





# Cambridge Senior Science

Physics

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#### **CAMBRIDGE** UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment, a department of the University of Cambridge.

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First published 2024

 $20 \ 19 \ 18 \ 17 \ 16 \ 15 \ 14 \ 13 \ 12 \ 11 \ 10 \ 9 \ 8 \ 7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1$ 

Cover designed by Loupe Studio Text designed by Shaun Jury Typeset by QBS Learning Printed in China by C&C Offset Printing Co., Ltd.

A catalogue record for this book is available from the National Library of Australia at www.nla.gov.au

ISBN 978-1-009-25901-9 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

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### About the authors

Dr Eddy de Jong has been involved with science and physics education at the secondary and tertiary level for many years. He has taught science at all levels, senior HSC/VCE Physics and university physics. He was involved in the Victorian Gifted Students Physics Network. Eddy has had a long association with Year 12 Physics, including involvement in course design; classroom teaching; and setting, vetting and marking the examinations. He is a successful author of numerous science and physics texts, including the Cambridge Victorian Science Years 7–10. He is passionate about seeing young minds engaging with science and physics and aims to instil in students a sense of curiosity while developing their critical thinking skills.

Dr Sydney Boydell has had a long association with VCE Physics, including involvement in course design; classroom teaching; and setting, vetting and marking the examinations. He has taught at VCE level in Australia and the UK, and has over 25 years experience teaching at first-year university level. He currently lectures in Science Education and is the author of two VCE physics titles in the Cambridge Checkpoints series as well as two HSC titles for New South Wales.

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Mary Macmillan (formerly Willox) has recently retired after more than 35 years' experience teaching physics in Melbourne and in the UK, including HSC/VCE, A-Level and International Baccalaureate. She has an ongoing interest in the broader curriculum, in particular teaching for effective learning, developing a range of generic life skills through classroom activities and encouraging students to see physics in their everyday lives. Following her passion for physics, Mary retrained mid-career as a radiographer, worked in magnetic resonance imaging at the Florey Institute in Melbourne, but soon returned to the fun of the physics classroom.

#### Authors' acknowledgement

The authors wish to thank Dr David Mills and Bruce Walsh for their help in checking the manuscripts for this series.









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The online material summarised in Appendix 1 will be found in the Interactive Textbook and teacher resources.

Answers to all questions are available in the Interactive Textbook and the teacher resources.



## Overview: How to use this resource

The Cambridge Education Australia and New Zealand website has more information and demos for this title.

This overview guides you through all the components of the **print and PDF textbooks**, the **Interactive Textbook (ITB)**, and the teacher resources in the **Online Teaching Suite (OTS)**. Users of the award-winning *Cambridge Science 7–10 for the Victorian Curriculum* will recognise some similarities with this senior science resource, including the hosting of the digital material on the Edjin platform, which was developed from *Cambridge HOTmaths* and is already being used successfully by thousands of teachers and students across Victoria.

#### **Print book features**

#### Learning objectives

In the Curriculum table at the start of each chapter, the Study Design dot points are translated into Learning objectives, describing what students should be able to do by the end of the chapter:



Learning objectives are turned into Success Criteria (achievement standards) at the end of the chapter and are assessed in the Chapter review and tracked in the Checklists

Relevant Study Design dot points are repeated at the start of each section in the chapter, and an overall curriculum grid is provided in the teacher resources.

#### **Concept maps**

Concept maps display each chapter's structure with annotations emphasising interconnectedness, providing a great memory aid. All concept maps included in this book can be found on pages xiv and xv.The versions in the ITB are hyperlinked and offer an alternative way of navigating through the course.

#### Links

The interconnectedness of topics in Physics is demonstrated through links between sections, displayed in the margins. In the ITB, these are hyperlinks that provide a second alternative way of navigating through the course.





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#### **Chapter sections**

Chapters are divided into numbered sections, each with a consistent set of features.

#### Engage

At the start of each section, these boxes provide points of interest for the topic, emphasising its place in Physics. This material, though not assessable, can be used as examples of applications.

#### Explain

This icon marks the start of essential content that is assessed.

#### Glossary

Scientific terms are highlighted in the text, definitions are given in the margin of the print and PDF textbooks, or on mouseover in the ITB, and the terms are listed at the start of each chapter and section.

#### Check-in questions

Each section in the chapter has one or more sets of check-in questions, for formative assessment. Full answers are provided in the digital resources.

#### Formula boxes

Important formulas are given in boxes.

#### Worked examples

Fully worked examples are given for questions involving application of formulas and numerical calculations.

Study Design coverage for section



### Newton's laws

#### Study Design:

• Investigate and apply theoretically and practically Newton's three laws of motion in situations where two or more coplanar forces act along a straight line and in two dimensions

#### Glossary terms in the section

Glossary: Free-body diagram Free fall Inertia Net force Normal force

#### **Maglev** trains

ENGAGE

Forces can broadly be classified as either contact or non-contact. When the north poles of two magnets are aligned, a pushing force will occur even though the two magnets are not in contact. The non-contact force of magnets is utilised in the design of the fastest trains in the world, magnetic levitation (maglev) trains. A series of superconducting magnetics and an electromagnetic drive system allows maglev trains to float above the track. Maglev trains are able to achieve speeds of more than 400 km h<sup>-1</sup> and are quieter and less subject to vibration than normal locomotives.

#### **EXPLAIN**

#### Newton's first law

Newton's first law states that an object in a state of rest or travelling at a constant velocity will remain in its state of motion unless acted upon by an unbalanced force

Newton's first law is sometimes referred to as the law of inertia. For example, if a car is at rest on the ground, the normal force provided by the ground pushing up on the car balances the force due to gravity on the car. Since the forces on the stationary car are balanced, it remains at rest. If a car is moving on a straight road at a constant velocity, then the forces on the car will also be balanced. The force of air resistance and rolling resistance is balanced by the driving force provided by the road on the car.



a body's ability to resist a

change in its state of motion

Terms in the glossary

Glossary definitions

#### Check-in questions – Set 1

- 1 When an object is in uniform circular motion, what is the direction of the net force?
- 2 When an object is in uniform circular motion, what is the direction of the velocity?
- 3 A 1300 kg car moves in a horizontal circle that has a radius of 30 m. If the centripetal force is  $9.75 \times 10^3$  N, calculate the velocity of the car.

#### Formula 1A–4 Acceleration on a frictionless slope

#### $a = g \sin \theta$

Where:

- a = Gravitational acceleration down the slope (m s<sup>-2</sup>)
- g = Gravitational field strength close to the surface of Earth, 9.8 Nkg<sup>-1</sup> (or m s<sup>-2</sup>)
- $\theta$  = Angle of the slope (°)

#### Worked example 1B–4 Banked turn

A 4500 kg truck travels on a banked track angled at 30° to the horizontal. The track has a circular radius of 75 m.

- a Explain how the banking allows the truck to maintain circular motion around the bend without relying on the friction force.
- **b** If the truck is travelling at 50.4 km h<sup>-1</sup>, calculate the centripetal force required to keep the truck in circular motion.

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#### Skills

Skills boxes in every section provide advice and guidance on how to answer and prepare for questions, especially in examinations. The ITB has video versions of these, which provide extra comments and an alternative medium of delivery.

#### Charts, diagrams and tables

Detailed charts integrating text and diagrams, and illustrated tables, feature throughout the print books. In the ITB, many of these are available as animated slide-show presentations for students to use, with copies for teachers to display on data projectors or whiteboards.

#### Section questions

Summative assessment is provided at the end of each section, with full answers provided in the digital resources.

#### Chapter reviews

**Summaries:** Students are encouraged to make their own set of summary notes, to help them assimilate the material. Model summaries are provided in the teacher resources, to be given to those who need help. Creating summaries can also be turned into an assessment task, with the models serving as the answer.

Checklists and Success criteria:

The learning objectives from the front of the chapter are listed again in the form of success criteria linked to the **multiple-choice** and **shortanswer questions** that follow. The checklists are printable from the ITB, and students can tick off their achievement manually. If they do the questions in the ITB, they are ticked automatically when the questions are marked or self-assessed.

#### 2A SKILLS

#### Understanding how variables relate to each other

When looking at a formula, it is a useful skill to be able to quickly determine how the variables in the formula relate to each other. For example, consider the formula for gravitational potential energy:

$$E_{g} = mg\Delta h$$

In this formula, the gravitational potential energy would be directly proportional to the mass, gravity and change in height.





Section	<b>1</b> A	questions	

#### Multiple-choice questions

1 A tennis ball is travelling through the air in a direction up and to the right, close to the surface of Earth. Assuming that air resistance is negligible, which of the diagrams below represents all of the forces acting on the ball?

#### **Chapter 1 review**

#### Summary

Create your own set of summary notes for this chapter on paper or in a digital document. A model summary is provided in the Teacher Resources, which can be used to compare with yours.

#### Checklist

In the Interactive Textbook, the success criteria are linked from the review questions and will be automatically ticked when answers are correct. Alternatively, print or photocopy this page and tick the boxes when you have answered the corresponding questions correctly.

_	- Succe	ss criteria – I am now able to:	Linked questions
	1A.1	Apply Newton's first law to draw free-body diagrams and determine if a net force is acting on a body	30,40,50, 70,80,100, 110 120
_	Short	answer questions	
	17 A v and belo pos	iolin string is played forming a 900 Hz sound standing wave. The string a strobe photo of the vibrating string shows regions of no vibration (no ow. The distance between the fixed ends is 30 cm. The dashed line show ition when it is not vibrating.	is fixed at the ends des), as shown s the string's
		Nodes	
		Fixed ends	

a Describe the mechanism leading to the formation of the standing wave. (1 mark)b Calculate the period of the standing wave. (2 marks)

#### Unit revision exercises

Each Unit has a revision exercise in the print book, with both multiple-choice and short-answer questions SBN 978-1-009-25901-9 Photocopying is restricted under law and this material must not be transferred to another party.

#### **Interactive Textbook features**

Course Content

The digital version of the textbook is hosted on the Edjin platform, offering easy navigation, excellent on-screen display and multimedia assets, as well as automarking of multiple-choice questions, and workspaces for other questions with self-assessment and confidence rating tools. The different kinds of digital assets are listed below:

- Printable **Worksheets** with extra questions and activities (and content in some cases) are provided for all chapters, marked by an icon in the margin, as shown on the right.
- Videos are provided for all chapters, and are of two kinds: concept videos demonstrate or illustrate important theory, while skills and example videos work through the textbook's skills and example boxes, providing extra explanation and guidance. Some videos are provided in the print pages as QR codes for immediate access and review.
- Animated slide-show presentations (in PowerPoint Show format) of many charts, diagrams and tables are provided and are marked by an icon in the margin as shown at right, enabling them to be explored interactively.
- **Answers** (suggested responses) to questions are provided as printable documents in the teacher resources and, if enabled by the teacher, below the ITB question workspaces.

States and states and

• 'Scorcher' quizzes on terms and definitions let students test themselves.







Messages	Velcome HS!	
Task manager     Tests and Exams     Offline textbook	Chapter 1 Newton's laws of motion	Next chapter Chapter 2 Force, energy and mass
Teacher resources	1A Newton's laws →	Introduction ->
Manage classes     Manage classes     My classes     Manage bookmarks     Q Search resources	Change Text	
	13 m	
Q Search resources	No assigned Texts and Exams	Create a test Type: Diagnostic    Status: All
Q Search resources	Tests and Exams No assigned Tests and Exams Tasks Create Task Type: All	Create a test Type: Diagnostic v Statut: All v Class: All v Go to all my tests and exams Classes Create class Classes: School classes v
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#### **Online Teaching Suite features (teacher resources)**

The OTS provides Edjin's learning management system, which allows teachers to set tasks, track progress and scores, prepare reports on individuals and the class, and give students feedback. The assets include:

- Curriculum Grid and teaching programs
- Editable and printable Chapter tests with answers
- Checklists with linkage to the success criteria for the chapter question sets and tests
- A **question bank** and test generator, with answers
- Practice exams and assessment tasks, with answers
- Editable versions of Worksheets in the Interactive Textbook, and answers to them
- Editable versions of the PowerPoint files in the Interactive Textbook
- Downloadable, editable and printable practicals
- Editable and printable chapter **summaries** (model answers for the chapter summary activity)
- **Teacher notes** on selected content with additional theory explanation and suggestions for further activities and resources
- Curated links to internet resources such as videos and interactives.
- Practice SAC tasks

#### **Exam generator**

The Online Teaching Suite includes a comprehensive bank of exam-style and actual VCAA exam questions to create custom trial exams to target topics that students are having difficulty with. Features include:

- Filtering by question-type, topic and degree of difficulty
- Answers provided to teachers
- VCAA marking scheme
- Multiple-choice questions that will be auto-marked if completed online
- Tests that can be downloaded and used in class or for revision.

# Overview: Aboriginal and Torres Strait Islander knowledge, cultures and history

The VCE Physics Study Design includes aspects of Aboriginal and Torres Strait Islander knowledge, cultures and history. This overview is a guide to coverage in this resource.

Aboriginal and Torres Strait Islander peoples' world views are highly integrated: each aspect of culture, history and society connects with all other aspects. Each community has their own personalised system of thinking, doing and knowing based on sharing culture and adapting to the environment around them.

In order to gain an understanding of any system, Indigenous or not, time and effort is needed to appreciate it. That time is limited in this course; and it is wrong to try and generalise the Indigenous culture of Australia, or even of Victoria. Instead, the coverage in the resource should be taken as a collection of examples, and students should read up on or engage with their local Indigenous community to understand their cultural aspects.

This series includes examples of Aboriginal and Torres Strait Islander knowledge, cultures and history and in the Unit 2 Options in the Interactive Textbook there is coverage of *Option 2.15: How can physics explain traditional artefacts, knowledge and techniques?* 

In addition, for students, the Interactive Textbook includes an introductory guide prepared by First Nations consultants advising on approaches to studying Aboriginal and Torres Strait Islander knowledge, cultures and history, with links to further reading.

For teachers, the teacher resources include a guide to approaches to teaching Aboriginal and Torres Strait Islander knowledge, cultures and history in the VCE Physics course, with links to internet resources.

#### Guide to terms used in this resource

Language is very important in discussing Indigenous issues, especially given the past history of deliberately offensive usage in Australia, where language was used to oppress and control.

Indigenous First Australians and First Peoples of any country	Respectful usage requires a capital I.
First Australians, First Nations or First Peoples Indigenous people of Australia	These terms have become more common in recent years, with 'Indigenous' as the adjective.
Aboriginal an Aboriginal person is someone who is of Aboriginal descent, identifies as being Aboriginal and is accepted as such by the Aboriginal community with which they originally identified	One of the reasons that 'First Nations' and allied forms have become more common is that the term 'Aboriginal' was sometimes used disrespectfully, and still is in some circles.
Aboriginal and Torres Strait Islander peoples the Australian Indigenous population includes Aboriginal People, Torres Strait Islander People, and people who have both Aboriginal and Torres Strait Islander heritage. The term 'Aboriginal and Torres Strait Islander' encompasses all three	While this is still used in official circles and is in the name or title of many organisations and documents, it is tending to be replaced by 'First Australians' and similar terms, especially in everyday use. This is partly because the abbreviation 'ATSI' is considered disrespectful by Indigenous people, who regard it as lazy not to use a full title. The abbreviation should not be used to refer to people.

Cambridge University Press & Assessment acknowledges the Australian Aboriginal and Torres Strait Islander peoples of this nation. We acknowledge the traditional custodians of the lands on which our company is located and where we conduct our business. We pay our respects to ancestors and Elders, past and present.



This map attempts to represent the language, social or nation groups of Aboriginal Australia. It shows only the general locations of larger groupings of people which may include clans, dialects or individual languages in a group. It used published resources from the eighteenth century -1994 and is not intended to be exact, nor the boundaries fixed. It is not suitable for native title or other

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AIATSIS Map of Indigenous Australia, showing the general locations of larger language, social or nation groups. To zoom in on detail especially in Victoria, access the map in the Interactive Textbook.

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### Concept maps for Units 3&4

This spread displays the concept maps for Chapters 1–9. Access the digital versions in the ITB and click on hyperlinks to explore the interconnections of the topics.



# Chapter 4 Electric and magnetic fields

#### Chapter 5 Generating electricity





**Chapter 6** Transformers and

# Chapter 7 Light: Wave-like or particle-like?

X١



#### **Chapter 8 Light and matter**



# Chapter 9 Einstein's special theory of relativity



HOW DO FIELDS EXPLAIN MOTION AND ELECTRICITY?

# CHAPTER 1

# NEWTON'S LAWS OF MOTION

#### Introduction

First published in 1687, Newton's laws of motion were a revolution in science. Before Newton's laws, people thought that a different set of principles applied here on Earth to those on the stars and planets. Newton's laws unified these ideas, stating that the set of principles that predict the motion of an apple dropping on Earth is the same as those that predict the motion of the stars and planets.

Although Newton's laws are limited when it comes to relativistic effects, they are still widely used today by engineers and scientists and they continue to be used to plan the trajectories of most artificial satellites and space probes. Newton's laws were used to plan the flight path of Apollo 11 that successfully landed humans on the Moon in 1969.

This chapter describes and applies Newton's laws to a number of different everyday situations. It then goes on to explore circular motion and projectile motion.

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#### Curriculum

#### Area of Study 1 Outcome 1 How do physicists explain motion in two dimensions?

Study Design	Learning intentions – at the end of this chapter I will be able to:	
<ul> <li>Newton's laws of motion</li> <li>Investigate and apply theoretically and practically Newton's three laws of motion in situations where two or more coplanar forces act along a straight line and in two dimensions</li> </ul>	<ul> <li>1A Newton's laws</li> <li>1A.1 Apply Newton's first law to draw free-body diagrams and determine if a net force is acting on a body</li> <li>1A.2 Apply Newton's second law, using the formula F<sub>net</sub> = ma to calculate the acceleration of a system or to calculate the force on different parts of the system</li> <li>1A.3 Apply Newton's third law to identify action-reaction force pairs using the convention 'force on A by B' or F<sub>on A by B</sub> = -F<sub>on B by A</sub></li> </ul>	
<ul> <li>Investigate and analyse theoretically and practically the uniform circular motion of an object moving in a horizontal plane:</li> <li><i>F</i><sub>net</sub> = <sup>mv<sup>2</sup></sup>/<sub>r</sub>, including:</li> <li>a vehicle moving around a circular road</li> <li>a vehicle moving around a banked track</li> <li>an object on the end of a string</li> <li>Model natural and artificial satellite motion as uniform circular motion (<i>See Chapter 3</i>)</li> <li>Investigate and apply theoretically Newton's second law to circular motion in a vertical plane (forces at the highest and lowest positions only)</li> </ul>	<ul> <li>1B Circular motion</li> <li>1B.1 Understand that when in uniform circular motion, the centripetal force and acceleration is directed to the centre of the circle and that the velocity is tangential to the centripetal force</li> <li>1B.2 Apply the formula F<sub>net</sub> = mv<sup>2</sup>/r and a = v<sup>2</sup>/r to solve questions relating to a vehicle moving around a flat circular section of a road and an object on the end of a string that is being swung in a horizontal circle</li> <li>1B.3 Draw a free-body diagram of all of the forces acting on a vehicle moving around a banked track and indicate the direction of the net force (centripetal force)</li> <li>1B.4 Be able to explain that the horizontal component of the normal force provides the centripetal force for a vehicle moving around a frictionless banked track</li> <li>1B.5 Apply the formula F<sub>net</sub> = mg tan θ to solve questions relating to circular motion on a smooth, banked track or road</li> <li>1B.6 Be able to draw free-body diagrams of objects moving in vertical circular motion and be able to identify the centripetal force</li> <li>1B.7 Be able to write equations that relate the normal/tension force and the gravity force to the centripetal force and apply these equations to solve problems relating to vertical circular motion</li> <li>1B.8 Solve quantitative and qualitative problems relating to vertical circular motion, including using the formulas v = 2πr/T and T = 1/f.</li> </ul>	

#### **Study Design**

- Investigate and analyse theoretically and practically the motion of projectiles near Earth's surface, including a qualitative description of the effects of air resistance
- Investigate and apply theoretically and practically the laws of energy and momentum conservation in isolated systems in one dimension

#### Learning intentions – at the end of this chapter I will be able to:

#### **1C Projectile motion**

- **1C.1** Describe the flight path of a projectile within Earth's gravitational field and describe the effect that air resistance will have on the projectile
- **1C.2** Solve projectile motion questions by considering the vertical and horizontal components of the projectile's velocity and using the equations of straight-line motion under constant acceleration using the formulas: v = u + at

 $v^2 = u^2 + 2as$ 

$$s = \frac{1}{2}(u+v)t = ut + \frac{1}{2}at^{2} = vt - \frac{1}{2}at^{2}$$

VCE Physics Study Design extracts © VCAA; reproduced by permission.

#### Glossary

Banked track Centripetal acceleration Centripetal force Free-body diagram Free fall Inertia Net force Normal force Projectile System Uniform circular motion

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#### **Concept map**

**First:** an object in a state of rest or travelling at a constant velocity will remain in its state of motion unless acted upon by an unbalanced force

#### Newton's laws of motion

**Second:** the acceleration experienced by a body is directly proportional to the net force on the body and inversely proportional to the mass of the body *Third:* every action force has an equal and opposite reaction force



See the Interactive Textbook for an interactive version of this concept map interlinked with all concept maps for the course.



### Newton's laws

#### Study Design:

 Investigate and apply theoretically and practically Newton's three laws of motion in situations where two or more coplanar forces act along a straight line and in two dimensions **Glossary:** 

Free-body diagram Free fall Inertia Net force Normal force System



#### ENGAGE Maglev trains

Forces can broadly be classified as either contact or non-contact. When the north poles of two magnets are aligned, a pushing force will occur even though the two magnets are not in contact. The non-contact force of magnets is utilised in the design of the fastest trains in the world, magnetic levitation (maglev) trains. A series of superconducting magnetics and an electromagnetic drive system allows maglev trains to float above the track. Maglev trains are able to achieve speeds of more than 400 km h<sup>-1</sup> and are quieter and less subject to vibration than normal locomotives.



**Figure 1A–1** Shanghai's maglev train departures for Pudong airport. This train transports passengers from Pudong International Airport to Shanghai's downtown area, a distance of about 50 km in just over 10 minutes.

A maglev train, like the one between Pudong International Airport and Shanghai in China, could transport passengers from Melbourne to Sydney in two hours. Currently, this is a ten-hour car trip and a one-hour aircraft flight.



#### EXPLAIN Newton's first law

Newton's first law states that an object in a state of rest or travelling at a constant velocity will remain in its state of motion unless acted upon by an unbalanced force.

Newton's first law is sometimes referred to as the law of **inertia**. For example, if a car is at rest on the ground, the **normal force** provided by the ground pushing up on the car balances the force due to gravity on the car. Since the forces on the stationary car are balanced, it remains at rest. If a car is moving on a straight road at a constant velocity, then the forces on the car will also be balanced. The force of air resistance and rolling resistance is balanced by the driving force provided by the road on the car.



**Figure 1A–2** The sum of the forces on a stationary car and a car moving at a constant velocity will be equal to zero. Note that the force due to gravity and the normal force are not action–reaction pairs.

It is counter-intuitive to our daily experience that an object will continue at a constant velocity as long as the sum of the forces acting on the object is zero. This is because, in our experience, objects will come to rest due to friction, as the force of friction causes a non-zero **net force** on objects, which directly opposes their state of motion.

#### Newton's second law

Newton's second law states that the acceleration experienced by a body is directly proportional to the net force on the body and inversely proportional to the mass of the body.

This law is often expressed in terms of the following formula.







Inertia a body's ability to resist a change in its state of motion. Inertia is dependent only upon the mass of the body.

Normal force

the force that a surface applies to a body in contact with it. The force is always applied perpendicular to the surface and prevents the body falling through the surface.

#### Net force

the vector sum of all of the forces acting on a body. The net force may also be referred to as the unbalanced force or the resultant force.

Free-body diagram a diagram that shows the relative magnitude and direction of all of the forces acting on a body When determining the net force on a body, it is often helpful to draw a **free-body diagram**, as shown in Figure 1A-2 on the previous page and in the left-hand diagram of Worked example 1A-1. The individual force vectors can be added to find the net force vector.

#### Worked example 1A–1 Calculating net force

Calculate the acceleration of a 66 000 kg aircraft travelling in a level horizontal flight that has a forward thrust of 350 kN and a backward frictional force of 300 kN.



#### Solution

The free-body diagram on the left and the vector diagram on the right show that the net force on the aircraft will be 50.0 kN forwards. The acceleration can then be determined:

$$a = \frac{F_{\text{net}}}{m} = \frac{50 \times 10^3}{66000} = 0.756 \text{ m s}^{-2} \text{ forwards}$$

Note that as the aircraft is travelling in a level horizontal flight, the gravitational force down and the lift force up are balanced.

## Θ

#### Worked example 1A–2 Calculating net force with a force at an angle

A 3 kg block is being pulled by a force of 30 N by a rope that makes an angle of 27° to the horizontal. A 20 N friction force acts. A diagram of this situation is shown below.



- a What is the magnitude of the net horizontal force on the block?
- **b** What is the magnitude of the horizontal acceleration of the block?

#### Solution

**a** The net horizontal force will be the sum of all of the horizontal forces on the block:  $30 \cos 27^{\circ} - 20 = 6.73 \text{ N}$ 

**b** 
$$a = \frac{F_{\text{net}}}{m} = \frac{6.73}{3} = 2.24 \text{ m s}^{-2}$$

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It is important to remember that when two or more bodies are moving with the same acceleration, the force on each of the bodies will not be the same unless the masses are the same. For example, if a 1 kg, 2 kg and 3 kg ball are all dropped from the same height, they will all accelerate towards the surface of Earth at the same rate:  $9.8 \text{ m s}^{-2}$  (if air resistance is ignored). However, to have the same acceleration, the forces on the balls must all be different.



**Figure 1A–3** At the surface of Earth, dropped balls all accelerate downwards at 9.8 m s<sup>-2</sup>. The gravitational force that the balls experience depends on their mass.

Another fundamental idea is that the only way to cause a body to accelerate is by applying a net force on the body. The body will then accelerate in the same direction as the direction of the net force. This means that the direction of the acceleration and the velocity might be different.

For example, if a car is approaching a red light and the car brakes, the car's velocity will be forwards but the acceleration, and therefore the net force on the car, will be backwards. Similarly, if you throw a ball vertically upwards, as soon as the ball leaves your hand, the ball will be accelerating down, as the only force acting on the projectile is the force due to gravity (if air resistance is ignored). The ball may be moving up, but its upwards velocity is constantly decreasing until the projectile is momentarily stationary at the maximum height, after which point the ball's velocity will be downwards and increasing in the same direction as the acceleration.

#### Inclined planes

When on a slope, a stationary object will have three forces acting on it: the force due to gravity, which acts vertically down, the normal force, N, which acts perpendicular to the surface and the friction force,  $F_{\rm f}$ , which acts parallel to the slope as shown in Figure 1A–4 on the next page. The magnitude of the normal force (Formula 1A–2) and the gravitational force down the slope (Formula 1A–3) can be calculated as follows.

#### Formula 1A–2 Normal force of an object on an inclined plane

 $N = mg \cos \theta$ 

Where:

N = Normal force on the object by the surface (N)

- m = Mass of the object (kg)
- g = Gravitational field strength close to the surface of Earth, 9.8 N kg<sup>-1</sup>
- $\theta$  = Angle of the slope (°)

Formula 1A-3 Gravitational force down the slope

$$F_{ds} = mg \sin \theta$$

Where:

- m = Mass of the object (kg)
- g = Gravitational field strength close to the surface of Earth, 9.8 N kg<sup>-1</sup>
- $F_{\rm ds}$  = Gravitational force component down the slope (N)
- $\theta$  = Angle of the slope (°)



If the friction force is less than the gravitational force parallel to the slope, then the object will accelerate down the slope. On a frictionless surface, the acceleration down the slope is given by Formula 1A–4.

#### Formula 1A-4 Acceleration on a frictionless slope

$$a = g \sin \theta$$

Where:

- a = Gravitational acceleration down the slope (m s<sup>-2</sup>)
- g = Gravitational field strength close to the surface of Earth, 9.8 N kg<sup>-1</sup> (or m s<sup>-2</sup>)
- $\theta$  = Angle of the slope (°)



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A 15 kg box slides down a slope inclined at 34°. The acceleration of the box is measured to be 0.397 m s<sup>-2</sup>.



- **a** Calculate the magnitude of the normal force acting on the box.
- **b** Calculate the gravitational force of the box parallel to the slope.
- **c** Assuming that the slope provides a constant frictional force that opposes the box's motion, calculate the magnitude of the constant frictional force acting on the box.

#### Solution

- **a**  $N = mg \cos \theta = (15)(9.8)(\cos 34) = 122$  N
- **b**  $F_{\rm ds} = mg \sin \theta = (15)(9.8)(\sin 34) = 82.2$  N
- c  $F_{\text{net}} = ma = (15)(0.397) = 5.955 \text{ N}$  $F_{\text{friction}} = F_{\text{ds}} - F_{\text{net}} = 82.2 - 5.955 = 76.2 \text{ N}$

#### Newton's third law

Newton's third law states that every action force has an equal and opposite reaction force.

When considering the action and reaction force pairs, it is important to remember that these two forces will always act on different bodies. For example, in Figure 1A–5 below, the fighter jet pushes out extremely hot gas from the exhausts, which is considered the action force. The reaction force is the hot gas pushing on the jet accelerating forwards.



**Figure 1A–5** The jets of a fighter jet push out extremely hot gas; the gas pushes back on the fighter jet and accelerates it forwards.

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Force on the road by tyre

Figure 1A-6 Newton's third law in action for a mountain bike

#### Check-in questions – Set 1

1 A ten-pin bowling ball is sliding to the right at a constant velocity. On the diagram, label all of the forces acting on the ball. Assume there is no friction acting on the ball.



- 2 A 3600 kg speedboat is propelled by a thrust force of 10800 N. The water and air provide a combined resistance of 3500 N. Calculate the acceleration of the speedboat.
- 3 If an aircraft is flying horizontally through the sky and the action force on the aircraft is the force due to gravity, then what is the reaction force?

#### Normal force

Imagine that you are free-falling close to Earth's surface. When you free fall, the force of gravity acts to accelerate you to the ground. The action force is the force due to gravity on you and the reaction force is an equal and opposite force pulling Earth up towards you.

Now, imagine that instead of falling you are standing on a tightrope. When standing on the tightrope you no longer accelerate towards the



Figure 1A-7 Earth acts to pull you, in free fall, towards it. The reaction force will be you pulling Earth up towards you. This is an example of Newton's third law.

ISBN 978-1-009-25901-9

Free fall when an object

is falling down and only the

force of gravity

acts on the object

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surface of Earth. You remain stationary because the forces on you are balanced. You apply a force onto the tightrope and the tightrope applies a normal force on you. This reaction force is referred to as the normal force and is often expressed as N or  $F_N$ . This means that there are two force pairs in this situation. The action force of the force due to gravity on you is paired with the reaction force of the force on Earth acting towards you; the action force that you apply on the tightrope is balanced by the reaction force of the tightrope applying a force on you.



**Figure 1A–8** As a person stands on the rope, the force on the person due to the gravitational attraction towards Earth is balanced with the force on Earth due to the gravitational attraction towards the person. The bottom diagram shows that as the person applies a force on the rope, the rope will apply a force on the person. The force on the person due to the rope is called the normal force.

Note that the force due to gravity and the two reaction forces are not an action–reaction pair, even though they are equal in magnitude and opposite in direction, as they act on the same body.

#### Vertical motion

As discussed on pages 12 and 13, your normal force is a result of how hard you are pushing on the ground. Your normal force is not a fixed value and can vary. Take the situation of being in a lift, when stationary or moving at a constant velocity your normal force is equal in magnitude to the force on you due to gravity. But if the lift accelerates up, you will feel heavier, as you have a greater normal force. If the lift accelerates down, you will feel lighter, as your normal force will decrease. If the cable of the lifts breaks, you will have no normal force at all, for a short time.





**Figure 1A–9** The two forces acting on a person in a lift that is accelerating up. As the lift accelerates up, the passenger of the lift will push down harder on the ground, the ground will then push harder back up, increasing the normal force. This increased normal force provides the net force necessary for the passenger to accelerate with the lift.

The normal force can be calculated using the following formula.

### Formula 1A–5 Normal force acting vertically

Where:

N = Normal force of the object by the surface of the lift (N)

m = Mass of the object (kg)

- $a_{\text{vertical}}$  = Acceleration in the vertical direction (m s<sup>-2</sup>)
  - g = Strength of the gravitational field, 9.8 N kg<sup>-1</sup> (or m s<sup>-2</sup>) on the surface of Earth

 $N = m(a_{\text{vertical}} + g)$ 

#### Worked example 1A-4 Normal force

A 72 kg person stands on a set of scales that measure the force they apply to the floor inside a lift. What is the force that the scales read when the lift is:

a at rest

- **b** moving at a constant velocity of 0.50 m  $\rm s^{-1}$  down
- c moving with an upward acceleration of 0.15  $m\,s^{-2}$
- **d** moving with a downward acceleration of 0.30 m s<sup>-2</sup>?

#### Solution

- **a**  $N = m(a_{\text{vertical}} + g) = (72)(0 + 9.8) = 706 \text{ N}$
- **b**  $N = m(a_{\text{vertical}} + g) = (72)(0 + 9.8) = 706 \text{ N}$
- **c**  $N = m(a_{\text{vertical}} + g) = (72)(0.15 + 9.8) = 716 \text{ N}$
- **d**  $N = m(a_{\text{vertical}} + g) = (72)(-0.3 + 9.8) = 684 \text{ N}$

#### Applying Newton's laws

Newton's laws can be applied to solve a variety of problems.

#### Worked example 1A–5 Applying Newton's laws to a block system

A 150 N force is used to push two blocks, A and B, on a frictionless surface. Block A has a mass of 5 kg and block B has a mass of 2 kg; when they are pushed both blocks move as one.



Frictionless surface

- **a** Calculate the acceleration of the two blocks.
- **b** What is the force on block B by block A?
- **c** What is the force on block A by block B?

#### Solution

a As the two blocks move as one, you need to add the masses together.

$$a = \frac{F_{\text{net}}}{m} = \frac{150}{5+2} = 21.4 \text{ m s}^{-2}$$

**b** As B is accelerating at 21.4 m s<sup>-2</sup>, you need to find the force that will accelerate it at this rate:

 $F_{\text{on B by A}} = ma = 2 \times 21.4 = 42.8 \text{ N to the right}$ 

**c** As every action has an equal and opposite reaction, the action force is the force that block A applies on block B. So, the reaction force will be the force that block B applied to block A. Therefore, the force is 42.8 N to the left.





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System

a collection of objects that interact with each other and are being studied

#### Worked example 1A-6 Applying Newton's laws to a pulley system

A 10 kg mass, initially at rest, lies on a frictionless table. It is attached to a 4.5 kg mass that hangs freely off the table, via a cable and frictionless pulley **system**.



- **a** What is the acceleration of the system?
- **b** Calculate the tension in the string.

#### Solution

**a** The first step is to determine the net force acting on the system. As the 4.5 kg mass is hanging freely, it will provide a net force that pulls the whole system. As the 4.5 kg mass falls, the pulley system will transfer the force to the 10 kg block, causing it to slide to the right.

Therefore:

 $F_{\rm net} = mg = 4.5 \times 9.8 = 44.1 \text{ N}$ 

Newton's second law can now be applied to calculate the acceleration of the system (note that as the whole system is being accelerated, the two masses are added):

$$a = \frac{F_{\text{net}}}{m} = \frac{44.1}{10 + 4.5} = 3.04 \text{ m s}^{-2}$$

**b** The tension in the string is a result of the string pulling on the 10 kg mass and accelerating it at  $3.04 \text{ m s}^{-2}$ . Therefore, the tension can be found by applying Newton's second law:

 $F_{\rm net} = ma = 10 \times 3.04 = 30.4 \text{ N}$ 

### lacksquare

#### Worked example 1A–7 Applying Newton's laws to a carriage system

An engine with a mass of 19000 kg is towing two carriages, each with a mass of 15000 kg. The engine is connected to both of the carriages by coupling A and the two carriages are connected by coupling B. As the engine leaves the station, it produces a driving force of 48 kN. Assume there is no rolling or air friction in this system. A diagram of this situation is shown below.



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- a Calculate the acceleration of the engine and carriage system.
- **b** Calculate the tension in coupling A.
- c Calculate the tension in coupling B.

#### Solution

**a** First, convert the units to SI units:  $48 \times 10^3$  N As the engine is towing both of the carriages, they are all part of the same system and will all accelerate at the same rate. This means that the masses can be added together to find the magnitude of the acceleration:

$$a = \frac{F_{\text{net}}}{m} = \frac{48 \times 10^3}{15\,000 + 15\,000 + 19\,000} = 0.980 \text{ m s}^{-2}$$

**b** Coupling A is towing two carriages. This means it is subject to the tension required to accelerate both carriages at  $0.980 \text{ m s}^{-2}$ :

 $F = ma = (15000 + 15000) \times 0.980 = 2.94 \times 10^4$  N

**c** Coupling B is towing one carriage. This means it is subject to the tension required to accelerate one carriage at 0.980 m s<sup>-2</sup>:

 $F = ma = 15000 \times 0.980 = 1.47 \times 10^4$  N

#### **1A SKILLS**

#### Determining the direction of a vector

In any question that involves a vector quantity, the direction of the vector must be considered and incorporated into any equation that you use. Direction can be incorporated into an equation with a positive and negative sign representing different directions. This means that whenever you encounter a problem involving vectors, one of the first things you must do is determine which direction will be positive and which direction will be negative. It does not matter which direction you make negative or positive; for example down can be negative and up can be positive or down can be positive and up can be negative. However, once you have decided which direction represents positive and negative, you must remain consistent throughout the question. When you have made this decision, write it down on the page as this will allow you to keep track of the meaning of a positive or negative answer.

#### Question

A model rocket launches up in the air and experiences a thrust force of 650 N and a constant drag force of 225 N. Calculate the net force acting on the rocket.

#### Solution

Determine which direction is positive:

positive direction = up

Add the vectors:

 $F_{\rm net} = 650 + (-225) = 425 \text{ N up}$ 

Note that the positive answer gave the direction: up.

You could also let up be negative and the solution would still be the same:

 $F_{\text{net}} = (-650) + 225 = -425 \text{ N}$ 

The negative indicates the direction: up.



#### Section 1A questions

#### **Multiple-choice questions**

1 A tennis ball is travelling through the air in a direction up and to the right, close to the surface of Earth. Assuming that air resistance is negligible, which of the diagrams below represents all of the forces acting on the ball?



2 A truck has four times the mass of a car, both vehicles are moving at  $10 \text{ m s}^{-1}$  to the right. Both see a red traffic light ahead and use the 50 m ahead to brake. Both the car and the truck have a constant deceleration and both come to a stop after 2 seconds. Which of the following statements about the acceleration of the truck and the car is correct?



- A The truck's deceleration is four times greater than the car's.
- **B** The car's deceleration is four times greater than the truck's.
- **C** The truck's deceleration is two times greater than the car's.
- **D** The truck and the car have equal deceleration.
- **3** An ultralight aeroplane of mass 500 kg flies in a horizontal straight line at a constant speed of 100 m s<sup>-1</sup>.

The horizontal resistance force acting on the aeroplane is 1500 N.

Which one of the following best describes the magnitude of the forward horizontal thrust on the aeroplane?

- A 1500 N
- **B** slightly less than 1500 N
- **C** slightly more than 1500 N
- **D** 5000 N

VCAA 2019

#### Use the following information to answer Questions 4 and 5.

A truck has a 0.25 kg ball attached to a string of negligible mass, the string is fixed to the roof of a truck and free to swing. When the truck is at rest, the string is vertical. When the truck accelerates uniformly, a passenger on board the truck sees the string make a 24° angle with the vertical, as shown in the diagram below.



**4** Which of the diagrams best represents all of the forces acting on the ball when the truck is accelerating?



- **5** What is the acceleration of the truck?
  - **A** 9.48  $m s^{-2}$
  - **B**  $4.36 \text{ m s}^{-2}$
  - $C 2.45 m s^{-2}$
  - **D**  $0.045 \text{ m} \text{ s}^{-2}$
- 6 A heavy ball is hanging from the ceiling by a string that has negligible mass. If the action force is the force due to gravity of the ball, what is the reaction force?



- **A** the tension force provided by the string
- **B** the ball pulling Earth up
- **C** the normal force on the ball
- **D** a compression force within the ball

#### Short-answer questions

- 7 A 200 kg object has a 50 N net force being applied to it. What is the magnitude of the acceleration of the object?
- 8 A 650 kg object is accelerating to the right at  $28 \text{ cm s}^{-2}$ . What is the magnitude of the net force acting on it (in N)?
- **9** A car is moving at a constant velocity, when suddenly the driver sees some debris on the road and brakes hard. The driver observes that a tennis ball in the passenger seat moves forwards until it hits the front end of the passenger compartment. Use your knowledge of Newton's laws of motion to explain this observation.
- 10 A student conducted an experiment in which two heavy objects were dropped from the first story of a building. Assume air resistance is negligible. One of these objects has a mass that is five times greater than the other. The student states that the gravitational force on each object must be the same as both objects hit the ground at the same time. Is this explanation correct? Justify your answer.
- 11 A piston pushes three blocks, A, B and C, to the right. On the diagram below, draw all of the forces acting on B. Make sure that they are all clearly labelled.



12 A locomotive that has a mass of  $35\,000$  kg can produce a driving force of  $2.50 \times 10^4$  N. It is pulling a train consisting of three empty carriages, each with a mass of 1500 kg, towards a mine. The train is connected to the carriages by a coupling, X. Couplings Y and Z hold the carriages together. A diagram of this situation is shown below (friction forces are considered to be negligible).



- a Calculate the acceleration of the locomotive.
- **b** Calculate the tension in coupling X during acceleration.
- **c** When the train reaches the coal mine, the three carriages are filled with equal masses of rock. The train then accelerates from rest at  $0.540 \text{ m s}^{-2}$ .



Calculate the mass of rock added to each carriage, given that the driving force of the train remains unchanged.

**d** Calculate the tension in coupling Y during acceleration when the carriages are filled with rock.

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Students set up an experiment as shown in the diagram below.
 M<sub>1</sub>, of mass 4.0 kg, is connected by a light string (assume it has no mass) to a hanging mass, M<sub>2</sub>, of 1.0 kg.

The system is initially at rest. Ignore mass of string and friction.



The masses are released from rest.

- **a** Calculate the acceleration of M<sub>1</sub>.
- **b** Calculate the magnitude of the tension in the string as the masses accelerate.

VCAA 2015

14 Two blocks, A and B, are being pushed with a force of 56 N to the right. Block A has a mass of 3 kg and the two blocks move to the right with an acceleration of  $4 \text{ m s}^{-2}$ . Assume the surface is frictionless.



- a Calculate the mass of block B.
- **b** What is the force that block A exerts on block B?
- c What is the force that block B exerts on block A?
- 15 A mother is having difficulty moving her pram on the soft sand of a beach. Considering the components of the forces involved, is it easier to push or pull the pram? Explain your answer.





### **Circular motion**

#### **Study Design:**

- Investigate and analyse theoretically and practically the uniform circular motion of an object moving in a horizontal plane:  $F_{\text{net}} = \frac{mv^2}{r}$ , including:
  - a vehicle moving around a circular road
  - a vehicle moving around a banked track
  - an object on the end of a string
- Model natural and artificial satellite motion as uniform circular motion (see Chapter 3)
- Investigate and apply theoretically Newton's second law to circular motion in a vertical plane (forces at the highest and lowest positions only)

#### **Glossary:**

Banked track Centripetal acceleration Centripetal force Uniform circular motion



#### ENGAGE

#### You couldn't live without a centrifuge

All centrifuges work on the same basic principle; as they spin, the denser matter is forced away from the axis of spin and the less dense matter is displaced closer to the axis of spin. This means that centrifuges are very effective at separating matter that has varying densities.

Currently, there is a range of different centrifuges. Centrifuges are used in medicine for blood and urine to detect pathologies such as anaemia, bone marrow failure and leukaemia. Centrifuges are also used to purify chemicals; for example, gas centrifuges are used for enriching uranium by separating uranium-235 from uranium-238. Ultrahigh-speed centrifuges rotate at 150 000 revolutions per minute (RPM) and are used extensively in biochemistry and molecular chemistry as they have the ability to separate proteins, ribosomes and viruses.



**Figure 1B–1** The centrifuge above is being used to analyse blood tests. The specimen tubes are placed in holders that position the tubes at an angle to the axis of spin, or even at 90° to it, with the top closer to the axis and the bottom further away. During centrifugation, the denser red blood cells settle at the bottom of the tube, while the less dense plasma rises to the top.
Large centrifuges are used to train pilots and astronauts by simulating the high accelerations they encounter when in flight. In the future, large centrifuges may be used on donut-shaped spacecrafts that spin to produce an artificial gravity.



**Figure 1B–2** In the future, spacecraft might use a spinning donut shape to produce artificial gravity.



### EXPLAIN Horizontal circular motion

**Uniform circular motion** is the movement of an object in a circular path at constant speed. While in circular motion, the magnitude of an object's velocity (its speed) is not changing but the direction of the velocity is constantly changing. A change in the velocity vector means that there must be an acceleration and therefore a net force acting on the object. This acceleration and net force are called the **centripetal acceleration** and **centripetal force** respectively; they are always perpendicular to the velocity vector

and always directed towards the centre of the circular path. The direction of the velocity will always be tangential to the circular path.

The centripetal force is only present if there is an unbalanced force on the object that is directed towards the centre of the object's circular path. This unbalanced force can be provided by a single force, such as when a satellite orbits Earth. In this case, the force due to gravity of the satellite acts to provide the centripetal force. The unbalanced force can also be the vector sum of other forces, such as when a car on a roller-coaster goes through a loop. In this case, the centripetal force is provided by the sum of the normal force and the force due to gravity.



**Figure 1B–3** Centripetal acceleration and force directed towards the centre of the object's circular path. The velocity vector is always perpendicular to the centripetal force.

#### Uniform circular motion

the movement of an object in a circular path at constant speed

### Centripetal

acceleration acceleration directed to the centre of the circular path of an object

#### **Centripetal force**

a net force directed to the centre of the circular path of an object In all cases, the centripetal acceleration required to maintain circular motion can be calculated by Formula 1B–1.

 $a_{\rm c} = \frac{v^2}{r}$ 

Where:

 $a_c$  = Acceleration of the object in circular motion (m s<sup>-2</sup>)

v = Velocity of the object in circular motion (m s<sup>-1</sup>)

r =Radius of the circle (m)

Using Newton's second law, F = ma, it follows that:

Formula 1B–2 Net force on an object in circular motion

$$F_{\rm c} = \frac{mv^2}{r}$$

Where:

 $F_c$  = Net force on the object in circular motion (N)

m = Mass of the object in circular motion (kg)

v = Velocity of the object in circular motion (m s<sup>-1</sup>)

r =Radius of the circle (m)

### Horizontal circular motion of car turning a corner

A car turns a corner because the wheels that steer it turn in the direction of the curve. This means that a component of the friction force between the tyres and the road acts towards the centre of the circular path that the car is taking. So, this component of friction is providing the centripetal force.



**Figure 1B–4** When cars turn around a corner that is on level ground, a component of the friction between the tyres and the road provides the centripetal force.

VIDEO 1B-1

VERTICAL CIRCULAR

MOTION

Horizontal circular motion problems are the most basic circular motion problems and the formulas  $a_c = \frac{v^2}{r}$  and  $F_c = \frac{mv^2}{r}$  should be used.

### Worked example 1B–1 Horizontal circular motion

A 1000 kg car is travelling at a speed of 32.4 km h<sup>-1</sup> when it goes around a roundabout that has a radius of 3.56 m. Calculate the centripetal force required to keep the car in circular motion.

### Solution

The first step is to convert the speed from  $\text{km}\,\text{h}^{-1}$  to  $\text{m}\,\text{s}^{-1}$ .

$$\frac{32.4}{3.6} = 9.00 \text{ m s}^-$$

Then use the centripetal force formula to solve the problem.

$$F_{\rm c} = \frac{mv^2}{r}$$
$$= \frac{1000 \times 9^2}{3.56} = 2.28 \times 10^4 \text{ N}$$

### Check-in questions – Set 1

- 1 When an object is in uniform circular motion, what is the direction of the net force?
- **2** When an object is in uniform circular motion, what is the direction of the velocity?
- **3** A 1300 kg car moves in a horizontal circle that has a radius of 30 m. If the centripetal force is  $9.75 \times 10^3$  N, calculate the velocity of the car.

### Vertical circular motion

When an object is moving in a vertical circle, it will always have at least two forces acting on it. These two forces are the force due to gravity and either a normal force or a tension force, if the object is attached to a string. Remember that the centripetal force is provided by the sum of all of the forces acting on the object in circular motion.

### Vertical circular motion: an object on the end of a string

Imagine you tie a ball to a string and swing it in a vertical circle. Where along the circular path do you think the string will most likely break? What will happen if you swing the ball too slowly? When you swing the ball, there are two main points of interest, the top and the bottom of the circle. At the bottom of



**Figure 1B–5** A fire dancer traces out vertical circles with two poi. Poi consist of two weighted objects that are swung in circular patterns. Poi originated from the indigenous Maori people of New Zealand.

the circle, the tension acts towards the centre of the circle but gravity acts in the opposite direction to the centre of the circle. At the top of the circle, both the tension of the string

and gravity act in the same direction; towards the centre of the circle.

In both of these situations, a formula for the centripetal force can be derived by adding the two forces acting on the ball. Note that it is helpful to denote any force that acts towards the centre of the circle as **positive** and any force pointing away from the centre of the circle as **negative**. Therefore, the formulas in Formula 1B-4 can be derived.



Figure 1B–6 A free-body diagram showing the forces acting on a mass that is attached to a string when it is at the bottom and top of a vertical circle

### Formula 1B–3 Centripetal force on an object on the end of a string

At the bottom of the circle:

$$\frac{F_{\rm c}}{r} = T - mg$$
$$\frac{mv^2}{r} = T - mg$$

At the top of the circle:

$$F_{c} = T + mg$$
$$\frac{mv^{2}}{r} = T + mg$$

Where:

- $F_c$  = Centripetal force on the object on the end of a string (N)
- T = Tension in the string (N)
- g = Strength of the gravitational field, 9.8 N kg<sup>-1</sup> on the surface of Earth
- m = Mass of the object on the end of a string (kg)
- $\nu$  = Velocity of the object on the end of a string (m s<sup>-1</sup>)
- r = Radius of the circle (m)

### Worked example 1B–2 Vertical circular motion

A 1.27 kg ball is attached to a 0.511 m string and swung in a vertical circle.

- **a** If the ball is swung at a constant speed of  $4 \text{ m s}^{-1}$ , calculate the tension in the string at the bottom of the circle.
- **b** Calculate the speed that the ball would need to be swung so that at the top of the circle there is no tension in the string.

Solution

**a** 
$$F_c = T - mg$$
  
 $T = \frac{mv^2}{r} + mg$   
 $= \frac{(1.27)(4)^2}{0.511} + (1.27)(9.8) = 52.2 \text{ N}$ 

b T = 0, therefore:  $F_c = 0 + mg$  $\frac{mv^2}{r} = mg$  $v = \sqrt{rg}$  $= \sqrt{(0.511)(9.8)} = 2.24 \text{ m s}^{-1}$ 

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### Why are roller-coasters teardrop shaped?

In the past, roller-coaster designers used perfectly circular loops, which required carts to enter them at speeds that were too high, involved too much force and resulted in injuries for riders. The teardrop shape has a much smaller radius at the top than at the bottom. This allows roller-coaster cars to enter the loops of equal height at a much lower speed, providing a safe, fun ride.



Figure 1B–7 This image shows the typical teardrop shape of modern roller-coaster design.

### Vertical circular motion: a vehicle moving on a hilly road

Have you ever been in a car when driving over a bump and for a moment felt like you have almost lifted off your seat? This feeling is a result of your normal force decreasing and is ultimately a result of circular motion. When a car moves over a bump or through a dip in the road, the car is moving in vertical circular motion. In both cases the car has two forces acting on it: the force due to gravity and the normal force provided by the road. Figure 1B–8 demonstrates that when a car moves over a bump in the road, the force due to gravity is directed towards the centre of the circle, while the normal force is directed away from the centre of the circle. When the car moves through a dip in the road, the force due to gravity is directed away from the centre of the circle, while the normal force is directed to the centre of the circle.



Figure 1B–8 A free-body diagram showing a car moving over a bump in the road and through a dip in the road



From the free-body diagrams, the formulas for the centripetal force can be derived, as shown in Formula 1B-4.

Formula 1B–4 Centripetal force on a vehicle on a hilly road

At the top of a bump in the road:

$$F_{c} = mg - N$$
$$\frac{mv^{2}}{r} = mg - N$$

At the bottom of a dip in the road:

$$F_{\rm c} = \rm N - mg$$
$$\frac{mv^2}{r} = \rm N - mg$$

Where:

 $F_c$  = Centripetal force (N)

N = Normal force (N)

g = Strength of the gravitational field, 9.8 N kg<sup>-1</sup> on the surface of Earth

m = Mass of the vehicle (kg)

 $\nu$  = Velocity of the vehicle (m s<sup>-1</sup>)

r = Radius of the circle (m)



### Worked example 1B–3 Car moving over a bump in the road

A 1250 kg car is travelling over a bump that has a circular radius of 15.0 m.

- **a** If the car travels over the bump at a speed of  $36 \text{ km h}^{-1}$ , calculate the centripetal force required to keep the car in circular motion.
- **b** Calculate the speed that the car would need to go over the bump for the occupants to experience no normal force.
- **c** Once over the bump, the car continues to travel down the road at  $97.2 \text{ km h}^{-1}$  and goes through a dip in the road that has a circular radius of 25.0 m. Calculate the normal force on the car as it travels through the dip.

### Solution

**a** 
$$v = \frac{36}{3.6} = 10 \text{ ms}^{-1}$$
  
 $F_c = \frac{mv^2}{r}$   
 $= \frac{(1250)(10)^2}{15} = 8.33 \times 10^3 \text{ N}$   
**b** No normal force means that  $N = 0$ :  
 $\frac{mv^2}{15} = mg - 0$   
**c**  $v = \frac{97.2}{3.6} = 27 \text{ ms}^{-1}$   
 $\frac{mv^2}{r} = N - mg$   
 $N = \frac{mv^2}{r} + mg$   
 $= \frac{(1250)(27)^2}{15} + (1250)(9.8)$   
 $= 4.30 \times 10^4 \text{ N}$ 

$$\frac{mv^2}{r} = mg - 0$$
$$v = \sqrt{rg}$$
$$= \sqrt{(15)(9.8)} = 12.1 \text{ ms}^{-1}$$

### Check-in questions – Set 2

1 A 2000 kg car travels over a hill that has a circular radius of 90 m.



- **a** Calculate the velocity that the car would need to travel so that the road provides no normal force to the car at the top of the hill.
- **b** If the car was travelling at 90 km  $h^{-1}$ , what would be the normal force on the car at the top of the hill?
- 2 During an air show, a 9100 kg fighter jet, travelling with a constant speed of 230 m s<sup>-1</sup>, does a circular loop as shown below.



If the centripetal force on the fighter jet is  $3.21 \times 10^6$  N as it performs the loop, calculate the:

- **a** radius of the loop
- **b** force that the air applies on the jet at the top of the loop.

### A vehicle moving around a banked track

Imagine a car going around a corner at a speed of  $40 \text{ km h}^{-1}$ ; it makes the turn with no trouble. What will happen if the same car was driving along a highway and going around the same corner at  $60 \text{ km h}^{-1}$ ,  $80 \text{ km h}^{-1}$  or even  $100 \text{ km h}^{-1}$ ? Some corners have a circular radius that is too small given the speed that vehicles are expected to go around them. In these cases, the corners may be banked. This means that the horizontal component of the normal force acts to help provide the centripetal force. Without this banking, friction may not be large enough to provide sufficient centripetal force and the car might begin to skid off the road. Banking is used on roads, in cycling velodromes and even on some indoor running tracks.



Figure 1B-9 Banked tracks are used in both cycling velodromes (left) and racetracks (right)

Banked track a track that has been built with a transverse incline so that the horizontal component of the vehicle's normal force can contribute to the centripetal force When a vehicle is moving on a **banked track** at just the right speed, where there is no tendency to slide up or down the banking so no sideways friction is involved, there are two forces acting on the vehicle; the force on the vehicle due to gravity and the normal force. Figure 1B–10 illustrates the free-body diagram of a vehicle moving on a banked track.

As shown in the free-body diagram in Figure 1B–10, the normal force always acts perpendicular to the surface. The normal force has also been broken up into its horizontal and vertical components. As the car is not accelerating in the vertical direction, the vertical forces must balance. This means that the vertical component of the normal force must





be equal to the magnitude of the force due to gravity. The horizontal component of the normal force is directed to the centre of the circle. It is not balanced by any other force and therefore is providing the centripetal force.

As the vectors form a rightangled triangle, the formula for the centripetal force, Formula 1B–5, can be derived:

$$F_{c} = mg \tan \theta$$
$$\frac{mv^{2}}{r} = mg \tan \theta$$

30

Formula 1B–5 Centripetal force on a vehicle on a banked track

$$F_c = mg \tan \theta$$

Where:

 $F_c$  = Centripetal force (N)

m = Mass of the object in circular motion (kg)

g = Strength of the gravitational field, 9.8 N kg<sup>-1</sup> on the surface of Earth

 $\theta$  = Angle of the banked slope (°)

Formula 1B–6 Centripetal acceleration on a vehicle on a banked track

$$u_{\rm c} = \frac{v^2}{r} = g \tan \theta$$

Where:

tere:  
$$a_c = \text{Centripetal acceleration (m s^{-2})}$$

a

 $\nu$  = Velocity of the object on the banked curve (m s<sup>-1</sup>)

r =Radius of the circular path (m)

- g = Strength of the gravitational field, 9.8 N kg<sup>-1</sup> on the surface of Earth
- $\theta$  = Angle of the banked slope (°)

### Worked example 1B–4 Banked turn

A 4500 kg truck travels on a banked track angled at 30° to the horizontal. The track has a circular radius of 75 m.

- a Explain how the banking allows the truck to maintain circular motion around the bend without relying on the friction force.
- **b** If the truck is travelling at 50.4 km  $h^{-1}$ , calculate the centripetal force required to keep the truck in circular motion.
- **c** Calculate the maximum speed that the truck can go around the banked track without relying on sideways friction between the tyres and the road surface.

### Solution

**a** When the track is banked correctly the horizontal component of the truck's normal force acts to provide all of the centripetal force. This means the truck is no longer relying on the sideways friction between the tyres and the road to remain in the circular path of the road.

**b** 
$$v = \frac{50}{3.6} = 14 \text{ m s}^{-1}$$
  
 $F_c = \frac{mv^2}{r} = \frac{(4500)(14)^2}{75} = 1.18 \times 10^4 \text{ N}$   
**c**  $v = \sqrt{rg \tan \theta} = \sqrt{(75)(9.8) \tan 30} = 20.6 \text{ m s}^{-1}$ 

### An object on the end of a string

When the ends of a string are attached to a free object and a fixed object, the free object can move in horizontal circular motion. Figure 1B–11 shows a string that is attached to a ball and a fixed beam; the free ball is able to undergo uniform circular motion in a horizontal plane.

-1

#### CHAPTER 1 NEWTON'S LAWS OF MOTION



There are two forces that act on the ball: the force on the ball due to gravity and the tension force of the string attached to the ball. As the tension force acts at an angle, it can be broken into vertical and horizontal components. The vertical component must be equal and opposite to the force due to gravity as the ball is not accelerating vertically. The horizontal component provides the centripetal force. Breaking down the tension force in this way will allow you to solve these problems.



**Figure 1B–11** A free-body diagram of a ball attached to a string that is moving in a horizontal circle with radius, *r* 

### lacksquare

### Worked example 1B–5 Circular motion of an object on a string

A 4.83 kg ball is attached to a fixed steel beam via a rope. The ball is spun in a horizontal circle of radius, *r*, such that the rope makes an angle of 32° with the beam. A diagram of this situation is shown below.



- **a** Calculate the tension in the rope.
- **b** What is magnitude of the net force on the ball?

#### Solution

**a** First, break the tension into its component vectors. Remember the vertical component of the tension force is equal to the force due to gravity as the ball has no motion in the vertical, indicating the forces are balanced.



From this right-angled triangle, it is clear that:

$$T = \frac{mg}{\sin 32} = \frac{(4.83)(9.8)}{\sin 32} = 89.3$$
 N

**b** The net force,  $F_c$ , from the triangle above:

$$F_{\rm c} = \frac{mg}{\tan 32} = \frac{(4.83)(9.8)}{\tan 32} = 75.8 \text{ N}$$

32

### **1B SKILLS**

## How to derive an equation for the centripetal force for bodies in vertical circular motion

One of the fundamental ideas of centripetal motion is that the centripetal force is a result of other forces, it is the net force that acts on a body. In order to derive your own equations for the centripetal force, you must first draw a diagram and consider all of the forces acting on the body in centripetal motion. Take the situation of a cart on a roller-coaster doing a loop, the diagram below shows all of the forces acting on the cart.



Once you have drawn a diagram showing all of the forces acting on the body, you can derive the equation for the centripetal force by adding any forces that are directed towards the centre of the circle and subtracting any forces that point away from the centre. This means that the equations for a cart doing a loop would be:

bottom of the loop:  $F_{\rm c} = N - F_{\rm g}$ top of the loop:  $F_{\rm c} = N + F_{\rm g}$ 

### Section 1B questions

### **Multiple-choice questions**

1 A ball is being swung anticlockwise in a vertical circle as shown in the diagram. If the string that holds the ball breaks at the position shown, determine the direction that the ball will travel the instant the string breaks.



- **2** A car is travelling around a horizontal circular track at a constant velocity and requires *F* newtons of centripetal force. If the velocity of the car doubles, what happens to the centripetal force required to keep the car in circular motion?
  - $A \quad \frac{F}{2}$  $B \quad 2F$  $C \quad \frac{F}{4}$  $D \quad 4F$



### Use the following information to answer Questions 3, 4 and 5.

A truck travels around a horizontal circular road that has a radius of 75 m at a constant speed of 15 m s<sup>-1</sup>. A diagram of this situation is shown on a portion of the road.



- **3** Which of the following is the magnitude of the acceleration of the truck?
  - **A**  $3 \text{ m s}^{-2}$
  - **B**  $1.5 \text{ m s}^{-2}$
  - $C 0.2 \text{ m s}^{-2}$
  - $\textbf{D} \quad 0 \ m \ s^{-2}$

### 4 Which direction is the closest to the direction of the acceleration of the truck?

- A A
- **B** B
- **C** C
- D D
- 5 An observer standing directly behind the truck, observes that a ball, hanging by a wire fixed to the roof of the truck, is hanging at an angle of *θ* to the vertical. What is the angle *θ*?
  - **A** 73.0°
  - **B** 34.0°
  - **C** 17.0°
  - **D** 10.0°



6 A toy truck travels on a track around a vertical loop of radius 1.6 m, as shown below. Assume that the toy truck is a point mass.



The minimum speed at which the toy truck must be moving at point X for it to stay on the track is closest to

- **A**  $1.6 \text{ m s}^{-1}$
- **B**  $3.2 \text{ m s}^{-1}$
- **C**  $4.0 \text{ m s}^{-1}$
- **D**  $16 \text{ m s}^{-1}$

### VCAA NHT 2019

### **Short-answer questions**

7 A 950 kg vehicle drives through a valley at a constant velocity of 79.2 km h<sup>-1</sup>. The valley has a circular radius of 40 m.



- **a** On the diagram provided, draw the two main forces acting on the car, given that air resistance is ignored.
- **b** Calculate the magnitude of the normal force on the car at the bottom of the valley.
- 8 An engineer is constructing a curved road with a radius of 75 m for cars to join a highway. The cars must be able to travel on the ramp at 72 km h<sup>-1</sup> without relying on sideways friction to help them turn. Determine the banking angle required.

- 9 A 3 kg ball is tied to a 52.1 cm string and then spun in a vertical circle.
  - **a** What is the speed that the ball has to achieve at the top of the circle so that the string has no tension at that point?
  - **b** The string will break when the tension is greater than 100 N. What is the greatest constant speed that the ball can be spun without the string breaking?
- 10 A group of students are taking turns hitting a ball that is tied to a pole. The ball has a mass of 464 g and is attached to a 1.5 m string. On a certain strike, the ball moves around in a horizontal circle and makes an angle of 30° to the vertical pole.



- a Calculate the radius of the horizontal circle.
- **b** What is the velocity of the ball?
- c Calculate the tension in the string.
- d Calculate the magnitude of the net force on the ball.
- 11 Members of the public can now pay to take so-called 'zero gravity' flights in specially modified jet aeroplanes that fly at an altitude of 8000 m above Earth's surface. A typical trajectory is shown in the diagram below. At the top of the flight, the trajectory can be modelled as an arc of a circle.



- a Calculate the radius of the arc that would give passengers a 'zero gravity' experience at the top of the flight if the jet is travelling at 180 m s<sup>-1</sup>. Show your working.
- **b** Is the force of gravity on a passenger zero at the top of the flight? Explain what 'zero gravity experience' means.

12 A 1750 kg car is moving around a circular banked track at 68.4 km h<sup>-1</sup>. The banking angle on the track is 24°. The car is travelling around the banked corner at exactly the speed designed for the corner and therefore does not rely on friction forces acting up or down the slope.



- a On the diagram provided, draw all of the forces acting on the car.
- **b** Calculate the circular radius of the banked track given that the car does not rely on friction when travelling at  $68.4 \text{ km h}^{-1}$ .
- **c** Explain how the car is able to move around the circular banked track without relying on friction.





### **Projectile motion**

### **Study Design:**

- Investigate and analyse theoretically and practically the motion of projectiles near Earth's surface, including a qualitative description of the effects of air resistance
- Investigate and apply theoretically and practically the laws of energy and momentum conservation in isolated systems in one dimension

**Glossary:** Projectile

### 0°

### **ENGAGE** Projectile motion in golf

As golfers approach the hole, they must consider the club that they will use and how that club will determine the motion of the ball. All golf clubs have heads that slope back slightly from the vertical. Different golf clubs have different sloping angles of the head.



Figure 1C-1 Golf clubs have heads at different sloping angles, depending on their purpose.

The angle of the head has two important functions. First, the ball will be launched perpendicular to the face of the club. This means that the greater the slope of the head, the higher and shorter the ball will travel. This is important when a golfer wants to clear an object or prevent the ball from rolling too far.



**Figure 1C–2** A golfer is using a club with a head that has a large slope angle to send the ball high and not too far. This means the ball will clear the sand bunker and will not roll too far away from the hole.

The second important function of the head's slope is the spin that it gives the ball. The greater the slope on the head, the greater the spin will be on the ball. Spin on a golf ball is essential as it provides lift and means that the flight path off the ball is less affected by wind, making the shot more reliable.

### EXPLAIN Projectile motion

A **projectile** is a body that is launched into the air and moves freely through space, close enough to the surface of Earth so that the gravity is constant. When air resistance is ignored, a projectile will follow a parabolic path. It is a common misconception that when a ball is moving upwards, the force on the ball is also upward. This is not the case. Projectile questions will give you an initial velocity of a projectile and will not consider how the projectile reached this velocity. For example, if you throw a ball, you are accelerating the ball when the ball is in contact with your hand; the moment the ball leaves your hand, the only force acting on the ball is the force due to gravity, which is downwards and constant. It is important to remember that all projectiles will have a constant downwards acceleration due to gravity throughout their flight. Air resistance is usually ignored. However, if it is not ignored, the air resistance force will always directly oppose the motion of the projectile.

When solving projectile motion problems, it is essential to always consider the horizontal and vertical motions separately. When air resistance is ignored, the horizontal component of the projectile's velocity will always remain constant. As the projectile is always being accelerated down by gravity, the vertical component of the projectile's velocity will be constantly changing.





Projectile

a body that is launched into the air and moves freely through space, close enough to the surface of Earth so that the only force acting on the body is a constant gravitational force







When answering projectile motion questions, remember the following points:

- Draw a diagram.
- Decide if the up direction will be positive or negative and stay consistent with this decision throughout the question.
- Always treat the horizontal and vertical components of the projectile's velocity separately.
- The total time can be used to find the horizontal distance travelled by  $d = v_{\rm h} t$ .
- When considering the vertical motion of the projectile, remember that there is a constant acceleration of 9.8 m s<sup>-2</sup> downwards due to gravity.
- When calculating the final velocity of a projectile, the horizontal and vertical velocity vectors need to be added together and a direction needs to be given.
- If you need to consider the qualitative effects of air resistance, remember that air resistance directly opposes the projectile's motion. So, the projectile will not reach as high or travel as far.

### Worked example 1C-1 Projectile motion of a projectile launched at an angle

A projectile launcher launches a projectile from ground level at  $37 \text{ m s}^{-1}$ , at an angle of  $33^{\circ}$  to the horizontal. Ignore the effects of air resistance. A diagram of this situation is shown.



- a Calculate the time of flight for the projectile, given that it returns to ground level.
- **b** Calculate the horizontal distance travelled by the projectile.

### Solution

**a** First, break the initial velocity vector into its initial horizontal and vertical components:

$$u_{\rm h} = 37 \cos 33^\circ$$

 $u_{\rm v} = 37 \sin 33^{\circ}$ 

The time taken for the projectile to reach its maximum height will be half of its flight time. In addition, the vertical component of velocity at the maximum height will be zero, therefore:

$$v_v = u_v + at$$
  
 $t = \frac{v_v - u_v}{a}$   
 $= \frac{0 - 37 \sin 33}{-9.8} = 2.06 \text{ s}$ 

total flight time =  $2.06 \times 2 = 4.12$  s

**b** As there is no air resistance, the horizontal component will not change throughout the projectile's flight. This means the horizontal distance travelled can be found by multiplying the total flight time by the horizontal velocity:

$$d = u_{\rm h} t$$
  
= (37 cos 33)(4.12) = 128

m

40

## Worked example 1C–2 Projectile motion of a projectile launched horizontally off a height

A cannonball is launched from a cannon at 40 m s<sup>-1</sup> horizontally. The cannon is set up 25 m above the sea.



- **a** Ignoring air resistance, what is the velocity of the cannonball when it hits the water?
- **b** On a copy of the diagram above, trace a possible path that the cannonball might take if air resistance was present.

### Solution

**a** When the cannonball hits the water, its velocity will be the resultant of the horizontal velocity of the ball and the vertical velocity that the ball has gained by falling. Finding the vertical velocity of the ball:

$$v_v^2 = u_v^2 + 2as$$
  

$$v_v = \sqrt{u_v^2 + 2as}$$
  

$$= \sqrt{(0)^2 + 2(9.8)(25)} = 22.1 \,\mathrm{m\,s^{-1}}$$

Now, add the horizontal and vertical velocity vectors to find the resultant:



$$R^2 = 40^2 + 22.14^2$$

$$R = \sqrt{40^2 + 22.14^2} = 45.7 \text{ m s}^{-1}$$

Now, find the angle:

$$\theta = \tan^{-1} \left( \frac{22.14}{40} \right) = 29.0^{\circ}$$

Therefore, the velocity when the cannonball hits the water is  $45.7 \text{ m s}^{-1}$  at  $29.0^{\circ}$  below the horizontal.

**b** Air resistance will apply a force on the cannonball that directly opposes its motion. Therefore, it will be slowed and will not have travelled as far in the horizontal direction before it hits the water.





### 1C SKILLS

### The force of gravity in projectile motion

Projectiles are objects that move through space with only the constant force of gravity acting on them. In this chapter, when calculating questions about projectiles, the effects of air resistance can be ignored. Ignoring air resistance means that the horizontal component of the velocity will remain the same throughout the projectile's flight but the vertical velocity will increase in the downward direction at a rate of 9.8 m s<sup>-1</sup> every second due to the force of gravity on the object. It is important to note that the acceleration on an object acts independently to the velocity of the object. For example, these four projectiles have equal mass.



Each projectile has a different initial velocity but they are all acted upon by the same gravitational force and therefore the acceleration on all four of these balls will be equal:  $9.8 \text{ m s}^{-2}$  down. When solving projectile motion questions, you must always draw a diagram, separate the velocity into its horizonal and vertical components and remember that the only force acting on the projectile is the downwards force of gravity. This means the vertical velocity will change over time but the horizontal velocity will not.

#### Question

A cannon fired a cannonball with a velocity of  $32.5 \text{ m s}^{-1}$  at an angle of  $20^{\circ}$  from the horizontal on the top of a 12.7 m tall cliff. Find the horizontal distance travelled by the cannonball before it hits the ground.



First, draw a diagram.



Then you need to decide which direction (in this case up or down) will be positive:

down = positive

As the total time can be used to find the horizontal distance travelled by  $d = v_{\rm h} t$ , then you need to find the total time.

To do this, you need to treat the horizontal and vertical components of the projectile's velocity separately.

 $v_{\rm h} = 32.5 \cos 20 = 30.5 \text{ m s}^{-1}$  $v_{\rm y} = 32.5 \sin 20 = 11.1 \text{ m s}^{-1}$ 

To solve for the time, you can think of the vertical motion as occurring in two stages. The first stage is the cannonball rising in the air and reaching its maximum height and the second stage is the cannonball dropping from its maximum height to the ground.

Time to reach max height:

$$v_{v} = u_{v} + at$$

$$t = \frac{v_{v} - u_{v}}{a}$$

$$= \frac{0 - (-11.1)}{9.8} = 1.13$$

Before you move on to finding the time it takes the ball to drop from its maximum height, you must first find out the maximum height:

$$v_v^2 = u_v^2 + 2a_v s_v$$

$$s_v = \frac{v_v^2 - u_v^2}{2a_v}$$

$$= \frac{0^2 - (-11.1)^2}{(2)(9.8)} = 6.29 \text{ m}$$

$$6.29 + 12.7 = 19.0 \text{ m}$$

S

Now that you know the height when the ball's vertical velocity is zero, you can determine the time that it takes for the ball to reach the ground from its maximum height:

$$s_{\rm v} = u_{\rm v}t + \frac{1}{2}a_{\rm v}t^2$$
  
As  $u = 0$ , then:  
$$t = \sqrt{\frac{2s_{\rm v}}{a_{\rm v}}}$$
$$= \sqrt{\frac{(2)(19.0)}{9.8}} = 1.97 \text{ s}$$
total time = 1.13 + 1.97 = 3.1  
 $d = v_{\rm h}t$ 

=(30.5)(3.10)=94.6 m

Note that there is an alternative method to get total time.

- 1 Treat the entire vertical motion as one. No need to treat the up and down as separate journeys.
- 2 Find the vertical component of velocity as it is about to hit the ground.

.0 s

$$v_v^2 = u_v^2 + 2as_v$$
  
= (-11.1)2 + 2(9.8)(12.7  
 $v_v = 19.29$ 

**3** Now find total time for vertical motion:

$$v_v = u_v + a_v t$$
  
19.29 = -11.1 + 9.8t  
 $t = 3.10 \text{ s}$ 

### Section 1C questions

### **Multiple-choice questions**

- 1 This graph shows the vertical displacement of a ball as a function of time when it is thrown directly upwards. If air resistance is negligible, which position of the graph represents when the ball is stationary?
  - A A
  - **B** B
  - **C** C
  - D D



- 2 Two students are sitting on a bridge that crosses a river and throwing stones into the river. The first student throws a stone up with an initial speed of *u*. The second student drops the stone so that it has no initial velocity. Which of the following statements is correct? Assume air resistance is negligible.
  - A Both stones will hit the water at the same time.
  - **B** The stone thrown by the first student will hit the water with a greater speed than the stone thrown by the second student.
  - **C** The stone dropped by the second student will hit the water with a greater speed than the stone thrown by the first student.
  - **D** It is impossible to tell because you do not know the mass of the two stones.

### Use the following information to answer Questions 3 and 4.

An archer fires an arrow with an angle to the horizontal. The arrow traces a parabolic path as shown in the diagram. Ignore air resistance.



- **3** At point A (the maximum height), which statement describes the kinetic energy of the arrow?
  - A It is a maximum compared to any other point in its trajectory.
  - **B** It is a minimum compared to any other point in its trajectory.
  - **C** It is the same as every other point in the trajectory.
  - **D** It has half of the energy compared to when it first left the arrow.
- **4** Which of the following arrows represents the direction of the net force on the arrow at point A (the maximum vertical height)?



**5** A small ball is rolling at constant speed along a horizontal table. It rolls off the edge of the table and follows the parabolic path shown in the diagram below. Ignore air resistance.



Which one of the following statements about the motion of the ball as it falls is correct?

- A The ball's speed increases at a constant rate.
- **B** The momentum of the ball is conserved.
- **C** The acceleration of the ball is constant.
- **D** The ball travels at constant speed.

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### Short-answer questions

- 6 A 5.0 kg ball rolls off a table at 0.25 m s<sup>-1</sup>. The table has a height of 0.60 m.
  - a How long after the ball leaves the surface of the table does it strike the ground?
  - **b** How far horizontally will the ball fall from the edge of the table?
- 7 A ball is launched from ground level at 50 m s<sup>-1</sup> at an angle of 55° to the horizontal. Assume air resistance is negligible.
  - a What is the maximum height reached by the ball?
  - **b** What is the smallest value of the speed during the ball's flight?
  - c What is the acceleration of the ball when it reaches its maximum height?
  - d What is the ball's time of flight, assuming that it lands at ground level?
- 8 A supply aircraft is travelling horizontally at 46.5 m s<sup>-1</sup> towards a drop zone. The aircraft is flying at an altitude of 350 m when it releases the supply package. The package lands in the correct place. Ignore the effects of air resistance.



- a Calculate the horizontal distance, *X*, that the aircraft was away from the drop zone when it released the supply package.
- **b** What adjustments in the distance of release of the supply package would have to be made if air resistance was considered? Explain your response.
- **9** A projectile is launched from the ground at an angle of  $39^{\circ}$  and at a speed of  $25 \text{ m s}^{-1}$ , as shown in the diagram below. The maximum height that the projectile reaches above the ground is labelled *h*.



- **a** Ignoring air resistance, show that the projectile's time of flight from the launch to the highest point is equal to 1.6 s. Give your answer to two significant figures. Show your working and indicate your reasoning.
- **b** Calculate the range, *R*, of the projectile. Show your working.

VCAA 2019

10 A stunt car moves with a constant velocity, v, when it drives over a ramp that is inclined at 18°. It takes 1.44 s for the stunt car to reach the maximum height. The ramp has a maximum height of 3.50 m. Ignore air resistance. Note: diagram is not to scale.



- a Calculate the maximum height achieved by the stunt car.
- **b** Calculate the magnitude of the initial velocity of the stunt car when it leaves the first ramp.
- 11 A person throws a ball through the air. When the person releases the ball from their hand, it is 1.5 m above the ground. The ball reaches a maximum height of 4.5 m above the ground and lands 21 m away from the person who threw the ball. For the following questions, assume air resistance is negligible. A diagram of the situation is shown.



- a Calculate the total flight time of the ball.
- **b** Calculate the velocity of the ball when it left the person's hand. Your answer must include both a magnitude and direction.



### **Chapter 1 review**

### Summary

Create your own set of summary notes for this chapter on paper or in a digital document. A model summary is provided in the Teacher Resources, which can be used to compare with yours.

### Checklist

In the Interactive Textbook, the success criteria are linked from the review questions and will be automatically ticked when answers are correct. Alternatively, print or photocopy this page and tick the boxes when you have answered the corresponding questions correctly.

Success criteria – I am now able to: Linked questions				
1A.1	Apply Newton's first law to draw free-body diagrams and determine if a net force is acting on a body	3 , 4 , 5 , 5 , 7 , 8 , 10 , 11 , 12 , 13 , 14		
1 <b>A</b> .2	Apply Newton's second law, using the formula $F_{net} = ma$ to calculate the acceleration of a system or to calculate the force on different parts of the system	4, 8, 9, 9, 10, 11, 12, 13, 14, 21		
1A.3	Apply Newton's third law to identify action–reaction force pairs using the convention 'force on A by B' or $F_{\text{on A by B}} = {}^{-}F_{\text{on B by A}}$	6□, 10□, 11□, 21□		
1B.1	Understand that when in uniform circular motion, the centripetal force and acceleration is directed to the centre of the circle and that the velocity is tangential to the centripetal force	1		
1B.2	Apply the formula $F_{\text{net}} = \frac{mv^2}{r}$ and $a = \frac{v^2}{r}$ to solve questions relating to a vehicle moving around a flat circular section of a road and an object on the end of a string that is being swung in a horizontal circle	2 , 18		
1B.3	Draw a free-body diagram of all of the forces acting on a vehicle moving around a banked track and indicate the direction of the net force (centripetal force)	15		
1B.4	Be able to explain that the horizontal component of the normal force provides the centripetal force for a vehicle moving around a frictionless banked track	22		
1B.5	Apply the formula $F_{net} = mg \tan \theta$ to solve questions relating to circular motion on a smooth, banked track or road	15 , 22		
1B.6	Be able to draw free-body diagrams of objects moving in vertical circular motion and be able to identify the centripetal force	5, 16, 17		
1B.7	Be able to write equations that relate the normal/tension force and the gravity force to the centripetal force and apply these equations to solve problems of vertical circular motion	16 , 17		
1B.8	Solve quantitative and qualitative problems relating to vertical circular motion, including using the formulas $v = \frac{2\pi r}{T}$ and $T = \frac{1}{f}$	16□, 17□, 18□		

1C.1 Describe the flight path of a projectile within Earth's gravitational 19 field and describe the effect that air resistance will have on the projectile 1C.2 Solve projectile motion questions by considering the vertical and 12, 14, 14, horizontal components of the projectile's velocity and using the 19 . 20 . equations of straight-line motion under constant acceleration using  $21\square$ the formulas: v = u + at $v^2 = u^2 + 2as$  $s = \frac{1}{2}(u+v)t = ut + \frac{1}{2}at^{2} = vt - \frac{1}{2}at^{2}$ 

### **Multiple-choice questions**

1 A car is travelling at a constant speed on a horizontal circular track. When the car is at the position show in the diagram, which arrow best represents the acceleration of the car?



- A A
- **B** B
- **C** C
- D
- **2** A car is travelling around a horizontal circular track with radius, r, at a constant speed. If the car travels around a different horizontal circular track that has a radius of 2r at the same constant speed, then the centripetal force is:
  - A double what it was previously.
  - **B** half of what it was previously.
  - **C** quadruple what it was previously.
  - **D** one-quarter of what it was previously.
- **3** A car is travelling north at a constant velocity. What is net force acting on the car?
  - **A** The net force is north.
  - **B** The net force is south.
  - **C** The net force is down.
  - **D** There is no net force acting on the car.
- **4** Two balls, ball A and ball B, are dropped from the top of a building that is close to the surface of Earth. Ball A has a mass of *m*, while ball B has a mass of 5*m*. Ignore the effects of air resistance. Which of the following statements is correct?
  - A Ball B will accelerate towards Earth five times faster than ball A.
  - **B** Ball B will have five times more force on it than ball A.
  - **C** Both ball A and ball B have the same force on them.
  - **D** Ball A will accelerate towards Earth five times faster than ball B.

**5** A ball that is hanging by a string is pulled to the side and then released at an angle close to the surface of Earth. Once the ball is released, which of the following diagrams shows all of the forces acting on the ball?



**6** Earth revolves around the Sun. The reason for this is that the Sun exerts an attractive force on Earth; this force acts as the centripetal force keeping Earth in orbit. Which of the following statements is true about the force that Earth exerts on the Sun?

A The force that Earth exerts on the Sun is less than the force that the Sun exerts on Earth.

- **B** The force that Earth exerts on the Sun is greater than the force that the Sun exerts on Earth.
- **C** The force that Earth exerts on the Sun is equal to the force that the Sun exerts on Earth.
- **D** Earth does not exert a force on the Sun.
- **7** A ball is thrown upwards. While the ball is travelling upwards, what is the net force on the ball? Ignore the effects of air resistance.
  - A upwards and constant
  - **B** upwards and decreasing
  - **C** downwards and constant
  - D downwards and decreasing
- 8 A medicine ball that has a mass of 20 kg and a basketball of mass 0.5 kg are both thrown vertically upwards at  $10 \text{ m s}^{-1}$ . Ignoring the effects of air resistance, which of the following is true?
  - A The basketball is lighter, so it will hit the ground last.
  - **B** The medicine ball has more mass, so it will hit the ground last.
  - **C** Both balls will hit the ground at the same time.
  - D The medicine ball has more mass, so it will hit the ground first.
- 9 Lisa is driving a car of mass 1000 kg at 20 m s<sup>-1</sup> when she sees a dog in the middle of the road ahead of her. She takes 0.50 s to react and then brakes to a stop with a constant braking force. Her speed is shown in the graph below.

Lisa stops before she hits the dog.



Which one of the following is closest to the magnitude of the braking force acting on Lisa's car during her braking time?

- **A** 6.7 N
- **B** 6.7 kN
- **C** 8.0 kN
- **D** 20.0 kN

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### Short-answer questions

- 10 For each of the statements below, state whether it is true or false and justify your answer.
  - a When a large object collides with a small object the large object will apply a larger force on the smaller object than the smaller object applies to the larger object. (2 marks)
  - **b** Force due to gravity and mass are the same thing.
  - c The only way to slow down a moving object is to apply a force that has some component that is unbalanced in the opposite direction to the direction of the moving object's velocity.
     (2 marks)
  - **d** On Earth, a stationary object has no forces acting on it.
- **11** Two blocks, A of mass 4.0 kg and B of mass 1.0 kg, are being pushed to the right on a smooth, frictionless surface by a 40 N force, as shown below.



- **a** Calculate the magnitude of the force on block B by block A ( $F_{\text{on B by A}}$ ). Show your working. (2 marks)
- **b** State the magnitude and the direction of the force on block A by block B ( $F_{on A by B}$ ).

(2 marks)

(2 marks)

(2 marks)

VCAA 2018

12 A 5 kg mass is attached to a 3.5 kg mass via a pulley system. The 5 kg mass, initially at rest, sits on a frictionless table 1.5 m away from a 1 kg block that is at rest. The 3.5 kg mass, also at rest, is 1.5 m away from the floor.



**a** What is the initial acceleration of the 5 kg block?

- (2 marks) (2 marks)
- **b** What is the speed of the 5 kg mass when it collides with the 1 kg mass?

**13** An engine of mass  $1.40 \times 10^4$  kg pulls a carriage that has a mass of  $9.00 \times 10^3$  kg. A cable making an angle of 24.0° to the horizontal connects the engine and the carriage. The engine and carriage system is accelerated from rest to 15.0 m s<sup>-1</sup> in three minutes. A diagram of this situation is shown.



- **a** Calculate the net force acting on the engine and carriage system. (3 marks)
- **b** Calculate the tension in the cable.

(2 marks)

14 A fire door is attached to a pulley system that is activated by excess heat. When excess heat is detected, a 15.5 kg mass attached to the fire door is released. This pulls the 50 kg fire door 1.37 m to cover an opening. A diagram of this situation is shown.



- **a** Calculate the acceleration of the fire door as the mass falls. (2 marks)
- **b** Calculate the time taken before the opening is fully covered by the fire door. (2 marks)
- **15** A bicycle and its rider have a total mass of 100 kg and travel around a circular banked track at a radius of 20 m and at a constant speed of 10 m s<sup>-1</sup>, as shown in the diagram below. The track is banked so that there is no sideways friction force applied by the track on the wheels.



**a** On a copy of the diagram below, draw all of the forces on the rider and the bicycle, considered as a single object, as arrows. Draw the net resultant force as a dashed arrow labelled  $F_{net}$ . (2 marks)



b Calculate the correct angle of bank for there to be no sideways friction force applied by the track on the wheels. Show your working.(2 marks)

VCAA 2017

В

**16** A 2000 kg car travels over a hill that has a circular radius of 90 m.



- **a** Calculate the velocity that the car would need to travel so that the road provided no normal force to the car at the top of the hill. (2 marks)
- **b** If the car was travelling at 90 km  $h^{-1}$ , what would be the normal force on the car at the top of the hill? (3 marks)
- **c** The car continues on the road at 90 km h<sup>-1</sup> and encounters a dip in the road. When the car is at the bottom of the dip in the road, the car experiences a normal force of  $3.64 \times 10^4$  N. Calculate the radius of curvature of the dip. (2 marks)
- **17** A ball with a mass of 100 g is attached to the end of a string and swung in a vertical circle at a frequency of 8 Hz. Assume that the ball is travelling at a constant speed.
  - a At which point, A or B, is the string most likely to break? Justify your answer. (2 marks)
  - **b** What is the period of rotation of the ball? (2 marks)
  - c If the velocity of the ball is 1.75 m s<sup>-1</sup>, calculate the radius of the circle.
     (2 marks)
  - d Calculate the tension at point A, given a velocity of  $1.75 \text{ m s}^{-1}$ . (2 marks)



**20** A room on the top floor of a building is on fire. When the firefighters arrive, they set up a firehose with its nozzle 1 m above ground level and 11.54 m horizontally from the centre of the window where the fire is. The water exits the hose at  $30.0 \text{ m s}^{-1}$ . The firefighter angles the hose at  $50.0^{\circ}$  so that the jet of water enters the top window at

a point 13 m above the level ground at the base of the building. This situation is shown in the diagram.



- a How long does it take water leaving the nozzle to enter the window of the room that is on fire?
- Calculate the magnitude of the velocity of the water as it enters the open window. (3 marks) b
- c The firefighters decide that it is safer to fight the fire from further away. Explain how the firefighters could fight the fire from further away without changing the angle of the hose or the speed that the water comes out of the hose. (2 marks)
- **21** A basketball player is trying to make the winning shot for their team. He shoots from a point on the court that is 4.57 m away from the hoop and the hoop is 3.05 m above the ground. When the basketball player releases the ball, the ball is 2.00 m above the ground. The ball leaves the basketball player's hands at a speed of 8 m s<sup>-1</sup>.



а	If it took the ball 1.303 s to travel the horizontal distance of 4.57 m, calculate the		
	angle from the horizontal that the ball was launched.	(3 marks)	
b	Calculate the maximum height above the ground that the ball reaches.	(3 marks)	
С	Determine if the basketball player scored the shot.	(3 marks)	
d	When the basketball player is standing on the ground, a 931 N force due to gravity		
	acts on him. If the action force is the force due to gravity on the basketball player,		
	then identify the reaction force and explain how it is produced.	(2 marks)	
А	car is travelling around a banked curve.		
а	a Explain how the banked curve causes centripetal acceleration, which reduces the car's		
	reliance on friction between the tyres and the road.	(2 marks)	
b	The banked curve has a radius of 180 m. The car is travelling at a constant speed		
	of 20 m s <sup><math>-1</math></sup> . Calculate the banking angle that will enable the car to travel around		
	the banked curve without relying on friction.	(3 marks)	
	a b c d A a b	<ul> <li>a If it took the ball 1.303 s to travel the horizontal distance of 4.57 m, calculate the angle from the horizontal that the ball was launched.</li> <li>b Calculate the maximum height above the ground that the ball reaches.</li> <li>c Determine if the basketball player scored the shot.</li> <li>d When the basketball player is standing on the ground, a 931 N force due to gravity acts on him. If the action force is the force due to gravity on the basketball player, then identify the reaction force and explain how it is produced.</li> <li>A car is travelling around a banked curve.</li> <li>a Explain how the banked curve causes centripetal acceleration, which reduces the oreliance on friction between the tyres and the road.</li> <li>b The banked curve has a radius of 180 m. The car is travelling at a constant speed of 20 m s<sup>-1</sup>. Calculate the banking angle that will enable the car to travel around the banked curve without relying on friction.</li> </ul>	

(2 marks)

# HOW DO FIELDS EXPLAIN MOTION AND ELECTRICITY?

## FORCE, ENERGY AND MASS

### Introduction

UNIT

CHAPTER

Energy is a fundamental idea in physics. It orchestrates all events that occur in the universe. Energy can be kinetic energy, the energy of movement, or potential energy, energy stored in a system. Even though energy is a foundation for everything we do, its nature can seem abstract and difficult to understand. We know that energy can be transformed and transferred but cannot be created or destroyed. In November 1905, Albert Einstein wrote his most famous equation,  $E = mc^2$ , showing that there is an equivalence between mass and energy.

This chapter examines how a force can do work before investigating kinetic energy and gravitational potential energy both quantitatively and qualitatively. You will then consider the physics of straight-line collisions between two objects and finish by developing an understanding of Hooke's law and elastic potential energy of springs.

### Curriculum

Area of Study 1 Outcome 1 How do physicists explain motion in two dimensions?

<b>Work and energy</b> Apply the formula $W = Fs \cos \theta$ Understand that work is only achieved by a force that is parallel to the direction of motion Understand and apply the knowledge that work
Understand that work is only achieved by a force that is parallel to the direction of motion Understand and apply the knowledge that work
Understand and apply the knowledge that work
is the area under a force–displacement graph
Apply the formulas for kinetic energy and
gravitational potential energy: $E_r = mg\Delta h, E_k = \frac{1}{2}mv^2$
$\Delta E_{\rm k} = E_{\rm final} - E_{\rm initial} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
Analyse transformations of energy between kinetic energy and gravitational potential energy qualitatively and quantitatively using the formula $E_{1} + E_{2} = E_{1} + E_{2}$

Study Design		Learning intentions – at the end of this chapter I will be able to:		
<ul> <li>Newton's laws of motion</li> <li>Investigate and apply theoret and practically the laws of enmomentum conservation in issystems in one dimension</li> <li>Relationships between force, enand mass</li> <li>Investigate and analyse theor practically impulse in an isola for collisions between objects a straight line: FΔt = mΔv</li> <li>Analyse transformations of erbetween kinetic energy, elast energy dissipated to the envir (considered as a combination sound and deformation of maximum kinetic energy at low spee E<sub>k</sub> = <sup>1</sup>/<sub>2</sub>mv<sup>2</sup>; elastic and inconservation of kinetic energing with reference to conservation of kinetic energing the servation of kinetic energing with reference to conservation of kinetic energing the servation of kinetic energing the serva</li></ul>	ically ergy and solated 2B.1 2B.2 2B.2 2B.2 2B.2 2B.2 2B.3 2B.3 2B.3	<b>Momentum and impulse</b> Apply the formula $p = mv$ to momentum of bodies Apply the law of conservati to solve problems relating to objects colliding in a straig external forces act on those Be able to determine math collision is elastic or inelast Apply the relationship $l = \Delta p = m\Delta v = mv - mu =$ collision-type problems Understand that the area up graph is the impulse	o calculate the on of momentum to two or more ht line when no e bodies ematically if a tic $F_{av}\Delta t$ to solve under a force–time	
Relationships between force, er and mass • Analyse transformations of er between kinetic energy, elast energy, gravitational potential energy dissipated to the envir (considered as a combination sound and deformation of ma • elastic potential energy: ar force–distance graph inclu- ideal springs obeying Hoo Law: $E_s = \frac{1}{2}kx^2$	nergy2C 2C.1nergy ic potential energy and ronment n of heat, aterial): rea under uding ke's2C.22C.32C.32C.42C.5	<b>Springs</b> Understand that, in an ideal gradient of a force–comprese graph is the spring constant under the graph is the elast Understand that an ideal s Hooke's law, and apply the $F = -kx$ and $E_s = \frac{1}{2}kx^2$ Analyse transformations of elastic potential energy, king gravitational potential energy, king gravitational potential energy graphically, qualitatively and qualitatively and pultitatively and for elastic potential oscillating spring Understand that, in a spring the net force on a mass attactic zero, the velocity will be at a the formula $mg = kx$ to a boor vertical spring system	I spring, the ssion/extension t and the area ic potential energy pring will obey e formulas energy between netic energy and in a spring system id quantitatively cally, vely the energy in a vertical and system, when ched to a spring is maximum. Apply dy on an ideal	
CE Physics Study Design extrac	cts © VCAA; reprod	uced by permission.		
Closed system Impuls Equilibrium position Inertia		Momentum Open system	System Work	

Equilibrium position Gravitational potential energy

l

t

t

Inertia Kinetic energy

n	Syste
em	Work

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### **Concept map**



Work done is force × displacement, which is equivalent

See the Interactive Textbook for an interactive version of this concept map interlinked with all concept maps for the course.

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# Work and energy

#### Study Design:

- Investigate and apply theoretically and practically the concept of work done by a force using:
  - ► work done = force × displacement
  - work done = area under force vs distance graph (one dimensional only)
- Analyse transformations of energy between kinetic energy, elastic potential energy, gravitational potential energy and energy dissipated to the environment (considered as a combination of heat, sound and deformation of material):
  - ► gravitational potential energy: E<sub>g</sub> = mg∆h or from area under a force-distance graph and area under a field-distance graph multiplied by mass

#### Glossary:

Closed system Gravitational potential energy Kinetic energy Open system System Work

# **ENGAGE** The physics of a bungee jump

Inspired by the land divers of Vanuatu who dive off towers with vines tied to their ankles, two New Zealanders are credited with the development of a safe bungee cord, and the world's first commercial bungee jumping in the 1980s in New Zealand. The safe bungee cord needs the correct elasticity so that the bungee jumper is not stopped abruptly. The slower stop is required to prevent injury. When jumping with a bungee cord, the gravitational force on the jumper will initially cause free-fall acceleration until the cord begins to stretch and exert an upward force on the jumper.

As the cord stretches, the upward force produced increases with the amount of stretch in the cord. As the cord extends, the force will eventually increase to become greater than the downward gravitational force. Therefore, the net force on the jumper will become upwards. This means that the jumper will begin to slow down.

The jumper will eventually slow down to a stop; at this point the cord is at its maximum stretch and will now begin to contract and propel the jumper back upwards. A bungee jump can also be thought of as a transformation of energy. The jumper will start with maximum gravitational potential energy and once they jump, their gravitational potential energy,



**Figure 2A–1** A bungee cord at full stretch at a site in Queenstown. This was the world's first commercially available bungee jumping site, opened in 1988 by the New Zealand pair that developed the safer bungee cord.

kinetic energy and elastic potential energy will all change throughout the jump. Today, bungee jumping is performed all over the world. Commercial bungee jumps can reach heights greater than 200 m, normally above water, and some extreme jumpers have even jumped from hot air balloons, falling close to a kilometre.

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# EXPLAIN Work

Work the amount of energy transferred from one object or system to

another

The amount of energy transferred to or from another object or transformed to or from another form by the action of a force is called **work**.

The work, *W*, done when a force, *F*, causes a displacement, *s*, in the direction of the force is given by W = Fs.

If the force, *F*, is at an angle,  $\theta$ , to the displacement *s*, as shown in Figure 2A–2, then the Formula 2A–1 applies.





Formula 2A–1 Work
$W = Fs \cos \theta$
Where:
W = Work(J)
F = Net force (N)
s = Displacement (m)
$\theta$ = Angle that the force makes with the direction of displacement (°)

Work is a scalar quantity. The SI unit of work is the joule. One joule of work is done when a force of magnitude of 1 newton causes a displacement of 1 metre in the same direction as the force.

Kinetic energy the energy due to movement



# Force perpendicular to displacement

It is possible to apply a constant force on an object and not transfer any energy to the object. This occurs when the applied force is perpendicular to displacement. For example, as shown in Figure 2A-3, when an object is in uniform circular motion, there is a net force on the object, but no energy is

The work done on an object can also

 $Fs = \Delta E_{\rm b}$ 

cause a change in its kinetic energy.





