Chapter 2 Newtonian theories of motion

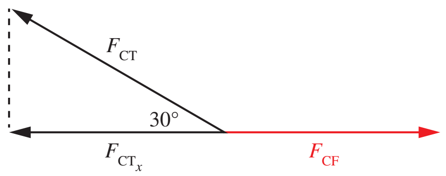
2.1 Newton's laws of motion

Worked example: Try yourself 2.1.1

APPLICATION OF NEWTON'S FIRST AND THIRD LAWS

The toddler adds extra blocks to the cart and drags it across the floor more slowly. The 5.5 kg cart travels at a constant speed of 0.65 m s^{-1}. The force of friction between the cart and the floor is 5.2 N and the handle is now at an angle of 30° above the horizontal.

a Calculate the net force on the cart.	
Thinking	**Working**
The cart has constant velocity. According to Newton's first law, the net force acting on an object with constant velocity is zero.	$F_{net} = 0\,N$

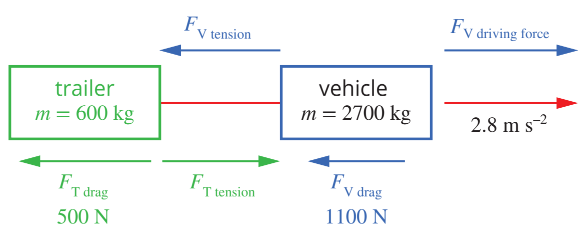
b Calculate the force that the toddler exerts on the cart.	
Thinking	**Working**
Draw a forces diagram.	
If the net force is zero then the horizontal forces must be in balance. Therefore the horizontal component of the force on the cart by the toddler, F_{CT_x}, is equal to the magnitude of the frictional force, F_{CF}.	$F_{CT_x} = F_{CT}\cos 30° = F_{CF}$ $F_{CT}\cos 30° = 5.2\,N$ $F_{CT} = \frac{5.2}{\cos 30°} = 6.0\,N\ 30°$ above the horizontal

c Determine the force that the cart exerts on the toddler.	
Thinking	**Working**
According to Newton's third law, the force on the cart by the toddler is equal and opposite to the force on the toddler by the cart. $F_{CT} = -F_{TC}$	Since the force on the cart is at an angle of 30° above the horizontal, the force of the cart on the toddler is 6.0 N at an angle of 30° below the horizontal.

Worked example: Try yourself 2.1.2

APPLICATION OF NEWTON'S LAWS

A vehicle towing a trailer accelerates at 2.8 m s^{-2} in order to overtake a car in front. The vehicle's mass is 2700 kg and the trailer's mass is 600 kg. The drag force on the vehicle is 1100 N and the drag force on the trailer is 500 N.

a Calculate the driving force of the engine.	
Thinking	**Working**
Draw a sketch showing all the forces acting.	

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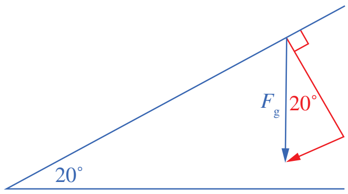
Since there is an acceleration, Newton's second law can be applied to the whole system. Note that the vehicle and the trailer are joined by the coupling and so the tension forces are not included at this stage. Consider the system as a whole.	$F_{system} = m_{system}a$ $F_{V\ driving\ force} - F_{V\ drag} - F_{T\ drag} = (m_V + m_T)a$ $F_{V\ driving\ force} - 1100 - 500 = (2700 + 600) \times 2.8$ $F_{V\ driving\ force} = 1.1 \times 10^4$ N in the direction of motion

b Calculate the magnitude of the tension in the coupling.	
Thinking	**Working**
Consider only one part of the system, for example the trailer, and once again apply Newton's second law.	$F_{T\ net} = m_T a$ $F_{T\ tension} - F_{T\ drag} = m_T a$ $F_{T\ tension} = 600 \times 2.8 + 500$ $= 2.2 \times 10^3$ N

Worked example: Try yourself 2.1.3

INCLINED PLANES

A skier of mass 85 kg travels down the same icy slope inclined at 20° to the horizontal. Assume that friction is negligible and that the acceleration due to gravity is 9.8 m s^{-2}.

a Determine the components of the force due to gravity on the skier perpendicular to the slope and parallel to the slope.	
Thinking	**Working**
Draw a sketch and include the values provided.	
Resolve the force due to gravity into the component perpendicular to the slope.	The perpendicular component is: $F_{\perp} = F_g \cos 20°$ $= 833 \cos 20°$ $= 7.8 \times 10^2$ N
Resolve the force due to gravity into the component parallel to the slope.	The parallel component is: $F_{\parallel} = F_g \sin 20°$ $= 833 \sin 20°$ $= 2.9 \times 10^2$ N

b Determine the normal force that acts on the skier.	
Thinking	**Working**
The normal force is equal in magnitude to the perpendicular component of the force due to gravity.	$F_N = 7.8 \times 10^2$ N

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c	Calculate the acceleration of the skier down the slope.
Thinking	**Working**
Apply Newton's second law, rearranged to make *a* the subject. The net force along the slope is the component of the force due to gravity parallel to the slope.	$a = \frac{F_{net}}{m}$ $= \frac{285}{85}$ $= 3.4\,\text{m}\,\text{s}^{-1}$ down the slope

KEY QUESTIONS

Knowledge and understanding

1 No. Phil's inertia made him remain stationary as the tram moved forward. This made it look like Phil was thrown backwards relative to the tram. This is an example of Newton's first law. Objects will remain at rest unless a net unbalanced force acts to change their motion.

2 $F_{net} = ma$

$= 5.3 \times 2.2$

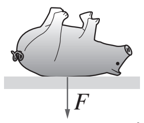
$= 11.66$

$= 12\,\text{N}$

3 **(a)**

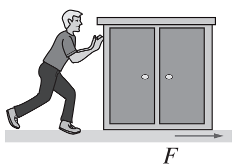
Force exerted on racquet by ball

(b)



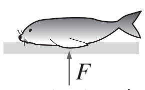
Force exerted on ground by pig

(c)



Force exerted on ground by wardrobe

(d)



Gravitational force of attraction that seal exerts on Earth

4 The forces are balanced, so the force due to air resistance is equal in magnitude to the force due to gravity on the ball.

$F_a = F_g = mg = 0.01 \times 9.8 = 0.098\,\text{N} = 9.8 \times 10^{-2}\,\text{N}$ upwards

5 **a** Constant speed, so the forces are balanced: $F_d = 45.0\,\text{N}$, where F_d is the driving force.

b $F_{net} = ma$

$F_d - F_f = ma$

$F_d - 45.0 = 80.0 \times 1.50$

$F_d = 165\,\text{N}$

6 **a** $u = 0$, $v = 7.5$, $t = 5.0$, $a = ?$

$v = u + at$

$7.5 = 0 + 5.0a$

$a = 1.5\,\text{m}\,\text{s}^{-2}$

b $F_{net} = ma = 80 \times 1.5 = 120\,\text{N}$ (or 1.2×10^2 N) forwards

c Constant speed, so the forces are balanced, i.e. $F_{net} = 0$. The frictional force will equal 60 N.

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7 a Constant speed, so the net force is zero.

b The horizontal component of the pulling force (tension) is in balance with the frictional force of 60 N.

$T_{\text{horizontal}} = F_{\text{friction}}$

$T\cos 25° = 60$

$T = \dfrac{60}{\cos 25°}$

$= 66\text{ N}$

c The rope is exerting a force of 66 N on Matt.

Analysis

8 The net force on the whole system (taking down as positive) is:

$F_{\text{net}} = ma = F_g + F_f$

$4.5 \times a = (3.5 \times 9.8) - 2.0$

$a = \dfrac{34.3 - 2.0}{4.5}$

$= 7.18$

$= 7.2\text{ m s}^{-2}$

$F_{\text{block on table}} = ma = T - F_f$

$1 \times 7.2 = T - 2.0$

$T = 9.2\text{ N}$

9 a The net force, F_{net}, is the difference between the driving force, F_d, and the resistive forces, F_r.

$F_{\text{net}} = F_d - F_r = ma$

$F_d - 1000 = (950 + 100) \times 0.800$

$= 840 + 1000$

$= 1840\text{ N}$

b The tension, F_t, can be calculated by considering the forces on the trailer:

$F_{\text{trailer}} = F_t - F_r = ma$

$F_t - 500 = 100 \times 0.800$

$= 80.0 + 500$

$= 580\text{ N}$

10 a A: the frictional force is opposite to the velocity

b C: perpendicular to the slope

c $F_{\text{net}} = 0$, so $F_f = F_g \sin\theta = 100 \times 9.8 \times \sin 30° = 490\text{ N}$ up the hill

d acceleration $a = g\sin\theta = 9.8 \times \sin 30° = 4.9\text{ m s}^{-2}$

e If there is no friction, acceleration is not affected by mass.

2.2 Circular motion in a horizontal plane

Worked example: Try yourself 2.2.1

CALCULATING SPEED

A water wheel has blades 2.0 m in length that rotate at a frequency of 10 revolutions per minute. At what speed do the tips of the blades travel? Express your answer in km h^{-1}.	
Thinking	**Working**
Calculate the period, T. Remember to express frequency in the correct units. Alternatively, recognise that 10 revolutions in 60 seconds means that each revolution takes 6 seconds.	10 revolutions per minute = $\dfrac{10}{60} = 0.167\text{ Hz}$ $T = \dfrac{1}{f} = \dfrac{1}{0.167}$ $= 6.0\text{ s}$

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Substitute r and T into the appropriate formula for speed and solve for v.	$v = \frac{2\pi r}{T}$ $= \frac{2 \times \pi \times 2.0}{6}$ $= 2.09$ $= 2.1\,m\,s^{-1}$
Convert $m\,s^{-1}$ into $km\,h^{-1}$ by multiplying by 3.6.	$2.09 \times 3.6 = 7.5\,km\,h^{-1}$

Worked example: Try yourself 2.2.2

CENTRIPETAL FORCES

An athlete in a hammer-throw event is swinging a ball of mass 7.0 kg in a horizontal circular path of radius 1.20 m. The ball is moving at $25.0\,m\,s^{-1}$.

a Calculate the magnitude of the acceleration of the ball.	
Thinking	**Working**
As the object is moving in a circular path, the centripetal acceleration is towards the centre of the circle. To find the magnitude of this acceleration, consider the variables that are given.	$v = 25.0\,m\,s^{-1}$ $r = 1.20\,m$ $a = ?$
Select the equation for centripetal acceleration that fits the values you have, substitute the values and solve the equation.	$a = \frac{v^2}{r}$ $= \frac{25.0^2}{1.20}$ $= 521\,m\,s^{-2}$
State the magnitude only, as no direction is required.	The acceleration of the ball is $521\,m\,s^{-2}$.

b Calculate the magnitude of the tensile force (tension) acting in the wire.	
Thinking	**Working**
Identify the unbalanced force that is causing the object to move in a circular path. Note the information that you have.	$m = 7.0\,kg$ $a = 521\,m\,s^{-2}$ $F_{net} = ?$
Select the appropriate equation for centripetal force, substitute the variables and solve the equation.	$F_{net} = ma$ $= 7.0 \times 521$ $= 3.6 \times 10^3\,N$
State the magnitude only, as no direction is required.	The force of tension in the wire is the unbalanced force that is causing the ball to accelerate. Tensile force $F_T = 3.6 \times 10^3\,N$

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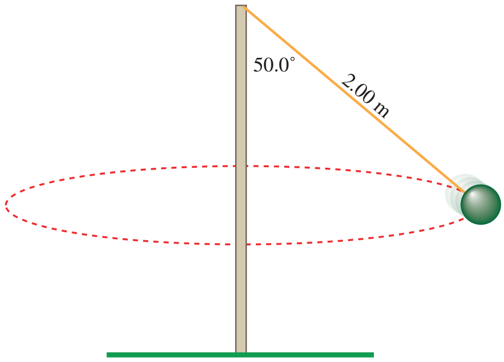
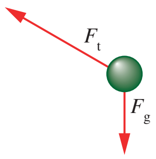
Worked example: Try yourself 2.2.3

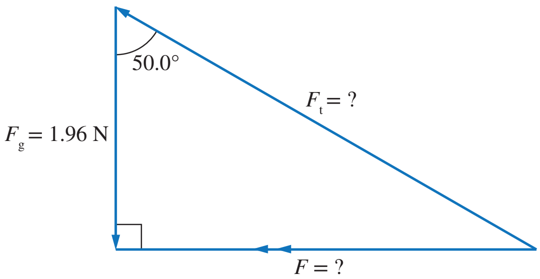
OBJECT ROTATING ON THE END OF A STRING

During a game of totem tennis, a ball of mass 200 g is swinging freely in a horizontal circular path. The cord is 2.00 m long and at an angle of 50.0° to the vertical.



a Calculate the radius of the ball's circular path.	
Thinking	**Working**
The radius of the circular path and the pole form a right angle. Use trigonometry to find the radius.	$r = 2.00 \sin 50.0° = 1.53\text{ m}$

b Draw and label the forces that are acting on the ball at the instant shown in the diagram.	
Thinking	**Working**
There are two forces acting: the tension in the cord, F_t, and gravity, F_g. These forces are unbalanced.	

c Determine the net force that is acting on the ball at this time.	
Thinking	**Working**
First calculate the force due to gravity, F_g.	$F_g = mg$ $= 0.200 \times 9.8$ $= 1.96\text{ N}$
The ball has an acceleration towards the centre of its circular path. This is horizontal and towards the left at this instant. The net force will also act in this direction at this instant. A force triangle and trigonometry can be used to determine the net force.	 $F_{net} = 1.96 \tan 50.0° = 2.34\text{ N}$ towards the centre of the circular path

d Calculate the size of the tensile force in the cord.	
Thinking	**Working**
Use trigonometry to find F_t.	$F_t = \dfrac{1.96}{\cos 50.0°}$ $= 3.05\text{ N}$

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KEY QUESTIONS

Knowledge and understanding

1 **a** A and D. The speed is constant but the velocity is changing (as the direction is constantly changing). The acceleration is directed towards the centre of the circle.

2 **a** $8.0\,m\,s^{-1}$

b $8.0\,m\,s^{-1}$ south

c $a = \frac{v^2}{r} = \frac{8.0^2}{9.2} = 7.0\,m\,s^{-2}$ towards the centre, i.e. west

3 $F_{net} = ma = 1200 \times 7.0 = 8.4 \times 10^3\,N$ west

4 **a** $8.0\,m\,s^{-1}$ north

b towards the centre, i.e. east

5 The force needed to give the car a larger centripetal acceleration could eventually exceed the maximum frictional force acting between the tyres and the road surface. At this time, the car would skid out of its circular path.

6 **a**
$$a = \frac{v^2}{r} = \frac{1.5^2}{2.5} = 0.90\,m\,s^{-2}$$

b The skater has an acceleration, so forces are unbalanced.

c
$$F_{net} = ma = 75 \times 0.90 = 67.5\,N$$

7 **a** $T = \frac{1}{f} = \frac{1}{2.5} = 0.40\,s$

b $v = \frac{2\pi r}{T} = \frac{2 \times \pi \times 1.2}{0.40} = 18.8 = 19\,m\,s^{-1}$

c $a = \frac{v^2}{r} = \frac{19^2}{1.2} = 3.0 \times 10^2\,m\,s^{-2}$

d $F_{net} = ma = 1.5 \times 3.0 \times 10^2 = 4.5 \times 10^2\,N$

8 **a** $r = 2.40 \cos 60° = 1.20\,m$

b There are two forces: the force due to gravity acting vertically downwards and the tension in the rope acting along the rope towards the top of the maypole.

c She has an acceleration directed towards point B, the centre of her circular path.

d Use a force triangle for the girl showing the net force towards B.

$$F_{net} = \frac{mg}{\tan 60°} = \frac{294}{1.73} = 170\,N \text{ towards B}$$

e
$$F_{net} = \frac{mv^2}{r}$$
$$170 = \frac{30.0 \times v^2}{1.20}$$
$$v = 2.61\,m\,s^{-1}$$

Analysis

9 **a**
$$F_{net} = \frac{mv^2}{r} = \frac{1500 \times 25^2}{30} = 3.1 \times 10^4\,N$$

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b The friction between the road surface and the car's tyres provides the centripetal force.

c The inertia of the passengers causes them to continue to move in a straight path while the car makes the turn. The passengers exert a force outwards on the side of the car as it moves around the turn.

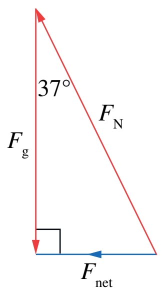
d If there is any ice or oil on the road, the friction between the road and the car's tyres will be reduced. This will reduce the centripetal force on the car. As a result, the car's inertia will cause the car to continue to move in a straight line at a tangent to the circle from the point where the car hits the oil or ice on the road.

2.3 Circular motion on banked tracks

Worked example: Try yourself 2.3.1

BANKED TRACKS

A curved section of track on an Olympic velodrome has radius of 40 m and is banked at an angle of 37° to the horizontal. A cyclist of mass 80 kg is riding on this section of track at the design speed. Assume that g is 9.8 m s^{-2}.

a Calculate the net force acting on the cyclist at this instant.	
Thinking	**Working**
Draw a force diagram and include all forces acting on the cyclist. The forces acting are the force due to gravity and the normal force from the track, and these are unbalanced. The net force is horizontal and towards the centre of the circular track.	
Calculate the force due to gravity, F_g.	$F_g = mg$ $= 80 \times 9.8$ $= 784$ N
Use the force triangle and trigonometry to calculate the net force, F_{net}.	$\tan\theta = \frac{F_{net}}{F_g}$ $\tan 37° = \frac{F_{net}}{784}$ $F_{net} = \tan 37° \times 784$ $= 591$ N
As force is a vector, a direction is needed in the answer.	Net force is 5.9×10^2 N towards the centre of the circle.

b Calculate the design speed for this section of the track.	
Thinking	**Working**
Note the relevant values.	$g = 9.8$ m s^{-2} $r = 40$ m $\theta = 37°$ $v = ?$
Use the design speed formula.	$v = \sqrt{rg\tan\theta}$ $= \sqrt{40 \times 9.8 \times \tan 37°}$ $= 17$ m s^{-1}

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Worked example: Try yourself 2.3.2

FINDING THE BANKING ANGLE

The curved portion of a highway needs to be banked to prevent cars from skidding off it. Assume that the banked track of the highway is designed for a top vehicle speed of 110 km h^{-1}. The banked track portion of the highway has a radius of 750 m. What is the value of the banking angle, θ, such that the forces keep the car on the highway without the need for friction? Assume that g is 9.8 m s^{-2}.

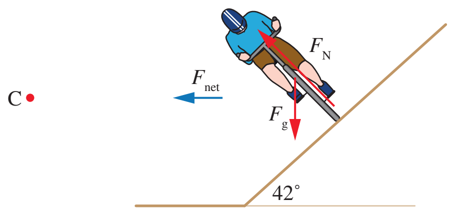
Thinking	Working
Recall the formula for finding the banking angle.	$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$
Convert the design speed from km h^{-1} to m s^{-1}.	$v = \frac{110\,\text{km h}^{-1}}{3.6}$ $= 30.6\,\text{m s}^{-1}$
Calculate the banking angle.	$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$ $= \tan^{-1}\left(\frac{30.6^2}{750 \times 9.8}\right)$ $= 7.3°$

KEY QUESTIONS

Knowledge and understanding

1 In all horizontal circular motion, the acceleration is directed horizontally towards the centre of the circle.

2



3 $\tan\theta = \frac{F_{net}}{F_g}$

$\theta = \tan^{-1}\left(\frac{780}{80 \times 9.8}\right) = 45°$

4 **a** $v = \sqrt{rg\tan\theta}$

$= \sqrt{35 \times 9.8 \times \tan 25°}$

$= 12.6\,\text{m s}^{-1}$

$= 12.6 \times 3.6$

$= 46\,\text{km h}^{-1}$

b $F_N = \frac{mg}{\cos 25°}$

$= \frac{735}{\cos 25°}$

$= 811$

$= 8.1 \times 10^2\,\text{N}$

c On a horizontal track, F_N is equal and opposite to the force due to gravity, so $F_N = mg = 735$ N. This is less than the normal force on the banked track (8.1×10^2 N).

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5 $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$= \tan^{-1}\left(\frac{55^2}{275 \times 9.8}\right)$

$= 48°$

6 $v = 90\,\text{km h}^{-1} = 25\,\text{m s}^{-1}$

$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$= \tan^{-1}\left(\frac{25^2}{450 \times 9.8}\right)$

$= 8.1°$

Analysis

7 The design speed depends on tan θ and the radius of the curve. Therefore the architect could make the banking angle larger or increase the radius of the track.

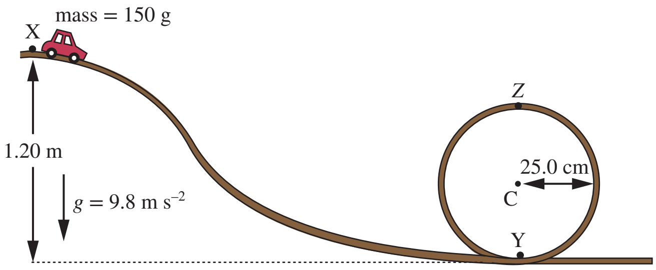
2.4 Circular motion in a vertical plane

Worked example: Try yourself 2.4.1

VERTICAL CIRCULAR MOTION

A student arranges a toy car track with a vertical loop of radius 25.0 cm, as shown.

A toy car of mass 150 g is released from rest at a height of 1.20 m (point X). The car rolls down the track and travels around the loop. Assume that g is $9.8\,\text{m s}^{-2}$ and ignore friction.



a Calculate the speed of the car as it reaches point Y at the bottom of the loop.	
Thinking	**Working**
Note all the variables given in the question.	At X: $m = 150\,\text{g} = 0.150\,\text{kg}$ $\Delta h = 1.20\,\text{m}$ $v = 0$ $g = 9.8\,\text{m s}^{-2}$
Approach the problem by considering that energy is conserved during the car's motion. Calculate the total mechanical energy first. Note that the initial speed is zero, so E_k at X is zero.	Mechanical energy, E_m, at X is: $E_m = E_k + E_g$ $= \frac{1}{2}mv^2 + mg\Delta h$ $= 0 + (0.150 \times 9.8 \times 1.20)$ $= 1.76\,\text{J}$

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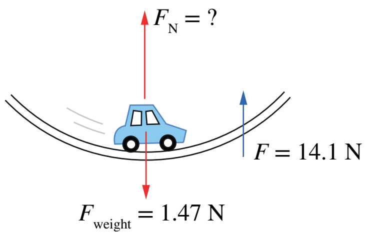
Use the conservation of energy ($E_m = E_k + E_g$) to determine the velocity at point Y. As the car rolls down the track, it loses its gravitational potential energy and gains kinetic energy. At the bottom of the loop (Y) the car has zero potential energy.	At Y: $E_m = 1.76\,J$ $\Delta h = 0$ $E_g = 0$ $E_m = E_k + E_g$ $= \frac{1}{2}mv^2 + mg\Delta h$ $1.76 = 0.5 \times 0.150v^2 + 0$ $v^2 = 23.5$ $v = 4.85$ $v = 4.9\,m\,s^{-1}$

b Calculate the normal force from the track at point Y.	
Thinking	**Working**
To solve for F_N, start by working out the net, or centripetal, force. At Y the car has a centripetal acceleration towards C (i.e. upwards), so the net centripetal force must also be vertically upwards at this point.	$F_{net} = \frac{mv^2}{r}$ $= \frac{0.150 \times 4.85^2}{0.250}$ $= 14.1\,N$ up
Calculate the force due to gravity, F_g, and add it to a force diagram.	$F_g = mg$ $= 0.150 \times 9.8$ $= 1.47\,N$ down At point Y: 
Work out the normal force using vectors. Note up as positive and down as negative in your calculations. The forces acting are unbalanced, as the car has a centripetal acceleration upwards (towards C). The upwards (normal) force must be larger than the downwards force.	$F_{net} = F_g + F_N$ $+14.1 = -1.47 + F_N$ $F_N = 14.1 + 1.47$ $= 16\,N$ up Note that the force the track exerts on the car is much greater (by about ten times) than the force due to gravity. If the car were travelling horizontally on a flat surface, the normal force would be just 1.47 N up.

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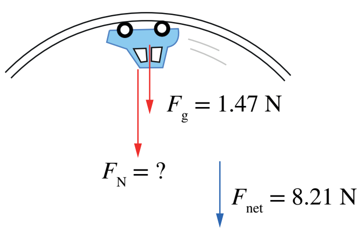
c What is the speed of the car as it reaches point Z?	
Thinking	**Working**
Calculate the velocity from the values you have, using $E_m = E_k + E_g$.	At Z: $m = 0.150\,\text{kg}$ $\Delta h = 2 \times 0.250 = 0.500\,\text{m}$ Mechanical energy is conserved, so use the value from part **a**: $E_m = 1.76\,\text{J}$. $E_m = E_k + E_g$ $= \frac{1}{2}mv^2 + mg\Delta h$ $1.76 = \frac{1}{2} \times 0.15v^2 + 0.150 \times 9.8 \times 0.500$ $= 0.075v^2 + 0.735$ $0.075v^2 = 1.76 - 0.735$ $v = \sqrt{13.67}$ $= 3.7\,\text{m}\,\text{s}^{-1}$

d What is the normal force acting on the car at point Z?	
Thinking	**Working**
To find F_N, start by working out the net, or centripetal, force. At Z the car has a centripetal acceleration towards C (i.e., downwards), so the net centripetal force must also be vertically downwards at this point.	$F_{net} = \frac{mv^2}{r}$ $= \frac{0.150 \times 3.70^2}{0.250}$ $= 8.21\,\text{N down}$
Work out the normal force using vectors. Note up as positive and down as negative in your calculations.	At point Z:  $F_{net} = F_g + F_N$ $-8.21 = -1.47 + F_N$ $F_N = -8.21 + 1.47$ $= -6.74$ $= 6.74\,\text{N down}$

KEY QUESTIONS

Knowledge and understanding

1 **a** It has a constant speed, so its centripetal acceleration ($a = \frac{v^2}{r}$) is also constant in magnitude.

b At the bottom of its path, the yo-yo has an upwards acceleration and so the net force is up. This indicates that the tension force is greater than F_g.

c At the top of its path, the yo-yo has a downwards acceleration and so the net force is down. This indicates that the tension force is less than F_g.

d At the bottom of its circular path.

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e At this point, the acceleration is downwards and the net force is down and equal to F_g.

$$F_{net} = \frac{mv^2}{r} = F_g$$
$$F_g = mg = \frac{mv^2}{r}$$
$$v = \sqrt{rg}$$
$$= \sqrt{1.25 \times 9.8}$$
$$= 3.5\,\text{m s}^{-1}$$

2 **a** The force due to gravity and the normal force from the road.

b $14.4\,\text{km h}^{-1} = 4\,\text{m s}^{-1}$. Therefore:

$$F_{net} = \frac{mv^2}{r}$$
$$= \frac{800 \times 4^2}{10}$$
$$= 1280\,\text{N (or } 1.3 \times 10^3\,\text{N)}$$

c Yes. When the driver is driving over a hump, the normal force is less than their force due to gravity (*mg*). Their apparent force due to gravity is given by the normal force that is acting, and so the driver feels lighter at this point.

d At point of lift-off, net force is down and equal to F_g.

$$F_{net} = \frac{mv^2}{r} = F_g$$
$$F_g = mg = \frac{mv^2}{r}$$
$$\frac{v^2}{r} = g$$
$$v = \sqrt{rg}$$
$$= \sqrt{10 \times 9.8}$$
$$= 9.9\,\text{m s}^{-1}$$
$$= 36\,\text{km h}^{-1}$$

3 **a** At X, the mechanical energy is:

$$E_m = E_k + E_g$$
$$= \frac{1}{2}mv^2 + mg\Delta h$$
$$= \frac{1}{2} \times 700 \times 1.75^2 + 700 \times 9.8 \times 70$$
$$= 1072 + 480\,200$$
$$= 481\,272\,\text{J}$$

At Y, E_g is zero, so the cart's kinetic energy is 481 272 J.

$$\frac{1}{2}mv^2 = 481\,272$$
$$\frac{1}{2} \times 700 \times v^2 = 481\,272$$
$$v = \sqrt{1375}$$
$$= 37\,\text{m s}^{-1}$$

b At Z, the mechanical energy is 481 272 J.

$$E_m = E_k + E_g$$
$$481\,272 = E_k + 700 \times 9.8 \times 34$$
$$481\,272 = E_k + 233\,240$$
$$E_k = 248\,032\,\text{J}$$
$$\frac{1}{2} \times 700v^2 = 248\,032$$
$$v = 27\,\text{m s}^{-1}$$

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c At Z, $F_g = mg = 700 \times 9.8 = 6860\,\text{N}$ down

$$\begin{aligned} F_{net} &= \frac{mv^2}{r} \\ &= \frac{700 \times 27^2}{17} \\ &= 30\,018\,\text{N down} \\ &= F_N + F_g \\ 30\,018 &= F_N + 6860 \\ F_N &= 2.3 \times 10^4\,\text{N down} \end{aligned}$$

d For the cart to just lose contact at Z, $F_{net} = F_N + F_g$, and $F_N = 0$.

$$\begin{aligned} F_{net} &= \frac{mv^2}{r} = F_g \\ F_g &= mg = \frac{mv^2}{r} \\ \frac{v^2}{r} &= g \\ v &= \sqrt{17 \times 9.8} \\ &= 13\,\text{m}\,\text{s}^{-1} \end{aligned}$$

4

$$\begin{aligned} F_{net} &= F_g + F_N \\ \frac{mv^2}{r} &= 80 \times 9.8 + F_N \\ \frac{80 \times 35^2}{100} &= 784 + F_N \\ F_N &= 980 - 784 \\ &= 2.0 \times 10^2\,\text{N down} \end{aligned}$$

5 **a**

$$\begin{aligned} a &= \frac{v^2}{r} \\ &= \frac{7.0^2}{3.0} \\ &= 16\,\text{m}\,\text{s}^{-2}\text{ up} \end{aligned}$$

b

$$\begin{aligned} F_{net} &= \frac{mv^2}{r} \\ &= \frac{72 \times 7.0^2}{3.0} \\ &= 1200 \\ &= 1.2 \times 10^3\,\text{N up} \end{aligned}$$

$$\begin{aligned} F_g &= mg \\ &= 72 \times 9.8 \\ &= 705.6 \\ &= 7.1 \times 10^2\,\text{N down} \end{aligned}$$

$$\begin{aligned} F_{net} &= F_N + F_g \\ 1.2 \times 10^3 &= F_N - 7.1 \times 10^2 \\ F_N &= 1.2 \times 10^3 + 7.1 \times 10^2 \\ &= 1.9 \times 10^3\,\text{N up} \end{aligned}$$

Analysis

6 $a = \frac{v^2}{r}$ and $a = 9g = 88.2\,\text{N}\,\text{kg}^{-1}$. Therefore:

$$\begin{aligned} v &= \sqrt{ra} \\ &= \sqrt{400 \times 88.2} \\ &= 188\,\text{m}\,\text{s}^{-1} \end{aligned}$$

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7 **a** If the toy car is just losing contact with the track, $F_N = 0$; thus $F_{net} = F_g$ and $a = 9.8\,\mathrm{m\,s^{-2}}$ down.

b $v = \sqrt{rg}$

$= \sqrt{0.40 \times 9.8}$

$= 2.0\,\mathrm{m\,s^{-1}}$

8 **a** $F_{net} = \dfrac{mv^2}{r}$

$= \dfrac{1500 \times 8.0^2}{10}$

$= 9600$

$= 9.6 \times 10^3$ N down

b $F_{net} = F_N + F_g$ (and taking down as negative)

$-9.6 \times 10^3 = F_N - 1500 \times 9.8$

$F_N = -9.6 \times 10^3 + 1500 \times 9.8$

$= 5100$ N

$= 5.1 \times 10^3$ N up

c $g = \dfrac{v^2}{r}$

$v = \sqrt{rg}$

$= \sqrt{10 \times 9.8}$

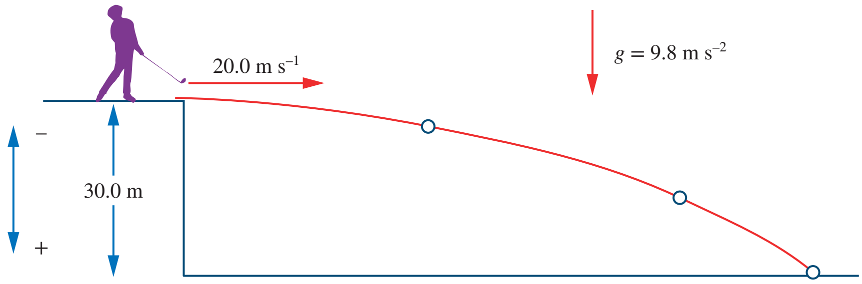
$= 9.9\,\mathrm{m\,s^{-1}}$

2.5 Projectiles launched horizontally

Worked example: Try yourself 2.5.1

PROJECTILE LAUNCHED HORIZONTALLY

A golf ball of mass 100 g is hit horizontally with a speed of $20.0\,\mathrm{m\,s^{-1}}$ from the top of a 30.0 m high cliff. Assume that $g = 9.8\,\mathrm{m\,s^{-2}}$ and ignore air resistance.



a Calculate the time the ball takes to land.	
Thinking	**Working**
Let the downward direction be positive. Write down the information relevant to the vertical component of the motion. Note that the instant the ball is hit, it is travelling only horizontally, so its initial vertical velocity is zero.	Down is positive. Vertically: $u = 0\,\mathrm{m\,s^{-1}}$ $s = 30.0$ m $a = 9.8\,\mathrm{m\,s^{-2}}$ $t = ?$
In the vertical direction, the ball has constant acceleration, so use an equation for uniform acceleration. Select the equation that best fits the information you have.	$s = ut + \frac{1}{2}at^2$

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Substitute values in the equation, rearrange it and solve for *t*.	$30.0 = 0 + 4.90t^2$ $t = \sqrt{\frac{30.0}{4.90}}$ $= 2.5\ \text{s}$

b Calculate the distance the ball travels from the base of the cliff, i.e. the range of the ball.	
Thinking	**Working**
Write down the information relevant to the horizontal component of the motion. As the ball is hit horizontally, the initial speed gives the horizontal component of the velocity throughout the flight.	Horizontally: $u = 20.0\ \text{m s}^{-1}$ $t = 2.47\ \text{s}$ from part **a** $s = ?$
Select the equation that best fits the information you have.	As the horizontal speed is constant (i.e. $u = v$), you can use: $v_{av} = \frac{s}{t}$
Substitute values in the equation, rearrange it and solve for *s*.	$20.0 = \frac{s}{2.47}$ $s = 20.0 \times 2.47$ $= 49.4\ \text{m}$

c Calculate the velocity of the ball as it lands.	
Thinking	**Working**
Find the horizontal and vertical components of the ball's speed as it lands. Write down the information relevant to both the vertical and horizontal components.	Horizontally: $u = v = 20.0\ \text{m s}^{-1}$ Vertically, with down as positive: $u = 0$ $a = 9.8\ \text{m s}^{-2}$ $s = 30.0\ \text{m}$ $t = 2.47\ \text{s}$ $v = ?$
To find the final vertical speed, use the equation for uniform acceleration that best fits the information you have.	$v = u + at$
Substitute values in the equation and solve for the variable you are looking for, in this case *v*.	Vertically: $v = u + at$ $= 0 + 9.8 \times 2.47$ $= 24.2\ \text{m s}^{-1}$ down

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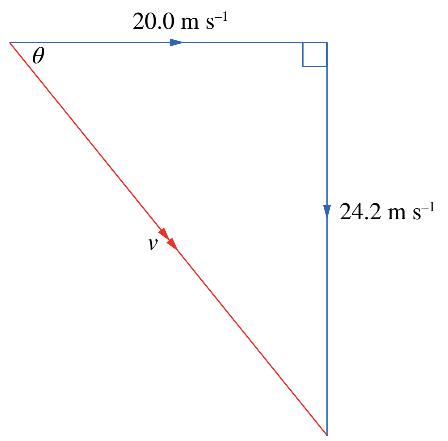
Add the components as vectors.	
Use Pythagoras's theorem to calculate the speed, *v*, of the ball.	$v = \sqrt{v_h^2 + v_v^2}$ $= \sqrt{20.0^2 + 24.2^2}$ $= \sqrt{986}$ $= 31.4\,\text{m s}^{-1}$
Use trigonometry to calculate the angle, *θ*.	$\theta = \tan^{-1}\left(\frac{24.2}{20.0}\right)^\circ$ $= 50.4^\circ$
Specify the velocity with its magnitude and a direction relative to the horizontal. Express the answer to 2 significant figures.	The final velocity of the ball is $31\,\text{m s}^{-1}$ at 50° below the horizontal.

KEY QUESTIONS

Knowledge and understanding

1 **a** Vertically with down as positive: $u = 0$, $a = 9.8$, $s = 1.7$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$1.7 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$t = \sqrt{\frac{1.7}{4.9}}$$

$$= 0.59\,\text{s}$$

b $v_{av} = \frac{s}{t}$

$$5.5 = \frac{s}{0.59}$$

$$s = 3.2\,\text{m}$$

c $a = 9.8\,\text{m s}^{-2}$ down

2 **a** Vertically with down as positive: $u = 0$, $a = 9.8$, $s = 2.5$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$

$$2.5 = 0 + \frac{1}{2} \times 9.8 \times t^2$$

$$t = \sqrt{\frac{2.5}{4.9}}$$

$$= 0.71\,\text{s}$$

b There is no difference in the time to fall for either ball: 0.71 s.

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c Ball X:

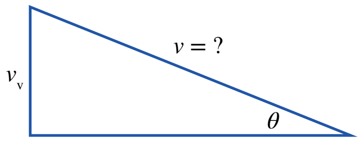
$$v_{av} = \frac{s}{t}$$
$$s = 7.5 \times 0.71$$
$$= 5.325\,\text{m}$$

Ball Y:

$$v_{av} = \frac{s}{t}$$
$$s = 12 \times 0.71$$
$$= 8.52\,\text{m}$$

The difference is 8.52 – 5.325 = 3.2 m

3 **a**



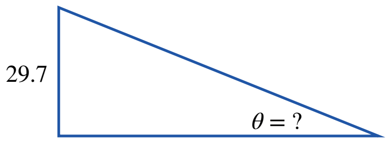
Vertically with down as positive: $u = 0$, $a = 9.8$, $s = 45$, $v = ?$

$$v^2 = u^2 + 2as$$
$$= 0 + 2 \times 9.8 \times 45$$
$$v = 29.7\,\text{m s}^{-1}$$

Horizontally: $u = v = 70\,\text{m s}^{-1}$

$$v = \sqrt{29.7^2 + 70^2}$$
$$= 76\,\text{m s}^{-1}$$

b



$$\tan\theta = \frac{29.7}{70}$$
$$\theta = 23°$$

4 **a** The horizontal velocity of the ball remains constant and $v_h = 6.5\,\text{m s}^{-1}$ forwards.

b Vertically with down as positive: $u = 0$, $a = 9.8$, $s = 1.0$, $v = ?$

$$v^2 = u^2 + 2as$$
$$= 0^2 + 2 \times 9.8 \times 1.0$$
$$v = 4.4\,\text{m s}^{-1}\text{ down}$$

c

$$v = \sqrt{4.4^2 + 6.5^2}$$
$$= 7.8\,\text{m s}^{-1}$$
$$\tan\theta = \frac{4.4}{6.5}$$
$$\theta = 34°$$
$$v = 7.8\,\text{m s}^{-1}\text{ at }34°\text{ below the horizontal}$$

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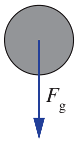
d Vertically with down as positive: $u = 0$, $a = 9.8$, $s = 1.0$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$1.0 = 0 + \frac{1}{2} \times 9.8 \times t^2$$
$$t = \sqrt{\frac{1}{4.9}}$$
$$= 0.45\,\text{s}$$

e Horizontally: $v = u = 6.5$, $t = 0.45$, $s = ?$

$$s = v_{av} \times t$$
$$= 6.5 \times 0.45$$
$$= 2.9\,\text{m}$$

f The only force acting is the force due to gravity, F_g.



Analysis

5 Find the time it takes the golf ball to land and then work backwards to find the horizontal speed at which it was hit. Take down as positive.

Vertically: $u = 0\,\text{m s}^{-1}$, $s = 75.0\,\text{m}$, $a = 9.8\,\text{m s}^{-2}$, $t = ?$

$$s = ut + \frac{1}{2}at^2$$
$$75 = 0 + \frac{1}{2} \times 9.8 \times t^2$$
$$t = \sqrt{15.3}$$
$$= 3.91\,\text{s}$$

Horizontally: $v = ?$, $t = 3.91\,\text{s}$ from above, $s = 100\,\text{m}$

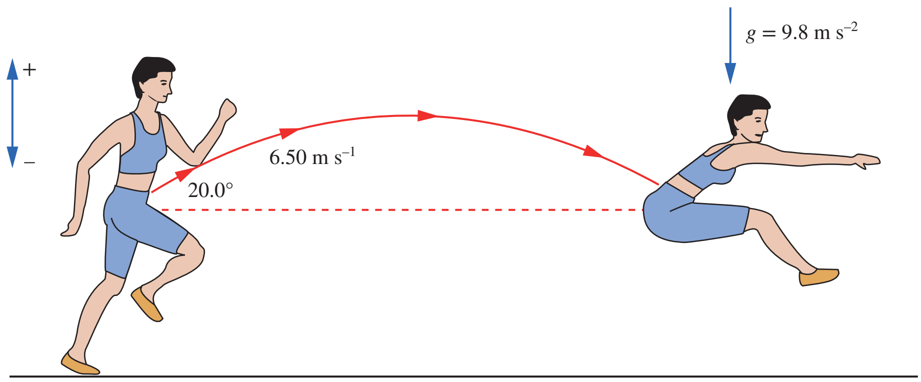
$$v_{av} = \frac{s}{t}$$
$$= \frac{100}{3.91}$$
$$= 26\,\text{m s}^{-1}$$

2.6 Projectiles launched obliquely

Worked example: Try yourself 2.6.1

LAUNCH A PROJECTILE AT AN ANGLE

A 50 kg athlete in a long-jump event leaps with a velocity of $6.50\,\text{m s}^{-1}$ at 20.0° to the horizontal.



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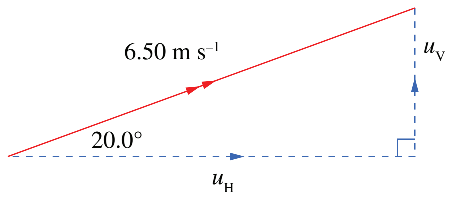
In answering the following questions, treat the athlete as a point mass, ignore air resistance and assume that $g = 9.8\,m\,s^{-2}$.

a What is the athlete's velocity at the highest point in the jump?

Thinking	Working
First find the horizontal and vertical components of the initial speed.	 Using trigonometry: $u_H = 6.50\cos 20.0° = 6.11\,m\,s^{-1}$ Taking up as positive: $u_V = 6.50\sin 20.0° = 2.22\,m\,s^{-1}$
Projectiles that are launched obliquely move only horizontally at their highest point. The vertical component of the velocity at this point is therefore zero. Thus the actual velocity is given by the horizontal component of the velocity throughout the motion.	At maximum height: $v = 6.11\,m\,s^{-1}$ horizontally to the right.

b What is the maximum height gained by the athlete's centre of mass during the jump?

Thinking	Working
To find the maximum height you must work with the vertical component of the velocity. Recall that the vertical component of velocity at the highest point is zero.	Vertically, taking up as positive: $u = 2.22$ $a = -9.8$ $v = 0$ $s = ?$
Substitute these values into an appropriate equation for uniform acceleration.	$v^2 = u^2 + 2as$ $0 = 2.22^2 + 2 \times 9.8 \times s$
Rearrange the equation and solve for *s*.	$s = \frac{2.22^2}{19.6}$ $= 0.25\,m$

c Assuming a return to the original height, what is the total time the athlete is in the air?

Thinking	Working
As the motion is symmetrical, the time required to complete it will be double that taken to reach the maximum height. First, the time it takes to reach the highest point must be found.	Vertically, taking up as positive: $u = 2.22\,m\,s^{-1}$ $a = -9.8\,m\,s^{-2}$ $v = 0$ $t = ?$
Substitute the relevant values into an appropriate equation for uniform acceleration.	$v = u + at$ $0 = 2.22 - 9.8t$
Rearrange the formula and solve for *t*, the time needed to reach maximum height.	$t = \frac{2.22}{9.8}$ $= 0.227\,s$
The time to complete the jump is double the time it takes to reach the maximum height.	Total time $= 2 \times 0.227 = 0.45\,s$

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CASE STUDY: ANALYSIS

The physics of shot putting

1 The initial horizontal speed is calculated as follows:

$u_h = u \times \cos 30° = 7.5 \times 0.866 = 6.5\,\text{m s}^{-1}$

2 The initial vertical speed is calculated as follows:

$u_v = u \times \sin 30° = 7.5 \times 0.5 = 3.8\,\text{m s}^{-1}$

3 The initial vertical speed, u, is $3.8\,\text{m s}^{-1}$. The acceleration (upwards) is $-9.8\,\text{m s}^{-2}$ (due to the force of gravity). The final vertical speed, v, is $0\,\text{m s}^{-1}$ (as the object has reached its maximum height). The time to reach the maximum height is calculated as follows:

$$v = u + at$$

$$t = \frac{v-u}{a} = \frac{0-3.8}{-9.8}$$

$$= 0.38\,\text{s}$$

4 The initial vertical speed, u, is $3.8\,\text{m s}^{-1}$. The acceleration (upwards) is $-9.8\,\text{m s}^{-2}$ (due to the force of gravity). The final vertical speed, v, is $0\,\text{m s}^{-1}$ (as the object has reached its maximum height). The maximum height is calculated as follows:

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2-u^2}{2\times a} = \frac{3.8^2}{2\times 9.8}$$

$$= 0.74\,\text{m}$$

The maximum height from the ground is the maximum height from the throw plus the height from where the shot put is launched (1.6 m). Therefore the maximum height is 2.3 m.

5 The speed at the maximum height is given only by the horizontal component of the velocity (the vertical component is zero, as it is the maximum height). This is $6.5\,\text{m s}^{-1}$.

6 To calculate the total distance the shot put travels, first calculate the time it takes to reach the maximum height and the time it takes to fall back to the ground.

Note that the motion is not symmetrical, as the object lands lower than the point from where it was launched.

The time to reach the maximum height was calculated in question **3**: $t = 0.38\,\text{s}$. The height the object falls is 2.3 m (from question **4**). To calculate the time for the object to fall:

$$s = ut + \frac{1}{2}at^2$$

$$u = 0\,\text{m s}^{-1}$$

$$s = 2.3\,\text{m}$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 2.3}{9.8}} = 0.69\,\text{s}$$

The total time the object is in the air is 0.38 s (while rising) + 0.69 s (while falling) = 1.07 s.

Ignoring air resistance, the horizontal speed is constant: $6.5\,\text{m s}^{-1}$. Therefore the total distance the shot put travels is $s = v \times t = 6.5 \times 1.07 = 7.0\,\text{m}$.

KEY QUESTIONS

Knowledge and understanding

1 The horizontal velocity remains constant throughout the javelin's flight (ignoring the effect of air resistance).

2 The optimal launch angle to give the greatest range for any ideal projectile is 45° (if air resistance is ignored, and if the start and end points are at the same height). Therefore Ollie is correct. James is incorrect in thinking that all the velocity of the water will be in the horizontal direction as he is not taking into account the acceleration due to gravity. The purely horizontal stream will have a larger horizontal velocity than the 45° stream but will have a shorter flight time and a shorter range.

3 At the highest point the ball has zero vertical velocity. The horizontal velocity is constant throughout the flight (when air resistance is ignored). So the overall velocity at the highest point is equal to its horizontal speed:

$v_H = v\cos\theta = 25\cos 40° = 19\,\text{m s}^{-1}$

4 **a** $v_H = v\cos\theta = 25\cos 30° = 22\,\text{m s}^{-1}$

b $v_V = v\sin\theta = 25\sin 30° = 13\,\text{m s}^{-1}$

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c The acceleration is constant and is due to the force of gravity. The acceleration is $9.8\,m\,s^{-2}$ down.

d At the highest point the ball has zero vertical velocity. The horizontal velocity is constant throughout the flight when air resistance is ignored. So the overall velocity at the highest point is equal to the horizontal velocity: $22\,m\,s^{-1}$.

Analysis

5 a $v_H = v\cos\theta = 12\cos 25° = 11\,m\,s^{-1}$

b $v_V = v\sin\theta = 12\sin 25° = 5.1\,m\,s^{-1}$

c Vertically, with up as positive: $u = 5.1$, $a = -9.8$, $v = 0$, $t = ?$

$$v = u + at$$
$$0 = 5.1 - 9.8t$$
$$t = 0.52\,s$$

d Vertically with up as positive: $u = 5.1$, $a = -9.8$, $v = 0$, $s = ?$

$$v^2 = u^2 + 2as$$
$$0 = 5.1^2 + 2 \times -9.8 \times s$$
$$5.1^2 = 19.6s$$
$$s = 1.3\,m$$

Total height = 1.3 + 1.8 = 3.1 m

e The speed is given by the horizontal component of the velocity (as the vertical velocity is zero at this point), that is, $11\,m\,s^{-1}$.

f To calculate the total distance the shot put travels, first calculate the time it takes to reach the maximum height and the time it takes to fall back to the ground.

Note that the motion is not symmetrical, as the object lands lower than from the point where it was launched.

The time to reach the maximum height was calculated in part **c** as 0.52 s. The height the object falls is 3.1 m (from part **d**). To calculate the time for the object to fall:

$$s = ut + \frac{1}{2}at^2$$
$$u = 0\,m\,s^{-1}$$
$$s = 3.1\,m$$
$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 3.1}{9.8}}$$
$$= 0.795\,s$$

The total time the object is in the air is 0.52 s (while rising) + 0.795 s (while falling) = 1.315 s.

The horizontal speed is constant (assuming no friction from air is considered) at $11\,m\,s^{-1}$. Therefore the total distance the shot put travels is:

$s = vt = 11 \times 1.315 = 14\,m$

6 a i $v_H = 22.0\cos 10.0° = 21.7\,m\,s^{-1}$ (and remains constant throughout the flight)

ii $21.7\,m\,s^{-1}$

iii $21.7\,m\,s^{-1}$

b i Taking up as positive: $v_V = 22\sin 10.0° = 3.82\,m\,s^{-1}$

ii Vertically: $u = 3.82$, $a = -9.8$, $t = 0.25$, $v = ?$

$$v = u + at$$
$$= 3.82 - 9.8 \times 0.25$$
$$= 1.37\,m\,s^{-1}$$

iii Vertically: $u = 3.82$, $a = -9.8$, $t = 0.50$, $v = ?$

$$v = u + at$$
$$= 3.82 - 9.8 \times 0.50$$
$$= -1.08$$
$$= 1.08\,m\,s^{-1} \text{ down}$$

c $$v = \sqrt{21.7^2 + (-1.08)^2}$$
$$= 21.7\,m\,s^{-1}$$

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d The flight of the ball is symmetrical. Therefore the ball will strike her racquet at the same speed with which it was launched (assuming that the machine is at the same height as her racquet): 21.7 m s^{-1} (and at an angle of 10.0° to the horizontal).

e Vertically, with up as positive: $u = 3.82$, $a = -9.8$, $v = 0$, $t = ?$

$v = u + at$
$0 = 3.82 - 9.8t$
$t = 0.389\ \text{s}$

Therefore the total time is $2 \times 0.389 = 0.780\ \text{s}$.

$v = \frac{s}{t}$
$s = 21.7 \times 0.780$
$= 16.9\ \text{m}$

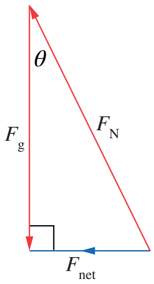
f Air resistance is a force that acts in the opposite direction to the velocity of the ball, thereby producing a horizontal and vertical deceleration of the ball during its flight. This means the maximum height of the ball is less than it would be with no air resistance. This also gives a shortened range of flight.

Chapter 2 Review

Knowledge and understanding

1 The bowling ball is increasing in speed at a constant rate, that is, with constant acceleration.

2 B. From the force triangle shown below you can see that $F_N > F_g$.



3 **a** $a = g \sin\theta$
$= 9.8 \sin 45.0°$
$= 6.9\ \text{m s}^{-2}$

b As $F_N = F_g \cos\theta$, the normal force must be less than the force due to gravity.

$F_N = F_g \cos\theta$
$= F_g \cos 45°$
$= 0.71 F_g$

4 The forces acting on the water when the bucket is directly overhead are the force of gravity and the normal force from the base of the bucket on the water. Both of these forces act downwards. There is no separate outwards or centrifugal force. Therefore Emma is correct.

This is an example of how the net force in circular motion is directed towards the centre of the circle. The water stays in the bucket because of inertia, and it moves in a circular path because of the normal force from the bucket, which is directed towards the centre of the circle. If the bucket were instantly removed, the water would leave the circle in a tangential path.

