Exam practice

Short-answer questions

Technology free: 20 questions

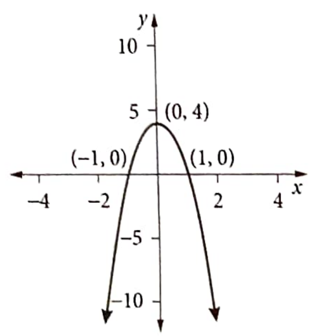
Solutions to this section start on page 205.

Question 1 (2 marks)

Sketch the graph of $y = \begin{cases} x^4, & x \geq 1 \\ -1, & x < 1 \end{cases}$.

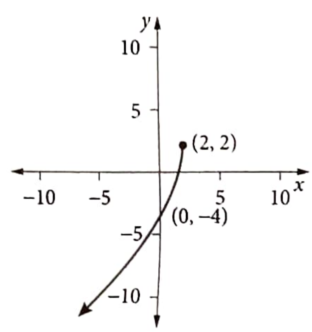
Question 2 (2 marks)

For the quadratic function shown, find the equation that defines the function.



Question 3 (3 marks)

For the square root function shown, determine the equation and domain that defines the function.



Question 4 (4 marks)

a Sketch the graph of $f(x) = 1 - 5x$, labelling all axial intercepts. 2 marks

b On the same set of axes, sketch the graph of the inverse function f^{-1}, labelling all axial intercepts clearly. 2 marks

Question 5 (3 marks)

State the amplitude, period and the range of the graph of $y = 3\sin\left(4x - \frac{\pi}{3}\right)$.

Question 6 (4 marks)

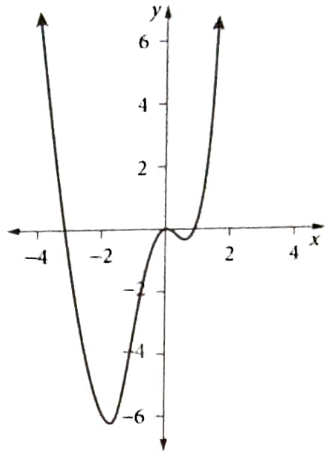
If the graph of $y = \sqrt{x}$ is reflected in the x-axis, dilated by a factor of 2 from the y-axis, and then translated 1 unit in the negative direction of the y-axis,

a give the equation of the image graph 1 mark

b sketch both $y = \sqrt{x}$ and the image graph on the same set of axes. 3 marks

Question 7 (2 marks)

A quartic function is shown.



Sketch the inverse relation on the same set of axes.

Question 8 (2 marks)

Sketch the graph of $y = 3e^{x+1} + 1$, labelling intercepts with the axes and giving the equations of any asymptotes.

Question 9 (2 marks)

Sketch the graph of $y = 3\log_e(2 - x)$ labelling intercepts with the axes and giving the equations of any asymptotes.

Question 10 (4 marks)

Sketch $y = \sin(x)$ and $y = \sin(2x)$ on the same set of axes for $0 \le x \le 2\pi$. Hence, using addition of ordinates, sketch the graph of $y = \sin(x) + \sin(2x)$.

Question 11 (4 marks)

If $f(x) = x^2$ and $g(x) = e^{x-2}$, state, with reason, whether or not the functions $f(g(x))$ and $g(f(x))$ exist.

Question 12 (4 marks)

a Sketch the graph of $y = 5\log_2(2 - x) + 1$, giving the equations of any asymptotes and labelling any axial intercepts. 2 marks

b State the domain and range of the graph. 2 marks

Question 13 (4 marks)

a Sketch the graph of $y = -2\tan(\pi x)$ for $-1 \le x \le 1$, giving the equations of any asymptotes and labelling any axial intercepts. 3 marks

b State the domain and range of the graph. 1 mark

Question 14 (4 marks)

a Sketch the graph of $y = 3^{-x}$ on a set of axes.

b On the same set of axes, sketch the graph of $y = -\log_3(x)$.

Question 15 (3 marks)

A decreasing exponential function of the form $y = ae^{-x} + b$ has a horizontal asymptote at $y = -2$ and passes through the origin. Write down the equation of the function.

Question 16 (4 marks)

a A cubic function of the form $y = a(x - b)(x - c)(x - d)$ has x-intercepts at $x = 3$ and $x = 2$, and passes through the points $(0, 5)$ and $(1, 1)$. Find the equation of the function. 2 marks

b Hence sketch the function, labelling axial intercepts. 2 marks

Question 17 (3 marks)

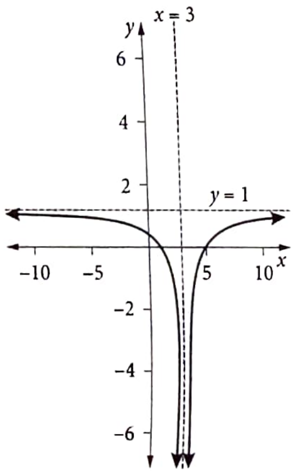
If $u(x) = 3x - 1$ and $g(u(x)) = [u(x)]^2$,

a find the rule for $h(x) = g(u(x))$ 2 marks

b find $h(2)$. 1 mark

Question 18 (6 marks)

The graph of $f(x) = \dfrac{-2}{(x-3)^2} + 1$ is shown below.



a Sketch $y = f(-x)$. 2 marks

b Sketch $y = f(1 - x)$. 2 marks

c Sketch $y = -f(-x)$. 2 marks

Question 19 (3 marks)

The coordinates of A and B are $(-6, 7)$ and $(2, -9)$, respectively. A line that is perpendicular to the line $y = -4x + 1$ passes through the midpoint of A and B.

Find the equation of the line.

Question 20 (5 marks)

a Sketch the graph of

$$y = \begin{cases} -4, & x \leq -1 \\ x + 1, & -1 < x < 3 \\ -\frac{2}{3}x + 6, & x \geq 3 \end{cases}$$

labelling the coordinates of any axial intercepts and endpoints. 3 marks

b Evaluate $f(-2), f(0)$ and $f(6)$. 1 mark

c Find the x value(s) for which $f(x) = 1$. 1 mark

Multiple-choice questions

Technology active: 50 questions

Solutions to this section start on page 208

Question 1

The linear function shown has a gradient of

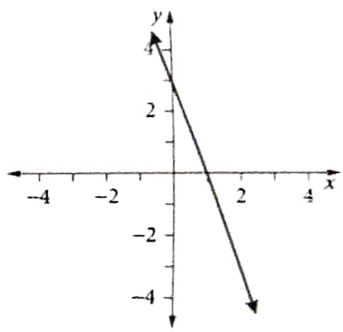
A -3

B -1

C 1

D 2

E 3



Question 2

The quadratic function shown could have a stationary point at

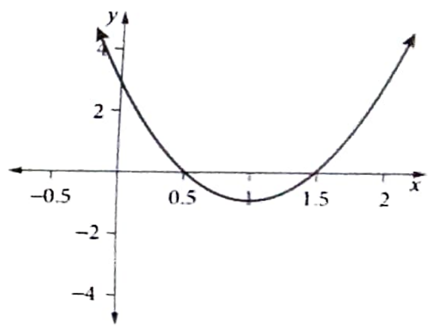
A $(0, 3)$

B $(0.5, 0)$

C $(0.5, 1.5)$

D $(1, -1)$

E $(1.25, -1)$



Question 3

Which of the following is **not** a function?

A $\{(x, y): y = x^2 - 3x\}$

B $\{(x, y): y = -\sqrt{3x - 1}\}$

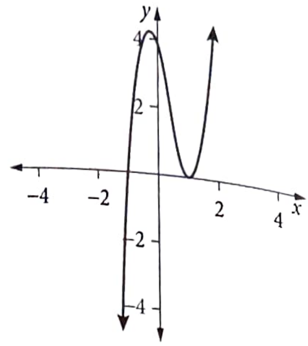
C $\{(x, y): y = 3x - 1\}$

D $\{(x, y): (0, 1), (1, 2), (2, 5), (4, 17)\}$

E $\{(x, y): y = \pm\sqrt{x + 3}\}$

Question 4

A possible equation for the graph shown could be



A $y = -(x+1)(x-1)(x+1)$

B $y = (x+1)(x-1)^2$

C $y = 4(x+1)(x-1)^2$

D $y = -(x+1)^2(x-1)(x-2)$

E $y = -2(x+1)(x-1)^2$

Question 5

The inverse of $\{(-1,4), (0,3), (1,1), (2,-1)\}$ is

A $\{-1,0,1,2\}$

B $\{4,3,1,-1\}$

C $\{(4,-1), (3,0), (1,1), (-1,2)\}$

D $\{(2,-1), (1,1), (0,3), (-1,4)\}$

E $\{(-1,4), (0,3), (1,1), (2,-1)\}$

Question 6

If $f(x) = 0.5x - 3$, then the rule of the inverse function f^{-1} is

A $f^{-1}(x) = \frac{x}{2} + 3$

B $f^{-1}(x) = \frac{x}{2} - 3$

C $f^{-1}(x) = \frac{1}{0.5x - 3}$

D $f^{-1}(x) = 2x + 6$

E $f^{-1}(x) = \frac{x+6}{2}$

Question 7

If $f: (0,3) \to R$, where $f(x) = 0.5x - 3$, then the inverse function f^{-1} is defined by

A $f^{-1}(x) = \frac{x}{2} + 3, x \in (0,3)$

B $f^{-1}(x) = \frac{x}{2} - 3, x \in (0,3)$

C $f^{-1}(x) = \frac{1}{0.5x - 3}, x \in (1.5,3)$

D $f^{-1}(x) = 2x + 6, x \in (0,3)$

E $f^{-1}(x) = 2x + 6, x \in (-3,-1.5)$

Question 8

The range of $\left\{(x,y): y = -\sqrt{9 - x^2}\right\}$ is given by

A $[-3,0]$

B $[0,3]$

C $(-3,0)$

D $[-9,0]$

E $(-\infty,3)$

Question 9

The graph of $y = \frac{1}{(2x-5)^2} + 2$ has asymptotes at

A $x = 2, y = \frac{5}{2}$

B $x = -\frac{2}{5}, y = -2$

C $x = -\frac{5}{2}, y = 2$

D $x = \frac{5}{2}, y = 2$

E $x = 5, y = 2$

Question 10

If $f(x) = \frac{1}{x-1} - 1$, then the maximal domain is

A $\{x: x \geq 1\}$

B $\{x: x < 1\}$

C $R\backslash\{0\}$

D $R\backslash\{1\}$

E $R\backslash\{-1\}$

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Question 11

The graph of $y = -e^{\frac{x}{2}} + 1$ cuts the x-axis at

A 1 **B** 0 **C** $\log_e\left(\frac{1}{2}\right)$ **D** $\log_e(2)$ **E** $\log_e(1)$

Question 12

The coordinates of the point of intersection of the graphs of $y = 3e^{-x} + 2$ and $y = 5$ are

A $(0,1)$ **B** $(0,3)$ **C** $(0,5)$ **D** $(0,7)$ **E** $(5,0)$

Question 13

If $f(x) = x + 2$ and $g(x) = x^2$, then $g(f(x))$ equals

A $x^2 + 2$

B x^4

C $(x+2)^4$

D $x + 4$

E $x^2 + 4x + 4$

Question 14

If $f(x) = 2x + 1$ and $g(x) = 3\log_e(x)$, when testing the existence of $f(g(x))$ it is true that

A $R \setminus \{0\} \subseteq R$

B $(-\infty, 1) \not\subseteq R \setminus \{0\}$

C $R \subseteq R$

D $(0, \infty) \not\subseteq R \setminus \{0\}$

E $R \subseteq R \setminus \{0\}$

Question 15

The period of $y = 2\sin(\pi x)$ is

A 1 **B** 2 **C** $\frac{\pi}{2}$ **D** π **E** 2π

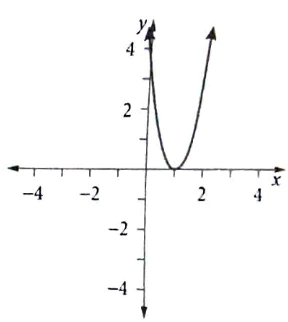
Question 16

The period of $y = -3\tan(\pi x)$ is

A 1 **B** 2 **C** $\frac{\pi}{2}$ **D** π **E** 2π

Question 17

The graph shown could be modelled by which of the following functions, where $a \in R$?



A $y = a(x-2)^2 + 3$

B $y = ax(x-1)^2$

C $y = a(x-1)^2 + 2$

D $y = a(x+1)^2 + 3$

E $y = 2a(x-1)^2$

Question 18

The transformations required to change $y = \sqrt{x}$ to $y = 2 + 3\sqrt{x-1}$ are

A dilation by a factor of 2 from the x-axis, translation of 1 unit to the right, and 3 units down

B dilation by a factor of 3 from the y-axis, translation of 1 unit to the right, and 2 units up

C dilation by a factor of 3 from the y-axis, translation of 1 unit to the left, and 2 units up

D dilation by a factor of 3 from the x-axis, translation of 1 unit to the right, and 2 units up

E dilation by a factor of $\frac{1}{3}$ from the y-axis, translation of 1 unit to the right, and 2 units up

Question 19

If $y = g(x)$ for $x \in [-1,3]$ and $y = h(x)$ for $x \in [0,4]$, then $y = 2g(x) - 4h(x)$ is defined for the domain

A $x \in (0,6)$

B $x \in [0,3]$

C $x \in [-1,4]$

D $x \in [-2,0]$

E $x \in (-1,4)$

Question 20

A piecewise function is given by

$$f(x) = \begin{cases} x+2, & x \le 0 \\ \dfrac{x^2}{4}, & 0 < x \le 1 \\ -3, & x > 1 \end{cases}$$

The value of $f(3)$ is

A -3

B $\frac{9}{4}$

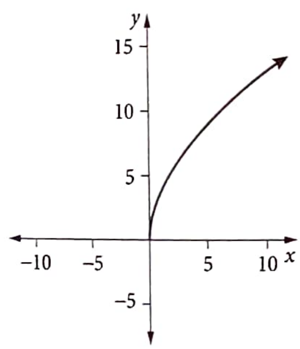
C 3

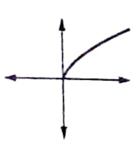
D 5

E 9

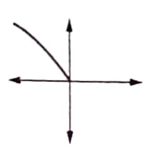
Question 21

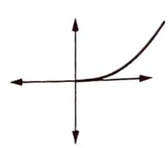
The graph of a function is shown. Which of the following is most likely to be the graph of the inverse function?

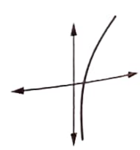


A


B


C


D


E


Question 22

If $f(x) = -\sqrt{2(x+2)} - 1$, then the range of the inverse function f^{-1} is

A $(-\infty, 1)$

B $(-\infty, -1)$

C $(-1, \infty)$

D $[-4, \infty)$

E $[-2, \infty)$

Question 23

The graph of $y = 2(2^{x-1}) - 8$ has x and y axes intercepts, respectively, as

A $x = 3, y = -7$

B $x = \frac{1}{2}, y = -4$

C $x = 1, y = -8$

D $x = \frac{1}{2}, y = 4$

E $x = 7, y = -3$

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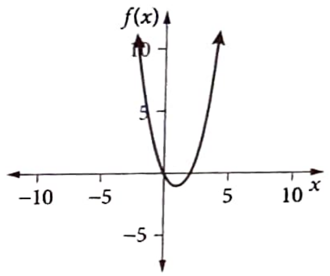
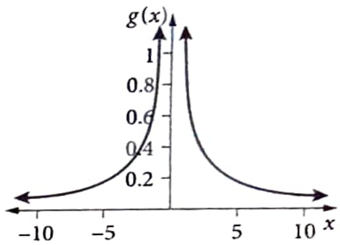
Question 24

Let $f(x) = x^2 - 2x$ and $g(x) = \frac{1}{x^2}$ for their maximal domains.





To find $f(g(x))$, the incorrect statement is

A $\text{dom } f(g(x)) = \text{dom } g = R \setminus \{0\}$

B $\text{dom } f(g(x)) = \text{dom } f = R$

C $\text{dom } f = R$, $\text{ran } f = [-1, \infty)$

D $\text{dom } g = R \setminus \{0\}$, $\text{ran } g = (0, \infty)$

E $f(g(x))$ is defined since $(0, \infty) \subseteq R$

Question 25

The line $2y - 6 = x$ passes through the point $\left(\frac{1}{2}, b\right)$. The value of b is

A $-\frac{11}{4}$ **B** $-\frac{5}{4}$ **C** $\frac{11}{4}$ **D** $\frac{13}{4}$ **E** 7

Question 26

The range of the piecewise function $f(x) = \begin{cases} x^2 + 2x, & x \le -1 \\ 2x^3, & x > -1 \end{cases}$ is

A $(-\infty, 1)$ **B** $(-2, \infty)$ **C** $(-1, \infty)$ **D** $[-4, \infty)$ **E** $[-2, \infty)$

Question 27

The rule of the image of $y = x^{\frac{2}{3}}$ when there is a dilation by a factor of 3 from the y-axis is

A $y = x$ **B** $y = x^{\frac{2}{3}}$ **C** $y = 3x^{\frac{2}{3}}$ **D** $y = \left(\frac{x}{3}\right)^{\frac{2}{3}}$ **E** $y = (3x)^{\frac{2}{3}}$

Question 28

The period and amplitude of the graph of $y = -3\sin(2x)$ are

	Period	Amplitude
A	π	−3
B	2	3
C	2π	−3
D	4π	3
E	π	3

Question 29

The rule of the function $y = -\frac{2}{\sqrt{x}}$ when it is reflected in the x-axis is

A $y = \frac{2}{\sqrt{x}}$ **B** $y = \frac{2}{\sqrt{-x}}$ **C** $y = -\frac{2}{\sqrt{-x}}$ **D** $y = \frac{1}{\sqrt{-x}}$ **E** $y = -\frac{1}{\sqrt{x}}$

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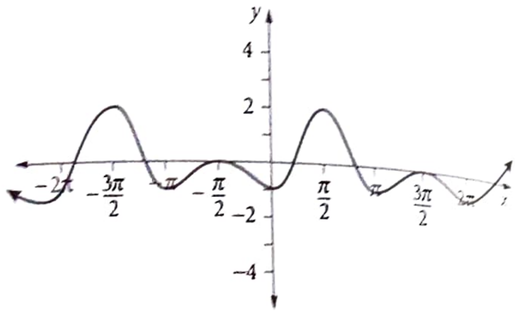
Question 30

The graph of $y = \sin(x) - \cos(2x)$ is shown.



The range of the graph is

A $(-\infty, 2)$ B $[-1.125, 2]$

C $(-1, \infty)$ D $[-1.125, \infty)$

E $[-1, 2)$

Question 31

The graph of $y = 3\tan(3x)$ for domain $x \in [0, \pi]$ has asymptotes at

A $x = -\frac{\pi}{2}, \frac{\pi}{2}$ B $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ C $x = \frac{\pi}{2}, \frac{3\pi}{2}$

D $x = \frac{\pi}{4}, \frac{3\pi}{4}$ E $x = \frac{\pi}{3}, \frac{2\pi}{3}$

Question 32

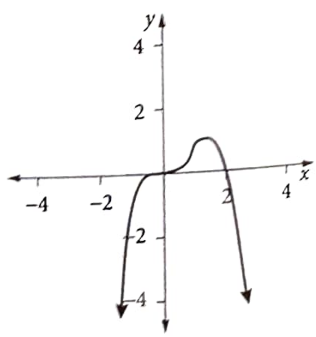
The image of the point $(-2, 3)$ under the transformations:

reflection in y-axis, then dilation by a factor of 5 from the x-axis, is

A $(2, -3)$ B $(-2, 3)$ C $(-10, 3)$ D $(2, 15)$ E $(-2, 30)$

Question 33

The equation of the graph shown, where a is positive, could be given by



A $y = -ax^3(x-2)$

B $y = -x^3(x-a)$

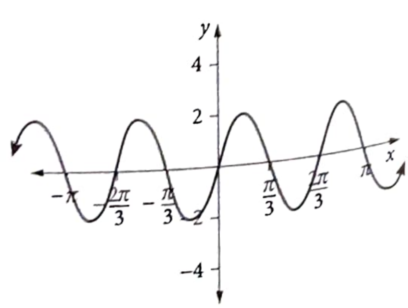
C $y = ax(x-2)^3$

D $y = x^3(x-a)$

E $y = ax^2(x-2)^2$

Question 34

The graph shown is of the form $y = a\sin(n(x-b))$.



The graph has the equation

A $y = 2\sin(3(x-\pi))$

B $y = -2\sin(3(x-\pi))$

C $y = -2\sin(3x)$

D $y = -2\sin(2(x-\pi))$

E $y = \sin(3(x-\pi))$

Question 35

If $f(x) = 3\log_e(x - 2)$, then which of the following statements is **not** true?

