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MATHEMATICAL METHODS CAS
Teach Yourself Series
Topic 10: Basic Probability

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Contents

Probability Basics

As it appears in Unit 2

Addition Rule for Probability

$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

Venn Diagrams are a useful tool to help solve problems.

Venn diagrams

Can be used to show the relationship between different sets. Common symbols are:

$$\xi: Universal\ set - everthing\ is\ included\ in\ it.$$
$$\cup: union - every\ thing\ inside\ the\ two\ sets$$
$$\cap: intersection - what\ is\ common\ between\ the\ two\ sets$$

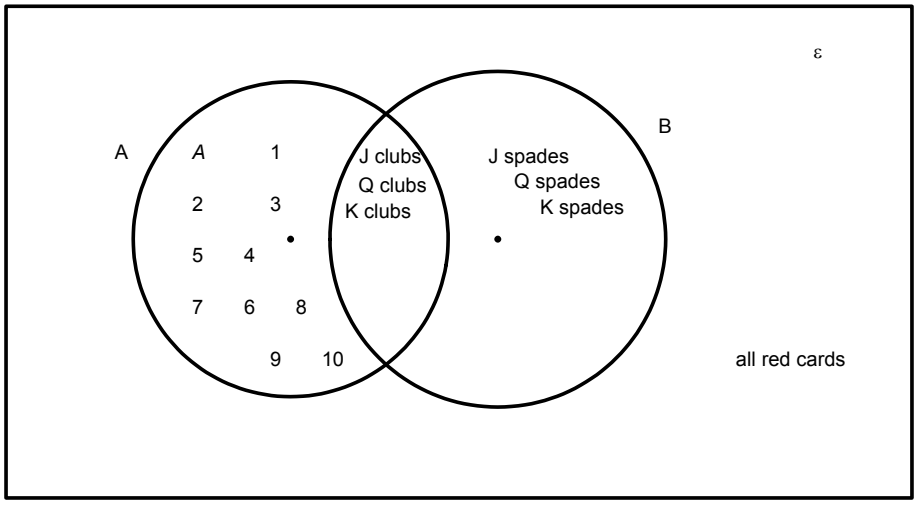
Example

Consider a standard pack of playing cards.
Let ξ: All cards
Let A: All clubs
Let B: Black picture cards

a. Draw a Venn Diagram displaying this information.
b. Find $\Pr(A \cap B)$.
c. Find $\Pr(A \cup B)$.

Answer

a.



b. $\Pr(A \cap B) = \frac{3}{52}$
c. $\Pr(A \cup B) = \frac{17}{52}$

Complementary events

The complement of A, A', are the elements not in A. It is important to note:

$$\Pr(A) + \Pr(A') = 1$$

Example

Consider a standard deck of playing cards.
Let A: be the set of clubs.
Let B: be the set of picture cards.

a. Explain what A' and B'.
b. Find the Pr (A') and Pr (B')

Answer

a. A': all spades, hearts, diamonds.
B': all aces, 2's, 3's,4's,5's,6's,7's,8's,9's,10's.

b. $\Pr(A') = \frac{3}{4}$ $\Pr(B') = \frac{10}{13}$

Independent Events

$Pr(A \cap B) = Pr(A) \times Pr(B)$

Often used with Tree Diagrams

Tree diagrams

These are used to help display the sample space When an experiment is taking place and is made up of a number of possibilities. Each branching lists the spate possibilities of the experiment. The probabilities are usually written on each branch.

