Chapter 1 Reviewing linear equations: **Assignment**

Name _____

1 Solve each of the following equations for x:

a $2 - 3x = 4$

b $\dfrac{2x - 3}{6} = 5$

c $2(3 - 2x) = 5x - 4$

d $\dfrac{5x - 2}{6} + \dfrac{2 - 4x}{5} = 1$

2 Solve the simultaneous equations:

$2x + 0.4y = 8$

$5x - 1.2y = 9$

3 Solve the inequality $5x - 4 > 21 - 5x$.

4 A man was 32 years old when his daughter was born. He is now five times as old as his daughter. How old is his daughter now?

5 I think of a pair of numbers. If I add 11 to the first, I obtain a number that is twice the second. If I add 20 to the second, I obtain a number which is twice the first. What are the numbers?

6 Make P the subject of the formula $I = \dfrac{PRT}{100}$.

7 Make x the subject of the formula $y = \dfrac{2 - x}{3 + 2x}$.

8 If $v = u + at$, $v = 12$, $u = 2$ and $a = 4$, find the value of t.

9 The perimeter of a square is not more than 80 cm. What is the largest possible area of the square?

10 Solve the inequality $1 + \dfrac{1 - 2x}{3} > 10$.

11 Solve each of the following literal equations for x:

a $a(x+b) = \dfrac{x+a}{a}$

b $\dfrac{m}{x} + \dfrac{n}{x} = 1$

c $m(x+n) = n(x+m) + m$

12 The sum of two numbers is 100 and their difference is 200. What are the two numbers?

13 Make m the subject of the formula $\dfrac{2}{m} - \dfrac{3}{n} = \dfrac{1}{p}$.

Answers

1 **a** $x = -\dfrac{2}{3}$ **b** $x = \dfrac{33}{2}$ **c** $x = \dfrac{10}{9}$ **d** $x = 28$

2 $x = 3, y = 5$

3 $x > 2.5$

4 The daughter is 8 years old.

5 The first number is 17 and the second number is 14.

6 $P = \dfrac{100I}{RT}$

7 $x = \dfrac{2-3y}{1+2y}$

8 $t = 2.5$

9 The largest possible area of the square is 400 cm^2.

10 $x < -13$

11 **a** $x = \dfrac{a - a^2b}{a^2 - 1}$

b $x = m + n$

c $x = \dfrac{m}{m-n}$

12 150 and -50

13 $m = \dfrac{2np}{3p+n}$

Chapter 2 Coordinate geometry and linear relations: **Assignment**

Name _____

1 Find the equation of the line with gradient 3 and which passes through the point with coordinates (1, 4).

2 For the line with equation $3x + 6y = 12$, find:

a the gradient of the line

b the x -axis intercept of the line

c the y -axis intercept of the line.

3 Find the equation of the line that is perpendicular to the line with equation $y = -2x + 6$ and passes through the point with coordinates (1, 6).

4 If 8 kilograms of potatoes and 5 kilograms of carrots cost \$28, and 2 kilograms of potatoes and 3 kilograms of carrots cost \$11.20, what is the cost of 1 kilogram of each item?

5 **a** Find the midpoint of the line segment joining the points with coordinates (3, 5) and(−2, 8).

b The point with coordinates (4, −6) is the midpoint of the line segment AB. The coordinates of the endpoints are (1, a) and (b, −4). Find the values of a and b.

c Find the distance between the points (1, −4) and (11, 8).

6 The cost, \$$C$, of electricity is determined by the number, n, of units used. The rule for determining the cost is of the form $C = pn + q$. It is known that the cost of 200 units of electricity is \$200 and of 500 units \$380. Find the values of p and q.

7 The points A, B and C have coordinates $A\,(0,\ 7)$, $B\,(6,\ -1)$ and $C\,(6,\ 9)$.

a Find the length of line segmentAC.

b Calculate the gradient of AC.

c Find the equation of line AC.

d $ACPB$ is a quadrilateral with BC its axis of symmetry. Find the coordinates of P.

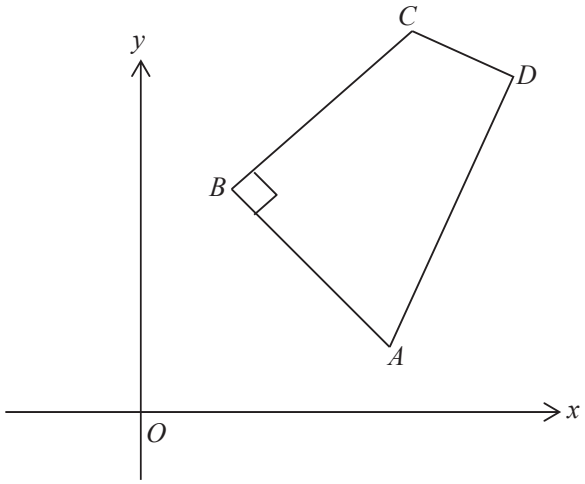
e Find the area of quadrilateral $ACPB$.

8 Find the magnitude of the acute angle between the lines with equations $y = 2x + 3$ and $y = -\frac{1}{3}x + 3$

9 Points A and B have coordinates $(7,\ 0)$ and $(0,\ 9)$. Find the midpoint of the line segment AB and the equation of the perpendicular bisector of AB.

10 $ABCD$ is a quadrilateral with angle ABC a right angle. D lies on the perpendicular bisector of AB. The coordinates of A and B are $(7,\ 2)$ and $(2,\ 5)$ respectively.

The equation of line AD is $y = 4x - 26$.



a Find the equation of the perpendicular bisector of the line segment AB.

b Find the coordinates of point D.

c Find the gradient of the line BC.

d Find the value of the second coordinate, c, of point $C\,(8,\, c)$.

e Find the area of quadrilateral $ABCD$.

Answers

1 $y = 3x + 1$

2 **a** Gradient: $m = -\frac{1}{2}$

b x-axis intercept: $x = 4$

c y-axis intercept: $y = 2$

3 $y = \frac{1}{2}x + \frac{11}{2}$

4 1 kilogram of potatoes costs \$2 and 1 kilogram of carrots costs \$2.40.

5 **a** The midpoint of line segment is $\left(\frac{1}{2}, \frac{13}{2}\right)$.

b $a = -8$, $b = 7$

c Distance between points is $2\sqrt{61}$.

6 $p = 0.6$ and $q = 80$

7 **a** Length of line segment AC is $2\sqrt{10}$.

b $m = \frac{1}{3}$

c $y = \frac{1}{3}x + 7$

d $P\,(12,\ 7)$

e Area $= 60$ units^2

8 Magnitude of acute angle between lines is $(\tan^{-1} 7)^\circ$.

9 Midpoint of $AB = \left(\frac{7}{2}, \frac{9}{2}\right)$; perpendicular bisector of AB is $y = \frac{7}{9}x + \frac{16}{9}$

10 **a** Equation of perpendicular bisector of AB is $y = \frac{5}{3}x - 4$.

b $D\left(\frac{66}{7}, \frac{82}{7}\right)$

c $m = \frac{5}{3}$

d $c = 15$

e Area $\approx 44.9\,\text{units}^2$

Chapter 3 Quadratics: **Assignment**

Name _____

1 Sketch the graphs of each of the following:

a $y = -x^2 + 8$

b $y = (x - 3)^2 - 5$

c $y = 5 - (x + 3)^2$

d $y = x^2 - x - 8$

2 Use the quadratic formula to solve each of the following:

a $x^2 - 6x - 2 = 0$

b $2x^2 - 3x - 7 = 0$

3 A rectangle has a perimeter of 80 m and the square of the length of the diagonal is 1000. Find its dimensions.

4 A parabola that has its vertex at the point with coordinates $(-1, 6)$ passes through the point $(2, 10)$. Find the equation of the parabola.

5 Solve the simultaneous equations for x and y:

$y = x^2 + 7x - 11$

$y = x - 1$

6 A lawn a metres long and b metres wide has a path of uniform width x metres around it.

a Find the area of the path in terms of a, b and x.

b **i** If $a = 28$ and $b = 50$ find the area of the path in terms of x.

ii If the area of the path is $160\,\text{m}^2$ find the value of x.

7 Consider the quadratic equation $2px^2 + 6x + 2 = 0$.

a Find the discriminant.

b Find the values of p for which there are two solutions.

c Find the values of p for which there are no solutions.

d Find the value of p for which there is one solution.

8 Using the discriminant, show that the graph of $y = 2x^2 + 6px - 2$ touches or crosses the x-axis for all values of p.

9 Consider the quadratic equation $(-2p + 1)\,x^2 + (p - 2)\,x + 6p = 0$.

a Find the discriminant.

b Show that the discriminant is a perfect square.

c For $p \neq \frac{1}{2}$, show that there are always two rational solutions and find these solutions.

10 Consider the quadratic equation $ax^2 + 10x + (a - 5) = 0$.

a Find the discriminant.

b Find the values of a for which there are two solutions.

c Find the values of a for which there are no solutions.

d Find the value of a for which there is one solution.

11 Consider the quadratic rule $a^2x^2 - 2ax - a + 1$.

a Find the discriminant.

b Find the values of a for which the graph $y = a^2x^2 - 2ax - a + 1$:

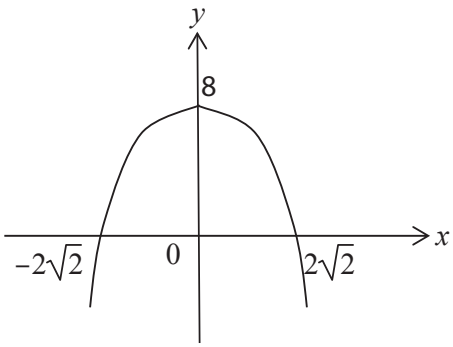
i crosses the x-axis

ii does not cross the x-axis.

c Show that $a^2x^2 - 2ax - a + 1 = (ax + \sqrt{a} \ - 1)(ax - \sqrt{a} - 1)$

Answers

1 **a**



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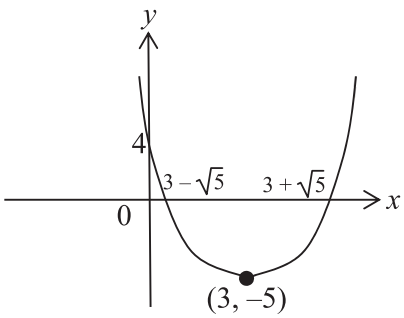

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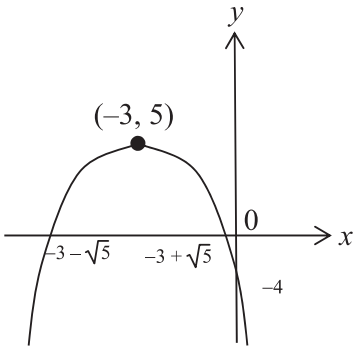

