ATARNotes

Maths Methods

ATARNotes October Lecture Series

Presented by:
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About Me

- Graduated 2023 with a 98.10 ATAR, studying Comp Sci Science at Uni Melb
- Maths and Science tutor at Tutesmart
- The subjects I did throughout VCE
    - Chemistry
    - Maths Methods
    - Physics
    - English
    - Further Maths

Overview

- Function, Relations and Matrices
- Transformations
- Logarithms and Exponents
- Circular Functions
- Differentiation and Application
- Integration and Application
- Probability Tools and Definitions
- The Binominal Distribution
- The Normal Distribution

- Relations can be defined by rules, e.g.

$$\{(x, y): y = x + 1, x \in \{1,2,3\}\}$$

  - {} – contains a set
  - : – such that
  - ∈ – is an element of the set
- So this relation is $\{(1,2), (2,3), (3,4)\}$.

- Function, Relations and Matrices
  - Understanding the basic properties of functions and their notation
  - Understand function transformations
  - Matrix operations
- Transformations
  - Understanding the geometric representation of transformations
  - Computing transformations in function and matrix notation
- Logarithms and Exponents
  - Understanding the geometric representation of the functions
  - Properties, rules and application of logarithms and exponentials
- Circular Functions
  - Understanding the geometric representation of the functions
  - Properties of complementary and supplementary angles

Understanding function notation

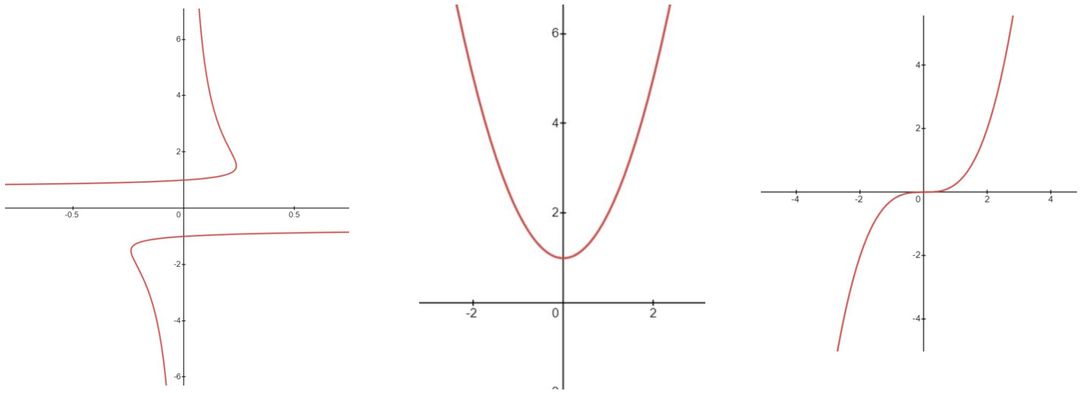
- A set is a collection of elements e.g. $\{1,2,5\}$
- An ordered pair (x, y) is a pair of elements; x is the first coordinate and y the second coordinate e.g. $(2,3)$
- A relation is a set of ordered pairs, e.g. $\{(1,2), (1,4), (3,2)\}$
  - The domain is the set of all x values i.e. $\{1,3\}$
  - The range is the set of all y values i.e. $\{2,4\}$
- Common sets you'll see are $\mathbb{Z}$ (integers) and $\mathbb{R}$ (real numbers)

Understanding function notation

- Functions are relations where each x value has only one corresponding y value:

$$f : [1,3) \to \mathbb{R}, f(x) = x^2 - 2$$

- The function is called f
- Domain is $[1,3)$
- The rule is $y = x^2 - 2$



- Note the vertical and horizontal line tests!

Importance of domain

- If the domain is not stated in the question stem, assume the domain is the largest for which the function makes sense (i.e. the maximal/implied domain).
- Always cross-reference solutions obtained with the implied domain.

Solve $4 - x = \sqrt{x - 2}$.

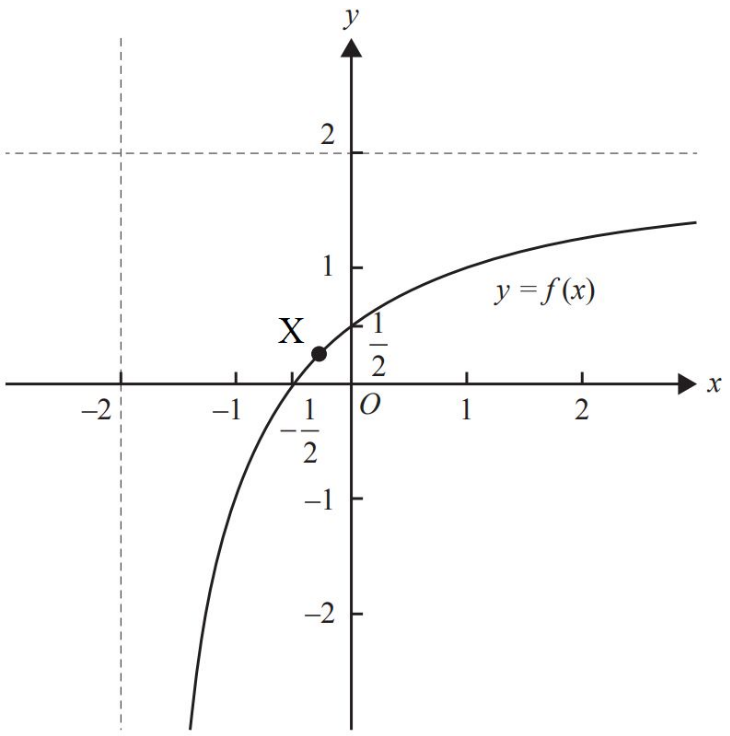
$$16 - 8x + x^2 = x - 2$$
$$x^2 - 9x + 18 = 0$$
$$(x - 6)(x - 3) = 0$$
$$x = 6 \; or \; x = 3$$

$$but \; 4 - 6 \neq \sqrt{6 - 2} \; (as \; 4 - x \geq 0 \; so \; x \leq 4)$$
$$so \; x = 3$$

CAS:

menu > 3 > 1
"... $4 - x = \sqrt{x - 2}, x)$"

- In questions involving graphs, cross-reference solutions that you obtain both with the graph as well as the given domain to choose which solutions to reject



Let $g: (-k, \infty) \to R$, $g(x) = \dfrac{kx+1}{x+k}$, where $k > 1$.

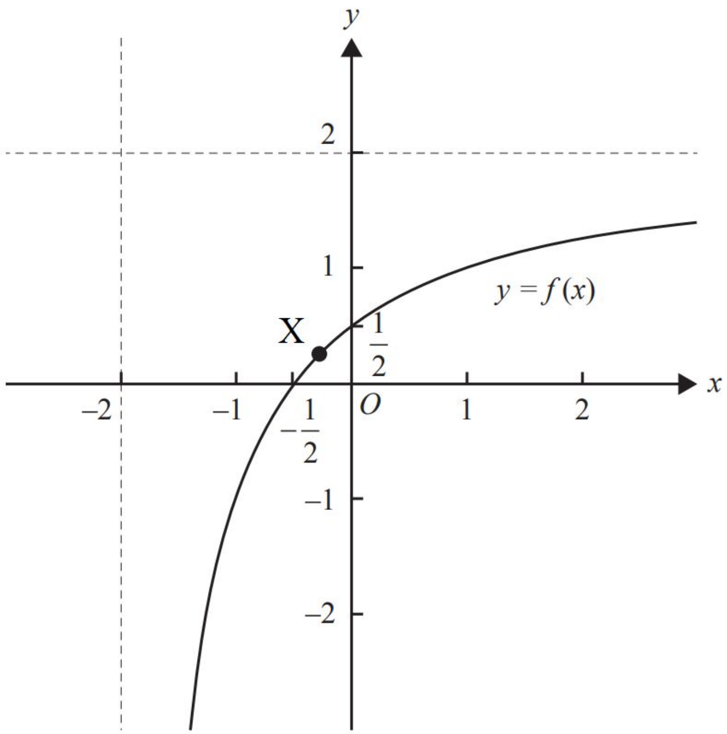
The graph of g for $k = 2$ is given on the left.

Let X be the point of intersection of the graphs of $y = g(x)$ and $y = -x$.

Find the coordinates of X in terms of k.

Adapted from VCAA
2016 exam 2

- In questions involving graphs, cross-reference solutions that you obtain both with the graph as well as the given domain to choose which solutions to reject



$$at\ intersection\ point\ X, g(x) = -x$$
$$so\ x = -k \pm \sqrt{k^2 - 1}$$
$$but\ x > -k$$
$$\therefore x = \sqrt{k^2 - 1} - k$$
$$so\ X = \left(\sqrt{k^2 - 1} - k, k - \sqrt{k^2 - 1}\right)$$

Adapted from VCAA 2016 exam 2

CAS:

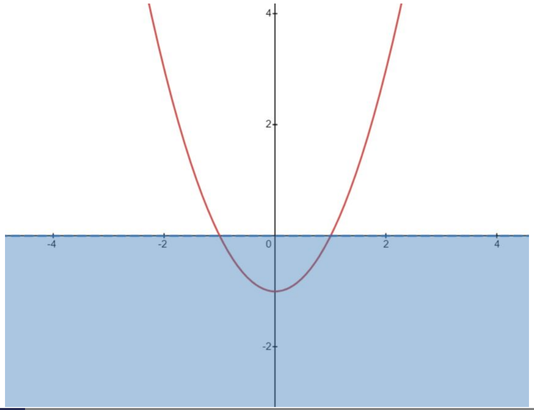
menu > 1 > 1, "... $g(x)=\frac{kx+1}{x+k}$"
menu > 3 > 1, "...$g(x) = -x, x$)"

- The composition of a function g with another function f is the operation

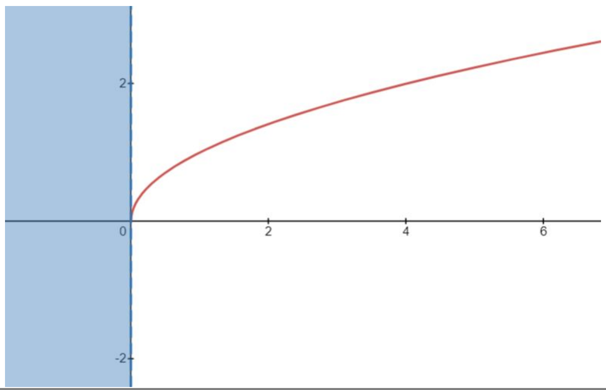
$$g \circ f(x) = g(f(x))$$

- Usually will have to invoke '$g \circ f$ is defined only when $ran\ f \subseteq dom\ g$'
- Always sketch for restriction questions!
  - The output of f is the input of g:

$f(x)$



$\mathrm{g}(x)$



Composite functions

Adapted from VCAA 2017 exam 1

Question 7 (5 marks)

Let $f : [0,\infty) \to R, f(x) = \sqrt{x+1}$.

a. State the range of f. 1 mark

b. Let $g : (-\infty, c] \to R, g(x) = x^2 + 4x + 3$, where $c < 0$.

i. Find the largest possible value of c such that the range of g is a subset of the domain of f. 2 marks

Adapted from VCAA 2017 exam 1

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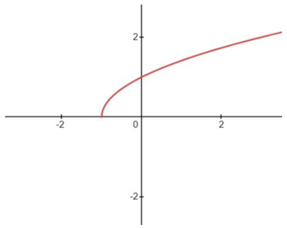

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$[1, \infty)$

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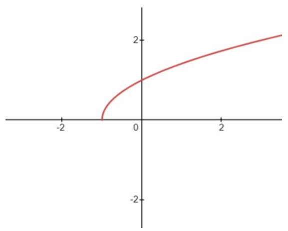

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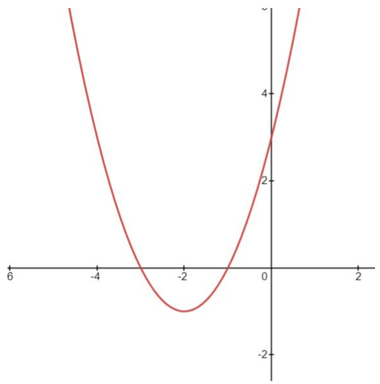


i. Find the largest possible value of c such that the range of g is a subset of the domain of f. 2 marks

$we\ require\ ran\ g \subseteq [0,\infty)$

$when\ g(x) = 0,$

$0 = x^2 + 4x + 3$

$\quad = (x+3)(x+1)$

$x = -3 \text{ or } x = -1$

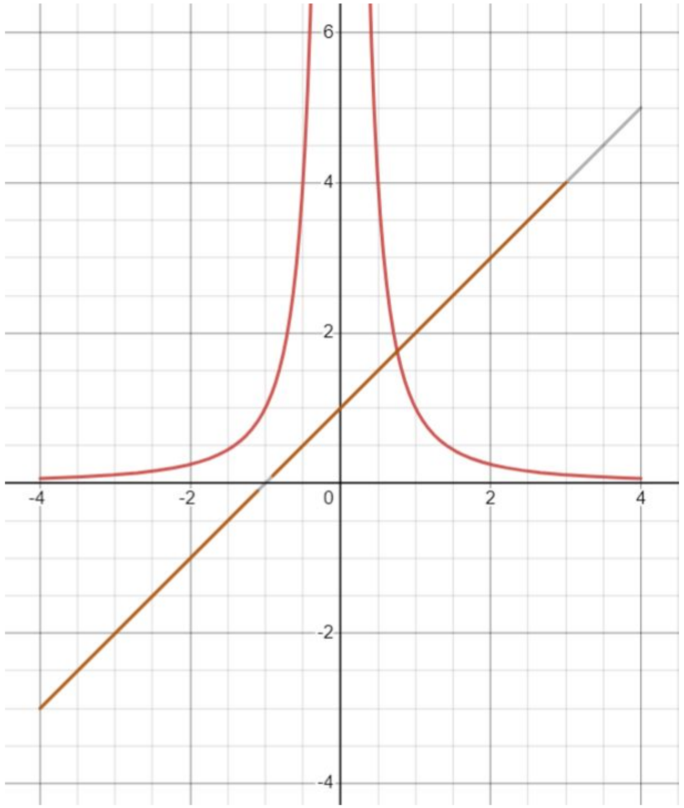
$so\ c = -3.$

Consider the functions $f(x) = \frac{1}{x^2}$ where $x \in [-4,0) \cup (0,4]$ and $g(x) = x + 1$ where $x \in (-4,4)$

a. Find the domain for which $h(x) = (f \circ g)(x)$ is defined

Consider the functions $f(x) = \frac{1}{x^2}$ where $x \in [-4,0) \cup (0,4]$ and $g(x) = x + 1$ where $x \in (-4,4)$

a. Find the domain for which $h(x) = (f \circ g)(x)$ is defined



$$\begin{aligned} &we\ require\ ran\ g \subseteq dom\ f \\ &\qquad \subseteq [-4,0) \cup (0,4] \\ &currently, ran\ g = (-3,5) \\ &\therefore we\ want\ ran\ g = (-3,0) \cup (0,4] \\ &\therefore dom\ g = (-4,-1) \cup (-1,3] \\ &\therefore dom\ h = (-4,-1) \cup (-1,3] \end{aligned}$$