A $3f(x + 1)$ has a vertical asymptote at $x = 1$

B $f(x - 1)$ has a vertical asymptote at $x = 3$

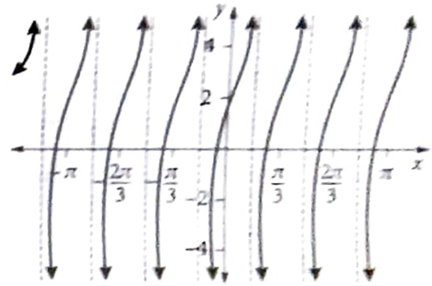
C $f(x) + 1$ has a vertical asymptote at $x = 2$

D $f(x)$ and $2f(x)$ have the same vertical asymptote at $x = 2$

E $f(x)$ and $f(3x)$ have the same vertical asymptote at $x = 2$

Question 36

The graph shown has the equation



A $y = \tan\left(\frac{x}{3}\right)$

B $y = 3\tan(3x)$

C $y = 3\tan(2x)$

D $y = \tan(3x) + 2$

E $y = \tan\left(\frac{x}{3}\right) + 2$

Question 37

If $f(x) = 10x - 2$ and $g(x) = f(f(x))$, then the rule for the inverse function g^{-1} is given by

A $g^{-1}(x) = \frac{x}{100} + \frac{11}{50}$

B $g^{-1}(x) = 10x - 2$

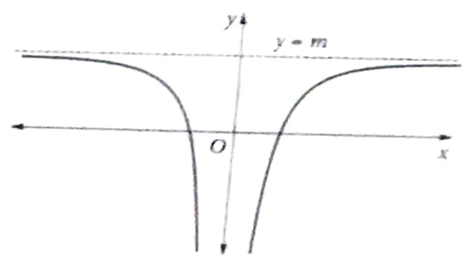
C $g^{-1}(x) = \frac{1}{10x - 2}$

D $g^{-1}(x) = 100x - 22$

E $g^{-1}(x) = \frac{1}{100x - 22}$

Question 38

A possible equation for the graph below, where $m, n > 0$, is



A $y = m + \frac{n}{x^2}$

B $y = m - \frac{n}{x}$

C $y = m - \frac{n}{x^2}$

D $y = \frac{n}{x^2} - m$

E $y = -\frac{n}{x^2} - m$

Question 39

The graph of $y = -\cos\left(4x - \frac{\pi}{2}\right)$ has been transformed from the graph of $y = \cos(x)$ by

A a reflection in the x-axis, dilation by a factor of $\frac{1}{4}$ from the y-axis and a horizontal translation of $\frac{\pi}{8}$ units to the right

B a reflection in the x-axis, dilation by a factor of $\frac{1}{4}$ from the y-axis and a horizontal translation of $\frac{\pi}{2}$ units to the right

C a reflection in the x-axis, dilation by a factor of 4 from the y-axis and a horizontal translation of $\frac{\pi}{8}$ units to the right

D a reflection in the x-axis, dilation by a factor of 4 from the y-axis and a horizontal translation of $\frac{\pi}{2}$ units to the right

E a reflection in the y-axis, dilation by a factor of $\frac{1}{4}$ from the x-axis and a horizontal translation of $\frac{\pi}{8}$ units to the right

Question 40

The function $f(x) = 3x^2$ is dilated by a factor of 2 from the x-axis and reflected in the y-axis and then translated 1 unit down. The image equation that results is

A $y = 6x^2$

B $y = 3x^2$

C $y = 3x^2 - 1$

D $y = 6x^2 - 1$

E $y = -6x^2 - 1$

Question 41

The graph of $y = \log_e(x)$ has been transformed to the graph of $y = \log_e(5 - 2x)$. The transformations required were

A reflection in the y-axis, dilation by a factor of 2 from the x-axis, translation of $\frac{5}{2}$ units to the left

B reflection in the y-axis, dilation by a factor of 2 from the x-axis, translation of $\frac{5}{2}$ units to the right

C reflection in the x-axis, dilation by a factor of 2 from the x-axis, translation of $\frac{5}{2}$ units to the right

D reflection in the x-axis, dilation by a factor of $\frac{1}{2}$ from the x-axis, translation of $\frac{5}{2}$ units to the left

E reflection in the y-axis, dilation by a factor of $\frac{1}{2}$ from the y-axis, translation of $\frac{5}{2}$ units to the right

Question 42

The transformation which maps the curve with equation $y = \sin(x)$ to the curve with equation $y = 2\sin(x - 3) + 1$ follows the mapping notation

A $(x, y) \rightarrow (x + 3, 2y + 1)$

B $(x, y) \rightarrow (x - 3, 2y + 1)$

C $(x, y) \rightarrow (x + 3, 2y - 1)$

D $(x, y) \rightarrow \left(x + 3, \frac{1}{2}y - 1\right)$

E $(x, y) \rightarrow \left(x - 3, \frac{1}{2}y - 1\right)$

Question 43

The range of $y = 11 - 4\cos(\pi(3 - x))$ is

A $[-11, 11]$

B $[7, 11]$

C $[7, 15]$

D $[15, 24]$

E $[3, \pi]$

Question 44

A model which fits the graph shown is of the form $f(x) = A\sin(nx) + B$.

The values of A, n and B, respectively are

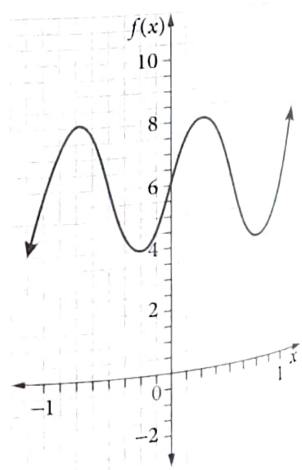
A $-2, 1, 4$

B $-2, 1, 6$

C $-2, 2\pi, 6$

D $2, 2\pi, 6$

E $2, 1, 8$



Question 45

For the transformations in the order

- dilate by 2 units from the y-axis
- reflect over the y-axis
- translate by 1 unit in the negative direction of the x-axis,

the image of the curve with equation $y = x^4$ is

A $y = \frac{(x+1)^4}{16}$ **B** $y = -\frac{(x+1)^4}{16}$ **C** $y = 2(2x+1)^4$

D $y = \frac{(x+1)^4}{4}$ **E** $y = -\frac{(x+1)^4}{4}$

Question 46

$y = ax^n e^{-kx} + b$ could be given as a possible model for the amount of medication remaining in the bloodstream after a dose of medication.

For the values $a = 2$ and $n = \frac{1}{2}$, the graph goes through the points $(0, 1)$ and $(2, 2)$. The values of k and b are

A $k = \frac{3}{4}, b = 1$ **B** $k = \frac{3}{4}\log_e(2), b = 1$ **C** $k = \frac{3}{4}\log_e(2), b = \log_e(2)$

D $k = \frac{3}{4}\ b = 2$ **E** no solution

Question 47

The graphs of $f^{-1}(x) = \sqrt{2x-1}$ and $f(x)$ are given.

The point(s) of intersection between f and f^{-1} is/are

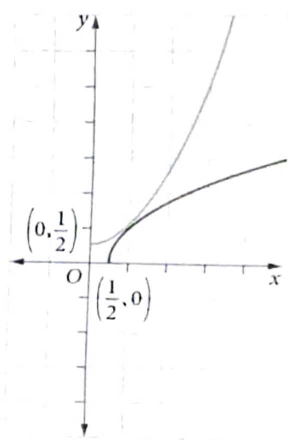
A $(-\sqrt{2}, \sqrt{2})$

B $(2+\sqrt{2}, \sqrt{2})$

C $(2, 2)$

D $(1, 1)$ and $\left(\frac{1}{2}, \frac{1}{2}\right)$

E $(1, 1)$



Question 48

The equation of the graph shown could be

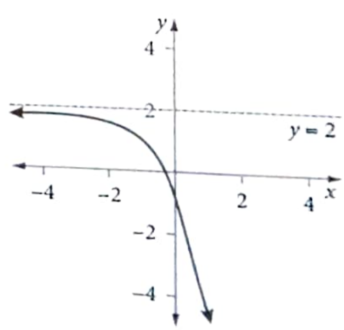
A $y = 3\log_e(x-2)$

B $y = -3\log_e(x) + 2$

C $y = -3e^{-x} + 2$

D $y = -3e^{x} + 2$

E $y = 3e^{-x} + 2$



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Question 49

The equation of the parabola that joins the points $(0,1)$, $(2,3)$, $(5,7)$ is

A $y = \frac{x^2 + 13x + 15}{15}$

B $y = \frac{x^2 + 13x + 1}{15}$

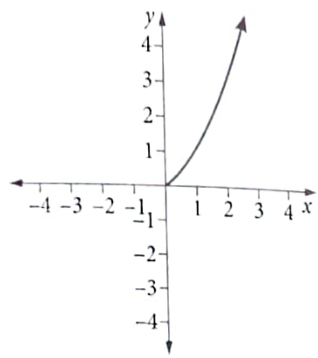
C $y = \frac{3x^2}{8} - \frac{x}{2} + \frac{1}{8}$

D $y = -\frac{x^2}{24} + \frac{7x}{6} - \frac{9}{8}$

E $y = -\frac{5x^2}{12} + \frac{17x}{4} - \frac{23}{6}$

Question 50

The equation of the graph shown could be



A $y = x^2$

B $y = x^{\frac{2}{3}}$

C $y = x^{\frac{3}{2}}$

D $y = 2x^2$

E $y = x^3$

Extended-answer questions

Technology active: 5 questions

Solutions to this section start on page 211

Question 1 (15 marks)

A polynomial is defined as $P(x) = ax^3 + bx^2 + 2x + 10$.

a It is known that $x - 2$ is a factor of $P(x)$ and that if $P(x)$ is divided by $x + 1$, the remainder is -3. Show that $a = \frac{5}{2}$ and $b = -\frac{17}{2}$. 3 marks

b Hence find the solutions for the equation $y = P(x) = 0$. 3 marks

c Sketch the graph of $y = P(x)$, labelling the coordinates of any axial intercepts. 4 marks

A section of the graph drawn in part c represents a proposed ski slope, where y metres is the height above ground level and x metres is the horizontal distance from the end of the ski slope. The ski slope ends at the point $(0, 10)$.

Consider $f: [0, 5] \rightarrow R, f(x) = \frac{5}{2}x^3 - \frac{17}{2}x^2 + 2x + 10$

d Find the maximum height of the ski slope. 2 marks

e How high will the engineers have to build the platform for the end of the slope? 1 mark

f The engineers do not permit the ski slope to be opened if the path goes underground. Will the ski slope go ahead? 2 marks

Question 2 (10 marks)

A jet of water from a hose is pointed over the fence by children into their neighbour's garden. The water stream from the hose starts at a distance from the ground of 2.88 metres. Amy, who is holding the hose, is standing at a horizontal distance of 14 metres from the fence. The water goes over the fence and lands on the other side in the neighbour's garden at a horizontal distance of 4 metres from the fence.

a Use the function $h(x) = a + b(x+6)^2$ to model the path of the water in the air, where a and b are constants and the variables, h and x, are in metres. Find the values of a and b. 3 marks

b Sketch the function $h(x) = a + b(x+6)^2$ that models the path of the water in the air, with your values of a and b. Use the domain $x \in [-14, 4]$. 2 marks

c The neighbours are sick of the children next door and have built a fence that is 5 metres high. Will the water go over the fence? Why/why not? 2 marks

d There are power lines above the children's garden that are 10 metres off the ground. Will the water hit the power lines? 1 mark

e The children's dog likes to jump up and lap at the water in the air. The dog, Scruffy, can jump 2 metres in the air to reach the water. Where does Scruffy manage to drink the water? 2 marks

Question 3 (12 marks)

a If $3x^3 + ax^2 + 5x = 3(x+b)^3 + c$, find all possible values of a, b and c. 4 marks

b Letting $f_1(x) = 3(x+b)^3 + c$, and using values from part **a**, where $b < 0$ and $c > 0$, find the x coordinates of the point(s) where $f(x) = 0$. 1 mark

c Letting $f_2(x) = 3(x+b)^3 + c$, and using values from part **a**, where $b > 0$ and $c < 0$, find the x coordinates of the point(s) where $f(x) = 0$. 1 mark

d Hence, find where $f_1(x) = f_2(x)$. 1 mark

e Sketch the graphs of $f_1(x)$ and $f_2(x)$ on the same axes, labelling point(s) of intersection. 3 marks

f Sketch the inverse of graph $f_1(x)$ on the same axes, labelling point(s) of intersection. 2 marks

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Question 4 (9 marks)

The temperature of a cup of tea cools according to the rule $T = T_0 \times 2^{-kt}$, where T is the temperature in degrees Celsius and t is time in hours. The original temperature of the cup of tea is 90°C.

a What is T_0? 1 mark

It takes 30 minutes for the temperature to halve.

b What fraction of the original temperature is the temperature of the cup of tea after 60 minutes? 3 marks

c What is the temperature of the cup of tea after 60 minutes? 1 mark

d A different drink's temperature follows the formula $T = T_0 \times 2^{-kt} + 20$, where T is the temperature in degrees Celsius and t is time in hours. It takes 20 minutes for the temperature to halve and the original temperature of the cup of tea is 90°C.

Find the value of k in this case, giving your answer correct to two decimal places. 2 marks

The formula is changed to suit another drink. The graph of $T = T_0 \times 2^{-kt}$ has a sequence of transformations applied to it, in the order given.

- The graph is translated 15 units in the negative direction of the t-axis.
- It is then translated 7 units in the negative direction of the T-axis.
- It is then dilated by a factor of 5 from the t-axis.
- It is then dilated by a factor of $\frac{1}{2}$ from the T-axis.
- The graph is then reflected in the T-axis.

e After all these transformations are applied, what is the new rule for $T(t)$? 2 marks

Question 5 (14 marks)

Consider the function $f(x) = ae^{x-2} + c$ where $a, c \in R$.

a State the type of function and the domain and range, in terms of c, of f. 3 marks

b Find the values of a and c if $f(0) = 2$ and $f(-2) = 0$. 2 marks

c Define the function f in the form f: dom $\rightarrow R, f(x) = \ldots$ 1 mark

d Sketch the graph of f labelling axial intercepts and the equation of the asymptote. 3 marks

e Find the rule and domain of the inverse f^{-1} of the function f. 2 marks

f Hence find the points of intersection, if any, between the graphs of f^{-1} and f. 1 mark

g Show your answer to part **f** on a graph. 2 marks

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Exam practice

Short-answer questions

Technology free: 22 questions

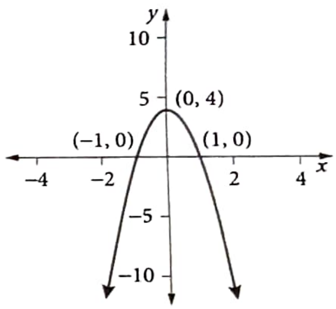
Solutions to this section start on page 215.

Question 1 (2 marks)

Solve for x: $\sqrt{9x - 4} = 2$

Question 2 (3 marks)

For the quadratic function shown, find



a the value of A so that the maximal domain, in the form of $[A, \infty)$, will allow the inverse function y^{-1} to exist — 1 mark

b the rule that defines the function y^{-1}. — 2 marks

Question 3 (1 mark)

Find the remainder if the polynomial $P(x) = 55 - 18x + 2x^2 - 3x^3 + x^4$ is divided by $x + 2$.

Question 4 (2 marks)

If $\log_e(y) = \log_e(x) + \log_e(p)$, write an equation relating x, y and p that does not involve logarithms.

Question 5 (1 mark)

Evaluate $\dfrac{\log_3(27)}{\log_3(9)}$.

Question 6 (2 marks)

Solve the equation $\sin(x°) = \dfrac{1}{\sqrt{2}}$ for $0° \le x° \le 180°$.

Question 7 (3 marks)

The height of water in a dam can be represented by a curve of the form $y = a\cos(kt) + c$, where t is in days after some rainfall. At $t = 0$, the height of water in the dam is 20 metres. At $t = 10$, the height of water in the dam is 60 metres. At $t = 20$, the height of water in the dam is back to 20 metres for the first time since $t = 0$.

a What is the value of k? — 1 mark

b What is the value of a? — 1 mark

c What is the value of c? — 1 mark

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Question 8 (1 marks)

Solve for x in the equation $\log_e(x^2) = 2$.

Question 9 (2 marks)

Find the rule that defines the inverse of the function $f(x) = 3 + \log_e(x + 2)$.

Question 10 (4 marks)

For $g: [D, \infty) \rightarrow R, g(x) = (x - 2)^2 + 3$,

a find the smallest value of D such that g^{-1} exists — 2 marks

b define $g^{-1}(x)$. — 2 marks

Question 11 (4 marks)

If $f(x) = x^2$ and $g(x) = \log_e(x - 2)$, state, with reasons, whether or not the functions $f(g(x))$ and $g(f(x))$ exist.

Question 12 (2 marks)

Solve the equation $x^3 - x^2 - x + 1 = 0$.

Question 13 (3 marks)

Find the value of x such that $9^x - 2(3^{x+1}) + 9 = 0$.

Question 14 (3 marks)

If $f(x) = x^3 + 1$ and $g(x) = \sqrt{x - 1}$, define $g(f(x))$ and $f(g(x))$.

Question 15 (3 marks)

Find the exact values of x in the domain $0 \le x \le 2\pi$ for which $2\sin^2(x) - \sin(x) - 1 = 0$.

Question 16 (3 marks)

A sinusoidal curve of the form $T = a\sin(nt) + b$ could be used to model the rise and fall of temperature on a newly-discovered planet. The temperature hits a maximum of 100°F after 1.5 months, and reaches the minimum of 40°F three months later. Find the rule that best models this data, where t is the time in months.

Question 17 (6 marks)

a Find the general solution to the equation $\sin\left(2\left(x - \frac{\pi}{2}\right)\right) = \frac{1}{\sqrt{2}}$. — 3 marks

b Hence, find all the exact solutions in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. — 3 marks

Question 18 (6 marks)

Find the value(s) of k for which these simultaneous linear equations have the following solutions:

$$(k - 1)x - 2y = 6$$
$$-x + ky = 3$$

a a unique solution — 2 marks

b no solutions — 2 marks

c infinitely many solutions — 2 marks

Question 19 (2 marks)

Find the quotient and remainder when the polynomial $Q(x) = x^3 - 9x^2 + 7x - 3$ is divided by $x - 3$.

Question 20 (3 marks)

If $3x^2 - 4x + 1 = a(x + b)^2 + c$, find the values of a, b and c.

Question 21 (3 marks)

Find the value(s) of k for which the following simultaneous linear equations have a unique solution.

$$kx - 9y = 5$$
$$x - ky = k$$

Question 22 (3 marks)

Find the value(s) of k for which the following simultaneous linear equations have infinitely many solutions.

$$kx - 9y = 5$$
$$x - ky = k$$

Multiple-choice questions

Technology active: 48 questions

Solutions to this section start on page 218.

Question 1

The linear function shown could have the equation

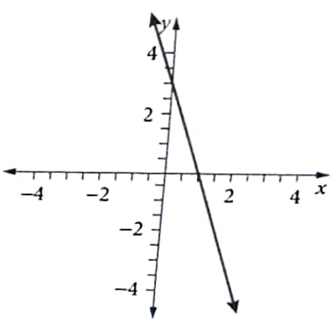
A $y = 2x + 4$

B $y = 2x - 4$

C $y = -2x + 4$

D $y = -3x + 3$

E $y = 3x + 3$



Question 2

The quadratic function shown could have the equation

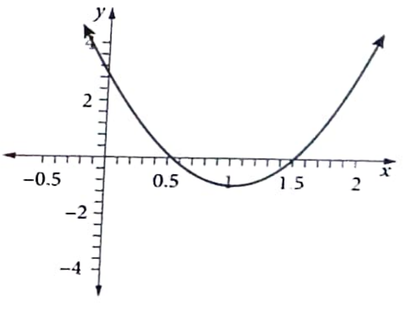
A $y = 2x^2 + 3$

B $y = (x - 0.5)(x + 1.5)$

C $y = 4(x - 1)^2 - 1$

D $y = (x - 1)^2 - 1$

E $y = x^2 - 1$



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Question 3

Which of the following is **not** a polynomial?