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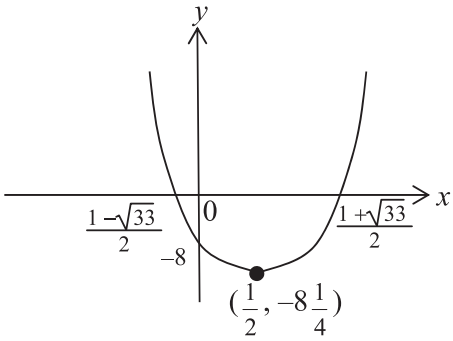


2 **a** $3-\sqrt{11}$ or $3+\sqrt{11}$

b $\dfrac{3+\sqrt{65}}{4}$ or $\dfrac{3-\sqrt{65}}{4}$

3 30 m by 10 m

4 $y = \frac{4}{9}(x+1)^2 + 6$

5 $\left(-3-\sqrt{19},\ -4-\sqrt{19}\right),\ \left(-3+\sqrt{19},\ -4+\sqrt{19}\right)$

6 **a** $A = 4x^2 + 2xb + 2xa$

b **i** $A = 4x^2 + 156x$

ii $x = 1$

7 **a** $36 - 16p$

b $p < \frac{9}{4}$

c $p > \frac{9}{4}$

d $p = \frac{9}{4}$

8 $36p^2 + 16 > 0$ for all p

9 **a** $49p^2 - 28p + 4$

b $(7p-2)^2$

c 2 and $\frac{3p}{1-2p}$

10 **a** $-4a^2 + 20a + 100$

b $\frac{5-5\sqrt{5}}{2} < a < \frac{5+5\sqrt{5}}{2}$

c $a > \frac{5+5\sqrt{5}}{2}$ or $a < \frac{5-5\sqrt{5}}{2}$

d $\frac{5+5\sqrt{5}}{2}$ or $\frac{5-5\sqrt{5}}{2}$

11 **a** $4a^3$

b **i** $a > 0$

ii $a < 0$

Chapter 4 Gallery of graphs: **Assignment**

Name _____

*Questions marked with an * involve circles and may be omitted*

1 Sketch the graphs of each of the following. Label axis intercepts and asymptotes:

a $y = \dfrac{2}{x}$

b $y = \dfrac{2}{x-1}$

c $y = \dfrac{2}{x} + 1$

d $y = \dfrac{2}{x+1} - 1$

2 Sketch the graphs of each of the following. Label axis intercepts and asymptotes:

a $y = \dfrac{2}{(x-1)^2}$

b $y = \dfrac{2}{(x+1)^2} - 1$

3 Sketch the graphs of each of the following:

a $y = -\sqrt{x+2}$

b $y = \sqrt{2x+1}$

c $y = \sqrt{x-2} - 2$

4 A rectangular hyperbola with rule of the form

$$y = \frac{a}{x-h} + k$$

has vertical asymptote $x = 3$, horizontal axis $y = -5$ and passes through the point $(0, 6)$. Find the values of a, h and k.

5 Show that the line $y = x + 8$ touches the hyperbola $y = -\dfrac{16}{x}$ and find the coordinates of this point.

6 Find the coordinates of the points of intersection of the parabola $(y-1)^2 = 4x$ and the line $y = x$.

7* State the coordinates of the centre and the length of the radius of the circle with the given equation:

a $(x-4)^2 + (y+2)^2 = 16$

b $x^2 + (y-2)^2 = 7$

c $x^2 + y^2 - 6x + 8y = 0$

8* Find the axis intercepts of the circle with equation $(x-2)^2 + (y+3)^2 = 45$.

9* Find the equation of the tangent to the circle with equation $x^2 + y^2 = 8$ at the point:

a $(2,\ 2)$

b $(-2,\ 2)$

c $(-2,\ -2)$

d $(2,\ -2)$

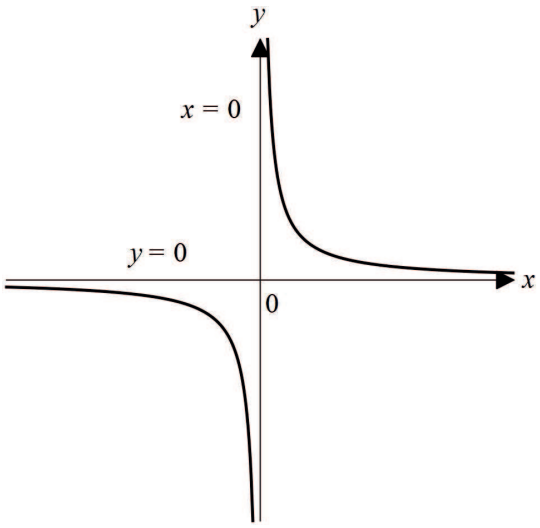
10* **a** Find the coordinates of the points of intersection of the line with equation $y = x$ and the circle $x^2 + y^2 = 1$.

b Find the y-coordinate of the points of intersection of the curve with equation $y = x^2$ and the circle $x^2 + y^2 = 1$.

c Find the x-coordinate of the point of intersection of the curve with equation $y = \sqrt{x}$ and the circle $x^2 + y^2 = 1$.

Answers

1 **a**



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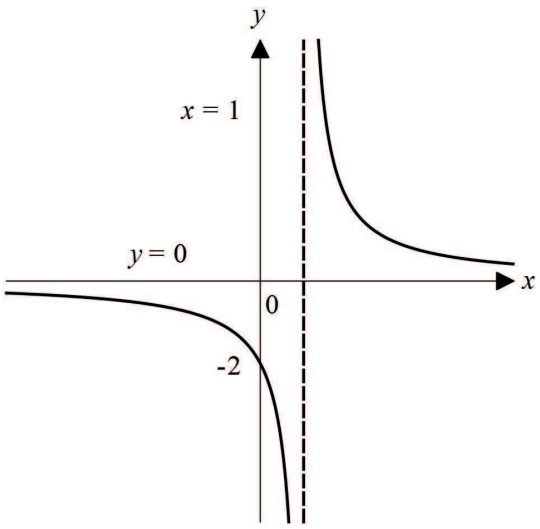

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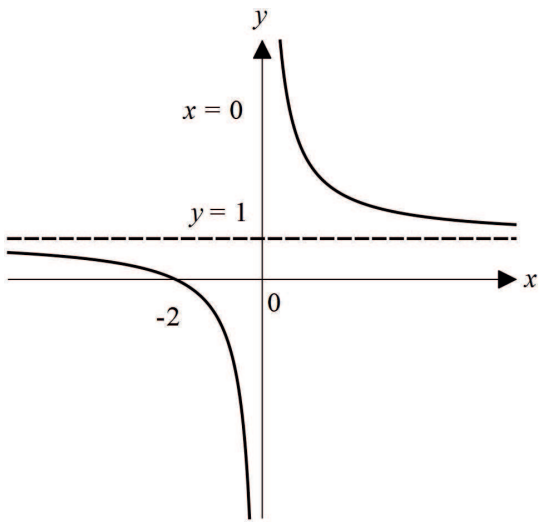

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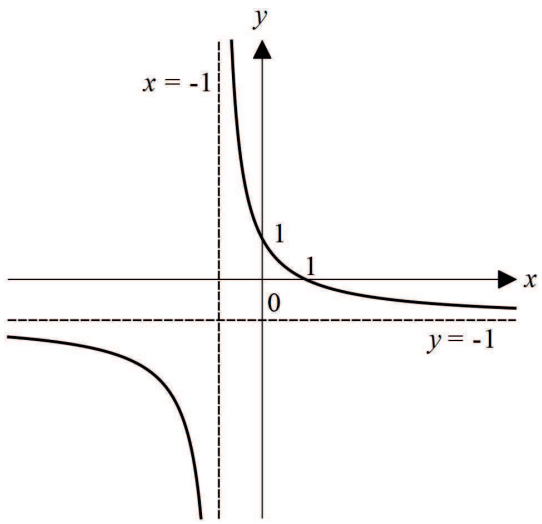
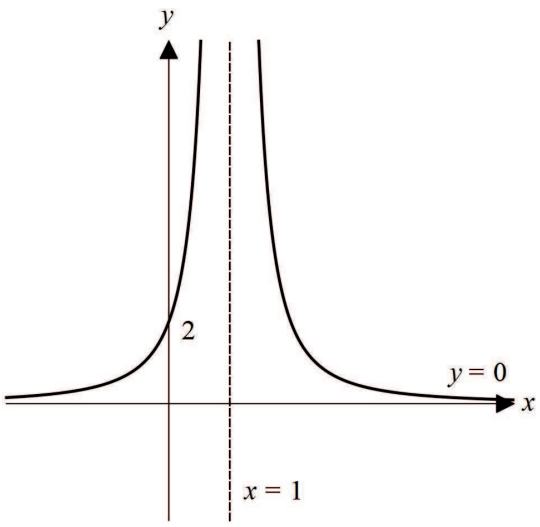


2 **a**



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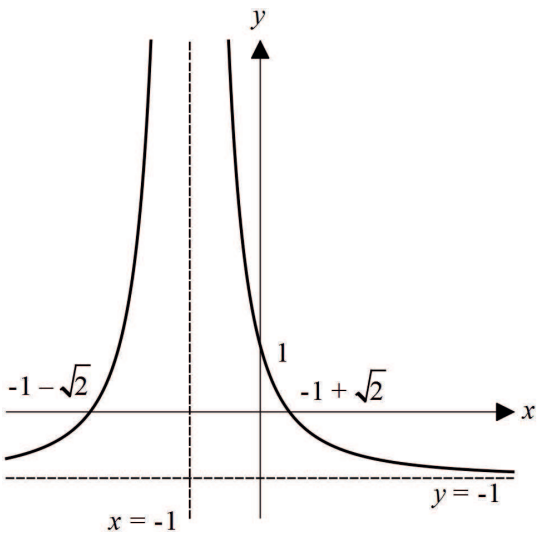
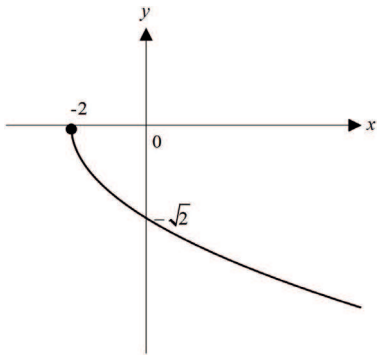


3 **a**



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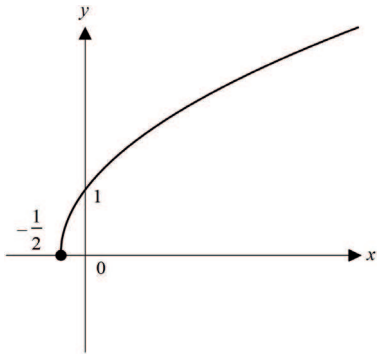

