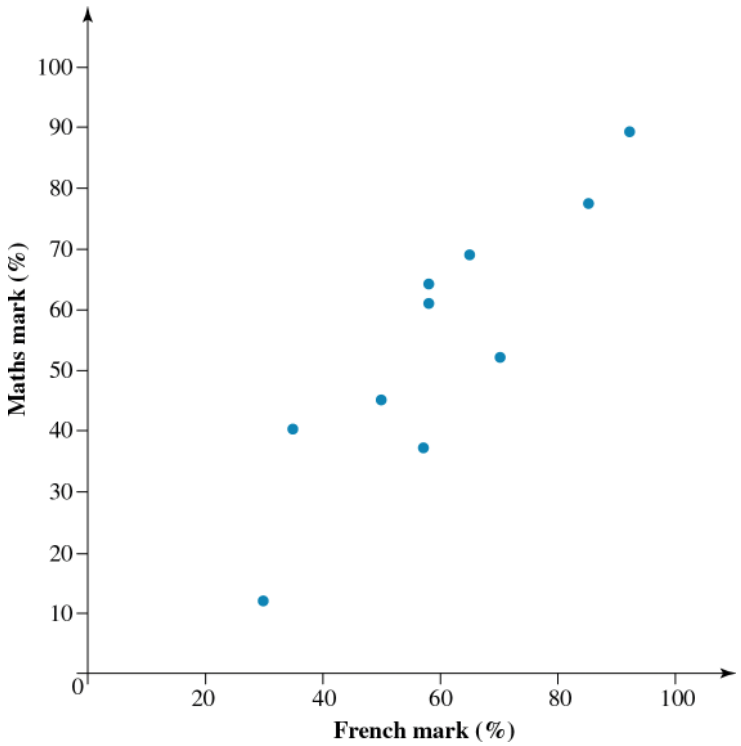
b. 0.98 **[1 mark]**

c. There is a strong (high correlation) positive relationship (average number of accidents increases as number of cars increases). **[1 mark]**

Question 10

a. French is the explanatory variable and Maths is the response variable; it does not matter. **[1 mark]**

b. **[1 mark]**



c. Positive **[1 mark]**

d. French marks are on average higher than Maths marks but are less spread. **[1 mark]**

e. $r = 0.888$, which means that there is a strong positive correlation between the marks of the two subjects, so Ms Rogers is likely to be correct. **[1 mark]**

Question 11

a. **i.** 100 **[1 mark]**

ii. 180 **[1 mark]**

iii. 83 **[1 mark]**

© John Wiley & Sons Australia, Ltd

iv. 125 **[1 mark]**

Note that the predictions are estimates, so answers do not need to be very accurate.

b. Gradient (b) of the line of good fit:

$b = 1.16$ **[1 mark]**

y intercept of the line of good fit:

$a = y - bx$

$= 53.67$ **[1 mark]**

$\therefore y = 53.67 + 1.16x$ **[1 mark]**

c. $y = 53.67 + 1.16x$

If $= 1200$,

$y = 53.67 + 1.16\,(1200)$

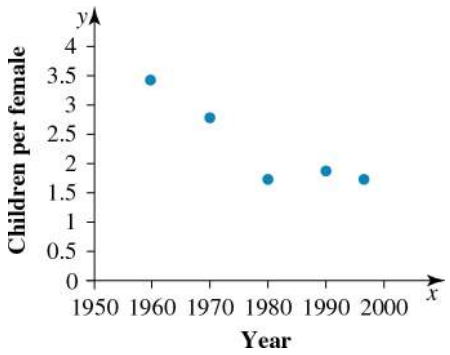
$= 53.67 + 1392$

$= 1445.67$

≈ 1446 **[1 mark]**

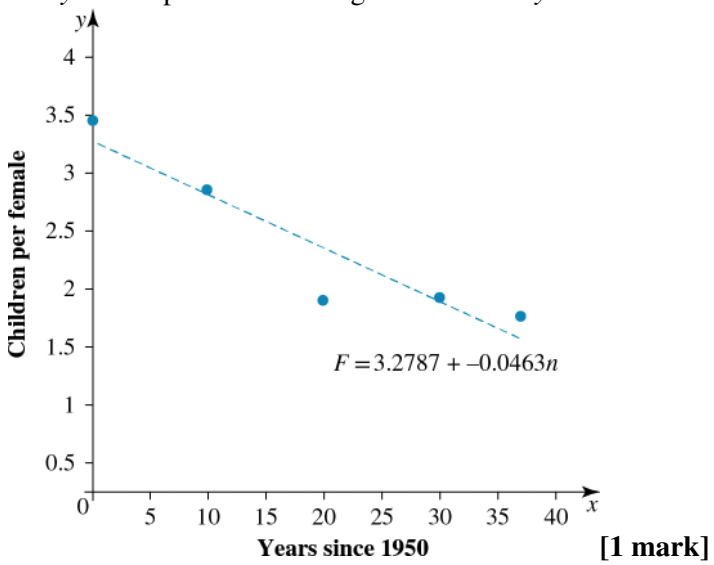
Question 12

a. **[1 mark]**



b. The gradient of the line of good fit is $b = r\frac{s_r}{s_x} = -0.046$

The y-intercept of the line of good fit is $a = \overline{y} - b\overline{x} = 3.28$



[1 mark]

c. $F = 3.28 - 0.046n$ **[1 mark]**

d. i. 2.13

ii. 1.44 **[1 mark]**

($n = 25$ and $n = 40$)

© John Wiley & Sons Australia, Ltd

e. The interpolation is more reliable than the extrapolation because data is known for values nearby. For extrapolation there is no guarantee that the pattern of falling fertility will continue. There are only five data points, which reduces reliability. **[1 mark]**

Question 13

In I, III and IV the point is close to the line of best fit. In II and V the point is away from the line of best fit.

Question 14

The correlation coefficient has a value of 0.25 (weak positive correlation).

Question 15

It would be reasonable to say that there is weak negative correlation between the variables; as -0.42 fits into the range of $-0.5 \leq r \leq -0.25$.

Question 16

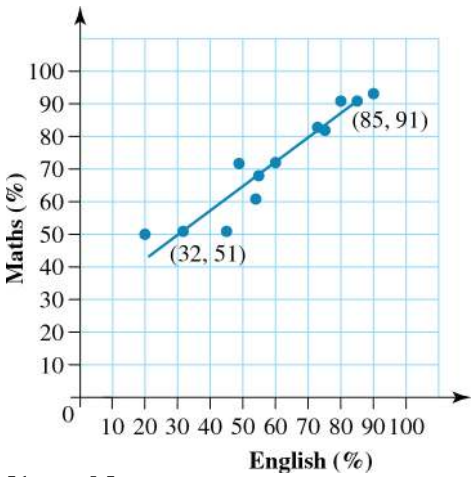
$2.5 \text{ hours} = 5 \text{ half} - \text{hourly increments}$

$C(\$) = 15 + 20n$
$C(\$) = 15 + 20(5)$
$C(\$) = 15 + 100$
$C(\$) = 115$

The total cost of the clean is $115.00.

Question 17

a.



[1 mark]

b. Using the points marked on the graph, calculate the gradient.

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{91 - 51}{85 - 32}$$
$$= \frac{40}{53}$$
$$= 0.75 \text{ (correct to 2 decimal places)} \quad \textbf{[1 mark]}$$

Substitute one point (32, 51) into the equation $y = a + bx$ to calculate a.

$y = a + bx$

$y = a + 0.75x$

$51 = a + 0.75\,(32)$

$51 = a + 24$

$a = 51 - 24$

$a = 27$ **[1 mark]**

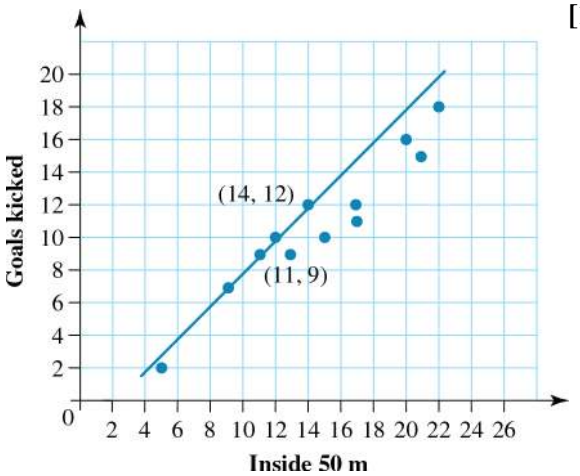
The equation of the line of good fit is $y = 27 + 0.75x$. **[1 mark]**

c. English $= 27 + 0.75$ (Maths) **[1 mark]**

d. For this set of students, there appears to be strong positive correlation between English results and Maths results. **[1 mark]**

Question 18

a. **[1 mark]**



b. $b = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{12 - 9}{14 - 11}$

$= \dfrac{3}{3}$

$= 1$ [1 mark]

Substitute one point (11, 9) into the equation $y = a + bx$ to calculate a.

$y = a + bx$

$y = a + 1x$

$11 = a + 1\,(9)$

$11 = a + 9$

$a = 11 - 9$

$a = 2$ **[1 mark]**

The equation of the line of good fit is $y = 2 + x$. **[1 mark]**

c. Number of goals $= 2 + 1 \times$ (Number of times inside 50 m) **[1 mark]**

d. Least squares line:: $y = -0.90 + 0.81x$ **[1 mark]**

e. The equations are different as the original calculation is based on a line of fit placed by eye. The gradients are quite similar. The least squares regression equation calculated using CAS would be a more reliable equation. **[1 mark]**

f. Based on the scatterplot, the relationship shows strong positive correlation. **[1 mark]**

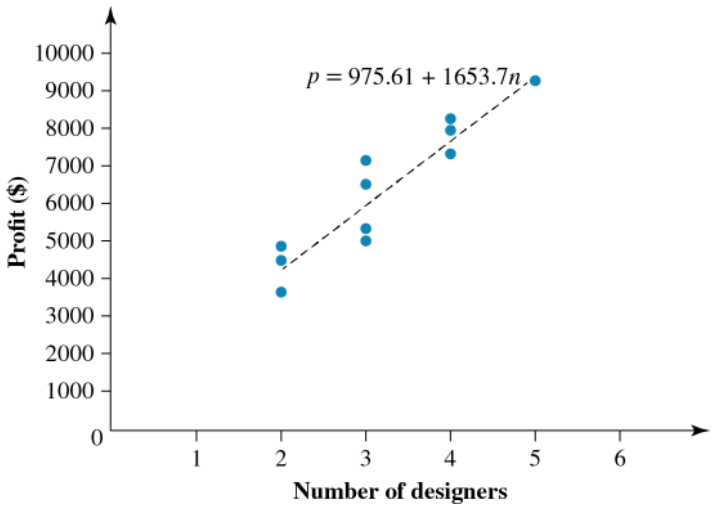
g. $r = 0.96$ which agrees with our finding in part f. **[1 mark]**

© John Wiley & Sons Australia, Ltd

h. It would not be advised to use these findings as the sample is very small. You would need to include many more data points in your analysis. **[1 mark]**

Question 19

a.



[1 mark]

b. $P = 1000 + 1700n$ **[1 mark]**
Actual line of good fit is $P = 976 + 1645n$

c. Using the estimated equation:
$n = 1$,

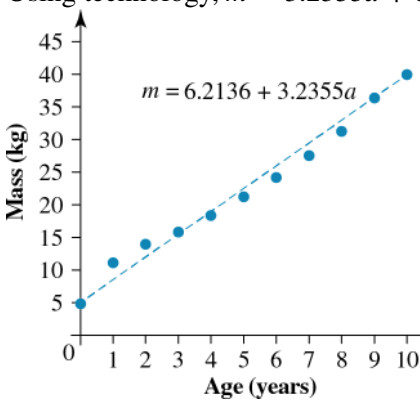
$P = 1000 + 1700\,(1)$
$= 1000 + 1700$
$= \$2700$ **[1 mark]**

d. Using the estimated equation:
$P = 18\,000$,

$P = 1000 + 1700n$
$18\,000 = 1000 + 1700n$
$1700n = 17\,000$
$n = 10$ **[1 mark]**

Question 20

a. Using technology, $m = 3.2355a + 6.2136$



[1 mark]

b. $42a - 13m + 81 = 0$
If $a = 10$,

$420 - 13m + 81 = 0$ or

$$m = \frac{420 + 81}{13}$$
$$= 38.5 \quad \textbf{[1 mark]}$$

c. $40 - 38.5 = 1.5$ kg **[1 mark]**

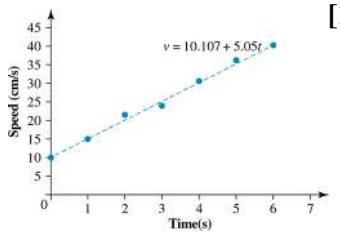
d. $m = 48$

$42a - 13 \times 48 + 81 = 0$

$$a = \frac{624 - 81}{42}$$
$$= 12.9 \quad \textbf{[1 mark]}$$

Question 21

a. **[1 mark]**



b. The graph is linear over the whole range. **[1 mark]**

c. $v = 10.1 + 5.05t$ **[1 mark]**

d. **i.** $v = 10.1 + 5.05t$

$v = 10.1 + 5.05 \times 3.5$

$= 27.78$ **[1 mark]**

ii. $v = 10.1 + 5.05 \times 10$

$= 60.6$ **[1 mark]**

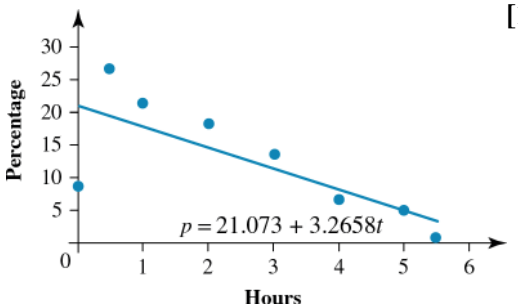
iii. $20 = 10.1 + 5.05t$, so

$$t = \frac{20 - 10.1}{5.05}$$
$$= 1.96 \quad \textbf{[1 mark]}$$

e. All of the predictions are equally likely to be reliable, because the experiment yields results close to the line no matter what the value of t. **[1 mark]**

Question 22

a. **[1 mark]**

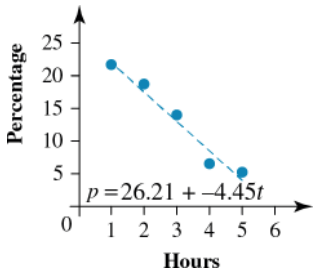


b. The scatterplot appears to be linear from 1 to 5 hours. **[1 mark]**

c. $p = 21.1 - 3.27h$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

d.



$p = 26.2 - 4.45h$ **[1 mark]**

e. i. $p = 26.2 - 4.45h$

$p = 26.2 - 4.45 \times 3.5$

$= 10.6\%$ **[1 mark]**

ii. $p = 26.2 - 4.45 \times 6$

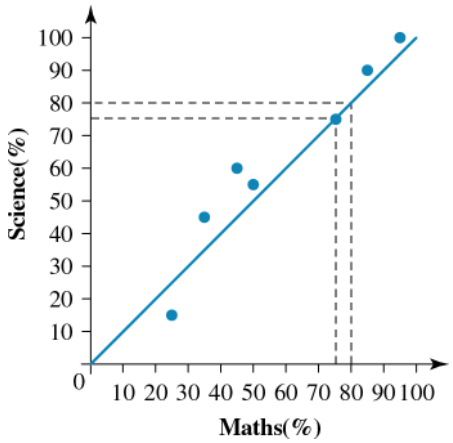
$= -0.5\%$ **[1 mark]**

iii. 0.27 hours **[1 mark]**

f. The percentage of 12 to 15 year olds who use a computer for 3.5 hours per day because this can be found from within the range of the data. **[1 mark]**

Question 23

a.



[1 mark]

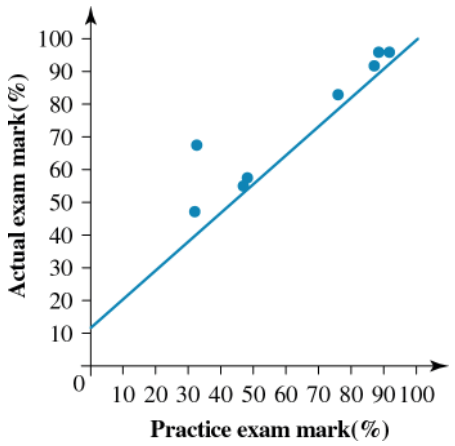
b. If the student achieves 75% for Science, it is estimated that they will achieve close to 75% for Maths also. **[1 mark]**

c. If a student achieves 80% for Maths, it is estimated that they will achieve close to 80% for Science also. **[1 mark]**

© John Wiley & Sons Australia, Ltd

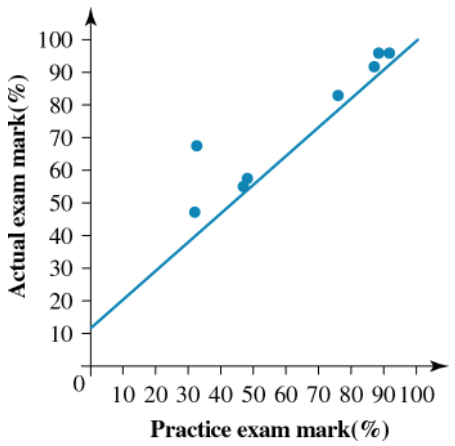
Question 24

a.



Award **1 mark** for the correct scaled axes.
Award **1 mark** for correct plotting of points.

b.



Award **1 mark** for the line of good fit.

c. There is one data point which seems out of place, that being the student who went from 32% on the practice exam to 68% on the actual exam. This one student certainly seemed to improve much more significantly than the other students. **[1 mark]**

d. Take two points on the line.
$(49, 58)$ and $(89, 95)$

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{95 - 58}{89 - 49}$$
$$= \frac{37}{40}$$
$$= 0.925 \quad \textbf{[1 mark]}$$

$y = a + 0.925x$
Substitute in $(49, 58)$ to calculate a. **[1 mark]**

$$y = a + 0.925x$$
$$58 = a + 0.925\,(49)$$
$$58 = a + 45.325$$
$$a = 58 - 45.325$$
$$a = 12.675 \quad \textbf{[1 mark]}$$

The equation of the line of good fit is $y = 12.675 + 0.925x$. **[1 mark]**

© John Wiley & Sons Australia, Ltd

e. The line of good fit shows that students did not perform as well on their actual exam compared to their practice exam. **[1 mark]**
A gradient of 0.925 means that for every mark achieved on the practice test, the student only achieves 0.925 marks on the actual exam. **[1 mark]**

© John Wiley & Sons Australia, Ltd

9 Graphs and networks

Topic	**9**	**Graphs and networks**
Subtopic	**9.2**	**Definitions and terms**

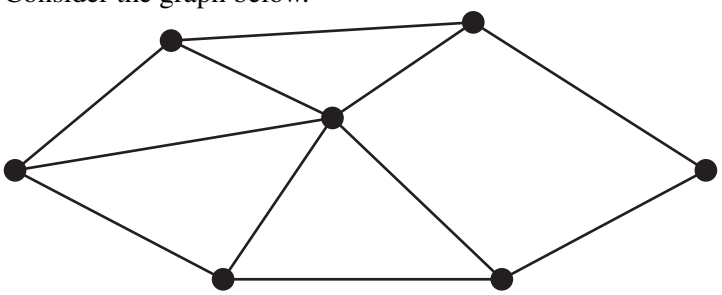
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2021, Further Mathematics Exam 1, Section B, Module 2, Q.1; © VCAA*

Question 1 (1 mark)

Consider the graph below.



The number of vertices with a degree of 3 is

A. 1
B. 2
C. 3
D. 4
E. 5

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 1, Section B, Module 2, Q.8; © VCAA*

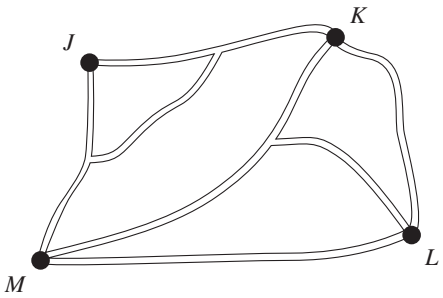
Question 2 (1 mark)

The adjacency matrix below shows the number of pathway connections between four landmarks: J, K, L and M.

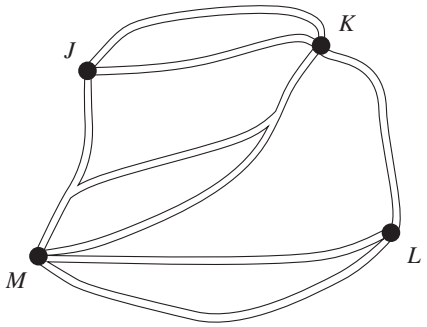
$$\begin{array}{c} \\ J \\ K \\ L \\ M \end{array} \begin{array}{c} \begin{array}{cccc} J & K & L & M \end{array} \\ \begin{bmatrix} 1 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \\ 0 & 1 & 0 & 2 \\ 1 & 2 & 2 & 0 \end{bmatrix} \end{array}$$

A network of pathways that could be represented by the adjacency matrix is

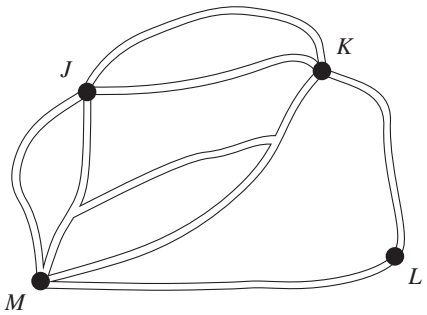
A.



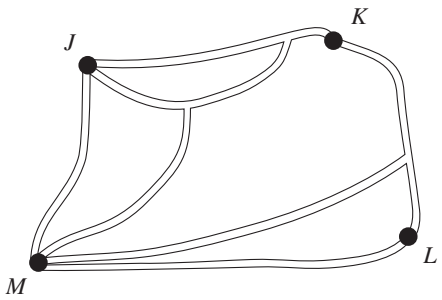
B.



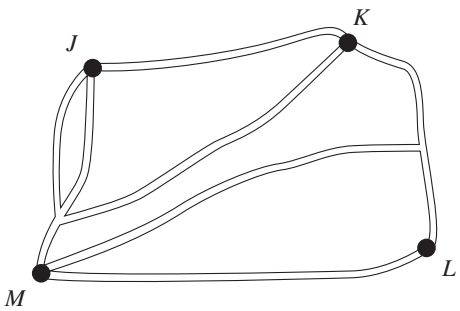
C.



D.



E.



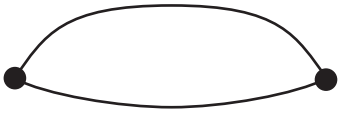
© John Wiley & Sons Australia, Ltd

Source: *VCE 2017, Further Mathematics Exam 1, Section B, Module 2, Q.1; © VCAA*

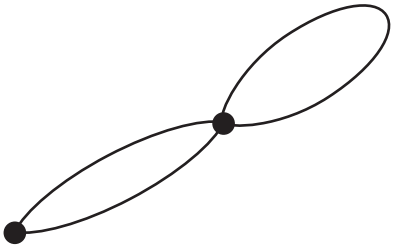
Question 3 (1 mark)

Which one of the following graphs contains a loop?

A.

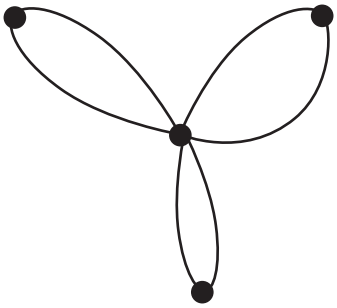


B.

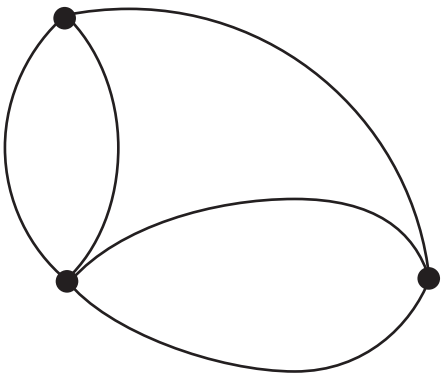


C.

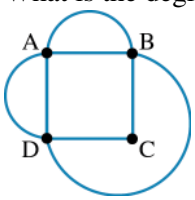
D.



E.

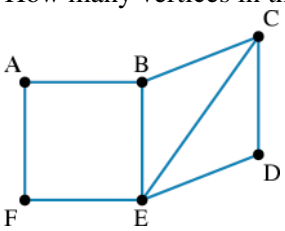


Question 4 (1 mark)
What is the degree of vertex A in this graph?



A. 1
B. 2
C. 3
D. 4
E. 5

Question 5 (1 mark)
How many vertices in this graph have degree 3?

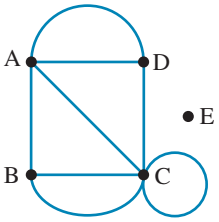


A. 1
B. 2
C. 3
D. 4
E. 5

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)

Which of the following describes this graph?



A. 5 vertices, 6 edges
B. 4 vertices, 8 edges
C. 5 vertices, 8 edges
D. 4 vertices, 7 edges
E. 5 vertices, 7 edges

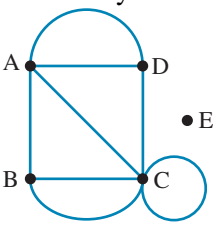
Question 7 (1 mark)

For the graph with adjacency matrix $\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$, what is the degree of vertex B?

A. 0
B. 1
C. 2
D. 3
E. 4

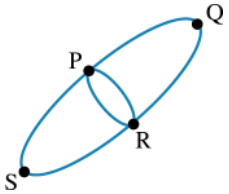
© John Wiley & Sons Australia, Ltd

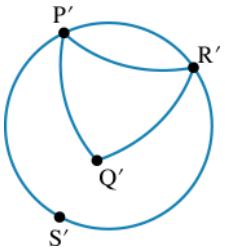
Question 8 (1 mark)
How many vertices in this graph are of odd degree?

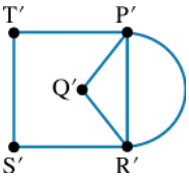


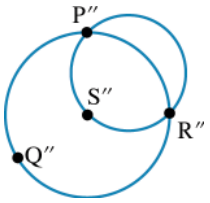
A. 5
B. 4
C. 3
D. 2
E. 1

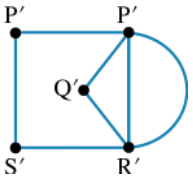
Question 9 (1 mark)
Which of these graphs is not isometric with all the others?

A.


B.


C.


D.


E.


© John Wiley & Sons Australia, Ltd

Topic	9	Graphs and networks
Subtopic	9.3	Planar graphs

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2020, Further Mathematics Exam 1, Section B, Module 2, Q.1; © VCAA*

Question 1 (1 mark)

A connected planar graph has seven vertices and nine edges.

The number of faces that this graph will have is

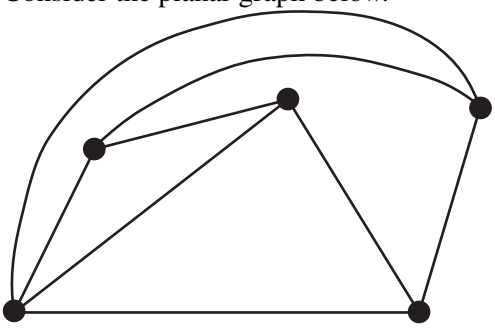
A. 1
B. 2
C. 3
D. 4
E. 5

© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics Exam 1, Section B, Module 2, Q.5; © VCAA*

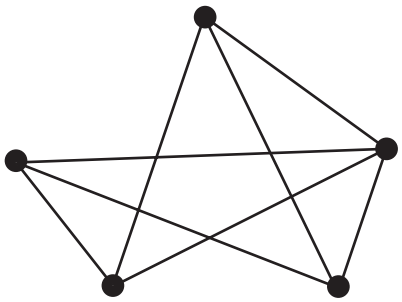
Question 2 (1 mark)

Consider the planar graph below.

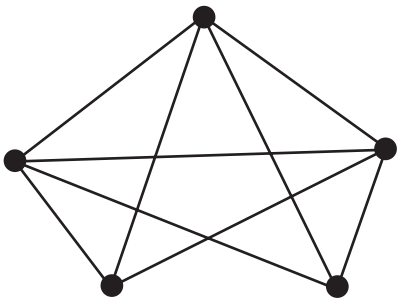


Which one of the following graphs can be redrawn as the planar graph above?

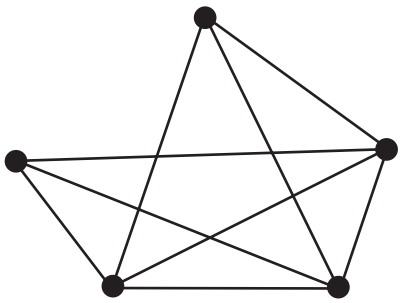
A.



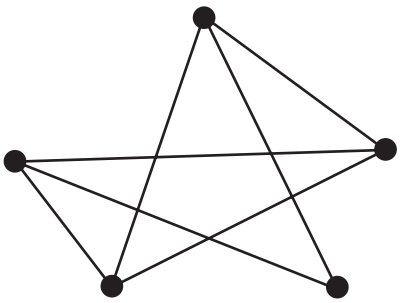
B.



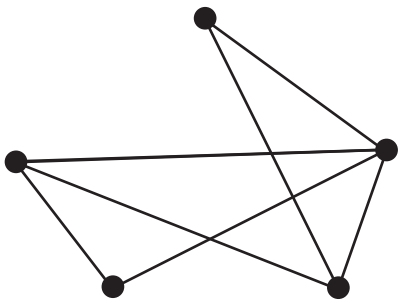
C.



D.



E.



© John Wiley & Sons Australia, Ltd

Source: *VCE 2013, Further Mathematics Exam 1, Section B, Module 5, Q.7; © VCAA*

Question 3 (1 mark)

A connected graph consists of five vertices and four edges.
Consider the following five statements.

- The graph is planar.
- The graph has more than one face.
- All vertices are of even degree.
- The sum of the degrees of the vertices is eight.
- The graph cannot have a loop.

How many of these statements are always true for such a graph?

A. 1
B. 2
C. 3
D. 4
E. 5

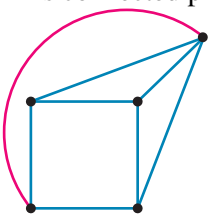
Question 4 (1 mark)

The number of faces in a connected plane network with 12 edges and 10 vertices is

A. 0
B. 6
C. 20
D. 4
E. 12

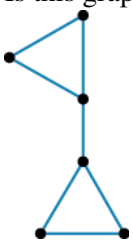
© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)
This connected plane network has



A. $v = 6; e = 9; f = 5$
B. $v = 9; e = 6; f = 5$
C. $v = 5; e = 9; f = 6$
D. $v = 5; e = 8; f = 5$
E. $v = 6; e = 8; f = 6$

Question 6 (1 mark)
Is this graph a connected planar graph; and why or why not?



A. No; $v = 6; f = 2; e = 7$
B. Yes; $v = 6; f = 2; e = 6$
C. No; $v = 6; f = 3; e = 7$
D. Yes; $v = 6; f = 2; e = 7$
E. Yes; $v = 6; f = 3; e = 7$

Question 7 (3 marks)
A connected planar graph had 13 edges and 7 vertices. Some of the edges were removed, resulting in another connected planar graph with 7 vertices and 6 faces. How many edges were removed?
Number of edges removed = □

© John Wiley & Sons Australia, Ltd

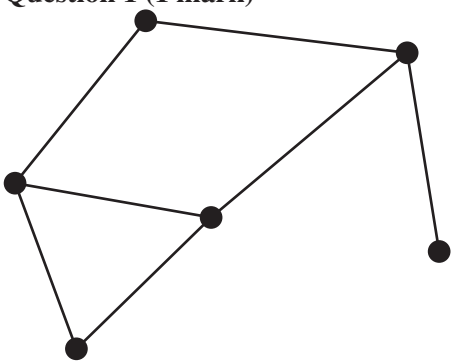
Topic	9	Graphs and networks
Subtopic	9.4	Connected graphs

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2021, Further Mathematics Exam 1, Section B, Module 2, Q.5;* © VCAA

Question 1 (1 mark)



Consider the following five statements about the graph above:

- The graph is planar.
- The graph contains a cycle.
- The graph contains a bridge.
- The graph contains an Eulerian trail.
- The graph contains a Hamiltonian path.

How many of these statements are true?

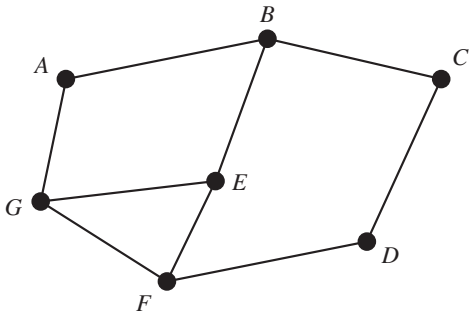
A. 1
B. 2
C. 3
D. 4
E. 5

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 1, Section B, Module 2, Q.2; © VCAA*

Question 2 (1 mark)

Consider the graph below.



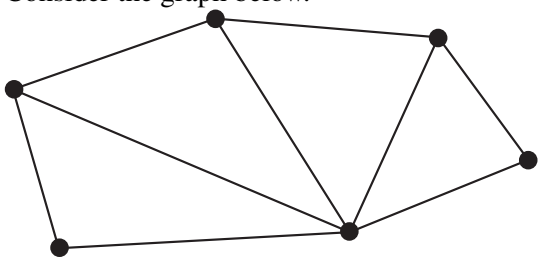
Which one of the following is not a Hamiltonian cycle for this graph?

A. *ABCDFEGA*
B. *BAGEFDCB*
C. *CDFEGABC*
D. *DCBAGFED*
E. *EGABCDFE*

Source: *VCE 2019, Further Mathematics Exam 1, Section B, Module 2, Q.2; © VCAA*

Question 3 (1 mark)

Consider the graph below.



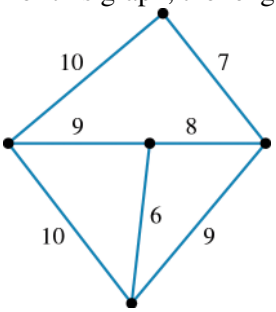
The minimum number of extra edges that are required so that an Eulerian circuit is possible in this graph is

A. 0
B. 1
C. 2
D. 3
E. 4

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)

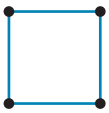
For this graph, the length of the shortest Hamiltonian circuit is



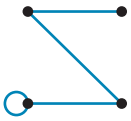
A. 23
B. 36
C. 41
D. 53
E. 56

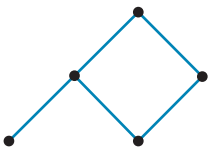
Question 5 (1 mark)

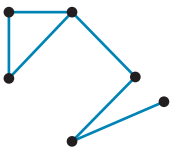
Which of the graphs is a tree?

A. 

B. 

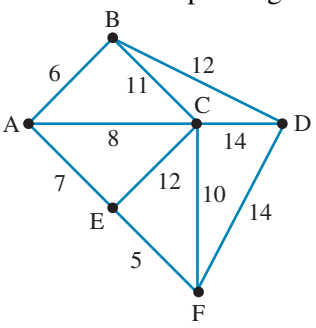
C. 

D. 

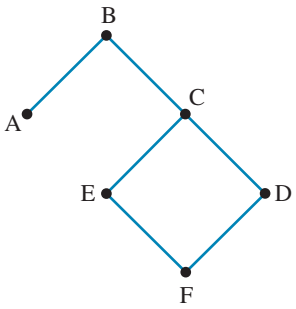
E. 

Question 6 (1 mark)

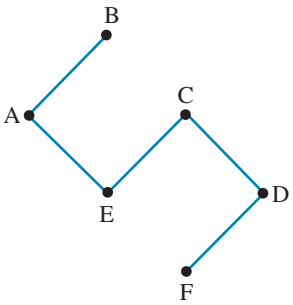
The minimum spanning tree for this network is



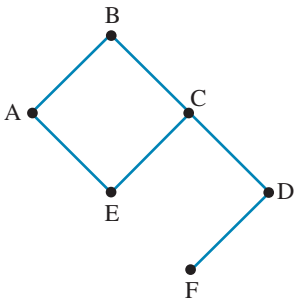
A.



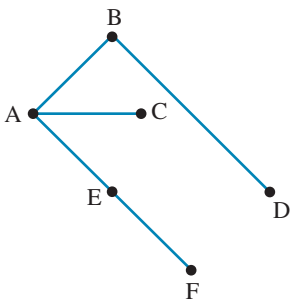
B.



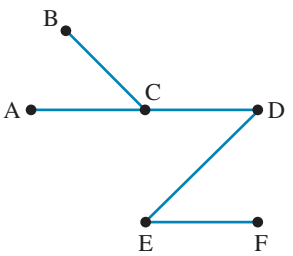
C.



D.



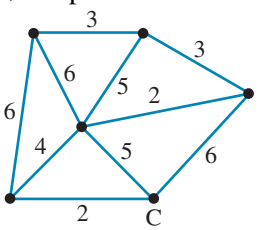
E.



© John Wiley & Sons Australia, Ltd

Question 7 (3 marks)

A number of office computers are to be connected to a central computer server, C, in the cheapest way. A graph is drawn showing the computers as vertices. The lengths of cables are shown in metres. Cabling costs $200 per metre.



a. Draw the minimum spanning tree that could be used to find the cheapest layout for the cabling. **(1 mark)**

b. What would be the total length of cabling needed?

Total length = □m **(1 mark)**

c. How much would the cabling cost?

The cabling would cost $ □ **(1 mark)**

© John Wiley & Sons Australia, Ltd

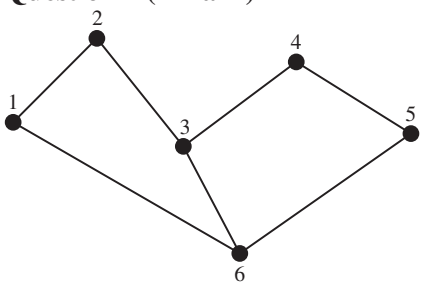
Topic	9	Graphs and networks
Subtopic	9.5	Weighted graphs and networks, and trees

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

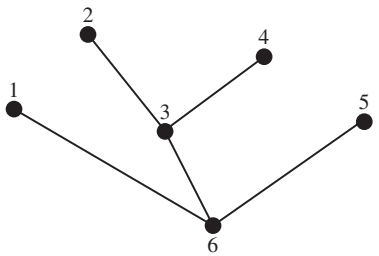
Source: *VCE 2020, Further Mathematics Exam 1, Section B, Module 2, Q.3;* © VCAA

Question 1 (1 mark)

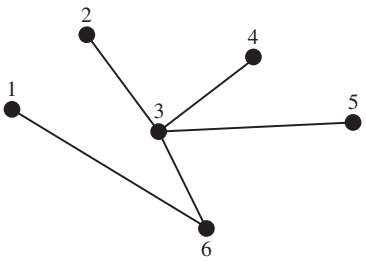


Which one of the following is not a spanning tree for the network above?

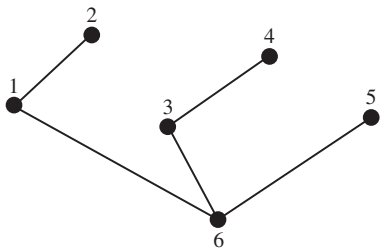
A.



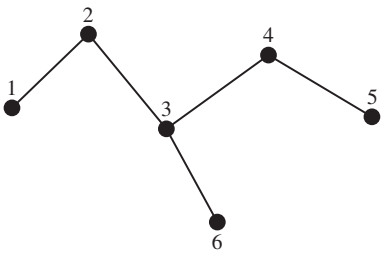
B.



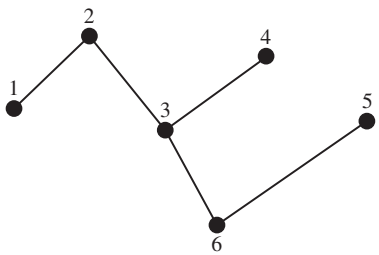
C.



