MATHEMATICAL METHODS (CAS)
Teach Yourself Series
Topic 2: Curve Sketching 2 – Semicircles, Circles, Rectangular Hyperbolae, Truncus, Reciprocal Curves and Square Root Curves

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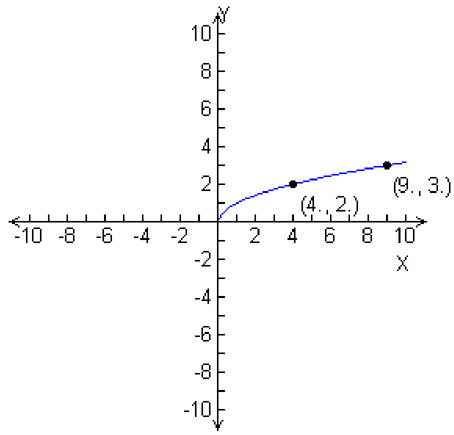
Contents

Curve Sketching 2

Square root graphs

As it appears in Unit 1

The basic square root function is given by $f(x) = \sqrt{x}$.



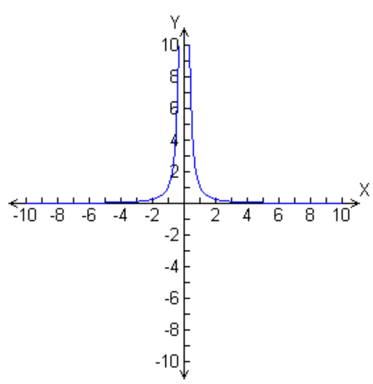
Dom: $R^+ \cup \{0\}$ Ran: $R^+ \cup \{0\}$

Truncus graphs

As it appears in Unit 1

These are functions written in the form of $f(x) = \frac{1}{x^2}, (x^{-2})$. The basic truncus looks like:

$f(x) = \frac{1}{x^2}$



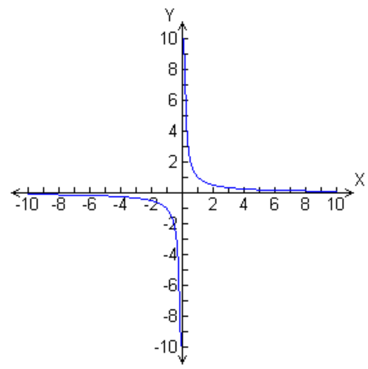
Note that these functions also have vertical and horizontal asymptotes.

Hyperbolae

As it appears in Unit 1

The basic hyperbola has the equation $f(x)=\frac{1}{x},(x^{-1})$. There are special features on a hyperbola called asymptotes. Refer to the graph below:

$f(x)=\frac{1}{x}$



Dom: $R\setminus\{0\}$ Ran: $R\setminus\{0\}$

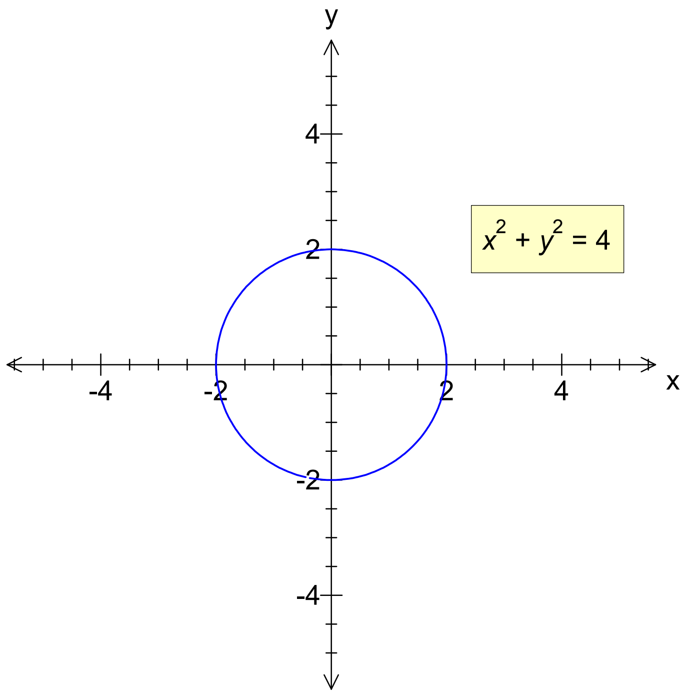
$x=0$ is an asymptote.

