1 Lines and linear relationships

Topic	1	Lines and linear relationships
Subtopic	1.2	Linear equations and inequations

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Question 1 (3 marks)
Solve the literal equation for x, expressing the answer in simplest form.

$$\frac{x-a}{b}-2=\frac{b-x}{a}$$

Question 2 (1 mark)
The solution to the equation $3(x-4) \leq 6$ is

A. $x \geq 6$
B. $x \leq 2$
C. $x \leq -2$
D. $x \geq -2$
E. $x \leq 6$

Question 3 (3 marks)
Solve for x.

$$\frac{x}{5}-\frac{2x-1}{3} \geq -2$$

Question 4 (1 mark)

The solution to the equation $3x + 12 = 4x - 16$ is

A. $x = -28$
B. $x = -4$
C. $x = 4$
D. $x = \pm 4$
E. $x = 28$

Question 5 (1 mark)

A rower travels upstream at 3 km/h and rows back to the starting point at 6 km/h. She takes 50 minutes for the total journey. If the distance she rowed upstream was x km, the equation that describes the time (in hours) to complete her journey is

A. $\frac{x}{3} + \frac{x}{6} = 50$
B. $\frac{3}{x} + \frac{6}{x} = 50$
C. $3x + 6x = 50$
D. $\frac{x}{3} = \frac{50}{2} + 6x$
E. $\frac{x}{3} + \frac{x}{6} = \frac{5}{6}$

Question 6 (1 mark)

Which of the following two equations represent parallel lines?

I. $y - 5 = 2x$
II. $4x - 2y = 3$
III. $2y = 2x - 1$
IV. $y = x + 5$

A. III, IV
B. I, III
C. II, III
D. I, II
E. I, IV

Topic	1	Lines and linear relationships
Subtopic	1.3	Systems of simultaneous linear equations

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Question 1 (1 mark)

If $3y = -4x - 4$ and $2x - \frac{y}{4} = 5$, the solution (x, y) equals

A. $(-2, -36)$
B. $(2, -4)$
C. $(-4, 52)$
D. $(-1, -28)$
E. $(1, -12)$

Question 2 (1 mark)

If the equations $x - 4 = 4y + 8$ and $3x - 6 = 2y + 20$ are solved simultaneously, the y-coordinate of their point of intersection would equal

A. 2
B. 14
C. -1
D. -1.4
E. -2

Question 3 (1 mark)

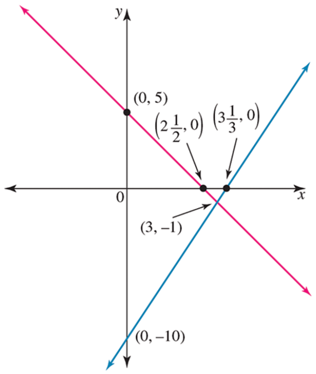
The solution to the simultaneous equations $y = x + 2$ and $\frac{1}{4}x + y = 0$ is

A. $(1, -1)$
B. $(4, -1)$
C. $(2, 0)$
D. $(0, 4, 2)$
E. $(-1.6, 0.4)$

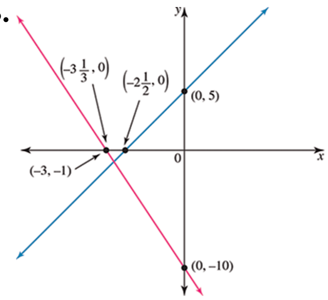
Question 4 (1 mark)

The graphical solution to the equations $y = 5 - 2x$ and $y = 3x - 10$ is

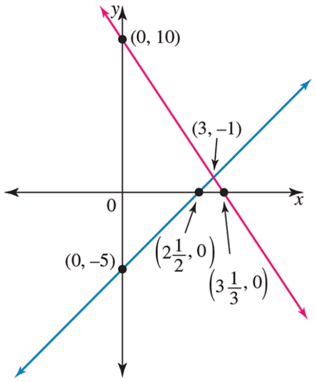
A.



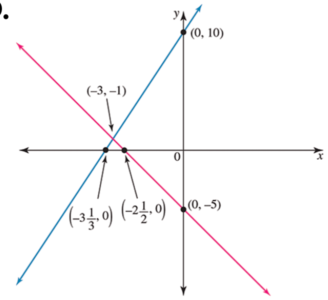
B.



C.

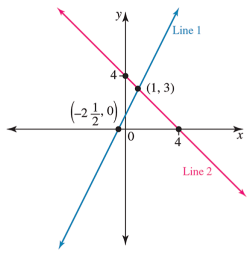


D.



E. None of the above

Question 5 (1 mark)
Which pair of equations is represented by this graph?



A. $4x + 4y = 0$ and $y = 2x + 1$
B. $y = -4x + 4$ and $y = \frac{1}{2}x + 1$
C. $y = x + 4$ and $y = 2x + 1$
D. $4y = 4x$ and $y = 2x + 1$
E. $y = 2x + 1$ and $y = -x + 4$

Question 6 (1 mark)
Sketch the following pair of lines and find their point of intersection.
$2x + 4y = 20$
$x + y = 6$

Question 7 (5 marks)
Two workmates are training to improve their fitness. Emma decides to go for a run and sets off at an average pace of 7 km per hour. Half an hour later Fiona decides to catch Emma by riding her bike at an average speed of 12 km per hour.
Draw a graph of the distance versus time for the two girls and find the solution for the distance they travelled and the time it took for them to meet.

Topic	1	Lines and linear relationships
Subtopic	1.4	Linear graphs and their equations

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Question 1 (1 mark)

The gradient of the line passing through the points $(-3, 5)$ and $(1, -4)$ is

A. $\frac{9}{2}$

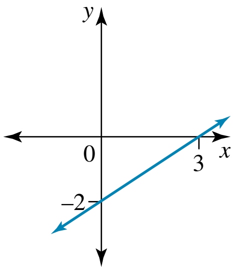
B. $-\frac{9}{4}$

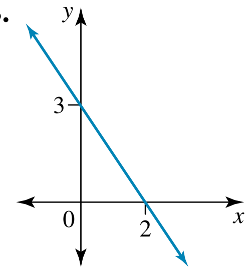
C. -1

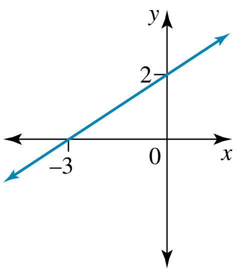
D. $-\frac{1}{4}$

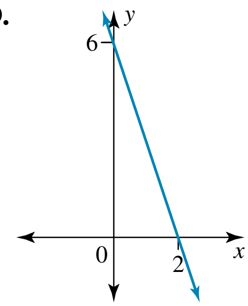
E. $\frac{1}{9}$

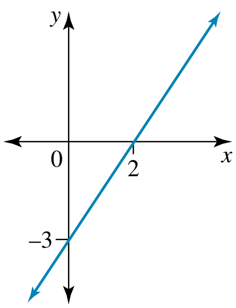
Question 2 (1 mark)
State which of the following graphs can represent the equation $-3y + 2x = -6$.

A.


B.


C.


D.


E.


Question 3 (2 marks)
Form the equation of the line passing through the points $(3, 1)$ and $(-1, 3)$.

Question 4 (1 mark)

The gradient, m, and y-intercept, c of the line with equation $3x + y - 5 = 0$ are

A. $m = 3, c = 5$
B. $m = 3, c = -5$
C. $m = -5, c = 3$
D. $m = -3, c = 5$
E. $m = -3, c = -5$

Question 5 (4 marks)

Sketch the graph of the line with equation $x + 3y = 12$, labelling the intercepts with the axes in your workbook.
Explain clearly whether the point (6, 12) lies on, above or below the line.

Topic	1	Lines and linear relationships
Subtopic	1.5	Intersections of lines and their applications

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Question 1 (1 mark)

The simultaneous equations $y = 2x - 1$ and $y = 2x + 1$

A. intersect at $(2, 0)$.
B. intersect at $(-1, 1)$.
C. do not intersect.
D. intersect at $\left(\frac{1}{2}, -\frac{1}{2}\right)$.
E. intersect at $(0, 2)$.

Question 2 (1 mark)

The value of a such that the lines $y = 4(2a + 1) + 2$ and $y = 2(a - 1)x - \frac{1}{2}$ will not intersect is

A. 1
B. -1
C. 4
D. 2
E. $-\frac{1}{2}$

Question 3 (5 marks)

Determine the values of a and b for which the system of equation $ax + y = b$ and $x + 3y = 2$ will have infinitely many solutions.

Topic	1	Lines and linear relationships
Subtopic	1.6	Straight lines and gradients

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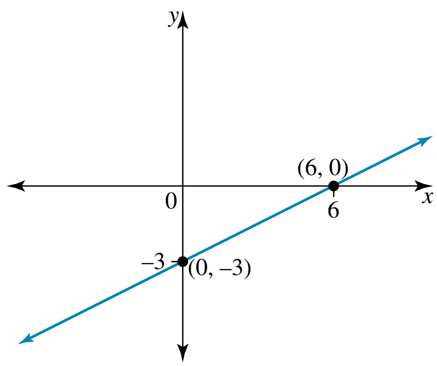
Question 1 (1 mark)

The equation of the line that would pass through the points shown on the graph is



A. $y = -\frac{x}{2} - 3$

B. $y = \frac{x}{2} - 3$

C. $y = 2x - 3$

D. $y = -2x - 3$

E. $y = -\frac{x}{2} + 3$

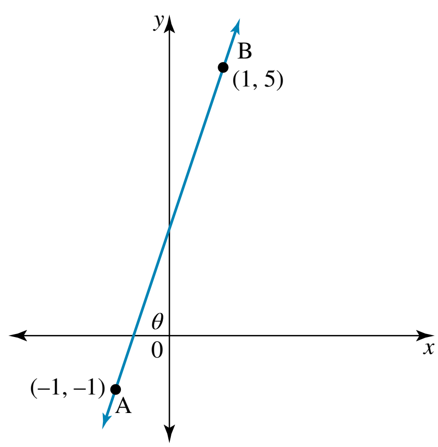
Question 2 (4 marks)

a. Line L_1 has equation $y = 3x$ and line L_2 has the equation $y = 3 - x$. Sketch the graphs for each line on the same axes, labelling them clearly. **(2 marks)**

b. Write down the equation of the line with the same gradient as L_1 and the same x-intercept as L_2. **(2 marks)**

Question 3 (1 mark)
To the nearest degree, the line with equation $6x + 10y - 3 = 0$ is inclined to the positive direction of the x-axis at an angle of:
A. 99°
B. 17°
C. 31°
D. 149°
E. 81°

Question 4 (1 mark)
The angle that the line shown makes with the positive direction of the x-axis is nearest to



A. 37°
B. 53°
C. 72°
D. 76°
E. 0.05°

Question 5 (3 marks)
a. Find the equation, in $y = mx + c$ form, of the straight line that makes an angle of 45° with the positive direction of the x-axis and passes through the point $(2, -1)$. **(2 marks)**

b. Sketch the line found in **part a.** **(1 mark)**

Question 6 (1 mark)

The equation of the line passing through the point $(3, -2)$ and perpendicular to the line with equation $2x - 3y + 1 = 0$ is

A. $2x - 3y = 0$
B. $3x + 2y - 5 = 0$
C. $3x - 2y - 13 = 0$
D. $2x + 3y = 0$
E. $2x - 3y - 12 = 0$

Question 7 (3 marks)

Find the equation of the line passing through the point $(-1, 2)$ and parallel to the line with equation $3x - 2y = 6$.

Topic	1	Lines and linear relationships
Subtopic	1.7	Bisection and lengths of line segments

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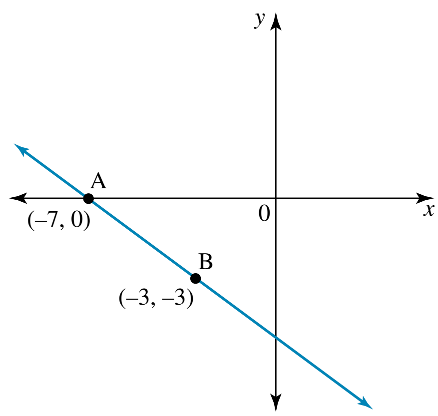
Question 1 (1 mark)

The distance between the points A and B on the graph shown is



A. 2
B. 13
C. 7
D. 5
E. 25

Question 2 (1 mark)

The midpoint of interval joining P $(-5, -3)$ and Q $(2, -1)$ is

A. $\left(-\frac{7}{2}, -2\right)$

B. $\left(-\frac{7}{2}, -1\right)$

C. $(-7, -4)$

D. $(-3, -4)$

E. $\left(-\frac{3}{2}, -2\right)$

Question 3 (4 marks)

For the points A (1, −5) and B (4, −2) find:

a. the gradient of the line joining A and B **(1 mark)**

b. the equation of the line that is perpendicular to AB and passes through A. **(3 marks)**

Topic	1	Lines and linear relationships
Subtopic	1.8	Review

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Question 1 (2 marks)

Sam is a very good student, and his latest test results show that the sum of his marks for Mathematical Methods, Chemistry and Physics is 256. Mathematical Methods is his best subject, and the difference between his Mathematical Methods and Chemistry marks is 8. His Chemistry mark was 1 mark higher than his Physics mark.

Determine his marks for each subject.

Question 2 (1 mark)

The gradient, m, and y-intercept, c, of the line with equation $3x + y - 5 = 0$ are

A. $m = 3, c = 5$
B. $m = 3, c = -5$
C. $m = -5, c = 3$
D. $m = -3, c = 5$
E. $m = -3, c = -5$

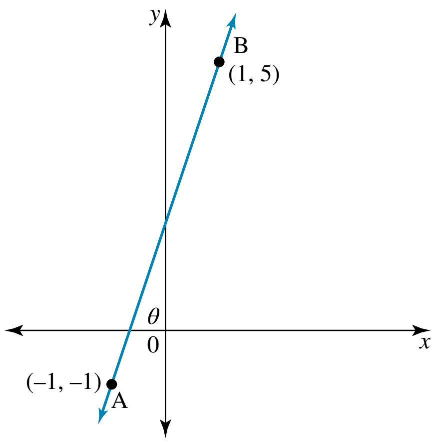
Question 3 (4 marks)

Find the value of a such that the system of equations has no solution.

$$2ax - 3y = 2a$$
$$8x - 10 = 2y$$

Question 4 (1 mark)

The angle that the line shown makes with the positive direction of the x-axis is nearest to



A. 37°
B. 53°
C. 72°
D. 76°
E. 0.05°

Question 5 (1 mark)

The equation of the line passing through the point $(3, -2)$ and perpendicular to the line with equation $2x - 3y + 1 = 0$ is

A. $2x - 3y = 0$
B. $3x + 2y - 5 = 0$
C. $3x - 2y - 13 = 0$
D. $2x + 3y - 1 = 0$
E. $2x - 3y - 12 = 0$

Question 6 (1 mark)

The solution to the equation $2x - 9 = 5x + 3$ is

A. $x = \frac{12}{7}$
B. $x = -4$
C. $x = 2$
D. $x = 4$
E. $x = -2$

Question 7 (1 mark)

Solve the literal equation for x, expressing the answer in simplest form.

$$\frac{x + a}{a} - 1 = \frac{x - b}{b}$$

A. $x = -\frac{ab}{a - b}$
B. $x = \frac{ab}{a - b}$
C. $x = \frac{3ab}{a - b}$
D. $x = \frac{ab}{a + b}$
E. $x = -\frac{3ab}{a - b}$

Question 8 (1 mark)

The solution to the simultaneous equations $y = \frac{x}{2}$ and $\frac{1}{2}x - 2y = -\frac{1}{4}$ is

A. $\left(\frac{1}{2}, \frac{1}{4}\right)$
B. $(2, 1)$
C. $\left(\frac{1}{4}, \frac{1}{2}\right)$
D. $\left(\frac{1}{2}, 1\right)$
E. $\left(\frac{1}{4}, 1\right)$

Question 9 (1 mark)

The gradient, m, and y-intercept, c, of the line with equation $4x - 2y + 6 = 0$ are

A. $m = -2$ and $c = 3$
B. $m = 4$ and $c = 6$
C. $m = 2$ and $c = 6$
D. $m = -4$ and $c = -6$
E. $m = 2$ and $c = 3$

Question 10 (1 mark)

For the system of linear equations $y = m_1x + c_1$ and $y = m_2x + c_2$, there will be an infinite number of solutions if

A. $m_1 = m_2$ and $c_1 = c_2$
B. $m_1 = m_2$ and $c_1 \neq c_2$
C. $m_1 = m_2$ and $c_1 = 0$
D. $m_1 \neq m_2$ and $c_1 = c_2$
E. $m_1 \neq m_2$ and $c_1 = 0$

Question 11 (1 mark)

The value of a such that the lines with equations $y = 3(a - 4)x - 1$ and $y = 2(3 - a)x + 4$ will not intersect is.

A. $\frac{5}{6}$
B. 3
C. 18
D. 6
E. $\frac{18}{5}$

Question 12 (1 mark)

The distance between the points $(2, 5)$ and $(-4, -1)$ correct to 2 decimal places, is

A. 3.46
B. 6.00
C. 4.47
D. 8.49
E. 6.49

Question 13 (1 mark)

The equation of the line that would pass through the points $(-2, 3)$ and $(-4, 8)$ is

A. $y = \frac{5x}{2} - 2$
B. $y = -\frac{5x}{2} + 2$
C. $y = -\frac{5x}{6} - \frac{4}{3}$
D. $y = -\frac{5x}{2} - 2$
E. $y = \frac{5x}{2} + 2$

Question 14 (1 mark)

The equation of the line passing through the point $(1, -4)$ and perpendicular to the line with equation $2y - 5x - 4 = 0$ is

A. $5y = -2x - 18$
B. $5y = -5x - 3$
C. $y = -2x - 18$
D. $2y = -5x + 22$
E. $5y = 2x + 18$

Question 15 (1 mark)

To the nearest degree, the line with equation $3x - 4y + 1 = 0$ is inclined to the positive direction of the x-axis at an angle of

A. 143°
B. 123°
C. 42°
D. 72°
E. 37°

Question 16 (3 marks)

Sketch the graph of the line with equation $3x - 4y = 2$, labelling all important points.

Question 17 (4 marks)

Hang is a very good student and her latest test results show that the sum of her marks for Mathematical Methods, Biology and Literature is 245. Mathematics Methods was her best subject, and the difference between her Mathematical Methods and Literature marks was 4. Her Biology mark was 2 marks lower than her Literature mark. What were her marks for each subject?

Answers and marking guide

1.2 Linear equations and inequations

Question 1

$$\frac{x-a}{b}-2=\frac{b-x}{a}$$

$$\frac{a(x-a)}{ab}-\frac{2ab}{ab}=\frac{b(b-x)}{ab} \qquad \textbf{[1 mark]}$$

$$ax-a^2-2ab=b^2-bx$$

$$ax+bx=b^2+2ab+a^2 \qquad \textbf{[1 mark]}$$

$$x(a+b)=(a+b)^2$$

$$x=a+b \qquad \textbf{[1 mark]}$$

Question 2

$3(x-4)\le 6$

$3x-12\le 6$

$3x\le 18$

$x\le 6$

The correct answer is **E**.

Question 3

$$\frac{x}{5}-\frac{2x-1}{3}\ge -2$$

$$\frac{3x-5(2x-1)}{15}\ge -2 \qquad \textbf{[1 mark]}$$

$$3x-10x+5\ge -30$$

$$-7x\ge -35 \qquad \textbf{[1 mark]}$$

$$x\le 5 \qquad \textbf{[1 mark]}$$

Question 4

$3x+12=4x-16$

$3x-4x=-16-12$

$-x=-28$

$x=28$

Question 5

$$\text{speed}=\frac{\text{distance}}{\text{time}}$$

$$\therefore \text{time}=\frac{\text{distance}}{\text{speed}}$$

Time taken upstream $=\dfrac{x}{3}$

Time taken downstream $=\dfrac{x}{6}$

50 minutes in hours is $\dfrac{50}{60}=\dfrac{5}{6}$

Total time taken $=\dfrac{x}{3}+\dfrac{x}{6}=\dfrac{5}{6}$

Question 6

Equation I:

$$y - 5 = 2x$$
$$y = 2x + 5$$
$$\therefore m = 2$$

Equation II:

$$4x - 2y = 3$$
$$2y = 4x - 3$$
$$y = 2x - \frac{3}{2}$$
$$\therefore m = 2 \text{ (the gradients are equal)}$$
$$\therefore \text{lines I and II are parallel.}$$

1.3 Systems of simultaneous linear equations

Question 1

$$3y = -4x - 4 \quad [1]$$
$$2x - \frac{y}{4} = 5 \quad [2]$$

In [1], move the term to the left-hand side. Multiply [2] by 2:

$$4x + 3y = -4$$
$$4x - \frac{y}{2} = 10$$

Subtract [2] from [1]:

$$3y + \frac{y}{2} = -14$$
$$\frac{7y}{2} = -14$$
$$7y = -28$$
$$y = -4$$

Substitute $y = -4$ into [1]:

$$-12 = -4x - 4$$
$$x = 2$$

$\therefore (x,\ y) = (2,\ -4)$

The correct answer is **B**.

Question 2

$$x - 4 = 4y + 8 \quad [1]$$
$$3x - 6 = 2y + 20 \quad [2]$$

Multiply [1] by 3:

$$3x - 12 = 12y + 24$$
$$3x - 6 = 2y + 20$$

Subtract the equations:

$$-6 = 10y + 4$$
$$10y = -10$$
$$y = -1$$

The correct answer is **C**.

Question 3

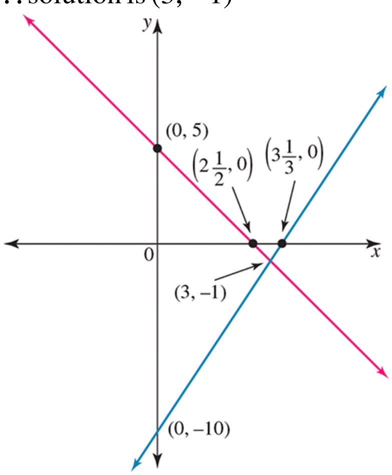
$$y = x + 2 \quad [1]$$
$$\frac{1}{4}x + y = 0 \quad [2]$$

Substitute [1] into [2]:

$$\frac{1}{4}x + (x + 2) = 0$$
$$\frac{5}{4}x = -2$$
$$5x = -8$$
$$x = \frac{-8}{5}$$
$$x = -1.6$$
Substitute $x = -1.6$ into [1]
$$y = -1.6 + 2$$
$$y = 0.4$$
Therefore, the solution is $(-1.6, 0.4)$.
The correct answer is **E**.

Question 4

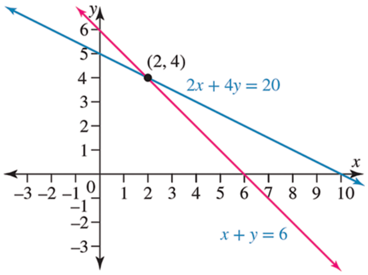
$$y = 5 - 2x \qquad (1)$$
$$y = 3x - 10 \qquad (2)$$
Substitute (1) into (2)
$$5 - 2x = 3x - 10$$
$$15 = 5x$$
$$x = 3$$
Substitute $x = 3$ into (1)
$$y = 5 - 2x$$
$$y = 5 - 2(3)$$
$$y = -1$$
$\therefore$ solution is $(3, -1)$



Question 5
Line 1:
$c = 1, m = \frac{1}{\frac{1}{2}} = 2$
$\Rightarrow y = 2x + 1$
Line 2:
$c = 4, m = -\frac{4}{4} = -1$
$\Rightarrow y = -x + 4$
Check point (1,3) by substituting for x

Line 1:
$y = 2x + 1$
$y = 2(1) + 1$
$y = 3$
$\therefore (1, 3)$ is on line 1.
Line 2:
$y = -x + 4$
$y = -1 + 4$
$y = 3$
$\therefore (1, 3)$ is on line 2
$\therefore$ the two equations are $y = 2x + 1$ and $y = -x + 4$

Question 6
Line 1:
$2x + 4y = 20$
x-intercept ($y = 0$)
$2x + 4(0) = 20$
$x = 10$
$\Rightarrow (10, 0)$
y-intercept ($x = 0$)
$2(0) + 4y = 20$
$y = 5$
$\Rightarrow (0, 5)$ [**1 mark**]
Line 2:
$x + y = 6$
x-intercept ($y = 0$)
$x + (0) = 6$
$x = 6$
$\Rightarrow (6, 0)$
y-intercept ($x = 0$)
$(0) + y = 6$
$y = 6$
$\Rightarrow (0, 6)$ **[1 mark]**
Point of intersection:
$2x + 4y = 20 \quad (1)$
$x + y = 6 \quad (2)$
Multiply (2) by 2
$2x + 2y = 12 \quad (3)$
Subtract (3) from (1)
$2y = 8$
$y = 4$
Substitute $y = 4$ into (2)
$x + (4) = 6$
$x = 2$
$\therefore$ point of intersection is (2, 4). **[1 mark]**



From the graph, the point of intersection is (2, 4). **[1 mark]**

Question 7

Write equations to represent both Emma's and Fiona's activity.

Emma:

$d = 7t$

Fiona:

$m = 12$

$d - 0 = 12(t - 0.5)$

$d = 12(t - 0.5)$

$d = 12t - 6$ **[1 mark]**

Solve simultaneously:

$12t - 6 = 7t$

$5t = 6$

$t = 1.2$ **[1 mark]**

Substitute $t = 1.2$ into the first equation:

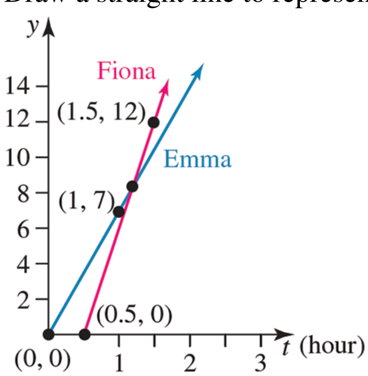
$d = 7(1.2)$

$d = 8.4$ **[1 mark]**

∴ The girls meet after 1 hour and 12 minutes and were 8.4 km from the starting point. **[1 mark]**

Draw a straight line to represent Emma's journey using points (0, 0) and (1, 7).

Draw a straight line to represent Fiona's journey using (0.5, 12).



[1 mark]

1.4 Linear graphs and their equations

Question 1

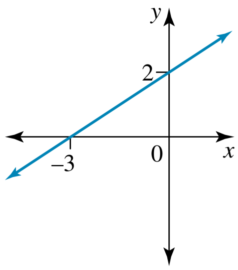
Let $(x_1, y_1) = (-3, 5)$.

Let $(x_2, y_2) = (1, -4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 5}{1 - (-3)} = -\frac{9}{4}$$

The correct answer is **B**.

Question 2



$-3y + 2x = -6$

x-intercept when $y = 0$:

$-3(0) + 2x = -6$

$x = -3$

Therefore, the x-intercept is $(-3, 0)$.

y-intercept when $x = 0$:

$-3y + 2(0) = -6$

$y = 2$

Therefore, the x-intercept is $(0, 2)$.

The correct answer is **C**.

Question 3

$y - y_1 = m(x - x_1)$

$$m = \frac{3-1}{-1-3}$$

$$= \frac{2}{-4} \qquad \textbf{[1 mark]}$$

$$y - 1 = -\frac{1}{2}(x - 3)$$

$$y - 1 = -\frac{1}{2}x + \frac{3}{2}$$

$$y = -\frac{1}{2}x + \frac{5}{2} \qquad \textbf{[1 mark]}$$

Question 4

$y = mx + c$

$3x + y - 5 = 0$

$y = -3x + 5$

$\therefore m = -3, c = 5$

Question 5

x-intercept $(y = 0)$:

$x + 3(0) = 12$

$x = 12$

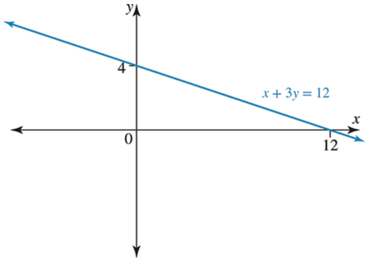
$\Rightarrow (12, 0)$

y-intercept $(x = 0)$:

$0 + 3y = 12$

$y = 4$

$\Rightarrow (0, 4)$ **[1 mark]**



[1 mark]

$$x + 3y = 12$$
$$(6) + 3(12) = 12$$
$$42 > 12$$

$\therefore$ point (6,12) lies above the line (it can also be seen by looking at the graph). **[1 mark]**

1.5 Intersections of lines and their applications

Question 1

$y = mx + c$

$y = 2x - 1$

$m = 2,\ c = -1$

$y = 2x + 1$

$m = 2,\ c = 1$

Therefore, the two lines have the same gradient but different y-intercepts, so they are parallel and will never intersect.

The correct answer is **C**.

Question 2

Parallel lines have the same gradient and do not intersect.

For the gradients to be equal:

$$4(2a + 1) = 2(a - 1)$$
$$8a + 4 = 2a - 2$$
$$a = -1$$

The correct answer is **B**.

Question 3

Linear equations that have equal gradients and equal y-intercepts have infinitely many solutions.

$ax + y = b$

$x + 3y = 2$

Rewrite the equations in $y = mx + c$ form:

$y = -ax + b$ [1]

$3y = -x + 2$ [2]

$y = -\frac{1}{3}x + \frac{2}{3}$

$m_1 = -a, \quad m_2 = -\frac{1}{3}$ **[1 mark]**

$-a = -\frac{1}{3}$

$\therefore a = \frac{1}{3}$ **[1 mark]**

$c_1 = b, \quad c_2 = \frac{2}{3}$ **[1 mark]**

$\therefore b = \frac{2}{3}$ **[1 mark]**

Therefore, the equations will have infinitely many solutions when $a = \frac{1}{3}$ and $b = \frac{2}{3}$. **[1 mark]**

1.6 Straight lines and gradients

Question 1

$$\text{Gradient, } m = \frac{0-(-3)}{6-0} = \frac{1}{2}$$

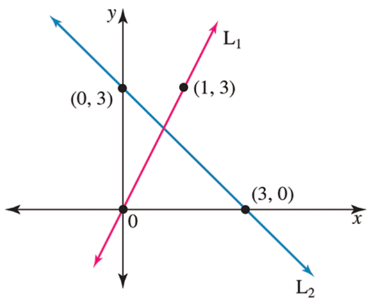
y-intercept, $c = -3$

$$y = mx + c$$
$$y = \frac{1}{2}x - 3$$

The correct answer is **B**.

Question 2

a



Line 1:
$y = 3x$
$x = 0,\ y = 0$
$x = 1,\ y = 3$
$\therefore$ points at $(0, 0)$ and $(1, 3)$
Line 2:
$y = 3 - x$
$x = 0,\ y = 3$
$y = 0,\ x = 3$
$\therefore$ points at $(0, 3)$ and $(3, 0)$
Award 1 mark for correctly drawing line 1.
Award 1 mark for correctly drawing line 2.

b L_1: $m = 3$
L_2: x-intercept is point $(3, 0)$ **[1 mark]**
Equation of a line:
$$y - y_1 = m(x - x_1)$$
$$y - 0 = 3x - 9$$
$$y = 3x - 9$$ **[1 mark]**

Question 3

$$6x + 10y - 3 = 0$$
$$10y = -6x + 3$$
$$y = -\frac{6}{10}x + \frac{3}{10}$$
$$m = -\frac{6}{10} = -0.6 = \tan\theta$$

Since $\tan\theta$ is negative,
$\theta = 180° - \tan^{-1}(0.6)$
$\approx 149°$
The correct answer is **D**.

Question 4

Gradient, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Let $(x_1, y_1) = (-1, -1)$ and $(x_2, y_2) = (1, 5)$

$$m = \frac{5-(-1)}{1-(-1)}$$
$$= \frac{6}{2}$$
$$= 3$$
$$m = \tan\theta$$
$$\therefore \theta = \tan^{-1}(m)$$
$$\theta = \tan^{-1}(3)$$
$$\approx 72°$$

Question 5

a. $m = \tan(\theta)$

$= \tan(45°)$

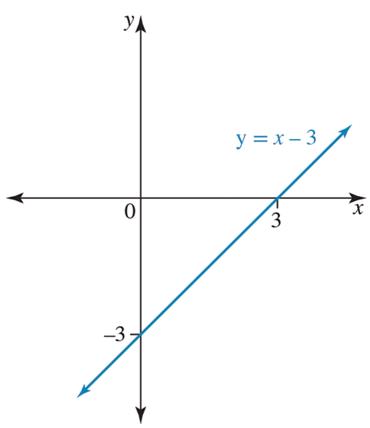
$= 1$ **[1 mark]**

$y - y_1 = m(x - x_1)$
Let $(2, -1)$ be (x_1, y_1)
$y - (-1) = 1(x - 2)$

$y + 1 = x - 2$

$y = x - 3$ **[1 mark]**

b. **[1 mark]**



Question 6

$2x - 3y + 1 = 0$

$3y = 2x + 1$

$y = \dfrac{2}{3}x + \dfrac{1}{3}$

Gradient: $m = \dfrac{2}{3}$

Gradient of perpendicular line: $m = -\dfrac{3}{2}$

Equation of the line through $(3, -2)$ and with $m = -\dfrac{3}{2}$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{3}{2}(x - 3)$$
$$y + 2 = -\frac{3}{2}x + \frac{9}{2}$$
$$y = -\frac{3}{2}x + \frac{5}{2}$$
$$2y = -3x + 5$$
$$2y + 3x - 5 = 0$$

Question 7

$$3x - 2y = 6$$

Write in $y = mx + c$ form to find m

$$-2y = -3x + 6$$
$$y = \frac{3}{2}x - 3$$
$$\therefore m = \frac{3}{2}$$ **[1 mark]**

Equation of a line through a point: $y - y_1 = m(x - x_1)$

$$m = \frac{3}{2}, (x_1, y_1) = (-1, 2)$$
$$y - 2 = \frac{3}{2}(x - (-1))$$ **[1 mark]**
$$y - 2 = \frac{3}{2}(x + 1)$$
$$2y - 4 = 3(x + 1)$$
$$2y - 4 = 3x + 3$$
$$2y - 3x - 7 = 0$$ **[1 mark]**

1.7 Bisection and lengths of line segments

Question 1

A: $(-7,\ 0) = (x_1,\ y_1)$

B: $(-3,\ -3) = (x_2,\ y_2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-3 - (-7))^2 + (-3 - 0)^2}$$
$$= \sqrt{16 + 9}$$
$$= 5$$

The correct answer is **D**.

Question 2

Let P $(-5, -3) = (x_1, y_1)$.

Let Q $(2,\ -1) = (x_2,\ y_2)$.

$$\text{Mid-point} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-5 + 2}{2}, \frac{-3 + (-1)}{2}\right)$$
$$= \left(-\frac{3}{2}, -2\right)$$

The correct answer is **E**.

Question 3

a Let A $(1, -5) = (x_1, y_1)$

Let B $(4, -2) = (x_2, y_2)$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2-(-5)}{4-1} \\ &= \frac{3}{3} \\ &= 1 \end{aligned}$$ **[1 mark]**

b For perpendicular lines, $m_1 m_2 = -1$.

The gradient of the perpendicular line to AB $= -1$. **[1 mark]**

The equation of a line through a point: $y - y_1 = m(x - x_1)$

$A = (x_1, y_1) = (1, -5), m = -1$ **[1 mark]**

$$\begin{aligned} y - (-5) &= -1(x-1) \\ y + 5 &= -x + 1 \\ y &= -x - 4 \end{aligned}$$

The equation of the line perpendicular to AB and passing through A is $y = -x - 4$. **[1 mark]**

1.8 Review

Question 1

Let Sam's Mathematical Methods mark be x, Chemistry mark be y, and Physics mark be z.

$$\begin{aligned} x + y + z &= 256 \quad [1] \\ x - y &= 8 \quad [2] \\ y &= z + 1 \quad [3] \end{aligned}$$ **[1 mark]**

Using CAS technology, Sam's Mathematical Methods mark is 91, his Chemistry mark is 83 and his Physics mark is 82. [1 mark]

Question 2

$$\begin{aligned} y &= mx + c \\ 3x + y - 5 &= 0 \\ y &= -3x + 5 \\ \therefore m &= -3, \ c = 5 \end{aligned}$$

The correct answer is **D**.

Question 3

A system of linear equations that has no solution has equations with equal gradients and different y-intercepts (parallel lines).

Rewrite the equations in $y = mx + c$ form:

$$\begin{aligned} 2ax + 3y &= 2a \\ 3y &= -2ax + 2a \\ y &= -\frac{2}{3}ax + \frac{2}{3}a \\ m_1 &= -\frac{2}{3}a, \quad c_1 = \frac{2}{3}a \end{aligned}$$ **[1 mark]**

$$\begin{aligned} 8x - 10 &= 2y \\ y &= 4x - 5 \\ m_2 &= 4, \quad c_2 = -5 \end{aligned}$$ **[1 mark]**

Equate the gradients:

$$-\frac{2}{3}a = 4$$
$$2a = -12$$
$$a = -6 \qquad \textbf{[1 mark]}$$
$$m_1 = -\frac{2}{3}(-6) = 4$$
$$c_1 = \frac{2}{3}(-6) = -4$$

$\Rightarrow m_1 = m_2$ and $c_1 \neq c_2$

Therefore, the lines are parallel and there is no solution. **[1 mark]**

Question 4

Gradient, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Let $(x_1, y_1) = (-1, -1)$ and $(x_2, y_2) = (1, 5)$.

$$m = \frac{5-(-1)}{1-(-1)} = \frac{6}{2} = 3$$
$$m = \tan\theta$$
$$\therefore \theta = \tan^{-1}(m)$$
$$\theta = \tan^{-1}(3)$$
$$\theta \approx 72°$$

The correct answer is **C**.

Question 5

$$2x - 3y + 1 = 0$$
$$3y = 2x + 1$$
$$y = \frac{2}{3}x + \frac{1}{3}$$

Gradient: $m = \dfrac{2}{3}$

Gradient of the perpendicular line: $m = -\dfrac{3}{2}$

Equation of the line through $(3, -2)$ and with $m = -\dfrac{3}{2}$:

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{3}{2}(x - 3)$$
$$y + 2 = -\frac{3}{2}x + \frac{9}{2}$$
$$y = -\frac{3}{2}x + \frac{5}{2}$$
$$2y = -3x + 5$$

$$2y + 3x - 5 = 0$$

The correct answer is **B**.

Question 6

$$\begin{aligned} 2x - 9 &= 5x + 3 \\ 2x - 5x &= 3 + 9 \\ -3x &= 12 \\ x &= -4 \end{aligned}$$

Question 7

$$\begin{aligned} \frac{x+a}{a} - 1 &= \frac{x-b}{b} \\ b(x+a) - ab &= a(x-b) \\ bx + ab - ab &= ax - ab \\ bx - ax &= -ab \\ x(b-a) &= -ab \\ x &= \frac{-ab}{b-a} \\ &= \frac{ab}{a-b} \end{aligned}$$

Question 8

$y = \frac{x}{2}$ (1)

$\frac{1}{2}x - 2y = -\frac{1}{4}$ (2)

Substitute (1) into (2)

$$\begin{aligned} \frac{1}{2}x - 2\left(\frac{x}{2}\right) &= -\frac{1}{4} \\ \frac{1}{2}x - x &= -\frac{1}{4} \\ -\frac{1}{2}x &= -\frac{1}{4} \\ x &= \frac{1}{2} \end{aligned}$$

substitute $x = \frac{1}{2}$ into (1)

$$\begin{aligned} y &= \frac{1}{2} \div 2 \\ &= \frac{1}{4} \end{aligned}$$

Therefore, the solution or point of intersection of the two equations is $\left(\frac{1}{2}, \frac{1}{4}\right)$

Question 9

$$\begin{aligned} 0 &= 4x - 2y + 6 \\ 2y &= 4x + 6 \\ y &= 2x + 3 \\ y &= mx + c,\ m = 2,\ c = 3 \end{aligned}$$

Question 10

An infinite number of solutions exist between two lines if the lines have the same gradient and the same y-intercept.

Question 11

Parallel lines have the same gradient and do not intersect.

For the gradients to be equal

$3(a-4) = 2(3-a)$

$$\begin{aligned} 3a - 12 &= 6 - 2a \\ 5a &= 18 \\ a &= \frac{18}{5} \end{aligned}$$

Question 12

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\begin{aligned} d &= \sqrt{(-4-2)^2 + (-1-5)^2} \\ &= \sqrt{36+36} \\ &= 8.49 \end{aligned}$$

Question 13

Gradient

$$\begin{aligned} m &= \frac{8-3}{-4-(-2)} \\ &= -\frac{5}{2} \end{aligned}$$

y-intercept

Substitute $(-2, 3)$

$y = -\frac{5}{2}x + c$

$3 = -\frac{5}{2} \times -2 + c$

$3 = 5 + c$

$c = -2$

$\therefore y = -\frac{5x}{2} - 2$

Question 14

$$\begin{aligned} 0 &= 2y - 5x - 4 \\ 2y &= 5x + 4 \\ y &= \frac{5}{2}x + 2 \end{aligned}$$

Gradient $m = \frac{5}{2}$

Gradient of perpendicular line: $m = -\frac{2}{5}$

Equation of line through $(1, -4)$ and with $m = -\frac{2}{5}$

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = -\frac{2}{5}(x - 1)$$
$$y + 4 = -\frac{2}{5}x + \frac{2}{5}$$
$$y = -\frac{2}{5}x - \frac{18}{5}$$
$$5y = -2x - 18$$

Question 15

$$0 = 3x - 4y + 1$$
$$4y = 3x + 1$$
$$y = \frac{3}{4}x + \frac{1}{4}$$
$$m = \frac{3}{4}$$
$$= \tan\theta$$
$$\theta = \tan^{-1}\left(\frac{3}{4}\right)$$
$$\approx 37°$$

Question 16

x-intercept ($y = 0$):

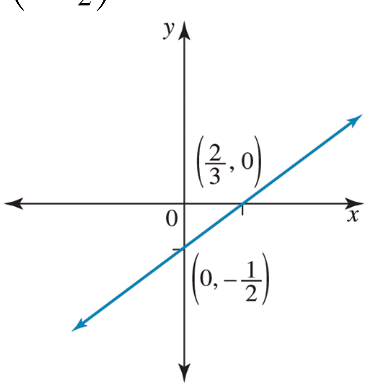
$$3x = 2$$
$$x = \frac{2}{3}$$

$\left(\frac{2}{3}, 0\right)$ **[1 mark]**

y-intercept ($x = 0$):

$$-4y = 2$$
$$y = -\frac{1}{2}$$

$\left(0, -\frac{1}{2}\right)$ **[1 mark]**



[1 mark]

Question 17

Let her Mathematical Methods mark be x, her Literature mark be y and her Biology mark be z.

$x + y + z = 245$..............(1)

$x - y = 4$..............(2)

$y = z + 2$............(3)

Subtracting (2) from (1) gives $2y + z = 241$.......(4)

Substitute (3) into (4)

$2(z + 2) + z = 241$

$2z + 4 + z = 241$

$3z + 4 = 241$

$3z = 237$

$z = 79$ **[1 mark]**

Substitute $z = 79$ into (3)

$y = 79 + 2$

$= 81$ **[1 mark]**

Substitute $z = 81$into (2)

$x - 81 = 4$

$x = 85$ **[1 mark]**

Hang's Mathematical Methods mark was 85, her Literature mark was 81 and her Biology mark was 79.

2 Algebraic foundations

Topic	2	Algebraic foundations
Subtopic	2.2	Algebraic skills

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Question 1 (1 mark)
The coefficient of x in the expansion of $x(x-2)(x+1)+(x-3)^2$ is
A. 12
B. 8
C. 4
D. -4
E. -8

Question 2 (1 mark)
The factors of $9(2x-1)^2-9$ are
A. $(1, 0)$
B. $2x-1, 9$
C. $\left(-\frac{1}{2}, 9\right)$
D. $36x, x-1$
E. $36x^2-36x$

Question 3 (1 mark)
Factorise $8(x + 3)^2 + 24(x + 3) + 16$.

Question 4 (1 mark)
$\dfrac{(a+b)^2}{a^3+b^3}$ is equivalent to

A. $\dfrac{(a+b)^2}{a^2-ab+b^2}$

B. $\dfrac{(a+b)^2}{a^2+b^2}$

C. $\dfrac{(a+b)^2}{a^2+ab+b^2}$

D. 1

E. $\dfrac{(a-b)^2}{a^2-b^2}$

Question 5 (1 mark)
The factorised form of $4(x - 2)^2 - 5(x - 2) - 6$ is

A. $(4x + 5)(x + 4)$
B. $(4x + 1)(x - 4)$
C. $(x - 2)(x - 4)$
D. $(4x - 5)(x - 4)$
E. $(4x + 5)(x - 4)$

Topic	2	Algebraic foundations
Subtopic	2.3	Pascal's triangle and binomial expansions

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Question 1 (1 mark)

The coefficient of x^3 in the expansion of $(x+2)^5$ is

A. 120
B. 40
C. 1
D. 8
E. 20

Question 2 (1 mark)

The coefficient of the term independent of in the expansion of $\left(x-\dfrac{2}{x}\right)^4$ is

A. 4
B. −6
C. 6
D. −24
E. 24

Question 3 (1 mark)

Expand $(x-2)^4-(x-2)^3$ and hence show that the coefficient of x is −44.

Question 4 (1 mark)

The coefficient of x^3y^3 in the expansion of $(x - 2y)^6$ is

A. 20
B. 15
C. −120
D. 120
E. −160

Question 5 (2 marks)

Expand $(1 - x)^3$. Hence, using a suitably chosen value of x, evaluate $(0.8)^3$ using the expansion.

Topic	2	Algebraic foundations
Subtopic	2.4	The binomial theorem

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Question 1 (1 mark)

The value of $\frac{4!+3!-2!}{4!-3!+2!}$ is

A. 1

B. -1

C. 20

D. $\frac{7}{5}$

E. $-\frac{7}{5}$

Question 2 (1 mark)

The value of $3\times\binom{5}{3}+4\times\binom{4}{2}$ is

A. 13
B. 16
C. 21
D. 54
E. 78

Question 3 (4 marks)

Identify which term in the expansion of $\left(3-4x^3\right)^7$ would contain x^{12} and express the coefficient of x^{12} as a product of its prime factors.

Question 4 (1 mark)

$\binom{3n}{3}$ is equal to

A. $\frac{n(n-1)}{3}$

B. $\frac{3n}{6}$

C. $\frac{n}{2}\left(9n^2+9n+2\right)$

D. $\frac{n}{3}\left(9n^2-9n-2\right)$

E. $\frac{n}{2}\left(9n^2-9n+2\right)$

Question 5 (3 marks)

Find the 4th term in the expansion of $\left(\frac{x^2}{2}+\frac{x}{3}\right)^8$.

Question 6 (1 mark)

The value of $5\times\binom{6}{3}+2\times\binom{4}{3}$ is

Topic	2	Algebraic foundations
Subtopic	2.5	Sets of real numbers

online only

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Question 1 (1 mark)

Identify which of the following represents a rational number.

A. $\sqrt{4+1}$

B. $\left(\sqrt{3+\sqrt{5}}\right)^2$

C. $\dfrac{2}{3-\sqrt{9}}$

D. $\sqrt{9}+\sqrt{16}$

E. $\sqrt{9-9\times 4}$

Question 2 (1 mark)

Determine the values of x for which $\dfrac{x-2}{x(x-1)(x+3)}$ would be undefined. State the reason.

Question 3 (1 mark)

The set of numbers described by $R\backslash[-2,5)$ is the same as

A. $\{x:-2\le x\le 5\}$

B. $\{x:x<-2\}\cup\{x:x\ge 5\}$

C. $\{x:x\le -2\}\cup\{x:x>5\}$

D. $\{x:-5\le x\le 2\}$

E. $\{x:x<5\}\cup\{x:x\ge -2\}$

Question 4 (1 mark)

The set of numbers described by $R\backslash(4, 8]$, is the same as

A. $\{x : x < 4\} \cup \{x : x \geq 8\}$

B. $\{x : 4 \leq x < 8\}$

C. $\{x : 4 < x \leq 8\}$

D. $\{x : x \leq 4\} \cup \{x : x \geq 8\}$

E. $\{x : x \leq 4\} \cup \{x : x > 8\}$

Question 5 (1 mark)

$\frac{1}{2}$ and $\sqrt{2}$ respectively belong to which of the following sets?

A. Q and R

B. Q and Z

C. R and Q

D. Z and R

E. Z and Q

Topic	2	Algebraic foundations
Subtopic	2.6	Surds

online only

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Question 1 (1 mark)

Simplify $5\sqrt{6} \times 3\sqrt{2} - \sqrt{3}\left(4\sqrt{3} + 2\right)$.

A. $28\sqrt{3} - 12$
B. $8\sqrt{12} - 12 - \sqrt{6}$
C. $32\sqrt{3} - 12$
D. $16\sqrt{6} - 12$
E. $9\sqrt{3} + 5\sqrt{2}$

Question 2 (1 mark)

Expand and simplify $\left(3\sqrt{2} - 2\sqrt{3}\right)\left(3\sqrt{2} + 2\sqrt{3}\right)$.

Question 3 (1 mark)

$\dfrac{\sqrt{2} - \sqrt{3}}{\sqrt{2} + \sqrt{3}}$ is equal to

A. $5 + 2\sqrt{6}$
B. $-5 - 2\sqrt{6}$
C. -1
D. $5 - 2\sqrt{6}$
E. $2\sqrt{6} - 5$

Question 4 (1 mark)

The surds in the set of numbers $\{2\sqrt{3}, 7\sqrt{5}, 4\sqrt{9}, 10\sqrt{6}, 12\sqrt{7}, 11\sqrt{64}\}$ are

A. all of these.

B. $4\sqrt{9}$ and $11\sqrt{64}$.

C. $2\sqrt{3}$ and $12\sqrt{7}$.

D. $2\sqrt{3}, 7\sqrt{5}$ and $10\sqrt{6}$.

E. $2\sqrt{3}, 7\sqrt{5}, 10\sqrt{6}$ and $12\sqrt{7}$.

Question 5 (1 mark)

$3\sqrt{128} + \sqrt{250} + \sqrt{112} - \sqrt{63}$ in simplest form equals

A. $24\sqrt{2} + 5\sqrt{10} + \sqrt{7}$

B. $24\sqrt{2} + 5\sqrt{10} - \sqrt{7}$

C. $38\sqrt{2} + 10\sqrt{5} + \sqrt{7}$

D. $38\sqrt{2} - 10\sqrt{5} + \sqrt{7}$

E. $2\sqrt{24} + 10\sqrt{5} + \sqrt{7}$

Question 6 (3 marks)

Simplify $\dfrac{6}{\sqrt{75}} + \sqrt{128} \div \sqrt{2}$. Give your answer in rationalised form.

Question 7 (3 marks)

Expand and simplify $\left(3x-\sqrt{5}\right)^{3}+\left(\sqrt{5x}+\sqrt{5}\right)^{2}$.

Question 8 (1 mark)

$\dfrac{6\sqrt{5}-4\sqrt{3}}{\sqrt{12}}$ is equal to

A. $\sqrt{15}-12$

B. $\dfrac{6\sqrt{5}-12}{6}$

C. $\sqrt{5}-2$

D. $\sqrt{15}-2$

E. $5-\sqrt{2}$

Topic	2	Algebraic foundations
Subtopic	2.7	Review

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Question 1 (1 mark)

$\frac{(a-b)^3}{a^3-b^3}$ simplifies to

A. $\frac{a-b}{a+b}$

B. $\frac{(a-b)^2}{(a+b)^2}$

C. $\frac{(a-b)^2}{a^2+ab+b^2}$

D. $-\frac{1}{ab}$

E. 1

Question 2 (1 mark)

Simplify $\frac{p^2-9q^2}{(p-3q)^2} \div \frac{(p+3q)^2}{4p-12q}$.

Question 3 (1 mark)

Select the sets to which π and $\sqrt{169}$ respectively belong.

A. R and N
B. Q and N
C. R and Z
D. R and Q
E. Q and Z

Question 4 (2 marks)

Illustrate the interval $(-3, 1]$ on a number line and write the set in alternative notation.

Question 5 (1 mark)

$\dfrac{5\sqrt{6}-2\sqrt{2}}{\sqrt{8}}$ is equal to

A. $\dfrac{5\sqrt{6}-2}{4}$
B. $\dfrac{5\sqrt{3}-2}{2}$
C. $\dfrac{5\sqrt{12}-4}{8}$
D. $5\sqrt{3}$
E. $\dfrac{3\sqrt{8}}{8}$

Question 6 (1 mark)

The coefficient of x^4 in the expansion of $(x-3)^6$ is

A. 90
B. -135
C. 135
D. -90
E. 115

Question 7 (4 marks)

Given that $x = 5\sqrt{3} - 3\sqrt{2}$ find

a. $x^2 =$ **(2 marks)**

b. $x^2 + 3x + 2 =$ **(2 marks)**

Question 8 (1 mark)

$\dfrac{\sqrt{2}+1}{3\sqrt{2}}$ is equal to

A. $\dfrac{1+\sqrt{2}}{2}$
B. $\dfrac{2+\sqrt{2}}{6}$
C. $\dfrac{1}{3}$
D. $1+\sqrt{2}$
E. $\dfrac{2\sqrt{6}+\sqrt{2}}{12}$

Question 9 (1 mark)

The coefficient of the term independent of x in the expansion of $\left(2x + \frac{1}{x}\right)^4$ is

A. 24
B. 8
C. 4
D. 12
E. 16

Question 10 (1 mark)

For what values of x would $\frac{x+4}{2x(x-4)(x+1)}$ be undefined?

A. $x = 0, 4, -1$
B. $x = 2, 4, -1, -4$
C. $x = 2, 4, -1$
D. $x = 0, 4, -1, -4$
E. $x = 2, -4, 1$

Question 11 (1 mark)

$\left(2\sqrt{5} - \sqrt{6}\right)\left(2\sqrt{5} + \sqrt{6}\right)$ expanded is

A. 4
B. $14 - 4\sqrt{30}$
C. 14
D. $14 + 4\sqrt{30}$
E. 24

Question 12 (4 marks)

Expand $(x+3)^4 - (x+3)^3$ and hence show that the coefficient of x is 81.

