

# Chapter 1 answers

## Section 1.1

### Worked example: Try yourself 1.1.1

#### GRAVITATIONAL ATTRACTION BETWEEN SMALL OBJECTS

Two bowling balls are sitting next to each other on a shelf so that the centres of the balls are 60 cm apart. Ball 1 has a mass of 7.0 kg and ball 2 has a mass of 5.5 kg. Calculate the force of gravitational attraction between them.

Thinking	Working
Recall the formula for Newton's law of universal gravitation.	$F_g = G \frac{m_1 m_2}{r^2}$
Identify the information required, and convert values into appropriate units when necessary.	$m_1 = 7.0 \text{ kg}$ $m_2 = 5.5 \text{ kg}$ $r = 0.60 \text{ m}$ $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Substitute the values into the equation.	$F_g = 6.67 \times 10^{-11} \times \frac{7.0 \times 5.5}{0.60^2}$
Solve the equation.	$F_g = 7.1 \times 10^{-9} \text{ N}$

### Worked example: Try yourself 1.1.2

#### GRAVITATIONAL ATTRACTION BETWEEN LARGE OBJECTS

Calculate the force of gravitational attraction between the Earth and the Moon, given the following data:  
 $m_{\text{Earth}} = 6.0 \times 10^{24} \text{ kg}$   
 $m_{\text{Moon}} = 7.3 \times 10^{22} \text{ kg}$   
 $r_{\text{Moon-Earth}} = 3.8 \times 10^8 \text{ m}$

Thinking	Working
Recall the formula for Newton's law of universal gravitation.	$F_g = G \frac{m_1 m_2}{r^2}$
Identify the information required.	$m_1 = 6.0 \times 10^{24} \text{ kg}$ $m_2 = 7.3 \times 10^{22} \text{ kg}$ $r = 3.8 \times 10^8 \text{ m}$ $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Substitute the values into the equation.	$F_g = 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 7.3 \times 10^{22}}{(3.8 \times 10^8)^2}$
Solve the equation.	$F_g = 2.0 \times 10^{20} \text{ N}$

**Worked example: Try yourself 1.1.3****ACCELERATION CAUSED BY A GRAVITATIONAL FORCE**

<p>The force of gravitational attraction between the Sun and the Earth is approximately <math>3.6 \times 10^{22}</math> N. Calculate the acceleration of the Earth and the Sun caused by this force. Compare these accelerations by calculating the ratio <math>\frac{a_{\text{Earth}}}{a_{\text{Sun}}}</math>.</p> <p>Use the following data:</p> $m_{\text{Earth}} = 6.0 \times 10^{24} \text{ kg}$ $m_{\text{Sun}} = 2.0 \times 10^{30} \text{ kg}$	
<b>Thinking</b>	<b>Working</b>
Recall the formula for Newton's second law of motion.	$F = ma$
Transpose the equation to make $a$ the subject.	$a = \frac{F}{m}$
Substitute values into this equation to find the accelerations of the Earth and the Sun.	$a_{\text{Earth}} = \frac{3.6 \times 10^{22}}{6.0 \times 10^{24}} = 6.0 \times 10^{-3} \text{ m s}^{-2}$ $a_{\text{Sun}} = \frac{3.6 \times 10^{22}}{2.0 \times 10^{30}} = 1.8 \times 10^{-8} \text{ m s}^{-2}$
Compare the two accelerations.	$\frac{a_{\text{Earth}}}{a_{\text{Sun}}} = \frac{6.0 \times 10^{-3}}{1.8 \times 10^{-8}} = 3.3 \times 10^5$ <p>The acceleration of the Earth is <math>3.3 \times 10^5</math> times greater than the acceleration of the Sun.</p>

**Worked example: Try yourself 1.1.4****GRAVITATIONAL FORCE AND WEIGHT**

<p>Compare the weight of a 1.0 kg mass on the Earth's surface calculated using the formulas <math>F_g = mg</math> and <math>F_g = G \frac{m_1 m_2}{r^2}</math>.</p> <p>Use the following dimensions of the Earth where necessary:</p> $g = 9.8 \text{ m s}^{-2}$ $m_{\text{Earth}} = 6.0 \times 10^{24} \text{ kg}$ $r_{\text{Earth}} = 6.4 \times 10^6 \text{ m}$	
<b>Thinking</b>	<b>Working</b>
Apply the weight equation.	$F_g = mg$ $= 1 \times 9.8$ $= 9.8 \text{ N}$
Apply Newton's law of universal gravitation.	$F_g = G \frac{m_1 m_2}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 1}{(6.4 \times 10^6)^2}$ $= 9.77 \text{ N}$ $= 9.8 \text{ N}$
Compare the two values.	Both equations give the same result to two significant figures.

**Worked example: Try yourself 1.1.5****APPARENT WEIGHT**

Calculate the apparent weight of a 90 kg person in an elevator which is accelerating downwards at $0.8 \text{ m s}^{-2}$ . Use $g = 9.8 \text{ m s}^{-2}$ .	
<b>Thinking</b>	<b>Working</b>
Calculate the weight of the person using $F_g = mg$ .	$F_g = mg = 90 \times 9.8 = 882 \text{ N}$
Calculate the force required to accelerate the person at $0.8 \text{ m s}^{-2}$ .	$F_{net} = ma = 90 \times 0.8 = 72 \text{ N}$
The net force that causes the acceleration consists of the normal reaction force (upwards) and the weight force (downwards). Since the elevator is accelerating downwards, $F_g > F_N$ . Notice that, as the person is partially falling in the direction of gravitational acceleration, there is less contact force and the person feels lighter than if standing still.	$F_{net} = 72$ $F_g - F_N = 72$ $882 - F_N = 72$ $F_N = 882 - 72$ Apparent weight = 810 N

**1.1 review**

1 The force of attraction between any two bodies in the universe is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

2  $r$  is the distance between the centres of the two objects.

$$3 \quad F_g = G \frac{m_1 m_2}{r^2}$$

$$= 6.67 \times 10^{-11} \times \frac{2.0 \times 10^{30} \times 6.4 \times 10^{23}}{(2.2 \times 10^{11})^2} = 1.8 \times 10^{21} \text{ N}$$

$$4 \quad F_g = m_{\text{Mars}} \times a_{\text{Mars}}$$

$$1.8 \times 10^{21} = 6.4 \times 10^{23} \times a_{\text{Mars}}$$

$$a_{\text{Mars}} = \frac{1.8 \times 10^{21}}{6.4 \times 10^{23}}$$

$$= 2.8 \times 10^{-3} \text{ m s}^{-2}$$

5 a Note: 1 million km =  $1 \times 10^6 \text{ km} = 1 \times 10^9 \text{ m}$

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$= 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 6.4 \times 10^{23}}{(9.3 \times 10^{10})^2}$$

$$= 3.0 \times 10^{16} \text{ N}$$

$$b \quad F_g = G \frac{m_1 m_2}{r^2}$$

$$= 6.67 \times 10^{-11} \times \frac{2.0 \times 10^{30} \times 6.0 \times 10^{24}}{(15.3 \times 10^{10})^2}$$

$$= 3.4 \times 10^{22} \text{ N}$$

c % comparison =  $(3.0 \times 10^{16}) \div (3.4 \times 10^{22}) \times 100 = 0.000088\%$ . The Mars–Earth force was 0.000088% of the Sun–Earth force.

6 The Moon has a smaller mass than the Earth and therefore experiences a larger acceleration from the same gravitational force.

$$7 \quad a = g = G \frac{M}{r^2}$$

$$g = 6.67 \times 10^{-11} \times \frac{3.3 \times 10^{23}}{(2\,500\,000)^2}$$

$$= 3.5 \text{ m s}^{-2}$$

$$8 \quad F_g = G \frac{m_1 m_2}{r^2}$$

$$= 6.67 \times 10^{-11} \times \frac{6.4 \times 10^{23} \times 65}{(3.4 \times 10^6)^2}$$

$$= 240 \text{ N}$$

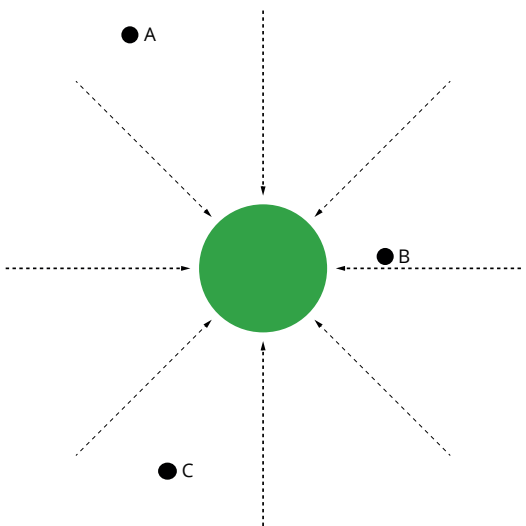
- 9 On Earth, weight is the gravitational force acting on an object near the Earth's surface whereas apparent weight is the contact force between the object and the Earth's surface. In many situations, these two forces are equal in magnitude but are in opposite directions. This is because apparent weight is a reaction force to the weight of an object resting on the ground. However, in an elevator accelerating upwards, the apparent weight of an object would be greater than its weight since an additional force would be required to cause the object to accelerate upwards.
- 10 a  $F_g = mg = 50 \times 9.8 = 490 \text{ N}$   
 When accelerating upwards at  $1.2 \text{ m s}^{-2}$ , the net force is  $F_{\text{net}} = ma = 50 \times 1.2 = 60 \text{ N}$ , and  $F_N > F_g$ .  
 $F_{\text{net}} = F_N - F_g = 60 \text{ N}$   
 $F_N = 60 + 490 = 550 \text{ N}$ . The person's apparent weight is 550 N.
- b When the person is moving at a constant speed, their apparent weight is equal to their weight.  
 $F_N = F_g = mg = 50 \times 9.8 = 490 \text{ N}$

## Section 1.2

### Worked example: Try yourself 1.2.1

#### INTERPRETING GRAVITATIONAL FIELD DIAGRAMS

The diagram below shows the gravitational field of a planet.

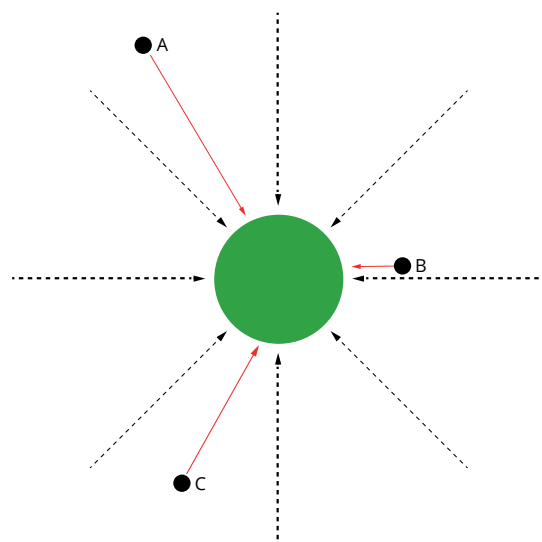


(a) Use arrows to indicate the direction of the gravitational force acting at points A, B and C.

#### Thinking

The direction of the field arrows indicates the direction of the gravitational force, which is inwards towards the centre of the planet.

#### Working

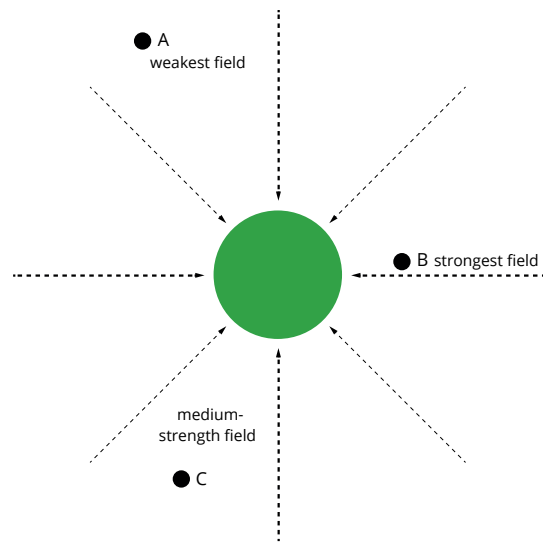


(b) Indicate the relative strength of the gravitational field at each point.

**Thinking**

The closer the field lines, the stronger the force.

**Working**



**Worked example: Try yourself 1.2.2**

**CALCULATING GRAVITATIONAL FIELD STRENGTH**

A student uses a spring balance to measure the weight of a piece of wood as 2.5 N. If the piece of wood is thought to have a mass of 260 g, calculate the gravitational field strength indicated by this experiment.

**Thinking**

Recall the equation for gravitational field strength.

**Working**

$$g = \frac{F_g}{m}$$

Substitute in the appropriate values.

$$g = \frac{2.5}{0.26}$$

Solve the equation.

$$g = 9.6 \text{ N kg}^{-1}$$

**Worked example: Try yourself 1.2.3**

**CALCULATING GRAVITATIONAL FIELD STRENGTH AT DIFFERENT ALTITUDES**

Commercial airlines typically fly at an altitude of 11 000 m. Calculate the gravitational field strength of the Earth at this height using the following data:

$$r_{\text{Earth}} = 6.38 \times 10^6 \text{ m}$$

$$m_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg}$$

**Thinking**

Recall the formula for gravitational field strength.

**Working**

$$g = G \frac{M}{r^2}$$

Add the altitude of the plane to the radius of the Earth.

$$\begin{aligned} r &= 6.38 \times 10^6 + 11\,000 \text{ m} \\ &= 6.391 \times 10^6 \text{ m} \end{aligned}$$

Substitute the values into the formula.

$$\begin{aligned} g &= G \frac{M}{r^2} \\ &= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{(6.391 \times 10^6)^2} \\ &= 9.75 \text{ N kg}^{-1} \end{aligned}$$

**Worked example: Try yourself 1.2.4****GRAVITATIONAL FIELD STRENGTH ON ANOTHER PLANET OR MOON**

Calculate the strength of the gravitational field on the surface of Mars.

$$m_{\text{Mars}} = 6.42 \times 10^{23} \text{ kg}$$

$$r_{\text{Mars}} = 3390 \text{ km}$$

Give your answer correct to two significant figures.

Thinking	Working
Recall the formula for gravitational field strength.	$g = G \frac{M}{r^2}$
Convert Mars' radius to m.	$r = 3390 \text{ km}$ $= 3.39 \times 10^6 \text{ m}$
Substitute values into the formula.	$g = G \frac{M}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{6.42 \times 10^{23}}{(3.39 \times 10^6)^2}$ $= 3.7 \text{ N kg}^{-1}$

**1.2 review**

1  $\text{N kg}^{-1}$

2  $g = \frac{F_g}{m} = \frac{1.4}{0.15} = 9.3 \text{ N kg}^{-1}$

- 3 The distance has been increased three times from 400 km to 1200 km so, in terms of the inverse square law, and the original distance,  $r$ :

$$F \propto \frac{1}{r^2}$$

$$\propto \frac{1}{(3r)^2}$$

$$\propto \frac{1}{(9r)^2}$$

$$\therefore \frac{1}{9} \text{ of the original}$$

4 a  $g = G \frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 2000) \times 10^3)^2}$$

$$= 5.67 \text{ N kg}^{-1}$$

b  $g = G \frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 10\,000) \times 10^3)^2}$$

$$= 1.48 \text{ N kg}^{-1}$$

c  $g = G \frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 20\,200) \times 10^3)^2}$$

$$= 0.56 \text{ N kg}^{-1}$$

d  $g = G \frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 35\,786) \times 10^3)^2}$$

$$= 0.22 \text{ N kg}^{-1}$$

- 5  $g = G \frac{M}{r^2}$   
 $= 6.67 \times 10^{-11} \times \frac{1 \times 10^{13}}{900^2}$   
 $= 0.0008 \text{ N kg}^{-1}$  or  $8 \times 10^{-4} \text{ N kg}^{-1}$
- 6  $g = G \frac{M}{r^2}$   
 Mercury:  $g = 6.67 \times 10^{-11} \times \frac{3.30 \times 10^{23}}{(2.44 \times 10^6)^2} = 3.7 \text{ N kg}^{-1}$   
 Saturn:  $g = 6.67 \times 10^{-11} \times \frac{5.69 \times 10^{26}}{(6.03 \times 10^7)^2} = 10.4 \text{ N kg}^{-1}$   
 Jupiter:  $g = 6.67 \times 10^{-11} \times \frac{1.90 \times 10^{27}}{(7.15 \times 10^7)^2} = 24.8 \text{ N kg}^{-1}$
- 7  $g = G \frac{M}{r^2} = 6.67 \times 10^{-11} \times \frac{3.0 \times 10^{30}}{(10 \times 10^3)^2} = 2 \times 10^{12} \text{ N kg}^{-1}$

- 8  $g_{\text{poles}} = G \frac{M}{r^2}$   
 $8 = 6.67 \times 10^{-11} \times \frac{M}{5\,000\,000^2}$   
 $M = 3 \times 10^{24} \text{ kg}$   
 $g_{\text{equator}} = G \frac{M}{r^2} = 6.67 \times \frac{3 \times 10^{24}}{6\,000\,000^2} = 5.6 \text{ N kg}^{-1}$

$8 \div 5.6 = 1.4$ . The gravitational field strength at the poles is 1.4 times that at the equator. (Alternatively, the inverse square law could also be used to find this relationship.)

- 9 Let  $x$  be the distance from the centre of the Earth where the Earth's gravity equals the Moon's gravity. Then:

$$g_{\text{Earth}} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{x^2}$$

$$g_{\text{Moon}} = \frac{6.67 \times 10^{-11} \times 7.3 \times 10^{22}}{(3.8 \times 10^8 + x)^2}$$

Equating these two expressions gives:

$$\frac{6 \times 10^{24}}{x^2} = \frac{7.3 \times 10^{22}}{(3.8 \times 10^8 + x)^2}$$

$$\frac{82.2}{x^2} = \frac{1}{(3.8 \times 10^8 + x)^2}$$

Taking square roots of both sides gives:

$$\frac{9.07}{x} = \frac{1}{(3.8 \times 10^8 + x)}$$

Inverting both sides gives:

$$\frac{x}{9.07} = 3.8 \times 10^8 + x$$

$$x = 3.45 \times 10^9 - 9.07x$$

$$10.07x = 3.45 \times 10^9$$

$$x = 3.4 \times 10^8 \text{ m}$$

- 10  $g$  is proportional to  $\frac{1}{r^2}$ , so if  $g$  becomes  $\frac{1}{100}$ th of its value,  $r$  must become 10 times its value so that  $\frac{1}{r^2}$  becomes  $\frac{1}{100}$ .  
 10 times  $r$  means a distance of 10 Earth radii.

## Section 1.3

## Worked example: Try yourself 1.3.1

## WORK DONE FOR A CHANGE IN GRAVITATIONAL POTENTIAL ENERGY

Calculate the work done (in MJ) to lift a weather satellite of 3.2 tonnes from the Earth's surface to the limit of the atmosphere, which ends at the Karman line (exactly 100 km up from the surface of the Earth). Assume $g = 9.8 \text{ N kg}^{-1}$ .	
<b>Thinking</b>	<b>Working</b>
Convert the values into the appropriate units.	$m = 3.2 \text{ tonnes} = 3200 \text{ kg}$ $h = 100 \text{ km} = 100 \times 10^3 \text{ m}$
Substitute the values into $E_g = mg\Delta h$ . Remember to give your answer in MJ to two significant figures.	$E_g = mg\Delta h$ $= 3200 \times 9.8 \times 100 \times 10^3$ $= 3.136 \times 10^9 \text{ J}$ $= 3.1 \times 10^3 \text{ MJ}$
The work done is equal to the change in gravitational potential energy.	$W = \Delta E = 3.1 \times 10^3 \text{ MJ}$

## Worked example: Try yourself 1.3.2

## SPEED OF A FALLING OBJECT

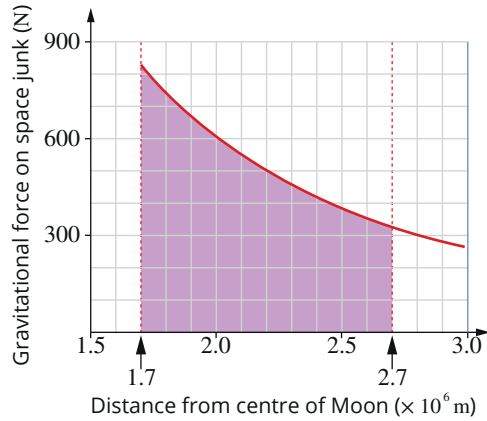
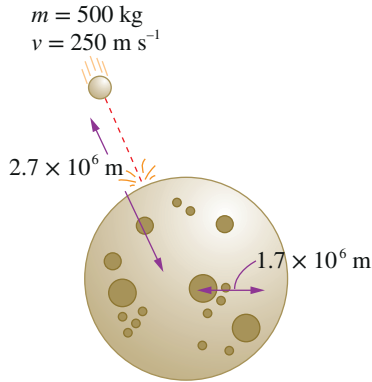
Calculate how fast a 450 g hammer would be going as it hit the ground if it was dropped from a height of 1.4 m on Earth, where $g = 9.8 \text{ N kg}^{-1}$ .	
<b>Thinking</b>	<b>Working</b>
Calculate the gravitational potential energy of the hammer on Earth.	$E_g = mg\Delta h$ $= 0.45 \times 9.8 \times 1.4$ $= 6.2 \text{ J}$
Assume that when the hammer hits the surface of the Earth, all of its gravitational potential energy has been converted into kinetic energy.	$E_k = E_g = 6.2 \text{ J}$
Use the definition of kinetic energy to calculate the speed of the hammer as it hits the ground.	$E_k = \frac{1}{2}mv^2$ $6.2 = \frac{1}{2} \times 0.45 \times v^2$ $\frac{6.2 \times 2}{0.45} = v^2$ $v = 5.2 \text{ m s}^{-1}$



**Worked example: Try yourself 1.3.3**

**CHANGE IN GRAVITATIONAL POTENTIAL ENERGY USING A FORCE-DISTANCE GRAPH**

A 500 kg lump of space junk is plummeting towards the Moon. The Moon has a radius of  $1.7 \times 10^6$  m. Using the force-distance graph, determine the decrease in gravitational potential energy of the junk as it falls to the Moon's surface.

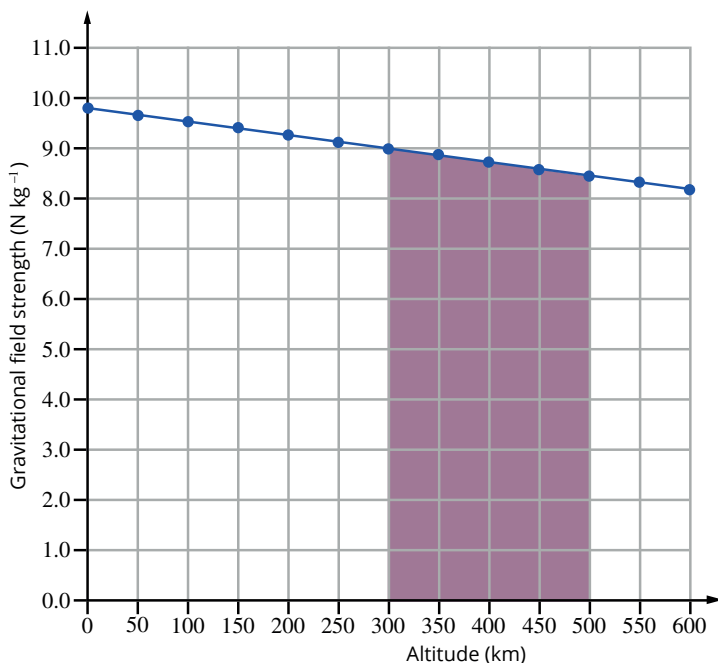


Thinking	Working
Count the number of shaded squares. (Only count squares that are at least 50% shaded.)	Number of shaded squares = 52
Calculate the area (energy value) of each square.	$E_{\text{square}} = 0.1 \times 10^6 \times 100$ $= 1 \times 10^7 \text{ J}$
To calculate the energy change, multiply the number of shaded squares by the energy value of each square.	$\Delta E_g = 52 \times (1 \times 10^7)$ $= 5.2 \times 10^8 \text{ J}$

**Worked example: Try yourself 1.3.4**

**CHANGE IN GRAVITATIONAL POTENTIAL ENERGY USING A GRAVITATIONAL FIELD STRENGTH-DISTANCE GRAPH**

A 3000 kg Soyuz rocket moves from an orbital height of 300 km above the Earth's surface to dock with the International Space Station at a height of 500 km. Use the graph of the gravitational field strength of the Earth below to determine the approximate change in gravitational potential energy of the rocket.



Thinking	Working
Count the number of shaded squares. Only count squares that are at least 50% shaded.	Number of shaded squares = 36
Calculate the energy value of each square.	$E_{\text{square}} = 50 \times 10^3 \text{ m} \times 1 \text{ N kg}^{-1}$ $= 5 \times 10^4 \text{ J kg}^{-1}$
To calculate the energy change, multiply the number of shaded squares by the energy value of each square and the mass of the rocket.	$\Delta E_{\text{g}} = 36 \times 5 \times 10^4 \times 3000$ $= 5.4 \times 10^9 \text{ J}$

### 1.3 review

- C. A stable orbit suggests that the object is in a uniform gravitational field, hence its gravitational potential energy does not change. Its speed will also remain the same in a stable orbit.
- $g$  increases from point A to point D.
- The meteor is under the influence of the Earth's gravitational field which will cause it to accelerate at an increasing rate as it approaches the Earth.
- A, B and C are all correct. The total energy of the system does not change.
- $$W = E_{\text{g}} = 3000000 \times 9.8 \times 67000$$

$$= 2.0 \times 10^{12} \text{ J}$$
- $$E_{\text{g}} = mg\Delta h$$

$$= 0.4 \times 6.1 \times 7000$$

$$= 17100 \text{ J}$$

$$E_{\text{k}} = \frac{1}{2}mv^2$$

$$17100 = \frac{1}{2} \times 0.4 \times v^2$$

$$v = \sqrt{\frac{2 \times 17100}{0.4}}$$

$$= 292 \text{ m s}^{-1}$$
- 100 km above the Earth's surface is a distance of  $6.4 \times 10^6 \text{ m} + 100000 \text{ m} = 6.5 \times 10^6 \text{ m}$ . According to the graph,  $F$  is between 9 N and 9.2 N at this height.
  - According to the graph, 5 N occurs at approximately  $9.0 \times 10^6 \text{ m}$  from the centre of the Earth. So, the height above the Earth's surface =  $9.0 \times 10^6 - 6.4 \times 10^6 = 2.6 \times 10^6 \text{ m}$  or 2600 km.
- Convert  $\text{km s}^{-1}$  to  $\text{m s}^{-1}$  then apply the rule:
$$E_{\text{k}} = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 1 \times 4000^2$$

$$= 8 \times 10^6 \text{ J}$$
  - $$\Delta E_{\text{k}} = \Delta E_{\text{g}}$$

$$\Delta E_{\text{g}} = \text{area under the graph}$$

$$\text{area} = 19 \text{ squares} \times 2 \times 0.5 \times 10^6$$

$$= 1.9 \times 10^7 \text{ J}$$
  - new  $E_{\text{k}} = \text{starting } E_{\text{k}} + \Delta E_{\text{k}} = 8 \times 10^6 + 1.9 \times 10^7 = 2.7 \times 10^7 \text{ J}$
  - new speed =  $\sqrt{2 \times 2.7 \times 10^7}$ 

$$= 7348 \text{ m s}^{-1} \text{ or } 7.3 \text{ km s}^{-1}$$
- 600 km above the Earth's surface =  $6.4 \times 10^6 + 600000 = 7.0 \times 10^6 \text{ m}$  or 7000 km  
Area under the graph between 7000 km and 8000 km is approximately 7 squares.  
As the satellite comes to a stop, the change in kinetic energy over the distance is the same as the  $E_{\text{k}}$  at its launch.  

$$E_{\text{k}} = \text{area under the graph} \times \text{mass of the satellite}$$

$$= 7 \text{ squares} \times 2 \times 0.5 \times 10^6 \times 240$$

$$= 1.7 \times 10^9 \text{ J}$$

- 10** 600 km above the Earth's surface =  $6.4 \times 10^6 + 600\,000 = 7.0 \times 10^6$  m or 7000 km  
 2600 km above the Earth's surface =  $6.4 \times 10^6 + 2\,600\,000 = 9.0 \times 10^6$  m or 9000 km.  
 The area under the graph between 7000 km and 9000 km is approximately 26 squares.

$$\begin{aligned}\Delta E_g &= \text{area under the graph} \times \text{mass of the satellite} \\ &= 26 \text{ squares} \times 1 \times 0.5 \times 10^6 \times 20\,000 \\ &= 2.6 \times 10^{11} \text{ J}\end{aligned}$$

## CHAPTER 1 REVIEW

$$1 \quad F_g = G \frac{m_1 m_2}{r^2}$$

$$\begin{aligned}&= 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 75}{(6.4 \times 10^6)^2} \\ &= 730 \text{ N}\end{aligned}$$

$$2 \quad F_g = G \frac{m_1 m_2}{r^2}$$

$$\begin{aligned}2.79 \times 10^{20} &= 6.67 \times 10^{-11} \times \frac{1.05 \times 10^{21} \times 5.69 \times 10^{26}}{r^2} \\ r^2 &= \frac{6.67 \times 10^{-11} \times 1.05 \times 10^{21} \times 5.69 \times 10^{26}}{2.79 \times 10^{20}} \\ &= 378\,000\,000 \text{ m} \\ &= 3.78 \times 10^8 \text{ m}\end{aligned}$$

$$3 \quad F = m a_{\text{Sun}}$$

$$\begin{aligned}a_{\text{Sun}} &= \frac{F}{m} \\ &= \frac{4.2 \times 10^{23}}{2.0 \times 10^{30}} = 2.1 \times 10^{-7} \text{ m s}^{-2}\end{aligned}$$

- 4 a** The force exerted on Jupiter by the Sun is equal to the force exerted on the Sun by Jupiter.  
**b** The acceleration of Jupiter caused by the Sun is greater than the acceleration of the Sun caused by Jupiter.

$$5 \quad g = G \frac{M}{r^2}$$

$$\begin{aligned}&= 6.67 \times 10^{-11} \times \frac{6.4 \times 10^{23}}{(3\,400\,000)^2} \\ &= 3.7 \text{ m s}^{-2}\end{aligned}$$

$$6 \quad \mathbf{a} \quad F_g = mg = 50 \times 9.8 = 490 \text{ N}$$

When accelerating downwards at  $0.6 \text{ m s}^{-2}$ , the net force is  $F_{\text{net}} = ma = 50 \times 0.6 = 30 \text{ N}$  and  $F_g > F_N$ .

$$F_g - F_N = 30$$

$$490 - F_N = 30$$

$$F_N = 490 - 30 = 460 \text{ N}$$

- b** When the person is moving at a constant speed, their apparent weight is equal to their weight:  $F_g = F_N = 490 \text{ N}$

$$7 \quad \mathbf{a} \quad F = G \frac{m_1 m_2}{r^2}$$

$$\begin{aligned}&= \frac{(6.67 \times 10^{-11} \times 1.9 \times 10^{27} \times 1000)}{(7.15 \times 10^7)^2} \\ &= 2.48 \times 10^4 \text{ N}\end{aligned}$$

- b** The magnitude of the gravitational force that the comet exerts on Jupiter is equal to the magnitude of the gravitational force that Jupiter exerts on the comet =  $2.48 \times 10^4 \text{ N}$ .

$$\mathbf{c} \quad \mathbf{a} = \frac{F_{\text{net}}}{m}$$

$$= \frac{2.48 \times 10^4}{1000}$$

$$= 24.8 \text{ m s}^{-2}$$

$$\mathbf{d} \quad \mathbf{a} = \frac{F_{\text{net}}}{m}$$

$$= \frac{2.48 \times 10^4}{1.90 \times 10^{27}}$$

$$= 1.31 \times 10^{-23} \text{ m s}^{-2}$$

- 8 D. At a height of two Earth radii above the Earth's surface, a person is a distance of three Earth radii from the centre of the Earth.

$$\text{Then } F = \frac{900}{3^2} = \frac{900}{9} = 100 \text{ N}$$

- 9 a D.  $F_{\text{net}} = F_{\text{N}} - F_{\text{g}} = ma$

$$F_{\text{N}} = 80 \times 30 + 80 \times 9.8 = 3184 \text{ N or } 3200 \text{ N}$$

- b B. From part (a), the apparent weight is greater than the weight of the astronaut.

- c C. True weight is unchanged during lift-off as  $g$  is constant.

- d A. During orbit, the astronaut is in free fall.

- e D.  $F_{\text{g}} = ma = 80 \times 8.2 = 656 \text{ N or } 660 \text{ N}$

- 10 When representing a gravitational field with a field diagram, the direction of the arrowhead indicates the *direction* of the gravitational force and the space between the arrows indicates the *magnitude* of the field. In gravitational fields, the field lines always point towards the sources of the field.

11  $g = \frac{F_{\text{g}}}{m} = \frac{600}{61.5} = 9.76 \text{ N kg}^{-1}$

12 a  $g = G \frac{M}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6378 \times 1000)^2}$$

$$= 9.79 \text{ N kg}^{-1}$$

b  $g = G \frac{M}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6357 \times 1000)^2}$$

$$= 9.85 \text{ N kg}^{-1}$$

$$\% = \frac{9.85}{9.79} \times 100 = 100.61\%$$

13 a  $g = G \frac{M}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 1.02 \times 10^{26}}{(2.48 \times 10^7)^2}$$

$$= 11.1 \text{ N kg}^{-1}$$

- b C. It will accelerate at a rate given by the gravitational field strength,  $g$ .

14  $G \frac{M}{(0.8R)^2} = G \frac{m}{(0.2R)^2}$

$$\frac{M}{0.64} = \frac{m}{0.04}$$

$$\frac{M}{m} = \frac{0.64}{0.04} = 16$$

- 15 a Increase in  $E_{\text{k}} = \text{area under the graph between } 3 \times 10^6 \text{ m and } 2.5 \times 10^6 \text{ m}$   
 $= 6 \text{ squares} \times 10 \times 0.5 \times 10^6 = 3 \times 10^7 \text{ J}$

b  $E_{\text{k(initial)}} = \frac{1}{2} mv^2 = \frac{1}{2} \times 20 \times 1000^2 = 1 \times 10^7 \text{ J}$

$$E_{\text{k(new)}} = 1 \times 10^7 + 3 \times 10^7 = 4 \times 10^7 \text{ J}$$

c  $v = \sqrt{\frac{2 \times E_{\text{k}}}{m}} = \sqrt{\frac{2 \times 4 \times 10^7}{20}} = 2000 \text{ m s}^{-1} \text{ or } 2 \text{ km s}^{-1}$

- d From the graph,  $F = 70 \text{ N} = mg$

$$g = \frac{70}{20} = 3.5 \text{ N kg}^{-1}$$

- 16  $300 \text{ km} = 300\,000 \text{ m or } 3 \times 10^5 \text{ m}$

From the graph,  $g = 9 \text{ N kg}^{-1}$  at this altitude.

- 17 D. The units on the graph are  $\text{N m kg}^{-1}$ , which are the same as  $\text{J kg}^{-1}$

- 18 C. As the satellite falls, its gravitational potential energy decreases. The units on the graph are  $\text{J kg}^{-1}$ , so therefore C is correct.

- 19 Increase in  $E_{\text{k}} = \text{area under the graph} \times \text{mass of the satellite}$

$$= 35 \text{ squares} \times 1 \times 1 \times 10^5 \times 1000 = 3.5 \times 10^9 \text{ J}$$

- 20 No. Air resistance will play a major part as the satellite re-enters the Earth's atmosphere.

## Chapter 2 answers

### Section 2.1

#### Worked example: Try yourself 2.1.1

USING  $F = qE$

Calculate the magnitude of the uniform electric field that creates a force of $9.00 \times 10^{-23}$ N on a proton. ( $q_p = +1.602 \times 10^{-19}$ C)	
<b>Thinking</b>	<b>Working</b>
Rearrange the relevant equation to make electric field strength the subject.	$F = qE$ $E = \frac{F}{q}$
Substitute the values for $F$ and $q$ into the rearranged equation and calculate the answer.	$E = \frac{9.00 \times 10^{-23}}{1.602 \times 10^{-19}}$ $= 5.62 \times 10^{-4} \text{ N C}^{-1}$

#### Worked example: Try yourself 2.1.2

WORK DONE ON A CHARGE IN A UNIFORM ELECTRIC FIELD

A student sets up a parallel plate arrangement so that one plate is at a potential of 36.0 V and the other earthed plate is positioned 2.00 m away. Calculate the work done to move an electron a distance of 75.0 cm towards the negative plate. ( $q_e = -1.602 \times 10^{-19}$ C) In your answer identify what does the work and what the work is done on.	
<b>Thinking</b>	<b>Working</b>
Identify the variables presented in the problem to calculate the electric field strength $E$ .	$V_2 = 36.0 \text{ V}$ $V_1 = 0 \text{ V}$ $d = 2.00 \text{ m}$
Use the equation $E = \frac{V}{d}$ to determine the electric field strength.	$E = \frac{V}{d}$ $= \frac{36.0 - 0}{2.00}$ $= 18.0 \text{ V m}^{-1}$
Use the equation $W = qEd$ to determine the work done. Note that $d$ here is the distance that the electron moves.	$W = qEd$ $= 1.602 \times 10^{-19} \times 18.0 \times 0.750$ $= 2.16 \times 10^{-18} \text{ J}$
Determine if work is done on the charge by the field or if work is done on the field.	Since the negatively charged electron would normally move away from the negative plate, work is done on the field.

### 2.1 review

- C. In an electric field, a force is exerted between two charged objects.
- B. The electric field direction is defined as being the direction that a positively charged test charge moves when placed in the electric field.

- 3 a True. Electric field lines start and end at  $90^\circ$  to the surface, with no gap between the lines and the surface.  
 b False. Field lines can never cross. If they did it would indicate that the field is in two directions at that point, which can never happen.  
 c False. Electric fields go from positively charged objects to negatively charged objects.  
 d True. Around small charged spheres called point charges you should draw at least eight field lines: top, bottom, left, right and in between each of these.  
 e True. Around point charges the field lines radiate like spokes on a wheel.  
 f False. Between two point charges the direction of the field at any point is the resultant field vector determined by adding the field vectors due to each of the two point charges.  
 g False. Between two oppositely charged parallel plates the field between the plates is evenly spaced and is drawn straight from the positive plate to the negative plate.

$$4 \quad F = qE$$

$$= 5.00 \times 10^{-3} \times 2.5$$

$$= 0.005 \times 2.5$$

$$= 0.0125$$

$$= 1.25 \times 10^{-2} \text{ N}$$

$$5 \quad F = qE$$

$$q = \frac{F}{E}$$

$$= \frac{0.025}{18}$$

$$= 0.00139 \text{ C}$$

$$= 1.39 \times 10^{-3} \text{ C}$$

$$= 1.39 \text{ mC}$$

$$6 \quad F = qE$$

$$= 1.602 \times 10^{-19} \times 3.25$$

$$= 5.207 \times 10^{-19} \text{ N}$$

and

$$F = ma$$

$$a = \frac{F}{m}$$

$$= \frac{5.207 \times 10^{-19}}{9.11 \times 10^{-31}}$$

$$= 5.72 \times 10^{11} \text{ m s}^{-2}$$

$$7 \quad E = \frac{V}{d}$$

$$4000 = \frac{V}{0.3}$$

$$V = 4000 \times 0.3 = 1200 \text{ V}$$

- 8 As the oil drop is stationary, the electric force must be equal to the gravitational force. Use the equations  $F = mg$  and  $F = qE$  to determine the force and the charge respectively. The number of electrons is found by dividing the charge by the charge of one electron.

$$F = mg$$

$$= 1.161 \times 10^{-14} \times 9.8$$

$$= 1.138 \times 10^{-13} \text{ N}$$

$$q = \frac{F}{E}$$

$$= \frac{1.138 \times 10^{-13}}{3.55 \times 10^4}$$

$$= 3.206 \times 10^{-18} \text{ C}$$

Number of electrons is found by dividing this value by the charge on one electron

$$= \frac{3.206 \times 10^{-18}}{1.602 \times 10^{-19}}$$

$$= 20 \text{ electrons}$$

- 9 a work done by the field  
 b no work is done  
 c work done on the field  
 d no work is done  
 e work done on the field  
 f work done by the field
- 10 a  $W = qEd$   
 $= 3.204 \times 10^{-19} \times 34 \times 0.01$   
 $= 1.09 \times 10^{-19} \text{ J}$
- b Work is done on the field if the charge is forced to go in a direction it would not naturally go. Alpha particles carry a positive charge. So work is done on the field since a positive charged particle is being moved towards a positive potential.

## Section 2.2

### Worked example: Try yourself 2.2.1

#### USING COULOMB'S LAW TO CALCULATE FORCE

Two small spheres A and B act as point charges separated by 75.0 mm in air. Calculate the force on each point charge if A has a charge of 475 nC and B has a charge of 833 pC. (Use $\epsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ .)	
<b>Thinking</b>	<b>Working</b>
Convert all values to SI units.	$q_A = 475 \times 10^{-9} = 4.75 \times 10^{-7} \text{ C}$ $q_B = 833 \times 10^{-12} = 8.33 \times 10^{-10} \text{ C}$ $r = 7.50 \times 10^{-2} \text{ m}$
State Coulomb's law.	$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$
Substitute the values $q_A$ , $q_B$ , $r$ and $\epsilon_0$ into the equation and calculate the answer.	$F = \frac{1}{4\pi \times 8.8542 \times 10^{-12}} \times \frac{4.75 \times 10^{-7} \times 8.33 \times 10^{-10}}{(7.50 \times 10^{-2})^2}$ $= 6.32 \times 10^{-4} \text{ N}$
Assign a direction based on a negative force being attraction and a positive force being repulsion.	$F = 6.32 \times 10^{-4} \text{ N repulsion}$

**Worked example: Try yourself 2.2.2****USING COULOMB'S LAW TO CALCULATE CHARGE**

Two small point charges are charged by transferring a number of electrons from $q_1$ to $q_2$ , and are separated by 12.7 mm in air. The charge on each point is equal and opposite to the other. Calculate the charge on $q_1$ and $q_2$ if there is an attractive force of 22.5 $\mu\text{N}$ between them. (Use $k = 9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$ .)	
<b>Thinking</b>	<b>Working</b>
Convert all values to SI units.	$F = 22.5 \times 10^{-6} = 2.25 \times 10^{-5} \text{ N}$ $r = 12.7 \times 10^{-3} = 1.27 \times 10^{-2} \text{ m}$
State Coulomb's law.	$F = k \frac{q_1 q_2}{r^2}$
Substitute the values for $F$ , $r$ and $k$ into the equation and calculate the answer. (Remember to indicate which charge is positive and which is negative in your final answer.)	$q_1 q_2 = \frac{Fr^2}{k}$ $= \frac{2.25 \times 10^{-5} \times (1.27 \times 10^{-2})^2}{9.0 \times 10^9}$ $= 4.03 \times 10^{-19}$  Since $q_1 = q_2$ : $q_1^2 = 4.03 \times 10^{-19}$ $q_1 = \sqrt{4.03 \times 10^{-19}}$ $q_1 = +6.35 \times 10^{-10} \text{ C}$ $q_2 = -6.35 \times 10^{-10} \text{ C}$

**Worked example: Try yourself 2.2.3****ELECTRIC FIELD OF A SINGLE POINT CHARGE**

Calculate the magnitude and direction of the electric field at point P at a distance of 15 cm to the right of a positive point charge, $Q$ , of $2.0 \times 10^{-6} \text{ C}$ .	
<b>Thinking</b>	<b>Working</b>
Determine the quantities known and unknown and convert to SI units as required.	$E = ?$ $Q = 2.0 \times 10^{-6} \text{ C}$ $r = 15 \text{ cm} = 0.15 \text{ m}$
Substitute the known values to find the magnitude of $E$ using $E = k \frac{Q}{r^2}$ .	$E = k \frac{Q}{r^2}$ $= 9.0 \times 10^9 \times \frac{2.0 \times 10^{-6}}{0.15^2}$ $= 8.0 \times 10^5 \text{ N C}^{-1}$
The direction of the field is defined as that acting on a positive test charge. Point P is to the right of the charge.	Since the charge is positive the direction will be away from the charge $Q$ or to the right.



## 2.2 review

- 1 When a positive charge is multiplied by a negative charge the force is negative, and a positive charge attracts a negative charge. When a negative charge is multiplied by a negative charge the force is positive, and a negative charge repels a negative charge.

Force	$q_1$ charge	$q_2$ charge	Action
a) positive	positive	positive	repulsion
b) negative	negative	positive	attraction
c) positive	negative	negative	repulsion
d) negative	positive	negative	attraction

- 2 D.  $24.0 \times 10^3 \text{ N C}^{-1}$ . Since the distance has been halved, by the inverse square law, the field will be four times the original.

$$\begin{aligned}
 3 \quad F &= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \\
 &= \frac{1}{4\pi \times 8.8542 \times 10^{-12}} \times \frac{+1.602 \times 10^{-19} \times -1.602 \times 10^{-19}}{(53 \times 10^{-12})^2} \\
 &= -8.22 \times 10^{-8} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad \text{Recall that } E &= k \frac{Q}{r^2} \\
 E &= 9 \times 10^9 \times \frac{3.0 \times 10^{-6}}{(0.05)^2} \\
 &= 1.1 \times 10^7 \text{ N C}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad F &= k \frac{q_1 q_2}{r^2} \\
 &= 9 \times 10^9 \times \frac{1.00 \times 1.00}{1000^2} \\
 &= 9000 \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 6 \quad F &= mg = 0.01 \times 9.8 = 0.098 \text{ N} \\
 &= k \frac{q_1 q_2}{r^2}
 \end{aligned}$$

$$0.098 = 9 \times 10^9 \times \frac{3.45 \times 10^{-9} \times 6.5 \times 10^{-3}}{r^2}$$

$$r^2 = 9 \times 10^9 \times \frac{3.45 \times 10^{-9} \times 6.5 \times 10^{-3}}{0.098}$$

$$= 2.059$$

$$r = \sqrt{2.059}$$

$$= 1.435 \text{ m}$$

- 7 The force is directly proportional to the product of the charges. The force is inversely proportional to the square of the distance between the point charges.