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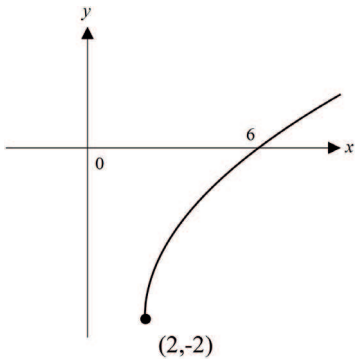


4 $h = 3, k = -5, a = -33$

5 $(-4, 4)$

6 $(3 - 2\sqrt{2}, 3 - 2\sqrt{2})$ and $(3 + 2\sqrt{2}, 3 + 2\sqrt{2})$

7* **a** $(4, -2)$, 4

b $(0, 2)$, $\sqrt{7}$

c $(3, -4)$, 5

8* $y = -3 \pm \sqrt{41}$, $x = 8$ or $x = -4$

9* **a** $y = -x + 4$

b $y = x + 4$

c $y = -x - 4$

d $y = x - 4$

10* **a** $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

b $y = \frac{\sqrt{5} - 1}{2}$

c $x = \frac{\sqrt{5} - 1}{2}$

Chapter 5 Functions and relations: **Assignment**

Name _____

1 For the function with rule $g(x) = 2 - x^2$ find:

a $g(2)$

b $g(-1)$

c $g(a)$

d $g(a+2)$

2 For the function with rule $f(x) = (x+2)^2$ find:

a $f(2)$

b $f(-1)$

c $f(a)$

d $f(a+2)$

3 **a** For $g: R \to R,\ g(x) = 4x - 3$, sketch the graph and state the range.

b For $g: [-2,\ 3] \to R,\ g(x) = 4x - 3$, sketch the graph and state the range.

c For $g: [-2,\ 3) \to R$, state the range.

d For $g: (-2,\ 3) \to R$, state the range.

4 **a** For $g: R \setminus \{0\} \to R, g(x) = \dfrac{2}{x} + 3$, sketch the graph and state the range.

b For $g: [1,\ 5] \to R,\ g(x) = \dfrac{2}{x} + 3$, sketch the graph and state the range.

c For $g: [1,\ 5) \to R$ state the range.

d For $g: (1,\ 5) \to R$ state the range.

5 Sketch the graph of each of the following functions and state its range:

a $f(x) = \begin{cases} 2x & \text{if} \quad x \geq 0 \\ -x+2 & \text{if} \quad x < 0 \end{cases}$

b $f(x) = \begin{cases} 2x+6 & \text{if} \quad x \geq 2 \\ -x+2 & \text{if} \quad x < 2 \end{cases}$

6 Find the inverse of each of the following functions. State the domain of the inverse function:

a $f : [-1,\ 4] \to R, \quad f(x) = 10 - 9x$

b $f : (-6,\ \infty) \to R, \quad f(x) = (x+7)^2$

c $f : [-1,\ \infty) \to R, \quad f(x) = (x+\ 1)^2 + 2$

7 Sketch the graph of each of the following relations and state its range:

a $(x+7)^2 + (y-7)^2 = 10$

b $y = x^2 + 2x - 3$

Answers

1 **a** -2

b 1

c $2 - a^2$

d $2 - (a+2)^2$

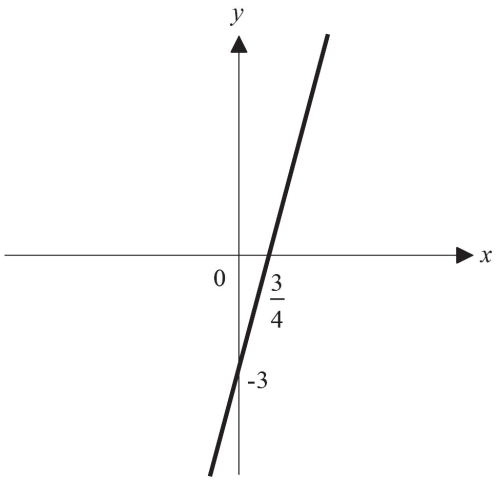
2 **a** 16

b 1

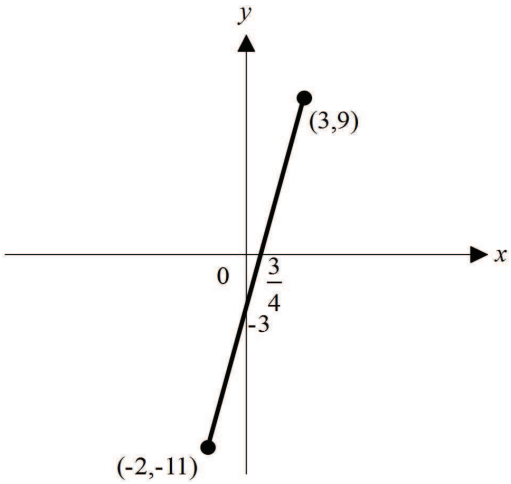
c $(a+2)^2$

d $(a+4)^2$

3 **a** Range = R



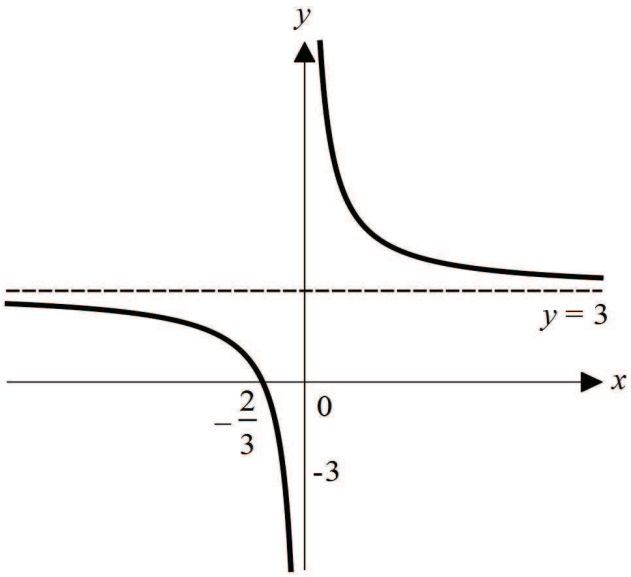
b Range = $[-11,\ 9]$



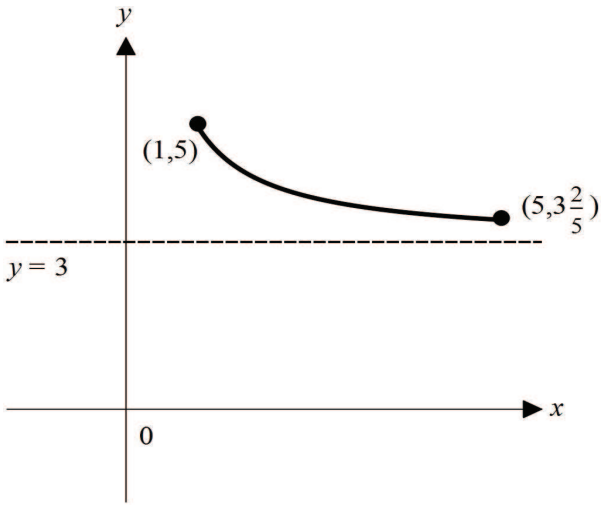
c $[-11,\ 9)$

d $(-11,\ 9)$

4 **a** Range $= R \setminus \{3\}$



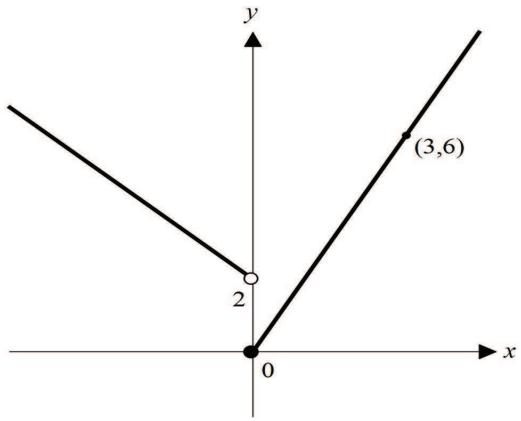
b Range $= \left[3\frac{2}{5},\ 5\right]$



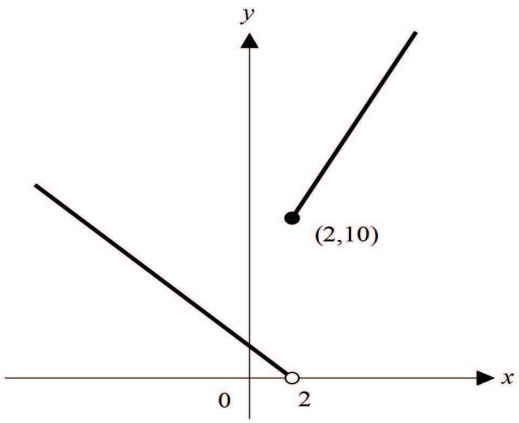
c $\left(3\frac{2}{5},\ 5\right]$

d $\left(3\frac{2}{5},\ 5\right)$

5 **a** Range $= [0,\ \infty)$



b Range $= (0,\ \infty)$

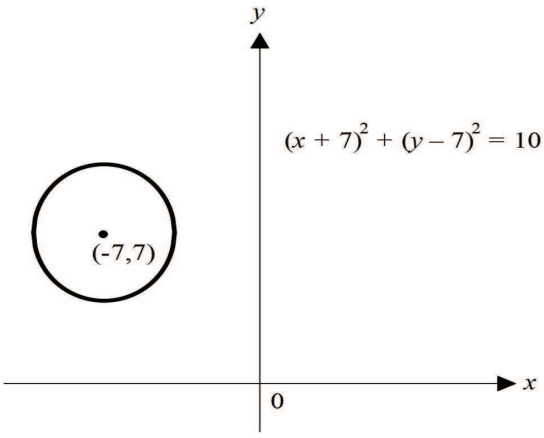


6 **a** $f^{-1}(x) = \dfrac{10 - x}{9}$, domain $= [-26,\ 19]$

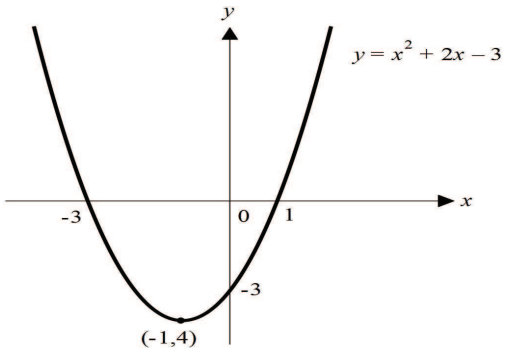
b $f^{-1}(x) = \sqrt{x} \quad - 7$, domain $= (1,\ \infty)$

c $f^{-1}(x) = -1 + \sqrt{x - 2}$, domain $= [2,\ \infty)$

7 **a** Range $= \left[7 - \sqrt{10},\ 7 + \sqrt{10}\right]$



b Range $= [-4,\ \infty)$



Chapter 6 Polynomials: **Assignment**

Name _____

1 Sketch the graphs of each of the following and label the axes intercepts:

a $y = (x+2)^3 + 5$

b $y = 2(x-1)^3 + 2$

c $y = -2(x+2)^3 + 6$

2 Sketch the graphs of each of the following. Label the axes intercepts and the turning points.

a $y = (x+1)^4 + 3$

b $y = -2(x-1)^4 + 2$

3 Divide $x^3 + 2x^2 - 3x + 6$ by $x - 2$.

4 Use the remainder theorem to find the remainder when the polynomial $P(x) = x^3 + 2x^2 - x + 3$ is divided by:

a $x - 3$

b $2x - 1$

c $2x + 1$

5 Find the value of a in the polynomial $ax^3 + 2x^2 + 3$, if the remainder is 3 when the polynomial is divided by $x - 2$.

