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Mathematical Methods (CAS) Teach Yourself Series

Topic 15: Set Notation for Probability

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Set notation for probability

This topic discusses the notations used in set theory which is a useful tool in probability. It is extremely important to identify these notations in order to understand the question properly.

Classification of sets

As it appears in Unit 2

A set is a collection of items (numbers) that belong to a well defined category. It is denoted by capital letters and is enclosed in braces { } The items in a set are referred to as elements and are denoted by small letters.

A set can be classified in two forms-

- Listed set – when the elements are in the form of a list where elements are written in braces seperated with commas. Ex. { 2, 4, 6, 8, 10}
- Described set – when the elements are described in words.
  Ex. $\{x : x \text{ is an even number less than } 12\}$

Note that the notation for the number of elements in any set A is $n(A)$

Ex 1. Change the listed set { 2, 3, 5, 7, 11, 13} into a described set

$\{x : x \text{ is a prime number less than } 17\}$

Ex 2.

Change the described set $\{x : x \text{ is a natural number between } 3 \text{ and } 11\}$ into a listed set

{ 4, 5, 6, 7, 8, 9, 10}

Review Questions

1. List these things correctly as a set: bread, milk, eggs, sausages, cereal in the

a. Listed form

_____

b. Described form

_____

_____

2. Write down the value of $n(A)$ for each of the following-

a. A = {cat, dog, frog}

_____

_____

b. A = {x: x is a non prime positive integer less than 13}

_____

_____

3. The number of elements in the set {integers that are not prime and not composite} is:

A. 0
B. 1
C. 2
D. 3
E. Infinite

4. The number of elements in the set {even multiples of 5 and less than 100} is:

A. 8
B. 9
C. 10
D. 11
E. infinite

Special sets and notation

As it appears in Unit 2

You need to be familiar with certain kinds of sets and their notations. They are defined in the following table-

NAME OF THE SET	DEFINITION
null set	no element (∅)
unit set	one element only
finite set	countable number of elements
infinite set	infinite number of elements
universal set	all elements referring to a particular problem
subset	part of another set(⊂)
disjoint set	sets with no elements in common
overlapping set	sets with some elements in common
complement of a set	elements from the universal set not in the set(X')
equal set	same elements
equivalent sets	same number of elements(↔)

Ex 1. Write a set that is equal to {e, a, p}

There are many ways to do this. All elements in the set above should be written, no matter what the order is. One such way is-

{p, a, e}

Ex 2. List three subsets of {x, y, z}

There are many ways to do this. One such way is-

{x, z}, {y, z}, {x}

Ex 3. If X = {8, 9, 10} Y = {6, 7, 8, 9, 10} Z = {8, 9} R = {6, 8, 10} S = {10} and T = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}, check whether the following statements are true or false-

(a) $S \subset X$ True (all elements of S are in X)
(b) $9 \in R$ False (the number 9 is not in R)
(c) $Z \not\subset R$ True (8 and 9 are not both part of R)
(d) $11 \notin Y$ True (11 is not part of Y)
(e) $\varnothing \subset S$ True (a null set is part of every set)
(f) $X \subset T$ True (all elements in X are in T)

Review Questions

5.

a. Write a set that is equivalent to {bicycle, car, bus, tram, aeroplane}

_____

_____

b. List three subsets of {a, e, o, u}

_____

_____

6. If ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9} A = {1, 3} B = {6, 7, 8, 9} C = {2, 4, 6} D = {1, 3, 5, 7, 9} E = {1} T = {2, 3, 4, 5, 6, 7, 8}, check the validity of each of the following statements-

a. $A \subset B$

_____

b. $9 \in D$

_____

c. $B \not\subset T$

_____

d. $11 \notin \xi$

_____

7. A subset of the set {prime factors of 20} is:

A. {1}
B. {1, 2, 4, 5, 10, 20}
C. {2, 4, 5}
D. {2, 5}
E. {2, 5, 20}

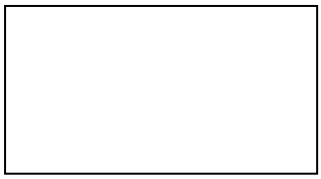
8. The complement of the set $\{m: m \text{ is less than or equal to } 4\}$ is:

A. $\{m: m \text{ is greater than or equal to } 4\}$
B. $\{m: m \text{ is not equal to } 4\}$
C. $\{m: m \text{ is greater than } 4\}$
D. $\{5, 6, 7, 8, 9, 10\}$
E. $\{1, 2, 3\}$

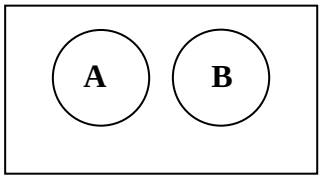
Venn diagrams

As it appears in Unit 2

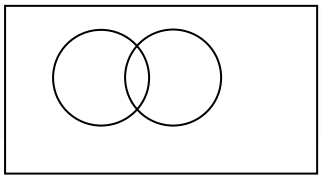
Venn diagrams help you to visually represent the relationship between sets. They are a very useful tool to understand worded problems in probability. Some of these relationships are represented below-



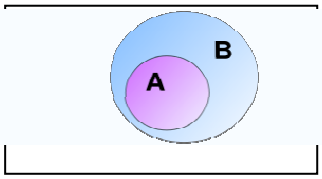
UNIVERSAL SET



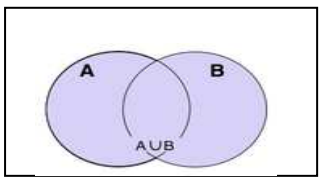
DISJOINT SETS



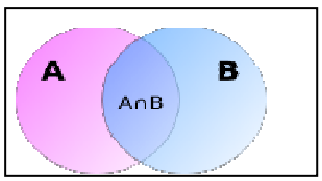
OVERLAPPING SETS



$A \subset B$



$A \cup B$



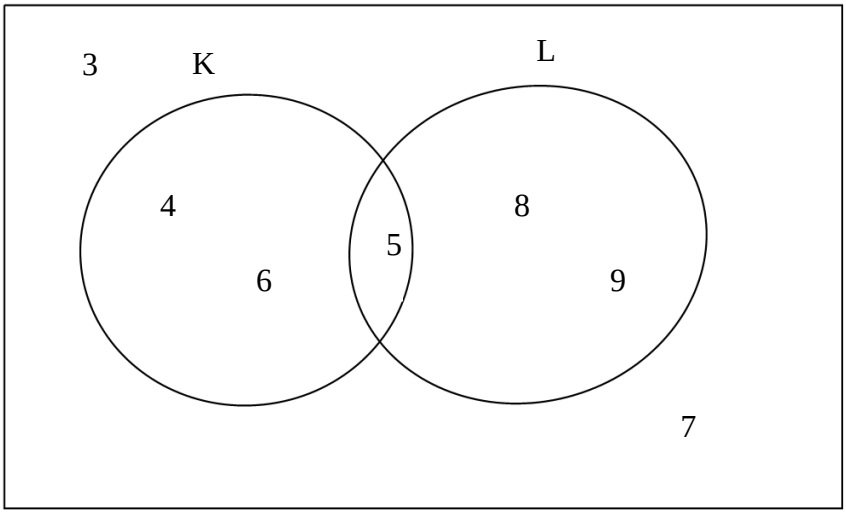
$A \cap B$

Ex 1. Draw a Venn diagram to show:

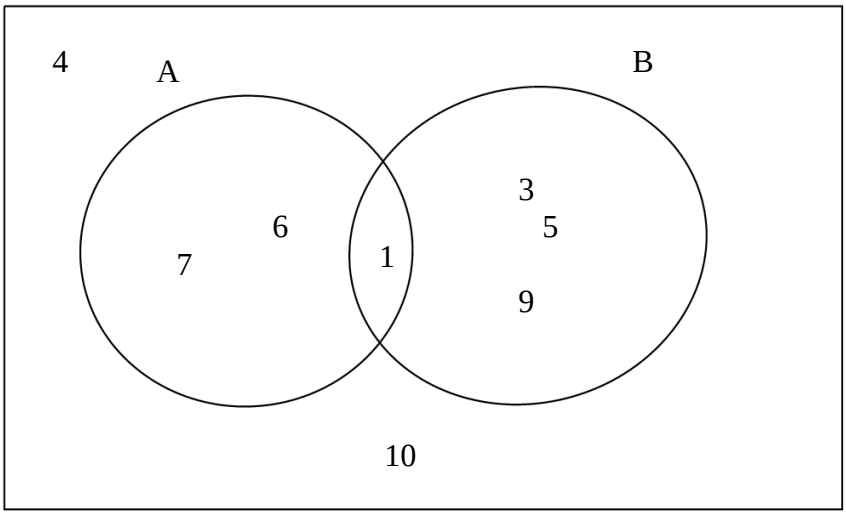
$\xi = \{3, 4, 5, 6, 7, 8, 9\}$

$K = \{4, 5, 6\}$

$L = \{5, 8, 9\}$



Ex 2. From the Venn diagram below list the following sets-



$A \cup B = \{1, 3, 5, 6, 7, 9\}$

$A \cap B = \{1\}$

$A' = \{4, 10, 3, 5, 9\}$

Review Questions

9. Draw a Venn diagram to show ξ = {a, b, c, d, e, f, g, h, i}, A = {a, e, g}, B = {b, c, d, e, g}

10. In a survey 80 Year 10 students were asked about their favorite fast food outlet. Sixty-two students like McDonalds and 45 like KFC. Thirty-nine students like both fast food outlets. Draw a Venn diagram to represent this information.

11. 120 Year 10 students are surveyed on subject satisfaction. 90 students like Maths and 105 students like English. 80 students like both subjects. Draw a Venn diagram to represent this information.

Karnaugh maps

As it appears in Unit 2

A Karnaugh map is another representation of relationships between two sets. It is a table that summarises all possible combinations of two sets. It looks like-

	B	B′	Total
A	$A \cap B$	$A \cap B'$	
A′	$A' \cap B$	$A' \cap B'$	
Total			

Ex 1. Copy and complete the following Karnaugh Map:

	A	B′	Total
A		104	390
B′	324		
Total			983

	A	B′	Total
A	**286**	104	390
B′	324	**269**	**593**
Total	**610**	**373**	983

Review Questions

12. Complete the following Karnaugh Map.

	X	X′	Total
Y	421		813
Y′	188		
Total			1274

13. The following Karnaugh map shows the number of swimming pools (P) and spas (S) in the local community.

	P	P'	Total
S		104	390
S'	324		
Total			983

a. How many households are there in the community?

_____

b. What number of these households has neither a spa nor a pool?

_____

c. How many have a spa and a pool?

_____

d. How many households do not have a spa?

_____

14. The following Karnaugh map shows the outcomes and probabilities of choosing one marble from each of two bags. Bag A contains 4 red and 2 blue marbles, while bag B contains 3 red and 3 blue marbles.

		Bag A	
		Red	Blue
Bag B	Red	$\frac{12}{36}$	$\frac{6}{36}$
	Blue	$\frac{12}{36}$	$\frac{6}{36}$

The probability of choosing one red and one blue marble is:

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

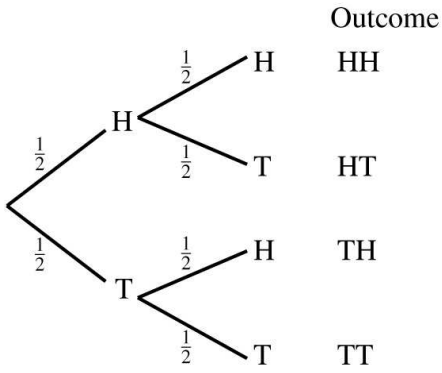
E. 1

Tree diagrams

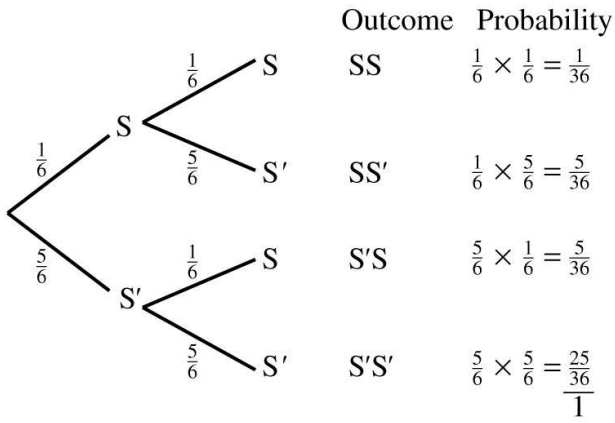
As it appears in Unit 2

Tree diagrams are another way to show the relationship between sets. A tree diagram is a branching diagram that lists all possible combinations of two or more sets (and their complements). This is particularly useful for displaying overlapping and disjoint sets.

Ex 1. Two coins are tossed. Draw a tree diagram to represent this information-



Ex 2. Two dice are rolled. Use a tree diagram to find the probability of rolling two sixes.



$$\Pr(\text{rolling two sixes}) = \frac{1}{36}$$

Review Questions

15. A 2-digit number is to be formed using the digits 1, 2 and 3. If the same number can be used twice,

a. draw a tree diagram to represent this information.

b. find the probability that the number formed is 31

_____

_____

c. find the probability that the number formed is greater than 30

_____

_____

d. find the probability that the number formed is odd

_____

_____

16. Three coins are tossed simultaneously-

a. Draw a tree diagram to list the sample space.

b. Use your diagram to find the probability of tossing at least one Head.

_____

_____

Solutions to Review Questions

1.

a. {*bread*, *milk*, *eggs*, *sausages*, *cereal*}

b. {x : *x is an item for breakfast*}

2.

a. 3

b. 7

3. *Answer:* B

Explanation:

1 is the only number which is not prime and not composite.

4. *Answer:* B

Explanation:

There are 19 multiples of 5 less than 100 and 9 of them are even.

5.

a. {car, bus, tram, bicycle, aeroplane}
Alternate solutions are possible.

b. {a, e}, {u, e}, {a, e, o}
Alternate solutions are possible.

6.

a. False

b. True

c. True

d. True

7. *Answer:* D

Explanation:

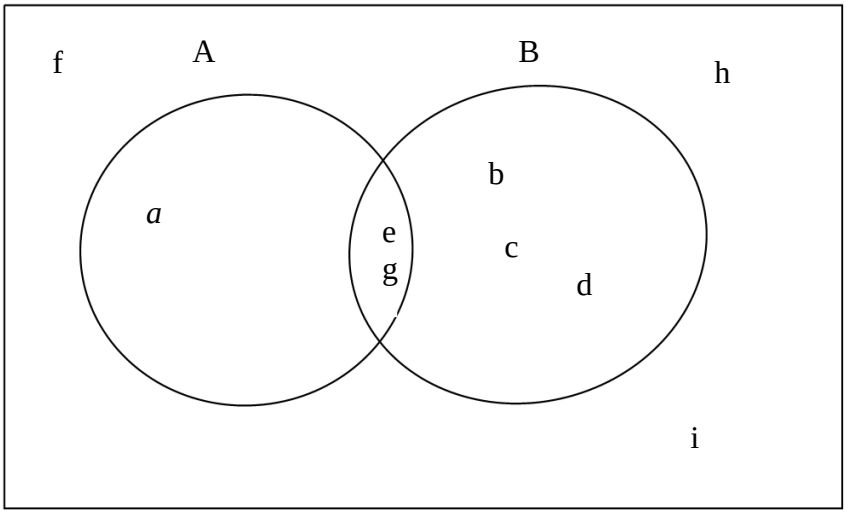
Factors of 20 are 1, 2, 4, 5, 10, 20 but only 2 and 5 are prime.