**a** If one of the charges is doubled to  $+2q$ , the force will **double** and **repel**.

**b** If both charges are doubled to  $+2q$ , the force will **quadruple** and **repel**.

**c** If one of the charges is changed to  $-2q$ , the force will **double** and **attract**.

**d** If the distance between the charges is halved to  $0.5r$ , the force will **quadruple** and **repel**.

- 8 There are two protons within the helium nucleus. Recall that a proton has a charge of  $q_p = +1.602 \times 10^{-19} \text{ C}$ . Use Coulomb's law to calculate the force on the protons.

$$\begin{aligned}
 F &= k \frac{q_1 q_2}{r^2} \\
 &= 9 \times 10^9 \times \frac{1.602 \times 10^{-19} \times 1.602 \times 10^{-19}}{(2.5 \times 10^{-15})^2}
 \end{aligned}$$

$$= 36.96 \text{ N}$$

$$= 37 \text{ N}$$

9 Determine the charge on either point using Coulomb's law:

$$F = k \frac{q_1 q_2}{r^2}$$

$$1 = 9 \times 10^9 \times \frac{q^2}{(0.30)^2}$$

$$q = \sqrt{\frac{1 \times 0.30^2}{9 \times 10^9}} = 3.16 \times 10^{-6} \text{ C}$$

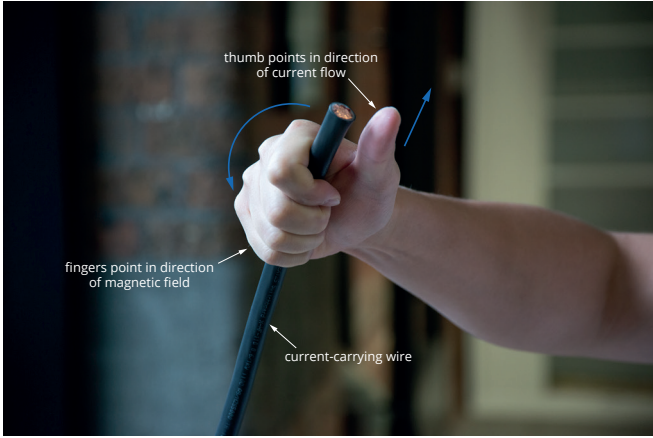
Since each electron has charge  $1.602 \times 10^{-19} \text{ C}$ , the number of electrons is:  $= \frac{3.16 \times 10^{-6}}{1.602 \times 10^{-19}}$

$= 1.97 \times 10^{13}$  electrons

## Section 2.3

### Worked example: Try yourself 2.3.1

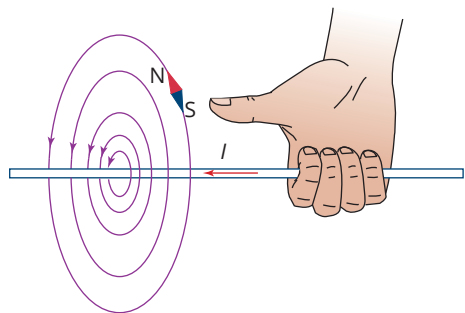
#### DIRECTION OF THE MAGNETIC FIELD

<p>A current-carrying wire runs along the length of a table. The conventional current direction, <math>I</math>, is running toward an observer standing at the near end. What is the direction of the magnetic field created by the current as seen by the observer?</p>	
<p><b>Thinking</b></p> <p>Recall that the right-hand grip rule indicates the direction of the magnetic field.</p>	<p><b>Working</b></p> <p>Point your thumb to the front in the direction of the current flow.</p> <p>Hold your hand with your fingers aligned as if gripping the wire.</p> 
<p>Describe the direction in simple terms of the field in terms of the reference object or wire so that the description can be readily understood by a reasonable reader.</p>	<p>The magnetic field direction is perpendicular to the wire. As the current travels along the wire, the magnetic field runs anticlockwise around the wire.</p>

### 2.3 review

- 1 B. A north pole is always associated with a south pole. The field around the magnet is known as a dipole field. All magnets are dipolar. This means that every magnet always has a north and a south pole.
- 2 A. A magnet suspended freely will behave like a compass. The north end of a magnet is attracted to a south pole and the Earth's geographic north currently coincides with a magnetic south pole.
- 3 C. If you consider the spacing of the magnetic field in the loop as shown by the crosses and dots, it is already non-uniform. Turning the current on and off creates a changing field around the loop but the loop's magnetic field is still non-uniform.
- 4 C. The poles are different so the force will be attractive. With increased distance, the force will decrease.

- 5 The direction of the magnetic field created by the current is perpendicular to the wire and runs up the front of the wire then down the back when looking from the front of the wire.



- 6 The end labelled A is the north pole. Use the right-hand grip rule to find the field in the conductor.
- 7 Based on the directions provided, the magnetic field would be east—away from the north pole of the left-hand magnet.
- 8 Based on the directions provided, the magnetic field would be west—away from the north pole of the right-hand magnet.
- 9 A magnetic field is a vector. If a point is equidistant from two magnets and the directions of the two fields are opposite, then the vector sum would be zero.
- 10 a A = east, B = south, C = west, D = north  
b A = west, B = north, C = east D = south

## Section 2.4

### Worked example: Try yourself 2.4.1

#### MAGNITUDE OF FORCE ON A POSITIVELY CHARGED PARTICLE

A single, positively charged particle with a charge of  $+1.6 \times 10^{-19}$  C travels at a velocity of  $50 \text{ m s}^{-1}$  perpendicular to a magnetic field,  $B$ , of strength  $6.0 \times 10^{-5}$  T.

What is the magnitude of the force the particle will experience from the magnetic field?

Thinking	Working
Check the direction of the velocity and determine whether a force will apply. Forces only apply on the component of the velocity perpendicular to the magnetic field.	Particle is moving perpendicular to the field. A force will apply and so $F = qvB$ .
Establish which quantities are known and which ones are required.	$F = ?$ $q = +1.6 \times 10^{-19} \text{ C}$ $v = 50 \text{ m s}^{-1}$ $B = 6.0 \times 10^{-5} \text{ T}$
Substitute values into the force equation.	$F = qvB$ $= 1.6 \times 10^{-19} \times 50 \times 6.0 \times 10^{-5}$
Express final answer in appropriate form with appropriate significant figures. Note that only magnitude has been requested so do not include direction.	$F = 4.8 \times 10^{-22} \text{ N}$

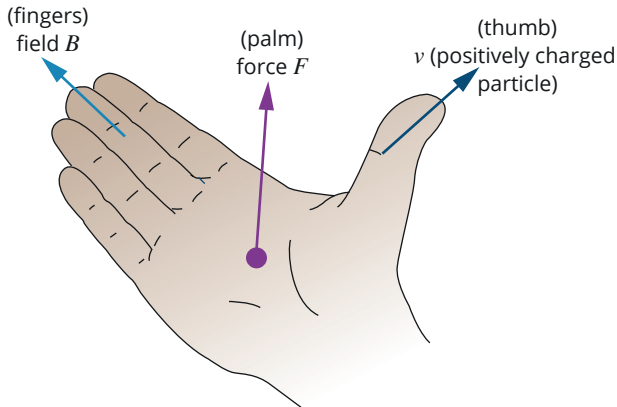
**Worked example: Try yourself 2.4.2**

**DIRECTION OF FORCE ON A NEGATIVELY CHARGED PARTICLE**

A single, negatively charged particle with a charge of  $-1.6 \times 10^{-19}$  C is travelling horizontally from left to right across a computer screen and perpendicular to a magnetic field,  $B$ , that runs vertically down the screen. In what direction will the force experienced by the charge act?

**Thinking**

The right-hand rule is used to determine the direction of the force on a positive charge.



**Working**

Align your hand so that your fingers are pointing downwards in the direction of the magnetic field.

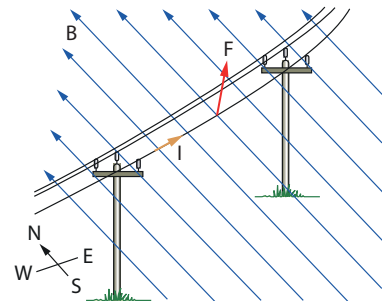
If the negatively charged particle is travelling from left to right, a positively charged particle would be moving in the opposite direction, i.e. from right to left. Align your thumb so it is pointing left, in the direction that a positive charge would travel.

Your palm is facing outwards, which is the direction of the force applied by the magnetic field on a negative charge.

**Worked example: Try yourself 2.4.3**

**MAGNITUDE OF THE FORCE ON A CURRENT-CARRYING WIRE**

Determine the magnitude of the force due to the Earth's magnetic field that acts on a suspended power line running east-west near the equator at the moment it carries a current of 50 A from west to east. Assume that the strength of the Earth's magnetic field at this point is  $5.0 \times 10^{-5}$  T.



**Thinking**

Check the direction of the conductor and determine whether a force will apply.

Forces only apply to the component of the wire perpendicular to the magnetic field.

Establish what quantities are known and what are required. Since the length of the power line hasn't been supplied, consider the force per unit length (i.e. 1 m).

Substitute values into the force equation and simplify.

Express final answer in an appropriate form with a suitable number of significant figures. Note that only magnitude has been requested; so do not include direction.

**Working**

As the current is running east-west and the Earth's magnetic field runs south-north, the current and the field are at right angles and a force will exist.

$$F = ?$$

$$n = 1$$

$$I = 50 \text{ A}$$

$$l = 1.0 \text{ m}$$

$$B = 5.0 \times 10^{-5} \text{ T}$$

$$F = nIlB$$

$$= 1 \times 50 \times 1.0 \times 5.0 \times 10^{-5} \text{ N}$$

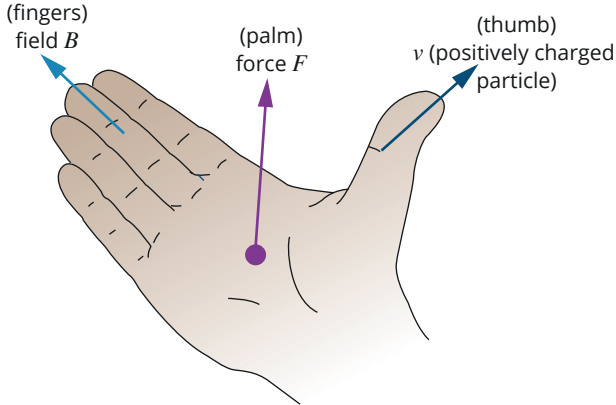
$$= 2.5 \times 10^{-3} \text{ N}$$

$$F = 2.5 \times 10^{-3} \text{ N per metre of power line}$$

**Worked example: Try yourself 2.4.4**

**DIRECTION OF THE FORCE ON A CURRENT-CARRYING WIRE**

A current balance is used to measure the force from a magnetic field on a wire of length 5.0 cm running perpendicular to the magnetic field. The conventional current direction in the wire is from left to right. The magnetic field can be considered to be running out of the page. What is the direction of the force on the wire?

Thinking	Working
<p>The right-hand rule is used to determine the direction of the force.</p> 	<p>Align your hand so that your fingers are pointing in the direction of the magnetic field.</p> <p>Align your thumb so it is pointing to the right in the direction of the current.</p> <p>Your palm is facing downwards.</p>
<p>State the direction in terms of the other directions included in the question. Make the answer as clear as possible to avoid any misunderstanding.</p>	<p>The force on the charge is acting vertically downwards.</p>

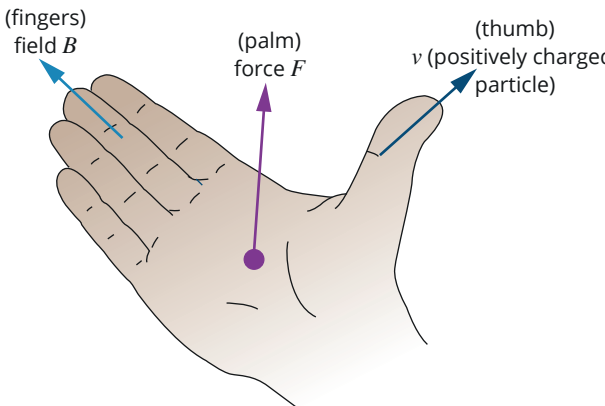
**Worked example: Try yourself 2.4.5**

**FORCE AND DIRECTION ON A CURRENT-CARRYING WIRE**

Santa's house sits at a point that can be considered the Earth's magnetic north pole (which behaves like the south pole of a magnet).

Assuming the strength of the Earth's magnetic field at this point is  $5.0 \times 10^{-5}$  T, calculate the magnetic force and its direction on the following:

(a) A 2.0 m length of wire carrying a conventional current of 10.0 A vertically up the outside wall of Santa's house.	
Thinking	Working
<p>Forces only apply to the components of the wire running perpendicular to the magnetic field.</p> <p>The direction of the magnetic field at the magnetic north pole will be almost vertically downwards.</p>	<p>The section of the wire running up the wall of the building will be parallel to the magnetic field, <math>B</math>. Hence, no force will apply.</p>
<p>State your answer. A numeric value is required. No direction is required with a zero answer.</p>	<p><math>F = 0</math> N</p>

(b) A 2.0 m length of wire carrying a conventional current of 10.0 A running horizontally right to left across the outside of Santa's house.	
<b>Thinking</b>	<b>Working</b>
Forces only apply to the components of the wire running perpendicular to the magnetic field. The direction of the magnetic field at the magnetic north pole will be almost vertically downwards.	The section of the wire horizontally through the building will be perpendicular to the magnetic field, $B$ . Hence, a force will apply.
Identify the known quantities.	$F = ?$ $n = 1$ $I = 10.0 \text{ A}$ $l = 2.0 \text{ m}$ $B = 5.0 \times 10^{-5} \text{ T}$
Substitute into the appropriate equation and simplify.	$F = nIlB$ $= 1 \times 10.0 \times 2.0 \times 5.0 \times 10^{-5}$ $= 1.0 \times 10^{-3} \text{ N}$
<p>The direction of the magnetic force is also required to fully specify the vector quantity. Determine the direction of the magnetic force using the right-hand rule.</p> 	<p>Align your hand so that your fingers are pointing in the direction of the magnetic field, i.e. vertically down.</p> <p>Align your thumb so it is pointing left in the direction of the current.</p> <p>Your palm should be facing outwards (out from the house). That is the direction of the force applied by the magnetic field on the wire.</p>
State the magnetic force in an appropriate form with a suitable number of significant figures and with the direction to fully specify the vector quantity.	$F = 1.0 \times 10^{-3} \text{ N}$ out from Santa's house.

## 2.4 review

- D. Orientating the right hand with the fingers pointing right and the thumb pointing inwards in the direction of the motion of the charge, the palm is pointing vertically down.
- a South (S). The palm of the hand will be pointing downwards, indicating that the force will be south based on the compass directions provided.

b The path followed is therefore C.

c Since  $v$  is constant and energy is a scalar quantity, the kinetic energy remains constant.

d Path A. The palm of the hand will be pointing upwards, indicating that the force will be north based on the compass directions provided. The particle will curve upwards, as the force changes direction with the changing direction of the negative particle.

e Particles with no charge, e.g. neutrons, could follow path B.
- D.

$$F = qvB$$

$$= 1.6 \times 10^{-19} \times 0.5 \times 2 \times 10^{-5}$$

$$= 1.6 \times 10^{-24} \text{ N}$$

- 4 0 N. The particle will experience a force of zero newton because the particle is moving parallel to the magnetic field. That is, there is no component of the motion perpendicular to the field.
- 5 The section of the wire PQ is running parallel to the magnetic field,  $B$ . Hence, no force will apply. You can confirm this using the right-hand rule. Try to align your hand so that your fingers are pointing in the direction of the magnetic field, i.e. to the right, and your thumb is also pointing to the right. It's impossible to do so while still keeping both thumb and fingers outstretched.
- 6  $F = nIB$   
 $= 1 \times 80 \times 100 \times 5.0 \times 10^{-5}$   
 $= 0.4 \text{ N}$   
 Direction: thumb points right (west to east), fingers point into the page (north), palm will face up  
 Therefore the force is 0.4 N upwards.
- 7  $F = qvB$   
 $= 1.6 \times 10^{-19} \times 2 \times 1.5 \times 10^{-5}$   
 $= 4.8 \times 10^{-24} \text{ N south}$
- 8 The force would double when the velocity doubles. The magnitude of the force becomes  $2F$ . The direction of the force is north.
- 9  $F = nIB$   
 $= 1 \times 2 \times 0.05 \times 2.0 \times 10^{-3}$   
 $= 2.0 \times 10^{-4} \text{ N}$   
 Direction is north.
- 10 a  $F = nIB$   
 $= 1 \times 50 \times 80 \times 4.5 \times 10^{-5}$   
 $= 0.18 \text{ N downwards}$   
 b Same as (a). The change in height has no effect on the perpendicular components of the magnetic field (south–north) and the wire's direction.

## Section 2.5

### 2.5 review

- 1 C. The direction of a field line at any point is the resultant field vector. At either end the field outside the plates is less. The horizontal component of the resultant field vector would point outwards.
- 2 B. In a static field, the strength of the field doesn't change with time. This is true of most gravitational and magnetic fields since mass of the object or the strength of the magnet is unchanging.
- 3  $F_g = G \frac{m_1 m_2}{r^2}$   
 $8 \times 10^{-37} = 6.67 \times 10^{-11} \times \frac{9.1 \times 10^{-31} \times 9.1 \times 10^{-31}}{(r)^2}$   
 $r = \sqrt{6.67 \times 10^{-11} \times \frac{9.1 \times 10^{-31} \times 9.1 \times 10^{-31}}{8 \times 10^{-37}}}$   
 $= 8.3 \times 10^{-18} \text{ m}$
- 4 a monopoles  
 b monopoles and dipoles  
 c dipoles
- 5 The field around a monopole is **radial**, **static** and **non-uniform**.  
 A monopole is a single point source associated with electrical and gravitational fields. The inverse square law applies to the radial fields around monopoles.
- 6 The charge on the right is negative. As the field lines run from one charge to the other, the force is one of attraction. Therefore charge must have the opposite sign.
- 7 D. Magnetic fields exist only as dipoles. The inverse square law is associated with the radial field around essentially point source monopoles.

8 The direction of a field at any point is defined as the **resultant** field vector determined by adding the **individual** field vectors due to each mass, charge or magnetic pole within the field.

$$\begin{aligned}
 9 \quad F &= k \frac{q_1 q_2}{r^2} \\
 &= 9 \times 10^9 \times \frac{+1.6 \times 10^{-19} \times -1.6 \times 10^{-19}}{(0.53 \times 10^{-10})^2} \\
 &= 8.2 \times 10^{-8} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 10 \quad F_g &= G \frac{m_1 m_2}{r^2} \\
 &= 6.67 \times 10^{-11} \times \frac{9.1 \times 10^{-31} \times 1.67 \times 10^{-27}}{(0.53 \times 10^{-10})^2} \\
 &= 3.6 \times 10^{-47} \text{ N}
 \end{aligned}$$

## CHAPTER 2 REVIEW

$$\begin{aligned}
 1 \quad F &= qE = 3.00 \times 10^{-3} \times 7.5 \\
 &= 0.003 \times 7.5 \\
 &= 0.0225 \text{ N}
 \end{aligned}$$

2 D

$$\begin{aligned}
 E &= k \frac{Q}{r^2} \\
 &= 9 \times 10^9 \times \frac{30 \times 10^{-6}}{(0.30)^2} \\
 &= 3 \times 10^6 \text{ N C}^{-1} \text{ upwards.}
 \end{aligned}$$

Since the electric field is defined as that acting on a positive test charge, the field direction would be upwards or away from the charge,  $Q$ .

3 The electrical potential is defined as the work done per unit charge to move a charge from infinity to a point in the electric field. The electrical potential at infinity is defined as zero. When you have two points in an electric field ( $E$ ) separated by a distance ( $d$ ) that is parallel to the field, the potential difference  $V$  is then defined as the change in the electrical potential between these two points.

$$4 \quad E = \frac{V}{d}$$

$$1000 = \frac{V}{0.025}$$

$$V = 1000 \times 0.025 = 25 \text{ V}$$

5 C. For a uniform electric field, the electric field strength is the same at all points between the plates.

6 When a positively charged particle moves across a potential difference from a positive plate towards an earthed plate, work is done by the **field** on the **charged particle**.

7 Use the work done in a uniform electric field,  $W = qEd$ , equation to determine the work done on the field.

$$\begin{aligned}
 W &= qEd \\
 &= 2.5 \times 10^{-18} \times 556 \times 3.0 \times 10^{-3} \\
 &= 4.17 \times 10^{-18} \text{ J}
 \end{aligned}$$

$$8 \quad E = \frac{V}{d} = \frac{15 \times 10^3}{0.12} = 125\,000 \text{ V m}^{-1}$$

$$\begin{aligned}
 F &= qE \\
 &= 1.6 \times 10^{-19} \times 125\,000 = 2 \times 10^{-14} \text{ N}
 \end{aligned}$$

9 a The distance between the charges is doubled to  $2r$ , so the force will **quarter** and **repel**.

b The distance between the charges is halved to  $0.5r$ , so the force will **quadruple** and **repel**.

c The distance between the charges is doubled and one of the charges is changed to  $-2q$ , so the force will **halve** and **attract**.



- 10** Recall that kinetic energy gained by the ion ( $E_k$ ) is equal to work done ( $W$ ). Therefore, the velocity can be calculated using the equation  $E_k = \frac{1}{2}mv^2$  when the kinetic energy is known.

$E_k$  can be calculated in two steps by using the work done on a charge in a uniform electric field equation,

$$W = qEd, \text{ and the equation to determine the electric field, } E = \frac{V}{d}.$$

$$E = \frac{V}{d} = \frac{1000}{0.020} = 50\,000 \text{ V m}^{-1}$$

$$W = qEd = 3 \times 1.602 \times 10^{-19} \times 50\,000 \times 0.020 = 4.806 \times 10^{-16} \text{ J}$$

$$E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 4.806 \times 10^{-16}}{3.27 \times 10^{-25}}}$$

$$= 5.42 \times 10^4 \text{ m s}^{-1}$$

**11**  $F = k \frac{q_1 q_2}{r^2}$

$$= 9 \times 10^9 \times \frac{5.00 \times 10^{-3} \times 4.00 \times 10^{-9}}{(2.00)^2}$$

$$= 0.045 \text{ N}$$

**12**  $F = mg$

$$F = k \frac{q_1 q_2}{r^2}$$

$$mg = k \frac{q_1 q_2}{r^2}$$

$$r^2 = k \frac{q_1 q_2}{mg}$$

$$= 9 \times 10^9 \times \frac{2.25 \times 10^{-3} \times 3.05 \times 10^{-3}}{3 \times 9.8}$$

$$= 2100$$

$$= \sqrt{2100}$$

$$r = 45.8 \text{ m}$$

- 13** Find the weight force of the ball using  $F = mg$ . Then substitute this value into the equation  $F = Eq$  to calculate the charge.

$$F = mg = 5.00 \times 10^{-3} \times 9.8 = 4.9 \times 10^{-2} \text{ N}$$

$$= qE$$

$$q = \frac{F}{E} = \frac{4.9 \times 10^{-2}}{300.0}$$

$$= +1.63 \times 10^{-4} \text{ C}$$

The charge must be positive to provide an upwards force in the vertically upwards field.

- 14** With the current turned off the loop is producing no field. The steady field in the region would be the only contributing field. It has a value of  $B$  into the page.

- 15** With the current doubled, the loop is producing double the field,  $2B$ . The steady field in the region would be contributing  $B$ . The total is  $3B$  into the page.

- 16** The field from the loop would exactly match that of the field in the region but in the opposite direction. The vector total would be zero.

- 17** D. The magnitude of the magnetic force on a conductor aligned so that the current is running parallel to a magnetic field is zero.

A component of the conductor's length must be perpendicular to a magnetic field for a force to be created.

- 18** **a** palm  
**b** fingers  
**c** thumb

- 19** For the electron beams to behave as shown in (a),  $v_1$  is **equal to**  $v_2$  and the region of the magnetic field,  $B_y$ , must be acting **into** the page.

20  $F = nIB$

$$0.800 = 1 \times I \times 3.2 \times 0.0900$$

$$I = \frac{0.800}{0.0900 \times 3.20} = 2.78 \text{ A}$$

21 In each case the force is found from  $F = nIB$  as the field is perpendicular to the current.

a  $F = 1 \times 1 \times 10^{-3} \times 5 \times 10^{-3} \times 1 \times 10^{-3}$

$$= 5.0 \times 10^{-9} \text{ N into the page (from the right-hand rule)}$$

b  $F = 1 \times 2 \times 1 \times 10^{-2} \times 0.1 = 2.0 \times 10^{-3} \text{ N into the page}$

22 The magnetic force exerted on the electron is:

$$F = qvB$$

$$= 1.6 \times 10^{-19} \times 7.0 \times 10^6 \times 8.6 \times 10^{-3}$$

$$= 9.6 \times 10^{-15} \text{ N}$$

23 Using the right-hand rule: fingers pointing to the right in the direction of the magnetic field; thumb pointing away in the direction of conventional current. The palm is pointing down indicating the direction of the force is downwards.

24 The east-west line would experience the greater magnetic force as it runs perpendicular to the Earth's magnetic field.

25 C. Magnetic fields are associated only with dipoles. Only monopoles generate radial fields.

26  $F = k \frac{q_1 q_2}{r^2}$

$$= 9 \times 10^9 \times \frac{-1.6 \times 10^{-19} \times -1.6 \times 10^{-19}}{(5.4 \times 10^{-12})^2}$$

$$= 7.9 \times 10^{-6} \text{ N}$$

27  $F_g = G \frac{m_1 m_2}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{9.1 \times 10^{-31} \times 9.1 \times 10^{-31}}{(5.4 \times 10^{-12})^2}$$

$$= 1.9 \times 10^{-48} \text{ N}$$

# Chapter 3 answers

## Section 3.1

### Worked example: Try yourself 3.1.1

#### CALCULATING APPARENT WEIGHT

A 79.0 kg student rides a lift down from the top floor of an office block to the ground. During the journey the lift accelerates downwards at  $2.35 \text{ m s}^{-2}$ , before travelling at a constant velocity of  $4.08 \text{ m s}^{-1}$  and then finally decelerating at  $4.70 \text{ m s}^{-2}$ .

(a) Calculate the apparent weight of the student in the first part of the journey while accelerating downwards at $2.35 \text{ m s}^{-2}$ .	
<b>Thinking</b>	<b>Working</b>
Ensure that the variables are in their standard units.	$m = 79.0 \text{ kg}$ $a = 2.35 \text{ m s}^{-2} \text{ down}$ $g = 9.80 \text{ m s}^{-2} \text{ down}$
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 79.0 \text{ kg}$ $a = -2.35 \text{ m s}^{-2}$ $g = -9.80 \text{ m s}^{-2}$
Apply the equation for apparent weight (the normal force).	$F_{\text{net}} = F_{\text{N}} + F_{\text{g}}$ $F_{\text{N}} = F_{\text{net}} - F_{\text{g}}$ $= ma - mg$ $= (79.0 \times -2.35) - (79.0 \times -9.80)$ $= -185.65 + 774.2$ $= 589 \text{ N}$
(b) Calculate the apparent weight of the student in the second part of the journey while travelling at a constant speed of $4.08 \text{ m s}^{-1}$ .	
<b>Thinking</b>	<b>Working</b>
Ensure that the variables are in their standard units.	$m = 79.0 \text{ kg}$ $a = 0 \text{ m s}^{-2} \text{ down}$ $g = 9.80 \text{ m s}^{-2} \text{ down}$
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 79.0 \text{ kg}$ $a = 0 \text{ m s}^{-2}$ $g = -9.80 \text{ m s}^{-2}$
Apply the equation for apparent weight (the normal force).	$F_{\text{net}} = F_{\text{N}} + F_{\text{g}}$ $F_{\text{N}} = F_{\text{net}} - F_{\text{g}}$ $= ma - mg$ $= (79.0 \times 0) - (79.0 \times -9.80)$ $= 0 + 774.2$ $= 774 \text{ N}$

(c) Calculate the apparent weight of the student in the last part of the journey while travelling downwards and decelerating at $4.70 \text{ m s}^{-2}$ .	
<b>Thinking</b>	<b>Working</b>
Ensure that the variables are in their standard units. Also consider that deceleration is a negative acceleration.	$m = 79.0 \text{ kg}$ $a = -4.70 \text{ m s}^{-2}$ down $g = 9.80 \text{ m s}^{-2}$ down
Apply the sign and direction convention for motion in one dimension. Up is positive and down is negative.	$m = 79.0 \text{ kg}$ $a = 4.70 \text{ m s}^{-2}$ $g = -9.80 \text{ m s}^{-2}$
Apply the equation for apparent weight (the normal force).	$F_{\text{net}} = F_{\text{N}} + F_{\text{g}}$ $F_{\text{N}} = F_{\text{net}} - F_{\text{g}}$ $= ma - mg$ $= (79.0 \times 4.70) - (79.0 \times -9.80)$ $= 371.3 + 774.2$ $= 1150 \text{ N}$

**Worked example: Try yourself 3.1.2****WORKING WITH KEPLER'S LAWS**

Determine the orbital speed of a satellite, assuming it is in a circular orbit of radius of 42 100 km around the Earth. Take the mass of the Earth to be $5.97 \times 10^{24} \text{ kg}$ and use $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ .	
<b>Thinking</b>	<b>Working</b>
Ensure that the variables are in their standard units.	$r = 42\,100 \text{ km} = 4.21 \times 10^7 \text{ m}$
Choose the appropriate relationship between the orbital speed, $v$ , and the data that has been provided.	$a = g = \frac{GM}{r^2} = \frac{v^2}{r}$
Make $v$ , the orbital speed, the subject of the equation.	$v = \sqrt{\frac{GM}{r}}$
Substitute in values and solve for the orbital speed, $v$ .	$v = \sqrt{\frac{GM}{r}}$ $= \sqrt{\frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{4.21 \times 10^7}}$ $= 3.08 \times 10^3 \text{ m s}^{-1}$

**Worked example: Try yourself 3.1.3****SATELLITES IN ORBIT**

Callisto is the second largest of Jupiter's moons. It is about the same size as the planet Mercury. Callisto has a mass of  $1.08 \times 10^{23}$  kg, an orbital radius of  $1.88 \times 10^6$  km and an orbital period of  $1.44 \times 10^6$  s (16.7 days).

(a) Use Kepler's third law to calculate the orbital radius (in km) of Europa, another moon of Jupiter, which has an orbital period of 3.55 days.	
<b>Thinking</b>	<b>Working</b>
Note down the values for the known satellite. You can work in days and km.	Callisto: $r = 1.88 \times 10^6$ km $T = 16.7$ days
$\frac{r^3}{T^2} = \text{constant}$ for all satellites of a central mass. Work out this ratio for the known satellite.	Europa: $\frac{r^3}{T^2} = \text{constant}$ $= \frac{(1.88 \times 10^6)^3}{16.7^2}$ $= 2.38 \times 10^{16}$
Use this constant value with the ratio for the satellite in question.	$\frac{r^3}{T^2} = \text{constant}$ $\frac{r^3}{3.55^2} = 2.38 \times 10^{16}$
Make $r^3$ the subject of the equation.	$r^3 = 3.55^2 \times 2.38 \times 10^{16}$ $= 3.00 \times 10^{17}$
Solve for $r$ .	$r = \sqrt[3]{3.00 \times 10^{17}}$ $= 6.70 \times 10^5$ km Europa has a shorter period than Callisto so you should expect Europa to have a smaller orbit than Callisto.

(b) Use the orbital data for Callisto to calculate the mass of Jupiter.	
<b>Thinking</b>	<b>Working</b>
Note down the values for the known satellite. You must work in SI units.	Callisto/Jupiter: $r = 1.88 \times 10^9$ m $T = 1.44 \times 10^6$ s $m = 1.66 \times 10^{23}$ kg $G = 6.67 \times 10^{-11}$ N m <sup>2</sup> kg <sup>-2</sup> $M = ?$
Select the expressions from the equation for centripetal acceleration that best suit your data. $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} = g$	$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$ These two expressions use the given variables $r$ and $T$ , and the constant $G$ , so that a solution may be found for $M$ .
Transpose to make $M$ the subject.	$M = \frac{4\pi^2 r^3}{GT^2}$
Substitute values and solve.	$M = \frac{4\pi^2 (1.88 \times 10^9)^3}{6.67 \times 10^{-11} \times (1.44 \times 10^6)^2}$ $= 1.90 \times 10^{27}$ kg

(c) Calculate the orbital speed of Callisto in $\text{km s}^{-1}$ .	
<b>Thinking</b>	<b>Working</b>
Note values you will need to use in the equation $v = \frac{2\pi r}{T}$ .	Callisto: $r = 1.88 \times 10^6 \text{ km}$ $T = 1.44 \times 10^6 \text{ s}$ $v = ?$
Substitute values and solve. The answer will be in $\text{km s}^{-1}$ if $r$ is expressed in km.	$v = \frac{2\pi r}{T}$ $= \frac{2\pi \times 1.88 \times 10^6}{1.44 \times 10^6}$ $= 8.20 \text{ km s}^{-1}$

### 3.1 review

- $F_g = mg$   
 $= 6.50 \times 9.80$   
 $= 63.7 \text{ N}$
- Normal force = weight force for an object at rest  
 $F_N = 150 \text{ N}$
- $F_{\text{net}} = F_N + F_g$   
 $F_N = F_{\text{net}} - F_g$   
 $= ma - mg$   
 $= (45.0 \times 2.02) - (45.0 \times -9.80)$   
 $= 90.9 + 441$   
 $= 532 \text{ N}$
- $F_{\text{net}} = F_N + F_g$   
 $F_N = F_{\text{net}} - F_g$   
 $= ma - mg$   
 $= (45.0 \times 0) - (45.0 \times -9.80)$   
 $= 0 + 441$   
 $= 441 \text{ N}$
- B. It is the only object that is on a surface and so it can experience an upwards normal force due to that surface.
- C. Satellites orbit around a central mass. The Earth does not orbit Mars. The Moon does not orbit the Sun and the Sun does not orbit the Earth.
- B. In order to be geostationary, the satellite must be in a high orbit.
- D. Increasing the mass of the satellite will not affect its orbital properties.
- $a = g = 0.22 \text{ m s}^{-2}$
  - $F_g = mg$   
 $= 2.3 \times 10^3 \times 0.22$   
 $= 506 \text{ N (or } 510 \text{ N to 2 significant figures)}$

10  $\frac{r^3}{T^2} = \text{constant}$  for satellites of Saturn, therefore the orbital period for each moon can be calculated.

For Atlas:

$$\frac{r^3}{T^2} = \frac{(1.37 \times 10^5)^3}{(0.60)^2}$$

$$= 7.14 \times 10^{15}$$

For Titan:

$$\frac{r^3}{T^2} = 7.14 \times 10^{15}$$

$$T^2 = \frac{r^3}{7.14 \times 10^{15}}$$

$$= \frac{(1.20 \times 10^6)^3}{7.14 \times 10^{15}}$$

$$= 242$$

$$T = \sqrt{242}$$

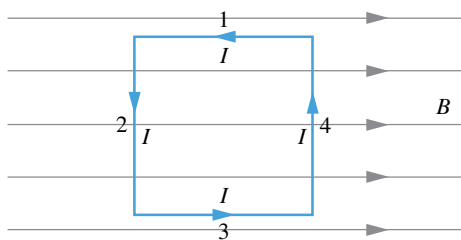
$$= 15.6 \text{ days}$$

## Section 3.2

### Worked example: Try yourself 3.2.1

#### TORQUE ON A COIL

A single square wire coil, with a side length of 4.0 cm, is free to rotate within a magnetic field,  $B$ , of strength  $1.0 \times 10^{-4}$  T. A current of 1.0 A is flowing through the coil. What is the torque on the coil?



Thinking	Working
Confirm that the coil will experience a force based on the magnetic field and current directions supplied.	Using the right-hand rule, confirm that a force applies on side 2 out of the page. A force applies to side 4 into the page. The coil will turn clockwise as viewed from in front of side 3. Sides 1 and 3 lie parallel to the magnetic field and no force will apply.
Calculate the magnetic force on one side.	$F = IB$ $= 1.0 \times 0.04 \times 1.0 \times 10^{-4}$ $= 4.0 \times 10^{-6} \text{ N}$
Determine the distance, $r$ , from the point of rotation that the magnetic force is applied.	length of side = 4.0 cm distance between axis of rotation and application of force $= \frac{1}{2} \times \text{side length}$ $r = 2.0 \text{ cm} = 0.020 \text{ m}$
Calculate the torque applied by the magnetic force on one side of the coil.	$\tau = r_{\perp}F$ $= 0.020 \times 4.0 \times 10^{-6}$ $= 8.0 \times 10^{-8} \text{ N m}$
Since two sides, 2 and 4, experience a magnetic force and hence a torque, the torque on one side should be multiplied by 2 to find the total torque. State also the direction of rotation.	Total torque $= 2 \times 8.0 \times 10^{-8}$ $= 1.6 \times 10^{-7} \text{ N m}$ The direction is clockwise as viewed from side 3.

### 3.2 review

- 1 A. Recall the equation  $\tau = r_{\perp}F$ . The maximum torque exists when the force is applied at right angles to the axis of rotation.
- 2  $F = nIB$   
 $= 1 \times 2.0 \times 0.05 \times 0.10$   
 $= 1.0 \times 10^{-2}$  N into the page
- 3  $F = nIB$   
 $= 1 \times 2.0 \times 0.05 \times 0.10$   
 $= 1.0 \times 10^{-2}$  N out of the page
- 4 The force will be 0 N.  
 Side PQ is parallel to the magnetic field.
- 5 Considering the direction of the forces acting on sides PS and QR, the coil would rotate in an anticlockwise direction.
- 6 D. The direction of the current does not affect the magnitude of the torque. This is the only option that doesn't affect either the distance to the axis of rotation or the magnetic force from the options available.
- 7  $\tau = r_{\perp}F \times 2$  sides  
 $= 2r_{\perp}F$   
 $= 2 \times \frac{0.02}{2} \times 1.0 \times 10^{-2}$   
 $= 2.0 \times 10^{-4}$  N m
- 8  $F = nIB$   
 $= 1 \times 1.0 \times 0.50 \times 0.20$   
 $= 0.1$  N
- 9 Current flows into brush P and around the coil from V to X to Y to W. So force on side VX is down, force on side YW is up, so rotation is anticlockwise.
- 10 D. As  $F = nIB$ , the coil will experience more force, and rotate faster, if the current and field strength are increased. Therefore, A and B are correct. Whether C is correct will depend on how the area is increased. But if the length in the field is increased, you would expect it to turn faster. If widened, it will experience more torque but that may not make it turn faster.

## Section 3.3

### Worked example: Try yourself 3.3.1

#### CALCULATING THE SPEED OF ACCELERATED CHARGED PARTICLES

Determine the final speed of a single electron, with a charge of magnitude $1.6 \times 10^{-19}$ C and a mass of $9.1 \times 10^{-31}$ kg when accelerating across a potential difference of 1.2 kV.	
<b>Thinking</b>	<b>Working</b>
Ensure that the variables are in their standard units.	$1.2 \text{ kV} = 1.2 \times 10^3 \text{ V}$
Establish what quantities are known and what are required.	$v = ?$ $q = 1.6 \times 10^{-19} \text{ C}$ $m = 9.1 \times 10^{-31} \text{ kg}$ $V = 1.2 \times 10^3 \text{ V}$
Substitute values into the equation and re-arrange to solve for the speed.	$qV = \frac{1}{2}mv^2$ $1.6 \times 10^{-19} \times 1.2 \times 10^3 = \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2$ $v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 1.2 \times 10^3}{9.1 \times 10^{-31}}}$ $= 2.1 \times 10^7 \text{ m s}^{-1}$



**Worked example: Try yourself 3.3.2****CALCULATING SPEED AND PATH RADIUS OF ACCELERATED CHARGED PARTICLES**

An electron gun releases electrons from its cathode, which are accelerated across a potential difference of 25 kV, over a distance of 20 cm between a pair of charged parallel plates. Assume that the mass of an electron is  $9.1 \times 10^{-31}$  kg and the magnitude of the charge on an electron is  $1.6 \times 10^{-19}$  C.

(a) Calculate the strength of the electric field acting on the electron beam.	
<b>Thinking</b>	<b>Working</b>
Ensure that the variables are in their standard units.	$25 \text{ kV} = 25 \times 10^3 = 2.5 \times 10^4 \text{ V}$ $20 \text{ cm} = 0.20 \text{ m}$
Apply the correct equation.	$E = \frac{V}{d}$
Solve for $E$ .	$E = \frac{2.5 \times 10^4}{0.20}$ $= 1.3 \times 10^5 \text{ V m}^{-1}$
(b) Calculate the speed of the electrons as they exit the electron gun assembly.	
<b>Thinking</b>	<b>Working</b>
Apply the correct equation.	$\frac{1}{2} mv^2 = qV$
Rearrange the equation to make $v$ the subject.	$v = \sqrt{\frac{2qV}{m}}$
Solve for $v$ .	$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 2.5 \times 10^4}{9.1 \times 10^{-31}}}$ $= 9.4 \times 10^7 \text{ m s}^{-1}$
(c) The electrons then travel through a uniform magnetic field perpendicular to their motion. Given that this field is of strength 0.3 T, calculate the expected radius of the path of the electron beam.	
<b>Thinking</b>	<b>Working</b>
Apply the correct equation.	$r = \frac{mv}{qB}$
Solve for $r$ .	$r = \frac{9.1 \times 10^{-31} \times 9.4 \times 10^7}{1.6 \times 10^{-19} \times 0.3}$ $= 1.8 \times 10^{-3} \text{ m}$

**3.3 review**

1 B. A charged particle moving in a magnetic field will experience a force.

$$\begin{aligned}
 2 \quad F &= qvB \\
 &= 1.6 \times 10^{-19} \times 1.0 \times 1.5 \times 10^{-5} \\
 &= 2.4 \times 10^{-24}
 \end{aligned}$$

The electron will experience a force of  $2.4 \times 10^{-24}$  N south.

$$\begin{aligned}
 3 \quad v &= \sqrt{\frac{2qV}{m}} \\
 &= \sqrt{\frac{2 \left( (1.6 \times 10^{-19}) \times (2.5 \times 10^3) \right)}{9.1 \times 10^{-31}}} \\
 &= 3.0 \times 10^7 \text{ m s}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad a \quad F &= qvB \\
 &= 1.6 \times 10^{-19} \times 7.0 \times 10^6 \times 8.6 \times 10^{-3} \\
 &= 9.6 \times 10^{-15} \text{ N}
 \end{aligned}$$

$$\begin{aligned}
 b \quad r &= \frac{mv}{qB} \\
 &= \frac{9.1 \times 10^{-31} \times 7.0 \times 10^6}{1.6 \times 10^{-19} \times 8.6 \times 10^{-3}} \\
 &= 4.6 \times 10^{-3} \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad r &= \frac{mv}{qB} \\
 B &= \frac{mv}{qr} \\
 \text{Radius} &= \frac{1}{2} \text{ diameter} = \frac{1}{2} \times 9.2 \times 10^{-2} = 4.6 \times 10^{-2} \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 B &= \frac{9.1 \times 10^{-31} \times 7.6 \times 10^6}{1.6 \times 10^{-19} \times 4.6 \times 10^{-2}} \\
 &= 9.4 \times 10^{-4} \text{ T}
 \end{aligned}$$

$$\begin{aligned}
 6 \quad r &= \frac{mv}{qB} \\
 v &= \frac{rqB}{m} \\
 &= 0.06 \times (1.76 \times 10^{11}) \times (1.5 \times 10^{-4}) \\
 &= 1.6 \times 10^6 \text{ m s}^{-1}
 \end{aligned}$$

- 7 A charged particle in a magnetic field will experience a force ( $F = qvB$ ). As force  $\propto$  velocity, the force will increase as the velocity increases. This will continue while the charge remains in the magnetic field, continuously accelerating the charge.

$$\begin{aligned}
 8 \quad E_k &= \frac{1}{2} mv^2 = qV \\
 v &= \sqrt{\frac{2qV}{m}} \\
 &= \sqrt{\frac{2 \times (1.6 \times 10^{-19}) \times (3.0 \times 10^3)}{9.1 \times 10^{-31}}} \\
 &= 3.25 \times 10^7 \text{ m s}^{-1}
 \end{aligned}$$

Because the forces acting on the electron due to the electric and magnetic fields are balanced, you can equate them.

$$F_E = qE \text{ and } F_B = qvB$$

$$qE = qvB$$

$$E = vB$$

$$= (3.25 \times 10^7) \times (1.6 \times 10^{-3})$$

$$= 5.20 \times 10^4 \text{ J}$$

$$= \frac{V}{d} \text{ so the plate separation, } d = \frac{V}{E}$$

$$d = \frac{3000}{5.20 \times 10^4}$$

$$= 5.8 \times 10^{-2} \text{ m or } 5.8 \text{ cm}$$

## CHAPTER 3 REVIEW

- 1  $F_{\text{net}} = F_{\text{N}} + F_{\text{g}}$   
 $F_{\text{N}} = F_{\text{net}} - F_{\text{g}}$   
 $= ma - mg$   
 $= (45.0 \times -3.15) - (45.0 \times -9.80)$   
 $= -141.75 + 441$   
 $= 299 \text{ N}$
- 2 D. Objects in orbit are in free fall. While still in orbit around the Earth, gravity is reduced, but is still significant in magnitude.
- 3 D. At this altitude, gravity is reduced and so will be less than  $9.8 \text{ N kg}^{-1}$ , hence, acceleration is less than  $9.8 \text{ m s}^{-2}$ . Note: B is not correct, because while the speed of the satellite would be constant, its velocity is not.
- 4 A. Apparent weightlessness is felt during free fall, when  $F_{\text{N}}$  is zero.
- 5  $\frac{r^3}{T^2} = \text{constant}$  for satellites of Earth, therefore the orbital period for each satellite can be calculated.

For X:

$$\frac{r^3}{T^2} = \text{constant} = k$$

For Y:

$$\frac{(5r)^3}{T_Y^2} = k$$

$$\frac{(5r)^3}{T_Y^2} = \frac{r^3}{T^2}$$

$$\frac{125r^3}{T_Y^2} = \frac{r^3}{T^2}$$

$$T_Y^2 = \frac{125r^3}{r^3} T^2$$

$$= 125T^2$$

$$T_Y = 11.2T$$

- 6 a  $a = \frac{GM}{r^2}$   
 $= \frac{6.67 \times 10^{-11} \times 1.02 \times 10^{26}}{(3.55 \times 10^8)^2}$   
 $= 0.0540 \text{ m s}^{-2}$
- b  $a = \frac{v^2}{r}$   
 $v = \sqrt{ar} = \sqrt{0.054 \times 3.55 \times 10^8}$   
 $= 4.38 \times 10^3 \text{ m s}^{-1}$
- c  $F_{\text{g}} = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2} = \frac{GMm}{r^2} = mg$   
 $\frac{4\pi^2 rm}{T^2} = \frac{GMm}{r^2}$   
 $T^2 = \frac{4\pi^2 r^3}{GM}$   
 $= \frac{4\pi^2 (3.55 \times 10^8)^3}{6.67 \times 10^{-11} \times 1.02 \times 10^{26}}$   
 $= 2.60 \times 10^{11}$   
 $T = \sqrt{2.60 \times 10^{11}} = 5.09 \times 10^5 \text{ s}$   
 $1 \text{ day} = 24 \times 60 \times 60 = 86\,400 \text{ s}$   
 $T = \frac{5.09 \times 10^5}{86\,400}$   
 $= 5.89 \text{ days}$

$$7 \quad \mathbf{a} \quad a = \frac{GM}{r^2} = g$$

$$g = \frac{GM}{r^2} \\ = \frac{6.67 \times 10^{-11} \times 7.0 \times 10^{20}}{(3.85 \times 10^5)^2} \\ = 0.315 \text{ N kg}^{-1}$$

$$\mathbf{b} \quad \frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v = \sqrt{\frac{GM}{r}} \\ = \sqrt{\frac{6.67 \times 10^{-11} \times 7.0 \times 10^{20}}{3.95 \times 10^5}} \\ = 344 \text{ m s}^{-1}$$

8 a down the page

b up the page

9 anticlockwise

10 a down the page

b up the page

c Zero torque acts as the forces are trying to pull the coil apart rather than turn it. The force is parallel to the coil, rather than perpendicular to it.

11 C. Reversing the direction of the current in the loop will ensure that the loop keeps travelling in the same direction. Use the right-hand rule to verify this.

12 The commutator's function is to reverse the current direction in the coil every half turn to keep the coil rotating in the same direction.

13 Electrons are released from a negative terminal or hot cathode of the evacuated tube and accelerate towards a positively charged anode. They can be detected as they hit a fluorescent screen at the rear of the tube. The electrons are accelerated by a high potential difference between the cathode and positively charged anode.

$$14 \quad v = \sqrt{\frac{2qV}{m}} \\ = \sqrt{\frac{2 \times (1.6 \times 10^{-19}) \times (10 \times 10^3)}{9.1 \times 10^{-31}}} \\ = 5.9 \times 10^7 \text{ m s}^{-1}$$

$$15 \quad r = \frac{mv}{qB} \\ = \frac{9.1 \times 10^{-31} \times 5.9 \times 10^7}{1.6 \times 10^{-19} \times 1.5} \\ = 2.2 \times 10^{-4} \text{ m}$$

16 a The electron will experience a force at right angles to its motion. This acts upwards in the initial moment and causes the electron to curve in an upwards arc from its starting position.

b The radius of the electron path is dependent upon its velocity and the magnitude of the magnetic field that is acting.