A $f(x) = 1 + 2x$

B $f(x) = -3 + 4x + 2x^2$

C $f(x) = 4x + 2x^2$

D $f(x) = 2x^2 - 8x^3$

E $f(x) = -\frac{1}{x} + x - 2$

Question 4

If $P(x) = -2x^3 + 7x - 5$, the following **correct** statement is

A $P(1) = 0$

B $(x + 1)$ is a factor

C $P(-1) = 0$

D $P(x) = -2(1)^3 + 7(1) - 5$

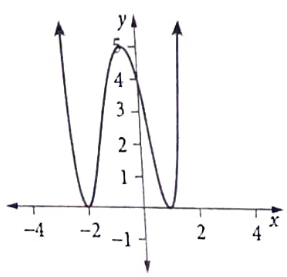
E $(x - 5)$ is a factor

Question 5

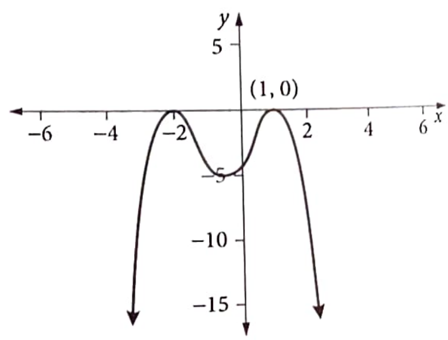
Let $Q(x) = -x^4 - 2x^3 + 3x^2 + 4x - 4$.

The graph that best represents this polynomial is

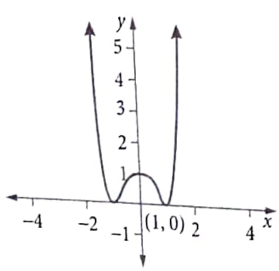
A



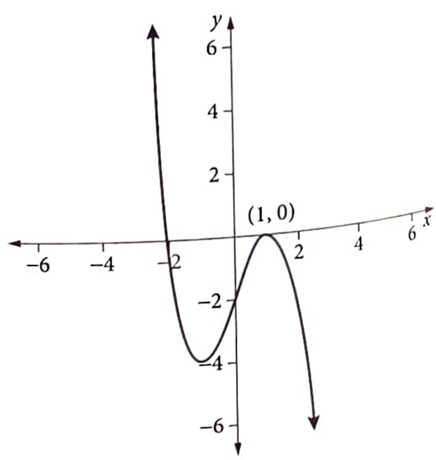
B



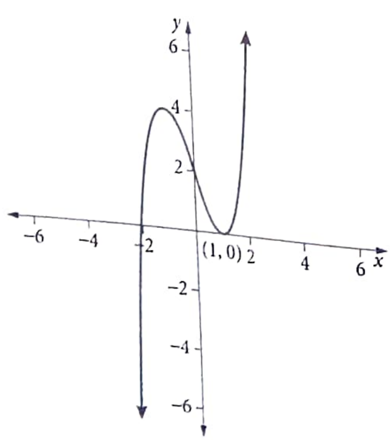
C



D



E



Question 6

One factor of the polynomial $Q(x) = -x^4 - 2x^3 + 3x^2 + 4x - 4$ is $(1 - x)$. The other factors are

A $(x-1)(x+2)$

B $(x-1)(1-x)(x+2)(x+2)$

C $(x-1)(1-x)(x-1)$

D $(x-1)(x+2)(x+2)$

E $-(x+2)$

Question 7

If $P(x) = -2(x-3)(x+2)(x-5)$, the y-intercept is at

A $(0,-60)$ **B** $(-60,0)$ **C** $(0,-30)$ **D** $(0,30)$ **E** $(0,60)$

Question 8

If $4x^2 - 3x + 1 = a(x-b)^2 + c$, the values of a, b and c are

A $a = 4, b = 3, c = 1$

B $a = 4, b = -\frac{3}{8}, c = \frac{7}{16}$

C $a = -4, b = \frac{3}{8}, c = -\frac{7}{16}$

D $a = 4, b = \frac{3}{16}, c = \frac{7}{16}$

E $a = 4, b = \frac{3}{8}, c = \frac{7}{16}$

Question 9

If $(ax + b)$ is a factor of $P(x)$, then

A $P(-b) = 0$ **B** $P\left(-\frac{b}{a}\right) = 0$ **C** $P\left(\frac{b}{a}\right) = 0$ **D** $P(a) = 0$ **E** $P\left(\frac{a}{b}\right) = 0$

Question 10

If we divide the polynomial $P(x) = x^3 + x^2 - 5x + 7$ by $(x+1)$, the remainder is

A 0 **B** 4 **C** 7 **D** 10 **E** 12

Question 11

If $f(x) + f(y) = f(xy)$, the rule for $f(x)$ could be

A $f(x) = x$

B $f(x) = x^2$

C $f(x) = e^x$

D $f(x) = \log_e(x)$

E $f(x) = \frac{1}{x}$

Question 12

If $f(x) \times f(y) = f(x+y)$, the rule for $f(x)$ could be

A $f(x) = x$

B $f(x) = x^2$

C $f(x) = e^x$

D $f(x) = \log_e(x)$

E $f(x) = \frac{1}{x}$

Question 13

Consider $f: (-\infty, A] \to R, f(x) = x^2 + 4x$. The maximal value of A for the inverse f^{-1} to exist is

A -4 **B** -2 **C** 0 **D** 2 **E** 4

Question 14

Consider $f: (-\infty, 1) \to R$, $f(x) = x^2 - 2x$. The rule and domain for the inverse function f^{-1} is

A $f^{-1}(x) = 1 + \sqrt{x+1}, x \in (-\infty, 1]$

B $f^{-1}(x) = 1 \pm \sqrt{x+1}, x \in (-\infty, 1]$

C $f^{-1}(x) = 1 - \sqrt{x+1}, x \in (-\infty, -1)$

D $f^{-1}(x) = 1 - \sqrt{x+1}, x \in (-1, \infty)$

E $f^{-1}(x) = 1 + \sqrt{x+1}, x \in (-1, \infty)$

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Question 15

For the function $f: (-\infty, 0] \to R, f(x) = x^2 - 4$, the point(s) of intersection, correct to two decimal places, between f and f^{-1} is/are

A $(-1.56, -1.56)$

B $(1.56, 1.56)$

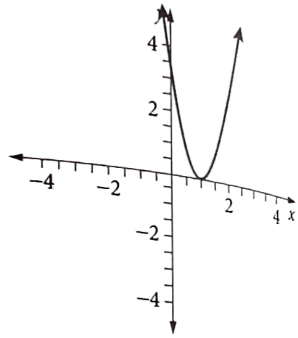
C $(-1.56, -1.56)$ and $(1.56, 1.56)$

D $(-2.30, 1.30)$

E $(2.56, 2.56)$

Question 16

The graph shown has the equation $y = 2a^2(x-1)^2$ where $a \in R$. The point(s) of intersection between this graph and the graph of $y = 2$ is/are



A $\left(\frac{a-1}{a}, 2\right), \left(\frac{a+1}{a}, 2\right)$

B $\left(\frac{a-1}{a}, 4\right), \left(\frac{a+1}{a}, 2\right)$

C $\left(\frac{a-1}{a}, \frac{a+1}{a}\right)$

D $\left(0, \frac{a+1}{a}\right), a+1, \left(0, \frac{a-1}{a}\right)$

E $\left(2, \frac{a-1}{a}\right), \left(2, \frac{a+1}{a}\right)$

Question 17

Written in radians, 25° is

A $\frac{\pi}{36}$

B $\frac{5\pi}{36}$

C 0.436 353

D 0.44

E $\frac{4500}{\pi}$

Question 18

Written in degrees, $\frac{\pi}{12}$ is

A $\frac{\pi^2}{2160}$

B 15°

C 25°

D 45°

E 90°

Question 19

$\sin\left(\frac{7\pi}{2}\right) =$

A −2

B −1

C 0

D $\frac{1}{2}$

E 1

Question 20

$\cos\left(\frac{7\pi}{2}\right) =$

A −2

B −1

C 0

D $\frac{1}{2}$

E 1

Question 21

If $f(x) = -2\sqrt{x+2} - 1$, then the point(s) of intersection between $f(x)$ and the function $g(x) = x + 1$ is/are

A $(-2, 0)$

B $(2, 0)$

C $(-2, -1)$

D $(-2, -1)$ and $(-2, 0)$

E $(-2, -1)$ and $(2, 3)$

Question 22

Consider the graph of $y = 2(2^{x-1}) - 8$. Its inverse graph has the rule

A $y^{-1} = \frac{\log_e(x+8)}{\log_e(2)}$

B $y^{-1} = \log_2(x+8)$

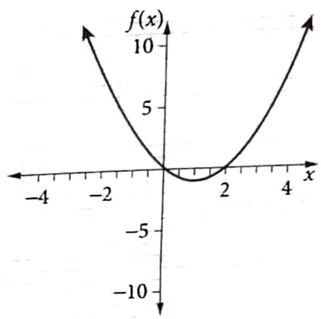
C $y^{-1} = \frac{\log_2(x+8)}{\log_2(2)}$

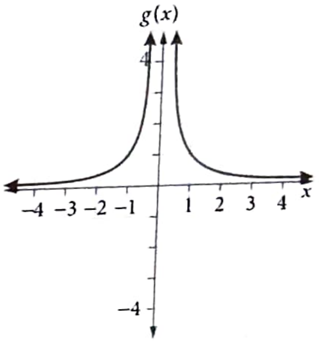
D $y^{-1} = \log_e\left(\frac{x}{2} + 4\right)$

E $y^{-1} = \log_2(x-8)$

Question 23

Let $f(x) = x^2 - 2x$ and $g(x) = \frac{1}{x^2}$ for a maximal domain.





$g(f(x))$ is

A $g(f(x)) = \frac{1}{(x^2-2x)^2}$ for $x \in (0, \infty)$

B $g(f(x)) = \frac{1}{(x^2-2x)^2}$ for $x \in R \setminus \{0, 2\}$

C $g(f(x)) = \frac{1}{(x^2-2x)^2}$ for $x \in R$

D $g(f(x)) = \frac{1}{x^2-2x}$ for $x \in R$

E $g(f(x)) = x^4 - 2x^2$ for $x \in (1, \infty)$

Question 24

The line $2y - 6x = 3$ has an x-intercept of

A $x = -3$

B $x = -\frac{1}{2}$

C $x = -\frac{1}{6}$

D $x = \frac{9}{2}$

E $x = 3$

Question 25

An equivalent expression for $\sin\left(\frac{7\pi}{6}\right)$ is

A $\sin\left(\frac{\pi}{6}\right)$

B $\sin\left(\frac{5\pi}{6}\right)$

C $\cos\left(\frac{\pi}{6}\right)$

D $\cos\left(\frac{\pi}{3}\right)$

E $-\sin\left(\frac{\pi}{6}\right)$

Question 26

An equivalent expression for $\log_x(25) = 2$ is

A $x^2 = 25$

B $2^x = 25$

C $\log_5(25) = 2$

D $\log_2(25) = x$

E $5^2 = 25$

Question 27

If $9^{x-2} \times 3^x = 9$, then x equals

A $x = 2$

B $x = 2.5$

C $x = 3$

D $x = 9$

E $x = 25$

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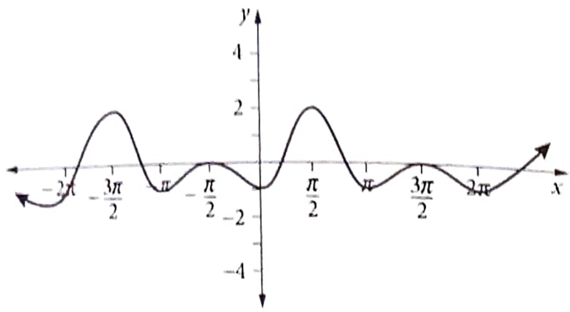
Question 28

The maximal domain for the function $f(x) = -\frac{2}{\sqrt{x}}$ is

A $x > 0$ **B** $x < -2$ **C** $x \geq -2$ **D** $x \leq 0$ **E** $x \geq 0$

Question 29

The graph of $y = \sin(x) - \cos(2x)$ is shown.



The solution(s) for the equation $y = 0$ for the domain $(\pi, 2\pi)$ is/are

A $x = \frac{\pi}{2}, \frac{3\pi}{2}$ **B** $x = \frac{\pi}{2}$ **C** $x = \frac{27\pi}{25}$ **D** $x = -\frac{\pi}{2}$ **E** $x = \frac{3\pi}{2}$

Question 30

A simplified version of the expression $2\log_3(x) - \log_3(x+1) + 2\log_3(x+4)$ is

A $\log_3\left(\frac{x^2+4x}{x+1}\right)$ **B** $\log_3\left(\frac{x^2(x+4)^2}{x+1}\right)$ **C** $\frac{x^2(x+4)^2}{x+1}$

D $\log_x\left(\frac{x^2+4x}{x+1}\right)$ **E** $\log_3\left(\frac{8x}{x+1}\right)$

Question 31

To make d the subject in the literal equation $ax^2 + b = cd$, the answer is

A $d = ax^2 + b$ **B** $d = ax^2 + b - c$ **C** $d = \frac{ax^2+b}{c}$

D $d = \frac{ax^2-b}{c}$ **E** $b = cd - ax^2$

Question 32

The solutions to the simultaneous equations

$$x + y = 5$$
$$2x - y = 1$$

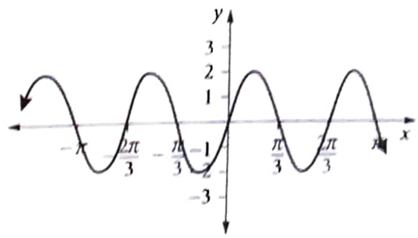
are

A $(2, 3)$ **B** $\left(\frac{1}{2}, \frac{9}{2}\right)$ **C** $(-2, 7)$ and $\left(\frac{3}{2}, \frac{7}{2}\right)$

D $(-2, 7)$ only **E** $\left(\frac{3}{2}, \frac{7}{2}\right)$ only

Question 33

The graph shown has the equation $f(x) = -2\sin(3(x - \pi))$.



The number of solutions for the equation $f(x) = 2$ in the domain $(-\pi, \pi)$ is

A 0 B 1 C 2 D 3 E 4

Question 34

The inverse of the function $f: R \rightarrow R, f(x) = -e^{x+2} - 3$ is

A $f^{-1}: R \rightarrow R, f^{-1}(x) = \log_e(3 - x) - 2$

B $f^{-1}: (3, \infty) \rightarrow R, f^{-1}(x) = \log_e(x - 3) - 2$

C $f^{-1}: (-\infty, -3) \rightarrow R, f^{-1}(x) = \log_e(-3 - x) - 2$

D $f^{-1}: R \rightarrow R, f^{-1}(x) = e^{x-2} - 3$

E $f^{-1}: (-\infty, 3) \rightarrow R, f^{-1}(x) = -\log_e(x - 3) - 2$

Question 35

For $f(x) = e^x$ which of the following is correct?

A $f(xy) = f(x) + f(y)$

B $f(x)f(y) = f(x + y)$

C $f(x + y) = f(x) + f(y)$

D $f(x - y) = f(x) - f(y)$

E $f(x) = f(y)$

Question 36

If $\log_a(y) = \frac{1}{2}$, then $2\log_a(y^3)$ equals

A $\frac{1}{8}$ B $\frac{1}{2}$ C 2 D 3 E 4

Question 37

If $(\log_5(x))^2 = \log_5(x^2)$, then x is equal to

A 1 or 25 B 0 or 3 C 1 only D 1 or 125 E 1 or 5

Question 38

If $N = Ae^{-kt}$ and $N = 4.12$ when $t = 2$, and $N = 2.62$ when $t = 5$, the values of A and k, respectively (correct to one decimal place), are

A $A = 5.6, k = 0.2$

B $A = 5.6, k = -0.2$

C $A = 5.4, k = 0.1$

D $A = 5.4, k = 0.2$

E $A = 1.5, k = 0.5$

Question 39

The solutions to the equation $\sin^2(2\pi x) = \frac{3}{4}$ for $x \in \left[0, \frac{1}{2}\right]$ are

A $x = \frac{\sqrt{3}}{2}$ B $x = 0.13, 0.37$ C $x = \frac{1}{2}$ D $x = \frac{1}{6}, \frac{1}{3}$ E $x = \frac{1}{3}$

Question 40

$x^{-3} + x^{-1}$ simplifies to

A $\frac{1}{x^2}$ B $\frac{1+x^2}{x^3}$ C $\frac{2x}{x^3}$ D x^2 E $\frac{1+x}{x^2}$

Question 41

A system of linear equations is

$$3x + 4y + 1 = 0$$
$$6x + 8y + 2 = 0$$

The graph of these equations has

A no points of intersection

B one point of intersection

C an infinite number of points of intersection

D two points of intersection

E three points of intersection.

Question 42

For what value(s) of k does the system of equations

$$kx + 4y = 1$$
$$(k - 1)x + ky = 2k$$

have no solutions?

A $k \in R\backslash\{2\}$ B $k \in R\backslash\{-2\}$ C $k = 4$

D $k = 2$ E $k \in R\backslash\{\pm 2\}$

Question 43

If $f(x) = 3\log_e(x - b)$ and $f(2) = 6$, then b equals

A $6 - e^{\frac{4}{3}}$ B $e^2 - 4$ C $e^{\frac{4}{3}} - 6$ D $2 - e^2$ E $4 - e^2$

Question 44

If $5^{x-2} = 10$, then x is equal to

A $\frac{1}{\log_{10}(5)} + 2$ B $\frac{1}{\log_e(5)} + 2$ C $\log_5(10) - 2$

D $2 + \log_{10}(5)$ E $\log_e(5) + \log_e(10)$

Question 45

The simultaneous linear equations

$$(k - 3)x - 5y = -2$$
$$2x - (2k + 2)y = -4$$

have infinitely many solutions for

A $k = 4$ B $k = -2$ C $k \in R\backslash\{4\}$

D $k \in R\backslash\{-2\}$ E $k \in R\backslash\{4, -2\}$

Question 46

The general solution to the equation $\sin(x) = 1$ is

A $\frac{\pi}{2}$

B $\frac{\pi}{2} + 2n\pi, n \in Z$

C $\frac{\pi}{2}, \frac{3\pi}{2}$

D $\frac{\pi}{2} + n\pi, n \in Z$

E $\frac{\pi}{2} + 2n\pi, n \in R$

Question 47

The equation of the inverse of the graph shown could be

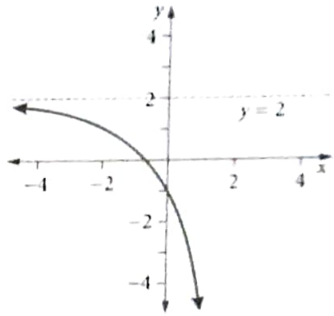
A $y = \log_e\left(\frac{2-x}{3}\right)$

B $y = -3\log_e(x) + 2$

C $y = 3e^x + 2$

D $y = -3e^x + 2$

E $y = 3e^{-x} + 2$



Question 48

The temperature of a hot chocolate cools according to the rule $T = T_0 \times 2^{-kt}$, where T is the temperature in degrees Celsius and t is the time in minutes. If it takes 20 minutes for the temperature to halve, what fraction of the original temperature is the temperature of the hot chocolate after 60 minutes?

A $\frac{1}{4}$ B $\frac{1}{5}$ C $\frac{3}{4}$ D $\frac{1}{16}$ E $\frac{1}{8}$

Extended-answer questions

Technology active: 6 questions

Solutions to this section start on page 222

Question 1 (10 marks)

Two lines are defined by $y = 3x - k$ and $y = 3kx + k$ for their maximal domains, where k is a real constant.

a What are the value(s) of k for the two lines to have no point of intersection? 3 marks

b If the lines $y = 3x - k$ and $y = 3kx + k$ are now perpendicular, show that the value of k is equal to $-\frac{1}{9}$. 1 mark

c Hence, using $k = -\frac{1}{9}$, find the coordinates of the point of intersection of the lines. 2 marks

d Sketch the graphs of the lines $y = 3x + \frac{1}{9}$ and $y = -\frac{1}{3}x - \frac{1}{9}$, labelling the point of intersection found in part c. 2 marks

e Find the area of the shape bounded by the line with the –ve gradient, the line with the +ve gradient and the x-axis. 2 marks

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Question 2 (15 marks)

Consider $f: [-2, 2] \rightarrow R, f(x) = ax^4 - 2bx^2 + c$

a It is known that both $x - 2$ and $2x + 1$ are factors of $f(x)$, and when $f(x)$ is divided by $x + 1$, the remainder is 6. Find a, b and c. 3 marks

b Hence, fully factorise $f(x)$. 1 mark

c Sketch the graph of $f(x)$ for $x \in [-2, 2]$, labelling exact x-intercepts as well as the turning points, correct to two decimal places. 3 marks

d Find a new function, $g(x)$, if it is the image of $f(x)$ after it is translated 2 units in the +ve direction of the x-axis, $\frac{8}{3}$ units in the +ve direction of the y-axis, and then dilated by 2 units from the y-axis. 3 marks

e Express this new function, $g(x)$, in fully factorised form, hence showing that one quadratic factor has rational roots and the other quadratic factor has irrational roots. 2 marks

f Sketch a graph of $g(x)$, labelling all axial intercepts. 2 marks

Question 3 (7 marks)

Sue is throwing a javelin in an athletics event. She starts with a long run up before she throws her javelin, modelled by the quadratic function below.

$$y = -0.25x^2 + 14.75x - 187$$

She runs for 19 metres with the javelin held above her head at a height of 3 metres from the ground.

