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MATHEMATICAL METHODS CAS
Teach Yourself Series
Topic 14: Normal Distribution

A: Level 14, 474 Flinders Street Melbourne VIC 3000
T: 1300 134 518 **W:** tssm.com.au **E:** info@tssm.com.au

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Normal Distribution

As it appears in Unit 4

Continuous random variables are those quantities that can take a range of values. Often they are grouped together into class intervals.
Examples of continuous random variables are: time, height, weight.
This information can then be graphed into histograms.

e.g.

If we use height as an example, a person measuring a height of 160 cm could actually be between 159.5 cm and 160.5 cm. Even if we wish to express our answers to two decimal places, a range of values is still possible. A person measuring 150.4 cm could be between 150.35 cm to 150.45 cm.

- Because of this property: $\Pr(X = x) = 0$ for all values of x for any continuous variable.

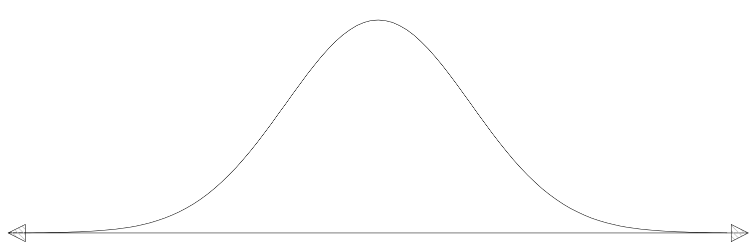
We can however work out $\Pr(159.5 \le X \le 160.5)$ as this is the range that represents a person who falls within this height. To work this out we define the probability distribution function (p.d.f.) of X, where X is the continuous random variable representing a person's height.

In general $\Pr(a \le X \le b) = \int_a^b f(x)dx$, where f(x) is a function that represents a continuous random variable.

One thing to remember which leads on from this discussion:

$\Pr(a \le X \le b) = \Pr(a < X \le b) = \Pr(a \le X < b) = \Pr(a < X < b)$
and
$\Pr(X < a) = \Pr(X \le a)$

The Normal Distribution is a probability distribution function of a continuous random variable. Its main characteristic is a symmetrical bell shape curve.



Due to this characteristic, the Mean, Median and Mode coincide at the maximum.
The confidence limits that have been learnt in DRV's also apply to the Normal Distribution.

The definition of the Normal Distribution is:

$$\Pr(a < X < b) = \int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{(x-\mu)}{\sigma}\right]^2}$$

where: μ = population mean

σ = population standard deviation

If a continuous random variable is a normal distribution we can write:

$X \sim N(\mu, \sigma^2)$

Some important rules for the Normal Distribution

$\approx$ 68 % of data falls within one standard deviation of the mean. ie. $\Pr(\mu - \sigma < X < \mu + \sigma) \approx 0.68$

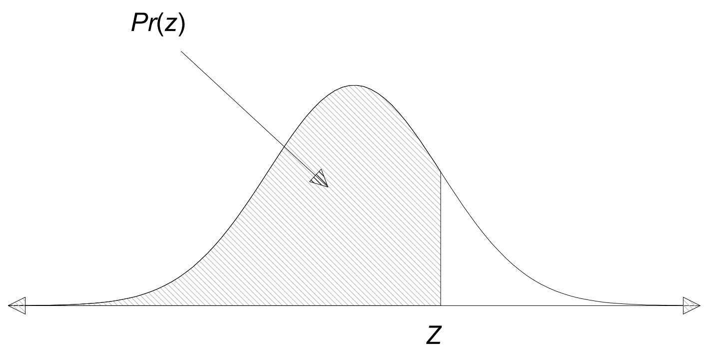
$\approx$ 95 % of data falls within two standard deviations of the mean. ie. $\Pr(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.95$

$\approx$ 99.7 % of data falls within three standard deviations of the mean. ie. $\Pr(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.997$

The Standard Normal Distribution

The Standard Normal Distribution is used where we convert all normally distributed data into the standard normal curve.
The Standard Normal Curve: $Z \sim N(0,1)$
This means that the normal distribution variable X has to converted into the Standard Normal variable Z.

If $X \sim N(\mu, \sigma^2)$ then $\frac{X-\mu}{\sigma} = Z \sim (0,1)$.
The probabilities are then: looked up in tables
Calculated on the calculator.
Note that the tabled or calculator values enable us to find the probability to the left of the z value.



