Topic 1 — Electromagnetic radiation and waves

1.2 Explaining waves as the transmission of energy

1.2 Exercise

1 A periodic wave is a disturbance that repeats itself at regular intervals; a single pulse is, as the name suggests, a disturbance without repeats.

2 For transverse waves, the disturbance is at right angles to the direction the waves travel (such as a pulse on a rope or ripple on the surface of water), whereas for longitudinal waves, the disturbance is parallel to the direction the waves are travelling (such as in sound waves or in a compression moving along a string).

3 $v = \frac{d}{t}$

$= \frac{996}{3.00}$

$= 332\ \text{m s}^{-1}$

4 As a transverse wave passes through a medium, particles in the medium will vibrate/oscillate up and down at right angles to the passage of the wave.

5 As a longitudinal wave passes through a medium, particles in the medium will vibrate/oscillate back and forth in the same direction as the passage of the wave.

1.2 Exam questions

1 Waves transmit **energy** [1 mark] without the net transfer of **matter**. [1 mark]

2 A medium [1 mark]

3 D

Sound waves are longitudinal waves, so the medium moves in the same direction as the wave.

4 B

A compression wave is longitudinal.

5 **a** A longitudinal wave is characterised by the disturbance being parallel to the direction of wave propagation. [1 mark]

b Sound waves [1 mark]

Note that sound waves need not necessarily be travelling in air and can propagate through any elastic material.

Other valid responses are also possible — for example, primary waves from an earthquake (P-waves).

c Light waves [1 mark]

Other names for electromagnetic waves are also valid responses — for example, radio waves, x-rays or microwaves.

Other valid responses are also possible — for example, waves in the strings of a musical instrument such as a guitar.

Water waves are not a clear-cut example, as these are commonly both longitudinal and transverse at the same time.

1.3 Properties of waves

Sample problem 1

$T = 2.0\ \text{ms}$

$= 2.0 \times 10^{-3}\ \text{s}$

$\lambda = 68\ \text{cm}$

$= 0.68\ \text{m}$

$v = f\lambda$

$\Rightarrow v = \frac{\lambda}{T}$

$= \frac{0.68\ \text{m}}{2.0 \times 10^{-3}\ \text{s}}$

$= 340\ \text{m s}^{-1}$

Practice problem 1

A wave will travel a distance equal to one wavelength in a time equal to its period. The speed of the wave is equal to:

$v = \frac{\lambda}{T}$

$= \frac{0.51}{1.5 \times 10^{-3}}$

$= 340\ \text{m s}^{-1}$

Sample problem 2

$f = 550\ \text{Hz},\ v = 335\ \text{m s}^{-1}$

$v = f\lambda$

$\Rightarrow \lambda = \frac{v}{f}$

$= \frac{335\ \text{m s}^{-1}}{550\ \text{Hz}}$

$= 0.609\ \text{m}$

Practice problem 2

Use the wave equation to find the speed. Thus:

$v = f\lambda$

$= 587 \times 0.571$

$= 335\ \text{m s}^{-1}$

1.3 Exercise

1 A wave will travel a distance of one wavelength during a time equal to one period.

2 Periodic sound waves travel at the same speed from a stationary source independent of how loud they are.

The loudness (i.e. intensity) of a sound is related to the amplitude of the longitudinal waves, which is a measure of the difference in air pressure between the compressions and rarefactions of the wave. The wavelength and speed of wave motion are not related to the amplitude of the wave.

For example, music from an orchestra, containing a variety of sound intensities, is heard synchronised at specific distances from the orchestra.

3 The marching band appears out of step with the music because the speed of light is significantly different to the speed of sound. Optical information arrives at an observer before audio information does.

4 Yes, you will, because the speed of sound is the same for low- and high-frequency sound waves produced by a stationary source, such as a group of musicians at a concert.

5 To interact with your ear, 200 wavelengths will take 200 periods:

$$T = \frac{1}{f} \Rightarrow 200 \times T = \frac{200}{f}$$
$$= \frac{200}{256}$$
$$= 0.780\text{ s}$$

6 $\lambda = \frac{v}{f} = vT$

$$= 340 \times 3.00 \times 10^{-3}$$
$$= 1.02\text{ m}$$

7 $v = f\lambda = \frac{\lambda}{T}$

$$= \frac{1.32}{4.00 \times 10^{-3}}$$
$$= 330\text{ m s}^{-1}$$

8 **a** $\lambda = \frac{v}{f}$

$$= \frac{340}{256}$$
$$= 1.33\text{ m}$$

b $\lambda = \frac{v}{f}$

$$= \frac{1.50 \times 10^3}{256}$$
$$= 5.86\text{ m}$$

9 In this instance, the source produces sound with a source frequency of 100 Hz. An observer moving towards the source will intercept more waves per second and so will measure a higher frequency; the observer moving towards the source measures 110 Hz. Likewise, an observer moving away from the source will intercept fewer waves per second and will measure a lower frequency; the observer moving away from the source measures 90 Hz.