Sue's run, then throw, can be described by the piecewise function below.

$$f(x) = \begin{cases} 3 & 0 \leq x < 19 \\ -0.25x^2 + 14.75x - 187 & 19 \leq x \leq A \end{cases}$$

a Why is it important to use the signs < 19 and $19 \leq$ in the written piecewise function? 1 mark

b Find $f(10)$. 1 mark

c Find $f(20)$. 1 mark

d Find the point A, correct to two decimal places. 1 mark

e Sketch the piecewise function that represents this run and throw labelling with coordinates, correct to two decimal places, all significant points: turning point, cusps, end points and axis intercepts. 3 marks

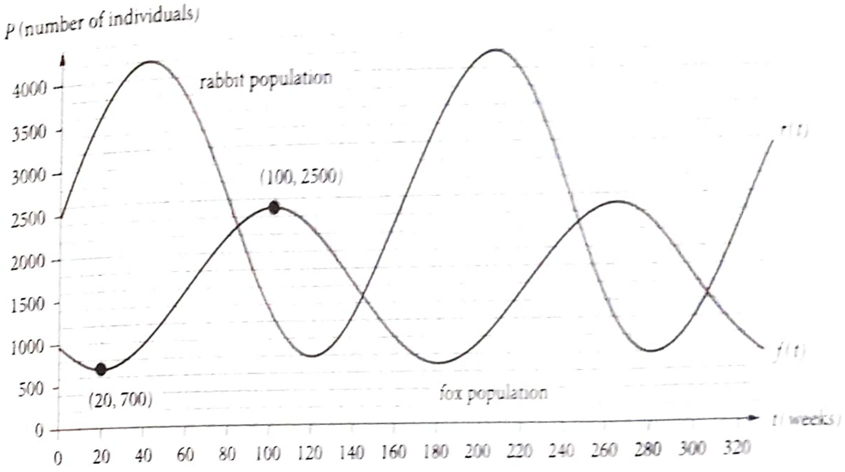
Question 4 (7 marks) ©VCAA 2022 2BQ2abcd ●●●

On a remote island, there are only two species of animals: foxes and rabbits. The foxes are the predators and the rabbits are their prey.

The populations of foxes and rabbits increase and decrease in a periodic pattern, with the period of both populations being the same, as shown in the graph below, for all $t \geq 0$, where time t is measured in weeks.

One point of minimum fox population, $(20, 700)$, and one point of maximum fox population, $(100, 2500)$, are also shown on the graph.

The graph has been drawn to scale.



The population of rabbits can be modelled by the rule $r(t) = 1700 \sin\left(\frac{\pi t}{80}\right) + 2500$.

a **i** 97% State the initial population of rabbits. 1 mark

ii 89% State the minimum and maximum population of rabbits. 1 mark

iii 84% State the number of weeks between maximum populations of rabbits. 1 mark

The population of foxes can be modelled by the rule $f(t) = a \sin(b(t - 60)) + 1600$.

b 75% Show that $a = 900$ and $b = \frac{\pi}{80}$. 2 marks

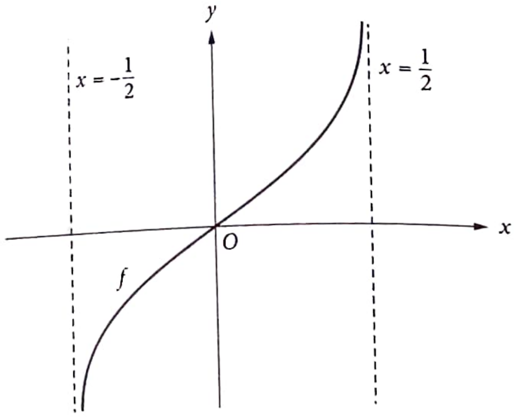
c 38% Find the maximum combined population of foxes and rabbits. Give your answer correct to the nearest whole number. 1 mark

d 57% What is the number of weeks between the periods when the combined population of foxes and rabbits is a maximum? 1 mark

Question 5 (8 marks) ©VCAA 2022 2BQ4abcd

Consider the function f, where $f: \left(-\frac{1}{2}, \frac{1}{2}\right) \to R,\ f(x) = \log_e\left(x + \frac{1}{2}\right) - \log_e\left(\frac{1}{2} - x\right)$.

Part of the graph of $y = f(x)$ is shown below.



a 82% State the range of $f(x)$. 1 mark

b **i** 88% Find $f'(0)$. 2 marks

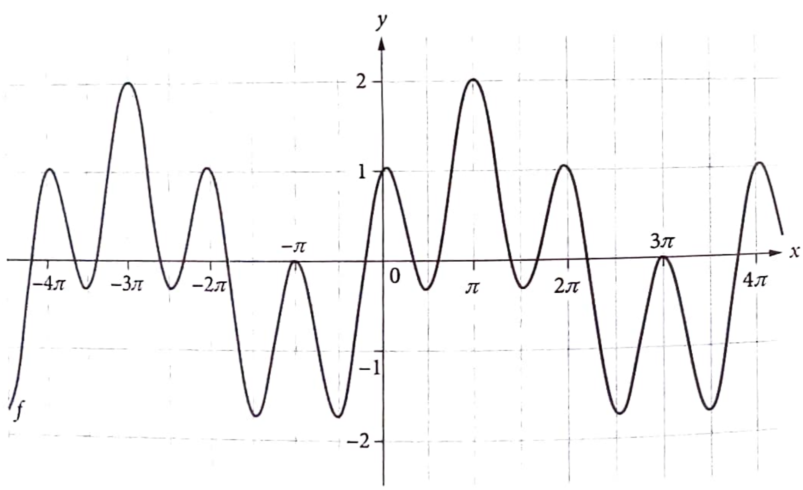
ii 57% State the maximal domain over which f is strictly increasing. 1 mark

c 72% Show that $f(x) + f(-x) = 0$. 1 mark

d 71% Find the domain and the rule of f^{-1}, the inverse of f. 3 marks

Question 6 (4 marks) ©VCAA 2021 2BQ5abcd

Part of the graph of $f: R \to R,\ f(x) = \sin\left(\frac{x}{2}\right) + \cos(2x)$ is shown below.



a 71% State the period of f. 1 mark

b 61% State the minimum value of f, correct to three decimal places. 1 mark

c 21% Find the smallest positive value of h for which $f(h - x) = f(x)$. 1 mark

Consider the set of functions of the form $g_a : R \to R,\ g_a(x) = \sin\left(\frac{x}{a}\right) + \cos(ax)$, where a is a positive integer.

d 68% State the value of a such that $g_a(x) = f(x)$ for all x. 1 mark

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Exam practice

Short-answer questions

Technology free: 20 questions

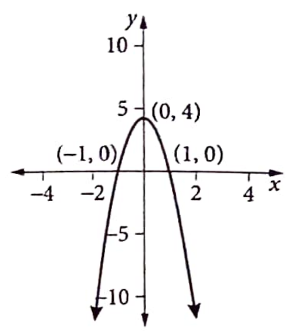
Solutions to this section start on page 224.

Question 1 (2 marks)

Find $\frac{dy}{dx}$ when $y = 2x\sin(3x)$.

Question 2 (5 marks)

For the quadratic graph shown, find



a the average rate of change between $x = -1$ and $x = 1$ 2 marks

b the average rate of change between $x = 0$ and $x = 1$ 2 marks

c the rate of change at $x = 1$. 1 mark

Question 3 (4 marks)

If $y = x\sin(x)$,

a find $\frac{dy}{dx}$. 2 marks

b Hence, find an anti-derivative for $\int 2x\cos(x)dx$. 2 marks

Question 4 (4 marks)

If $y = x\log_e(x)$,

a find $\frac{dy}{dx}$. 2 marks

b Hence, find $\int_1^2 2\log_e(x)dx$. 2 marks

Question 5 (3 marks)

Find $\frac{dy}{dx}$ when $y = \frac{\log_e(2x)}{x^2}$.

Question 6 (2 marks)

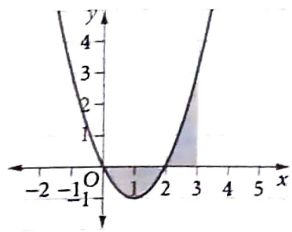
Find the area enclosed by the curve and the x-axis for the graph $y = -x^2 + 2x$.

Question 7 (3 marks)

Our biorhythms can be represented by sinusoidal curves of the form $y = \sin(kt)$, where t is in days. What is the value of k if the gradient of the curve equals $-\pi$ at $t = 1$?

Question 8 (4 marks)

The area enclosed by the curve of $y = x^2 - 2x$, the x-axis and $x = 0$ and $x = 3$ is shown.



a Find the area as described. 2 marks

b Evaluate $\int_0^3 (x^2 - 2x)\,dx$. 2 marks

Question 9 (3 marks)

Suppose that the velocity (v m/s) of a particle, at time t seconds ($t \geq 0$), of a body is given by $v(t) = t^2 - 6t + 8$. The particle is initially 2 metres to the right of a fixed point, 0.

Calculate the distance travelled between 1 and 3 seconds.

Question 10 (4 marks)

If $g: [k, 4] \rightarrow R, g(x) = -(x-2)^2 + 3$,

a find the smallest value of k such that g^{-1} exists. 2 marks

b find the derivative of $g^{-1}(x)$. 2 marks

Question 11 (3 marks)

Find the derivative of $\sqrt{2x^2 + 4}$ at $x = 2$.

Question 12 (4 marks)

Find the coordinates of any stationary points in the function $y = x^3 - x^2 - x - 2$.

Question 13 (4 marks)

If $f(x) = x^3 + ax^2 + b$ has a stationary point at $(2, -3)$, calculate

a the values of a and b. 2 marks

b the coordinates of any other stationary points. 2 marks

Question 14 (2 marks)

If $f(x) = \sin\left(\frac{x}{5}\right)$, find the gradient of $f(x)$ at $x = \frac{5\pi}{4}$.

Question 15 (4 marks)

If $f(x) = x^3 + 1$ and $g(x) = \sqrt{x - 1}$,

a define $g(f(x))$ and $f(g(x))$ if they exist. 2 marks

b find the derivative(s) of $g(f(x))$ and $f(g(x))$ if they exist. 2 marks

Question 16 (4 marks)

If $y = (5x + 2)^4$, calculate

a the gradient of the tangent to the curve at $x = 0$. 2 marks

b the equation of the tangent to the curve at $x = 0$. 2 marks

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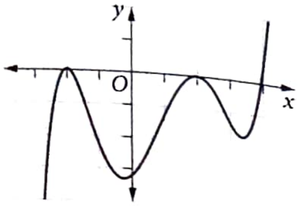
Question 17 (2 marks)

Sketch the graph of the gradient function $y = f'(x)$ if $y = f(x)$ is shown in the graph.



Question 18 (3 marks)

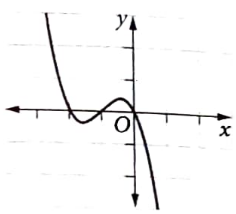
a Find the gradient of the chord PQ to the function $f(x) = 3x^2 + 2$ if the x-coordinates of P and Q are -2 and $-2 + h$, respectively. 2 marks

b Hence, find the gradient of the tangent at P. 1 mark

Question 19 (3 marks)

The graph of $y = f'(x)$ is shown.

Sketch what the graph of the anti-derivative, $y = f(x)$, might look like.



Question 20 (4 marks)

The temperature, T°C, on a mountain is related to height (h metres) by the rule

$$T = -0.002h^3 + 30,\ h \geq 0.$$

a Calculate the average rate of change of temperature over the first 10 metres. 2 marks

b Calculate the rate of change of temperature at $h = 10$. 2 marks

Multiple-choice questions

Technology active: 50 questions

Solutions to this section start on page 226.

Question 1

The derivative of the function $f(x) = 2x^2 + 4x$ is

A $f'(x) = 2x + 4$

B $f'(x) = 2x^2 + 4x$

C $f'(x) = 4x$

D $f'(x) = 2x + 4x$

E $f'(x) = 4x + 4$

Question 2

An anti-derivative of the function $f(x) = 2x^2 + 4x$ is

A $y = 2x + 4$

B $y = \frac{x^3}{3} + 4x^2$

C $y = \frac{2x^3}{3} + 2x^2$

D $y = \frac{2x^3}{3} + 4x$

E $y = 4x + 4$

Question 3

If $f(x) = 3\sqrt{x}$, then $f'(3)$ equals

A $\frac{\sqrt{3}}{2}$

B $2\sqrt{3}$

C $\sqrt{3}$

D $\frac{2}{\sqrt{3}}$

E $-\frac{1}{\sqrt{3}}$

Question 4

If $f(x) = 2\sqrt{x}$, then $f'(x)$ equals

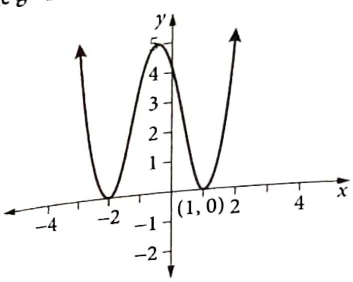
A $\frac{1}{\sqrt{x}}$ **B** $2\sqrt{x}$ **C** $\sqrt{x}$ **D** $\frac{2}{\sqrt{x}}$ **E** $-\frac{1}{\sqrt{x}}$

Question 5

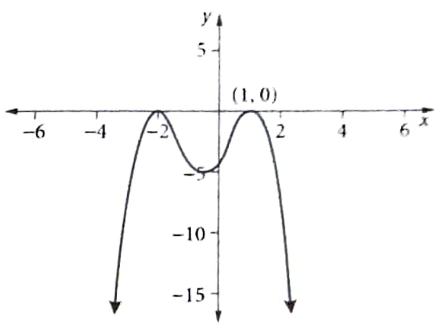
Let $g(x) = -\frac{1}{4}(x^4 - 6x^2 + 8x)$.

The graph that best represents $g'(x)$ is:

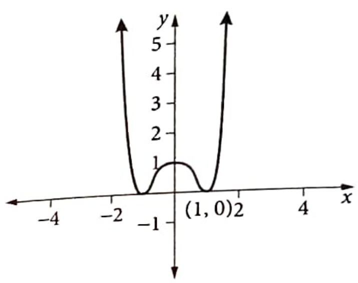
A



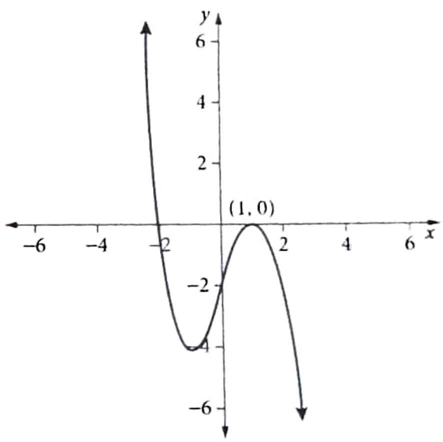
B



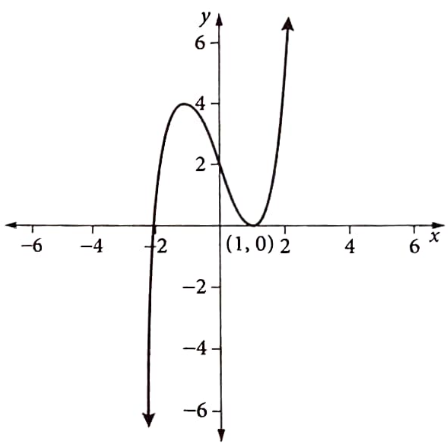
C



D



E



Question 6

The derivative of the function $f(x) = -x^4 - 2x^3 + 3x^2 + 4x - 4$ can be expressed as

A $(x-1)(x+2)$ **B** $(x-1)(1-x)(x+2)(x+2)$ **C** $(x-1)(1-x)(x-1)$

D $2(x+2)(x-1)(2x+1)$ **E** $-2(x+2)(x-1)(2x+1)$

Question 7

If $g(x) = -2(x-3)(x+2)(x-5)$, an anti-derivative can be expressed as

A $-\frac{x^4}{2} + 4x^3 + x^2 - 60x + 10$

B $-\frac{x^4}{2} + 4x^3 + x^2$

C $-6x^2 + 24x + 2$

D $-6x^2 + 24x$

E $-12x + 24$

Question 8

If $f(x) = 2x^3 - 3x^2 + 4$, the x value where the gradient of the curve is a minimum is

A $-\frac{3}{2}$

B 0

C $\frac{1}{2}$

D 1

E $\frac{3}{2}$

Question 9

$\frac{d}{dx}(-\sin(4x))$ equals

A $4\cos(4x)$

B $-4\cos(3x)$

C $-4x\cos(4x)$

D $-4\cos(4x)$

E $-\frac{1}{4}\cos(4x)$

Question 10

If $f(x) = e^{-3x}$, then $f'(x)$ equals

A $-3xe^{-3x}$

B $-3e^{-3x-1}$

C $-3e^{-3x}$

D $-3e^{-2x}$

E $-3e^{-4x}$

Question 11

If $f(x) = e^{-5x}$, then $f'(5)$ equals

A $-5xe^{-5}$

B $-5e^{-5x}$

C $-5e^{-25}$

D $-5e^{25}$

E $5e^{25}$

Question 12

The velocity (v m/s) of a particle at time t(s) is given by $v = t^3 - t^2 + 4$, $t \geq 0$. The rate of change of velocity at $t = 3$ is

A 1 m/s^2

B 8 m/s^2

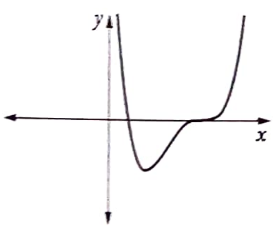
C 16 m/s^2

D 21 m/s^2

E 40 m/s^2

Question 13

The graph shown has



A one stationary point

B two stationary points

C the shape of a parabola

D a local maximum

E a negative gradient for $x \geq 0$

Question 14

Consider $f: (-4, 2] \rightarrow R, f(x) = x^2 + 4x$. The maximum value of f is

A -4

B 0

C 2

D 12

E ∞

Question 15

Consider $f: (-4, 2] \rightarrow R, f(x) = x^2 + 4x$. The minimum value of f is

A -4

B 0

C 2

D 12

E ∞

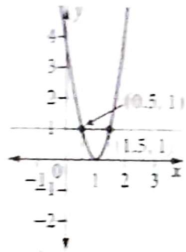
Question 16

The maximum height (in metres) of a ball thrown in the air with the height given by $h(t) = -t^2 + 2t + 1$, $t \geq 0$, is

A 1 m B 2 m C 12 m D 13 m E 14 m

Question 17

The graph shown has the equation $y = 4(x-1)^2$.
The area bounded by the curve and the line $y = 1$ can be expressed as



A $\int_0^2 4(x-1)^2 dx - 1$

B $\int_{\frac{1}{2}}^{\frac{3}{2}} 4(x-1)^2 dx - 1$

C $1 - \int_{\frac{1}{2}}^{\frac{3}{2}} 4(x-1)^2 dx$

D $\int_{\frac{1}{2}}^{\frac{3}{2}} 1 - 4(x-1)^2 dx$

E $\int_{\frac{1}{2}}^{\frac{3}{2}} 4(x-1)^2 - 1 dx$

Question 18

The population, P, of a certain town at t years is given by $P(t) = 1000e^{1-t}$, $t \geq 0$. The rate at which the population is decreasing after 10 years is approximated by

A −1000 B −0.123 C 0.123 D $1000e^{-9}$ E 1000

Question 19

If $y = 2\sin^2(3x)$, then $\frac{dy}{dx}$ is equal to

A $12\sin(3x)\cos(3x)$ B $2\sin(3x)\cos(3x)$ C $6x\sin(3x)\cos(3x)$

D $6\sin(3x)\cos(3x)$ E $3\cos^2(3x)$

Question 20

For the curve $y = 4x^3 - 3x^2 - 2$, the coordinates of the point(s) at which the gradient is zero are

A $(0,-2), \left(\frac{1}{2}, -\frac{9}{4}\right)$ B $(0,0), \left(\frac{1}{2}, 0\right)$ C $(0,-2)$

D $(0,-2), (2,19)$ E $(0,-1), \left(\frac{1}{2}, -\frac{5}{4}\right)$

Question 21

Consider $f(x) = ax^2 - bx$. The gradient of the tangent to the curve at the point at $(1, 1)$ is equal to −1. The values of a and b, respectively, are

A −3, −2 B −2, −3 C −3, 2 D −2, 3 E 1, −1

Question 22

If $f(x) = -2\sqrt{x+2} - 1$, then the gradient at $x = 2$ is

A −2 B −1 C −0.5 D 0.5 E 2

Question 23

If $y = \log_e(3x + 3)$, the value of x where the gradient is equal to 1 is

A $-\frac{1}{3}$ B 0 C $\frac{1}{5}$ D $\frac{3}{5}$ E 1

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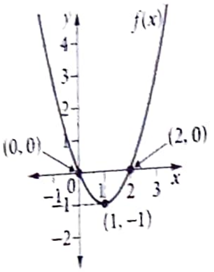
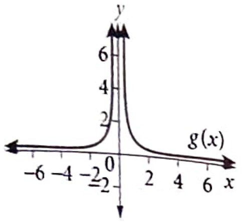
Question 24

Let $f(x) = x^2 - 2x$ and $g(x) = \frac{1}{x^2}$ for a maximal domain.