The basic process in solving problems:

1. Draw a diagram showing all information in terms of X.
2. Convert into Z. Use $Z = \frac{X-\mu}{\sigma}$
3. Draw a diagram in terms of Z.
4. Use the tables or calculator to find the probability.

Types of problems:

Find the probability given X or Z
Find the X or Z given the probability.
Finding μ and or σ when given the relevant information.

Remember the symmetrical properties of the curve

Calculator skills

Normal pdf
Normal cdf
Inverse normal

Review Questions

1. A variable X is known to be normally distributed with a mean $\mu = 20$ and a standard deviation $\sigma = 3$

a. Sketch a diagram that represents the area for $\Pr(X>26)$

b. Find this probability.

2. Let X be a normally distributed random variable with a mean $\mu = 4$ and a standard deviation $\sigma = 0.5$. Find:

a. $\Pr(X < 4.5)$

b. $\Pr(Z<3.5)$

3. Based on past experience, Jon Benson can run the 100 m dash with a mean of 9.99 seconds and a standard deviation of 0.1 seconds.

a. Find the probability that Jon can run a race in less than 9.8 seconds.

b. Find the probability that Jon can run 2 out of 5 races in less than 9.8 seconds.

4. A school decides that they wish to fail the bottom 20% of students who sit their English exam. The scores are normally distributed with a mean of 60 and a standard deviation of 15. Find the mark a student would need to get to pass the English exam.

5. The weights of fish caught at a farm are normally distributed with a mean of 2.0 kg and a standard deviation 0f 0.4 kg.

a. Find the probability that if a trout is caught it weighs less than 1.5 kg.

b. Fish that are in the top 15% of weights are classified as delux fish. Calculate the minimum weight that a fish would have to be to be classified as delux.

c. A particular hotel have placed an order with the fish farm and want to have fish that are between 1.0 kg and 2.5 kg. Calculate the proportion of fish in the fish farm that meet the requirements of the hotel.

6. Jenny is a keen long jumper. Her distances recorded are normally distributed with a mean of 7.2 m and a standard deviation of 0.2 m.

a. Calculate the probability that a jump will exceed 7.25 m.

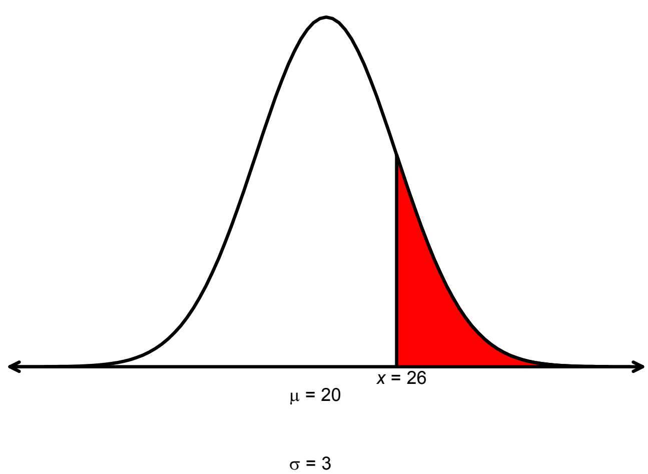
b. At the last event a record was set of 7.42 m. Calculate the probability that Jenny will break the record for any jump.

c. Jenny's friend, John, is also a keen long jumper. His mean length is 7.9 m. If the probability for any jump that John will exceed 8.25 m is 0.125 calculate the standard deviation if we assume that the distances he jumps are normally distributed.

Solutions to Review Questions

1.

a.



b. Use normalcdf(26,∞,20,3) where normalcdf(lower limit, upper limit ,mean, standard deviation)
0.0228

2.

a. Use normalcdf(-∞,4.5,4,0.5) where normalcdf(lower limit, upper limit ,mean, standard deviation)
0.8413

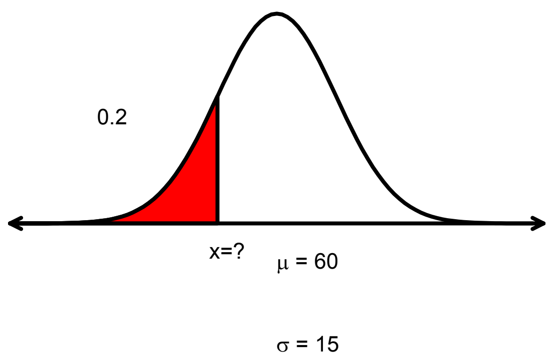
b. Use normalcdf(-∞,3.5,0,1) where normalcdf(lower limit, upper limit ,mean, standard deviation)
0.9998

