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MATHEMATICAL METHODS CAS
Teach Yourself Series
Topic 7: Rates of Change

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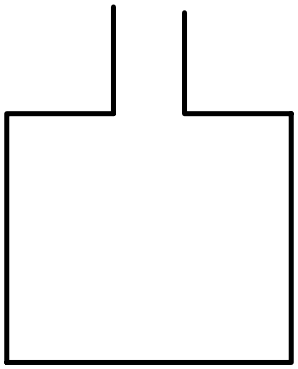
Rates of Change

Relationships

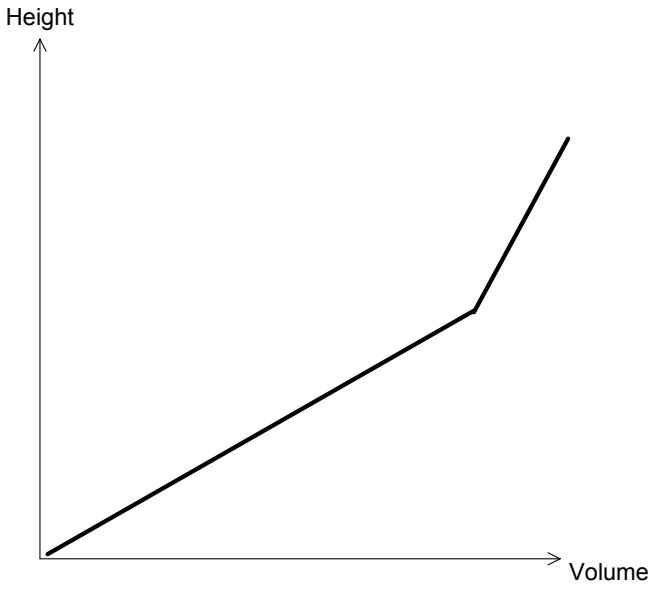
When one variable is related to another we often are interested to know how one changes when the other changes. For example how does the height of water change when a volume of water is poured into a container?

Example

For the following shaped container, draw a graph of how the height of water would change with respect to volume of water changing when the water is being poured into the container at a constant rate.

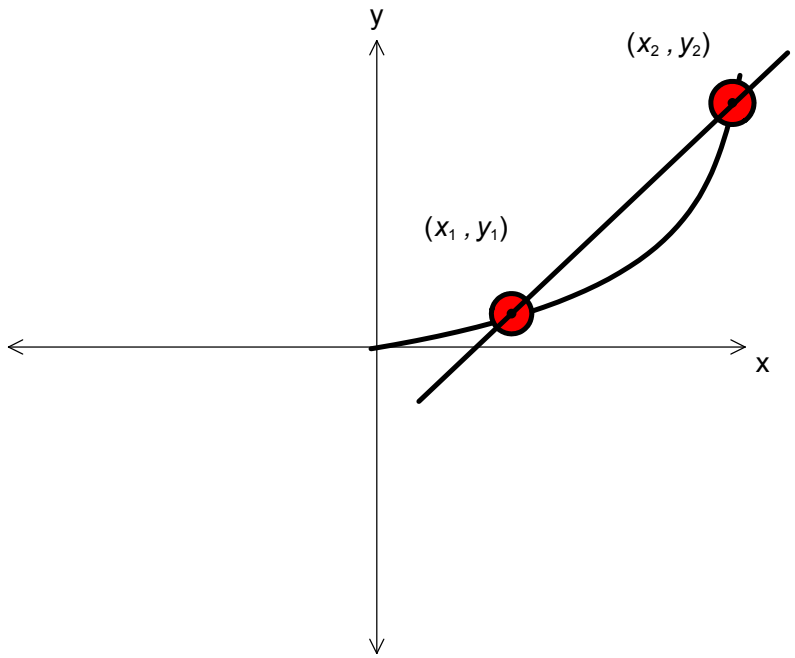


Answer



Average Rate of Change – Constant Rate of change

This is the rate of change between two points. It is the gradient of a secant (chord).



$$\text{average rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Example

Find the average rate of change between $x = 1$ and $x = 3$ of the curve $\boldsymbol{f(x) = x^2}+2$.