D.



E.



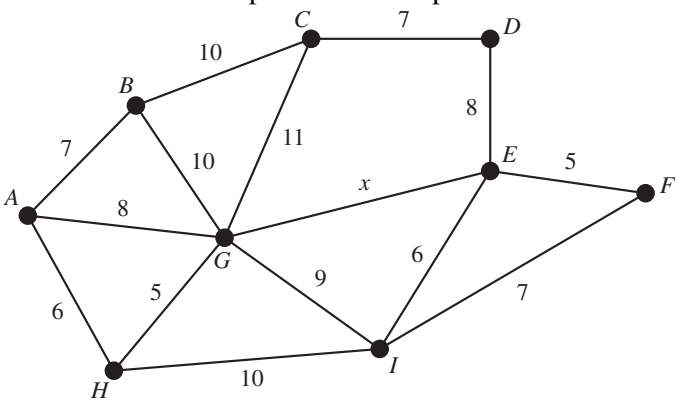
© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics Exam 1, Section B, Module 2, Q.5;* © VCAA

Question 2 (1 mark)

The network below shows the distances, in metres, between camp sites at a camping ground that has electricity.

The vertices *A* to *I* represent the camp sites.



The minimum length of cable required to connect all the camp sites is 53 m.

The value of *x*, in metres, is at least

A. 5

B. 6

C. 8

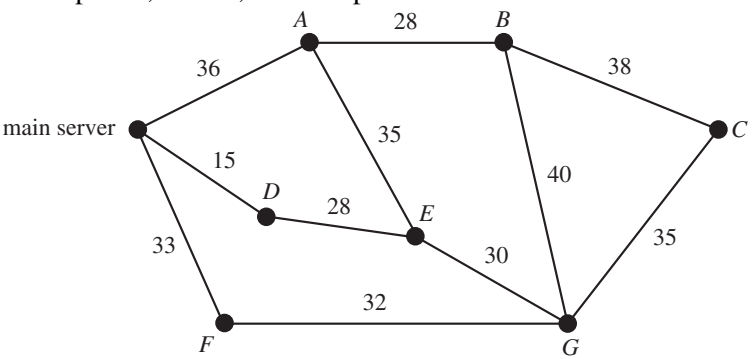
D. 9

E. 11

Source: *VCE 2019, Further Mathematics Exam 1, Section B, Module 2, Q.5*; © VCAA

Question 3 (1 mark)

The following diagram shows the distances, in metres, along a series of cables connecting a main server to seven points, *A* to *G*, in a computer network.



The minimum length of cable, in metres, required to ensure that each of the seven points is connected to the main server directly or via another point is

A. 175
B. 203
C. 208
D. 221
E. 236

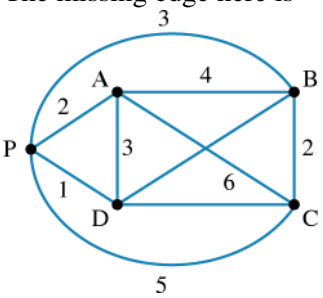
© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)

The adjacency matrix shows the cost of running water pipes (in $100 000 s) around a network PABCD.

	P	A	B	C	D
P		2	3	5	1
A	2		4	6	3
B	3	4		2	1
C	5	6	2		1
D	1	3	1	1	

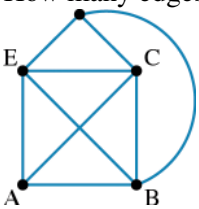
The missing edge here is



A. 1
B. 2
C. 3
D. 4
E. 6

Question 5 (1 mark)

How many edges are there in the minimum spanning tree?

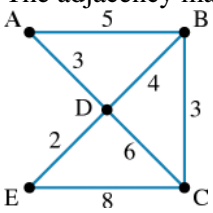


A. 4
B. 5
C. 6
D. 7
E. 8

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)

The adjacency matrix for this network is shown below.



	A	B	C	D	E
A		5		3	
B	5		3	·	
C		3		6	8
D	3	4	6		2
E			8	2	

The missing number, marked by the •, in the adjacency matrix is

A. 6
B. 5
C. 4
D. 3
E. 2

© John Wiley & Sons Australia, Ltd

Topic	9	Graphs and networks
Subtopic	9.6	Review

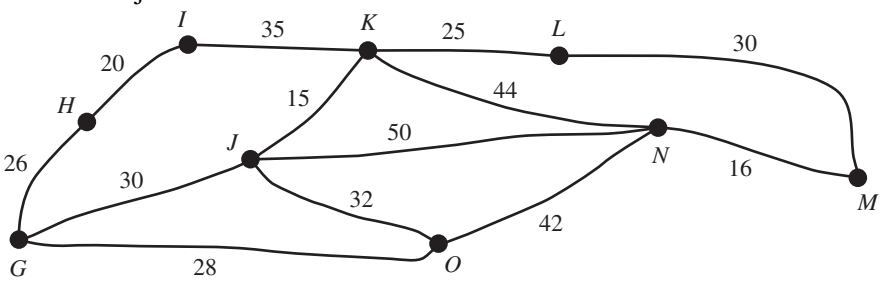
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2021, Further Mathematics Exam 2, Section B, Module 2, Q.2;* © VCAA

Question 1 (2 marks)

George lives in Town *G* and Maggie lives in Town *M*.
The diagram below shows the network of main roads between Town *G* and Town *M*.
The vertices *G*, *H*, *I*, *J*, *K*, *L*, *M*, *N* and *O* represent towns.
The edges represent the main roads. The numbers on the edges indicate the distances, in kilometres, between adjacent towns.



a. What is the shortest distance, in kilometres, between Town *G* and Town *M*?
The shortest distance is ☐ km. **(1 mark)**

b. George plans to travel to Maggie's house. He will pass through all the towns shown above.
George plans to take the shortest route possible.
Which town will George pass through twice? **(1 mark)**

☐ *G*
☐ *H*
☐ *I*
☐ *J*
☐ *K*
☐ *L*
☐ *M*
☐ *N*
☐ *O*

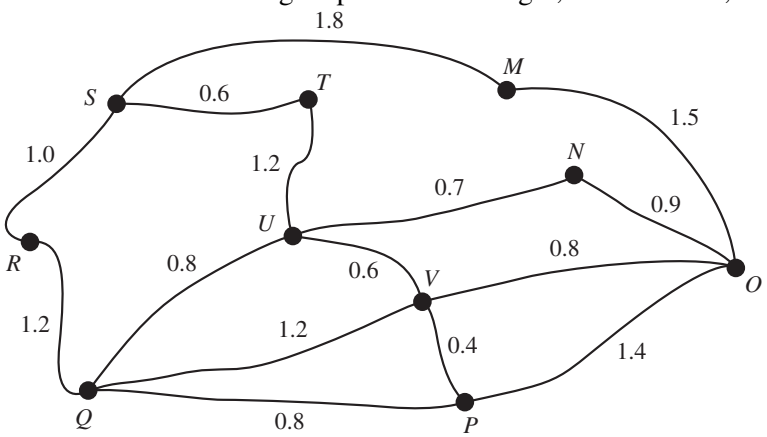
Source: *VCE 2020, Further Mathematics Exam 2, Section B, Module 2, Q.3*; © VCAA

Question 2 (4 marks)

A local fitness park has 10 exercise stations: *M* to *V*.

The edges on the graph below represent the tracks between the exercise stations.

The number on each edge represents the length, in kilometres, of each track.



The Sunny Coast cricket coach designs three different training programs, **all starting at exercise station *S*.**

Training program number	Training details
1	The team must run to exercise station *O*.
2	The team must run along all tracks just once.
3	The team must visit each exercise station and return to exercise station *S*.

a. What is the shortest distance, in kilometres, covered in training program1?

The shortest distance is ☐ km. **(1 mark)**

b. Answer the following.

i. What mathematical term is used to describe training program 2? **(1 mark)**

ii. At which exercise station would training program 2 finish? **(1 mark)**

c. To complete training program 3 in the minimum distance, one track will need to be repeated.

Complete the following sentence by filling in the blanks provided.

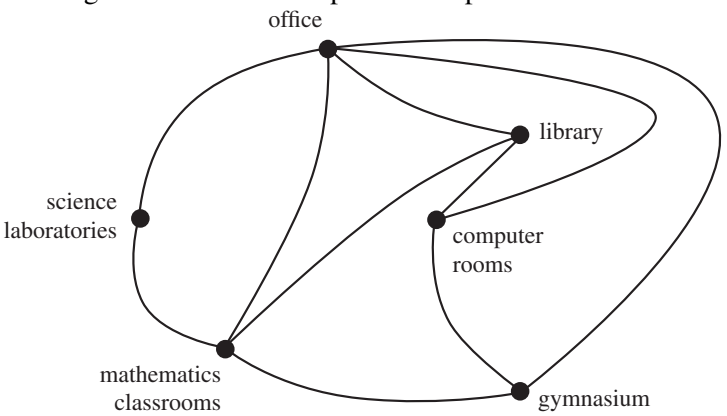
This track is between exercise station ☐ and exercise station ☐. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics Exam 2, Section B, Module 2, Q.1;* © VCAA

Question 3 (3 marks)

Fencedale High School has six buildings. The network below shows these buildings represented by vertices. The edges of the network represent the paths between the buildings.

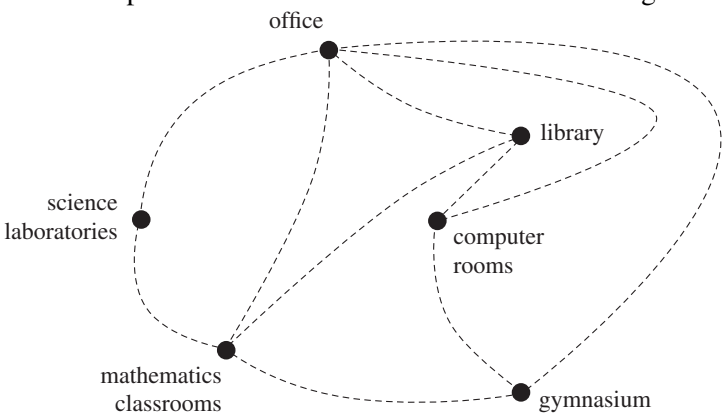


a. Which building in the school can be reached directly from all other buildings?
The building that can be reached directly from all other buildings is the ☐ **(1 mark)**

b. A school tour is to start and finish at the office, visiting each building only once.

i. What is the mathematical term for this route? **(1 mark)**

ii. Draw in a possible route for this school tour on the diagram below. **(1 mark)**

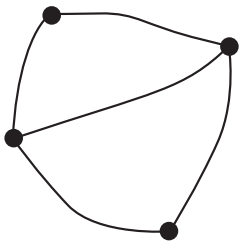


Source: *VCE 2019, Further Mathematics Exam 1, Section B, Module 2, Q.4*; © VCAA

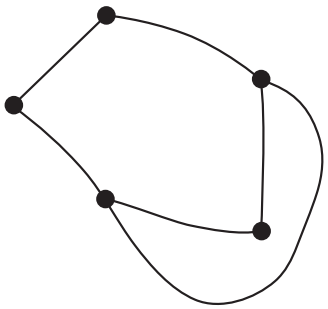
Question 4 (1 mark)

Two graphs, labelled Graph 1 and Graph 2, are shown below.

Graph 1



Graph 2



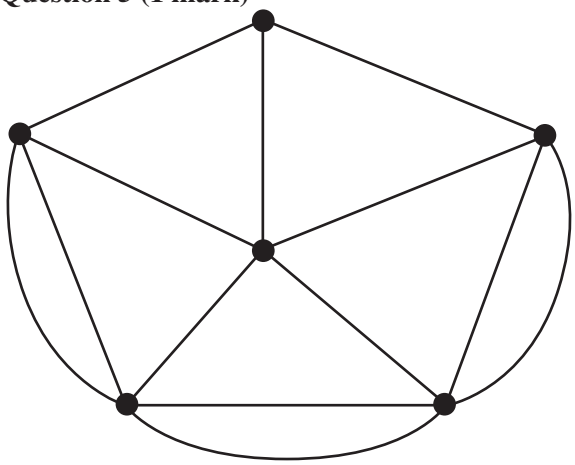
Which one of the following statements is **not** true?

A. Graph 1 and Graph 2 are isomorphic.
B. Graph 1 has five edges and Graph 2 has six edges.
C. Both Graph 1 and Graph 2 are connected graphs.
D. Both Graph 1 and Graph 2 have three faces each.
E. Neither Graph 1 nor Graph 2 are complete graphs.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2017, Further Mathematics Exam 1, Section B, Module 2, Q.6; © VCAA*

Question 5 (1 mark)



An Eulerian trail for the graph above will be possible if only one edge is removed.
In how many different ways could this be done?

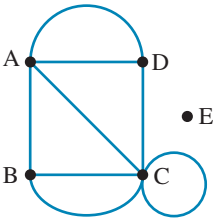
A. 1
B. 2
C. 3
D. 4
E. 5

Question 6 (1 mark)

If a graph has four vertices, what is the least number of edges it could have so that it is connected?
The least number of edges is □.

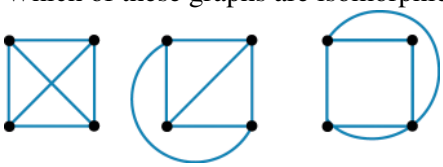
© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)
Which vertices in this graph have degree 4?



☐ Vertex **A**
☐ Vertex **B**
☐ Vertex **C**
☐ Vertex **D**
☐ Vertex **E**

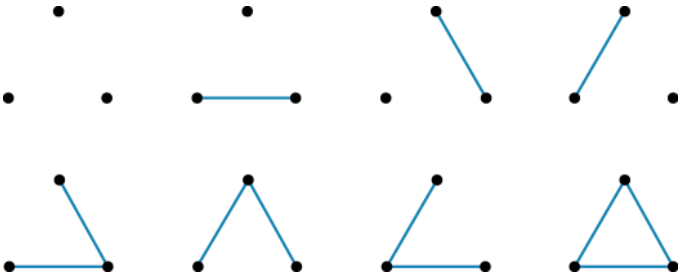
Question 8 (2 marks)
Which of these graphs are isomorphic?



© John Wiley & Sons Australia, Ltd

Question 9 (2 marks)

The diagram shows the different numbers of edges that can be drawn with three vertices.



Some of these are isomorphic.

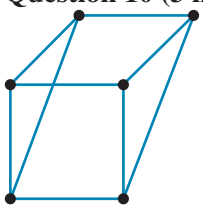
a. Draw the ones that are unique. **(1 mark)**

b. How many unique graphs can be drawn with three vertices?

Then number of unique graphs is ☐. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 10 (3 marks)



a. Redraw the graph so there is no crossing over. **(1 mark)**

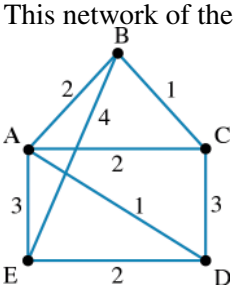
b. How many vertices, faces and edges does it have? **(1 mark)**

c. Is Euler's formula obeyed? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 11 (3 marks)

This network of the roads between five suburbs shows the distances in kilometres.



a. Draw the minimum spanning tree. **(1 mark)**

b. How many edges are in the minimum spanning tree? **(1 mark)**

Number of edges = ☐

c. Find the shortest distance around the network if the suburbs are visited only once. **(1 mark)**

The shortest distance = ☐ km

© John Wiley & Sons Australia, Ltd

Question 12 (3 marks)
The adjacency matrix shows the lengths (in metres) of possible links from computers A, B and C to the main server S:

	S	A	B	C
S		6	3	5
A	6		7	9
B	3	7		4
C	5	9	4	

a. Use Prim's algorithm to find the minimum spanning tree. **(1 mark)**

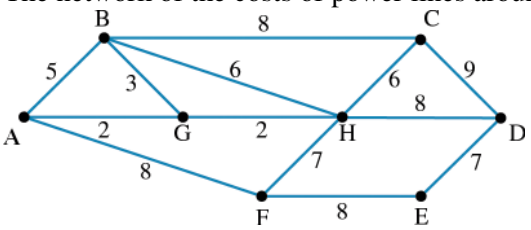
b. What is the total length of the network? **(1 mark)**
Total length of the network = ☐ m

c. What is the total cost of the network, if cables cost $100 per metre? **(1 mark)**
The total network cost is $ ☐.

© John Wiley & Sons Australia, Ltd

Question 13 (2 marks)

The network of the costs of power lines around eight sites is shown.



The cost of installing electricity ($ millions) depends on the route the lines take.

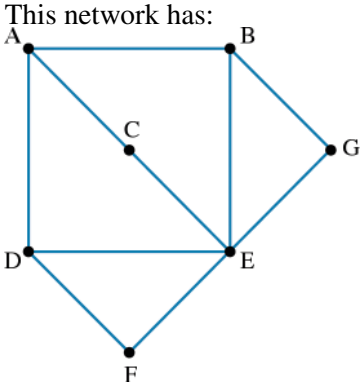
a. Draw the route of the lines for minimum cost, starting at the generator at H. **(1 mark)**

b. What would be the total minimum cost for the installation? **(1 mark)**

The total minimum cost is $ ☐ milllion.

Question 14 (1 mark)

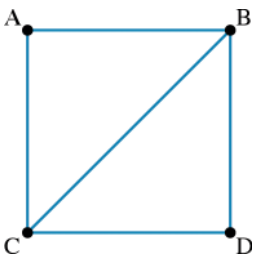
This network has:

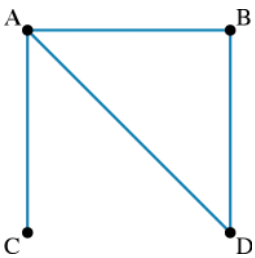


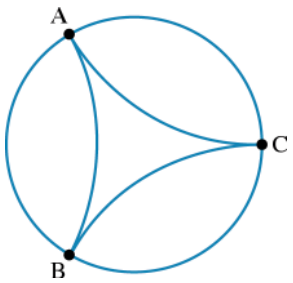
A. 7 vertices and 9 edges
B. 7 vertices and 10 edges
C. 6 vertices and 9 edges
D. 6 vertices and 10 edges
E. 7 vertices and 11 edges

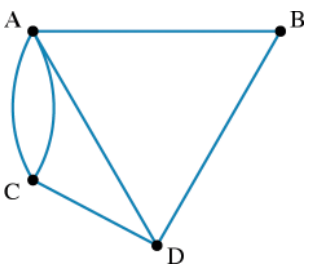
Question 15 (1 mark)

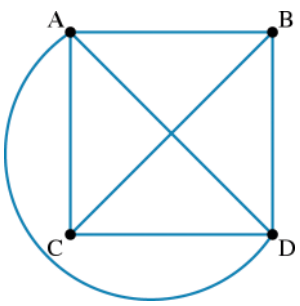
The connected graph with 4 vertices and 6 edges is:

A.


B.


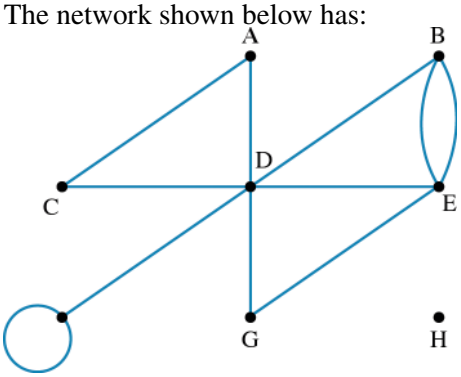
C.


D.


E.


© John Wiley & Sons Australia, Ltd

Question 16 (1 mark)
The network shown below has:



A. 7 vertices, 11 edges and 6 faces.
B. 7 vertices, 11 edges and 5 faces.
C. 8 vertices, 11 edges and 5 faces.
D. 8 vertices, 12 edges and 6 faces.
E. 8 vertices, 11 edges and 6 faces.

Question 17 (1 mark)
The correct form of Euler's formula is:
A. $v + f + e = 2$
B. $v - f - e = 2$
C. $v + f - e = 2$
D. $v + f = 2 - e$
E. $f - e = 2 + v$

Question 18 (1 mark)
A connected planar graph with 4 edges and 3 vertices will have:
A. 2 faces.
B. 3 faces.
C. 4 faces.
D. 5 faces.
E. 1 face.

© John Wiley & Sons Australia, Ltd

Question 19 (1 mark)

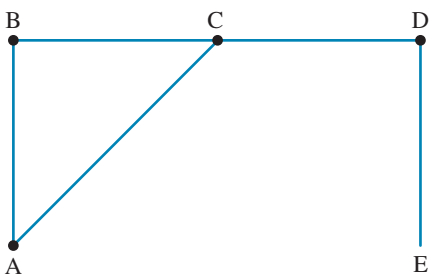
The number of vertices a connected planar graph must have, if it has 11 edges and 6 faces is:

A. 7
B. 6
C. 8
D. 9
E. 5

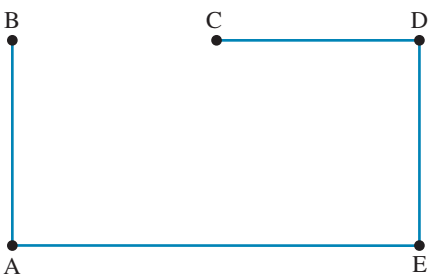
Question 20 (1 mark)

The network which contains an Eulerian circuit is:

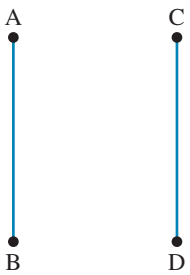
A.



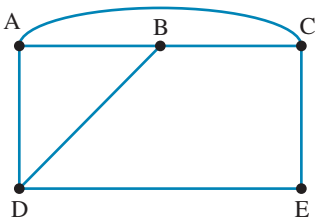
B.



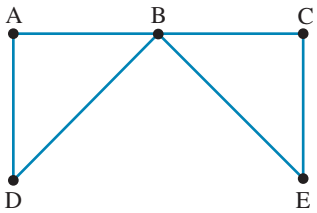
C.



D.



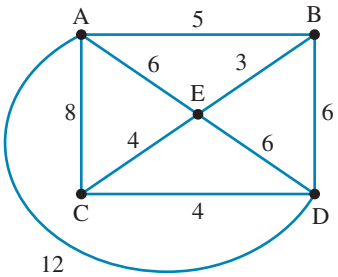
E.



© John Wiley & Sons Australia, Ltd

Question 21 (1 mark)

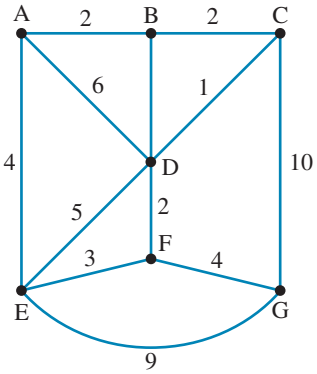
Using the network shown below, the shortest path from vertex A to D is:



A. A – E – D
B. A – C – D
C. A – B – D
D. A – D
E. A – E – B – D

Question 22 (1 mark)

Using the network shown below, the shortest path from vertex A to G is:

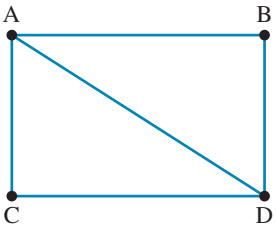


A. A – B – C – G
B. A – D – F – G
C. A – E – F – G
D. A – B – D – F – G
E. A – E – G

© John Wiley & Sons Australia, Ltd

Question 23 (1 mark)

The matrix representation of the network below is:



A. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

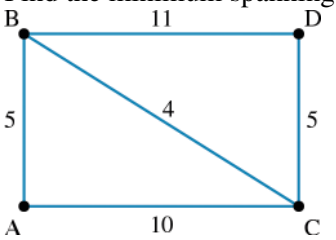
C. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

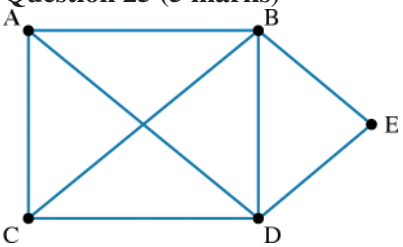
Question 24 (2 marks)

Find the minimum spanning tree of the network below using matrices.



© John Wiley & Sons Australia, Ltd

Question 25 (5 marks)



Answer the following.

a. Redraw the network so that it is planar. **(1 mark)**

b. Determine how many vertices, edges and faces the planar network has. **(1 mark)**

c. Does this network satisfy Euler's formula? **(1 mark)**

d. Does the planar network contain an Euler trail? **(1 mark)**

e. Does the planar network contain an Euler circuit? **(1 mark)**

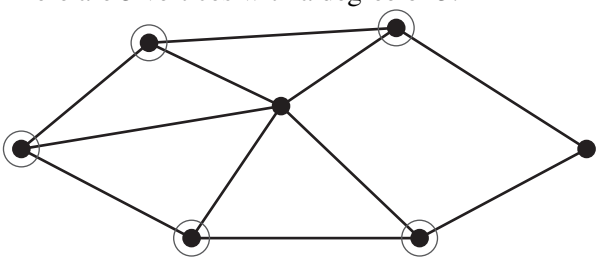
© John Wiley & Sons Australia, Ltd

Answers and marking guide

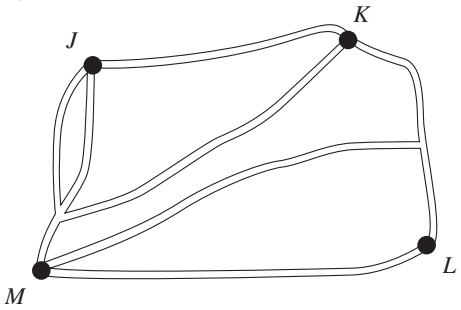
9.2 Definitions and terms

Question 1

There are 5 vertices with a degree of 3:



Question 2



Question 3

A loop joins a vertex to itself, so option B has the only loop.

Question 4

deg (A) = 4, as there are four edges connected to vertex A

Question 5

Two vertices have degree 3: vertices B and C each have three edges connected to them.

Question 6

There are 5 vertices and 8 edges. E is a vertex even though it is not connected.

Question 7

The degree of a vertex is the sum of the row of the adjacency matrix.
Therefore, the degree of vertex *B* is:

$$\deg(B) = 1 + 1$$
$$= 2$$

Question 8

The degree of a vertex is the number of edges connected to it.
deg (A) = 4 (even)
deg (B) = 3 (odd)
deg (C) = 6 (even)
deg (D) = 3 (odd)
deg (E) = 0 (neither)
Therefore, two vertices (B and D) are of odd degree.

Question 9

P does not map 1:1 onto P′′.

© John Wiley & Sons Australia, Ltd

9.3 Planar graphs

Question 1

$v + f - e = 2$

$7 + f - 9 = 2$

$f = 4$

Question 2

The planar graph has 8 edges, so the original must have 8 edges too.

Question 3

Using Euler's formula, the number of faces for this graph is
$f = 2 - v + e = 2 - 5 + 4 = 1$.
Because there is only 1 face, the graph can always be redrawn so that no edges meet.
Therefore the graph is always planar, so statement (1) is true.
As there is only 1 face, statement (2) is not true.
Because the number of edges is 1 less than the number of vertices, at least 2 vertices will not be directly connected by the edge and will hence have degree 1 (odd degree). Therefore, statement (3) is not true.
The sum of degrees of vertices = number of edges $\times 2 = 4 \times 2 = 8$. Therefore, statement (4) is true.
For the graph with v vertices to be connected, the minimum number of edges is $e = v - 1$, which is the case for this graph: $4 = 5 - 1$.
Because the graph has the minimum number of edges, a loop is not possible. (If an edge is used to form a loop, there will not be enough edges left to connect all vertices.) Therefore, statement (5) is true.
Statements (1), (4) and (5) are true. Therefore, the number of statements that are always true for this graph is 3.

Question 4

$v + f - e = 2$

$10 + f - 12 = 2$

$f = 4$

Question 5

$v = 5; e = 8; f = 5$
Outside the figure counts as a face.

Question 6

If $v = 6$; $f = 3$; $e = 7$
$v + f - e = 2$
$6 + 3 - 7 = 2$
$\therefore$ Euler's formula is satisfied.

Question 7

$v + f - e = 2$

$7 + f - 13 = 2$

$f = 8$ **[1 mark]**
The new graph has the same number of vertices but two fewer faces.

© John Wiley & Sons Australia, Ltd

$v + f - e = 2$

$7 + 6 - e = 2$

$e = 11$ **[1 mark]**

∴ Two edges were removed. **[1 mark]**

9.4 Connected graphs

Question 1

There is no Eulerian trail, because it is not possible to get to and from the vertex on the right without passing over the edge twice. The other 4 statements are true.

Question 2

A Hamiltonian cycle passes through each vertex only once and starts and finishes at the same vertex. There is no edge connecting *E* and *D*.

Question 3

For an Eulerian circuit to be possible, all vertices must be of even degree. There are four vertices with odd degree vertices, so two more edges are needed.

Question 4

A Hamiltonian circuit is one that touches each vertex just once.
The shortest follows the route: $9 + 6 + 9 + 7 + 10 = 41$

Question 5

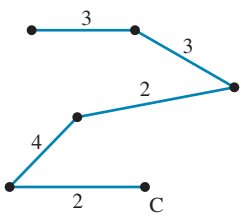
Option B is a tree as it contains no circuits, loops or multiple edges and contains only one region.
Option A contains a circuit.
Option C contains a loop.
Option D contains a circuit.
Option E contains a circuit.

Question 6

Option D is the minimum spanning tree for this network.
Option A is not a tree.
Option B is not a minimum.
Option C is not a tree.
Option E has some edges that are not in the original network.

Question 7

a. **[1 mark]**



b. $3 + 3 + 2 + 4 + 2 = 14$ m **[1 mark]**

c. Total cost of cabling $= \$200 \times 14$

$= \$2800$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

9.5 Weighted graphs and networks, and trees

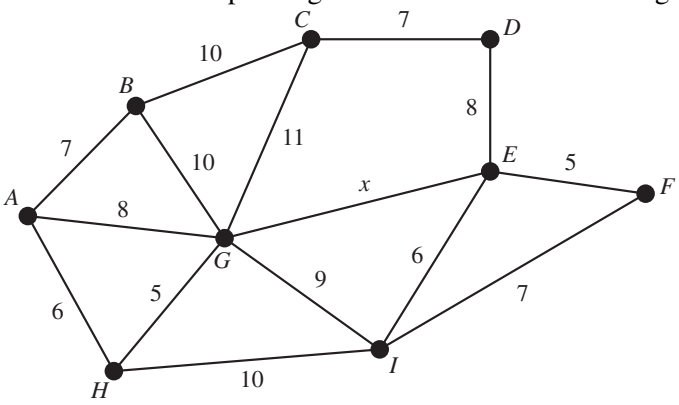
Question 1

A spanning tree includes all the vertices and some of the edges of the original network, and no loops, multiple edges or cycles.

Option B has an edge connecting vertices 3 and 5 that is not present in the original network.

Question 2

Draw a minimum spanning tree that has a minimum length of 53 m.



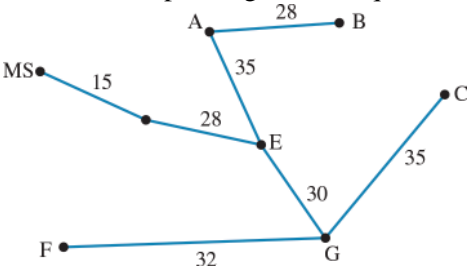
$x = 53 - (7 + 6 + 5 + 6 + 5 + 8 + 7)$

$= 53 - 44$

$= 9$

Question 3

A minimum spanning tree is required.

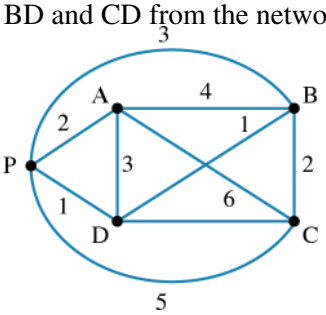


Add up the edges:

$15 + 28 + 30 + 32 + 35 + 35 + 28 = 203$

Question 4

BD and CD from the network is 1 in the matrix.



Question 5

There are $n = 5$ vertices, so $n - 1 = 4$ edges

Question 6

The missing number is 4, BD from the matrix is 4 on the network.

© John Wiley & Sons Australia, Ltd

9.6 Review

Question 1

a. Shortest distance is for $G - O - N - M$ for 86 km **[1 mark]**

b. George will pass through K twice ($G - H - I - K - L - K - J - O - N - M$) **[1 mark]**

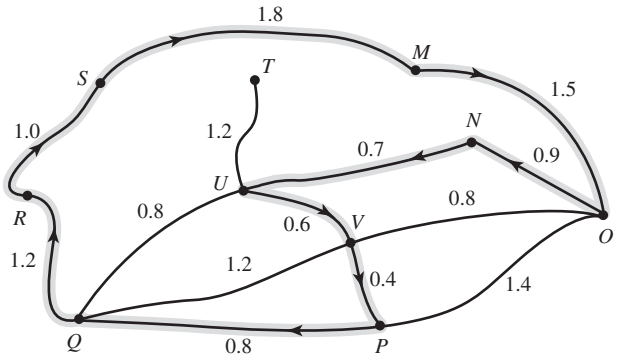
Question 2

a. Shortest distance = 0.6 + 1.2 + 0.6 + 0.8 = 3.2 kilometres **[1 mark]**

b. i. Eulerian trail **[1 mark]**

ii. Eulerian trails start and finish at vertices with an odd degree. The training program starts at S, with a degree of 3, and will finish at P, also with a degree of 3. **[1 mark]**

c. This track is between exercise station S and exercise station T. **[1 mark]**

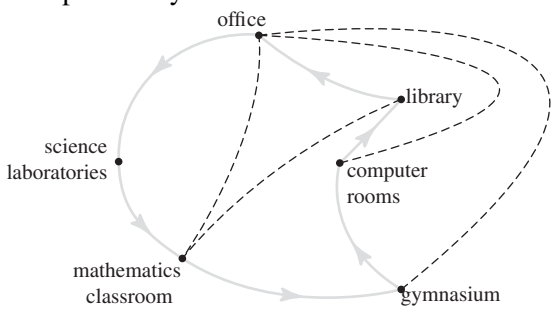


Question 3

a. The office **[1 mark]**

b. i. Hamiltonian cycle **[1 mark]**

ii. One possibility:



[1 mark]

Question 4

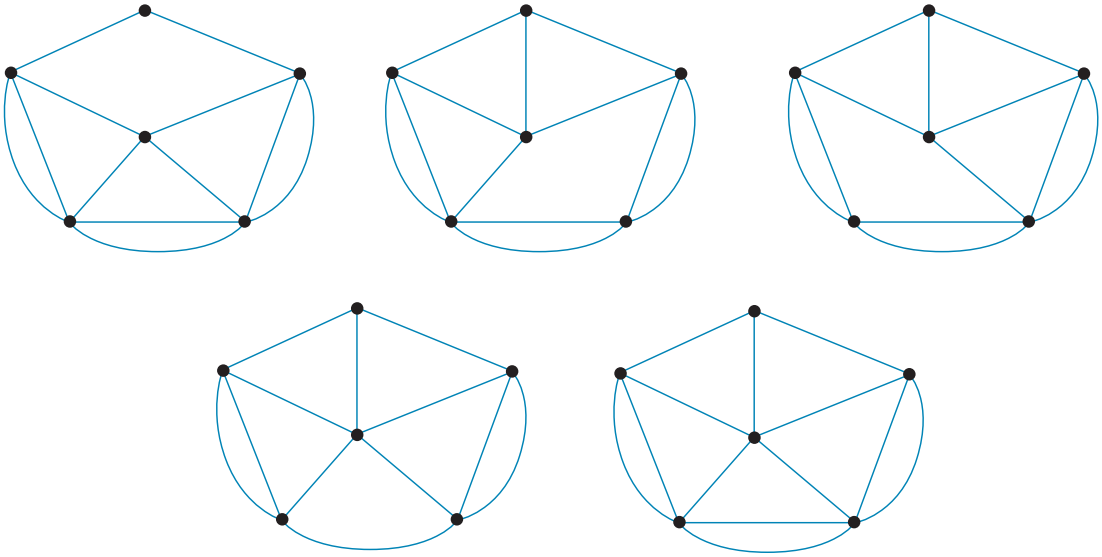
Isomorphic means the equivalent number of vertices and edges.

Graph 1 has four vertices and five edges, while Graph 2 has five vertices and six edges. Therefore, Graphs 1 and 2 are not isomorphic.

Question 5

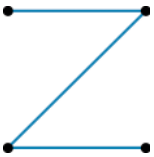
An Eulerian trail exists if the graph has two vertices with an odd degree and the degree of the other vertices are even. The given graph has six vertices of which four are of an odd degree and two are of an even degree. Removing any edge between two vertices that are of an odd degree will change the network to an Eulerian trail. There are five different ways in which this can be done.

© John Wiley & Sons Australia, Ltd



Question 6

The least number of edges needed to connect four vertices is 3. **[1 mark]**



Question 7

deg (A) = 4

deg (B) = 3

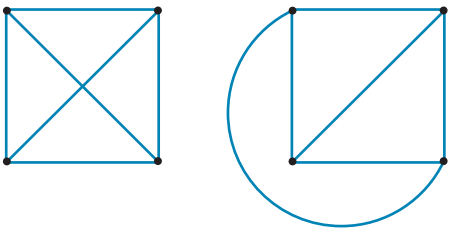
deg (C) = 6 (loop counts as 2)

deg (D) = 3

deg (E) = 0

Therefore, vertex A has degree 4. **[1 mark]**

Question 8



Award **1 mark** for each correct graph.

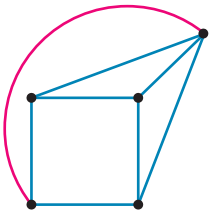
Question 9

a.  **[1 mark]**

b. 4 **[1 mark]**

Question 10

a. **[1 mark]**



b. 5 vertices, 5 faces and 8 edges **[1 mark]**

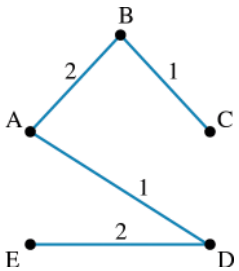
c. $v - f - e = 2$

$5 + 5 - 8 = 2$

Therefore Euler's formula is obeyed. **[1 mark]**

Question 11

a. **[1 mark]**



b. There are four edges in the minimum spanning tree. **[1 mark]**

c. $2 + 1 + 2 + 1 = 6$ km **[1 mark]**

Start at E and then choose the smallest edge at each vertex. You can start at another vertex and use a different layout, but you will use the same number of edges and get the same shortest distance.

Question 12

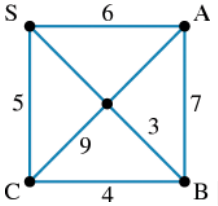
a.

	S(1)	A	B	C
S		6	3	5
A	6		7	9
B	3*	7		4
C	5	9	4	

	S(1)	A	B (2)	C
S		6	3	5
A	6		7	9
B	3*	7		4
C	5	9	4	

	S(1)	A	B (2)	C (3)
S		6	3	5
A	6		7	9
B	3*	7		4
C	5	9	4*	

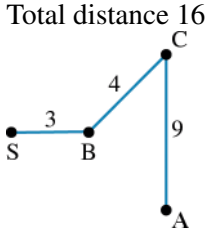
Or can use the network



[1 mark]

© John Wiley & Sons Australia, Ltd

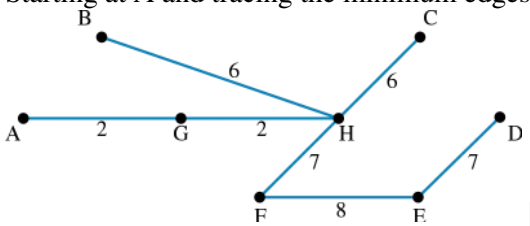
b. Total distance 16 metres **[1 mark]**



c. The total cost is $1600. **[1 mark]**

Question 13

a. Starting at A and tracing the minimum edges gives the minimum cost tree.



[1 mark]

b. $2 + 2 + 6 + 6 + 7 + 8 + 7 = \$38$ million **[1 mark]**

Question 14

7 vertices and 10 edges.