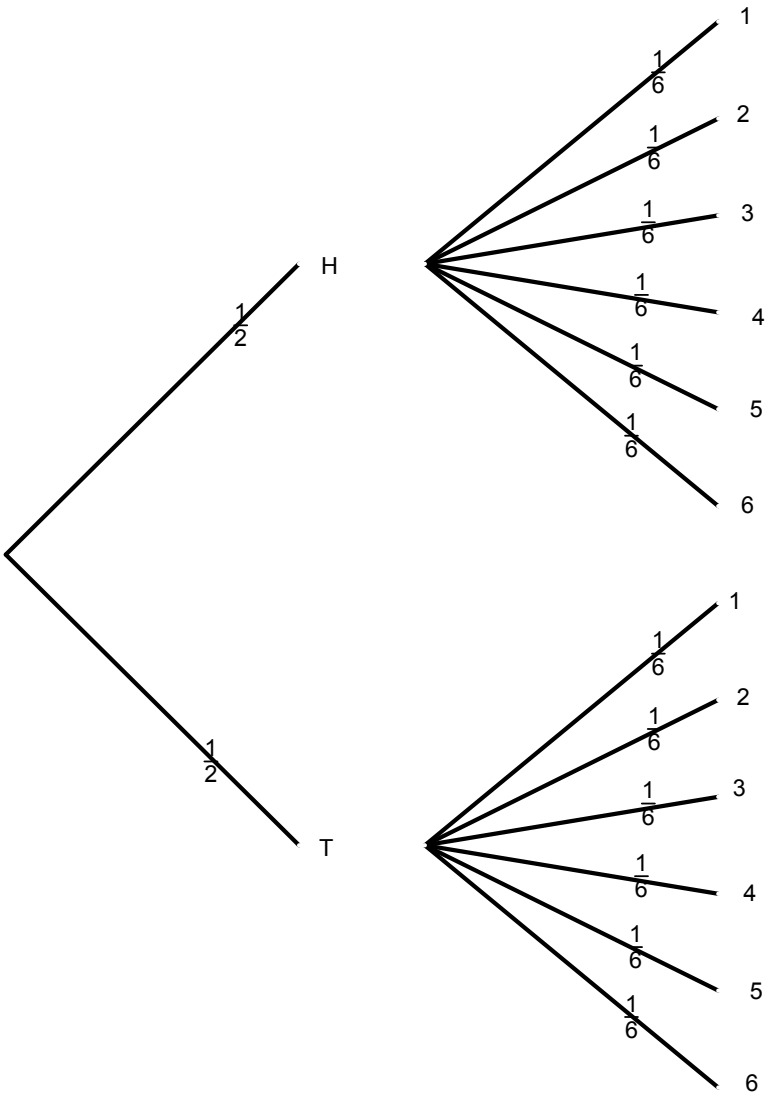
Example

A coin is tossed and a die is rolled.

a. Display the possibilities on a Tree Diagram.
b. Calculate the probability of throwing a Head and rolling a 6.

Answer

a.



b. $Pr(H \cap 6) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

Example

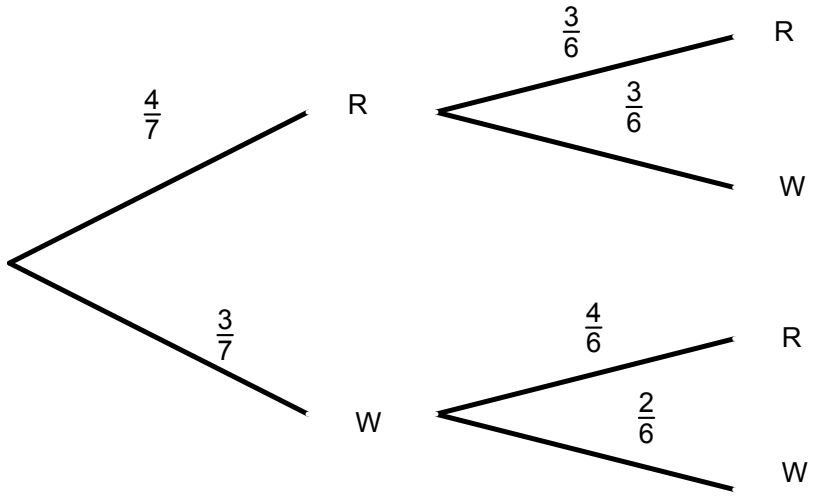
A container has 4 red marbles and 3 white marbles. Two marbles are selected without replacement.

a. Display the possibilities on a Tree Diagram.

b. Calculate the probability of drawing two red marbles.