If $h = g(f(x))$, the derivative $h'(x)$ is equal to

A $h'(x) = \dfrac{-4x+4}{(x^2-2x)^2}$ for $x \in (-\infty, 0) \cup (0, 2) \cup (2, \infty)$

B $h'(x) = \dfrac{1}{(x^2-2x)^2}$ for $x \in (2, \infty)$

C $h'(x) = \dfrac{-4x+4}{(x^2-2x)^3}$ for $x \in R$

D $h'(x) = \dfrac{-4x+4}{(x^2-2x)^3}$ for $x \in (-\infty, 0) \cup (0, 2) \cup (2, \infty)$

E $h'(x) = x^4 - 2x^2$ for $x \in (1, \infty)$

Question 25

The line $2y - 6x = 3$ has a gradient of

A -3 B $-\frac{1}{2}$ C $-\frac{1}{6}$ D 3 E $\frac{9}{2}$

Question 26

The average rate of change of $y = x^2 - 2x$ between $x = 1$ and $x = 3$ is

A -1 B 1 C 2 D 3 E 4

Question 27

The equation of the tangent to the curve $y = 2x^3$ at $x = 2$ is

A $x + 24y + 32 = 0$ B $y = 24x - 24$ C $y = 24$

D $y = 24x - 32$ E $y = 12x - 16$

Question 28

The gradient of the secant from $x = 2$ to $x = 2 + h$ in the function $y = 3x^2 + 2x - 1$ is found by

A $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{2}$ B $\lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{2}$ C $\dfrac{f(2+h) - f(2)}{h}$

D $\dfrac{f(2) - f(2-h)}{h}$ E $6x + 2$

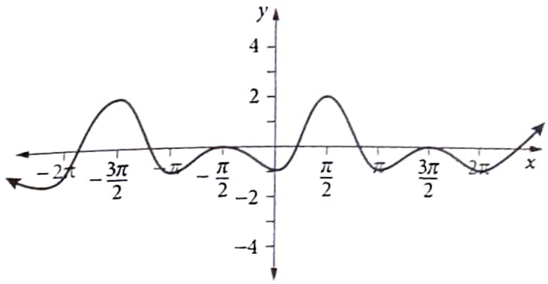
Question 29

If $y = \tan(x)$, the rate of change of y with respect to x at $x = a$, where $\frac{\pi}{2} < a < \pi$, is:

A $\sec^2(a)$ B $-\tan(a)$ C $\tan(a)$ D $\sec^2(0)$ E $-\dfrac{1}{\cos^2(a)}$

Question 30

The graph of $y = \sin(x) - \cos(2x)$ is shown.



The absolute maximum points in the domain shown are at

A $x = \frac{\pi}{2}, \frac{3\pi}{2}$

B $x = \frac{\pi}{2}$

C $x = -\frac{3\pi}{2}, \frac{\pi}{2}$

D $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

E $x = \frac{3\pi}{2}$

Question 31

The derivative of $g(x) = \sqrt{x}(x^p - 1)$ with respect to x is

A $\frac{1}{2}x^{-\frac{1}{2}}(px^{p-1} - 1)$

B $\left(p + \frac{1}{2}\right)x^{p-\frac{1}{2}} - \frac{3\sqrt{x}}{2}$

C $\frac{p}{2}x^{p-\frac{3}{2}} + \frac{1}{2\sqrt{x}}$

D $\frac{1}{2}(2p + 1)x^{p-\frac{1}{2}} - \frac{1}{2\sqrt{x}}$

E $\sqrt{x}(px^{p-1} - 1)$

Question 32

The derivative of $f(x) = \sin^n(x)$ is

A $n\sin^{n-1}(x)$

B $\frac{n\tan^n(x)}{\cos(x)}$

C $n\sin(x)\cos^{n-1}(x)$

D $n\cos(x)\sin^{n-1}(x)$

E $n\cos^2(x)\sin^n(x)$

Question 33

Which of the following is *not* true for the curve of $y = f(x)$, where $f(x) = x^{\frac{1}{8}}$?

A The domain is $[0, \infty)$.

B The range is $R^+ \cup \{0\}$.

C The curve passes through $(16, \sqrt{2})$.

D The function is strictly decreasing for all $x \in R^+$.

E The gradient is undefined at $x = 0$.

Question 34

If $y = x\log_{10}(2x^2 - 1)$, the gradient of the tangent at $x = 2$ is closest to

A 0.50

B 1.06

C 1.83

D 1.84

E undefined

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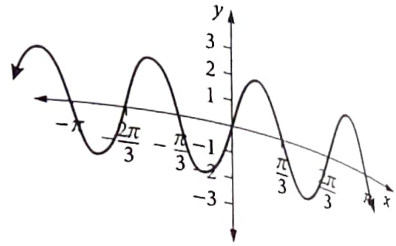
Question 35

The graph shown has the equation $f(x) = -2\sin(3(x - \pi))$

The number of stationary points in the domain $(-\pi, \pi)$ is



A 1 **B** 2 **C** 3

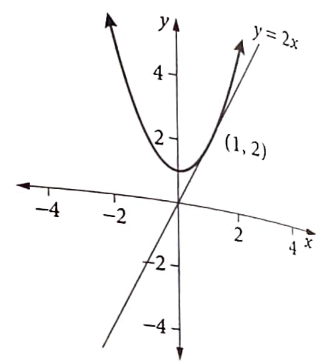
D 4 **E** 6

Question 36

The parabola shown has the equation $y = x^2 + 1$.

The tangent to the graph at the point $(1, 2)$ has the equation $y = 2x$.

The instantaneous rate of change of the curve $y = x^2 + 1$ at $x = 1$ is

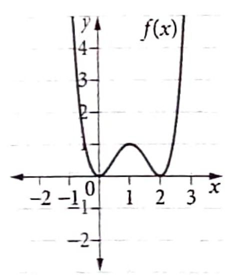


A 1 **B** 2 **C** 3

D 4 **E** 6

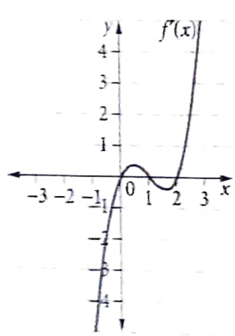
Question 37

Consider the graph below of $y = f(x)$.

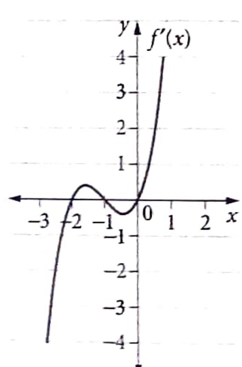


The graph of $f'(x)$ could look like

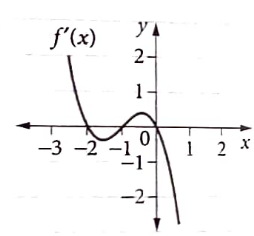
A



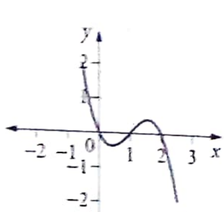
B



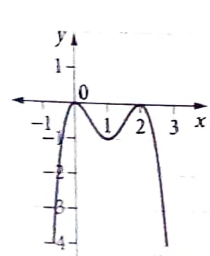
C



D



E



Question 38

The average rate of change between the points $(x, f(x))$ and $(x+h, f(x+h))$ in any function is found by

A $x+h$

B $\frac{f(x+h)-f(x)}{h}$

C $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

D $\lim_{h\to 0}\frac{h(2x+h)}{h}$

E $2x$

Question 39

If $N = Ae^{-kt}$, an anti-derivative of N could be

A Ae^{-kt}

B $-Ake^{-kt}$

C $-Ake^{kt}$

D $-\frac{A}{k}e^{-kt}$

E $\frac{A}{k}e^{-kt}$

Question 40

The equation of the tangent to the function $h(x) = \tan(2\pi x) + 1$ at $x = 0$ is

A $y = 2\pi x + 1$

B $y = 2\pi x$

C $y = \pi x + 1$

D $y = 2\pi$

E $y = 2\pi x - 2\pi + 1$

Question 41

The gradient graph for the function $f(x) = x^{-2} + x^{-1}$ has

A an asymptote at $x = 0$

B an asymptote at $y = 0$

C asymptotes at both $x = 0$ and $y = 0$

D a defined value at $x = 0$

E a derivative at $x = 0$.

Question 42

The function $g(x) = 2x + x^2$ has

A a constant rate of change of 2

B an average rate of change of 3 between $x = 0$ and $x = 1$

C an instantaneous rate of change of 3 at $x = 1$

D a chord gradient of 2 between $x = 0$ and $x = 1$

E a derivative of 3 at $x = 0$.

Question 43

For the function $y = e^{3x}\sin(x)$, we get $\frac{dy}{dx} =$

A $3e^{3x}\cos(x)$

B $3e^{3x}$

C $e^{3x}\cos(x) + 3\sin(x)$

D $3e^{3x}(\cos(x) + 3\sin(x))$

E $e^{3x}(\cos(x) + 3\sin(x))$

Question 44

If $f(x) = 3\log_e(x-b)$, then $f'(4)$ equals

A $-\frac{3}{4-b}$

B $\frac{3}{4-b}$

C $3\log_e(4-b)$

D $\frac{3}{x-b}$

E $4-b$

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Question 45

For the function $y = \dfrac{x}{\log_e(x)}$, $\dfrac{dy}{dx}$ equals

A $\dfrac{\log_e(x)}{\log_e(x-1)}$

B $\dfrac{\log_e(1)-1}{\log_e(x-1)}$

C $\dfrac{\log_e(x)-1}{(\log_e(x))^2}$

D $\dfrac{\log_e(x-1)}{(\log_e(x))^2}$

E $\log_e(5)+\log_e(10)$

Question 46

To find the gradient of the function $y = \cos(3x^2+1)$, what would u be in the chain rule $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$?

A $\cos(u)$

B $\cos(x)$

C $3x^2+1$

D $6x$

E $-\sin(u)$

Question 47

To find the gradient of the function $y = \cos(3x^2+1)$, what would $\dfrac{du}{dx}$ be in the chain rule $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$?

A $\cos(u)$

B $\cos(x)$

C $3x^2+1$

D $6x$

E $-\sin(u)$

Question 48

Evaluate $\int_0^1 (-3f(x)+2x)\,dx$, where $\int_0^1 f(x)\,dx = 2$.

A -6

B -5

C 1

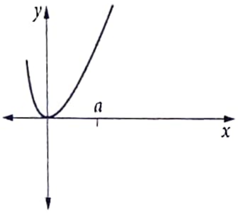
D 2

E 7

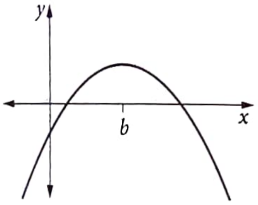
Question 49

The the graph of $f(x) = x\log_e(x+1)$ for all $x > -1$ has the general shape of

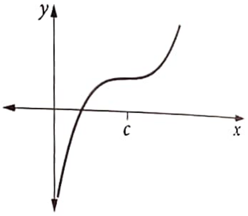
A



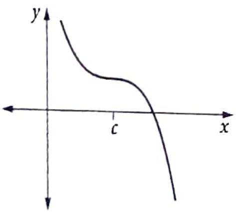
B



C



D



E none of the above

Question 50

$\lim\limits_{\delta x_i \to 0} \sum\limits_{i=1}^{n} f(x_i)\,\delta x_i$ can be expressed as

A $\lim\limits_{\delta x_i \to 0} \sum\limits_{0}^{n} f(x)\,\delta x$

B $\sum\limits_{0}^{n} f(x)\,\delta x$

C $\int_0^i f(x_i)\,dx$

D $\int_a^b f(x)\,dx$

E $\int_0^i f(x)\,dx$

Extended-answer questions

Technology active: 13 questions

Solutions to this section start on page 231

Question 1 (10 marks)

a Find $\frac{dy}{dx}$ if $y = 3x\cos(x)$. 1 mark

b Hence find an expression for $\int x\sin(x)\,dx$. 3 marks

c Evaluate $\int_0^{\pi} x\sin(x)\,dx$. 1 mark

Consider the curve of $f(x) = x\sin(x)$.

d Sketch the graph of $f: [0, \pi] \rightarrow R, f(x) = x\sin(x)$, labelling the coordinates of the endpoints and the local maximum point, correct to three decimal points. 2 marks

e Find the area under the curve $f(x) = x\sin(x)$ for $x \in [0, \pi]$. 1 mark

f Solve the equation $f'(x) = 0$ for $x \in [0, \pi]$, giving your answer correct to three decimal places. 2 marks

Question 2 (14 marks)

A particular rock concert emits sound (measured in decibels) that ranges from 90 dB to quite a high level. The function that models the sound at the concert follows the function of $f(t) = 10\sin(\pi t) + 100$, where $t \geq 0$, t hours after the start of the concert at 7 pm.

a What is the average noise level, in dB, at the rock concert from 7 pm to 11 pm? 1 mark

b The concert is so good that the band returns for encores, and the concert finishes at 11:45 pm. What is the average sound, in dB, correct to two decimal places, at the rock concert from 7 pm to 11:45 pm? 1 mark

c For $f(t) = 10\sin(\pi t) + 100$, what is the amplitude and period of the function? 2 marks

d Sketch the graph $f(t) = 10\sin(\pi t) + 100$ for the domain $x \in [0, 4]$, showing the coordinates of the endpoints and the maximum and minimum turning points. 3 marks

The level of sound, in decibels, at which a person may sustain hearing loss is 95 dB.

e For what period of time during the 4-hour concert will the decibel level be dangerous for patrons? 2 marks

Normal conversation is held between 60 dB and 65 dB.

f Give a reason as to why normal conversation cannot be heard during the 4-hour concert. 1 mark

g Find $\frac{d}{dt}(10\sin(\pi t) + 100)$. 1 mark

h Hence solve the equation $f'(t) = 0$. 2 marks

The loudest recommended exposure with hearing protection is 140 dB. Even short-term exposure can cause permanent hearing loss at this level.

i Using your information from part **h**, explain why there is no possibility of permanent hearing loss for the concert patrons. 1 mark

Question 3 (6 marks)

The velocity of a particle is described by the function $v(t) = 3t^3 - t - 2$, where $v(t)$ is in m/s and t is in seconds, $t \geq 0$.

a By finding the discriminant of a quadratic factor in $v(t)$, show that its graph has only one t-intercept. 2 marks

b Find when the particle momentarily stops in its journey. 1 mark

c What distance does the particle travel in the first 5 seconds? 3 marks

Question 4 (16 marks)

The function of $f(x) = x^3 + ax^2 + bx + c$ has a stationary point at $(1, 300)$.

a Find the values of a and b in terms of c. 3 marks

A company models its monthly profits using the function $f(x) = x^3 + ax^2 + bx + c$, where $f(x)$ is the monthly profit in \$ and x is the day of the month, where, for example, $x = 1$ represents the 1st of the month and $x = 20$ represents the 20th day of the month. The mathematicians in the company find that profits are satisfactory when $a = b$.

b Find the value of c for which $a = b$. 1 mark

c For the domain $x \geq 1$, and using your values of a, b and c, find $f'(x)$, and hence show that the maximum or minimum point(s) occur at the end of the domain. 3 marks

d Find the minimum and maximum profits for a 30-day month, and the days of the month on which they occur. 3 marks

e Sketch the graph of $f: [1, 30] \rightarrow R, y = f(x)$, labelling the coordinates of the endpoints. 3 marks

f Find the value of c for which $a = 2b$. 1 mark

g Using the values of c found in part **f**, describe what this does to the profit of the company. 2 marks

Question 5 (10 marks)

The pitch of a roof in the northern states of America is, by standard, a certain value so that the heavy snow in the area doesn't weigh too heavily on the roof. In general, as the pitch of the roof increases, the snow slides off more easily, and the load on the roof decreases. But a steeper roof is more expensive to build. To encourage snow to slide off, the roof should have a minimum pitch of 3:12, meaning that it drops more than 3 feet for every 12 feet of roof, but a pitch of 4:12 is better.

a What is the gradient of a 3:12 roof? 1 mark

b What is the gradient of a 4:12 roof? 1 mark

c If $\tan(45°) = 1$ and equals the ratio $\frac{12}{12}$, what approximate ratio in the form of $\frac{x}{12}$ does $\tan(14°)$ equal? 1 mark

d If $\tan(45°) = 1 =$ the ratio $\frac{12}{12}$, what approximate ratio in the form of $\frac{x}{12}$ does $\tan(18.5°)$ equal? 1 mark

e A straight line of roofing has a pitch of 4:12 and goes through the point $(0, 0)$. What is the equation of the line? 1 mark

f A straight line of roofing has a pitch of 3:12 and goes through the point $(2, 10)$. What is the equation of the line? 2 marks

Let $\tan(\theta) = \frac{x}{12}$ so that $\theta = \tan^{-1}\left(\frac{x}{12}\right)$, where $\theta°$ is the angle of the roof to the horizontal and x is the vertical height of the roof, in inches.

g Find $\frac{d}{dx}(\theta)$ and hence find the value of x for which the angle of the roof is at its maximum. 3 marks

Question 6 (14 marks)

Let $f(x) = x + 2$ and $g(x) = 2x^2 + ax + 4$ for maximal domains, where a is a real constant.

If $f(x) = g(x)$, find the value(s) of a for which there is

a exactly one solution to the equation $f(x) = g(x)$. 2 marks

b more than one solution to the equation $f(x) = g(x)$. 2 marks

c no solution to the equation $f(x) = g(x)$. 1 mark

d For the case of exactly one solution to the equation $f(x) = g(x)$, where $a > 0$, sketch the graph of $h(x) = f(x) - g(x)$. 3 marks

e Find $h'(x)$, and hence find the coordinates of any stationary point in the graph of $h(x)$. 3 marks

f Find the simplified transformed equation for $h_T(x) = -h(2x - 3) + 7$. 1 mark

g Describe in words the step-by-step transformation to get to the image $h_T(x)$. 2 marks

Question 7 (11 marks)

Consider the function $f(x) = (x - 1)(x^2 - k)$, where k is a real constant.

a Find the rule for $g(x)$ if $g(x) = -f(2x - 1) + 3$. Give your answer in the form of $ax^3 + bx^2 + cx + d$. 2 marks

b Find the equation of the tangent line to the curve $g(x)$, when $x = 1$, in terms of k. 3 marks

c Find the equation of the line that is perpendicular to the tangent found in part **b**, also going through the point when $x = 1$, in terms of k. 3 marks

d Find the value(s) of k, if it is possible, such that the tangent found in part **b** is also a tangent to $f(x) = (x - 1)(x^2 - k)$ at $x = 1$. 3 marks

Question 8 (13 marks)

Paddy inadvertently throws his cricket ball over the fence into the parkland behind his house. The ball follows the path of a parabola with the equation $h(x) = -x^2 + 10x + 1$, where $h(x)$ metres is the vertical height of the ball from the ground, at a horizontal distance, x metres, from Paddy's hand.

a At what height from the ground does the ball leave Paddy's hand? 1 mark

b What is the maximum height that the ball will reach? 1 mark

c The fence into the parkland has a height of 2 metres and is at a horizontal distance of 2 metres from Paddy. Will his ball hit the fence? Why/why not? 2 marks

A jogging path has been built around the parkland and Imogen is jogging just as Paddy throws the ball. She sees the ball and stops jogging. Imogen is 1.5 metres tall and the ball hits her on the head.

d Where is Imogen standing, correct to two decimal places, when the ball hits her? 1 mark

Imogen picks up the ball, runs away with it and puts it on the ground at the coordinates (30, 0).

e What is the shortest distance from the ball on the ground to Paddy's hand, still at 1 metre vertically from the ground? 2 marks

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f Paddy finds a way to crawl through the fence and runs to the ball. How far does Paddy run? 1 mark

g Find the equation of the tangent to the curve $h(x)$ at $x = 2$. 2 marks

h i Hence state an integral to find the area between the tangent at $x = 2$, the curve $h(x)$ and the y-axis. 2 marks

ii Evaluate this area. 1 mark

Question 9 (13 marks)

The function f is defined by $f: [0, 2\pi] \rightarrow R$, where $f(x) = 3e^{\frac{x}{10}}g(x)$, where $g(x) = \sin(2x)$.

a Show that the solutions to the equation $g(x) = 0$ for $x \in [0, 2\pi]$ are $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$. 2 marks

b Hence state the solutions to the equation $f(x) = 0$. 1 mark

c Using the product rule, find $f'(x)$ and hence find the coordinates, correct to two decimal places, of the stationary points of the graph of $f(x)$. 3 marks

d Sketch the graph of $f(x)$, labelling axial intercepts and endpoints. 3 marks

e Let the points $A(a, f(a))$ and $B(b, f(b))$ be the first two points of intersection between the graphs of $f(x)$ and $y = 3e^{\frac{x}{10}}$. The line that joins A and B has a gradient of m.