Functions can be summed or multiplied:

$$fg(x) = f(x)g(x)$$
$$(f + g)(x) = f(x) + g(x)$$

- The domain is $dom\ g \cap dom\ f$
- Usually appears in both exams - in exam 1 it will ask for the domain and range; in exam 2 you will be asked to analyse the function’s behaviour

CAS:

Ensure you have defined the functions

“$domain(f(x), x)$"

Inverse functions

- Inverse functions 'undo' the operation of a given function:

$$f^{-1}(f(x)) = x$$

- $dom\ f^{-1} = ran\ f, ran\ f^{-1} = dom\ f$
- Function f reflected in the line $y = x$

- 5 step process to find f^{-1}:

$$\text{For } f: [2, \infty) \to \mathbb{R}, f(x) = \sqrt{x-2} + 1, \text{find } f^{-1}(x).$$

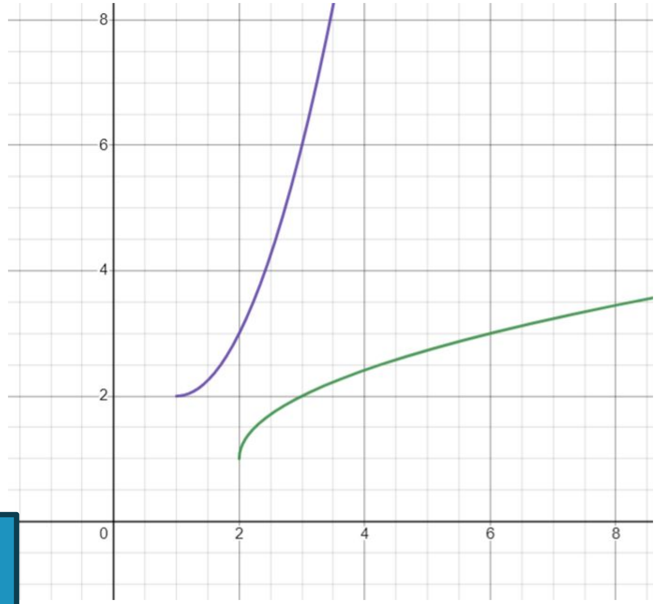
(1) Let $y = \sqrt{x-2} + 1$

(2) The inverse has rule $x = \sqrt{y-2} + 1$

(3) So $y = x^2 - 2x + 3$

(4) Note that $dom f^{-1} = ran\ f = [1, \infty)$

(5) So $f^{-1}: [1, \infty) \to R, f^{-1}(x) = x^2 - 2x + 3$



CAS:

Menu > 1 > 1, "…$f(x) = \sqrt{x-2} + 1 | x > 2$"

Menu > 3 > 1, "… $f(y) = x, y)$"

Inverse functions

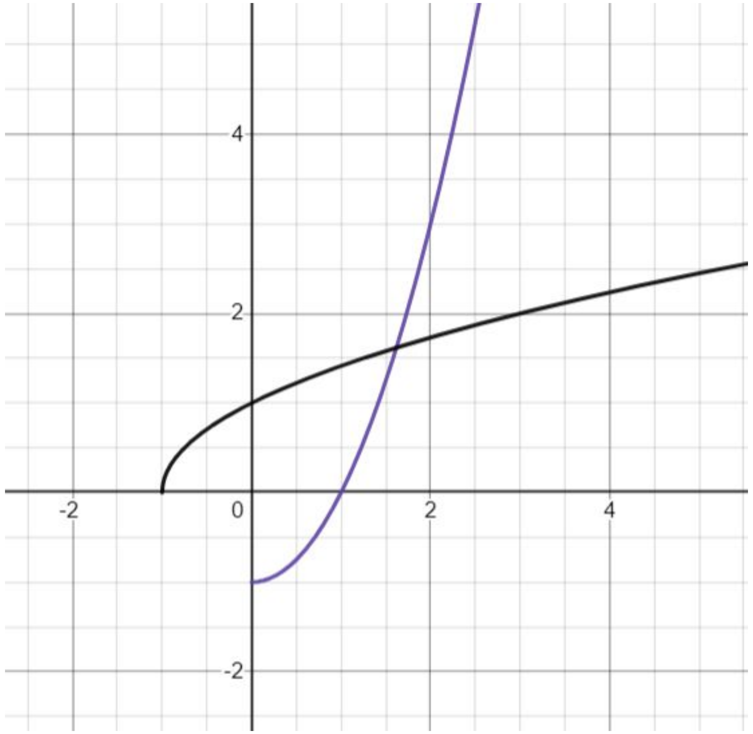
- Let f: $[0,\infty) \rightarrow \mathbb{R}, f(x) = x^2 - 1$ and f^{-1}: $[-1,\infty) \rightarrow \mathbb{R}, f^{-1}(x) = \sqrt{x+1}$. Find the points of intersection.

$$\text{at intersection points,}$$
$$f(x) = g(x)$$
$$x^2 - 1 = \sqrt{x-1}$$
$$x^4 - 2x^2 + 1 = x - 1$$

???

Inverse functions

- Let f: $[0,\infty) \to \mathbb{R}, f(x) = x^2 - 1$ and f^{-1}: $[-1,\infty) \to \mathbb{R}, f^{-1}(x) = \sqrt{x+1}$. Find the points of intersection.



$$\textit{at intersection points,}$$
$$f(x) = x$$
$$x^2 - 1 = x$$
$$0 = x^2 - x - 1$$
$$x = \frac{1 \pm \sqrt{5}}{2}$$
$$\textit{but } x > -1$$
$$\textit{so } x = \frac{1 + \sqrt{5}}{2}$$
$$\therefore \textit{intersection at } \left(\frac{1+\sqrt{5}}{2}, \frac{1}{-}\right.$$

CAS:

Menu > 3 > 1,
"… $f(x) = x, x)$"

- We apply transformations to *stretch, squeeze,* or *shift* the graphs of functions.
- Need to memorise both **definitions** and **procedures** for transforming functions ☺️

- Reflections ⇒ flipping a graph
- Dilations ⇒ stretching a graph
- Translations ⇒ shifting a graph

Dilations from the x-axis

Dilation by factor k away from the x-axis is described by

$$(x, y) \to (x, ky)$$

Replace y with y/k

$$y = f(x) \quad \to \quad y = kf(x)$$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Dilations from the y-axis

Dilation by factor k away from the y-axis is described by

$$(x, y) \to (kx, y)$$

Replace x with x/k

$$y = f(x) \quad \to \quad y = f\left(\frac{x}{k}\right)$$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Reflection in the x-axis

$$(x, y) \to (x, -y)$$

Replace y with $-y$

$$y = f(x) \quad \to \quad -y = f(x)$$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

Reflection in the y-axis

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$(x, y) \to (-x, y)$$

Replace x with $-x$

$$y = f(x) \quad \to \quad y = f(-x)$$

Vertical translations

- Translation by k units in the positive direction of the y-axis

$$(x, y) \to (x, y + k)$$

Replace y with $(y - k)$

$$y = f(x) \quad \to \quad y - k = f(x)$$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ k \end{bmatrix}$$

Horizontal translations

- Translation by h units in the positive direction of the x-axis

$$(x, y) \to (x + h, y)$$

Replace x with $(x - h)$

$$y = f(x) \quad \to \quad y = f(x - h)$$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ 0 \end{bmatrix}$$

- $h/k > 0 \Rightarrow$ shift in the positive direction
- $h/k < 0 \Rightarrow$ shift in the negative direction

Example

The graph of $\text{g}(x)$ has been shifted left 3 units, then dilated by a factor of 2 from the y $axis$. If $\text{g}(x) = \log_e(3x - 1)$ find $\text{f}(x)$, the transformed function.

$$g(x+3) = \log_e(3(x+3) - 1)$$

$$g\left(\frac{x}{2} + 3\right) = \log_e\left(3\left(\frac{x}{2} + 3\right) - 1\right)$$

$$\therefore f(x) = \log_e\left(\frac{3x}{2} + 8\right)$$

- $$\text{Let } g(x) = 3\left(\frac{x}{2} - 1\right)^2 + 1 \text{ and } f(x) = x^2$$

Determine the set of transformations that take $f(x)$ to $g(x)$.

$$y' = 3\left(\frac{x'}{2} - 1\right)^2 + 1 \qquad\qquad y = x^2$$

Rearrange to replace

$$\frac{1}{3}(y' - 1) = \left(\frac{x'}{2} - 1\right)^2 \qquad\qquad y = (x)^2$$

so $y = \frac{1}{3}(y' - 1)$ and $x = \frac{x'}{2} - 1$

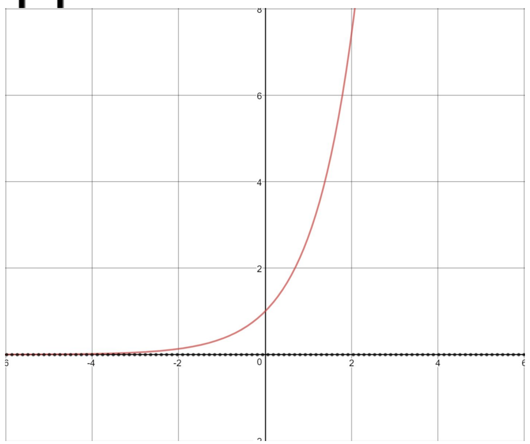
$$y' = 3y + 1 \qquad\qquad x' = 2x + 1$$

$$(x, y) \to (2x + 1, 3y + 1)$$

- Introducing Euler's constant $e = 2.71828182846\ldots$

$$e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^{n}$$

- Has some special properties and application in calculus and finance
- Base function $f(x) = e^x$
- Has a horizontal asymptote $y = 0$
- Special property: $\frac{d}{dx}(e^x) = e^x$



- $$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

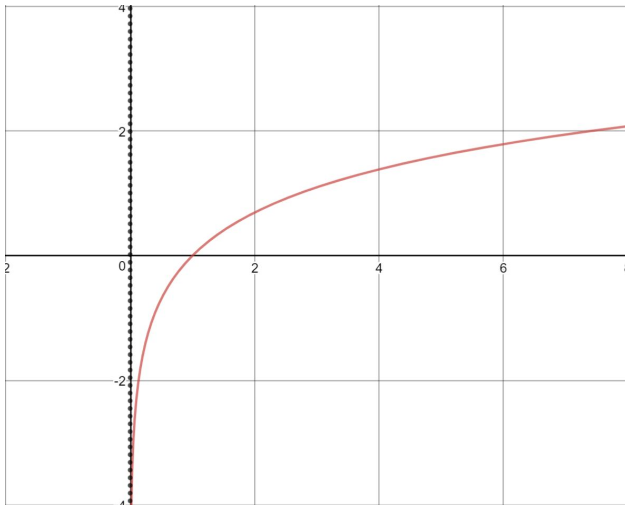
$$(a^m)^n = a^{mn}$$

$$a^m = e^{m\ln(a)}$$

Application of these rules tend to appear in exam one questions which require students to solve equations after finding a common base

- Essentially the inverse function of the exponent

$$\log_b(b^a) = a$$



- Base function $f(x) = \log_e(x) = \ln(x)$
- Has a vertical asymptote $x = 0$

Expect to see solve questions, similar to exponentials. Properties are more diverse, students often make mistakes with the implied domain

Logarithmic Laws

- $$\log_a(m) + \log_a(n) = \log_a(mn)$$

$$\log_a(m) - \log_a(n) = \log_a\left(\frac{m}{n}\right)$$

$$\log_a(m^n) = n\log_a(m)$$

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)} = \frac{\ln(x)}{\ln(a)}$$

$$\log_a(a) = 1; \log_a(1) = 0$$

The change of base rule, whilst uncommon, is often forgotten.