$y=0$ is an asymptote.

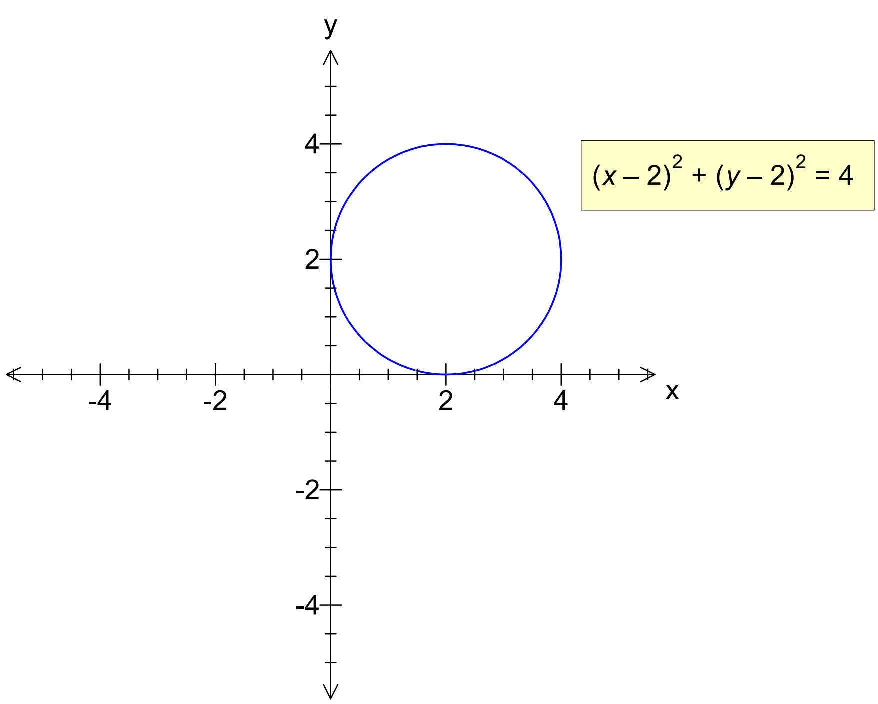
Circles

As it appears in Unit 1

The basic equation is $x^2 + y^2 = r^2$. This is a circle centered at the origin with a radius of r.



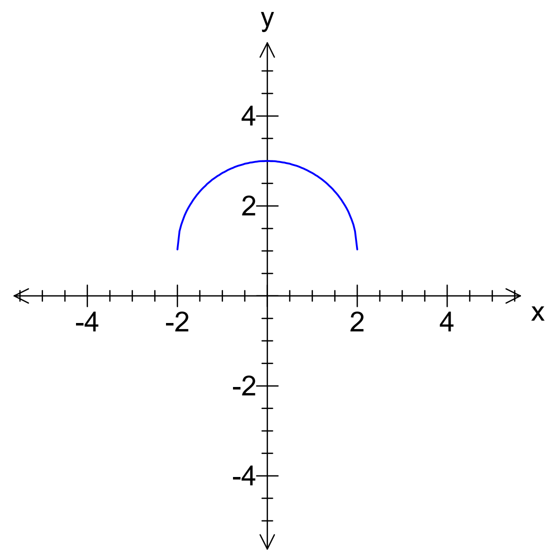
This equation can be translated in the x and y direction: $(x-h)^2 + (y-k)^2 = r^2$. This is a circle centered at (h,k) with a radius of r.