Question 15

Option D is the only connected graph with 4 vertices and 6 edges. **[1 mark]**

Question 16

This network has 8 vertices, 11 edges and 6 faces.

Question 17

Eulers formula is $v + f - e = 2$.

Question 18

$v + f - e = 2$

$v = 3$ and $e = 4$

$3 + f - 4 = 2$

$3 + f = 6$

$f = 6 - 3$

$f = 3$

Question 19

$v + f - e = 2$

$f = 6$ and $e = 11$

$v + 6 - 11 = 2$

$v + 6 = 13$

$v = 13 - 6$

$v = 7$

Question 20

Option E contains a Eulerian circuit as it is possible to make a complete loop starting and finishing at the same vertex, while only crossing each edge once.

© John Wiley & Sons Australia, Ltd

Question 21
The shortest path is
A – B – D
5 + 6 = 11

Question 22
The shortest path is
A – E – F – G
4 + 3 + 4 = 11

Question 23
Vertex A connects with each of the other vertices.
Vertex B connects with A and D.
Vertex C connects with A and D.
Vertex D connects with each of the other vertices.
Therefore the matrix representation of this network is:

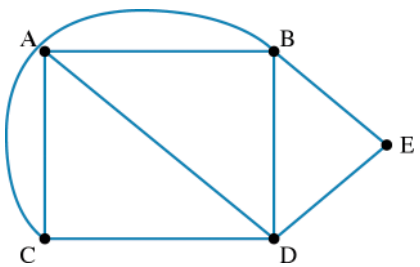
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Question 24
The matrix representation of this network is

$$\begin{bmatrix} 0 & 5 & 10 & \infty \\ 5 & 0 & 4 & 11 \\ 10 & 4 & 0 & 5 \\ \infty & 11 & 5 & 0 \end{bmatrix}$$ **[1 mark]**

The shortest length in row A excluding itself is 5.
The shortest length in row B excluding itself, and 5 from above, is 4.
The shortest length in row C excluding itself and 4 from above is 5.
These three lengths connect all the vertices, and are therefore combined to be the minimum spanning tree.
The minimum spanning tree is therefore
A – B – C – D
5 + 4 + 5 = 14 **[1 mark]**

Question 25
a.



By moving the edge BC to the outside of the network, the network becomes planar. **[1 mark]**

b. The network has 5 vertices, 8 edges and 5 faces. **[1 mark]**

© John Wiley & Sons Australia, Ltd

c. $v = 5$

$e = 8$

$f = 5$

$v + f - e = 2$

$5 + 5 - 8 = 2$

This is true. Therefore the network is a connected planar graph, as it satisfies Euler's formula. **[1 mark]**

d. Yes: C – B – D – A – C – D – E – B – A

This trail connects every vertex, and only uses each edge exactly once. **[1 mark]**

e. No: it is impossible to finish at the same vertex at which you started. **[1 mark]**

© John Wiley & Sons Australia, Ltd

10 Variation

Topic	1	Variation
Subtopic	10.2	Direct and inverse variation

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2021, Further Mathematics Exam 1, Section A, Core, Q.1;* © VCAA

Question 1 (1 mark)

Identify which of these sets of variables is an example of direct proportion.

A. The number of hours spent studying and the mark received on a test
B. A person's height and weight
C. The number of hours exercising and weight
D. The number of hours worked and the pay received at a constant hourly rate
E. The amount of money in an account and the length of time

Question 2 (1 mark)

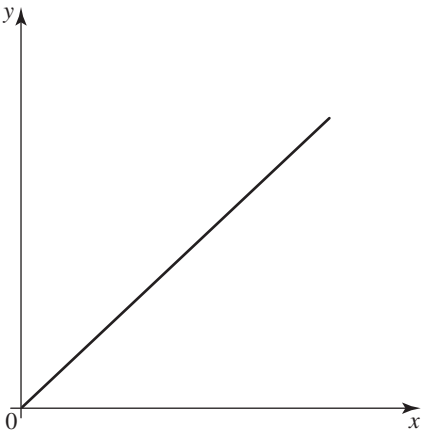
State which of the following correctly describes the characteristics of inverse proportion.

A. As one variable increases, the other must decrease.
B. • As one variable increases, the other must also increase.
• The graph relating the two variables must be a hyperbola.
C. As one variable increases, the other must also increase.
D. The variables must not be related.
E. • As one variable increases, the other must decrease.
• The graph relating the two variables must be a hyperbola.

© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)

Explain, using the figure, why y and x are directly proportional.



Question 4 (1 mark)

Which of the tables illustrates $b \propto 2a^2$

A.

b	8	6	4	2
a	4	3	2	1

B.

b	4	3	2	1
a	3	6	9	18

C.

b	6	5	4	3
a	4	3	2	1

D.

b	32	18	8	2
a	4	3	2	1

E.

b	28	18	8	2
a	4	3	2	1

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)

If m varies jointly as p and the square root of q, the relationship can be expressed as:

A. $m \propto p + \sqrt{q}$

B. $m \propto p\sqrt{p+q}$

C. $m \propto p\sqrt{q}$

D. $m \propto \sqrt{pq}$

E. $m \propto \dfrac{p}{\sqrt{q}}$

Question 6 (3 marks)

The profit made by sweet manufacturer Sugary Sweets varies directly with the number of hours their machinery is operating.

a. If the company makes a profit of \$15 000 when the machinery operates for 24 hours, what is the relationship between the profit, P, and the number of hours the machine is operating, n? **(1 mark)**

b. How long does the machinery operate for a profit of \$100 000 to be made? **(1 mark)**

c. What would be the profit if the machinery operated 24 hours per day for 7 days? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Topic	10	Variation
Subtopic	10.3	Data transformations

online only

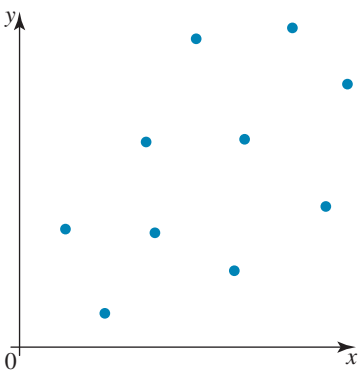
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Source: *VCE 2012, Further Mathematics Exam 1, Section A, Q.1;* © VCAA

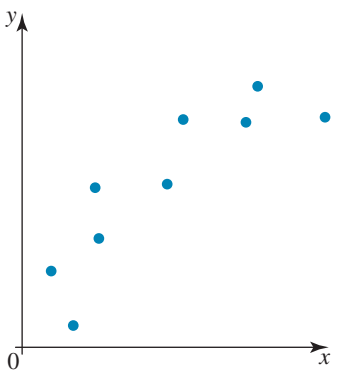
Question 1 (1 mark)

State which of the following scatterplots would benefit most from an $x \rightarrow x^2$ transformation.

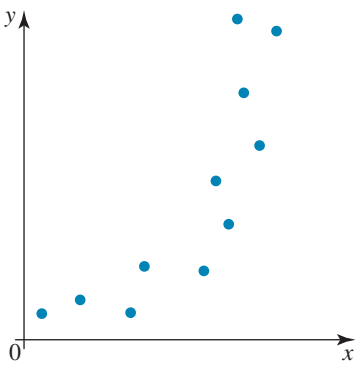
A.



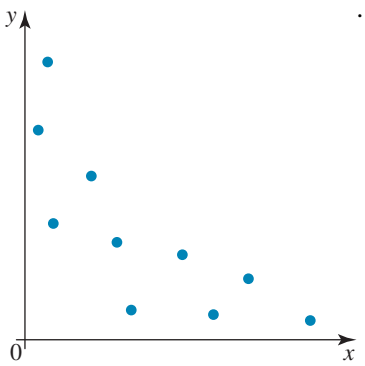
B.



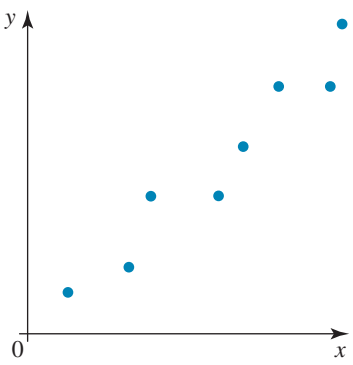
C.



D.



E.



© John Wiley & Sons Australia, Ltd

Question 2 (1 mark)

The table shows the corresponding values of x and y such that $y = a + \frac{b}{x}$.

x	1	2
y	6	4

The respective values of a and b are

A. 3, 3
B. 2, 4
C. 2, −6
D. 1, 6
E. 3, 10

Question 3 (1 mark)

The cost of printing advertising flyers is partly constant and varies partly as the area to be printed.
If it costs 45 cents to print one flyer in a square pattern of side length 10 cm, and 90 cents to print one flyer in a square pattern of side length 21 cm, find:

a. the fixed cost per flyer **(1 mark)**

b. the equation relating the cost of printing one flyer, C, to the side length of the square pattern to be printed, 1 **(1 mark)**

c. the cost of printing 1000 flyers in a square pattern of side length 14.8 cm.

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)

The relationship between two variables, a and b, can be represented by a part linear variation. If $a = 5$ when $b = 14$, the relationship could be

A. $a = b^2 - 9$

B. $b - 7 = 1.4a$

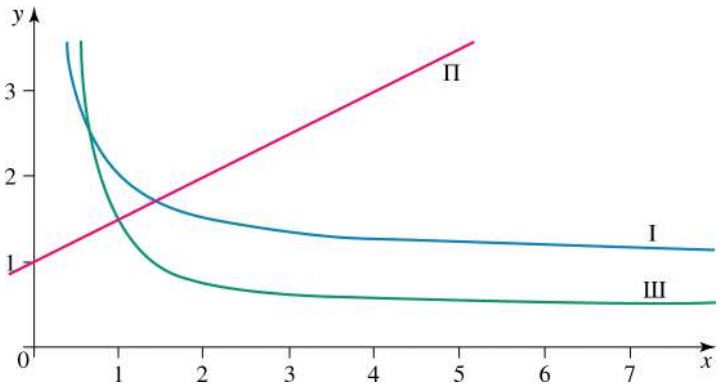
C. $a = \frac{l}{b} + 4.5$

D. $a = b^2 - \frac{9}{14}b$

E. $b = 2a + 5$

Question 5 (1 mark)

Which of the following graph(s) represent(s) part inverse variation?



A. I and III

B. I, II and III only

C. I only

D. II only

E. III only

© John Wiley & Sons Australia, Ltd

Question 6 (3 marks)

The velocity of a ball, v, thrown straight up in the air is partly constant and varies partly as the time elapsed, t, since it was thrown.

a. Write down a relationship between v and t using constants $f$and g. **(1 mark)**

b. The velocity of a ball is 30.2 m/s after 1 second and 20.4 m/s after 2 seconds. Find the relationship between v and t. **(1 mark)**

c. After how many seconds is the velocity of this ball zero?

© John Wiley & Sons Australia, Ltd

Topic	10	Variation
Subtopic	10.4	Orders of magnitude

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

The order of magnitude of the number of seconds in a day is

A. 2
B. 5
C. 24
D. 60
E. 86 400

Question 2 (1 mark)

An earthquake registers 3.0 on the Richter scale. A second earthquake registers 6.0.
Select how many times bigger the amplitude of the second earthquake is than the amplitude of the first earthquake.

A. 1000
B. 100
C. 10
D. 6
E. 2

Question 3 (1 mark)

An order of magnitude estimate of the number of cells in the human body (100 000 000 000 000) is

A. 100
B. 15
C. 14
D. 13
E. 10

© John Wiley & Sons Australia, Ltd

Question 4 (2 marks)
By how many orders of magnitude do a kilogram and a nanogram differ?

Question 5 (2 marks)
The heart pumps a total of approximately 7000 L of blood around the body as it beats 100 000 times in a day. Find an estimate of the number of litres moved during each beat of the heart.

© John Wiley & Sons Australia, Ltd

Topic	10	Variation
Subtopic	10.5	Non-linear data modelling

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)
It is known that y varies directly with the square of x.
If $y = 300$ when $x = 25$, select which of the following is a correct equation connecting the variables.
A. $y = 300\,x^2$
B. $y = x^2$
C. $y = 0.48\,x^2$
D. $y = 25\,x^2$
E. $y = 12\,x^2$

Question 2 (1 mark)
It is known that y varies inversely with x. If $y = 50$ when $x = 2$, select which of the following is a correct equation connecting the variables.
A. $y = \frac{50}{x}$
B. $y = \frac{2}{x}$
C. $y = \frac{25}{x}$
D. $y = \frac{100}{x}$
E. $y = \frac{1}{x}$

Question 3 (1 mark)
It is known that y varies directly with the square of x. If $y = 90$ when $x = 5$, select which of the following is a correct equation connecting the variables.
A. $y = 3.6\,x^2$
B. $y = 90\,x^2$
C. $y = 18\,x^2$
D. $y = 5\,x^2$
E. $y = x^2$

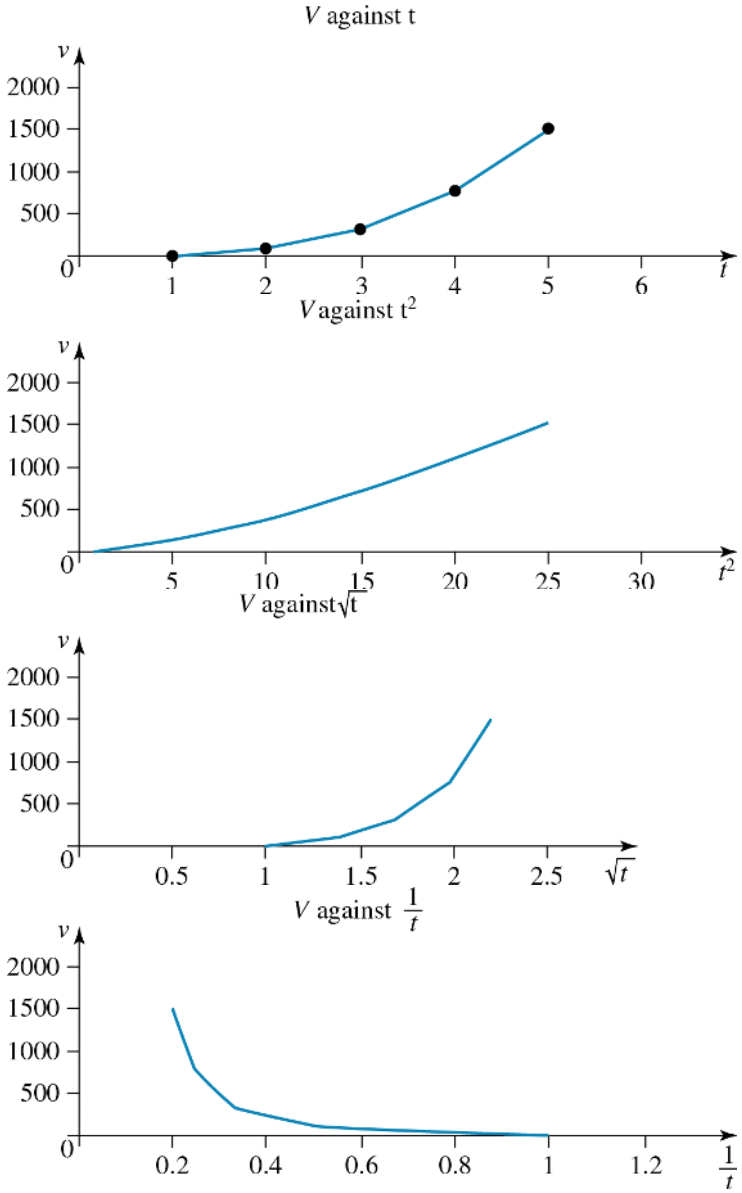
© John Wiley & Sons Australia, Ltd

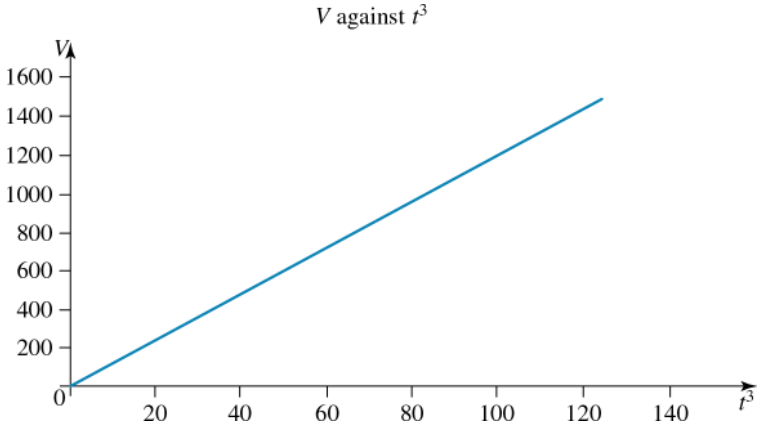
Question 4 (1 mark)

In an experiment, Anna measured the volume of a solid, v, at time t, as it was inflated. The data is shown in the table:

t (s)	1	2	3	4	5
V (cm^3)	12	96	324	768	1500

Anna plotted a series of graphs.





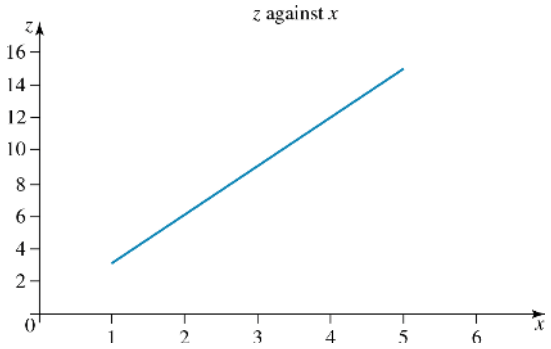
A. $V = 12t$
B. $V = t^3$
C. $V = 12t^2$
D. $V = 12t^3$
E. $V = 3t^2$

Question 5 (1 mark)
The force of gravitational attraction between two bodies, F, varies inversely as the square of their distance apart, d. To show that this is correct by drawing a straight line graph, Tony should plot:
A. F against d
B. F against d^2
C. F against d^3
D. F against $\frac{1}{d}$
E. $\frac{1}{d^2}$

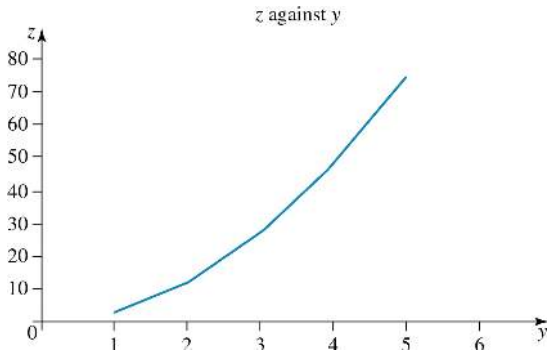
© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)

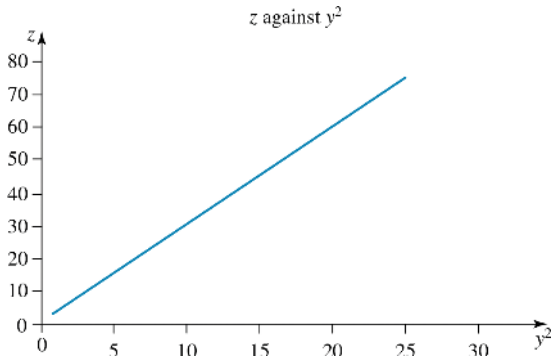
The graph of z against x (keeping y constant) is:



The graph of z against y (keeping x constant) is:



The graph of z against y^2 (keeping x constant) is:



The graph of z to x and y is most likely to be

A. $z = x^2y^2$
B. $z = 3xy^2$
C. $z = xy$
D. $z = 3xy$
E. $z = x^2y$

© John Wiley & Sons Australia, Ltd

Question 7 (4 marks)

The energy, E joule, of a moving body (its kinetic energy) depends on its mass, m kg, and its velocity, v metres per second (m/s).

When E is plotted against m and v is held constant at 10 (m/s), the graph is a straight line with gradient 50.

When E is plotted against v^2 and m is held constant at 3 kg, the graph is a straight line.

When E is plotted against mv^2, the graph is a straight line with gradient 0.5.

a. What is the variation relationship between E, m and v?

b. What is the relationship between E, m and v?

c. What is the kinetic energy of a student with mass 50 kg running at 8 m/s?

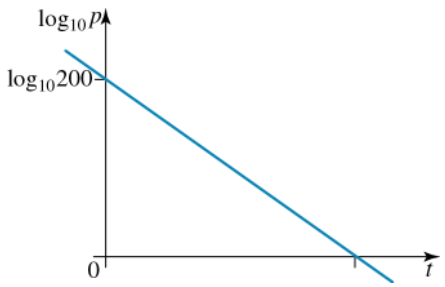
d. What is the kinetic energy of the same student running at half the speed?

© John Wiley & Sons Australia, Ltd

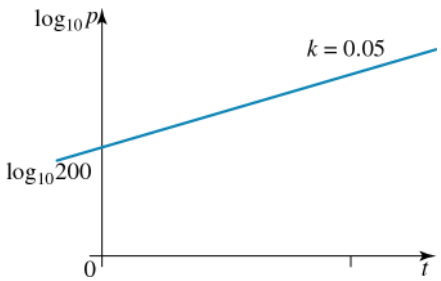
Question 8 (4 marks)

The population of fish in a pond, p, after t months is given by $p = 200 \times 10^{0.05t}$.
If $\log_{10} p$ is plotted against t, the graph will be

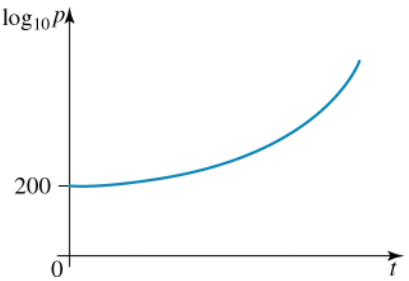
A.



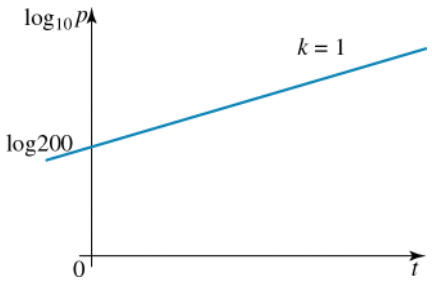
B.



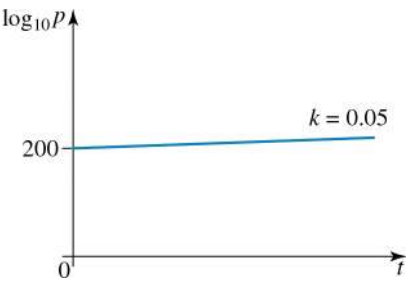
C.



D.



E.



Question 9 (4 marks)

The population, P, of a small town at time t years after the year 2000 is modelled by $\log_{10} P = \log_{10} 1200 - 0.2t$. Which of the following is/are true?

I. The population was 1200 in the year 2000.
II. The population was greater in the year 2010 than in the year 2000.
III. The population is changing exponentially.
IV. In the year 2005, the population was 125 000.

A. I only
B. II only
C. III only
D. I and III
E. II and IV

© John Wiley & Sons Australia, Ltd

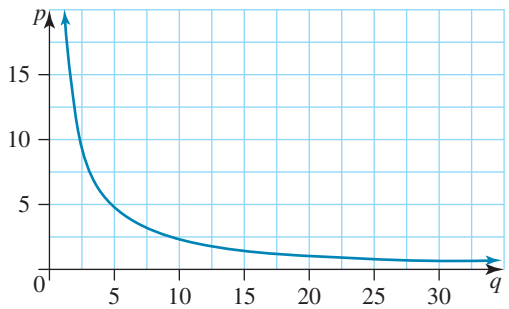
Topic	10	Variation
Subtopic	10.6	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)
Select by how many orders of magnitude 1 gigabyte and 1 megabyte differ.
A. 1
B. 2
C. 3
D. 4
E. 5

Question 2 (1 mark)
Given the following graph of p against q, select the *true* statement from the following.



A. p varies inversely as q
B. p varies directly as q
C. p varies as q^2
D. p varies inversely as q^2
E. There is no relationship between q and p.

© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)

It is known that y varies inversely with x.

If $y = 75$ when $x = 2$, select which of the following is a correct equation connecting the variables.

A. $y = \frac{75}{x}$

B. $y = \frac{2}{x}$

C. $y = \frac{37.5}{x}$

D. $y = \frac{150}{x}$

E. $y = \frac{1}{x}$

Question 4 (1 mark)

Select which of these sets of variables is an example of direct proportion.

A. The temperature and the number of ice creams sold

B. A person's age and height

C. The amount of fuel in a car's tank and the distance that can be travelled at a constant rate

D. The price of a daily train ticket and the length of time travelled

E. The number of hours spent watching TV and the number of hours spent studying

© John Wiley & Sons Australia, Ltd

Question 5 (5 marks)

The time it takes to travel from Melbourne to Sydney depends on the average speed of travel. The values of time, t (hours), and average speed, v (kilometres per hour), are shown in the table.

v (km/h)	100	200	300	400	500	600	700	800
t (h)	8.80	4.40	2.93	2.20	1.76	1.47	1.26	1.10

a. Sketch the graph of time taken against speed. **(1 mark)**

b. Select the *true* statements from the following.

I. The graph is a straight line.

II. As the speed increases, the time taken decreases.

III. As the speed increases, the time taken increases.

IV. When the speed is doubled, the time taken is halved. **(1 mark)**

c. Write the variation relationship between t and v. **(1 mark)**

d. Determine what graph should be plotted to find the relationship between t and v. **(1 mark)**

e. Determine the relationship between t and v. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 6 (3 marks)

Determine the relationship between, $\$I$, varies jointly as the rate of interest, $r\%$ per annum., and the length of the term, m months.

a. If the interest is \$200 when the rate is 4% per annum and the term is 6 months, what is the relationship between I, r and m? **(1 mark)**

b. What interest rate is needed to earn \$500 for a term of 18 months? **(1 mark)**

c. By what percentage does the interest earned change if the term and the interest rate are both decreased by 10%? **(1 mark)**

Question 7 (1 mark)

The area of bacteria in a dish, A cm^2, is growing exponentially over time, t days. This growth is modelled as $\log_{10} A = \log_{10} 2 + 0.2t$. The equation relating A and t is

A. $A = 0.1 + 10^{2t}$

B. $A = 2 \times 10^{0.2t}$

C. $A = 2 \times 10^{-0.1t}$

D. $A = 0.1 \times 10^{-2t}$

E. $A = 2 + 0.1t$

© John Wiley & Sons Australia, Ltd

Question 8 (3 marks)

Daffy's Flowers grow daffodils that start flowering in autumn. On the first day of flowering they picked a few flowers and the number they picked increased as time went on.

The number of flowers picked, N, at time t days after the first flowering is modelled by $\log_{10} N = \log_{10} 5 + 0.035t$.

a. How many flowers did they pick on the first day of flowering? **(1 mark)**

b. How many flowers did they pick 100 days after the first flowering? **(1 mark)**

c. Complete the table

t	1	100
N		

and hence write the equation in the form $N = N_0 10^{kt}$. **(1 mark)**

Question 9 (4 marks)

All the box office tickets sold out 30 days before the night of a particular concert. The original ticket price was \$150, but the price on eBay, \$$P$, went up as shown in the table, where t is the number of days since the box office sold out.

***t*(*days*)**	0	10	20	30
P(\$)	150	299	597	1191

a. How much will tickets cost if they are bought on eBay on the day of the concert? **(1 mark)**

b. Plot $\log_{10} P$ against t and use it to find the gradient and y-intercept. **(2 marks)**

c. Hence, find the equation of growth of ticket price over time. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 10 (1 mark)

What three characteristics need to be fulfilled for two variables to be considered directly proportional?

A. As one variable increases, the other must decrease. The graph relating the two variables must go through the origin. The graph relating the two variables must be linear.As one variable increases, the other must decrease. The graph relating the two variables must go through the origin. The graph relating the two variables must be linear.

B. As one variable increases, the other must also increase. The graph relating the two variables must go through the origin. The graph relating the two variables must be linear.

C. As one variable increases, the other must also increase. The graph relating the two variables must be negative. The graph relating the two variables must be linear.

D. As one variable increases, the other must also increase. The graph relating the two variables must go through the origin. The graph relating the two variables must be a quadratic. As one variable increases, the other must also increase. The graph relating the two variables must go through the origin. The graph relating the two variables must be a quadratic.

E. As one variable increases, the other must also increase. The graph relating the two variables must go through the origin. The graph relating the two variables must be random.

Question 11 (1 mark)

It is known that y varies directly with x. If $y = 20$ when $x = 10$, which of the following is a correct equation connecting the variables?

A. $y = x$

B. $y = 2x$

C. $y = 200x$

D. $y = 20x$

E. $y = 10x$

Question 12 (1 mark)

It is known that y varies inversely with x. If $y = 50$ when $x = 2$, which of the following is a correct equation connecting the variables?

A. $y = \frac{50}{x}$

B. $y = \frac{2}{x}$

C. $y = \frac{25}{x}$

D. $y = \frac{100}{x}$

E. $y = \frac{1}{x}$

© John Wiley & Sons Australia, Ltd

Question 13 (1 mark)

It is known that y varies directly with the square of x. If $y = 90$, when $x = 5$ which of the following is a correct equation connecting the variables?

A. $y = 3.6\,x^2$
B. $y = 90\,x^2$
C. $y = 18\,x^2$
D. $y = 5\,x^2$
E. $y = \ x^2$

Question 14 (1 mark)

It is known that y varies directly with the square of x. If $y = 300$ when $x = 25$ which of the following is a correct equation connecting the variables?

A. $y = 300\,x^2$
B. $y = \ x^2$
C. $y = 0.48\,x^2$
D. $y = 25\,x^2$
E. $y = 12\,x^2$

Question 15 (3 marks)

Rod and Angelo are using a calculator to find the value of $\left(-5 - \sqrt{6^2 - 5 \times 4}\right) \div 9$. Angelo pressed the correct keys on the calculator but Rod left out the brackets. Find:

a. Angelo's answer **(1 mark)**

b. Rod's answer **(1 mark)**

c. the difference between the two answers (Angelo's answer minus Rod's answer). **(1 mark)**

© John Wiley & Sons Australia, Ltd

Answers and marking guide

10.2 Direct and inverse variation

Question 1

As the number of hours worked increases, the pay received will also increase.
When hours worked is zero, pay received will also be zero.
As the hourly rate is consistent, the graph connecting these two variables will be linear.
Therefore, these variables are directly proportional.

Question 2

- As one variable increases, the other must decrease.
- The graph relating the two variables must be a hyperbola.

Question 3

As x increases, y also increases. **[1 mark]**
The graph relating y and x goes through the origin. **[1 mark]**
The graph relating y and x is linear. **[1 mark]**
As these variables satisfy all three conditions, y and x are directly proportional.

Question 4

$b = 2a^2$
The b values are double the square of a values.

Question 5

'and' indicates joint variation, so multiplication of p and $\sqrt{q}$, therefore $m \propto p\sqrt{q}$

Question 6

a. $P = kn$
Substitute $n = 24$ and $P = 15\,000$
$15\,000 = k \times 24$
$k = 625$
$\therefore P = 625n$ **[1 mark]**

b. $P = 625n$
Substitute $P = 100\,000$
$100\,000 = 625n$
$n = 160$ hours **[1 mark]**

c. $P = 625n$
Substitute $n = (24 \times 7)$
$P = 625 \times (24 \times 7)$
$= \$105\,000$ **[1 mark]**

10.3 Data transformations

Question 1

Option C would benefit most from an $x \rightarrow x^2$ transformation, based on the shape of the $y \longleftrightarrow x$ graph.

Question 2

$$y = a + \frac{b}{x}$$

$$6 = a + \frac{b}{1}$$

© John Wiley & Sons Australia, Ltd

$6 = a + b$ (1)

$4 = a + \frac{b}{2}$ (2)

Solving simultaneous equations

$8 = 2a + b$ (3)

Subtract (1) from (3):

$a = 2$

Substitute $a = 2$ into (1):

$6 = 2 + b$

$b = 4$

$\therefore a = 2, b = 4$

Question 3

a. $C = F + kl^2$

$45 = F + k10^2$ (1)

$90 = F + k21^2$ (2)

Solve the equations simultaneously.

Substract (1) from (2):

$45 = 341k$

$k = \frac{45}{341}$

≈ 0.132

$\therefore F \approx 31.80$ cents **[1 mark]**

b. $C = F + kl^2$

$= 31.80 + 0.132l^2$ **[1 mark]**

c. If $l = 14.8$,

$C = 31.80 + 0.132 \times 14.8^2$

$= 31.80 + 28.91328$

$= 60.71328$

$1000C = 1000 \times 60.713\,28$ cents

$= 60713.28$ cents

$= \$607.13$ **[1 mark]**

Don't round off before completing calculations, since you will be multiplying by 1000 .

Question 4

Part variation means that there is addition in the relationship. Linear variation means that there are no squares, cubes, roots etc; the equation can represent a straight line. Substituting for a and b in each alternative shows that $b - 7 = 1.4a$ is satisfied.

Question 5

A straight line graph represents direct variation.

Question 6

a. $v = f + gt$ **[1 mark]**

b. $v = f + gt$

$30.2 = f + g(1)$ [1]

$20.4 = f + 2g$ [2]

Solving simultaneous equations

$v = 40 - 9.8t$ [1 mark]

© John Wiley & Sons Australia, Ltd

c. $v = 40 - 9.8t$

If $v = 0$,

$0 = 40 - 9.8t$

$9.8t = 40$

$t = 4.08$ s **[1 mark]**

10.4 Orders of magnitude

Question 1

1 day $= 24$ hours

$= 24 \times 60$ minutes

$= 24 \times 60 \times 60$ seconds

$= 86\,400$ seconds

$= 8.64 \times 10^4 \approx 10^5$ seconds

Question 2

$10^6 = 1000 \times 10^3$

Question 3

$100\,000\,000\,000\,000 = 10^{14}$

Question 4

1 kg $= 1000$ g

$= 10^3$ g

1 ng $= 10^{-9}$ g **[1 mark]**

They differ by a factor of 10^{12}, so they differ by 12 orders of magnitude. **[1 mark]**

Question 5

7000 L in 100 000 beats $\approx \dfrac{10^4}{10^5}$ **[1 mark]**

$= 10^{-1}$ **[1 mark]**

10.5 Non-linear data modelling

Question 1

$y = kx^2$

$y = 300$ when $x = 25$

$300 = k \times 25^2$

$300 = k \times 625$

$k = \dfrac{300}{625}$

$k = 0.48$

Therefore, $y = 0.48\,x^2$

Question 2

$y = \dfrac{k}{x}$

$y = 50$ when $x = 2$

$50 = \dfrac{k}{2}$

$k = 50 \times 2$

$k = 100$

Therefore, $y = \dfrac{100}{x}$

© John Wiley & Sons Australia, Ltd

Question 3

$y = kx^2$

$y = 90 \quad \text{when} \quad x = 5$

$90 = k \times 5^2$

$90 = k \times 25$

$k = \frac{90}{25}$

$k = 3.6$

Therefore, $y = 3.6\,x^2$

Question 4

The plot of V against t^3 is a straight line with gradient 12.

$\therefore V = 12t^3$

Question 5

F is inversely proportion to the square of the distance, so $F \propto \frac{1}{d^2}$.

Question 6

From the graphs, $z \propto x$ and $z \propto y^2$

$\therefore z = 3xy^2$

Question 7

a. The plots show that :

$E \propto m$

$E \propto v^2$

$E \propto mv^2$ **[1 mark]**

b. The graph E against mv^2 has a gradient 0.5, so constant of proportionality is 0.5

$\therefore E = 0.5mv^2$ **[1 mark]**

c. Substitute given values into the equation:

$E = 0.5mv^2$

$= 0.5 \times 50 \times 8^2$

$= 25 \times 64$

$= 1600\,\text{joule}$ **[1 mark]**

d. Substitute given values into the equation:

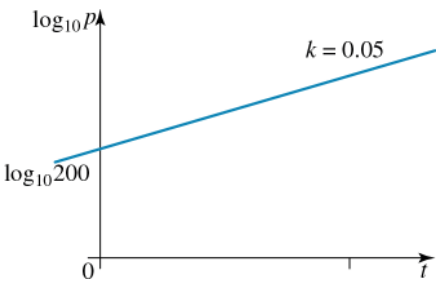
$E = 0.5mv^2$

$= 0.5 \times 50 \times 4^2$

$= 25 \times 16$

$= 400\,\text{joule}$ **[1 mark]**

Question 8



© John Wiley & Sons Australia, Ltd

$y = y_0 \times 10^{kx}$ can be written in the form $\log_{10} y = \log_{10} y_0 + kx$,and the graph of $\log_{10} y$ against x has gradient k and y-intercept $\log_{10} y_0$.
So $p = 200 \times 10^{0.05t}$ can be written in the form $\log_{10} p = \log_{10} 200 + 0.05t$, and the graph of $\log_{10} p$ against t has a gradient of 0.05 and y-intercept of $\log_{10} 200$.

Question 9
$\log_{10} y = \log_{10} y_0 + kx$ can be written in the form $y = y_0 \times 10^{kx}$, which is changing exponentially with an initial value of y_0. So $\log_{10} P = \log_{10} 1200 - 0.2t$ can be written in the form $P = 1200 \times 10^{-0.2t}$, which is changing exponentially with an initial value of 1200.

10.6 Review

Question 1
1 Gb $= 10^9$ bytes
and 1 Mb $= 10^6$ bytes
$\frac{10^9}{10^6}$ gives a difference of 3 orders of magnitude.

Question 2
$p \propto \frac{1}{q}$

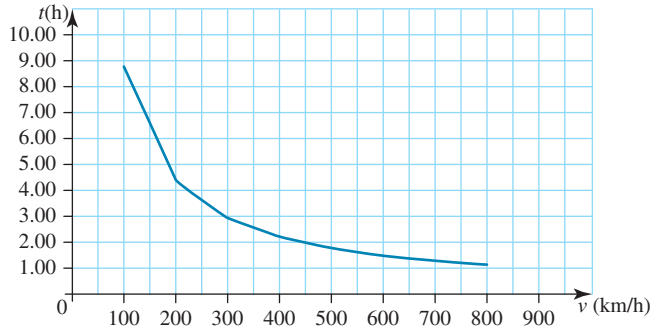
$p = \frac{25}{q}$
$\therefore p$ varies inversely as q

Question 3
$y = \frac{k}{x}$
$y = 75$ when $x = 2$
$75 = \frac{k}{2}$
$k = 75 \times 2$
$k = 150$
Therefore, $y = \frac{150}{x}$

Question 4
As the amount of fuel in the car's tank increases, the distance that can be travelled will also increase.
When the fuel tank is empty, the car will not be able to travel any distance.
As the car is travelling at a constant rate, the graph connecting these two variables will be linear.
Therefore, these variables are directly proportional.

Question 5

a. **[1 mark]**



b. II and IV **[1 mark]**

c. $t \propto \frac{1}{v}$ **[1 mark]**

d. t against $\frac{1}{v}$ **[1 mark]**

e. $t = \frac{880}{v}$ **[1 mark]**

Question 6

a. $I = krm$

Substitute $I = 200$, $r = 4$ and $m = 6$

$200 = k \times 4 \times 6$

$k = \frac{200}{24}$

$= \frac{25}{3}$

$\therefore I = \left(\frac{25}{3}\right) rm$ **[1 mark]**

b. $I = krm$

Substitute $I = 500$ and $m = 18$

$500 = \left(\frac{25}{3}\right) r \times 18$

$\frac{500}{18} = \left(\frac{25}{3}\right) r$

$r = \frac{1500}{18 \times 25}$

$r = 3\frac{1}{3}\%$ **[1 mark]**

c. Since interest rate is decreased by 10%, r is multiplied by 0.9. Likewise the term is multiplied by 0.9. Therefore in the formula $I = krm$ is multiplied by $0.9 \times 0.9 = 0.81$

I is multiplied by 0.81 or a reduction of 19%. **[1 mark]**

Question 7

$\log_{10} y = \log_{10} y_0 + kx$ can be written in the form $y = y_0 \times 10^{kx}$ Here, $\log_{10} A = \log_{10} 2 + 0.2t$ can be written in the form $A = 2 \times 10^{0.2t}$

Question 8

a. $y = y_0 \times 10^{kx}$ can be written in the form $\log_{10} y = \log_{10} y_0 + kx$, and the graph of $\log_{10} y$ against x has gradient K and y-intercept $\log_{10} y_0$.