Show that $m = \dfrac{f(b) - f(a)}{\pi}$. 2 marks

f Hence, find an expression for the gradient, n, of points C and D, where C and D are the first 2 points of intersection between the graphs of $f(x)$ and $y = -3e^{\frac{x}{10}}$. 2 marks

Question 10 (8 marks)

A section of a rollercoaster is modelled by a function with the equation $p(x) = a(bx^2 - cx)^2$.

a Show that the x-intercepts of the function are at $x = 0$ and $x = \frac{c}{b}$. 2 marks

b Show that the stationary points of the function are at $x = 0$, $x = \frac{c}{2b}$ and $x = \frac{c}{b}$. 2 marks

This section of the rollercoaster has the domain $x \in [-1, 5]$, where x and y are respectively the horizontal and vertical displacement, in metres, from the point $(0, 0)$. The point $(1, 0)$ is where the safety officer sits.

c We know that $p'\left(\frac{10}{3}\right) = 0$. Show that $\frac{10}{3} = \frac{c}{b}$. 1 mark

d It is also true that $p(5) = 62.5$ and $p'(0) = 0$. Find the values of a, b and c. 1 mark

e The safety officer looks directly above him and decides that the rollercoaster is inoperable if the gradient is greater than 5. Does he shut down the rollercoaster? 2 marks

Question 11 (21 marks)

Consider the function $f: [-1, 3] \rightarrow R, f(x) = 3x^2 - x^3$.

a Find the coordinates of the stationary points of f. 2 marks

b Sketch the graph of f, labelling endpoints and stationary points with their coordinates. 3 marks

c What is the maximum value of f and for what x value(s) does the maximum exist? 2 marks

d What is the minimum value of f and for what x value(s) does the minimum exist? 2 marks

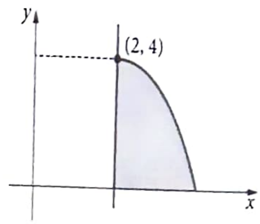
e For what x values is the graph of f strictly increasing? 1 mark

f A tangent is drawn to the curve at $x = \frac{7}{4}$. What is the equation of this tangent? 2 marks

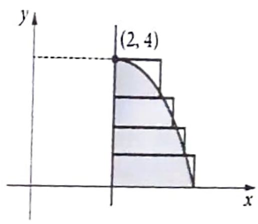
g **i** An area is formed that is enclosed by the curve of f and the tangent. Write down the integral for finding this area. 3 marks

ii Hence evaluate this area. 1 mark

A different area is formed when a line is drawn parallel to the y-axis, starting at the point $(2, 0)$ and ending at $(2, 4)$, as shown in the diagram.



An approximation to the area between the line $x = 2$ and the curve to the right side of the line is found by a series of rectangles of width 1 unit, as shown.



h Find an approximation to the area using the four rectangles. Give your answer correct to two decimal places. 3 marks

i Find the ratio between the shaded area found by integration and your approximated area from part **h**. 2 marks

Question 12 (12 marks)

The population of the common loon in one particular lake in the Great Lakes region of North America varies according to the rule $p(t) = 10\,000 - 5000\cos\left(\frac{\pi t}{6}\right)$, where p is the population of the common loon and t is the number of months after 1 January 2014 with $t \in [0, 12]$.

Lucas is the conservation officer in charge of maintaining the population health of the loon birds.

a State the period and amplitude of the function p. 2 marks

b Find the maximum number of common loons in this lake and state when this maximum occurs. 2 marks

c Find the minimum number of common loons in this lake and state when this minimum occurs. 1 mark

In his first year of work, starting on 1 January 2014, Lucas reported that the loon population is healthy when the rate of population change is greater than 1000 birds per month.

d Find the fraction of time, over the first 12 months of his job, when Lucas decides that the population is healthy. Give your answer as an interval for t, correct to two decimal places. 2 marks

e Sketch the graph of $p(t)$, labelling axial intercepts and endpoints. 2 marks

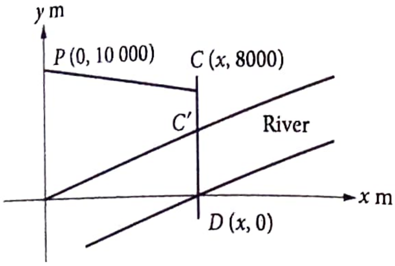
Lucas moves to another lake for the second year of his job, starting on 1 January 2015.

This time, the population of the yellow-billed loon varies according to the rule $y(t) = 5000\sin\left(\frac{\pi t}{4}\right) + 80\,000$, where y is the population of the yellow-billed loon and t is the number of months after 1 January 2015. The standard for a healthy loon population stays the same.

f Find the ratio of time when the yellow-billed loon population is healthy compared to when the common loon population is healthy. 3 marks

Question 13 (8 marks)

Toby Jones is hiking through the woods towards a river and is deciding whether he will swim or walk to his destination. He is able to walk at a rate of 6 metres per second and swim at a rate of k metres per second, where k is a constant. Toby has a 'mud map' that he is following, sketched below.



In the diagram above, P is Toby's starting point, C is the coffee shop and D is his destination on the opposite bank of the river. P has coordinates $(0, 10000)$. D has coordinates $(x, 0)$. C has coordinates $(x, 8000)$. C' is where Toby hits the river. Let x be the horizontal distance between the origin and D. Let $(1000 - \sqrt{x})$ metres be the distance between C and C'.

a State an expression for $T(x)$, the time in seconds that Toby walks and swims for if he goes from P to C and then from C to D. 2 marks

b Find $T'(x)$ and hence find the minimum time that Toby walks if $k = 10$. Give your answer to the nearest second. 2 marks

c What is the restriction on x in this derivative? 1 mark

d Find the time taken, to the nearest second, if Toby decides to walk in a line directly from P to the nearest bank of the river. Assume that this direct line is at right angles to the bank of the river. 3 marks

Exam practice

Short-answer questions

Technology free: 20 questions

Solutions to this section start on page 239.

Question 1 (2 marks)

IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. It is known that $\Pr(Z > 1.5) = 0.0668$. What is the score, x, that a randomly selected student will score for the IQ test if $\Pr(X < x) = 0.0668$?

Question 2 (2 marks)

If X is a normal variable with $\mu = 10$ and $\sigma = 5$, when $X = 18$, what does the standard normal variable z equal?

Question 3 (2 marks)

For the following discrete random variable Z, calculate the mean.

z	1	3	5
$\Pr(Z = z)$	0.5	0.3	0.2

Question 4 (2 marks)

The following table represents a discrete probability distribution:

y	−2	−1	0	1
$\Pr(Y = y)$	0.4	0.3	0.1	0.2

a Calculate the mean. 1 mark

b Find $\Pr(Y \le 0)$. 1 mark

Question 5 (1 mark)

If $E(X) = 2$, state the value for $E(3X - 2)$.

Question 6 (3 marks)

The following table represents a discrete probability distribution with a mean of 1. Find the values for a and b.

x	0	1	2	3
$\Pr(X = x)$	a	b	0.1	0.1

Question 7 (1 mark)

The probability of catching the common cold is 0.2. At an insurance company there are 500 employees. How many of these would you expect to catch the common cold?

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Question 8 (4 marks)

A binomial random variable has a mean of 12 and a probability of success of 0.2.

a How many trials were conducted? 1 mark

b What is the variance? 1 mark

c For this binomial random variable, write an expression for $\Pr(X = 1)$. 2 marks

Question 9 (4 marks)

Let X = number of boys in a two-child family.

Assume that the probability of a boy for each birth is $\frac{1}{2}$.

a Complete the table for the probability distribution. 3 marks

x	0	1	2
$\Pr(X = x)$			

b Calculate $\Pr(X \le 1)$. 1 mark

Question 10 (3 marks)

The probability that Sam (S) catches a plane on time is 0.8, but if it is raining (R) on the way to the airport, it is known that the probability decreases to 0.5. If it rains 20% of the time, calculate the probability that it rains if Sam catches a plane on time.

Question 11 (4 marks)

A multiple-choice test has four questions, with five alternative solutions for each question, with only one alternative being correct for each question. If a student guesses the answers to all four questions, calculate the probability that

a exactly four questions are correct 1 mark

b up to one question is correct 1 mark

c at least three questions are correct 2 marks

Question 12 (1 mark)

A normally distributed variable has $\mu = 18$ and $\sigma = 2.7$. Find the range between which approximately 95% of the values lie.

Question 13 (3 marks)

A particular clothing store has mean Saturday sales of 80 dresses, normally distributed with a standard deviation of 5, and mean Sunday sales of 50 dresses, normally distributed with a standard deviation of 6. The store sells well on a particular Saturday, with 90 dresses sold, and on the following Sunday it sells 60 dresses. On which of these two days did it do better, relative to usual sales?

Question 14 (5 marks)

For a random variable X with probability density function defined by

$$f(x) = \begin{cases} 12x^2(1-x) & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

a sketch the graph of $f(x)$, shading on the graph the area that corresponds to $\Pr\left(X < \frac{1}{2}\right)$ 3 marks

b calculate $\Pr\left(X < \frac{1}{2}\right)$. 2 marks

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Question 15 (4 marks)

If

$$f(x) = \begin{cases} kx & 0 \le x \le 1 \\ 2 - x & 1 < x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

a find k such that $f(x)$ is a PDF 2 marks

b find the mean of the PDF. 2 marks

Question 16 (3 marks)

Consider a random variable X with a probability density defined by

$$f(x) = \begin{cases} 2k \sin(\pi x) & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

where k is a real constant.

Find k such that $f(x)$ is a PDF.

Question 17 (3 marks)

A sample of 20 people waiting in a queue at the MCG on a particular Saturday afternoon were asked how much money they intended to pay in total for tickets. The smallest amount was $10, the average amount was $50 and the greatest was $100.

Identify

a the population 1 mark

b three population parameters 1 mark

c three sample statistics. 1 mark

Question 18 (4 marks)

It is given that $\hat{p} = \frac{X}{4}$, where $\Pr(X = x) = {}^4C_x \, 0.7^x \, 0.3^{4-x}$ and X is binomial.

a Complete the discrete random variable probability table for a sample of 4. 3 marks

x	0	1	2	3	4
$\hat{p} = \frac{x}{4}$				$\frac{3}{4}$	
$\Pr(X = x)$		${}^4C_1 0.7^1 0.3^3$			

b Show that $E(\hat{p}) = p$. 1 mark

Question 19 (2 marks)

If you were checking whether taking the bus to work increases the chances of catching a cold from other passengers, you might ask one sample of people to not catch the bus for 5 days, another to catch one bus for the week and a third sample to take buses every day.

For each sample, the variable would be a random binomial variable with success being 'catching a cold'. You would then look at the frequencies of people in each of the three sample groups.

From 100 people who didn't catch the bus, 20 caught colds during one week. What is the sample proportion of colds?

Question 20 (3 marks)

Find the approximate 95% CI for the proportion p of students, using a random sample of 100 students, when it is found that the sample proportion $\hat{p}$ is 0.5.

Multiple-choice questions

Technology active: 50 questions

Solutions to this section start on page 242

Question 1

A fair coin is tossed twice and the outcomes are recorded. The sample space of the outcomes is

A $\{H, T\}$ **B** $\{H, HT, T\}$ C $\{HH, HT, TT\}$

D $\{HH, HT, TH, TT\}$ **E** $\{T, TH, H\}$

Question 2

A fair die is thrown and the outcomes are recorded. The sample space of the outcomes is

A $\{11, 22, 33, 44, 55, 66\}$ **B** $\{1, 2, 3, 4, 5, 6\}$ C $\{1, 3, 5\}$

D $\{2, 4, 6\}$ **E** $\{H, T\}$

Question 3

Which one of the following random variables is **not** discrete?

A The number of people in your hall for a school assembly.

B The shoe size of students in your class.

C The number of people ahead of you in the queue at your school canteen.

D The height of footballers in a team.

E The number of apples you ate for lunch this week.

Question 4

Which one of the following random variables is discrete?

A The weight of students in your class.

B The shoe size of students in your class.

C The height of people ahead of you in the queue at your school canteen.

D The height of footballers in a team.

E The volume of apples you ate for lunch this week.

Question 5

For a discrete probability distribution, X, if $\text{Var}(X) = 2$, then $\text{Var}(2X + 1)$ equals

A $\sqrt{2}$ **B** 3 C 4 D 5 E 8

Question 6

For a discrete probability distribution, X, if $\text{Var}(X) = 4$, then $\text{Var}(-X)$ equals

A -4 **B** -1 C 2 D 4 E 9

Question 7

For a discrete probability distribution, X, if $E(X) = 1.1$, then $E(-X - 2)$ equals

A -3.1 **B** -1.1 **C** -0.9 **D** -0.79 **E** 0.9

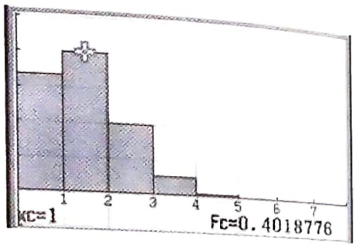
Question 8

Four coins are tossed. The probability of getting four tails is

A $\frac{1}{16}$ **B** $\frac{1}{8}$ **C** $\frac{1}{4}$ **D** $\frac{1}{2}$ **E** $\frac{15}{16}$

Question 9

For the graph of the binomial distribution shown, which of the following is an **incorrect** statement?



A The probability of success, p, is less than 0.5.

B The graph is skewed to the right.

C The expected value is approximately 1.

D The graph is skewed negatively.

E The graph is not symmetrical.

Question 10

A fair die is rolled 60 times. The expected number of sixes is

A 5 **B** 10 **C** 15 **D** 20 **E** 30

Question 11

For a binomial experiment, it is **not** true to say

A there are n independent trials

B the probability of success, p, is the same for each trial

C there are only two possible outcomes for each trial

D binomial trials form a continuous probability distribution

E the sum of the probabilities of all the possible outcomes is 1.

Question 12

If a binomial variable X has the probability function $\Pr(X = x) = {}^5C_x(0.8)^{5-x}(0.2)^x$, where $x = 0, 1 \ldots 5$, then the probability of success is

A 0.2 **B** 0.8 **C** $5 - x$ **D** 1 **E** x

Question 13

Consider the following table of a discrete probability distribution.

x	-1	0	1	2	3
$\Pr(X = x)$	0.3	0.1	0.3	0.2	0.1

The mean of X is

A 0 **B** 0.7 **C** 1 **D** 1.81 **E** 2.3

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Question 14

Consider the following table of a discrete probability distribution.

x	−1	0	1	2	3
$\Pr(X = x)$	0.3	0.1	0.3	0.2	0.1

The expected value of X^2 is

A 0 **B** 0.7 **C** 1 **D** 1.81 **E** 2.3

Question 15

Consider the following table of a discrete probability distribution.

x	−1	0	1	2	3
$\Pr(X = x)$	0.3	0.1	0.3	0.2	0.1

The variance of X is

A 0 **B** 0.7 **C** 1 **D** 1.81 **E** 2.3

Question 16

Consider the following table of a discrete probability distribution.

x	−1	0	1	2	3
$\Pr(X = x)$	0.3	0.1	0.3	0.2	0.1

E(X) equals

A 0.1 **B** 0.3 **C** 0.6 **D** 0.7 **E** 0.8

Question 17

A discrete random variable X is given below.

x	−1	0	1
$\Pr(X = x)$	$2a$	$0.5a$	a

The value of a and the mean of the distribution respectively equal

A $a = \frac{2}{7}, \frac{2}{7}$ **B** $a = -\frac{2}{7}, \frac{2}{7}$ **C** $a = 0, 0$ **D** $a = \frac{2}{7}, -\frac{2}{7}$ **E** $a = -\frac{2}{7}, \frac{2}{7}$

Question 18

The graph shown has $E(X^2) =$

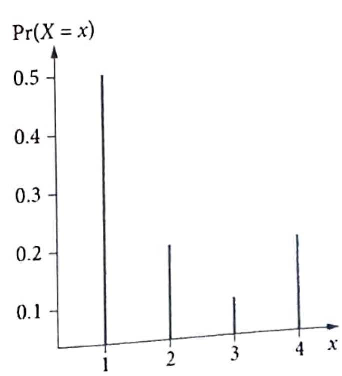
A 1

B 2

C 2.5

D 3

E 5.4



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Question 19

For the graph shown in question 18, the mean of X is

A 0.5 **B** 1 **C** 2 **D** 3 **E** 4

Question 20

Which one of the following is a binomial trial?

A Drawing 6 marbles from a bag of marbles without replacement and recording the number of yellow marbles.

B Rolling a die 10 times and recording each number.

C Selecting 4 students from a group of 10 students to form a committee.

D Drawing 6 cards from a deck of cards, with replacement, and recording the number of picture cards.

E Selecting 4 jellybeans from a jar containing 2 different-colour jelly beans and eating them after each selection.

Question 21

If X is a binomial random variable with $n = 40, p = \frac{3}{4}$, then the mean and standard deviation, respectively, are:

A $15, \frac{15}{4}$ **B** $5, \frac{\sqrt{15}}{2}$ **C** $15, \frac{\sqrt{15}}{2}$ **D** $30, \sqrt{\frac{15}{2}}$ **E** $40, \frac{\sqrt{30}}{2}$

Question 22

If X has a binomial distribution and $\Pr(X = x) = {}^4C_x(0.3)^x(0.7)^{4-x}$, then $\Pr(X \geq 3)$ equals

A $4(0.3)(0.7)^3 + (0.7)^4$

B $4(0.3)^3(0.7) + (0.3)^4$

C $4(0.3)^3(0.7)$

D $1 - (0.7)^4 - 4(0.3)(0.7)^3$

E $1 - 4(0.3)(0.7)^3 - 6(0.3)^2(0.7)^2$

Question 23

X and Y are discrete random variables. If $\text{Var}(X) = 4$ and $Y = 2X + 3$, then $\text{Var}(Y)$ equals

A 2 **B** 4 **C** 11 **D** 16 **E** 19

Question 24

X and Y are discrete random variables. If $E(X) = 4$ and $Y = 2X + 3$, then $E(Y)$ equals

A 2 **B** 4 **C** 11 **D** 16 **E** 19

Question 25

A random variable X has the following probability distribution:

x	0	2	3	5
$\Pr(X = x)$	0.2	0.1	0.5	0.2

$E((X - 1)^2)$ is equal to

A 1.7 **B** 2.89 **C** 5.5 **D** 6.29 **E** 7.3

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Question 26

Two dice are rolled. Let X be the sum of the numbers on the uppermost faces. If a discrete distribution table is set up with all the outcomes and their probabilities, it would look like

A

x	2	3	4	5	6	7	8	9	10	11	12
$\Pr(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

B

x	1	2	3	4	5	6	7	8	9	10	11	12
$\Pr(X = x)$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{5}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

C

x	2	4	6	8	10	12
$\Pr(X = x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

D

x	2	3	4	5	6	7	8	9	10	11	12
$\Pr(X = x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$	$\frac{5}{6}$	$\frac{4}{6}$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

E

x	1	2	3	4	5	6	7	8	9	10	11	12
$\Pr(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$	$\frac{5}{6}$	$\frac{4}{6}$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$

Question 27

A random variable X is defined by the probability density function

$$f(x) = \begin{cases} k(x-1)^2 & 1 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

The value of k is

A $\frac{3}{8}$ **B** $\frac{\sqrt{6}}{4}$ **C** 1 **D** $\frac{8}{3}$ **E** 3

Question 28

X is a discrete random variable, and if $\text{Var}(X) = 6$ and $\text{E}(X) = 2$, then $\text{E}(X^2)$ equals

A 1 **B** 4 **C** 7 **D** 9 **E** 10

Question 29

The probability that Amy (A) drives to work is 0.7. Independently of Amy, the probability that her sister Claire (C) walks to work is 0.5. Which of the following is **incorrect**?

A $\Pr(A \cap C) = 0.35$

B A and C are independent events.