PIER 2 FORCE

2A WORK AND ENERGY

being transferred to it. This is because when in uniform circular motion, the force vector is perpendicular to the velocity vector, which means that there is no component of force that acts in the direction of displacement. This explains why the kinetic energy of an object in uniform circular motion does not increase.

## Energy transfers, transformations and the law of conservation of energy

Energy can be transferred from one object to another or transformed from one form to another, but it cannot be created or destroyed. However, whether energy is conserved in a **system** consisting of several objects depends on if it is open or closed. In a **closed system**, the total amount of energy will remain the same, this means that energy or mass cannot be transferred to the surrounding environment, therefore the total amount of energy (and mass) will remain constant. Many calculation questions in Units 3&4 Physics will occur in closed systems. The exception to this is inelastic collisions. Inelastic collisions occur in an **open system**, where heat and sound energy are lost to the surrounding environment.



**Figure 2A–4** Left: The 27 km long Large Hadron Collider, located on the Swiss–French border, collides beams of protons into each other at close to the speed of light in a highly evacuated vacuum chamber. These subatomic collisions are considered to be elastic, and the system is assumed to be a closed one. Right: A car crash is a common inelastic collision that you will study in Unit 3 Physics.

# Friction and energy

Imagine that you are pushing a box a distance of 10 m on a thick carpet that provides a lot of friction to the movement of the box. To push this box at a constant velocity, the 50 N that you push the box with will be equal to the 50 N friction force in the opposite direction. The amount of work done to push the box 10 m can be calculated by:

$$W = Fs$$
  
= (50)(10)  
= 500 J

However, as soon as you stop pushing the box, it will be stationary, which means the box has zero kinetic energy. This energy has been converted into thermal energy, that is stored in both the carpet and the box and some sound energy. The thermal energy and sound energy is a by-product of the friction between the box and the carpet. The amount of energy converted into other forms can be calculated using the law of conservation of energy. Note that friction cannot be thought of as doing negative work on the box as the carpet is not giving energy to the box.

#### System

a collection of objects that can interact with each other and are being studied

#### **Closed system**

a system that does not allow the transfer of mass or energy to the surrounding environment

#### Open system

a system that allows the transfer of mass or energy to the surrounding environment



#### Worked example 2A–1 Calculating work

A person pushes a 1500 kg car a distance of 12.6 m in the direction of displacement, at a constant velocity, with a constant force of 900 N. A constant friction force of 900 N acts in the opposite direction.

- **a** Calculate the work done on the car by the person.
- **b** What is the increase in kinetic energy of the car?
- **c** Determine the energy lost due to the presence of a friction force.

#### Solution

**a** 
$$W = Fs \cos \theta$$

- $= (900)(12.6) \cos(0)$
- $=900 \times 12.6$
- $= 1.13 \times 10^4 \text{ J}$
- **b** Zero, the kinetic energy of the car will not increase (it is travelling at a constant velocity).
- **c** Since no energy is gained by the car, the energy lost due to the friction force must be equal to the work done by the person, therefore  $1.13 \times 10^4$  J.

### Worked example 2A–2 Calculating work at an angle

A child pulls a cart a distance of 250 m. The child pulls the cart via a string that makes an angle of  $31.0^{\circ}$  to the horizontal. The string has a constant tension of 29.4 N throughout the journey. Calculate the amount of work done.



#### Solution

The equation to calculate the force that is parallel to the direction of displacement is:



62

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## Force-distance graphs

Force–distance graphs are often used as they visually show the force as a function of the distance travelled.

If a box is pushed with a constant force of 100 N for 7.00 m, it could be graphed as shown in Figure 2A-5.



Figure 2A-5 A constant 100 N force being applied to push a box 7 m

Force–distance graphs are useful tools to calculate the amount of work done as the area under the graph gives the amount of work done.

For the graph given in Figure 2A–6, the work would be calculated as follows:



**Figure 2A–6** The area under a force–displacement graph gives the amount of work done on an object.

A graph is also useful when the force is not constant. Figure 2A–7 shows a force on an object that is decreasing at a constant rate as the object is pushed 4 m.



Figure 2A-7 The net force displacing an object is decreasing down to zero

To calculate the amount of work done at any point, you need to calculate the area under the graph up to that point. Therefore, the amount of work done to push the object 4 m according to the graph in Figure 2A–7 is:

work = area under graph = 0.5 × 4 × 80

= 160 J

Sometimes you may be asked to draw your own graph and calculate the work done from that graph.

```
lacksquare
```

#### Worked example 2A–3 Calculating work from a force–distance graph

A person pushes a car a distance of 300 m. The car is initially pushed with a force of 950 N, this force is then reduced at a constant rate so that when the car has been pushed 300 m, there is no net force on the car. Draw a graph of the force plotted against the distance and calculate the amount of work done on the car by the person pushing.

#### Solution



# Check-in questions – Set 1

- 1 A person is moving furniture in their house. They apply a force of 496 N parallel to the direction of motion to move the couch at a constant velocity and move it 10 cm to the right. A frictional force of 496 N opposes the motion.
  - **a** Calculate the amount of work done to move the couch.
  - **b** Determine the change in kinetic energy of the couch.
  - **c** Determine the energy lost due to the friction force.
- **2** A person pulls a golf cart 324 m horizontally from the tee to the green. If the person pulls with an average force of 250 N in the direction of the handle, and the handle makes an angle of 60° to the horizontal, calculate the amount of work done.



**3** In order to test the strength of two materials, A and B, a group of students decide to see how much work is required to compress the material by 10 mm. The students applied an increasing force to material A, but then became tired and their force slowly reduced to zero as they approached 10 mm of compression. For material B, the students applied a constant force of 450 N throughout the compression.



Based on the graph, which material took more energy to compress by 10 mm?

### Energy

#### Gravitational potential energy

When a weightlifter lifts a weight above their head, they are doing work against gravity to raise the weight.

If a weightlifter lifts a weight that has a mass of 160 kg from the ground to 1.90 m above the ground, assuming that the bar moves at a relatively constant speed, the amount of work done would be calculated as follows:

$$W = Fs$$
  
=  $(mg)(\Delta h)$   
=  $mg\Delta h$   
=  $(160)(9.8)(1.9)$   
= 2979.2 J



Figure 2A-8 Weightlifters do work against gravity to lift their weights.

#### Gravitational potential energy the amount of energy an object has stored due to its position in a gravitational field

66

For problems that are set close to the surface of Earth, it is assumed that the strength of Earth's gravitational field is constant at a value of 9.8 N kg<sup>-1</sup>. The work done to lift the weight can be expressed in terms of the **gravitational potential energy**. Gravitational potential energy is calculated using either the change in gravitational potential energy from one point to the other or by defining a point of zero gravitational potential energy and calculating the gravitational potential energy with reference to that point.

#### Formula 2A-2 Gravitational potential energy

 $E_{g} = mg\Delta h$ 

Where:

 $E_{g}$  = Gravitational potential energy (J)

m = Mass (kg)

g = Strength of the gravitational field, 9.8 N kg<sup>-1</sup> on the surface of Earth

 $\Delta h$  = Change in height (m)

# Worked example 2A–4 Gravitational potential energy

An 80 kg builder climbs a 4.6 m ladder. Calculate the change in gravitational potential energy.

Solution

 $E_{\rm g} = mg\Delta h$ = (80)(9.8)(4.6) = 3606.4 J

67

#### **Kinetic energy**

When an object is in motion relative to another, that object has kinetic energy. The kinetic energy of a body can be calculated using Formula 2A–3.

 $E_{\rm k} = \frac{1}{2}mv^2$ 

Formula 2A–3 Kinetic energy

Where:

 $E_{\rm k}$  = Kinetic energy (J) m = Mass (kg)  $\nu$  = Velocity (m s<sup>-1</sup>)

Speed is often given in terms of kilometres per hour  $(\text{km h}^{-1})$ . When given a value of speed or velocity in terms of kilometres per hour, it is useful to first convert it to metres per second. This can be done by dividing the km h<sup>-1</sup> value by 3.6, which means that converting from m s<sup>-1</sup> to km h<sup>-1</sup> is done by multiplying the m s<sup>-1</sup> value by 3.6. Most highways have a speed limit of 100 km h<sup>-1</sup>, which is just over 27.7 m s<sup>-1</sup>.



Figure 2A–9 The conversion factor between kilometres per hour and metres per second is 3.6.

#### Worked example 2A–5 Kinetic energy

If a tennis ball that has a mass of 58 g is travelling at a speed of 27 m s<sup>-1</sup>, what is the kinetic energy of the tennis ball?

Solution

$$E_{k} = \frac{1}{2}mv^{2}$$
$$= \frac{1}{2} \times 0.058 \times 27$$
$$= 21.1 \text{ J}$$

To alter the speed of the tennis ball in Worked example 2A–5, work would have to be done on the tennis ball. When work is done on a point-like object and no other forces are present, the work will change the kinetic energy of the object. This can be proven as:

$$v^{2} = u^{2} + 2as$$

$$s = \frac{v^{2} - u^{2}}{2a}$$

$$W = Fs \text{ (when } \theta = 0^{\circ}\text{)}$$

$$= ma\left(\frac{v^{2} - u^{2}}{2a}\right)$$

$$= \Delta E_{k} = \frac{1}{2}mv^{2} - \frac{1}{2}mu^{2}$$

# lacksquare

# Worked example 2A–6 Change in kinetic energy

A cyclist is travelling along a flat road at 63 km h<sup>-1</sup>. The cyclist applies the brakes, slowing down to a speed of 54 km h<sup>-1</sup>. What is the decrease in the kinetic energy of the bicycle, given that the cyclist and the bicycle have a combined mass of 93.2 kg?

#### Solution

First, convert the speed into SI units.

$$v = \frac{54}{3.6} = 15 \,\mathrm{m \, s^{-1}}, u = \frac{63}{3.6} = 17.5 \,\mathrm{m \, s^{-1}}$$
$$W = \Delta E_{\mathrm{k}}$$
$$= \frac{1}{2} m v^{2} - \frac{1}{2} m u^{2}$$
$$= \frac{1}{2} (93.2) (15)^{2} - \frac{1}{2} (93.2) (17.5)^{2}$$
$$= -3.79 \times 10^{3} \,\mathrm{J}$$

Note that the negative sign indicates that the energy was lost from the bicycle and given to the surrounding environment as heat and sound.

The kinetic energy–velocity graph is not linear but quadratic. It will take much more energy to change the velocity of an object by  $1 \text{ m s}^{-1}$  if the object's initial speed is greater. For example, changing the velocity of a 2 kg object from  $0 \text{ m s}^{-1}$  to  $1 \text{ m s}^{-1}$  takes 1 J of energy. But changing the velocity of a 2 kg object from  $3 \text{ m s}^{-1}$  to  $4 \text{ m s}^{-1}$  takes 7 J of energy.





**Figure 2A–10** The kinetic energy–velocity graph of a 2 kg object. It takes far less energy to change the velocity of an object from  $0 \text{ ms}^{-1}$  to  $1 \text{ ms}^{-1}$  than it does to change the velocity of the same object from  $3 \text{ ms}^{-1}$  to  $4 \text{ ms}^{-1}$ .

# Check-in questions – Set 2

- 1 When a diver climbs from ground level to the top of the 10 m diving board, their gravitational potential energy increases by  $5.68 \times 10^3$  J. What is the mass of the diver?
- 2 Calculate the kinetic energy of a 465 kg motorbike travelling with a velocity of 117 km  $h^{-1}$ .
- **3** When a 70 kg runner runs at a speed of  $4 \text{ m s}^{-1}$ , they have kinetic energy of 560 J. If the runner doubled their kinetic energy to 1120 J, what is their new speed?

# Gravitational potential energy, kinetic energy and the law of conservation of energy

Often you will be asked to solve problems that involve energy transformations between gravitational potential energy and kinetic energy. To visualise the conservation of energy, imagine a ball starting from point A on the ramp in Figure 2A–11.



# Figure 2A-11 A ball starts at point A on the ramp with no initial speed.

Taking ground level as the point of zero gravitational potential energy, then at point A the ball has no kinetic energy but maximum gravitational potential energy. As the ball rolls down the ramp, the gravitational potential energy is transformed into kinetic energy. Point B is the point defined as having zero gravitational potential energy, the gravitational potential energy from point A has all been transformed into kinetic energy. Once passing point B, the ball begins to rise up the ramp, transforming its kinetic energy into gravitational potential energy as stored gravitational potential energy and the other half is kinetic energy. If there is no friction or air resistance, there will be no energy lost, so the ball will reach point D, which is equal in height to point A. At point D, the ball will have transformed all of the remaining kinetic energy into gravitational potential energy.

Figure 2A–12 As the skier moves down the mountain, gravitational potential energy is being









If the system is closed, then in any situation where gravitational potential energy and kinetic energy are being transformed, the following equation holds true:

$$E_{\rm k initial} + E_{\rm g initial} = E_{\rm k final} + E_{\rm g final}$$
$$\frac{1}{2}mv_{\rm i}^2 + mg\Delta h_{\rm i} = \frac{1}{2}mv_{\rm f}^2 + mg\Delta h_{\rm f}$$

#### Worked example 2A–7 Energy transformations

A swimmer is standing on top of a 10 m diving platform before stepping off. Calculate the velocity of the swimmer when they hit the surface of the water. Assume that the swimmer hits the water with their feet first and ignore the effects of air resistance.

#### Solution

In this question, all the gravitational potential energy has been transformed into kinetic energy. Therefore:

$$\frac{1}{2}mv_{i}^{2} + mg\Delta h_{i} = \frac{1}{2}mv_{f}^{2} + mg\Delta h_{f}$$
$$\frac{1}{2}mv_{i}^{2} + mg\Delta h_{i} = \frac{1}{2}mv_{f}^{2} + mg\Delta h_{f}$$
$$\frac{1}{2}(0)^{2} + (9.8)(10) = \frac{1}{2}v_{f}^{2} + (9.8)(0)$$
$$v = 14 \text{ m s}^{-1}$$



# Worked example 2A-8 Energy transformation down a slope

A snowboarder is travelling down a slope. The snowboarder moves past point A, 50 m above ground level, at  $1.20 \text{ m s}^{-1}$ . Calculate the velocity of the snowboarder at point B, 13.6 m above the ground. Assume friction and air resistance are negligible.



# **2A SKILLS**

#### Understanding how variables relate to each other

When looking at a formula, it is a useful skill to be able to quickly determine how the variables in the formula relate to each other. For example, consider the formula for gravitational potential energy:

$$E_{g} = mg\Delta h$$

In this formula, the gravitational potential energy would be directly proportional to the mass, gravity and change in height.

When one variable is directly proportional to another, then as one variable becomes larger or smaller the linked variable also becomes larger or smaller. Direct proportionality can be represented using the symbol  $\infty$ . For example, as an object's position increases in height, the gravitational potential energy also increases:  $E_g \propto \Delta h$ .

But what about when the variables have powers or are the denominator in an equation? For example, consider the formula for centripetal force:

$$F_{\rm c} = \frac{mv^2}{r}$$

The centripetal force is directly proportional to:

- the mass,  $F_{\rm c} \propto m$
- the velocity squared,  $F_c \propto v^2$ •
- one over the radius (i.e. the inverse of the radius),  $F_{\rm c} \propto \frac{1}{2}$

The radius, r, is on the denominator and therefore the centripetal force is directly proportional to one over the radius,  $\frac{1}{r}$ , or inversely proportional to the radius, *r*. When one variable is inversely proportional to another, then as one variable increases, the other decreases.

Understanding proportionality can help to determine how an increase in one variable will affect the other variables. For example, if variable A is directly proportional to variable B, then variable A divided by variable B will equal a constant (assuming that the other variables in the equation are unchanged):

$$k = \frac{A}{B}$$

Returning to the centripetal force formula above, it can be said that:

$$k = \frac{F_{\rm c}}{v^2}$$

If variable Y was inversely proportional to variable Z, then Y times Z would be equal to a constant:

$$k = YZ$$

In the case of the centripetal force formula above, it can be seen that:

$$k = F_{c}r$$

Using these proportionality equations can help to solve problems like that on the following page.





#### Question

A car turns a corner in a circular path, travelling at a constant speed of  $\nu$ . If the car was to turn the same corner with a speed of  $2\nu$ , by what factor will the centripetal force increase?

**A**  $\frac{1}{2}$  **B** 2 **C** 4 **D** 8

#### Solution

The correct option is C:

$$k = \frac{F_c}{v^2} = \frac{xF_c}{(2v)^2} = \frac{xE_k}{2^2v^2} = \frac{xE_k}{4v^2}$$

Therefore, x = 4, so the kinetic energy will increase by a factor of 4.

# **Section 2A questions**

#### **Multiple-choice questions**

1 A ball is rolled down a ramp, as shown in the diagram below. Which position represents the position of greatest kinetic energy?



- A A
- **B** B
- **C** C
- D D
- 2 A child slides down a 2 m long slide. The force moving the child down the slide is 300 N and the frictional force provided by the slide is 50 N. The amount of work given to the child to change her speed is
  - **A** 600 J
  - **B** 500 J
  - **C** 300 J
  - D 250 J

**3** The gravitational field strength at the surface of Mars is  $3.7 \text{ N kg}^{-1}$ .

Which one of the following is closest to the change in gravitational potential energy when a 10 kg mass falls from 2.0 m above Mars's surface to Mars's surface?

- **A** 3.7 J
- **B** 7.4 J
- **C** 37 J
- **D** 74 J

#### VCAA NHT 2019

- **4** If the velocity of an object increases from a velocity of *v* to a velocity of 5*v*, the kinetic energy will be
  - **A** 5 times greater.
  - **B** 15 times greater.
  - **C** 25 times greater.
  - **D** 125 times greater.
- 5 A 1130 kg car is travelling at a speed of 99 km  $h^{-1}$  on a highway. The driver sees roadworks ahead and slows to a velocity of 45 km  $h^{-1}$ . Calculate the change in kinetic energy of the car.
  - A 8475 J
  - **B** 30.5 kJ
  - **C** 339 kJ
  - D 4393 kJ
- 6 A roller-coaster cart starts 90 m above the ground and then drops to ground level before completing a loop that is 70 m tall. When at the edge of the drop off, at point A, the roller-coaster cart has a velocity of 2 m s<sup>-1</sup>.



The speed at the top of the loop, point B, is

- A  $18.5 \text{ m} \text{ s}^{-1}$
- **B**  $19.9 \text{ m s}^{-1}$
- C 42.0 m s<sup>-1</sup>
- **D**  $392 \text{ m} \text{s}^{-1}$

# **Short-answer questions**

7 A person pushes a couch a distance of 50.0 cm with a constant force of 350 N. Calculate the amount of work done to move the couch, assuming that the force applied is parallel to the direction of movement.

8 A factory worker is pushing a machine suspended from an overhead track with a force of 240 N at an angle of 27.0° from the horizontal. Calculate the amount of work done to move the box 400 m.



- 9 If a car crashes at a certain speed, it can be equivalent to the same car falling from a certain height. For example, if a car crashes when it is travelling at 40 km h<sup>-1</sup>, it is equivalent to the same car falling from a height of 6.30 m. For the speeds below, calculate an equivalent height that the car is falling from.
  - **a**  $60 \text{ km } \text{h}^{-1}$
  - ${\color{blue}b} 80 \; km \, h^{-1}$
  - ${\rm c} \ \ 100 \ \, km \ h^{-1}$
- 10 On a copy of the set of axes below, sketch a graph of the force due to gravity as a function of vertical displacement as a 1 kg ball is dropped from a 10 m tall building. Assume the ball does not bounce when it hits the ground and that the effect of air resistance is negligible.



**11** A rock of mass 2.0 kg is thrown horizontally from the top of a vertical cliff 20 m high with an initial speed of 25 m s<sup>-1</sup>, as shown in the diagram below.



Base of cliff

- a Calculate the time taken for the rock to reach the sea. Show your working.
- **b** Calculate the horizontal distance from the base of the cliff to the point where the rock reaches the sea. Show your working.
- **c** Calculate the kinetic energy of the rock as it reaches the surface of the sea. Show your working.

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- 12 A factory worker is pushing an empty trolley with a force of 50 N while walking down the warehouse. The worker walks 20 m down the aisle before stopping and loading an item onto the trolley. The worker then pushes with a force of 150 N for another 30 m before stopping and loading another item onto the trolley. The worker then pushes with a force of 300 N to the end of the aisle, 50 m away.
  - a Draw a force-displacement graph of the worker's interaction with the trolley.
  - **b** Calculate the work done by the worker in their 100 m journey.





# Momentum and impulse

#### **Study Design:**

- Investigate and apply theoretically and practically the laws of energy and momentum conservation in isolated systems in one dimension
- Investigate and analyse theoretically and practically impulse in an isolated system for collisions between objects moving in a straight line:  $F\Delta t = m\Delta v$
- Analyse transformations of energy between kinetic energy, elastic potential energy, gravitational potential energy and energy dissipated to the environment (considered as a combination of heat, sound and deformation of material):
  - ► kinetic energy at low speeds: E<sub>k</sub> = <sup>1</sup>/<sub>2</sub>mv<sup>2</sup>; elastic and inelastic collisions with reference to conservation of kinetic energy

**Glossary:** Impulse Inertia Momentum

# 00

**ENGAGE** 

## 'Ah, Houston, we've had a problem'

Apollo 13, destined for the Moon, encountered a major problem when, during a routine oxygen stir, a suspected electrical short caused an explosion of an oxygen tank that disabled the service module (the third stage of the launch rocket). At that moment, any hope of landing on the Moon was abandoned and all of the efforts were focused on getting the astronauts back to Earth safely. All three astronauts were forced to take shelter in the lunar module (LM), which was only intended to support two people for two days. Since the main engine was on the service module, it was too risky to use as it may have caused another explosion, instantly killing all of the astronauts.

The team at mission control came up with a brilliant solution. They would modify their course to a free-return trajectory, using the lunar module engine. The Moon is moving at a significant speed around Earth; if the spacecraft approached the Moon close enough, the gravitational force on the spacecraft by the Moon would be sufficient to provide the roughly circular motion around it and also increase the speed of the spacecraft as it travels with the Moon. This manoeuvre is termed



**Figure 2B–1** The plan to use a gravitational slingshot manoeuvre to return the astronauts of Apollo 13 back safely to Earth

a 'gravitational slingshot' and it works by transferring some of the Moon's orbital momentum to the spacecraft.

Using the gravitational slingshot, Apollo 13 was able to successfully return to Earth.



Figure 2B-2 The astronauts of Apollo 13 landed safely in the Pacific ocean.

As humanity contiunes to venture out into space, gravitational slingshots will become common practice. This is because they are an excellent way to gain an additional speed boost, change course and slow a spacecraft down (when done in the opposite direction).

p = mv

# EXPLAIN

Where:

## Momentum

Momentum is an object's mass times by its velocity.

# Formula 2B–1 Momentum

 $p = \text{Momentum (kg m s^{-1})}$ m = Mass (kg) $\nu = \text{Velocity (m s^{-1})}$ 

Momentum is a vector quantity and as such it must have both magnitude and direction. The direction of the momentum vector and velocity vector will always be the same. Therefore, momentum can only be changed when a net force is applied.

# Worked example 2B-1 Calculating momentum

Calculate the momentum of a 2450 kg car driving at 81 km  $h^{-1}$  north.

Solution

First, convert units into SI units.

$$\frac{81}{3.6} = 22.5 \text{ m s}^{-1}$$

$$p = mv$$

$$= (2450)(22.5)$$

$$= 5.51 \times 10^4 \text{ kg m s}^{-1} \text{ north}$$

```
Momentum
the product of
an object's mass
and velocity
```

Momentum is always conserved when two or more objects collide and there is no net external force acting on the objects.

$$\sum p_{\text{before}} = \sum p_{\text{after}}$$

$$m_1 u_1 + m_2 u_2 + \dots = m_1 v_1 + m_2 v_2 + \dots$$

Where:

 $m_1$  = Mass of the first object (kg)

 $m_2$  = Mass of the second object (kg)

 $u_1$  = Initial velocity of the first object (m s<sup>-1</sup>)

 $u_2$  = Initial velocity of the second object (m s<sup>-1</sup>)

 $v_1$  = Final velocity of the first object (m s<sup>-1</sup>)

 $v_2$  = Final velocity of the second object (m s<sup>-1</sup>)

For one-dimensional collisions, it is important to use a positive and negative sign to indicate the direction of straight-line motion. If a diagram is not given, it is advisable to draw your own.

# Θ

#### Worked example 2B–2 Collision problem

In a physical education class, two students, Mark and Max, hold an inflatable fit ball to their chest and run towards each other at full pace. Mark, who is 86 kg, is running with a velocity of  $4.35 \text{ m s}^{-1}$  north. Max, who is 62 kg, is running with a velocity of  $5.55 \text{ m s}^{-1}$  south. After the collision, Mark has a velocity of  $3.64 \text{ m s}^{-1}$  south. Calculate the velocity of Max after the collision. Assume that there is no external horizontal force by ground on the two people during the collision.



#### Solution

Draw a diagram of what you think will happen after the collision.

79



Let north be positive.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$
(86)(4.35) + (62)(-5.55) = (86)(-3.64) + (62)v\_2
$$v_2 = 5.53 \text{ m s}^{-1}$$

The positive velocity indicates that Max is moving north.

In some cases, two or more objects may collide and then combine and move as one. The resulting object will have a mass that is equal to the combined mass of the two colliding objects. This can be expressed mathematically as:

$$m_1u_1 + m_2u_2 + \ldots = m_3v_3$$

where,  $m_3 = m_2 + m_1$ .

## Worked example 2B–3 Sticky collisions

A 200 g toy train is moving with a velocity of  $1.22 \text{ m s}^{-1}$  south when a child throws a 150 g piece of plasticine south at it. The plasticine travels with a velocity of  $2.45 \text{ m s}^{-1}$  south and when it hits the toy train, the two objects combine and move as one. Calculate the velocity of the plasticine and train combination after the collision.



First, convert units into SI units.

$$m_1 = 0.20 \text{ kg}, m_2 = 0.150 \text{ kg}$$

 $m_1 u_1 + m_1 u_2 = m_3 v_3$ 

Let south be positive.

 $(0.2)(1.22) + (0.15)(2.45) = (0.2 + 0.15)v_3$ 

 $v_3 = 1.75 \text{ m s}^{-1}$ 

The positive velocity indicates the direction: south.

In other cases, a single object may break apart into two or more other objects in what is known as an explosive collision. In this case, the law of conservation of momentum still applies. This can be expressed mathematically as:

$$m_1 u_1 = m_2 v_2 + m_3 v_3 + \dots$$

Where,  $m_1 = m_2 + m_3 + \dots$ .



# Worked example 2B-4 Explosive collisions

Shen is riding a bicycle west at  $18 \text{ km h}^{-1}$ , when he throws a 500 g Australian rules football with a velocity of 36 km h<sup>-1</sup> west. Calculate the final velocity of Shen, given that he and the bicycle have a combined mass of 59 kg.

Solution

First, convert units into SI units.

$$u_1 = \frac{18}{3.6} = 5 \text{ m s}^{-1}, u_2 = \frac{36}{3.6} = 10 \text{ m s}^{-1}, m_2 = 0.500 \text{ kg}$$

Take west to be positive.

$$m_1 u_1 = m_2 v_2 + m_3 v_3$$
  
(59 + 0.500)(5) = (10)(0.500) + (59) v\_3  
$$v_3 = 4.96 \text{ m s}^{-1} \text{ west}$$



a body's ability to resist a

state of motion. Inertia is

dependent only upon the mass of the body.

change in its

Inertia

#### Inertia

It is important to note that momentum and inertia are two different concepts. An object that is stationary has no momentum, but it has inertia. Inertia is linked to Newton's first law of motion and is defined as a body's tendency to resist a change in its state of motion (this could be stationary or moving at a constant velocity) when acted upon by a net force. The inertia of an object depends only on the mass; the greater the mass the greater the inertia. For example, an 80 kg person travelling at  $0.175 \text{ m s}^{-1}$  would have the same momentum as a 0.02 kg bullet travelling at 700 m s<sup>-1</sup>, but the inertia of the person is much greater.



# Check-in questions – Set 1

- 1 A male African elephant that has a mass of 6400 kg is walking with a velocity of  $2 \text{ m s}^{-1}$  north. What is the magnitude and direction of the elephant's momentum?
- 2 On an ice hockey rink, a keen physics student observes that there are many collisions.
  - **a** In one collision, two hockey pucks of equal mass collide head on, each with a speed of 7.25 m s<sup>-1</sup>. One of the hockey pucks moves away with a velocity of 3.99 m s<sup>-1</sup> south after the collision. Calculate the magnitude and direction of the velocity of the other hockey puck after the collision.
  - **b** In another collision, an 85.7 kg hockey player, who is initially at rest, hits the stationary 154 g hockey puck. The hockey puck moves north with a velocity of 22.5  $m s^{-1}$  north after the collision. Calculate the magnitude and direction of the velocity of the hockey player after the collision.
  - **c** Later on the ice hockey rink, the 85.7 kg hockey player is travelling 8.45 m s<sup>-1</sup> south when they collide with a 93.4 kg player, who is travelling at 4.22 m s<sup>-1</sup> north. After the collision, the two players move as one. Calculate the magnitude and direction of the velocity of the hockey players after the collision.

#### Change in momentum and impulse

When an object changes its velocity, its momentum also changes. An increase in velocity means an increase in momentum, a decrease in velocity means a decrease in momentum. This change in momentum can be expressed mathematically, as shown in Formula 2B–3.

#### Formula 2B–3 Change in momentum

 $\Delta p = mv - mu$ 

Where:

```
\Delta p = \text{Change in momentum } (\text{kg m s}^{-1} \text{ or N s})

m = \text{Mass (kg)}

v = \text{Final velocity (m s}^{-1})

u = \text{Initial velocity (m s}^{-1})
```

The change in momentum is a vector quantity and therefore always needs a magnitude and direction.

#### Worked example 2B–5 Calculating change in momentum

An archer fires a 32.0 g arrow at a stationary target. The arrow travels at 79.0 m s<sup>-1</sup> east towards the target. When the arrow hits the target, it takes 0.1 ms to stop. Calculate the change in momentum of the arrow when it hits the target.

Solution

First, convert units to SI units.

m = 0.032 kg

Let the direction east be positive.

 $\Delta p = mv - mu$ = 0.032 × 0 - 0.032 × 79.0 = -2.53 kg m s<sup>-1</sup>

The negative sign indicates the direction of the change in momentum: west.

Remember the only way to change an object's velocity (and therefore its momentum) is to apply a net force on the object. If you want to determine the average force during the time that the force is applied, you can use the following formula:

$$F_{\rm av} = ma$$
$$= \frac{m\Delta v}{\Delta t}$$

Therefore, replace  $m\Delta v$  with  $\Delta p$ :

$$F_{av} = \frac{\Delta p}{\Delta t}$$
$$\Delta p = F_{av} \Delta t$$

This means that change in momentum can also be referred to as **impulse**, *I*. Impulse is a vector quantity; it requires both a magnitude and direction. Impulse has units of Ns and is equivalent to the change in momentum. Therefore, impulse can be expressed mathematically as shown in Formula 2B–4.

# Formula 2B–4 Impulse

$$I = F_{av}\Delta t = \Delta p = mv - mu$$

#### Where:

Impulse the change in

momentum of an

object, caused by a force acting

for a certain amount of time

I =Impulse (kg m s<sup>-1</sup> or N s)  $F_{av} =$ Average force applied (N)

 $\Delta t$  = Time that the force is applied (s)

 $\Delta p$  = Change in momentum (kg m s<sup>-1</sup> or N s)

m = Mass (kg)

v = Final velocity (m s<sup>-1</sup>)

u = Initial velocity (m s<sup>-1</sup>)

#### Worked example 2B–6 Impulse

A driver is travelling in a car at 54 km  $h^{-1}$  north, when they slow to 9 km  $h^{-1}$  to drive over a speed bump. The driver takes 3.45 s to brake and the car and its contents have a mass of 1650 kg.

- a Calculate the change in momentum of the car.
- **b** Calculate the average force on the car as it slowed down. Include a direction in your answer.

#### Solution

a Convert all units into SI units.

$$u = \frac{54}{3.6} = 15 \,\mathrm{m \, s^{-1}}$$
,  $v = \frac{9}{3.6} = 2.5 \,\mathrm{m \, s^{-1}}$ 

Let the direction north be positive.

 $\Delta p = mv - mu$ = (1650)(2.5) - (1650)(15) = -2.06 × 10<sup>4</sup> kg m s<sup>-1</sup>

The negative sign indicates the direction: south.

**b** 
$$\Delta p = I = F_{av} \Delta t$$

$$F_{\rm av} = \frac{-2.06 \times 10^4}{3.45}$$
  
= -5.98 × 10<sup>3</sup> N

The negative sign indicates the direction: south.

# **Elastic and inelastic collisions**

Collisions may be elastic or inelastic. Elastic collisions occur between all gas molecules. However, perfectly elastic collisions are quite rare for larger objects, as often some of the energy is converted into heat, sound and deformation of the material. If a collision is 100% inelastic, then all of the energy is converted into heat, sound and deformation of the material, and the two colliding objects will be stationary after the collision. To determine if a collision is elastic or inelastic, the total kinetic energy of all of the objects involved in the collision must be calculated before and after the collision. If the total kinetic energy before and after the collision is equal, then it is an elastic collision. If the total kinetic energy before the collision is greater than the total kinetic energy after the collision, then it is an inelastic collision.

### Worked example 2B–7 Inelastic collisions

Two carts on a track, each with a mass of 0.150 kg, collide head on with each other at 4.5 m s<sup>-1</sup>. After the collision, the carts move away from each other at 1.25 m s<sup>-1</sup>. A diagram of this situation is shown below.



a Show that the momentum of the carts is conserved.

**b** Show that this collision is inelastic.

#### Solution

**Before:** 

After:

**a** Let movement to the right be positive.

 $\Sigma$  momentum before = (4.5)(0.15) + (-4.5)(0.15) = 0

 $\Sigma$  momentum after = (1.25)(0.15) + (-1.25)(0.15) = 0

Since  $\Sigma$  momentum before =  $\Sigma$  momentum after, momentum is conserved.

**b** Note that while momentum is a vector quantity, kinetic energy is a scalar and so there are no negatives.

$$\Sigma E_{\text{k before}} = \frac{1}{2} (0.15) (4.5)^2 + \frac{1}{2} (0.15) (4.5)^2 = 3.04 \text{ J}$$

$$\Sigma E_{\text{k after}} = \frac{1}{2} (0.15) (1.25)^2 + \frac{1}{2} (0.15) (1.25)^2 = 0.234 \text{ J}$$
Since  $\Sigma E_{\text{k before}} > \Sigma E_{\text{k ofter}}$  this collision is not elastic.

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#### Graphing the change in momentum (impulse)

The area under a force-time graph is the impulse. This can be proven as the area under any section of a graph is the product of the average *y*-value and the *x*-value. Since the value on the *y*-axis is the force and the value on the *x*-axis is the time, the area under the graph is the product of the average force and the time, which is equal to the impulse.

lacksquare

#### Worked example 2B-8 Calculating impulse from a force-time graph

A box is pushed with a force of 75 N for 15 s as shown in the graph on the right. Calculate the magnitude of the impulse given to the box in this time.

#### Solution

The impulse would be calculated by finding the area under the graph:

 $I = 75 \times 15$ 

 $= 1.13 \times 10^3 \,\mathrm{N\,s}$ 



If the force applied to an object is increasing or decreasing at a constant rate, then a diagonal line will be produced on a force–time graph. The line will have a constant, not zero, gradient. The area under the graph, the impulse, can be calculated using the formula for the area of triangle.

# Worked example 2B–9 Calculating impulse from a force–time graph when the force is changing

A car is accelerated by an initial force of 35 kN, the force then decreases at a constant rate to zero over 8 s. Calculate the magnitude of the impulse given to the car by the engine.

Solution

First, convert all units into SI units.

$$F = 35 \times 10^{3} \text{ N}$$

$$I = \text{area under the graph} = \frac{1}{2}bh$$

$$= \frac{1}{2} (8)(35 \times 10^{3})$$

$$= 1.40 \times 10^{5} \text{ N s}$$



In some cases, it may be appropriate to use several shapes to calculate the area under the graph.

# Worked example 2B–10 Calculating impulse from a more complex force–time graph

A person pushes a 30 kg sled through the snow. A force–time graph of the situation is shown in the diagram below. Calculate the magnitude of the change in momentum given to the sled in the 20 s journey. Assume friction is negligible.



#### Solution

First, recall that impulse is equal to the change in momentum.

To calculate the area under the graph, and therefore the impulse, the graph needs to be broken up into a rectangle and a triangle.

$$=\frac{1}{2}(4)(27.5) + (16)(27.5)$$
  
= 495 N s

The area of a trapezium can be helpful in finding the impulse when a force is applied that gradually reduces or increases. The area of a trapezium could be used in this case.

$$\frac{1}{2}(20 + 16)(27.5) = 495$$
 Ns



# Worked example 2B–11 Calculating impulse from a force–time graph with different units

Dog sledding is sometimes used in snowy areas as a relatively fast and novel way of moving around the snow. In dog sledding, several dogs are attached to a sleigh via a harness. The sleigh carries the passenger and glides through the snow. As the dogs pull the sleigh through the snow, they will often get tired. A force-time graph of a 30-minute dog sleigh journey is shown to the right. Calculate the magnitude of the impulse that the dogs have given to the sleigh in the 30-minute journey.



#### Solution

First, convert the units into SI units.

 $F_{\text{initial}} = 4000 \text{ N}, F_{\text{final}} = 3000 \text{ N}, t = 30 \times 60 = 1800 \text{ s}$ I = area of a trapezium $= \frac{4000 + 3000}{2} \times 1800$  $= 6.30 \times 10^6 \text{ N s}$ 



# Check-in questions – Set 2

- 1 A ball that has a mass of 0.344 kg is thrown south with a velocity of 8.11 m s<sup>-1</sup> against a brick wall. The ball collides with the wall and then rebounds with a velocity of 6.26 m s<sup>-1</sup> north. Calculate the magnitude and direction of the change in momentum of the ball.
- **2** A person pushes a car that has broken down with an average force of 500 N east for one minute. The impulse given to the person by the car is
  - A 500 Ns east
  - **B** 30000 N s east
  - C 500 Ns west
  - **D** 30000 N s west
- 3 Two cars, A and B, are crashed at the same speed and orientation, head-on into a wall. The cars differ only in the design of how the front of their car crumples during a head-on collision. The results of the crash tests showing the force applied to each car are shown on the graph on the right.
  - a Calculate the magnitude of the change in momentum of the crash of car A.



- **b** Does car A or car B have the most effective crumple zone? Justify your answer.
- 4 Two bumper cars of equal mass collide head-on. The bumper cars are both travelling at  $4 \text{ m s}^{-1}$  before the collision and rebound in opposite directions travelling at  $3.75 \text{ m s}^{-1}$  after the collision. Is this collision elastic or inelastic? Explain your answer.

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# **2B SKILLS**

#### Equating change in momentum and impulse

The equivalent equations for impulse and change in momentum are shown below:

$$I = F_{av}\Delta t = \Delta p = mv - mu$$

Knowing that impulse and momentum are equal to each other can help you to find the average force, time, mass, final velocity or initial velocity of an object in a collision. It is important to recall the units for impulse, N s, and change in momentum, kg m s<sup>-1</sup>, and realise that these two units are interchangeable. It is also useful to recall that the impulse, and therefore the change in momentum, can be calculated from the area under a force–time graph.

#### Question

A group of scientists are testing a new material for a super high bounce ball. The ball has a mass of 10 g and is fired horizontally north into a wall at 17.5 m s<sup>-1</sup>. The scientists place a detector on the wall to measure the amount of force on the ball as a function of time when it is in contact with the detector. A graph of the scientists' results is shown on the right.

- **a** Calculate the magnitude of the average force on the ball.
- **b** Calculate the final velocity of the ball after the collision.

#### Solution

**a** The average force can be determined by re-arranging the formula  $I = F_{av} \Delta t$ . Before the average force can be calculated, you first must calculate the impulse by calculating the area under the force-time graph.

$$I = (0.5)(10 \times 10^{-2})(5)$$
  
= 0.25 N s  
$$F_{av} = \frac{I}{t}$$
  
=  $\frac{0.25}{10 \times 10^{-2}}$   
= 2 50 N



b The velocity of the ball can be determined by equating the impulse with the change in momentum. Take south to be positive.

$$I = \Delta p = mv - mu$$
$$v = \frac{I + mu}{m}$$
$$= \frac{0.25 + (0.01)(-17.5)}{0.01}$$
$$= 7.5 \text{ m s}^{-1}$$

The positive answer indicates the direction: south.

# Section 2B questions

# **Multiple-choice questions**

- 1 What is the momentum of a 75 kg runner who is running at 21.6 km  $h^{-1}$ ?
  - C  $450 \text{ kg m s}^{-1}$ A 5832 kg m s<sup>-1</sup>
  - **B** 1630 kg m s<sup>-1</sup>

**D**  $315 \text{ kg m s}^{-1}$ 

Use the following information to answer Questions 2 and 3.

Block A has a mass of 5 kg and is moving towards block B, that has a mass of 2.8 kg. Block A moves horizontally towards the stationary block B with a velocity of  $4 \text{ m s}^{-1}$  to the right. Once the blocks collide, they move as one to the right.





- **A**  $7.14 \text{ m s}^{-1}$
- **B**  $2.74 \text{ m s}^{-1}$
- C  $2.56 \text{ m s}^{-1}$
- **D**  $0.90 \text{ m s}^{-1}$
- **3** What is the impulse on block A by block B?
  - A 7.2 Ns to the right
  - **B** 7.2 Ns to the left
  - **C** 7.6 Ns to the right
  - **D** 7.6 Ns to the left
- 4 A railway wagon X of mass 10 tonnes, moving at  $6.0 \text{ m s}^{-1}$ , collides with a stationary railway wagon Y of mass 5.0 tonnes. After the collision the wagons are joined together and move off as one. The situation is shown on the right. The final speed of the joined railway wagons after the collision is closest to
  - A  $2.0 \text{ m s}^{-1}$
  - **B** 3.0 m s<sup>-1</sup>
  - **C**  $4.0 \text{ m s}^{-1}$
  - **D**  $6.0 \text{ m s}^{-1}$
- **5** A swimmer is completing a 100 m swim. The force-time graph on the right shows the force applied by the swimmer to the water throughout the race.

The impulse that the swimmer gives to the water is

- **A**  $7.50 \times 10^4 \text{ Ns}$
- **B**  $3.46 \times 10^4$  N s
- **C**  $2.25 \times 10^4 \text{ Ns}$
- **D**  $2.10 \times 10^4$  Ns









ISBN 978-1-009-25901-9

Eddy de Jong et al

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Before the collision

3

2.5

2

1.5

1

0.5

0

Force (kN)

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6 A team of engineers are modifying the crumple zone on the front of a new car to make it safer. Initial testing was conducted with no improvements made to the crumple zone, and the force-time graph shown on the right was produced.

The engineers then crashed the car that had the improved crumple zone design into the wall. All other crash variables from the initial test, including the speed and mass of the car, were the same. Which graph represents the data gathered from the car with improved the crumple zone?



## **Short-answer questions**

7 Two blocks are moving towards each other with a speed of  $2.5 \text{ m s}^{-1}$  on a frictionless surface. Both blocks have a mass of 1.35 kg. After they collide, the two blocks move away from each other at a speed of  $0.9 \text{ m s}^{-1}$ .

Aaron argues that the law of conservation of momentum has been broken, since the magnitude of the total momentum is not conserved. Maya argues that the law of conservation is not broken and says that the total momentum is the same

2.5 ms <sup>-1</sup>	2.5 m s <sup>-1</sup> 1.35 kg
0.9 m s <sup>-1</sup>	0.9 m s <sup>-1</sup>

before and after the collision. Who is correct? Justify your answer.

8 Two bumper cars approach each other. A yellow bumper car, that has a total mass of 250 kg, is moving due north at a velocity of  $2 \text{ m s}^{-1}$ . It collides with a 150 kg red bumper car moving at 1.75 m s<sup>-1</sup> south. When the two bumper cars collide, they move as one.



- a Calculate the velocity of the red bumper car after the collision.
- **b** Calculate the impulse given to the red bumper car during the collision.
- c Calculate the amount of energy lost to thermal energy and sound energy in the collision.
- 9 A proton in an accelerator detector collides head-on with a stationary alpha particle, as shown below. After the collision, the alpha particle travels at a speed of  $4.0 \times 10^6$  m s<sup>-1</sup>. The proton rebounds at  $6.0 \times 10^6$  m s<sup>-1</sup>.



Find the speed of the proton before the collision, modelling the mass of the alpha particle, 4m, to be equal to four times the mass of the proton, m. Show your working. Ignore relativistic effects.

VCAA 2019

- 10 A 1500 kg car is travelling at 50.4 km h<sup>-1</sup> north when the driver sees a tree branch on the road. The car brakes and slows down to 21.6 km h<sup>-1</sup> north in 1.45 seconds.
  - a What is the change in momentum of the car?
  - **b** What is the impulse applied to the car?
  - **c** Calculate the average braking force.
- 11 Before physical education class, two students are trying to make a soccer ball and basketball collide. The first student throws a 625 g basketball and gives it a velocity of 6 m s<sup>-1</sup> south and the second student kicks a 450 g soccer ball and gives it a velocity of 7.88 m s<sup>-1</sup> north. After the collision, the basketball has a velocity of 4.22 m s<sup>-1</sup> north. Calculate the velocity of the soccer ball after the collision.
- 12 A cannon is at rest on a hill when it fires a 5.50 kg cannon ball with a velocity of 130 m s<sup>-1</sup> west. The cannon and cannonball have a combined mass of 565 kg. Calculate the velocity of the cannon when it releases the cannonball.



# Springs

#### Study Design:

- Analyse transformations of energy between kinetic energy, elastic potential energy, gravitational potential energy and energy dissipated to the environment (considered as a combination of heat, sound and deformation of material):
  - elastic potential energy: area under force-distance

graph including ideal springs obeying Hooke's Law:

$$E_{\rm s} = \frac{1}{2}kx^2$$

#### **Glossary:** Equilibrium position

# Ø

# ENGAGE

# A shoe that can make you run 4% faster

On 6 May 2017, Kenyan distance runner, Eliud Kipchoge, attempted what many people thought was impossible; to run the 42.2 km marathon distance in under two hours. To do this, he would be required to maintain a speed of  $21.1 \text{ km h}^{-1}$ . This is equivalent to running Melbourne's Tan Track in 10 minutes and 48 seconds, just over 11 times, nonstop. To help, Eliud wore Nike shoes with improved elastic properties. For almost fifty years, running shoes had used ethylene vinyl-acetate (EVA) in their midsoles, returning up to 65% of the runner's energy as they compress and expand. Nike replaced the traditional EVA midsole with a new material called Pebax, which was lighter than EVA and returned up to 87% of the runner's energy. Another innovation rarely seen at the time was the use of a carbon-fibre plate that ran the length of the shoe. This plate rocked the runner onto their toes faster and helped the Pebax to compress and expand quickly.

Various independent studies, as well as running data from the fitness app Strava, confirmed that the enhanced elastic properties of the shoes can improve running performance by up to 4%. In his first attempt in 2017, Eliud missed the sub two-hour mark by just 25 seconds. Other shoe brands released versions with carbon-fibre plates in them and in the 2020 World Athletics Half Marathon Championships, 92% of the shoes worn had a carbon-fibre plate in them. Eliud Kipchoge did eventually break the two-hour marathon in October 2019. However, this was not officially recognised as a world record.



**Figure 2C–1** Kenya's Eliud Kipchoge (white jersey) breaking the two-hour marathon on 12 October 2019 in Vienna



# EXPLAIN

# Hooke's law

In order to bend, stretch, twist or compress an object, work must be done on it. If the object is not damaged, it will return to its original shape once the external force has been removed. This means that an object is storing energy; this stored energy is known as elastic potential energy.

If you take three different springs, A, B and C, and stretch them all by 0.300 m, it will require a different amount of force to stretch each of them to that length. When the applied force is plotted against the extension, a line graph is given. The graph in red on the right shows the force applied plotted against the extension for the three different springs.

We can analyse these lines on the graph through the formula y = mx + c. When the spring is not extended, there will be no force on the spring. This means



Figure 2C–2 The force–extension graphs of three different ideal springs

the graph will always pass through the origin and therefore the values for *c* will always be zero. The gradient of the graph is the  $\frac{\text{rise}}{\text{run}}$ , which will be  $\frac{\text{newtons}}{\text{metres}}$  (giving the unit N m<sup>-1</sup>). The gradient represents the stiffness of the spring and the amount of force require to

stretch the spring by one metre. Since the stiffness of springs will vary, the gradients of the lines will also vary. A stiffer spring will require more force to stretch it by one metre, so a stiffer spring will have a steeper gradient. The stiffness of an ideal spring is constant; this value is known as the spring constant and is given the value *k*.

lacksquare

#### Worked example 2C-1 Calculating the spring constant

Calculate the spring constant for spring A in Figure 2C–2.

Solution

gradient =  $\frac{\text{rise}}{\text{run}}$ =  $\frac{8-0}{0.3-0}$ = 26.7 N m<sup>-1</sup>

Robert Hooke recognised the linear relationship that related the force applied to extend or compress an ideal spring with the resulting extension or compression of the spring. He summarised this relationship in what is now known as Hooke's law, shown in Formula 2C-1.

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#### Formula 2C–1 Hooke's law

F = -kx

Where:

F = Force (N)

k =Spring constant (N m<sup>-1</sup>)

x =Compression or extension of the spring (m)

There is a negative sign to indicate that springs restoring force acts in the opposite direction to the compression or extension. Usually the negative sign can be ignored in calculations as long as you remain aware of the direction of the forces on the spring and the direction of the spring's restoring force. It is worth noting that this relationship holds true for many objects other than springs that are stretched or compressed.

## Worked example 2C-2 Using Hooke's law

A car drives over a bump in the road and the ideal spring that is used in the car suspension is exposed to a force of  $2.56 \times 10^3$  N. If the spring constant is  $1.50 \times 10^4$  N m<sup>-1</sup>, calculate the amount that the spring is compressed.

Solution

$$F = kx$$
$$x = \frac{F}{k}$$
$$= \frac{(2.56 \times 10^{3})}{(1.50 \times 10^{4})}$$
$$= 0.171 \,\mathrm{m}$$

# **Elastic potential energy**

As we previously learned, the area under a force–displacement graph is the work done (W = Fs). When an ideal spring is stretched or compressed, the force–extension graph will also give the amount of work done.

For example, an ideal spring that has a spring constant of  $800 \text{ Nm}^{-1}$ is stretched to 1.00 m. The graph in Figure 2C–3 can be used to calculate the amount of work done to stretch the spring.

To calculate the amount of energy



Figure 2C–3 The force–extension graph of an ideal spring with a spring constant of 800  $N\,m^{-1}$ 

used to extend the spring, which will be equal to the stored elastic potential energy, find the area under the graph as shown in Figure 2C-4.



**Figure 2C–4** The area under the graph represents the elastic potential energy.

The elastic potential energy in Figure 2C–4 would be calculated as follows:

elastic potential energy =  $0.5 \times 1 \times 800$ 

= 400 J

The formula for the elastic potential energy can also be derived from the graph. The area under the graph is half the product of the *y*-value multiplied by the *x*-value (the area of a triangle).

area under the graph = elastic potential energy =  $\frac{1}{2}Fx$ 

The force on a spring is equal to *kx*. If *kx* is substituted for force in the formula above, the following is obtained:

elastic potential energy =  $\frac{1}{2}(kx)x$ which gives Formula 2C–2.

 $E_{\rm s} = \frac{1}{2}kx^2$ 

## NOTE

terms.

Elastic potential energy, spring

interchangeable

Q/

Formula 2C–2 Elastic potential energy

potential energy and strain potential energy are all

Where:

 $E_{\rm s}$  = Elastic potential energy (J)

k =Spring constant (N m<sup>-1</sup>)

x = Compression or extension of the spring (m)



**Figure 2C–5** As the spring extends, it takes more and more energy to extend it by the same amount. The energy required to extend the spring by 1 m, from 1 m to 2 m, is much less than the energy required to extend a spring from 4 m to 5 m. The energy required is indicated by the area under the graph, highlighted in green.

According to Hooke's law, F = kx, the more a spring is extended or compressed, the more force will be required to continue to extend or compress. For example, to extend or compress a spring by 1 m when it is originally at its rest length, does not take as much energy to extend the same spring by 1 m when it is already extended or compressed. This is because the more a spring is extended, the greater the restoring force becomes and therefore more energy must be applied to overcome this force. This is shown on the graph in Figure 2C–5, with a spring that has a spring constant of 100 N m<sup>-1</sup>.

That progressively more energy is required, at higher degrees of extension or compression, to extend or compress a spring by the same amount, can also be proved with Formula 2C-2,

the elastic potential energy formula,  $E_s = \frac{1}{2}kx^2$ . The elastic potential energy,  $E_s$ , is directly proportional to the extension or compression squared,  $x^2$ , which means a change in the extension will result in a quadratic change in the elastic potential energy.
So, when finding the change in elastic potential energy, you can either find the area under the graph or subtract the elastic potential energy of the final length of the spring from the elastic potential energy of the initial length of the spring. This can be expressed mathematically as shown in Formula 2C–3.

Formula 2C–3 Change in elastic potential energy

$$\Delta E_{\rm s} = \frac{1}{2}kx_{\rm f}^2 - \frac{1}{2}kx_{\rm i}^2$$

Where:

 $E_{s}$  = Elastic potential energy (J)

k =Spring constant (N m<sup>-1</sup>)

 $x_{\rm f}$  = Final length of the spring (m)

 $x_{i}$  = Initial length of the spring (m)

# Worked example 2C-3 Calculating change in elastic potential energy

Calculate the amount of energy used to stretch a spring, with a spring constant of  $150 \text{ Nm}^{-1}$ , from 10 cm to 14 cm. *Solution* 

$$x_{i} = 0.10 \text{ m}, x_{f} = 0.14$$
  

$$\Delta E_{s} = \frac{1}{2} k x_{f}^{2} - \frac{1}{2} k x_{i}^{2}$$
  

$$= \frac{1}{2} (150) (0.14)^{2} - \frac{1}{2} (150) (0.10)^{2}$$
  

$$= 0.72 \text{ J}$$

# Check-in questions – Set 1

- 1 A force of 225 N is applied to a spring that has a spring constant of 1000 N m<sup>-1</sup>. What is the extension of the spring?
- **2** A spring with a spring constant of  $5500 \text{ Nm}^{-1}$  is stretched so that it stores 248 J of energy. How much was the spring extended by?
- **3** The force–extension graph for a spring is shown below.



- **a** Use the graph to calculate the spring constant.
- **b** Use the graph to calculate the elastic potential energy when the spring is extended by 2 cm.
- 4 A spring with a spring constant of  $6800 \text{ Nm}^{-1}$  is stretched from 0.02 m to 0.05 m. Calculate the increase in the elastic potential energy of the spring.

## Horizontal spring systems

Imagine a spring system consisting of a spring that has the same spring constant in compression and extension. The spring is attached to a wall at one end, has a mass attached to the other end and rests on a frictionless surface. When the spring is at rest, it will

be in position B in Figure 2C–6. If the spring is compressed to position A and then released, it



Figure 2C–6 A horizontal spring system showing a spring that oscillates from point A (fully compressed) through point B (at rest) to point C (fully extended). Note that the 1 m compression/ extension will change based on the total energy of the spring system and the spring constant of the spring. Figures 2C-7 and 2C-8 relate to this system.

will naturally oscillate. During the oscillation, the net force and therefore the acceleration of the mass that is attached to the spring will always be directed towards point B. Point B is known as the **equilibrium position**, it is the point in the system where the net force on the mass is zero. As the mass moves from point A to B to C and back again, elastic potential energy and kinetic energy are being constantly transformed into each other. Therefore, the following expression will hold true for any horizontal spring after any amount of time has passed (assuming no energy is lost):

spring energy before + kinetic energy before = spring energy after + kinetic energy after

$$\frac{1}{2}kx_{\text{before}}^{2} + \frac{1}{2}mv_{\text{before}}^{2} = \frac{1}{2}kx_{\text{after}}^{2} + \frac{1}{2}mv_{\text{after}}^{2}$$

Figure 2C-7 shows the acceleration-spring extension and energy-spring extension graphs as the mass moves from point A to B to C. Figure 2C-7 describes the energy changes accompanying this movement.



Equilibrium position the position in a system where the net force on the oscillating object is zero

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Figure 2C-7 The net force/acceleration-spring extension and energy-spring extension graphs for one oscillation of the spring system in Figure 2C-6. The negative spring extension corresponds with spring compression.

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#### Point A

At point A, the spring has maximum compression (shown in the graph in Figure 2C-7 as negative extension). At this point of maximum compression, the net force and acceleration will be at a maximum because the spring force is at a maximum.

100% of the energy in the system will be elastic potential energy and the kinetic energy will be zero as the mass will be momentarily stationary.



#### Point B

As the mass accelerates to point B the elastic potential energy will decrease and the kinetic energy will increase.

When the mass does reach point B the spring has no compression or extension, therefore it will have zero elastic potential energy. No spring compression or extension means no horizontal spring force on the mass and therefore no horizontal acceleration. Since the mass will no longer be accelerated in this direction the velocity and therefore the kinetic energy is at a maximum and contains 100% of the energy of the system.



#### Point C

As the mass moves away from point B towards point C it will begin to extend. The extension causes the spring potential energy to increase. As the spring extends the spring force increases meaning that the net force and acceleration will also increase which slows the mass down, decreasing the kinetic energy.

At point C, the spring has maximum extension which means 100% of the systems energy is elastic potential energy. The net force and acceleration will also be at an equal maximum to point A but in an opposite direction.

Point C will also be a momentary stationary point, meaning the that the kinetic energy is zero. From point C the mass will then move back to point B and then to point A.

Figure 2C-8 Elastic potential energy and kinetic energy changes in the horizontal spring system shown in Figure 2C-6

# Vertical spring systems

In a vertical spring system, gravitational potential energy, kinetic energy and elastic potential energy are all being transformed into each other as the spring oscillates. The following expression will hold true for all vertical springs:

total energy at position 1			=	= total energy at position 2			n 2			
gravitational potential energy at position 1	+	elastic potential energy at position 1	+	kinetic energy at position 1	=	gravitational potential energy at position 2	+	elastic potential energy at position 2	+	kinetic energy at position 2
$mg\Delta h_1$	+	$\frac{1}{2}kx_{1}^{2}$	+	$\frac{1}{2}mv_1^2$	=	$mg\Delta h_2$	+	$\frac{1}{2}kx_{2}^{2}$	+	$\frac{1}{2}mv_2^2$

A common example is an ideal spring that has a mass at one end and is attached to a fixed support at the other end. When the spring is in position A in Figure 2C-9, the spring has no extension. The mass is then dropped from position A. Assuming that friction is negligible, the system will oscillate from this top position to a bottom position, C. Note that the 2 m total extension will change based on the stiffness of the spring and the mass attached to the spring.



**Figure 2C–9** A vertical ideal spring with a mass attached will oscillate from its top to bottom position continually if friction is ignored. Note that the 2 m total extension will change based on the spring system. Figures 2C–10 and 2C–11 relate to this system.

The net force, acceleration, elastic potential energy, gravitational potential energy and kinetic energy of the mass will all change throughout the oscillation, and the change in these quantities can be represented by the graphs shown in Figure 2C-10.



**Figure 2C–10** The net force/acceleration–spring extension and energy– spring extension graphs for one half-oscillation of the vertical spring system in Figure 2C–9



#### Point A

At point A, the spring is unstretched, meaning that the elastic potential energy is zero. At point A there is no spring force but there is the force due to gravity on the mass, which acts to accelerate it down at  $9.8 \text{ m s}^{-2}$ . At this point the mass is momentarily stationary meaning that the kinetic energy is zero.

If we take point C to be the point of zero gravitational potential energy, then point A is when 100% of the energy is stored as gravitational potential energy.





As the mass begins to fall due to the force of gravity, the spring will begin to stretch, hence the elastic potential energy will increase. As the velocity of the mass is increasing, the kinetic energy is also increasing and since the height is decreasing, the gravitational potential energy is also decreasing.

At point B, the mass is in the equilibrium position as the downward force due to gravity is balanced by the upward spring force (mg = kx). Point B is the point of maximum velocity and therefore maximum kinetic energy (25% of the total energy). The gravitational potential energy has now halved.





#### Point C

As the spring moves past point B, the upward spring force becomes greater than the downwards force due to gravity, meaning the net force and the acceleration is now directed up. As the spring extends, the upwards net force increases due to an increasing spring force. Hence, the elastic potential energy will also increase and the velocity will decrease.

At point C, the acceleration will be  $9.8 \text{ m s}^{-2}$  up; the spring potential energy will be a maximum and will account for 100% of the energy of the system. The mass will momentarily be stationary, meaning that the kinetic energy will be zero and the gravitational potential energy is defined as zero at this point. The mass will then accelerate to point B.

As is passes point B it will begin to decelerate and become stationary at point A.

Figure 2C-11 Elastic potential energy and kinetic energy changes in the vertical spring system shown in Figure 2C-9

Note that the total energy at any given distance is the same as any other point. This can be seen from the graph in Figure 2C-10 as at any point the sum of the gravitational potential energy, elastic potential energy and kinetic energy will equal 100%.





# C

## Worked example 2C-4 Vertical spring systems

A spring that has a 1.25 kg mass at one end is attached to a fixed support and left to oscillate freely. When oscillating, the maximum extension in the spring is found to be 18.0 cm. The diagrams on the right compare the spring when it is unextended and when it is at a point of maximum extension.

**a** On a copy of the diagram showing the spring when it is unextended, draw all of the forces acting on the mass.



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- **b** Calculate the spring constant of this ideal spring.
- c Calculate the maximum velocity of the mass as it is oscillating.

#### Solution

- **a** Since the spring has no extension, the only force acting on the ball will be the force due to gravity.
- **b** Take the bottom of the oscillation to be zero gravitational potential energy:

gravitational potential energy at position 1 + elastic potential energy at position 1

- + kinetic energy at position 1 = gravitational potential energy at position 2
- + elastic potential energy at position 2 + kinetic energy at position 2

$$mg\Delta h_1 + \frac{1}{2}kx_1^2 + \frac{1}{2}mv_1^2 = mg\Delta h_2 + \frac{1}{2}kx_2^2 + \frac{1}{2}mv_2^2$$

At maximum spring extension, the kinetic energy of the spring is zero, therefore:

$$mg\Delta h = \frac{1}{2}kx^2$$

You also know that in this case the change in height will be equal to the extension in the spring, therefore:

$$mgx = \frac{1}{2}kx^{2}$$

$$(1.25)(9.8)(0.18) = \frac{1}{2}k(0.18)^{2}$$

$$k = \frac{2(1.25)(9.8)(0.18)}{0.18^{2}}$$

$$mg = kx$$

$$(1.25)(9.8) = (136.1)x$$

$$x = 0.090007 \text{ m}$$

$$mg\Delta h = \frac{1}{2}mv^{2} + \frac{1}{2}kx^{2}$$

$$mg\Delta h - \frac{1}{2}kx^{2} = \frac{1}{2}mv^{2}$$

$$(1.25)(9.8)(0.090007) - (0.5)(136.1)(0.090007)^{2} = (0.5)(1.25)v^{2}$$

$$v = 0.939 \text{ m s}^{-1}$$

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# Worked example 2C–5 Complex vertical spring systems

A 3.77 kg mass is dropped from a height of 5.20 m onto an 11.6 m industrial spring. When the spring is at its point of maximum compression, the length of the spring is 5.45 m. A diagram of this situation is shown on the right.

- **a** Show that the spring constant is  $22.17 \text{ Nm}^{-1}$ .
- **b** When is the velocity of the ball at a maximum?
- **c** Calculate the compression of the spring when the velocity is at a maximum.
- d Calculate the maximum velocity of the ball.

# Solution

a change in gravitational potential energy = elastic potential energy

$$mg\Delta h = \frac{1}{2}kx^{2}$$
(3.77)(9.8)(5.2+6.15) =  $\frac{1}{2}k(6.15)^{2}$ 

$$= 22.17 \text{ N} \text{ m}^{-1}$$

**b** The velocity of the ball is at a maximum when the downward force of gravity is equal to the upward spring force. This is because at this point the acceleration is zero and the ball has been accelerated downwards as much as possible.

k

c 
$$kx = mg$$
  
(22.17) $x = (3.77)(9.8)$ 

$$x = 1.67 \text{ m}$$

**d** kinetic energy = potential energy change – elastic energy

$$E_{\rm k} = mg\Delta h - \frac{1}{2}k\Delta x^2 = (3.77)(9.8)(1.67 + 5.2) - \frac{1}{2}(22.17)(1.67)^2 = 222.9 \text{ J}$$
  
222.9 = (0.5)(3.77)v<sup>2</sup>  
v = 10.9 m s<sup>-1</sup>

# 2C SKILLS

## **Understanding graphs**

In this course, there are two main elements of graphs that you need to understand:

- the gradient and area of the graph
- the area under the graph.

The gradient of a graph is the steepness of the graph; the bigger the gradient, the steeper the graph. Consider the graph on the right.

To get a sense of the gradient, you can imagine that the graph is a hill. What would it be like to walk up and down this graph? Segment AB has a positive gradient with a moderate steepness, segment BC has a positive gradient with quite





11.6 m

3.77 kg

5.20 m

a steep (large) gradient. Segment CD is flat and therefore has a gradient of zero. Segment DE has a steep (large) negative gradient. The gradient is calculated by the change in the *y*-value ( $\Delta y$ ) divided by the change in the *x*-value ( $\Delta x$ ):

gradient = 
$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

This means that the unit for the gradient will always be the *y*-axis unit divided by the *x*-axis unit. Consider the graph of a spring below right.

The gradient for this line is:

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\Delta F}{\Delta x}$$
$$= \frac{4000 - 1000}{0.4 - 0.1}$$
$$= 10\ 000$$

The unit for this would be:

unit = 
$$\frac{\text{force}}{\text{extension}} = \frac{N}{m} = N \text{ m}^{-1}$$



Understanding that the gradient represents the

quantity on the *y*-axis divided by the quantity on the *x*-axis can help with substituting it into the formula; this helps to prove that the gradient of this line is the spring constant.

$$k = \frac{F}{x} =$$
gradient

Finding the area under a graph involves multiplying the *x*- and *y*-axis. If you wanted to find the area under the force–extension graph above right, it could be calculated by finding the area of the triangle.

area 
$$= \frac{1}{2}Fx$$
$$= \left(\frac{1}{2}\right)(5000)(0.5)$$
$$= 1250 \text{ J}$$



The unit for the area under the graph can be calculated by multiplying the unit on the *y*-axis with the unit on the *x*-axis:

unit = Fx = Nm = J

Knowing that the area under the graph is the product of the *x*- and *y*-axis is useful to understand how the area under the graph relates to known formulas. For example, consider the graph above right:

area under graph = Fx

#### You know that:

W = Fs

So, the work done on the spring, and therefore its stored elastic potential energy, is equal to the area under a force–extension graph.

#### Question

A group of students are investigating how the elastic potential energy changes with the extension of the spring. The students select a spring and extend it by 1 cm, 2 cm, 3 cm, 4 cm and 5 cm and record the average force required to stretch the spring to each length. The students then multiply the average force by the change in length to get the elastic potential energy. The students square the extension and plot it on a graph against the elastic potential energy. The students' graph is shown below.



- **a** What is the gradient of the graph? Include a unit in the answer.
- **b** Use the gradient of the line of best fit to calculate the spring constant of the spring.

#### Solution

**a** The gradient of the line must take two distant points from the line of best fit. The example solution below has taken the points (0, 0) and  $(19 \times 10^{-4}, 120)$ :

gradient = 
$$\frac{\text{rise}}{\text{run}}$$
  
=  $\frac{120 - 0}{(19 - 0) \times 10^{-4}}$   
=  $6.32 \times 10^4 \text{ J m}^{-2}$   
unit =  $\frac{\text{J}}{\text{m}^2}$  =  $\text{J m}^{-2}$ 

**b** First, you must understand what the gradient represents:

gradient = 
$$\frac{\text{rise}}{\text{run}} = \frac{E_s}{x^2}$$

Once you understand the significance of the gradient, you can use it with the formula for the elastic potential energy to calculate the spring constant:

$$E_{s} = \frac{1}{2}kx^{2}$$

$$k = \frac{2E_{s}}{x^{2}}$$

$$= 2 \times \text{gradient}$$

$$= (2)(6.32 \times 10^{4})$$

$$= 1.26 \times 10^{5} \text{ N m}^{-1}$$

# Section 2C questions

# **Multiple-choice questions**

Use the following information to answer Questions 1, 2 and 3.

A ball with a mass of 1 kg is travelling at 3 m s<sup>-1</sup> as it approaches an ideal spring, which has a spring constant of  $250 \text{ N m}^{-1}$ . The spring is fixed to a wall as shown below. Friction can be neglected.



The ball comes to rest for an instant when the spring is compressed by a total amount, *X*. After this instant, the spring rebounds.

- **1** The value of *X* is:
  - A 20 cm
  - **B** 19 cm
  - **C** 3.6 cm
  - **D** 1.2 cm



2 Which graph best represents the kinetic energy of the ball as a function of the spring's compression?



- 3 What is the magnitude of the momentum of the ball when the spring has been compressed by an amount  $\frac{1}{2}X$ ?
  - A 14.75 kg ms<sup>-1</sup>
  - **B** 6.74 kg ms<sup>-1</sup>
  - **C**  $2.60 \text{ kg ms}^{-1}$
  - **D**  $1.50 \text{ kg ms}^{-1}$

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#### Use the following information to answer Questions 4 and 5.

A model car is on a track and moving to the right. At the end of the track it hits and compresses a spring to bring it to a stop, as shown in the diagram below. The spring is considered ideal.

The car compresses the spring to 0.50 m when it comes to rest. The force–distance graph for the spring is also shown below.

Assume that friction is negligible.



- 4 Based on the graph above, what is the best estimate of the spring constant, k?
  - ${\rm A} \ 100 \ N \ m^{-1}$
  - $B 200 N m^{-1}$
  - $C 400 N m^{-1}$
  - $D 800 N m^{-1}$
- 5 What is the initial kinetic energy of the car?
  - **A** 25 J
  - **B** 50 J
  - **C** 100 J
  - **D** 200 J



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# Short-answer questions

- 6 How far will an ideal spring stretch if it is stretched with an external force of 140 N and has a spring constant of 300 N m<sup>-1</sup>?
- 7 The graph below shows the extension of two different ideal springs. Prove that spring A is stiffer than B. Hint: calculate the spring constant of the two springs and then compare them.



8 The force–extension graph below shows how the force applied to extend two springs, A and B, varies with the amount of extension. Use the graph to calculate the difference in elastic potential that the two springs have when they are both extended by 1 m.



- **9** A 5.00 kg mass is hung from a spring, dropped from the spring's unstretched length and allowed to oscillate up and down freely. A diagram of the situation is shown on the right.
  - a The position shown is the spring at its point of maximum extension. On a copy of the diagram, draw all of the forces that are acting on the mass and their approximate magnitudes.
  - **b** The maximum extension of the spring is 80.0 cm. Show that the spring constant is  $122.5 \text{ Nm}^{-1}$ .
  - **c** Calculate the maximum velocity that the mass will reach as it is oscillating.

-MMMMM-

10 A ball of mass 2.0 kg is dropped from a height of 2.0 m above a spring, as shown below. The spring has an uncompressed length of 2.0 m. The ball and the spring come to rest when they are at a distance of 0.50 m below the top of the uncompressed position of the spring.



- **a** Using g = 9.8 N kg<sup>-1</sup>, show that the spring constant, *k*, is equal to 392 N m<sup>-1</sup>. Show your working.
- **b** Determine the acceleration of the ball when it reaches its maximum speed. Explain your answer.
- **c** Calculate the compression of the spring when the ball reaches its maximum speed. Show your working.

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- **11** The suspension on a particular make of car, with a mass of 900 kg, is made up of four springs that each has a spring constant of  $55.1 \text{ kN m}^{-1}$ .
  - a Calculate the amount of compression in the springs when the car is stationary.
  - **b** The owners of the car are going on a camping trip. They load the car with camping equipment that has a combined mass of 120 kg. Calculate the new compression in the springs.
  - **c** Calculate the gain in elastic potential energy when the car is loaded with the camping gear.

# **Chapter 2 review**

# Summary

Create your own set of summary notes for this chapter on paper or in a digital document. A model summary is provided in the Teacher Resources, which can be used to compare with yours.

# Checklist

In the Interactive Textbook, the success criteria are linked from the review questions and will be automatically ticked when answers are correct. Alternatively, print or photocopy this page and tick the boxes when you have answered the corresponding questions correctly.

Succe	ss criteria – I am now able to:	Linked questions
2A.1	Apply the formula $w = Fs \cos \theta$	18 , 20
2A.2	Understand that work is only achieved by a force that is parallel to the direction of motion	80,90,18
2A.3	Understand and apply the knowledge that work is the area under a	17
	force–displacement graph	_
2A.4	Understand that energy is a conserved quantity	10
2A.5	Apply the formulas for kinetic energy and gravitational potential	50, 140, 200,
	energy: $E_{\rm g} = mg\Delta h, E_{\rm k} = \frac{1}{2}mv^2, \Delta E_{\rm k} = E_{\rm final} - E_{\rm initial} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	21
2A.6	Analyse transformations of energy between kinetic energy and gravitational potential energy qualitatively and quantitatively using the formula $E_{\rm k\ initial} + E_{\rm g\ initial} = E_{\rm k\ final} + E_{\rm g\ final}$	23
2B.1	Apply the formula $p = mv$ to calculate the momentum of bodies	14 , 16 , 19
2B.2	Apply the law of conservation of momentum to solve problems	20,40,70,
	relating to two or more objects colliding in a straight line when no external forces act on those bodies	14 , 16 , 19
2B.3	Be able to determine mathematically if a collision is elastic or inelastic	c 7□, 16□
2B.4	Apply the relationship $I = \Delta p = m\Delta v = mv - mu = F_{av}\Delta t$ to solve collision-type problems	6□, 11□, 14□, 22□
2B.5	Understand that the area under a force-time graph is the impulse	15
2C.1	Understand that, in an ideal spring, the gradient of a force– compression/extension graph is the spring constant and the area under the graph is the elastic potential energy	13
2C.2	Understand that an ideal spring will obey Hooke's law, and apply	$1 \square, 2 \square, 3 \square,$
	the formulas $F = -kx$ and $E_s = \frac{1}{2}kx^2$	12, 13, 20
2C.3	Analyse transformations of energy between elastic potential energy,	22
	kinetic energy and gravitational potential energy in a spring system	
	graphically, qualitatively and quantitatively	
2C.4	Be able to describe graphically, quantitatively and qualitatively the energy transformations that occur in a vertical and horizontal	22
20.5	Understand that in a spring system, when the net force on a mass	22
	attached to a spring is zero, the velocity will be at a maximum. Apply the formula $mg = kx$ to a body on an ideal vertical spring system	

# **Multiple-choice questions**

**1** Which graph best shows the relationship between the force on and extension of a spring?



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Use the following information to answer Questions 2 and 3.

An ideal spring, that has a spring constant of  $340 \text{ Nm}^{-1}$ , is compressed a distance of 0.05 m. As the spring is released, it pushes a 500 g block that slides smoothly on a horizontal frictionless surface.



- **2** What is the magnitude of the maximum acceleration of the block once the spring is released?
  - **A**  $34 \text{ m} \text{s}^{-2}$
  - **B**  $17 \text{ m} \text{s}^{-2}$
  - **C**  $8.5 \text{ m s}^{-2}$
  - **D**  $3.2 \text{ m s}^{-2}$
- **3** Which graph is the best representation of the relationship between the acceleration of the block as a function of the distance travelled by the block?



#### Use the following information to answer Questions 4, 5 and 6.

An arrow, with mass M, is fired at a cylinder with a velocity of v. The cylinder has a mass of 30M and can slide smoothly along the frictionless horizontal rails. When the arrow hits the cylinder, it pierces it and the cylinder and arrow move as one.



4 The velocity of the arrow–cylinder system after the collision is



5 The kinetic energy of the arrow-cylinder system is  $M^{2}$ 

$$A \quad \frac{Mv^2}{62}$$

$$\mathbf{B} \quad \frac{M\nu}{8}$$

$$C \quad \frac{Mv^2}{2}$$

**D** 
$$Mv^2$$

- **6** What impulse did the arrow give to the cylinder?
  - **Α** 31*M*ν
  - **B** 30*Mv*
  - **c**  $\frac{30M\nu}{31}$
  - $D \quad \frac{31M\nu}{31}$

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7 Two 2.5 kg blocks slide horizontally along a frictionless surface. The two blocks are both travelling at 4.8 m s<sup>-1</sup>, in opposite direction to when they collided. They rebound, each travelling at 3.6 m s<sup>-1</sup> in opposite directions.



Before collision

After collision

В

Which of the following statements is true?

- **A** The momentum is not conserved and the collision is elastic.
- **B** The momentum is not conserved and the collision is not elastic.
- **C** The momentum is conserved and the collision is elastic.
- **D** The momentum is conserved and the collision is not elastic.
- **8** Which of the following statements is true?
  - **A** When an object is in circular motion, neither the momentum nor the kinetic energy of the object will change.
  - **B** When an object is in circular motion, both the momentum and kinetic energy of the object will change.
  - **C** When an object is in circular motion, the momentum will change but no work will be done on the object.
  - **D** When an object is in circular motion, the momentum will not change and no work will be done on the object.
- **9** In which of the following situations is no work done?
  - **A** A person takes an elevator from the ground floor to the second floor.
  - **B** A person holds their books.
  - **C** A person walks up a flight of stairs.
  - **D** A person carries shopping to their car.
- **10** A meteoroid moves through space
- towards Earth, as seen in the diagram on the right. In which of the positions shown would the meteoroid have the greatest amount of kinetic energy?
  - A A
  - B B
  - **C** C
  - DD
- 11 A golf club strikes a stationary golf ball of mass 0.040 kg. The golf club is in contact with the ball for one millisecond. The ball moves off at 50 m s<sup>-1</sup>.

The average force exerted by the club on the ball is closest to

- **A** 2.0 N
- **B**  $1.0 \times 10^3$  N
- **C**  $2.0 \times 10^3$  N
- **D**  $1.0 \times 10^{6}$  N

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## **Short-answer questions**

- **12** For each of the following springs, calculate the unknown value.
  - **a** An ideal spring has a spring constant of  $30 \text{ Nm}^{-1}$  and is stretched 25.0 cm. Calculate the magnitude of the restorative force that the spring is applying. (2 marks)
  - **b** A spring is compressed by 8.00 cm with a force of 100 N. Calculate the spring constant for this ideal spring.
     (2 marks)
  - **c** A force of 250 N is applied to an ideal spring that has a spring constant of  $3.00 \times 10^3$  N. Calculate the extension in the spring. (2 marks)
- **13** An ideal spring is stretched and the force–extension graph is shown on the right.
  - **a** What is the spring constant of this ideal spring? (2 marks)
  - b If the spring is stretched by 10.0 m, calculate the amount of elastic potential energy stored in the spring. (2 marks)
  - **c** If the spring had an initial elastic potential energy of *X* J and it was stretched such that the new energy was 2*X* J, by what factor has the spring increased in length? (2 marks)
- 14 A person who has a mass of 90 kg is running at a velocity of 5 m s<sup>-1</sup> west when they jump onto a stationary



(2 marks)

skateboard. The skateboard and the person move west as one. The skateboard has a mass of 5.64 kg.



#### **a** Calculate the velocity of the person and the skateboard.

- **b** Calculate the impulse that the skateboard applies to the person. (3 marks)
- **c** When the person jumps on the skateboard, they are decelerating for 1.5 seconds. Calculate the average force that the skateboard applies to the person. (2 marks)
- d If the initial velocity of the person was increased from 5 m s<sup>-1</sup> to 10 m s<sup>-1</sup>, by what factor would the energy increase?
   (2 marks)

**15** A company that designs running shoes is testing two new shoes by measuring the force generated while a runner's foot is in contact with the ground. The solid line represents the results for shoe A and the broken line represents the results for shoe B.



Based on the graph, which shoe is the superior shoe? Justify your answer.



**16** Students are using two trolleys, Trolley A of mass 4.0 kg and Trolley B of mass 2.0 kg, to investigate kinetic energy and momentum in collisions.

Before the collision, Trolley A is moving to the right at 5.0 m s<sup>-1</sup> and Trolley B is moving to the right at 2.0 m s<sup>-1</sup>, as shown below. The trolleys collide and lock together and move as one, also shown below.



Determine, using calculations, whether the collision is elastic or inelastic. Show your working and justify your answer. (3 marks)

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**17** A worker is pushing a trolley around a warehouse. The force–distance graph of this situation is shown below.



Calculate the work done by the worker.

(2 marks)

**18** A shopper is pushing a shopping trolley with a force of 500 N at an angle of  $40^{\circ}$  from the horizontal.



**a** Calculate the horizontal component of the force.

(2 marks)

(2 marks)

- **b** If the shopper pushes the trolley a distance of 501 m, how much work have they done?
- **19** A 93 kg person, initially stationary, standing on a frozen lake throws a 57.0 g tennis ball north<br/>at 25.6 m s<sup>-1</sup>. Calculate the magnitude and direction of the person's velocity after they have<br/>thrown the tennis ball, ignoring friction.(3 marks)



**20** A child bouncing on a trampoline applies an average force of 4590 N to the centre of the trampoline from their highest bounce. At its lowest point, the trampoline bed is displaced by 0.125 m.



- **a** Calculate the work done to displace the trampoline bed. (2 marks)
- b If the trampoline has eight identical springs attaching the trampoline bed to the frame, and each of these springs is extended by a maximum of 5 cm when the child is at the lowest point in their jump, calculate the spring constant of each spring. Assume the fabric of the bed is not elastic, and the springs absorb all the force applied by the child. (3 marks)
- c Calculate the child's maximum height above the ground from which they fell to the trampoline bed, given that at the lowest point the child is 0.450 m above the ground.
   (3 marks)
- A 1220 kg car coming up to a school zone is moving at 72 km h<sup>-1</sup>. Calculate the change in kinetic energy if the car slows to 45 km h<sup>-1</sup>. (3 marks)
- **22** An 85.6 kg person is standing 100 m above the ground, about to perform a bungee jump. The bungee cord has an unstretched length of 40 m. At the bottom of the jump, the bungee jumper is 10 m away from the ground.
  - **a** Calculate the spring constant of the bungee cord.

- (3 marks)
- b The owners of the bungee company are discussing the use of different bungee cords with different spring constants. Assuming that the bungee is calibrated correctly, so that the jumper does not hit the ground, would a bungee cord with a lower or higher spring constant be preferable. Justify your answer. (3 marks)
- **c** When is the bungee jumper travelling at their maximum velocity? Justify your answer. (2 marks)
- d Assuming no energy is lost, sketch the graph of the elastic potential energy against distance. Graph from when the jumper jumps from the platform to when they are at the lowest point of their jump. Assume that the jumper does not lose energy during the jump or gain any height when they jump. (3 marks)
- **23** A child throws a 56.9 g tennis ball with a velocity of  $13.7 \text{ m s}^{-1}$  off the top of a 20 m tall bridge. Calculate the velocity of the ball when it strikes the water at the bottom of the bridge. (3 marks)

# HOW DO FIELDS EXPLAIN MOTION AND ELECTRICITY?

# GRAVITY

Aboriginal and Torres Strait Islander peoples should be aware this chapter contains images of people who have, or may have, passed away.

#### Introduction

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This chapter introduces us to Newton's law of universal gravitation. In 1666, the bubonic plague raged through England and Europe killing millions of people. Isaac Newton, then aged 24 and on leave from Cambridge University at his Woolsthorpe farm property, contemplated the matter of falling apples and the Moon.

One of his first biographers, William Stukeley, recalls Newton telling him about the apple:

'After dinner, the weather being warm, we went into the garden and drank tea, under the shade of some apple trees ... he told me, he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion'd by the fall of an apple, as he sat in contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself ...'

Newton's surprising discovery was not that Earth's gravity created a force on an apple that pulled it straight down to Earth, but that the apple's gravity created an equal force on Earth to pull Earth up. Equally surprising was Newton's realisation that the Moon was falling towards Earth, just like the apple!

Our starting point in our understanding of gravity is the law of gravitation developed by Isaac Newton.

In this chapter, you will learn how to represent a gravitational field using a field model for gravitation. You will investigate gravitational fields, including the directions and shapes of fields, and gravitational fields about a point mass with reference to the direction of the field, the use of the inverse square law to determine the magnitude of the field and the potential energy changes associated with a point mass moving in the field.

You will analyse how gravity accelerates masses by using gravitational field and gravitational force

concepts such as  $g = G \frac{M}{r^2}$  and  $F_g = G \frac{m_1 m_2}{r^2}$ .

You will explore potential energy changes in a uniform gravitational field using the formula  $E_g = mg\Delta h$  and determine the change in gravitational potential energy from the area under a force–distance graph and the area under a field–distance graph multiplied by mass in non-uniform gravitational fields.

# Curriculum

# Area of Study 2 Outcome 2 How do things move without contact?

Study Design	Learning intentions – at the end of this chapter I will be able to:
<ul> <li>Fields and interactions</li> <li>Describe gravitation, magnetism and electricity using a field model</li> <li>Investigate and compare theoretically and practically gravitational, magnetic and electric fields, including directions and shapes of fields, attractive and repulsive effects, and the existence of dipoles and monopoles</li> <li>Investigate and compare theoretically and practically gravitational fields and electrical fields about a point mass or charge (positive or negative) with reference to:</li> <li>the direction of the field</li> <li>the shape of the field</li> <li>the use of the inverse square law to determine the magnitude of the field</li> <li>Identify fields as static or changing, and as uniform or non-uniform <i>Electric and magnetic fields are covered in Chapter 4.</i></li> <li>Effects of fields</li> <li>Analyse the use of gravitational fields to accelerate mass, including:</li> <li>gravitational field concepts</li> <li>Application of field concepts</li> <li>Apply the concepts of force due to gravity and normal force including in relation to satellites in orbit where the orbits are assumed to be uniform and circular <i>Orbiting satellites are covered in 3C.</i></li> </ul>	<ul> <li>3A Gravity and gravitational fields</li> <li>3A.1 Understand both theoretically and practically the direction, shape and attractive effect of a gravitational field; identify fields as uniform or non-uniform; investigate the existence of monopoles and dipoles and gravity as a monopole</li> <li>3A.2 Understand both theoretically and practically the direction, shape and attractive effect of a gravitational field about a point mass and be able to use the inverse square law to determine the magnitude of the field</li> <li>3A.3 Analyse the use of gravitational field and gravitation and g</li></ul>
<ul> <li>Fields and interactions</li> <li>Investigate and compare theoretically and practically gravitational fields and electrical fields about a point mass or charge (positive or negative) with reference to: <ul> <li>potential energy changes (qualitative) associated with a point mass or charge moving in the field</li> </ul> </li> <li>Effects of fields <ul> <li>Analyse the use of gravitational fields to accelerate mass, including:</li> <li>potential energy changes in a uniform gravitational field: <i>E</i><sub>g</sub> = <i>mg</i>\Delta<i>h</i></li> </ul> </li> <li>Analyse the change in gravitational potential energy from area under a force vs distance graph and area under a field vs distance graph multiplied by mass</li> </ul>	3BGravitational potential energy3B.1Determine gravitational potential energy changes (qualitative) associated with a point mass and potential energy changes in a uniform gravitational field using $E_g = mg\Delta h$ 3B.2Analyse the change in gravitational potential energy from both the area under a force-distance graph and the area under a field-distance graph multiplied by mass

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#### Application of field concepts

- Apply the concepts of force due to gravity and normal force including in relation to satellites in orbit where the orbits are assumed to be uniform and circular
- Model satellite motion (artificial, Moon, planet) as  $v^2 = 4\pi^2 r$

uniform circular orbital motion:  $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$ 

# Learning intentions – at the end of this chapter I will be able to:

- **3C** Orbiting satellites
   **3C.1** Apply the concepts of force due to gravity, *F*<sub>g</sub>, and the normal force, *F*<sub>N</sub>, including in relation to satellites in orbit where the orbits are assumed to be both uniform and circular
- **3C.2** Model satellite motion for artificial satellites, our Moon and planets as uniform circular orbital motion using

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

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#### Glossary

Acceleration due to gravity Artificial satellite Centripetal acceleration Centripetal force Dipole Field Field line g Geostationary satellite Gravimeter Gravitational constant, G Gravitational field Gravitational field strength Gravitational force Gravitational monopole Gravitational potential energy Inverse square law Monopole Newton's law of universal gravitation Newton's third law Non-uniform gravitational field Normal force Orbital period Orbital radius Orbital speed Period Point mass Scalar field Uniform gravitational field Vector field Work

# Concept map



3A Gravity and gravitational fields



Change in gravitational potential energy can be analysed as the area under a force–distance graph and the area under a field–distance graph multiplied by mass



The concepts of force due to gravity and normal force can be applied to satellites in orbit

#### **3C Orbiting satellites**



See the Interactive Textbook for an interactive version of this concept map interlinked with all concept maps for the course.



# Gravity and gravitational fields

#### **Study Design:**

- Describe gravitation, magnetism and electricity using a field model
- Investigate and compare theoretically and practically gravitational magnetic and electric fields, including directions and shapes of fields, attractive and repulsive effect, and the existence of dipoles and monopoles
- Investigate and compare theoretically and practically gravitational fields and electrical fields about a point mass or charge (positive or negative) with reference to:
  - ► the direction of the field
  - ► the shape of the field
  - the use of the inverse square law to determine the magnitude of the field
- Identify fields as static or changing, and as uniform or non-uniform
- Analyse the use of gravitational fields to accelerate mass, including:
  - gravitational field and gravitational force

concepts: 
$$g = G \frac{M}{r^2}$$
 and  $F_g = G \frac{m_1 m_2}{r^2}$ 

• Apply the concepts of force due to gravity and normal force including in relation to satellites in orbit where the orbits are assumed to be uniform and circular

#### **Glossary:**

Acceleration due to gravity Centripetal acceleration Centripetal force Dipole Field Field line g Gravimeter Gravitational constant, G Gravitational field Gravitational field strength Gravitational force Gravitational monopole Inverse square law Monopole Newton's law of universal gravitation Newton's third law Non-uniform gravitational field Normal force Orbital radius Period Point mass Scalar field Uniform gravitational field Vector field

# 0°

# ENGAGE

# The nature of gravity

One of the enduring goals of physics is to fully understand the nature of gravity – the gravitational force that holds you to Earth, holds the Moon in orbit around Earth, and holds Earth in orbit around the Sun. This same force reaches out through the whole of our Milky Way Galaxy, holding together the billions of stars in our galaxy. In our Local Group, there are three large galaxies: the Triangulum Galaxy, the Milky Way Galaxy and the Andromeda Galaxy (Figure 3A–1), all held together by gravity. In the centre of our galaxy, there is a supermassive black hole, containing the mass of



**Figure 3A–1** The Andromeda Galaxy is the closest galaxy to the Milky Way.

4.3 million solar masses where the gravity is so strong that even light cannot escape. In the known universe, there are approximately 10<sup>11</sup> galaxies, all ruled by gravity. Although the gravitational force is still not fully understood, the starting point is an understanding

of the law of gravitation, developed by Isaac Newton. An understanding of gravity is fundamental to understanding the universe.

There have been numerous theories of gravitation since ancient times. The first sources that discuss gravitation are the works of the ancient Greek philosopher, Aristotle (400 BC). In the 7th century, the ancient Indian astronomer, Brahmagupta, wrote of gravity as an attractive force. In the early 17th century, Galileo Galilei had found that all objects tend to accelerate equally in free fall under the influence of gravity. By the middle of the 17th century, Isaac Newton had formulated his law of universal gravitation.

In Australia, Aboriginal and Torres Strait Islander peoples have held a practical understanding of gravity for approximately 50 000 years. Throwing spears and boomerangs uses a knowledge of both contact and non-contact forces. In addition to Aboriginal and Torres Strait Islander peoples' mastery of the practical physics of projectile motion (Figure 3A–2), they were aware of the Moon's influence on tides.

For example, the Yolngu people of the Northern Territory have detailed stories that explain the phases of the Moon and accurately link the Moon with the changing tides. Since the Yolngu people rely on the sea for many resources, they have developed an in-depth knowledge of tides, including the ability to interpret the phases of the Moon in ways that allow the prediction of ocean phenomena, such as the time and height of the next tide. In contrast, the great 17th century Italian astronomer Galileo Galilei did not believe that the Moon was in any way connected to tidal phenomena.



**Figure 3A–2** Aboriginal and Torres Strait Islander peoples' mastery of practical physics principles, for example in throwing boomerangs, is part of traditional hunting techniques. Members of the Tjapukai Dance Theatre, pictured here, recreate these techniques in their performances.



# EXPLAIN Gravitational fields

Attraction between masses occurs without the need for physical contact. This is called 'action at a distance'. In 1849, the famous English scientist Michael Faraday, while working on understanding the nature of electromagnetism, proposed the concept of a field. Faraday's 'field' referred to 'a region in the vicinity of a magnet affected by some force'.

VIDEO 3A–1 GRAVITY AND GRAVITATIONAL FIFI DS



#### Field

a region where an object feels a force, such as gravitational, electric or magnetic; more precisely defined as a physical quantity that has a value at each point in space

#### Scalar field

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an assignment of a scalar to each point in a region in the space

Vector field an assignment of a vector to each point in a region in the space

## Field line

a line drawn to represent the strength of a field with arrows to indicate direction. Field lines that are closer together indicate a stronger field.

#### Monopole

an object that is either the source of a field (meaning the field lines point away from it) or the sink of a field (meaning the field lines point towards it)

#### Dipole

a source and a sink of a field that are paired together

# Gravitational monopole

mass, as it always has the field lines pointing towards it, because gravity is an attractive force A field is more precisely defined as a physical quantity that has a value at each point in space. For example, in various sports, 'heat maps' are used as an indicator of the range and frequency of a player's movement during a game (Figure 3A–3). They provide coaches with valuable analytical information. This is an example of a scalar field.

In contrast, gravitational, electric and magnetic fields are all **vector fields**; they give both an indication of the value of the strength of the field at each point in space and also the direction of the field at that point. For example, the arrows in the two-dimensional representation of the magnetic field of a bar magnet (Figure 3A–4) show the direction of the field and the density of the lines (how close together the lines are) indicates the relative strength of the field.

The conventions for vector fields are that the **field lines** are drawn with arrows to indicate the direction of the field; field lines that are closer together indicate a stronger field and field lines further apart indicate a weaker field. Field lines never intersect or touch, if they did it would indicate a point where the field would point in two different directions simultaneously.

A **monopole** is an object that is either the source of a field (meaning the field lines point away from it, see Figure 3A–5 left) or the sink of a field (meaning the field lines point towards it, see Figure 3A–5 right), but not both. A **dipole** describes a source and a sink that are paired together, as in the diagram of the magnetic field of the bar magnet (Figure 3A–4). Mass is a **gravitational monopole** and looks like Figure 3A–5 on the right, as gravity is always an attractive force.



**Figure 3A–3** An impression of a heat map depicting the range and frequency of a soccer player's movement during a game. Warm colours (red, orange, yellow) represent areas with higher frequency of occupation while the cool colours (green, blue) represent areas with lower frequency of occupation.



**Figure 3A–4** The arrows in the two-dimensional representation of the magnetic field of a bar magnet show the direction of the magnetic field, while the density of the lines (how close together the lines are) indicates the relative strength of the magnetic field.



**Figure 3A–5** A monopole is an object that is either the source of a field (left), meaning the field lines point away from it, or the sink of a field (right), meaning the field lines point towards it. Mass is a gravitational monopole, as gravity is always an attractive force (right).

For a mass, M, there is a gravitational field in the space around it (Figure 3A–6 left) such that if a second mass, m, is placed in that space, it will experience a gravitational force,  $F_{a}$ , towards mass *M*. This is shown in Figure 3A–6 below.





A gravitational field is the force field that exists in the space around every mass or group of masses. This field extends out in all directions, but the magnitude of the gravitational field decreases as the distance from the object increases. Because the gravitational field gets weaker the further out it is from the mass, the field is described as a non-uniform gravitational field.

The gravitational field strength is

measured in units of force per mass, newtons per kilogram (N kg<sup>-1</sup>). It is also given its own special symbol, g. Figure 3A–7 shows the circles of equal gravitational field strength encircling Earth.

Close to Earth's surface, the

gravitational field can be approximated as being uniform. A uniform gravitational field is shown in Figure 3A–8. A uniform field is a field that has a constant strength and direction at all points. When representing a uniform field, all lines are evenly spaced and parallel

to each other. As Figure 3A-8 shows, the parallel property of the forces is an approximation that holds for force lines near each other. In fact, they point very slightly towards each other, but this can be ignored for most localised practical purposes.



Figure 3A-7 Circles of equal gravitational field

of the force of gravity (arrows)

strength (dashed lines) encircling Earth and direction

Figure 3A-8 An example of a uniform gravitational field close to Earth's surface



a gravitational field where the strength and direction of the field vary

#### Gravitational

field strength the strength of gravity measured in newtons per kilogram (N kg<sup>-1</sup>)

# symbol used to

represent the gravitational field strength

#### Uniform gravitational field

a field that has a constant strength and direction at all points

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# Gravitational

field

the region around an object where other objects will experience a gravitational force

Gravitational force the force of attraction acting between two

objects that have mass

ISBN 978-1-009-25901-9

Eddy de Jong et al

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The average magnitude of the uniform gravitational field strength measured across the surface of Earth is 9.81 N kg<sup>-1</sup>. On the surface of Earth's Moon, the average value of the gravitational field strength is  $1.61 \text{ N kg}^{-1}$ . Table 3A-1 shows the average surface gravitational field strength, *g*, for the planets, our Moon and the Sun in our solar system, in ascending order.



**Figure 3A–9** Astronaut Buzz Aldrin stands beside the lunar module on the Moon, where the gravitational field strength is only  $1.61 \text{ N kg}^{-1}$  (~ one-sixth of that on the surface of Earth)

The gravitational field, *g*, varies by about 0.5% depending on the location on Earth's surface. It is 9.77  $N kg^{-1}$  at the top of Mount Everest and 9.83 N kg<sup>-1</sup> at the poles. Local variations of a few parts per million are caused by variations in the density of Earth's crust. For example, the field is slightly stronger than average over substantial subterranean lead deposits. Large caverns that may be filled with natural gas have a slightly weaker gravitational field. Geologists and prospectors of oil and minerals make extremely precise measurements of Earth's gravitational field using a gravimeter to predict what may be beneath the surface.

Table 3A-1Surface gravitational field strength forthe planets, our Moon and the Sun in our SolarSystem, in ascending order

Body	g (N kg <sup>-1</sup> )
Moon	1.6
Mercury	3.6
Mars	3.7
Uranus	8.7
Venus	8.9
Saturn	9.0
Earth	9.8
Neptune	11.0
Jupiter	23.1
Sun	274

Source: https://cambridge.edu.au/redirect/10075

Gravimeter

used for petroleum and mineral prospecting, seismology, geodesy and other geophysical research. Its fundamental purpose is to map the gravity field in space and time.

## Using gravity mapping to study climate change

NASA has mapped Earth's gravity (Figure 3A–10) using the GRACE (Gravity Recovery and Climate Experiment) twin spacecraft that flew in tandem for 15 years around Earth in a polar orbit with a **period** of 99 minutes and completed almost 80 000 orbits.

Its mission was to study key changes in the planet's waters, ice sheets and the solid Earth.