5 **a** The only force acting on the block on the table is tension:

$F_T = m_1 a = 5a$

The forces acting on the falling block are tension and the force due to gravity:

$F_g - F_T = m_2 a$

Substitute the expression for F_T into the equation for F_g:

$10 \times 9.8 - 5a = 10a$
$98 = 15a$
$a = 6.5\ \text{m s}^{-2}$

b $F_T = 5a = 5 \times 6.5 = 33\ \text{N}$

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6 **a** The perpendicular component is:

$$F_{\perp} = F_g \cos 15°$$
$$= (90 \times 9.8) \times \cos 15°$$
$$= 8.5 \times 10^2\,\text{N}$$

The parallel component is

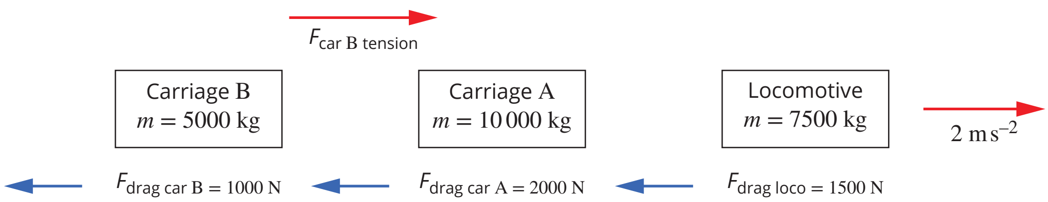
$$F_{\parallel} = F_g \sin 15°$$
$$= (90 \times 9.8) \times \sin 15°$$
$$= 2.3 \times 10^2\,\text{N}$$

b The normal force is equal in magnitude to the perpendicular component: $F_N = 852\,\text{N}$.

c Apply Newton's second law. The net force along the incline is the component of F_g parallel to the slope.

$$a = \frac{F_{net}}{90} = \frac{228}{90}$$
$$= 2.5\,\text{m s}^{-2} \text{ down the slope}$$

7 **a**



$$F_{system} = m_{system}a$$
$$F_{thrust} - F_{drag\ loco} - F_{drag\ car\ A} - F_{drag\ car\ B} = (m_{loco} + m_{car\ A} + m_{car\ B}) \times a$$
$$F_{thrust} - 1500 - 2000 - 1000 = (7500 + 10000 + 5000) \times 2$$
$$F_{thrust} = 5.0 \times 10^4 \text{ in the direction of motion}$$

b

$$F_{net\ car\ B} = m_{car\ B}a$$
$$F_{tension\ car\ B} - F_{drag\ car\ B} = m_{car\ B}a$$
$$F_{tension\ car\ B} = 5000 \times 2 + 2000$$
$$= 1.2 \times 10^4\,\text{N}$$

8

$$F_{net} = \text{thrust} - \text{drag forces} = m_{total}a$$
$$\text{thrust} - (800 + 700) = (1000 + 200) \times 2.5$$
$$\text{thrust} = 3000 + 1500 = 4500$$
$$= 4.5 \times 10^3\,\text{N}$$

9 **a**

$$a = g \sin\theta$$
$$= 9.8 \sin 40°$$
$$= 6.3\,\text{m s}^{-2}$$

b $u = 0\,\text{m s}^{-1}$, $s = 3.5$ m, $a = 6.3\,\text{m s}^{-2}$, $v = ?$

$$v^2 = u^2 + 2as$$
$$v^2 = 0 + 2 \times 6.3 \times 3.5$$
$$v = 6.6\,\text{m s}^{-1}$$

10 **a**

$$F_N = mg \cos\theta$$
$$= (57 + 3) \times 9.8 \times \cos 65°$$
$$= 248$$
$$= 2.5 \times 10^2\,\text{N}$$

b

$$a = g \sin\theta$$
$$= 9.8 \sin 65°$$
$$= 8.9\,\text{m s}^{-2} \text{ down the ramp}$$

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c $F_{net} = ma$

$= (57 + 3) \times 8.88$

$= 533$

$= 5.3 \times 10^2$ N down the ramp

d $u = 0$, $s = 5.0$, $a = 8.88$, $v = ?$

$v^2 = u^2 + 2as$

$= 0 + 2 \times 8.88 \times 5.0$

$= 89$

$v = 9.4\,\text{m s}^{-1}$ (speed only)

e $F_{net} = 0$ so forces parallel to the incline are balanced.
$F_f = mg \sin\theta = 5.3 \times 10^2$ N up the ramp

11 a $v = \dfrac{2\pi r}{T}$

$= \dfrac{2\pi \times 0.800}{1.36}$

$= 3.70\,\text{m s}^{-1}$

b $a = \dfrac{v^2}{r}$

$\dfrac{3.70^2}{0.800}$

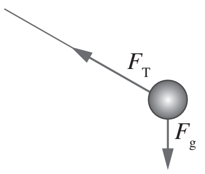
$= 17.1\,\text{m s}^{-2}$ towards the centre of the circle

c $F_{net} = ma$

$= 0.0250 \times 17.1$

$= 0.428$ N (only magnitude is needed)

d



e $F_t = \dfrac{mg}{\sin 30.0°}$

$= \dfrac{0.0250 \times 9.80}{0.50}$

$= 0.49$ N

12 a $a = \dfrac{v^2}{r}$

$= \dfrac{7.5^2}{15}$

$= 3.8\,\text{m s}^{-2}$ towards the centre of the circle

b The centripetal force is created by the friction between the tyres and the ground. Thus friction keeps the toy car moving in its circular path.

13 $v = \sqrt{rg \tan\theta}$

$= \sqrt{30 \times 9.8 \times \tan 40°}$

$= 16\,\text{m s}^{-1}$

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14 a i At top:

$$F_{net} = \frac{mv^2}{r}$$
$$= \frac{50 \times 5.0^2}{10}$$
$$= 125\,N\ \text{down}$$
$$F_N = F_g - 125$$
$$= 490 - 125$$
$$= 365\,N\ \text{up}$$

ii At bottom:

$$F_{net} = \frac{mv^2}{r}$$
$$= \frac{50 \times 5.0^2}{10}$$
$$= 125\,N\ \text{up}$$
$$F_N = F_g + 125$$
$$= 490 + 125$$
$$= 615\,N\ \text{up}$$

b D. At the top of the ride, $F_N < F_g$, so he would feel lighter than usual.

15 a $v_{av} = \frac{s}{t}$

$$s = v_{av} \times t$$
$$= 3.75 \times 1.5$$
$$= 5.6\,m$$

b 9.8 m s^{-2} downwards (due to gravity)

16 a 15.0 m s^{-1}. As there are no forces acting horizontally, the horizontal velocity is constant.

b Vertically, with down as positive: $u = 0$, $s = 1.27$, $a = 9.80$, $v = ?$

$$v^2 = u^2 + 2as$$
$$= 0 + 2 \times 9.8 \times 1.27$$
$$v = 4.99\,m\,s^{-1}$$

c $v = \sqrt{15^2 + 4.99^2}$

$$= 15.8\,m\,s^{-1}\ \text{(only the speed is required)}$$

Application and analysis

17 a $u_h = 18.5 \cos 46°$

$$= 12.9\,m\,s^{-1}$$

b $u_v = 18.5 \sin 46°$

$$= 13.3\,m\,s^{-1}$$

c Vertically, with up as positive: $u = 13.3$, $a = –9.8$, $v = 0$, $s = ?$

$$v^2 = u^2 + 2as$$
$$0 = 13.3^2 + 2 \times -9.8 \times s$$
$$= 177 - 19.6s$$
$$s = 9.025\,m$$

The total height from the ground is 1.7 + 9.03 = 10.7 m.

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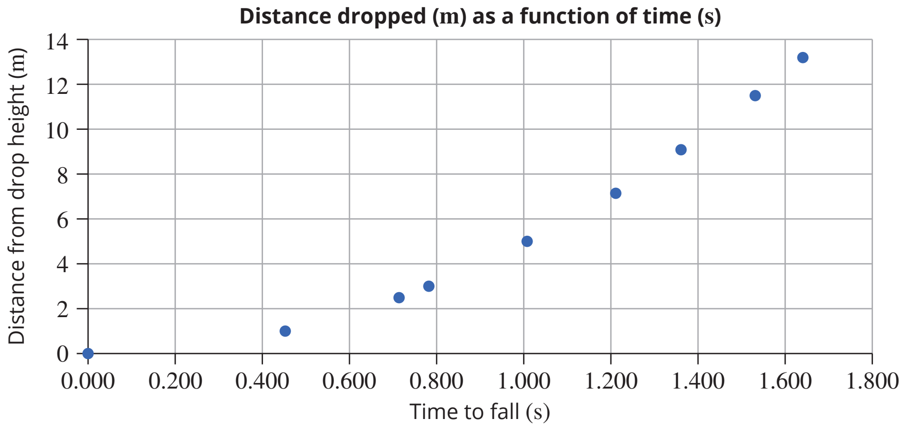
18 **a** The object drops at a constant acceleration due to the force of gravity (with an acceleration downwards of 9.8 m s^{-2}).

b Your plot should look like the following:



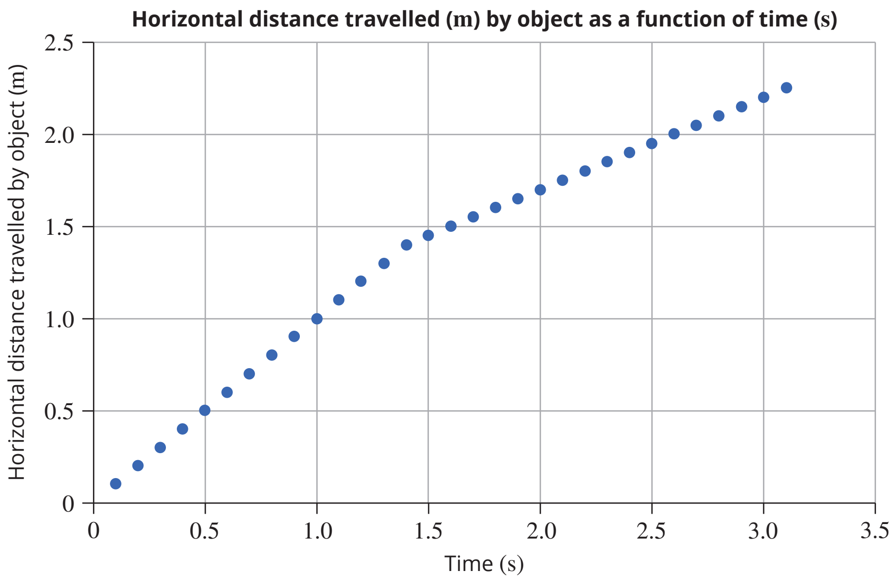
c Predict the time to fall 15 m.

This could be done by examining the graph, or by using the equation $s = ut + \frac{1}{2}at^2$.

We know that $u(t) = 0$, therefore the time taken to fall 15 m can be found using:

$$t = \sqrt{\frac{2 \times 15}{9.8}}$$
$$= 1.75\ \text{s}$$

19 **a**



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b To obtain the horizontal velocity, determine the horizontal distance travelled in each 0.1 s time increment by taking the difference between successive data points. For example, in the first 0.1 s the object has travelled 0.1 m so the velocity is 1 m s^{-1}. This data is shown below in the 'Horizontal velocity' column.

Horizontal distance travelled (m)	Time (s)	Height (m)	Horizontal distance travelled each 0.1 s (m)	Horizontal velocity (m s^{-1})
0.10	0.1	15.0	0.10	1.0
0.20	0.2	15.2	0.10	1.0
0.30	0.3	15.6	0.10	1.0
0.40	0.4	16.2	0.10	1.0
0.50	0.5	17.0	0.10	1.0
0.60	0.6	18.0	0.10	1.0
0.70	0.7	19.2	0.10	1.0
0.80	0.8	18.0	0.10	1.0
0.90	0.9	17.0	0.10	1.0
1.00	1.0	16.2	0.10	1.0
1.10	1.1	15.6	0.10	1.0
1.20	1.2	15.2	0.10	1.0
1.30	1.3	15.0	0.10	1.0
1.40	1.4	14.3	0.10	1.0
1.45	1.5	13.6	0.05	0.5
1.50	1.6	12.8	0.05	0.5
1.55	1.7	12.2	0.05	0.5
1.60	1.8	11.5	0.05	0.5
1.65	1.9	10.9	0.05	0.5
1.70	2.0	10.1	0.05	0.5
1.75	2.1	9.5	0.05	0.5
1.80	2.2	8.7	0.05	0.5
1.85	2.3	8.2	0.05	0.5
1.90	2.4	7.6	0.05	0.5
1.95	2.5	6.5	0.05	0.5
2.00	2.6	5.8	0.05	0.5
2.05	2.7	5.1	0.05	0.5
2.10	2.8	4.3	0.05	0.5
2.15	2.9	3.7	0.05	0.5
2.20	3.0	2.9	0.05	0.5
2.25	3.1	2.0	0.05	0.5

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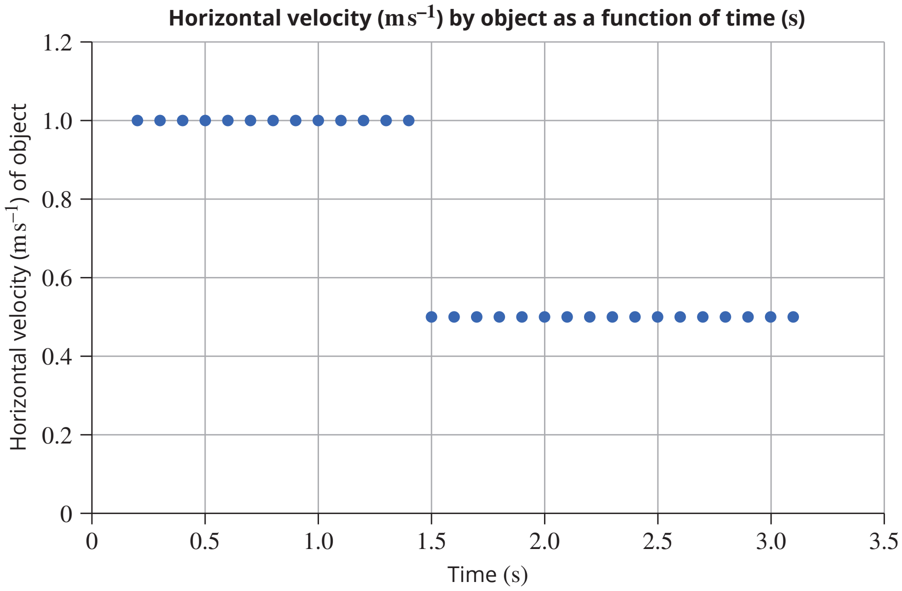
c



Observations:

The velocity is constant at 1 $m\,s^{-1}$ from initial launch of the object to the point where the object reaches its maximum height.

The velocity is constant at 0.5 $m\,s^{-1}$ from the object's maximum height to the point where it reaches the ground.

d To obtain the horizontal acceleration, take the difference between the data points of the horizontal velocity and divide by the time increment for each successive data point. This data is shown below in the 'Horizontal acceleration' column.

Horizontal distance travelled (m)	Time (s)	Height (m)	Horizontal velocity ($m\,s^{-1}$)	Horizontal acceleration ($m\,s^{-2}$)
0.10	0.1	15.0	1.0	0
0.20	0.2	15.2	1.0	0
0.30	0.3	15.6	1.0	0
0.40	0.4	16.2	1.0	0
0.50	0.5	17.0	1.0	0
0.60	0.6	18.0	1.0	0
0.70	0.7	19.2	1.0	0
0.80	0.8	18.0	1.0	0
0.90	0.9	17.0	1.0	0
1.00	1.0	16.2	1.0	0
1.10	1.1	15.6	1.0	0
1.20	1.2	15.2	1.0	0
1.30	1.3	15.0	1.0	0
1.40	1.4	14.3	1.0	0
1.45	1.5	13.6	0.5	–5
1.50	1.6	12.8	0.5	0
1.55	1.7	12.2	0.5	0
1.60	1.8	11.5	0.5	0
1.65	1.9	10.9	0.5	0
1.70	2.0	10.1	0.5	0
1.75	2.1	9.5	0.5	0
1.80	2.2	8.7	0.5	0
1.85	2.3	8.2	0.5	0
1.90	2.4	7.6	0.5	0
1.95	2.5	6.5	0.5	0

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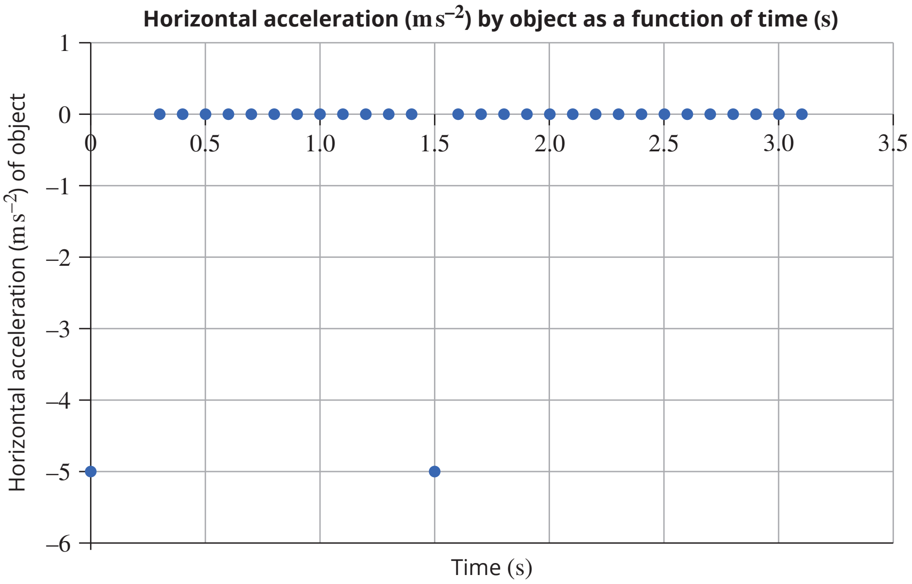
Horizontal distance travelled (m)	Time (s)	Height (m)	Horizontal velocity ($m\,s^{-1}$)	Horizontal acceleration ($m\,s^{-2}$)
2.00	2.6	5.8	0.5	0
2.05	2.7	5.1	0.5	0
2.10	2.8	4.3	0.5	0
2.15	2.9	3.7	0.5	0
2.20	3.0	2.9	0.5	0
2.25	3.1	2.0	0.5	0

e



Observations:

There is one outlier in the data which would need to be explained in the experiment report (i.e. whether it originates from an error or a mistake).

The effect of air resistance on the horizontal component is negligible as the acceleration is zero.

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Chapter 3 The relationship between force, energy and mass

3.1 Conservation of momentum

Worked example: Try yourself 3.1.1

CONSERVATION OF MOMENTUM

In a safety-rating test of head-on collisions, a car of mass 1200 kg travelling east at 22.0 m s^{-1} crashes into a bus of mass 7000 kg travelling west at 15.0 m s^{-1}. Assume that the car and bus lock together on impact. You can ignore the effect of friction.

a Calculate the final common velocity of the vehicles.	
Thinking	**Working**
First assign a direction that will be considered positive.	In this case we will consider vectors directed eastwards to be positive. $m_c = 1200\,\text{kg}$ $u_c = 22.0\,\text{m s}^{-1}$ $m_b = 7000\,\text{kg}$ $u_b = -15.0\,\text{m s}^{-1}$
Apply the law of conservation of momentum.	$\Sigma p_{\text{initial}} = \Sigma p_{\text{final}}$ $m_c u_c + m_b u_b = (m_c + m_b)v$ $(1200 \times 22.0) + (7000 \times -15.0) = (1200 + 7000)v$ $(-78600) = (8200)v$ $v = \frac{(-78600)}{(8200)}$ $= -9.58537$ $= 9.59\,\text{m s}^{-1}$ west

b Calculate the change in momentum of the car.	
Thinking	**Working**
The change in momentum of the car is its final momentum minus its initial momentum.	$\Delta p_c = p_{\text{final}} - p_{\text{initial}}$ $= m_c(v - u)$ $= (1200)(-9.58537 - 22.0)$ $= -37902.4$ $= 3.79 \times 10^4\,\text{kg m s}^{-1}$ west

c Calculate the change in momentum of the bus.	
Thinking	**Working**
The change in momentum of the bus is its final momentum minus its initial momentum.	$\Delta p_b = p_{\text{final}} - p_{\text{initial}}$ $= m_b(v - u)$ $= (7000)(-9.58537 - (-15.0))$ $= 37902.4$ $= 3.79 \times 10^4\,\text{kg m s}^{-1}$ east

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d Verify that the momentum of the system is constant.	
Thinking	**Working**
The total change in the momentum of a system is the vector sum of the change of momentum of its parts. This should be zero from the conservation of momentum.	$\Delta p_c + \Delta p_b = (-3.79 \times 10^4) + (3.79 \times 10^4) = 0$ Therefore the momentum of the system is constant (i.e. conserved) as expected.

Worked example: Try yourself 3.1.2

REBOUNDING

In a child's toy, a blue marble rolls along a track and collides with a red marble rolling the other way. The blue marble has a mass of 0.003 20 kg and is travelling south at 0.800 m s^{-1} as it hits the red marble. The red marble has a mass of 0.001 50 kg and is travelling north at 1.00 m s^{-1} when it hits the blue marble. After the collision the blue marble is now travelling towards the south at 0.450 m s^{-1}. Assume that the two marbles bounce off each other on impact and ignore the effect of friction.

a Calculate the sum of the momentum of the two marbles before they hit.	
Thinking	**Working**
First assign a direction that will be considered positive.	In this case we will consider vectors directed towards the north to be positive. $m_b = 0.00320\,\text{kg}$ $u_b = -0.800\,\text{m s}^{-1}$ $v_b = -0.450\,\text{m s}^{-1}$ $m_r = 0.00150\,\text{kg}$ $u_b = 1.00\,\text{m s}^{-1}$
Use the equation of momentum for each marble and substitute the values.	$\Sigma p_{\text{initial}} = p_r + p_b$ $= m_b u_b + m_r u_r$ $= (0.00320)(-0.800) + (0.00150)(1.00)$ $= (-0.00256) + (0.00150)$ $= -0.00106$ $= 1.06 \times 10^{-3}\,\text{kg m s}^{-1}$ south

b Calculate the final velocity of the red marble.	
Thinking	**Working**
The sum of the momentum after the collision is equal to the sum of the momentum before the collision.	$\Sigma p_{\text{final}} = \Sigma p_{\text{initial}}$ $m_b v_b + m_r v_r = \Sigma p_{\text{initial}}$ $(0.00320)(-0.450) + (0.00150)v_r = (-0.00106)$ $v_r = \dfrac{(-0.00106) + (0.00144)}{(0.00150)}$ $= 0.253333$ $= 0.253\,\text{m s}^{-1}$ north

c Calculate the change in momentum of the blue marble.	
Thinking	**Working**
The change in momentum of the blue marble is its final momentum minus its initial momentum.	$\Delta p_b = p_{\text{final}} - p_{\text{initial}}$ $= m_b(v_b - u_b)$ $= (0.00320)(-0.450 - (-0.800))$ $= 0.00112\,\text{kg m s}^{-1}$ $= 1.12 \times 10^{-3}\,\text{kg m s}^{-1}$ north

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d Calculate the change in momentum of the red marble.	
Thinking	**Working**
The change in momentum of the red marble is its final momentum minus its initial momentum.	$\Delta p_r = p_{final} - p_{initial}$ $= m_r(v_r - u_r)$ $= (0.00150)(0.253333 - 1.00)$ $= -0.00112$ $= 1.12 \times 10^{-3}\ \text{kg m s}^{-1}$ south

Worked example: Try yourself 3.1.3

EXPLOSIVE MOMENTUM

Two ice dancers are standing still in the centre of an ice rink facing each other with their palms together. They then begin their routine by pushing with their hands. After pushing away, ice dancer A, of mass 62.0 kg, travels towards the north at 2.20 m s^{-1}. Ice dancer B, of mass 98.0 kg, travels towards the south. You can ignore the effect of friction.

a Calculate the sum of the momentum of the two ice dancers before they push away.	
Thinking	**Working**
Assign a direction that will be considered positive.	In this case we will consider vectors directed towards the north to be positive. $m_A = 62.0\ \text{kg}$ $u_A = 0$ $v_A = 2.20\ \text{m s}^{-1}$ $m_B = 98.0\ \text{kg}$ $u_B = 0\ \text{m s}^{-1}$
Use the equation of momentum for the combined mass of the dancers.	$\Sigma p_{initial} = p_A + p_B$ $= (m_A + m_B)u$ $= (62.0 + 98.0)(0)$ $= (160)(0)$ $= 0\ \text{kg m s}^{-1}$

b Calculate the final velocity of ice dancer B.	
Thinking	**Working**
The sum of the momentum after the dancers push away is equal to the sum of their momentum before the push.	$\Sigma p_{final} = \Sigma p_{initial}$ $m_A v_A + m_B v_B = \Sigma p_{initial}$ $(62.0)(2.20) + (98.0)v_2 = 0$ $(98.0)v_2 = -(136.4)$ $v_2 = -1.39184$ $= 1.39\ \text{m s}^{-1}$ south

c Calculate the change in momentum of ice dancer A.	
Thinking	**Working**
The change in momentum of dancer A is their final momentum minus their initial momentum.	$\Delta p_A = p_{final} - p_{initial}$ $= m_A(v_A - u_A)$ $= (62.0)(2.20 - 0)$ $= 136.4$ $= 136\ \text{kg m s}^{-1}$ north

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d Calculate the change in momentum of ice dancer B.	
Thinking	**Working**
The change in momentum of dancer B is their final momentum minus their initial momentum.	$\Delta p_B = p_{final} - p_{initial}$ $= m_B(v_B - u_B)$ $= (98.0)(-1.39184 - 0)$ $= -136.4$ $= 136\ kg\,m\,s^{-1}$ south

KEY QUESTIONS

Knowledge and understanding

1 Before you step forward to get off the stand-up paddle board, the sum of the initial momentum of you and the board can be assumed to be zero. Therefore the total momentum after you step forward must also be zero. This means that your forward momentum must be matched by the board's backward momentum. But as the board is likely to be less than your mass, the board will move rapidly backwards while you move much more slowly forward. You may end up quite wet. On the other hand, the mass of a river ferry is very large in comparison to you, so its backwards velocity will be extremely small in comparison to your forward velocity. In this case you are more likely to land on the dock without incident.

2 As the two toy cars are moving in opposite directions, one of the cars will have a positive momentum and the other car will have a negative momentum. Therefore the sum of their initial momenta adds to zero to match their overall final momentum of zero.

Analysis

3

$$\begin{aligned} \Sigma p_{final} &= \Sigma p_{initial} \\ (m_g + m_o)v &= m_g u_g + m_o u_o \\ (25.0 + 50.0)v &= (25.0)(3.50) + (50.0)(-6.00) \\ (75.0)v &= (-212.500) \\ v &= -2.83333 \\ &= 2.83\,m\,s^{-1} \text{ west} \end{aligned}$$

4

$$\begin{aligned} \Sigma p_{final} &= \Sigma p_{initial} \\ (m_p + m_d)v &= m_p u_p + m_d u_d \\ (11000 + 16000)v &= (11000)(7.50) + (16000)(3.50) \\ (27000)v &= (138500) \\ v &= 5.12963 \\ &= 5.13\,m\,s^{-1} \text{ north} \end{aligned}$$

5 a i

$$\begin{aligned} p_c &= m_c u_c \\ &= (1000)\left(\frac{36.0}{3.6}\right) \\ &= 10000 \\ &= 1.00 \times 10^4\,kg\,m\,s^{-1} \text{ east} \end{aligned}$$

ii

$$\begin{aligned} p_w &= m_w u_w \\ &= (2000)\left(\frac{-18.0}{3.6}\right) \\ &= -10000 \\ &= 1.00 \times 10^4\,kg\,m\,s^{-1} \text{ west} \end{aligned}$$

iii

$$\begin{aligned} \Sigma p &= p_c + p_w \\ &= (1.00 \times 10^4) + (-1.00 \times 10^4) \\ &= 0\,kg\,m\,s^{-1} \end{aligned}$$

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b i

$$\Sigma p_{final} = \Sigma p_{initial}$$
$$(m_c + m_w)v = \Sigma p_{initial}$$
$$(1000 + 2000)v = 0$$
$$(3000)v = 0$$
$$v = 0\,m\,s^{-1}$$

ii The vector momentum of the two vehicles before the collision has combined and is conserved (as it is in any collision). The sum of the vector momentum of the system before the collision becomes the sum of the vector momentum of the system after the collision.

iii $\Delta p_c = p_{final} = p_{initial}$

$$= m_c(v_c - u_c)$$
$$= (1000)(0 - 10.0)$$
$$= -10\,000$$
$$= 1.00 \times 10^4\,kg\,m\,s^{-1} \text{ west}$$

iv $\Delta p_w = p_{final} = p_{initial}$

$$= m_w(v_w - u_w)$$
$$= (2000)(0 - (-5.00))$$
$$= 10\,000$$
$$= 1.00 \times 10^4\,kg\,m\,s^{-1} \text{ east}$$

6

$$\Sigma p_{final} = \Sigma p_{initial}$$
$$m_p v_p + m_g v_g = m_p u_p + m_g u_g$$
$$(0.155)(-3.00) + (0.132)v_g = (0.155)(5.00) + (0.132)(0)$$
$$(0.132)v_g = (0.775) + (0) - (-0.465)$$
$$v_g = \frac{(1.24)}{(0.132)}$$
$$= 9.39394$$
$$= 9.39\,m\,s^{-1} \text{ to the right}$$

7

$$\Sigma p_{final} = \Sigma p_{initial}$$
$$m_y v_y + m_o v_o = m_y u_y + m_o u_o$$
$$(71.0)v_y + (65.0)(1.40) = (71.0)(4.20) + (65.0)(-5.30)$$
$$(71.0)v_y = (298.200) + (-344.500) - (91.0000)$$
$$v_y = \frac{(-137.300)}{(71.0)}$$
$$= -1.93380$$
$$= 1.93\,m\,s^{-1} \text{ west}$$

8

$$\Sigma p_{final} = \Sigma p_{initial}$$
$$m_i v_i + m_s v_s = m_i u_i + m_s u_s$$
$$(4.20 \times 10^5)v_i + (3.20 \times 10^4)(5.00) = (4.20 \times 10^5)(0) + (3.20 \times 10^4)(-5.00)$$
$$(4.20 \times 10^5)v_i = (0) + (-1.600\,00 \times 10^5) - (1.600\,00 \times 10^5)$$
$$v_i = \frac{(-3.20\,000 \times 10^5)}{(4.20 \times 10^5)}$$
$$= -0.761905$$
$$= 0.762\,m\,s^{-1} \text{ south}$$

9

$$\Sigma p_{final} = \Sigma p_{initial}$$
$$m_b v_b + m_c v_c = (m_b + m_c)u$$
$$(10.0)(505) + (1000)v_c = (1000 + 10.0)(0)$$
$$(1000)v_c = (0) - (5050)$$
$$v_c = \frac{(-5.050)}{(1000)}$$
$$= -5.050$$
$$= 5.05\,m\,s^{-1} \text{ west}$$

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10

$$\Sigma p_{\text{final}} = \Sigma p_{\text{initial}}$$
$$m_a v_a + m_t v_t = (m_a + m_t)u$$
$$(235)(0.300) + (46.0)v_t = (235 + 46.0)(-0.750)$$
$$(46.0)v_t = (-210.750) - (70.5000)$$
$$v_t = \frac{(-281.250)}{(46.0)}$$
$$= -6.11413$$
$$= 6.11\,\text{m s}^{-1} \text{ away from the spaceship}$$

3.2 Impulse

Worked example: Try yourself 3.2.1

CALCULATING THE IMPULSE

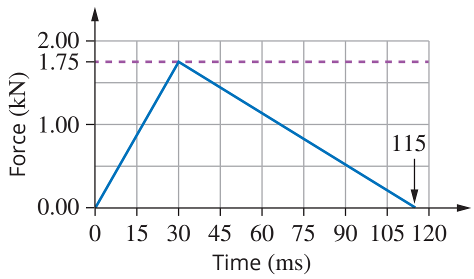
Calculate the impulse of the braking system on the 1480 kg sports car if the vehicle was travelling at 95.5 km h^{-1} in a north-easterly direction before coming to an abrupt halt.

Thinking	Working
Convert the speed to m s^{-1}.	$95.5\,\text{km h}^{-1} = \frac{95.5}{3.6}\,\text{m s}^{-1} = 26.5278\,\text{m s}^{-1}$
Calculate the change in momentum. The negative sign indicates that the change in momentum, and therefore the impulse, is in the direction opposite to the initial momentum, as would be expected.	$\Delta p = m(v - u)$ $= (1480)(0 - 26.5278)$ $= -3.92611 \times 10^4$ $= 3.93 \times 10^4\,\text{N s}$ south-west
The impulse is equal to the change in momentum.	impulse $= 3.93 \times 10^4\,\text{N s}$ south-west

Worked example: Try yourself 3.2.2

IMPULSE OF RUNNING SHOES

A running-shoe company plots the following force vs time graph for an alternative design intended to reduce the peak force on the heel. Calculate the magnitude of the impulse.



Thinking	Working
Recall that impulse = $F\Delta t$. This is the area under the force vs time graph.	impulse $= \frac{1}{2} \times \text{base} \times \text{height}$ $= \frac{1}{2} \times 115 \times 10^{-3} \times 1.75 \times 10^3$ $= 101\,\text{N s}$

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Worked example: Try yourself 3.2.3

BRAKING FORCE

The same 2520 kg truck travelling at 30.0 m s^{-1} needs to stop in 1.50 s because a vehicle in front has suddenly stopped. Calculate the magnitude of the average braking force required to stop the truck in that time.

Thinking	Working
Calculate the change in momentum. The negative sign indicates that the change in momentum, and therefore the braking force, is in the direction opposite to the initial momentum, as would be expected.	$\Delta p = m(v - u)$ $= 2520(0 - 30)$ $= -75\,600\ \text{kg m s}^{-1}$
Transpose $\Delta p = F\Delta t$ to find the force. The sign of the momentum can be ignored, since you are finding just the magnitude of the average force.	$F = \dfrac{\Delta p}{\Delta t}$ $= \dfrac{75\,600}{1.50}$ $= 50\,400$ $= 5.04 \times 10^4\ \text{N}$

CASE STUDY: ANALYSIS

Car safety and crumple zones

1 $p_d = m_d u_d$

$$= (90.0)\left(\frac{60.0}{3.6}\right)$$

$$= 1.50 \times 10^3\ \text{kg m s}^{-1}\ \text{north}$$

2 $\Delta p_d = p_{final} - p_{initial}$

$$= (0) - (1500)$$

$$= -1500$$

$$= 1.50 \times 10^3\ \text{kg m s}^{-1}\ \text{south}$$

3 The impulse of the car is the change in momentum of the car. The change in momentum does not change if the time over which the driver came to a stop were extended, as it is independent of time. The change in momentum is only dependent on the mass, initial velocity and final velocity.

4 The impulses experienced by the car driver and the tank driver will be the same, as they both have the same mass, the same initial velocity and the same final velocity.

5 Use the value for impulse found in question **2**.

$$I_c = \Delta p_c = F_c \Delta t_c$$

$$F_c = \frac{I_c}{\Delta t_c}$$

$$= \frac{(-1500)}{(985 \times 10^{-3})}$$

$$= -1522.84$$

$$= 1.52 \times 10^3\ \text{N south}$$

6 $I_t = \Delta p_t = F_t \Delta t_t$

$$F_t = \frac{I_t}{\Delta t_t}$$

$$= \frac{(-1500)}{(81.5 \times 10^{-3})}$$

$$= -18\,404.9$$

$$= 1.84 \times 10^4\ \text{N south}$$

7 There is an inverse relationship between the force acting and the time taken for the accident to occur. The longer the time, the smaller the force that acts.

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KEY QUESTIONS

Knowledge and understanding

1 Speed of the ball $= \frac{155}{3.6} = 43.0556\,\text{m s}^{-1}$

impulse $= \Delta p = m(v - u) = 0.165(0 - 43.0556) = -7.10\,\text{N s}$

The magnitude of the impulse is 7.10 N s.

2 Final speed of ball is:

$$\frac{144}{3.6} = 40.0\,\text{m s}^{-1}$$

$$F = \frac{\Delta p}{\Delta t} = \frac{(0.0570)(40.0 - 0)}{(0.0600)}$$

$$= 38.0\,\text{N}$$

Note: The force may also be determined by calculating the acceleration and then using the relationship $F = ma$.

3 Impulse $= \Delta p$

Taking up as positive:

$$\Delta p = (0.625)(24.5 - (-32.0))$$
$$= 35.3125\,\text{kg m s}^{-1}$$
$$= F\Delta t$$
$$F = \frac{35.3125}{0.0165}$$
$$= 2140.15$$
$$= 2.14 \times 10^3\,\text{N}$$

Note: The force may also be determined by calculating the acceleration and then using the relationship $F = ma$.

4 **a** Speed $= \frac{50.0}{3.6} = 13.8889\,\text{m s}^{-1}$

$$p = mv$$
$$= 100\,000 \times 13.8889$$
$$= 1\,388\,890$$
$$= 1.39 \times 10^6\,\text{kg m s}^{-1}$$

b 1.39×10^6 N s. Since the final momentum is zero, the magnitude of the impulse is equal to the magnitude of the initial momentum. The object with which the train collides, and the time it takes to stop, does not affect the impulse.

Analysis

5 From least to most: balls A, C and B

Remember to consider the initial and final momentum of each ball. The ball with the greatest change in momentum is the ball with the greatest final momentum in the opposite direction, since they all have the same initial momentum.

6 Wearing good runners will reduce the force, while being barefoot will increase the force. Runners have cushioned soles which compress, increasing the time over which the change in momentum occurs.

Landing on concrete will increase the force compared to landing on grass, as grass and soil compress much more than concrete, increasing the time over which the change in momentum occurs.

Jumping from a lower branch will decrease the force.

Dropping the backpack before jumping will decrease the force, as this will decrease her mass and thus her change in momentum/impulse.

Any factor that increases the time of deceleration will decrease the force. Similarly, any factor that decreases the impulse—by decreasing the change in velocity or the mass—will decrease the force.

7

$$I_B = I_A$$
$$F_B t_B = F_A t_A$$
$$F_B(0.00400) = F_A(0.0896)$$
$$F_B = \frac{(0.0896)}{(0.00400)} F_A$$
$$= 22.4 \times F_A$$

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8 **a** impulse = area under force–time graph

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 200 \times 10^{-3} \times 2000$$

$$= 200\,\text{N s}$$

b $F_{av} = \frac{\Delta p}{\Delta t}$

$$= \frac{200}{200 \times 10^{-3}}$$

$$= 1.00 \times 10^{3}\,\text{N}$$

9 First find the initial and final velocity by equating the initial potential with the final kinetic energy.

$$mgh = \frac{1}{2}mv^2$$

$$v_i = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 2.51} = 7.01\,\text{m s}^{-1}$$

$$v_f = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1.46} = 5.35\,\text{m s}^{-1}$$

$$\Delta p = m\Delta v$$

$$= 0.0575 \times (-5.35 - 7.01)$$

$$= -0.711\,\text{kg m s}^{-1}$$

$$= F\Delta t$$

$$F = \frac{\Delta p}{\Delta t}$$

$$= \frac{0.711}{0.055}$$

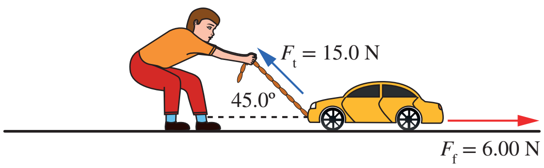
$$= 13\,\text{N}$$

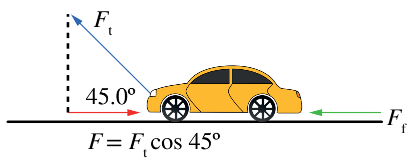
3.3 Work done

Worked example: Try yourself 3.3.1

FORCE APPLIED AT AN ANGLE TO THE DISPLACEMENT

A boy moves a toy car by pulling on a cord that is attached to the car at 45.0° to the horizontal. The boy applies a force of 15.0 N and pulls the car for 10.0 m along a path against a frictional force of 6.00 N.



a Determine the work done by the boy pulling on the cord.	
Thinking	**Working**
Draw the diagram of the forces in action.	
Find the component of the tension in the rope that is in the direction of the displacement (shown by the red arrow).	$F = 15.0 \times \cos 45.0° = 10.6066\,\text{N}$

Find the work done by the boy.	$W = Fs$ $= 10.6066 \times 10.0$ $= 106.066$ $= 106\ \text{J}$

b Calculate the work done on the toy car.	
Thinking	**Working**
The work done on the car is the net force acing on it multiplied by the displacement. (This is also the increase in the kinetic energy of the car.)	$W = Fs$ $= (F - F_f)s$ $= (10.6066 - 6.00) \times 10.0$ $= 46.0660$ $= 46.1\ \text{J}$

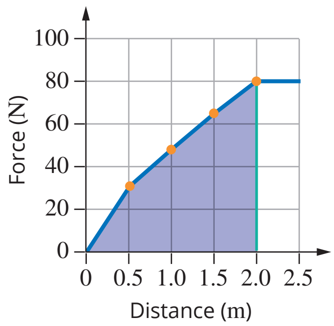
c Calculate the energy transformed into heat and sound due to the frictional force.	
Thinking	**Working**
The energy transformed into heat and sound due to the frictional force is the difference between the work done by the boy and the energy gained by the toy car.	$E = 106.066 - 46.0660$ $= 60.0\ \text{J}$
This is equal to the work done against friction, which can also be calculated from the frictional force.	$W_f = F_f s$ $= 6.00 \times 10.0$ $= 60.0\ \text{J}$

Worked example: Try yourself 3.3.2

CALCULATING WORK DONE FROM A GRAPH

The force required to elongate a piece of rubber tubing is represented in the graph below. Calculate the work done when the tubing is stretched by 2.0 m.

Force vs distance stretched of rubber tubing



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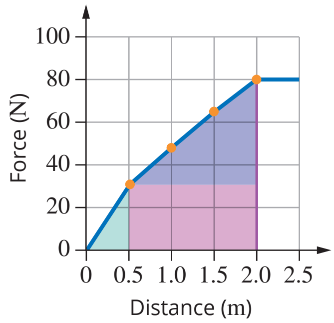
Thinking	Working
The work done is the area under the force vs distance graph. This may be found by calculation or by counting squares. In this case it is best to divide the area into triangles and rectangles and sum the individual areas.	
Add the areas together to calculate the work done.	area = $(\frac{1}{2} \times 0.50 \times 30) + (\frac{1}{2} \times 1.5 \times 50) + (30 \times 1.5)$ work done = 90 J

KEY QUESTIONS

Knowledge and understanding

1 Student answers will vary. An example is where someone standing on a horizontal travelator is holding a suitcase above the ground. No work is done on the suitcase while the case is being held.

2 Work is only done if the force, or any component of the force, is in the same direction as the displacement. The attractive force of gravity on the Earth is at right angles to the motion of the Earth. Therefore the force and any displacement are perpendicular. Hence no work is done on the Earth by the Sun.

3 **a** $F_{horizontal} = F\cos\theta$

$= 30.0\cos 60.0°$

$= 15.0\,\text{N}$

$W = Fs$

$= 15.0 \times 2.40$

$= 36.0\,\text{J}$

b $W = F_f s$

$= 10.0 \times 2.40$

$= 24.0\,\text{J}$

c $W = Fs$

$= (15.0 - 10.0) \times 2.40$

$= 12.0\,\text{J}$

4 The work represented by one square is $10 \times 0.001 = 0.01\,\text{J}$.

There are 27 squares (approx.) under the curve up to 7 mm compression.

$W = 27 \times 0.01 = 0.27\,\text{J}$

5 Note that the work is being done against gravity, hence:

$W = Fs = mg\Delta h$

$= 155 \times 9.8 \times 1.20$

$= 1822.8$

$= 1.8 \times 10^3\,\text{J}$

6 The net force on the mower must be zero, since it is travelling at constant speed.

The force required to oppose friction must be 68.0 cos 60.0° = 34.0 N.

$W = Fs$

$= 34.0 \times 15.0$

$= 510\,\text{J}$

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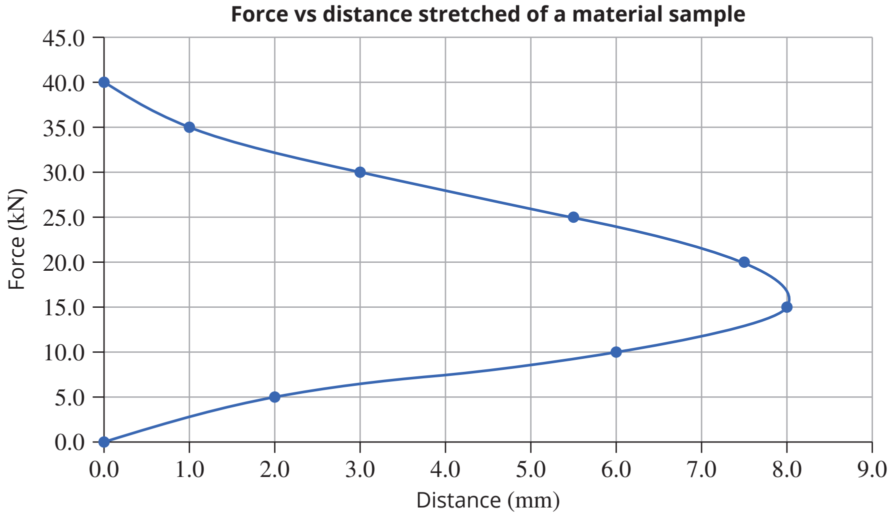
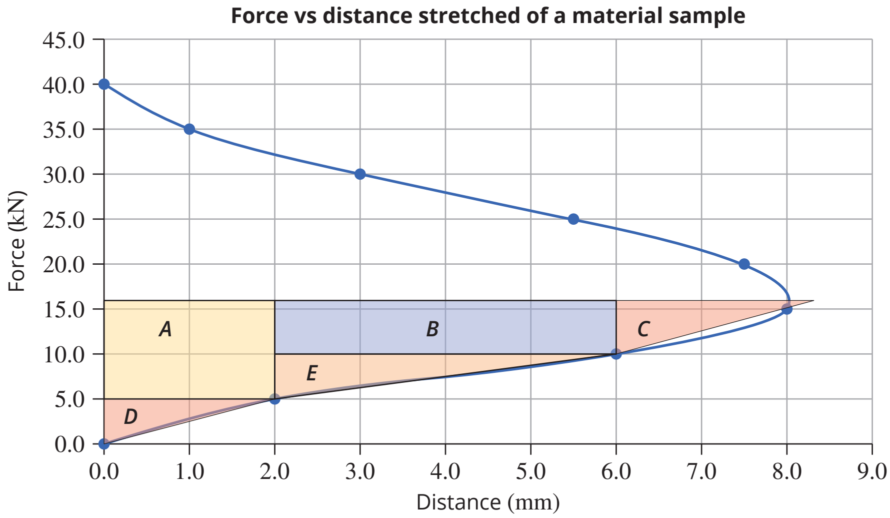
Analysis

7 **a**



b



W = area under graph

$= A + B + C + D + E$

$= (2.0\times10^{-3})(11000) + (4.0\times10^{-3})(6000) + \frac{1}{2}(2.3\times10^{-3})(6000) + \frac{1}{2}(2.0\times10^{-3})(5000) + \frac{1}{2}(4.0\times10^{-3})(5000)$

$= 22.0 + 24.0 + 6.90 + 5.0 + 10.0$

$= 68\text{ J}$

8 Vertically, taking up as positive:

$u = \frac{108}{3.6} = 30.0\text{ m s}^{-1}$

$u_v = 30.0\sin 45.0°$

$= 21.2\text{ m s}^{-1}$

$v = 0$ (at the top), $a = -9.8$, $s = ?$

$v^2 = u^2 + 2as$

$0 = 21.2^2 - 19.60s$

$s = 22.9\text{ m up}$

s_{total} = 22.9 m up + 22.9 m down + 1.9 m down (to the ground) = 47.7 m

$W = F_g \times s_{total}$

$= 0.806 \times 9.8 \times 47.7$

$= 376.77$

$= 3.8 \times 10^2\text{ J}$

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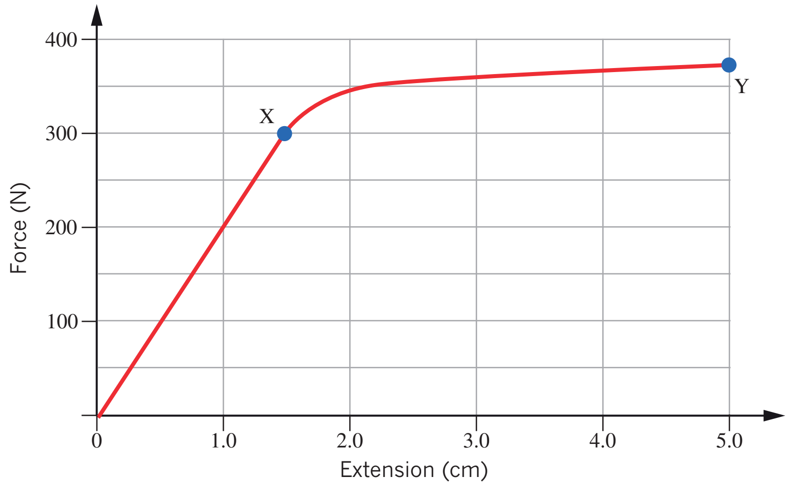
3.4 Elastic potential energy

Worked example: Try yourself 3.4.1

CALCULATING THE SPRING CONSTANT, ELASTIC POTENTIAL ENERGY AND WORK

An alloy sample is tested under tension, giving the force vs extension graph shown below. X indicates the elastic limit and Y indicates the breaking point.



a Calculate the spring constant, k, for the sample.	
Thinking	**Working**
The spring constant is the gradient of the first linear section of the graph (in units N m^{-1}).	$k = \frac{\Delta F}{\Delta x}$ $= \frac{300}{0.015}$ $= 20000$ $= 2.0 \times 10^4\ \text{N m}^{-1}$

b Calculate the elastic potential energy that the alloy can store before permanent deformation begins.	
Thinking	**Working**
The elastic potential energy is the area under the curve up to the elastic limit.	$E_s = \frac{1}{2} \times \text{height} \times \text{base of triangle}$ $= \frac{1}{2} \times 300 \times 0.015$ $= 2.3\ \text{J}$
This value can also be obtained using the formula for elastic potential energy.	$E_s = \frac{1}{2}kx^2$ $= \frac{1}{2} \times 2.0 \times 10^4 \times (0.015)^2$ $= 2.3\ \text{J}$

c Calculate the work done to break the sample.	
Thinking	**Working**
Add up the number of squares under the curve up to the breaking point.	number of squares = 29 (approx.)
Calculate the energy per square. This is given by the area of a single square. Remember to convert cm to m.	energy for one square $= 50 \times 0.01$ $= 0.5\ \text{J}$
Multiply the energy per square by the number of squares.	work = energy per square × number of squares $= 0.5 \times 29$ $= 14.5\ \text{J}$ (approx.)

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KEY QUESTIONS

Knowledge and understanding

1 From least to most stiff: C, B, A. Stiffness is indicated by the gradient of a force vs distance graph, where the distance is either compression or extension.

2 a A high spring constant value. This would reduce the oscillation of the panel as it is fixed in place.
b A medium spring constant value. Allowing for a little give as the rope tightens reduces the chances that the rope will break.
c A low spring constant value. This enables the net to be stretched over a variety of items to keep them secure.

3 a spring constant: $k = \frac{\Delta F}{\Delta x}$

stiff spring constant: $\frac{20}{0.1} = 200\,\text{N m}^{-1}$

weak spring constant: $\frac{10}{0.2} = 50\,\text{N m}^{-1}$

b stiff spring: $E_s = \frac{1}{2} \times 200 \times 0.20^2 = 4.0\,\text{J}$

weak spring: $E_s = \frac{1}{2} \times 50 \times 0.20^2 = 1.0\,\text{J}$

energy difference = 4.0 − 1.0 = 3.0 J

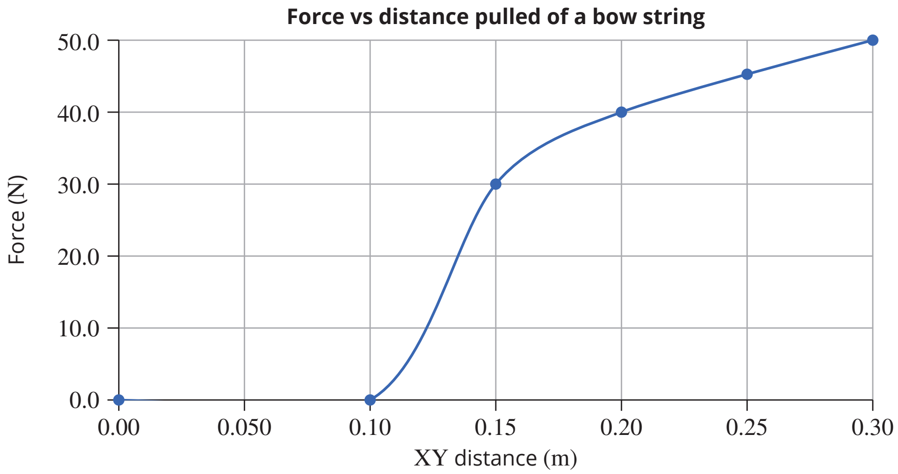
4 $x = \frac{F}{k} = \frac{4.00}{50.0} = 0.0800\,\text{m}$ or 8.00 cm

5 a $F = kx$
$= 128 \times 0.250$
$= 32.0\,\text{N}$

b $E_s = \frac{1}{2}kx^2$
$= \frac{1}{2} \times 128 \times 0.250^2$
$= 4.00\,\text{J}$

Analysis

6 a



b Each square = 0.05 × 10 = 0.5 J. Therefore elastic potential energy = 14 squares × 0.5 = 7 J.
c The work done by the archer is what becomes the elastic potential energy. Therefore the work done is 7 J.
d No. Hooke's law is not obeyed, as the force vs distance graph is not a straight line between 10.0 cm and 30.0 cm.
e The elastic limit is at the point where the distance is 0.15 m and the force is 30 N.

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3.5 Kinetic and potential energy

Worked example: Try yourself 3.5.1

ELASTIC OR INELASTIC COLLISION?