17 a The strength of the electric field between the charged plates is given by:

$$E = \frac{V}{d} \\ = \frac{500}{3.5 \times 10^{-2}} \\ = 1.4 \times 10^4 \text{ V m}^{-1}$$

b Because the strength of the electric and magnetic fields is balanced, you can say that:

$$F_B = F_E \\ qvB = qE \\ v = \frac{E}{B} \\ = \frac{1.4 \times 10^4}{1.5 \times 10^{-3}} \\ = 9.3 \times 10^6 \text{ m s}^{-1}$$

$$\begin{aligned} 18 \quad v &= \sqrt{\frac{2qV}{m}} \\ &= \sqrt{\frac{2 \left( (1.6 \times 10^{-19}) \times (4.5 \times 10^3) \right)}{9.1 \times 10^{-31}}} \\ &= 4.0 \times 10^7 \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} 19 \quad r &= \frac{mv}{qB} \\ B &= \frac{mv}{qr} \\ \text{radius} &= \frac{1}{2} \times \text{diameter} \\ &= \frac{1}{2} \times 8.4 \times 10^{-2} \\ &= 4.2 \times 10^{-2} \text{ m} \\ B &= \frac{9.1 \times 10^{-31} \times 4.3 \times 10^6}{1.6 \times 10^{-19} \times 4.2 \times 10^{-2}} \\ &= 5.8 \times 10^{-4} \text{ T} \end{aligned}$$

$$\begin{aligned} 20 \text{ a} \quad F &= qvB \\ &= 1.6 \times 10^{-19} \times 6.4 \times 10^6 \times 9.1 \times 10^{-3} \\ &= 9.3 \times 10^{-15} \text{ N} \end{aligned}$$

$$\begin{aligned} \text{b} \quad r &= \frac{mv}{qB} \\ &= \frac{9.1 \times 10^{-31} \times 6.4 \times 10^6}{1.6 \times 10^{-19} \times 9.1 \times 10^{-3}} \\ &= 4.0 \times 10^{-3} \text{ m} \end{aligned}$$

## Chapter 4 answers

### Section 4.1

#### Worked example: Try yourself 4.1.1

##### MAGNETIC FLUX

A student places a horizontal square coil of wire of side length 4.0 cm into a uniform vertical magnetic field of 0.050 T. How much magnetic flux 'threads' the coil?	
<b>Thinking</b>	<b>Working</b>
Calculate the area of the coil perpendicular to the magnetic field.	$\text{side length} = 4.0 \text{ cm}$ $= 0.04 \text{ m}$ $\text{area of the square} = (0.04 \text{ m})^2$ $= 0.0016 \text{ m}^2$
Calculate the magnetic flux.	$\phi_B = B_{\perp} A$ $= 0.050 \times 0.0016$ $= 0.00008 \text{ Wb}$
State the answer in an appropriate form.	$\phi_B = 8.0 \times 10^{-5} \text{ Wb}$

#### Worked example: Try yourself 4.1.2

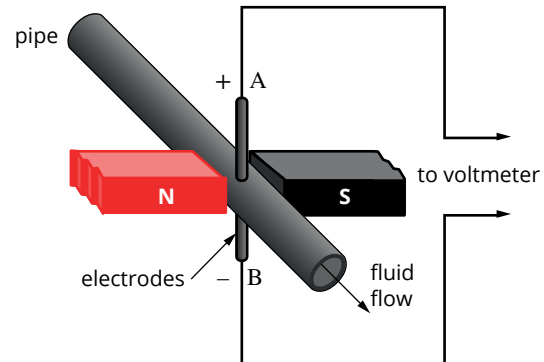
##### ELECTROMOTIVE FORCE ACROSS AN AIRCRAFT'S WINGS

Will a moving airplane develop a dangerous emf between its wing tips solely from the Earth's magnetic field? A fighter jet with a wing span of 25 m is flying at a speed of 2000 km h <sup>-1</sup> at right angles to the Earth's magnetic field of 5.0 × 10 <sup>-5</sup> T.	
<b>Thinking</b>	<b>Working</b>
Identify the quantities required in the correct units.	$\varepsilon = ?$ $l = 25 \text{ m}$ $B = 5.0 \times 10^{-5} \text{ T}$ $v = 2000 \text{ km h}^{-1}$ $= 2000 \times \frac{1000}{3600}$ $= 556 \text{ m s}^{-1}$
Substitute into the appropriate formula and simplify.	$\varepsilon = lvB$ $= 25 \times 556 \times 5.0 \times 10^{-5}$ $= 0.695 \text{ V}$
State your answer as a response to the question.	$\varepsilon = 0.695 \text{ V}$ This is a very small emf and would not be dangerous.

**Worked example: Try yourself 4.1.3**

**FLUID FLOW MEASUREMENT**

The rate of fluid flow within a vessel can be measured using the induced emf when the fluid contains charged ions. A small magnet and sensitive voltmeter calibrated to measure speed are used. This can be applied to measure fluid flow in small pipes. If the diameter of a particular small pipe is 1.00 cm, the strength of the magnetic field applied is 0.10 T and the measured emf is 0.50 mV, what is the speed of the fluid flow?



Thinking	Working
Identify the quantities required and put them into SI units.	$\epsilon = 0.50 \text{ mV} = 5.0 \times 10^{-4} \text{ V}$ $l = 1.00 \text{ cm} = 0.0100 \text{ m}$ $v = ?$ $B = 0.10 \text{ T}$
Rearrange the appropriate formula, substitute and simplify.	$\epsilon = lvB$ $v = \frac{\epsilon}{lB}$ $= \frac{5.0 \times 10^{-4}}{0.0100 \times 0.10}$ $= 0.50 \text{ m s}^{-1}$
State your answer with the correct units.	$v = 0.50 \text{ m s}^{-1}$

**4.1 review**

- 1 A. There is no change in magnetic flux in this scenario and so there cannot be an induced emf.
- 2 0 Wb. Since the plane of the coil is parallel to the magnetic field there is no flux passing through the coil.
- 3  $\phi_B = B_{\perp}A = 2.0 \times 10^{-3} \times 0.04^2 = 3.2 \times 10^{-6} \text{ Wb}$
- 4 The magnetic flux decreases from  $3.2 \times 10^{-6} \text{ Wb}$  to 0 after one-quarter of a turn. Then it increases again to  $3.2 \times 10^{-6} \text{ Wb}$  through the opposite side of the loop after half a turn. Then it decreases to 0 again after three-quarters of a turn. After a full turn it is back to  $3.2 \times 10^{-6} \text{ Wb}$  again.
- 5  $\phi_B = B_{\perp}A = 1.6 \times 10^{-3} \times \pi \times 0.05^2 = 1.3 \times 10^{-5} \text{ Wb}$
- 6  $\epsilon = lvB = 0.12 \times 0.150 \times 0.800 = 0.0144 \text{ V}$  or  $1.44 \times 10^{-2} \text{ V}$
- 7  $\epsilon = lvB$   
 $v = \frac{\epsilon}{lB} = \frac{0.100}{0.132 \times 0.90} = 0.84 \text{ m s}^{-1}$
- 8  $\epsilon = lvB$   
 $l = \frac{\epsilon}{vB} = \frac{0.080}{1.6 \times 0.50} = 0.10 \text{ m}$
- 9 As the rod is held vertically and dropped downwards through a vertically upward field, the rod and magnetic field are oriented parallel to each other. No emf will be produced, therefore the correct answer is 0 V. (As the rod has some width, there would be an emf created across this width, but the question specifically dismisses this by stating it is of 'very small diameter'.)
- 10  $\epsilon = lvB = 20 \times 1000 \times 2.5 \times 10^{-5} = 0.50 \text{ V}$

## Section 4.2

## Worked example: Try yourself 4.2.1

## INDUCED EMF IN A COIL

A student winds a coil of area  $50 \text{ cm}^2$  with 10 turns. She places it horizontally in a vertical uniform magnetic field of  $0.10 \text{ T}$ .

(a) Calculate the magnetic flux perpendicular to the coil.	
<b>Thinking</b>	<b>Working</b>
Identify the quantities to calculate the magnetic flux through the coil and convert to SI units where required.	$\phi_B = B_{\perp}A$ $B = 0.10 \text{ T}$ $A = 50 \text{ cm}^2 = 50 \times 10^{-4} \text{ m}^2$
Calculate the magnetic flux and state with appropriate units.	$\phi_B = B_{\perp}A = 0.10 \times 50 \times 10^{-4}$ $= 5.0 \times 10^{-4} \text{ Wb}$

(b) Calculate the magnitude of the average induced emf in the coil when the coil is removed from the magnetic field in a time of $1.0 \text{ s}$ .	
<b>Thinking</b>	<b>Working</b>
Identify the quantities for determining the induced emf.	$\varepsilon = -N \frac{\Delta\phi_B}{\Delta t}$ $N = 10 \text{ turns}$ $\Delta\phi_B = \phi_2 - \phi_1$ $= 0 - 5.0 \times 10^{-4}$ $= \text{a change of } 5.0 \times 10^{-4} \text{ Wb}$ $t = 1.0 \text{ s}$
Calculate the magnitude of the average induced emf, ignoring the negative sign that indicates the direction. Use appropriate units.	$\varepsilon = -N \frac{\Delta\phi_B}{\Delta t}$ $= 10 \times \frac{5.0 \times 10^{-4}}{1.0}$ $= 5.0 \times 10^{-3} \text{ V}$

## Worked example: Try yourself 4.2.2

## NUMBER OF TURNS IN A COIL

A coil of cross-sectional area $2.0 \times 10^{-3} \text{ m}^2$ experiences a change in the strength of a magnetic field from $0$ to $0.20 \text{ T}$ over $1.00 \text{ s}$ . If the magnitude of the average induced emf is measured as $0.40 \text{ V}$ , how many turns must be on the coil?	
<b>Thinking</b>	<b>Working</b>
Identify the quantities to calculate the magnetic flux through the coil when in the presence of the magnetic field and convert to SI units where required.	$\phi_B = B_{\perp}A$ $B = 0.20 \text{ T}$ $A = 2.0 \times 10^{-3} \text{ m}^2$
Calculate the magnetic flux when in the presence of the magnetic field.	$\phi_B = B_{\perp}A$ $= 0.20 \times 2.0 \times 10^{-3}$ $= 4.0 \times 10^{-4} \text{ Wb}$



Identify the quantities from the question required to complete Faraday's law.	$\varepsilon = -N \frac{\Delta\phi_B}{\Delta t}$ $N = ?$ $\Delta\phi_B = \phi_2 - \phi_1$ $= 4.0 \times 10^{-4} - 0$ $= \text{a change of } 4.0 \times 10^{-4} \text{ Wb}$ $t = 1.0 \text{ s}$ $\varepsilon = 0.40 \text{ V}$
Rearrange Faraday's law and solve for the number of turns on the coil. Ignore the negative sign.	$\varepsilon = -N \frac{\Delta\phi_B}{\Delta t}$ $N = \frac{\varepsilon \Delta t}{\Delta\phi_B}$ $= \frac{0.40 \times 1.0}{4.0 \times 10^{-4}}$ $= 1000 \text{ turns}$

## 4.2 review

- $\phi_B = B_{\perp} A = 2.0 \times 10^{-3} \times 0.02 \times 0.03 = 1.2 \times 10^{-6} \text{ Wb}$
- Zero flux threads the loop when the loop is parallel to the magnetic field.
- $\Delta\phi_B = 1.2 \times 10^{-6} \text{ Wb}$   

$$\varepsilon = -N \frac{\Delta\phi_B}{\Delta t} = \frac{1.2 \times 10^{-6}}{0.040} = 3.0 \times 10^{-5} \text{ V}$$
- C. The speed of the magnet reduces the time over which the change occurs but there is no change in the strength of the magnetic field or the area of the coil, hence the total flux (area under the curve) is the same.
- $\phi_B = 80 \times 10^{-3} \times 10 \times 10^{-4}$   
 $= 8 \times 10^{-5} \text{ Wb}$   

$$\varepsilon = -\frac{\Delta\phi_B}{\Delta t} = \frac{8 \times 10^{-5}}{0.020} = 4 \times 10^{-3} \text{ V}$$
- The effect of using multiple coils is similar to placing cells in series—the emf of each of the coils adds together to produce the total emf.  
 $\Delta\phi_B = 500 \times 4 \times 10^{-3} \text{ V} = 2 \text{ V}$
- $\phi_B = 5.0 \times 10^{-3} \times 200 \times 10^{-4}$   
 $= 1.0 \times 10^{-4} \text{ Wb}$   

$$\varepsilon = -N \frac{\Delta\phi_B}{\Delta t} = 30 \times \frac{1.0 \times 10^{-4}}{0.50} = 6.0 \times 10^{-3} \text{ V}$$
- The student must induce an emf of 1.0 V in the wire by somehow changing the magnetic flux through the coil at an appropriate rate. A change in flux can be achieved by changing the strength of the magnetic field or by changing the area of the coil. The magnetic field can be changed by changing the position of the magnet relative to the coil. The area can be changed by changing the shape of the coil or by rotating the coil relative to the magnetic field. To calculate the required rate of change of flux to produce 1.0 V:

$$\frac{\Delta\phi_B}{\Delta t} = \frac{\varepsilon}{N}$$

$$\frac{\Delta\phi_B}{\Delta t} = \frac{1.0}{100} = 0.01 \text{ Wb s}^{-1}$$

For example, if the shape was changed from  $0.01 \text{ m}^2$  to  $0.02 \text{ m}^2$  in a time of 0.1 s, then:

$$\frac{\Delta\phi_B}{\Delta t} = \frac{(100 \times 10^{-3} \times 0.02) - (100 \times 10^{-3} \times 0.01)}{0.1} = \frac{0.001}{0.1} = 0.01 \text{ Wb s}^{-1}$$

$$\begin{aligned}
 9 \quad \varepsilon &= -N \frac{\Delta\phi_B}{\Delta t} \\
 &= -N \frac{\Delta BA_{\perp}}{\Delta t} \\
 A_{\perp} &= \frac{-\varepsilon \Delta t}{N \Delta B} \\
 &= \frac{0.020 \times 0.050}{1 \times 0.10} \\
 &= 0.010 \text{ m}^2
 \end{aligned}$$

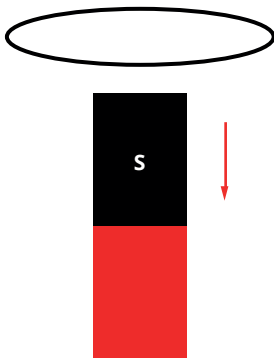
$$\begin{aligned}
 10 \quad \varepsilon &= -N \frac{\Delta\phi_B}{\Delta t} \\
 \Delta t &= -N \frac{\Delta\phi_B}{\varepsilon} = 100 \times \frac{0.40 \times 50 \times 10^{-4}}{1600 \times 10^{-3}} \\
 &= 0.125 \text{ s}
 \end{aligned}$$

## Section 4.3

### Worked example: Try yourself 4.3.1

#### INDUCED CURRENT IN A COIL FROM A PERMANENT MAGNET

The south pole of a magnet is moved downwards away from a horizontal coil held above it. In which direction will the induced current flow in the coil?



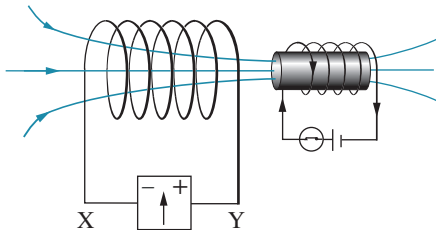
Thinking	Working
Consider the direction of the change in magnetic flux.	The magnetic field direction will be downwards towards the south pole. The downwards flux from the magnet will decrease as the magnet is moved away from the coil. So the change in flux is decreasing downwards.
What will oppose the change in flux?	The magnetic field that opposes the change would act downwards.
Determine the direction of the induced current required to oppose the change.	In order to oppose the change, the current direction would be clockwise when viewed from above (using the right-hand grip rule).

**Worked example: Try yourself 4.3.2**

**INDUCED CURRENT IN A COIL FROM AN ELECTROMAGNET**

What is the direction of the current induced in the solenoid when the electromagnet is:

- (i) switched on
- (ii) left on
- (iii) switched off?

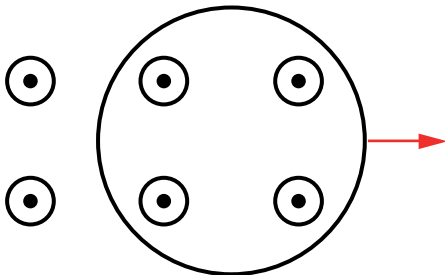


Thinking	Working
<p>Consider the direction of the change in magnetic flux for each case.</p>	<p>(i) Initially there is no magnetic flux through the solenoid. When the electromagnet is switched on, the electromagnet creates a magnetic field directed to the right. So the change in flux through the solenoid is increasing to the right.</p> <p>(ii) While the current in the electromagnet is steady, the magnetic field is constant and the flux through the solenoid is constant.</p> <p>(iii) In this case, initially there is a magnetic field from the electromagnet directed to the right. When the electromagnet is switched off, there is no longer a magnetic field so the change in flux through the solenoid is decreasing to the right.</p>
<p>What will oppose the change in flux for each case?</p>	<p>(i) The magnetic field that opposes the change in flux through the solenoid is directed to the left.</p> <p>(ii) There is no change in flux and so no opposition is needed and there will be no magnetic field created by the solenoid.</p> <p>(iii) The magnetic field that opposes the change in flux through the solenoid is directed to the right.</p>
<p>Determine the direction of the induced current required to oppose the change for each case.</p>	<p>(i) In order to oppose the change, the current will flow through the solenoid in the direction from Y to X (through the meter from X to Y), using the right-hand grip rule.</p> <p>(ii) There will be no induced emf or current in the solenoid.</p> <p>(iii) In order to oppose the change, the current will flow through the solenoid in the direction from X to Y (through the meter from Y to X), using the right-hand grip rule.</p>

**Worked example: Try yourself 4.3.3**

**FURTHER PRACTICE WITH LENZ'S LAW**

A coil is moved to the right and out of a magnetic field that is directed out of the page. In what direction will the induced current flow in the coil while the magnet is moving?



Thinking	Working
Consider the direction of the change in magnetic flux.	Initially, the magnetic flux passes through the full area of the coil and out of the page. Moving the coil out of the field decreases the magnetic flux. So the change in flux is decreasing out of the page.
What will oppose the change in flux?	The magnetic field that opposes the change would act out of the page again.
Determine the direction of the induced current required to oppose the change.	In order to oppose the change, the current direction would be anticlockwise (using the right-hand grip rule).

**Worked example: Try yourself 4.3.4**

**PEAK AND RMS AC CURRENT VALUES**

A 1000 W kettle is connected to a 240 V AC power outlet. What is the peak power use of the kettle?

Thinking	Working
Note that the values given in the question represent rms values. Power is $P = VI$ so both $V$ and $I$ must be known to calculate the power use. The voltage $V$ is given, and the current $I$ can be calculated from the rms power supplied.	$P_{\text{rms}} = V_{\text{rms}} I_{\text{rms}}$ $I_{\text{rms}} = \frac{P_{\text{rms}}}{V_{\text{rms}}}$ $= \frac{1000}{240}$ $= 4.17 \text{ A}$
Substitute in known quantities and solve for peak power.	$P_p = \sqrt{2}V_{\text{rms}} \times \sqrt{2}I_{\text{rms}} = 2V_{\text{rms}}I_{\text{rms}}$ $= 2 \times V_{\text{rms}} \times I_{\text{rms}}$ $= 2 \times 240 \times 4.17$ $= 2000 \text{ W}$

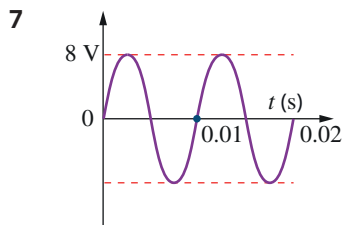
**4.3 review**

- 1 C. The magnetic field of the induced current will always oppose the original change.
- 2 a A. When the external magnetic field is switched off this represents a change in flux through the coil that is decreasing out of the page. In order to oppose this change, the induced current will create a magnetic field out of the page.  
 b A. When the external magnetic field is reversed this represents a change in flux through the coil that is decreasing out of the page, followed by increasing into the page. In order to oppose this change, the induced current will create a magnetic field out of the page.

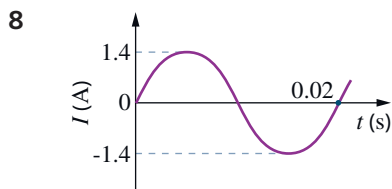
- 3 a Anticlockwise. The magnetic flux is increasing in strength downwards, so the field opposing that change will be upwards. Using the right-hand grip rule, the direction of the induced current will be anticlockwise.
- b Any combination of four factors that address the three ways of creating an induced emf or the relationship between emf and current, such as:
- 1) strength of the magnet
  - 2) speed of the magnet
  - 3) area/diameter of the ring
  - 4) orientation of the ring
  - 5) type of copper making up the ring
  - 6) resistance of the circuit containing the coil.

- 4 B. Applying Lenz's law, the back emf opposes the change in magnetic flux that created it, so the induced back emf will be in the opposite direction to the emf creating it. The net emf used by the motor is then less than the supplied voltage.
- 5 B. When the coil begins rotating the flux is a maximum and decreases initially, having the shape of graph D. The current graph (like the induced emf graph) will be zero initially and will increase, having the pattern shown in graph B.

6  $V_p = 8.0 \text{ V}$   
 $V_{p-p} = 2 \times V_p = 2 \times 8.0 = 16 \text{ V}$   
 $V_{\text{rms}} = \frac{V_p}{\sqrt{2}} = \frac{8.0}{\sqrt{2}} = 5.66 \text{ V}$



Doubling the magnetic field strength will double the emf, as will doubling the frequency. Halving the radius reduces the area by one-quarter, and so reduces the emf by one-quarter. Thus the emf will remain the same magnitude overall. Doubling the frequency of rotation, however, does reduce the period of the output graph by half.



9  $V_{\text{rms}} = \frac{10}{\sqrt{2}} = 7.07 \text{ V}$   
 $I_{\text{rms}} = \frac{6}{\sqrt{2}} = 4.24 \text{ A}$   
 $P_{\text{rms}} = V_{\text{rms}} \times I_{\text{rms}}$   
 $= 7.07 \times 4.24 = 29.98 \text{ W or } 30 \text{ W}$

10  $I_{\text{rms}} = \frac{P_{\text{rms}}}{V_{\text{rms}}} = \frac{600}{240} = 2.5 \text{ A}$   
 $I_p = \sqrt{2} \times I_{\text{rms}} = \sqrt{2} \times 2.50 = 3.54 \text{ A}$

## Section 4.4

## Worked example: Try yourself 4.4.1

## TRANSFORMER EQUATION—VOLTAGE

A transformer is built into a phone charger to reduce the 240 V supply voltage to the required 6 V for the charger. If the number of turns in the secondary coil is 100, what is the number of turns required in the primary coil?	
<b>Thinking</b>	<b>Working</b>
State the relevant quantities given in the question. Choose a form of the transformer equation with the unknown quantity in the top left position.	$V_2 = 6 \text{ V}$ $V_1 = 240 \text{ V}$ $N_2 = 100 \text{ turns}$ $N_1 = ?$ $\frac{N_1}{N_2} = \frac{V_1}{V_2}$
Substitute the quantities into the equation, rearrange and solve for $N_1$ .	$\frac{N_1}{100} = \frac{240}{6}$ $N_1 = \frac{100 \times 240}{6}$ $= 4000 \text{ turns}$

## Worked example: Try yourself 4.4.2

## TRANSFORMER EQUATION—CURRENT

A phone charger with 4000 turns in the primary coil and 100 turns in its secondary coil draws a current of 0.50 A. What is the current in the primary coil?	
<b>Thinking</b>	<b>Working</b>
State the relevant quantities given in the question. Choose a form of the transformer equation with the unknown quantity in the top left position.	$I_2 = 0.50 \text{ A}$ $N_2 = 100 \text{ turns}$ $N_1 = 4000 \text{ turns}$ $I_1 = ?$ $\frac{I_1}{I_2} = \frac{N_2}{N_1}$
Substitute the quantities into the equation, rearrange and solve for $I_1$ .	$\frac{I_1}{0.50} = \frac{100}{4000}$ $I_1 = \frac{0.50 \times 100}{4000}$ $= 0.0125 \text{ A}$

## Worked example: Try yourself 4.4.3

## TRANSFORMERS—POWER

The power drawn from the secondary coil of the transformer by a phone charger is 3 W. What power is drawn from the mains supply if the transformer is an ideal transformer?	
<b>Thinking</b>	<b>Working</b>
The energy efficiency of a transformer can be assumed to be 100%. The power in the secondary coil will be the same as that in the primary coil.	The power drawn from the mains supply is the power in the primary coil, which will be the same as the power in the secondary coil: $P = 3 \text{ W}$

**Worked example: Try yourself 4.4.4****TRANSMISSION-LINE POWER LOSS**

300 MW is to be transmitted from the Hazelwood power station to Melbourne along a transmission line with a total resistance of $1.0 \Omega$ . What would be the total transmission power loss if the voltage along the line was now to be 500 kV?	
<b>Thinking</b>	<b>Working</b>
Convert the values to SI units.	$P = 300 \text{ MW} = 300 \times 10^6 \text{ W}$ $V = 500 \text{ kV} = 500 \times 10^3 \text{ V}$
Determine the current in the line based on the required voltage.	$P = VI \therefore I = \frac{P}{V}$ $I = \frac{300 \times 10^6}{500 \times 10^3}$ $= 600 \text{ A}$
Determine the corresponding power loss.	$P = I^2R$ $= 600^2 \times 1$ $= 3.6 \times 10^5 \text{ W or } 0.36 \text{ MW}$

**Worked example: Try yourself 4.4.5****VOLTAGE DROP ALONG A TRANSMISSION LINE**

Power is to be transmitted from the Loy Yang power station to Melbourne along a transmission line with a total resistance of $1.0 \Omega$ . The current is 600 A. What voltage would be needed at the Loy Yang end of the transmission line to achieve a supply voltage of 500 kV?	
<b>Thinking</b>	<b>Working</b>
Determine the voltage drop along the transmission line.	$\Delta V = IR$ $= 600 \times 1.0$ $= 600 \text{ V}$
Determine the initial supply voltage.	$V_{\text{initial}} = V_{\text{supplied}} + \Delta V$ $= 500 \times 10^3 + 600$ $= 500.6 \text{ kV}$

**4.4 review**

- B. The power equation is  $P = VI$  and the '2' indicates the secondary coil.
- D. A change in flux through the secondary coil is required for an emf to be induced in the second coil, but a DC input to the primary coil will create a constant flux. Therefore the voltage output is zero.
- $$\frac{N_2}{N_1} = \frac{V_2}{V_1}$$

$$\frac{N_2}{800} = \frac{12}{240}$$

$$N_2 = \frac{12 \times 800}{240}$$

$$= 40 \text{ turns}$$
- a In an ideal transformer there should be no power loss, so  $P_1 = P_2$ .

b  $\frac{I_2}{I_1} = \frac{N_1}{N_2}$
- a  $\frac{V_2}{V_1} = \frac{N_2}{N_1}$

$$\frac{V_2}{8.0} = \frac{200}{20}$$

$$V_2 = \frac{8.0 \times 200}{20}$$

$$= 80 \text{ V}$$

- b** Assuming an ideal transformer, the power output at the secondary coil must be equal to the power input at the primary coil.  
 $P_1 = V_1 I_1 = 8.0 \times 2.0 = 16 \text{ W}$
- c**  $I_2 = \frac{P_2}{V_2} = \frac{16}{80} = 0.20 \text{ A}$
- 6 a**  $\frac{N_2}{N_1} = \frac{V_2}{V_1}$   
 $\frac{N_2}{800} = \frac{12}{240}$   
 $N_2 = \frac{800 \times 12}{240}$   
 $= 40 \text{ turns}$
- b**  $\frac{I_1}{I_2} = \frac{N_2}{N_1}$   
 $I_1 = \frac{I_2 N_2}{N_1}$   
 $= \frac{2.0 \times 40}{800}$   
 $= 0.10 \text{ A}$   
 $I_p = I_{\text{rms}} \times \sqrt{2}$   
 $= 0.10 \times \sqrt{2}$   
 $= 0.14 \text{ A}$
- c**  $P_1 = V_1 \times I_1$   
 $= 240 \times 0.1$   
 $= 24 \text{ W}$
- d** A DC supply operates at a constant voltage, hence there is no changing flux through the secondary coil so no output voltage will be produced and the transformer will not operate.
- 7**  $I = \frac{P}{V} = \frac{5.0 \times 10^3}{500} = 10 \text{ A}$   
 $P_{\text{loss}} = I^2 R = 10^2 \times 4.0 = 400 \text{ W}$
- 8**  $I = \frac{P}{V} = \frac{500 \times 10^6}{250 \times 10^3} = 2000 \text{ A}$   
 $P_{\text{loss}} = I^2 R = 2000^2 \times 10 = 4 \times 10^7 \text{ W or } 40 \text{ MW}$
- 9 a**  $I = \frac{P}{V} = \frac{500 \times 10^6}{100 \times 10^3} = 5000 \text{ A}$
- b**  $V_{\text{drop}} = I \times R$   
 $V_{\text{drop}} = 5000 \times 2 = 10\,000 \text{ V or } 10 \text{ kV}$   
 $V_{\text{supplied}} = 100 - 10 = 90 \text{ kV}$
- 10** B is correct. A is incorrect because the  $\Delta V$  in the formula indicates the voltage drop in the transmission lines; it does not refer to the voltage being transmitted.

## CHAPTER 4 REVIEW

- 1 a**  $B$  changes from  $8.0 \times 10^{-4} \text{ T}$  to  $16 \times 10^{-4} \text{ T}$  which is a change of  $8.0 \times 10^{-4} \text{ T}$ .  
 $\phi_B = \Delta B \times A = 8.0 \times 10^{-4} \times 40 \times 10^{-4}$   
 $= 3.2 \times 10^{-6} \text{ Wb}$   
 $\epsilon = -N \frac{\Delta \phi_B}{\Delta t} = \frac{3.2 \times 10^{-6}}{1.0 \times 10^{-3}} = 3.2 \times 10^{-3} \text{ V or } 3.2 \text{ mV}$
- b** Clockwise. Doubling the magnetic field strength increases the flux through the coil out of the page. The induced magnetic field will act into the page to oppose the increasing magnetic flux out of the page. Using the right-hand grip rule, the induced current direction is clockwise around the coil.



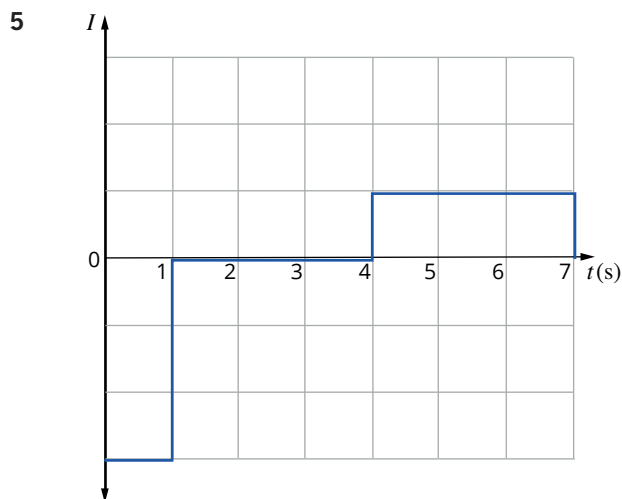
2 a  $\phi_B = 20 \times 10^{-3} \times \pi \times 0.04^2$   
 $= 1 \times 10^{-4} \text{ Wb}$   
 $\epsilon = -N \frac{\Delta\phi_B}{\Delta t} = 40 \frac{1 \times 10^{-4}}{0.10} = 0.04 \text{ V}$

b From Y to X. As the coil is removed, the magnetic flux through the coil changes from being directed downwards to no magnetic flux. To oppose this change the coil must create a magnetic field that is directed downwards again. Using the right-hand grip rule, this means the current must flow clockwise around the coil when viewed from above.

3 a  $\epsilon = l v B = 0.20 \times 2.0 \times 10 \times 10^{-3} = 4 \times 10^{-3} \text{ V}$  or 4.0 mV

b From X to Y. As the rod moves to the right the area of the loop decreases so the magnetic flux through the loop, which is directed out of the page, decreases. In order to oppose this change the loop will create a magnetic field directed out of the page again. Using the right-hand grip rule, the current will flow through the rod from X to Y.

4  $\epsilon = l v B = 8.0 \times 4.0 \times 5.0 \times 10^{-5} = 1.6 \times 10^{-3} \text{ V}$  or 1.6 mV



A change in the emf in  $S_1$  produces a current in  $S_2$ . So no current flows in  $S_2$  between  $t = 1 \text{ s}$  and  $t = 4 \text{ s}$ . An increase in emf at a constant rate ( $t = 0$  to  $t = 1 \text{ s}$ ) would produce a constant current and a decrease in emf at a lower rate ( $t = 4$  to  $t = 7 \text{ s}$ ) would produce a lower current in the opposite direction. Either the graph shown or its inversion is correct.

6  $\frac{I_2}{I_1} = \frac{V_1}{V_2}$   
 $\frac{I_2}{3.0} = \frac{14}{42}$   
 $I_2 = 1.0 \text{ A}$

7  $\frac{N_1}{N_2} = \frac{V_1}{V_2}$   
 $\frac{N_1}{30} = \frac{14}{42}$   
 $N_1 = 10$

There are 10 turns in the primary coil.

8 A. The spikes in the voltage output occur when the input voltage rises and falls i.e. when it *changes*.

9 a  $V_{\text{rms}} = \frac{V_p}{\sqrt{2}}$   
 $= \frac{25}{\sqrt{2}}$   
 $= 18 \text{ V}$

b  $P_p = I_p V_p = 15 \times 25 = 375 \text{ W}$

10 C

$$\begin{aligned}
 P_{\text{rms}} &= V_{\text{rms}} I_{\text{rms}} \\
 &= \frac{I_{\text{p-p}}}{2\sqrt{2}} \times \frac{V_{\text{p-p}}}{2\sqrt{2}} \\
 &= \frac{I_{\text{p-p}} \times V_{\text{p-p}}}{8}
 \end{aligned}$$

$$I_{\text{p-p}} \times V_{\text{p-p}} = 8 \times P_{\text{rms}} = 8 \times 60 = 480 \text{ W}$$

Option C is the only option that meets this requirement.

11 In a quarter of a turn  $\Delta\phi_B = 80 \times 10^{-3} \times 10 \times 10^{-4} = 8 \times 10^{-5} \text{ Wb}$ 

Frequency is 50 Hz so quarter of a turn takes  $\frac{1}{4} \times 0.02 = 0.005 \text{ s}$

$$\begin{aligned}
 \varepsilon &= -N \frac{\Delta\phi_B}{\Delta t} = 500 \times \frac{8 \times 10^{-5}}{0.005} \\
 &= 8 \text{ V}
 \end{aligned}$$

12 Doubling the frequency halves the  $\Delta t$  in Faraday's law so it doubles the average emf to 16 V.

13 Any two of:

- Using a DC power supply means that the voltage cannot be stepped up or down with transformers.
- Hence there will be significant power loss along the  $8 \Omega$  power lines.
- Damage to any appliances operated in the shed that are designed to operate on 240 V AC and not on 240 V DC.

14 As the coil area is reduced, the flux into the page will decrease. To oppose this the induced current will try to increase the flux again in the same direction. Using the right-hand grip rule the direction of the induced current will be clockwise.

15 AB and CD. Both the sides AB and CD cut across lines of flux as the coil rotates.

16  $P = VI$ 

$$150 \times 10^3 = 10000 \times I$$

$$I = \frac{150 \times 10^3}{10000}$$

$$= 15 \text{ A}$$

17 Calculate the voltage drop:

$$V = IR$$

$$= 15 \times 2.0$$

$$= 30 \text{ V}$$

Calculate the final voltage: Initial voltage – voltage drop

$$V = 10000 - 30 = 9970 \text{ V}$$

18  $P = I^2R$ 

Using current calculated from Question 16,  $I = 15 \text{ A}$

$$P = 15^2 \times 2.0$$

$$= 450 \text{ W}$$

19 Without the first transformer, voltage in the transmission lines,  $V = 1000 \text{ V}$ 

Calculate  $I$ :

$$P = VI$$

$$150 \text{ kW} = 1000I$$

$$I = \frac{150 \times 10^3}{1000} = 150 \text{ A}$$

Power loss in the lines:

$$P = I^2R$$

$$= 150^2 \times 2.0$$

$$= 45 \text{ kW}$$

Power supplied = 150 kW – 45 kW = 105 kW

This represents a 30% power loss—bad idea!

20 Anticlockwise. Initially there is no flux through the coil. As the coil begins to rotate, the amount of flux increases and to the left. To oppose this change, an induced magnetic field will be directed to the right. Using the right-hand grip rule this creates an anticlockwise current in the coil for the orientation shown in the diagram.

## Chapter 5 answers

### Section 5.1

#### Worked example: Try yourself 5.1.1

##### APPLICATION OF NEWTON'S FIRST AND THIRD LAWS

The toddler adds extra blocks to the cart and drags it across the floor more slowly. The 5.5 kg cart travels at a constant speed of  $0.65 \text{ m s}^{-1}$ . The force of friction between the cart and the floor is 5.2 N and the handle is now at an angle of  $30^\circ$  above the horizontal.

(a) Calculate the net force on the cart.	
<b>Thinking</b>	<b>Working</b>
The cart has constant velocity. According to Newton's first law, the net force acting on the cart is zero.	$F_{\text{C net}} = 0 \text{ N}$
(b) Calculate the force that the toddler exerts on the cart.	
<b>Thinking</b>	<b>Working</b>
Draw a diagram.	
If the net force is zero then the horizontal forces must be in balance. Therefore the horizontal component of the force on the cart by the toddler, $F_{\text{CT}x}$ , is equal to the magnitude of the frictional force, $F_{\text{CF}}$ .	$F_{\text{CF}} = F_{\text{CT}} \cos 30^\circ = 5.2 \text{ N}$ $F_{\text{CT}} = \frac{5.2}{\cos 30^\circ} = 6.0 \text{ N}$
(c) Calculate the force that the cart exerts on the toddler.	
<b>Thinking</b>	<b>Working</b>
Apply Newton's third law to find the force on the toddler by the cart.	The force of the cart on the toddler is 6.0 N at an angle of $30^\circ$ below the horizontal.

## Worked example: Try yourself 5.1.2

### APPLICATION OF NEWTON'S LAWS

A vehicle towing a trailer accelerates at  $2.8 \text{ m s}^{-2}$  in order to overtake a car in front. The vehicle's mass is  $2700 \text{ kg}$  and the trailer's mass is  $600 \text{ kg}$ . The drag forces on the vehicle are  $1100 \text{ N}$ , and the drag forces on the trailer are  $500 \text{ N}$ .

(a) Calculate the thrust of the engine.	
<b>Thinking</b>	<b>Working</b>
Draw a sketch showing all the forces acting.	
Since there is an acceleration, Newton's second law may be applied to the whole system. Note that the vehicle and the trailer are considered to be joined by the coupling and so the tension forces are not included at this stage. Consider the system as a whole.	$F_{\text{system}} = m_{\text{system}} a$ $F_{V \text{ thrust}} - F_{V \text{ drag}} = (m_v + m_t)a$ $F_{V \text{ thrust}} - 1100 - 500 = (2700 + 600)2.8$ $F_{V \text{ thrust}} = 1.1 \times 10^4 \text{ N in the direction of motion}$

(b) Calculate the magnitude of the tension in the coupling.	
<b>Thinking</b>	<b>Working</b>
Consider only one part of the system, for example the trailer, once again applying Newton's second law.	$F_{T \text{ net}} = m_t a$ $F_{T \text{ tension}} - F_{T \text{ drag}} = m_t a$ $F_{T \text{ tension}} = 600 \times 2.8 + 500$ $= 2.2 \times 10^3 \text{ N}$

## Worked example: Try yourself 5.1.3

### INCLINED PLANES

A much heavier skier of mass  $85 \text{ kg}$  travels down the same icy slope inclined at  $20^\circ$  to the horizontal. Assume that friction is negligible and that the acceleration due to gravity is  $9.8 \text{ m s}^{-2}$ .

(a) Determine the components of the weight of the skier perpendicular to the slope and parallel to the slope.	
<b>Thinking</b>	<b>Working</b>
Draw a sketch including the values provided.	
Resolve the weight into a component perpendicular to the slope.	The perpendicular component is: $F_{\perp} = F_g \cos 20^\circ$ $= 833 \cos 20^\circ = 783 \text{ N}$
Resolve the weight into a component parallel to the slope.	The parallel component is: $F_{\parallel} = F_g \sin 20^\circ$ $= 833 \sin 20^\circ = 285 \text{ N}$

(b) Determine the normal force that acts on the skier.	
<b>Thinking</b>	<b>Working</b>
The normal force is equal in magnitude to the perpendicular component.	$F_N = 783 \text{ N}$
(c) Calculate the acceleration of the skier down the slope.	
<b>Thinking</b>	<b>Working</b>
Apply Newton's second law. The net force along the incline is the component of the weight parallel to the slope.	$a = \frac{F_{\text{net}}}{m}$ $= \frac{285}{85}$ $= 3.4 \text{ m s}^{-2} \text{ down the slope}$

## 5.1 review

1 No. Phil's inertia made him stay where he was (stationary) as the tram moved forwards. This made it look like Phil was thrown backwards relative to the tram. This is an example of Newton's first law. Objects will remain at rest unless a net unbalanced force acts to change the motion.

2 Forces are balanced, so air resistance force is equal in magnitude to the weight of the ball.

$$F_a = F_g = mg = 0.01 \times 9.8 = 0.098 \text{ N upwards}$$

3 a Constant speed, so forces are balanced:  $F_d = 45 \text{ N}$ .

b  $F_{\text{net}} = ma = 80 \times 1.5 = 120 \text{ N}$

$$F_d - 45 = 120$$

$$F_d = 165 \text{ N}$$

4 a  $u = 0, v = 7.5, t = 5.0, a = ?$

$$v = u + at$$

$$7.5 = 0 + 5.0a$$

$$a = 1.5 \text{ m s}^{-2}$$

b  $F_{\text{net}} = ma = 80 \times 1.5 = 120 \text{ N}$

c Constant speed, so forces are balanced, i.e.  $F_{\text{net}} = 0$ . The frictional force will equal 60 N.

5 a Constant speed so net force is zero.

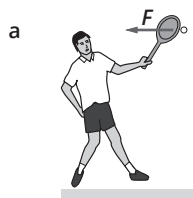
b The horizontal component of the pulling (tension) force is in balance with the frictional force of 60 N.

$$T_h = T \cos 25^\circ = 60$$

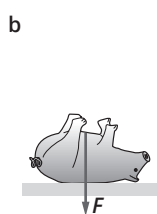
$$T = \frac{60}{\cos 25^\circ} = 66 \text{ N}$$

c The rope is exerting a force of 66 N on Matt.

6



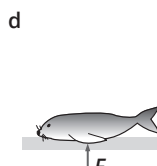
Force exerted on racquet by ball



Force exerted on ground by pig



Force exerted on ground by wardrobe



Gravitational force of attraction that seal exerts on Earth

- 7 The net force on the whole system is  $F_{\text{g (falling weight)}} - F_{\text{fr}} = 4.9 - 1.5 = 3.4 \text{ N}$   
 The acceleration of the system is thus  $a = \frac{F}{m} = \frac{3.4}{2.5} = 1.4 \text{ m s}^{-2}$   
 The 2.0 kg block has the same acceleration, hence  
 $F_{\text{net}} = 2.0 \times 1.4 = 2.8 \text{ N}$   
 $T - 1.5 = 2.8$   
 $T = 4.3 \text{ N}$
- 8 a  $F_{\text{net}} = \text{thrust} - \text{drag forces} = m_{\text{total}} a$   
 $\text{thrust} - 600 - 1025 = (2000 + 250) \times 1.5$   
 $\text{thrust} = 3375 + 600 + 1025$   
 $= 5000 = 5.0 \times 10^3 \text{ N}$
- b  $F_{\text{net tree}} = \text{tension} - \text{drag}_{\text{tree}} = m_{\text{tree}} a$   
 $\text{tension} - 1025 = 250 \times 1.5$   
 $\text{tension} = 375 + 1025 = 1400 \text{ N}$   
 The rope will not break as the tension is less than the breaking strength.
- 9 a A, the frictional force is opposite to the velocity.  
 b C, perpendicular to the slope.  
 c  $F_{\text{net}} = 0$  so  $F_{\text{f}} = F_{\text{g}} \sin \theta = 100 \times 9.8 \times \sin 30^\circ = 490 \text{ N}$  up the hill.  
 d Acceleration  $a = g \sin \theta = 9.8 \times \sin 30^\circ = 4.9 \text{ m s}^{-2}$   
 e Acceleration is not affected by mass if there is no friction.
- 10 A, B and D. The weight and normal force act on the same body, so they cannot be an action–reaction pair. All other options are true of normal reaction forces.
- 11 A.  $F_{\text{N}} = F_{\text{g}} \cos \theta$ , thus the normal force is less than the weight. It is a component of the *weight* that causes the acceleration.

## Section 5.2

### Worked example: Try yourself 5.2.1

#### CALCULATING SPEED

A water wheel has blades 2.0 m in length that rotate at a frequency of 10 revolutions per minute. At what speed do the tips of the blades travel? Express your answer in $\text{km h}^{-1}$ .	
<b>Thinking</b>	<b>Working</b>
Calculate the period, $T$ . Remember to express frequency in the correct units. Alternatively, recognise that 10 revolutions in 60 seconds means that each revolution takes 6 seconds.	10 revolutions per minute $= \frac{10}{60} = 0.167 \text{ Hz}$ $T = \frac{1}{f}$ $= \frac{1}{0.167} = 6 \text{ s}$
Substitute $r$ and $T$ into the formula for speed and solve for $v$ .	$v = \frac{2\pi r}{T}$ $= \frac{2 \times \pi \times 2.0}{6}$ $= 2.09 \text{ m s}^{-1}$
Convert $\text{m s}^{-1}$ into $\text{km h}^{-1}$ by multiplying by 3.6.	$2.09 \times 3.6 = 7.5 \text{ km h}^{-1}$

**Worked example: Try yourself 5.2.2****CENTRIPETAL FORCES**

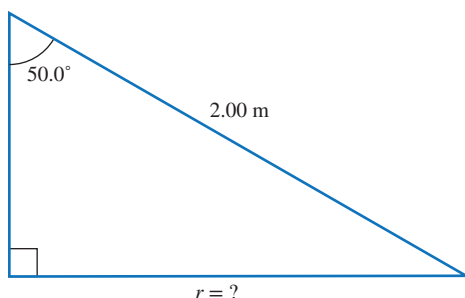
An athlete in a hammer throw event is swinging the ball of mass 7.0 kg in a horizontal circular path. The ball is moving at  $25 \text{ m s}^{-1}$  in a circle of radius 1.2 m.

(a) Calculate the magnitude of the acceleration of the ball.	
<b>Thinking</b>	<b>Working</b>
As the object is moving in a circular path, the centripetal acceleration is towards the centre of the circle. To find the magnitude of this acceleration, write down the other variables that are given.	$v = 25 \text{ m s}^{-1}$ $r = 1.2 \text{ m}$ $a = ?$
Find the equation for centripetal acceleration that fits the information you have, and substitute the values.	$a = \frac{v^2}{r}$ $= \frac{25^2}{1.2}$ $= 521 \text{ m s}^{-2}$
Calculate the magnitude only, so no direction is needed in the answer.	Acceleration of ball is $521 \text{ m s}^{-2}$

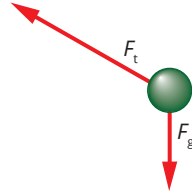
(b) Calculate the magnitude of the tensile force acting in the wire.	
<b>Thinking</b>	<b>Working</b>
Identify the unbalanced force that is causing the object to move in a circular path. Write down the information that you are given.	$m = 7.0 \text{ kg}$ $a = 521 \text{ m s}^{-2}$ $F_{\text{net}} = ?$
Select the equation for centripetal force, and substitute the variables you have.	$F_{\text{net}} = ma$ $= 7.0 \times 521$ $= 3.6 \times 10^3 \text{ N}$
Calculate the magnitude only, so no direction is needed in the answer.	The force of tension in the wire is the unbalanced force that is causing the ball to accelerate. Tensile force $F_T = 3.6 \times 10^3 \text{ N}$

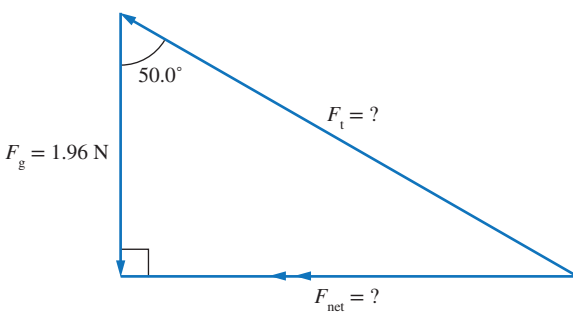
**Worked example: Try yourself 5.2.3****OBJECT ON THE END OF A STRING**

During a game of Totem Tennis, the ball of mass 200 g is swinging freely in a horizontal circular path. The cord is 2.00 m long and is at an angle of  $50.0^\circ$  to the vertical shown in the diagram.