This equation can be transposed to make y the subject:

$y = \pm\sqrt{r^2 - (x-h)^2} + k$.

This gives us semi circles:



Reciprocal Functions

As it appears in Unit 3

These are graphs that are written in the form of $y = \frac{1}{f(x)}$. To sketch these use the basic process of:

1. Sketch $y = f(x)$ first.
2. Mark in vertical asymptotes. These are where $f(x) = 0$.
3. Now sketch $y = \frac{1}{f(x)}$ on all side of asymptotes. You need to take note of the reciprocal behaviour of the function.

Calculator skills

Graphing functions – setting up an appropriate window
Defining functions and evaluating them.
Factor
Prop Frac – to convert a hyperbola into a form that can be graphed.

Review Questions

1. State the maximal domain and the corresponding range of each of these functions:
   a. $f(x) = \sqrt{x} + 4$.
   b. $g(x) = \dfrac{1}{x-2}$.
   c. $h(x) = \dfrac{1}{(x+2)^2} + 3$.

2. Sketch the graphs of each of the following. Label all important features. State the maximal domain and range of each of the functions.
   a. $f(x) = \sqrt{x-4}$.
   b. $f(x) =$

c. $f(x) = \frac{4}{(x-3)^2} - 4$.

d. $f(x) = 3 - \frac{2}{x-4}$.

e. $f(x) = \frac{5}{x+6}f.$

f. $f(x) = 6 + \sqrt{4x-1}$.

3. Sketch the following graphs stating the domain and range:

a. $x^2 + y^2 = 9$.

b. $x^2 + (y + 5)^2 = 25$.

c. $(x - 7)^2 + (y + 7)^2 = 49$.

d. $y = \sqrt{9 - x^2} - 3$.

e. $y = \sqrt{\left(25 - (x - 5)^2\right)}$.

4. If $f(x) = x^2 + 4$, Sketch $\frac{1}{f(x)}$.

5. If $f(x) = x^2 + 6x + 8$, Sketch $\frac{1}{f(x)}$.

Solutions to Review Questions

1.

a. R^+. Range is $[4, \infty)$

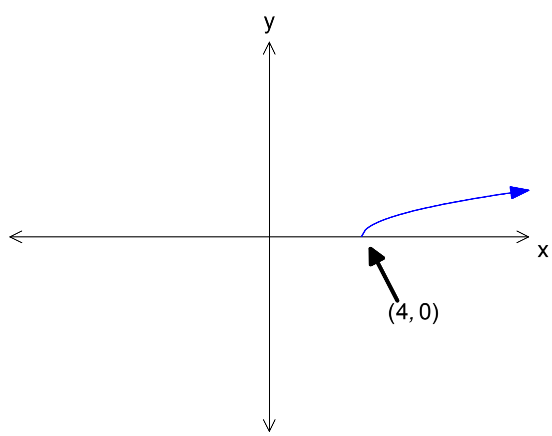
b. $R\backslash\{2\}$ Range is $R\backslash\{0\}$

c. $R\backslash\{-2\}$ Range is $(3, \infty)$

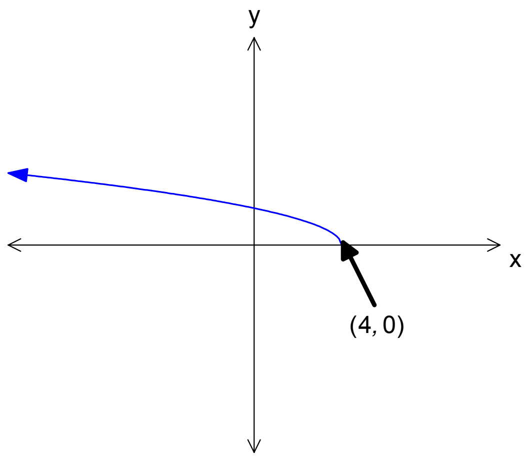
Sketch a graph of each to work out domain.

2.

