MATHEMATICAL METHODS CAS
Teach Yourself Series
Topic 13: Continuous Probability Distributions

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Continuous Random Variables

As it appears in Unit 4

These are probabilities that are calculated by finding area under curves that are described by functions. The areas between values on the x axis correspond to the probabilities.

Key ideas:

$$\int_a^b f(x)dx = 1$$

$$\mu = \int_a^b x.f(x)dx$$

Median: $\int_a^{med} f(x)dx = 0.5$

Mode: Max value that lies between *a* and *b* for *f*(*x*). If this is a max stationary point - solve $f'(x)=0$.

$$\sigma^2 = \int_a^b x^2.f(x)dx - \mu^2 = \int_a^b (x-\mu)^2 f(x)dx$$

$$\sigma = \sqrt{\sigma^2}$$

Percentiles

The value along the x axis that corresponds to an area under the curve is called a percentile.

$\int_a^p f(x)dx = q$, a is the starting value of the continuous random variable.

That is to find the 80th percentile q = 0.80, and you find the value of p.

Finding the Interquartile Range

The interquartile range is the difference between the upper and lower quartiles.

The upper quartile is found by: $\int_a^{q_3} f(x)dx = 0.75$

The lower quartile is found by: $\int_a^{q_1} f(x)dx = 0.25$

Interquartile range = $q_3 - q_1$.

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Example

Let a continuous random variable be defined as $\boldsymbol{f(x)} = \begin{cases} \boldsymbol{ke^{-x}, x \geq 0} \\ \boldsymbol{0, elsewhere} \end{cases}$

a. Show find the value of *k*.
b. Sketch *f*(*x*).

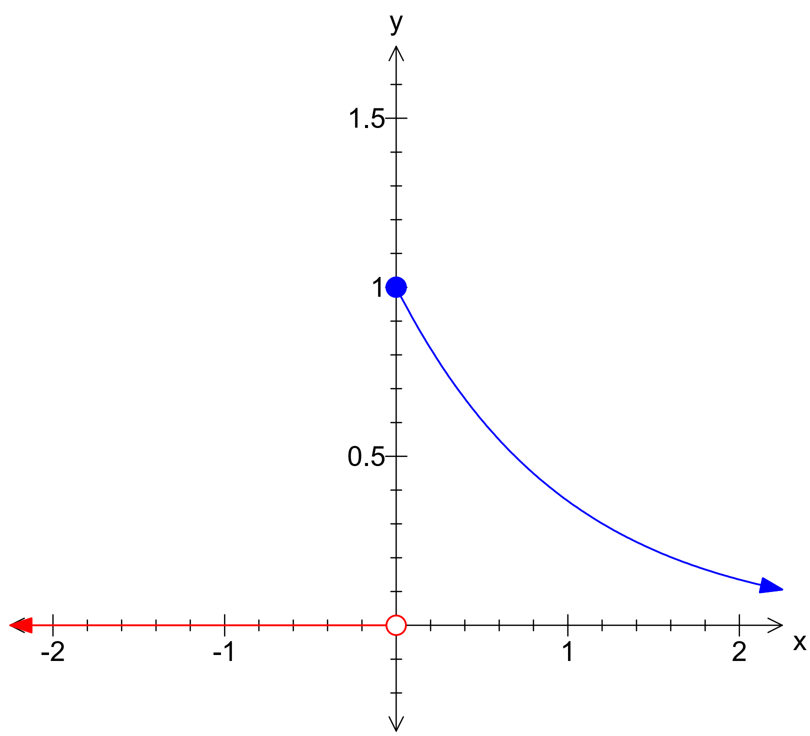
Answer

a.

$$\int_0^\infty ke^{-x}\,dx = 1$$
$$\left[\frac{-k}{e^x}\right]_0^\infty = 1$$
$$k = 1$$

b.

From part a, $\boldsymbol{f(x)} = \begin{cases} \boldsymbol{e^{-x}, x \geq 0} \\ \boldsymbol{0, elsewhere} \end{cases}$



Example

Let a continuous random variable be defined as $\boldsymbol{f(x)} = \begin{cases} 12\boldsymbol{x^2(1-x), 0 \leq x \leq 1} \\ \boldsymbol{0, elsewhere} \end{cases}$,.

- **a.** Show that this is a continuous random variable.
- **b.** Calculate Pr(X>0.2).
- **c.** Find the expected value of the continuous random variable, E(X).

Answer

a.