Answers and marking guide

2.2 Algebraic skills

Question 1

$$\begin{aligned} x(x-2)(x+1)+(x-3)^2 &= x\left(x^2-x-2\right)+x^2-6x+9 \\ &= x^3-x^2-2x+x^2-6x+9 \\ \text{Coefficient of } x &= -2-6 \\ &= -8 \end{aligned}$$

The correct answer is **E**.

Question 2

$$\begin{aligned} 9(2x-1)^2-9 &= 9\left[(2x-1)^2-1\right] \\ &= 9\left[(2x-1+1)(2x-1-1)\right] \\ &= 9(2x)(2x-2) \\ &= 18x(2x-2) \\ &= 36x(x-1) \end{aligned}$$

Therefore, the factors are $36x$ and $x-1$.
The correct answer is **D**.

Question 3

$8(x+3)^2+24(x+3)+16$

Let $(x+3)=a$.

$$\begin{aligned} 8a^2+24a+16 &= 8\left(a^2+3a+2\right) \\ &= 8(a+2)(a+1) \end{aligned}$$

Substitute $(x+3)=a$.

$$\begin{aligned} &= 8(x+3+2)(x+3+1) \\ &= 8(x+5)(x+4) \end{aligned}$$ **[1 mark]**

Question 4

$$\begin{aligned} \frac{(a+b)^3}{a^3+b^3} &= \frac{\cancel{(a+b)}(a+b)^2}{\cancel{(a+b)}\left(a^2-ab+b^2\right)} \\ &= \frac{(a+b)^2}{a^2-ab+b^2} \end{aligned}$$

Question 5

$4(x-2)^2-5(x-2)-6$

Let $(x-2)=a$

$4a^2-5a-6=(4a+3)(a-2)$

Substitute $(x-2)=a$

$=(4(x-2)+3)(x-2-2)$

$=(4x-5)(x-4)$

2.3 Pascal's triangle and binomial expansions

Question 1

For $(x+2)^5$, the power of the binomial is 5.

Therefore, the binomial coefficients are in row 5.

1 5 10 10 5 1

$(x+2)^5=x^5+5x^4(2)^1+10x^3(2)^2\ldots$

The required coefficient of x^3 is $10\times 2^2=40$.

The correct answer is **B**.

Question 2

For $\left(x-\frac{2}{x}\right)^4$, the power of the binomial is 4.

Therefore, the binomial coefficients are in row 4.

1 4 6 4 1

$\left(x-\frac{2}{x}\right)^4 = x^4 - 4x^3\left(\frac{2}{x}\right) + 6x^2\left(\frac{2}{x}\right)^2 - 4x\left(\frac{2}{x}\right)^3 ...$

The coefficient of term independent of x is the x^0 term, that is $6x^2\left(\frac{2}{x}\right)^2$.

The coefficient $= 6 \times (2)^2 = 24$.

The correct answer is **E**.

Question 3

$(x-2)^3$

The power of the binomials is 4. Therefore, use row 4 coefficients.

$$(x-2)^4 = x^4 - 4x^3(2) + 6x^2(2)^2 - 4x(2)^3 + 2^4$$
$$(x-2)^4 = x^4 - 8x^3 + 24x^2 - 32x + 16$$

$(x-2)^3$

The power of the binomials is 3. Therefore, use row 3 coefficients.

$$(x-2)^3 = x^3 - 3x^2(2) + 3x(2)^2 - (2)^3$$
$$(x-2)^3 = x^3 - 6x^2 + 12x - 8$$

$$(x-2)^4 - (x-2)^3 = \left(x^4 - 8x^3 + 24x^2 - 32x + 16\right) - \left(x^3 - 6x^2 + 12x - 8\right)$$
$$= x^4 - 9x^3 + 30x^2 - 44x + 24$$

Therefore, the coefficient of x is -44. [1 mark]

Question 4

For $(x-2y)^6$, the power of the binomial is 6.

Therefore, the binomial coefficients are in row 6.

1 6 15 20 15 6 1

$(x-2y)^6 = x^6 - 6x^5(2y) + 15x^4(2y)^2 - 20x^3(2y)^3 ...$

Required coefficient of x^3y^3 is $-20 \times 2^3 = -160$.

Question 5

$(1-x)^3$ The power of the binomial is 3. Therefore, using row 3 coefficients.

$(1-x)^3 = 1 - 3(1)^2x + 3(1)x^2 - x^3$

$(1-x)^3 = 1 - 3x + 3x^2 - x^3$ **[1 mark]**

Let $x = 0.2$

$$(1-(0.2))^3 = 1 - 3(0.2) + 3(0.2)^2 - (0.2)^3$$
$$= 1 - 0.6 + 0.12 - 0.008$$
$$= 0.512$$ **[1 mark]**

2.4 The binomial theorem

Question 1

$\dfrac{4! + 3! - 2!}{4! - 3! + 2!}$

$= \dfrac{24 + 6 - 2}{24 - 6 + 2}$

$= \dfrac{28}{20}$

$= \dfrac{7}{5}$

The correct answer is **D**.

Question 2

$3 \times \binom{5}{3} + 4 \times \binom{4}{2}$

$= 3 \times \dfrac{5!}{3! \times 2!} + 4 \times \dfrac{4!}{2! \times 2!}$

$= 3 \times \dfrac{5 \times 4}{2} + 4 \times \dfrac{4 \times 3}{2}$

$= 3 \times 10 + 4 \times 6$

$= 30 + 24$

$= 54$

The correct answer is **D**.

Question 3

$t_{r+1} = \binom{n}{r} x^{n-r} y^r$

$t_{r+1} = \binom{7}{r} 3^{7-r} \left(-4x^3\right)^r$ [**1 mark**]

For x^{12}, we require $r = 4$.

$r + 1 = 5$, so the 5th term contains x^{12}.

$t_5 = t_{4+1} = \binom{7}{4} 3^{7-4} \left(-4x^3\right)^4$ [**1 mark**]

The coefficient $= \dfrac{7!}{4!\,3!} \times 3^3 \times (-4)^4$ [**1 mark**]

$= \dfrac{7 \times 6 \times 5}{3 \times 2 \times 1} \times 3^3 \times \left[(-2)^2\right]^4$

$= \dfrac{7 \times 3 \times 2 \times 5}{3 \times 2 \times 1} \times 3^3 \times 2^8$

$= 7 \times 5 \times 3^3 \times 2^8$ [**1 mark**]

Therefore, the coefficient of x^{12} is $7 \times 5 \times 3^3 \times 2^8$.

Question 4

$$\binom{3n}{3} = \frac{(3n)!}{3!\,(3n-3)}$$
$$= \frac{\cancel{3}n\,(3n-1)\times(3n-2)\times\cancel{(3n-3)}\times\cancel{(3n-4)}\times\ldots}{\cancel{3}\times 2\times 1\times\cancel{(3n-3)}\times\cancel{(3n-4)}\times\ldots}$$
$$= \frac{n}{2}(3n-1)(3n-2)$$
$$= \frac{n}{2}\left(9n^2-9n+2\right)$$

Question 5

General term of $(x+y)^n$ is t_{r+1}.

$$t_{r+1} = \binom{n}{r}x^{n-r}y^r$$

Since the power of the binomial is 8, $n = 8$.

$$t_{r+1} = \binom{8}{r}\left(\frac{x^2}{2}\right)^{8-r}\left(\frac{x}{3}\right)^r$$ **[1 mark]**

For the 4th term, $r + 1 = 4, \therefore r = 3$ **[1 mark]**

$$t_4 = \binom{8}{3}\left(\frac{x^2}{2}\right)^{8-3}\left(\frac{x}{3}\right)^3$$
$$= \frac{8!}{3!\times 5!}\times\frac{x^{10}}{2^5}\times\frac{x^3}{3^3}$$
$$= \frac{8\times 7\times 6}{3\times 2\times 1}\times\frac{x^{10}}{32}\times\frac{x^3}{27}$$
$$= \frac{7x^{13}}{108}$$ [1 **mark**]

Question 6

$$5\times\binom{6}{3}+2\times\binom{4}{3} = 5\times\frac{6!}{3!\times 3!}+2\times\frac{4!}{3!\times 1}$$
$$= 5\times\frac{6\times 5\times 4\times 3!}{3!\times 3!}+2\times\frac{4\times 3!}{3!\times 1}$$
$$= \frac{5\times 6\times 5\times 4}{3\times 2\times 1}+\frac{2\times 4}{1}$$
$$= 5\times 5\times 4+8$$
$$= 108$$

2.5 Sets of real numbers

Question 1

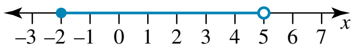
$$\sqrt{9}+\sqrt{16} = 3+4$$
$$= 7, \text{ which is a rational number}$$

The correct answer is **D**.

Question 2

If $x = 0, 1, -3$, the expression is undefined since, for any of these values, the denominator is 0. Division by 0 is not possible. Therefore, any value that makes the denominator of a fraction 0 causes the expression to be undefined. **[1 mark]**

Question 3



The number line shows the numbers to be excluded, from −2 up to but not including 5.
$\{x : x < -2\} \cup \{x : x \geq 5\}$
The correct answer is **B**.

Question 4
The required numbers are all the numbers up to 4, including 4, and all the numbers above 8.8 is not included as it has a square bracket, so is part of the excluded numbers.

Question 5
R is the set of real numbers and both $\sqrt{2}$ and $\frac{1}{2}$ belong to the set of real numbers.
Q is the set of rational numbers; $\frac{1}{2}$ is a fraction and, therefore, a rational number.

2.6 Surds

Question 1

$$\begin{aligned}5\sqrt{6} \times 3\sqrt{2} - \sqrt{3}\left(4\sqrt{3} + 2\right) &= 15\sqrt{12} - 12 - 2\sqrt{3}\\ &= 15\sqrt{4 \times 3} - 12 - 2\sqrt{3}\\ &= 30\sqrt{3} - 12 - 2\sqrt{3}\\ &= 28\sqrt{3} - 12\end{aligned}$$

The correct answer is **A**.

Question 2

$$\begin{aligned}&\left(3\sqrt{2} - 2\sqrt{3}\right)\left(3\sqrt{2} + 2\sqrt{3}\right)\\ &= \left(3\sqrt{2}\right)^2 - \left(2\sqrt{3}\right)^2\\ &= 9 \times 2 - 4 \times 3\\ &= 6\end{aligned}$$

[1 mark]

Question 3

$$\begin{aligned}\frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} + \sqrt{3}} &= \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} + \sqrt{3}} \times \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}}\\ &= \frac{2 - \sqrt{6} - \sqrt{6} + 3}{2 - 3}\\ &= \frac{5 - 2\sqrt{6}}{-1}\\ &= 2\sqrt{6} - 5\end{aligned}$$

The correct answer is **E**.

Question 4
$2\sqrt{3}, 7\sqrt{5}$, $10\sqrt{6}$ and $12\sqrt{7}$ are all surds as the numbers under the $\sqrt{\ }$ sign are not square numbers.
$4\sqrt{9}$ and $11\sqrt{64}$ can be simplified.

$$4\sqrt{9} = 4 \times 3 = 12$$

$$11\sqrt{64} = 11 \times 8 = 88$$

Question 5

$$\begin{aligned} 3\sqrt{128} + \sqrt{250} + \sqrt{112} - \sqrt{63} &= 3\sqrt{64 \times 2} + \sqrt{25 \times 10} + \sqrt{16 \times 7} - \sqrt{9 \times 7} \\ &= 24\sqrt{2} + 5\sqrt{10} + 4\sqrt{7} - 3\sqrt{7} \\ &= 24\sqrt{2} + 5\sqrt{10} + \sqrt{7} \end{aligned}$$

Question 6

$$\begin{aligned} \frac{6}{\sqrt{75}} + \sqrt{128} \div \sqrt{2} &= \frac{6}{\sqrt{75}} + \sqrt{128} \times \frac{1}{\sqrt{2}} \\ &= \frac{6}{\sqrt{25 \times 3}} + \sqrt{64 \times 2} \times \frac{1}{\sqrt{2}} \quad \textbf{[1 mark]} \\ &= \frac{6}{5\sqrt{3}} + 8\cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}} \quad \textbf{[1 mark]} \\ &= \frac{6}{5\sqrt{3}} + 8 \\ &= \frac{6}{5\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} + 8 \\ &= \frac{6\sqrt{3}}{15} + 8 \quad \textbf{[1 mark]} \end{aligned}$$

Question 7

$$\begin{aligned} &\left(3x - \sqrt{5}\right)^3 + \left(\sqrt{5x} + \sqrt{5}\right)^2 \\ &= 27x^3 - 3(3x)^2\sqrt{5} + 3(3x)\left(\sqrt{5}\right)^2 - \left(\sqrt{5}\right)^3 + 5x + 10\sqrt{x} + 5 \quad \textbf{[1 mark]} \\ &= 27x^3 - 27\sqrt{5}x^2 + 45x - 5\sqrt{5} + 5x + 10\sqrt{x} + 5 \quad \textbf{[1 mark]} \\ &= 27x^3 - 27\sqrt{5}x^2 + 50x + 10\sqrt{x} - 5\sqrt{5} + 5 \quad \textbf{[1 mark]} \end{aligned}$$

Question 8

$$\begin{aligned} \frac{6\sqrt{5} - 4\sqrt{3}}{\sqrt{12}} &= \frac{6\sqrt{5} - 4\sqrt{3}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\sqrt{3}\left(6\sqrt{5} - 4\sqrt{3}\right)}{2 \times 3} \\ &= \frac{6\sqrt{15} - 12}{6} \\ &= \frac{6\left(\sqrt{15} - 2\right)}{6} \\ &= \sqrt{15} - 2 \end{aligned}$$

2.7 Review

Question 1

$$\frac{(a-b)^3}{a^3-b^3} = \frac{\cancel{(a-b)}(a-b)^2}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}$$

$$= \frac{(a-b)^2}{a^2+ab+b^2}$$

The correct answer is **C**.

Question 2

$$\frac{p^2-9q^2}{(p-3q)^2} \div \frac{(p+3q)^2}{4p-12q}$$

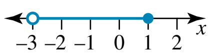
$$= \frac{(p-3q)(p+3q)}{(p-3q)(p-3q)} \times \frac{4(p-3q)}{(p+3q)(p+3q)}$$

$$= \frac{4}{p+3q}$$ **[1mark]**

Question 3

R is the set of real numbers, and both π and $\sqrt{169}\,(=13)$ belong to the set of real numbers.
Z is the set of integers and $\sqrt{169}\,(=13)$ is an integer.
The correct answer is **C**.

Question 4



[1 mark]

$\{x: -3 < x \leq 1\}$ **[1 mark]**

Question 5

$$\frac{5\sqrt{6}-2\sqrt{2}}{\sqrt{8}} = \frac{5\sqrt{6}-2\sqrt{2}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2}\left(5\sqrt{6}-2\sqrt{2}\right)}{2\times 2}$$

$$= \frac{5\sqrt{12}-4}{4}$$

$$= \frac{10\sqrt{3}-4}{4}$$

$$= \frac{2\left(5\sqrt{3}-2\right)}{4}$$

$$= \frac{5\sqrt{3}-2}{2}$$

The correct answer is **B**.

Question 6

For $(x-3)^6$, the power of the binomial is 6. Therefore, the binomial coefficients are in row 6.

1 6 15 20 15 6 1

$(x-3)^6 = x^6 + 6x^5(-3) + 15x^4(-3)^2 + 20x^3(-3)^3 + ...$

Required coefficient of x^4 is $15 \times (-3)^2 = 135$.

Question 7

a. $x = 5\sqrt{3} - 3\sqrt{2}$

$$x^2 = \left(5\sqrt{3} - 3\sqrt{2}\right)^2$$
$$= 25 \times 3 - 30\sqrt{6} + 9 \times 2 \quad \textbf{[1 mark]}$$
$$= 75 - 30\sqrt{6} + 18$$
$$= 93 - 30\sqrt{6} \quad \textbf{[1 mark]}$$

b. $x^2 + 3x + 2 = 93 - 30\sqrt{6} + 3\left(5\sqrt{3} - 3\sqrt{2}\right) + 2$ **[1 mark]**

$$= 93 - 30\sqrt{6} + 15\sqrt{3} - 9\sqrt{2} + 2$$
$$= 95 - 30\sqrt{6} + 15\sqrt{3} - 9\sqrt{2} \quad \textbf{[1 mark]}$$

Question 8

$$\frac{\sqrt{2}+1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}\left(\sqrt{2}+1\right)}{3 \times 2}$$
$$= \frac{2+\sqrt{2}}{6}$$

Question 9

For $\left(2x + \frac{1}{x}\right)^4$, the power of the binomial is 4. Therefore, the binomial coefficients are in row 4.

1 4 6 4 1

$$\left(2x + \frac{1}{x}\right)^4 = (2x)^4 + 4(2x)^3\left(\frac{1}{x}\right) + 6(2x)^2\left(\frac{1}{x}\right)^2 + 4(2x)\left(\frac{1}{x}\right)^3 + \ldots$$

The coefficient of the term independent of x is the x^0 term; that is, $6(2x)^2\left(\frac{1}{x}\right)^2$.

Coefficient $= 6 \times (2)^2 = 24$

Question 10

Division by 0 is not possible. Therefore, any value that makes the denominator of a fraction 0 causes the expression to be undefined.

$\frac{x+4}{2x(x-4)(x+1)}$ is undefined if $x = 0, 4, -1$.

Question 11

$$\left(2\sqrt{5} - \sqrt{6}\right)\left(2\sqrt{5} + \sqrt{6}\right) = \left(2\sqrt{5}\right)^2 - \left(\sqrt{6}\right)^2 \Rightarrow \text{DOTS}$$
$$= 4 \times 5 - 6$$
$$= 14$$

Question 12

$(x+3)^4$: the power of the binomial is 4; therefore, use row 4 coefficients.

$$(x+3)^4 = x^4 + 4x^3(3) + 6x^2(3)^2 + 4x(3)^3 + 3^4$$
$$= x^4 + 12x^3 + 54x^2 + 108x + 81 \quad \textbf{[1 mark]}$$

$(x+3)^3$: the power of the binomial is 3; therefore, use row 3 coefficients.

$$(x+3)^3 = x^3 + 3x^2(3) + 3x(3)^2 + (3)^3$$
$$= x^3 + 9x^2 + 27x + 27 \quad \textbf{[1 mark]}$$

$$(x+3)^4 - (x+3)^3 = x^4 + 12x^3 + 54x^2 + 108x + 81 - \left(x^3 + 9x^2 + 27x + 27\right)$$
$$= x^4 + 11x^3 + 45x^2 + 81x + 54 \quad \textbf{[1 mark]}$$

Coefficient of $x = 108 - 27 = 81$ **[1 mark]**

3 Quadratic relationships

Topic	3	Quadratic relationships
Subtopic	3.2	Quadratic equations with rational roots

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Question 1 (1 mark)

The factorised form of $12a^2 - a - 20$ is

A. $(4a - 5)(3a - 4)$
B. $(4a + 5)(3a - 4)$
C. $(4a - 5)(3a + 4)$
D. $(12a + 5)(a - 4)$
E. $(12a - 4)(a + 5)$

Question 2 (1 mark)

The solution(s) to $3x^2 - 243 = 0$ is/are

A. -81
B. $+81$
C. 9
D. ± 9
E. -9

Question 3 (3 marks)

Solve the equation for a.

$2(a - 1)^2 - 7(a - 1) + 6 = 0$

Question 4 (1 mark)

The expansion of $(3x - 4y)^2$ is

A. $9x^2 - 12xy + 16y^2$
B. $9x^2 + 24xy - 16y^2$
C. $9x^2 - 24xy + 16y^2$
D. $9x^2 - 24xy - 16y^2$
E. $9x^2 + 12xy - 16y^2$

Question 5 (4 marks)

Solve for x.

$$\left(x + \frac{1}{x}\right)^2 + 4\left(x + \frac{1}{x}\right) + 4 = 0$$

Question 6 (1 mark)

The factorised form of $12a^2 + 8a - 15$ is

A. $(4a - 3)(3a + 5)$
B. $(4a + 3)(3a - 5)$
C. $(2a - 3)(6a + 5)$
D. $(2a + 3)(6a - 5)$
E. $(4a - 3)(3a + 5)$

Question 7 (1 mark)

Solve the following equation for a.

$3(a - 2)^2 + 10(a - 2) - 8 = 0$

A. $a = \frac{8}{3}, -6$

B. $a = \frac{7}{3}, 2$

C. $a = \frac{2}{3}, -4$

D. $a = \frac{7}{2}, -2$

E. $a = \frac{8}{3}, -2$

Topic	3	Quadratic relationships
Subtopic	3.3	Quadratics over ***R***

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Question 1 (1 mark)

The discriminant of $3x^2 + 5x - 1$ is equal to

A. 37
B. 13
C. 15
D. -15
E. 45

Question 2 (1 mark)

The solutions to the equation $2x^2 - 7x + 1 = 0$ are

A. $\dfrac{7 \pm \sqrt{57}}{4}$
B. $\dfrac{-7 \pm \sqrt{41}}{2}$
C. $\dfrac{2 \pm \sqrt{41}}{2}$
D. $\dfrac{2 \pm \sqrt{41}}{4}$
E. $\dfrac{7 \pm \sqrt{41}}{4}$

Question 3 (4 marks)

Determine the value(s) of m for which the quadratic $mx^2 + (m + 3)x = -3$ will have real roots.

Question 4 (1 mark)
If the equation $ax^2 + bx + c = 0$ has two unequal solutions, then $b^2 - 4ac$ is
A. ≥ 0
B. > 0
C. $= 0$
D. < 0
E. ≤ 0

Question 5 (3 marks)
Solve $x^2 - 6x + 2 = 0$.

Question 6 (1 mark)
The equation $y = 2x^2 - 2x + 5$ has
A. two rational solutions.
B. no real solutions.
C. one real solution.
D. two irrational solutions.
E. one rational solution.

Question 7 (1 mark)
The solutions to the equation $x^2 - 8x + 3 = 0$ are
A. $x = 4 \pm \sqrt{13}$
B. $x = -4 \pm \sqrt{13}$
C. $x = 4 \pm \sqrt{52}$
D. $x = 4 \pm 2\sqrt{13}$
E. $x = -4 \pm \sqrt{52}$

Topic	3	Quadratic relationships
Subtopic	3.4	Applications of quadratic equations

online only

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Question 1 (1 mark)
A farmer wants to fence off a small section of natural vegetation alongside a river. He has 100 metres of fencing materials to make the three sides of the rectangular section abutting the river. If the length of the side parallel to the river is x metres, the area of the section can be written as

A. $50x - \frac{x^2}{2}$
B. $50x + \frac{x^2}{2}$
C. $100x - x^2$
D. $100x + x^2$
E. $50x + 2x^2$

Question 2 (1 mark)
A right-angled triangle has a hypotenuse of $(5x + 4)$ cm and two sides $5x$ cm and $(x - 4)$ cm, respectively. The value of x is

A. 16
B. 25
C. 32
D. 8
E. 48

Question 3 (4 marks)
The product of two numbers is -483. If the difference between the two numbers is 44, determine the smaller of the two numbers.

Question 4 (1 mark)

The volume of a square pyramid of fixed height is directly proportional to the square of the length of its base. When the length is l_1 cm, the volume is v_1 cm^3. The length of the base when the volume is v_2 cm^3 is

A. $l_1\sqrt{\frac{v_2}{v_1}}$

B. $l_1\sqrt{\frac{v_1}{v_2}}$

C. $l_1\frac{v_1}{v_2}$

D. $l_1 v_1 v_2$

E. $l_1\sqrt{v_1 v_2}$

Question 5 (4 marks)

Elira decided to go into the bicycle business. She imported a container of bicycles for a cost of $5000. She marked up the price of each bike by $25. At the end of 2 weeks she had sold all but two of the bicycles and had made $6000 from the sales. Determine how many bicycles Elira imported.

Topic	3	Quadratic relationships
Subtopic	3.5	Graphs of quadratic polynomials

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Question 1 (1 mark)

The graph of $y = (x - 2)^2 + 3$ is the graph of $y = x^2$

A. translated 2 units to the right and 3 down.
B. translated 2 units to the right and 3 up.
C. translated 2 units to the left and 3 up.
D. translated 2 units to the left and 3 down.
E. translated 3 units to the right and 2 down.

Question 2 (1 mark)

The turning point of the graph of the quadratic function $f(x) = x^2 - 6x + 7$ is

A. $(-3, 2)$
B. $(-3, -2)$
C. $(3, -2)$
D. $(3, 7)$
E. $(3, -7)$

Question 3 (5 marks)

Sketch the graph of $y = 2x^2 + x - 3$, labelling all key points.

Question 4 (1 mark)

The graph of $-4x^2 - x - 3$ has

A. no x-intercepts.
B. one x-intercept.
C. two x-intercepts.
D. three x-intercepts.
E. one x-intercept and one y-intercept.

Question 5 (4 marks)

Express $y = -2x^2 + 2x + 1$ in the form $y = a(x + h)^2 + k$ and hence state the coordinates of the vertex.

Question 6 (1 mark)

The graph of $y = (x + 4)^2 - 1$ is the graph of $y = x^2$

A. translated 4 units to the right and 1 unit down.
B. translated 4 units to the left and 1 unit up.
C. translated 4 units to the left and 1 unit down.
D. translated 4 units to the right and 1 unit up.
E. translated 1 unit to the left and 4 units down.

Question 7 (1 mark)

A parabola has a turning point of $(2, 5)$ and a y-intercept of -3. The equation of this parabola is

A. $y = -2(x + 2)^2 + 5$
B. $y = -(x - 2)^2 + 5$
C. $y = -2(x - 2)^2 + 5$
D. $y = 2(x - 2)^2 + 5$
E. $y = 5(x - 2)^2 + 2$

Topic	3	Quadratic relationships
Subtopic	3.6	Determining the rule of a quadratic polynomial from a graph

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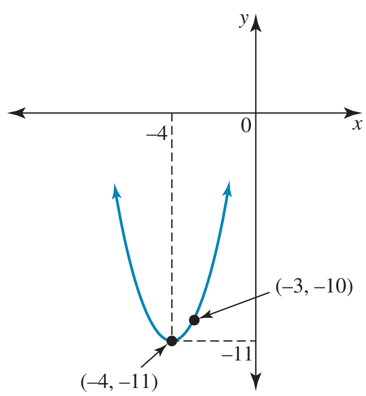
Question 1 (1 mark)

The equation of the parabola shown could be



A. $y = x^2 + 8x + 5$
B. $y = x^2 - 8x - 5$
C. $y = x^2 + 8x + 16$
D. $y = x^2 - 8x + 16$
E. $y = x^2 - 8x + 5$

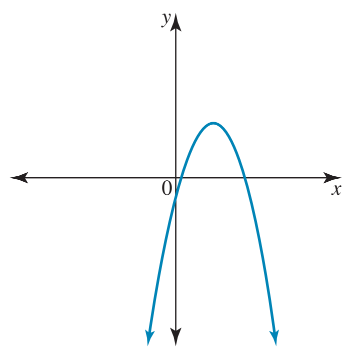
Question 2 (2 marks)

Determine the equation of the parabola in the form $y = ax^2 + bx + c$ that has a y-intercept of -12 and x-intercepts of 4 and -3.

Topic 3 | Subtopic 3.6 Determining the rule of a quadratic polynomial from a graph

Question 3 (1 mark)
A parabola has a turning point of $(2, 3)$ and a y-intercept of 1. The equation of this parabola could be
A. $y = (x + 2)^2 - 3$
B. $y = (x + 2)^2 + 3$
C. $y = \frac{1}{2}(x + 2)^2 + 3$
D. $y = -\frac{1}{2}(x - 2)^2 + 3$
E. $y = (x - 2)^2 + 3$

Question 4 (1 mark)
The equation of the graph shown may be



A. $y = -(x + 2)^2 - 3$
B. $y = (x + 2)^2 + 3$
C. $y = (x - 2)^2 + 3$
D. $y = -(x - 2)^2 + 3$
E. $y = -(x + 2)^2 + 3$

Question 5 (4 marks)
Given that the points $(-1, -3)$, $(2, 21)$ and $(3, 37)$ lie on a parabola with equation $y = ax^2 + bx + c$, find the values of a, b and c.

Topic	3	Quadratic relationships
Subtopic	3.7	Quadratic inequations

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Question 1 (1 mark)

The solution to the inequality $(4x - 3)(2x + 1) > 0$ is

A. $\{x : x > 1\}$
B. $\{x : -0.75 < x < 0.5\}$
C. $\{x : -0.5 < x < 0.75\}$
D. $\{x : x < 0.75\} \cup \{x : x > 0.5\}$
E. $\{x : x < -0.5\} \cup \{x : x > 0.75\}$

Question 2 (1 mark)

The solution to the inequality $-2x^2 - x + 3 \geq 0$ is

A. $x \in \left(-\infty, -\frac{3}{2}\right) \cup (1, \infty)$
B. $x \in \left(-\frac{3}{2}, 1\right)$
C. $x \in \left(-\infty, -\frac{3}{2}\right] \cup [1, \infty)$
D. $x \in \left(-1, \frac{3}{2}\right)$
E. $x \in \left[-1, \frac{3}{2}\right]$

Question 3 (3 marks)
Determine the values of m for which the line $y = mx + 24$ will be a tangent to the parabola $x^2 + 4x + 33$.

Question 4 (5 marks)
For what values of k will there be at least one intersection between the line $y = kx + 4$ and the parabola $y = x^2 - 8x + 12$?

Question 5 (1 mark)
The solution to the inequality $(5x - 2)(x + 3) < 0$ is

A. $x \in (-\infty, -3] \cup \left[\frac{2}{5}, \infty\right)$

B. $x \in \left(-3, \frac{2}{5}\right)$

C. $x \in (-\infty, -3) \cup \left(\frac{2}{5}, \infty\right)$

D. $x \in \left[-3, \frac{2}{5}\right]$

E. $x \in \left[\frac{2}{5}, -3\right]$

Topic	3	Quadratic relationships
Subtopic	3.8	Quadratic models and applications

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Question 1 (1 mark)

Jill has 80 metres of wire to enclose a rectangular-shaped pen for her new alpaca pair. If she can use the back of her house as one side, the maximum area, in square metres, that she can create for her new animals is

A. 400
B. 800
C. 1000
D. 1200
E. 1600

Question 2 (1 mark)

An artist decides to include a 40 cm length rod in her latest sculpture. She wants to make two squares from the rod and so cuts it into two parts. If x cm is the side length of one of the squares, the equation for the sum of the areas of the two squares is

A. $A = 2x^2 - 20x + 100$
B. $A = 2x^2 - 20x + 40$
C. $A = x^2 - 20x + 100$
D. $A = x^2 - 20x + 40$
E. $A = 4x^2 - 20x + 40$

Question 3 (3 marks)

Nick is standing on a 20-metre cliff overlooking a beach. Nick throws a ball vertically up into the air. Ruby is lying on the beach below watching and later calculated that the motion of the ball can be described as $h = 1.5 + 4t - 0.2t^2$, where h is the height in metres above the cliff from where Nick threw the ball and t is the time in seconds.

Find the greatest height the ball reached above Ruby and how many seconds it took to reach that height.

Question 4 (1 mark)

Edie has 120 metres of wire to enclose a rectangular pen for her new pet menagerie. If she can use the back of her house as one side, what is the maximum area she can create for her new animals?

A. $1800\,\text{m}^2$
B. $1200\,\text{m}^2$
C. $3600\,\text{m}^2$
D. $900\,\text{m}^2$
E. $2400\,\text{m}^2$

Question 5 (3 marks)

The product of two even numbers is 176. If the difference between the two numbers is 14. determine the numbers.

Topic	3	Quadratic relationships
Subtopic	3.9	Review

online only

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Question 1 (2 marks)
Solve for x.

$$\left(x+\frac{1}{x}\right)^2+4\left(x+\frac{1}{x}\right)+4=0$$

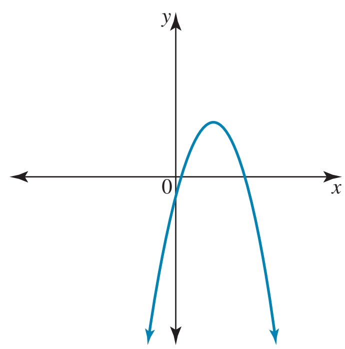
Question 2 (1 mark)
If the equation $ax^2+bx+c=0$ has two unequal solutions, then b^2-4ac is

A. ≥ 0
B. > 0
C. $= 0$
D. < 0
E. ≤ 0

Question 3 (1 mark)
The graph of $-4x^2-x-3$ has

A. no x-intercepts.
B. one x-intercept.
C. two x-intercepts.
D. three x-intercepts.
E. one x-intercept and y-intercept.

Question 4 (1 mark)
The equation of the graph shown may be



A $y = -(x + 2)^2 - 3$
B $y = (x + 2)^2 + 3$
C $y = (x - 2)^2 + 3$
D $y = -(x - 2)^2 + 3$
E $y = -(x + 2)^2 + 3$

Question 5 (3 marks)
Determine the values of k for which there will be at least one intersection between the line $y = kx + 4$ and the parabola $y = x^2 - 8x + 12$.

Question 6 (1 mark)

The graphs of the equations $y = 2 - x^2$ and $y = 2x$ will

A. intersect at two points.
B. not intersect.
C. intersect at one point only.
D. intersect at three points.
E. It is not possible to determine if they will intersect.

Question 7 (1 mark)

If two numbers (x, y) are added together, they give 4. The same two numbers form a quantity (q) in the equation $q = 3x^2 + 2xy + 2y^2$. The value of (x, y) to give the minimum value of q is

A. $\left(\frac{3}{2}, \frac{5}{2}\right)$
B. $\left(\frac{2}{3}, \frac{10}{3}\right)$
C. $\left(\frac{7}{4}, \frac{9}{4}\right)$
D. $\left(\frac{1}{2}, \frac{7}{2}\right)$
E. $\left(\frac{4}{3}, \frac{8}{3}\right)$

Question 8 (4 marks)

Research for a company selling soft toys found that the number sold each week, y, can be expressed as $y = -11x^2 + 330x$, where x is the number of weeks after the release of the toy.

In your workbook, draw a graph of the number of sales each week versus number of weeks, and find the most profitable week.

Question 9 (1 mark)

The turning point of the graph of the quadratic function $f(x) = x^2 + 2x - 8$ is

A. $(-1, 9)$
B. $(1, -5)$
C. $(-2, -8)$
D. $(-1, -11)$
E. $(-1, -9)$

Question 10 (1 mark)

The equation of the parabola in the form $y = ax^2 + bx + c$ that has a y-intercept of 8 and x-intercept 2 and $-\frac{1}{2}$ is

A. $y = -4x^2 + 12x + 8$
B. $y = -8x^2 - 12x - 8$
C. $y = -8x^2 + 3x - 2$
D. $y = -8x^2 + 12x + 8$
E. $y = 8x^2 + 3x - 2$

Answers and marking guide

3.2 Quadratic equations with rational roots

Question 1

$$\begin{aligned}(4a+5)(3a-4) &= 12a^2 - 16a + 15a - 20 \\ &= 12a^2 - a - 20 \\ &= (4a+5)(3a-4)\end{aligned}$$

The correct answer is **B**.

Question 2

$$\begin{aligned}3x^2 - 243 &= 0 \\ x^2 - 81 &= 0 \\ (x-9)(x+9) &= 0 \\ x &= \pm 9\end{aligned}$$

The correct answer is **D**.

Question 3

Let $(a-1) = x$. **[1 mark]**

$$\begin{aligned}2x^2 - 7x + 6 &= 0 \\ (2x-3)(x-2) &= 0\end{aligned}$$

$2x - 3 = 0,\ x - 2 = 0$

$x = \frac{3}{2}, \quad x = 2$ **[1 mark]**

Substitute back for $x = a - 1$.

$a - 1 = \frac{3}{2}, \quad a - 1 = 2$

$\therefore a = \frac{5}{2}, \quad a = 3$ **[1 mark]**

Question 4

$$\begin{aligned}(3x-4y)^2 &= (3x)^2 - 2(3x)(4y) + (-4y)^2 \\ &= 9x^2 - 24xy + 16y^2\end{aligned}$$

Question 5

Let $\left(x + \frac{1}{x}\right) = a$.

$a^2 + 4a + 4 = 0$ **[1 mark]**

$(a+2)^2 = 0$

$a = -2$ **[1 mark]**

Substituting back:

$x + \frac{1}{x} = -2$ **[1 mark]**

$$\begin{aligned}x^2 + 1 &= -2x \\ x^2 + 2x + 1 &= 0 \\ (x+1)^2 &= 0\end{aligned}$$

$x = -1$ **[1 mark]**

Question 6

$(2a+3)(6a-5)$

$= 12a^2 - 10a + 18a - 15$

$= 12a^2 + 8a - 15$

Question 7

Let $(a-2)=x$

$3x^2+10x-8=0$

$(3x-2)(x+4)=0$

$$3x-2=0, \quad x+4=0$$

$$x=\frac{2}{3}, \quad x=-4$$

$$a-2=\frac{2}{3}, \quad a-2=-4$$

$$a=\frac{8}{3}, -2$$

3.3 Quadratics over *R*

Question 1

$\Delta=b^2-4ac$

$a=3,\ b=5,\ c=-1$

$$\Delta=(5)^2-4(3)(-1)$$
$$=37$$

The correct answer is **A**.

Question 2

$2x^2-7x+1=0$

$a=2,\ b=-7,\ c=1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-7)\pm\sqrt{(-7)^2-4(2)(1)}}{2(2)}$$
$$=\frac{7\pm\sqrt{49-8}}{4}$$
$$=\frac{7\pm\sqrt{41}}{4}$$

The correct answer is **E**.

Question 3

$mx^2+(m+3)x=-3$

$mx^2+(m+3)x+3=0$ **[1 mark]**

For all roots, $\Delta\geq 0$.

$\Delta=b^2-4ac$

$a=m, b=(m+3), c=+3$

$\Delta=(m+3)^2-4m(+3)$ **[1 mark]**

$=m^2+6m+9-12m$

$=m^2-6m+9$

$=(m-3)^2$ **[1 mark]**

As Δ is a perfect square, $\Delta\geq 0$.

Therefore, the quadratic will have real roots for any real value of m. **[1 mark]**

Question 4

$b^2-4ac>0$ indicates there are two real roots.

Question 5

Equation does not factorise so use completing the square method.

$$x^2 - 6x + 2 = 0$$
$$x^2 - 6x + 9 - 9 + 2 = 0 \quad \textbf{[1 mark]}$$
$$(x-3)^2 - 7 = 0 \quad \textbf{[1 mark]}$$
$$(x-3)^2 = 7$$
$$x - 3 = \pm\sqrt{7}$$
$$x = 3 - \sqrt{7},\ 3 + \sqrt{7} \quad \textbf{[1 mark]}$$

Question 6

$$\Delta = b^2 - 4ac$$
$$a = 2,\ b = -2,\ c = 5$$
$$\Delta = (-2)^2 - 4(2)(5)$$
$$= -36$$

As $\Delta < 0$, there are no real solutions.

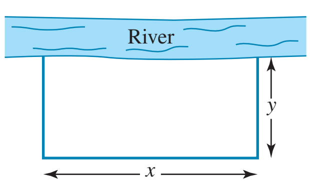
Question 7

$$x^2 - 8x + 3 = 0$$
$$a = 1, \quad b = -8, \quad c = 3$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-8) \pm \sqrt{\left(\sqrt{-8^2}\right) - 4(1)(3)}}{2(1)}$$
$$= \frac{8 \pm \sqrt{64 - 12}}{2}$$
$$= \frac{8 \pm \sqrt{52}}{2}$$
$$= \frac{8 \pm \sqrt{13}}{2}$$
$$\therefore x = 4 \pm \sqrt{13}$$

3.4 Applications of quadratic equations

Question 1

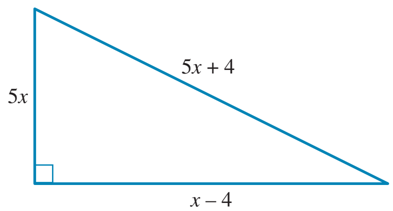
Let the length of each of the other sides be y.



$$2y + x = 100$$
$$y = 50 - \frac{x}{2}$$
$$\text{Area} = xy$$
$$= x\left(50 - \frac{x}{2}\right)$$
$$= 50x - \frac{x^2}{2}$$

The correct answer is **A**.

Question 2



Use Pythagoras' theorem:

$$h^2 = a^2 + b^2$$
$$(5x+4)^2 = (5x)^2 + (x-4)^2$$
$$25x^2 + 40x + 16 = 25x^2 + x^2 - 8x + 16$$
$$x^2 - 48x = 0$$
$$x(x-48) = 0$$
$$x = 0, x = 48$$

$x = 0$ is not a solution in this context.

$\therefore x = 48$

The correct answer is **E**.

Question 3

Let x be the first number. Then the second number is $x + 44$. **[1 mark]**

$$x(x+44) = -483$$
$$x^2 + 44x + 483 = 0 \quad \textbf{[1 mark]}$$
$$(x+23)(x+21) = 0$$
$$x = -23, -21 \quad \textbf{[1 mark]}$$

Two pairs of numbers: $-23, 21$ and $-21, 23$

The smallest number is -23. **[1 mark]**

Question 4

$$v \propto l^2$$
$$v = kl^2$$
$$k = \frac{v}{l^2}$$
$$k = \frac{v_1}{l_1^2}$$
$$v_2 = kl_2^2$$
$$l_2 = \sqrt{\frac{v_2}{k}}$$
$$= \sqrt{\frac{v_2 l_1^2}{v_1}}$$
$$= l_1\sqrt{\frac{v_2}{v_1}}$$

Question 5

The cost of importing x bicycles is \$5000.

Let y be the initial cost of each bicycle.

$\therefore$ the cost of each bicycle is $y = \frac{\$5000}{x}$ **[1 mark]**

Income from bicycles sold:

$$\begin{aligned}
\text{Sale price} \times \text{number sold} &= \$6000\\
(y+25)(x-2) &= 6000\\
\left(\frac{5000}{x}+25\right)(x-2) &= 6000 \qquad \textbf{[1 mark]}\\
5000-\frac{10\,000}{x}+25x-50 &= 6000\\
\frac{10\,000}{x}+25x &= 1050\\
25x^2-1050x-10\,000 &= 0\\
x^2-42x-400 &= 0 \qquad \textbf{[1 mark]}\\
(x-50)(x+8) &= 0\\
x &= 50 \text{ or } -8
\end{aligned}$$

-8 is unrealistic.

Therefore, 50 bicycles were imported. **[1 mark]**

3.5 Graphs of quadratic polynomials

Question 1

$y = a(x-h)^2 + k$

h is a horizontal translation; k is a vertical translation.

$y = (x-2)^2 + 3$

$\therefore$ translated 2 unit to the right and 3 up

The correct answer is **B**.

Question 2

Turning point: $x = \dfrac{-b}{2a}$

$a = 1, b = -6$

$$\begin{aligned}
x &= \frac{-(-6)}{2(1)}\\
&= 3
\end{aligned}$$

Substitute $x = 3$.

$$\begin{aligned}
f(3) &= 3^2 - 6(3) + 7\\
&= -2
\end{aligned}$$

$\therefore$ turning point $(3, -2)$

The correct answer is **C**.

Question 3

$y = 2x^2 + x - 3$

y-intercept, $x = 0$:

$y = 2(0)^2 + (0) - 3$

$y = -3$

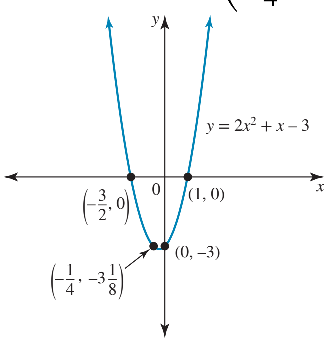
y-intercept $(0, -3)$ **[1 mark]**

x-intercept, $y = 0$:

$$\begin{aligned}
(0) &= 2x^2 + x - 3\\
(2x+3)(x-1) &= 0\\
x &= \frac{-3}{2}, 1
\end{aligned}$$

x-intercepts $\left(\dfrac{-3}{2}, 0\right), (1, 0)$ **[1 mark]**

Equation of axis of symmetry:
$x = \frac{-b}{2a}$
$a = 2,\ b = 1,\ c = -3$
$x = -\frac{1}{2(2)}$
$x = -\frac{1}{4}$ **[1 mark]**
y-coordinate of axis of symmetry:
$y = 2\left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right) - 3$
$y = \frac{1 - 2 - 24}{8}$
$y = -\frac{25}{8}$
$y = -3\frac{1}{8}$
$\therefore$ turning point $= \left(-\frac{1}{4}, -3\frac{1}{8}\right)$ **[1 mark]**



Question 4
$\Delta = b^2 - 4ac$
For $-4x^2 - x - 3$, $a = -4, b = -1, c = -3$
$\Delta = (-1)^2 - 4(-4)(-3)$
$\Delta = -47$
Δ is negative so no x-intercepts.

Question 5
$y = -2x^2 + 2x + 1$
$= -2\left(x^2 - x - \frac{1}{2}\right)$ **[1 mark]**
$= -2\left(x^2 - x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - \frac{1}{2}\right)$ **[1 mark]**
$= -2\left[\left(x - \frac{1}{2}\right)^2 - \frac{3}{4}\right]$
$= -2\left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$ **[1 mark]**
Vertex: $\left(\frac{1}{2}, \frac{3}{2}\right)$ **[1 mark]**

Question 6

$y = a(x - h)^2 + k$

h is a horizontal translation; k is a vertical translation.

Therefore, it is translated 4 units to the left and 1 unit down.

Question 7

As the information given is the TP and another point, use the turning point form of the quadratic equation.

$$\begin{aligned} y &= a(x-h)^2 + k \\ &= a(x-2)^2 + 5 \end{aligned}$$

Substitute $(0, -3)$.

$$\begin{aligned} -3 &= a(-2)^2 + 5 \\ -8 &= 4a \\ a &= -2 \end{aligned}$$

$\therefore y = -2(x-2)^2 + 5$

3.6 Determining the rule of a quadratic polynomial from a graph

Question 1

$$y = a(x-h)^2 + k$$

$(h, k) = (-4, -11)$

$$y = a(x+4)^2 - 11$$

Substitute $(-3, -10)$.

$$-10 = a(-3+4)^2 - 11$$

$\therefore a = 1$

$$\begin{aligned} y &= (x+4)^2 - 11 \\ &= x^2 + 8x + 16 - 11 \\ &= x^2 + 8x + 5 \end{aligned}$$

The correct answer is **A**.

Question 2

Using x-intercepts: $y = a\,(x-4)\,(x+3)$

Substitute the y-intercept, $(0, -12)$.

$$\begin{aligned} -12 &= a\,(-4)\,(3) \\ -12 &= -12a \\ a &= 1 \end{aligned}$$ **[1 mark]**

Equation: $y = 1\,(x-4)\,(x+3)$

$y = x^2 - x - 12$ **[1 mark]**

Question 3

Turning point $(2, 3)$

$$\Rightarrow y = a(x-2)^2 + 3$$

$$(0, 1) \Rightarrow 1 = a(-2)^2 + 3$$

$$\begin{aligned} -2 &= 4a \\ a &= -\frac{1}{2} \\ y &= -\frac{1}{2}(x-2)^2 + 3 \end{aligned}$$

The correct answer is **D**.

Question 4

Options are in the form $y = a(x-h)^2 + k$.

Parabola is inverted so a is negative.

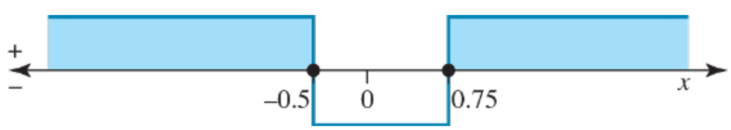
Vertex has positive values for x and y.

Positive values of x-intercepts
$y = -(x-2)^2 + 3$ is the only valid option.

Question 5
$y = ax^2 + bx + c$
Using $(-1, -3)$:
$-3 = a(-1)^2 + b(-1) + c$
$-3 = a - b + c$ [1]
Using $(2, 21)$:
$21 = a(2)^2 + b(2) + c$
$21 = 4a + 2b + c$ [2]
Using $(3, 37)$:
$37 = a(3)^2 + b(3) + c$
$37 = 9a + 3b + c$ [3] [**1 mark**]
Subtract [1] from [2]:
$24 = 3a + 3b$ [4]
Subtract [2] from [3]:
$16 = 5a + b$ [5]
Multiply [5] $\times$ 3:
$48 = 15a + 3b$ (6)
Subtract [4] from [6]:
$24 = 12a$
$\therefore a = 2$ [**1 mark**]
Substitute $a = 2$ into [4]
$24 = 3 \times 2 + 3b$
$3b = 18$
$b = 6$ [**1 mark**]
Substitute $a = 2$ and $b = 6$ into [1]:
$-3 = 2 - 6 + c$
$c = 1$ [**1 mark**]
The equation is $y = 2x^2 + 6x + 1$ [**1 mark**].

3.7 Quadratic inequations

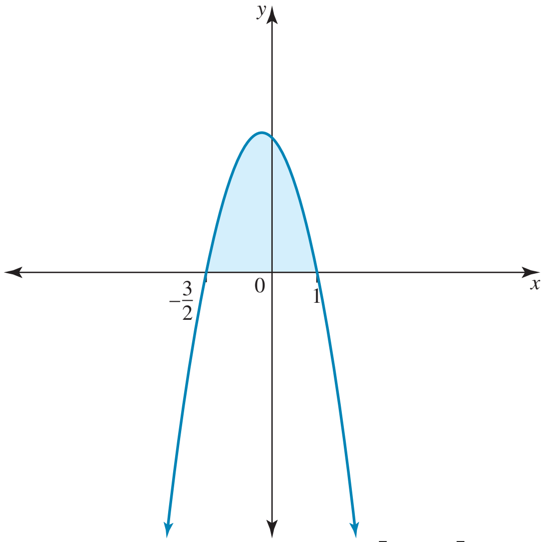
Question 1
$(4x-3)(2x+1) > 0$
Solve $(4x-3)(2x+1) = 0$ to find the zeros.
$x = 0.75, \; x = -0.5$



The solution is $\{x : x < -0.5\} \cup \{x : x > 0.75\}$
The correct answer is **E**.

Question 2
Solve
$-2x^2 - x + 3 = 0$
$-\left(2x^2 + x - 3\right) = 0$
$-(x-1)(2x+3) = 0$
$x = 1, \; -\dfrac{3}{2}$

Sketch the graph to find the given region.



For $-(x-1)(2x+3) \geq 0, x \in \left[-\frac{3}{2}, 1\right]$.

The correct answer is **E**.

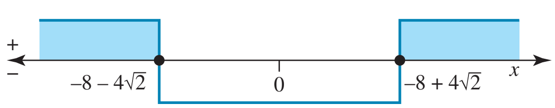
Question 3

$$x^2 + 4x + 33 = mx + 24$$
$$x^2 + (4-m)x + 9 = 0$$
$a = 1,\ b = (4-m),\ c = 9$ **[1 mark]**
$$\Delta = b^2 - 4ac$$
$$= (4-m)^2 - 4(1)(9)$$
$$= (4-m)^2 - 36$$
For the line to be a tangent, $\Delta = 0$ **[1 mark]**
$$(4-m)^2 - 36 = 0$$
$$(4-m-6)(4-m+6) = 0$$
$$(-m-2)(10-m) = 0$$
$m = -2,\ m = 10$ **[1 mark]**

The line $y = mx + 24$ will be a tangent to the parabola $x^2 + 4x + 33$ for $m = -2$ or $m = 10$.

Question 4

$x^2 - 8x + 12 = kx + 4$ **[1 mark]**
$x^2 - x(8+k) + 8 = 0$ **[1 mark]**
$$\Delta = b^2 - 4ac$$
$$a = 1,\ b = (-8-k,)\ c = 8$$
$$\Delta = (-8-k)^2 - 4(1)(8)$$
$$= (8+k)^2 - 32$$
$\Delta \geq 0$ for one or two intersections
$\left(8+k-\sqrt{32}\right)\left(8+k+\sqrt{32}\right) = 0$ **[1 mark]**
$$\left(8+k-4\sqrt{2}\right)\left(8+k+4\sqrt{2}\right) = 0$$
Zeros are $-8+4\sqrt{2}, -8-4\sqrt{2}$ **[1 mark]**



[1 mark]

For at least one intersection, $k \in \left(-\infty, -8 - 4\sqrt{2}\right) \cup \left[-8 + 4\sqrt{2}, \infty\right]$ (also accept $k \in \left(-\infty, -8 - \sqrt{32}\right) \cup \left[-8 + \sqrt{32}, \infty\right]$).

Question 5

$(5x - 2)(x + 3) < 0$

Solve $(5x - 2)(x + 3) = 0$ to find the zeros.

$x = \frac{2}{5}, -3$

Draw sign diagram.

Solution: $x \in \left(-3, \frac{2}{5}\right)$

3.8 Quadratic models and applications

Question 1

Let the length of side opposite the back of her house be y and let the other two sides be x.

80 metres of wire to enclose three sides:

$$2x + y = 80$$
$$y = 80 - 2x$$
$$\text{Area} = xy$$
$$= x(80 - 2x)$$
$$= 80x - 2x^2$$

Turning point:

$$x = \frac{-b}{2a}$$
$$a = -2,\ b = 80$$
$$x = \frac{-80}{-4}$$
$$= 20$$
$$\therefore y = 80(20) - 2(20)^2$$
$$= 1600 - 800$$
$$= 800$$

$\therefore$ maximum area is 800 m^2.

The correct answer is **B**.

Question 2

Square 1:

The length of the sides is $4x$

Square 2:

The sum of the four sides is $40 - x$

$\therefore$ one side is $\frac{40 - x}{4} = 10 - x$

$\text{Area} = x^2 + (10 - x)^2$

$\text{Area} = x^2 + 100 - 20x + x^2$

$\text{Area} = 2x^2 - 20x + 100$

The correct answer is **A**.

Question 3

$h = 1.5 + 4t - 0.2t^2$

$a = -0.2,\ b = 4,\ c = 1.5$

Turning point:

Axis of symmetry has equation $t = -\dfrac{b}{2a}$

$$\therefore t = -\frac{4}{2(-0.2)}$$
$$= \frac{4}{0.4}$$
$$= 10 \quad \textbf{[1 mark]}$$
$$h = 1.5 + 4t - 0.2t^2$$
$$= 1.5 + 4(10) - 0.2(10)^2$$
$$= 1.5 + 40 - 20$$
$$= 21.5 \quad \textbf{[1 mark]}$$

Add on height of cliff $= 20$ metres

The height of the ball above Ruby was 41.5 metres and it took 10 seconds to reach this height. **[1 mark]**

Question 4

Let the length of side opposite the back of her house be y and the other two sides x.