A 209 g softball with initial velocity 9.00 m s^{-1} to the right collides with a stationary baseball of mass 112 g. After the collision, both balls move to the right and the softball has a speed of 3.00 m s^{-1}. Using appropriate calculations, show whether the collision is elastic or inelastic.

Thinking	Working
Use conservation of momentum to find the final velocity of the 112 g baseball.	Taking to the right as positive and labelling the baseball 'ball 2': $\Sigma p_{\text{initial}} = \Sigma p_{\text{final}}$ $p_{\text{initial ball 1}} + p_{\text{initial ball 2}} = p_{\text{final ball 1}} + p_{\text{final ball 2}}$ $(0.209 \times 9.00) + 0 = (0.209 \times 3.00) + (0.112 \times v_{\text{ball 2}})$ $v_{\text{v ball 2}} = 11.2\,\text{m s}^{-1}$
Calculate the total initial kinetic energy before the collision.	Before: $E_k = \frac{1}{2}mu^2$ $= \frac{1}{2} \times 0.209 \times 9.00^2 + 0$ $= 8.46\,\text{J}$
Calculate the total final kinetic energy of the joined balls.	After: $E_k = \frac{1}{2}mv^2_{\text{ball 1}} + \frac{1}{2}mv^2_{\text{ball 2}}$ $= \frac{1}{2} \times 0.209 \times 3.00^2 + \frac{1}{2} \times 0.112 \times 11.2^2$ $= 0.90 + 7.2$ $= 7.96\,\text{J}$
Compare the kinetic energy before and after the collision to determine whether the collision is elastic or inelastic.	The kinetic energy after the collision is less than the kinetic energy before the collision. Therefore the collision is inelastic.

Worked example: Try yourself 3.5.2

CHANGES IN GRAVITATIONAL POTENTIAL ENERGY USING A FORCE VS DISTANCE GRAPH

Using the graph in Figure 3.5.2, calculate the gravitational potential energy gained if the 10 kg object is moved from the surface of the Earth to 2.0 × 10^7 m above the centre of the Earth.

Thinking	Working
Find the energy represented per square in the graph.	One square represents 10.0 × 0.25 × 10^7 = 2.5 × 10^7 J.
Identify the two values of distance that are relevant to the question.	The relevant distances are the radius of the Earth, 6.4 × 10^6 m, and the distance of the object, 2.0 × 10^7 m from the centre of the Earth.
Count the squares under the curve between the two distances and multiply the total by the energy per square.	18 squares (approx.) × 2.5 × 10^7 = 4.5 × 10^8 J
Potential energy gained = work done	4.5 × 10^8 J

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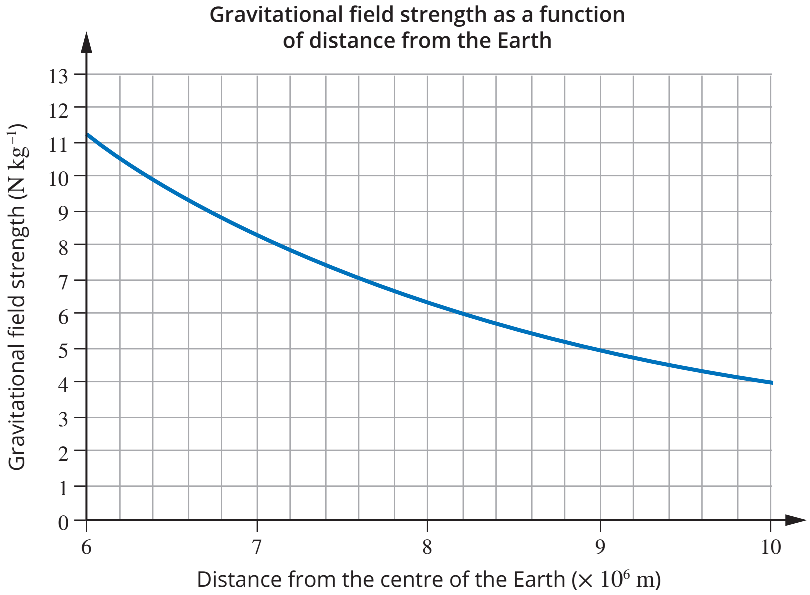
Worked example: Try yourself 3.5.3

CHANGES IN GRAVITATIONAL POTENTIAL ENERGY USING A GRAVITATIONAL FIELD STRENGTH GRAPH

A satellite of mass 1100 kg is in an elliptical orbit around the Earth. At its closest approach (perigee) it is just 600 km above the Earth's surface. Its furthest point (apogee) is 2600 km from the Earth's surface. The Earth has a radius of 6.4×10^6 m. The gravitational field strength of the Earth is shown in the graph.



a Calculate the change in potential energy of the satellite as it moves from its perigee to its apogee.	
Thinking	**Working**
Convert the distances given as altitudes to distances from the centre of the Earth.	perigee = $6.4 \times 10^6 + 600 \times 10^3 = 7.0 \times 10^6$ m apogee = $6.4 \times 10^6 + 2600 \times 10^3 = 9.0 \times 10^6$ m
Find the energy represented by each square in the graph.	One square represents: $1.0 \times 0.20 \times 10^6 = 2.0 \times 10^5$ J kg^{-1}
Count the squares under the curve for the relevant area and multiply the total by the energy per kg represented by each square.	64 squares (approx.) $\times 2.0 \times 10^5 = 1.3 \times 10^7$ J kg^{-1}
Calculate the potential energy gained by the satellite by multiplying the work done by the mass of the satellite.	Enerygy gained: $E_g = 1.3 \times 10^7 \times 1100$ $= 1.4 \times 10^{10}$ J (approx.)

b The satellite is moving with a speed of 8.0 km s^{-1} at its perigee. How fast will it be travelling at its apogee?	
Thinking	**Working**
First calculate the satellite's kinetic energy at its perigee.	$E_{k\,p} = \frac{1}{2} m v_p$ $= \frac{1}{2} \times 1100 \times (8.0 \times 10^3)^2$ $= 3.5 \times 10^{10}$ J
The gain in gravitational potential energy at the apogee is at the expense of kinetic energy. Calculate the kinetic energy of the satellite at its apogee.	$E_{k\,a} = E_{k\,p} - E_g$ $= 3.5 \times 10^{10} - 1.4 \times 10^{10}$ $= 2.1 \times 10^{10}$ J

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Calculate the speed of the satellite at its apogee.	$E_{k\,a} = \frac{1}{2}mv_a^2$ $v_a = \sqrt{\frac{2E_{k\,a}}{m}}$ $= \sqrt{\frac{2 \times 2.1 \times 10^{10}}{1100}}$ $= 6.2\,km\,s^{-1}$

KEY QUESTIONS

Knowledge and understanding

1 A and E. As potential energy decreases, kinetic energy increases. However, there are energy losses because the meteor is burning up.

2 A, C and D. The cars travel at constant speed and so have constant kinetic energy. As the descending car loses gravitational energy, the ascending car gains gravitational energy, and the motor applies a force over a distance to drag the cable, thus doing work.

3
$$E_g = mg\Delta h$$
$$= 115 \times 9.8 \times 2228$$
$$= 2.5 \times 10^6\ J$$

4
$$E_k = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 0.283 \times (9.50)^2$$
$$= 12.8\ J$$

5
$$E_g = mg\Delta h$$
$$= 3.00 \times 9.8 \times 45.0$$
$$= 1.3 \times 10^3\ J$$

Analysis

6 B and D.

A is not correct. It would only be correct if the Earth's gravitational force did not vary with distance from the centre of the Earth. You can only use the equation $E_g = mg\Delta h$ in regions where the strength of the gravitational field is approximately constant.

B is correct because the velocity is proportional to $\frac{1}{\sqrt{r}}$, so a higher r means a lower speed.

C is not correct as kinetic energy is inversely proportional to the radius.

D is correct. The higher the altitude, the more work has to be done against the gravitational force, and hence the more gravitational potential energy relative to the surface of the Earth.

7 **a**
$$E_k = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 500 \times 250^2$$
$$= 1.6 \times 10^7\ J$$

b The shaded area = 53 squares (approx.).
The energy represented by one square = $100 \times 0.1 \times 10^6 = 1.0 \times 10^7$ J
The loss in potential energy = the gain in kinetic energy = 5.3×10^8 J

c The total kinetic energy on landing = $1.6 \times 10^7 + 5.3 \times 10^8 = 5.5 \times 10^8$ J

$$5.5 \times 10^8 = \frac{1}{2} \times 500 \times v^2$$
$$v = \sqrt{\frac{2 \times 5.5 \times 10^8}{500}}$$
$$= 1.5 \times 10^3\ m\,s^{-1}$$

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8 First convert the altitudes to distances from the centre of the Earth:
$6.4 \times 10^6 + 6.0 \times 10^5 = 7.0 \times 10^6$ m and $6.4 \times 10^6 + 2.6 \times 10^6 = 9.0 \times 10^6$ m
There are approximately 25 squares under the curve between these two distances.
The energy per kg for one square is $1.0 \times 0.5 \times 10^6 = 5 \times 10^5$ J kg^{-1}.
Calculate the gain in potential energy using $\Delta E_g = mg\Delta h$:

$$\begin{aligned}\Delta E_g &= mg\Delta h\\ &= 25 \times 5.0 \times 10^5 \times 20 \times 10^3\\ &= 2.5 \times 10^{11}\ \text{J}\end{aligned}$$

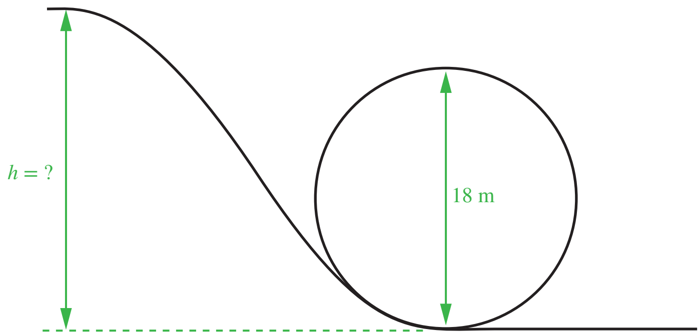
Therefore 2.5×10^{11} J of work is done to increase the orbital radius of the space junk.

3.6 Conservation of energy

Worked example: Try yourself 3.6.1

APPLYING THE LAW OF CONSERVATION OF ENERGY

Use the law of conservation of energy to determine the height of the lift hill required to ensure that the speed of a rollercoaster car at the top of the 18 m loop is 25 m s^{-1}. Assume that the velocity of the car at the top of the hill is zero just before it begins to roll down the hill, friction is negligible and that $g = 9.8$ m s^{-2}.



Thinking	Working
Equate the total mechanical energy, E_m, of the car before rolling down the hill to the total mechanical energy at the top of the loop.	$E_{m\ before} = E_{m\ at\ top\ of\ loop}$
Expand the equation and then cancel m from both sides.	$\frac{1}{2}mu^2 + mg\Delta h = \frac{1}{2}mv^2 + mg\Delta h$ $\frac{1}{2}u^2 + g\Delta h = \frac{1}{2}v^2 + g\Delta h$
Substitute the given values into the equation.	$\frac{1}{2}(0)^2 + (9.8)\Delta h = \frac{1}{2}(25)^2 + (9.8)(18)$
Rearrange the equation and solve for Δh.	$(9.8)\Delta h = \frac{1}{2}(25)^2 + (9.8)(18)$ $\Delta h = \frac{312.5 + 176.4}{9.8}$ $\Delta h = \frac{488.9}{9.8}$
Present your answer with the correct number of significant figures and the correct unit.	$\Delta h = 50$ m

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Worked example: Try yourself 3.6.2

USING THE CONSERVATION OF ENERGY TO ANALYSE PROJECTILE MOTION

An arrow of mass 35 g is fired into the air at 80 m s^{-1} from a height of 1.4 m above the ground. Calculate the speed of the arrow when it is 30 m above the ground. Assume that $g = 9.8$ m s^{-2}.

Thinking	Working
Equate the total mechanical energy, E_m, of the arrow as it is released to the total mechanical energy at a height of 30.0 m.	$E_{\text{m before}} = E_{\text{m at 30m}}$
Expand the equation and then cancel m from both sides.	$\frac{1}{2}mu^2 + mg\Delta h = \frac{1}{2}mv^2 + mg\Delta h$ $\frac{1}{2}u^2 + g\Delta h = \frac{1}{2}v^2 + g\Delta h$
Substitute the given values into the equation.	$\frac{1}{2}(80)^2 + (9.8)(1.4) = \frac{1}{2}v^2 + (9.8)(30)$
Rearrange the equation and solve for v.	$(3200) + (11.2) = \frac{1}{2}v^2 + (294.0)$ $v = \sqrt{2(3200 + 11.2 - 294.0)}$ $= \sqrt{5834.4}$
Present your answer with the correct number of significant figures and the correct unit.	$v = 76\text{ m s}^{-1}$

KEY QUESTIONS

Knowledge and understanding

1 D is correct as, according to the law of conservation of energy, energy can neither be created nor destroyed. Thus no energy can be gained or lost from the system.

2 **a** Both sticks would land at the same time. Mass is not a factor in the rate at which objects fall.

b The brown stick. The heaviest stick would have the greatest gravitational potential energy according to the equation $E_g = mg\Delta h$.

c Both sticks would land with the same speed if dropped from the same height.

d The brown stick. The heaviest stick would have the greatest kinetic energy according to the equation $E_k = \frac{1}{2}mv^2$.

3
$$E_g = E_k$$
$$mg\Delta h = \frac{1}{2}mv^2$$
$$\Delta h = \frac{1}{2}\frac{v^2}{g}$$
$$= \frac{1}{2}\frac{(45.5)^2}{9.8}$$
$$= 1.1 \times 10^2\text{ m}$$

4
$$E_g = E_k$$
$$mg\Delta h = \frac{1}{2}mv^2$$
$$v = \sqrt{2g\Delta h}$$
$$= \sqrt{2(9.8)(10.0)}$$
$$= 14\text{ m s}^{-1}$$

5
$$E_g = E_k$$
$$mg\Delta h = \frac{1}{2}mv^2$$
$$v = \sqrt{2g\Delta h}$$
$$= \sqrt{2(9.8)(2.10)}$$
$$= 6.4\text{ m s}^{-1}$$

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6 **a** $E_k = \frac{1}{2}mv^2$

$= \frac{1}{2}(0.1984)(21.7)^2$

$E_k = 46.7\text{ J}$

b $E_g = E_k = 46.7\text{ J}$

c $E_g = mg\Delta h$

$h = \frac{(46.7)}{(0.1984)(9.8)}$

$h = 24\text{ m}$

Analysis

7 First, calculate the vertical height:

$\Delta h = 12.0\sin 35.0°$

$= 6.88292\text{ m}$

$E_g = E_k$

$mg\Delta h = \frac{1}{2}mv^2$

$v = \sqrt{2g\Delta h}$

$= \sqrt{2(9.8)(6.88292)}$

$= 12\text{ m s}^{-1}$

8 **a** The chain breaks when the ball is 85.0 cm above the ground and 17.0 cm below its starting position.

$E_g = E_k$

$mg\Delta h = \frac{1}{2}mv^2$

$v = \sqrt{2g\Delta h}$

$= \sqrt{2(9.8)(0.170)}$

$= 1.8\text{ m s}^{-1}$

b $E_g = E_k$

$mg\Delta h = \frac{1}{2}mv^2$

$\Delta h = \frac{1}{2}\frac{v^2}{g}$

$= \frac{1}{2}\frac{(1.82538)^2}{(9.8)}$

$= 0.170\text{ m}$

$h = (0.850) + (0.170)$

$= 1.02$

$= 1.0\text{ m}$

c $E_g = E_k$

$mg\Delta h = \frac{1}{2}mv^2$

$v = \sqrt{2(9.8)(1.02)}$

$= 4.47124$

$= 4.5\text{ m s}^{-1}$

d $E_g = E_k$

$mg\Delta h = \frac{1}{2}mv^2$

$v = \sqrt{2(9.8)(0.850 + 0.170)}$

$= 4.47124$

$= 4.5\text{ m s}^{-1}$

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9 **a** $E_k = \frac{1}{2}mv^2$

$E_k = \frac{1}{2}(75.0)(6.27)^2$

$= 1.47423 \times 10^3$

$= 1.47 \times 10^3$ J

b $E_g = E_k$

$= 1.47 \times 10^3$ J

c $E_g = mg\Delta h$

$\Delta h = \frac{E_g}{mg}$

$= \frac{(1.47423 \times 10^3)}{(75.0)(9.8)}$

$= 2.00576$

$= 2.0$ m

Chapter 3 Review

Knowledge and understanding

1 B, C, A, D is the correct order, from highest to lowest momentum.

2 C is correct. The unit kg m s^{-1} converts to $(\text{kg m s}^{-2}) \times (\text{s}) = \text{N s}$.

3 The other separated mass has a momentum of 345 kg m s^{-1} north.

4 B and D. A is incorrect, as force is the rate of change of momentum. C is incorrect, as impulse is a vector.

5 Impulse is the change in momentum of an object. In a car crash, the impulse of a driver impacting the dashboard will be constant regardless of the time over which the impulse occurs. Airbags are designed to increase the duration of the collision. Increasing the duration of the collision decreases the force, which is likely to reduce the severity of injury.

6 The person exerts a force on the wall but the wall undergoes no displacement ($s = 0$), so no work is done.

7 When an object has access to energy it has the *capacity* to do work, whereas work occurs when there is an energy transfer or energy is transformed. (It can be transferred from one object to another or transformed from one form to another.)

8 The gradient of an F vs x graph is $\frac{\Delta F}{\Delta x}$. The units of the gradient are $\frac{\text{N}}{\text{m}} = \text{N m}^{-1}$ which is the unit for k, the spring constant.

The area under an F vs x graph is found by $\Delta F \times \Delta x$. The units of the area are $\text{N m} = \text{kg m s}^{-2}$, which is equivalent to the unit for joule. Therefore the area represents the elastic potential energy.

9 D. The kinetic energy before each collision is more than after the collision, with some of the energy being transformed into heat. This would not be the case for a perfectly elastic collision. While it is true that the racquet gives the ball kinetic energy, and the impulse is positive, these do not explain the heat.

10 No work is done on the backpack as it did not rise up in the gravitational field to increase its gravitational potential energy along the horizontal path. Further, it did not increase its kinetic energy along the way as the speed was constant. Therefore, as no energy was transferred to the backpack, no work was done on it.

11 45.5 J. The apple will have the same gravitational potential energy at the beginning of its fall as it has kinetic energy at the end of its fall.

12 As the tennis ball goes higher, its kinetic energy is doing work on the gravitational field. At the top of its flight, when the speed is zero, the kinetic energy is also zero. At this point, all the ball's kinetic energy is now stored in the gravitational field and becomes available to do work on the tennis ball on its way down to the ground.

13 $\Sigma p_{\text{final}} = \Sigma p_{\text{initial}}$

$m_r v_r + m_b v_b = (m_r + m_b)u$

$(70.0)(2.50) + (495)v_b = (70.0 + 495)(0)$

$175 + 495v_b = 0$

$v_b = \frac{(-175)}{(495)}$

$= -0.353535$

$= 0.354\ \text{m s}^{-1}$ in the opposite direction to the rower

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14

$$\Sigma p_{\text{final}} = \Sigma p_{\text{initial}}$$

$$m_s v_s + m_g v_g = (m_s + m_g)u$$

$$(1.00\times10^4)v_s + (5.00)(6.00\times10^3) = (1.00\times10^4 + 5.00)(0)$$

$$(1.00\times10^4)v_s + (3.00\times10^4) = 0$$

$$v_s = \frac{(-3.00\times10^4)}{(1.00\times10^4)}$$

$$= -3.00$$

$= 3.00\,\text{m}\,\text{s}^{-1}$ in the opposite direction to the gas

15 Take the direction away from the batter as positive:

$$v_i = \frac{-104}{3.6} = -28.8889\,\text{m}\,\text{s}^{-1}$$

$$v_f = \frac{20}{3.6} = 5.55556\,\text{m}\,\text{s}^{-1}$$

$$\Delta p = m\Delta v$$

$$= m(v-u)$$

$$= 0.165\times(5.55556-(-28.8889))$$

$$= 5.68333$$

$$= 5.68\,\text{kg}\,\text{m}\,\text{s}^{-1}$$

16 $\Delta p = m(v-u)$

$$= (65.0+15.0)\times(0-(-12.0))$$

$$= 960\,\text{kg}\,\text{m}\,\text{s}^{-1}$$

$$F = \frac{\Delta p}{\Delta t}$$

$$= \frac{960}{2.00}$$

$$= 480\,\text{N}$$

17 a $F_h = F\cos 60.0°$

$$= 316\cos 60.0°$$

$$= 158\,\text{N}$$

$$W = F_h s$$

$$= 158\times245$$

$$= 3.87100\times10^4$$

$$= 3.87\times10^4\,\text{J}$$

b $F_{net} = F + F_f$

$$= 158-105$$

$$= 53.0\,\text{N}$$

$$E_k = W$$

$$= 53.0\times245$$

$$= 1.29850\times10^4\,\text{J}$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{2\times1.29850\times10^4}{152}}$$

$$= 13.0712$$

$$= 13.1\,\text{m}\,\text{s}^{-1}$$

18 Area = 2 squares × 0.5 × 20.0 = 20 J
Work = 20 × 150 repetitions = 3 × 10^3 J

19 Energy per square = 2.0 × 1.0 × 10^{-3} = 2.0 × 10^{-3} J
Area = 16 squares
Elastic potential energy = 16 × 2.0 × 10^{-3} = 0.032 J

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20 $v = \frac{80.0}{3.6} = 22.2222\,\text{m s}^{-1}$

$$E_k = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(232)(22.2222)^2$$
$$= 5.72840 \times 10^4$$
$$= 5.73 \times 10^4\ \text{J}$$

21 $W = E_{k\ final} - E_{k\ initial}$

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$
$$= \frac{1}{2}(1540)(28.0)^2 - \frac{1}{2}(1540)(17.0)^2$$
$$= (6.03680 \times 10^5) - (2.22530 \times 10^5)$$
$$= 3.81150 \times 10^5$$
$$= 3.81 \times 10^5\ \text{J}$$

22 **a** $E_g = mg\Delta h$

$$= 0.0570 \times 9.8 \times 8.20$$
$$= 4.58052$$
$$= 4.6\ \text{J}$$

b $E_g = mg\Delta h$

$$= 0.0570 \times 9.8 \times 4.10$$
$$= 2.29026$$
$$= 2.3\ \text{J}$$

Alternatively, as $E_g \propto \Delta h$, if h is halved, then E_g is also halved: $\frac{4.6}{2} = 2.3\,\text{J}$.

23 $E_g = mg\Delta h$

$$= 65.0 \times 9.8 \times (8848 - 5150)$$
$$= 2.35563 \times 10^6$$
$$= 2.4 \times 10^6\ \text{J}$$

Application and analysis

24 **a** Yes, momentum is conserved in all collisions.

b Inelastic. 20 J of kinetic energy has been transformed into heat and sound energy.

c Total initial $E_k = \frac{1}{2} \times 4.00 \times 3.00^2 + \frac{1}{2} \times 4.00 \times 3.00^2 = 36.0\,\text{J}$

20.0 J is transformed into heat and sound, so the final E_k is 36.0 – 20.0 = 16.0 J.

From symmetry, the balls will have the same final speeds and the same kinetic energy: 8.00 J. For each ball:

$$E_k = \frac{1}{2}mv^2$$
$$8.00 = 0.5 \times 4.00 \times v^2$$
$$v^2 = 4.00$$
$$v = 2.00\,\text{m s}^{-1}$$

The balls will each travel with a speed of $2.00\,\text{m s}^{-1}$ in opposite directions to their initial vectors.

25 Note: Δx = 10.0% of 134 m = 134 × 0.100

$$mg\Delta h = \frac{1}{2}kx^2$$
$$k = \frac{2mg\Delta h}{x^2}$$
$$= \frac{2(80.0)(9.8)(134)}{(134 \times 0.100)^2}$$
$$= 1.17015 \times 10^3$$
$$= 1.2 \times 10^3\,\text{N m}^{-1}$$

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26 $4 \times h$. Equating the gravitational potential energy with the kinetic energy, $mg\Delta h = \frac{1}{2}mv^2$, we can see that h is proportional to v^2. Thus $(2v)^2$ gives $4v^2$. Therefore the new height is $4 \times h$.

27 A. Both stones have the same gravitational potential energy, as both were thrown from the same height. They also have the same kinetic energy, as each has the same initial speed. They will thus have the same kinetic energy on landing and therefore land at the same speed.

28 **a** Determine the radii of the two orbits from the centre of the Earth:

$R_1 = 6.37 \times 10^6 + 1.13 \times 10^6 = 7.50 \times 10^6$ m

$R_2 = 6.37 \times 10^6 + 2.13 \times 10^6 = 8.50 \times 10^6$ m

There are 6.5 squares under the curve.

The energy per kg for one square is $2 \times (0.5 \times 10^6) = 1 \times 10^6$ J kg^{-1}.

The gain in potential energy = $6.5 \times (1 \times 10^6) \times (11.0 \times 10^3) = 7.2 \times 10^{10}$ J.

b
$$v = \sqrt{\frac{GM}{r}}$$
$$= \sqrt{\frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{8.50 \times 10^6}}$$
$$= 6.850 \times 10^3 \text{ m s}^{-1}$$
$$E_k = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 11.0 \times 10^3 \times (6.850 \times 10^3)^2$$
$$= 2.5807 \times 10^{11}$$
$$= 2.58 \times 10^{11} \text{ J}$$

29 An altitude of 631 km is $6.37 \times 10^6 + 631\,000 = 7.001\,00 \times 10^6$ m. There are approximately 28 squares full or more than half full under the curve between 6.37×10^6 and 7.00×10^6.

The energy per kg for one square is $0.2 \times 10^6 \times 1 = 2 \times 10^5$ J kg^{-1}.

The gain in potential energy is:
$$= 28 \times (2 \times 10^5) \times (1.10 \times 10^7)$$
$$= 6.16001 \times 10^{13}$$
$$= 6.2 \times 10^{13} \text{ J}$$

30 **a**
$$\Sigma p_i = \Sigma p_f$$
$$p_{\text{i truck}} + p_{\text{i car}} = p_{\text{f truck}} + p_{\text{f car}}$$
$$(0.264 \times 0.300) + (0.112 \times 0.200) = 0.264v_f + (0.112 \times 0.300)$$
$$0.10160 = 0.264v_f + 0.03360$$
$$0.264v_f = 0.06800$$
$$v_f = \frac{0.06800}{0.264}$$
$$= 0.257576$$
$$= 0.258 \text{ m s}^{-1}$$

b
$$E_{k\,i} = \frac{1}{2}m_t u_t^2 + \frac{1}{2}m_c u_c^2$$
$$= \frac{1}{2} \times 0.264 \times 0.300^2 + \frac{1}{2} \times 0.112 \times 0.200^2$$
$$= 0.01188 + 0.002240$$
$$= 0.01412$$
$$= 0.0141 \text{ J}$$

c
$$E_{k\,f} = \frac{1}{2}m_t v_t^2 + \frac{1}{2}m_c v_c^2$$
$$= \frac{1}{2} \times 0.264 \times 0.257576^2 + \frac{1}{2} \times 0.112 \times 0.300^2$$
$$= 0.0087576 + 0.0050400$$
$$= 0.0137976$$
$$= 0.0138 \text{ J}$$

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d **i** The total kinetic energy before the collision is more than the total kinetic energy after the collision.

ii The kinetic energy of the system of toys is not conserved.

iii The total energy of the system of toys is conserved.

iv The total momentum of the system of toys is conserved.

v The collision is not perfectly elastic because kinetic energy is not conserved.

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Unit 3 Area of Study 1

How do physicists explain motion in two dimensions?

Multiple-choice questions

1 C. Action–reaction pairs always act on different objects. One force acts on the floor and the other force acts on the ball. These forces are equal in magnitude and opposite in direction, as described by Newton's third law.

2 C. The only force acting is the gravitational force.

3 C. $18\,\text{m}\,\text{s}^{-2}$ up

$$a = \frac{v^2}{r} = \frac{6.0^2}{2.0} = 18\,\text{m}\,\text{s}^{-2} \text{ up (that is, towards the centre of the circle)}$$

4 D. $1.5 \times 10^3\,\text{N}$

$$F_{net} = F_g + F_N$$
$$F_N = F_{net} - F_g$$
$$= 55 \times 18 - (55 \times -9.8)$$
$$= 1.5 \times 10^3\,\text{N}$$

5 B. The skater feels heavier than they do when stationary. This is because the normal force is 1.5×10^3 N, which is almost three times greater than the force due to gravity. Thus the skater would feel almost three times heavier than usual.

6 A. Since the collision is inelastic, kinetic energy is not conserved. Thus $E_{k\ after} < E_{k\ before}$. This is shown in graphs A and C. During the collision, some of the kinetic energy is converted into spring potential energy, and some of this is then restored to kinetic energy, that is, E_k dips slightly over the time of the collision. This is shown only in graph A.

7 D. Momentum is conserved (i.e. is constant) throughout the interaction, as represented by the flat line in graph D.

8 D. For momentum to be conserved, what is lost by the tennis ball is gained by the bowling ball. The tennis ball's change in momentum will be back towards the thrower. The bowling ball's change in momentum will be away from the thrower. So the changes in momentum are in opposite directions.

9 D. The forces exerted by each ball on the other make an action–reaction pair and must be equal and opposite according to Newton's third law.

10 **a** C. Position C is when the spring has the greatest extension (x = maximum), which means that the greatest elastic potential energy is at this position (since $E_s = \frac{1}{2}kx^2$).

b A. Position A is when the mass has the greatest height (Δh = maximum), which means that the greatest gravitational potential energy is at this position (since $E_g = mg\Delta h$).

c B and D. The mass is momentarily at rest at positions A and C ($v = 0\,\text{m}\,\text{s}^{-1}$), which means that there is no kinetic energy at these positions. Positions B and D are at the midpoint where the mass will have the highest velocity (v = maximum) and hence the greatest kinetic energy (since $E_k = \frac{1}{2}mv^2$).

11 D. 39 cm

From the conservation of energy:

$$E_A = E_C$$
$$mg\Delta h = \frac{1}{2}kx^2$$

Since the distance the mass falls and the distance the spring extends are the same, $\Delta h = x$.

$$mgx = \frac{1}{2}kx^2$$
$$mg = \frac{1}{2}kx$$
$$x = \frac{2mg}{k} = \frac{2 \times 2.0 \times 9.8}{100}$$
$$= 39\,\text{cm}$$

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12 D. 20 cm

When the spring stops oscillating, the net force is 0 N, so:

$mg = kx$

$x = \frac{mg}{k} = \frac{2.0 \times 9.8}{100}$

$= 20\,\text{cm}$

Short-answer questions

13 a unbalanced, balanced

b $v = \frac{2\pi r}{T} = \frac{2\pi \times 5.00}{2.50} = 12.6\,\text{m s}^{-1}$

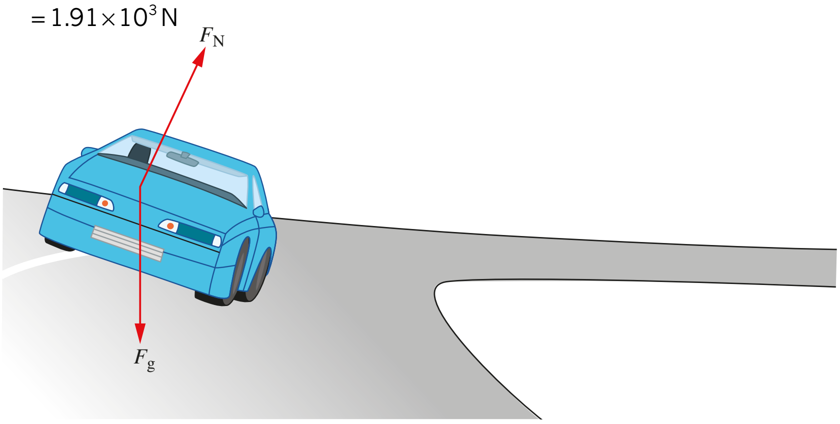
c $a = \frac{v^2}{r} = \frac{12.6^2}{5.00} = 31.8\,\text{m s}^{-2}$

d $F_N = F_{net} = ma$

$= 60.0 \times 31.8$

$= 1.91 \times 10^3\,\text{N}$

14 a



b $\tan\theta = \frac{v^2}{rg}$

$\theta = \tan^{-1}\frac{v^2}{rg}$

$= \tan^{-1}\frac{40^2}{150 \times 9.8}$

$= 47°$

15 a Take up as positive. Then vertically: $v = 0$ (at the top), $a = -9.8$, $t = 1.0$, $s = ?$

$s = vt - \frac{1}{2}at^2$

$= 0 - 0.5 \times 9.8 \times 1.0^2$

$= 4.9\,\text{m}$

b $9.8\,\text{m s}^{-2}$ down

c horizontally: $u = ?$, $t = 2.0$, $s = 8.0$

$v_{av} = \frac{s}{t}$

$= \frac{8.0}{2.0}$

$= 4.0\,\text{m s}^{-1}$

Take up as positive. Therefore vertically: $v = 0$ (at the top), $a = -9.8$, $t = 1.0$, $u = ?$

$v = u + at$

$0 = u - 9.8 \times 1.0$

$u = 9.8\,\text{m s}^{-1}$

Use Pythagoras's theorem to find the actual speed at launch:

$u = \sqrt{4.0^2 + 9.8^2}$

$= 11\,\text{m s}^{-1}$ to 2 significant figures

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16 **a** The ball bearing just maintains contact with the track, so $F_N = 0$, $F_{net} = F_g$ and $a = 9.8\,\text{m s}^{-2}$ down.

b At point C, $F_N = 0$, so $F_{net} = F_g$:

$$ma = mg$$
$$a = g$$
$$\frac{v^2}{r} = g$$
$$v = \sqrt{rg} = \sqrt{0.50 \times 9.8}$$
$$= 2.2\,\text{m s}^{-1}$$

c Total energy at point C = $E_k + E_g$

$$E_k + E_g = \frac{1}{2}mv^2 + mg\Delta h$$
$$= \frac{1}{2} \times 0.025 \times 2.2^2 + 0.025 \times 9.8 \times 1.0$$
$$= 0.3055\,\text{J}$$

The total energy at point C is equal to the total energy at point B.

Total energy at point B = E_k:

$$E_k = \frac{1}{2}mv^2 = 0.3055\,\text{J}$$
$$0.3055 = \frac{1}{2}0.025v^2$$
$$v = \sqrt{\frac{2 \times 0.3055}{0.025}}$$
$$= 4.9\,\text{m s}^{-1}$$

17 **a** $F_{net} = F_{pull} - F_f$

$$= 100 - 30$$
$$= 70\,\text{N}$$

$$W_{\text{on trolley}} = F \times s$$
$$= 70 \times 20$$
$$= 1400$$
$$= 1.4 \times 10^3\,\text{J}$$

b $W_{\text{on load}} = \Delta E_k = 1400 = 1.4 \times 10^3\,\text{J}$

c $\Delta E_k = 1.4 \times 10^3\,\text{J}$ and because the trolley starts from rest, $E_{k\,\text{final}} = 1.4 \times 10^3\,\text{J}$

$$\frac{1}{2}mv^2 = 1.4 \times 10^3$$
$$v = \sqrt{\frac{2 \times 1.4 \times 10^3}{200}}$$
$$= 3.7\,\text{m s}^{-1}$$

d E_k lost = E_s gained

$$E_s = \frac{1}{2}kx^2$$
$$= \frac{1}{2} \times 1500 \times 0.18^2$$
$$= 24\,\text{J}$$

18 **a**

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$
$$120 \times 6.0 + 45 \times 0 = (120 + 45)v$$
$$720 = 165v$$
$$v = 4.4\,\text{m s}^{-1}$$

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b **i** $\Delta p = p_{final} - p_{initial}$
$= (120 \times 4.4) - (120 \times 6.0)$
$= -192\,kg\,m\,s^{-1}$

The ruckman loses $1.9 \times 10^2\,kg\,m\,s^{-1}$.

ii In this collision, the momentum gain of the bag is equal to the momentum loss of the ruckman, i.e. $1.9 \times 10^2\,kg\,m\,s^{-1}$.

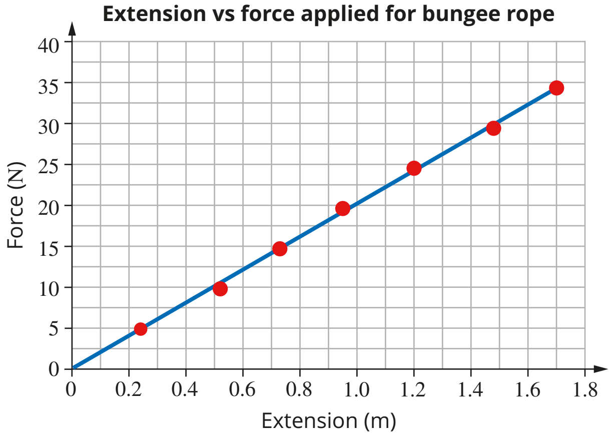
c $F = \dfrac{\Delta p}{\Delta t} = \dfrac{192}{120 \times 10^{-3}}$
$= 1.6 \times 10^3\,N$

d $E_{k\ before} = \dfrac{1}{2} \times 120 \times 6.0^2 = 2160\,J$

$E_{k\ after} = \dfrac{1}{2} \times (120 + 45) \times 4.4^2 = 1597\,J$

Since kinetic energy is not conserved, the collision is inelastic.

19 **a**



b k = gradient
$= \dfrac{rise}{run} = \dfrac{34.3}{1.7} \approx 20\,N\,m^{-1}$

c $E_s = \dfrac{1}{2}kx^2$
$= \dfrac{1}{2} \times 20 \times 15^2$
$= 2.3 \times 10^3\,J$

d Equating E_s and E_k:

$E_k = 2.3 \times 10^3\,J$

$\dfrac{1}{2} \times 60 \times v^2 = 2.3 \times 10^3$

$v = \sqrt{\dfrac{2 \times 2.3 \times 10^3}{60}}$
$= 8.7\,m\,s^{-1}$

20 Aristotle’s ideas agree with our everyday observations. We experience objects as slowing or stopping without an external force to keep them going, and we cannot see that there are actually forces (e.g. gravitational or frictional) acting to slow them down. In a space station we would often experience objects moving with constant velocity as they floated around the ship in freefall without friction from surfaces to slow their motion.

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Chapter 4 Gravity

4.1 Newton's law of universal gravitation

CASE STUDY: ANALYSIS

Measuring the gravitational constant, *G*

1 $F_g = G\frac{m_1 m_2}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{158 \times 0.730}{0.230^2}$

$= 1.45 \times 10^{-7}\,\text{N}$

2 $\frac{6.75 \times 10^{-11} - 6.67 \times 10^{-11}}{6.67 \times 10^{-11}} \times 100 = 1.20\%$

Worked example: Try yourself 4.1.1

GRAVITATIONAL ATTRACTION BETWEEN SMALL OBJECTS

Two bowling balls are sitting next to each other on a shelf. The centres of the balls are 60 cm apart. Ball 1 has a mass of 7.0 kg and ball 2 has a mass of 5.5 kg. Calculate the force of gravitational attraction between them.

Thinking	Working
Recall the equation for Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$
Note the information provided and convert values into appropriate units where necessary.	$G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$ $m_1 = 7.0\,\text{kg}$ $m_2 = 5.5\,\text{kg}$ $r = 0.60\,\text{m}$
Substitute the values into Newton's equation.	$F_g = 6.67 \times 10^{-11} \times \frac{7.0 \times 5.5}{0.60^2}$
Solve the equation.	$F_g = 7.1 \times 10^{-9}\,\text{N}$

Worked example: Try yourself 4.1.2

GRAVITATIONAL ATTRACTION BETWEEN MASSIVE OBJECTS

Calculate the force of gravitational attraction between the Sun and the Earth given the following data:
$m_{Sun} = 1.99 \times 10^{30}\,\text{kg}$
$m_{Earth} = 5.98 \times 10^{24}\,\text{kg}$
$r_{Sun\text{–}Earth} = 1.50 \times 10^{11}\,\text{m}$

Thinking	Working
Recall the equation for Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$
Note the information provided.	$m_1 = 1.99 \times 10^{30}\,\text{kg}$ $m_2 = 5.98 \times 10^{24}\,\text{kg}$ $r = 1.50 \times 10^{11}\,\text{m}$ $G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$
Substitute the values into Newton's equation.	$F_g = 6.67 \times 10^{-11} \times \frac{5.98 \times 10^{24} \times 1.99 \times 10^{30}}{(1.50 \times 10^{11})^2}$
Solve the equation.	$F_g = 3.53 \times 10^{22}\,\text{N}$

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Worked example: Try yourself 4.1.3

ACCELERATION CAUSED BY A GRAVITATIONAL FORCE

The force of gravitational attraction between the Sun and the Earth is approximately 3.5×10^{22} N. Calculate the acceleration of the Earth and the Sun caused by this force. Compare these accelerations by calculating the ratio $\frac{a_{Earth}}{a_{Sun}}$.

Use the following data:

$m_{Earth} = 5.98 \times 10^{24}$ kg

$m_{Sun} = 1.99 \times 10^{30}$ kg

Thinking	Working
Recall the equation for Newton's second law of motion.	$F_{net} = ma$
Transpose the equation to make *a* the subject.	$a = \frac{F_{net}}{m}$
Substitute values into this equation to find the accelerations of the Earth and the Sun.	$a_{Earth} = \frac{3.5 \times 10^{22}}{5.98 \times 10^{24}} = 5.85 \times 10^{-3}\ \text{m s}^{-2}$ $a_{Sun} = \frac{3.5 \times 10^{22}}{1.99 \times 10^{30}} = 1.76 \times 10^{-8}\ \text{m s}^{-2}$
Compare the two accelerations.	$\frac{a_{Earth}}{a_{Sun}} = \frac{5.85 \times 10^{-3}}{1.76 \times 10^{-8}} = 3.33 \times 10^{5}$ The acceleration of the Earth is approximately 3.3×10^{5} times greater than the acceleration of the Sun.

CASE STUDY: ANALYSIS

Extrasolar planets

1 A hot Jupiter is an exoplanet that is at least as large as Jupiter and which orbits its host star much closer than Mercury orbits the Sun. Because of their large mass and relatively close proximity to their host star, hot Jupiters exert a relatively large gravitational pull on their host star. The resultant wobble of the host star is detectable by astronomers.

2 **a** $F_g = G\frac{m_1 m_2}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{1.68 \times 10^{28} \times 3.62 \times 10^{30}}{(1.95 \times 10^{11})^2}$$

$$= 1.07 \times 10^{26}\ \text{N}$$

b $a = \frac{F}{m}$

$$= \frac{1.07 \times 10^{26}}{3.62 \times 10^{30}}$$

$$= 2.96 \times 10^{-5}\ \text{m s}^{-2}$$

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Worked example: Try yourself 4.1.4

GRAVITATIONAL FORCE

Compare the force due to gravity on a 1.0 kg mass on the Earth's surface calculated using the equations $F_g = mg$ and $F_g = G\frac{m_1 m_2}{r^2}$. Use the following data: $g = 9.8\ \text{m s}^{-2}$ $m_{Earth} = 5.98 \times 10^{24}\ \text{kg}$ $r_{Earth} = 6.37 \times 10^6\ \text{m}$

Thinking	Working
Apply the equation $F_g = mg$.	$F_g = mg$ $= 1.0 \times 9.8$ $= 9.8\ \text{N}$
Apply Newton's law of universal gravitation.	$F_g = G\frac{m_1 m_2}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{1.0 \times 5.98 \times 10^{24}}{(6.37 \times 10^6)^2}$ $= 9.83$ $= 9.8\ \text{N}$
Compare the two values.	Both equations give the same result to two significant figures.

Worked example: Try yourself 4.1.5

FORCES DURING VERTICAL ACCELERATION

Calculate the force due to gravity and the normal force acting on a 90 kg person in a lift that is accelerating downwards at $0.80\ \text{m s}^{-2}$. Describe that person's experience. Assume that $g = 9.8\ \text{m s}^{-2}$.

Thinking	Working
Calculate the force due to gravity on the person using $F_g = mg$.	$F_g = mg = 90 \times 9.8 = 882\ \text{N}$
Calculate the force required to accelerate the person downwards at $0.80\ \text{m s}^{-2}$.	$F_{net} = ma = 90 \times 0.80 = 72\ \text{N}$
The net force that causes the acceleration results from the normal force (upwards) and the force due to gravity (downwards). Since the lift is accelerating downwards, $F_g > F_N$. Note that as the person is partially falling in the direction of gravitational acceleration, there is less contact force and the person feels lighter than if standing still.	$F_{net} = 72$ $F_g - F_N = 72$ $882 - F_N = 72$ $F_N = 882 - 72$ $= 810$ $= 8.1 \times 10^2\ \text{N}$ The person will feel lighter than when they are standing on the ground.

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KEY QUESTIONS

Knowledge and understanding

1 Newton's law of universal gravitation contains a constant, G, which is a very small number: $6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$. The effect of this constant is that when objects have a small mass, the gravitational force will be so small as to be insignificant. Only when at least one of the objects is massive is the gravitational force significant.

In addition, due to the inverse square relationship, as the distance between objects increases, the gravitational force between them decreases. When the value of r^2 is close to (or exceeds) the magnitudes of the masses, the force will be very small.

2 r is the distance between the centres of the two objects, measured in metres.

3 **a** The gravitational force between the two masses is doubled.
b The gravitational force between the two masses is one quarter of the original force.
c The gravitational force between the two masses is decreased by a factor of 16 (4^2).

4 **a** $F_g = G\frac{m_1 m_2}{r^2}$

$= 6.67\times10^{-11}\times\frac{2.0\times10^{30}\times4.9\times10^{24}}{(1.1\times10^{11})^2}$

$= 5.4\times10^{22}\,\mathrm{N}$

b $F_g = m_{\text{Venus}}\times a_{\text{Venus}}$

$5.4\times10^{22} = 4.9\times10^{24}\times a_{\text{Venus}}$

$a_{\text{Venus}} = \frac{5.4\times10^{22}}{4.9\times10^{24}}$

$= 1.1\times10^{-2}\,\mathrm{m\,s^{-2}}$

5 Deimos has a smaller mass than Mars and therefore experiences a larger acceleration from the same gravitational force.

6 **a** $F_g = G\frac{m_1 m_2}{r^2}$

$= 6.67\times10^{-11}\times\frac{150\times150}{(1.00)^2}$

$= 1.50\times10^{-6}\,\mathrm{N}$

b $F = ma$

$a = \frac{F}{m}$

$= \frac{1.50\times10^{-6}}{150}$

$= 1.0\times10^{-8}\,\mathrm{m\,s^{-2}}$ towards each other

Note that the acceleration of each astronaut is the same because their masses are the same.

7 $F_g = G\frac{m_1 m_2}{r^2}$

$= 6.67\times10^{-11}\times\frac{6.40\times10^{23}\times80.0}{(3.40\times10^{6})^2}$

$= 295\,\mathrm{N}$

8 **a** $F_g = mg = 60 \times 9.8 = 588\,\mathrm{N}$

When accelerating upwards at $1.40\,\mathrm{m\,s^{-2}}$, the net force is $F_{net} = ma = 60.0 \times 1.40 = 84\,\mathrm{N}$, and $F_N > F_g$.

$F_{net} = F_N - F_g = 84\,\mathrm{N}$

$F_N = 84 + 588 = 6.7 \times 10^2\,\mathrm{N}$

b When the person is moving at a constant speed, the normal force is equal to the force due to gravity on them.

$F_N = F_g = mg = 60.0 \times 9.8 = 5.9 \times 10^2\,\mathrm{N}$

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Analysis

9 **a** Note: 1 million km = 1 × 10^6 km = 1 × 10^9 m

$$F_g = G\frac{m_1m_2}{r^2}$$

$$= 6.67\times10^{-11}\times\frac{1.90\times10^{27}\times5.68\times10^{26}}{(734\times10^9)^2}$$

$$= 1.34\times10^{20}\text{ N}$$

b $F_g = G\frac{m_1m_2}{r^2}$

$$= 6.67\times10^{-11}\times\frac{1.99\times10^{30}\times1.90\times10^{27}}{(778\times10^9)^2}$$

$$= 4.17\times10^{23}\text{ N}$$

c The mass of the Sun is much greater than the mass of Jupiter. It is this difference that accounts for the difference in the forces calculated.

d Comparison of forces is: $\frac{4.17\times10^{23}}{1.34\times10^{20}} = 3112$. The force between the Sun and Jupiter is 3112 times that between Jupiter and Saturn.

Comparison of masses: $\frac{1.99\times10^{30}}{5.68\times10^{26}} = 3504$. The Sun's mass is 3504 times the mass of Saturn.

The force between the Sun and Jupiter is approximately 3000 times more than the force between Jupiter and Saturn. The Sun–Saturn mass ratio is also approximately 3000. The discrepancy (3112 versus 3504) can be attributed to the difference in the two distances.

10 **a** Note: 1 million km = 1 × 10^6 km = 1 × 10^9 m

$$F_g = G\frac{m_1m_2}{r^2}$$

$$= 6.67\times10^{-11}\times\frac{5.98\times10^{24}\times6.39\times10^{23}}{(5.6\times10^{10})^2}$$

$$= 8.1\times10^{16}\text{ N}$$

b $F_g = G\frac{m_1m_2}{r^2}$

$$= 6.67\times10^{-11}\times\frac{5.98\times10^{24}\times6.39\times10^{23}}{(2.25\times10^{11})^2}$$

$$= 5.03\times10^{15}\text{ N}$$

c % comparison $= \frac{(5.03\times10^{15})}{(8.1\times10^{16})\times100} = 6.2\%$. The average force between the Earth and Mars is about 6% of the force during the close approach in August 2003.

11 **a** $g_{\text{Mercury}} = G\frac{m_{\text{Mercury}}}{(r_{\text{Mercury}})^2}$

b The smaller radius of Mercury has the effect of increasing the gravitational acceleration at the surface. (All other things being equal, a radius of one-third would *increase* the gravitational acceleration by a factor of nine.)

c The much smaller mass of the planet would have the effect of decreasing the gravitational acceleration at the surface.

d $g_{\text{Mercury}} = G\frac{m_{\text{Mercury}}}{(r_{\text{Mercury}})^2}$

$$= 6.67\times10^{-11}\times\frac{3.29\times10^{23}}{(2440\times10^3)^2}$$

$$= 3.69\text{ m s}^{-2}\text{ towards the surface of Mercury}$$

e
$$F_g = m\times g_{\text{Earth}}$$
$$735 = m\times9.8$$
$$m = 75\text{ kg}$$
$$F_g = m\times g_{\text{Mercury}}$$
$$= 75\times3.69$$
$$= 2.8\times10^2\text{ N towards the surface of Mercury}$$

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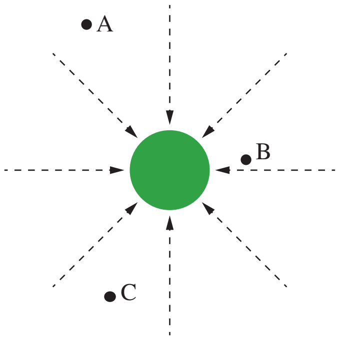
4.2 Gravitational fields

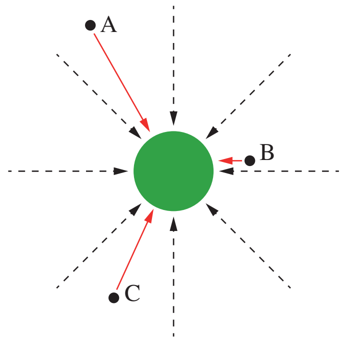
Worked example: Try yourself 4.2.1

INTERPRETING GRAVITATIONAL FIELD DIAGRAMS

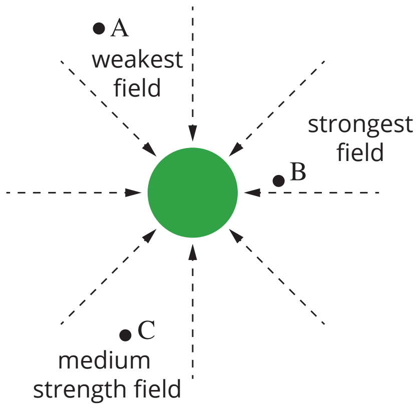
The diagram below represents the gravitational field of a planet.



a Draw arrows to indicate the direction of the gravitational force acting at points A, B and C.

Thinking	Working
The direction of the field arrows indicates the direction of the gravitational force, which is inwards towards the centre of the planet.	

b State the relative strength of the gravitational field at each point.

Thinking	Working
The closer the field lines, the stronger the force. So the field is strongest at point B, next strongest at point C and weakest at point A.	

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Worked example: Try yourself 4.2.2

CALCULATING GRAVITATIONAL FIELD STRENGTH

A student uses a spring balance to measure the force due to gravity on a piece of wood as 2.5 N. If the piece of wood has a mass of 260 g, calculate the gravitational field strength indicated by this experiment.

Thinking	Working
Recall the equation for gravitational field strength.	$g = \frac{F_g}{m}$
Substitute the appropriate values.	$g = \frac{2.5}{0.26}$
Solve the equation.	$g = 9.6\ \text{N kg}^{-1}$

Worked example: Try yourself 4.2.3

CALCULATING GRAVITATIONAL FIELD STRENGTH AT DIFFERENT ALTITUDES

Commercial airlines typically fly at an altitude of 12 000 m. Calculate the gravitational field strength of the Earth at this height using the following data: $r_{Earth} = 6.37 \times 10^6\ \text{m}$ $m_{Earth} = 5.98 \times 10^{24}\ \text{kg}$

Thinking	Working
Recall the equation for gravitational field strength.	$g = G\frac{M}{r^2}$
Add the altitude of the plane to the radius of the Earth.	$r = 6.37 \times 10^6 + 12000$ $= 6.382 \times 10^6\ \text{m}$
Substitute values into the equation and solve it.	$g = G\frac{M}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{5.98 \times 10^{24}}{(6.382 \times 10^6)^2}$ $= 9.79\ \text{N kg}^{-1}$

Worked example: Try yourself 4.2.4

GRAVITATIONAL FIELD STRENGTH ON ANOTHER PLANET OR MOON

Calculate the strength of the gravitational field on the surface of Mars. $m_{Mars} = 6.42 \times 10^{23}\ \text{kg}$ $r_{Mars} = 3390\ \text{km}$ Compare your answer with the Earth's average gravitational field strength ($9.8\ \text{N kg}^{-1}$).