6 Factorise each of the following polynomials:

a $2x^3 + 5x^2 - x - 6$

b $2x^3 + x^2 - 7x - 6$

c $2x^4 - x^3 - 8x^2 + x + 6$

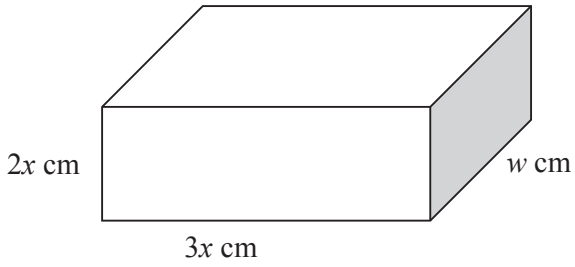
7 Solve each of the following equations for x:

a $2x^3 + 5x^2 - x - 6 = 0$

b $2x^4 - x^3 - 8x^2 + x + 6 = 0$

8 The rule for a cubic function is of the form $y = ax^3 + bx$. The graph passes through the points $(2,\ 0)$ and $(5,\ 6)$. Find the values of a and b.

9 A piece of wire 2000 cm long is used to make the edges of a cuboid with dimensions as shown.



a Find w in terms of x.

b Find the volume, $V\,\text{cm}^3$, of the cuboid in terms of x.

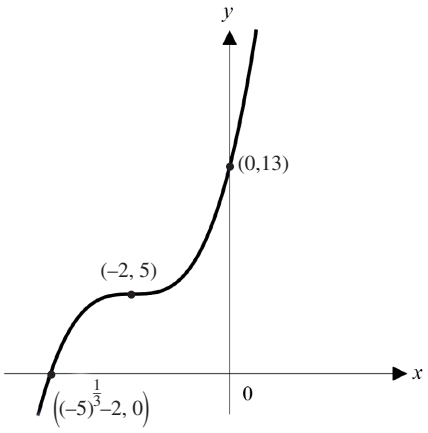
c State the possible values of x.

d Find the volume if $x = 50$.

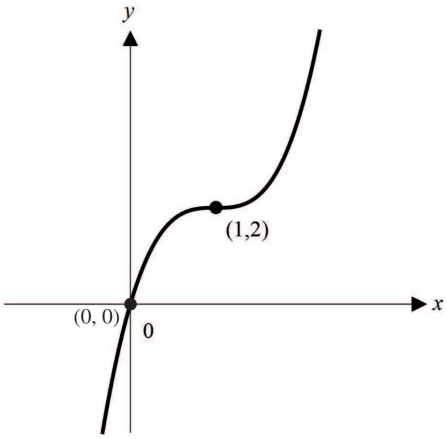
e Use a calculator to find the maximum volume possible.

Answers

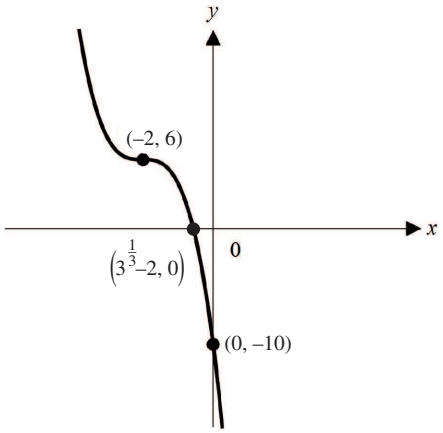
1 a



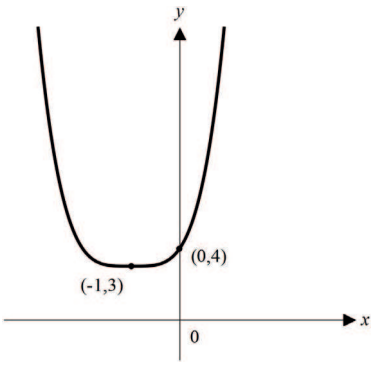
b



c



2 **a**



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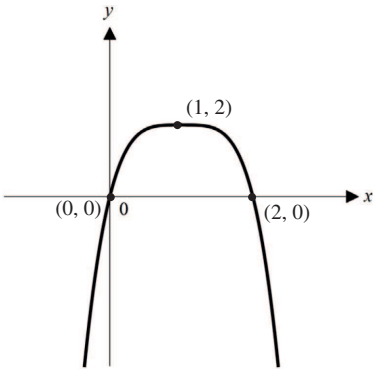


3 $x^2 + 4x + 5 + \dfrac{16}{x-2}$

4 **a** 45

b $\dfrac{25}{8}$

c $\dfrac{31}{8}$

5 -1

6 **a** $(x-1)\ (x+2)\ (2x+3)$

b $(x+1)\ (x-2)\ (2x+3)$

c $(x-2)\ (x-1)\ (x+1)\ (2x+3)$

7 **a** $x = 1$ or -2 or $-\dfrac{3}{2}$

b $x = 2$ or 1 or -1 or $-\dfrac{3}{2}$

8 $a = \frac{2}{35}$, $b = -\frac{8}{35}$

9 **a** $w = 500 - 5x$

b $V = 6x^2(500 - 5x)$

c $0 < x < 100$

d $3\,750\,000 \text{ m}^3$

e $\frac{40\,000\,000}{9} \text{ cm}^3$

Cambridge Senior Mathematical Methods AC/VCE Units 1 & 2

Chapter 7 Transformations: **Assignment**

Name _____

1 **a** State the coordinates of the image point obtained when $(6, 3)$ is translated according to the rule $(x, y) \to (x-2, y+5)$

b State the image of (x, y) under the translation $(x, y) \to (x-2, y+5)$

c Hence find the equation of the image of $y = \dfrac{1}{x}$ under the translation $(x, y) \to (x-2, y+5)$

d Find the equation of the image of $y = \dfrac{1}{x^2} + 4$ under the translation $(x, y) \to (x-2, y+5)$

2 A transformation is defined by the matrix defined by the rule $(x, y) \to (-x, 8y)$. Find the equation of the image of the graph of $y = x^2$ under this transformation.

3 A transformation is defined by the matrix $(x, y) \to (-2x, 3y)$. Find the equation of the image of the graph of $y = x^2 + 2x + 3$ under this transformation.

4 Find a sequence of transformations which maps $y = x^2$ to $y = -2(x+1)^2 + 3$.

5 Find a sequence of transformations which maps $y = x^2$ to $y = (3x-6)^2 + 5$.

6 State the rule for each of the following transformations.

a A translation of 1 unit in the negative direction of the x-axis and 3 units in the positive direction of the y-axis

b A translation of 3 units in the positive direction of the x-axis and 3 units in the negative direction of the y-axis followed by a reflection in the x-axis.

c A dilation of factor 4 from the y-axis followed by a translation of one unit in the negative direction of the x-axis and 2 units in the positive direction of the y-axis.

d The transformation defined in part **c** followed by a reflection in the x-axis.

7 Find the image of $y = x^2 + 2$ under the transformation defined by each of the following:

a a translation of 1 unit in the negative direction of the x-axis and 3 units in the positive direction of the y-axis.

b a translation of 3 units in the positive direction of the x-axis and 4 units in the negative direction of the y-axis followed by a reflection in the x-axis

c a dilation of factor 4 from the y-axis followed by a translation a translation of 1 unit in the negative direction of the x-axis and 2 units in the positive direction of the y-axis.

d the transformation defined in part **c** followed by a reflection in the x-axis

8 Describe the transformation which maps

a $\{(x, y) : y = x^2\}$ to $\left\{(x, y) : y = 2(x-3)^2 + 5\right\}$

b $\{(x, y) : y = 2^x\}$ to $\{(x, y) : y = 3 \times 2^x - 4\}$

c $\left\{(x, y) : y = 2(x-5)^2 + 3\right\}$ to $\{(x, y) : y = x^2\}$

d $\left\{(x, y) : y = 2(x+3)^2\right\}$ to $\{(x, y) : y = 2x^2\}$

9 Let $y = f(x)$ where

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Sketch the graph of:

a $y = f(x-5)$

b $y = f\left(\frac{x}{2}\right)$

c $y = -f\left(\frac{x}{2}\right) + 3$

d Solve the equation $-f(x) + 2 = f(2(x-1))$ for x.

Answers

1 **a** $(4, 8)$

b $(x-2,\ y+5)$

c $y = \dfrac{1}{x+2} + 5$

d $y = \dfrac{1}{(x+2)^2} + 9$

2 $y = 8x^2$

3 $y = \dfrac{3}{4}x^2 - 3x + 9$

4 A dilation of factor 2 from the x axis and a reflection in the x axis, followed by a translation 1 unit in the negative direction of the x axis and 3 units in the positive direction of the y axis.

5 A dilation of factor $\dfrac{1}{3}$ from the y axis, followed by a translation 2 units in the positive direction of the x axis and 5 units in the positive direction of the y axis.