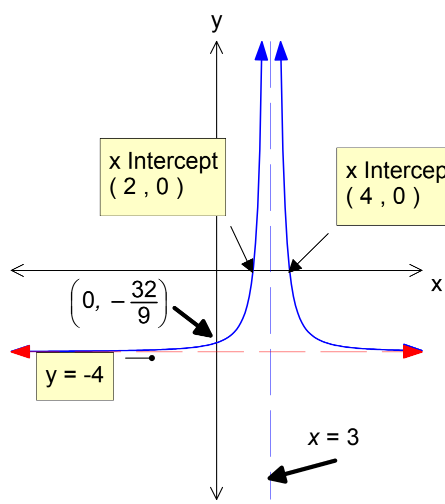
a. Dom: $[4, \infty)$
Ran: $R^+ \cup \{0\}$



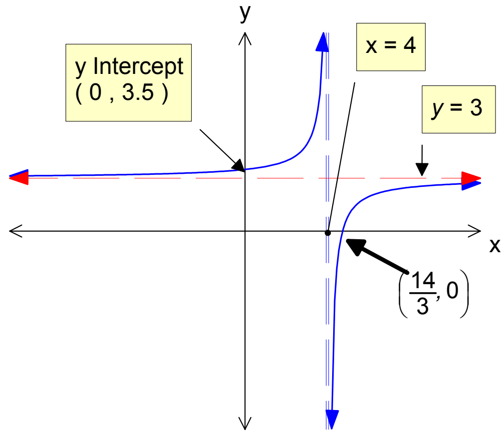
b. Dom: $(-\infty, 4]$
Ran: $R^+ \cup \{0\}$



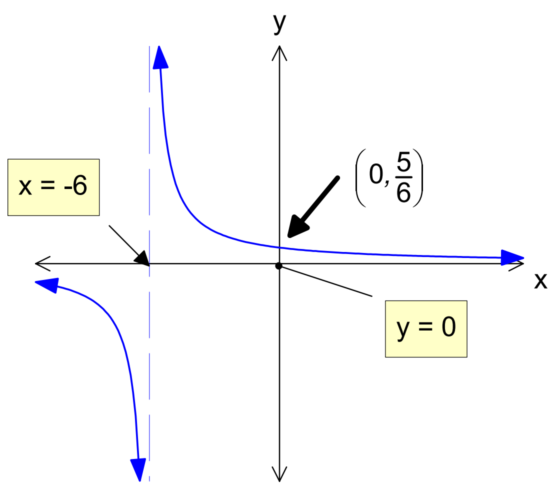
c. Dom: $R\backslash\{3\}$
Ran: $(-4, \infty)$