Show that the integral equals 1.

$$\int_0^1 12x^2(1-x)\,dx$$
$$= [4x^3 - 3x^4]_0^1$$
$$= 1$$

b.

$$\Pr(X > 0.2) = \int_{0.2}^1 f(x)dx$$
$$= [4x^3 - 3x^4]_{0.2}^1$$
$$= 0.9728$$

c.

$$E(x) = \int_0^1 xf(x)\,dx$$
$$\int_0^1 x \times 12x^2(1-x)\,dx$$
$$= 0.6$$

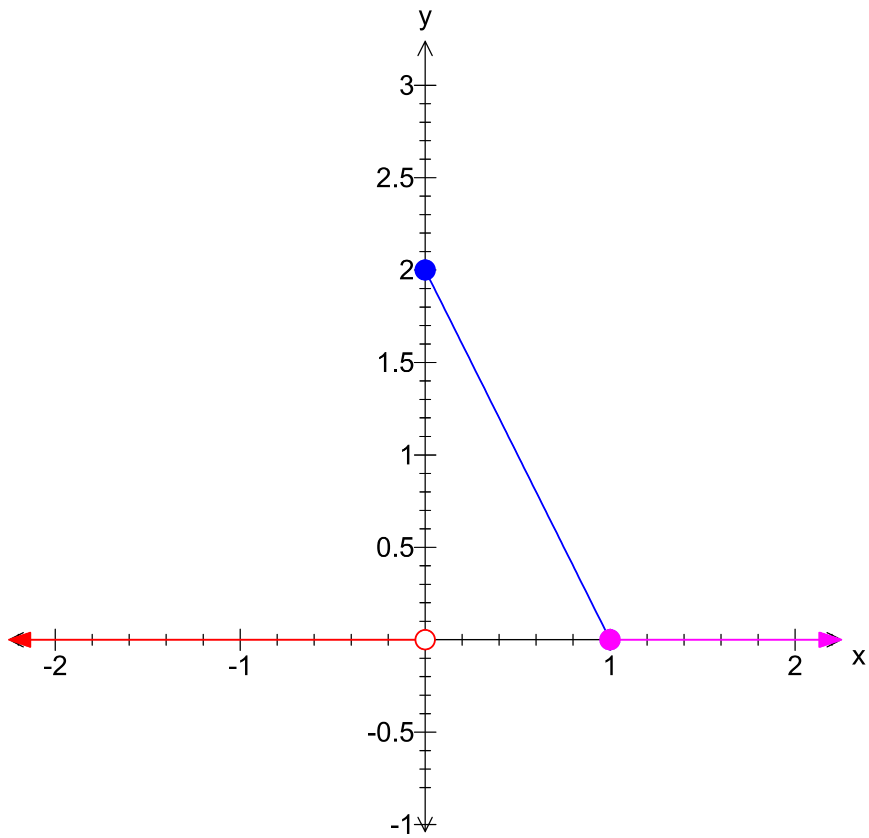
Example

The probability of the life span of a certain brand of cheap AAA batteries is described by the function: $\boldsymbol{f(x)} = \begin{cases} \boldsymbol{2(1-x), 0 \leq x \leq 1} \\ \boldsymbol{0, elsewhere} \end{cases}$, x is measured in hours.

- **a.** Sketch *f*(*x*).
- **b.** Calculate the probability that a battery will last for more than 45 minutes.
- **c.** Calculate the interquartile range of the battery life span.
- **d.** Calculate the median life span.

Answer

a.



b.

$$\Pr\left(X > \frac{45}{60}\right) = \int_{\frac{45}{60}}^{1} 2(1-x)dx$$

$$= \frac{1}{16} = 0.0625$$

c.

Lower Quartile

$$\int_{0}^{q_1} 2(1-x)dx = 0.25$$

$[2x - x^2]_0^{q_1}$=0.25

$$q_1 = 0.134$$

Upper Quartile

$$\int_{0}^{q_3} 2(1-x)dx = 0.75$$

$[2x - x^2]_0^{q_3}$=0.75

$$q_3 = 0.5$$

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Interquartile Range

$$q_3 - q_1 = 0.366$$

d.

$$\int_0^m 2(1-x)dx = 0.5$$

$[2x - x^2]_0^m$=0.5

$$m = 0.293$$

Calculator skills

Define functions
Graphing functions
Evaluate definite integral
Evaluate $f'(x) = 0$.

Review Questions

1. Show by calculation if the following are continuous probability distributions:

a. $f(x) = \begin{cases} \frac{1}{2}\left(x^2 + 4\right) & 0 < x < 1 \\ 0 & x \leq 0 \text{ or } x \geq 1 \end{cases}$

b. $f(x) = \begin{cases} \frac{1}{2} & 2 \leq x \leq 4 \\ 0 & x < 2 \text{ or } x > 4 \end{cases}$

2. The probability density function of the age of puppies, X years, brought into a vet is given by:

$$f(x) = \begin{cases} \frac{3}{4}x(2-x) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

If 60 puppies are brought in on a particular day, how many are expected to be less than 8 months old?

