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MATHEMATICAL METHODS CAS
Teach Yourself Series
Topic 12: Binomial Distribution

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Contents

Binomial Probability Distribution

A binomial experiment is one that:

1. Has two outcomes; success and failure.
2. Probability of success, p, and probability of failure, q (1 – p).
3. The experiment consists of a number of trials, n, which are independent of one another.
4. p remains constant for all trials.
5. Can be written as: X ~ (n,p)
6. Key words/concepts: Sampling **with replacement**.

Examples of binomial experiments are:

Throwing a fair die 10 times, recording the number of 6's.

The number of heads showing, when a biased coin is thrown 5 times.

The number of patients cured of a disease when given an antibiotic.

The formula to calculate the binomial distribution is:

$$\Pr(X = x) = \left({}^{n}C_{x}\right)p^{x}q^{n-x}$$

Mean can be calculated by:

$$E(X) = \mu = np$$

Variance can be calculated by:

$$VAR(X) = \sigma^2 = npq$$

Standard Deviation can be calculated by:

$$SD(X) = \sigma = \sqrt{npq}$$

Calculator skills

Binom pdf
Binomcdf

Example

An aspiring Olympic shooter has the probability of 0.7 of hitting the target. If the shooter has 10 attempts shooting at the target find:

a. The probability that he hits exactly 3 targets.
b. The probability that he hits at least 3 targets.
c. The probability that he hits exactly 3 targets given that he hits at least 3 targets.
d. The expected number of targets that the shooter will hit.

Answers

a.

$$n = 10$$
$$p = 0.7$$
$$x = 3$$
$$Pr(X = 3) = \binom{10}{3} 0.7^3 \times 0.3^7 = 0.009$$

Could use binompdf on calculator.

b.

$$n = 10$$
$$p = 0.7$$
$$x \geq 3$$
$$Pr(X \geq 3) = 0.998$$

Use Binomcdf on calculator or $Pr(X \geq 3) = 1 - Pr(X \leq 2)$

c.

Conditional Probability where :$Pr(X = 3|X \geq 3) = \frac{Pr(X=3)}{Pr(X\geq 3)} = \frac{0.009}{0.998} \approx 0.009$

d.

$$\mu = np = 7$$

Example

The probability that a boy answers any maths question correctly is 0.55. Find the number of questions that the boy has to answer if the probability of him answering at least 2 questions correctly is more than 0.9.

Answer

$$n = ?$$
$$p = 0.55$$
$$x \geq 2$$
$$Pr(X \geq 2) = 0.9$$
$$Pr(X \geq 2) = 1 - Pr(X = 0) - Pr(X = 1)$$
$$0.1 = 0.45^n + n \times 0.55^1 \times 0.45^{n-1}$$

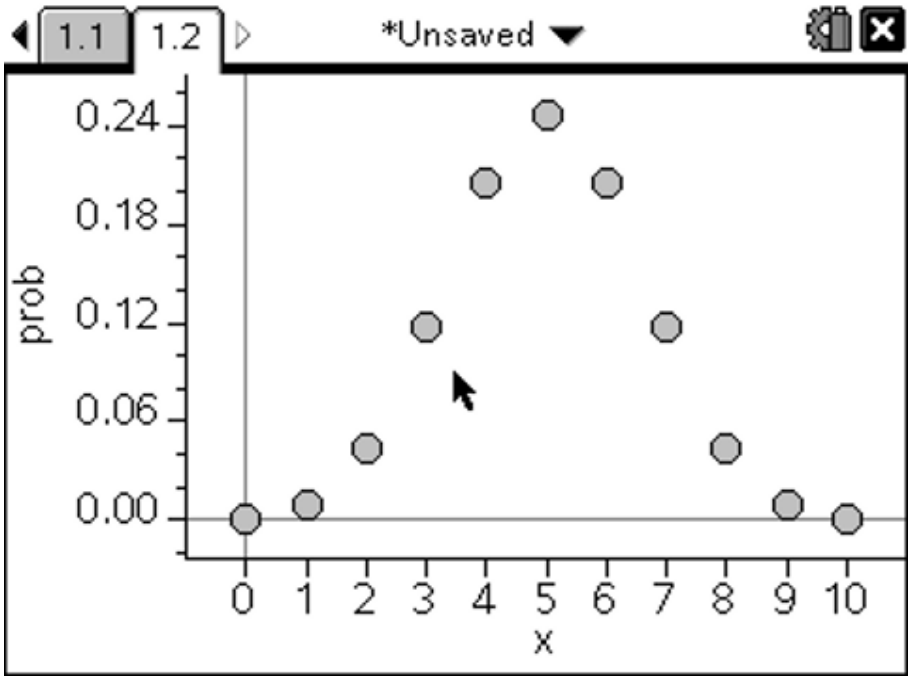
Use the calculator to solve this equation