Thinking	Working
Recall the equation for gravitational field strength.	$g = G\frac{M}{r^2}$
Convert the radius of Mars to metres.	$r = 3390\ \text{km} = 3.39 \times 10^6\ \text{m}$
Substitute values into the equation and solve it.	$g = G\frac{M}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{6.42 \times 10^{23}}{\left(3.39 \times 10^6\right)^2}$ $= 3.73\ \text{N kg}^{-1}$

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Compare the field strength with the Earth's average gravitational field strength by calculating the ratio $\frac{g_{Earth}}{g_{Mars}}$.	$\frac{g_{Earth}}{g_{Mars}} = \frac{9.8}{3.73}$ $= 2.6$ At their surfaces, the Earth's gravitational field strength is approximately two-and-a-half times that of Mars.

KEY QUESTIONS

Knowledge and understanding

1 **a** $9.8\,N\,kg^{-1}$. An average value for g is used in calculations because there is variation in the Earth's gravitational field strength depending on location, so it makes sense to use an average. The Earth has a smaller radius at the poles than at the equator, so gravitational field strength at the poles is greater than that at the equator. As well, gravitational field strength varies due to differing densities of rocks at different locations and due to altitude.

b Gravitational field lines from a single body always point towards the centre of mass of that body. Because the Earth is so large, small distances can be considered to be flat. Therefore we can approximate these field arrows as being parallel at the surface over a small area.

2 $g = \frac{F_g}{m} = \frac{1.5}{0.200} = 7.5\,N\,kg^{-1}$

3 The distance has been increased four times from 300 km to 1200 km. Thus from the inverse square law:

$$F \propto \frac{1}{r^2}$$
$$\propto \frac{1}{(4r)^2}$$
$$\propto \frac{1}{16r^2}$$
$$= \frac{1}{16} \text{ of the original}$$

4 $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{5.98 \times 10^{24}}{((6370 + 400) \times 10^3)^2}$$
$$= 8.70\,N\,kg^{-1}$$

5 $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{1.0 \times 10^{13}}{(900 + 100)^2}$$
$$= 6.7 \times 10^{-4}\,N\,kg^{-1}$$

6 $g = G\frac{M}{r^2}$

a Titan: $6.67 \times 10^{-11} \times \frac{1.35 \times 10^{23}}{(2.57 \times 10^6)^2} = 1.36\,N\,kg^{-1}$

b Dione: $6.67 \times 10^{-11} \times \frac{1.10 \times 10^{21}}{(5.61 \times 10^5)^2} = 0.233\,N\,kg^{-1}$

c Tethys: $6.67 \times 10^{-11} \times \frac{6.17 \times 10^{20}}{(5.31 \times 10^5)^2} = 0.146\,N\,kg^{-1}$

Analysis

7 **a** $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{3.0 \times 10^{30}}{(10 \times 10^3)^2}$$
$$= 2.0 \times 10^{12}\,N\,kg^{-1}$$

b $\frac{g_{neutron\ star}}{g_{Earth}} = \frac{2.0 \times 10^{12}}{9.8} = 2.0 \times 10^{11}$. The gravitational field strength of a neutron star is 200 billion times that at the surface of the Earth.

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c $g = G\frac{M}{r^2}$

$$9.8 = 6.67\times10^{-11}\times\frac{3.0\times10^{30}}{r^2}$$

$$r = \sqrt{\frac{6.67\times10^{-11}\times3.0\times10^{30}}{9.8}}$$

$$= 4.5\times10^{9}\,\text{m}$$

8 **a** $\frac{6000}{5000} = 1.2$

$$g \propto \frac{1}{(1.2)^2}$$

$$\propto \frac{1}{1.44}$$

$$\propto 0.7$$

Thus g at the equator is approximately 0.7 that at the poles, i.e. 5.6 N kg^{-1}.

b $g_{\text{poles}} = G\frac{M}{r^2}$

$$8 = 6.67\times10^{-11}\times\frac{M}{5\,000\,000^2}$$

$$M = 3.0\times10^{24}\,\text{kg}$$

$$g_{\text{equator}} = G\frac{M}{r^2}$$

$$= 6.67\times10^{-11}\times\frac{3\times10^{24}}{6\,000\,000^2}$$

$$= 5.6\,\text{N}\,\text{kg}^{-1}$$

9 Let x be the distance from the centre of the Earth to where the Earth's gravity equals the Moon's gravity. Then:

$$g_{\text{Earth}} = \frac{6.67\times10^{-11}\times5.98\times10^{24}}{x^2}$$

$$g_{\text{Moon}} = \frac{6.67\times10^{-11}\times7.3\times10^{22}}{\left(3.8\times10^{8} - x\right)^2}$$

Equating these two expressions gives:

$$\frac{5.98\times10^{24}}{x^2} = \frac{7.3\times10^{22}}{\left(3.8\times10^{8} - x\right)^2}$$

$$\frac{81.9}{x^2} = \frac{1}{\left(3.8\times10^{8} - x\right)^2}$$

Taking the square root of both sides gives:

$$\frac{9.05}{x} = \frac{1}{\left(3.8\times10^{8} - x\right)}$$

Inverting both sides gives:

$$\frac{x}{9.05} = 3.8\times10^{8} - x$$

$$x = 3.44\times10^{9} - 9.05x$$

$$10.05x = 3.44\times10^{9}$$

$$x = 3.4\times10^{8}\,\text{m}$$

10 Since g is proportional to $\frac{1}{r^2}$, if g becomes $\frac{1}{100}$th its value, r must become 10 times its value. 10 times r is a distance of 10 Earth radii.

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4.3 Work in a gravitational field

Worked example: Try yourself 4.3.1

WORK DONE FOR A CHANGE IN GRAVITATIONAL POTENTIAL ENERGY

Calculate the work done (in MJ) to lift a weather satellite of 3.2 tonnes from the Earth's surface to the limit of the atmosphere, which ends at the Karman line (exactly 100 km up from the surface of the Earth). Assume that $g = 9.8\,\text{N kg}^{-1}$.

Thinking	Working
Convert the values into the appropriate units.	m = 3.2 tonnes = 3200 kg Δh = 100 km = 100×10^3 m
Substitute the appropriate values into $E_g = mg\Delta h$. Remember to give your answer in MJ to two significant figures.	$E_g = mg\Delta h$ $= 3200 \times 9.8 \times 100 \times 10^3$ $= 3.136 \times 10^9$ J $= 3.1 \times 10^3$ MJ
The work done is equal to the change in gravitational potential energy.	$W = \Delta E = 3.1 \times 10^3$ MJ

Worked example: Try yourself 4.3.2

SPEED OF A FALLING OBJECT

Calculate how fast a 450 g hammer is going as it hits the ground after being dropped from a height of 1.4 m on the Earth, where $g = 9.8\,\text{N kg}^{-1}$.

Thinking	Working
Calculate the gravitational potential energy of the hammer. Change the units of measurement where necessary.	$E_g = mg\Delta h$ $= 0.45 \times 9.8 \times 1.4$ $= 6.17$ J
Assume that when the hammer hits the ground, all its gravitational potential energy has been converted into kinetic energy.	$E_k = E_g = 6.17$ J
Use the equation for kinetic energy to calculate the speed of the hammer as it hits the ground.	$E_k = \frac{1}{2}mv^2$ $6.17 = \frac{1}{2} \times 0.45 \times v^2$ $v^2 = \frac{6.17 \times 2}{0.45}$ $v = 5.2\,\text{m s}^{-1}$

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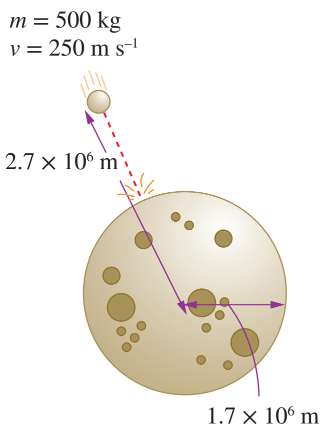
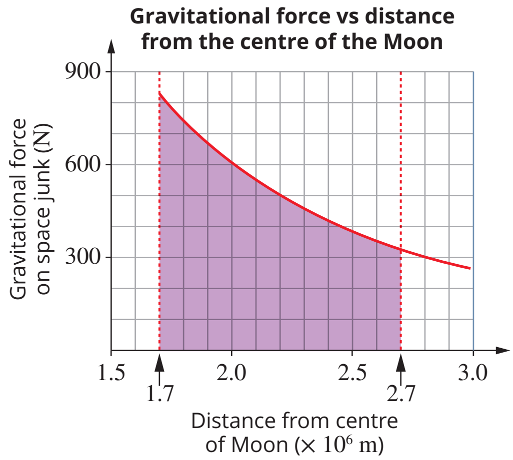
Worked example: Try yourself 4.3.3

CHANGE IN GRAVITATIONAL POTENTIAL ENERGY USING A FORCE VS DISTANCE GRAPH

A 500 kg lump of space junk is plummeting towards the Moon. It falls a distance of 1.0×10^6 m and then strikes the surface of the Moon. Using the diagram and the force vs distance graph shown, determine the approximate decrease in gravitational potential energy of the space junk as it hits the Moon's surface.



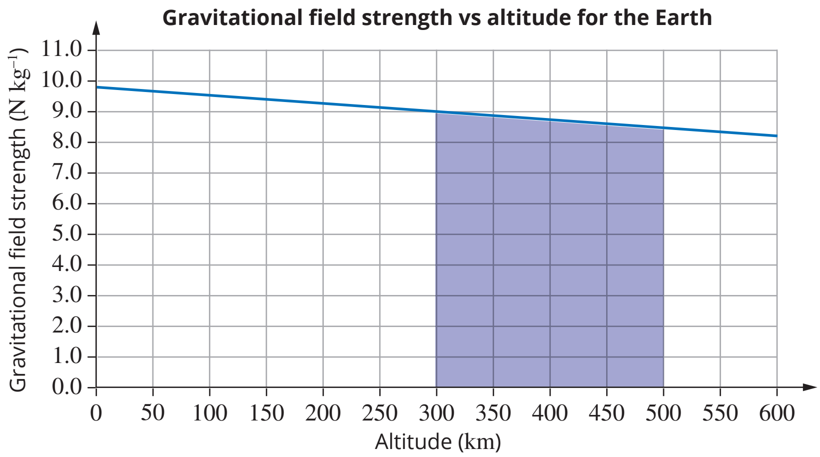


Thinking	Working
Count the number of shaded squares. (Only count squares that are at least 50% shaded.)	Number of shaded squares = 52
Calculate the area (energy value) of each square.	$E_{square} = 0.1 \times 10^6 \times 100$ $= 1 \times 10^7$ J
To calculate the change in energy, multiply the number of shaded squares by the energy value of each square.	$\Delta E_g = 52 \times 1 \times 10^7$ $= 5.2 \times 10^8$ J

Worked example: Try yourself 4.3.4

CHANGE IN GRAVITATIONAL POTENTIAL ENERGY USING A GRAVITATIONAL FIELD STRENGTH VS DISTANCE GRAPH

A 3000 kg Soyuz rocket moves from an orbital height of 300 km above the Earth's surface to dock with the International Space Station at a height of 500 km. Using the graph below, determine the approximate change in the gravitational potential energy of the rocket.



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Thinking	Working
Count the number of shaded squares. Only count squares that are at least 50% shaded.	Number of shaded squares = 36
Calculate the energy value of each square.	$E_{square} = 50 \times 10^3\ \text{m} \times 1\ \text{N kg}^{-1}$ $= 5 \times 10^4\ \text{J kg}^{-1}$
To calculate the change in energy, multiply the number of shaded squares by the energy value of each square and the mass of the rocket.	$\Delta E_g = 36 \times (5 \times 10^4) \times 3000$ $= 5.4 \times 10^9\ \text{J}$

KEY QUESTIONS

Knowledge and understanding

1 $W = E_g = mg\Delta h$

$$E_g = 75 \times 9.8 \times 285$$
$$= 2.1 \times 10^5\ \text{J}$$

2 $W = E_g = mg\Delta h$

$$E_g = 2000000 \times 9.8 \times 70000$$
$$= 1.4 \times 10^{12}\ \text{J}$$

3 $W = E_g = mg\Delta h$

$$= 1.5 \times 3.7 \times 2.2$$
$$= 12.21\ \text{J}$$
$$E_k = \frac{1}{2}mv^2$$
$$12.21 = \frac{1}{2} \times 1.5 \times v^2$$
$$v^2 = \sqrt{\frac{2 \times 12.21}{1.5}}$$
$$v = 4.0\ \text{m s}^{-1}$$

4 **a** The gravitational field strength, g, increases along the path from point A to point D.

b The meteor is under the influence of the Earth's gravitational field, which increases from point A to point D. This will cause the meteor to accelerate at an increasing rate as it approaches the Earth.

c **i** The kinetic energy of the meteor as it travels from A to D will **increase.**

ii The gravitational potential energy of the meteor as it travels from A to D will **decrease.**

iii The total energy of the meteor as it travels from A to D will **stay the same**.

Analysis

5 **a** $E_g = mg\Delta h$

$$= 0.6 \times 3.7 \times 7000$$
$$= 15540\ \text{J or } 1.6 \times 10^4\ \text{J}$$
$$E_k = \frac{1}{2}mv^2$$
$$15540 = \frac{1}{2}0.6v^2$$
$$v = \sqrt{\frac{2 \times 15540}{0.6}}$$
$$= 228\ \text{m s}^{-1}$$
$$= 2.3 \times 10^2\ \text{m s}^{-1}$$

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b $E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.6 \times 189^2$

$= 10716.3\,J$

$E_g = mg\Delta h$

$10716.3 = 0.6 \times 9.8 \times \Delta h$

$\Delta h = 1822.5\,m$ or $1.8\,km$ (approx.)

6 **a** 100 km above the Earth's surface is a distance of 6.4×10^6 m + 100 000 m = 6.5×10^6 m from the centre of the Earth. According to the graph, F is between 9 N and 9.2 N at this distance.

b According to the graph, 5 N occurs at approximately 9.0×10^6 m from the centre of the Earth. Thus the height above the Earth's surface = $9.0 \times 10^6 - 6.4 \times 10^6 = 2.6 \times 10^6$ m or 2600 km.

c Convert km s^{-1} to m s^{-1} by multiplying by 1000, then apply the following equation:

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 1 \times 4000^2$

$= 8.0 \times 10^6\,J$

d $E_g = mg\Delta h$ = area under the graph

= 19 squares $\times 2 \times 0.5 \times 10^6$

$= 1.9 \times 10^7\,J$

e New E_k = starting $E_k + \Delta E_k = 8 \times 10^6 + 1.9 \times 10^7 = 2.7 \times 10^7$ J

f $E_k = \frac{1}{2}mv^2$

new speed $= \sqrt{\frac{2 \times 2.7 \times 10^7}{1.0}}$

$= 7348\,m\,s^{-1}$ or $7.3\,km\,s^{-1}$

7 **a** 600 km above the Earth's surface = $6.4 \times 10^6 + 600000 = 7.0 \times 10^6$ m from the Earth's centre

8000 km from the Earth's centre is 8.0×10^6 m on the graph.

The area under the graph between 7×10^6 m and 8.0×10^6 m is approximately 7 squares.

As the satellite comes to a stop, the change in kinetic energy over the distance is the same as the kinetic energy at its launch.

Note that the graph is drawn for a 1.0 kg object and the satellite has mass 240 kg. Therefore:

E_k = area under graph $\times$ mass of the satellite

= 7 squares $\times 2 \times 0.5 \times 10^6 \times 240$

$= 1.7 \times 10^9\,J$

b Work done = area under the graph × mass of the satellite. Since $2.64 \times 10^9 = 2 \times 0.5 \times 10^6 \times 240$ × number of squares, the number of squares = 11.

Referring to the graph and counting 11 squares from the Earth's radius, the satellite reaches approximately 8×10^6 m. This is equal to 1.6×10^6 m or 1600 km above the Earth's surface.

8 100 km above the Earth's surface = $6.4 \times 10^6 + 100000 = 6.5 \times 10^6$ m from the Earth's centre.

2600 km above the Earth's surface = $6.4 \times 10^6 + 2600000 = 9.0 \times 10^6$ m from the Earth's centre.

The area under the graph between 6500 km and 9000 km is approximately 35 squares.

ΔE_g = area under graph $\times$ mass of the satellite

= 35 squares $\times 1 \times 0.5 \times 10^6 \times 20000$

$= 3.5 \times 10^{11}\,J$

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Chapter 4 Review

Knowledge and understanding

1 $F_g = G\frac{m_1m_2}{r^2}$

$= 6.67\times10^{-11}\times\frac{5.98\times10^{24}\times100}{(6.37\times10^6)^2}$

$= 983\text{N}$

2 $F_g = G\frac{m_1m_2}{r^2}$

$= 6.67\times10^{-11}\times\frac{1.05\times10^{21}\times5.69\times10^{26}}{r^2}$

$r^2 = \frac{6.67\times10^{-11}\times1.05\times10^{21}\times5.69\times10^{26}}{2.79\times10^{20}}$

$r = 377930000$

$= 3.78\times10^8\text{ m}$

3 $F_g = G\frac{m_1m_2}{r^2}$

$1.0\times10^{-3} = 6.67\times10^{-11}\times\frac{2.0\times10^4\times2.0\times10^3}{r^2}$

$r^2 = \frac{6.67\times10^{-11}\times2.0\times10^4\times2.0\times10^3}{1.0\times10^{-3}}$

$r = 1.633$

$= 1.6\text{m}$ between centres

4 **a** The force exerted on Jupiter by the Sun is equal to the force exerted on the Sun by Jupiter.

b As the Sun is larger than Jupiter, the acceleration of Jupiter caused by the Sun is greater than the acceleration of the Sun caused by Jupiter.

5 $F_{net} = m_{star}\times a_{star}$

$a_{star} = \frac{F_{net}}{m_{star}}$

$= \frac{2.1\times10^{23}}{1.0\times10^{30}}$

$= 2.1\times10^{-7}\text{ ms}^{-2}$

6 **a** $F_g = G\frac{m_1m_2}{r^2}$

$= \frac{(6.67\times10^{-11}\times1.9\times10^{27}\times1000)}{(7.15\times10^7)^2}$

$= 2.48\times10^4\text{ N}$

b The magnitude of the gravitational force that the comet exerts on Jupiter is equal to the magnitude of the gravitational force that Jupiter exerts on the comet: 2.48×10^4 N.

c $a = \frac{F_{net}}{m}$

$= \frac{2.48\times10^4}{1000}$

$= 24.8\text{ ms}^{-2}$

d $a = \frac{F_{net}}{m}$

$= \frac{2.48\times10^4}{1.90\times10^{27}}$

$= 1.31\times10^{-23}\text{ ms}^{-2}$

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7 $g = G\frac{M}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{6.4 \times 10^{23}}{(3\,400\,000)^2}$

$= 3.7\,\text{m}\,\text{s}^{-2}$

8 **a** $F_g = mg = 50 \times 9.8 = 490\,\text{N}$

When accelerating downwards at $0.6\,\text{m}\,\text{s}^{-2}$, the net force is $F_{net} = ma = 50 \times 0.6 = 30\,\text{N}$.

$F_{net} = 30$ and $F_g > F_N$

$F_g - F_N = 30$

$490 - F_N = 30$

$F_N = 490 - 30$

$= 460\,\text{N}$

$= 4.6 \times 10^2\,\text{N}$ to 2 significant figures

b When the person is moving at a constant speed, the normal force is equal to the force due to gravity:

$F_g = F_N = 490\,\text{N}$ or $4.9 \times 10^2\,\text{N}$ to 2 significant figures

9 **a** $F_g = 80 \times 9.8 = 784\,\text{N}$

$F_{net} = 80 \times 30 = 2400\,\text{N}$

When accelerating upwards:

$F_{net} = F_N - F_g = 2400$

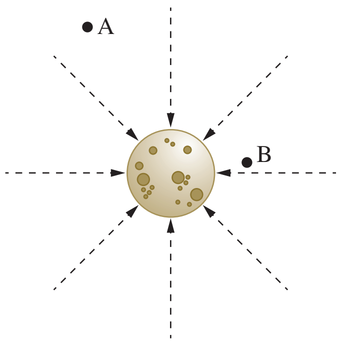
$F_N - 784 = 2400$

$F_N = 3184$ N or 3.2×10^3 N to 2 significant figures

b Since the astronaut is in free fall during orbit, the normal force acting on the astronaut is 0 N.

c $F_g = mg = 80 \times 8.2 = 656\,\text{N}$ or $6.6 \times 10^2\,\text{N}$

10 **a** In a gravitational field, the field lines always point towards the source of the field, hence:



b The gravitational field strength is greater at B because the field lines are closer at B than at A.

11 **a** $F_g = mg$

$m = \frac{F}{g}$

$= \frac{1.86}{3.72}$

$= 0.500\,\text{kg}$ or $500\,\text{g}$

b Mass does not change with location, so the object will have a mass of 500 g on the Earth.

12 $g = \frac{F_g}{m} = \frac{700}{71.4} = 9.80\,\text{N}\,\text{kg}^{-1}$

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13 a $g = G\frac{M}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{4.00 \times 10^{24}}{(4800 \times 10^3)^2}$

$= 11.58\,\text{N kg}^{-1}$

b $g = G\frac{M}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{4.00 \times 10^{24}}{(4750 \times 10^3)^2}$

$= 11.82\,\text{N kg}^{-1}$

$\% = \frac{11.82}{11.58} \times 100 = 102.07\%$

The field strength at the poles is 102.07% of that at the equator, or 2.07% more.

14 a $g = G\frac{M}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{1.02 \times 10^{26}}{(2.48 \times 10^7)^2}$

$= 11.1\,\text{N kg}^{-1}$

b The ice will accelerate at a rate given by the gravitational field strength, g, therefore it will accelerate at $11.1\,\text{m s}^{-2}$.

15 $g_{\text{Sun}} = G\frac{m_{\text{Sun}}}{r^2}$

$9.8 = 6.67 \times 10^{-11} \times \frac{1.99 \times 10^{30}}{r^2}$

$r^2 = 6.67 \times 10^{-11} \times \frac{1.99 \times 10^{30}}{9.8}$

$r = 3.7 \times 10^9\,\text{m}$

At almost 4 million kilometres from the Sun, the gravitational field strength is the same magnitude as at the Earth's surface.

16 At a height of two Earth radii above the Earth's surface, a person is a distance of three Earth radii from the centre of the Earth. Thus:

$F_g = \frac{900}{3^2} = \frac{900}{9} = 100\,\text{N}$

17 a The increase in kinetic energy is the area under the graph between $3 \times 10^6\,\text{m}$ and $2.5 \times 10^6\,\text{m}$ = 6 squares $\times 10 \times 0.5 \times 10^6 = 3 \times 10^7\,\text{J}$.

b $E_{\text{k initial}} = \frac{1}{2}mv^2 = \frac{1}{2} \times 20 \times 1000^2 = 1 \times 10^7\,\text{J}$

$E_{\text{k new}} = 1 \times 10^7 + 3 \times 10^7 = 4 \times 10^7\,\text{J}$

c $v = \sqrt{\frac{2 \times E_k}{m}} = \sqrt{\frac{2 \times 4.0 \times 10^7}{20}}$

$= 2000\,\text{m s}^{-1}$ or $2.0\,\text{km s}^{-1}$

d From the graph, $F_g = 70\,\text{N} = mg$, thus:

$g = \frac{70}{20} = 3.5\,\text{N kg}^{-1}$

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Application and analysis

18 **a** Approximately 5×10^6 m or 5000 km

b The gravitational field strength at ground level is close to $10\,N\,kg^{-1}$ (or $9.8\,N\,kg^{-1}$) and the gravitational field strength at an altitude of 6500 km is approximately $2.5\,N\,kg^{-1}$.

As the Earth's radius is close to 6500 km, the distance to the centre of the Earth at an altitude of 6500 km is about twice that from ground level. Using the inverse square law, we would expect the gravitational field strength at 6500 km to be a quarter of that at ground level. This is confirmed by the graph.

19 As the spacecraft experiences no net gravitational force, the gravitational field strength of *M* must be equal to (and opposite) that of *m* at this point.

$$G\frac{M}{(0.8R)^2} = G\frac{m}{(0.2R)^2}$$

Both *G* and *R* cancel each other out. Therefore:

$$\frac{M}{0.64} = \frac{m}{0.04}$$

$$\frac{M}{m} = \frac{0.64}{0.04} = 16$$

20 **a** An altitude of *r* represents a distance of twice the Earth's radius (from the centre of the Earth). An altitude of 2*r* is three times the Earth's radius.

$$\frac{F_{g\ at\ r}}{F_{g\ at\ 2r}} = \frac{\frac{1}{4}}{\frac{1}{9}} = \frac{9}{4}$$

The force due to gravity at *r* is $\frac{9}{4}$ times that at 2*r*.

b The ratio of the value of *g* will be the same as the ratio for F_g, so the value of *g* at *r* is $\frac{9}{4}$ times that of *g* at 2*r*.

21 **a** 500 km = 500 000 m or 5×10^5 m. From the graph, *g* is approximately $8.5\,N\,kg^{-1}$ at this altitude.

b The shaded region represents the gain in kinetic energy per kilogram of the satellite as it drifts towards the Earth. The shaded region also represents the loss of gravitational potential energy per kilogram of the satellite as it drifts towards the Earth.

c The increase in kinetic energy is the area under the graph × the mass of the satellite = 54 squares $\times 1 \times 1 \times 10^5 \times 2000 = 1.08 \times 10^{10}$ J.

d No. Air resistance will play a major part as the satellite re-enters the Earth's atmosphere.

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Chapter 5 Electric and magnetic fields

5.1 Electric fields

Worked example: Try yourself 5.1.1

USING $F = qE$

Calculate the magnitude of the uniform electric field that creates a force of 9.00×10^{-23} N on a proton. Assume that $q_p = +1.6 \times 10^{-19}$ C.

Thinking	Working
Rearrange the relevant equation to make electric field strength the subject.	$F = qE$ $E = \frac{F}{q}$
Substitute the values for F and q into the rearranged equation and solve for E.	$E = \frac{9.00 \times 10^{-23}}{1.6 \times 10^{-19}}$ $= 5.6 \times 10^{-4}\ \text{NC}^{-1}$

CASE STUDY: ANALYSIS

Gravitational force and electric force

1

$$F = qE$$
$$F = mg$$
$$mg = qE$$
$$m = \frac{qE}{g}$$
$$= \frac{1.6 \times 10^{-19} \times 80 \times 10^{3}}{9.8}$$
$$= 1.3 \times 10^{-15}\ \text{g}$$

Worked example: Try yourself 5.1.2

WORK DONE ON A CHARGE IN A UNIFORM ELECTRIC FIELD

A student sets up two parallel plates. One plate is at a potential of 36.0 V and the other plate, which is earthed, is positioned 2.00 m away. Calculate the work done to move an electron a distance of 75.0 cm towards the negative plate. Assume that $q_e = -1.6 \times 10^{-19}$ C. In your answer identify what does the work and what the work is done on.

Thinking	Working
Identify the variables presented in the problem that are needed to calculate the electric field strength, E.	$V_2 = 36.0\ \text{V}$ $V_1 = 0\ \text{V}$ $d = 2.00\ \text{m}$
Use the equation $E = \frac{V}{d}$ to determine the electric field strength.	$E = \frac{V}{d}$ $= \frac{36.0 - 0}{2.00}$ $= 18.0\ \text{V m}^{-1}$
Use the equation $W = qEd$ to determine the work done. Note that d here is the distance that the electron moves.	$W = qEd$ $= 1.6 \times 10^{-19} \times 18.0 \times 0.750$ $= 2.2 \times 10^{-18}\ \text{J}$
Determine if work is done on the charge by the field or if work is done by the charge on the field.	Since the negatively charged electron would normally move away from the negative plate, work is done on the field.

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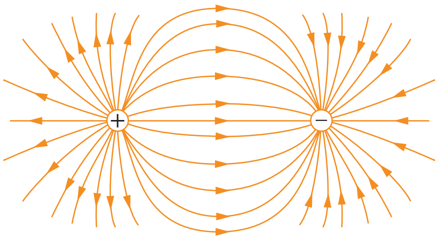
KEY QUESTIONS

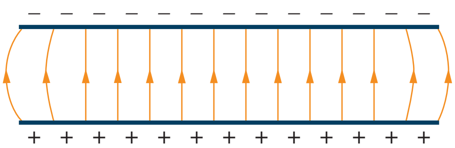
Knowledge and understanding

1 A drawing similar to the one below:

A positive and a negative charge



2 A drawing similar to the one below:



3 **a** True. Electric field lines start and end at 90° to the surface, with no gap between the lines and the surface.
b False. Field lines can never cross. If they did it would indicate that the field is in two directions at that point, which can never happen.
c False. Electric fields go from positively charged objects to negatively charged objects.
d True. Around small charged spheres called point charges you should draw at least eight field lines: top, bottom, left, right and in between each of these lines.
e True. Around point charges the field lines radiate like spokes on a wheel.
f False. Between two point charges, the direction of the field at any point is the resultant field vector determined by adding the field vectors due to each of the two point charges.
g False. Between two oppositely charged parallel plates, the field between the plates is evenly spaced and is drawn straight from the positive plate to the negative plate.

4 **a** work is done by the field
b no work is done
c work is done on the field
d no work is done
e work is done by the field

5 $F = qE$

$= 5.00 \times 10^{-3} \times 2.5$

$= 0.005 \times 2.5$

$= 0.0125$

$= 1.25 \times 10^{-2}\,\text{N}$

6 $F = qE$

$q = \dfrac{F}{E}$

$= \dfrac{0.025}{18}$

$= 0.00139\,\text{C}$

$= 1.39\,\text{mC}$

7 $F = qE$

$= 1.6 \times 10^{-19} \times 3.25$

$= 5.2 \times 10^{-19}\,\text{N}$

and

$F = ma$

$a = \dfrac{F}{m}$

$= \dfrac{5.2 \times 10^{-19}}{9.1 \times 10^{-31}}$

$= 5.7 \times 10^{11}\,\text{m}\,\text{s}^{-2}$

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Analysis

8 $E = \frac{V}{d}$

$4000 = \frac{V}{0.3}$

$V = 4000 \times 0.3 = 1200\,\text{V}$

9 **a** $W = qEd$

$= 3.204 \times 10^{-19} \times 34 \times 0.01$

$= 1.09 \times 10^{-19}\,\text{J}$

b Work is done on the field if the charge is forced to go in a direction it would not naturally go. Alpha particles carry a positive charge. So work is done on the field, since a positive particle is being moved towards a positive potential.

10 As the oil drop is stationary, the electric force must be equal to the gravitational force. Use the equations $F = mg$ and $F = qE$ to determine the force and the charge respectively. The number of electrons is found by dividing the total charge by the charge of one electron.

$F = mg$

$= 1.161 \times 10^{-14} \times 9.8$

$= 1.138 \times 10^{-13}\,\text{N}$

$q = \frac{F}{E}$

$= \frac{1.138 \times 10^{-13}}{3.55 \times 10^{4}}$

$= 3.206 \times 10^{-18}\,\text{C}$

The number of electrons is found by dividing this value by the charge on one electron:

$= \frac{3.206 \times 10^{-18}}{1.6 \times 10^{-19}}$

= 20 electrons

5.2 Coulomb's law

Worked example: Try yourself 5.2.1

USING COULOMB'S LAW TO CALCULATE CHARGE

Two small point charges (q_1 and q_2) are charged by transferring a number of electrons from q_1 to q_2. The point charges are separated by 12.7 mm in air and their charges are equal and opposite. Calculate the charge on q_1 and q_2 if there is an attractive force of 22.5 μN between them. Assume that $k = 8.99 \times 10^9\,\text{N m}^2\,\text{C}^{-2}$.

Thinking	Working
Convert the variables to SI units.	$F = 22.5 \times 10^{-6} = 2.25 \times 10^{-5}\,\text{N}$ $r = 12.7 \times 10^{-3} = 1.27 \times 10^{-2}\,\text{m}$
State Coulomb's law.	$F = k\frac{q_1 q_2}{r^2}$
Substitute the values for F, r and k into the equation and calculate the answer. (Remember to indicate which charge is positive and which is negative in your final answer.)	$q_1 q_2 = \frac{Fr^2}{k}$ $= \frac{2.25 \times 10^{-5} \times (1.27 \times 10^{-2})^2}{8.99 \times 10^{9}}$ $= 4.04 \times 10^{-19}$ Since $q_1 = q_2$: $q_1 q_2 = 4.04 \times 10^{-19}$ $q_1 = \sqrt{4.04 \times 10^{-19}}$ $q_1 = +6.35 \times 10^{-10}\,\text{C}$ $q_2 = -6.35 \times 10^{-10}\,\text{C}$

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Worked example: Try yourself 5.2.2

ELECTRIC FIELD OF A SINGLE POINT CHARGE

Calculate the magnitude and direction of the electric field at a point P that is 15 cm to the right of a positive point charge, Q, of 2.0×10^{-6} C.

Thinking	Working
Determine the known and unknown quantities and convert to SI units as required.	$E = ?$ $Q = 2.0 \times 10^{-6}$ C $r = 15$ cm $= 0.15$ m
Substitute the known values to find the magnitude of E using the equation $E = k\frac{Q}{r^2}$.	$E = k\frac{Q}{r^2}$ $= 8.99 \times 10^9 \times \frac{2.0 \times 10^{-6}}{0.15^2}$ $= 8.0 \times 10^5\ \text{NC}^{-1}$
The direction of the field is defined as that acting on a positive test charge. Point P is to the right of the charge.	Since the charge is positive, the direction will be away from the charge, i.e. to the right.

KEY QUESTIONS

Knowledge and understanding

1 The force is directly proportional to the product of the charges. The force is inversely proportional to the square of the distance between the point charges.

- **a** If one of the charges is doubled to $+2q$, the force will double and the charges repel.
- **b** If both charges are doubled to $+2q$, the force will quadruple and the charges repel.
- **c** If one of the charges is changed to $-2q$, the force will double and the charges attract.
- **d** If the distance between the charges is halved to $0.5r$, the force will quadruple and the charges repel.

2 The question is regarding the electric field, E, between two charges. Using Coulomb's law:

$$F = k\frac{q_1 q_2}{r^2}$$

We do not have information on the values of the charges q_1 and q_2. However, we do know that the distance between the two charges changed from 30 cm to 15 cm and therefore the value of r decreased by a factor of 2. If r decreases by a factor of 2, r^2 decreases by a factor of 4. Therefore the electric field, E, would *increase* by a factor of 4, from $6.0 \times 10^3\ \text{NC}^{-1}$ to $24 \times 10^3\ \text{NC}^{-1}$ (that is, $2.4 \times 10^4\ \text{NC}^{-1}$). Since the distance has been halved, by the inverse square law the field will be four times the original.

3

$$F = k\frac{q_1 q_2}{r^2}$$
$$= 8.99 \times 10^9 \times \frac{1.00 \times 1.00}{1000^2}$$
$$= 8.99 \times 10^3\ \text{N}$$

4 There are two protons in the helium nucleus. Recall that a proton has a charge of $+1.6 \times 10^{-19}$ C. Use Coulomb's law to calculate the force on the protons.

$$F = k\frac{q_1 q_2}{r^2}$$
$$= 8.99 \times 10^9 \times \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(2.5 \times 10^{-15})^2}$$
$$= 36.8$$
$$= 37\ \text{N}$$

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Analysis

5 $F = mg = 0.01 \times 9.8 = 0.098\,\text{N}$

$$F = k\frac{q_1q_2}{r^2}$$

$$0.098 = 8.99\times10^9 \times \frac{3.45\times10^{-9}\times6.5\times10^{-3}}{r^2}$$

$$r^2 = 8.99\times10^9 \times \frac{3.45\times10^{-9}\times6.5\times10^{-3}}{0.098}$$

$$= 2.057$$

$$r = \sqrt{2.057}$$

$$= 1.43\,\text{m}$$

6 Determine the charge on either point using Coulomb's law:

$$F = k\frac{q_1q_2}{r^2}$$

$$1 = 8.99\times10^9 \times \frac{q^2}{0.30^2}$$

$$q = \sqrt{\frac{1\times0.30^2}{8.99\times10^9}}$$

$$= 3.2\times10^{-6}\,\text{C}$$

Note that if you are calculating the charges for q_1 and q_2, each point charge will be equal and opposite, as electrons are being transferred from one point to the other. Since each electron has a charge of 1.6×10^{-19} C, the number of electrons is:

$$\frac{3.2\times10^{-6}}{1.6\times10^{-19}}$$

$$= 2.0\times10^{13} \text{ electrons}$$

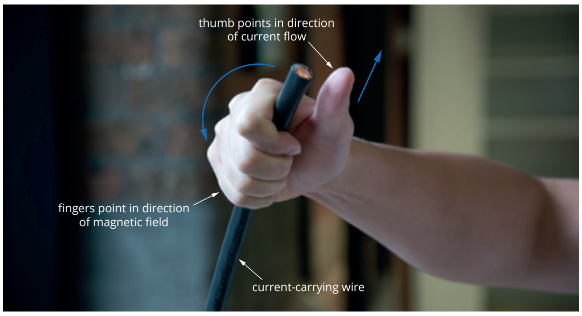
7 In a photocopier, electrostatic charge is applied to a cylindrical drum that is coated with a photoconductive material. This material starts conducting when exposed to light. When the light from the photocopier strikes the document being copied, the white areas reflect the light onto the surface of the photoconductive drum. The areas of the cylindrical drum that are exposed to light start conducting and the areas not exposed to light (black portions of the original document) remain negatively charged. The toner in the photocopier is positively charged, so when it is applied to the drum, the toner is attracted to and sticks to the areas that are negatively charged (black areas).

5.3 The magnetic field

Worked example: Try yourself 5.3.1

DIRECTION OF THE MAGNETIC FIELD

A current-carrying wire runs along the length of a table. The direction of the conventional current, *I*, is towards an observer. What is the direction of the magnetic field created by the current as seen by the observer?

Thinking	Working
Recall that the right-hand grip rule explains how to find the direction of the magnetic field.	Point your thumb towards yourself (as the observer) in the direction of the current flow. Hold your hand with your fingers aligned as if gripping the wire. 
Describe the direction of the field in relation to the wire in simple terms so that the description can be readily understood by a reasonable reader.	The magnetic field direction is perpendicular to the wire. As the current travels along the wire, the magnetic field runs anticlockwise around the wire.

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KEY QUESTIONS

Knowledge and understanding

1 No matter how many times you cut or break a magnet and how small the pieces are, each new piece will be a separate magnet with two poles. As magnets always have two poles, they are said to be dipolar.

2 A magnet suspended freely will behave like a compass. Its north end will point towards the magnetic North Pole as this is effectively the south pole of an imaginary bar magnet within the Earth.

3 With increasing distance, the force between the two magnets decreases. This is the case regardless of the type of force between them: attractive or repulsive.

4 The end labelled B is the south pole. Use the right-hand grip rule to find the direction of the field in the conductor.

Analysis

5 **a** Based on the directions provided, the magnetic field at point A would be directed to the east, i.e. away from the north pole of the magnet at the left. The magnetic field at point C would be directed to the west, i.e. away from the north pole of the magnet at the right.

b A magnetic field is a vector. If a point is equidistant from two magnets and the directions of the two fields are opposite, then the vector sum would be zero.

6 A = east, B = south, C = west, D = north

5.4 Forces on charged objects due to magnetic fields

Worked example: Try yourself 5.4.1

MAGNITUDE OF FORCE ON A POSITIVELY CHARGED PARTICLE

A single positively charged particle with a charge of $+1.6 \times 10^{-19}$ C travels at a velocity of $50\,\text{m s}^{-1}$ perpendicular to a magnetic field of strength 6.0×10^{-5} T.

What is the magnitude of the force the particle will experience from the magnetic field?

Thinking	Working
Check the direction of the particle's velocity and determine whether a force will apply. Forces only apply on the component of the velocity perpendicular to the magnetic field.	The particle is moving perpendicular to the field and so a force will apply: $F = qvB$.
Establish which quantities are known and which are required.	$F = ?$ $q = +1.6 \times 10^{-19}$ C $v = 50\,\text{m s}^{-1}$ $B = 6.0 \times 10^{-5}$ T
Substitute the values into the force equation.	$F = qvB$ $= 1.6 \times 10^{-19} \times 50 \times 6.0 \times 10^{-5}$
Express your final answer in an appropriate form with the correct number of significant figures. Note that only magnitude has been requested, so do not include direction.	$F = 4.8 \times 10^{-22}$ N

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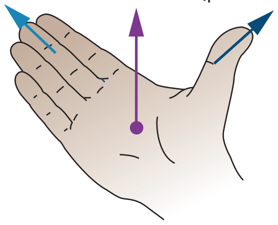
Worked example: Try yourself 5.4.2

DIRECTION OF FORCE ON A NEGATIVELY CHARGED PARTICLE

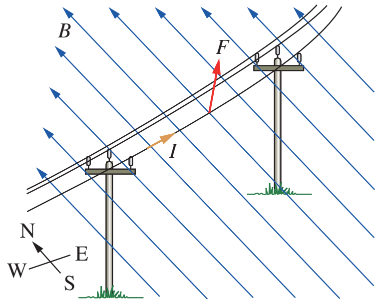
A single negatively charged particle with a charge of -1.6×10^{-19} C is travelling horizontally from left to right across a computer screen and perpendicular to a magnetic field that runs vertically down the screen. In what direction will the force experienced by the charge act?

Thinking	Working
The right-hand force rule is used to determine the direction of the force on a positive charge.  	Align your hand so that your fingers are pointing downwards in the direction of the magnetic field. If the negatively charged particle is travelling from left to right, a positively charged particle is moving in the opposite direction, i.e. from right to left. Align your thumb so that it is pointing left, in the direction that a positive charge would travel. Your palm is facing outwards from the screen, which is the direction of the force applied by the magnetic field on a negative charge.

Worked example: Try yourself 5.4.3

MAGNITUDE OF THE FORCE ON A CURRENT-CARRYING WIRE

Determine the magnitude of the force per metre due to the Earth's magnetic field that acts on a single suspended power line running east–west at the moment it carries a current of 50 A. Assume that the strength of the Earth's magnetic field at this point is 5.0×10^{-5} T.



Thinking	Working
Check the direction of the conductor and determine whether a force will apply. Forces only apply to the component of the wire that is perpendicular to the magnetic field.	As the current is running east–west and the Earth's magnetic field runs south–north, the current and the field are at right angles and a force will exist.
Establish the quantities that are known and what is required. Since the length of the power line hasn't been supplied, consider the force per unit length (i.e. 1 m).	$F = ?$ $n = 1$ $I = 50$ A $l = 1.0$ m $B = 5.0 \times 10^{-5}$ T
Substitute the values into the force equation and simplify it.	$F = nIlB$ $= 1 \times 50 \times 1.0 \times 5.0 \times 10^{-5}$ $= 2.5 \times 10^{-3}$ N
Express your final answer in an appropriate form with the correct number of significant figures. Note that only magnitude has been requested, so do not include direction.	$F = 2.5 \times 10^{-3}$ N per metre of power line

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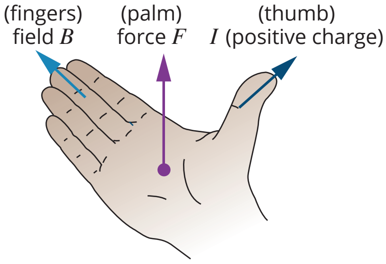
Worked example: Try yourself 5.4.4

DIRECTION OF THE FORCE ON A CURRENT-CARRYING WIRE

A current balance is used to measure the force from a magnetic field on a wire of length 5.0 cm running perpendicular to the field. The conventional current direction in the wire is from left to right. The magnetic field can be considered to be running towards you (the observer). What is the direction of the force on the wire?

Thinking	Working
The right-hand force rule is used to determine the direction of the force. 	Align your hand so that your fingers are pointing in the direction of the magnetic field, i.e. towards you. Align your thumb so it is pointing to the right in the direction of the current. Your palm is facing downwards.
State the direction in terms of the other directions given in the question. Make the answer as clear as possible to avoid any misunderstanding.	The force on the charge is acting vertically downwards.

Worked example: Try yourself 5.4.5

FORCE AND DIRECTION ON A CURRENT-CARRYING WIRE

Santa's house sits at a point that can be considered the Earth's magnetic North Pole (which behaves like the south pole of a magnet).

Assuming the strength of the Earth's magnetic field at this point is 5.0×10^{-5} T, calculate the magnetic force and its direction on the following current-carrying wires.

a a 2.0 m length of wire carrying a conventional current of 10.0 A vertically up the outside wall of Santa's house

Thinking	Working
Forces only apply to the components of the wire running perpendicular to the magnetic field. The direction of the magnetic field at the magnetic North Pole will be almost vertically downwards.	The section of the wire running up the wall of the building will be parallel to the magnetic field, *B*. Hence, no force will apply.
State your answer. A numeric value is required. No direction is required with a zero answer.	$F = 0$ N

b a 2.0 m length of wire carrying a conventional current of 10.0 A running horizontally right to left across the back wall of Santa's house

Thinking	Working
Forces only apply to the components of the wire running perpendicular to the magnetic field. The direction of the magnetic field at the magnetic north pole will be almost vertically downwards.	The section of the wire horizontally through the building will be perpendicular to the magnetic field, *B*. Hence a force will apply.
Identify the known quantities.	$F = ?$ $n = 1$ $I = 10.0$ A $l = 2.0$ m $B = 5.0 \times 10^{-5}$ T

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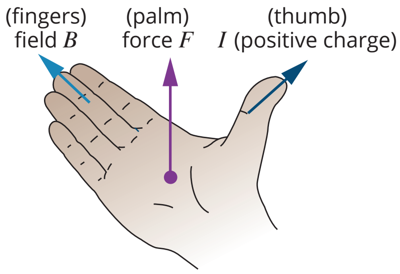
Substitute the values into the appropriate equation and simplify.	$F = nIlB$ $= 1 \times 10.0 \times 2.0 \times 5.0 \times 10^{-5}$ $= 1.0 \times 10^{-3}$ N
The direction of the magnetic force is also required to fully specify the vector quantity. Determine the direction of the magnetic force using the right-hand force rule. 	Align your hand so that your fingers are pointing in the direction of the magnetic field, i.e. vertically down. Align your thumb so that it is pointing left in the direction of the current. Your palm should be facing outwards (out from the house). That is the direction of the force applied by the magnetic field on the wire.
State the magnetic force in an appropriate form with a suitable number of significant figures. Include the direction to fully specify the vector quantity.	$F = 1.0 \times 10^{-3}$ N out from the back wall of Santa's house

KEY QUESTIONS

Knowledge and understanding

1 **a** South (S). The palm of the hand will be pointing downwards, indicating that the force will be south (based on the compass directions provided).

b The path followed is C.

c Since *v* is constant and energy is a scalar quantity, the kinetic energy remains constant.

d Path A. The palm of your hand will be pointing upwards, indicating that the force will be north (based on the compass directions provided). The particle will curve upwards as the force changes direction with the changing direction of the negative particle.

e Particles with no charge, e.g. neutrons, could follow path B.

2 Align your hand so that your fingers are pointing in the direction of the magnetic field, i.e. towards you. Align your thumb so that it is pointing left, in the direction of the current. Your palm should be facing upwards. That is the direction of the force applied by the magnetic field on the wire.

3 Recall that $F = nIlB$. Therefore the force is directly proportional to the number of individual wires (n). If the number of wires doubles, the force will double; if the number of wires quadruples, the force will quadruple.

4 0 N. The particle will experience a force of zero newtons because the particle is moving parallel to the magnetic field. In other words there is no component of the motion perpendicular to the field.

5 The section of the wire marked QR is perpendicular to the magnetic field, *B*, and the direction of the force is out of the page. You can confirm this using the right-hand force rule. Align your hand so that your fingers are pointing in the direction of the magnetic field, i.e. to the right, and your thumb is pointing down along the plane of the page. Your palm should be facing out of the page.

6 $F = nIlB$

$= 1 \times 100 \times 200 \times 5.0 \times 10^{-5}$

$= 1.0$ N

Direction: thumb points left (east to west), fingers point north, palm will face down.

Therefore the force is 1.0 N downwards.

7 $F = qvB$

$= 1.6 \times 10^{-19} \times 2 \times 1.5 \times 10^{-5}$

$= 4.8 \times 10^{-24}$ N south

8 The force would double when the velocity doubles. The magnitude of the force becomes 2*F*. The direction of the force is north.

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Analysis

9 $F = nIlB$

$= 1 \times 2 \times 0.05 \times 2 \times 10^{-3}$

$= 2.0 \times 10^{-4}$ N

With reference to the compass provided, the direction is north.

10 **a** $F = nIlB$

$= 1 \times 50 \times 80 \times 4.5 \times 10^{-5}$

$= 0.18$ N downwards

b Same as for part **a**. The change in height has no effect on the perpendicular components of the magnetic field (south–north) and the wire's direction.

5.5 Comparing fields—a summary

KEY QUESTIONS

Knowledge and understanding

1 B. The strength of a static field does not change with time. This is true of most gravitational and magnetic fields since the mass of the object, or the strength of the magnet, is unchanging.

2 **a** monopoles
b monopoles and dipoles
c dipoles

3 The field around a monopole is radial, static and non-uniform.

A monopole is a single point source associated with electrical and gravitational fields. The inverse square law applies to the radial fields around monopoles.

4 The charge on the right is negative. As the field lines run from one charge to the other, the force is one of attraction. Therefore the charge on the right must have the opposite sign to the charge on the left.

5 D. Magnetic fields are only associated with dipoles. The inverse square law only applies to a radial field around point-source monopoles.

6 The direction of a field at any point is defined as the resultant field vector determined by adding the individual field vectors due to each mass, charge or magnetic pole within the field.

Analysis

7 The direction of a field line at any point is the resultant field vector. At either end the field outside the plates is less than between the plates. The horizontal component of the resultant field vector would point outwards.

8

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$8.0 \times 10^{-37} = 6.67 \times 10^{-11} \times \frac{9.1 \times 10^{-31} \times 9.1 \times 10^{-31}}{r^2}$$

$$r = \sqrt{6.67 \times 10^{-11} \times \frac{9.1 \times 10^{-31} \times 9.1 \times 10^{-31}}{8.0 \times 10^{-37}}}$$

$$r = 8.3 \times 10^{-18}\text{ m}$$

9 **a** $F = k\frac{q_1 q_2}{r^2}$

$$= 8.99 \times 10^9 \times \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(0.53 \times 10^{-10})^2}$$

$$= 8.2 \times 10^{-8}\text{ N}$$

b $F_g = G\frac{m_1 m_2}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{9.1 \times 10^{-31} \times 1.67 \times 10^{-27}}{(0.53 \times 10^{-10})^2}$$

$$= 3.6 \times 10^{-47}\text{ N}$$

c The gravitational force of attraction is significantly less than the electrical force of attraction between the two particles. The electrical force of attraction is approximately 2×10^{39} times greater than the gravitational force of attraction.

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Chapter 5 Review

Knowledge and understanding

1 *Electrical potential* is defined as the work done per unit charge to move a charge from infinity to a point in the electric field. The electrical potential at infinity is defined as zero. When you have two points in an electric field (E) separated by a distance (d) that is parallel to the field, the *potential difference* (V) is defined as the change in the electrical potential between these two points.

2 **a** Work is done by the field.
b Work is done on the charged particle.

3 Because the plates are parallel, *a uniform* electric field exists between them. Thus the electric field strength is the same at all points between the plates.

4 **a** With the current turned off, the loop is producing no field. The steady field in the region would be the only contributing field. It has a value of B into the page.
b With the current increased by a factor of four, the loop is producing four times the field ($4B$). The steady field in the region would be contributing to B, thus the total is $5B$ into the page.

5 **a** The distance between the charges is increased by a factor of 3 (to $3r$). This will decrease the force by a factor of 9.
b The distance between the charges is reduced by a factor of 4 to $0.25r$. This will increase the force by a factor of 16.

6 The magnitude of the magnetic force on a conductor aligned so that the current is running parallel to a magnetic field is zero. A component of the conductor's length must be perpendicular to a magnetic field for a force to be created.

7 The palm is the direction of the force applied by the magnetic field, the fingers are pointing in the direction of the magnetic field and the thumb is pointing in the direction of the conventional current.

8
$$\begin{aligned} F &= qE \\ &= 3.00\times10^{-3}\times7.5 \\ &= 0.003\times7.5 \\ &= 0.0225\,\text{N} \end{aligned}$$

9 D

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= 8.99\times10^{9}\times\frac{3\times10^{-5}}{0.30^2} \\ &= 3\times10^{6}\,\text{NC}^{-1}\ \text{upwards} \end{aligned}$$

Since the electric field is defined as that acting on a positive test charge, the field direction would be upwards (i.e. away from the charge, Q).