6 **a** $(x, y) \to (x-1, y+3)$

b $(x, y) \to (x+3, 3-y)$

c $(x, y) \to (4x-1, y+2)$

d $(x, y) \to (4x-1, -y-2)$

7 **a** $y = x^2 + 2x + 6$

b $y = -x^2 + 6x - 7$

c $y = \dfrac{1}{16}(x+1)^2 + 4$

d $y = \dfrac{-1}{16}(x+1)^2 - 4$

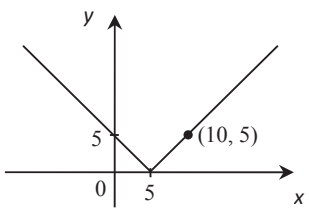
8 **a** Dilation of factor 2 from the x-axis, followed by a translation 3 units in the positive direction of the x-axis and 5 units in the positive direction of the y-axis

b Dilation of factor 3 from the x-axis followed by a translation 4 units in the negative direction of the y-axis

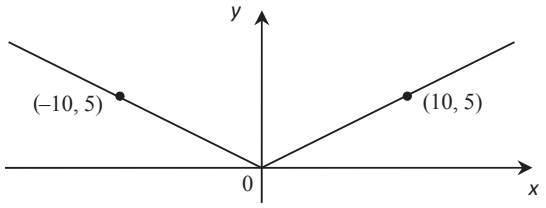
c Dilation of factor $\frac{1}{2}$ from the x-axis, followed by a translation 5 units in the negative direction of the x-axis and $\frac{3}{2}$ units in the negative direction of the y-axis

d A translation 3 units in the positive direction of the x-axis

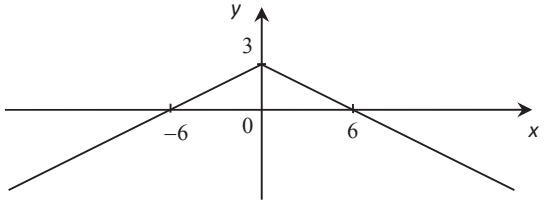
9 **a**



b



c



d $x = 0, \frac{4}{3}$

Chapter 9 Probability: **Assignment**

Name _____

1 A group of 200 Year 11 students at Bayview Secondary College were asked to indicate their subject choices for Year 12. It was found that 135 chose a mathematics subject (M), 84 chose a language (L), and 55 chose both mathematics and a language.

a Draw a Venn diagram to show this situation, and use the diagram to determine the number of students who chose either a language or mathematics or both. Hence find $\Pr(M \cup L)$.

b From the Venn diagram write down the following probabilities:

i $\Pr(M)$

ii $\Pr(L)$

iii $\Pr(M \cap L)$

c Use the addition rule to determine the value of $\Pr(M \cup L)$.

d Use the information in this question to complete the following Karnaugh map:

	L	L'	
M			
M'			
			1

2 Another group of 100 Year 11 students at Mountainview Secondary College were also asked to indicate their subject choices for Year 12. Here it was found that 75 chose a mathematics subject (M), 44 chose a language (L), and 25 chose both mathematics and a language.

a Use the information in this question to complete the following Karnaugh map.

	L	L'	
M			
M'			
			1

b Use the Karnaugh map from part **a** to determine:

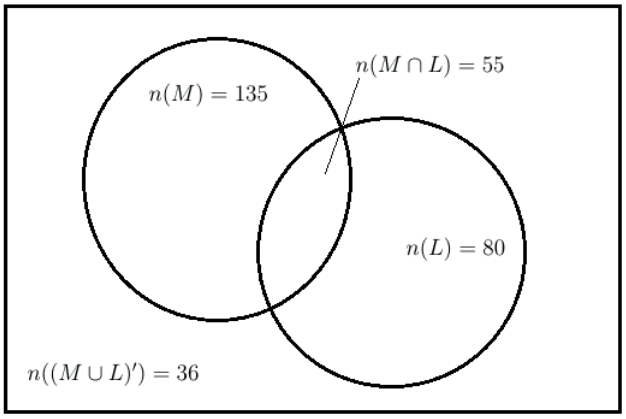
i the probability that a student chose mathematics and did not choose a language

ii the probability that a student chose neither mathematics nor a language.

3 Bayview and Mountainview Secondary Colleges decide to amalgamate. Find the probability that a student at the combined school chose mathematics.

Answers to Chapter 9 Assignment

1 **a**



$n(M \cup L) = 164 \quad \Pr(M \cup L) = \frac{41}{50} = 0.82$

b **i** $\frac{27}{40} = 0.675$

ii $\frac{21}{50} = 0.42$

iii $\frac{11}{40} = 0.275$

c $\Pr(M \cup L) = 0.675 + 0.42 - 0.275 = 0.82$, as before

d

	L	L'	
M	0.275	0.40	0.675
M'	0.145	0.18	0.325
	0.42	0.58	1

2 **a**

	L	L'	
M	0.25	0.5	0.75
M'	0.19	0.06	0.25
	0.44	0.56	1

b **i** 0.5

ii 0.06

3 0.7

Chapter 10 Counting methods: **Assignment**

Name _____

1 Ten students are to be seated in a row of ten seats. There are four boys and six girls.

a Find the number of ways in which they can be seated:

i if there are no restrictions

ii if the boys must sit together and the girls must sit together

iii if the boys must sit together.

b If the students are seated randomly, find the probability that:

i the boys will sit together

ii there will be a boy at each end of the row

iii a boy named Matthew and a girl named Helene are **not** sitting together.

2 **a** Suppose that there are three people at a party. If each person shakes hands with each other person once, how many handshakes are there?

b Suppose that there are four people at a party. If each person shakes hands with each other person once, how many handshakes are there?

c Suppose that there are five people at a party. If each person shakes hands with each other person once, how many handshakes are there?

d Suppose that there are n people at a party. If each person shakes hands with each other person once, how many handshakes are there?

e Suppose that there are 45 handshakes. How many people are there at the party?

Answers

1 **a** **i** 3 628 800

ii 34 560

iii 120 960

b **i** $\frac{1}{30}$

ii $\frac{2}{15}$

iii $\frac{4}{5}$

2 **a** 3

b 6

c 10

d $\frac{n(n-1)}{2}$

e 10

Chapter 11 Discrete probability distributions: **Assignment**

Name _____

1 In 1693 Samuel Pepys asked Isaac Newton whether it is more likely to get at least one 1 in six rolls of a die, or at least two 1s in 12 rolls of a die. This is known as Pepys' problem.

a Use simulation to estimate the answer to this problem using empirical data.

b Use the binomial distribution to solve Pepys' problem.

2 Which is more likely: at least one 1 with four throws of a fair die, or at least one 'double 1' in 24 throws of two fair dice? This is known as DeMere's problem.

a Use simulation to estimate the answer to this problem using empirical data.

b Use the binomial distribution to solve DeMere's problem.

Answers

1 **a** Pr(at least one 1 in six rolls) = 0.6651

Pr(at least two 1s in 12 rolls) = 0.6187

2 **a** Pr(at least one 1 in four rolls of a die) = 0.5177

Pr(at least two 1s in 24 rolls of two dice) = 0.4914

Chapter 13 Exponential functions and logarithms: **Assignment**

Name _____

1 Simplify the expression $6w^5y^3 \div 15(wy)^4$.

2 The population of a town increases by 6% every year. In January 2006 the population was 5500.

a Find the population of the town:

i in January 2007

ii after n years.

b Find the year in which the population will reach 11 000.

3 The value of a particular piece of equipment, initially valued at \$60 000 depreciates by 8% every year.

a Find the value of the piece of equipment after 1 year.

b Find the value of the piece of equipment after n years.

c After how many years, correct to two decimal places, will the value be less than \$20 000?

4 Simplify each of the following, giving your answer with positive index:

a $\dfrac{2a^2(2b)^3}{2ab^2}$

b $\dfrac{a^2b^3}{ab} \times \dfrac{a^2b^5}{a^2b^2}$

c $\dfrac{(2a)^2 \times 8b^3}{16a^2b^2}$

d $\dfrac{2a^2b^3}{8a^2b^2} \div \dfrac{16(ab)^2}{2ab}$

5 Write $\dfrac{2^n \times 8^n}{2^{2n} \times 16}$ in the form 2^{an+b}

6 Write $2^{-x} \times 3^{-x} \times 6^{2x} \times 3^{2x} \times 2^{2x}$ as a power of 6.

7 Simplify each of the following:

a $2^{\frac{1}{3}} \times 2^{\frac{1}{6}} \times 2^{-\frac{2}{3}}$

b $a^{\frac{1}{4}} \times a^{\frac{2}{5}} \times a^{-\frac{1}{10}}$

c $\left(2^{\frac{1}{43}}\right)^2 \times \left(2^{\frac{1}{5}}\right)^5$

d $\left(2^{\frac{1}{3}}\right)^2 \times 2^{\frac{1}{3}} \times 2^{-\frac{2}{5}}$

8 Simplify each of the following:

a $\sqrt[3]{a^3 b^2}$

b $\sqrt[5]{a^3 b^2} \times \sqrt[5]{a^2 b^{-1}}$

c $\sqrt[5]{a^3 b^2} \div \sqrt[5]{a^2 b^{-1}}$

d $\dfrac{\sqrt{a^3 b^2}}{a^2 b^{-1} c^{-5}} \times \dfrac{\sqrt{a^{-4} b^2}}{a^3 b^{-1}} \times \sqrt{a^3 b^{-1}}$

9 Solve each of the following equations for x:

a $2^x = \dfrac{1}{16}$

b $2^{x-4} = 256$

c $2^x = 0.65$

d $\log_2(x-2) = 6$

e $\log_3(2x-4) - \log_3(6) = 3$

f $\log_{10}(x-4) + \log_{10}(x+4) = 3$

10 Sketch the graphs of each of the following. Give the equations of asymptotes and the axes intercepts:

a $y = 2^x - 4$

b $y = \log_2(2x)$

c $y = \log_2(x - 4)$

d $y = \log_2(x - 4) - 3$

11 **a** Find a sequence of transformations which transform the graph of $y = \log_2(x)$ to the graph of $y = -3\ \log_2(5 + x)$.

b Sketch the graph of $y = -3\ \log_2(5 + x)$.

c Solve the equation $-3\ \log_2(5 + x) = 10$ for x.