GRACE measured changes in the local pull of gravity as water shifts around Earth due to changing seasons, weather and climate processes. Among its innovations, GRACE has monitored the loss of ice mass from Earth's ice sheets, improved understanding of the processes responsible for sea level rise and ocean circulation, provided insights into where global groundwater resources may be shrinking or growing and where dry soils are contributing to drought, and monitored changes in the solid Earth.



**Figure 3A–10** NASA's mapping of variations in Earth's gravitational field. Mountain ranges, such as the Himalayas, have a high concentration of mass and therefore a stronger gravity field. Warm colours (red, orange, yellow) represent areas with stronger gravity. Cool colours (green, blue) represent areas with weaker gravity.

# Check-in questions – Set 1

- 1 Why are gravitational fields designated as vector fields? Give an example.
- **2** What is a uniform gravitational field?
- **3** Why did NASA use the GRACE satellites to map minute changes in Earth's gravity?

Period the time it takes for one complete orbit

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Inverse square law the relationship between two variables where one is proportional to the reciprocal of the square of the other

#### The inverse square law

In a three-dimensional space, point source radiators of energy (e.g. light from the Sun) that spread uniformly in all directions, follow an inverse square law (Figure 3A–11). Gravity is no exception.



**Figure 3A–11** Light radiating out from a source is shown passing through 'frames' at single, double and triple distances; the individual frames are the same size, and so they represent units of area. Close to the source, at a radius of r = 1, the light energy intensity is *E*, but further away, at r = 3, the light energy intensity is now one-ninth as strong  $(\frac{E}{3^2})$ .

The strength of a gravitational field, g, decreases with the square of the distance, r, from the mass, M, that is creating the gravitational field (Figure 3A–12). This was demonstrated by Isaac Newton in 1666 by analysing the data Johannes Kepler has amassed concerning the orbits of the planets around the Sun. This is expressed in Formula 3A–1.

# Formula 3A–1 Gravitational field strength

$$g \propto \frac{M}{r^2}$$
 or  $g = G \frac{M}{r^2}$ 

Where:

g = Gravitational field strength (N kg<sup>-1</sup> or m s<sup>-2</sup>) G = Universal gravitational constant, 6.67 × 10<sup>-11</sup> N m<sup>2</sup> kg<sup>-2</sup> M = Mass (kg) r = Distance from the mass (m)

Substituting data values of 6.37 × 10<sup>6</sup> m for the radius of Earth,  $R_{\rm E}$ , and 5.97 × 10<sup>24</sup> kg for the mass of Earth into Formula 3A–1,  $g = G \frac{M}{r^2}$ , gives:

$$g = (6.67 \times 10^{-11}) \frac{(5.97 \times 10^{24})}{(6.37 \times 10^{6})^{2}}$$
  
= 9.81 N kg<sup>-1</sup>

which is a familiar number and is equivalent to the average gravitational field strength measured at Earth's surface.

Gravitational constant, G the universal gravitational constant. It has a value of

 $6.67 \times 10^{-11}$ 

N m<sup>2</sup> kg<sup>-2</sup>

Treating the mass of Earth as being a **point mass**, then at Earth's surface, which is at radius  $R_{\rm E}$  from the centre of Earth, the gravitational field is *g* and equal to 9.81 N kg<sup>-1</sup>. Using the inverse square law at two Earth radii  $(2R_{\rm E})$ ,  $g = \frac{9.81}{4} = 2.45$  N kg<sup>-1</sup> and at three Earth radii  $(3R_{\rm E})$ ,  $g = \frac{9.81}{9} = 1.09$  N kg<sup>-1</sup>.

Point mass an ideal situation in which all of the mass on an object is considered to be concentrated at a single point



**Figure 3A–12** Gravity from a point mass varies with the inverse square law as you move away from it. This diagram is similar to Figure 3A–11, and the individual frames are, again, the same size. At Earth's radius,  $R_E$ , the gravitational field, g, is 9.81 N kg<sup>-1</sup>. Further away, at a distance of  $3R_E$ , the gravitational field is now one-ninth as strong

 $(\frac{g}{3^2})$ , so g is 1.09 N kg<sup>-1</sup>.

The International Space Station (Figure 3A–13) is in orbit at a height of 408 km above Earth's surface. It is in a weaker gravitational field than Earth's surface gravitational field of 9.81 N kg<sup>-1</sup>. The formula  $g = G \frac{M}{r^2}$  can be used to determine the exact value of the gravitational field strength at the International Space Station. This is shown in Worked example 3A–1.



**Figure 3A–13** The International Space Station is in orbit at a height of 408 km above Earth's surface and therefore in a slightly weaker gravitational field.

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#### Worked example 3A–1 Calculating gravitational field strength

Calculate the magnitude of the gravitational field at the International Space Station (ISS), in orbit at a height of 408 km above Earth's surface. Use the following data:

Radius of Earth,  $R_{\rm E} = 6.37 \times 10^6$  m Mass of Earth,  $M_{\rm E} = 5.97 \times 10^{24}$  kg  $G = 6.67 \times 10^{-11}$  N m<sup>2</sup> kg<sup>-2</sup>

Solution

Determine the **orbital radius** in metres.

orbital radius of ISS =  $(6.37 \times 10^6) + (4.08 \times 10^5) = 6.778 \times 10^6$  m

Substitute the data into the equation for the gravitational field strength.

$$g = G \frac{M}{r^2}$$
  
=  $\left(6.67 \times 10^{-11}\right) \frac{\left(5.97 \times 10^{24}\right)}{\left(6.778 \times 10^6\right)^2}$   
= 8.67 N kg<sup>-1</sup>

Figure 3A–14 shows the graph of the gravitational field strength plotted against distance above Earth's surface to a distance of ~6 Earth radii ( $6R_E$ ). Earth is treated as a point mass here. Note that at  $2R_E$ , the gravitational field strength is now one-quarter of that at Earth's surface (2.45 Nkg<sup>-1</sup>) and at  $3R_F$ , it is one-ninth (1.09 Nkg<sup>-1</sup>), as shown previously.





**Figure 3A–14** Graph of the gravitational field–distance for Earth (value of acceleration due to gravity, *g*, plotted against distance, *R*, above Earth's surface, in units of Earth radii,  $R_{\rm E}$ )

# Force due to gravity

The gravitational force acting on an object can be determined by multiplying the mass of the object by the gravitational field strength at that point ( $F_g = mg$ ).

In a uniform gravitational field close to Earth's surface where  $g = 9.8 \text{ N kg}^{-1}$ , it is easy to calculate the force acting on any mass. See Worked example 3A–2.

Orbital radius the distance that the satellite is from the centre of mass of the body it is orbiting
#### Worked example 3A–2 Calculating gravitational force

Calculate the gravitational force acting on a 60.0 kg physics student close to Earth's surface.

$$F_{g} = mg$$
  
= (60)(9.8  
= 588 N

If the gravitational force is the only force acting on an object ( $F_g = F_{net}$ ), then mg = ma. This means that g = a (that is, the gravitational field strength and the acceleration are equivalent). For this reason, g also gives the size of the acceleration due to gravity; the unit of m s<sup>-2</sup> is considered an equivalent unit to Nkg<sup>-1</sup>.

If no other forces such as air resistance are acting, all objects close to Earth's surface will fall at the same acceleration rate regardless of their mass. Galileo originally discovered this in the mid-17th century. This is easily demonstrated by dropping a coin and a feather in an air chamber and then in a vacuum chamber (Figure 3A–15). In air, the coin easily beats the feather because of the air resistance acting on the feather. However, when virtually all the air is removed from the chamber, both land at the same time. This was demonstrated on the airless Moon when astronaut David Scott dropped a feather and a hammer at the same time. Both objects fell at the same rate and landed on the Moon's surface at the same time. A video showing this experiment can be found by searching 'David Scott does the feather hammer experiment on the moon' on the internet.

#### The normal force in gravitational fields

We do not directly feel the force of gravity,  $F_{\rm g}$ . Instead, we feel normal forces that give us a sense of gravity. The **normal force**,  $F_{\rm N'}$  is what keeps us from sinking into the ground; we experience the force of gravity and yet if we are on firm ground we aren't pulled by gravity into Earth's crust. This is because the object we stand upon provides an equal but opposite force on our feet:  $F_{\rm N} = -F_{\rm g}$  (Figure 3A–16).



**Figure 3A–15** Removing the effect of air resistance shows that a coin and feather both accelerate at a rate of  $9.8 \text{ m s}^{-2}$ .



**Figure 3A–16** We do not fall through the solid floor because the normal force,  $F_{\rm N}$ , provides an equal and opposite force to the gravity force,  $F_{\rm g}$ , pulling us down:  $F_{\rm N} = -F_{\rm g}$ .



in a gravitational field. Equivalent to the gravitational field strength; measured in m s<sup>-2</sup>

Acceleration due

#### NOTE

Aristotle's teaching that heavier objects alwavs fall faster than light ones was accepted for almost 2000 years. It is an idea still commonly held today. What is not commonly understood is that it is only the effects of air resistance that produce a variation in the rate at which objects can accelerate down towards Earth.

Normal force

the force that a surface applies to a body in contact with it. The force is always applied perpendicular to the surface and prevents the body falling through the surface.

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# Check-in questions – Set 2

1 An asteroid is observed at a distance of 9.00  $R_{\rm F}$  above the surface of Earth. Calculate the strength of the gravitational field acting on the asteroid.

Use g = 9.81 N kg<sup>-1</sup> at Earth's surface.

- 2 Two different masses, a 1.0 kg steel ball and a 2.0 kg steel ball, are dropped from a small height above the ground and are observed to land at the same time.
  - **a** Why is this so?
  - **b** What is an important consideration that makes them land at the same time?
- 3 Explain in terms of forces why do we not fall through a strong solid floor.

#### Newton's law of universal gravitation

Combining the general formula for gravitational field strength (Formula 3A–1),  $g = G \frac{M}{r^2}$ , with the formula for the gravitational force acting on a mass,  $F_g = mg$ , gives Formula 3A–2.

#### Newton's law of universal gravitation the attractive gravitational force between two masses, whose centres of mass are a distance r apart, is directly proportional to the product of their masses and inversely proportional to the square of the distance between the masses

#### Newton's third law

when object A exerts a force on object B. object B exerts an equal and opposite reaction force on object A



 $F_{\rm g} = G \frac{m_1 m_2}{r^2}$ 

Formula 3A–2 Newton's law of universal gravitation

Where:

 $F_{\sigma}$  = Force due to gravity (N)

- $\ddot{G}$  = Universal gravitational constant, 6.67 × 10<sup>-11</sup> N m<sup>2</sup>kg<sup>-2</sup>
- $m_1$  = Mass of the first object (kg)
- $m_2$  = Mass of the second object (kg)
- r = Distance between the centres of mass of  $m_1$  and  $m_2$  (m)

#### This is known as Newton's law of universal

gravitation, which establishes the magnitude of the gravitational force acting between any two masses  $(m_1 \text{ and } m_2)$  whose centres of mass are a distance r apart (Figure 3A-17).

Note that the forces acting on both masses are exactly the same magnitude but in opposite directions. This is a direct consequence of Newton's third law. It is also why Newton's discovery was not just that Earth's gravity created a force on an apple that pulled it straight down to Earth, but that the apple's gravity created an equal force on Earth to pull Earth straight up! The apple will accelerate down to Earth at 9.81 m s<sup>-2</sup>, while Earth will accelerate up towards the apple with an acceleration of  $1.67 \times 10^{-24}$  m s<sup>-2</sup>. This interaction



Figure 3A-17 Newton's law of universal gravitation states that the magnitude of the gravitational force acting between any two masses,  $m_1$  and  $m_2$ , whose centres of mass are a distance, r, apart is given by  $F_{\rm g} = G \frac{m_1 m_2}{r^2}$ 

is not practically detectable from the point of view of seeing Earth accelerate upwards towards the apple. However, considering the interaction between the Moon and Earth; the force of Earth on the Moon causes it to orbit Earth, while the equal size force of the Moon on Earth is noticeable in the tides in large bodies of water on Earth.

Newton believed that the force of attraction between Earth and the apple, and Earth and the Moon was the same kind of force. He reasoned that the Moon continuously falls towards Earth instead of travelling off into space (Figure 3A-18). Earth's gravity provides a centripetal force on the Moon that maintains its circular path around Earth. The Moon's centripetal acceleration towards Earth can be calculated by observing its orbit. Newton calculated the centripetal acceleration of the Moon from observation. He used the value of 60 for the ratio of the radius, r, of the Moon's orbit to Earth's radius, which gave him a figure equivalent to  $3.82 \times 10^8$  m. He used 27 days, 7 hours, 43 minutes for the Moon's period of orbit converted into  $2.36 \times 10^6$  seconds. Therefore, the centripetal acceleration of the Moon as calculated by Newton is:

$$a = \frac{4\pi^2 r}{T^2}$$
  
=  $\frac{4\pi^2 (3.82 \times 10^8)}{(2.36 \times 10^6)^2}$   
=  $2.72 \times 10^{-3} \text{ m s}^{-2}$ 

This value of the centripetal acceleration of the Moon that Newton calculated helped him to convince his colleagues that the force of gravity on the apple and the Moon was the same force, but just diluted by distance in the case of the Moon. In fact, Newton reasoned that it had to be an inverse square dependence on separation, because:

$$\frac{g_{\text{moon}}}{g_{\text{apple}}} = \frac{2.72 \times 10^{-3}}{9.8}$$
$$= \frac{1}{3600}$$
$$= \frac{1}{60^2}$$

Path of Moon Moon if it did not 'fall' Path of 'falling' Moon Figure 3A–18 The gravitational pull of Earth



In a similar manner, the motion of the planets around the Sun, and the interaction of the planets on each other, could then be understood using Newton's law of universal gravitation.

The gravitational attractions of the Sun on the planets and the planets on each other explained almost all the features of their motions. However, the motion of Uranus could not be explained by the gravitational interactions caused by Saturn, Jupiter and the other planets. Two 19th-century astronomers, John Couch Adams and Urbain-Jean-Joseph Le Verrier, independently deduced the existence of an unseen eighth planet that could explain the observed discrepancies. Using Newton's law of universal gravitation, they independently calculated the position of this 'new' planet. The new planet, which could only be observed using telescopes, was found in 1846 in almost the same predicted position and was named Neptune, after the Roman god of the sea.



Figure 3A–19 The planet Neptune's very existence was discovered using Newton's law of universal gravitation.

Centripetal



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**Centripetal force** 

a net force directed to the

centre of the

circular path of an object

Newton's gravitational discoveries gave a theoretical underpinning to Kepler's detailed planetary observations. Table 3A–2 gives some modern values for various bodies in our solar system.

Body	Mass (kg)	Radius (m)	Mean orbital radius (m)	Orbital period (s)
Sun	$1.99 \times 10^{30}$	$6.96 \times 10^{8}$	_	-
Mercury	$3.30 \times 10^{23}$	$2.44 \times 10^{6}$	$5.79 \times 10^{10}$	$7.60 \times 10^{6}$
Venus	4.87× 10 <sup>24</sup>	$6.05 \times 10^{6}$	$1.08 \times 10^{11}$	$1.94 \times 10^{7}$
Earth	$5.97 \times 10^{24}$	$6.37 \times 10^{6}$	$1.50 \times 10^{11}$	$3.16 \times 10^{7}$
Mars	$6.42 \times 10^{23}$	$3.40 \times 10^{6}$	$2.28 \times 10^{11}$	$5.94 \times 10^{7}$
Jupiter	$1.90 \times 10^{27}$	$7.15 \times 10^{7}$	$7.78 \times 10^{11}$	$3.74 \times 10^{8}$
Saturn	$5.68 \times 10^{26}$	$6.03 \times 10^{7}$	$1.43 \times 10^{12}$	$9.29 \times 10^{8}$
Uranus	$8.68 \times 10^{25}$	$2.59 \times 10^{7}$	$2.87 \times 10^{12}$	$2.64 \times 10^{9}$
Neptune	$1.02 \times 10^{26}$	$2.48 \times 10^{7}$	$4.50 \times 10^{12}$	$5.17 \times 10^{9}$
Pluto	$1.30 \times 10^{22}$	$1.88 \times 10^{6}$	$5.91 \times 10^{12}$	$7.82 \times 10^{9}$

Table 3A-2 Solar system data on mass, radius of body, mean radius of orbit and period of revolution

However, precise measurements of the motion of the innermost planet, Mercury, showed it 'wobbled' faster than could be accounted for from the gravitational interactions of all the other known planets. Astronomers searched in vain for other bodies that could explain this discrepancy that threatened to undermine Newton's law of universal gravitation. The issue was only resolved in 1915 with Einstein's publication of his theory of general relativity. Einstein's theory precisely predicted the observed behaviour of Mercury's orbit and refined Newton's law of universal gravitation.



**CHAPTER 9** 

#### Check-in questions – Set 3

- 1 Earth exerts a gravitational force on the Moon, keeping it in its orbit. With reference to Newton's third law, the reaction to this force is the
  - A centripetal force on the Moon.
  - **B** nearly circular orbit of the Moon.
  - **C** gravitational force on Earth by the Moon.
  - **D** tides due to the Moon.
- **2** a Calculate the magnitude of the attractive force between two physics students, modelled as point masses, each of mass 60 kg, sitting 1.0 m apart.
  - **b** How does this compare to the magnitude of the attractive force of one of these students to Earth? Use mass of Earth,  $M_{\rm E} = 6.0 \times 10^{24}$  kg.

VIDEO 3A-2 SKILLS: USING THE INVERSE SQUARE LAW FOR GRAVITY

# **3A SKILLS**

#### Using the inverse square law for gravity

The inverse square law for gravity is still very useful in determining the gravitational field strength at another location if the gravitational field strength at a particular location is known, but the mass of the object creating the field is not known.

For example, to find the gravitational field strength of a satellite that has been placed in orbit around the planet Mars, only Mars' surface gravitational field strength and the ratio of radii is needed. The satellite is launched into orbit around Mars at an altitude of 300 km above the surface of Mars.

Mars' surface gravitational field strength is 3.72 Nkg<sup>-1</sup>. The radius of Mars is 3390 km.

To determine the magnitude of Mars' gravity at the orbital height of the satellite, use the ratio:

$$g \propto \frac{M}{r^2}$$

First, find the radius of orbit,  $r_0$ :

 $r_0 = 3390 + 300 = 3690 \text{ km}$ 

Using the radius of Mars,  $r_{\rm m}$ , and the radius of the orbiting satellite,  $r_{\rm o}$ , determine the gravitational field strength by substituting into:

$$g \propto \frac{M}{r^2}$$

Notice the mass of Mars, *M*, can be ignored as it cancels out in the equations.

As  $\frac{g_o}{g_m} = (\frac{r_m}{r_o}) = \left(\frac{3390}{3690}\right)^2$ , the gravitational field strength at the radius of the orbiting

satellite,  $r_0$ , is:

$$g_{\rm o} = 3.72 \left(\frac{3390}{3690}\right)^2 = 3.14 \,\,\mathrm{N\,kg^{-1}}$$



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# Section 3A questions

## **Multiple-choice questions**

- 1 Which one of the following correctly describes Earth's gravitational field?
  - **A** It is a repulsive dipole.
  - **B** It is an attractive dipole.
  - **C** It is a repulsive monopole.
  - **D** It is an attractive monopole.
- 2 How far above the Moon's surface will the gravitational force on a mass be one-quarter of that on the surface? Take the Moon's radius as  $R_{M}$ .

$$\begin{array}{c} \mathbf{A} \quad \frac{R_{\rm M}}{2} \\ \mathbf{B} \quad R_{\rm M} \end{array}$$

- $C 2R_{\rm M}$
- **D**  $4R_{\rm M}$
- **3** Use the data in the table below to answer the following question.

Mass of Mercury	$3.34 \times 10^{23} \text{ kg}$
Radius of Mercury	$2.44 \times 10^{6}$ m
Universal gravitational constant, G	$6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

The gravitational field strength at the surface of Mercury is close to

- **A**  $9.00 \times 10^{6} \, N \, kg^{-1}$
- **B** 9.81 N kg<sup>-1</sup>
- C 3.74 N kg<sup>-1</sup>
- **D**  $3.74 \times 10^{-2} \,\mathrm{N\,kg^{-1}}$

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- 4 Which one of the following correctly describes the force that keeps the Moon in orbit around Earth?
  - A centripetal force
  - **B** centrifugal force
  - **C** gravitational force
  - **D** centripetal force and gravitational force acting together
- 5 Jupiter's moon Ganymede is its largest satellite.

Ganymede has a mass of  $1.5 \times 10^{23}$  kg and a radius of  $2.6 \times 10^{6}$  m.

Which one of the following is closest to the magnitude of Ganymede's surface gravity?

- **A**  $0.8 \text{ m s}^{-2}$
- **B** 1.5 m s<sup>-2</sup>
- $C 3.8 \text{ m s}^{-2}$
- **D** 9.8 m s<sup>-2</sup>

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#### **Short-answer questions**

- 6 Draw a diagram depicting the gravitational field monopole surrounding a spherical mass.
- 7 Determine each of the gravitational forces acting on a 60 kg person when standing on the surface of Earth, the Moon, Mars and Neptune. Use the gravitational field strengths for the planets and the Moon shown in Table 3A–1.

**3A GRAVITY AND GRAVITATIONAL FIELDS** 

- 8 The diagram shows a satellite in orbit above Earth at an altitude of 250 km. Take the radius of Earth as 6400 km, the mass of Earth as  $6.0 \times 10^{24}$  kg and the value of *G* as  $6.67 \times 10^{-11}$  N m<sup>2</sup>kg<sup>-2</sup>.
  - a Calculate the magnitude of the acceleration due to gravity at this orbital height.
  - **b** Calculate the orbital speed of the satellite.
  - c Calculate the period of the satellite.
- 9 A box of mass 20 kg sits at rest on the floor as shown.
  - a What is the magnitude and direction of the normal force acting on the box? Use g = 9.81 Nkg<sup>-1</sup> at Earth's surface.



- **b** Draw on a copy of the diagram where the gravitational force  $(F_g)$  and the normal force  $(F_N)$  act on the box.
- 10 The astronaut David Scott simultaneously dropped a feather and a hammer from the same height while standing on the Moon. Both objects landed on the Moon's surface at exactly the same time. The mass of the hammer was 2000 times the mass of the feather.
  - **a** Explain why they both landed at the same time on the Moon's surface, despite the gravitational force acting on the hammer being 2000 times greater than the gravitational force acting on the feather.
  - **b** Why would this experiment not work if you tried it on the school oval? Which object would land first and why?
- **11** The gravitational field at the surface of the Sun is 242 N kg<sup>-1</sup>. A comet is at a distance from the centre of the Sun by a factor of 100 times the radius of the Sun. What is the strength of the gravitational field from the Sun at the comet's location?
- 12 Calculate the gravitational field strength on the surface of Mars if it has a mass of  $6.4 \times 10^{23}$  kg and a radius of 3400 km.
- 13 a Use Newton's law of universal gravitation (Formula 3A–2) and the data in Table 3A–2 to calculate the magnitude of the force that Earth exerts on the Moon.
  - **b** How does your answer compare to the magnitude of the force that the Moon exerts on Earth? Explain.
- 14 A spacecraft with astronauts on board is in orbit around Mars at an altitude of  $1.6 \times 10^6$  m above the surface of Mars.

The mass of Mars is  $6.4\times10^{23}\,\rm kg$  and its radius is  $3.4\times10^6$  m.

Take the universal gravitational constant, *G*, to be  $6.7 \times 10^{-11}$  N m<sup>2</sup> kg<sup>-2</sup>.

The mass of the spacecraft is  $2.0\times 10^4$  kg.

- **a** Calculate the period of orbit of the spacecraft around Mars. Show your working.
- **b** The altitude of the spacecraft above the surface of Mars is doubled so that the spacecraft is now in a new stable orbit.

Will the speed of the spacecraft be greater, the same or lower in this new orbit? Explain your reasoning.

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# Gravitational potential energy

#### **Study Design:**

- Investigate and compare theoretically and practically gravitational fields and electrical fields about a point mass or charge (positive or negative) with reference to:
  - potential energy changes (qualitative) associated with a point mass or charge moving in the field
- Analyse the use of gravitational fields to accelerate mass, including:
  - ► potential energy changes in a uniform gravitational field:  $E_g = mg\Delta h$
- Analyse the change in gravitational potential energy from area under a force vs distance graph and area under a field vs distance graph multiplied by mass

#### **Glossary:**

Gravitational potential energy Work

#### ENGAGE Roller-c

# Roller-coasters

Roller-coasters entertain riders by providing them with stomach-churning accelerations, high speeds, dramatic climbs, huge drops and twists through inverted loops, barrel rolls, and banked turns. Roller-coasters are designed with both high precision and safety to juggle kinetic energy, gravitational potential energy and energy losses due to friction.

One such ride is the Kingda Ka, the tallest roller-coaster in the world and the fastest in North America. The roller-coaster leaves the station going from 0 to 200 km h<sup>-1</sup> in 3.5 seconds (maximum kinetic energy at this point) and then shoots almost straight up to a height of 142 m (maximum gravitational potential energy at this point – see the coaster heading to the top in Figure 3B–1). Once at the top, the roller-coaster plummets straight back down in a 270-degree spiral and finishes off with a 40 m camel hump.



**Figure 3B–1** Kingda Ka, the tallest rollercoaster in the world, juggles kinetic energy, gravitational potential energy and energy losses due to friction to provide safe entertaining rides.

### EXPLAIN

# Gravitational potential energy in a uniform field

The concept of gravitational potential energy,  $E_{\rm g}$ , will be familiar to you from Chapter 2.

Land diving is a ritual performed by the men of the southern part of Pentecost Island, Vanuatu. Men jump off wooden towers up to 30 m high from jumping platforms constructed at the 10 m, 20 m and 30 m positions. They dive with just two tree vines wrapped around the ankles, designed to pull them up before they crash into the ground (Figure 3B–2). The Pentecost Islander land divers aim to get as close as they can to the ground without actually hitting it. Therefore, the higher the jump, the greater the gravitational potential energy a land diver will have. This is shown in Formula 3B–1.

 $E_g = mg\Delta h$ 

# Formula 3B–1 Gravitational potential energy

- $E_g$  = Gravitational potential energy (J)
- m = Mass (kg)
- g = Acceleration due to gravity, 9.8 m s<sup>-2</sup> on the surface of Earth
- $\Delta h$  = Change in height (m)



**Figure 3B–2** Pentecost Islander land divers jump off wooden towers up to 30 m high, with just two tree vines wrapped around the ankles designed to pull them up before they crash into the ground.



Gravitational potential energy the amount of energy an object has stored due to its position in a gravitational field

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#### Worked example 3B–1 Calculating gravitational potential energy

Three land divers climb the tower shown in Figure 3B–2.

Kaikoa, of mass 80 kg, goes to the 10 m platform. Aukia, of mass 40 kg, goes to the 20 m platform. Laki, of mass 60 kg, goes to the 30 m platform.

Use  $g = 10 \text{ m s}^{-2}$  at the surface of Earth.

- a Calculate the gravitational potential energy for each land diver.
- **b** Explain the  $E_g$  values for Kiakoa and Aukia, given that Aukia is at twice the height of Kaikoa.

Solution

- **a** Use  $E_g = mg\Delta h$ . Kaikoa:  $E_g = 80 \times 10 \times 10 = 8000$  J Aukia:  $E_g = 40 \times 10 \times 20 = 8000$  J Laki:  $E_g = 60 \times 10 \times 30 = 18000$  J
- **b** Although Aukia is at twice the height of Kaikoa, he is half of his mass. Therefore, they both have the same gravitational potential energy.

Formula 3B–1 works well in uniform gravitational fields, such as those close to Earth's surface ( $g = 9.8 \text{ m s}^{-2}$ ) or close to the Moon's surface ( $g = 1.6 \text{ m s}^{-2}$ ).

The formula for gravitational potential energy comes from the concept of the **work** done against the force of gravity being converted into potential energy:

#### Formula 3B–2 Change in gravitational potential energy

$$\Delta E_{\sigma} = W = Fd = mga$$

Where:

 $\Delta E_g$  = Change in gravitational potential energy (J)

 $\vec{W} = Work done (J)$ 

 $F_{\sigma}$  = Force due to gravity (N)

- d = Distance moved in the gravitational field (m)
- m = Mass (kg)
- g = Acceleration due to gravity, 9.8 m s<sup>-2</sup> on the surface of Earth

The force–distance graph for the land diver Laki (F = mg = 600 N) as he climbs the tower distance of 30 m is shown in Figure 3B–3. The area under the graph ( $600 \times 30 = 18000$  J) indicates work done and therefore also gives the change in gravitational potential energy.



**Figure 3B–3** The area under the force–distance graph (600 N × 30 m = 18000 J) indicates the work done and is therefore also the change in Laki's gravitational potential energy,  $\Delta E_g$ .

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Work the amount of energy

of energy transferred from one object or system to another

# Calculating work done and change in gravitational potential energy for non-uniform gravitational fields

For a mass that moves large enough distances away from Earth, where the gravitational field can no longer be considered uniform, the change in gravitational potential energy has to be determined by estimating the area under a gravitational force–distance graph.

When the force due to gravity is the only force acting on an object, the work done, W, and the change in gravitational potential energy,  $\Delta E_{\rm g}$ , are equal in magnitude.  $\Delta E_{\rm g}$  is found as the area under a force–distance graph using a method of approximation by counting squares.

Figure 3B–4 shows the force acting on a 20 kg mass as it moves from a position 8000 km to a position 16000 km from the centre of Earth.

To estimate  $\Delta E_{\rm g}$ , it is necessary to count squares, adding up estimated fractions of squares. The area is ~12.5 squares.

Each square represents 20 N  $\times$  2  $\times$  10  $^{6}$  m = 4.0  $\times$  10  $^{7}$  J or 40 MJ.

Total  $\Delta E_{\rm g}$  = 12.5  $\times$  4.0  $\times$  10^7 = 50  $\times$  10^7 J or 500 MJ.

Usually, there is an allowance for a small variation in the number of squares counted, so a range of values from  $45 \times 10^7$  J to  $55 \times 10^7$  J is acceptable.

#### Using area under a gravitational field-distance graph multiplied by mass

Similarly, the change in gravitational potential energy can be calculated from the area under a

field–distance graph multiplied by the mass of the object that is moving through the field (Figure 3B–5). Both of these methods of determining gravitational potential energy are necessary when considering the motion of objects like rockets that travel to high altitudes or natural satellites, such as the Moon, and artificial satellites, such as the International Space Station.

Gravitational field–distance graphs are independent of the mass of an object within the field, and only depend on the mass of the gravitational body creating the field.

The same procedure is used for determining change in gravitational potential energy for Figure 3B–4, except that it is important that the mass of the object in the gravitational field is accounted for in the calculations.



**Figure 3B–5** Graph of the gravitational field strength plotted against distance above Earth's surface ( $R_E$  represents the radius of Earth)



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## Check-in questions – Set 1

1 Jane lifts a 20.0 kg barbell in the gym vertically up by a distance of 2.00 m. Calculate the gain in gravitational potential energy for the barbell.

Use g = 9.81 N kg<sup>-1</sup> at Earth's surface.

- **2** Use the gravitational field–distance graph for Earth (Figure 3B–5) to answer the questions below.
  - a A 20 kg mass moves from a point  $12 \times 10^6$  m from the centre of Earth to a point  $20 \times 10^6$  m from the centre of Earth. Estimate the change in gravitational potential energy for the 20 kg mass.
  - **b** How would your answer change if it were a 2.0 tonne satellite that was moved instead of a 20 kg mass?

VIDEO 3B –1 Skills: Interpreting Gravitational Force–distance Graphs

# **3B SKILLS**

#### Interpreting gravitational force-distance graphs

#### Question

A 700 kg spacecraft is launched from Earth on a journey to a distant planet. The graph of the gravitational force acting on the 700 kg spacecraft plotted against height above Earth's surface is shown below.



- **a** From the graph, determine the approximate value of the gravitational force acting on the spacecraft at a height of  $1.5 \times 10^7$  m.
- **b** Estimate the minimum launch energy needed for the spacecraft to effectively escape Earth's gravitational attraction.

#### Solution

**a** ~500 N. See the dotted line on the graph below. Note that  $1.5 \times 10^7$  m is the same as  $15 \times 10^6$  m.



**b** The required launch energy is calculated by determining the total area under the graph, representing the change in gravitational potential energy,  $\Delta E_{\rm g}$ . Counting the highlighted squares under the graph results in ~13 squares. Each square represents work done of  $3 \times 10^9$  J (1000 N ×  $3 \times 10^6$  m). Therefore, total  $\Delta E_{\rm g} = 13 \times 3 \times 10^9 = 3.9 \times 10^{10}$  J.

Usually, there is an allowance for a variation in the number of squares counted, so a range of values from  $3.3 \times 10^{10}$  J to  $4.4 \times 10^{10}$  J, is acceptable.



# Section 3B questions

#### **Multiple-choice questions**

- 1 Which one of the following statements concerning the formula  $E_g = mg\Delta h$  is correct? A It only works for uniform fields.
  - **B** It only works on the surface of Earth.
  - **C** It only works for non-uniform fields.
  - **D** It works for both uniform and non-uniform gravitational fields.
- 2 Which one or more of the following statements is correct?

 $\Delta E_{\sigma}$  can be found directly from the area under a

- A uniform gravitational field–distance graph.
- **B** uniform gravitational force–distance graph and a non-uniform gravitational force–distance graph.
- **C** non-uniform gravitational field–distance graph.
- **D** uniform gravitational field–distance graph and a non-uniform gravitational field–distance graph.
- **3** The gravitational field strength at the surface of Mars is 3.7 N kg<sup>-1</sup>.

Which one of the following is closest to the change in gravitational potential energy when a 10 kg mass falls from 2.0 m above Mars's surface to Mars's surface?

- **A** 3.7 J
- **B** 7.4 J
- **C** 37 J
- **D** 74 J

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#### **Short-answer questions**

- 4 A 75 kg person jumps from a height of 40 m above a river using a bungee rope. Calculate the gravitational potential energy of the bungee jumper before they jump, assuming that  $E_{\rm g}$  is taken as 0 at the surface of the water. Use g = 9.81 N kg<sup>-1</sup> at Earth's surface.
- 5 Explain why, as a mass moves a large enough distance away from Earth, the change in gravitational potential energy has to be determined by estimating the area under a gravitational force-distance graph rather than using  $E_g = mg\Delta h$ .
- 6 The diagram on the right shows how Earth's gravitational field varies as a function of distance from Earth's centre ( $R_E$ represents the radius of Earth).
  - a Estimate the size of the gravitational force acting on a 200 kg satellite in orbit at a distance of  $9.0 \times 10^3$  km from the centre of Earth.
  - Estimate the amount of energy required to move this satellite from Earth's surface to its orbit height.



**7** The spacecraft *Juno* has been put into orbit around Jupiter. The table below contains information about the planet Jupiter and the spacecraft *Juno*. The diagram below shows gravitational field strength (N kg<sup>-1</sup>) as a function of distance from the centre of Jupiter.



- a Calculate the gravitational force acting on *Juno* by Jupiter when *Juno* is at a distance of  $2.0 \times 10^8$  m from the centre of Jupiter. Show your working.
- **b** Use the graph to estimate the magnitude of the change in gravitational potential energy of the spacecraft *Juno* as it moves from a distance of  $2.0 \times 10^8$  m to a distance of  $1.0 \times 10^8$  m from the centre of Jupiter. Show your working.

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# **Orbiting satellites**

#### **Study Design:**

- Apply the concepts of force due to gravity and normal force including in relation to satellites in orbit where the orbits are assumed to be uniform and circular
- Model satellite motion (artificial, Moon, planet) as uniform circular orbital motion:

$$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

#### **Glossary:**

Artificial satellite Geostationary satellite Orbital period Orbital speed



#### ENGAGE

## Newton, the apple and 'heavenly motion'

Newton was troubled by the lack of explanation for the planet's orbits around the Sun.

He was familiar with the work of Nicolaus Copernicus, who in 1542 published 'On the Revolution of the Heavenly Orbs', outlining an explanation for the observations of planetary motion with the Sun at the centre. According to Copernicus' explanation, the planets moved in circular orbits about the Sun.

In 1609, Johannes Kepler, using the accurate planetary data collected by the Danish astronomer, Tycho Brahe, determined that the orbits of the planets around the Sun were actually elliptical, that equal areas were swept out by the planets in equal times (Figure 3C–1), and that the average radius



**Figure 3C–1** Kepler determined that the planets orbiting the Sun swept out equal areas in equal times.

cubed over the period squared,  $\frac{R^3}{T^2}$ , for each planet was constant.

Newton reasoned that there must be some cause for the elliptical motion discovered by Kepler. Even more troubling to him was the circular motion of the Moon about Earth. Newton knew from his investigation of forces that there must be some sort of force at work. Both the motion of the Moon in a circular path, and of the planets in elliptical paths, required that there be some force towards Earth and the Sun respectively.

It was Newton's ability to relate the cause for the 'heavenly motion' (the orbit of the Moon about Earth), to the cause for Earthly motion (the falling of an apple to Earth), that eventually led him to his concept of universal gravitation and for us to understand how satellites can be put into orbit.

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## EXPLAIN Satellites

Newton's *System of the World* (published in 1687) contains an illustration of a thought experiment showing the various paths of a cannonball that is fired horizontally from a high mountain in a region with no air resistance (Figure 3C–2).



Newton's reasoning is as follows: a cannonball shot with a certain velocity would land at point D. Cannonballs shot with more and more velocity would land at E, F and G. If shot with an even greater velocity, the cannonball would completely encircle Earth and return to the mountain from which it was projected. The cannonball would become a satellite of Earth, just like the moon is a satellite of Earth.



more velocity (p. 400); and, because the cclestial motions are scarcely retarded by the little or no resistance of the spaces in which they are performed, to keep up the parity of cases, let us suppose either that there is no air about the earth, or at least that it is endowed with little or no power of resisting; and for the same reason that the body projected with a less velocity describes the lesser are VD, and with a greater velocity the greater are VE. and, augmenting the velocity, it goes farther and farther to F and G, if the velocity was still more and more augmented, it would reach at last quite beyond the circumference of the earth, and return to the mountain from which it was projected.

And since the areas which by this motion it describes by a radius drawn to the centre of the earth are (by Prop. 1, Book 1, *Princip. Math.*) proportional to the times in which they are described, its velocity, when it returns to the mountain, will be no less than it was at first; and, retaining the same velocity, it will describe the same curve over and over, by the same law

#### Figure 3C-2 Newton's thought experiment in his 1687 work, System of the World

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From Newton's thought experiment, we can see that the force due to gravity,  $F_{\rm g}$ , is the only force acting on a satellite in orbit. For example, the Moon, in its orbit around Earth, is constantly falling towards Earth due to the force of gravity. It constantly 'misses' hitting Earth due to its velocity, which is not only perpendicular to the gravitational field, (Figure 3C–3) but large enough to ensure that a continual circular orbit is achieved.

1B CIRCULAR MOTION

In Chapter 1 you learned that for an object moving in a circular path, the net force is the centripetal force and it always acts radially inwards (towards the centre of the circle).

For a satellite (mass *m*) undergoing



**Figure 3C–3** The Moon in its orbit around Earth is constantly falling towards Earth due to the force of gravity,  $F_g$ . It constantly 'misses' hitting Earth due to its velocity, which is not only perpendicular to the gravitational field, but large enough to ensure that a continual circular orbit is achieved.

uniform circular orbital motion, the force due gravity,  $F_{g}$ , from a body of mass M is the centripetal force,  $F_{c}$ .

#### Formula 3C-1 Force due to gravity

$$F_{\rm g} = G \frac{mM}{r^2}$$

Where:

 $F_{\sigma}$  = Force due to gravity (N)

 ${G}$  = Universal gravitational constant, 6.67 × 10<sup>-11</sup> N m<sup>2</sup>kg<sup>-2</sup>

m = Mass of satellite (kg)

M = Mass of body (kg)

r = Distance between the centres of mass of m and M (m)

#### Formula 3C–2 Centripetal force

$$F_{\rm c} = \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2}$$

Where:

 $F_c$  = Centripetal force (N)

m = Mass of satellite (kg)

 $v = \text{Orbital speed } (\text{m s}^{-1})$ 

r = Distance between the centres of mass of m and M (m)

T =Orbital period (s)

Equating Formula 3C–1 and 3C–2 gives a number of useful formulas.

Using  $G\frac{mM}{r^2} = \frac{mv^2}{r}$  gives an important expression for the **orbital speed**, shown in Formula 3C–3.

Orbital speed the speed a satellite has in a given orbit

## Formula 3C–3 Orbital speed

$$v = \sqrt{\frac{GM}{r}}$$

Where:

 $v = Orbital \text{ speed } (m s^{-1})$ 

G = Universal gravitational constant, 6.67 × 10<sup>-11</sup> N m<sup>2</sup> kg<sup>-2</sup>

M = Mass of body (kg)

r = Distance between the centres of mass of m and M (m)

Note that the orbital speed of a satellite is independent of the satellite's mass, m.

Using  $G\frac{mM}{r^2} = \frac{m4\pi^2 r}{T^2}$  gives important expressions for both the orbital radius, *r*, and the orbital period, *T*.

Rearranging  $4\pi^2 r^3 = GMT^2$ , gives the orbital radius, shown in Formula 3C–4. Finding the orbital radius, Formula 3C–4, involves taking a cube root.

# Formula 3C–4 Orbital radius

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

Where:

r = Orbital radius (m) G = Universal gravitational constant, 6.67 × 10<sup>-11</sup> N m<sup>2</sup> kg<sup>-2</sup> M = Mass of body (kg)T = Orbital period (s)

Finding the orbital period, Formula 3C–5, involves taking a square root.

## Formula 3C–5 Orbital period

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

Where:

T = Orbital period (s)r = Orbital radius (m) $G = \text{Universal gravitational constant, 6.67 \times 10^{-11} \,\text{N}\,\text{m}^2\text{kg}^{-2}}$ M = Mass of body (kg)

Note that just like the orbital speed, both the orbital radius and the orbital period of a satellite are independent of the satellite's mass, *m*.

If a satellite's orbital radius increases, its period, *T*, will increase while its speed, *v*, will decrease as  $T \propto \sqrt{r^3}$  and  $v \propto \frac{1}{\sqrt{r}}$ .

Orbital period the time a satellite takes to complete one orbit The formula  $4\pi^2 r^3 = GMT^2$  can also be rearranged to find the mass of an object that has an orbiting satellite, as shown below. Therefore, the mass of a planet such as Jupiter (Figure 3C-4) can be easily calculated by observing one of its moons (see Worked example 3C-1).

Formula 3C-6 Mass of a body with an orbiting satellite

$$M = \frac{4\pi^2 r^3}{GT^2}$$

Where:

M = Mass of body (kg)

- G = Universal gravitational constant, 6.67 × 10<sup>-11</sup> N m<sup>2</sup>kg<sup>-2</sup>
- r = Distance between the centres of mass of body and satellite (m)

T =Orbital period (s)



**Figure 3C–4** The mass of Jupiter can be easily calculated by observing the motions of its moons. This artist's impression shows Jupiter and its four largest moons, Io, Europa, Ganymede and Callisto.

#### Worked example 3C-1 Mass of Jupiter

Calculate the mass of Jupiter by observing one of its moons, Callisto.

Take the period of Callisto to be 16.7 days ( $1.443 \times 10^6$  s) and the mean radius of the orbit of Callisto to be  $1.883 \times 10^9$  m.

Use  $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ .

Solution

Substitute into the equation for mass.

$$M = \frac{4\pi^2 r^3}{GT^2}$$
$$= \frac{4\pi^2 (1.883 \times 10^9)^3}{(6.67 \times 10^{-11}) (1.443 \times 10^6)^2}$$

 $= 1.90 \times 10^{27} \text{ kg}$ 

Therefore, the mass of Jupiter is  $1.90 \times 10^{27}$  kg, which is the value given in Table 3A–2.

#### Artificial satellites

The space age began in 1957 when Sputnik 1 (Figure 3C–5) became the world's first artificial Earth satellite. It was launched into an elliptical low Earth orbit by the former Soviet Union on 4 October 1957. It orbited for three weeks before its batteries ran out. The satellite then silently continued to orbit the planet for two months before it fell back into the atmosphere on 4 January 1958.

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**Figure 3C–5** Sputnik-1 prior to launch. Its mass was 84 kg, its orbital period was 96.2 minutes and it completed 1440 orbits before burning up in Earth's atmosphere.

Currently, there are about 5000 active artificial satellites in orbit around Earth, used for a multitude of purposes including weather, navigation, the Global Positioning System (GPS), commercial enterprise, astronomy, research and communications. Figure 3C–6 shows a modern communications satellite that uses solar energy to power its batteries and run its electronic systems.

#### **Geostationary satellites**

A geostationary orbit is a special case where a satellite remains at the same location relative to Earth's surface; it must orbit directly above the equator and travel



**Figure 3C–6** A modern communications satellite that uses solar energy to power its batteries and run its electronic systems

in the same direction as Earth's rotation (Figure 3C–7). If the orbit of a satellite is inclined or polar, then it will not remain at the same point relative to a particular point on Earth. So, for the satellite to be stationary with respect to a particular point on Earth, its orbit has to be equatorial.

**Geostationary satellites** are very useful and used extensively in telecommunications and as weather satellites (as they are always above the same point on Earth).

The period of the orbit must equal the period of rotation of Earth (24 hours), which means that it completes one revolution around Earth in exactly the same amount of time that Earth completes one rotation on its own axis.

Using  $r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$  and substituting gives  $r = 4.2 \times 10^7$  m. So, altitude =  $4.2 \times 10^7 - 0.64 \times 10^7 = 3.6 \times 10^7$  m.

#### Artificial satellite any human-

made structure such as Sputnik, Hubble Space Telescope, International Space Station, Mars Orbiter etc. that is placed in orbit around a planet (like Earth or Mars) or Earth's Moon

#### Geostationary satellite a satellite whose orbit is such that it remains directly above the same point of Earth at all times

This means all geostationary satellites above a fixed point on Earth have a period of 24 hours and are at an altitude of 36000 km above Earth's equator.

This method can also be applied to satellites in synchronous orbits around the Sun, Moon or other planets.

#### NOTE

There are approximately 400 geostationary satellites currently in orbit around Earth. The average cost of a geostationary satellite is US\$250 million and it has a life span of about 7–10 years. There are only 1800 spots available for geostationary satellites to be parked in orbit around Earth at any one time because the satellites have to be safely distanced two degrees, or 1000 km, apart in order to avoid collisions and interference. A graveyard orbit, also called a junk or disposal orbit, is an orbit that lies away from common operational orbits. One significant graveyard orbit is a supersynchronous orbit well beyond the geostationary orbit. Some satellites are moved into such orbits at the end of their operational life to reduce the probability of colliding with operational spacecraft and generating space debris.



**Figure 3C–7** A geostationary orbit means the satellite remains at the same location relative to Earth's surface and it completes one revolution around Earth in exactly the same amount of time that Earth completes one rotation on its axis. A minimum of three geostationary satellites cover the whole globe.





#### The normal force on astronauts in orbiting satellites

As previously explored, we do not directly feel the force of gravity,  $F_g$ . Instead, we feel normal forces that give us a sense of gravity. The normal force,  $F_N$ , is what keeps us from falling through the floor. We experience the force of gravity because the floor we are standing on provides an equal but opposite force on our feet:  $F_N = -F_g$ .

It is a common misconception that gravity does not act on astronauts in orbit. In fact, the gravitational field strength at the International Space Station, which is at an altitude of 408 km above Earth's surface, is  $8.67 \text{ Nkg}^{-1}$ ; this is approximately 88% of the gravitational field strength on Earth's surface. This means that both the astronauts and the ISS are accelerating towards Earth at  $8.67 \text{ ms}^{-2}$ .

However, orbiting astronauts do not feel gravity acting on them because gravity is the only force (constantly) acting on them and they experience no normal force ( $F_{\rm N} = 0$ ) as they continually fall towards Earth under the influence of the gravitational field. As the International Space Station orbits Earth, it is accelerating at exactly the same rate as the astronauts, so objects inside do not appear to fall relative to the space station.

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**Figure 3C–8** NASA engineer astronaut Anne McClain on the International Space Station performing a time perception experiment. Both Anne and the ISS are continually falling towards Earth with an acceleration of 8.67 ms<sup>-2</sup> due to Earth's gravity.

# **3C SKILLS**

#### **Tackling general satellite questions**

Satellite questions can involve manipulation of one or more of the formulas given earlier in this section:

$$\begin{split} F_c &= \frac{mv^2}{r} = \frac{m4\pi^2 r}{T^2} & \text{Centripetal force (Formula 3C-2)} \\ F_g &= G \frac{mM}{r^2} = \frac{mv^2}{r} & \text{Force due to gravity (Formula 3C-1) and centripetal force} \\ F_g &= G \frac{mM}{r^2} = \frac{m4\pi^2 r}{T^2} & \text{Universal law of gravitation (Formula 3A-2) and} \\ entripetal force (Formula 3C-2) & \text{V} = \sqrt{\frac{GM}{r}} & \text{Orbital speed (Formula 3C-3)} \\ v &= \sqrt{\frac{GMT^2}{4\pi^2}} & \text{Orbital radius (Formula 3C-4)} \\ T &= \sqrt{\frac{4\pi^2 r^3}{GM}} & \text{Orbital period (Formula 3C-5)} \\ M &= \frac{4\pi^2 r^3}{GT^2} & \text{Mass of an object with an orbiting satellite (Formula 3C-6)} \end{split}$$



It is often useful to have these formulas, with worked examples, on your two double-sided A4 sheets that you can take into the examination, as they do not appear in the formula sheet that comes with the physics exam.

Note that unless data is given, as in the example question below, you will need to use the data given on the formula sheet that comes with the physics exam.

#### Question

The Ionospheric Connection Explorer (ICON) space weather satellite, constructed to study Earth's ionosphere, was launched in October 2019. ICON studies the link between space weather and Earth's weather at its orbital altitude of 600 km above Earth's surface. Assume that ICON'S orbit is a circular orbit. Use  $R_{\rm E} = 6.37 \times 10^6$  m.

- **a** Calculate the orbital radius of the ICON satellite.
- **b** Calculate the orbital period of the ICON satellite, correct to three significant figures. Show your working.
- **c** Explain how the ICON satellite maintains a stable circular orbit without the use of propulsion engines.

Adapted from VCAA 2020

#### Solution

**a** Ensure all units are in SI units (metres).

 $r = 6.37 \times 10^6 + 600 \times 10^3$ 

 $= 6.97 \times 10^{6} \text{ m}$ 

**b** Choose the correct formula and substitute.

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}} = \sqrt{\frac{4\pi^2 (6.97 \times 10^6)^3}{(6.67 \times 10^{-11})(5.98 \times 10^{24})}}$$
  
= 5788 s

$$= 5.79 \times 10^3 \,\mathrm{s}$$

Your answer has to be given to three significant figures (the same number as the data) for full marks.

You should also check if your answer is sensible: 5788 seconds = 96.5 minutes.

The shortest period a satellite could theoretically have orbiting Earth at 'treetop' height is ~85 minutes. So, the answer of 95.5 minutes is sensible. Answers less than ~85 minutes mean the satellite is orbiting in solid Earth; obviously an impossibility.

**c** The only force acting on the satellite, once it is positioned in a stable circular orbit around Earth, is the gravitational force towards Earth. This force is constant in magnitude and always directed towards Earth's centre.

## Section 3C questions

#### **Multiple-choice questions**

- 1 Which of the following best describes the force acting on a satellite as it orbits Earth?
  - A The satellite's rocket motors provide the force that keep it moving around Earth.
  - **B** The gravitational force of Earth.
  - **C** The centripetal force of the satellite.
  - **D** No forces are required; the satellite just orbits Earth like the Moon orbits Earth.

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- 2 When a spacecraft orbits Earth, its orbital period is **not** a function of the
  - A mass of Earth.
  - **B** mass of the spacecraft.
  - **C** velocity of the spacecraft.
  - D height of the spacecraft above Earth.

VCAA NHT 2021

3 Engineers want to put a satellite into orbit where the gravitational field is one-quarter of its value at Earth's surface. Take the radius of Earth as  $R_{\rm E}$ . The altitude above Earth's surface of this satellite will be

- **A**  $R_{\rm E}$
- **B**  $2R_{\rm F}$
- $C 3R_{\rm F}$
- **D**  $4R_{\rm F}$
- 4 A geostationary satellite has a period of
  - A 0 seconds, as it is stationary.
  - **B** 43200 s
  - **C** 86400 s
  - **D** 864000 s

#### **Short-answer questions**

- **5** Explain how Newton's thought experiment, that involved projecting cannonballs horizontally from a high mountain with negligible air resistance and increasingly faster speeds, lead to the concept of satellite motion. Use a diagram in your answer.
- 6 The dwarf planet Pluto has an orbital period of 248 Earth years (1 Earth year = 365 days). Assume the orbit is exactly circular.

The mass of the Sun is  $2.0 \times 10^{30}$  kg.

 $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ 

- a Calculate Pluto's orbital period in seconds.
- **b** Calculate Pluto's distance from the centre of the Sun. Ignore the radius of the Sun in metres. Show your working.
- A piece of a comet with mass
  2.0 × 10<sup>12</sup> kg collides with the planet
  Jupiter. The graph on the right
  shows part of the gravitational field
  strength around Jupiter plotted
  against distance.
  - a Calculate the force of gravity acting on the fragment of the comet when it is a distance of  $2 \times 10^8$  m from Jupiter.
  - b Estimate the magnitude of the change in gravitational potential energy of the comet piece as it moves from  $5 \times 10^8$  m to  $2 \times 10^8$  m from Jupiter.





# **Chapter 3 review**

# Summary

Create your own set of summary notes for this chapter on paper or in a digital document. A model summary is provided in the Teacher Resources, which can be used to compare with yours.

# Checklist

In the Interactive Textbook, the success criteria are linked from the review questions and will be automatically ticked when answers are correct. Alternatively, print or photocopy this page and tick the boxes when you have answered the corresponding questions correctly.

Succe	ess criteria – I am now able to:	Linked questions
3A.1	Understand both theoretically and practically the direction, shape and attractive effect of a gravitational field; identify fields as uniform or non-uniform; investigate the existence of monopoles and dipoles and gravity as a monopole	11 , 21
3A.2	Understand both theoretically and practically the direction, shape and attractive effect of a gravitational field about a point mass and be able to use the inverse square law to determine the magnitude of the field	6□, 17□, 20□
3A.3	Analyse the use of gravitational fields to accelerate mass, including gravitational field and gravitational force concepts using $g = G \frac{M}{r^2}$ and $F_g = G \frac{m_1 m_2}{r^2}$	1 , 2 , 3 , 4 , 9 ,14 , 15 ,16 , 20
3B.1	Determine gravitational potential energy changes (qualitative) associated with a point mass and potential energy changes in a uniform gravitational field using $E_{\sigma} = mg\Delta h$	13 , 21
3B.2	Analyse the change in gravitational potential energy from both the area under a force–distance graph and the area under a field– distance graph multiplied by mass	18 , 21
3C.1	Apply the concepts of force due to gravity, $F_{g}$ , and the normal force, $F_{N'}$ including in relation to satellites in orbit where the orbits are assumed to be both uniform and circular	12
3C.2	Model satellite motion for artificial satellites, our Moon and planets as uniform circular orbital motion using $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$	5□, 7□, 8□, 10□, 15□, 16□, 17□, 18□, 19□

# **Multiple-choice questions**

- 1 An object with a mass of 48 kg as measured on Earth is taken to the Moon. What is the mass of the object on the Moon's surface if the acceleration due to gravity on the Moon is one-sixth of that on Earth?
  - **A** 8 kg
  - **B** 48 kg
  - **C** 288 kg
  - **D** 480 kg

- **2** Two objects, one with a mass of *M* and one with a mass of 4*M*, are attracted to each other by a gravitational force, *F*. If the magnitude of the force on mass 4*M* is *F*, what is the magnitude of the force on mass *M* in terms of *F*?
  - <u>۲</u>
  - 4
  - $\mathbf{B} \stackrel{F}{=}$
  - $\frac{1}{2}$ C F
  - D F
- **3** An apple hangs on a tree on Earth. Which one of the following statements concerning the gravitational force on Earth and the apple is correct?
  - A The force on the apple is greater than the force on Earth because Earth is more massive.
  - **B** The force on Earth is greater than the force on the apple because Earth is more massive.
  - **C** The force on the apple is less than the force on Earth by a factor of the inverse square of their relative masses.
  - **D** The forces on the apple and Earth are equal.
- 4 The planet Phobetor has a mass four times that of Earth. Acceleration due to gravity on the surface of Phobetor is  $18 \text{ m s}^{-2}$ .

If Earth has a radius R, which one of the following is closest to the radius of Phobetor?

- **A** *R*
- **B** 1.5*R*
- **C** 2*R*
- **D** 4*R*

#### VCAA 2021

**5** The International Space Station (ISS) is travelling around Earth in a stable circular orbit, as shown in the diagram below.



Which one of the following statements concerning the momentum and the kinetic energy of the ISS is correct?

- **A** Both the momentum and the kinetic energy vary along the orbital path.
- **B** Both the momentum and the kinetic energy are constant along the orbital path.
- **C** The momentum is constant, but the kinetic energy changes throughout the orbital path.
- **D** The momentum changes, but the kinetic energy remains constant throughout the orbital path.

VCAA 2020

- **6** A rocket moves away from the surface of Earth. As the distance, *R*, from the centre of Earth increases, what happens to the force of gravity on the rocket?
  - **A** The force increases directly proportional to  $R^2$ .
  - **B** The force remains constant.
  - **C** The force decreases directly proportional to  $\frac{1}{R}$ .
  - **D** The force decreases directly proportional to  $\frac{1}{R^2}$ .
- 7 A satellite is orbiting Earth at an altitude of two Earth radii  $(2R_E)$  above the surface. What is the centripetal acceleration of the satellite compared to what it would be if orbiting just above Earth's surface?



- D the same
- **8** A satellite is orbiting a planet with an orbital radius *r*. Which of the following diagrams correctly represents the direction of the velocity and acceleration of the satellite?



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**9** The magnitude of the acceleration due to gravity at Earth's surface is *g*.

Planet Y has twice the mass and half the radius of Earth. Both planets are modelled as uniform spheres.

Which one of the following best gives the magnitude of the acceleration due to gravity on the surface of Planet Y?

A 
$$\frac{1}{2}g$$

- **C** 4g
- **D** 8g
- 10 A satellite is orbiting a planet of radius *R*, mass *M* and an orbital radius *r*. Which of the following represents the orbital velocity of the satellite?





## Short-answer questions

11 The diagram below shows a representation of the gravitational field around a mass.What does the direction and density of the gravitational field lines represent? (2 marks)



- **12 a** Calculate the magnitude and direction of the gravitational force,  $F_{g}$ , acting on an 80 kg astronaut when standing on the surfaces of Earth and the Moon. Take the Moon's gravity as one-sixth Earth's gravity. (2 marks)
  - **b** Calculate the magnitude and direction of the normal force,  $F_{N'}$  acting on an 80 kg astronaut when standing on the surfaces of Earth and the Moon. (2 marks)
  - **13** Jac lifts a 30 kg barbell in the gym straight up by a distance of 3.0 m. Calculate the gain in gravitational potential energy for the barbell. Use  $g = 10 \text{ N kg}^{-1}$  at Earth's surface. (2 marks)

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14 The diagram shows the Moon in orbit around Earth where the tangential velocity,  $\nu$ , is perpendicular to the gravitational force,  $F_{o}$ .



Explain how the Moon would move if the tangential velocity *v*:

- a increased(2 marks)b decreased(2 marks)c became zero.(2 marks)
- **15** The diagram shows a satellite in orbit above Earth at an altitude of 320 km.



Use the following data:

b

Radius of Earth,  $R_{\rm E} = 6400$  km

Mass of Earth,  $M_{\rm E}\,{=}\,6.0\times10^{24}\,{\rm kg}$ 

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$

- a Calculate the magnitude of the acceleration due to gravity at this orbital height. (2 marks)
  - (2 marks)

(3 marks)

**c** Calculate the period of the satellite.

Calculate the orbital speed of the satellite.

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3 2 1 0 3 7 2 4 5 6 8 0 1 9 10 Height above Earth's surface (10<sup>5</sup> m)

9

- **a** Calculate the period of the satellite in seconds. Take the mass of Earth to be  $6.0 \times 10^{24}$  kg and the universal gravitational constant, G, to be  $6.7 \times 10^{-11}$  N m<sup>2</sup> kg<sup>-2</sup>. Show your working. (3 marks)
- **b** The controllers of the satellite use its motors to move the satellite into a higher orbit.
  - i Will this increase, decrease or have no effect on the speed of the satellite? Justify (3 marks) your answer.
  - ii Will this increase, decrease or have no effect on the gravitational potential energy of the satellite? Take the surface of Earth as the zero of gravitational potential energy. Justify your answer. (3 marks)

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VCAA NHT 2018
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- **17** A planet in another solar system has a mass *n* times the mass of Earth and a diameter of *m* times the diameter of Earth. Calculate the acceleration of a body falling near the surface of the planet as a multiple of Earth's surface gravity, g. (3 marks)
- **18** A space telescope of mass 1.2 t is placed in orbit at a height of 610 km above Earth's surface. The diagram below shows the graph of the gravitational field strength plotted against height above Earth's surface.





(1 mark)

- What does the blue shaded area represent in the graph of the gravitational field С strength-height above Earth's surface? (2 marks)
- **d** Calculate the change in gravitational potential energy for the space telescope of mass 1.2 t as it is placed in orbit at a height of 610 km above Earth's surface. (3 marks)

**19** A spacecraft is in orbit around Mars at an altitude of  $1.6 \times 10^6$  m above the surface of Mars. The mass of Mars is  $6.4 \times 10^{23}$  kg and its radius is  $3.4 \times 10^6$  m.

Take the universal gravitational constant, *G*, to be  $6.7 \times 10^{-11}$  N m<sup>2</sup> kg<sup>-2</sup>.

The mass of the spacecraft is  $2.0 \times 10^4$  kg.

- a Calculate the strength of the gravitational field acting on the spacecraft. Show your working.(2 marks)
- **b** Calculate the force acting on the spacecraft in its orbit around Mars. (2 marks)
  - Adapted from VCAA NHT 2019
- **20** A planet of radius *r* has two satellites,  $S_1$  and  $S_2$ , in circular orbits.  $S_1$  moves just above the planet's surface and  $S_2$  is in an orbit of radius 4r.
  - **a** What is the value of the ratio acceleration of  $S_1$ : acceleration of  $S_2$  (2 marks)
  - **b** What is the value of the ratio orbital speed of  $S_1$ : orbital speed of  $S_2$  (2 marks)
  - **c** What is the value of the ratio period of  $S_1$ : period of  $S_2$
  - **d** Using the symbols T = period of S<sub>1</sub>, r = radius of orbit of S<sub>1</sub>, and G = universal gravitational constant, write an expression for the mass of the planet. (3 marks)
- **21** Assume that a journey from approximately 2 Earth radii  $(2R_E)$  down to the centre of Earth is possible. The radius of Earth  $(R_E)$  is  $6.37 \times 10^6$  m. Assume that Earth is a sphere of constant density.

A graph of gravitational field strength versus distance from the centre of Earth is shown below.



**a** What is the numerical value of Y?

(1 mark)

(2 marks)

- **b** Explain why gravitational field strength is 0 N kg<sup>-1</sup> at the centre of Earth. (2 marks)
- c Calculate the increase in potential energy for a 75 kg person hypothetically moving from the centre of Earth to the surface of Earth. Show your working. (2 marks)

VCAA 2019

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HOW DO FIELDS EXPLAIN MOTION AND ELECTRICITY?

# CHAPTER

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# ELECTRIC AND MAGNETIC FIELDS

#### Introduction

Bees don't just recognise flowers by their colour and scent; they can also detect a plant's minute electric fields. These electric fields form from an imbalance of charge between the ground and the atmosphere. They are unique to each species of plant and flower. The bees not only sense these patterns, they change the charge and the electric field of whatever flower they land upon. Research shows that the electrical potential in the stem of a petunia increases by around 25 millivolts when a bee lands upon it. This change in the electric field can also provide a signal to another bee that a flower has recently been visited and might therefore be short of nectar.

Every object with an electric charge has an associated electric field. Electric field diagrams are a useful way of interpreting these fields. This chapter explores concepts such as electric field strength, electric force and electric potential energy changes in both uniform and non-uniform electric fields.

You will also discover the concepts of magnetic fields, examine how they are created and look at the pattern of magnetic fields around bar magnets, current-carrying wires and coils. The presence of magnetic fields around current-carrying wires, and the forces on current-carrying wires caused by external magnetic fields, demonstrate an important relationship between electricity and magnetism and lead to an understanding of the operation of simple DC motors.

You will then investigate the forces on charged particles due to electric and magnetic fields, leading to an understanding of particle accelerators and their use in electron microscopes, mass spectrometers, synchrotrons and medical linear accelerators.

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#### Curriculum

Area of Study 2 Outcome 2 How do things move without contact?

#### **Study Design**

#### **Fields and interactions**

- Describe gravitation, magnetism and electricity using a field model
- Investigate and compare theoretically and practically gravitational, magnetic and electric fields, including directions and shapes of fields, attractive and repulsive effect, and the existence of dipoles and monopoles
- Investigate and compare theoretically and practically gravitational fields and electric fields about a point mass or charge (positive or negative) with reference to:
  - the direction of the field
  - the shape of the field
  - the use of the inverse square law to determine the magnitude of the field
  - potential energy changes (qualitative) associated with a point mass or charge moving in the field
- Identify fields as static or changing, and as uniform or non-uniform
   *Gravitational fields and mass are covered in Chapter 3. Magnetic fields are covered in 4B.*

#### **Effects of fields**

- Analyse the use of an electric field to accelerate a charge, including:
  - electric field and electric force concepts:

$$E = k \frac{Q}{r^2}$$
 and  $F = k \frac{Q_1 Q_2}{r^2}$ 

- ► potential energy changes in a uniform electric field: W = qV,  $E = \frac{V}{d}$
- the magnitude of the force on a charged particle due to a uniform electric field: F = qE

# Learning intentions – at the end of this chapter I will be able to:

- 4A Electric fields and forces
- **4A.1** Describe electricity using a field model
- **4A.2** Investigate and compare theoretically and practically electric fields, including direction and shapes of fields, attractive and repulsive fields, and the existence of dipoles and monopoles
- **4A.3** Investigate and compare theoretically and practically electric fields about a point charge (positive or negative) with reference to:
  - ► the direction of the field
  - ► the shape of the field
- **4A.4** Use the inverse square law to determine the magnitude of an electric field
- **4A.5** Determine potential energy changes (qualitative) associated with a point charge moving in an electric field
- **4A.6** Identify electric fields as static or changing, and as uniform or non-uniform
- **4A.7** Quantitatively determine the magnitude of the electric field strength around

point charges,  $E = k \frac{Q}{r^2}$ , and between

parallel plates,  $E = \frac{v}{d}$ 

- **4A.8** Quantitatively determine the magnitude of the force between two fixed charged objects using Coulomb's law,  $F = k \frac{q_1 q_2}{2}$
- **4A.9** Understand work done and potential energy changes in a uniform electric field, W = qV

**4A.10** Quantitatively determine the forces acting on charged particles in a uniform electric field, F = qE

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#### Study Design

#### Fields and interactions

- Describe gravitation, magnetism and electricity using a field model
- Investigate and compare theoretically and practically gravitational, magnetic and electric fields, including directions and shapes of fields, attractive and repulsive effect, and the existence of dipoles and monopoles
- Investigate and apply theoretically and practically a field model to magnetic phenomena, including shapes and directions of fields produced by bar magnets, and by current-carrying wires, loops and solenoids
- Identify fields as static or changing, and as uniform or non-uniform
   Note: Gravitational fields are covered in Chapter 3. Electric fields are covered in 4A.

#### **Effects of fields**

- Analyse the use of a magnetic field to change the path of a charged particle, including:
  - the magnitude and direction of the force apaplied to an electron beam by a magnetic field: F = qvB, in cases where the directions of v and B are perpendicular or parallel
  - ► the radius of the path followed by an electron in a magnetic field:  $qvB = \frac{mv^2}{r}$ , where  $v \ll c$

#### Application of field concepts

- Describe the interaction of two fields, allowing that electric charges, magnetic poles and current-carrying conductors can either attract or repel, whereas masses only attract each other
- Investigate and analyse theoretically and practically the force on a current-carrying conductor due to an external magnetic field, F = nIIB, where the directions of *I* and *B* are either perpendicular or parallel to each other
- Investigate and analyse theoretically and practically the operation of simple DC motors consisting of one coil, containing a number of loops of wire, which is free to rotate about an axis in a uniform magnetic field and including the use of a split ring commutator

# Learning intentions – at the end of this chapter I will be able to:

- 4B Magnetic fields and forces
- **4B.1** Describe magnetism using a field model
- **4B.2** Investigate and compare theoretically and practically magnetic fields, including directions and shapes of fields, attractive and repulsive effects, and the existence of dipoles; identify magnetic fields as static or changing, and as uniform or non-uniform
- **4B.3** Investigate and apply theoretically and practically a vector field model to magnetic phenomena, including shapes and directions of fields produced by bar magnets, and by current-carrying wires, loops and solenoids
- **4B.4** Describe the interaction of two fields, allowing that electric charges, magnetic poles and current-carrying conductors can either attract or repel
- **4B.5** Investigate and analyse theoretically and practically the force on a currentcarrying conductor due to an external magnetic field, F = nIIB, where the directions of *I* and *B* are either perpendicular or parallel to each other
- **4B.6** Investigate and analyse theoretically and practically the operation of simple DC motors consisting of one coil, containing a number of loops of wire, which is free to rotate about an axis in a uniform magnetic field and including the use of a split ring commutator
- **4B.7** Investigate, qualitatively, the effect of current, external magnetic field and the number of loops of wire on the torque of a simple motor
#### Study Design

- Investigate, qualitatively, the effect of current, external magnetic field and the number of loops of wire on the torque of a simple motor
- Model the acceleration of particles in a particle accelerator (including synchrotrons) as uniform circular motion (limited to linear acceleration by a uniform electric field and direction change by a uniform magnetic field)

# Learning intentions – at the end of this chapter I will be able to:

- **4B.8** Analyse the use of a magnetic field to change the path of a charged particle, including:
  - the magnitude and direction of the force applied to an electron beam by a magnetic field, F = qvB, in cases where the directions of v and B are perpendicular or parallel
  - the radius of the path followed by an electron in a magnetic field,  $mv^2$

$$qvB = \frac{mv}{r}$$
, where  $v \ll c$ 

**4B.9** Model the acceleration of particles in a particle accelerator (including synchrotrons) as uniform circular motion (limited to linear acceleration by a uniform electric field and direction change by a uniform magnetic field)

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## Glossary

Coulomb (C) Coulomb's law Electric dipole Electric field Electric force Electric potential energy Electromagnet Electromagnetism Electron gun Field Field lines Inverse square law Low speed Magnetic dipole Magnetic field Non-contact force Non-uniform electric field Particle accelerator Point charge Right-hand grip rule Right-hand slap rule Solar wind Solenoid Static field Synchrotron Torque Uniform electric field Uniform magnetic field

## Concept map



*Electricity using a field model, and its effects to accelerate a charge* 

4A Electric fields and forces



## 4B Magnetic fields and forces



See the Interactive Textbook for an interactive version of this concept map interlinked with all concept maps for the course.



# **Electric fields and forces**

#### Study Design:

- Describe gravitation, magnetism and electricity using a field model
- Investigate and compare theoretically and practically gravitational, magnetic and electric fields, including directions and shapes of fields, attractive and repulsive effect, and the existence of dipoles and monopoles
- Investigate and compare theoretically and practically gravitational fields and electrical fields about a point mass or charge (positive or negative) with reference to:
  - ► the direction of the field
  - ► the shape of the field
  - the use of the inverse square law to determine the magnitude of the field
  - potential energy changes (qualitative) associated with a point mass or charge moving in the field
- Identify fields as static or changing, and as uniform or non-uniform
- Analyse the use of an electric field to accelerate a charge, including:
  - electric field and electric force concepts: a a

$$E = k \frac{Q}{r^2}$$
 and  $F = k \frac{Q_1 Q_2}{r^2}$ 

- ► potential energy changes in a uniform electric field: W = qV,  $E = \frac{V}{d}$
- the magnitude of the force on a charged particle due to a uniform electric field: F = qE

#### Glossary:

Coulomb (C) Coulomb's law Electric dipole Electric field Electric force Electric monopole Electric potential energy Field Field line Inverse square law Low speed Non-contact force Non-uniform electric field Point charge Uniform electric field



## ENGAGE

# Sharks and platypuses use electric fields to 'see'

Many creatures use electric fields to communicate, sense predators, sense prey or, in the case of the electric eel, even stun their prey with powerful electric shocks. How, or why, this ability came about is still under scientific investigation.

A number of shark species have electroreceptors that allow them to detect minuscule changes in electric fields. Figure 4A–1 shows the electroreceptors as tiny pores on the snout of a tiger shark.



**Figure 4A–1** The many tiny pores on the snout of this tiger shark are electroreceptors called 'ampullae of Lorenzini'. These enable sharks to detect minuscule electric fields.

Electroreception is normally limited to aquatic environments. In the case of ocean environments the resistivity of salt water is low enough for the electric fields generated from biological sources to be easily detected. Sharks can detect extremely weak electric fields, less than one hundredth-millionth of a volt per centimetre (0.01  $\mu$ V cm<sup>-1</sup>).

The only known mammals to have electroreceptors are the platypus and echidna.

While the platypus has 40 000 electroreceptors (on its bill), echidnas have only 400-2000. The platypus can detect electric fields in the order of three-tenths of a millivolt per centimetre (0.3 mV cm<sup>-1</sup>) and uses the electroreceptors in its 'duck' bill to hunt prey in deep murky waters, without the use of sight, sound or smell. Figure 4A-2 shows a diagram of the electric field strength detection of a platypus. As the prey gets closer, the signal strength of the electric field becomes larger, giving the platypus useful information on the location and movement of the prey.





## EXPLAIN Electric fields

#### **Electric field strength**

**Electric fields** are produced by electric charges and can be modelled by drawing electric field line diagrams, similar to the gravitational field lines introduced in Chapter 3. Figure 4A–3 (left) shows electric field lines around a positive charge. Figure 4A–3 (centre) shows electric field lines around a negative charge and Figure 4A–3 (right) shows the gravitational field around a mass.



**Figure 4A–3** Electric field around a fixed positive charge, +Q, (left), a fixed negative charge, -Q, (centre) and a gravitational field around a mass, M (right)

#### Electric field

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a physical field that creates a force on all charged particles within the field. Produced by charged particles and changing magnetic fields. A changing electric field also produces a changing magnetic field.

#### Field line

a line drawn to represent the strength of a field with arrows to indicate direction. Field lines that are closer together indicate a stronger field.

VIDEO 4A–1 Electric Fields and Forces



Field

a region where an object feels a force, such as gravitational, electric or magnetic; more precisely defined as a physical quantity that has a value at each point in space As we discovered in Chapter 3, it was Michael Faraday who proposed the concept of a 'field'.

In the case of an electric charge, there is an electric field in the space around the charge such that if a second charge is placed in that space, it will experience an electric force.

Just as the gravitational field, *g*, is the force per unit mass (N kg<sup>-1</sup>), so the electric field, *E*, is the force per unit charge (N  $C^{-1}$ ), where the charge, *q*, is measured in **coulomb** (C):

$$g = \frac{F}{m}$$
 and  $E = \frac{F}{q}$ 

This can be expressed as Formula 4A–1.

Formula 4A–1 Force on a charged particle in an electric field

F = qE

Where:

F = Force on a charged particle in a electric field (N)

q = Charge of the particle (C)

E = Strength of the electric field (N C<sup>-1</sup>)

By convention, the direction of the electric field is given by the direction of the force on a small positive unit test charge placed in the field.

The strength of force between two charges Q and q is given by:

$$F = k \frac{Qq}{r^2}$$

So, the strength of the electric field can be given as:

$$E = \frac{F}{q} = k \frac{Q}{r^2}$$

where *k* is known as the Coulomb constant and has a value of  $k = 8.99 \times \text{Nm}^2 \text{C}^{-2}$ .

This is the strength of the electric field around a point charge.

The unit of electric field strength is newtons per coulomb (N $C^{-1}$ ).

An equivalent but more commonly used unit is volts per metre ( $Vm^{-1}$ ).

This is the **uniform electric field** between two parallel, oppositely charged, metal plates that are a distance d apart and with a voltage drop of V across them. This is explained in more detail later in this chapter.

There are many similarities between the expressions for gravitational forces and fields and electrostatic forces and fields. Forces between point masses, and the force between point charges, are both proportional to the square of the distance between them. The forces involve the product of the masses (gravity) and product of the charges (electric) respectively. The gravitational field, *g*, is dependent upon the point mass, *M*, and the electric field, *E*, is dependent upon the point charge, *Q*.

FIELDS



#### Point charge

an ideal situation in which all of the charge on an object is considered to be concentrated at a single point

## Uniform electric field

an electric field in which the value of the field strength remains the same at all points

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**3A GRAVITY AND** 

GRAVITATIONAL

Electric force a force that exists between charged particles and may be attractive (unlike charges) or repulsive (like charges) Table 4A–1 shows the comparison between expressions for gravitational and **electric forces** and fields around point masses and point charges respectively.

Table 4A-1 Comparison between expressions for gravitational and electric forces and fields

Gravitational force and field (masses <i>M</i> and <i>m</i> )	Electric force and field (charges $Q$ and $q$ )
$F = G \frac{M m}{r^2}$	$F = k \frac{Q q}{r^2}$
F = mg	F = qE
$g = G \frac{M}{r^2}$	$E = k \frac{Q}{r^2}$



## Worked example 4A–1 Equating forces in a uniform electric field

An oil drop with an excess charge of three electrons is balanced in a uniform electric field between two parallel charged metal plates as shown. The magnitude of the electric field is  $10 \text{ kV m}^{-1}$ .

Calculate the mass of the oil drop. The charge on an electron is  $1.6 \times 10^{-19}$  C.

Use  $g = 10 \text{ N kg}^{-1}$ .

Solution

The force due to gravity ( $F_g = mg$ ) is balanced by the electric force ( $F_e = qE$ ). Therefore:

 $F_{\rm g} = F_{\rm e}$  mg = qE  $m \times 10 = 3 \times 1.6 \times 10^{-19} \times 10^4$  $m = 4.8 \times 10^{-16} \,\rm kg$ 



#### **Electric field patterns**

For an electric field, the strength of the electric force is defined by F = Eq, while the direction of the force is in the direction of where it acts on a stationary positive charge.

The force diagram for electric forces around a point charge is similar to the gravitational force diagram around a point mass; except that with electric forces it is possible to distinguish between the attractive and repulsive forces, as shown in Figure 4A–4.



**Figure 4A–4** Electric force vectors around a fixed positive charge (left) and a fixed negative charge (right). The arrows show the direction of the force.

In electric field diagrams, the force vectors are formed into continuous lines. These lines are called lines of force or electric field lines. The arrows indicate the direction of the field.

Figure 4A–5 shows two different pictorial representations of the same electric field created by a positive point charge, *Q*. The left-hand part shows numerous individual arrows with each arrow representing the force vector on a test charge, *Q*. The right-hand part shows the standard representation using continuous lines. Field lines are essentially a map of infinitesimal force vectors.

The distance between adjacent field lines indicates the strength of the electric field, *E*. So around a point charge, as shown below right, the field lines are closer together nearer the charge and get further apart the further from the charge you get. This indicates that the electric field strength decreases as the distance from the point charge increases.



Figure 4A-5 Two equivalent representations of the electric field due to a positive charge, Q.

Figure 4A–6 shows standard electric field patterns for four different situations.



**Figure 4A–6** Electric field patterns for a single negative charge (top left), a pair of opposite charges (top right), a pair of positive charges (bottom left) and two parallel plates with opposite charges (bottom right)

When drawing two-dimensional electric field lines patterns around, and between, charged objects there are a few rules that need to be followed:

- electric field lines run from from positive charges to negative charges
- electric field lines start and end at 90° to conducting surfaces, with no gap between the lines and the surface
- field lines can never cross; if they did cross, it would mean that the force is acting in two different directions at that point, which is not possible
- drawings are usually two-dimensional representations of three-dimensional fields.

## Worked example 4A–2 Identifying charges from field patterns

The diagram (below) shows the electric field pattern for four charges of equal magnitude arranged in a square.



Determine the polarity of each of the charges.

Solution

The electric field lines start at positive charges and end at negative charges.





#### **Uniform electric fields**

Between two parallel oppositely charged metal plates, a uniform electric field is established in the central region between the plates (Figure 4A–7). The field strength is constant at all points within the region of the uniform electric field, so the field lines are parallel. The strength of the electric field is given by Formula 4A–2.

#### Formula 4A-2 Electric field strength between parallel plates

$$E = \frac{V}{d}$$

Where:

E = Electric field strength for a uniform electric field (V m<sup>-1</sup>)

- V = Electric potential difference (voltage) between the plates (V)
- d = Distance between the plates (m)



**Figure 4A–7** A uniform electric field created by two parallel oppositely charged metal plates

Figure 4A–8 shows a photograph of small seeds aligning in an almost-uniform electric field created between two parallel oppositely charged plates using a common physics laboratory apparatus. The voltage used is 100 V and the distance between the two parallel plates is 2 cm.



**Figure 4A–8** Apparatus showing an almost-uniform electric field between two parallel oppositely charged plates



#### Worked example 4A–3 Calculating the strength of a uniform electric field

Calculate the magnitude of the electric field strength for the apparatus shown in the photograph in Figure 4A.8.

Solution  $E = \frac{V}{d}$   $= \frac{100}{0.02}$   $= 5000 \text{ V m}^{-1}$ 

Figure 4A–9 shows a photograph of a  $3 \times 10^8$  V (300 million volt) lightning bolt discharging at a height of 10 km above the ground. Superimposed on the image are the negative charges at the bottom of the cloud that have attracted positive charges to the top of the surface of Earth. This charge distribution generates an electric field of the order of 30 000 V m<sup>-1</sup>. Lightning is a naturally occurring electrostatic discharge that occurs as the electrical arc overcomes the insulating properties of air.



Electric monopole

a single electric point charge, such as an electron or proton in which all the field lines point inward (-) or outward (+)

Electric dipole a pair of equal and opposite electric charges, the centres of which are separated

#### Figure 4A–9 An electric field created by a thunderstorm

## Electric monopoles and dipoles

Electric charges can exist as both monopoles and dipoles. A single positive charge (Figure 4A–10 left) or a single negative charge exists as an **electric monopole**. An **electric dipole** consists of two equal point charges of opposite sign, separated by a short distance (Figure 4A–10 right).

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Figure 4A-10 Left: An electric monopole. Right: An electric dipole

However, electric dipoles do occur in nature, some of which are very important.

## Electric dipoles in nature: water

Electric dipoles can occur when electrons are shared in the bonds between atoms in molecules. For example, in the water molecule,  $H_2O$ , the single oxygen atom more strongly attracts the shared electrons than do each of the two hydrogen atoms. This makes the oxygen end of the molecule slightly more negatively charged and the hydrogen end slightly more positively charged (Figure 4A–11). The water molecule is therefore called a dipolar molecule.

Figure 4A–12 (left) shows the effect of bringing a negatively charged comb near a stream of neutral (uncharged) tap water. The stream of water is attracted to the comb. This can be explained by understanding that the water molecules are electric dipoles (Figure 4A–12 right) and their positive charged ends are closer to the comb than the negative ends.



Oxygen atom with partial negative charge

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atoms with partial positive charges

**Figure 4A–11** In a molecule of water, H<sub>2</sub>O, the oxygen atom more strongly attracts the shared electrons than do each of the hydrogen atoms, so it has a partial negative charge, while the hydrogen atoms have partial positive charges, producing an electric dipole molecule.

**Figure 4A–12** Left: A trickle stream of water is bent by a negatively charged comb. Right: The electric dipolar nature of the water molecules explains why the stream of water is bent by the comb: the positive charges on one side of the water molecules are attracted to the negative charges on the comb.

#### CHAPTER 4 ELECTRIC AND MAGNETIC FIELDS





Dipoles are also commonly associated with magnetic fields, where the ends of a bar magnet have different polarities (north and south). These will be explored in Section 4B.

Among the important consequences of the polarity of water for living organisms is water's ability to dissolve a variety of substances, more than any other liquid, and its strong surface tension. This not only allows insects to walk on water (Figure 4A-13 left) but also allows water to form drops and to travel through tiny roots, stems and capillaries. Water is the only substance that exists as a gas, liquid and solid at temperatures found on Earth. Because of the polarity of the water molecule, the solid state is less dense than the liquid state. When water gets colder than  $4^{\circ}$ C, it begins expanding and becomes less dense. As a result, at close to freezing, colder water floats to the top, and the warmer water sinks to the bottom. This means that ice floats, rivers and lakes freeze from the top down and warmer water is trapped underneath (Figure 4A-13 right). This has profound implications for life on Earth.



**Figure 4A–13** Left: Insects can walk on water because of the strong surface tension created by the dipolar nature of the H<sub>2</sub>O molecule. Right: A girl casts a line through a hole created in the surface of a frozen river during the annual ice fishing festival in Hwacheon, South Korea. The polarity of water explains why ice floats and forms at the top of a body of water, allowing aquatic animals and plants to avoid freezing.

## Check-in questions – Set 1

**1** a Two equal negative charges are placed as shown.



Copy the diagram and draw a representation of the electric field lines between and surrounding the two charges.

**b** One of the negative charges, -Q, is now replaced with the same magnitude positive charge, +Q, as shown.



Copy the diagram and draw a representation of the electric field lines between and surrounding the two charges.

**2** Explain, using a diagram, the difference between an electric monopole and an electric dipole.

4A ELECTRIC FIELDS AND FORCES

**3** The diagram shows two parallel charged plates in a vacuum.

The voltage between the plates is 100 V and the distance between the plates is 10 cm.

- **a** Copy the diagram and sketch the electric field lines that exist between the two charged plates.
- **b** The best description of the field in the region between X and Y is that it is uniform/non-uniform and static/changing. Select the correct words.
- c i Describe what happens if an electron is placed at point X.ii Describe what happens if an electron is placed at point Y.
- d i Describe what happens if a proton is placed at point X.ii Describe what happens if a proton is placed at point Y.
- e Calculate the magnitude and direction of the electric field between the two plates.

## Electric fields around point charges

The strength of an electric field due to a point charge can be determined from Formula 4A–3.

Formula 4A–3 Electric field strength near a point charge

$$E = k \frac{Q}{r^2}$$

Where:

E = Electric field strength due to a point charge (N C<sup>-1</sup> or V m<sup>-1</sup>)

 $k = \text{Coulomb's constant} (8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2})$ 

Q = Electric charge generating the field (C)

r = Distance from the centre of the charge (m)

The electric field strength follows an **inverse square law**, just as the gravitational field strength does  $(g = G\frac{M}{r^2})$ , except that the strength of an electric field is determined by the charge, *Q*, rather than the mass, *M*, of that object.

If the point charge is positive, the direction of the electric field is radially outwards. If the point charge is negative, the direction of the electric field is radially inwards.

Electric field-distance graphs are analogous to gravitational field distance graphs and have a similar shape due to the inverse square law. The main difference is that electric charges come in positive and negative varieties, while mass comes in only one variety. Therefore, electric field-distance graphs show separate curves for positive and electric charges. These are the same shape but inverted with respect to each other. Figure 4A–14 (top) shows the electric field-distance graph for a positive point charge, while



**Figure 4A–14** Top left: A positive electric charge and the direction of the electric field *E* away from it at point P which is at distance *r*. Top right: The electric field–distance graph (*E* versus *r*) for a positive charge. Bottom left: A negative electric charge and the direction of the electric field *E* towards it at point P which is at distance *r*. Bottom right: The electric field–distance graph for a negative charge, with negative values of *E*.

Figure 4A–14 (bottom) shows the electric field–distance graph for a negative point charge. Note that at point P, the electric field is to the right in the top diagram and to the left in the bottom diagram. Note that the radial direction away from the charge is assumed as the positive direction for the electric field.



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law the relationship between two variables where one is proportional to the reciprocal

of the square of

the other

Inverse square





## Worked example 4A–4 Calculating the field strength of a point charge

Calculate the electric field strength and direction at a distance of 2.0 m from a charge of  $+2.0 \times 10^{-4}$  C. Use  $9.0 \times 10^{-9}$  as the value for *k*.

Solution

$$E = k \frac{Q}{r^2}$$
  
= (9.0 × 10<sup>9</sup>)  $\frac{(2.0 × 10^{-4})}{(2.0)^2}$ 

 $= 4.5 \times 10^5 \,\mathrm{N}\,\mathrm{C}^{-1}$  radially outwards from the positive point charge



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## **Electric forces**

Electric forces, like gravitational forces discussed in Chapter 3, are another example of non-contact forces – forces that act on an object without coming physically in contact with it. Unlike gravitational forces, which are always attractive, electric forces can be either attractive or repulsive.

Like charges repel and unlike charges attract (Figure 4A–15). If the charges are the same size (magnitude) and the same distance apart, then these attractive and repulsive forces are the same size, F, and point in the directions as shown in Figure 4A-15.

The symbol *q* is often used to indicate the amount of charge on an object and the SI unit of electrical charge is the coulomb (C). At a given fixed distance between two charges, the size of the force depends on the size of the charges on each object (Figure 4A–16).

Changing the charge on one object affects the force acting on both the objects. This is shown in the middle diagram in Figure 4A-16, where the left-hand charge is +2qand the right-hand charge is +q and the

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Figure 4A–15 Like charges repel, unlike charges attract. The arrows indicate the direction of the force on each of the charges.



Figure 4A–16 The force between two charges depends on the size of the charges.

force on each charge is now 2F. Note that for each of the three diagrams in Figure 4A-16, the force on each of the paired objects remains equal and opposite to the force on the other paired object. This is a consequence of Newton's third law.

It can also be seen from Figure 4A-16 that the force between any two charged objects (for example,  $q_1$  and  $q_2$ ) is directly proportional to the product of the charges on each object:

$$F \propto q_1 q_2$$

ISBN 978-1-009-25901-9 Eddy de Jong et al © Cambridge University Press & Assessment 2023

Non-contact

force a force that acts on an object without coming physically in contact with it



What happens to the force when the distance between two charged objects is increased?

Newton demonstrated that the gravitational force between two masses varies with the product of the masses and the inverse square of the distance of separation:

$$F \propto \frac{m_1 m_2}{r^2}$$

In 1767, the English scientist Joseph Priestley, having confirmed an experiment undertaken by Benjamin Franklin the previous year, and knowing of Newton's inverse square law for gravity, published his finding that the electric force follows an inverse square law. However, Priestley's scientific paper was largely ignored by other scientists.

It was not until 1788–1789, when the French scientist Charles-Augustin de Coulomb published a series of eight papers on different aspects of his carefully constructed experiments on electrically charged objects, that the scientific community accepted that the electric force satisfied the inverse square law.

#### Coulomb's law

Combining the concepts of force being proportional to the product of the charges,  $F \propto q_1 q_2$ , and that force follows an inverse square law,  $F = \frac{1}{r^2}$ , gives:

$$F \propto \frac{q_1 q_2}{r^2}$$

This is known as **Coulomb's law** and describes the force between two charges at rest.

When the charges are in motion, additional forces are involved. These are discussed in later chapters. Coulomb's law applies to situations where the charged objects are much smaller in size than the distance between them. Such objects are referred to as point charges.

Coulomb's law the force between two charges at rest is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them

#### CHAPTER 4 ELECTRIC AND MAGNETIC FIELDS

If the charged object is a metal sphere, the entire charge can be considered to be concentrated at the centre of the sphere. This holds true provided the size of the objects is small compared to the distance of separation.

As shown in Figure 4A–17, doubling the distance reduces the force, *F*,

by a factor of four, so that is in now one-quarter as large  $\left(\frac{F}{4}\right)$ ; tripling the distance reduces the force by a factor of nine, so it is now one-ninth

as large  $\left(\frac{F}{9}\right)$ .

If Coulomb's law is written as an equation, it becomes Formula 4A–4.



**Figure 4A–17** Coulomb's law. The effect on the force between two charges as the distance is increased follows an inverse square law.

#### Formula 4A–4 Coulomb's law

$$F = k \frac{q_1 q_2}{r^2}$$

Where:

F = Force between two charged particles (N)

 $k = \text{Coulomb's constant} (8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2})$ 

 $q_1, q_2$  = Charge of each particle (C)

r = Distance between charged particles (m)

The force between the two charges depends on the nature of the medium between them and this affects the value of Coulomb's constant. If the medium is a vacuum, or dry air, and the force is measured in newtons, the separation in metres and the charge in coulombs is then  $k = 8.988 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$ . (When doing calculations using Coulomb's law to two significant figures, use  $k = 9.0 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$ .)

## Worked example 4A–6 Using Coulomb's law

One coulomb is a very large charge.

- a Calculate the force between two charges each of magnitude 1.0 C that are 3.0 m apart. Use  $k = 8.99 \times 10^9$  N m<sup>2</sup> C<sup>-2</sup>.
- **b** Calculate the magnitude of a mass (in kg and t) that would have the same gravitational force acting on it at Earth's surface. Use  $g = 10 \text{ N kg}^{-1}$ .
- **c** What sort of an object with that mass could be hypothetically lifted off Earth's surface with an electrostatic force of that size?

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#### Solution

a Use Coulomb's law.

$$F = k \frac{q_1 q_2}{r^2}$$
  
=  $\left(8.99 \times 10^9\right) \frac{(1.0)(1.0)}{(3.0)^2}$   
=  $1.0 \times 10^9 \text{ N}$ 

**b** Gravitational force =  $mg = 1.0 \times 10^9$  N

$$m = 1.0 \times 10^8 \text{ kg}$$
  
= 100 000 t

**c** The electrostatic force calculated would be more than enough, for example, to lift a modern ocean cruise liner 3.0 m into the air!

## Check-in questions – Set 2

- 1 The diagram shows a charge *Q*. Point X is 10 cm from the centre of *Q* and point Y is 20 cm from the centre of *Q*. The charge on  $Q = +2.0 \times 10^{-8}$  C.
  - Q X Y
  - a Calculate the magnitude and direction of the electric field at points X and Y.
  - **b** How would your answers change if *Q* was a negative charge of twice the magnitude?
- 2 Two positive charges on metal spheres are at a distance of 30 cm apart as shown.  $Q_1 = 2.0 \times 10^{-8}$  C and  $Q_2 = 8.0 \times 10^{-8}$  C.



- **a** Determine how far from the middle of charge  $Q_1$ , on the line joining  $Q_1$  to  $Q_2$ , that the magnitude of the electric field is zero.
- **b** Sketch a diagram of how the electric field varies between  $Q_1$  and  $Q_2$  on the line in the region between the two metal spheres.





#### Electric potential energy

the amount of work needed to move a unit charge from a reference point to a specific point against an electric field

## **Electric potential energy**

**Electric potential energy** is a form of energy that is stored in an electric field. In Chapter 3 it was demonstrated that gravitational potential energy can be stored in a gravitational field. Gravitational potential energy can be stored in a uniform gravitational field near the surface of Earth (Figure 4A–18 left) or in a changing gravitational field as we move away from the surface of Earth (Figure 4A–19 left).

Similarly, electric potential energy can be stored in a uniform electric field between two oppositely charged parallel plates (Figure 4A–18 right) as discussed earlier in this chapter, or in a changing electric field as we move away from a stationary charge (Figure 4–19 right).



**Figure 4A–18** Gravitational and electric potential energy changes occur (left) when a mass is moved in a uniform gravitational field, and (right), when a charge, *q*, is moved in a uniform electric field.

#### Non-uniform gravitational field



#### Non-uniform electric field



**Figure 4A–19** Gravitational and electric potential energy changes (left) when a mass is moved in a non-uniform gravitational field (left) and (right) when a positive charge is moved in a **non-uniform electric field** produced by a negative charge.

Non-uniform electric field an electric field where the strength and direction of the field vary

#### Electric potential energy changes in a non-uniform electric field

As noted earlier, the electric field, *E*, surrounding a point charge, *Q*, is given by:

$$E = k \frac{Q}{r^2}$$

The electric force, *F*, surrounding a point charge, *Q*, is given by:

$$F = k \frac{Q q}{r^2} = qE$$

This means that electric force diagrams (Figure 4A–20) follow the same shape as electric field diagrams (Figure 4A–14). Note that the radial direction away from the charge is assumed as the positive direction for the electric field.

Note that in Figure 4A–20 (top), a positive test charge would have a force acting to the right, while in Figure 4A–20 (bottom), a positive test charge would have a force acting to the left.

In Chapter 3, we learned that the area under a gravitational force– distance graph corresponds to the work done, W, and the change in potential energy,  $\Delta E_{g}$ , of a mass in a gravitational field around a point mass. This is also true for charged particles in an electric field around a point charge.

For example, Figure 4A-21shows an electric force–distance graph for the force between charge +Q at the origin, and a point charge +q placed at a distance r from it. If a second positive charge, +q, is placed at point B, a distance from a central positive charge as shown in Figure 4A-21, then to move it from point B to point A, work needs to be done to move it against the repulsive electrical force.



**Figure 4A–20** Top left: A positive electric charge and the direction of the electric force, *F*, away from it at point P, which is at distance *r*. Top right: Electric force–distance graph (*F* versus *r*) for a positive charge. Bottom left: A negative electric charge and the direction of the electric force, *F*, towards it at point P, which is at distance *r*. Bottom right: Electric force–distance graph for a negative charge, with negative values of *F*.

F  $F \propto \frac{1}{r^2}$  Area = work doneA = B



At point A, the second positive charge and field will have stored the work done as a change in electric potential energy. The amount of extra potential energy will be equal to the shaded area under the electric force–distance graph. If a positive charge is placed at point A, electric potential energy will be converted to kinetic energy as the charge moves away. At point B the charge will have kinetic energy equal to the change in electric potential energy (the shaded area) and it will continue to accelerate away to the right as the potential energy reduces.

**3B** GRAVITATIONAL POTENTIAL ENERGY

LINK



## Worked example 4A-7 The electric potential energy of a charge

A charge +q is pushed from position B to position A in Figure 4A–21 (on the previous page).

- **a** Calculate the electric potential energy,  $E_{pA}$ , at point A if the electrical potential energy at point B = 0.1 J and the shaded area under the graph = 0.3 J.
- **b** Calculate the kinetic energy,  $E_{k}$ , at point B ( $E_{kB}$ ) if the charge is released at point A.

Solution

**a** Electric potential at A is electric potential at B plus the work done in moving a positive charge from B to A. So:

 $E_{\rm pA} = 0.1 + 0.3 = 0.4 \, \text{J}$ 

**b** The kinetic energy at point B is the change in electric potential energy between points A and B, which is the shaded area under the graph. So:

 $E_{\rm kB} = 0.3 \, \rm J$ 

## Electric potential

The amount of electric potential energy (joules) required to move a coulomb of charge in an electric field is known as the electric potential, or voltage, V, and is measured in volts; 1 volt = 1 joule per coulomb (J C<sup>-1</sup>).

By convention, when considering the electric potential from a point charge, the potential is zero at infinity. This means that when a calculation problem specifies that a charged particle moves away 'to infinity', the potential is regarded as falling to zero.

#### Three-dimensional electric potential diagrams

The real world of electric potentials exists in the three dimensions of space. Figure 4A–22 shows a three-dimensional representation of the electric potential of a particular dioxin molecule. Dioxins are a group of highly toxic chemical compounds that are harmful to human and animal health. They can cause problems with reproduction, development and the immune system.