120 metres of wire to enclose three sides.

$2x + y = 120$

$y = 120 - 2x$

Area $= xy$

$= x(120 - 2x)$

$= 120x - 2x^2$

Turning point for maximum area

$x = \dfrac{-b}{2a}$

$a = -2, \quad b = 120$

$x = \dfrac{-120}{-4}$

$= 30$

$A = 120(30) - 2(30)^2$

$= 1800$

The maximum area is 1800 m^2.

Question 5

Let x be the first number.

The second number is $x + 14$. **[1 mark]**

$x(x + 14) = 176$

$x^2 + 14x = 176$

$x^2 + 14x - 176 = 0$

$(x - 8)(x + 22) = 0$

$x - 8, \quad -22$ **[1 mark]**

If $x - 8$ the second number is 22.

If $x - 22$, the second number is -8 .

There are two pairs of numbers: 8, 22 and -22, 8. **[1 mark]**

3.9 Review

Question 1

Let $\left(x+\frac{1}{x}\right)=a$.

$$a^2+4a+4=0$$
$$(a+2)^2=0$$
$$a=-2 \quad \textbf{[1mark]}$$

Substituting back,

$$x+\frac{1}{x}=-2$$
$$x^2+1=-2x$$
$$x^2+2x+1=0$$
$$(x+1)^2=0$$
$$x=-1 \quad \textbf{[1 mark]}$$

Question 2

$b^2-4ac>0$ indicates there are two real roots.

The correct answer is **B**.

Question 3

$\Delta=b^2-4ac$

For $-4x^2-x-3,\ a=-4,\ b=-1,\ c=-3$.

$\Delta=(-1)^2-4(-4)(-3)$

$\Delta=-47$

Δ is negative, so there is no x-intercept.

The correct answer is **A**.

Question 4

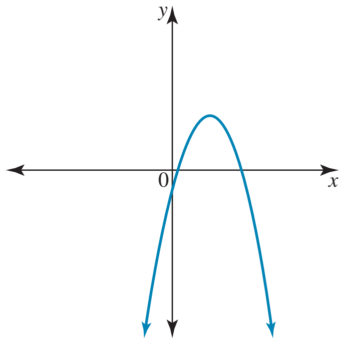
Options are in the form $y=a(x-h)^2+k$.

The parabola is inverted, so a is negative.

The vertex has positive values for x and y.

For positive values of the x-intercepts,

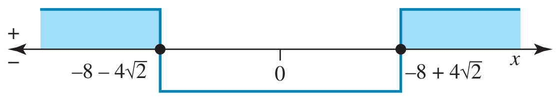
$y=-(x-2)^2+3$ is the only valid option.



Question 5

$$x^2 - 8x + 12 = kx + 4$$
$$x^2 - x(8 + k) + 8 = 0$$
$$\Delta = b^2 - 4ac$$
$$a = 1,\ b = (-8 - k),\ c = 8$$
$$\Delta = (-8 - k)^2 - 4(1)(8)$$
$$= (8 + k)^2 - 32 \qquad \textbf{[1 mark]}$$
$$\Delta \geq 0 \text{ for one or two intersections } \textbf{[1 mark]}$$
$$\left(8 + k - \sqrt{32}\right)\left(8 + k + \sqrt{32}\right) = 0$$
$$\left(8 + k - 4\sqrt{2}\right)\left(8 + k + 4\sqrt{2}\right) = 0$$

The zeros are $-8 + 4\sqrt{2}, -8 - 4\sqrt{2}$.



For at least one intersection, $k \in \left(-\infty, -8 - 4\sqrt{2}\right] \cup \left[-8 + 4\sqrt{2}, \infty\right)$. **[1 mark]**

$\left(\text{Also accept } k \in \left(-\infty, -8 - \sqrt{32}\right] \cup \left[-8 + \sqrt{32}, \infty\right)\right)$

Question 6

$y = 2 - x^2$ and $y = 2x$

For intersection:

$$2 - x^2 = 2x$$
$$x^2 + 2x - 2 = 0$$
$$a = 1,\ b = 2,\ c = -2$$
$$\Delta = b^2 - 4ac$$
$$\Delta = (2)^2 - 4(1)(2)$$
$$= 12$$
$$\Delta > 0$$

$\therefore$ there are two intersection points.

Question 7

$$x + y = 4$$
$$y = 4 - x$$

Substitute into equation

$$q = 3x^2 + 2xy + 2y^2$$
$$q = 3x^2 + 2x(4 - x) + 2(4 - x)^2$$
$$q = 3x^2 + 8x - 2x^2 + 32 - 16x + 2x^2$$
$$q = 3x^2 - 8x + 32$$

Turning point gives the minimum q value:

$$x = \frac{-b}{2a}$$
$$b = -8,\ a = 3$$
$$x = -\frac{-8}{6}$$

$$= \frac{4}{3}$$
$$y = 4 - \frac{4}{3}$$
$$= \frac{8}{3}$$
$$\therefore (x, y) = \left(\frac{4}{3}, \frac{8}{3}\right)$$

Question 8

Let $y = -11x^2 + 330x$, where y is the number of sales per week.

x -intercepts when $y = 0$

$$y = -11x^2 + 330x$$
$$-11x^2 + 330x = 0$$
$$-11x(x - 30) = 0$$
$$x = 0, 30$$

x-intercepts: $(0, 0)$, $(30, 0)$

y-intercept: $(0, 0)$ **[1 mark]**

Turning point gives maximum number of sales.

Axis of symmetry has equation $x = -\frac{b}{2a}$

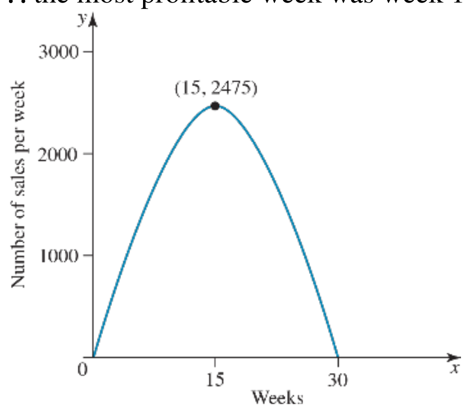
$$\therefore x = -\frac{330}{2(-11)}$$
$$= \frac{330}{22}$$
$$= 15$$

Substitute $x = 15$ in $y = -11x^2 + 330x$

$$y = -11(15)^2 + 330(15)$$
$$= 2475$$

Turning point is $(15, 2475)$ **[1 mark]**

$\therefore$ the most profitable week was week 15. **[1 mark]**



[1 mark]

Question 9

$$x = \frac{-b}{2a}$$
$$a = 1, \quad b = 2$$
$$x = \frac{-2}{2(1)}$$
$$= -1$$
$$f(1) = (-1)^2 + 2(-1) - 8$$
$$= -9$$
$$\text{TP} = (-1, -9)$$

Question 10

As the information given contains intercepts, use the intercept form of the quadratic equation.

$y = a(x-2)(2x+1)$

Substitute y-intercept $(0, 8)$

$$\begin{aligned} 8 &= a(-2)(1) \\ -2a &= 8 \\ a &= -4 \end{aligned}$$

$$\begin{aligned} \therefore y &= -4(x-2)(2x+1) \\ &= -4\left(2x^2 - 3x - 2\right) \\ &= -8x^2 + 12x + 8 \end{aligned}$$

4 Cubic polynomials

Topic	4	Cubic polynomials
Subtopic	4.2	Polynomials

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Question 1 (1 mark)

Given $p(x) = x^3 - 2x^2 - 4x + 2$, use the long division method to divide $p(x)$ by $(x - 1)$, and state the quotient and the remainder.

Question 2 (1 mark)

Select the expression that is not a polynomial.

A. $x^4 - x^3 + \frac{3x}{7} - 1$

B. $5x^4 + 3x^3$

C. $-3x^3 + x^2 - 2\sqrt{x}$

D. $4x$

E. $3x^2 + \frac{x}{\sqrt{5}}$

Question 3 (1 mark)
The degree, coefficient of the leading term and constant term of the polynomial with equation $5x + 6 - 3x^2 - x^3 - 5$ are, in order

A. $2, -3, 6$
B. $3, -1, -5$
C. $1, 5, 6$
D. $3, -1, 1$
E. $3, 1, -5$

Question 4 (1 mark)
The quotient when $4x - 1$ is divided by $x + 3$ is

A. -13
B. 3
C. 4
D. -1
E. -9

Question 5 (1 mark)
Calculate the values of a, b and c for which $4x^2 - 3x + 1 = ax(x-2) + b(x-2) + c$, and hence express $\frac{4x^2 - 3x + 1}{x - 2}$ in the form $p(x) + \frac{a}{x-2}$, where $p(x)$ is a polynomial and $a \in R$.

Topic	4	Cubic polynomials
Subtopic	4.3	The remainder and factor theorems

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Question 1 (1 mark)

When $2x^3 - 7x^2 + x - 2$ is divided by $(x - 1)$, the remainder is

A. -12

B. -6

C. 8

D. 6

E. -8

Question 2 (1 mark)

The cubic polynominal $p(x) = x^3 - kx + 3$ is exactly divisible by $(x + 3)$. The value of k must be

A. 8

B. -8

C. 3

D. -3

E. 5

Question 3 (1 mark)

Determine the polynominal $p(x) = ax^3 + bx + 20$ that is exactly divisible by $(x - 1)$ and, when divided by $(x - 2)$, leaves a remainder of -14.

Question 4 (1 mark)

If $p(-3) = 0$, then

A. -3 is the remainder when $p(x)$ is divided by $(x + 3)$.
B. $(x + 3)$ is a factor of $p(x)$.
C. $(x - 3)$ is a factor of $p(x)$
D. 3 is the remainder when $p(x)$ is divided by $(x + 3)$.
E. $(x + 3)$ is the quotient when $p(x)$ is divided by $(x - 3)$

Question 5 (1 mark)

Solve the equation $6x^3 - 17x^2 = 5x - 6$.

Question 6 (1 mark)

When $x^3 - 5x^2 + 2x + 8$ is divided by $(x + 2)$, the remainder is

A. 16
B. -24
C. -2
D. 0
E. 8

Topic	4	Cubic polynomials
Subtopic	4.4	Graphs of cubic polynomials

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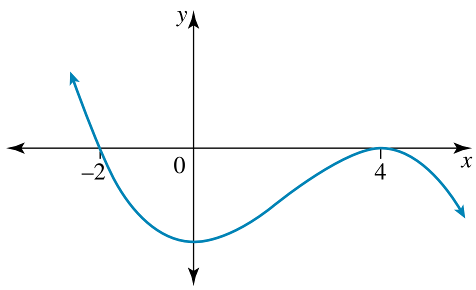
Question 1 (1 mark)

The equation of the graph shown, where a is a positive constant, could be



A. $y = a(x + 2)(x - 4)^2$
B. $y = a(x + 2)^2(x - 4)$
C. $y = -a(x + 2)(x - 4)^2$
D. $y = a(x + 4)^2(x - 2)$
E. $y = -a(x + 4)(x - 2)^2$

Question 2 (3 marks)

Sketch the graph of $y = (x + 4)(x + 1)(x - 3)$. (Do not attempt to find the turning points.)

Question 3 (1 mark)

For the function $y = -2(3 - x)^3 - 1$, determine which of the following statements is false.

A. The function has been translated 1 unit downwards.
B. The y-intercept is -55.
C. The function has been translated 3 units to the right.
D. The function has been dilated by a factor of 2.
E. The function is a negative cubic.

Question 4 (1 mark)

The graph with equation $y = -3(x+5)^3 - 4$ has a stationary point of inflection with coordinates

A. $(-5, -4)$
B. $(5, -4)$
C. $(-15, -4)$
D. $(15, 4)$
E. $(-5, 4)$

Question 5 (1 mark)

Consider $p(x) = 3x^3 + kx^2 + 4$.

a. Given that $(3x + 2)$ is a factor of $p(x)$, find $p\left(-\frac{2}{3}\right)$ and hence express $p(x)$ as a product of linear factors.

b. sketch $p(x)$.

c. Does the point $(-1, -5)$ lie on the graph? Justify your answer.

Question 6 (1 mark)

The solutions to the equation $8x^3 + 4x^2 = 18x + 9$ are

A. $x = -\frac{1}{2}, -\frac{2}{3}, \frac{2}{3}$
B. $x = -1, -3, 3$
C. $x = -\frac{1}{2}, -\frac{3}{2}, \frac{3}{2}$
D. $x = \frac{1}{2}, -3, 3$
E. $x = \frac{1}{2}, \frac{2}{3}, 3$

Topic	4	Cubic polynomials
Subtopic	4.5	Equation of cubic polynomials

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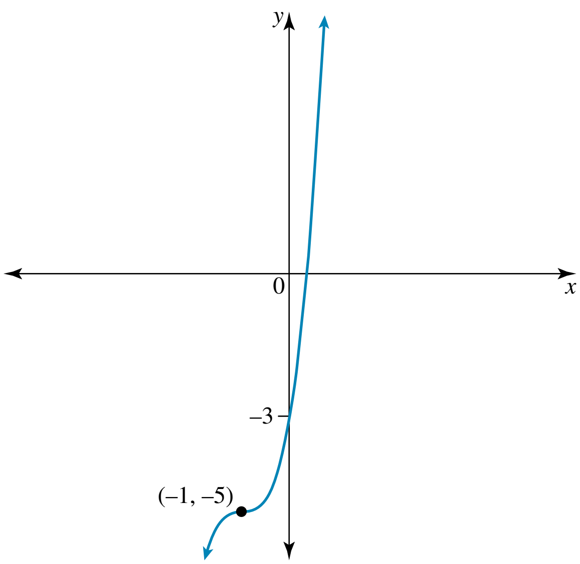
Question 1 (1 mark)

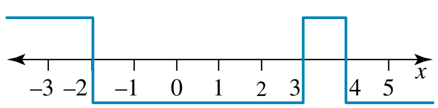
The equation of the graph shown could be



A. $y = (x + 1)^3 - 5$
B. $y = 2(x + 1)^3 - 5$
C. $y = \frac{1}{2}(x + 1)^3 - 5$
D. $y = (x - 1)^3 - 5$
E. $y = (x + 1)^3 - 3$

Question 2 (1 mark)

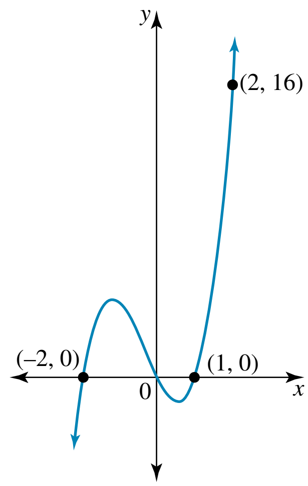
The equation of the cubic function that has the sign diagram shown below is



A. $y = (x + 2)(x - 3)(x - 4)$
B. $y = (x - 2)(x + 3)(x + 4)$
C. $y = (x + 2)^2 (x - 3)(x + 4)$
D. $y = -(x + 2)(x - 3)(x - 4)$
E. $y = (x + 2)^2 (x - 3)(x + 4)$

Question 3 (3 marks)

Determine the equation of the graph below in both of the forms $y = a(x - b)(x - c)(x - d)$ and $y = ax^3 + bx^2 + cx + d$



Question 4 (1 mark)

The equation of the graph formed when the graph of $y = 2x^3$ is reflected in the x-axis and translated 2 units to the left and 3 units upwards is

A. $y = -2(x + 2)^3 + 3$
B. $y = 2(x + 2)^3 + 3$
C. $y = -2(x - 2)^3 + 3$
D. $y = -2(x + 3)(x - 2)^2$
E. $y = 2(x + 2)^3 + 3$

Question 5 (4 marks)

Find the equation of the cubic function $y = ax^3 + bx^2 + cx + d$ given that $a = 1,\ d = 36$ and the curve passes through the points $(-1, 42)$ and $(1, 20)$.

Question 6 (1 mark)

The equation of a cubic graph with x-intercept at 2, -3 and -4, and a y-intercept of 1 is

A. $y = -\frac{1}{24}(x-2)(x+3)(x+4)$

B. $y = -(x-2)(x+3)(x+4)$

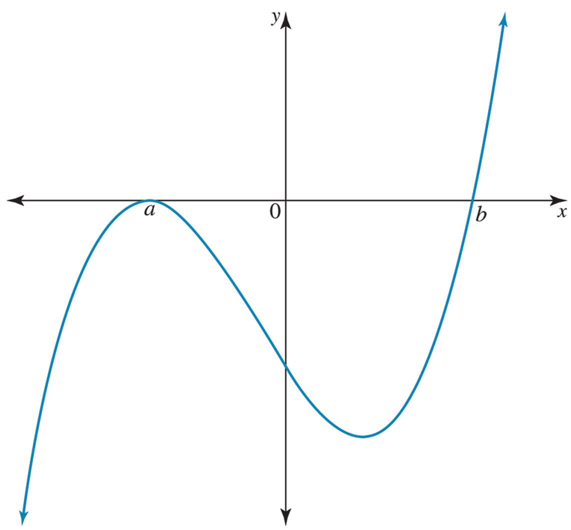
C. $y = (x-2)(x+3)(x+4)$

D. $y = -\frac{1}{12}(x-2)(x+3)(x+4)$

E. $y = \frac{1}{24}(x+2)(x-3)(x-4)$

Question 7 (1 mark)

The equation of the graph below could be



A. $y = (x-a)^2(x+b)$

B. $y = (x-a)^2(x-b)$

C. $y = (x+a)^2(x-b)$

D. $y = (x+a)^2(x+b)$

E. $y = (x+a)(x+b)^2$

Topic	4	Cubic polynomials
Subtopic	4.6	Cubic models and applications

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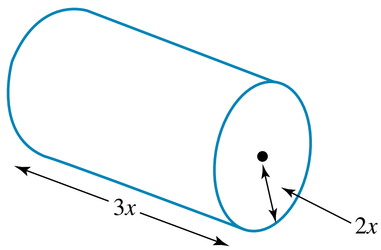
Question 1 (1 mark)

An expression for the volume of the cylinder shown, with radius $2x$ and length $3x$, would be



A. $4\pi x^3$
B. $12\pi x^3$
C. $4\pi x^2 + 3x$
D. $3r^2 + \pi r^3$
E. $6\pi x^3$

Question 2 (1 mark)

A rectangular shaped box with a square base has a volume of 60 cm^3. The base of the box has a width 2 cm larger than the height. If x is the height of the box, an expression for the volume is

A. $x^3 + 2x^2 + 2x - 60 = 0$
B. $60 - 2x^3 - 2x^2 - 2x = 0$
C. $x^3 + 4x^2 + 4x - 60 = 0$
D. $x^3 + 4x^2 + 4x + 60 = 0$
E. $60 - x^3 + 4x^2 + 4x = 0$

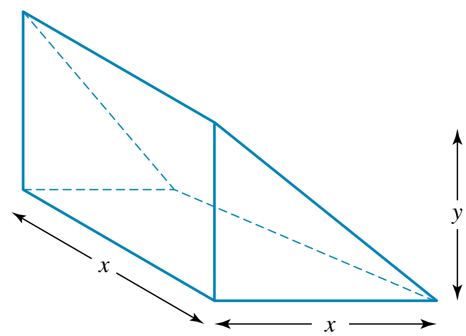
Question 3 (1 mark)

The volume of a container is given by $V = 2.7x^2 - 3x^3$. The restrictions on the values that x can take are

A. $x > 0.9$
B. $x > 0$
C. $0 \leq x \leq 0.9$
D. $0 < x < 0.9$
E. $x < 0.9$

Question 4 (4 marks)

A triangular box consists of sides x mm in length and height y mm. If the length, width and height added together equal 18 mm, find an expression for the volume and determine the restrictions on the values that x can take. Sketch a graph of x versus volume.



Question 5 (3 marks)

Taylah and Shani set up a business building and selling garden sheds. After looking at their competitors, they decide to sell each shed for \$300. Their accountant advises them that the cost of building the sheds is $x^3 + 25x + 1000$ dollars, where x represents the number of sheds built each month.

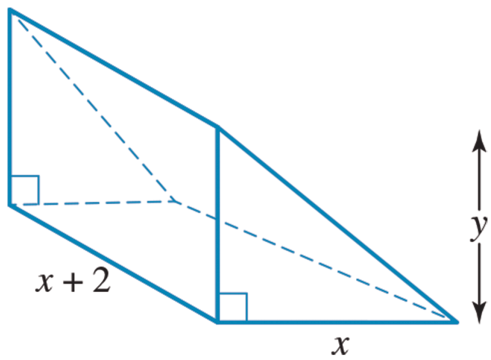
Work out an equation for the profit they make on selling sheds per month.

Calculate the profit they would make if they sold 2, 6, 8, 12 and 16 sheds per month.

They aim to increase production to build 20 sheds per month. Is this a good business decision? What would you advise them to do?

Question 6 (4 marks)

The triangular box below consists of sides x mm and $x + 2$ mm in length and width and a height of y mm. If the length, width and height added together equal 24 mm, find an expression for the volume, and determine the restrictions on the values that x can take.



Topic	4	Cubic polynomials
Subtopic	4.7	Review

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Question 1 (3 marks)

Calculate the values of a, b and c for which $4x^2 - 3x + 1 = ax(x-2) + b(x-2) + c$, and hence express $\frac{4x^2 - 3x + 1}{x - 2}$ in the form $p(x) + \frac{k}{x-2}$, where $p(x)$ is a polynomial and $k \in R$.

Question 2 (4 marks)

Solve the equation $6x^3 - 17x^2 = 5x - 6$.

Question 3 (4 marks)

Consider $p(x) = 3x^3 + kx^2 + 4$.

a. Given that $(3x + 2)$ is a factor of $p(x)$, find k and hence express $p(x)$ as a product of linear factors. **(2 marks)**

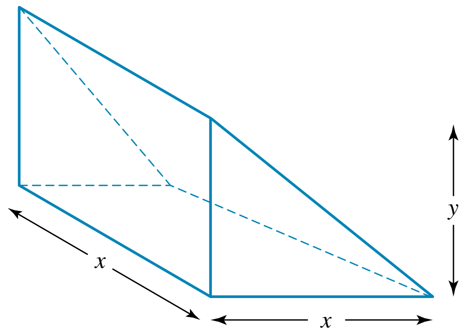
b. Sketch $p(x)$. **(2 marks)**

Question 4 (4 marks)

Find the equation of the cubic function $y = ax^3 + bx^2 + cx + d$ given that $a = 1,\ d = 36$ and the curve passes through the points $(-1, 42)$ and $(1, 20)$.

Question 5 (4 marks)

A triangular box consists of sides x mm in length and height y mm. If the length, width and height added together equal 18 mm, find an expression for the volume and determine the restrictions on the values that x can take. Sketch a graph of x versus volume.



Question 6 (1 mark)

Which of the following is not a polynomial?

A. $-3x^2 + x^4 - \frac{3x}{2}$

B. $-3x + \frac{4}{x}$

C. $x^2 + \frac{x}{\sqrt{2}}$

D. $-2x$

E. $4x - 3x^2$

Question 7 (1 mark)

If $p(4) = 0$, then

A. $(x + 4)$ is a factor of $p(x)$.

B. 4 is the remainder when $p(x)$ is divided by $(x - 4)$.

C. 4 is the remainder when $p(x)$ is divided by $(x + 4)$.

D. $(x - 4)$ is a factor of $p(x)$.

E. $(x - 4)$ is the remainder when $p(x)$ is divided by $(x + 4)$.

Question 8 (1 mark)

The cubic polynomial $p(x) = x^3 - 2x^2 - kx - 4$ is exactly divisible by $(x + 1)$. The value of k must be

A. -7
B. 3
C. -1
D. -3
E. 7

Question 9 (1 mark)

The graph with equation $y = -(x - 3)^3 + 2$ has a stationary point of inflection with coordinates

A. $(-3, 2)$
B. $(3, -2)$
C. $(3, 2)$
D. $(-3, -2)$
E. $(2, 3)$

Question 10 (1 mark)

For the function $y = 2(1 - x)^3 + 3$, which of the following statements is false?

A. The function is a negative cubic.
B. The function has been translated 1 unit to the right.
C. The function has been dilated by a factor of 2.
D. The stationary point of inflection is $(3, 1)$.
E. The function has been shifted 3 units up.

Question 11 (1 mark)

The equation of the graph formed when the graph $y = -x^3$ is reflected in the y-axis, translated 3 units right and 1 unit down is

A. $y = (x - 3)^3 - 1$
B. $y = (x + 3)^3 - 1$
C. $y = -(x - 3)^3 - 1$
D. $y = -(x + 3)^3 - 1$
E. $y = (x + 3)^3 + 1$

Question 12 (3 marks)

Sketch the graph of $y = 2x^3 - x^2 - 12x - 9$ in your workbook. Do not work out the turning points.

Question 13 (1 mark)

The graph of $y = (2 - x)\left(x^2 - 2x - 8\right)$ has

A. one stationary point of inflection.
B. one minimum turning point.
C. one maximum turning point.
D. one maximum and one minimum turning point.
E. no stationary points.

Answers and marking guide

4.2 Polynomials

Question 1

$$\begin{array}{r} x^2 - x - 5 \\ x - 1 \overline{\big) x^3 - 2x^2 - 4x + 2} \\ \underline{x^3 - x^2 \quad\quad\quad\quad} \\ -x^2 - 4x \quad\quad \\ \underline{-x^2 + x \quad\quad\;\;} \\ -5x + 2 \\ \underline{-5x + 5} \\ -3 \end{array}$$

$\therefore x^2 - x - 5 - \dfrac{3}{x-1}$

$\therefore$ quotient $= x^2 - x - 5$, remainder $= -3$ **[1 mark]**

Question 2

A polynomial is an expression that contains positive whole number powers of x.

$\sqrt{x} = x^{\frac{1}{2}}$ – the power is not a positive whole number.

$\therefore\ -3x^3 = x^2 - 2\sqrt{x}$ is not a polynomical.

The correct answer is **C**.

Question 3

$5x + 6 - 3x^2 - x^3 - 5$

Rearrange the polynomial in ascending powers and simplify:

$-x^3 - 3x^2 + 5x + 1$

The highest power is 3.

$\therefore$ degree is 3.

The leading term is $-x^3$.

$\therefore$ coefficient is -1.

The constant term $= 6 - 5 = 1$.

$\therefore 3, -1, 1$

The correct answer is **D**.

Question 4

$$\frac{4x-1}{x+3} = \frac{4(x+3) - 13}{x+3}$$
$$= 4 - \frac{13}{x+3}$$

Therefore, the quotient is 4.

Question 5

$4x^2 - 3x + 1 = ax(x-2) + b(x-2) + c$

$4x^2 - 3x + 1 = ax^2 - 2ax + bx - 2b + c$ [**1 mark**]

Equate coefficients

$$a = 4$$
$$-2a + b = -3$$
$$-2(4) + b = -3$$
$$\therefore b = 5$$
$$-2b + c = 1$$
$$-2(5) + c = 1$$
$$\therefore c = 11$$
$$\therefore a = 4, b = 5, c = 11 \qquad \textbf{[1 mark]}$$
$$4x^2 - 3x + 1 = 4x(x-2) + 5(x-2) + 11$$
$$\frac{4x^2 - 3x + 1}{x-2} = 4x + 5 + \frac{11}{x-2} \qquad \textbf{[1 mark]}$$

4.3 The remainder and factor theorems

Question 1

Let $p(x) = 2x^3 - 7x^2 + x - 2$.

Remainder $= p(1)$

$$p(1) = 2(1)^3 - 7(1)^2 + 1 - 2$$
$$= -6$$

The correct answer is **B**.

Question 2

$p(x) = x^3 - kx + 3$

If $(x+3)$ is factor of $p(x)$, $p(-3) = 0$.

$$p(-3) = (-3)^3 - (-3)k + 3$$
$$0 = -27 + 3k + 3$$
$$0 = -24 + 3k$$
$$24 = 3k$$
$$\therefore k = 8$$

The correct answer is **A**.

Question 3

$p(x) = ax^3 + bx + 20$

$p(x)$ is exactly divisible by $(x-1)$, so $p(1) = 0$.

$a + b + 20 = 0$

$a + b = -20 \qquad [1]$

The remainder when divided by $(x-2)$ is -14, so $p(2) = -14$.

$8a + 2b + 20 = -14$

$8a + 2b = -34$ **[1 mark]**

$4a + b = -17 \qquad [2]$

Subtract [1] from [2]:

$3a = 3$

$a = 1$

Substitute $a = 1$ into [1]:

$1 + b = -20$

$b = -21$

$\therefore p(x) = x^3 - 21x + 20$ **[1 mark]**

Question 4

Factor theorem:

If $(x+3)$ is a factor of $p(x)$, $p(-3) = 0$.

$\therefore (x+3)$ is a factor of $p(x)$.

Question 5

$$6x^3 - 17x^2 = 5x - 6$$

$6x^3 - 17x^2 - 5x + 6 = 0$

Let $p(x) = 6x^3 - 17x^2 - 5x + 6$.

$p(3) = 0, \therefore (x - 3)$ is a factor. **[1 mark]**

$$\begin{array}{r} 6x^2 + x - 2 \\ x-3\overline{\smash{)}6x^3 - 17x^2 - 5x + 6} \\ \underline{6x^3 - 18x^2} \qquad\qquad \\ x^2 - 5x \qquad \\ \underline{x^2 - 3x} \qquad \\ -2x + 6 \\ \underline{-2x + 6} \\ 0 \end{array}$$

$p(x) = 6x^3 - 17x^2 - 5x + 6 = 0$

$0 = (x - 3)(6x^2 + x - 2)$ **[1 mark]**

Now factorise the quadratic factor to fully factorise the cubic.

$0 = (x - 3)(2x - 1)(3x + 2)$ **[1 mark]**

$\therefore x = 3, \frac{1}{2}, -\frac{2}{3}$ **[1 mark]**

Question 6

Let $p(x) = x^3 - 5x^2 + 2x + 8$

Remainder $= p(-2)$

$p(-2) = (-2)^3 - 5(-2)^2 + 2(-2) + 8$

$= -8 - 20 - 4 + 8$

$= -24$

4.4 Graphs of cubic polynomials

Question 1

$-a$, since the cubic graph slopes downwards.

x-intercept at -2 and turning point at $x = 4$ indicate that $(x + 2)$ is a factor and $(x - 4)$ is a factor with multiplicity of 2 at $x = 4$.

$\therefore y = -a(x + 2)(x - 4)^2$

The correct answer is **C**.

Question 2

The graph will have a positive cubic shape.

x-intercepts at $y = 0$:

$(x + 4)(x + 1)(x - 3) = 0$

$x = -4, -1, 3$

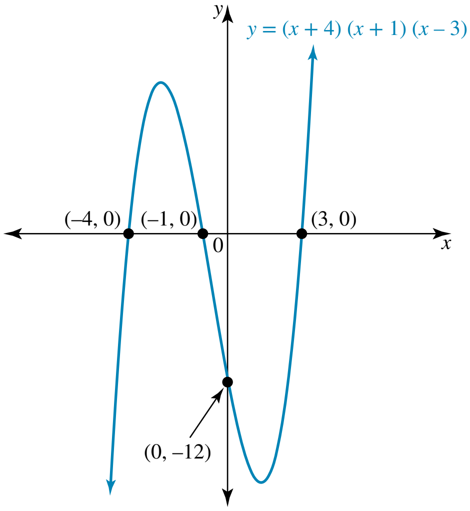
$\therefore x$-intercepts are $(-4, 0), (-1, 0), (3, 0)$. **[1 mark]**

y-intercepts at $x = 0$:

$y = (4)(1)(-3)$

$y = -12$

$\therefore y$-intercept is $(0, -12)$. **[1 mark]**



Award 1 mark for correct graph.

Question 3

$y = -2(3 - x)^3 - 1$

$\quad = -2(-(-3 + x))^3 - 1$

$\quad = -2 \times -1(x - 3)^3 - 1$

$\quad = 2(x - 3)^3 - 1$

The function is a positive cubic, since the coefficient of the leading term is positive. The statement that the function is a negative cubic is false.

The correct answer is **E**.

Question 4

$y = -3(x + 5)^3 - 4$

Equation is in the form: $y = a(x - h)^3 + k$

$\therefore h = -5, k = -4$

Inflection point is $(-5, -4)$.

Question 5

a. $p(x) = 3x^3 + kx^2 + 4$

$(3x + 2)$ is a factor $\therefore p\left(-\frac{2}{3}\right) = 0.$

Substitute into $p(x)$

$$0 = 3\left(-\frac{2}{3}\right)^3 + k\left(-\frac{2}{3}\right)^2 + 4$$

$$0 = -\frac{8}{9} + \frac{4k}{9} + \frac{36}{9}$$

$$0 = \frac{4k + 28}{9}$$

$$0 = 4k + 28$$

$$4k = -28$$

$k = -7$ **[1 mark]**

$p(x) = 3x^3 - 7x^2 + 4$

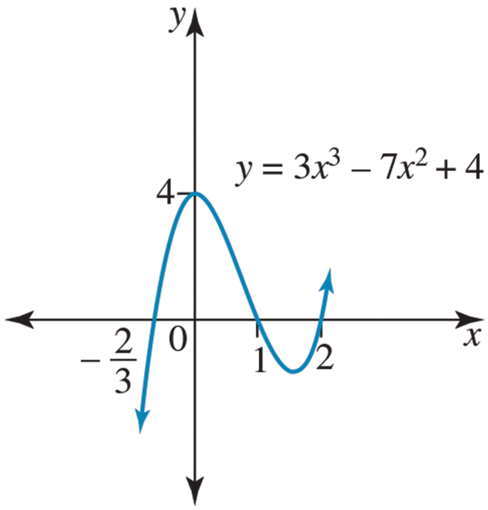
$p(x) = (3x + 2)\left(x^2 - 3x + 2\right)$

$p(x) = (3x + 2)(x - 1)(x - 2)$ **[1 mark]**

b. $p(x) = (3x + 2)(x - 1)(x - 2)$

x-intercepts at $\left(-\frac{2}{3}, 0\right), (1, 0), (2, 0)$

y-intercepts at $(0, 4)$



c. $p(x) = (3x + 2)(x - 1)(x - 2)$

$x = -1$

$p(-1) = (-3 + 2)(-1 - 2)(-1 - 1)$

$p(-1) = -1 \times -3 \times -2$

$p(-1) = -6$

The point $(-1, -6)$ lies on the graph. $\therefore$ the point $(-1, -5)$ does not. **[1 mark]**

Question 6

$8x^3 + 4x^2 = 18x + 9$

$$\begin{aligned} 0 &= 8x^3 + 4x^2 - 18x - 9 \\ &= 4x^2(2x + 1) - 9(2x + 1) \\ &= (2x + 1)\left(4x^2 - 9\right) \\ &= (2x + 1)(2x - 3)(2x + 3) \end{aligned}$$

$x = -\frac{1}{2}, -\frac{3}{2}, \frac{3}{2}$

4.5 Equation of cubic polynomials

Question 1

The graph is a cubic shape of the form $y = a(x - h)^3 + k$.

The point of inflection is $(-1, -5)$.

$\therefore y = a(x + 1)^3 - 5$

y-intercept $(0, -3)$

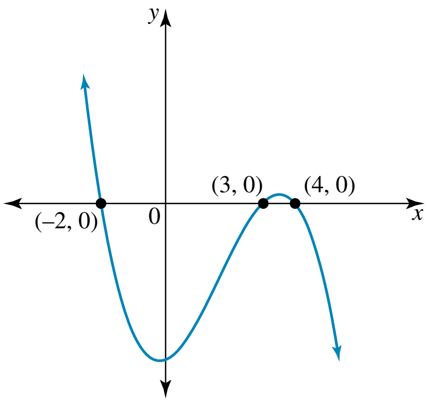
$-3 = a(x + 1)^3 - 5$

$a = 2$

The equation is $y = 2(x + 1)^3 - 5$.

The correct answer is **B**.

Question 2
The graph corresponding to the sign diagram is shown below.



It is a negative cubic graph with x-intercepts at $(-2, 0), (3, 0), (4, 0)$.
Therefore, the equation is $y = -(x + 2)(x - 3)(x - 4)$.
The correct answer is **D**.

Question 3
$y = a(x - b)(x - c)(x - d)$
x-intercepts $(-2, 0), (0, 0), (1, 0)$
$y = ax(x - 1)(x + 2)$ **[1 mark]**
Find a using $(2, 16)$:
$16 = 2a(1)(4)$
$16 = 8a$
$a = 2$ **[1 mark]**
$\therefore y = 2x(x - 1)(x + 2)$
$y = 2x\left(x^2 + x - 2\right)$
$y = 2x^3 + 2x^2 - 4x$ **[1 mark]**

Question 4
$y = a(x - h)^3 + k$
Reflected in the x-axis. $\therefore a$ is a negative.
$h = -2 \therefore$ translated 2 units left.
$k = +3 \therefore$ translated 3 units upwards
$\therefore y = -2(x + 2)^3 + 3$

Question 5
$y = ax^3 + bx^2 + cx + d$
$a = 1,\ d = 36$
$y = x^3 + bx^2 + cx + 36$ [**1 mark**]
Using $(-1, 42)$
$42 = -1 + b(-1)^2 + c(-1) + 36$
$7 = b - c$ (1) [**1 mark**]
using $(1, 20)$
$20 = 1 + b(1)^2 + c(1) + 36$
$-17 = b + c$ (2) [**1 mark**]
Add (1) and (2)
$2b = -10$
$b = -5$
Substitute $b = -5$ into (2)

$-17 = -5 + c$

$c = -12$

$\therefore$ the equation is $y = x^3 - 5x^2 - 12x + 36$. **[1 mark]**

Question 6

General form: $y = a(x - b)(x - c)(x - d)$

Substitute the x-intercept $y = a(x - 2)(x + 3)(x + 4)$

Substitute the y-intercept:

$1 = a(-2)(3)(4)$

$1 = -24a$

$a = -\frac{1}{24}$

$\therefore y = -\frac{1}{24}(x - 2)(x + 3)(x + 4)$

Question 7

General form of a cubic is $y = (x - a)(x - b)(x - c)$.

As there is a repeated factor at $x = a$, the equation becomes $y = (x - a)^2(x - b)$

4.6 Cubic models and applications

Question 1

Volume of cylinder = area of circular end × length

$$\begin{aligned}\text{Volume} &= \pi r^2 \times l \\ &= \pi(2x)^2 \times 3x \\ &= 12\pi x^3\end{aligned}$$

The correct answer is **B**.

Question 2

$$\begin{aligned}\text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ 60 &= (x + 2)(x + 2)(x) \\ x\left(x^2 + 4x + 4\right) &= 60 \\ x^3 + 4x^2 + 4x &= 60 \\ x^3 + 4x^2 + 4x - 60 &= 0\end{aligned}$$

The correct answer is **C**.

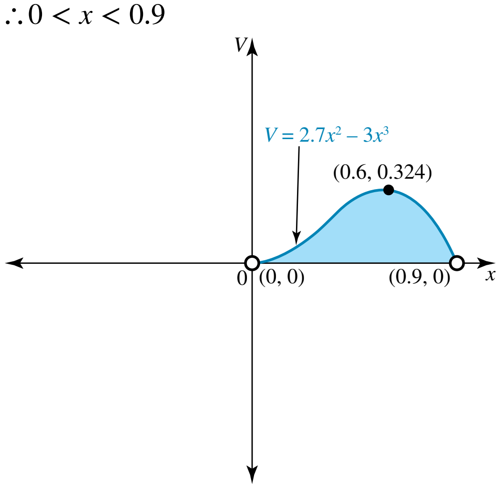
Question 3

$V > 0$ and $x > 0$

$2.7x^2 - 3x^3 > 0$

$-3x^2(x - 0.9) > 0$

$\therefore 0 < x < 0.9$



The correct answer is **D**.

Question 4

$2x + y = 18$

$y = 18 - 2x$ **[1 mark]**

Volume of the triangular box $= \frac{1}{2}$ length × width × height

$V = \frac{1}{2}(x)(x)(y)$

$V = \frac{1}{2}x^2(18 - 2x)$

$V = 9x^2 - x^3$ **[1 mark]**

Restriction of volume:

$V > 0$ and $x > 0$

$9x^2 - x^3 > 0$

$x^2(9 - x) > 0$

$\therefore$ as volume and length must be positive, $0 < x < 9$. **[1 mark]**

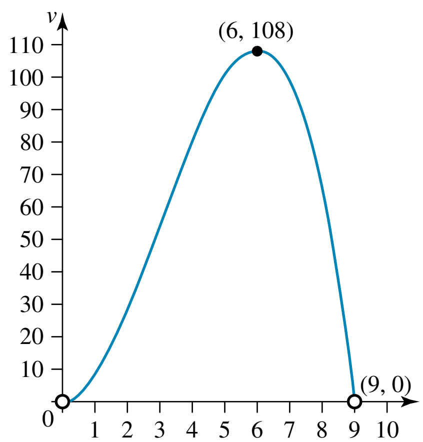
$v = 9x^2 - x^3$

This is a cubic function and, when graphed, the intercepts are (0, 0) and (9, 0).Unlike a quadratic function, the maximum value does not occur halfway between the intercepts.

Using CAS, we can find the maximum TP.

TP = (6, 108)

[1 mark]



Question 5

Profit (P) = selling price − cost of production

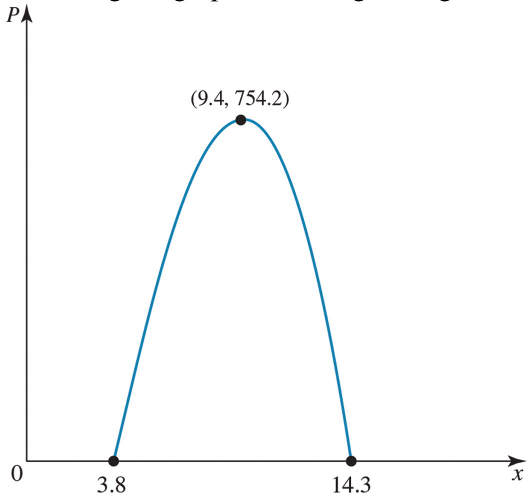
$P = 300x - (x^3 + 25x + 1000)$

$P = -x^3 + 275x - 1000$ **[1 mark]**

Sheds per month	Profit
2	−458
6	434
8	688
12	572
16	−696

[1 mark]

Sketching the graph of P using CAS gives:



Producing more than 14 sheds per month is not a good decision as they will make a loss. Producing 9 or 10 sheds per month gives maximum profit. **[1 mark]**

Question 6

$$x + x + 2 + y = 24$$
$$2x + y + 2 = 24$$
$$y = 22 - 2x \quad [\textbf{1 mark}]$$

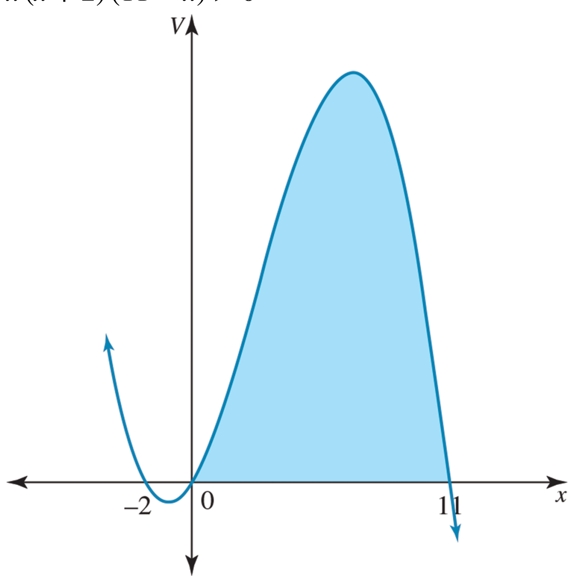
Volume of the triangular box $= \dfrac{1}{2}$ length $\times$ width $\times$ height

$$V = \frac{1}{2}(x)(x+2)(y)$$
$$= \frac{1}{2}(x)(x+2)(22-2x)$$
$$= \frac{1}{2}x(x+2)2(11-x) \qquad [\textbf{1 mark}]$$

$\therefore V = x(x+2)(11-x)$ **[1 mark]**

Restriction of volume

$$V > 0$$
$$x(x+2)(11-x) > 0$$



As volume and length must be positive, $x \in (0, 11)$. **[1 mark]**

4.7 Review

Question 1

$4x^2 - 3x + 1 = ax(x-2) + b(x-2) + c$

$4x^2 - 3x + 1 = ax^2 - 2ax + bx - 2b + c$ **[1 mark]**

Equate coefficients:

$$a = 4$$
$$-2a + b = -3$$
$$-2(4) + b = -3$$
$$\therefore b = 5$$
$$-2b + c = 1$$
$$-2(5) + b = 1$$
$$\therefore c = 11$$
$$\therefore b = 4, b = 5, c = 11$$ **[1 mark]**

$4x^2 - 3x + 1 = 4x(x-2) + 5(x-2) + 11$

$\dfrac{4x^2 - 3x + 1}{x-2} = 4x + 5 + \dfrac{11}{x-2}$ **[1 mark]**

Question 2

$$6x^3 - 17x^2 = 5x - 6$$

$6x^3 - 17x^2 - 5x + 6 = 0$

Let $P(x) = 6x^3 - 17x^2 - 5x + 6$.

$P(3) = 0$, $\therefore (x-3)$ is a factor. **[1 mark]**

$$\begin{array}{r|l} & 6x^2 + x - 2 \\ \hline x - 3 & 6x^3 - 17x^2 - 5x + 6 \\ & 6x^3 - 18x^2 \\ \hline & \quad x^2 - 5x \\ & \quad x^2 - 3x \\ \hline & \quad\quad -2x + 6 \\ & \quad\quad -2x + 6 \\ \hline & \quad\quad\quad 0 \end{array}$$

$P(x) = 6x^3 - 17x^2 - 5x + 6 = 0$

$0 = (x-3)(6x^2 + x - 2)$ **[1 mark]**

Now factorise the quadratic factor to fully factorise the cubic.

$0 = (x-3)(2x-1)(3x+2)$ **[1 mark]**

$\therefore x = 3, \dfrac{1}{2}, -\dfrac{2}{3}$ **[1 mark]**

Question 3

a. $p(x) = 3x^3 + kx^2 + 4$

$(3x+2)$ is a factor. $\therefore p\left(-\dfrac{2}{3}\right) = 0$.

Substitute into $p(x)$:

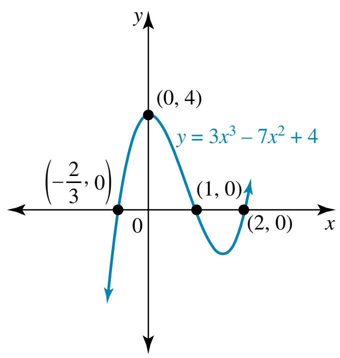
$$0 = 3\left(-\frac{2}{3}\right)^3 + k\left(-\frac{2}{3}\right)^2 + 4$$
$$0 = -\frac{8}{9} + \frac{4k}{9} + \frac{36}{9}$$
$$0 = \frac{4k + 28}{9}$$
$$0 = 4k + 28$$
$$4k = -28$$
$$k = -7$$ **[1 mark]**

$$p(x) = 3x^3 - 7x^2 + 4$$
$$p(x) = (3x + 2)\left(x^2 - 3x + 2\right)$$
$$p(x) = (3x + 2)(x - 1)(x - 2)$$ **[1 mark]**

b. $p(x) = (3x + 2)(x - 1)(x - 2)$

x-intercepts at $\left(-\frac{2}{3}, 0\right)$, $(1, 0)$, $(2, 0)$

y-intercepts at $(0, 4)$ **[1 mark]**



[1 mark]

Question 4

$y = ax^3 + bx^2 + cx + d$

$a = 1,\ d = 36$

$y = x^3 + bx^2 + cx + 36$ **[1 mark]**

Using $(-1, 42)$:

$42 = -1 + b(-1)^2 + c(-1) + 36$

$7 = b - c$ **[1 mark]**

Using $(1, 20)$:

$20 = 1 + b(1)^2 + c(1) + 36$

$-17 = b + c$ [2] **[1 mark]**

Add [1] and [2]:

$2b = -10$

$b = -5$

Substitute $b = -5$ into [2]:

$-17 = -5 + c$

$c = -12$

Therefore, the equation is $y = x^3 - 5x^2 - 12x + 36$. **[1 mark]**

Question 5

$2x + y = 18$

$y = 18 - 2x$ **[1 mark]**

Volume of the triangular box $= \frac{1}{2}$ lenght × width × height

$V = \frac{1}{2}(x)(x)(y)$

$V = \frac{1}{2}x^2(18 - 2x)$

$V = 9x^2 - x^3$ **[1 mark]**

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$x^2(9 - x) > 0$

Therefore, as volume and length must be positive,

$0 < x < 9.$ **[1 mark]**

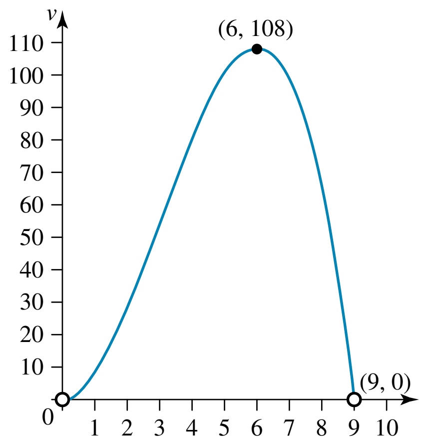
$V = 9x^2 - x^3$

This is a cubic function and, when graphed, the intercepts are $(0, 0)$ and $(9, 0)$.

Unlike a quadratic function, the maximum value does not occur halfway between the intercepts.

Using CAS, we can find the maximum TP.

TP = (6, 108)



[1 mark]

Question 6

A polynomial is an expression that contains positive whole number powers of x.

$\frac{4}{x} = 4x^{-1}$; therefore, the power is not a positive whole number.

$\therefore -3x + \frac{4}{x}$ is not a polynomial.

Question 7

Factor theorem: if $(x - 4)$ is a factor of $p(x)$, $p(4) = 0$.

Question 8

$p(x) = x^3 - 2x^2 - kx - 4$

If $x + 1$ is a factor of $p(x)$, $p(-1) = 0$.

$p(-1) = (-1)^3 - 2(-1)^2 - k(-1) - 4$

$0 = -1 - 2 + k - 4$

$k = 7$

Question 9

$y = -(x-3)^3 + 2$

Equation is in the form $y = a(x-h)^3 + k$

$\therefore h = 3, k = 2$

Stationary point of inflection: $(3, 2)$

Question 10

$y = 2(1-x)^3 + 3$

$\quad = 2(-(x-1))^3 + 3$

$\quad = -2(x-1)^3 + 3$

The stationary point of inflection is $(1, 3)$, not $(3, 1)$.

Question 11

$y = -x^3$

Reflected in the y-axis: $\Rightarrow -(-x)^3 = x^3$

Translated 3 units right: $\Rightarrow (x-3)^3$

Translated 1 unit down: $\Rightarrow (x-3)^3 - 1$

$\therefore y = (x-3)^3 - 1$

Question 12

y-intercept $= -9$ **[1 mark]**

Let $p(x) = 2x^3 - x^2 - 12x - 9$

$P(-1) = 0$: therefore $(x+1)$ is a factor.

By inspection

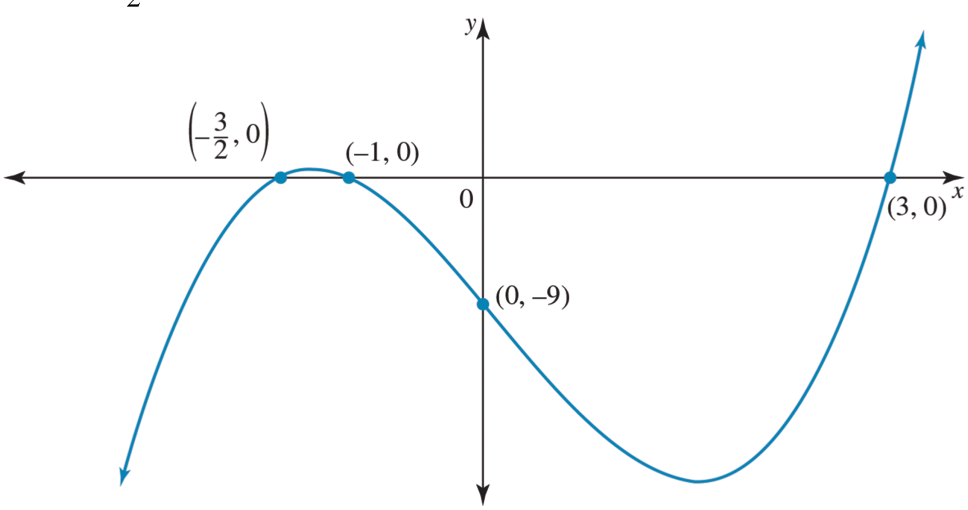
$y = (x+1)\left(2x^2 - 3x - 9\right)$

$\quad = (x+1)(2x+3)(x-3)$

x-intercepts:

$0 = (x+1)(2x+3)(x-3)$

$x = -1, -\dfrac{3}{2}, 3$ **[1 mark]**



[1 mark]

$p(-1) = 0$

Question 13

$y = (2-x)\left(x^2 - 2x - 8\right)$

$\quad = (x-2)(x+2)(x-4)$

There are three x-intercepts at $x = 4, 2$ and -2. Therefore, there are two turning points.

As the graph is a negative cubic, the first turning point must be a minimum and the second one must be a maximum, in order to satisfy the long-term behaviour requirements of a negative cubic polynomial.

5 Quartic polynomials

Topic	5	Quartic polynomials
Subtopic	5.2	Quartic polynomials

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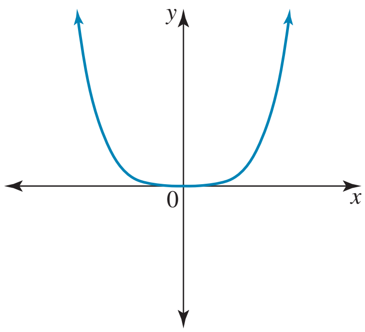
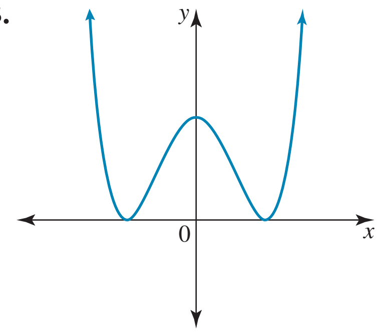
Question 1 (1 mark)

Which of the following options could **not** be the graph of $y = ax^4 + bx^3 + cx^2 + dx + e$ with $a > 0$?