(a) Calculate the radius of the ball's circular path.	
<b>Thinking</b>	<b>Working</b>
The centre of the circular path is not the top end of the cord, but is where the pole is level with the ball. Use trigonometry to find the radius.	$r = 2.00 \sin 50.0^\circ = 1.53 \text{ m}$

(b) Draw and identify the forces that are acting on the ball at the instant shown in the diagram.	
<b>Thinking</b>	<b>Working</b>
There are two forces acting—the tension in the cord, $F_t$ , and gravity, $F_g$ . These forces are unbalanced.	

(c) Determine the net force that is acting on the ball at this time.	
<b>Thinking</b>	<b>Working</b>
First calculate the weight force, $F_g$ .	$F_g = mg$ $= 0.200 \times 9.8$ $= 1.96 \text{ N}$
The ball has an acceleration that is towards the centre of its circular path. This is horizontal and towards the left at this instant. The net force will also lie in this direction at this instant. A force triangle and trigonometry can be used here.	 $F_{\text{net}} = 1.96 \tan 50.0^\circ = 2.34 \text{ N towards the left}$

(d) Calculate the size of the tensile force in the cord.	
<b>Thinking</b>	<b>Working</b>
Use trigonometry to find $F_t$ .	$F_t = \frac{1.96}{\cos 50.0^\circ}$ $= 3.05 \text{ N}$

## 5.2 review

- B. A sideways force of *friction* between the road and tyres is enabling the car to travel in a circle.
- $$T = \frac{1}{f}$$

$$= \frac{1}{5}$$

$$= 0.2 \text{ s}$$
- A and D. The speed is constant, but the velocity is changing as the direction is constantly changing. The acceleration is directed towards the centre of the circle.
- 8.0 m s<sup>-1</sup>
  - 8.0 m s<sup>-1</sup> south
  - $a = \frac{v^2}{r} = \frac{(8.0)^2}{9.2} = 7.0 \text{ m s}^{-2}$  towards the centre, i.e. west
- $F_{\text{net}} = ma = 1200 \times 7.0 = 8.4 \times 10^3 \text{ N west}$



- 6 a  $8.0 \text{ m s}^{-1}$  north  
b towards the centre, i.e. east
- 7 The force needed to give the car a larger centripetal acceleration will eventually exceed the maximum frictional force that could act between the tyres and the road surface. At this time, the car would skid out of its circular path.
- 8 a  $a = \frac{v^2}{r}$   
 $= \frac{(2.0)^2}{1.5}$   
 $= 2.67 \text{ m s}^{-2}$   
 b The skater has an acceleration so forces are unbalanced.  
 c  $F_{\text{net}} = ma = 50 \times 2.7$   
 $= 135 \text{ N}$
- 9 a  $v = 50 \text{ km h}^{-1}$   
 $= \frac{50}{3.6} = 13.89 \text{ m s}^{-1}$   
 $= \frac{2\pi r}{T}$   
 $T = \frac{2\pi r}{v} = \frac{2 \times \pi \times 62}{13.89} = 28 \text{ s}$   
 b  $F_{\text{net}} = \frac{mv^2}{r}$   
 $= \frac{1.6 \times 13.89^2}{62}$   
 $= 5.0 \text{ N}$
- 10 a  $T = \frac{1}{f}$   
 $= \frac{1}{2.0}$   
 $= 0.5 \text{ s}$   
 b  $v = \frac{2\pi r}{T}$   
 $= \frac{2 \times \pi \times 0.80}{0.5}$   
 $= 10 \text{ m s}^{-1}$   
 c  $a = \frac{v^2}{r}$   
 $= \frac{(10)^2}{0.8}$   
 $= 125 \text{ m s}^{-2}$   
 d  $F_{\text{net}} = ma$   
 $= 2.5 \times 125$   
 $= 310 \text{ N}$
- 11 a  $r = 2.4 \cos 60^\circ = 1.2 \text{ m}$   
 b The forces are her weight acting vertically and the tension in the rope acting along the rope towards the top of the maypole.  
 c She has an acceleration directed towards point B, the centre of her circular path.  
 d Use a force triangle for the girl, showing the net force towards B.  
 $F_{\text{net}} = \frac{mg}{\tan 60^\circ} = \frac{294}{1.73} = 170 \text{ N towards B}$   
 e  $F_{\text{net}} = \frac{mv^2}{r}$   
 $170 = \frac{30 \times v^2}{1.2}$   
 $v = 2.6 \text{ m s}^{-1}$

## Section 5.3

### Worked example: Try yourself 5.3.1

#### BANKED CORNERS

A curved section of track on an Olympic velodrome has radius of 40 m and is banked at an angle of  $37^\circ$  to the horizontal. A cyclist of mass 80 kg is riding on this section of track at the design speed.

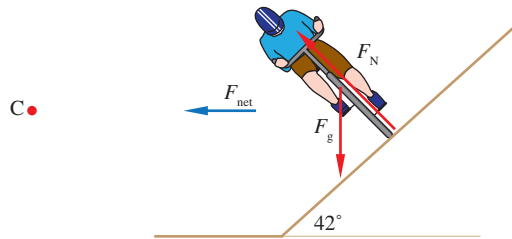
(a) Calculate the net force acting on a cyclist at this instant as they are riding at the design speed.	
<b>Thinking</b>	<b>Working</b>
Draw a force diagram and include all forces acting on the cyclist.	
Calculate the weight force, $F_g$ .	$F_g = mg$ $= 80 \times 9.8$ $= 784 \text{ N}$
Use the force triangle and trigonometry to work out the net force, $F_{\text{net}}$ .	$\tan \theta = \frac{F_{\text{net}}}{F_g}$ $\tan 37^\circ = \frac{F_{\text{net}}}{784}$ $F_{\text{net}} = 0.75 \times 784$ $= 590 \text{ N}$
As force is a vector, a direction is needed in the answer.	Net force is 590 N towards the centre of the circle
(b) Calculate the design speed for this section of the track.	
<b>Thinking</b>	<b>Working</b>
Write down all the known values.	$m = 80 \text{ kg}$ $r = 40 \text{ m}$ $\theta = 37^\circ$ $F_g = 784 \text{ N}$ $F_{\text{net}} = 590 \text{ N}$ $v = ?$
Use the design speed formula.	$v = \sqrt{rg \tan \theta}$ $= \sqrt{40 \times 9.8 \times \tan 37^\circ}$ $= 17 \text{ m s}^{-1}$

## 5.3 review

- In all circular motion, the acceleration is directed towards the centre of the circle.
- The design speed depends on  $\tan \theta$  and the radius of the curve, therefore the architect could make the banking angle larger or increase the radius of the track.
- The car will travel higher up the banked track as the greater speed means that a greater radius is required in the circular path. When travelling faster than the design speed the normal force is not sufficient to keep the car moving in a circle and causes the car to move outward from the centre.

- 4 On the horizontal track, the car is depending on the force of **friction** to turn the corner. The **normal** force is equal to the **weight** of the car so these vertical forces are **balanced**. When driving on the banked track, the **normal** force is not vertical and so is not balanced by the **weight** force. In both cases, the forces acting on the car are unbalanced.

5



$$\begin{aligned}
 6 \quad v &= \sqrt{rg \tan \theta} \\
 &= \sqrt{28 \times 9.8 \times \tan 33^\circ} \\
 &= 13.3 \text{ m s}^{-1} \\
 &= 13.3 \times 3.6 \\
 &= 48 \text{ km h}^{-1}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad \text{a} \quad F_N &= \frac{mg}{\cos 33^\circ} \\
 &= \frac{539}{\cos 33^\circ} \\
 &= 640 \text{ N}
 \end{aligned}$$

- b On a horizontal track,  $F_N$  is equal and opposite to the weight force, so  $F_N = mg = 539 \text{ N}$ . This is less than the normal force on the banked track (643 N).

$$\begin{aligned}
 8 \quad \theta &= \tan^{-1} \left( \frac{v^2}{rg} \right) \\
 &= \tan^{-1} \left( \frac{40^2}{150 \times 9.8} \right) \\
 &= 47^\circ
 \end{aligned}$$

$$\begin{aligned}
 9 \quad \text{a} \quad F_{\text{net}} &= \frac{mv^2}{r} \\
 &= \frac{1200 \times 18^2}{80} \\
 &= 4860 \\
 &= 4.9 \text{ kN}
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad \theta &= \tan^{-1} \left( \frac{v^2}{rg} \right) \\
 &= \tan^{-1} \left( \frac{18^2}{80 \times 9.8} \right) \\
 &= 22^\circ
 \end{aligned}$$

- 10 Since the angle of bank ( $\theta$ ) is fixed, an increasing  $v$  increases  $r$  for constant  $\theta$  as  $\theta = \tan^{-1} \left( \frac{v^2}{rg} \right)$ .

A greater radius will make the car travel higher up the banked track. The driver would have to turn the front wheels slightly towards the bottom of the bank.

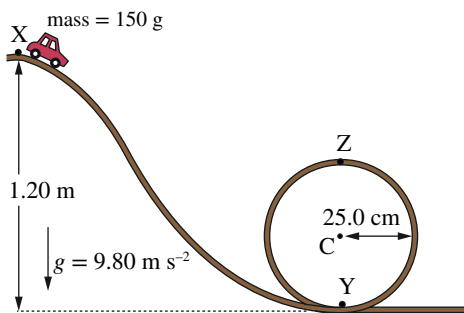
## Section 5.4

### Worked example: Try yourself 5.4.1

#### VERTICAL CIRCULAR MOTION

A student arranges a toy car track with a vertical loop of radius 25.0 cm, as shown.

A toy car of mass 150 g is released from rest at a height of 1.20 m at point X. The car rolls down the track and travels around the loop. Assume  $g$  is  $9.80 \text{ m s}^{-2}$ , and ignore friction for the following questions.



(a) Calculate the speed of the car as it reaches the bottom of the loop, point Y.

Thinking	Working
Note all the variables given to you in the question.	At X: $m = 150 \text{ g} = 0.150 \text{ kg}$ $\Delta h = 1.20 \text{ m}$ $v = 0$ $g = 9.80 \text{ m s}^{-2}$
Use an energy approach to calculate the speed. Calculate the total mechanical energy first.	The initial speed is zero, so $E_k$ at X is zero. Mechanical energy, $E_m$ , at X is: $E_m = E_k + E_g$ $= \frac{1}{2}mv^2 + mg\Delta h$ $= 0 + (0.150 \times 9.80 \times 1.20)$ $= 1.76 \text{ J}$
Use conservation of energy ( $E_m = E_k + E_g$ ) to determine the velocity at point Y. As the car rolls down the track, it loses its gravitational potential energy and gains kinetic energy. At the bottom of the loop (Y), the car has zero potential energy.	At Y: $E_m = 1.76 \text{ J}$ $h = 0$ $E_g = 0$ $E_m = E_k + E_g$ $= \frac{1}{2}mv^2 + mg\Delta h$ $1.76 = 0.5 \times 0.150v^2 + 0$ $v^2 = 23.5$ $v = \sqrt{23.5}$ $= 4.85 \text{ m s}^{-1}$

(b) Calculate the normal reaction force from the track at point Y.

**Thinking**

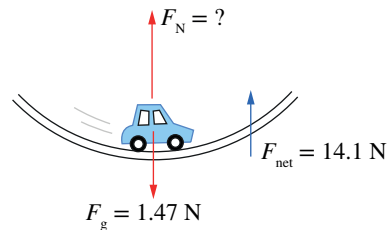
To solve for  $F_N$ , start by working out the net, or centripetal, force. At Y, the car has a centripetal acceleration towards C (i.e., up), so the net (centripetal) force must also be vertically up at this point.

**Working**

$$\begin{aligned} F_{\text{net}} &= \frac{mv^2}{r} \\ &= \frac{0.150 \times 4.85^2}{0.250} \\ &= 14.1 \text{ N up} \end{aligned}$$

Calculate the weight force,  $F_g$ , and add it to a force diagram.

At point Y



$$\begin{aligned} F_g &= mg \\ &= 0.150 \times 9.80 \\ &= 1.47 \text{ N down} \end{aligned}$$

Work out the normal force using vectors. Note up as positive and down as negative for your calculations. These forces are unbalanced, as the car has a centripetal acceleration upwards (towards C). The upwards (normal) force must be larger than the downwards force.

$$\begin{aligned} F_N &= 14.1 + 1.47 \\ &= 15.6 \text{ N up} \end{aligned}$$

(c) What is the speed of the car as it reaches point Z?

**Thinking**

Calculate the velocity from the values you have, using  $E_m = E_k + E_g$ .

**Working**

At Z:  
 $m = 0.150 \text{ kg}$   
 $\Delta h = 2 \times 0.250 = 0.500 \text{ m}$   
 Mechanical energy is conserved, so use the value from part (a).  
 At Z:  

$$E_m = E_k + E_g$$

$$= \frac{1}{2}mv^2 + mg\Delta h$$

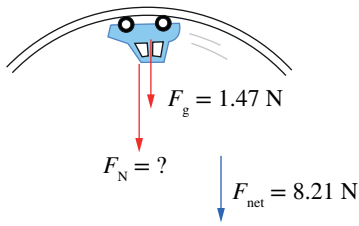
$$1.76 = \frac{1}{2} \times 0.15 \times v^2 + 0.150 \times 9.8 \times 0.500$$

$$1.76 = 0.075 \times v^2 + 0.735$$

$$0.075v^2 = 1.76 - 0.735$$

$$v = \sqrt{13.67}$$

$$= 3.70 \text{ m s}^{-1}$$

(d) What is the normal force acting on the car at point Z?	
<b>Thinking</b>	<b>Working</b>
To find $F_N$ , start by working out the net, or centripetal, force. At Z, the car has a centripetal acceleration towards C (i.e., down), so the net (centripetal) force must also be vertically down at this point.	$F_{\text{net}} = \frac{mv^2}{r}$ $= \frac{0.150 \times 3.70^2}{0.250}$ $= 8.21 \text{ N down}$
Work out the normal force using vectors. Note up as positive and down as negative for your calculations.	<p>At point Z</p>  $F_{\text{net}} = F_g + F_N$ $-8.21 = -1.47 + F_N$ $F_N = -8.2 + 1.47$ $= -6.73$ $= 6.73 \text{ N down}$

## 5.4 review

- It has a constant speed so its centripetal acceleration  $a = \frac{v^2}{r}$  is also constant in magnitude.
  - At the bottom of its path, the yo-yo has an upwards acceleration and so the net force is up. This indicates that the tension force is greater than  $F_g$ .
  - At the top of its path, the yo-yo has a downwards acceleration and so the net force is down. This indicates that the tension force is less than  $F_g$ .
  - At the bottom of its circular path.
- At this point
 
$$a = \frac{v^2}{r} = g \text{ so,}$$

$$v = \sqrt{rg}$$

$$= \sqrt{1.5 \times 9.80}$$

$$= 3.8 \text{ m s}^{-1}$$
- The weight force from gravity and the normal force from the road.
  - $14.4 \text{ km h}^{-1} = 4 \text{ m s}^{-1}$ 

$$F_{\text{net}} = \frac{mv^2}{r} = \frac{800 \times 4^2}{10} = 1280 \text{ N (or } 1.3 \times 10^3 \text{ N)}$$
  - Yes. When the driver is moving over a hump, the normal force is less than her weight  $mg$ . Her apparent weight is given by the normal force that is acting and so the driver feels lighter at this point.
  - At point of lift-off,  $F_N = 0$  and  $a = g$ 

$$a = \frac{v^2}{r} = g \text{ so,}$$

$$v = \sqrt{rg}$$

$$= \sqrt{10 \times 9.80}$$

$$= 9.9 \text{ m s}^{-1}$$

$$= 36 \text{ km h}^{-1}$$

- 4 a At X, mechanical energy is:

$$\begin{aligned}
 E_m &= E_k + E_g \\
 &= \frac{1}{2}mv^2 + mg\Delta h \\
 &= 0.5 \times 500 \times 2.00^2 + 500 \times 9.80 \times 50.0 \\
 &= 1000 + 245\,000 \\
 &= 246\,000 \text{ J}
 \end{aligned}$$

At Y:  $E_g$  is zero so its kinetic energy is 246 000 J

$$\begin{aligned}
 \frac{1}{2}mv^2 &= 246\,000 \\
 0.5 \times 500 \times v^2 &= 246\,000 \\
 v &= \sqrt{984} \\
 &= 31.4 \text{ m s}^{-1}
 \end{aligned}$$

- b At Z, mechanical energy = 246 000 J

$$\begin{aligned}
 E_m &= E_k + E_g \\
 246\,000 &= E_k + 500 \times 9.80 \times 30.0 \\
 246\,000 &= E_k + 147\,000 \\
 E_k &= 99\,000 \text{ J} \\
 0.5 \times 500v^2 &= 99\,000 \\
 v &= 19.9 \text{ m s}^{-1}
 \end{aligned}$$

- c At Z:  $F_g = mg = 500 \times 9.8 = 4900 \text{ N down}$

$$F_{\text{net}} = \frac{mv^2}{r} = \frac{500 \times 19.9^2}{15} = 13\,200 \text{ N down}$$

$$\begin{aligned}
 F_{\text{net}} &= F_N + F_g \\
 13\,200 &= F_N + 4900
 \end{aligned}$$

$$F_N = 8300 \text{ N down}$$

- 5 For the cart to just lose contact at Z,  $F_N = 0$

$$a = \frac{v^2}{r} = g \text{ so,}$$

$$\begin{aligned}
 v &= \sqrt{rg} \\
 &= \sqrt{15.0 \times 9.80} \\
 &= 12.1 \text{ m s}^{-1}
 \end{aligned}$$

- 6  $F_{\text{net}} = F_N + F_g$

$$\frac{mv^2}{r} = F_N + 80 \times 9.80$$

$$\frac{80 \times 35^2}{100} = F_N + 784$$

$$F_N = 980 - 784 = 196 \text{ N down}$$

- 7  $F_{\text{net}} = \frac{mv^2}{r} = F_N + F_g$ , and  $F_N = 0$  when losing contact with seat

$$\frac{80 \times v^2}{100} = 0 + 80 \times 9.8$$

$$v^2 = \frac{784 \times 100}{80}$$

$$= 980$$

$$v = 31.3 \text{ m s}^{-1}$$

Alternatively, use

$$a = \frac{v^2}{r} = g \text{ so,}$$

$$\begin{aligned}
 v &= \sqrt{rg} \\
 &= \sqrt{100 \times 9.80} \\
 &= 31.3 \text{ m s}^{-1}
 \end{aligned}$$

- 8**  $a = \frac{v^2}{r}$  so,  
 $v = \sqrt{rg}$   
 $= \sqrt{400 \times 88.2}$   
 $= 188 \text{ m s}^{-1}$
- 9 a**  $a = \frac{v^2}{r}$   
 $= \frac{6.0^2}{2.0}$   
 $= 18 \text{ m s}^{-2}$  up
- b**  $F_{\text{net}} = \frac{mv^2}{r}$   
 $= \frac{55 \times 6.0^2}{2.0}$   
 $= 990 \text{ N up}$   
 $F_{\text{g}} = mg$   
 $= 55 \times 9.8$   
 $= 540 \text{ N down}$   
 $F_{\text{net}} = F_{\text{N}} + F_{\text{g}}$  (and take down as negative)  
 $990 = F_{\text{N}} - 540$   
 $F_{\text{N}} = 990 + 540$   
 $= 1530 \text{ N up}$
- 10 a** If the ball is just losing contact with track,  $F_{\text{N}} = 0$  so  $F_{\text{net}} = F_{\text{g}}$  and therefore  $a = 9.8 \text{ m s}^{-2}$  down.
- b**  $v = \sqrt{rg}$   
 $= \sqrt{0.5 \times 9.8}$   
 $= 2.2 \text{ m s}^{-1}$
- 11 a**  $F_{\text{net}} = \frac{mv^2}{r}$   
 $= \frac{500 \times 6.0^2}{5.0}$   
 $= 3600$   
 $= 3.6 \times 10^3 \text{ N down}$
- b**  $F_{\text{net}} = F_{\text{N}} + F_{\text{g}}$  (and taking down as negative)  
 $-3600 = F_{\text{N}} - 4900$   
 $F_{\text{N}} = -3600 + 4900$   
 $= 1300 \text{ N}$   
 $= 1.3 \times 10^3 \text{ N up}$
- c**  $v = \sqrt{rg}$   
 $= \sqrt{5.0 \times 9.8}$   
 $= 7.0 \text{ m s}^{-1}$

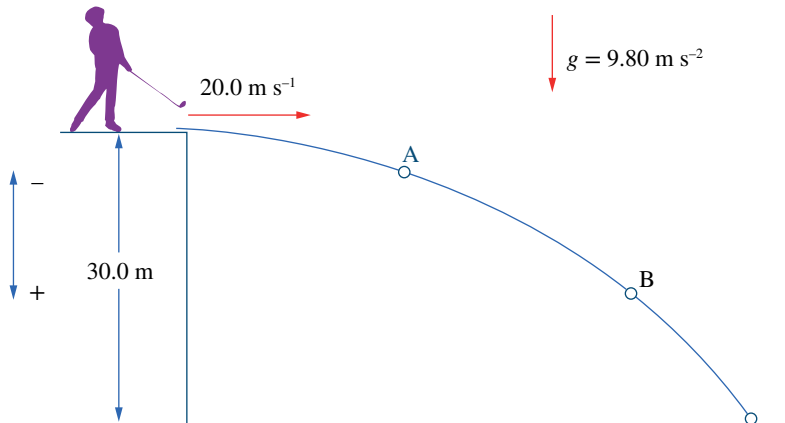


## Section 5.5

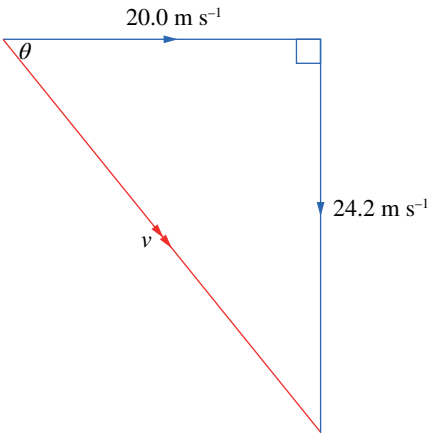
### Worked example: Try yourself 5.5.1

#### PROJECTILE LAUNCHED HORIZONTALLY

A golf ball of mass 100 g is hit horizontally from the top of a 30.0 m high cliff with a speed of 20.0 m s<sup>-1</sup>. Using  $g = 9.80 \text{ m s}^{-2}$  and ignoring air resistance, calculate the following values.



(a) Calculate the time that the ball takes to land.	
<b>Thinking</b>	<b>Working</b>
Let the downward direction be positive. Write out the information relevant to the vertical component of the motion. Note that the instant the ball is hit, it is travelling only horizontally, so its initial vertical velocity is zero.	Down is positive. Vertically: $u = 0 \text{ m s}^{-1}$ $s = 30.0 \text{ m}$ $a = 9.80 \text{ m s}^{-2}$ $t = ?$
In the vertical direction, the ball has constant acceleration, so use equations for uniform acceleration. Select the equation that best fits the information you have.	$s = ut + \frac{1}{2}at^2$
Substitute values, rearrange and solve for $t$ .	$30.0 = 0 + 4.90t^2$ $t = \sqrt{\frac{30.0}{4.90}}$ $= 2.47 \text{ s}$
(b) Calculate the distance that the ball travels from the base of the cliff, i.e. the range of the ball.	
<b>Thinking</b>	<b>Working</b>
Write out the information relevant to the horizontal component of the motion. As the ball is hit horizontally, the initial speed gives the horizontal component of the velocity throughout the flight.	Horizontally: $u = 20.0 \text{ m s}^{-1}$ $t = 2.47 \text{ s}$ from part (a) $s = ?$
Select the equation that best fits the information you have.	As horizontal speed is constant, you can use $v_{av} = \frac{s}{t}$
Substitute values, rearrange and solve for the variable $s$ .	$20.0 = \frac{s}{2.47}$ $s = 20.0 \times 2.47$ $= 49.4 \text{ m}$

(c) Calculate the velocity of the ball as it lands.	
<b>Thinking</b>	<b>Working</b>
Find the horizontal and vertical components of the ball's speed as it lands. Write out the information relevant to both the vertical and horizontal components.	Horizontally: $u = v = 20.0 \text{ m s}^{-1}$ Vertically, with down as positive: $u = 0$ $a = 9.8 \text{ m s}^{-2}$ $s = 30.0 \text{ m}$ $t = 2.47 \text{ s}$ $v = ?$
To find the final vertical speed, $v_v$ , use the equation for uniform acceleration that best fits the information you have.	Therefore, use $v = u + at$
Substitute values, rearrange and solve for the variable you are looking for, in this case $v$ .	$v_v = u + at$ $= 0 + 9.80 \times 2.47$ $= 24.2 \text{ m s}^{-1} \text{ down}$
Add the components as vectors.	
Use Pythagoras' theorem to work out the actual speed, $v$ , of the ball.	$v = \sqrt{v_h^2 + v_v^2}$ $= \sqrt{20.0^2 + 24.2^2}$ $= \sqrt{986}$ $= 31.4 \text{ m s}^{-1}$
Use trigonometry to solve for the angle, $\theta$ .	$\theta = \tan^{-1}\left(\frac{24.2}{20.0}\right)$ $= 50.4^\circ$
Indicate the velocity with magnitude and direction relative to the horizontal.	Final velocity of ball is $31.4 \text{ m s}^{-1}$ at $50.4^\circ$ below the horizontal.

## 5.5 review

1 A and D. Assuming zero air resistance, the only force acting on the stone is gravity, and as a result of the gravity, its vertical speed increases. The resultant overall speed, which is a combination of its horizontal and vertical components, will therefore also increase.

2 a  $v_{av} = \frac{s}{t}$

$$2 = \frac{s}{0.75}$$

$$s = 1.5 \text{ m}$$

- b** Vertically, with down as positive:  $u = 0$ ,  $a = 9.8$ ,  $t = 0.75$ ,  $v = ?$   
 $v = u + at$   
 $= 0 + 9.8 \times 0.75$   
 $= 7.35 \text{ m s}^{-1}$
- c**  $v = \sqrt{2.0^2 + 7.35^2}$   
 $= 7.6 \text{ m s}^{-1}$
- 3 a** Vertically with down as positive:  $u = 0$ ,  $a = 9.8$ ,  $s = 1.2$ ,  $t = ?$   
 $s = ut + \frac{1}{2}at^2$   
 $1.2 = 0 + \frac{1}{2} \times 9.8 \times t^2$   
 $t = \sqrt{\frac{1.2}{4.9}}$   
 $= 0.49 \text{ s}$
- b**  $v_{av} = \frac{s}{t}$   
 $4 = \frac{s}{0.49}$   
 $s = 2.0 \text{ m}$
- c**  $a = 9.8 \text{ m s}^{-2}$  down
- 4 a** Vertically, with down as positive:  $u = 0$ ,  $a = 9.8$ ,  $s = 4.9$ ,  $t = ?$   
 $s = ut + \frac{1}{2}at^2$   
 $4.9 = 0 + \frac{1}{2} \times 9.8 \times t^2$   
 $t = 1.0 \text{ s}$
- b** Horizontally:  $u = 20 \text{ m}$ ,  $t = 1$ ,  $s = ?$   
 $s = v_{av} \times t$   
 $= 20 \times 1.0$   
 $= 20 \text{ m}$
- c** The acceleration of the ball is constant at any time during its flight, and is equal to the acceleration due to gravity  
 $= 9.8 \text{ m s}^{-2}$  down
- d** After 0.80 s, the ball has two components of velocity:  
 $v_x = 20 \text{ m s}^{-1}$   
and  $v_y = u + at = 0 + 9.8 \times 0.80 = 7.84 \text{ m s}^{-1}$   
The speed of the ball at 0.80 s is given by:  
 $\sqrt{20^2 + 7.84^2} = 21.5 \text{ m s}^{-1}$
- e** The ball will hit the ground 1.0 s after it is struck.  
 $v_x = 20 \text{ m s}^{-1}$   
and  $v_y = u + at = 0 + 9.8 \times 1.0 = 9.8 \text{ m s}^{-1}$   
The speed of the ball at 1.0 s is given by:  
 $\sqrt{20^2 + 9.8^2} = 22.3 \text{ m s}^{-1}$
- 5 a** Vertically with down as positive:  $u = 0$ ,  $a = 9.8$ ,  $s = 80$ ,  $v = ?$   
 $v^2 = u^2 + 2as$   
 $= 0 + 2 \times 9.8 \times 80$   
 $v = 39.6 \text{ m s}^{-1}$   
Horizontally:  $25 \text{ m s}^{-1}$   
 $v = \sqrt{25^2 + 39.6^2} = 47 \text{ m s}^{-1}$
- b**  $\tan \theta = \frac{39.6}{25}$   
 $\theta = 58^\circ$
- 6** B and C. No atmosphere means no drag, and so the balls travelled in parabolic paths and went much further than they would on Earth.
- 7** The hockey ball travels further. A polystyrene ball is much lighter and is therefore more strongly affected by air resistance than the hockey ball.

- 8 a Vertically with down as positive:  $u = 0$ ,  $a = 9.8$ ,  $s = 2.0$ ,  $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$2.0 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$t = \sqrt{\frac{2.0}{4.9}}$$

$$= 0.64 \text{ s}$$

- b There is no difference in the time to fall for either ball. Therefore, 0.64 s.

- c Ball X:

$$v_{av} = \frac{s}{t}$$

$$5 = \frac{s}{0.64}$$

$$s = 3.2 \text{ m}$$

Ball Y:

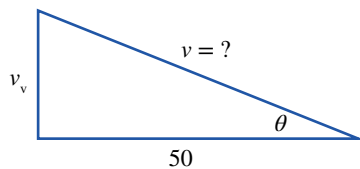
$$v_{av} = \frac{s}{t}$$

$$10 = \frac{s}{0.64}$$

$$s = 6.4 \text{ m}$$

Difference is  $6.4 - 3.2 = 3.2 \text{ m}$

- 9 a



Vertically with down as positive:  $u = 0$ ,  $a = 9.8$ ,  $s = 20$ ,  $v = ?$

$$v^2 = u^2 + 2as$$

$$= 0 + 2 \times 9.8 \times 20$$

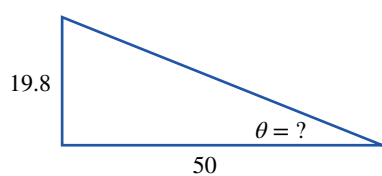
$$v = 19.8 \text{ m s}^{-1}$$

Horizontally:  $u = v = 50 \text{ m s}^{-1}$

$$v = \sqrt{19.8^2 + 50^2}$$

$$= 54 \text{ m s}^{-1}$$

- b



$$\tan \theta = \frac{19.8}{50}$$

$$\theta = 22^\circ$$

- 10 a The horizontal velocity of the ball remains constant and  $v_h = 10 \text{ m s}^{-1}$  forwards.

- b Vertically with down as positive:  $u = 0$ ,  $a = 9.8$ ,  $s = 1.0$ ,  $v = ?$

$$v^2 = u^2 + 2as$$

$$= 0^2 + 2 \times 9.8 \times 1.0$$

$$v_v = 4.4 \text{ m s}^{-1} \text{ down}$$

- c  $v_v = \sqrt{4.4^2 + 10^2} = 10.9 \text{ m s}^{-1}$

$$\tan \theta = \frac{4.4}{10}$$

$$\theta = 24^\circ$$

$v = 10.9 \text{ m s}^{-1}$  at  $24^\circ$  below the horizontal

d Vertically with down as positive:  $u = 0, a = 9.8, s = 1.0, t = ?$

$$s = ut + \frac{1}{2}at^2$$

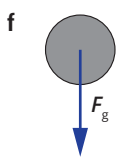
$$1.0 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$t = \sqrt{\frac{1}{4.9}}$$

$$t = 0.45 \text{ s}$$

e Horizontally:  $v = u = 10, t = 0.45, s = ?$

$$\begin{aligned} s &= v_{av} \times t \\ &= 10 \times 0.45 \\ &= 4.5 \text{ m} \end{aligned}$$

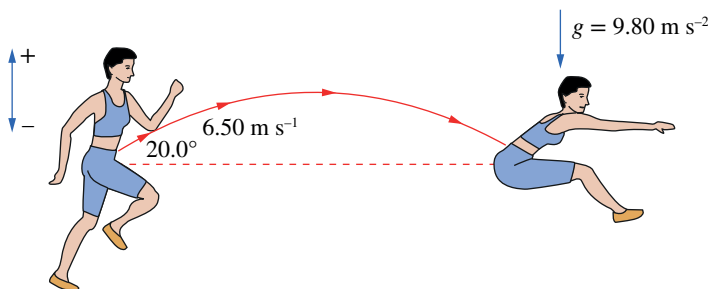


## Section 5.6

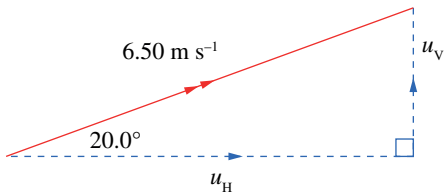
### Worked example: Try yourself 5.6.1

#### LAUNCH AT AN ANGLE

A 50 kg athlete in a long-jump event leaps with a velocity of  $6.50 \text{ m s}^{-1}$  at  $20.0^\circ$  to the horizontal.



For the following questions, treat the athlete as a point mass, ignore air resistance and use  $g = 9.80 \text{ m s}^{-2}$ .

(a) What is the athlete's velocity at the highest point?	
<p><b>Thinking</b></p> <p>First find the horizontal and vertical components of the initial speed.</p>	<p><b>Working</b></p>  <p>Using trigonometry:  <math>u_H = 6.50 \cos 20.0^\circ</math>                      Taking up as positive:  <math>u_v = 6.50 \sin 20.0^\circ</math></p>
<p>Projectiles that are launched obliquely move only horizontally at the highest point. The vertical component of the velocity at this point is therefore zero. The actual velocity is given by the horizontal component of the velocity throughout the motion.</p>	<p>At maximum height: <math>v = 6.11 \text{ m s}^{-1}</math> horizontally to the right.</p>

(b) What is the maximum height gained by the athlete's centre of mass during the jump?	
<b>Thinking</b>	<b>Working</b>
To find the maximum height that is gained, you must work with the vertical component. Recall that at the maximum height, the vertical component of velocity is zero.	Vertically, taking up as positive: $u = 2.22$ $a = -9.80$ $v = 0$ $s = ?$
Substitute these values into an appropriate equation for uniform acceleration.	$v^2 = u^2 + 2as$ $0 = 2.22^2 + -9.8 \times s$
Rearrange and solve for $s$ .	$s = \frac{2.22^2}{19.6}$ $= 0.25 \text{ m}$

(c) Assuming a return to the original height, what is the total time the athlete is in the air?	
<b>Thinking</b>	<b>Working</b>
As the motion is symmetrical, the time required to complete the motion will be double that taken to reach the maximum height. First, the time it takes to reach the highest point must be found.	Taking up as positive Vertically: $u = 2.22$ $a = -9.80$ $v = 0$ $t = ?$
Substitute these values into an appropriate equation for uniform acceleration.	$v = u + at$ $0 = 2.22 - 9.80t$
Solve for $t$ needed to reach maximum height.	$t = \frac{2.22}{9.80}$ $= 0.227 \text{ s}$
The time to complete the motion is double the time it takes to reach the maximum height.	Total time = $2 \times 0.227$ $= 0.45 \text{ s}$

## 5.6 review

- B. A javelin travels fastest at launch, then slows on the way up, is *slowest* at the highest point, then speeds up on the way down. At the highest point, the vertical component of the velocity is zero.
- The optimal launch angle to give the greatest range for any projectile on the ground is  $45^\circ$ .
- At the highest point the ball has zero vertical velocity. The horizontal velocity is constant throughout the flight when air resistance is ignored. So the overall velocity at the highest point is equal to the horizontal speed:  
 $v_H = v \cos \theta = 20 \cos 30^\circ = 17.3 \text{ m s}^{-1}$
- $v_H = v \cos \theta = 15 \cos 25^\circ = 13.6 \text{ m s}^{-1}$
  - $v_V = v \sin \theta = 15 \sin 25^\circ = 6.34 \text{ m s}^{-1}$
  - The acceleration is constant and is due to the force of gravity. The acceleration is  $9.8 \text{ m s}^{-2}$  down.
  - At the highest point the ball has zero vertical velocity. The horizontal velocity is constant throughout the flight when air resistance is ignored. So the overall velocity at the highest point is equal to the horizontal velocity:  $13.6 \text{ m s}^{-1}$ .

- 5
- a**  $v_H = v \cos \theta = 8 \cos 60^\circ = 4.0 \text{ m s}^{-1}$
- b**  $v_V = v \sin \theta = 8 \sin 60^\circ = 6.9 \text{ m s}^{-1}$
- c** Vertically, with up as positive:  $u = 6.9, a = -9.8, v = 0, t = ?$   
 $v = u + at$   
 $0 = 6.9 - 9.8t$   
 $t = 0.70 \text{ s}$
- d** Vertically with up as positive:  $u = 6.9, a = -9.8, v = 0, s = ?$   
 $v^2 = u^2 + 2as$   
 $0 = 6.9^2 + 2 \times -9.8 \times s$   
 $6.9^2 = 19.6s$   
 $s = 2.4 \text{ m}$   
 Total height =  $2.4 + 1.5 = 3.9 \text{ m}$
- e** The speed is given by the horizontal component of the velocity (as the vertical velocity is zero at this point), so  $4.0 \text{ m s}^{-1}$ .
- 6
- a**  $v_H = 28 \cos 30^\circ = 24.2 \text{ m s}^{-1}$  (and remains constant throughout the flight)
- b**  $24.2 \text{ m s}^{-1}$
- c**  $24.2 \text{ m s}^{-1}$
- 7 Taking up as positive
- a**  $v_V = 28 \sin 30^\circ = 14.0 \text{ m s}^{-1}$
- b** Vertically:  $u = 14, a = -9.8, t = 1, v = ?$   
 $v = u + at$   
 $= 14 - 9.8 \times 1.0$   
 $= 4.20 \text{ m s}^{-1}$
- c** Vertically:  $u = 14, a = -9.8, t = 2 \text{ s}, v = ?$   
 $v = u + at$   
 $= 14 - 9.8 \times 2$   
 $= -5.60 = 5.60 \text{ m s}^{-1} \text{ down}$
- 8  $v = \sqrt{5.60^2 + 24.2^2}$   
 $= 24.8 \text{ m s}^{-1}$
- 9 The flight of the ball is symmetrical. Therefore the ball will strike the ground at the same speed as that when it was launched:  $28 \text{ m s}^{-1}$  (at an angle of  $30^\circ$  to the horizontal).
- 10 Vertically with up as positive:  $u = 14 \text{ m}, a = -9.8, v = 0, t = ?$   
 $v = u + at$   
 $0 = 14 - 9.8t$   
 $t = 1.43 \text{ s}$   
 Total time is therefore  $2 \times 1.43 = 2.86 \text{ s}$   
 $v_{av} = \frac{s}{t}$   
 $24.2 = \frac{s}{2.86}$   
 $s = 69.2 \text{ m}$
- 11 C. Air resistance is a force that would be acting in the opposite direction to the velocity of the ball, thereby producing a horizontal and vertical deceleration of the ball during its flight.

## Section 5.7

## Worked example: Try yourself 5.7.1

## CONSERVATION OF MOMENTUM

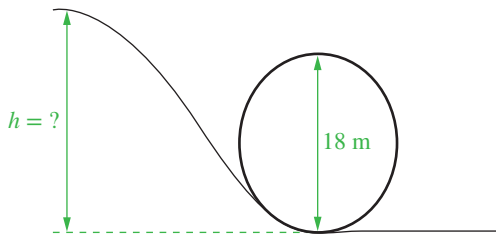
In a head-on collision on a freeway, a car of mass  $1.0 \times 10^3$  kg travelling east at  $20 \text{ m s}^{-1}$  crashes into a bus of mass  $5.0 \times 10^3$  kg travelling west at  $8.0 \text{ m s}^{-1}$ . Assume the car and bus lock together on impact, and ignore friction.

(a) Calculate the final common velocity of the vehicles.	
<b>Thinking</b>	<b>Working</b>
First assign a direction that will be considered positive.	In this case we consider vectors directed eastwards to be positive.
Use the law of conservation of momentum.	$\Sigma p_i = \Sigma p_f$ $m_c u_c + m_b u_b = (m_c + m_b)v$ $1.0 \times 10^3 \times 20 = 5.0 \times 10^3 \times -8.0 = (1.0 \times 10^3 + 5.0 \times 10^3) \times v$ $-20 \times 10^3 = 6 \times 10^3 v$ $v = -3.33 \text{ m s}^{-1}$ so, $v = 3.3 \text{ m s}^{-1}$ west
(b) Calculate the change in momentum of the car.	
<b>Thinking</b>	<b>Working</b>
The change in momentum of the car is its final momentum minus its initial momentum.	$\Delta p_c = p_{c(\text{final})} - p_{c(\text{initial})}$ $= m_c(v - u_c)$ $= 1.0 \times 10^3(-3.33 - 20)$ $= -2.3 \times 10^4 \text{ kg m s}^{-1}$ That is, $\Delta p_c = 2.3 \times 10^4 \text{ kg m s}^{-1}$ west
(c) Calculate the change in momentum of the bus.	
<b>Thinking</b>	<b>Working</b>
The change in momentum of the bus is its final momentum minus its initial momentum.	$\Delta p_b = p_{b(\text{final})} - p_{b(\text{initial})}$ $= m_b(v - u_b)$ $= 5.0 \times 10^3(-3.33 - -8.0)$ $= -2.3 \times 10^4 \text{ kg m s}^{-1}$ That is, $\Delta p_b = 2.3 \times 10^4 \text{ kg m s}^{-1}$ east
(d) Verify that the momentum of the system is constant.	
<b>Thinking</b>	<b>Working</b>
The total change in momentum is the <i>vector sum</i> of the momentum changes of the parts. This is expected to be zero from the conservation of momentum.	$\Sigma p_c + \Sigma p_b = -2.3 \times 10^4 + 2.3 \times 10^4 = 0$ Therefore the momentum of the system is constant, as expected.



**Worked example: Try yourself 5.7.2****CONSERVATION OF ENERGY**

Use the law of conservation of energy to determine the height of the lift hill required to ensure that the speed of the car at the top of the 18.0 m loop is 25.0 m s<sup>-1</sup>. Assume that the velocity of the car at the top of the lift hill is zero; then it begins to roll down the hill. Give your answer correct to three significant figures.



Thinking	Working
Calculate the total mechanical energy of the car at the top of the loop in terms of the mass $m$ .	$E_m = mg\Delta h + \frac{1}{2}mv^2$ $= (m \times 9.8 \times 18) + (\frac{1}{2} \times m \times 25^2)$ $= 176.4m + 312.5m$ $= 488.9m \text{ J}$
Use conservation of energy to find the height required for the lift hill, remembering that the total potential energy at the lift hill is equal to the total energy at the top of the loop	$mg\Delta h = 488.9m$ $\Delta h = \frac{488.9}{9.8}$ $= 49.9 \text{ m}$

**5.7 review**

- The billiard balls form an isolated system. Momentum is conserved, so if the momentum is zero after the collision, it was initially zero as well. This is possible because the two balls were initially travelling in opposite directions and their momentum vectors cancelled out to give zero.
- Let motion towards the east be positive.
 
$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$175 \times 3.5 + 100 \times (-5.0) = 275 \times v$$

$$v = \frac{112.5}{275}$$

$$= 0.41 \text{ m s}^{-1} \text{ east}$$
- $$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$110 \times 15 + 90 \times -5 = 200 \times v$$

$$1200 = 200v$$

$$v = 6.0 \text{ m s}^{-1} \text{ (speed only so no direction needed)}$$
- sports car's speed =  $\frac{36}{3.6} = 10 \text{ m s}^{-1}$   

$$p = mv = 1.0 \times 10^3 \times 10 = 1.0 \times 10^4 \text{ kg m s}^{-1} \text{ east}$$
  - station wagon's speed =  $\frac{18}{3.6} = 5 \text{ m s}^{-1}$   

$$p = mv = 2.0 \times 10^3 \times 5.0 = 1.0 \times 10^4 \text{ kg m s}^{-1} \text{ west}$$
  - Total momentum =  $1.0 \times 10^4 \text{ kg m s}^{-1} \text{ east} + 1.0 \times 10^4 \text{ kg m s}^{-1} \text{ west} = 0$
- $\Sigma p_i = \Sigma p_f$   
From Question 4c,  $\Sigma p_i = 0$  so  $\Sigma p_f = 0$ , i.e. common velocity = 0
  - It hasn't gone anywhere. The vehicles had a total of zero momentum before the collision and so there still is zero momentum after the collision.
  - The change in momentum of the sports car is  $\Delta p_c = p_{c(\text{final})} - p_{c(\text{initial})} = 0 - 1.0 \times 10^4 \text{ kg m s}^{-1} \text{ east}$   

$$= 1.0 \times 10^4 \text{ kg m s}^{-1} \text{ west}$$
  - The change in momentum of the station wagon is  $\Delta p_w = p_{w(\text{final})} - p_{w(\text{initial})} = 0 - 1.0 \times 10^4 \text{ kg m s}^{-1} \text{ west}$   

$$= 1.0 \times 10^4 \text{ kg m s}^{-1} \text{ east}$$

- 6 Using the conservation of momentum, where the direction of positive velocity is to the right:

$$\Sigma p_i = \Sigma p_f$$

Taking to the right as positive:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$0.2 \times 9.0 + 0.1 \times 0 = 0.2 \times 3.0 + 0.1 \times v$$

$$1.8 = 0.6 + 0.1v$$

$$0.1v = 1.2$$

$$v = 12 \text{ m s}^{-1}$$

i.e. the velocity of the 100 g ball after the collision is 12 m s<sup>-1</sup> to the right

- 7 Before firing,  $\Sigma p_i = 0$  as neither the cannon nor shell are moving.

Taking the shell's direction of travel after firing as positive:

$$\Sigma p_i = \Sigma p_f$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$10.0 \times 0 + 1000 \times 0 = 10.0 \times 500 + 1000 \times v$$

$$0 = 5000 + 1000v$$

$$v = -5$$

and so the recoil velocity of the cannon is  $v = 5.0 \text{ m s}^{-1}$  in the opposite direction to the shell.