$\log_{10} N = \log_{10} 5 + 0.035t$

Can be written as:

$N = 5 \times 10^{0.035t}$

When $t = 0$,

$N = 5 \times 10^0$

$= 5$ **[1 mark]**

b. When $t = 100$,

$N = 5 \times 10^{0.035 \times 100}$

$= 5 \times 10^{3.5}$

$\approx 15\,811$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

c.

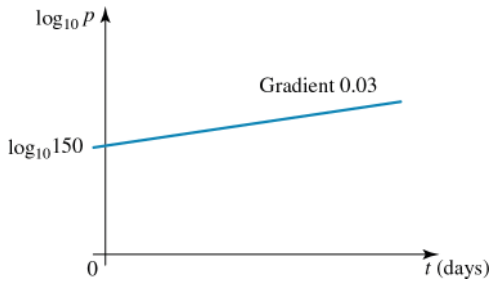
t	1	100
N	5	15 811

$N = 5 \times 10^{0.035t}$ **[1 mark]**

Question 9

a. Reading from the table: $P = \$1191$ when $t = 30$ days **[1 mark]**

b. $y = y_0 \times 10^{kx}$ can be written in the form $\log_{10} y = \log_{10} y_0 + kx$and the graph of $\log_{10} y$aagainst x has gradient k and y-intercept $\log_{10} y_0$. In this case, for $\log_{10} P = \log_{10} P_o + kt$ the graph of $\log_{10} P$against t has gradient 0.03 and y-intercept $\log_{10} 150$.
Gradient 0.03 and y-intercept $\log_{10} 150$ **[1 mark]**



c. $P = 150 \times 10^{0.03t}$ **[1 mark]**

Question 10

- As one variable increases, the other must also increase.
- The graph relating the two variables must go through the origin.
- The graph relating the two variables must be linear.

Question 11

$y = kx$

$y = 20 \quad \text{when} \quad x = 10$

$20 = k \times 10$

$k = \frac{20}{10}$

$k = 2$

Therefore, $y = 2x$

Question 12

$y = \frac{k}{x}$

$y = 50 \quad \text{when} \quad x = 2$

$50 = \frac{k}{2}$

$k = 50 \times 2$

$k = 100$

Therefore $y = \frac{100}{x}$

© John Wiley & Sons Australia, Ltd

Question 13

$y = kx^2$

$y = 90 \quad \text{when} \quad x = 5$

$90 = k \times 5^2$

$90 = k \times 25$

$k = \frac{90}{25}$

$k = 3.6$

Therefore $y = 3.6\,x^2$

Question 14

$y = kx^2$

$y = 300 \quad \text{when} \quad x = 25$

$300 = k \times 25^2$

$300 = k \times 625$

$k = \frac{300}{625}$

$k = 0.48$

Therefore $y = 0.48\,x^2$

Question 15

a. Angelo: $\left(-5 - \sqrt{6^2 - 5 \times 4}\right) \div 9 = \left(-5 - \sqrt{36 - 20}\right) \div 9$

$= (-5 - 4) \div 9$

$= -1$ **[1 mark]**

b. Rod: $-5 - \sqrt{6^2 - 5 \times 4} \div 9 = -5 - \sqrt{36 - 20} \div 9$

$= -5 - 4 \div 9$

$= -5 - \frac{4}{9}$

$= -5\frac{4}{9}$ **[1 mark]**

c. Difference: $-1 - -5\frac{4}{9} = -1 + 5\frac{4}{9}$

$= 4\frac{4}{9}$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

11 Space and measurement

Topic	11	Space and measurement
Subtopic	11.2	Scientific notation, significant figures and rounding

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

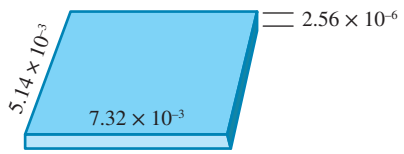
Which of these numbers is expressed with three significant figures?

A. 250
B. 25
C. 2.5
D. 0.0250
E. 0.025

Question 2 (1 mark)

A film of gold on a computer chip measures 5.14×10^{-3} metres wide, 7.32×10^{-3} metres long and 2.56×10^{-6} metres thick.

The volume of gold on the chip, in scientific notation, is closest to



A. 4×10^{-11} m^3
B. 1.5×10^{-11} m^3
C. 1.5×10^{-53} m^3
D. 4×10^{-53} m^3
E. 9.63×10^{-11} m^3

© John Wiley & Sons Australia, Ltd

Question 3 (3 marks)

Amy is calculating the surface area of Earth in square kilometres using the formula $A = 4\pi r^2$ as an approximation. She knows that Earth is not really spherical and that the radius measurement in kilometres depends on where it is measured. Also pi can be used with different levels of accuracy.

	Surface area of Earth		
Value of pi	**Radius at equator 6378.1 km**	**Radius at poles 6356.8 km**	**Average value**
3.141 59	511 201 530.5	507 792 863	509 497 196.8
3.141 592 654	511 201 962.3	507 793 291.9	509 497 627.1
pi from computer	511 201 962.3	507 793 291.9	509 497 627.1
3.142 857 143 $\left(\pi \approx \frac{22}{7}\right)$	511 407 720.8	507 997 678.4	509 702 699.6

a. Which value of pi in the table is likely to be most accurate?

The value of pi that is likely to be most accurate is **(1 mark)**

b. What is the most accurate average value of the surface area?

☐ 509 497 196.8
☐ 509 497 627.1
☐ 509 702 699.6 **(1 mark)**

c. Write the most accurate average value of the surface area in scientific notation.

The most accurate average value of the surface area is ☐ km^2. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Topic	11	Space and measurement
Subtopic	11.3	Pythagoras' theorem

online only

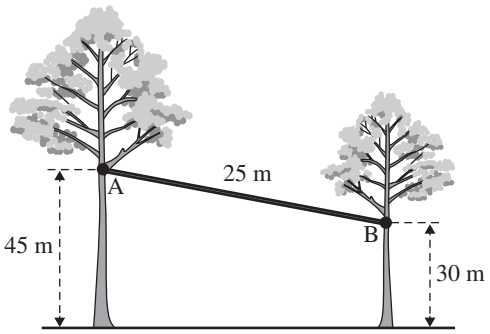
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2020, Further Mathematics 1, Section B, Module 3, Q.3; © VCAA*

Question 1 (1 mark)

Two trees stand on horizontal ground.

A 25 m cable connects the two trees at point A and point, as shown in the diagram below.



Point A is 45 m above the ground and point B is 30 m above the ground.

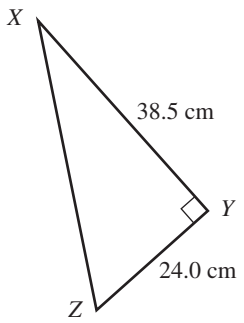
The horizontal distance, in metres, between point A and point B is

A. 10
B. 15
C. 20
D. 30
E. 35

Source: *VCE 2017, Further Mathematics 1, Section B, Module 3, Q.2; © VCAA*

Question 2 (1 mark)

A right-angled triangle, XYZ, has side lengths $XY = 38.5$ cm and $YZ = 24.0$ cm, as shown in the diagram below.



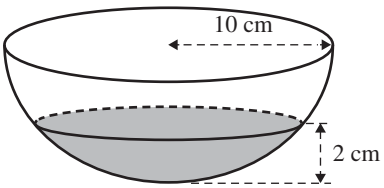
The length of XZ, in centimetres, is closest to

A. 24.8
B. 30.1
C. 38.8
D. 45.4
E. 62.5

Source: *VCE 2017, Further Mathematics 1, Section B, Module 3, Q.6; © VCAA*

Question 3 (1 mark)

A hemispherical bowl of radius 10 cm is shown in the diagram below.



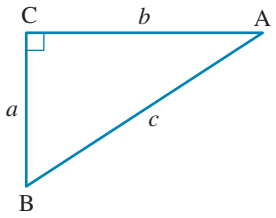
The bowl contains water with a maximum depth of 2 cm.
The radius of the surface of the water, in centimetres, is

A. 2
B. 6
C. 8
D. 9
E. 10

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)

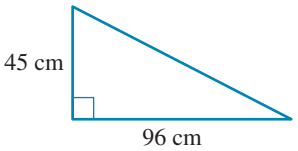
In ΔABC, the formula relating a, b and c is:



A. $a = \sqrt{c^2 - b^2}$
B. $c = a + b$
C. $b = \sqrt{c^2 + a^2}$
D. $a = c - b$
E. $c = b - a$

Question 5 (1 mark)

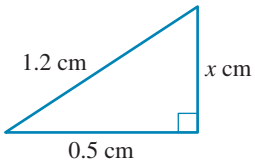
The hypotenuse has a length of:



A. 51 cm
B. 106 cm
C. 141 cm
D. 38 cm
E. 11 241 cm

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)
The value of x is:



A. 0.7
B. 1.179
C. 1.091
D. 1.044
E. 1.3

Question 7 (1 mark)
Which of the following are **not** Pythagorean triads?
A. 8, 15, 17
B. 3, 4, 5
C. 3, 5, 7
D. 5, 12, 13
E. 6, 8, 10

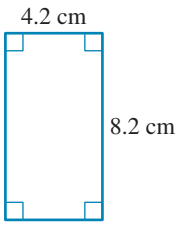
© John Wiley & Sons Australia, Ltd

Topic	11	Space and measurement
Subtopic	11.4	Perimeter and area of polygons and triangles

online only

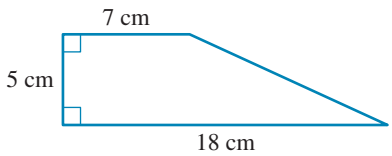
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)
The perimeter of this figure is



A. 34.44 cm
B. 68.88 cm
C. 12.4 cm
D. 24.8 cm
E. 8.0 cm

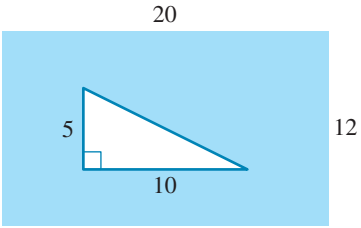
Question 2 (1 mark)
The area of this figure is



A. 62.5 cm^2
B. 90 cm^2
C. 126 cm^2
D. 35 cm^2
E. 150 cm^2

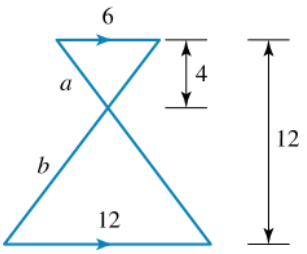
© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)
A right-angled triangle is inside a rectangle, with measurements shown in centimetres.



The shaded area is
A. 240 cm^2
B. 25 cm^2
C. 95 cm
D. 190 cm^2
E. 215 cm^2

Question 4 (3 marks)
A shape consists of two isosceles triangles.



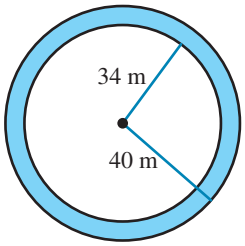
If all measurements are in metres, find:
a. the area of the whole shape
The area of the whole shape is ☐ m^2. **(1 mark)**

b. the lengths of sides labeled a and b
$a =$ ☐ m; $b =$ ☐ m **(1 mark)**

c. the perimeter of the whole shape.
Perimeter = ☐ m **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 5 (3 marks)
The circular running track is shown shaded. The outer radius is 40 m and the inner radius is 34 m.



a. How wide is the running track?
The running track is ☐ m wide. **(1 mark)**

b. Manny is running on the outside and Lucy is running on the inside of the track. How much further will Manny run if they run a full circuit? **(1 mark)**

c. How far apart should the starting lines for the race be staggered so that all athletes run the same distance around the track? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Topic	11	Space and measurement
Subtopic	11.5	Perimeter and area of composite shapes and sectors

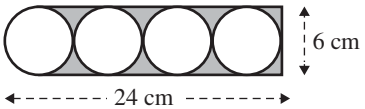
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2021, Further Mathematics 1, Section B, Module 3, Q.6; © VCAA*

Question 1 (1 mark)

A child's toy has the following design.



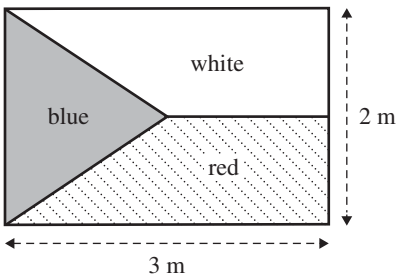
The area of the shaded region, in square centimetres, is closest to

A. 13
B. 27
C. 31
D. 45
E. 113

Source: *VCE 2020, Further Mathematics 1, Section B, Module 3, Q.2; © VCAA*

Question 2 (1 mark)

A flag consists of three different coloured sections: red, white and blue.
The flag is 3 m long and 2 m wide, as shown in the diagram below.



The blue section is an isosceles triangle that extends to half the length of the flag.
The area of the blue section, in square metres, is

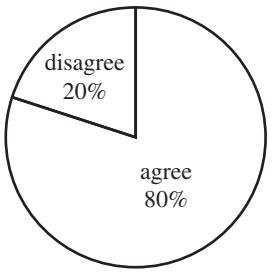
A. 0.75
B. 1.5
C. 2
D. 3
E. 6

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics 1, Section B, Module 3, Q.5; © VCAA*

Question 3 (1 mark)

The pie chart below displays the results of a survey.



Eighty per cent of the people surveyed selected 'agree'.
Twenty per cent of the people surveyed selected 'disagree'.
The radius of the pie chart is 16 mm.
The area of the sector representing 'agree', in square millimetres, is closest to

A. 80
B. 161
C. 483
D. 643
E. 804

Question 4 (1 mark)

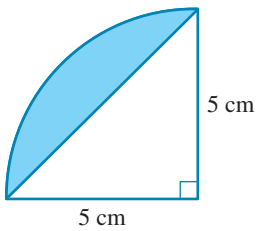
The circumference of a circle of diameter 20 cm is

A. 15.71 cm
B. 31.42 cm
C. 62.83 cm
D. 314.2 cm
E. 628.3 cm

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)
A circle and a square of side length 20 cm have the same area. The radius of the circle is
A. 10 cm
B. 11.28 cm
C. 20 cm
D. 22.56 cm
E. 127.3 cm

Question 6 (1 mark)
The shaded area is



A. 7.13 cm^2
B. 12.5 cm^2
C. 19.63 cm^2
D. 5.37 cm^2
E. 26.76 cm^2

© John Wiley & Sons Australia, Ltd

Topic	11	Space and measurement
Subtopic	11.6	Volume

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2019, Further Mathematics 1, Section B, Module 3, Q.7; © VCAA*

Question 1 (1 mark)

A can of dog food is in the shape of a cylinder. The can has a circumference of 18.85 cm and a volume of 311 cm^3.

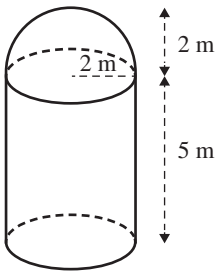
The height of the can, in centimetres, is closest to

A. 2.8
B. 3.0
C. 6.0
D. 11.0
E. 16.5

Source: *VCE 2016, Further Mathematics 1, Section B, Module 3, Q.5; © VCAA*

Question 2 (1 mark)

A water tank in the shape of a cylinder with a hemispherical top is shown below.



The volume of water that this tank can hold, in cubic metres, is closest to

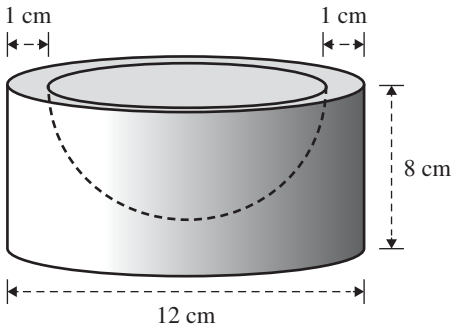
A. 80
B. 88
C. 96
D. 105
E. 121

© John Wiley & Sons Australia, Ltd

Source: *VCE 2015, Further Mathematics 1, Section B, Module 2, Q.6; © VCAA*

Question 3 (1 mark)

A cylindrical block of wood has a diameter of 12 cm and a height of 8 cm.
A hemisphere is removed from the top of the cylinder, 1 cm from the edge, as shown below.

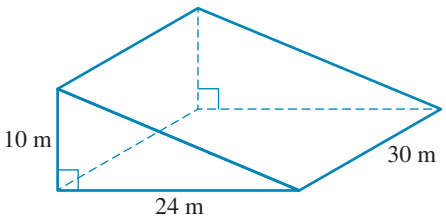


The volume of the block of wood, in cubic centimetres, after the hemisphere has been removed is closest to

A. 452
B. 606
C. 643
D. 1167
E. 1357

Question 4 (1 mark)

The volume of the triangular prism is

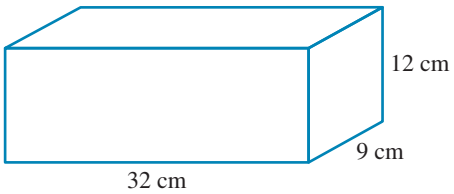


A. 3600 m^3
B. 7200 m^3
C. 2400 m^3
D. 7500 m^3
E. 720 m^3

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)

This carton can be completely filled with 36 boxes.



The dimensions of a single box are

A. 16 cm × 3 cm × 4 cm
B. 8 cm × 3 cm × 4 cm
C. 8 cm × 3 cm × 6 cm
D. 4 cm × 3 cm × 3 cm
E. 16 cm × 3 cm × 6 cm

© John Wiley & Sons Australia, Ltd

Topic	11	Space and measurement
Subtopic	11.7	Surface area

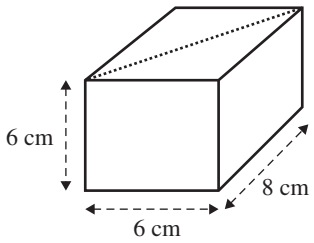
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2020, Further Mathematics 1, Section B, Module 3, Q.8; © VCAA*

Question 1 (1 mark)

A cake is in the shape of a rectangular prism, as shown in the diagram below.



The cake is cut in half to create two equal portions.
The cut is made along the diagonal, as represented by the dotted line.
The total surface area, in square centimetres, of one portion of the cake is

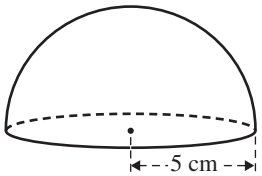
A. 132
B. 180
C. 192
D. 212
E. 264

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics 1, Section B, Module 3, Q.3; © VCAA*

Question 2 (1 mark)

An ice cream dessert is in the shape of a hemisphere. The dessert has a radius of 5 cm.



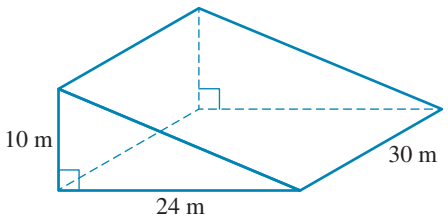
The top and the base of the dessert are covered in chocolate.

The total surface area, in square centimetres, that is covered in chocolate is closest to

A. 52
B. 157
C. 236
D. 314
E. 942

Question 3 (1 mark)

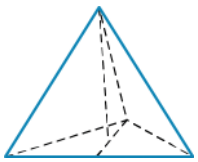
The surface area of the triangular prism, including the base, is



A. 1260 m^2
B. 1710 m^2
C. 1770 m^2
D. 2040 m^2
E. 3600 m^2

© John Wiley & Sons Australia, Ltd

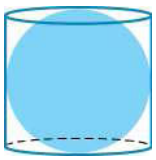
Question 4 (1 mark)
A pyramid with a height of 3 cm has a triangular base of side length 2 cm and height $\sqrt{3}$ cm.



The volume is:
A. 3 cm^3
B. $\sqrt{3}\text{ cm}^3$
C. $2\sqrt{3}\text{ cm}^3$
D. $4\sqrt{3}\text{ cm}^3$
E. 9 cm^3

Question 5 (1 mark)
The volume of a sphere is 800 cm^3. The radius to the nearest centimetre is
A. 4 cm
B. 3 cm
C. 6 cm
D. 7 cm
E. 5 cm

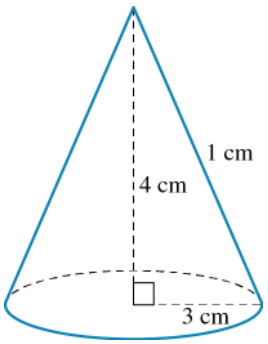
Question 6 (1 mark)
A spherical ball with radius 14 cm just fits inside a cylindrical can.



Using $\pi \approx \frac{22}{7}$, the volume of the can is:
A. 4312 cm^3
B. 8624 cm^3
C. $12\,936\text{ cm}^3$
D. $34\,496\text{ cm}^3$
E. $17\,248\text{ cm}^3$

© John Wiley & Sons Australia, Ltd

Question 7 (3 marks)
For the cone below, find, correct to the nearest whole number:



a. the slant height, l cm **(1 mark)**

b. the area of the curved surface of the cone **(1 mark)**

c. the total surface area of the cone, including the base. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Topic	11	Space and measurement
Subtopic	11.8	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2021, Further Mathematics 1, Section B, Module 3, Q.5; © VCAA*

Question 1 (1 mark)

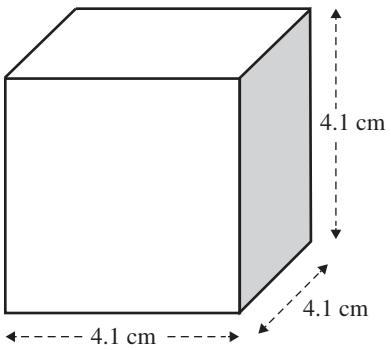
A cone and a cylinder both have a radius of r centimetres.
The height of the cone is 12 cm.
If the cylinder and the cone have the same volume, then the height of the cylinder, in centimetres, is

A. 4
B. 6
C. 8
D. 12
E. 36

Source: *VCE 2021, Further Mathematics 2, Section B, Module 3, Q.1; © VCAA*

Question 2 (4 marks)

The game of squash is played with a spherical ball that has a radius of 2 cm.
Squash balls may be sold in cube-shaped boxes.
Each box contains one ball and has a side length of 4.1 cm, as shown in the diagram below.



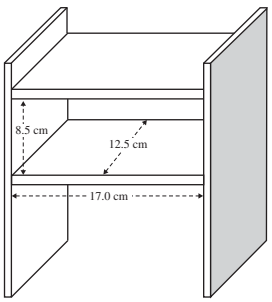
a. Show that the volume of one squash ball, rounded to two decimal places, is 33.51 cm^3. **(1 mark)**

b. Calculate the empty space, in cubic centimetres, that surrounds the ball in the box.
Round your answer to two decimal places.
☐ cm^3 **(1 mark)**

© John Wiley & Sons Australia, Ltd

c. Calculate the total surface area, in square centimetres, of one box.
TSA = ☐ cm^2 **(1 mark)**

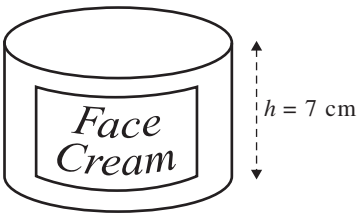
d. Retail shops store the cube-shaped boxes in a space within a display unit.
The space has a length of 17.0 cm and a width of 12.5 cm. Due to the presence of a shelf above, there is a maximum height of 8.5 cm available. This is shown in the diagram below.



Calculate the maximum number of cube-shaped boxes that can fit into the space within the display unit.
☐ **(1 mark)**

Source: *VCE 2020, Further Mathematics 2, Section B, Module 3, Q.1; © VCAA*

Question 3 (6 marks)
Khaleda manufactures a face cream. The cream comes in a cylindrical container.
The area of the circular base is 43 cm^2. The container has a height of 7 cm, as shown in the diagram below.



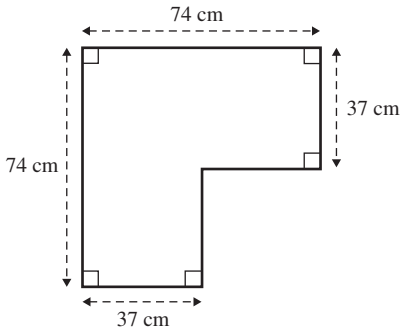
a. What is the volume of the container, in cubic centimetres?
Volume = ☐ cm^3 **(1mark)**

b. Write a calculation that shows that the radius of the cylindrical container, rounded to one decimal place, is 3.7 cm. **(1 mark)**

c. What is the total surface area of the container, in square centimetres, including the base and the lid? Round your answer to the nearest square centimetre.

☐ cm^2 **(1 mark)**

d. The diagram below shows the dimensions of a shelf that will display the containers.



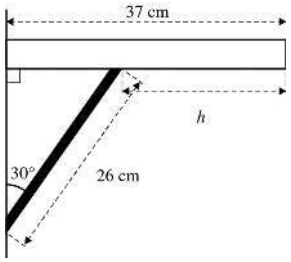
What is the perimeter of the shelf, in centimetres?

Perimeter = ☐ cm **(1 mark)**

e. The shelf will display the containers in a single layer. Each container will stand upright on the shelf. What is the maximum number of containers that can fit on the shelf?

The maximum number of cans that can fit on the shelf is ☐. **(1 mark)**

f. The shelf sits against a wall at a 90° angle.
The shelf is supported by a 26 cm bracket that forms a 30° angle with the wall, as shown in the diagram below.



Find the value of h, the distance between the edge of the shelf and the bracket, in centimetres.

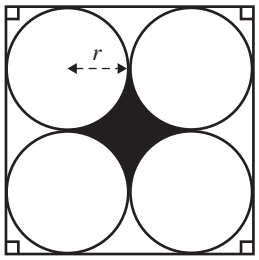
☐ cm **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2019, Further Mathematics 1, Section B, Module 3, Q.8; © VCAA*

Question 4 (1 mark)

Four identical circles of radius r are drawn inside a square, as shown in the diagram below.
The region enclosed by the circles has been shaded in the diagram.



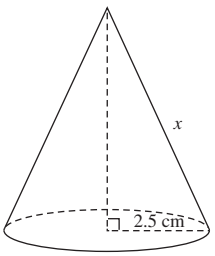
The shaded area can be found using

A. $4r^2 - 2\pi r$
B. $4r^2 - \pi r^2$
C. $4r - \pi r^2$
D. $2r^2 - \pi r^2$
E. $2r - 2\pi r$

Source: *VCE 2018, Further Mathematics 1, Section B, Module 3, Q.8; © VCAA*

Question 5 (1 mark)

A cone with a radius of 2.5 cm is shown in the diagram below.



The slant edge, x, of this cone is also shown.
The volume of this cone is 36 cm^3.
The surface area of this cone, including the base, can be found using the rule $Surface\ area = \pi r(r + x)$.
The total surface area of this cone, including the base, in square centimetres, is closest to

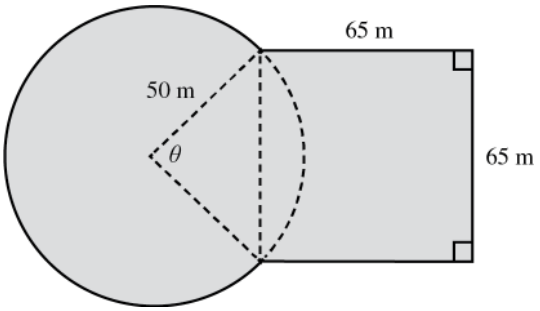
A. 20
B. 42
C. 63
D. 67
E. 90

© John Wiley & Sons Australia, Ltd

Source: *VCE 2017, Further Mathematics 2, Section B, Module 3, Q.3; © VCAA*

Question 6 (3 marks)

The hostel buildings are arranged around a grassed area.
The grassed area is shown shaded in the diagram below.

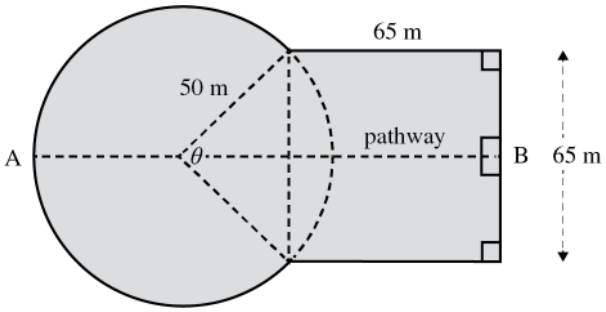


The grassed area is made up of a square overlapping a circle.
The square has side lengths of 65 m.
The circle has a radius of 50 m.
An angle, θ, is also shown on the diagram.

a. Use the cosine rule to show that the angle θ, correct to the nearest degree, is equal to 81°. **(1 mark)**

b. What is the perimeter, in metres, of the entire grassed area?
Round your answer to the nearest metre.
Perimeter = ☐ m **(1 mark)**

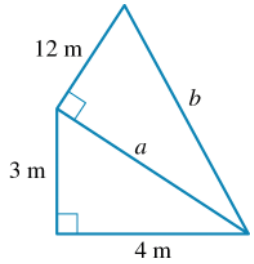
c. The hostel's management is planning to build a pathway from point A to point B, as shown on the diagram below.



Calculate the length, in metres, of the planned pathway.
Round your answer to the nearest metre.
☐ m **(1 mark)**

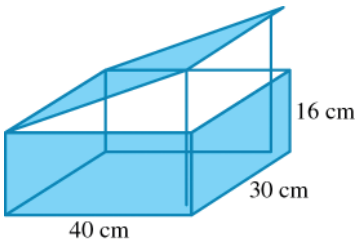
© John Wiley & Sons Australia, Ltd

Question 7 (2 marks)
Find the values of a and b.



$a = \square$; $b = \square$

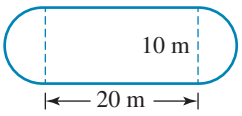
Question 8 (2 marks)
A box is 40 cm long, 30 cm wide and 16 cm high. The lid is propped open by vertical rods 32 cm long, placed at the corners.



a. How high is the raised edge of the lid above the top of the box?
The lid is $\square$ cm above the top of the box. **(1 mark)**

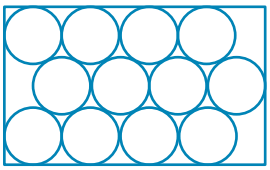
b. Find the horizontal distance from the 'hinge' of the lid to the position of the rods.
$\square$ cm **(1 mark)**

Question 9 (1 mark)
What is the length of the gutter to be built around the edge of this pool?



$\square$ m

Question 10 (4 marks)



Twelve pizzas, each in the shape of a circle of radius 5 cm, are packed on a tray measuring 45 cm long and 27.32 cm wide.

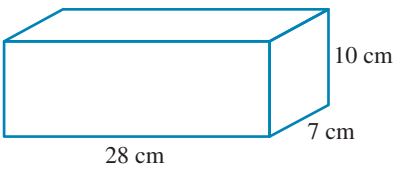
a. What is the area of the tray, correct to 1 decimal place?
Area = ☐ cm^2 **(1 mark)**

b. What is the area of a pizza, correct to 2 decimal places?
Area of a pizza = ☐ cm^2 **(1 mark)**

c. What is the percentage of the tray not covered by pizzas, correct to 2 decimal places?
Percentage of the tray not covered in pizzas = ☐ %. **(2 marks)**

Question 11 (2 marks)

A container in the shape of a rectangular prism is filled to the brim with water.

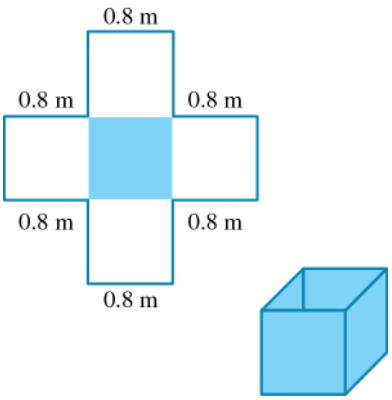


a. Find the volume of water in the container.
Volume = ☐ cm^3 **(1 mark)**

b. The water is poured from this container into a cylindrical tin of diameter 14 cm. Find, correct to 1 decimal place, how far up the side of the tin the water will reach. Use $\pi = \frac{22}{7}$.
☐ cm **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 12 (3 marks)
A sheet of metal 2.4 m by 2.4 m has four square corners cut out from it.



a. Find the area of the metal remaining.
☐ m^2 **(1 mark)**

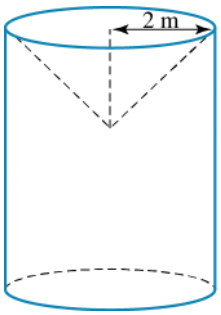
b. If the sides are folded up to form an open box, find the volume of the box correct to 3 decimal places.
Volume = ☐ m^3 **(1 mark)**

c. If the sheet metal costs $100 per square metre, including wastage, and labour costs to cut it out and fold the metal up are $55, how much will this box cost to produce?
The box will cost $ ☐ to produce. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 13 (3 marks)

An inverted cone in the top of a cylinder has height and base radius equal to 2 m.



Use $\pi = \frac{22}{7}$

If the cylinder is 7 m high:

a. what is the volume of the cylinder?

☐ m^3 **(1 mark)**

b. what is the volume of the cone?

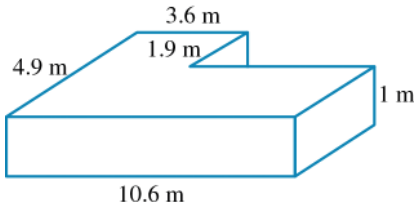
☐ m^3 **(1 mark)**

c. what volume of the cylinder is not occupied by the cone (correct to one decimal place)?

☐ cm^3 **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 14 (3 marks)
Two children wish to find how much water is in their swimming pool. They take the measurements of the water, as shown in the diagram.



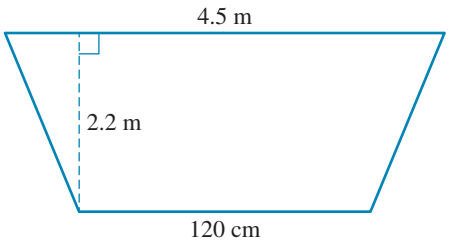
a. What is the volume of water in the pool, in metres cubed?
☐ m^3 **(1 mark)**

b. What is the capacity of the pool in litres?
Capacity = ☐ L **(1 mark)**

c. If 1 m^3 of water weighs 1 tonne, how many tonnes of water are in the pool, correct to 2 decimal places?
Weight of water = ☐ tonnes. **(1 mark)**

Question 15 (1 mark)
The distance between Brisbane and Darwin is best measured in:
A. mm
B. cm
C. m
D. km
E. ha

Question 16 (1 mark)
The area of this shape, in square centimetres, is:



A. 6.27 cm^2
B. 627 cm^2
C. 62 700 cm^2
D. 136.95 cm^2
E. 11.88 cm^2

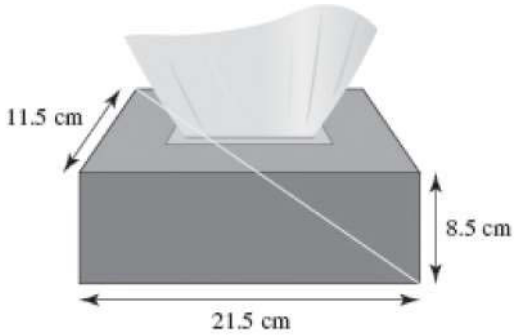
Question 17 (1 mark)
The volume of a cylinder with a radius of 4 cm and a height of 12 cm is best described by:
A. $V = \pi \times 4^2 \times 12$
B. $V = \pi r^2 \times H$
C. $V = \pi \times 12^2 \times 4$
D. $V = L \times W \times H$
E. $V = L^3$

Question 18 (1 mark)
A ladder leans against a wall, with the foot of the ladder 2.4 m from the wall. If the ladder is 8 m in length, it meets the wall at a height of:

A. 8.35 m
B. 31.36 m
C. 9.6 m
D. 5.32 m
E. 7.63 m

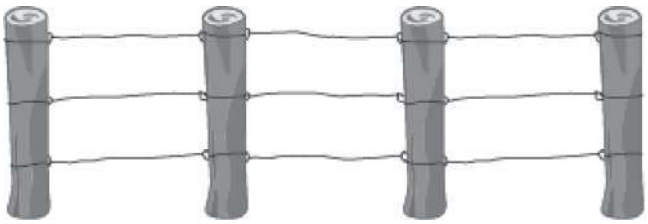
© John Wiley & Sons Australia, Ltd

Question 19 (1 mark)
Ben decides to convert a tissue box into a pencil tub. If the dimensions of the tissue box are as shown, the longest item that he can place into this tub is:



A. 23.11 cm
B. 25.82 cm
C. 19.75 cm
D. 20.05 cm
E. 24.62 cm

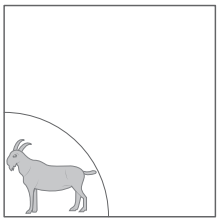
Question 20 (1 mark)
A farmer wishes to fence his paddock with three wires running through fence posts. If the length of the paddock is 20m and the width is 15m, the total length of wire required is:



A. 70 m
B. 210 m
C. 140 m
D. 45 m
E. 135 m

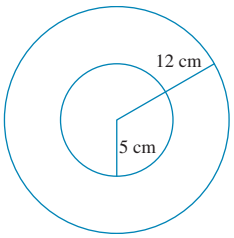
© John Wiley & Sons Australia, Ltd

Question 21 (1 mark)
A goat is tethered to the corner of a square paddock of side length 3.5 m. The goat has a tether which is 1.75 m long. The area of grass that cannot be eaten by the goat is:



A. 12.25 m^2
B. 9.62 m^2
C. 9.84 m^2
D. 2.41 m^2
E. 14.66 m^2

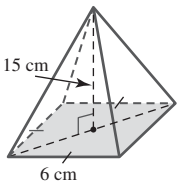
Question 22 (1 mark)
The area of this annulus is closest to:



A. 374 cm^2
B. 169π cm^2
C. 144 cm^2
D. 25 cm^2
E. 119 cm^2

© John Wiley & Sons Australia, Ltd

Question 23 (1 mark)
The total surface area of this square pyramid is closest to:

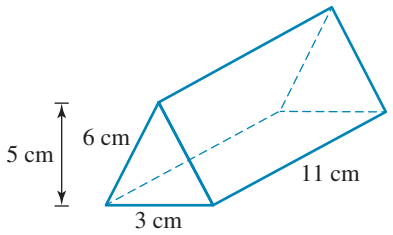


A. 184 cm^2
B. 36 cm^2
C. 220 cm^2
D. 82 cm^2
E. 148 cm^2

Question 24 (1 mark)
The volume of the three-dimensional figure shown is best calculated using:

A. $V = \frac{1}{2}\pi r^3 H$
B. $V = \frac{4}{3}\pi r^3$
C. $V = \frac{1}{2}\pi r^2 H$
D. $V = \frac{2}{3}\pi r^3$
E. $V = \frac{1}{3}\pi r^2 H$

Question 25 (3 marks)
A student decides to construct a vase using perspex, in the shape of a triangular prism.



a. Calculate the total surface area, correct to the nearest whole number, of perspex that will be required (assuming no overlap).
☐ cm^2 **(1 mark)**

b. Calculate the volume of the vase, given that it will be filled to $\frac{2}{3}$ of the marked height.
☐ cm^3 **(1 mark)**

c. Calculate the actual capacity of the vase.
☐ mL **(1 mark)**

Question 26 (9 marks)
A cake maker is making a wedding cake for a very special customer. It is a two-tiered cake of the shape shown.

a. If the circumference of the bottom tier is 47.12 cm, calculate the diameter of the bottom tier.
☐ cm **(1 mark)**

b. The height of the bottom tier is to be 8 cm. Given that there will be no fondant applied to the bottom of the tier, calculate the surface area of fondant required for the bottom tier.
☐ cm^2 **(1 mark)**

© John Wiley & Sons Australia, Ltd

c. The top tier has a diameter of 8 cm and will have a height of 6 cm. Calculate the surface area of fondant required to cover this tier; remember that the bottom of this tier will not be covered.
☐ cm^2 **(1 mark)**

d. If a 5 kilogram packet of fondant can be rolled to a square of size 800 cm^2, will the cake maker have enough fondant for this design? **(2 marks)**

e. Calculate the total volume of cake in this wedding cake.
☐ cm^3 **(1 mark)**

f. If a slice of cake is approximately 15 cm^3, will this cake be sufficient to feed the 150 guests, assuming that they all have one slice? **(3 marks)**

Question 27 (1 mark)
The radius of a circle is 2.9 cm. A square has the same area as this circle. The length of the diagonal of the square is closest to
A. 5.2
B. 7.3
C. 10.3
D. 17.9
E. 26.4

© John Wiley & Sons Australia, Ltd

Answers and marking guide

11.2 Scientific notation, significant figures and rounding

Question 1

0.0250

The first two zeros are not significant but the last zero is.