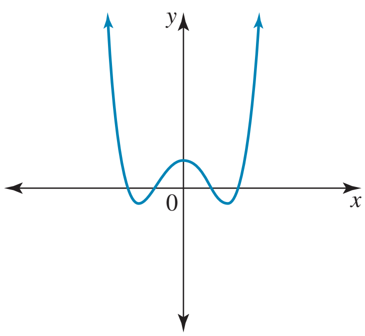
A.



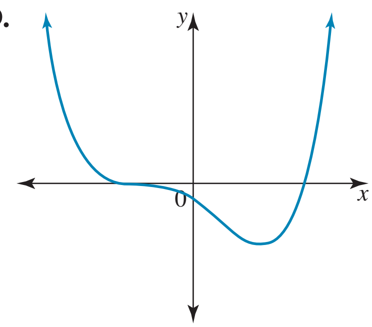
B.



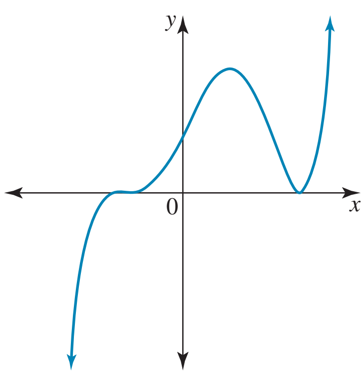
C.



D.

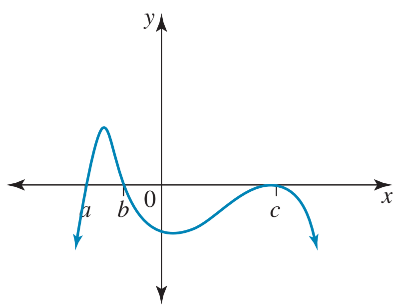


E.



Question 2 (1 mark)

A possible equation for the graph shown, given that $a,\ b,\ c > 0$, would be

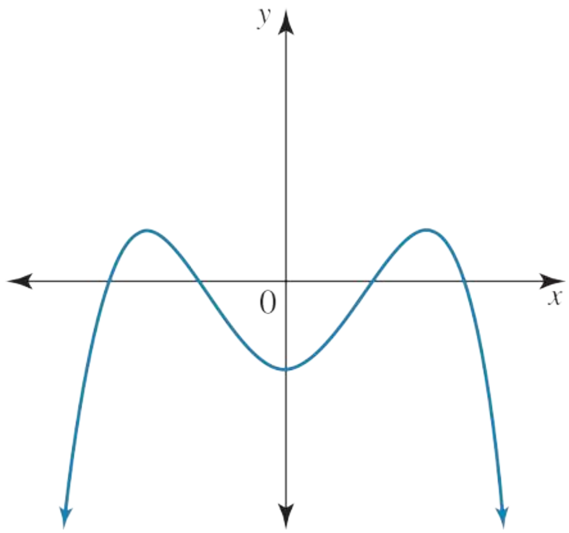


A. $f(x) = -(x-a)(x-b)(x-c)^2$
B. $f(x) = (x-a)(x-b)(x-c)^2$
C. $f(x) = (x-a)(x-b)(x+c)^2$
D. $f(x) = (x+a)(x+b)(x-c)^2$
E. $f(x) = -(x+a)(x+b)(x-c)^2$

Question 3 (4 marks)

The equation of a quartic graph is $y = -(x+1)^4 + 16$.
Sketch the graph, showing all key points.

Question 4 (1 mark)
The graph below is of the function $y = ax^4 + bx^3 + cx^2 + dx + e$.



Which of the following options is most **correct**?

A. $a > 0$, $c > 0$ and $e < 0$
B. $a > 0$, $c < 0$ and $e > 0$
C. $a < 0$, $b = 0$, $c < 0$, $d = 0$ and $e > 0$
D. $a < 0$, $b = 0$, $c > 0$, $d = 0$ and $e < 0$
E. $a < 0$, $b = 0$, $c < 0$, $d = 0$ and $e < 0$

Question 5 (4 marks)

Given that $(x + 1)$ and $(x - 3)$ are factors of the polynomial $x^4 - x^3 - 7x^2 + x + 6$, find the other linear factors and hence solve the inequation $x^4 - x^3 - 7x^2 + x + 6 < 0$.

Question 6 (1 mark)

The equation of a quartic graph is $y = -(x - 2)^4 + 3$. What is its turning point?

A. $(2, -3)$
B. $(-2, 3)$
C. $(16, 3)$
D. $(-2, -3)$
E. $(2, 3)$

Topic	5	Quartic polynomials
Subtopic	5.3	Families of polynomials

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Question 1 (1 mark)

For the family of graphs $y = ax^3$, where $a > 0$, select the statement that is **not** true.

A. As $x \to \pm\infty, y \to \pm\infty$.
B. Each graph passes through $(-1, -a)$.
C. Each graph has a stationary point of inflection at $(0, 0)$.
D. All graphs are essentially similar shapes.
E. The greater the value of a, the wider the graph.

Question 2 (1 mark)

For the family of graphs of $y = x^n$, where $n \in N$ and $n = 2$ or 4, select the statement that is **not** true.

A. As $x \to \pm\infty, y \to \infty$.
B. Each graph passes through the point $(1, 1)$.
C. Each graph has a stationary point of inflection at $(0, 0)$.
D. All graphs are essentially similar shapes.
E. The larger the power, the narrower the graph.

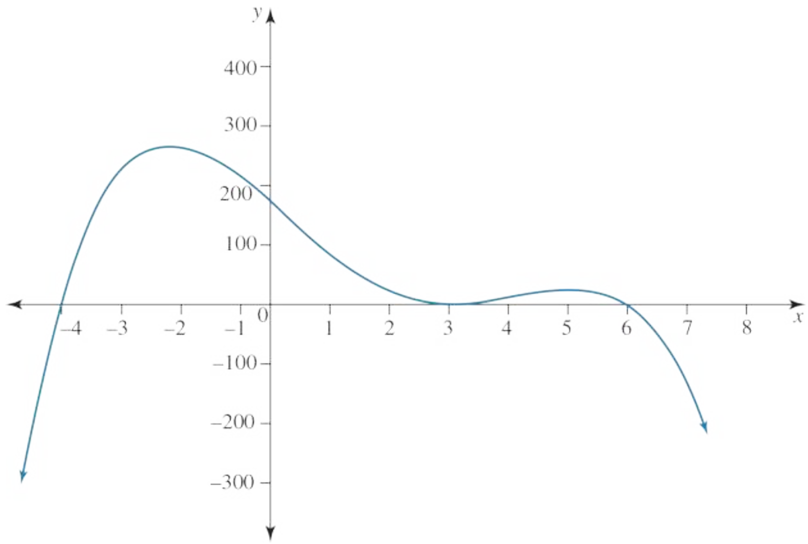
Question 3 (3 marks)

Find x- and y-intercepts and any turning points/points of inflection, and sketch the graph of $y = (x-3)^4 - 1$.

Topic 5 | Subtopic 5.3 Families of polynomials

Question 4 (4 marks)

State the degree of the graph below, and write the equation for the polynomial, given that the point (2, 18) lies on the graph.



Topic	5	Quartic polynomials
Subtopic	5.4	Numerical approximation of roots of polynomial equations

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Question 1 (1 mark)

For the graph $y = x(x-4)(x+5)$, a maximum turning point occurs between

A. $x = -5$ and $x = 0$.
B. $x = 0$ and $x = 4$.
C. $x = 4$ and $x = 20$.
D. $x = 0$ and $x = 5$.
E. $x = 0$ and $x = -4$.

Question 2 (3 marks)

Consider the cubic polynomial $p(x) = x^3 - 2x^2 + 8x - 5$.

a. Show that the equation $x^3 - 2x^2 + 8x - 5$ has a root that lies between $x = 0$ and $x = 1$. **(1 mark)**

b. State a first estimate of the root and then use the method of bisection to calculate a more accurate estimate. **(2 marks)**

Question 3 (1 mark)

The graph of $y = (x + 2)\left(x^2 - 5x + 4\right)$ has

A. one maximum turning point.
B. one minimum turning point.
C. one maximum and one minimum turning point.
D. one maximum and two minimum turning points.
E. two maximum and one minimum turning point.

Question 4 (4 marks)

Consider the cubic polynomial $p(x) = 3x^3 - 5x^2 - 4x + 4$.

a. Show that the equation $3x^3 - 5x^2 - 4x + 4 = 0$ has a root that lies between $x = 0$ and $x = 1$. **(1 mark)**

b. State a first estimate of the root, and then use the method of bisection to calculate a more accurate estimate. **(3 marks)**

Topic	5	Quartic polynomials
Subtopic	5.5	Review

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Question 1 (2 marks)

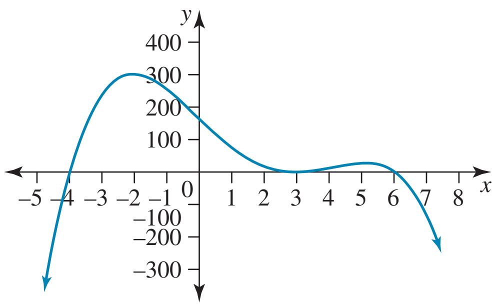
Given that $(x+1)$, $(x-3)$, $(x+2)$ and $(x-1)$ are factors of the polynomial $x^4 - x^3 - 7x^2 + x + 6$, solve the inequation $x^4 - x^3 - 7x^2 + x + 6 < 0$.

Question 2 (1 mark)

The family of graphs $y = a(x-h)^n + k$, where n is an even positive integer, has

A. one stationary point of inflection.
B. two stationary points of inflection.
C. one turning point.
D. two turning points.
E. one stationary point of inflection and one turning point.

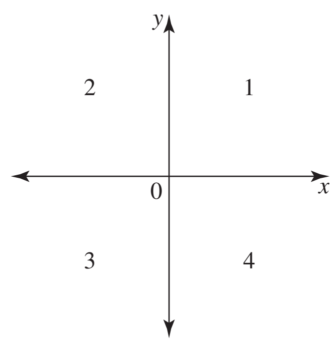
Question 3 (4 marks)
State the degree of the graph shown and write the equation for the polynomial, given that the point (2, 18) lies on the graph.



Question 4 (3 marks)
State the y-intercept for the polynomial $y = 2x^3 - x^2 - 10x + 7$ and the other points on the graph that have the same y-coordinate as the y-intercept. Between which two of these points does the minimum turning point lie?

Question 5 (1 mark)

The Cartesian plane consists of four quadrants as shown.

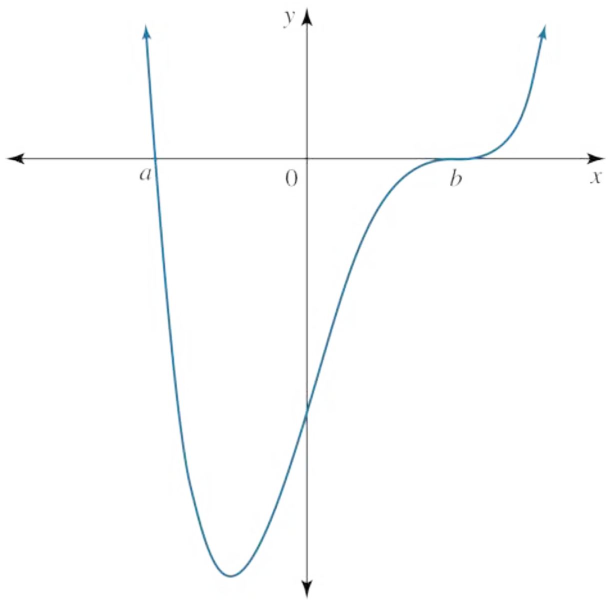


The extremities (or extreme ends) of the graph $y = -2x(x-3)(x+5)^2$ as $x \to \pm\infty$ lie in quadrants

A. 1 and 2
B. 1 and 3
C. 1 and 4
D. 3 and 4
E. 2 and 4

Question 6 (1 mark)

A possible equation for the graph shown, given $a, b > 0$, would be



A. $y = (x + a)(x - b)^3$
B. $y = (x - a)(x - b)^2$
C. $y = (x - a)(x - b)^3$
D. $y = (x + a)(x + b)^3$
E. $y = (x + a)^3(x - b)$

Question 7 (1 mark)

A quartic graph has x-intercepts of -2, 1 and 3, with the intercept at 3 having multiplicity 2. It also has y-intercept of 2. The equation of the graph is

A. $y = -(x+2)(x-1)(x-3)^2$

B. $y = (x+2)(x-1)(x-3)^2$

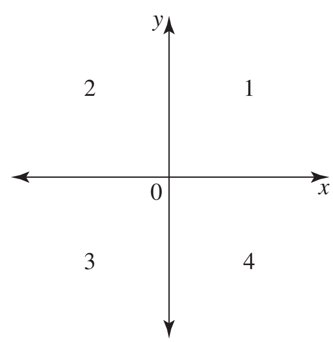
C. $y = -\frac{1}{3}(x+2)(x-1)(x-3)^2$

D. $y = -\frac{1}{6}(x+2)(x-1)(x+3)^2$

E. $y = -\frac{1}{9}(x+2)(x-1)(x-3)^2$

Question 8 (1 mark)

The Cartesian plane consists of four quadrants as shown.



The extremities (or extreme ends) of the graph $y = x(x-1)^3(x+2)^3$ as $x \to \pm\infty$ lie in quadrants

A. 1 and 3.

B. 2 and 4.

C. 1 and 2.

D. 3 and 4.

E. 2 and 3.

Question 9 (1 mark)

For the graph $y = x(x-1)^2(x+4)$, a maximum turning point occurs between

A. $x = 0$ and $x = 1$.
B. $x = 4$ and $x = 0$.
C. $x = 1$ and $x = 4$.
D. $x = -1$ and $x = 0$.
E. $x = 4$ and $x = -1$.

Question 10 (3 marks)

Given that $(x-1)$ and $(x-3)$ are factors of the polynomial $2x^4 - 3x^3 - 11x^2 + 3x + 9$, find the other linear factors and hence solve the equation $2x^4 - 3x^3 - 11x^2 + 3x + 9 = 0$.

Answers and marking guide

5.2 Quartic polynomials

Question 1

$y = ax^4 + bx^3 + cx^2 + dx + e$ with $a > 0$

As $x \to \pm\infty, y \to \infty$.

Option **E** does not satisfy this; however, all other graphs do.

The correct answer is **E**.

Question 2

$f(x) = -(x - a)(x - b)(x - c)^2$

The shape is a negative quartic.

$\therefore -x^4$ term

The intercepts are at a, b and c. (*Note:* a and b are positive but represent negative values due to their placement on the x-axis.)

The intercept at c has multiplicity 2.

The correct answer is **E**.

Question 3

$y = -(x + 1)^4 + 16$

The equation is of the form $y = a(x - h)^4 + k$.

$a = -1,\ h = -1,\ k = 16$ **[1 mark]**

The turning point is $(-1, 16)$.

y-intercept ($x = 0$):

$y = -(1)^4 + 16$

$= 15$

$\therefore (0, 15)$ **[1 mark]**

x-intercept(s) ($y = 0$):

$-(x + 1)^4 + 16 = 0$

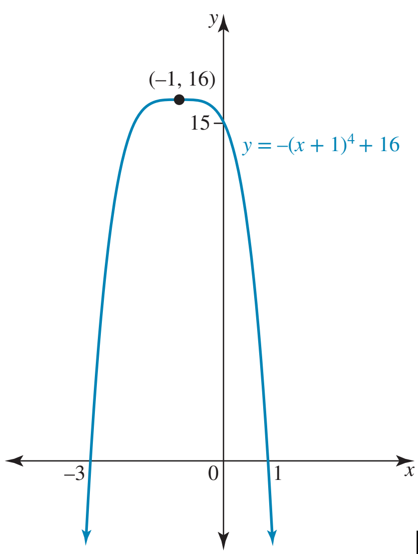
$(x + 1)^4 = 16$

$x + 1 = \sqrt[4]{16}$

$x + 1 = \pm 2$

$x = 1, -3$

$\therefore (1, 0), (-3, 0)$ **[1 mark]**



[1 mark]

Question 4
The graph is an even function, and symmetrical about the y-axis,
so
$f(-x) = f(x) \Rightarrow b = d = 0$
The graph crosses the y-axis, below the x-axis.
$\Rightarrow e < 0$
as $x \rightarrow \pm - \infty \Rightarrow a < 0$.
Consider the graph of

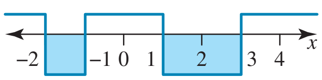
$$\begin{aligned} y &= -(x+1)(x-1)(x+2)(x-2) \\ &= -\left(x^2-1\right)\left(x^2-4\right) \\ &= -\left(x^4-5x^2+4\right) \\ &= -x^4+5x^2-4 \Rightarrow c > 0 \end{aligned}$$

Question 5
$(x+1)(x-3) = x^2 = 2x - 3$ **[1 mark]**
$x^4 - x^3 - 7x^2 + x + 6 = \left(x^2-2x-3\right)\left(x^2+bx-2\right)$
Equate the coefficients of the x^3 term.

$$\begin{aligned} bx^2 - 2x &= -1x^3 \\ \therefore b - 2 &= -1 \\ b &= 1 \end{aligned}$$

$$\begin{aligned} x^4 - x^3 - 7x^2 + x + 6 &= \left(x^2-2x-3\right)\left(x^2+2x-2\right) \\ &= (x+1)(x-3)(x+2)(x-1) \end{aligned}$$ **[1 mark]**

Use a sign diagram to solve the inequation.



[1 mark]

$x^4 - x^3 - 7x^2 + x + 6 < 0$
$\{x: -2 < x - 1\} \cup \{x: 1 < x < 3\}$ **[1 mark]**

Question 6
$y = -(x-h)^4 + k$
$\text{TP} = (h, k)$
$\therefore \text{TP} = (2, 3)$

5.3 Families of polynomials

Question 1
'The greater the value of a, the wider the graph' is incorrect. It should read, 'The greater the value of a, the narrower the graph.'
The correct answer is **E**.

Question 2
'Each graph has a stationary point of inflection at $(0, 0)$' is incorrect. It should read, 'Each graph has a minimum turning point at $(0, 0)$.'
The correct answer is **C**.

Question 3

The equation is in turning point form, giving the turning point as $(3, -1)$. **[1 mark]**

x-intercept $(y = 0)$:

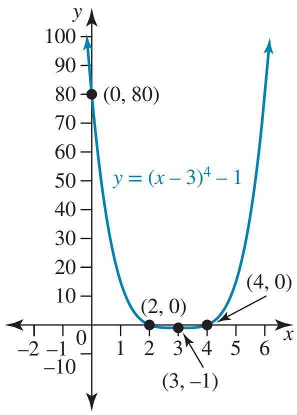
$$(x-3)^4 - 1 = 0$$
$$(x-3)^4 = 1$$
$$x - 3 = \pm 1$$
$$x = 2, x = 4$$

$\therefore$ x-intercept at $(2, 0)$, $(4, 0)$ [**1 mark**]

y-intercept $(x = 0)$:

$$y = (-3)^4 - 1$$
$$y = 80$$

$\therefore$ y-intercept at $(0, 80)$



[1 mark]

Question 4

The graph:

cuts the axis at $x = -4$

touches the axis at $x = 3$

cuts the axis $x = 6$. **[1 mark]**

$$y = a(x+4)(x-3)^2(x-6)$$

Degree is 4. **[1 mark]**

Work out a:

Using point $(2, 18)$

$$18 = a(6)(-1)^2(-4)$$
$$18 = 24a$$
$$a = -\frac{3}{4}$$ [**1 mark**]

$$\therefore y = -\frac{3}{4}(x+4)(x-3)^2(x-6)$$ [**1 mark**]

5.4 Numerical approximation of roots of polynomial equations

Question 1

The graph is a positive odd cubic polynomial $\left(x^3\right)$ with x-intercepts at $x = -5, 0, 4$ and two turning points between these points. The first turning point must be a maximum and the second one must be a minimum in order to satisfy the long-term behaviour requirements of a positive cubic polynomial.

The correct answer is **A**.

Question 2

a. $p(x) = x^3 - 2x^2 + 8x - 5$

$p(0) = -5$

$p(0) < 0$

$p(1) = 1 - 2 + 8 - 5$

$\quad = 2$

$p(1) > 0$

$\therefore p(0) < 0, p(1) > 0$

$\Rightarrow$ The root is between $x = 0$ and $x = 1$. **[1 mark]**

b. The root is closer to $p(1)$, so a first estimate of the root is $x = 1$. **[1 mark]**

Take the midpoint of $x = 0$ and $x = 1$. $\therefore x = \frac{1}{2}$

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) - 5$$

$$\begin{aligned} p\left(\frac{1}{2}\right) &= \frac{1}{8} - \frac{1}{2} + 4 - 5 \\ &= -\frac{3}{8} - 1 \\ &= -1\frac{3}{8} \end{aligned}$$

The root is closer to $p\left(\frac{1}{2}\right)$, so a more accurate estimate of the root is $x = \frac{1}{2}$. **[1 mark]**

Question 3

$y = (x + 2)\left(x^2 - 5x + 4\right)$

$y = (x + 2)(x - 1)(x - 4)$

There are three x-intercepts at $x = -2$, $x = 1$ and $x = 4$. Therefore, there are two turning points. The first turning point must be a maximum and the second one must be a minimum in order to satisfy the long-term behaviour requirements of a positive odd cubic polynomial.

The correct answer is **C**.

Question 4

a. $p(x) = 3x^3 - 5x^2 - 4x + 4$

$p(0) = 4$

$p(0) > 0$

$p(1) = 3 - 5 - 4 + 4$

$\quad = -2$

$p(1) < 0$

Therefore, $p(0) > 0,\ p(1) < 0 \Rightarrow$ root is between $x = 0$ and $x = 1$. **[1 mark]**

b. The root is closer to $p(1)$, so a first estimate of the root is $x = 1$. **[1 mark]**

Take the midpoint of $x = 0$ and $x = 1$; therefore, $x = \frac{1}{2}$.

$$\begin{aligned} p\left(\frac{1}{2}\right) &= 3\left(\frac{1}{2}\right)^3 - 5\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 4 \\ &= \frac{3}{8} - \frac{5}{4} - 2 + 4 \\ &= \frac{3}{8} - \frac{10}{8} + \frac{16}{8} \\ &= \frac{9}{8} \text{ [1 mark]} \end{aligned}$$

The root is closer to $p\left(\frac{1}{2}\right)$, so a more accurate estimate of the root is $x = \frac{1}{2}$. **[1 mark]**

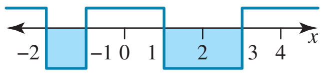
5.5 Review

Question 1

$x^4 - x^3 - 7x^2 + x + 6$

$= (x + 1)(x - 3)(x + 2)(x - 1)$ **[1 mark]**

Use a sign diagram to solve the inequation.



$x^4 - x^3 - 7x^2 + x + 6 < 0$

$\{x: -2 < x < -1\} \cup \{x: 1 < x < 3\}$ **[1 mark]**

Question 2

The graphs have one turning point at (h, k).

The correct answer is **C**.

Question 3

The graph:

cuts the axis at $x = -4$

touches the axis at $x = 3$

cuts the axis at $x = 6$. **[1 mark]**

$y = a(x + 4)(x - 3)^2(x - 6)$

The degree is 4. **[1 mark]**

Work out a using point (2, 18).

$18 = a(6)(-1)^2(-4)$

$18 = -24a$

$a = -\frac{3}{4}$ **[1 mark]**

$\therefore y = -\frac{3}{4}(x + 4)(x - 3)^2(x - 6)$ **[1 mark]**

Question 4

$y = 2x^3 - x^2 - 10x + 7$

y-intercept $(x = 0)$

$y = 7$ **[1 mark]**

Other points on the graph with $y = 7$:

$7 = 2x^3 - x^2 - 10x + 7$

$2x^3 - x^2 - 10x = 0$

$x\left(2x^2 - x - 10\right) = 0$

$x(x + 2)(2x - 5) = 0$

$\therefore y = 7$ when $x = -2, 0, 2.5$. **[1 mark]**

The graph is a positive odd cubic with two turning points. The second turning point is a minimum turning point and lies somewhere between (0, 7) and (2.5, 7). **[1 mark]**

Question 5

The curve is a negative quartic, so as $x \to \pm\infty,\ y \to -\infty$.

The curve starts in quadrant 3 and ends in quadrant 4.

The correct answer is **D**.

Question 6
The shape is a positive quartic, with intercepts at a and b.
The intercept at b has multiplicity 3.
Note: a is positive but represents a negative value due to its placement on the x-axis.

Question 7
$y = a(x+2)(x-1)(x-3)^2$ from the x-intercepts.
Substitute $(0, 2)$ to find a.
$2 = a(2)(-1)(-3)^2$
$2 = 18a$
$a = -\frac{1}{9}$
$\therefore y = -\frac{1}{9}(x+2)(x-1)(x-3)^2$

Question 8
The graph starts in quadrant 3 and ends in quadrant 1 to satisfy the long-term behaviour requirements of a positive odd function.

Question 9
The graph is a positive, quartic polynomial $\left(x^4\right)$ with x-intercepts at $x = -4, 0$ and 1, and the intercept at 1 has multiplicity 2.
There are two turning points between these points. The first turning point must be a minimum and the second one must be a maximum in order to satisfy the long-term behaviour requirements of a positive quartic polynomial.

Question 10
$(x-1)(x-3) = x^2 - 4x + 3$
$2x^4 - 3x^3 - 11x^2 + 3x + 9 = \left(x^2 - 4x + 3\right)\left(2x^2 + bx + 3\right)$ **[1 mark]**
$-3 = b - 8$
$b = 5$
$2x^4 - 3x^3 - 11x^2 + 3x + 9 = \left(x^2 - 4x + 3\right)\left(2x^2 + 5x + 3\right)$
$= (x-1)(x-3)(2x+3)(x+1)$ [**1 mark**]
$0 = (x-1)(x-3)(2x+3)(x+1)$
$x = 1, 3, -1, -\frac{3}{2}$ [**1 mark**]

6 Functions and relations

Topic	6	Functions and relations
Subtopic	6.2	Functions and relations

online only

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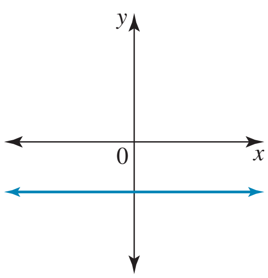
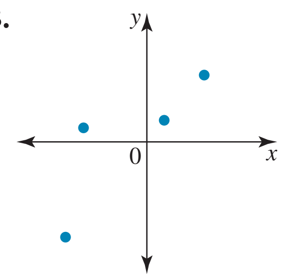
Question 1 (1 mark)

State which of the following relations is not a function.

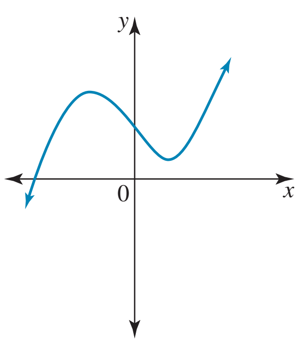
A.



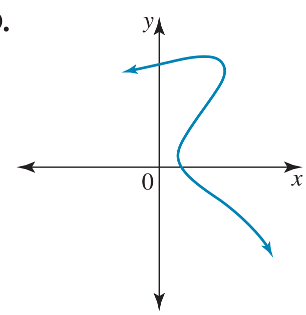
B.



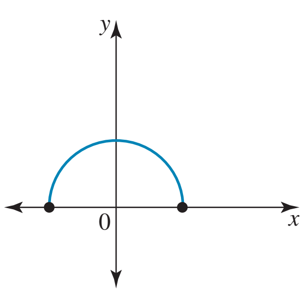
C.



D.



E.



Question 2 (1 mark)

The type of correspondence for a function can be

A. many-to-one and many-to-many.
B. many-to-one and one-to-one.
C. many-to-one and one-to-many.
D. one-to-one and many-to-many.
E. one-to-many and one-to-one.

Question 3 (5 marks)

if $f: R \rightarrow R, f(x) = 2x^3 - 3x^2$

a. find the image of -1 **(1 mark)**

b. sketch the graph of $y = f(x)$ **(2 marks)**

c. state the domain and range of the function f **(1 mark)**

d. state the type of correspondence of f. **(1 mark)**

Question 4 (1 mark)

The range defined by $f: [-5, 0) \to R$, where $f(x) = x^2 - 16$, is best given by

A. $[-16, 9)$
B. $[-16, \infty)$
C. $(-16, 9]$
D. $[-16, 9]$
E. $(-16, \infty)$

Question 5 (3 marks)

$f(x) = x^2 - 6x$ and $g(x) = f(1 - x)$

Express the rule for g as a polynominal in x and calculate any values for which $f(x) = g(x)$.

Question 6 (1 mark)

The range defined by $f: [-2, 4) \to R$ where $f(x) = x^2 - 25$ is best given by

A. $(-21, -9]$
B. $[-25, -9)$
C. $(-25, -9]$
D. $[-21, -9)$
E. $(-25, -9)$

Topic	6	Functions and relations
Subtopic	6.3	The rectangular hyperbola and the truncus

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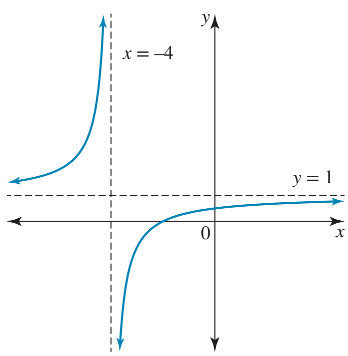
Question 1 (4 marks)

a. Sketch $y = \dfrac{1}{(x-2)^2}$, showing axial intercepts and asymptotes. **(2 marks)**

b. State the domain, range and type of correspondence. **(2 marks)**

Question 2 (1 mark)

A possible equation for the graph shown is



A. $y = \dfrac{2}{x+4} - 1$

B. $y = \dfrac{2}{x+4} + 1$

C. $y = \dfrac{-2}{x-4} + 1$

D. $y = \dfrac{-2}{x+4} + 1$

E. $y = \dfrac{2}{x-1} + 4$

Question 3 (1 mark)

For the graph of the function $y = \frac{1}{x} - 3$, state which of the following is *not* true.

A. The function has one-to-one correspondence.
B. The domain is $R\backslash\{0\}$.
C. The graph passes through the point $(1, -2)$.
D. There are asymptotes at $x = 3,\ y = 0$.
E. The range is $R\backslash\{-3\}$.

Question 4 (3 marks)

Identify the asymptotes of the hyperbola with equation $y = \frac{3x + 2}{x + 3}$.

Question 5 (1 mark)

For the graph of the function $y = \frac{1}{x + 2} - 1$, which of the following is *not* true?

A. One of the asymptotes is $x = 2$.
B. The graph has a one-to-one correspondence.
C. As $x \to \infty,\ y \to -1$.
D. The range is $R\backslash(-1)$.
E. The domain is $R\backslash(-2)$.

Topic	**6**	**Functions and relations**
Subtopic	**6.4**	**The square root function**

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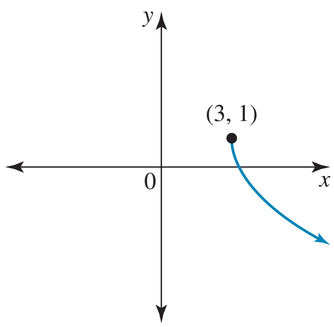
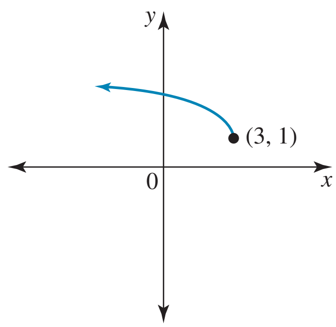
Question 1 (1 mark)

The graph of the function $f(x) = 1 + \sqrt{3 - x}$ could be

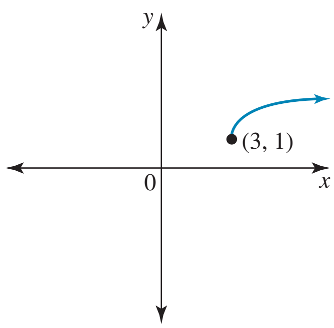
A.



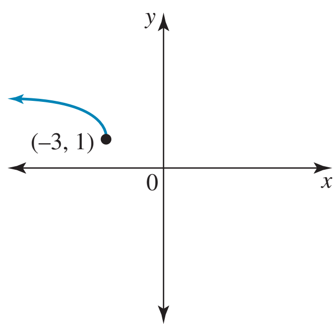
B.



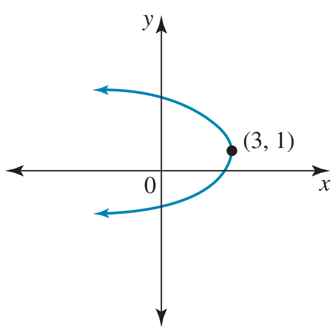
C.



D.



E.



Question 2 (5 marks)
Sketch $y = 3\sqrt{x + 4} - 3$ and state the domain and range.

Question 3 (1 mark)
The coordinates of the end point of the square root function with equation $y - 2 = \sqrt{3 - x}$ are

A. (2, 3)
B. (2, −3)
C. (3, 2)
D. (−3, 2)
E. (−3, −2)

Question 4 (5 marks)
Sketch $y = 3\sqrt{x + 4} - 3$ and state the domain and range.

Question 5 (1 mark)
The relations with the following equations are all functions except

A. $y = \sqrt{4 - x}$
B. $y = \pm\sqrt{4 - x}$
C. $y = 4 - x$
D. $y = -\sqrt{4 - x}$
E. $y = \sqrt{x + 4}$

Topic	6	Functions and relations
Subtopic	6.5	Other functions and relations

online only

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Question 1 (1 mark)

The maximal domain of the function with the rule $f(x) = 3 + \dfrac{1}{2x-1}$ is

A. $R\backslash\left\{\frac{1}{3}\right\}$
B. $R\backslash\left\{\frac{1}{2}\right\}$
C. $R\backslash\left\{-\frac{1}{2}\right\}$
D. $R\backslash\{1\}$
E. $R\backslash\{-1\}$

Question 2 (1 mark)

The inverse function $f^{-1}(x)$ of $f:(2, \infty) \to R$, where $f(x) = x - 2$, is given by

A. $f^{-1}:[2, \infty) \to R$ where $f^{-1}(x) = x + 2$.
B. $f^{-1}:R^{+} \to R$ where $f^{-1}(x) = x - 2$.
C. $f^{-1}:R^{+} \to R$ where $f^{-1}(x) = x + 2$.
D. $f^{-1}:R \to R$ where $f^{-1}(x) = x + 2$.
E. $f^{-1}:[2, \infty)$ where $f^{-1}(x) = x - 2$.

Question 3 (1 mark)

The function $f:(-\infty, a] \to R, f(x) = x^2 + 4x - 5$ has an inverse function if

A. $a < 2$
B. $a = 2$
C. $a \geq 0$
D. $a \leq -2$
E. $a \geq -5$

Question 4 (1 mark)

The inverse function $f^{-1}(x)$ of $f:(-\infty, 1) \to R$, where $f(x) = x + 3$, is given by

A. $f^{-1}:(-\infty, 4) \to R$, where $f^{-1}(x) = x + 3$.
B. $f^{-1}:R \to R$, where $f^{-1}(x) = x - 3$.
C. $f^{-1}:(-\infty, 4) \to R$, where $f^{-1}(x) = x - 3$.
D. $f^{-1}:(-\infty, 1) \to R$, where $f^{-1}(x) = x - 3$.
E. $f^{-1}:(-\infty, 1) \to R$, where $f^{-1}(x) = x + 3$.

Question 5 (1 mark)

The function $f:[a, \infty) \to R$, $f(x) = x^2 + 3x - 4$ has an inverse function if

A. $a \leq -\frac{3}{2}$
B. $a \geq -\frac{3}{2}$
C. $a \geq -3$
D. $a \geq -2$
E. $a > -\frac{3}{2}$

Topic	6	Functions and relations
Subtopic	6.6	Transformations of functions

online only

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Question 1 (1 mark)

The image of the point $(-6, 3)$ dilated parallel to the x-axis by a factor of $\frac{1}{3}$ is

A. $(-2, 3)$
B. $(2, 3)$
C. $(-6, 1)$
D. $(-6, -1)$
E. $(-6, -9)$

Question 2 (3 marks)

Form the image of the graph $y = \sqrt{x}$ dilated by a factor of 2 parallel to the y-axis, reflected in the x-axis and translated 3 units to the left.

Question 3 (3 marks)

Describe the transformation of $y = f(x)$ required to obtain $y = -3 - 4f(x - 1)$.

Question 4 (1 mark)

The image of the point $(4, -3)$ dilated by a factor of $\frac{1}{4}$ from the x-axis is

A. $(1, -3)$
B. $(-1, -3)$
C. $\left(4, \frac{3}{4}\right)$
D. $\left(-4, \frac{3}{4}\right)$
E. $\left(4, -\frac{3}{4}\right)$

Question 5 (1 mark)

Which of the following equations best represents the equation of a function $f(x)$ reflected in the x-axis, dilated by factor 2 parallel to the x-axis, and translated 4 units parallel to the y-axis in the positive direction?

A. $-f\left(\frac{x}{2}\right) + 4$
B. $-f(2x) + 4$
C. $f\left(-\frac{x}{2}\right) + 4$
D. $f(-2x) + 4$
E. $f(2x) - 4$

Topic	6	Functions and relations
Subtopic	6.7	Review

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (3 marks)

$f(x) = x^2 - 6x$ and $g(x) = f(1 - x)$.

Express the rule for g as a polynomial in x and calculate any values for which $f(x) = g(x)$.

Question 2 (1 mark)

The domain and range respectively of the function $f(x) = 2 - \dfrac{5}{(4-x)^2}$ are given by

A. $R\backslash\{2\}, R\backslash\{4\}$
B. $R\backslash\{4\}, R\backslash\{2\}$
C. $R\backslash\{4\}, (-\infty, 2)$
D. $R\backslash\{-4\}, (2, \infty)$
E. $R\backslash\{-4\}, (-\infty, 2)$

Question 3 (5 marks)

Sketch the graph of $y = -1 + \sqrt{4 + x}$ and its inverse by reflecting the given function in the line $y = x$.

Question 4 (3 marks)

Consider the function for which $f(x) = \begin{cases} x^2, & \text{if } x < 1 \\ 3 - x, & \text{if } x \geq 1 \end{cases}$.

a. Evaluate $f(-1)$, $f(1)$ and $f(2)$. **(1 mark)**

b. Sketch the graph of $y = f(x)$. State the domain, range and any other values of x for which the function is discontinuous. **(2 marks)**

Question 5 (1 mark)

State which of the following equations best represents the equation of a function $f(x)$ reflected in the y-axis, reflected in the x-axis and translated 2 units parallel to the y-axis in the positive direction.

A. $-f(-x)+2$
B. $f(-x)+2$
C. $f(-x)-2$
D. $-f(-x)-2$
E. $-f(-x-2)$

Question 6 (1 mark)

The asymptotes of the hyperbola with equation $y=\dfrac{4x-1}{x-2}$ are

A. $x=-2$ and $y=4$.
B. $x=2$ and $y=1$.
C. $x=-2$ and $y=7$.
D. $x=2$ and $y=4$.
E. $x=2$ and $y=-1$.

Question 7 (1 mark)

The maximal domain of the function with the rule $f(x)=1-\dfrac{1}{\sqrt{4-x}}$ is

A. $(-\infty, 4)$
B. $[4, \infty)$
C. $(-\infty, 4]$
D. $[(4, \infty)$
E. $[-\infty, 4)$

Question 8 (1 mark)

The image of the point (4, 1) dilated parallel to the y axis by a factor of 2 is

A. $\left(4, \frac{1}{2}\right)$
B. (8, 1)
C. (2, 1)
D. (4, 2)
E. (4, 1)

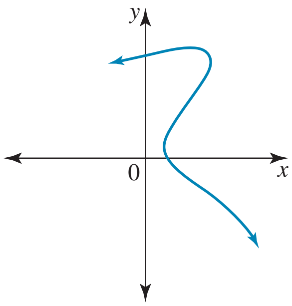
Question 9 (5 marks)

Sketch $y = \sqrt{2 - x} - 1$ and state the domain and range.

Answers and marking guide

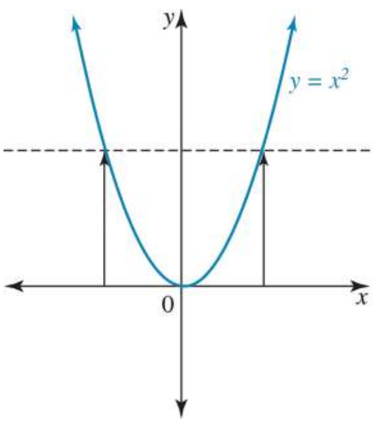
6.2 Functions and relations

Question 1

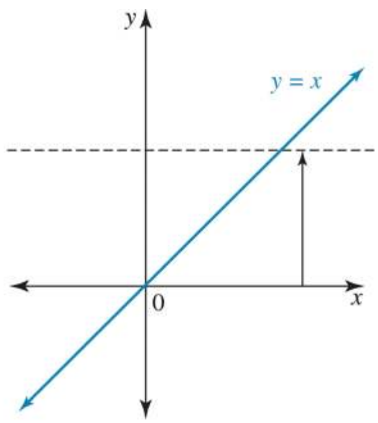


A vertical line crosses the graph in three places. Therefore, the graph is a relation but not a function. A vertical line should cross the graph of a function a maximum of one time.
The correct answer is **D**.

Question 2
The horizontal line test is used to determine the type of correspondence.



Many-to-one correspondence



One-to-one correspondence

The correct answer is **B**.

Question 3

a. $f(x) = 2x^3 - 3x^2$

$f(-1) = 2(-1)^3 - 3(-1)^2$

$= -5$ **[1 mark]**

b. Calculate intercepts:

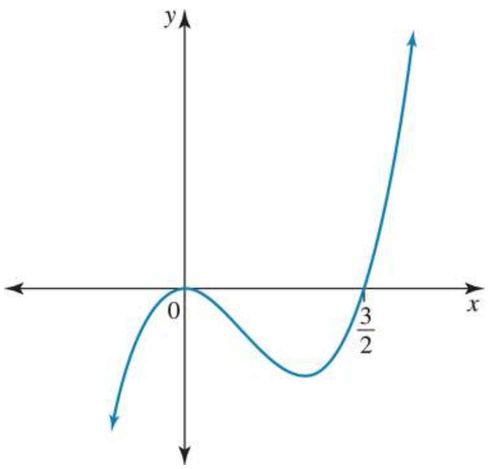
x-intercepts $(y = 0)$:

$2x^3 - 3x^2 = 0$

$x^2(2x - 3) = 0$

$x = 0, \frac{3}{2}$

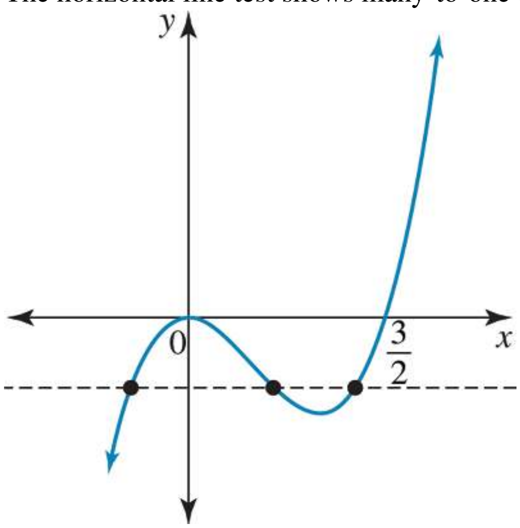
Turning point at $x = 0$ (repeated factor) **[1 mark]**



[1 mark]

c. Domain = R, range = R **[1 mark]**

d. The horizontal line test shows many-to-one correspondence. **[1 mark]**



Question 4

$f: [-5, 0) \to R, f(x) = x^2 - 16$

$x = -5, f(-5) = (-5)^2 - 16 = 9$

$x = 0, f(0) = -16$

Always consider the turning points,. In this case, the TP = $(0, -16)$, one of the end points.

Range is $(-16, 9]$. (Note the correct use of ( ] to match the domain.)

The correct answer is **C**.

Question 5

$$f(x) = x^2 - 6x$$

$$f(1-x) = (1-x)^2 - 6(1-x)$$

$$= 1 - 2x + x^2 - 6 + 6x$$

$$= x^2 + 4x - 5 \qquad \textbf{[1 mark]}$$

$$f(x) = g(x)$$

$$-10x = -5$$

$$x^2 - 6x = x^2 + 4x - 5$$

$x = \frac{1}{2}$ **[1 mark]**

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right)$$
$$= \frac{1}{4} - 3$$
$$= -2\frac{3}{4}$$

$f(x) = g(x), \left(\frac{1}{2}, -2\frac{3}{4}\right)$ **[1 mark]**

Question 6

$x = -2, f(-2) = (-2)^2 - 25 = -21$

$x = 4, f(4) = (4)^2 - 25 = -9$

The turning point must also be considered. In this case, the turning point = (0, −25)

Therefore, the lowest point is −25 and the highest point is −9.

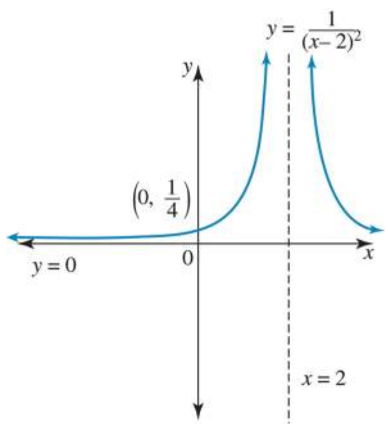
Range is [−25, −9). Note correct use of [ ) to match domain.

The correct answer is **B**.

6.3 The rectangular hyperbola and the truncus

Question 1

a.



[1 mark]

$\frac{1}{(x-2)^2}$ is a truncus with asymptotes at $y = 0, x = 2$.

y-intercepts: when $x = 0$,

$y = \frac{1}{(0-2)^2} = \frac{1}{4}$. **[1 mark]**

b. Domain $R\backslash(2)$, range $(0, \infty)$ **[1 mark]**

A horizontal line test would show many-to-one correspondence. **[1 mark]**

Question 2

The graph is a basic hyperbola $\left(y = \frac{1}{x}\right)$ shape but reflected, hence negative, with asymptotes $x = -4, y = +1$.

2 is a dilation factor.

The correct answer is **D**.

Question 3

$y = \frac{a}{(x-h)} + k$ is a rectangular hyperbola shape with asymptotes with equations $y = k,\ x = h$.

$y = \frac{1}{(x-0)} - 3$

Asymptotes occur at $x = 0,\ y = -3$.
The correct answer is **D**.

Question 4

Reduce $\dfrac{3x+2}{x+3}$ to a proper form.

$$
\begin{array}{r}
3 \\
x-3\,\overline{)\,3x+2} \\
3x+9 \\
\hline
-7
\end{array}
$$

$y = \dfrac{3x+2}{x+3}$

$ = 3 - \dfrac{7}{x+3}$ **[1 mark]**

Asymptotes:

$x = -3$ [**1 mark**]

$y = 3$ [**1 mark**]

Question 5

$y = \dfrac{a}{(x-h)} + k$ is a rectangular hyperbola shape with asymptotes with equations $y = k$ and $x = h$.

$y = \dfrac{1}{x+2} - 1$

Asymptotes occur at $x = -2$ and $y = -1$.
The correct answer is **A**.

6.4 The square root function

Question 1

$f(x) = a\sqrt{x-h} + k$, vertex $= (h, k)$

$\sqrt{\ }$ shape reflected in the y-axis; the end point is (3, 1).
The correct answer is **B**.

Question 2

The end point is $(-4, -3)$. **[1 mark]**

y-intercept $(x = 0)$

$y = 3\sqrt{4} - 3$

$y = 3 \Rightarrow (0, 3)$ [**1 mark**]

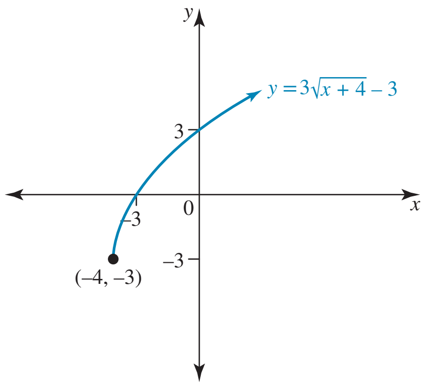
x-intercepts $(y = 0)$

$0 = 3\sqrt{x+4} - 3$

$3 = 3\sqrt{x+4}$

$1 = x + 4$

$x = -3 \Rightarrow (-3, 0)$ [**1 mark**]



[1 mark]

Domain $[-4, \infty)$, range $[-3, \infty)$ **[1 mark]**

Question 3

$y - 2 = \sqrt{3 - x}$

$y = \sqrt{-(x-3)} + 2$

$\therefore$ End point:$(3, 2)$.

The correct answer is **C**.

Question 4

End point is $(-4, -3)$. **[1 mark]**

y-intercept $(x = 0)$

$y = 3\sqrt{4} - 3$

$y = 3 \Rightarrow (0, 3)$ **[1 mark]**

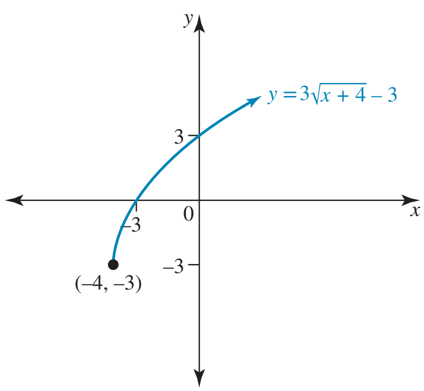
x-intercepts $(y = 0)$

$0 = 3\sqrt{x+4} - 3$

$3 = 3\sqrt{x+4}$

$1 = x + 4$

$x = -3 \Rightarrow (-3, 0)$ **[1 mark]**



[1 mark]

Domain $[-4, \infty)$, range $[-3, \infty)$ **[1 mark]**

Question 5

$y = \pm\sqrt{4 - x}$ is a sideways parabola, and so a vertical line cuts the graph twice. Therefore, this relation is not a function.

The correct answer is **B**.

6.5 Other functions and relations

Question 1

For the function $f(x) = 3 + \frac{1}{2x - 1}$ to exist

$2x - 1 \neq 0$

$2x \neq 1$

$x \neq \frac{1}{2}$

$\therefore x$ can be any value except $\frac{1}{2}$.

$\therefore R \backslash \left\{\frac{1}{2}\right\}$

The correct answer is **B**.

Question 2

Original function: $y = x - 2$

Interchanging x and $y \Rightarrow x = y - 2$

$\therefore y = x + 2$

$\therefore f^{-1}(x) = x + 2$

Domain of $f(x) = (2, \infty)$, range of $f(x) = R^+$

Domain of $f^{-1}(x) = R^+$, range $= (2, \infty)$

$\therefore f^{-1}: R^+ \to R$, where $f^{-1}(x) = x + 2$

The correct answer is **C**.

Question 3

To ensure that the inverse exists as a function, the domain of the original function is restricted, so that $f(x)$ is a one-to-one function.

For $x^2 + 4x - 5$, a turning point exists at $x = -\frac{4}{2} = -2$.

To ensure one-to-one correspondence, $a \leq -2$.

$\therefore$ if $a \leq -2$, the inverse function exists.

The correct answer is **D**.

Question 4

Original function is $y = x + 3$.

Interchanging x and $y \Rightarrow x = y + 3$

$\therefore y = x - 3$

$\therefore f^{-1}(x) = x - 3$

Domain of $f(x) = (-\infty, 1)$, range of $f(x) = (-\infty, 4)$

Domain of $f^{-1}(x) = (-\infty, 4)$, range $= (-\infty, 1)$

$\therefore f^{-1}: (-\infty, 4) \to R$, where $f^{-1}(x) = x - 3$

The correct answer is **C**.

Question 5

To ensure the inverse exists as a function, the domain of the original function is restricted so that $f(x)$ is a one-to-one function.

For $f(x) = x^2 + 3x - 4$, a turning point exists at $x = \frac{-b}{2x} = -\frac{3}{2}$.

To ensure one-to-one correspondence, $a \geq -\frac{3}{2}$.

$\therefore$ if $a \geq -\frac{3}{2}$, the inverse function exists.

The correct answer is **B**.

6.6 Transformations of functions

Question 1

$(x, y) \to (ax, y)$ when dilated by a factor of $\frac{1}{3}$ parallel to the x-axis

$\therefore (-6, 3) \to \left(-6 \times \frac{1}{3}, 3\right)$

$\therefore (-6, 3) \to (-2, 3)$

The correct answer is **A**.

Question 2

$y = \sqrt{x}$

Dilated by a factor of 2 parallel to the y-axis:

$y = 2\sqrt{x}$ **[1 mark]**

Reflected in the x-axis:

$y = -2\sqrt{x}$ **[1 mark]**

Translated 3 units to the left:

$y = -2\sqrt{(x+3)}$ **[1 mark]**

Question 3

$y = -3 - 4f(5x - 1)$

$y = -4f(x - 1) - 3$ [**1 mark**]

Dilation of factor 4 from the x-axis, followed by a reflection in the x-axis **[1 mark]**

Then a horizontal translation 1 unit to the right and a vertical translation 3 units downwards **[1 mark]**

Question 4

$(x, y) \to (x, ay)$ when dilated by a factor of a from the x-axis or parallel to the y-axis.

$\therefore (4, 3) \to \left(4, -3 \times \frac{1}{4}\right)$

$\therefore (4, 3) \to \left(4, -\frac{3}{4}\right)$

The correct answer is **E**.

Question 5

$f(x)$ reflected in the x-axis is $-f(x)$.

$f(x)$ dilated by factor 2 parallel to the x-axis is $f\left(\frac{x}{2}\right)$.