Answers

1 $\frac{2w}{5y}$

2 **a** **i** 5830

ii $5500(1.06)^n$

b 2017

3 **a** \$55 200

b $\$60\,000(0.92)^n$

c 13.18 years

4 **a** $8ab$

b ab^5

c $2b$

d $\frac{1}{32a}$

5 2^{2n-4}

6 6^{3x}

7 **a** $2^{-1/6}$

b $a^{11/20}$

c $2^{45/43}$

d $2^{3/5}$

8 **a** $ab^{2/3}$

b $ab^{1/5}$

c $a^{1/5}b^{3/5}$

d $\frac{b^{7/2}c^5}{a^4}$

9 **a** -4

b 12

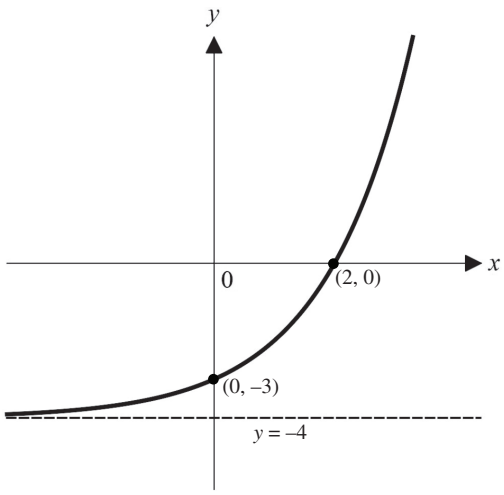
c $\log_2(0.65)$

a 66

b 83

c $2\sqrt{254}$

10 **a**



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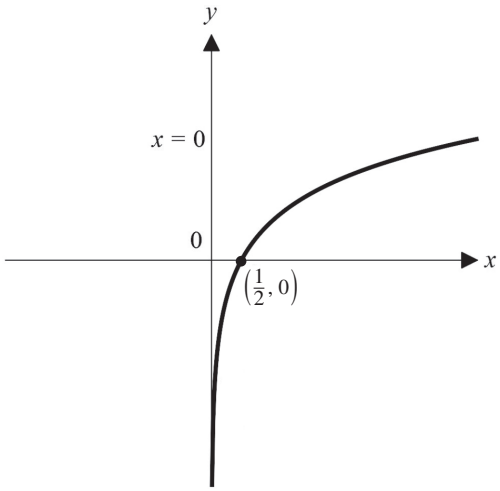

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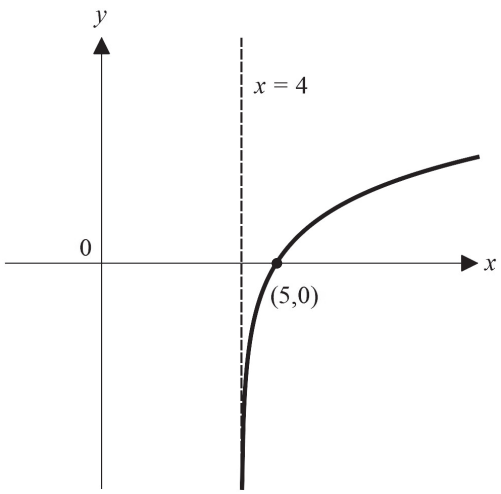

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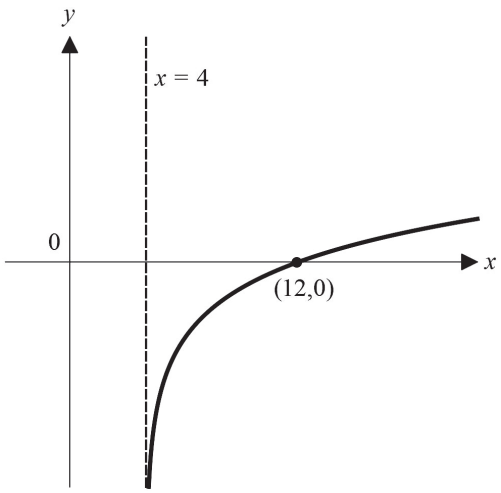
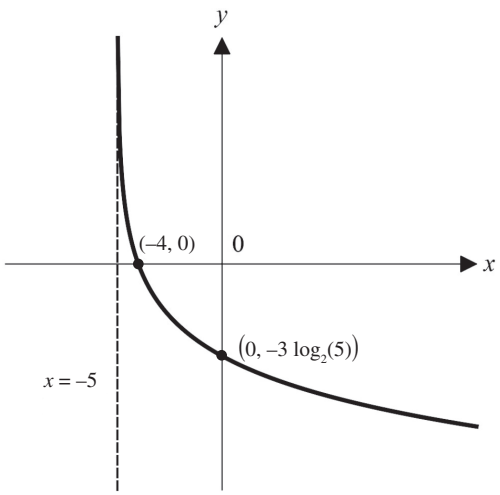


11 **a** Reflection in the x-axis, dilation of factor 3 from the x-axis, translation of 5 units in the negative direction of the x-axis

b



c $x = 2^{-10/3} - 5$

Chapter 14 Circular functions: **Assignment**

Name _____

1 Convert each of the following to radians:

a 120°

b 135°

c 225°

2 Convert each of the following to degrees:

a $\frac{13\pi}{4}$

b $\frac{23\pi}{9}$

c $\frac{31\pi}{4}$

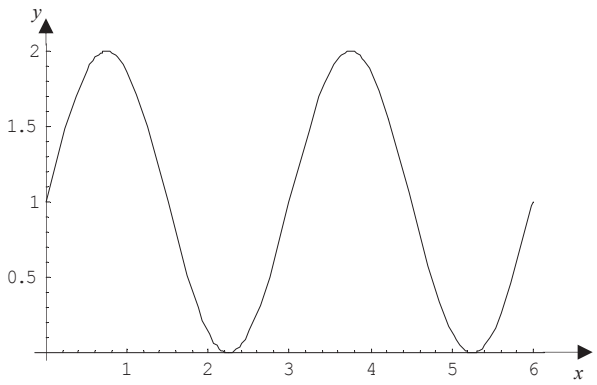
3 Write down the period, amplitude and range of each of the following:

a $f(t) = 3\sin(\pi t)$

b $f(t) = 5\cos\left(\frac{2\pi t}{3}\right) + 6$

c $f(x) = -5\sin\left(\frac{3\pi x}{5}\right) - 7$

4 The graph shown has rule $y = \sin(nx) + c$. Find the values of n and c.



5 Solve each of the following equations for x, where $0 \leq x \leq 2\pi$.

a $\sin\left(2x + \frac{\pi}{3}\right) = 0.5$

b $\cos\left(x + \frac{\pi}{4}\right) = \frac{-\sqrt{3}}{2}$

c $\tan(2x) = 1$

6 Solve the equation $2\sin(2\pi x) = -1$ for x $[0,\ 2]$.

7 Find the general solution to each of the following equations.

a $\sin(x) = \frac{1}{2}$

b $2\cos(x) = \sqrt{2}$

Answers

1 **a** $\frac{2\pi}{3}$

b $\frac{3\pi}{4}$

c $\frac{5\pi}{4}$

2 **a** 585°

b 460°

c 1395°

3 **a** Period = 2, amplitude = 3 and range = $[-3, 3]$

b Period = 3, amplitude = 5 and range = $[1, 11]$

c Period = $\frac{10}{3}$, amplitude = 5 and range = $[-12, -2]$

4 $n = \frac{2\pi}{3}$, $c = 1$

5 **a** $\frac{\pi}{4}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{23\pi}{12}$

b $\frac{7\pi}{12}, \frac{11\pi}{12}$

c $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

6 $\frac{7}{12}, \frac{11}{12}, \frac{19}{12}, \frac{23}{12}$

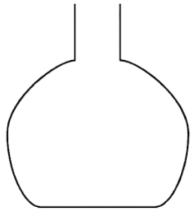
7 **a** $x = \frac{\pi}{6} + 2k\pi$ or $x = \frac{5\pi}{6} + 2k\pi$, where k is an integer

b $x = 2k\pi \pm \frac{\pi}{4}$, where k is an integer

Chapter 16 Rates of change: **Assignment**

Name _____

1 Water is poured at a constant rate into the flask shown. Sketch a graph showing how the depth of water changes with respect to time.



2 Assuming a constant speed, find the speed of a car that travels a distance of 140 km in 2 hours.

3 A hiker covers a distance of 23 km over a period of 4 hours. Assuming she walks at a constant speed, calculate her average speed over the 4 hours.

4 A shearer shears 120 sheep in 9 hours and is paid $300.

a Find the rate at which he shears sheep per hour.

b Find how much he earns per sheep.

c Find how much he earns per hour.

5 A cyclist spends 2 hours cycling from Swifts Creek to Tambo Crossing. The ride can be described in three stages:

Stage 1 He rides at a constant speed of 20 km/h for 45 minutes.

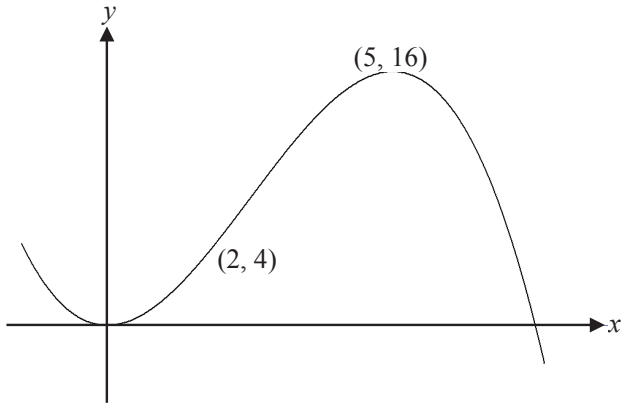
Stage 2 He rides at a constant speed of 15 km/h for 45 minutes.

Stage 3 He rides at a constant speed of 25 km/h for 30 minutes.

Draw a distance-time graph that illustrates this motion.

6 Find the average rate of change of the function $f(x) = 2x^2 - x$ as x changes from 3 to 5.

7 Find the average rate of change of the function depicted in the graph below for the interval $[2, 5]$.



8 A candle burns with a steady flame and gradually diminishes in height. The height, h cm, of the candle after burning for t minutes is given by the rule $h = 18 - \frac{1}{4\pi}t$.

Find the average rate of change of the height of the candle in the first 3 minutes after it has been lit, correct to two decimal places.

9 By considering the chord joining the points where $x = 1$ and $x = 1.01$, estimate the gradient of the curve $y = 2x^3 + x$ at $x = 1$.

10 Let $s(t) = 5t^2 - 3t$ be the displacement function of a particle moving in a straight line, where t is in seconds and s is in metres.

a Find the average velocity for the time interval $[0, 1]$.

b Find the average velocity for the time interval $[0.9, 1]$.

c Find the average velocity for the time interval $[0.99, 1]$.

d Estimate the instantaneous velocity for $t = 1$.

11 For $y = 2\sin(x)$, find the average rate at which y changes with respect to x over the interval $\left[0, \frac{\pi}{4}\right]$.

12 Consider $y = 3^x$.

a Find the average rate at which y changes with respect to x over each of the following intervals:

i $[0,\ 1]$

ii $[0,\ 0.5]$

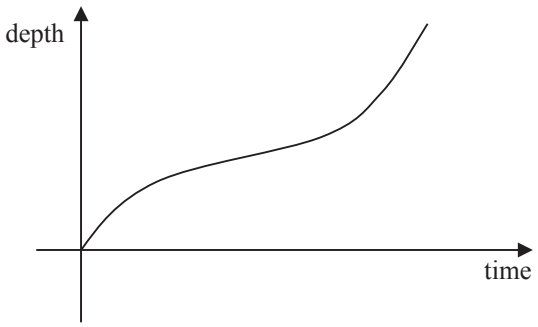
iii $[0,\ 0.1]$

iv $[0,\ 0.01]$

b Estimate the instantaneous rate of change of y with respect to x when $x = 0$.