3.

a. Use normalcdf(-∞,9.8,9.99,0.1) where normalcdf(lower limit, upper limit ,mean, standard deviation)
0.0287

b. Use binomial distribution: n=5
p=0.0287
x=2
0.0075

4. Need to use inverse norm.



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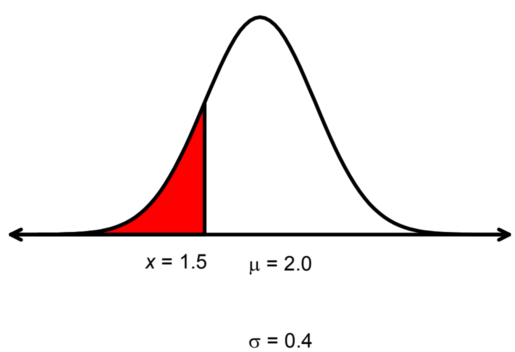
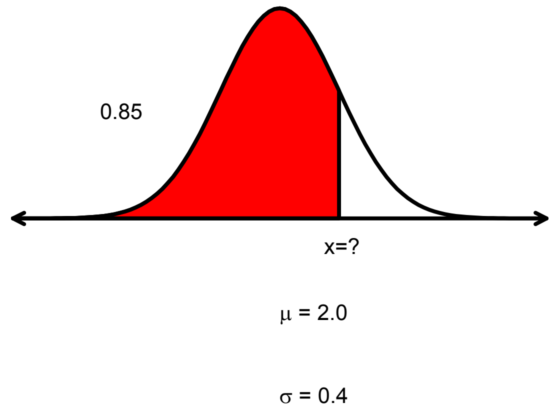
5.

a. Use normalcdf



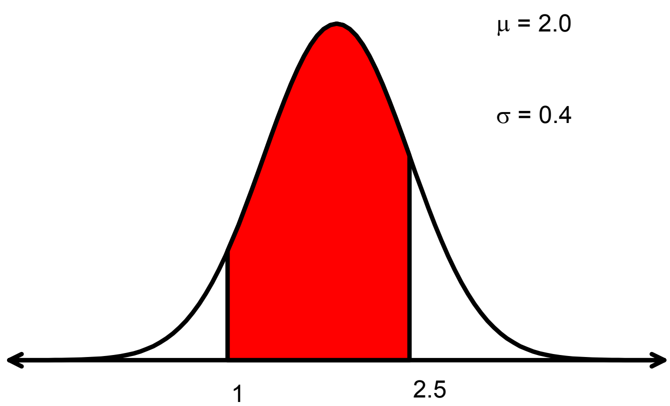
0.1056

b. Use inverse normal



2.41 kg

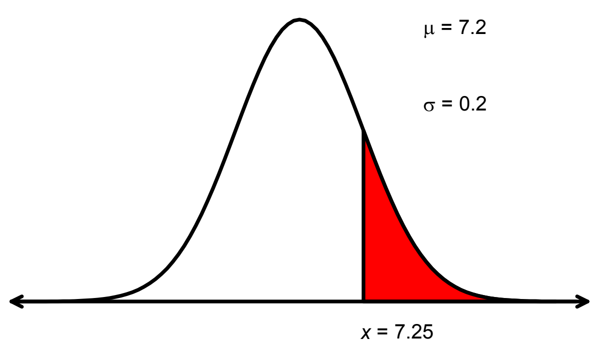
c. Use normalcdf



0.8881

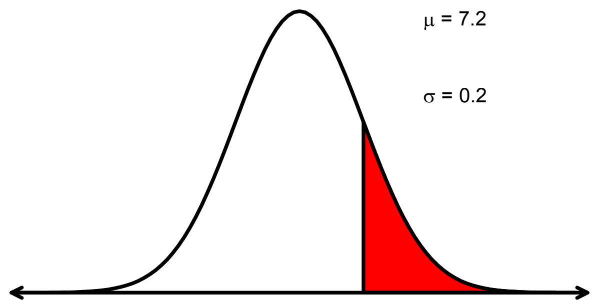
6.

a. Use normalcdf.



0.4013

b. Use normal cdf.



0.1357

c. Use inverse norm to calculate $z = 1.1503$

$$z = \frac{x - \mu}{\sigma}$$

$$1.1503 = \frac{8.25 - 7.9}{\sigma}$$

3.29