- 8 Taking to the right as positive:

$$\Sigma p_i = \Sigma p_f$$

$$0.1 \times 40 + 0.08 \times 0 = (0.1 + 0.08) \times v$$

$$4 = 0.18v$$

$$v = 22 \text{ m s}^{-1} \text{ to the right}$$

- 9  $E_k = \frac{1}{2}mv^2$

$$= \frac{1}{2} \times 0.0400 \times (370)^2$$

$$= 2740 \text{ J}$$

- 10  $E_g = mg\Delta h$

$$= 0.0400 \times 9.8 \times 1000$$

$$= 392 \text{ J}$$

- 11 a  $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.157 \times (20.5)^2 = 33.0 \text{ J}$

b  $E_g = E_k = 33.0 \text{ J}$

c  $E_g = mg\Delta h$

$$33.0 = 0.157 \times 9.8 \times \Delta h$$

$$\Delta h = 21.4 \text{ m}$$

## CHAPTER 5 REVIEW

1 B. The ball will increase in speed at a constant rate, that is, with constant acceleration.

2 a The only force acting on the block on the table is tension:

$$F_T = m_1 a = 5a$$

The forces acting on the falling block are tension and weight:

$$F_g - F_T = m_2 a$$

Substitute the expression for  $F_T$  into the equation above:

$$10 \times 9.8 - 5a = 10a$$

$$98 = 15a$$

$$a = 6.5 \text{ m s}^{-2}$$

b  $F_T = 5a = 5 \times 6.5 = 32.5 \text{ N}$

3  $F_{\text{net}} = \text{thrust} - \text{drag forces} = m_{\text{total}} a$

$$\text{thrust} - (800 + 700) = (1000 + 200) \times 2.5$$

$$\text{thrust} = 3000 + 1500$$

$$= 4500 = 4.5 \times 10^3 \text{ N}$$

4 a  $a = g \sin \theta$   
 $= 9.8 \sin 30^\circ$   
 $= 4.9 \text{ m s}^{-2}$

b As  $F_N = F_g \cos \theta$ , the normal reaction force must be less than the weight force.

$$F_N = F_g \cos \theta$$

$$F_N = F_g \cos 30^\circ$$

$$F_N = 0.87 F_g$$

5 a  $a = g \sin \theta$   
 $= 9.8 \sin 30^\circ$   
 $= 4.9 \text{ m s}^{-2}$

b  $u = 0 \text{ m s}^{-1}$ ,  $s = 2.5 \text{ m}$ ,  $a = 4.9 \text{ m s}^{-2}$ ,  $v = ?$

$$v^2 = u^2 + 2as$$

$$= 0 + 2 \times 4.9 \times 2.5$$

$$v = 24.5 \text{ m s}^{-1}$$

6 a  $F_N = mg \cos \theta$   
 $= 57 \times 9.8 \times \cos 65^\circ$   
 $= 236 \text{ N}$

b  $a = g \sin \theta$   
 $= 9.8 \sin 65^\circ$   
 $= 8.88 \text{ m s}^{-2}$  down the ramp

c  $F_{\text{net}} = ma = 57 \times 8.88$   
 $= 506 \text{ N}$  down the ramp

d  $u = 0$ ,  $s = 5.0$ ,  $a = 8.88$ ,  $v = ?$

$$v^2 = u^2 + 2as$$

$$= 0 + 2 \times 8.88 \times 5.0$$

$$= 89$$

$$v = 9.4 \text{ m s}^{-1}$$
 (speed only)

e  $F_{\text{net}} = 0$  so forces parallel to incline are balanced.

$$F_f = mg \sin \theta = 506 \text{ N}$$
 up the ramp

7 a Convert speed to  $\text{m s}^{-1}$ : speed =  $\frac{100}{3.6} = 27.78 \text{ m s}^{-1}$

$$\text{Length of the slide: } s = \frac{50}{\sin 70^\circ} = 53.2 \text{ m}$$

To find  $a$ , use  $v^2 = u^2 + 2as$

$$a = \frac{v^2 - u^2}{2s} = \frac{(27.78)^2}{2 \times 53.20} = 7.25 \text{ m s}^{-2}$$

$$F_{\text{net}} = ma$$

$$= 70.0 \times 7.25 = 508 \text{ N}$$

$$\begin{aligned} \mathbf{b} \quad F_{\text{net}} &= mg \sin 70^\circ - F_{\text{friction}} \\ F_{\text{friction}} &= 70.0 \times 9.80 \times \sin 70^\circ - 508 \\ &= 137 \text{ N} \end{aligned}$$

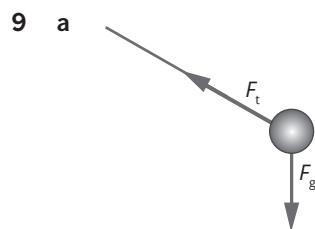
**c** The reaction to the force of the slide on the teenager due to friction is the force of the teenager on the slide.

**d** The reaction of the weight force on the teenager is the force of gravitational attraction from the teenager on the Earth.

$$\begin{aligned} \mathbf{8} \quad \mathbf{a} \quad v &= \frac{2\pi r}{T} \\ &= \frac{2\pi \times 0.800}{1.36} \\ &= 3.70 \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad a &= \frac{v^2}{r} \\ &= \frac{3.70^2}{0.800} \\ &= 17.1 \text{ m s}^{-2} \text{ towards the centre of the circle} \end{aligned}$$

$$\begin{aligned} \mathbf{c} \quad F_{\text{net}} &= ma \\ &= 0.0250 \times 17.1 = 0.430 \text{ N (size only needed)} \end{aligned}$$



**b** Use a force triangle for the ball.

$$\begin{aligned} F_T &= \frac{mg}{\sin 30.0^\circ} \\ &= \frac{0.0250 \times 9.80}{0.50} \\ &= 0.49 \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{10} \quad \mathbf{a} \quad a &= \frac{v^2}{r} \\ &= \frac{5^2}{10} \\ &= 2.5 \text{ m s}^{-2} \text{ towards the centre of the circle} \end{aligned}$$

**b** The centripetal force is created by the friction between the tyres and the ground.

$$\begin{aligned} \mathbf{11} \quad \mathbf{a} \quad v &= \frac{2\pi r}{T} \\ &= \frac{2 \times \pi \times 3.84 \times 10^8}{27.3 \times 24 \times 60 \times 60} \\ &= 1.02 \times 10^3 \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} \mathbf{b} \quad F &= \frac{mv^2}{r} \\ &= \frac{7.36 \times 10^{22} \times (1.02 \times 10^3)^2}{3.84 \times 10^8} \\ &= 1.99 \times 10^{20} \text{ N} \end{aligned}$$

$$\begin{aligned} 12 \text{ Orbital radius} &= 6.37 \times 10^6 \text{ m} + 3.6 \times 10^4 \text{ m} \\ &= 6.406 \times 10^6 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Orbital distance} &= 2\pi r \\ &= 2\pi \times 6.406 \times 10^6 \text{ m} \\ &= 4.025 \times 10^7 \text{ m} \end{aligned}$$

$$\begin{aligned} T &= (23 \times 60 \times 60) + (56 \times 60) + 5 \\ &= 86165 \text{ s} \end{aligned}$$

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{4.025 \times 10^7}{86165} \\ &= 467 \text{ m s}^{-1} \end{aligned}$$

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \frac{(467 \times 10^2)^2}{6.406 \times 10^6} \\ &= 3.40 \times 10^{-2} \text{ m s}^{-2} \end{aligned}$$

13 a  $10 \text{ m s}^{-1}$  south

b  $10 \text{ m s}^{-1}$ . Note direction is not required as question asked for speed.

$$\begin{aligned} \text{c } v &= \frac{2\pi r}{T} \\ T &= \frac{2\pi r}{v} \\ &= \frac{2 \times \pi \times 20}{10} \\ &= 13 \text{ s} \end{aligned}$$

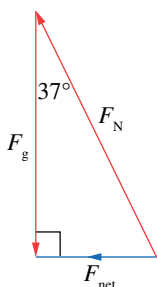
$$\begin{aligned} \text{d } a &= \frac{v^2}{r} \\ &= \frac{10^2}{20} \\ &= 5.0 \text{ m s}^{-2} \text{ west} \end{aligned}$$

$$\begin{aligned} \text{e } F_{\text{net}} &= ma \\ &= 1500 \times 5 \\ &= 7.5 \times 10^3 \text{ N west} \end{aligned}$$

14 Recall the equation for force on a moving charge. This provides the centripetal force, so is equal to  $\frac{mv^2}{r}$ .

$$\begin{aligned} qvB &= \frac{mv^2}{r} \\ r &= \frac{mv}{qB} \\ &= \frac{1.67 \times 10^{-27} \times 3.50 \times 10^6}{1.60 \times 10^{-19} \times 0.25} \\ &= 0.146 \text{ m} \end{aligned}$$

15 A. From the triangle,  $F_N > F_g$ .



16 Use a force triangle with weight, normal and net force (acting horizontally).

$$\begin{aligned} v &= \sqrt{rg \tan \theta} \\ &= \sqrt{30 \times 9.8 \times \tan 40^\circ} \\ &= 15.7 \text{ m s}^{-1} \end{aligned}$$

17 a (i) At top:

$$F_{\text{net}} = \frac{mv^2}{r}$$

$$= \frac{50 \times 5.0^2}{10}$$

$$= 125 \text{ N down}$$

$$F_{\text{N}} = F_{\text{g}} - 125$$

$$= 490 - 125$$

$$= 365 \text{ N up}$$

(ii) At bottom:

$$F_{\text{net}} = \frac{mv^2}{r}$$

$$= \frac{50 \times 5.0^2}{10}$$

$$= 125 \text{ N up}$$

$$F_{\text{N}} = F_{\text{g}} + 125$$

$$= 490 + 125$$

$$= 615 \text{ N up}$$

b D. At the top of the ride,  $F_{\text{N}} < F_{\text{g}}$  so he would feel lighter than usual.

18 The forces acting on the water when the bucket is directly overhead are the force of gravity (weight) and the normal force from the base of the bucket on the water.

Both of these forces are downwards acting forces.

The acceleration of the water is towards the centre of the circle, i.e. downwards and is greater than the acceleration due to gravity.

19 a  $v_{\text{av}} = \frac{s}{t}$

$$s = v_{\text{av}} \times t$$

$$= 2.5 \times 1$$

$$= 2.5 \text{ m}$$

b  $9.8 \text{ m s}^{-2}$  downwards (due to gravity)

20 a  $10 \text{ m s}^{-1}$ . As there are no forces acting horizontally, the horizontal velocity is constant.

b Vertically with down as positive:  $u = 0$ ,  $s = 0.97$ ,  $a = 9.8$ ,  $v = ?$

$$v^2 = u^2 + 2as$$

$$= 0 + 2 \times 9.8 \times 0.97$$

$$v = 4.4 \text{ m s}^{-1}$$

c  $v = \sqrt{10^2 + 4.4^2}$   
 $= 11 \text{ m s}^{-1}$  (speed only so no direction required)

21 a  $u_{\text{H}} = 16 \cos 50^\circ$   
 $= 10.3 \text{ m s}^{-1}$

b  $u_{\text{V}} = 16 \sin 50^\circ$   
 $= 12.3 \text{ m s}^{-1}$

c Vertically with up as positive:  $u = 12.3$ ,  $a = -9.8$ ,  $v = 0$ ,  $s = ?$

$$v^2 = u^2 + 2as$$

$$0 = 12.3^2 + 2 \times -9.8 \times s$$

$$s = 7.7 \text{ m}$$

Total height from ground is  $1.2 + 7.7 = 8.9 \text{ m}$

22 a  $E_{\text{k}} = \frac{1}{2}mv^2$   
 $= \frac{1}{2} \times 0.1 \times 6.0^2$   
 $= 1.8 \text{ J}$

b  $E_{\text{g}} = mg\Delta h$   
 $= 0.1 \times 9.8 \times 2$   
 $= 1.96 \text{ J}$

$$\text{c } E_m = 1.8 + 1.96 = 3.76 \text{ J}$$

$$\text{Upon landing } E_m = E_k = \frac{1}{2}mv^2$$

$$3.76 = \frac{1}{2} \times 0.1 \times v^2$$

$$v = 8.7 \text{ m s}^{-1}$$

**23 a** Both the boy and the sled are stationary so their momentum = 0

$$\text{b } p_{\text{boy (final)}} = m \times v = 50 \times 4.0 = 200 \text{ kg m s}^{-1} \text{ east}$$

**c** Since momentum is conserved, the momentum of sled = 200 kg m s<sup>-1</sup> west to maintain a total momentum of 0 after the jump.

$$\text{24 } p_{\text{sled}} = 200 \text{ kg m s}^{-1} \text{ west} = m \times v = 200v$$

$$v = 1.0 \text{ m s}^{-1} \text{ west}$$

**25** Taking to the west as positive:

$$m_b u_b + m_s u_s = (m_b + m_s)v$$

$$50 \times 4.4 + 200 \times 1 = (50 + 200) \times v$$

$$420 = 250v$$

$$v = 1.68 \text{ m s}^{-1}$$

$$\text{26 a } p_{\text{s (initial)}} = 200 \text{ kg m s}^{-1} \text{ west}$$

$$p_{\text{s (final)}} = 200 \times 1.68$$

$$= 336 \text{ kg m s}^{-1} \text{ west}$$

$$\Delta p_s = 336 - 200$$

$$= 136 \text{ kg m s}^{-1} \text{ west}$$

$$\text{b } p_{\text{b (initial)}} = 50 \times 4.4$$

$$= 220 \text{ kg m s}^{-1} \text{ west}$$

$$p_{\text{b (final)}} = 50 \times 1.68$$

$$= 84 \text{ kg m s}^{-1} \text{ west}$$

$$\Delta p_b = 84 - 220$$

$$= -136 \text{ west or } 136 \text{ kg m s}^{-1} \text{ east (opposite to } \Delta p_s)$$

## Chapter 6 answers

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### Section 6.1

#### 6.1 review

- 1 D. They believed that all waves needed to travel in some sort of medium, so just as air is the medium for sound they invented another to be the medium for light.
- 2 A and D. An aircraft taking off is accelerating, as is a car going around a curve. These would be non-inertial frames of reference as they are accelerating.
- 3 A hanging pendulum in the spaceship will move from its normal vertical position when the spaceship accelerates.
- 4 The speed of the ball is greater for Jana than it is for Tom.  
The speed of the sound is greater forwards than it is backwards for Jana, while for Tom it is the same forwards and backwards.  
The speed of light is the same for Jana and Tom.
- 5
  - a  $340 + 30 = 370 \text{ m s}^{-1}$
  - b  $340 - 40 = 300 \text{ m s}^{-1}$
  - c  $340 + 20 = 360 \text{ m s}^{-1}$
  - d  $340 \text{ m s}^{-1}$
- 6 A. In order for the same events to be simultaneous in one inertial frame and not simultaneous in another inertial frame, time must act differently in each inertial frame of reference.
- 7
  - a In Anna and Ben's frame:  $t = \frac{5}{0.2} = 25 \text{ m s}^{-1}$ , so in Chloe's frame  $v = 10 - 25 = 15 \text{ m s}^{-1}$  backwards
  - b  $d = vt = 15 \times 0.2 = 3 \text{ m}$  backwards
  - c  $0.2 \text{ s}$
- 8
  - a  $t = \frac{d}{v} = \frac{5}{50} = 0.1 \text{ s}$
  - b  $50 \text{ m s}^{-1}$  in all frames
  - c  $d = vt = 10 \times 0.1 = 1 \text{ m}$
  - d  $50 \text{ m s}^{-1}$  as always
  - e The light had to travel  $\approx 4 \text{ m}$ , so  $t = \frac{4}{50} = 0.08 \text{ s}$  (approx.)
- 9 Atomic clocks enabled extremely short events to be timed to many decimal places. Differences in time for the same event to occur, when measured by observers in different inertial frames of reference, indicate that time is not uniform between the two inertial frames. These measurements support Einstein's special theory of relativity.
- 10 Muons have **very short** lives. On average, muons live for approximately  $2.2 \mu\text{s}$ . Their speeds are measured as they travel through the atmosphere. A muon's speed is **very similar** to the speed of light. According to Newtonian laws, muons **should not** reach the Earth's surface. However, many **do**.



## Section 6.2

## Worked example: Try yourself 6.2.1

## TIME DILATION

Assume *Gedanken* conditions exist in this example. A stationary observer on Earth sees a very fast scooter passing by, travelling at  $2.98 \times 10^8 \text{ m s}^{-1}$ . On the wrist of the rider is a watch on which the stationary observer sees 60.0 s pass. Calculate how many seconds pass by on the stationary observer's clock during this observation. Use  $c = 3.00 \times 10^8 \text{ m s}^{-1}$ .

Thinking	Working
Identify the variables: the time for the stationary observer is $t$ , the proper time for the moving clock is $t_0$ and the velocities are $v$ and the constant $c$ .	$t = ?$ $t_0 = 60.0 \text{ s}$ $v = 2.98 \times 10^8 \text{ m s}^{-1}$ $c = 3.00 \times 10^8 \text{ m s}^{-1}$
Use Einstein's time dilation formula and the Lorentz factor.	$t = t_0 \gamma$ $= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$
Substitute the values for $t_0$ , $v$ , and $c$ into the equation and calculate the answer $t$ .	$t = \frac{60.0}{\sqrt{1 - \frac{(2.98 \times 10^8)^2}{(3.00 \times 10^8)^2}}}$ $= \frac{60.0}{0.11528}$ $= 520 \text{ s}$

## 6.2 review

- In a device called a **light** clock, the **oscillation** of light is used as a means of measuring **time**, as the speed of light is **constant** no matter from which inertial frame of reference it is viewed.
- 'Proper time' is the time measured at rest with respect to the event. Proper times are always less than any other times.

$$\begin{aligned}
 3 \quad t &= t_0 \gamma \\
 &= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 &= \frac{1.05}{\sqrt{1 - \frac{(1.75 \times 10^8)^2}{(3 \times 10^8)^2}}} \\
 &= \frac{1.05}{0.81223} \\
 &= 1.29 \text{ s}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad t &= t_0 \gamma \\
 &= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 75.0 &= \frac{t_0}{\sqrt{1 - \frac{(2.30 \times 10^8)^2}{(3 \times 10^8)^2}}} \\
 t_0 &= 75.0 \times 0.642 \\
 &= 48.15 \text{ s}
 \end{aligned}$$

$$5 \quad t = t_0 \gamma$$

$$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$5.50 = \frac{t_0}{\sqrt{1 - \frac{(2.75 \times 10^8)^2}{(3 \times 10^8)^2}}}$$

$$t_0 = 5.50 \times 0.3996$$

$$= 2.20 \text{ s}$$

$$6 \quad t = \frac{t_0}{\gamma} = \frac{1}{\sqrt{1 - 0.5^2}} = 1.15 \text{ s}$$

7 a Simply the height of the clock, 1 m

$$b \quad t = \frac{d}{v} = \frac{1}{3.0 \times 10^8} = 3.33 \times 10^{-9} \text{ s}$$

$$c \quad d = vt = ct_c$$

d As the distance the ship moves in Chloe's frame is  $0.9ct_c$  and the height of the clock is 1 m, the distance  $d$  which the light travels is given by

$$d^2 = (0.9ct_c)^2 + 1^2 = 0.81c^2t_c^2 + 1.$$

As this also equals  $c^2t_c^2$  (from part c), we find that

$$0.81c^2t_c^2 + 1 = c^2t_c^2$$

$$0.19c^2t_c^2 = 1 \text{ and so}$$

$$t_c^2 = \frac{1}{0.19c^2}, \text{ giving } t_c = 7.6 \times 10^{-9} \text{ s.}$$

$$e \quad \frac{t_c}{t_A} = \frac{7.6}{3.3} = 2.3 \text{ which is the same as } \gamma \text{ for } v = 90\% \text{ of } c.$$

$$8 \quad a \quad t = t_0 \gamma$$

$$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{2.2 \times 10^{-6}}{\sqrt{1 - \frac{(0.992c)^2}{c^2}}}$$

$$= \frac{2.2 \times 10^{-6}}{\sqrt{1 - 0.992^2}}$$

$$= 1.74 \times 10^{-5} \text{ s or } 17.4 \mu\text{s}$$

b Non-relativistic:

$$d = vt = 0.992 \times 3 \times 10^8 \times 2.2 \times 10^{-6} = 655 \text{ m}$$

Relativistic:

$$d = vt = 0.992 \times 3 \times 10^8 \times 1.74 \times 10^{-5} = 5178 \text{ m}$$

$$9 \quad t = \frac{d}{v}$$

$$= \frac{2.50 \times 10^{-2}}{2.83 \times 10^8}$$

$$= 8.83 \times 10^{-11} \text{ s}$$

So the moving particle lasts for  $8.83 \times 10^{-11} \text{ s}$ .

$$t = t_0 \gamma$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$8.83 \times 10^{-11} = \frac{t_0}{\sqrt{1 - \frac{(2.83 \times 10^8)^2}{c^2}}}$$

$t_0 = 2.93 \times 10^{-11} \text{ s}$ . So the particle lives for  $2.93 \times 10^{-11} \text{ s}$  in the rest frame. This is reasonable, as the 'normal' lifetime should be shorter than when observed to be travelling at high speeds.

10 The equator clock is moving faster relative to the poles. It is also accelerating and hence will run slower. The effect is well below what we can detect as the speed of the equator is 'only' about  $460 \text{ m s}^{-1}$ , which is about 1.5 millionths of  $c$ .

## Section 6.3

## Worked example: Try yourself 6.3.1

## LENGTH CONTRACTION

Assume <i>Gedanken</i> conditions exist in this example. A stationary observer on Earth sees a very fast scooter travelling by at $2.98 \times 10^8 \text{ m s}^{-1}$ . The stationary observer measures the scooter's length as 45.0 cm. Calculate the proper length of the scooter, measured when the scooter is at rest. Use $c = 3.00 \times 10^8 \text{ m s}^{-1}$ .	
<b>Thinking</b>	<b>Working</b>
Identify the variables: the length measured by the stationary observer is $L$ , the proper length for the scooter is $L_0$ and the velocities are $v$ and the constant $c$ .	$L_0 = ?$ $L = 0.450 \text{ m}$ $v = 2.98 \times 10^8 \text{ m s}^{-1}$ $c = 3.00 \times 10^8 \text{ m s}^{-1}$
Use Einstein's length contraction formula and the Lorentz factor.	$L = \frac{L_0}{\gamma}$ $= L_0 \sqrt{1 - \frac{v^2}{c^2}}$
Substitute the values for $L$ , $v$ and $c$ into the equation and calculate the answer, $L_0$ .	$L_0 = \frac{0.450}{\sqrt{1 - \frac{(2.98 \times 10^8)^2}{(3.00 \times 10^8)^2}}}$ $= \frac{0.450}{0.11528}$ $= 3.90 \text{ m}$

## Worked example: Try yourself 6.3.2

## LENGTH CONTRACTION FOR DISTANCE TRAVELLED

Assume <i>Gedanken</i> conditions exist in this example. A stationary observer on Earth sees a very fast train approaching a tunnel at a speed of $0.986c$ . The stationary observer measures the tunnel's length as 123 m long. Calculate the length of the tunnel as seen by the train's driver.	
<b>Thinking</b>	<b>Working</b>
Identify the variables: the length seen by the driver is $L$ , the proper length for the tunnel is $L_0$ and the velocity is $v$ .	$L = ?$ $L_0 = 123 \text{ m}$ $v = 0.986c \text{ m s}^{-1}$
Use Einstein's length contraction formula and the Lorentz factor.	$L = \frac{L_0}{\gamma}$ $= L_0 \sqrt{1 - \frac{v^2}{c^2}}$
Substitute the values for $L_0$ and $v$ into the equation. Cancel $c$ and calculate the answer, $L$ .	$L = 123 \sqrt{1 - \frac{0.986^2 \times c^2}{c^2}}$ $= 123 \times \sqrt{1 - (0.986)^2}$ $= 123 \times 0.16675$ $= 20.5 \text{ m}$

## 6.3 review

- The length that a stationary observer measures in their own frame of reference. That is, the object (or distance) that is being measured is at rest with the observer.
- A. Width and height are not affected as they are at right angles to the direction of motion, so a stationary observer will see a moving object with a contracted length.

- 3 Correct to three significant figures:

$$\begin{aligned}
 L &= \frac{L_0}{\gamma} \\
 &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 &= 1.00 \times \sqrt{1 - \frac{(1.75 \times 10^8)^2}{(3.00 \times 10^8)^2}} \\
 &= 1.00 \times 0.81223 \\
 &= 0.812 \text{ m}
 \end{aligned}$$

4  $L = \frac{L_0}{\gamma}$

$$\begin{aligned}
 &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 &= 5.25 \times \sqrt{1 - \frac{(2.30 \times 10^8)^2}{(3.00 \times 10^8)^2}} \\
 &= 5.25 \times 0.64205 \\
 &= 3.37 \text{ m}
 \end{aligned}$$

5 a  $\gamma = \frac{3.50}{1.50} = 2.33$

thus  $\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2.33} = 0.429$

$$\frac{v^2}{c^2} = 1 - 0.184$$

$$v^2 = c^2 \times 0.816$$

$$v = 0.9c \text{ or } 2.71 \times 10^8 \text{ m s}^{-1}$$

b  $L = \frac{L_0}{\gamma}$

$$\begin{aligned}
 &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 &= 1.50 \times \sqrt{1 - \frac{(2.71 \times 10^8)^2}{(3.00 \times 10^8)^2}} \\
 &= 1.50 \times 0.42894 \\
 &= 0.643 \text{ m}
 \end{aligned}$$

The fast-moving garage appears even shorter than its proper length to the car driver.

- 6 Proper time,  $t_0$ , because the observer can hold a stopwatch in one location and start it when the front of the carriage is in line with the watch and stop it when the back of the carriage is in line with it.
- 7 C. When the speed increases towards the speed of light, the distance travelled decreases.

8 a  $\gamma = \frac{800}{400} = 2$

thus  $\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2} = 0.5$

$$\frac{v^2}{c^2} = 1 - 0.25$$

$$v^2 = c^2 \times 0.75$$

$$v = 0.866c \text{ or } 2.60 \times 10^8 \text{ m s}^{-1}.$$

b  $\gamma = 2$  so  $\frac{L}{L_0} = \frac{1}{\gamma}$

$$= \frac{1}{2}$$

$$= 0.5$$

Alternatively, recognise that if the track length has been halved, then Dan appears half his thickness as well.

$$\begin{aligned}
 9 \quad L &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 &= 23.5 \times \sqrt{1 - \frac{(660)^2}{(3.00 \times 10^8)^2}} \\
 &= 23.5 \times 1.0000 \\
 &= 23.5 \text{ m}
 \end{aligned}$$

At this speed, there is no difference in length.

$$\begin{aligned}
 10 \text{ a} \quad L &= L_0 \sqrt{1 - \frac{v^2}{c^2}} \\
 &= 2.75 \times \sqrt{1 - \frac{(0.900)^2 c^2}{c^2}} \\
 &= 2.75 \times \sqrt{1 - (0.900)^2} \\
 &= 2.75 \times 0.43589 \\
 &= 1.20 \text{ m}
 \end{aligned}$$

b The length of the fishing rod is the proper length = 2.75 m.

## CHAPTER 6 REVIEW

1 No object can travel at or beyond the speed of light, so the value of  $\frac{v^2}{c^2}$  will always be less than 1.

The number under the square root sign will also, therefore, be a positive number less than one.

The square root of a positive number less than one will always be less than one as well.

Note, however, when  $v$  is very small that  $\frac{v^2}{c^2}$  is also very small and so the number under the square root sign will be very close to one. The result is a number very close to one. Some calculators may not be able to distinguish a number so close to one, but this is just due to the limitations of the calculator.

2 The speed is 0.000167 of  $c$  and so  $\gamma \approx 1 + \frac{v^2}{2c^2} = 1 + \frac{(0.000167c)^2}{2c^2} = 0.00000014 = 1.4 \times 10^{-8}$

3 A (postulate 2) and C (postulate 1)

4 At the poles. The Earth has a very small circular acceleration which is negligible for most purposes, however at the poles it is even less.

5 C. There is no 'fixed space' in which to measure absolute velocities; we can only measure them relative to some other frame of reference.

6 Space and time are interdependent—motion in space reduces motion in time.

7 Both observers will see the light travel at  $3 \times 10^8 \text{ m s}^{-1}$ . According to Einstein's second postulate, the speed of light will always be the same no matter what the motion of the light source or observer.

8 A and B. We are in the same frame in either case. C and D may be true, but they are not sufficient conditions as we must be in the same frame. (C did not specify with respect to what we were stationary.)

9 B. Crews A and B will see each other normally as there is no relative velocity between them. They will both see C and the Earthlings moving in slow motion as the Earth has a high relative velocity.

10 You could not tell the difference between (i) and (iii), but in (ii) you could see whether an object like a pendulum hangs straight down.

11 In your frame of reference time proceeds normally. Your heart rate would appear normal. As Mars is moving at a high speed relative to you, people on Mars appear to be in slow motion as time for them, as seen by you, will be dilated.

$$\begin{aligned}
 12 \quad t &= t_0 \gamma \\
 &= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \\
 &= \frac{20.0}{\sqrt{1 - \frac{(2.00 \times 10^8)^2}{(3 \times 10^8)^2}}} \\
 &= 26.8 \text{ s}
 \end{aligned}$$

**13 a**  $t = t_0\gamma$

$$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$1.5 = \frac{t_0}{\sqrt{1 - \frac{(2.25 \times 10^8)^2}{(3 \times 10^8)^2}}}$$

$$t_0 = 1.5 \times 0.6614$$

$$= 0.992 \text{ s}$$

**b** 0.992 s (the swimmer sees the pool clock as  $t_0$ )

**14 C.** The remaining twin ages faster *during both the acceleration and deceleration phases* because the twin that travels experiences non-inertial frames of reference when accelerating and decelerating. This is when the travelling twin sees the twin at home ageing rapidly.

**15 a**  $L = \frac{L_0}{\gamma}$  and  $\frac{L}{L_0} = \frac{1}{2}$

$$\text{Thus } \sqrt{1 - \frac{v^2}{c^2}} = 0.5$$

$$\frac{v^2}{c^2} = 1 - 0.25$$

$$v^2 = c^2 \times 0.75$$

$$v = 0.866c \text{ or } 2.598 \times 10^8 \text{ m s}^{-1}.$$

**b** No, it can't have doubled to over  $c$ ! The contraction has doubled so this time  $\gamma = 4$ .

$$\text{Then } \sqrt{1 - \frac{v^2}{c^2}} = 0.25$$

$$\frac{v^2}{c^2} = 1 - 0.0625$$

$$v^2 = c^2 \times 0.9375$$

$$v = 0.968c \text{ or } 2.90 \times 10^8 \text{ m s}^{-1}.$$

**16 a**  $t = t_0\gamma$

$$= \frac{1.00}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1.00}{\sqrt{1 - \frac{(2.4 \times 10^8)^2}{(3 \times 10^8)^2}}}$$

$$= 1.67 \text{ s}$$

**b** Length:

$$L = \frac{L_0}{\gamma}$$

from part a,  $\gamma = 1.67$

$$L = \frac{3.00}{1.67}$$

$$= 1.80 \text{ m}$$

The height is unchanged at 1.0 m

**17 a**  $t = \frac{d}{v} = \frac{5}{0.9} = 5.6 \text{ years}$

**b**  $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}} = 2.29$

$$t_0 = \frac{t}{\gamma}$$

$$= \frac{5.6}{2.29}$$

$$= 2.45 \text{ years}$$

**c** Raqu sees the distance as only

$$L = \frac{L_0}{\gamma} = \frac{5}{2.29} = 2.183 \text{ ly}$$

**18 a** At  $8000 \text{ m s}^{-1}$ ,  $\frac{v}{c} = 2.7 \times 10^{-5}$  and  $\gamma$  will have a value of

$$\gamma \approx 1 + \frac{v^2}{2c^2} = 1 + \frac{(2.7 \times 10^{-5})^2}{2} = 1 + 3.6 \times 10^{-10}.$$

The difference (in mm) will therefore be  $4 \times 10^9 \times 3.6 \times 10^{-10} = 1.4 \text{ mm}$ —hardly a problem!

**b** No, as the motion is perpendicular to the north–south direction this dimension is not affected.

**19 a**  $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.995c)^2}{c^2}}} = \frac{1}{\sqrt{1 - (0.995)^2}} = 10.01$

**b** No, they don't experience any difference in their own time frame.

**c**  $t = \frac{25}{0.995} = 25.1 \text{ years}$

About 25.1 years from our frame of reference.

**d** 2.51 years as  $\gamma = 10$

**e** No! They see the distance between Earth and Vega foreshortened because of the high relative speed, so to them the distance is only about 2.5 ly.

**20** Earth observer: the observer will not measure the proper time of the muon's life span. Instead he or she will see that the muon's time is slow according to the equation  $t = t_0\gamma$  where  $t_0$  is the rest lifespan of the muon. The result is that the observer sees the muon live a much longer time,  $t$ , and therefore makes it to the surface.

Muon: the muon will see the Earth approach at a very high speed (approx.  $0.992c$ ) and will see the distance contracted. It will not be 15 km, but instead be much shorter according to the equation  $L = \frac{L_0}{\gamma}$ . The distance the muon travels is  $L$ .

# Chapter 7 answers

## Section 7.1

### Worked example: Try yourself 7.1.1

#### CALCULATING THE IMPULSE AND AVERAGE FORCE

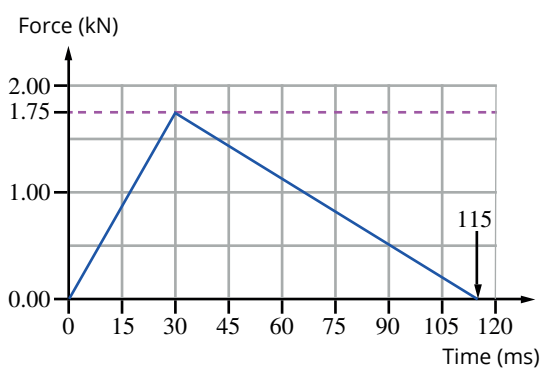
Prior to the accident, the driver had stopped to refuel. Calculate the impulse of the braking system on the 1485 kg car if the vehicle was travelling at  $95.5 \text{ km h}^{-1}$  in a north-easterly direction and the driver took 12.5 s to come to a halt. Also find the average braking force.

Thinking	Working
Convert speed to $\text{m s}^{-1}$ .	$95.5 \text{ km h}^{-1} = \frac{95.5}{3.6} \text{ m s}^{-1} = 26.5 \text{ m s}^{-1}$
Calculate the change in momentum. The negative sign indicates that the change in momentum, and therefore the impulse, is in the direction opposite to the initial momentum, as would be expected.	$\Delta p = m(v - u)$ $= 1485(0 - 26.5)$ $= 3.94 \times 10^4 \text{ kg m s}^{-1}$
The impulse is equal to the change in momentum.	impulse = $3.94 \times 10^4 \text{ kg m s}^{-1}$ south-west
Transpose $\Delta p = F_{\text{ave}} \Delta t$ to find the force.	$F_{\text{ave}} = \frac{\Delta p}{\Delta t}$ $= \frac{-3.94 \times 10^4}{12.5}$ $= 3.15 \times 10^3 \text{ N south-west}$

### Worked example: Try yourself 7.1.2

#### RUNNING SHOES

A running-shoe company plots the following force–time graph for an alternative design intended to reduce the peak force on the heel. Calculate the magnitude of the impulse.



Thinking	Working
Recall that impulse = $F_{\text{ave}} \Delta t$ . When the force is not constant, this is the area under the force–time graph.	impulse $= \frac{1}{2} \text{ base} \times \text{height}$ $= \frac{1}{2} \times 115 \times 10^{-3} \times 1.75 \times 10^3$ $= 101 \text{ N s}$



**Worked example: Try yourself 7.1.3****BRAKING FORCE**

The same 2500 kg truck travelling at 30.0 m s<sup>-1</sup> needs to stop in 1.5 s because a vehicle up ahead stops suddenly. Calculate the magnitude of the braking force required to stop the truck.

Thinking	Working
<p>The change in momentum, <math>\Delta p = F_{\text{ave}} \Delta t</math>. Calculate the change in momentum. The negative sign indicates that the change in momentum, and therefore the braking force, is in the direction opposite to the initial momentum, as would be expected.</p>	$\begin{aligned}\Delta p &= m(v - u) \\ &= 2500(0 - 30.0) \\ &= -7.5 \times 10^4 \text{ kg m s}^{-1}\end{aligned}$
<p>Transpose <math>\Delta p = F_{\text{ave}} \Delta t</math> to find the force. The sign of the momentum can be ignored since you are finding the magnitude of the force.</p>	$\begin{aligned}F_{\text{ave}} &= \frac{\Delta p}{\Delta t} \\ &= \frac{7.5 \times 10^4}{1.5} \\ &= 5.0 \times 10^4 \text{ N}\end{aligned}$

**7.1 review**

- 1 Ball A, ball C, ball B.

Remember to choose a direction for positive velocity and consider the initial and final momentum of each ball. The ball with the greatest change in momentum is the ball with the greatest final momentum, since they all have the same initial momentum.

- 2 Final speed of ball =  $\frac{144}{3.6}$   
= 40 m s<sup>-1</sup>

$$F = \frac{\Delta p}{\Delta t} = \frac{0.057(40 - 0)}{0.060}$$

$$= 38 \text{ N}$$

- 3 Speed of ball =  $\frac{155}{3.6} = 43.06 \text{ m s}^{-1}$

$$\text{impulse} = \Delta p = m(v - u) = 0.160(0 - 43.06) = -6.89 \text{ N s}$$

The magnitude of the impulse is 6.89 N s.

- 4 A, C and D. Any factor that increases the time of deceleration will decrease the force. Similarly, any factor that decreases the impulse—by decreasing the change in velocity or the mass—will decrease the force.

- 5 impulse =  $\Delta p$

taking up as positive

$$\begin{aligned}\Delta p &= 0.625(24.5 - (-32.0)) \\ &= 35.31 \text{ kg m s}^{-1}\end{aligned}$$

$$\Delta p = F \Delta t$$

$$F = \frac{35.31}{0.0165} = 2.14 \times 10^3 \text{ N}$$

- 6 a Speed =  $\frac{50.0}{3.6} = 13.89 \text{ m s}^{-1}$

$$\begin{aligned}p &= mv \\ &= 100 \times 10^3 \times 13.89 \\ &= 1.39 \times 10^6 \text{ kg m s}^{-1}\end{aligned}$$

- b  $1.39 \times 10^6 \text{ kg m s}^{-1}$ . Since the final momentum is zero, the magnitude of the impulse is equal to the magnitude of the initial momentum. The object with which the train collides and time it takes to stop does not affect the impulse.

7 Jacinta is correct. Because the higher pressure ball bounces back further, its change in momentum is more. Since the impulse is the force applied multiplied by the interaction time, it would be expected that the higher the impulse, the higher the force. You would also expect that the interaction time for the harder ball would be less, further increasing the force, and hence the pain of the player. Sarah is quite right that both balls have the same initial momentum, but she has not taken the final momentum into account.

8 impulse = area under force–time graph.

$$= \frac{1}{2} \text{ base} \times \text{height}$$

$$= \frac{1}{2} \times 200 \times 10^{-3} \times 2 \times 10^3$$

$$= 200 \text{ N s}$$

9  $F_{\text{ave}} = \frac{\Delta p}{\Delta t}$

$$\text{average force} = \frac{2.0 \times 10^2}{200 \times 10^{-3}} = 1.0 \times 10^3 \text{ N}$$

10 First find the initial and final velocity using energy considerations.

$$mgh = \frac{1}{2} mv^2$$

$$v_i = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 2.51} = 7.01 \text{ m s}^{-1}$$

$$v_f = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1.46} = -5.35 \text{ m s}^{-1}$$

$$\Delta p = m\Delta v = 0.0575 \times (-5.35 - 7.01) = 0.711 \text{ kg m s}^{-1}$$

$$\Delta p = F_{\text{ave}} \Delta t$$

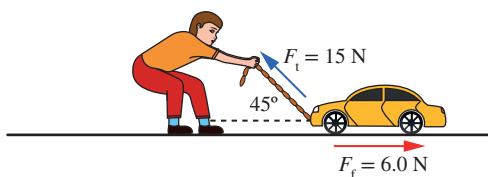
$$F_{\text{ave}} = \frac{\Delta p}{\Delta t} = \frac{0.711}{0.055} = 12.9 \text{ N}$$

## Section 7.2

### Worked example: Try yourself 7.2.1

#### WHEN THE FORCE APPLIED IS AT AN ANGLE TO THE DISPLACEMENT

A boy drives a toy car by pulling on a cord that is attached to the car at 45° to the horizontal. The boy applies a force of 15.0 N and pulls the car for 10.0 m down a pathway against a frictional force of 6.0 N.



(a) Determine the work done on the car by the boy pulling on the cord.	
<b>Thinking</b>	<b>Working</b>
Complete the diagram of the forces in action.	
Find the component of the force applied by the boy, $F_b$ , that is in the direction of the displacement, i.e. $F_{bc}$ .	$F_{bc} = 15.0 \times \cos 45^\circ = 10.6 \text{ N}$
Find the work done by the boy. (This includes work done on the car, and work done against friction.)	$W = F_{bc}s$ $= 10.6 \times 10.0$ $= 106 \text{ J}$

(b) Calculate the work done on the toy car.

**Thinking**

The work done on the car is the net force multiplied by the displacement. This is the increase in the kinetic energy of the car.

**Working**

$$\begin{aligned} W &= F_{\text{net}}s \\ &= (F_{\text{bc}} - F_f)s \\ &= (10.6 - 6.00) \times 10.0 \\ &= 46 \text{ J} \end{aligned}$$

(c) Calculate the energy transformed to heat and sound due to the frictional force.

**Thinking**

The energy transformed to heat and sound due to the frictional force is the difference between the work done by the boy and the energy gained by the toy car.

**Working**

$$\text{Energy} = 106 \text{ J} - 46.0 \text{ J} = 60.0 \text{ J}$$

This is equal to the work done against friction, which could also be calculated from the frictional force.

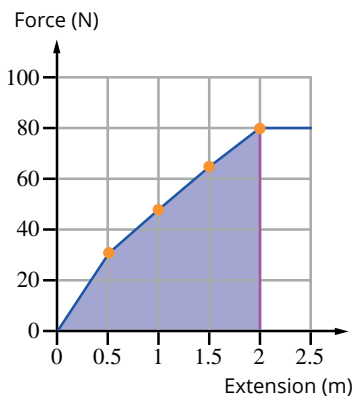
$$\begin{aligned} W_f &= F_f s \\ &= 6.00 \times 10.0 \\ &= 60.0 \text{ J} \end{aligned}$$

**Worked example: Try yourself 7.2.2**

**CALCULATING WORK FROM A FORCE-DISTANCE GRAPH**

The force required to elongate a piece of rubber tubing was recorded in the graph below. Calculate the work done when the rubber was stretched by 2.0 m.

**Force vs extension of rubber tubing**



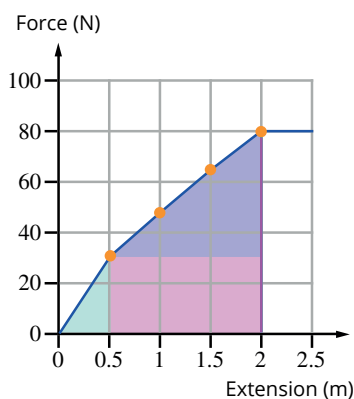
**Thinking**

Once again the work done is the area under the curve for an extension of 2.0 m.  
Divide the area into 2 triangles and a rectangle.

**Working**

$$\begin{aligned} \text{area} &= \frac{1}{2} \times 0.50 \times 30 + \frac{1}{2} \times 1.5 \times 50 + 30 \times 1.5 \\ \text{work done} &= 90 \text{ J} \end{aligned}$$

**Force vs extension of rubber tubing**



## 7.2 review

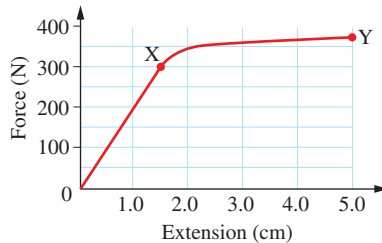
- 1 A. No work is done in A because the force applied by Janet to hold the suitcase up is perpendicular to the displacement of the suitcase as she and the suitcase move on the travelator.  
James does work on the bag when he climbs the stairs.  
Jeremy does work as he lifts the suitcase, but not by holding it up.  
Jason applies a force to the suitcase and the displacement is in the same direction.
- 2  $F_{\text{horizontal}} = F \cos 60^\circ$   
 $= 30 \cos 60^\circ$   
 $= 15 \text{ N}$   
 $W = F_{\text{net}} s$   
 $= 15 \times 2.4$   
 $= 36 \text{ J}$
- 3  $W = F_t s$   
 $= 10 \times 2.4$   
 $= 24 \text{ J}$
- 4  $W = F_{\text{net}} s$   
 $= (15 - 10) \times 2.4$   
 $= 12 \text{ J}$
- 5 Work represented by one square is  $10 \times 1 \times 10^{-3} = 10^{-2} \text{ J}$   
 There are  $27 \pm 0.5$  squares under the curve up to 7 mm compression.  
 $W = 27 \times 10^{-2}$   
 $= 0.27 \text{ J}$
- 6 D. Work is only done if the force is in the same direction as the displacement.
- 7  $W = Fs$   
 $= 150 \times 9.8 \times 1.20$   
 $= 1.8 \times 10^3 \text{ J}$
- 8 The magnetic force is at right angles to the velocity and displacement, hence the magnetic force does no work on the particle. It simply provides a centripetal force to keep the proton in a circular path.
- 9 vertically, taking up as positive:  
 $u = 30 \sin 45^\circ = 21.2 \text{ m s}^{-1}$   
 $v = 0$  (at the top),  $a = -9.8$ ,  $s = ?$   
 $v^2 = u^2 + 2as$   
 $0 = 21.2^2 - 19.6s$   
 $s = 22.9 \text{ m up}$   
 $s_{\text{total}} = 22.9 \text{ m up} + 22.9 \text{ m down} + 1.9 \text{ m down (to the ground)} = 47.7 \text{ m}$   
 $W = F_g \times s_{\text{total}}$   
 $= 0.8 \times 9.8 \times 47.7$   
 $= 374 \text{ J}$
- 10 The net force on the mower must be zero, since it is travelling at constant speed.  
 The force required to oppose friction must be  $68 \cos 60^\circ = 34 \text{ N}$ .  
 $W = F_{\text{net}} s$   
 $= 34 \times 15$   
 $= 510 \text{ J}$

## Section 7.3

### Worked example: Try yourself 7.3.1

#### CALCULATING THE SPRING CONSTANT, STRAIN POTENTIAL ENERGY AND WORK

An alloy sample is tested under tension, giving the graph shown below where X indicates the elastic limit and Y the breaking point.



(a) Calculate the spring constant  $k$  for the sample.

Thinking	Working
The spring constant is the gradient of the linear section of the force extension curve in units $\text{N m}^{-1}$ .	$k = \frac{\Delta F}{\Delta x}$ $= \frac{300}{1.5 \times 10^{-2}}$ $= 2.0 \times 10^4 \text{ N m}^{-1}$

(b) Calculate the strain potential energy that the alloy can store before permanent deformation begins.

Thinking	Working
The strain potential energy is the area under the curve up to the elastic limit.	$E_s = \frac{1}{2} \text{height} \times \text{base of triangle}$ $= \frac{1}{2} \times 300 \times 1.5 \times 10^{-2}$ $= 2.25 \text{ J}$
This value can also be obtained using the formula for strain potential energy.	$E_s = \frac{1}{2} k \Delta x^2$ $= \frac{1}{2} \times 2.0 \times 10^4 \times (1.5 \times 10^{-2})^2$ $= 2.25 \text{ J}$

(c) Calculate the work done to break the sample.

Thinking	Working
Estimate the number of squares under the curve up to the breaking point.	Number of squares = 29
Calculate the energy per square. The energy per square is given by the area of each square.	Energy for one square = $50 \times 1.0 \times 10^{-2}$ = 0.5 J
Multiply the number of squares by the energy per square.	Work = energy per square $\times$ number of squares = $0.5 \times 29 = 14.5 \text{ J}$

### 7.3 review

- 1 C, B, A. Stiffness is determined by the gradient on the force–extension graph.
- 2 Spring constant,  $k = \frac{\Delta F}{\Delta x}$   
Stiff spring constant =  $\frac{20}{0.1} = 200 \text{ N m}^{-1}$   
Weak spring constant =  $\frac{10}{0.2} = 50 \text{ N m}^{-1}$
- 3 For the stiff spring:  $E_s = \frac{1}{2} \times 200 \times 0.20^2 = 4.0 \text{ J}$   
For the weak spring:  $E_s = \frac{1}{2} \times 50 \times 0.20^2 = 1.0 \text{ J}$   
Energy difference =  $4.0 - 1.0 = 3.0 \text{ J}$
- 4  $\Delta x = \frac{F}{k} = \frac{4.0}{50} = 0.08 \text{ m}$  or  $8.0 \text{ cm}$
- 5  $F = k\Delta x$   
 $= 120 \times 0.25$   
 $= 30 \text{ N}$
- 6  $E_s = \frac{1}{2} k\Delta x^2$   
 $= \frac{1}{2} \times 120 \times 0.25^2$   
 $= 3.75 \text{ J}$
- 7 Each square =  $0.05 \times 10 = 0.5 \text{ J}$   
Strain potential energy =  $14 \text{ squares} \times 0.5 = 7 \text{ J}$
- 8 The work done by the archer is what becomes the strain potential energy. Therefore work done by archer =  $7 \text{ J}$ .
- 9 No. Hooke's law is not obeyed as the force vs distance graph is not a straight line (not linear).  
OR  
Yes. Hooke's law is obeyed up to a stretch of  $0.05 \text{ m}$  (i.e. a distance of  $0.15 \text{ m}$ ) on the graph where the line changes from being linear.
- 10 The elastic limit is at the point where the distance is  $0.15 \text{ m}$  and the force is  $30 \text{ N}$ .