d. Dom: R\{4}
Ran: R\{3}

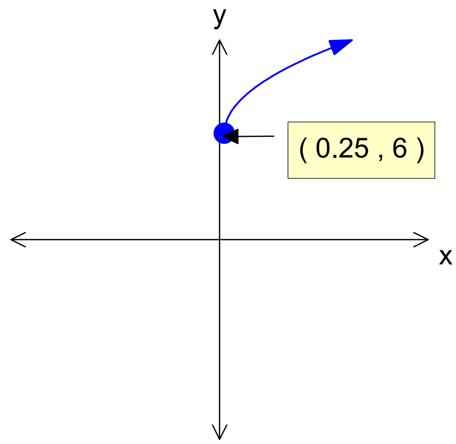


e. Dom: R\{-6}
Ran: R\{0}



f. Dom: $\left[\frac{1}{4}, \infty\right)$

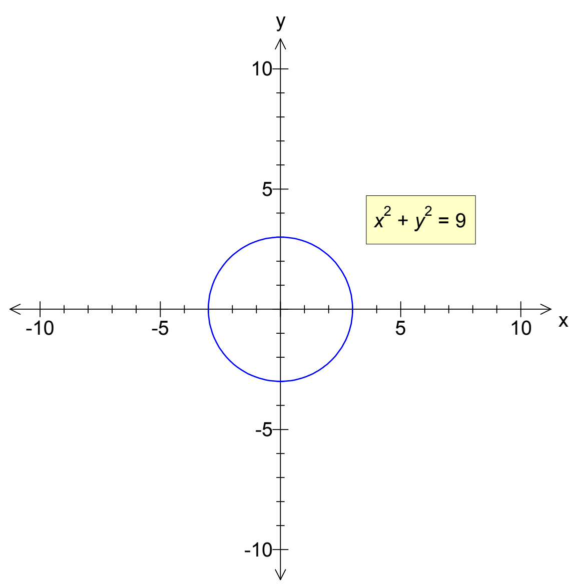
Ran: $[6, \infty)$



3.

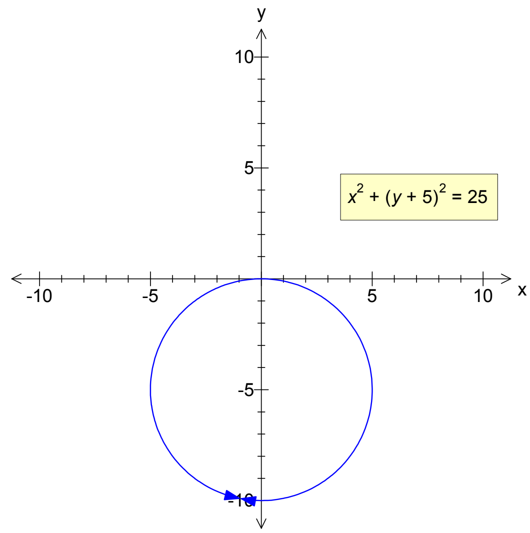
a.

Domain: [-3, 3] Range: [-3, 3]



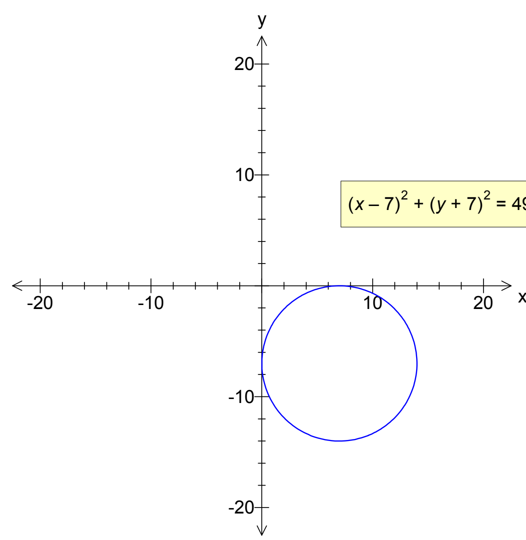
b.

Domain: [-5, 5] Range: [-10,0]



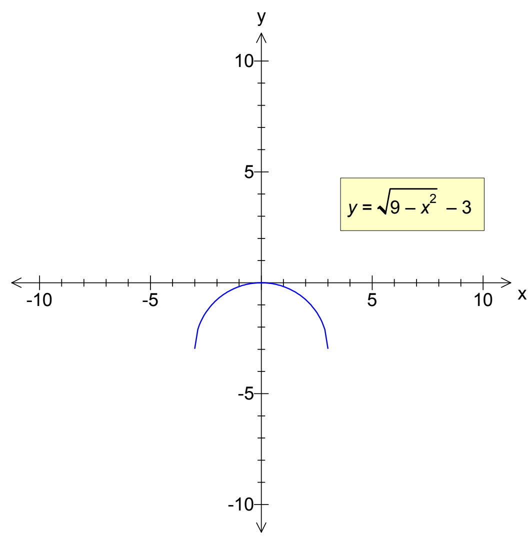
c.