Question 2

$5.14 \times 10^{-3} \times 7.32 \times 10^{-3} \times 2.56 \times 10^{-6} = 96.3 \times 10^{-12}$

$= 9.63 \times 10^{-11}\ \text{m}^3$

Question 3

a. Pi from the computer **[1 mark]**

b. $509\,497\,627.1\ \text{km}^2$ **[1 mark]**

c. $5.094\,976\,271 \times 10^8\ \text{km}^2$ **[1 mark]**

11.3 Pythagoras' theorem

Question 1

Horizontal distance AB $= \sqrt{25^2 - 15^2} = 20$

Question 2

$(XZ)^2 = (XY)^2 + (YZ)^2$

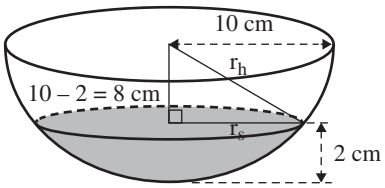
$= 38.5^2 + 24.0^2$

$\therefore XZ = \sqrt{38.5^2 + 24.0^2}$

$\simeq 45.37$

Question 3

Use Pythagoras' Theorem in the hemisphere.



Let r_h = radius of the hemisphere and left

r_S = radius of the surface of the water

$a^2 + b^2 = c^2$

$8^2 + (r_S)^2 = (r_h)^2$

$8^2 + (r_S)^2 = 10^2$

$\therefore r_S = \sqrt{10^2 - 8^2}$

$= 6$

VCAA Examination Report **note:**

This question provided a diagram of a hemispherical bowl containing water. In order to calculate the required radius of the surface of that water, students needed to apply Pythagoras's theorem to the triangle.

Question 4

$c^2 = a^2 + b^2$

$a^2 = c^2 - b^2$

$a = \sqrt{c^2 - b^2}$

© John Wiley & Sons Australia, Ltd

Question 5

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 45^2 + 96^2 \\ &= 11\,241 \\ c &= \sqrt{11\,241} \\ &\approx 106 \end{aligned}$$

Question 6

$$\begin{aligned} 1.2^2 &= x^2 + 0.5^2 \\ 1.2^2 - 0.5^2 &= x^2 \\ 1.44 - 0.25 &= x^2 \\ x &= \sqrt{1.19} \\ &\approx 1.091 \end{aligned}$$

Question 7

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 3^2 + 5^2 \\ &= 34 \\ c &= \sqrt{34} \approx 5.83 \\ \therefore c &\neq 7 \end{aligned}$$

11.4 Perimeter and area of polygons and triangles

Question 1

$$\begin{aligned} 2l + 2w &= 2\,(4.2) + 2\,(8.2) \\ &= 8.4 + 16.4 \\ &= 24.8\text{ cm} \end{aligned}$$

Question 2

Total area = area of rectangle + area of triangle

$$\begin{aligned} &-lw + \frac{1}{2}hb \\ &= (5 \times 7) + \left(\frac{1}{2} \times (18 - 7) \times 5\right) \\ &= 35\text{ cm}^2 + 27.5\text{ cm}^2 \\ &= 62.5\text{ cm}^2 \end{aligned}$$

Question 3

$$\begin{aligned} \text{Area} &= lw - \frac{1}{2}hb \\ &= (20 \times 12) - \left(\frac{1}{2} \times 10 \times 5\right) \\ &= 240\text{ cm}^2 - 25\text{ cm}^2 \\ &= 215\text{ cm}^2 \end{aligned}$$

Question 4

a. Height of larger triangle:

$12 - 4 = 8$ m

Area of each triangle:

$$A = \frac{1}{2}bh$$

$$\text{Total area} = \frac{1}{2}bh + \frac{1}{2}bh$$
$$= \left(\frac{1}{2} \times 12 \times 8\right) + \left(\frac{1}{2} \times 6 \times 4\right)$$
$$= 48\,\text{m}^2 + 12\,\text{m}^2$$
$$= 60\,\text{m}^2 \textbf{ [1 mark]}$$

b. Use Pythagoras' theorem or Pythagorean triads to find each hypotenuse:

$a = \sqrt{3^2 + 4^2}$

$= 5\,\text{m}$ **[1 mark]**

$b = \sqrt{6^2 + 8^2}$

$= 10\,\text{m}$ **[1 mark]**

c. Perimeter $= 6 + 5 + 5 + 12 + 10 + 10$

$= 48\,\text{m}$ **[1 mark]**

Question 5

a. $40\,\text{m} - 34\,\text{m} = 6 \quad \text{m}$ **[1 mark]**

b. $2\pi r_2 - 2\pi r_1 = 2\pi\,(40 - 34)$

$= 37.7\,\text{m}$ **[1 mark]**

c. Staggering of start equals the difference in the distance around the circuit between the inside and outside of the track.

37.7 m **[1 mark]**

11.5 Perimeter and area of composite shapes and sectors

Question 1

The area of the rectangle is: $6 \times 21 = 126\,\text{cm}^2$

The radius of each circle is 3 cm.

There are 3.5 circles in the rectangle, so the area of all the circles is:

$\pi \times 3^2 \times \frac{7}{2} = \frac{63\pi}{2}\,\text{cm}^2$

Therefore, the area of the shaded region is: $126 - \frac{63\pi}{2} = 27.0398 \simeq 27\,\text{cm}^2$

Question 2

$A = \frac{1}{2} \times 2 \times 1.5 = 1.5\,\text{m}^2$

Question 3

Area of the circle $= \pi \times 16^2 = 804.2477\,\text{mm}^2$

$80\% \times 804.2477 = 643.398 = 643\,\text{mm}^2$

Question 4

$C = \pi d$

$= \pi \times 20$

$\approx 62.83\,\text{cm}$

© John Wiley & Sons Australia, Ltd

Question 5

$$\begin{aligned}\text{Area of circle} &= \pi r^2\\ \text{Area of square} &= l^2\\ &= 20^2\\ &= 400\,\text{cm}^2\\ \pi r^2 &= 400\,\text{cm}^2\\ r^2 &= \frac{400}{\pi}, \text{ so}\\ r &= \sqrt{\frac{400}{\pi}}\\ &\approx 11.28\,\text{cm}\end{aligned}$$

Question 6

$$\begin{aligned}\frac{1}{4}\pi r^2 - \frac{1}{2}bh &= \frac{1}{4}\pi 5^2 - \frac{1}{2}\times 5\times 5\\ &\approx 19.63 - 12.5\\ &= 7.13\,\text{cm}^2\end{aligned}$$

11.6 Volume

Question 1

$$C = 2\pi r$$

$$r = \frac{18.85}{2\pi} = 3.0\,cm$$

$$\begin{aligned}V &= \pi r^2 h\\ 311 &= \pi \times 3^2 \times h\\ h &= 10.99937\\ &= 11\,cm\end{aligned}$$

Question 2

$$V = \left(\pi r^2 h\right) + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$\begin{aligned}&= \left(\pi \times 2^2 \times 5\right) + \frac{1}{2}\left(\frac{4}{3}\times \pi \times 2^3\right)\\ &= \frac{76\pi}{3}\\ &= 79.587\\ &= 80\,\text{m}^3\end{aligned}$$

Question 3

$$\begin{aligned}V_{\text{cylinder}} - V_{\text{hemisphere}} &= \pi r_c^2 h - \frac{2}{3}\pi r_h^3\\ &= \left[\pi \times (6)^2 \times 8\right] - \left[\frac{2}{3}\times \pi \times (5)^3\right]\\ &= 288\pi - \frac{250\pi}{3}\\ &= 642.9793\\ &\approx 643\,\text{cm}^3\end{aligned}$$

© John Wiley & Sons Australia, Ltd

Question 4

Volume of a prism $=$ area of base $\times$ height

$=$ area of traiangle $\times$ height

$= \left(\frac{1}{2} \times 24 \times 10\right) \times 30$

$= 3600\,\text{m}^3$

Question 5

The volume of 36 boxes must equal the volume of the carton.
The lengths of the boxes must be factors of the lengths of the carton.
Hence, 8 cm $\times$ 3 cm $\times$ 4 cm are the dimensions of a single box.

11.7 Surface area

Question 1

The length of the diagonal is $\sqrt{6^2 + 8^2} = 10$.
So, the total surface area will be
$(6 \times 6) + (6 \times 8) + 2 \times \left(\frac{1}{2} \times 6 \times 8\right) + (10 \times 6) = 192\,\text{cm}^2$.

Question 2

$SA = \pi r^2 + \frac{1}{2}\left(4\pi r^2\right)$

$= \pi(5)^2 + 2\pi(5)^2$

$= 235.6194$

$= 236\,cm^2$

***VCAA Examination Report* note:**

A significant proportion of students chose option B, forgetting to include the circular base.

Question 3

Find the missing side length:

$c^2 = a^2 + b^2$

$c^2 = 10^2 + 24^2$

$c = 26$

$\text{SA} = (24 \times 30) + \left(\frac{1}{2} \times 24 \times 10\right) + \left(\frac{1}{2} \times 24 \times 10\right) + (10 \times 30) + (26 \times 30)$

$= 720 + 120 + 120 + 300 + 780$

$= 2040\,\text{m}^2$

Question 4

$V = \frac{1}{3}Ah$

$= \frac{1}{3}\left(\frac{1}{2} \times 2 \times \sqrt{3}\right) 3$

$= \sqrt{3}\,\text{cm}^3$

Question 5

$\frac{4}{3}\pi r^3 = 800$

$r^3 = \frac{800 \times 3}{4\pi}$

© John Wiley & Sons Australia, Ltd

$= 190.986$

$r = \sqrt[3]{190.986}$

$= 5.7588$

$\therefore 6$ cm to the nearest centimetre

Question 6

Volume $= \pi \times 14^2 \times 28$

$\approx \frac{22}{7} \times 196 \times 28$

$= 17\,248\,\text{cm}^3$

Question 7

a. $c^2 = a^2 + b^2$

$l^2 = 3^2 + 4^2$

$l^2 = 25$

$l = \sqrt{25}$

$= 5$ cm **[1 mark]**

b. $A = \pi rl$

$= \pi \times 3 \times 5$

$= 15\pi$

$\approx 47\,\text{cm}^2$ **[1 mark]**

c. $A = 15\pi + \pi \times 3^2$

$= 24\pi$

$= 75\,\text{cm}^2$ **[1 mark]**

11.8 Review

Question 1

The cylinder and the cone have the same volume and the same radius:

$\pi r^2 h_{cylinder} = \frac{1}{3}\pi r^2 h_{cone}$

$h_{cylinder} = \frac{1}{3} h_{cone}$

The height of the cone is 12 cm:

$h_{cylinder} = \frac{1}{3} \times 12$

$= 4$cm

Question 2

a. $V = \frac{4}{3} \times \pi \times r^3$

$= \frac{4}{3} \times \pi \times 2^3$ [**1 mark** – this is a SHOW THAT question. Must see the substitution]

$= 33.51$

b. $V_{box} = 4.1^3 = 68.92\,\text{cm}^3$

$V_{empty} = 68.91 - 33.51 = 35.41\,\text{cm}^3$ [**1 mark** – rounding to 2 decimal places applies]

c. $TSA = 6 \times 4.1^2 = 100.86\,\text{cm}^2$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

d. length: $\frac{17}{4.1} = 4.146 \rightarrow$ so 4 boxes will fit.
width: $\frac{12.5}{4.1} = 3.049 \rightarrow$ so 3 boxs will fit.
height: $\frac{8.5}{4.1} = 2.073 \rightarrow$ so 2 boxes will fit.
Therefore, the maximum number of boxes that can fit in the display unit is $4 \times 3 \times 2 = 24$ boxes. **[1 mark]**

Question 3

a. Volume $= 43\,\text{cm}^2 \times 7\,\text{cm} = 301\,\text{cm}^3$ **[1 mark]**

b. Area $= \pi r^2$

$43 = \pi r^2$

$r = \sqrt{\frac{43}{\pi}}$ **[1 mark]**

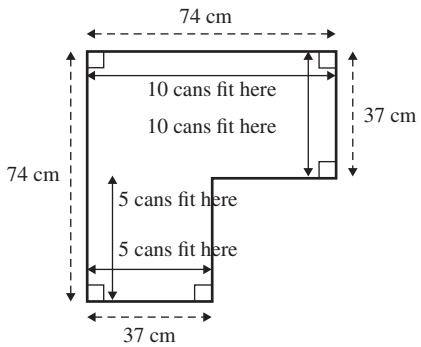
$r = 3.6996...$

$= 3.7$

c. TSA $= 2 \times 43 + 2\pi \times 3.7 \times 7 = 248.734... = 249\,\text{cm}^2$ **[1 mark]**

d. $74\,\text{cm} \times 4 = 296\,\text{cm}$ **[1 mark]**

e. We know that the radius of each can is 3.7 cm, which means the diameter will be 7.4 cm.
Break the shape into two sections:

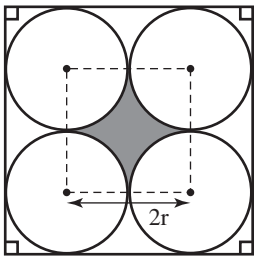


So $10 \times 5 + 5 \times 5 = 75$ cans can fit on the shelf. **[1 mark]**

f. $h = 37 - 26 \times \sin 30° = 24\,\text{cm}$ **[1 mark]**

Question 4

Form a square with the vertices at the centre of each circle and side length of $2r$.



The four quarter circles inside the smaller square make up one whole circle. Take this circle away from the square.
i.e. Area of the small square – area of a circle (or 4 quarter circles)
$= (2r^2) - \pi r^2$
$= 4r^2 - \pi r^2$

© John Wiley & Sons Australia, Ltd

***VCAA Examination Report* note:**

$= (2r \times 2r) - \left(4 \times \frac{1}{4}\pi r^2\right) = 4r^2 - \pi r^2$

Question 5

The volume of a cone is $\frac{1}{3}\pi r^2 h$. Given the volume equation, $36 = \frac{1}{3}\pi \times 2.5^2 \times h$, h can be found.

$h = \frac{3 \times 36}{\pi \times 2.5^2} = 5.5\text{ cm}$

x can then be found by using Pythagoras' theorem.

$x = \sqrt{r^2 + h^2} = \sqrt{2.5^2 + 5.5^2} = 6.04\text{ cm}$

The surface area of a cone $= \pi r(r + x)$.

Substituting in $x = 6.04$, the surface area is $\pi \times 2.5 \times (2.5 + 6.04) \approx 67\text{ cm}^2$

Question 6

a. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos\theta = \frac{50^2 + 50^2 - 65^2}{2 \times 50 \times 50}$

$= \frac{775}{5000}$

$\theta = \cos^{-1}\left(\frac{775}{5000}\right)$

$\simeq 81.08°$

The angle θ is equal to 81° to the nearest degree (as required). **[1 mark]**

***VCAA Examination Report* note:**

Most students were able to write the equation, but some made errors in the rule by placement of a 65 where a 50 should have been or vice versa.

b. Perimeter = major arc length + three square side lengths

$= \left(\frac{360 - 81}{360}\right)° \times 2 \times \pi \times 50 + 3 \times 65$

$= \left(\frac{279}{360}\right)° \times 2 \times \pi \times 50 + 195$

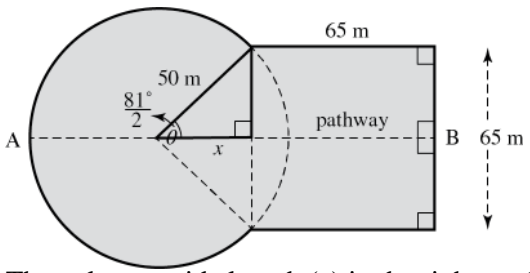
$\simeq 438.47$

The perimeter is 438 m (to the nearest metre). **[1 mark]**

***VCAA Examination Report* note:**

Students who could apply the formula for arc length, generally answered this question well.

c.



The unknown side length (x) in the right-angled triangle shown represents the part of the pathway from the centre of the circle to the start of the segment.

$\cos\left(\frac{81}{2}\right)° = \frac{x}{50}$

$x = 50 \times \cos\left(\frac{81}{2}\right)°$ **[1 mark]**

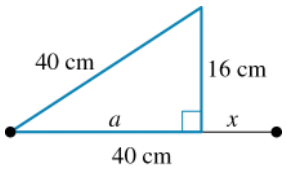
$x = 38\text{ m}$

© John Wiley & Sons Australia, Ltd

Total length of the pathway from point A to point B
= Radius of circle + (x) + length of square
$= 50 + 50 \times \cos\left(\frac{81}{2}\right)^\circ + 65 \simeq 153.02$ m
The length of the pathway is 153 m (to the nearest metre). **[1 mark]**
***VCAA Examination Report* note:**
This question was not answered well. Some students made an incorrect assumption about the distance in the square/circle overlap.

Question 7
This question uses Pythagorean triads:
The first triangle is 3, 4, 5 and so $a = 5$ m. **[1 mark]**
The second triangle is 5, 12, 13 and so $b = 13$ m. **[1 mark]**

Question 8
a. If the rod is 32 cm and the box is 16 cm:
32 cm − 16 cm = 16 cm **[1 mark]**
b.



$40^2 - 16 = a^2$
$a = \sqrt{1600 - 256}$
$= \sqrt{1344}$
$= 36.66$
∴ The distance from the hinge to the rods is 36.66 cm. **[1 mark]**

Question 9
Perimeter = circumference of 2 semicircles + $2l$ **[1 mark]**
$= (2 \times \pi \times 5) + (2 \times 20)$
$= 31.42 + 40$
$= 71.42$ m **[1 mark]**

Question 10
a. Area = length × width
$= 45 \times 27.32$
$= 1229.4\text{ cm}^2$ **[1 mark]**
b. Area = πr^2
$= \pi \times 5^2$
$= 78.54\text{ cm}^2$ **[1 mark]**
c. Area = area of tray − 12 × area of pizza
$= 1229.4 - 12 \times 78.54$
$= 286.92\text{ cm}^2$ **[1 mark]**
As a percentage of the tray:
$\text{Percentage} = \frac{286.92}{1229.4} \times \frac{100}{1}$
$\approx 23.34\%$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

Question 11

a. $V = 28 \times 10 \times 7$

$= 1960\,\text{cm}^3$ **[1 mark]**

b. $\pi r^2 h = 1960$

$\frac{22}{7} \times 7^2 \times h = 1960$ **[1 mark]**

$h = \frac{1960 \times 7}{22 \times 49}$

$\approx 12.7\,\text{cm}$ **[1 mark]**

Question 12

a. Area $= 2.4^2 - 4 \times 0.8^2$

$= 5.76 - 2.56$

$= 3.2\,\text{m}^2$ **[1 mark]**

b. Volume $= l^3$

$= 0.8^3$

$= 0.512\,\text{m}^3$ **[1mark]**

c. Cost $= 2.4^2 \times \$100 + 55$

$= \$576 + 55$

$= \$631$ **[1 mark]**

Question 13

a. Volume of cylinder $= \pi \times 2^2 \times 7$

$= \frac{22}{7} \times 28$

$= 88\,\text{m}^3$ **[1 mark]**

b. Volume of cone $= \frac{1}{3}\pi \times 2^2 \times 2$

$= \frac{1}{3} \times \frac{22}{7} \times 8$

$= 8.38\,\text{m}^3$ **[1 mark]**

c. Volume = Volume of cylinder − Volume of cone

$= 88 - 8.38$

$= 79.6\,\text{cm}^3$ (to one decimal place) **[1 mark]**

Question 14

a. Volume $= (3 \times 10.6 \times 1) + (1.9 \times 3.6 \times 1)$

$= 31.8 + 6.84$

$= 38.64\,\text{m}^3$ **[1 mark]**

b. $1\,\text{m}^3 = 1000\,\text{L}$

Capacity of pool $= 38\,640\,\text{L}$ **[1 mark]**

c. 1000 L weighs 1 tonne

Mass of water $= 38.64$ tonne

Question 15

The distance between Brisbane and Darwin is best measured in kilometres.

Question 16

$120\,\text{cm} = 1.2\,\text{m}$

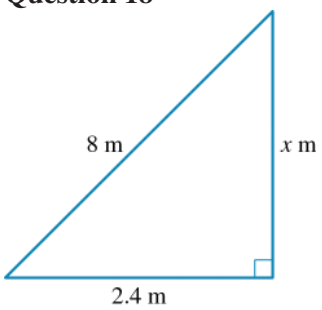
$A = \frac{1}{2}(a + b)h$

© John Wiley & Sons Australia, Ltd

$= \frac{1}{2} \times (1.2 + 4.5) \times 2.2$
$= 6.27\,\text{m}^2$
$= 6.27 \times 100^2\,\text{cm}^2$
$= 62\,700\,\text{cm}^2$

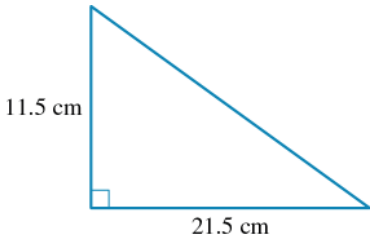
Question 17
The formula for the volume of a cylinder is $V = \pi r^2 \times H$, but given that we know the radius and the height, we can substitute those values in, so for this cylinder, the volume is best described by $V = \pi \times 4^2 \times 12$.

Question 18

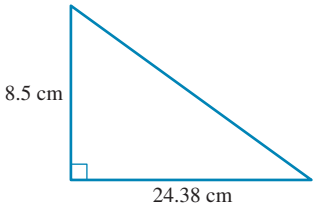


$c^2 = a^2 + b^2$
$b^2 = c^2 - a^2$
$b^2 = 8^2 - 2.4^2$
$b^2 = 64 - 5.76$
$b^2 = 58.24$
$b = \sqrt{58.24}$
$b = 7.63\,\text{m}$

Question 19
First find the diagonal across the bottom of the box.



$c^2 = a^2 + b^2$
$c^2 = 21.5^2 + 11.5^2$
$c^2 = 594.5$
$c = \sqrt{594.5}$
$c = 24.38\,\text{cm}$ (correct to 2 decimal places)
Then use this to find the length of the longest item.



© John Wiley & Sons Australia, Ltd

$c^2 = a^2 + b^2$
$c^2 = 24.38^2 + 11.5^2$
$c^2 = 666.6344$
$c = \sqrt{666.6344}$
$c = 25.82$ cm (correct to 2 decimal places)

Question 20

$P = 2L + 2W$
$= 2 \times 20 + 2 \times 15$
$= 40 + 30$
$= 70$ m
Total length of wire $= 70$ m $\times 3$
$= 210$ m

Question 21

$A_{\text{paddock}} = l^2$
$= 3.5^2$
$= 12.25$ m^2
$r = 1.75$ m^2
$A_{\text{sector of circle}} = \frac{\theta}{360}\pi r^2$
$= \frac{90}{360} \times \pi \times 1.75^2$
$= 2.41$ m^2
$A_{\text{grass not eaten}} = 12.25$ m$^2 - 2.41$ m^2
$= 9.84$ m^2

Question 22

$A = \pi(R^2 - r^2)$
$= \pi(12^2 - 5^2)$
$= \pi(119)$
$= 373.85$ cm^2(correct to 2 decimal places)
This is closest to Option A.

Question 23

Slanted height of triangle $(c^2) = a^2 + b^2$
$= 3^2 + 15^2$
$= 9 + 225$
$= 234$
Slanted height $= \sqrt{234}$
$= 15.30$ cm (correct to 2 decimal places)
TSA = area of square + area of four triangles
$= 6^2 + 4\left(\frac{1}{2} \times 6 \times 15.30\right)$
$= 6^2 + 4(45.9)$
$= 36 + 183.6$
$= 219.6$ cm^2

Question 24

The correct formula used to calculate the volume of a cone is $V = \frac{1}{3}\pi r^2 H$.

© John Wiley & Sons Australia, Ltd

Question 25

a. $\text{TSA} = \left(\frac{1}{2} \times 5 \times 3\right) + (3 \times 11) + (6 \times 11) + (6 \times 11)$ **[1 mark]**

$= (7.5) + (33) + (66) + (66)$

$= 175.5\,\text{cm}^2$ **[1 mark]**

$\approx 173\,\text{cm}^2$ **[1 mark]**

b. $\frac{2}{3} \times 11 = \frac{22}{3}$ **[1 mark]**

$= 7.333...$

≈ 7.3 **[1 mark]**

hjg $V = \text{Area of base} \times \text{height}$

$= \left(\frac{1}{2} \times 3 \times 5\right) \times 7.3$ **[1 mark]**

$= 7.5 \times 7.3$

$= 54.75 \quad \text{cm}^3$ **[1 mark]**

c. $V = \text{Area of base} \times \text{height}$

$= \left(\frac{1}{2} \times 3 \times 5\right) \times 11$ **[1 mark]**

$= 7.5 \times 11$

$= 82.5\,\text{cm}^3$ **[1 mark]**

$1\,\text{cm}^3 = 1\,\text{mL}$

$= 82.5\,\text{mL}$ **[1 mark]**

Question 26

a. $C = 2\pi r$ **[1 mark]**

$47.12 = 2\pi r$

$r = \frac{47.12}{2\pi}$

$r = 7.5\,\text{cm}$ **[1 mark]**

$\text{Diameter} = 2r$

$= 2 \times 7.5$

$= 15\,\text{cm}$ **[1 mark]**

b. $\text{SA} = \pi r^2 + 2\pi rh$ **[1 mark]**

$= \pi \times 7.5^2 + 2 \times \pi \times 7.5 \times 8$ **[1 mark]**

$= 176.71 + 376.99$

$= 533.70\,\text{cm}^2$ **[1 mark]**

c. $\text{SA} = \pi r^2 + 2\pi rh$

$= \pi \times 4^2 + 2 \times \pi \times 4 \times 6$ **[1 mark]**

$= 50.27 + 150.80$

$= 201.07\,\text{cm}^2$ **[1 mark]**

d. $800\,\text{cm}^2 - (553.70 + 201.07) = 800\,\text{cm}^2 - (754.77)$

$= 45.23\,\text{cm}^2$ remaining **[1 mark]**

Therefore yes, she has enough fondant for this design. **[1 mark]**

e. Total volume $= V_{\text{bottom}} + V_{\text{top}}$ **[1 mark]**

$= \pi \times 7.5^2 \times 8 + \pi \times 4^2 \times 6$ **[1 mark]**

$= 1413.72 + 301.59$

$= 1715.31\,\text{cm}^3$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

f. Number of average serves $= \dfrac{1715.31\text{ cm}^3}{15\text{ cm}^3}$ **[1 mark]**

$= 114.35$

≈ 114 serves **[1 mark]**

This cake is therefore not large enough to serve the 150 guests at the wedding. **[1 mark]**

Question 27

Area of circle $= \pi(2.9)^2$

$= 26.4208\text{ cm}^2$

Area of square $= l^2$

$= 26.4208\text{ cm}^2$

Side length of square $= l$

$= \sqrt{26.4208}$

$= 5.1401\text{ cm}$

Let d = diagonal of the square

Using Pythagoras' theorem:

$5.14^2 + 5.14^2 = d^2$

$d = \sqrt{5.14^2 + 5.14^2}$

$= 7.269\text{ cm}$

© John Wiley & Sons Australia, Ltd

12 Applications of trigonometry

Topic	12	Applications of trigonometry
Subtopic	12.2	Trigonometric ratios

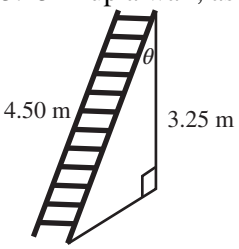
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2014, Further Mathematics 1, Section B, Module 2, Q.1; © VCAA*

Question 1 (1 mark)

The top of a ladder that is 4.50 m long rests 3.25 m up a wall, as shown in the diagram below.

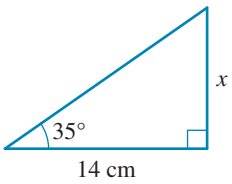


The angle, θ, that the ladder makes with the wall is closest to

A. 36°
B. 44°
C. 46°
D. 50°
E. 54°

Question 2 (1 mark)

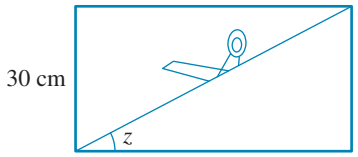
The value of x in the diagram is



A. 11.4 cm
B. 20 cm
C. 24.4 cm
D. 9.8 cm
E. 8 cm

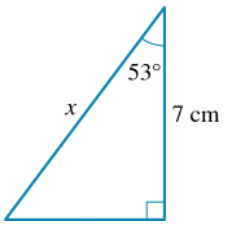
© John Wiley & Sons Australia, Ltd

Question 3 (1 mark)
A diagonal cut 45 cm long is made in a rectangular piece of cardboard 30 cm wide.



Calculate the angle, z, that the cut makes with the longer side of the cardboard.
A. 33.7°
B. 48.6°
C. 56.3°
D. 48.2°
E. 41.8°

Question 4 (1 mark)
The value of x in the diagram is



A. 11.63 cm
B. 10.64 cm
C. 8.76 cm
D. 5.28 cm
E. 4.21 cm

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)

Exactly how high up a wall will a ladder 4 metres long reach, if it rests at an angle of 45° to the horizontal

A. $2\sqrt{2}$ m

B. $4\sqrt{2}$ m

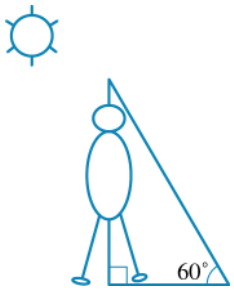
C. $4\sqrt{3}$ m

D. $\sqrt{3}$ m

E. $\sqrt{2}$ m

Question 6 (1 mark)

Winnie is 1.5 m tall. At 11 am, she casts a shadow as shown. How long is the shadow?



A. $1.5\sqrt{2}$ m

B. $1.5\sqrt{3}$ m

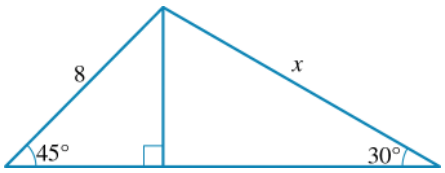
C. $\frac{\sqrt{3}}{2}$ m

D. $\frac{1.5}{\sqrt{2}}$ m

E. $\frac{3}{\sqrt{3}}$ m

© John Wiley & Sons Australia, Ltd

Question 7 (1 mark)
The exact value of x is



A. $4\sqrt{2}$
B. $\frac{1}{2}$
C. $8\frac{\sqrt{2}}{\sqrt{3}}$
D. $4\sqrt{6}$
E. $8\sqrt{2}$

Question 8 (1 mark)
The triangle ABC has side lengths 7.2 cm, 12 cm and 9.6 cm. The triangle is
A. right-angled.
B. obtuse-angled.
C. cannot be drawn.
D. insufficiently described to find its angles.
E. has no right angle.

© John Wiley & Sons Australia, Ltd

Topic	12	Applications of trigonometry
Subtopic	12.3	Angles of elevation and depression, and bearings

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

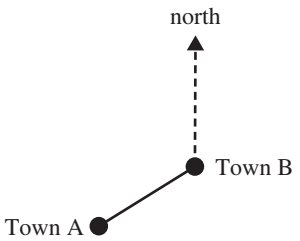
Source: *VCE 2019, Further Mathematics 1, Section B, Module 3, Q.2; © VCAA*

Question 1 (1 mark)

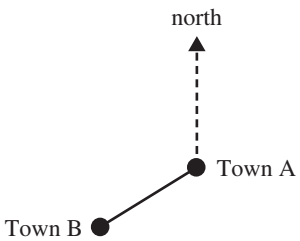
Town B is located on a bearing of 060° from Town A.

The diagram that could illustrate this is

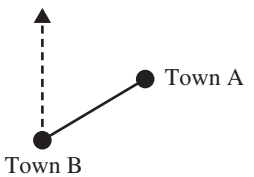
A.



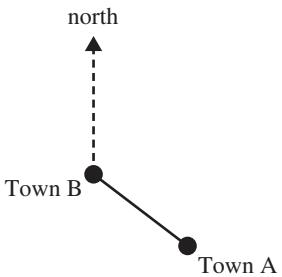
B.



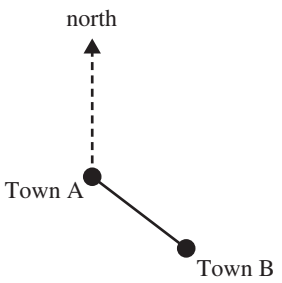
C.



D.



E.



© John Wiley & Sons Australia, Ltd

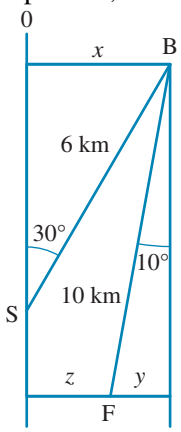
Question 2 (1 mark)

If the angle of elevation of the top of a cliff from the deck of a ship is 56°, which of the following is true?

A. The angle of depression of the cliff is 56° from the deck of the ship.
B. The angle of depression of the cliff is 34° from the deck of the ship.
C. The angle of elevation of the deck of a ship is 56° from the cliff.
D. The angle of depression of the deck of a ship is 56° from the cliff.
E. The angle of depression of the deck of a ship is 34° from the cliff.

Question 3 (3 marks)

A yacht race starts at point *S*. The yachts sail for 6 km on a bearing of 030° to a buoy at point *B*. They then turn to a bearing of 190° and sail to the finish at point *F*, 10 km away.



a. Calculate how far east of *S* buoy *B* is. **(1 mark)**

☐ km

b. Calculate how far east of buoy *B* the finish, *F* is. **(1 mark)**

☐ km

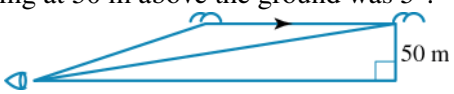
c. Calculate how far east of the start, *S*, the finish, *F* is. **(1 mark)**

☐ km

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
A jogger ran on a bearing of 095° for 1 hour, at which time he was 0.9 km south of his starting point. How fast did he run?
A. 0.90 km/h
B. 0.90 km
C. 10.33 km/h
D. 0.08 km
E. 0.18 km/h

Question 5 (4 marks)
The angle of elevation of a bird flying at 50 m above the ground was 5°.



a. How far away was the bird horizontally? (1 mark)
☐ m

b. Two minutes later, the bird was still flying at a height of 50 m but its angle of elevation was 4°.
How far away was the bird horizontally at that moment? **(1 mark)**
☐ m

c. Hence, find:
i. the distance travelled by the bird during the two minutes **(1 mark)**
☐ m

ii. how fast the bird flew, in metres per minute. **(1 mark)**
☐ metres per minute

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)
If the outward course of a yacht was on a bearing of 040°, then the bearing on the inward course would be:
A. 320°
B. 220°
C. 140°
D. 050°
E. 040°

Question 7 (1 mark)
A ship's navigator could not hear some of his radioed directions:
Sail 15 km due north, turn XXX and sail 20 km on that course. Then turn and sail due west and return to your starting point.
A. '138.6° towards the west'.
B. '41.4° towards the west'.
C. '41.4° towards the east'.
D. '48.6° towards the west'.
E. '138.6° towards the east'.

© John Wiley & Sons Australia, Ltd

Topic	12	Applications of trigonometry
Subtopic	12.4	The sine rule

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2017, Further Mathematics 1, Section B, Module 3, Q.7; © VCAA*

Question 1 (1 mark)

A triangle ABC has:

- one side, $\overline{AB}$, of length 4 cm
- one side, $\overline{BC}$, of length 7 cm
- one angle, $\angle ACB$, of 26°.

Which one of the following angles, correct to the nearest degree, could not be another angle in triangle ABC?

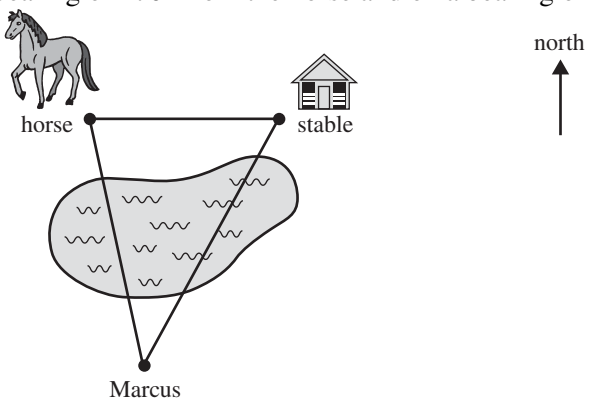
A. 24°
B. 50°
C. 104°
D. 130°
E. 144°

© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics 1, Section B, Module 3, Q.6; © VCAA*

Question 2 (1 mark)

Marcus is on the opposite side of a large lake from a horse and its stable. The stable is 150 m directly east of the horse. Marcus is on a bearing of 170° from the horse and on a bearing of 205° from the stable.

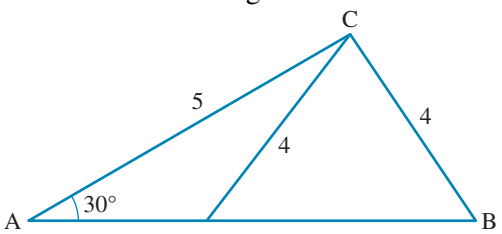


The straight-line distance, in metres, between Marcus and the horse is closest to

A. 45
B. 61
C. 95
D. 192
E. 237

Question 3 (1 mark)

In the diagram, the angle(s) opposite the side with length 5 units is/are



A. 38.68°
B. 51.32°
C. 141.32°
D. 111.32° and 68.68°
E. 38.68° and 141.32°

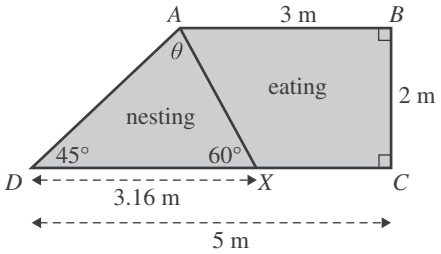
© John Wiley & Sons Australia, Ltd

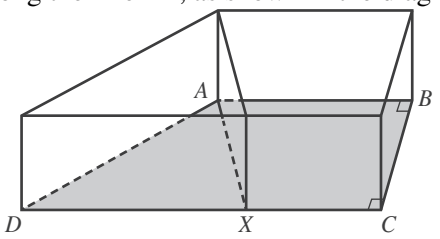
Source: *VCE 2014, Further Mathematics 2, Module 2, Q.2; © VCAA*

Question 4 (5 marks)

The chicken coop has two spaces, one for nesting and one for eating.

The nesting and eating spaces are separated by a wall along the line AX, as shown in the diagrams below.





$DX = 3.16$ m, $\angle ADX = 45°$ and $\angle AXD = 60°$.

a. Write down a calculation to show that the value of θ is 75°. **(1 mark)**

b. The sine rule can be used to calculate the length of the wall AX. **(1 mark)**

Fill in the missing numbers below.

$$\frac{AX}{\sin(\_\_\_\_\_)^\circ} = \frac{\_\_\_\_\_}{\sin(\_\_\_\_\_)^\circ}$$

c. What is the length of AX?

Write your answer in metres, correct to two decimal places. **(1 mark)**

☐ m

d. Calculate the area of the floor of the nesting space, ADX.

Write your answer in square metres, correct to one decimal place. **(1 mark)**

☐ m^2

Wire mesh will cover the roof of the eating space.

The area of the walls along the lines AB,BC and CX will also be covered with wire mesh.