Dioxins have no common uses. They mostly exist in our environment as dangerous by-products of industrial processes: for example, bleaching paper pulp, pesticide manufacture and combustion processes such as waste incineration.

Three-dimensional electric potential diagrams allow scientists to better understand the nature of dioxins and therefore possibly mitigate against their harmful effects.



**Figure 4A–22** A three-dimensional representation of the electric potential of the dioxin known as –2,3,7,8– tetrachlorodibenzo para dioxin (TCDD). Dioxins are a group of highly toxic chemical compounds that are harmful to health.

## Electric potential energy changes in a uniform electric field

In Chapter 3, we learned that gravitational potential energy is stored in a field when a mass, m, is raised up a height, h, in a constant gravitational field,  $g(E_g = mg\Delta h)$ . Similarly, electric potential energy can be stored in a constant uniform electric field.

The electric field strength is given by the equations  $E = \frac{F}{q}$  and  $E = \frac{V}{d}$  for a uniform electric field.

Equating the two expressions and rearranging them enables us to derive a formula for work done, *W*. The latter is force times the distance moved, *Fd*. So, the work done by an external force to make a charged particle move a distance, *d*, against a potential difference, *V*, can be found:

$$E = \frac{F}{q} = \frac{V}{d}$$
$$W = Fd = qV$$

This is expressed in Formula 4A–5.

Formula 4A–5 Work on a particle in an electric field

$$W = qV$$

Where:

W = Work done on a charged particle in a uniform electric field (J) by an external force

q =Charge of the particle (C)

V = Change in electrical potential from the start to the end point of the particle in the electric field (V)

Figure 4A–23 shows a uniform electric field created between two charged metal plates, with plate A being positive and plate B being negative. The electric potential difference,  $\Delta V$ , equals  $V_{\rm AB}$ .

The positive charge, q, has a constant force acting down on it as it moves from A to B over a distance, d. The work done, W, on the charge is  $qV_{AB}$ .



**Figure 4A–23** Work, *W*, being done on a positive charge, *q*, as it travels a distance, *d* in a uniform electric field created between two parallel metal plates with a potential difference across the plates,  $\Delta V$ .





## CHAPTER 4 ELECTRIC AND MAGNETIC FIELDS

Low speed v << c, so relativistic effects can be ignored

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What happens when a **low speed** charged particle enters a uniform electric field at right angles to the field? Figure 4A-24 shows such an example. A positively charged particle, q, is injected with a horizontal speed, v, into a constant uniform electric field, E. The force acting on the charged particle in this case will always be constant and straight down, and its magnitude is given by F = qE.

This will cause the particle to follow a parabolic path while between the two horizontal charged plates. This is analogous to the parabolic path followed by a mass, *m*, projected horizontally in a uniform gravitational field, *g* (Figure 4A–25).



**Figure 4A–24** The path of a positively charged particle, *q*, entering a uniform electric field, *E*, at right angles is a parabola while in the field.







## Check-in questions – Set 3

1 The diagram below shows a positive charge *Q*. Point X is 10 cm from the centre of *Q* and point Y is 20 cm from the centre of *Q*.





- **a** What is the direction of the electric field at point X?
- **b** What is the direction of the electric field at point Y?
- c Compare the size of the electric field at points X and Y.
- **d** Which point, X or Y, has the greater electric potential energy?
- 2 Two parallel pates are 10 cm apart. An electron starting from rest at the surface of the negative plate is accelerated between the plates through a potential difference of 100 V. The charge on the electron is  $1.6 \times 10^{-19}$  C.

Calculate the:



- a magnitude of electric field between the two plates
- **b** magnitude of the electric force acting on the electron while in the uniform electric field
- c work done on the electron while travelling between the two plates
- d kinetic energy of the electron just before it arrives at the positive plate.



## **4A SKILLS**

Analysing the electric field, the work done and the change in kinetic energy on a charged particle between two oppositely charged parallel metal plates

#### Question

A uniform electric field is generated between two parallel pates 10.0 cm apart. A proton, initially at rest, moves from the positive plate to the negative plate. The voltage drop between the plates is 100 V. The mass of the proton is  $1.67 \times 10^{-27}$  kg.

#### Calculate the:

- a magnitude of electric field between the two plates
- **b** magnitude of the electric force acting on the proton while in the uniform electric field
- **c** work done on the proton while travelling between the two plates
- **d** kinetic energy of the proton just before it arrives at the negative plate
- **e** speed of the proton just before it arrives at the negative plate.

Note that the charge of the proton is not given in the original information. This is the same charge as carried by the electron, but positive not negative:  $+1.6 \times 10^{-19}$  C.

#### Solution

**a** To find the electric field, use:

$$E = \frac{V}{d}$$
$$= 1000 \text{ V m}^{-1}$$

**b** To find the magnitude of the force on the proton, use:

$$F = qE$$
  
= (1.6 × 10<sup>-19</sup>)(1000)  
= 1.6 × 10<sup>-16</sup> N

**c** To find the work done on the proton while travelling between the two plates, use:

$$W = qV$$
  
= (1.6 × 10<sup>-19</sup>)(100)  
= 1.6 × 10<sup>-17</sup> J

**d** The work done is also the change in kinetic energy, so the answer is the same as part c.

$$W = \Delta E_{\rm k}$$
$$= 1.6 \times 10^{-17} \,\rm J$$

**e** To find the speed of the proton just before it arrives at the negative plate, use:

$$qV = \Delta E_{\rm k} = \frac{1}{2} mv^2$$
$$v = \sqrt{\frac{2E_{\rm k}}{m}}$$
$$= 1.38 \times 10^5 \,\rm{m\,s^{-1}}$$





## **Section 4A questions**

## **Multiple-choice questions**

1 The following is an electric field diagram drawn by a physics student for the electric field surrounding two positive charges.



The electric field diagram is

- A correct.
- **B** incorrect as the field lines cross.
- **C** incorrect as all the field lines should be straight not curved.
- **D** incorrect as the field lines should have arrows pointing towards the positive charges.
- 2 The diagram below shows two positive charges of magnitude Q and 2Q.



P

Which vector best represents the direction of the electric field at point P, which is equidistant from both of the charges?



**3** Two large charged plates with equal and opposite charges are placed close together, as shown in the diagram below.



A distance of 5.0 mm separates the plates. The electric field between the plates is equal to  $1000 \text{ NC}^{-1}$ .

Which one of the following is closest to the voltage difference between the plates?

- **A** 5.0 V
- **B** 200 V
- **C** 5000 V
- **D** 5000000 V

VCAA 2017

#### Use the following information to answer Questions 4 and 5.

The diagram shows a charge Q. Point X is 10 cm from the centre of Q and point Y is 20 cm from the centre of *Q*. The charge on  $Q = 8.0 \times 10^{-8}$  C.



- **4** The electric field strength at point X is closest to
  - A 7.2 NC<sup>-1</sup>
  - **B**  $7.2 \times 10^1 \text{ N C}^{-1}$
  - C  $7.2 \times 10^3 \, N \, C^{-1}$
  - **D**  $7.2 \times 10^4 \text{ N C}^{-1}$
- 5 Which one of the following correctly shows the electric field strength at Y, if the electric field strength at X is  $E_0$ ?

$$A \quad \frac{E_0}{4}$$
$$B \quad \frac{E_0}{2}$$
$$C \quad E_1$$

**D** 
$$2E_0$$

Use the following information to answer Questions 6 and 7.

Two positive charges, 4Q and Q, are placed a distance, r, apart as shown below. The force between them has a magnitude of *F*.



- 6 Which one of the following statements is true?
  - A The charge with a greater magnitude exerts a larger force on the small charge.
  - **B** The charge with a greater magnitude exerts a smaller force on the small charge.
  - **C** The forces on each charge are the same magnitude and opposite in direction.
  - **D** The forces on each charge are the same magnitude and point in the same direction.
- 7 The distance between the two charges is doubled from r to 2r.

Which one of the following is closest to the force between the two charges?



- **C** *F*
- **D** 2F



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#### CHAPTER 4 ELECTRIC AND MAGNETIC FIELDS

8 Three charges (-Q, +2Q, -2Q) are placed at the vertices of an isosceles triangle, as shown below.



Which one of the following arrows best represents the direction of the net force on the charge -Q?



VCAA 2019

9 A metal sphere has a charge of  $1.0 \times 10^{-8}$  C on it. A small sphere with a charge of  $1.0 \times 10^{-9}$  C is placed 30 cm from it.

Assume both can be considered point charges.

Take  $k = 9.0 \times 10^9$ .

Which one of the following best gives the magnitude of the force on the small sphere?

- $\textbf{A} \quad 1.1 \times 10^{-14} \text{ N}$
- **B**  $1.0 \times 10^{-6}$  N
- **C**  $3.0 \times 10^{-6}$  N
- **D**  $3.0 \times 10^{-5}$  N

VCAA 2017

- **10** A charge, which is initially at rest, is accelerated through a potential difference of 500 V and increases its kinetic energy by  $4.0 \times 10^{-5}$  J. Which one of the following is closest to the magnitude of the charge?
  - **A**  $4.0 \times 10^{-8} \text{ C}$
  - **B**  $8.0 \times 10^{-8} \text{ C}$
  - **C**  $4.0 \times 10^{-7} \text{ C}$
  - **D**  $8.0 \times 10^{-7} \text{ C}$

#### **Short-answer questions**

**11** Two equal negative charges are placed as shown.



**a** Copy the diagram and draw a representation of the electric field lines between and surrounding the two charges.

One of the negative charges, -Q, is now replaced with the same magnitude positive charge, +Q, as shown.



**b** Copy the diagram and draw a representation of the electric field lines between and surrounding the two charges.

12 The diagram shows two equal positive stationary point charges placed near each other.



Copy the diagram and sketch the shape and direction of the electric field lines. Use at least eight field lines.

VCAA 2019

**13** The diagram shows a charge, *Q*. Point X is 10 cm from the centre of *Q* and point Y is 20 cm from the centre of *Q*. The charge on  $Q = +2.0 \times 10^{-8}$  C.



- **a** Calculate the magnitude and direction of the electric field at points X and Y.
- **b** How would your answers change if *Q* was a negative charge of twice the magnitude?
- **14 a** Describe one difference between electrostatic forces acting on charges and gravitational forces acting on masses.
  - **b** Describe one important similarity between electrostatic forces and gravitational forces. Hint: distance.
- **15** Three charges are arranged in a line, as shown.



Draw an arrow at point X to show the direction of the resultant electric field at X. If the resultant electric field is zero, write the letter 'N' at X.

VCAA2017

Х

Q

**16** Two negative charges on insulated metal spheres are at a distance of 20 cm apart as shown.  $Q_1 = -1.0 \times 10^{-8}$  C and  $Q_2 = -9.0 \times 10^{-8}$  C. Use  $k = 9.0 \times 10^{9}$  N m<sup>2</sup> C<sup>-2</sup>.



Calculate the distance from charge  $Q_1$ , where the electric force is zero.

- 17 Four equal positive charges, *Q*, are placed at the corner of a square as shown. A positive test charge is placed at point X in the centre of the square.
  - **a** What is the net force acting on the positive test charge placed at point X?
  - **b** Explain your answer to part a.
- **18** The diagram shows a positive charge, *Q*. Point X is 10 cm from the centre of *Q* and point Y is 20 cm from the centre of *Q*.



Υ

Х

- a What is the direction of the electric field at point X?
- **b** What is the direction of the electric field at point Y?
- **c** Compare the size of the electric field at points X and Y.
- **d** Which point, X or Y, has the greater electric potential energy?



**19** The diagram shows the force–distance graph for a positive charge.



Estimate the change in electric potential energy if the distance between the points A and B is 20 cm.

**20** The electron gun section of a particle accelerator accelerates electrons between two plates that are 10 cm apart and have a potential difference of 5000 V between them.

Mass of electron	$9.1  imes 10^{-31} \text{ kg}$
Charge on electron	(−) 1.6 × 10 <sup>−19</sup> C

- a Calculate the electric field between the plates. Include an appropriate unit.
- **b** Calculate the magnitude of the force on an electron between the plates.
- **c** Calculate the speed of the electrons as they exit the electron gun. Ignore any relativistic effects. Assume that the initial speed of the electrons is zero.

#### VCAA NHT 2018

21 A charged particle carrying charge *Q* and travelling at speed *v* enters a uniform electric field, *E*. The charged particle is deflected vertically a distance *d* while travelling through the uniform electric field as shown.



- a Is the charged particle positive or negative? Justify your answer.
- **b** Is the force acting on the charged particle constant or changing? Justify your answer.
- **c** Is the speed of the charged particle as it leaves the uniform electric field less, the same, or greater than when it entered the uniform electric field? Justify your answer.

A particle carrying the same charge, Q, enters the field travelling at a speed  $2\nu$ .

**d** Does the deflection distance, *d*, decrease, stay the same, or increase? Justify your answer.

Another charged particle, carrying charge  $\frac{Q}{2}$ , enters the uniform electric field travelling at speed *v*.

e Does the deflection distance, *d*, decrease, stay the same, or increase? Justify your answer.



# **Magnetic fields and forces**

#### Study Design:

- Describe gravitation, magnetism and electricity using a field model
- Investigate and compare theoretically and practically gravitational, magnetic and electric fields, including directions and shapes of fields, attractive and repulsive effects, and the existence of dipoles and monopoles
- Investigate and apply theoretically and practically a field model to magnetic phenomena, including shapes and directions of fields produced by bar magnets, and by current-carrying wires, loops and solenoids
- Identify fields as static or changing, and as uniform or non-uniform
- Analyse the use of a magnetic field to change the path of a charged particle, including:
  - the magnitude and direction of the force applied to an electron beam by a magnetic field:
     F = qvB, in cases where the directions of v and B are perpendicular or parallel
  - ► the radius of the path followed by an electron in a magnetic field:  $qvB = \frac{mv^2}{r}$ , where  $v \ll c$
- Describe the interaction of two fields, allowing that electric charges, magnetic poles and currentcarrying conductors can either attract or repel, whereas masses only attract each other
- Investigate and analyse theoretically and practically the force on a current-carrying conductor due to an external magnetic field, *F* = *nIIB*, where the directions of *I* and *B* are either perpendicular or parallel to each other
- Investigate and analyse theoretically and practically the operation of simple DC motors consisting of one coil, containing a number of loops of wire, which is free to rotate about an axis in a uniform magnetic field and including the use of a split ring commutator
- Investigate, qualitatively, the effect of current, external magnetic field and the number of loops of wire on the torque of a simple motor
- Model the acceleration of particles in a particle accelerator (including synchrotrons) as uniform circular motion (limited to linear acceleration by a uniform electric field and direction change by a uniform magnetic field)

#### **Glossary:**

Electromagnet Electromagnetism Electron gun Magnetic dipole Magnetic field Particle accelerator Right-hand grip rule Right-hand slap rule Solar wind Solenoid Static field Synchrotron Torque Uniform magnetic field



# ENGAGE

## Low-level flying

Imagine you are flying along at over 600 km h<sup>-1</sup>, without noise or vibration, not at 10 000 m altitude in an aircraft, but in a newly designed passenger train, 'flying' only a few millimetres above a specially fabricated train track. The secret of this mode of travel is magnetism. Such trains are supported and propelled by magnetic fields. One of the most advanced of these experimental magnetic levitation (maglev) trains is shown in Figure 4B–1. This is China's new experimental maglev bullet train, which travels at speeds of 600 km h<sup>-1</sup> on a magnetic 'cushion'.



Figure 4B–1 China's new experimental maglev bullet train, which can travel at 600 km  $h^{-1}$  on a magnetic 'cushion'

On the other hand, you may want to travel at high speeds in a more conventional train – one that still uses wheels and electric motors! The Train à Grande Vitesse (TGV) in France uses conventional 22 000 V DC (direct current) motors to transport people at 330 km h<sup>-1</sup> on a regular basis (and has recorded a maximum speed of 515 km h<sup>-1</sup>).

Australian engineers are examining the TGV as a model for the 'Very Fast Train' (VFT), which has been suggested as an alternative to aircraft for inter-capital express travel throughout Australia. How do such trains, either using conventional electric motors or linear induction maglev-style propulsion, work? Can the same technology be applied to personal transport? Could we have low flying 'maglev' cars? Will the advent of new high temperature superconducting materials give rise to radically new designs in electrical motors? And will these provide us with cheaper, more efficient, lighter and more reliable electric motors and electric cars?



VIDEO 4B-1

MAGNETIC FIELDS AND

FORCES

## Magnetism

**EXPLAIN** 

You will be familiar with the fact that a permanent magnet attracts paper clips, nails, keys and other objects made of iron (Figure 4B–2).

All magnets, no matter what their shape (e.g. bar, horseshoe or cylindrical), have two ends or faces called poles, where the magnetic effect is strongest. By convention, these poles are called the north and south poles respectively (the technical term for this is a **magnetic dipole**).

Whenever a magnet is broken, the individual pieces are each (weaker) magnets, also having two poles. This process of creating smaller and weaker magnets, by continually breaking up magnetic materials, can continue right down to subatomic particles. Even protons and electrons have magnetic dipoles (Figure 4B–3).



**Figure 4B–2** The neodymium–iron–boron magnet is the strongest permanent magnet material in wide-scale use today.



generates a magnetic field in which the field is considered to emanate from two opposite poles, as in the north and south poles of a magnet

**Magnetic dipole** 

any object that

## **Magnetic fields**

The region around a magnet, where

magnetic effects can be experienced, is called the magnetic field and can be demonstrated using iron filings (Figure 4B–4 left).

have magnetic dipoles.

The direction of the field, at a point, is taken to be the direction of force that would be on a magnet's north pole if it was placed at that point (Figure 4B–4 right).

Arrows are used to indicate the direction of the force. Note that both of these figures show a two-dimensional representation of the magnetic field. The magnetic field (which is given the symbol B) is a vector quantity, since it has both magnitude and direction. The SI unit of magnetic field is the tesla (T).



**Figure 4B–4** All permanent magnets have a north and south pole. The magnetic field surrounding a magnet can be visualised using iron filings (left) and represented using arrows (right).

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Magnetic field the property

of the space around a magnet that causes an object in that space to experience a force due only to the presence of the magnet

#### Measuring magnetic fields

The strength of a magnetic field is measured in a unit called tesla (T).

The strength of Earth's magnetic field at its surface is small, about one ten-thousandth of a tesla (0.1 mT). A typical fridge magnet has a strength of approximately 30 mT. The strength of a magnetic field of a typical school bar magnet is about 0.5 T, the strength of a very strong physics laboratory magnet is about 5.0 T.

Physicists have developed very sensitive magnetic detectors and can measure magnetic fields as low as  $8 \times 10^{-15}$  T. Apart from pure research into the mechanisms of magnetisation and the tracing and mapping of geological mineral ore bodies from low-flying helicopters, the measurement of weak magnetic fields also enables medical diagnosticians to examine the very weak magnetic fields generated, for example, by the heart and the brain.

The magnetic field exists in three dimensions. A close-up photograph of the iron filings pattern between the north pole of a bar magnet and the south pole of a horseshoe magnet shows the three-dimensional nature of the magnetic field (below).



**Figure 4B–5** The three-dimensional nature of magnetic field lines can be seen when iron filings are gathered between the north pole of a bar magnet and the south pole of a horseshoe magnet.

In 1846, Michael Faraday introduced the concept of magnetic field lines to explain how magnets could attract and repel each other.

Faraday's rules for magnetic field lines are listed below:

- Each magnetic field line is a continuous loop that leaves the north end of the magnet, enters at the south end and passes through the magnet back to the north end.
- Field lines do not intersect.
- The closeness of the magnetic field lines represents the strength of the magnetic field.

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Magnetic field diagrams can be used to explain why magnets attract each other and repel each other as shown in Figure 4B–4. The forces between magnets can be attractive, when unlike poles are placed near each other (Figure 4B–6 left) or repulsive, when like poles are placed near each other (Figure 4B–6 right). Note that between the poles in Figure 4B–7, there is a section of almost uniform magnetic field.





**Figure 4B–6** Magnetic field lines shown by iron filings for attraction: north–south magnets (left) and repulsion: north–north magnets (right)

U-shaped magnets can exhibit a greater region of uniform magnetic field (Figure 4B–7). The magnetic field is not changing and is therefore said to be a **static field**.

#### **Magnetic dipoles**

All of the magnetic field diagrams show a north pole and a corresponding south pole; in other words magnetic fields are examples of dipole fields. This is different to electric and gravitational fields, where monopoles exist (see Worked example 4B–1). To date, there is no evidence for the existence of magnetic monopoles (see note).



**Figure 4B–7** A two-dimensional view of the magnetic field between the poles of a U-shaped magnet. Note the region of uniform magnetic field.

Static field a field that is not changing

## NOTE

#### The magnetic monopole

To date, it has not been possible to isolate a single magnetic pole. However, since Paul Dirac's 1931 theoretical prediction that they should exist, physicists have been searching for them continuously. The guiding principle for the prediction is the symmetry of nature. For example, the existence of the negative electron suggested the existence of the positive electron (positron). This positron was subsequently discovered and is now the basis of an invaluable medical diagnostic technique known as positron emission tomography (PET). So far, no one has been able to find an isolated magnetic pole (that is, without its corresponding opposite magnetic pole). The absence of experimental confirmation of the existence of magnetic monopoles creates a serious theoretical threat to the idea of an underlying symmetry between our understanding of electricity and of magnetism.

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# $\mathbf{E}$

## Worked example 4B–1 Dipoles and monopoles

Gravitation, magnetism and electricity can be explained using a field model. According to our understanding of physics and current experimental evidence, these three field types can be

Field type	Only monopoles	Only dipoles	Monopoles and dipoles
Gravitation			
Magnetism			
Electricity			

associated with only monopoles, only dipoles or both monopoles and dipoles. Copy and complete the table by ticking the appropriate box to indicate whether each field type can be associated with only monopoles, only dipoles or both monopoles and dipoles.

#### Solution

There is only one variety of mass; it doesn't come in negative or positive like electric charge, or north or south like magnetic poles. Therefore, gravitation is associated only with monopoles.

In magnetism there are north and south poles that exist together in north–south pairs with no evidence that they exist on their own. Therefore, magnetism is associated only with dipoles.

Electric charges come in positive and negative varieties, which can exist separately as monopoles or in positive–negative pairs. Therefore, electricity is associated with both monopoles and dipoles.

Field type	Only monopoles	Only dipoles	Monopoles and dipoles
Gravitation	<ul> <li>✓</li> </ul>		
Magnetism		~	
Electricity			<b>v</b>

## Representing a magnetic field in three dimensions

Representing a magnetic field often requires a three-dimensional view. To achieve this on a flat two-dimensional page, a convention is adopted called the arrow convention. The symbol of a circle with a dot in the middle is often used to represent a magnetic field



Figure 4B–8 Magnetic fields going into the page as viewed from the left and coming out of the page as viewed from the right, using the arrow convention.



used to represent a magnetic field coming out of a page (arrow head coming towards you) and a circle with a diagonal cross is often used to represent a magnetic field going *into the page* (arrow tail going away from you). Note that in some diagrams, just the arrow heads (dots) and tails (crosses) are used, without the surrounding circles.

Figure 4B–8 demonstrates this. The eye on the left sees the magnetic field going into the page (arrow tails) and the eye on the right sees the magnetic field coming out of the page (arrow heads).

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- **b** Explain the arrow convention used in representing magnetic fields.

## Electromagnetism

In 1820, the Danish physicist Hans Christian Ørsted noticed that the needle of a compass, placed next to an electrical wire carrying a current, created forces on the needle (Figure 4B–9 left) that eventually turned the needle of the compass so that it was perpendicular to the wire (Figure 4B–9 right).



**Figure 4B–9** In 1820, Hans Ørsted discovered that an electric current creates its own magnetic field that forces the compass needle to turn perpendicular to the current-carrying wire.



#### Electromagnetism

the branch of physics that considers the connection between electricity and magnetism

## Right-hand grip rule

a mnemonic used to determine the direction of the magnetic field produced by a current-carrying wire. The thumb indicates the direction of the current and the curled grip represents the field lines. Ørsted had discovered a connection between electricity and magnetism – **electromagnetism**. He found that a wire carrying a current shows an associated magnetic field (Figure 4B–10 left).

A straight current-carrying wire creates a circular magnetic field around it. The direction of the circular field is determined by using the **right-hand grip rule** (Figure 4B–10 right). For this rule, the thumb indicates the direction of current and the curled grip indicates the direction of the field lines. The strength of a magnetic field around a current-carrying wire is proportional to the magnitude of current flowing in the wire and decreases radially outward from the wire.





**Figure 4B–10** Left: Ørsted's discovery of the existence of a magnetic field, *B*, (green lines) around a wire carrying an electric current, *I*. Right: The right-hand grip rule, where *I* is the current-carrying wire and *B* is the magnetic field. Recall the convention for current direction is from the positive to negative terminal.

The arrow convention is used to indicate the direction of the current if the wires go into or come out of the page. A cross represents the tail of an arrow, while the dot represents the head of an arrow. Applying the right-hand grip rule creates magnetic fields surrounding the currents as shown in Figure 4B–11.



Figure 4B–11 Magnetic field for current-carrying wires with current coming out of the page (left) and current going into the page (right)
The arrow convention is also used for the magnetic field going into or coming out of the page as shown in Figure 4B–12.

Two current-carrying wires arranged parallel to each other will each have their own magnetic field. The direction of the magnetic field around each wire is given by the right-hand grip rule. If the two wires are brought closer together, they interact. The interaction can result in either an attraction or repulsion of the wires. When the currents are in the same direction, the wires attract (Figure 4B–13 top). When the currents are in opposite directions, the wires repel (Figure 4B–13 bottom).



**Figure 4B–12** The direction of the magnetic field, *B*, in and out of the page, can be determined using the right-hand grip rule for a current-carrying wire, where *I* is the current and *B* is the magnetic field.

Other current-carrying wire geometries can concentrate the magnetic field. For example, a single loop of wire concentrates the magnetic field in the centre of the loop (Figure 4B–14 left). By making a coil of wire with lots of loops, it is possible to create a very intense magnetic field. Such a coil of wire is called a solenoid. When an electric current flows in the coil, a magnetic field is produced in and around the solenoid. Figure 4B–14 (right) shows that the magnetic field created by the solenoid is similar to that of a bar magnet.







#### Solenoid

a coil of wire that creates a magnetic field when current is passed through it that is similar to a bar magnet

**Figure 4B–14** Left: A single loop of wire concentrates the magnetic field in the centre of the loop. Right: A coil of wire with many loops (solenoid) creates an even stronger magnetic field.

#### CHAPTER 4 ELECTRIC AND MAGNETIC FIELDS

Electromagnet a coil of wire, consisting of an iron or steel core, through which a current is passed In 1823, an English electrical engineer, William Sturgeon, placed an iron rod inside a solenoid (Figure 4B–15). This increased the strength of the magnetic field produced by the solenoid. Sturgeon had invented the **electromagnet**, which could be turned on and off, made stronger by increasing the current and constructed into large configurations. Modern electromagnets (Figure 4B–16) can produce magnetic fields up to a thousand times stronger than the magnetic field produced in a solenoid using the same current.



**Figure 4B–15** An electromagnet made by placing an iron rod inside a solenoid. The magnetic field surrounding the electromagnet is similar to that of the solenoid, but can be up to one thousand times stronger.



**Figure 4B–16** 500 kg of hot metal chips are transported into an industrial furnace by an electromagnet. The furnace then creates cast metal for wind turbines.

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# Check-in questions – Set 2

1 The diagram shows an electromagnet and its associated magnetic field. A physics student is looking at it, as shown by the eye at the right-hand end.



- **a** From the student's viewpoint, in which direction is the current in the electromagnetic coil going (clockwise or anticlockwise)?
- **b** Give three reasons why electromagnets are more versatile than permanent magnets for industrial uses.
- **2** A small compass is placed next to a solenoid, as shown in the diagram. Determine the direction (P, Q, R, or S) that the north pole of the compass would point to when the solenoid is connected to the battery.



## Forces on current-carrying wires in magnetic fields

With the development of electromagnets, very strong magnetic fields could be created. This meant that the reverse of Ørsted's discovery could be investigated: what is the effect of a magnetic field on a current-carrying wire? Ørsted's experiment demonstrated that

the magnetic field due to the current exerts a force on the magnetic field of the compass. So, according to Newton's third law, the compass should exert an equal and opposite force on the current-carrying wire.

If a straight current-carrying wire, *I*, is placed between the poles of two magnets, such a force, *F*, is observed (Figure 4B–17). The direction of the force on the wire is given by a rule known as the **right-hand slap rule** (or right-hand palm rule), shown in Figure 4B–19 on page 207.



**Figure 4B–17** In an external magnetic field, a current-carrying conductor experiences a force.



## **Right-hand**

**1A** NEWTON'S

I AWS

slap rule a mnemonic used to determine the direction of the force on a currentcarrying wire in a magnetic field. The thumb indicates the direction of the current, the fingers represent the magnetic field and the palm faces in the direction of the force.



No magnetic force is produced if the current-carrying wire is parallel or anti-parallel to the magnetic field (Figure 4B–18). For the purposes of this course, only current-carrying wires that are perpendicular or parallel to the magnetic field are considered.



Figure 4B–18 No magnetic force is produced if the currentcarrying wire is parallel or anti-parallel to the magnetic field.

Experimental observations of the magnetic force applied to the current-carrying wire show that there is a larger force on the wire if:

- the strength of the magnetic field increases
- the magnetic field acts on a larger current in the wire
- the magnetic field acts on a longer section of wire
- there are more wires in the magnetic field.

Combined, these findings can be expressed as Formula 4B-1.



- I = Current in each whe (A)
- l = Length of each wire (m)
- B = Strength of the magnetic field (T)

# lacksquare

## Worked example 4B–2 Calculating the magnetic force on a wire

Calculate the magnitude of the force acting on a straight wire of length 2.0 m carrying a current of 3.0 A in a magnetic field of strength 1.5 T if the wire is perpendicular to the magnetic field.

Solution

As the current-carrying wire is perpendicular to the magnetic field, there will be a magnetic force acting.

Use the magnetic force equation, F = IlB (as it is a single wire, n = 1).

Substitute the values into the magnetic force equation.

F = (3.0)(2.0)(1.5)

= 9.0 N

#### **Right-hand slap rule**

The right-hand slap rule is a simple mnemonic, shown in Figure 4B–19, that can be used to determine the direction of the force on a current-carrying wire in a magnetic field. Using your right hand, with fingers outstretched and flat, point the thumb in the direction the current is moving and the outstretched fingers in the direction of the magnetic field. The direction of the resulting force on the current-carrying wire is the direction in which your palm is pointing.



**Figure 4B–19** The right-hand slap rule: point the thumb of your right hand in the direction of the current and the fingers in the direction of the magnetic field. The force on the current-carrying wire will point out from the palm.

# Check-in questions – Set 3

1 Rutherford used a magnetic field to investigate the charge on particle rays being emitted from radioactive polonium, as shown in the diagram below. The particle rays are labelled as X, Y and Z.



Determine the charge of each particle ray and explain how you determined the charge.

- **2** Calculate the magnitude of the force acting on a straight wire of length 10.0 cm carrying a current of 200 mA in a magnetic field of strength 0.50 T if the wire is:
  - a perpendicular to the magnetic field
  - **b** parallel to the magnetic field.

# **DC motors**

If a loop of wire is placed in a magnetic field, it will experience forces when a current is passed through it. Figure 4B–20 (left) shows the forces on a loop of current-carrying wire in a magnetic field. You can easily check the direction of the force using the right-hand slap rule. The loop tends to rotate about the horizontal axis, around the pivot point. The forces on the sides, if we call positive up, are +*IlB* (on the left) and –*IlB* (on the right) per loop. These forces generate a turning moment (torque).

The greater the number of loops, the greater the overall turning effect.

Taking a closer look at the front of the loops in Figure 4B–20 (left), we can see that the battery providing the electric (direct) current is connected via carbon brushes to a split ring commutator. This reverses the current exactly each half cycle, providing the necessary pair of forces to keep the device spinning. In essence, this is an electrical direct current (DC) motor.



**Figure 4B–20** A pair of forces are created when a loop of current-carrying wire is placed in a magnetic field. Because of the design geometry, this pair of forces produce a turning moment (torque) around the pivot point. This effect is the basis for all simple direct current (DC) motors.

When the plane of the loop has turned to the vertical, the pair of forces wants to pull the loop apart; only the clockwise angular momentum of the spinning loop keeps it turning at this point (otherwise it would be in a 'dead spot' and would not rotate at all).

You might expect the pair of forces at this stage would now cause the loops to reverse direction and head back to the vertical position. This would be the case except that the current is reversed when the loops reach the vertical position. This is done via the split ring commutator that sits in front of the motor. It reverses the battery's electric current to the loops exactly each half cycle.

The magnetic field can be provided by a permanent magnet or by an electromagnet. A practical commercial DC motor, an electric drill for example (Figure 4B–21), not only has many loops (perhaps hundreds or thousands) in a single coil (called an armature winding), but it also has many groups of these coils wound at various angles on thin iron strips (laminations). The two ends of the armature windings are connected to many strips of copper called segments, which make up the split ring commutator (see Figure 4B–22).

Torque a measure of how much of a force acting on an object is causing it to rotate

#### 4B MAGNETIC FIELDS AND FORCES

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Figure 4B–21 Cross-section showing an electric motor in a drill

**Figure 4B–22** A typical multi-coil DC electric motor showing the main features. The stator coils produce an electromagnet that provides the magnetic field. The split ring commutator supplies current to the armature windings to ensure maximum torque will be experienced.

#### In 1821, Michael Faraday created the world's first DC electric motor (Figure 4B–23).



Figure 4B–23 Left: Michael Faraday, 1821. Right: The first electric motors – Michael Faraday, 1821 (*Quarterly Journal of Science, Vol XIII, 1821*)

For his motor, Faraday created a circuit comprised of a wire, a battery and a dish of mercury (a very good electrical conductor but can produce dangerous vapours). The wire was arranged so that one end hung free in the mercury bath. When current ran through the circuit, it generated a circular magnetic field around the wire (seen in the far right-hand side of Figure 4B–23 right). The wire's magnetic field interacted with the magnetic field of a permanent magnet fixed in the centre of the mercury bath. This caused the wire to rotate around the magnet. Faraday had converted electrical energy into rotational kinetic energy (movement).

#### Torque

To make an object start to rotate requires a force. However, the direction of this force and where it is applied are also important. For example, if you take your pen and apply a force to its end in the direction indicated in Figure 4B–24a, you will not get any turning effect.

Now, compare the situations shown in Figure 4B–24 (a–c). If you now apply the same force, *F*, to the pen but at different distances ( $r_1$  and  $r_2$ ) from the pivot point or hinge, the effect will be different. The closer you get to the hinge, the less effect the force has in turning the pen.

Note that this effect is often used in levers. In other words, the turning effect is dependent upon not only the magnitude and direction of the force, but also on where the force is applied relative to the hinge. The turning effect is proportional to the product of the force multiplied by the length of the lever arm (i.e.  $Fr_1$  in Figure 4B–24b or  $Fr_2$  in Figure 4B–24c).

If you apply a force at an angle to the pen, (Figure 4B–24d) only the component of the force,  $F \sin \theta$ , perpendicular to the lever arm has a turning effect.

This turning effect is given by the product of the lever arm distance and the force applied. It is called the moment of the force or torque and is abbreviated by the Greek lowercase letter tau,  $\tau$ .

 $\tau = rF$  (where *r* and *F* are perpendicular)

 $\tau = rF \sin \theta$  (where *r* and *F* are at an angle  $\theta$ )

The SI unit for torque is newton metre (Nm).



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#### Torque in electric motors

For electrical motors, the torque will be dependent upon the force (given by F = IlB for one loop and F = nIl for n loops of wire) and the lever arm of the wire from the centre of rotation. When the loop is parallel to the magnetic field, the torque is a maximum. When the loop is perpendicular to the field, the force has no component perpendicular to the lever arm and so the torque is zero. The graph of the torque,  $\tau$ , plotted against time for one complete rotation is shown in Figure 4B–25.



**Figure 4B–25** The graph of torque plotted against time for one complete rotation of a simple electric motor. The spinning of the motor (its angular momentum) keeps it moving through the positions where the torque is zero (position c). At point e, it swaps to a and the cycle starts again.



# Check-in questions – Set 4

- 1 Explain the role of the split ring commutator in a DC motor.
- 2 A rectangular loop of wire, ABCD, with the dimensions as shown on the right, is in a magnetic field of strength 0.20 T. The current in the loop is 1.0 A.

Calculate the magnitude of the:

a magnetic force acting on side AB when it is in the position shown



- **b** magnetic force acting on side BC when it is in the position shown
- c torque about the axis of rotation that acts on side CD in the position shown.

# Magnetic force on moving charges

The fact that electric currents can produce magnetic fields led physicists to believe that all magnetic effects were due to electric currents. In fact, to explain permanent magnetism, it is theorised that each electron has its own magnetic dipole associated with it, a result of the motion of the electrons within the atoms themselves.

Just like current carrying wires, moving electric charges can interact with external magnetic fields. This effect is detectable and the basis of a number of modern technologies including mass spectrometers, uranium isotope enrichment plants, medical cyclotrons, synchrotrons (such as the Australian Synchrotron in Melbourne) and cathode ray oscilloscopes (CROs).

Instead of the moving charges being contained in a current-carrying wire, individual charged particles move through a magnetic field often in an evacuated chamber.

The force acting on each charged particle depends on the strength of the charge, q, the velocity at which it moves, v, and the strength of the magnetic field, B, and the direction of v relative to B.

This relationship is expressed in Formula 4B-2.

#### Formula 4B–2 Magnetic force on moving charge

$$F = qvB$$

Where:

F = Magnetic force on the charged particle (N)

q = Charge of the particle (C)

v = Velocity of the particle (m s<sup>-1</sup>)

B = Strength of the magnetic field (T)

Note: The direction of the force acting on the charged particle is given by the right-hand slap rule.

Synchrotron a type of particle

accelerator where the particle beam travels in a closed-loop path

#### Direction and velocity of charged particles in a magnetic field

For the purposes of this course, only the motion of charged particles that are travelling perpendicular or parallel to the magnetic field are considered. In addition, the velocity of the particles considered are such that their velocity is much less than the speed of light ( $\nu << c$ ), meaning that relativistic effects are not considered in this part of the course.

Figure 4B–26 (left) shows a positive charge travelling to the right through a magnetic field into the page. The force acting on the charge will be, at that instant, in the upward direction perpendicular to both the velocity and the magnetic field. A fraction of time later, because the charge has now changed direction under the influence of the force, the velocity vector is in a different direction. Therefore, the new force will be perpendicular to this new velocity vector (Figure 4B–26 right). This regular change will cause the particle to travel in the arc of a circle (if the force is large enough it will cause the particle to complete a circular pattern, see Figure 4B–30 on page 215). Note that if the charge were negative, the force on the negative charge would be down. Note also that a negative charge going to the left is equivalent to a positive charge going to the right.



**Figure 4B–26** The positive charge travelling to the right at velocity *v* through the magnetic field acting into the page has a magnetic force acting on it, given by F = qvB. Using the right-hand slap rule, the direction of the force at this instant is up the page. The force always acts perpendicular to the direction of the movement and so causes the particle to follow the arc of a circle, or even a complete circular path.

#### Cosmic rays and discovery of the positron

In 1932, Carl Anderson was photographing the tracks of cosmic rays in a cloud chamber when he noticed that some particles, created when the cosmic rays interacted with a lead plate, were deflected the opposite way to the deflection of the electrons (Figure 4B–27). As the particles had the same deflection characteristics as electrons, he reasoned that they were positively charged anti-electrons or positrons. This confirmed a theoretical prediction, made by Paul Dirac in 1931, of the existence of an anti-electron that would have the same mass as an electron but the opposite charge. Dirac postulated these two particles would mutually annihilate upon interaction and produce gamma rays. This is the basis of medical gamma ray diagnosis based on positron emission tomography (PET) scans. Carl Anderson was awarded the Nobel Prize in 1936 for his discovery of the positron.



**Figure 4B–27** Photograph taken by Carl Anderson in 1932 of positron and electron tracks in a Wilson cloud chamber produced by a cosmic shower. The electrons and positrons travel from right to left in the photograph and the magnetic field is into the page. This bends the electrons up, while the positrons are bent down.



8B SIMILARITIES BETWEEN LIGHT AND MATTER

#### Solar wind a stream of charged particles from the Sun, consisting mainly of electrons, protons and alpha particles

#### Charged particles interacting with Earth's magnetic field

Earth's magnetosphere is not as symmetrical as that of the bar magnet. This is because Earth's magnetic field interacts with the solar wind emanating from the Sun (Figure 4B–28). Earth's magnetic field also forces charged solar particles (consisting mainly of electrons, protons and alpha particles with kinetic energies between 0.5 keV and 10 keV) towards the North Pole and South Pole, where they create brilliant aurorae (Figure 4B–29).



**Figure 4B–28** The comet-like shape of Earth's magnetosphere (blue) is created by the interaction of the magnetic field with the solar wind (the yellow lines streaming away from the Sun). The magnetosphere is very important for life on Earth as it deflects most of the charged particle component of the incoming solar radiation, which is ionising radiation with the potential to damage cells. Some of the charged particles are deflected towards the poles, where they produce brilliant displays known as aurorae.



Figure 4B–29 Aurora Australis, reflected in the Southern Ocean, Antarctica, caused by the interaction of charged particles from the Sun with Earth's magnetic field

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# Charged particles: moving in circles

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Figure 4B–30 shows a positively charged particle, q, travelling in a circle of radius, r, with speed, v, in a uniform magnetic field, B, where the velocity is perpendicular to the magnetic field.

×

a

×

Figure 4B-30 Circular motion of a charged particle in a uniform magnetic field

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The magnetic force, F = qvB, provides the centripetal force,  $F = \frac{mv^2}{r}$ , required for the motion of the particle in a circular path.

# Formula 4B–3 Equation for charged particles moving in circles

×

×

$$qvB = \frac{mv^2}{r}$$

Where:

- q =Charge of the particle (C)
- $\nu$  = Speed of the particle (m s<sup>-1</sup>)
- B = Strength of the (uniform) magnetic field (T)
- m = Mass of the particle (kg)
- r =Radius of the circular motion (m)

This means that the radius is given by  $r = \frac{mv}{qB}$ .

This implies that:

- the heavier the mass, the larger the radius.
- the faster the object, the larger the radius.
- the stronger the magnetic field, the smaller the radius.
- the larger the charge, the smaller the radius.





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9D MASS AND ENERGY EQUIVALENCE

#### Particle accelerator a machine

that uses electromagnetic fields to propel or bend the path of charged particles

# **Particle accelerators**

**Particle accelerators** involve the acceleration of charged particles in uniform electric and magnetic fields, including the change of speed caused by electric fields and the change of direction caused by magnetic fields.

Practical applications of particle accelerators include electron microscopes, mass spectrometers, synchrotrons and medical linear accelerators. Figure 4B–31 (left) shows an electron microscope image of a gall midge (note the 50  $\mu$ m scale on the image). Figure 4B–31 (right) shows a medical linear accelerator used in treatment of cancerous tumours.



**Figure 4B–31** Left: An electron microscope image of a gall midge. Right: A medical linear accelerator (LINAC) delivers high-energy X-rays or electrons to a precise region of the patient's cancerous tumour.



Electron gun a device that uses a heated cathode to provide free electrons for linear accelerators At the heart of most particle accelerators is an **electron gun**. Figure 4B–32 shows a schematic diagram of a simple electron gun. The acceleration voltage can be varied so that the electrons can be given a variety of velocities and energies, and in the case of electron microscopes, very precise selectable wavelengths.





If the acceleration voltage for the electron gun is V and the charge being accelerated is q, the following equations apply:

$$qV = \Delta E_{\rm k} = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2E_{\rm k}}{m}} = \sqrt{\frac{2qV}{m}}$$

#### Worked example 4B–3 Calculations in a particle accelerator

An electron, initially at rest, is accelerated from the cathode to the anode in an electron gun through a voltage of 1000 V. Take the charge on an electron as  $1.60 \times 10^{-19}$  C and the mass of an electron as  $9.11 \times 10^{-31}$  kg.

Calculate:

- a the kinetic energy of the electron as it arrives at the anode
- **b** the speed of the electron as it arrives at the anode.

Solution

**a** 
$$E_{\rm k} = qV$$
  
= (1.60 × 10<sup>-19</sup>) (1000)  
= 1.60 × 10<sup>-16</sup> J  
**b**  $v = \sqrt{\frac{2E_{\rm k}}{m}}$   
= 1.87 × 10<sup>7</sup> m s<sup>-1</sup>

Previously it was shown that a charged particle travelling at speed *v* perpendicular to a uniform **electric** field followed a **parabolic path**, with the following equations applying for the force, *F*, and the vertical deflection, *s*:



**Figure 4B–33** An electron moving perpendicular to a uniform electric field moves in a circular or parabolic path

It was also shown that a charged particle travelling perpendicular to a uniform **magnetic** field follows a **circular path**, or an arc of a circle, with the following equations applying:

$$F = qvB = \frac{mv^2}{r}$$
 and  $r = \frac{mv}{qB}$ 

This means we can now analyse how particle accelerators utilise electron guns with electric fields to accelerate the electrons to a specified speed, and how they utilise crossed electric and magnetic fields to achieve specific tasks (for example, mass in spectrometers, electron microscopes and synchrotrons). Crossed electric and magnetic fields refer to fields that are perpendicular to each other. This arrangement can produce useful results.

#### Thompson's experiment on the charge to mass ratio of the electron

An exploration of J.J. Thomson's classic 1897 experiment to determine the ratio of the charge of an electron to its mass is useful.

Thomson used a cathode ray tube to produce an electron beam of a type known as cathode rays. The beam could be deflected by an electric field and a crossed magnetic field within an evacuated tube (Figure 4B–34).





**Figure 4B–34** A schematic representation of J.J. Thomson's apparatus for finding the charge to mass ratio  $(\frac{q}{m})$  for cathode rays (electrons). In 1897, Thomson used cathode rays that could be deflected both by an electric field and a crossed magnetic field (N and S) within an evacuated tube. The electrons caused the phosphorescent coating to glow where the beam hit, and deflections could be read off the scaling marks. First the electric field was turned on to produce a deflection, then the magnetic field was applied and adjusted until the beam returned to the centre line, and the strength of the fields was measured.

The magnetic field was applied such that it was perpendicular to both the electric field and cathode rays, each forming one part of an x, y and z axes. This is shown in Figure 4B–35.

Initially, Thomson applied only the electric field, which deflected the beam up as shown in Figure 4B–34. This electric deflection was measured. Then, the magnetic field was varied until the beam





returned to the original straight-line path. At this point in the experiment, the magnetic force,  $F_{\rm B}$ , and the electric force,  $F_{\rm E}$ , were equal in magnitude but opposite in direction, hence the net force is zero.



**Figure 4B–36** Path of an electron where the magnetic force,  $F_{\rm B}$ , is balanced by the electric force,  $F_{\rm F}$ 

As the electric and magnetic forces were balanced in Thomson's cathode ray tube, the velocity of the rays could be easily calculated using:

$$qvB = qE$$
$$v = \frac{E}{B}$$

By turning off the electric field, *E*, Thomson measured the deflection of the cathode rays in the magnetic field, *B*, alone. The beam bent into a circular arc and he measured the radius of this deflection, *r*.

By equating the centripetal force to the magnetic force and using the calculated velocity, Thomson was able to obtain the charge-to-mass ratio using the following equations.

$$qvB = \frac{mv^2}{r}$$
$$\frac{q}{m} = \frac{v}{Br}$$
$$v = \frac{E}{B}$$
$$\frac{q}{m} = \frac{E}{rB^2}$$

Thomson (Figure 4B–37) was later awarded the Nobel Prize in Physics (1906) for his work on the conduction of electricity in gases. His research led to other important discoveries about the electron and eventually led physicists down the path of developing electron microscopes, mass spectrometers, synchrotrons and linear accelerators.



Figure 4B–37 J.J. Thomson in 1897 with his cathode ray tube at the Cavendish Laboratory, Cambridge University

#### CHAPTER 4 ELECTRIC AND MAGNETIC FIELDS





#### Mass spectrometers

In modern mass spectrometers, charged ions are accelerated in an evacuated chamber and bent by a magnetic field (Figure 4B–38).



**Figure 4B–38** A schematic diagram of a mass spectrometer showing how the magnetic field separates out the different masses of the sample after they have been accelerated by the electric field plates.

Mass spectrometry is an analytical tool useful for measuring the mass-to-charge ratio of one or more molecules present in a sample. These measurements can be used to calculate the exact molecular weight of the sample components. Typically, mass spectrometers can identify unknown compounds via molecular weight determination, quantify known compounds and then determine the structure and chemical properties of molecules.

Recently, pharmacists in Martin Luther University Halle-Wittenberg (MLU) have succeeded in detecting minute amounts of the coronavirus SARS-CoV–2 from patients' gargle samples by using mass spectrometry. This technique opens up the possibility of identifying new strains as the SARS-CoV-2 virus mutates.

#### Synchrotrons

Particle accelerators, such as the Australian Synchrotron in Melbourne (Figure 4B–39), accelerate subatomic particles to near light speeds, where special relativity is essential for understanding the behaviour of the particles. As the particles in a synchrotron are accelerated, the strength of the magnetic field is increased to keep the radius of the orbit approximately constant.

## CHAPTER 9 LINK

The Australian Synchrotron accelerates electrons through an equivalent of  $3 \times 10^9$  V (3.0 GeV). At this energy, the electrons travel at 99.99999% of the speed of light. Because of the relativistic effects that occur at these near-light speeds, the 'effective' mass of the electrons is about 6000 times that of when they are at rest. Because they are being accelerated as they travel around a 70 m diameter circle, the electrons emit electromagnetic radiation. It is this light, ranging from infrared through to X-ray wavelengths, that is used for the research projects being conducted at the synchrotron.



AND ENERGY EQUIVALENCE



Figure 4B-39 Quadruple magnets of the linear accelerator of the Australian Synchrotron

# Check-in questions – Set 5

1 The diagram below shows an electron, e<sup>-</sup>, travelling at a speed,  $\nu$ , of  $2.0 \times 10^6$  m s<sup>-1</sup> moving into a region where there are crossed electric, *E*, and magnetic fields, *B*. The strength of the magnetic field is 0.1 T.



- **a** What is the direction of the electric field?
- **b** What is the direction of the magnetic field?
- c What is the direction of the electric force on the electron travelling to the right?
- **d** What is the direction of the magnetic force on the electron travelling to the right?

The strength of the electric field and the magnetic field are adjusted so that the electric and magnetic forces acting on the electron completely balance each other out and the electron travels straight across undeflected.

e Calculate the new strength of the electric field.





Magnetic field of strength  $B = 5.0 \times 10^{-2}$  T  $\times \times \times \times$  $\mathbf{X}$ 50 cm X X X XX  $v = 4.2 \times 10^7 \text{ m s}^{-1}$  $\begin{array}{c} \times \times \times \times \times \\ \overline{\times} \times \times \times \times \end{array}$ Electron U source  $\times \times \times$ X X  $\times \times \times$ 

**2** Electrons are accelerated from rest between two plates that are 50 cm apart, as shown.

The electrons emerge from the second plate at a speed,  $\nu$ , of  $4.2 \times 10^7$  m s<sup>-1</sup>. Ignore relativistic effects.

Mass of electron	$9.1 \times 10^{-31} \text{ kg}$
Charge on electron	$-1.6 \times 10^{-19} \text{ C}$

a Calculate the voltage between the two plates. Show your working.

The electrons enter a region of uniform magnetic field of strength,  $B = 5.0 \times 10^{-2}$  T, that is at right angles to their path.

- **b** Calculate the magnitude of the force on each electron. Show your working.
- **c** Will the path of the electrons in this region of uniform magnetic field be a straight line, part of a parabola or part of a circle? Give a reason for your answer.

VCAA 2019

# **4B SKILLS**

VIDEO 4B-2

PARTICLES TRAVELLING

IN UNIFORM MAGNETIC FIELDS

SKILLS: ANALYSING CHARGED

#### Analysing charged particles travelling in uniform magnetic fields

#### Question

An electron, e, enters a magnetic field, *B*, of strength 20.0 mT at a speed of  $2.0 \times 10^6$  m s<sup>-1</sup> as shown.



- **a** What is the direction of the force acting on the electron when it enters the magnetic field?
- **b** Calculate the magnitude of the magnetic force acting on the electron when it enters the magnetic field.
- **c** Calculate the radius of the path that the electron takes in the magnetic field. Use  $9.1 \times 10^{-31}$  kg for the mass and  $1.6 \times 10^{-19}$  C for the charge on the electron.

#### Solution

- **a** To find the direction of the force acting on the electron when it enters the magnetic field, use the right-hand slap rule. This gives the force on the electron as being down.
- **b** To calculate the magnitude of the magnetic force acting on the electron when it enters the magnetic field, use:

$$F = qvB$$
  
= (1.6 × 10<sup>-19</sup>)(2.0 × 10<sup>6</sup>)(2.0 × 10<sup>-2</sup>)  
= 6.4 × 10<sup>-15</sup> N

**c** To calculate the radius of the path that the electron takes in the magnetic field, use:

$$r = \frac{m\nu}{qB}$$
  
=  $\frac{(9.1 \times 10^{-31})(2.0 \times 10^6)}{(1.6 \times 10^{-19})(2.0 \times 10^{-2})}$   
= 5.7 × 10<sup>-4</sup> m

# Section 4B questions

#### **Multiple-choice questions**

- 1 A bar magnet is divided in two pieces. Which one of the following statements is true?
  - A The bar magnet is demagnetised.
  - **B** The magnetic field of each separated piece becomes stronger.
  - **C** The magnetic poles are separated.
  - **D** Two new bar magnets are created.
- **2** A group of students is considering how to create a magnetic monopole. Which one of the following is correct?
  - A Break a bar magnet in half.
  - **B** Pass a current through a long solenoid.
  - **C** Pass a current through a circular loop of wire.
  - **D** It is not known how to create a magnetic monopole.

VCAA 2017

**3** A straight long wire carrying an electric current, *I*, to the right is placed in a uniform magnetic field, *B*, directed into the page as shown.



What is the direction of the magnetic force on the current?

- A left
- **B** right
- **C** to the bottom of the page
- **D** to the top of the page

4 Millikan, a famous scientist, measured the size of the electron charge by balancing an upwards electric force with a gravitational force on a small oil drop. In a repeat of this experiment, an oil drop with a charge of  $9.6 \times 10^{-19}$  C was placed in an electric field of  $10^4$  V m<sup>-1</sup>.

Which one of the following is closest to the electrical force on the oil drop?

- **A**  $9.6 \times 10^{-14} \text{ N}$
- **B**  $9.6 \times 10^{-15}$  N
- **C**  $9.6 \times 10^{-22}$  N
- **D**  $9.6 \times 10^{-23}$  N
- **5** A negative charge moving with a constant velocity, *v*, enters a region of a uniform magnetic field, *B*, pointing out the page as shown.

What is the direction of the magnetic force on the charge?

- A left
- **B** right
- **C** to the bottom of the page
- **D** to the top of the page

#### Use the following information to answer Questions 6 and 7.

A powerline carries a current of 1000 A DC in the direction east to west. At the point of measurement, Earth's magnetic field is horizontally north and its strength is  $5.0 \times 10^{-5}$  T.

- **6** Which one of the following best gives the direction of the electromagnetic force on the powerline?
  - A horizontally west
  - **B** horizontally north
  - **C** vertically upwards
  - **D** vertically downwards
- 7 The magnitude of the force on each metre of the powerline is best given by
  - A  $5.0 \times 10^3$  N
  - **B**  $5.0 \times 10^2$  N
  - **C**  $5.0 \times 10^{-2}$  N
  - $\textbf{D} \quad 5.0\times 10^{-5} \; N$

# 8 An electron moves in a circular path with radius *r* in a magnetic field as shown.

If the speed of the electron is increased, which of the following correctly shows the effects of this change on both the force acting on the electron and the radius of the electron's path?

- A The force on the electron increases and the radius of the path decreases.
- **B** The force on the electron increases and the radius of the path increases.
- **C** The force on the electron decreases and the radius of the path decreases.
- **D** The force on the electron decreases and the radius of the path increases.





## **Short-answer questions**

**9** The diagram shows a U-shaped magnet.



- **a** Copy the diagram and draw the magnetic field lines between the poles of the U-shaped magnet.
- **b** Describe the nature of the magnetic field between the two poles of the U-shaped magnet.
- 10 Calculate the magnitude of the magnetic force acting on a straight wire of length 40.0 cm carrying a current of 500 mA in a magnetic field of strength 0.20 T if the wire is:
  - a perpendicular to the magnetic field
  - **b** parallel to the magnetic field.
- **11** The diagram below shows a schematic diagram of a simple DC motor.

It consists of two magnets, a single 9.0 V DC power supply, a split ring commutator and a rectangular coil of wire consisting of 10 loops.

The total resistance of the coil of wire is 6.0  $\Omega$ .

The length of the side JK is 12 cm and the length of the side KL is 6.0 cm.

The strength of the uniform magnetic field is 0.50 T.



- **a** Determine the size and the direction (A–F) of the force acting on the side JK.
- **b** What is the size of the force acting on the side KL in the orientation shown in the diagram? Explain your answer.

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**12** A proton,  $p^+$ , of charge  $+1.6 \times 10^{-19}$  C enters a magnetic field, *B*, of strength 1.5 T at a speed,  $\nu$ , of  $3.0 \times 10^5$  m s<sup>-1</sup> as shown.



- **a** What is the direction of the force acting on the proton when it enters the magnetic field?
- **b** Calculate the magnitude of the magnetic force acting on the proton when it enters the magnetic field.
- **13** Electron microscopes use a high-precision electron velocity selector consisting of an electric field, *E*, perpendicular to a magnetic field, *B*.

Electrons travelling at the required velocity,  $\nu_0$ , exit the aperture at point Y, while electrons travelling slower or faster than the required velocity,  $\nu_0$ , hit the aperture plate, as shown in the diagram below.



- **a** Show that the velocity of an electron that travels straight through the aperture to point Y is given by  $v_0 = \frac{E}{B}$ .
- **b** Calculate the magnitude of the velocity,  $v_0$ , of an electron that travels straight through the aperture to point Y if  $E = 500 \text{ kV m}^{-1}$  and B = 0.25 T. Show your working.
- **c** i At which of the points (X, Y or Z) in the diagram could electrons travelling faster than  $\nu_0$  arrive?
  - ii Explain your answer to part c i.

VCAA 2020

14 An electron, initially at rest, is accelerated from the cathode to the anode in an electron gun through a voltage of 2000 V. Take the charge on an electron as  $1.60 \times 10^{-19}$  C and the mass of an electron as  $9.11 \times 10^{-31}$  kg. Ignore relativistic effects.

Calculate:

- a the kinetic energy of the electron as it arrives at the anode
- **b** the speed of the electron as it arrives at the anode.

**15** An electric field accelerates a proton between two plates. The proton exits into a region of uniform magnetic field at right angles to its path, directed out of the page, as shown below.



Mass of proton	$1.7 \times 10^{-27} \text{ kg}$
Charge on proton	$+1.6 \times 10^{-19} \text{ C}$
Accelerating voltage	10 kV
Distance between plates	20 cm
Strength of magnetic field	$2.0 \times 10^{-2} \text{ T}$

- a Calculate the strength of the electric field between the plates.
- **b** Calculate the speed of the proton as it exits the electric field. Show your working.
- **c** With a different accelerating voltage, the proton exits the electric field at a speed of  $1.0 \times 10^6 \text{ m s}^{-1}$ . Calculate the radius of the path of this proton in the magnetic field. Show your working.

VCAA 2018



# **Chapter 4 review**

# Summary

Create your own set of summary notes for this chapter on paper or in a digital document. A model summary is provided in the Teacher Resources, which can be used to compare with yours.

# Checklist

In the Interactive Textbook, the success criteria are linked from the review questions and will be automatically ticked when answers are correct. Alternatively, print or photocopy this page and tick the boxes when you have answered the corresponding questions correctly.

Success criteria – I am now able to: Linked questions		
4A.1	Describe electricity using a field model	4 , 7
4A.2	Investigate and compare theoretically and practically electric fields, including direction and shapes of fields, attractive and repulsive fields, and the existence of dipoles and monopoles	1 , 2 , 9
4A.3	<ul> <li>Investigate and compare theoretically and practically electric fields</li> <li>about a charge (positive or negative) with reference to:</li> <li>the direction of the field</li> <li>the shape of the field</li> </ul>	1
4 <b>A</b> .4	Use the inverse square law to determine the magnitude of an electric field	3
4 <b>A</b> .5	Determine potential energy changes (qualitative) associated with a point charge moving in an electric field	20
4 <b>A</b> .6	Identify electric fields as static or changing, and as uniform or non- uniform	4 , 20
4A.7	Quantitatively determine the magnitude of the electric field strength around point charges, $E = k \frac{Q}{r^2}$ , and between parallel plates, $E = \frac{V}{d}$	5□, 6□, 10□, 14□, 15□, 16□, 18□
4 <b>A</b> .8	Quantitatively determine the magnitude of the force between two fixed charged objects using Coulomb's law, $F = k \frac{q_1 q_2}{r^2}$	d 13
4 <b>A</b> .9	Understand work done and potential energy changes in a uniform electric field, $W = qV$	15□, 16□, 17□, 19□
4A.10	Quantitatively determine the forces acting on charged particles in a uniform electric field, $F = qE$	8 , 16 , 18
4B.1	Describe magnetism using a field model	12
4B.2	Investigate and compare theoretically and practically magnetic fields, including directions and shapes of fields, attractive and repulsive effects, and the existence of dipoles; identify magnetic fields as static o changing, and as uniform or non-uniform	21 🗌
4B.3	Investigate and apply theoretically and practically a vector field model to magnetic phenomena, including shapes and directions of fields produced by bar magnets, and by current-carrying wires, loops and solenoids	21

Success criteria – I am now able to:		Linked questions
4B.4	Describe the interaction of two fields, allowing that electric charges, magnetic poles and current-carrying conductors can either attract or repel	10 , 21
4B.5	Investigate and analyse theoretically and practically the force on a current-carrying conductor due to an external magnetic field, $F = nIlB$ where the directions of <i>I</i> and <i>B</i> are either perpendicular or parallel to each other	11 🗌 , 21 🗌
4B.6	Investigate and analyse theoretically and practically the operation of simple DC motors consisting of one coil, containing a number of loop of wire, which is free to rotate about an axis in a uniform magnetic fiel and including the use of a split ring commutator	23 🗌 s d
4B.7	Investigate, qualitatively, the effect of current, external magnetic field and the number of loops of wire on the torque of a simple motor	25
4B.8	<ul> <li>Analyse the use of a magnetic field to change the path of a charged particle, including:</li> <li>► the magnitude and direction of the force applied to an electron beam by a magnetic field, F = qvB, in cases where the directions of and B are perpendicular or parallel</li> </ul>	22 🗌 , 24 🗌 v
	► the radius of the path followed by an electron in a magnetic field, $qvB = \frac{mv^2}{r}$ , where $v \ll c$	
4B.9	Model the acceleration of particles in a particle accelerator (including synchrotrons) as uniform circular motion (limited to linear acceleration by a uniform electric field and direction change by a uniform magnetic field)	16 , 22 , 24

# **Multiple-choice questions**

**1** Which one of the following diagrams best represents the electric field around a single positive charge?





**2** Which one of the following diagrams shows the electric field pattern surrounding two equal, positive point charges?



- 5 A small sphere has a charge of  $2.0 \times 10^{-6}$  C on it. Take  $k = 8.99 \times 10^{9}$  N m<sup>2</sup> C<sup>-2</sup>. The strength of the electric field due to this charge at a point 3.0 m from the sphere is best given by

VCAA 2018

Use the following information to answer Questions 6, 7 and 8.

Two parallel plates separated by 2.5 cm have a potential difference of 100 V.

Point P is located midway between the two plates as shown below. The charge on a proton is +1.6  $\times$  10<sup>-19</sup> C.



- **6** Which one of the following is closest to the magnitude of the electric field at point P?
  - $\textbf{A} \quad 40 \ V \ m^{-1}$
  - $B 250 V m^{-1}$
  - $C 400 V m^{-1}$
  - $D 4000 V m^{-1}$

A proton is placed at point P.

- 7 The direction of the force on the proton at point P is
  - A left
  - **B** right
  - C up
  - **D** down
- 8 The magnitude of the force on the proton at point P is closest to
  - **A**  $6.4 \times 10^{-19} \text{ N}$
  - $\textbf{B} \quad 6.4\times 10^{-16} \; N$
  - **C** 1000 N
  - **D** 4000 N
- 9 Which one of the following is the best definition of an electric dipole?
  - **A** a pair of equal and like charges, usually separated by a small distance
  - **B** a pair of unequal and like charges, usually separated by a small distance
  - **C** a pair of equal and unlike charges, usually separated by a small distance
  - **D** a pair of unequal and unlike charges, usually separated by a small distance

**10** Which one of the following is closest to the magnitude of the electric field at point P due to the two fixed charges as shown?



- **A**  $3.0 \times 10^3 \,\mathrm{N}\,\mathrm{C}^{-1}$
- **B**  $9.4 \times 10^3 \text{ N C}^{-1}$
- $\textbf{C} \ \ 2.8 \times 10^4 \ N \ C^{-1}$
- $\textbf{D} \ 9.4 \times 10^4 \ N \ C^{-1}$
- **11** A wire carrying a current, *I*, of 6.0 A passes through a magnetic field, *B*, of strength  $1.4 \times 10^{-5}$  T, as shown below. The magnetic field is exactly 1.0 m wide.



The magnitude of the force on the wire is closest to

- **A** 0 N
- **B**  $2.3 \times 10^{-6}$  N
- $\textbf{C} \quad 8.4\times10^{-5} \; N$
- $\textbf{D} \quad 4.3\times 10^5 \; N$

VCAA NHT 2021

- **12** Which one of the following statements is correct?
  - **A** Only electrostatic and magnetic fields are vector fields.
  - **B** Only magnetic and gravitational fields are vector fields.
  - **C** Only electrostatic and gravitational fields are vector fields.
  - D Electrostatic, magnetic and gravitational fields are all vector fields.

#### **Short-answer questions**

**13** According to one model of the atom, the electron in the ground state of a hydrogen atom moves around the stationary proton in a circular orbit with a radius of 53 pm ( $53 \times 10^{-12}$  m).

Show that the magnitude of the force acting between the proton and the electron at this separation is equal to  $8.2 \times 10^{-8}$  N. Take  $k = 9.0 \times 10^{9}$  N m<sup>2</sup> C<sup>-2</sup> and the magnitude of the electron and proton charges as  $1.6 \times 10^{-19}$  C. Show all the steps of your working. (3 marks)

VCAA 2017

14 Two parallel plates are connected to a battery and thus have a constant potential difference of 100 V between them. They are initially 10.0 cm apart. The distance, *d*, is is initially doubled to 20.0 cm and then quadrupled to 40.0 cm.



- **a** Calculate the electric field strength values for each of the separation distances. (3 marks)
- b Copy the graph below and plot the electric field strength, *E*, against distance, *d*, for this experiment. Put units and a scale on each axis. Show the data points and draw the line of best fit. (4 marks)



15 Two charged parallel plates are 10.0 cm apart and have a voltage drop of 200 V across the plates.



- **a** Calculate the strength of the electric field between the two plates. (1 mark)
- b A positive charge of 0.2 C moves from the top plate to the lower plate under the influence of the electric field. Calculate the work done by the electric field on the charge as it moves from the top plate to the lower plate. (3 marks)

**16** A simple linear particle accelerator can be made using two parallel metal plates.

A positively charged ion, Q, of magnitude  $3.2 \times 10^{-19}$  C is placed in between the plates as shown. The voltage, V, between the plates is 60 V and the distance, d, between the plates is 10.0 cm.



- **a** Calculate the magnitude of the electric field between the two charged plates. (1 mark)
- **b** Calculate the magnitude of the electric force acting on *Q*. (2 marks)
- **c** Calculate the work done on Q as it moves from the top plate to the bottom plate. (2 marks)
- **d** Calculate the kinetic energy Q has when it arrives at the bottom plate. (2 marks)
- 17 A particle of mass *m* and charge *q* is accelerated from rest through a potential difference,*V*, in a uniform electric field. Assume that the only force acting on the particle is due tothe electric field associated with this potential difference. Show that the final particle velocity is

$$v = \sqrt{\frac{2qV}{m}}.$$
 (3 marks)

**18** A small sphere carrying a charge of  $-2.7 \,\mu\text{C}$  is placed between charged parallel plates as shown. The potential difference between the plates is set at 15.5 V, which just holds the sphere stationary. The electric field between the plates is uniform.



- a In which direction (up, down, right, left) will the sphere move if the voltage is increased? (1 mark)
- b Calculate the value of the electric force that is holding the sphere stationary if the plates are 2.0 mm apart. Show your working. (3 marks)

VCAA NHT 2021

**19** Two parallel metal plates are 1 mm apart. A potential difference of 100 V is applied as shown. Use mass of electron =  $9.1 \times 10^{-31}$  kg and mass of proton =  $1.67 \times 10^{-27}$  kg.



- a How much kinetic energy will an electron gain travelling between the two plates if it is at rest when it leaves the negative plate? (2 marks)
- b How much kinetic energy will a proton gain travelling between the two plates if it is at rest when it leaves the positive plate? (2 marks)
- **c** What is the ratio of the speed of the electron,  $v_e$ , to the speed of the proton,  $v_p$ , when they both hit their target plates? Give your answer to two significant figures. (3 marks)
- **20** A positive charge, *Q*, is placed as shown in the diagram. A small positive test charge is moved from position X to position Y.



- **a** Is the electric field produced by *Q* a uniform or non-uniform electric field? Explain. (2 marks)
- **b** Describe the electric potential energy changes as the small positive test charge moves from X to Y.
- **21** The diagram below shows two solenoids and a small magnetic compass. The same current flows through both solenoids. Ignore Earth's magnetic field.



Use the directions A-F below to answer the following questions.

**a** The small magnetic compass is placed at point X in the diagram on the top left.



Which of the directions, A–F, best shows how the compass will point? Give a reason for your answer. (2 marks)

(3 marks)

**b** A wire carrying a current, *I*, vertically upwards is placed midway between the two solenoids as shown in the diagram.

Which of the options, A–G, below best shows the direction of the electromagnetic force, if any, acting on the wire? Explain your answer. (2 marks)

**A** upwards **B** downwards **C** into page **D** out of page E left **F** right **G** force will be zero VCAA 2017 × × × × × **22** A positron and an electron are fired one × at a time into a strong uniform magnetic field in an evacuated chamber. They Positron Electron are fired at the same speed but from opposite sides of the chamber. Their initial velocities are perpendicular to the × magnetic field and opposite in direction

to each other, as shown. A positron has the same mass as an electron  $(9.1 \times 10^{-31} \text{ kg})$ , the same magnitude of electric charge as an electron  $(-1.6 \times 10^{-19} \text{ C})$  but is positively charged  $(+1.6 \times 10^{-19} \text{ C})$ . On a copy of the diagram, sketch and label the respective paths that the positron and the electron will take while in the uniform magnetic field. (3 marks)

VCAA NHT 2022

**23** Students build a simple electric motor consisting of a single coil and a split ring commutator as shown. The magnetic field between the pole pieces is a constant 0.02 T.



- **a** Calculate the magnitude of the force on the side WX. (2 marks)
- b From the position of an observer at the split ring commutator, will the coil rotate in a clockwise or anticlockwise? Explain your answer. (2 marks)
- **c** Explain the role of the split ring commutator in the operation of the electric motor.

(2 marks) VCAA NHT 2017 **24** An electron is accelerated from rest by a potential difference of  $V_0$ . It emerges at a speed of  $2.0 \times 10^7 \text{ m s}^{-1}$  into a magnetic field, *B*, of strength  $2.5 \times 10^{-3}$  T and follows a circular arc as shown.



- **a** Calculate the value of the accelerating voltage,  $V_0$ . Show your working. (2 marks)
- **b** Explain why the path of the electron in the magnetic field follows a circular arc. (2 marks)
- **c** Calculate the radius of the circular arc travelled by the electron. Show your working.

(3 marks)

(1 mark)

VCAA NHT 2021

**25** The diagram shows a simple DC motor.



The force, *F*, acting on side JK is given by F = nBIl where *n* is the number of loops on the coil, *B* is the strength of the external magnetic field, *I* is the current in the coil, *l* is the length of the side of the coil (in this case JK).

The torque,  $\tau$ , acting on side JK is given by  $\tau = Fd$ , where *d* is the length of the lever arm between the coil and the axis of rotation.

Explain what would happen to the torque if:

а	the number of loops on the coil is increased	(1 mark)
b	the strength of the external magnetic field is increased	(1 mark)

**c** the current in the coil is increased.

# HOW DO FIELDS EXPLAIN MOTION AND ELECTRICITY?

# CHAPTER 5

UNIT

# **GENERATING ELECTRICITY**

# Introduction

The production, distribution and use of electricity has had a major impact on the way that humans live. As we transition from using fossil fuels for transport, household heating/cooling and industrial processes, producing and distributing electricity will become even more important.

There are many ways to generate electricity. For example, you may be familiar with the use of chemical reactions in batteries; or chemical reactions in the bodies of humans and other animals that generate electric currents; or the use of solar cells to generate electricity from solar energy. Heat from a nuclear reactor can also be used to generate electricity. This technique is used on spacecraft travelling a long way from the Sun that are unable to use solar power. However, nuclear energy is controversial because often the heat is generated from a long-lived nuclear radioisotope such as plutonium.

However, most of the electricity that we consume at home or use in industry and transportation is generated by rotating a strong magnetic field near large coils of wire. We call these special devices electrical generators or dynamos, or alternators if they produce alternating current. You may have generated electricity from such mechanical movement yourself, either by cranking a small dynamo-powered torch, emergency radio or phone charger; these are often used by campers and bushwalkers.

Many electric vehicles use their brakes to generate electricity that recharges their large batteries, while traditional ICE vehicles (internal combustion engines) have a belt that drives an alternator to power a 12 V battery that runs lights, radios, air conditioning and other devices.

Some enterprising people have connected their portable TVs directly to the dynamo output from a bicycle. To watch the television, you must make the electricity for it! This scheme not only makes viewers much more discriminating in their choice of content and amount of screentime but has the bonus of keeping them fit.

What are the principles behind the generation of electricity using electrical generators? In this chapter, you will use empirical evidence and models of electric, magnetic and electromagnetic effects to explain how electricity is produced.
# Curriculum

Area of Study 3 Outcome 3 How are fields used in electricity generation?

Study Design	Learning intentions – at the end of this chapter I will be able to:
<ul> <li>Generation of electricity</li> <li>Describe the production of electricity using photovoltaic cells and the need for an inverter to convert power from DC to AC for use in the home (not including details of semiconductors action or inverter circuitry)</li> </ul>	<ul> <li>5A Solar panels: generating electricity from photovoltaic cells</li> <li>5A.1 Describe the way DC potential difference is produced in a photovoltaic cell (but not details of semiconductor action, pn junctions etc.)</li> <li>5A.2 Discuss the energy transformations that occur when a photovoltaic cell converts sunlight to electricity</li> <li>5A.3 Recognise the need for, and describe the basic function of, an inverter to convert DC to AC for household use (not details of wiring etc.)</li> <li>5A.4 Analyse the connections of photovoltaic panels in series and parallel in terms of potential difference, current and power production</li> </ul>
<ul> <li>Calculate magnetic flux when the magnetic field is perpendicular to the area, and describe the qualitative effect of differing angles between the area and the field:</li> <li>Φ<sub>B</sub> = B<sub>⊥</sub>A</li> <li>Investigate and analyse theoretically and practically the generation of electromotive force (emf) including AC voltage and calculations using induced emf:</li> <li>ε = -N ΔΦ<sub>B</sub>/Δt, with reference to:</li> <li>rate of change of magnetic flux</li> <li>number of loops through which the flux passes</li> <li>direction of induced emf in a coil</li> </ul>	<ul> <li>58 Generating emf by varying the magnetic flux</li> <li>58.1 Recognise a variety of situations where changes in magnetic fields and/or motion of conductors result in induced potential difference (called emf, electromotive force, ε), i.e. where electromagnetic induction occurs</li> <li>58.2 Recall that magnetic flux is the strength of a magnetic field for a given area, with symbol Φ<sub>B</sub> and units of weber (Wb). Calculate its value when the field is perpendicular to the area using the formula Φ<sub>B</sub> = B<sub>⊥</sub>A and qualitatively describe the effect of varying the angle between B and A</li> <li>58.3 Explain electromagnetic induction in terms of <i>changing</i> the magnetic flux, ΔΦ<sub>B</sub>, near a conductor to generate a potential difference (an emf, ε)</li> <li>58.4 Use Faraday's law of electromagnetic induction, ε = -N ΔΦ<sub>B</sub>/Δt, in a variety of situations to analyse the effect on emf induced when the rate of change of magnetic flux and/or the number of loops through which the flux passes are varied</li> <li>58.5 Recognise and explain the effect of the negative sign in Faraday's law (Lenz's law) in terms of conservation of energy. Use Lenz's law to predict the direction of the induced emf (and hence induced current) when magnetic flux is changed (using the right-hand rule where necessary); explain that the magnetic field of the induced current is in the opposite direction to the <i>change in magnetic flux</i> that produced it (Lenz's law)</li> </ul>

Study Design	Learning intentions – at the end of this chapter I will be able to:
	<ul> <li>5B.6 Explain that an induced emf will produce a current if there is a complete circuit with some parts external to the field, and predict the direction of that current. Use Ohm's law and other circuit theory to perform calculations to determine the induced current associated with an induced emf when there is a complete circuit external to the magnetic field</li> <li>5B.7 Predict the shape and values of various ε vs time graphs, given a graph of Φ<sub>B</sub> versus time (using the negative gradient) and vice versa</li> </ul>
• Explain the production of DC voltage in DC generators and AC voltage in alternators, including the use of split ring commutators and slip rings respectively AC generators with slip rings are covered in 5D.	<ul> <li>5C DC generators: producing DC voltage</li> <li>5C.1 Recognise a DC generator as a loop of wire connected to a split ring commutator which, when rotated uniformly in a constant magnetic field, produces a sinusoidal DC voltage (emf, ε) given by Faraday's law and Lenz's law:</li> <li>ε = -N ΔΦ<sub>B</sub>/Δt</li> <li>5C.2 Predict and explain the sinusoidal shape of the Φ<sub>B</sub> vs time graph and the ε vs time graph produced from</li> </ul>
	<ul> <li>a DC generator, including the function of the split ring commutator</li> <li>5C.3 Calculate the emf, ε, induced in a rotating coil using Faraday's law by considering the change in magnetic flux, ΔΦ<sub>B</sub>, when the coil rotates through 90°, 180° or other simple angles</li> </ul>
	<b>5C.4</b> Determine the direction of both the induced $\varepsilon$ and $I$ in a coil rotating in a magnetic field using Lenz's law
	<ul> <li>5C.5 Analyse the effect on emf of varying the rate of rotation, the number of loops, and/or the direction of rotation of a DC generator coil; show this by calculation and/or graphically</li> </ul>
	<b>5C.6</b> Use Ohm's law and other circuit theory to perform calculations to determine the induced current associated with an induced emf from a coil rotating in a magnetic field

#### **Study Design**

 Explain the production of DC voltage in DC generators and AC voltage in alternators, including the use of split ring commutators and slip rings respectively

#### Learning intentions – at the end of this chapter I will be able to:

**5D** Generators with slip rings produce sinusoidal AC 5D.1 Recognise an AC generator as a loop of wire connected to slip rings, which, when rotated uniformly in a constant magnetic field produces a sinusoidal AC voltage (emf,  $\varepsilon$ ), given by Faraday's law and Lenz's law:  $arepsilon=-Nrac{\Delta \Phi_{\!B}}{2}$ 5D.2 Predict and explain the sinusoidal shape of the  $\Phi_{\scriptscriptstyle B}$  vs time graph and the  $\varepsilon$  vs time graph produced from an AC generator, including the function of the slip rings 5D.3 Calculate the emf,  $\varepsilon$ , induced in a rotating coil using Faraday's law by considering the change in magnetic flux,  $\Delta \Phi_{\rm B}$ , when the coil rotates through 90°, 180° or similar angles Analyse the effect on emf of varying the rate of rotation, 5D.4 the number of loops, and/or the direction of rotation of an AC generator coil; show this by calculation and/or graphically 5D.5 Use Ohm's law and other circuit theory to perform calculations to determine the induced current associated with an induced emf from an AC generator 5D.6 Recognise the set-up of an AC generator as 'a motor run backwards', i.e., an AC motor where mechanical energy is transformed to produce electrical energy; recognise a DC generator as 'a DC motor run backwards' Compare the structure and function of AC generators 5D.7 (with slip rings) to DC generators (with split ring commutator)

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### Glossary

Alternating current (AC) circuit Alternator DC generator Direct current (DC) circuit Electric generator Electromagnet Electromagnetic induction emf (electromotive force), ε Faraday's law of electromagnetic induction Galvanometer Induced current Inverter

Lenz's law Magnetic flux,  $\Phi_B$ Photovoltaic effect Photovoltaic (pv) cell pn junction Rate of change Semiconductor Sinusoidal Slip ring Split ring commutator Voltage



# **Concept map**

Empirical evidence and models of electric, magnetic and electromagnetic effects explain how electricity is produced



See the Interactive Textbook for an interactive version of this concept map interlinked with all concept maps for the course.



# **Solar panels: generating electricity** from photovoltaic cells

**Study Design:** 

photovoltaic cells and the need for an inverter to convert power from DC to AC for use in the home (not including details of semiconductors action or inverter circuitry)

Describe the production of electricity using

#### **Glossary:**

Alternating current (AC) circuit Direct current (DC) circuit Inverter Photovoltaic effect Photovoltaic (pv) cell pn junction Semiconductor Sinusoidal Voltage

# **ENGAGE Improving efficiency of solar panels**

In 2022, the most efficient solar panels, when new, convert up to about 25% of the sunlight falling on them into electrical energy with a power output of up to 420 W. The efficiency decreases with age of the panel and increasing temperature amongst other factors. (A very useful invention combines hot water service pipes to cool the photovoltaic panels on hot days, while heating the water for free.)

To catch the maximum amount of sunlight, the orientation of the panels is important. This requires an average position usually facing north, as the direction of the incident rays varies throughout the day and the year, as well as with latitude. The invention of microinverters attached to each panel, rather than as a box on the wall taking all power produced, has allowed panels on both sides of east-west-facing roofs to be productive. If a single panel is affected by shade from a tree, the production from the others can still be collected, whereas with a single inverter this would reduce the output significantly.

The silicon in solar panels is very shiny, so the surface is textured to reduce reflection. Also, a coating is used to create an anti-reflective layer. This technique is familiar from camera lenses and spectacles. The thickness of the coating is chosen so that light reflecting off the top layer of the coating is 180° out of phase with light reflected from the lower surface, causing destructive interference resulting in zero net reflected energy, which reduces energy loss.

In solar farms, efficiency can be increased by using low power electric motors to move the panels and track the sun during the day.

The technology of solar cells is improving rapidly, with mono- or poly-crystalline, PERC (Passive Emitter and Rear Contact), thin film and even wearable/flexible experimental development occurring here in Australia.





pn junction the boundary within a semiconductor between two different types of semiconductor materials.

VIDEO 5A-1

**Photovoltaic** (pv) cell

also known as a solar cell. It

converts light

the photovoltaic effect.

Semiconductor a material that will conduct

electricity only

conducting properties are

between a

conductor and

an insulator, depending on

conditions.

under particular conditions. Its

energy into electricity via

SOLAR PANELS

# **EXPLAIN**

# How do solar panels work?

benefit with less pollution and fewer disadvantages.

The detail of **pn junctions** is beyond the scope of this course. What is important is that when solar energy (light and thermal) falls on the pn junction, some of it is transformed into electrical energy. Solar panels that are commercially available for household and industrial use in 2022 are up to 25% efficient, meaning that 25% of the incoming energy is converted to electrical energy. Some sunlight is reflected but most of the rest is converted to unwanted thermal energy. Panel efficiency is affected by many factors including temperature, amount of solar energy received, cell type and cell connections.

As humans, we are in the process of transitioning our supply system from using fixed

coal-fired to renewable methods to generate electricity. We are also progressing from

generation on demand to storage in batteries and gravity systems. These developments

will change the nature of the grid. Underlying physics principles are applied to greater

Each solar panel is made of an array of photovoltaic (pv) cells. Each pv cell is made of two semiconductor slices, usually silicon, called a p-type (p for positive) and n-type (n for negative). A small amount of other materials, such as phosphorus or boron, are added to the silicon to create either an excess or a lack of electrons. This is called 'doping' the silicon. Then the two slices are joined to create a pn junction.





**Photovoltaic** effect when a photovoltaic cell is exposed to sunlight, electrons move from the n-type side and become trapped by the pn junction, creating a potential difference across the junction and generating current

Figure 5A-1 A section through a solar panel. Two silicon-based semiconductors are joined to create a pn junction. One side of the junction has excess negative electrons (n-type) and the other has a lack of electrons, known as holes (p-type). Electrons move from the n-type to the p-type and become trapped by the pn junction, creating a potential difference.

Bonding the two different semiconductors together sets up a region of electric field across the junction between the n- and the p-type silicon. Light incident on the n-type semiconductor can cause more free electrons to be emitted (photovoltaic effect). A simplified explanation is that the electric field at the junction traps the electrons on the p-type side, creating a potential difference across the slice. If connected to an external circuit, current will flow.



Figure 5A-2 Schematic symbol for a photovoltaic (pv) cell



#### Worked example 5A–1 Power, voltage and current of solar panels

A solar panel produces a maximum voltage of 14.7 V at a current of 27.2 A. What is its maximum power output?

#### Solution

Use the power formula, P = IV.

 $P = 27.2 \times 14.7$ = 399.8 W ≈ 400 W

# Check-in questions – Set 1

- **1** What does pv stand for?
- 2 What are solar panels typically made of?
- **3** What is the (desired) energy transformation taking place in a pv (solar) panel on a sunny day?

# Series or parallel?

The way solar panels are wired together will depend on the type of inverter and other control systems in place. As with batteries (recall from Unit 1), solar panels have a positive and a negative terminal.

When wired in series (sometimes called a 'string' of panels), the total potential difference adds up, while the total current is the same through each panel.

When wired in parallel, the potential difference is the same, but the current from each panel adds to give the total current. Figure 5A-3 shows both types of circuit (as a diagram rather than an electrical circuit).

Often panels will be wired with a combination set-up, with strings of panels in parallel.

The choice between wiring panels in series or parallel is based on what output voltage and current is required. Different combinations of the same panels can produce a variety of different total values. The overall desired power output is also a major consideration.







Total voltage =  $14.7 \text{ V} \times 3 = 44.1 \text{ V}$ Total current = 27.2 A Total power =  $44.1 \text{ V} \times 27.2 \text{ A} = 1199.5 \text{ W}$ 







ISBN 978-1-009-25901-9

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Voltage another name

for potential difference.

derived from its unit, volt

# Check-in questions – Set 2

- 1 What is the maximum voltage (potential difference) obtainable from two 12 V pv panels?
- **2** Draw a circuit diagram to show how to wire your answer to Question 1. Include a resistor as a load.
- 3 What is the maximum current obtainable from two 12 V 100 W pv panels wired in parallel?
- 4 Draw a circuit diagram to show how to wire your answer to Question 3
- 5 Calculate the power produced in each of these arrangements, series and parallel.

## DC or AC? Inverters for solar panels

Recall from Unit 1 that in a **direct current (or DC) circuit**, the polarity of the potential difference stays constant. In other words, the positive terminal is always positive and the negative terminal is always negative, as in the case of a battery. When a battery is connected into a circuit, the current always flows along the wire in the same direction.

In an **alternating current (or AC) circuit**, the polarity of the potential difference changes in a regular way, so that the charge first flows in one direction then in the other. The Australian mains electricity supply is AC, changing polarity first one way, then back again, 50 times per second – a frequency of 50 Hz. The magnitude of the potential difference varies as a **sinusoidal** wave (a sine wave shape) with time; the graph of potential difference plotted against time is a sine wave of frequency 50 Hz.





The choice of AC or DC is determined by the situation in which electricity is needed and how it is produced. Any battery-powered supply will be DC, as are solar pv panels. Many caravans have LED lights and other low power devices that can operate from 12 or 24 V DC, either battery or solar panel. AC is chosen for the mains supply from the grid because an alternating potential difference is needed for the transformers, which form an essential part of the mains supply system. A laboratory power supply can usually provide both AC and DC.

Buildings with solar panels also need a device called an inverter to change the DC generated into AC, which can be used to power AC appliances in the house or fed back into the grid, earning a payment from the power company. The inverter might be a single

UNIT 1 5B ELECTRICAL ENERGY AND POTENTIAL DIFFERENCE

5D GENERATORS WITH SLIP RINGS PRODUCE SINUSOIDAL AC

#### Direct current

(DC) circuit the polarity of the potential difference stays constant; the current always flows in the same direction

#### Alternating current (AC) circuit

the polarity of the potential difference changes in a regular way; the current changes direction at a rate that is measured in hertz (Hz)

#### Sinusoidal

having a sine wave shape; the regular, repeating form of a sine function



box located near the meter box; string inverters connect a set of panels – a string – to one inverter; or there could be individual 'micro-inverters', one on each solar panel. Although cost-effective, the string inverter gives reduced output if any individual panel experiences issues, such as shading. Microinverters can be more expensive to install, but shading or damage to one panel will not affect the power that can be drawn from the others.





Figure 5A-5 An inverter for a domestic solar panel installation, with isolating switches

### NOTE

Early inverters built in the nineteenth century were mechanical, often a spinning motor used to continually change the connection of the DC source forwards and backwards. Today, the switching inverter is made from solid state semiconductors (transistors made from silicon or gallium arsenide) with no moving parts. Inverters also contribute to the stability of the power grid, as do their giant cousins, the syncons. Fossil fuel generators have inbuilt frequency control as they spin, but there are no moving parts in a solar panel. Syncons (short for synchronising condensers) are big spinning machines that maintain the frequency of AC at 50 Hz. This is particularly necessary when a large proportion of power is being produced from solar.

# Check-in questions – Set 3

- 1 What is the main function of an inverter?
- 2 Why is this main function of an inverter important for an array of pv panels?

### **ACTIVITY 5A–1 SURGE PROTECTION**

You probably have your computer or tv plugged into a power point via a surge protector or a power board with surge protection built in. Many laptop power supplies include surge protection. Research the way surge protectors work and write a short paragraph describing how they work. Include labelled diagram(s) in your answer.



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#### Worked example 5A-2 Calculations with inverters for solar panels

On a caravan trip in the holidays, you want to power your laptop during the sunny part of the day. Your laptop charger is plugged into the caravan inverter and draws 50 W continuously.

- **a** If the current drawn by the laptop charger is rated at 4 A at 12 V, what is the maximum power provided to the laptop from the charger?
- **b** You also need to consider the power drawn by the inverter itself. The process of converting DC to AC costs energy. A 1000 W inverter with a 12 V input will draw around 1 A when it is just on standby, without any load. If the inverter is plugged in on standby, how much power will it consume?
- **c** When operating at maximum power generation, each solar panel produces a current of 3 A at a potential difference of 19 V. How many panels operating in full sunlight would you need to supply the inverter with the laptop plugged in?

Solution

- **a** P = VI
  - $= 12 \times 4$

= 48 W can be supplied by the charger

- **b** P = VI
  - $= 12 \times 1$

= 12 W is consumed by the inverter on standby.

**c** Total power required to charge laptop:

 $P_{\text{total}} = 48 + 12 = 60 \text{ W}$ 

Power produced by one solar panel =  $VI = 19 \times 3 = 58$  W. These are maximum figures, so expect to get less than this and buy two panels at least! You may want to run some LED lights at the same time. Note that in real life, the solar panels would usually recharge a battery, which would then power the inverter.

VIDEO 5A-2 SKILLS: CURRENT, VOLTAGE AND POWER OF SOLAR PANELS

# 5A SKILLS

#### Current, voltage and power of solar panels in series and parallel

You may be asked questions requiring you to calculate current, voltage and power for solar panels, and to compare them for series and parallel circuits.

#### Question

Four solar panels, each with power 400 W, voltage 14.7 V and producing a current of 27.2 A are wired as shown. Calculate total voltage, current and power.



#### Solution

You should start by analysing the circuit to consider whether the panels are wired in series, in parallel or a combination of the two. There are two pairs of panels wired in series, and the pairs are wired in parallel. Considering the panels wired in series first, their voltages should be added, while the current is the same through each panel. It is helpful to annotate each panel in the question diagram with its power, voltage and current rating.

Now, consider the two pairs of panels that are wired in parallel. The current from each pair is added to give the total current:

total current = 27.2 + 27.2 = 54.4 A

The voltage from each pair is the same as the total voltage:

total voltage = 29.4 V

The total power is total voltage × total current:

total power =  $29.4 \times 54.4 = 1599.4 \text{ W} \approx 1600 \text{ W}$ 

# Section 5A questions

### **Multiple-choice questions**

- 1 Solar panels often require an inverter because
  - A it keeps the cost down.
  - **B** it turns electricity backwards and upside down.
  - **c** many appliances run on AC, but solar panels only produce DC.
  - D many appliances run on DC, but solar panels only produce AC.
- 2 The pv cells of a solar panel are made up of
  - A sunlight.
  - **B** glass and copper.
  - **C** slices of copper doped with semiconductors like silicon.
  - **D** slices of doped semiconductor like silicon, with copper connections.
- 3 The main function of an inverter used in a solar panel array is to
  - A reverse the generated current.
  - **B** convert DC to AC for household use.
  - **C** allow AC from the mains grid to enter the panel.
  - **D** convert series panels to parallel when high current is required.
- 4 When solar cells are connected in parallel,
  - A the potential difference produced by each cell adds up.
  - **B** the currents produced by each cell add up.
  - **C** the power produced by each cell is reduced.
  - **D** they are easier to install on a rooftop.



#### CHAPTER 5 GENERATING ELECTRICITY

- 5 When solar cells are connected in series,
  - A the potential difference produced by each cell adds up.
  - **B** the currents produced by each cell add up.
  - **C** the power produced by each cell is reduced.
  - **D** they are easier to install on a rooftop.
- 6 To get maximum power from an array of solar pv panels, the best connection would be
  - A in series.
  - **B** in parallel.
  - **C** mostly in series with one or two panels in parallel.
  - **D** in a combination of strings (consisting of panels in series) in parallel with each other.

#### Short-answer questions

- **7** What is the most usual meaning of the abbreviation 'pv' when discussing electricity generation?
- 8 A particular solar cell in bright sunlight is able to produce maximum power of 300 W.
  - **a** If the maximum potential difference produced in this circumstance is 20 V, calculate the maximum possible output current. (Remember the power formula, P = VI).
  - **b** If 10 of these cells were placed in series, what would the potential difference be?
  - c If 10 of these cells were placed in parallel, what would the current be?
  - **d** How many of these cells would be needed to produce 5 kW?
- **9** The following is a map of the photovoltaic power potential for Australia. The colours represent estimated annual energy potential of a solar power plant, from blue, the lowest at 949 kWh/m<sup>2</sup>, to dark red, the highest at 1972 kWh/m<sup>2</sup>.



- a What energy transformation takes place when light energy is absorbed by a pv panel?
- **b** Why is it often said that the future of electric power in Australia is solar?
- **c** Compare the possibilities of powering the entire grid from solar in Victoria compared to South Australia. What advantages and disadvantages does each state have (use the map or other general knowledge you have)?
- **10** Give two advantages and two disadvantages of using solar photovoltaic panels to produce electricity for household use.



# Generating emf by varying the magnetic flux

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#### **Study Design:**

- Calculate magnetic flux when the magnetic field is perpendicular to the area, and describe the qualitative effect of differing angles between the area and the field:  $\Phi_B = B_{\perp} A$
- Investigate and analyse theoretically and practically the generation of electromotive force (emf) including AC voltage and

calculations using induced emf:  $\varepsilon = -N \frac{\Delta \Phi_B}{\Delta t}$ , with reference to:

- rate of change of magnetic flux
- number of loops through which the flux passes
- direction of induced emf in a coil

#### **Glossary:**

Alternator Electric generator Electromagnetic induction emf (electromotive force),  $\varepsilon$ Faraday's law of electromagnetic induction Galvanometer Induced current Lenz's law Magnetic flux,  $\Phi_{R}$ Rate of change

# **ENGAGE** King Island wave generators

One of the difficulties with the transition to using renewable sources for all our electricity generation is what to do when 'the wind don't blow and the sun don't shine'. Of course, storage in large- and small-scale batteries and in pumped hydro is one of the solutions. Another is a more reliable source of energy: wave power. Wave energy as

an energy source varies much more slowly than wind or solar power. It is also highly predictable over periods of several days.

Early wave technology relied on the oscillating water column (OWC) for generation. A pipe-like artificial blowhole is fixed so it is open underneath the waterline.

Figure 5B-1 Top: A cross-section through the main components of an oscillating water column (OWC) generator using a Wells turbine. Waves push water up into the water column inside the concrete structure, forcing air out through a cylindrical opening and turning the blades of a Wells turbine. As the water column falls with the wave trough, air is sucked in again through the turbine. A Wells turbine has blades that are shaped so that it rotates in the same direction, no matter which direction the air flows past it. A belt or chain drive from the turbine turns a generator connected to the grid. Bottom: An alternative layout for an OWC



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As waves pass the OWC, the water rises and falls inside the pipe, forcing the air to pass a turbine at the top of the pipe. This turbine generates electricity in the usual way. A problem with this style of wave generator is that the turbine spins both ways, as wave pushes the air up and then down as they pass. It also requires anchoring to the sea floor and takes up valuable coastline. The version shown in Figure 5B–1 uses a 'Wells' turbine that rotates in the same direction, with air both entering and leaving the structure.

An alternative to this, located on King Island and called UniWave 200, was developed in Melbourne by a company called Wave Swell. It has a valve system that diverts air there is through the turbine when there is a rising wave, then through a separate inlet valve when the wave recedes. This makes the turbine robust and efficient as it does not need the more complex blades of a Wells turbine since it only turns one way. Wave Swell systems sit in shallow water, where all the moving parts (turbine and valves) are well above the water line. Being in shallow water also makes them easier to service and cheaper to connect to the grid. A number of these systems could be deployed along a stretch of beach to prevent erosion, or within a sea wall or as an artificial reef to create a safe harbour. The electricity they create can be used for desalination or hydrogen production as well as to power the grid.



**Figure 5B–2** The Uniwave OWC generator positioned in shallow water off King Island, Tasmania, where it generates electricity from wave power with 40–50% efficiency.



**EXPLAIN** 

**Electromagnetic induction** 

it generate electricity?

In Chapter 4, we learned that electric current-carrying wires placed in magnetic fields experience a force perpendicular to both the field and the current, F = BIl. This is the basis of all electric motors and was first applied for this purpose by Michael Faraday in 1821. Does the reverse also work? If a magnet is forced to move relative to some coils of wire, will



**Figure 5B–3** Left: When the magnet is pushed into the wire coil, the galvanometer indicates the direction and strength of the induced current. Centre: if there is no relative movement between the magnet and the coil of wire, no current is created. Right: As the magnet is pulled out of the coil of wire at the same rate as in the left diagram, the galvanometer indicates that the current produced is of the same magnitude but opposite direction.