Answer:

$$\boldsymbol{f(1) = 3 \text{ and } f(3) = 11}$$

$$\boldsymbol{\text{average rate of change} = \frac{11 - 3}{3 - 1} = 4}$$

Example

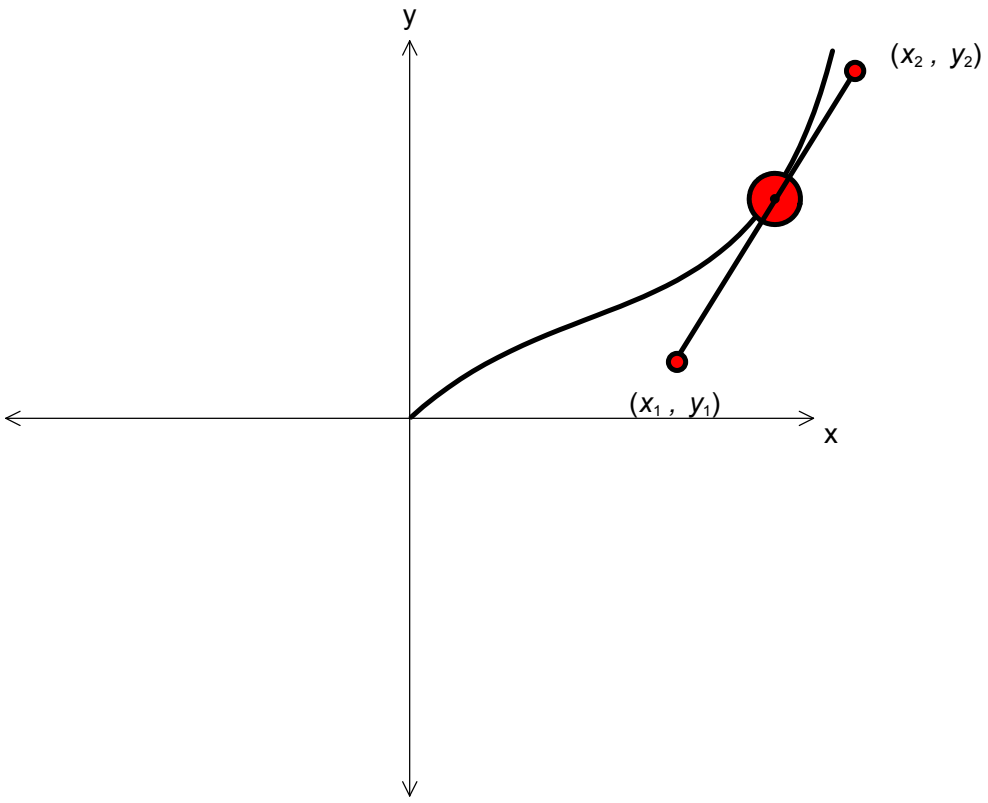
Find the average rate of change between $x = 1$ and $x = 1+h$ of the curve $\boldsymbol{f(x) = x^2 + 4x}$.

Answer:

$$f(1) = 5 \text{ and } f(1 + h) = h^2 + 6h + 5$$
$$\text{average rate of change} = \frac{h^2 + 6h + 5 - 5}{1 + h - 1}$$
$$\text{average rate of change} = h + 6$$

Instantaneous Rate of Change

This is the rate of change at a point. It is the gradient of a tangent on a graph.