## Section 7.4

## Worked example: Try yourself 7.4.1

## ELASTIC OR INELASTIC COLLISION?

A 200 g snooker ball with initial velocity $9.0 \text{ m s}^{-1}$ to the right collides with a stationary snooker ball of mass 100 g. After the collision, both balls are moving to the right and the 200 g ball has a speed of $3.0 \text{ m s}^{-1}$ . Show calculations to test whether or not the collision is inelastic.	
<b>Thinking</b>	<b>Working</b>
Use conservation of momentum to find the final velocity of the 100 g ball.	Taking to the right as positive: $p_i(\text{ball 1}) + p_i(\text{ball 2}) = p_f(\text{ball 1}) + p_f(\text{ball 2})$ $= 0.2 \times 9.0 + 0$ $= 0.2 \times 3.0 + 0.1 \times v_f(\text{ball 2})$ $v_f = 12 \text{ m s}^{-1}$
Calculate the initial kinetic energy before the collision.	$E_{\text{ki}} = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2$ $= \frac{1}{2} \times 0.20 \times 9.0^2 + 0$ $E_{\text{ki}} = 8.1 \text{ J}$
Calculate the final kinetic energy of the balls after the collision.	$E_{\text{kf}} = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$ $= \frac{1}{2} \times 0.2 \times 3.0^2 + \frac{1}{2} \times 0.10 \times 12^2$ $E_{\text{kf}} = 0.90 + 7.2$ $= 8.1 \text{ J}$
Compare the kinetic energy before and after collision to determine whether or not the collision is inelastic.	The kinetic energy after the collision is the same as the kinetic energy before the collision. The collision is perfectly elastic.

## Worked example: Try yourself 7.4.2

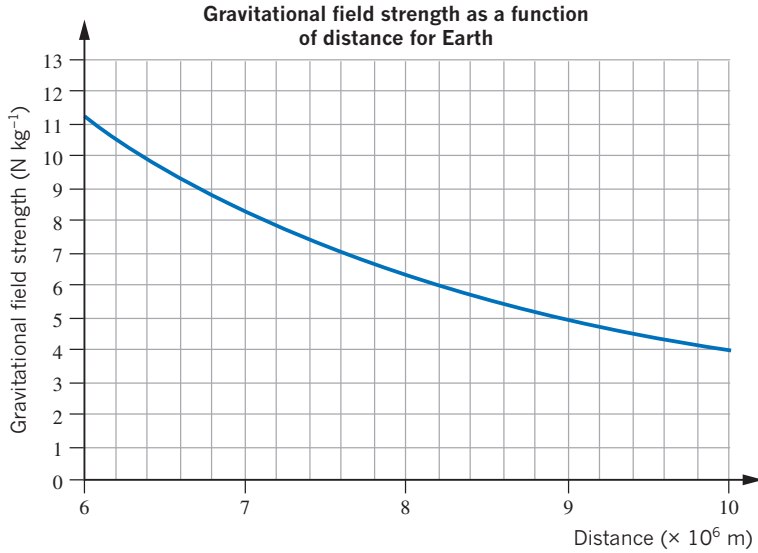
## CHANGES IN GRAVITATIONAL POTENTIAL ENERGY USING A FORCE GRAPH

Use the graph in Figure 7.4.2 to estimate the gravitational potential energy of the 10 kg object relative to the surface of the Earth, for the 10 kg object at $2.0 \times 10^7 \text{ m}$ .	
<b>Thinking</b>	<b>Working</b>
Find the energy represented by each square in the graph.	One square represents $10.0 \times 0.25 \times 10^7 = 2.5 \times 10^7 \text{ J}$
Identify the two values of distance that are relevant to the question.	The relevant distances are the radius of the Earth, which is $6.4 \times 10^6 \text{ m}$ , and the distance of the object, which is at $2.0 \times 10^7 \text{ m}$ from the centre.
Count the squares under the curve for the relevant area between the two distance values identified above, and multiply by the energy.	Work done = potential energy gained $18 \text{ squares (approx.)} \times 2.5 \times 10^7 = 4.5 \times 10^8 \text{ J}$

**Worked example: Try yourself 7.4.3**

**CHANGES IN GRAVITATIONAL POTENTIAL ENERGY USING A GRAVITATIONAL FIELD STRENGTH VS DISTANCE GRAPH**

A satellite of mass 1100 kg is in an elliptical orbit around the Earth. At its closest approach (perigee), it is just 600 km above the Earth’s surface. Its furthest point (apogee) is 2600 km from the Earth’s surface. The Earth has a mass of  $6.0 \times 10^{24}$  kg and a radius of  $6.4 \times 10^6$  m. The gravitational field strength of the Earth is shown in the graph.



(a) Calculate the change in potential energy of the satellite as it moves from the closest point (perigee) to the furthest point (apogee) from the Earth.

Thinking	Working
Convert distances given as altitudes to distances from the centre of the Earth.	Perigee = $6.4 \times 10^6 + 600 \times 10^3 = 7.0 \times 10^6$ m Apogee = $6.4 \times 10^6 + 2600 \times 10^3 = 9.0 \times 10^6$ m
Find the energy represented by each square.	One square represents $1.0 \times 0.20 \times 10^6 = 2.0 \times 10^5$ J kg <sup>-1</sup>
Count the squares under the curve for the relevant area, and multiply by the energy per kg represented by each square.	Work done per kg = potential energy gained per kg of mass 64 squares (approx.) $\times 2.0 \times 10^5 = 1.3 \times 10^7$ J kg <sup>-1</sup>
Calculate the potential energy gained by the satellite by multiplying by the mass.	Energy gained = $E_g$ $= 1.3 \times 10^7 \times 1100$ $= 1.4 \times 10^{10}$ J



(b) The satellite was moving with a speed of $8.0 \text{ km s}^{-1}$ at its closest point to Earth. How fast was it travelling at its furthest point?	
<b>Thinking</b>	<b>Working</b>
First calculate the kinetic energy at the closest point (perigee).	$E_{\text{kp}} = \frac{1}{2}mv_{\text{p}}^2$ $= \frac{1}{2} \times 1100 \times (8.0 \times 10^3)^2$ $= 3.5 \times 10^{10} \text{ J}$
The gain in gravitational potential energy at the apogee is at the expense of kinetic energy. Calculate the kinetic energy at the furthest point (apogee).	$E_{\text{ka}} = E_{\text{kp}} - E_{\text{g}}$ $= 3.5 \times 10^{10} - 1.4 \times 10^{10}$ $= 2.1 \times 10^{10} \text{ J}$
Calculate the speed of the satellite at the apogee.	$E_{\text{ka}} = \frac{1}{2}mv_{\text{a}}^2$ $v_{\text{a}} = \sqrt{\frac{2E_{\text{ka}}}{m}}$ $= \sqrt{\frac{2 \times 2.1 \times 10^{10}}{1100}}$ $= 6.2 \text{ km s}^{-1}$

## 7.4 review

- $E_{\text{g}} = mg\Delta h$   
 $= 115 \times 9.8 \times 2228 = 2.51 \times 10^6 \text{ J}$
- A and E. As potential energy decreases, kinetic increases, but there are losses because the meteor is burning up.
- $E_{\text{k}} = \frac{1}{2}mv^2$   
 $= \frac{1}{2} \times 500 \times 250^2$   
 $= 1.6 \times 10^7 \text{ J}$
- shaded area =  $53 \pm 1$  squares  
energy for one square =  $100 \times 0.1 \times 10^6 = 1.0 \times 10^7 \text{ J}$   
loss in potential energy = gain in kinetic energy =  $5.3 \times 10^8 \text{ J}$
- Total kinetic energy on landing =  $1.6 \times 10^7 + 5.3 \times 10^8 = 5.5 \times 10^8 \text{ J}$   
 $5.5 \times 10^8 = \frac{1}{2} \times 500 \times v^2$   
 $v = \sqrt{\frac{2 \times 5.5 \times 10^8}{500}}$   
 $= 1.5 \times 10^3 \text{ m s}^{-1}$
- The altitudes convert to distances of:  
 $6.4 \times 10^6 + 6 \times 10^5 = 7.0 \times 10^6 \text{ m}$  and  $6.4 \times 10^6 + 2.6 \times 10^6 = 9.0 \times 10^6 \text{ m}$   
There are 25 squares under the curve between these two distances  
The energy per kg for one square is  $1.0 \times 0.5 \times 10^6 = 5 \times 10^5 \text{ J kg}^{-1}$   
The gain in potential energy =  $25 \times 5.0 \times 10^5 \times 20 \times 10^3$   
 $= 2.5 \times 10^{11} \text{ J}$
- A, C and D. The cars travel at constant speed and so have constant kinetic energy. As the descending car loses gravitational energy, the ascending car gains energy, and the motor applies a force over a distance to drag the cable, thus doing work.
- B and D. A is not correct. It would only be correct if the Earth's gravitational force did not vary with distance from the centre of the Earth. You can only use  $E_{\text{g}} = mg\Delta h$  in regions where the field strength is approximately constant.  
B is correct because the velocity is proportional to  $\frac{1}{\sqrt{r}}$ , so larger  $r$  means lower speed.  
C is not correct as kinetic energy is inversely proportional to radius.  
D is correct. The higher the altitude, the more work has to be done against the gravitational force, and hence the more gravitational potential energy relative to the surface of the Earth.

$$\begin{aligned}
 9 \quad E_g &= E_k \\
 mgh &= \frac{1}{2}mv^2 \\
 v &= \sqrt{2gh} \\
 &= \sqrt{2 \times 9.8 \times 2.1} \\
 &= 6.4 \text{ m s}^{-1}
 \end{aligned}$$

## Section 7.5

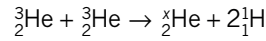
### Worked example: Try yourself 7.5.1

#### RELATIVISTIC MOMENTUM

(a) Calculate the momentum, as seen by a stationary observer, provided to an electron with a rest mass of $9.11 \times 10^{-31}$ kg, as it goes from rest to a speed of $0.985c$ . Assume <i>Gedanken</i> conditions exist in this example.	
<b>Thinking</b>	<b>Working</b>
Identify the variables: the rest mass is $m$ , and the velocity of the electron is $v$ .	$\Delta p = ?$ $m = 9.11 \times 10^{-31} \text{ kg}$ $v = 0.985c$
Use the relativistic momentum formula.	$p = \gamma mv$
Substitute the values for $m$ and $v$ into the equation and calculate the answer $p$ .	$p = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} mv$ $= \frac{1}{\sqrt{1 - \frac{0.985^2 c^2}{c^2}}} \times 9.11 \times 10^{-31} \times 0.985 \times 3.00 \times 10^8$ $= 1.56 \times 10^{-21} \text{ kg m s}^{-1}$
(b) If three times the relativistic momentum from part (a) is applied to the electron, calculate the new final speed of the electron in terms of $c$ .	
<b>Thinking</b>	<b>Working</b>
Identify the variables: the rest mass is $m$ , and the relativistic momentum of the electron is $p$ .	$p = 3 \times (1.56 \times 10^{-21})$ $= 4.68 \times 10^{-21} \text{ kg m s}^{-1}$ $m = 9.11 \times 10^{-31} \text{ kg}$ $v = ?$
Use the relativistic momentum formula, rearranged.	$p = \gamma mv$ $p = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} mv$ $v = \frac{p}{m \sqrt{1 + \frac{p^2}{m^2 c^2}}}$
Substitute the values for $m$ and $p$ into the rearranged equation and calculate the answer $v$ .	$v = \frac{p}{m \sqrt{1 + \frac{p^2}{m^2 c^2}}}$ $= \frac{4.68 \times 10^{-21}}{9.11 \times 10^{-31} \sqrt{1 + \frac{(4.68 \times 10^{-21})^2}{(9.11 \times 10^{-31})^2 (3.00 \times 10^8)^2}}}$ $= 2.995 \times 10^8 \text{ m s}^{-1}$ $= 0.998c$

**Worked example: Try yourself 7.5.2****FUSION**

A further fusion reaction in the Sun fuses two helium nuclides. A helium nucleus and two protons are formed and 30 MeV of energy is released.



(a) What is the value of the unknown mass number $x$ ?	
<b>Thinking</b>	<b>Working</b>
Analyse the mass numbers.	$3 + 3 = x + 2$ $x = 4$ A helium-4 nucleus is formed.
(b) How much energy released in joules?	
<b>Thinking</b>	<b>Working</b>
1 eV = $1.6 \times 10^{-19}$ J	$30 \text{ MeV} = 30 \times 10^6 \times 1.6 \times 10^{-19}$ $= 4.8 \times 10^{-12} \text{ J}$
(c) Calculate the mass defect for this reaction.	
<b>Thinking</b>	<b>Working</b>
Use $\Delta E = \Delta mc^2$ .	$\Delta m = \frac{\Delta E}{c^2} = \frac{4.8 \times 10^{-12}}{(3.0 \times 10^8)^2}$ $= 5.3 \times 10^{-29} \text{ kg}$

**7.5 review**

$$1 \quad p = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}mv$$

$$= \frac{1}{\sqrt{1 - \frac{(775)^2}{(3.00 \times 10^8)^2}}} \times 1230 \times 775$$

$$= 9.53 \times 10^5 \text{ kg m s}^{-1}$$

$$2 \quad p = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}mv$$

$$= \frac{1}{\sqrt{1 - \frac{(0.850)^2 c^2}{c^2}}} \times 1.99264824 \times 10^{-26} \times 0.850 \times 3.00 \times 10^8$$

$$= 9.65 \times 10^{-18} \text{ kg m s}^{-1}$$

$$3 \quad \text{since } v \ll c; p = mv$$

$$= 1.99264824 \times 10^{-26} \times 800$$

$$= 1.59 \times 10^{-23} \text{ kg m s}^{-1}$$

$$4 \quad E_k = \gamma mc^2$$

$$= \frac{1}{\sqrt{1 - \frac{(0.750)^2 c^2}{c^2}}} \times 0.0123 \times (3.00 \times 10^8)^2$$

$$= 1.67 \times 10^{15} \text{ J}$$

$$5 \quad E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(0.0123)(0.750 \times 3.00 \times 10^8)^2$$

$$= 3.11 \times 10^{14} \text{ J}$$

6 B. Relativistic kinetic energy depends on the momentum of the arrow. For the very fast arrow, the relativistic momentum is larger than the classical momentum.

$$\begin{aligned}
 7 \quad E_{\text{total}} &= \gamma mc^2 \\
 &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} m_0 c^2 \\
 &= \frac{1}{\sqrt{1 - \frac{(2.55 \times 10^8)^2}{(3.00 \times 10^8)^2}}} (210)(3.00 \times 10^8)^2 \\
 &= 3.59 \times 10^{19} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 8 \quad E &= mc^2 \\
 &= (4.00 \times 10^6 \times 10^3)(3.00 \times 10^8)^2 \\
 &= 3.60 \times 10^{26} \text{ J per second}
 \end{aligned}$$

For a full day:

$$\begin{aligned}
 &= (3.60 \times 10^{26})(24 \times 60 \times 60) \\
 &= 3.11 \times 10^{31} \text{ J}
 \end{aligned}$$

9 C. There is the same number of nucleons, but less mass.

10 C. The greater the impulse, the greater the increase in the momentum. At speeds near that of light, this can be interpreted as an increase in the mass of the object, and so the velocity only increases a very small amount.

## CHAPTER 7 REVIEW

1 B and D. A is incorrect, as force is the rate of change of momentum. C is incorrect, as impulse is a vector.

2 Take the direction away from the batsman as positive:

$$v_i = -100 \div 3.6 = -27.8 \text{ m s}^{-1}$$

$$v_f = 20 \div 3.6 = 5.56 \text{ m s}^{-1}$$

$$\Delta p = m\Delta v$$

$$= m(v_f - v_i)$$

$$= 0.16 \times (5.56 - (-27.8))$$

$$= 5.3 \text{ kg m s}^{-1}$$

3 D. The kinetic energy before each collision is more than after the collision, with some of the energy being transformed into heat. This would not be the case for a perfectly elastic collision. While it is true that the racquet gives the ball kinetic energy, and the impulse is positive, these do not explain the heat.

4 a Yes, momentum is conserved in all collisions.

b Inelastic; 20 J of kinetic energy has been transformed into heat and sound energy.

$$c \quad \text{Total initial } E_k = \frac{1}{2} \times 4.0 \times 3.0^2 + \frac{1}{2} \times 4.0 \times 3.0^2 = 36 \text{ J}$$

20 J is transformed into heat and sound, so total final  $E_k = 36 - 20 = 16 \text{ J}$

From symmetry, the balls will have the same final speeds and the same kinetic energies of 8 J each. For each ball:

$$E_k = \frac{1}{2} mv^2$$

$$= 0.5 \times 4.0 \times v^2$$

$$= 8$$

$$v^2 = 4$$

$$v = 2 \text{ m s}^{-1}$$

The balls will travel with speeds of 2 m s<sup>-1</sup>.

- 5  $\Delta p = m(v - u)$   
 $= 65 + 15 \times (0 - (-12))$   
 $= 960 \text{ kg m s}^{-1}$
- $F = \frac{\Delta p}{\Delta t}$   
 $= \frac{960}{2.0}$   
 $= 480 \text{ N}$
- 6 a  $W = 3.6 \times 10^4 \text{ J}$   
 $F_h = F \cos 60^\circ$   
 $= 300 \cos 60^\circ$   
 $= 150 \text{ N}$   
 $W = F_h x$   
 $= 150 \times 240$   
 $= 3.6 \times 10^4 \text{ J}$
- b  $F_{\text{net}} = 150 - 105 = 45 \text{ N}$   
 $W = 45 \times 240 = 1.08 \times 10^4 \text{ J}$
- $v = \sqrt{\frac{2E}{m}}$   
 $= \frac{2 \times 1.08 \times 10^4}{150}$   
 $= 12 \text{ m s}^{-1}$
- 7 a Determine the limits imposed by the two orbits:  
 $R_1 = 6.4 \times 10^6 + 1\,100\,000 = 7.5 \times 10^6 \text{ m}$   
 $R_2 = 6.4 \times 10^6 + 2\,100\,000 = 8.5 \times 10^6 \text{ m}$   
 There are 6.3 squares under the curve  
 The energy/kg for one square is  $2 \times 0.5 \times 10^6 = 1 \times 10^6 \text{ J kg}^{-1}$   
 The gain in potential energy =  $6.3 \times 1 \times 10^6 \times 11 \times 10^3 = 6.9 \times 10^{10} \text{ J}$
- b  $v = \sqrt{\frac{GM}{r}}$   
 $= \sqrt{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{8.5 \times 10^6}}$   
 $= 6.86 \times 10^3 \text{ m s}^{-1}$   
 $E_k = \frac{1}{2}mv^2$   
 $= \frac{1}{2} \times 11 \times 10^3 \times (6.86 \times 10^3)^2$   
 $= 2.6 \times 10^{11} \text{ J}$
- 8 Area = 2 squares  $\times 10 = 20 \text{ J}$   
 Work =  $20 \times 150 = 3 \times 10^3 \text{ J}$
- 9 Energy/square =  $2.0 \times 1.0 \times 10^{-3} = 2.0 \times 10^{-3} \text{ J}$   
 Area =  $16 \times 2.0 \times 10^{-3} = 3.20 \times 10^{-2} \text{ J}$
- 10 A. Both stones have the same gravitational potential energy, being thrown from the same height, and the same kinetic energy because they have the same initial speed. They will thus have the same kinetic energy on landing. They will therefore land at the same speed.
- 11  $p_i(\text{truck}) + p_i(\text{car}) = p_f(\text{truck}) + p_f(\text{car})$   
 $0.20 \times 0.30 + 0.10 \times 0.20 = 0.20v_f + 0.10 \times 0.30$   
 $0.08 = 0.2v_f + 0.03$   
 $0.2v_f = 0.05$   
 $v_f = 0.05 \div 0.2$   
 $= 0.25 \text{ m s}^{-1}$

$$\begin{aligned}
 12 \quad E_{ki} &= \frac{1}{2}m_t v_{ti}^2 + \frac{1}{2}m_c v_{ci}^2 \\
 &= \frac{1}{2} \times 0.200 \times 0.300^2 + \frac{1}{2} \times 0.100 \times 0.200^2 \\
 &= 0.009 + 0.002 \\
 &= 1.10 \times 10^{-2} \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 13 \quad E_{kf} &= \frac{1}{2}m_t v_{tf}^2 + \frac{1}{2}m_c v_{cf}^2 \\
 &= \frac{1}{2} \times 0.200 \times 0.250^2 + \frac{1}{2} \times 0.100 \times 0.300^2 \text{ J} \\
 &= 6.25 \times 10^{-3} + 4.5 \times 10^{-3} \\
 &= 1.1 \times 10^{-2} \text{ J}
 \end{aligned}$$

14 a The total kinetic energy before the collision is **more than** the total kinetic energy after the collision.

b The kinetic energy of the system of toys **is not** conserved.

c The total energy of the system of toys **is** conserved.

d The total momentum of the system of toys **is** conserved.

e The collision **is not** perfectly elastic because **kinetic energy** is not conserved.

$$\begin{aligned}
 15 \quad mgh &= \frac{1}{2}k(\Delta x)^2 \\
 k &= \frac{2mgh}{(\Delta x)^2} \\
 &= \frac{2 \times 80 \times 9.8 \times 110}{(10)^2} \\
 &= 1.7 \times 10^3 \text{ N m}^{-1}
 \end{aligned}$$

16 500 km altitude is  $6.4 \times 10^6 + 500000 = 6.9 \times 10^6$  m. There are 23 squares under the curve.

The energy per kg for one square is  $0.2 \times 10^6 \times 1 = 2 \times 10^5$  J kg<sup>-1</sup>

The gain in potential energy =  $23 \times 2 \times 10^5 \times 11 \times 10^6$   
 $= 5.1 \times 10^{13}$  J

17 B. As objects approach the speed of light,  $c$ , their inertia gets larger and larger and they become more and more difficult to accelerate.

$$\begin{aligned}
 18 \quad E_{\text{total}} &= E_k + E_0 \\
 E_k &= E_0 = mc^2 \\
 E_k &= (\gamma - 1) mc^2 = mc^2 \\
 \therefore \gamma &= 2
 \end{aligned}$$

$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2$$

$$1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$\frac{v^2}{c^2} = \frac{3}{4}$$

$$\begin{aligned}
 v &= \sqrt{0.75c^2} = 0.87c \\
 &= \sqrt{0.75} \times 3.00 \times 10^8 \\
 &= 2.60 \times 10^8 \text{ m s}^{-1}
 \end{aligned}$$

19 From the previous question  $\gamma = 2$

$$\begin{aligned}
 \text{relativistic mass} &= \gamma m \\
 &= 2m
 \end{aligned}$$

$$m_r = 2 \times 1.67 \times 10^{-27} = 3.34 \times 10^{-27} \text{ kg}$$

$$\begin{aligned}
 20 \quad E_k &= (\gamma - 1)mc^2 \\
 &= \frac{1}{\sqrt{1 - \frac{(0.960)^2 c^2}{c^2}}} - 1 (5.30 \times 10^3)(3.00 \times 10^8)^2 \\
 &= 1.23 \times 10^{21} \text{ J}
 \end{aligned}$$

## Chapter 8 answers

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### Section 8.1

#### 8.1 review

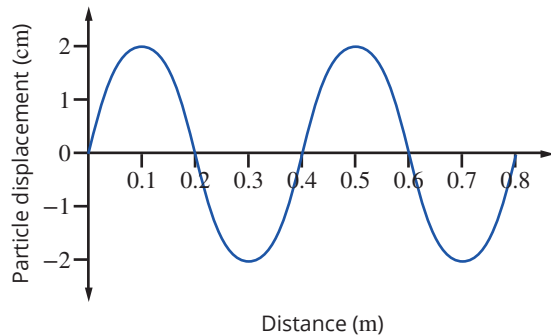
- 1 The particles oscillate back and forth or up and down around a central or average position and pass on the energy carried by the wave. They do not move along with the wave.
- 2 **a** False: Longitudinal waves occur when particles of the medium vibrate in the *same* direction or *parallel* to the direction of the wave.  
**b** True.  
**c** True.  
**d** True.
- 3 Point B is moving downwards.
- 4 Mechanical waves: sound, ripples on a pond, vibrations in a rope. (Light does not require the particles of a medium to propagate and is therefore not a mechanical wave.)
- 5 A has moved right and B has moved left. As the sound wave moves to the right, particles ahead of the compression must move to the left initially to meet the compression and then move forward to carry the compression to the right. Therefore, particle B has moved to the left of its undisturbed position and particle A has now moved to the right of its undisturbed position.
- 6 C and D. Only energy is transferred by a wave therefore the statements saying that air particles have travelled to Lee are incorrect. Energy has been transferred from the speaker to Lee and it is the air particles that have passed this energy along through the air.
- 7 In a transverse wave the motion of the particles is at right angles (perpendicular) to the direction of travel of the wave itself.
- 8 Longitudinal: a and d  
Transverse: b and c
- 9 Mechanical waves move energy via the interaction of particles. The molecules in a solid are closer together than those in a gas. A smaller movement is needed to transfer energy and, hence, the energy of the wave is usually transferred more quickly in a solid when compared with other states of matter.
- 10 The energy travels towards the right, that is, the energy is transferred away from the tuning fork towards X.

## Section 8.2

### Worked example: Try yourself 8.2.1

#### DISPLACEMENT–DISTANCE GRAPH

The displacement–distance graph below shows a snapshot of a transverse wave as it travels along a spring towards the right. Use the graph to determine the wavelength and the amplitude of this wave.

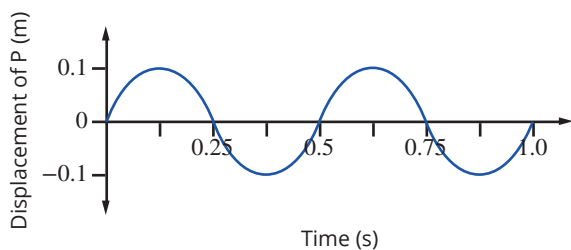


Thinking	Working
Amplitude on a displacement–distance graph is the distance from the average position to a crest or trough.	The amplitude is 2 cm or 0.02 m
Wavelength is the distance for one complete cycle. Any two consecutive points at the same position on the wave could be used.	The wavelength is 0.4 m.

### Worked example: Try yourself 8.2.2

#### DISPLACEMENT–TIME GRAPHS

The displacement–time graph below shows the motion of a single part of a rope as a wave passes travelling to the right. Use the graph to find the amplitude, period and frequency of the wave.



Thinking	Working
Amplitude on a displacement–time graph is the displacement from the average position to a crest or trough. Note the displacement of successive crests and/or troughs on the wave and carefully note units on the vertical axis.	The amplitude is 0.1 m.
Period on a displacement–time graph is the time it takes to complete one cycle and can be identified in the graph as the time between two successive points on the graph that are in phase.	The period is 0.5 s.
Frequency can be calculated using $f = \frac{1}{T}$ , measured in hertz (Hz).	$f = \frac{1}{T} = \frac{1}{0.5} = 2$ The frequency is 2 Hz.



**Worked example: Try yourself 8.2.3****THE WAVE EQUATION**

A longitudinal wave has a wavelength of $4.0 \times 10^{-7}$ m and a speed of $3.0 \times 10^8$ m s <sup>-1</sup> . What is the frequency, $f$ , of the wave?	
<b>Thinking</b>	<b>Working</b>
The wave equation states that $v = f\lambda$ . Knowing both $v$ and $\lambda$ , the frequency, $f$ , can be found. Rewrite the wave equation in terms of $f$ .	$v = f\lambda$ $f = \frac{v}{\lambda}$
Substitute the known values and solve.	$f = \frac{v}{\lambda}$ $= \frac{3.0 \times 10^8}{4.0 \times 10^{-7}}$ $= 7.5 \times 10^{14}$ Hz

**Worked example: Try yourself 8.2.4****THE WAVE EQUATION**

A longitudinal wave has a wavelength of $4.0 \times 10^{-7}$ m and a speed of $3.0 \times 10^8$ m s <sup>-1</sup> . What is the period, $T$ , of the wave?	
<b>Thinking</b>	<b>Working</b>
Rewrite the wave equation in terms of $T$ .	$v = f\lambda$ and $f = \frac{1}{T}$ $v = \frac{\lambda}{T}$ $T = \frac{\lambda}{v}$
Substitute the known values and solve.	$T = \frac{\lambda}{v}$ $= \frac{4.0 \times 10^{-7}}{3.0 \times 10^8}$ $= 1.3 \times 10^{-15}$ s

**8.2 review**

- C and F
  - wavelength
  - B and D
  - amplitude
- Wavelength is the length of one complete wave cycle. Any two points at the same position on the wave could be used. In this case  $\lambda = 1.6$  m.  
Amplitude is the displacement from the average position to a crest or trough. In this case, amplitude = 20 cm.
- period = 0.4 s
  - $f = \frac{1}{T} = \frac{1}{0.4} = 2.5$  Hz
- $f = 5$  Hz, amplitude = 0.3 m,  $\lambda = 1.3$  m,  $v = ?$   
 $v = f\lambda = 5 \times 1.3 = 6.5$  m s<sup>-1</sup>
- True.
  - False: The period of a wave is *proportional* to its wavelength.
  - True.
  - False: The wavelength *and* frequency of a wave determine its speed.

- 6 a wavelength = 4 cm; amplitude = 0.5 cm  
b  $T = 2$  s,  $\lambda = 4$  cm,  $v = ?$   
$$v = \frac{\lambda}{T} = \frac{4}{2} = 2 \text{ cm s}^{-1} \text{ or } 0.02 \text{ m s}^{-1}$$
  
c red
- 7  $T = \frac{1}{f} = \frac{1}{2 \times 10^5} = 5 \times 10^{-6}$  s
- 8 As the speed of each vehicle is the same and there is no relative motion of the medium, the frequency observed would be the same as that at the source.
- 9 The apparent frequency increases when travelling towards you and decreases when travelling away from you.

## Section 8.3

### 8.3 review

- 1 The wave is reflected and there is a  $180^\circ$  change in phase.
- 2 Since a wave is reflected back into the same medium, the only property that will change is amplitude. This is because some of the energy of the wave has been absorbed by the second medium from which the wave was reflected. (Note: velocity will change in direction but speed will not change, because it is a scalar quantity.)
- 3 a True.  
b False: As the pulses pass through each other, the interaction *does not* permanently alter the characteristics of each pulse.  
c True.
- 4 B. Each pulse travels 3 m in 3 s. Adding their amplitudes together means they will look like C, but the result is they will cancel each other out as in B.
- 5 An object subjected to forces varying with its natural oscillating frequency will oscillate with increasing amplitude. This could continue until the structure can no longer withstand the internal forces and fails.
- 6  $\theta_i = 90^\circ - 38^\circ = 52^\circ$   
 $\theta_r = \theta_i = 52^\circ$
- 7 B. If maximum energy is transferred then the amplitude will increase. The frequency is unchanged.
- 8 Normal walking results in a frequency of 1 Hz or 1 cycle per second i.e. two steps per second. This frequency may result in an increase in the amplitude of oscillation of the bridge over time, which could damage the structure.
- 9 C. The object must have been convex, that is, curved outwards.

## Section 8.4

## Worked example: Try yourself 8.4.1

## FUNDAMENTAL FREQUENCY

A standing wave in a string is found to have a wavelength of 0.50 m for the fundamental frequency of vibration. Assume that the tension of the string is not changed and that the string is fixed at both ends.

(a) What is the length of the string?	
<b>Thinking</b>	<b>Working</b>
Identify wavelength of the string ( $\lambda$ ) in metres and the harmonic number ( $n$ ).	$\lambda = 0.5 \text{ m}$ $n = 1$
Recall that for any frequency $\lambda = \frac{2l}{n}$ . Rearrange to find $l$ .	$\lambda = \frac{2l}{n}$ $l = \frac{n\lambda}{2}$
Substitute the value for the wavelength from the question and solve for $l$ .	$l = \frac{1 \times 0.5}{2}$ $= 0.25 \text{ m}$

(b) What is the wavelength of the third harmonic?	
<b>Thinking</b>	<b>Working</b>
Identify length of the string ( $l$ ) in metres and the harmonic number ( $n$ )	$l = 0.25 \text{ m}$ $n = 3$
Recall that for any frequency $\lambda = \frac{2l}{n}$ . Substitute the values from the question and solve for $\lambda$ .	$\lambda = \frac{2l}{n}$ $= \frac{2 \times 0.25}{3}$ $= 0.17 \text{ m}$

## 8.4 review

- It is a common misconception that standing waves somehow remain stationary. It is only the pattern made by the amplitude along the rope that stays still at the nodes. The rope is still moving, especially at the antinodes.
- A transverse wave moving along a slinky spring is reflected from a fixed end. The interference that occurs during the superposition of this reflected wave and the original wave creates a standing wave. This standing wave consists of locations called nodes, where the movement of the spring is cancelled out, and antinodes where maximum movement of the spring occurs.
- $\lambda = \frac{2l}{n} = \frac{2 \times 0.4}{1} = 0.8 \text{ m}$
- Rearranging  $\lambda = \frac{2l}{n}$  gives  $l = \frac{n\lambda}{2} = \frac{4 \times 0.75}{2} = 1.5 \text{ m}$
- This wave will have a frequency four times that of the fundamental frequency, which means that it will have a wavelength  $\frac{1}{4}$  of the fundamental wavelength due to the inverse relationship between frequency and wavelength.
- The wavelength of the standing wave in the diagram is 5 m. The wavelength of the fundamental frequency is twice the length of the string. Therefore, a string length of 2.5 m would produce a standing wave with wavelength 5 m.
- $f = \frac{nv}{2l} \rightarrow l = \frac{nv}{2f} = \frac{1 \times 387}{2 \times 350} = 0.55 \text{ m}$   
new length =  $\frac{2}{3} \times 0.55 = 0.37 \text{ m}$   
new wavelength =  $2 \times \text{new length} = 2 \times 0.37 = 0.74 \text{ m}$

$$8 \quad f = \frac{nv}{2l} = \frac{1 \times 300}{2 \times 0.5} = 300 \text{ Hz}$$

$$9 \quad f = \frac{nv}{2l} = \frac{2 \times 300}{2 \times 0.5} = 600 \text{ Hz}$$

$$10 \quad f = \frac{nv}{2l} = \frac{3 \times 300}{2 \times 0.5} = 900 \text{ Hz}$$

## CHAPTER 8 REVIEW

- The particles on the surface of the water move up and down as the waves radiate outwards carrying energy away from the point on the surface of the water where the stone entered the water.
- Similarities: both are waves, both carry energy away from the source, both are caused by vibrations.  
Differences: transverse waves involve particle displacement at right angles to the direction of travel of the wave; longitudinal waves involve particle displacement parallel to the direction of travel of the wave.
- U is moving down and V is momentarily stationary (and will then move downwards).
- $f = 10.0 \text{ Hz}$ ,  $\lambda = 30.0 \text{ mm} = 0.0300 \text{ m}$ ,  $v = ?$   
 $v = f\lambda = 10 \times 0.03 = 0.300 \text{ m s}^{-1}$
- $f = 32\,000$ ,  $v = 1400$ ,  $\lambda = ?$   
 $v = f\lambda$  rearranges to  $\lambda = \frac{v}{f}$   
 $\lambda = 1400 \div 32\,000 = 0.044 \text{ m}$
- $v = 1500$ ,  $f = 300$ ,  $\lambda = ?$   
 $v = f\lambda$  rearranges to  $\lambda = \frac{v}{f}$   
 $\lambda = 1500 \div 300 = 5 \text{ m}$
- $v = 340$ ,  $f = 300$ ,  $\lambda = ?$   
 $v = f\lambda$  rearranges to  $\lambda = \frac{v}{f}$   
 $\lambda = 340 \div 300 = 1.1 \text{ m}$
- 256 Hz. Since there is no relative motion between the source and the observer, the apparent frequency would be unchanged.
- C and D. Since the frequency rose and fell, the bike must have travelled past you. It must have come towards you and then moved away from you.
- By inspecting the wave equation  $v = f\lambda$  since wavelength decreases and the velocity must stay the same, the frequency must increase. This ensures the product of the wavelength and frequency still equals the velocity, which has remained unchanged. (Note: velocity is constant as it is a property of the medium.)
- At the fixed end of a string or rope, a wave undergoes a phase change.
- transmission
  - reflection
  - absorption
- The green wave represents the superposition of the blue and the red waves.
- Sound waves are longitudinal mechanical waves where the particles only move back and forth around an equilibrium position, parallel to the direction of travel of the wave. When these particles move in the direction of the wave, they collide with adjacent particles and transfer energy to the particles in front of them. This means that kinetic energy is transferred between particles in the direction of the wave through collisions. Therefore, the particles cannot move along with the wave from the source as they lose their kinetic energy to the particles in front of them during the collisions.
- All objects/materials have a resonant frequency. If the object is made to vibrate at this frequency, the amplitude of the object's vibrations will increase with time. If a building or bridge was subjected to wind that made it vibrate at its natural frequency, this vibration may increase in amplitude so much that the structure is damaged or collapses.
- $f = \frac{nv}{4l} = \frac{1 \times 340}{4 \times 0.85} = 100 \text{ Hz}$

$$17 \quad f = \frac{nv}{4l} = \frac{3 \times 340}{4 \times 0.85} = 300 \text{ Hz}$$

18 The fundamental frequency is given by:

$$f_1 = \frac{1}{T} = \frac{1}{4.0} = 0.25 \text{ Hz}$$

The frequency of the second harmonic is given by:

$$f_2 = 2 \times f_1 = 2 \times 0.25 = 0.50 \text{ Hz}$$

19 Calculate the wavelength of the wave using the wave equation:

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$= \frac{78}{428}$$

$$= 0.182 \text{ m}$$

Since the separation of antinodes and of nodes in a standing wave in a string with fixed ends is half the wavelength, then:

$$d = \frac{\lambda}{2} = \frac{0.182}{2}$$

$$= 0.091 \text{ m or } 9.1 \text{ cm}$$

- 20 All of the options are correct. The light rays striking all of these surfaces will obey the law of reflection as it always holds regardless of the shape of the reflector.
- 21 For a wave that is propagated by a medium, relative motion between the source, observer and medium can all cause the Doppler effect.

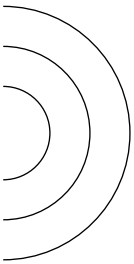
# Chapter 9 answers

## Section 9.1

### Worked example: Try yourself 9.1.1

#### APPLYING HUYGENS' PRINCIPLE

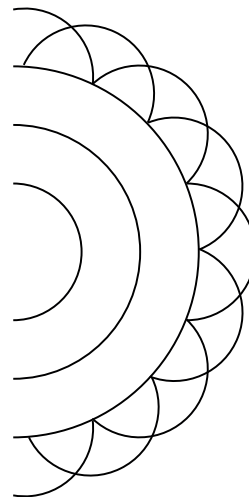
On the circular waves shown below, sketch some of the secondary wavelets on the outer wavefront and draw the appearance of the new wave formed after 1 period.



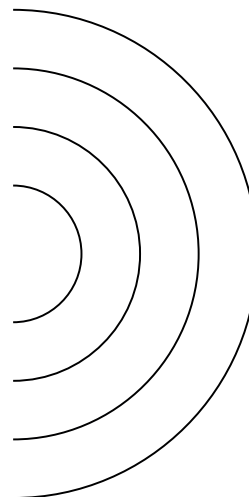
#### Thinking

Sketch a number of secondary wavelets on the advancing wavefront.

#### Working



Sketch the new wavefront.



**Worked example: Try yourself 9.1.2****CALCULATING REFRACTIVE INDEX**

The speed of light in crown glass is $1.97 \times 10^8 \text{ m s}^{-1}$ . Given that the speed of light in a vacuum is $3.00 \times 10^8 \text{ m s}^{-1}$ , calculate the refractive index of crown glass.	
<b>Thinking</b>	<b>Working</b>
Recall the definition of refractive index.	$n = \frac{c}{v}$
Substitute the appropriate values into the formula and solve.	$n = \frac{3.00 \times 10^8}{1.97 \times 10^8} = \frac{3.00}{1.97} = 1.52$

**Worked example: Try yourself 9.1.3****SPEED OF LIGHT CHANGES**

A ray of light travels from water ( $n = 1.33$ ) where it has a speed of $2.25 \times 10^8 \text{ m s}^{-1}$ into glass ( $n = 1.85$ ). Calculate the speed of light in glass.	
<b>Thinking</b>	<b>Working</b>
Recall the formula.	$n_1 v_1 = n_2 v_2$
Substitute the appropriate values into the formula and solve.	$1.33 \times 2.25 \times 10^8 = 1.85 \times v_2$ $\therefore \frac{1.33 \times 2.25 \times 10^8}{1.85} = v_2$ $\therefore v_2 = 1.62 \times 10^8 \text{ m s}^{-1}$

**Worked example: Try yourself 9.1.4****USING SNELL'S LAW**

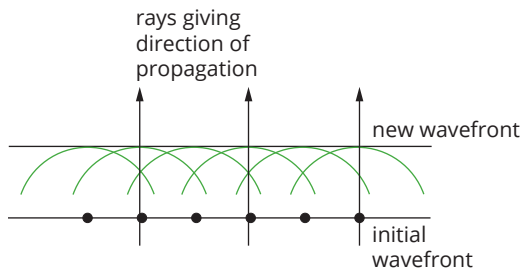
A ray of light in air strikes a piece of flint glass ( $n = 1.62$ ) at angle of incidence of $50^\circ$ to the normal. Calculate the angle of refraction of the light in the glass.	
<b>Thinking</b>	<b>Working</b>
Recall Snell's law.	$n_1 \sin \theta_1 = n_2 \sin \theta_2$
Recall the refractive index of air.	$n_1 = 1.00$
Substitute the appropriate values into the formula to find a value for $\sin \theta_2$ .	$1.00 \times \sin 50^\circ = 1.62 \times \sin \theta_2$ $\sin \theta_2 = 0.4729$
Calculate the angle of refraction.	$\therefore \theta_2 = \sin^{-1} 0.4729 = 28.2^\circ$

**Worked example: Try yourself 9.1.5****CALCULATING CRITICAL ANGLE**

Calculate the critical angle for light passing from diamond into air.	
<b>Thinking</b>	<b>Working</b>
Recall the equation for critical angle.	$\sin \theta_c = \frac{n_2}{n_1}$
Substitute the refractive indexes of diamond and air into the formula.	$\sin \theta_c = \frac{1.00}{2.42} = 0.4132$
Solve for $\theta_c$ .	$\theta_c = \sin^{-1} 0.4131 = 24.4^\circ$

## 9.1 review

- 1 a wave model  
b wave model  
c particle model
- 2 C. Newton's esteemed reputation meant that his theory was regarded as correct.
- 3 The new wavefront should be a straight line across the front of the secondary wavelets.



- 4 Therefore, the speed of light in seawater will be **slower than** in pure water.
- 5 Recall the definition of refractive index:  $n = \frac{c}{v}$   
Rearrange to get  

$$v = \frac{c}{n}$$

$$= \frac{3.00 \times 10^8}{1.38}$$

$$= 2.17 \times 10^8 \text{ m s}^{-1}$$
- 6  $n_1 v_1 = n_2 v_2$   
 $1.33 \times 2.25 \times 10^8 = n_2 \times 2.29 \times 10^8$   
 $n_2 = 1.31$
- 7 Recall Snell's law:  $n_1 \sin \theta_1 = n_2 \sin \theta_2$   
 $1.33 \times \sin 44^\circ = 1.60 \times \sin \theta_2$   
 $\sin \theta_2 = \frac{1.33 \times \sin 44^\circ}{1.60}$   
 $= 0.5774$   
 $\theta_2 = \sin^{-1} 0.5774$   
 $= 35.3^\circ$
- 8 Total internal reflection occurs when light passes from a more-dense medium into a less-dense medium and refracts away from the normal.
  - a no
  - b yes
  - c yes
  - d no
- 9 D. Significant diffraction occurs when  $\frac{\lambda}{w}$  is approximately 1 or greater.  $700 \text{ nm} \approx 10^{-6} \text{ m}$  and  $0.001 \text{ mm} = 0.001 \times 10^{-3} \text{ or } 10^{-6} \text{ m}$ .
- 10 Polarisation is a phenomenon in which transverse waves are restricted in their direction of vibration. Polarisation can only occur in transverse waves and cannot occur in longitudinal waves. Since light can be polarised, it must be a transverse wave.



## Section 9.2

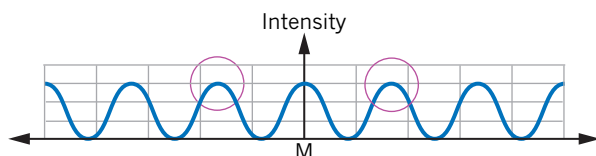
## Worked example: Try yourself 9.2.1

## CALCULATING WAVELENGTH FROM FRINGE SEPARATION

Green laser is directed through a pair of thin slits that are 25 $\mu\text{m}$ apart. The slits are 1.5 m from a screen on which bright fringes are 3.3 cm apart. Use this information to calculate the wavelength of green light in nm.	
<b>Thinking</b>	<b>Working</b>
Recall the equation for fringe separation.	$\Delta x = \frac{\lambda L}{d}$
Transpose the equation to make $\lambda$ the subject.	$\lambda = \frac{\Delta x d}{L}$
Substitute values into the equation and solve.	$\lambda = \frac{0.033 \times 25 \times 10^{-6}}{1.5} = 5.5 \times 10^{-7} \text{ m}$
Express your answer using the unit specified.	$\lambda = 550 \text{ nm}$

## 9.2 review

- D. Light passed through the double slits to hit the screen. Young's double-slit experiment produced an interference pattern of alternating bright and dark lines on the screen.
- C and D. As laser light is monochromatic and coherent, it is more likely to produce the interference pattern expected in Young's experiment.
- A and D. When crests meet troughs, the addition of these out-of-phase waves means that they cancel to form a node.
- The central antinode occurs where both waves have travelled the same distance, i.e. the path difference is 0. The next antinodes on either side occur when the path difference is  $1\lambda$ .