- $$\text{Solve } 3 \times 16^{3x} - 1 + 2 \times 4^{3x} = 0$$

$$3(4^2)^{3x} - 1 + 2(4^{3x}) = 0$$

$$3(4^{3x})^2 + 2(4^{3x}) - 1 = 0$$

$$(3(4^{3x}) - 1)(4^{3x} + 1) = 0$$

$$3(4^{3x}) - 1 = 0 \text{ and } 4^{3x} + 1 = 0$$

1

Solving exponential equations

- Find values of k such that the graphs of $y = x^2e^{kx}$ and $y = xe^{kx}(kx + 2)$ have exactly one point of intersection. (Adapted from VCAA 2018 exam 1)

$$\text{at intersection point,}$$

$$x^2e^{kx} = xe^{kx}(kx + 2)$$

$$x^2e^{kx} - xe^{kx}(kx + 2) = 0$$

$$xe^{kx}(x - kx + 2) = 0$$

$$\therefore xe^{kx} = 0 \text{ or } x - kx + 2 = 0$$

$$\Rightarrow x = 0 \left(e^{kx} \neq 0\right) \qquad x = \frac{2}{k - 1}$$

$$\therefore k = 1.$$

CAS:

Menu > 3 > 1 " $x^2e^{kx} - xe^{kx}(kx + 2), x)$"

Solving logarithm equations

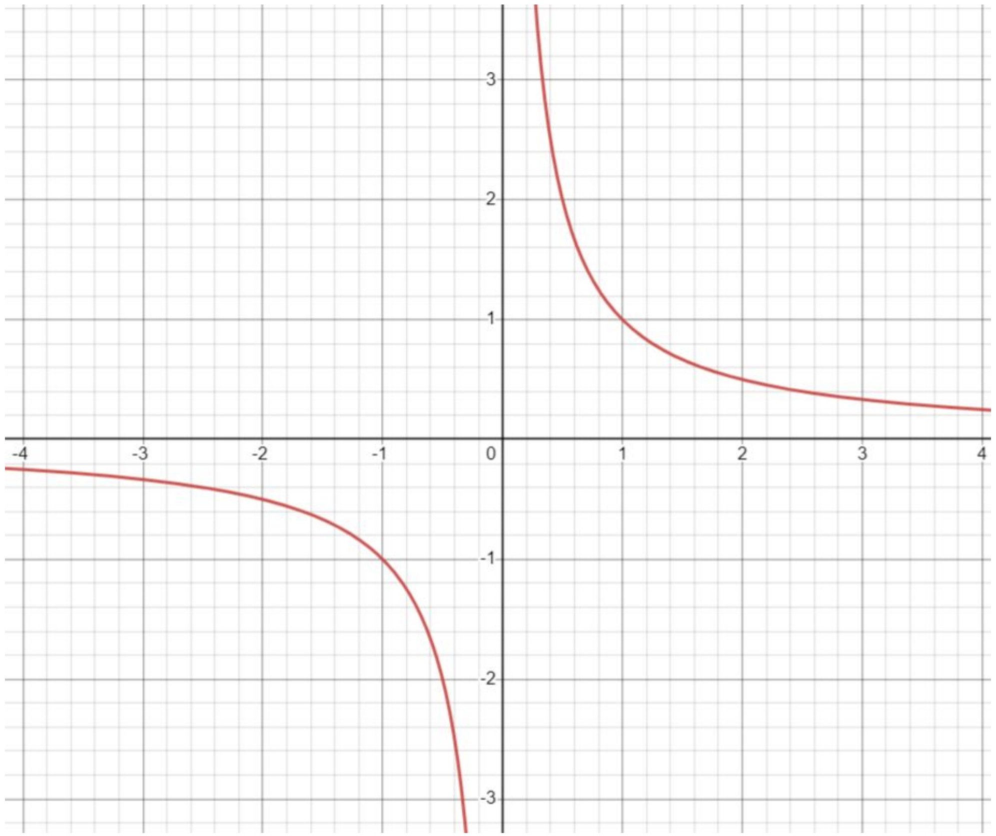
Consider the function $f(x) = \frac{1}{2}\log_e(\frac{1}{x})$.

a. Find the maximal domain of $f(x)$.

$$we\ require\ \frac{1}{x} > 0$$
$$\therefore x > 0\ i.e.\ dom\ f = (0, \infty)$$

CAS:

- "$domain(\frac{1}{2}\ln\left(\frac{1}{x}\right), x)$"



Solve $2\log_2(x+5) - \log_2(x+9) = 1$.

$$\log_2\left(\frac{(x+5)^2}{x+9}\right) = \log_2(2)$$
$$\Rightarrow \frac{(x+5)^2}{x+9} = 2$$
$$x^2 + 10x + 25 = 2x + 18$$
$$x^2 + 8x + 7 = 0$$
$$(x+7)(x+1) = 0$$
$$so\ x = -7\ or\ x = -1$$
$$but\ \log_2(x+5)\ is\ undefined\ for\ x = -7$$
$$\therefore x = -1.$$

CAS:

Menu > 3 > 1, "… $2\,log_2(x+5) - \log_2(x+9) = 1, x)$"

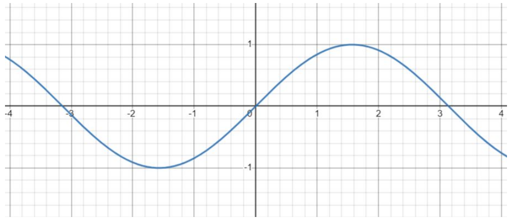
Circular Functions

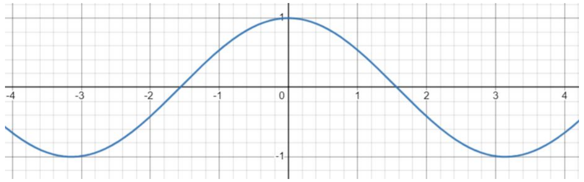
- Our second irrational constant $\pi = 3.14159265359\ldots$

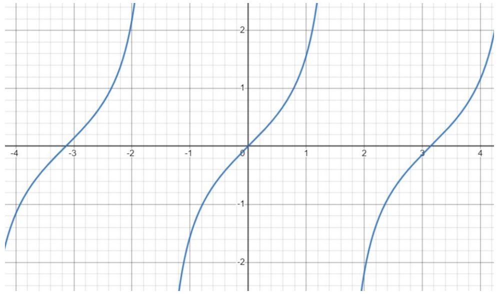
$$\pi^c = 180^\circ$$

- We have three basic functions

$$f(x) = \sin(x) = \frac{O}{H} \qquad f(x) = \cos(x) = \frac{A}{H} \qquad f(x) = \tan(x) = \frac{O}{A}$$

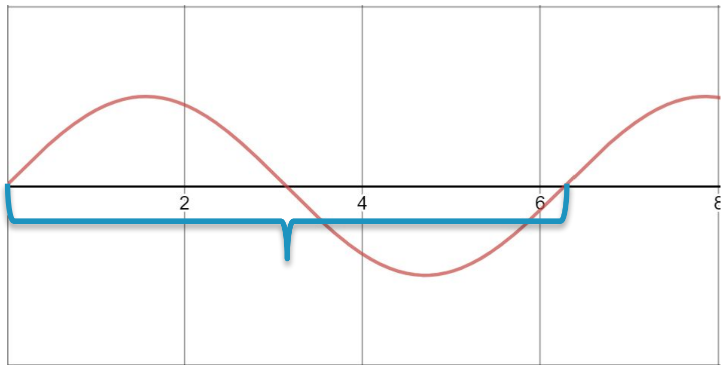


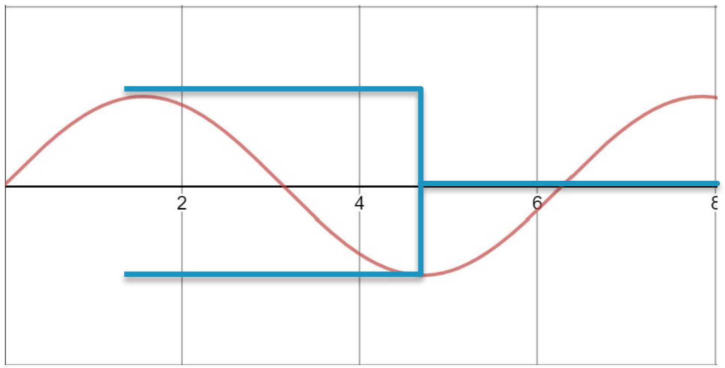




Functions

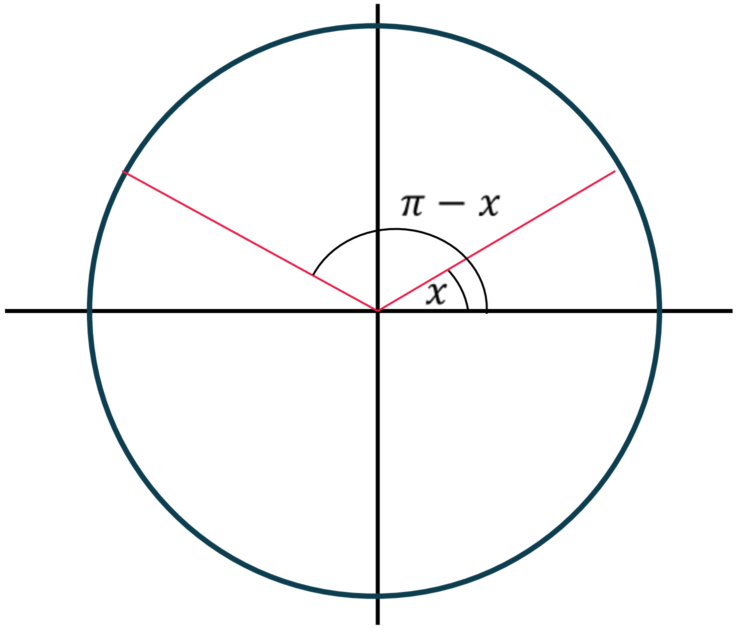
- Perhaps the biggest source of careless mistakes!





- Common exam one and multiple choice question, usually asked at the beginning for free marks.

- Be very familiar with symmetry rules (e.g. $\sin(\pi - x) = \sin(x)$)...



- We are dealing with sin, so we want to look at the y-coordinate of the corresponding points on the unit circle

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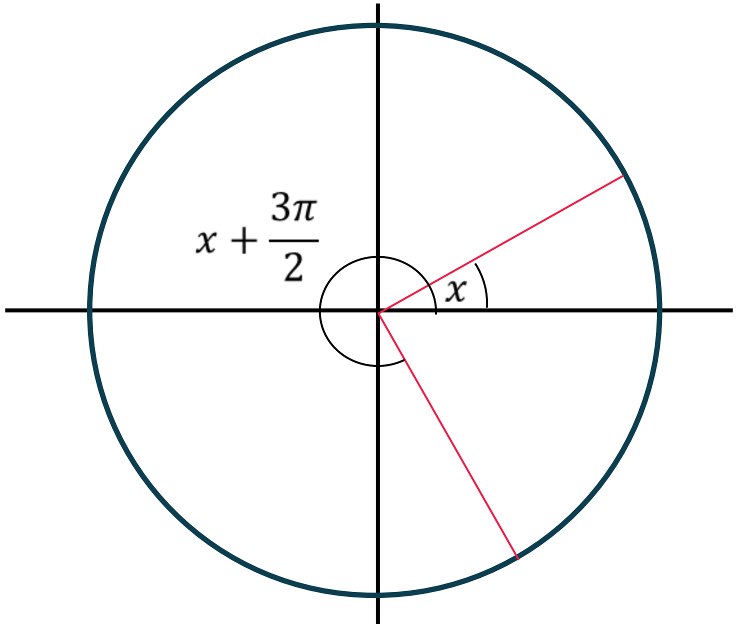
- ...complementary rules (e.g. $\sin\left(x+\frac{3\pi}{2}\right)=-\cos(x)$)...



- We are adding a multiple of $\frac{\pi}{2}$, so we know complementary rules apply ($\sin\left(x+\frac{3\pi}{2}\right)\to\cos(\mathrm{x})$)
- Sin is negative in the quadrant we arrive at, so we place a negative sign out front

- …and the Pythagorean identity

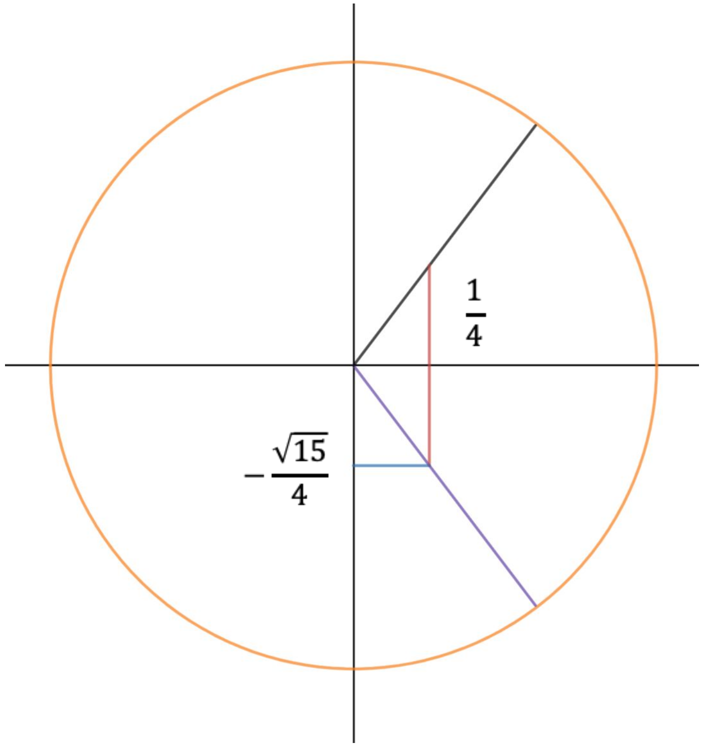
$$\sin^2(x) + \cos^2(x) = 1$$

- These three sets of rules are crucial to solving equations as well as transformation questions!