10
$$\begin{aligned} E &= \frac{V}{d} \\ 1000 &= \frac{V}{0.025} \\ V &= 1000\times0.025 \\ &= 25\,\text{V} \end{aligned}$$

11 Use the equation for work done in a uniform electric, $W = qEd$, to determine the work done on the field.

$$\begin{aligned} W &= qEd \\ &= 5.00\times10^{-18}\times650\times5.00\times10^{-3} \\ &= 1.63\times10^{-17}\,\text{J} \end{aligned}$$

12 $E = \frac{V}{d} = \frac{15\times10^{3}}{0.12} = 125\,000\,\text{V}\,\text{m}^{-1}$

$$\begin{aligned} F &= qE \\ &= 1.6\times10^{-19}\times125\,000 \\ &= 2.0\times10^{-14}\,\text{N} \end{aligned}$$

13
$$\begin{aligned} F &= k\frac{q_1q_2}{r^2} \\ &= 8.99\times10^{9}\times\frac{7.50\times10^{-3}\times2.00\times10^{-9}}{4.00^2} \\ &= 8.43\,\text{mN} \end{aligned}$$

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14 $F = nIlB$

$1.350 = 1 \times I \times 3.7 \times 0.1100$

$$I = \frac{1.35}{0.11 \times 3.7} = 3.3\,\text{A}$$

15 In each case the force is found from $F = nIlB$ as the field is perpendicular to the current.

a $F = 1 \times 10^{-3} \times 5 \times 10^{-3} \times 1 \times 10^{-3}$

$= 5.0 \times 10^{-9}\,\text{N}$ towards you (from the right-hand force rule)

b $F = 1 \times 2.0 \times 1 \times 10^{-2} \times 0.1 = 2.0 \times 10^{-3}\,\text{N}$ away from you

16 The magnetic force exerted on the electron is:

$$\begin{aligned} F &= qvB \\ &= 1.6 \times 10^{-19} \times 7.0 \times 10^{6} \times 8.6 \times 10^{-3} \\ &= 9.6 \times 10^{-15}\,\text{N} \end{aligned}$$

Application and analysis

17 The east–west line would experience the greater magnetic force as it runs perpendicular to the Earth's magnetic field.

18 Recall that the kinetic energy gained by the ion (E_k) is equal to work done (W). Therefore the velocity can be calculated using the equation $E_k = \frac{1}{2}mv^2$ when the kinetic energy is known.

$$\begin{aligned} W &= qV \\ &= 5 \times 1.6 \times 10^{-19} \times 1000 \\ &= 8.0 \times 10^{-16}\,\text{J} \end{aligned}$$

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{2E_k}{m}} \\ &= \sqrt{\frac{2 \times 8.0 \times 10^{-16}}{3.27 \times 10^{-25}}} \\ &= 7.0 \times 10^{4}\,\text{m s}^{-1} \end{aligned}$$

19 $F = mg = k\dfrac{q_1q_2}{r^2}$

$$\begin{aligned} r^2 &= k\frac{q_1q_2}{mg} \\ &= 8.99 \times 10^{9} \times \frac{3.50 \times 10^{-3} \times 5.01 \times 10^{-3}}{4.5 \times 9.8} \\ &= 3574.595 \\ r &= \sqrt{3574.595} \\ &= 59.8\,\text{m} \end{aligned}$$

20 Find the force due to gravity acting on the ball using $F = mg$. Then substitute the value obtained into the equation $F = Eq$ to calculate the charge.

$$\begin{aligned} F &= mg = 7.50 \times 10^{-3} \times 9.8 \\ &= 7.35 \times 10^{-2}\,\text{N} \\ F &= qE \\ q &= \frac{F}{E} = \frac{7.35 \times 10^{-2}}{450} \\ &= +1.63 \times 10^{-4}\,\text{C} \end{aligned}$$

The charge must be positive to provide an upwards force in the vertically upwards field.

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21 a $F = k\frac{q_1 q_2}{r^2}$

$= 8.99\times10^{9}\times\frac{-1.6\times10^{-19}\times-1.6\times10^{-19}}{(5.4\times10^{-12})^2}$

$= 7.9\times10^{-6}\,\text{N}$

b $F_g = G\frac{m_1 m_2}{r^2}$

$= 6.67\times10^{-11}\times\frac{9.1\times10^{-31}\times9.1\times10^{-31}}{(5.4\times10^{-12})^2}$

$= 1.9\times10^{-48}\,\text{N}$

c The gravitational force of attraction is significantly less than the electrical force of attraction between the two particles. The electrical force of attraction is approximately 4×10^{42} times greater than the gravitational force of attraction.

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Chapter 6 Application of field concepts

6.1 Satellite motion

Worked example: Try yourself 6.1.1

CALCULATING NORMAL FORCE

A 68.0 kg student rides a lift down from the top floor of an office block to the ground floor. During the journey the lift accelerates downwards at 1.50 m s^{-2} before travelling at a constant velocity of 3.08 m s^{-1} and then finally decelerating at 3.80 m s^{-2} until it reaches the ground floor. Assume that g = 9.8 m s^{-2}.

a Calculate the normal force acting on the student in the first part of the journey, i.e. while accelerating downwards at 1.50 m s^{-2}.

Thinking	Working
Ensure that the variables are in their standard units.	m = 68.0 kg a = 1.50 m s^{-2} down g = 9.8 m s^{-2} down
Apply the sign and direction convention for motion in one dimension: up is positive and down is negative.	m = 68.0 kg a = −1.50 m s^{-2} g = −9.8 m s^{-2}
Apply the appropriate equations to calculate the normal force.	$F_{net} = F_N + F_g$ $F_N = F_{net} - F_g$ $= ma - mg$ $= (68.0 \times -1.50) - (68.0 \times -9.8)$ $= -102.0 + 666.4$ $= 564.4$ $= 5.6 \times 10^2$ N

b Calculate the normal force acting on the student in the second part of the journey, i.e. while travelling at a constant speed of 3.08 m s^{-1}.

Thinking	Working
Ensure that the variables are in their standard units.	m = 68.0 kg a = 0 m s^{-2} down g = 9.8 m s^{-2} down
Apply the sign and direction convention for motion in one dimension: up is positive and down is negative.	m = 68.0 kg a = 0 m s^{-2} g = −9.8 m s^{-2}
Apply the appropriate equations to calculate the normal force.	$F_{net} = F_N + F_g$ $F_N = F_{net} - F_g$ $= ma - mg$ $= (68.0 \times 0) - (68.0 \times -9.8)$ $= 0 + 666.4$ $= 6.7 \times 10^2$ N to 2 significant figures

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c Calculate the normal force of the student in the last part of the journey, i.e. while travelling downwards and decelerating at $3.80\,m\,s^{-2}$.	
Thinking	**Working**
Ensure that the variables are in their standard units. Also consider that deceleration is acceleration that is opposite to the direction of motion.	$m = 68.0\,kg$ $a = 3.80\,m\,s^{-2}$ up $g = 9.8\,m\,s^{-2}$ down
Apply the sign and direction convention for motion in one dimension: up is positive and down is negative.	$m = 68.0\,kg$ $a = 3.80\,m\,s^{-2}$ $g = -9.8\,m\,s^{-2}$
Apply the appropriate equations to calculate the normal force.	$F_{net} = F_N + F_g$ $F_N = F_{net} - F_g$ $= ma - mg$ $= (68.0 \times 3.80) - (68.0 \times -9.8)$ $= 258.4 + 666.4$ $= 924.8$ $= 9.2 \times 10^2\,N$

CASE STUDY: ANALYSIS

Four satellites

1 A geostationary orbit is where the satellite has an orbital period of 24 hours, so it will appear as though the satellite is stationary with respect to the Earth's surface. This is ideal for weather satellites because it permits them to study the weather and climate of a single region in depth for an extended period.

2 The formula for acceleration due to gravity is $g = \frac{GM}{r^2}$, where M is the mass of the Earth (5.98×10^{24} kg) and r is the distance of the satellites from the centre of the Earth. In each case, r will be the radius of the Earth (6.37×0^{6} m) plus the altitude of the apogee. Thus we have:

$$g_1 = \frac{GM}{r_1^2} = \frac{(6.67\times10^{-11})\times(5.98\times10^{24})}{(3.58\times10^{7}+6.37\times10^{6})^2} = 0.224\,m s^{-2}$$

$$g_2 = \frac{GM}{r_2^2} = \frac{(6.67\times10^{-11})\times(5.98\times10^{24})}{(5.99\times10^{5}+6.37\times10^{6})^2} = 8.21\,m s^{-2}$$

$$g_3 = \frac{GM}{r_3^2} = \frac{(6.67\times10^{-11})\times(5.98\times10^{24})}{(8.33\times10^{5}+6.37\times10^{6})^2} = 7.69\,m s^{-2}$$

3 First compute the average between the perigee and apogee for each satellite using $r_{av} = \frac{\text{perigee} + \text{apogee}}{2}$ (taking care to include the Earth's radius). Then use the formula for circumference ($2\pi r_{av}$) to find the total distance covered by the satellite in one period.

$$\text{circumference}_1 = 2\pi \times \frac{(3.5791\times10^{7}+6.37\times10^{6})+(3.5795\times10^{7}+6.37\times10^{6})}{2} = 2.65\times10^{8}\,m$$

$$\text{circumference}_2 = 2\pi \times \frac{(5.91\times10^{5}+6.37\times10^{6})+(5.99\times10^{5}+6.37\times10^{6})}{2} = 4.38\times10^{7}\,m$$

$$\text{circumference}_3 = 2\pi \times \frac{(8.24\times10^{5}+6.37\times10^{6})+(8.33\times10^{5}+6.37\times10^{6})}{2} = 4.52\times10^{7}\,m$$

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4 Converting the orbital periods in the table for each satellite into seconds gives $T_1 = 86\,400\,\text{s}$, $T_2 = 96.6 \times 60 = 5796\,\text{s}$, $T_3 = 101.4 \times 60 = 6084\,\text{s}$.

5 $$\text{speed}_1 = \frac{\text{circumference}_1}{\text{period}_1} = \frac{2.65\times10^8}{86\,400} = 3.07\times10^3\,\text{m}\,\text{s}^{-1}$$

$$\text{speed}_2 = \frac{\text{circumference}_2}{\text{period}_2} = \frac{4.38\times10^7}{5796} = 7.56\times10^3\,\text{m}\,\text{s}^{-1}$$

$$\text{speed}_3 = \frac{\text{circumference}_3}{\text{period}_3} = \frac{4.53\times10^7}{6084} = 7.45\times10^3\,\text{m}\,\text{s}^{-1}$$

Worked example: Try yourself 6.1.2

WORKING WITH KEPLER'S LAWS

Determine the orbital speed of a satellite that is in a circular orbit of radius 42 100 km around the Earth. Assume that the mass of the Earth is 5.98×10^{24} kg and that G is $6.67 \times 10^{-11}\,\text{N}\,\text{m}^2\,\text{kg}^{-2}$.

Thinking	Working
Convert the radius to standard units.	$r = 42\,100\,\text{km} = 4.21 \times 10^7\,\text{m}$
Choose the appropriate relationship between the orbital speed, v, and the data that has been provided.	$a = g = \frac{GM}{r^2} = \frac{v^2}{r}$
Make v the subject of the equation.	$v = \sqrt{\frac{GM}{r}}$
Substitute the given values and solve for the orbital speed, v.	$v = \sqrt{\frac{GM}{r}}$ $= \sqrt{\frac{6.67\times10^{-11}\times5.98\times10^{24}}{4.21\times10^7}}$ $= 3.08\times10^3\,\text{m}\,\text{s}^{-1}$

Worked example: Try yourself 6.1.3

SATELLITES IN ORBIT

Callisto is the second largest of Jupiter's moons. It is about the same size as the planet Mercury. Callisto has a mass of 1.08×10^{23} kg, an orbital radius of 1.88×10^6 km and an orbital period of 1.44×10^6 s (i.e. 16.7 days).

a Use Kepler's third law to calculate the orbital radius (in km) of Europa, another moon of Jupiter, which has an orbital period of 3.55 days.

Thinking	Working
Note the values for the known satellite. You can work in days and km, as the question only requires a ratio.	Callisto: $r = 1.88 \times 10^6\,\text{km}$ $T = 16.7$ days
For all satellites orbiting the same central mass, $\frac{r^3}{T^2}$ is constant. Calculate this ratio for the satellite whose radius and period is known.	Europa: $\frac{r^3}{T^2} = \frac{(1.88\times10^6)^3}{16.7^2}$ $= 2.38\times10^{16}$
Use this constant value as the ratio to apply to the satellite in question. Make sure that T is in days to match the value used in the previous step.	$\frac{r^3}{T^2} = \text{constant} = 2.38\times10^{16}$ $\frac{r^3}{3.55^2} = 2.38\times10^{16}$

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Make r^3 the subject of the equation.	$r^3 = 3.55^2 \times 2.38 \times 10^{16}$ $= 3.00 \times 10^{17}$
Solve for r. The unit for r is km as the original ratio was calculated using km.	$r = \sqrt[3]{3.00 \times 10^{17}}$ $= 6.70 \times 10^5$ km Europa has a shorter period than Callisto, so Europa has a smaller orbit than Callisto.

b Use the orbital data for Callisto to calculate the mass of Jupiter.	
Thinking	**Working**
Note the values for the known satellite. You must work in SI units to find the mass in kg.	Callisto: $r = 1.88 \times 10^9$ m $T = 1.44 \times 10^6$ s $m = 1.66 \times 10^{23}$ kg $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} Jupiter: $M = ?$
Select the expressions from the equations for centripetal acceleration that best suit your data. $a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} = g$ Use the third and fourth expressions. These use the given variables, r and T, and the constant G, so that a solution can be found for M.	$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$
Transpose the expressions to make M the subject.	$M = \frac{4\pi^2 r^3}{GT^2}$
Substitute the values and solve for M.	$M = \frac{4\pi^2(1.88 \times 10^9)^3}{6.67 \times 10^{-11} \times (1.44 \times 10^6)^2}$ $= 1.90 \times 10^{27}$ kg

c Calculate the orbital speed of Callisto in km s^{-1}.	
Thinking	**Working**
Note the values you will need to use in the equation $v = \frac{2\pi r}{T}$.	Callisto: $r = 1.88 \times 10^6$ km $T = 1.44 \times 10^6$ s $v = ?$
Substitute the values and solve the equation. The answer will be in km s^{-1} if r is expressed in km.	$v = \frac{2\pi r}{T}$ $= \frac{2\pi \times 1.88 \times 10^6}{1.44 \times 10^6}$ $= 8.20$ km s^{-1}

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KEY QUESTIONS

Knowledge and understanding

1 $F_g = mg$
$= 7.20 \times 9.8$
$= 71\text{ N}$

2 Normal force = the force due to gravity for an object at rest: $F_N = 220\text{ N}$

3 $F_{net} = F_N + F_g$
$F_N = F_{net} - F_g$
$= ma - mg$
$= (55.0 \times 2.72) - (55.0 \times -9.8)$
$= 149.6 + 539$
$= 6.9 \times 10^2\text{ N}$

4 $F_{net} = F_N + F_g$
$F_N = F_{net} - F_g$
$= ma - mg$
$= (55.0 \times 0) - (55.0 \times -9.8)$
$= 0 + 539$
$= 5.4 \times 10^2\text{ N}$

5 B. It is the only object that is on a surface and so it can experience an upwards normal force from that surface.

6 D. Satellites orbit around a central mass. The Earth does not orbit Mars, the Moon does not orbit the Sun and the Sun does not orbit the Earth.

Analysis

7 **a** A geostationary satellite orbits at the same rate that the Earth turns, that is, its period is 24 hours.

For satellites in a circular orbit around a central body of mass M: $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$.

First convert the period from hours into seconds: 24 hours = 24 × 60 × 60 = 86 400 seconds.

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$= \sqrt[3]{\frac{(6.67 \times 10^{-11}) \times (5.98 \times 10^{24}) \times (86\,400)^2}{4\pi^2}}$$

$$= 4.23 \times 10^7\text{ m}$$

b Let T_N and r_N denote the period and radius of the Navstar satellite, and T_G and r_G denote the period and radius of the geostationary satellite. Recall that $\frac{r^3}{T^2}$ is a constant for all satellites that orbit a common central body and thus is the same for the Navstar satellite and the geostationary satellite.

$$\frac{r_G^3}{T_G^2} = \frac{r_N^3}{T_N^2}$$

$$\left(\frac{r_N}{r_G}\right)^3 = \left(\frac{T_N}{T_G}\right)^2$$

$$\frac{r_N}{r_G} = \left(\frac{T_N}{T_G}\right)^{\frac{2}{3}}$$

$$= \left(\frac{12}{24}\right)^{\frac{2}{3}}$$

$$= 0.63$$

$$\therefore r_N = 0.63 \times T_G$$

$$= 0.63 \times 4.23 \times 10^7$$

$$= 2.66 \times 10^7\text{ m}$$

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8 **a** $a = g = 0.22\,\text{m s}^{-2}$

b $F_g = mg$

$= 3.2 \times 10^3 \times 0.22$

$= 704\,\text{N}$ (or $7.0 \times 10^2\,\text{N}$ to 2 significant figures)

9 $\frac{r^3}{T^2}$ is constant for all the moons of Saturn, therefore the orbital period for each moon can be calculated from just one moon.

For Atlas:

$\frac{r^3}{T^2} = \frac{(1.37 \times 10^5)^3}{(0.60)^2}$

$= 7.14 \times 10^{15}$

For Titan:

$\frac{r^3}{T^2} = 7.14 \times 10^{15}$

$T^2 = \frac{r^3}{7.14 \times 10^{15}}$

$= 242$

$T = \sqrt{242}$

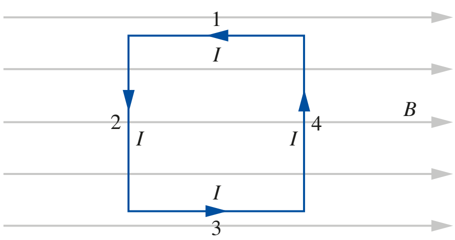
$= 15.6$ days

6.2 DC motors

Worked example: Try yourself 6.2.1

FORCE ON A COIL

A single square of wire with 4.0 cm sides is free to rotate in a magnetic field, B, of strength 1.0×10^{-4} T. A 1.0 A current is flowing through the coil in the direction indicated by the blue arrows. What is the force acting on each side of the coil?



Thinking	Working
Confirm that the coil will experience a force based on the magnetic field and current directions supplied.	Using the right-hand force rule, confirm that a force applies on side 2 out of the page and a force applies to side 4 into the page. The coil will turn clockwise as viewed from in front of side 3. Sides 1 and 3 lie parallel to the magnetic field and so no force will apply.
Calculate the magnitude of the magnetic force on the sides perpendicular to the field.	$F = nIlB$ $= 1 \times 1.0 \times 0.040 \times 1.0 \times 10^{-4}$ $= 4.0 \times 10^{-6}\,\text{N}$
State the magnitude and direction of the force on each side.	The force acting on side 2 will be equal to and opposite the force acting on side 4. Hence $F_2 = 4.0 \times 10^{-6}\,\text{N}$ out of the page and $F_4 = 4.0 \times 10^{-6}\,\text{N}$ into the page.

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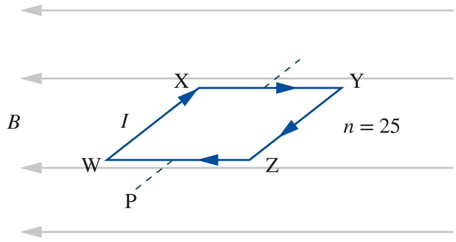
Worked example: Try yourself 6.2.2

A SIMPLE DC MOTOR

A DC motor is constructed using a square coil with 25 turns and sides of length 7.50 cm. It is placed in a magnetic field of strength 0.250 T. The coil carries a current of 200 mA in the direction of the blue arrows.



a Viewed from point P, in which direction will the motor rotate: clockwise or anticlockwise?	
Thinking	**Working**
Apply the right-hand force rule to the sides perpendicular to *B* to determine the direction of the magnetic force.	Aligning your thumb with the current in direction YZ and fingers in the direction of the magnetic field indicates that a downwards force will apply on side YZ. Similarly, applying the right-hand force rule on side WX confirms that an upwards force applies to the side. Thus viewed from P the coil will rotate clockwise.

b Calculate the magnitude of the torque acting on side WX.	
Thinking	**Working**
Write down the formula for magnetic force and convert the relevant quantities in SI units.	$F = nIlB$ $n = 25$ $I = 200\text{ mA} = 0.200\text{ A}$ $l = 7.50\text{ cm} = 0.075\text{ m}$ $B = 0.250\text{ T}$
Substitute the values and calculate the total force.	$F = nIlB$ $= 25 \times 0.200 \times 0.075 \times 0.250$ $= 0.09375\text{ N}$
Using the side lengths given, calculate the torque.	$\tau = r_{\perp}F$ $= 0.075 \times 0.09375$ $= 0.0070\text{ Nm}$

c The number of turns in the coil is reduced to 10. What will be the new magnitude of the force acting on side WX?	
Thinking	**Working**
Note how magnetic force depends on the number of coils.	Since $F = nIlB$, if the number of coils, n, is reduced to 10, the magnetic force will be reduced to $\frac{10}{25} = 0.4$ of its original value, that is: $0.4 \times 0.09375 = 0.0375\text{ N}$.

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KEY QUESTIONS

Knowledge and understanding

1 A. The maximum torque occurs when the force is applied perpendicular to (i.e. at right angles to) the axis of rotation.

2 The force acting on each side of the coil will be equal in magnitude, but the resulting torques from each side will be opposite in direction. Since the total torque is the sum of the individual torques, the net torque will be zero and hence the coil will not turn.

3 **a** $F = nIlB$

$= 1 \times 3.5 \times 0.060 \times 0.25$

$= 5.3 \times 10^{-2}$ N into the page

b $F = nIlB$

$= 1 \times 3.5 \times 0.060 \times 0.25$

$= 5.3 \times 10^{-2}$ N out of the page

c The force will be 0 N. This is because side PQ is parallel to the magnetic field.

d Considering the direction of the forces acting on sides PS and QR, the coil will rotate anticlockwise.

e D. The direction of the current does not affect the magnitude of the torque. This is the only option that doesn't affect the torque from the options available.

Analysis

4 **a** We can compute the force acting on each side separately. Using $F = nIlB$, we have:

$F_{left} = 1 \times 2.75 \times 0.25 \times 0.75 = 0.52$ N

$F_{right} = 1 \times 2.75 \times 0.25 \times 1.25 = 0.86$ N

b Using $\tau = r_{\perp}F$, we can compare the quantities to see which is greater. Since both sides of the coil are the same radial distance from the axis of rotation, then the one with a greater force has the greater individual torque. Hence τ_{right} contributes more to the total torque.

c Using $\tau = r_{\perp}F$, the force acting on the coils will be the same, but the radial distance of each side has doubled. Hence the total torque will double.

5 **a** $F = nIlB$

$= 15 \times 1.0 \times 0.50 \times 0.20$

$= 1.5$ N

b Current flows into brush P and around the coil from V to X to Y to W. The force on side VX is down and the force on side YW is up, so the rotation is anticlockwise.

c The commutator keeps the motor rotating in the same direction by reversing the polarity of the current after each half rotation. The net effect is that the left and right sides of the coil always have their direction of current pointing either upwards or downwards, regardless of whether the coil has turned. If the coil were directly connected to the battery, it would rotate half a revolution before experiencing a force in the other direction. It would continue to oscillate backwards and forwards rather than rotating smoothly.

d Since $F = nIlB$, if the number of turns in the coil is increased to 30, the force will double. However, by decreasing the side length from 0.50 m to 0.125 m, the force will decrease to one quarter. Overall the coil will experience half the amount of force.

6.3 Particle accelerators

Worked example: Try yourself 6.3.1

CALCULATING THE SPEED OF ACCELERATED CHARGED PARTICLES

Determine the final speed of a single electron, with a charge of 1.6×10^{-19} C and a mass of 9.1×10^{-31} kg, when accelerating from rest across a potential difference of 1.2 kV.

Thinking	Working
Ensure that the variables are in their standard units.	$1.2\ \text{kV} = 1.2 \times 10^3\ \text{V}$
Establish what quantities are known and what is required.	$v = ?$ $q = 1.6 \times 10^{-19}$ C $m = 9.1 \times 10^{-31}$ kg $V = 1.2 \times 10^3$ V

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Substitute the values into the electron gun equation and rearrange it to solve for the speed.	$qV = \frac{1}{2}mv^2$ $1.6 \times 10^{-19} \times 1.2 \times 10^3 = \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2$ $v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 1.2 \times 10^3}{9.1 \times 10^{-31}}}$ $= 2.1 \times 10^7\ \text{m s}^{-1}$

Worked example: Try yourself 6.3.2

CALCULATING SPEED AND PATH RADIUS OF ACCELERATED CHARGED PARTICLES

An electron gun releases a beam of electrons from its cathode. They are accelerated across a potential difference of 2.5 kV between a pair of charged parallel plates 20 cm apart. Assume that the mass of an electron is 9.1×10^{-31} kg and the magnitude of its charge is 1.6×10^{-19} C.

a Calculate the strength of the electric field acting on the electron beam.

Thinking	Working
Ensure that the variables are in their standard units.	$2.5\ \text{kV} = 2.5 \times 10^3\ \text{V}$ $20\ \text{cm} = 0.20\ \text{m}$
Apply the correct equation.	$E = \frac{V}{d}$
Solve for *E*.	$E = \frac{2.5 \times 10^3}{0.20}$ $= 1.3 \times 10^4\ \text{V m}^{-1}$

b Calculate the speed of the electrons as they leave the electron gun.

Thinking	Working
Apply the correct equation.	$\frac{1}{2}mv^2 = qV$
Rearrange the equation to make *v* the subject.	$v = \sqrt{\frac{2qV}{m}}$
Solve for *v*.	$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 2.5 \times 10^3}{9.1 \times 10^{-31}}}$ $= 3.0 \times 10^7\ \text{m s}^{-1}$

c The electrons then travel through a uniform magnetic field perpendicular to their motion. If this field is of strength 0.3 T, calculate the expected radius of the path of the electron beam.

Thinking	Working
Apply the correct equation.	$r = \frac{mv}{qB}$
Solve for *r*.	$r = \frac{9.1 \times 10^{-31} \times 3.0 \times 10^7}{1.6 \times 10^{-19} \times 0.3}$ $= 5.7 \times 10^{-4}\ \text{m}$

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KEY QUESTIONS

Knowledge and understanding

1 C. A charged particle moving in a magnetic field will experience a force.

2 $F = qvB$

$= 1.6 \times 10^{-19} \times 2.0 \times 3.2 \times 10^{-4}$

$= 1.0 \times 10^{-22}$ N

The electron will experience a force of 1.0×10^{-22} N south.

3 $\frac{1}{2}mv^2 = qV$

$v = \sqrt{\frac{2qv}{m}}$

$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 2.5 \times 10^3}{9.1 \times 10^{-31}}}$

$= 3.0 \times 10^7\ \text{m s}^{-1}$

4 **a** $F = qvB$

$= 1.6 \times 10^{-19} \times 4.1 \times 10^6 \times 9.8 \times 10^{-3}$

$= 6.4 \times 10^{-15}$ N

b $r = \frac{mv}{qB}$

$= \frac{(9.1 \times 10^{-31}) \times (4.1 \times 10^6)}{(1.6 \times 10^{-19}) \times (9.8 \times 10^{-3})}$

$= 2.4 \times 10^{-3}$ m

5 $r = \frac{mv}{qB}$

$B = \frac{mv}{qr}$

$= \frac{9.1 \times 10^{-31} \times 7.6 \times 10^6}{1.6 \times 10^{-19} \times 4.6 \times 10^{-2}}$

$= 9.4 \times 10^{-4}$ T

Note: The radius, r, is half the diameter: $0.5 \times 9.2 \times 10^{-2} = 4.6 \times 10^{-2}$ m

6 $r = \frac{mv}{qB}$

$v = \frac{rqB}{m}$

$= 0.1 \times \left(1.76 \times 10^{11}\right) \times \left(4.0 \times 10^{-4}\right)$

$= 7.0 \times 10^6\ \text{m s}^{-1}$

7 A charged particle in a magnetic field will experience a force ($F = qvB$). As force is proportional to velocity, the force will increase as the velocity increases. This will continue while the charge remains in the magnetic field, continuously accelerating the charge.

8 $E_k = \frac{1}{2}mv^2 = qV$

$v = \sqrt{\frac{2qV}{m}}$

$= \sqrt{\frac{2 \times (1.6 \times 10^{-19}) \times (4.5 \times 10^3)}{9.1 \times 10^{-31}}}$

$= 3.98 \times 10^7\ \text{m s}^{-1}$

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Because the forces acting on the electron due to the electric and magnetic fields are balanced, you can equate them:

$F_E = qE$ and $F_B = qvB$

$qE = qvB$

$E = vB$

$= 3.98 \times 10^7 \times 2.3 \times 10^{-3}$

$= 9.15 \times 10^4$ J

$= \frac{V}{d}$

$d = \frac{4500}{9.15 \times 10^4}$

$= 4.9 \times 10^{-2}$ m or 4.9 cm

Analysis

9 **a** $W = qV = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2qV}{m}}$

$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 4.5 \times 10^4}{9.1 \times 10^{-31}}}$

$= 1.3 \times 10^8$ m s^{-1}

b Using the formula for bending radius:

$r = \frac{mv}{qB}$

$= \frac{(9.1 \times 10^{-31}) \times (1.26 \times 10^8)}{(1.6 \times 10^{-19}) \times (1.0 \times 10^{-3})}$

$= 0.72$ m

Since this is less than 5 m, the bending radius is too small for the particle accelerator. The electrons are curving inwards too soon. Because the radius of the path travelled by an electron in a uniform magnetic field is proportional to its velocity, if we wish to increase the radius we must increase the velocity of the electrons. To have a higher velocity the electrons must be given more kinetic energy, which means they must be accelerated across a larger potential difference. Hence Sally should increase the potential difference of the electron gun.

c Rearranging the formula for the bending radius:

$v = \frac{qrB}{m}$

$= \frac{(1.6 \times 10^{-19}) \times 5.0 \times (1.0 \times 10^{-3})}{9.1 \times 10^{-31}}$

$= 8.8 \times 10^8$ m s^{-1}

This is the velocity the electrons need to be accelerated to in order to maintain a bending radius of 5 m. Given that $E = \frac{V}{d}$ and kinetic energy $= \frac{1}{2}mv^2 = qV$, it follows that:

$qEd = \frac{1}{2}mv^2$

$E = \frac{mv^2}{2qd}$

$= \frac{(9.1 \times 10^{-31}) \times (8.8 \times 10^8)^2}{2 \times (1.6 \times 10^{-19}) \times 0.40}$

$= 5.5 \times 10^6$ V m^{-1}

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Chapter 6 Review

Knowledge and understanding

1 $F_{net} = F_N + F_g$

$$\begin{aligned} F_N &= F_{net} - F_g \\ &= ma - mg \\ &= (38.0 \times -2.95) - (38.0 \times -9.8) \\ &= -112.1 + 372.4 \\ &= 2.6 \times 10^2\ \text{N} \end{aligned}$$

2 D. Objects in orbit are in free fall. While in orbit around the Earth, gravity is reduced, but it is still significant.

3 D. At this altitude, gravity is reduced and so will be less than 9.8 N kg^{-1}. Hence acceleration is less than 9.8 m s^{-2}. Note: B is not correct because, while the speed of the satellite would be constant, its velocity is not.

4 A. No normal force is felt during free fall.

5 $\frac{r^3}{T^2}$ is constant for all satellites of Earth. Therefore the orbital period for each satellite can be calculated.

For X:

$$\frac{r^3}{T_X^2} = \text{constant} = k$$

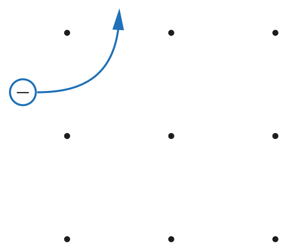
For Y:

$$\begin{aligned} \frac{(5r)^3}{T_Y^2} &= k \\ &= \frac{r^3}{T_X^2} \\ \frac{125r^3}{T_Y^2} &= \frac{r^3}{T_X^2} \\ T_Y^2 &= \frac{125r^3}{r^3}T_X^2 \\ &= 125T_X^2 \\ T_Y &= 11.2T_X \end{aligned}$$

6 The commutator's function is to reverse the direction of the current in the coil after every half turn. This keeps the coil rotating in the same direction.

7 Electrons are released from a negative terminal (or hot cathode) of the evacuated tube and accelerate towards a positively charged anode. They can be detected as they hit a fluorescent screen at the rear of the tube. The electrons are accelerated by a large potential difference between the cathode and anode.

8 **a** The electron will experience a force at right angles to its motion. This acts upwards and causes the electron to curve in an upwards arc from its starting position.



b The radius of the electron's path is dependent on its velocity and the magnitude of the magnetic field that is acting on it.

9 $r = \frac{mv}{qB}$

$$\begin{aligned} B &= \frac{mv}{qr} \text{ (where } r = \text{half the diameter)} \\ &= \frac{9.1 \times 10^{-31} \times 4.3 \times 10^6}{1.6 \times 10^{-19} \times 4.2 \times 10^{-2}} \\ &= 5.8 \times 10^{-4}\ \text{T} \end{aligned}$$

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10 **a** $F = qvB$

$= 1.6 \times 10^{-19} \times 6.4 \times 10^{6} \times 9.1 \times 10^{-3}$

$= 9.3 \times 10^{-15}$

b $r = \dfrac{mv}{qB}$

$= \dfrac{9.1 \times 10^{-31} \times 6.4 \times 10^{6}}{1.6 \times 10^{-19} \times 9.1 \times 10^{-3}}$

$= 4.0 \times 10^{-3}\,\text{m}$

Application and analysis

11 **a** $a = \dfrac{GM}{r^2}$

$= \dfrac{6.67 \times 10^{-11} \times 1.02 \times 10^{26}}{(3.55 \times 10^{8})^2}$

$= 0.0540\,\text{m s}^{-2}$

b $a = \dfrac{v^2}{r}$

$v = \sqrt{ar}$

$= \sqrt{0.054 \times 3.55 \times 10^{8}}$

$= 4.38 \times 10^{3}\,\text{m s}^{-1}$

c $F_g = \dfrac{mv^2}{r} = \dfrac{4\pi^2 rm}{T^2} = \dfrac{GMm}{r^2} = mg$

$\dfrac{4\pi^2 rm}{T^2} = \dfrac{GMm}{r^2}$

$T^2 = \dfrac{4\pi^2 r^3}{GM}$

$= \dfrac{4\pi^2 (3.55 \times 10^{8})^3}{6.67 \times 10^{-11} \times 1.02 \times 10^{26}}$

$= 2.60 \times 10^{11}$

$T = \sqrt{2.60 \times 10^{11}}$

$= 5.09 \times 10^{5}\,\text{s}$

$1\text{ day} = 24 \times 60 \times 60 = 86\,400\,\text{s}$

$T = \dfrac{5.09 \times 10^{5}}{86\,400}$

$= 5.90\text{ days}$

12 **a** $a = \dfrac{GM}{r^2} = g$

$g = \dfrac{GM}{r^2}$

$= \dfrac{6.67 \times 10^{-11} \times 9.38 \times 10^{20}}{(4.70 \times 10^{5})^2}$

$= 0.283\,\text{N kg}^{-1}$

b $\dfrac{mv^2}{r} = \dfrac{GMm}{r^2}$

$v = \sqrt{\dfrac{GM}{r}}$

$= \sqrt{\dfrac{6.67 \times 10^{-11} \times 9.38 \times 10^{20}}{4.80 \times 10^{5}}}$

$= 361\,\text{m s}^{-1}$

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13 **a** down the page

b up the page

14 anticlockwise

15 **a** down the page

b up the page

c There is zero torque, as the forces are trying to pull the coil apart rather than turn it. The force is parallel to the coil rather than perpendicular to it.

16 C. Reversing the direction of the current in the loop will ensure that the loop keeps travelling in the same direction. Use the right-hand force rule to verify this.

17 Using $F = nIlB$, we have $n = 8$, $I = 30\,\text{mA} = 0.030\,\text{A}$, $l = 15\,\text{cm} = 0.15\,\text{m}$, $B = 0.25\,\text{T}$.

$F = 8 \times 0.030 \times 0.15 \times 0.25 = 9.0 \times 10^{-3}\,\text{N}$

18 Using $\tau = r_{\perp}F$ for each side, we can see that each torque depends linearly on the perpendicular radius. So if we double the side lengths, then each torque will double as well. Since the total torque is equal to the sum of all the torques, the total torque is also linearly dependent on the length of the sides of the square coil.

19 **a** $\frac{1}{2}mv^2 = qV$

$$v = \sqrt{\frac{2qV}{m}}$$
$$= \sqrt{\frac{2 \times (1.6 \times 10^{-19}) \times (10 \times 10^3)}{9.1 \times 10^{-31}}}$$
$$= 5.9 \times 10^7\,\text{m}\,\text{s}^{-1}$$

b $r = \frac{mv}{qB}$

$$= \frac{9.1 \times 10^{-31} \times 5.9 \times 10^7}{1.6 \times 10^{-19} \times 1.5}$$
$$= 2.2 \times 10^{-4}\,\text{m}$$

20 **a** The strength of the electric field between the charged plates is given by:

$$E = \frac{V}{d}$$
$$= \frac{500}{3.5 \times 10^{-2}}$$
$$= 1.4 \times 10^4\,\text{V}\,\text{m}^{-1}$$

b Because the strengths of the electric field and the magnetic field are balanced, it follows that:

$$F_B = F_E$$
$$qvB = qE$$
$$v = \frac{E}{B}$$
$$= \frac{1.4 \times 10^4}{1.5 \times 10^{-3}}$$
$$= 9.3 \times 10^6\,\text{m}\,\text{s}^{-1}$$

21 $\frac{1}{2}mv^2 = qV$

$$v = \sqrt{\frac{2qV}{m}}$$
$$= \sqrt{\frac{2 \times \left(1.6 \times 10^{-19}\right) \times \left(4.5 \times 10^3\right)}{9.1 \times 10^{-31}}}$$
$$= 4.0 \times 10^7\,\text{m}\,\text{s}^{-1}$$

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Unit 3 Area of Study 2

How do things move without contact?

Multiple-choice questions

1 A. During launch the normal force acting on the astronaut will be greater than usual. She will feel a force pushing her downwards, but the force due to gravity on her has not increased.

2 B. The gravitational force will be constant during the launch.

3 B. In a stable orbit there is no normal force acting on the astronaut ($F_N = 0$) so she will experience apparent weightlessness. However, the gravitational force is still acting to keep her and the spacecraft in orbit.

4 C. In deep space the astronaut would be far away from planets or other large masses that could exert a significant force of gravity on her, so she will experience very little force due to gravity.

5 D. 2.0 N

$$F = \frac{kq_1q_2}{r^2}$$
$$= \frac{8.99\times10^9\times5.0\times10^{-6}\times7.0\times10^{-6}}{0.40^2}$$
$$= 2.0\,\text{N}$$

6 B. $6.9 \times 10^9\,\text{N C}^{-1}$ to the left, away from the charge

$$E = \frac{kq}{r^2}$$
$$= \frac{8.99\times10^9\times9.4\times10^{-6}}{(3.5\times10^{-3})^2}$$
$$= 6.9\times10^9\,\text{NC}^{-1}\text{ to the left}$$

7 A. Both have fields in direction A.

8 B. There is a field in the BC direction from the left-hand current, and in the AB direction from the right-hand current. Vertically, the fields cancel out at point Q. (The field from the left-hand current is downwards, while the field from the right-hand current is upwards.)

9 G. The field in directions A and C cancel each other.

10 C and D. C will increase the area of the coils and D will increase the magnetic field through the coils, both of which will increase the torque in the motor. A will reduce the force on the coils and B will reduce the current through the coils, both of which will decrease torque.

11 B. 60 N

At 300 km, $g \approx 3.0\,\text{N kg}^{-1}$

$F_g = mg = 20 \times 3.0 = 60\,\text{N}$

12 C. 36 MJ

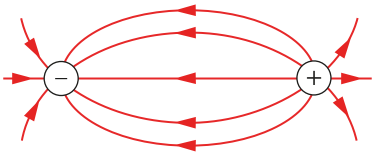
Area ≈ 9 squares = $9 \times 1.0 \times 2.0 \times 10^5 = 1.8 \times 10^6\,\text{J kg}^{-1}$

ΔE_k = area × mass = $1.8 \times 10^6 \times 20 = 3.6 \times 10^7\,\text{J}$

$\Delta E_k = 36\,\text{MJ}$

Short-answer questions

13



14 **a** E_g = area under graph between 7.0×10^6 m and 6.5×10^6 m. Counting squares below the graph gives 8.5 squares. The area of each square is $1.0 \times 10^4 \times 0.5 \times 10^6 = 5 \times 10^9$ J. Therefore:

$$E_g = 8.5\times5\times10^9$$
$$= 4.25\times10^{10}$$
$$= 4.3\times10^{10}\,\text{J}$$

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b At an altitude of 600 km (a height of 7000 km):

$$E_k = \frac{1}{2}mv^2$$
$$= 0.5 \times 10\,000 \times 1500^2$$
$$= 1.125 \times 10^{10}\text{ J}$$

At an altitude of 100 km altitude (height of 6500 km):

$$E_k = 1.125 \times 10^{10} + 4.25 \times 10^{10}$$
$$= 5.375 \times 10^{10}\text{ J}$$

From this it follows that:

$$\frac{1}{2}mv^2 = 5.375 \times 10^{10}$$
$$0.5 \times 10\,000v^2 = 5.375 \times 10^{10}$$
$$v = 3.3 \times 10^3\text{ m s}^{-1}$$

c **i** $r = 6400 + 3600 = 10\,000$ km $= 10 \times 10^6$ m

$F_g = 4.0 \times 10^4$ N (from the graph)

ii $r = 6.0 \times 10^5 + 6.4 \times 10^6 = 7.0 \times 10^6$ m

$F_g = 8.1 \times 10^4$ N (from the graph)

d At 600 km, $F_g = 8.1 \times 10^4$ N, so $a = \frac{F_g}{m} = 8.1\text{ m s}^{-2}$

At 100 km, $F_g = 9.2 \times 10^4$ N, so $a = \frac{F_g}{m} = 9.2\text{ m s}^{-2}$

The acceleration increases from 8.1 m s^{-2} to 9.2 m s^{-2}.

15 **a** $E = \frac{V}{d}$

$$= \frac{240}{1.6 \times 10^{-3}}$$
$$= 1.5 \times 10^5\text{ N C}^{-1}\text{(or V m}^{-1}\text{) downwards}$$

b $F_e = F_g$

$$= -1.96 \times 10^{-14} \times 9.8$$
$$= 1.92 \times 10^{-13}$$
$$= 1.9 \times 10^{-13}\text{ N upwards (acting against gravity to keep the drop stationary)}$$

c $q = \frac{F}{E}$

$$= \frac{1.92 \times 10^{-13}}{1.5 \times 10^5}$$
$$= 1.3 \times 10^{-18}\text{ C}$$

16 **a** The force is to the left, due to magnetic attraction to the soft-iron core.

b The force is more strongly to the left, as the right end of the electromagnet is now a south pole.

c The force is to the right as the right end of the electromagnet is now a north pole.

17 **a** $F = nIlB = 1 \times 100 \times 1.0 \times 1.0 \times 10^{-5} = 1.0 \times 10^{-3}$ N

b The right-hand rule tells us that a current from west to east will experience an upwards force.

c The force due to gravity acting on 1 m of cable is $mg = 0.05 \times 9.8 = 0.49$ N. For the magnetic force to equal this:

$$I = \frac{F}{nlB} = \frac{0.49}{1 \times 1.0 \times 1.0 \times 10^{-5}} = 4.9 \times 10^4\text{ A}$$

So there is not much chance of magnetic levitation for power cables!

18 **a** The field is from N to S, so the right-hand rule shows that the force on side AB is upwards and the force on side CD is downwards.

b In the position shown (with the coil horizontal), the direction of the forces on sides AB and CD are at right angles to the radius and the torque is at a maximum.

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c The torque becomes zero when the coil is in the vertical position. It continues to rotate for two reasons: (i) its momentum will carry it past the true vertical position and (ii) at the vertical position, the commutator reverses the direction of the current through the coil. When the current reverses, the forces reverse, and thus the coil continues to rotate for another half turn, at which point the current reverses again and the rotation continues.

d $I = \frac{F}{nlB} = \frac{40}{100 \times 0.2 \times 0.5} = 4.0\,\text{A}$

e $\tau = r_{\perp}F$
$= 0.20 \times 40$
$= 8.0\,\text{Nm}$

19 a The increase in kinetic energy of the electron as it travels from one plate to another is:

$$E_k = \frac{1}{2}mv^2 = qV$$

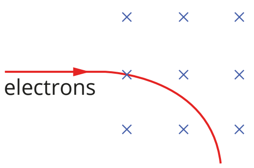
So:

$$v^2 = \frac{2qV}{m}$$

$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 3.0 \times 10^3}{9.1 \times 10^{-31}}}$$

$$= 3.2 \times 10^7\,\text{m s}^{-1}$$

b



c $r = \frac{mv}{qB}$

$$= \frac{9.1 \times 10^{-31} \times 3.2 \times 10^7}{1.6 \times 10^{-19} \times 1.2}$$

$$= 1.5 \times 10^{-4}\,\text{m}$$

20 a $g = G\frac{M}{r^2}$

$$= 6.67 \times 10^{-11} \times \frac{1.90 \times 10^{27}}{(1.07 \times 10^9)^2}$$

$$= 1.11 \times 10^{-1}\,\text{N kg}^{-1}$$

b $F_g = mg$

$$= 5.3 \times 10^3 \times 1.11 \times 10^{-1}$$

$$= 5.9 \times 10^2\,\text{N}$$

c $T = \sqrt{\frac{4\pi^2 r^3}{GM}}$

$$= \sqrt{\frac{4\pi^2(1.07 \times 10^9)^3}{6.67 \times 10^{-11} \times 1.90 \times 10^{27}}}$$

$$= 6.18 \times 10^5\,\text{s}$$

d The mass of the satellite has no effect on its orbital period.

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Chapter 7 Electromagnetic induction and transmission of electricity

7.1 Inducing an EMF in a magnetic field

Worked example: Try yourself 7.1.1

MAGNETIC FLUX

A student places a horizontal square coil of wire with sides of length 4.0 cm into a uniform vertical magnetic field of 0.050 T. How much magnetic flux passes through the coil?

Thinking	Working
Calculate the area of the coil perpendicular to the magnetic field.	side length = 4.0 cm = 0.04 m area of the square = $(0.04)^2 = 0.0016\ m^2$
Calculate the magnetic flux.	$\Phi_B = B_{\perp}A$ $= 0.050 \times 0.0016$ $= 0.00008$ Wb
State the answer in an appropriate form.	$\Phi_B = 8.0 \times 10^{-5}$ Wb

Worked example: Try yourself 7.1.2

MAGNETIC FLUX AT AN ANGLE

A student places a horizontal square coil of wire with sides of length 5.0 cm into a uniform vertical magnetic field of 0.10 T. The plane of the coil is parallel to the magnetic field. How much magnetic flux passes through the coil?

Thinking	Working
All the magnetic field lines pass through the coil when the coil is perpendicular to the magnetic field. None of the magnetic field lines pass through the coil when the coil is parallel to the magnetic field.	Since the plane of the coil is parallel to the magnetic field, none of the magnetic field lines pass through the coil. Therefore there is no magnetic flux through the coil.

Worked example: Try yourself 7.1.3

ELECTROMOTIVE FORCE ACROSS AN AIRCRAFT'S WINGS

Consider a fighter jet with a wing span of 25 m, flying at a speed of 2000 km h^{-1} at right angles to the Earth's magnetic field (5.0×10^{-5} T). Will the jet develop a dangerous EMF between its wing tips solely from the Earth's magnetic field?

Thinking	Working
Identify the quantities required in their correct units.	$\varepsilon = ?$ $l = 25$ m $B = 5.0 \times 10^{-5}$ T $v = 2000$ km h^{-1} = 556 m s^{-1}
Substitute the values into the appropriate formula and calculate ε.	$\varepsilon = lvB$ $= 25 \times 556 \times 5.0 \times 10^{-5}$ $= 0.70$ V
State your answer as a response to the question.	$\varepsilon = 0.70$ V (to 2 significant figures) This is a very small EMF and would not be dangerous.

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KEY QUESTIONS

Knowledge and understanding

1 **a** Yes, an EMF will be induced as there is a change in the magnetic flux.
b Yes, an EMF will be induced as there is a change in the magnetic flux.
c There is no change in magnetic flux and so there is no induced EMF.
d Yes, an EMF will be induced as there is a change in the magnetic flux.

2 0 Wb. Since the plane of the coil is parallel to the magnetic field, there is no flux passing through the coil.

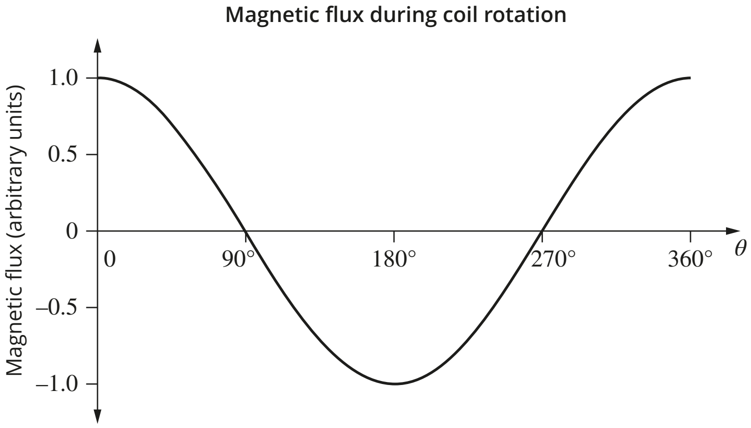
3 $\Phi_B = B_\perp A$
$= 0.20 \times 0.010$
$= 2.0 \times 10^{-3}$ Wb

4 $\varepsilon = lvB$
$= 0.15 \times 0.11 \times 0.60$
$= 9.9 \times 10^{-3}$ V

5 You can increase the magnetic flux through a coil either by:
- increasing the magnetic field strength over the surface area and/or
- increasing the area of the loop, which could also include adjusting the angle so that the maximum magnetic field passed through the loop.

Analysis

6 A plot of $y = \cos\theta$ such as the following:



7 **a** The area of the coil perpendicular to the magnetic field is 6.0 cm × 6.0 cm = 0.06 m × 0.06 m = 0.0036 m^2. The magnetic flux $\Phi_B = B_\perp A = 2.0 \times 10^{-3} \times 0.0036 = 7.2 \times 10^{-6}$ Wb.
b The magnetic flux decreases from 7.2×10^{-6} Wb to 0 after one quarter of a turn. Then it increases again to 7.2×10^{-6} Wb through the opposite side of the loop after half a turn. Then it decreases to 0 again after three quarters of a turn. After a full turn it is back to 7.2×10^{-6} Wb.

7.2 Induced EMF from a changing magnetic flux

Worked example: Try yourself 7.2.1

INDUCED EMF IN A COIL

A student winds a coil of area 50 cm^2 with 10 turns. She places it horizontally in a vertical uniform magnetic field of 0.10 T.

a Calculate the magnetic flux perpendicular to the coil.	
Thinking	**Working**
Identify the quantities required to calculate the magnetic flux through the coil. Convert them to SI units where required.	$B = 0.10$ T $A = 50\ cm^2 = 50 \times 10^{-4}\ m^2$
Calculate the magnetic flux. Give your answer in the appropriate units.	$\Phi_B = B_\perp A$ $= 0.10 \times 50 \times 10^{-4}$ $= 5.0 \times 10^{-4}$ Wb

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b Calculate the magnitude of the average induced EMF in the coil when the coil is removed from the magnetic field in 1.0 s.	
Identify the quantities required for determining the induced EMF.	$N = 10$ turns $\Delta\Phi_B = \Phi_2 - \Phi_1$ $= 0 - 5.0 \times 10^{-4}$ $= 5 \times 10^{-4}$ Wb $\Delta t = 1.0$ s
Calculate the magnitude of the average induced EMF, ignoring the negative sign that indicates the direction.	$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$ $= 10 \times \frac{5.0 \times 10^{-4}}{1.0}$ $= 5.0 \times 10^{-3}$ V

Worked example: Try yourself 7.2.2

NUMBER OF TURNS IN A COIL

A coil of cross-sectional area 2.0×10^{-3} m^2 experiences a change in the strength of a magnetic field from 0 to 0.20 T in 1.0 s. If the magnitude of the average induced EMF is 0.40 V, how many turns must be on the coil?

Thinking	**Working**
Identify the quantities required to calculate the magnetic flux through the coil when in the presence of the magnetic field.	$\Phi_B = B_\perp A$ $B = 0.20$ T $A = 2.0 \times 10^{-3}$ m^2
Calculate the magnetic flux when the coil is in the presence of the magnetic field.	$\Phi_B = B_\perp A$ $= 0.20 \times 2 \times 10^{-3}$ $= 4 \times 10^{-4}$ Wb
From the question, identify the quantities required by Faraday's law.	$N = ?$ $\Delta\Phi_B = \Phi_2 - \Phi_1$ $= 4.0 \times 10^{-4} - 0$ $= 4 \times 10^{-4}$ Wb $\Delta t = 1.0$ s $\varepsilon = 0.40$ V
Rearrange Faraday's law and solve for the number of turns on the coil. Ignore the negative sign.	$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$ $N = \frac{\varepsilon\Delta t}{\Delta\Phi_B}$ $= \frac{0.4 \times 1.0}{4 \times 10^{-4}}$ $= 1000$ turns

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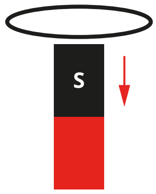
Worked example: Try yourself 7.2.3

INDUCED CURRENT IN A COIL FROM A PERMANENT MAGNET

The south pole of a magnet is moved downwards away from a horizontal coil held directly above it. In which direction will the current be induced in the coil?



Thinking	Working
Consider the direction of the change in magnetic flux.	The direction of the magnetic field will be downwards towards the south pole. Downwards flux from the magnet will decrease as the magnet is moved away from the coil. So the change in flux is decreasing downwards.
What will oppose the change in flux?	The magnetic field that opposes the change would act downwards.
Determine the direction of the induced current required to oppose the change.	In order to oppose the change, the direction of the current would be clockwise when viewed from above (from the right-hand grip rule).