3. The probability density function of X is given by:

$$f(x) = \begin{cases} \sin(2x) & 0 \le x \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

Find the mean, median and mode of X.

4. The life of a brand of sparklers, in seconds, can be modeled by the probability function:

$$f(x) = \begin{cases} \frac{c}{x^2} & x \ge 100 \\ 0 & \textit{elsewhere} \end{cases} .$$

a. Find the value of c.

b. Find the median life of a sparkler.

c. Draw a sketch of $f(x)$.

5. A continuous probability density function is modeled by the function

$$f(x) = \begin{cases} ax(2-x) & 0 \le x \le 2 \\ 0 & elsewhere \end{cases}$$

a. Find the value of a.

b. Calculate $Pr\left(X < \frac{1}{2}\right)$.

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Solutions to Review Questions

1.

a. $\int_0^1 f(x) \neq 1$

No it is not

b. $\int_2^4 f(x) = 1$

Yes it is

2.

$$\int_0^{\frac{8}{12}} f(x) = \frac{7}{27}$$

$$60 \times \frac{7}{27} = \frac{140}{9} \approx Answer$$

Approximately 16 puppies.

3.

Mean : $\int_0^{\frac{\pi}{2}} x.\,f(x)dx$

Median : $\int_0^{med} f(x)dx = 0.5$

Mode: Solution to $f\,'(x) = 0$.

Mean= $\frac{\pi}{4}$ Median = $\frac{\pi}{4}$ Mode=$\frac{\pi}{4}$

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4.

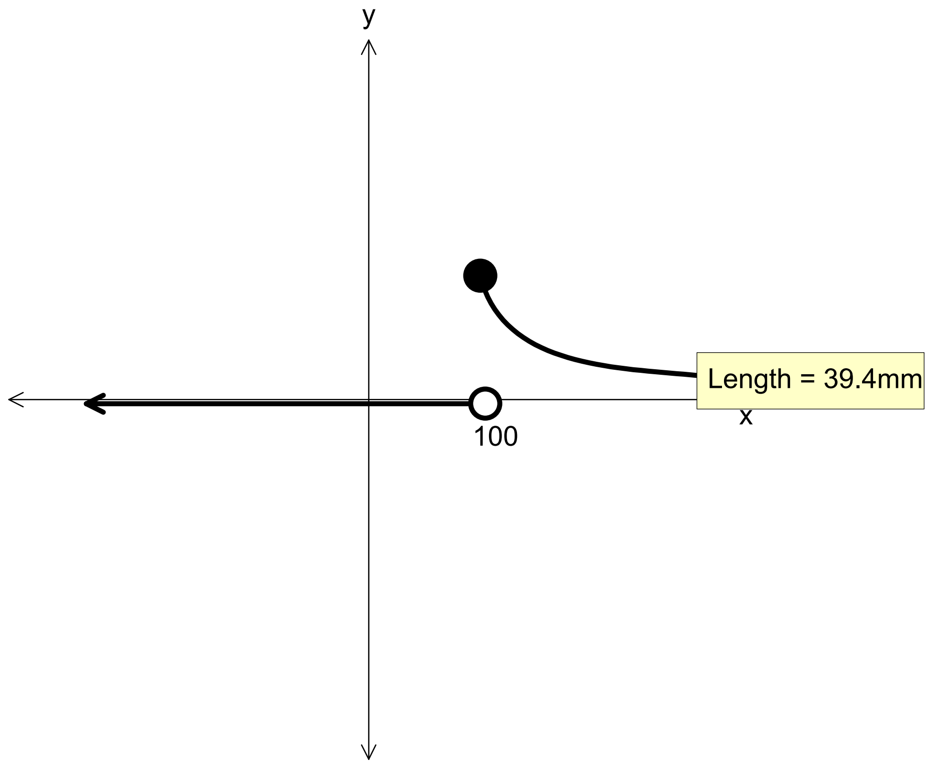
a. Integrate between x=100 and x=∞.

100

b. Solve the equation: $\int_{100}^{med} f(x)dx = 0.5$

200

c.



5.

a. Solve the equation: $\int_{0}^{2} f(x)dx = 1$

$\frac{3}{4}$

b. Evaluate: $\int_{0}^{\frac{1}{2}} f(x)dx$

$\frac{5}{32}$

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