Answer

a.



b. $Pr(R \cap R) = \frac{4}{7} \times \frac{3}{6} = \frac{2}{7}$

Conditional Probability

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Karnaugh Diagrams

	B	B'	
A	Pr(A ∩ B)	Pr(A ∩ B')	Pr(A)
A'	Pr(A' ∩ B)	Pr(A' ∩ B')	Pr(A')
	Pr(B)	Pr(B')	1

To use this in problems you draw up the table, fill in the information that is in the question, fill in the rest of the table.

Conditional Probability

$$Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)}$$

The key concept is the the original sample space is reduced. This is what the condition does.

Review Questions

1. A kindergarten teacher has found over the years that 35% of children can tie their shoelaces, 30% can use a pair of scissors, and 20% can do both. Find the probability that a randomly selected child:

a. can neither tie their shoelaces nor use scissors

b. can tie their shoelaces but cannot use scissors

2. The probability that Penny will phone Fred is 0.3, and the probability that Hugh will phone Fred is 0.2. If these events are independent, what is the probability that either Penny or Hugh or both will phone Fred?

3. A coin is biased such that the probability that a head will fall for any throw is 1/3. The coin is tossed twice. Draw a tree diagram showing the possible outcomes and their corresponding probabilities.

4. A die is 'fixed' so that certain numbers will appear more often. The probability that a 6 appears is twice the probability of a 5 and 3 times the probability of a 4. The probabilities of 3, 2 and 1 are unchanged from a normal die.

Find:

a. The probability distribution of this fixed die.

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b. The probability of getting a 'double' with two of these dice. Compare with the 'normal' probability of getting a double.

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5. Two fair dice are rolled and the numbers facing up are noted.

a. List the sample space.

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b. Find the probability of a double – A pair of the same numbers facing up.

c. Find the probability of the sum of the numbers facing up is at least 7.

d. Find the probability of a double given the numbers facing up add to at least 7.

6. A card is selected from a standard deck of playing cards. It is replaced and a second card is drawn. Find the probability that:

a. Both cards are hearts

b. Both cards are aces

c. Both cards are picture cards.

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7. If A and B are independent events, Pr(A) = 0.5 and Pr(B)=0.2 find $Pr(A \cup B)$.

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8. In a recent survey of pensioners, 85% favoured greater powers of arrest for police, 60% favoured longer sentences for convicted criminals and 50% were infavour of both.

a. What was the percentage of pensioners that were infavour of either or both proposals.

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b. What was the percentage of pensioners that were not infavour of either proposal.

_____

_____

_____

_____

Solutions to Review Questions

1.

a. Construct Probability table(Karnaugh Diagram) or Venn diagram.

0.55

b. Use Probability table(Karnaugh Diagram) or Venn diagram from previous question.

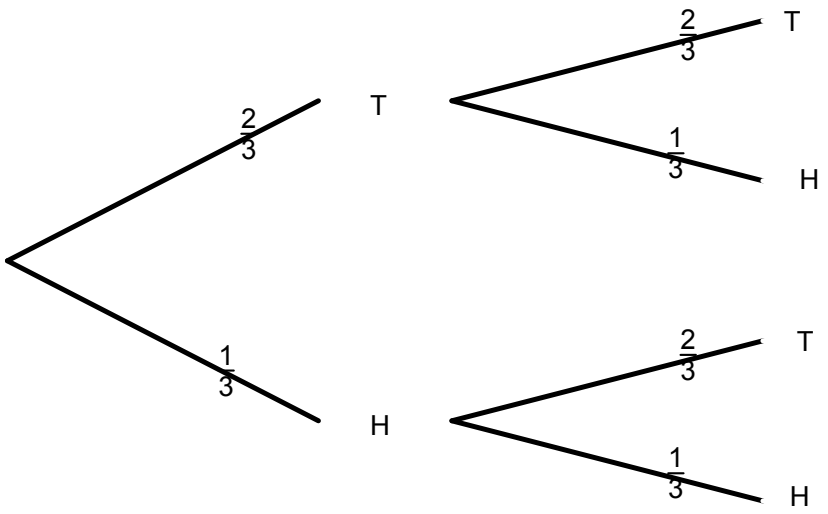
0.15

2.