$$\textit{instantaneous rate of change} = \frac{y_2 - y_1}{x_x - x_1}$$

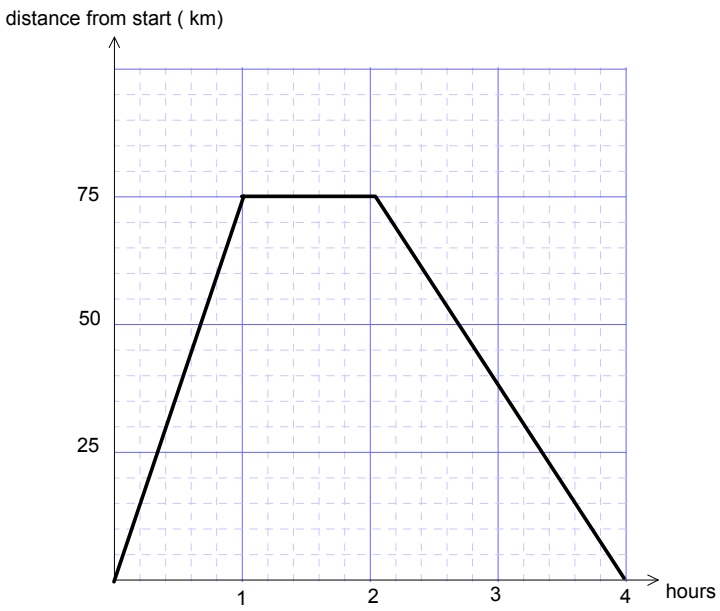
Common Rate

$$\textit{speed} = \frac{\textit{distance}}{\textit{time}}$$

Speed = rate of change (gradient) of a distance vs time graph.

Example

The following graph describes the distance a car moves from its starting position.



Using the graph above find the following:

a. The distance the car is from the start after 1 hour

b. The distance the car is from the start after 4 hours.

c. The average speed between t=0 and t=1
t=1 and t=2
t=2 and t=4

Answer:

Reading from the graph, the car is 75km from the start.

Reading from the graph the car is back at the starting position.

For each section the gradient of the graph is calculated.

For t=0 to t= 1:

$$\boldsymbol{speed = \frac{75}{1} = 75\ km/h}$$

For t=1 to t=2

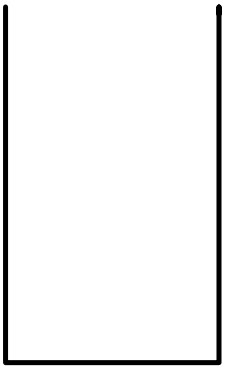
speed = 0 (it is not moving)

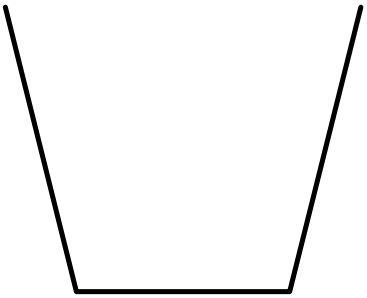
For t=2 to t=4

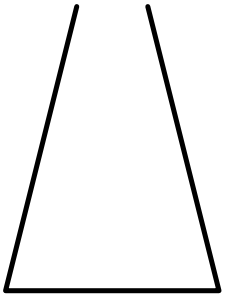
$$\boldsymbol{speed = -\frac{75}{2} = -37.5\ km/h}$$

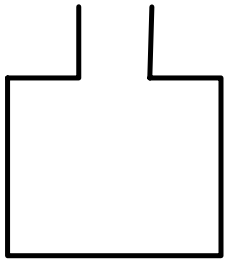
Review Questions

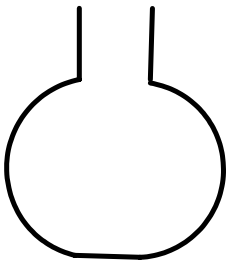
1. For the following shaped containers, draw a graph of height of water against volume:

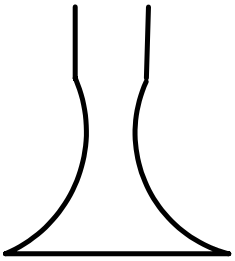




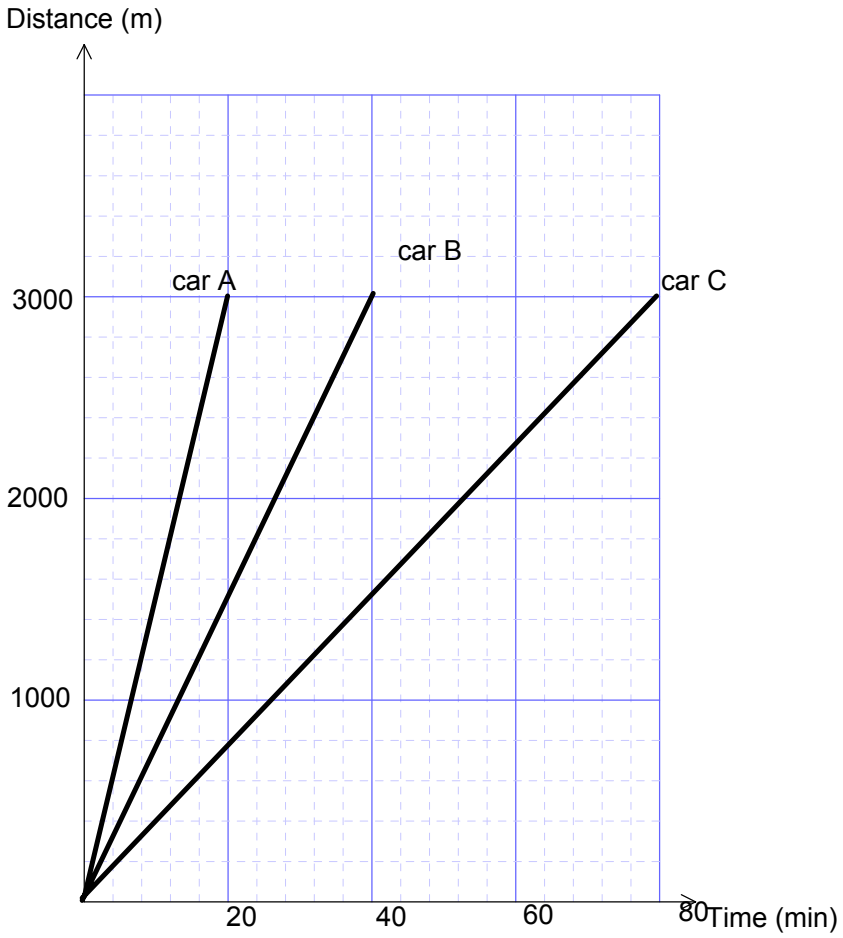








2. Three cars had their motion recorded and the graphs are shown below:



a. How far did each car travel?

_____

_____

_____

_____

b. How long did each car take to complete the distance travelled.

_____

_____

_____

_____

c. Calculate the speed of each car.

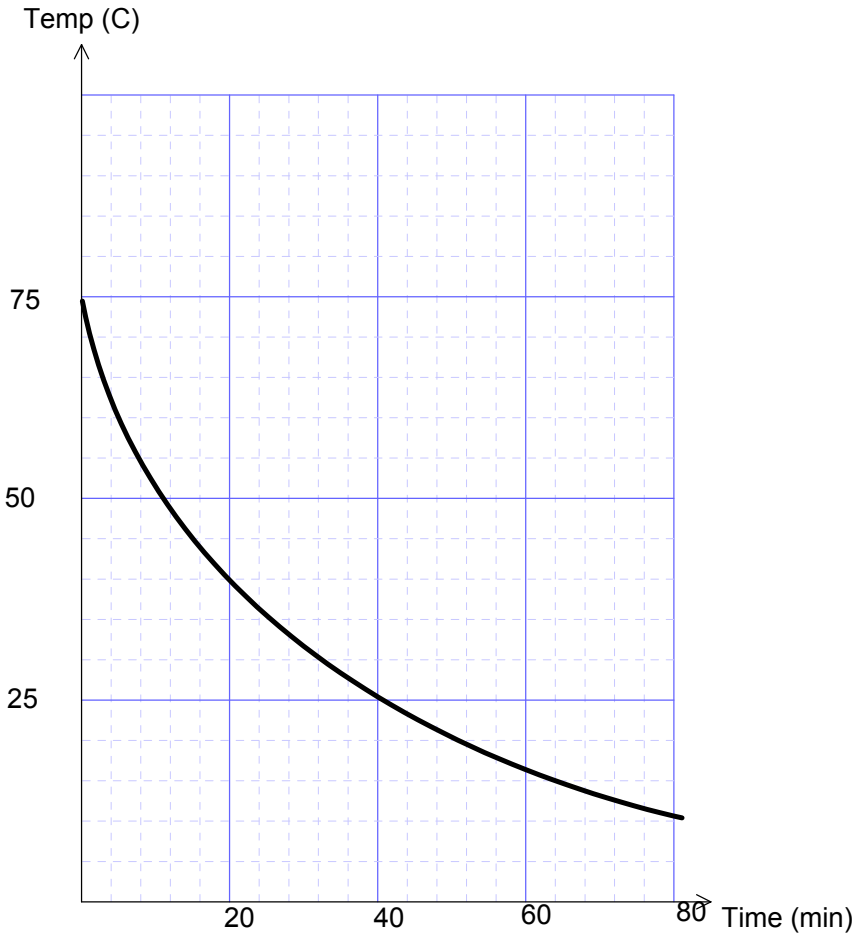
d. How can you tell from the gradient which car had the greatest rate of change?