Domain: [0,14] Range:[-14,0]



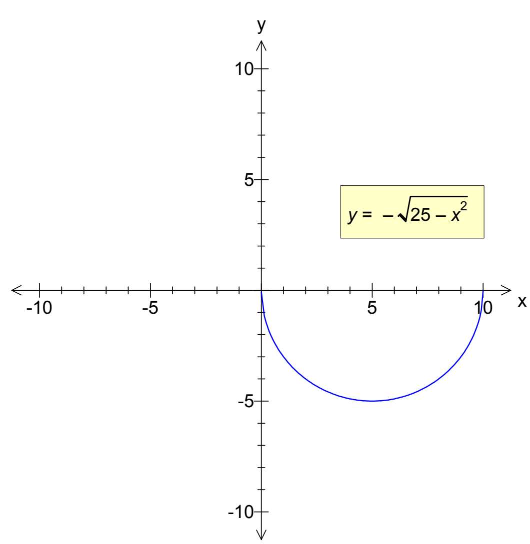
d.

Domain: [-3,3]   Range:[-3,0]

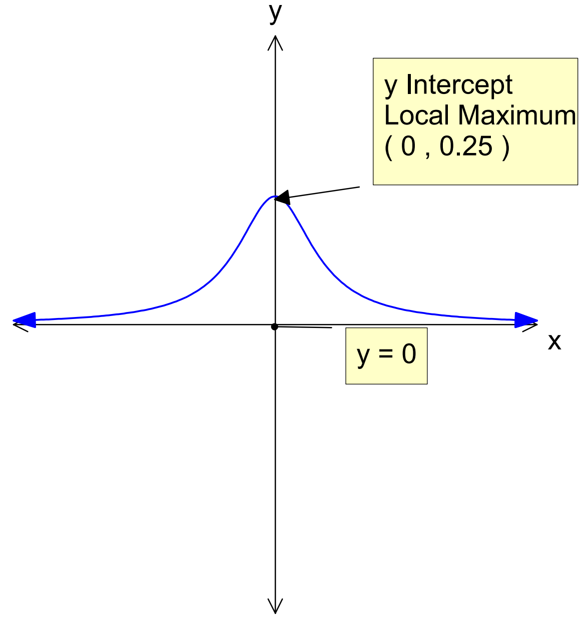


e.

Domain: [0,10]   Range:[-5,0]

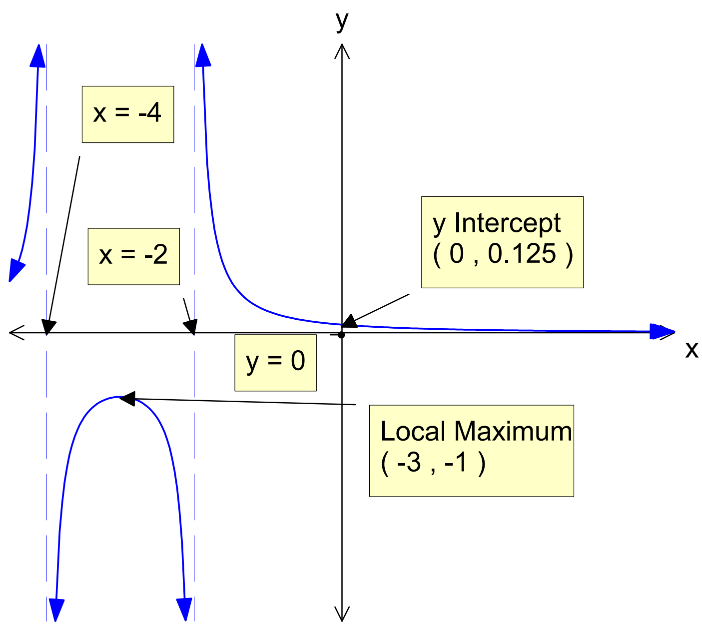


4. If $f(x) = x^2 + 4$, Sketch $\frac{1}{f(x)}$



Sketch $y = x^2 + 4$ first.
Then reciprocate the y values.

5.



Sketch $x^2 + 6x + 8$ first, then reciprocate y values.
x intercepts on first graph become vertical asymptotes.