Translated 4 units parallel to the y-axis in the positive direction indicates vertical translation, +4 units.

$\therefore -f\left(\frac{x}{2}\right) + 4$

The correct answer is **A**.

6.7 Review

Question 1

$$\begin{aligned} f(x) &= x^2 - 6x \\ f(1-x) &= (1-x)^2 - 6(1-x) \\ &= 1 - 2x + x^2 - 6 + 6x \\ &= x^2 + 4x - 5 \qquad \textbf{[1 mark]} \\ f(x) &= g(x) \\ x^2 - 6x &= x^2 + 4x - 5 \\ 10x &= -5 \\ x &= \frac{1}{2} \qquad \textbf{[1 mark]} \end{aligned}$$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 6\left(\frac{1}{2}\right)$$
$$= \frac{1}{4} - 3$$
$$= -2\frac{3}{4}$$

$f(x) = g(x), \left(\frac{1}{2}, -2\frac{3}{4}\right)$ **[1 mark]**

Question 2

$y = \dfrac{a}{(x-h)^2} + k$ is a truncus shape with asymptotes with equations $y = k, x = h$.

Horizontal asymptote at $y = 2$, vertical asymptote at $x = 4$

The graph is reflected in the x-axis, so $y < 2$.

The correct answer is **C**.

Question 3

$y = -1 + \sqrt{4 + x}$

End point $= (-4, -1)$ **[1 mark]**

x-intercept $(y = 0)$:

$0 = -1 + \sqrt{4 + x}$

$1 = \sqrt{4 + x}$

$1 = 4 + x$

$x = -3$

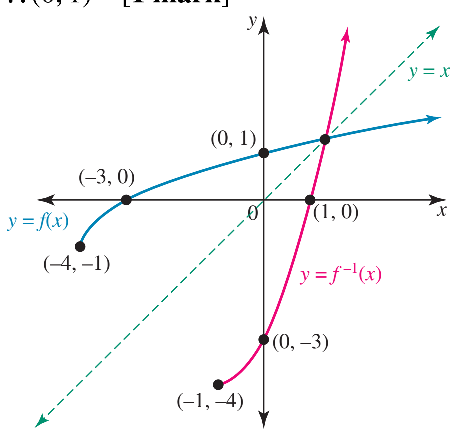
$\therefore (-3, 0)$ **[1 mark]**

y-intercept $(x = 0)$:

$y = -1 + \sqrt{4 + 0}$

$y = 1$

$\therefore (0, 1)$ **[1 mark]**



[1 mark]

Reflect each point in the line $y = x$ to obtain the graph of the inverse function. **[1 mark]**

$\therefore$ x-intercept $= (1, 0)$, y-intercept $= (0, -3)$ and end point $= (-1, -4)$.

Question 4

$$f(x) = \begin{cases} x^2, & \text{if } x < 1 \\ 3 - x, & \text{if } x \geq 1 \end{cases}$$

a. $f(-1)$

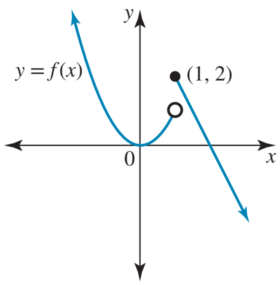
Since $x = -1$ lies in the domain section $x < 1$, use the rule $f(x) = x^2$.

$\therefore f(-1) = 1$

$f(-1)$

Since $x = 1$ and $x = 2$ lie in the domain section $x \geq 1$, use the rule $f(x) = 3 - x$.
$\therefore f(2) = 1$ and $f(1) = 2$ **[1 mark]**
Since $x = 2$ lies in the domain section $x \geq 1$, use the rule $f(x) = 3 - x$.
$\therefore f(2) = 1$

b. Sketch $y = x^2, x < 1$ (parabola open end point $(1, 1)$).
Sketch $y = 3 - x, x \geq 1$ (straight line, closed end point $(1, 2)$).



[1 mark]

Domain R, range R **[1 mark]**
The function is not continuous at $x = 1$ because there is a break in the graph.

Question 5

$f(x)$ reflected in the y-axis: $f(-x)$
$f(x)$ reflected in the x-axis: $-f(x)$
Translated 2 units parallel to the y-axis in the positive direction indicates vertical translation +2 units upwards.
$\therefore\ -f(-x) + 2$
The correct answer is **A**.

Question 6

Express $y = \dfrac{4x-1}{x-2}$ in standard form.

$$\begin{aligned} y &= \frac{4x-1}{x-2} \\ &= \frac{4(x-2)+7}{x-2} \\ &= \frac{4(x-2)}{x-2} + \frac{7}{x-2} \\ &= 4 + \frac{7}{x-2} \end{aligned}$$

Asymptotes are $x = 2$ and $y = 4$.

Question 7

For the function $f(x) = 1 - \dfrac{1}{\sqrt{4-x}}$ to exist, $4 - x > 0$.

$$\begin{aligned} 4 - x &> 0 \\ 4 &> x \\ x &< 4 \end{aligned}$$

$\therefore x \in (-\infty, 4)$
The correct answer is **A**.

Question 8

$(x, y) \to (x, ay)$ when dilated by a factor of a parallel to the y-axis.
$\therefore (4, 1) \to (4, 1 \times 2)$
$\therefore (4, 1) \to (4, 2)$
The correct answer is **D**.

Question 9

End point $(2, -1)$ **[1 mark]**

y-intercept $(x = 0)$

$y = \sqrt{2} - 1 \Rightarrow \left(0, \sqrt{2} - 1\right)$ **[1 mark]**

x-intercept $(y = 0)$

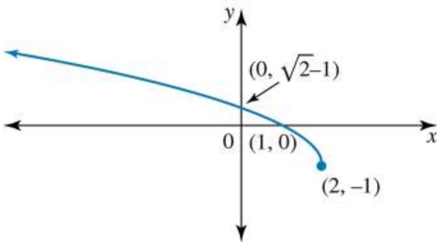
$0 = \sqrt{2 - x} - 1$

$1 = \sqrt{2 - x}$

$1 = 2 - x$

$x = 1$

$\Rightarrow (1, 0)$ **[1 mark]**



[1 mark]

Domain is $(-\infty\ 2]$ and range is $[-1\ \infty)$. **[1 mark]**

7 Probability

Topic	7	Probability
Subtopic	7.2	Probability review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (3 marks)

a. Draw a simple tree diagram to show the possible outcomes when an unbiased coin is tossed three times. **(1 mark)**

b. Calculate the probability of obtaining Heads on the first two throws and Tails on the third. **(1 mark)**

c. Calculate the probability of obtaining either exactly three Heads or three Tails. **(1 mark)**

Question 2 (1 mark)

Using the following probability table, the incorrect statement is

	B	B'	
A	0.2		0.4
A'		0.2	
	0.6		1

A. $\Pr(A \cap B) = 0.2$
B. $\Pr(A' \cap B) = 0.4$
C. $\Pr(A \cup B) = 0.9$
D. $\Pr(A) + \Pr(B) = 1$
E. $\Pr(A) < \Pr(B)$

Question 3 (3 marks)

At a school with 55 VCE students, 30 are enrolled in Mathematical methods and 17 are enrolled in Chemistry.

The Chemistry teacher noted that, in his class, 5 students were not studying Mathematical Methods.

a. Draw a Venn diagram showing the enrolments. **(1 mark)**

b. The principal randomly selects a student walking down the corridor and asks if they are studying Mathematical Methods but not Chemistry.

Calculate the probability that the student says 'yes'. **(1 mark)**

c. The principal randomly selects a student walking down the corridor and asks if they are studying Mathematical Methods but not Chemistry.

Calculate the probability that the student is not doing Mathematical Methods or Chemistry. **(1 mark)**

Question 4 (1 mark)

A die is biased so that the number 3 has a probability of $\frac{1}{3}$ and the other numbers are equally probable. The probability that a 5 is thrown is

A. $\frac{1}{3}$

B. $\frac{1}{5}$

C. $\frac{2}{15}$

D. $\frac{3}{18}$

E. $\frac{5}{36}$

Question 5 (1 mark)

A die is biased so that the number 4 has a probability of $\frac{1}{4}$ and the other numbers are equally probable. The probability that a 2 is thrown is

A. $\frac{1}{10}$

B. $\frac{1}{8}$

C. $\frac{1}{5}$

D. $\frac{2}{15}$

E. $\frac{3}{20}$

Topic	7	Probability
Subtopic	7.3	Conditional probability

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Question 1 (1 mark)

If $\Pr(A) = 0.7, \Pr(B) = 0.3$ and $\Pr(A \cup B) = 0.8$, then $\Pr(A \cap B)$ is

A. 1.0
B. 0.5
C. 0.3
D. 0.2
E. 0

Question 2 (1 mark)

If $\Pr(A) = 0.7, \Pr(B) = 0.3$ and $\Pr(A \cup B) = 0.8$, then $\Pr(B \mid A)$ is

A. $\frac{2}{7}$
B. $\frac{3}{7}$
C. $\frac{7}{8}$
D. $\frac{3}{8}$
E. $\frac{2}{3}$

Question 3 (1 mark)

Two classes completed a difficult Mathematics test and the results are shown in the table below. The probability that a randomly selected student was from class 1 and had passed the test was

	Passed	Failed
Class 1	15	10
Class 2	13	12

A. $\frac{2}{10}$

B. $\frac{3}{10}$

C. $\frac{3}{5}$

D. $\frac{2}{5}$

E. $\frac{5}{10}$

Question 4 (4 marks)

Two unbiased dice are rolled and the sum of the topmost numbers is noted. Given that the sum is less than 6, use a lattice diagram to find the probability that the sum is an even number.

Question 5 (1 mark)

A and B are mutually exclusive events. If $\Pr(A) = 0.5$ and $\Pr(B) = 0.2$, then $\Pr(A \cup B)$ is

A. 0.3

B. 0.7

C. 0.8

D. 0.4

E. 0.5

Topic	7	Probability
Subtopic	7.4	Independence

online only

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Question 1 (1 mark)

If A and B are independent events and $\Pr(A) = 0.4$ and $\Pr(B) = 0.5$, then $\Pr(A \cup B)$ is

A. 1
B. 0.2
C. 0.9
D. 0.7
E. 0

Question 2 (1 mark)

Two independent events, A and B, are such that $\Pr(A) = 0.7$, $\Pr(B) = 0.8$ and $\Pr(A \cup B) = 0.94$.
$\Pr(A \cap B)$ is equal to

A. 0
B. 0.7
C. 0.56
D. 0.14
E. 0.24

Question 3 (3 marks)

The probabilities that three swimmers — Sue, Bill and Fred — can swim 100 metres in less than 1 minute are 0.7, 0.8 and 0.4 respectively.

a. Calculate the probability that all the swimmers will swim 100 metres in less than 1 minute, assuming they are not in the same race and their times are independent of each other. **(1 mark)**

b. Calculate the probability that at least one swimmer will break 1 minute. **(2 marks)**

Question 4 (3 marks)
Two double-headed coins and one fair coin were used in a game where one is randomly selected and tossed. Draw a probability tree diagram and calculate the probability of throwing a Tail.

Question 5 (1 mark)
If A and B are independent events and $\Pr(A) = 0.3$ and $\Pr(B) = 0.7$, then $\Pr(A \cup B)$ is

A. 0.79
B. 0.21
C. 0.49
D. 0.41
E. 0.35

Topic	7	Probability
Subtopic	7.5	Counting techniques

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Question 1 (1 mark)
Five students have made an appointment to see their teacher. The number of ways the appointments can be arranged if Sam, a struggling student, must go first is
A. 120
B. 60
C. 24
D. 16
E. 12

Question 2 (1 mark)
The number of arrangements of the letters in the word 'FINDER' where the vowels are together is
A. 720
B. 360
C. 240
D. 120
E. 60

Question 3 (4 marks)
A review panel is to be established. It is to consist of the principal and four other teachers selected from the 7 male and 9 female staff members.

a. Calculate how many different panels can be formed. **(1 mark)**

b. Calculate how many panels can be formed that consist of at least 3 female teachers. **(1 mark)**

c. Calculate the probability that only 1 male is on the panel. **(2 marks)**

Question 4 (1 mark)
The number of different three-digit numbers that can be formed using $\{4, 6, 8, 10, 12\}$ if each digit can be used only once is

A. 24
B. 120
C. 15
D. 60
E. 46

Question 5 (1 mark)
The number of arrangements of the letters in the word 'BREAK' where the vowels are together is

A. 24
B. 12
C. 48
D. 36
E. 32

Topic	7	Probability
Subtopic	7.6	Binomial coefficients and Pascal's triangle

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Question 1 (3 marks)

Find the term independent of x in the expansion of $\left(2x+\frac{3}{x^3}\right)^4$.

Question 2 (1 mark)

The value of $\binom{7}{5}$ is

A. 42
B. 210
C. 21
D. 2530
E. 35

Question 3 (1 mark)

The notation for the ninth coefficient in the 14th row of Pascal's triangle is

A. ${}^{15}C_8$
B. ${}^{14}C_8$
C. ${}^{15}C_9$
D. ${}^{14}C_9$
E. ${}^{9}C_{14}$

Question 4 (1 mark)
The sixth term in $(3c + 2d)^6$ is
A. $64d^6$
B. $576cd^5$
C. $144c^3 d^3$
D. $(6cd)^6$
E. $288cd^5$

Question 5 (3 marks)
Simplify the expression $(1 - 2x)^4 - 4(1 - 2x)^3 + 6(1 - 2x)^2 - 4(1 - 2x) + 1$.
Hint: Use the binomial expansion
$(a + b)^n = \sum_{r-0}^{n} \binom{n}{r} a^{n-r} b^r$
where $a = 1 - 2x$ and $b = -1$

Question 6 (1 mark)
The value of $\binom{8}{6}$ is
A. 14
B. 56
C. 32
D. 42
E. 28

Topic	7	Probability
Subtopic	7.7	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

A and B are mutually exclusive events. If $\Pr(A) = 0.6$ and $\Pr(B) = 0.3$, then $\Pr(A \cup B)$ is

A. 0
B. 0.18
C. 0.9
D. 0.3
E. cannot be determined

Question 2 (3 marks)

A local shop sells mixed bags containing a dozen small muffins. Within each bag, five muffins have a chocolate surprise inside and the others don't. Sue randomly selects a muffin and then Margaret randomly selects a muffin.

Draw a probability tree diagram and work out the probability that Sue's muffin doesn't contain a chocolate surprise while Margaret's does contain a chocolate surprise.

Question 3 (1 mark)

A box contained 6 red balls, 9 blue balls and 5 orange balls. One ball was chosen randomly, the color noted then replaced, and another ball was chosen. The probability that a blue and a red ball were selected is

A. 0.45
B. 0.27
C. 0.41
D. 0.03
E. 0.66

Question 4 (4 marks)
Emma has a special collection of bottles. Eight are red, five are blue and seven are green. She likes to arrange them in threes on a shelf so that one of each colour is displayed with green in the middle.

a. If the bottles are all different in shape and size, determine how many arrangements she can make. **(2 marks)**

b. If she randomly placed three bottles together ignoring the colour, calculate the probability that they will all be green. **(2 marks)**

Question 5 (1 mark)
The number of different three-digit numbers that can be formed using $\{3, 4, 5, 7\}$ if each digit can be used only once is

A. 30
B. 24
C. 12
D. 10
E. 8

Question 6 (1 mark)

If $\Pr(A) = 0.6, \Pr(B) = 0.25$ and $\Pr(A \cup B) = 0.75$, then $\Pr\left(A|B\right)$ is

A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{2}{5}$

D. $\frac{1}{3}$

E. $\frac{1}{5}$

Question 7 (1 mark)

A box contained 4 red balls, 8 blue balls and 3 yellow balls. One ball was randomly chosen, the colour noted then replaced and another one chosen. The probability that a blue and a yellow ball were selected is

A. $\frac{22}{15}$

B. $\frac{11}{15}$

C. $\frac{8}{75}$

D. $\frac{3}{25}$

E. $\frac{16}{75}$

Question 8 (1 mark)

Two double-headed coins and two fair coins were used in a game where one was randomly selected and tossed. The probability of throwing a head was

A. $\frac{2}{3}$

B. $\frac{5}{8}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

E. $\frac{3}{8}$

Question 9 (1 mark)

The fifth term in the expansion of $(2c - d)^7$ is

A. $280\, c^3\, d^4$

B. $140\, c^3\, d^4$

C. $280\, c^3\, d^5$

D. $140\, c^4\, d^3$

E. $560\, c^4\, d^3$

Question 10 (3 marks)

At a school with 70 VCE students, 35 are enrolled in Mathematical Methods and 25 are enrolled in Biology. The Biology teacher noted that, in his class, 12 students were not studying Mathematical Methods.

a. Draw a Venn diagram showing the enrolments. **(1 mark)**

b. The principal randomly selects a student walking down the corridor and asks if they are studying Mathematical Methods but not Biology.
What is the probability that the student says yes? **(1 mark)**

c. The principal randomly selects a student walking down the corridor and asks if they are studying neither Mathematical Methods nor Biology.
What is the probability that the student is not doing Mathematical Methods or Biology? **(1 mark)**

Question 11(3 marks)

The probabilities that three swimmers — Matt, Lizzie and Edie — can swim 100 metres in under 1 minute are 0.6, 0.75 and 0.3, respectively.

a. What is the probability that all the swimmers will swim 100 metres in under 1 minute, assuming they are not in the same race and their times are independent of each other? **(1 mark)**

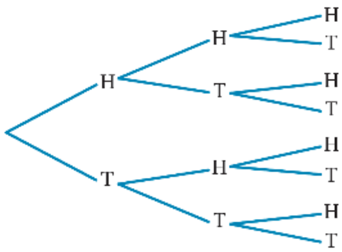
b. What is the probability that at least one swimmer will break 1 minute? **(2 marks)**

Answers and marking guide

7.2 Linear equations and inequations

Question 1

a.



Total number of outcomes = 8. **[1 mark]**

b. There is only one outcome for HHT.

$\therefore \Pr(\text{HHT}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$ [**1 mark**]

c. $\Pr(\text{HHH}) + \Pr(\text{TTT}) = \frac{1}{8} + \frac{1}{8}$

$= \frac{1}{4}$ **[1 mark]**

Question 2

Completed probability table:

	B	B''	
A	0.2	0.2	0.4
A''	0.4	0.2	0.6
	0.6	0.4	1

Read the other results from the completed probability table.

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A \cup B) = 0.6 + 0.4 - 0.2$

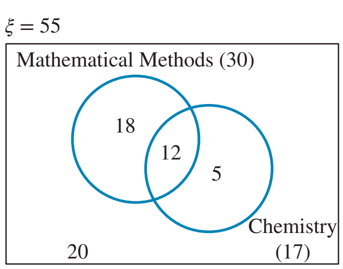
$= 0.8$

$\therefore \Pr(A \cup B) \neq 0.9$

The correct answer is **C**.

Question 3

a. **[1 mark]**



b. From the diagram, studying Mathematical Methods but not Chemistry: $n = 18$.

$\Pr(\text{MM not C}) = \frac{18}{55}$ **[1 mark]**

c. $\Pr(\text{not MM not C}) = \dfrac{20}{55}$

$= \dfrac{4}{11}$ **[1 mark]**

Question 4

Probability of not being a 3:

$\Pr(3') = 1 - \Pr(3)$

$= 1 - \dfrac{1}{3}$

$= \dfrac{2}{3}$

As there are five equally probable numbers:

$\Pr(5) = \dfrac{1}{5} \times \dfrac{2}{3}$

$= \dfrac{2}{15}$

Question 5

Probability of not being a 4

$\Pr(4') = 1 - \Pr(4)$

$= 1 - \dfrac{1}{4}$

$= \dfrac{3}{4}$

As there are 5 equally probable numbers

$\Pr(2) = \dfrac{1}{5} \times \dfrac{3}{4} = \dfrac{3}{20}$

7.3 Conditional probability

Question 1

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B)$

$= 0.7 + 0.3 - 0.8$

$= 0.2$

The correct answer is **D**.

Question 2

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B)$

$= 0.7 + 0.3 - 0.8$

$= 0.2$

$\Pr\left(B|A\right) = \dfrac{\Pr(A \cap B)}{\Pr(A)}$

$\Pr\left(B|A\right) = \dfrac{0.2}{0.7}$

$= \dfrac{2}{7}$

The correct answer is **A**.

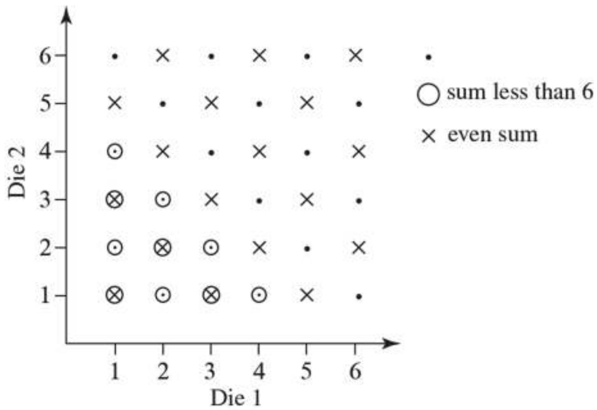
Question 3

$$n\,(\text{Total}) = 50$$
$$n\,(\text{class }1 + \text{passed}) = 15$$
$$\Pr\,(\text{class }1) = \frac{15}{50} = \frac{3}{10}$$

The correct answer is **B**.

Question 4

Let A be the event that the sum is an even number.
Let B be the event that the sum is less than 6.



[1 mark]

$$n\,(\text{Total}) = 6 \times 6 = 36$$
$$n\,(A) = 18$$
$$n\,(B) = 10$$
$$n\,(A \cap B) = 4$$
$$\Pr\,(A) = \frac{18}{36}$$
$$\Pr\,(B) = \frac{10}{36}$$
$$\Pr\,(A \cap B) = \frac{4}{36}$$ **[1 mark]**

$$\Pr\left(A|B\right) = \frac{\Pr\,(A \cap B)}{\Pr\,(B)}$$
$$\Pr\left(A|B\right) = \frac{\frac{4}{36}}{\frac{10}{36}}$$ **[1 mark]**

$$\Pr\left(A|B\right) = \frac{4}{10} = \frac{2}{5}$$ **[1 mark]**

(Also inferred from the lattice diagram)

Question 5

If the events A and B are mutually exclusive, then they cannot occur simultaneously. For mutually exclusive events, $n\,(A \cap B) = 0$, therefore, $\Pr\,(A \cap B) = 0$.

$$\Pr\,(A \cup B) = \Pr\,(A) + \Pr\,(B) = 0.5 + 0.2 = 0.7$$

7.4 Independence

Question 1

$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$
$= 0.4 \times 0.5$
$= 0.2$
$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$= 0.4 + 0.5 - 0.2$
$= 0.7$

The correct answer is **D**.

Question 2

Independent events:
$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$
$= 0.7 \times 0.8$
$= 0.56$

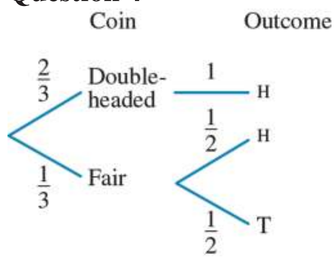
The correct answer is **C**.

Question 3

a. $\Pr(B) = 0.8, \Pr(S) = 0.7, \Pr(F) = 0.4$
$\Pr(S \cap B \cap F) = \Pr(S) \times \Pr(B) \times \Pr(F)$
$\Pr(S \cap B \cap F) = 0.7 \times 0.8 \times 0.4$
$= 0.224$ **[1 mark]**

b. Calculate:
1 – Pr(none swims under 1 minute)
$\Pr(B') = 0.2, \Pr(S') = 0.3, \Pr(F') = 0.6$
$\Pr(S' \cap B' \cap F') = \Pr(S') \times \Pr(B') \times \Pr(F')$
$\Pr(S' \cap B' \cap F') = 0.2 \times 0.3 \times 0.6$
$= 0.036$ **[1 mark]**
Probability that at least one swimmer will break 1 minute:
$1 - \Pr(S' \cap B' \cap F') = 1 - 0.036$
$= 0.964$ **[1 mark]**

Question 4



[1 mark]

There is only one way of throwing a Tail — first selecting the fair coin and then flipping a Tail. Multiply along the branches to determine the probability.

$\Pr(\text{Tail}) = \frac{1}{3} \times \frac{1}{2}$ **[1 mark]**
$= \frac{1}{6}$ **[1 mark]**

Question 5

$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

$\Pr(A \cap B) = 0.3 \times 0.7 = 0.21$

$$\begin{aligned}\Pr(A \cup B) &= \Pr(A) + \Pr(B) - \Pr(A \cap B)\\ &= 0.3 + 0.7 - 0.21\\ &= 0.79\end{aligned}$$

7.5 Counting techniques

Question 1

Sam is not counted in calculations because we know he must be first. The other appointments can then be arranged in any order.

$$\begin{aligned}4! &= 4 \times 3 \times 2 \times 1\\ &= 24\end{aligned}$$

The correct answer is **C**.

Question 2

Count the IE as one unit.

Therefore, there are 5! ways to arrange the letters.

The IE can also be EI.

$$\begin{aligned}5! \times 2 &= 5 \times 4 \times 3 \times 2 \times 1 \times 2\\ &= 240\end{aligned}$$

The correct answer is **C**.

Question 3

a. $$\begin{aligned}{}^{16}C_4 &= \frac{16!}{4! \times 12!}\\ &= \frac{16 \times 15 \times 14 \times 13 \times \cancel{12!}}{4 \times \cancel{12!}}\\ &= \frac{43\,680}{24}\\ &= 1820\end{aligned}$$ **[1 mark]**

b. 3 females (and 1 male) or 4 females

$${}^{9}C_3 \times {}^{7}C_1 + {}^{9}C_4 = \frac{9!}{3! \times 6!} \times 7 + \frac{9!}{4! \times 5!}$$ **[1 mark]**

$$\begin{aligned}&= 588 + 126\\ &= 714\end{aligned}$$

c. Calculations from parts **a** and **b**:

$$\Pr(\text{only 1 male}) = \frac{\Pr(3 \text{ females} + 1 \text{ male})}{\text{total number of panels}}$$ **[1 mark]**

$$\begin{aligned}&= \frac{588}{1820}\\ &= 0.3231\end{aligned}$$ **[1 mark]**

Question 4

Five digits

$5 \times 4 \times 3 \times 2 \times 1 = 120$

Question 5

Count the EA as one unit.

Therefore, there are 4! number of ways to arrange the letters. The EA can also be AE.

$$\begin{aligned}&4! \times 2\\ &= 4 \times 3 \times 2 \times 1 \times 2\\ &= 48\end{aligned}$$

7.6 Binomial coefficients and Pascal's triangle

Question 1

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r}b^r$$

$$\left(2x+\frac{3}{x^3}\right)^4 = \sum_{r=0}^{4} \binom{4}{r} (2x)^{4-r}\left(\frac{3}{x^3}\right)^r \quad \textbf{[1 mark]}$$

$$= \sum_{r=0}^{4} \binom{4}{r} 2^{4-r}x^{4-r}3^{2r}x^{-3r}$$

Powers of $x = 0$ for the term independent of x

$4 - r - 3r = 0$

$r = 1$ (2nd term)

$$\therefore \text{2nd term} = \binom{4}{1} 2^3x^3 3^1 x^{-3} \quad \textbf{[1 mark]}$$

$$= 4 \times 8 \times 3$$

$$= 96$$

Question 2

$$\binom{7}{5} = \binom{7}{2}$$

$$= \frac{7 \times 6 \times \cancel{5!}}{2 \times \cancel{5!}}$$

$$= 21$$

The correct answer is **C**.

Question 3

The values of n and r are required. Remember, numbering starts at 0.

$n = 14$

$r = 9 - 1$

$= 8$

$\therefore {}^nC_r = {}^{14}C_8$

The correct answer is **B**.

Question 4

$n = 6$

$r = 6 - 1$

$= 5$

$$n\text{th term} = \binom{n}{r} q^{n-r}p^r$$

$$\text{6th term} = \binom{6}{5} (3c)^{6-5}(2d)^5$$

$$= 6 \times 3c \times 32d^5$$

$$= 576cd^5$$

Question 5

Binomial expansion

$$(a+b)^n = \sum_{r=0}^{n} a^{n-r}b^r \quad \textbf{[1 mark]}$$

$$(1-2x)^4 - 4(1-2x)^3 + 6(1-2x)^2 - 4(1-2x) + 1$$

Let $a = 1 - 2x$

$a^4 - 4a^3 + 6a^2 - 4a + 1$

This is recognisable as a power of 4 expansion with coefficients 1, 4, 6, 4 and 1

$\therefore (a-1)^4$ **[1 mark]**

Substitute for a

$= (1 - 2x - 1)^4$

$= 16x^4$ **[1 mark]**

Question 6

$$\binom{8}{6} = \frac{8!}{6! \times 2!}$$
$$= \frac{8 \times 7 \times 6!}{6! \times 2 \times 1}$$
$$= \frac{56}{2}$$
$$= 28$$

7.7 Review

Question 1

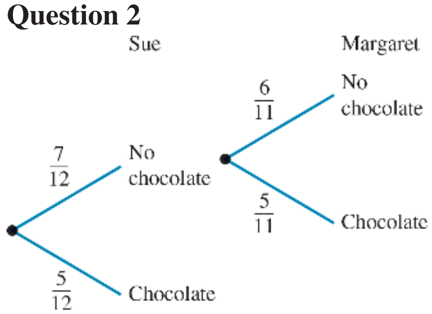
If the events A and B are mutually exclusive, then they cannot occur simultaneously.

For mutually exclusive events, $n(A \cap B) = 0$ and, therefore, $\Pr(A \cap B) = 0$.

$$\Pr(A \cup B) = \Pr(A) + \Pr(B)$$
$$\therefore \Pr(A \cup B) = 0.6 + 0.3$$
$$= 0.9$$

The correct answer is **C**.

Question 2



[1 mark]

No chocolate surprise for Sue:

$\Pr(\text{no C}) = \frac{7}{12}$

Chocolate surprise for Margaret:

$\Pr(\text{C}) = \frac{5}{11}$ **[1 mark]**

$$\Pr(\text{no C} \cap \text{C}) = \frac{7}{12} \times \frac{5}{11}$$
$$= \frac{35}{132}$$ **[1 mark]**

Question 3

It could have been red then blue, or blue then red.

$$\Pr(\text{red}) = \frac{6}{20}$$
$$\Pr(\text{blue}) = \frac{9}{20}$$
$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$
$$\Pr(\text{red} \cap \text{blue}) = \frac{6}{20} \times \frac{9}{20} \times 2$$
$$= 0.27$$

The correct answer is **B**.

Question 4

a. The colours can be arranged in 2 ways: RGB or BGR. **[1 mark]**

$$2 \times {}^8C_1 \times {}^5C_1 \times {}^7C_1 = 2 \times 8 \times 7 \times 5$$
$$= 560$$

b. Total of 20 bottles

The total number of ways of choosing three bottles from 20 available is ${}^{20}C_3$.

The number of ways of selecting three green bottles is ${}^7C_3$.

$$\Pr(3G) = \frac{{}^7C_3}{{}^{20}C_3}$$
$$= \frac{7!}{3!\,4!} \div \frac{20!}{3!\,17!} \quad \textbf{[1 mark]}$$
$$= \frac{7 \times 6 \times 5 \times \cancel{4!}}{\cancel{3!} \times \cancel{4!}} \times \frac{\cancel{3!} \times \cancel{17!}}{20 \times 19 \times 18 \times \cancel{17}}$$
$$= \frac{7 \times 6 \times 5}{20 \times 19 \times 18}$$
$$= 0.0307 \quad \textbf{[1 mark]}$$

Question 5

Four digits:

$4 \times 3 \times 2 \times 1 = 24$ The correct answer is **B**.

Question 6

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$
$$\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B)$$
$$= 0.6 + 0.25 - 0.75$$
$$= 0.1$$
$$\Pr(A \cap B) = 0.1$$
$$\Pr\left(A|B\right) = \frac{\Pr(A \cap B)}{\Pr(B)}$$
$$= \frac{0.1}{0.25}$$
$$= \frac{10}{25}$$
$$= \frac{2}{5}$$

Question 7

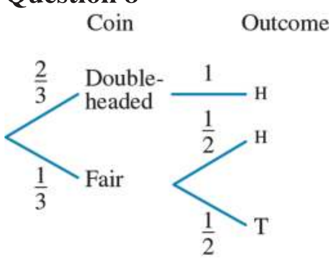
It could have been yellow then blue or blue then yellow.

$\Pr(\text{yellow}) = \frac{3}{15} = \frac{1}{5}, \Pr(\text{blue}) = \frac{8}{15}$

$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

$$\begin{aligned}\Pr(\text{yellow} \cap \text{blue}) &= \frac{1}{5} \times \frac{8}{15} \times 2 \\ &= \frac{16}{75}\end{aligned}$$

Question 8



$$\begin{aligned}\Pr(\text{H}) &= \Pr(\text{double-headed coin}) + \Pr(\text{fair coin, then a Head}) \\ &= \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{2} + \frac{1}{4} \\ &= \frac{3}{4}\end{aligned}$$

Question 9

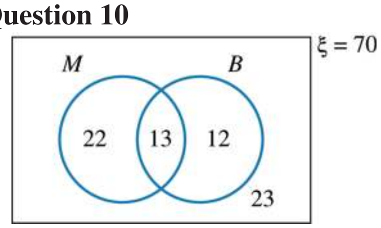
$n = 7$

$(r+1)$th term $\therefore r = 4$

nth term $= \binom{n}{r} q^{n-r} p^r$

$$\begin{aligned}\text{5th term} &= \binom{7}{4} (2c)^3 (-d)^4 \\ &= 35 \times 8c^3 \times d^4 \\ &= 280c^3 d^4\end{aligned}$$

Question 10

a. 

b. From the diagram, $n = 22$ students studying Mathematical Methods but not Biology.

$\Pr(\text{M not B}) = \frac{22}{70} = \frac{11}{35}$ **[1 mark]**

c. $\Pr(\text{not M or B}) = \frac{23}{70}$ **[1 mark]**

Question 11

a. $\Pr(M) = 0.6, \Pr(L) = 0.75, \Pr(E) = 0.3$

$$\begin{aligned}\Pr(M \cap L \cap E) &= \Pr(M) \times \Pr(L) \times \Pr(E) \\ &= 0.6 \times 0.75 \times 0.3 \\ &= 0.135\end{aligned}$$ **[1 mark]**

b. Calculate 1− (probability of none swimming under 1 minute)

$\Pr(M') = 0.4, \Pr(L') = 0.25, \Pr(E') = 0.7$

$$\begin{aligned}\Pr(M' \cap L' \cap E') &= \Pr(M') \times \Pr(L') \times \Pr(E') \\ &= 0.4 \times 0.25 \times 0.7 \\ &= 0.07\end{aligned}$$ **[1 mark]**

Probability that at least one swimmer will break 1 minute

$1 - \Pr(M' \cap L' \cap E')$

$= 1 - 0.07$

$= 0.93$ **[1 mark]**

8 Trigonometric functions

Topic	8	Trigonometric functions
Subtopic	8.2	Trigonometric ratios

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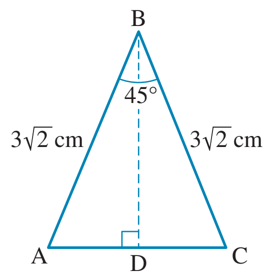
Question 1 (3 marks)

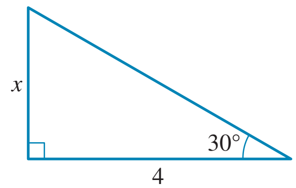
For the diagram shown



a. calculate the perpendicular height of triangle ABC (correct to 2 decimal places) **(1 mark)**

b. determine the area of ΔABC in exact form. **(2 marks)**

Question 2 (1 mark)
For the right-angled triangle shown, the exact value of x would be



A. $2\sqrt{3}$
B. $4\sqrt{3}$
C. $\frac{4\sqrt{3}}{3}$
D. 4
E. 2

Question 3 (1 mark)
$4\sin(60°)\cos(60°)$ is equal to
A. 1
B. $\sqrt{3}$
C. $\sqrt{2}$
D. $\frac{1}{2}$
E. 0

Question 4 (1 mark)

$2\sin(30°)\cos(30°)$ is equal to

A. $\frac{\sqrt{2}}{2}$

B. $\sqrt{3}$

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{2}$

E. $\sqrt{\frac{1}{2}}$

Question 5 (1 mark)

Which of the following statements is false?

A. $\cos^2(60°) = 1 - \sin^2(60°)$

B. $\tan(60°) = \cos(60°) + \sin(30°)$

C. $\sin^2(30°) + \cos^2(30°) = 1$

D. $\sin(45°) = \cos(45°)$

E. $\sin(30°) = \cos(60°)$

Topic	8	Trigonometric functions
Subtopic	8.3	Circular measure

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Question 1 (1 mark)

The angle 300° is equivalent to

A. $\frac{9\pi}{4}$

B. $\frac{7\pi}{4}$

C. 3π

D. $\frac{3\pi}{2}$

E. $\frac{5\pi}{3}$

Question 2 (1 mark)

The angle that is equivalent to $\frac{7\pi}{6}$ is

A. 120°

B. 210°

C. 100°

D. 150°

E. 240°

Question 3 (1 mark)

The angle (to the nearest degree) subtended by an arc of length 3.5 cm at the centre of a circle with radius 5 cm is

A. 82°
B. 68°
C. 50°
D. 40°
E. 35°

Question 4 (1 mark)

The angle that is equivalent to $\frac{5\pi}{4}$ is

A. 225°
B. 135°
C. 215°
D. 250°
E. 223°

Question 5 (1 mark)

The radian measure $\frac{5\pi}{3}$ is

A. in the second quadrant.
B. in the fourth quadrant.
C. in the first quadrant.
D. in the third quadrant.
E. between the third and fourth quadrants.

Topic	8	Trigonometric functions
Subtopic	8.4	Unit circle definitions

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Question 1 (1 mark)

The radian measure $\frac{5\pi}{4}$ is

A. in the first quadrant.
B. in the second quadrant.
C. in the third quadrant.
D. in the fourth quadrant.
E. exactly between the first and second quadrants.

Question 2 (1 mark)

The angle $-\frac{11\pi}{2}$ is

A. exactly between the first and second quadrants.
B. exactly between the third and fourth quadrants.
C. exactly between the third and fourth quadrants.
D. in the fourth quadrant.
E. in the second quadrant.

Question 3 (1 mark)

If $0 < x < 2\pi$, $\sin(x) < 0$ and $\cos(x) < 0$, state which of the following is true.

A. $0 < x < \dfrac{\pi}{2}$

B. $\dfrac{\pi}{2} < x < \pi$

C. $\pi < x < \dfrac{3\pi}{2}$

D. $\dfrac{3\pi}{2} < x < 2\pi$

E. $0 < x < \pi$

Question 4 (4 marks)

By considering two cycles of the unit circle, express the Cartesian point (0, 1) as four different trigonometric points, two with a positive value for θ and two with a negative value for θ.

Topic	8	Trigonometric functions
Subtopic	8.5	Symmetry properties

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Question 1 (1 mark)

The value of $\cos\left(\frac{3\pi}{4}\right)$ is

A. $\frac{1}{2}$

B. $-\frac{\sqrt{2}}{2}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{\sqrt{3}}{2}$

E. $\frac{2}{\sqrt{2}}$

Question 2 (1 mark)

If $\sin(\theta) = -\frac{1}{\sqrt{2}}$ and $0° \leq \theta \leq 360°$, then θ equals

A. $225°$
B. $135°, 225°$
C. $135°$
D. $225°, 315°$
E. $45°$

Question 3 (2 marks)

Calculate the exact value of $4\sin\left(\frac{5\pi}{6}\right) - 2\cos\left(\frac{\pi}{3}\right) + \tan\left(\frac{5\pi}{4}\right) + \sin\left(\frac{7\pi}{3}\right)$.

Question 4 (1 mark)

$\cos(\pi - \theta)$ is equal to

A. $-\sin(\theta)$
B. $-\cos(\theta)$
C. $\cos(\theta)$
D. $\sin(\theta)$
E. $\cos(\pi)$

Question 5 (1 mark)

$\sin\left(\frac{4\pi}{3}\right)$ is equal to

A. $-\frac{\sqrt{3}}{2}$
B. $-\frac{1}{2}$
C. $\frac{\sqrt{3}}{2}$
D. $\frac{\sqrt{2}}{2}$
E. $\frac{1}{2}$

Question 6 (1 mark)

If $\tan(\theta) = -\frac{1}{\sqrt{3}}$ and $0° \leq \theta \leq 360°$, then θ equals

A. $150°, 330°$
B. $120°, 300°$
C. $30°, 210°$
D. $135°, 315°$
E. $90°, 135°$

Topic	8	Trigonometric functions
Subtopic	8.6	Graphs of the sine and cosine functions

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Question 1 (1 mark)

State which of the following is not true of both $f(x) = \sin(x)$ and $g(x) = \cos(x)$.

A. The period is 2π.
B. The domain is all real numbers.
C. The range is $-1 \leq y \leq 1$.
D. They pass through the point $(0, 0)$.
E. The amplitude is 1.

Question 2 (1 mark)

For the domain $-3\pi \leq x \leq 3\pi$, the values of x for which $\sin(x) = 0$ are

A. $0, \pm\pi, \pm 2\pi, \pm 3\pi$
B. $0, \pm\pi, \pm 2\pi$
C. $0, \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$
D. $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$
E. $0, \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}$

Question 3 (1 mark)

For the domain $-2\pi \le x \le 3\pi$, the values of x for which $\cos(x) = -1$ are

A. $0, \pm 2\pi$

B. $\pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}$

C. $\pm \frac{\pi}{2}, \pm \frac{3\pi}{2}$

D. $\pm \pi, \pm 3\pi$

E. $\pm \pi, 3\pi$

Question 4 (1 mark)

Sketch the graph of $y = -3\sin(2x)$ for $x \in [0, 2\pi]$.

Question 5 (1 mark)

For the domain $-2\pi \le x \le 4\pi$ the values of x for which $\sin(x) = 1$ are

A. $-2\pi, -\pi, 0, \pi, 2\pi, 3\pi, 4\pi$

B. $-\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}$

C. $-\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}$

D. $-\frac{3\pi}{2}, 0, \frac{\pi}{2}$

E. 0

Topic	8	Trigonometric functions
Subtopic	8.7	Review

online only

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Question 1 (4 marks)

Evaluate $\dfrac{\tan(30°)\sin(60°)}{\cos(60°)\sin(30°) - \tan(45°)} + \dfrac{\sin(45°)\cos(45°)}{\tan(45°)}$.

Question 2 (1 mark)

Identify which of the following statements is false.

A. $\tan(60°) = \dfrac{\sin(60°)}{\cos(60°)}$

B. $\sin^2(45°) + \cos^2(45°) = 1$

C. $\cos^2(60°) = 1 - \sin^2(60°)$

D. $\tan(90°) = \tan(45°) + \tan(45°)$

E. $\sin(60°) = \cos(30°)$

Question 3 (1 mark)

$\sin(\pi + \theta)$ is equal to

A. $\sin(\theta)$

B. $-\sin(\theta)$

C. $\cos(\theta)$

D. $\tan(\theta)$

E. $\cos(\pi + \theta)$

Question 4 (4 marks)

Given $\sin(\theta) = 0.61$, $\cos(t) = 0.48$, $\tan(x) = 1.6$ and θ, x, t are all acute, use the symmetry properties to obtain the values of $\sin(\pi + \theta)$, $\cos(\pi - t)$, $\tan(2\pi + x)$ and $\cos(-t)$.

Question 5 (4 marks)

Sketch the graphs of $y = \cos(x)$ and $y = 3 - x$ on the same set of axes. Explain why the equation $\cos(x) = 3 - x$ has only one root, and give an interval in which the root lies.

Question 6 (1 mark)

The angle $-\frac{9}{2}\pi$ is

A. exactly between the first and second quadrants.
B. exactly between the second and third quadrants.
C. exactly between the third and fourth quadrants.
D. exactly between the fourth and first quadrants.
E. in the third quadrant.

Question 7 (4 marks)

Given that $\sin(\theta) = 0.25, \cos(\alpha) = 0.72, \tan(x) = 1.3$ and θ, x and α are all acute, use the symmetry properties to obtain values of $\sin(\pi - \theta), \cos(2\pi - \alpha), \tan(\pi - x)$, and $\sin(-\theta)$.

Answers and marking guide

8.2 Trigonometric ratios

Question 1

a. $\cos(22.5°) = \dfrac{BD}{3\sqrt{2}}$

$\therefore BD = 3.92$ cm **[1 mark]**

b. $\text{Area of } \Delta\, ABC = \dfrac{1}{2} \times BA \times BC \times \sin(45°)$ **[1 mark]**

$= \dfrac{1}{2} \times 3\sqrt{2} \times 3\sqrt{2} \times \dfrac{1}{\sqrt{2}}$

$= \dfrac{9\sqrt{2}}{2}\text{ cm}^2$ **[1 mark]**

Question 2

$\tan(30°) = \dfrac{1}{\sqrt{3}}$

From the diagram:

$\tan(30°) = \dfrac{x}{4}$

$\tan(30°) = \dfrac{x}{4}$

$\dfrac{1}{\sqrt{3}} = \dfrac{x}{4}$

$\therefore x = \dfrac{4}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3}$

The correct answer is **C**.

Question 3

$\sin(60°) = \dfrac{\sqrt{3}}{2}, \cos(60°) = \dfrac{1}{2}$

$4\sin(60°)\cos(60°) = 4 \times \dfrac{\sqrt{3}}{2} \times \dfrac{1}{2}$

$= \sqrt{3}$

The correct answer is **B**.

Question 4

$\sin(30°) = \dfrac{1}{2}, \cos(30°) = \dfrac{\sqrt{3}}{2}$

$2\sin(30°)\cos(30°) = 2 \times \dfrac{1}{2} \times \dfrac{\sqrt{3}}{2}$

$= \dfrac{\sqrt{3}}{2}$

Question 5

$\cos(60°) + \sin(30°) = \dfrac{1}{2} + \dfrac{1}{2}$

$\tan(60°) = \sqrt{3}$

8.3 Circular measure

Question 1

$300° \times \frac{\pi}{180°} = \frac{5\pi}{3}$

The correct answer is **E**.

Question 2

$\frac{7\pi}{6} \times \frac{180°}{\pi} = 210°$

The correct answer is **B**.

Question 3

$$\begin{aligned} l &= r\theta \\ \therefore \theta &= \frac{l}{r} \\ \theta &= \frac{3.5}{5} \\ &= 0.7 \end{aligned}$$

Convert to degrees:

$0.7 \times \frac{180°}{\pi} \cong 40°$

The correct answer is **D**.

Question 4

$\frac{5\pi}{4} \times \frac{180°}{\pi} = 225°$

Question 5

$\frac{5\pi}{3} = 2\pi - \frac{\pi}{3}$

$\therefore$ fourth quadrant

8.4 Unit circle definitions

Question 1

$\frac{5\pi}{4} = \pi + \frac{\pi}{4}$

$\therefore$ third quadrant

The correct answer is **C**.

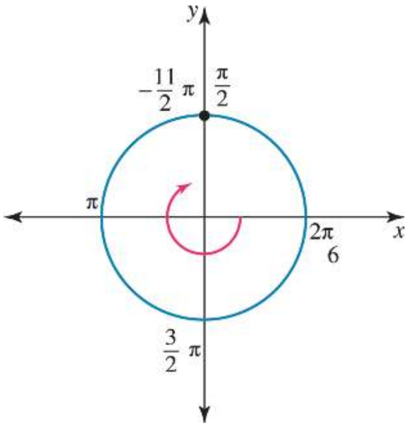
Question 2

Delete multiples of 2π to find the position of $\frac{-11\pi}{2}$.

$$\begin{aligned} \frac{-11\pi}{2} &= -\left(\frac{8\pi}{2} + \frac{3\pi}{2}\right) \\ &= -\left(4\pi + \frac{3\pi}{2}\right) \\ &= -\frac{3\pi}{2} \end{aligned}$$

The angle $-\frac{3\pi}{2}$ is in the same position as the angle $\frac{\pi}{2}$.

Therefore, it is exactly between the first and second quadrants.



The correct answer is **A**.

Question 3

$\sin(x) < 0$, so the y-coordinate is negative.
$\cos(x) < 0$, so the x-coordinate is negative.
$\therefore$ the third quadrant is required.

$\therefore \pi < x < \frac{3\pi}{2}$

The correct answer is **C**.

Question 4

The point (0, 1) occurs at $\left[\frac{\pi}{2}\right]$ on the unit circle.

Positive angles:

$\left[\frac{\pi}{2}\right]$ (First cycle) **[1 mark]**

$\left[\frac{5\pi}{2}\right]$ (Second cycle, add 2π) **[1 mark]**

Negative angles:

$\left[\frac{-3\pi}{2}\right]$ (First cycle) **[1 mark]**

$\left[\frac{-7\pi}{2}\right]$ (Second cycle, add -2π) **[1 mark]**

$\therefore$ the four points are $\left[\frac{\pi}{2}\right], \left[\frac{5\pi}{2}\right], \left[-\frac{3\pi}{2}\right]$ and $\left[-\frac{7\pi}{2}\right]$.

8.5 Symmetry properties

Question 1

$$\cos\left(\frac{3\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right)$$
$$= -\cos\left(\frac{\pi}{4}\right)$$
$$= -\frac{\sqrt{2}}{2}$$

The correct answer is **B**.

Question 2

$\sin(\theta)$ is negative. $\therefore \theta$ is in the third or fourth quadrant.

The basic angle for which $\sin(\theta) = \dfrac{1}{\sqrt{2}}$ is 45.

Third quadrant:

$$\begin{aligned}\theta &= 180° + 45° \\ &= 225°\end{aligned}$$

Fourth Quadrant:

$$\begin{aligned}\theta &= 360° - 45° \\ &= 315°\end{aligned}$$

$\therefore \theta = 225°, 315°$

The correct answer is **D**.

Question 3

$$4\sin\left(\frac{5\pi}{6}\right) - 2\cos\left(\frac{\pi}{3}\right) + \tan\left(\frac{5\pi}{4}\right) + \sin\left(\frac{7\pi}{3}\right)$$

$$= 4\sin\left(\frac{\pi}{6}\right) - 2\cos\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)$$

$$= 4 \times \frac{1}{2} - 2 \times \frac{1}{2} + 1 + \frac{\sqrt{3}}{2}$$ **[1 mark]**

$$= 2 - 1 + 1 + \frac{\sqrt{3}}{2}$$

$$= 2 + \frac{\sqrt{3}}{2}$$ **[1 mark]**

$$= \frac{4 + \sqrt{3}}{2}$$

Question 4

$\pi - \theta$ is in the second quadrant.

$\cos(\pi - \theta) = -\cos(\theta)$as $\cos(\theta)$ is negative in the second quadrant.

Question 5

$$\begin{aligned}\sin\left(\frac{4\pi}{3}\right) &= \sin\left(\pi + \frac{\pi}{3}\right) \\ &= -\sin\left(\frac{\pi}{3}\right) \\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

Question 6

$\tan(\theta)$ is negative.

$\therefore \theta$ is in the second or fourth quadrant.

The basic angle for $\tan(\theta) = \dfrac{1}{\sqrt{3}}$ is 30°.

Second quadrant: $\theta = 180° - 30° = 150°$

Fourth quadrant: $\theta = 360° - 30° = 330°$

$\therefore \theta = 150°, 330°$

8.6 Graphs of the sine and cosine functions

Question 1

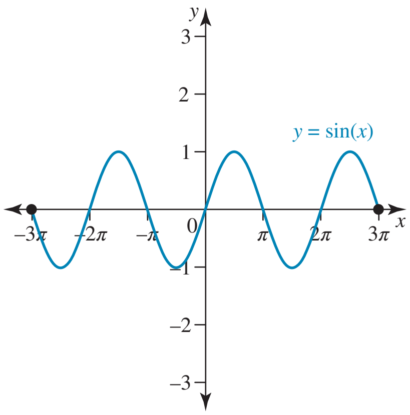
$y = \sin(x)$ passes through $(0, 0)$.

$y = \cos(x)$ passes through $\left(\frac{\pi}{2}, 0\right)$ and $(0, 1)$.

$\therefore$ option D is true for $f(x) = \sin(x)$ but false for $g(x) = \cos(x)$.

The correct answer is **D**.

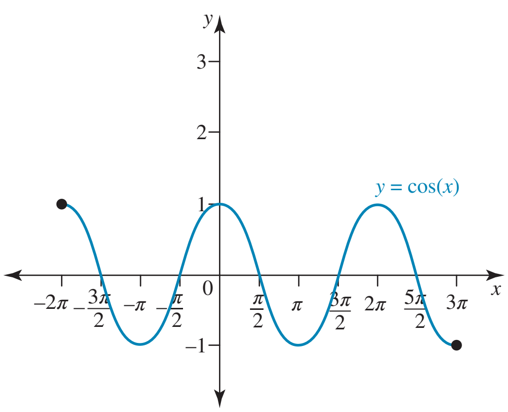
Question 2



The graph of $y = \sin(x)$ has x-intercepts at $0, \pm\pi, \pm2\pi, \pm3\pi$.

The correct answer is **A**.

Question 3



The graph shows solution $\pm\pi, 3\pi$.

The correct answer is **E**.

Question 4

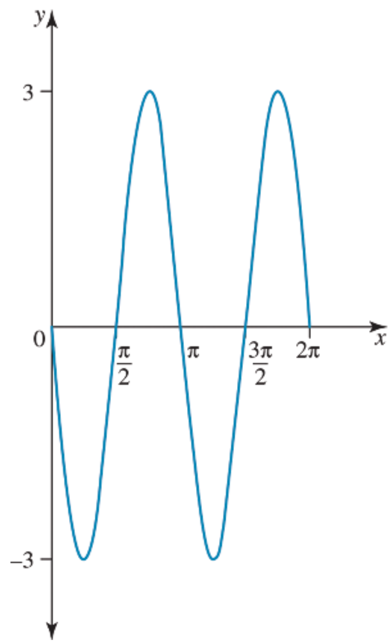
Amplitude $= 3$

Period $= \frac{2\pi}{2} = \pi$ **[1 mark]**

x-intercepts are $x = 0, \pi$ and 2π for $y = \sin(x)$. **[1 mark]**

$\therefore$ for $y = \sin(2x)$, intercepts are $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ and 2π. **[1 mark]**

The graph is inverted due to the negative sign. **[1 mark]**



Question 5

Using the unit circle, $\sin(x) = 1$ when the y-coordinate $= 1$

Moving anticlockwise, this occurs at $\frac{\pi}{2}$, then one cycle later at $2\pi + \frac{\pi}{2} = \frac{5\pi}{2}$.