3. Calculate the average rate of change of the function $f(x) = x^2 + 4$ between the following points:

a. $x = 1$ and $x = 3$.

b. $x = 1$ and $x = 1 + h$.

4. The following information was collected when the temperature of a cup of coffee was taken:



a. What was the initial temperature of the coffee?

_____

_____

_____

_____

b. What was the temperature of the coffee after 80 minutes?

_____

_____

_____

_____

c. What information does the gradient of the graph give us?

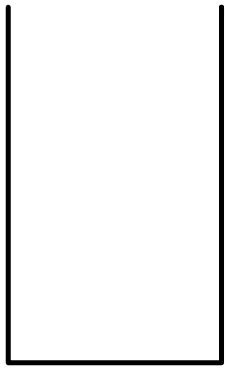
d. What was the temperature of the coffee after 40 minutes?

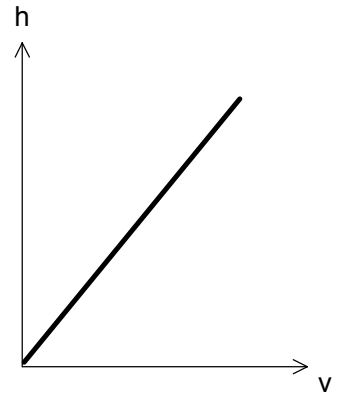
e. What was the average rate of cooling over the first 40 minutes?

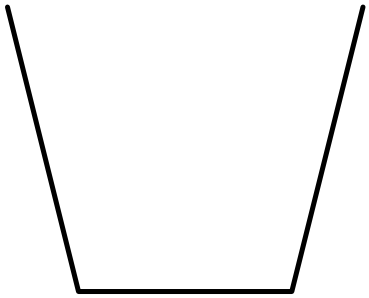
f. What was the rate of cooling at the 40 minute mark?

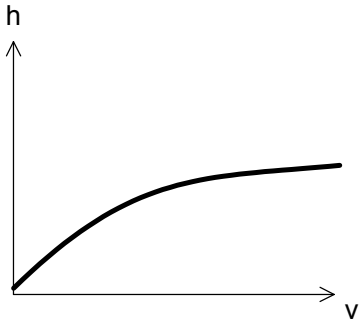
Solutions to Review Questions

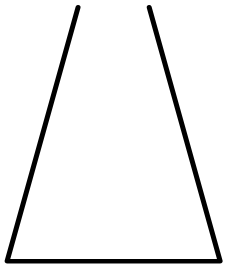
1.

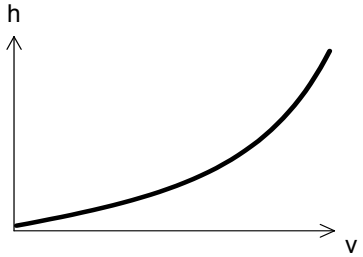


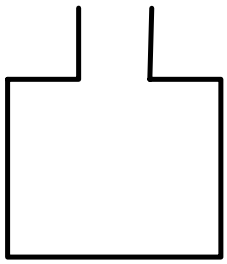


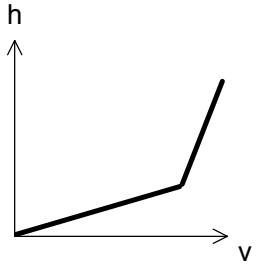


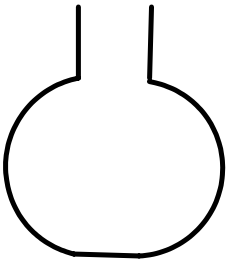


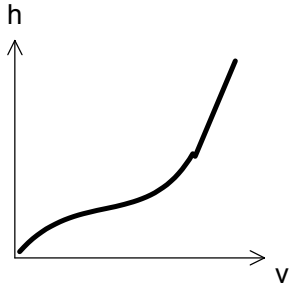


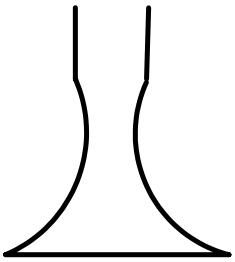


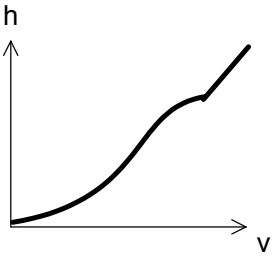












2.

a.