- Up until Young's experiment, most scientists supported a particle or 'corpuscular' model of light. Young's experiment demonstrated interference patterns, which are characteristic of waves. This led to scientists abandoning the particle theory and supporting a wave model of light.
- Recall the equation for fringe separation:  $\Delta x = \frac{\lambda L}{d}$ 
  - increase
  - decrease
  - increase
- $$\text{pd} = \left(n - \frac{1}{2}\right)\lambda$$

For the fifth dark fringe,  $n = 5$

$$\text{pd} = \left(5 - \frac{1}{2}\right)\lambda$$

Therefore, the fifth dark fringe occurs where the path difference is  $4.5\lambda = 4.5 \times 580 \text{ nm} = 2610 \text{ nm}$  or  $2.61 \times 10^{-6} \text{ m}$
- Constructive interference occurs when the path difference is a whole number multiple of the wavelength. Destructive interference occurs when the path difference is an odd number multiple of half the wavelength.
  - destructive
  - constructive
  - destructive

$$9 \quad pd = n\lambda$$

For the second bright fringe,  $n = 2$

$$pd = 2\lambda$$

Therefore, the second bright fringe occurs where the path difference is

$$2 \times 700 = 1400 \text{ nm}$$

$$10 \quad \Delta x = \frac{\lambda L}{d}$$

$$\lambda = \frac{\Delta x d}{L}$$

$$= \frac{0.037 \times 40 \times 10^{-6}}{3.25}$$

$$= 4.55 \times 10^{-7} \text{ m}$$

$$= 455 \text{ nm}$$

## Section 9.3

### Worked example: Try yourself 9.3.1

#### USING THE WAVE EQUATION FOR LIGHT

A particular colour of red light has a wavelength of 600 nm. Calculate the frequency of this colour.	
Thinking	Working
Recall the wave equation for light.	$c = f\lambda$
Transpose the equation to make frequency the subject.	$f = \frac{c}{\lambda}$
Substitute in values to determine the frequency of this wavelength of light.	$f = \frac{3.0 \times 10^8}{600 \times 10^{-9}}$ $= 5.0 \times 10^{14} \text{ Hz}$

### 9.3 review

- B. Mechanical waves require a medium whereas light waves can travel through a vacuum.
- D. Light is electromagnetic radiation that is composed of changing electric and magnetic fields. Electric and magnetic waves oscillate at  $90^\circ$  to each other, so in an electromagnetic wave the changing electric and magnetic fields are orientated perpendicular to each other.
- D. Electromagnetic radiation with a wavelength of 200 nm would be classified as ultraviolet light since this part of the spectrum is shorter wavelength than visible light.
- From shortest to longest wavelength: X-rays, visible light, infrared radiation, FM radio waves.
- Use  $c = f\lambda$   
Transpose to make frequency the subject.
  - red,  $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{656 \times 10^{-9}} = 4.57 \times 10^{14} \text{ Hz}$
  - yellow,  $f = 5.09 \times 10^{14} \text{ Hz}$
  - blue,  $f = 6.17 \times 10^{14} \text{ Hz}$
  - violet,  $f = 7.56 \times 10^{14} \text{ Hz}$
- $\frac{3 \times 10^8 - 299\,792\,458}{299\,792\,458} \times 100\% = 0.07\%$

7 Use  $c = f\lambda$

Transpose to make wavelength the subject.

$$\begin{aligned}\lambda &= \frac{c}{f} \\ &= \frac{3 \times 10^8}{6.0 \times 10^{14}} \\ &= 5 \times 10^{-7} \\ &= 500 \text{ nm}\end{aligned}$$

8 Use  $c = f\lambda$

Transpose to make wavelength the subject.

$$\begin{aligned}\lambda &= \frac{c}{f} \\ &= \frac{3 \times 10^8}{7.0 \times 10^7} \\ &= 4.2857 \\ &= 4.3 \text{ m}\end{aligned}$$

9 Use  $c = f\lambda$

Transpose to make frequency the subject.

$$\begin{aligned}f &= \frac{c}{\lambda} \\ &= \frac{3 \times 10^8}{200 \times 10^{-12}} \\ &= 1.5 \times 10^{18} \text{ Hz}\end{aligned}$$

10 Frequency of microwave oven is 2.45 GHz from text on page 323.

Use  $c = f\lambda$

Transpose to make wavelength the subject.

$$\begin{aligned}\lambda &= \frac{c}{f} \\ &= \frac{3 \times 10^8}{2.45 \times 10^9} \\ &= 0.122 \text{ m}\end{aligned}$$

## CHAPTER 9 REVIEW

- A. This shows the bending of the edges of the waves as they pass through a gap.
- Since  $\Delta x = \frac{\lambda L}{d}$ , the diffraction pattern would spread out more from blue to green. The green light ( $\lambda = 525 \text{ nm}$ ) has a longer wavelength than blue light ( $\lambda = 460 \text{ nm}$ ). Green's longer wavelength results in more widely spaced fringes and a wider overall pattern.
- D. Polarisation is a phenomenon in which transverse waves are restricted in their direction of vibration. Polarisation can only occur in transverse waves and cannot occur in longitudinal waves. Since light can be polarised, it must be a transverse wave.
- Both snow and water reflect light. This reflected light is known as glare. The light reflected from water and snow is partially polarised. Both snowboarders and sailors are likely to wear polarising sunglasses as these will absorb the polarised glare from the snow or water respectively.
- $c = f\lambda$   
 $= 4.5 \times 10^{14} \times 500 \times 10^{-9}$   
 $= 2.25 \times 10^8 \text{ m s}^{-1}$
- As light travels from quartz ( $n = 1.46$ ) to water ( $n = 1.33$ ), its speed **increases** which causes it to refract **away from** the normal.
- A: incident ray  
 B: normal  
 C: reflected ray  
 D: boundary between media  
 E: refracted ray

$$8 \quad n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$n_2 = \frac{n_1 \sin \theta_1}{\sin \theta_2}$$

$$= \frac{1.00 \times \sin 43^\circ}{\sin 28.5^\circ}$$

$$= 1.429$$

$$\text{Since } n = \frac{c}{v}$$

$$v = \frac{c}{n}$$

$$= \frac{3.00 \times 10^8}{1.429}$$

$$= 2.1 \times 10^8 \text{ m s}^{-1}$$

$$9 \quad \text{Use Snell's law: } n_1 \sin \theta_1 = n_2 \sin \theta_2$$

angle  $a$

$$1.00 \times \sin 40^\circ = 1.50 \times \sin a$$

$$\sin a = \frac{1.00 \times \sin 40^\circ}{1.5}$$

$$= 0.4285$$

$$a = \sin^{-1}(0.4285) = 25.4^\circ$$

angle  $b$

Since  $a$  and  $b$  are corresponding angles,  $a = b = 25.4^\circ$

angle  $c$

$$1.50 \times \sin 25.4^\circ = 1.33 \times \sin c$$

$$\sin c = \frac{1.50 \times \sin 25.4^\circ}{1.33}$$

$$= 0.4837$$

$$c = \sin^{-1} 0.4837 = 28.9^\circ$$

- 10 a The angle of incidence is measured with respect to the normal which is drawn at a right angle to the glass-air boundary.

$$\theta_1 = 90 - 58.0 = 32.0^\circ$$

$$b \quad n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1.52 \times \sin 32^\circ = 1.00 \times \sin \theta_2$$

$$\sin \theta_2 = \frac{n_1 \sin \theta_1}{\sin \theta_2}$$

$$= \frac{1.52 \times \sin 32^\circ}{1.00}$$

$$= 0.8055$$

$$\theta_2 = \sin^{-1} 0.8055 = 53.7^\circ$$

$$c \quad \Delta\theta = \theta_2 - \theta_1$$

$$= 53.7 - 32$$

$$= 21.7^\circ$$

$$d \quad v = \frac{c}{n}$$

$$= \frac{3 \times 10^8}{1.52}$$

$$= 1.97 \times 10^8 \text{ m s}^{-1}$$

- 11 a red light

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1.00 \times \sin 30^\circ = 1.50 \times \sin \theta_{\text{red}}$$

$$\sin \theta_{\text{red}} = \frac{n_1 \sin \theta_1}{n_2}$$

$$= \frac{1.00 \times \sin 30^\circ}{1.50}$$

$$= 0.3333$$

$$\theta_2 = \sin^{-1} 0.3333 = 19.5^\circ$$

**b** violet light

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1.00 \times \sin 30^\circ = 1.53 \times \sin \theta_{\text{violet}}$$

$$\sin \theta_{\text{violet}} = \frac{n_1 \sin \theta_1}{n_2}$$

$$= \frac{1.00 \times \sin 30^\circ}{1.53}$$

$$= 0.3268$$

$$\theta_2 = \sin^{-1} 0.3268 = 19.1^\circ$$

**c**  $\Delta\theta = \theta_2 - \theta_1 = 19.5 - 19.1 = 0.4^\circ$

**d**  $v = \frac{c}{n} = \frac{3 \times 10^8}{1.53} = 1.96 \times 10^8 \text{ m s}^{-1}$

**12**  $\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$

**a**  $\theta_c = \sin^{-1} \left( \frac{1.00}{1.31} \right) = 49.8^\circ$

**b**  $\theta_c = \sin^{-1} \left( \frac{1.00}{1.54} \right) = 40.5^\circ$

**c**  $\theta_c = \sin^{-1} \left( \frac{1.00}{2.16} \right) = 27.6^\circ$

**13** B, D, A, C. The bigger the difference in refractive indexes, the bigger the angle of deviation. The air–water boundary has the smallest difference in refractive indices so it will produce the smallest angle of deviation. The air–diamond boundary has the biggest difference in refractive indices so it will produce the biggest angle of deviation.

**14 a**  $\Delta x = \frac{\lambda L}{d}$

$$\lambda = \frac{\Delta x d}{L}$$

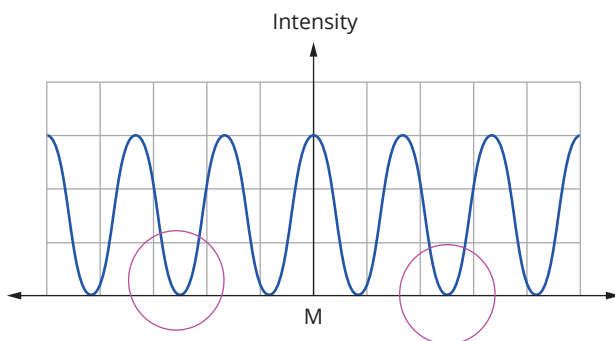
$$= \frac{0.031 \times 75 \times 10^{-6}}{4.0}$$

$$= 5.81 \times 10^{-7}$$

$$= 581 \text{ nm}$$

**b** 581 nm is closest to yellow (according to Table 9.1.3)

**15** A path difference of  $1\frac{1}{2}\lambda$  corresponds to the second dark band on each side of the central maximum at M.



**16** In order of decreasing wavelength: radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays

**17 a** microwaves

**b** infrared waves

**c** X-rays

**18** Since  $c = f\lambda$

$$\lambda = \frac{c}{f}$$

$$= \frac{3 \times 10^8}{612 \times 10^3}$$

$$= 490 \text{ m}$$

- 19** Young performed his famous experiment in 1803, in which he observed an interference pattern in light. Young shone monochromatic light on a pair of narrow slits. Light passed through the slits and formed a pattern of bright and dark lines/fringes/bands on a screen. Young compared this to interference patterns he had observed, and he identified that these lines corresponded to regions of constructive and destructive interference. This could only be explained by considering light to be a wave.
- 20** A microwave oven is tuned to produce electromagnetic waves with a frequency of 2.45 GHz. This is the resonant frequency of water molecules. When food is bombarded with radiation at this frequency, the water molecules within the food start to vibrate. The energy of the water molecules is then transferred to the rest of the food, heating it up.

# Chapter 10 answers

## Section 10.1

### Worked example: Try yourself 10.1.1

#### USING PLANCK'S EQUATION

Calculate the energy in joules of a quantum of infrared radiation that has a frequency of $3.6 \times 10^{14}$ Hz.	
<b>Thinking</b>	<b>Working</b>
Recall Planck's equation.	$E = hf$
Substitute in the appropriate values to solve.	$E = 6.63 \times 10^{-34} \times 3.6 \times 10^{14}$ $= 2.4 \times 10^{-19}$ J

### Worked example: Try yourself 10.1.2

#### CONVERTING TO ELECTRON-VOLTS

A quantum of light has $2.4 \times 10^{-19}$ J. Convert this energy to electron-volts.	
<b>Thinking</b>	<b>Working</b>
Recall the conversion rate for joules to electron-volts.	$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
Divide the value expressed in joules by $1.6 \times 10^{-19} \text{ J eV}^{-1}$ to convert to electron-volts.	$2.4 \times 10^{-19} \div (1.6 \times 10^{-19})$ $= 1.5 \text{ eV}$

### Worked example: Try yourself 10.1.3

#### CALCULATING QUANTUM ENERGIES IN ELECTRON-VOLTS

Calculate the energy (in eV) of a quantum of infrared radiation that has a frequency of $3.6 \times 10^{14}$ Hz. Use $h = 4.14 \times 10^{-15} \text{ eV s}$ .	
<b>Thinking</b>	<b>Working</b>
Recall Planck's equation.	$E = hf$
Substitute in the appropriate values and solve for $E$ .	$E = 4.14 \times 10^{-15} \times 3.6 \times 10^{14}$ $= 1.5 \text{ eV}$

### Worked example: Try yourself 10.1.4

#### CALCULATING THE WORK FUNCTION OF A METAL

Calculate the work function (in J and eV) for gold, which has a threshold frequency of $1.2 \times 10^{15}$ Hz.	
<b>Thinking</b>	<b>Working</b>
Recall the formula for work function.	$\phi = hf_0$
Substitute the threshold frequency of the metal into this equation.	$\phi = 6.63 \times 10^{-34} \times 1.2 \times 10^{15}$ $= 8.0 \times 10^{-19} \text{ J}$
Convert this energy from J to eV.	$\phi = \frac{8.0 \times 10^{-19}}{1.6 \times 10^{-19}}$ $= 5.0 \text{ eV}$

**Worked example: Try yourself 10.1.5****CALCULATING THE KINETIC ENERGY OF PHOTOELECTRONS**

Calculate the kinetic energy (in eV) of the photoelectrons emitted from lead by ultraviolet light which has a frequency of  $1.5 \times 10^{15}$  Hz. The work function of lead is 4.14 eV. Use  $h = 4.14 \times 10^{-15}$  eV s.

Thinking	Working
Recall Einstein's photoelectric equation.	$E_{k \max} = hf - \phi$
Substitute values into this equation.	$E_{k \max} = 4.14 \times 10^{-15} \times 1.5 \times 10^{15} - 4.25$ $= 6.21 - 4.14$ $= 2.07 \text{ eV}$

**10.1 review**

- $E = \frac{hc}{\lambda} = 6.63 \times 10^{-34} \times 3 \times 10^8 \div (656 \times 10^{-9}) = 3.03 \times 10^{-19} \text{ J}$
  - $E = \frac{hc}{\lambda} = 6.63 \times 10^{-34} \times 3 \times 10^8 \div (589 \times 10^{-9}) = 3.38 \times 10^{-19} \text{ J}$
  - $E = \frac{hc}{\lambda} = 6.63 \times 10^{-34} \times 3 \times 10^8 \div (486 \times 10^{-9}) = 4.09 \times 10^{-19} \text{ J}$
  - $E = \frac{hc}{\lambda} = 6.63 \times 10^{-34} \times 3 \times 10^8 \div (397 \times 10^{-9}) = 5.01 \times 10^{-19} \text{ J}$
- In the photoelectric effect, a metal surface may become positively charged if light shining on it causes electrons to be released.
- True.
  - False: When light sources of the same intensity but different frequencies are used, the higher frequency light has a higher stopping voltage, but it produces the same maximum current as the lower frequency.
  - True.
- $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.0 \times 10^{15} = 4.1 \text{ eV}$
  - $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.1 \times 10^{15} = 4.6 \text{ eV}$
  - $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.5 \times 10^{15} = 6.2 \text{ eV}$
- D.

The threshold frequency is:

$$f_0 = \frac{\phi}{h}$$

$$= \frac{3.66}{4.14 \times 10^{-15}}$$

$$= 8.84 \times 10^{14} \text{ Hz}$$

In order to release photoelectrons, the light must have a frequency higher than the threshold frequency. Therefore,  $9.0 \times 10^{14}$  Hz is the only frequency that will release photoelectrons.

- $$E_{k \max} = 4.14 \times 10^{-15} \times 9.0 \times 10^{14} - 3.66$$

$$= 0.066 \text{ eV}$$
  - $$E = \frac{hc}{\lambda}$$

$$= 4.14 \times 10^{-15} \times 3 \times 10^8 \div (475 \times 10^{-9}) = 2.61 \text{ eV}$$

$$E_{k \max} = 2.61 - 2.36$$

$$= 0.25 \text{ eV}$$
  - C and D.
- $$\phi = hf_0 = \frac{hc}{\lambda_0}$$
- $$\lambda_0 = \frac{hc}{\phi} = \frac{4.14 \times 10^{-15} \times 3.00 \times 10^8}{1.81}$$
- $$= 6.86 \times 10^{-7} \text{ m}$$
- $$= 686 \text{ nm}$$

Photons with wavelengths shorter than the threshold wavelength—i.e. violet light and ultraviolet radiation—will cause photoelectrons to be emitted.



- 9 a True.  
 b False: The stopping voltage is reached when the photocurrent is reduced completely to zero.  
 c True.  
 d True.
- 10  $E_{k \text{ max}} = hf - \phi$   
 $= \frac{hc}{\lambda} - \phi$   
 $0.80 = \frac{4.14 \times 10^{-15} \times 3.0 \times 10^8}{500 \times 10^{-9}} - \phi$   
 $\phi = 2.48 - 0.80$   
 $= 1.68 \text{ eV}$

## Section 10.2

### Worked example: Try yourself 10.2.1

#### CALCULATING THE DE BROGLIE WAVELENGTH

Calculate the de Broglie wavelength of a proton travelling at $7.0 \times 10^5 \text{ m s}^{-1}$ . The mass of a proton is $1.67 \times 10^{-27} \text{ kg}$ .	
<b>Thinking</b>	<b>Working</b>
Recall de Broglie's equation.	$\lambda = \frac{h}{mv}$
Substitute the appropriate values into the equation and solve it.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 7.0 \times 10^5}$ $= 5.7 \times 10^{-13} \text{ m}$

### Worked example: Try yourself 10.2.2

#### CALCULATING THE DE BROGLIE WAVELENGTH OF A MACROSCOPIC OBJECT

Calculate the de Broglie wavelength of a person with $m = 66 \text{ kg}$ running at $36 \text{ km h}^{-1}$ .	
<b>Thinking</b>	<b>Working</b>
Convert velocity to SI units.	$v = 36 \div 3.6 = 10 \text{ m s}^{-1}$
Recall de Broglie's equation.	$\lambda = \frac{h}{mv}$
Substitute the appropriate values into the equation and solve it.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{66 \times 10}$ $= 1.0 \times 10^{-36} \text{ m}$

**Worked example: Try yourself 10.2.3****WAVELENGTH OF ELECTRONS FROM AN ELECTRON GUN**

Find the de Broglie wavelength of an electron that has been accelerated from rest through a potential difference of 50 V. The mass of an electron is $9.11 \times 10^{-31}$ kg and the magnitude of the charge on an electron is $1.6 \times 10^{-19}$ C.	
<b>Thinking</b>	<b>Working</b>
Calculate the kinetic energy of the electron from the work done on it by the electric potential.	$W = qV$ $= 1.6 \times 10^{-19} \times 50$ $= 8.0 \times 10^{-18} \text{ J}$
Calculate the velocity of the electron.	$E_k = \frac{1}{2}mv^2$ $v = \sqrt{\frac{2 \times E_k}{m}}$ $= \sqrt{\frac{2 \times 8.0 \times 10^{-18}}{9.11 \times 10^{-31}}}$ $= 4.2 \times 10^6 \text{ m s}^{-1}$
Use de Broglie's equation to calculate the wavelength of the electron.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 4.2 \times 10^6}$ $= 1.7 \times 10^{-10} \text{ m}$ $= 0.17 \text{ nm}$

**Worked example: Try yourself 10.2.4****CALCULATING PHOTON MOMENTUM**

Calculate the momentum of a photon of blue light with a wavelength of 450 nm.	
<b>Thinking</b>	<b>Working</b>
Convert 450 nm to m.	$450 \text{ nm} = 450 \times 10^{-9} \text{ m}$
Transpose de Broglie's equation to make momentum ( $p$ ) the subject.	$\lambda = \frac{h}{p}$ $p = \frac{h}{\lambda}$
Substitute in values and solve for $p$ .	$p = \frac{6.63 \times 10^{-34}}{450 \times 10^{-9}}$ $= 1.47 \times 10^{-27} \text{ kg m s}^{-1}$

**10.2 review**

- $$\lambda = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 1.0 \times 10^6}$$

$$= 7.3 \times 10^{-10} \text{ m}$$
- $$\lambda = \frac{h}{mv}$$

$$4.0 \times 10^{-9} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times v}$$

$$v = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 4.0 \times 10^{-9}}$$

$$= 1.8 \times 10^5 \text{ m s}^{-1}$$

3 B. Wave behaviour of matter is linked to the mass and the velocity (that is, momentum) of the matter. So only moving particles exhibit wave behaviour.

$$4 \quad \mathbf{a} \quad \lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{8.6 \times 10^{18}} = 3.5 \times 10^{-11} \text{ m}$$

$$\mathbf{b} \quad \lambda = \frac{h}{mv}$$

$$3.5 \times 10^{-11} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times v}$$

$$v = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.5 \times 10^{-11}} = 2.1 \times 10^7 \text{ m s}^{-1}$$

5 The wavelength of a cricket ball is so small that its wave-like behaviour could not be seen by a cricket player.

6 An electron microscope cannot observe individual atoms because the radius of an atom is smaller than the wavelength of an electron.

$$7 \quad \lambda = \frac{hc}{E}$$

$$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^8}{6.63 \times 10^{-14}}$$

$$= 3.0 \times 10^{-12} \text{ m}$$

Speed of the proton to exhibit this wavelength:

$$v = \frac{h}{m\lambda}$$

$$= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 3.0 \times 10^{-12}}$$

$$= 1.32 \times 10^5 \text{ m s}^{-1}$$

$$8 \quad W = qV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{m\sqrt{\frac{2qV}{m}}}$$

$$= \frac{h}{\sqrt{2qVm}}$$

$$9 \quad \lambda = \frac{h}{mv}$$

$$\lambda mv = h$$

$$mv = \frac{h}{\lambda}$$

$$p = \frac{h}{\lambda}$$

10 An electron microscope can resolve images in finer detail than an optical microscope because a high-speed electron has a shorter wavelength than a light wave.

## Section 10.3

## Worked example: Try yourself 10.3.1

## SPECTRAL ANALYSIS

In the Sun's absorption spectrum, one of the dark 'Fraunhofer' lines corresponds to a frequency of $6.9 \times 10^{14}$ Hz. Calculate the energy (in joules) of the photon that corresponds to this line.	
<b>Thinking</b>	<b>Working</b>
Recall Planck's equation.	$\Delta E = hf$
Substitute in the appropriate values and solve.	$\Delta E = 6.63 \times 10^{-34} \times 6.9 \times 10^{14}$ $= 4.6 \times 10^{-19} \text{ J}$

## Worked example: Try yourself 10.3.2

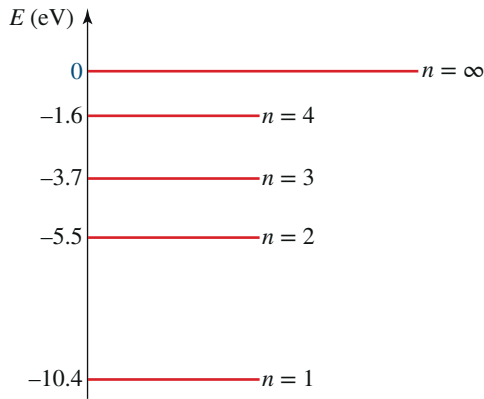
## USING THE BOHR MODEL OF THE HYDROGEN ATOM

Calculate the wavelength (in nm) of the photon produced when an electron drops from the $n = 3$ energy level of the hydrogen atom to the $n = 1$ energy level. Identify the spectral series to which this line belongs. Use Figure 10.3.8 to calculate your answer.	
<b>Thinking</b>	<b>Working</b>
Identify the energy of the relevant energy levels of the hydrogen atom.	$E_3 = -1.5 \text{ eV}$ $E_1 = -13.6 \text{ eV}$
Calculate the change in energy.	$\Delta E = E_3 - E_1$ $= -1.5 - (-13.6)$ $= 12.1 \text{ eV}$
Calculate the wavelength of the photon with this amount of energy.	$\lambda = \frac{hc}{E}$ $= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^8}{12.1}$ $= 1.03 \times 10^{-7} \text{ m}$ $= 103 \text{ nm}$
Identify the spectral series.	The electron drops down to the $n = 1$ energy level. Therefore, the photon must be in the Lyman series.

**Worked example: Try yourself 10.3.3**

**ABSORPTION OF PHOTONS**

Some of the energy levels for atomic mercury are shown in the diagram below.

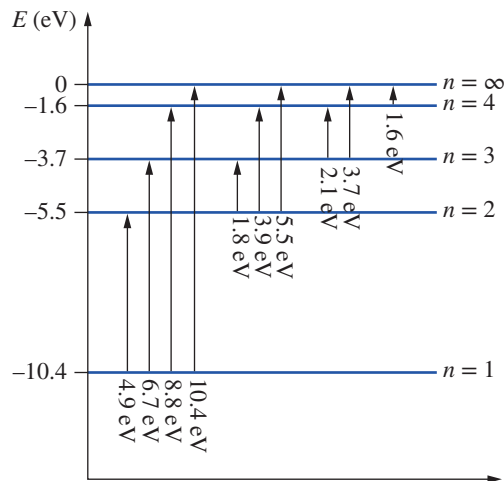


Light with photon energies 6.7, 9.0 and 11.0 eV is incident on some mercury gas. What could happen to as a result of the incident light?

**Thinking**

Check whether the energy of each photon corresponds to any differences between energy levels by determining the difference in energy between each pair of levels.

**Working**



Compare the energy of the photons with the energies determined and comment on the possible outcomes.

A photon of 6.7 eV corresponds to the energy required to promote an electron from the ground state to the second excited state ( $n = 1$  to  $n = 3$ ). The photon may be absorbed.

A photon of 5.0 eV cannot be absorbed.

A photon of 11.0 eV may ionise the mercury atom. The ejected electron will leave the atom with 0.6 eV of kinetic energy.

## 10.3 review

1 The electrons in a sample become excited when the substance is heated or an electric current flows through it. As the electrons return to their ground state, a photon is emitted.

$$\begin{aligned} 2 \quad \Delta E &= hf \\ &= 6.63 \times 10^{-34} \times 6.0 \times 10^{14} \\ &= 4.0 \times 10^{-19} \text{ J} \end{aligned}$$

$$\begin{aligned} 3 \quad \Delta E &= \frac{hc}{\lambda} \\ \lambda &= \frac{hc}{\Delta E} \\ &= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^8}{0.42} \\ &= 2.96 \times 10^{-6} \text{ m} = 3.0 \times 10^{-6} \text{ m} \end{aligned}$$

4 a Light Emitting Diode

b Light Amplification by Stimulated Emission of Radiation

$$\begin{aligned} 5 \quad \Delta E &= \frac{hc}{\lambda} \\ \therefore \lambda &= \frac{hc}{\Delta E} \\ &= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^8}{1.84} \\ &= 6.75 \times 10^{-7} \text{ m} \\ &= 675 \text{ nm} \end{aligned}$$

$$\begin{aligned} 6 \quad \Delta E &= E_4 - E_1 \\ &= -0.85 - (-13.6) \\ &= 12.75 \text{ eV} \end{aligned}$$

$$\begin{aligned} 7 \quad \Delta E &= \frac{hc}{\lambda} \\ \therefore \lambda &= \frac{hc}{\Delta E} \\ &= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^8}{12.75} \\ &= 9.74 \times 10^{-8} \text{ m or } 97.4 \text{ nm} \end{aligned}$$

8 de Broglie proposed a model where electrons were viewed as matter waves with wavelengths that formed standing waves within an atomic orbit circumference. A bowed violin string forms standing waves between the bridge of the violin and the violinist's finger.

9 High-energy orbits of multi-electron atoms, the continuous emission spectrum of solids and the two close spectral lines in hydrogen that are revealed at high resolution.

$$\begin{aligned} 10 \quad \Delta E &= \frac{hc}{\lambda} = E_5 - E_2 \\ \frac{4.14 \times 10^{-15} \times 3.0 \times 10^8}{434 \times 10^{-9}} &= E_5 - (-3.4) \\ E_5 &= 2.86 - 3.4 \\ &= -0.54 \text{ eV} \end{aligned}$$

## Section 10.4

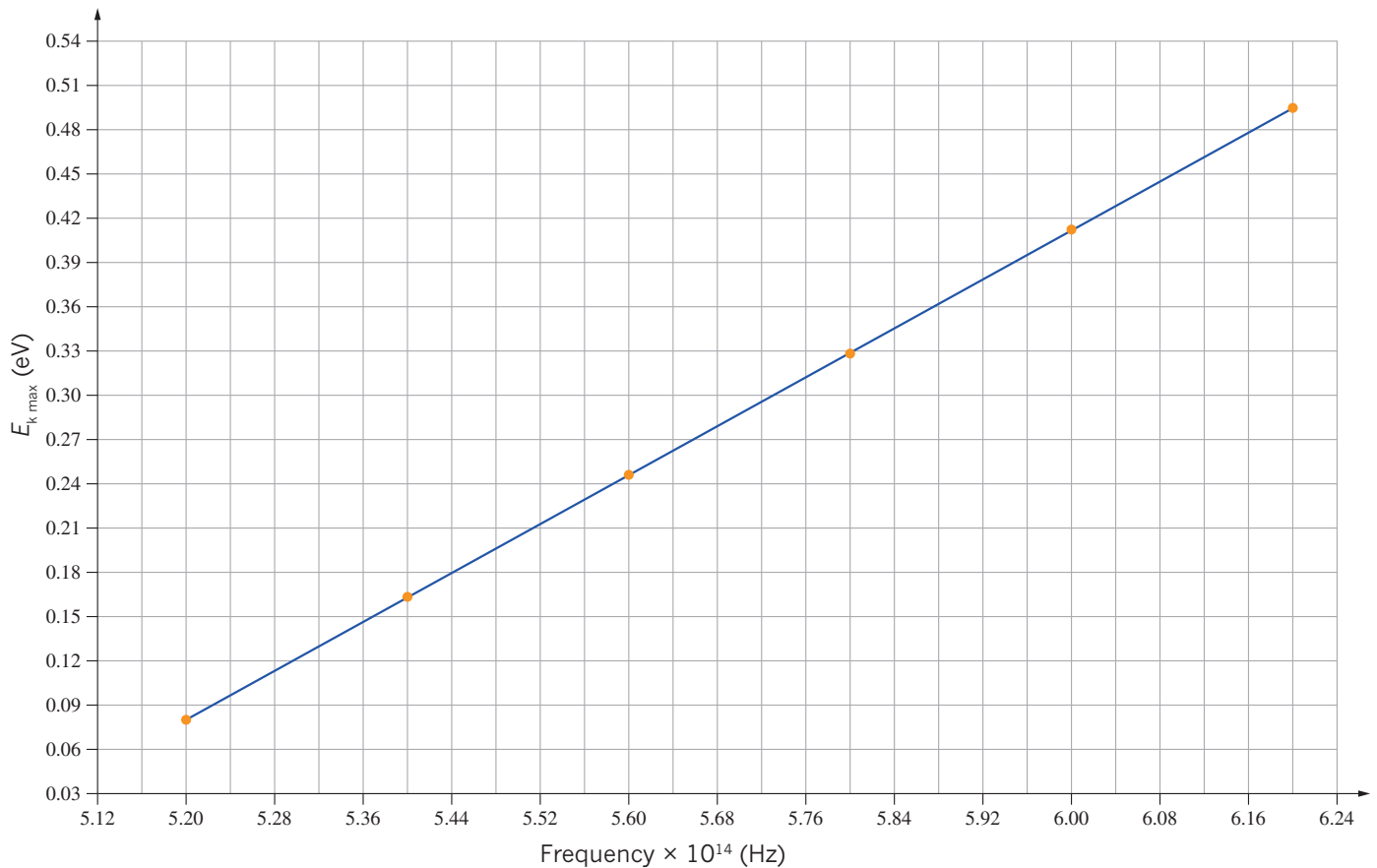
## 10.4 review

- 1 According to Heisenberg's principle, if the uncertainty about the position of a particle were decreased, then the uncertainty about the speed of the particle would *increase*.
- 2 The location of electrons can't be restricted to specific orbital paths. While electrons exist at particular energy levels, it is impossible to know the precise location of the electron. The movement of electrons cannot be predicted based on a previously known position.
- 3 Newtonian physics describes the position and velocity of an object as 'known', so its future position can be predicted. Quantum mechanics proposes that the dual nature of particles, particularly fundamental particles, prevents you from knowing the position and velocity of an object at the same time. So the assumptions that Newtonian physics makes do not fit with what happens at the sub-atomic level, making classical physics inappropriate at this scale.
- 4 The photon would transfer energy to the electron, changing its momentum and hence changing its path.
- 5 The uncertainty in the position of everyday objects is virtually zero. For the normal-sized world around us, the inclusion of Planck's constant,  $h$ , in the measure of uncertainty means that the level of uncertainty in determining the position of everyday objects is extremely small—in fact, virtually insignificant.
- 6 When there is no slit in front of a light source there are no constraints on the path of the light. The uncertainty in the position of a photon becomes large and hence the uncertainty in momentum becomes small.
- 7 Increasing the width of the slit will increase the uncertainty in the position of the electron. As a consequence, the uncertainty of the momentum of the electron will decrease. Fringes on the diffraction pattern will move closer together.
- 8 While an uncertainty can always be calculated, when applied to large objects the uncertainty is insignificant. The uncertainty principle is applied to sub-atomic particles in the study of quantum mechanics.
- 9 D. According to Heisenberg's uncertainty principle, it is impossible to precisely measure the momentum and position of a particle simultaneously.

## CHAPTER 10 REVIEW

- 1  $E = hf = 4.14 \times 10^{-15} \times 6.0 \times 10^{14} = 2.5 \text{ eV}$
- 2  $E = 5.0 \times 1.6 \times 10^{-19} \text{ J} = 8.0 \times 10^{-19} \text{ J}$
- 3 photoelectrons
- 4  $\phi = hf_0$   
 $f_0 = \frac{\phi}{h}$   
 $= \frac{5.0}{4.14 \times 10^{-15}}$   
 $= 1.2 \times 10^{15} \text{ Hz}$
- 5  $\phi = hf_0$   
 $= 4.14 \times 10^{-15} \times 1.5 \times 10^{15}$   
 $= 6.2 \text{ eV}$   
 $E_{k \text{ max}} = 4.14 \times 10^{-15} \times 2.2 \times 10^{15} - 6.2$   
 $= 2.9 \text{ eV}$
- 6 The stopping voltage is equivalent to the maximum kinetic energy of the photoelectrons, so  $E_{k \text{ max}} = 1.95 \text{ eV}$ .
- 7 The work function is given by the y-intercept of the  $E_{k \text{ max}}$  versus frequency graph. Approximate values are:  
Rb = 2.1 eV, Sr = 2.5 eV, Mg = 3.4 eV, W = 4.5 eV

8 a



$$\begin{aligned}
 \text{b gradient} = h &= \frac{\text{rise}}{\text{run}} \\
 &= \frac{0.494 - 0.080}{6.20 \times 10^{14} - 5.20 \times 10^{14}} \\
 &= \frac{0.414}{1.00 \times 10^{14}} \\
 &= 4.1 \times 10^{-15} \text{ eV s}
 \end{aligned}$$

c The x-intercept on the graph will give an approximate value of  $5.0 \times 10^{14}$  Hz.

d No. The frequency of red light is below the threshold frequency for rubidium.

Frequency of the red light:

$$\begin{aligned}
 f &= \frac{c}{\lambda} \\
 &= \frac{3.00 \times 10^8}{680 \times 10^{-9}} \\
 &= 4.41 \times 10^{14} \text{ Hz}
 \end{aligned}$$

This is less than the threshold frequency of  $5.0 \times 10^{14}$  Hz so no photoelectrons will be emitted.

$$\begin{aligned}
 \text{9 a } E &= \frac{hc}{\lambda} \\
 &= \frac{4.14 \times 10^{-15} \times 3.00 \times 10^8}{260 \times 10^{-12}} \\
 &= 4777 \text{ eV} \\
 &= 4.78 \text{ keV}
 \end{aligned}$$

b The electrons have a de Broglie wavelength which is similar to the wavelength of the X-rays. This is evidence for the dual nature of light and matter.

$$\begin{aligned}
 \text{c } p &= \frac{h}{\lambda} \\
 &= \frac{6.63 \times 10^{-34}}{260 \times 10^{-12}} \\
 &= 2.55 \times 10^{-24} \\
 &= 2.6 \times 10^{-24} \text{ kg m s}^{-1}
 \end{aligned}$$



- 10 a** The detector observed a sequence of maximum and minimum intensities.  
**b** As the electron beam is diffracted, the electrons are exhibiting wave-like behaviour. Electrons are not light but, like light, a beam of electrons can be diffracted.

$$\begin{aligned} \text{11 Electron: } \lambda &= \frac{h}{mv} \\ &= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 7.5 \times 10^6} \\ &= 9.7 \times 10^{-11} \text{ m} \end{aligned}$$

$$\text{Blue light: } \lambda = 470 \times 10^{-9} = 4.7 \times 10^{-7} \text{ m}$$

$$\text{X-ray: } c = f\lambda \text{ so } \lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{5 \times 10^{17}} = 6 \times 10^{-10} \text{ m}$$

$$\text{Proton: } \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.7 \times 10^{-21}} = 3.9 \times 10^{-13} \text{ m}$$

$\therefore$  B, blue light, has the longest wavelength

$$\begin{aligned} \text{12 } \lambda &= \frac{h}{mv} \\ &= \frac{6.63 \times 10^{-34}}{0.040 \times 1.0 \times 10^3} \\ &= 1.658 \times 10^{-35} \\ &= 1.7 \times 10^{-35} \text{ m} \end{aligned}$$

- 13** No—the wavelength is much smaller than the size of everyday objects.

The wavelength of the bullet travelling at  $1.0 \times 10^3 \text{ m s}^{-1}$  is many times smaller than the radius of an atom. Significant diffraction only occurs when wavelength and gap (or object) size is approximately equal i.e. when  $\lambda \geq w$ .

- 14** Energy levels in an atom cannot assume a continuous range of values but are restricted to certain discrete values, i.e. the levels are quantised.

$$\begin{aligned} \text{15 } \Delta E &= E_3 - E_1 \\ &= -1.5 - (-13.6) \\ &= 12.1 \text{ eV} \end{aligned}$$

$$\Delta E = hf$$

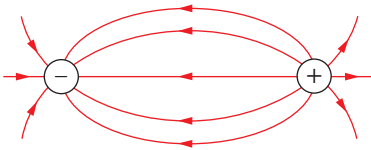
$$\begin{aligned} f &= \frac{\Delta E}{h} \\ &= 12.1 \div (4.14 \times 10^{-15}) \\ &= 2.9 \times 10^{15} \text{ Hz} \end{aligned}$$

- 16** Bohr's work on the hydrogen atom convinced many scientists that a particle model was needed to explain the way light behaves in certain situations. It built significantly on the work done by Planck and Einstein.
- 17** The emission spectrum of hydrogen appears as a series of coloured lines. The absorption spectrum of hydrogen appears as a full visible spectrum with a number of dark lines. The colours that are missing in the absorption spectrum match the colours that are visible in the emission spectrum.
- 18** As the filament heats up, the free electrons in the tungsten atoms collide, accelerate and emit photons. A wide range of photon wavelengths are emitted due to a wide range of different collisions (some weak, some strong).
- 19** For the product of the uncertainty in position and the uncertainty in momentum to remain constant then as the uncertainty in position is decreased, the uncertainty in momentum would increase.
- 20** It is likely that the photon would knock the electron off course and hence the electron's position would be subject to greater uncertainty.

## Unit 3 Area of Study 1 Review

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### Question 1



### Question 2

$$\begin{aligned}
 F &= \frac{kq_1q_2}{r^2} \\
 &= \frac{9 \times 10^9 \times 5 \times 10^{-6} \times 7 \times 10^{-6}}{0.4^2} \\
 &= 2.0 \text{ N attraction}
 \end{aligned}$$

### Question 3

$$\begin{aligned}
 E &= \frac{kq}{r^2} \\
 &= \frac{9 \times 10^9 \times 9.4 \times 10^{-6}}{(3.5 \times 10^{-3})^2} \\
 &= 6.9 \times 10^9 \text{ N C}^{-1} \text{ to the left (away from the charge)}
 \end{aligned}$$

### Question 4

$$E = \frac{V}{d} = \frac{400}{0.038} = 1.05 \times 10^4 \text{ V m}^{-1}$$

### Question 5

$$F = Eq = 1.05 \times 10^4 \times 1.6 \times 10^{-19} = 1.68 \times 10^{-15} \text{ N}$$

### Question 6

$$W = Vq = 400 \times 1.6 \times 10^{-19} = 6.4 \times 10^{-17} \text{ J}$$

OR

$$W = Fs = 1.68 \times 10^{-15} \times 0.038 = 6.4 \times 10^{-17} \text{ J}$$

### Question 7

$$\begin{aligned}
 W &= \frac{1}{2} mv^2 \\
 6.4 \times 10^{-17} &= \frac{1}{2} \times 9.11 \times 10^{-31} \times v^2 \\
 v &= 1.2 \times 10^7 \text{ m s}^{-1}
 \end{aligned}$$

### Question 8

$$\begin{aligned}
 E &= \frac{V}{d} \rightarrow V = E \times d \\
 V &= 300 \times 0.12 \\
 &= 36 \text{ V}
 \end{aligned}$$

**Question 9**

$$\begin{aligned}
 E &= \frac{V}{d} \\
 &= \frac{240}{(1.6 \times 10^{-3})} \\
 &= 1.5 \times 10^5 \text{ N C}^{-1} \text{ (or V m}^{-1}\text{) downwards}
 \end{aligned}$$

**Question 10**

$$\begin{aligned}
 q &= \frac{mgd}{V} \\
 &= \frac{1.96 \times 10^{-14} \times 9.8 \times 1.6 \times 10^{-3}}{240} \\
 &= 1.28 \times 10^{-18} \text{ C}
 \end{aligned}$$

**Question 11**

$$\begin{aligned}
 q &= nq_e \\
 n &= \frac{1.28 \times 10^{-18}}{(1.6 \times 10^{-19})} \\
 &= 8 \text{ electrons}
 \end{aligned}$$

**Question 12**

A. The source of the electrons is the heated filament at A.

**Question 13**

$$\begin{aligned}
 F &= \frac{qV}{d} \\
 &= \frac{1.6 \times 10^{-19} \times 15 \times 10^3}{12 \times 10^{-2}} \\
 &= 2 \times 10^{-14} \text{ N}
 \end{aligned}$$

**Question 14**

$$\begin{aligned}
 v^2 &= \frac{2qV}{m} \\
 &= \frac{2 \times 1.6 \times 10^{-19} \times 28 \times 10^3}{9.11 \times 10^{-31}} \\
 v &= 9.9 \times 10^7 \text{ m s}^{-1}
 \end{aligned}$$

**Question 15**

$$\begin{aligned}
 E &= \frac{V}{d} \\
 &= \frac{28 \times 10^3}{0.20} \\
 &= 1.4 \times 10^5 \text{ V m}^{-1}
 \end{aligned}$$

**Question 16**

- a A. Both have fields in direction A.
- b B. There is a field in the BC direction from the left-hand current, and in the AB direction from the right-hand current.
- c G. The field in directions A and C cancel.

**Question 17**

- a The force is to the left, due to magnetic induction in the soft iron.
- b The force is more strongly to the left as the right end of the electromagnet is now a south pole.
- c The force is to the right as the right end of the electromagnet is now a north pole.

**Question 18**

$$F = IIB = 100 \times 1 \times 1 \times 10^{-5} = 1 \times 10^{-3} \text{ N}$$

**Question 19**

The right-hand rule tells us that a current from west to east will experience an upwards force.

**Question 20**

The weight of 1 m of cable is  $mg = 0.05 \times 9.8 = 0.49 \text{ N}$ . For the magnetic force to equal this:  
 $I = \frac{F}{B} = \frac{0.49}{(1 \times 10^{-5})} = 4.9 \times 10^4 \text{ A}$ . (Not much chance of magnetic levitation for power cables!)

**Question 21**

The change of force is from  $1 \times 10^{-3} \text{ N}$  up to  $1 \times 10^{-3} \text{ N}$  down—a change of  $2 \times 10^{-3} \text{ N}$  down.

**Question 22**

B. The horizontal component of the current is now less and so there will be a smaller force per metre of cable.

**Question 23**

$$F = IIB = 1.0 \times 0.05 \times 1.0 = 0.05 \text{ N}$$

**Question 24**

The right-hand rule tells us that it is to the right.

**Question 25**

$$F = IIB = 1.0 \times 0.01 \times 1.0 = 0.01 \text{ N}$$

**Question 26**

The direction of the force on side PQ is to the left.

**Question 27**

- a Side AB is parallel to the field. Hence there will be no force on it.
- b Side DC is parallel to the field. Hence there will be no force on it.

**Question 28**

- a  $F = nIIB = 100 \times 0.2 \times 0.1 \times 0.25 = 0.5 \text{ N}$  out of the page
- b  $F = nIIB = 100 \times 0.2 \times 0.1 \times 0.25 = 0.5 \text{ N}$  into the page

**Question 29**

The coil will rotate through  $90^\circ$  until the plane of the loop is perpendicular to the field (and the page). It may swing back and forth until it settles in this position.

**Question 30**

A, B and C. A will produce a greater total force, B will increase the current, and C will result in a stronger magnetic field through the coil. D would reduce the current.

**Question 31**

- The field is from N to S, so the right-hand rule shows that the force on side AB is upwards and that on side CD is downwards.
- In the position shown (with the coil horizontal), the direction of the forces on the sides AB and CD are at right angles to the radius and the turning effect is maximum.
- The torque becomes zero when the coil is in the vertical position. It continues to rotate for two reasons: (i) Its momentum will carry it past the true vertical position. (ii) At the vertical position the commutator reverses the direction of the current through the coil and so the forces reverse, thus it continues to rotate for another half turn, at which point the current reverses again and the rotation continues.

**Question 32**

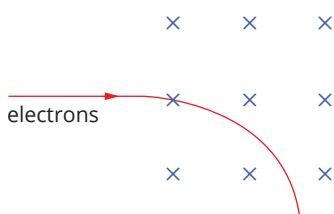
- $I = \frac{F}{nIB} = \frac{40}{(100 \times 0.2 \times 0.5)} = 4.0 \text{ A}$
- The shorter side will halve the force, twice the current will double the force and half the turns halves the force. The net effect is to halve the force, so  $F = 20 \text{ N}$ .
- The force increases with the field, so the new force is  $F = 40 \times \frac{8}{5} = 64 \text{ N}$ .

**Question 33**

$$r = \frac{mv}{eB}$$

$$= \frac{9.11 \times 10^{-31} \times 4.2 \times 10^6}{1.6 \times 10^{-19} \times 1.2}$$

$$= 2 \times 10^{-5} \text{ m}$$

**Question 34**

**Question 35**

The increase in kinetic energy of the electron as it travels from one plate to another is:

$$E_k = \frac{1}{2} mv^2 = qV$$

$$\text{So } v^2 = \frac{2qV}{m}$$

$$v = 3.25 \times 10^7 \text{ m s}^{-1}$$

Because the forces acting on the electron due to the electric and magnetic fields are balanced, we know that the electric force is equivalent to the magnetic force:

$$F_B = F_E$$

$$qvB = qE$$

$$qvB = q \frac{V}{d}$$

$$d = \frac{V}{vB}$$

$$= \frac{3000}{3.25 \times 10^7 \times 1.6 \times 10^{-3}}$$

$$= 5.8 \times 10^{-2} \text{ m}$$

**Question 36**

$$F = \frac{Gm_1m_2}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 24 \times 81}{0.72^2}$$

$$= 2.5 \times 10^{-7} \text{ N}$$

**Question 37**

C. During launch the normal force acting on the astronaut will be greater than usual and so the apparent weight will be greater.

**Question 38**

D. The gravitational force will be constant during the launch.

**Question 39**

A. In a stable orbit, there is no normal force acting ( $N = 0$ ) on the astronaut so they will experience apparent weightlessness.

**Question 40**

B. In deep space, there are no planets or large masses to exert a force of gravity on the astronaut so they will experience weightlessness since  $g = 0$ ,  $W = 0$ .