We know that a bar magnet has an associated three-dimensional magnetic field. We also know that individual electrons have their own magnetic fields (dipoles) associated with them. If one end of a bar magnet is inserted into a coil of insulated copper wire connected to a sensitive current measuring device (often known as a **galvanometer**), a small electrical current is generated because of the interaction between the magnetic field of the bar magnet and the magnetic fields of the electrons. When the bar magnet is stationary, no current is generated – even if the bar magnet is inside the coil. When the bar magnet is being removed from the coil, current is again generated but in the opposite direction (see Figure 5B–3). Why does this happen? To understand this phenomenon, we will use the principle of forces acting on moving charges explored in Chapter 4, F = qvB.

If a copper rod is pulled through a magnetic field, as shown in Figure 5B–4 (left), then the positively and negatively-charged particles in the rod, both moving to the right with velocity v, will have a force acting on them of F = qvB.

As the diagram illustrates, this force has a tendency to push the positive charges up and the negative charges down. Remember that the electrons are free to move but the positive ions are held firmly in the lattice. This creates a net negative charge at the bottom end, while the top end of the rod becomes positively charged due to lack of electrons. In other words, it creates a potential difference (an induced emf) between the ends of the rod. As shown in Figure 5B–4 (centre), by connecting the rod or thinner wire to a galvanometer and completing the circuit, it is possible (by moving the wire left or right through the magnetic field) to see the effect of this induced emf as a current flow through the galvanometer. When inducing currents in this way, a magnetic force is generated that *opposes* the applied force generating it. This is shown in Figure 5B–4 (right).

Similarly, the changing magnetic field pattern around the wires of the coil in Figures 5B–4 (left and right) has induced an electric current in the circuit.

Galvanometer a device that measures the magnitude and direction of very small currents

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emf (electromotive force),  $\varepsilon$ potential difference between the terminals of a source when no current flows to an external circuit; measured in volts (V)

CHAPTER 5 GENERATING ELECTRICITY



**Figure 5B–4** Left: Moving a copper rod through a magnetic field creates equal and opposite forces on the negative and positive charges in the rod (F = qvB). This creates an induced emf (potential difference) between the two ends of the rod as the electrons move towards the bottom end. Centre: when a copper rod is connected to a circuit containing a galvanometer, the effect of the induced emf is to produce a current flow (induced current) as the electrons are free to move within the metal. This can be detected on the galvanometer. Right: To continuously pull a conductor through a magnetic field at a constant velocity, *v*, requires an applied force,  $F_{applied}$ , which is equal and opposite to the magnetic force,  $F_{magnetic}$ , generated as a result of the induced current set up in the wire.

#### Electromagnetic induction the process of generating ap

generating an electric current with a changing magnetic field near a wire, or by moving a metal wire in a steady magnetic field

Magnetic flux,  $\Phi_B$ the total

magnetic field

(or total number of magnetic

field lines) that passes through

a given area; proportional to

the area and field strength

This phenomenon is called **electromagnetic induction** and it is the basis of most electrical generators, both thermal and renewable (other than pv panels). Wind and wave generators, coal-, gas- and nuclear-fired power stations and even geothermal power stations rely on electromagnetic induction. Electromagnetic induction was discovered by Michael Faraday in 1831, ten years after his discovery of the principles of the electric motor. (An American, Joseph Henry, is also credited with the independent discovery of this effect in 1831).

# Check-in questions – Set 1

- 1 What must occur to produce an electric current from just a magnet and a coil of wire?
- 2 Is an electric current produced while a magnet is stationary near or inside a coil?



# Magnetic flux, $\Phi_{B}$

A more useful way of understanding electromagnetic induction uses the concept of **magnetic flux**,  $\Phi_{B}$ , shown in Figure 5B–5. The magnetic flux, symbol  $\Phi_{B}$ , can be thought of as the total number of magnetic field lines through a given area. For instance, in Figure 5B–5 (left) the magnetic flux through loop P (8 units) is twice the magnetic flux through Loop Q (4 units). Note that the area of loop P is twice that of loop Q, so the magnetic flux through a loop is proportional to the area, A, of the loop.

That is:

 $\Phi_{B} \alpha A$ 



**Figure 5B–5** Left: The magnetic flux,  $\Phi_B$ , through loop P (8 units) is twice the magnetic flux, through loop Q (4 units). Note that the area of loop P is exactly twice the area of loop Q and as  $\Phi_B = QP$ , this implies the magnetic flux through P is double that through Q. Right: If the same loop Q is now placed in a magnetic field four times as strong as previously, then the magnetic flux will also increase by a factor of four (again because  $\Phi_B = QP$ ).

Also, the magnetic flux through the loop will depend on the strength of the magnetic field,  $B_{\perp}$  (*B* at right angles to the area *A*). For instance, if loop Q from Figure 5B–5 (centre) is placed in a magnetic field that is four times as strong as previously (Figure 5B–5 right), then the magnetic flux will increase by a factor of four. That is:

$$\Phi_B \alpha B_\perp$$

In Figure 5B–3, when the bar magnet is brought closer to the coil, the magnetic field in the coil increases and so the magnetic flux through the area of the coil increases. This induces an electrical current. When the bar magnet is stationary, there is no changing magnetic flux, and no **induced current** is detected on the galvanometer. Removing the bar magnet causes the magnetic flux to decrease and again produces a current.

It follows that:

$$\Phi_{R} \alpha B_{\downarrow} A$$

 $\Phi_{R} = B_{\perp}A$ 

This can be expressed as Formula 5B–1.

#### Formula 5B–1 Magnetic flux through a loop

Where:

 $\Phi_{\rm B} = \text{Magnetic flux (T m<sup>2</sup> or Wb)}$ 

 $B_{\perp}$  = Magnetic field strength perpendicular to the face of the loop (T)

A =Area of the loop (m<sup>2</sup>)

The unit for  $\Phi_B$  is the tesla metre squared (T m<sup>2</sup>), which is given the special name weber (Wb). It follows from Formula 5B–1 that:

$$B = \frac{\Phi_B}{A}$$

Induced current an electric current that results from an induced emf,  $\varepsilon$ , if there is a complete circuit





Faraday's law of electromagnetic induction the magnitude of the induced emf is directly proportional to the rate of change of magnetic flux

Experiments with magnets, coils and galvanometers show that a change in magnetic flux induces an electric current. The exact relationship between the changing magnetic flux and the induced electric current is detailed in Faraday's law of electromagnetic induction on the following page.

This is why *B* is often referred to as the magnetic flux density, the magnetic flux through each square metre.

In studying electromagnetic induction effects and the principles of electricity generation, it is far more useful to investigate the changes that occur in magnetic flux with respect to time,  $\frac{\Delta \Phi_{\rm B}}{\Delta t}$ . This will be our next focus.

# Check-in questions – Set 2

- 1 Write down the formula that relates magnetic flux, area of a loop and magnetic field strength. Make a note of the units for each quantity.
- 2 The diagram shows a uniform magnetic field and two loops of wire.



- **a** Loop 1 has side lengths 1.0 m and 0.5 m. The magnetic field strength is  $5 \times 10^{-3}$  T. Calculate the magnetic flux,  $\Phi_{R}$ , through loop 1.
- **b** Loop 2 is a square of side length 0.5 m. The magnetic field strength is  $5 \times 10^{-3}$  T. Calculate the magnetic flux,  $\Phi_{\scriptscriptstyle B}$ , through loop 2.
- 3 If the magnetic flux through a loop (such as the end of an MRI scanner) of area  $1.50 \text{ m}^2$ is 2.25 Wb, what is the magnitude of the magnetic flux density in the region of the loop?

# Induced emf, *ɛ*

Any device, such as a battery, solar cell or an electric generator, which transforms one type of energy (chemical, light, mechanical, etc.) into electrical energy is a source of electromotive force or emf. Note that electromotive force is not really a 'force' and is not measured in newtons; rather it is a term that is used to describe the potential difference between the terminals of a source when no current flows to an external circuit. To avoid confusion, we will consistently use the abbreviation emf and/or its symbol,  $\varepsilon$  when referring to electromotive force. The SI unit of emf is the volt (V).

Faraday's experiments demonstrated that a changing magnetic flux through a coil of wire made a current flow (an induced current) as if there were a source of potential difference in the circuit. To describe this more accurately, we say that 'an induced emf is produced by a changing magnetic flux. The potential difference (emf,  $\varepsilon$ ) exists whether or not there is a complete circuit for an induced current to flow.

Electric

generator also known as a dynamo, a device that converts kinetic (mechanical) energy into electrical energy

# Faraday's law of electromagnetic induction

Faraday noticed that the faster the bar magnet was inserted into the coil (refer again to Figure 5B–3 left and right) and thus the smaller the time interval,  $\Delta t$ , taken for the flux to change, the larger the momentary deflection of the galvanometer – that is, the larger the current produced. He also noticed that if the bar magnet was replaced by a stronger bar magnet, making a greater change in magnetic flux, the galvanometer deflection increased even if the speed of insertion was the same.

What Faraday had noticed was a **rate of change** of magnetic flux. It is calculated by dividing the change in the magnetic flux by the change in time (duration) over which the change happens.

Faraday summed up these two results as follows:

the magnitude of the induced emf is directly proportional to the rate of change of magnetic flux

If the single wire loop instead consists of *N* closely-wrapped turns or loops to make a coil, then the induced emfs in each loop add together to give the basis of Faraday's law of electromagnetic induction, written mathematically as:

$$\varepsilon = N \frac{\Delta \Phi_B}{\Delta t}$$

where  $\varepsilon$  is the induced emf (V),  $\Delta \Phi_B$  is the change in magnetic flux (Wb),  $\Delta t$  is the time taken for the flux to change (s) and N is the number of loops in the coil. See Formula 5B–2 on page 259 for a complete statement of Faraday's law.

# Check-in questions – Set 3

1 The diagram shows a bar magnet moving into a coil. Suggest two actions that would increase the magnitude of the induced emf,  $\varepsilon$ , in this set-up.



**2** What would the magnitude of the induced current be if both the magnet and the coil in the diagram from Question 1 move to the right at constant velocity?

Rate of change the rate at which one quantity is changing with respect to time

## Direction of induced current: Lenz's law

When a magnet is used as shown in Figure 5B–6, the increase in applied magnetic flux from the approaching magnet causes an induced current.



**Figure 5B–6** A diagram of Lenz's law. The change in magnetic flux from an approaching magnet induces a current in a wire. As the north pole of the magnet is pushed into the loop, the induced current creates a magnetic field that opposes the change in magnetic flux creating it. The magnetic flux of the applied *B* field to the right increases as the magnet approaches, so the induced *B* field is to the left.

A second magnetic field is produced through the coil by this induced current; this *induced* magnetic flux also exists throughout the loop area. What is the direction of this induced magnetic flux? In 1834, Heinrich Lenz suggested that:

the magnetic flux of the induced current through the loop opposes the change in the applied magnetic flux that produced it

This law, known as Lenz's law, can be thought of as common sense. It can be deduced from energy conservation. *If* the induced flux *did not oppose* the applied magnetic flux change that caused the emf in the first place, then the induced magnetic flux *would assist* the overall flux change, causing the emf to build up, causing the flux to further build up and so on; unlimited energy would be obtained for very little expenditure. This is not plausible as it would contradict the law of conservation of energy.

Lenz's law implies that there should be a minus sign in Faraday's law, to show this 'opposing the change':

$$\varepsilon \alpha - N \frac{\Delta \Phi_B}{\Delta t}$$
$$\varepsilon = -kN \frac{\Delta \Phi_B}{\Delta t}$$

Using energy conservation arguments, it is possible to demonstrate that the constant of proportionality, k, in the above equation has a value of one. Consequently, Faraday's law can be written as Formula 5B–2.

Lenz's law the magnetic flux of the induced current through the loop opposes the *change* in the applied magnetic flux that produced it Formula 5B–2 Faraday's law of electromagnetic induction and Lenz's law

$$\varepsilon = -N \frac{\Delta \Phi_B}{\Delta t}$$

Where:

 $\varepsilon =$ Induced emf (V)

N = Number of turns (or loops) of wire

 $\Delta \Phi_{B}$  = Change in magnetic flux through the coil (Wb)

 $\Delta t$  = Time taken for the change to occur (s)

The negative sign indicates that the induced emf produces a magnetic flux that *opposes the change* in magnetic flux used to produce it.

This is the more generalised form of Faraday's law of electromagnetic induction and is extremely important in the design of electric generators, dynamos and alternators.



**Figure 5B–7** As the north pole of the magnet is pushed into the coil, the induced current produced creates a magnetic flux that opposes the change in magnetic flux creating it. This is known as Lenz's law. (Note that the far side of the coil is not shown, so the field lines can be seen more clearly.)

#### Check-in questions – Set 4

- 1 What do the following terms stand for in Faraday's law and what are their standard units:  $\varepsilon$ , *N*,  $\Delta \Phi_{R}$ ,  $\Delta t$ ?
- 2 a Calculate the change in magnetic flux through a coil of 100 turns, with an area of 0.5 m<sup>2</sup>, when the magnetic field strength changes from 0.05 to 0.10 T?
  - **b** Calculate the magnitude of the induced emf in the coil if it takes 0.5 s for the change in flux to occur.
- **3** Using the principles of conservation of energy, explain why there needs to be a minus sign in Faraday's law.



Alternator an electrical generator that produces

electrical energy

in AC output



WORKSHEET 5B-1 Generating EMF

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#### ACTIVITY 6B-1 GENERATING EMF FROM BODY MOVEMENT

Suggest a design for the heels of children's shoes that light up when they walk or jump. Hint: Use electromagnetic induction (or a 'linear generator') as the basis.

If you research this topic, you will find five or six different methods for producing emf from body movement, many of them using nanoelectronics (TENG, Peng and others). As wearable electronics become cheaper and more broadly available, this topic is sparking much research to enable on-the-go recharging (not just sparkly lights).

#### Worked example 5B–1 Calculating magnetic flux and applying Faraday's law

Magnetic field is vertically downwards Horizontal square conducting metal loop

A single loop of wire is fully within an area of magnetic field.

- **a** Calculate the magnetic flux through the loop if the area of the loop is 0.4 m<sup>2</sup> and the magnetic field strength is 1.0 T.
- **b** The loop is pulled slowly and steadily toward the right, so that it leaves the area of magnetic field totally in 5 s. What is the emf, ε, generated in the loop?
- c Describe the direction of induced current and show your reasoning.

Solution

- **a**  $\Phi_B = B \times A$ , so:  $\Phi_B = 1.0 \times 0.4$ 
  - $P_B = 1.0 \times 0.$ = 0.4 Wb
- **b** Flux reduces to zero from 0.4 Wb, so  $\Delta \Phi_B = -0.4$  Wb.

Using Faraday's law:

$$\varepsilon = \frac{-\Delta \Phi_B}{\Delta t}$$
$$= \frac{-(-0.4)}{5}$$
$$= 0.008 \text{ V}$$

The meaning of the minus sign in Faraday's law is dealt with in part c.

C The minus sign means that ε will be in a direction such that its associated induced current produces a flux (a magnetic field) that opposes the original change in flux (Lenz's law). In this case the flux downwards is reduced, so the induced current produces a downwards flux to oppose that change. The current is therefore clockwise as seen from the top (using the right-hand grip rule.)

# **5B SKILLS**



Analysing diagrams from different viewpoints is a valuable skill in physics, especially in this topic. Practice thinking about this example from the front, the top and the side.



The view then seen by each eye is shown here (excluding the galvanometer).



A view from the left-hand side (LHS) behind the magnet is more useful here, as it can show the change of magnetic flux through the coil. If we represent some of the field lines from the magnet ( $B_{applied}$ ) as  $\times \times \times$  in the coil, there is no need to try to draw the actual magnet.



Thinking about it from this viewpoint, you can clearly see that the amount of flux through the loop increases as the bar magnet gets closer to the loop. We usually show more field lines closer together, in this case crosses, indicating magnetic field *into* the page.

Using Faraday's law and Lenz's law, the loop will carry an induced current whose magnetic flux opposes that change; the magnetic flux of the induced current must be opposite in direction to the original increasing flux, as shown below.





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As the area of the loop is constant, the change in magnetic flux,  $\Delta \Phi_{B}$ , is closely related to the change in the magnetic field,  $\Delta B_{applied}$ .



Using the right-hand hand grip rule for current and magnetic field directions gives the direction of the induced current in the loop; in this case anticlockwise.

Overall, if the magnetic field strength to the *right increases*, then the magnetic field of the induced current will be to the *left*. This means that if we bring a north pole closer, the induced current creates a north pole facing the opposite way to oppose the change in flux, bringing the total flux in that area back to its original value.

There's a neat memory trick that relates the direction of current in a coil and the direction of the magnetic field it produces. Recall that for a north pole the magnetic field lines point away from the pole and for a south pole they point towards the pole.

So, viewing the original coil above from the left-hand end shows the following pattern.



Note the way a north pole, N, can be imagined between the arrows showing anticlockwise (ACW) current.

A south pole, S, seen from the right-hand end, would be associated with a clockwise (CW) current.



Look closely and you will notice that it is not possible to mix these up. Arrows pointing anticlockwise current can only show north, not south, and vice versa for clockwise current. This memory trick is useful, similar to the right-hand grip rule, but remember these are just memory tricks to help us recall the way things happen. They aren't reasons why they happen!



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# Section 5B questions

# Multiple-choice questions

1 A student drags a small metal rod towards the right, at constant velocity, through a magnetic field, *B*, acting into the page as shown.



Which of the following statements most accurately describes what will occur?

- A Nothing will occur.
- **B** For a short time, the student will notice a force on the rod that accelerates it towards the right.
- **C** For a short time, the student will notice a force on the rod that accelerates it towards the left.
- **D** The student will notice a steady force on the rod that opposes the motion of the rod.

Use the following information to answer Questions 2, 3, 4 and 5.

A coil of wire is positioned in the magnetic field, *B*, of a bar magnet. The direction of the field (but not necessarily magnitude) remains constant. The north pole of the bar magnet steadily approaches the coil as shown. Any current induced in the coil will be registered on the galvanometer.

- 2 Which of the following best describes the change in magnetic flux as the bar magnet approaches the coil?
  - A increases to the right
  - **B** decreases to the right
  - **C** increases to the left
  - **D** decreases to the left
- **3** Which of the following best describes the magnitude and direction of the induced magnetic field, *B*<sub>induced</sub>, in the coil due to the induced current?
  - A increases to the right
  - **B** decreases to the right
  - **C** increases to the left
  - **D** decreases to the left
- 4 Which one of the following best describes the current through the galvanometer while the magnet is moving steadily?
  - A steady current to the left
  - **B** steady current to the right
  - **C** increasing current to the left
  - **D** increasing current to the right



#### **Short-answer questions**

- **5** Explain the magnitude and direction of the current through the galvanometer when the magnet stops moving.
- 6 Calculate the magnetic flux for a field of:
  - **a** 2 T perpendicular to an area of  $1 \text{ m}^2$
  - **b** 2 T perpendicular to an area of  $2 \text{ m}^2$
  - **c** 6 T perpendicular to an area of  $1 \text{ m}^2$
  - d 6 T perpendicular to an area of  $2 \text{ m}^2$ .
- 7 State Faraday's law of electromagnetic induction and explain what each term represents.
- 8 What is Lenz's law? Why is it said that Lenz's law is common sense? Hint: answer in terms of conservation of energy.
- **9 a** Draw diagrams showing:
  - i a north pole entering a coil of wire
  - ii a north pole leaving a coil of wire
  - iii a south pole entering a coil of wire
  - iv a south pole leaving a coil of wire.
  - **b** For each of your diagrams, i to iv, indicate clearly the applied magnetic field,  $B_{\text{applied}}$ , and the induced magnetic field,  $B_{\text{induced}}$ .
  - **c** Which pairs of diagrams could possibly produce the same induced emf between the ends of the coil?



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# DC generators: producing DC voltage

#### **Study Design:**

 Explain the production of DC voltage in DC generators and AC voltage in alternators, including the use of split ring commutators and slip rings respectively

#### **Glossary:**

DC generator Electromagnet Split ring commutator

### ENGAGE Werner von Siemens

In 1866, Werner von Siemens, a Prussian ex-artillery officer, is credited with having invented the first electric generator, in which current to the field winding is supplied by the generator itself. Modern generators are still based on this principle. Siemens co-founded the company still known by his name: Siemens AG. Today it is one of the largest electrical engineering companies in the world.



In the 1850s, Siemens laid the first telegraph cables in

Europe, under the Mediterranean and to India. He improved the electrical insulation of cables using gutta-percha, a rubbery substance obtained from trees in Asia and the Pacific, which insulated the conducting metal from moisture. This revolutionised the use of electricity, as the new insulated cables could carry signals with much less interference and short-circuiting. The telegraph from Adelaide to the port of Darwin was laid cross-country in 1872, connecting Australia instantly to Europe (via undersea cable to Singapore) for the first time.



In the 21st century, Siemens products and others inspired or made possible by von Siemens' inventions are used across the world and range from medical diagnostic imaging systems, mobile phones, hearing aids, mass transit systems, ground movement radar for airfields and power generating equipment. The company also continues to design, build and operate telecommunications networks.



# EXPLAIN

## Generators

Probably the most important practical result of Faraday's great discovery of electromagnetic induction was the development of the electric generator or dynamo.

A dynamo or electric generator is a device that converts kinetic (mechanical) energy into electrical energy. This is the opposite of what a motor does. Indeed, a generator is basically the inverse of an electric motor (described in Section 4B and shown in Figure 4B–22). In the context of the regenerative braking techniques used on modern electric vehicles and in trams, electric motors can be used as generators when a vehicle slows down, producing electrical energy that can be fed back into the battery for future use, or through the overhead power lines (in the case of trams).

When producing DC output, electrical generators are usually referred to as **DC generators** or DC dynamos. When producing AC output, electrical generators are usually called AC generators or alternators. The principles of generation are both very similar. Coils of wire are rotated through magnetic fields (or vice versa) and the changing magnetic flux created by this rotation induces an emf. This is an extension of the electromagnetic induction described in Section 5B, where the magnetic field changed near a coil.

Imagine a coil of wire rotating in an area where there is a uniform magnetic field. Such a set-up is shown in Figure 5C–1. As the coil spins, the flux passing through the coil varies, as the *effective* area of the coil is smaller when the face of the coil is not perpendicular to the magnetic field. Note that a force turning the coil must be maintained. If not, the coil will stop turning and no emf or current will be produced. (This should be obvious from consideration of conservation of energy.)

#### NOTE

See 5C Skills at the end of this section to learn the valuable skill of analysing diagrams from different viewpoints.



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4B MAGNETIC FIELDS AND FORCES

> DC generator also known as a DC dynamo, an electric generator that produces electrical energy in DC output

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Figure 5C–1 A rectangular wire loop of area, A, is rotated on its axis (the handle isn't shown) in a magnetic field, B, directed from left to right; north pole is left of the diagram, south pole to the right. When the magnetic field is parallel to the rectangular loop area, the magnetic flux  $\Phi_{\rm P}$  is a minimum (position **a**).



When the magnetic field is perpendicular to the rectangular loop area, the magnetic flux  $\Phi_B$  is a maximum (position **b**), and so on for the whole cycle. In general, for rotating loops where the angle between the normal to area A and the magnetic field B is  $\theta$ , the expression for the magnetic flux is given by  $\Phi_{B} = AB \sin \theta$ . The letters **a**–**e** indicating the position of the loop are also used on the graphs of its rotation in Figures 5C-2 and 5C-3.

Figure 5C–2 The graph of magnetic flux variation for one whole cycle of the rotating loop shown in Figure 5C-1, with letters **a**-**e** referring to the same position of the loop. The magnetic flux variation follows the shape of a sine wave. This can be seen from the area through which the magnetic field acts in position **a** to **e** of Figure 5C–1, or can be directly plotted from the equation of magnetic flux:  $\Phi_B = AB \sin \theta$ .

Figure 5C–3 The graph of induced emf variation for one whole cycle of the rotating loop shown in Figure 5C-2. It follows the shape of a cosine wave. This can be seen from knowing that the induced emf is given by the negative gradient of the graph of magnetic flux,  $\Phi_{\!R}$  plotted against time, t (Faraday's law).

At position **a**, when the flux is zero, it is changing most rapidly, from entering one side of the coil to entering the opposite side as the coil spins. Therefore, the magnitude of the induced emf,  $\varepsilon$ , is maximum.



negative.

Examining the graphs of Figures 5C–2 and 5C–3 closely reveals that the  $\varepsilon$  vs time graph has the shape of the negative gradient of the magnetic flux versus time graph.

induced emf.

zero.

# Check-in questions – Set 1

Refer to Figure 5C-2 and 5C-3 to answer the following questions.

- 1 Which positions of the coil, **a** to **e**, show maximum magnetic flux magnitude through the coil?
- **2** Which positions of the coil, **a** to **e**, show minimum flux magnitude through the coil?
- 3 Which positions of the coil, **a** to **e**, show maximum emf magnitude generated in the coil?
- 4 Which positions of the coil, **a** to **e**, show minimum emf generated in the coil?

#### DC generators (DC dynamos)

The simplest DC generator consists of a loop of wire between magnetic poles. To obtain an output current, the ends of the wire are connected to a circuit via a **split ring commutator**. Note how similar this set-up is to a DC motor. In a motor, electrical energy is the input, transforming to kinetic

energy out. In this DC generator, kinetic energy is put in (by rotating the loop via the driveshaft) and the output is electrical energy. The spinning loop always produces emf; there will also be a current if there is a complete circuit for it to flow around.

Figure 5C–4 shows a very simple DC electrical generator (or dynamo). It has a single coil that is rotated in the magnetic field provided by a set of permanent magnets, called the field magnets. The ends of the coil are joined to a ring of copper with two gaps in it (known as a split ring). Spring-loaded carbon brushes touch the split ring as it rotates to ensure that there is an electrical connection from the rotating coil to the external load. The split ring and the brushes together are called a commutator.

Drive shaft turned by an external mechanical force

**Figure 5C–4** A very simple DC generator – a single coil rotating in a magnetic field produced by a set of permanent magnets. The split ring commutator ensures that the carbon brush contacts are reversed every half cycle, so the induced emf is always positive on the right-hand terminal in this diagram.

Split ring commutator a device used in DC generators to obtain a DC ouput, i.e. to rectify the AC emf (and current) produced

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**Figure 5C–5** The graph shows the emf generated in the coil of a simple single coil DC generator. The emf generated in the coil is the blue line and the output is the red line.

Because of the split ring, the emf produced in the coil and the emf at the brushes look quite different (Figure 5C–5). Although the current is varying, it is in one direction and produces DC in an external circuit. To smooth the DC, a capacitator could be placed in parallel with the output. An alternative way of smoothing the output is to use a greater number of coils and more splits in the ring. For example, just by increasing the number of coils to two (called a two coil DC generator), as shown in Figure 5C–6 (left), a much smoother DC output is produced (Figure 5C–6 right).



**Figure 5C–6** Left: A very simple two coil DC generator with the two coils at right angles to each other. There are now two sets of split rings connected to the spring-loaded carbon brushes. Right: This graph shows the output emf (blue line) of each pair of coils and the total DC emf output (red line) of a simple two coil DC generator. This is a smoother output than the emf output from the single coil DC generator.

The coil is known as the armature or rotor winding. In Figure 5C–4 and Figure 5C–6 (left), each coil is represented by a single turn of wire but in a practical DC generator it would contain a large number of turns (e.g. 60–1000 turns) wound on to a soft iron core (Figure 5C–7). Many such armature windings (eight pairs are shown in Figure 5C–7) are then placed into the magnetic field. Note that real DC generators (such as those used in cars before alternators became popular) use **electromagnets** (labelled field windings in Figure 5C–7) to create the magnetic field, rather than permanent magnets. This design feature has two advantages. First, the field produced by an electromagnet can be much stronger than that produced by a permanent magnet. Secondly, some of the current produced by the generator can be fed straight back into the field windings (so-called 'self-excitation').



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Electromagnet a coil of wire, consisting of an iron or steel core, through which a current is passed





However, most commercial electrical generators use substantially different geometries to the simple one shown in Figure 5C–7 and involve many more windings of coils. They often use rotating magnets rather than coils. Some of these more practical and useful geometries for large scale generation are discussed further in the section on dynamos, AC generators and alternators in Section 5D.



**Figure 5C–7** Real DC generators use many windings per coil and many coils per generator to create maximum emf outputs. The rotor is connected to the commutator, which in this case, contains multiple pairs of split rings. Also, the magnetic field for the field windings is produced using self-excited electromagnets. An almost completely smooth DC output is obtained.



# Check-in questions – Set 2

- 1 Draw a diagram to show a loop of wire rotating in a magnetic field.
- **2** When a coil spins in a magnetic field, the flux,  $\Phi_{B'}$  through it changes. Explain why this is and what the positions are when the flux is maximum and minimum.
- **3** When is the *rate of change* in flux,  $\Delta \Phi_{B'}$  maximum and minimum? Describe the positions of the coil in the field.



#### Worked example 5C-1 Voltage output of a DC generator

A model motor is set up as a DC generator, with the output connected to a voltmeter and oscilloscope via a split ring commutator, as shown below. The coil is rotated by hand.



The oscilloscope (also called a cathode ray oscilloscope or CRO) acts just like a voltmeter, with the added advantage of showing a graph of emf plotted against time on the screen.

Which one of the following graphs best shows the voltage output as viewed on the oscilloscope as the coil rotates steadily? (At t = 0, the coil is horizontal, as shown in the diagram above.)



#### Solution

At t = 0, the coil is horizontal, so the magnetic flux is zero but changing rapidly. Therefore, the emf will be maximum at t = 0 and will vary sinusoidally. The split ring commutator reverses the direction of the current (and emf) every half cycle, so the correct answer is B. 271



VIDEO 5C-1 SKILLS: VIEWING A ROTATING COIL FROM DIFFERENT ANGLES

# 5C SKILLS

#### Viewing a rotating coil from different angles to visualise $\Delta {m arPsi}_{_{B}}$

To practise analysing diagrams from different viewpoints, a useful situation is to visualise is a loop turning in a uniform magnetic field, shown in Figure 5C-1.



Direction of rotation

It's difficult to show the area of the coil perpendicular to the field in this view, but visualising the view *from the top* makes this much clearer. (We only show the edge of the loop closest to the viewer, BC.)



Using the top view, it's easier to see that when the area of the coil is perpendicular to the field lines, *B*, it has maximum effective area and therefore maximum flux,  $\Phi_B$ . It is also easier to see that the flux is changing quite slowly from **b** to **c** to **d**, so  $\Delta \Phi_B$  is close to zero. Also, when the coil spins through positions **a** and **e**, where it's parallel to the field lines, the effective area is small but changing rapidly. The flux effectively enters first through one side of the coil, then the other, as the coil spins. This change in flux,  $\Delta \Phi_B$ , corresponds to the greatest (peak) emf.

The corresponding graphs of magnetic flux versus time and emf versus time are shown below.



Remember that the relationship between these two graphs is a negative gradient, as given by Faraday's law. In a DC generator, the split ring commutator has the effect of rectifying the emf output, so any values below the axis would become positive. The output is sinusoidally varying *direct* current.

# Section 5C questions

### **Multiple-choice questions**

Use the following information to answer Questions 1, 2, 3 and 4.

The diagram shows a rotating coil of 50 turns and area 0.020 m<sup>2</sup>. The uniform magnetic field is 2.0 T.



Orientation A





Orientation C

- 1 Which of the following is a possible value of the magnetic flux through the coil when it is in orientation C?
  - **A** 0 Wb
  - **B** 0.03 Wb
  - **C** 0.04 Wb
  - **D** 1.5 Wb
- **2** Which of the following is closest to the magnitude of the change in magnetic flux through the coil when it moves 180° from orientation B?
  - A 0 Wb
  - **B** 0.04 Wb
  - **C** 0.08 Wb
  - **D** 40 Wb
- **3** Which of the following is closest to the change in flux through the coil when it moves 360° from orientation B?
  - A 0 Wb
  - **B** 0.08 Wb
  - **C** 40 Wb
  - **D** 80 Wb
- **4** Which of the following is closest to the size of the change in magnetic flux through the rotating coil when it moves 90° from orientation A?
  - A 0 Wb
  - **B** 2.0 Wb
  - **C** 50 Wb
  - **D** 200 Wb

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#### Use the following information to answer Questions 5, 6 and 7.

The following diagrams show a square loop being moved between the poles of a magnet. In the space between the poles there is a uniform magnetic field.



The loop moves at a steady speed from position 1 to position 3.

The loop is connected to a sensitive microammeter. The area of the loop is much less than the area of the magnetic field.

You may assume that the only magnetic field present is located directly between the north and south poles.

**5** Which of the following graphs best shows how the flux through the square loop varies with time as it moves from position 1 through to position 3?



VCAA 2012

6 Which of the following graphs best shows how the magnitude of the emf in the square loop varies with time as it moves from position 1 through to position 3?


- 7 Which of the following best describes the direction of the current in the loop as it moves from position 2 to position 3, viewed from the south pole?
  - **A** a steady anticlockwise current
  - **B** a steady clockwise current
  - **C** initially no current, then a steady anticlockwise current, then no current
  - **D** initially no current, then a steady clockwise current, then no current

Use the following information to answer Questions 8 and 9.

A simple DC motor has permanent magnets and one loop for a coil.



- 8 If spun by hand, it produces an output voltage. The best reason for this is that
  - **A** the flux through the coil changes.
  - **B** the attraction of the two permanent magnets to each other forces the electrons around the coil.
  - **C** this action will cause an equal and opposite reaction.
  - **D** friction generates heat, which accelerates the electrons.
- **9** The connection between the coil and the output is a split ring commutator. This means that the output voltage is best described as a
  - A sinusoidal voltage.
  - **B** steady DC voltage.
  - **C** varying AC voltage.
  - **D** DC voltage whose size varies with time.



#### **Short-answer questions**

- **10** A coil is rotating in a uniform magnetic field.
  - a Explain carefully why a current is induced in the coil.
  - **b** Explain how the induced current depends on the rate of rotation of the coil, the number of turns of the coil and the strength of the magnetic field.

Use the following information to answer Questions 11 and 12.

A model is set up as a DC generator, with the output connected to a voltmeter and oscilloscope via a commutator (as shown below) with a square coil of side length 4.0 cm and 10 turns, and a uniform magnetic field of  $2.0 \times 10^{-3}$  T.

The coil is rotated by hand.



11 The coil makes one complete revolution per second. Calculate the magnitude of the average voltage as shown on the voltmeter during onequarter revolution. Show your working and give your answer in mV.

Adapted from VCAA 2015

12 The coil now spins backwards, but all other variables remain the same as in Question 11. What effect, if any, will this have on the output seen on the oscilloscope? Explain your answer.

Adapted from VCAA 2015





## **Generators with slip rings produce sinusoidal AC**

#### **Study Design:**

• Explain the production of DC voltage in DC generators and AC voltage in alternators, including the use of split ring commutators and slip rings respectively **Glossary:** Slip ring

#### ENGAGE

#### Wind turbines generate AC power

Wind is currently the cheapest source of large-scale renewable energy. Solar energy causes changes in air temperature and pressure that result in wind currents. The energy from wind can turn propellor-like blades on a rotor. This is not a new idea. In Persia, over 3000 years ago, the wind was used for energy to mill (or grind) corn, hence the name 'windmill'. In Europe, windmills have also been used for centuries. In Australia, one of the most common sights in the outback is an old-style windmill pumping water from a well into a dam or tank.

Wind has kinetic energy. This enables the wind to do work turning the angled blades of the windmill, which turn the turbine. In a twenty-first century windmill, the turbine turns the generator via a series of gears creating electrical energy. Of course, wind turbines cannot take all the kinetic energy from the wind – think what would happen if the air stopped after passing through the blades! The maximum theoretical percentage of the wind's energy that can be extracted using a turbine is 59%, but modern windmills mostly manage an efficiency of 40-50%.

A wind turbine is made up of four main parts: the base, tower, generator and blades. The longer the blades, the more kinetic energy they can 'harvest' from the wind; even gentle breezes can



**Figure 5D–1** In Australia, the outback windmill lifting bore water to the surface is a common sight. This one is purely mechanical, but there are also windmills that drive a generator instead of a pump to supply DC power to remote homes and stations.

produce energy. A gearing system within the generator allows the blades to turn slowly, while the alternator rotor moves fast. A brake is necessary to control motion in extreme winds and for maintenance when the blades must be stopped. The tower is equipped with a yaw motor and sensors to point the nose into the wind as it changes direction. Inverters and transformers convert the power to high voltage for transmission by the grid interconnectors.





#### Slip ring

a form of commutator used in alternators to connect the rotating coil to the non-rotating terminals and therefore transfer the alternating current (AC) produced in the coil



#### **EXPLAIN**

Figure 5D–3 A simple

and induced emf.

#### How do alternators work?

giant pylons extending to the sea floor.

Figure 5D-2 The most common form of wind turbine, in which the kinetic energy of wind spins

an electric generator, which produces electricity. Commonly located on hilltops or near the ocean, wind turbines have also been built in the ocean ('offshore'), either floating on the surface or using

Figure 5D–3 shows a very simple, single coil AC electrical generator (or alternator), consisting of a loop of wire that can be rotated in a magnetic field. It is very similar to the design of the DC generator studied in Section 5C and shown in Figure 5C-4, except the commutator is subtly different. Now the spring-loaded carbon brushes press against two copper slip rings. These slip rings rotate with the axle and are insulated from each other. Each slip ring is separately connected to one end of the coil. The orientation of the single coil relative to the magnetic field is shown in Figure 5D-4 (middle) for a whole cycle. Figure 5D–4 (bottom) shows how both the magnetic flux,  $\Phi_{\rm B}$ , and the induced emf,  $\varepsilon$ , change as a function of time.







Figure 5D-4 Top: The front view of the coil in Figure 5D–3 showing its orientation over a whole cycle in the magnetic field. Middle and bottom: Graphs of the corresponding variation in magnetic flux,  $\Phi_{\rm B}$ , and induced emf,  $\varepsilon$ , plotted against time. Note that the induced emf is given by the negative gradient of the magnetic flux graph.

It may surprise you that the maximum emf occurs when  $\Phi_B$  is zero (e.g., after one quarter of a cycle at  $t = \frac{T}{A}$ ). However, it is at this point on the graph that  $\Phi_B$  is *changing most rapidly* (you can see this by looking at the slope or gradient on the  $\Phi_{R}$  vs time graph at this point). Recall Faraday's law and Lenz's law (Formula 5B-2):

$$\varepsilon = -N \frac{\Delta \Phi_B}{\Delta t}$$

The induced emf is the negative gradient of the graph of  $\Phi_{B}$  plotted against time (multiplied by the number of loops in the turning coil).

The induced emf produced is alternating regularly in time with the rotation of the coil. As with real life DC generators, real alternators consist of many loops or windings in each coil. Several coils make up the rotating armature.

#### Check-in questions – Set 1

1 Consider an AC generator. Label parts a-d, and describe the output at e.



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**5B** GENERATING

EMF BY VARYING THE MAGNETIC FLUX



The figure shows a simple alternator. A coil (UVWX), 0.30 m by 0.40 m, consists of 20 turns of wire. It is in a uniform magnetic field of strength 1.5 T and can rotate as shown.



- **a** With the coil oriented as in the figure, what is the magnitude of the magnetic field through each turn of the coil?
- **b** The coil is rotated at a constant rate of 50 revolutions per second in the direction shown. What is the average voltage developed across the resistor R when the coil rotates through 90 degrees from the orientation shown in the figure?

The graphs (A–D) below show possible variations of the magnetic through the coil as a function of time as it rotates. They begin at time t = 0, when the coil is oriented vertically as in the above figure.

**c** Which of the graphs best shows the variation of the magnetic flux through the coil as a function of time? Take the direction from N to S in the above figure as positive.



**d** Assuming the same conditions, which of the graphs (A–D) best shows the variation of the current flowing from U to V in the coil, as a function of time? Explain the logic of your choice.

VCAA 2003

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#### Solution

**a** Use the magnetic flux definition:

$$P_B = BA$$
  
= 1.5 × (0.30 × 0.40)  
= 0.18 Wb

**b** First, find the time taken for 90° or  $\frac{1}{4}$  of a revolution:

360° takes 
$$\frac{1}{50}$$
 s so 90° takes  $\frac{1}{200}$  s or 0.005 sec

Then, use Faraday's law (Formula 5B-2):

$$\varepsilon = -N \frac{\Delta \Phi_B}{\Delta t}$$
$$= 20 \times \frac{0.18}{0.005}$$
$$= 720 \text{ V}$$

- **c** The positive direction is from north to south, so the flux began as a positive maximum. Answer B is the only possibility.
- **d** From t = 0, the flux through the loop started as a maximum to the right (N  $\rightarrow$  S) and was then decreasing as the coil rotated.

By Lenz's law, extra flux was needed to the right  $(N \rightarrow S)$  to oppose that change.

So, using the right-hand grip rule, the current had to go from  $V \rightarrow U$ .

This is the negative direction. Also, the initial current had to be zero because the rate of change of flux at that time was zero. (The gradient of the  $\Phi$ -t graph is zero.) Therefore, the answer is D.







#### Check-in questions – Set 2

Use the following information to answer these questions.

A simple AC generator with a coil of one loop of wire is rotating in a magnetic field of strength *B*. The graph below shows the output emf plotted against time.



- 1 How long is one period of rotation of the coil?
- **2** What effect would doubling the magnetic field strength have on the output ε if the rate of rotation is kept constant?
- **3** If, instead, the rate of rotation is doubled and the magnetic field strength is kept constant, what two effects will this have on the output ε?
- **4** Make a copy of the graph above and sketch on it the combined effects described in Question 3.



#### **5D SKILLS**

#### Varying parameters for alternating current generators

The graph of induced emf plotted against time, shown in Figure 5D–4 (bottom), follows a sinusoidal shape. You may have realised that the formula for the magnetic flux as a function of time, t, is given by:

$$\phi_{B} = BA \sin \omega t$$

for a coil of area, *A*, in a uniform magnetic field, *B*, rotating at a constant angular velocity,  $\omega$ , where  $\omega = 2\pi f$  and *f* is the frequency of rotation. If the coil had *N* turns, then the emf would be given by:

$$\varepsilon = -N \frac{\Delta \phi_B}{\Delta t} = -N \frac{\Delta (BA \sin 2\pi ft)}{\Delta t}$$

Using differential calculus to take the derivative gives a useful formula:

\$

$$\epsilon = NBA \cos 2\pi ft$$

Calculations using this formula are beyond the scope of this course, but the results from this simple mathematics, as well as common sense, shows that:

- if the area of the coil doubles (A' = A  $\times$  2), the new emf  $\epsilon'$  will also be double the original  $\epsilon$
- if the magnetic field strength, *B*, is increased, the ε' will increase proportionally
- the number of loops in the coil is also directly proportional to the output emf,  $\varepsilon$ , as expected.

What may be less obvious is that if the coil is rotated twice as fast, *both* the frequency of rotation and therefore the angular velocity,  $\omega (= 2\pi f)$ , and *also* the time,  $\Delta t$ , change. The effect is to double the  $\varepsilon$  *and* halve the period of the output emf.

Note that the study design makes reference to the effects of these changes, although not to the mathematical derivation.

#### Section 5D questions

#### **Multiple-choice questions**

 In the diagram on the right, the solid line represents the graph of output emf produced by an AC generator, *ε*, plotted against time. A single change is made to the AC generator and its operation, and the new graph of output emf, *ε*, plotted against time is shown as a dashed line.



Which one of the following best describes the change made to the AC generator?

- A The area of the coil was doubled.
- **B** The speed of rotation was halved.
- **C** The speed of rotation was doubled.
- **D** The number of turns of the wire in the coil was doubled.

VCAA NHT 2021

VIDEO 5D-1 SKILLS: VARYING PARAMETERS FOR ALTERNATING CURRENT GENERATORS

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#### **Short-answer questions**

- 2 An AC generator with 100 turns of wire makes half a turn (180°) every 20 ms. The area of the coil is  $2.0 \times 10^{-3}$  m<sup>2</sup> and the permanent magnets produce a magnetic field of 2.0 mT. What is the rms output,  $\varepsilon$ , of the dynamo as it turns through 180°?
- 3 A slip ring generator is turned twice as quickly as before in the same magnetic field. What effect would this have on the output emf if all other parameters remain the same?
- 4 The magnetic field surrounding a model generator is tripled by adding extra magnets. What effect would this have on the output, ε?
- **5** a Explain the principles of an AC dynamo (generator) using a simplified diagram and labels.
  - **b** Discuss the main difference in construction between your answer to part a and a DC generator.



## **Chapter 5 review**

#### Summary

Create your own set of summary notes for this chapter on paper or in a digital document. A model summary is provided in the Teacher Resources, which can be used to compare with yours.

#### Checklist

In the Interactive Textbook, the success criteria are linked from the review questions and will be automatically ticked when answers are correct. Alternatively, print or photocopy this page and tick the boxes when you have answered the corresponding questions correctly.

Succe	ess criteria – I am now able to:	Linked questions
5A.1	Describe the way DC potential difference is produced in a photovoltaic cell (but not details of semiconductor action, pn junctions etc.)	11
5A.2	Discuss the energy transformations that occur when a photovoltaic cell converts sunlight to electricity	11
5A.3	Recognise the need for, and describe the basic function of, an inverter to convert DC to AC for household use (not details of wiring etc.)	11
5A.4	Analyse the connections of photovoltaic panels in series and parallel in terms of potential difference, current and power production	12 , 13
5B.1	Recognise a variety of situations where changes in magnetic fields and/or motion of conductors result in induced potential difference (called emf, electromotive force, $\varepsilon$ ), i.e. where electromagnetic induction occurs	14, 16, 18, 21
5B.2	Recall that magnetic flux is the strength of a magnetic field for a given area, with symbol $\Phi B$ and units of weber (Wb). Calculate its value when the field is perpendicular to the area using the formula $\Phi_B = B_{\perp}A$ and qualitatively describe the effect of varying the angle between <i>B</i> and <i>A</i>	1, 15, 19, 20
5B.3	Explain electromagnetic induction in terms of <i>changing</i> the magnetic flux, $\Delta \Phi_B$ , near a conductor to generate a potential difference (an emf, $\varepsilon$ )	18
5B.4	Use Faraday's law of electromagnetic induction, $\varepsilon = -N \frac{\Delta \Phi_B}{\Delta t}$ , in a variety of situations to analyse the effect on emf induced when the rate of change of magnetic flux and/or the number of loops through which the flux passes are varied	70,140,180,190
5B.5	Recognise and explain the effect of the negative sign in Faraday's law (Lenz's law) in terms of conservation of energy. Use Lenz's law to predict the direction of the induced emf (and hence induced current) when magnetic flux is changed (using the right-hand rule where necessary); explain that the magnetic field of the induced current is in the opposite direction to the <i>change in magnetic flux</i> that produced it (Lenz's law)	14 , 18

Succe	ess criteria – I am now able to:	Linked questions
5B.6	Explain that an induced emf will produce a current if there is a complete circuit with some parts external to the field, and predict the direction of that current. Use Ohm's law and other circuit theory to perform calculations to determine the induced current associated with an induced emf when there is a complete circuit external to the magnetic field	14 , 16 , 18
5B.7	Predict the shape and values of various $\varepsilon$ vs time graphs, given a graph of $\Phi_B$ vs time (using the negative gradient) and vice versa	5
5C.1	Recognise a DC generator as a loop of wire connected to a split ring commutator which, when rotated uniformly in a constant magnetic field, produces a sinusoidal DC voltage (emf, $\varepsilon$ ) given by Faraday's law and Lenz's law: $\varepsilon = -N \frac{\Delta \Phi_B}{\Delta t}$	10,60,80,110
5C.2	Predict and explain the sinusoidal shape of the $\Phi_B$ vs time graph and the $\varepsilon$ vs time graph produced from a DC generator, including the function of the split ring commutator	3□, 4□, 6□, 8□, 11□, 17□, 19□, 20□
5C.3	Calculate the emf, $\varepsilon$ , induced in a rotating coil using Faraday's law by considering the change in magnetic flux, $\Delta \Phi_{B}$ , when the coil rotates through 90°, 180° or other simple angles	2□,4□
5C.4	Determine the direction of both the induced $\varepsilon$ and <i>I</i> in a coil rotating in a magnetic field using Lenz's law	10 , 19 , 20
5C.5	Analyse the effect on emf of varying the rate of rotation, the number of loops, and/or the direction of rotation of a DC generator coil; show this by calculation and/or graphically	17
5C.6	Use Ohm's law and other circuit theory to perform calculations to determine the induced current associated with an induced emf from a coil rotating in a magnetic field	10 , 19 , 20
5D.1	Recognise an AC generator as a loop of wire connected to slip rings, which, when rotated uniformly in a constant magnetic field produces a sinusoidal AC voltage (emf, $\varepsilon$ ), given by Faraday's law and Lenz's law: $\varepsilon = -N \frac{\Delta \Phi_B}{\Delta t}$	10 , 19 , 20
5D.2	Predict and explain the sinusoidal shape of the $\Phi_B$ vs time graph and the $\varepsilon$ vs time graph produced from an AC generator, including the function of the slip rings	4□, 10□
5D.3	Calculate the emf, $\varepsilon$ , induced in a rotating coil using Faraday's law by considering the change in magnetic flux, $\Delta \Phi_{B}$ , when the coil rotates through 90°, 180° or similar angles	19□, 20□
5D.4	Analyse the effect on emf of varying the rate of rotation, the number of loops, and/or the direction of rotation of an AC generator coil; show this by calculation and/or graphically	90, 19
5D.5	Use Ohm's law and other circuit theory to perform calculations to determine the induced current associated with an induced emf from an AC generator	17

Succe	Linked questions	
5D.6	Recognise the set-up of an AC generator as 'a motor run	11
	backwards', i.e., an AC motor where mechanical energy is	
	transformed to produce electrical energy; recognise a DC	
	generator as 'a DC motor run backwards'	
5D.7	Compare the structure and function of AC generators (with	8, 19
	slip rings) to DC generators (with split ring commutator)	· · · · · · · · · · · · · · · · · · ·

#### **Multiple-choice questions**

Use the following information to answer Questions 1, 2, 3 and 4.

The diagram below shows a simple DC generator. A coil (UVWX) 0.30 m by 0.40 m, consists of 20 turns of wire. It is in a uniform magnetic field of strength 0.25 T and can rotate as shown.



- 1 With the coil oriented as in the above diagram, what is the magnitude of the magnetic flux through each turn of the coil?
  - **A** 0.03 Wb
  - **B** 0.12 Wb
  - **C** 0.25 Wb
  - **D** 0.48 Wb

VCAA 2003

- **2** What is the induced emf, *ε*, developed across the resistor R when the coil in the above diagram takes 0.2 s to rotate through 90 degrees from the orientation shown in the figure?
  - **A** 0.60 V
  - **B** 0.15 V
  - **C** 3.0 V
  - **D** 6.0 V

VCAA 2003

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#### The following information relates to Questions 3 and 4.

Below are graphs of possible variations of the magnetic flux through the coil as a function of time as it rotates. They all begin at time t = 0, when the coil is oriented as in the diagram on the previous page.



- **3** Which of the above graphs best shows the variation of the magnetic flux through the coil as a function of time? Take the direction from N to S in the figure as positive.
  - **A** A
  - **B** B
  - **C** C
  - **D** D

VCAA 2003

- **4** Assuming the same conditions as for Question 3, which of the above graphs best shows the variation of the current flowing from U to V in the coil, as a function of time?
  - **A** A
  - **B** B
  - **C** C
  - DD

VCAA 2003

**5** The graph below shows the change in magnetic flux, *Φ*, through a coil of wire as a function of time, *t*.



Which one of the following graphs best represents the induced emf,  $\varepsilon$ , across the coil of wire as a function of time, t?



VCAA 2017

**6** A simple DC generator consits of two magnets that produce a uniform magnetic field, in which a square loop of wire of 100 turns rotates at constant speed, and a commutator, as shown in the diagram below.



Which one of the following best shows the display observed on the oscilloscope?



7 A magnet approaches a coil with six turns, as shown in the diagram below. During time interval  $\Delta t$ , the magnetic flux changes by 0.05 Wb and the average induced emf is 1.2 V.



Which one of the following is closest to the time interval  $\Delta t$ ?

- **A** 0.04 s
- **B** 0.01 s
- **C** 0.25 s
- **D** 0.50 s

VCAA 2021

8 The diagram below shows a simple electrical generator consisting of a rotating wire loop in a magnetic field, connected to an external circuit with a light globe, a split ring commutator and brushes. The direction of rotation is shown by the arrow.



Which one of the following best describes the function of the split ring commutator in the external circuit?

- **A** It delivers a DC current to the light globe.
- **B** It delivers an AC current to the light globe.
- **C** It ensures the force on the side of the loop nearest the north pole is always up.
- **D** It ensures the force on the side of the loop nearest the north pole is a always down.

VCAA 2021

**9** The coil of an AC generator completes 50 revolutions per second. A graph of output voltage plotted against time for this generator is shown below.







VCAA 2019

**10** An electrical generator is shown in the diagram below. The generator is turning clockwise.



The voltage between P and Q and the magnetic flux through the loop are both graphed as a function of time, with voltage vs time shown as a solid line and magnetic flux vs time shown as a dashed line. Which one of the following graphs best shows the relationships for this electrical generator?



#### Short-answer questions

11	A of	A solar installer asks their client whether they prefer a single inverter for the whole string of pv panels or a microinverter for each panel.	
	а	What is the function of an inverter and why is it necessary?	(1 mark)
	b	Explain one advantage and one disadvantage of each type of inverter.	(2 marks)
12 A caravan owner runs a small DC refrigerator from a 12 V battery and wants to in solar panels on the roof of the van to recharge the battery that runs the refrigerato		11	
	а	If the refrigerator uses 1.2 A and runs 24 hours per day, how many 300 W solar pa be needed? State any assumptions you make and show your calculations clearly.	nels will (2 marks)
	b	They decide to add a microwave to the caravan, which will draw 93 A of current for approximately 10 minutes per day. Will extra pv panels be needed?	or (2 marks)
	С	A 12 V DC coffee pod machine draws up to 90 A for five minutes per day. Will ext pv panels be needed?	tra (2 marks)
13	а	Draw a circuit diagram to show how to connect 4 pv solar panels together to prod maximum current.	uce (2 marks)
	b	Draw a circuit diagram to show how to connect 4 pv solar panels together to prod maximum potential difference.	uce (2 marks)
14	а	Draw a diagram showing how to generate an induced emf by pulling a conductor a magnetic field. Label the end of the conductor that would become positive.	through (2 marks)
	b	Draw a diagram showing how to generate an induced current by pulling a conduct through a magnetic field,. Indicate the direction of the current.	tor (2 marks)
	С	What would happen to the current if you:	
		i pushed the conductor the other way	(1 mark)
		ii used a stronger magnet to create the magnetic field	(1 mark)
		iii pulled or pushed the conducting wire much faster through the field?	(1 mark)
	d	Why does creating a current in this manner produce a force that opposes the force	e
		producing it?	(1 mark)
15	Ex sy:	xplain carefully the difference between magnetic field and magnetic flux. What is the mbol and SI unit for each?	ne (2 marks)
16	а	What is meant by electromagnetic induction? Describe one important application of it.	ı (2 marks)
	b	Does electromagnetic induction always produce an induced current?	(1 mark)
17	а	Sketch a graph showing the output current plotted against time from an eight coil	AC
		generator like the one shown in Figure 5D–3.	(3 marks)
	b	Why is this output current almost as good as direct current from a battery?	(1 mark)

**18** A horizontal square conducting metal loop of one turn is placed in a uniform, steady magnetic field between two poles of an electromagnet, as shown below. The plane of the loop is perpendicular to the magnetic field.



 a Students discuss different methods of causing a current to flow in the loop: Ahmed says to move the loop directly upwards in the field towards the N pole. Beth suggests move the loop sideways (to the left) but keep it completely inside the field. Charlie wants to try moving the loop sideways (to the right) so that it moves out of the magnetic field.

Dot says that rotating the loop about a horizontal axis will cause a current.

- i Choose one or more of these suggestions that would cause a current to flow. (1 mark)
- ii Explain whose ideas would work best and why. (3 marks)
- b The uniform field of the magnet between its poles is initially equal to 0.050 T. The current in the electromagnet is then adjusted so that the field reduces to zero in 10 ms. An average current of 0.020 A flows in the loop. The area of the loop is 0.080 m<sup>2</sup>. Calculate the resistance of the loop, in Ω. Show your working. (2 marks)
- c The magnetic field is reduced to zero.
   Make a copy of the diagram of the loop viewed from above. Indicate on it the direction of the resulting induced current in the loop. Explain your reasoning. (3 marks)



**19** Students in a Physics practical class investigate the piece of electrical equipment shown below. It consists of a single rectangular loop of wire that can be rotated within a uniform magnetic field. The loop has dimensions  $0.50 \text{ m} \times 0.25 \text{ m}$  and is connected to the output terminals with slip rings. The loop is in a uniform magnetic field of strength 0.40 T.



- a Which of the following best describes the piece of electrical equipment shown in the above diagram? (1 mark)
  - **A** alternator
  - **B** DC generator
  - **C** DC motor
  - **D** AC motor
- b i What is the magnitude of the flux through the loop when it is in the position shown in the diagram? (1 mark)
  - ii Explain your answer to part i.

The students connect the output terminals of the piece of electrical equipment to an oscilloscope. One student rotates the loop at a constant rate of 20 revolutions per second, while the others observe the oscilloscope.

- **c** Calculate the period of rotation of the loop.
- **d** Calculate the maximum flux through the loop. Show your working. (2 marks)
- **e** The loop starts in the position shown in the diagram above. What is the average emf measured across the output terminals for the first quarter turn? Show your working. (2 marks)
- f State two ways that the amplitude of the voltage across the output terminals can be increased.(2 marks)
- **g** The graph below shows the output voltage graph shown on the oscilloscope for two cycles.

The students now replace the slip rings in the top diagram with a split ring commutator. On a copy of the graph, sketch with a dashed line the output that the students will now observe on the oscilloscope. Show two complete revolutions.



(1 mark)

(1 mark)

**20** The alternator in the figure below has a rectangular coil with sides of  $0.30 \text{ m} \times 0.40 \text{ m}$  and 10 turns. The coil rotates four times a second in a uniform magnetic field. The magnetic flux through the coil in the position shown is 0.20 Wb.



- **a** Calculate the magnitude of the magnetic field. Include an appropriate unit. (2 marks)
- b Calculate the magnitude of the average emf, *ε*, generated in a quarter of a turn. Show all the steps of your working. (3 marks)
- **c** The figure below shows a graph of the output emf, *ε*, plotted against time of the alternator for two complete cycles.



The two slip rings in the figure are now replaced with a split ring commutator. On a copy of the axes provided below, plot the emf,  $\varepsilon$ , against time graph of this new arrangement for two complete cycles. (3 marks)



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(2 marks)

**21** Comment on the statement that 'electricity is a very efficient and clean source of energy'.

#### HOW DO FIELDS EXPLAIN MOTION AND ELECTRICITY?

# CHAPTER 6

UNIT

## TRANSFORMERS AND TRANSMISSION OF ELECTRICITY

#### Introduction

Availability of cheap electricity has totally changed our lifestyles over the past 100 years. It is becoming even more important as we transition to a decarbonised economy over the next decade or two. The aim of this course is to examine the production of electricity (see Chapter 5) and its transmission over large distances, to homes and businesses. Models of electromagnetic effects and some empirical evidence are considered.

You may have noticed that mobile phones and computers become warm when electricity flows in circuits. Some of the electrical energy is transformed into thermal energy and may also cause vibration when it is transformed to kinetic energy, and humming, when this is transformed to sound energy. This loss of electrical power and energy by conversion to thermal energy, sound energy and kinetic energy is a major issue in the distribution of electricity. The solution is to transmit electricity at high voltage and low current. This means that the voltage has to be stepped up before being sent over powerlines and then stepped down before entering houses, to prevent overloading of machines and appliances, and to avoid electrical shock and the need for thick insulation. The technical solution to stepping up and stepping down voltage is the transformer, a critical device for minimising electrical power loss.

The use of electromagnetic induction in transformers is a major reason why our electric power grid is AC, not DC. From Chapter 5, you know that alternating current is often produced by rotating conductors in a magnetic field and that a split ring commutator can be used to generate direct current from a rotating coil. AC has some advantages over DC for transmission in most but not all circumstances, as discussed later in this chapter.

#### Curriculum

Area of Study 3 Outcome 3 How are fields used in electricity generation?

Study Design	Learning intentions – at the end of this chapter I will be able to:		
<ul> <li>Fransmission of electricity</li> <li>Compare sinusoidal AC voltages produced as a result of the uniform rotation of a loop in a constant magnetic field with reference to frequency, period, amplitude, peak-to-peak voltage (V<sub>p-p</sub>) and peak-to-peak current (I<sub>p-p</sub>)</li> <li>Compare alternating voltage expressed as the root-mean-square (rms) to a constant DC voltage developing the same power in a resistive component</li> </ul>	<ul> <li>6A Peak and rms values of sinusoidal AC voltages</li> <li>6A.1 Explain peak voltage, V<sub>p</sub>, and peak-to-peak voltage, V<sub>p-p</sub>, including on graphs of induced emf plotted against time and through calculation using V<sub>rms</sub> = V<sub>p</sub>/√2</li> <li>6A.2 Explain peak current, I<sub>p</sub>, and peak-to-peak current, I<sub>p-p</sub>, including on graphs of induced current plotted against time and through calculation using I<sub>rms</sub> = (I<sub>p</sub>/√2)</li> <li>6A.3 Recognise that a given AC voltage expressed as V<sub>rms</sub> produces the same power in a resistive load as a constant DC voltage with the same value; calculate power using P<sub>rms</sub> = V<sub>rms</sub>I<sub>rms</sub> = (V<sub>p</sub>/p)/2</li> </ul>		
• Analyse transformer action with reference to electromagnetic induction for an ideal transformer: $\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$	<ul> <li>6B Transformers: electromagnetic induction at work</li> <li>6B.1 Recognise transformers as a method to transfer electrical power from one circuit to a separate circuit via a varying magnetic flux, causing an induced emf, <i>ε</i></li> <li>6B.2 Explain the functions of a transformer in terms of electromagnetic induction, including the roles of primary and secondary coils and the soft iron core; identify and explain whether a transformer steps-up or steps-down the voltage</li> <li>6B.3 Recall that in <i>ideal</i> transformers (only), power input to the primary coil is equal to power output from the secondary coil, <i>P</i><sub>in</sub> = <i>P</i><sub>out</sub>. Discuss why real-life transformers are <i>not</i> 'ideal'</li> <li>6B.4 Use the formula <i>N</i><sub>1</sub>/<i>N</i><sub>2</sub> = <i>V</i><sub>1</sub>/<i>V</i><sub>2</sub> = <i>I</i><sub>2</sub>/<i>I</i><sub>1</sub> (known as the turns ratio) to perform calculations relating to step-up and step-down transformers, input and output currents and potential differences</li> <li>6B.5 Perform calculations using the turns ratio, Ohm's law, power formulas and other circuit theory to determine the currents, potential differences and power associated with the primary and secondary circuits of a transformer</li> </ul>		

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#### **Study Design**

• Analyse the supply of power by considering transmission losses across transmission lines

### Learning intentions – at the end of this chapter I will be able to:

6C	Minimising power transmission losses
6C.1	Calculate electrical power and electrical energy
	transformed to heat in a wire using $P = l^2 R$ ,
	and hence $E = I^2 R t$
6C.2	Analyse power losses in an AC electrical supply
	system (with and without transformers) using
	$P_{\text{loss in wires}} = l^2 R$
6C.3	Perform calculations using Ohm's law and
	other circuit theory to determine the currents,
	potential differences and power associated with a
	transmission of electricity over longer distances,
	including models of such systems
6C.4	Explain, using supporting calculations, the most
	efficient methods for supplying electricity to a
	load that is distant from the generator; include
	energy losses, the advantages of high-voltage over
	low-voltage transmission, and the advantages of
	AC rather than DC transmission
6C.5	Analyse and evaluate an electricity distribution
	system; design an efficient transfer model for
	supplying electricity to a load that is distant

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#### Glossary

Electromagnetic induction Peak-to-peak value Peak value Root-mean-square (rms) Step-down transformer Step-up transformer Transformer Transmission line Transmission loss

from the generator

#### Concept map

Sinusoidal AC voltage and current is produced by rotation of a loop in a magnetic field

Compare calculated values for peak, peak-to-peak and rms AC voltage and current to DC

6A Peak and rms values of sinusoidal AC voltages



Transformers transfer electrical power to a separate circuit via a varying magnetic flux, inducing an EMF

**6B Transformers: electromagnetic induction at work** 



*Efficient methods for supplying electricity to distant loads; compare high-voltage and low-voltage transmission; compare AC to DC* 

#### 6C Minimising power transmission losses



See the Interactive Textbook for an interactive version of this concept map interlinked with all concept maps for the course.



## Peak and rms values of sinusoidal AC voltages

#### **Study Design:**

- Compare sinusoidal AC voltages produced as a result of the uniform rotation of a loop in a constant magnetic field with reference to frequency, period, amplitude, peak-to-peak voltage (V<sub>p-p</sub>) and peak-to-peak current (I<sub>p-p</sub>)
- Compare alternating voltage expressed as the root-mean-square (rms) to a constant DC voltage developing the same power in a resistive component

#### **Glossary:**

Peak-to-peak value Peak value Root-mean-square (rms)

### Q<sup>O</sup>

**ENGAGE** 

#### Single-phase and three-phase AC supplies

A knowledge or understanding of three-phase power does not form part of this course, but you may have come across references to it. The type of alternating current discussed in Chapter 5, and to be developed further in this chapter, is single-phase AC, shown in the graph below left: there is one sine waveform and one active wire connects electrical devices and appliances to the mains. However, there are other possible AC supply systems using two or more phases. The most common such system is three-phase AC. Three-phase AC uses three active wires to deliver three AC voltages that are out of step with each other, as shown graphically in Figure 6A–1 (right).



**Figure 6A–1** Left: Voltage–time graph for one cycle of single-phase AC. Right: Voltage–time graph for one cycle of three-phase AC. The time axis can also be considered in terms of phase angle, with 360° corresponding to one period, *T*. The phases are 120° out of phase.



As you learned in Chapter 5, the sine waveform relates to a rotating coil. One complete rotation (360°) is one cycle or wavelength of the sine wave and so the angle position of the coil at any given time can be marked on the graph as shown. The phases are 120° apart; this is called the phase angle.

You can see from the graph that three-phase power delivers the maximum voltage for more of the time than single-phase. Greater power can be delivered because each phase supplies energy, not just one. This gives three-phase AC supply an advantage over single-phase AC, especially for high-power applications, such as driving heavy-duty electric motors, large-scale heating and cooling systems, stage lighting and a variety of energy-hungry machines used in businesses and industries. If you ever buy an industrial or high-power version of a machine for home use, it may require installation of a three-phase AC supply that you don't yet have connected.

#### **EXPLAIN**

#### Peak and rms values for voltage

Consider a single-phase sinusoidal 230 V 50 Hz AC (like that delivered by the mains to your home). The graph of voltage plotted against time is shown in Figure 6A–2. Note the period,

*T*, is 20 ms, as we would expect for AC with frequency 50 Hz (from  $f = \frac{1}{\pi}$ ).





**Figure 6A–2** Emf–time graph for two cycles of an alternating 230 V AC 50 Hz mains supply. The actual peak voltage swings between +325 V and –325 V. The value of 230 V only represents the root-mean-square (rms) value.

**Figure 6A–3** Root-mean-square value of emf–time graph for an alternating 230 V AC 50 Hz mains supply. This is where the value of 230 V for main AC comes from. Note that the graph axes are the same for AC as in Figure 6A–2.

Contrary to what you might expect, 240 V is not the maximum voltage that comes into your home; it is only the effective value. The actual mains electricity delivered to your home reaches a **peak value** of  $230 \times \sqrt{2}$ , meaning that it oscillates from +325 V to -325 V, or a change of almost 650 V every half-cycle.

This peak value is not a true indication of the effectiveness of the voltage in being able to do work (for example, run lights, heaters or motors), as it is only maintained for an instant. When using an AC power supply, we need a method to sensibly calculate the effective power available. A straight average is not useful – the mathematical average of the AC power supply shown in Figure 6A-2 is zero, because the negative values cancel out the positive. Instead, we use a mathematical averaging technique that gives the same effective power dissipation as an equivalent DC voltage. The effective value of AC voltage is known as the **root-mean-square** or **rms** value. This is shown in Figure 6A-3.

#### Peak value

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VIDEO 6A-1 Peak and RMS

VALUES

the maximum value reached in one cycle of an alternating variable, such as AC voltage or current; the amplitude of the sine wave

#### Root-meansquare (rms)

the effective mean of a sinusoidal variable, such as AC voltage, current or power. The rms value gives the same effect as a constant DC voltage or current in a resistive load.

#### Formula 6A–1 AC voltage

Where:

 $V_{\rm rms}$  = Root-mean-square voltage (V)

 $I_{\rm rms}$  = Root-mean-square current (A)

 $V_{\rm p}$  = Peak voltage (V)

When referring to AC voltages, the rms value is usually quoted. For a fixed resistive load, there is a similar relationship for the peak current and the rms current, expressed in Formula 6A–2.

 $V_{\rm rms} = \frac{V_{\rm p}}{\sqrt{2}}$ 



 $I_{\rm p}$  = Peak current (A)

Where:

$$I_{\rm rms} = \frac{I_{\rm p}}{\sqrt{2}}$$

Peak-to-peak value

a measurement of the value of a sinusoidal variable (such as AC voltage, current or power) from the top of one peak to the bottom of the next trough, i.e. double the amplitude Occasionally, the **peak-to-peak** voltage or current is useful. For example, when reading off an oscilloscope screen it is more accurate to measure from the top of one peak to the bottom of the next, rather than trying to centre the trace and measure the peak. Clearly,  $V_{p-p} = 2 \times V_p$ .



**Figure 6A–4** An oscilloscope connected to an alternator displays voltage on a screen like a graph. The peak-to-peak voltage  $V_{p-p}$  is indicated.

The effective power in an AC circuit with only resistive components is given by:

$$P_{\text{effective}} = V_{\text{rms}} I_{\text{rms}}$$

So substituting for  $V_{\rm rms}$  and  $I_{\rm rms}$  gives:

$$P_{\text{effective}} = \frac{V_{\text{p}}}{\sqrt{2}} \times \frac{I_{\text{p}}}{\sqrt{2}}$$
$$= \frac{V_{\text{p}}I_{\text{p}}}{2}$$

The effective power in the AC circuit is half of the peak power, given by Formula 6A–3.

## Formula 6A–3 Effective power in an AC circuit $P_{effective} = V_{rms}I_{rms} = \frac{1}{2}V_{p}I_{p}$ Where: $P_{effective} = Effective power in an AC circuit (W)$ $V_{rms} = \text{Root-mean-square voltage (V)}$ $I_{rms} = \text{Root-mean-square current (A)}$ $V_{p} = \text{Peak voltage (V)}$ $I_{p} = \text{Peak current (A)}$







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#### Worked example 6A–1 Calculations for an AC circuit

Students test the output of a power supply by connecting it to an oscilloscope (CRO), shown in Figure 6A–5.

The power supply is labelled 50 Hz 20  $V_{\rm rms}$ .

- **a** Sketch the trace they will see on the oscilloscope and label  $V_{\rm p}$  and  $V_{\rm p-p}$ .
- **b** With a load resistor of 100  $\Omega$  in series with the power supply, what is the peak current in the resistor?
- **c** What effective (rms) power would be drawn from the power supply while connected to the 100  $\Omega$  resistor?

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#### Solution

**a**  $V_{\rm p} = V_{\rm rms} \times \sqrt{2} = 28.3$  V. This will be the peak of the sinusoidal trace.



#### Check-in questions – Set 1

- 1 Explain why the mathematical average of AC power is not the effective power delivered.
- 2 What does rms stand for and how is it calculated?
- 3 Calculate the effective AC voltage,  $V_{\rm rms}$ , obtained from an AC generator, with  $V_{\rm p} = 339$  V.
- 4 Calculate the effective AC current,  $I_{\rm rms}$ , when the peak current,  $I_{\rm p}$ , is 5.0 A.
- 5 a Calculate the effective power used by a reverse cycle air conditioner labelled 240 V 10 A.
  - **b** What would be the peak power used by this appliance?
  - c What are the implications of this peak power value for the construction of the circuit?

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WORKSHEET 6A-1

PEAK AND RMS VALUES

UNIT 1

DOC

LINK

#### ACTIVITY 6A-1 SAFETY IMPLICATIONS OF RMS AND PEAK CURRENT

Recall from Unit 1 that fuses can be constructed in several different ways:

- wires that burn out quickly (normal fuses)
- wires that burn out slowly (slow 'blow' fuses)
- circuit breakers.

Research these types and answer the following questions.

- 1 What is the use of each of these safety devices?
- **2** Describe how each type works and discuss their relative advantages and disadvantages.

Use the concepts of peak and rms values in your discussion.

#### 6A SKILLS

#### Patterns of peak and rms values

It is easy to get confused between the versions of voltage, current and power in this section, but they all follow the same pattern. Using a an emf-time graph for one or more cycles of alternating current like the one below may help.



It is clear from the diagram that the:

- peak-to-peak value is 2 × peak value
- rms value must always be less than the peak value, because the peak value is divided by  $\sqrt{2}$  (= 1.414); dividing by 1.414 will always give a smaller number.

Remember the connection between frequency and period, which for AC in Australia is

$$\frac{1}{50 \text{ Hz}} = 20 \text{ ms.}$$

You could add the effects of changing parameters on the output from the AC generators considered in Chapter 5 to this diagram, as a memory aid or for your exam formula sheet – for example, the effect of changing magnetic field strength or frequency of rotation.





#### **Section 6A questions**

#### **Multiple-choice questions**

- 1 The mains voltage in a particular part of Australia is AC with a voltage of 240  $V_{rms}$ . Which one of the following is closest to the peak-to-peak voltage,  $V_{p-p}$ , for this mains voltage?
  - A 170 V
  - **B** 340 V
  - **C** 480 V
  - **D** 680 V

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#### **Short-answer questions**

- **2** A coil of 200 turns of wire, each of effective area (that is, the area cut by the magnetic field) of 2.0 m<sup>2</sup>, is rotated through a magnetic field of 0.5 T. It takes 3.3 s for one complete rotation.
  - a Calculate the rms emf produced.
  - **b** Calculate the peak emf produced.
  - **c** If the external circuit load is 10  $\Omega$ , calculate the maximum (peak) current.
- 3 Sketch a graph of the alternating current produced by an alternator as a function of time. Label the peak voltage, peak-to-peak voltage, average voltage, rms voltage, and period. How would you calculate the frequency from this graph?
- **4 a** What is the voltage and frequency of the AC supplied to your home?
  - **b** Draw a graph of the voltage as a function of time. Show the  $\varepsilon_{\rm rms}$  and  $\varepsilon_{\rm p}$  values.
- **5** An electrical engineer is checking the electrical input to a factory, using an oscilloscope (CRO) to show the variation of voltage with time. It produces the following signal.



- **a** Determine the frequency of the AC (alternating current) observed. Show your working.
- **b** Determine the peak-to-peak voltage,  $V_{p-p}$ .
- **c** Determine the peak input voltage,  $V_{\rm p}$ .
- d Calculate the rms input voltage,  $V_{\rm rms}$ .
- **e** If the machinery in the factory draws 100  $A_{rms}$ , calculate the effective power,  $P_{effective}$ , supplied to the factory.
- f Calculate the peak power,  $P_{\rm p}$ .

VCAA 2015



## Transformers: electromagnetic induction at work

#### **Study Design:**

• Analyse transformer action with reference to electromagnetic induction for an ideal  $N_1 = V_1 = I_2$ 

transformer: 
$$\frac{1}{N_2} = \frac{1}{V_2} = \frac{1}{I_1}$$

#### **Glossary:**

Electromagnetic induction Transformer Step-down transformer Step-up transformer 307



#### ENGAGE Regenerative braking

# Ever since the first Toyota Prius arrived in the late 1990s, the concept of regenerative braking has become well known as a method of increasing range in hybrid and electric vehicles (EVs). Regenerative braking is a system that recharges your battery by braking. But did you know that regenerative braking isn't limited to electric cars? These days, you can find it in everything from electric bicycles and skateboards to electric scooters.



**Figure 6B–1** An electric motor drives the wheels when supplied with electrical power from the battery pack but to slow the car down it reverses operation and becomes an electric generator, putting electrical energy back into the battery.

Moving vehicles have a lot of kinetic energy. When brakes are solely friction-based, that kinetic energy is transformed to thermal energy in the brake pads. However, regenerative braking uses the motor in an electric vehicle as a generator, converting much of the kinetic energy lost when decelerating back into stored energy in the vehicle's battery.

The electric motor in a hybrid or electric car runs in two directions – one to drive the wheels and move the car, and the other to recharge the battery. When you lift your foot off the accelerator pedal and onto the brake, the motor swaps directions and starts to put energy back into the battery. You can feel the car start to slow down as soon as you take your foot off the accelerator. This will feel different in different models of electric vehicle because manufacturers program how much regenerative braking occurs when you lift your foot off the pedal. (All cars still have normal brakes, so if you push the pedal hard enough, the hydraulic system will kick in to stop the car quickly, depending on your speed.)

Some electric vehicles even have an automatic cruise control system that uses brake regeneration. The car in front is monitored by sensors and the brake regeneration is used to match that car's speed on the road. The next time it accelerates, the electric motor uses much of the energy previously stored from regenerative braking, instead of tapping into its own battery reserves.

Regenerative braking doesn't make electric vehicles more efficient, because some energy is transformed to thermal energy in each braking and regenerating event, but it does make vehicles less inefficient. The most efficient way to drive any vehicle would be to accelerate to a constant speed and never touch the brake pedal. Because braking removes kinetic energy and requires extra energy input to get back up to speed, getting the best range means simply never slowing down in the first place; not very practical in real-life driving. Regenerative braking takes the inefficiency of braking and makes the process less wasteful. Some drivers have reported recapturing as much as 32% of their total energy use while driving down and then back uphill. This could increase the range of the vehicle by a third. Others have reported recapturing between 15-20% of their total kW h during normal trips, a great everyday use of electromagnetic induction.



### **EXPLAIN**

#### **Transformers**

In Chapter 5, we learned how electricity could be generated by oscillating or turning permanent or electromagnets inside coils of wire, or by turning coils of wire inside magnetic fields. The operation of these generators follows Faraday's law of electromagnetic induction (Formula 5B-2):

$$\varepsilon = -N \frac{\Delta \boldsymbol{\varphi}_{B}}{\Delta t}$$

Magnetic flux,  $\Phi_{R}$ , is changed by the physical movement of the coil relative to magnetic field, through an area bounded by wire. But is it possible to induce an emf with no moving parts? From the formula for Faraday's law, all that is needed is to set up a changing magnetic flux.

Consider two coils placed side by side as shown in Figure 6B–2.



Figure 6B-2 A changing current in the primary coil will create a changing magnetic flux, B, through coil and therefore induce an emf in the secondary coil. This induction of emf is the basis of all transformers.

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Eddy de Jong et al

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Suppose a current passes through the primary coil to produce a magnetic field *B*. The magnetic flux produced also passes through the secondary coil. If the current through the primary coil changes, the flux will also change. This changing flux links through the secondary coil and will induce an emf in it. The process of inducing an emf in one circuit by a change of a current in another is called **electromagnetic induction**. The device that uses electromagnetic induction to vary voltage is called a **transformer**.

#### Structure of a transformer

When both coils are wound on a 'soft' iron core, the emf induced is enhanced. The presence of iron increases the overall magnetic flux linking the coils. If the primary and secondary coils are wound onto two sides of a continuous soft iron core, as shown in Figure 6B–3, the magnetic flux is even stronger. The circuit symbol for a transformer with an iron core is shown in the bottom right of this diagram.

#### NOTE

In a 'soft iron core', 'soft' means iron that has been heat-treated to make it more ductile, and has not had carbon added as is done in steel-making to make it harder and stronger. Soft iron has desirable magnetic properties for transformers, including being easily magnetised and demagnetised, and more permeable to the magnetic flux. This means it is very good at carrying the magnetic field between the coils. There are other considerations in the most efficient design of soft iron cores for transformers to minimise energy losses, but these are not included in this course.



**Figure 6B–3** Structure of a transformer. By winding the primary and secondary windings on to a common soft iron core, the magnetic flux between them, and therefore the efficiency of the transformer, is enhanced. This is a step-down transformer, as the number of turns on the primary coil is greater than on the secondary. Inset at bottom left is the turns ratio. Inset at bottom right is the circuit symbol for a transformer with an iron core. It could also be used if both coils are wound on a single iron bar.

An alternating current (Figure 6A–2 in Section 6A) changes magnitude and direction sinusoidally. So, the effect of applying *alternating* current to the primary coil is to induce an *alternating* emf of the same frequency in the secondary coil.



#### Electromagnetic induction

the process of generating an electric current with a changing magnetic field near a wire, or by moving a metal wire in a steady magnetic field

#### Transformer

a device that changes voltage through electromagnetic induction in an alternating current

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6A PEAK AND RMS VALUES OF SINUSOIDAL AC VOLTAGES

#### The turns ratio

But what about the relative sizes of the voltages in the primary and secondary coils? This can be determined (in an ideal case) by varying the number of turns on the primary core compared to the secondary. That is, by using the turns ratio, given in Formula 6B–1.

-	CD 1		
Formula	6B-1	lurns	ratio
i onnaia			Tutio

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Where:

 $V_1$  = Primary voltage (V)

 $V_2$  = Secondary voltage (V)

- $N_1$  = Number of turns per unit length on the primary coil
- $N_2$  = Number of turns per unit length on the secondary coil

#### Step-up

transformer a transformer that increases voltage in the secondary coil compared to the input voltage in the primary coil

#### Step-down

transformer a transformer that decreases voltage in the secondary coil compared to the input voltage in the primary coil

Figure 6B-4 Left:

A transformer used for teaching purposes showing the soft iron core and coils. **Right: Mains** transformers become hot in operation and are inside a metal container with cooling fins and/or tubes on the outside to increase the surface area. allowing air to flow over the metal and transfer the thermal energy away from the coils inside.

By simply changing the turns ratio, that is, the number of turns on either the primary or secondary coil, the voltage in the secondary coil can be made greater or smaller than that of the primary coil. If the secondary voltage is greater than the primary voltage, the transformer is called a **step-up transformer**. If the secondary voltage is smaller than the primary voltage, the transformer is a **step-down transformer**. Note that it is the *ratio*, not the actual number, of turns in each coil that matters. You may have seen a transformer high up on a power pole in your street, stepping down the voltage to 230 V for use in local houses.

The main function of a transformer is to change the magnitude of an AC voltage. A transformer can *only* operate on a varying current. If the primary coil is connected to a steady DC, no emf will be induced in the secondary coil, because the steady DC does not produce any change in the magnetic flux.



#### 'Ideal transformers'

In an 'ideal' transformer, no electrical energy is transformed to thermal energy. So:

power input to the primary coil  $P_1$  = power output from the secondary coil  $P_2$ 

$$\begin{aligned} \boldsymbol{P}_1 &= \boldsymbol{P}_2 \\ \boldsymbol{V}_1 \boldsymbol{I}_1 &= \boldsymbol{V}_2 \boldsymbol{I}_2 \end{aligned}$$

where  $I_1$  and  $I_2$  represent the primary and secondary currents respectively. This can be combined with the turns ratio to show that if the voltage is stepped up, the current is stepped down, as expressed in Formula 6B–2.
#### Formula 6B–2 Turns ratio for an ideal transformer

Where:

- $\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$
- $N_1$  = Number of turns per unit length on the primary coil
- $N_2$  = Number of turns per unit length on the secondary coil

 $V_1$  = Primary voltage (V)

 $V_2$  = Secondary voltage (V)

$$I_1$$
 = Primary current (A)

 $I_2$  = Secondary current (A)

The output power from the secondary coil,  $P_2$ , can never be greater than the input power,  $P_1$ , because energy must be conserved. The typical efficiency of high-quality transformers is usually 98–99%, the other 1–2% being lost (waste) thermal energy. You may have felt this small amount of waste heat on small transformers you use at home, charging your phone or laptop. On large transformers used by electric power utilities and industries, the excess heat is removed by circulating oil (convection) or using air-cooled radiators with fins (as shown in Figure 6B–4). You may also notice a humming noise as some electrical energy is transformed into sound energy.

## Check-in questions – Set 1

- 1 Why are two individual coils required for a transformer, and what is the function of the soft iron core?
- **2** Draw the symbol for a transformer.
- **3** What determines whether a transformer is a 'step-up' rather than a 'step-down' transformer?
- 4 Alternating current is used for the input to the primary coil. Explain why this is and why a DC input would not work.

### Worked example 6B–1 Calculating rms voltage of a transformer

The ratio of number of turns in an ideal step-up transformer is  $\frac{N_1}{N_2} = \frac{1}{250}$ . An alternating rms voltage of 12 V is supplied to the primary coil.

Calculate the rms secondary voltage.

Solution

Use the turns ratio inverted, as you need to find a value for  $V_2$ , the rms secondary voltage,  $\frac{V_2}{V_1} = \frac{N_2}{N_1}$ .

Given  $V_1 = 12$  V, which is the rms voltage and the answer required is also rms, substitute this and the inverted value for turns ratio given in the question to find  $V_2$ .

$$\frac{V_2}{12} = \frac{250}{1}$$
$$V_2 = 12 \times 250$$
$$= 3000 \text{ V}_{rm}$$

Note: this value could be converted to  $V_{\rm p}$  or  $V_{\rm p-p}$  if required.



WORKSHEET 6B-1 TRANSFORMERS

## NOTE

Recall from Unit 2 Chapter 9 that energy efficiency is the ratio of the energy output to the energy input of a system, usually expressed as a percentage. The energy or power efficiency of a transformer can be calculated in the same way, but while this may be useful to know you won't be asked to do efficiency percentage calculations for transformers.



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## **6B SKILLS**

#### Annotating and drawing diagrams to answer questions

If you are confused by all the information given in a question, draw a diagram like the one below.

Go through the information piece by piece and label each value given with the symbol you would use in a formula. Then add it to the diagram, either on the primary or secondary side of the transformer. See the example below.

#### Question

A phone charger with 2000 turns in the primary coil and 50 turns in its secondary coil draws a current of 4.0 A. Calculate the current in the primary coil.

#### Solution

Step 1 Draw a diagram. This ensures that you have understood the situation properly.



**Step 2** Annotate the question. This ensures that you don't miss a vital piece of information. For example:



If you weren't sure whether the 4.0 A should be  $I_1$  or  $I_2$ , read the whole question and see what information is missing. You are required to calculate  $I_1$ , so the 4 A value must be  $I_2$ . It's not usually enough to just highlight – you must write the symbols to get a good grasp of the information given and to help you quickly find the best formula for Step 3.

**Step 3** Add the values to the diagram from Step 1, taking care to place them properly where they belong.



**Step 4** By now the formula you need has probably popped into your head. If not, check previous examples, the formula sheet or your notes. In this case, you need the transformer turns ratios using currents:

$$\frac{I_1}{I_2} \!=\! \frac{N_2}{N_1}$$

**Step 5** Substitute the values into the turns ratio formula and solve to find  $I_1$ .

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$
$$\frac{I_1}{4.0} = \frac{50}{2000}$$
$$I_1 = \frac{50 \times 4}{2000}$$
$$= 0.1 \text{ A}$$

If you get mixed up recalling the transformer turns ratio formula, remember that transformers are named step-up or step-down according to what they do to the voltage, so the  $N_1$  must match  $V_1$  in the formula, either on top or bottom of the fraction. (This connects well with what you studied in Chapter 5, where we saw that adding more loops to a generator produced greater emf,  $\varepsilon$ , because there was effectively greater change in magnetic flux,  $\Delta \Phi_B$ .) For current, it's the opposite, because a step-up transformer must reduce the current to step-up the voltage while transferring the same amount of power (P = VI). Look again at the complete formula and check that this makes sense to you.

# LINK CHAPTER 5

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## Section 6B questions

#### **Multiple-choice questions**

- 1 The primary and secondary coils of a step-up transformer have 5000 turns and 7500 turns respectively. If the primary coil is connected to a 240 V AC source, what is the voltage across the secondary coil?
  - **A** 160 V
  - **B** 240 V
  - **C** 360 V
  - **D** 720 V
- 2 A transformer is to be designed to step down the mains voltage from 240 V to 12 V. If the primary coil has 1200 turns, how many turns should the secondary coil have?
  - **A** 60
  - **B** 600
  - **C** 240
  - **D** 2400

3 The ratio of number of turns in an ideal step-up transformer is  $\frac{N_1}{N_2} = \frac{1}{250}$ . An alternating rms voltage of 14.0 V is supplied to the primary coil.

The rms secondary voltage will be closest to

- A 56 mV
- **B** 560 V
- **C** 3500 V
- **D** 4760 V

4 The ratio of number of turns in an ideal voltage step-up transformer is  $\frac{N_1}{N_2} = \frac{1}{250}$ . An alternating rms current of 30.0 mA is supplied to the primary coil.

The rms secondary current will be closest to

- A 0 mA
- **B** 0.12 mA
- **C** 7.5 mA
- **D** 7.5 A
- 5 Two concentric loops of conducting wire are placed on a flat horizontal surface. The outer loop contains an open switch and a battery cell. The inner loop consists of a single closed loop of wire. The diagram below shows the arrangement of the two loops, as viewed from above.



Which one of the following best describes the induced current in the inner loop once the switch is closed in the outer loop, as viewed from above?

- A a steady clockwise current
- **B** a steady anticlockwise current
- **C** a momentary clockwise current
- D a momentary anticlockwise current

VCAA NHT 2022

#### **Short-answer questions**

- **6 a** What are some of the factors affecting the emf output of the secondary coil of a transformer?
  - **b** Explain the main difference between a step-up and a stepdown transformer.
  - **c** If a transformer is not 'ideal', where does the 'lost' energy go?
- 7 a Why do solenoids used in electromagnets or transformers have soft iron cores?
  - **b** Why are the cores laminated?
- 8 a Explain, in terms of the relative currents, why the primary coil of a step up transformer (22 kV to 440 kV) is made up of much thicker copper wire than that of the secondary coil.
  - **b** If the power to be transformed is 500 MW, calculate the current in the primary and secondary transformers.
- **9** a Why do transformers operate on AC and not on DC?
  - **b** How could you design a DC current to activate a transformer (for example, to change the battery's 12 V DC to 15 kV for the spark plugs in a car)?

- **10** An ideal transformer is shown on the right.
  - **a** Explain:
    - i why the core is made of iron
    - ii why an electromotive force, emf, is not induced at the output when a constant direct voltage is at the input.
  - b An alternating voltage of peak value 150 V is applied across the 1200 turns of the primary coil. The variation with time, *t*, of the emf, *ε*, induced across the secondary coil is shown below.





Use data from the graph to:

- i calculate the number of turns of the secondary coil
- ii state one time (in ms) when the magnetic flux linking the secondary coil is a maximum.
- **c** A resistor is connected between the output terminals of the secondary coil. The mean power dissipated in the resistor is 1.2 W. For this question, assume that the varying voltage across the resistor is equal to the varying emf,  $\varepsilon$ , shown in the graph above.
  - i Calculate the resistance of the resistor.
  - ii Copy the graph below and sketch the variation with the time, *t*, of the power, *P*, dissipated in the resistor for t = 0 to t = 22.5 ms.





## Minimising power transmission losses

#### **Study Design:**

 Analyse the supply of power by considering transmission losses across transmission lines **Glossary:** Transmission line Transmission loss

## ENGAGE

## Standalone power grids

While most electricity in Australia is supplied from large-scale generators connected by high-voltage powerlines, increased access and advances to renewable energy resources and battery storage are making the development of smaller, standalone grids possible.

In Victoria, Australia's first mini-grid project was established in 2017. It began in a volunteer-run community group; the goal was to power the small Victorian town of Yackandandah with 100 per cent renewable energy by 2022. The project was originally given the appropriate name TRY, Totally Renewable Yackandandah. The homes in each mini-grid neighbourhood use rooftop solar and battery storage, which is monitored and managed by a computer device called an 'ubi'. The Victorian-made ubi computer system links homes and businesses, creating the mini-grid network that enables electricity trading. The project, renamed Indigo Power, is now a model adopted by other communities keen to take control of their future energy needs and make real change to reduce carbon emissions.



**Figure 6C–1** TRY community battery being craned into position in June 2021. The battery – which was Victoria's first behind-the-meter, community-owned solar-and-battery system – now stores solar energy from a 64 kW solar array. It has the capacity to power up to 40 Yackandandah households, including through the evening peak.

Yackandandah will be 100% renewable by 2024. Australia needs \$100 billion of investment in new forms of energy to hit its target of net zero emissions by 2050.



## Efficient power transmission using high-voltage AC

Electrical power is transmitted by high-voltage transmission lines from power stations to our homes (Figure 6C-2).



**Figure 6C–2** In Australia, we typically use 500 kV (extra high voltage, historically known as EHT) transmission lines for the transmission of electrical power over large distances, from the power station to the cities. Using such high voltages saves considerable electrical energy being lost as thermal energy in the transmission lines (see Table 6C–1).

Power stations are often a large distance from the consumer, so the resistance of the wires cannot be neglected. When a current passes through a powerline, electrical energy is 'lost' (wasted) as thermal energy because of the heating effect of the current.

Although the domestic Australian consumer, for safety and convenience (as well as historical reasons), is supplied with 230 V AC at 50 Hz mains electricity, the voltage generated by large AC generators is typically produced at 11 kV to 33 kV. In peak demand times in Victoria, there may be a total of 10 000 MW of electrical power needed for distribution. How can this electric power be efficiently transmitted, with very little heat losses in the powerlines, from the power station (typically hundreds of kilometres from the cities) to consumers?

The answer is to substantially increase the transmission voltages by using step-up transformers as power leaves the power station. In Australia, the voltages are typically stepped up to 200 kV, 330 kV or 500 kV. The voltage is then lowered again using step-down transformers at substations close to where the supply is required. This is typically 11 kV or 22 kV for factories, train or tram supply, and 415 V or 240 V for smaller factories, schools and domestic consumers. A schematic diagram of a typical transmission system is shown in Figure 6C-3.

## Transmission line

a specialised powerline that carries electrical energy at high voltage between geographical locations, sometimes over very long distances

#### CHAPTER 6 TRANSFORMERS AND TRANSMISSION OF ELECTRICITY



**Figure 6C–3** Diagrammatic representation of a large-scale electricity transmission and distribution system. Electricity is generated by alternators at thermal and hydroelectric power stations at high voltages (11 kV to 33 kV). It is stepped up to a still higher voltage (from 220 kV to 500 kV) by transformers. Power that is supplied by wind and solar power stations at lower voltages is also stepped up. It is then transmitted through a network of powerlines to towns and cities. There it is stepped down by transformers to 11–66 kV for factories and other large users, and to 415–230 V for offices and homes. Note that there are smaller transformers not shown here in housing, office and factory areas, to step voltages down to the requirements of various machines and process, all the way down to the lowest voltage, 230 V, where it is needed.

Electricity distribution networks are generally constructed with two or more paths between major substations, so that if one fails the electricity can be re-routed via the other path. The networks of the eastern and southern states, including Tasmania and the ACT, are connected to form a grid known as the national grid. See Figure 6C–6 for a diagram of Victoria's main high-voltage grid and the interconnections with other states. Most powerlines are conducting cables supported by pylons with glass or porcelain insulators preventing earthing. Power is fed into the grid at different points from power stations, solar farms, wind farms or big batteries.

#### NOTE

Recall from Unit 1 that there are some useful relationships between potential difference (V), current (I), resistance (R) known as Ohm's law (V = IR) and expressions for electric power (P):  $V^2$ 

 $P = VI = I^2 R = \frac{V^2}{R}$ 

#### Power loss in transmission lines

Transforming electricity to higher voltages means using smaller currents (since P = VI) given that, at any instant, there is a certain amount of power that must be transmitted. For any resistance in the transmission circuit, using higher voltage means the **transmission loss** will be considerably less.

loss electric power transformed to thermal energy, sound energy and kinetic energy as current passes through the resistance of the transmission wires; calculated from  $P_{\text{loss}} = l^2 R$ . It is sometimes expressed as a percentage of total power transmitted.

Transmission



Numerically, the power loss is proportional to the square of the current:

 $P = I^2 R$ 

So, for every factor of ten that the voltage is increased, the current will decrease by a factor of ten and the power loss will be reduced by a factor of one hundred. This is the main reason that transformers are used by the electric power utilities, and why AC (that can be easily transformed) is preferred to DC for electrical transmission over large distances.



**Figure 6C–4** A substation with step-down transformers to convert HV (high-voltage) AC, such as 200 kV, to lower voltage, 415 or 230 V, for distribution to local houses and businesses.

#### Worked example 6C-1 Power loss in wires I

The diagram shows the power supply to a shed that is 400 m distant from the farmhouse. The electricity is to operate a 230 V 1500 W piece of machinery. The connecting wires have a total resistance,  $R_{total}$ , of 50  $\Omega$ . At the farmhouse, the electrician must ensure that the required input voltage,  $V_{in}$ , is provided to the connecting wires for the machinery to operate at 230 V.



- the connecting wires? Show your working.
- **b** Calculate the rms voltage of  $V_{in}$ .

#### Solution

**a** Machinery operates at V = 230 V, P = 1.5 kW. Use the power formula.

P = IV $I = \frac{P}{V}$  $= \frac{1500}{230}$ = 6.52 A

As it is a series circuit, this current through the machinery is the same as the current in the wires. So, power lost in the wires can be calculated from:

$$P_{\text{loss}} = I^2 R$$
$$= (6.52)^2 \times 5$$
$$= 213 \text{ W}$$

This is over 200 W, which is considerable, more than 10% of the power for the machinery.

**b**  $V_{\text{in}}$  at the house must be larger than 230 V, so that even with the drop in potential across the wires, 230 V is still provided at the shed.

Calculate the potential drop across the wires using Ohm's law and the current from part a.

 $V_{drop}$  across connecting wires =  $IR_{wires}$ = 6.52 × 5 = 32.6 V So:  $V_{in} = V_{machinery} + V_{wires}$ = 230 + 32.6 = 262.6

#### Worked example 6C–2 Power loss in wires II

Refer to the question information in Worked example 6C-1 Power loss in wires I.

To reduce the power loss in the connecting wires, the electrician changes the input voltage,  $V_{\rm in}$ , and installs a 15:1 step-down transformer at the shed. After these changes, the machinery still operates at 230 V<sub>rms</sub> and 1500 W.



Solution

**a** First, obtain  $I_{\text{wires}}$  from the turns ratio.  $I_2$  is the same as part a of Worked example 6C–1, as the machinery is still operating at 230 V, 1.5 kW.

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$
$$I_1 = \frac{1}{15} \times 6.52$$
$$\cong 0.43 \text{ A}$$

This is the current in the wires, which is much smaller than the 6.52 A without the transformer.

So, power loss in wires is:

$$P_{\text{loss}} = I^2 R$$
  
= (0.43)<sup>2</sup> × 5  
= 0.925 W

Therefore, the power loss in wires is about 1 W, compared to over 200 W without the transformer.

**b**  $V_{\text{in}}$  at house =  $V_{\text{drop}}$  in wires + 230 V (for machinery)

Use Ohm's law.

 $V_{\rm drop \ in \ wires} = IR$  $= 0.43 \times 5$  $= 2.15 \text{ V} \cong 2.2 \text{ V}$ So:  $V_{\rm in} = 2.2 + 230$ 

= 232 V

### Worked example 6C-3 Power loss in wires III

Refer to the question information in Worked example 6C-1 Power loss in wires I. Often, the electrician would change the value of  $V_{in}$  (at the house) to this new value by installing a step-up transformer on the farmhouse. The input to the farmhouse transformer is 230  $V_{\rm rms}$ .