- Given that $\cos(a) = \frac{1}{4}$, find $\sin(a)$ and $\sin\left(a + \frac{\pi}{2}\right)$ for $a \in (-\frac{\pi}{2}, 0)$

$$\sin(a) = -\sqrt{1-\cos^2(a)} = -\frac{\sqrt{15}}{4}$$

$$\sin\left(a+\frac{\pi}{2}\right) = \cos(a) = \frac{1}{4}$$



Be careful, these values are positive in 2 quadrants, make sure you know

Solving can be quite tricky – so it's best to adopt some systematic steps

- Rearrange so trig ratio is on one side, e.g. $\sin(ax+b)=c$
- Modify the domain to suit the angle written in brackets
- Find the ***basic angle***
- Determine based on the trig ratio where the additional solutions lie
- Solve for the angle in brackets, considering the domain
- Hence, solve for x

- **Solve:** $\sin\left(2\theta - \frac{\pi}{3}\right) = -\frac{1}{2}, \theta \in [0, 2\pi]$

- Rearrange so the trig ratio is on one side
  - Already done
- Modify the domain to suit the angle written in brackets

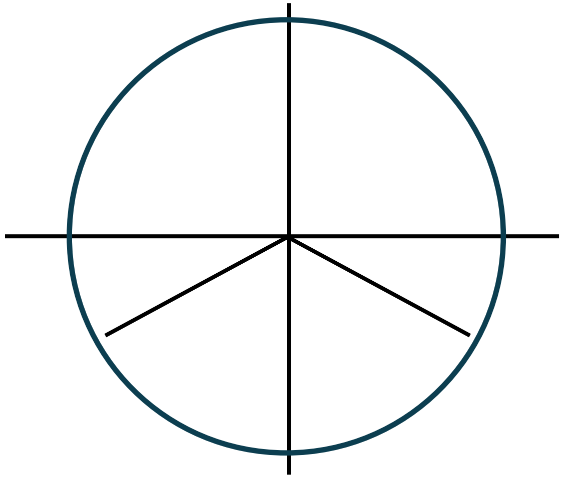
$$\theta \in [0,2\pi]$$
$$2\theta \in [0,4\pi]$$
$$2\theta - \frac{\pi}{3} \in \left[-\frac{\pi}{3}, \frac{11\pi}{3}\right]$$

Solve: $\sin\left(2\theta - \frac{\pi}{3}\right) = -\frac{1}{2}, \theta \in [0, 2\pi]$

- Find the basic angle

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

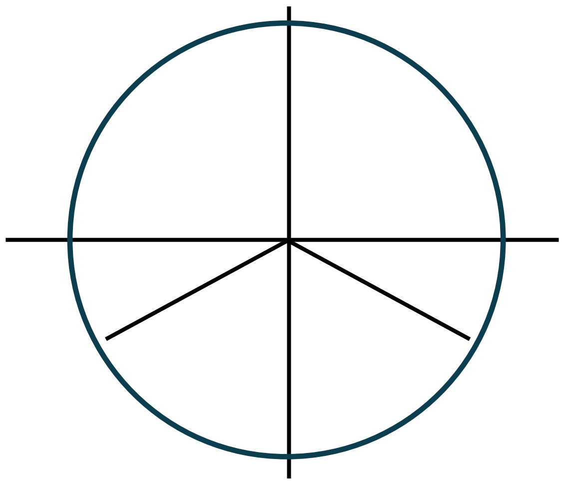
- Determine based on the trig ratio where the additional solutions lie



- We can see some solutions are $-\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ etc.

Solve: $\sin\left(2\theta - \frac{\pi}{3}\right) = -\frac{1}{2}, \theta \in [0, 2\pi]$

- Solve for the angle in brackets, considering the domain



$$2\theta - \frac{\pi}{3} \in \left[-\frac{\pi}{3}, \frac{11\pi}{3}\right]$$

$$2\theta - \frac{\pi}{3} = -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}$$

CAS:

Menu > 3 > 1, "…$\sin\left(2x - \frac{\pi}{3}\right) = -\frac{1}{2}, x)|0 \leq x \leq 2\pi$"

- $$\text{Solve } \sin^3(x) + \cos^2(x)\sin(x) = 0 \quad \text{for } x \in [2\pi, 4\pi]$$

$$\sin(x)(\sin^2(x) + \cos^2(x)) = 0$$

$$\sin(x) = 0$$

$$x = 2\pi, 3\pi, 4\pi$$

What if $x \in R$?

$$x = n\pi, \qquad n \in Z$$

- When the domain is unspecified or when specifically asked, the **general solution** is required
- When you arrive at the stage $f(x) = a$, where f is a trig function, then:

$$x = 2n\pi \pm \cos^{-1}(a)$$

$$x = n\pi + \tan^{-1}(a)$$

$$x = 2n\pi + \sin^{-1}(a) \text{ and } x = (2n+1)\pi - \sin^{-1}(a)$$

$\boldsymbol{n \in Z}$ … don’t forget this!

- Sketching graphs is significantly more important in exam one, examiners are looking for scale, curvature, and asymptotic behaviour when applicable.
- Exam 1 questions will very rarely require you to sketch a complicated trigonometric function; many of the steps described in the next slides can be skipped
- Important part is getting the domain and general shape correct
- There are many ways to sketch trigonometry graphs, use the method you are comfortable with

- For $f(x) = a\tan(nx + c) + d$:
- Period is $\frac{\pi}{n}$
- Asymptotes given by $nx + c = \frac{(2k+1)\pi}{2}, \mathrm{k} \in \mathbb{Z}$
- x and y intercepts
- Endpoint coordinates
- Careful that domain excludes asymptotes; range is always $\mathbb{R}$

The function must approach the asymptotes as they tend towards them, the general shape must be curvy and you must have the correct end points.

Sketching Tan function

Sketch $y = 3\tan\left(2\left(x - \frac{\pi}{3}\right)\right) + \sqrt{3}$ for $x \in [0, 2\pi]$

1. Period $= \frac{\pi}{2}$
2. Asymptotes

$$2\left(x - \frac{\pi}{3}\right) = \frac{(2k+1)\pi}{2}$$
$$x = \frac{(2k+1)\pi}{4} + \frac{\pi}{3} = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12} \text{ etc.}$$

3. x intercept: solve $0 = 3\tan\left(2\left(x - \frac{\pi}{3}\right)\right) + \sqrt{3} \Leftrightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

   y intercept: solve $y = 3\tan\left(2\left(0 - \frac{\pi}{3}\right)\right) + \sqrt{3} \Leftrightarrow y = 4\sqrt{3}$

Sketch $y = 3\tan\left(2\left(x - \frac{\pi}{3}\right)\right) + \sqrt{3}$ for $x \in [0, 2\pi]$

4. Endpoint coordinates

$$when\ x = 2\pi,$$
$$y = 3\tan\left(2\left(2\pi - \frac{\pi}{3}\right)\right) + \sqrt{3}$$
$$y = 4\sqrt{3}$$

Sketch $y = 3\tan\left(2\left(x - \frac{\pi}{3}\right)\right) + \sqrt{3}$ for $x \in [0, 2\pi]$

Sketch $y = 3\tan\left(2\left(x - \frac{\pi}{3}\right)\right) + \sqrt{3}$ for $x \in [0, 2\pi]$

CAS:

If this is exam 2, sketch the graph using CAS

Key Take Aways

- Sketching
  - Shape
  - Scale
  - Asymptotic behaviour
  - Labelling equations of asymptotes and coordinates
- Solving
  - Consider underlying domains
  - Do not put equal signs between quantities that are not equal
- Transformations
  - Carefully consider which is the original function and which is the image
  - The numbers used in function notation are inconsistent with matrix notation and mapping

- Differentiation and Application
  - Understanding the implications of a derivative
  - Derivation and basic derivatives in function and Leibnitz notation
  - Chain, product and quotient rules
  - Tangents and normals
  - Rates of change and stationary points
- Integration and Applications
  - Understanding the implications of an integral
  - Basic antiderivatives and the fundamental theorem of calculus
  - Area bounded by curves and average value

- Differentiation allows us to find a function f' that tells us the instantaneous rate of change at any point on a given function f. By first principles:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- This formula is derived from the gradient formula $m = \frac{y_2 - y_1}{x_2 - x_1}$
- So the derivative f' describes the slope of a tangent to a function f at this particular point

-

$$\text{Let } y = e^{4x^2} \text{ find } \frac{dy}{dx}.$$

$$\frac{dy}{dx} = \frac{d}{dx}(4x^2) \times e^{4x^2}$$

$$= 8xe^{4x^2}$$

CAS:

Menu > 4 > 1, "$\frac{d}{dx}(e^{4x^2})$"

These types of questions, in conjunction with our rules have appeared on exam one every single year for the past decade, it is safe to expect it to do so again.

Calculus

Derivatives

Function Notation	Leibniz notation

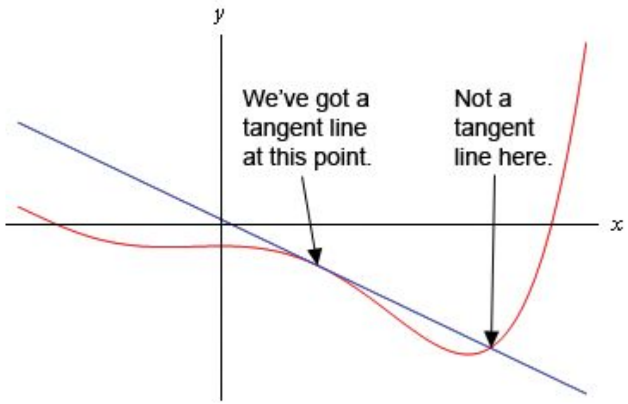
- ‘Show that’ questions involving proving a derivative will require use of these rules:

Let $y = e^{2x^2}$. Show that $\frac{dy}{dx} = 4xe^{2x^2}$.

$Let\ u = 2x^2. Then\ y = e^u.$

$\therefore by\ the\ chain\ rule$ dy $=$ dy du

- A tangent to a function is a line that shares a common point and gradient at said point; it 'touches' the graph of the function.



Find the tangent to the graph of $y = x^2$ at $(2,4)$

1. Let the tangent at $(2,4)$ have equation $y = mx + c$
2. Then $m = f'(2) = 2(2) = 4$, so $y = 4x + c$
3. At $(2,4)$, $4 = 4(2) + c$, so $c = -4$
4. The equation of the tangent is thus $y = 4x - 4$

CAS:

Note: if you are more confident using a formula, find the tangent using this:

$$y_{tangent|x=a} = f'(a)(x-a) + f(a)$$

- Solving difficult questions involving tangents involves a familiarity of its connection with the derivative:

Consider a function f for which $f'(x) = -e^{kx}$. The tangent at $(1, -e^3)$ passes through the origin. Find the value of k.

$$Tangent\ has\ gradient\ \frac{-e^3 - 0}{1 - 0} = -e^3$$
$$\therefore f'(1) = -e^3$$
$$\therefore -e^k = -e^3$$
$$\therefore k = 3.$$

- Consider $h: (0, \infty) \to \mathbb{R}, h(x) = \frac{k}{x} - 1$ and $h^{-1}: (-1, \infty) \to \mathbb{R}, h^{-1}(x) = \frac{k}{x+1}$, where $k > 0$.

Let θ be the acute angle made by the tangent to h at its x-intercept and the x-axis. Let α be the acute angle made by the tangent to h^{-1} at its y-intercept and the x-axis. Find the values of k such that $\theta > \alpha$.

- Consider $h: (0, \infty) \to \mathbb{R}, h(x) = \frac{k}{x} - 1$ and $h^{-1}: (-1, \infty) \to \mathbb{R}, h^{-1}(x) = \frac{k}{x+1}$, where $k > 0$.

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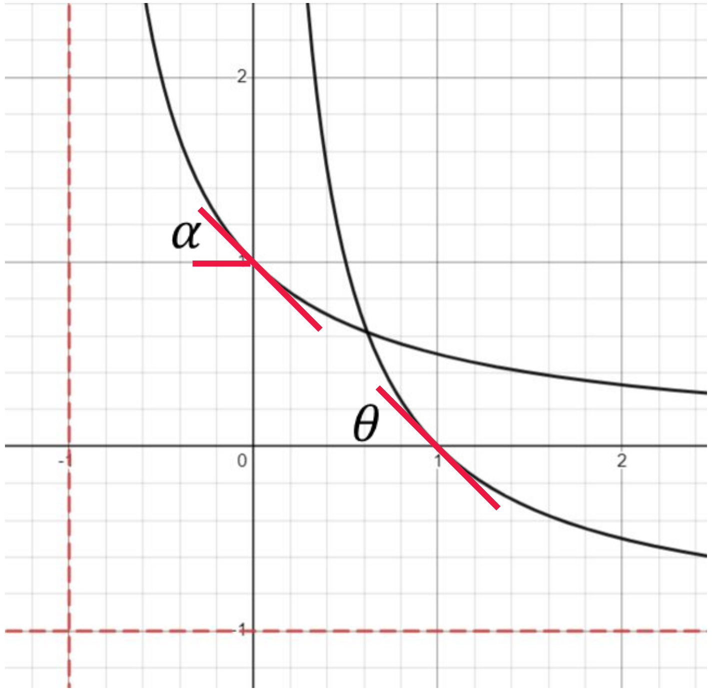

- Consider $h\colon (0,\infty) \to \mathbb{R}, h(x) = \frac{k}{x} - 1$ and $h^{-1}\colon (-1,\infty) \to \mathbb{R}, h^{-1}(x) = \frac{k}{x+1}$, where $k > 0$.

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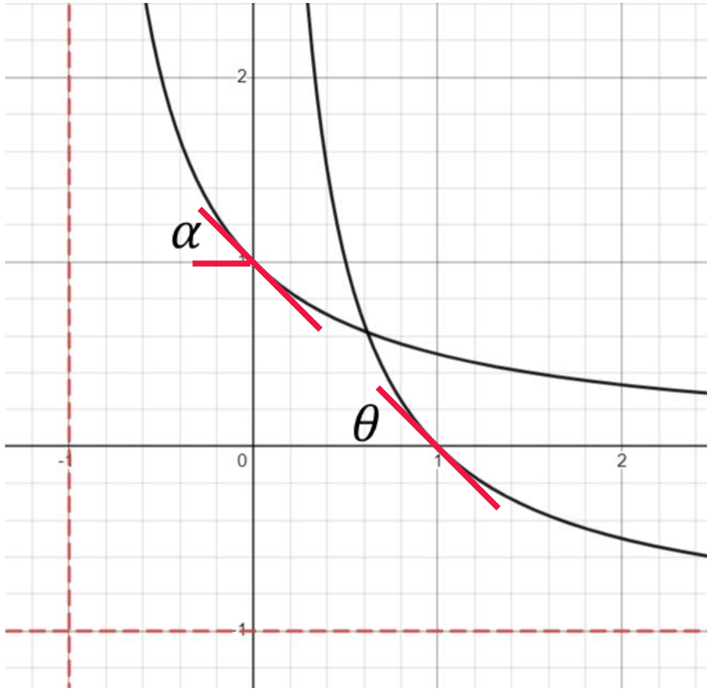
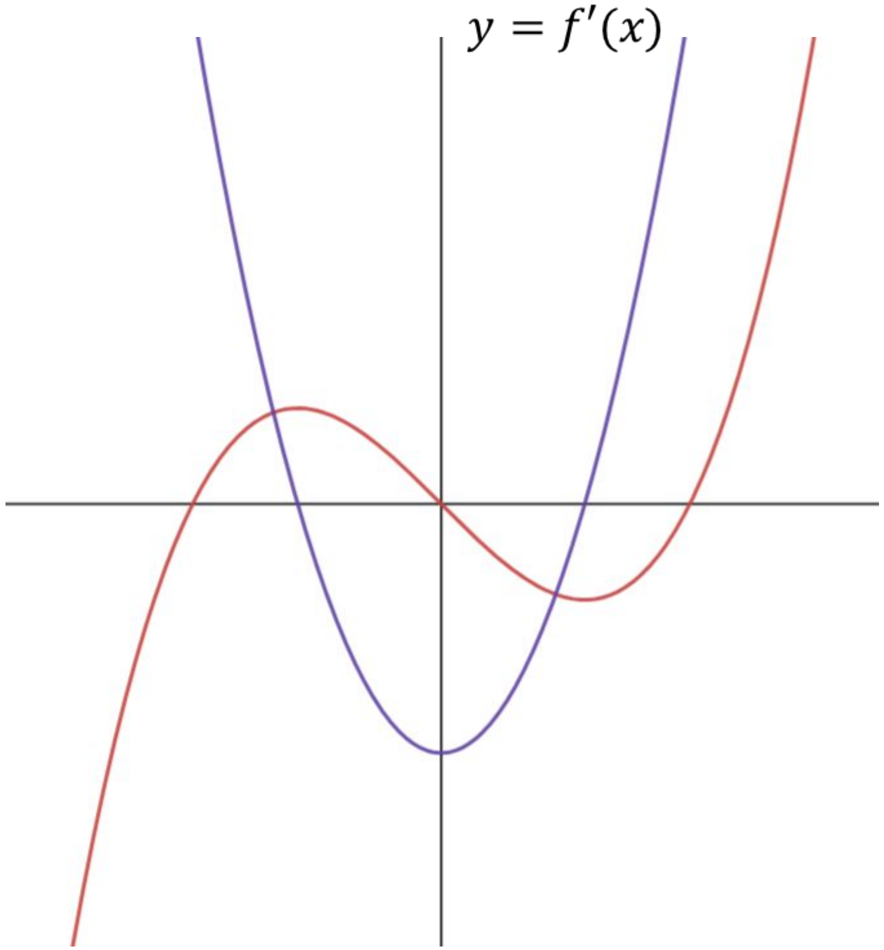