**Question 41**

$E_g$  = area under graph between  $7.0 \times 10^6 \text{ m}$  and  $6.5 \times 10^6 \text{ m}$

Counting squares gives 8.5 squares

area of each square =  $1.0 \times 10^4 \times 0.5 \times 10^6 = 5 \times 10^9 \text{ J}$

$$E_g = 8.5 \times 5 \times 10^9$$

$$= 4.25 \times 10^{10} \text{ J}$$

**Question 42**

At 600 km altitude (height of 7000 km):  $E_k = \frac{1}{2}mv^2 = 0.5 \times 10\,000 \times 1500^2 = 1.125 \times 10^{10} \text{ J}$

So  $E_k$  at 100 km altitude (height of 6500 km) =  $1.125 \times 10^{10} + 4.25 \times 10^{10} = 5.375 \times 10^{10} \text{ J}$

$$\frac{1}{2}mv^2 = 5.375 \times 10^{10}$$

$$0.5 \times 10\,000 v^2 = 5.375 \times 10^{10}$$

$$v = 3.3 \times 10^3 \text{ m s}^{-1}$$

**Question 43**

**a**  $r = 6400 + 3600 = 10\,000 \text{ km} = 10 \times 10^6 \text{ m}$

$$W = 4.0 \times 10^4 \text{ N (from graph)}$$

**b**  $r = 6.0 \times 10^5 + 6.4 \times 10^6 = 7.0 \times 10^6 \text{ m}$

$$W = 8.1 \times 10^4 \text{ N (from graph)}$$

**Question 44**

At 600 km,  $F_g = 8.1 \times 10^4 \text{ N}$ , so  $a = \frac{F_g}{m} = 8.1 \text{ m s}^{-2}$

At 100 km,  $F_g = 9.2 \times 10^4 \text{ N}$ , so  $a = \frac{F_g}{m} = 9.2 \text{ m s}^{-2}$

The acceleration increases from  $8.1 \text{ m s}^{-2}$  to  $9.2 \text{ m s}^{-2}$ .

**Question 45**

$$F = \frac{GMm}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 1.2 \times 10^4}{(6.73 \times 10^6)^2}$$

$$= 1.06 \times 10^5 \text{ N}$$

**Question 46**

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$= \sqrt{\frac{4 \times \pi^2 \times (6.73 \times 10^6)^3}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}}$$

$$= 5.5 \times 10^3 \text{ s}$$

**Question 47**

The mass of the satellite has no effect on its orbital period.

**Question 48**

At 300 km,  $g \approx 3.0 \text{ N kg}^{-1}$

$$F_g = mg = 20 \times 3.0 = 60 \text{ N}$$

**Question 49**

$$\text{Area} \approx 9 \text{ squares} = 9 \times 1.0 \times 2.0 \times 10^5 = 1.8 \times 10^6 \text{ J kg}^{-1}$$

$$\Delta E_k = \text{area} \times \text{mass} = 1.8 \times 10^6 \times 20 = 3.6 \times 10^7 \text{ J}$$

**Question 50**

Determine the energy associated with each grid square by multiplying each area by the mass of 20 kg. Calculate the altitude at which the total area starting from zero height is equal to 40 MJ.



## Unit 3 Area of Study 2 Review

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### How are fields used to move electrical energy?

#### Question 1

Since the plane of the loop is parallel to the magnetic field direction, no (zero) flux threads the loop.

#### Question 2

Rotate the loop or the magnetic field so they are no longer parallel.

#### Question 3

The maximum flux threads the loop when the plane of the loop and the magnetic field direction are perpendicular (or at right angles) to each other.

#### Question 4

$$\begin{aligned}\Phi_B &= B_{\perp}A \\ &= 0.50 \times 0.2 \times 0.1 \\ &= 0.01 \text{ Wb or } 10^{-2} \text{ Wb}\end{aligned}$$

#### Question 5

As the loop enters the magnetic field there is a flux increasing down through the loop. Lenz's law states the induced current in the loop will oppose the change in flux that causes it. Therefore there will be an induced field (or flux) up through the loop. Using the right-hand grip rule with the thumb pointing up, the fingers curl in the direction of the induced current from Y to X.

#### Question 6

The loop moves at a speed of  $5 \text{ cm s}^{-1}$ , and with side length 20 cm, it is halfway into the field when it has travelled 10 cm, which takes 2 s.

$$\begin{aligned}\varepsilon &= N \frac{\Delta\Phi_B}{\Delta t} \\ &= \frac{1 \times 0.40 \times 0.2 \times 0.1}{2} \\ &= 4 \times 10^{-3} \text{ V}\end{aligned}$$

#### Question 7

$$\begin{aligned}I &= \frac{V}{R} \\ &= \frac{4 \times 10^{-3}}{0.5} \\ &= 8 \times 10^{-3} \text{ A}\end{aligned}$$

#### Question 8

$$\begin{aligned}P &= VI \\ &= 4 \times 10^{-3} \times 8 \times 10^{-3} \\ &= 3.2 \times 10^{-5} \text{ W}\end{aligned}$$

**Question 9**

The source of this power is the external force that is moving the loop into the magnetic field.

**Question 10**

The loop is moving at a speed of  $5 \text{ cm s}^{-1}$ , so after 5 seconds it has moved 25 cm and has been totally within the magnetic field for 1 second. Since there is now no flux change there will be no emf induced in the loop at this moment.

**Question 11**

As the loop emerges from the magnetic field there is a flux decreasing down through the loop. Lenz's law states the induced current in the loop will oppose the change in flux that causes it. Therefore there will be an induced field (or flux) down through the loop. Using the right-hand grip rule with the thumb pointing down, the fingers curl in the direction of the induced current from X to Y.

**Question 12**

$$\begin{aligned}\Phi_B &= B_{\perp} A \\ &= 1.0 \times 10^{-3} \times 100 \times 10^{-3} \times 50 \times 10^{-3} \\ &= 5 \times 10^{-6} \text{ Wb}\end{aligned}$$

**Question 13**

No (zero) flux threads the loop in the new position as the plane of the loop is now parallel to the magnetic field.

OR

$\theta = 90^\circ$  and  $\cos \theta = 0$  so no (zero) flux threads the loop in the new position.

**Question 14**

$$\begin{aligned}\varepsilon &= N \frac{\Delta \Phi_B}{\Delta t} \\ &= \frac{1 \times 5 \times 10^{-6}}{2 \times 10^{-3}} \\ &= 2.5 \times 10^{-3} \text{ V}\end{aligned}$$

**Question 15**

$$\begin{aligned}I &= \frac{V}{R} \\ &= \frac{2.5 \times 10^{-3}}{2.0} \\ &= 1.25 \times 10^{-3} \text{ A}\end{aligned}$$

**Question 16**

No. Once the loop is stationary, there is no change in flux and therefore no emf generated and no current flows in the loop.

**Question 17**

The emf, and hence the current, depends on the rate of change. If the rate is increased by 4, then the current will also increase by 4:  $I = 200 \mu\text{A}$ .

**Question 18**

The emf generated must have been  $V = IR = 50 \times 10^{-6} \times (595 + 5) = 3 \times 10^{-2} \text{ V}$ .

$$\varepsilon = N \frac{\Delta\Phi_B}{\Delta t} \quad \text{and} \quad \Phi_B = B_{\perp} A \quad \text{and} \quad A = \pi r^2$$

$$\therefore \varepsilon = N \frac{B_{\perp} \pi r^2}{\Delta t}$$

$$\begin{aligned} \rightarrow B &= \frac{\varepsilon \Delta t}{N \pi r^2} \\ &= \frac{3 \times 10^{-2} \times 2}{(100 \times \pi \times 0.03^2)} \\ &= 0.21 \text{ T} \end{aligned}$$

**Question 19**

$$T = \frac{1}{f} = \frac{1}{100} = 0.01 \text{ s}$$

The graph is a sine wave with peak amplitude of 0.9 V and a period of 0.01 s (10 ms).

**Question 20**

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}} = \frac{0.9}{\sqrt{2}} = 0.64 \text{ V}$$

**Question 21**

The output graph would have half the period and twice the amplitude. The rms voltage would be 1.3 V.

**Question 22**

$$\begin{aligned} f &= \frac{3000}{60} \\ &= 50 \text{ Hz} \end{aligned}$$

**Question 23**

$$T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ s}$$

A quarter turn will take  $\frac{0.02}{4} = 0.005 \text{ s}$

$$\begin{aligned} \varepsilon &= N \frac{\Delta\Phi_B}{\Delta t} \\ &= \frac{200 \times 0.5 \times 100 \times 10^{-4}}{0.005} \\ &= 200 \text{ V} \end{aligned}$$

**Question 24**

B. This is twice the frequency and so the amplitude will be double and the period will halve.

**Question 25**

D. This has twice the amplitude but the same period and so could be obtained by doubling  $N$ .

**Question 26**

C. This time the period has halved (the frequency has doubled) but the amplitude remains the same. Thus a combination of the other quantities must have halved ( $B$  has doubled, but  $N$  has reduced to one-quarter).

**Question 27**

$$f = \frac{1}{T} = \frac{1}{2 \times 10^{-3}} = 500 \text{ Hz}$$

**Question 28**

$$V_{\text{p-p}} = 20 \text{ V}$$

**Question 29**

$$V_{\text{rms}} = \frac{V_{\text{peak}}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.1 \text{ V}$$

**Question 30**

$$I_{\text{rms}} = \frac{I_{\text{peak}}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.71 \text{ A}$$

**Question 31**

$$\begin{aligned} P_{\text{rms}} &= V_{\text{rms}} \times I_{\text{rms}} \\ &= 7.1 \times 0.71 \\ &= 5 \text{ W} \end{aligned}$$

**Question 32**

An alternator has a pair of slip rings instead of a split ring commutator.

**Question 33**

AC is generated in the coils of an alternator. Each slip ring connects to each end of the coil. The slip rings maintain the AC generated in the coil at the output.

**Question 34**

$$I_2 = \frac{I_1 \times V_1}{V_2} = \frac{2.0 \times 600}{3000} = 0.4 \text{ A}$$

**Question 35**

$$\begin{aligned} V_{\text{p-p}} &= 2 \times 3000 \\ &= 6000 \text{ V} \end{aligned}$$

**Question 36**

$$N_1 = \frac{N_2 \times V_1}{V_2} = \frac{1000 \times 600}{3000} = 200 \text{ turns}$$

**Question 37**

$$P_{2 \text{ rms}} = V_{2 \text{ rms}} \times I_{2 \text{ rms}} = \frac{3000}{\sqrt{2}} \times 0.4 = 849 \text{ W}$$

**Question 38**

$$P_{2 \text{ peak}} = V_{2 \text{ peak}} \times I_{2 \text{ peak}} = 3000 \times 0.4 \times \sqrt{2} = 1697 \text{ W}$$

**Question 39**

C. The alternating current in the primary produces a changing magnetic flux, which induces an emf in the secondary coils (as well as the primary coils).

**Question 40**

C. The self-induced emf is known as a back emf and opposes the mains emf.

**Question 41**

With little or no current in the power line there was almost no voltage drop. When the house appliances were turned on, there was a higher current in the power line and hence a voltage drop along the line, leaving a low voltage at the house.

**Question 42**

As the generator was supplying 4000 W at 250 V, the current in the line was  $I = 4000 \div 250 = 16$  A. The voltage drop along the line was therefore  $\Delta V = IR = 16 \times 2 = 32$  V and so the voltage at the house was  $250 - 32 = 218$  V. The power lost is  $P_{\text{loss}} = I^2R = 16^2 \times 2 = 512$  W and so the power at the house was  $4000 - 512 = 3488$  W OR  $P_{\text{house}} = VI = 218 \times 16 = 3488$  W

**Question 43**

At the generator end a 1:20 step-up transformer is required ( $5000 \div 250 = 20$ ). There will be 20 times as many turns in the secondary as in the primary. At the house end a 20:1 step-down transformer is required.

**Question 44**

$$I = \frac{P}{V} = \frac{4000}{5000} = 0.8 \text{ A}$$

**Question 45**

The voltage drop is  $V = IR = 0.8 \times 2 = 1.6$  V.

**Question 46**

The power loss is  $P = I^2R = 0.8^2 \times 2 = 1.28$  W.

**Question 47**

The voltage at the house will be  $\frac{5000 - 1.6}{20} = 249.92$  V.

**Question 48**

The power at the house will be  $4000 - 1.28 = 3998.72$  W.

**Question 49**

The power loss before the transformers were added was 512 W (Question 42) which was 12.8% of the power generated (4000 W), and the power loss with the transformers was 1.28 W (Q46), which is about 0.03% of the power generated.

**Question 50**

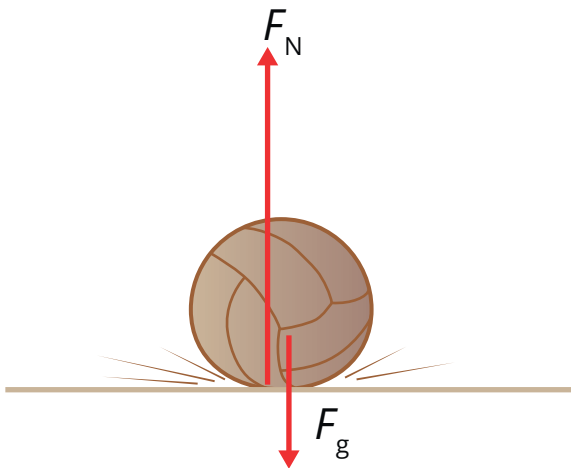
The reason is that the power loss in the power line depends on the square of the current ( $P = I^2R$ ). Since the current was reduced by a factor of 20 and the resistance remains constant, the power loss decreased by a factor of  $20^2$  or 400.

## Unit 3 Area of Study 3 Review

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### Question 1

$F_N$  must be greater than  $F_g$



### Question 2

C. Action/reaction pairs always act on different objects. One force acts on the floor and the other force acts on the ball. These forces are equal in magnitude as described in Newton's third law.

### Question 3

unbalanced, balanced

### Question 4

$$v = \frac{2\pi r}{T} = \frac{2\pi \times 5}{2.5}$$
$$= 12.6 \text{ m s}^{-1}$$

### Question 5

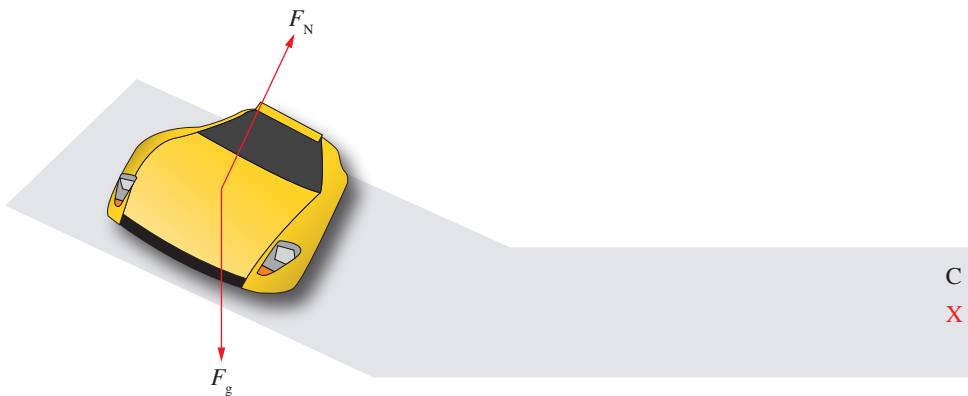
$$a = \frac{v^2}{r} = \frac{12.6^2}{5}$$
$$= 31.8 \text{ m s}^{-2}$$

### Question 6

$$F_N = F_{\text{net}} = ma$$
$$= 60 \times 31.6$$
$$= 1.9 \times 10^3 \text{ N}$$

### Question 7

$$T = \frac{10}{6} = 1.67 \text{ s}$$
$$f = \frac{1}{T} = \frac{1}{1.67} = 0.6 \text{ Hz}$$

**Question 8****Question 9**

$$\begin{aligned}\tan \theta &= \frac{v^2}{rg} \\ \theta &= \tan^{-1} \frac{v^2}{rg} \\ &= \tan^{-1} \left( \frac{40^2}{150 \times 9.8} \right) \\ &= 47^\circ\end{aligned}$$

**Question 10**

$$a = \frac{v^2}{r} = \frac{6^2}{2} = 18 \text{ m s}^{-2} \text{ up}$$

**Question 11**

$$\begin{aligned}F_N &= F_g + F_{\text{net}} \\ &= mg + ma \\ &= 55 \times 9.8 + 55 \times 18 \\ &= 1.5 \times 10^3 \text{ N}\end{aligned}$$

**Question 12**

The apparent weight is given by the normal force of  $1.5 \times 10^3 \text{ N}$ . This is almost three times larger than the weight force and so the skater would feel much heavier than usual.

**Question 13**

Take up as positive.

vertically:  $v = 0$  (at the top),  $a = -9.8$ ,  $t = 1.0$ ,  $s = ?$

$$\begin{aligned}s &= vt - \frac{1}{2} at^2 \\ &= 0 - 0.5 \times -9.8 \times 1.0^2 \\ &= 4.9 \text{ m}\end{aligned}$$

**Question 14**

$9.8 \text{ m s}^{-2}$  down

**Question 15**

horizontally:  $u = ?$ ,  $a = 0$ ,  $t = 2$ ,  $s = 8$

$$s = ut + \frac{1}{2} at^2$$

$$8 = u \times 2$$

$$u = 4 \text{ m s}^{-1}$$

taking up as positive

vertically:  $v = 0$  (at the top),  $a = -9.8$ ,  $t = 1$ ,  $u = ?$

$$v = u + at$$

$$0 = u - 9.8 \times 1$$

$$u = 9.8 \text{ m s}^{-1}$$

Use Pythagoras to find the actual speed at launch:

$$\begin{aligned} u &= \sqrt{4^2 + 9.8^2} \\ &= 10.6 \text{ m s}^{-1} \end{aligned}$$

**Question 16**

C. The only force acting is the gravitational force.

**Question 17**

A. Since the collision is inelastic, kinetic energy is not conserved and  $E_{k \text{ after}} < E_{k \text{ before}}$  as shown in graphs A and C. During the collision, some of the kinetic energy is converted into spring potential energy, and then some of this is restored to kinetic energy, so the  $E_k$  graph dips slightly over the time of the collision as shown in graph A.

**Question 18**

D. Momentum is conserved (i.e. is constant) in all collisions as shown by the flat line in graph D.

**Question 19**

D. For momentum to be conserved, what is lost by the tennis ball is gained by the bowling ball. The tennis ball's change in momentum will be backwards. The bowling ball's change in momentum will be forwards. So their directions are opposite.

**Question 20**

D. The forces exerted by each ball on the other make an action/reaction pair and must be equal and opposite according to Newton's third law.

**Question 21**

The ball bearing just maintains contact with the track so  $N = 0$ , and  $F_{\text{net}} = F_g$  so  $a = 9.8 \text{ m s}^{-2}$  down.



**Question 22**

at point C,  $F_N = 0$ , so  $F_{\text{net}} = F_g$

$$ma = mg$$

$$a = g$$

$$\frac{v^2}{r} = g$$

$$v = \sqrt{rg} = \sqrt{0.5 \times 9.8} = 2.2 \text{ m s}^{-1}$$

**Question 23**

apparent weight =  $F_N$  and at point C,  $F_N = 0$   $\therefore$  apparent weight = 0

**Question 24**

Total energy at point C =  $E_k + E_g$

$$= \frac{1}{2} mv^2 + mgh$$

$$= \frac{1}{2} \times 0.025 \times 2.22^2 + 0.025 \times 9.8 \times 1.0$$

$$= 0.3066 \text{ J}$$

Total energy at point B =  $E_k = \frac{1}{2} mv^2 = 0.3066 \text{ J}$

$$0.3066 = \frac{1}{2} \times 0.025 v^2$$

$$v = \sqrt{\frac{2 \times 0.3066}{0.025}}$$

$$= 5.0 \text{ m s}^{-1}$$

**Question 25**

net force =  $100 - 30 = 70 \text{ N}$

$W$  on cart =  $F \times s$

$$= 70 \times 20$$

$$= 1400 \text{ J}$$

**Question 26**

$$\Delta E_k = 1400 \text{ J}$$

**Question 27**

$$\Delta E_k = 1.4 \times 10^3 \text{ J}$$

$$\frac{1}{2} mv^2 = 1.4 \times 10^3$$

$$v = \sqrt{\frac{2 \times 1.4 \times 10^3}{200}}$$

$$= 3.7 \text{ m s}^{-1}$$

**Question 28**

$$\begin{aligned} P &= \frac{W \text{ (by prospector)}}{\text{time}} \\ &= \frac{100 \times 20}{10} \\ &= 200 \text{ W} \end{aligned}$$

**Question 29**

$$\begin{aligned} E_s &= \frac{1}{2}k\Delta x^2 \\ &= 0.5 \times 1500 \times 0.18^2 \\ &= 24.3 \text{ J} \end{aligned}$$

**Question 30**

$$\begin{aligned} m_1u_1 + m_2u_2 &= (m_1 + m_2)v \\ 120 \times 6 + 45 \times 0 &= (120 + 45)v \\ 720 &= 165v \\ v &= 4.4 \text{ m s}^{-1} \end{aligned}$$

**Question 31**

$$\begin{aligned} \Delta p &= p_f - p_i \\ &= (120 \times 4.4) - (120 \times 6) \\ &= -192 \text{ kg m s}^{-1} \end{aligned}$$

The ruckman loses  $192 \text{ kg m s}^{-1}$ .

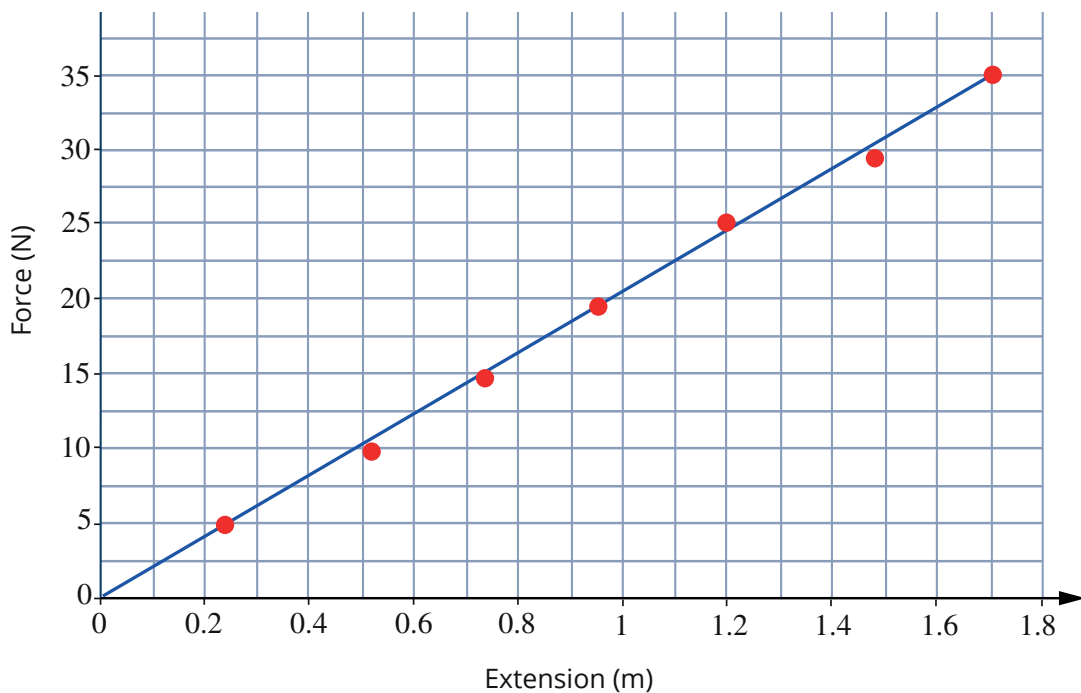
**Question 32**

In this collision, the momentum gain of the bag is equal to the momentum loss of the ruckman, i.e.  $192 \text{ kg m s}^{-1}$ .

**Question 33**

$$\begin{aligned} E_k \text{ before} &= \frac{1}{2} \times 120 \times 6^2 = 2160 \text{ J} \\ E_k \text{ after} &= \frac{1}{2} (120 + 45) \times 4.4^2 = 1597 \text{ J} \end{aligned}$$

Since kinetic energy is not conserved, the collision is inelastic.

**Question 34****Question 35**

$$k = \text{gradient}$$

$$= \frac{\text{rise}}{\text{run}} = \frac{34.3}{1.70} \approx 20 \text{ N m}^{-1}$$

**Question 36**

$$\begin{aligned} E_s &= \frac{1}{2} k \Delta x^2 \\ &= 0.5 \times 20 \times 15^2 \\ &= 2.25 \times 10^3 \text{ J} \end{aligned}$$

**Question 37**

C. As the stretched bungee rope returns to its original length, the extension  $\Delta x$  decreases, and since  $F = k\Delta x$ , the force becomes smaller.  $F = ma$ , so as  $F$  becomes smaller, acceleration will become smaller. There is still an acceleration, so the velocity will continue to increase.

**Question 38**

$$\begin{aligned} E_k &= \frac{1}{2} mv^2 \\ &= 2250 \text{ J} \end{aligned}$$

$$0.5 \times 60 \times v^2 = 2250$$

$$\begin{aligned} v &= \sqrt{\frac{2250}{0.5 \times 60}} \\ &= 8.7 \text{ m s}^{-1} \end{aligned}$$

**Question 39**

A (postulate 2) and C (postulate 1)

**Question 40**

A or C. If the other craft is further away, its velocity away from Earth is  $4 \times 10^6 - 0.4 \times 10^6 = 3.6 \times 10^6 \text{ m s}^{-1}$ . If it is between us and Earth, it is  $4 \times 10^6 + 0.4 \times 10^6 = 4.4 \times 10^6 \text{ m s}^{-1}$ .

**Question 41**

C.  $\gamma$  must be  $> 1$ , so A and B are not correct. The speed is much less than  $c$ , so D is not correct. C is the only feasible answer.

OR

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{50\,000^2}{300\,000\,000^2}}} = 1.000\,000\,014$$

**Question 42**

C. It was the elegance of Maxwell's equations that convinced Einstein that they, and their implications about light, were correct.

**Question 43**

Aristotle's ideas agreed with our everyday observations. We experience objects as slowing or stopping without an external force to keep them going, and we cannot see that there are actually forces (e.g. gravitational or frictional) acting to slow them down. In a space station we would often experience objects moving with constant velocity with no external force, as objects 'floated' around the ship.

**Question 44**

In your frame of reference time proceeds normally. As Mars is moving at a high speed relative to you, people on Mars appear to be in slow motion as time for them, as seen by you, will be dilated.

**Question 45**

$$\begin{aligned} \text{time} &= \frac{\text{distance}}{\text{speed}} \\ &= \frac{5}{0.9} \\ &= 5.6 \text{ years} \end{aligned}$$

**Question 46**

At  $0.9c$  Raqu's time will seem to be shortened by a factor  $\gamma = 2.3$ , thus it will seem to take her only 2.4 years.

**Question 47**

Relative to her, the distance appeared to be foreshortened by the factor  $\gamma$ , thus the distance she travelled was much less than 5 light years.

**Question 48**

The mass difference is  $4 \times 1.673 \times 10^{-27} - 6.645 \times 10^{-27} = 4.7 \times 10^{-29} \text{ kg}$

$$\begin{aligned} E &= mc^2 = 4.7 \times 10^{-29} \times (3 \times 10^8)^2 \\ &= 4.23 \times 10^{-12} \text{ J} \end{aligned}$$

**Question 49**

As the total energy produced by the Sun each second is  $3.9 \times 10^{26} \text{ J}$ , and the last answer gives us the energy produced for each helium atom, the number of helium atoms must be given by

$$\frac{3.9 \times 10^{26}}{4.2 \times 10^{-12}} = 9.3 \times 10^{37} \text{ every second.}$$

**Question 50**

The mass lost by the Sun each second is given by  $m = 9.3 \times 10^{37} \times 4.7 \times 10^{-29} = 4.37 \times 10^9 \text{ kg}$

In one day this will be  $4.37 \times 10^9 \times 24 \times 60 \times 60 = 3.8 \times 10^{14} \text{ kg}$ .

## Unit 4 Area of Study 1 Review

### How can waves explain the behaviour of light?

#### Question 1

C. Sound is a longitudinal wave of pressure variations.

#### Question 2

amplitude of X = 2 squares

amplitude of Y = 1 square

$$A_x:A_y = 2:1$$

#### Question 3

period  $T$  of X = 4 squares

period  $T$  of Y = 2 squares

$$\text{frequency} = \frac{1}{\text{period}}$$

$$\text{if } T_x:T_y = 4:2 \text{ or } 2:1$$

$$f_x:f_y = 1:2$$

#### Question 4

There is only one trough of low pressure on graph X, hence only one rarefaction.

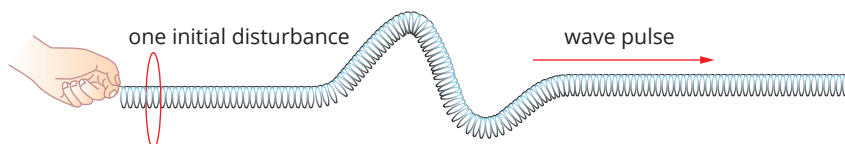
#### Question 5

The graph of pressure variation has three positive peaks corresponding to three compressions (areas of higher than normal pressure).

#### Question 6

D. In a compression, the particles have moved closest together so the pressure is a maximum. In a rarefaction, the particles have moved the furthest apart so the pressure is a minimum.

#### Question 7



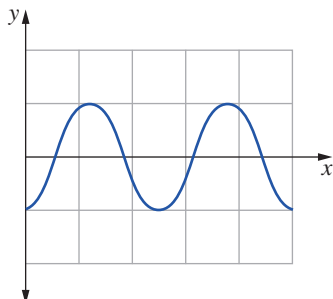
#### Question 8

A mechanical wave involves energy being transferred from one location to another, without any net transfer of matter. Mechanical waves cannot transfer energy through a vacuum.

**Question 9**

Displacement of the medium perpendicular to the direction of travel of the wave produces a transverse wave.

Displacement of the medium parallel to the direction of travel of the wave produces a longitudinal wave.

**Question 10****Question 11**

A wavelength is defined as one complete cycle, which could be measured from 0.5 to 4.5 m, from 1.5 to 5.5 m or from 2.5 to 6.5 m. All of these combinations give a wavelength of 4.0 m.

**Question 12**

A. The energy and the wave are moving away from the source from left to right.

**Question 13**

C. The speed of sound is determined by the medium through which the sound wave travels.

**Question 14**

$$v = f\lambda$$

$$\lambda = \frac{v}{f} = v \times T = 340 \times 3.9 \times 10^{-3} = 1.3 \text{ m}$$

**Question 15**

The wavelength is the distance along the wave of one complete cycle of the wave. This occurs every 4 cm for this wave.

The amplitude is the maximum displacement of the wave from its average position. This is 2 cm for this wave.

**Question 16**

When the wave reaches the fixed end a force is exerted on the fixed end. A reaction force acts on the rope and some energy is absorbed by the fixed end. Energy is transformed into heat within the fixed end. The reflection of the wave results in a decrease in amplitude and a phase change.

**Question 17**

When two or more waves meet and combine, the resulting waveform will be the vector addition of the individual waves due to the principle of superposition. Although there is a different displacement as the waves are superimposed, passing through each other does not alter the shape, amplitude or speed of the individual waves.

**Question 18**

The superposition of waves 1 and 2 results in a wave with an amplitude of twice the original amplitude and a wavelength the same as the original wavelength.

**Question 19**

After the pulses have passed through each other, they will have the same characteristics as before the interaction.

**Question 20**

$$\begin{aligned} f &= \frac{v}{4l} \\ &= \frac{330}{4 \times 0.75} \\ &= 110 \text{ Hz} \end{aligned}$$

**Question 21**

$$\begin{aligned} f &= \frac{nv}{4l} \\ &= \frac{3 \times 330}{4 \times 0.75} \\ &= 330 \text{ Hz} \end{aligned}$$

**Question 22**

$$\begin{aligned} f &= \frac{nv}{4l} \\ &= \frac{5 \times 330}{4 \times 0.75} \\ &= 550 \text{ Hz} \end{aligned}$$

**Question 23**

A = node, B = antinode

**Question 24**

The two ropes in the image show the maximum and minimum positions of the rope as it oscillates.

**Question 25**

The image shows three loops of the rope so it is the third harmonic.

**Question 26**

Due to the Doppler effect, the apparent frequency will increase when the source is moving towards the detector.



**Question 27**

Being pushed in the direction of motion once, at the correct moment each oscillation will drive the swing at its resonant frequency and will result in a gain in amplitude.

**Question 28**

$$3 \times 10^8 \text{ m s}^{-1}$$

**Question 29**

$$v = f\lambda$$

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{10^{16}}$$

$$= 3 \times 10^{-8} \text{ m}$$

**Question 30**

From the diagram, they are ultraviolet waves.

**Question 31**

Any one of:

- UV lamps are used to sterilise surgical equipment in hospitals
- UV lamps are used to sterilise food and drugs
- UV rays help the body to produce vitamin D
- any other suitable use of UV.

**Question 32**

$$v = f\lambda \text{ so } \lambda = \frac{v}{f} = \frac{3 \times 10^8}{3 \times 10^{15}} = 10^{-7} \text{ m}$$

From the diagram this is in the infrared region.

**Question 33**

C and D. The amount of diffraction  $\propto \frac{\lambda}{w}$  and since  $v = f\lambda$ ,  $\lambda = \frac{v}{f}$  so the amount of diffraction  $\propto \frac{v}{fw}$ . Since  $f$  and  $w$  are on the bottom, if they are decreased, the amount of diffraction will increase.

**Question 34**

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1 \times \sin 30^\circ = 2.42 \sin \theta_2$$

$$\theta_2 = \sin^{-1} \left( \frac{0.5}{2.42} \right)$$

$$= 11.9^\circ$$

**Question 35**

$$n_1 v_1 = n_2 v_2$$

$$1 \times 3 \times 10^8 = 2.42 \times v_2$$

$$v_2 = \left( \frac{3 \times 10^8}{2.42} \right)$$

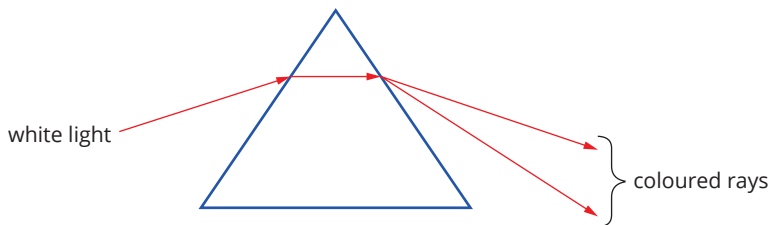
$$= 1.24 \times 10^8 \text{ m s}^{-1}$$

**Question 36**

Different colours have different wavelengths (or frequencies).

**Question 37**

The students would need to pass white light through a triangular glass prism as shown.

**Question 38**

The white light will separate into the component colours which, from top to bottom, will be red, orange, yellow, green, blue, indigo, violet.

**Question 39**

In polarised light, the electromagnetic variations occur in only one direction at right angles to the direction of propagation.

**Question 40**

Sound is a longitudinal wave and the variations occur parallel to the direction of propagation so no right-angle variation can occur.

**Question 41**

Maximum light intensities occur when the polarising axes are parallel.

**Question 42**

During a full 360° rotation, the polarising axes go from parallel, to perpendicular at 90°, to parallel at 180°, to perpendicular at 270° and back to parallel at 360°. After the start there are two instances of perpendicular polarising axes where the crossed polaroids transmit zero light and two instances of parallel polarising axes where the transmitted light intensity will be a maximum.

**Question 43**

Since  $\Delta x = \frac{\lambda L}{d}$ , if  $d$  is halved,  $\Delta x$  will be doubled.

**Question 44**

Since  $\Delta x = \frac{\lambda L}{d}$ , if  $L$  is doubled,  $\Delta x$  will be doubled.

**Question 45**

Since  $\Delta x = \frac{\lambda L}{d}$  and  $\lambda = \frac{v}{f}$ , then  $\Delta x = \frac{vL}{df}$  and if  $f$  is halved,  $\Delta x$  will be doubled.

**Question 46**

There will be a wider central band.

**Question 47**

The wave model and the particle (or corpuscular) model.

**Question 48**

Young's experiment resulted in bright and dark bands or fringes being seen on a screen.

These can only be due to interference effects. The ability to interfere with one another constructively and destructively is a property of waves. So his work supported the wave model of light. (The particle model could not explain the interference effects observed; it would predict just two bright bands.)

**Question 49**

A and B show maximum constructive interference. At position A, two crests add to give maximum constructive interference. At position B, two troughs add to give maximum constructive interference. At positions C and D, a trough and a crest add to give maximum destructive interference.

**Question 50**

For the third dark band,  $pd = 2.5 \lambda$ . For the fourth dark band,  $pd = 3.5 \lambda$ . That is, the path difference is always one whole wavelength greater for each consecutive dark band. This value has been stated as equal to 500 nm in this example, therefore  $\lambda = 500$  nm.

## Unit 4 Area of Study 2 Review

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### How are light and matter similar?

#### Question 1

B.  $V_0$  is proportional to the energy of the incident photons. Since blue light has a higher frequency than yellow light, its photons have more energy.

#### Question 2

C. The retarding potential difference works against the electrons as they try to reach the collector.

A and B are incorrect because the potential difference between the emitter and the collector does not affect these quantities.

#### Question 3

A. The colour of the incident light is indicated by the value of  $V_0$ , while the intensity of the incident light is indicated by the size of the current.

#### Question 4

$$E_{k\max} = \frac{hc}{\lambda} - \phi$$
$$1.21 = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{200 \times 10^{-9}} - \phi$$
$$\phi = 6.21 - 1.21 = 5 \text{ eV}$$

#### Question 5

- Only certain frequencies of light will emit photoelectrons.
- There is no time difference between the emission of photoelectrons by light of different intensities.
- The maximum kinetic energy of the ejected photoelectrons is the same for different light intensities of the same frequency.

#### Question 6

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$
$$= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times \frac{0.01}{100} \times 3 \times 10^8}$$
$$= 2.42 \times 10^{-8} \text{ m}$$

#### Question 7

A series of bright and dark fringes.

#### Question 8

The high-speed electrons are exhibiting wave-like behaviour.

**Question 9**

$$\lambda = \frac{h}{mv} \rightarrow v = \frac{h}{m\lambda}$$

$$v = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 2.42 \times 10^{-9}}$$

$$= 164 \text{ m s}^{-1}$$

**Question 10**

$$\Delta E = E_4 - E_1$$

$$= -1.6 - (-10.4)$$

$$= 8.8 \text{ eV}$$

$$\Delta E = hf \rightarrow f = \frac{\Delta E}{h}$$

$$f = \frac{8.8}{4.14 \times 10^{-15}}$$

$$= 2.13 \times 10^{15} \text{ Hz}$$

$$c = f\lambda \rightarrow \lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{2.13 \times 10^{15}}$$

$$= 1.41 \times 10^{-7} \text{ m}$$

**Question 11**

$$\Delta E = E_2 - E_1$$

$$= -5.5 - (-10.4)$$

$$= 4.9 \text{ eV}$$

$$\Delta E = hf \rightarrow f = \frac{\Delta E}{h}$$

$$f = \frac{4.9}{4.14 \times 10^{-15}}$$

$$= 1.18 \times 10^{15} \text{ Hz}$$

$$c = f\lambda \rightarrow \lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{1.18 \times 10^{15}}$$

$$= 2.53 \times 10^{-7} \text{ m}$$

**Question 12**

$$\Delta E = E_4 - E_3$$

$$= -1.6 - (-3.7)$$

$$= 2.1 \text{ eV}$$

$$\Delta E = hf \rightarrow f = \frac{\Delta E}{h}$$

$$f = \frac{2.1}{4.14 \times 10^{-15}}$$

$$= 5.07 \times 10^{14} \text{ Hz}$$

$$c = f\lambda \rightarrow \lambda = \frac{c}{f}$$

$$\lambda = \frac{3 \times 10^8}{5.07 \times 10^{14}}$$

$$= 5.91 \times 10^{-7} \text{ m}$$

**Question 13**

$$\begin{aligned}
 W &= qV \\
 &= 1.60 \times 10^{-19} \times 65 \\
 &= 1.04 \times 10^{-17} \text{ J}
 \end{aligned}$$

**Question 14**

$$\begin{aligned}
 \Delta E_k &= \frac{1}{2} mv^2 \\
 1.04 \times 10^{-17} &= \frac{1}{2} \times 9.11 \times 10^{-31} \times v^2 \\
 2.28 \times 10^{-17} &= v^2 \\
 v &= 4.78 \times 10^6 \text{ m s}^{-1}
 \end{aligned}$$

**Question 15**

$$\begin{aligned}
 \lambda &= \frac{h}{mv} \\
 &= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 4.8 \times 10^6} \\
 &= 1.52 \times 10^{-10} \text{ m}
 \end{aligned}$$

**Question 16**

Neils Bohr would state that if incident light had an energy value less than the minimum energy difference between the lowest and next orbital levels within the hydrogen atom, the light would not result in any orbital changes. Therefore the light would not be absorbed by the atom.

**Question 17**

$$\begin{aligned}
 \lambda &= \frac{h}{p} = \frac{h}{mv} \\
 &= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 1.75 \times 10^7} \\
 &= 4.16 \times 10^{-11} \text{ m} \\
 &= 0.0416 \text{ nm}
 \end{aligned}$$

**Question 18**

There would be circular bands or fringes of specific spacing around a common central point.

**Question 19**

As the accelerating voltage is increased, the electron speed would increase. Therefore the electron has more momentum. As  $\lambda = \frac{h}{p}$ , the electron's wavelength is reduced. The amount of diffraction depends on  $\frac{\lambda}{w}$  and so less diffraction occurs. Less diffraction means the overall pattern is smaller; that is, the circular bands are more closely spaced.

**Question 20**

de Broglie would say that the electrons (with their associated wavelengths) were diffracted as they passed through the gaps between the atoms in the crystal, creating a diffraction pattern.

**Question 21**

In addition to their particle properties, electrons have a de Broglie wavelength. The orbit must fit an integral number of wavelengths so that a standing wave is formed ( $2\pi r = n\lambda$ ). Only energy levels corresponding to these wavelengths exist.

**Question 22**

B, C, D, E. The photoelectric effect treats light as having particle-like properties as well as wave properties.

**Question 23**

An electronvolt is the energy that a single electron would gain after being moved through a potential of 1 V.

**Question 24**

If electrons receive more than enough energy to release them from the atom, any excess energy results in extra kinetic energy of the electron.

**Question 25**

The discovery that light can display both particle and wave properties was repeated when electrons were found to have wave properties, when moving very fast, as well as particle properties.

**Question 26**

C. The de Broglie wavelength of a particle is given by  $\lambda = \frac{h}{p}$  and therefore depends only on the momentum of the particle.

**Question 27**

In the particle model, the energy of the incident photons is set by their frequency according to  $E = hf$ . Each incident photon interacts with only one electron; therefore, the energy of the emitted electrons will depend only on the frequency of the incident light. Electron energy was not altered by altering the intensity because this only varies the number of photons, not their energy. Therefore, the energy of the emitted electrons is not affected, only the number emitted.

**Question 28**

The wave model predicts that altering the intensity of the light corresponds to waves of greater amplitude. Hence, the wavefronts should deliver more energy to the electrons and, therefore, the emerging electrons should have higher energy. (This is not observed.)

**Question 29**

Photon energy > ionisation energy, i.e. the photon has enough energy to free the electron.

**Question 30**

$$14.0 - 13.6 = 0.4 \text{ eV}$$

$$\begin{aligned} 0.4 \text{ eV} &= 0.4 \times 1.6 \times 10^{-19} \text{ J} \\ &= 6.4 \times 10^{-20} \text{ J} \end{aligned}$$

**Question 31**

$$\Delta E_k = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 6.4 \times 10^{-20}}{9.11 \times 10^{-31}}}$$

$$= 3.74 \times 10^5 \text{ ms}^{-1}$$

$$p = mv$$

$$= 9.11 \times 10^{-31} \times 3.74 \times 10^5$$

$$= 3.41 \times 10^{-25} \text{ kg m s}^{-1}$$

**Question 32**

$$\lambda = \frac{h}{p}$$

$$= \frac{6.63 \times 10^{-34}}{3.41 \times 10^{-25}}$$

$$= 1.94 \times 10^{-9} \text{ m}$$

**Question 33**

Since there is no energy level 10.0 eV above the ground state, the photon cannot be absorbed.

**Question 34**

$$E_{3-2} = -3.7 - (-5.5) = 1.8 \text{ eV}$$

$$E_{3-1} = -3.7 - (-10.4) = 6.7 \text{ eV}$$

$$E_{2-1} = -5.5 - (-10.4) = 4.9 \text{ eV}$$

Therefore 1.8 eV, 4.9 eV and 6.7 eV photons will be present in the emission spectrum.

**Question 35**

$E = \frac{hc}{\lambda}$  so the shortest wavelength has the highest energy

$$\lambda = \frac{hc}{E} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{6.7}$$

$$= 1.85 \times 10^{-7} \text{ m}$$

Therefore the highest energy photon, 6 eV, will have the shortest wavelength.

**Question 36**

$$\text{Incident energy} = 30.4 \text{ eV} + 10.4 \text{ eV} = 40.8 \text{ eV}$$

$$\lambda = \frac{hc}{E} = \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{40.8}$$

$$= 3.04 \times 10^{-8} \text{ m}$$

**Question 37**

$E_{k \text{ max}}$  represents the maximum kinetic energy with which the electrons are emitted.

$f$  is the frequency of the light incident on the metal plate (usually after passing through a filter, so it is not sufficient to call this the frequency of light from the source).

$\phi$  is the work function, which is the minimum energy required to eject an electron (it is a property of the metal).



**Question 38**

$E_{k \max}$  is not altered.

**Question 39**

More photoelectrons are ejected each second, therefore more current is flowing.

**Question 40**

Since three full wavelengths fit into a circumference,  $n = 3$ .

**Question 41**

$$\begin{aligned} p &= \frac{h}{\lambda} \\ &= \frac{6.63 \times 10^{-34}}{3.0 \times 10^{-12}} \\ &= 2.21 \times 10^{-22} \text{ kg m s}^{-1} \end{aligned}$$

**Question 42**

To produce diffraction patterns with the same fringe separation, they must have equivalent wavelengths.

**Question 43**

$$\begin{aligned} \lambda &= \frac{c}{f} = \frac{3 \times 10^8}{8.3 \times 10^{18}} \\ &= 3.6 \times 10^{-11} \text{ m} \end{aligned}$$

**Question 44**

$\lambda = 3.6 \times 10^{-11} \text{ m}$ , since they must have an equivalent wavelength to the X-ray photons.

**Question 45**

$$\begin{aligned} p &= \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{3.6 \times 10^{-11}} \\ &= 1.8 \times 10^{-23} \text{ kg m s}^{-1} \end{aligned}$$

**Question 46**

No. The energy of the X-rays is given by  $E = \frac{hc}{\lambda}$  and the energy of the electrons is given by  $\Delta E_k = \frac{1}{2} mv^2$ .

**Question 47**

The photoelectric effect supports the particle (photon) model of light because:

- 1 It predicts a minimum frequency (threshold frequency and energy) before electrons are emitted. (The wave model predicts that any frequency should work.)
- 2 The energy of the emitted electrons depends only on the frequency of the incident light. (The wave model predicts that increasing the intensity of light would increase the energy of the emitted electrons.)
- 3 It also explains an absence of any time delay before electrons are emitted when weak light sources are used. (This time delay is suggested by the wave model.)

**Question 48**

- a B. Vapour lamps involve the excited electrons dropping one of more energy levels and radiating photons as they do so.
- b A. As the filament of an incandescent light bulb heats up, the free electrons in the tungsten atoms collide, accelerate and emit photons.
- c C. The material that LEDs are made from contain a conduction band and a valence band.
- d D. The photons in laser light all have the same wavelength and frequency and are in phase with each other i.e. they are coherent.

**Question 49**

In the single slit diffraction experiment, as the slit is made narrower the position of the particle becomes more precisely known. As a consequence, the direction, and therefore the momentum of the particle becomes less precisely known, because with a narrower slit the diffraction pattern becomes wider.

**Question 50**

$\Delta p$  is the uncertainty of a particle's momentum. If this value gets smaller it means the momentum (or velocity) of a particle is known more precisely. As a consequence, as the right-hand side of the relation remains constant, the uncertainty in a particle's position,  $\Delta x$ , becomes greater.