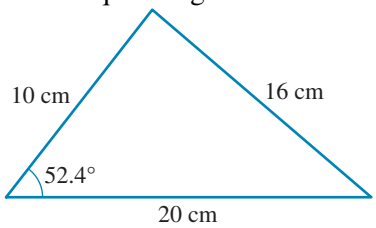
What total area, in square metres, will be covered by wire mesh?

Write your answer, correct to the nearest square metre.

☐ m^2

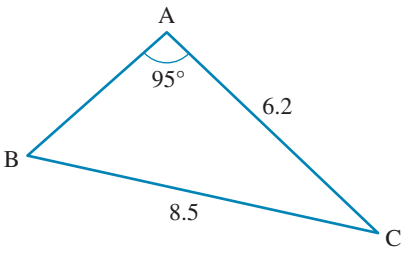
© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)
The largest angle in this triangle is θ. Which equation gives the correct value for θ?



A. $\sin(\theta) = \dfrac{20 \sin(52.4°)}{16}$
B. $\sin(\theta) = \dfrac{16 \sin(52.4°)}{20}$
C. $\sin(\theta) = \dfrac{20 \sin(52.4°)}{10}$
D. $\sin(\theta) = \dfrac{10 \sin(52.4°)}{20}$
E. $\sin(\theta) = \dfrac{10 \sin(52.4°)}{16}$

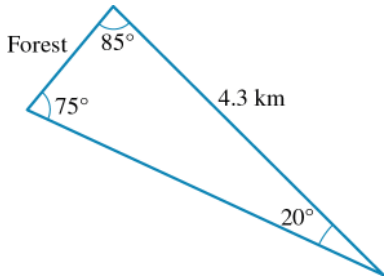
Question 6 (1 mark)
In the triangle, side c (opposite angle C) is



A. 38.39
B. 46.61
C. 5.3
D. 6.2
E. 8.47

© John Wiley & Sons Australia, Ltd

Question 7 (3 marks)
A triangular area of flat land is bounded on one side by a forest and on the other sides by fences, as shown in the diagram.

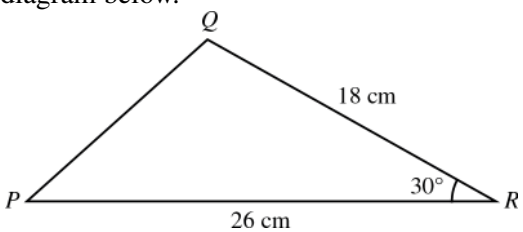


a. Find the length of the frontage of the land at the forest. **(1 mark)**
☐ km

b. Find the area of the land. **(1 mark)**
☐ km^2

c. What length of fencing is needed? **(1 mark)**
☐ km

Question 8 (1 mark)
A triangle *PQR* is shown in the diagram below.



The length of the side *QR* is 18 cm.
The length of the side *PR* is 26 cm.
The angle *QRP* is 30°.
The area of triangle *PQR*, in square centimetres, is closest to
A. 117
B. 162
C. 171
D. 234
E. 468

© John Wiley & Sons Australia, Ltd

Topic	12	Applications of trigonometry
Subtopic	12.5	The cosine rule

online only

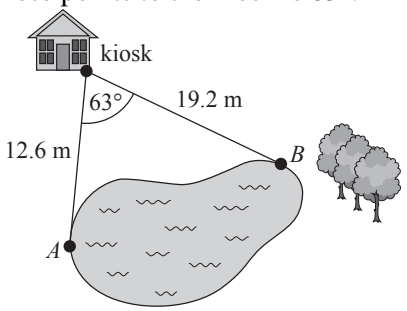
To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2013, Further Mathematics 1, Section B, Module 2, Q.2; © VCAA*

Question 1 (1 mark)

The distances from a kiosk to points A and B on opposite sides of a pond are found to be 12.6 m and 19.2 m respectively.

The angle between the lines joining these points to the kiosk is 63°.



The distance, in m, across the pond between points A and B can be found by evaluating

A. $\frac{1}{2} \times 12.6 \times 19.2 \times \sin(63°)$

B. $\frac{19.2 \times (63°)}{12.6}$

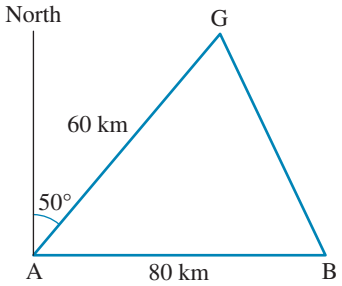
C. $\sqrt{12.6^2 + 19.2^2}$

D. $\sqrt{12.6^2 + 19.2^2 - 2 \times 12.6 \times 19.2 \cos(63°)}$

E. $\sqrt{s(s - 12.6)(s - 19.2)(s - 63)}$, where $s = \frac{1}{2}(12.6 + 10.2 + 6.3)$

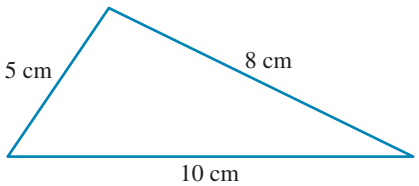
© John Wiley & Sons Australia, Ltd

Question 2 (1 mark)
The towns Alferton (A) and Betaton (B) are 80 km apart. Gamerton (G) is 60 km from Alferton, on a bearing of 050°. Betaton is due east of Alferton.



The distance between Gamerton and Betaton is
A. 131.17 km
B. 79.52 km
C. 61.88 km
D. 51.44 km
E. 51.42 km

Question 3 (1 mark)
The smallest angle of the triangle is θ.



Which equation would give the correct value of θ?
A. $\cos(\theta) = \dfrac{5^2 + 10^2 - 8^2}{2 \times 5 \times 10}$
B. $\cos(\theta) = \dfrac{8^2 + 10^2 - 5^2}{2 \times 8 \times 10}$
C. $\cos(\theta) = \dfrac{5^2 + 8^2 - 10^2}{2 \times 5 \times 8}$
D. $\cos(\theta) = \dfrac{8^2 + 10^2 - 5^2}{2 \times 10 \times 5}$
E. $\cos(\theta) = \dfrac{8^2 + 5^2 - 10^2}{2 \times 5 \times 10}$

© John Wiley & Sons Australia, Ltd

Source: *VCE 2018, Further Mathematics 1, Section B, Module 3, Q.6; © VCAA*

Question 4 (1 mark)

Aaliyah is bushwalking.

She walks 5.4 km from a starting point on a bearing of 045° until she reaches a hut. From this hut, she walks 2.8 km on a bearing of 300° until she reaches a river.

From the river, she turns and walks back directly to the starting point.

The total distance that she walks, in kilometres, is closest to

A. 8.2

B. 13.2

C. 13.6

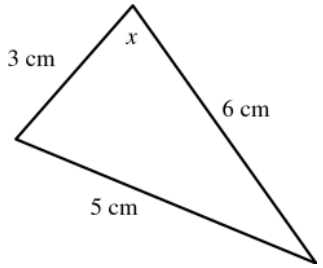
D. 14.1

E. 14.9

Source: *VCE 2015, Further Mathematics 1, Section B, Module 2, Q.3; © VCAA*

Question 5 (1 mark)

The lengths of the sides of a triangle are 3 cm, 6 cm and 5 cm, as shown below.



The angle, x, can be found using

A. $\cos(x) = \dfrac{3^2 + 6^2 - 5^2}{2 \times 3 \times 6}$

B. $\cos(x) = \dfrac{3^2 + 6^2 + 5^2}{2 \times 3 \times 6}$

C. $\cos(x) = \dfrac{3^2 + 5^2 - 6^2}{2 \times 3 \times 6}$

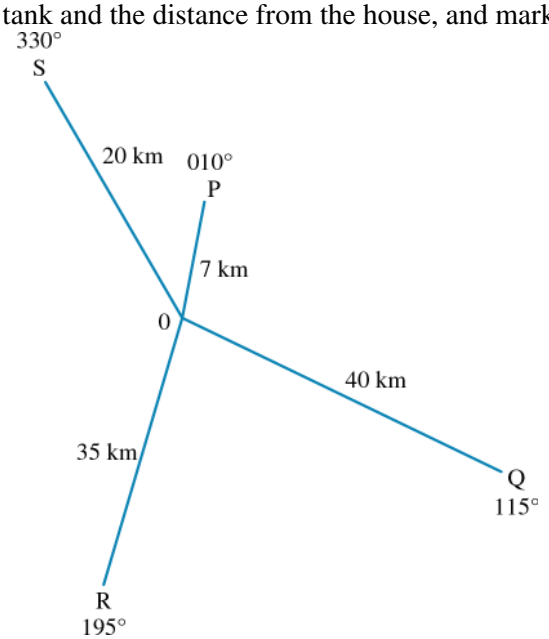
D. $\cos(x) = \dfrac{3}{5}$

E. $\cos(x) = \dfrac{3}{6}$

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)
An orienteering course starts at point X, running 5.2 km on a bearing of 225° to point Y. From point Y, the next mark on the course, Z, is on a bearing of 150°, 4 km away. The distance from Z back to X is
A. 53.8 km
B. 32.3 km
C. 7.34 km
D. 5.7 km
E. 7.0 km

Question 7 (3 marks)
Sally the surveyor makes a radial survey of the water tanks on a farm relative to the farmhouse. She measures the bearings of each tank and the distance from the house, and marks them on her map.



a. Find the size of the angle QOR. **(1 mark)**
□°

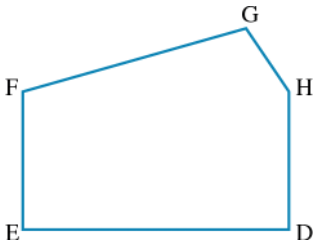
b. Hence, find the distance between the tanks at Q and R. **(1 mark)**
□ km

c. The pipe running from the tank at R to the farmhouse at O cannot be used. How far will water have to flow from the tank at R to reach the farmhouse via a pipeline to the tank at Q? **(1 mark)**

☐ km

Question 8 (1 mark)

A cross-section of the front of a factory is shown below. The width of the factory is $ED = 20$ m. The two sections of the factory roof have lengths $FG = 17.4$ m and $GH = 5.8$ m.



The size of $\angle FGH$ is closest to:

A. 108.4
B. 90.0
C. 71.6
D. 55.7
E. 16.0

© John Wiley & Sons Australia, Ltd

Topic	12	Applications of trigonometry
Subtopic	12.6	Area of triangles

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Question 1 (1 mark)

A triangle has side lengths of 24 cm, 26 cm and 20 cm. The area of the triangle is closest to

A. 322 cm^2

B. 2791 cm^2

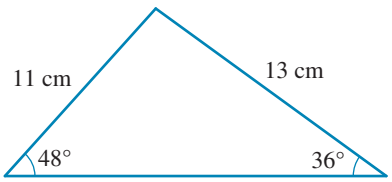
C. 51975 cm^2

D. 228 cm^2

E. 53 cm^2

Question 2 (1 mark)

The area of the triangle shown is:



A. 71.11 cm^2

B. 53.13 cm^2

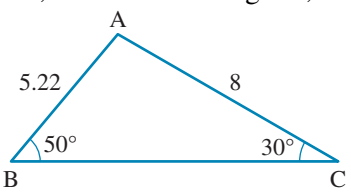
C. 42.03 cm^2

D. 11.93 cm^2

E. 142.22 cm^2

Question 3 (1 mark)

The area of a triangular sheet of cardboard, shown in the diagram, is



A. 20.88 sin (50°)

B. 20.88 sin (30°)

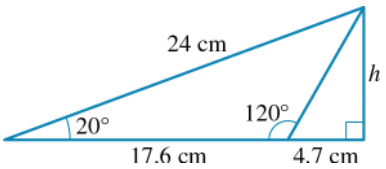
C. 20.88 sin (100°)

D. 10.44 sin (50°)

E. 10.44 sin (100°)

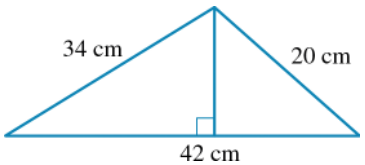
© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)
The area of the shaded triangle is 72.23 cm^2.



The height, *h*, of the triangle is
A. 8.31 cm
B. 6.48 cm
C. 30.7 cm
D. 3.2 cm
E. 6.02 cm

Question 5 (3 marks)
For the triangle shown, find:



a. the perimeter **(1 mark)**
☐ cm

b. the area **(1 mark)**
☐ cm^2

c. the height. **(1 mark)**
☐ cm

© John Wiley & Sons Australia, Ltd

Topic	12	Applications of trigonometry
Subtopic	12.7	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au.**

Source: *VCE 2021, Further Mathematics 1, Section B, Module 3, Q.8; © VCAA*

Question 1 (1 mark)

Rod and Lucia went on a bushwalk.
They walked 1400 m from the car park to reach a lookout that was directly east of the car park.
From the lookout, Rod returned to the car park via a cafe and Lucia returned to the car park via a swimming hole.

- The bearing of the swimming hole from the lookout is 290°.
- The bearing of the cafe from the lookout is 240°.
- The swimming hole is 950 m from the car park.
- The cafe is 700 m from the car park.
- The swimming hole is closer to the lookout than it is to the car park.

In relation to the total distance each of them individually walked from the lookout back to the car park, which one of the following statements is true?

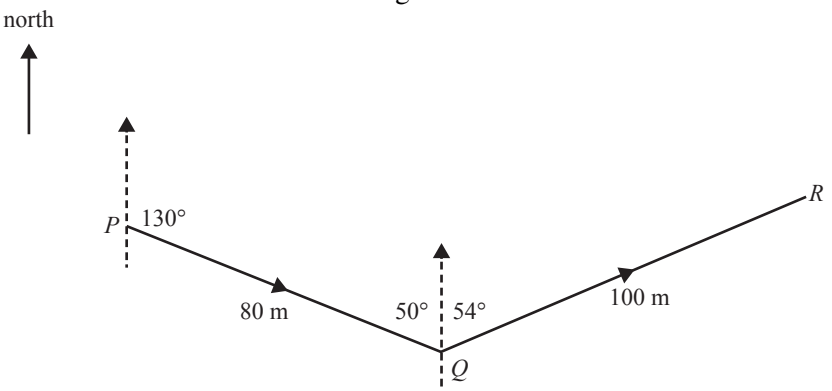
A. Rod and Lucia walked the same distance.
B. Rod walked 467 m further than Lucia, to the nearest metre.
C. Rod walked 717 m further than Lucia, to the nearest metre.
D. Lucia walked 924 m further than Rod, to the nearest metre.
E. Lucia walked 1174 m further than Rod, to the nearest metre.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics 2, Module 3, Q.4; © VCAA*

Question 2 (2 marks)

During a game of golf, Salena hits a ball twice, from P to Q and then from Q to R.
The path of the ball after each hit is shown in the diagram below.



After Salena's first hit, the ball travelled 80 m on a bearing of 130° from point P to point Q.
After Salena's second hit, the ball travelled 100 m on a bearing of 054° from point Q to point R.

a. Another ball is hit and travels directly from P to R.
Use the cosine rule to find the distance travelled by this ball.
Round your answer to the nearest metre. **(1 mark)**

☐ m

b. What is the bearing of R from P?
Round your answer to the nearest degree. **(1 mark)**

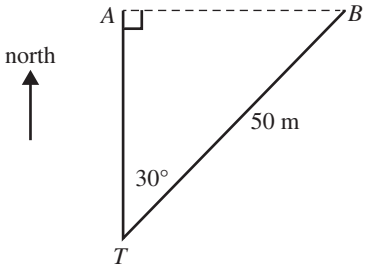
☐°

© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics 2, Module 3, Q.2; © VCAA*

Question 3 (2 marks)

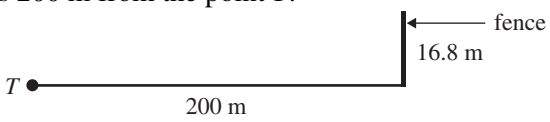
Salena practises golf at a driving range by hitting golf balls from point T.
The first ball that Salena hits travels directly north, landing at point A.
The second ball that Salena hits travels 50 m on a bearing of 030°, landing at point B.
The diagram below shows the positions of the two balls after they have landed.



a. How far apart, in metres, are the two golf balls? **(1 mark)**

☐ *m*

A fence is positioned at the end of the driving range.
The fence is 16.8 m high and is 200 m from the point T.



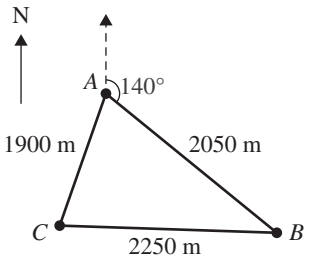
b. What is the angle of elevation from T to the top of the fence? **(1 mark)**
Round your answer to the nearest degree.

☐°

Source: *VCE 2014, Further Mathematics 1, Section B, Module 2, Q.7; © VCAA*

Question 4 (1 mark)

A cross-country race is run on a triangular course. The points *A, B* and *C* mark the corners of the course, as shown below.



The distance from *A* to *B* is 2050 m.
The distance from *B* to *C* is 2250 m.
The distance from *A* to *C* is 1900 m.
The bearing of *B* from *A* is 140°.
The area within the triangular course *ABC*, in square metres, can be calculated by evaluating

A. $\sqrt{3100 \times 1200 \times 1050 \times 850}$

B. $\sqrt{3100 \times 2250 \times 2050 \times 1900}$

C. $\sqrt{6200 \times 4300 \times 4150 \times 3950}$

D. $\frac{1}{2} \times 2050 \times 2250 \times \sin(140°)$

E. $\frac{1}{2} \times 2050 \times 2250 \times \sin(40°)$

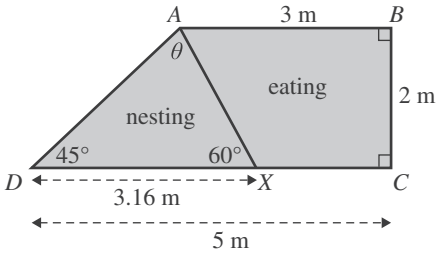
© John Wiley & Sons Australia, Ltd

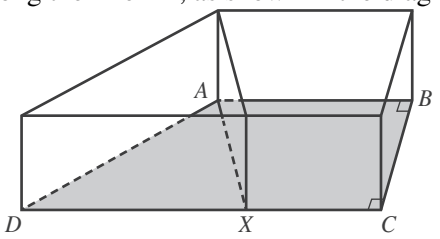
Source: *VCE 2014, Further Mathematics 2, Module 2, Q.2; © VCAA*

Question 5 (5 marks)

The chicken coop has two spaces, one for nesting and one for eating.

The nesting and eating spaces are separated by a wall along the line AX, as shown in the diagrams below.





$DX = 3.16$ m, $\angle ADX = 45°$ and $\angle AXD = 60°$.

a. Write down a calculation to show that the value of θ is 75°. **(1 mark)**

b. The sine rule can be used to calculate the length of the wall AX. **(1 mark)**

Fill in the missing numbers below.

$$\frac{AX}{\sin \_\_\_\_\_^\circ} = \frac{\_\_\_\_\_}{\sin \_\_\_\_\_^\circ}$$

c. What is the length of AX? **(1 mark)**

Write your answer in metres, correct to two decimal places.

☐ m

d. Calculate the area of the floor of the nesting space, ADX. **(1 mark)**

Write your answer in square metres, correct to one decimal place.

☐ m^2

e. The height of the chicken coop is 1.8 m. **(1 mark)**

Wire mesh will cover the roof of the eating space.

The area of the walls along the lines AB, BC and CX will also be covered with wire mesh.

What total area, in square metres, will be covered by wire mesh?

Write your answer, correct to the nearest square metre.

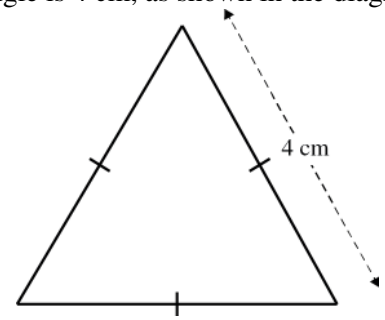
☐ m^2

© John Wiley & Sons Australia, Ltd

Source: *VCE 2021, Further Mathematics 1, Section B, Module 3, Q.4; © VCAA*

Question 6 (1 mark)

The side length of an equilateral triangle is 4 cm, as shown in the diagram below.



Which one of the following is **not** a correct calculation for the area of this triangle?

A. $\frac{\sqrt{3} \times 4^2}{4}$

B. $2 \times \sqrt{12}$

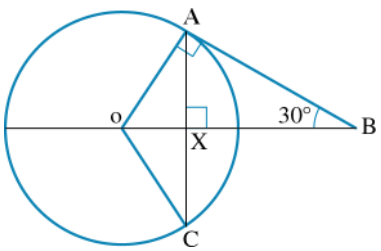
C. $\frac{1}{2} \times 4 \times 4$

D. $\frac{4^2}{2} \times \sin(60°)$

E. $\sqrt{6(6-4)^3}$

Question 7 (4 marks)

The radius of the circle in the diagram is 10 cm.



Find the exact value of the length of:

a. *AB* **(1 mark)**

☐ cm

b. *OB* **(1 mark)**

☐ cm

© John Wiley & Sons Australia, Ltd

c. *OX* **(1 mark)**

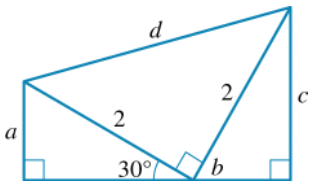
☐ cm

d. *AC* **(1 mark)**

☐ cm

Question 8 (4 marks)

For the diagram shown, find the exact value of:



a. *a* **(1 mark)**

☐

b. *b* **(1 mark)**

☐°

c. *c* **(1 mark)**

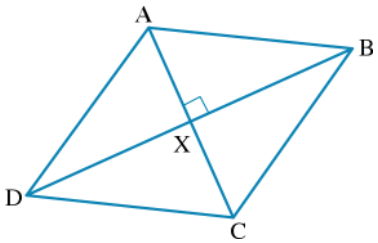
☐

d. *d* **(1 mark)**

☐

Question 9 (3 marks)

ABCD is a rhombus with diagonal $AC = 20$ cm and $\angle BAX = 50°$.



a. What is the length of *AX*? **(1 mark)**

☐ cm

b. Find the side length *AB* of the rhombus. **(1 mark)**

☐ cm

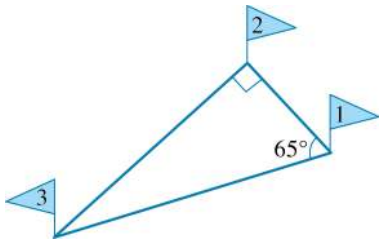
c. Find the length of *XB*. **(1 mark)**

☐ cm

© John Wiley & Sons Australia, Ltd

Question 10 (3 marks)

The third hole of a golf course is to be built so that the first three holes form a right-angled triangle, as shown.



distance between Flag 1 and Flag 2 is 540 m. The angle at Flag 1 is 65°.

a. What is the distance between Flag 2 and Flag 3? **(1mark)**

☐ m

b. What is the distance between Flag 3 and Flag 1? **(1 mark)**

☐ m

c. How many kilometres will a golfer walk if she walks from Flag 1 to Flag 2 to Flag 3 and back to Flag 1? **(1 mark)**

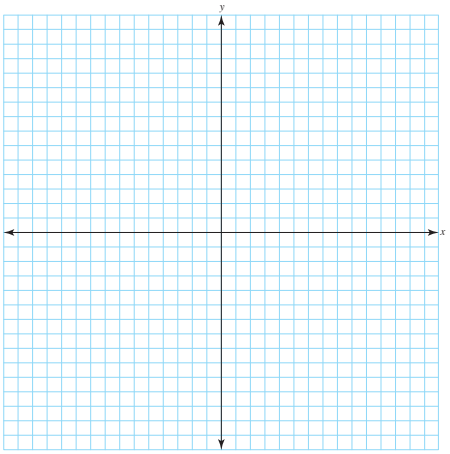
☐ km

© John Wiley & Sons Australia, Ltd

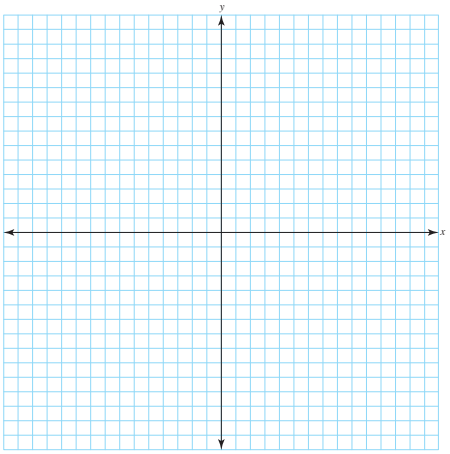
Question 11 (3 marks)

The last known position of a light aircraft was 75 km from Pawsey and 65 km from Queenston. Pawsey is 38 km west of Queenston.

a. Draw a diagram showing the **two** possible positions of the aircraft. **(1 mark)**



b. Polly, in Pawsey, said that the aircraft was on a bearing of 030° from her house. **(1 mark)**
Show the last known position of the aircraft on your diagram.



c. Find the bearing of the aircraft from Queenston at that time. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 12 (3 marks)

A tutor asked a student to solve ΔABC using the sine rule. The student was given the dimensions of the triangle, as follows:

$A = 30°$

$b = 5\,\text{cm}$

$a = 2\,\text{cm}$

a. Find the angle B and comment on the result. **(1 mark)**

b. The tutor realised that there may be a problem with the dimensions given and changed them as follows: **(1 mark)**

$A = 20°$

$b = 5\,\text{cm}$

$a = 2\,\text{cm}$

Find the angle B and comment on the result.

c. In a final attempt, the tutor gave the following dimensions for the triangle: **(1 mark)**

$A = 36.87°$

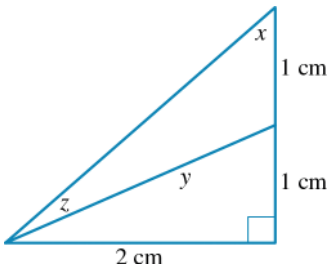
$b = 5\,\text{cm}$

$a = 3\,\text{cm}$

Find the angle B and comment on the result.

© John Wiley & Sons Australia, Ltd

Question 13 (3 marks)
For the diagram below, find:



a. angle x **(1 mark)**

□°

b. the exact length of y **(1 mark)**

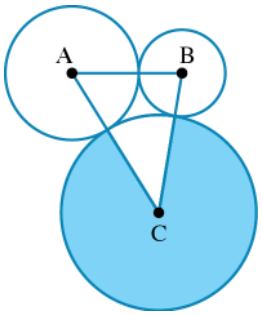
□ cm

c. the exact value of the sine of angle z. **(1 mark)**

□

© John Wiley & Sons Australia, Ltd

Question 14 (2 marks)
Three circles, with centres *A*, *B* and *C*, are in contact. The radii of the circles are 1.5 cm, 1 cm and 2.5 cm, respectively.



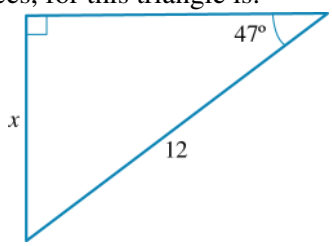
a. Find the size of the angle *ACB*. **(1 mark)**

□°

b. Find the area of the shaded region of the circle with centre at *C*. **(1 mark)**

□ cm^2

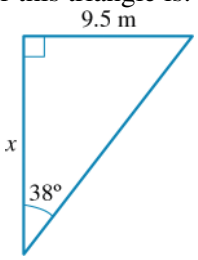
Question 15 (1 mark)
The value of *x*, correct to 2 decimal places, for this triangle is:



A. 16.41
B. 8.78
C. 8.18
D. 12.87
E. 17.60

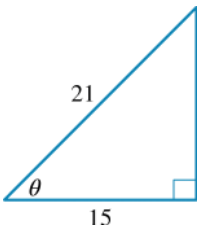
© John Wiley & Sons Australia, Ltd

Question 16 (1 mark)
The value of x, correct to 2 decimal places, for this triangle is:



A. 7.42 m
B. 12.06 m
C. 15.43 m
D. 5.85 m
E. 12.16 m

Question 17 (1 mark)
The value of θ, in degrees, for this triangle is:



A. 44.42°
B. 45.58°
C. 35.53°
D. 54.45°
E. 42.44°

© John Wiley & Sons Australia, Ltd

Question 18 (1 mark)
Which of the following statements is *incorrect*?

A. $\cos(30°) = \frac{\sqrt{3}}{2}$

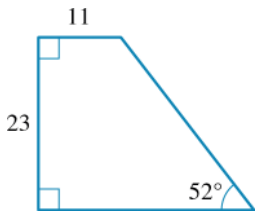
B. $\sin(30°) = \frac{1}{2}$

C. $\cos(60°) = \frac{1}{2}$

D. $\tan(60°) = \frac{1}{\sqrt{3}}$

E. $\sin(60°) = \frac{\sqrt{3}}{2}$

Question 19 (1 mark)
The perimeter of this block of land is:



A. 45
B. 92.16
C. 74.19
D. 62.97
E. 81.16

© John Wiley & Sons Australia, Ltd

Question 20 (1 mark)
From a vertical surf life saving tower 15 m high, the angle of depression to the breaking surf is 9.5°. How far, correct to 2 decimal places, must the surf life saver run before she reaches the surf?
A. 90.88 m
B. 89.64 m
C. 15.21 m
D. 2.51 m
E. 2.48 m

Question 21 (1 mark)
In triangle ABC: $a = 11.2, c = 7.5$ and $A = 48°$. The value of b is:
A. 14.73
B. 29.84°
C. 102.16°
D. 13.48
E. 8.32

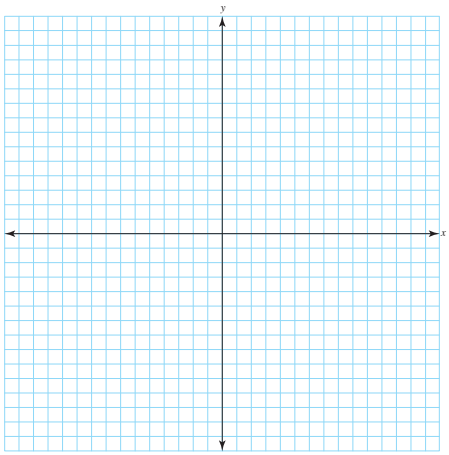
Question 22 (1 mark)
Given that $a = 2.8$, $b = 5.2$ and $C = 54°$, find the length of the third side of the triangle, correct to 1 decimal place, is:
A. 21.3
B. 7.5
C. 4.2
D. 32.3
E. 6.2

© John Wiley & Sons Australia, Ltd

Question 23 (4 marks)

A paddleboard rider sets out on a journey to explore a wilderness lake. The lake traverses 8.5 km in an easterly direction and then 3.2 km in a southerly direction. At this point, the rider decides to camp for the night.

a. Represent this scenario as a diagram. **(1 mark)**



b. Calculate the angle travelled from the starting point to the nearest degree and minute. **(1 mark)**

☐ °

c. Calculate the bearing from the starting point as:

i. a true bearing **(1 mark)**

☐ °

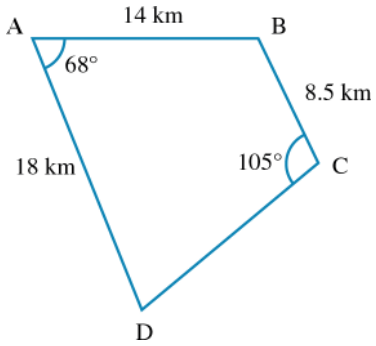
ii. a compass bearing. **(1 mark)**

S ☐ °E

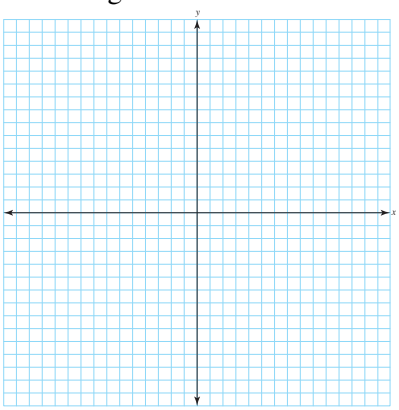
© John Wiley & Sons Australia, Ltd

Question 24 (7 marks)

A newly married couple is looking at the best way to use their block of land. To make best use of their land, they need to be able to calculate its area. To do this they need to complete some calculations, so that they have all of the required measurements.



a. The couple measure the distance from B to D and find it to be 27 km exactly. Break the block of land into two triangles which can be used to solve for all missing side lengths and angles. **(2 marks)**



b. Find the missing angles in triangle ABD. **(1 mark)**

$B = \square°$

$D = \square°$

c. Find the missing side length and angles in triangle CBD. **(1 mark)**

$D = \square°$

$B = \square°$

$a = \square$ km

© John Wiley & Sons Australia, Ltd

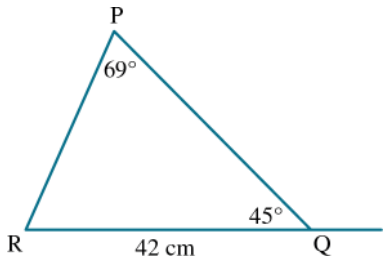
d. Can you think of a way to check that your angle calculations are correct? Explain your reasoning. **(2 marks)**

e. Calculate the total area of the block of land. **(1 mark)**

☐ km^2

Question 25 (1 mark)

The length of PQ in the given triangle is closest to



A. 9.7
B. 36.8
C. 41.1
D. 42.9
E. 181.6

© John Wiley & Sons Australia, Ltd

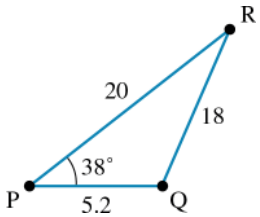
Question 26 (1 mark)

The information listed below is sufficient to create two separate triangles.

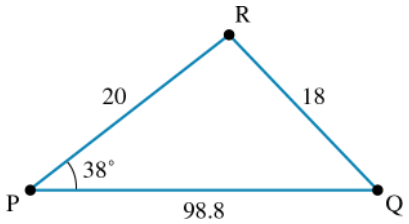
$\angle P = 38°, p = 18, q = 20$

One possible triangle solution is:

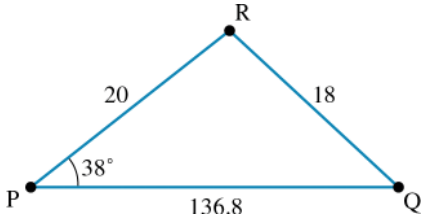
A.



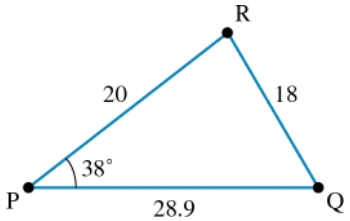
B.



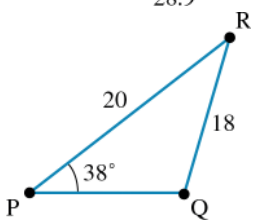
C.



D.

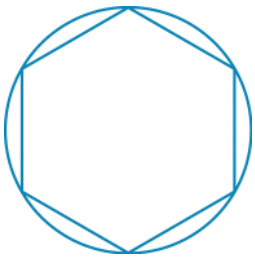


E.



© John Wiley & Sons Australia, Ltd

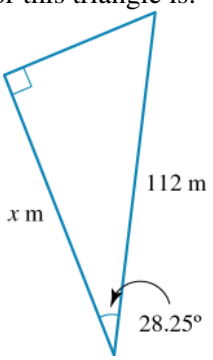
Question 27 (2 marks)
A hexagon is drawn inside a circle of radius 4 cm.



a. Find the area of the hexagon. **(1 mark)**
☐ cm^2

b. Find the area of the circle not occupied by the hexagon. **(1 mark)**
☐ cm^2

Question 28 (1 mark)
The value of x, correct to 2 decimal places, for this triangle is:



A. 127.14 m
B. 53.01 m
C. 98.66 m
D. 60.18 m
E. 208.44 m

© John Wiley & Sons Australia, Ltd

Question 29 (1 mark)
From a vertical surf life saving tower 15 m high, the angle of depression to the breaking surf is 9.5°. How far, correct to 2 decimal places, must the surf life saver run before she reaches the surf?
A. 90.88 m
B. 89.64 m
C. 15.21 m
D. 2.51 m
E. 2.48 m

© John Wiley & Sons Australia, Ltd

Answers and marking guide

12.2 Trigonometric ratios

Question 1

$$\cos(\theta) = \frac{3.25}{4.5}$$
$$\theta = \cos^{-1}\left(\frac{3.25}{4.5}\right)$$
$$= 43.8°$$
$$\approx 44°$$

Question 2

$$\tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$$
$$\tan(35°) = \frac{x}{14}$$
$$x = \tan(35°) \times 14$$
$$\approx 9.8\,\text{cm}$$

Question 3

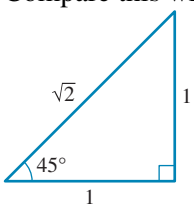
$$\sin(\theta) = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$
$$\sin(z) = \frac{30}{45}$$
$$z = \sin^{-1}\left(\frac{30}{45}\right)$$
$$\approx 41.8°$$

Question 4

$$\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$
$$\cos(53°) = \frac{7}{x}$$
$$x = \frac{7}{\cos(53°)}$$
$$\approx 11.63\,\text{cm}$$

Question 5

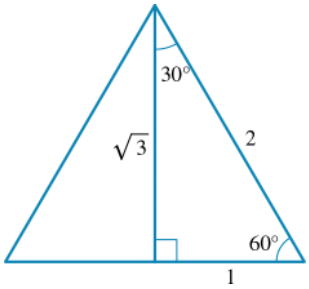
Draw the diagram, and label the angle and length.
Compare this with



Question 6

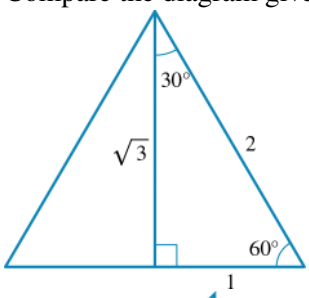
Compare the diagram given with

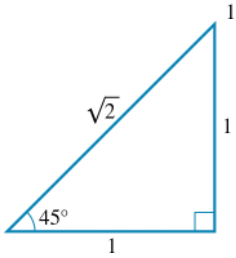
© John Wiley & Sons Australia, Ltd



Question 7

Compare the diagram given with these two diagrams.





Question 8

Three sides give sufficient information to define a triangle.

$a^2 + b^2 = c^2$

$9.6^2 + 7.2^2 = 144$

$= 12^2$

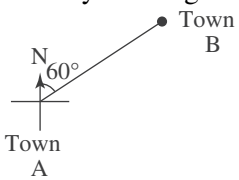
Therefore ΔABC is right-angled.

12.3 Angles of elevation and depression, and bearings

Question 1

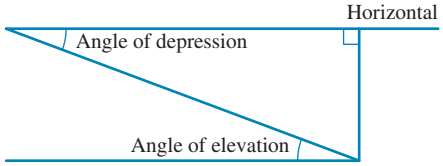
The true bearing of 60° would start from North and move in a clockwise direction.

Start by drawing Town A and a compass rose, then add the bearing to Town B.



Question 2

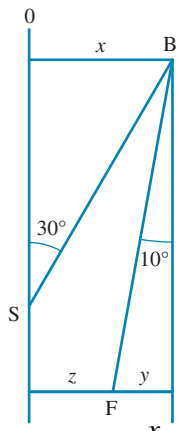
Note the equality of angles of elevation and depression.



© John Wiley & Sons Australia, Ltd

Question 3

a.

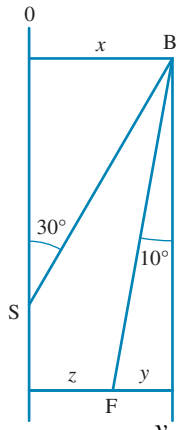


$\sin(30°) = \frac{x}{6}$

$x = 6\sin(30°)$

$x = 3$ km **[1 mark]**

b.