#### Solution

**a** On the house transformer:

$$V_{1} = 230 \text{ V}$$

$$V_{2} = 232 \text{ V}$$

$$\frac{N_{1}}{N_{2}} = \frac{V_{1}}{V_{2}}$$

$$= \frac{230}{232}$$

$$\approx 0.99 \text{ (or } \frac{99}{100} \text{ or } 99:100)$$

**b** It is probably not worth the expense of installing the farmhouse transformer, as this increase in voltage required is so small, less than a factor of 0.01 or 1%. There is unlikely to be any negative effect on either the shed machinery or household appliances if the system has only the shed transformer installed.

In situations where transformers are used from power generation stations to substations and on to domestic housing, the differences in voltage are significantly greater over longer distances, so transformers are used at both ends of the transmission lines.

#### Efficient power transmission using low-resistance transmission lines

Transferring electricity over long distances is an expensive operation. Aside from engineering costs and the land itself, one of the main costs is the wire used to carry the electricity. To keep the energy losses as small as possible means keeping the resistance low.

The resistance of wires is affected by several factors including the total length, crosssectional area and resistivity of the wire.

Each of the three factors can help to reduce the total resistance of the transmission line system and thereby reduce the power losses in transmission due to heating of the lines.

- **Reduce the length of wire**, *l*. This is not practical if the fuel source (oil, gas, coal or nuclear energy) for a thermal power station, or water source for a hydroelectric power station, is located away from the cities. In addition, most people who live in cities do not want the associated pollution or dangers of such power stations close to where they live. Therefore, the length is usually not a variable that can practically be adjusted.
- Reduce the resistivity,  $\rho$ , of the wire by using a better conductor. Traditionally, gold, silver and copper are very good conductors (having resistivities in the order of  $10^{-9}$  or  $10^{-8} \Omega$ m). However, apart from the use of very thin gold strands in microprocessor connections, the bulk usage of any of these metals is very expensive (prohibitively so for gold and silver). Therefore, most countries have adopted aluminium as the conductor for transmission wires. Although aluminium has a higher resistivity, it is obviously much cheaper than gold, silver or copper and considerably lighter (meaning in turn less cost for the supporting pylon structures). To increase the strength of the aluminium conductors, the aluminium strands are wound around a core of galvanised steel wires.
- Increase the cross-sectional area, *A*, of the conductor being used. Doubling the radius gives four times the cross-sectional area and one-quarter of the resistance. However, it also increases the mass by a factor of four. Therefore, this method of decreasing resistance is only practical until the extra mass results in the need to construct sturdier, and overall uneconomic, support structures.

#### NOTE

While it is not required by this course, it may be useful to note the way these factors are related by the formula:

$$R = \frac{\rho l}{A}$$

where R is the resistance  $(\Omega)$ ,  $\rho$  is the resistivity  $(\Omega m)$ , l the length (m) and A the crosssectional area (m<sup>2</sup>) of a metal wire.

#### Worked example 6C–4 Power loss in transmission lines

What are the typical percentage power savings that can be made on fixed, low-resistance transmission lines by using high voltages? For these examples, assume a total power of 300 MW is transmitted from a distance of 120 km away, with a total transmission line resistance of 5  $\Omega$ , and possible transmission voltages of 500 kV, 220 kV and 66 kV.

#### Solution

With transmission voltage  $V_1 = 500$  kV, the current through the 5  $\Omega$  transmission wires is given by:

$$I = \frac{P}{V}$$
$$= \frac{300 \times 1}{500 \times 1}$$
$$= 600 \text{ A}$$

 $0^{6}$ 

The power loss in the wires is then given by:

$$P_{\text{loss}} = I^2 R$$
$$= 600^2 \times S$$
$$= 1.8 \text{ MW}$$

Next, convert this to a percentage.

$$\%P_{\text{loss}} = \frac{P_{\text{loss}}}{P_{\text{transmitted}}} \times 100$$
$$= \frac{1.8 \times 10^6}{300 \times 10^6} \times 100$$
$$= 0.6\%$$

Repeat the whole calculation for V = 220 kV and 66 kV and check your answers with those in Table 6C-1, which shows a summary of this analysis.

Table 6C-1 Calculations of power loss in transmission lines at various transmission voltages

Transmission voltage	500 kV	220 kV	66 kV
Calculate current from <i>P</i> = <i>VI</i>	/= 600 A	/=1364 A	/=4545 A
Calculate power loss from $P_{loss} = l^2 R$	$P_{\rm loss} = 1.8 \; {\rm MW}$	$P_{\rm loss} = 9.3 \; {\rm MW}$	$P_{\rm loss} = 103 \; {\rm MW}$
Transmission losses as %	0.6 %	3.1%	34.3 %

The calculations show that 500 kV power transmission is the most economical (around 0.6% losses), while power losses of 2–3% (as is the case with 220 kV transmission) are economically acceptable.





#### Disadvantages of very high voltage transmission lines

You may wonder why all power is not transmitted at 500 kV or why we do not push the voltages even higher (say one million or ten million volts) and reduce ohmic thermal losses to minute fractions of a percentage. The reason is that extra high voltage (EHV) can cause the air surrounding the conductor to break down; the so-called corona phenomena. Air molecules break down and the ions conduct, when the voltage gradient reaches 800 V mm<sup>-1</sup>. So, 500 kV transmission lines need to be at least 600–700 mm away from the pylons, in ideal conditions, to avoid sparking (a 10000 kV transmission line would throw lethal arcs to the ground almost continuously). The air leakage from a 500 kV line creates additional power losses – TV, cellular phone and radio interference, visual coronas (visible as an eerie glowing around the wires at night) and audible crackling effects. This is more prevalent in humid climates.



**Figure 6C–5** Transmission overhead powerlines range in voltage from 66 kV up to 500 kV, installed on tall towers or steel poles. The dimensions are the width of the corridor in which safety regulations apply and which the power company uses for maintenance and restricts vegetation height to 3 m. Cables may also be run underground, which gives a smaller footprint once constructed, but involves more disruption and can be more expensive to install. A maintenance pit is required every 400–500 m of underground cabling.



**Figure 6C–6** A map of Victoria's electricity grid, showing transmission lines of different voltages, with interconnectors to South Australia, New South Wales and Tasmania. These connections between states help to balance power demand with availability. This is becoming even more important with the growth in renewable sources of energy and storage.

You may have heard your local power pole transformer (with a power input of around 200 kW, and which transforms 11 kV to 415 V or 240 V) crackle during the wet winter months. In Australia, 500 kV appears to be the current practical limit for the transmission of electrical power. Note that because of the lower resistance over shorter distances (called sub-transmission areas), it is acceptable, safer and more economical to use 66 kV, 22 kV or 11 kV. This is because the length, *l*, of the wires is much smaller. In Australia, it is estimated that overall rate of power loss for the whole transmission system amounts to about 10% of generation, which is a substantial amount of energy.

#### ACTIVITY 6C-1 HVDC TRANSMISSION

With the surge of interest in higher temperature superconductors and the subsequent technological breakthroughs enabling the construction of higher current-carrying wires, substantial changes to the conventional transmission of power are possible. At times, low-current high-voltage DC is more suitable for power transmission than AC.



**Figure 6C–7** High-voltage cables connect areas of electricity generation with power consumers across the country.

Thomas Edison, the famous American inventor, thought DC transmission would be better than AC in the long run. In fact, it is already used for power transmission purposes between Tasmania and Victoria (the 290 km BassLink cable), and between the North and South islands of New Zealand. More DC interconnectors are planned.

Research the physics of high-voltage DC (HVDC) interconnectors and explore the benefits of such a powerline from the Northern Territory to Singapore.

### Check-in questions – Set 1

- 1 Why isn't electrical power delivered to households at the same voltage it is generated at by the power station?
- **2** The supply voltage to large factories is usually high (11 kV or 22 kV). The voltage is then lowered for distribution and use within the factory. Explain why such a high voltage is supplied directly to the factory.
- **3** Using Figure 6C–3, describe how electrical energy is supplied to your house. Explain the use of the various step-up and step-down transformers.







### 6C SKILLS

#### Questions involving transformers and power loss in wires

A typical problem in this section of the course requires you to analyse a power supply system that includes transformers, power loss in wires, and the most efficient method to distribute electricity – the one with the least power loss. Watch out for differences between each question – they look similar and work out easily, but a small change could confuse you. In the diagram for the example below, the power station is on the right of the diagram, not the usual left-hand side. So, you might be tempted to assume that  $T_1$  would be a step-*up* transformer because it takes the power from the power station ... but look more closely!  $T_1$  has 220 kV input and transforms it *down* to 10 kV, so it must be a step-*down* transformer, and so is  $T_2$ . Read the question below and see if you can solve it.

#### Question

The Smith family and the Jones family are farmers near Warragul. Their electricity supply comes more than 100 km, from a power station in the LaTrobe valley. It is carried by transmission lines, at a voltage of 220 kV<sub>rms</sub>. Near the town, a switchyard transformer,  $T_1$ , steps the voltage down to 10 kV<sub>rms</sub> for the local area. A 10 kV<sub>rms</sub> line runs to the Jones' farm, where there is a transformer,  $T_2$ , that provides 240  $V_{\rm rms}$  for the farms. A 240  $V_{\rm rms}$  line then runs 2 km to the Smith's farm. A sketch of the situation is shown below. For this question, consider all transformers to be ideal.



- **a** Explain why the supply from the power station to the local area is chosen to be  $220 \text{ kV}_{\text{rms}}$  rather than  $240 \text{ V}_{\text{rms}}$ . Use numerical estimates to support your answer.
- **b** Assume that the input voltage to transformer  $T_1$  is 220 kV<sub>rms</sub> and the output is 10 kV<sub>rms</sub>.

What is the value of the ratio  $\frac{\text{number of turns on the primary coil}}{\text{number of turns on the secondary coil}}$ 

**c** The supply and return lines between transformer  $T_2$  and the Smith's farm have a total resistance of 0.0004  $\Omega$  m<sup>-1</sup>, that is each metre of transmission line has a resistance of  $4 \times 10^{-4} \Omega$ .

At a particular time, 20 A of current is being supplied to the Smith's farm. Assume that the potential at the secondary for transformer  $T_2$  is 240 V<sub>rms</sub>.

What is the voltage at the Smith's farm?

VCAA 2002

A good answer will include the following information.

**a** The power loss in the transmission lines is calculated using the formula  $P = I^2 R$ . Consequently, using low line currents can reduce the power loss. The transmitted power, given by P = VI, is a given value and so high-transmission voltages result in low line currents and less power loss in the lines. For example, when the transmission voltage is 220 kV compared to 240 V, the currents are in the ratio  $\frac{1}{1920}$  and so the power losses are in the ratio  $\left(\frac{1}{1920}\right)^2$ . A typical answer to this question involves mentioning the power loss in the wires,

 $P = I^2 R$ , and the consequent need for low currents to reduce power loss. It may discuss that low I means higher V without specifically referring to the power, P = VIas a fixed quantity. The most common error in answering this type of question is in not making a numerical comparison for transmission at 220 kV and 240 V as requested.

**b** Use the turns ratio formula,  $\frac{V_1}{V_2} = \frac{N_1}{N_2}$ , to get an answer of 22 for the ratio  $\frac{N_1}{N_2}$ . Take care to get the 1 and 2 values in the right place!

The length of the supply and return lines is 4000 m, so this represents a total С resistance of 1.6  $\Omega = 4000 \times 4 \times 10^{-4}$ .

Using Ohm's law gives a potential drop across the transmission wires.

V = IR $= 20 \times 1.6$ = 32 V

Hence, the voltage at the Smith's farm is:

240 - 32

= 208 V

Note that part c would usually be worth three marks and there are three distinct steps in the answer.



### Section 6C questions

328

#### **Multiple-choice questions**

#### Use the following information to answer Questions 1-6.

Two students, Charlie and Devina, are running an experiment using a powerpack inside the physics lab to light a globe outside the door. The insulated wires are quite long and seem to get quite hot when the light is on, so they rig up transformers beside the powerpack and outside the room beside the light. The wires easily pass through the gap between the door and doorframe.

The set-up is as shown below.



Transformer 1

- 1 The best reason for using the transformers is that
  - A high voltages can mean lower currents and lower energy losses in long wires.
  - B high voltages can mean higher currents and lower energy losses in long wires.
  - C high voltages can mean lower currents and higher energy losses in long wires.
  - D low voltages can mean lower currents and lower energy losses in long wires.
- 2 The best description of the two transformers is
  - A transformer 1 steps down the voltage and transformer 2 steps up the voltage.
  - **B** transformer 1 steps up the voltage and transformer 2 steps down the voltage.
  - **C** transformer 1 steps up the current and transformer 2 steps down the current.
  - **D** transformer 1 steps up the current and transformer 2 steps down the current.
- 3 The current required for the light globe to function normally is closest to
  - **A** 0.75 A
  - **B** 1.3 A
  - **C** 3.0 A
  - **D** 12 A
- 4 Assuming the light functions normally and is close to transformer 2, what is the input voltage required to transformer 2?
  - **A** 0.67 V
  - **B** 0.15 V
  - **C** 20 V
  - **D** 60 V
- **5** The current in the long wires between the two transformers and the potential drop across them (potential difference between the output of transformer 1 and the input to transformer 2) are closest to
  - **A** 0.065 A and 40 V
  - ${\ensuremath{\mathsf{B}}}$   $\ensuremath{\,0.065\ensuremath{\,A}}$  and  $\ensuremath{\,0.39\ensuremath{\,V}}$
  - **C** 26 A and 18 V
  - **D** 3.0 A and 0.39 V

- 6 The voltage required from the powerpack to supply the input to transformer 1 is closest to
  - **A** 3.02 V
  - **B** 20.0 V
  - **C** 60.4 V
  - **D** 121 V

#### **Short-answer questions**

- 7 a What are some advantages of high-voltage power transmissions?
  - **b** What is the main advantage of high-voltage AC power transmission over the equivalent DC power transmission?
- 8 Use the following information concerning Yallourn W power station in Victoria to determine the efficiency of generating electrical energy from coal.

1 tonne of Yallourn brown coal releases 8.7 gigajoule (GJ) of heat, of which 2.0 GJ is used in drying the coal. The remaining 6.75 GJ is used to drive the generators to produce 2.3 GJ of electrical energy.

Where does all the other energy go?

- **9** A hospital generator, 800 m from the hospital, generates 40 kW of power at 250 V AC for use in an emergency. The transmission lines over which the power is transmitted have a total resistance of  $0.2 \Omega$ .
  - **a** Draw a diagram to show this set-up.
  - **b** How much power is lost if the power is transmitted at 250 V?
  - **c** What would the voltage be at the hospital? Could the hospital staff use normal 240 V AC electrical appliances? Why or why not?
  - d What would happen if the generator were sited 4 km away and the total resistance of the lines was now 1.0  $\Omega$ . Answer parts b and c again for this case.
  - **e** How much power would be lost if the voltage was stepped up by a transformer at the generator to 10.0 kV and down again to 240 V at the hospital end?



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**10** The electric power for Melbourne trams is supplied at a DC voltage of 600 V. The current flows from the overhead wire through the tram motor and returns through the metal rails. Because of the voltage drop that occurs in the overhead wire, the wire is made up of separate 3.0 km sections. One of these sections is shown in the figure below. A separate 600 V supply is connected to *one end only* of each section.



3.0 km

Tram 2 is accelerating and is drawing a current of 500 A. Tram 1 is drawing a current of 200 A.

- **a** What is the current in sections P, Q and R of the wire? The voltage at the position of tram 2 is 540 V.
- **b** How much electrical power is tram 2 using?
- **c** What is the resistance of 1.0 km of the overhead wire?
- d What is the voltage at the position of tram 1?

VCAA 2003



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# **Chapter 6 review**

## Summary

Create your own set of summary notes for this chapter on paper or in a digital document. A model summary is provided in the Teacher Resources, which can be used to compare with yours.

## Checklist

In the Interactive Textbook, the success criteria are linked from the review questions and will be automatically ticked when answers are correct. Alternatively, print or photocopy this page and tick the boxes when you have answered the corresponding questions correctly.

Success criteria – I am now able to: Linked questions				
6A.1	Explain peak voltage, $V_{\rm p}$ , and peak-to-peak voltage, $V_{\rm p-p}$ , including on graphs of induced emf plotted against time and through	11 🗌 , 14 🗍		
	calculation using $V_{\rm rms} = \frac{v_{\rm p}}{\sqrt{2}}$			
6A.2	Explain peak current, $I_p$ , and peak-to-peak current, $I_{p-p}$ , including on graphs of induced current plotted against time and through	90,14		
	calculation using $I_{\rm rms} = \frac{I_{\rm p}}{\sqrt{2}}$			
6A.3	Recognise that a given AC voltage expressed as $V_{\rm rms}$ produces the same power in a resistive load as a constant DC voltage with the	14		
	same value; calculate power using $P_{\rm rms} = V_{\rm rms} I_{\rm rms} = \frac{V_{\rm p} I_{\rm p}}{2}$			
6B.1	Recognise transformers as a method to transfer electrical power from one circuit to a separate circuit via a varying magnetic flux, causing an induced emf $\varepsilon$	1, 7		
6B.2	Explain the functions of a transformer in terms of electromagnetic induction, including the roles of primary and secondary coils and the soft iron core; identify and explain whether a transformer steps-up or steps-down the voltage	1 , 8 , 9 , 12		
6B.3	Recall that in <i>ideal</i> transformers (only), power input to the primary coil is equal to power output from the secondary coil, $P_{in} = P_{out}$ . Discuss why real-life transformers are <i>not</i> 'ideal'	14 , 15		
6B.4	Use the formula $\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$ (known as the turns ratio) to perform calculations relating to step-up and step-down transformers, input and output currents and potential differences	2 , 3 , 4 , 6 , 8 , 11 , 12 , 14 , 16 , 20		
6B.5	Perform calculations using the turns ratio, Ohm's law, power formulas and other circuit theory to determine the currents, potential differences and power associated with the primary and secondary circuits of a transformer	3□, 5□, 9□, 16□, 17□, 19□		
6C.1	Calculate electrical power and electrical energy transformed to heat in a wire using $P = I^2 R$ , and hence $E = I^2 R t$	14 🗌 , 15 🗍 , 17 🗍 , 20 🗌		
6C.2	Analyse power losses in an AC electrical supply system (with and without transformers) using $P_{\text{loss in wires}} = I^2 R$	13□, 15□, 16□, 17□, 18□		

Succe	Linked questions	
6C.3	Perform calculations using Ohm's law and other circuit theory to determine the currents, potential differences and power associated with a transmission of electricity over longer distances, including models of such systems	13□, 16□, 18□
6C.4	Explain, using supporting calculations, the most efficient methods for supplying electricity to a load that is distant from the generator; include energy losses, the advantages of high-voltage over low-voltage transmission, and the advantages of AC rather than DC transmission	10□, 17□, 18□, 20□
6C.5	Analyse and evaluate an electricity distribution system; design an efficient transfer model for supplying electricity to a load that is distant from the generator	13 , 16 , 18

1000 turns

Х

 $40 V_{rms} AC$  (

## **Multiple-choice questions**

Use the following information to answer Questions 1, 2 and 3.

A 40  $\mathrm{V}_{\mathrm{rms}}\mathrm{AC}$  generator and an ideal transformer are used to supply power. The diagram shows the generator circuit diagram on the left, and a diagram of the transformer supplying 240  $\rm V_{rms}$  to a resistor with a resistance of 1200  $\Omega$ .

**1** Which of the following correctly identifies the parts labelled X and Y, and the function of the transformer?

	Part X	Part Y	Function of transformer
Α	Primary coil	Secondary coil	Step-down
В	Primary coil	Secondary coil	Step-up
-			

Dart V

Step-down С Secondary coil Primary coil Step-up D Secondary coil Primary coil

VCAA 2019

1200 Ω

6000 turns

Transformer

2 Which of the following is closest to the rms voltage across the secondary coil?

**A** 6.7 V

Part Y

- **B** 40 V
- **C** 240 V
- **D** 4000 V

**3** Which one of the following is closest to the rms current in the primary circuit?

**A** 0.04 A

Use the following information to answer Questions 4, 5 and 6.

Fifteen Christmas tree lights, each rated at 12 V AC 0.5 W, are connected in parallel and run from a transformer. The 800-turn primary coil is connected to a mains supply of 240 V 50 Hz.

- **4** How many turns are required in the secondary coil of the transformer for the lights to operate as rated?
  - **A** 40
  - **B** 160
  - **C** 400
  - **D** 1600
- **5** Which of the following is closest to the rms current in the secondary coil when the lights are operating as rated?
  - **A** 0.42 A
  - **B** 0.63 A
  - **C** 6.0 A
  - **D** 90 A
- **6** Which of the following is closest to the rms current in the primary coil when the lights are operating as rated?
  - **A** 0.031 A
  - **B** 0.050 A
  - **C** 0.13 A
  - **D** 0.63 A
- **7** An ideal transformer has an input DC voltage of 240 V, 2000 turns in the primary coil and 80 turns in the secondary coil.

The output voltage is closest to

- **A** 0 V
- **B** 9.6 V
- $\textbf{C} \quad 6.0\times 10^3\,\mathrm{V}$
- $\textbf{D} \ 3.8\times 10^7 \ V$

VCAA 2020

Use the following information to answer Questions 8 and 9.

Students doing a VCE Physics practical investigation use a step-down transformer with AC rms voltage 240 V transformed to AC rms 12 V.

8 Which one of the following best gives the ratio of the number of turns,  $\frac{N_{\text{primary}}}{N_{\text{secondary}}}$ ?

**A**  $\frac{1}{4}$  **B**  $\frac{1}{20}$  **C**  $\frac{4}{1}$ **D**  $\frac{20}{1}$ 

VCAA 2017

**9** The transformer delivers  $P_{\rm rms}$  of 48 W to a resistor.

Assume that the transformer is ideal. Which one of the following best gives the peak current in the secondary coil?

- **A** 0.2 A
- **B** 4.0 A
- **C** 5.7 A
- **D** 11.3A

VCAA 2017

Secondary

= 12 V AC

Primarv

**10** Electrical power stations are often situated far from the cities that require the power that they generate. Which one of the following best describes the reason for the high-voltage transmission of electrical energy?

 $V_1 = 240 \text{ V AC}$ 

- A Transformers can be used to increase the voltage in the cities.
- **B** High voltages reduce the energy losses in the transmission lines.
- **C** High voltages provide the large currents needed for efficient transmission.
- **D** High voltages can reduce the overall total resistance in the transmission lines.

VCAA NHT 2021

### **Short-answer questions**

**11** A mobile phone charger uses a step-down transformer to convert 240 V AC to 5 V AC. The mobile phone draws a current of 3.0 A while charging.

Assume that the transformer is ideal and all figures given are rms values.

- **a** Calculate the peak voltage of the input to the transformer. (2marks)
- **b** Calculate the ratio of turns in the primary (input) to turns in the secondary (output). (2 marks)
- **c** Find the current drawn from the mains during charging. (2 marks)
- 12 A transformer in a laptop computer cord transforms mains 230V AC electricity to 5 V AC at 1.25 A.
  - **a** Is this a step-up or step-down transformer? Explain your answer. (2 marks)
  - **b** Calculate the current through the primary coil.
  - **c** If the secondary coil has 100 turns, calculate the number of turns on the primary coil.

(2 marks)

(2 marks)

(2 marks)

VCAA 2021

**13** Two students, Alan and Becky, are constructing a model of an electricity transmission system to demonstrate power loss in transmission lines. The purpose of the model is to operate a 2.0 V, 4.0 W lamp as the load.

They first set up the model as shown in the figure below. Each of the transmission lines has a resistance of 2.0  $\Omega$ . Ignore the resistance of other connecting wires.



The power supply is adjusted so that the lamp is operating correctly (2.0 V, 4.0 W).

- **a** Calculate the current in the wires.
- **b** Calculate the voltage output from the power supply. (2 marks)
- c Calculate the total power loss in the transmission lines. Show your working. (3 marks)

The students wish to demonstrate how this power loss can be reduced by using a higher transmission voltage involving a step-up and a step-down transformer. They place a step-up transformer,  $T_1$  (1:10), at the power supply end and a step-down transformer,  $T_2$  (10:1), at the lamp end, as shown in the figure below. Assume the transformers are ideal; that is, there is no power loss in them. The students adjust the voltage of the power supply so that the lamp operates correctly.



d Calculate the current in the transmission lines in this situation.

(3 marks) (2 marks)

VCAA 2015

- **e** Calculate the power loss in the transmission lines.
- f Describe the real situation that this model could represent. Explain why the higher transmission voltage is used in terms of power losses you calculated in Alan and Becky's model.
   (4 marks)
- **14** A rechargeable electric toothbrush uses a transformer circuit, as shown in the figure on the right. The toothbrush has a cylindrical recess in its base so it sits on the end of the iron core that protrudes from the base of the charger. The secondary coil of the transformer circuit is inside the toothbrush and at the other end of the iron core is the primary coil in the charger that is connected to the mains power supply. The mains power is 240  $\mathrm{V}_{\mathrm{rms}}$ and the toothbrush recharges its battery at 12  $V_{rms}$ . The average power delivered by the transformer to the toothbrush is 0.90 W. For this question, assume that the transformer is ideal.



Transformer circuit to recharge toothbrush

- **a** Calculate the peak voltage in the secondary coil. Show your working.(2 marks)**b** Determine the ratio of the number of turns,  $\frac{N_{\text{primary}}}{N_{\text{secondary}}}$ .(1 mark)
- c Calculate the rms current in the primary coil while the toothbrush is charging.Show your working. (2 marks)

VCAA 2020

335

**15** The figure below shows a generator at an electrical power station that generates 100 MW<sub>rms</sub> of power at 10 kV<sub>rms</sub> AC. Transformer T<sub>1</sub> steps the voltage up to 500 kV<sub>rms</sub> AC for transmission through transmission wires that have a total resistance,  $R_{\rm T}$ , of 3.0  $\Omega$ . Transformer T<sub>2</sub> steps the voltage down to 50 kV<sub>rms</sub> AC at the substation. For this question, assume that both transformers are ideal.



- **a** The current in the transmission lines is 200 A. Calculate the total electrical power loss in the transmission wires. (2 marks)
- **b** Transformer  $T_1$  stepped the voltage up to 250 kV<sub>rms</sub> AC instead of 500 kV<sub>rms</sub> AC. By what factor would the power loss in the transmission lines increase? (2 marks) VCAA 2017
- **16** Students construct a model to show the transmission of electricity in transmission lines. The apparatus is shown in the figure below.



The students use two transformers,  $T_1$  and  $T_2$ , with ratios of 1:8 and 8:1 respectively, and a 2.0  $V_{\rm rms}$  AC power supply. Assume that the transformers are ideal. The students use a light globe that operates correctly when there is a voltage of 2.0 V across it. The wires of the transmission lines have a total resistance of 4.0  $\Omega$ . The students measure the current in these wires to be 0.50 A.

- **a** Calculate the power loss in the wires. (2 marks)
- **b** Calculate the voltage across the light globe.
- c The light globe does not operate correctly, as it should with a voltage of 2.0 V across it. Describe one change the students could make to the model to make the light globe operate correctly.
   (2 marks)

(4 marks)

(1 mark)

VCAA NHT 2018

- **17** A homeowner on a large property creates a backyard entertainment area. The entertainment area has a low-voltage lighting system. To operate correctly, the lighting system requires a voltage of 12  $V_{rms}$ . The lighting system has a resistance of 12 Ω.
  - **a** Calculate the power drawn by the lighting system.

To operate the lighting system, the homeowner installs an ideal transformer at the house to reduce the voltage from 240 V<sub>rms</sub> to 12 V<sub>rms</sub>. The homeowner then runs a 200 m long heavy-duty outdoor extension lead, which has a total resistance of 3  $\Omega$ , from the transformer to the entertainment area.

- b The lights are a little dimmer than expected in the entertainment area. Give one possible reason for this and support your answer with calculations. (4 marks)
- C Using the same equipment, what changes could the homeowner make to improve the brightness of the lights? Explain your answer.
   (2 marks) VCAA 2019

**18** Ruby and Max are investigating the transmission of electric power using a model system, as shown in Figure a. The circuit is shown in Figure b.



#### Figure b

Ruby and Max use an 18 V DC power supply, as shown in Figure b. The two transmission lines have a total resistance of 3.0  $\Omega$ . Assume that the resistance of the globes is constant at 9.0  $\Omega$  and that the other connecting wires have zero resistance.

- **a** Calculate the power delivered to globe A. (2 marks)
- **b** Calculate the total voltage drop over the transmission lines. Show your working. (2 marks)
- c Calculate the power delivered to globe B. Show your working. (3 marks)

Ruby has noticed that the voltage supply to houses is AC and that there are transformers involved (on street poles and at the fringes of the city). Ruby and Max next investigate the use of transformers to reduce power losses in transmission. Ruby and Max have two transformers available – a 1:10 step-up transformer and a 10:1 step-down transformer.

- d Redraw the circuit in Figure b with an 18 V AC supply and with the transformers correctly connected. Label the transformers as step-up and step-down. (2 marks)
- e Explain why the transformers would reduce the transmission losses. Your answer should include reference to key physics formulas and principles. (3 marks)

VCAA 2016

**19** A transformer in a substation is used to convert the high voltage of a transmission line to the voltage needed for a factory. The transformer has a ratio of 45:1 of primary to secondary turns:

$$\frac{N_{\text{primary}}}{N_{\text{secondary}}} = \frac{45}{1}$$

Electricians use an oscilloscope to test the output (lower voltage) side of the transformer. They observe the following signal, as shown in the figure below.



- **a** Determine the frequency of the AC (alternating current) observed. Show your working.
  - (2 marks) Determine the rms voltage of the incoming (high voltage) input to the transformer.

```
(2 marks)
VCAA 2015
```

(3 marks)

(2 marks)

**20** The suburban train network has a series of transformer substations along the railway route every four or five kilometres. The substations buy electricity from the grid and receive it at 33 kV or 66 kV. They transform it down to 1500 V (and eventually convert it to DC).

- **a** Why are the substations placed so close together?
- **b** What turns ratio,  $\frac{N_{\text{primary}}}{N_{\text{secondary}}}$ , do they need for the transformer (assume 33 kV to

1.5 kV for this calculation).

ii the current in the primary coil.

Show your working.

b

- **c** If the overhead lines carry an average 1.5 kV at 500 A, calculate:
  - i the power required to be supplied by the secondary coil of the transformer (2 marks)

(2 marks)



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## **Unit 3 Revision exercise**

## **Multiple-choice questions**

A cricket ball is moving to the right through the air at 90 km h<sup>-1</sup>.
 Which one of the following diagrams correctly shows all the forces acting on the cricket ball if there is substantial air resistance acting?



Which one of the following is closest to the gravitational force of attraction between Enceladus and Saturn? Use  $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ .

- **A** 0 N
- **B** 1300 N
- $\textbf{C} \quad 4.9\times 10^8 \; N$
- $\textbf{D} \quad 7.2\times 10^{19}\,N$

VCAA 2023

**3** A book rests on a table. A normal reaction force from the table pushes vertically upward on the book as shown in the diagram below.



The action-reaction force pair to this force (according to Newton's third law) is

- **A** a force, F = mg, acting on the book.
- **B** a normal reaction force on the table from the book.
- **C** the force of gravity acting downwards on the book.
- **D** a compression force on the book pushing the pages together.

4 Melissa launches a ball from height h above the ground at a speed of 8.0 m s<sup>-1</sup> and at an angle of 30° above the horizontal. The time of the ball's flight is 1.0 s. The diagram below shows the trajectory of the ball.



Ignoring air resistance, which one of the following is closest to the horizontal distance that the ball landed from Melissa?

- **A** 4.6 m
- **B** 5.0 m
- **C** 6.9 m
- **D** 8.0 m

VCAA NHT 2021

5 A planet of mass  $10^{20}$  kg is orbiting a dwarf star at a radius of  $10^8$  m. The planet's period is  $10^6$  s.

Which one of the following is closest to the speed of the planet in its circular orbit?

- **A**  $60 \text{ m s}^{-1}$
- ${\pmb B} \ \ 600 \ m \ s^{-1}$
- $C 6000 \text{ m s}^{-1}$
- ${\color{blue} \textbf{D}} \quad 60\,000 \ m \, s^{-1}$
- **6** Which one of the following correctly describes the force that keeps Earth in orbit around the Sun.
  - A centripetal force
  - **B** centrifugal force
  - **C** gravitational force
  - **D** centripetal force and gravitational force acting together
- 7 The gravitational field strength at the surface of a uniform spherical planet of radius R is  $g \operatorname{Nkg}^{-1}$ . At a distance of 2R above the planet's surface, the strength of the planet's gravity will be
  - $A \quad \frac{g}{2}$  $B \quad \frac{g}{3}$  $C \quad \frac{g}{4}$  $D \quad \frac{g}{9}$

8 Two large parallel conducting plates are separated by 20 cm.

A potential difference of 400 V is applied across the plates.

Which one of the following best describes the electric field in the space between the plates?

- **A** There is a uniform field of  $20 \text{ Vm}^{-1}$  from the + plate to the plate.
- **B** There is a uniform field of  $20 \text{ Vm}^{-1}$  from the plate to the + plate.
- ${\bm C}$  There is a uniform field of 2000  $V\,m^{-1}$  from the + plate to the plate.
- ${\bm D}$  There is a uniform field of 2000  $V\,m^{-1}$  from the plate to the + plate.
- **9** The diagram below shows an ideal transformer in which the primary coil is connected to a battery and a switch. An ammeter is connected to the secondary coil.

When the switch is closed, the pointer on the ammeter momentarily deflects. How could the deflection on the ammeter be made larger?



- A decrease the number of primary coils
- **B** decrease the number of secondary coils
- **C** increase the number of secondary coils
- **D** place a resistor in series with the ammeter

VCAA NHT 2021

#### Use the following information to answer Questions 10 and 11.



- **11** The speed of the positron is changed to  $7.0 \times 10^5$  m s<sup>-1</sup>. Which one of the following best gives the value of the radius, *r*, for this speed?
  - $A \quad \frac{r}{4}$  $B \quad \frac{r}{2}$  $C \quad r$  $D \quad 2r$

- VCAA 2020
- **12** The graph below shows the change in magnetic flux,  $\Phi$ , through a coil of wire as a function of time, *t*.



Which one of the following graphs best represents the induced emf,  $\varepsilon$ , across the coil of wire as a function of time, *t*?



VCAA 2017

3/3

#### Use the following information to answer Questions 13 and 14.

A DC motor with permanent magnets is shown. The coil of the motor has 20 turns. The dimensions of the coil (which is free to turn) are shown. The direction of the magnetic field is shown and is of strength 0.5 Wb, and the current in the wire is 4.0 A.



- 13 With the coil in the position shown and with the current flowing as shown, the magnitude and direction of the force acting on side WX is
  - **A** 16 N up **B** 16 N down **C** 1600 N up **D** 1600 N down
- **14** Which one of the following best describes the primary function of the split ring commutator?
  - A It prevents the wires from the power supply becoming tangled.
  - **B** It increases the size of the turning force on the coil.
  - **C** It converts the DC from the battery into an AC voltage.
  - **D** It ensures that the coil rotates continuously in one direction.
- **15** Electrical generators may use slip rings or split ring commutators when generating electricity. When operating at equal frequencies, the output voltages of these two types of generators can be displayed together on an oscilloscope screen.

The output of the split ring commutator is displayed as a dotted line.

The output of the slip rings is displayed as a solid line.

Which one of the following diagrams best represents the two outputs?



VCAA NHT 2021

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#### Short-answer questions

**16** Two blocks, A and B, are being pushed with a force of 60 N to the right as shown below. Block A has a mass of 4.0 kg and block B has a mass of 6.0 kg. Assume the surface is frictionless.



- **a** Calculate the acceleration of block B.
- **b** What is the force that block A exerts on block B? (2 marks)
- **c** What is the force that block B exerts on block A?
- **17** A cyclist of mass 50 kg is riding along the path shown in the diagram. From point A to B the path is a straight line, from B to C the path is circular, and from C to D it is a straight line. The cyclist rides at a constant speed of 10 m s<sup>-1</sup> throughout. Between points B and C, the cyclist is accelerating at 6.0 m s<sup>-2</sup>.



- a Explain how the cyclist can be accelerating between B and C when they are travelling at constant speed. (2 marks)
- **b** Calculate the radius of the circular section between B and C. (2 marks)
- **c** Calculate the magnitude of the force acting on the cyclist in sections AB, BC and CD. (2 marks)
- **18** The diagram below shows the Sun, the Moon and Earth.

The mass of the Sun is approximately  $3.3 \times 10^5$  times the mass of Earth.

The distance from the Sun to the Moon is approximately 390 times the distance from Earth to the Moon.



Calculate the magnitude of the Sun's gravitational force on the Moon and the magnitude of Earth's gravitational force on the Moon. (3 marks)

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(2 marks)

(2 marks)

**19** Two small charges, A and B, are placed 12.0 cm apart in a straight line, as shown below. The charge of 2q at point A exerts a force of  $4.0 \times 10^{-8}$  N on the charge q at point B.



- a Calculate the magnitude and direction of the force of charge *q* at point B on charge 2*q* at point A. (2 marks)
- **b** Determine the distance from point A at which the electric field is zero between points A and B. (2 marks)
- **20** Two electrons, e<sub>1</sub> and e<sub>2</sub>, are emitted, one after the other, from point P in a uniform magnetic field, as shown below.

Both electrons travel perpendicular to the magnetic field, but in opposite directions. Throughout their journey, both electrons remain within the magnetic field.

Electron  $e_1$  travels at twice the speed of  $e_2$ . Relativistic effects can be ignored as both electrons are travelling at low speeds. Electrostatic effects at point P can be ignored as the two electrons are emitted at different times.

$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$
$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$
$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$
$\otimes$	$\otimes$	2v e1		$\otimes$	$\otimes$
$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$
$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$
$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$
$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$

(1 mark)

- a Which one of the following three outcomes occurs?
  Outcome 1: Electron e, returns to point P in the shortest time.
  - Outcome 2: Electron e, returns to point P in the shortest time.
  - Outcome 3: Both electrons take the same time to return to point P.
- **b** Explain your answer to part a.

(3 marks) VCAA NHT 2023 **21** The diagram below shows a schematic of an electron gun.



An electron, initially at rest, is accelerated from the cathode to the anode in the electron gun through a voltage of 1000 V. Take the charge on an electron as  $1.60 \times 10^{-19}$  C and the mass of an electron as  $9.11 \times 10^{-31}$  kg.

**a** Calculate the kinetic energy of the electron as it arrives at the anode. (2 marks)

**b** Calculate the speed of the electron as it arrives at the anode. (2 marks)

**22** Two physics students hold a coil of wire in a constant uniform magnetic field, as shown in the left-hand diagram. The ends of the wire are connected to a sensitive ammeter. The students then change the shape of the coil by pulling each side of the coil in the horizontal direction, as shown in the right-hand diagram. They notice a current register on the ammeter.



- a Will the magnetic flux through the coil increase, decrease or stay the same as the students change the shape of the coil? (1 mark)
- b Explain, using physics principles, why the ammeter registered a current in the coil and determine the direction of the induced current. (3 marks)
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**c** The students then push each side of the coil together, as shown in the diagram below, so that the coil returns to its original circular shape, as shown, and then changes to the shape shown in the right-hand diagram below.



Describe the direction of any induced currents in the coil during these changes. Give your reasoning. (2 marks)

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### HOW HAVE CREATIVE IDEAS AND INVESTIGATION REVOLUTIONISED THINKING IN PHYSICS?

## CHAPTER **7**

UNIT

3/18

# LIGHT: WAVE-LIKE OR PARTICLE-LIKE?

### Introduction

Light plays a crucial role in human life. Through photosynthesis, light sustains all humans and our understanding of light has led to astonishing and wide-ranging applications. Human exploration of the nature of light has been ongoing for at least 60 000 years – Aboriginal and Torres Strait Islander peoples used their knowledge of light to spear fish and construct effective housing.

During the 17th century, Isaac Newton published his particle-like model for light, which, despite challenges, was quickly adopted by the scientific community. In the early 19th century, Thomas Young observed wave-like interference in light. Then in 1862, James Clerk Maxwell proposed that waves made up of vibrating electric and magnetic fields would travel in a vacuum as a transverse wave at  $3.0 \times 10^8$  m s<sup>-1</sup> without the need for a medium; light became established as an electromagnetic wave. However, in the early 20th century, quantum effects that could not be explained by a wave-like model indicated that light had particle-like properties. This chapter explores these dual aspects of light's nature. Next, Chapter 8 considers the properties of matter, showing that it too has wave-like and particle-like properties. Finally, Chapter 9 explores Albert Einstein's special theory of relativity. That chapter concludes by looking at the famous relationship between energy and mass,  $E = \Delta mc^2$ .

### Curriculum

### Area of Study 1 Outcome 1

How has understanding about the physical world changed?

Study Design	Learning intentions – at the end of this chapter I will be able to:
<ul> <li>Light as a wave</li> <li>Describe light as a transverse electromagnetic wave which is produced by the acceleration of charges, which in turn produces changing electric fields and associated changing magnetic fields</li> <li>Identify that all electromagnetic waves travel at the same speed, <i>c</i>, in a vacuum</li> <li>Explain the formation of a standing wave resulting from the superposition of a travelling wave and its reflection</li> <li>Analyse the formation of standing waves (only those with nodes at both ends is required)</li> <li>Investigate and explain theoretically and practically diffraction as the directional spread of various frequencies with reference to different gap width or obstacle size, including the qualitative effect of changing the  \$\frac{\lambda}{W}\$ ratio, and apply this to limitations of imaging using electromagnetic waves</li> <li>Explain the results of Young's double slit experiment with reference to: <ul> <li>evidence for the wave-like nature of light</li> <li>constructive and destructive interference of coherent waves in terms of path differences: <i>n</i>\u03c0 and \$\bigg(n + \frac{1}{2}\u03c0 \u03c0 \u03c0 and \$\u03c0 and \$\u03c</li></ul></li></ul>	7AWave-like properties of light7A.1Explain that accelerating charges produce transverse electromagnetic waves7A.2Describe electromagnetic waves as perpendicular linked oscillating electric and magnetic fields travelling at <i>c</i> in a vacuum7A.3Explain that the superposition of a travelling wave and its reflection results in a standing wave7A.4Analyse standing waves with nodes at each end in terms of speed, wavelength, amplitude, frequency, nodes and antinodes7A.5Investigate and explain diffraction with reference to different gap or obstacle width and frequency7A.6Explain qualitatively the effect of changing the $\frac{\lambda}{w}$ ratio applied to the limitations of imaging with electromagnetic waves7A.7Explain the results of Young's double slit experiment related to the constructive and destructive interference of coherent waves in terms of path differences: $n\lambda$ and $\left(n + \frac{1}{2}\right)\lambda$ 7A.9Explain the results of Young's double slit experiment with reference to the effect of wavelength, distance of screen and slit separation on interference patterns: when $L >> d$ , using $\Delta x = \frac{\lambda L}{d}$

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### Study Design

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#### Light as a particle

• Apply the quantised energy of photons:

$$E = hf = \frac{hc}{\lambda}$$

- Analyse the photoelectric effect with reference to:
  - evidence for the particle-like nature of light
  - experimental data in the form of graphs of photocurrent versus electrode potential, and of kinetic energy of electrons versus frequency
  - kinetic energy of emitted photoelectrons:
     E<sub>k max</sub> = hf φ, using energy units of joule and electron-volt
  - effects of intensity of incident irradiation on the emission of photoelectrons
- Describe the limitation of the wave model of light in explaining experimental results related to the photoelectric effect

## Learning intentions – at the end of this chapter I will be able to:

- **7B Particle-like properties of light 7B.1** Use the formula  $E = hf = \frac{hc}{\lambda}$  for the quantised energy of photons **7B.2** Analyse the experimental results of photelectric experiments for evidence for the particle-like nature of light **7B.3** Analyse the results of photoelectric experimental graphs of photocurrent
- versus electrode potential, and of kinetic energy of electrons versus frequency
  7B.4 Analyse the results of photelectric experiments with respect to the kinetic energy of emitted photoelectrons:
- *E*<sub>k max</sub> = hf φ, using energy units of joule and electronvolt
  7B.5 Analyse the results of photoelectric
- experiments with respect to the effects of intensity of incident irradiation on the emission of photoelectrons
- **7B.6** Describe the limitations of the wave model of light in explaining experimental results related to the photoelectric effect

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### Glossary

- Amplitude Antinode Coherent Collector electrode Constructive interference Destructive interference Diffraction Dispersion Electromagnetic wave Electroscope Harmonic series
- In phase Longitudinal wave Microwave Node Photocell Photocell Photoelectric effect Photoelectron Photon Photon Polarisation Quanta
- Quantisation Refraction Refractive index Standing wave Stopping potential Superposition Threshold frequency Transverse wave Wavelength Wavelet Work function ( $\phi$ )

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See the Interactive Textbook for an interactive version of this concept map interlinked with all concept maps for the course.

Electroscope collapses

as electrons leave zinc

Electroscope with

negative charge



## Wave-like properties of light

#### **Study Design:**

- Describe light as a transverse electromagnetic wave which is produced by the acceleration of charges, which in turn produces changing electric fields and associated changing magnetic fields
- Identify that all electromagnetic waves travel at the same speed, *c*, in a vacuum
- Explain the formation of a standing wave resulting from the superposition of a travelling wave and its reflection
- Analyse the formation of standing waves (only those with nodes at both ends is required)
- Investigate and explain theoretically and practically diffraction as the directional spread of various frequencies with reference to different gap width or obstacle size, including the qualitative effect of

changing the  $\frac{\lambda}{w}$  ratio, and apply this to limitations

of imaging using electromagnetic waves

- Explain the results of Young's double slit experiment with reference to:
  - evidence for the wave-like nature of light
  - ► constructive and destructive interference of coherent waves in terms of path differences:  $n\lambda$ and  $\left(n + \frac{1}{2}\right)\lambda$  respectively, where n = 0, 1, 2, ...
  - effect of wavelength, distance of screen and slit

separation on interference patterns:  $\Delta x = \frac{\lambda L}{d}$ when L >> d

#### **Glossary:**

Amplitude Antinode Coherent Constructive interference Destructive interference Diffraction Dispersion Electromagnetic wave Harmonic series In phase Longitudinal wave Microwave Node Polarisation Refraction Refractive index Standing wave Superposition Transverse wave Wavelength Wavelet



### ENGAGE

### Understanding light over time

Humankind's quest to understand the nature and properties of light, and all of the electromagnetic spectrum, has a long and continuing history. From an early understanding of the properties of reflection, refraction and vision through to the insights of Greek and Islamic scientists and the



## International Day of Light

16 May

inventions of telescopes and spectacles, the modern era has produced a multitude of applications. These accomplishments are now recognised on the International Day of Light on 16 May each year; the anniversary of the first successful laser.

The laser is an icon of the modern era of light-based technologies and it has found applications in an extraordinary number of fields. In medicine, it is used in retinal repair, cauterising and laser 'scalpels' that can make incisions as narrow as  $5 \times 10^{-4}$  m. Some dentists use lasers instead of drills for many procedures, including gum shaping. We have all used DVD readers and barcode scanners at the supermarket; you may have seen laser printers and surveying and distance measuring in operation using lasers. One application of distance measurement is the monitoring of the Earth–Moon distance, currently accurate to 15 cm. Cutting and welding with lasers is now common in heavy engineering as well as garment construction. Optical fibres use infrared laser light, and 3D laser scanning can capture information in digital form to share globally. LIDAR (Light Detection and Ranging) mapping can capture detailed topography. It is a remote sensing method that uses light in the form of a pulsed laser to measure ranges. LIGO (Laser Interferometer Gravitational-wave Observatory) detectors use laser interferometry to capture the subtleties of gravitational waves from the universe. Finally, lasers can now enable autonomous weeding in agriculture, and poultry farmers can detect the sex of unhatched eggs after only nine days of incubation.





## EXPLAIN

### **Electromagnetic waves**

Following Maxwell's pioneering work in 1826, when he showed mathematically that the laws of electricity and magnetism predicted the possibility of **electromagnetic waves**, the scientific community of the time became convinced that the nature of light was now understood. Accelerating charges produce changing (oscillating) electric fields, which in turn produce changing (oscillating) magnetic fields. The electric and magnetic oscillating fields were arranged as shown in Figure 7A–2, perpendicular to each other and also to the direction of travel. They form a **transverse wave**.

## Electromagnetic wave

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a transverse wave consisting of perpendicular oscillating electric and magnetic fields that travel at  $3.00 \times 10^8 \,\mathrm{m\,s^{-1}}$ in a vacuum with a range of wavelengths from  $10^{-18} \,\mathrm{m}$  to more than  $10^4 \,\mathrm{km}$ 

#### Transverse wave

a wave in which vibrations are perpendicular to the direction of travel







Figure 7A-2 Snapshot of an electromagnetic wave. The magnetic and electric fields oscillate and are perpendicular to each other and to the direction of travel, forming a transverse wave.

All such electromagnetic waves travel at the speed of light, c, in a vacuum and obey the general wave equation given by Formula 7A-1.

#### Formula 7A–1 The wave equation

 $c = f\lambda$ 

Wavelength the distance between repeated parts of a wave shape, measured in metres (m)

#### Where:

c = Speed of light in a vacuum,  $3.0 \times 10^8 \text{ m s}^{-1}$ f = Frequency (Hz)  $\lambda =$  Wavelength (m)

In a medium they travel more slowly. The speed is the given by the relationship in Formula 7A-2.

Formula 7A–2 The wave equation inside a medium

 $v = \frac{c}{c}$ n

#### **Refractive index**

a measure of how much slower light travels through a medium compared to a vacuum; given the symbol n

Where:

v = Speed of light inside the medium (m s<sup>-1</sup>)

- *n* = **Refractive index** of the medium
- c = Speed of light in a vacuum,  $3.00 \times 10^8 \text{ m s}^{-1}$



Figure 7A–3 James Clerk Maxwell's work had a huge impact on physics, setting the basis for electromagnetic theory and putting Einstein on the path towards special relativity. Einstein is reported as saying: 'There would be no modern physics without Maxwell's electromagnetic equations; I owe more to Maxwell than to anyone.'

ISBN 978-1-009-25901-9

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### Measuring the speed of light

The speed of light was not measured for many years; many thought it was impossible because it travelled 'instantaneously'. Then, in 1676, Rømer used the orbit time of Io, a moon of Jupiter, to estimate the speed of light. The time of Io's orbit is constant, but measured from Earth, there was a puzzling variation. When Earth is closer to Jupiter, the orbit time of Io is smaller than when Earth is further from Jupiter. Rømer estimated that the largest time difference was about 22 minutes (we now know it's closer to 16.7 minutes). Rømer saw that this was due to the time light took to cross the diameter of Earth's orbit around the Sun  $(3.00 \times 10^{11} \text{ m})$ . Using the modern value of the orbit time this gives a speed of  $3.00 \times 10^8 \text{ m s}^{-1}$ .

### Check-in questions – Set 1

- 1 What is the speed of all electromagnetic waves in a vacuum?
- 2 Will red light travelling in clear glass travel faster or slower than in a vacuum?
- **3** What is at right angles to both the oscillating electric field and the oscillating magnetic field in an electromagnetic wave?

### Standing waves in a stretched string

**Standing waves** in a stretched string can be produced when equal waves, travelling in opposite directions, meet. This can be demonstrated by fixing one end of a string and vibrating the other end, so that the reflected wave overlaps with the original wave. When the frequency of the wave meets certain conditions, a standing wave (sometimes called a stationary wave) is formed, as in Figure 7A–4.



Figure 7A-4 Standing wave in a stretched string

The superposition of the wave with the reflected wave forms a regular pattern of nodes (points of minimal oscillation), shown at points N, and antinodes (points of maximum oscillation midway between nodes), shown at points A. Note that the two ends of the standing wave in this example are both nodes. (There are other types of standing wave, with antinodes at the ends, but these are beyond the scope of this book. Every point on the standing wave (except the nodes) oscillates vertically; the wave shape does not travel to the left or to the right. At the antinodes, the amplitude of the oscillation is at a maximum.

The conditions for the formation of this kind of standing wave, with nodes at both ends, is that the length of the string must be equal to a whole number of half wavelengths. It follows that the regular spacing of the nodes will be  $\frac{\lambda}{2}$ , and it will be the same for the antinodes.

#### Standing wave a wave that oscillates in

oscillates in time but whose amplitude profile does not move in space

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#### Superposition

the principle that when two or more waves overlap, their displacements add together

#### Node

point of minimal disturbance in an interference pattern

#### Antinode

point of maximum disturbance in an interference pattern

#### Amplitude

the maximum distance of an oscillation from its midpoint

#### Another type of standing waves with nodes at each end

Standing waves with nodes at each end can also form with **longitudinal waves**. The principles are the same; there must be a series of nodes spaced at a whole number of half wavelengths apart, and midway between the nodes as regularly spaced antinodes, also spaced the same distance apart. An example of a longitudinal standing wave would be a sound wave in a column or pipe of air. If the pipe is open at both ends, then the pressure at each end can be considered as a node.

# Formation of standing waves in a stretched string with nodes at both ends

A rope is attached to a post at one end and the other is shaken up and down to produce a standing wave, shown here in multiple positions. When the wave has a peak on the left, it has a trough on the right. The peak goes down on the left and up on the right, until the wave looks reversed. The diagrams in Figure



Figure 7A–5 Standing waves in multiple positions

7A–6 explain how the addition of the forward wave and the reflected wave interact to form the standing wave pattern. Figure 7A–6 shows a string being shaken at the left-hand end. The right-hand end is fixed and reflects the wave back. At one instant (Figure 7A–6a) the incident and reflected waves will be in in step (in phase), reinforce each other, and produce the shape on the left. This is an example of constructive interference. The centre point is not displaced, it is a node. Later the two waves will be out of step (out of phase) (Figure 7A–6b) and produce the flat shape on the left. This is an example of destructive interference. Later still, the shape shown in Figure 7A–6c will be produced. The principle of superposition – the adding up of waves as they overlap – shows how nodes and antinodes are formed. The antinode positions are shown with the letter A.



Figure 7A-6 Diagrams a to c show particular positions of the string in Figure 7A-5 as it is shaken up and down (left), forming nodes and antinodes (A) in a standing wave. The right-hand diagrams explain this as the superposition of forward and reflected waves.



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Longitudinal wave a wave in which

direction of travel

vibrations are parallel to the

7A WAVE-LIKE PROPERTIES OF LIGHT

# Analysis of standing waves with nodes at both ends

The simplest standing wave that can be formed is when the distance between the two

nodes is  $\frac{\lambda}{2}$  of the length (*l*) of the rope or

stretched string. For the case of a longitudinal wave, such as a sound wave in a pipe of air open at both ends, the result is the same. This standing wave, the first harmonic, is often called the fundamental. The frequency of the fundamental (often written as  $f_1$ ) can be found from the wave equation:

$$f_1 = \frac{\nu}{\lambda} = \frac{\nu}{2l}$$

The next most simple case (shown in Figure 7A–7) is when the length of the rope is  $l = \lambda$  and the frequency  $f_2$  is given by:

$$f_2 = \frac{v}{l}$$

Fourth harmonic  $f_4$ Figure 7A–7 The first four harmonics of a standing wave in a

There is a series of these standing waves, each

with its own frequency; the first four are shown in Figure 7A–7. These standing waves are known as a harmonic series.

The equation for the *n*th harmonic is shown in Formula 7A–3.

Formula 7A–3 Harmonic wave equation

$$f_{\rm n} = \frac{nv}{2l}$$

Where:

n = An integer  $\nu = \text{Speed of the forward wave (m s^{-1})}$ l = Length of the stretched string (m)

From this it follows that the frequencies of the harmonics are a multiple of the fundamental  $f_1$ . They are easy to demonstrate with a jump rope or many musical instruments. They can also be set up using light reflected in a laser as shown in Figure 7A–8.



Figure 7A-8 Laser standing wave



**Figure 7A–7** The first four harmonics of a standing wave in a stretched string

Harmonic series the set of frequencies consisting of a fundamental (the first harmonic) and the harmonics related to it by an integer multiple of the fundamental

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### Check-in questions – Set 2

- 1 When are standing waves produced?
- 2 What are nodes and antinodes in a standing wave?
- **3** What is the other name given to the first harmonic?
- 4 Can longitudinal waves form standing waves or only transverse waves?
- 5 What is the principle of superposition?

and the beam will spread over 180°.

### **Diffraction of light**

When sound waves and water waves pass through a narrow opening, or around an obstacle, they spread out. This phenomenon is called **diffraction** and also occurs with light, although it is a little harder to observe.



Figure 7A–9 Diffraction of a laser beam through a narrow slit

The width of the beam on the screen will be proportional to  $\frac{\lambda L}{w}$ . If  $\lambda \ll w$ , the diffraction will be negligible. On the other hand, if  $\lambda \ge w$ , the slit will behave like a point source of light,



**Figure 7A–10** Effect of slit width (*w*) on diffraction. The three diagrams show wavefronts (red) of light with different wavelengths moving from left to right through slits of different sizes. The labels indicate the relative sizes of wavelength and slits.

In between these extremes, it will depend on the sensitivity of the observing method or instrument and the size of *L*. As a rough guide, when  $\frac{\lambda}{w} = 0.01$ , the beam will spread out at about 1°. Sometimes single slit diffraction will be described using a graph of intensity plotted against position on the screen, as shown in Figure 7A–11.

Diffraction the spreading of a wave when passing through a narrow opening or passing around an object

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Position on the screen

**Figure 7A–11** Intensity representation of diffraction from a single slit, with a diagram showing diffraction and (right) the resulting graph of light intensity against position on the screen.



**Figure 7A–12** Diffraction around objects. Wavefronts are shown moving from top to bottom, for different ratios  $\frac{\lambda}{h}$ .

### Separating close objects

You may have been travelling at night along a straight road when you see a vehicle approaching some distance away. At first, all you can see is one headlight and you can't tell whether it is a car or a motorbike. If it's a car, you will be able to see two headlights when it comes close enough. The original images were overlapping on your retina due to diffraction as the light passed through the pupil of your eye. The situations are shown in Figure 7A–13.





Distant headlightsIntermediate headlightsFigure 7A-13Separating headlights of a vehicle as they approach



Close headlights

#### CHAPTER 7 LIGHT: WAVE-LIKE OR PARTICLE-LIKE?





For two light sources, the diffraction pattern can be represented graphically as in Figure 7A-14 These light intensity versus position graphs are the same type as shown in Figure 7A-11, but with the usual orientation, having intensity as the vertical axis.



**Figure 7A–14** Graphs of light intensity against position showing the diffraction pattern of two light sources (such as the headlights in Figure 7A–13), with dashed curves for the pattern for each source shown separately: left: at distance, with overlapping images; centre: at intermediate distance with images just separated; right: close, with separated images.

#### Diffraction and telescopes

Diffraction occurs in astronomical telescopes, even those in space, like the Hubble or James Webb telescopes. The bottom star of the Southern Cross, Alpha Crucis, is in fact a double star; a small telescope can resolve the two, but seen with the naked eye through an aperture (the pupil) of diameter about 5 mm, it appears to be a single star because the diffracted images of the two stars overlap each other due to the small size of the human pupil.



Figure 7A-15 The Southern Cross with Alpha Crucis (overlapping)

Figure 7A-16 Alpha Crucis (separated)

We can use the  $\frac{\lambda}{w}$  ratio to understand why even a small telescope can separate this double star when the eye cannot. The eye has a much smaller aperture (*w*) than telescopes. In fact, the bigger the telescope, the smaller the diffraction. There are many terrestrial reflecting telescopes with diameters greater than 10 m. The other way to reduce the diffraction is to observe at shorter wavelengths (for example, in the ultraviolet rather than the visible wavelength range). The Hubble Space Telescope, shown in Figure 7A–17, has an aperture of only 2.4 m but observes in ultraviolet light as well as in visible light. The more recent

James Webb Space Telescope, shown in the same figure, observes in infrared and visible light with an aperture of 6.5 m. Although they have smaller apertures than the biggest terrestrial telescopes, being above the atmosphere means neither of these space telescopes suffers from image distortion caused by light passing through turbulent air, the same phenomenon that makes stars twinkle.



Figure 7A-17 Hubble Space Telescope (left) and James Webb Space Telescope (right)

### Another approach to diffraction

More than a century before the wave model for light was becoming established, the scientist Huygens proposed a principle for the propagation of light. He suggested that every point on a light wavefront acts like a source of secondary **wavelets**, and the envelope of all these wavelets forms the new wavefront. These are shown in Figure 7A–18 for a plane wave.

#### Wavelet

a wave-like phenomenon that begins with zero amplitude

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Points on original wavefront

Figure 7A–18 Propagation of plane wavefront using Huygens wavelets model

If the plane wavefront reaches an opening, the wavefront will spread, or diffract, as shown in Figure 7A–19.



Figure 7A–19 Huygens wavelet explanation of diffraction

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### Check-in questions – Set 3

- 1 If the distance between the observing screen and the aperture is fixed, what two factors does the spread of diffraction depend on?
- 2 What will cause diffraction to spread out through 180°?
- **3** Which part of the visible electromagnetic spectrum gives the best separation of closely-spaced distant light-emitting objects?

### Worked example 7A–1 Problems using diffraction

Microwave an electromagnetic wave with wavelength between 1 mm and 1 m Students working with 3.0 cm wavelength **microwaves** set up a single slit experiment as shown below. They direct the microwaves at a slit of width, *w*.



The width of the diffraction pattern measured by the receiver is 20 cm. They then change the microwave frequency so that  $\lambda = 6.0$  cm. What effect will this have on the width of the diffraction pattern?

### Solution

For a fixed distance to the receiver, the width of the diffraction pattern is proportional to

 $\frac{\pi}{w}$ . As the width of the slit, *w*, is also fixed, from the relation you can see that increasing

the wavelength will also increase the width of the diffraction pattern. So, the effect is a broader diffraction pattern.

### The Poisson spot

Very careful observations of the shadow cast by a circular object reveal a bright spot in the very centre of the shadow, called the Poisson spot.



Figure 7A–20 The experimental arrangement for viewing the Poisson spot (left) and the wave explanation of Poisson spot (right)

7A WAVE-LIKE PROPERTIES OF LIGHT

Light that diffracts from the edge of the object is all at an equal distance from the centre of the shadow, which means that the wavefronts of light meet at the centre at the same time, adding constructively together to create a bright spot.

The appearance of the shadow and the central spot can be seen in Figure 7A–21. Note the interference fringes around the edges; these come from light diffracting from the edges resulting in path differences. These will be referred to in Chapter 8.

The spot is named after Poisson, who thought that light was particlelike and that such a bright spot in the middle of a shadow should be impossible. Unfortunately, he had backed the wrong theory and the spot (first demonstrated by Arago) became clear evidence that light's behaviour was wave-like.

### Young's double slit experiment

Debate about the nature of light had been around since at least the 5th century BCE. At the start of the 19th century, Newton's particle-like theory was supported by many leading scientists, possibly because of Newton's great reputation.

Newton believed that light was composed of coloured particles, which he called 'corpuscles', that travelled at great speed in straight lines. His theory explained reflection and refraction (provided that the particles travelled faster in media like glass and water), explained the dispersion of white light into the colours of the spectrum and had the advantage over wave-like theories that no medium was required for light to travel through.

However, there were proponents for light's wave-like nature, (including Huygens, Hooke, Fresnel and Young), and it was Young's famous double slit experiment that turned the tide of scientific opinion. It established light's wave-like nature as the accepted theory in the early 19th century, especially after Maxwell's explanation of its nature and speed. This scientific agreement lasted about a century until the birth of quantum physics in the early 20th century.

In his experiment, Young allowed a narrow beam of sunlight to be cut in two by a card, forming two light sources as shown in Figure 7A–22.



Figure 7A-22 Young's original double slit experiment



Figure 7A–21 The Poisson spot appearing at the centre of a shadow formed by a circular object



### Between light And matter

**8B** SIMILARITIES

#### Refraction

the change in direction of a wave moving from one medium (or vacuum) to another medium (or vacuum) caused by the wave changing speed

#### Dispersion

the separation of white light into its different colours

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When the light from these sources overlapped (because of diffraction), a series of dark and bright bands were produced, with the bright fringes edged with colours, as in Figure 7A–23.

These experimental results could not be explained by Newton's coloured particles theory; a wave-like model was needed. Modern versions of Young's experiment generally use monochromatic laser light instead of a sunlit pinhole, resulting in a fringe pattern like that shown in Figure 7A–24. Note that laser light is not needed to observe these interference patterns; the laser just produces a brighter pattern.



**Figure 7A–23** Interference fringe pattern from white light



Figure 7A-24 Interference fringe pattern with monochromatic red light

### Analysis of Young's double slit experiment

A more modern arrangement with a laser and two slits is shown in Figure 7A-25.





**Figure 7A–25** Young's double slit experiment with monochromatic light from a laser with a diverging beam. The separation of the horizontal slits, *d*, is approximately 0.5 mm and the slit to screen distance, *L*, could be several metres. The green curve shows the intensity of the light falling on the screen where the two beams overlap, the peaks forming light bands and the troughs forming dark ones.

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A green ( $\lambda \sim 550$  nm) laser pointer is commonly used. Suitable slit separations, *d*, are approximately 0.5 mm, and the slit to screen distance, *L*, could be several metres. Several methods of producing the double slits can be found, including modifying Young's method with a single fine wire or hair cutting the middle of the laser beam.

At point C, the centre of the pattern, there is a bright band. This is because the light waves from slits  $S_1$  and  $S_2$  travel exactly the same distance and arrive in phase. The two light intensities add up, by the principle of superposition. This is constructive interference.

There is also a bright band at point X, but here the distance  $S_2X$  is greater than the distance  $S_1X$  by a whole number of wavelengths and the waves arrive in phase. Looking at the intensity pattern it is clear that the path difference  $S_2X - S_1X$  must be equal to  $3\lambda$ , as point X is the third maximum from the centre.

Dark bands are formed when waves from the slits arrive out of phase with each other and cancel (similar to when a compression in a sound wave meets a rarefaction, or a crest in a water wave meets a trough). This is **destructive interference**.

Destructive interference occurs when there is a path difference of  $\frac{\lambda}{2}$ ,  $\frac{3\lambda}{2}$ ,  $\frac{5\lambda}{2}$  and so on.

For example, at point Y, the second dark band from the centre, the path difference

$$= S_1 Y - S_2 Y = \frac{3\lambda}{2}.$$

In summary, bright bands are formed when the path difference =  $n\lambda$ , and dark bands are

formed when the path difference =  $\left(n + \frac{1}{2}\right)\lambda$ , where *n* is an integer greater than or equal to 0 (i.e. 0, 1, 2, 3... and so on).

The spacing between the bright bands on the screen,  $\Delta x$ , can be found by using Formula 7A–4.

### Formula 7A-4 Interference pattern width

$$\Delta x = \frac{L\lambda}{d}$$

Where:

 $\Delta x$  = Distance between bright bands (dark bands)

L = Distance between the slits and the screen

d = Distance between the slits

 $\lambda$  = Wavelength of light used

This formula holds as long as  $d \ll L$  and is useful as it relates the spread of interference patterns to wavelength, screen distance and slit separation.

#### Coherent and incoherent waves

Two wave sources are said to be **coherent** if they are monochromatic and there is no phase difference between them (or a fixed phase difference). Light from an incoherent source, such as an incandescent globe or even an LED, emit light waves with random phase relationships to each other. Light from a laser is coherent; all the light is monochromatic and in phase. Young's double slit experiment only displays clear light and dark bands if the two light sources are at least partially coherent. The light and dark bands are clearest

In phase

if two waves coincide with peaks and troughs matching, they are said to be in phase

#### Constructive

interference occurs when two identical waves are in phase and meet, doubling the amplitude of one wave

## Destructive interference

occurs where two identical waves are completely out of phase and meet, cancelling each other

#### Coherent

light sources that are monochromatic with a fixed or zero phase difference between them if a fully coherent light source, like a laser, is used. If two tiny LEDs (or tiny incandescent globes) were set up to show an interference pattern, the result would be a failure. However, passing the light from one LED through a single narrow slit introduces a degree of coherence, enough to show the wave-like results if this light is then shone onto a double slit. The problem does not exist with water waves in a ripple tank or sound from two loudspeakers (if the speakers are connected to the same sound source) because such sources are coherent.

lacksquare

### Worked example 7A–2 Using Young's double slit experiment

Students set up a Young's double slit experiment and observe the interference pattern as shown below. The paths of the light rays are not shown. C is the central bright band of the interference pattern.



- **a i** Give reasons why the light intensity at point P is a minimum.
  - ii Determine the path difference  $S_2P S_1P$  for point P.
- **b** They measure the average spacing between the bright bands to be 2.0 mm, the length, *L*, between the slits and screen to be 1.6 m and the slit spacing, *d*, to be 0.5 mm. Calculate the wavelength of the laser light.

#### Solution

- **a i** As the band is dark, point P is a point of destructive interference in the diffraction pattern. These points are also points of minimum light intensity as the light waves effectively cancel each other.
  - ii From above, you know that dark bands occur at points where the path difference  $S_2P S_1P = \left(n + \frac{1}{2}\right)\lambda$  (for n = 0, 1, 2, ...). As point P is the second dark band from the central point, C, you have that n = 1. Therefore, the path difference must be

$$S_2 P - S_1 P = \frac{3}{2}\lambda.$$

b Calculate the wavelength of light from the band spacing using Formula 7A- 4.
 Rearrange to make λ the subject:

$$\lambda = \frac{\Delta x}{L}d$$

From the question  $\Delta x = 2.0$  mm, L = 1.6 m and d = 0.5 mm. Substitute these values into the above equation:

$$\lambda = \frac{0.002}{1.6} \times 0.0005 = 625 \times 10^{-9} \,\mathrm{m} = 625 \,\mathrm{nm}$$

Hence, the wavelength of light is measured to be 625 nm using the above apparatus.

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### Check-in questions – Set 4

- 1 Where does constructive interference occur in a double slit interference pattern?
- **2** Where does destructive interference occur in a double slit interference pattern?
- 3 What does it mean to say that two equal waves are in phase at a point in space?
- 4 What is meant by the term 'coherence'?

### Evidence for the wave-like nature of light

It is clear that the interference patterns observed in Young's double slit experiment provided powerful experimental evidence for the wave-like nature of light, as it duplicated the behaviour of other well established wave motions such as sound waves and water waves.

For example, a double slit experiment can be set up with sound by replacing the two slits with two loudspeakers producing the same frequency note in phase (i.e. in step, with peaks and troughs matching), as shown in Figure 7A–26. The experiment is best done outdoors in a park or on a school oval to minimise confusing reflections. The observers can move along the line AB with either a sound level meter or with one ear blocked. They will encounter a series of nodes (points of minimal sound intensity) and antinodes (points of maximum sound intensity). At the point where they are equidistant from the two loudspeakers (C), there should be an antinode.



Figure 7A–26 Young's experiment with sound waves: two loudspeakers playing the same note

The experiment can also be performed with water waves in a ripple tank, shown in Figure 7A–27, with point C equidistant from the dippers.

Two 'dippers' are connected to an oscillator so that they produce circular wave fronts that are in phase and are of equal frequency, wavelength and amplitude. These wavefronts overlap and interfere. At the far end of the tank, nodes and antinodes can be seen. At the centre of the tank, at point *C*, equidistant from dippers  $S_1$  and  $S_2$ , is the central antinode. The path difference for the waves





arriving here is zero, so constructive interference occurs. Either side of the central antinode

are two nodes; these have path differences of  $\frac{\lambda}{2}$ , so destructive interference occurs here.

The pattern is the same as for the two slit experiment with light.





WORKSHEET 7A-1 WAVE-LIKE DOC PROPERTIES OF LIGHT

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Polarisation a property of transverse waves describing the orientation of their oscillations; in transverse waves, this is perpendicular to the wave motion direction

### Other evidence for the wave-like nature of light

As well as the results of Young's experiment, proponents of the wave-like model claimed that it explains why light travels in straight lines, why light diffracts, why light beams pass through each other without being affected, reflection and refraction, polarisation and the dispersion of white light into the colours of the spectrum. But the medium through which these waves passed remained unexplained. Even Maxwell's brilliance left this somewhat of a mystery. However, the followers of Newton's particle-like model claimed that, without the need for a medium, they could explain most of the list above, except Young's double slit experiment, polarisation, diffraction and refraction. This last issue was because Newton's particle-like model required light to travel faster in water than in air – and experiments showed that it travels slower.

#### The genius of Thomas Young

Young is famous for his double slit experiment (and the invention of the ripple tank) but his overall contributions to human knowledge are breathtaking. In the field of human vision, he was the first to work out how the eye focuses and how it perceives colour with three different kinds of cells responding to different frequencies. He was the first to work out the equation for the surface tension of water. He was the first to measure the size of a blood corpuscle. In engineering, 'Young's modulus' is based upon his work and he did extensive research on both the strength of materials and hydraulics



(including blood circulation). Outside of the fields of science and engineering, he was a medical practitioner, made possibly the major contribution to the translation of the Rosetta stone, was fluent in Latin, Greek, Hebrew, Italian and French, and familiar with Chaldean and Syriac.

A 2006 biography of Thomas Young is titled, The Last Man Who Knew Everything.

VIDEO 7A-2 SKILLS: EVIDENCE FOR THE WAVE-LIKE NATURE OF LIGHT

### **7A SKILLS**

#### Evidence for the wave-like nature of light

When asked to explain why the results of Young's double slit experiment give evidence for the wave-like nature of light, you need to compare the behaviour of light shown by this experiment with the parallel behaviour of other undisputed wave motions, such as sound waves and water waves. At the time, most scientists did not believe that light was wave-like in its nature, but there was no such dispute about sound waves and water waves.

Depending on the wording of the question, you may also need to indicate that the two light sources formed by the two slits need to be at least partially coherent for the wave behaviour to be observed; these days most people use a laser, which ensures full coherence, but Young did not have access to such a device! Two slits in front of a narrow light source, such as the narrow beam of sunlight used by Young, ensures enough coherence to show interference bands.

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### **Section 7A questions**

### **Multiple-choice questions**

- 1 Which of the following best describes the nature of light rays?
  - A transverse electric and magnetic fields parallel to the direction of the rays
  - **B** changing electric and magnetic fields perpendicular to the direction of the rays
  - **C** longitudinal electric and magnetic fields at right angles to the rays
  - D self-generating electric fields perpendicular to the direction of the rays
- 2 Electromagnetic waves can be produced by which of the following?
  - A changing electric fields generated by accelerating charges
  - B charges moving at constant speeds with no acceleration
  - **C** relativistic neutrons
  - D transverse high intensity electric and magnetic fields
- 3 Which of the following is a correct statement about the speed of light?
  - A It is always equal to *c*.
  - **B** In a vacuum, it depends on the frequency.
  - **C** It is affected by local magnetic fields.
  - **D** It depends on the medium it is travelling through.

#### Use the following information to answer Questions 4 and 5.

Tyler plucks a guitar string, causing it to vibrate as shown in the diagram below. Two extreme positions of the resulting standing wave in the string are shown. In the diagram, the amplitude of the vibrations has been exaggerated.



- **4** Which one of the following statements best indicates how to interpret the motion of the guitar string?
  - A It is the result of two equal waves travelling along the string in the same direction.
  - **B** It is the result of two equal waves travelling along the string in opposite directions.
  - **C** It is the result of two unequal waves travelling along the string in the same direction.
  - **D** It is the result of two unequal waves travelling along the string in opposite directions.

#### Adapted from VCAA 2017

- **5** S is a point on the guitar string. For the instant immediately after that shown above, the direction in which point S on the guitar string will move is
  - A upwards.
  - **B** to the left.
  - **C** to the right.
  - D downwards.

Adapted from VCAA 2017

6 An experiment investigating the diffraction of light is sketched schematically below.



Which of the following would best increase the width of the diffracted beam on the screen?

- A doubling the value of *L* and doubling the value of *w*
- **B** doubling the value of  $\lambda$  and halving the value of *w*
- **C** halving the value of  $\lambda$  and doubling the value of *w*
- **D** doubling the value of *L* and halving the value of  $\lambda$
- 7 When diffraction of white light takes place around an object, you should observe
  - A all the colours of the spectrum diffracting by the same amount.
  - **B** red frequency light diffracting more than blue frequency light.
  - **C** blue frequency light diffracting more than red frequency light.
  - **D** that the size of the object will determine which colours diffract the most.
- 8 The formation of light and dark interference fringes on a screen when coherent monochromatic light passes through a double slit is due to
  - A light from the slits travelling different distances when they arrive on the screen.
  - **B** the inability of the slits to diffract light into the darker areas.
  - **C** the light from each slit interfering before they arrive at the screen.
  - **D** different frequencies of light interacting constructively and destructively.
- **9** Students set up a two slit interference experiment with a green laser as shown in the diagram below.



Which of the following would occur if the green laser was replaced by a red laser?

- A The pattern would be replaced by broader fringes at the same spacing.
- **B** The spacing,  $\Delta x$ , would remain the same and the fringes would become thinner.
- **C** The spacing,  $\Delta x$ , would decrease and there would be little change in the fringes.
- **D** The spacing,  $\Delta x$ , would increase and there would be little change in the fringes.

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### **Short-answer questions**

- **10** Describe how electromagnetic waves can be produced by electrons.
- **11** Assess the following statement:

It is possible for light to travel at speeds less than or greater than *c*.

**12** Students experiment with the diffraction of light. A beam of monochromatic light falls on a small circular aperture. A diffraction pattern forms on the screen as shown in the diagram.



The students then change some parameters of the experiment, one at a time. Describe the effect that each step should produce.

- a First, they increase the distance between the aperture and the screen.
- **b** Next, they reduce the diameter of the circular aperture.
- **c** Finally, they increase the frequency of the light.

Adapted from VCAA 2014

**13** A Young's double slit experiment is carried out with a narrow beam of white light. The apparatus is shown in the diagram below.



The resulting fringe pattern is shown on the right.

Students notice the coloured edges to the bright bands.

- a Explain why the white central band does not have coloured edges.
- **b** Explain why the bands on the left have a red edge on the left and the bands on the right have a red edge on the right.



14 Physics students studying interference set up a double slit experiment using a 610 nm laser, as shown in the diagram below.



A section of the interference pattern is shown below. There is a bright band at point C, the centre point of the pattern.



- **a** Explain why point C is a bright band rather than a dark band.
- **b** Another point on the pattern to the right of point C is further from  $S_1$  than  $S_2$  by a distance of  $2.14 \times 10^{-6}$  m. Mark this point on a copy of the diagram by writing an X above the point. Use a calculation to justify your answer.

VCAA 2018



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## Particle-like properties of light

### **Study Design:**

- Apply the quantised energy of photons:  $E = hf = \frac{nc}{r}$
- Analyse the photoelectric effect with reference to:
  - evidence for the particle-like nature of light
  - experimental data in the form of graphs of photocurrent versus electrode potential, and of kinetic energy of electrons versus frequency
  - kinetic energy of emitted photoelectrons:
     E<sub>k max</sub> = hf φ, using energy units of joule and electron-volt
  - effects of intensity of incident irradiation on the emission of photoelectrons
- Describe the limitation of the wave model of light in explaining experimental results related to the photoelectric effect

#### **Glossary:**

Collector electrode Electroscope Photocell Photocurrent Photoelectric effect Photoelectron Photon Quanta Quantisation Stopping potential Threshold frequency Work function ( $\phi$ )

### ENGAGE The quantum world

Today we live in a world increasingly dominated by quantum devices. It would be hard to imagine life without digital devices, silicon chips, lasers, optical fibres, GPS and MRI technology. Yet these are all dependent on applications of quantum theory.

Late in the 19th century, before any quantum physics had developed, most physicists thought that all the major problems of physics were solved. It was thought only a few issues remained unsolved. One of these was the emission of electromagnetic radiation from hot objects like stars; this was called the blackbody problem. Another was the emission of electrons from metal surfaces when struck by light with a high enough frequency; this was called the photoelectric effect. Both of these could not be solved using the physics of the day. The first of these, the blackbody problem, was explained by Planck by



Figure 7B–1 Just like Schrödinger's famous thought experiment, particles can exist in two states at the same time

assuming that energy had to come in discrete quantities, which he called quanta. This was at variance with the then contemporary wave-like view of electromagnetic radiation. Einstein was able to explain the experimental results of the photoelectric effect by using Planck's idea and applying it to light falling on metal surfaces.

However, the quantum world is strange. Waves can behave like particles and particles can behave like waves. Schrödinger's famous thought experiment of a cat in a box containing a life-threatening substance can be thought to be alive and dead at the same time – until the box is opened. Likewise, a particle can exist in two states at the same time – until it is measured.

Just as strange, quantum particles can appear to exhibit connection at a distance even when there is no physical interaction.



VIDEO 7B-1 PARTICI F-I IKF PROPERTIES OF LIGHT

#### Photoelectric effect

the phenomenon in which light can release an electron from a metal surface

#### Quantisation

the concept that a physical quantity can have only certain discrete allowable values

### **EXPLAIN**

### The photoelectric effect

The **photoelectric effect** is the phenomenon in which shining light onto a metal surface causes electrons to be emitted from the surface. This phenomenon puzzled the physics community for many years until Einstein explained it using the quantisation of light. Einstein's explanation of the mechanism of the photoelectric effect proved a landmark in the development of quantum physics. Hertz is credited with observing the effect in 1887 as a side effect of his work on radio waves. Lenard identified that electrons were being emitted from metal surfaces in 1902. The effect can be demonstrated with very simple equipment as shown in Figure 7B–2.





Figure 7B-2 Demonstration of photoelectric effect with a zinc plate electroscope and a UV lamp. When the lamp is off and the zinc plate is negatively charged, the gold leaf moves away from the stand due to the repulsion of the negative charges. When UV radiation shines on the plate, it drives off the electrons, causing the gold leaf to move back towards the stand as the negative charges leave via the plate.

#### Electroscope

a simple scientific instrument used to detect the presence of electric charge on a body, with a light gold or aluminium leaf that moves due to the electrostatic force between charges



A clean zinc plate is placed on the top of the **electroscope**. Then the electroscope is charged negatively. If left alone it will slowly discharge, but if the UV light is shone on the zinc plate it collapses quickly, indicating that electrons are escaping from the plate.

Slightly more sophisticated equipment is shown in Figure 7B–3. Here potassium is used as the metal surface. When the light source shines onto the potassium surface, electrons are emitted and are attracted across the evacuated tube. This completes the circuit and a current flows around it.

#### Figure 7B-3 Simple photoelectric effect apparatus

ISBN 978-1-009-25901-9

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Simple experiments like these indicated that there was a **threshold frequency** required for the emission of **photoelectrons**. That is, below this frequency, no electrons were emitted. Furthermore, the threshold frequency varied between different metal surfaces. For example, zinc required ultraviolet light, while potassium would emit electrons at visible light frequencies.

### Check-in questions – Set 1

- 1 What kind of light is required for the emission of photoelectrons from a metal surface?
- **2** Why does UV light falling on a zinc plate on a negatively charged electroscope discharge the electroscope?
- 3 What causes current to flow around the circuit shown in Figure 7B–3?

### Measuring the energy of the photoelectrons

To measure the energy of the photoelectrons, more complex apparatus is required. A diagram of this is shown in Figure 7B–4.



Figure 7B-4 Schematic diagram of a photoelectric experiment

The light source is a hot filament producing a continuous spectrum of white light. The filter allows nearly monochromatic light to pass into the evacuated glass tube and fall on the metal plate, which emits the photoelectrons. The glass tube, the **photocell**, is evacuated to allow free movement of the photoelectrons and protect the metal surface from oxidation and to allow released photoelectrons to pass across the photocell to the **collector electrode**.

Notice that the DC voltage source is connected so that it opposes (retards) movement of the electrons, this is known as a retarding potential. The electrons will only cause a current in the ammeter if they have sufficient kinetic energy to overcome this potential difference of the DC supply.

The energies of emitted electrons are conveniently measured in units of electronvolts (eV). 1 eV is the energy an electron gains or loses from a 1 volt potential difference. In joules,  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ . Typically, we use  $10^3 \text{ eV} = 1 \text{ keV}$  and  $10^6 \text{ eV} = 1 \text{ MeV}$ .

Note that  $1 \text{ J} = 6.25 \times 10^{18} \text{ eV}$ .

#### Threshold frequency

the minimum frequency required to release a photoelectron from a metal surface

#### Photoelectron

an electron released from a metal by the photoelectric effect

#### Photocell

an evacuated glass tube with a metal plate and collector electrode used in photelectric investigations and applications

Collector electrode the electrode designed to capture

photoelectrons

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To measure the kinetic energy of the most energetic emitted electrons, the DC supply is set to 0 V and then gradually increased. As this happens, the less energetic electrons will be turned back. (Some electrons will have less energy because they are emitted from a depth in the metal and are more firmly bound to the metal.) The ammeter current will therefore reduce. Eventually the current will just reach zero. At this point, the reading on the voltmeter gives the maximum kinetic energy of the photoelectrons in eV. Hence a reading of 1.2 V means a maximum electron kinetic energy of 1.2 eV. A graph that plots a typical photoelectric current (or **photocurrent**) against the DC supply potential difference is shown in Figure 7B–5.

Photocurrent the current through a conducting photocell caused by the photoelectric

effect





Stopping potential the potential required to just stop the most energetic photoelectrons The most significant point on this graph is the point at  $-V_0$ , the **stopping potential**, because it gives, in eV, the maximum kinetic energy of the most energetic photoelectrons.

From the graph, you will also notice that when the potential difference of the DC supply is attractive, rather than retarding, the graph levels off, indicating that all the emitted photoelectrons are being collected.

### Check-in questions – Set 2

- **1** Consider the apparatus in Figure 7B–4.
  - a What is the role of the filter?
  - **b** What is the role of the metal plate?
  - c What is the role of the collector electrode?
  - **d** What is the role of the voltmeter?
- **2** Consider the graph in Figure 7B–5.
  - **a** What is the name and significance of the  $-V_0$ ?
  - **b** What is the significance of the flat section of the graph?
- 3 Why is the photocell evacuated?

### The variation of photoelectron energy with intensity of the light

To the surprise of those investigating the kinetic energy of the photoelectrons, the intensity of the light falling on the metal plate had no effect on photoelectron kinetic energy or on the threshold frequency. For the same frequency, more intense light (as long as it was above the threshold frequency) did produce more photoelectrons, but their kinetic energy did not increase. Less intense light produced fewer photoelectrons of the same kinetic energy, but the stopping potential was the same. This is shown in the graph in Figure 7B–6.

In addition, there was no time delay of photoelectron emission when extremely low intensity light was used.



Figure 7B–6 Effect of intensity on photocurrent, when using the same frequency of light

These experimental results seemed to be completely at variance with the well-established wave-like model of light because it seemed obvious that more intense light should be able to transfer more energy to the electrons in the metal plate. It was also very surprising that there was no delay when very faint light was used, because it was expected that it would take some time for sufficient light energy to be transferred to electrons in the metal surface.

### The variation of photoelectron energy with the frequency of the light

In addition, the frequency of the light was found to affect the maximum photoelectron kinetic energy ( $E_{k max}$ ). The higher the frequency was above the threshold frequency, the greater was the photoelectron kinetic energy. A graph of Millikan's early experimental results of photoelectric emissions from potassium is shown in Figure 7B–7.

Results for other metals were very similar to that for potassium; a straight line with the same gradient. The only difference was that the threshold frequency was different for different metals. An example of a graph with the photoelectron kinetic energy



**Figure 7B–7** Early photoelectric measurements by Millikan, plotted as stopping potential against frequency. Below the frequency indicated,  $43.9 \times 10^{13}$  Hz, no electrons were ejected.

plotted against frequency for four different metals is shown in Figure 7B-8.

It should be clear from Figure 7B–8 that different metals differ in the kinetic energy that

the photoelectrons gain from the same light. For example, the photoelectrons from metal D and light of frequency  $6.4 \times 10^{14}$  Hz will be much less than those from the other metals in the graph with the same incident light. It means that some metals make it harder for electrons to be released as photoelectrons than others.



Figure 7B-8 Photoelectron kinetic energy plotted against frequency for four different metals

### Check-in questions – Set 3

- 1 How does the intensity of the light of fixed frequency affect the stopping potential?
- 2 How does the intensity of the light of fixed frequency affect the current in the photocell?
- 3 Is there a delay in the emission of photoelectrons at very low light intensities?
- **4** Describe the shape of the graphs of maximum photoelectron kinetic energy against frequency.

### Einstein's explanation of the photoelectric effect

Within a few years of these experimental results, in 1905 (often known as the Annus Mirabilis – the Year of Miracles), Einstein published a controversial but simple explanation of them. Taking an idea from Planck's work on blackbody radiators, he proposed that light was composed of **quanta** of energy, and each one had an energy given by Formula 7B–1.

#### Formula 7B–1 Energy of a photon

$$E = hf = \frac{hc}{\lambda}$$

Where:

- E = Energy of quanta (photon) (J or eV)
- h = Planck's constant, 6.63  $\times$   $10^{-34}\,\mathrm{J}\,\mathrm{s}$  (or 4.14  $\times$   $10^{-15}\,\mathrm{eV}\,\mathrm{s})$
- f = Frequency of light shone onto the metal surface in (Hz)
- c =Speed of light,  $3.0 \times 10^8 \,\mathrm{m \, s^{-1}}$
- $\lambda$  = Wavelength of quanta (photon) (m)



**Figure 7B–9** Einstein in 1921 when he received the Nobel Prize for his explanation of the photoelectric effect

At the time, to say that light had particlelike properties was very controversial as it was at odds with the well-established wave-like model of light. In fact, it was described as being 'a bold, not to say a reckless hypothesis'. It took nearly 20 years for the scientific community to accept light quanta as an established theory and more than 20 years to describe them with the word '**photon**'.

Einstein proposed that each photoelectron was an electron close to the surface of the metal that had been given the energy of one of these light quanta. With this energy, the electron now had enough energy to escape from the metal surface with extra available as kinetic energy. His equation is summed up in Formula 7B–2.

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#### Quanta

plural of quantum, used by Einstein to describe the packets of light energy in his explanation of the photoelectric effect, later called photons

Photon a packet of electromagnetic radiation; the word that replaced the term 'quanta'

### Formula 7B-2 The photoelectric effect

$$E_{\rm k\,max} = hf - \phi$$

Where:

$$\begin{split} E_{\rm k\,max} &= {\rm Maximum\ kinetic\ energy\ of\ photoelectrons\ emitted\ (J\ or\ eV)}\\ h &= {\rm Planck's\ constant,\ 6.63\times 10^{-34}\ J\ s\ (or\ 4.14\times 10^{-15}\ eV\ s)} \end{split}$$

f = Frequency of light shone onto the metal surface in (Hz)

 $\phi$  = Work function (J or eV)

The work function,  $\phi$ , is the energy required to release a photoelectron from a metal surface and depends only on the metal used; it is independent of the light source. This equation

also predicts the threshold frequency,  $f_{\text{threshold}} = \frac{\phi}{h}$ , as it is the frequency where the

photoelectron would be released when  $E_{\rm k\,max}$  is zero. The work function and the threshold frequency are therefore a property of the metal surface.

Einstein's equation can be represented graphically, for two different metal surfaces, in Figure 7B–10.







With this graph, you can measure Planck's constant, h, from the gradient in J s (joule second) or eV s (electronvolt-second), read off the threshold frequency ( $f_{\text{threshold 1}}$  or  $f_{\text{threshold 2}}$ ) in Hz and read off the work function ( $\phi_1$  or  $\phi_2$ ) in joules or electronvolts.

Work function ( $\phi$ ) the minimum energy required to release a photoelectron from a metal surface. It varies from metal to metal.



### Worked example 7B–1 Calculations from a photoelectric experiment



to pass through it. Calculate the energy of these photons in electronvolts.

- **b** The work function of caesium is 2.10 eV. Calculate  $E_{k \max}$  of the photoelectrons in joules when the green light falls on the caesium surface.
- **c** The filter is now changed to allow violet photons with energy of 2.80 eV to fall on the caesium surface. Calculate the maximum speed of the photoelectrons emitted. Use  $9.1 \times 10^{-31}$  kg for electron mass.

#### Solution

**a** To calculate the energy of the photons, use Formula 7B–1, E = hf. Using  $h = 4.14 \times 10^{-15}$  eV s and  $f = 5.45 \times 10^{14}$  Hz gives:

 $E = 4.14 \times 10^{-15} \times 5.45 \times 10^{14} = 2.26 \text{ eV}$ 

**b** Using Formula 7B–2:

 $E_{\rm kmax} = hf - \phi = 2.26 - 2.10 = 0.16 \text{ eV}$ 

By definition, 1 eV =  $1.60 \times 10^{-19}$  J, so multiplying 0.16 eV by  $1.60 \times 10^{-19}$  J gives:

$$E_{\rm k\,max} = 0.16 \times 1.60 \times 10^{-19} = 2.56 \times 10^{-20}$$
 J

**c** To calculate the maximum speed of photoelectrons emitted, first find the maximum kinetic energy of photoelectrons emitted under the violet light. Using Formula 7B–2 with  $E_{\text{violet}} = hf = 2.80$  eV and noting that the work function is independent of light used, gives:

$$E_{\rm kmax} = hf - \phi = 2.80 - 2.10 = 0.70 \text{ eV}$$

Converting this to joules gives:

 $E_{\rm kmax} = 0.70 \times 1.60 \times 10^{-19} = 1.12 \times 10^{-19} \,\rm J$ 

Now, calculate the maximum electron speed from the maximum kinetic energy, since

by definition,  $E_{k \max} = \frac{1}{2}mv_{\max}^2$ . Rearranging this for  $v_{\max}$  gives:

$$v_{\text{max}} = \sqrt{\frac{2E_{\text{k max}}}{m}}$$
$$= \sqrt{\frac{2 \times 1.12 \times 10^{-19}}{9.1 \times 10^{-31}}} = 4.96 \times 10^5 \text{ m s}^{-1}$$

Hence, the maximum speed of the photoelectrons emitted is  $4.96 \times 10^5$  m s<sup>-1</sup>.

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It is not surprising that it took nearly 20 years for the scientific community to accept Einstein's explanation of the photoelectric effect as it was thought that classical physics had solved all the major questions of the physical world. The only remaining significant problems – blackbody radiation, the photoelectric effect and the stability of the hydrogen atom - would need the strange new ideas of the quantum world before they were understood.

### **7B SKILLS**

Limitations of the wave model's explanations of the photoelectric effect

You need to be able to explain the key features of photoelectric experiments that gave such strong support for Einstein's particle-like (photon) model compared to the wave-like model for light.

How does light energy liberate electrons from metal surfaces and give them kinetic energy?

This could be explained with a wave-like model. Energy from the light wavefront would transfer energy to the electrons in the metal surface, eventually giving them enough energy to escape with some kinetic energy.

- For dim light sources, why is there no time delay between the light striking the • surface and the liberation of photoelectrons? This was very difficult to explain using a wave-like model, a dim light would mean a wavefront with little energy and it should take time to build up enough energy to liberate the electrons.
- Why is there a threshold frequency for each metal surface below which photoelectrons are never emitted, no matter how intense the light is? This was also very difficult to explain using a wave-like model, for reasons previously given.
- Why is the rate at which photoelectrons are emitted at fixed frequency, proportional to the light intensity?

This was more plausible, although it was puzzling that the energy of the photoelectrons did not increase in the brighter light.

Why was the energy of the photoelectrons linked to the frequency of the light (and • not to the intensity of the light)?

Although the energy intensity of wavefronts is connected with both amplitude and frequency, this still caused problems for the wave-like model, as even a low frequency below the threshold should be able to liberate electrons with sufficient amplitude.

### NOTE

### Questions involving intensity

The intensity of the light striking a photocell should be carefully understood as the product of the number of photons per second and the energy of a single photon, hf. This means that when the frequency is changed but the intensity is stated as being constant, the number of photons must also change. For constant intensity, increasing the frequency must decrease the number of photons and decreasing the frequency must increase the number of photons.





VIDEO 7B-2

SKILLS LIMITATIONS OF THE WAVE

MODEL'S **EXPLANATIONS** OF THE

PHOTOELECTRIC EFFECT

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## Section 7B questions

### **Multiple-choice questions**

1 A photocell is sketched below.



This can be used to determine  $E_{k max}$  of photoelectrons. Which of the following best describes the key measurement for this?

- A the maximum current through the ammeter
- **B** the maximum voltage at maximum current
- C the smallest voltage to just achieve maximum current
- **D** the smallest voltage to just achieve minimum current
- **2** Einstein's explanation of the photoelectric effect contributed a particular insight, namely that
  - A more intense light carries more energy.
  - **B** the energy carried by light can be transferred to electrons.
  - **C** different frequency light has different wavelengths.
  - **D** the energy that light carries is in discrete quantities.
- **3** The diagram below shows a plot of maximum kinetic energy,  $E_{k max}$ , against frequency, *f*, for various metals capable of emitting photoelectrons.



Which one of the metals in the diagram has the greatest work function?

- A Metal A
- **B** Metal B
- **C** Metal C
- D Metal D

Adapted from VCAA 2020

© Cambridge University Press & Assessment 2023 ISBN 978-1-009-25901-9 Eddy de Jong et al Photocopying is restricted under law and this material must not be transferred to another party.
- 4 Photoelectric experiments involve shining light onto metal surfaces. Measurements are made of the number of emitted electrons and their maximum kinetic energy for a range of frequencies and light intensities. Which one of the following would not be an experimental finding?
  - **A** The ability to eject electrons from a metal depended on the frequency of light.
  - **B** The stopping potential for photoelectrons was independent of the light intensity.
  - **C** The maximum photoelectron kinetic energy depended only on the light intensity.
  - **D** At frequencies below the threshold frequency, no electrons were ejected.

VCAA 2021

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#### **Short-answer questions**

- 5 Calculate the frequency (in hertz) and energy (in joules) of a 550 nm photon.
- 6 Einstein's equation for the photoelectric effect can be written as:

$$E_{\rm kmax} = hf - f$$

Outline the physical meaning of each term.

7 The diagram below shows some experimental results reported by Millikan in 1912.



#### Millikan's photoelectric measurements

- **a** What is the significance of the stopping potential with regard to the photoelectric effect?
- **b** What is the significance of the gradient of the plotted line with regard to the photoelectric effect?
- **c** What is the significance of the intersection of the plotted line with the frequency axis, regarding the photoelectric effect?
- 8 Outline an aspect of the photoelectric effect that the wave model of light could qualitatively explain and an aspect that the wave model had difficulty in explaining.
- **9** Describe how the near instantaneous emission of photoelectrons at very low light levels and the linear relationship between  $E_{k \max}$  and frequency supported the particle-like model underlying the Einstein equation,  $E_{k \max} = hf f$ .

**10** To investigate the photoelectric effect, two students, Kym and Sai, set up an experiment. The apparatus is shown below.



With the light source on and a filter in place, the students measure the  $E_{\rm k\,max}$  of the photoelectrons by gradually changing the variable DC voltage source until the current measured by the ammeter just falls to zero. They record this stopping potential for each frequency of the incident light and plot the results in a graph of stopping potential,  $V_{\rm s}$ , against frequency, *f*, of the incident light, as shown below.



With light of frequency  $6.0 \times 10^{14}$  Hz, the ammeter always shows zero, even when the DC voltage source is set to 0 V. Sai wants to repeat the experiment for this frequency with a much brighter light source and also wants to expose the metal to the light for a much longer time. Kym says photoelectrons will never be ejected with this frequency of light.