C $\Pr(A \cup C) = 0.85$

D A and C are mutually exclusive events

E $\Pr(A \mid C) = 0.7$

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Question 30

A binomial probability distribution has a mean of 20 and a variance of 10. The probability of success, p, and the number of trials, n, are

A $p = \frac{1}{2}, n = 36$

B $p = \frac{1}{2}, n = 40$

C $p = \frac{1}{6}, n = 40$

D $p = \frac{1}{6}, n = 180$

E $p = \frac{1}{2}, n = 4$

Question 31

A bag of marbles has 5 red and 5 blue marbles in it. I reach into the bag and randomly select a marble and set it aside. I reach into the bag again and select another marble. The probability of selecting one of each colour is

A $\frac{25}{90}$

B $\frac{5}{18}$

C $\frac{4}{9}$

D $\frac{1}{2}$

E $\frac{5}{9}$

Question 32

The probability that Sue, a cricketer, will score one run each time she faces a bowler is 0.7. If Sue has 6 shots at facing a bowler in every over, and she never takes more than one run at a time, the probability that she will score at least 2 runs in an over can be expressed as

A $1 - (0.3)^6 - 6(0.3)^5(0.7)^1$

B $1 - (0.3)^6 - 6(0.3)^5(0.7)^1 - 15(0.3)^4(0.7)^2$

C $1 + (0.3)^6 + 6(0.3)^5(0.7)^1$

D $(0.3)^6 + 6(0.3)^5(0.7)^1$

E $(0.3)^6$

Question 33

A binomial variable, X, has the probability function $\Pr(X = x) = {}^3C_x(0.2)^{3-x}(0.8)^x$ where $x = 0, 1, 2, 3$.

The probability distribution is

A

x	0	1	2	3
$\Pr(X = x)$	$(0.8)^3$	$3(0.2)^2(0.8)$	$3(0.2)^2(0.8)$	$(0.2)^3$

B

x	0	1	2	3
$\Pr(X = x)$	$(0.2)^3$	$3(0.2)^2(0.8)$	$3(0.2)(0.8)^2$	$(0.8)^3$

C

x	0	1	2	3
$\Pr(X = x)$	0.008	0.009	0.081	0.512

D

x	0	1	2	3
$\Pr(X = x)$	0.729	0.2	0.07	0.001

E

x	0	1	2	3
$\Pr(X = x)$	0.001	0.12	0.15	0.729

Question 34

If the mean of a binomial distribution is 3 and the standard deviation is 1.5, then the number of trials, n, and the probability of success, p, are respectively

A $n = 3, p = \frac{3}{2}$

B $n = 10, p = 0.9$

C $n = 15, p = 0.6$

D $n = 12, p = \frac{1}{4}$

E $n = 12, p = \frac{3}{4}$

Question 35

Anna takes a short maths test every week. There are always 20 questions on the test and Anna has found that on average she usually answers 15 of them correctly. The probability that she correctly answers exactly 18 questions on a particular test is closest to

A 0.000 003

B 0.000 182

C 0.0669

D 0.07

E 0.202

Question 36

If $f(x) = 1.5(1 - x^2)$, $0 \le x \le 1$ represents a probability density function, then $\Pr(X \le 0.3)$ equals

A $\frac{1127}{2000}$

B $\frac{1703}{2000}$

C $\frac{5157}{80\,000}$

D $\frac{873}{200}$

E $\frac{873}{2000}$

Question 37

If a random variable X has a probability density function $f(x) = 1.5(x - 1)^2$, $0 \le x \le 2$, and 0 elsewhere, then the mean of X is

A 0

B 1

C $\frac{8}{5}$

D 0 and 2

E 2

Question 38

If $f(x)$ is a probability density function such that

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 < x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

then the mean of X is

A $\frac{1}{2}$

B 1

C $\frac{7}{6}$

D $\frac{5}{4}$

E 2

Question 39

If $f(x)$ is a probability density function such that

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 < x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

then variance of X is

A $\frac{1}{6}$

B 1

C $\frac{7}{6}$

D $\frac{5}{4}$

E 2

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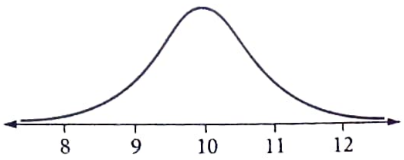
Question 40

If X is a normal random variable with $\mu = 10$ and $\sigma = \frac{1}{3}$, then the graph of this distribution is most likely to be

A



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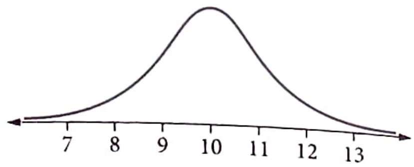

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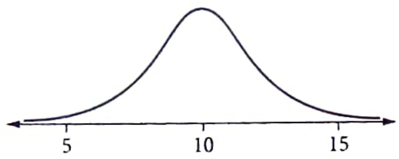
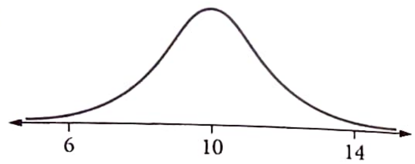
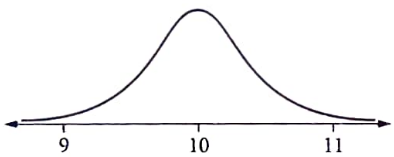


D



E



Question 41

Heights of students, in cm, in a particular classroom are normally distributed with a mean of 150 and a standard deviation of 8. The probability that a randomly selected student is taller than 166 cm is approximately

A 0.425 **B** 0.475 **C** 0.84 **D** 0.925 **E** 0.975

Question 42

If Z is the standard normal variable, $\Pr(Z < 1.2)$ is closest to

A 0.03 **B** 0.05 **C** 0.11 **D** 0.86 **E** 0.88

Question 43

If $\Pr(Z \le k) = 0.8$ and Z is a standard normal variable, then k is closest to

A −1.282 **B** −0.842 **C** 0.524 **D** 0.726 **E** 0. 842

Question 44

If X is normally distributed with a mean of 15 and a standard deviation of 2, and $\Pr(X < m) = 0.48$, then m is closest to

A 12.1 **B** 14.8 **C** 14.9 **D** 16.8 **E** 17.9

Question 45

IQ scores are currently normally distributed with a mean of 100 and a standard deviation of 15. When a randomly selected student sits the IQ test, the probability that he scores higher than 102 is closest to

A 0.102 **B** 0.288 **C** 0.356 **D** 0.447 **E** 0.553

Question 46

The weight of bags of lollies is normally distributed. The bags of lollies have a mean weight of 100 g. If a bag weighs 95 g or less, it is unacceptable. Tests show that 3% of bags were unacceptable. The standard deviation of the weight (in grams) of the bags is

A 2.054 **B** 2.435 **C** 2.658 **D** 5.941 **E** 10.03

Question 47

In a binomial distribution it is found that 70% of 17-year-olds have their L plates. For a sample size of 20, the probability that the sample proportion is equal to the population proportion (0.7) is closest to

A 0.0002 **B** 0.006 **C** 0.1916 **D** 0.9196 **E** 14

Question 48

In a binomial distribution it is found that 70% of 17-year-olds have their L plates.

For a sample size of 20, the probability that the percentage of 17-year-olds who have their L plates lies within one standard deviation of the population proportion is closest to

A 0.7795 **B** 0.7796 **C** 0.9165 **D** 0.9166 **E** 0.9752

Question 49

The approximate 95% CI for the proportion p of primary school children, using a random sample of 2000 children when it is found that the sample proportion $\hat{p}$ is 0.6, is

A $(0.042, 0.042)$ **B** $(0.579, 0.621)$ **C** $(0.670, 0.730)$

D $(0.6, 0.95)$ **E** $(0.690, 0.710)$

Question 50

It is known that 60% of families in a district of over 20 000 families own a piano. If you choose a sample of 1000 families, the probability that the proportion of families in the sample is between 58% and 62% is closest to

A 0% **B** 1.65% **C** 58% **D** 60% **E** 80.31%

Extended-answer questions

Technology active: 18 questions

Solutions to this section start on page 248

Question 1 (24 marks)

A binomial random variable distribution has probability p for success.

a **i** For three independent trials state an expression, in terms of p, that would find $\Pr(X > 2)$. 2 marks

ii For three independent trials state an expression, in terms of p, that would find $\Pr(X = 0)$. 2 marks

b **i** For what value of p is $\Pr(X > 2) = \Pr(X = 0)$? 1 mark

ii Explain why you would expect this answer for p in this situation. 1 mark

For a 3-child family, the distribution table is shown below, where X = number of boys in a family and the probability of having a boy for each birth is $p = 0.5$.

c Complete the discrete distribution table below in terms of p. 4 marks

x	0	1	2	3
$\Pr(X = x)$				

d What is the mean number of boys in a family for this situation? 3 marks

The situation is changed and for a particular genetic community, a 3-child family, where Y = number of boys in a family, the probability of having a boy is 0.55.

e Complete the discrete distribution table below, correct to four decimal places. 2 marks

y	0	1	2	3
$\Pr(Y = y)$				

f **i** What is the mean and variance of the number of boys in a family for this situation? Give your answers correct to three decimal places. 3 marks

ii Find the probability that the outcome falls within one standard deviation of the mean. 2 marks

iii In a family where the probability of a boy is 0.55, what is the probability, correct to three decimal places, that there will be at least 2 boys in the family? 1 mark

g Counting each family of 3 children as a trial, and with the probability of a boy for each birth being 0.55, how many trials will be needed for the probability of at least 2 boys in at least 2 families to be at least 0.6? 3 marks

Question 2 (16 marks) ©VCAA 2018 2BQ4

Doctors are studying the resting heart rate of adults in two neighbouring towns: Mathsland and Statsville. Resting heart rate is measured in beats per minute (bpm).

The resting heart rate of adults in Mathsland is known to be normally distributed with a mean of 68 bpm and a standard deviation of 8 bpm.

a 87% Find the probability that a randomly selected Mathsland adult has a resting heart rate between 60 bpm and 90 bpm. Give your answer correct to three decimal places. 1 mark

The doctors consider a person to have a slow heart rate if the person's resting heart rate is less than 60 bpm. The probability that a randomly chosen Mathsland adult has a slow heart rate is 0.1587. It is known that 29% of Mathsland adults play sport regularly. It is also known that 9% of Mathsland adults play sport regularly and have a slow heart rate.

Let S be the event that a randomly selected Mathsland adult plays sport regularly and let H be the event that a randomly selected Mathsland adult has a slow heart rate.

b **i** 57% Find $\Pr(H|S)$, correct to three decimal places. 1 mark

ii 44% Are the events H and S independent? Justify your answer. 1 mark

c **i** 71% Find the probability that a random sample of 16 Mathsland adults will contain exactly one person with a slow heart rate. Give your answer correct to three decimal places. 2 marks

ii 41% For random samples of 16 Mathsland adults, $\hat{P}$ is the random variable that represents the proportion of people who have a slow heart rate.

Find the probability that $\hat{P}$ is greater than 10%, correct to three decimal places. 2 marks

iii 11% For random samples of n Mathsland adults, $\hat{P}_n$ is the random variable that represents the proportion of people who have a slow heart rate.

Find the least value of n for which $\Pr\left(\hat{P}_n > \frac{1}{n}\right) > 0.99$. 2 marks

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The doctors took a large random sample of adults from the population of Statsville and calculated an approximate 95% confidence interval for the proportion of Statsville adults who have a slow heart rate. The confidence interval they obtained was (0.102, 0.145).

d **i** 45% Determine the sample proportion used in the calculation of this confidence interval. 1 mark

ii 11% Explain why this confidence interval suggests that the proportion of adults with a slow heart rate in Statsville could be different from the proportion in Mathsland. 1 mark

Every year at Mathsland Secondary College, students hike to the top of a hill that rises behind the school. The time taken by a randomly selected student to reach the top of the hill has the probability density function M with the rule

$$M(t) = \begin{cases} \dfrac{3}{50}\left(\dfrac{t}{50}\right)^2 e^{-\left(\frac{t}{50}\right)^3} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

where t is given in minutes.

e 58% Find the expected time, in minutes, for a randomly selected student from Mathsland Secondary College to reach the top of the hill. Give your answer correct to one decimal place. 2 marks

Students who take less than 15 minutes to get to the top of the hill are categorised as 'elite'.

f 56% Find the probability that a randomly selected student from Mathsland Secondary College is categorised as elite. Give your answer correct to four decimal places. 1 mark

g 8% The Year 12 students at Mathsland Secondary College make up $\frac{1}{7}$ of the total number of students at the school. Of the Year 12 students at Mathsland Secondary College, 5% are categorised as elite. Find the probability that a randomly selected non-Year 12 student at Mathsland Secondary College is categorised as elite. Give your answer correct to four decimal places. 2 marks

Question 3 (7 marks) ©VCAA 2013 2BQ2ab

FullyFit is an international company that owns and operates many fitness centres (gyms) in several countries. At every one of FullyFit's gyms, each member agrees to have his or her fitness assessed every month by undertaking a set of exercises called S. There is a five-minute time limit on any attempt to complete S and if someone completes S in less than three minutes, they are considered fit.

At FullyFit's Melbourne gym, it has been found that the probability that any member will complete S in less than three minutes is $\frac{5}{8}$. This is independent of any other member.

In a particular week, 20 members of this gym attempt S.

a **i** 77% Find the probability, correct to four decimal places, that at least 10 of these 20 members will complete S in less than three minutes. 2 marks

ii 58% Given that at least 10 of these 20 members complete S in less than three minutes, what is the probability, correct to three decimal places, that more than 15 of them complete S in less than three minutes? 3 marks

b 62% Paula is a member of FullyFit's gym in San Francisco. She completes S every month as required, but otherwise does not attend regularly and so her fitness level varies over many months. Paula finds that if she is fit one month, the probability that she is fit the next month is $\frac{3}{4}$, and if she is not fit one month, the probability that she is not fit the next month is $\frac{1}{2}$.

If Paula is not fit in one particular month, what is the probability that she is fit in exactly two of the next three months? 2 marks

Question 4 (7 marks) ©VCAA 2016 2BQ3abh MODIFIED

A school has a class set of 22 new laptops kept in a recharging trolley. Provided each laptop is correctly plugged into the trolley after use, its battery recharges.

On a particular day, a class of 22 students uses the laptops. All laptop batteries are fully charged at the start of the lesson. Each student uses and returns exactly one laptop. The probability that a student does **not** correctly plug their laptop into the trolley at the end of the lesson is 10%. The correctness of any student's plugging-in is independent of any other student's correctness.

a 80% Determine the probability that at least one of the laptops is **not** correctly plugged into the trolley at the end of the lesson. Give your answer correct to four decimal places. 2 marks

b 51% A teacher observes that at least one of the returned laptops is not correctly plugged into the trolley. Given this, find the probability that fewer than five laptops are **not** correctly plugged in. Give your answer correct to four decimal places. 2 marks

c The laptop supplier also provides laptops to businesses. The probability density function for battery life, x (in minutes), of a laptop after six months of use in a business is

$$f(x) = \begin{cases} \dfrac{(210 - x)e^{\frac{x-210}{20}}}{400} & 0 \le x \le 210 \\ 0 & \text{elsewhere} \end{cases}$$

i 67% Find the **mean** battery life, in minutes, of a laptop with six months of business use, correct to two decimal places. 1 mark

ii 66% Find the value of m, correct to the nearest integer, for which $\Pr(X < m) = 0.5$. 2 marks

Question 5 (11 marks) ©VCAA 2015 2BQ3

Mani is a fruit grower. After his oranges have been picked, they are sorted by a machine according to size. Oranges classified as **medium** are sold to fruit shops and the remainder are made into orange juice.

The distribution of the diameter, in centimetres, of medium oranges is modelled by a continuous random variable, X, with probability density function.

$$f(x) = \begin{cases} \dfrac{3}{4}(x-6)^2(8-x) & 6 \le x \le 8 \\ 0 & \text{elsewhere} \end{cases}$$

a i 82% Find the probability that a randomly selected medium orange has a diameter greater than 7 cm. 2 marks

ii 54% Mani randomly selects three medium oranges. Find the probability that exactly one of the oranges has a diameter greater than 7 cm. Express the answer in the form $\frac{a}{b}$, where a and b are positive integers. 2 marks

b 73% Find the mean diameter of medium oranges, in centimetres. 1 mark

For oranges classified as large, the quantity of juice obtained from each orange is a normally distributed random variable with a mean of 74 mL and a standard deviation of 9 mL.

c 57% What is the probability, correct to three decimal places, that a randomly selected large orange produces less than 85 mL of juice, given that it produces more than 74 mL of juice? 2 marks

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Mani also grows lemons, which are sold to a food factory. When a truckload of lemons arrives at the food factory, the manager randomly selects and weighs four lemons from the load. If one or more of these lemons is underweight, the load is rejected. Otherwise it is accepted.

It is known that 3% of Mani's lemons are underweight.

d i 56% Find the probability that a particular load of lemons will be rejected. Express the answer correct to four decimal places. 2 marks

ii 42% Suppose that instead of selecting only four lemons, n lemons are selected at random from a particular load.

Find the smallest integer value of n such that the probability of at least one lemon being underweight exceeds 0.5 2 marks

Question 6 (13 marks) ©VCAA 2012 2BQ3abicd

Steve, Katerina and Jess are three students who have agreed to take part in a psychology experiment. Each student is to answer several sets of multiple-choice questions. Each set has the same number of questions, n, where n is a number greater than 20. For each question there are four possible options (A, B, C or D), of which only one is correct.

a Steve decides to guess the answer to every question, so that for each question he chooses A, B, C or D at random. Let the random variable X be the number of questions that Steve answers correctly in a particular set.

i 70% What is the probability that Steve will answer the first three questions of this set correctly? 1 mark

ii 60% Find, to four decimal places, the probability that Steve will answer at least 10 of the first 20 questions of this set correctly. 2 marks

iii 55% Use the fact that the variance of X is $\frac{75}{16}$ to show that the value of n is 25. 1 mark

If Katerina answers a question correctly, the probability that she will answer the next question correctly is $\frac{3}{4}$. If she answers a question incorrectly, the probability that she will answer the next question incorrectly is $\frac{2}{3}$.

In a particular set, Katerina answers Question 1 incorrectly.

b 44% Calculate the probability that Katerina will answer questions 3, 4 and 5 correctly. 3 marks

c 19% The probability that Jess will answer any question correctly, independently of her answer to any other question, is p ($p > 0$). Let the random variable Y be the number of questions that Jess answers correctly in any set of 25.

If $\Pr(Y > 23) = 6\Pr(Y = 25)$, show that the value of p is $\frac{5}{6}$. 2 marks

d 19% From these sets of 25 questions being completed by many students, it has been found that the time, in minutes, that any student takes to answer each set of 25 questions is another random variable, W, which is **normally distributed** with mean a and standard deviation b.

It turns out that, for Jess, $\Pr(Y \geq 18) = \Pr(W \geq 20)$ and also $\Pr(Y \geq 22) = \Pr(W \geq 25)$.

Calculate the values of a and b, correct to three decimal places. 4 marks

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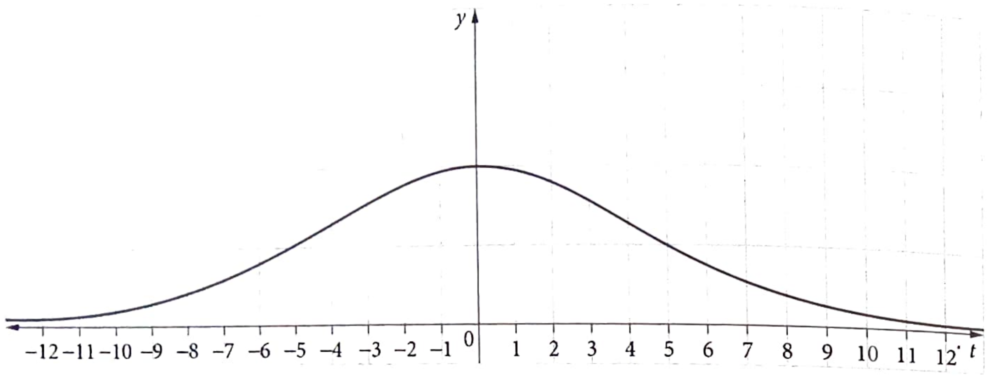
CHAPTER 4 – EXAM PRACTICE

Question 7 (12 marks) ©VCAA 2020 2BQ3

A transport company has detailed records of all its deliveries.