Worked example: Try yourself 7.2.4

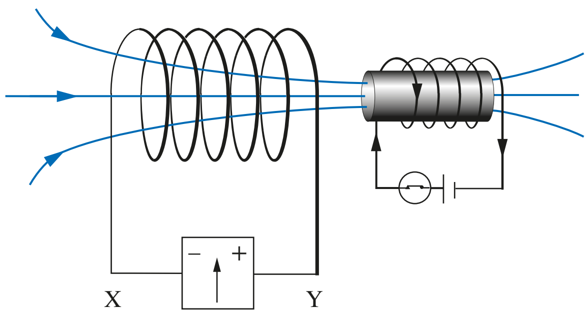
INDUCED CURRENT IN A COIL FROM AN ELECTROMAGNET

What is the direction of the current induced in the solenoid shown below when the electromagnet is:

a switched on

b left on

c switched off.



Thinking	Working
Consider the direction of the change in magnetic flux in each case.	**a** Initially there is no magnetic flux through the solenoid. When the electromagnet is switched on, it creates a magnetic field directed to the right. So the change in flux through the solenoid is increasing to the right. **b** While the current in the electromagnet is steady, the magnetic field is constant and the flux through the solenoid is constant. **c** Initially there is a magnetic field from the electromagnet directed to the right. Then there is no longer a magnetic field, so the change in flux through the solenoid is decreasing to the right.

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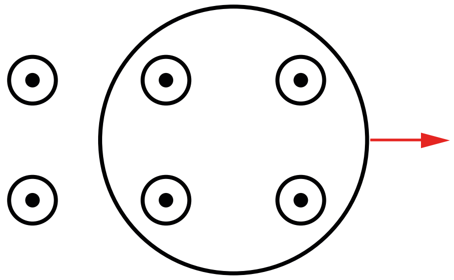
What will oppose the change in flux in each case?	**a** The magnetic field that opposes the change in flux through the solenoid is directed to the left. **b** There is no change in flux and so no opposition is needed. (There will be no magnetic field created by the solenoid.) **c** The magnetic field that opposes the change in flux through the solenoid is directed to the right.
Determine the direction of the induced current required to oppose the change in each case.	**a** In order to oppose the change, the current must be produced in the solenoid from Y to X (or through the meter from X to Y), from the right-hand grip rule. **b** There will be no induced EMF or current in the solenoid. **c** In order to oppose the change, the current must be produced in the solenoid from X to Y (or through the meter from Y to X), from the right-hand grip rule.

Worked example: Try yourself 7.2.5

FURTHER PRACTICE WITH LENZ'S LAW

A coil is moved to the right and out of a magnetic field that is directed out of the page. In what direction will the current be induced in the coil while the coil is moving?



Thinking	Working
Consider the direction of the change in magnetic flux.	Initially the magnetic flux passes through the full area of the coil and out of the page. Moving the coil out of the field decreases the magnetic flux. So the change in flux is decreasing out of the page.
What will oppose the change in flux?	The magnetic field that opposes the change would act out of the page.
Determine the direction of the induced current required to oppose the change.	In order to oppose the change, the current direction must be anticlockwise (from the right-hand grip rule).

KEY QUESTIONS

Knowledge and understanding

1 **a** $\Phi_B = B_\perp A$

$= 1.2 \times 10^{-3} \times 0.07 \times 0.03$

$= 2.5 \times 10^{-6}$ Wb

b No flux passes through the loop when the loop is parallel to the magnetic field.

c $\Delta\Phi_B = 2.5 \times 10^{-6}$ Wb

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

$$= \frac{2.5 \times 10^{-6}}{0.060}$$

$$= 4.2 \times 10^{-5}\ \text{V}$$

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2 **a** $\Phi_B = 65\times10^{-3}\times15\times10^{-4}$

$= 9.75\times10^{-5}$ Wb

$\varepsilon = \frac{\Delta\Phi_B}{\Delta t}$

$= \frac{9.75\times10^{-5}}{0.025}$

$= 3.9\times10^{-3}$ V in each turn of wire

b The effect of using multiple coils is similar to placing cells in series; that is, the EMF of each coil is added to derive the total EMF.

$\varepsilon = 600 \times 3.9 \times 10^{-3}\,\text{V} = 2.3\,\text{V}$

3 $\Phi_B = 10.0\times10^{-3}\times225\times10^{-4}$

$= 2.25\times10^{-4}$ Wb

$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$

$= 10\times\frac{2.25\times10^{-4}}{0.25}$

$= 9.0\times10^{-3}$ V

Analysis

4 C. The speed of the magnet reduces the time over which the change occurs, but there is no change in the strength of the magnetic field or the area of the coil. Hence the total flux (i.e. the area under the curve) is the same.

5 The student must induce an EMF of 1.0 V in the wire by changing the magnetic flux through the coil at an appropriate rate. A change in flux can be achieved by changing the strength of the magnetic field or by changing the area of the coil. The magnetic field can be changed by changing the position of the magnet relative to the coil. The area can be changed by changing the shape of the coil or by rotating the coil relative to the magnetic field.

To calculate the required rate of change of flux to produce 1.0 V:

$\frac{\Delta\Phi_B}{\Delta t} = \frac{\varepsilon}{N}$

$= \frac{1.0}{100} = 0.01\,\text{Wb s}^{-1}$

For example, if the shape is changed from 0.01 m^2 to 0.02 m^2 in 0.1 s, then:

$\frac{\Delta\Phi_B}{\Delta t} = \frac{(100\times10^{-3}\times0.02)-(100\times10^{-3}\times0.01)}{0.1}$

$= \frac{0.001}{0.1} = 0.01\,\text{Wb s}^{-1}$

6 $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$

$= -N\frac{\Delta BA_\perp}{\Delta t}$

$A_\perp = \frac{-\varepsilon\Delta t}{N\Delta B}$

$= \frac{0.020\times0.050}{1\times0.10}$

$= 0.010\,\text{m}^2$

7 $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$

$\Delta t = -N\frac{\Delta\Phi_B}{\varepsilon}$

$= 100\times\frac{0.40\times50\times10^{-4}}{1600\times10^{-3}}$

$= 0.13$ s

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7.3 Applications of Lenz's law

Worked example: Try yourself 7.3.1

PEAK AND RMS AC CURRENT VALUES

A 1000 W kettle is connected to a 240 V AC power outlet. What is the peak power use of the kettle?

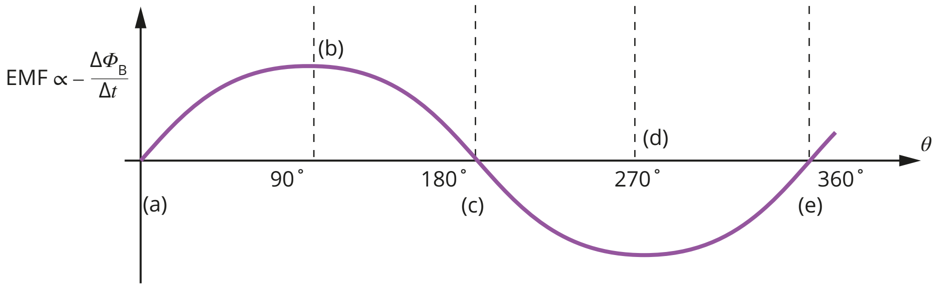
Thinking	Working
Note that the values given in the question represent rms values. Power is $P = VI$ so both V and I must be known to calculate the power use. The voltage is given and the current can be calculated from the rms power supplied.	$P_{rms} = V_{rms} I_{rms}$ $I_{rms} = \frac{P_{rms}}{V_{rms}}$ $= \frac{1000}{240} = 4.17\text{ A}$
Substitute the known quantities into the appropriate equation and solve for peak power.	$P_p = \sqrt{2} V_{rms} \times \sqrt{2} I_{rms}$ $= 2V_{rms} I_{rms}$ $= 2 \times 240 \times 4.17$ $= 2000\text{ W}$ $= 2.00 \times 10^3\text{ W}$

KEY QUESTIONS

Knowledge and understanding

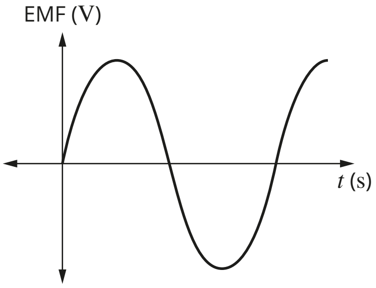
1 The resulting output of all three phases maintains an EMF near the maximum voltage more continuously than output from a single coil.

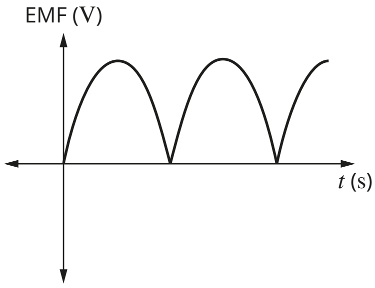
2 A graph similar to the one shown below.



3 The differences between a slip ring and a split ring commutator are:
- A slip ring is a continuous ring with no breaks or cuts.
- A split ring commutator has at least two breaks.
- A slip ring provides a continuous transfer of current and is generally used in AC motors and generators.
- A split ring commutator reverses the polarity of the current and is used in DC motors and generators.

4 EMF using split rings





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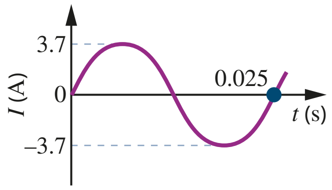
5 **a**



b The peak current is calculated as follows:

$I_p = I_{rms} \times \sqrt{2}$

$= 3.7\,\text{A}$

The peak-to-peak current is calculated as follows:

$I_{p\text{–}p} = 2I_p$

$= 2 \times 3.7$

$= 7.4\,\text{A}$

c The period is calculated as follows:

$T = \frac{1}{f}$

$= \frac{1}{40}$

$= 0.025\,\text{s}$

Analysis

6 B. When the coil begins rotating, the flux is a maximum and decreases initially, having the shape of graph D. The graph of the current (like the graph of induced EMF) will be zero initially and then increase, having the pattern shown in graph B.

7 **a** $V_p = 8.0\text{ V}$

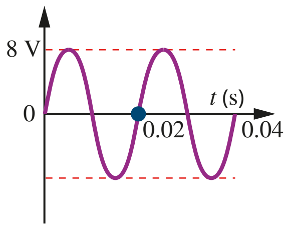
$V_{p\text{–}p} = 2 \times V_p = 2 \times 8.0 = 16\,\text{V}$

$V_{rms} = \frac{V_p}{\sqrt{2}}$

$= \frac{8.0}{\sqrt{2}} = 5.7\,\text{V}$

b The period of the waveform is 0.02 seconds

c



Halving the magnetic field strength will halve the EMF, as will halving the frequency. Doubling the radius increases the area to four times its original, and so increases the EMF to four times its original. Thus the EMF will remain the same magnitude overall. Halving the frequency of rotation, however, does double the period of the output.

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7.4 Producing electricity—photovoltaic cells

CASE STUDY: ANALYSIS

Small-scale solar production

Calculate the electrical power that needs to be supplied to the pump, given an efficiency of 60%:

$$P = \frac{150}{0.6} = 250\,\text{W}$$

Thus one 400 W panel will be enough to operate the pump.

KEY QUESTIONS

Knowledge and understanding

1 D. Approximately 20%.

2 The photons from sunlight cause an electron to move away from a silicon atom, setting the electron loose and causing it to move in the semiconductor material.

3 The solar panel contains a semiconductor material, usually made from silicon. The top layer of the semiconductor has an excess of electrons (*n-type* silicon) and the bottom layer a deficit of electrons (*p-type* silicon).

4 An inverter converts the DC current generated by the solar cell into AC current that can be used by household and industrial appliances (or fed into the electricity transmission grid).

Analysis

5 To maximise the use of solar energy, it would be best to run appliances that consume large amounts of power (e.g. those for heating and cooling), during peak daylight hours even if they are not required. Appliances that require less power (e.g. dishwashers and washing machines) can be run during the shoulder times (that is, morning and late afternoon).

7.5 Supplying electricity—transformers and large-scale power distribution

Worked example: Try yourself 7.5.1

TRANSFORMER EQUATION—VOLTAGE

A transformer built into a phone charger reduces the 240 V supply voltage to the required 6 V. If the number of turns in the secondary coil is 100, what is the number of turns in the primary coil?

Thinking	Working
Note the relevant quantities given in the question.	$V_2 = 6\,\text{V}$ $V_1 = 240\,\text{V}$ $N_2 = 100$ turns $N_1 = ?$
Substitute the given quantities into the transformer equation and solve for N_1.	$\frac{N_1}{N_2} = \frac{V_1}{V_2}$ $\frac{N_1}{100} = \frac{240}{6}$ $N_1 = \frac{100 \times 240}{6}$ $= 4000$ turns

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Worked example: Try yourself 7.5.2

TRANSFORMER EQUATION—CURRENT

A phone charger with 4000 turns in its primary coil and 100 turns in its secondary coil provides a current of 0.50 A. What is the current in the primary coil?

Thinking	Working
Note the relevant quantities given in the question.	$I_2 = 0.50\,\text{A}$ $N_2 = 100$ turns $N_1 = 4000$ turns $I_1 = ?$
Recall the transformer equation written in terms of current. Substitute the given quantities into the equation and solve for I_1.	$\frac{N_1}{N_2} = \frac{I_2}{I_1}$ $\frac{I_1}{0.50} = \frac{100}{4000}$ $I_1 = \frac{0.50 \times 100}{4000}$ $= 0.013\,\text{A}$

Worked example: Try yourself 7.5.3

TRANSFORMERS—POWER

The power drawn from the secondary coil of the transformer in a phone charger is 3 W. What power is drawn from the mains supply if the transformer is an ideal transformer?

Thinking	Working
The energy efficiency of an ideal transformer is 100%. Hence the power in the secondary coil will be the same as that in the primary coil.	The power drawn from the mains supply is the power in the primary coil, which will be the same as the power in the secondary coil: $P = 3\,\text{W}$.

Worked example: Try yourself 7.5.4

TRANSMISSION-LINE POWER LOSS

300 MW is to be transmitted from a power station to Melbourne along a transmission line with a total resistance of 1.0 Ω. What would be the total power loss if the voltage along the line is 500 kV?

Thinking	Working
Convert the power and voltage to SI units.	$P = 300\,\text{MW} = 300 \times 10^6\,\text{W}$ $V = 500\,\text{kV} = 500 \times 10^3\,\text{V}$
Determine the current in the line based on the given voltage.	$P = V \times I$ $I = \frac{P}{V}$ $= \frac{300 \times 10^6}{500 \times 10^3}$ $= 600\,\text{A}$
Determine the corresponding power loss.	$P = I^2R$ $= 600^2 \times 1.0$ $= 3.6 \times 10^5\,\text{W}$ or $0.36\,\text{MW}$

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Worked example: Try yourself 7.5.5

VOLTAGE DROP ALONG A TRANSMISSION LINE

Power is to be transmitted from a power station to Melbourne along a transmission line with a total resistance of 1.0 Ω. The current is 600 A. What voltage would be needed at the power station to achieve a supply voltage of 500 kV?

Thinking	**Working**
Determine the voltage drop along the transmission line.	$\Delta V = IR$ $= 600 \times 1.0$ $= 600\text{ V}$
Determine the initial supply voltage.	$V_{\text{initial}} = V_{\text{supplied}} + \Delta V$ $= 500 \times 10^3 + 600$ $= 5.0 \times 10^5\text{ V}$

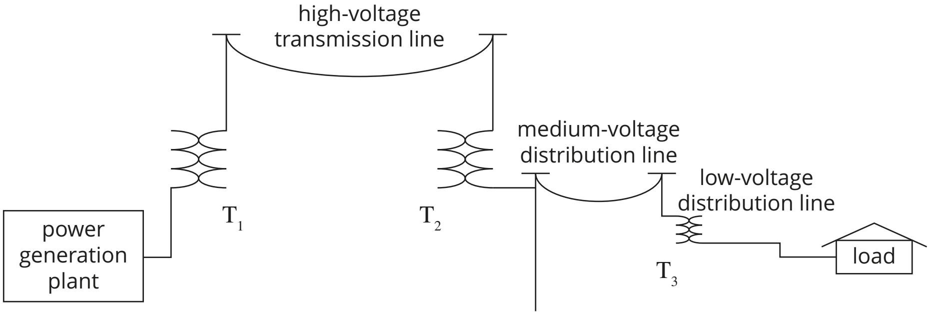
Worked example: Try yourself 7.5.6

POWER LOSS THROUGH A TRANSMISSION NETWORK

Consider the power transmission system shown below. The power plant generates AC voltage at 40 kV, which is stepped up to 600 kV using transformer T_1. Transformer T_2 steps the 600 kV voltage down to 60 kV and transformer T_3 steps the 60 kV voltage down to 240 V for the load.

The resistance of the high-voltage transmission line is 2.5 Ω, the resistance of the medium-voltage distribution line is 2.5 Ω and the resistance of the low-voltage distribution line is 7.5 Ω.

Assume that the transformers are 100% efficient and that the load consumes 2 MW of power.



a Calculate the turns ratio of transformers T_1, T_2 and T_3.	
Thinking	**Working**
Rearrange the transformer equation and calculate the turns ratios.	$\frac{N_2}{N_1} = \frac{V_2}{V_1}$ For T_1: $V_1 = 40\text{ kV}$ and $V_2 = 600\text{ kV}$, therefore: $\frac{V_2}{V_1} = \frac{600 \times 10^3}{40 \times 10^3} = 15$ For T_2: $V_1 = 600\text{ kV}$ and $V_2 = 60\text{ kV}$, therefore: $\frac{V_2}{V_1} = \frac{60 \times 10^3}{600 \times 10^3} = 0.10$ For T_3: $V_1 = 60\text{ kV}$ and $V_2 = 240\text{ V}$, therefore: $\frac{V_2}{V_1} = \frac{240}{60 \times 10^3} = 4.0 \times 10^{-3}$

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b Determine the power loss through the high-voltage transmission and medium-voltage distribution lines.	
Thinking	**Working**
Calculate the current through each part of the system. Recall that the load consumes 2 MW of power. Work back from the load to find the current in the other sections.	At the load: $P = VI$ $I = \frac{P}{V}$ $= \frac{2 \times 10^6}{240}$ $= 8.33\,\text{kA}$ In the medium-voltage distribution line: $\frac{I_1}{I_2} = \frac{N_2}{N_1}$ $I_1 = \frac{N_2}{N_1} \times I_2$ $= 0.004 \times 8.33\,\text{kA} = 33.3\,\text{A}$ In the high-voltage transmission line: $\frac{I_1}{I_2} = \frac{N_2}{N_1}$ $I_1 = \frac{N_2}{N_1} \times I_2$ $= 0.10 \times 33.3 = 3.33\,\text{A}$
Calculate the power losses using the currents calculated in the last step.	The power loss through the high-voltage transmission line is: $P = I^2 \times R = (3.33)^2 \times 2.5 = 28\,\text{W}$ The power loss through the medium-voltage distribution line is: $P = I^2 \times R = (33.3)^2 \times 2.5 = 2.8\,\text{kW}$

c Calculate the power loss through the medium-voltage distribution line if it carried 120 kV instead of 60 kV.	
Thinking	**Working**
Determine the effect of the change on the turns ratio and therefore on the current ratio. Recall that the power loss is proportional to I^2. If I decreases, so will the power loss.	If the voltage of the medium-voltage distribution line were increased to 120 kV from 60 kV, the turns ratio for that transformer (T_2) would increase by a factor of 2. Therefore the current in the medium-voltage distribution line would decrease by a factor of 2. If the current decreased by a factor of 2, the power loss would decrease by a factor of 4. Therefore the power loss through the medium-voltage distribution line would decrease from 2.7 kW to 675 W.

d Describe how the power loss could be minimised throughout the system.	
Thinking	**Working**
Recall that power loss is proportional to I^2 and that voltage and current are inversely proportional in this type of system.	Power loss through the transmission network can be minimised by reducing the effective resistance of the transmission lines or by using higher voltages to transmit power. It is for this reason that power is distributed at such high voltages across the country to major centres or loads.

KEY QUESTIONS

Knowledge and understanding

1 B. The power equation is $P = VI$ and the '2' indicates the secondary coil.

2 It is best to transmit power at high voltage and low current (since power loss is proportional to the square of the current).

3 D. A change in flux through the secondary coil is required for an EMF to be induced in the coil, but a DC input to the primary coil will create a constant flux. Therefore the voltage output is zero.

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4 $\frac{N_2}{N_1} = \frac{V_2}{V_1}$

$\frac{N_2}{600} = \frac{24}{240}$

$N_2 = \frac{24 \times 600}{240}$

$= 60\,\text{turns}$

5 **a** $\frac{V_2}{V_1} = \frac{N_2}{N_1}$

$\frac{V_2}{10} = \frac{500}{50}$

$V_2 = \frac{10 \times 500}{50}$

$= 100\,\text{V}$

b Assuming an ideal transformer, the power output from the secondary coil must be equal to the power input at the primary coil.

$P_1 = V_1 I_1 = 10 \times 5.0 = 50\,\text{W}$

c $I_2 = \frac{P_2}{V_2}$

$= \frac{50}{10} = 0.50\,\text{A}$

6 $I = \frac{P}{V} = \frac{9.9 \times 10^3}{700} = 14.14\,\text{A}$

$P_{\text{loss}} = I^2 R$

$= 14.14^2 \times 3.6$

$= 7.2 \times 10^2\,\text{W}$

7 **a** $I = \frac{P}{V} = \frac{720 \times 10^6}{100 \times 10^3} = 7.20 \times 10^3\,\text{A}$

b $V_{\text{drop}} = IR$

$= 7200 \times 2.5 = 18\,000\,\text{V or } 1.8 \times 10^4\,\text{V}$

$V_{\text{supplied}} = 100 - 18 = 8.2 \times 10^4\,\text{V} = 82\,\text{kV}$

Analysis

8 There are advantages to both AC and DC high-voltage (HV) transmission. The solar production facility generates DC voltages; however, these are unlikely to be sufficiently high to achieve low power losses on a HVDC transmission network. Therefore the generated DC voltage would need to be stepped up before transmission. This would require DC-to-AC converters, transformers and then AC-to-DC converters. Thus extra equipment would be needed, which adds costs and additional maintenance requirements. In addition, the output from the generation plant feeds into the Victorian electricity network which uses HVAC for transmission. Therefore, considering the costs and ease of integration with the rest of the state's electricity network, HVAC is the preferable option.

9 **a** Using the transformer equation:

$\frac{N_1}{N_2} = \frac{V_1}{V_2}$

$\frac{250}{N_2} = \frac{50\,000}{220\,000}$

$N_2 = \frac{250 \times 220\,000}{50\,000}$

$= 1100\,\text{turns}$

b An ideal transformer has no power loss, therefore $V_1 \times I_1 = V_2 \times I_2$.

The input power $= V_1 \times I_1 = 50\,\text{kV} \times 75\,\text{A} = 3.75\,\text{MW}$

The output power $= V_2 \times I_2 = 220\,\text{kV} \times 14.5\,\text{A} = 3.19\,\text{MW}$

The output power is less than the input power, therefore this is not an ideal transformer.

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10 The resistance for the 13 km section of the high voltage transmission line would be:

$$R = 0.34 \times 13 \\ = 4.42\,\Omega$$

The power loss using the rms current of 1 kA would therefore be:

$$P = I^2R \\ = 1000^2 \times 4.42 \\ = 4.4\,\text{MW}$$

The peak current would be $1 \times \sqrt{2} = 1.41\text{kA}$.

Therefore the peak power loss would be:

$$P = I^2R \\ = 1410^2 \times 4.42 \\ = 8.8\,\text{MW}$$

Chapter 7 Review

REVIEW QUESTIONS

Knowledge and understanding

1 Only two conductors (live and return) are required.
DC can be more economical for transmission over very long distances (especially greater than 500 km).
High voltage DC (HVDC) can be used to connect different AC power networks (e.g. different AC frequencies).

2 A split ring commutator connects the armature of the motor to the external power supply. Its function is to reverse the connection between the rotor coil and power supply each half cycle. This ensures that the current in the armature of the motor produces a torque in one direction as the rotor coil rotates.

3 Anticlockwise. Initially there is no flux through the coil. As the coil begins to rotate, the amount of flux increases and is directed to the left. To oppose this change, an induced magnetic field will be directed to the right. As the right-hand grip rule will show, this creates an anticlockwise current in the coil relative to the orientation shown in the diagram.

4 As the coil area is reduced, the flux into the page will decrease. To oppose this, the induced current will act to increase the flux again in the same direction. The right-hand grip rule shows that the direction of the induced current will be clockwise.

5 AB and CD. Both sides cut across lines of flux as the coil rotates.

6 **a** B changes from 8.0×10^{-4} T to 32×10^{-4} T, a change of 24.0×10^{-4} T.

$$\Delta\Phi_B = \Delta B \times A \\ = 24.0 \times 10^{-4} \times 25 \times 10^{-4} \\ = 6.0 \times 10^{-6}\,\text{Wb}$$

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} = \frac{6.0 \times 10^{-6}}{3.5 \times 10^{-3}} \\ = 1.7 \times 10^{-3} = 1.7\,\text{mV}$$

b Anticlockwise. Halving the magnetic field strength decreases the flux through the coil out of the page. The induced magnetic field will be out of the page to oppose the decreasing magnetic flux out of the page. The right-hand grip rule shows that the direction of the induced current is anticlockwise around the coil.

7 **a** $$\Phi_B = 35 \times 10^{-3} \times \pi \times 0.05^2 \\ = 2.7 \times 10^{-4}\,\text{Wb}$$

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} \\ = 65 \times \frac{2.7 \times 10^{-4}}{0.15} \\ = 0.12\,\text{V}$$

b From Y to X. As the coil is removed, the magnetic flux through it changes from being directed downwards to no magnetic flux. To oppose this change the coil must create a magnetic field that is directed downwards again. The right-hand grip rule shows that the current must be clockwise around the coil when viewed from above.

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8 **a** $\varepsilon = lvB = 0.45 \times 2.5 \times 35 \times 10^{-3} = 3.9 \times 10^{-2}\,\text{V} = 39\,\text{mV}$

b From X to Y. As the rod moves to the right, the area of the loop decreases. Thus the magnetic flux through the loop, which is directed out of the page, decreases. In order to oppose this change, the loop will produce a magnetic field directed out of the page. The right-hand grip rule shows that the current will be from X to Y.

9 $\varepsilon = lvB = 12.0 \times 6.5 \times 7.5 \times 10^{-5} = 5.9 \times 10^{-3}\,\text{V}$ or $5.9\,\text{mV}$

10 **a** $\frac{I_2}{I_1} = \frac{V_1}{V_2}$

$\frac{I_2}{3.5} = \frac{18}{54}$

$I_2 = 1.2\,\text{A}$

b $\frac{N_1}{N_2} = \frac{V_1}{V_2}$

$\frac{N_1}{45} = \frac{18}{54}$

$N_1 = 15$

There are 15 turns in the primary coil.

11 **a** The maximum magnetic flux the coil experiences is in a quarter of a turn:

$$\Delta\Phi_B = 120 \times 10^{-3} \times 10 \times 10^{-4}$$
$$= 1.2 \times 10^{-4}\,\text{Wb}$$

The frequency is 50 Hz, so a quarter of a turn takes $\frac{1}{4} \times 0.02 = 0.005\,\text{s}$.

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$
$$= 750 \times \frac{1.2 \times 10^{-4}}{5 \times 10^{-3}}$$
$$= 18\,\text{V}$$

b By Faraday's law, doubling the frequency halves Δt, which doubles the average EMF to 36 V.

12 **a** Without the first transformer, voltage in the transmission lines is 1000 V.

Calculate I:

$$P = VI$$
$$250 \times 10^3 = 1000 \times I$$
$$I = \frac{250 \times 10^3}{1000}$$
$$= 250\,\text{A}$$

When the voltage is stepped up to 10 000 V, the current is reduced to 25 A.

b The power loss in the lines is:

$$P = I^2R$$
$$= 25^2 \times 2$$
$$= 1\,\text{kW}$$

c If the voltage was not stepped up, the current in the transmission line would be 250 A. The power lost in the transmission line would be $I^2R = (250)^2 \times 2 = 125\,\text{kW}$. Power supplied to the load would then be $250\,\text{kW} - 125\,\text{kW} = 125\,\text{kW}$. This is a 50% power loss—a bad idea!

Application and analysis

13 C. The magnetic flux through a coil is proportional to the area of the coil and the strength of the magnetic field. It is not dependent on the number of turns of the coil. The magnetic flux is calculated using:

$$\Phi = B \times A = 0.05 \times 0.2$$
$$= 0.01\,\text{Wb}$$
$$= 10\,\text{mWb}$$

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14 D. The magnitude of the average induced EMF is calculated using:

$\Phi_2 = B \times A = 5 \times 10^{-4} \times 0.075 = 3.75 \times 10^{-5}$ Wb

$\Phi_1 = B \times A = 5 \times 10^{-4} \times 0 = 0$ Wb

$$\begin{aligned} \varepsilon &= -N\frac{\Phi_2 - \Phi_1}{\Delta t} \\ &= -2 \times \frac{3.75 \times 10^{-5}}{0.25} \\ &= 3 \times 10^{-4}\ \text{V} \end{aligned}$$

15 A. The output voltage is calculated using the transformer equation:

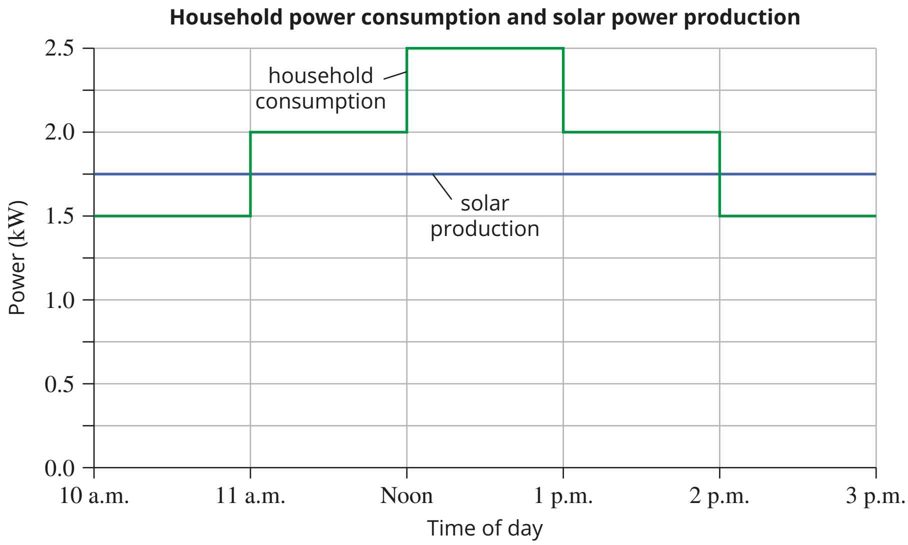
$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{200}{V_2} = \frac{1500}{60}$$

$$\begin{aligned} V_2 &= \frac{200 \times 60}{1500} \\ &= 8\ \text{V} \end{aligned}$$

16 The power loss is proportional to the square of the current. Therefore the power loss in transmission line B would be 9 times as much as in transmission line A.

17 **a** The data can be plotted as follows:



The energy for each one-hour period is as follows:

Time period	Energy used (kWh)	Solar energy produced (kWh)	Energy to/from grid (kWh)
10 a.m. to 11 a.m.	1.5	1.75	0.25 to grid
11 a.m. to noon	2.0	1.75	0.25 from grid
noon until 1 p.m.	2.5	1.75	0.75 from grid
1 p.m. to 2 p.m.	2.0	1.75	0.25 from grid
2 p.m. to 3 p.m.	1.5	1.75	0.25 to grid

b The total consumption of electricity from the grid is 1.25 kWh. At \$0.25 per kWh, that would cost \$0.31.

c The house's total consumption of electricity between 10 a.m. and 3 p.m. is 9.5 kW. At \$0.25 per kWh, the cost would be \$2.38. With the solar panels, the price for electricity between 10 a.m. and 3 p.m. is \$0.3125. The cost savings is \$2.07.

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18 **a** It is an AC generator, as it has slip rings.

b The magnetic field flows from the north to the south pole of the magnets (from left to right in the diagram), which would induce a current in the rotor loop wire from contact Y to contact X. Therefore contact Y would be at a higher potential than contact X.

c The induced EMF is momentarily zero when the rotor coil is perpendicular to the magnetic field. The induced EMF is proportional to the rate of change of magnetic flux through the coil. The rate of change of magnetic flux is zero when the plane of the coil is perpendicular to the magnetic field.

19 **a** As the generator is ideal, the output power must equal the input power. The power provided to the circuit is 4.5 kW.

$$P = VI$$

$$I = \frac{P}{V}$$

$$= \frac{4.5 \times 10^3}{240}$$

$$= 18.75$$

$$= 19\,\text{A (to 2 significant figures)}$$

$$R = \frac{V}{I}$$

$$= \frac{240}{18.75}$$

$$= 12.8$$

$$= 13\,\Omega\ \text{(to 2 significant figures)}$$

b If the total resistance of the circuit is quadrupled to 51.2 Ω, then:

$$P = \frac{V^2}{R}$$

$$= \frac{240^2}{51.2}$$

$$= 1.1\,\text{kW}$$

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Unit 3 Area of Study 3

How are fields used in electricity generation?

Multiple-choice questions

1 D. $\Phi_B = B_\perp A$

2 C

$$\Phi_B = B_\perp A$$
$$= 60 \times 10^{-3} \times 0.05^2$$
$$= 0.15\,\text{mWb}$$

3 D. As no field lines pass through the loop, the flux is zero.

4 A. As the induction of EMF depends on the rate of change of magnetic flux, the field lines will need to thread through the coil. But the flux within the coil needs to be changed by moving the magnet, which will change the number of field lines threading through the coil.

5 C. The alternating current in the primary windings produces a changing magnetic flux, which induces an EMF in the secondary windings (as well as in the primary windings).

6 B. As the magnet enters the coil, the magnetic field direction from the magnet will be away from the north pole, i.e. downwards. Therefore the change in flux is increasing downwards. To oppose the change, the current direction would be anticlockwise when viewed from above. As the magnet exits the coil, the magnetic field direction from the magnet will be away from the north pole, i.e. downwards. Therefore, the change in flux is decreasing downwards. To oppose the change, the current direction would be clockwise when viewed from above.

7 C. Initially, the magnetic flux passes through the full area of the coil and into the page. Moving the coil out of the field decreases the magnetic flux. So the change in flux is decreasing into the page. The magnetic field that opposes the change would act into the page again. To oppose the change, the current direction would be clockwise (from the right-hand grip rule).

8 D.

$$\frac{N_1}{N_2} = \frac{V_1}{V_2}$$
$$\frac{N_1}{80} = \frac{240}{10}$$
$$N_1 = \frac{240 \times 80}{10}$$
$$= 1920 \text{ turns}$$

9 B. The output of photovoltaic cells is DC but most household appliances require AC current. (Some devices used in the home run on DC power, but they have their own transformers built in.)

Short-answer questions

10 **a** Since the plane of the loop is parallel to the direction of the magnetic field, no flux threads the loop.

b Rotate the loop or the magnetic field so that they are no longer parallel.

c The maximum flux threads the loop when the plane of the loop and the direction of the magnetic field are perpendicular (that is, at right angles to each other).

d $\Phi_B = B_\perp A$

$$= 0.50 \times 0.20 \times 0.10$$
$$= 0.010\,\text{Wb or } 1.0 \times 10^{-2}\,\text{Wb}$$

11 **a** As the loop enters the magnetic field there is a flux increasing down through the loop. Lenz's law states that the induced current in the loop will oppose the change in flux that causes it. Therefore there will be an induced field (or flux) up through the loop. Using the right-hand grip rule, align your fingers so that they are pointing up on the inside of the loop. Your thumb will point in the direction of the induced current, that is, from Y to X.

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b The loop moves at 5.0 cm s^{-1} and, with a side length 20 cm, it is halfway into the field when it has travelled 10 cm, which takes 2.0 s.

$$\varepsilon = N\frac{\Delta\Phi_B}{\Delta t}$$
$$= 1\times\frac{0.40\times0.20\times0.10}{2.0}$$
$$= 4.0\times10^{-3}\text{ V}$$

c $$I = \frac{V}{R}$$
$$= \frac{4.0\times10^{-3}}{0.50}$$
$$= 8.0\times10^{-3}\text{ A}$$

d $$P = VI$$
$$= 4.0\times10^{-3}\times8.0\times10^{-3}$$
$$= 3.2\times10^{-5}\text{ W}$$

e The source of the power is the external force that is moving the loop into the magnetic field.

f As the loop is moving at 5.0 cm s^{-1}, after 5.0 seconds it has moved 25 cm and has been totally within the magnetic field for 1.0 second. Since there is now no flux change there will be no EMF induced in the loop at this moment.

g As the loop emerges from the magnetic field there is flux decreasing down through the loop. Lenz's law states that the induced current in the loop will oppose the change in flux that causes it. Therefore there will be an induced field (or flux) down through the loop. Using the right-hand grip rule, align your fingers so that they are pointing downwards on the inside of the loop. Your thumb will point in the direction of the induced current, that is, from X to Y.

12 a $$\Phi_B = B_\perp A$$
$$= 1.0\times10^{-3}\times100\times10^{-3}\times50\times10^{-3}$$
$$= 5.0\times10^{-6}\text{ Wb}$$

b No flux threads the loop in the new position as the plane of the loop is now parallel to the magnetic field. Alternatively, $\theta = 90°$ and $\cos\theta = 0$, so no flux threads the loop in the new position.

c To find the magnitude:

$$\varepsilon = N\frac{\Delta\Phi_B}{\Delta t}$$
$$= 1\times\frac{5.0\times10^{-6}}{2.0\times10^{-3}}$$
$$= 2.5\times10^{-3}\text{ V}$$

d $$I = \frac{V}{R}$$
$$= \frac{2.5\times10^{-3}}{2.0}$$
$$= 1.3\times10^{-3}\text{ A}$$

e No. Once the loop is stationary there is no change in flux. Therefore no EMF is generated and no current flows in the loop.

13 a The EMF, and hence the current, depends on the rate of change. If the rate is increased by 4, then the current will also increase by 4. Thus $I = 200\ \mu\text{A}$.

b The EMF generated is $V = IR = 50\times10^{-6}\times(595 + 5.0) = 3.0\times10^{-2}$ V.

$$\varepsilon = N\frac{\Delta\Phi}{\Delta t}\text{ and }\Phi_B$$
$$= B_\perp A\text{ where }A = \pi r^2$$
$$\therefore \varepsilon = N\frac{B_\perp \pi r^2}{\Delta t}$$
$$B = \frac{\varepsilon\Delta t}{N\pi r^2}$$
$$= \frac{3.0\times10^{-2}\times2.0}{100\times\pi\times0.030^2}$$
$$= 0.21\text{ T}$$

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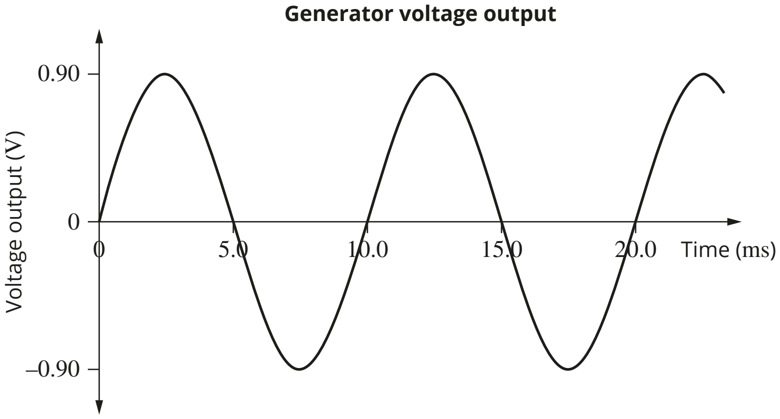
14 a $T = \frac{1}{f} = \frac{1}{100} = 0.0100\,\text{s} = 1.00 \times 10^{-2}\,\text{s}$

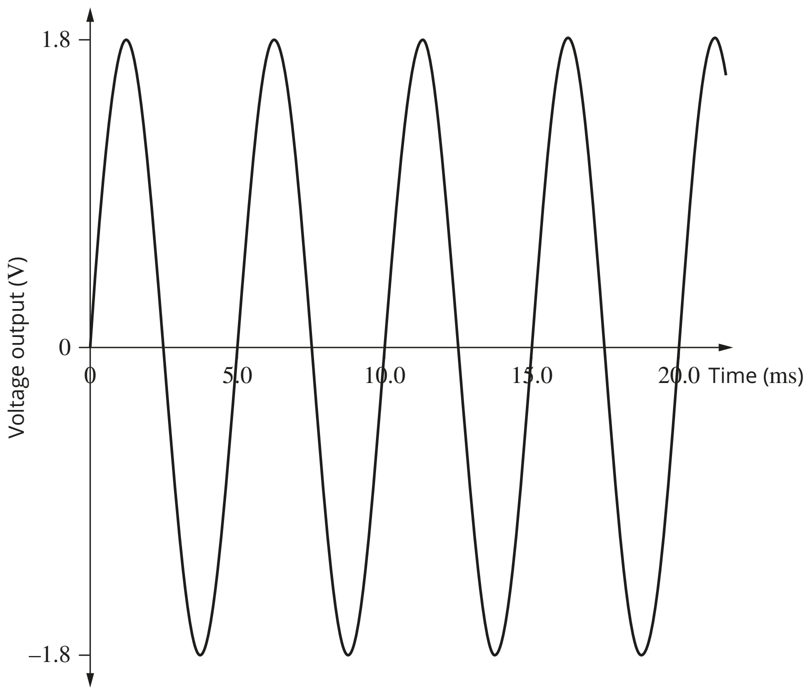
The graph is a sine wave with peak amplitude of 0.90 V and a period of 0.0100 s (i.e. 10.0 ms).

Generator voltage output



b The output graph would have half the period and twice the amplitude.

Generator voltage output



15 An alternator has a pair of slip rings instead of a split ring commutator.

16 AC is generated in the coils of an alternator. Each slip ring connects to each end of the coil. The slip rings are continuous and so maintain the AC generated in the coil at the output. Carbon brushes press against the slip rings to allow a constant output to the circuit without a fixed point of connection.

17 a $I_2 = \frac{I_1 \times V_1}{V_2} = \frac{2.0 \times 600}{3000} = 0.40\,\text{A}$

b $V_{\text{p-p}} = 2 \times 3000 = 6000\,\text{V}$

c $N_1 = \frac{N_2 \times V_1}{V_2} = \frac{1000 \times 600}{3000} = 200$ turns

d $P_{2\text{ rms}} = V_{2\text{ rms}} \times I_{2\text{ rms}}$

$= \frac{3000}{\sqrt{2}} \times 0.40$

$= 849\text{ W} = 8.5 \times 10^2\,\text{W}$

e $P_{2\text{ peak}} = V_{2\text{ peak}} \times I_{2\text{ peak}}$

$= 3000 \times 0.40 \times \sqrt{2}$

$= 1697\,\text{W}$ or $1.7 \times 10^3\,\text{W}$

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18 **a** With little or no current in the power line there was almost no voltage drop. When the house appliances were turned on, there was a higher current in the power line and hence a voltage drop along the line, leaving a lower voltage at the house.

b As the generator was supplying 4000 W at 250 V, the current in the line was $I = \frac{4000}{250} = 16.0\,\text{A}$.

The voltage drop along the line was therefore $\Delta V = IR = 16.0 \times 2.0 = 32\,\text{V}$ and so the voltage at the house was $250 - 32 = 2.2 \times 10^2\,\text{V}$.

The power lost is $P_{loss} = I^2R = 16.0^2 \times 2.0 = 512\,\text{W}$. Thus the power at the house was:
$4000 - 512 = 3488 = 3.5 \times 10^3\,\text{W}$.

Alternatively, $P_{house} = VI = 218 \times 16.0 = 3488\,\text{W}$.

c At the generator end a 1:20 step-up transformer is required (since 5000 ÷ 250 = 20). There will be 20 times as many turns in the secondary circuit as in the primary circuit. At the house end a 20:1 step-down transformer is required.

d $I = \frac{P}{V} = \frac{4000}{5000} = 0.8000\,\text{A}$

e The voltage drop is $V = IR = 0.8000 \times 2.0 = 1.6\,\text{V}$.

f The power loss is $P = I^2R = 0.8000^2 \times 2.0 = 1.3\,\text{W}$.

g The voltage at the house will be $\frac{5000 - 1.60}{20} = 249.92\,\text{V} = 2.5 \times 10^2\,\text{V}$.

h The power at the house will be $4000 - 1.28 = 3998.72 = 4.0 \times 10^3\,\text{W}$.

i The power loss before the transformers were added was 512 W (from part **b**) which was 12.8% of the power generated (4000 W), and the power loss with the transformers was 1.3 W (from part **f**), which is about 0.03% of the power generated.

j The reason is that power loss in the power line depends on the square of the current ($P = I^2R$). Since the current was reduced by a factor of 20 and the resistance remained constant, the power loss decreased by a factor of 20^2 or 400.

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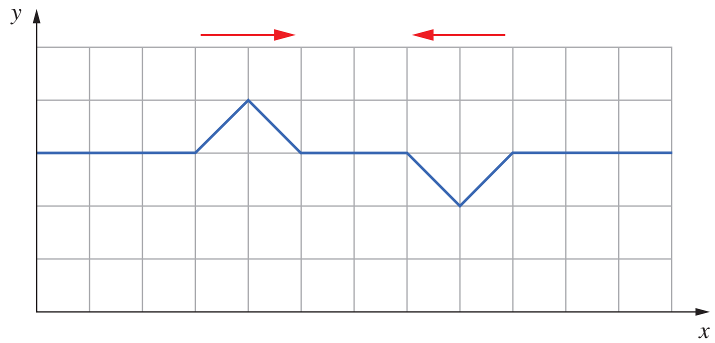
Chapter 8 Light as a wave

8.1 Wave interactions

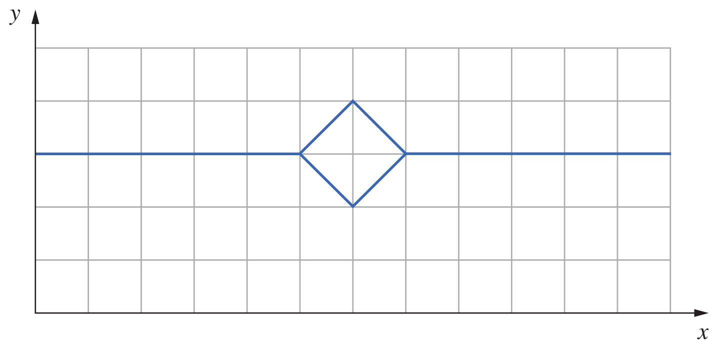
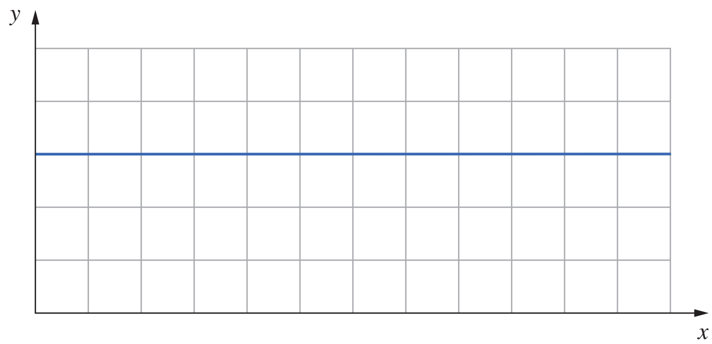
Worked example: Try yourself 8.1.1

WAVE SUPERPOSITION

Two wave pulses are travelling towards each other parallel to the *x*-axis at 1 m s^{-1}. The directions are shown by the red arrows. The *x*- and *y*-axes have a scale of 1 m for each box.



What is the amplitude of the combined pulse when they interact 2 seconds after the instant shown above?

Thinking	Working
Draw a diagram of the two pulses after 2 seconds.	
Draw a new diagram with the waves superimposed.	
The amplitude is the height of the resultant wave.	The amplitude of the combined pulse wave is zero.

KEY QUESTIONS

Knowledge and understanding

1 **a** True

b False. As the pulses pass through each other, the interaction does not permanently alter the characteristics of either pulse.

c True

2 Resonance occurs when the applied frequency is equal to the object's natural frequency of vibration.

3 B. The amplitude of the vibration will increase due to constructive interference from the forcing vibration. Note that the frequency stays the same when resonance occurs.

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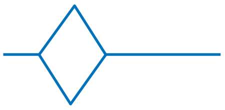
4 To push a child on the swing you need to work out the frequency of the swing (i.e. how regularly the swing goes backwards and forwards) and then push at exactly the same rate. This will enable the maximum transfer of energy from the person pushing the swing to the swing, thereby increasing the amplitude of the swing.

5 Each pulse travels 3 m in 3 s.

The superimposed waves will look like this:



Adding their amplitudes together results in the waves being cancelled:

Analysis

6 The bridge resonates at 1.5 Hz. A pedestrian moving at a frequency of 1.5 Hz would be completing 1.5 cycles per second. Since one cycle is 2 steps, that pedestrian would be taking 3 steps per second. This closely corresponds on the graph to fast running. Thus a pedestrian running fast may cause an increase in the amplitude of the bridge's oscillation which, over time, could damage it.

7 Pendulum D would swing with maximum amplitude. Pendulum D is the same length as the first pendulum. Therefore its natural frequency of vibration is the same as the first pendulum. The frequency of vibration of the first pendulum becomes the applied frequency for pendulum D; thus there is maximum energy transfer.

8 As the vibration of the truck is noticeably larger when the truck is stationary, the natural frequency of vibration of the body of the truck must be close to 100 Hz, the frequency of vibration of the motor. As the truck accelerates to a higher speed, the frequency of vibration of the motor increases and is no longer at the resonant frequency of the truck body. Therefore the amplitude of vibration becomes much less.

8.2 Standing waves in strings

Worked example: Try yourself 8.2.1

FUNDAMENTAL FREQUENCY

A standing wave in a string fixed at both ends has a wavelength of 0.50 m for the fundamental frequency of vibration.

a What is the length of the string?	
Thinking	**Working**
Note the wavelength of the string (λ) in metres and the harmonic number (n).	$\lambda_1 = 0.50\,\text{m}$ $n = 1$
Recall that for any frequency, $\lambda_n = \frac{2l}{n}$. Rearrange the equation to make l the subject.	$\lambda_n = \frac{2l}{n}$ $l = \frac{n\lambda_n}{2}$
Substitute the given values and solve for l.	$l = \frac{1 \times 0.50}{2}$ $= 0.25\,\text{m}$

b What is the wavelength of the third harmonic?	
Thinking	**Working**
Note the length of the string (l) in metres and the harmonic number (n).	$l = 0.25\,\text{m}$ $n = 3$
Recall that for any frequency, $\lambda_n = \frac{2l}{n}$. Substitute the given values and solve for λ.	$\lambda_3 = \frac{2l}{3}$ $= \frac{2 \times 0.25}{3}$ $= 0.17\,\text{m}$

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CASE STUDY: ANALYSIS

Physics of the guitar

1 For the low-E string, f_1= 82.41 Hz and the length of the string is 0.650 m.

For the fundamental:

$\lambda_1 = 2l = 2 \times 0.650 = 1.30\,m$

$v = f_1\lambda_1 = 82.41 \times 1.30 = 107\,m\,s^{-1}$

2 The wavelength of the fundamental frequency for each string remains the same. The fundamental frequency decreases for each string as the guitarist moves from the higher-pitched strings to the lower-pitched strings. Therefore, by the equation $v = f_1\lambda_1$, the speed of the wave along the string must also decrease. Thus, from the relationship $v = \sqrt{\frac{T}{\mu}}$, either the mass per unit length must increase (as it is an inverse relationship) or the tension must decrease. A thicker string of the same material means a greater mass per unit length and a lower pitch. Using a low-density material (such as nylon) for the higher-pitched strings and denser steel for the lower-pitched strings also gives a higher mass per unit length for the lower-pitched strings. Guitarists tune guitars by adjusting the tension in the strings.

3 For the D string, the fundamental frequency is 146.83 Hz and the wavelength is 1.30 m.

$v_D = f_1\lambda_1 = 146.83 \times 1.30 = 190.88\,m\,s^{-1}$

If the string is shortened to $\frac{2}{3}$ of the original length:

$l = \frac{2}{3} \times 0.650 = 0.433\,m$ and

$\lambda = 2l = 2 \times 0.433 = 0.866\,m$

Then:

$f_1 = \frac{v_D}{\lambda_D} = \frac{190.88}{0.866} = 220\,Hz$

Note that the final answer is expressed to 3 significant figures, as the wavelength is expressed to 3 significant figures.

4 The high-E string has a frequency of 329.63 Hz.

Waves travel along the low-E string with a velocity of 107 m s^{-1} (from question **1**).

Matching this to the frequency of the high-E string corresponds to a wavelength of $\frac{v}{f} = \frac{107}{329.63} = 0.325\,m$.

Therefore, since this is the fundamental, $l = \frac{\lambda_1}{2} = \frac{0.325}{2} = 0.163\,m$.

Note that the velocity of the waves along the high-E string will be greater than the velocity of the waves along the low-E string, as discussed in question **2**.

5 For the low-E string, $v = 107\,m\,s^{-1}$ (from question **1**).

When $n = 3$:

$\lambda_3 = \frac{2l}{3} = \frac{2 \times 0.650}{3} = 0.433\,m$ and

$f_3 = \frac{v_E}{\lambda_3} = \frac{107}{0.433} = 247\,Hz$

This is almost the same as the fundamental frequency of the B string. Thus the $n = 3$ vibration of the low-E string should resonate with the B string and cause it to vibrate.