- a i Who is correct Sai or Kym?
  - ii What explanation might Sai give to support her opinion that by waiting longer and using a brighter light source, photoelectrons could be ejected from the metal with light of a frequency of  $6.0 \times 10^{14}$  Hz?
- **b** Use the graph to calculate Planck's constant. Show your working.
- **c** Determine the work function of the metal from the graph. Give your reasoning.

VCAA 2018

## **Chapter 7 review**

## Summary

Create your own set of summary notes for this chapter on paper or in a digital document. A model summary is provided in the Teacher Resources, which can be used to compare with yours.

## Checklist

In the Interactive Textbook, the success criteria are linked from the review questions and will be automatically ticked when answers are correct. Alternatively, print or photocopy this page and tick the boxes when you have answered the corresponding questions correctly.

Succe	ess criteria – I am now able to:	Linked questions
7A.1	Explain that accelerating charges produce transverse	1 🗌 , 3 🗌 , 4 🗌
	electromagnetic waves	
7A.2	Describe electromagnetic waves as perpendicular linked	1 , 2 , 5
	oscillating electric and magnetic fields travelling at <i>c</i> in a	
74.2	vacuum	
7A.3	reflection results in a standing wave	17
7A.4	Analyse standing waves with nodes at each end in terms of	17 , 18 , 19
	speed, wavelength, amplitude, frequency, nodes and antinodes	
7A.5	Investigate and explain diffraction with reference to different gap or obstacle width and frequency	6, 7, 8, 20
7A.6	Explain qualitatively the effect of changing the $rac{\lambda}{w}$ ratio applied	6 <b>,</b> 7 <b>,</b> 8
	to the limitations of imaging with electromagnetic waves	
7A.7	Explain the evidence for the wave-like nature of light provided by the results of Young's double slit experiment	10 , 21
7A.8	Explain the results of Young's double slit experiment related to the constructive and destructive interference of coherent waves	20 , 21
	in terms of path differences: $n\lambda$ and $\left(n+\frac{1}{2}\right)\lambda$	
7A.9	Explain the results of Young's double slit experiment with reference to the effect of wavelength, distance of screen and slit	9□, 20□
	separation on interference patterns: when $L >> d$ , using $\Delta x = \frac{\lambda L}{d}$	
7B.1	Use the formula $E = hf = \frac{hc}{\lambda}$ for the quantised energy of photons	22
7B.2	Analyse the experimental results of photelectric experiments	10, 11, 23
	for evidence for the particle-like nature of light	,,, <b>_</b>
7B.3	Analyse the results of photoelectric experimental graphs of	14 ], 16
	photocurrent versus electrode potential, and of kinetic energy of electrons versus frequency	

Succe	ess criteria – I am now able to:	Linked questions
7B.4	Analyse the results of photelectric experiments with respect	12 🗌 , 13 🗌
	to the kinetic energy of emitted photoelectrons: $E_{k \max} = hf - \phi$ ,	
	using energy units of joule and electronvolt	
7B.5	Analyse the results of photoelectric experiments with respect	11 🗌 , 12 🗌 , 15 🗌
	to the effects of intensity of incident irradiation on the	
	emission of photoelectrons	
7B.6	Describe the limitations of the wave model of light	23
	in explaining experimental results related to the	
	photoelectric effect	

## **Multiple-choice questions**

Use the following information to answer Questions 1 and 2.

Many people in remote settings now carry a PLB (personal locator beacon) transmitter. Some operate at 406 MHz, using electromagnetic radiation.

- 1 Which of the following statements about this PLB radiation is correct?
  - **A** It requires a medium for its transmission.
  - **B** It is a longitudinal wave motion.
  - **C** It is a mixture of transverse and longitudinal waves.
  - **D** It does not require a medium for its transmission.
- 2 PLB radiation and visible light are similar because
  - **A** they both travel at *c* in a vacuum.
  - **B** they have the same wavelengths.
  - **C** PLB radiation is a longitudinal wave motion.
  - **D** visible light is a longitudinal wave motion.
- **3** Electromagnetic waves are best described as
  - A longitudinal.
  - **B** transverse waves.
  - **C** transverse and longitudinal.
  - **D** rays, not waves.
- 4 Electromagnetic waves can be generated by
  - A strong, electrostatic fields.
  - **B** strong, static magnetic fields.
  - **C** constant high velocity charges.
  - **D** circulating charged particles.
- 5 The electric and magnetic fields in electromagnetic waves are
  - A parallel, oscillating and linked.
  - **B** transverse, linked and oscillating.
  - **C** transverse, linked and static.
  - **D** parallel, static and linked.

**6** When cars with two headlights are a long way away on a straight road, it is hard to tell that there are two headlights; our eyes cannot separate the two objects.



Far away

Close

If the headlights gave out violet light, they would be easier to separate, because the

- **A** wavelength of violet light is less than the average wavelength of white light.
- **B** wavelength of violet light is longer than the average wavelength of white light.
- **C** speed of violet light is faster than the speed of white light.
- **D** speed of violet light is slower than the speed of white light.
- 7 A beam of electromagnetic radiation with  $f = 1 \times 10^{15}$  Hz is directed at a hole of diameter 0.1 mm. A circular diffraction pattern is formed on a screen 10 m from the hole, of width 6 mm. When the hole is changed for one of diameter 0.2 mm, and the radiation is changed to  $f = 5 \times 10^{14}$  Hz, the pattern on the screen is still likely to be circular, with a width close to:
  - **A** 3 mm
  - **B** 6 mm
  - **C** 12 m
  - **D** 60 mm
- **8** When small water waves are directed at a barrier with a small gap, diffraction results, as shown in the idealised diagram below.



Students decrease the period of the water waves. The speed of the waves is the same. Which of the following best describes what they are now likely to see in the waves to the right of the gap?

- A The pattern will be unchanged.
- **B** The pattern will spread further.
- **C** The pattern will become narrower.
- **D** The wavelength will increase, but the spread will remain the same.

- A student sets up Young's double slit experiment. A pattern of bright and dark bands is observed on the screen, as shown in the diagram on the right.
   Which one of the following actions will increase the distance, Δx, between the adjacent dark bands in this interference pattern?
  - A increase the distance between the slits and the screen
  - **B** decrease the wavelength of the light
  - **C** increase the slit separation
  - **D** decrease the distance between the slits and the screen



Adapted from VCAA 2018

- **10** Which one of the following reasons best explains why Young's double slit experiment, such as the one described in Question 9, gave evidence for the wave-like nature of light rather than a particle-like nature?
  - **A** A wave-like nature is needed to explain why the light passes through the double slits.
  - **B** Overlapping waves can produce regions of zero disturbance.
  - **C** A wave-like nature is only needed to explain the bright bands.
  - **D** Without a wave-like nature, the light from the two slits would form a random pattern on the screen.

Adapted from VCAA 2018

- **11** Photoelectrons are emitted almost immediately from a metal surface after light of a suitable frequency strikes it. The best explanation for this is that
  - A only one photon is required to liberate one electron.
  - **B** very few photons are required to liberate electrons.
  - **C** the electrons resonate strongly to the frequency of the incident photons.
  - **D** even weak light sources contain lots of energy to liberate electrons.

Adapted from VCAA 2018

- **12** When light above the threshold frequency strikes a particular metal surface, photoelectrons are emitted. If the light intensity is increased but the frequency of the light is unchanged, which of the following is correct?
  - **A** The number of photoelectrons emitted remains the same and  $E_{k max}$  of the photoelectrons remains the same.
  - **B** The number of photoelectrons emitted remains the same and  $E_{\rm k\ max}$  of the photoelectrons increases.
  - **C** The number of photoelectrons emitted increases and  $E_{\rm k\ max}$  of the photoelectrons remains the same.
  - **D** The number of photoelectrons emitted increases and  $E_{k max}$  of the photoelectrons increases.
- **13** A metal surface has a work function of 2.0 eV. The minimum energy of an incoming photon required to eject a photoelectron is
  - **A**  $3.2 \times 10^{-19} \text{ J}$
  - $\textbf{B} \quad 1.6\times 10^{-19} \text{ J}$
  - **C**  $8.0 \times 10^{-20}$  J
  - $\textbf{D} \ 4.0\times 10^{-20} \ J$

Adapted from VCAA 2018

14 The results of a photoelectric experiment are displayed in  $E_{k max}$  the graph on the right. It shows  $E_{k max}$  of the photoelectrons versus the frequency f of light falling on the metal surface. A second experiment is conducted with another metal surface replacing the original. The new metal surface has a larger work function. The original data is shown as a solid line and the results of the second experiment are shown as a dashed line. Which one of the following graphs shows the results from the second experiment?



Adapted from VCAA 2019

- **15** Students conduct a photoelectric effect experiment. They shine light of known frequency onto a metal and measure the  $E_{k max}$  of the emitted photoelectrons. They increase the intensity of the incident light, but keep the frequency unchanged. The effect of this increase would most likely be
  - A lower maximum kinetic energy of the emitted photoelectrons.
  - B increase maximum kinetic energy of the emitted photoelectrons.
  - **C** fewer emitted photoelectrons but of higher maximum kinetic energy.
  - **D** more emitted photoelectrons but of the same maximum kinetic energy.

Adapted from VCAA 2020

**16** When photons with energy, *E*, strike a metal surface, electrons may be emitted. The maximum kinetic energy,  $E_{k max}$ , of the emitted electrons is given by  $E_{k max} = E - \Phi$ , where  $\Phi$  is the work function of the metal surface. Which one of the following graphs best shows the relationship between the maximum kinetic energy of these electrons,  $E_{k max}$  and the photon wavelength,  $\lambda$ ?



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## Short-answer questions

**17** A violin string is played forming a 900 Hz sound standing wave. The string is fixed at the ends and a strobe photo of the vibrating string shows regions of no vibration (nodes), as shown below. The distance between the fixed ends is 30 cm. The dashed line shows the string's position when it is not vibrating.



- **a** Describe the mechanism leading to the formation of the standing wave. (1 mark)
- **b** Calculate the period of the standing wave.
- c Calculate the speed of a transverse wave along the violin string. Show your working.
  - (2 marks)

(2 marks)

- **d** The violin player finds that they can play two other lower frequency notes on the 30 cm length string, with the string otherwise unchanged. Calculate the frequency of these two notes, showing your working. (2 marks) VCAA 2016
- **18** Yasmin and Paul set up the following experiment in a large open area. They connect two speakers that face each other, as shown below.



The speakers are 10 m apart and connected to the same signal generator. They produce a sound with  $\lambda = 1.0$  m. The speakers are in phase. Yasmin stands in the middle, 5.0 m from speaker A and 5.0 m from speaker B.

- **a** Is she standing on a node or an antinode? Give a reason for your answer. (2 marks)
- **b** She then moves towards speaker B and hears a sequence of soft and loud regions. She stops at the second soft region. How far is she from speaker B? (3 marks)
- **19** Students create a standing wave in an elastic cord 2.4 m long. They take video shots of the vibrating cord. The following diagram shows the cord at one extreme of its vibration (maximum amplitude). The dashed line shows the 'at rest' position of the elastic cord. The frequency of the wave is 100 Hz.



- **a** State the wavelength of the standing wave. (1 mark)(2 marks) **b** Sketch the position of the cord at a time 7.5 ms after that shown. (2 marks)
- Calculate the speed of a transverse wave along the cord. С

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**20** Students perform a Young's double slit experiment with the experimental set-up on the right. Two narrow slits (A and B) let light from a laser pass through them. The result is a pattern of light (white) and dark (black) bands on a screen. The bands are numbered. The spacing between the bands,  $\Delta x$ , is also shown.



- a One of the bands is equidistant from slit A and slit B. Explain why it is not possible for bands 1, 3, 5, 7, 9 or 11 to be that band.
   (2 marks)
- **b** The students measure that the distance from slit A to band 11 is longer than the distance from slit B to band 11 by 1.5  $\mu$ m. Use this information to calculate the wavelength of the light from the laser. (Assume that band 6 is the central band.) (3 marks)
- **c** Identify two changes to the apparatus that would increase the spacing,  $\Delta x$ . (2 marks)
- **d** Explain why narrow slits (A and B) are necessary if the light from the slits is to spread out and overlap after passing through them. (2 marks)

Adapted from VCAA 2018

**21** Physics students studying interference set up a double slit experiment using light from a 610 nm laser, as shown below.



A section of the interference pattern observed by the students is shown below. There is a bright band at point *C*, at the centre of the pattern.



- **a** Explain why point C is located in a bright band rather than in a dark band. (2 marks)
- **b** Another point on the pattern to the right of point C is further from  $S_1$  than  $S_2$  by a distance of  $2.14 \times 10^{-6}$  m. Mark this point on the diagram by writing an X above the point. Use a calculation to show your reasoning. (3 marks)
- c Explain why this experiment, first performed by Thomas Young, provided evidence for the wave-like nature of light. (2 marks)
- **22 a** Calculate the frequency of a photon of wavelength 610 nm. (1 mark)
  - **b** Calculate the energy of a photon of wavelength 610 nm in electronvolts and joules. (2 marks)
- **23** Some aspects of photoelectric effect experiments, particularly with regard to the rapid emission of photoelectrons at low light intensities and the relationship between maximum photoelectron energy ( $E_{k max}$ ) and light frequency, gave strong support to a particle-like nature of light. Outline the reasons for this support as evidence against a wave-like model of light.

(3 marks)

## HOW HAVE CREATIVE IDEAS AND INVESTIGATION REVOLUTIONISED THINKING IN PHYSICS?

## LIGHT AND MATTER

## Introduction

UNIT

**CHAPTER** 

R

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This chapter begins by observing that a basic property of all electrons includes wave behaviour. Electrons, like photons dealt with in Chapter 7, will diffract when passing through narrow gaps, or around small objects such as atoms. To understand the conditions for this diffraction to occur, we need to know the size of the electron's wavelength. The wave properties of electrons also help explain why atoms have stable energy states that do not collapse (as predicted by Newtonian physics). The stable energy states are then used to explain the absorption and emission spectra of elements. Finally, two slit experiments with single photons (and also with single electrons) yield convincing evidence for the dual nature of light and matter.

#### Curriculum

#### Area of Study 1 Outcome 1 How has understanding about the physical world changed?

Study Design	Learning intentions – at the end of this chapter I will be able to:		
<ul> <li>Matter as particles or waves</li> <li>Interpret electron diffraction patterns as evidence for the wave-like nature of matter</li> <li>Distinguish between the diffraction patterns produced by photons and electrons</li> <li>Calculate the de Broglie wavelength of matter: λ = h/p</li> </ul>	8AMatter as particles or waves8A.1Recognise electron diffraction patterns8A.2Understand that the diffraction patterns of matter are evidence of the wave-like nature of matter, including electrons8A.3Distinguish between diffraction patterns produced by photons and electrons8A.4Calculate the de Broglie wavelength of matter particles using $\lambda = \frac{h}{p}$		

#### **Study Design**

#### Similarities between light and matter

- Discuss the importance of the idea of quantisation in the development of knowledge about light and in explaining the nature of atoms
- Compare the momentum of photons and of matter of the same wavelength including

calculations using:  $p = \frac{h}{\lambda}$ 

- Explain the production of atomic absorption and emission line spectra, including those from metal vapour lamps
- Interpret spectra and calculate the energy of absorbed or emitted photons: *E* = *hf*
- Analyse the emission or absorption of a photon by an atom in terms of a change in the electron energy state of the atom, with the difference in the states' energies being equal to the photon energy:  $E = hf = \frac{hc}{r}$
- Interpret the single photon and the electron double slit experiment as evidence for the dual nature of light and matter

## Learning intentions – at the end of this chapter I will be able to:

- 8B Similarities between light and matter8B.1 Discuss how the idea of quantisation
- developed our knowledge about light and helped explain the nature of atoms
- **8B.2** Use  $p = \frac{h}{\lambda}$  to calculate the momentum of photons
- **8B.3** Compare the momentum of photons and matter with the same wavelength
- **8B.4** Explain the production of atomic emission line spectra, including those from metal vapour lamps
- **8B.5** Explain the formation of atomic absorption line spectra
- **8B.6** Calculate the energy of emitted or absorbed photons from spectra using  $\Delta E = hf$
- **8B.7** Analyse the emission or absorption of photons by atoms in terms of a change in the energy state of atoms using  $\Delta E = hf$
- **8B.8** Interpret the results of the double slit experiment with single photons and electrons as evidence for the dual nature of light and matter

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## Glossary

Absorption line spectrum Atomic or subatomic sizes Blackbody Constructive interference de Broglie wavelength Destructive interference Diffraction Diffraction grating Electromagnetic wave Electron Electrostatic force Emission line spectrum Energy state Exoplanet Gas discharge tube

- Graphite Ground state Helium Hydrogen Infrared light Ion Metal vapour lamp Momentum Neutron Nucleus Outer-shell electron Photoelectric effect Photon Polar aurora Polycrystalline
- Prism Proton Quanta Quantisation Quantised Quantum number Spectroscope Superposition Tungsten filament incandescent globe Ultraviolet light Wave-particle duality X-ray

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## **Concept map**



See the Interactive Textbook for an interactive version of this concept map Interlinked with all concept maps for the course.



## Matter as particles or waves

#### Study Design:

- Interpret electron diffraction patterns as evidence for the wave-like nature of matter
- Distinguish between the diffraction patterns produced by photons and electrons
- Calculate the de Broglie wavelength of matter:

$$\lambda = -\frac{r}{r}$$

#### **Glossary:**

Atomic or subatomic size Constructive interference de Broglie wavelength Destructive interference Diffraction Electron Graphite Momentum Neutron Photoelectric effect Photon Polycrystalline Proton Quanta Wave-particle duality X-ray

## **ENGAGE** Louis de Broglie

In his 1924 PhD thesis, French physicist Louis de Broglie suggested that electrons, a constituent of matter, have wave-like properties. This was a revolutionary idea, which won him the 1929 Nobel Prize in Physics. In the years following, there was debate about what kind of waves de Broglie had proposed. He himself thought of them as 'guiding waves' (pilot waves) for electrons. Within two years, Erwin Schrödinger had formulated the wave function of electrons mathematically and this was interpreted as a 'probability density' of finding an electron in a specific location. This has become the dominant interpretation in quantum mechanics.



**Figure 8A–1** Louis de Broglie won the 1929 Nobel Prize for suggesting that matter such as electrons have wave-like properties.



# 04.1





7A WAVE-LIKE PROPERTIES OF LIGHT

#### Diffraction

the spreading of a wave when passing through a narrow opening or passing around a small object

#### Photoelectric effect

the phenomenon in which light can release an electron from a metal surface

#### Quanta

plural of quantum, used by Einstein to describe the packets of light energy in his explanation of the photoelectric effect

#### Photon

packet of electromagnetic radiation; the word that replaced the term 'quanta'

#### Momentum

the product of an object's mass and velocity

#### Wave-particle duality

the concept that light and matter can have both wave-like and particle-like properties



## EXPLAIN What is light?

Before discussing the wave-like properties of matter, it is useful to review the wave-like properties of light, since comparisons between the two have to be made. In Chapter 7, we saw that light can be successfully modelled as transverse waves made of oscillating electric and magnetic fields, travelling at  $c (3.00 \times 10^8 \text{ m s}^{-1})$ . At the end of the 19th century, this was accepted by as correct by the scientific community based on Young's double slit experiment results and other phenomena, including diffraction. However, in 1905, Einstein's Nobel prize-winning explanation of the photoelectric effect challenged this model, proposing successfully that light transfers energy in quanta or packets, each with an energy of *hf*. These quanta are now known as photons and are accepted by the scientific community. Photons are also now known to have momentum, despite having zero mass. Hence, we have two apparently competing models for light, a wave model and a particle (or photon) model. This is referred to as wave–particle duality and it means that we have to apply the appropriate model to the appropriate situation.

## Diffraction with visible and X-ray photons

Chapter 7 introduced the phenomenon of light diffraction. A standard simplified representation of this is the diffraction of light when it passes through a narrow slit, as shown in Figure 8A–2.



#### Figure 8A-2 Detail of single diffraction pattern

A more detailed description of a single slit diffraction pattern shows weak interference fringes either side of the broader central maximum.



**Figure 8A–3** Single slit diffraction showing broad central maximum of the diffraction, with weaker bright and dark side bands caused by interference effects.

8A MATTER AS PARTICLES OR WAVES



Closer observation of the diffraction pattern reveals fainter side bands as shown in Figure 8A–3. When the slit is replaced by a small circular aperture, the image changes to that shown in Figure 8A–4.



**Figure 8A–4** Diffraction through a circular aperture yields a circular interference pattern; a bright central maximum is surrounded by less bright circular bright and dark bands caused by interference effects.

This pattern is very similar to the diffraction through a **polycrystalline** solid with **X-rays**, as shown in Figure 8A–5.

The pattern is clearly a wave phenomenon; the dark circles are regions of **destructive interference** and the bright circles are regions of **constructive interference**. The angular

spread of the diffraction pattern depends on the ratio  $\frac{\lambda}{w}$  (as discussed in Chapter 7). Both the spacing of atomic layers in the solid and the wavelength of the X-rays are much smaller than the diameter of the circular aperture and the wavelength of the green laser in Figure 8A–4. A common wavelength used in X-ray diffraction is 0.15 nm and the spacing of atomic layers in graphite (as an

example) is 0.33 nm. This combination gives  $\frac{\lambda}{w} = 0.45$ , ensuring an observable diffraction pattern (as  $\lambda < w$ ).

## Check-in questions – Set 1

- 1 Why are X-ray wavelengths used for diffraction experiments with crystalline and polycrystalline materials?
- 2 How are the light and dark bands in diffraction patterns formed?

## de Broglie's prediction

In 1909, Einstein had asserted that light photons carried momentum, given by  $p = \frac{h}{\lambda}$ . In 1924

de Broglie suggested that particles of matter, such as electrons, had a 'guiding wave' associated

with them, with a wavelength given by the same relationship,  $\lambda = \frac{h}{n}$ . This means that an

electron travelling at  $4.9 \times 10^6$  ms<sup>-1</sup> has a momentum of  $4.46 \times 10^{-24}$  kg ms<sup>-1</sup> and therefore an associated **de Broglie wavelength** of 0.15 nm, the same as a typical common wavelength used in X-ray diffraction experiments. In 1927, Clinton Davisson and Lester Germer observed electron diffraction, as did George Thomson, confirming both the phenomenon as well as the accuracy of de Broglie's formula. Davisson and Thomson shared the 1937 Nobel Prize for Physics.



**Figure 8A–5** X-ray diffraction through a polycrystalline solid produces a similar pattern to diffraction through a circular aperture.



7A WAVE-LIKE PROPERTIES OF LIGHT

#### Polycrystalline

a solid material that consists of many small grains or crystals. The size of the grains may vary from nanometers to millimeters.

#### X-ray

the region of the electromagnetic spectrum between UV and gamma rays. X-rays have the second shortest wavelengths and second highest frequencies.

## Destructive

occurs where two identical waves are completely out of phase and meet, cancelling each other

## Constructive interference

occurs when two identical waves are in phase and meet, doubling the amplitude of one wave



Graphite crystalline form of the element carbon

Electron

the lightest stable subatomic particle known. It carries a negative charge of  $1.60 \times 10^{-19}$  coulomb, which is considered the basic unit of electric charge.

de Broglie wavelength wavelength of matter particles,

given by  $\lambda = \frac{h}{r}$ 

An electron diffraction pattern is shown in Figure 8A–6.



Figure 8A-6 An electron diffraction pattern produced by sending electrons though a small hole in a screen. Compare this to the similar set-up for photons on Figure 8A-4.

Formula 8A–1 Momentum of photons

$$p = \frac{h}{\lambda}$$

Where:

 $\lambda$  = Photon wavelength (m)

 $h = \text{Planck's constant } 6.63 \times 10^{-34} \text{ J s}$ 

 $p = Photon momentum (kg m s^{-1})$ 

### Formula 8A–2 de Broglie wavelength of matter particle

$$\lambda = \frac{h}{p} = \frac{h}{m\nu}$$

Where:

- $\lambda = \text{de Broglie wavelength}(m)$
- $h = \text{Planck's constant } 6.63 \times 10^{-34} \text{ J s}$
- p = Momentum of matter particle (kg m s<sup>-1</sup>)
- m = Mass of matter particle (kg)
- v = Speed of matter particle (m s<sup>-1</sup>)

#### de Broglie wavelengths of matter

The de Broglie wavelength applies to all matter, small or large, though it is only useful when wave behaviour is significant. In the example on the previous page, the electron travelling at  $4.9 \times 10^6 \text{ m s}^{-1}$  will show diffraction when passing through a crystal lattice with spacings similar to the wavelength. However, a cricket ball of mass 150 g travelling at 20 m s<sup>-1</sup> will have a de Broglie wavelength of only  $2.2 \times 10^{-34}$  m, which is tiny (even when compared to a proton, which has a diameter of  $10^{-15}$  m). Typically, diffraction is



measurable only when the ratio  $\frac{\lambda}{w}$  is 0.01 or greater.

Eddy de Jong et al

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#### Proton

a positively charged particle in the nucleus of an atom



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## Check-in questions – Set 2

- 1 Does the de Broglie wavelength apply only to **atomic- or subatomic-sized** particles?
- 2 Does the de Broglie wavelength increase or decrease with increasing particle momentum?

## Worked example 8A–1 Calculating the de Broglie wavelength

Find the de Broglie wavelength of a neutron ( $m = 1.67 \times 10^{-27}$  kg) travelling at 2000 m s<sup>-1</sup>.

Solution

To find the momentum use:

$$p = m\nu$$
  
= (1.67 × 10<sup>-27</sup>)(2000)

$$= 3.34 \times 10^{-24} \text{ kg m s}^{-1}$$

Then use:

$$\lambda = \frac{h}{p}$$
  
=  $\frac{6.63 \times 10^{-34}}{3.34 \times 10^{-24}}$   
=  $1.99 \times 10^{-10}$  m  
=  $0.199$  nm

## **8A SKILLS**

#### Using the correct value for Planck's constant, h, for photons and matter

There are two values that can be used for Planck's constant, *h*: either  $4.14 \times 10^{-15}$  eV s or  $6.63 \times 10^{-34}$  Js. If photon energy is in eV, use the eV s value for photons with  $E = hf = \frac{hc}{\lambda}$ . However, when matter is involved, use the Js value when calculating the de Broglie wavelength,  $\lambda = \frac{h}{n}$ .

## **Section 8A questions**

## **Multiple-choice questions**

- 1 Amy is interested in the diffraction patterns of electrons and X-rays through small apertures. She knows that these patterns have their spacing controlled
  - **A** only by the size of the diffracting aperture.
  - **B** by the size of the aperture and the energy of the X-rays or electrons.
  - **C** only by the momentum of the X-rays or electrons.
  - **D** by the size of the aperture and the momentum of the X-rays or electrons.

#### Atomic or

subatomic size atoms are of the order of  $10^{-10}$  m; subatomic could be taken to be the size of a proton  $(10^{-15}$  m) or smaller



an uncharged particle in the nucleus of an atom

- 2 Amy reasoned that neutrons could also show diffraction patterns, if the
  - A diffracting aperture was comparable with the speed of the neutrons.
  - **B** diffracting aperture was comparable with the ratio  $\frac{h}{p_{\text{neutron}}}$
  - **c** ratio,  $\frac{h}{p}$ , of the neutrons was very much smaller than the size of the diffracting aperture.
  - **D** neutrons were moving near the speed of light.
- **3** Electrons (mass  $9.1 \times 10^{-31}$  kg) are accelerated to a speed of  $1.0 \times 10^7$  m s<sup>-1</sup>. The best estimate of the de Broglie wavelength of these electrons is
  - **A**  $4.5 \times 10^{-6}$  m
  - **B**  $7.3 \times 10^{-8}$  m
  - C  $7.3 \times 10^{-11} \text{ m}$
  - **D**  $4.5 \times 10^{-12} \text{ m}$

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#### Short-answer questions

- 4 Electrons passing through a crystal produce a diffraction pattern whose spacing is very close to that produced by 56 pm X-rays passing through the same crystal. Estimate the speed of the electrons.
- **5** The pattern produced when electrons are passed through a polycrystalline material is shown below. What features of this image suggest that electrons have wave-like properties?



6 Students are discussing the results of diffraction experiments carried out with X-rays and neutrons using the same polycrystalline material. They notice that both X-rays and neutrons give a similar pattern of light and dark rings, but that the rings produced by the X-rays have smaller diameters. What conclusion can they draw from these observations about the relative momenta of the X-rays and neutrons used?



## Similarities between light and matter

#### Study Design:

- Discuss the importance of the idea of quantisation in the development of knowledge about light and in explaining the nature of atoms
- Compare the momentum of photons and of matter of the same wavelength including

calculations using:  $p = \frac{h}{\lambda}$ 

- Explain the production of atomic absorption and emission line spectra, including those from metal vapour lamps
- Interpret spectra and calculate the energy of absorbed or emitted photons: *E* = *hf*
- Analyse the emission or absorption of a photon by an atom in terms of a change in the electron energy state of the atom, with the difference in the states' energies being equal to the photon

energy: 
$$E = hf = \frac{hc}{\lambda}$$

• Interpret the single photon and the electron double slit experiment as evidence for the dual nature of light and matter

#### **Glossary:**

Absorption line spectrum Blackbody Diffraction grating Electromagnetic wave Electrostatic force Emission line spectrum Energy state Exoplanet Gas discharge tube Ground state Helium Hydrogen Infrared light lon Metal vapour lamp Nucleus Outer-shell electron Polar auroras Prism Quantisation Quantised Quantum number Spectroscope Superposition **Tungsten filament** incandescent globe Ultraviolet light

## 0°

**ENGAGE** 

## What is quantum physics?

Quantum physics underlies everything in the Universe, but before the discovery of its quantum nature early in the 20th century, humanity made no active use of it. Today, it has been estimated that around 35% of the economy in modern countries comes from technology that makes use of quantum physics. Every time we



have an MRI scan, navigate using GPS, take a digital photograph or use a mobile phone, the internet or a laser scanner at the checkout we are making active use of quantum physics. Quantum physics is the physics of the tiny – the physics of the building blocks of the Universe – of atoms, photons and electrons and the bewildering array of



Quantisation the concept that a physical quantity can have only certain discrete allowable values

elementary particles, such as the Higgs boson. At its heart, quantities such as charge, mass, energy and momentum are *quantised* – they can only have certain values and cannot have values in-between these allowed values. You have briefly met this idea in your study of the photoelectric effect in Section 8A. In this section you will explore the quantisation of energy, which is essential for the existence of atoms.

## The momentum of photons

We are used to thinking of momentum as being a property of moving masses, so the idea that photons, which have no mass at all, have momentum seems impossible. Maxwell predicted that electromagnetic waves carried momentum. Einstein showed that a consequence of Planck's law of blackbody radiation was that light quanta must carry momentum. An example of the momentum of photons can be seen in one of the 'tails' of comets as they go around the sun. The ion tail (see Figure 8B-1) is caused by the stream of charged particles emitted by the Sun's photons transferring momentum to the dust and gas released by the Sun's heating of the comet; the dust tail is caused by the stream of photons from the Sun transferring their momentum to the emitted dust and gas.





Some spacecraft have been launched that use 'space sails' to assist in their propulsion, as shown in Figure 8B-2.



Figure 8B-2 Schematic diagram of how a 'space sail' uses photon momentum to generate thrust Note that the sail is at 90° to the reflecting photons.

**EXPLAIN** 

VIDEO 8B-1 SIMILARITIES BETWEEN LIGHT AND MATTER

#### Electromagnetic wave

a transverse wave consisting of perpendicular oscillating electric and magnetic fields that travel at  $3.00 \times 10^8 \text{ ms}^{-1}$ in a vacuum with a range of wavelengths from 10-18 m to 100 km

Photons from the Sun have momentum, given by  $\frac{h}{\lambda}$  (to the right). After reflection they have  $\frac{h}{\lambda}$  (to the left). Each reflecting photon thus contributes  $\Delta p = \frac{2h}{\lambda}$  to the sail, directed to the right, due to conservation of momentum. If there are *n* photons reflecting off the space sail every second, there will be an impulse of *F* × 1, which will equal  $\frac{2h}{\lambda} \times n$ . Therefore,  $F = \frac{2nh}{\lambda}$ .

## Check-in questions – Set 1

- 1 What causes the dust tails of comets to point away from the Sun?
- **2** Is momentum conserved in interactions between photons and other particles, such as electrons?
- **3** How much momentum is transferred from a photon of wavelength  $\lambda$  when it reflects at 90° to a surface (i.e. the photon turns through 180°)?

#### Worked example 8B–1 Calculations involving the momentum of photons

Find the thrust on a large space sail, which is reflecting  $10^{21}$  photons per second of wavelength 550 nm.

#### Solution

Each photon carries a momentum of  $\frac{h}{\lambda}$ . If the photons reflect from the sail at right angles to it, each photon transfers a momentum of  $\frac{2h}{\lambda}$ . If there are *n* photons, the impulse is  $F\Delta t = \frac{2nh}{\lambda}$ . Given  $\Delta t = 1$  s,  $n = 10^{21}$  and  $\lambda = 550$  nm:

$$F = \frac{2nh}{\lambda\Delta t}$$
  
=  $\frac{2(10^{21})(6.34 \times 10^{-34})}{(550 \times 10^{-9})(1)}$   
=  $2.3 \times 10^{-6} \text{ N}$ 



## **Photon-electron collisions**

A detailed investigation of photon–electron collisions was carried out by Compton in 1923. He was investigating why X-ray wavelengths lengthened when scattering off graphite; classical theory predicted that such collisions should be elastic, meaning there should be no loss in energy or wavelength. Instead, the collisions were inelastic. Compton's experiment is shown in Figure 8B–3 below.



**Figure 8B–3** Schematic of Compton's apparatus for investigating the collisions of X-ray photons with a graphite target, with the scattered photons having a different wavelength ( $\lambda$ ')

Tungsten filament incandescent globe an electric light with a tungsten wire filament heated until it glows white hot

Outer-shell electron the outermost electrons in an atom

Infrared light

electromagnetic radiation with a wavelength range of 700 nm to 1 mm

#### Ultraviolet light

electromagnetic radiation with a wavelength range of 100 nm to 400 nm

#### Spectroscope

an optical instrument that displays the wavelength spectrum of a light source

#### Prism

a triangular glass prism that separates light of different wavelengths by refraction

#### Diffraction grating

an optical device of many parallel slits that separates light of different wavelengths by interference effects

## Worked example 8B-2 Calculations involving photon-electron collisions

only be obeyed if the X-ray photons had momentum, given by  $p = \frac{h}{2}$ .

An X-ray photon collides head-on with an **outer-shell electron** in a graphite atom. The electron can be modelled as stationary and free from the graphite atom. The photon has momentum of  $6.6 \times 10^{-23}$  kg m s<sup>-1</sup> before the collision. Relativistic effects can be ignored. The electron gains momentum of  $1.1 \times 10^{-22}$  kg m s<sup>-1</sup> in the same direction as the incident photon. Calculate the magnitude of the photon momentum after the collision.

It was clear that the scattered photon had less energy and momentum. From his results and

calculations, Compton was able to show that conservation of energy and momentum could

Solution

Momentum is conserved, so:

 $P_{\text{before}} = P_{\text{after}}$   $(6.6 \times 10^{-23}) = (1.1 \times 10^{-22}) + P_{\text{photon after}}$   $P_{\text{photon after}} = -4.4 \times 10^{-23} \text{ kg m s}^{-1}$ 

The minus sign gives the direction and is not required for the question.

## **Emission spectra**

All objects (except perhaps dark matter) above absolute zero (0 K) emit electromagnetic radiation. The visible spectrum of light from a **tungsten filament incandescent globe** is shown in Figure 8B–4. Wavelengths greater than 700 nm of **infrared light** from the globe are not visible (but can be felt). **Ultraviolet light** (wavelengths less than 400 nm) is also not visible. A spectrum like this can be obtained by observing the emitted electromagnetic radiation though a **spectroscope**, which allows the electromagnetic radiation to pass through a **prism** or **diffraction grating**; this separates the wavelengths present in the radiation.

The spectrum is continuous; there are no gaps. The spectrum is close to the **blackbody** spectrum of a perfect emitter. Only wavelengths from around 400 nm to 700 nm are visible to the human eye. Not all spectra are continuous like this. A **gas discharge tube** filled with a gas such as **hydrogen** will emit electromagnetic radiation when sufficient current is passed through it, as shown in Figure 8B–5.



Figure 8B-4 Visible spectrum of a tungsten incandescent light globe, shown at right



**Figure 8B–5** The violet glow from a hydrogen discharge tube is very different from the light produced by an incandescent light globe.

When this atomic hydrogen discharge is viewed though a spectroscope, a quite different spectrum from the incandescent globe is observed. This is an **emission line spectrum** and the visible part is shown in Figure 8B–6.





Energy is being supplied to the hydrogen atoms by the current through the tube. The excited atoms then radiate electromagnetic radiation – but only at specific wavelengths and energies, which are related by  $E = \frac{hc}{\lambda} = hf$ . Unlike the continuous spectrum of the incandescent globe, the energies emitted do not depend on the temperature of the gas in the tube.

#### Blackbody

an object that is a perfect absorber and emitter of electromagnetic radiation

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#### Gas discharge tube

a sealed tube filled with gases through which an electrical discharge is passed, exciting gas atoms into excited states. These decay and display characteristic spectra.

#### Hydrogen

the simplest element, made of a proton and an orbiting electron; element number 1

## Emission line spectrum

a spectrum with lines of a unique wavelength showing the composition of light emitted by hot gases

Helium element number 2; a nucleus with two protons and two neutrons surrounded by two orbiting electrons

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If a discharge tube is filled with another gas, for example **helium**, a quite different line spectrum results, as shown in Figure 8B–7.





These line spectra are characteristic of the atoms in the discharge tube. Each atom has its own distinct spectrum. In fact, the element helium was discovered in the Sun by this method, during a solar eclipse in the Sun's atmosphere, before the element was discovered as being present on Earth.

#### Another representation of an emission line spectrum

Line spectrum can also be shown graphically as shown in Figure 8B–8. This shows wavelengths from the near infrared to ultraviolet wavelengths. These are nearly all visible wavelengths.





#### Worked example 8B–3 Problems involving emission spectra

What is the photon energy of the transition with the greatest energy in Figure 8B–8? Give your answer in both eV and in J to three significant figures.

#### Solution

The greatest energy level difference,  $\Delta E$ , corresponds to the transition with the greatest value of  $\frac{hc}{2}$ . This is the 389 nm photon, as it is the peak with the smallest wavelength. Use:

$$\Delta E = \frac{hc}{\lambda}$$

$$= \frac{\left(6.63 \times 10^{-34}\right) \left(3.00 \times 10^{8}\right)}{389 \times 10^{-9}}$$

$$= 5.11 \times 10^{-19} \text{ J}$$

$$= 3.19 \text{ eV (since 1 J = 6.25 \times 10^{18} \text{ eV})}$$

#### A different type of discharge tube – a metal vapour lamp

No metals are gases at room temperature, so it is necessary to vaporise them in order to observe their spectrum. Well known types are mercury and sodium vapour lamps. These **metal vapour lamps** work by establishing a hot electric arc that vaporises the metal, enabling the atoms to be excited into higher **energy states** and produce a spectrum from the energy state transitions, just like the hydrogen and helium spectra.

#### A natural discharge 'lamp'

When high speed, charged particles from the solar 'wind' are channelled by magnetic fields near the poles, they strike atoms in the higher regions of the atmosphere and excite them into high energy states. Photons from the subsequent decay transitions cause the famous **polar auroras** – aurora borealis in the northern hemisphere and aurora australis in the southern hemisphere. The different colours correspond to the excitation of different atoms – mainly oxygen and nitrogen.





#### Metal vapour lamp

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a gas discharge lamp in which metals are vaporised through a hot electric arc, exciting gas atoms into higher energy states to produce characteristic spectra

#### **Energy state**

the possible energy state of atoms, often described as an energy level

#### Polar aurora

charged particles from the Sun excite atoms near the poles into high energy states. They then decay, emitting photons.

### Check-in questions – Set 2

- **1** What is an emission line spectrum?
- 2 What determines the wavelengths present in a gas discharge spectrum?
- **3** What is the difference between a simple gas discharge lamp and a metal vapour lamp?

### The energy states of atoms

Using the line spectrum of hydrogen, Niels Bohr was the first to suggest an explanation of the way in which the lines of the spectrum occur at their specific wavelengths. He suggested that the electrons in the atom can only occupy certain fixed energy states, and that light of a specific energy was emitted when the electron moved from one state to a lower state.

Mathematically, this can be written as  $\Delta E = \frac{hc}{\lambda} = hf$ , where  $\Delta E$  is the energy difference

between the states. This is another example of quantisation, a phenomenon that cannot be explained in terms of classical Newtonian physics.

#### **Energy state diagrams**

The energy states of atoms are routinely described in terms of diagrams like that shown in Figure 8B–10.



Ground state

the lowest energy state of an atom. It has a quantum number of 1.

**Quantum number** a number used to order the energy states in an atom

lon

a charged atom due to the loss or gain of an electron Figure 8B-10 Part of the energy state diagram for the hydrogen atom

Normally the atom will be in the **ground state**; this is its stable state and has the **quantum number** '1'. The higher energy states are labelled with quantum numbers as shown in the above diagram. When an atom receives enough additional energy to raise it to a higher level, an electron can transition into one of the higher energy states. (Note that if a hydrogen atom receives 13.6 eV or more, it will lose an electron and become an ion.)

8B SIMILARITIES BETWEEN LIGHT AND MATTER

The states above the ground state are not stable, and they decay into lower energy states until the ground state is reached. Each transition from a higher state to a lower state means a loss of energy; this happens by the emission of a photon of energy equal to the difference in the energy states. The transition can be shown on the energy state diagram by means of an arrow. As an example, two transitions in a hydrogen atom are shown in Figure 8B–11.

The decay path shown in Figure 8B–11 is from the n = 4 state to the n = 2 state, emitting a 2.6 eV photon and then from the n = 2 state to the ground state, emitting a 10.2 eV photon. Of course, the n = 4 state might have decayed via the n = 3 state, and then the n = 2 state, and then to the ground state. However, it will always finish at the ground state. Figure 8B–12 shows all possible decays of a hydrogen atom, initially in the n = 5 state.

There will also be possible transitions from n = 6 and higher states (if excited) not shown in the diagram. Taken together, all the emitted photons from the possible transitions make up the line spectrum that we see from a gas discharge tube containing hydrogen. The line spectra from other gases can also be observed and analysed. Each element will have its own unique spectrum, and this can be used to detect the presence or absence of particular elements in a sample of material. It can even be used to detect the elements present in stars, comets and even exoplanets and their atmospheres.



**Figure 8B–11** A possible decay path from the n = 4 state to the ground state, via the n = 2 state





Exoplanet a planet that orbits a star outside the solar system

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### Worked example 8B–4 Calculations with energy state diagrams

Some of the energy states of sodium are shown below. Part of the emission spectrum of sodium vapour includes a photon of energy 1.51 eV.



- **a** Which energy states are involved in the transition producing a 1.51 eV photon?
- **b** What is the greatest frequency photon that can be emitted when the atom decays from the n = 4 level to the ground state?
- **c** Is it possible for cool sodium vapour to absorb photons of 591 nm? Justify your answer with a calculation.

#### Solution

- **a** The photon energy is equal to  $\Delta E$ . There is a gap of 1.51 eV between the n = 4 and n = 2 states. Hence, these are the states involved.
- **b** The greatest frequency photon will be the photon with the greatest energy. This corresponds to the transition from n = 4 state to the ground state. This has an energy of 3.61 eV. Now use E = hf, rearranging with f as the subject.

$$f = \frac{3.61}{4.14 \times 10^{-15}}$$
$$= 8.72 \times 10^{14} \text{ Hz}$$

**c** Calculate the energy in eV of a 591 nm photon. Use  $E = \frac{hc}{\lambda}$ .

$$E = \frac{hc}{\lambda} = 4.14 \times 10^{-15} \times \frac{3 \times 10^8}{591 \times 10^{-9}} = 2.1 \text{ eV}$$

This corresponds to a transition from the ground state to the 2.10 eV state. Hence, the absorption is possible.

## **Absorption spectra**

Absorption line spectra (plural of spectrum) occur when photons with specific energies are absorbed from an otherwise continuous spectrum. A wellknown example are the dark lines in the Sun's spectrum, known as Fraunhofer lines, named after an early observer.

Several decades after their discovery, it was noticed that the wavelengths of many of the lines were the same as the emission lines of elements in the solar atmosphere (such as hydrogen and helium) and in Earth's atmosphere (such as oxygen and nitrogen). The dark lines indicate that on their journey through the atmospheres of the Sun and Earth, photons at this wavelength have been absorbed by an atom. This absorption is only possible because the photon energy is equal to the energy gap between atomic energy states. An example is the 656 nm dark line. This has been identified as a transition of hydrogen atoms in the n =2 state, absorbing a photon of 1.9 eV and making an upward transition to the n = 3state. This is shown in the hydrogen energy state diagram in Figure 8B-14.

These lines occur when a cooler gas is between a wide spectrum of photons (like the Sun) and an observer or detector. The dark line occurs because the photons are absorbed and then re-emitted in random directions. Since almost all of these random directions are not in the original direction, they appear to have been completely absorbed, showing a black line in the spectrum.

## Another form of the energy state diagram

Sometimes the energy state diagrams are drawn with the ground state as a negative quantity, as shown in Figure 8B–15 for the first four states of the hydrogen atom.

The spacing between the states is identical to the other representation. The negative



wavelength seen against the background of a continuous spectrum

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Absorption line spectrum

a spectrum with dark lines

of a unique

**Figure 8B–13** The dark lines observed in the solar spectrum are called Fraunhofer absorption lines.



**Figure 8B–14** The 656 nm absorption line is caused by a transition from the n = 2 state to the n = 3 state of hydrogen.





values indicate that the electron is bound to the atom, unable to 'escape' from it unless it is given a total energy greater than 0 eV. So, an atom in the ground state (n = 1) needs 13.6 eV to be ionised, an atom in the n = 2 state needs 3.4 eV to be ionised, and so on.

## Check-in questions – Set 3

- 1 What is the quantum number of the ground state?
- **2** How many different photon energies can possibly be emitted when a large number of atoms transition from the *n* = 4 to the *n* = 1 state?
- 3 What is the difference between emission and absorption spectra?
- **4** How do you find the wavelength of a transition, given  $\Delta E$ , the difference in two energy states?

#### Why are the atomic energy states stable?

Atoms are often pictured as 'planetary', with electrons orbiting the **nucleus**, as shown in Figure 8B–16. An atom of hydrogen using this representation is shown.

This is essentially a classical model, where the **electrostatic force** between the proton and the electron keeps the electron in a stable circular (or elliptical) orbit. This model has two weaknesses. First, classical physics predicts that the electron orbit should be able to have a continuous range of values, like satellites orbiting Earth. However, this does not happen.





Instead, the energy of the electrons in atoms is **quantised**. Second, classical physics predicts that accelerating charged particles should lose energy by emitting electromagnetic radiation and their orbits should collapse. Clearly this does not happen as the actual orbits are stable. To explain the electron behaviour in atoms, their wave properties must be used.

## Check-in questions – Set 4

- 1 What are two main problems that classical physics encountered when it attempted to explain the stable structure of atoms?
- 2 What property of electrons enables them to form stable energy states?

#### Photographs photon by photon

We are so used to high quality images that we do not think of images being built up photon by photon, as in Figure 8B–17.

0.001 photons/pixel	0.01 photons/pixel	0.1 photons/pixel	
1 photons/pixel	10 photons/pixel	 100 photons/pixel	
	and the second sec	and the second second	
	- (A)	130	
1000 photons/pixel	10000 photons/pixel	100 000 photons/pixel	
1000 photons/pixel	10000 photons/pixel	100 000 photons/pixel	
1000 photons/pixel	10 000 photons/pixel	100 000 photons/pixel	

**Figure 8B–17** Development of a photograph, photon by photon

#### Nucleus

the solid centre of an atom where most of the mass of an atom is concentrated

#### Electrostatic force

the attractive force between the protons in the nucleus and the orbiting electrons

#### Quantised

when a quantity or variable can only have certain (discrete) values

## Double slit photon experiments at very low light levels

Double slit experimental results are routinely and successfully interpreted using a wave model for light, using the concepts of **superposition**, constructive interference and destructive interference, which are covered in Chapter 7 and Section 8A. A typical set-up is shown in Figure 7A–25.

The explanation of the interference pattern relies on waves from one slit interfering (by superposition) with waves from the other slit. However, it is possible to arrange for the intensity of the light to be so small, using a very faint light, that only one photon passes through one or other of the double slits. Does this mean that there will not be an interference pattern? Figure 8B–18 shows the result, over time, of a double slit experiment with only one photon passing through the slits at any time.



Figure 8B–18 Double slit experiment performed photon by photon. An interference pattern appears as more photons are detected.

Some experimenters placed detectors just in front of each slit, to detect which slit the photons pass through. When this is done, the photon is detected at one slit or the other, never at both, and the interference pattern vanishes. This is particle-like behaviour. However, if it is not detected, it maintains its wave-like behaviour; it travels to the screen and does not go to a dark region in the pattern. This wave-like behaviour responds to the presence of both slits. It is as if the photon is interfering with itself; photons do not interfere with another photon. This exemplifies the dual nature of light: particle-like behaviour and wave-like behaviour.



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Superposition the principle

that when two

or more waves overlap, their

displacements

add together

## Double slit experiments with electrons

Carrying out double slit experiments with electrons is more difficult than with light, as the wavelengths are so much smaller. Nonetheless, in 1961, Claus Jönsson sent a beam with wavelength  $3.1 \times 10^{-11}$  m through slits 300 nm wide. It produced an interference pattern, confirming the wave-like nature of electrons. Moreover, in 2013, Roger Bach and his team were able to reduce the electron beam in a similar experiment to one electron at a time, producing, as with the single photon experiment, an interference pattern. Thus electrons, as well as photons, have been shown to have a dual nature with particle-like behaviour and wave-like behaviour.



Figure 8B–19 A double slit experiment performed with single electrons also creates an interference pattern.



## Check-in questions – Set 5

- 1 If a double slit experiment with light is carried out with single photons passing through the double slits, one at a time, will the interference pattern disappear?
- **2** Will a double slit interference experiment with electrons, with suitable slit spacing and wavelengths, produce an interference pattern?

## **Quantisation and light**

Einstein's revolutionary explanation of the photoelectric effect involved challenging the prevailing theory of the wave nature of light and was an early example of quantisation. He used the word quanta; we now use the word photon. Chapter 7 deals with this in some detail. It does not mean that we can dispense with the wave properties of light, as the single photon diffraction and interference experimental results show. Until we detect a photon, it retains its wave properties, and we have to describe its position in terms of probabilities.

## Quantisation and the nature of atoms

Several quantities associated with atoms are quantised. These include the charge on the nucleus, the number of nucleons (protons and neutrons) in the nucleus, the number of electrons in the atom and the charge on each electron. All electrons have a charge of  $-1.602 \times 10^{-19}$  C, and all protons have a charge of  $+1.602 \times 10^{-19}$  C. Of particular interest in this chapter is the quantisation of energy states, which arises from the wave properties of electrons and is responsible for the line emission and absorption spectra studied in this chapter. These spectra show that electrons in atoms can only exist in stable discrete energy states; their energies are quantised. This can be accurately explained only by using the Schrödinger equation of quantum mechanics. However, sometimes models are used to simulate this phenomenon, such as electrons circling nuclei in stable standing matter waves. While this is not an accurate representation, it can sometimes assist an approach to the complexities of quantum mechanics.

## **8B SKILLS**

#### **Explain-type questions**

VCAA examinations and internal school SACs frequently require explanations rather than numerical or letter choice responses. An approach to these questions can involve some of the following steps:

- **1** Where possible and/or appropriate, start with a diagram, graph or sketch.
- 2 Construct your explanation as a series of single idea dot points.
- **3** Always quote any relevant formulas.
- **4** If possible, give numerical values of quantities.
- **5** Summarise your answer, even if it means repeating most of the original question.

#### Using the correct formulas for photons and matter

Be clear about the formula to use for momentum and energy. See the table below.

Matter	Photons
$\lambda = \frac{h}{p}$	$\lambda = \frac{h}{p}$
p = mv	$p = \frac{E}{c}$
$p = \sqrt{2mE_k}$	$c = f\lambda$
$E_{\rm k} = \frac{1}{2}mv^2$	$E = hf = \frac{hc}{\lambda}$

Recall that there are two values that can be used for Planck's constant, *h*: either

 $4.14 \times 10^{-15}$  eV s or  $6.63 \times 10^{-34}$  J s. Only use the eV s value for  $E = hf = \frac{hc}{\lambda}$ ; as the energy should be in units of eV.

## Section 8B questions

## **Multiple-choice questions**

- A searchlight beams out green light of wavelength 520 nm. The momentum of a single 520 nm photon is closest to
  - **A**  $3.9 \times 10^{-19} \text{ kg m s}^{-1}$
  - **B**  $1.3 \times 10^{-27} \text{ kg m s}^{-1}$
  - **C**  $6.6 \times 10^{-37} \text{ kg m s}^{-1}$
  - **D**  $3.4 \times 10^{-60} \text{ kg m s}^{-1}$
- **2** The searchlight power in Question 1 is 5.0 kW. Its photons are focused onto a perfectly reflecting mirror. The average force on the mirror is closest to
  - A  $1.7 \times 10^{-5}$  N
  - **B**  $3.3 \times 10^{-5}$  N
  - **C**  $5.0 \times 10^3 \,\mathrm{N}$
  - **D**  $1.0 \times 10^4 \text{ N}$



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- **3** The spacing between atoms in a certain metal crystal is  $3 \times 10^{-9}$  m. Which of the following would be most likely to show wave-like behaviour when fired at a thin layer of the crystal? Justify your choice.
  - **A** 60 eV electrons
  - **B** X-rays with a frequency of  $10^{19}$  Hz
  - C  $10^{-6}$  kg dust particles moving at 0.01 m s<sup>-1</sup>
  - D gamma rays of energy 2.5 MeV
- 4 Quantised energy levels within atoms can best be explained by
  - A electrons behaving as individual particles with varying energies.
  - **B** atoms having energy requirements that can only be satisfied by electrons.
  - **C** electrons behaving as waves, with each energy level representing a diffraction pattern.
  - D wave-like properties of electrons, with stable standing waves being formed.

Adapted from VCAA 2017

- **5** In an experiment, an atom has only four possible energy levels available. In increasing levels of energy, they are  $E_0$ ,  $E_1$ ,  $E_2$  and  $E_3$ . An atom is excited to  $E_3$ . Which of the following is a possible energy for an emitted photon?
  - **A**  $E_3$  **B**  $E_3 - E_0$ **C**  $E_2$
  - **D**  $\vec{E_0}$
- 6 Which of the following options best explains the observation that atoms are generally only stable at very specific energies?
  - A wave-like properties of electrons
  - **B** particle-like properties of electrons
  - **C** wave-like properties of photons
  - D particle-like properties of photons

#### **Short-answer questions**

7 The diagram below shows some energy levels in a neon atom.



Use the data in the diagram to calculate two photon frequencies that could be absorbed by neon atoms in the lowest state shown in the diagram.

8 Calculate the de Broglie wavelength of 1.9 eV electrons. Ignore relativistic effects.

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9 Below is an energy level diagram for a hydrogen atom.

List the possible photon energies following emissions from the n = 4 state.

Adapted from VCAA 2019

10 Scientists are carrying out Young's double slit interference at light levels so low that there is only one photon passing through the double slits at any one time. They are surprised to see that a clear interference pattern builds up, despite the lack of interfering photons in the apparatus. Explain why this is possible.



## **Chapter 8 review**

## Summary

Create your own set of summary notes for this chapter on paper or in a digital document. A model summary is provided in the Teacher Resources, which can be used to compare with yours.

## Checklist

In the Interactive Textbook, the success criteria are linked from the review questions and will be automatically ticked when answers are correct. Alternatively, print or photocopy this page and tick the boxes when you have answered the corresponding questions correctly.

Succe	ss criteria – I am now able to:	Linked questions
8A.1	Recognise electron diffraction patterns	1 , 4
8A.2	Understand that the diffraction patterns of matter are evidence of the wave-like nature of matter, including electrons	3 , 13
8A.3	Distinguish between diffraction patterns produced by photons and electrons	3□, 14□, 15□
8A.4	Calculate the de Broglie wavelength of matter particles using $\lambda = \frac{h}{p}$	2 , 4 , 5 , 12 , 13 , 16
8B.1	Discuss how the idea of quantisation developed our knowledge about light and helped explain the nature of atoms	23 , 24
8B.2	Use $p = \frac{h}{\lambda}$ to calculate the momentum of photons	6 , 19
8B.3	Compare the momentum of photons and matter with the same wavelength	12 , 19
8B.4	Explain the production of atomic emission line spectra, including those from metal vapour lamps	10
8B.5	Explain the formation of atomic absorption line spectra	11 , 20
8B.6	Calculate the energy of emitted or absorbed photons from spectra using $\Delta E = \frac{hc}{\lambda}$	17 , 20
8B.7	Analyse the emission or absorption of photons by atoms in terms of a change in the energy state of atoms using $\Delta E = hf$	7□, 8□, 17□, 18□, 20□
8B.8	Interpret the results of the double slit experiment with single photons and electrons as evidence for the dual nature of light and matter	9 , 21 , 22
# **Multiple-choice questions**

1 When X-rays of wavelength  $1.5 \times 10^{-11}$  m are directed at some graphite, a pattern of light and dark lines are observed, part of which are shown in the image to the right.

Which of the following best describes this pattern?

- **A** a classic double slit interference pattern
- **B** electron paths circling the graphite nuclei
- **C** an energy level diagram of graphite atoms
- **D** a diffraction pattern with light and dark constructive and destructive interference
- **2** Identify the correct statement about de Broglie wavelengths.
  - A Neutrons cannot have a de Broglie wavelength because they are uncharged.
  - **B** Protons cannot have a de Broglie wavelength because they never diffract.
  - **C** The de Broglie wavelength of electrons increases with increasing speed.
  - **D** As neutrons speed up, their de Broglie wavelength reduces.
- **3** An electron diffraction pattern can be distinguished from an X-ray diffraction by
  - A increasing the speed of the electrons only; the pattern will become more spread out.
  - **B** applying a magnetic field to the beams; the X-rays will be deflected but the electrons will not.
  - **C** decreasing the speed of the electrons only; the pattern will become more spread out.
  - D increasing the speed of the X-rays only; the pattern will become more spread out.
- 4 Electrons pass through a very fine metal grid, forming a diffraction pattern. If the speed of the electrons is doubled using the same grid spacing, the pattern spacing would
  - A increase.
  - **B** decrease.
  - **C** not change.
  - **D** not be able to be determined from the information given.

Adapted from VCAA 2019

- 5 Protons are travelling at a speed of  $2.0 \times 10^3$  m s<sup>-1</sup>. The best estimate of the de Broglie wavelength of these protons is
  - **A**  $1.2 \times 10^{-10} \text{ m}$
  - **B**  $2.0 \times 10^{-10} \text{ m}$
  - **C**  $1.2 \times 10^{-7}$  m
  - $\textbf{D} \ 2.0\times 10^{-7} \ m$

- VCAA 2020
- **6** Which of the following is closest to the momentum of a 3.0 MeV gamma ray?
  - **A**  $1.6 \times 10^{-21} \text{ kg m s}^{-1}$
  - $\textbf{B} \ 1.0 \times 10^{-2} \, kg \, m \, s^{-1}$
  - **C**  $1.6 \times 10^{-27} \text{ kg m s}^{-1}$
  - **D**  $1.6 \times 10^{-24} \text{ kg m s}^{-1}$



7 Part of the energy level diagram for an unknown atom is shown below.



Which one of the arrows shows a change of energy level corresponding to the absorption of a photon of highest frequency?

- **A** A
- **B** B
- **C** C
- **D** D
- 8 The diagram below shows some of the energy levels for the electrons within an atom. The arrows labelled A, B, C and D indicate transitions between the energy levels and their lengths indicate the relative size of the energy change.



Which transition results in the emission of a photon with the most energy?

- **A** A
- **B** B
- **C** C
- DD

**9** The centre of a dark region in a single photon double slit interference pattern for light happens because

- A any photons that arrive there annihilate each other at that point.
- **B** some photons involved arrive there but are not able to be detected.
- **C** the energy of the photons is greatly reduced due to interference.
- **D** no photons travel to that point; the probability of them going there is zero.

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- **10** Which of the following best describes the formation of emission spectra from gas discharge and metal vapour tubes?
  - **A** A current of energetic electrons excites gas atoms into higher energy states, which then decay, emitting photons.
  - **B** Energetic electrons collide with gas atoms, each collision absorbs photons from higher states.
  - **C** A current flows through the discharge tube; this current creates photons that are then emitted from the tube.
  - **D** Many atoms are already in higher energy states; the current then stimulates their decay, emitting photons.
- **11** Which of the following best describes an absorption spectrum?
  - **A** the bright lines in an otherwise dim spectrum formed by the absorption of photons
  - **B** the dark lines in an otherwise continuous spectrum formed by the absorption of photons of energy equal to the gap in atomic energy levels
  - **C** re-emitted light from photons formed by absorption of photon energy from a bright spectral background
  - **D** the emission of photons descending from higher to lower energy states in a low intensity spectrum

## Short-answer questions

**12** X-rays are fired at metal foils. Those passing through the foils are diffracted to form rings centred on the axis. The pattern is roughly sketched below. It is not to scale.



The dark circles indicate where the maximum number of X-rays arrived

When electrons are fired at the same foils, the same kinds of patterns are formed, with the same spacings. The X-ray photons have a wavelength of  $8.35 \times 10^{-10}$  m and a momentum of  $7.94 \times 10^{-25}$  kg m s<sup>-1</sup>.

- a Calculate a value for Planck's constant, *h*, from the data, in Js and eVs. Show all your reasoning.(3 marks)
- **b** Calculate the speed of the electrons. Ignore relativistic effects. (3 marks)
- **13 a** Calculate the de Broglie wavelength of a small 50 g ball travelling at 20 m s<sup>-1</sup>. (2 marks) **b** Would it be possible to observe diffraction effects with this ball travelling at this speed?
  - (1 mark)
- **14** Distinguish between diffraction patterns produced by X-rays and those produced by electrons. (2 marks)
- 15 Compare quantitatively the diffraction effects of 500 nm photons passing through a 0.05 mm slit with electrons with speed 0.01*c* passing through a 0.5 mm slit. (5 marks)
- 16 Calculate the speed that neutrons would have if they were to have the same de Broglie wavelength as 10 keV electrons. (3 marks)

**17** The diagram below shows the first six energy states of a mercury atom. It is not to scale.



- **a** List the photon energies that could be emitted when the atom is in the n = 3 state. (2 marks)
- **b** A mercury atom is in the 10.4 eV state. What is the longest wavelength photon that could be emitted when the atoms decays? What region of the electromagnetic spectrum will it occupy? (3 marks)
- **c** A mercury atom is in the n = 5 state. It decays to the ground state, emitting four photons. One has an energy of 2.2 eV. What are the energies of the other three? (3 marks)
- **18** The diagram below shows the first four energy states of a hydrogen atom.



*n* = 2 — \_\_\_\_\_\_\_ –3.40 eV



- **a** A photon of wavelength  $4.87 \times 10^{-7}$  m is emitted by an excited hydrogen atom within the<br/>Sun. Identify the transition occurring within hydrogen.(2 marks)
- **b** A photon of wavelength  $1.03 \times 10^{-7}$  m is absorbed by a hydrogen atom near the Sun's surface. Identify the transition occurring within the hydrogen atom. (2 marks)
- **c** A hydrogen atom near the Sun's surface is in the n = 2 state. What energy photon could it absorb for it to move to the n = 3 state? (2 marks)
- **19** The de Broglie wavelength of an electron and the wavelength of an X-ray photon are measured to be 0.42 nm.
  - **a** Calculate the energy of the electron in electronvolts (eV). (2 marks)
  - **b** Calculate the energy of the X-ray photon in kiloelectronvolts (keV). (2 marks)

**20** Close examination of the solar spectrum reveals dark lines, known as Fraunhofer lines, at specific wavelengths. A teacher is looking at a section of the solar spectrum, shown below, with their students.



The dark lines are an example of an absorption line spectrum.

Explain the origin of the dark lines.

а

(2 marks)

- **b** There is a dark line at 656 nm. Calculate the energy, in electronvolts (eV), of a 656 nm photon.
  - (1 mark)
- **c** The teacher believes that this wavelength corresponds to an atomic transition in the hydrogen spectrum. Use the diagram below to identify the most likely transition. (2 marks)



**d** Suggest a reason why it is not surprising that hydrogen should be involved in the solar line absorption spectrum. (2 marks)

**21** Students are carrying out a Young's double slit interference experiment. The interference pattern is obtained, as shown schematically below.



They explain the results in terms of waves from the two slits arriving on the screen and apply the superposition principle. When the path difference equals  $n\lambda$ , it is the centre of a bright band, and when the path difference equals  $(n + \frac{1}{2})\lambda$  it is the centre of a dark band, producing

the interference pattern above. They then modify the apparatus using a dim light source, so dim that only one photon at a time passes through the double slits. They cannot now apply the superposition principle, yet the interference pattern still forms on the screen, although it builds up slowly, one photon at a time.

Explain how the dark and light pattern forms on the screen with the very dim light source. (3 marks)

- 22 In the 1970s, Giorgio Merli and his team performed a double slit interference experiment with one electron passing through the double slit at a time. They found that even when only one electron passed through the double slits, over time an interference pattern was built up on the screen. If an attempt was made to determine which slit the electrons passed through, the interference pattern disappeared. Account for these two experimental findings in terms of the nature of electrons. (3 marks)
- 23 Outline the quantisation involved by Einstein in his explanation of the mechanism of the photoelectric effect. (3 marks)
- 24 Outline how observations of line emission spectra provide evidence for the quantisation of the energy states of atoms. (3 marks)



# HOW HAVE CREATIVE IDEAS AND INVESTIGATION REVOLUTIONISED THINKING IN PHYSICS?

# EINSTEIN'S SPECIAL THEORY OF RELATIVITY

### Introduction

**CHAPTER** 

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When Albert Einstein was five years old, his father gave him a compass. Einstein was captivated by the compass and would lay awake at night wondering about the nature of the invisible force that pushed the needle to point north. By the age of sixteen, Einstein had taught himself integral and differential calculus and used this knowledge to study Maxwell's equations of electromagnetism. Einstein's endless curiosity came at a cost; he found the regimented teaching methods of school stifling and often came into conflict with teachers and professors. The conflict was so great that none of his professors were willing to give him a good reference. In 1900, Einstein graduated from the Zürich polytechnic school with a maths and physics diploma. Although well qualified, Einstein couldn't get a job as a high school physics teacher. Through a friend's connection, Einstein ended up working as patent clerk in Bern, Switzerland.

In his spare time, Einstein began to work on several papers. In 1905, he published five papers, one of which, 'On electrodynamics of moving bodies', outlined his special theory of relativity. This single paper represented a radical shift in the prevailing thinking at the time. It challenged Newton's ideas; that there is no limit to the speed objects could go and that there is an absolute reference frame from which time and motion could be measured. Einstein argued that two observers that are moving relative to each other would not agree on the amount of time that passes, the length of objects or even the order that events take place. Einstein also wrote his most famous equation,  $E = mc^2$ , relating energy and mass to each other.

It took three years after his papers were published for Einstein to be recognised as a leading scientist. In 1908, he was finally offered a job as a lecturer at the University of Bern. Einstein was not satisfied with his theory of special relativity, which discussed only the special case of objects moving at a constant velocity. So, he began working on a more general theory, and in 1915 released his general theory of relativity, which proposed that acceleration and gravity are equivalent. In the paper, Einstein described how a massive object can curve the fabric of space-time. Einstein proposed that the trajectory of massless light particles could be affected by a gravitational field (the curvature of space-time), additionally he predicted that time will pass more slowly in a gravitational field.

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In 1919, Sir Arthur Eddington took a photo during a solar eclipse and showed that the apparent position of a few stars were different from their true position. The shift in the stars position could only be explained by the trajectory of light bending around the Sun.

While all of Einstein's work is significant and intriguing, this chapter will focus on Einstein's special theory of relativity. This chapter will investigate how the two postulates of special relativity led Einstein to predict very different results than classical physics for objects moving close to the speed of light. These predictions include that two people in different frames of reference will not agree on the time between events or the length of objects. These different predictions helped to correctly build particle accelerators, global positioning systems (GPS), nuclear power plants and explain the detection of muons on Earth.

### Curriculum

#### Area of Study 1 Outcome 1 How has understanding about the physical world changed?

<ul> <li>Einstein's special theory of relativity</li> <li>Describe Einstein's two postulates for his special theory of relativity that: <ul> <li>the laws of physics are the same in all inertial (non-accelerated) frames of reference</li> <li>the speed of light has a constant value for all observers regardless of their motion or the motion of the source</li> </ul> </li> <li>Interpret the null result of the Michelson-Morley experiment as evidence in support of Einstein's special theory of relativity</li> <li>Describe the limitation of classical mechanics when considering motion approaching the speed of light</li> <li>Compare Einstein's special theory of relativity with the principles of classical physics</li> </ul>	Study Design	Learning intentions – at the end of this chapter I will be able to:		
	<ul> <li>Einstein's special theory of relativity</li> <li>Describe Einstein's two postulates for his special theory of relativity that: <ul> <li>the laws of physics are the same in all inertial (non-accelerated) frames of reference</li> <li>the speed of light has a constant value for all observers regardless of their motion or the motion of the source</li> </ul> </li> <li>Interpret the null result of the Michelson-Morley experiment as evidence in support of Einstein's special theory of relativity</li> <li>Describe the limitation of classical mechanics when considering motion approaching the speed of light</li> <li>Compare Einstein's special theory of relativity with the principles of classical physics</li> </ul>	<ul> <li>9A What is relativity?</li> <li>9A.1 Discuss the nature of inertial reference frames</li> <li>9A.2 Identify inertial and non-inertial reference frames</li> <li>9A.3 Discuss the idea of relativity</li> <li>9A.4 Recall and apply the two postulates of special relativity</li> <li>9A.5 Evaluate the Michelson and Morley attempt to measure the aether and justify why the null result is indicative of the non-existence of the aether</li> <li>9A.6 Compare and explain the different predictions of Einstein's special theory of relativity with the predictions of classical physics</li> </ul>		

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#### **Study Design**

- Describe proper time (t<sub>0</sub>) as the time interval between two events in a reference frame where the two events occur at the same point in space
- Describe proper length (*L*<sub>0</sub>) as the length that is measured in the frame of reference in which objects are at rest
- Model mathematically time dilation and length contraction at speeds approaching *c* using the equations:

$$t = t_0 \gamma \text{ and } L = \frac{L_0}{\gamma} \text{ where}$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

- Explain and analyse examples of special relativity including that:
  - particle accelerator lengths must be designed to take the effects of special relativity into account
  - time signals from GPS satellites must be corrected for the effects of special relativity due to their orbital velocity

- Explain and analyse examples of special relativity including that:
  - muons can reach Earth even though their half-lives would suggest that they should decay in the upper atmosphere

# Learning intentions – at the end of this chapter I will be able to:

### 9B Time dilation and length contraction

- **9B.1** Define a proper time interval (*t*<sub>0</sub>) as the time interval between two events, where the two events occur at the same point in space
- **9B.2** Identify that the time interval between two events, where the events occur in two different points in space, is the dilated time interval (*t*)
- **9B.3** Understand that proper length  $(L_0)$  is the length measured when stationary relative to the object being measured
- **9B.4** Identify when a length interval is a proper length  $(L_0)$

**9B.5** Recall that objects will only contract in length in the relative direction of travel

**9B.6** Be able to apply the length contraction formula  $L = \frac{L_0}{L_0}$  and the time dilation

formula  $t = t_0 \gamma$  to various situations

- **9B.7** Explain why a particle accelerator must consider the effects of length contraction in its design
- **9B.8** Understand and explain why time signals from GPS satellites must be corrected for the effects of special relativity due to their high orbital velocity

**9B.9** Understand what happens to dilated time and contracted length with an increasing speed

#### 9C Muon decay

- **9C.1** Explain that from an Earth reference frame, the mean half-lives of the muons are dilated, which allows them to travel more distance than would be predicted by classical physics
- **9C.2** Explain that from the muons reference frame, the length from their point of origin to the surface of Earth is contracted, which means they can travel a greater distance than would be predicted by classical physics

3

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40 2-

Study Design	Learning intentions – at the end of this chapter I will be able to:		
<ul> <li>Relationship between energy and mass</li> <li>Interpret Einstein's prediction by showing that the total 'mass-energy' of an object is given by:</li> <li>E<sub>tot</sub> = E<sub>k</sub> + E<sub>0</sub> = γmc<sup>2</sup> where E<sub>0</sub> = mc<sup>2</sup>, and where kinetic energy can be calculated by:</li> </ul>	<b>9D Mass and energy equivalence</b> <b>9D.1</b> Understand that mass and energy and different forms of the same thing and use the formula $E = mc^2$ to convert between rest energy and mass <b>9D.2</b> Apply the formulas of total energy		
• Apply the energy–mass relationship to mass conversion in the Sun, to positron– electron annihilation and to nuclear transformations in particle accelerators (details of the particular nuclear processes are not required)	$(E_{tot} = E_k + E_0 = \gamma mc^2), \text{ rest energy}$ $(E_0 = mc^2) \text{ and kinetic energy}$ $(E_k = (\gamma - 1)mc^2) \text{ to various situation}$ <b>9D.3</b> Apply the formula $\Delta E = \Delta mc^2$ to a nuclear fusion reaction, positron–electron annihilation and the nuclear transformations in particle accelerators		
	<ul> <li>9D.4 Understand how particle accelerator can produce new elements</li> <li>9D.5 Plot the value of the Lorentz factor (γ) against relative speed on a graph and interpret its effects on energy</li> </ul>		

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## Glossary

- Coherent Dilated time (*t*) Event Inertial reference frame Lorentz factor Mass defect Medium
- Muon Nuclear fusion Nuclear transmutation Positron Postulate Product Proper length  $(L_0)$
- Proper time  $(t_0)$ Reactant Reference frame Relativity Strong nuclear force

and mass

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# **Concept map**

Einstein based his special theory of relativity on two postulates.

- the laws of physics are the same in all inertial frames of reference
- the speed of light has a constant value for all observers regardless of their motion or the motion of the source



See the Interactive Textbook for an interactive version of this concept map interlinked with all concept maps for the course.



# What is relativity?

#### Study Design:

- Describe Einstein's two postulates for his special theory of relativity that:
  - the laws of physics are the same in all inertial (non-accelerated) frames of reference
  - the speed of light has a constant value for all observers regardless of their motion or the motion of the source
- Interpret the null result of the Michelson–Morley experiment as evidence in support of Einstein's special theory of relativity
- Describe the limitation of classical mechanics when considering motion approaching the speed of light
- Compare Einstein's special theory of relativity with the principles of classical physics

#### **Glossary:**

Coherent Event Inertial reference frame Medium Postulate Reference frame Relativity

**ENGAGE** 

# Evidence that Einstein was right

As time passes, more evidence is emerging to support Einstein's theories. In 2015, the Laser Interferometer Gravitational-wave Observatory (LIGO) first detected gravitational waves, predicted by the theory of general relativity, produced by two black holes merging 1.3 billion years ago. In April 2019, the first photo of a black hole was taken. The photo was taken of the supermassive black hole at the centre of the galaxy Messier 87. The light is bent enough to create a bright ring around the black hole, but the black hole itself is still invisible (i.e., black). The dark region in the centre of the ring is the black hole's event horizon, beyond which no light can escape.



**Figure 9A–1** The image on the left is a representation of two black holes merging to create gravitational waves. The image on the right is humanity's first ever image of a black hole. The black hole belongs to the Messier 87 galaxy, more than 50 million light-years away. It has a mass 6.5 billion times greater than our Sun.

Einstein's theories of relativity led to a fundamental change in physics but he was awarded only one Nobel Prize: in 1922 for his paper on the photoelectric effect, which was one of the five papers he published in 1905. In this paper on the photoelectric effect, Einstein described light as packets of discrete energy instead of a wave. Describing light as quantised was a big step towards the development of quantum mechanics.



Note that general relativity is not part of this course.

# EXPLAIN

### **Relativity and reference frames**

Sitting on an Italian shoreline watching boats go by, the 17th century 'father' of modern science, Galileo Galilei, had an interesting thought: if a ball was dropped from the mast of a ship that was moving at a constant velocity, how would it look to a person on the ship?

Galileo determined that the crew on board the ship would see the ball fall straight down and land directly under the position that it was dropped. However, Galileo noted that from his vantage point on the shore, the ball falls in a parabola.



#### Reference frame a coordinate system

whose quantities, such as distances and time, can be measured

#### Inertial reference frame

a non-accelerating reference frame (i.e. a reference frame that is at a constant velocity or stationary)

#### Relativity

the dependence of physical phenomena on the relative motion between the thing being observed and the observer **Figure 9A–2** An impression of Galileo watching a ball dropped from the top of the mast of a fastmoving sailing boat. The orange arrow represents the motion of the boat while the ball is falling and the start and end positions of the boat are shown. Galileo would see the path of the ball as a parabola, represented by the red arrow. A sailor on the boat would see the path of the ball as represented by the pink arrow.

Galileo wondered who was correct? He concluded that both observers are correct in their own reference frame. Galileo noted that the ball obeys the same laws of physics in both reference frames. Observations on the ship, such as the falling ball, cannot determine if the boat was moving at a constant velocity or was stationary. Newton used Galileo's ideas and gave the name 'inertial reference frame' to a frame of reference that was at a constant velocity or stationary.

The concept of **relativity** in physics began by relating the motion of the thing being observed to the motion of the observer. It has since been extended to consider the nature and behaviour of light, space, time and gravity.

## Michelson-Morley experiment

Up until the late 1800s, it was thought that light waves, like other waves, need a **medium** to travel through. Scientists at the time theorised that space was filled with a medium called the luminiferous aether and electromagnetic waves travelled through this aether. In the late 1800s, the Michelson–Morley experiment was performed to try to provide evidence for the existence of the aether.

The Michelson-Morley experiment used a light interferometer to detect changes in the speed of light. An interferometer works by sending out a single coherent light beam at a halfsilvered mirror. This mirror splits the beam equally and the two beams travel down two perpendicular arms that are equal in length. When the light reaches the end of the arm, it is reflected off a mirror and travels back down the arm. The two beams are then recombined and detected at the detector. The recombined waveform is carefully studied for any changes between the two beams of light.

If the aether did exist, then it was expected that the two beams of light would no longer be coherent when they arrive at the detector. This is because Earth is moving through space and therefore moving relative to the aether. This creates an aether wind, as seen by a person on Earth, just like how driving through still air creates a wind for the person in the car. Depending



**Figure 9A–3** An interferometer can be used to compare the speed of light in different directions. Light travelling to mirror 1 from the beam splitter and being reflected to the detector travels exactly the same distance as light travelling to mirror 2 from the beam splitter and being reflected back to the beam splitter then down to the detector. However, if the aether exists, whatever direction it moves relative to the inferometer, these two beams of light will travel to the mirrors and back in directions that are different by 90°.

on the arm's orientation relative to the aether wind, the beams will make it back to the half-silvered mirror at different times. For example, imagine that the beam heading to Mirror 2 in Figure 9A–3 is travelling with and against the aether and the beam travelling to Mirror 1 is travelling across the aether. The time it takes for the two light beams to return to the half-silvered mirror is different due to motion relative to the aether, which means the recombined light will no longer be coherent. However, when this experiment was conducted, it was found that irrespective of the time of day, season or orientation of the perpendicular arms relative to Earth, the recombined light would always remain in phase. This null result was strong evidence that light did not require a medium to travel through.

To the surprise of many at the time, the Michelson–Morley experiment demonstrated that there was no aether and that the speed of light is independent to the motion of Earth about the Sun. Most people began looking for flaws in Maxwell's equations, which predicted that the speed of light was constant for all observers. However, Einstein chose to accept Maxwell's equations and started to think about what might happen if the speed of light was constant for all observers.

#### Medium

a substance that allows waves to travel through it

#### Coherent

light sources that are monochromatic and in phase

# Check-in questions – Set 1

- 1 Explain, using an example, what a frame of reference is.
- **2** If you are in deep space on a rocket with no windows, is there any way to tell if you are stationary or moving at a constant velocity? Explain your answer.
- **3** Explain how the results of the Michelson–Morley experiment support Einstein's special theory of relativity.

#### WORKSHEET 9A–1 RELATIVITY

Postulate established fact used as a basis for reasoning

#### Event

something that happens at a specific time and place. Each reference frame will measure the location and the time of the event, often differently.

#### VIDEO 9A-1 SKILLS: IDENTIFYING EVENT AND FRAMES OF REFERENCE

9B TIME DILATION AND LENGTH CONTRACTION

# Einstein's postulates of special relativity

There are two **postulates** of special relativity:

- 1 All inertial reference frames are equivalent the laws of physics are the same in all inertial reference frames.
- **2** The speed of light in a vacuum is constant. Every observer will measure the speed of light in a vacuum as the same constant value.

These two ideas are the basis of Einstein's special theory of relativity. They demonstrate that there is no absolute frame of reference with respect to which position and velocity are defined. Only relative positions and velocities between objects are meaningful.

Special relativity explains many things that classical physics cannot account for. In special relativity, two observers, moving relative to each other:

- don't agree about how much time passes between events
- don't agree on the distance between two points in space
- in particular cases, don't agree on the chronological order of events.

The fact that the speed of light is constant to all observers cannot be explained by classical physics. Classical physics would predict that velocities are additive. For example, if a train was travelling at half the speed of light (0.5c) and turned its headlights on, classical physics would predict that the light would travel at 1.5c relative to Earth, not at c.

# 9A SKILLS

### Identifying the event and the frames of reference in a question

In any question based on special relativity, it is essential that the reference frames and event being viewed are identified by the student. Understanding what the event is, as well as the frames of reference, will help you to define the proper time, dilated time, proper length and contracted length (all terms that will be covered in Section 9B). For example:

### Question

Galileo Galilei is sitting on an Italian shoreline watching boats go by him at constant speeds, when he sees a cannonball being dropped from the mast of a boat. The crew of the boat also see the cannonball being dropped. In this situation, what are the two frames of reference and the event?

#### Solution

The frames of reference are Galileo's frame of reference on the shore and crew of the boat's frame of reference. The event is the cannonball being dropped and then hitting the deck.

# **Section 9A questions**

## **Multiple-choice questions**

1 A rocket moves away from Earth at 0.500*c*. While travelling, the rocket sends a radio signal back to Earth. Which of the following statements is true?



- A An observer on Earth would measure the speed of the radio signal to be 0.500*c*.
- **B** An observer on Earth would measure the speed of the radio signal to be *c*.
- **C** An observer on Earth would measure the speed of the radio signal to be 1.500*c*.
- **D** An observer on Earth would measure the speed of the radio signal to be 0.750*c*.
- **2** Isaac is standing on a platform and watches Albert pass him on a train that is moving at a constant speed of 100 km h<sup>-1</sup>. Which of the following statements is correct?
  - A Isaac is in a non-inertial reference frame as he is stationary.
  - **B** Albert is in a non-inertial reference frame as he is moving relative to Isaac.
  - **C** Albert is moving in Isaac's reference frame, but is stationary in his own reference frame.
  - **D** Isaac can be certain he is stationary and not moving at a constant velocity because, in his frame of reference, the train is moving.
- 3 Which of the following is not an inertial reference frame?
  - A a rocket that is travelling at a constant velocity of 0.5*c*
  - **B** a rocket that is stationary relative to the platform it has landed on. The platform is moving in a straight line at a constant speed through space.
  - **C** a satellite orbiting a planet at a constant speed
  - D a car travelling down a straight road with a constant velocity
- **4** A student sits inside a windowless box that has been placed on a smooth-riding train carriage. The student conducts a series of motion experiments to investigate frames of reference. Which one of the following observations is correct?
  - A The results when the train accelerates are identical to the results when the train is at rest.
  - **B** The results when the train accelerates differ from the results when the train is in uniform motion in a straight line.
  - **C** The results when the train is at rest differ from the results when the train is in uniform motion in a straight line.
  - **D** The results when the train accelerates are identical to the results when the train is in uniform motion in a straight line.

VCAA 2017

**5** In a thought experiment, two trains are moving towards each other. One train is moving at 0.450*c* to the right and the other train is moving at 0.700*c* to the left. A diagram of this situation is shown below.



If the train travelling at 0.700*c* turns its headlights on, which of the following set of predictions is correct about the value of the speed that each train will measure? Note: for the purpose of this question, think of light as a ball being thrown from the train on the right.

	Classical predictions	Special relativity predictions
Α	Both trains will measure the speed of light to be $2.15c$	Both trains will measure the speed of light to be $c$
B	Both trains will measure the speed of light to be $c$	Both trains will measure the speed of light to be 1.15 <i>c</i>
С	The train travelling at $0.700c$ will measure the speed of light as $c$ , while the train travelling at $0.450c$ will measure the speed of light to be $2.15c$ .	Both trains will measure the speed of light to be <i>c</i>
D	The train travelling at 0.700 <i>c</i> will measure the speed of light as 0.250 <i>c</i> , while the train travelling at 0.450 <i>c</i> will measure the speed of light to be 1.15 <i>c</i>	Both trains will measure the speed of light to be <i>c</i>

6 Students use sound to test the ideas of the Michelson–Morley experiment. They conduct an experiment on an outdoor basketball court on a windy day.

Student A stood at the western end and created a loud pulse of sound. Student B stood 30.0 m away at the eastern end with a sound detector, as shown below.



They found that the sound travelling towards the eastern end took 0.0857 s to reach student B.

Student B, at the eastern end, then created a loud pulse of sound. This time, the sound travelling towards the western end took 0.0909 s to reach student A.

Which one of the following best explains their observations?

- **A** The wind was blowing to the east at  $10 \text{ m s}^{-1}$ .
- **B** The wind was blowing to the east at  $20 \text{ m s}^{-1}$ .
- **C** The wind was blowing to the west at  $20 \text{ m s}^{-1}$ .
- **D** The speed of sound is the same in all inertial reference frames.

#### **Short-answer questions**

- 7 What is the difference between an inertial and a non-inertial reference frame?
- 8 If you are on a train with no windows, that is travelling in a straight line on a perfectly smooth track, could you perform a test to tell if the train was moving at a constant velocity or was stationary? Justify your answer.
- 9 Jani is stationary in a spaceship travelling at constant speed.

Does this mean that the spaceship must be in an inertial frame of reference? Justify your answer.

VCAA 2018

10 What is the second postulate of Einstein's theory of special relativity regarding the speed of light? Explain how the second postulate differs from the concept of the speed of light in classical physics.

VCAA 2019

11 When you are flying at a constant velocity in a commercial jet, it may appear to you that the airplane is stationary and Earth is moving beneath you. Is this point of view valid? Discuss briefly. (Note: strictly speaking, Earth is not an inertial reference frame, but it may be considered inertial for the purpose of this question.)



VCAA 2013



# Time dilation and length contraction

#### **Study Design:**

- Describe proper time (*t*<sub>0</sub>) as the time interval between two events in a reference frame where the two events occur at the same point in space
- Describe proper length  $(L_0)$  as the length that is measured in the frame of reference in which objects are at rest
- Model mathematically time dilation and length contraction at speeds approaching *c* using the equations:

$$t = t_0 \gamma$$
 and  $L = \frac{L_0}{\gamma}$  where  
 $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ 

- Explain and analyse examples of special relativity including that:
  - particle accelerator lengths must be designed to take the effects of special relativity into account
  - time signals from GPS satellites must be corrected for the effects of special relativity due to their orbital velocity

#### **Glossary:**

Dilated time (t) Lorentz factor Proper length ( $L_0$ ) Proper time ( $t_0$ )

# Ø<sup>0</sup>

**ENGAGE** 

## An experimental answer to the twin paradox

The twin paradox has become a famous example that points to an apparent contradiction in special relativity.

Imagine that two identical twins, Sophie and Amanda, are on Earth and synchronise their accurate clocks. For the purpose of this thought experiment, we will consider Earth to be an inertial frame of reference. Sophie goes on a spaceship that travels close to the speed of light, while Amanda remains on Earth. Sophie's spaceship travels to a star a few light-years away before returning to Earth. Upon her return, Sophie has aged a few years, but Amanda is an old woman. How can this be?

Special relativity states that all inertial observers' frames of reference are equally valid. So, it is true that when Amanda looks up, sees the spaceship travelling at a constant velocity and measures one second on Sophie's clock, Amanda will measure more than one second on her own clock. It is also true that if Sophie looks out of her spaceship, she sees Earth moving away from her at a constant velocity. So, when Sophie measures one second pass by on Amanda's clock, more than one second will pass by on Sophie's clock.

The answer to this apparent contradiction is that Amanda remains in a single inertial frame of reference while Sophie is essentially in two different inertial frames of reference. Neglecting the acceleration phases, Sophie's journey to the star is one inertial reference frame and her return journey is another inertial reference frame.

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**Figure 9B–1** Sophie arrives back after her interstellar trip having aged a few years but her identical twin, Amanda, is now an old woman. Amanda is the one who has aged because Sophie's spaceship is essentially in two different inertial frames of reference. Neglecting the acceleration phases, Sophie's journey to the star is one inertial reference frame, and her return journey is another inertial reference frame.



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# EXPLAIN

# VIDEO 9B-1 TIME DILATION AND LENGTH CONTRACTION





**Figure 9B–2** The light clock, showing a light beam being emitted, bouncing off the mirror and returning to the detector

#### Figure 9B–3

The left diagram shows what the passengers on board the rocket see when looking at the rocket's light clock. The right diagram shows what the observer on Earth sees when looking at the light clock on board the rocket.

#### Proper time $(t_0)$ the time interval between two events, where the two events occur at the same point in space

Dilated time (t) the time interval between two events, where the two events will occur at a different points in space

# **Time dilation**

Imagine that you are standing on the surface of Earth (assumed to be an inertial reference frame) and you see a rocket pass you by at a constant but significant fraction of the speed of light. The rocket has a special clock on board called a light clock. The light clock works by firing a light beam towards a mirror; the light then bounces off the mirror and hits a detector.

Both the observer on Earth and a passenger on the rocket agree to measure the time that it takes for the light to leave the detector on the rocket, hit the mirror and return to the detector; these are events that both observers are measuring. To a passenger on the rocket, the light clock is stationary relative to them. So, the light goes up and down, and the events of the light leaving the box and then arriving back to the box occur at the same point in space relative to them. However, the light clock on the rocket is moving relative to an observer on Earth. The Earthbound observer will see the event of the light leaving the source and the event of the light returning to the detector occuring at two different points in space. Because of the postulates of special relativity, both observers measure the speed of the light as *c*.



Figure 9B–3 illustrates that the distance travelled by the light is 2*D*, as seen by the passenger on board the rocket, and 2*L* as seen by the observer on Earth. The observer on Earth sees the light of the rocket's light clock trace a diagonal path due to the relative motion of the rocket. It follows that the amount of time that passes between the two events is equal to  $t_0 = \frac{2D}{c}$ , as seen by the passenger on board the rocket, and the amount of time that passes  $\frac{2L}{c}$ 

between the two events is equal to  $t = \frac{2L}{c}$ , as seen by the observer on Earth. These two

values of time demonstrate that the passenger on board the rocket will measure less time between the two events than the observer on Earth. The passenger on board the rocket is measuring the **proper time** ( $t_0$ ), as the proper time is defined as the time interval between two events, where the events occur in the same point in space. However, the observer on Earth measures the **dilated time** (t), which is defined as the time interval between two events, where the two events occur at different points in space. The proper time, dilated time and the relative velocity of the reference frames are related by Formula 9B–1.



$$t = \gamma t_0 = \frac{1}{\sqrt{1 - \frac{\nu^2}{c^2}}} t_0$$

Where:

- $t_0$  = Proper time, the time interval between two events, where the two events occur at the same point in space. Time measured in other inertial frames is never shorter than proper time.
- $t = \text{Dilated time, the time interval between two events, where the two events will$ occur at different points in space. This is always longer than the proper time.
- $\nu$  = Relative velocity of the reference frame to the event (m s<sup>-1</sup>)
- c = Speed of light,  $3.0 \times 10^8 \text{ m s}^{-1}$

$$\gamma$$
 = Lorentz factor,  $\frac{1}{\sqrt{1 - \frac{\nu^2}{c^2}}} = \left(1 - \frac{\nu^2}{c^2}\right)^{-\frac{1}{2}}$ 

The factor  $\frac{1}{\sqrt{1-\frac{\nu^2}{c^2}}}$  is a term that is so common in special relativity that it is often replaced

by the Greek letter gamma, y. This factor is known as the Lorentz factor.

#### Deriving the time dilation formula

Note: knowing how to derive the time dilation formula is not required for this course.

The time dilation formula can be derived with reference to the image shown in Figure 9B–4.

By applying Pythagoras' theorem, the following formula can be derived (Note: *vt* is the distance the rocket travels in the reference frame of the observer on Earth):



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# NOTE

Any unit for time can be used in this formula. For example, if you give proper time in minutes, the dilated time will be in minutes. If the proper time is given in years, then the dilated time will also be in years.

#### Lorentz factor

the factor by which time, length and energy will change due to their relative velocity. It is represented by the symbol gamma, γ, and given by 1

#### Evidence of special relativity: atomic clocks in relative motion

Atomic clocks use atoms to keep extremely precise time. If two accurate and synchronised atomic clocks are put in two different inertial reference frames that are moving relative to each other, the clocks will become asynchronised (will no longer display the same time). The difference in the time can be determined by the equation mentioned on the previous page:  $t = \gamma t_0$ .

The theory that atomic clocks in relative motion will disagree on the passage of time was tested experimentally by Joseph Hafele and Richard Keating. The pair flew caesium atomic clocks on regular commercial flights in 1971. One set of clocks flew eastward, another westward and one set was left on the US Naval Observatory. After flying around the world once, all three sets of clocks were compared. It was found that all of the sets of clocks disagreed by the amount that was predicted by Einstein's special and general theories of relativity (gravity also affects time, but an understanding of this is not part of this course).

# lacksquare

### Worked example 9B–1 Time dilation

A rocket has an accurate atomic clock that ticks every 0.100 seconds, as seen by the crew of the rocket. An observer on Earth sees the rocket pass by at a speed of 0.866*c* ( $\gamma$ = 2) relative to them. How long are the ticks of the rockets clock from the observer's frame of reference?

#### Solution

First, identify who is measuring a proper time interval. In this case, the crew of the rocket are measuring proper time. This is because the crew of the rocket observe the ticks of the clock occurring at the same point in space. Hence, the observer on Earth is measuring the dilated time of the clock. Using the formula for dilated time gives:

 $t = \gamma t_0$ = 2 × 0.1 = 0.200 seconds

So, the observer on Earth measures each clock tick to be 0.200 seconds in their reference frame.



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### Worked example 9B–2 Time dilation, finding proper time

An observer on Earth sees a rocket pass at a speed of 0.700*c* ( $\gamma$  = 1.40). A flash of light occurs on the rocket, which the observer on Earth measures to last for a period of 5 seconds. How long does the flash of light last in the frame of reference of the crew of the rocket?

#### Solution

To the observer on Earth, the flash of light is occurring at different points in space. This means the observer on Earth measures dilated time and the crew of the rocket will measure proper time. Therefore, rearranging the formula  $t = \gamma t_0$ :

$$t_0 = \frac{t}{\gamma}$$
$$= \frac{5}{1.4}$$
$$= 3.57 \text{ seconds}$$

It is sometimes useful to rearrange the Lorentz factor as follows:

$$\gamma = \frac{1}{\sqrt{1 - \frac{\nu^2}{c^2}}}$$
$$1 - \frac{\nu^2}{c^2} = \frac{1}{\gamma^2}$$
$$\frac{\nu}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

The term  $\frac{v}{c}$  is called the velocity ratio, which compares an object's velocity v to the speed of light *c*. It turns up in many equations, as does the squared velocity ratio,  $\frac{v^2}{c^2}$ . The value of the velocity ratio indicates if relativistic effects such as time dilation, length contraction and mass changes are likely to be significant for objects at high velocities. It is not possible to give a threshold for the velocity ratio in any given situation above which relativistic effects should be taken into account; it depends on the context and application. However, as a rule of thumb, one might say that if  $\frac{\nu}{c} > 0.1$  (i.e. if  $\nu$  is above  $3 \times 10^8$  ms<sup>-1</sup>) then relativistic effects should be checked.

#### Formula 9B–2 Lorentz factor rearranged

$$\frac{\gamma}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

Whe

nce frame to the event  $(m s^{-1})$ 

ere:  

$$v = \text{Relative velocity of the refere}$$
  
 $c = \text{Speed of light, } 3.0 \times 10^8 \text{ m s}^{-1}$ 

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#### CHAPTER 9 EINSTEIN'S SPECIAL THEORY OF RELATIVITY

# lacksquare

# Worked example 9B–3 Finding velocity from time dilation

In a laboratory, a scientist observes a moving particle decay after 45.2 ps  $(10^{-12} \text{ s})$ . When the same particles are observed stationary relative to the observer, they decay after 3.44 ps. Calculate the speed of the particles relative to the scientist.

## Solution

When the particles are stationary relative to the scientist, the creation and decay is occurring at the same point in space. Therefore, this is a measure of proper time. Hence,  $t = 45.2 \times 10^{-12}$  s and  $t_0 = 3.44 \times 10^{-12}$  s. Using  $t = \gamma t_0$  gives:

$$(45.2 \times 10^{-12}) = \gamma (3.44 \times 10^{-12})$$
  
 $\gamma = \frac{45.2}{3.44}$   
= 13.14

Finally, using the formula above gives:

$$\frac{\nu}{c} = \sqrt{1 - \frac{1}{13.14^2}}$$
$$\nu = 0.997c$$

# Check-in questions – Set 1

- 1 Which of the following statements about proper time is correct?
  - **A** Two observers in different inertial reference frames can both measure the proper time of one event.
  - **B** Proper time is the longest interval of time.
  - **C** Proper time is the shortest interval of time.
  - **D** Proper time can be either the shortest or longest interval of time depending on the reference frame.
- 2 Suppose a cosmic ray collides with a nucleus in Earth's upper atmosphere to produce a fundamental particle called a muon, that has a velocity of 0.950c. Muon's are known to decay after  $1.52 \ \mu$ s, from measurements in the laboratory when the muons are stationary relative to the observing scientist. How much time passes before the travelling muon decays, as measured by an earthbound observer?

# The effect of increasing speed

If you refer to Figure 9B–3, you can see that a person on Earth who looks at a light clock on a rocket that is moving relative to them will see the light from the clock trace a diagonal path.

If the rocket were to speed up, then the length of the diagonal path taken by the light, as seen by the observer on Earth, would increase. As light is travelling a greater distance, the observer on Earth must measure more time for the light to complete its journey to the mirror and back to the detector. Therefore, as relative speed increases, the time dilation also increases. See the 9D Skills box for more on the relationship between velocity and relativistic effects.



**Figure 9B–5** A spaceship passing an observer on Earth at lower and higher velocities. The length of the diagonal path taken by the light is longer at the higher velocity. As light is travelling a greater distance at the higher velocity, the observer on Earth must measure more time on the spaceship's light clock for the light to complete its journey to the mirror and back to the detector. Therefore, as relative speed increases, the time dilation also increases.

# Length contraction

Einstein's special theory of relativity predicts that the length of a travelling object will be contracted (shortened) in the direction of motion as measured by an observer in a different inertial frame of reference. To illustrate this point, imagine a rocket moving at a speed,  $\nu$ , relative to an observer who is standing on a platform. The observer measures the platform to have a length of  $L_0$ , the **proper length**. The proper length is the length of an object measured by an observer who is at rest with respect to the object (is in the same inertial reference frame as the object being measured).