Construct a tree diagram, first branching showing Penny calling Fred, second branch showing Hugh calling Fred.

0.48

3.



4.

a.

x	**1**	**2**	**3**	**4**	**5**	**6**
Pr (X=x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{11}$	$\frac{3}{22}$	$\frac{3}{11}$

Pr(6) + Pr(5) +Pr(4)= $\frac{1}{2}$

$x + \frac{x}{2} + \frac{x}{3} = \frac{1}{2}$

b.

Multiply: the probability of getting a 6 x the probability of getting a 6 for both.

Pr(Double with Bias)= $\frac{9}{121}$ Pr (Double with fair)= $\frac{1}{36}$

5.

a.

	1	2	3	4	5	6
1	**1, 1**	**2, 1**	**3, 1**	**4, 1**	**5, 1**	**6, 1**
2	**1, 2**	**2, 2**	**3, 2**	**4, 2**	**5, 2**	**6, 2**
3	**1, 3**	**2, 3**	**3, 3**	**4, 3**	**5, 3**	**6, 3**
4	**1, 4**	**2, 4**	**3, 4**	**4, 4**	**5, 4**	**6, 4**
5	**1, 5**	**2, 5**	**3, 5**	**4, 5**	**5, 5**	**6, 5**
6	**1, 6**	**2, 6**	**3, 6**	**4, 6**	**5, 6**	**6, 6**

b.

Get the answer from the sample space above = $\frac{6}{36}$

$\frac{1}{6}$

c.

Get the answer from the sample space above.

$\frac{21}{36} = \frac{7}{12}$

d.

Only want the doubles that are above a sum of 7. This is conditional probability.

$\frac{3}{21} = \frac{1}{7}$

6.

a. Heart and heart = $\frac{1}{4} \times \frac{1}{4}$

$\frac{1}{16}$

b. Ace and Ace = $\frac{1}{13} \times \frac{1}{13}$

$\frac{1}{169}$

c. Picture and Picture = $\frac{3}{13} \times \frac{3}{13}$

$\frac{9}{169}$

7.

$Pr(A \cap B) = 0.5 \times 0.2$

$= 0.1$ because A and B are independent.

Then use addition law.

0.6

8.

a.

Draw a Venn Diagram. Start with the intersection.

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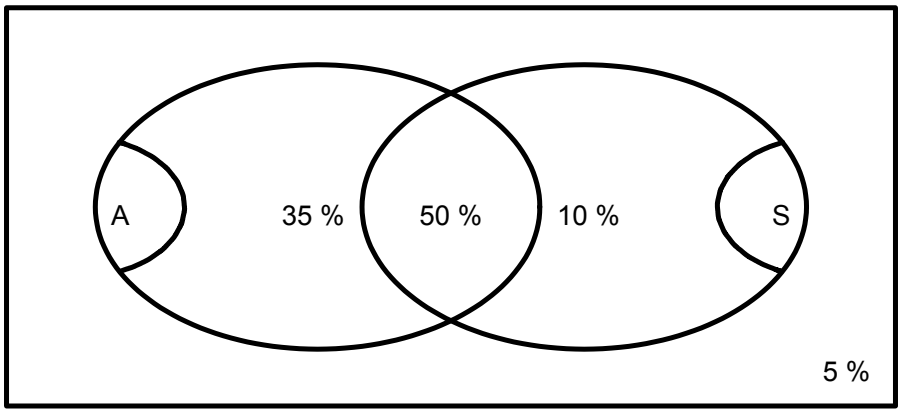

b. Refer to Venn Diagram.
5 %