Moving clockwise, $\sin(x) = 1$ at $-\frac{3\pi}{2}$.

8.7 Review

Question 1

$$\frac{\tan(30°)\sin(60°)}{\cos(60°)\sin(30°) - \tan(45°)} + \frac{\sin(45°)\cos(45°)}{\tan(45°)}$$

$= \dfrac{\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}}{\frac{1}{2} \times \frac{1}{2} - 1} + \dfrac{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}}{1}$ **[1 mark]**

$= \dfrac{\frac{1}{2}}{-\frac{3}{4}} + \frac{1}{2}$ **[1 mark]**

$= -\frac{2}{3} + \frac{1}{2}$ **[1 mark]**

$= -\frac{1}{6}$ **[1 mark]**

Question 2

$\tan(45°) + \tan(45°) = 1 + 1$
$= 2$

$\tan(90°)$ is undefined.

The correct answer is **D**.

Question 3

$\pi + \theta$ is in the third quadrant.

$\sin(\pi + \theta) = -\sin(\theta)$ as $\sin(\theta)$ is negative in the third quadrant.

The correct answer is **B**.

Question 4

$\sin(\theta) = 0.61,\ \sin(\pi + \theta) = -\sin(\theta) = -0.61$ **[1 mark]**

$\cos(t) = 0.48,\ \cos(\pi - t) = -\cos(t) = -0.48$ **[1 mark]**

$\tan(x) = 1.6, \tan(2\pi + x) = \tan(x) = 1.6$ **[1 mark]**

$\cos(-t) = \cos(t) = 0.48$ **[1 mark]**

Question 5

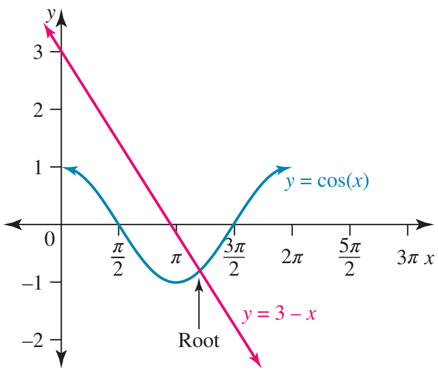
$y = \cos(x)$

One cycle of the graph has domain $[0, 2\pi]$.

The axis intercepts are $(0, 1)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{2}, 0\right)$.

$y = 3 - x$

The axis intercepts are $(0, 3)$ and $(3, 0)$.



Award 1 mark for correctly drawing each of the two graphs.

The two graphs intersect at one point only, so the equation $\cos(x) = 3 - x$ has only one root. **[1 mark]**

From the graph, it can be seen that the root lies between π and $\frac{3\pi}{2}$

$\therefore$ the interval in which the root lies is $\left[\pi, \frac{3\pi}{2}\right]$. **[1 mark]**

Question 6

Delete multiples of 2π to find the position of $-\frac{9}{2}\pi$.

$$\begin{aligned} -\frac{9}{2}\pi &= -\left(\frac{8\pi}{2} + \frac{\pi}{2}\right) \\ &= -\left(4\pi + \frac{\pi}{2}\right) \\ &= -\frac{\pi}{2} \end{aligned}$$

The angle $-\frac{\pi}{2}$ is the same position as the angle $\frac{3\pi}{2}$.

Therefore, it lies exactly between the third and fourth quadrants.

Question 7

$\sin(\pi - \theta) = \sin(\theta) = 0.25$ **[1 mark]**

$\cos(2\pi - \alpha) = \cos(\alpha) = 0.72$ **[1 mark]**

$\tan(\pi - x) = -\tan(x) = -1.3$ **[1 mark]**

$\sin(-\theta) = -\sin(\theta) = -0.25$ **[1 mark]**

9 Trigonometric functions and applications

Topic	9	Trigonometric functions and applications
Subtopic	9.2	Trigonometric equations

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Question 1 (1 mark)

The equation $\sin(x) = \frac{1}{2}, 0 \leq x \leq 2\pi$ has two solutions in quadrants

A. 1 and 2.
B. 1 and 3.
C. 2 and 3.
D. 2 and 4.
E. 3 and 4.

Question 2 (1 mark)

Consider the equation $\sin(\theta) = -\frac{1}{2}, -180° \leq \theta \leq 720°$.

The number of solutions for $\theta°$ is

A. 2
B. 3
C. 4
D. 5
E. 6

Question 3 (3 marks)

Solve the equation $2\cos(x) + \sqrt{3} = 0, 0 \leq x \leq 2\pi$, and obtain exact values for x.

Topic	9	Trigonometric functions and applications
Subtopic	9.3	Transformations of sine and cosine graphs

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Question 1 (1 mark)

The amplitude of the graph $y = 4 - 2\sin(3x)$is

A. 3
B. 4
C. -3
D. 2
E. -2

Question 2 (1 mark)

The maximum and minimum points of the graph $y = 2\sin(3x) - 1$ are

A. -1 and 1.
B. -2 and 0.
C. -3 and 1.
D. 2 and 3.
E. -2 and 3.

Question 3 (1 mark)

The period of the graph $y = \frac{2}{3}\sin\left(\frac{3}{2}x\right) - \frac{1}{4}$ is

A. $\frac{3\pi}{2}$
B. $\frac{4\pi}{3}$
C. $\frac{2\pi}{3}$
D. 2π
E. 4π

Question 4 (4 marks)
Sketch the graph of $y = 4\sin(x) + 2, 0 \le x \le 2\pi$.

Question 5 (1 mark)
The amplitude of the graph $y = 2 - \sin(4x)$is

A. 2
B. 1
C. 4
D. -1
E. -2

Question 6 (1 mark)
The period of the graph $y = 2\sin(3\pi x) - 4$ is

A. $\frac{2}{3}$
B. $\frac{2\pi}{3}$
C. 3
D. 3π
E. 2π

Topic	9	Trigonometric functions and applications
Subtopic	9.4	Applications of sine and cosine functions

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Question 1 (1 mark)

The minimum and maximum values for $y = 7 - 4\cos(3x)$ are

A. 0 and 7
B. 3 and 11
C. 4 and 7
D. 4 and 14
E. -3 and 5

Question 2 (1 mark)

The temperature, T degrees Celsius, during one day in November is given by $T = 20 - 4\sin\left(\frac{\pi}{12}t\right)$, where t is the time in hours after midnight. The minimum temperature during the day and the time at which it occurred were

A. 20 °C and midnight.
B. 16 °C and 6 am.
C. 14 °C and 8 am.
D. 12 °C and 10 am.
E. 14 °C and 4 am.

Question 3 (4 marks)

An ecologist studying the population of kangaroos in a national park calculated that the population roughly followed the function $p = 1000 + 500\sin\left(\frac{\pi t}{4}\right)$, where t is the time in months and P is the number of kangaroos in the population.

Determine:

a. the initial population **(1 mark)**

b. the highest and lowest population sizes in the first 2 years **(2 marks)**

c. the population at the end of 2 years. **(1 mark)**

Question 4 (1 mark)

For the equation $y = a\sin(\pi x) + k$, the maximum value of y is 15 and the minimum value is 6.
The values of a and k, respectively, are

A. 9 and 6.
B. 4.5 and 10.5.
C. 5 and 10.
D. 6 and 9.5.
E. 6.5 and 9.

Question 5 (1 mark)

The minimum and maximum values for $y = 5 - 3\sin(2x)$ are

A. 0 and 5.
B. 3 and 5.
C. −2 and 8.
D. 2 and 8.
E. −2 and −8.

Topic	9	Trigonometric functions and applications
Subtopic	9.5	The tangent function

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Question 1 (1 mark)

Select the statement that is *not* true about the graph $y = \tan(x)$ for the domain $[0, 2\pi]$.

A. The asymptotes are one period apart.

B. The x-intercepts occur at $x = 0, \pi, 2\pi$.

C. Two cycles are completed.

D. The range is $[-1, 1]$.

E. The mean position is $y = 0$.

Question 2 (1 mark)

The graph $y = \tan(4x), 0 \leq x \leq \dfrac{\pi}{2}$ has vertical asymptotes at

A. $x = \dfrac{\pi}{8}, \dfrac{3\pi}{8}$

B. $x = \dfrac{\pi}{6}, \dfrac{3\pi}{6}$

C. $x = \dfrac{\pi}{4}, \dfrac{\pi}{2}$

D. $x = 0, \dfrac{\pi}{2}$

E. $x = \dfrac{2\pi}{9}, \dfrac{4\pi}{9}$

Question 3 (5 marks)

Sketch the graph of $y = 2\tan(x), 0 \leq x \leq 2\pi$.

Question 4 (1 mark)

When comparing the graph $y = 4\tan(x) + 1$ with $y = \tan(x)$, which statement is false?

A. The x-intercepts lie midway between the asymptotes.
B. The positions of the asymptotes remain the same.
C. The graph is narrower.
D. The asymptotes are one period apart.
E. Range R.

Question 5 (1 mark)

Which statement is *not* true about the graph $y = \tan(x)$ for the domain $[0, 2\pi]$?

A. The first asymptote occurs at $x = \frac{\pi}{4}$
B. The range is R.
C. The x-intercepts are $0, \pi$ and 2π.
D. There is no amplitude for tan graphs.
E. The function is not defined for $x = \frac{\pi}{2}, \frac{3\pi}{2}$

Topic	9	Trigonometric functions and applications
Subtopic	9.6	Trigonometric identities and properties

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Question 1 (1 mark)

State which of the following is true.

A. $\tan(x) = \dfrac{\cos(x)}{\sin(x)}$

B. $\sin^2(x) = 1 + \cos^2(x)$

C. $\sin(x) + \cos(x) = 1$

D. $\cos(x) = \sqrt{1 - \sin^2(x)}$

E. $5\cos^2(x) = 5\sin^2(x) + 5$

Question 2 (1 mark)

Given $\sin(\theta) = \dfrac{3}{5}, 0 < \theta < \dfrac{\pi}{2}$, $\cos(\theta)$ is

A. $\dfrac{4}{5}$

B. $\dfrac{5}{8}$

C. $\dfrac{3}{4}$

D. $\dfrac{2}{5}$

E. $\dfrac{3}{8}$

Question 3 (4 marks)

Given $\tan(x) = -\dfrac{2}{5}, 0 \leq x \leq \pi$, deduce the exact values of $\sin(x)$ and $\cos(x)$.

Question 4 (4 marks)

Express $\frac{10 - 10\cos^2(\theta)}{3\sin(\theta)\cos(\theta)}$ in terms of $\tan(\theta)$ only.

Question 5 (1 mark)

Applying symmetry and complementary properties, $\cos\left(\frac{3\pi}{2} - \theta\right)$ can be written as

A. $\cos(\theta)$
B. $-\sin(\theta)$
C. $-\cos(\theta)$
D. $\sin(\theta)$
E. $-\cos\left(\frac{3\pi}{2}\right)$

Topic	9	Trigonometric functions and applications
Subtopic	9.7	Review

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Question 1 (4 marks)

Solve the following equation for x.

$\sin(2x) = \frac{1}{2}, 0 \leq x \leq 2\pi$

Question 2 (1 mark)

For $\sin(x) = \sqrt{3}\cos(x)$, the solutions of x, where $0 \leq x \leq 2\pi$, are

A. $\frac{\pi}{6}, \frac{4\pi}{6}$

B. $\frac{\pi}{\sqrt{3}}, \frac{4\pi}{\sqrt{3}}$

C. $\frac{\pi}{4}, \frac{3\pi}{4}$

D. $\frac{\pi}{3}, \frac{4\pi}{3}$

E. $\frac{\pi}{2}, \frac{5\pi}{2}$

Question 3 (3 marks)

Find the period, amplitude and range for the graph of $y = -3\cos(5x) + 7$.

Question 4 (1 mark)

For the equation $y = r\cos(\pi x) + c$, the maximum value of y is 19 and the minimum value is 8. The values of r and c are

A. 5.5 and 13.5
B. 8.0 and 11.0
C. 4.0 and 11.5
D. 3.5 and 10.5
E. 8.0 and 19.0

Question 5 (3 marks)

A windmill manufacturer attaches a flashing light on the far end of the rotor blade of a particular model to help scare birds from flying into the blades. The height of the light is given by $h = 80 - 25\cos\left(\frac{\pi}{2}t\right)$, where h is the height in metres above ground level after t seconds.

a. Determine how far above the ground the light is initially. **(1 mark)**

b. Calculate how many revolutions the rotor blade will complete in a 2-minute time interval. **(1 mark)**

c. Calculate the area that the windmill blade sweeps over (correct to 2 decimal places). **(1 mark)**

Question 6 (4 marks)

Sketch the graph of $y = \tan(x) + 1$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Question 7 (1 mark)

The equation $\cos(x) = \frac{1}{2}, 0 \le x \le 2\pi$ has two solutions in quadrants

A. 1 and 2.
B. 1 and 3.
C. 2 and 3.
D. 1 and 4.
E. 2 and 4.

Question 8 (1 mark)

$\sqrt{3}\sin(x) = \cos(x)$. The solutions for x, where $0 \le x \le 2\pi$, are

A. $\frac{\pi}{3}, \frac{4\pi}{3}$
B. $\frac{\pi}{6}, \frac{5\pi}{6}$
C. $\frac{\pi}{6}, \frac{7\pi}{6}$
D. $\frac{\pi}{3}, \frac{2\pi}{3}$
E. $\frac{\pi}{3}, \frac{7\pi}{3}$

Question 9 (3 marks)

Sketch the graph of $y = 3\cos(x) - 1.5, 0 \le x \le 2\pi$.

Question 10 (1 mark)

The graph $y = \tan(2x)$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ has vertical asymptotes at

A. $x = -\frac{\pi}{2}, \frac{\pi}{2}$

B. $x = -\frac{\pi}{8}, \frac{\pi}{8}$

C. $x = -\pi, \pi$

D. $x = -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}$

E. $x = -\frac{\pi}{4}, \frac{\pi}{4}$

Question 11 (4 marks)

An ecologist studying the population of koalas in a national park calculated that the population roughly followed the function $P = 800 + 400\sin\left(\frac{\pi t}{3}\right)$, where t is the time in months and P is the number of koalas in the population.

Find the initial population, the largest and smallest population sizes in the first three years, and the population at the end of 3 years.

Answers and marking guide

9.2 Trigonometric equations

Question 1

Sine is positive in quadrants 1 and 2.

The correct answer is **A**.

Question 2

Sine is negative in quadrants 3 and 4.

Between $0°$ and $-180°$, there are 2 solutions.

Between $0°$ and $720°$, there are 4 solutions.

The total number of solutions is 6.

The correct answer is **E**.

Question 3

$$2\cos(x) = -\sqrt{3}$$

$$\cos(x) = -\frac{\sqrt{3}}{2}$$

Cosine is negative in quadrants 2 and 3. **[1 mark]**

The base is $\frac{\pi}{6}$.

Since $x \in [0, 2\pi]$, there will be two solutions. **[1 mark]**

$$\therefore x = \pi - \frac{\pi}{6} \text{ or } x = \pi + \frac{\pi}{6}$$

$$\therefore x = \frac{5\pi}{6} \text{ or } \frac{7\pi}{6}$$ [**1 mark**]

9.3 Transformations of sine and cosine graphs

Question 1

The coefficient 2 is the amplitude.

The correct answer is **D**.

Question 2

Amplitude $= 2$

Vertical translation 1 unit downwards

$\therefore k = -1$

Instead of the graph oscillating between 2 and -2, it oscillates between -3 and 1.

The correct answer is **C**.

Question 3

$$\text{Period} = \frac{2\pi}{n} = \frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$$

The correct answer is **B**.

Question 4

$a = 4, n = 1, k = 2$

Amplitude = 4, Period = 2π **[1 mark]**

The graph oscillates between $y = 2 - 4 = -2$ and $y = 2 + 4 = 6$.

so it has the range $[-2, 6]$.

It will have x-intercepts.

x-intercepts: Let $y = 0$

$\therefore 4\sin(x) + 2 = 0$

$\therefore \sin(x) = -\frac{1}{2}$

Base $\frac{\pi}{6}$, quadrants 3 and 4

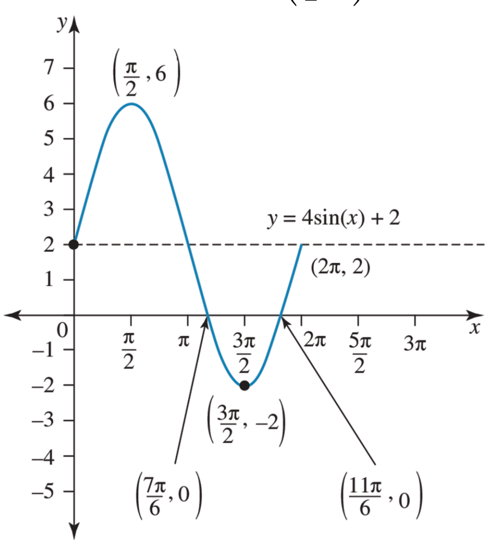
$\therefore x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$\therefore x = \frac{7\pi}{6}, \frac{11\pi}{6}$

x-intercepts = $\left(\frac{7\pi}{6}, 0\right), \left(\frac{11\pi}{6}, 0\right)$ **[1 mark]**

y-intercepts = $(0, 2)$ as the graph has shifted up 2 units from the normal strating position of $(0, 0)$.

The maximum point is $\left(\frac{\pi}{2}, 6\right)$ and the minimum point is $\left(\frac{3\pi}{2}, -2\right)$. **[1 mark]**



[1 mark]

Question 5

$y = 2 - \sin(4x)$

Coefficient of sin is -1, so amplitude = $|-1| = 1$.

Note that the amplitude is always positive.

Question 6

Period = $\frac{2\pi}{n} = \frac{2\pi}{3\pi} = \frac{2}{3}$

9.4 Applications of sine and cosine functions

Question 1

$y = 7 - 4\cos(3x)$

Minimum value:

largest positive value of $\cos(3x) = 1$

$y = 7 - 4\cos(3x)$

$y = 7 - 4 \times (1)$

$y = 3$

Maximum value: largest negative value of $(3x) = -1$

$y = 7 - 4\cos(3x)$

$y = 7 - 4 \times (-1)$

$y = 11$

∴ the minimum value is 3 and the maximum value is 11.

The correct answer is **B**.

Question 2

The minimum temperature occurs when $\sin\left(\frac{\pi}{12}t\right) = 1$ (because the graph is reflected).

Minimum temperature: $20 - 4 \times 1 = 16\,°\text{C}$

$\sin\left(\frac{\pi}{12}t\right) = 1$

Solving:

$\frac{\pi}{12}t = \frac{\pi}{2}$

$\therefore t = 6$

The minimum temperature was 16 °C and it occurred at 6:00 am.

The correct answer is **B**.

Question 3

a. Initial population when $t = 0$:

$p = 1000 + 500\sin(0)$

$= 1000$ **[1 mark]**

b. Highest population when $\sin\left(\frac{\pi t}{4}\right) = 1$:

$p = 1000 + 500 \times (1)$

$= 1500$

∴ the highest population is 1500. **[1 mark]**

Lowest population when $\sin\left(\frac{\pi t}{4}\right) = -1$:

$p = 1000 + 500 \times (-1)$

$= 500$

∴ the lowest population is 500. **[1 mark]**

c. At the end of 2 years (24 months):

$$p = 1000 + 500\sin\left(\frac{\pi t}{4}\right)$$

$$= 1000 + 500\sin\left(\frac{\pi \times 24}{4}\right)$$

$$= 1000 + 500\sin(6\pi)$$

As $\sin(6\pi) = 0$

$p = 1000$

∴ population after 2 years is 1000. **[1 mark]**

Question 4

$$\text{Amplitute} = \frac{\max - \min}{2}$$
$$= \frac{15 - 6}{2}$$
$$= 4.5$$

$\therefore a = 4.5$

$$k = \text{mean position}$$
$$= \text{minimum} + \text{amplitude}$$
$$= 6 + 4.5$$
$$= 10.5$$

Question 5

$y = 5 - 3\sin(2x)$

Minimum value:

Largest positive value of $\sin(2x) = 1$

$$y = 5 - 3\sin(2x)$$
$$= 5 - 3 \times 1$$
$$= 2$$

Maximum value:

Largest negative value of $\sin(2x) = -1$

$$y = 5 - 3\sin(2x)$$
$$= 5 - 3 \times -1$$
$$= 8$$

Minimum value is 2, and maximum value is 8.

9.5 The tangent function

Question 1

The range is R, so the statement that the range is $[-1, 1]$ is incorrect.

The correct answer is **D**.

Question 2

For $y = \tan(4x)$, $n = 4$

The period is $\frac{\pi}{n}$

$\therefore$ the period is $\frac{\pi}{4}$.

An asymptote occurs when:

$$4x = \frac{\pi}{2}$$
$$\therefore x = \frac{\pi}{8}$$

The other asymptotes in the domain are formed by adding multiples of the period to $x = \frac{\pi}{8}$.

$$x = \frac{\pi}{8} + \frac{\pi}{4}$$
$$= \frac{3\pi}{8}$$

The equations of the asymptotes in the given domain are $x = \frac{\pi}{8}, \frac{3\pi}{8}$.

The correct answer is **A**.

Question 3

$y = 2\tan(x),\ 0 \le x \le 2\pi$

The period is π. **[1 mark]**

Asymptotes:

An asymptote occurs when $x = \frac{\pi}{2}$.

For the domain $[0, 2\pi]$ and period, other asymptotes occur when

$$x = \frac{\pi}{2} + \pi$$
$$= \frac{3\pi}{2} \quad \textbf{[1 mark]}$$

x-intercepts:

$2\ \tan(x) = 0$

$x = 0,\ \pi,\ 2\pi$ **[1 mark]**

A dilation of factor 2 from the x-axis makes the graph of $y = 2\tan(x)$ narrower than $y = \tan(x)$.

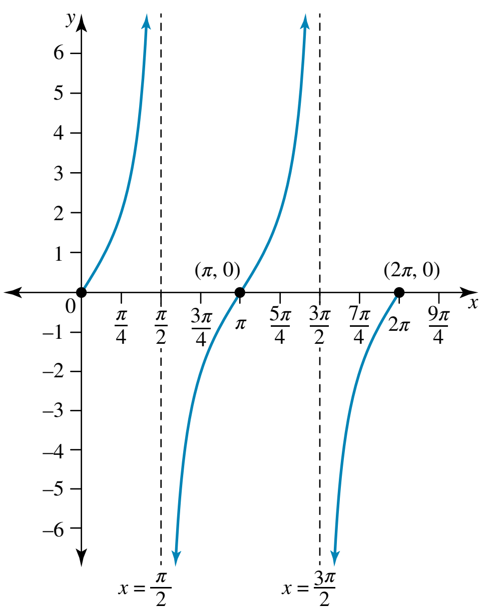
When

$$x = \frac{\pi}{4},$$
$$y = 2\tan\left(\frac{\pi}{4}\right)$$
$$= 2$$
$$\Rightarrow \left(\frac{\pi}{4}, 2\right)$$

End points:

When $x = 0$,
$$y = 2\tan(0)$$
$$= 0$$
$$\Rightarrow (0, 0)$$

when $x = 2\pi$,
$$y = 2\tan(2\pi)$$
$$= 0$$
$$\Rightarrow (2\pi, 0) \quad \textbf{[1 mark]}$$



[1 mark]

Question 4

A vertical translation of $y = \tan(x)$ upwards by 1 unit does not affect the positions of the vertical asymptotes. However, the positions of the x-intercepts would no longer lie midway between the asymptotes, since $y = 0$ is no longer the mean position. The x-intercepts are located by solving the equation $\tan(x) + k = 0$.

Question 5

The first asymptote occurs at $x = \frac{\pi}{2n} = \frac{\pi}{2}$.

9.6 Trigonometric identities and properties

Question 1

The relationship $\cos^2(\theta) + \sin^2(\theta) = 1$ is known as the Pythagorean identity. It is a true statement for any value of θ.

$\cos^2(x) = 1 - \sin^2(x)$

$\cos(x) = \sqrt{1 - \sin^2(x)}$

The correct answer is **D**.

Question 2

Since $\sin(\theta) = \frac{3}{5}$, the hypotenuse of a right-angled triangle is 5 and the side opposite θ is 3. The remaining side is 4, since 3, 4, 5 form a right-angled triangle; $\therefore \cos(\theta) = \frac{4}{5}$.

$\cos(\theta) = \frac{4}{5}, \left(0 < \theta < \frac{\pi}{2}\right)$

The correct answer is **A**.

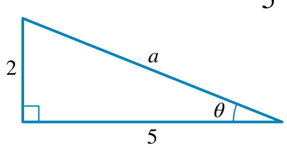
Question 3

$\tan(x) = -\frac{2}{5}, 0 \le x \le \pi$

The sign of $\tan(x)$ indicates either quadrant 2 or 4.

The condition that $0 \le x \le \pi$ makes it quadrant 2. **[1 mark]**

Quadrant 1: $\tan(x) = \frac{2}{5} = \frac{\text{opposite}}{\text{adjacent}}$



[1 mark]

Using Pythagoras' theorem,

$2^2 + 5^2 = a^2$

$\therefore a = \sqrt{29}$ (positive root required) **[1 mark]**

In quadrant 2, sine is positive and cosine is negative.

$\sin(x) = +\frac{2}{\sqrt{29}}$ and $\cos(x) = -\frac{5}{\sqrt{29}}$ **[1 mark]**

Question 4

There is a common factor in the numerator.

$\frac{10 - 10\cos^2(\theta)}{3\sin(\theta)\cos(\theta)} = \frac{10(1 - \cos^2(\theta))}{3\sin(\theta)\cos(\theta)}$ **[1 mark]**

Since $1 - \cos^2(\theta) = \sin^2(\theta)$:

$\frac{10\left(1 - \cos^2(\theta)\right)}{3\sin(\theta)\cos(\theta)} = \frac{10\sin^2(\theta)}{3\sin(\theta)\cos(\theta)}$ **[1 mark]**

Cancelling $\sin(\theta)$

$$\frac{10\sin^2(\theta)}{3\ \sin(\theta)\cos(\theta)} = \frac{10\ \sin(\theta)}{3\ \cos(\theta)} \qquad \textbf{[1 mark]}$$

$$= \frac{10}{3}\tan(\theta) \qquad \textbf{[1 mark]}$$

Question 5

The three trigonometric points $\left[\frac{\pi}{2}+\theta\right]$, $\left[\frac{3\pi}{2}-\theta\right]$ and $\left[\frac{3\pi}{2}+\theta\right]$ each have the same base, $\frac{\pi}{2}-\theta$ while the three trigonometric points $[\pi-\theta]$, $[\pi+\theta]$ and $[2\pi-\theta]$ each have the base θ.

$\cos\left(\frac{3\pi}{2}-\theta\right) = \cos\left(\frac{\pi}{2}-\theta\right)$, as cos is negative in the third quadrant.

$= -\sin(\theta)$, using the complementary property.

9.7 Review

Question 1

$\sin(2x) = \frac{1}{2}, 0 \le x \le 2\pi$

Multiply the end points of the domain of x by 2.

$\therefore \sin(2x) = \frac{1}{2}, 0 \le 2x \le 4\pi$ **[1 mark]**

Sine is positive in quadrants 1 and 2.

The base is $\frac{\pi}{6}$ **[1 mark]**

As $2x \in [0, 4\pi]$, each of the two revolutions will generate 2 solutions, giving a total of 4 values for $2x$.

$\therefore 2x = \frac{\pi}{6}, \pi-\frac{\pi}{6}, 2\pi+\frac{\pi}{6}, 3\pi-\frac{\pi}{6}$

$\therefore 2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ **[1 mark]**

Divide each of the solutions by 2 to obtain the solutions for x within the original domain of $0 \le x \le 2\pi$.

$\therefore x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ **[1 mark]**

Question 2

Divide both sides by $\cos(x)$.

$\frac{\sin(x)}{\cos(x)} = \sqrt{3}$

$\therefore \tan(x) = \sqrt{3}$

Tangent is positive in quadrants 1 and 3.

The base is $\frac{\pi}{3}$.

$\therefore x = \frac{\pi}{3}, \pi+\frac{\pi}{3}$

$\therefore x = \frac{\pi}{3}, \frac{4\pi}{3}$

The correct answer is **D**.

Question 3

$y = -3\cos(5x) + 7$

$a = 3, n = 5, c = 7$

Period:

$\frac{2\pi}{n} = \frac{2\pi}{5}$ **[1 mark]**

Amplitude:

$a = 3$ and -3 (negative indicates that the graph is inverted). **[1 mark]**

Range:
The graph oscillates between $y = 7 - 3$ and $y = 7 + 3$.
$\therefore$ range is [4, 10]. [1 mark]

Question 4

$$\begin{aligned}\text{Amplitude} &= \frac{\text{max} - \text{min}}{2}\\ &= \frac{19 - 8}{2}\\ &= 5.5\\ &\therefore r = 5.5\\ c &= \text{mean position}\\ &= \text{min} + \text{amplitude}\\ &= 8 + 5.5\\ &= 13.5\end{aligned}$$

The correct answer is **A**.

Question 5

a. $t = 0$

$$\begin{aligned}h &= 80 - 25\cos\left(\frac{\pi}{2}t\right)\\ &= 80 - 25\cos(0)\\ &= 80 - 25 \times 1\\ &= 55 \text{ metres} \quad \textbf{[1 mark]}\end{aligned}$$

b.

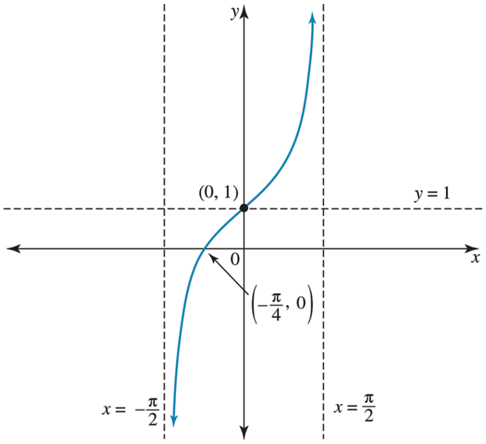
$$\begin{aligned}\text{Period} &= \frac{2\pi}{n}\\ &= \frac{2\pi}{\frac{\pi}{2}}\\ &= 4\end{aligned}$$

Two minutes = 120 seconds
Therefore, the rotor blades complete 30 revolutions in 2 minutes. **[1 mark]**

c. Area covered by rotor blades:
Length of rotor blades = 25 m
Area = $\pi(25)^2$
The area is 1963.50 m^2. **[1 mark]**

Question 6

$y = \tan(x) + 1$
There is a vertical translation up to 1 unit. **[1 mark]**
x-intercepts: Let $y = 0$
$\therefore \tan(x) = -1, -\frac{\pi}{2} < x < \frac{\pi}{2}$
$\therefore x = -\frac{\pi}{4}$
$\left(-\frac{\pi}{4}, 0\right)$ is the x-intercepts. **[1 mark]**
y-intercepts = (0, 1)
The translation does not alter the period of asympotates.
Period is π.
Asymptotes: $x = \pm\frac{\pi}{2}$ **[1 mark]**



[1 mark]

Question 7

Cosine is positive in quadrants 1 and 4.

Question 8

Divide both sides by $\cos(x)$ and convert to $\tan(x)$.

$$\frac{\sqrt{3}\sin(x)}{\cos(x)} = 1$$
$$\sqrt{3}\tan(x) = 1$$
$$\tan(x) = \frac{1}{\sqrt{3}}$$

Tangent is positive in quadrants 1 and 3.

Base angle is $\frac{\pi}{6}$.

$$\therefore x = \frac{\pi}{6}, \pi + \frac{\pi}{6}$$
$$\therefore x = \frac{\pi}{6}, \frac{7\pi}{6}$$

Question 9

amplitude = 3, period = 2π. **[1 mark]**

mean position: $y = -1.5$

The graph oscillates between $y = -1.5 + 3 = 1.5$ and $y = -1.5 - 3 = -4.5$ so it has range $[-4.5, 1.5]$.

It will have x-intercepts.

x-intercepts: Let $y = 0$

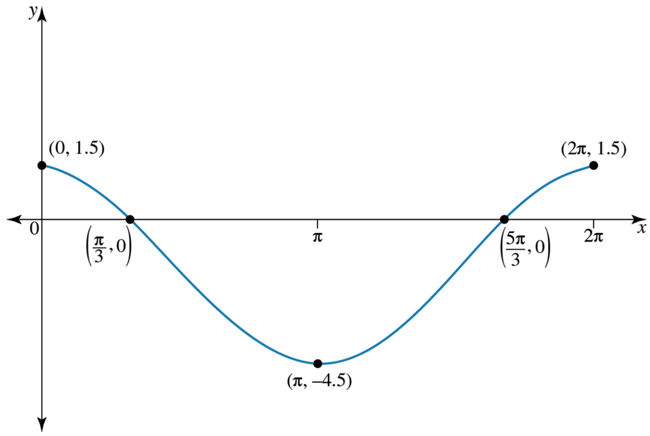
$$0 = 3\cos(x) - 1.5, \; 0 \le x \le 2\pi$$
$$1.5 = 3\cos(x)$$
$$\cos(x) = \frac{1}{2}$$

Base angle = $\frac{\pi}{3}$, and cosine is positive in quadrants 1 and 4.

$$\therefore x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$$
$$= \frac{\pi}{3}, \frac{5\pi}{3}$$

x-intercepts are $\left(\frac{\pi}{3}, 0\right)$ and $\left(\frac{5\pi}{3}, 0\right)$. **[1 mark]**

y-intercept = (0, 1.5) the graph has shifted down 1.5 units from a starting position of (0, 3) given the amplitude is 3.



[1 mark]

Question 10

For $y = \tan(2x)$, $n = 2$.

Period is $\frac{\pi}{n} = \frac{\pi}{2}$

The first asymptote is when $x = \frac{\pi}{2n} = \frac{\pi}{4}$.

The other asymptotes in the domain are formed by adding multiples of the period to $x = \frac{\pi}{4}$.

$$x = \frac{\pi}{4} + \frac{\pi}{2}$$
$$= \frac{3\pi}{4}$$

(out of domain)

$$x = \frac{\pi}{4} - \frac{\pi}{2}$$
$$= -\frac{\pi}{4}$$

The equations of the asymptotes in the given domain are $x = -\frac{\pi}{4}, \frac{\pi}{4}$.

Question 11

Initial population when $t = 0$

$P = 800 + 400\sin(0)$

$= 800$ **[1 mark]**

Largest population when $\sin\left(\frac{\pi t}{3}\right) = 1$

$P = 800 + 400 \times (1)$

$= 1200$

Largest population is 1200. **[1 mark]**

Smallest population when $\sin\left(\frac{\pi t}{3}\right) = -1$

$P = 800 + 400 \times (-1)$

$= 400$

Smallest population is 400. **[1 mark]**

At the end of 3 years (36 months)

$$\begin{aligned}P &= 800 + 400\sin\left(\frac{\pi t}{3}\right)\\ &= 800 + 400\sin\left(\frac{36\pi}{3}\right)\\ &= 800 + 400\sin(12\pi)\\ &= 800 + 400\times 0\\ &= 800\end{aligned}$$

Population after 3 years is 800. **[1 mark]**

10 Exponential functions and logarithms

Topic	10	Exponential functions and logarithms
Subtopic	10.2	Indices as exponents

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

The value of $3^{-4} \times \left(\frac{8}{27}\right)^{-\frac{1}{3}}$ is

A. $\frac{1}{8}$

B. $\frac{1}{12}$

C. $\frac{3}{32}$

D. $\frac{1}{54}$

E. $\frac{8}{81}$

Question 2 (1 mark)

If $27^{3-2x} = 729^{x}$, then x is equal to

A. 6

B. $\frac{3}{4}$

C. $\frac{1}{3}$

D. 1

E. $\frac{7}{9}$

Question 3 (4 marks)

Solve for x.

$27^x \div 9^{1-x} = 3\sqrt{3}$

Question 4 (4 marks)

Multiply the following numbers, expressing the answer in scientific notation and stating the number of significant figures the answer contains.

0.098 735 and 1.52×10^{-4}

Question 5 (1 mark)

If $16^{2-x} = 256^x$, then x is equal to

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. $\frac{5}{4}$

D. $\frac{4}{5}$

E. $\frac{3}{4}$

Question 6 (1 mark)

If $8^{2x} \div 4^{3-x} = 2\sqrt{2}$, then x is equal to

A. $\frac{13}{8}$

B. $\frac{15}{8}$

C. $\frac{13}{16}$

D. $\frac{8}{13}$

E. $\frac{15}{16}$

Topic	10	Exponential functions and logarithms
Subtopic	10.3	Indices as logarithms

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

$\log_6(42) - \log_6(7)$ equals

A. $\log_{12}(35)$

B. $\log_6(35)$

C. -1

D. 1

E. 6

Question 2 (1 mark)

$y = a^x$ in logarithmic form is

A. $\log_y(a) = x$

B. $\log_x(y) = a$

C. $\log_a(y) = x$

D. $\log_y(x) = a$

E. $\log_a(x) = y$

Question 3 (3 marks)

Solve the equation $\log_{12}(x) + \log_{12}(x + 1) = 1$ for x and check the validity of the solutions.

Question 4 (2 marks)

State the exact solution to $7^x = 9$, and calculate its value to 3 decimal places.

Question 5 (1 mark)

$\log_5(40) - \log_5(8)$ equals

A. -1

B. $\log_5(32)$

C. 5

D. 1

E. -5

Question 6 (1 mark)

The solution(s) to the equation $\log_8(x) + \log_8(x - 2) = 1$ are

A. $x = 4, -2$

B. $x = 2, 0$

C. $x = 4$

D. $x = 2$

E. $x = 0$

Topic	10	Exponential functions and logarithms
Subtopic	10.4	Graphs of exponential functions

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

State which one of the following is *not* a key feature of the graph $y = 2^x$ or any such function $y = a^x$ where $a > 1$.

A. Horizontal asymptote with equation $y = 0$

B. y-intercept $(0, 1)$

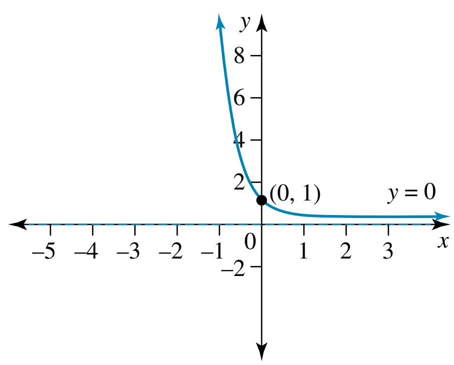
C. Shape of 'exponential growth'

D. Domain R

E. Range R

Question 2 (1 mark)

A possible equation for the graph shown is



A. $y = 10^x$

B. $y = 10^{-x}$

C. $y = 2^x$

D. $y = 2^{2x}$

E. $y = -2^x$

Question 3 (4 marks)
Sketch the graph of $y = 9 \times 3^{3x-2}$ and state its range.

Question 4 (5 marks)
Sketch the graph $y = 2^x - 8$ and state its range.

Question 5 (1 mark)
Which one of the following is *not* a key feature of the graph of $y = 3^{-x}$ and any such function $y = a^{-x}$ where $a > 1$?

A. Shape of exponential 'decay'
B. Domain R
C. y-intercept $(0, 1)$
D. Vertical asymptote at $x = 0$
E. Range $(0, \infty)$

Question 6 (1 mark)
The domain and range, respectively, of the graph $y = -3 \times 10^x$ are

A. R and $(-\infty, 0)$.
B. R and $(0, \infty)$.
C. R and R.
D. $(-\infty, 0)$ and R.
E. $(0, \infty)$ and R.

Topic	10	Exponential functions and logarithms
Subtopic	10.5	Applications of exponential functions

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

At a secondhand car dealer, the depreciation of the value of a car is calculated using $V = V_0 \times 3^{-0.1t}$, where t is the age of the car (in years). The number of years it would take to lose two-thirds of the purchase value is

A. 3
B. 5
C. 10
D. 15
E. 20

Question 2 (1 mark)

The decay of a radioactive substance is modelled by the equation $Q(t) = 15 \times 5^{-0.0025t}$, where Q kg is the amount of the substance present at time t years. The initial amount of the substance was

A. 5
B. 15
C. 25
D. 3
E. 0.0025

Question 3 (1 mark)

Sue calculates that a cup of tea cools at a rate of $T = 90 \times 3^{-0.01t}$, where T (°C) is the temperature of the tea after t minutes. After 30 minutes, the tea cools by

A. 34.21 °C
B. 25.27 °C
C. 22.32 °C
D. 14.32 °C
E. 64.73 °C

Question 4 (3 marks)

The decay of a radioactive substance is modelled by the equation $Q(t) = 12 \times 4^{-0.015t}$, where Q kg is the amount of the substance present at time t years.

a. Determine the initial amount of the substance. **(1 mark)**

b. How long does it take for the substance to reduce to half its initial amount? Give your answer to the nearest year. **(2 marks)**

Topic	10	Exponential functions and logarithms
Subtopic	10.6	Inverses of exponential functions

online only

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Question 1 (1 mark)

State which of the following is *not* true about the inverse of $y = a^x, a > 1$.

A. It has a vertical asymptote with equation $x = 0$.
B. The point $(a, 1)$ lies on the graph.
C. The y-intercept is $(0, 1)$.
D. The domain is R^+.
E. The range is R.

Question 2 (1 mark)

Consider the function $f: R \to R,\ f(x) = 4 \times 2^{-x} + 5$. The domain of its inverse function is

A. $(4, \infty)$
B. $(5, \infty)$
C. $(\infty, 4)$
D. $(\infty, 5)$
E. R

Question 3 (4 marks)

Sketch the graph of $y = \log_2(x + 4)$ and state its domain.

Question 4 (1 mark)

Consider the function, $f: R \to R, f(x) = 2 \times 5^{-x} - 3$. The domain of its inverse function is

A. $(3, \infty)$
B. $(-3, \infty)$
C. R
D. $(-\infty, 3)$
E. $(-\infty, 0)$

Topic	10	Exponential functions and logarithms
Subtopic	10.7	Review

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

$52.2 \times 10^{-4} \div \left(5 \times 10^{8}\right)$ expressed in standard form is

A. 262

B. 10.44

C. 2.62×10^{-32}

D. 1.044×10^{-11}

E. 2.62×10^{-12}

Question 2 (1 mark)

If $\log_{10}(2x - 3) = 0$, then x is equal to

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. 2

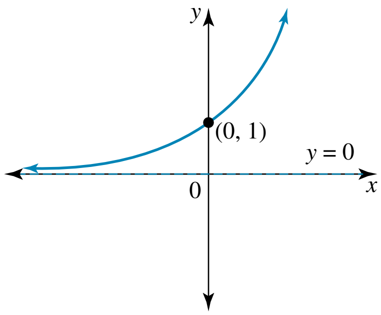
D. 1

E. 0

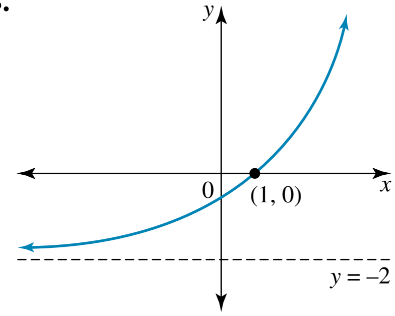
Question 3 (1 mark)

The graph of $y = -4 \times 5^x$ is best represented by

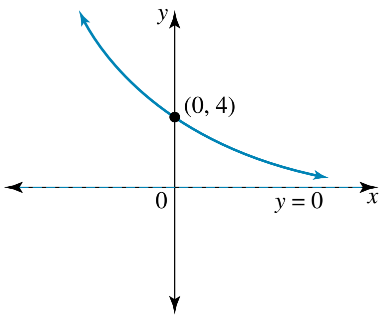
A.



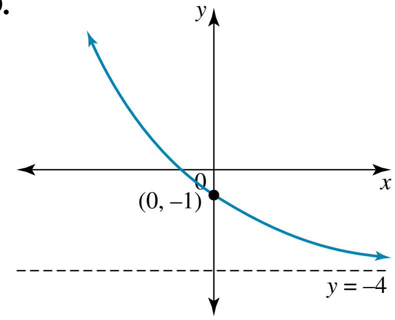
B.



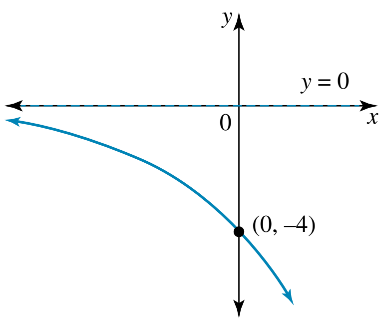
C.



D.



E.



Question 4 (4 marks)

Find the values of a, b and the equation if $y = a\log_9(bx)$ contains the points $(3, 0)$ and $(27, 4)$.

Question 5 (1 mark)

Simplify $\log_{20}\left(16^x \times 5^{2x}\right)$ using the inverse relationship between exponentials and logarithms.

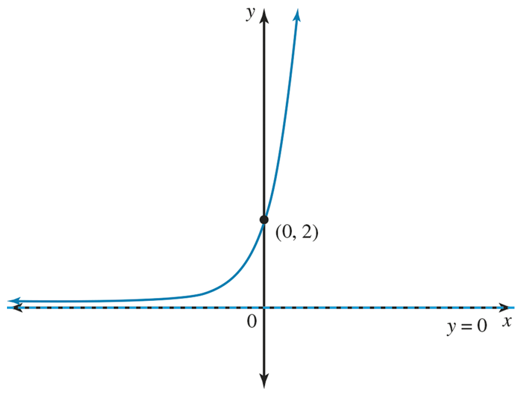
A. 20
B. x
C. $2x$
D. $16x$
E. $5x$

Question 6 (1 mark)

$m = n^x$ in logarithmic form is

A. $n = \log_x(m)$
B. $x = \log_n(m)$
C. $m = \log_n(x)$
D. $x = \log_m(n)$
E. $n = \log_x(x)$

Question 7 (1 mark)
A possible equation for the graph below is



A. $y = 2^x$
B. $y = 5^{2x}$
C. $y = 2^{5x}$
D. $y = 2^{2x}$
E. $y = 2 \times 5^x$

Question 8 (4 marks)
Find the value of a and b if $y = a\log_4(bx)$ contains the points $(2, 0)$ and $(8, 2)$.

Answers and marking guide

10.2 Indices as exponents

Question 1

$$\begin{aligned}3^{-4}\times\left(\frac{8}{27}\right)^{-\frac{1}{3}} &= 3^{-4}\times\frac{2^{3\times\frac{1}{3}}}{3^{3\times\frac{1}{3}}}\\ &= 3^{-4}\times\frac{2^{-1}}{3^{-1}}\\ &= 3^{-3}\times 2^{-1}\\ &= \frac{1}{27}\times\frac{1}{2}\\ &= \frac{1}{54}\end{aligned}$$

The correct answer is **D**.

Question 2

$$\begin{aligned}27^{3-2x} &= 729^{x}\\ 3^{3(3-2x)} &= 3^{6x}\\ 9-6x &= 6x\\ 9 &= 12x\\ x &= \frac{9}{12}\\ &= \frac{3}{4}\end{aligned}$$

The correct answer is **B**.

Question 3

$27^{x}\div 9^{1-x} = 3\sqrt{3}$

$3^{3x}\div 3^{2(1-x)} = 3\times 3^{\frac{1}{2}}$

$3^{3x-2(1-x)} = 3^{\frac{3}{2}}$ **[1 mark]**

$5x-2 = \frac{3}{2}$

$5x = \frac{7}{2}$

$x = \frac{7}{10}$ **[1 mark]**

Question 4

0.098 735 and $1.52\times 10^{-4} = 9.8735\times 10^{-2}\times 1.52\times 10^{-4}$ **[1 mark]**

$= 15.007\,72\times 10^{-6}$ **[1 mark]**

$= 1.5007\,72\times 10^{-5}$ **[1 mark]**

7 significant figures **[1 mark]**

Question 5

$$16^{2-x} = 256^x$$
$$4^{2(2-x)} = 4^{4x}$$
$$4 - 2x = 4x$$
$$4 = 6x$$
$$x = \frac{2}{3}$$

Question 6

$$8^{2x} \div 4^{3-x} = 2\sqrt{2}$$
$$2^{3\times 2x} \div 2^{2(3-x)} = 2^1 \times 2^{\frac{1}{2}}$$
$$6x - (6 - 2x) = \frac{3}{2}$$
$$8x - 6 = \frac{3}{2}$$
$$8x = \frac{15}{2}$$
$$x = \frac{15}{16}$$

10.3 Indices as logarithms

Question 1

$$\log_6(42) - \log_6(7) = \log_6 \frac{42}{7}$$
$$= \log_6 6$$
$$= 1$$

The correct answer is **D**.

Question 2

The statements $n = a^x$ and $x = 1 \log_a(n)$ are equivalent.

$\therefore y = a^x \Rightarrow x = \log_a(y)$

The correct answer is **C**.

Question 3

$$\log_{12}(x) + \log_{12}(x+1) = 1$$
$$\log_{12}(x(x+1)) = 1$$
$$\log_{12}(x(x+1)) = \log_{12} 12$$ **[1 mark]**
$$x^2 + x = 12$$
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$$\therefore x = -4,\ x = 3$$ **[1 mark]**

As $x > 0$, $x = -4$ is not valid solution.

$$\therefore x = 3$$ **[1 mark]**

Question 4

$$7^x = 9$$
$$\therefore x = \log_7(9)$$

The exact solution is $x = \log_7(9)$. **[1 mark]**

$$\text{Since,}\quad \log_a(p) = \frac{\log_{10}(p)}{\log_{10}(a)}$$
$$\text{Then,}\quad \log_7(9) = \frac{\log_{10}(9)}{\log_{10}(7)}$$
$$\therefore x = \frac{\log_{10}(9)}{\log_{10}(7)}$$
$$= 1.129 \text{ to 3 decimal places}$$ **[1 mark]**

Or, using CAS, $x = 1.129$ (to 3 decimal places).

Question 5

$\log_5(40) - \log_5(8)$

$= \log_5\left(\frac{40}{8}\right)$

$= \log_5(5)$

$= 1$

Question 6

$\log_8(x) + \log_8(x-2) = 1$

$\log_8(x(x-2)) = \log_8(8)$

$x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

As $x > 2, x = -2$ is not a valid solution.

$\therefore x = 4$

10.4 Graphs of exponential functions

Question 1

For $x \in R, y > 0$

$\therefore$ range R^+

The correct answer is **E**.

Question 2

$y = a^{-x}$ where $a > 1$:

- horizontal asymptote with equation, $y = 0$
- y-intercept $(0, 1)$
- shape of 'exponential decay'
- domain R
- range R^+
- one-to-one correspondence

$y = 10^{-x}$

The correct answer is **B**.

Question 3

Intercepts:

y-intercept ($x = 0$)

$$\begin{aligned} y &= 9 \times 3^{3x-2} \\ &= 9 \times 3^{0-2} \\ &= 9 \times 3^{-2} \\ &= 9 \times \frac{1}{9} \\ &= 1 \end{aligned}$$

y-intercept is $(0, 1)$ **[1 mark]**

x-intercept ($y = 0$)

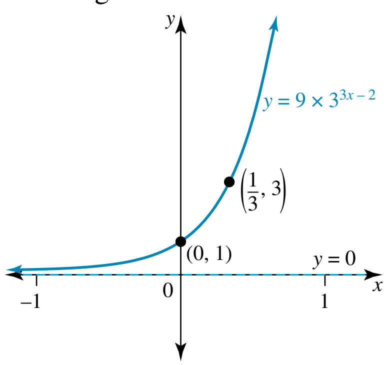
No intercept; the x-axis is an asymptote.

The second point can be found by substituting $x = \frac{1}{3}$.

$$\begin{aligned} y &= 9 \times 3^{1-2} \\ &= 9 \times 3^{-1} \\ &= 9 \times \frac{1}{3} \\ &= 3 \end{aligned}$$

$\left(\frac{1}{3}, 3\right)$ **[1 mark]**

The range is R^+. **[1 mark]**



[1 mark]

Question 4

The vertical translation 8 units down affects the asymptote.

The asymptote is $y = -8$. **[1 mark]**

Intercept:

y-intercept ($x = 0$)

$$\begin{aligned} y &= 2^0 - 8 \\ &= 1 - 8 \\ &= -7 \end{aligned}$$

$(0, -7)$ **[1 mark]**

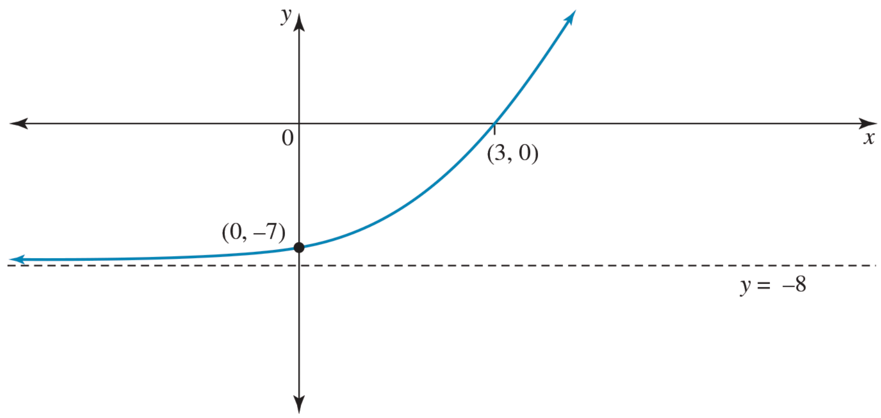
x-intercept ($y = 0$)

$$\begin{aligned} 0 &= 2^x - 8 \\ 2^x &= 8 \\ x &= 3 \end{aligned}$$

$(3, 0)$ **[1 mark]**

As the coefficient of x is positive, the shape is 'growth'.

Range is $(-8, \infty)$. **[1 mark]**



[1 mark]

Question 5
There is a horizontal asymptote, rather than a vertical asymptote, at $y = 0$.