Answers

1



2 70 km/h

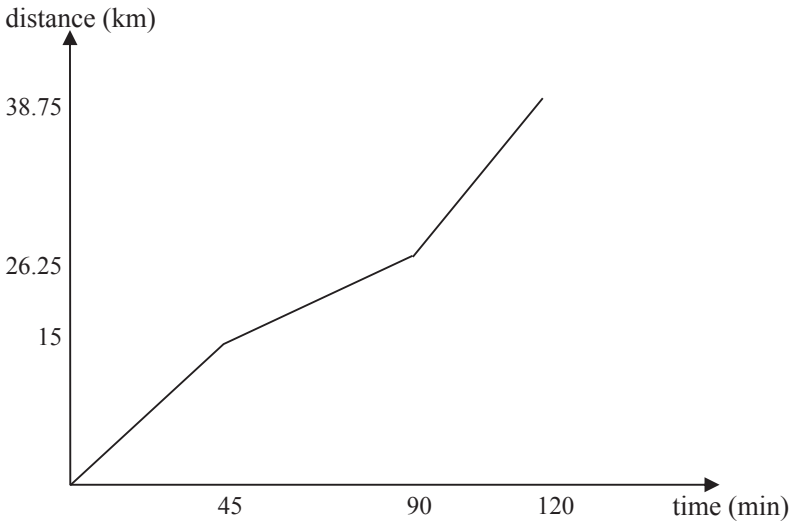
3 5.75 km/h

4 **a** $13\frac{1}{3}$ sheep per hour

b \$2.50 per sheep

c $\$33\frac{1}{3}$ per hour

5



6 15

7 4

8 −0.08 cm/minute

9 ≈ 7

10 **a** 2 m/s

b 6.5 m/s

c 6.95 m/s

d 7 m/s

11 $\frac{4\sqrt{2}}{\pi}$

12 **a** **i** 2

ii 1.46

iii 1.16

iv 1.1

b 1.1

Chapter 17 Differentiation and antidifferentiation: **Assignment**

Name _____

1 Consider a curve with equation $y = 2x^2 + x$.

a If P is the point $(1,\ 3)$ and Q is the point $\left((1+h)\ ,\ 2(1+h)^2 + (1+h)\right)$. Find the gradient of chord PQ.

b Find the gradient of PQ when $h = 0.1$.

c Find the gradient of the curve at P.

2 For the function $f(x) = 2x^2$, find $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

3 Evaluate the following limits:

a $\lim\limits_{x \to 0} \dfrac{(x+3)^2 - 9}{x}$

b $\lim\limits_{h \to 0} \dfrac{h^3 - 2h^2 + h}{h}$

c $\lim\limits_{x \to 2} \dfrac{x^3 + 8}{x + 2}$

4 Find the derivative of each of the following:

a $y = 8x^3 - 3x + 4$

b $y = 2x(x^2 - 2x^3)$

c $y = (2x + 3)(x + 1)$

d $y = -x^4 + 3x(x^3 - x)$

e $y = \dfrac{x^5 - 2x^3 + x^2}{x}$

f $y = \dfrac{6x^3 - 2x^2}{3x}$

g $y = 7x^4 - \dfrac{1}{x^2} + 7$

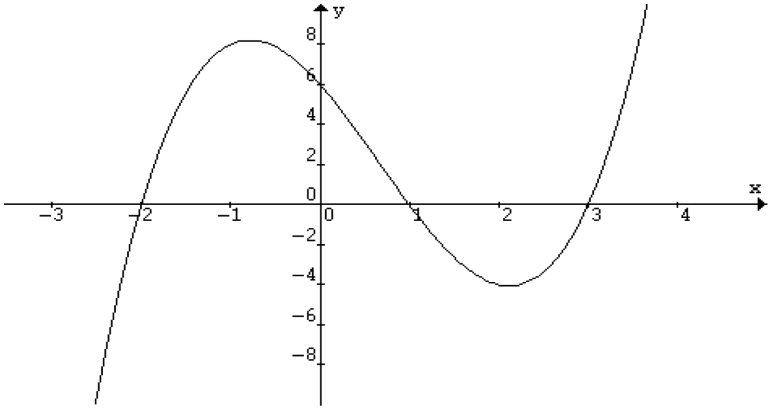
h $y = 3x^{-2} + 2\sqrt{x}$

5 Let $y = x^4 + x^3 + x^{-2} + 8$.

a Find the average rate of change of y between $x = 1$ and $x = 2$.

b Find the gradient of the curve at $x = 2$.

6 For the graph shown, sketch the graph of the gradient function.



7 If $y = x^2 - 3x - 18$ find the interval(s) for which $\frac{dy}{dx} > 0$.

8 The function $s(t) = -3t^3 + 6t^2 - 3$ represents the displacement of a particle moving along a straight line, where t is in seconds and s is in metres.

a Find the **position** of the particle after 3 seconds.

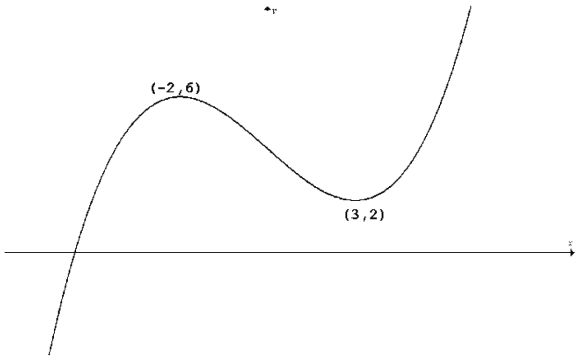
b Find the **velocity** of the particle at that time.

9 The curve with equation $y = ax^2 + bx$ has a gradient of 5 at the point $(1, -2)$.

a Find the values of a and b.

b Find the coordinates of the point where the gradient is 0.

10 For the graph of $f : R \to R$, find:



a $\{x : f'(x) > 0\}$

b $\{x : f'(x) < 0\}$

c $\{x : f'(x) = 0\}$

11 Find the coordinates of the points on the curve $y = x^2 + 5x + 3$ at which the tangent:

a makes an angle of 45° with the positive direction of the x-axis

b is parallel to the line $y = 3x + 4$.

12 Consider the equation $y = x\left(x^2 - 9\right)$.

a Find the gradient at the points at which the curve crosses the x-axis.

b Find the coordinates of the point on the curve at which the gradient $= 0$.

Answers

1 **a** $5 + 2h$

b 5.2

c 5

2 $4x$

3 **a** 6

b 1

c 4

4 **a** $24x^2 - 3$

b $6x^2 - 16x^3$

c $4x + 5$

d $8x^3 - 6x$

e $4x^3 - 4x + 1$

f $4x - \frac{2}{3}$

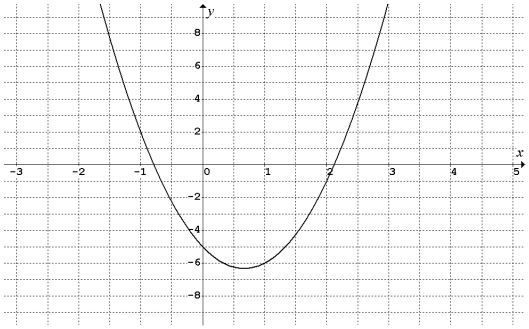
g $28x^3 + \frac{2}{x^3}$

h $-\frac{6}{x^3} + \frac{1}{\sqrt{x}}$

5 **a** $21\frac{1}{4}$

b $43\frac{3}{4}$

6



7 $x > \frac{3}{2}$

8 **a** $s = -30\,\text{m}$

b $v = -45\,\text{m/s}$

9 **a** $a = 7,\ b = -9$

b $\left(\frac{9}{14},\ -\frac{81}{28}\right)$

10 **a** $(-\infty, -2) \cup (3, \infty)$

b $(-2, 3)$

c $x = -2$ and $x = 3$

11 **a** $(-2, -3)$

b $(-1, -1)$

12 **a** At $x = 0$ gradient $= -9$; at $x = -3$ gradient $= 18$; at $x = 3$ gradient $= 18$

b $\left(-\sqrt{3},\ 6\sqrt{3}\right)$ and $\left(\sqrt{3},\ -6\sqrt{3}\right)$

Chapter 18 Applications of differentiation and antidifferentiation of polynomials: **Assignment**

Name _____

1 Find the coordinates of the stationary point of each of the following curves:

a $y = x^2 + 4x - 12$

b $y = 2t^3 - 5t^2 - 4t + 13$

2 Let $y = x^4 + x^3 + x^{-2} + 8$.

a Find the average rate of change of y between $x = 1$ and $x = 2$.

b Find the gradient of the curve at $x = 2$.

3 Find the equation of the tangent and the normal to the curve $y = x^3 - 2x^2 + 4$ at the point where $x = 2$.

4 A particle moves from rest in a straight line so that after t seconds it is s metres from a fixed point O on the line, where $s = t^3 - 6t^2 + 12$.

a Find the position of the particle after 3 seconds.

b Find the time and position when the particle comes to rest again.

c Find the total distance travelled in the first 4 seconds.

5 The line $y = ax + 3$ is tangent to the parabola $y = x^2 + x + b$ when $x = 1$. Find the values of the constants a and b.

6 Water is being poured into a tank so that the volume, V mL, of water in the tank at time t minutes is given by $V(t) = \frac{1}{3}\left(8t^2 - \frac{t^3}{2}\right)$, $0 \leq t \leq 10$.

a Find the volume of water in the tank at time:

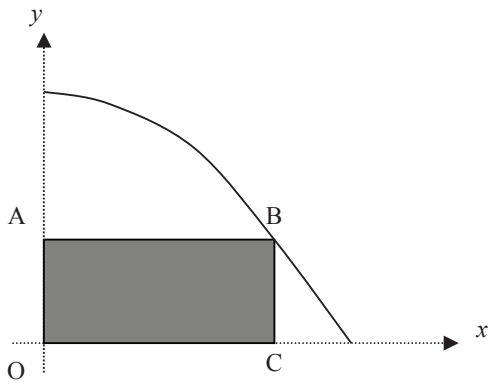
i $t = 0$

ii $t = 10$

b Find the rate of flow of water into the tank at any time t.

c Find the rate of flow of water into the tank at $t = 5$.

7 The diagram shows the graph of $y = 4 - 3x^2$, for $x \geq 0$ and $y \geq 0$. B is a point on the graph and $OABC$ is a rectangle. Find the value of x for which the area of $OABC$ is a maximum.



8 Consider the function $y = x^3 - x^2 - 5x - 3$.

a Factorise and find the x- and y-intercepts for the graph.

b Find the coordinates of the turning points.

c Sketch the graph of the function, labelling all the important features.

9 The derivative of a curve is $\frac{dy}{dx} = -2x + 8$. Find the equation of the tangent to this curve at the point $(-2, 3)$.