3000 metres

Read from graph

b.

Car A 20 min

Car B 40 min

Car C 80 min

Read from graph

c.

Car A 150 m/min

Car B 75 m/min

Car C 37.5 m/min

Gradient of each line.

d. Slope of the graph – Steeper the graph the greater the rate of change.

3.

a.

Average rate of change = $\frac{f(3) - f(1)}{3 - 1}$

4

b.

Average rate of change = $\frac{f(1 + h) - f(1)}{h}$. Expand the numerator and take out a common factor of h.

$h + 2$

4.

a.

75 degrees

Read off graph

b.

10 degrees

Read off graph

c.

Unit in "y" direction divided by unit in "x" direction.

Rate of change of cooling in deg/minute

d.

25 degrees.

Read off graph

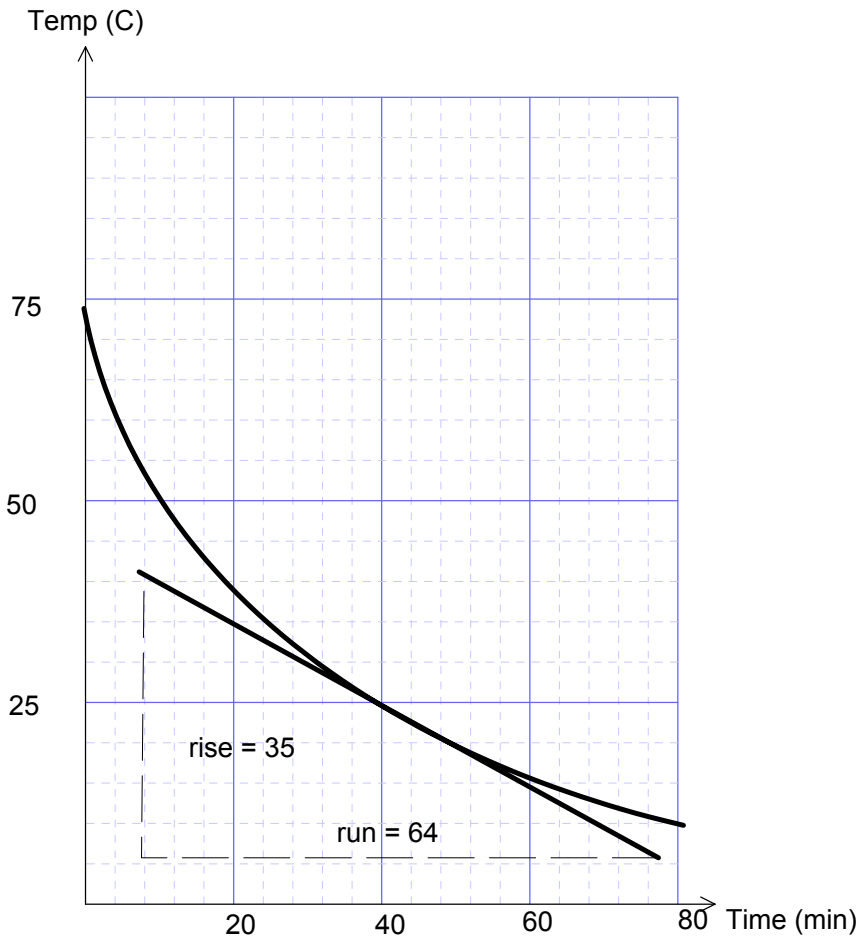
e.

Gradient of graph over the first 4 minutes – Average rate of change. Answer is positive if you want rate of cooling.

1.25 degrees/minute

f.

Gradient of tangent at the 40 minute mark. Answer will vary a little due to where you locate the tangent.



$gradient = \frac{35}{64} \approx 0.55$ degrees/minute