$$h'(x) = -\frac{k}{x^2}, (h^{-1})'(x) = -\frac{k}{(x+1)^2}$$

h has x-intercept k, h^{-1} has y-intercept at $x = 0$.

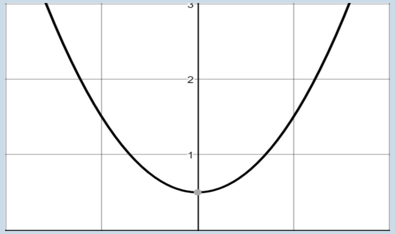
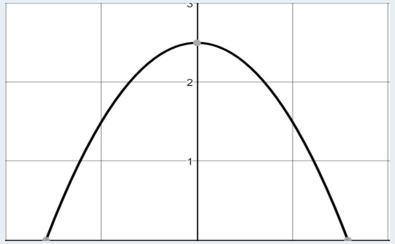
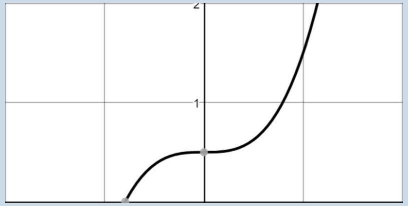
$\Rightarrow$ tangent to h has gradient $h'(k) = -\frac{1}{k}$, tangent to h^{-1} has gradient $(h^{-1})'(0) = -k$.



- when $f'(x) > 0, f$ is increasing
- when $f'(x) < 0, f$ is decreasing
- when $f'(x) = 0$, a stationary point occurs

Stationary points

Stationary points occur where $f'(x) = 0$

Nature	Graph
Local minimum	
Local maximum	
Stationary point of inflection	

- While sign diagrams can be used to determine a stationary point's nature, enough context clues will be provided in the paper for you to deduce this anyway
- The absolute maximum/minimum will always occur at an endpoint or when $f'(x) = 0$ – let's see an example in the next slide

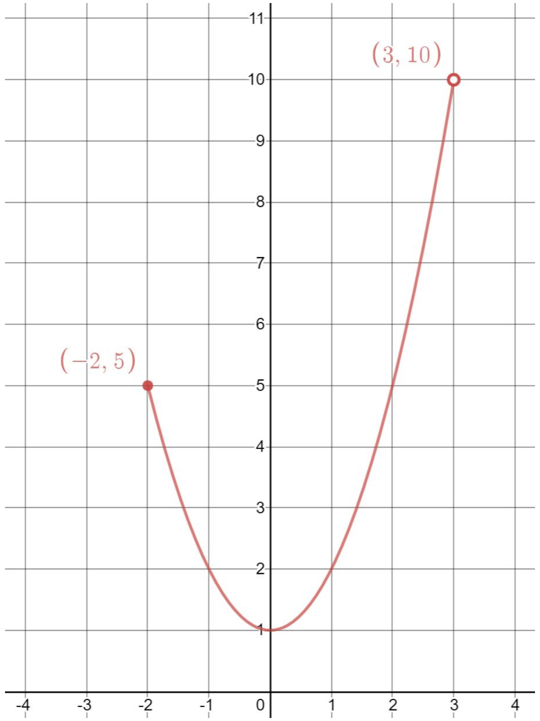
CAS:

Graph page → menu > 6
Allows you to analyse graph features!

Absolute Max and Min

Let $f: [-2,3) \to R, f(x) = x^2 + 1$

Find the range of f



$$f'(x) = 2x$$

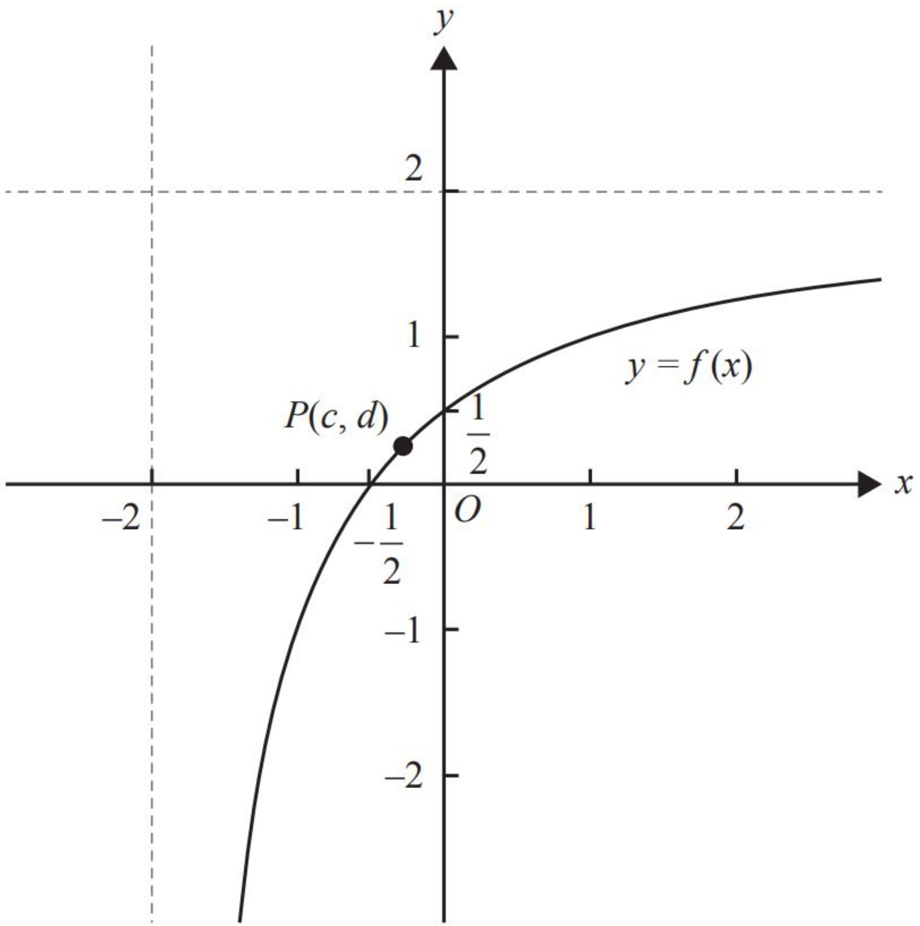
$$f'(x) = 0 \text{ when } x = 0$$

$$f(0) = 1$$

$$f(-2) = 5 \text{ and } f(3) = 10$$

$$\therefore ran\ f \in [1, 10)$$

Let $f: R\backslash\{-2\} \to R,\ f(x) = \dfrac{2x+1}{x+2}$.

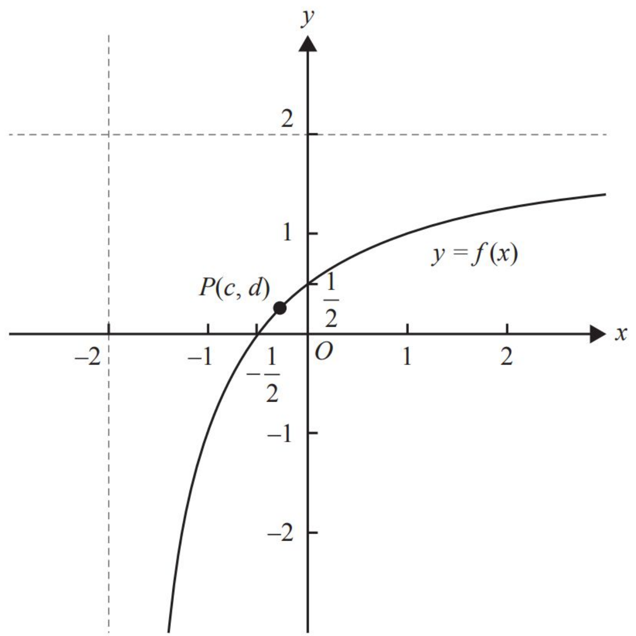


The point $P(c, d)$ is on the graph of f.

Find the exact values of c and d such that the distance of this point to the origin is a minimum, and find this minimum distance.

Adapted from VCAA 2016 exam 2

Let $f: R\backslash\{-2\} \to R,\ f(x) = \dfrac{2x+1}{x+2}$.



The point $P(c, d)$ is on the graph of f.

Find the exact values of c and d such that the distance of this point to the origin is a minimum, and find this minimum distance.

$d = f(c), so$

$OP = \sqrt{d^2 + c^2} = \sqrt{\left(f(c)\right)^2 + c^2}$

Minimum distance occurs when $\frac{d}{dc}(OP) = 0, i.e.$

$\frac{d}{dc}\left(\sqrt{\left(f(c)\right)^2 + c^2}\right) = 0$

$\Rightarrow c = -2 \pm \sqrt{3}.$ but c > −2 so c $= -2 + \sqrt{3}$.

$d = f(c) = 2 - \sqrt{3}$

$minimum\ distance = OP = \sqrt{d^2 + c^2} = 2\sqrt{2} - 6$

Adapted from VCAA 2016 exam 2

CAS:

Menu > 1 > 1, "… $f(x) = \frac{2x+1}{x+2}$"

Antidifferentiation

- Antidifferentiation allows us to find the function f given the derivative f'. That is:

$$\int f'(x)dx = f(x) + c$$

- Finds the area under a given curve
- Always remember the dx and $+c$!

Why do we need $+c$?

- What is the derivative of $2x$?

$$\frac{d(2x)}{dx} = 2$$

- What is the derivative of $2x + 1$?

$$\frac{d}{dx}(2x + 1) = 2$$

- So, what is the antiderivative of 2?

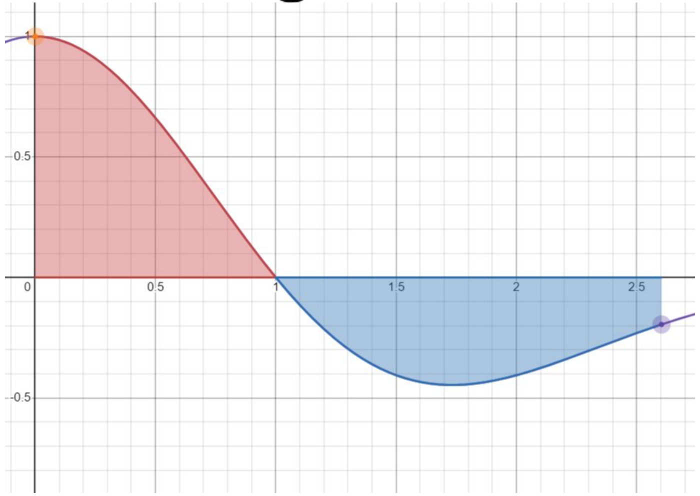
Antidifferentiation

Fundamental Theorem of Calculus

For a function $f(x)$ with antiderivative $F(x)$, then:

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

- Gives the signed area under graph of f over the interval $x \in [a, b]$

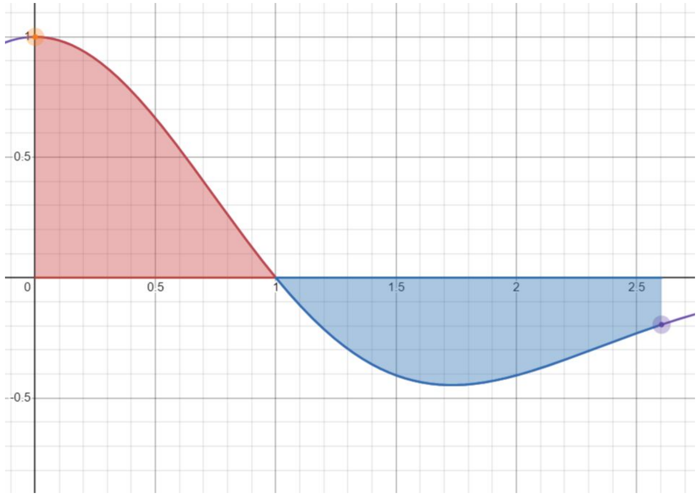


$$\int_a^b f(x)\,dx = A_1 + A_2$$

Finding the total area

Steps to finding the total area contained under f over an interval $[a, b]$

1. Find the x intercepts of f
2. Set up multiple definite integrals, each with the appropriate sign and consider x intercepts when determining bounds
3. Evaluate the integrals