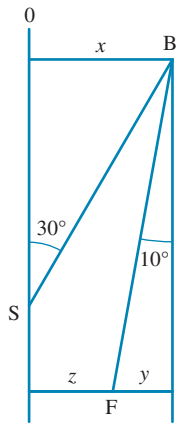


$\sin(10°) = \frac{y}{10}$

$y = 10\sin(10°)$

$y \approx 1.74$ km **[1 mark]**

c.



$z = x - y$

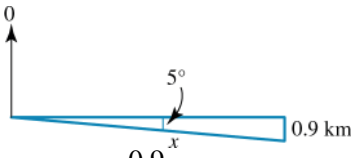
$= 3 - 1.74$

$= 1.26$ km **[1 mark]**

Question 4

095° clockwise from 0 means 5° from bearing 090°.

© John Wiley & Sons Australia, Ltd



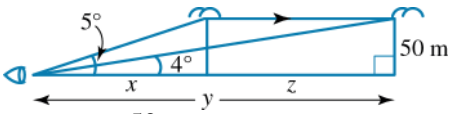
$$\sin(5°) = \frac{0.9}{x}$$

$$x \approx 10.326$$

$$\approx 10.33 \text{ km/h}$$

Question 5

a.

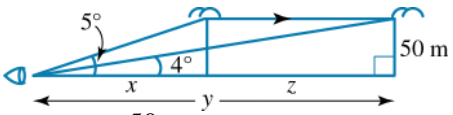


$$\tan(5°) = \frac{50}{x}$$

$$x = \frac{50}{\tan(5°)}$$

$$x \approx 571.503 \text{ m } \textbf{[1 mark]}$$

b.

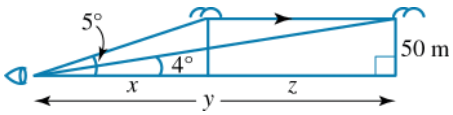


$$\tan(4°) = \frac{50}{y}$$

$$y = \frac{50}{\tan(4°)}$$

$$y \approx 715.033 \text{ m } \textbf{[1 mark]}$$

c. i.



$$z = y - x$$

$$= 715.033 - 571.503$$

$$= 143.53 \text{ m } \textbf{[1 mark]}$$

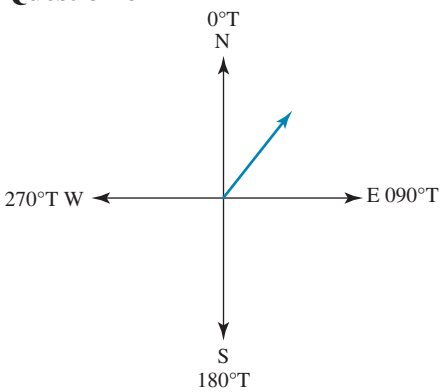
ii. Speed = 143.53 per 2 minutes

= 71.765 per minute

≈ 71.8 metres per minute **[1 mark]**

Be careful of round-off error.

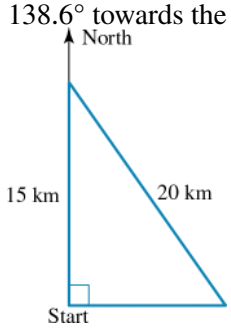
Question 6



© John Wiley & Sons Australia, Ltd

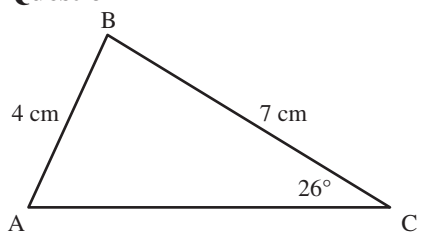
Question 7

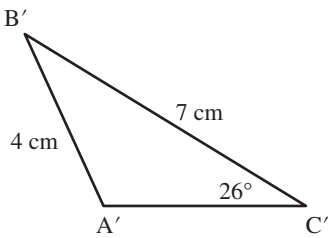
138.6° towards the east.



12.4 The sine rule

Question 1





From the first triangle, we can find angle A using the sine rule.

$$\frac{\sin(26)^\circ}{4} = \frac{\sin(A)^\circ}{7}$$

Solving for A, $A = 50^\circ$ (to the nearest degree)

Using the angle sum of a triangle, $A + B + C = 180^\circ$

So, $50^\circ + B + 26^\circ = 180^\circ$,

$B = 104^\circ$

From the ambiguous case, $A' = 180^\circ - 50^\circ = 130^\circ$ (to the nearest degree)

Again, using the angle sum of a triangle, A' + B' + C' = 180°

So, $130^\circ + B' + 26^\circ = 180^\circ$,

$B' = 24^\circ$

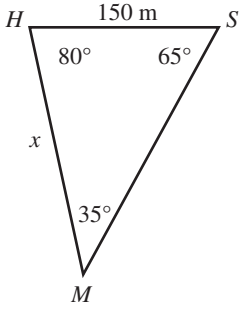
So the angle that could not be an angle in the triangle ABC is 144°.

***VCAA Examination Report* note:**

Given that this question required students to draw their own diagrams from given information, it was very well completed, with over half of the students answering it correctly.

© John Wiley & Sons Australia, Ltd

Question 2



$$\frac{x}{\sin 65°} = \frac{150}{\sin 35°}$$
$$x = 237.0149$$
$$= 237 \text{ m}$$

Question 3

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$
$$\frac{5}{\sin(B)} = \frac{4}{\sin(30)}$$
$$\sin(B) = \frac{5\sin(30)}{4}$$
$$B = \sin^{-1}\left(\frac{5\sin(30)}{4}\right)$$
$$\approx 38.68° \text{ and}$$
$$180 - 38.68° = 141.32°$$
$$\therefore B \text{ is } 38.68° \text{ and } 141.32°$$

Question 4

a. Angles in a triangle add to 180°.

$\theta = 180 - (45 + 60)$

$= 75°$.

[1 mark for first line, not answer]

***VCAA Assessment Report* note:**

A suitable calculation that resulted in 75 was required.

b. $\frac{d}{\sin D} = \frac{a}{\sin A}$

$\frac{AX}{\sin(45°)} = \frac{3.16}{\sin(75°)}$ **[1 mark]**

c. $\frac{AX}{\sin(45°)} = \frac{3.16}{\sin(75°)}$

$AX = \frac{3.16}{\sin(75°)} \times \sin(45°)$

$= 2.31 \text{ m}$ **[1 mark]**

d. $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 3.16 \times 2$

$= 3.2 \text{ m}^2$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

e. $CX = 5 - 3.16$
$= 1.84\,\text{m}$
$A = A_{top} + A_{side}AB + A_{side}CB + A_{side}CX$ **[1 mark]**
$= \frac{1}{2}(3 + 1.84) \times 2 + 3 \times 1.8 + 2 \times 1.8 + 1.84 \times 1.8$
$= 17.15\,\text{m}^2$
$= 17\,\text{m}^2$ **[1 mark]**

***VCAA Assessment Report* note:**
Many students misread the question and covered the nesting and eating spaces. Some others who covered only the eating space as required, then incorrectly included the wall *AX*.
A method mark was available in this two-mark question if the final answer was incorrect. Many calculations were set out poorly and a method mark could not be allocated. Students are encouraged to clearly label each step in an extended calculation and to draw supporting diagrams where applicable.

Question 5
Largest angle is opposite the longest side:
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$
$$\frac{20}{\sin(\theta)} = \frac{16}{\sin(52.4°)}$$
$$\sin(\theta) = \frac{20\sin(52.4°)}{16}$$

Question 6
First, find angle *C* and then find side *c*.
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$
$$\frac{8.5}{\sin(95°)} = \frac{6.2}{\sin(B)}$$
$$\sin(B) = \frac{6.2\ \sin(95°)}{8.5}$$
$$B = \sin^{-1}\left(\frac{6.2\ \sin(95°)}{8.5}\right)$$
$$B = 46.605°$$
$$C = 180° - (95° + 46.605°)$$
$$= 38.395°$$
$$\frac{8.5}{\sin(95°)} = \frac{c}{\sin(38.395°)}$$
$$c = \frac{8.5\sin(38.395°)}{\sin(95°)}$$
$$\approx 5.299$$

Question 7
a. $x = \frac{4.3\sin(20°)}{\sin(75°)}$
$\approx 1.5226\,\text{km}$ **[1 mark]**
b. $A = 0.5 \times 1.5226 \times 4.3\sin(85°)$
$\approx 3.26\,\text{km}^2$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

c. Length of fencing $= 4.3 + \dfrac{4.3\sin(85°)}{\sin(75°)}$

≈ 8.73 km **[1 mark]**

Note that (from the question) the frontage on the forest is not fenced, so the fencing is on two sides only.

Question 8

Using the sine rule:

Area $= \dfrac{1}{2}ab\sin(C) = \dfrac{1}{2} \times 26 \times 18 \times \sin 30 = 117$

The answer is A.

12.5 The cosine rule

Question 1

We are given two side lengths and an included angle. To find the third side, use the cosine rule.

$AB = \sqrt{12.6^2 + 19.2^2 - 2 \times 12.6 \times 19.2\cos(63°)}$

Question 2

$\angle GAB = 90° - 50°$

$= 40°$

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$x^2 = 60^2 + 80^2 - 2 \times 60 \times 80\cos(40°)$

$= 10\,000 - 9600\cos(40°)$

$x = \sqrt{(10\,000 - 9600\cos(40°))}$

≈ 51.44 km

Question 3

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$5^2 = 8^2 + 10^2 - 2 \times 8 \times 10\cos(\theta)$

$-\cos(\theta) = \dfrac{5^2 - 8^2 - 10^2}{2 \times 8 \times 10}$

$\cos(\theta) = \dfrac{8^2 + 10^2 - 5^2}{2 \times 8 \times 10}$

Question 4

To find the distance of the final walking distance, the angle at the hut must be calculated.

Through use of alternate angles, the angle of depression from the hut to the start point is 45°, and the angle of elevation from the hut to the river is $300 - 270 = 30°$. The angle at the hut is $45 + 30 = 75°$.

The final distance can be found by using the cosine rule $a^2 = b^2 + c^2 - 2bc\cos\theta$.

Let a be the final walking distance.

$a^2 = 5.4^2 + 2.8^2 - 2 \times 5.4 \times 2.8\cos(75) = 29.17$

$a = 5.4$

The total distance walked is $a + b + c = 5.4 + 5.4 + 2.8 = 13.6$ km.

Question 5

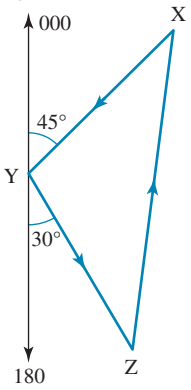
The cosine rule needs to be used: $a^2 = b^2 + c^2 - 2bc\cos(A)$, which rearranges to give:

$\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$.

So $\cos(x) = \dfrac{3^2 + 6^2 - 5^2}{2 \times 3 \times 6}$

© John Wiley & Sons Australia, Ltd

Question 6



$\angle XYZ = 180° - (45° + 30°)$
$= 105°$

The distance ZX is found using:

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$x^2 = 4^2 + 5.2^2 - 2 \times 4 \times 5.2\cos(105°)$

$x = \sqrt{(43.04 - 41.6\cos(105°))}$
$\approx 7.34\,\text{km}$

Question 7

a. $\angle QOR = 195° - 115°$
$= 80°$ **[1 mark]**

b. $a^2 = b^2 + c^2 - 2bc\cos(A)$

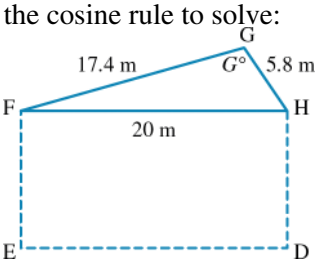
$x^2 = 35^2 + 40^2 - 2 \times 35 \times 40\cos(80°)$

$x = \sqrt{2825 - 2800\cos(80°)}$
$\approx 48.36\,\text{km}$ **[1 mark]**

c. $RQ + QO = 48.36 + 40$
$= 88.36\,\text{km}$ **[1 mark]**

Question 8

Redrawing the diagram to highlight and include all relevant information produces a triangle that requires the cosine rule to solve:



Using the cosine rule:

$$\cos(G°) = \frac{17.4^2 + 5.8^2 - 20^2}{2 \times 17.4 \times 5.8}$$
$= -0.315$
$G° = \cos^{-1}(-0.315)$
$= 108.4°$

© John Wiley & Sons Australia, Ltd

12.6 Area of triangles

Question 1

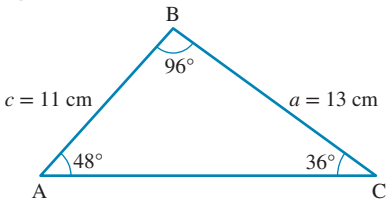
Using Heron's formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$
$$= \frac{1}{2}(20+24+26)$$
$$= 35$$

$$\text{Area} = \sqrt{35(35-20)(35-24)(35-26)}$$
$$= \sqrt{35 \times 15 \times 11 \times 9}$$
$$= \sqrt{51975}$$
$$= 227.98$$
$$\approx 228 \text{ cm}^2$$

Question 2



$$B = 180° - (48° + 36°)$$
$$= 96°$$

$$\text{Area} = \frac{1}{2}ac\sin(B)$$
$$= \frac{1}{2} \times 13 \times 11 \times \sin(96°)$$
$$= 71.11 \text{ cm}^2$$

Question 3

$$\text{Area} = \frac{1}{2}bc\sin(A)$$
$$= \frac{1}{2} \times 8 \times 5.22 \sin(100°)$$
$$= 20.88\sin(100°)$$

© John Wiley & Sons Australia, Ltd

Question 4

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$a^2 = 24^2 + 17.6^2 - 2 \times 24 \times 17.6\cos(20°)$$
$$= 885.76 - 844.8\cos(20°)$$
$$a = \sqrt{885.76 - 844.8\cos(20°)}$$
$$= 9.59\text{ cm}$$
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenues}}$$
$$\sin(60°) = \frac{h}{9.59}$$
$$h = 9.59\sin(60°)$$
$$= 8.31\text{ cm}$$

Question 5

a. $P = 34 + 20 + 42$
$= 96\text{ cm}$ **[1 mark]**

b. Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$$s = \frac{1}{2}(34 + 20 + 42)$$
$$= 48 \textbf{ [1 mark]}$$
$$\text{Area} = \sqrt{48(48-34)(48-20)(48-42)}$$
$$= \sqrt{48 \times 14 \times 28 \times 6}$$
$$= \sqrt{112896}$$
$$= 336\text{ cm}^2 \textbf{ [1 mark]}$$

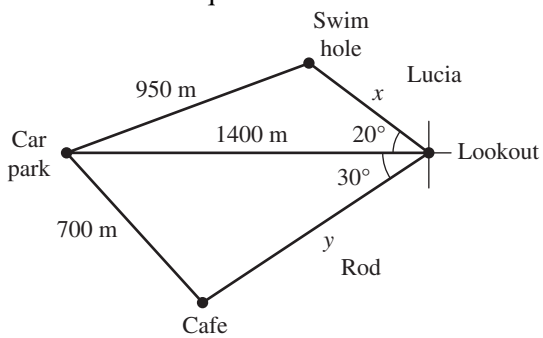
c. Area $= \frac{1}{2}hb$

$$336 = \frac{1}{2} \times 42 \times h$$
$$h = \frac{336}{21}$$
$$= 16\text{ cm } \textbf{[1 mark]}$$

12.7 Review

Question 1

The trick to this question is to be able to draw the diagram as close to scale as possible:



To find how far Lucia walked:

$950^2 = x^2 + 1400^2 - 2 \times x \times 1400 \times \cos 20°$

© John Wiley & Sons Australia, Ltd

$x \simeq 495$ m
Therefore, Lucia walked $1400 + 495 + 950 = 2845$ m
To find how far Rod walked:
$700^2 = y^2 + 1400^2 - 2 \times y \times 1400 \times \cos 30°$
$y \simeq 1212$ m
Therefore, Rod walked $1400 + 1212 + 700 = 2845$ m
So, Rod walked 467 m further than Lucia.

Question 2

a. distance $PR = \sqrt{80^2 + 100^2 - 2\,(80)\,(100)\cos 104°}$ **[1 mark]**
$= 142.375$
$= 142$ m **[1 mark]**

b. $\dfrac{\sin(\angle RPQ)}{100} = \dfrac{\sin 104°}{142}$

$\angle RPQ = \sin^{-1}\left(\dfrac{100 \times \sin 104°}{142}\right)$
$= 43.1025955$
$= 43°$

$\therefore$ bearing of R from $P = 130° - 43° = 087°$ **[1 mark]**

***VCAA Assessment Report* note:**
Some students found the angle PRQ, which did not readily help to give the required bearing.

Question 3

a. $\sin 30° = \dfrac{\text{distance}}{50}$

distance $= 50 \times \sin 30°$
$= 25$ m **[1 mark]**

b. $\tan\theta = \dfrac{16.8}{200}$

$\theta = \tan^{-1}\dfrac{16.8}{200}$
$= 4.80157°$
$= 5°$ **[1 mark]**

Question 4

$s = \dfrac{1}{2}(2250 + 1900 + 2050)$
$= 3100$
$A = \sqrt{3100\,(3100 - 1900)\,(3100 - 2050)\,(3100 - 2250)}$
$= \sqrt{3100 \times 1200 \times 1050 \times 850}$

Question 5

a. Angles in a triangle add to 180°.
$\theta = 180 - (45 + 60)$
$= 75°$.
[1 mark for first line, not answer]
VCAA Assessment Report note:
A suitable calculation that resulted in 75 was required.

© John Wiley & Sons Australia, Ltd

b. $\dfrac{d}{\sin D} = \dfrac{a}{\sin A}$

$\dfrac{AX}{\sin(45°)} = \dfrac{3.16}{\sin(75°)}$ **[1 mark]**

c. $\dfrac{AX}{\sin(45°)} = \dfrac{3.16}{\sin(75°)}$

$AX = \dfrac{3.16}{\sin(75°)} \times \sin(45°)$

$= 2.31\text{ m}$ **[1 mark]**

d. $A = \dfrac{1}{2}bh$

$= \dfrac{1}{2} \times 3.16 \times 2$

$= 3.2\text{ m}^2$ **[1 mark]**

e. $CX = 5 - 3.16$

$= 1.84\text{ m}$

$A = A_{top} + A_{side}AB + A_{side}CB + A_{side}CX$ **[1 mark]**

$= \dfrac{1}{2}(3 + 1.84) \times 2 + 3 \times 1.8 + 2 \times 1.8 + 1.84 \times 1.8$

$= 17.15\text{ m}^2$

$= 17\text{ m}^2$ **[1 mark]**

VCAA Assessment Report note:

Many students misread the question and covered the nesting and eating spaces. Some others who covered only the eating space as required, then incorrectly included the wall *AX*.

A method mark was available in this two-mark question if the final answer was incorrect. Many calculations were set out poorly and a method mark could not be allocated. Students are encouraged to clearly label each step in an extended calculation and to draw supporting diagrams where applicable.

Question 6

The perpendicular height of the triangle is: $\sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}\text{ cm}$

The calculation for C is incorrect, as it used an incorrect perpendicular height.

Question 7

a. $AB = \dfrac{10}{\tan(30°)}$

$= \dfrac{10}{\frac{1}{\sqrt{3}}}$

$= 10\sqrt{3}\text{ cm}$ **[1 mark]**

b. Using Pythagoras' theorem:

$c^2 = a^2 + b^2$

$OB^2 = (10)^2 + \left(10\sqrt{3}\right)^2$

$OB = \sqrt{400}$

$= 20\text{ cm}$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

c. $\cos(\theta) = \frac{A}{H}$

$\cos(60°) = \frac{OX}{10}$

$OX = 10\cos(60°)$

$= 5$ cm **[1 mark]**

d. $AC = 2\sin(30°) \times \frac{10}{\tan(30°)}$

$= 2 \times \frac{1}{2} \times 10\sqrt{3}$

$= 10\sqrt{3}$ cm **[1 mark]**

Question 8

a. $\sin(\theta) = \frac{O}{H}$

$\sin(30°) = \frac{a}{2}$

$a = 2\sin(30°)$

$= 1$ **[1 mark]**

b. $b = 180° - (90° + 30°)$

$= 60°$ **[1 mark]**

c. $\sin(\theta) = \frac{O}{H}$

$\sin(60°) = \frac{c}{2}$

$c = 2\sin(60°)$

$= \sqrt{3}$ **[1 mark]**

d. $c^2 = a^2 + b^2$

$d^2 = (2)^2 + (2)^2$

$d = \sqrt{(2)^2 + (2)^2}$

$= 2\sqrt{2}$ **[1 mark]**

Question 9

a. $AX = \frac{1}{2} \times AC$

$= \frac{1}{2} \times 20$

$= 10$ cm **[1 mark]**

b. $\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$

$\cos(50°) = \frac{10}{AB}$

$AB = \frac{10}{\cos(50°)}$

≈ 15.56 cm **[1 mark]**

c. 11.92 cm **[1 mark]**

Pythagoras' theorem:

© John Wiley & Sons Australia, Ltd

$c^2 = a^2 + b^2$

$b^2 = 15.56^2 - 10^2$

$b = \sqrt{142.1136}$

≈ 11.92 cm

Or

$$\sin(\theta) = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\sin(50°) = \frac{XB}{15.56}$$

$XB = 15.56 \times \sin(50°)$

≈ 11.92 cm

Question 10

a. Let the distance between Flag 2 and Flag 3 be a.

$$\tan(\theta) = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan(65°) = \frac{a}{540}$$

$a = 540 \times \tan(65°)$

≈ 1158.03 m **[1 mark]**

b. Let the distance between Flag 3 and Flag 1 be b.

Use Pythagoras' theorem or:

$$\cos(\theta) = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\cos(65°) = \frac{540}{b}$$

$$b = \frac{540}{\cos(65°)}$$

≈ 1277.75 m **[1 mark]**

c. Distance = (Flag 1 to Flag 2) + (Flag 2 to Flag 3) + (Flag 3 to Flag 1)

$= 540 + 1158.03 + 1277.75$

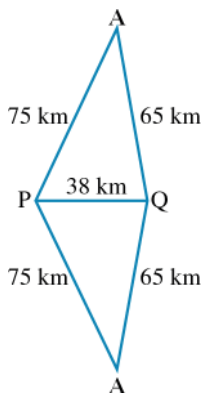
$= 1698.03 + 1277.75$

≈ 2.976 km **[1 mark]**

Be careful of round-off errors when finding the distance.

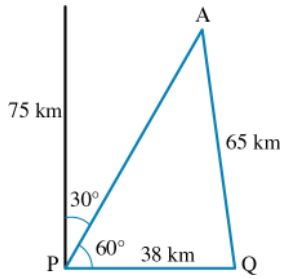
Question 11

a.



[1 mark]

b.



[1 mark]

c. 000° or due north **[1 mark]**

$$\cos(Q) = \frac{65^2 + 38^2 - 75^2}{2 \times 65 \times 38}$$

$Q \approx 90°$

Or use the sine rule.

Question 12

a. $\sin(B) > 1$, so B cannot be found. There is no triangle with these dimensions. **[1 mark]**

b. $B \approx 58.77°$ or $121.23°$. There are two possible triangles with these dimensions. **[1 mark]**

$\sin(B) \approx 0.8551$, so $B \approx 58.77°$ or $121.23°$.

c. $\sin(B) = 1$, so $B = 90°$ and the triangle is right-angled. **[1 mark]**

Question 13

a. The larger triangle is an isosceles and right-angled triangle, so angle x is 45°. **[1 mark]**

b. Use Pythagoras' theorem:

$c^2 = a^2 + b^2$

$y^2 = 1^2 + 2^2$

$y = \sqrt{5}$ cm **[1 mark]**

c.

$$\frac{\sqrt{5}}{\sin(45)} = \frac{1}{\sin(z)}$$

$$\sqrt{5}\sin(z) = \frac{1}{\sqrt{2}}$$

$$\sin(z) = \frac{1}{\sqrt{10}}$$ **[1 mark]**

Question 14

a.

$$a^2 = b^2 + c^2 - 2bc\cos(C)$$

$$2.5^2 = 3.5^2 + 4^2 - 2 \times 3.5 \times 4\cos(C)$$

$$\cos(C) = \frac{3.5^2 + 4^2 - 2.5^2}{2 \times 3.5 \times 4}$$ **[1 mark]**

$$C = \cos^{-1}\left(\frac{3.5^2 + 4^2 - 2.5^2}{2 \times 3.5 \times 4}\right)$$

$\approx 38.21°$ **[1 mark]**

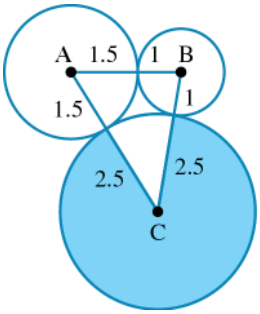
b. $A =$ fraction of the area of the circle

$$= \frac{360 - 38.21}{360} \times \pi \times 2.5^2$$ **[1 mark]**

$= 0.8939 \times \pi \times 2.5^2$

$\approx 17.55\,\text{cm}^2$ **[1 mark]**

© John Wiley & Sons Australia, Ltd



Question 15

$$\sin(\theta) = \frac{o}{h}$$
$$\sin(47°) = \frac{x}{12}$$
$$x = 12 \times \sin(47°)$$
$$x = 8.78$$

Question 16

$$\tan(\theta) = \frac{o}{a}$$
$$\tan(38°) = \frac{9.5}{x}$$
$$x = \frac{9.5}{\tan(38°)}$$
$$x = 12.16\text{ m}$$

Question 17

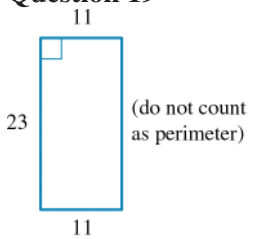
$$\cos(\theta) = \frac{a}{h}$$
$$\cos(\theta) = \frac{15}{21}$$
$$\cos(\theta) = 0.714285714...$$
$$\theta = \cos^{-1}(0.714\,285\,714...)$$
$$\theta = 44.42° \text{ (correct to 2 decimal places)}$$

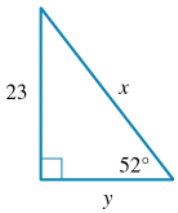
Question 18

statement D is incorrect.

$$\tan(30°) = \frac{1}{\sqrt{3}} \text{ and } \tan(60°) = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Question 19





First find x.

$$\sin(\theta) = \frac{o}{h}$$
$$\sin(52°) = \frac{23}{x}$$
$$x = \frac{23}{\sin(52°)}$$
$$x = 29.19 \text{ (correct to 2 decimal places)}$$

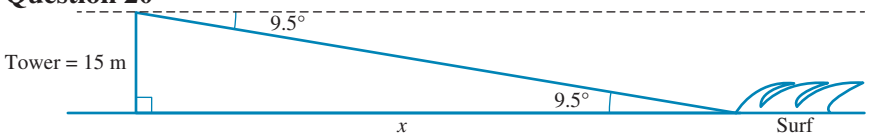
Then find y.

$$\tan(\theta) = \frac{o}{a}$$
$$\tan(52°) = \frac{23}{y}$$
$$y = \frac{23}{\tan(52°)}$$
$$y = 17.97 \text{ (correct to 2 decimal places)}$$

then calcilate the perimeter.

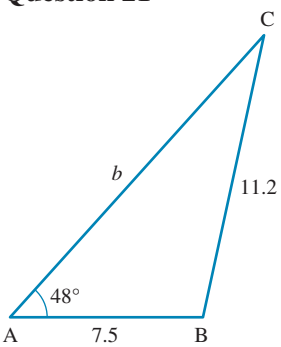
$$\text{Perimeter} = 11 + 23 + 11 + 29.19 + 17.97$$
$$= 92.16$$

Question 20



$$\tan(\theta) = \frac{0}{a}$$
$$\tan(9.5°) = \frac{15}{x}$$
$$x = \frac{15}{\tan(9.5°)}$$
$$x = 89.64 \text{ m (correct to 2 decimal places.)}$$

Question 21



$$\frac{\sin(C)}{c} = \frac{\sin(A)}{a}$$
$$\frac{\sin(C)}{7.5} = \frac{\sin(48°)}{11.2}$$
$$\sin(C) = 7.5 \times \left(\frac{\sin(48°)}{11.2}\right)$$
$$\sin(C) = 0.4976...$$
$$C = \sin^{-1}(0.4976...)$$
$$C = 29.8°$$
$$B = 180° - (29.84° + 48°)$$
$$= 102.16°$$

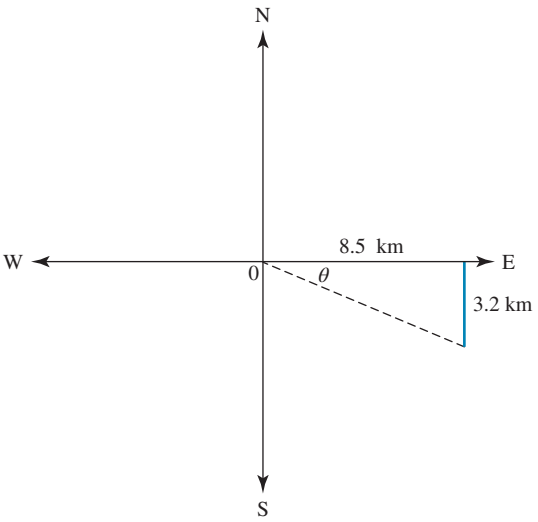
© John Wiley & Sons Australia, Ltd

$$\frac{b}{\sin(B)} = \frac{a}{\sin(A)}$$
$$\frac{b}{\sin(102.16°)} = \frac{11.2}{\sin(48°)}$$
$$b = \sin(102.16°) \times \left(\frac{11.2}{\sin(48°)}\right)$$
$$b = 14.73$$

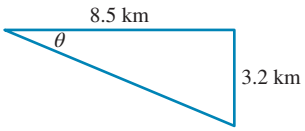
Question 22

$c^2 = a^2 + b^2 - 2ab\cos(C)$
$c^2 = 2.8^2 + 5.2^2 - 2 \times 2.8 \times 5.2\cos(54°)$
$c^2 = 34.88 - 17.12$
$c^2 = 17.76$
$c = \sqrt{17.76}$
$c = 4.2$ (correct to 1 decimal place)

Question 23



a. **[1 mark]**



b.

$\tan(\theta) = \frac{o}{a}$ **[1 mark]**
$\tan(\theta) = \frac{3.2}{8.5}$
$\tan(\theta) = 0.3764...$
$\theta = \tan^{-1}(0.3764...)$ **[1 mark]**
$\theta = 20.63°$ **[1 mark]**

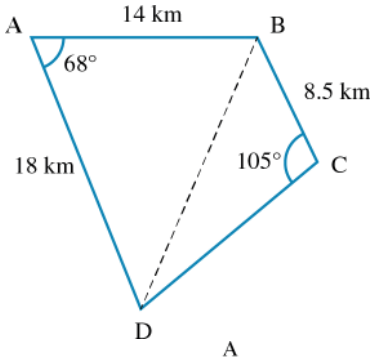
c. i. True bearing $= 90° + 20.63°$
$= 111.63°$ **[1 mark]**

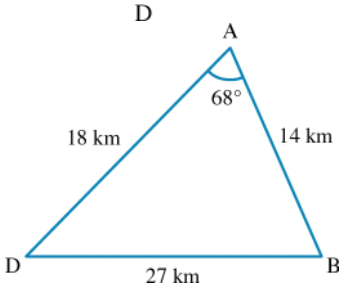
ii. $90° - 20.63° = 69.37°$ **[1 mark]** compass bearing = S69.37°E **[1 mark]**

© John Wiley & Sons Australia, Ltd

Question 24

a.





Award 1 mark for each of the two triangles.

b. $\cos(B) = \dfrac{a^2 + c^2 - b^2}{2ac}$ **[1 mark]**

$\cos(B) = \dfrac{27^2 + 14^2 - 18^2}{2 \times 27 \times 14}$

$\cos(B) = \dfrac{601}{756}$

$\cos(B) = 0.7949...$

$B = \cos^{-1}(0.7949...)$

$B = 37.35°$ **[1 mark]**

$D = 180° - (68° + 37.35°)$

$= 74.65°$ **[1 mark]**

c. $\dfrac{\sin(D)}{d} = \dfrac{\sin(C)}{c}$ **[1 mark]**

$\dfrac{\sin(D)}{8.5} = \dfrac{\sin(105°)}{27}$

$\sin(D) = 8.5 \times \left(\dfrac{\sin(105°)}{27}\right)$

$\sin(D) = 0.3040...$

$D = \sin^{-1}(0.3040...)$ **[1 mark]**

$D = 17.70°$ **[1 mark]**

$B = 180° - (105° + 17.70°)$

$= 57.30°$ **[1 mark]**

$a^2 = b^2 + c^2 - 2bc\cos(A)$ **[1 mark]**

$a^2 = 8.5^2 + 27^2 - 2 \times 8.5 \times 27\cos(57.30°)$

$a^2 = 801.25 - 247.97$

$a^2 = 553.28$ **[1 mark]**

$a = \sqrt{553.28}$

$a = 23.52$ km **[1 mark]**

d. All angles should add to be 360° as the block is a quadrilateral. **[1 mark]**

© John Wiley & Sons Australia, Ltd

$68° + 37.35° + 74.65° + 105° + 17.70° + 57.3° = 360°$ **[1 mark]**

e. $A_{\text{triangle } ABD} = \frac{1}{2}bc\ \sin(A)$

$= \frac{1}{2} \times 18 \times 14 \times \sin(68°)$

$= 116.83\ \text{km}^2$ (correct to 2 decimal places) **[1 mark]**

$A_{\text{triangle } CBD} = \frac{1}{2}ab \sin(C)$

$= \frac{1}{2} \times 23.52 \times 8.5 \times \sin(105°)$

$= 96.55\ \text{km}^2$ (correct to 2 decimal places) **[1 mark]**

Total area $= 116.83\ \text{km}^2 + 96.55\ \text{km}^2$

$= 213.38\ \text{km}^2$ **[1 mark]**

Question 25

The angle $\angle PQR = 180 - (45 + 69) = 66°$

Using the sine rule:

$$\frac{PQ}{\sin(66°)} = \frac{42}{\sin(69°)}$$

Solving using the CAS calculator gives:

$PQ = 41.099$ cm

Question 26

Using the sine rule to find $\angle Q$ first.

$$\frac{\sin(Q)}{20} = \frac{\sin(38°)}{18}$$

$$\sin(Q) = \frac{\sin(38°)}{18} \times 20$$

$$= 0.6841$$

$$Q = \sin^{-1}(0.6841)$$

There are two possible solutions to this equation:

$Q = 43.2°$ and $Q = (180 - 43.2) = 136.8°$ representing the ambiguous case.

If $Q = 43.2°$, $R = 180 - (38 + 43.2) = 98.8°$ and $r = \frac{18}{\sin(38°)} \times \sin(98.8°) = 28.9$

If $Q = 136.8°$, $R = 180 - (136.8 + 38) = 5.2°$ and $r = \frac{18}{\sin(38°)} \times \sin(5.2°) = 2.6$

Question 27

a. Area of one triangle $= \frac{1}{2}bc\ \sin(A)$

$= \frac{1}{2} \times 4 \times 4 \times \sin(60°)$

$= 6.9282$ **[1 mark]**

Area of hexagon $= 6 \times \frac{1}{2}bc \sin(A)$

$= 6 \times 6.9282$

$= 41.5692\ \text{cm}^2$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

b. Area of circle $= \pi r^2$

$= \pi \times 16$

$= 50.2655 \text{ cm}^2$ **[1 mark]**

Area between circle and hexagon $= 50.2655 - 41.5692$

$= 8.6963$

$\approx 8.7 \text{ cm}^2$ **[1 mark]**

Question 28

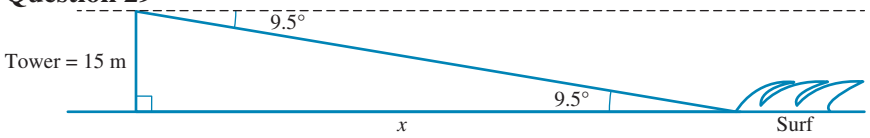
$$\cos(\theta) = \frac{a}{h}$$

$$\cos(28.25°) = \frac{x}{112}$$

$$x = 112 \times \cos(28.25°)$$

$x = 98.66$ m (correct to 2 decimal places)

Question 29



$$\tan(\theta) = \frac{0}{a}$$

$$\tan(9.5°) = \frac{15}{x}$$

$$x = \frac{15}{\tan(9.5°)}$$

$x = 89.64$ m (correct to 2 decimal places.)

© John Wiley & Sons Australia, Ltd

13 Similarity and scale

Topic	13	Similarity and scale
Subtopic	13.2	Similar objects

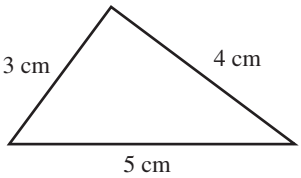
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at www.jacplus.com.au.

Source: *VCE 2019, Further Mathematics 1, Section B, Module 3, Q.4;* © VCAA

Question 1 (1 mark)

Triangle *M*, shown below, has side lengths of 3 cm, 4 cm and 5 cm.



Four other triangles have the following side lengths:

- Triangle *N* has side lengths of 3 cm, 6 cm and 8 cm.
- Triangle *O* has side lengths of 4 cm, 8 cm and 12 cm.
- Triangle *P* has side lengths of 6 cm, 8 cm and 10 cm.
- Triangle *Q* has side lengths of 9 cm, 12 cm and 15 cm.

The triangles that are similar to triangle *M* are

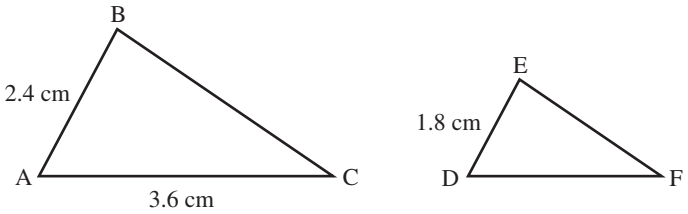
A. triangle *N* and triangle *O*.
B. triangle *N*, triangle *O* and triangle *P*.
C. triangle *O* and triangle *P*.
D. triangle *O* and triangle *Q*.
E. triangle *P* and triangle *Q*.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2016, Further Mathematics 1, Section B, Module 3, Q.2;* © VCAA

Question 2 (1 mark)

Triangle *ABC* is similar to triangle *DEF*.



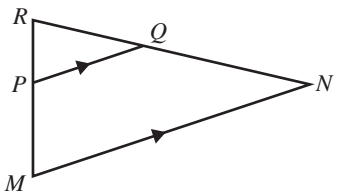
The length of *DF*, in centimetres, is

A. 0.9
B. 1.2
C. 1.8
D. 2.7
E. 3.6

Source: *VCE 2013, Further Mathematics 1, Section B, Module 2, Q.6;* © VCAA

Question 3 (1 mark)

In triangle *MNR*, point *P* lies on side *MR* and point *Q* lies on side *NR*.
The lines *PQ* and *MN* are parallel.
The length of *RQ* is 4 cm, the length of *QN* is 6 cm and the length of *PQ* is 5 cm.



The length of *MN*, in cm, is equal to

A. 7.5
B. 8.3
C. 12.0
D. 12.5
E. 15.0

© John Wiley & Sons Australia, Ltd

Question 4 (1 mark)

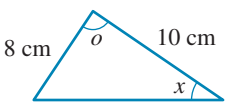
Of the statements below, which ones are true?

I. If two shapes are congruent and one is cut out, it will fit exactly over the other.
II. Corresponding angles of two congruent shapes are in the same ratio.
III. The lengths of corresponding sides of two similar shapes are in the same ratio.
IV. Corresponding angles of pairs of similar shapes are equal.
V. If two shapes are similar and one is cut out, it will fit exactly over the other.

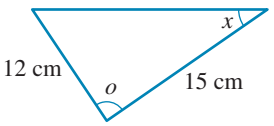
A. I, III, IV
B. I, IV, V
C. II, III, IV
D. II, IV, V
E. III, IV, V

Question 5 (1 mark)

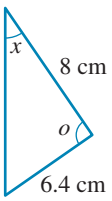
For these figures, the similar triangles are the ones labelled:



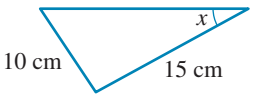
P



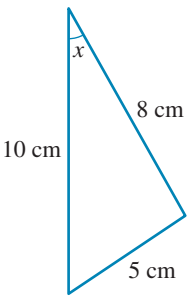
Q



R



S



T

A. P, S, T
B. Q, R, S
C. P, Q, R
D. Q, S, T
E. R, S, T

© John Wiley & Sons Australia, Ltd

Question 6 (1 mark)

Two triangles are described as follows.