$$n = 5.4 = 6$$

Graphs of Binomial Distributions

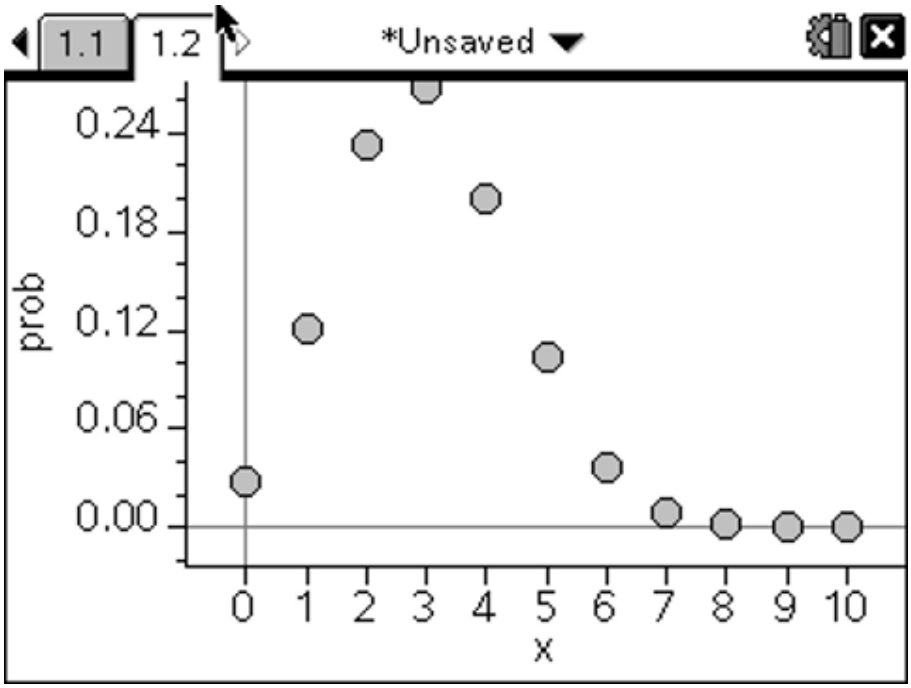
When p = 0.5, the probability distribution is symmetrical.