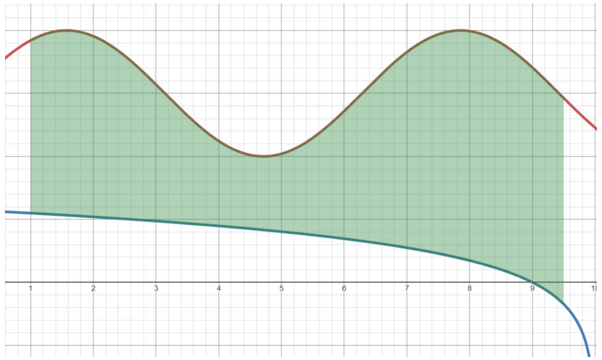
1. x intercept at c

2. $area = \int_a^c f(x)\,dx - \int_c^b f(x)\,dx$

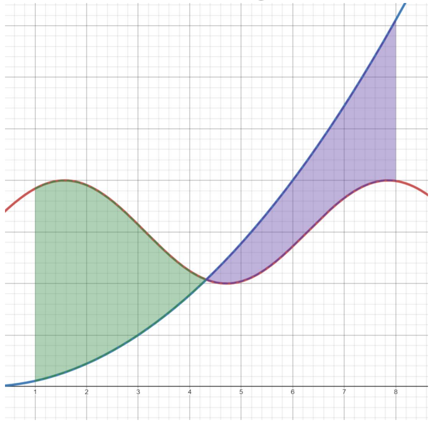
Finding the total area

- Remember that the area bounded by two functions f and g over an interval $x \in [a, b]$ is given by

$$\int_a^b f(x) - g(x)\, dx$$



- Note that, in this case $f(x) > g(x)$ over the whole interval. If f and g intersect, you need to set up a new integral with appropriate bounds (follow the same 3 steps):



1. Intersection occurs at $x = c$

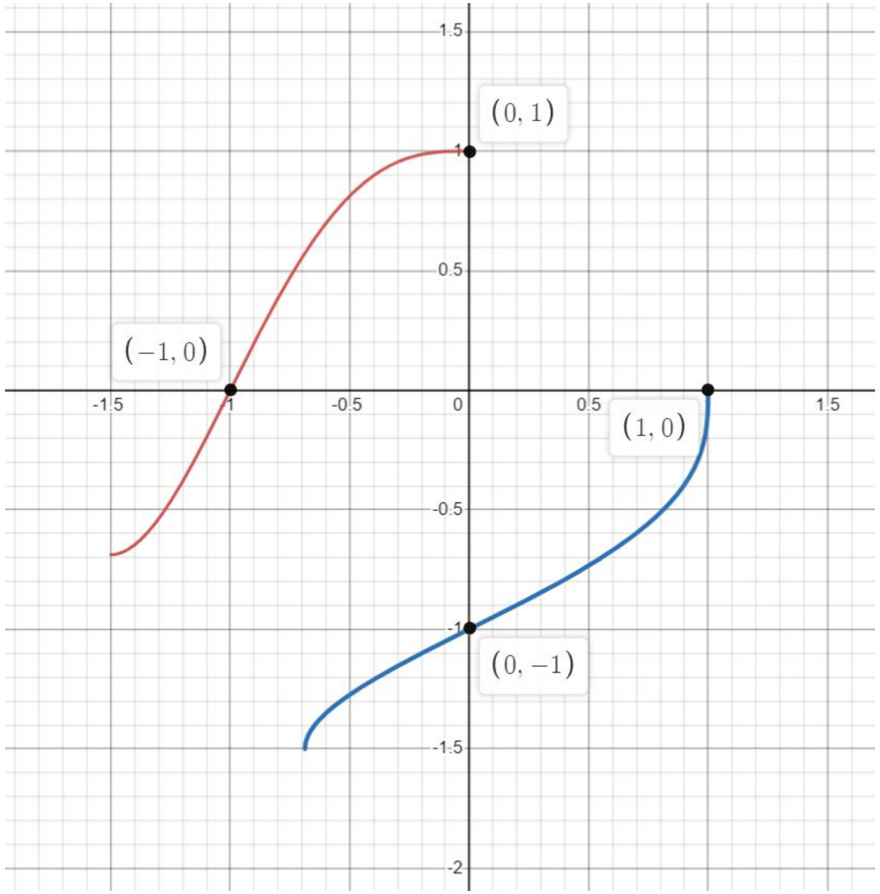
2. $area = \int_a^c f(x) - g(x)\, dx + \int_c^b g(x) - f(x)\, dx$

Area between curves

- Questions where you have to find the bounded area are extremely common in both exam 1 and 2 and are worth many marks!
- Being able to visualise the problem graphically is often what differentiates the high-scoring students
  - We will go through three examples – but there are many different ways this is tested, and you will have to build up your experience with it through practice exams

Area between curves

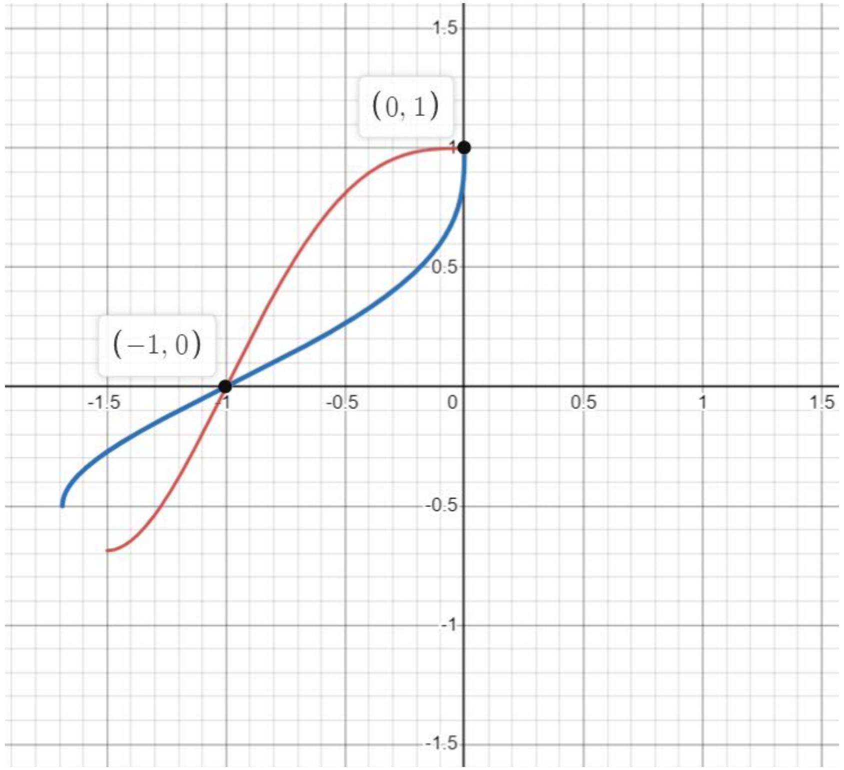
The graph of $g: \left[-\frac{3}{2}, 0\right] \to \mathbb{R}, g(x) = (x+1)(x^3 + x^2 - x + 1)$ and its inverse g^{-1} is given below.



The graph of g^{-1} is translated left one unit and up one unit. Find the area enclosed by g and the transformed graph of g^{-1}.

Area between curves

The graph of $g: \left[-\frac{3}{2}, 0\right] \to \mathbb{R}, g(x) = (x+1)(x^3 + x^2 - x + 1)$ and its inverse g^{-1} is given below.



The graph of g^{-1} is translated left one unit and up one unit. Find the area enclosed by g and the transformed graph of g^{-1}.

The bounded area is symmetric about the line $y = x + 1$.

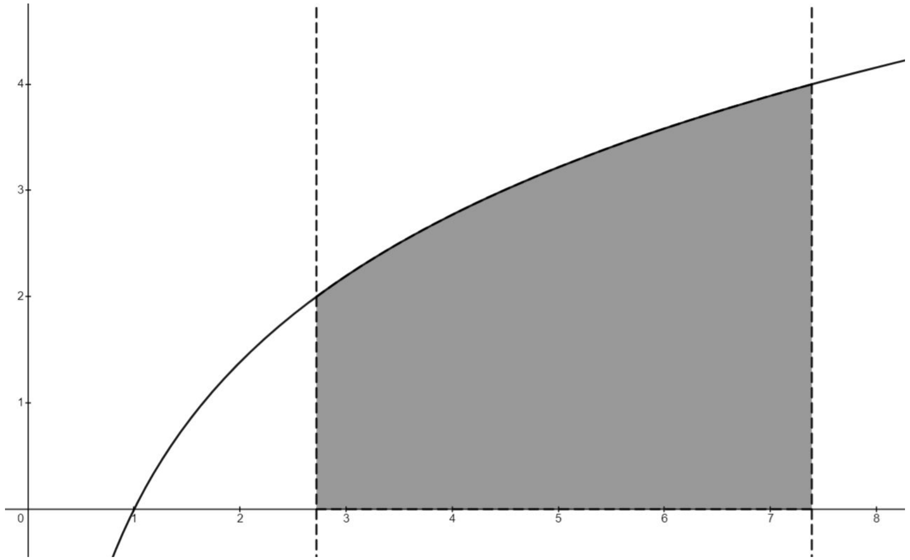
$$\therefore area = 2\int_{-1}^{0} g(x) - (x+1)\,dx = \frac{2}{5}\ units^2$$

CAS:

Menu > 3 > 1, "$g(x) = (x + 1 \ldots + 1, x)$"

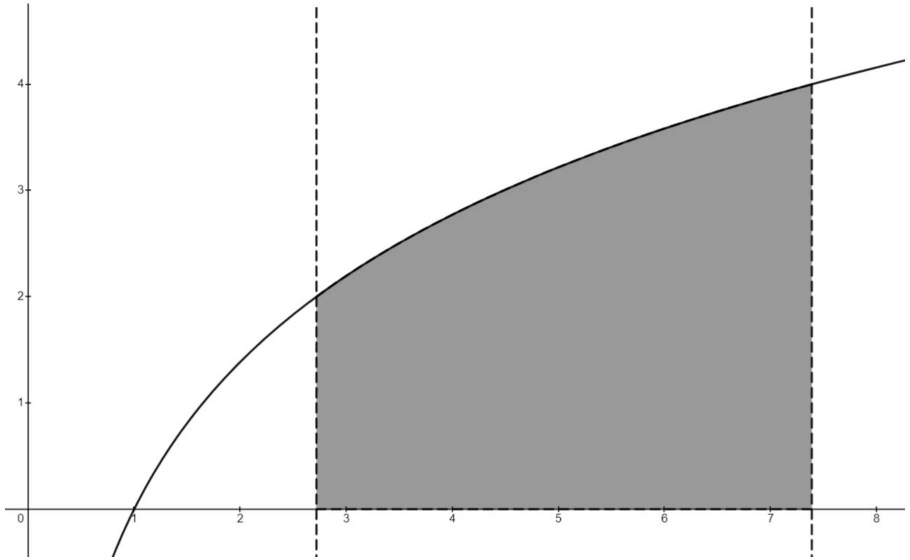
Menu > 4 > 3, "$2\int_{-1}^{0} g(x) - (x+1)\,dx$"

The area bounded by the x-axis, the graph of $y = 2\ln(x)$, and the lines $x = e$ and $x = e^2$ is shown.



Find the area of the shaded region.

The area bounded by the x-axis, the graph of $y = 2\ln(x)$, and the lines $x = e$ and $x = e^2$ is shown.



Find the area of the shaded region.

The inverse of $y = 2\ln(x)$ is $y = e^{\frac{x}{2}}$.

$$dotted\ area = 2(e^2 - e) = 2e^2 - 2e$$

$$striped\ area = \int_2^4 e^2 - e^{\frac{x}{2}}\,dx = 2e$$

$$\therefore\ total\ area = 2e^2 - 2e + 2e = 2e^2$$

CAS:

Menu > 4 > 3, "$\int_e^{e^2} 2\ln(x)\,dx$"

Calculus

Find the area enclosed between the curve $2\sin\left(\frac{\pi x}{8}\right)$, the x axis, the y axis, and the line $x = 40$.