The number of minutes a delivery is made before or after its scheduled delivery time can be modelled as a normally distributed random variable, T, with a mean of zero and a standard deviation of four minutes. A graph of the probability distribution of T is shown below.



a 68% If $\Pr(T \le a) = 0.6$, find a to the nearest minute. 1 mark

b 49% Find the probability, correct to three decimal places, of a delivery being no later than three minutes after its scheduled delivery time, given that it arrives after its scheduled delivery time. 2 marks

c 24% Using the model described, the transport company can make 46.48% of its deliveries over the interval $-3 \le t \le 2$. It has an improved delivery model with a mean of k and a standard deviation of four minutes. Find the values of k, correct to one decimal place, so that 46.48% of the transport company's deliveries can be made over the interval $-4.5 \le t \le 0.5$. 3 marks

A rival transport company claims that there is a 0.85 probability that each delivery it makes will arrive on time or earlier. Assume that whether each delivery is on time or earlier is independent of other deliveries.

d 51% Assuming that the rival company's claim is true, find the probability that on a day in which the rival company makes eight deliveries, fewer than half of them arrive on time or earlier. Give your answer correct to three decimal places. 2 marks

e Assuming that the rival company's claim is true, consider a day in which it makes n deliveries.

i 24% Express, in terms of n, the probability that one or more deliveries will **not** arrive on time or earlier. 1 mark

ii 23% Hence, or otherwise, find the minimum value of n such that there is at least a 0.95 probability that one or more deliveries will **not** arrive on time or earlier. 1 mark

f 4% An analyst from a government department believes the rival transport company's claim is only true for deliveries made before 4 pm. For deliveries made after 4 pm, the analyst believes the probability of a delivery arriving on time or earlier is x, where $0.3 \le x \le 0.7$. After observing a large number of the rival transport company's deliveries, the analyst believes that the overall probability that a delivery arrives on time or earlier is actually 0.75. Let the probability that a delivery is made after 4 pm be y. Assuming that the analyst's beliefs are true, find the minimum and maximum values of y. 2 marks

Question 8 (8 marks) ©VCAA 2009 2BQ3abcid

The Bouncy Ball Company (BBC) makes tennis balls whose diameters are normally distributed with mean 67 mm and standard deviation 1 mm. The tennis balls are packed and sold in cylindrical tins that each hold four balls. A tennis ball fits into such a tin if the diameter of the ball is less than 68.5 mm.

a 77% What is the probability, correct to four decimal places, that a randomly selected tennis ball produced by BBC fits into a tin? 2 marks

BBC management would like each ball produced to have diameter between 65.6 m and 68.4 mm.

b 77% What is the probability, correct to four decimal places, that the diameter of a randomly selected tennis ball made by BBC is in this range? 2 marks

c 42% What is the probability, correct to four decimal places, that the diameter of a tennis ball which fits into a tin is between 65.6 mm and 68.4 mm? 1 mark

BBC management wants engineers to change the manufacturing process so that 99% of all balls produced have a diameter between 65.6 mm and 68.4 mm. The mean is to stay at 67 mm, but the standard deviation is to be changed.

d 35% What should the new standard deviation be (correct to two decimal places)? 3 marks

Question 9 (12 marks) ©VCAA 2014 2BQ4a-f

Patricia is a gardener and she owns a garden nursery. She grows and sells basil plants and coriander plants. The heights, in centimetres, of the basil plants that Patricia is selling are distributed normally with a mean of 14 cm and a standard deviation of 4 cm. There are 2000 basil plants in the nursery.

a 43% Patricia classifies the tallest 10 percent of her basil plants as super. What is the minimum height of a super basil plant, correct to the nearest millimetre? 1 mark

Patricia decides that some of her basil plants are not growing quickly enough, so she plans to move them to a special greenhouse. She will move the basil plants that are less than 9 cm in height.

b 54% How many basil plants will Patricia move to the greenhouse, correct to the nearest whole number? 2 marks

The heights of the coriander plants, x centimetres, follow the probability density function $h(x)$, where

$$h(x) = \begin{cases} \frac{\pi}{100}\sin\left(\frac{\pi x}{50}\right) & 0 \le x \le 50 \\ 0 & \text{otherwise} \end{cases}$$

c 73% State the mean height of the coriander plants. 1 mark

Patricia thinks that the smallest 15% of her coriander plants should be given a new type of plant food.

d 33% Find the maximum height, correct to the nearest millimetre, of a coriander plant if it is to be given the new type of plant food. 2 marks

Patricia also grows and sells tomato plants that she classifies as either tall or regular. She finds that 20% of her tomato plants are tall. A customer, Jack, selects n tomato plants at random.

e 30% Let q be the probability that at least one of Jack's n tomato plants is tall. Find the minimum value of n so that q is greater than 0.95. 2 marks

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In another section of the nursery, a craftsman makes plant pots. The pots are classified as smooth or rough. The craftsman finishes each pot before starting on the next. Over a period of time, it is found that if one plant pot is smooth, the probability that the next one is smooth is 0.7, while if one plant pot is rough, the probability that the next one is rough is p, where $0 < p < 1$. The value of p stays fixed for a week at a time, but can vary from week to week.

The first pot made each week is always a smooth pot.

f **i** 54% Find, in terms of p, the probability that the third pot made in a given week is smooth. 2 marks

ii 57% In one particular week, the probability that the third pot made is smooth is 0.61. Calculate the value of p in this week. 2 marks

Question 10 (6 marks) ©VCAA 2018N 2BQ2bgh

Rebecca's Robotics manufactures three types of components for robots: sensors, motors and controllers. The manufacturing processes for each type of component are independent.

It is known that 8% of all of the sensors manufactured are defective.

A random sample of 50 sensors is selected and it is found that the proportion of defective sensors in this sample is 0.08.

a Determine an approximate 90% confidence interval for the proportion of defective sensors, correct to four decimal places. 2 marks

The weight, w, in grams, of controllers is modelled by the following probability density function.

$$C(w) = \begin{cases} \dfrac{3}{640\,000}(330 - w)^2(w - 290) & 290 \le w \le 330 \\ 0 & \text{elsewhere} \end{cases}$$

b Determine the mean weight, in grams, of the controllers. 2 marks

c Determine the probability that a randomly selected controller weighs less than the mean weight of the controllers. Give your answer correct to four decimal places. 2 marks

Question 11 (15 marks) ©VCAA 2011 2BQ2

In a chocolate factory, the material for making each chocolate is sent to one of two machines, machine A or machine B.

The time, X seconds, taken to produce a chocolate by machine A, is normally distributed with mean 3 and standard deviation 0.8.

The time, Y seconds, taken to produce a chocolate by machine B, has the following probability density function.

$$f(y) = \begin{cases} 0 & y < 0 \\ \dfrac{y}{16} & 0 \le y \le 4 \\ 0.25e^{-0.5(y-4)} & y > 4 \end{cases}$$

a Find correct to four decimal places

i 77% $\Pr(3 \le X \le 5)$ 1 mark

ii 66% $\Pr(3 \le Y \le 5)$ 3 marks

b 52% Find the mean of Y, correct to three decimal places. 3 marks

c **i** 53% Find where the upper 50% of Y exists 1 mark

ii 32% Find the value of a, correct to two decimal places, such that $\Pr(Y \le a) = 0.7$. 2 marks

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d 57% It can be shown that $\Pr(Y \le 3) = \frac{9}{32}$. A random sample of 10 chocolates **produced by machine *B*** is chosen. Find the probability, correct to four decimal places, that exactly 4 of these 10 chocolates took 3 or less seconds to produce. 2 marks

All of the chocolates produced by machine *A* and machine *B* are stored in a large bin. There is an equal number of chocolates from each machine in the bin.

It is found that if a chocolate, produced by either machine, takes longer than 3 seconds to produce then it can easily be identified by its darker colour.

e 19% A chocolate is selected at random from the bin. It is found to have taken longer than 3 seconds to produce. Find, correct to four decimal places, the probability that it was produced by machine *A*. 3 marks

Question 12 (14 marks) ©VCAA 2007 2BQ5

In the Great Fun amusement park there is a small train called Puffing Berty which does a circuit of the park. The continuous random variable *T*, the time in minutes for a circuit to be completed, has a probability density function *f* with rule

$$f(t) = \begin{cases} \frac{1}{100}(t-10) & \text{if } 10 \le t < 20 \\ \frac{1}{100}(30-t) & \text{if } 20 \le t \le 30 \\ 0 & \text{otherwise} \end{cases}$$

a 45% Sketch the graph of $y = f(t)$. 2 marks

b 59% Find the probability that the time taken by Puffing Berty to complete a full circuit is less than 25 minutes. Give the exact value. 2 marks

c 49% Find $\Pr(T \le 15 \mid T \le 25)$. Give the exact value. 2 marks

The train must complete six circuits between 9.00 am and noon. The management prefers Puffing Berty to complete a circuit in less than 25 minutes.

d 48% Find the probability, correct to four decimal places, that of the 6 circuits completed, at least 4 of them take less than 25 minutes each. 2 marks

For scheduling reasons the management wants to know the time, *b* minutes, for which the probability of exactly 3 or 4 out of the 6 circuits completed each taking less than *b* minutes, is maximised.

Let $\Pr(T < b) = p$

Let Q be the probability that exactly 3 or 4 circuits completed each take less than *b* minutes.

e 28% Show that $Q = 5p^3(1-p)^2(4-p)$. 2 marks

f i 19% Find the maximum value of Q and the value of *p* for which this occurs. (Give the exact value.) 2 marks

ii 11% Find, correct to one decimal place, the value of *b* for which this maximum occurs. 2 marks

Question 13 (6 marks) ©VCAA 2008 2BQ1c ●●

Sharelle is the goal shooter for her netball team. The time in hours that Sharelle spends training each day is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \frac{1}{64}(6-x)(x-2)(x+2) & \text{if } 2 \le x \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

a 61% Sketch the probability density function, and label the local maximum with its coordinates, correct to two decimal places. 2 marks

b 73% What is the probability, correct to four decimal places, that Sharelle spends less than 3 hours training on a particular day? 2 marks

c 58% What is the mean time (in hours), correct to four decimal places, that she spends training each day? 2 marks

Question 14 (19 marks) ©VCAA 2017 2BQ3 ●●●

The time Jennifer spends on her homework each day varies, but she does some homework every day. The continuous random variable T, which models the time, t, in minutes, that Jennifer spends each day on her homework, has a probability density function f, where

$$f(t) = \begin{cases} \frac{1}{625}(t-20) & 20 \le t < 45 \\ \frac{1}{625}(70-t) & 45 \le t \le 70 \\ 0 & \text{elsewhere} \end{cases}$$

a 62% Sketch the graph of f. 3 marks

b 73% Find $\Pr(25 \le T \le 55)$. 2 marks

c 64% Find $\Pr(T \le 25 \mid T \le 55)$. 2 marks

d 36% Find a such that $\Pr(T \ge a) = 0.7$, correct to four decimal places. 2 marks

e The probability that Jennifer spends more than 50 minutes on her homework on any given day is $\frac{8}{25}$. Assume that the amount of time spent on her homework on any day is independent of the time spent on her homework on any other day.

i 66% Find the probability that Jennifer spends more than 50 minutes on her homework on more than three of seven randomly chosen days, correct to four decimal places. 2 marks

ii 65% Find the probability that Jennifer spends more than 50 minutes on her homework on at least two of seven randomly chosen days, given that she spends more than 50 minutes on her homework on at least one of those days, correct to four decimal places. 2 marks

Let p be the probability that on any given day Jennifer spends more than d minutes on her homework.

Let q be the probability that on two or three days out of seven randomly chosen days she spends more than d minutes on her homework.

f 36% Express q as a polynomial in terms of p. 2 marks

g **i** 30% Find the maximum value of q, correct to four decimal places, and the value of p for which this maximum occurs, correct to four decimal places. 2 marks

ii 9% Find the value of d for which the maximum found in part **g i** occurs, correct to the nearest minute. 2 marks

Question 15 (14 marks) ©VCAA 2019N 2BQ3

Concerts at the Mathsland Concert Hall begin L minutes after the scheduled starting time. L is a random variable that is normally distributed with a mean of 10 minutes and a standard deviation of four minutes.

a What proportion of concerts begin before the scheduled starting time, correct to four decimal places? 1 mark

b Find the probability that a concert begins more than 15 minutes after the scheduled starting time, correct to four decimal places. 1 mark

If a concert begins more than 15 minutes after the scheduled starting time, the cleaner is given an extra payment of $200. If a concert begins up to 15 minutes after the scheduled starting time, the cleaner is given an extra payment of $100. If a concert begins at or before the scheduled starting time, there is no extra payment for the cleaner.

Let C be the random variable that represents the extra payment for the cleaner, in dollars.

c i Using your responses from part **a** and part **b**, copy and complete the following table, correct to three decimal places.

c	0	100	200
$\Pr(C = c)$			

1 mark

ii Calculate the expected value of the extra payment for the cleaner, to the nearest dollar. 1 mark

iii Calculate the standard deviation of C, correct to the nearest dollar. 1 mark

d The owners of the Mathsland Concert Hall decide to review their operation. They study information from 1000 concerts at other similar venues, collected as a simple random sample. The sample value for the number of concerts that start more than 15 minutes after the scheduled starting time is 43.

i Find the 95% confidence interval for the proportion of concerts that begin more than 15 minutes after the scheduled starting time. Give values correct to three decimal places. 1 mark

ii Explain why this confidence interval suggests that the proportion of concerts that begin more than 15 minutes after the scheduled starting time at the Mathsland Concert Hall is different from the proportion at the venues in the sample. 1 mark

The owners of the Mathsland Concert Hall decide that concerts must not begin before the scheduled starting time. They also make changes to reduce the number of concerts that begin after the scheduled starting time. Following these changes, M is the random variable that represents the number of minutes after the scheduled starting time that concerts begin.

The probability density function for M is

$$f(x) = \begin{cases} \dfrac{8}{(x+2)^3} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

where x is the time, in minutes, after the scheduled starting time.

e Calculate the expected value of M. 2 marks

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f **i** Find the probability that a concert now begins more than 15 minutes after the scheduled starting time. 1 mark

ii Find the probability that each of the next nine concerts begins no more than 15 minutes after the scheduled starting time and the 10th concert begins more than 15 minutes after the scheduled starting time. Give your answer correct to four decimal places. 2 marks

iii Find the probability that a concert begins up to 20 minutes after the scheduled starting time, given that it begins more than 15 minutes after the scheduled starting time. Give your answer correct to three decimal places. 2 marks

Question 16 (18 marks) ©VCAA 2017N 2BQ3

A company supplies schools with whiteboard pens. The total length of time for which a whiteboard pen can be used for writing before it stops working is called its use-time. There are two types of whiteboard pens: Grade A and Grade B. The use-time of Grade A whiteboard pens is normally distributed with a mean of 11 hours and a standard deviation of 15 minutes.

a Find the probability that a Grade A whiteboard pen will have a use-time that is greater than 10.5 hours, correct to three decimal places. 1 mark

The use-time of Grade B whiteboard pens is described by the probability density function

$$f(x) = \begin{cases} \frac{x}{576}(12 - x)(e^{\frac{x}{6}} - 1) & 0 \le x \le 12 \\ 0 & \text{otherwise} \end{cases}$$

where x is the use-time in hours.

b Determine the expected use-time of a Grade B whiteboard pen. Give your answer in hours, correct to two decimal places. 2 marks

c Determine the standard deviation of the use-time of a Grade B whiteboard pen. Give your answer in hours, correct to two decimal places. 2 marks

d Find the probability that a randomly chosen Grade B whiteboard pen will have a use-time that is greater than 10.5 hours, correct to four decimal places. 2 marks

A worker at the company finds two boxes of whiteboard pens that are not labelled, but knows that one box contains only Grade A whiteboard pens and the other box contains only Grade B whiteboard pens. The worker decides to randomly select a whiteboard pen from one of the boxes. If the selected whiteboard pen has a use-time that is greater than 10.5 hours, then the box that it came from will be labelled Grade A and the other box will be labelled Grade B. Otherwise, the box that it came from will be labelled Grade B and the other box will be labelled Grade A.

e Find the probability, correct to three decimal places, that the worker labels the boxes incorrectly. 2 marks

f Find the probability, correct to three decimal places, that the whiteboard pen selected was Grade B, given that the boxes have been labelled incorrectly. 2 marks

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As a whiteboard pen ages, its tip may dry to the point where the whiteboard pen becomes defective (unusable). The company has stock that is two years old and, at that age, it is known that 5% of Grade A whiteboard pens will be defective.

g A school purchases a box of Grade A whiteboard pens that is two years old and a class of 26 students is the first to use them. If every student receives a whiteboard pen from this box, find the probability, correct to four decimal places, that at least one student will receive a defective whiteboard pen. 2 marks

h Let $\hat{P}_A$ be the random variable of the distribution of sample proportions of defective Grade A whiteboard pens in boxes of 100. The boxes come from the stock that is two years old. Find $(\hat{P}_A > 0.04 \mid \hat{P}_A < 0.08)$. Give your answer correct to four decimal places. Do not use a normal approximation. 3 marks

i A box of 100 Grade A whiteboard pens that is two years old is selected and it is found that six of the whiteboard pens are defective. Determine a 90% confidence interval for the population proportion from this sample, correct to two decimal places. 2 marks

Question 17 (10 marks) ©VCAA 2021 2BQ4abcdeg ●●●

A teacher coaches their school's table tennis team. The teacher has an adjustable ball machine that they use to help the players practise. The speed, measured in metres per second, of the balls shot by the ball machine is a normally distributed random variable W. The teacher sets the ball machine with a mean speed of 10 metres per second and a standard deviation of 0.8 metres per second.

a 78% Determine $\Pr(W \geq 11)$, correct to three decimal places. 1 mark

b 64% Find the value of k, in metres per second, which 80% of ball speeds are below. Give your answer in metres per second, correct to one decimal place. 1 mark

The teacher adjusts the height setting for the ball machine. The machine now shoots balls high above the table tennis table. Unfortunately, with the new height setting, 8% of balls do not land on the table.

Let $\hat{P}$ be the random variable representing the sample proportion of balls that do not land on the table in random samples of 25 balls.

c 45% Find the mean and the standard deviation of $\hat{P}$. 2 marks

d 49% Use the binomial distribution to find $\Pr(\hat{P} > 0.1)$, correct to three decimal places. 2 marks

The teacher can also adjust the spin setting on the ball machine. The spin, measured in revolutions per second, is a continuous random variable X with the probability density function

$$f(x) = \begin{cases} \dfrac{x}{500} & 0 \leq x < 20 \\ \dfrac{50 - x}{750} & 20 \leq x \leq 50 \\ 0 & \text{elsewhere} \end{cases}$$

e 21% Find the maximum possible spin applied by the ball machine, in revolutions per second. 1 mark

f 39% Find the standard deviation of the spin, in revolutions per second, correct to one decimal place. 3 marks

Question 18 (17 marks) ©VCAA 2019 2BQ4

The Lorenz birdwing is the largest butterfly in Town A.

The probability density function that described its life span, X, in weeks, is given by

$$f(x) = \begin{cases} \frac{4}{625}(5x^3 - x^4) & 0 \le x \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

a 80% Find the mean life span of the Lorenz birdwing butterfly. 2 marks

b 57% In a sample of 80 Lorenz birdwing butterflies, how many butterflies are expected to live longer than two weeks, correct to the nearest integer? 2 marks

c 66% What is the probability that a Lorenz birdwing butterfly lives for at least four weeks, given that it lives for at least two weeks, correct to four decimal places? 2 marks

The wingspans of Lorenz birdwing butterflies in Town A are normally distributed with a mean of 14.1 cm and a standard deviation of 2.1 cm.

d 81% Find the probability that a randomly selected Lorenz birdwing butterfly in Town A has a wingspan between 16 cm and 18 cm, correct to four decimal places. 1 mark

e 61% A Lorenz birdwing butterfly is considered to be **very small** if its wingspan is in the smallest 5% of all the Lorenz birdwing butterflies in Town A. Find the greatest possible wingspan, in centimetres, for a **very small** Lorenz birdwing butterfly in Town A, correct to one decimal place. 1 mark

Each year, a detailed study is conducted on a random sample of 36 Lorenz birdwing butterflies in Town A. A Lorenz birdwing butterfly is considered to be **very large** if its wingspan is greater than 17.5 cm. The probability that the wingspan of any Lorenz birdwing butterfly in Town A is greater than 17.5 cm is 0.0527, correct to four decimal places.

f **i** 73% Find the probability that three or more of the butterflies, in a random sample of 36 Lorenz birdwing butterflies from Town A, are **very large**, correct to four decimal places. 1 mark

ii 33% The probability that n or more butterflies, in a random sample of 36 Lorenz birdwing butterflies from Town A, are **very large** is less than 1%. Find the smallest value of n, where n is an integer. 2 marks

iii 49% For random samples of 36 Lorenz birdwing butterflies in Town A, $\hat{P}$ is the random variable that represents the proportion of butterflies that are **very large**. Find the expected value and the standard deviation of $\hat{P}$, correct to four decimal places. 2 marks

iv 25% What is the probability that a sample proportion of butterflies that are **very large** lies within one standard deviation of 0.0527, correct to four decimal places? Do not use a normal approximation. 2 marks

g 28% The Lorenz birdwing butterfly also lives in Town B.

In a particular sample of Lorenz birdwing butterflies from Town B, an approximate 95% confidence interval for the proportion of butterflies that are very large was calculated to be (0.0234, 0.0866), correct to four decimal places.

Determine the sample size used in the calculation of this confidence interval. 2 marks

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