Question 6
The graph has been reflected in the x-axis, so the range is $(-\infty, 0)$. The domain is R as there is no restriction on the x values.

10.5 Applications of exponential functions

Question 1
$V = V_0 \times 3^{-0.1t}$
$t = 0, V = V_0$ the purchase value
When $\frac{2}{3}$ of the value is lost, $\frac{1}{3}$ remains.
A third of the purchase value is $\frac{V_0}{3}$.
$\frac{V_0}{3} = V_0 \times 3^{-0.1t}$
$\frac{1}{3} = 3^{-0.1t}$
$3^{-1} = 3^{-0.1t}$
$-1 = -0.1t$
$t = 10$ years
The correct answer is **C**.

Question 2
$t = 0$
$Q(t) = 15 \times 5^0$
$= 15$
The correct answer is **B**.

Question 3
$T = 90 \times 3^{-0.01t}$
$= 90 \times 3^{-0.01 \times 30}$
$= 64.73\,°C$
The initial temperature is $90\,°C$.
Drop in temperature $= 90\,°C - 64.73°C$
$= 25.27\,°C$
The correct answer is **B**.

Question 4

a. $Q(t) = 12 \times 4^{-0.015t}$

$t = 0$

$Q(0) = 12 \times 4^{0}$

$= 12\,\text{kg}$ **[1 mark]**

b. Half the initial amount = 6 kg

$6 = 12 \times 4^{-0.015t}$ **[1 mark]**

Solve on CAS.

$t = 33.33$

$= 33$ years **[1 mark]**

10.6 Inverses of exponential functions

Question 1

Original function: $y = a^x$, y-intercept $(0,\ 1)$, no x-intercept

Inverse function: $y = \log_a x$, x-intercept $(1, 0)$, no y-intercept

y-intercept at $(0, 1)$ is for the inverse function.

The correct answer is **C**.

Question 2

$f(x) = 4 \times 2^{-x} + 5$

y-intercept $(0, 9)$

Asymptote at $y = 5$

As is the y-intercept lies above the asymptote, the range is $(5.\infty)$,

The domain of the inverse function is the range of the given function,

$\therefore$ the domain of the inverse is $(5.\infty)$,

The correct answer is **B.**

Question 3

$y = \log_2(x + 4)$

Horizontal translation 4 units to the left.

Asymptote at $x = -4$ **[1 mark]**

Domain $(x : x > -4)$

y-intercept $(x = 0)$

$y = \log_2(0 + 4)$

$y = \log_2 (2)^2$

$= 2\log_2(2)$

$= 2$

$(0, 2)$ **[1 mark]**

x-intercept $(y = 0)$

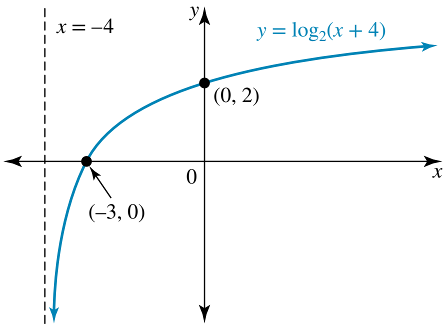
$0 = \log_2(x + 4)$

$\therefore x + 4 = 2^0$

$x = -3$

(-30) **[1 mark]**

Check: the point $(1, 0) \to (-3, 0)$ under the horizontal translation. **[1 mark]**



Question 4

$f(x) = 2 \times 5^{-x} - 3$

y-intercept $(0, -1)$

Asymptote at $y = -3$

As y-intercept lies above the asymptote, range is $(-3, \infty)$.

The domain of the inverse function is the range of the given function.

Domain of inverse is $(-3, \infty)$.

10.7 Review

Question 1

$$52.2 \times 10^{-4} \div \left(5 \times 10^{8}\right) = \frac{52.2 \times 10^{-4}}{5 \times 10^{8}}$$
$$= 10.44 \times 10^{-12}$$
$$= 1.044 \times 10^{-11}$$

The correct answer is **D**.

Question 2

$\log_{10}(2x - 3) = 0$

First law $\log_a(1) = 0$

$\therefore \log_{10}(2x - 3) = \log_{10} 1$

$2x - 3 = 1$

$2x = 4$

$x = 2$

The correct answer is **C**.

Question 3

y-intercept $(x = 0)$

$y = -4 \times 5^0$

$y = -4$

$(0, 4)$

Decay shaped graph

Asymptote $y = 0$

$\therefore$ option E

The correct answer is **E**.

Question 4

$y = a\log_9(bx)$

$9^y = (bx)^a$ **[1 mark]**

Point $(3, 0)$

$9^0 = b^a 3^a$

$b^a = \dfrac{1}{3^a}$ **[1 mark]**

Point $(27, 4)$

$9^4 = 27^e 3^e$

Substitute $b^a = \dfrac{1}{3^e} = 3^{-e}$

$$9^4 = 27^e \times 3^{-e}$$

$$\left(3^2\right)^4 = 3^{-e} \times \left(3^3\right)^e$$

$$3^8 = 3^{-e} \times 3^{3e}$$

$$3^8 = 3^{2e}$$

$$\therefore a = 4$$

Substitute $a = 4$ to find b.

$$4 = 4\log_9(27b)$$

$$1 = \log_9(27b)$$

$$9 = 27b$$

$$\frac{1}{3} = b$$ **[1 mark]**

$\therefore a = 4,\ b = \dfrac{1}{3},\ y = 4\log_9\left(\dfrac{1}{3}x\right)$ **[1 mark]**

Question 5

$$\log_{20}\left(16^x \times 5^{2x}\right) = \log_{20}\left(\left(4^2\right)^x \times 5^{2x}\right)$$
$$= \log_{20}\left(4^{2x} \times 5^{2x}\right)$$
$$= \log_{20}\left(20^{2x}\right)$$
$$= 2x$$

The correct answer is **C**.

Question 6

$x = \log_n(m)$

The statements $m = n^x$ and $x = \log_n(m)$ are equivalent.

$\therefore m = n^x \Rightarrow x = \log_n(m)$

Question 7

$y = k \times a^x$ where $a > 1$:

- horizontal asymptote with equation $y = 0$
- y-intercept $(0, k)$
- shape of 'exponential growth'.

The only equation that gives a y-intercept of $(0, 2)$ is $y = 2 \times 5^x$.

Question 8

$y = a\log_4(bx)$

$4^y = (bx)^a$ **[1 mark]**

point $(2, 0)$

$4^0 = b^a\, 2^a$

$b^a = \dfrac{1}{2^a}$ **[1 mark]**

point $(8, 2)$

$4^2 = b^a 8^a$

Substitute $b^a = \dfrac{1}{2^a} = 2^{-a}$

$$\begin{aligned} 4^2 &= 2^{-a} \times 8^a \\ 2^{2\times 2} &= 2^{-a} \times 2^{3a} \\ 4 &= 2a \\ a &= 2 \end{aligned}$$ **[1 mark]**

Substitute $a = 2$ into $b^a = 2^{-a}$ to find b.

$$\begin{aligned} b^2 &= 2^{-2} \\ \left(b^2\right)^{\frac{1}{2}} &= \left(2^{-2}\right)^{\frac{1}{2}} \\ b &= 2^{-1} \\ &= \frac{1}{2} \end{aligned}$$ **[1 mark]**

11 Introduction to differential calculus

Topic	11	Introduction to differential calculus
Subtopic	11.2	Rates of change

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

A gardener observed that, during spring, their tomato bush grew from 5 cm to 40 cm over a 2-week period. They estimated that the average daily growth of the tomato bush was

A. 2 cm/day
B. 35 cm/day
C. 2.5 cm/day
D. 4.3 cm/day
E. 20 cm/day

Question 2 (6 marks)

A small business calculated that its yearly profit followed the equation Profit $p(t) = 200\,(10t - t^2)$, where t is in years.

a. Sketch the graph of profit against time for the first 10 years. **(3 marks)**

b. Calculate the average rate of change of the profit over the first 3 years and compare it with the rate of change of profit from years 6 to 9. Comment on the different rates of change. **(3 marks)**

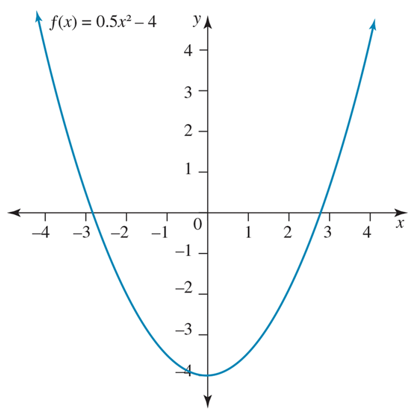
Question 3 (1 mark)

The average rate of change of the function $f(x) = 3 - x^2$ between $x = 1$ and $x = 5$ is

A. 3
B. -3
C. 6
D. -6
E. 5

Question 4 (3 marks)

For the curve below with equation $y = 0.5x^2 - 4$, estimate the gradient of the curve at point P(2, −2) by choosing a point on the curve close to P and calculating the average rate of change between P and this point.



Question 5 (1 mark)

Yumi is a keen gardener and she observed that during spring that her corn grew from 4 cm to 36 cm over an 8-week period. She estimated that the average weekly growth of the corn was

A. 8 cm/week
B. 2 cm/week
C. 4.5 cm/week
D. 4 cm/week
E. 6 cm/week

Topic	11	Introduction to differential calculus
Subtopic	11.3	Gradients of secants

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

A secant is a line that

A. touches a curve at a point.
B. passes through two points on a curve.
C. is a tangent to the curve.
D. is parallel to the curve.
E. is at right angles to the curve.

Question 2 (4 marks)

a. For the curve with the equation $y = x^2 - 4x$, express the gradient of the secant through the points on the curve where $x = 6$ and $x = 6 + h$ in terms of h. **(2 marks)**

b. Use $h = 0.01$ to obtain an estimate of the gradient of the tangent to the curve at $x = 6$. **(1 mark)**

c. Deduce the gradient of the tangent to the curve at the point where $x = 6$. **(1 mark)**

Question 3 (1 mark)

The gradient of the secant passing through the points on the graph $y = (x - 3)(x + 1)(x - 4)$ with x-coordinates 0 and h, where $h \neq 0$, is

A. $h^2 - 6h + 5$
B. $h^2 + 6h + 5$
C. $h^2 - 5h + 6$
D. $h^2 + 5h + 6$
E. $2h^2 - 5h + 6$

Question 4 (1 mark)

A tangent is a line that

A. passes through two points on a curve.
B. is parallel to the curve.
C. is at right angles to the curve.
D. runs perpendicular to the curve.
E. cuts through the curve only once.

Question 5 (1 mark)

The gradient of the secant passing through the points on the graph $y = (x - 2)(x + 4)$ with x-coordinates 0 and h, $h \neq 0$, is

A. $3h$
B. $h + 3$
C. $h + 4$
D. $2h + 2$
E. $h + 2$

Topic	11	Introduction to differential calculus
Subtopic	11.4	The derivative function

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

The gradient of the tangent to the curve $y = f(x)$ at the point where $x = 3$ is

A. $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

B. $\dfrac{f(x+h) - f(x)}{h}$

C. $\lim\limits_{h \to 0} \dfrac{f(3+h) - f(3)}{3}$

D. $\lim\limits_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$

E. $\dfrac{f(3+h) - f(3)}{h}$

Question 2 (4 marks)

Use the limit definition the derivative of a function to differentiate $3x^3 - 2x$ with respect to x.

Question 3 (3 marks)

Find $\frac{dy}{dx}$ using the central difference approximation for the function $y = 2 - x^2$.

Question 4 (1 mark)

The gradient of the tangent to the curve $y = f(x)$ at the point where $x = -2$ is

A. $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

B. $\dfrac{f(-2+h) - f(-2)}{h}$

C. $\lim\limits_{h \to 0} \dfrac{f(-2+h) - f(-2)}{h}$

D. $\dfrac{f(x+h) - f(x)}{h}$

E. $\dfrac{f(2+h) - f(2)}{h}$

Topic	11	Introduction to differential calculus
Subtopic	11.5	Differentiation of polynomials by rule

online only

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Question 1 (1 mark)

If $f(x) = 2x^2 - 3x + 5$, then $f'(2)$ is

A. -3
B. 2
C. 5
D. 4
E. -4

Question 2 (1 mark)

The coordinates of the point on the curve $f(x) = 3x^2 - 12x + 8$ where the gradient is 0 are

A. $(3, 8)$
B. $(2, -4)$
C. $(3, -12)$
D. $(6, 12)$
E. $(0, 0)$

Question 3 (2 marks)

Sam inflates a spherical balloon. If the volume of a sphere is $\frac{4\pi r^3}{3}$ where r is the radius, find the rate the volume is increasing when the radius is 15 cm.

Question 4 (1 mark)

The derivative of $\dfrac{x^3 - 2x + 18}{4}$ is

A. $\dfrac{1}{4}(3x^2 - 2)$

B. $\dfrac{1}{4}3x^2 - 2$

C. $3x^2 - 2$

D. $\dfrac{1}{4}(3x^2 - 4)$

E. $\dfrac{1}{4}(4x^2 + 3)$

Question 5 (1 mark)

If $f(x) = -x^2 + 4x + 1$, then $f'(-3)$ is

A. 2

B. -2

C. 5

D. -20

E. 10

Topic	11	Introduction to differential calculus
Subtopic	11.6	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)
The point D lies on the curve with equation $y = x^3 - x$. An expression for the y-coordinate of D, given that its x-coordinate is $3 + h$, is
A. $h^3 - h$
B. $h^3 + 9h^2 + 28h + 30$
C. $h^3 + 9h^2 + 26h + 24$
D. $h^3 + 9h^2 + 26h$
E. $h^3 + 9h^2 + 24h$

Question 2 (4 marks)
Calculate the gradient of the chord joining the points $x = 2.9$ and $x = 3$ on the curve with equation $y = \frac{x^3}{3} + 1$ and estimate the gradient of the tangent to the curve at the point where $x = 3$. Explain how you could improve your estimate.

Question 3 (1 mark)
The gradient of a secant for the function $f(x) = 2x^2 - 4$ is given by
A. $2h^2 - 4$
B. $2(x + h)^2 - 4$
C. $2h^2 - x^2$
D. $2(2x + h)$
E. $4(x + h) - 2$

Question 4 (1 mark)

The derivative of $\dfrac{4x^3 - 3x + 15}{3}$ is

A. $4x^2 - 1$

B. $\dfrac{4}{3}x^3 - x + 5$

C. $12x - 3$

D. $-3x + 15$

E. $3x^2 - 3$

Question 5 (4 marks)

Sketch, on the same set of axes, the graphs of $y = f(x)$ and $y = f'(x)$ given that $f(x) = 2 - x^2$.

Question 6 (1 mark)

The average rate of change of the function $f(x) = 3x^2 - 4$ between $x = 0$ and $x = 4$ is

A. 12
B. 16
C. 4
D. 8
E. 2

Question 7 (1 mark)

The coordinates of the point on the curve $f(x) = -2x^2 + 6x - 3$ where the gradient is 0 are

A. $\left(\frac{2}{3}, \frac{3}{2}\right)$
B. $\left(\frac{3}{2}, \frac{3}{2}\right)$
C. $\left(\frac{2}{3}, \frac{17}{9}\right)$
D. $(2, 1)$
E. $\left(\frac{3}{2}, \frac{17}{9}\right)$

Question 8 (1 mark)

Kari deflates a spherical balloon. If the volume of a sphere is $\frac{4\pi r^3}{3}$ where r is the radius, the rate the volume is changing when the radius is 8 cm is

A. 256π cm^3/cm
B. 64π cm^3/cm
C. -256π cm^3/cm
D. -64π cm^3/cm
E. -84π cm^3/cm

Question 9 (4 marks)

Answer the following.

a. For the curve with the equation $y = 2x^2 - 3x$ express the gradient of the secant through the points on the curve where $x = 4$ and $x = 4 + h$ in terms of h. **(2 marks)**

b. Use $h = 0.01$ to obtain an estimate of the gradient of the tangent to the curve at $x = 4$. **(1 mark)**

c. Deduce the gradient of the tangent to the curve at the point where $x = 4$. **(1 mark)**

Answers and marking guide

11.2 Rates of change

Question 1

$$\text{Average rate of growth} = \frac{\text{change in growth (cm)}}{\text{time (days)}}$$
$$= \frac{40-5}{14}$$
$$= 2.5\ \text{cm/day}$$

The correct answer is **C**.

Question 2

$p = 200t\,(10-t)$

a. p-intercept, $t = 0$

$p = 0 \Rightarrow (0, 0)$

t-intercept, $p = 0$

$0 = 200t\,(10-t)$

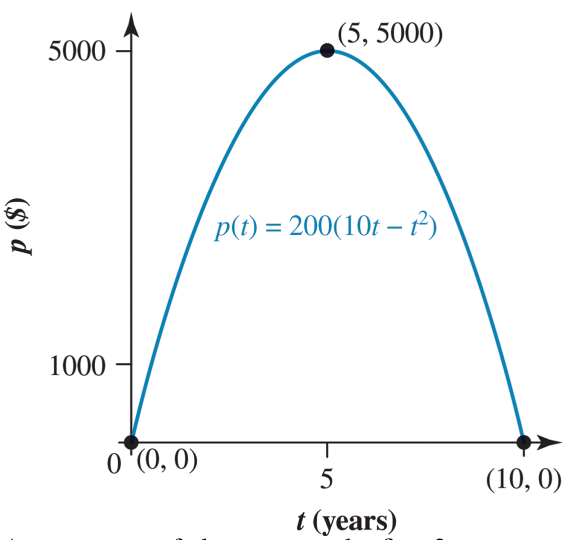
$p = 0, 10 \Rightarrow (0, 0), (10, 0)$ **[1 mark]**

$\therefore$ T.P, $t = 5$

$\therefore p = 200 \times 5\,(10-5)$

$= 5000 \Rightarrow (5.5000)$ **[1 mark]**

[1 mark]



b. Average rate of change over the first 3 years

$$= \frac{p(3) - p(0)}{3-0}$$
$$= \frac{200 \times 21 - 0}{3}$$
$= 1400$ **[1 mark]**

Average rate of change years 6 to 9

$$= \frac{p(9) - p(6)}{9-6}$$
$$= \frac{200 \times (90-81) - 200\,(60-36)}{3}$$
$$= \frac{1800 - 4800}{3}$$
$= -1000$ **[1 mark]**

In the first 3 years, the rate of change of profit was positive (\$1400 per year) but, during years 6 to 9, the rate of change of profit was in decline at −\$1000 per year. **[1 mark]**

Question 3

Average rate of change is calculated by:

$$\frac{f(b)-f(a)}{b-a} = \frac{f(5)-f(1)}{5-1}$$
$$= \frac{3-25-(3-1)}{4}$$
$$= -\frac{24}{4}$$
$$= -6$$

The correct answer is **D**.

Question 4

Let $x = 1.9$ be a point close to $(2, -2)$.

$y = 0.5x^2 - 4$

$y = 0.5(1.9)^2 - 4$

$y = -2.195$

$\therefore (1.9, -2.195)$ **[1 mark]**

Points $(1.9, -2.195)$ and $(2, -2)$

$$\text{Average rate of change} = \frac{-2-(-2.195)}{2-1.9}$$
$$= \frac{0.195}{0.1}$$
$$= 1.95$$ **[1 mark]**

The gradient of the curve at point P is approximately equal to 1.95. **[1 mark]**
(Answers may vary depending on point chosen).

Question 5

$$\text{Average rate of growth} = \frac{\text{change in growth (cm)}}{\text{time (weeks)}}$$
$$= \frac{36-4}{8}$$
$$= 4 \text{ cm/week}$$

11.3 Gradients of secants

Question 1

The correct answer is **B**.

Question 2

a. $y = x^2 - 4x$

Point $x = 6$

$y = 36 - 24$

$= 12$

$\Rightarrow (6, 12)$

Point $x = 6 + h$

$$y = (6+h)^2 - 4(6+h)$$
$$y = 36 + 12h + h^2 - 24 - 4h$$
$$y = h^2 + 8h + 12$$
$$\Rightarrow \left(6+h,\ h^2+8h+12\right)$$ **[1 mark]**

$$\text{Gradient: } \frac{y_2 - y_1}{x_2 - x_1} = \frac{h^2 + 8h + 12 - 12}{6 + h - 6}$$
$$= \frac{h^2 + 8h}{h}$$
$$= \frac{\not{h}(h+8)}{\not{h}}$$
$$= h + 8,\ h \neq 0$$ **[1 mark]**

b. When $h = 0.01$

Gradient $= h + 8$

$= 8.01$ **[1 mark]**

c. The gradient of the tangent to the curve at the point where $x = 6$ is 8. **[1 mark]**

Question 3

Point $x = 0$

$y = (x-3)(x+1)(x-4)$

$y = (-3)(1)(-4)$

$y = 12$

$\therefore (0,\ 12)$

Point $x = h$

$y = (h-3)(h+1)(h-4)$

$y = \left(h^2 - 2h - 3\right)(h-4)$

$y = h^3 - 4h^2 - 2h^2 + 8h - 3h + 12$

$y = h^3 - 6h^2 + 5h + 12$

$\therefore$ point $\left(h,\ h^3 - 6h^2 + 5h + 12\right)$ and $(0,\ 12)$

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{h^3 - 6h^2 + 5h + 12 - 12}{h - 0}$$
$$= \frac{h^3 - 6h^2 + 5h}{h}$$
$$= \frac{\not{h}\left(h^2 - 6h + 5\right)}{\not{h}},\ h \neq 0$$
$$= h^2 - 6h + 5$$

The correct answer is **A**.

Question 4

A tangent is a line that runs parallel to the curve. It just touches the curve at this point as well.

Question 5

Point $x = 0$

$y = (x-2)(x+4)$

$= (-2)(4)$

$= -8$

$(0, -8)$

Points $x = h$

$y = (h-2)(h+4)$

$= h^2 + 2h - 8$

$(h, h^2 + 2h - 8)$

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{h^2 + 2h - 8 - (-8)}{h - 0}$$

$$= \frac{h^2 + 2h}{h}$$

$$= \frac{h(h+2)}{h}$$

$$= h + 2$$

11.4 The derivative function

Question 1

For the function $y = f(x)$, its gradient function is defined as the limiting value of the difference quotient as $h \to 0$.

$$\text{Gradient} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

At $x = 3$

$$\text{Gradient} = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

The correct answer is **D**.

Question 2

Let $f(x) = 3x^3 - 2x$

$f(x+h) = 3(x+h)^3 - 2(x+h)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ **[1 mark]**

$$= \lim_{h \to 0} \frac{3(x+h)^3 - 2(x+h) - [3x^3 - 2x]}{h}$$

$$= \lim_{h \to 0} \frac{3(x^3 + 3x^2h + 3xh^2 + h^3) - 2(x+h) - [3x^3 - 2x]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^3} + 9x^2h + 9xh^2 + 3h^3 \cancel{-2x} - 2h - \cancel{3x^3} + \cancel{2x}}{h}$$

$$= \lim_{h \to 0} \frac{9x^2h + 9xh^2 + 3h^3 - 2h}{h}$$ **[1 mark]**

$$= \lim_{h \to 0} \frac{\cancel{h}(9x^2 + 9xh + 3h^2 - 2), (h \neq 0)}{\cancel{h}}$$ **[1 mark]**

$$= \lim_{h \to 0} 9x^2 + 9xh + 3h^2 - 2$$

Substitute $h = 0$

$f'(x) = 9x^2 - 2$ **[1 mark]**

Question 3

$y = 2 - x^2$

Using the central difference approximation,

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

$$f'(x) \approx \frac{[2-(x+h)^2] - [2-(x-h)^2]}{2h} \quad \textbf{[1 mark]}$$

$$\approx \frac{[2 - x^2 - 2xh - h^2] - [2 - x^2 + 2xh - h^2]}{2h} \quad \textbf{[1 mark]}$$

$$\approx \frac{2 - x^2 - 2xh - h^2 - 2 + x^2 - 2xh + h^2}{2h}$$

$$\approx \frac{-4xh}{2h}$$

$$\approx -2x \quad \textbf{[1 mark]}$$

Question 4

For the function $y = f(x)$, its gradient function is defined as the limiting value of the difference quotient as $h \to 0$.

$$\text{Gradient function} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

At $x = -2$

$$\text{Gradient} = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h}$$

11.5 Differentiation of polynomials by rule

Question 1

$f(x) = 2x^2 - 3x + 5$

$f'(x) = 4x - 3$

$f'(2) = 4(2) - 3$

$= 5$

The correct answer is **C**.

Question 2

$$f(x) = 3x^2 - 12x + 8$$

$$f'(x) = 6x - 12$$

$$0 = 6x - 12$$

$$6x = 12$$

Zero gradient at $x = 2$

$$f(x) = 3x^2 - 12(2) + 8$$

$$f(2) = 3(2)^2 - 12(2) + 8$$

$$= 12 - 24 + 8$$

$$= -4$$

$$\therefore (2, -4)$$

The correct answer is **B**.

Question 3

Let volume be V

$$V = \frac{4\pi r^3}{3}$$

$$\frac{dV}{dr} = 4\pi r^2 \quad \textbf{[1 mark]}$$

$$r = 15$$

$$\frac{dV}{dr} = 4\pi \times 15^2$$

$$\frac{dV}{dr} = 900\pi \text{ cm}^3/\text{cm} \quad \textbf{[1 mark]}$$

Question 4

$$f(x) = \frac{x^3 - 2x + 18}{4}$$

$$f'(x) = \frac{1}{4}(3x^2 - 2)$$

Question 5

$$\begin{aligned} f(x) &= x^2 + 4x + 1 \\ f'(x) &= -2x + 4 \\ f'(-3) &= -2(-3) + 4 \\ &= 10 \end{aligned}$$

11.6 Review

Question 1

$y = x^3 - x$

Point D

$x = 3 + h$

$y = (3 + h)^3 - (3 + h)$

$y = 3^3 + 3(3)^2 h + 3(3)h^2 + h^3 - (3 + h)$

$y = 27 + 27h + 9h^2 + h^3 - 3 - h$

$y = h^3 + 9h^2 + 26h + 24$

The correct answer is **C**.

Question 2

At point $x = 2.9$,

$$\begin{aligned} y &= \frac{x^3}{3} + 1 \\ &= \frac{(2.9)^3}{3} + 1 \\ &= 9.1297 \end{aligned}$$

$$\therefore (2.9, 19.1297) \quad \textbf{[1 mark]}$$

At point $x = 3$,

$$\begin{aligned} y &= \frac{3^3}{3} + 1 \\ &= 10 \end{aligned}$$

$$\therefore (3, 10)$$

$$\text{Gradient of chord} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{10 - 9.1297}{3 - 2.9}$$
$$= 8.703$$ **[1 mark]**

Estimated gradient of the tangent to the curve = 9 **[1 mark]**

Improve the estimate by using a point closer to 3 such as $x = 2.99$. **[1 mark]**

Question 3

The gradient of a secant or the average rate of change of the function between the two points is calculated using

$$\frac{f(x+h) - f(x)}{h} = \frac{2(x+h)^2 - 4 - \left(2x^2 - 4\right)}{h}$$
$$= \frac{2x^2 + 4xh + 2h^2 - 4 - 2x^2 + 4}{h}$$
$$= \frac{4xh + 2h^2}{h}$$
$$= \frac{\cancel{h}(4x + 2h)}{\cancel{h}}, \; h \neq 0$$
$$= 2(2x + h)$$

The correct answer is **D**.

Question 4

$$f(x) = \frac{4x^3 - 3x + 15}{3}$$
$$f'(x) = \frac{12x^2 - 3}{3}$$
$$= 4x^2 - 1$$

The correct answer is **A**.

Question 5

$f(x) = 2 - x^2$

$f'(x) = -2x$ **[1 mark]**

Graph $y = 2 - x^2$ and $y = -2x$.

For $y = 2 - x^2$:

x-intercepts ($y = 0$):

$$0 = 2 - x^2$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$
$$\therefore \left(-\sqrt{2}, 0\right), \left(\sqrt{2}, 0\right)$$

y-intercepts ($x = 0$):

$$y = 2$$
$$\therefore (0, 2)$$ **[1 mark]**

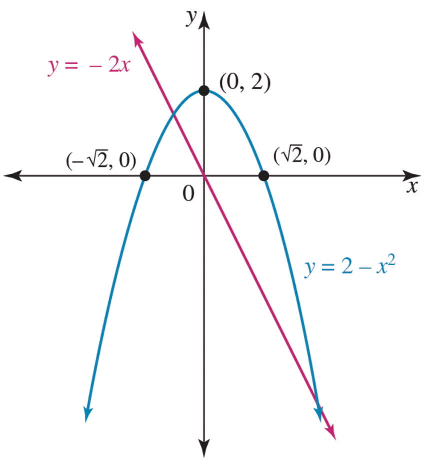
For $y = -2x$:

x- and y-intercepts:

$(0, 0)$

Other point:

$(1, -2)$ **[1 mark]**



[1 mark]

Question 6

Average rate of change is calculated by $\dfrac{f(b)-f(a)}{b-a}$.

$$\begin{aligned}\frac{f(4)-f(0)}{4-0} &= \frac{3\times 16-4-(0-4)}{4}\\ &= \frac{48}{4}\\ &= 12\end{aligned}$$

Question 7

$$\begin{aligned}f(x) &= -2x^2+6x-3\\ f'(x) &= -4x+6\\ 0 &= -4x+6\\ 4x &= 6\\ x &= \frac{3}{2}\end{aligned}$$

$$\begin{aligned}f\left(\frac{3}{2}\right) &= -2\left(\frac{3}{2}\right)^2+6\left(\frac{3}{2}\right)-3\\ &= -\frac{9}{2}+9-3\\ &= \frac{3}{2}\end{aligned}$$

$$\therefore \left(\frac{3}{2},\frac{3}{2}\right)$$

Question 8

Let volume be V.

$$V=\frac{4\pi r^3}{3}$$

$\dfrac{dV}{dr} = -4\pi r^2$ (Need to add the negative sign as the balloon is decreasing in size.)

$$r = 8$$

$$\frac{dV}{dr} = -4\pi\times 8^2$$

$$\frac{dV}{dr} = -256\pi \text{ cm}^3/\text{cm}$$

Question 9

a. $y = 2x^2 - 3x$

Point $x = 4$

$= 2 \times 16 - 3 \times 4$

$= 20$

$(4, 20)$

Point $x = 4 + h$

$y = 2(4 + h)^2 - 3(4 + h)$

$= 2\left(16 + 8h + h^2\right) - 12 - 3h$

$= 32 + 16h + 2h^2 - 12 - 3h$

$= 20 + 13h + 2h^2$

$\left(4 + h, 2h^2 + 13h + 20\right)$ **[1 mark]**

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2h^2 + 13h + 20 - 20}{4 + h - 4}$$

$$= \frac{2h^2 + 13h}{h}$$

$$= \frac{h(2h + 13)}{h}$$

$$= 2h + 13$$

Gradient $= 2h + 13, h \neq 0$ **[1 mark]**

b. When $h = 0.01$

Gradient $= 13.02$ **[1 mark]**

c. The gradient of the tangent to the curve at the point where $x = 4$ is 13. **[1 mark]**

12 Differentiation and applications

Topic	12	Differentiation and applications
Subtopic	12.2	Limits, continuity and differentiability

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

Evaluate $\lim_{x \to 3} f(x)$ for the function $f(x) = \dfrac{x^2 - 9}{x - 3}, x \neq 3$.

A. 3
B. 0
C. 6
D. 9
E. Does not exist

Question 2 (1 mark)

The value of v such that $y = \begin{cases} x + 2, & x < 1 \\ v - x, & x \geq 1 \end{cases}$ is a continuous function is

A. 4
B. 0
C. 1
D. 2
E. undefined.

Question 3 (3 marks)

Determine the values of a and b such that $f(x) = \begin{cases} ax^2 + 4, & x \leq 1 \\ 6x + b, & x > 1 \end{cases}$ is smooth and continuous at $x = 1$.

Question 4 (1 mark)

What is the limit for $\lim_{x \to -2} (4x + 5)$?

A. 4
B. 13
C. 5
D. -3
E. Does not exist

Question 5 (1 mark)

What is the limit for $\lim_{x \to -2} \left(\frac{1}{x + 2}\right)$?

A. -2
B. 2
C. 0
D. 4
E. Does not exist

Question 6 (1 mark)

At $x = 2$, the function $f(x) = \begin{cases} (x-2)^2, & x < 2 \\ x - 2, & x \geq 2 \end{cases}$ is

A. continuous.
B. discontinuous at $x = 2$.
C. discontinuous at $x = -2$.
D. discontinuous at $x = 0$.
E. undefined.

Question 7 (1 mark)

Given $f(x) = \begin{cases} 2x, & x \leq 1 \\ 2x^2, & x > 1 \end{cases}$ at $x = 1$, the function is

A. continuous and differentiable.
B. not continuous but differentiable.
C. continuous but not differentiable.
D. not continuous and not differentiable.
E. undefined.

Question 8 (1 mark)

The value of $\lim_{x \to 3} \left(\frac{x^3 - 27}{x - 3} \right)$ is

A. 27
B. 0
C. undefined
D. 9
E. 3

Topic	12	Differentiation and applications
Subtopic	12.3	Coordinate geometry applications of differentiation

online only

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Question 1 (1 mark)

The gradient of the line perpendicular to the tangent $(2, 3)$ on the curve $y = 3 + 7x - 2x^2$ is

A. 1
B. 2
C. 3
D. -1
E. -2

Question 2 (4 marks)

Calculate the coordinates of the point of intersection of the tangents drawn to the x-intercepts of the curve $y = x(6 - x)$.

Question 3 (3 marks)

Determine the domain interval over which the function $f(x) = \frac{1}{3}x^3 + 5x^2 + 24x - 16$ is decreasing.

Question 4 (1 mark)

Which of these statements about tangents is *not* true?

A. The tangent is a straight line.
B. Tangents that are parallel have the same gradient.
C. The gradient of a horizontal tangent is 0.
D. $m = \tan(\theta)$ provides the angle of inclination of the tangent to the horizontal.
E. The gradient of a vertical tangent is 1.

Question 5 (1 mark)

The gradient of the tangent at the point (2, 3) on the curve $y = 3x^2 - 9$ is

A. 3
B. 12
C. 18
D. 9
E. 0

Topic	12	Differentiation and applications
Subtopic	12.4	Curve sketching

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Question 1 (4 marks)

The point $(2, -16)$ is a stationary point on the curve $y = ax^2 + bx$. Calculate the values of a and b, and sketch the curve.

Question 2 (3 marks)

A polynomial with equation $y = f(x)$ has the following properties:

- $f(x)$ has a y-intercept at $(0, -1)$ and x-intercepts at $(-4, 0)$ and $(-1, 0)$.
- $f(x)$ is a decreasing function for $\{x : x < -3\} \cup \{x : x > -1\}$.
- $f'(-3) = 0$ and $f'(-1) = 0$.

Sketch a possible shape of the graph $f(x)$.

Question 3 (5 marks)

Consider the function $f(x) = -x^3 + 3x^2 - 4$ for $-2 \leq x \leq 3$.

a. Find the exact coordinates of any stationary points. **(3 marks)**

b. Sketch the graph of $y = -x^3 + 3x^2 - 4$ for $x \in [-2, 3]$ given that an x-intercept occurs at $(-1, 0)$. **(1 mark)**

c. State the global minimum and global maximum values. **(1 mark)**

Question 4 (1 mark)

A stationary point occurs only when

A. there is a local maximum or minimum.

B. there is a point of inflection.

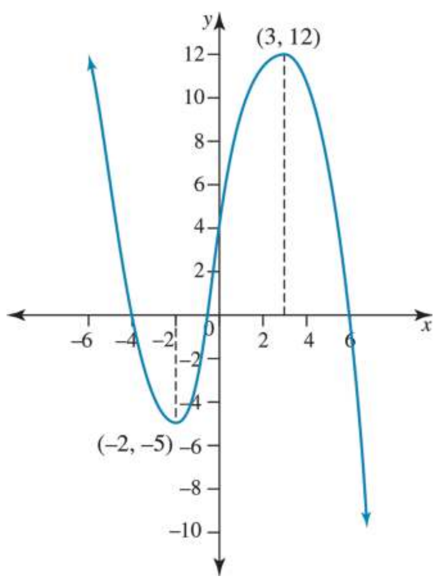
C. the gradient to the left of the point is the same as the gradient to the right of the point.

D. $f(x) = 0$

E. $f'(x) = 0$

Question 5 (1 mark)

Which of the following is an incorrect statement about the curve shown?



A. The curve is increasing at the point where it cuts the y-axis.

B. The maximum value of the function is 12.

C. The curve intersects the x-axis in three places.

D. The curve displays two points of zero gradient.

E. The function is a cubic polynomial.

Question 6 (5 marks)

Find the stationary points for $y = 2x^3 - 3x^2 - 12x + 18$, and justify their nature.

Question 7 (1 mark)

The graph of the function whose rule is $f(x) = 2x^3 + 5x^2 - 2$ has stationary points where x is

A. 0 and $-\frac{5}{3}$.

B. 2 and $-\frac{5}{3}$.

C. 2 and $\frac{5}{3}$.

D. 0 and $-\frac{3}{5}$.

E. 0 and 2.

Topic	12	Differentiation and applications
Subtopic	12.5	Optimisation problems

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Question 1 (1 mark)

The total surface area of a closed cylinder is $100\,\text{cm}^2$, where the base radius is r cm and the height is h cm. An expression for h in terms of r is

A. $h = \dfrac{50 - \pi r^2}{\pi r}$

B. $h = 100r$

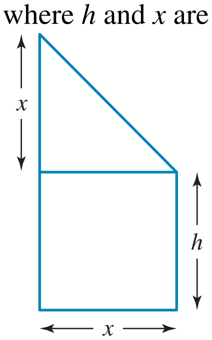
C. $h = \dfrac{100 - \pi r^2}{\pi}$

D. $h = \dfrac{50r - \pi r^2}{\pi}$

E. $h = 100\pi r^2$

Question 2 (1 mark)

The shape shown has a perimeter of 200 cm, where h and x are lengths of the sides.



The area of this shape in terms of x is

A. $50x - x^2\left(\dfrac{1+\sqrt{2}}{5}\right)$

B. $x^2\left(\dfrac{1-\sqrt{2}}{2}\right)$

C. $100x - x^2\left(\dfrac{1+\sqrt{2}}{2}\right)$

D. $100x - x^2\sqrt{2}$

E. $50x + x^2\left(\dfrac{3+\sqrt{2}}{2}\right)$

Topic 12 | Subtopic 12.5 Optimisation problems

Question 3 (6 marks)

A rectangular box with an open top is to be constructed from a rectangular sheet of cardboard measuring 10 cm by 10 cm. Equal squares of side length x cm are cut out of the four corners and the flaps are folded up to form the box.

a. Express the volume of the box as a function of x. **(2 marks)**

b. Determine the dimensions of the box with the greatest volume, and determine this maximum volume to the nearest whole number. **(4 marks)**

Question 4 (1 mark)

The profit ($) of a small business can be modelled by $p = 1000n(4 - n)$, where r is the number of sales staff selling the products. The number of staff needed to maximise the profit is

A. 2
B. 4
C. 6
D. 8
E. 10

Question 5 (4 marks)

Sue decides to build a rectangular vegetable garden bed using the corner formed by the back and side fences. There is 20 metres of fencing available for enclosing the other two sides of the garden bed. Draw a diagram of the garden bed, and express the area in terms of x and y (the lengths of the two fenced sides). Use calculus to obtain the dimensions of the garden for maximum area, and state the maximum area.

Topic	12	Differentiation and applications
Subtopic	12.6	Rates of change and kinematics

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (5 marks)

A particle moves in a straight line such that its position, x metres, from a fixed origin at time t seconds is modelled by $x = 2t^2 - 8t + 9, t \geq 0$.

a. Find its initial velocity and the time and position that the particle is momentarily at rest. **(3 marks)**

b. Calculate the average velocity after the first 3 seconds. **(2 marks)**

Question 2 (1 mark)

A particle moves in a straight line such that its position, x metres, from a fixed origin at time t seconds is given by $x = 2t^2 - 15t + 23, t \geq 0$. Its speed after it has travelled for 3 seconds is

A. 6 m/s
B. −6 m/s
C. 3 m/s
D. −3 m/s
E. 0 m/s

Question 3 (1 mark)
Oil dropped into water spreads in a circular shape on the surface of the water. When the radius of the spill is 3 metres, the rate of change of the area with respect to the radius is
A. $\pi\ \text{cm}^2/\text{cm}$
B. $2\pi\ \text{cm}^2/\text{cm}$
C. $4\pi\ \text{cm}^2/\text{cm}$
D. $6\pi\ \text{cm}^2/\text{cm}$
E. $9\pi\ \text{cm}^2/\text{cm}$

Question 4 (1 mark)
A particle moves in a straight line such that its displacement, x metres, from a fixed origin at time t seconds is $x = 5t^2 - 2t + 3, t \geq 0$. Its initial position is
A. 5
B. -2
C. 3
D. 0.2
E. 0

Topic	12	Differentiation and applications
Subtopic	12.7	Derivatives of power functions (extending)

online only

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Question 1 (1 mark)

If $y = 4x\sqrt{x}$, $\dfrac{dy}{dx}$ is

A. $6\sqrt{x}$

B. $4\sqrt{x}$

C. $\dfrac{6}{\sqrt{x}}$

D. $\dfrac{4}{\sqrt{x}}$

E. $2x$

Question 2 (4 marks)

A function f is defined as $f: [0, \infty] \to R, f(x) = 3 - 2\sqrt{x}$.

a. Define the derivative function f'. **(1 mark)**

b. Obtain the gradient of the graph of $y = f(x)$ as the graph cuts the x-axis. **(3 marks)**

Question 3 (4 marks)

Consider $f(x) = 2x + \frac{1}{3x}$.

a. State the domain of the function. **(1 mark)**

b. Form the rule for its gradient function, stating its domain. **(1 mark)**

c. Find the coordinates of the points on the curve where the tangent has a gradient of -1. **(2 marks)**

Question 4 (1 mark)

If $y = \frac{x^4 + 3x - 6}{3x^2}$, $\frac{dy}{dx}$ is

A. $\frac{x^3}{3} + x^{-2} - 3x^{-3}$

B. $x - 2x^{-2} + x^{-3}$

C. $\frac{2x}{3} - x^{-2} + 4x^{-3}$

D. $\frac{2x}{5} - x^{-3} + 4x^{-3}$

E. $\frac{2x}{5} + 2x^{-2} + 2x^{-3}$

Question 5 (1 mark)

If $f(x) = 6\sqrt{x}$, $\dfrac{dy}{dx}$ is

A. $\dfrac{3}{\sqrt{4x}}$

B. $\dfrac{3}{\sqrt{x}}$

C. $3\sqrt{x}$

D. $\dfrac{3\sqrt{x}}{2}$

E. $12x$

Question 6 (1 mark)

If $y = \dfrac{x^4 - 4x^2 + 8x}{2x^3}$, $\dfrac{dy}{dx}$ is

A. $\dfrac{x}{2} + \dfrac{2}{x^2} - \dfrac{8}{x^3}$

B. $\dfrac{1}{2} + \dfrac{2}{x^2} - \dfrac{8}{x^3}$

C. $\dfrac{1}{2} - \dfrac{2}{x^2} + \dfrac{8}{x^3}$

D. $\dfrac{1}{2} + \dfrac{2}{x^2} - \dfrac{8}{x}$

E. $\dfrac{1}{2} + \dfrac{2}{x} - \dfrac{8}{x^2}$

Topic	12	Differentiation and applications
Subtopic	12.8	Review

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

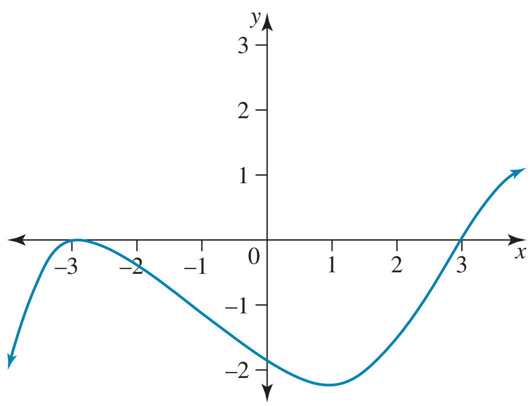
Question 1 (1 mark)

Given $f(x) = \begin{cases} x^2 - 2, & x \leq 1 \\ 2x - 3, & x > 1 \end{cases}$ at $x = 1$, the function is

A. continuous and differentiable.
B. not continuous but differentiable.
C. continuous but not differentiable.
D. not continuous and not differentiable.
E. undefined.

Question 2 (1 mark)

For the graph shown, the gradient is positive for



A. $x \in (-\infty, 3)$
B. $x \in (-3, 3)$
C. $x \in (-\infty, -3) \cup (1, \infty)$
D. $x \in (-\infty, -3) \cup (3, \infty)$
E. $x \in (-3, 1)$

Question 3 (1 mark)

If $f'(x) < 0$ where $x < 3$, $f'(x) = 0$ at $x = 3$ and $f'(x) < 0$ when $x > 3$, then at point $x = 3$, $f(x)$ has

A. a gradient of 3.

B. a negative gradient for all x.

C. a discontinuity at $x = 3$.

D. a point of inflection at $x = 3$.

E. a turning point at $x = 3$.

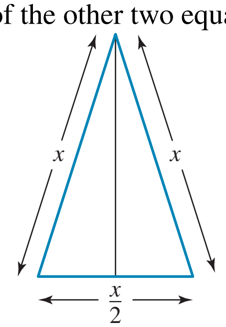
Question 4 (4 marks)

Morgan decides to build a rectangular vegetable garden bed using the corner formed by the back and side fences. There is 20 metres of fencing available for enclosing the other two sides of the garden bed.

Draw a diagram of the garden bed, and express the area in terms of x and y (the lengths of the two fenced sides). Use calculus to obtain the dimensions of the garden for maximum area, and state the maximum area.

Question 5 (3 marks)

A triangle has a base that is half the length of the other two equal sides (cm) as shown.



a. Express the area of the triangle in terms of x. **(2 marks)**

b. Determine the rate with respect to x at which the area is changing when $x = 8$ cm. **(1 mark)**

Question 6 (3 marks)

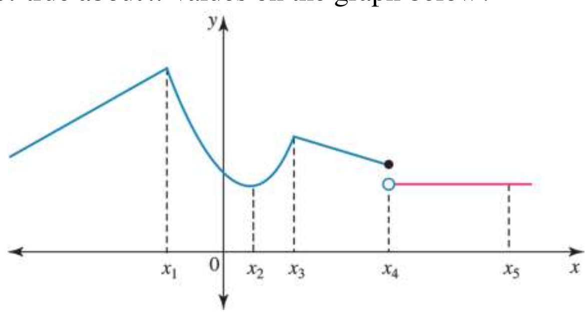
Calculate $\lim_{x \to 4} \left(\frac{x^3 - 64}{x - 4} \right)$.

Question 7 (4 marks)

Determine the values of a and b such that $y = \begin{cases} ax + b, & x < -1 \\ 6, & -1 \le x \le 4 \\ bx + 2a, & x < 4 \end{cases}$ is a continuous function.

Question 8 (1 mark)

Which of the statements is *not* true about x-values on the graph below?



A. Continuous at x_1
B. Differentiable at x_2
C. Not differentiable at x_3
D. Not continuous at x_4
E. Not differentiable at x_5

Question 9 (5 marks)

Consider the function defined by the rule $f(x) = \begin{cases} 4 - 2x, & x < 0 \\ 4 - x^2, & x \geq 0 \end{cases}$.

a. Determine whether the function is differentiable at $x = 0$. **(2 marks)**

b. Sketch the graph of $y = f(x)$. **(1 mark)**

c. Form the rule for $f'(x)$ and state its domain. **(1 mark)**

d. Calculate $f'(5)$. **(1 mark)**

Question 10 (1 mark)

The value of v such that $y = \begin{cases} x + 4, & x < 2 \\ 2v - x, & x \geq 2 \end{cases}$ is a continuous function is

A. 2
B. −4
C. 4
D. −2
E. −8

Question 11 (1 mark)

Given $f(x) = \begin{cases} -x, & x \le -1 \\ 2x^2 - 1, & x > -1 \end{cases}$ at $x = -1$, the function is

A. continuous and differentiable.
B. not continuous and not differentiable.
C. not continuous but differentiable.
D. continuous but not differentiable.
E. smooth and differentiable.

Question 12 (5 marks)

A rectangular box with an open top is to be constructed from a rectangular sheet of cardboard measuring 15 cm by 15 cm. Equal squares of side length x cm are cut out of the four corners and the flaps folded up to form the box.

Express the volume of the box as a function of x. Determine the dimensions of the box with the greatest volume, and determine this maximum volume.

Answers and marking guide

12.2 Limits, continuity and differentiability

Question 1

$$f(x) = \frac{x^2 - 9}{x - 3}, x \neq 3$$

$$f(x) = \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$$

$$\lim_{x \to 3} f(x) = \lim_{x \to 3} f(x+3)$$

$$f(3) = 3 + 3$$
$$= 6$$

The correct answer is **C**.

Question 2

$$L^- = \lim_{x \to 1^-} f(x)$$
$$= \lim_{x \to 1} (x + 2)$$
$$L^+ = \lim_{x \to 1} f(x)$$
$$= \lim_{x \to 1} (v - x)$$

Since $L^- = L^+$ to be continuous

$$x + 2 = v - x, \text{ when } x = 1$$
$$1 + 2 = v - 1$$
$$v = 4$$

The correct answer is **A**.

Question 3

To be continuous at $x = 1$, the two branches must have the same end point when $x = 1$.

$$ax^2 + 4 = 6x + b$$
$$a(1)^2 + 4 = 6(1) + b$$
$$a = 2 + b \quad [1]$$ **[1 mark]**

To be smooth and continuous, the derivatives of the two branches must be equal when $x = 1$.

From the left:	From the right:
$f(x) = ax^2 + 4$	$f(x) = 6x + b$
$\therefore f'(x) = 2ax$	$\therefore f'(x) = 6$
$\therefore f'(1) = 2a$	$\therefore f'(1) = 6$

Hence, $2a = 6$, giving $a = 3$. **[1 mark]**

Substitute $a = 3$ in equation [1].

$$a = 2 + b$$
$$3 = 2 + b$$
$$\therefore b = 1$$ **[1 mark]**

$$\therefore a = 3, b = 1$$

Question 4

$$\lim_{x \to -2} (4x + 5) = 4 \times -2 + 5$$
$$= -3$$

The correct answer is **D**.

Question 5

The function $f(x) = \dfrac{1}{x+2}$ is not well behaved at $x = -2$ since its denominator would be 0. There is an asymptote on the graph of $f(x)$ at $x = -2$.
The correct answer is **E**.

Question 6

Limit from the left of $x = 2$:

$$\begin{aligned} L^- &= \lim_{x \to 2^-} f(x) \\ &= \lim_{x \to 2} \left((x-2)^2\right) \\ &= 0 \end{aligned}$$

Limit from the right of $x = 2$:

$$\begin{aligned} L^+ &= \lim_{x \to 2^+} f(x) \\ &= \lim_{x \to 2} (x-2) \\ &= 0 \end{aligned}$$

Since $L^- = L^+$, $\lim\limits_{x \to 2} f(x) = 0$ and $f(x)$ is continuous at $x = 2$.

The correct answer is **A**.

Question 7

Limit from the left of $x = 1$:

$$\begin{aligned} L^- &= \lim_{x \to 1^-} f(x) \\ &= \lim_{x \to 1} (2x) \\ &= 2 \end{aligned}$$

Limit from the right of $x = 1$:

$$\begin{aligned} L^+ &= \lim_{x \to 1^+} f(x) \\ &= \lim_{x \to 1} \left(2x^2\right) \\ &= 2 \\ &\therefore \text{function is continuous} \end{aligned}$$

$$f(x) = \begin{cases} 2x, & x \le 1 \\ 2x^2, & x > 1 \end{cases}$$

Derivative from the left of $x = 1$:

$$\begin{aligned} f(x) &= 2x \\ &\therefore f'(x) = 2(1) \\ &\therefore f'(1^-) = 2 \end{aligned}$$

Derivative from the right of $x = 1$:

$$\begin{aligned} f(x) &= 2x^2 \\ &\therefore f'(x) = 4x \\ &\therefore f'\left(1^+\right) = 4 \times 1 \\ &= 4 \end{aligned}$$

Since the derivative from the left does not equal the derivative from the right, the function is not differentiable at $x = 1$.
The correct answer is **C**.

Question 8

$$\lim_{x\to3}\left(\frac{x^3-27}{x-3}\right)=\lim_{x\to3}\left(\frac{\cancel{(x-3)}(x^2+3x+3^2)}{\cancel{x-3}}\right)$$
$$=\lim_{x\to3}(x^2+3x+9)$$
$$=27$$

The correct answer is **A**.

12.3 Coordinate geometry applications of differentiation

Question 1

$$y=3+7x-2x^2$$
$$\therefore \frac{dy}{dx}=7-4x$$

At the point $(2,3)$,

$$\frac{dy}{dx}=7-4\times2$$
$$=-1$$

For perpendicular lines, $m_1, m_2=-1$.

$$m_2=-\frac{1}{m_1}$$

The gradient of the perpendicular is 1.

The correct answer is **A**.

Question 2

$$y=x(6-x)$$
$$y=6x-x^2$$

$\frac{dy}{dx}=6-2x$ **[1 mark]**

x-intercepts $(y=0)$:

$$0=x(6-x)$$

$\therefore x=0,6$

$\therefore$ points $(0,0)$ and $(6,0)$ **[1 mark]**

Gradient at $(0,0)$:

$$\frac{dy}{dx}=6-2\times0$$
$$=6$$

Equation of tangent 1: $m=6$, point $(0,0)$

$y=mx+c$

$y=6x$

Gradient at $(6,0)$:

$$\frac{dy}{dx}=6-2\times6$$
$$=-6$$

$$y=mx+c$$
$$y=-6x+c$$

Substituting $(6,0)$:

$0=-36+c$

$c=36$

Equation of tangent 2:

$y=-6x+36$ **[1 mark]**

Solve the equations simultaneously to point of intersection.

$$6x = -6x + 36$$
$$12x = 36$$
$$x = 3$$

Substitute $x = 3$ into $y = 6x$.