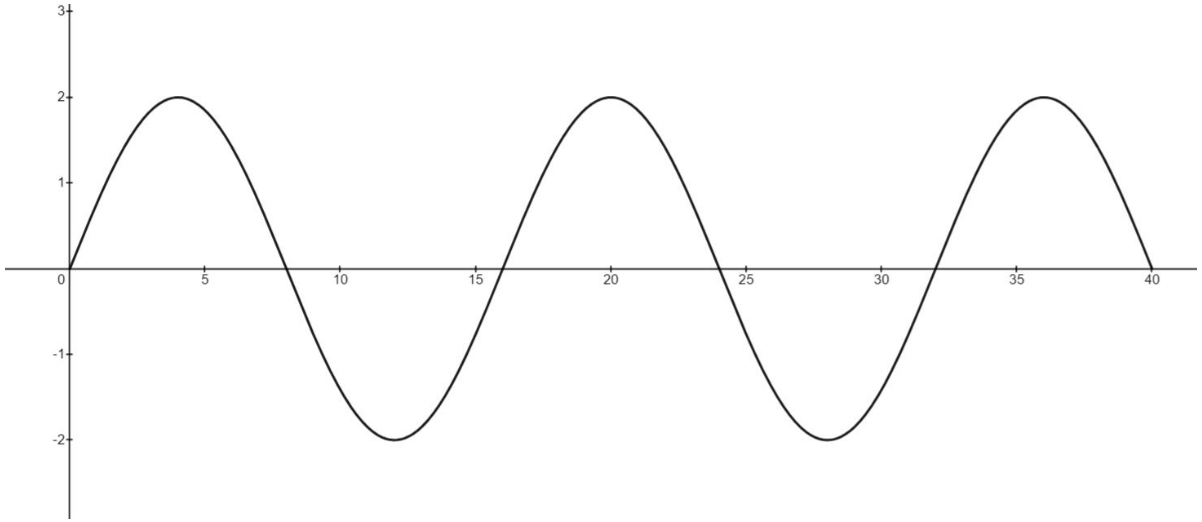
$$\Rightarrow x = 0, 8, 16, 24, 32, 40$$

$$area = \int_0^8 2\sin\left(\frac{\pi x}{8}\right)dx - \int_8^{16} 2\sin\left(\frac{\pi x}{8}\right)dx + \int_{16}^{24} 2\sin\left(\frac{\pi x}{8}\right)dx \ldots ?$$

Calculus

Find the area enclosed between the curve $2\sin\left(\frac{\pi x}{8}\right)$, the x axis, the y axis, and the line $x = 40$.

$$\Rightarrow x = 0, 8, 16, 24, 32, 40$$



$$area = 5\int_0^8 2\sin\left(\frac{\pi x}{8}\right)dx$$

160

Definite Integrals

$$\int_a^a f(x)dx = 0$$

$$\int_a^b af(x)dx = a\int_a^b f(x)dx$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

Manipulations using integration rules pop up on every exam 2 in the MC section

Key Take Aways

- Differentiation and integration go hand-in-hand – they are crucial to understanding the ins and outs of how a function behaves
- Understanding these processes from a graphical perspective will allow you to answer the tougher differentiator questions

- The Binominal Distribution
  - Understanding the properties of Bernoulli sequences and the relationship with the binomial distribution
  - Calculating probabilities, measure of centre and measures of spread
  - Utilising technology
- The Normal Distribution
  - Understanding the underlying properties of the normal distribution

- When a task is performed, a range of possible outcomes can result.
  - Consider rolling a dice – the outcomes are the numbers 1 through 6
- A group of one or more outcomes is an event, and can be represented as a capital letter, e.g. A
  - Rolling a 1, or rolling an even number

- Probability is the chance of an event happening, given by the ratio of outcomes in that event to the total number of possible outcomes

$$\mathrm{Pr(A)} = \frac{\mathrm{n(A)}}{n(\epsilon)}$$

- These are called simple probabilities – often, the question stem will already give you the probability of an event and ask you to use it to find more complex probabilities
  - This is where probability rules and knowledge of distributions is important

- Two events A and B are conditional if the occurrence of one event impacts the chances of the other occurring:

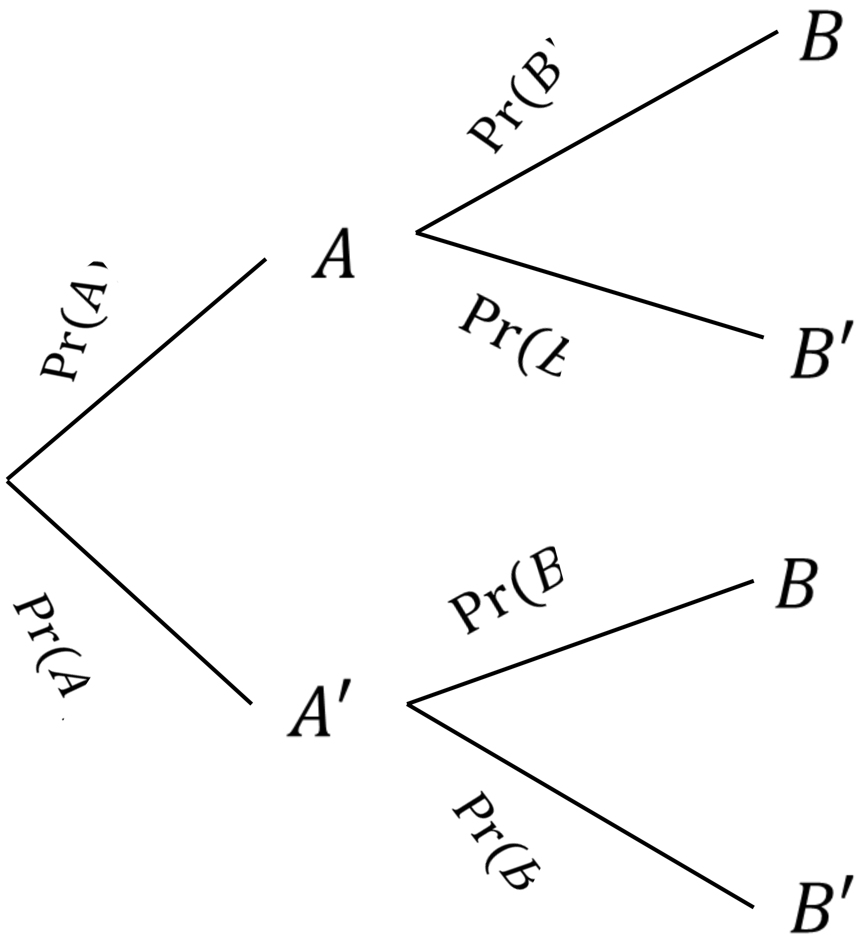
$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(\mathrm{B})}$$

  - Order of events matters!

- The events are independent if the occurrence of one event has no bearing on the chances of the other occurring:

$$\Pr(A|B) = \Pr(A)$$

- What if order matters? Draw a tree diagram



$$\Pr(A \cap B) = \Pr(A)\Pr(B|A)$$

$$\Pr(B) = \Pr(A \cap B) + \Pr(A' \cap B)$$

- What if order doesn't matter? Use independent event rules:
  - Note that you can still draw a tree diagram if it helps

- Here are some additional tools to help you with exploring probabilities (whether order matters or not)

Addition rule: $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

Conditional probability rule: $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

Probability table:

The probability that it rains today and not tomorrow is p. The probability that it rains tomorrow is p^3 and the probability that it rains neither today nor tomorrow is p^2. Whether it rains today is independent of whether it rains tomorrow. Find the probability that it rains today and tomorrow.

The probability that it rains today and not tomorrow is p. The probability that it rains tomorrow is p^3 and the probability that it rains neither today nor tomorrow is p^2. Whether it rains today is independent of whether it rains tomorrow. Find the probability that it rains today and tomorrow.

$$let\ A = rains\ today$$
$$B = rains\ tomorrow$$

$$then\ \Pr(A \cap B') = p$$

The probability that it rains today and not tomorrow is p. The probability that it rains tomorrow is p^3 and the probability that it rains neither today nor tomorrow is p^2. Whether it rains today is independent of whether it rains tomorrow. Find the probability that it rains today and tomorrow.

$$\begin{aligned} &let\ A = rains\ today \\ &\quad B = rains\ tomorrow \end{aligned}$$

$$then\ \Pr(A \cap B') = p$$

$$\begin{aligned} &p^3 + p^2 + p = 1 \Rightarrow p = 0.5437 \\ &\Pr(A \cap B) = \Pr(A)\Pr(B) \\ &a = (a + p)p^3, \qquad where\ p = 0.5437 \\ &a = \Pr(A \cap B) = 0.10 \end{aligned}$$

CAS:

Menu > 3 > 1, "… $p^3 + p^2 + p = 1, p)$"
Menu > 3 > 1, "… $a = (a + p)p^3, a)|p = 0.5437$

- For a random variable X, we represent each outcome as a value
- A probability distribution is a "list" of all possible values for X and the probability X takes that value
  - Each possible value is denoted x
- Properties are:

$$0 \leq \Pr(X = x) \leq 1$$

and

$$\sum \Pr(X = x) = 1$$

- $E(X) = \sum x\Pr(X = x), Var(X) = E(X^2) - \left(E(X)\right)^2$

Binomial Distribution

- What if we repeat a task associated with an event multiple times? A binomial distribution results.
- A distribution is said to be binomial if it meets the following criteria:
  - Each trial has only two outcomes, success or failure
  - Each trial has probability of success p
  - Trials are independent; success of a trial has no bearing on the success of any other trials (i.e. order doesn't matter)
  - Watch out for these features when choosing to use the binomial distribution in your answer

- A binomial random variable X describes the number of successes x out of n trials, where a success has a probability p:
  - Only use this formula in exam 1 – you have your CAS to help in exam 2

$$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$\binom{n}{x} = {}^nC_x = \frac{n!}{x!\,(n-x)!}$$

- $E(X) = np, Var(X) = np(1-p), sd(X) = \sqrt{Var(X)}$
- Questions relating to binomial variables often follow similar formats

- Define your random variable before using it in calculations

Each year, a detailed study is conducted on a random sample of 36 Lorenz birdwing butterflies in Town A. A Lorenz birdwing butterfly is considered to be **very large** if its wingspan is greater than 17.5 cm. The probability that the wingspan of any Lorenz birdwing butterfly in Town A is greater than 17.5 cm is 0.0527, correct to four decimal places.

f. **i.** Find the probability that three or more of the butterflies, in a random sample of 36 Lorenz birdwing butterflies from Town A, are **very large**, correct to four decimal places.

Let X = number of very large butterflies in the sample

Then $X \sim Bi(36, 0.0527)$

CAS:

Menu > 5 > 5 > B

$n = 36, p = 0.0527,$

$lower = 3, upper = 36$

ii. The probability that n or more butterflies, in a random sample of 36 Lorenz birdwing butterflies from Town A, are **very large** is less than 1%.

Find the smallest value of n, where n is an integer.

$$\Pr(X \geq n) < 0.01$$
$$\therefore n = 7$$

CAS:

Menu > 1 > 1, "... $b(n) =$", Menu > 5 > 5 > 3

$n = 36, p = 0.0527,$

$lower = n, upper = 36$

Adapted from VCAA 2019 exam 2

- What if a random variable X can take any value? Then we have a continuous random variable.
- The function modelling probabilities associated with X is a probability density function p:

$$\Pr(a < X < b) = \int_a^b p(x)\,dx$$

- p is usually a piece-wise function, where it is clear that the domain is a finite interval:

$$p(x) = \begin{cases} 3 - x^2, & -1 \leq x \leq 3 \\ 0, & elsewhere \end{cases}$$

- Questions relating to continuous random variables will require you to exploit its properties:

$$\int_c^d p(x)\,dx = 1, \qquad where\ dom\ p \in [c,d]$$

$$p(x) \geq 0, \qquad for\ x \in \mathbb{R}$$

$$E(X) = \int_c^d xp(x)\,dx$$

$$Var(X) = \int_c^d x^2 p(x)\,dx - \left(\int_c^d xp(x)\,dx\right)^2$$

$$sd(X) = \sqrt{Var(X)}$$

The spin, measured in revolutions per second, is a continuous random variable X with the probability density function

$$f(x) = \begin{cases} \dfrac{x}{500} & 0 \le x < 20 \\ \dfrac{50 - x}{750} & 20 \le x \le 50 \\ 0 & \text{elsewhere} \end{cases}$$

e. Find the maximum possible spin in revolutions per second.

Adapted from VCAA 2021 exam 2

The spin, measured in revolutions per second, is a continuous random variable X with the probability density function

$$f(x) = \begin{cases} \dfrac{x}{500} & 0 \le x < 20 \\ \dfrac{50 - x}{750} & 20 \le x \le 50 \\ 0 & \text{elsewhere} \end{cases}$$

e. Find the maximum possible spin in revolutions per second.

- 50

Adapted from VCAA 2021 exam 2

The spin, measured in revolutions per second, is a continuous random variable X with the probability density function

$$f(x)=\begin{cases}\dfrac{x}{500} & 0\le x<20\\[2ex] \dfrac{50-x}{750} & 20\le x\le 50\\[2ex] 0 & \text{elsewhere}\end{cases}$$

Find the standard deviation of the spin, in revolutions per second, correct to one decimal place.

Adapted from VCAA 2021 exam 2

The spin, measured in revolutions per second, is a continuous random variable X with the probability density function

$$f(x) = \begin{cases} \dfrac{x}{500} & 0 \le x < 20 \\ \dfrac{50-x}{750} & 20 \le x \le 50 \\ 0 & \text{elsewhere} \end{cases}$$

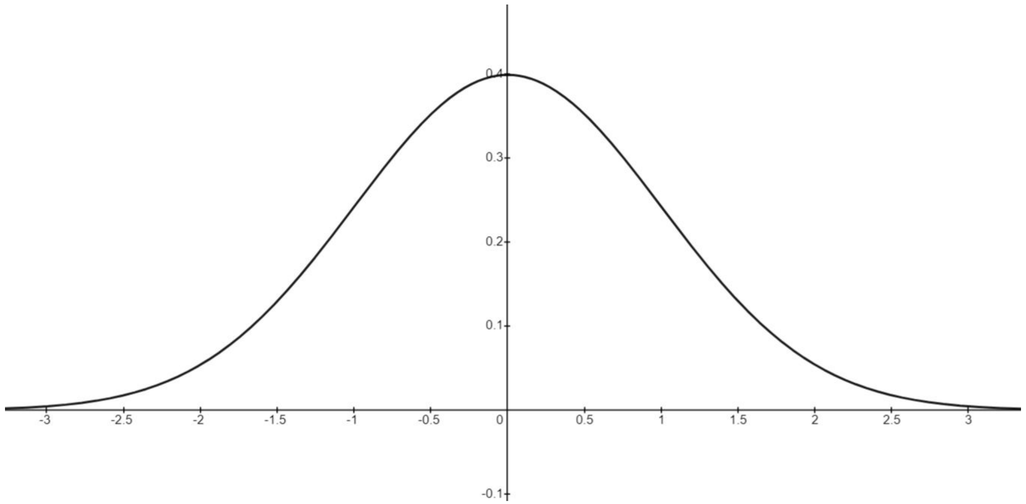
Find the standard deviation of the spin, in revolutions per second, correct to one decimal place.