8. *Answer:* C

Explanation:

Read all the choices carefully.

9.



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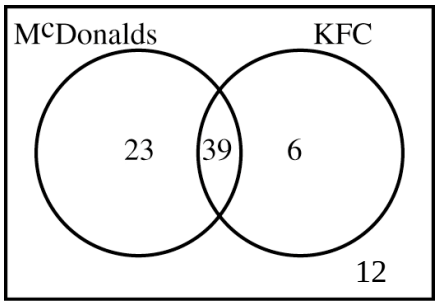

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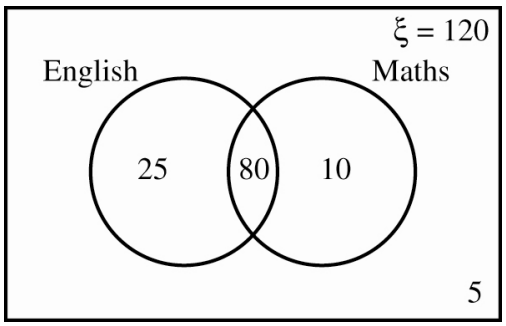


12.

	X	X′	Total
Y	421	**392**	813
Y′	188	**273**	**461**
Total	**609**	**665**	1274

13.

	P	P′	Total
S	**286**	104	390
S′	324	**269**	**593**
Total	**610**	**373**	983

a. 983

b. 269

c. 286

d. 593

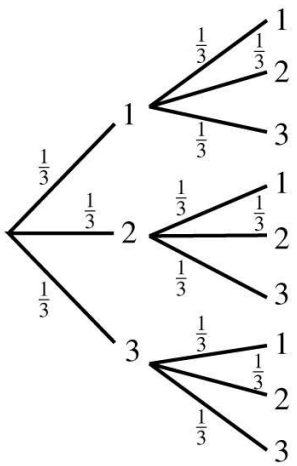
14. *Answer:* C

Explanation:

$$\frac{6}{36}+\frac{12}{36}=\frac{18}{36}=\frac{1}{2}$$

15.

a.



Outcome	Probability
11	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
12	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
13	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
21	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
22	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
23	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
31	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
32	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
33	$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$
	1

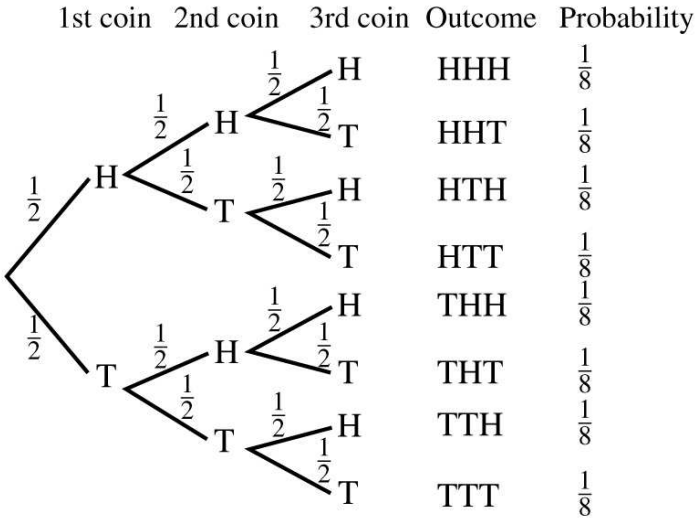
b. $\frac{1}{9}$

c. $\frac{1}{3}$

d. $\frac{2}{3}$

16.

a.



b. $\Pr(\textit{at least one head}) = 1 - \Pr(\textit{no head}) = 1 - \frac{1}{8} = \frac{7}{8}$