Therefore, the point of intersection (3, 18). **[1 mark]**

Question 3

$f'(x) = x^2 + 10x + 24$

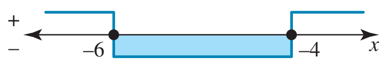
$f'(x) = (x + 4)(x + 6)$ **[1 mark]**

For a decreasing function, $f'(x) < 0$.

$\therefore (x + 4)(x + 6) < 0$

Zeros are $x = -6, x = -4$. **[1 mark]**

Sign diagram of $f'(x)$



$\therefore f'(x) < 0$ for $-6 < x < -4$

The function is decreasing over the interval $x \in (-6, -4)$. **[1 mark]**

Question 4

The gradient of a vertical tangent is undefined.

The correct answer is **E**.

Question 5

$y = 3x^2 - 9$

$$\therefore \frac{dy}{dx} = 6x$$

At the point (2, 3)

$$\therefore \frac{dy}{dx} = 6 \times 2$$
$$= 12$$

The correct answer is **B**.

12.4 Curve sketching

Question 1

Point $(2, -16)$ is a stationary point, so $\frac{dy}{dx} = 0$ at this point.

$$y = ax^2 + bx$$
$$\frac{dy}{dx} = 2ax + b$$
$$x = 2, \frac{dy}{dx} = 0$$

$\therefore 0 = 2a(2) + b$

$\therefore b = -4a$ [1] **[1 mark]**

Point $(2, -16)$ is a point on the curve.

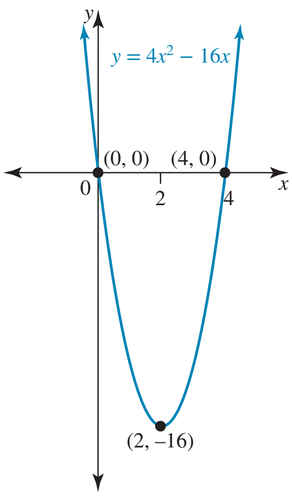
$-16 = a(2)^2 + b$ [2]

$= 4a + 2b$ [2] **[1 mark]**

Substitute equation [1] into equation [2].

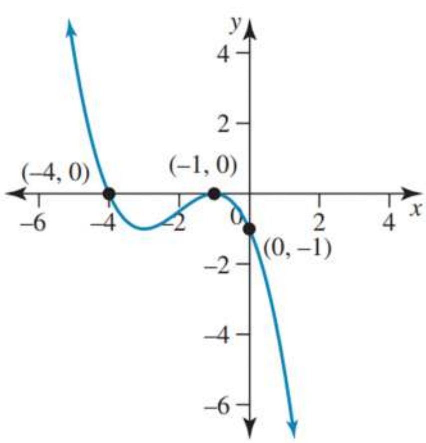
$$-16 = 4a + 2(-4a)$$
$$= -4a$$
$$a = 4$$

Substitute $a = 4$ into [1] to find b.
$b = -4(4)$
$= -16$
$\therefore a = 4, b = -16$
$\therefore y = 4x^2 - 16x$
y-intercepts $(x = 0)$:
$y = 0$
$(0, 0)$ is a y-intercepts.
x-intercepts $(y = 0)$:
$0 = 4x^2 - 16x$
$0 = 4x(x - 4)$
$x = 0, 4$
$(0, 0)$ and $(4, 0)$ are x-intercepts **[1 mark]**



[1 mark]

Question 2
$f(x)$ has a y-intercept at -1 and x-intercepts at -4 and -1.
$f(x)$ is decreasing for $x < -3$ and increasing for $x \in (-3, -1)$, so $x = -3$ must be a minimum turning point.
[1 mark]
$f'(-1) = 0$ so $x = -1$ is also a turning point and, since $f(x)$ is decreasing for $x > -1$, it must be a maximum turning point. **[1 mark]**



[1 mark]

Question 3

a. $f(x) = -x^3 + 3x^2 - 4$

$f'(x) = -3x^2 + 6x$ **[1 mark]**

For stationary points, $f'(x) = 0$.

$-3x^2 + 6x = 0$

$-3x(x - 2) = 0$

$\therefore x = 0,\ x = 2$ **[1 mark]**

$f(0) = -4$

$\therefore (0, -4)$ is a stationary point.

$f(2) = -(2)^3 + 3(2)^2 - 4$

$f(2) = 0$

$\therefore$ (2, 0) is a turning point and also an x-intercept. **[1 mark]**

b. End points:

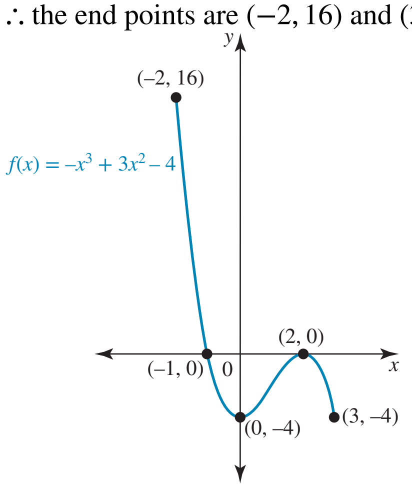
$f(-2) = -(-2)^3 + 3(-2)^2 - 4$

$= 16$

$f(3) = -(3)^3 + 3(3)^2 - 4$

$= -4$

$\therefore$ the end points are $(-2, 16)$ and $(3, -4)$.



[1 mark]

c. The maximum value in the given domain is 16, and the minimum value is -4. **[1 mark]**

Question 4

By definition, a stationary point occurs when $f'(x) = 0$.

It may be one of three types: a local maximum, a local minimum or a stationary point of inflection.

The correct answer is **E**.

Question 5

It is incorrect to say the maximum value of the function is 12. The graph shows that there is a local maximum at (3, 12) but, as $x \to -\infty, y \to \infty$, the global maximum is not 12.

The correct answer is **B**.

Question 6

$y = 2x^3 - 3x^2 - 12x + 18$

$\frac{dy}{dx} = 6x^2 - 6x - 12$ **[1 mark]**

For stationary point,

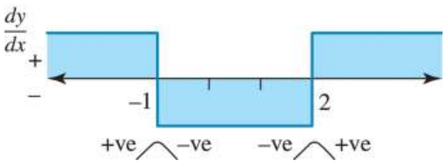
$6x^2 - 6x - 12 = 0$

$6\left(x^2 - x - 2\right) = 0$

$6(x - 2)(x + 1) = 0$

$x = 2, -1$ **[1 mark]**

When $x = 2$,
$y = 2(2)^3 - 3(2)^2 - 12(2) + 18$
$y = -2$
$\therefore (2, -2)$ is a stationary point. **[1 mark]**
When $x = -1$,
$y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 18$
$y = 25$
$\therefore (-1, 25)$ is a stationary point. **[1 mark]**
Type of stationary point



[1 mark]

$(-1, 25)$ is a local maximum turning point.
$(2, -2)$ is a local minimum turning point.

Question 7
$f(x) = 2x^3 + 5x^2 - 2$
$f'(x) = 6x^2 + 10x$
For stationary points, $f'(x) = 0$
$0 = 6x^2 + 10x$
$= 2x(3x + 5)$
$x = 0, -\frac{5}{3}$
$\therefore$ stationary points at $x = 0, -\frac{5}{3}$
The correct answer is **A**.

12.5 Optimisation problems

Question 1
Surface area of cylinder $= 2\pi r^2 + 2\pi rh$
$100 = 2\pi r^2 + 2\pi rh$
$2\pi rh = 100 - 2\pi r^2$
$h = \frac{100 - 2\pi r^2}{2\pi r}$
$h = \frac{50 - \pi r^2}{\pi r}$
The correct answer is **A**.

Question 2
Side lengths of triangle at top are x, x and $\sqrt{2}x$ (via Pythagoras' theorem).
The total perimeter is
$200 = 2h + 2x + \sqrt{2}x$
$2h = 200 - 2x - \sqrt{2}x$
$h = \frac{200 - 2x - \sqrt{2}x}{2}$
$= 100 - x - \frac{\sqrt{2}}{2}x$
Total area = area of rectangle + area of triangle

$$\text{Area} = xh + \frac{1}{2}x^2$$

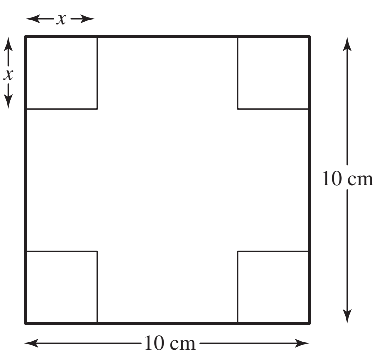
Substitute $h = 100 - x - \frac{\sqrt{2}}{2}x$.

$$\begin{aligned}\text{Area} &= x\left(100 - x - \frac{\sqrt{2}}{2}x\right) + \frac{1}{2}x^2 \\ &= 100x - x^2 - \frac{\sqrt{2}}{2}x^2 + \frac{1}{2}x^2 \\ &= 100x - x^2\left(1 + \frac{\sqrt{2}}{2} - \frac{1}{2}\right) \\ &= 100x - x^2\left(\frac{1+\sqrt{2}}{2}\right)\end{aligned}$$

The correct answer is **C**.

Question 3

a. **[1 mark]**



$$\begin{aligned}V &= lwh \\ &= (10-2x)(10-2x)(x) \\ &= x(10-2x)^2 \\ &= x\left(100 - 40x + 4x^2\right) \\ &= 100x - 40x^2 + 4x^3\end{aligned}$$ **[1 mark]**

b. $\frac{dV}{dx} = 100 - 80x + 12x^2$

When $\frac{dV}{dx} = 0$,

$$\begin{aligned}0 &= 100 - 80x + 12x^2 \\ 0 &= 25 - 20x + 3x^2 \\ 0 &= (3x-5)(x-5)\end{aligned}$$ **[1 mark]**

$$x = 5, \frac{5}{3}$$

$x = 5$ is unrealistic as the lengths would be 0, so no volume.

$\therefore x = \frac{5}{3}$ is a maximum. **[1 mark]**

$$\begin{aligned}\therefore \text{length and width} &= 10 - 2 \times \frac{5}{3} \\ &= \frac{20}{3}\end{aligned}$$

Height $= \frac{5}{3}$ **[1 mark]**

Maximum volume:

$$V = x(10 - 2x)^2$$
$$= \frac{5}{3}\left(10 - 2 \times \frac{5}{3}\right)^2$$

$\therefore$ maximum volume is 74.07 cm^2. [**1 mark**]

The maximum volume is 74 cm^3 (to the nearest whole number).

Question 4

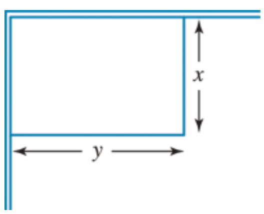
$$p = 1000n(4 - n)$$
$$= 4000n - 1000n^2$$
$$\frac{dp}{dn} = 4000 - 2000n$$
$$= 0 \text{ for maximum profit}$$
$$0 = 4000 - 2000n$$
$$2000n = 4000$$
$$n = 2$$
$$\therefore p = 1000 \times 2(4 - 2)$$
$$= 4000$$
$$\therefore \text{TP} = (2, 4000)$$

The function is a negative parabola, so the point (2, 4000) is a maximum turning point.

$\therefore$ 2 sales staff maximises profit.

The correct answer is **A**.

Question 5



[1 mark]

Available fencing:

$$20 = x + y$$
$$y = 20 - x \qquad \textbf{[1 mark]}$$

Area:

$$A = xy$$

Substitute $y = 20 - x$

$$A = x(20 - x)$$
$$= 20x - x^2$$

Maximum area:

$$\frac{dA}{dx} = 20 - 2x$$
$$= 0, \text{ for maximum area} \qquad \textbf{[1 mark]}$$
$$0 = 20 - 2x$$
$$2x = 20$$
$$\therefore x = 10$$
$$\therefore y = 20 - 10 = 10$$

Dimensions of the garden bed are 10×10 m, and the area is 100 m^2. **[1 mark]**

12.6 Rates of change and kinematics

Question 1

a. $x = 2t^2 - 8t + 9, t \geq 0$

$v = 4t - 8$

Initial velocity $t = 0$

$v = -8$

The initial velocity is -8 m/s. **[1 mark]**

Particle at rest: $\frac{dx}{dt} = 0$

$0 = 4t - 8$

$t = 2$

The particle is at rest after 2 seconds. **[1 mark]**

Position when the particle is at rest:

$x = 2t^2 - 8t + 9$

$x = 2(2)^2 - 8(2) + 9$

$x = 1$

Therefore, the particle is momentarily at rest after 2 seconds at the position 1 metre to the right of the origin. **[1 mark]**

b. Average velocity is the average rate of change of displacement.

For the first 3 seconds, when $t = 3$:

$x = 2t^2 - 8t + 9$

$x = 2(3)^2 - 8(3) + 9$

$x = 3$

$(t_1, x) = (0, 9), (t_2, x_2) = (3, 3)$ **[1 mark]**

$$\text{Average velocity} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{3 - 9}{3 - 0} = -2$$

The average velocity for the first 3 seconds is -2 m/s. **[1 mark]**

Question 2

$x = 2t^2 - 15t + 23, t \geq 0$

$\frac{dx}{dt} = 4t - 15$

When $t = 3$,

$$\frac{dx}{dt} = 4(3) - 15 = -3$$

$\therefore$ speed is 3 m/s

(Speed is not concerned with the direction of motion and is never negative.)

The correct answer is **C**.

Question 3

Area $= \pi r^2$

Rate of change:

$\frac{dA}{dr} = 2\pi r$

$r = 3$

$\frac{dA}{dr} = 6\pi \text{cm}^2/\text{cm}$

The correct answer is **D**.

Question 4

$$x = 5t^2 - 2t + 3$$

Initial position, $t = 0$

$$\therefore x = 5(0)^2 - 2(0) + 3$$
$$x = 3$$

The correct answer is **C**.

12.7 Derivatives of power functions

Question 1

$$y = 4x\sqrt{x}$$
$$= 4x^{\frac{3}{2}}$$
$$\frac{dy}{dx} = \left(4 \times \frac{3}{2}\right) x^{\frac{3}{2}-1}$$
$$= 6x^{\frac{1}{2}}$$
$$= 6\sqrt{x}$$

The correct answer is **A**.

Question 2

a. $f(x) = 3 - 2\sqrt{x}$

$$= 3 - 2x^{\frac{1}{2}}$$
$$f'(x) = -\left(2 \times \frac{1}{2}\right) x^{\frac{1}{2}-1}$$
$$= -x^{-\frac{1}{2}}$$
$$= -\frac{1}{\sqrt{x}} \qquad \textbf{[1 mark]}$$

b. Graph cuts the x-axis when $y = 0$

$$y = 3 - 2\sqrt{x}$$
$$2\sqrt{x} = 3$$
$$\sqrt{x} = \frac{3}{2}$$
$$x = \frac{9}{4} \qquad \textbf{[1 mark]}$$
$$f'\left(\frac{9}{4}\right) = -\frac{1}{\sqrt{\frac{9}{4}}}$$
$$= -\frac{1}{\frac{3}{2}}$$
$$= -\frac{2}{3} \quad \textbf{[1 mark]}$$

$\therefore$ gradient at $y = 0$ is $-\frac{2}{3}$ **[1 mark]**

Question 3

a. The denominator cannot equal 0 as the function is undefined.

When $x = 0$, $3x = 0$ and $\frac{1}{3x}$ is undefined. Therefore, the domain is $R \setminus (0)$. **[1 mark]**

b. $f(x) = 2x + \frac{1}{3x}$

$= 2x + \frac{1}{3}x^{-1}$

$f'(x) = 2 + \left(\frac{1}{3} \times -1\right) x^{-1-1}$

$= 2 - \frac{1}{3}x^{-2}$

$= 2 - \frac{1}{3x^2}$, domain $R \setminus (0)$ **[1 mark]**

c. Gradient is -1

$f'(x) = -1$

$-1 = 2 - \frac{1}{3x^2}$

$\frac{1}{3x^2} = 2 + 1$

$1 = 9x^2$

$x^2 = \frac{1}{9}$

$x = \pm\frac{1}{3}$

$f(x) = 2x + \frac{1}{3x}$ **[1 mark]**

$f\left(\frac{1}{3}\right) = 2x + \frac{1}{3} + \frac{1}{3 \times \frac{1}{3}}$

$= \frac{2}{3} + 1 = \frac{5}{3}$

$f\left(-\frac{1}{3}\right) = 2 \times -\frac{1}{3} + \frac{1}{3 \times -\frac{1}{2}}$

$= -\frac{2}{3} - 1$

$= -\frac{5}{3}$

Coordinates of the points on the curve where the tangent has a gradient of -1 are $\left(\frac{1}{3}, \frac{5}{3}\right)$ and $\left(-\frac{1}{3}, -\frac{5}{3}\right)$. **[1 mark]**

Question 4

$y = \frac{x^4 + 3x - 6}{3x^2}$

$= \frac{x^4}{3x^2} + \frac{3x}{3x^2} - \frac{6}{3x^2}$

$= \frac{1}{3}x^2 + x^{-1} - 2x^{-2}$

$\frac{dy}{dx} = \frac{2x}{3} - x^{-2} + 4x^{-3}$

The correct answer is **C**.

Question 5

$$f(x) = 6\sqrt{x}$$
$$= 6(x)^{\frac{1}{2}}$$
$$\frac{dy}{dx} = 3x^{-\frac{1}{2}}$$
$$= \frac{3}{\sqrt{x}}$$

The correct answer is **B**.

Question 6

$$y = \frac{x^4 - 4x^2 + 8x}{2x^3}$$
$$= \frac{x^4}{2x^3} - \frac{4x^2}{2x^3} + \frac{8x}{2x^3}$$
$$= \frac{1}{2}x - 2x^{-1} + 4x^{-2}$$
$$\frac{dy}{dx} = \frac{1}{2} + 2x^{-2} - 8x^{-3}$$
$$= \frac{1}{2} + \frac{2}{x^2} - \frac{8}{x^3}$$

The correct answer is **B**.

12.8 Review

Question 1

Limit from the left of $x = 1$:

$$L^- = \lim_{x \to 1^-} f(x)$$
$$= \lim_{x \to 1} \left(x^2 - 2\right)$$
$$= -1$$

Limit from the right of $x = 1$:

$$L^+ = \lim_{x \to 1^+} f(x)$$
$$= \lim_{x \to 1} (2x - 3)$$
$$= -1$$

$\therefore$ function on is continuous.

$$f(x) = \begin{cases} x^2 - 2, & x \le 1 \\ 2x - 3, & x > 1 \end{cases}$$

Derivative from the left of $x = 1$:

$$f(x) = x^2 - 2$$
$$\therefore f'(x) = 2x$$
$$\therefore f'(1^-) = 2$$

Derivative from the right of $x = 1$:

$$f(x) = 2x - 3$$
$$\therefore f'(x) = 2$$
$$\therefore f'(1^+) = 2$$

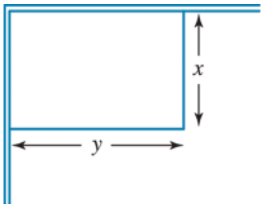
Since the derivative from the left equals the derivative from the right, the function is differentiable at $x = 1$.

The correct answer is **A**.

Question 2
The gradient is positive where the function is increasing; that is, it slopes up towards the right.
$\therefore x \in (-\infty, -3) \cup (1, \infty)$
The correct answer is **C**.

Question 3
The gradient changes from negative to zero and back to negative, so it doesn't change from negative to positive. This means there is a stationary point of inflection at $x = 3$.
The correct answer is **D**.

Question 4



[1 mark]

Available fencing:
$$20 = x + y$$
$$y = 20 - x \qquad \textbf{[1 mark]}$$
Area:
$$A = xy$$
Substitute $y = 20 - x$
$$A = x(20 - x)$$
$$= 20x - x^2$$
Maximum area:
$$\frac{dA}{dx} = 20 - 2x$$
For maximum area, $\frac{dA}{dx} = 0$ **[1 mark]**
$$0 = 20 - 2x$$
$$2x = 20$$
$$\therefore x = 10$$
$$\therefore y = 20 - 10 = 10$$
The dimensions of the garden bed are 10×10 m and the area is 100 m^2. **[1 mark]**

Question 5
a. Height of the triangle, using Pythagoras' theorem:
$$h^2 + \left(\frac{x}{4}\right)^2 = x^2$$
$$h^2 + \frac{x^2}{16} = x^2$$
$$h^2 = \frac{15x^2}{16}$$
$$h = \pm\sqrt{\frac{15}{16}}x$$
$$= \frac{\sqrt{15}}{4}x, (h > 0) \qquad \textbf{[1 mark]}$$

Area of a triangle is $\frac{1}{2} \times \text{base} \times \text{height}$

$A = \frac{1}{2} \times \left(\frac{x}{2}\right) \times \frac{\sqrt{15}}{4}x$

$A = \frac{\sqrt{15}}{16}x^2$ [**1 mark**]

b. $\frac{dA}{dx} = \frac{\sqrt{15}}{8}x$

When $x = 8$.

$\frac{dA}{dx} = \frac{\sqrt{15}}{8} \times 8$

$= \sqrt{15}$

The rate that the area is changing when $x = 8$ is $\sqrt{15}\,\text{cm}^2/\text{cm}$. **[1 mark]**

Question 6

$\lim_{x \to 4}\left(\frac{x^3 - 64}{x - 4}\right) = \lim_{x \to 4}\left(\frac{\cancel{(x - 4)}(x^2 + 4x + 4^2)}{\cancel{x - 4}}\right)$ [**1 mark**]

$= \lim_{x \to 4}(x^2 + 4x + 16)$ [**1 mark**]

$= 16 + 16 + 16$

$= 48$ [**1 mark**]

Question 7

To be continuous,

$ax + b = 6$ when $x = -1$, and $bx + 2a = 6$ when $x = 4$. **[1 mark]**

At $x = -1$,

$ax + b = 6$

$-a + b = 6$

$b = a + 6$ (1) [**1 mark**]

At $x = 4$,

$bx + 2a = 6$

$4b + 2a = 6$ (2) [**1 mark**]

Substitute (1) into (2):

$4(a + 6) + 2a = 6$

$6a + 24 = 6$

$6a = -18$

$a = -3$

Substitute $a = -3$ into (1):

$\therefore b = -3 + 6$

$b = 3$ [**1 mark**]

Question 8

The statement that the graph is not differentiable at x_5, is untrue. x_5 falls on a straight line and is both continuous and differentiable.

The correct answer is **E**.

Question 9

a. For the function to be differential at $x = 0$, the function needs to be smooth and continuous at this point.

Limit from the left of $x = 0$:

$$\begin{aligned} L^- &= \lim_{x \to 0^-} f(x) \\ &= \lim_{x \to 0} (4 - 2x) \\ &= 4 \end{aligned}$$

Limit from the right of $x = 0$:

$$\begin{aligned} L^+ &= \lim_{x \to 0^+} f(x) \\ &= \lim_{x \to 0} \left(4 - x^2\right) \\ &= 4 \end{aligned}$$

$\therefore$ function is continuous **[1 mark]**

$$f(x) = \begin{cases} 4 - 2x, & x < 0 \\ 4 - x^2, & x \geq 0 \end{cases}$$

Derivative from the left of $x = 0$:

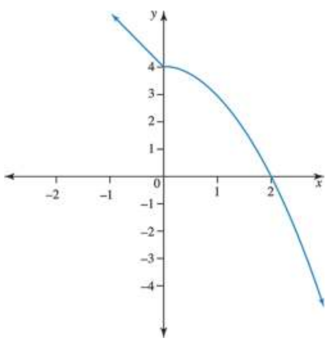
$$\begin{aligned} f(x) &= 4 - 2x \\ \therefore f'(x) &= -2 \\ \therefore f'(0^-) &= -2 \end{aligned}$$

Derivative from the right of $x = 0$:

$$\begin{aligned} f(x) &= 4 - x^2 \\ \therefore f'(x) &= -2x \\ \therefore f'\left(0^+\right) &= 0 \end{aligned}$$

Since the derivative from the left does not equal the derivative from the right, the function is not smooth at $x = 0$ and therefore is not differentiable at $x = 0$. **[1 mark]**

b.



c. $f'(x) = \begin{cases} -2, & x < 0 \\ -2x, & x > 0 \end{cases}$ **[1 mark]**

d. At $x = 5$

$f'(x) = -2x$

$f'(5) = -10$ **[1 mark]**

Question 10

$$\begin{aligned} L^- &= \lim_{x \to 2^-} f(x) \\ &= \lim_{x \to 2} (x + 4) \end{aligned}$$

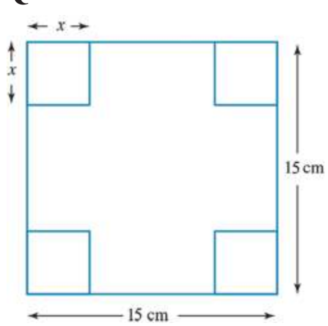
$$\begin{aligned} L^+ &= \lim_{x \to 2^+} f(x) \\ &= \lim_{x \to 2} (2v - x) \end{aligned}$$

Since $L^- = L^+$ to be continuous

$x + 4 = 2v - x$ when $x = 2$
$2 + 4 = 2v - 2$
$2v = 8$
$v = 4$
The correct answer is **C**.

Question 11
Limit from the left of $x = -1$:
$$L^- = \lim_{x \to -1^-} f(x)$$
$$= \lim_{x \to -1} (-x)$$
$$= 1$$
Limit from the right of $x = -1$:
$$L^+ = \lim_{x \to -1^+} f(x)$$
$$= \lim_{x \to -1} \left(2x^2 - 1\right)$$
$$= 1$$
$\therefore$ function is continuous.
$$f(x) = \begin{cases} -x, & x \le -1 \\ 2x^2 - 1, & x > -1 \end{cases}$$
Derivative from the left of $x = -1$:
$f(x) = -x$
$\therefore f'(x) = -1$
$\therefore f'(-1^-) = -1$
Derivative from the right of $x = -1$:
$f(x) = 2x^2 - 1$
$\therefore f'(x) = 4x$
$\therefore f'\left(-1^+\right) = 4 \times -1$
$= -4$
Since the derivative from the left does not equal the derivative from the right, the function is not differentiable at $x = -1$.
The correct answer is **C**.

Question 12



$$V = lwh$$
$$= (15 - 2x)(15 - 2x)(x)$$
$$= x(15 - 2x)^2$$
$$= x\left(225 - 60x + 4x^2\right)$$
$$= 225x - 60x^2 + 4x^3$$ [**1 mark**]

$$\frac{dV}{dx} = 225 - 120x + 12x^2$$

At maximum volume, $\frac{dV}{dx} = 0$.

$0 = 225 - 120x + 12x^2$
$= 3\left(75 - 40x + 4x^2\right)$
$= 3(2x - 15)(2x - 5)$

$x = \frac{15}{2}, \frac{5}{2}$ **[1 mark]**

$x = \frac{15}{2}$ is unrealistic as the volume would be 0.

$\therefore x = \frac{5}{2}$ gives a maximum. **[1 mark]**

Length, width

$= 15 - 2 \times \frac{5}{2}$
$= 15 - 5$
$= 10\,\text{cm}$

$h = \frac{5}{2}\,\text{cm}$ **[1 mark]**

Maximum volume

$V = x(15 - 2x)^2$
$= \frac{5}{2}\left(15 - 2 \times \frac{5}{2}\right)^2$
$= \frac{5}{2} \times 10^2$
$= 250\,\text{cm}^3$ **[1 mark]**

13 Anti-differentiation and introduction to integral calculus

Topic	13	Anti-differentiation and introduction to integral calculus
Subtopic	13.2	Anti-derivatives

online only

To answer questions online and to receive **immediate feedback** and **sample responses** for every question, go to your learnON title at **www.jacplus.com.au**.

Question 1 (1 mark)

Given that $\frac{dy}{dx} = 6x^{11}$, then

A. $y = \frac{x^{10}}{5} + c$

B. $y = 12x^{12} + c$

C. $y = 5x^{10} + c$

D. $y = \frac{x^{12}}{2} + 3x + c$

E. $y = \frac{x^{12}}{2} + c$

Question 2 (1 mark)

Given that $f'(x) = 3x^5 - 3x^8$, an expression for $f(x)$ is

A. $\frac{1}{2}x^6 - \frac{1}{3}x^9 + c$

B. $3x^6 - 9x^9 + c$

C. $\frac{1}{6}x^6 - \frac{1}{9}x^9 + c$

D. $15x^4 - 24x^7 + c$

E. $2x^6 - 8x^9 + c$

Question 3 (1 mark)

Calculate $\int \frac{7x^6 - 4x^5}{2x^3}\,dx$.

A. $7x^4 - 4x^3 + c$

B. $8x^4 - 3x^3 + c$

C. $\frac{8}{7}x^4 - \frac{3}{2}x^3 + c$

D. $\frac{7}{9}x^6 - \frac{2}{5}x^5 + c$

E. $\frac{7}{8}x^4 - \frac{2}{3}x^3 + c$

Question 4 (1 mark)

Given $f'(x) = 4x^3 - 2x^5$, an expression for $f(x)$ is

A. $x^4 - \frac{1}{3}x^6 + c$

B. $12x^2 - 10x^4 + c$

C. $\frac{4}{3}x^4 - \frac{2}{5}x^6 + c$

D. $x^4 - \frac{2}{5}x^6 + c$

E. $x^3 - \frac{2}{3}x^5 + c$

Question 5 (1 mark)

Determine $F(x)$, where $F'(x) = f(x)$ and $f(x) = \frac{2}{\sqrt{3x^4}} + \frac{3}{\sqrt{3a^2}}$.

Topic	13	Anti-differentiation and introduction to integral calculus
Subtopic	13.3	Anti-derivative functions and graphs

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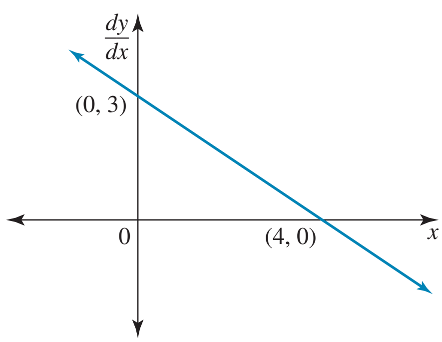
Question 1 (1 mark)
The gradient function of a curve is given by $f'(x) = -6x^2 + 2$. If the curve passes through the point $(-1, 8)$, its equation is
A. $-3x^3 + 2x + 6$
B. $-2x^2 + x + 4$
C. $-2x^3 + 2x + 8$
D. $-x^3 + 2x + 8$
E. $-2x^3 + 2x - 1$

Question 2 (3 marks)
The gradient of a curve is given by $\frac{dy}{dx} = 5x + b$, where b is a constant. Given that the curve has a stationary point at $(4, 6)$, determine its equation.

Question 3 (3 marks)
The graph of the gradient function of a particular curve is shown. Given that $(8, 9)$ lies on the curve with this gradient, determine the equation of the curve.



Question 4 (1 mark)

Given that $\frac{dA}{dt} = \frac{3}{\sqrt{t}}$ and $A = 22$ when $t = 4$, the value of A when $t = 16$ is

A. 88
B. 5
C. 34
D. 3
E. 22

Question 5 (1 mark)

The gradient function of a curve is given by $f'(x) = 4x^2 - 3$. If the curve passes through the point (3, 19), its equation is

A. $f(x) = \frac{4}{3}x^3 - 3x$
B. $f(x) = 8x - 5$
C. $f(x) = 3x^3 - 4x - 8$
D. $f(x) = \frac{4}{3}x^3 - 3x + 8$
E. $f(x) = \frac{4}{3}x^3 - 3x - 8$

Topic	13	Anti-differentiation and introduction to integral calculus
Subtopic	13.4	Applications of anti-differentiation

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Question 1 (1 mark)

A particle moves in a straight line so that its acceleration, in m/s^2, at time t seconds is given by $a = 2t + 1, t \geq 0$. If $v = 10\,\text{m/s}$ when $t = 0$, then the particle's velocity when $t = 2$ is

A. $0\,\text{m/s}$
B. $5\,\text{m/s}$
C. $10\,\text{m/s}$
D. $16\,\text{m/s}$
E. $20\,\text{m/s}$

Question 2 (3 marks)

A particle moves in a straight line so that its acceleration, in m/s^2, at time t seconds is given by $a = 16, t \geq 0$. If $t = 0$ when $v = 2\,\text{m/s}$ and $x = 2\,\text{m}$, calculate the particle's position when $t = 3$.

Question 3 (3 marks)

While riding home from work, Angela discovers a puncture in a tyre on her bicycle. If the volume of her tyre was $2880\,\text{cm}^3$ when fully pumped, and the air is escaping at a rate of $0.4t\,\text{cm}^3/\text{s}$, where t is the number of seconds since the puncture occurred, determine how many minutes it will take for the tyre to be fully flat (assuming volume goes to 0).

Question 4 (1 mark)

Which of these statements is *not* correct?

A. The derivative of velocity gives acceleration.
B. Acceleration is the rate of change of speed.
C. The antiderivative of velocity gives displacement.
D. Velocity is the rate of change of displacement.
E. The derivative of displacement gives velocity.

Question 5 (1 mark)

A particle moves in a straight line so that its velocity at time t seconds is given by $v = 4, t \geq 0$. Initially, the particle is 3 metres to the left of a fixed origin. The time (in seconds) it takes to reach the origin is

A. $\frac{3}{4}$
B. 2
C. $\frac{4}{3}$
D. 3
E. 4

Topic	13	Anti-differentiation and introduction to integral calculus
Subtopic	13.5	The definite integral

online only

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Question 1 (4 marks)

The velocity of a particle moving in a straight line is given by $v = t^3 - 2, t \geq 0$.

Assume units for distance are in metres and time in seconds.

a. Find the particle's velocity and acceleration when $t = 2$. **(2 marks)**

b. Sketch the velocity–time graph and shade the area that represents the distance the particle travels over the interval $t \in [2, 3]$. **(1 mark)**

c. Use a definite integral to calculate the distance. **(1 mark)**

Question 2 (2 marks)

Sketch a graph to show the area represented by $\int_{-2}^{2} 3x^2 dx$ and calculate the area.

Question 3 (1 mark)

Evaluate $\int_0^3 (x+2)^2 dx$.

A. 4
B. 16
C. 24
D. 30
E. 39

Question 4 (1 mark)

Evaluate $\int_1^2 \frac{2x^5 + 3x^4 - 24}{x^4} dx$.

A. −19
B. −1
C. 0
D. 3
E. 5

Question 5 (1 mark)

The value of $\int_{-1}^{3} 4x + 2dx$ is

A. 11
B. 18
C. 24
D. 20
E. 21

Topic	13	Anti-differentiation and introduction to integral calculus
Subtopic	13.6	Review

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Question 1 (2 marks)

Calculate the anti-derivative (primitive function) of $(x-2)(2x+5)(x+2)$.

Question 2 (1 mark)

The gradient of a curve is directly proportional to the square of x, and, at the point $(3, 9)$ on the curve, the gradient is -18. The constant of proportionality is

A. 9
B. -2
C. 1
D. 3
E. -9

Question 3 (1 mark)

State which of the following is **not** correct.

A. The anti-derivative of acceleration gives velocity.
B. Acceleration is the rate of change of velocity.
C. Velocity is the rate of change of distance.
D. The derivative of velocity gives acceleration.
E. The anti-derivative of velocity gives position.

Question 4 (1 mark)

A particle moves in a straight line so that its velocity at time t seconds is given by $v = 2, t \geq 0$. Initially, the particle is 6 metres to the left of a fixed origin. The time (in seconds) that it takes to reach the origin is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 5 (1 mark)

Evaluate $\int_{-2}^{2} 3x^2 dx$.

A. 0
B. 6
C. 12
D. 16
E. 24

Question 6 (1 mark)

$\int \frac{5x^5 + 3x^2}{3x^4} dx =$

A. $\frac{5}{3} - 2x + c$
B. $\frac{5}{6}x^2 - \frac{3}{x^3} + c$
C. $\frac{5}{3} - \frac{1}{x} + c$
D. $\frac{5}{6}x^2 - \frac{1}{x} + c$
E. $\frac{5}{3}x - \frac{1}{x} + c$

Question 7 (1 mark)

Given $\frac{dA}{dt} = \frac{4}{\sqrt{t}}$ and $A = 20$ when $t = 9$, the value of A when $t = 25$ is

A. 40
B. 24
C. 44
D. 36
E. 32

Question 8 (1 mark)

The gradient of a curve is directly proportional to the cube of x, and, at the point (2, 12) on the curve, the gradient is −24. The constant of proportionality is

A. 3
B. −3
C. 2
D. −2
E. 1

Question 9 (1 mark)

A particle moves in a straight line so that its acceleration, in m/s^2, at time t seconds is given by $a = t - 2,\ t \geq 0$. if $v = 8$ m/s when $t = 0$, then the particle's velocity when $t = 4$ is

A. 4 m/s
B. −8 m/s
C. 24 m/s
D. 8 m/s
E. −4 m/s

Question 10 (1 mark)

The value of $\int_1^4 \frac{x^4 + 4x^3 - 8}{x^3} dx$ is

A. $15\frac{3}{4}$

B. $31\frac{3}{4}$

C. $16\frac{1}{4}$

D. $12\frac{3}{4}$

E. $14\frac{1}{2}$

Question 11 (4 marks)

The gradient of a curve is given by $\frac{dy}{dx} = 4x + b$, where b is a constant. Given the curve has a stationary point at (2, 10), determine its equation.

Question 12 (3 marks)

While riding home from work, Alicia discovers a puncture in the tyre on her bicycle. If the volume of her tyre was $2550\,\text{cm}^3$ when fully pumped, and the air is escaping at a rate of $0.5t\,\text{cm}^3/s$ where t is the number of seconds since the puncture occurred, how many minutes will it take for the tyre to be fully flat (assuming volume goes to zero)? Give your answer correct to 2 decimal places.

Answers and marking guide

13.2 Anti-derivatives

Question 1

If $\frac{dy}{dx} = x^n, n \in N,$ then $y = \frac{1}{n+1}x^{n+1} + c,$
where c is an arbitrary constant.

$$\frac{dy}{dx} = 6x^{11}$$
$$y = \frac{1}{11+1} \times 6x^{11+1} + c$$
$$y = \frac{1}{2}x^{12} + c$$
$$y = \frac{x^{12}}{2} + c$$

The correct answer is **E**.

Question 2

$$f(x) = \frac{3}{5+1}x^{5+1} - \frac{3}{8+1}x^{8+1} + c$$
$$= \frac{1}{2}x^6 - \frac{1}{3}x^9 + c$$

The correct answer is **A**.

Question 3

$$\int \frac{7x^6 - 4x^5}{2x^3}dx = \int \left(\frac{7x^6}{2x^3} - \frac{4x^5}{2x^3}\right) dx$$
$$= \int \left(\frac{7x^3}{2} - 2x^2\right) dx$$
$$= \frac{7x^{3+1}}{(3+1)\times 2} - \frac{2}{2+1}x^{2+1} + c$$
$$= \frac{7}{8}x^4 - \frac{2}{3}x^3 + c$$

The correct answer is **E**.

Question 4

$$f(x) = \frac{4}{3+1}x^{3+1} - \frac{2}{5+1}x^{5+1} + c$$
$$= x^4 - \frac{1}{3}x^6 + c$$

The correct answer is **A**.

Question 5

$$F'(x) = f(x) = \frac{2}{\sqrt{3x^4}} + \frac{3}{\sqrt{3}a^2}$$
$$= \frac{2}{\sqrt{3}}x^{-4\times\frac{1}{2}} + \frac{3}{\sqrt{3}a^2}$$
$$F'(x) = \frac{2}{\sqrt{3}}x^{-2} + \frac{3}{\sqrt{3}a^2}$$

$$F(x) = \int F'(x)dx$$

$$= \int \left(\frac{2}{\sqrt{3}} x^{-2} + \frac{3}{\sqrt{3a^2}} dx \right) \quad \textbf{[1 mark]}$$

$$= \frac{1}{-2+1} \times \frac{2}{\sqrt{3}} x^{-2+1} + \frac{3}{\sqrt{3a^2}} x + c$$

$$= -\frac{2}{\sqrt{3}} x^{-1} + \frac{3}{\sqrt{3a^2}} x + c$$

$$= -\frac{2}{\sqrt{3x}} + \frac{3}{\sqrt{3a^2}} x + c \quad \textbf{[1 mark]}$$

13.3 Anti-derivative functions and graphs

Question 1

$f'(x) = -6x^2 + 2$

$f(x) = \int \left(-6x^2 + 2\right) dx$

$f(x) = -2x^3 + 2x + c$

Point $(-1, 8)$

$8 = -2(-1)^3 + 2(-1) + c$

$8 = 2 - 2 + c$

$c = 8$

$f(x) = -2x^3 + 2x + 8$

The correct answer is **C**.

Question 2

$\frac{dy}{dx} = 5x + b$

Stationary point $\frac{dy}{dx} = 0$, when $x = 4$

$0 = 5 \times 4 + b$

$\therefore b = -20$ **[1 mark]**

$\frac{dy}{dx} = 5x - 20$

$y = \int (5x - 20)\, dx$

$y = \frac{5x^2}{2} - 20x + c$ **[1 mark]**

Substitute point $(4, 6)$.

$6 = \frac{5(4)^2}{2} - 20 \times 4 + c$

$6 = 40 - 80 + c$

$\therefore c = 46$

The equation is $y = \frac{5x^2}{2} - 20x + 46$. **[1 mark]**

Question 3

Equation of gradient function:

Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - 0} = -\frac{3}{4}$ **[1 mark]**

$\frac{dy}{dx} = mx + c_1$

$\frac{dy}{dx} = -\frac{3}{4}x + 3$

$y = \int \left(-\frac{3}{4}x + 3\right) dx$

$y = \frac{3}{8}x^2 + 3x + c_2$ **[1 mark]**

Point $(8, 9)$

$9 = -\frac{3}{8}(8)^2 + 3(8) + c_2$

$9 = -24 + 24 + c_2$

$c_2 = 9$

The equation is $y = -\frac{3}{8}x^2 + 3x + 9$. **[1 mark]**

Question 4

$\frac{dA}{dt} = \frac{3}{\sqrt{t}}$

$= 3t^{-\frac{1}{2}}$

$A = \int 3t^{-\frac{1}{2}} dt$

$= 6t^{\frac{1}{2}} + c$

$= 6\sqrt{t} + c$

When $A = 22,\ t = 4$

$22 = 6 \times \sqrt{4} + c$

$c = 10$

$A = 6\sqrt{t} + 10$

When $t = 16$

$A = 6 \times \sqrt{6} + 10$

$= 34$

The correct answer is **C**.

Question 5

$f'(x) = 4x^2 - 3$

$f(x) = \frac{4}{3}x^3 - 3x + c$

Point $(3,\ 19)$

$19 = \frac{4}{3}(3)^3 - 3(3) + c$

$= 36 - 9 + c$

$c = -8$

$f(x) = \frac{4}{3}x^3 - 3x - 8$

The correct answer is **E**.

13.4 Applications of anti-differentiation

Question 1

$a = 2t + 1,\ t \geq 0$

$v = \int (2t + 1)\, dt$

$v = t^2 + t + c$

$v = 10$ when $t = 0$

$\therefore v = t^2 + t + 10$

$t = 2$

$v = 2^2 + 2 + 10$

$v = 16$

The correct answer is **D**.

Question 2

$a = 16,\ t \geq 0$

$v = \int (16)\, dt$

$v = 16t + c_1$

When $t = 0,\ v = 2$ m/s.

$\therefore v = 16t + 2$ [**1 mark**]

$x = \int (16t + 2)$

$x = 8t^2 + 2t + c_2$

When $t = 0, v = 2$ m.

$x = 8t^2 + 2t + 2$ **[1 mark]**

Position at $t = 3$:

$x = 8(3)^2 + 2(3) + 2$

$x = 72 + 6 + 2$

$x = 80$

The position at $t = 3$ is 80 m. **[1 mark]**

Question 3

$\frac{dV}{dt} = -0.4t$ [**1 mark**]

$V = \int (-0.4t)\, dt$

$V = -0.2t^2 + c$

When $t = 0, V = 2880$.

$\therefore V = -0.2t^2 + 2880$ [**1 mark**]

When $V = 0$.

$0 = -0.2t^2 + 2880$

$0.2t^2 = 2800$

$t^2 = 14400$

$t = \pm 120$

$\therefore$120 seconds $(t > 0)$

$t = 2$ minutes [**1 mark**]

Question 4

Speed is not concerned with the direction of motion. It should read 'Acceleration is the rate of change of velocity'.
The correct answer is **B**.

Question 5

$v = 4$

$x = \int v\,dt$

$= 4t + c$

$t = 0, x = -3$

$-3 = 4(0) + c$

$c = -3$

$\therefore x = 4t - 3$

$0 = 4t - 3$

$4t = 3$

$t = \frac{3}{4}$

The correct answer is **A**.

13.5 The definite integral

Question 1

a. $v = t^3 - 2$

When $t = 2$

$v = 2^3 - 2$

$= 6$

The velocity is 6 m/s. **[1 mark]**

$a = \frac{dv}{dt}$

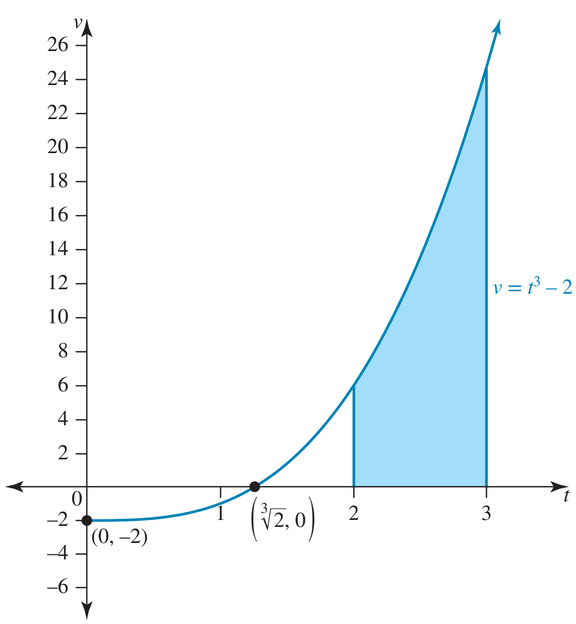
$= 3t^2$

$= 3(2)^2$

$= 12$

The acceleration is 12 m/s^2. **[1 mark]**

b. **[1 mark]**



c. Area $= \int_{2}^{3} \left(t^3 - 2\right) dt$

$= \left[\frac{t^4}{4} - 2t\right]_2^3$

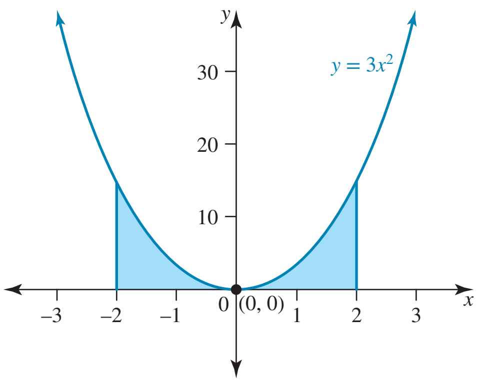
$= \left(\frac{81}{4} - 2 \times 3\right) - \left(\frac{16}{4} - 4\right)$

$= 14.25 - 0$

$= 14.25$ [**1 mark**]

The distance travelled by the particle over the time interval [2, 3] is 14.25 metres. **[1 mark]**

Question 2



[1 mark]

$$\int_{-2}^{2} 3x^2 dx = \left[x^3\right]_{-2}^{2}$$

$= (2)^3 - (-2)^3$

$= 16$

∴ the area is 16 square units. [**1 mark**]

Question 3

$$\int_{0}^{3} (x+2)^2 dx = \int_{0}^{3} \left(x^2 + 4x + 4\right) dx$$

$= \left[\frac{x^3}{3} + 2x^2 + 4x\right]_0^3$

$= (9 + 18 + 12) - (0)$

$= 39$

The correct answer is **E**.

Question 4

$$\int_1^2 \frac{2x^5+3x^4-24}{x^4}dx = \int_1^2 \left(\frac{2x^5}{x^4}+\frac{3x^4}{x^4}-\frac{24}{x^4}\right)dx$$
$$= \int_1^2 \left(2x+3-24x^{-4}\right)dx$$
$$= \left[x^2+3x+8x^{-3}\right]_1^2$$
$$= \left[x^2+3x+\frac{8}{x^3}\right]_1^2$$
$$= (4+6+1)-(1+3+8)$$
$$= -1$$

The correct answer is **B**.

Question 5

$$\int_{-1}^{3} 4x+2dx = \left[2x^2+2x\right]_{-1}^{3}$$
$$= 2(3)^2+2(3)-\left(2(-1)^2+2(-1)\right)$$
$$= 18+6-2+2$$
$$= 24$$

The correct answer is **C**.

13.6 Review

Question 1

Let $f'(x) = (x-2)(2x+5)(x+2)$.
$= \left(x^2-4\right)(2x+5)$
$= 2x^3+5x^2-8x-20$ **[1 mark]**

$$f(x) = \int f'(x)\,dx$$
$$= \frac{1}{3+1}\times 2x^{3+1}+\frac{1}{2+1}\times 5x^{2+1}-\frac{1}{1+1}\times 8x^{1+1}-20x+c$$
$$= \frac{1}{2}x^4+\frac{5}{3}x^3-4x^2-20x+c$$ **[1 mark]**

Question 2

$$\frac{dy}{dx} \propto x^2$$
$$\frac{dy}{dx} = kx^2$$

The gradient is -18.

When $x = 3$,

$$-18 = k(3)^2$$
$$9k = -18$$
$$k = -2$$

The correct answer is **B**.

Question 3

Distance is not concerned with the direction of motion. It should read: 'Velocity is the rate of change of position with respect to time'.
The correct answer is **C**.

Question 4

$v = 2$

$x = \int v\,dt$

$x = 2t + c$

$t = 0,\ x = -6$ (6 m left of the origin)

$x = 2t - 6$

origin, $x = 0$

$0 = 2t - 6$

$2t = 6$

$\therefore\ t = 3$

The correct answer is **D**.

Question 5

$$\int_{-2}^{2} 3x^2\,dx = \left[x^3\right]_{-2}^{2}$$

$$= (2)^3 - (-2)^3$$

$$= 16$$

The correct answer is **D**.

Question 6

$$\int \frac{5x^5 + 3x^2}{3x^4}dx = \int \left(\frac{5}{3}x + x^{-2}\right)dx$$

$$= \frac{5}{3 \times 2}x^{1+1} + \frac{x^{-2+1}}{-2+1} + c$$

$$= \frac{5}{6}x^2 - x^{-1} + c$$

$$= \frac{5}{6}x^2 - \frac{1}{x} + c$$

The correct answer is **D**.

Question 7

$$\frac{dA}{dt} = \frac{4}{\sqrt{t}}$$

$$= 4t^{-\frac{1}{2}}$$

$$A = \int 4t^{-\frac{1}{2}}dt$$

$$= \frac{4t^{-\frac{1}{2}+1}}{\frac{1}{2}} + c$$

$$= 8\sqrt{t} + c$$

When $A = 20,\ t = 9$.

$$20 = 8 \times \sqrt{9} + c$$
$$= 24 + c$$
$$c = -4$$
$$\therefore A = 8\sqrt{t} - 4$$
When $t = 25$
$$A = 8\sqrt{25} - 4$$
$$= 36$$
The correct answer is **D**.

Question 8

$$\frac{dy}{dx} \propto x^3$$
$$\frac{dy}{dx} = kx^3$$
Gradient is -24 when $x = 2$.
$$-24 = k(2)^3$$
$$8k = -24$$
$$k = -3$$
The correct answer is **B**.

Question 9

$a = t - 2,\ t \geq 0$
$$v = \int (t - 2)dt$$
$$= \frac{t^2}{2} - 2t + c$$
$v = 8$ when $t = 0$
$$\therefore v = \frac{t^2}{2} - 2t + 8$$
When $t = 4$
$$v = \frac{4^2}{2} - 2(4) + 8$$
$$= 8\text{ m/s}$$
The correct answer is **D**.

Question 10

$$\int_1^4 \frac{x^4 + 4x^3 - 8}{x^3} dx = \int_1^4 \left(\frac{x^4}{x^3} + \frac{4x^3}{x^3} - \frac{8}{x^3}\right) dx$$
$$= \int_1^4 \left(x + 4 - 8x^{-3}\right) dx$$
$$= \left[\frac{x^2}{2} + 4x - \frac{8x^{-2}}{-2}\right]_1^4$$
$$= \left[\frac{x^2}{2} + 4x + \frac{4}{x^2}\right]_1^4$$
$$= \frac{4^2}{2} + 4(4) + \frac{4}{4^2} - \left(\frac{1}{2} + 4(1) + \frac{4}{1^2}\right)$$
$$= 8 + 16 + \frac{1}{4} - \frac{1}{2} - 4 - 4$$
$$= 15\frac{3}{4}$$
The correct answer is **A**.

Question 11

$\frac{dy}{dx} = 4x + b$ [**1 mark**]

Stationary point: $\frac{dy}{dx} = 0$ when $x = 2$

$0 = 4 \times 2 + b$

$\therefore b = -8$ **[1 mark]**

$\frac{dy}{dx} = 4x - 8$

$$\begin{aligned} y &= \int (4x - 8)dx \\ &= \frac{4x^2}{2} - 8x + c \\ &= 2x^2 - 8x + c \end{aligned}$$ [**1 mark**]

Substitute point $(2, 10)$

$$\begin{aligned} 10 &= 2(2)^2 - 8 \times 2 + c \\ &= 8 - 16 + c \\ c &= 18 \end{aligned}$$

Equation is $y = 2x^2 - 8x + 18$. **[1 mark]**

Question 12

$\frac{dV}{dt} = -0.5t$

$$\begin{aligned} V &= \int -0.5\,t dt \\ &= -0.25t^2 + c \end{aligned}$$ [**1 mark**]

When $t = 0,\ V = 2550.$

$\therefore V = -0.25t^2 + 2550$ **[1 mark]**

When $V = 0$

$$\begin{aligned} 0 &= -0.25t^2 + 2550 \\ 0.25t^2 &= 2550 \\ t^2 &= 10\,200 \\ t &= \pm 101.00 \end{aligned}$$

$\therefore t = 101$ seconds, $t > 0$

Find time in minutes

$$\begin{aligned} t &= 101/60 \\ &= 1.68 \text{ min} \end{aligned}$$