- $$\sigma = \sqrt{\int_0^{50} x^2 f(x)\,dx - \left(\int_0^{50} x f(x)\,dx\right)^2}$$
  $= 10.7$
  - Note that we use σ instead of $sd(X)$ as we haven't defined the random variable X

Adapted from VCAA 2021 exam 2

Introducing the normal distribution

- The standard normal distribution is a specific type of continuous probability density shown below

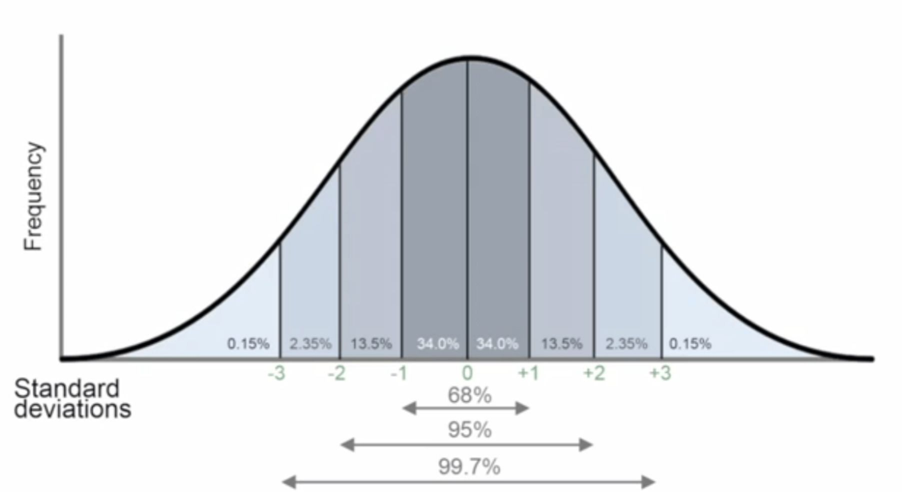


- It has the special property that $\mu = 0$, $\sigma = 1$

Standardisation properties

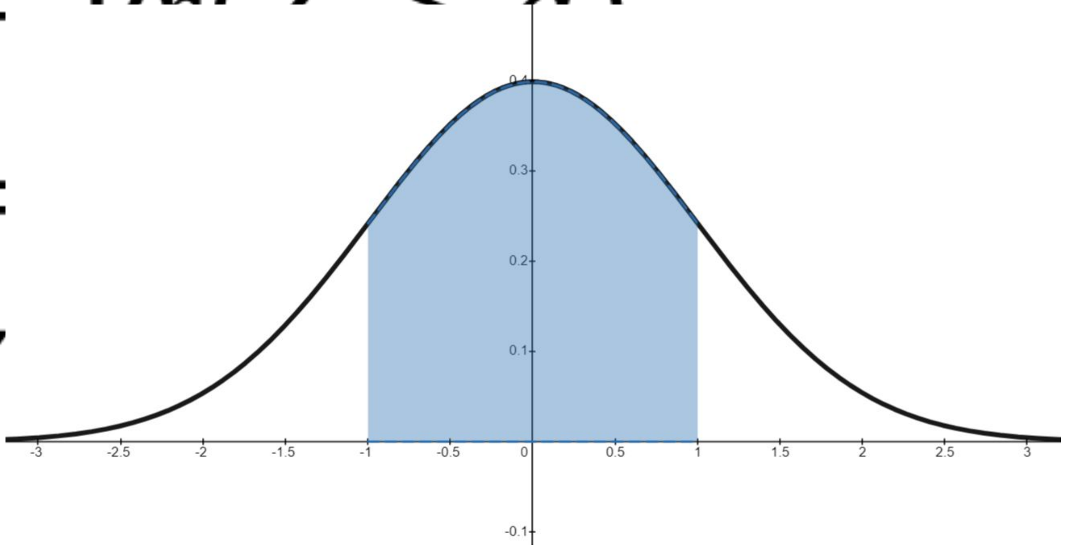
- We can compare two normal distributions with different means and standard deviations by standardising them – transforming them back to the standard normal distribution:
  - The z-score tells us the distance of x in standard deviations from the mean

$$z = \frac{x - \mu}{\sigma}$$



Symmetry properties

- $\Pr(Z > a) = 1 - \Pr(Z < a)$
- $\Pr(Z < -a) = \Pr(Z > a)$
- $\Pr(Z > -a) = 1 - \Pr(Z > a)$
- $\Pr(-a < Z < a) =$
- $\Pr(Z < a) = \Pr(Z$



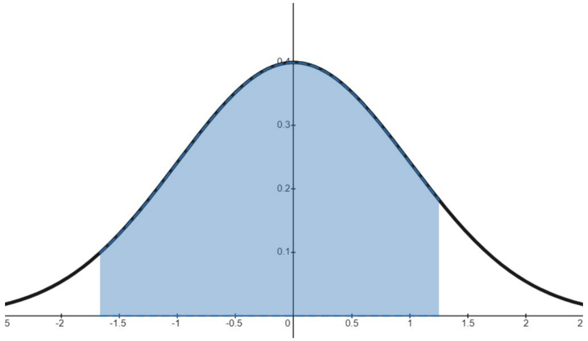
Using standardisation and symmetry properties

- Let X be a normally distributed variable where $\mu = 1.5$ and $\sigma = 0.4$. It is known that $\Pr\left(Z > \frac{5}{3}\right) = 0.048$ and $\Pr\left(Z > \frac{5}{4}\right) = 0.106$. Find k such that $\Pr(k < X < 2) = 0.846$.

$$\Pr(k < X < 2) = 0.846$$

$$\Pr\left(\frac{k-1.5}{0.4} < Z < \frac{2-1.5}{0.4}\right) = 0.846$$

$$\Pr\left(Z < \frac{k-1.5}{0.4}\right) = 0.048$$



Finding normal variable probabilities

The battery life of a certain brand of laptops is normally distributed, with a mean of 3 hours and 10 minutes and a standard deviation of 6 minutes.

Find the probability that an initially fully-charged laptop dies after 3 hours of use.

Finding normal variable probabilities

The battery life of a certain brand of laptops is normally distributed, with a mean of 3 hours and 10 minutes and a standard deviation of 6 minutes.

Find the probability that an initially fully-charged laptop dies after 3 hours of use.

$$Let\ X_1 = battery\ life\ of\ a\ laptop$$
$$Then\ X_1 \sim N\left(\frac{19}{6}, \frac{1}{100}\right)$$
$$\Pr(X_1 < 3) = 0.0478$$

CAS:

Menu > 5 > 5 > 2

$lower = -\infty, upper = 3$

$\mu = \frac{19}{6}, \sigma = \frac{1}{10}$

- Note that we state the variance ($\frac{1}{100}$ in this case) when defining the random variable, as opposed to stating the standard deviation ($\frac{1}{10}$ in this case)

Finding normal variable probabilities

For a different brand of laptops, whose battery life is also normally distributed, it is known that 12% work for more than 3 hours and 10 minutes, whereas 10% die before 2 hours 50 minutes of use.
Find the mean and standard deviation of battery life for this brand of laptop.

$$Let\ X_2 = battery\ life\ of\ a\ laptop$$

$$Then\ 0.12 = \Pr\left(X_2 > \frac{19}{6}\right) \text{ and } 0.1 = \Pr\left(X_2 < \frac{17}{6}\right)$$

$$\therefore 0.12 = \Pr\left(Z > \frac{\frac{19}{6} - \mu}{\sigma}\right)\ and\ 0.1 = \Pr\left(Z < \frac{\frac{17}{6} - \mu}{\sigma}\right)$$

$$\Rightarrow \frac{\frac{19}{6} - \mu}{\sigma} = 1.17498\ and\ \frac{\frac{17}{6} - \mu}{\sigma} = -1.28155$$

$$\Rightarrow \mu = 3.0072\ hours\ and\ \sigma = 0.1357\ hours$$

CAS:

Menu > 5 > 5 > 3

$area = 0.12, \mu = 0, \sigma = 1$

Finding normal variable probabilities

For a different brand of laptops, whose battery life is also normally distributed, it is known that 12% work for more than 3 hours and 10 minutes, whereas 10% die before 2 hours 50 minutes of use.
Find the mean and standard deviation of battery life for this brand of laptop.

- The inverse normal function is very limited – instead, you can use definite integrals to solve the problem:

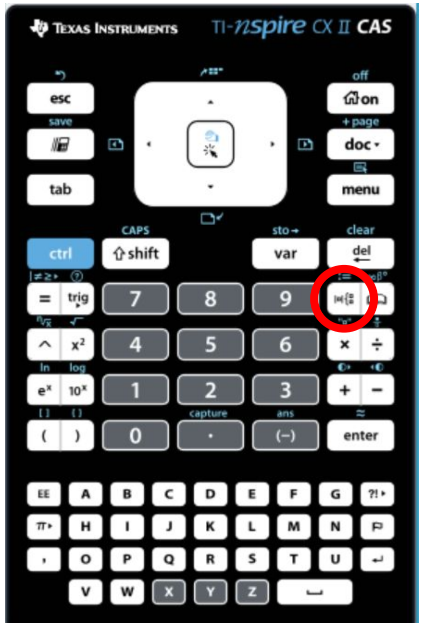
CAS:

Menu > 3 > 7 > 1, variables a, b

Key takeaways

- For probability questions involving two events A and B, be very familiar with the different tools and formulas available to explore probabilities
- Memorise all expected value, variance, and std. dev. formulas (even though they’re on the formula sheet!) and apply them wherever you see the chance
- Visualise the normal distribution to ensure you use symmetry properties correctly

- Ctrl > Doc > 1 – opens calculator; Ctrl > Doc > 2 – opens graphing
- Always define functions at the beginning of a question using menu > 1 > 1
- To access the derivative, integral and piecewise functions, click this button:



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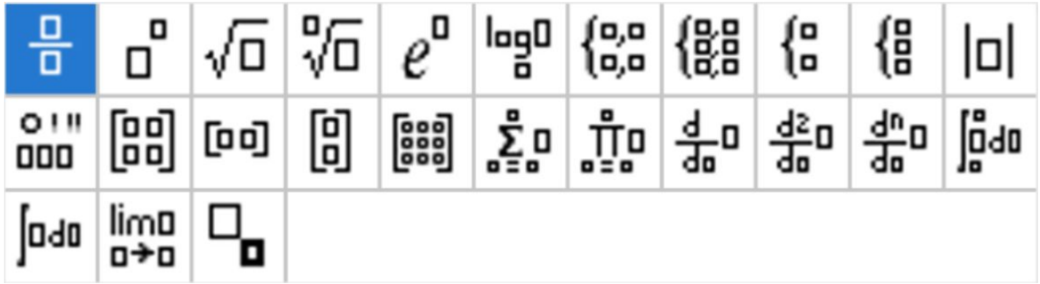
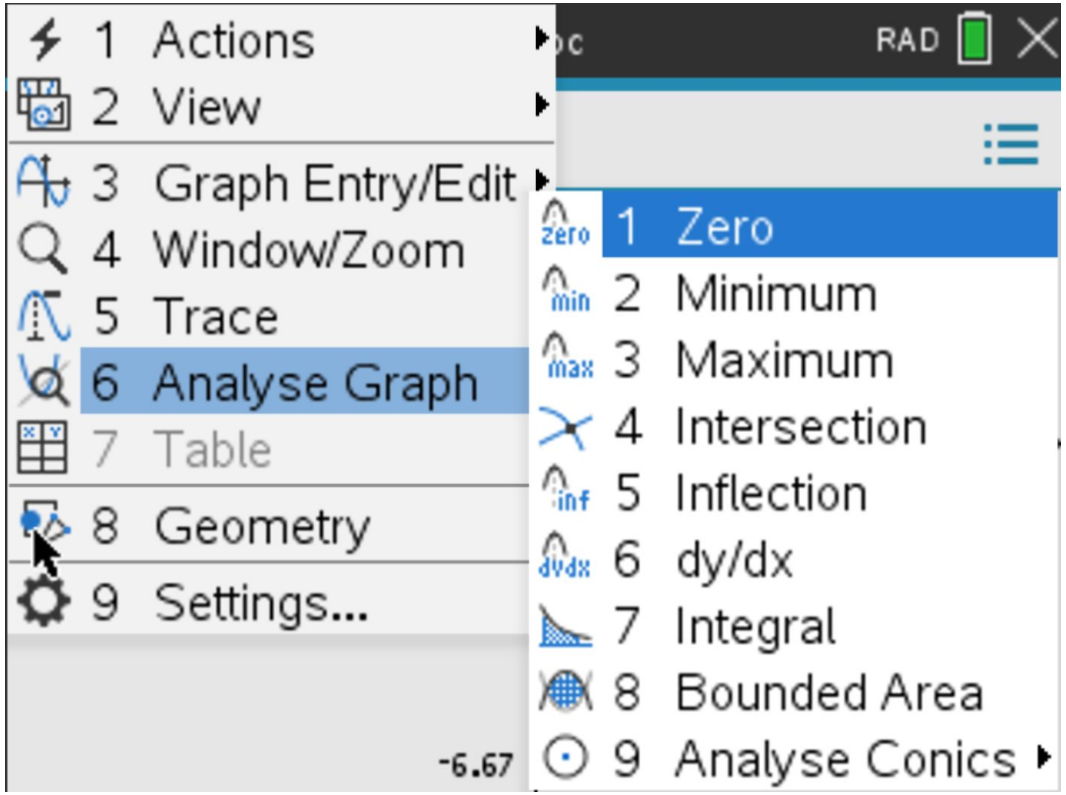
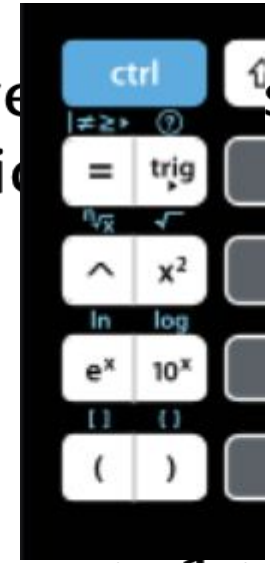


- When sketching graphs, remember menu > 6 to analyse maxima, minima, intersection points and intercepts



Calculator tips

- Buttons on the left hand side of your keypad give s to trig functions, e^x and $\ln(x)$, x^2 and $\sqrt{x}$ functi



- Your most used functions will be the define (menu > 1 > 1) and solve (menu > 3 > 1) functions, followed by the probability functions (menu > 5 > 5)
    - Remember menu > 3 > 7 > 1 for solving systems of equations!
- Menu > 3 gives access to a whole bunch of arithmetic operations like factoring and expanding – won’t need

Final tips for the exam

- Use reading time ***effectively***
  - Spend some time familiarising yourself with each scenario for the extended-response questions
  - For exam 1, go on to spend the rest of your reading time memorising exactly how you'll solve the first few questions

- Carefully read the long-worded questions to understand exactly what you are being asked (especially for probability)

Final tips for the exam

- Leave questions which confused you in reading time until the end

- Don’t rely on your notes! The highest-scoring students don’t look through their bound reference even once.

Please leave us a review

SCAN

Thanks so much!

ATARNotes