KEY QUESTIONS

Knowledge and understanding

1 **a** False: The frequency of the wave stays the same.

b False: The speed of the wave stays the same.

c False: There is no phase shift when reflected from an end that is free to move. The 180° phase change occurs when the wave is reflected from a fixed end.

d True

2 A transverse wave moving along a rope is reflected from a fixed end. The interference that occurs during the superposition of the reflected wave and the original wave creates a standing wave. This standing wave consists of locations called nodes (where the movement of the rope is cancelled out) and antinodes (where maximum movement of the rope occurs).

It is a common misconception that these standing waves remain stationary. It is only the pattern made by the nodes and antinodes along the rope that stays still. The rope is still moving, especially at the antinodes.

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3 When the wave is reflected, there is a 180° change in phase (and thus a crest is reflected as a trough and vice versa).

4 $\lambda = \frac{2l}{n} = \frac{2 \times 0.40}{1} = 0.80\,\text{m}$

$f = \frac{v}{\lambda_1} = \frac{58}{0.80} = 73\,\text{Hz}$

5 Rearranging $\lambda = \frac{2l}{n}$ gives $l = \frac{n\lambda}{2} = \frac{4 \times 0.750}{2} = 1.50\,\text{m}$

6 a $f_1 = \frac{nv}{2l} = \frac{1 \times 300}{2 \times 0.50} = 300\,\text{Hz}$

b $f_2 = \frac{nv}{2l} = \frac{2 \times 300}{2 \times 0.50} = 600\,\text{Hz}$

c $f_3 = \frac{nv}{2l} = \frac{3 \times 300}{2 \times 0.50} = 900\,\text{Hz}$

Analysis

7 This wave will have a frequency four times that of the fundamental frequency. This means that it will have a wavelength $\frac{1}{4}$ of the fundamental wavelength (due to the inverse relationship between frequency and wavelength).

8 a Recall the formula $l = n\frac{\lambda}{2}$. There are 4 half-wavelengths in the diagram. Therefore $n = 4$.

b For the fifth harmonic $n = 5$, so there will be 5 half-wavelengths. Therefore $\lambda = \frac{2l}{n} = \lambda_5 = \frac{2}{5} \times 10 = 4.0\,\text{m}$.

9 $f_1 = 350\,\text{Hz}$ and $v = 387\,\text{m s}^{-1}$. Therefore the original fundamental wavelength is $\lambda_1 = \frac{v}{f_1} = \frac{387}{350} = 1.106\,\text{m}$.

Shortening the length of the string by two thirds will also shorten the fundamental wavelength by two thirds:

$\lambda_{1\ \text{new}} = \frac{2}{3} \times \lambda_1 = \frac{2}{3} \times 1.106 = 0.737\,\text{m}$.

8.3 Evidence for the wave model of light

Worked example: Try yourself 8.3.1

USING THE WAVE EQUATION FOR LIGHT

A particular colour of red light has a wavelength of 600 nm. Calculate the frequency of this colour.

Thinking	Working
Recall the wave equation for light.	$c = f\lambda$
Transpose the equation to make frequency the subject.	$f = \frac{c}{\lambda}$
Substitute the appropriate values to determine the frequency.	$f = \frac{3.0 \times 10^8}{600 \times 10^{-9}}$ $= 5.0 \times 10^{14}\,\text{Hz}$

CASE STUDY: ANALYSIS

Heating up food in a microwave

1 The microwave oven is a resonant cavity. The microwave electromagnetic radiation is reflected from the cavity walls and forms a standing wave. The hot spots are where the antinodes occur.

2 $2.45\,\text{GHz} = 2.45 \times 10^9\,\text{Hz}$

3 $f = 2.45 \times 10^9\,\text{Hz}$, $\lambda = 0.06 \times 2 = 0.12\,\text{m}$

$c = f\lambda$

$= 2.45 \times 10^9 \times 0.12$

$= 2.94 \times 10^8\,\text{m s}^{-1}$

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4 The accepted value is $c = 2.997\ 924\ 58 \times 10^8\,\text{m s}^{-1}$.

$$\Delta c = \frac{2.99792458 \times 10^8 - 2.94 \times 10^8}{2.99792458 \times 10^8} \times 100 = 2\%$$

5 The marshmallows may have melted unevenly. There would be uncertainty in measuring the position of the antinodes.

Worked example: Try yourself 8.3.2

CALCULATING WAVELENGTH FROM FRINGE SEPARATION

A green laser is directed through a pair of thin slits that are 25.0 μm apart. The slits are 1.50 m from a screen on which bright fringes are 3.30 cm apart. What is the wavelength of the green light in nm?

Thinking	Working
Recall the equation for fringe separation.	$\Delta x = \frac{\lambda L}{d}$
Transpose the equation to make λ the subject.	$\lambda = \frac{\Delta x d}{L}$
Substitute the given values and solve for λ. (Note: 1 μm = 1×10^{-6} m)	$\lambda = \frac{0.0330 \times 25.0 \times 10^{-6}}{1.50}$ $= 5.50 \times 10^{-7}$ m
Express your answer using convenient units (in this case nm, where 1 nm = 1×10^{-9} m).	$\lambda = 550$ nm

CASE STUDY: ANALYSIS

X-ray diffraction

1 The wavelength of X-rays is similar in size to the lattice spacing of the crystal (i.e. the spacing between the atoms).

2 $f = 4.23 \times 10^{18}$ Hz

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{4.23 \times 10^{18}}$$
$$= 7.09 \times 10^{-11}\,\text{m}$$
$$= 0.71\,\text{Å}$$

3 The peaks in an X-ray diffraction pattern occur where the path difference is equal to one wavelength or multiples of one wavelength (which is where constructive interference occurs).

4 For the left peak, $2\theta = 12°$ so $\theta = 6°$

At $\theta = 6°$ and $\lambda = 0.71 \times 10^{-10}$ m:

$$d = \frac{n\lambda}{2\sin\theta} = \frac{1 \times 0.71 \times 10^{-10}}{2\sin 6°} = 3.4 \times 10^{-10}\,\text{m} = 3.4\,\text{Å}$$

For the right peak, $2\theta = 27°$ so $\theta = 13.5°$

At $\theta = 13.5°$:

$$d = \frac{n\lambda}{2\sin\theta} = \frac{1 \times 0.71 \times 10^{-10}}{2\sin 13.5°} = 1.52 \times 10^{-10}\,\text{m} = 1.5\,\text{Å}$$

KEY QUESTIONS

Knowledge and understanding

1 Light does not require a medium in which to travel. Accelerating charges produce varying magnetic fields. A varying magnetic field produces a varying electric field, which produces a varying magnetic field and so on. Therefore light is self-propagating and we can see light from far away (providing there is no obstruction or heavy gravitational field which can divert the light).

2 **a** D

b Significant diffraction occurs when $\frac{\lambda}{w}$ is approximately 1 or greater. Red light has a wavelength of 700 nm (which is approximately 10^{-6} m) and a diffraction opening of 0.001 mm is 0.001×10^{-3} or 10^{-6} m.

3 If light were a particle it would be expected to create two bright bands on the screen behind the slits. However, an interference pattern with alternating bright and dark lines is seen, which is characteristic of wave behaviour.

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4 C and D. As laser light is monochromatic and coherent, all waves line up and are in phase, so it is more likely to produce the interference pattern observed in Young's experiment. Torch light has many wavelengths and the waves are not in phase.

5 A and D. When crests meet troughs, the addition of these out-of-phase waves means that complete destructive interference occurs and they cancel to form a node.

6 Recall the equation for fringe separation: $\Delta x = \frac{\lambda L}{d}$.

- **a** increase
- **b** decrease, as the wavelength is shorter
- **c** increase

Analysis

7 $\text{pd} = \left(n + \frac{1}{2}\right)\lambda$

For the fifth dark fringe, $n = 4$. (Recall that numbering starts from $n = 0$.). Hence $\text{pd} = (4 + \frac{1}{2})\lambda$.

Therefore the fifth dark fringe occurs where the path difference is $4.5\lambda = 4.5 \times 580\,\text{nm} = 2610\,\text{nm}$ or $2.61 \times 10^{-6}\,\text{m}$.

8 Constructive interference occurs when the path difference is a whole-number multiple of the wavelength. Destructive interference occurs when the path difference is an odd-number multiple of half the wavelength.

- **a** destructive
- **b** constructive
- **c** destructive

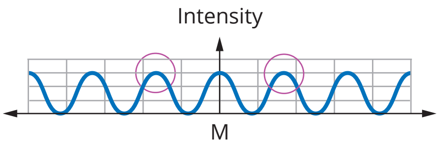
9 $\text{pd} = n\lambda$. Recall that the central maximum is where $n = 0$. So for the second bright fringe, $n = 2$ and $\text{pd} = 2\lambda$. Therefore the second bright fringe occurs where the path difference is $2 \times 700 = 1400\,\text{nm}$.

10 $\Delta x = \frac{\lambda L}{d}$

$$\lambda = \frac{\Delta x d}{L} = \frac{0.037 \times 40 \times 10^{-6}}{3.25} = 4.55 \times 10^{-7}\,\text{m} = 455\,\text{nm}$$

11 The diagram shows diffraction occurring. Increasing the frequency of the wave decreases its wavelength. Once the wavelength is smaller than the slit width, the diffraction effects would become less significant.

12 The central antinode occurs where both waves have travelled the same distance, i.e. the path difference is 0. The next antinodes on either side occur when the path difference is 1λ.



Chapter 8 Review

Knowledge and understanding

1 The green wave represents the superposition of the blue and the red waves.

2 This is known as destructive interference. The two wave pulses must have the same frequency (or wavelength) and the same amplitude and be out of phase by 180°. In these circumstances a maximum positive displacement of one wave coincides with a maximum negative displacement of the other wave.

3 The motor produces different frequencies of vibrations depending on its speed. At speeds where the car is vibrating more strongly, the motor frequency can be assumed to be at a similar frequency to the natural frequency of the car. This is known as resonance. The amplitude of the vibration will depend on how well the wheels are aligned.

4 **a** A node is where the amplitude of the standing wave is zero.

b An antinode is where the amplitude of the standing wave varies from maximum positive through to zero through to maximum negative.

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5 **a** The lowest frequency is the longest wavelength, which is the fundamental.

$$f_n = \frac{nv}{2l}$$

$$f_1 = \frac{1 \times 400}{2 \times 0.950}$$

$$= 211\,\text{Hz}$$

b $f_3 = 3 \times f_1 = 3 \times 211 = 633\,\text{Hz}$

6 The fundamental frequency is given by $f_1 = \frac{1}{T} = \frac{1}{4.0} = 0.25\,\text{Hz}$.

The frequency of the second harmonic is given by $f_2 = 2 \times f_1 = 2 \times 0.25 = 0.50\,\text{Hz}$.

7 Calculate the wavelength of the wave using the wave equation:

$$v = f\lambda$$

$$\lambda = \frac{v}{f}$$

$$= \frac{95.0}{540}$$

$$= 0.176\,\text{m}$$

Since the distance between an antinode and a node in a standing wave in a string with fixed ends is a quarter of a wavelength, then:

$$d = \frac{\lambda}{4} = \frac{0.176}{4}$$

$$= 0.044\,\text{m or } 4.4\,\text{cm}$$

8 **a** The strings on a violin are fixed at both ends. Therefore $l = \frac{\lambda_1}{2} = \frac{0.710}{2} = 0.355\,\text{m}$.

b If $n = 5$, $\lambda_5 = \frac{2l}{n} = \frac{2 \times 0.355}{5} = 0.142\,\text{m}$

Alternatively, $\lambda_5 = \frac{\lambda_1}{5} = \frac{0.710}{5} = 0.142\,\text{m}$

9 The speed of light is $3.0 \times 10^8\,\text{m}\,\text{s}^{-1}$. Start by converting 2.537 million years to seconds:

$t = 2.537 \times 10^6 \times 365.25 \times 24 \times 60 \times 60 = 8.0 \times 10^{13}\,\text{s}$.

Since $c = \frac{d}{t}$, $d = ct = 3.0 \times 10^8 \times 8.0 \times 10^{13} = 2.4 \times 10^{22}\,\text{m}$.

This distance is so large that astronomers use the term 'light-years' to express astronomical distance. The Andromeda galaxy is 2.537 million light-years from the Earth.

10 A varying magnetic field is produced by accelerating oscillating charged particles. The varying magnetic field induces a varying electric field which in turn produces a magnetic field. This sequence repeats indefinitely.

11 The phenomenon is diffraction. The figure shows the edges of the waves bending as the waves pass through the gap. Narrowing the gap would make the effect stronger.

12 D. The frequency and the speed of light are both known. Solve for the wavelength of green light:

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{5.66 \times 10^{14}} = 5.3 \times 10^{-7}\,\text{m}$$

Convert to mm by multiplying by 1000: $\lambda = 5.3 \times 10^{-4}\,\text{mm}$.

13 The green light ($\lambda = 525\,\text{nm}$) has a longer wavelength than blue light ($\lambda = 460\,\text{nm}$). Since $\Delta x = \frac{\lambda L}{d}$, the diffraction pattern spacing, Δx, would spread out more when changing from a blue laser to a green laser.

14 **a** $\Delta x = \frac{\lambda L}{d}$

$$\lambda = \frac{\Delta x d}{L}$$

$$= \frac{0.031 \times 75.0 \times 10^{-6}}{4.00}$$

$$= 5.81 \times 10^{-7}$$

$$= 581\,\text{nm}$$

b 581 nm is closest to yellow.

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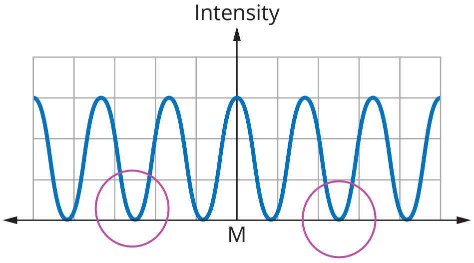
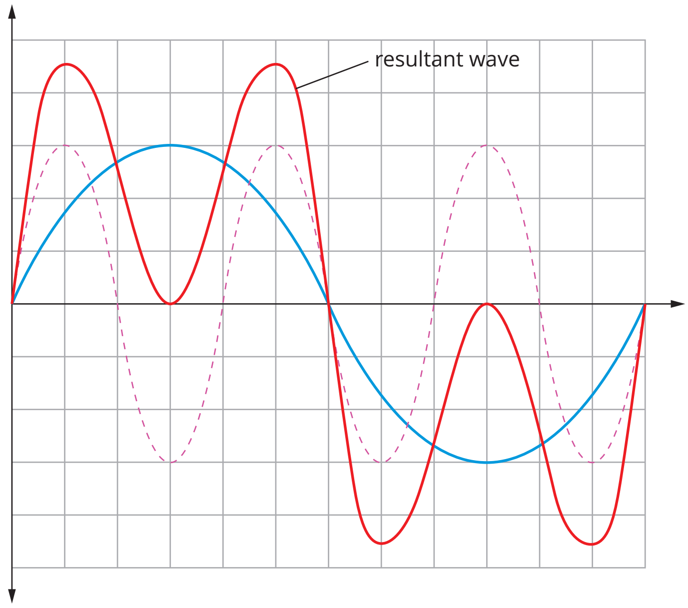
15 A path difference of $1\frac{1}{2}\lambda$ corresponds to the second dark band on each side of the central maximum (at M), circled on the diagram below.

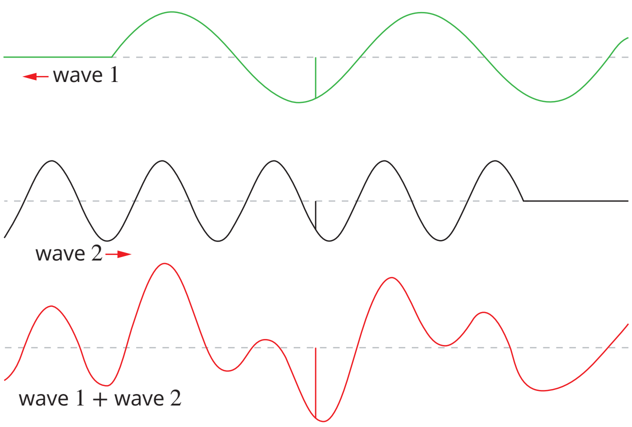


Application and analysis

16 The resultant wave pattern, shown by the red line, is determined using the principle of superposition.



17 The resultant wave from the interference of two waves travelling in opposite directions is shown below.



18

The shorter wavelength shows a smaller diffraction effect because the width of the gap is much greater than the wavelength. The larger wavelength shows significant diffraction because the width of the gap is similar to the wavelength.

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19 This would occur if the natural frequency of the human ear is 2500 Hz. If the applied frequency of the signal generator is the same as the natural frequency of the ear canal, there would be maximum transfer of energy from the generator to the ear, which is perceived as a louder sound.

20 **a** The principle of superposition states that the resultant wave pattern from two overlapping waves is the vector sum of the individual displacements of the waves. The resultant wave has an amplitude greater than that of each individual wave. This is constructive interference. The whole wave shows constructive interference, as both waves have the same period (if the horizontal axis is time) or wavelength (if the horizontal axis is linear position).

b A $\frac{\lambda}{2}$ (or 180°) shift in phase of one of the waves would result in a resultant wave pattern with an amplitude less than the individual amplitudes of wave A and wave B. This would be destructive interference.

21 The spacing between the ruts is $d = 0.10$ m and $v = 50\text{ km h}^{-1} = \frac{50}{3.6} = 13.9\text{ m s}^{-1}$. Therefore $f = \frac{v_{car}}{d} = \frac{13.9}{0.10} = 139\text{Hz}$.

22 The wavelength of light is very small, in the order of 10^{-7} m. Significant diffraction only occurs when $\frac{\lambda}{w} \geq 1$. The wavelength of light is too small to diffract around most everyday objects.

23 The visible light used in optical instruments varies in wavelength from 400 nm to 700 nm. For objects with spacings of 400 nm or less, diffraction effects will limit the ability of the microscope or telescope to resolve the image.

24 A microwave oven is tuned to produce electromagnetic waves with a frequency of 245 GHz. This is the resonant frequency of water molecules. When food is bombarded with radiation at this frequency, the water molecules in the food start to vibrate. The energy of the water molecules is transferred to the rest of the food, heating it up.

25 **a** False. Visible light waves have a wavelength range from 400 nm to 750 nm and require an opening with a width of 400 nm to 750 nm for diffraction to occur.

b Determine the wavelength:

$$\lambda = \frac{v}{f} = \frac{0.5}{0.2} = 2.5\text{m}$$

This is similar in size to the opening, so the statement is true.

c Determine the wavelength:

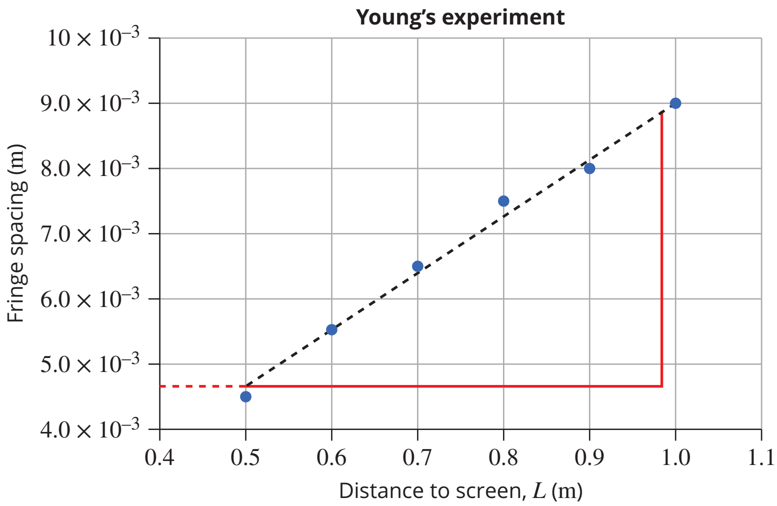
$$\lambda = \frac{v}{f} = \frac{400}{261.6} = 1.53\text{m}$$

This is similar in size to the opening, so the statement is true.

d False. Red light has a longer wavelength than violet light; therefore red light will diffract through a larger gap than violet light.

e True

26 Plot the data in the table to obtain the graph below.



Draw line of best fit. The gradient construction lines (shown in red above) should be drawn from the line of best fit. From the line of best fit:

$$\text{gradient} = \frac{8.9\times10^{-3} - 4.7\times10^{-3}}{0.98 - 0.5} = 0.00875$$

From $\Delta x = \frac{\lambda}{d}L$, the gradient $= \frac{\lambda}{d}$.

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Rearranging to make the slit separation the subject of the equation gives:

$$d = \frac{\lambda}{\text{gradient}} = \frac{550 \times 10^{-9}}{0.00875} = 63 \times 10^{-6} = 63\,\mu\text{m}$$

Answers between 60 to 65 μm are acceptable.

27 Count the number of wavelengths between S_1 and position A:

$d_1 = 5$ wavelengths $= 5\lambda$

Count the number of wavelengths between S_2 and position A:

$d_2 = 6$ wavelengths $= 6\lambda$

Path difference $= 6\lambda - 5\lambda = \lambda$

Thus constructive interference occurs at position A. This is called an antinodal point.

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Chapter 9 The dual nature of light and matter

9.1 The photoelectric effect

Worked example: Try yourself 9.1.1

USING PLANCK'S EQUATION

Calculate the energy in joules of a quantum of infrared radiation that has a frequency of 3.6×10^{14} Hz.

Thinking	Working
Recall Planck's equation.	$E = hf$
Substitute the appropriate values and solve for E.	$E = 6.63 \times 10^{-34} \times 3.6 \times 10^{14}$ $= 2.4 \times 10^{-19}$ J

Worked example: Try yourself 9.1.2

CONVERTING TO ELECTRON VOLTS

A quantum of light has 2.4×10^{-19} J of energy. Convert this energy to electron volts.

Thinking	Working
Recall the rate for converting joules to electron volts and vice versa.	1 eV = 1.6×10^{-19} J
Divide the value expressed in joules by 1.6×10^{-19} to convert it to electron volts.	$E = \frac{2.4 \times 10^{-19}}{1.6 \times 10^{-19}}$ $= 1.5$ eV

Worked example: Try yourself 9.1.3

CALCULATING QUANTUM ENERGIES IN ELECTRON VOLTS

Calculate the energy in eV of a quantum of infrared radiation that has a frequency of 3.6×10^{14} Hz. Assume that $h = 4.14 \times 10^{-15}$ eV s.

Thinking	Working
Recall Planck's equation.	$E = hf$
Substitute in the given values and solve for E.	$E = 4.14 \times 10^{-15} \times 3.6 \times 10^{14}$ $= 1.5$ eV

Worked example: Try yourself 9.1.4

CALCULATING THE WORK FUNCTION OF A METAL

Calculate the work function (in J and eV) of gold, which has a threshold frequency of 1.2×10^{15} Hz.

Thinking	Working
Recall the formula for work function.	$\phi = hf_0$
Substitute the threshold frequency of the metal and solve for ϕ.	$\phi = 6.63 \times 10^{-34} \times 1.2 \times 10^{15}$ $= 8.0 \times 10^{-19}$ J
Convert the energy from J to eV.	$\phi = \frac{8.0 \times 10^{-19}}{1.6 \times 10^{-19}}$ $= 5.0$ eV

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Worked example: Try yourself 9.1.5

CALCULATING THE KINETIC ENERGY OF PHOTOELECTRONS

Calculate the kinetic energy in eV of the photoelectrons emitted from lead by ultraviolet light with a frequency of 1.50×10^{15} Hz. The work function of lead is 4.14 eV. Assume that $h = 4.14 \times 10^{-15}$ eV s.

Thinking	Working
Recall Einstein's photoelectric equation.	$E_{k\ max} = hf - \phi$
Substitute the given values and solve for $E_{k\ max}$.	$E_{k\ max} = 4.14 \times 10^{-15} \times 1.50 \times 10^{15} - 4.14$ $= 2.07$ eV

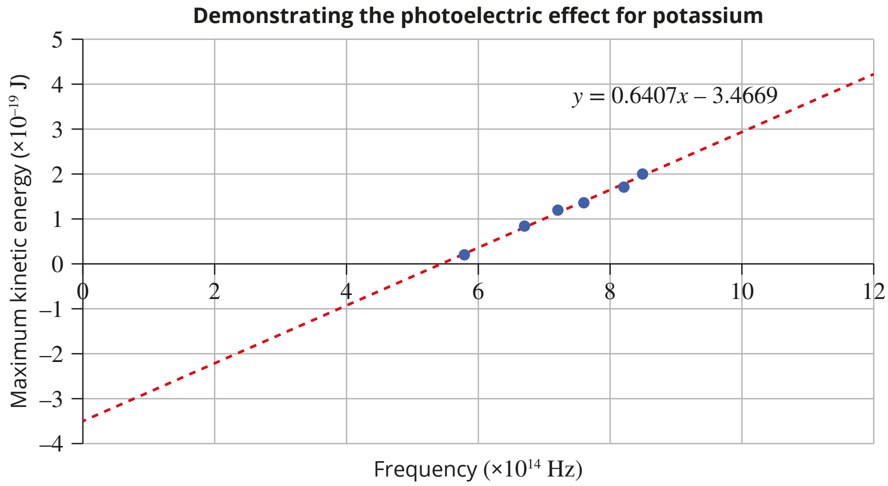
CASE STUDY: ANALYSIS

Lenard's experiment

1 The frequencies can be calculated using the formula $f = \frac{c}{\lambda}$. The maximum kinetic energy can be converted to joules using 1 eV = 1.6×10^{-19} J.

λ (nm)	Maximum kinetic energy (eV)	$f (\times 10^{14}$ Hz)	Maximum kinetic energy ($\times 10^{-19}$ J)
517	0.14	5.8	0.22
448	0.53	6.7	0.85
414	0.75	7.2	1.20
395	0.86	7.6	1.38
366	1.09	8.2	1.74
353	1.25	8.5	2.00

2



3 From the graph in question **2**:

a $m = 6.4 \times 10^{-34}$ J s

b $y = (0.64 \times 10^{-33})x - 3.5$

c x-intercept = 5.4×10^{14} Hz

4 From the values derived in question **3**:

a The gradient of the line is Planck's constant, h:
$h = 6.4 \times 10^{-34}$ J s

b The x-intercept of the line is the threshold frequency, f_0:
$f_0 = 5.4 \times 10^{14}$ Hz

c The y-intercept of the line is the work function, ϕ:
$\phi = 3.5 \times 10^{-19}$ J

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KEY QUESTIONS

Knowledge and understanding

1 In the photoelectric effect, a metal surface may become positively charged if light shining on it with a frequency above the threshold frequency of the metal causes electrons to be released. This leaves electron-deficient atoms, hence the positive charge. If the incident light is below the threshold frequency for that metal, the photoelectric effect will not occur.

2 **a** True

b False. When light sources of the same intensity but different frequencies are used, the higher frequency light has a higher stopping voltage, but it produces the same maximum current as the lower-frequency light.

c True

3 **a** True

b False. The stopping voltage is reached when the photocurrent is reduced to zero.

c True

d True

4 $E_{k\,max} = 4.14 \times 10^{-15} \times 8.00 \times 10^{14} - 3.15$

$= 0.162\,\text{eV}$

Analysis

5 **a** $E = \frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{656 \times 10^{-9}}$

$= 3.0 \times 10^{-19}\,\text{J}$

b $E = \frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{589 \times 10^{-9}}$

$= 3.4 \times 10^{-19}\,\text{J}$

c $E = \frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{486 \times 10^{-9}}$

$= 4.1 \times 10^{-19}\,\text{J}$

d $E = \frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{397 \times 10^{-9}}$

$= 5.0 \times 10^{-19}\,\text{J}$

6 **a** $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.0 \times 10^{15} = 4.1\,\text{eV}$

b $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.1 \times 10^{15} = 4.6\,\text{eV}$

c $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.5 \times 10^{15} = 6.2\,\text{eV}$

7 D. The threshold frequency is:

$f_0 = \frac{\phi}{h}$

$= \frac{3.66}{4.14 \times 10^{-15}}$

$= 8.84 \times 10^{14}\,\text{Hz}$

In order to release photoelectrons, the light must have a frequency higher than the threshold frequency. Therefore frequencies greater than 8.84×10^{14} Hz (e.g. 9.0×10^{14} Hz) are the only frequencies that will release photoelectrons.

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8 $E_k = \frac{hc}{\lambda} - 3.77$

$$= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{310 \times 10^{9}} - 3.77$$

$$= 4.01 - 3.77$$

$$= 0.24\,\text{eV}$$

9 C and D

$$\phi = hf_0 = \frac{hc}{\lambda_0}$$

$$\lambda_0 = \frac{hc}{\phi}$$

$$= 6.86 \times 10^{-7}\,\text{m}$$

$$= 686\,\text{nm}$$

Photons with wavelengths shorter than the threshold wavelength—e.g. violet light and ultraviolet radiation—will cause photoelectrons to be emitted.

10 $E_{k\,max} = hf - \phi$

$$= \frac{hc}{\lambda} - \phi$$

$$0.60 = \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{530 \times 10^{-9}} - \phi$$

$$\phi = 2.34 - 0.60$$

$$= 1.74\,\text{eV}$$

11 **a** $f = \frac{v}{\lambda}$

$$= \frac{3.0 \times 10^{8}}{585 \times 10^{-9}}$$

$$= 5.61 \times 10^{14}\,\text{Hz}$$

b $E = hf$

$$= 6.63 \times 10^{-34} \times 5.61 \times 10^{14}$$

$$= 3.72 \times 10^{-19}\,\text{J}$$

c Total energy = 5000 × energy per photon

$$= 5000 \times 3.72 \times 10^{-19}$$

$$= 1.86 \times 10^{-15}\,\text{J}$$

9.2 The quantum nature of light and matter

Worked example: Try yourself 9.2.1

CALCULATING THE DE BROGLIE WAVELENGTH

Calculate the de Broglie wavelength of a proton travelling at $7.0 \times 10^{5}\,\text{m}\,\text{s}^{-1}$. The mass of a proton is $1.67 \times 10^{-27}\,\text{kg}$.

Thinking	Working
Recall de Broglie's equation.	$\lambda = \frac{h}{mv}$
Substitute the appropriate values and solve for λ.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 7.0 \times 10^{5}}$ $= 5.7 \times 10^{-13}\,\text{m}$

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Worked example: Try yourself 9.2.2

CALCULATING THE DE BROGLIE WAVELENGTH OF A MACROSCOPIC OBJECT

Calculate the de Broglie wavelength of a 66 kg person running at 36 km h^{-1}.

Thinking	Working
Convert velocity to SI units.	$v = \frac{36}{3.6}$ $= 10\,\text{m s}^{-1}$
Recall de Broglie's equation.	$\lambda = \frac{h}{mv}$
Substitute the appropriate values and solve for λ.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{66 \times 10}$ $= 1.0 \times 10^{-36}\,\text{m}$

Worked example: Try yourself 9.2.3

WAVELENGTH OF ELECTRONS EMITTED FROM AN ELECTRON GUN

Find the de Broglie wavelength of an electron that has been accelerated from rest through a potential difference of 50 V. The mass of an electron is 9.1×10^{-31} kg and the charge on an electron is 1.6×10^{-19} C.

Thinking	Working
Calculate the kinetic energy of the electron from the work done on it by the electric potential. Recall from earlier chapters that $W = qV = E_k$.	$W = qV$ $= 1.6 \times 10^{-19} \times 50$ $= 8.0 \times 10^{-18}\,\text{J}$
Calculate the velocity of the electron.	$E_k = \frac{1}{2}mv^2$ $v = \sqrt{\frac{2E_k}{m}}$ $= \sqrt{\frac{2 \times 8.0 \times 10^{-18}}{9.1 \times 10^{-31}}}$ $= 4.2 \times 10^{6}\,\text{m s}^{-1}$
Use de Broglie's equation to calculate the wavelength of the electron.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 4.2 \times 10^{6}}$ $= 1.7 \times 10^{-10}$ $= 0.17\,\text{nm}$

CASE STUDY: ANALYSIS

Electron microscope

1 High-energy electrons colliding with gas atoms will produce anomalies in the resulting images.

2 By de Broglie's equation $\left(\lambda = \frac{h}{mv}\right)$ the wavelength is inversely proportional to the velocity.

3 While an electron is in any part of a magnetic field its path will be circular.

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4 Remember from Chapter 6 that the force on an electron is proportional to both its velocity and the strength of the magnetic field ($F = qvB$). Therefore if we reduce the velocity (v) we must increase the magnetic field (B) by the same factor in order to keep the electron beam travelling in the same path.

$$qvB = \frac{mv^2}{r}$$
$$B = \frac{mv}{qr}$$

5 Convert the energy in eV to J:

$E_k = 150 \times 10^3 \times 1.60 \times 10^{-19} = 240 \times 10^{-16}$ J

Use the equation for kinetic energy to calculate the velocity:

$$E_k = \frac{1}{2}mv^2$$
$$v^2 = \frac{2E_k}{m}$$
$$= \frac{2 \times 240 \times 10^{-16}}{9.1 \times 10^{-31}}$$
$$v = \sqrt{52.7 \times 10^{15}}$$
$$= 2.3 \times 10^8 \text{ m s}^{-1}$$

Worked example: Try yourself 9.2.4

CALCULATING PHOTON MOMENTUM

Calculate the momentum of a photon of blue light with a wavelength of 450 nm.

Thinking	Working
Recall the formula for the momentum of a photon.	$p = \frac{h}{\lambda}$
Convert 450 nm to m.	450 nm = 450×10^{-9} m
Substitute the appropriate values and solve for p.	$p = \frac{h}{\lambda}$ $= \frac{6.63 \times 10^{-34}}{450 \times 10^{-9}}$ $= 1.47 \times 10^{-27}$ kg m s^{-1}

KEY QUESTIONS

Knowledge and understanding

1 The wavelength of a cricket ball is so small that its wave-like behaviour could not be seen by a cricket player.

2 The wavelength of light is larger than the radius of the atom so it cannot be reflected from the atom. An electron microscope can observe individual atoms because the wavelength of the electron is very small and comparable to the radius of the atom. When it strikes the atom it is scattered and can create an image of the atom.

3 B. The wavelike behaviour of matter is determined by its mass and velocity (that is, momentum). Therefore only *moving* particles exhibit wave-like behaviour.

4 Classical physics requires an object to have a rest mass in order to calculate its momentum. Therefore it cannot determine the momentum of a photon because a photon has no rest mass.

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Analysis

5 $W = qV = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2qV}{m}}$$

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{m\frac{\sqrt{2qV}}{\sqrt{m}}}$$

$$= \frac{h}{\sqrt{2qVm}}$$

6 The image from a light microscope is produced by reflecting white light from the object. White light comprises many wavelengths. Those that are reflected to, and seen by, the observer depend on the colour of the object itself.

The image from an electron microscope is produced by a beam of electrons with specific wavelengths that lie outside the visible spectrum. Where the beam passes easily through parts of the object, a white region will result. Areas that are more difficult to pass through will result in shades of grey to black.

7 $\lambda = \frac{h}{mv}$

$$v = \frac{h}{m\lambda}$$

$$= \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times2.5\times10^{-9}}$$

$$= 2.9\times10^{5}\ \text{m s}^{-1}$$

8 **a** $\lambda = \frac{3.0\times10^{8}}{9.6\times10^{17}}$

$$= 3.1\times10^{-10}\ \text{m}$$

b $\lambda = \frac{h}{mv}$

$$3.1\times10^{-10} = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times v}$$

$$v = \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times3.1\times10^{-10}}$$

$$= 2.4\times10^{6}\ \text{m s}^{-1}$$

9 $\lambda = \frac{hc}{E}$

$$= \frac{6.63\times10^{-34}\times3.0\times10^{8}}{3.65\times10^{-13}}$$

$$= 5.45\times10^{-13}\ \text{m}$$

The speed of a proton necessary to exhibit this wavelength is:

$$v = \frac{h}{m\lambda}$$

$$= \frac{6.63\times10^{-34}}{1.67\times10^{-27}\times5.45\times10^{-13}}$$

$$= 7.3\times10^{5}\text{m s}^{-1}$$

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9.3 Light and matter

Worked example: Try yourself 9.3.1

SPECTRAL ANALYSIS

In the Sun's absorption line spectrum, one of the dark Fraunhofer lines corresponds to a frequency of 6.9×10^{14} Hz. Calculate the energy (in joules) of the photons that corresponds to this line.

Thinking	**Working**
Recall Planck's equation.	$E = hf$
Substitute the appropriate values and solve for *E*.	$E = 6.63 \times 10^{-34} \times 6.9 \times 10^{14}$ $= 4.6 \times 10^{-19}$ J

Worked example: Try yourself 9.3.2

USING THE BOHR MODEL OF THE HYDROGEN ATOM

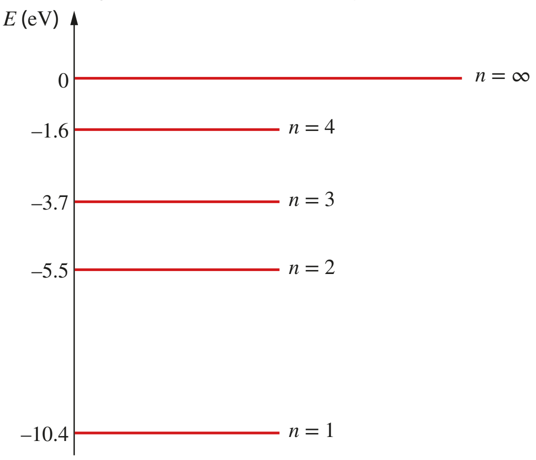
Calculate the wavelength (in nm) of the photon produced when an electron drops from the $n = 3$ energy level of a hydrogen atom to the $n = 1$ energy level. Identify the spectral series to which the corresponding spectral line belongs. Use Figure 9.3.10 to calculate your answer.

Thinking	**Working**
Note the energy of the relevant energy levels of the hydrogen atom.	$E_3 = -1.5$ eV $E_1 = -13.6$ eV
Calculate the difference in energy.	$\Delta E = E_3 - E_1$ $= -1.5 - (-13.6)$ $= 12.1$ eV
Calculate the wavelength of the photon with this amount of energy.	$\lambda = \frac{hc}{E}$ $= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{12.1}$ $= 1.03 \times 10^{-7}$ m $= 103$ nm
Identify the spectral series.	The electron drops to the $n = 1$ energy level. Therefore the spectral line belongs to the Lyman series.

Worked example: Try yourself 9.3.3

ABSORPTION OF PHOTONS

Some of the energy levels for atomic mercury are shown in the diagram below.



Light with photon energies 6.7 eV, 9.0 eV and 11.0 eV passes through some mercury gas. What could happen as a result of the incident light?

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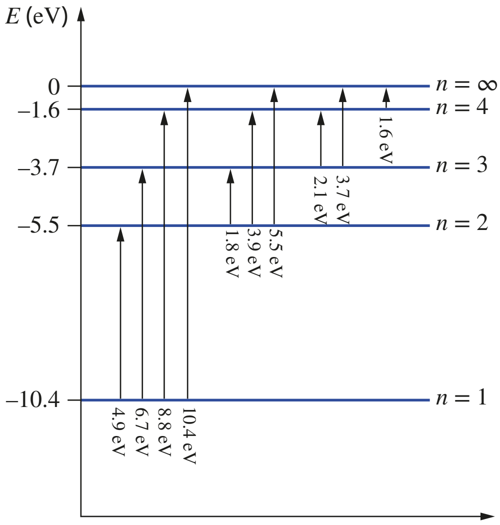
Thinking	**Working**
Calculate the difference in energy between each level.	
Check whether a given photon energy corresponds with any energy difference.	A photon of 6.7 eV corresponds to the energy required to promote an electron from the ground state to the second excited state ($n = 1$ to $n = 3$). The photon may be absorbed. A photon of 9.0 eV cannot be absorbed. A photon of 11.0 eV may ionise the mercury atom. The ejected electron will leave the atom with 0.6 eV of kinetic energy.

KEY QUESTIONS

Knowledge and understanding

1 **a** Energy levels are restricted to certain discrete values.
  b The ground state is when the electrons in the atom are not in an excited state. It is the lowest energy state.
  c Excited states are the quantised energy states to which electrons can be excited by the addition of energy.
  d Ionisation energy is the minimum energy needed to overcome the forces keeping the electron in the atom.

2 The electrons in an element in the gaseous state become excited to a higher energy level when the gas is heated or an electric current flows through it. On returning to their ground state, the electrons emit the energy gained as a photon of light.

3 Bohr's model could not explain the spectra of multi-electron atoms, the continuous emission spectrum of compounds and the two close spectral lines in hydrogen that are only revealed at high resolution.

4 An electron ordinarily occupies the lowest energy orbit, so absorption is observed when electrons move mainly from the ground state, while emission includes all possible downward transitions from a higher excited state to a lower excited state.

5 $E = hf$

$= 6.63 \times 10^{-34} \times 5.3 \times 10^{14}$

$= 3.5 \times 10^{-19}$ J

6 $E = \dfrac{hc}{\lambda}$

$\lambda = \dfrac{hc}{E}$

$= \dfrac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{1.84}$

$= 6.75 \times 10^{-7}$ m

$= 675$ nm

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Analysis

7 **a** The energies of incident photons that correspond to the exact differences between energy levels in the lithium atom are absorbed if the photons collide with electrons in a gas atom and excite them to a higher energy level. The wavelengths that correspond to these energies are removed from the incident beam, leaving a dark line on the spectrum of the emergent beam.

b
$$E = \frac{hc}{\lambda}$$
$$= \frac{6.63\times10^{-34}\times3.0\times10^{8}}{452\times10^{-9}}$$
$$= 4.40\times10^{-19}\ \text{J}$$
$$= \frac{4.40\times10^{-19}}{1.60\times10^{-19}}\ \text{eV}$$
$$= 2.75\ \text{eV}$$

Electrons absorbed, and removed, the 452 nm photons from the incident beam and were excited from the $n = 3$ level to the $n = 4$ level.

8 **a**
$$\Delta E = E_3 - E_1$$
$$= -1.5 - (-13.6)$$
$$= 12.1\ \text{eV}$$

b
$$E = \frac{hc}{\lambda}$$
$$\lambda = \frac{hc}{\Delta E}$$
$$= \frac{4.14\times10^{-15}\times3.0\times10^{8}}{12.1}$$
$$= 1.03\times10^{-7}\ \text{m}$$
$$= 103\ \text{nm}$$

9
$$E = \frac{hc}{\lambda} = E_5 - E_2$$
$$\frac{4.14\times10^{-15}\times3.0\times10^{8}}{410\times10^{-9}} = E_5 - (-3.4)$$
$$3.03 = E_5 - (-3.4)$$
$$E_5 = 3.03 - 3.4$$
$$= -0.37\ \text{eV}$$

Chapter 9 Review

Knowledge and understanding

1 **a** The detector observed a sequence of maximum and minimum intensities.

b As the electron beam is diffracted, the electrons are exhibiting wave-like behaviour. Electrons are not light but, like light, a beam of electrons can be diffracted.

2 The energy levels in an atom cannot be any value within a continuous range but are restricted to certain discrete values, i.e. the levels are quantised.

3 Bohr's work on the hydrogen atom and his idea of electrons revolving around the nucleus in orbits with specific energies convinced many scientists that a particle model was needed to explain the way light behaves in certain situations. It built significantly on the work of Planck and Einstein.

4 The emission line spectrum of sodium appears as a series of coloured lines. The absorption line spectrum of sodium appears as a full visible spectrum with a number of dark lines. The colours that are missing in the absorption line spectrum match the colours that are visible in the emission line spectrum.

5 The Sun's spectrum is an absorption line spectrum with the dark lines being the same as those in the emission line spectra of hydrogen and helium. This indicates that these gases are present in the Sun's atmosphere.

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6 The photoelectric effect is explained by the particle nature of light and electron diffraction patterns are explained by the wave nature of particles. The interference pattern in Young's double slit experiment is used as evidence of the wave nature of light, so an electron diffraction pattern produced in a similar way suggests that electrons have a wave property.

7 From de Broglie's equation, momentum, p, equals $\frac{h}{\lambda}$. It is possible for particles and photons to have the same momentum provided both the particle and photon have the same wavelength.

8 photoelectrons

9 $E = hf = 4.14 \times 10^{-15} \times 6.0 \times 10^{14} = 2.5\,\text{eV}$

10 $E = 5.0 \times 1.6 \times 10^{-19}\,\text{J} = 8.0 \times 10^{-19}\,\text{J}$

11 $\phi = hf_0$

$$f_0 = \frac{\phi}{h}$$
$$= \frac{5.0}{4.14 \times 10^{-15}}$$
$$= 1.2 \times 10^{15}\,\text{Hz}$$

12

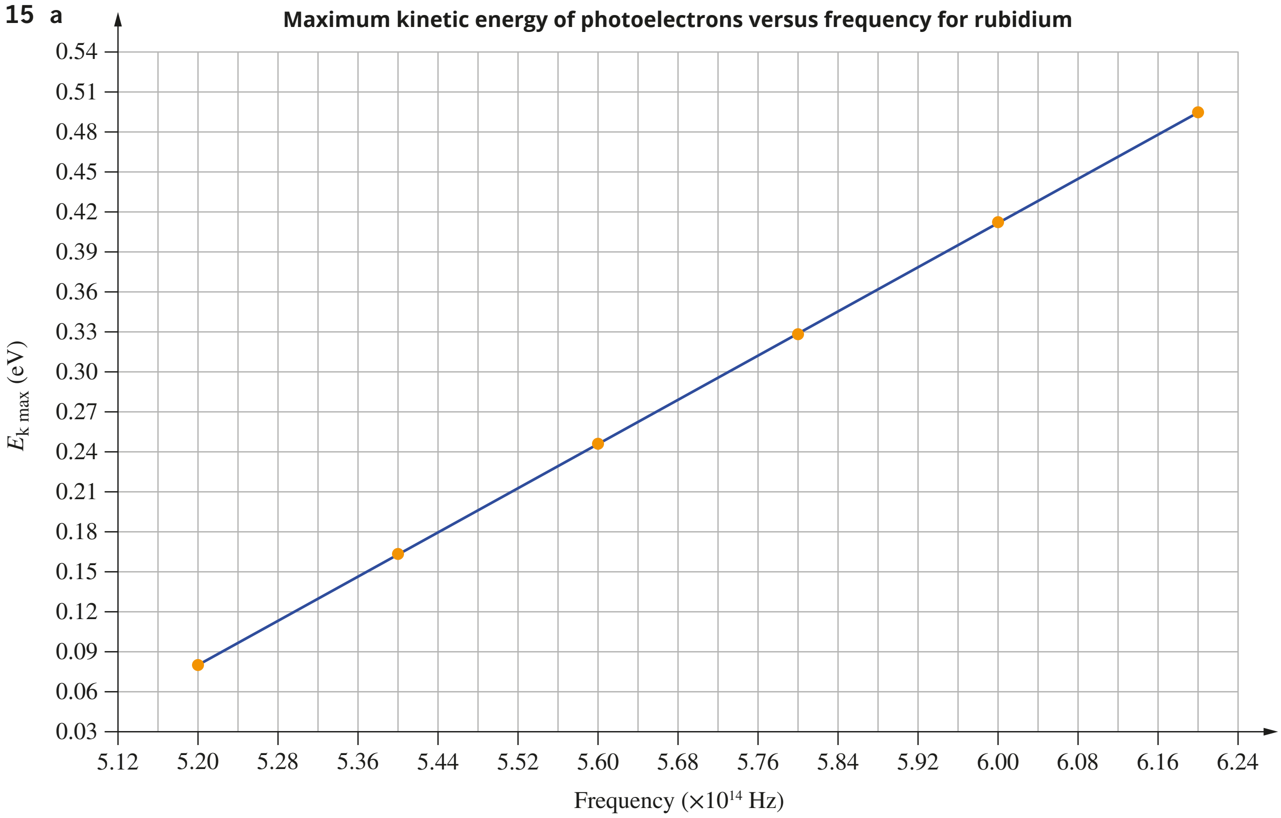
$$\phi = hf_0$$
$$= 4.14 \times 10^{-15} \times 1.5 \times 10^{15}$$
$$= 6.2\,\text{eV}$$
$$E_{\text{k max}} = 4.14 \times 10^{-15} \times 2.2 \times 10^{15} - 6.2$$
$$= 2.9\,\text{eV}$$

13 The stopping voltage is equivalent to the maximum kinetic energy of the photoelectrons, so $E_{\text{k max}} = 1.95\,\text{eV}$.

Application and analysis

14 The work function is given by the y-intercept of the $E_{\text{k max}}$ versus frequency graph. Approximate values are: Rb = 2.1 eV, Sr = 2.5 eV, Mg = 3.4 eV, W = 4.5 eV

15 a



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b gradient $= h = \frac{\text{rise}}{\text{run}}$

$$= \frac{0.494 - 0.080}{6.20 \times 10^{14} - 5.20 \times 10^{14}}$$

$$= \frac{0.414}{1.00 \times 10^{14}}$$

$$= 4.14 \times 10^{-15}\ \text{eV s}$$

c The x-intercept is at approximately 5.0×10^{14} Hz. This is the threshold frequency.

d No. The frequency of red light is below the threshold frequency for rubidium. Its frequency is:

$$f = \frac{c}{\lambda}$$

$$= \frac{3.0 \times 10^{8}}{680 \times 10^{-9}}$$

$$= 4.41 \times 10^{14}\ \text{Hz}$$

This is less than the threshold frequency of 5.0×10^{14} Hz, so no photoelectrons will be emitted.

16 **a** $E = \frac{hc}{\lambda}$

$$= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{260 \times 10^{-12}}$$

$$= 4777\ \text{eV}$$

$$= 4.78\ \text{keV}$$

b The electrons have a de Broglie wavelength that is similar to the wavelength of the X-rays. This is evidence of the dual nature of light and matter.

c $p = \frac{h}{\lambda}$

$$= \frac{6.63 \times 10^{-34}}{260 \times 10^{-12}}$$

$$= 2.55 \times 10^{-24}\ \text{kg m s}^{-1}$$

17 Electron:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 7.5 \times 10^{6}}$$

$$= 9.7 \times 10^{-11}\ \text{m}$$

Blue light:

$\lambda = 470 \times 10^{-9} = 4.7 \times 10^{-7}$ m

X-ray:

$c = f\lambda$

$$\lambda = \frac{c}{f}$$

$$= \frac{3.0 \times 10^{8}}{5 \times 10^{17}}$$

$$= 6 \times 10^{-10}\ \text{m}$$

Proton:

$$\lambda = \frac{h}{p}$$

$$= \frac{6.63 \times 10^{-34}}{1.7 \times 10^{-21}}$$

$$= 3.9 \times 10^{-13}\ \text{m}$$

Therefore blue light has the longest wavelength (option B).

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18 a $\lambda = \dfrac{h}{mv}$

$= \dfrac{6.63 \times 10^{-34}}{0.040 \times 1.0 \times 10^{3}}$

$= 1.658 \times 10^{-35}$

$= 1.7 \times 10^{-35}\,\text{m}$

b No. The wavelength is much smaller than the size of everyday objects.
The wavelength of the bullet travelling at $1.0 \times 10^{3}\,\text{m s}^{-1}$ is many times smaller than the radius of an atom.
Significant diffraction only occurs when the wavelength is equal to or greater than the size of the gap (or object), i.e. when $\lambda \geq w$.

19 a 6 spectral lines:
n_4 to n_1
n_4 to n_2, n_2 to n_1
n_4 to n_3, n_3 to n_1
n_4 to n_3, n_3 to n_2, n_2 to n_1
As there are two repeated transitions listed, there are only six unique lines.

b $\Delta E = E_4 - E_1$

$= -0.85 - (-13.6)$

$= 12.75\,\text{eV}$

$= hf$

$f = \dfrac{\Delta E}{h}$

$= \dfrac{12.75}{4.14 \times 10^{-15}}$

$= 3.08 \times 10^{15}\,\text{Hz}$

c This is the ultraviolet region.

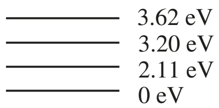
20 a $\dfrac{1}{2}mv^2 = 3.72 \times 1.60 \times 10^{-19}$

$v^2 = \dfrac{1.60 \times 10^{-19} \times 3.72 \times 2}{9.11 \times 10^{-31}}$

$= 1.307 \times 10^{12}$

$v = 1.14 \times 10^{6}\,\text{m s}^{-1}$

b



When high-energy incident electrons collide with ground state electrons, they can impart some of their energy to these electrons. If the energy absorbed by any ground state electron corresponds to the exact energy difference between a higher state and the ground state, the ground state electron will be excited to that state. These excited electrons will quickly fall back to the ground state via one or more steps, emitting a photon of light equal to the energy difference of that step. In this example electrons can be excited to all levels between the ground state and up to, but not including, the energy of the incident electrons, as these must retain some kinetic energy to escape.
The excited states are:
3.72 – 1.61 = 2.11 eV
3.72 – 0.52 = 3.20 eV
3.72 – 0.10 = 3.62 eV
The electron emerging from the tube at 3.72 eV has the same energy as the incident electron, so it must have passed straight through without transferring any of its energy to electrons in the sodium vapour.

c Six spectral lines with energies of 3.62 eV, 3.62 – 2.11 eV, 3.62 – 3.20 eV, 3.20 eV, 3.20 – 2.11 eV, and 2.11 eV are possible, as the excited electron returns to the ground state via one or more steps.

d $E = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{589 \times 10^{-9} \times 1.6 \times 10^{-19}} = 2.11\,\text{eV}$. The electron transition will be from the second energy level to the first energy level (i.e. to the ground state).

e lowest frequency = lowest energy: transition is $n = 4$ to $n = 3$

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Chapter 10 Einstein's special theory of relativity

10.1 Einstein's special relativity

KEY QUESTIONS

Knowledge and understanding

1 They believed that all waves needed to travel in a medium, so just as air is the medium for sound, they invented the aether to be the medium for light.