10 A rectangular box, made of thin sheet metal and without a lid, is of length $2x$ cm, width x cm and height h cm.

a Write down, in terms of x and h, the area of sheet metal required to make the box.

b Given that the area of sheet metal is 600 cm^2, show that $h = \frac{600 - 2x^2}{6x}$.

c Hence show that the volume, V cm^3, of the box is given by $V = 200x - \frac{2x^3}{3}$.

d Find $\frac{dV}{dx}$ and find the value of x for which V is a maximum.

e Hence calculate the volume of the largest such box that can be constructed using 600 cm^2 of sheet metal.

11 A particle moves in a straight line so that its position x cm relative to O at time t seconds is given by $x = t^2 - 9t + 8, \ t \geq 0$.

a Find its initial velocity.

b Find when and where its velocity equals zero.

c Determine its average velocity for the first 4 seconds.

d Determine its average speed for the first 4 seconds.

12 **a** Let $f : [-3, 5] \to R, \ f(x) = x^2 + 2$. Find the maximum and minimum value of the function.

b Let $f : [-2, 1] \to R, \ f(x) = x^3 + 2x + 6$. Find the maximum and minimum value of the function for its domain

Answers

1 **a** $(-2, -16)$

b $\left(-\frac{1}{3}, \frac{370}{27}\right)$ and $(2, 1)$

2 **a** 21.25

b 43.75

3 Tangent: $y = 4x - 4$; normal: $4y + x = 18$

4 **a** -15 m

b $t = 4$ s, $s = -20$ m

c 32 m

5 $a = 3$ and $b = 4$

6 **a** **i** 0

ii 100

b $V'(t) = \frac{1}{3}\left(16t - \frac{3t^2}{2}\right)$

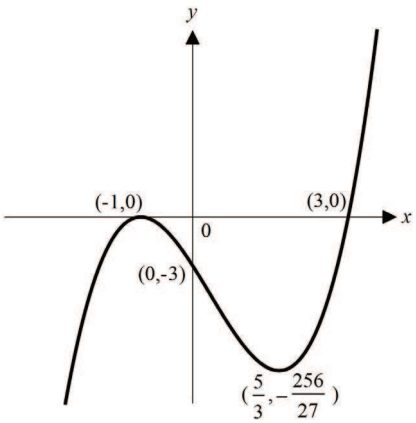
c $14\frac{1}{6}$ mL/min

7 $x = \frac{2}{3}$

8 **a** $(x+1)^2(x-3)$ x-int: $(-1, 0)$ and $(3, 0)$; y-int: $(0, -3)$

b $(-1, 0)$ and $\left(\frac{5}{3}, -\frac{256}{27}\right)$

c



Need to label two turning points and all intercepts

9 $y = 12x + 27$

10 **a** Area $= 2x^2 + 6xh$

d $\frac{dV}{dx} = 200 - 2x^2$, V is maximum for $x = 10$

e Max volume $= \frac{4000}{3}$ cm^3

11 **a** -9 cm/s

b $t = \frac{9}{2}$ s, $x = -\frac{49}{4}$ cm

c -5 cm/s

d 5 cm/s

12 **a** Maximum value $= 27$; minimum value $= 2$

b Minimum value $= -6$; maximum value $= 9$

Chapter 20 Further differentiation and antidifferentiation: **Assignment**

Name _____

1 Differentiate each of the following with respect to x:

a $\left(2x^3 - 5x^2\right)^2$

b $\left(3x^4 - 5x^2\right)^8$

c $\sqrt{x^2 + 4}$

2 Consider $y = \dfrac{x+8}{2x}$.

a Find $\dfrac{dy}{dx}$.

b Find the gradient of the curve with equation $y = \dfrac{x+8}{2x}$ at the point where the graph cuts the x-axis.

c Find the equation of the tangent to the curve of $y = \dfrac{x+8}{2x}$ at the point where it crosses the x-axis.

3 Find the equation of the normal to the curve with equation $y = \left(4x^2 - 1\right)^2$ at the point where:

a $x = \dfrac{1}{2}$

b $x = 1$

4 The height, h m, of a golf ball t seconds after it has been hit is given by the rule $h(t) = -t^3 + t^2 + 12t$ for $0 \leq t \leq 4$.

a Find the height of the ball 2 seconds after it has been hit.

b Let the rate at which the ball's height is changing be defined as its vertical velocity.

i Find the ball's vertical velocity 2 seconds after the ball has been hit.

ii Will the ball continue to gain height after 2 seconds?

c Find the vertical velocity of the ball at the moment it hits the ground.

d How many seconds after it has been hit does the ball reach a vertical velocity of 11 m/s?

5 The point $(1,\ 1)$ lies on the curve with equation $y = \sqrt{x}$.

a Find the gradient of the curve at this point.

b Find the equation of the tangent to the curve at this point.

c Find the coordinates of the point on the graph where the gradient is 1, and find the equation of the tangent to the curve at this point.

6 Sketch the graph of $y = 4x + \frac{1}{x}$. Give the coordinates of the turning points and the equation of the asymptote(s).

Answers

1 **a** $4x^3(2x-5)(3x-5)$

b $16x^{15}(3x^2-5)^7(6x^2-5)$

c $\dfrac{x}{\sqrt{x^2+4}}$

2 **a** $-\dfrac{4}{x^2}$

b $-\dfrac{1}{16}$

c $y=-\dfrac{1}{16}(x+8)$

3 **a** $x=\dfrac{1}{2}$

b $y=\dfrac{-x}{48}+\dfrac{433}{48}$

4 **a** 20 m

b **i** 4 m/s

ii Yes

c −28 m/s

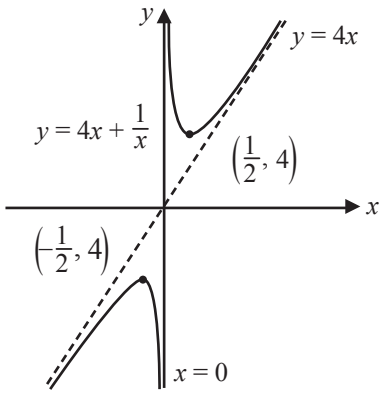
d 1 second

5 **a** $\dfrac{1}{2}$

b $y=\dfrac{1}{2}x+\dfrac{1}{2}$

c $\left(\dfrac{1}{4},\dfrac{1}{2}\right)$, $y=x+\dfrac{1}{4}$

6



Cambridge Senior Mathematical Methods AC/VCE Units 1 & 2

Chapter 21 Integration: **Assignment**

Name _____

1 Find an antidertive of each of the following:

a $5x - 4$

b $6x^3 - 2x^2$

c $4x^2 + 6x - 1$

d $15x^4 + 8x^3 + 2x$

2 Find:

a $\int 8 - x\,dx$

b $\int 8x^3 - 2x^2 + 4\,dx$

c $\int 16x^3 + 4x^2 - 5x + 4\,dx$

d $\int (x-4)^2\,dx$

3 Find y in terms of x if:

a $\frac{dy}{dx} = 6$ and $y = 4$ when $x = 1$

b $\frac{dy}{dx} = 4x^{-1/2}$ and $y = 1$ when $x = 9$

c $\frac{dy}{dx} = \frac{1}{x^3}$ and $y = 16$ when $x = 1$

d $\frac{dy}{dx} = 2\sqrt{x}$ and $y = 1$ when $x = 9$

4 A curve with equation $y = f(x)$ passes through the point $(4,\ 16)$ and $f'(x) = x^2 - 8a$, where a is a positive constant. The curve has a stationary point where $x = 2$. Find $f(x)$.

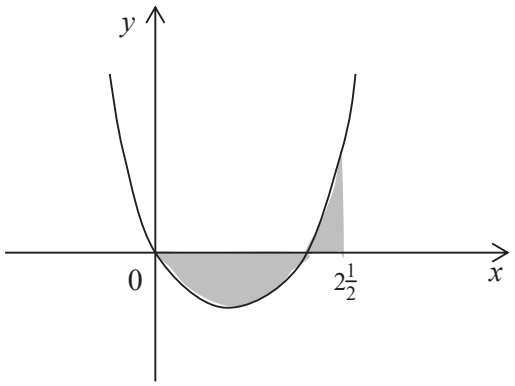
5 Evaluate each of the following:

a $\int_1^2 3x^2 + 6x \, dx$

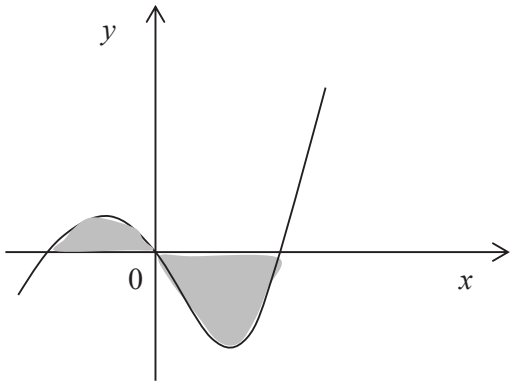
b $\int_{-2}^4 2x + 4 \, dx$

c $\int_1^5 3 - x \, dx$

6 Part of the graph of $y = x^2 - 2x$ is shown. Find the area of shaded region.



7 Part of the graph of $y = x(x-3)\ (x+1)$ is shown. Find the area of shaded region.



8 A body starts from O and moves in a straight line. After t seconds $(t \geq 0)$ its velocity (v cm/s) is given by $v = 3t - 6$.

a Find its position x in terms of t.

b Find its position after 3 seconds.

c What is the distance travelled in the first 3 seconds?

d Determine its average velocity in the first 3 seconds

e Find its average speed in the first 3 seconds

Answers

1 **a** $\frac{5x^2}{2} - 4x$

b $\frac{3x^4}{2} - \frac{2x^3}{3}$

c $\frac{4x^3}{3} + 3x^2 - x$

d $3x^5 + 2x^4 + x^2$

2 **a** $8x - \frac{x^2}{2} + c$

b $2x^4 - \frac{2x^3}{3} + 4x + c$

c $4x^4 + \frac{4x^3}{3} - \frac{5x^2}{2} + 4x + c$

d $\frac{1}{3}(x-4)^3 + c$

3 **a** $y = 6x - 2$

b $y = 8x^{\frac{1}{2}} - 23$

c $y = \frac{33}{2} - \frac{1}{2x^2}$

d $y = \frac{4}{3}x^{\frac{3}{2}} - 35$

4 $f(x) = \frac{x^3}{3} - 4x + \frac{32}{3}$

5 **a** 16

b 36

c 0

6 $\frac{13}{8}$

7 $\frac{71}{6}$

8 **a** $x = \dfrac{3t^2}{2} - 6t$

b $-\dfrac{9}{2}$ cm

c $\dfrac{15}{2}$ cm

d $-\dfrac{3}{2}$ cm/s

e $\dfrac{5}{2}$ cm/s