Example **n=10 & p=0.5**



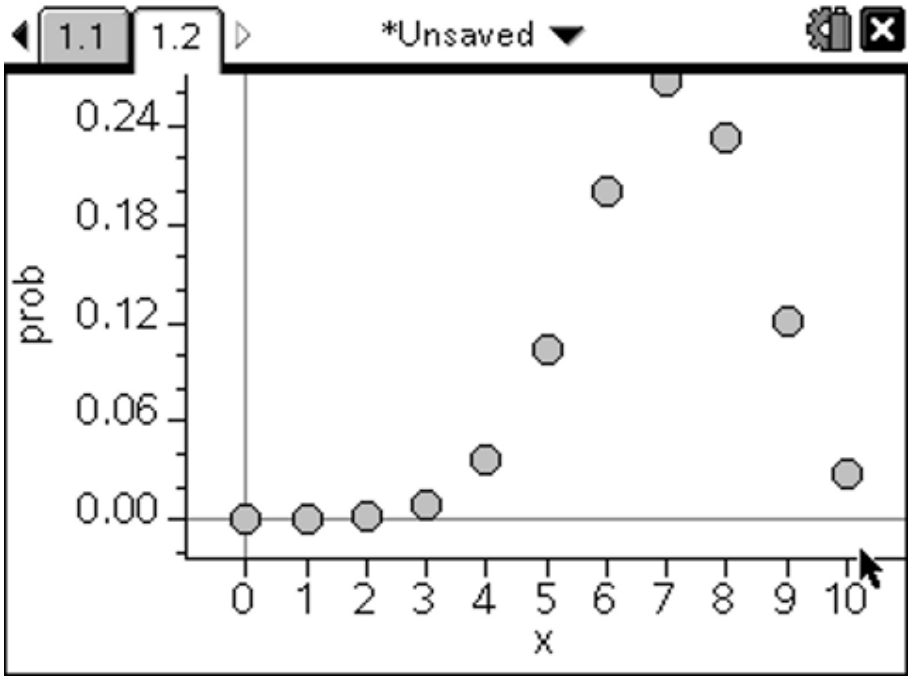
When p < 0.5, the probability distribution is skewed to the right.

Example **n=10 & p=0.3**



When p > 0.5, the probability distribution is skewed to the left.

Example **n=10 & p=0.7**



Review Questions

1. Suppose that X is the number of female children born into a family. If the distribution of X is binomial, with probability of success of 0.48, then find the probability that a family with five children will have two female children.

2. In order to assess the quality of computer games being produced, the manufacturer selects 10 games at random before they are put into boxes for delivery to shops, and inspects them.

If it is known that 8% of games are defective, find, correct to four decimal places:

a. the probability that one of the games in the sample will be defective

b. the probability that at least one of the games in the sample will be defective

c. the probability that exactly one of the games in the sample will be defective, **given** that at least one of the games is defective

d. the expected number of defective games in the sample

e. the standard deviation of the number of defective games in the sample

3. The probability of having to stop at a particular traffic light is 0.3. What is the probability of having to stop 3 out of 5 days of work?

4. The chances of an archer scoring a bulls eye is 0.4. He fires 5 arrows at the target. Assume the probability of scoring a bulls eye remains constant and each result is independent of the other.

a. Calculate the probability that the archer scores five bulls eyes.

b. Calculate the probability that he scores at least 4 bulls eyes.

c. Calculate the probability that the archer scores five bulls eyes given that he scores at least 4 bulls eyes.

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5. The probability of winning a prize in a game is 0.45. What is the least number of games that must be played to ensure that the probability of winning at least twice is more than 0.95?

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Solutions to Review Questions

1. Use binomial pdf: $p = 0.48$
$n=5$
$x=2$

0.324

2.

a. Use binomial pdf: $p = 0.08$
$n=10$
$x=1$

0.3777

b. Use binomial cdf: $p = 0.08$
$n=10$
$x \geq 1$

0.5656

c. Use definition of conditional probability. Answer of part" a" divided by answer from part "b".

0.6678

d. $\mu = np$

0.8

e. $\sigma = \sqrt{np(1-p)}$)

0.8579

3. Use binomial pdf: $p = 0.3$

$n=5$
$x=3$

0.1323

4.

a. Use binomial pdf: $p = 0.4$

$n=5$
$x=5$

0.01024

b. Use binomial cdf: $p = 0.4$

$n=5$
$x \geq 4$

0.08704

c. Conditional Probability. The problem simplifies to Answer in part a divided by answer in part b.
0.1176

5. Have to calculate n in the binomial formula.

$Pr(X \geq 2) = 0.95$

$Pr(X < 2) = 0.05$

$p = 0.45$

$(0.55)^n + n \times 0.45 \times (0.55)^{n-1} < 0.5$

Solve the equation $(0.55)^n + n \times 0.45 \times (0.55)^{n-1} = 0.5$ on the calculator.

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