2 Place a hanging pendulum in both spaceships. The pendulum in the accelerating spaceship will move from its normal vertical position when the spaceship accelerates.

3 The speed of the ball is greater for Jana than it is for Tom.

The speed of the sound in the carriage is greater on the approach than it is on departure for Jana, while for Tom it is the same both forwards and backwards.

The speed of light is the same for Jana and Tom.

4 Two examples are a person standing on a train that has a constant velocity and a passenger sitting on a plane that is travelling with constant velocity.

5 Two examples are a person sitting on a Ferris wheel that is rotating at a constant speed and a driver in a racing car accelerating away from the start line.

6 Einstein's first postulate is that the laws of physics are the same in all inertial frames of reference. On a train or in a plane moving at constant velocity, an object dropped would hit the ground vertically below the point from which it was dropped. Both situations show that the laws of physics apply equally in inertial frames of reference. On a rotating Ferris wheel, the normal force acting on a person would be different at the top of the ride than at the bottom, while if a driver were to drop a ball inside an accelerating racing car, it would hit the floor at a point not vertically below the point from which it is dropped. Both these situations show that in non-inertial frames of reference, the laws of physics don't appear to be consistent with the laws in inertial frames of reference.

7 Max's sister is in circular motion, so she is in a non-inertial frame of reference. Thus she sees the path of the ball curve. Max sees the path of the ball move in a straight line, as the ball is in his inertial frame of reference. He should have thrown the ball slightly in front of his sister for her to catch it.

Analysis

8 **a** $340 + 30 = 370\,\text{m}\,\text{s}^{-1}$

b $340 - 40 = 300\,\text{m}\,\text{s}^{-1}$

c $340 + 20 = 360\,\text{m}\,\text{s}^{-1}$

d $340 + 10 = 350\,\text{m}\,\text{s}^{-1}$

9 **a** In Alex and Bill's frame of reference $v = \frac{7.00}{1.59} = 4.40\,\text{m}\,\text{s}^{-1}$ north, so in Carla's frame of reference

$v = (-6.00) + (4.40) = -1.60 = 1.60\,\text{m}\,\text{s}^{-1}$ south

b $d = vt = (-1.60)(1.59) = -2.54 = 2.54\,\text{m}$ south

c $1.59\,\text{s}$

10 **a** $t = \frac{7.00}{70.0} = 0.100\,\text{s}$

b $70.0\,\text{m}\,\text{s}^{-1}$, as it is in all frames of reference

c $s = vt = (6.00)(0.100) = 0.600\,\text{m}$ south

d $70.0\,\text{m}\,\text{s}^{-1}$, as it is in all frames of reference

10.2 Einstein's *Gedanken* train

KEY QUESTIONS

Knowledge and understanding

1 The three dimensions of space and the fourth dimension of time are interdependent, i.e. the passage of time can be affected by motion in space. It is most noticed when the velocity involved is very fast. Similarly, distances in space are also affected by the motion of an object through space at high speeds.

2 The effects of relativity occur in situations that we cannot normally observe. In a similar way, observations of simultaneous events would not be possible using our senses. Thus we must use thought experiments in which we have special abilities of observation.

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3 Galilean relativity applies to objects moving in inertial frames of reference. The outside observer would see the balls moving at different speeds in their frame of reference. So the ball travelling forwards is going faster by the amount required to cover the extra distance that the front wall moves away in the same time that the ball moving backwards is slowed to cover the shorter distance as the back wall moves forward.

4 Both Amaya and Clare will see the light travel at $3.0 \times 10^8\,m\,s^{-1}$. According to Einstein's second postulate, the speed of light will always be the same no matter what the motion of the light source or observer.

5 The fundamental property that causes the lack of simultaneity is the constancy of the speed of light viewed from any frame of reference.

6 Muons have **very short** lives. They are created approximately 15 km up in the atmosphere. As they travel down through the atmosphere the muon's speed is **very similar to** the speed of light. According to Newtonian laws, muons **should not** reach the Earth's surface. However, **many do**.

Analysis

7 Stephen is in a different frame of reference to Barry. Stephen will be moving towards the light source on the left and away from the light source on the right. Thus Stephen sees the light from the left arrive first and the light from the right arrive some time later. He then deduces that the flashes of light were sent at different times.

8 Answers will vary. An example is a person sitting in a plane travelling at high speed and sending flashes of light simultaneously towards the front and back of the plane. If a stationary observer on the ground could see this light, they will see the flash of light strike the back of the plane first, whereas the person on the plane will see the flashes of light strike the front and back at the same time.

9 Atomic clocks enable extremely short durations to be timed to many decimal places. Differences in time for the same event to occur, when measured by observers in different inertial frames of reference, indicate that time is not uniform between the two inertial frames. The ability to time these very small differences enables researchers to gather evidence to support Einstein's special theory of relativity.

10 **a** $10 \times (23 \times 10^{-24}) = 2.3 \times 10^{-22}\,s$

b $d = vt = (0.993) \times (3.0 \times 10^8) \times (2.3 \times 10^{-22}) = 6.9 \times 10^{-14}\,m$

c To still exist over 8 times the distance from their source, the hydrogen-7 atoms must be living for a longer time. By existing for a longer time, they can travel further.

10.3 Time dilation

Worked example: Try yourself 10.3.1

TIME DILATION

A stationary observer on the Earth sees a very fast scooter passing by, travelling at $2.98 \times 10^8\,m\,s^{-1}$. On the wrist of the rider is a watch on which the stationary observer measures 60.0 s passing. Calculate how many seconds pass by on the stationary observer's clock during this observation. Assume that $c = 3.00 \times 10^8\,m\,s^{-1}$.

Thinking	Working
Identify the variables: the time for the stationary observer is t, the proper time for the moving clock is t_0 and the velocities are v and c.	$t = ?$ $t_0 = 60.0\,s$ $v = 2.98 \times 10^8\,m\,s^{-1}$ $c = 3.00 \times 10^8\,m\,s^{-1}$
Use Einstein's time dilation formula and the Lorentz factor.	$t = \gamma t_0 = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$
Substitute the appropriate values and solve for t.	$t = \gamma t_0 = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$ $= \frac{60.0}{\sqrt{1-\frac{(2.98\times10^8)^2}{(3.00\times10^8)^2}}}$ $= \frac{60.0}{0.11528}$ $= 5.20 \times 10^2\,s$

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KEY QUESTIONS

Knowledge and understanding

1 In a device called a **light** clock, the **oscillation** of light is used to measure **time**, as the speed of light is **constant** no matter from which inertial frame of reference it is viewed.

2 Proper time, t_0. This is because, from this observer's frame of reference, the observer can hold their stopwatch stationary in one location, start it when the front of the space probe is in line with the stopwatch and stop it when the back of the probe is in line with the stopwatch.

3
$$t = \gamma t_0$$
$$= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$= \frac{1.05}{\sqrt{1-\frac{(1.75\times10^8)^2}{(3.00\times10^8)^2}}}$$
$$= \frac{1.05}{0.81223}$$
$$= 1.29\,\text{s}$$

4
$$t = \gamma t_0$$
$$= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$75.0 = \frac{t_0}{\sqrt{1-\frac{(2.30\times10^8)^2}{(3.00\times10^8)^2}}}$$
$$t_0 = 75.0\times0.642$$
$$= 48.2\,\text{s}$$

5
$$t = \gamma t_0$$
$$= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$5.50 = \frac{t_0}{\sqrt{1-\frac{(2.75\times10^8)^2}{(3.00\times10^8)^2}}}$$
$$t_0 = 5.50 \times 0.3996$$
$$= 2.20\,\text{s}$$

6
$$t = \gamma t_0$$
$$= \frac{1}{\sqrt{1-0.50^2}}\times1$$
$$= 1.2\,\text{s}$$

Analysis

7 **a** The height of the clock: 1.0 m

b $t_N = \frac{d}{v} = \frac{1.0}{3.00\times10^8}$

$= 3.3\times10^{-9}\,\text{s}$

c $d = ct_G$

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d As the distance the ship moves in Gemma's frame of reference is $0.90ct_G$ and the height of the clock is 1.0 m, the distance, *d*, which the light travels is given by:

$$d^2 = (0.90ct_G)^2 + 1.0^2 = 0.81c^2t_G^2 + 1.0$$

As this also equals $c^2t_G^2$ (from part **c**), it follows that:

$$0.81c^2t_G^2 + 1.0 = c^2t_G^2$$

$$0.19c^2t_G^2 = 1.0$$

Thus:

$$t_G^2 = \frac{1.0}{0.19c^2}$$

$$t_G = 7.6 \times 10^{-9}\ \text{s}$$

e $\frac{t_G}{t_N} = \frac{7.6 \times 10^{-9}}{3.3 \times 10^{-9}} = 2.3$ which is the same as γ for $v = 90\%$ of c.

8 **a** $t = \gamma t_0$

$$= \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{(2.20 \times 10^{-6})}{\sqrt{1 - \frac{(0.9992c)^2}{c^2}}}$$

$$= \frac{2.20 \times 10^{-6}}{\sqrt{1 - 0.9992^2}}$$

$$= 5.50110 \times 10^{-5}\ \text{s or } 55.0\ \mu\text{s}$$

b Non-relativistic:

$d = vt = 0.9992 \times 3.00 \times 10^8 \times 2.20 \times 10^{-6} = 660$ m

Relativistic:

$d = vt = 0.9992 \times 3.00 \times 10^8 \times 5.50110 \times 10^{-5} = 1.649 \times 10^4$ m or 16.5 km

9 **a** $t = \frac{d}{v}$

$$= \frac{15.5 \times 10^{-2}}{2.93 \times 10^8}$$

$$= 5.29010 \times 10^{-10}\ \text{s}$$

The moving particles last for 5.29×10^{-10} s.

b

$$t = \gamma t_0$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$5.29010 \times 10^{-10} = \frac{t_0}{\sqrt{1 - \frac{(2.93 \times 10^8)^2}{(3.00 \times 10^8)^2}}}$$

$$t_0 = 1.13611 \times 10^{-10}\ \text{s}$$

Hence the particle lives for 1.14×10^{-10} s in the rest frame of reference. This is expected, as the lifetime for the particle at rest should be shorter than when observed to be travelling at high speeds.

10 The equator clock is moving faster relative to the poles, so special relativity effects mean that the clock will run slower relative to an observer at the poles. It is also accelerating and hence will run slower due to general relativity effects. The combined effect is well below what we can detect with any clock, as the speed of the equator is only about 460 m s^{-1}, which is about 1.5 millionth the speed of light.

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10.4 Length contraction

Worked example: Try yourself 10.4.1

LENGTH CONTRACTION

A stationary observer on the Earth sees a very fast scooter travelling by at $2.98 \times 10^8\,\text{m s}^{-1}$. The stationary observer measures the scooter's length as 22.0 cm. Calculate the proper length of the scooter, i.e. the length measured when the scooter is at rest. Assume that $c = 3.00 \times 10^8\,\text{m s}^{-1}$.

Thinking	Working
Identify the variables: the length measured by the stationary observer is L, the proper length of the scooter is L_0 and the velocities are v and c.	$L_0 = ?$ $L = 0.220\,\text{m}$ $v = 2.98 \times 10^8\,\text{m s}^{-1}$ $c = 3.00 \times 10^8\,\text{m s}^{-1}$
Use Einstein's length contraction formula and the Lorentz factor.	$L_0 = L\gamma$ $= \dfrac{L}{\sqrt{1-\dfrac{v^2}{c^2}}}$
Substitute the appropriate values and solve for L_0.	$L_0 = \dfrac{0.220}{\sqrt{1-\dfrac{(2.98\times10^8)^2}{(3.00\times10^8)^2}}}$ $= \dfrac{0.220}{0.11528}$ $= 1.91\,\text{m}$

Worked example: Try yourself 10.4.2

LENGTH CONTRACTION FOR DISTANCE TRAVELLED

A stationary observer on the Earth sees a very fast train approaching a tunnel at a speed of 0.986c. The stationary observer measures the tunnel's length as 123 m. Calculate the length of the tunnel as seen by the train's driver.

Thinking	Working
Identify the variables: the length seen by the driver is L, the proper length of the tunnel is L_0 and the velocity is v.	$L = ?$ $L_0 = 123\,\text{m}$ $v = 0.986c$
Use Einstein's length contraction formula and the Lorentz factor.	$L = \dfrac{L_0}{\gamma}$ $= L_0\sqrt{1-\dfrac{v^2}{c^2}}$
Substitute the appropriate values and solve for L.	$L = 123\sqrt{1-\dfrac{0.986^2\times c^2}{c^2}}$ $= 123\sqrt{1-(0.986)^2}$ $= 123 \times 0.16675$ $= 20.5\,\text{m}$

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CASE STUDY: ANALYSIS

How length contraction affects linear particle accelerators

1 $L = L_0\sqrt{1-\frac{v^2}{c^2}}$

$= (16.0)\sqrt{1-\frac{(0.9999422)^2c^2}{c^2}}$

$= (16.0)(0.003399995)$

$= 0.054399921$

$= 0.0544\,\text{m}$ or $5.44\,\text{cm}$

2 $L = L_0\sqrt{1-\frac{v^2}{c^2}}$

$= (216)\sqrt{1-\frac{(0.9999999855)^2c^2}{c^2}}$

$= (216)(1.702938637\times10^{-4})$

$= 0.036783474$

$= 0.0368\,\text{m}$ or $3.68\,\text{cm}$

3 $C = 2\pi r$

$r = \frac{C}{2\pi}$

$= \frac{216}{2\pi}$

$= 34.4\,\text{m}$

This is the same as the radius that would be seen in the electrons' frame of reference, as it is perpendicular to their motion.

KEY QUESTIONS

Knowledge and understanding

1 To measure proper length, the object being measured must be at rest relative to the observer.

2 Width and height are not affected as they are at right angles to the direction of motion, but the stationary observer will see the spaceship with a contracted length.

3 $L = \frac{L_0}{\gamma}$

$= L_0\sqrt{1-\frac{v^2}{c^2}}$

$= 1.00\times\sqrt{1-\frac{(1.75\times10^8)^2}{(3.00\times10^8)^2}}$

$= 1.00\times0.81223$

$= 0.812\,\text{m}$

4 $L = \frac{L_0}{\gamma}$

$= L_0\sqrt{1-\frac{v^2}{c^2}}$

$= 5.25\times\sqrt{1-\frac{(2.30\times10^8)^2}{(3.00\times10^8)^2}}$

$= 5.25\times0.64205$

$= 3.37\,\text{m}$

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5 **a** $\gamma = \dfrac{3.50}{1.50} = 2.33$

Thus:

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2.33} = 0.429$$

$$\frac{v^2}{c^2} = 1 - 0.184$$

$$v^2 = c^2 \times 0.816$$

$$v = 0.9c \text{ or } 2.71\times10^8\,\text{m s}^{-1}$$

b $L = \dfrac{L_0}{\gamma}$

$$= L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$= 1.50\times\sqrt{1-\frac{(2.71\times10^8)^2}{(3.00\times10^8)^2}}$$

$$= 1.50\times0.42894$$

$$= 0.643\,\text{m}$$

To the car driver the fast-moving garage appears even shorter than its proper length.

Analysis

6 **a** $\gamma = \dfrac{800.0}{400.0} = 2.000$

Thus:

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2.000} = 0.5000$$

$$1-\frac{v^2}{c^2} = 0.25000$$

$$\frac{v^2}{c^2} = 1 - 0.2500$$

$$v^2 = c^2 \times 0.7500$$

$$v = 0.8660c \text{ or } 2.598\times10^8\,\text{m s}^{-1}$$

b $\gamma = 2.000$

$$\frac{L}{L_0} = \frac{1}{\gamma}$$

$$= \frac{1}{2.000} = 0.5000$$

So Emily appears half as thin as she normally would, but only in the direction of her motion. Alternatively, recognise that if the track length has been halved, then Emily appears half her thickness as well.

7 $L = L_0\sqrt{1-\dfrac{v^2}{c^2}}$

$$= 35.0\times\sqrt{1-\frac{(745)^2}{(3.00\times10^8)^2}}$$

$$= 35.0\times1.0000000$$

$$= 35.0\,\text{cm}$$

At this speed there is no difference in length.

8 **a** $L = L_0\sqrt{1-\dfrac{v^2}{c^2}}$

$$= 2.55\times\sqrt{1-\frac{(0.934)^2c^2}{c^2}}$$

$$= 2.55\times\sqrt{(1-(0.934)^2)}$$

$$= 2.55\times0.35727$$

$$= 0.911\,\text{m}$$

b The length of the fishing rod is the proper length: 2.55 m.

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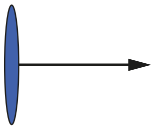
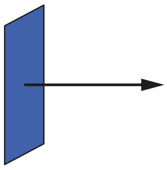
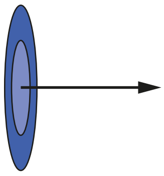
9. **a** an almost flat circular disc with a radius of 125 m



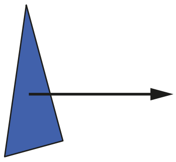
b an almost flat square with sides of 141 m



c an almost flat circular ring 30.0 m in radius



d an almost flat triangle with a base of 50.0 m and an apex 80.0 m above the base



10.5 Einstein's mass-energy relationship

Worked example: Try yourself 10.5.1

TOTAL ENERGY OF AN OBJECT

Calculate the total energy of an electron speeding through the Australian Synchrotron if its rest mass is 9.11×10^{-31} kg and it is travelling at a speed of 2.9979×10^{8} m s^{-1}. Assume that $c = 3.00 \times 10^{8}$ m s^{-1} and that *Gedanken* conditions apply for this question.

Thinking	Working
Identify the relevant variables: the question asks for the total energy E_{tot}, the mass of the electron is m, the speed is v and the speed of light is c.	$E_{tot} = ?$ $m = 9.11 \times 110^{-31}$ kg $v = 2.9979\ 10^{8}$ m s^{-1} $c = 3.00 \times 10^{8}$ m s^{-1}
Use Einstein's total energy formula and the Lorentz factor.	$E_{tot} = \gamma mc^2$ $= \dfrac{mc^2}{\sqrt{1-\dfrac{v^2}{c^2}}}$
Substitute the appropriate values and solve for E_{tot}.	$E_{tot} = \dfrac{mc^2}{\sqrt{1-\dfrac{v^2}{c^2}}}$ $= \dfrac{(9.11\times10^{-31})(3.00\times10^{8})^2}{\sqrt{1-\dfrac{(2.9979\times10^{8})^2}{(3.00\times10^{8})^2}}}$ $= \dfrac{(8.19900\times10^{-14})}{(0.037410)}$ $= 2.19\times10^{-12}$ J

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Worked example: Try yourself 10.5.2

FUSION

A fusion reaction in the Sun fuses two helium nuclides. A helium nucleus and two protons are formed, and 30.0 MeV of energy is released.

$$^{3}_{2}He + ^{3}_{2}He \rightarrow ^{x}_{2}He + 2^{1}_{1}H$$

a What is the value of the unknown mass number x?	
Thinking	**Working**
Analyse the mass numbers.	$3 + 3 = x + (2 \times 1)$ $x = 4$ A helium-4 nucleus is formed.

b How much energy is released in joules?	
Thinking	**Working**
Recall that 1 eV = 1.60×10^{-19} J.	$30.0\text{ MeV} = (30.0 \times 10^{6})(1.60 \times 10^{-19})$ $= 4.80 \times 10^{-12}$ J

c Calculate the mass defect for this reaction.	
Thinking	**Working**
Use $\Delta E = \Delta mc^2$.	$\Delta m = \frac{\Delta E}{c^2}$ $= \frac{(4.80 \times 10^{-12})}{(3.00 \times 10^{8})^2}$ $= 5.33333 \times 10^{-29}$ $= 5.33 \times 10^{-29}$ kg

Worked example: Try yourself 10.5.3

TRANSFORMATIONS AND CONSERVATIONS

Two protons each with 82.0 MeV of kinetic energy collide to produce a neutral pion (π^0) with a mass–energy equivalence of 134.976 8 MeV. One of the two protons remains intact after the collision, while the fate of the other proton is unknown. The total kinetic energy available to the particles after the collision is 29.022 90 MeV. Use $c = 3.00 \times 10^8$ m s^{-1} and the data in the table below to answer the following questions.

Particle	Mass (kg)
proton/antiproton	1.672622×10^{-27}
neutron/antineutron	1.674922×10^{-27}
electron/positron	9.109384×10^{-31}

a Calculate the total energy of the reactants before the collision.	
Thinking	**Working**
Use Einstein's equation to calculate the total mass–energy of the two protons before the collision (in joules).	$E = mc^2$ $= 2(1.672622 \times 10^{-27})(3.00 \times 10^{8})^2$ $= 3.010720 \times 10^{-10}$ J
Convert the energy to MeV.	$E = \frac{(3.010720 \times 10^{-10})}{(1.60 \times 10^{-19})}$ $= 1.881700 \times 10^{9}$ eV $= 1.881700 \times 10^{3}$ MeV

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Add the kinetic energy of the colliding protons.	$E_{initial} = (1.881700 \times 10^3) + 2(82.0)$ $= 2.045700 \times 10^3$ $= 2.05 \times 10^3$ MeV

b Determine the total energy of the known particles present after the collision and hence identify the particle that one of the protons transforms into.	
Thinking	**Working**
Use Einstein's equation to calculate the total mass–energy of the proton after the collision (in joules).	$E = mc^2$ $= (1.672622 \times 10^{-27})(3.00 \times 10^8)^2$ $= 1.505360 \times 10^{-10}$ J
Convert the energy to MeV.	$E = \frac{(1.505360 \times 10^{-10})}{(1.60 \times 10^{-19})}$ $= 9.408499 \times 10^8$ eV $= 9.408499 \times 10^2$ MeV
Add the mass–energy equivalents of the known particles to the kinetic energy of the products.	$E = (9.408499 \times 10^2) + (134.9768) + (29.02290)$ $= 1.104850 \times 10^3$ MeV
Subtract the total energy of the known products in eV from the total energy of the reactants to find the mass–energy of the unknown particle.	$E_{unknown} = (2.045700 \times 10^3) - (1.104850 \times 10^3)$ $= 9.408504 \times 10^2$ MeV $= 9.408504 \times 10^8$ eV
Convert the mass–energy of the unknown particle to mass in kilograms and identify the name of the particle from the table provided. Note that the energy in eV will need to be converted to joules.	$E = mc^2$ $m = \frac{(9.408504 \times 10^8)(1.60 \times 10^{-19})}{(3.00 \times 10^8)^2}$ $= 1.672623 \times 10^{-27}$ kg This is close to the mass of a proton, so the unknown particle is still a proton.

c Show that charge is conserved during the collision.	
Thinking	**Working**
Write the overall reaction for the collision.	$p^+ + p^+ \longrightarrow p^+ + p^+ + \pi^0$
Determine the sum of the charges before and after the collision.	$(2 \times +) \longrightarrow (2 \times +) + (1 \times 0)$ $2+ \longrightarrow 2+$ Hence charge is conserved.

KEY QUESTIONS

Knowledge and understanding

1 As the velocity of an object increases, so does its momentum, but at a rate much greater than predicted by classical mechanics. This is due to the increase in the mass of the object. As the object approaches the speed of light, the mass increases and approaches infinity.

2 In this fusion reaction some mass goes missing when the two particles fuse and create the two products. This missing mass (the mass defect) is converted into energy according to Einstein's equation $E = mc^2$.

3 Einstein's equation suggests that matter and energy are interchangeable. The mass in kilograms of the electron can be completely converted into the equivalent amount of energy. For an electron, 0.510 MeV is the energy-equivalent of its mass.

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Analysis

4 **a** $p = \gamma mv$

$$= \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{(1230)(775)}{\sqrt{1-\frac{(775)^2}{(3.00\times10^8)^2}}}$$

$$= \frac{(9.5325\times10^5)}{\sqrt{1.00000}}$$

$$= 9.53\times10^5\ \text{kg m s}^{-1}$$

b Given that the value of γ is essentially 1, at this speed no relativistic effects would be noticed. Thus the value for the classical momentum would be identical to the relativistic momentum to many decimal places.

5 $p = \gamma mv$

$$= \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{(1.99264824\times10^{-26})(0.950)(3.00\times10^8)}{\sqrt{1-\frac{(0.950)^2c^2}{c^2}}}$$

$$= \frac{(5.67905\times10^{-18})}{\sqrt{0.097500}}$$

$$= 1.82\times10^{-17}\ \text{kg m s}^{-1}$$

6 **a** $E_k = (\gamma - 1)mc^2$

$$= \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)mc^2$$

$$= \left(\frac{1}{\sqrt{1-\frac{0.840^2c^2}{c^2}}}-1\right)(15.1\times10^{-3})(3.00\times10^8)^2$$

$$= (1.843024-1)(1.35900\times10^{15})$$

$$= 1.15\times10^{15}\ \text{J}$$

b $E_k = \frac{1}{2}mv^2$

$$= \frac{1}{2}(15.1\times10^{-3})(0.840\times3.00\times10^8)^2$$

$$= 4.79\times10^{14}\ \text{J}$$

c According to Einstein, the momentum of the arrow at relativistic speeds increases much more rapidly than classical theory predicts. Since $E_k = \frac{1}{2}mv^2$, $E_k = \frac{1}{2}mv\times v$ and $E_k = \frac{1}{2}pv$. The relativistic momentum shown in the third version of the equation has a significant effect on the kinetic energy of the arrow.

7 $E = mc^2$

$$= (4.00\times10^9)(3.00\times10^8)^2$$

$$= 3.60\times10^{26}\ \text{J per second}$$

$$= (3.60\times10^{26})(24\times60\times60)$$

$$= 3.11\times10^{31}\ \text{J per day}$$

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8 $^{1}_{1}\text{H} + ^{1}_{1}\text{H} \longrightarrow ^{2}_{1}\text{H} + ^{0}_{1}\text{e}^{+} + ^{0}_{0}\nu$

$$\Delta m = m_{\text{reactants}} - m_{\text{products}}$$
$$= (2 \times 1.672622 \times 10^{-27}) - (3.34358 \times 10^{-27} + 9.10938 \times 10^{-31} + 2.14000 \times 10^{-37})$$
$$= (3.345244 \times 10^{-27}) - (3.34449 \times 10^{-27})$$
$$= 7.54000 \times 10^{-31}\text{ kg}$$
$$E = mc^2$$
$$= (7.54000 \times 10^{-31})(3.00 \times 10^{8})^2$$
$$= 6.78600 \times 10^{-14}\text{ J}$$
$$= \frac{6.78600 \times 10^{-14}}{1.60 \times 10^{-19}}$$
$$= 4.24125 \times 10^{5}\text{ eV}$$
$$= 0.424125 \times 10^{6}$$
$$= 0.424\text{ MeV}$$

9 **a** $m_{\text{total}} = 2(1.672622 \times 10^{-27})$

$$= 3.345244 \times 10^{-27}\text{ kg}$$
$$E = mc^2$$
$$= (3.345244 \times 10^{-27})(3.00 \times 10^{8})^2$$
$$= 3.010720 \times 10^{-10}\text{ J}$$
$$= \frac{(3.010720 \times 10^{-10})}{(1.60 \times 10^{-19})}$$
$$= 1.881700 \times 10^{9}\text{ eV}$$
$$= 1.881700 \times 10^{3}\text{ MeV}$$
$$E_{\text{total}} = (1.881700 \times 10^{3}) + 2 \times (105.0)$$
$$= 2.091700 \times 10^{3}$$
$$= 2.09\text{ MeV}$$

b $E_{\text{gamma}} = \dfrac{2.091700 \times 10^{3}}{2}$

$$= 1.045850 \times 10^{3}$$
$$= 1.05 \times 10^{3}\text{ MeV}$$

c

$$p^{+} + p^{-} \longrightarrow \gamma^{0} + \gamma^{0}$$
$$(1 \times +) + (1 \times -) \longrightarrow (2 \times 0)$$
$$0 \longrightarrow 0$$

Hence charge is conserved.

10 **a** $E = mc^2$

$$= (2 \times 1.674922 \times 10^{-27}) \times (3 \times 10^{8})^2$$
$$= 3.014860 \times 10^{-10}\text{ J}$$
$$= \frac{(3.014860 \times 10^{-10})}{(1.60 \times 10^{-19})}$$
$$= 1.884287 \times 10^{9}\text{ eV}$$
$$= 1.884287 \times 10^{3}\text{ MeV}$$
$$E_{\text{initial}} = (1.884287 \times 10^{3}) + 2(145.0000)$$
$$= 2.174287 \times 10^{3}\text{ MeV}$$

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b $E_{known} = 2(139.57039) + 4(2.072649)$

$= 2.874314 \times 10^2$ MeV

$E_{unknown} = (2.174287 \times 10^3) - (2.874314 \times 10^2)$

$= 1.8868556 \times 10^3$ MeV

$= 1.8868556 \times 10^9$ eV

$= (1.8868556 \times 10^9)(1.60 \times 10^{-19})$

$= 3.018969 \times 10^{-10}$ J

$E = mc^2$

$m_{unknown} = \frac{E}{c^2}$

$= \frac{(3.018969 \times 10^{-10})}{(3.00 \times 10^8)^2}$

$= 3.354410 \times 10^{-27}$ kg

$m_{each} = \frac{(3.354410 \times 10^{-27})}{2}$

$= 1.677205 \times 10^{-27}$ kg

This is close to the mass of a proton, so the two particles produced are protons.

c $n^0 + n^0 \longrightarrow p^+ + p^+ + \pi^- + \pi^-$

$(2 \times 0) \longrightarrow (2 \times +) + (2 \times -)$

$0 \longrightarrow 0$

Hence charge is conserved.

Chapter 10 Review

Knowledge and understanding

1 A and D. An aircraft taking off is accelerating, as is a car going around a curve. These are non-inertial frames of reference, as they are accelerating.

2 At the poles. The Earth has a very small centripetal acceleration which is negligible for most purposes; however, at the poles it is zero.

3 The observer on the track would see the ray of light strike the back wall before the ray of light strikes the front wall. This is because (a) the rays of light travel at the same speed forwards and backwards, (b) the front of the carriage is moving forward, thus extending the distance that the forward-moving ray must travel before it hits the front wall and (c) the back wall has moved forward, thus decreasing the distance that the rear-moving ray must travel before it hits the back wall. The ray that travels the shorter distance takes less time to hit the wall than the ray that travels the longer distance.

4 A. For the same events to be simultaneous in one inertial frame of reference and not simultaneous in another inertial frame of reference, time must pass differently in each inertial frame of reference.

5 A (postulate 2) and C (postulate 1)

6 C. There is no fixed space in which to measure absolute velocities. We can only measure velocity relative to some other frame of reference.

7 A and B. We are in the same frame of reference as the event in both cases. C and D may be true, but they are not sufficient conditions, as we must also be in the same frame of reference. (C did not specify with respect to what we were stationary.)

8 You could not tell the difference between (i) and (iii), but in (ii) you could see whether an object, such as a pendulum, hangs straight down.

9 Space and time are interdependent: motion in space alters an object's passage through time as measured by a stationary observer. Thus the three dimensions of physical space must be considered relative to the one dimension of time, as they are not independent.

10 Crews A and B will see each other normally as there is no relative velocity between them. They will both see the people on Earth moving in slow motion, as the Earth has a high relative velocity.

11 In your frame of reference time proceeds normally. Your heart rate would appear normal. As Mars is moving at a high speed relative to you, people on Mars appear to be in slow motion, as time for them—as seen by you—will be dilated.

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12 Both observers will see the light travel at $3.0 \times 10^8\,\text{m s}^{-1}$. According to Einstein's second postulate, the speed of light will always be the same whatever the motion of the light source or observer.

Application and analysis

13 $t = \gamma t_0$

$$= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{20.0}{\sqrt{1-\frac{(2.00\times10^8)^2}{(3.00\times10^8)^2}}}$$

$$= 26.8\,\text{s}$$

14 **a** $t = \gamma t_0$

$$t_0 = t\sqrt{1-\frac{v^2}{c^2}}$$

$$= 21.5\sqrt{1-\frac{(2.12\times10^8)^2}{(3.00\times10^8)^2}}$$

$$= 21.5\times0.707547$$

$$= 15.2\,\text{s}$$

b 15.2 s. The swimmer sees their own watch as t_0 and the pool clock as t. This is due to the fact that, in the frame of reference of the swimmer, it is the pool and its clock that are moving while the swimmer and their watch are stationary.

15 **a** $\gamma = 2$ as the observed length is half the proper length. Thus:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 2$$

$$\frac{1}{2} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{1}{4} = 1-\frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1-\frac{1}{4}$$

$$v^2 = 0.75c^2$$

$$v = 0.866c$$

b The contraction has doubled, so in this instance $\gamma = 4$. Thus:

$$\frac{1}{4} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{v^2}{c^2} = 1-\frac{1}{16}$$

$$v^2 = 0.9375c^2$$

$$v = 0.968c$$

16 **a** $t = \gamma t_0$

$$= t_0\times\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= 4.00\times\frac{1}{\sqrt{1-\frac{(2.38\times10^8)^2}{(3.00\times10^8)^2}}}$$

$$= 4.00\times1.64261$$

$$= 6.57\,\text{s}$$

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b $L = \frac{L_0}{\gamma}$ and, from part **a**, $\gamma = 1.64261$

$= \frac{18.3}{1.64261}$

$= 11.1\,\text{m}$

The length is observed to be 11.1 m and the width is unchanged at 1.07 m.

17 **a** $t = \frac{d}{v} = \frac{(5.96)}{(0.900)} = 6.62$ years

b $t = \gamma t_0$

$t_0 = \frac{t}{\gamma}$

$= t\sqrt{1 - \frac{v^2}{c^2}}$

$= 6.62\sqrt{1 - \frac{(0.900c)^2}{c^2}}$

$= 6.62 \times 0.435890$

$= 2.89$ years

c An observer on the Earth, who is stationary in the star's frame of reference, measures the proper length as 5.96 light-years. However, that distance is contracted for Amelia:

$L = \frac{L_0}{\gamma} = (5.96) \times \sqrt{1 - \frac{(0.900c)^2}{c^2}}$

$= 2.60$ light-years

As she travelled a shorter distance, this means it took her less time to travel to the star.

18 **a** $L = \frac{L_0}{\gamma} = (145) \times \sqrt{1 - \frac{(0.885c)^2}{c^2}}$

$= 145 \times 0.465591$

$= 67.5\,\text{km}$

The difference (in km) will therefore be $\Delta L = 145 - 67.5107 = 77.5\,\text{km}$.

b No. Since the motion is perpendicular to the depth, this dimension is not affected.

19 **a** $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{0.997^2 c^2}{c^2}}}$

$= \frac{1}{\sqrt{1 - 0.997^2}}$

$= 12.9196 \approx 13$ times

b No, they do not experience any difference in the rate at which time passes in their own frame of reference.

c $t = \frac{10.89}{0.997} = 10.92$ years

d $t_0 = \frac{t}{\gamma} = \frac{10.92}{12.9196} = 0.845$ years

e No. The moving explorer travels a contracted distance between the Earth and Ross-128, which is about 13 times shorter than we see on the Earth. The speed of the spaceship is constant in both frames of reference, so the explorer travels only 0.845 light-years to the star.

20 The observer on the Earth will not measure the proper time of the muon's lifetime. Instead they will see that the muon's lifetime is slow according to the equation $t = t_0\gamma$, where t_0 is the lifetime of a muon at rest. The result is that the observer sees the muon live a much longer time, t, and therefore makes it to the surface.

The muon will see the Earth approach at a very high speed (approx. 0.992c) and will see the distance contracted. It will not be 15.0 km. Instead it will travel a much shorter distance according to the equation $L = \frac{L_0}{\gamma}$. As the distance the muon travels is shorter, it can make the journey to the Earth's surface during its proper lifetime.

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21 a $E_k = (\gamma - 1)mc^2$

$= \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)mc^2$

$= \left(\frac{1}{\sqrt{1-\frac{0.925^2c^2}{c^2}}} - 1\right)(30.0\times10^{-3})(3.00\times10^{8})^2$

$= (2.63181 - 1)(2.70000\times10^{15})$

$= 4.41\times10^{15}$ J

b $E_k = \frac{1}{2}mv^2$

$= \frac{1}{2}(30.0\times10^{-3})(0.925\times3.00\times10^{8})^2$

$= 1.16\times10^{15}$ J

22 ${}^{3}_{2}He + {}^{3}_{2}He \longrightarrow {}^{4}_{2}He + {}^{1}_{1}p^{+} + {}^{1}_{1}p^{+}$

$\Delta m = m_{reactants} - m_{products}$

$= (2\times5.00823\times10^{-27}) - (6.64648\times10^{-27} + 2\times1.67262\times10^{-27})$

$= (1.00165\times10^{-26}) - (9.99172\times10^{-27})$

$= 2.47400\times10^{-29}$ kg

$E = mc^2$

$= (2.47400\times10^{-29})(3.00\times10^{8})^2$

$= 2.226600\times10^{-12}$ J

$E = \frac{2.226600\times10^{-12}}{1.60\times10^{-19}}$

$= 1.391625\times10^{7}$ eV

$= 1.391625\times10^{1}$ MeV

$= 13.9$ MeV

23 a $E = mc^2$

$= (2\times1.674922\times10^{-27})\times(3\times10^{8})^2$

$= 3.014860\times10^{-10}$ J

$= \frac{(3.014860\times10^{-10})}{(1.60\times10^{-19})}$

$= 1.884287\times10^{9}$ eV

$= 1.884287\times10^{3}$ MeV

$= (1.884287\times10^{3}) + 2(155)$

$E_{before} = 2.194287\times10^{3}$ MeV

$E_{after} = \frac{(1.674922\times10^{-27} + 1.672622\times10^{-27})\times(3\times10^{8})^2}{1.60\times10^{-19}\times10^{6}} + 139.6$

$= 1.882994\times10^{3} + 139.6$

$= 2.022594\times10^{3}$ MeV

$E_k = E_{before} - E_{after}$

$= 2.194287\times10^{3} - 2.022594\times10^{3}$

$= 1.716935\times10^{2}$

$= 1.72\times10^{2}$ MeV

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b $n^0 + n^0 \longrightarrow +n^0 + p^+ + \pi^-$

$(2\times 0) \longrightarrow (1\times 0) + (1\times +) + (1\times -)$

$0 \longrightarrow 0$

Hence charge is conserved.

24 a $m_{total} = 2\times(9.10938356\times 10^{-31})$

$= 1.8218767\times 10^{-30}$ kg

$E = mc^2$

$= (1.8218767\times 10^{-30})(3.00\times 10^{8})^2$

$= 1.6396890\times 10^{-13}$ J

$= \dfrac{(1.6396890\times 10^{-13})}{(1.60\times 10^{-19})}$

$= 1.02480565\times 10^{6}$ eV

$= 1.02480565$ MeV

$E_{total} = (1.02480565) + 2\times(42.0000)$

$= 85.0248$ MeV

b $E_{gamma} = \dfrac{8.5024806\times 10^{1}}{2}$

$= 4.2512403\times 10^{1}$

$= 42.5124$ MeV

c $e^+ + e^- \longrightarrow \gamma^0 + \gamma^0$

$(1\times +) + (1\times -) \longrightarrow (2\times 0)$

$0 \longrightarrow 0$

Hence charge is conserved.

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Unit 4 Area of Study 1

How has understanding about the physical world changed?

Multiple-choice questions

1 A. Gamma rays are a form of electromagnetic radiation. The electric and magnetic fields in electromagnetic radiation are perpendicular to each other and are both perpendicular to the direction of propagation of the radiation.

2 B and E. The spectra in A and C are continuous, and the spectrum in D is an absorption spectrum.

3 C. The de Broglie wavelength of a particle is given by $\lambda = \frac{h}{p}$ and therefore depends only on the momentum of the particle.

4 B

Electron:

$$\lambda = \frac{h}{mv}$$
$$= \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times7.5\times10^{6}}$$
$$= 9.7\times10^{-11}\text{m}$$

UV light:

$$\lambda = 150 \text{ nm} = 150\times10^{-9} = 1.5\times10^{-7}\text{m}$$

X-ray:

$$f = \frac{c}{\lambda}$$
$$\lambda = \frac{c}{f}$$
$$= \frac{3.0\times10^{8}}{3.0\times10^{17}}$$
$$= 1.0\times10^{-9}\text{ m}$$

Proton:

$$\lambda = \frac{h}{p}$$
$$= \frac{6.63\times10^{-34}}{1.7\times10^{-21}}$$
$$= 3.9\times10^{-13}\text{m}$$

Comparing these, UV has the longest wavelength.

5 B. V_0 is proportional to the energy of the incident photons. Since blue light has a higher frequency than yellow light, its photons have more energy.

6 C. The reverse potential difference works against the electrons as they try to reach the collector.

A and B are incorrect because the potential difference between the emitter and the collector does not affect these quantities.

D is incorrect, as the photoelectrons crossing the gap are what complete the circuit when not at the stopping voltage. When at the stopping voltage, it is not the presence of the gap that accounts for zero current.

7 A. The colour of the incident light is indicated by the value of V_0, as the stopping voltage is proportional to the energy of the photons (which in turn is proportional to the frequency of the photons), while the intensity of the incident light is indicated by the size of the current.

8 C

$$E_{\text{k max}} = \frac{hc}{\lambda} - \phi$$
$$1.21 = \frac{4.14\times10^{-15}\times3.0\times10^{8}}{200\times10^{-9}} - \phi$$
$$\phi = 6.21 - 1.21 = 5.0 \text{ eV}$$

9 A (postulate 2) and C (postulate 1)

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10 A. Inertial frames move at a constant velocity.

11 C. γ must be > 1, so A and B are not correct. The speed is much less than c, so D is not correct. C is the only feasible answer.

You could also calculate γ:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{50\,000^2}{300\,000\,000^2}}} = 1.000\,000\,014$$

12 D. It undergoes length contraction in the dimension that is parallel to the direction of motion. The width and height are perpendicular to the direction of motion, so they do not change.

13 A. In one second 4.0×10^9 kg of mass is being converted to energy.

$$\begin{aligned} E &= mc^2 \\ &= 4.0 \times 10^9 \times (3.0 \times 10^8)^2 \\ &= 3.6 \times 10^{26}\,\text{J} \end{aligned}$$

14 C. It was believed that light needed a medium through which to propagate. The Michelson–Morley experiment sought to measure the velocity of the Earth with respect to the aether by measuring the speed of light on the Earth when travelling with and against the proposed aether. As the velocity was constant in both directions, there was no evidence for an aether being present; thus the experiment provided evidence for the theory of special relativity.

15 A. The speed of light in a vacuum is constant.

Short-answer questions

16 When two or more waves meet and combine, the resulting waveform will be the vector addition of the displacement of the individual particles in the waves due to the principle of superposition. Although there is a different displacement as the waves are superimposed, passing through each other does not permanently alter the shape, amplitude or speed of the individual waves.

17 **a** A = node, B = antinode

b The two images show the maximum and minimum positions of the rope as it oscillates.

c The image shows three loops of the rope, so it is the third harmonic.

d The oscillating hand makes the incident waves. Those waves approach the hook. As the hook is a fixed end, the wave is inverted (i.e. it undergoes a 180° phase change) and is reflected back towards the hand. The continuous incident waves superimpose with the reflected waves, forming a series of nodes and antinodes in the standing wave.

e $f_n = \frac{nv}{2l}$

As this is the third harmonic:

$$\begin{aligned} f_3 &= \frac{3 \times 2.0}{2 \times 0.800} \\ &= 3.8\,\text{Hz} \end{aligned}$$

18 **a** Since $\Delta x = \frac{\lambda L}{d}$, if d is halved, Δx will be doubled.

b Since $\Delta x = \frac{\lambda L}{d}$, if L is doubled, Δx will be doubled.

c There will be no effect, as the brightness (intensity) of the light does not affect the interference pattern.

d Since $\Delta x = \frac{\lambda L}{d}$, if λ is decreased, Δx will decrease.

e There will be a wider central band.

19 **a** The two models are the wave model and the particle (or corpuscular) model.

b Young's experiment resulted in bright and dark bands or fringes being seen on a screen. These can only be due to interference effects. A property of waves is that they can interfere with one another (constructively and destructively), so his work supported the wave model of light. (The particle model could not explain the interference effects observed; it could predict just two bright bands.)

20 A and B show maximum constructive interference. At position A, two crests add to give maximum constructive interference, as the waves are arriving in phase. At position B, two troughs add to give maximum constructive interference, as again the waves are arriving in phase. At positions C and D, a trough and a crest add to give maximum destructive interference, as the waves are arriving 180° out of phase.

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21 Destructive interference occurs where there is a path difference of $\left(n+\frac{1}{2}\right)\lambda$ where $n = 0, 1, 2$ … For the third dark band, $n = 2$ and pd = 2.5λ. For the fourth dark band, $n = 3$ and pd = 3.5λ. That is, the path difference is always one whole wavelength greater for each consecutive dark band. As this value is 500 nm in this example, $\lambda = 500$ nm.

22 Only certain frequencies of light will emit photoelectrons.

There is no time difference between the emission of photoelectrons by light of different intensities.

The maximum kinetic energy of the ejected photoelectrons is the same for different light intensities of the same frequency.

23 **a** A series of bright fringes/bands (indicating that electrons were detected) and dark fringes/bands (indicating that no electrons were detected). The pattern is called an interference pattern.

b The high-speed electrons are exhibiting wave-like behaviour.

c $\lambda = \frac{h}{p} = \frac{h}{mv}$

$$= \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times\frac{0.1}{100}\times3.0\times10^{8}}$$

$$= 2.4\times10^{-9}\text{ m}$$

d **i** $\lambda_{neutron} = \lambda_{electron}$

$\lambda = \frac{h}{mn} \longrightarrow v = \frac{h}{m\lambda}$

$$v = \frac{6.63\times10^{-34}}{1.67\times10^{-27}\times2.4\times10^{-9}}$$

$$= 163.5$$

$$= 1.6\times10^{2}\text{ m s}^{-1}$$

ii As the pattern is the same and the spacing of the bands (Δx) depends on $\frac{\lambda L}{d}$, $\lambda_{neutron} = \lambda_{electron}$ given that L, d and the slit width remain constant. So the answer remains the same as for part **i**: $v = 1.6 \times 10^2$ m s^{-1}.

24 **a** **i** $\Delta E = E_4 - E_1$

$$= -1.6 - (-10.4)$$

$$= 8.8 \text{ eV}$$

$\Delta E = hf \rightarrow f = \frac{\Delta E}{h}$

$$f = \frac{8.8}{4.14\times10^{-15}}$$

$$= 2.1\times10^{15}\text{ Hz}$$

ii $\Delta E = E_2 - E_1$

$$= -5.5 - (-10.4)$$

$$= 4.9 \text{ eV}$$

$\Delta E = hf \rightarrow f = \frac{\Delta E}{h}$

$$f = \frac{4.9}{4.14\times10^{-15}}$$

$$= 1.2\times10^{15}\text{ Hz}$$

iii $\Delta E = E_4 - E_3$

$$= -1.6 - (-3.7)$$

$$= 2.1 \text{ eV}$$

$\Delta E = hf \rightarrow f = \frac{\Delta E}{h}$

$$f = \frac{2.1}{4.14\times10^{-15}}$$

$$= 5.1\times10^{14}\text{ Hz}$$

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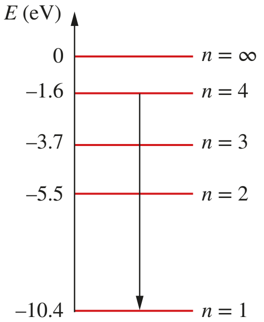
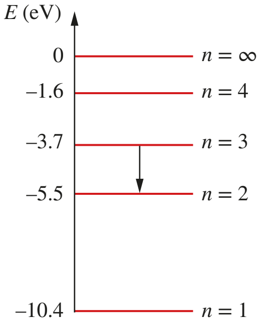
b As $E = hf$, a higher frequency photon will be produced from the largest energy jump. As a photon is being emitted, the transition must be from $n = 4$ to $n = 1$.



c As $E = hf = \frac{hc}{\lambda}$, a larger wavelength photon will be produced from the smallest energy jump. The transition must be from $n = 3$ to $n = 2$, as this corresponds to the smallest jump possible (1.8 eV).



d $E_{3-2} = -3.7 - (-5.5) = 1.8\,\text{eV}$
$E_{3-1} = -3.7 - (-10.4) = 6.7\,\text{eV}$
$E_{2-1} = -5.5 - (-10.4) = 4.9\,\text{eV}$
Therefore 1.8 eV, 4.9 eV and 6.7 eV photons will be present in the emission spectrum.

25 **a** $W = \Delta E_k = \frac{1}{2}mv^2$ and $W = qV$ (where V = voltage; take care not to confuse this with velocity, v)

From this we can get:

$$qV = \frac{1}{2}mv^2$$

$$1.6\times10^{-19}\times65 = \frac{1}{2}\times9.1\times10^{-31}\times v^2$$

$$v = \sqrt{\frac{2\times1.6\times10^{-19}\times65}{9.1\times10^{-31}}}$$

$$= 4.8\times10^{6}\,\text{m s}^{-1}$$

b $\lambda = \frac{h}{mv}$

$$= \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times4.8\times10^{6}}$$

$$= 1.5\times10^{-10}\,\text{m}$$

26 Bohr stated that if incident light has an energy value less than the minimum energy difference between the lowest and next orbital levels within the hydrogen atom, the light would not cause any orbital changes. Therefore the light would not be absorbed by the atom.

27 **a** $\lambda = \frac{h}{p} = \frac{h}{mv}$

$$= \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times1.75\times10^{7}}$$

$$= 4.16\times10^{-11}\,\text{m}$$

$$= 0.0416\,\text{nm}$$

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b According to de Broglie, the electrons were diffracted as they passed through the gaps between the atoms in the crystal, thus creating a diffraction pattern. This pattern would be circular bands or fringes of specific spacing around a common central point. Dark bands are due to destructive interference; bright bands are due to constructive interference.

c As the accelerating voltage is increased, the electron speed would increase. Therefore the electron has more momentum. As $\lambda = \frac{h}{p}$, the electron's wavelength is reduced. The amount of diffraction depends on $\frac{\lambda}{w}$ and so less diffraction occurs. Less diffraction means the overall pattern is smaller, that is, the circular bands are more closely spaced.

28 After discovering that light displays both particle and wave properties, physicists discovered that matter also has wave properties (when moving very fast) as well as particle properties. Both matter and light display wave–particle duality, but matter waves are not experienced in our macroscopic, low-speed world.

29 a In the particle model, the energy of the incident photons is set by their frequency (according to $E = hf$). Each incident photon interacts with only one electron. Therefore the energy of the emitted electrons will depend only on the frequency of the incident light. Electron energy is not altered by altering the intensity because this only varies the number of photons, not their energy. Therefore the energy of the emitted electrons is not affected, only the number emitted.

b The wave model predicts that altering the intensity of light corresponds to waves of greater amplitude. Hence the wavefronts should deliver more energy to the electrons and the emerging electrons should thus have higher energy. (This is not observed.)

30 a $E_{k\ max}$ represents the maximum kinetic energy with which electrons are emitted.

f is the frequency of the light incident on the metal plate (usually after the light passes through a filter, so it is not sufficient to call this the frequency of light *from the source*).

ϕ is the work function, which is the minimum energy required to eject an electron. (This is a property of the metal.)

b $E_{k\ max}$ does not change.

c More photoelectrons are ejected each second, therefore there is a higher photocurrent.

31 a Both the X-rays and the electrons have produced diffraction patterns with the same fringe separation. The amount of diffraction is proportional to $\frac{\lambda}{w}$. Given that the X-rays and electrons are both diffracting through the same sample of crystal, they are diffracting through the same width, w. Therefore they must have equivalent wavelengths.

b Given w is constant, $\lambda_{X\text{-ray}} = \lambda_{electron}$

$$\lambda_{X\text{-ray}} = \frac{c}{f} = \frac{3.0 \times 10^{8}}{8.3 \times 10^{18}}$$
$$= 3.6 \times 10^{-11}\,\text{m}$$
$$= \lambda_{electron} \text{ as found in part } \mathbf{a}$$

Finding the momentum:

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{3.6 \times 10^{-11}}$$
$$= 1.8 \times 10^{-23}\,\text{kg}\,\text{m}\,\text{s}^{-1}$$

(This would also be the momentum of the X-ray.)

c No. The energy of the X-rays, being a form of electromagnetic radiation that travels at c, is given by $E = \frac{hc}{\lambda}$ and the energy of the electrons is given by $\Delta E_k = \frac{1}{2}mv^2$, as electrons have mass and do not travel at c.

d Because optical microscopes use visible light, they will encounter considerable diffraction around objects with widths similar to the wavelength of light, resulting in blurriness.

As electrons have wavelengths equal to X-rays, which are smaller than visible light, less diffraction will occur with widths similar to the wavelength of light, resulting in less blurry images.

32 a

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$
$$= \frac{5}{0.9}$$
$$= 5.6 \text{ years}$$

b At 0.9c, Raqu's time will seem to be shortened by a factor $\gamma = 2.3$, thus it will seem to take her only 2.4 years.

c Relative to Raqu, the distance appeared to be shortened by the factor γ, thus the distance she travelled was much less than 5 light-years.

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33 **a** The mass difference is $4\times1.673\times10^{-27}-6.645\times10^{-27}=4.7\times10^{-29}$ kg.

$$E=mc^2$$
$$=4.7\times10^{-29}\times(3.0\times10^8)^2$$
$$=4.2\times10^{-12}\text{ J}$$

b As the total energy produced by the Sun each second is 3.9×10^{26} J, and part **a** gives us the energy produced for each helium atom, the number of helium atoms must be given by $\frac{3.9\times10^{26}}{4.2\times10^{-12}}=9.3\times10^{37}$ every second.

c The mass lost by the Sun each second is given by $m=9.3\times10^{37}\times4.7\times10^{-29}=4.37\times10^{9}$ kg.
In one day this will be $4.37\times10^{9}\times24\times60\times60=3.8\times10^{14}$ kg.

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