#### **Proper length**

(L<sub>0</sub>) a length interval measured when stationary relative to the object or space



**Figure 9B–6** A rocket moving with a velocity, v, past an observer on a platform that has a length,  $L_0$ , as measured by the observer.

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#### CHAPTER 9 EINSTEIN'S SPECIAL THEORY OF RELATIVITY

Considering both reference frames, it can be said that:

- the observer on the platform is stationary and the rocket is moving at a speed, *v*, relative to the observer on the platform
- the rocket is stationary and the observer and the platform are moving at a speed, *v*, relative to the crew on board the rocket.

This means that each observer measures the same value for the other's speed:

#### $v_{\rm observer and platform} = v_{\rm rocket}$

The relative speeds can be calculated by timing how long it takes the nose of the rocket to move from one end of the platform to the other. Let's explore how this speed is calculated by each observer.





Figure 9B–7 Velocity can be calculated by measuring the time it takes the rocket's nose to pass from one end of the platform to the other.

The speed of the rocket, as measured by the observer on the platform, is given by the proper length of the platform divided by the time it takes the rocket to travel that distance:

$$v = \frac{L_0}{t}$$

The observer on the platform has measured the dilated time, as in the observer's reference frame, when the nose of the rocket passes from one end of the platform to the other, this event occurs in two different points in space.

The speed of the observer on the platform, as viewed by the crew on board the rocket, is given by:

$$v = \frac{L}{t_0}$$

The crew on board the rocket are measuring the proper time because the event of the nose of the rocket passing from one end of the platform to the other occur in the same point in space in their reference frame (since the platform moves to line up with the nose of the rocket). Note that L is the length of the platform as observed by the crew of the rocket. Therefore:

$$\frac{L_0}{t} = \frac{L}{t_0}$$

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For the equation to be true, the crew on board the rocket, who measure a shorter time interval,  $t_0$ , must measure the length of the platform, *L*, as contracted (shortened) in the direction of travel. This can be rearranged to:

$$L = t_0 \frac{L_0}{t}$$

Since  $t = \gamma t_0$ , therefore:

$$L = \frac{L_0}{\gamma}$$

A consequence of this equation is that as the velocity increases, the Lorentz factor,  $\gamma$ , increases and the length contracts further as seen by the observer. See the 9D Skills box for more on the relationship between velocity and relativistic effects.

Formula 9B–3 Length contraction

$$L = \frac{L_0}{\gamma} = L_0 \sqrt{1 - \frac{\nu^2}{c^2}}$$

Where:

- L = Contracted length, the length of the moving object or space as measured when there is relative motion to the object or space being measured. It will be less than the proper length.
- $L_0$  = Proper length, the length of the object or space as measured when at rest relative to the object or space being measured.
- v = Relative velocity of the reference frame to the event (m s<sup>-1</sup>)
- c = Speed of light,  $3.0 \times 10^8 \text{ m s}^{-1}$

$$\gamma$$
 = Lorentz factor =  $\frac{1}{\sqrt{1 - \frac{\nu^2}{c^2}}} = \left(1 - \frac{\nu^2}{c^2}\right)^{-\frac{1}{2}}$ 

When answering these types of questions, it is important to remember that the observed contraction is only in the direction of travel.

## Check-in questions – Set 2

- 1 An observer sees a moving rocket, with a length of 14.0 m, in the observer's reference frame in the direction of travel. When measured at rest, the rocket has a length of 60.0 m. Calculate the velocity of the rocket (in terms of *c*) relative to the observer.
- **2** A rocket is measured to be 100 m long, when measured by a passenger on board. An observer sees the rocket moving past them at a relative speed of 0.500*c*.
  - a What is the object being measured?
  - **b** Who is stationary relative to the object being measured?
  - c Who sees the object that is being measured as moving?
  - d Who measures the proper length and who measures the contracted length?
  - e Calculate the length of the rocket as seen by the observer.

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NOTE

Any unit for length can be used. For example, if you give proper length in metres, the contracted length will be in metres. If the proper length is given in light-years, then the contracted length will also be in light-years.

### **Evidence of special relativity**

# Applying special relativity to determine the length of particle accelerators in the particle's reference frame

Linear accelerators (linac for short) use a number of high voltage stages to accelerate charged particles in a straight line. The charged particles move through a series of tubes. The voltage of the tubes is continually switched so that as the charged particle leaves one tube, it becomes repelled by the tube it just left and attracted to the tube it is moving towards.

The tubes are called drift tubes as no acceleration occurs when the particles are moving through the tubes; acceleration only occurs between the tubes. The drift tubes are of increasing length to account for the increasing velocity and to ensure that when the voltage changes it occurs exactly as the particle reaches the end of the tube. Since the particles are travelling at a significant fraction of the speed of light when they travel through the drift tubes, the length of the tubes will be decreased in the particle's frame of reference. Engineers need to account for the effects of special relativity when designing the length of the particle accelerators; otherwise the timing of the alternating voltage would be incorrect.



**Figure 9B–8** A linear accelerator (linac). A positively charged particle such as a proton is fired from a source through a series of tubes. The tubes are connected to a very high-frequency alternating emf so that they rapidly cycle between a positive charge, negative charge and no charge. Here the first tube is positively charged and repels the proton that has just passed through it, while the second tube is negatively charged and attracts the approaching proton. As soon as it enters, the tube will lose its charge and the proton will drift though it. This situation is shown in the third tube.



### Worked example 9B-4 Length contraction of a drift tube

In a laboratory, a proton is travelling at 0.995*c* ( $\gamma$  = 10.0) when it passes through a drift tube of a linear accelerator that is 8.55 cm long in the particle's frame of reference. Calculate the length of the drift tube as seen by a scientist in the laboratory.

#### Solution

In the particle's reference frame, the particle is stationary and the drift tube is moving. This means that the drift tube will contract in the particle's frame of reference, so the

8.55 cm measurement is a contracted length. Rearranging  $L = \frac{L_0}{2}$  gives:

 $L_0 = L\gamma$  $= 8.55 \times 10$ = 85.5 cm

### Worked example 9B–5 Time and space in different reference frames

A spacecraft travels from Earth to a planet 9.34 light-years away, as measured by the person on the rocket. The spacecraft is travelling at 0.700c ( $\gamma = 1.40$ ) away from Earth. Calculate the length of the trip as seen by an observer on Earth. Then calculate the time it takes to complete the journey in both frames of reference. (Assume Earth and the planet aren't moving relative to each other and that the spacecraft's speed is constant for the whole trip.)

#### Solution

First, calculate the distance the spacecraft has travelled between Earth and the planet relative to a stationary observer on Earth. Now, relative to the observer on Earth, the distance to the planet isn't changing and so the observer on Earth is stationary relative to the length being measured. In the spacecraft's reference frame, the planet is moving towards them, while Earth is moving away at 0.700*c* and so the measurement is not stationary in that frame. Hence, the observer on Earth is measuring the proper length of the trip, while the spacecraft on the ship is measuring the dilated length.

Now that we know that the distance measured by the observer on Earth is the proper

length, the length contraction formula  $L = \frac{L_0}{\gamma}$  can be used with  $\gamma = 1.40$  and L = 9.34

light-years to give:

$$L_0 = L\gamma$$
  
= 9.34 × 1.40  
= 13.1 light-years

This is the distance measured between Earth and the planet as measured by the observer on Earth.

Now, we want to find out how long the trip takes in each frame of reference. First, calculate how long this journey will take in the frame of reference of the passenger on the spacecraft. Since we have the speed and distance as measured by the passenger, use d = vt to give:

$$t = \frac{d}{v}$$
  
=  $\frac{9.34 \times 365 \times 24 \times 60^2 \times 3.0 \times 10^8}{0.700 \times 3 \times 10^8}$   
=  $4.21 \times 10^8$  s or 13.3 yr

Alternatively, divide 9.34 light-years by 0.7.

Finally, to find the length of the trip as measured by the observer on Earth, use d = vt, since the speed is measured the same and we know the proper distance from above. This gives:

$$t = \frac{d}{\nu}$$
  
=  $\frac{13.1 \times 365 \times 24 \times 60 \times 60 \times 3.0 \times 10^8}{0.7 \times 3.0 \times 10^8}$   
= 5.89 × 10<sup>8</sup> s or 18.7 yr

Alternatively, divide 13.1 light-years by 0.7.



WORKSHEET 9B-1

TIME DILATION

AND LENGTH CONTRACTION

#### How accurate would GPS be without relativity?

The Global Positioning System (GPS) works by using a network of 30 satellites that each have a fixed orbit around Earth. Each of the GPS satellites carry on board an accurate atomic clock and will continually broadcast radio signals to Earth, which contain information about the time on the clock. GPS receivers on Earth can then calculate the distance to the satellite by taking the time difference between the broadcast time in the signal and the time on the receiver, using  $s = c\Delta t$ . At any point in time or place on Earth, at least four GPS satellites can always be seen by a receiver and by calculating and comparing the distance to each satellite, the position of the receiver can be accurately determined.

The satellites that are responsible for GPS travel at about 4 km per second. At these high velocities, the clock in the GPS satellite will slow down by about 7  $\mu$ s per day compared to a clock on Earth due to time dilation. This is a problem because the distance calculation relies on a comparison between the time measured by the clock on the satellite and the clock at the receiver. Since the clock on the satellite runs at a different rate to the one on Earth, because of time dilation, the calculations would be thrown off if this difference was not accounted for. A working GPS is achieved by electronically adjusting the rate at which the satellite clocks tick as well as by building in mathematical correction to receivers.

In the real world, general relativity (which is not covered in this course) also predicts that, as an object moves further away from a gravitational field, the time will pass faster. The GPS satellites orbit approximately 20000 km above the surface of Earth, which means that  $45 \mu$ s more will pass on the satellite's clock compared to the clock on Earth in a day. The net result of this effect, together with the one from special relativity, means that  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth as  $48 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more will pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more methed pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more methed pass on the satellite's clock compared to the clock on Earth at  $38 \mu$ s more methed pass on t

If the effects of relativity were not accounted for, it would only take 2 minutes before the GPS would be highly inaccurate, and the GPS would be off by about 10 km by the end of one day.



**Figure 9B–9** Global Positioning System (GPS) satellites account for relativistic effects. Without accounting for these effects, GPS would become useless in minutes.

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### **9B SKILLS**

#### The Lorentz factor timesaver

Many questions in special relativity will give the velocity in terms of the speed of light, *c*. This means that when the value for velocity is placed in the Lorentz factor equation, the *c*'s on the numerator and denominator will cancel. For example:

#### Question

A rocket is moving away from Ben at 0.977*c* when the rocket flashes a high-powered light for 5 seconds, as measured by the crew of the rocket. Calculate the amount of time that the high-powered light is on in Ben's frame of reference.

#### Solution

Both observers are measuring the event of the light flashing. The crew of the rocket see the flashes occurring at the same point in space and therefore they will measure the proper time for this event. Ben will measure the flashes occuring at different points in space due to the relative motion of the rocket, and thus, he will measure the dilated time. Therefore:

$$t = \frac{t_0}{\sqrt{1 - \frac{\nu^2}{c^2}}}$$
  
=  $\frac{5}{\sqrt{1 - \frac{(0.977c)^2}{c^2}}}$   
=  $\frac{5}{\sqrt{1 - \frac{0.977^2c^2}{c^2}}}$   
=  $\frac{5}{\sqrt{1 - \frac{0.977^2c^2}{c^2}}}$ 

= 23.4 seconds

Notice that since the velocity was given as a decimal fraction of *c*, the *c* terms cancel in the Lorentz factor.

# **Section 9B questions**

### **Multiple-choice questions**

- 1 A scientist observes a particle moving at 0.998c ( $\gamma = 15.8$ ) in a particle accelerator. The particle decays after 300 seconds, as seen by the scientist. If the particle were stationary relative to the scientist, how long would it take before it decays?
  - **A** 13.0 s
  - **B** 19.0 s
  - **C** 299 s
  - **D**  $4.74 \times 10^3$  s





- A An observer on Earth will measure their clock on Earth tick more slowly than the clock on the spacecraft.
- **B** An observer on Earth will measure the clock in the spacecraft tick faster than the clock on Earth.
- **C** A passenger on the spacecraft will measure the clock on Earth tick more slowly than the clock on the spacecraft.
- **D** A passenger on the spacecraft will measure the clock on the spacecraft tick by more slowly than the clock on Earth.
- **3** A scientist records that a particle moving in an accelerator decays after 700 s. When stationary in the laboratory, the same type of particle decays after 500 s. What is the speed of the particle relative to the scientist when the particle is moving in the accelerator?

<b>A</b> $0.4c$ <b>C</b> $0.6$	БC
--------------------------------	----

- **B** 0.5*c* **D** 0.7*c*
- 4 A person is standing on a platform that has a length of *L*. A train approaches the platform at a relativistic velocity. Which of the following describes the relative length measurement of the platform?

	To the observer on the platform	To the observer in the train
Α	L is shorter	No change
B	L is longer	L is shorter
С	No change	L is longer
D	No change	<i>L</i> is shorter

**5** Joanna is an observer in Spaceship A, watching Spaceship B fly past at a relative speed of 0.943c ( $\gamma = 3.00$ ). She measures the length of Spaceship B from her frame of reference to be 150 m.



Which one of the following is closest to the proper length of Spaceship B?

Α	50 m	С	450 m
В	150 m	D	900 m

VCAA 2019

- 6 In a thought experiment, a man is carrying a 10 m ladder horizontally when he runs at relativistic speeds through a barn that is 9 m long and open at both ends. Which of the following statements is true?
  - A It will never be possible to fit the ladder in the barn.
  - **B** It is possible for the ladder to fit inside the barn from the frame of reference of the man carrying the ladder.
  - **C** It is possible for the ladder to fit inside the barn from the frame of reference of a stationary observer in the barn.
  - **D** All observers will agree that the ladder fits inside the barn.

#### **Short-answer questions**

- 7 A neutron lives 618 s when at rest relative to an observer. How fast is the neutron moving relative to an observer who measures its life span to be 3100 s?
- 8 A futuristic car is measured by its driver to be 3.87 m long, however an observer measures the car to be 3.53 m long in the direction of travel.
  - a Who is correct? Explain your answer
  - **b** Calculate how fast the car is travelling.
- **9** Quasars are among the most distant and brightest objects in the universe. One quasar (3C446) has a brightness that changes rapidly with time.

Scientists observe the quasar's brightness over a 20-hour time interval in Earth's frame of reference. The quasar is moving away from Earth at a speed of 0.704c ( $\gamma = 1.41$ ).

Calculate the time interval that would be observed in the quasar's frame of reference. Show your working.

VCAA 2018

- 10 Two physics students, Beck and Barry, are arguing about a consequence of special relativity. In a thought experiment, Beck says it is possible for a 400 m long train to fit into a tunnel that is 300 m. Barry disagrees stating that because the 400 m train is so much bigger than the tunnel, one part of the train will always be outside the tunnel. Who is correct? Justify your answer.
- 11 To an observer on Earth, the distance between Earth and Proxima Centauri (the closest star to Earth) is 4.25 light-years. If a spacecraft is moving at 0.870*c*, how long would it take to travel this distance as measured by an Earthbound observer and as measured by the crew on the spacecraft?



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# **Muon decay**

#### **Study Design:**

- Explain and analyse examples of special relativity including that:
  - muons can reach Earth even though their half-lives would suggest that they should decay in the upper atmosphere

**Glossary:** Muon

# 2

ENGAGE

## Muons and the particle zoo

Muons originating from cosmic ray collisions in the atmosphere were discovered in 1936 by the American physicist Carl Anderson, who had previously won a Nobel Prize for his discovery of the positron. By the 1950s, so many new particles had been observed from cosmic ray collisions and particle accelerator tracks that they were referred to by some physicists as the 'particle zoo'. Gradually, physicists were able to determine which particles were fundamental (could not be broken down further) and which were composites of them. Physicists classified the fundamental particles based on their properties into an ordered scheme that became known as the Standard Model. The current form of the model is summarised in Figure 9C-1.





Organising knowledge into patterns, classifications and models based on structures or relationships is a key technique in science and many discoveries come from attempting to fill in the gaps in such organisational systems, often using notions of symmetry.
# EXPLAIN

# Evidence of special relativity: muon decay

**Muons** are small fundamental particles with extremely short half-lives. They are smaller than atoms (but 207 times the mass of electrons) and have a mean lifetime of  $2.20 \times 10^{-6}$  s when stationary. They are formed when cosmic rays, mainly high energy protons, hit atoms approximately 5000 m above Earth's surface. Muons travel at around 0.995*c* and are often detected hitting the surface of Earth. However, classical physics predicts the following:

 $s = t\nu$ = 2.20 × 10<sup>-6</sup> × 0.995 × 3.00 × 10<sup>8</sup>

= 656.7 m

The average muon can only travel 656.7 m before their lifetime is over and they decay. So how can they possibly hit the surface of Earth, which is 5000 m away?

The answer is in the effects of special relativity. When muons are measured in the labaroatory, the muons are stationary relative to the scientist who observes them on Earth. This means that both the muon and the scientist agree on how long it takes for the muon to decay (as they have no relative motion). This is a proper time measurement for both observers.

However, when the muons are created in Earth's upper atmosphere, they are travelling at 0.995*c* relative to the scientist on Earth. So, the time for the moving muons is dilated in the observer on Earth's reference frame. This means that the muon, which is stationary in its own reference frame, will measure its lifetime as  $2.20 \times 10^{-6}$  s (proper time,  $t_0$ ) but the scientist will measure a dilated lifetime. We can calculate the life of the muon as measured by the scientist on Earth as follows:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$= \frac{2.20 \times 10^{-6}}{\sqrt{1 - 0.995^2}}$$
$$= 2.20 \times 10^{-5} \text{ s}$$

Thus, the scientist will observe the moving muons lifetime to be  $2.20 \times 10^{-5}$  s; ten times longer than that observed in the laboratory. This is the reason why the muons can reach the surface of Earth. Repeating the calculation from before:

$$s = tv$$
  
= 2.20 × 10<sup>-5</sup> × 0.995 × 3.00 × 10<sup>8</sup>  
= 6575 m

Hence, muons can travel 6575 m, as observed by a scientist on Earth.

One of the fundamental consequences of special relativity is that all inertial reference frames are equally valid, so how can this greater distance be explained from the reference frame of the muon? The muon still decays after  $2.20 \times 10^{-6}$  s, measured in its own reference frame.

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Muon a small, fundamental particle; a type

of lepton

In the frame of reference of the muon, Earth is travelling towards it at 0.995*c*, this will mean that the distance between the muon and Earth must be contracted. We can calculate how much the length has contracted by:

$$\gamma = \frac{1}{\sqrt{1 - 0.995^2}} = 10.01$$
$$L = \frac{L_0}{\gamma} = \frac{5000}{10.01} = 499 \text{ m}$$

Therefore, from the muon's frame of reference, the distance to Earth is only 499 m, which it can easily cover in its lifetime.



**Figure 9C–2** The distance that classical physics would predict that the muons can travel (purple) and the distance that special relativity predicts that the muons can travel (red), from the reference frame of an Earthbound scientist.





**Figure 9C–3** The length contraction of space in the frame of reference of the muon. The purple arrow shows the distance travelled by Earth in the muon's reference frame, while the space from the muon's point of creation to the surface of Earth has contracted vertically.

WORKSHEET 9C–1 MUON DECAY

## 9C SKILLS

#### Being clear on your reference frame

A question on special relativity will sometimes ask you to explain an observation from a specific reference frame. For example:

#### Question

Muons are an elementary particle that are created in the upper atmosphere, which is about 5000 m from Earth's surface. The muons have a half-life of  $2.20 \times 10^{-6}$  seconds when observed stationary in a laboratory and travel at a speed of 0.995c relative to an observer on the surface of Earth. Classical physics would predict that the muons can only travel 657 m in their lifetime, however many muons are detected on the surface of Earth. Using special relativity, explain why the muons are able to reach the ground, from the muons' frame of reference.

#### Solution

In the muons' frame of reference, Earth is travelling towards it at 0.995*c*, this will mean that the distance between the muon and Earth must be contracted. We can calculate the length from the upper atmosphere to Earth's surface in the muons' frame, a contracted length:

$$\gamma = \frac{1}{\sqrt{1 - 0.995^2}}$$
$$= 10.01$$
$$L = \frac{L_0}{\gamma}$$
$$= \frac{5000}{10.01}$$
$$= 499 \text{ m}$$

Since 657 m is greater than 449 m, the muons will reach the surface of Earth.

Note that the question states 'from the muons' frame of reference', this means that only answers discussing length contraction can be awarded marks. Time dilation will occur in the observer on Earth's reference frame and consequently discussions of time dilation do not address the question that was asked. It is essential when answering these types of questions to always be clear on the frame of reference that you are discussing.





# Section 9C questions

## **Multiple-choice questions**

#### Use the following information to answer Questions 1, 2 and 3.

A building that is 26.5 m high to the top of its antenna and 9.80 m wide is struck by lightning in a thunderstorm. At the top of the building, a particle called a tau is created, which moves at a velocity of 0.816c ( $\gamma = 1.73$ ) vertically down relative to the inertial frame of the building. When observed at rest, a tau exists for  $2.9 \times 10^{-13}$  seconds.

- 1 What is the height of the building as seen from the tau's frame of reference?
  - **A** 45.8 m
  - **B** 26.5 m
  - **C** 19.2 m
  - **D** 15.3 m
- How long does the tau exist from the frame of reference of a person inside the building?
   A 5.02 × 10<sup>-13</sup> s
  - **B**  $2.90 \times 10^{-13}$  s
  - **C**  $1.68 \times 10^{-13}$  s
  - D  $1.73 \times 10^{-14} \text{ s}$
- **3** Which of the following statements is correct?
  - A Only classical physics predicts that the tau will reach the ground.
  - B Only special relativity predicts that the tau will reach the ground.
  - **C** Both classical physics and special relativity predicts that the tau will reach the ground.
  - **D** Neither classical physics nor special relativity predicts that the tau will reach the ground.
- 4 In a laboratory, a scientist measures a distance of 100 m from the point of creation to the point of decay of a particle. In the particle's reference frame, the same distance is 64.5 m. The velocity of the particle is
  - **A** 0.853*c*
  - **B** 0.764*c*
  - **C** 0.622*c*
  - **D** 0.557*c*
- **5** Which of the following is the correct reason that muons are able to reach the surface of Earth?
  - A The muons have a longer half-life in their own reference frame.
  - **B** The muons have a longer half-life in the scientist's reference frame.
  - **C** The length between the point where the muons are created and the surface of Earth is contracted in the muons and the scientist's reference frame.
  - **D** The length between the point where the muons are created and the surface of Earth is contracted in the scientist's reference frame.



# **Short-answer questions**

- 6 A muon is a fundamental particle that is created when a high-energy cosmic ray from the Sun hits a nucleus in the upper atmosphere. In one specific situation, a high-energy cosmic ray hits a nucleus that is 6000 m above Earth's surface. The collision creates a muon that travels at 0.998*c* ( $\gamma$ = 15.8) towards the surface of Earth. Muons are known to have a mean half-life of 2.20 × 10<sup>-6</sup> s when stationary.
  - a Calculate the distance from the point where the muon is created to the surface of Earth in the muon's frame of reference.
  - **b** Calculate the lifetime of the muon as measured by an observer on Earth.
  - **c** Is the muon able to reach the surface of Earth? You must justify your answer and include at least one calculation.
- 7 A scientist is performing experiments with a charged pion. When the scientist creates the charged pions in the laboratory, they are stationary relative to them. The scientist measures the mean lifetime of the pions to be  $2.6 \times 10^{-8}$  s. The scientist then fires the charged pions at a target by accelerating them through a linear accelerator. The charged pions then travel at a constant velocity through a drift chamber before hitting the target.
  - **a** In the charged pion's reference frame, the drift chamber is three times shorter than that measured by the scientist. Calculate the velocity of the charged pion as it exits the drift chamber.
  - **b** Calculate the lifetime of the charged pion, as measured by the scientist, when they are moving through the drift chamber.
  - **c** Explain from the scientist's reference frame, why the scientist is able to detect far more charged pions than would be predicted by classical physics.
- 8 Tests of relativistic time dilation have been made by observing the decay of short-lived particles. A muon, travelling from the edge of the atmosphere to the surface of Earth, is an example of such a particle.

To model this in the laboratory, another elementary particle with a shorter half-life is produced in a particle accelerator. It is travelling at 0.99875*c* ( $\gamma$ = 20). Scientists observe that this particle travels 9.14 × 10<sup>-5</sup> m in a straight line with a constant speed from the point where it is made to the point where it decays into other particles.

- a Calculate the lifetime of the particle in the scientists' frame of reference.
- **b** Calculate the distance that the particle travels in the laboratory, as measured in the particle's frame of reference.
- **c** Explain why the scientists would observe more particles at the end of the laboratory measuring range than classical physics would expect.







# Mass and energy equivalence

#### **Study Design:**

- Interpret Einstein's prediction by showing that the total 'mass-energy' of an object is given by:  $E_{\text{tot}} = E_{\text{k}} + E_0 = \gamma mc^2$  where  $E_0 = mc^2$ , and where kinetic energy can be calculated by:  $E_{\text{k}} = (\gamma - 1)mc^2$
- apply the energy–mass relationship to mass conversion in the Sun, to positron–electron annihilation and to nuclear transformations in particle accelerators (details of the particular nuclear processes are not required)

#### **Glossary:**

Mass defect Nuclear fusion Nuclear transmutation Positron Product Reactant Strong nuclear force

## ENGAGE

### **Our Sun is losing mass**

Since our Sun is constantly giving off energy, it must also be constantly losing mass. Our Sun gives off approximately  $3.8-3.9 \times 10^{26}$  J of light energy every second, which means that our Sun is losing approximately four billion kilograms every second. This means that every second, our Sun is losing the same mass as approximately three million cars, and every two minutes our Sun is losing the same amount of mass as the mass of the entire human race combined. In just over one-and-a-half days, our Sun would have lost a mass that is equal to the combined mass of all of the life on Earth. Amazingly, losing mass at this rate for its ten-billion-year life will result in our Sun only losing 0.07% of its total mass.



Figure 9D–1 The Sun loses mass equivalent to three million cars each second.

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# **EXPLAIN**

## **Relationship between mass and energy**

A jack-in-the-box is a child's toy that utilises elastic potential energy. To prepare the jackin-the-box, a spring is compressed and the lid closed. The lever is then cranked on the side. This turns a gear inside the jack-in-the-box, which releases the lid and the jack springs out. Surprisingly, this toy can help to illustrate the famous equation,  $E = mc^2$ .







Figure 9D–2 A jack-in-the-box can illustrate an example of the conversion of mass and energy. When the box is closed, the spring is compressed (right). The compressed spring has elastic potential energy and this stored energy manifests itself as a tiny increase in mass.

Imagine that you had a set of extremely sensitive weighing scales. You use these scales to weigh the jack-in-the-box when it is inside the box with the spring compressed, and then when it is released and the spring expands. If you did so, you would find that the mass of the jack-in-the-box is greater when it is in the box. This is because when the spring is in the compressed state in the box, its elastic potential energy gives it greater mass. Although the effect is tiny, this simple fact reveals something profound about the world around us, that mass and energy can be converted into one another; they are equivalent.

The energy that is released when mass is converted to energy can be calculated using Einstein's equation, Formula 9D-1.



# Formula 9D–1 Resting mass–energy relationship

$$E_0 = mc^2$$

Where:

 $E_0$  = Energy of an object at rest (J)

m =Change in mass (kg)

c = Speed of light,  $3.0 \times 10^8 \text{ m s}^{-1}$ 



#### CHAPTER 9 EINSTEIN'S SPECIAL THEORY OF RELATIVITY



#### Worked example 9D–1 Mass–energy equivalence

A muon is found to have a rest energy of 105 MeV. What is the mass of the muon in kilograms?

Solution

First, convert the energy from electronvolts (eV) to joules (J).

 $105 \times 10^{6} \times 1.60 \times 10^{-19} = 1.68 \times 10^{-11} \text{ J}$ 

Rearrange Formula 9D–1,  $E_0 = mc^2$ , to give:

$$m = \frac{E_0}{c^2}$$
$$= \frac{1.68 \times 10^{-11}}{c^2}$$
$$= 1.87 \times 10^{-28} \text{ kg}$$

Nuclear fusion

the process of joining together two or more small nuclei to form a larger, more stable nucleus

#### Positron

a positively charged antiparticle that has the same mass and magnitude of charge as the electron The mass–energy relationship is clearly demonstrated in the process of nuclear fusion within the Sun, during positron–electron annihilation and in transformations (decays) in particle accelerators.

As the velocity of an object increases, the energy of the object will also increase. In this case, the total energy of the object will be the combination of the relativistic kinetic energy and the rest energy,  $E_{tot} = E_k + E_0$ . The total energy is given by Formula 9D–2.

Formula 9D–2 Total relativistic energy

$$E_{\rm tot} = E_{\rm k} + E_0 = \frac{mc^2}{\sqrt{1 - \frac{\nu^2}{c^2}}} = \gamma mc^2$$

Where:

 $E_{tot} = Total (relativistic) energy (J)$ 

 $E_{\rm k}$  = Total (relativistic) kinetic energy (J)

 $E_0 = \text{Rest energy given by } mc^2$  (J)

m = Rest mass, the mass measured in the stationary frame of reference (kg)

 $v = \text{Speed} (\text{m} \text{s}^{-1})$ 

c = Speed of light,  $3.0 \times 10^8 \text{ m s}^{-1}$ 

$$\gamma = \text{Lorentz factor,} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

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#### Worked example 9D-2 Total relativistic energy

Find the total energy of a helium nucleus that has a rest mass of  $6.68 \times 10^{-27}$  kg and is moving at 0.200*c*.

#### Solution

First, calculate the Lorentz factor.

$$\gamma = \frac{1}{\sqrt{1 - 0.2^2}}$$
$$= 1.0206$$

Then, substitute this value and the given rest mass into Formula 9D-2.

$$E_{tot} = \gamma mc^2$$
  
= 1.0206 × 6.68 × 10<sup>-27</sup> × c<sup>2</sup>  
= 6.14 × 10<sup>-10</sup> J

Note that if the object is stationary then the equation can be simplified as follows:

$$E_{\rm tot} = \frac{mc^2}{\sqrt{1 - \frac{0^2}{c^2}}} = \frac{mc^2}{\sqrt{1}} = E_0$$

## Check-in questions – Set 1

- 1 The rest energy of a ping-pong ball that has a mass of 2.7 g is
  - $\textbf{A} \quad 2.43\times 10^{17} \text{ J}$
  - **B**  $2.43 \times 10^{14}$  J
  - C  $8.1 \times 10^8 \text{ J}$
  - **D** 810000 J
- **2** A linear accelerator accelerates a proton to a velocity of 0.6*c* ( $\gamma = 1.25$ ). The total energy of the proton is (note: the rest mass of a proton is  $1.67 \times 10^{-27}$  kg)
  - **A**  $3.76 \times 10^{-11} \text{ J}$
  - $\textbf{B} \quad 2.49\times 10^{-11} \text{ J}$
  - **C**  $1.50 \times 10^{-10}$  J
  - **D**  $1.88 \times 10^{-10}$  J



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#### Calculating kinetic energy using mass–energy equivalence

The kinetic energy of an object in motion can be calculated by subtracting the rest energy from the total energy when it is moving.

#### Formula 9D–3 Relativistic kinetic energy

$$E_{\rm k} = E_{\rm tot} - E_0 = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 = (\gamma - 1)mc^2$$

Where:

$$\begin{split} E_{\rm k} &= {\rm Total} \; ({\rm relativistic}) \; {\rm kinetic \; energy} \; ({\rm J}) \\ E_{\rm tot} &= {\rm Total} \; ({\rm relativistic}) \; {\rm energy} \; ({\rm J}) \\ E_0 &= {\rm Rest \; energy \; given \; by \; } mc^2 \; ({\rm J}) \\ m &= {\rm Rest \; mass} \; ({\rm kg}) \\ c &= {\rm Speed \; of \; light}, \; 3.0 \times 10^8 \; {\rm m \, s^{-1}} \end{split}$$

This is very different to the previously given equation for kinetic energy,  $E_k = \frac{1}{2}mv^2$ .

This equation only works at speeds much less than the speed of light, but at relativistic speeds it breaks down and Formula 9D–3 is required to be used. A good rule of thumb is to use Formula 9D–3 whenever the speed of the object is greater than about 10% the speed of light, or  $\nu > 0.1c$ .

#### Worked example 9D–3 Relativistic kinetic energy

A proton that has a rest mass of  $1.67 \times 10^{-27}$  kg is accelerated to a velocity of 0.999*c* ( $\gamma$ = 22.4) inside a linear accelerator. Calculate the kinetic energy of the proton.

Solution

Since the proton is travelling at relativistic speeds, use Formula 9D–3,  $E_{\rm k} = (\gamma - 1) mc^2$ . Substitute with the given values.

$$E_{\rm k} = (\gamma - 1) \ mc^2$$
  
= (22.4 - 1) × 1.67 × 10<sup>-27</sup> c<sup>2</sup>  
= 3.22 × 10<sup>-9</sup> J

# D

# Worked example 9D–4 Total relativistic energy

An electron is moving at a constant velocity relative to an observer. The electron has a rest mass of  $9.1 \times 10^{-31}$  kg and a kinetic energy of  $2.59 \times 10^{-14}$  J. Calculate the total energy of the electron.

Solution

From Formula 9D–2, total energy is given by  $E_{tot} = E_k + E_0$ .

Substitute with the given values.

$$\begin{split} \mathcal{E}_{\text{tot}} &= E_{\text{k}} + E_{0} \\ &= 2.59 \times 10^{-14} + 9.1 \times 10^{-31} \times c^{2} \\ &= 1.08 \times 10^{-13} \,\text{J} \end{split}$$

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## Nuclear fusion in the Sun

Nuclear fusion involves two or more atomic nuclei joining to make a larger nucleus. Energy can be released when this happens. While the fusion process in stars is very complex, we will focus on the hydrogen fusion that is occurring in our Sun. The fusion of hydrogen to helium can take several steps within a star. During each fusion, mass is lost and energy is released. Fusion reactions occur at very high pressure and temperatures. At these high temperatures, the electrons are stripped from the atom, leaving the nuclei exposed. The high pressures and temperatures also result in some pairs of hydrogen nuclei having sufficient kinetic energy to overcome the electrostatic repulsion between them, allowing them to get close enough for the strong nuclear

force to bind the nuclei together. When this happens, the mass of the products are less than the mass of the reactants. The difference in mass between the reactants and the products is known as the mass defect,  $\Delta m$ . This difference in mass occurs because some of the mass is converted to energy and released. This mass defect is released as energy in the form of electromagnetic radiation and sometimes kinetic energy of the particles. The amount of energy given off in a fusion reaction can be calculated using the mass defect and the equation  $E = \Delta mc^2$ .



reaction, where a hydrogen-2 nucleus and hydrogen-3 nucleus (the reactants) collide to produce a helium-4 nucleus and a neutron (the products that together are lighter than the reactants, by the mass defect) and energy that is converted from the mass defect.

#### Worked example 9D–5 Energy through fusion

In a simple fusion reaction, a hydrogen-3 isotope collides with a hydrogen-2 isotope to form a helium-4 isotope, a neutron and a gamma ray. The mass of the hydrogen-3 isotope is  $5.01 \times 10^{-27}$  kg, the mass of the hydrogen-2 isotope is  $3.34 \times 10^{-27}$  kg, the mass of the helium-4 isotope is  $6.64 \times 10^{-27}$  kg and the mass of the neutron is  $1.67 \times 10^{-27}$  kg. The equation for this fusion reaction is shown below:

$${}_{1}^{3}H + {}_{1}^{2}H \rightarrow {}_{2}^{4}He + {}_{0}^{1}n + energy$$

**a** Calculate the mass defect of this equation.

- m<sub>products</sub>

**b** Calculate the energy released per fusion reaction.

#### Solution

$$\Delta m = m_{\rm reactants}$$

$$= (5.01 \times 10^{-27} + 3.34 \times 10^{-27}) - (6.64 \times 10^{-27} + 1.67 \times 10^{-27})$$

$$= 4.00 \times 10^{-29} \text{ kg}$$

**b** 
$$E = mc^2$$

$$= (4.00 \times 10^{-29})c^2$$

 $= 3.60 \times 10^{-12} \text{ J}$ 

# Strong nuclear force

the fundamental force that binds nucleons (protons and/ or neutrons) together in a nucleus of an atom. It acts only over very short distances (10<sup>-15</sup> m).

#### Product

a substance formed as a part of a reaction

#### Reactant

the substance present before the reaction that is used up to make the products

#### Mass defect

the difference in mass between the products and the reactants of a reaction



#### Nuclear transformations in particle accelerators

Particle accelerators work by accelerating charged particles across an electric field. Magnetic fields are often used with the electric field to keep the beam of charged particles moving in a circular path. When a single beam of particles collides with a stationary target or when two beams of particles collide, the nucleus of the atoms involved in the collisions can change. When two nuclei hit each other, they can shear off parts of the nucleus and also fuse in many different ways, producing a range of different nuclei. The process of changing one chemical element into another is called **nuclear transmutation**.

Nuclear transmutation turning one chemical element into another

When atoms are transmuted in particle accelerators, the total mass of the particles before the collision will have a different mass compared to the total mass of the particles after the collision. This difference in mass is again referred to as the mass defect. The change in this mass is accounted for by the energy released or absorbed as a result of the collision. This energy change can again be calculated by the formula  $E = \Delta mc^2$ , where  $\Delta m$  is the mass defect in kilograms.

D

# Worked example 9D–6 Energy due to the transmutation of atoms in a particle accelerator

In a particle accelerator, a beam of carbon atoms is fired at a piece of bismuth foil. In one particular collision, a high-speed carbon atom fragments the bismuth atom, causing it to release four protons and fifteen neutrons. This produces an unstable gold atom and the carbon atom remains intact. The combined mass of the bismuth and carbon atom before the collision is  $3.67 \times 10^{-25}$  kg and the mass of the products after the collision is  $3.60 \times 10^{-25}$  kg. Calculate the energy released in this collision.

#### Solution

First, find the mass defect.

 $\Delta m = m_{\text{reactant}} - m_{\text{products}}$ = 3.67 × 10<sup>-25</sup> - 3.60 × 10<sup>-25</sup> = 0.07 × 10<sup>-25</sup> kg

Now, use the mass defect to calculate the energy released.

 $E = \Delta mc^2$ = (0.07 × 10<sup>-25</sup>)(3 × 10<sup>8</sup>)<sup>2</sup> = 6.30 × 10<sup>-10</sup> J

#### Turning a base metal into gold

For centuries, alchemists poured their efforts into the transmutation of so-called 'base' metals (meaning cheap metal) into gold. By the 1720s, most people had given up on the idea and by the 1800s the field of alchemy became the field of chemistry. It wasn't until the 1980s when the dream of nuclear transmutation was realised in a bismuth-to-gold experiment conducted at the Lawrence Berkeley National Laboratory (LBNL) in California.

To do this, the LBNL's particle accelerator accelerated beams of carbon and neon close to light speed and shot these beams into foils of bismuth. When the beams collide with the bismuth sheets, they cause a nuclear transformation, changing the nucleus of the atom. The team found that a number of gold isotopes were created among a number of other transmuted elements.

However, all of the gold that was detected was radioactive and it would cost about 35 quadrillion dollars to produce a kilogram of gold.



**Figure 9D–4** Alchemy was an early science with the primary goal of turning cheaper metals into gold. Although this turned out to be not possible by chemical means, many useful chemicals and techniques were discovered by alchemists.

#### **Positron–electron annihilation**

A positron, also known as an antielectron, is made of anti-matter and is the anti-particle of an electron. An anti-particle has the same mass as its corresponding particle, but its charge is the opposite. If an anti-particle comes into contact with its corresponding particle, the two will annihilate each other and their mass will be turned into energy (in the form of an electromagnetic wave). A positron has a charge of  $+1.6 \times 10^{-19}$  C and the same mass as an electron. If a



positron collides with an electron, the two particles will annihilate each other, producing two gamma rays of equal energy that travel in opposite directions.

When annihilation occurs between a particle and anti-particle pair, all of the mass of each particle is converted completely into energy. Due to the conservation of energy, the amount of energy before the annihilation must be the same as after, and this also includes the conversion of mass into energy. For an electron–positron pair colliding at low speeds, the energy of each gamma ray emitted is given my  $E = m_e c^2$ ;  $m_e$  is the rest mass of an electron (and positron).

#### Worked example 9D–7 Energy from a matter–anti-matter collision

An electron and positron travelling at low speeds collide with one another. Calculate the total energy released in the annihilation. Use  $9.1 \times 10^{-31}$  kg for the electron's mass.

#### Solution

During annihilation all mass is converted to energy and an electron and positron have the same mass. Therefore, the total change in mass for the system is given by:

 $\Delta m = 2 \times 9.1 \times 10^{-31}$ 

 $= 1.82 \times 10^{-30} \text{ kg}$ 

Substitute the total change in mass into  $E = \Delta mc^2$  to give the total energy released.

 $E = \Delta mc^2$ = 1.82 × 10<sup>-30</sup> ×  $c^2$ = 1.64 × 10<sup>-13</sup> J

#### Positron Emission Tomography (PET) scans

A PET scan produces functional (working) images of areas of the body, mainly the brain. The PET scan works when the patient is given a positron-emitting isotope. When the emitted positrons encounter electrons in the body, they annihilate each other and release two gamma rays.

During the scan, the patient's head or body part being scanned is positioned in a large donut-shaped ring of gamma ray detectors. Since the gamma rays are emitted with the

same energy but in opposite directions, a pair of detectors placed on opposite sides to each other will detect the two rays.

Figure 9D-6 Positron Emission Tomography (PET) uses electron-positron annihilation to image the internal structures and functions of the body. A patient positioned in the PET scanner (top) is given a positron-emitting isotope. When the emitted positrons encounter electrons in the body, they annihilate each other and release two gamma rays in opposite directions. Gamma ray detectors in the donut-shaped ring detect them and a computer program creates the image (bottom) based on the difference in timing of the gamma rays received on opposite sides. The left image is a PET scan of a healthy brain, and the right is of a patient suffering from Alzheimer's disease.



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# 9D SKILLS

#### **Understanding the Lorentz factor**

The Lorentz factor,  $\gamma$ , is used in formulas for time dilation ( $t = \gamma t_0$ ), length contraction

 $(L = \frac{L_0}{r_0})$ , total energy  $(E_{tot} = \gamma mc^2)$  and kinetic energy  $(E_k = (\gamma - 1) mc^2)$ . When the

velocity is low,  $\gamma$  tends to one. This means that at low speeds, the dilated time will tend to the proper time, the contracted length will tend to the proper length, the total energy will tend to the rest energy and the kinetic energy will tend to zero. As the velocity

increases, the value of  $\frac{v^2}{c^2}$  tends to one. This means that the value of  $\gamma$  tends to infinity. This can be represented as shown in Figure 9D–7.



Figure 9D–7 As an object approaches the speed of light, the value for  $\gamma$  approaches infinity

Therefore, as the velocity of a body, such as a rocket, increases relative to an observer:

- the time in the rocket will slow down as measured by the observer (time dilation increases)
- the length will contract as seen by an observer (length contraction increases)
- the total energy will tend to infinity and the kinetic energy will also tend to infinity
- the relativistic mass of the body will tend to infinity due to the equivalence of mass and energy, though the rest mass of the body will not change.

# **Section 9D questions**

# **Multiple-choice questions**

- 1 If the value of the Lorentz factor,  $\gamma$ , increases, then the kinetic energy will
  - A increase.
  - B decrease.
  - **C** stay the same.
  - **D** There is not enough information provided to answer the question.
- 2 Which of the following has the greatest amount of total energy?
  - A an electron at rest
  - **B** an electron moving at 0.999*c*
  - **C** a proton at rest
  - **D** a proton moving at 0.999*c*



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#### CHAPTER 9 EINSTEIN'S SPECIAL THEORY OF RELATIVITY

- 3 A moving neutron has a rest mass of  $1.67 \times 10^{-27}$  kg and a kinetic energy of  $2.33 \times 10^{-11}$  J. Which of the following is the total energy of the moving neutron?
  - **A**  $1.74 \times 10^{-10}$  J
  - **B**  $1.50 \times 10^{-10}$  J
  - **C**  $2.33 \times 10^{-11}$  J
  - **D**  $3.50 \times 10^{-11} \text{ J}$
- 4 Which one of the following statements about the kinetic energy,  $E_k$ , of a proton travelling at relativistic speed is the most accurate?
  - A The difference between the proton's relativistic  $E_k$  and its classical  $E_k$  cannot be determined.
  - **B** The proton's relativistic  $E_k$  is greater than its classical  $E_k$ .
  - **C** The proton's relativistic  $E_k$  is the same as its classical  $E_k$ .
  - **D** The proton's relativistic  $E_k$  is less than its classical  $E_k$ .

VCAA 2018

5 According to Einstein's special relativity theory, the rest energy is  $mc^2$  for a particle of

rest mass *m* and the kinetic energy of the particle is  $(\gamma - 1) mc^2$ , where  $\gamma = \frac{1}{\sqrt{1-1}}$ 



Which one of the curves in the above diagram best gives the relationship of kinetic energy to rest energy  $\frac{E_k}{\text{rest energy}}$  as a function of speed  $\nu$ ?

- A curve A
- **B** curve B
- **C** curve C
- D curve D

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## **Short-answer questions**

- 6 What is the rest energy of an electron that has a mass of  $9.10 \times 10^{-31}$  kg? Give your answer in J and MeV.
- 7 If the rest energies of a proton and a neutron are 938.3 and 939.6 MeV respectively, what is the difference in their masses (in kg)?
- 8 With reference to the process of nuclear fusion, explain why energy can be released when two small nuclei join together, and why it is difficult to make two nuclei come together.
- 9 When two  ${}_{1}^{1}$ H nuclei fuse together, there is a mass defect of  $1.05 \times 10^{-29}$  kg. Calculate the total amount of energy given off from this reaction. Give your answer in eV.
- **10** Calculate the work done to accelerate a proton from 0.4*c* ( $\gamma = 1.091$ ) to 0.987*c* ( $\gamma = 6.22$ ), given that the rest mass of the proton is  $1.67 \times 10^{-27}$  kg.
- 11 When a stationary neutron meets its anti-particle, an anti-neutron (also stationary), the two particles annihilate each other and two gamma rays are formed. Given that the mass of the neutron and anti-neutron are both  $1.67 \times 10^{-27}$  kg, calculate the energy of one of the gamma rays.



# **Chapter 9 review**

# Summary

Create your own set of summary notes for this chapter on paper or in a digital document. A model summary is provided in the Teacher Resources, which can be used to compare with yours.

# Checklist

In the Interactive Textbook, the success criteria are linked from the review questions and will be automatically ticked when answers are correct. Alternatively, print or photocopy this page and tick the boxes when you have answered the corresponding questions correctly.

Succe	ss criteria – I am now able to:	Linked questions
9A.1	Discuss the nature of inertial reference frames	3 🗌 , 26 🗌
9A.2	Identify inertial and non-inertial reference frames	3
9A.3	Discuss the idea of relativity	18
9A.4	Recall and apply the two postulates of special relativity	1 , 4
9A.5	Evaluate the Michelson and Morley attempt to measure the aether and justify why the null result is indicative of the non-existence of the aether	10
9A.6	Compare and explain the different predictions of Einstein's special theory of relativity with the predictions of classical physics	4
9B.1	Define a proper time interval $(t_0)$ as the time interval between two events, where the two events occur at the same point in space	2 , 5 , 11 , 13 , 18 , 20 , 21
9B.2	Identify that the time interval between two events, where the events occur in two different points in space, is the dilated time interval $(t)$	5, 18, 20, 21
9B.3	Understand that proper length ( $L_0$ ) is the length measured when stationary relative to the object being measured	6□, 7□, 16□, 18□, 20□, 21□
9B.4	Identify when a length interval is a proper length ( $L_0\!)$	9□, 16□, 18□, 20□, 21□
9B.5	Recall that objects will only contract in length in the relative direction of travel	5, 7, 9, 9, 13, 16, 20, 21, 27
9B.6	Be able to apply the length contraction formula $L = \frac{L_0}{\gamma}$ and the time dilation formula $t = t_0 \gamma$ to various situations	2∟, 6∟, 9∟, 13∟, 16□, 17□, 18□, 19□, 20□, 21□, 25□, 26□
9B.7	Explain why a particle accelerator must consider the effects of length contraction in its design	26
9B.8	Understand and explain why time signals from GPS satellites must be corrected for the effects of special relativity due to their high orbital velocity	26
9B.9	Understand what happens to dilated time and contracted length with an increasing speed	7 , 15 , 26

Succe	ess criteria – I am now able to:	Linked questions
9C.1	Explain that from an Earth reference frame, the mean half-lives of the muons are dilated, which allows them to travel more distance than would be predicted by classical physics	8
9C.2	Explain that from the muons reference frame, the length from their point of origin to the surface of Earth is contracted, which means they can travel a greater distance than would be predicted by classical physics	80,19
9D.1	Understand that mass and energy are different forms of the same thing and use the formula $E = mc^2$ to convert between rest energy and mass	12
9D.2	Apply the formulas of total energy $(E_{tot} = E_k + E_0 = \gamma mc^2)$ , rest energy $(E_0 = mc^2)$ and kinetic energy $(E_k = (\gamma - 1)mc^2)$ to various situations	14 , 15 , 22 , 24 , 25
9D.3	Apply the formula $\Delta E = \Delta mc^2$ to a nuclear fusion reaction, positron–electron annihilation and the nuclear transformations in particle accelerators	12 , 23 , 27
9D.4	Understand how particle accelerators can produce new elements	27
9D.5	Plot the value of the Lorentz factor ( $\gamma$ ) against relative speed on a graph, and interpret its effects on energy and mass	26

# **Multiple-choice questions**

1 A rocket is moving away from Earth at 0.999*c* when it sends back a radio signal. A person on Earth will measure the speed of the radio signal to be



- 2 A nanocraft is an ultra-lightweight spacecraft that has a solar sail, which uses photons to push it forwards. A nanocraft travelling at 20.0% of the speed of light sends a signal back to Earth, which lasts four (4.00) seconds in the frame of reference of the nanocraft. Calculate the length of the signal as seen by an observer on Earth.
  - A 3.92 seconds
  - **B** 4.00 seconds
  - **C** 4.08 seconds
  - **D** 4.47 seconds

ISBN 978-1-009-25901-9

- **3** Which of the following situations could be considered an inertial reference frame?
  - **A** a man falling during a bungee jump
  - **B** a ballerina spinning around at a constant speed
  - **C** a person driving on a banked track
  - **D** None of the above can be considered an inertial reference frame.
- **4** In a thought experiment, two trains move past a tower that is emitting light. The first train is moving away from the tower at 0.900*c* and the second train is moving toward the light tower at 0.250*c*. A diagram of this situation is shown below.



Which of the following set of predictions is correct?

	Classical predictions	Special relativity predictions
Α	The light has a velocity of <i>c</i> as measured by both observers.	The light has a velocity of $c$ as measured by both observers.
В	The light has a velocity of $0.1c$ as measured by the train moving away from the tower and $1.25c$ as measured by the train moving towards the tower.	The light has a velocity of <i>c</i> as measured by both observers.
С	The light has a velocity of 1.9 <i>c</i> as measured by the train moving away from the tower and 1.25 <i>c</i> as measured by the train moving towards the tower.	The light has a velocity of <i>c</i> as measured by both observers.
D	The light has a velocity of 1.9 <i>c</i> as measured by the train moving away from the tower and 0.75 <i>c</i> as measured by the train moving towards the tower.	The light has a velocity of 0.1 <i>c</i> as measured by the train moving away from the tower and 1.25 <i>c</i> as measured by the train moving towards the tower.

- 5 Two rockets, Niels and James, are travelling towards each other. Rocket James has an accurate atomic clock that ticks at a time interval of 5.0  $\mu$ s. The crew on Niels measure the time interval of one tick on James' atomic clock to be 6.25  $\mu$ s. The relative velocity of the rockets are
  - **A** 1.25*c*
  - **B** 0.80*c*
  - **C** 0.70*c*
  - **D** 0.60*c*

- **6** An observer on Earth notes that when a rocket passes them it is one eighth of its length when it was stationary relative to them. What is the value of  $\gamma$ ?
  - **A** 0.125
  - **B** 0.250
  - **C** 4
  - **D** 8
- 7 Which of the following graphs best represents the length, *L*, of an object at different velocities (as a percentage of the speed of light, *c*), as seen by an observer who is in a different reference frame than the object?



- 8 Classical physics cannot account for the abundance of muons created in the upper atmosphere that reach the surface of Earth each day. The short mean lifetime of muons means that they should decay after only travelling a short distance in the upper atmosphere. The special theory of relativity accounts for this observation by reasoning that
  - **A** the muons live longer in their frame of reference and the length is contracted in an observer on Earth's frame of reference.
  - **B** the muons live longer in an observer on Earth's frame of reference and the length is contracted in the muon's own frame of reference.
  - **C** the muons live longer in their frame of reference and the length is contracted in their frame of reference.
  - **D** the muons live longer in an observer on Earth's frame of reference and the length is contracted in an observer on Earth's frame of reference.

- - **9** A rocket passes by an observer with a velocity that gives a Lorentz factor of  $\gamma = 1.5 \times 10^{-11}$ . When measured at rest, the rocket has a length of 20 m. Which of the following represents the length of the rocket as seen by the observer?

**A** (20) (1.5 × 10<sup>-11</sup>)  
**B** 
$$\frac{20}{1.5 \times 10^{-11}}$$
  
**C**  $\frac{20}{\sqrt{1.5 \times 10^{-11}}}$   
**D** (20)( $\sqrt{1.5 \times 10^{-11}}$ )

10 Michelson and Morley conducted an important experiment in 1887 on the propagation of light. However, the experiment failed to show an effect that was expected by many physicists of that time. A number of theories were proposed to explain this result.

Which one of the following statements best describes the most important step in the knowledge of physics that followed from this Michelson–Morley result?

- A Einstein's first postulate on the laws of physics removed the problem of the Michelson–Morley result.
- **B** Einstein's second postulate on the speed of light directly explained the Michelson–Morley result.
- **C** The concept of an 'aether' (a medium that allowed light to propagate) was upheld.
- **D** Maxwell's equations were shown to be incorrect.

#### VCAA 2016

**11** Two rockets, Albert and Marie, move in the same direction. Albert is moving at 0.9*c* and Marie is moving at 0.2*c*, both relative to the inertial frame of the galaxy. A diagram of this situation is shown below.



If a pulse of light is sent from Albert to Marie, how can a proper time interval be measured from the time the signal was emitted to the time it was received?

- **A** No observer can measure proper time in this instance.
- **B** Use measurements from the crew on board Albert.
- **C** Use measurements from the crew on board Marie.
- **D** Use measurements from an observer who is in the inertial frame of the galaxy.
- 12 On average, the Sun emits  $3.8 \times 10^{26}$  J of energy each second in the form of electromagnetic radiation, which originates from the nuclear fusion reactions taking place in the Sun's core. The corresponding loss in the Sun's mass each second would be closest to

**A**  $2.1 \times 10^9$  kg **B**  $4.2 \times 10^9$  kg

- **C**  $8.4 \times 10^9$  kg
- **D**  $2.1 \times 10^{12}$  kg

VCAA 2017

# **Short-answer questions**

_	1	
13	An observer measures a tau neutrino decaying after 4.99 ps when moving at 0.845 <i>c</i> ( $\gamma$ = 1.87) relative to them. How long does the particle live as viewed in the laboratory when it is	
	stationary to an observer?	(2 marks)
14	Explain why a particle with mass cannot travel at the speed of light.	(2 marks)
15	Relativistic effects such as time dilation and length contraction are present for cars as aircraft. Why do we not observe them?	nd (2 marks)
16	A spacecraft, 350 m long as measured on board, moves by Earth at 0.500 <i>c</i> . What is it measured by an observer on Earth?	s length as (2 marks)
17	The length of a spaceship is measured to be exactly one-third of its rest length as it p by an observing station.	asses
	What is the speed of this spaceship, as determined by the observing station, expressed multiple of $c$ ?	d as a (3 marks) CAA 2017
18	Suppose an astronaut is moving at a constant velocity relative to Earth at a significan fraction of the speed of light.	t
	<ul> <li>a Does the astronaut observe the rate of their clocks to have slowed? Explain.</li> <li>b What change in the rate of Earth's clocks does the astronaut observe? Explain.</li> <li>c What does the astronaut observe about the distance between stars that lie on line neullal to the astronaut/operation?</li> </ul>	(2 marks) (2 marks) s
	<ul><li>d The rocket is travelling at 0.750<i>c</i>, as seen by an observer on Earth. Use this examp to explain the principle of relativity.</li></ul>	(1 mark) le (2 marks)
19	A muon is created in Earth's atmosphere and travels across the sky. The muon travels 0.988 <i>c</i> relative to an observer on Earth and travels 526 m in its frame of reference bef decaying. How far does the muon travel according to the observer on Earth?	s at fore (2 marks)
20	The Orion Nebula is a nebula situated in the Milky Way, south of Orion's Belt in the constellation of Orion. It is one of the brightest nebulae and is visible to the naked ey night sky. The Orion Nebula is 1344 light-years from Earth, as measured by an obser Earth. A space probe is sent to observe the Orion Nebula and is moving at 0.998 <i>c</i> tow <b>a</b> What is the distance from Earth to the Orion Nebula as measured by the	e in the ver on vards it.
	<ul> <li>b How long does the journey take in the frame of reference of the space probe?</li> <li>c How long will it take the space probe to complete the journey from the observer of Earth's point of view?</li> <li>d If 1347 years passes for a person on Earth, how much time has passed in the frame reference of the space probe as measured by a person on Earth?</li> </ul>	(2 marks) (2 marks) on (2 marks) e of (2 marks)
21	A spacecraft is passing by Earth at 0.750 <i>c</i> .	
	<ul> <li>a If a person on the spacecraft looked at Earth, what differences may they measure is time passing on Earth and spatial dimensions of Earth?</li> <li>b An observer on Earth measures five hours pass by on their accurate clock on Earth When the observer looks at the accurate clock on the spacecraft, how much time is on that clock?</li> </ul>	in (2 marks) h. has passed (3 marks)
22	Beta decay is a nuclear decay in which an electron is emitted. If the electron is given 0.750 MeV of kinetic energy, what is the total energy of the electron?	(3 marks)

- 23 When a positron and an electron meet, they annihilate, converting all of their mass into energy. Find the energy released during a positron-electron annihilation, assuming negligible kinetic energy before the annihilation. Take the mass of an electron to be  $9.11 \times 10^{-31}$  kg. (2 marks) **24** At rest, a tau particle has 1.777 GeV of total energy. If the tau is moving at 0.625*c*  $(\gamma = 1.281)$ , calculate its kinetic energy. (3 marks) **25** What is the kinetic energy (in MeV) of a muon that lives for  $1.40 \times 10^{-16}$  s when observed moving and  $0.840 \times 10^{-16}$  s when at rest relative to an observer, given that the rest energy is 135 MeV? (4 marks) **26** Special relativity must be considered in the design of many tools such as particle accelerators and the Global Positioning System (GPS). **a** Special relativity is applied to objects in inertial reference frames. What is an inertial reference frame? (1 mark)**b** Sketch a graph of the Lorentz factor, using a vertical axis for  $\gamma$  labelled from 0 to 10, and a horizontal axis for velocity  $\nu$  labelled from 0 to *c*, with intervals of 0.1*c*. (3 marks)
  - Use the graph you sketched for part b and the length contraction formula to explain С what would happen to the length of the drift tubes in a particle accelerator as the velocity of the particle approached the speed of light. (3 marks)
  - **d** Explain how GPS considers special relativity in its design. (3 marks)
- **27** In a particle accelerator, a beam of nitrogen and helium ions are accelerated in opposite directions and collide head on. When they collide, an oxygen atom and hydrogen atom are produced according to the following equation:

$$^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$$

The mass of the nitrogen is  $2.33 \times 10^{-26}$  kg, the mass of the helium is  $6.64 \times 10^{-27}$  kg, the mass of the oxygen is  $2.65 \times 10^{-26}$  kg and the mass of the hydrogen is  $1.67 \times 10^{-27}$  kg.

- Why do the particles need to be charged in order to accelerate them? а (1 mark)(1 mark)
- **b** What does 'nuclear transmutation' mean?
- **c** Explain how the collision of the beam of nitrogen ions and helium ions are able to create new particles. (2 marks)
- **d** Calculate the mass defect of the reaction. (2 marks)
- Calculate the energy given off in this reaction. (2 marks)



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# **Unit 4 Revision exercise**

# **Multiple-choice questions**

1 Diffraction results when water waves are directed at a barrier containing a small gap, as shown in the diagram below.



Students increase the frequency of the water waves.

Which one of the following statements best describes what they are now likely to see in the waves to the right of the gap?

- **A** The pattern will be unchanged.
- **B** The pattern will spread further.
- **C** The pattern will become narrower.
- **D** The wavelength will increase, but the spread will remain the same.
- 2 A standing wave is produced on a flexible string, as shown in the diagram below. The diagram shows the wave at two different times the solid line represents what the wave looks like at a particular time and the dotted line represents what the wave looks like exactly half a cycle later.



Which one of the following is closest to the wavelength of the standing wave?

- **A** 0.5 m
- **B** 1.0 m
- **C** 1.5 m
- **D** 2.0 m

VCAA NHT 2023

- **3** Which one of the following best describes what happens when the intensity of the source of light for an interference pattern is greatly reduced?
  - **A** The number of photons is greatly reduced.
  - **B** The speed of the photons is greatly reduced.
  - **C** The energy of the photons is greatly reduced.
  - **D** The frequency of the photons is greatly reduced.
- **4** When light of a specific frequency strikes a particular metal surface, photoelectrons are emitted.

If the light intensity is increased but the frequency of the light remains the same, which of the following is correct?

- **A** Number of photoelectrons emitted remains the same, maximum kinetic energy of the photoelectrons remains the same.
- **B** Number of photoelectrons emitted remains the same, maximum kinetic energy of the photoelectrons increases.
- **C** Number of photoelectrons emitted increases, maximum kinetic energy of the photoelectrons remains the same.
- **D** Number of photoelectrons emitted increases, maximum kinetic energy of the photoelectrons increases.

#### VCAA NHT 2018

- 5 Light of wavelength 300 nm is just able to cause the photoelectric emission of electrons from a lead surface. If light of twice this wavelength were incident on a lead surface, then
  - **A** no photoelectric emission would occur.
  - **B** half as many electrons would be ejected per second.
  - **C** the same number of electrons would be ejected per second, with twice the energy.
  - **D** the same number of electrons would be ejected per second, with more energy but not necessarily twice as much energy.

#### VCAA NHT 2023

- 6 Identify which one of the following statements about de Broglie wavelengths is true.
  - **A** Neutrons cannot have a de Broglie wavelength because they carry charge.
  - **B** Protons cannot have a de Broglie wavelength because they carry charge.
  - **C** The de Broglie wavelength of electrons increases with increasing speed.
  - **D** The de Broglie wavelength of neutrons decreases with increasing speed.
- 7 An atom has only four possible energy levels available in an experiment. They are energies  $E_0$ ,  $E_1$ ,  $E_2$  and  $E_3$ , in increasing energy. An atom is excited to energy state  $E_3$ . Which one of the following is a possible energy for an emitted photon?
  - **A**  $E_3$

**B** 
$$E_0$$

**C** 
$$E_2 - E_3$$

**D** 
$$E_{3} - E_{0}$$

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8 Part of the energy level diagram for an unknown atom is shown below.



Which one of the arrows shows a change of energy level corresponding to the absorption of a photon of the lowest frequency?

- **A** Arrow A
- **B** Arrow B
- $\mathbf{C}$  Arrow C
- **D** Arrow D
- **9** A pion and its antiparticle, each at rest, annihilate and produce only two photons with a total energy of  $4.5 \times 10^{-11}$  J. The masses of the pion and its antiparticle are the same.

The rest mass of the pion is closest to

- **A**  $1.3 \times 10^{-28}$  kg
- **B**  $2.5 \times 10^{-28} \text{ kg}$
- $\textbf{C} \quad 5.0\times10^{-28}\,kg$
- **D**  $7.5 \times 10^{-20} \text{ kg}$

#### VCAA NHT 2023

VCAA 2017

- **10** Which of the following is closest to the speed of a particle produced in an accelerator with a Lorentz factor,  $\gamma$ , equal to 11.5? Use  $c = 3.000 \times 10^8 \text{ m s}^{-1}$ .
  - $\textbf{A} \ \ 2.989 \times 10^8 \ m \ s^{-1}$
  - **B**  $2.867 \times 10^8 \text{ m s}^{-1}$
  - $\textbf{C} \ \ 2.997 \times 10^8 \ m \ s^{-1}$
  - $\textbf{D} \ \ 2.739 \times 10^8 \ m \, s^{-1}$

#### **Short-answer questions**

11 Juan conducts an experiment using a shallow tray of water waves. He uses two point sources of wavelength 10 cm and frequency 5 Hz to investigate a two point source interference pattern, as shown on the right.



He observes lines of maxima and minima in the resultant pattern, as shown above. The lines on the diagram represent wave crests. Point P is on a nodal line of minima. Juan measures the distance from source X to point P as 16.0 cm. Determine the distance from source Y to point P. (2 marks)

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12 Students carry out a Young's double slit experiment using the experimental set-up shown below. Laser light passes through two closely spaced narrow slits and forms a pattern of light and dark bands on a screen. The bands are numbered – the even numbers are bright bands and the odd numbers are dark bands. The band spacing is *X*. Band 6 is equidistant from each of the two slits.



- **a** Using a wave model of light, explain why band 3 is dark. (2 marks)
- **b** The two slits are 1.00 m from the screen. The spacing between the two slits is 0.100 mm. The wavelength of the laser is 600 nm.

Calculate the band spacing, *X*, in millimetres.

(2 marks)

c The whole apparatus is now immersed in an insulating liquid of refractive index 1.2. The spacing of the bands changes.
 Explain why the spacing of the bands changes and include a calculation of the new band spacing.
 (2 marks)

VCAA NHT 2023

**13** The diagram below shows a plot of maximum kinetic energy,  $E_{k max}$ , against frequency, *f*, for various metals capable of emitting photoelectrons.



a Which material has the smallest work function. Explain your answer. (2 marks)b Calculate the work function, in eV, for metal D. (2 marks)

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**14** In an experiment on the photoelectric effect, Sam shines ultraviolet light onto a zinc plate and ejects photoelectrons, as shown below.



**a** The work function of zinc is 4.30 eV.

Calculate the minimum frequency of the ultraviolet light that could eject a photoelectron. (2 marks)

 b Sam wants to produce a greater photocurrent – that is, to emit more photoelectrons. He considers using a much brighter red light instead of the original ultraviolet light source used in part a.

Is Sam's idea likely to produce a greater photocurrent? Explain your answer. (2 marks) VCAA NHT 2023

15 The diagram below shows some of the energy levels in a neon atom.



- a Use the data in the diagram to calculate two photon frequencies that could be absorbed by neon atoms in the lowest state shown in the diagram. (2 marks)
- **b** Draw an arrow on a copy of the diagram to show how a photon of energy  $2.9 \times 10^{-18}$  J is produced. (2)
- 16 A beam of electrons, each with a momentum  $4.60 \times 10^{-24} \text{ kg m s}^{-1}$ , is passed through a salt crystal to produce a diffraction pattern, as shown on the right.
  - **a** Calculate the de Broglie wavelength of the electrons. (2 marks)
  - Explain why electron diffraction patterns from salt crystals provide evidence for the wave-like nature of matter. (3 marks)

VCAA NHT 2023

(2 marks)

17 Students are using a simple pendulum to measure the local gravitational field, g.A fishing line supports a small mass from a sturdy stand.



The length of the fishing line supporting the mass is 95 cm  $\pm$  1 cm. The 1 cm figure represents the measurement uncertainty of the length of the fishing line. The students measure the *period* of the pendulum as it swings through a very small angle. They measure the time for 20 complete swings, and they repeat the measurement five times.

a Explain why the students repeat the measurement five times. (2 marks)
 They then repeat the experiment for five different small masses, and the results with measurement uncertainties are shown in the table below.

Mass (g)	Angle of swing (°)	Period (average of 5 swings)
50	$4 \pm 1$	$1.99 \pm 0.06$
75	4 ± 1	2.02 ± 0.06
100	4 ± 1	1.96 ± 0.06
125	4 ± 1	2.03 ± 0.06
150	4 ± 1	1.97 ± 0.06

b Identify the controlled, independent and dependent variables in this experiment.(3 marks)

**c** One of the students has made the hypothesis that larger masses would result in a longer period. Evaluate the results of the experiment with this hypothesis. (2 marks)

Their research (using reliable internet sites confirmed by their teacher) shows that, provided the swing amplitude is small, the connection between the period, *T*, length, *l*, and gravitational field, *g*, should be:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

d They use the formula above to determine a value for the local gravitational field.What value will they obtain? Show your working. (4 marks)

# Appendix 1: Overview of online extra material

# Practical investigation into fields, motion or light

This appendix provides guidelines and advice for designing and carrying out the student-designed practical investigation.

# Curriculum

# Study Design: Unit 4 Area of Study 2 How is scientific inquiry used to investigate fields, motion or light?

Students undertake a student-designed scientific investigation in either Unit 3 or Unit 4, or across both Units 3 and 4. The investigation involves the generation of primary data relating to fields, motion or light. The investigation draws on knowledge and related key science skills developed across Units 3 and 4 and is undertaken by students in the laboratory and/or in the field.

When undertaking the investigation students are required to apply the key science skills to develop a question, state an aim, formulate a hypothesis and plan a course of action to answer the question, while complying with safety and ethical guidelines. Students then undertake an investigation to generate primary quantitative data, analyse and evaluate the data, identify limitations of data and methods, link experimental results to scientific ideas, discuss implications of the results, and draw and evaluate a conclusion in response to the question. Students are expected to design and undertake an investigation involving one continuous independent variable. The presentation format for the investigation is a scientific poster constructed according to the structure outlined in the Study Design. A logbook is maintained by the students for record, assessment and authentication purposes.

# Key science skills

- Develop aims and questions, formulate hypotheses and make predictions
- Plan and conduct investigations
- Comply with safety and ethical guidelines
- Generate, collate and record data
- Analyse and evaluate data and investigation methods
- Construct evidence-based arguments and draw conclusions
- Analyse, evaluate and communicate scientific ideas

# Scientific investigation

- Scientific investigation methodologies
- Logbooks

# Key Science Skills in Senior Science

This section is an additional resource for generic key science skills used across all senior science subjects.

## Access in the Interactive Textbook

These optional resources are presented for students in downloadable PDF format. Navigate to the offline textbook (the PDF textbook) where they will be found in the contents list. Teachers' copies are located in the Teacher resources pane.

# Appendix 2: Formulas and data

# Formula and data sheet

This formula and data sheet is based on the 2024–2027 Study Design, and when the official VCAA Formula Sheet is published, students should refer to that for assessment and examinations.

Motion and related energy transformations	
velocity; acceleration	$v = \frac{\Delta s}{\Delta t}; \ a = \frac{\Delta v}{\Delta t}$
equations for constant acceleration	v = u + at $s = ut + \frac{1}{2}at^{2}$
	$s = vt - \frac{1}{2}at^{2}$ $v^{2} = u^{2} + 2as$ $s - \frac{1}{2}(v + u)t$
Newton's second law	$\frac{5}{2}(v+u)t$
circular motion	$a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$
Hooke's law	$F = -k\Delta x$
elastic potential energy	$\frac{1}{2}k(\Delta x)^2$
gravitational potential energy near the surface of Earth	mg∆h
kinetic energy	$\frac{1}{2}mv^2$
Newton's law of universal gravitation	$F = G \frac{m_1 m_2}{r^2}$
gravitational field	$g = G \frac{M}{r^2}$
impulse	$F\Delta t$
momentum	mv
Lorentz factor	$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$
time dilation	$t = t_{o}\gamma$
length contraction	$L = \frac{L_{o}}{\gamma}$
rest energy	$E_{\rm rest} = mc^2$
relativistic total energy	$E_{\rm total} = \gamma m c^2$
relativistic kinetic energy	$E_{\rm k} = (\gamma - 1)mc^2$

Fields and application of field concepts	
electric field between charged plates	$E = \frac{V}{d}$
energy transformations of charges in an electric field	$\frac{1}{2}mv^2 = qV$
field of a point charge	$E = \frac{kq}{r^2}$
force on an electric charge	F = qE
Coulomb's law	$F = \frac{kq_1q_2}{r^2}$
magnetic force on a moving charge	F = qvB
magnetic force on a current carrying conductor	F = nIIB
radius of a charged particle in a magnetic field	$r = \frac{mv}{qB}$

Generation and transmission of electricity		
voltage; power	$V = RI; P = VI = I^2 R$	
resistors in series	$R_{\rm T} = R_1 + R_2$	
resistors in parallel	$\frac{1}{R_{\rm T}} = \frac{1}{R_{\rm I}} + \frac{1}{R_{\rm 2}}$	
ideal transformer action	$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$	
AC voltage and current	$V_{\rm RMS} = \frac{1}{\sqrt{2}} V_{\rm peak} \qquad I_{\rm RMS} = \frac{1}{\sqrt{2}} I_{\rm peak}$	
electromagnetic induction	EMF: $\varepsilon = -N \frac{\Delta \Phi_{\rm B}}{\Delta t}$ flux: $\Phi_{\rm B} = B_{\perp}A$	
transmission losses	$V_{\rm drop} = I_{\rm line} R_{\rm line}$ $P_{\rm loss} = I^2_{\rm line} R_{\rm line}$	

Wave concepts	
wave equation	$v = f\lambda$
constructive interference	path difference = $n\lambda$
destructive interference	path difference = $\left(n + \frac{1}{2}\right)\lambda$
fringe spacing	$\Delta x = \frac{\lambda L}{d}$ , when $L >> d$

The nature of light and matter	
photoelectric effect	$E_{\rm kmax} = hf - \phi$
photon energy	E = hf
photon momentum	$p = \frac{h}{\lambda}$
de Broglie wavelength	$\lambda = \frac{h}{p}$

Data	
acceleration due to gravity at Earth's surface	$g = 9.8 \text{ m s}^{-2}$
mass of the electron	$m_{\rm e} = 9.1 \times 10^{-31}  \rm kg$
magnitude of the charge of the electron	$e = 1.6 \times 10^{-19} \text{ C}$
Planck's constant	$h = 6.63 \times 10^{-34} \text{ J s}$ $h = 4.14 \times 10^{-15} \text{ eV s}$
speed of light in a vacuum	$c = 3.0 \times 10^8 \text{ m s}^{-1}$
universal gravitational constant	$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
mass of Earth	$M_{\rm E} = 5.98 \times 10^{24}  \rm kg$
radius of Earth	$R_{\rm E} = 6.37 \times 10^6 {\rm m}$
Coulomb constant	$k = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

Prefixes/Units			
$p = pico = 10^{-12}$	n = nano = 10 <sup>-9</sup>	$\mu = \text{micro} = 10^{-6}$	$m = milli = 10^{-3}$
$k = kilo = 10^{3}$	$M = mega = 10^{6}$	$G = giga = 10^9$	t = tonne = 10³ kg

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# Units

# **SI units**

Internationally, the scientific community around the world agree to use the SI system of units. This is based on seven fundamental quantities and their units, from which all the other quantities can be calculated. It is essential that all quantities in physics are stated with a numerical value and a unit, such as:

boiling point of water = 373.15 K

Current is one such fundamental quantity, and the ampere is its fundamental unit. Until 2019, one ampere was defined as 'the constant current which, if maintained in two straight parallel conductors of infinite length of negligible circular cross section and placed one metre apart in a vacuum, would produce between these conductors a force equal to  $2 \times 10^{-7}$  newton per metre of length'. In 2018 the General Conference on Weights and Measures (CGPM) agreed that on May 20, 2019, the ampere would henceforth be defined such that the elementary charge would be equal to  $1.602176634 \times 10^{-19}$  coulomb. Note: ten significant figures in this definition!

The seven fundamental SI quantities are mass (in kilograms, kg), time (in seconds, s), temperature (in kelvins, K), distance (in metres, m), electric current (in amperes, A), amount of substance (in mole, mol) and luminous intensity (in candela, cd). The definitions of the kilogram, the kelvin and the mole were all changed at the same time as current in 2019. Their new definitions are based on fixed numerical values of the Planck constant (h), the Boltzman constant (k) and the Avogadro constant ( $N_A$ ) respectively.

All other units are derived from these seven base units.

Quantity	Unit
Mass	kilogram, kg
Length	metre, m
Time	second, s
Thermodynamic temperature	kelvin, K
Electric current	ampere, A
Amount of substance	mole, mol
Luminous intensity	candela, cd
# Prefixes

Knowing and converting between the following SI prefixes is an essential skill in this course.

Name	Value (x)	Examples
tera (T)	1012	THz, terahertz
giga (G)	109	GHz, gigahertz
mega (M)	106	MHz, megahertz
kilo (k)	10 <sup>3</sup>	km, kilometre
(Base unit, no prefix)	101	metre, hertz, gram
centi	10-2	cm, centimetres
milli (m)	10-3	mm, millimetre
micro (µ)	10-6	µm, micrometre
nano (n)	10-9	nm, nanometre
pico (p)	10-12	pm, picometre

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# Periodic table of the elements

2 He 4.0 Helium	10 Ne Neon	18 Ar 39.9 Argon	36 Kr 83.8 Krypton	54 Xe 131.3 Xenon	86 Rn (222) Radon	118 Og (294) Oganesson	71 Lu 175.0 Luteitum	103 Lr (262) Lawrencium
	9 F 19.0 Fluorine	17 CI 35.5 Chlorine	35 Br 79.9 Bromine	53 I 126.9 Iodine	85 At (210) Astatine	117 Ts (294) Tennessine	70 Yb 173.1 Yucrbium	102 No (259) Nobelium
	8 0 16.0 Oxygen	16 S 32.1 Sulfur	34 Se 79.0 Selenium	52 Te 127.6 Tellurium	84 Po (210) Polonium	116 Lv (292) Livermorium	69 Tm 168.9 Thulium	101 Md (258) Mendelevium
	7 N 14.0 Nitrogen	15 P 31.0 Phosphorus	33 As 74.9 Arsenic	51 Sb 121.8 Antimony	83 Bi 209.0 Bismuth	115 Mc (289) Moscovium	68 Er 167.3 Erbium	100 Fm (257) Fermium
\$         \$         6           B         C         10.8           10.8         12.0         Boron           Boron         Carbon         23.0           Al         Si         24.0           27.0         28.1         28.1           Almoving         23.1         23.1	14 Si 28.1 Silicon	32 Ge 72.6 Germaniun	50 Sn 118.7 Tin	82 Pb 207.2 Lead	114 Fl (289) Flerovium	67 Ho 164.9 Holmium	99 Es (252) Einsteinium	
	13 Al 27.0 Aluminium	31 Ga 69.7 Gallium	49 In 114.8 Indium	81 TI 204.4 Thallium	113 Nh (280) Nihonium	66 Dy I62.5 Dysprosium	98 Cf (251) Californium	
			<b>30</b> Zn 65.4 Zinc	48 Cd 112.4 Cadmium	80 Hg 200.6 Mercury	112 Cn (285) Copernicium	65 Tb 158.9 Terbium	97 Bk (247) Berkelium
	of element	29 Cu 63.5 Copper	47 Ag 107.9 Silver	79 Au 197.0 Gold	111 Rg (272) Roentgenium	64 Gd 157.3 Gadolinium	96 Cm (247) Curium	
nber 79 Symbo Au nass 197.0 Name	28 Ni 58.7 Nickel	46 Pd 106.4 Palladium	78 Pt 195.1 Platinum	110 Ds (271) Darmstadtium	63 Eu 152.0 Europium	95 Am (243) Americium		
	N.	27 Co 58.9 Cobalt	45 Rh 102.9 Rhodium	77 Ir 192.2 Iridium	109 Mt (268) Meitnerium	62 Sm 150,4 Samarium	94 Pu (244) Plutonium	
	Atomic nu lative atomic		26 Fe 55.8 Iron	44 Ru 101.1 Ruthenium	76 Os 190.2 Osmium	108 Hs (267) Hassium	61 Pm (145) Promethium	93 Np (237) Neptunium
Rei		25 Mn 54.9 Manganese	43 Te (98) Technetium	75 Re 186.2 Rhenium	107 Bh (264) Bohrium	60 Nd 144.2 Neodymium	92 U Uranium	
		24 Cr 52.0 Chromium	42 Mo 96.0 Molybdenum	74 W 183.8 Tungsten	106 Sg (266) Seaborgium	59 Pr 140.9 Praseodymium	91 Pa 231.0 Protactinium	
			23 V 50.9 Vanadium	41 Nb 92.9 Niobium	73 Ta 180.9 Tantalum	105 Db (262) Dubnium	58 58 Ce 140.1 Cerium	90 Th 2332.0 Thorium
		18	22 Ti 47.9 Titanium	40 Zr 91.2 Zirconium	72 Hf 178.5 Hafnium	104 Rf (261) Rutherfordium	57 La 138.9 Lanthanum	89 Ac (227) Actinium
		19	21 Sc 45.0 Scandium	39 Y 88.9 Yttrium	57–71 Lanthanoids	<b>89–103</b> Actinoids		
	4 Be 9.0 Beryllium	12 Mg 24.3 Magnesium	20 Ca 40.1 Calcium	38 Sr 87.6 Strontium	56 Ba 137.3 Barium	88 Ra (226) Radium		
1 H 1.0 Hydrogen	3 Li 6.9 Lithium	11 Na 23.0 Sodium	19 K 39.1 Potassium	37 Rb 85.5 Rubidium	55 Cs 132.9 Caesium	87 Fr (223) Francium		

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# Glossary

# Absorption line spectrum

a spectrum with dark lines of a unique wavelength seen against the background of a continuous spectrum

# Acceleration due to gravity

rate at which a falling object will accelerate in a gravitational field. Equivalent to the gravitational field strength; measured in ms<sup>-2</sup>

# Alternating current (AC) circuit

the polarity of the potential difference changes in a regular way; the current changes direction at a rate that is measured in hertz (Hz)

#### Alternator

an electrical generator that produces electrical energy in AC output

# **Amplitude**

the maximum distance of an oscillation from its midpoint

# Antinode

point of maximum disturbance in an interference pattern

# **Artificial satellite**

any human-made structure such as Sputnik, Hubble Space Telescope, International Space Station, Mars Orbiter etc. that is placed in orbit around a planet (like Earth or Mars) or Earth's Moon

#### Atomic or subatomic size

atoms are of the order of 10<sup>-10</sup> m; subatomic could be taken to be the size of a proton (10<sup>-15</sup> m) or smaller

#### Banked track

a track that has been built with a transverse incline so that the horizontal component of the vehicle's normal force can contribute to the centripetal force

# Blackbody

an object that is a perfect absorber and emitter of electromagnetic radiation

#### **Centripetal acceleration**

acceleration directed to the centre of the circular path of an object

#### **Centripetal force**

a net force directed to the centre of the circular path of an object

#### **Closed system**

a system that does not allow the transfer of mass or energy to the surrounding environment

# Coherent

light source that are monochromatic with a fixed or zero phase difference between them

#### **Collector electrode**

the electrode designed to capture photoelectrons

#### **Constructive interference**

occurs when two identical waves are in phase and meet, doubling the amplitude of one wave

# Coulomb (C)

the SI unit for charge. 1 C is equivalent to the combined charge of  $6.2 \times 10^{18}$  protons (or electrons) or the amount of charge that passes a point when a current of 1 A flows for a time of 1 s.

# Coulomb's law

the force between two charges at rest is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them

# **DC** generator

also known as a DC dynamo, an electric generator that produces electrical energy in DC output

#### de Broglie wavelength

wavelength of matter particles, given by  $\lambda = \frac{h}{\lambda}$ 

#### **Destructive interference**

occurs where two identical waves are completely out of phase and meet, cancelling each other

#### Diffraction

the spreading of a wave when passing through a narrow opening or passing around an object

#### **Diffraction grating**

An optical device of many parallel slits that separates light of different wavelengths by interference effects

# Dilated time (t)

the time interval between two events, where the two events will occur at a different points in space

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# Dipole

a source and a sink of a field that are paired together

#### Direct current (DC) circuit

the polarity of the potential difference stays constant; the current always flows in the same direction

#### Dispersion

the separation of white light into its different colours

#### **Electric dipole**

a pair of equal and opposite electric charges, the centres of which are separated

# **Electric field**

a physical field that creates a force on all charged particles within the field. Produced by charged particles and changing magnetic fields. A changing electric field also produces a changing magnetic field.

#### **Electric force**

a force that exists between charged particles and may be attractive (unlike charges) or repulsive (like charges)

#### **Electric generator**

also known as a dynamo, a device that converts kinetic (mechanical) energy into electrical energy

#### **Electric monopole**

a single electric point charge, such as an electron or proton in which all the field lines point inward (-) or outward (+)

## **Electric potential energy**

the amount of work needed to move a unit charge from a reference point to a specific point against an electric field

#### **Electromagnet**

a coil of wire, consisting of an iron or steel core, through which a current is passed

#### **Electromagnetic induction**

the process of generating an electric current with a changing magnetic field near a wire, or by moving a metal wire in a steady magnetic field

#### **Electromagnetic wave**

a transverse wave consisting of perpendicular oscillating electric and magnetic fields that travel at 3.00  $\times$  10  $^{8}\,m\,s^{\text{--1}}$  in a vacuum with a range of wavelengths from 10<sup>-18</sup> m to more than 10<sup>4</sup> km

#### **Electromagnetism**

the branch of physics that considers the connection between electricity and magnetism

#### Electron

the lightest stable subatomic particle known. It carries a negative charge of  $1.60 \times 10^{-19}$ coulomb, which is considered the basic unit of electric charge.

#### **Electron** gun

a device that uses a heated cathode to provide free electrons for linear accelerators

#### Electroscope

a simple scientific instrument used to detect the presence of electric charge on a body, with a light gold or aluminium leaf that moves due to the electrostatic force between charges

#### **Electrostatic force**

the attractive force between the protons in the nucleus and the orbiting electrons

#### emf (electromotive force), ε

potential difference between the terminals of a source when no current flows to an external circuit: measured in volts (V)

#### **Emission line spectrum**

a spectrum with lines of a unique wavelength showing the composition of light emitted by hot gases

#### **Energy state**

the possible energy state of atoms, often described as an energy level

#### Equilibrium position

the position in a system where the net force on the oscillating object is zero

#### **Event**

something that happens at a specific time and place. Each reference frame will measure the location and the time of the event, often differently.

#### Exoplanet

a planet that orbits a star outside our solar system

# Faraday's law of electromagnetic induction

the magnitude of the induced emf is directly proportional to the rate of change of magnetic flux

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# **Field**

a region where an object feels a force, such as gravitational, electric or magnetic; more precisely defined as a physical quantity that has a value at each point in space

# **Field line**

a line drawn to represent the strength of a field with arrows to indicate direction. Field lines that are closer together indicate a stronger field.

# Free-body diagram

a diagram that shows the relative magnitude and direction of all of the forces acting on a body

# Free fall

when an object is falling down and only the force of gravity acts on the object

symbol used to represent the gravitational field strength

#### Galvanometer

a device that measures the magnitude and direction of very small currents

#### Gas discharge tube

a sealed tube filled with gases through which an electrical discharge is passed, exciting gas atoms into excited states. These decay and display characteristic spectra.

#### **Geostationary satellite**

a satellite whose orbit is such that it remains directly above the same point of Earth at all times

#### Graphite

crystalline form of the element carbon

#### Gravimeter

used for petroleum and mineral prospecting, seismology, geodesy and other geophysical research. Its fundamental purpose is to map the gravity field in space and time.

#### Gravitational constant, G

the universal gravitational constant; it has a value of  $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ 

#### **Gravitational field**

the region around an object where other objects will experience a gravitational force

#### Gravitational field strength

the strength of gravity measured in newtons per kilogram (N kg<sup>-1</sup>)

### **Gravitational force**

the force of attraction acting between two objects that have mass

#### Gravitational monopole

mass, as it always has the field lines pointing towards it because gravity is an attractive force

#### Gravitational potential energy

the amount of energy an object has stored due to its position in a gravitational field

# **Ground state**

the lowest energy state of an atom. It has a quantum number of 1.

#### Harmonic series

the set of frequencies consisting of a fundamental (the first harmonic) and the harmonics related to it by an integer multiple of the fundamental

#### Helium

element number 2; a nucleus with two protons and two neutrons surrounded by two orbiting electrons

#### **Hydrogen**

the simplest element, made of a proton and an orbiting electron; element number 1

#### Impulse

the change in momentum of an object, caused by a force acting for a certain amount of time

#### Induced current

an electric current that results from an induced emf,  $\varepsilon$ , if there is a complete circuit

#### Inertia

a body's ability to resist a change in its state of motion. Inertia is dependent only upon the mass of the body.

#### Inertial reference frame

a non-accelerating reference frame (i.e. a reference frame that is at a constant velocity or stationary)

#### Infrared light

electromagnetic radiation with a wavelength range of 700 nm to 1 mm

# In phase

if two waves coincide with peaks and troughs matching, they are said to be in phase

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#### Inverse square law

the relationship between two variables where one is proportional to the reciprocal of the square of the other

#### Inverter

a device that changes DC into AC by use of an electronic circuit

#### lon

a charged atom due to the loss or gain of an electron

#### **Kinetic energy**

the energy due to movement

# Lenz's law

the magnetic flux of the induced current through the loop opposes the *change* in the applied magnetic flux that produced it

#### Longitudinal wave

a wave in which vibrations are parallel to the direction of travel

#### Lorentz factor

the factor by which time, length and energy will change due to their relative velocity. It is represented by the symbol gamma,  $\gamma$ , and

given by 
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

#### Low speed

 $v \ll c$ , so relativistic effects can be ignored

#### **Magnetic dipole**

any object that generates a magnetic field in which the field is considered to emanate from two opposite poles, as in the north and south poles of a magnet

#### Magnetic field

the property of the space around a magnet that causes an object in that space to experience a force due only to the presence of the magnet

#### Magnetic flux, $\Phi_{R}$

the total magnetic field (or total number of magnetic field lines) that passes through a given area; proportional to the area and field strength

# Mass defect

the difference in mass between the products and the reactants of a reaction

#### Medium

a substance that allows waves to travel through it

#### Metal vapour lamp

a gas discharge lamp in which metals are vaporised through a hot electric arc, exciting gas atoms into higher energy states to produce characteristic spectra

# **Microwave**

an electromagnetic wave with wavelength between 1 mm and 1 m

# Momentum

the product of an object's mass and velocity

#### Monopole

an object that is either the source of a field (meaning the field lines point away from it) or the sink of a field (meaning the field lines point towards it)

#### Muon

a small, fundamental particle; a type of lepton

#### Net force

the vector sum of all of the forces acting on a body. The net force may also be referred to as the unbalanced force or the resultant force.

# Neutron

an uncharged particle in the nucleus of an atom

# Newton's law of universal gravitation

the attractive gravitational force between two masses, whose centres of mass are a distance r apart, is directly proportional to the product of their masses and inversely proportional to the square of the distance between the masses

# Newton's third law

when object A exerts a force on object B, object B exerts an equal and opposite reaction force on object A

#### Node

point of minimal disturbance in an interference pattern

#### Non-contact force

a force that acts on an object without coming physically in contact with it

78-1-009-25901-9

# Non-uniform electric field

an electric field where the strength and direction of the field vary

#### Non-uniform gravitational field

a gravitational field where the strength and direction of the field varies

# Normal force

the force that a surface applies to a body in contact with it. The force is always applied perpendicular to the surface and prevents the body falling through the surface.

# **Nuclear fusion**

the process of joining together two or more small nuclei to form a larger, more stable nucleus

#### **Nuclear transmutation**

turning one chemical element into another

#### **Nucleus**

the solid centre of an atom where most of the mass of an atom is concentrated

#### Open system

a system that allows the transfer of mass or energy to the surrounding environment

#### **Orbital period**

the time a satellite takes to complete one orbit

#### **Orbital radius**

the distance that the satellite is from the centre of mass of the body it is orbiting

**Orbital speed** the speed a satellite has in a given orbit

#### **Outer-shell electron**

the outermost electrons in an atom

# **Particle accelerator**

a machine that uses electromagnetic fields to propel or bend the path of charged particles

# Peak-to-peak value

a measurement of the value of a sinusoidal variable (such as AC voltage, current or power) from the top of one peak to the bottom of the next trough, i.e. double the amplitude

#### Peak value

the maximum value reached in one cycle of an alternating variable, such as AC voltage or current; the amplitude of the sine wave

## Period

the time it takes for one complete orbit

#### Photocell

an evacuated glass tube with a metal plate and collector electrode used in photelectric investigations and applications

#### Photocurrent

the current through a conducting photocell caused by the photoelectric effect

#### **Photoelectric effect**

the phenomenon in which light can release an electron from a metal surface

# Photoelectron

an electron released from a metal by the photoelectric effect

#### Photon

a packet of electromagnetic radiation; the word that replaced the term 'quanta'

# **Photovoltaic effect**

when a photovoltaic cell is exposed to sunlight, electrons move from the n-type side and become trapped by the pn junction, creating a potential difference across the junction and generating current

#### Photovoltaic (pv) cell

also known as a solar cell. It converts light energy into electricity via the photovoltaic effect.

#### pn junction

the boundary within a semiconductor between two different types of semiconductor materials

#### Point charge

an ideal situation in which all of the charge on an object is considered to be concentrated at a single point

#### Point mass

an ideal situation in which all of the mass on an object is considered to be concentrated at a single point

#### Polar aurora

charged particles from the Sun excite atoms near the poles into high energy states. They then decay, emitting photons.

#### Polarisation

a property of transverse waves describing the orientation of their oscillations; in transverse waves, this is perpendicular to the wave motion direction

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# **Polycrystalline**

a solid material that consists of many small grains or crystals. The size of the grains may vary from nanometers to millimeters.

## Positron

a positively charged anti-particle that has the same mass and magnitude of charge as the electron

# **Postulate**

established fact used as a basis for reasoning

#### **Prism**

a triangular glass prism that separates light of different wavelengths by refraction

#### **Product**

a substance formed as a part of a reaction

#### **Projectile**

a body that is launched into the air and moves freely through space, close enough to the surface of Earth so that the only force acting on the body is a constant gravitational force

#### Proper length $(L_0)$

a length interval measured when stationary relative to the object or space

#### Proper time $(t_0)$

the time interval between two events, where the two events occur at the same point in space

#### Proton

a positively charged particle in the nucleus of an atom

#### Quanta

plural of quantum, used by Einstein to describe the packets of light energy in his explanation of the photoelectric effect, later called photons

#### **Quantisation**

the concept that a physical quantity can have only certain discrete allowable values

#### Quantised

when a quantity or variable can only have certain (discrete) values

# Quantum number

a number used to order the energy states in an atom

## Rate of change

the rate at which one quantity is changing with respect to time

#### Reactant

a substance present before the reaction that is used up to make the products

#### **Reference frame**

a coordinate system whose quantities, such as distances and time, can be measured

# Refraction

the change in direction of a wave moving from one medium (or vacuum) to another medium (or vacuum) caused by the wave changing speed

#### **Refractive index**

a measure of how much slower light travels through a medium compared to a vacuum; given the symbol n

#### Relativity

the dependence of physical phenomena on the relative motion between the thing being observed and the observer

#### **Right-hand grip rule**

a mnemonic used to determine the direction of the magnetic field produced by a current-carrying wire. The thumb indicates the direction of the current and the curled grip represents the field lines.

#### **Right-hand slap rule**

a mnemonic used to determine the direction of the force on a current-carrying wire in a magnetic field. The thumb indicates the direction of the current, the fingers represent the magnetic field and the palm faces in the direction of the force.

#### Root-mean-square (rms)

the effective mean of a sinusoidal variable, such as AC voltage, current or power. The rms value gives the same effect as a constant DC voltage or current in a resistive load.

#### Scalar field

an assignment of a scalar to each point in a region in the space

#### Semiconductor

a material that will conduct electricity only under particular conditions. Its conducting properties are between a conductor and an insulator, depending on conditions.

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# Sinusoidal

having a sine wave shape; the regular, repeating form of a sine function

#### Slip ring

a form of commutator used in alternators to connect the rotating coil to the non-rotating terminals and therefore transfer the alternating current (AC) produced in the coil

#### Solar wind

a stream of charged particles from the Sun, consisting mainly of electrons, protons and alpha particles

# Solenoid

a coil of wire that creates a magnetic field when current is passed through it that is similar to a bar magnet

#### Spectroscope

an optical instrument that displays the wavelength spectrum of a light source

# Split ring commutator

a device used in DC generators to obtain a DC ouput, i.e. to rectify the AC emf (and current) produced

#### Standing wave

a wave that oscillates in time but whose amplitude profile does not move in space

## Static field

a field that is not changing

#### **Step-down transformer**

a transformer that decreases voltage in the secondary coil compared to the input voltage in the primary coil

#### Step-up transformer

a device that increases voltage in the secondary coil compared to the input voltage in the primary coil

#### **Stopping potential**

the potential required to just stop the most energetic photoelectrons

#### Strong nuclear force

the fundamental force that binds nucleons (protons and/or neutrons) together in a nucleus of an atom. It acts only over very short distances  $(10^{-15} \text{ m})$ .

#### Superposition

the principle that when two or more waves overlap, their displacements add together

## Synchrotron

a type of particle accelerator where the particle beam travels in a closed-loop path

#### System

a collection of objects that interact with each other and are being studied

# **Threshold frequency**

the minimum frequency required to release a photoelectron from a metal surface

#### Torque

a measure of how much of a force acting on an object is causing it to rotate

#### Transformer

a device that changes voltage through electromagnetic induction in an alternating current

#### Transmission line

a specialised powerline that carries electrical energy at high voltage between geographical locations, sometimes over very long distances

#### **Transmission loss**

electric power transformed to thermal energy, sound energy and kinetic energy as current passes through the resistance of the transmission wires; calculated from  $P_{loss} = l^2 R$ . It is sometimes expressed as a percentage of total power transmitted.

#### **Transverse wave**

a wave in which vibrations are perpendicular to the direction of travel

#### Tungsten filament incandescent globe

an electric light with a tungsten wire filament heated until it glows white hot

#### **Ultraviolet light**

electromagnetic radiation with a wavelength range of 100 nm to 400 nm

#### **Uniform circular motion**

the movement of an object in a circular path at constant speed

#### **Uniform electric field**

an electric field in which the value of the field strength remains the same at all points

#### Uniform gravitational field

a field that has a constant strength and direction at all points

#### Uniform magnetic field

a magnetic field in which the value of the field strength remains the same at all points

#### Vector field

an assignment of a vector to each point in a region in the space

#### Voltage

another name for potential difference, derived from its unit, volt

#### Wavelength

the distance between repeated parts of a wave shape, measured in metres (m)

#### Wavelet

a wave-like phenomenon that begins with zero amplitude

#### Wave-particle duality

the concept that light and matter can have both wave-like and particle-like properties

# Work

the amount of energy transferred from one object or system to another

#### Work function ( $\Phi$ )

the minimum energy required to release a photoelectron from a metal surface. It varies from metal to metal.

#### X-ray

the region of the electromagnetic spectrum between UV and gamma rays. X-rays have the second shortest wavelengths and second highest frequencies.

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