ΔXYZ is an isosceles triangle with $XY = 10$ cm and the angle at $Y = 42°$.

ΔPQR is an isosceles triangle with $PQ = 10$ cm and the angle at $Q = 42°$.

The two triangles can be

I. Congruent

II. Similar

III. Neither congruent nor similar

IV. Equilateral

V. Scalene.

A. I

B. I and III

C. I and II

D. I and IV

E. V

© John Wiley & Sons Australia, Ltd

Topic	13	Similarity and scale
Subtopic	13.3	Linear scale factors

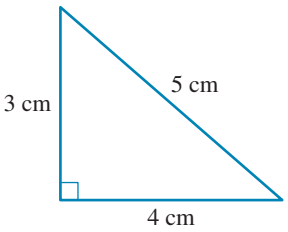
online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

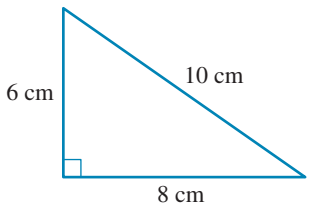
13.3 Linear scale factors

Question 1 (1 mark)

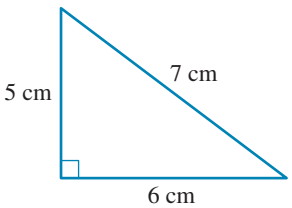
When enlarged by a factor of 2, this shape becomes:



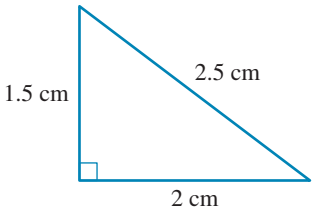
A.



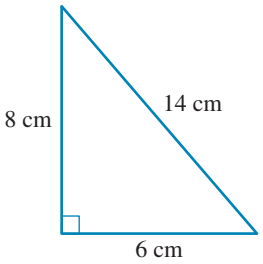
B.



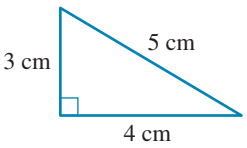
C.



D.

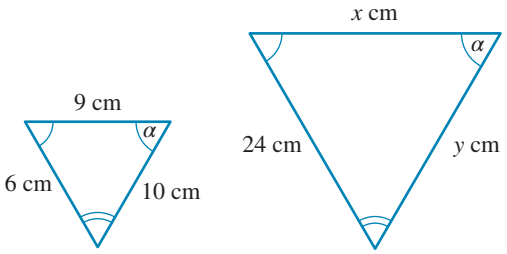


E.



© John Wiley & Sons Australia, Ltd

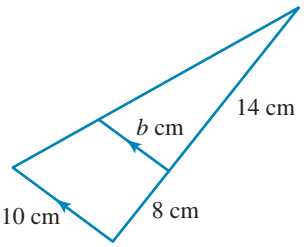
Question 2 (1 mark)
The missing lengths in this diagram are:



A. $x = 18$ cm, $y = 20$ cm
B. $x = 40$ cm, $y = 36$ cm
C. $x = 36$ cm, $y = 40$ cm
D. $x = 18$ cm, $y = 30$ cm
E. $x = 30$ cm, $y = 18$ cm

Question 3 (1 mark)
When considering similar triangles, which statement is *incorrect*?
A. Whether enlarged or reduced, similar triangles have identical angles.
B. When enlarged by a scale factor of 3, all side lengths will be multiplied by 3.
C. When enlarged by a factor of 2, the size of each angle in the new triangle will be double the size of the corresponding angle in the original triangle.
D. When reduced by a scale factor of 2, all side lengths will be multiplied by $\frac{1}{2}$
E. When reduced by a scale factor of 2, all side lengths will be divided by 2.

Question 4 (1 mark)
The value of the pronumeral, b, correct to two decimal places, is



A. 1.29
B. 5.71
C. 11.2
D. 17.6
E. 6.36

© John Wiley & Sons Australia, Ltd

Topic	13	Similarity and scale
Subtopic	13.4	Area and volume scale factors

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

13.4 Area and volume scale factors

Source: *VCE 2021, Further Mathematics 1, Section B, Module 3, Q.3;* © VCAA

Question 1 (1 mark)

A photograph was enlarged by an area scale factor of 9.
The length of the original photograph was 12 cm.
The original photograph and the enlarged photograph are similar in shape.
The length of the enlarged photograph, in centimetres, is

A. 4
B. 9
C. 27
D. 36
E. 108

Source: *VCE 2016, Further Mathematics 1, Section B, Module 3, Q.8;* © VCAA

Question 2 (1 mark)

A string of seven flags consisting of equilateral triangles in two sizes is hanging at the end of a racetrack, as shown in the diagram below.

The edge length of each black flag is twice the edge length of each white flag.
For this string of seven flags, the total area of the black flags would be

A. two times the total area of the white flags.
B. four times the total area of the white flags.
C. $\frac{4}{3}$ times the total area of the white flags.
D. $\frac{16}{3}$ times the total area of the white flags.
E. $\frac{16}{9}$ times the total area of the white flags.

© John Wiley & Sons Australia, Ltd

Source: *VCE 2013, Further Mathematics 1, Section B, Module 2, Q.4;* © VCAA

Question 3 (1 mark)

A cafe sells two sizes of cupcakes with a similar shape.

The large cupcake is 6 cm wide at the base and the small cupcake is 4 cm wide at the base.

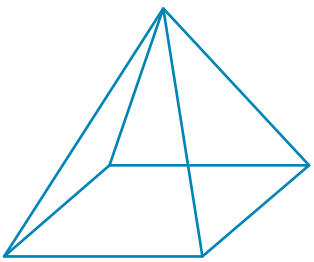
The price of a cupcake is proportional to its volume.

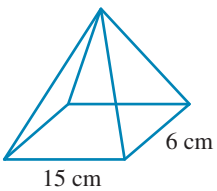
If the large cupcake costs $5.40, then the small cupcake will cost

A. $1.60
B. $2.32
C. $2.40
D. $3.40
E. $3.60

Question 4 (1 mark)

Two similar pyramids have a scale factor of 2 : 1. The area of the base of the larger pyramid is



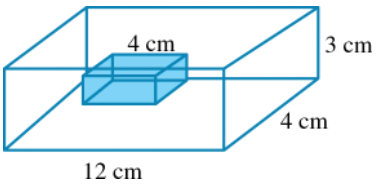


A. 90 cm^2
B. 180 cm^2
C. 504 cm^2
D. 360 cm^2
E. 810 cm^2

© John Wiley & Sons Australia, Ltd

Question 5 (1 mark)
The ratio of the volumes of two similar cylinders is 27 : 1. The area of the base of the smaller cylinder is 16π cm^2. The radius of the larger cylinder is
A. 12π cm
B. 36 cm
C. 12 cm
D. 36π cm
E. 48 cm

Question 6 (3 marks)
The shaded rectangular prism is placed inside a similar larger rectangular prism, as shown.



a. Find the scale factor relating the two prisms. **(1 mark)**

b. Find the total surface area of the larger prism. **(1 mark)**

c. Find the total surface area of the shaded prism. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Topic	13	Similarity and scale
Subtopic	13.5	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

13.5 Review

Source: *VCE 2020, Further Mathematics 1, Section B, Module 3, Q.9 ;* © VCAA

Question 1 (1 mark)

Shot-put is an athletics field event in which competitors throw a heavy spherical ball (a shot) as far as they can.

The size of the shot for men and the shot for women is different.

The diameter of the shot for men is 1.25 times larger than the diameter of the shot for women.

The ratio of the total surface area of the women's shot to the total surface area of the men's shot is

A. 1 : 4

B. 1 : 25

C. 4 : 5

D. 5 : 4

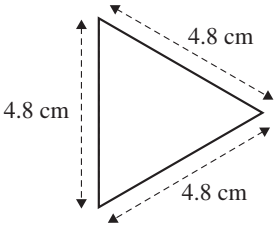
E. 16 : 25

© John Wiley & Sons Australia, Ltd

Source: *VCE 2020, Further Mathematics 2, Section B, Module 3, Q.2;* © VCAA

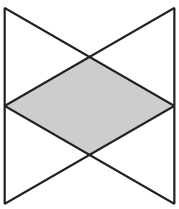
Question 2 (4 marks)

Khaleda has designed a logo for her business.
The logo contains two identical equilateral triangles.
The side length of each triangle is 4.8 cm, as shown in the diagram below.



a. Write a calculation to show that the area of one of the triangles, rounded to the nearest centimetre, is 10 cm^2. **(1 mark)**

b. In the logo, the two triangles overlap, as shown below. Part of the logo is shaded and part of the logo is not shaded.



What is the area of the entire logo?
Round your answer to the nearest square centimetre.
Area = ☐ cm^2. **(1 mark)**

c. What is the ratio of the area of the shaded region to the area of the non-shaded region of the logo?
The ratio of the shaded area is ☐. **(1 mark)**

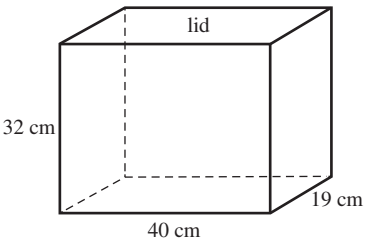
d. The logo is enlarged and printed on boxes for shipping
The enlarged logo and the original logo are similar in shape.
The area of the enlarged logo is four times the area of the original logo.
What is the height, in centimetres, of the enlarged logo?
Heigth = ☐ cm. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2017, Further Mathematics 2, Section B, Module 3, Q.1;* © VCAA

Question 3 (3 marks)

Miki is planning a gap year in Japan.
She will store some of her belongings in a small storage box while she is away.
This small storage box is in the shape of a rectangular prism.
The diagram below shows that the dimensions of the small storage box are 40 cm × 19 cm × 32 cm.



The lid of the small storage box is labelled on the diagram above.

a. i. What is the surface area of the lid, in square centimetres?
The surface area of the lid is ☐ cm^2 **(1 mark)**

ii. What is the total outside surface area of this storage box, including the lid and base, in square centimetres?
The total outside surface area is ☐ cm^2 **(1 mark)**

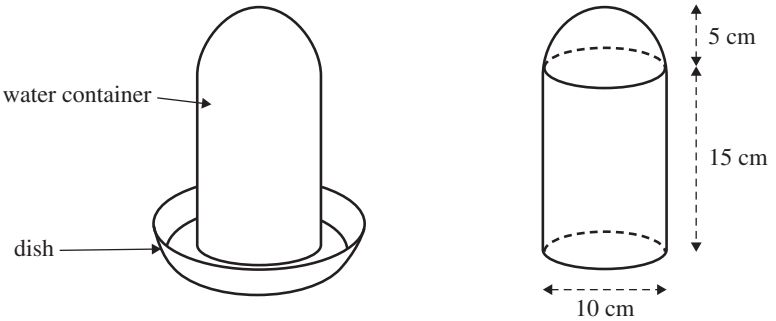
b. Miki has a large storage box that is also a rectangular prism.
The large storage box and the small storage box are similar in shape.
The volume of the large storage box is eight times the volume of the small storage box.
The length of the small storage box is 40 cm.
What is the length of the large storage box, in centimetres? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Source: *VCE 2014, Further Mathematics 2, Module 2, Q.3;* © VCAA

Question 4 (5 marks)

The chicken coop contains a circular water dish.
Water flows into the dish from a water container.
The water container is in the shape of a cylinder with a hemispherical top.
The water container and the dish are shown in the diagrams below.



The cylindrical part of the water container has a diameter of 10 cm and a height of 15 cm. The hemisphere has a radius of 5 cm.

a. What is the surface area of the hemispherical top of the water container?
Write your answer, correct to the nearest square centimetre. **(1 mark)**

b. What is the maximum volume of water that the water container can hold?
Write your answer, correct to the nearest cubic centimetre. **(2 marks)**

c. The eating space of the chicken coop also has a feed container.
The feed container is similar in shape to the water container.
The volume of the water container is three-quarters of the volume of the feed container.
The surface area of the water container is 628 cm^2.
What is the surface area of the feed container?
Write your answer, correct to the nearest square centimetre. **(2 marks)**

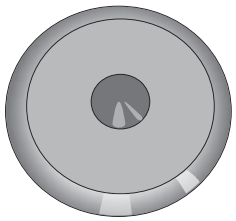
© John Wiley & Sons Australia, Ltd

Source: *VCE 2013, Further Mathematics 2, Module 2, Q.4;* © VCAA

Question 5 (2 marks)

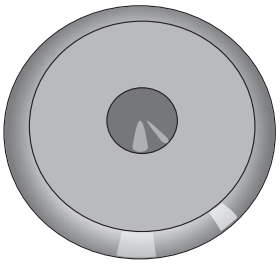
Competitors in the intermediate division of the discus use a smaller discus than the one used in the senior division, but of a similar shape. The total surface area of each discus is given below.

intermediate discus



total surface area 500 cm^2

senior discus



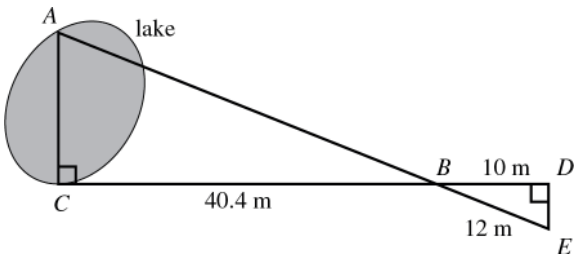
total surface area 720 cm^2

By what value can the volume of the intermediate discus be multiplied to give the volume of the senior discus?

Source: *VCE 2014, Further Mathematics 1, Section B, Module 2, Q.8;* © VCAA

Question 6 (1 mark)

The distance, *AC*, across a small lake can be calculated using the measurements shown in the diagram below.



In this diagram, *BCA* and *BDE* are right-angled triangles, where $CB = 40.4\ m$, $BD = 10$ m and $BE = 12\ m$. The distance between the points *A* and *C*, in metres, is closest to

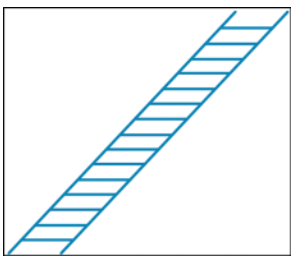
A. 22.4
B. 26.8
C. 33.6
D. 48.5
E. 177.8

© John Wiley & Sons Australia, Ltd

Source: *VCE 2013, Further Mathematics 1, Section B, Module 2, Q.5;* © VCAA

Question 7 (1 mark)
The scale used on a map is 1 : 50 000.
On this map, a distance of 4 km would be represented by
A. 2.0 cm
B. 5.0 cm
C. 8.0 cm
D. 12.5 cm
E. 20.0 cm

Question 8 (3 marks)
A ladder 4.8 m long has rungs 30 cm apart, starting at 30 cm from each end. It rests against a wall at a point 3.6 m from the ground.



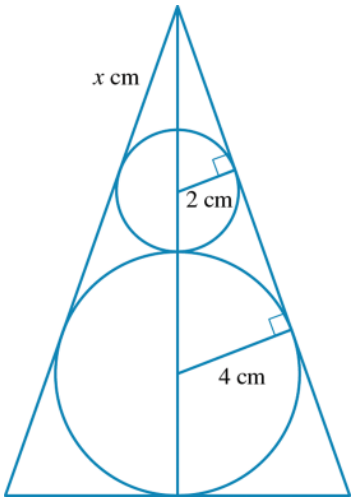
a. How many rungs are there on the ladder? **(1 mark)**

b. How high above the ground will Pam the painter be if she is halfway up the ladder? **(1 mark)**

c. How many rungs from the top is she when she is 2.7 m above the ground? **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 9 (3 marks)
Spheres of radius 2 cm and 4 cm, respectively, fit neatly inside a cone.



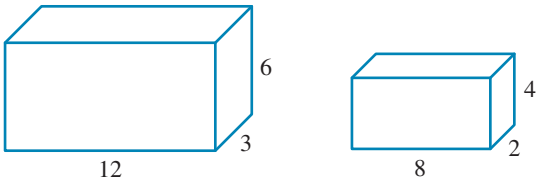
a. If x is the distance from the vertex of the cone to the top of the smaller sphere, use similar triangles to write down a relationship that could be used to find the value of x. **(1 mark)**

b. Find the value of x. **(1 mark)**

c. Hence, find the height of the cone. **(1 mark)**

© John Wiley & Sons Australia, Ltd

Question 10 (3 marks)
For the solids shown, what is:

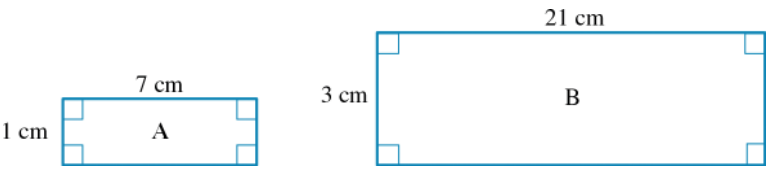


a. the scale factor?
Scale factor = ☐ **(1 mark)**

b. the ratio of surface areas? **(1 mark)**

c. the ratio of volumes? **(1 mark)**

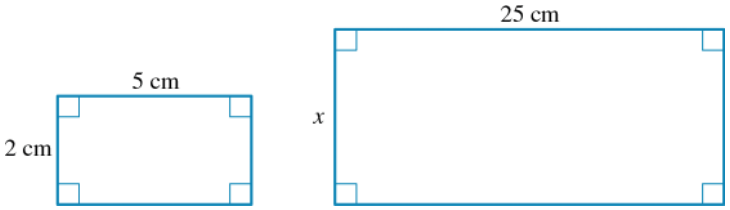
Question 11 (1 mark)
Identify the correct statement in relation to these rectangles.



A. Rectangle A has been enlarged by a scale factor of 2 to create rectangle B.
B. The perimeter of rectangle A is 8 cm.
C. When enlarging rectangle A to form rectangle B, the area of the rectangles will stay the same.
D. The angles in rectangle A will increase in the same ratio as the sides.
E. Rectangle A has been enlarged by a scale factor of 3 to create rectangle B.

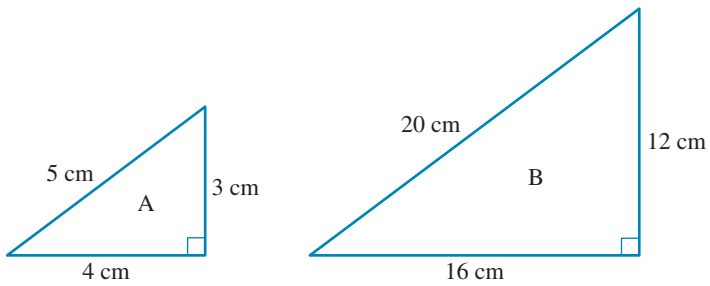
© John Wiley & Sons Australia, Ltd

Question 12 (1 mark)
The value of x in the rectangle below is:



A. 22 cm
B. 10 cm
C. 0.4 cm
D. 20 cm
E. 25 cm

Question 13 (1 mark)
Identify the *incorrect* statement in regards to these similar triangles.



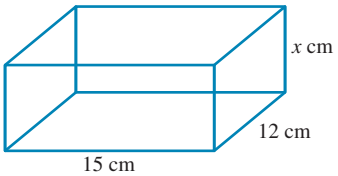
A. Triangle A and triangle B are similar triangles.
B. The scale factor by which triangle A has been enlarged to make triangle B is 4.
C. The area of triangle B will be 4 times the area of triangle A.
D. The angles in both triangle A and triangle B are equivalent.
E. The area of triangle B will be 16 times the area of triangle A.

Question 14 (1 mark)
Which of the following statements is *false*?
A. All regular pentagons are similar.
B. All squares are similar.
C. All equilateral triangles are similar.
D. All isosceles triangles are similar.
E. All circles are similar.

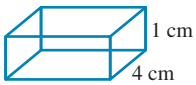
© John Wiley & Sons Australia, Ltd

Question 15 (1 mark)

Examine the cuboids shown in this diagram.



Cuboid A



Cuboid B

Which of the following statements is *incorrect*?

A. The scale factor of length is 3.

B. $x = 3$ cm

C. The total surface area of cuboid A will be 9 times the the total surface area of cuboid B.

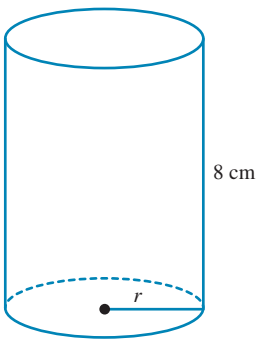
D. The volume of cuboid B is $\frac{1}{9}$ the volume of cuboid A.

E. The volume of cuboid B will be $\frac{1}{3}$ of the volume of cuboid A.

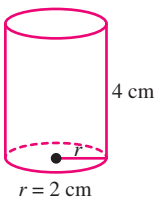
Question 16 (1 mark)

Consider the two cylinders. Which statement is *false*?

Cylinder A



Cylinder B



A. The scale factor is 2.

B. Volume will be in the ratio of $2^3 : 1$.

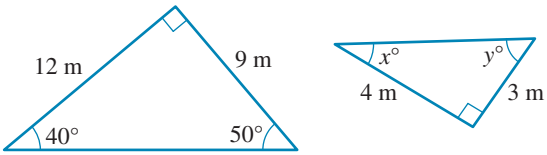
C. $\frac{\text{height of cylinder A}}{\text{height of cylinder B}} = \frac{\text{radius of cylinder A}}{\text{radius of cylinder B}}$

D. The volume of cylinder B will be half the volume of cylinder A.

E. They are similar.

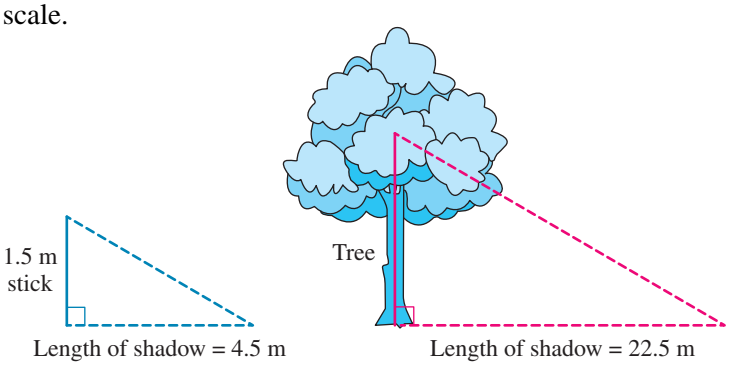
© John Wiley & Sons Australia, Ltd

Question 17 (1 mark)
The missing angles in these similar triangles are:



A. $x = 40°, y = 50°$
B. $x = 50°, y = 40°$
C. $x = 45°, y = 45°$
D. $x = 30°, y = 60°$
E. $x = 60°, y = 30°$

Question 18 (8 marks)
A tree and a 1.5 m stick both cast shadows at a certain time of day. Note that this diagram is not drawn to scale.



a. Given that these triangles are similar, what can be said about the angles in the two triangles? **(1 mark)**

b. Calculate the scale factor. **(1 mark)**
Scale factor = ☐

c. Calculate the height of the tree.
Height = ☐ m. **(1 mark)**

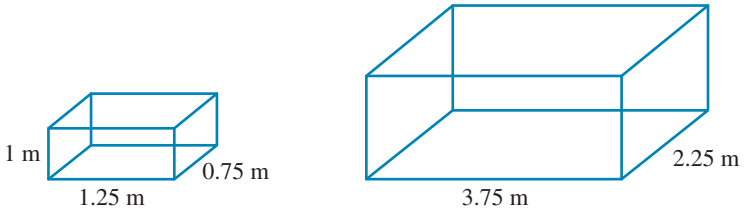
© John Wiley & Sons Australia, Ltd

d. If we were to calculate the area of each triangle, in what ratio would the areas be? **(1 mark)**

e. Complete a calculation to check your prediction for part **d**. **(4 marks)**

Question 19 (9 marks)

Consider these two fish tanks that are similar solids.



a. Calculate the scale factor.

Scale factor = ☐ **(1 mark)**

b. Calculate the height of the larger fish tank.

Height = ☐ m. **(1 mark)**

c. In what ratio would you expect the volumes of the two fish tanks to be?

Ratio of the volumes = ☐ **(1 mark)**

© John Wiley & Sons Australia, Ltd

d. Given that each tank is only filled to $\frac{4}{5}$ capacity, calculate:

i. the volume used for the smaller tank. **(2 marks)**

ii. the volume used for the larger tank. **(2 mark)**

e. Is the actual ratio what was expected? Explain your reasoning. **(2 marks)**

Question 20 (1 mark)

A model of pyramid has a width of 4 cm and a height of 6 cm.
If the real pyramid has a width of 60 m, what will its height be?

A. 90 cm
B. 240 cm
C. 90 m
D. 240 cm
E. 2400 cm

Question 21 (1 mark)

A rectangular pool has a width of 25 m and a length of 50 m.
If a scale model was created with a length of 5 cm, what is the scale model's width?

A. 0.0025 m
B. 0.025 m
C. 0.25 m
D. 2.5 m
E. 25 m

© John Wiley & Sons Australia, Ltd

Answers and marking guide

13.2 Similar objects

Question 1

Similar triangles have corresponding side lengths in the same ratio.
Triangle P is produced by multiplying the side lengths of triangle M by 2.
Triangle Q is produced by multiplying the side lengths of triangle M by 3.

Question 2

$$\frac{DF}{3.6} = \frac{1.8}{2.4}$$
$$DF = \frac{1.8 \times 3.6}{2.4}$$
$$= 2.7 \text{ cm}$$

Question 3

Triangles RPQ and RMN are similar (using the AAA test). Therefore, corresponding side lengths are in the same ratio.

$$\frac{RQ}{RN} = \frac{PQ}{MN}$$
$$\frac{4}{4+6} = \frac{5}{MN}$$
$$\frac{4}{10} = \frac{5}{MN}$$
$$MN = 12.5$$

Question 4

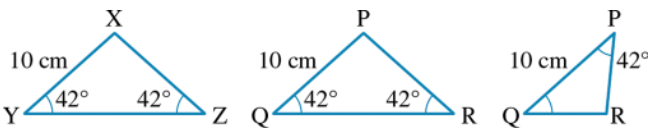
I, III, IV
II is false because corresponding angles of two congruent shapes are equal.
V is false because if two shapes are similar and one is cut out, it will be bigger or smaller than the other.

Question 5

P, Q, R
P, Q and R are all similar (AAA) because corresponding sides are in the same ratio. S and T are not similar to each other or to P, Q or R because corresponding sides are not in the same ratio.

Question 6

I and III



13.3 Linear scale factors

Question 1

When enlarged by a factor of 2, all side lengths are doubled. Remember that the side lengths must correspond.

Question 2

$x = 36$ cm, $y = 40$ cm

© John Wiley & Sons Australia, Ltd

Question 3
The angles in a triangle enlarged by a factor of 2 will be the same size as in the original. All other comments are correct.

Question 4
6.36
$\frac{14}{b} = \frac{14+8}{10}$ from the similar triangles.

13.4 Area and volume scale factors

Question 1
Area scale factor = $9:1$
Length scale factor = $\sqrt{9}:\sqrt{1} = 3:1$
Length of enlarged photograph: $3 \times 12\,\text{cm} = 36\,\text{cm}$

Question 2
If we let the area of each white flag be 1 unit squared, then the area of each black flag is 4 units squared.
$\frac{\text{total area of black flags}}{\text{total area of white flags}} = \frac{4 \times 4}{1 \times 3} = \frac{16}{3}$
***VCAA Assessment Report* note:**
Many students, in choosing option D, ignored the fact that there were different numbers of black and white flags in the string of flags.

Question 3
The side ratio is $6:4 = 3:2$; the volume ratio is $3^3 : 2^3 = 27:8$.
As the prices are proportional to volumes,
$27:8$
$5.40:x$
$27x = 43.2$
$x = 1.60$
Therefore, the small cupcake will cost $1.60.

Question 4
Area of base of smaller pyramid is $15\,\text{cm} \times 6\,\text{cm} = 90\,\text{cm}^2$
Area of base of larger pyramid must be $2^2 = 4$ times the area of base of smaller pyramid.
$90\,\text{cm}^2 \times 4 = 360\,\text{cm}^2$.

Question 5
12 cm

Question 6
a. Corresponding side lengths are 12 cm : 4 cm, so the ratio is 3 : 1. **[1 mark]**
b. TSA $= 2(12 \times 4 + 12 \times 3 + 3 \times 4)$
$= 192\,\text{cm}^2$ **[1 mark]**
c. Ratio of areas is found using the square of the scale factor, namely 9 : 1.
TSA of shaded prism $= \frac{192}{9}$
$= 21.3\,\text{cm}^2$

© John Wiley & Sons Australia, Ltd

13.5 Review

Question 1

Length: $\frac{w}{m} = \frac{1}{1.25} = \frac{4}{5}$

Area: $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$

Question 2

a. Each angle in an equilateral triangles is 60°.

Area $= \frac{1}{2} \times 4.8 \times 4.8 \times \sin 60° = 9.9766 ... = 10\,\text{cm}$ **[1 mark]**

b. Each triangle can be divided into 4 smaller triangles. The two triangles together will consist of 8 smaller triangles, but 2 of them overlap in the logo, so the logo has only 6 of the small triangles, or $\frac{6}{8} = \frac{3}{4}$ of the area of the two larger triangles.

Therefore, $\frac{3}{4} \times \left(10\,\text{cm}^2 + 10\,\text{cm}^2\right) = 15\,\text{cm}^2$. **[1 mark]**

c. Two shaded small triangles: 4 non-shaded small triangles, give a ratio of 1 : 2 **[1 mark]**

d. original area : enlarged area

4 : 1

Therefore, the length will be in the ratio $\sqrt{4} : \sqrt{1} = 2 : 1$.

If the original height of the logo is 4.8 cm, then the enlarged logo will be $2 \times 4.8 = 9.6\,\text{cm}$ **[1 mark]**

Question 3

a. i. Surface area $= l \times w$

$= 40 \times 19$

$= 760\,\text{cm}^2$ **[1 mark]**

ii. Surface area $= 2\,(l \times w + l \times h + w \times h)$

$= 2\,(40 \times 19 + 40 \times 32 + 19 \times 32)$

$= 2\,(760 + 1280 + 608)$

$= 2 \times 2648$

$= 5\,296\,\text{cm}^2$ **[1 mark]**

b. The scale factor of the volume is 8, or 2^3. Therefore, the scale factor of the lengths is 2. The length of the small storage box is 40 cm, so the length of the large storage box is $2 \times 40 = 80\,\text{cm}$. **[1 mark]**

***VCAA Examination Report* note:**

This question was not answered well. Many students did not calculate the linear scale factor of 2 and gave an answer of 320 cm.

Question 4

a. $SA = \frac{1}{2} \times 4\pi r^2$

$= \frac{1}{2} \times 4\pi \times 5^2$

$= 157.08$

$= 157\,\text{cm}^2$ **[1 mark]**

***VCAA Assessment Report* note:**

Some students calculated the surface area of a full sphere, while others used an incorrect radius or formula.

© John Wiley & Sons Australia, Ltd

b. $V = V_{\text{cylinder}} + V_{\text{hemisphere}}$

$= \pi r^2 h + \frac{1}{2} \times \frac{4}{3} \pi r^3$

$= \pi \times 5^2 \times 15 + \frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$

$= 1439.9\,\text{cm}^3$

$= 1440\,\text{cm}^3$

[**1 mark** is awarded for correct formula and correct radius]

[**1 mark** for correct answer]

***VCAA Assessment Report* note:**

A method mark was available in this two-mark question if the final answer was incorrect. Many calculations were poorly set out and a method mark could not be allocated. Some students used the wrong formula for the volume of a sphere, while others did not halve this for a hemisphere.

c. Volume ratio:

$V_{feed} : V_{Water}$

$1 : \frac{3}{4}$

Length ratio:

$\sqrt[3]{1} : \sqrt[3]{\frac{3}{4}}$

$= 1 : 0.90856$

SA ratio:

$1^2 : 0.90856^2$

$= 1 : 0.82582$

[**1 mark** is awarded for the surface area ratio]

If $SA_{water} = 628$

$\therefore SA_{feed} = 628 \times \frac{1}{0.825482}$

$= 760.77$

$= 761\,\text{cm}^2$ **[1 mark]**

***VCAA Assessment Report* note:**

This question was not answered well. Many students only converted the volume factor $\frac{3}{4}$ of or $\frac{4}{3}$into a linear factor, while others calculated the area factor as $\sqrt{\text{volume factor}}$, which was inappropriate. Many students did not attempt conversion of the volume factor.

Some students rounded too early within the question, such as rounding the linear factor to 1.1 before squaring it again to produce the incorrect surface area of $759.88\,\text{cm}^2$.

Question 5

The ratio of the areas is $720 : 500 = 36 : 25$.

The ratio of the radii is $\sqrt{36} : \sqrt{25} = 6 : 5$.

The ratio of the volumes is $6^3 : 5^3 = 216 : 125$. **[1 mark]**

Therefore, to give the volume of the senior discus, the volume of the intermediate discus must be multiplied by or $\frac{216}{115}$ or 1.728. **[1 mark]**

***VCAA Assessment Report* note:**

The ratio of dimensions of the larger discus to the smaller discus was required. The value of $k > 1$ since a larger volume is scaled up from the smaller volume. **[1 mark]**

© John Wiley & Sons Australia, Ltd

A common error was to work with the reciprocal of the area ratio. This would give a value less than one for the linear and the volume ratios. If this is multiplied with the smaller discus dimensions, the result would be a smaller discus, not a larger one.
This question was very poorly answered. Many students did not attempt this question.

Question 6

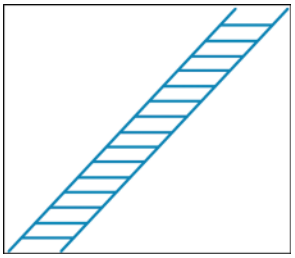
$$DE^2 + 10^2 = 12^2$$
$$DE^2 = 12^2 - 10^2$$
$$DE = \sqrt{44}$$
$$= 6.63$$
$$\frac{CB}{BD} = \frac{AC}{DE}$$
$$\frac{40.4}{10} = \frac{AC}{6.63}$$
$$AC = \frac{40.4}{10} \times 6.63$$
$$= 26.8 \text{ m}$$

Question 7

$4 \text{ km} = 400\,000 \text{ cm}$
Let the distance on the map $= x$.
$1 : 50\,000$
$x : 400\,000$
$50\,000x = 400\,000$
$x = 8 \text{ cm}$

Question 8

a. 16 lots of 30 cm make 4.8 m, which means there are 15 rungs. **[1 mark]**
b. Using similar triangles, halfway up the ladder is halfway to the top = half of 3.6 m = 1.8 m (labelled x) **[1 mark]**
c. 2.4 m above the ground is 3/4 of the way up the ladder, which is 12 rungs (labelled y). **[1 mark]**



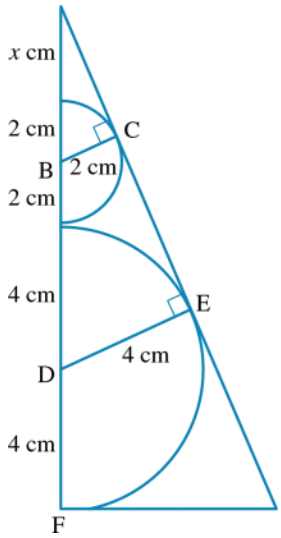
Question 9

a. $(x + 2) : (x + 2 + 2 + 4) = 1 : 2$
$(x + 2) : (x + 8) = 1 : 2$ **[1 mark]**
b. $\frac{x+2}{x+8} = \frac{1}{2}$
$2(x + 2) = x + 8$
$2x + 4 = x + 8$
$x = 4$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

c. Label the diagram

ΔABC and ΔADE are similar, with a scale factor of 1 : 2.



AF = Total height

$= x + 2 + 2 + 4 + 4$

$= 4 + 12$

$= 16$ cm **[1 mark]**

Question 10

a. All sides are in the same ratio: 12 : 8 = 6 : 4 = 3 : 2 or 1.5 **[1 mark]**

b. All areas of faces are in the same ratio: 72 : 32 = 36 : 16 = 18 : 8 = 2.25 **[1 mark]**

c. Volumes are in the ratio $12 \times 3 \times 6 : 8 \times 2 \times 4 = 216 : 64 = 3.375$ **[1 mark]**

Question 11

Rectangle A has been enlarged by a scale factor of 3 to create rectangle B.

Ratio of lengths $= \frac{21}{7} = \frac{3}{1}$

Scale factor = 3

Question 12

Ratio of lengths $= \frac{25}{5}$

$= 5$

Scale factor = 5

$x = 5 \times 2$

$= 10$ cm

Question 13

The statement which is incorrect is 'The area of triangle B will be 4 times the area of triangle A.' The area of triangle B will be 4^2 times the area of triangle A.

Question 14

Statement D is false. Not all isosceles triangles are similar.

Question 15

Statement E is incorrect.

The volume of cuboid B will be $\frac{1}{3^2} = \frac{1}{27}$ the volume of cuboid A.

© John Wiley & Sons Australia, Ltd

Question 16
Statement D is false as the volume of cylinder B is one-eighth that of cylinder A.

Question 17
$x = 40°, y = 50°$

Question 18
a. The angles in the two triangles will be equal. **[1 mark]**

b. Scale factor $= \frac{22.5}{4.5}$
$= 5$ **[1 mark]**

c. Height of the tree $= 5 \times 1.5$
$= 7.5$ **[1 mark]**

d. Area would be in the ratio of $1 : k^2$ or the area of the larger triangle will be 5^2 times the area of the smaller triangle. **[1 mark]**

e. $A_{\text{small triangle}} = \frac{1}{2}bh$
$= \frac{1}{2} \times 4.5 \times 1.5$
$= 3.375$ m **[1 mark]**

$A_{\text{large triangle}} = \frac{1}{2}bh$
$= \frac{1}{2} \times 22.5 \times 7.5$
$= 84.375$ m **[1 mark]**

ratio $= \frac{84.375}{3.375}$
$= \frac{25}{1}$ **[1 mark]**

That is, the larger triangle has an area $25(5^2)$ times the area of the small triangle. **[1 mark]**

Question 19
a. Scale factor $= \frac{3.75}{1.25}$ **[1 mark]**
$= \frac{2.25}{0.75}$
$= 3$ **[1 mark]**

b. Height of the larger tank $= 3 \times 1$
$= 3$ m **[1 mark]**

c. You would expect the volume to be in the ratio of $1 : 3^3$ or $1 : 27$. **[1 mark]**
That is, you would expect the volume of the larger tank to be 27 times the volume of the smaller tank.

d. i. Volume of smaller tank $= \frac{4}{5} \times (lwh)$ **[1 mark]**
$= \frac{4}{5} \times (1.25 \times 0.75 \times 1)$
$= 0.75\ \text{m}^3$ **[1 mark]**

© John Wiley & Sons Australia, Ltd

ii. Volume of larger tank $= \frac{4}{5} \times (lwh)$ **[1 mark]**

$= \frac{4}{5} \times (3.75 \times 2.25 \times 3)$

$= 20.25\ \text{m}^3$ **[1 mark]**

e. Actual ratio $= \frac{20.25}{0.75}$

$= \frac{27}{1}$ **[1 mark]**

The actual ratio is what was expected, despite the tanks only being filled to $\frac{4}{5}$ capacity. The larger tank has a volume of 27 times that of the smaller tank. **[1 mark]**

Question 20

$\frac{4}{6} = \frac{60}{h}$

$h = \frac{60 \times 6}{4}$

$= 90\ \text{m}$

Question 21

$\frac{50}{25} = \frac{5}{w}$

$w = \frac{25 \times 5}{50}$

$h = 2.5\ \text{cm}$

$= 0.025\ \text{m}$

© John Wiley & Sons Australia, Ltd