1.3 Exam questions

1 C

Amplitude is found from the displacement axis (8 cm).
Frequency is found using:

$$c = f\lambda$$
$$18 = f \times \left(6 \times 10^{-2}\right)$$
$$f = 300\text{ Hz [1 mark]}$$

2 The speed of a wave can be calculated using the frequency of the wave and the wavelength of the wave.
The wavelength (λ) is given as 1.40 m.
The period (and hence the frequency) of the wave can be determined given the time take for point P to move from maximum displacement to zero. This is equivalent to $\frac{1}{4}$ of the period.

Therefore, the period $= 4 \times 0.120$
$= 0.480\text{ s}$ [1 mark]

$$\text{Frequency} = \frac{1}{\text{period}}$$
$$f = \frac{1}{0.480}$$
$$= 2.08\text{ Hz [1 mark]}$$

Then:

$$v = f \times \lambda$$
$$= 2.08 \times 1.40$$
$$= 2.92\text{ m s}^{-1}\text{ [1 mark]}$$

3 $v = \frac{\lambda}{T}$

$$\Rightarrow T = \frac{\lambda}{v}$$
$$= \frac{510 \times 10^{-9}}{3 \times 10^8}\text{ [1 mark]}$$
$$= 1.7 \times 10^{-15}\text{ s [1 mark]}$$

4 $f = \frac{\text{cycles}}{\text{time}}$

$$= \frac{26}{2 \times 60}\text{ [1mark]}$$
$$= 0.22\text{ Hz [1 mark]}$$

5 $T = \frac{1}{f}$

$$= \frac{1}{261.6}\text{ [1 mark]}$$
$$= 3.82 \times 10^{-3}\text{ s [1 mark]}$$

1.4 Energy from the Sun

Sample problem 3

a Temperature of hot iron:

$$T_{(\text{kelvin})} = T_{(\text{Celsius})} + 273$$
$$= 480\text{ °C} + 273$$
$$= 753\text{ K}$$

Temperature of cold iron:

$$T_{(\text{kelvin})} = T_{(\text{Celsius})} + 273$$
$$= 20\text{ °C} + 273$$
$$= 293\text{ K}$$

$$\frac{P_{\text{hot}}}{P_{\text{cold}}} = \left(\frac{T_{\text{hot}}}{T_{\text{cold}}}\right)^4$$
$$= \left(\frac{753}{293}\right)^4$$
$$\approx 44$$

The hot iron emits 44 times as much energy every second as it does when it is at room temperature.

b $\frac{P_{\text{hot}}}{P_{\text{cold}}} = \left(\frac{T_{\text{hot}}}{T_{\text{cold}}}\right)^4$

$$10 = \left(\frac{T_{\text{hot}}}{293}\right)^4$$
$$10^{\frac{1}{4}} = \frac{T_{\text{hot}}}{293}$$

$\Rightarrow T_{hot} = 293 \times 10^{\frac{1}{4}}$

$10^{\frac{1}{4}} \approx 1.778$

$\Rightarrow T_{hot} = 293 \times 1.778$
$\approx 521\,K$

Temperature of hot iron $= 521 - 273$
$= 248\,°C$

The iron would need to be at 248 °C to emit 10 times the energy it does every second at 20 °C

Practice problem 3

Using ratio:

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^4$$

$$\Rightarrow P_1 = P_2 \times \left(\frac{T_1}{T_2}\right)^4$$

$$= 3.846 \times 10^{26} \times \left(\frac{8000}{5778}\right)^4$$

$$= 1.413 \times 10^{27}\,W$$

A star of similar size would radiate 1.413×10^{27} W of heat if its surface temperature was 8000 K.

Sample problem 4

a $\lambda_{max} T = \text{constant}$

$$\Rightarrow \lambda_{max} = \frac{\text{constant}}{T}$$

Constant $= 2.90 \times 10^{-3}$ mK, $T = 11\,000$ K

$$\lambda_{max} = \frac{\text{constant}}{T}$$
$$= \frac{2.90 \times 10^{-3}\,mK}{11\,000\,K}$$
$$= 2.64 \times 10^{-7}\,m$$

The light coming from a star whose surface temperature is 11 000 K has a peak intensity at a wavelength of 2.64×10^{-7} m.

b The peak wavelength, 264 nm, is beyond the violet end of the visible specturm, so it is in the ultraviolet section of the electromagnetic spectrum.

Practice problem 4

Using λ_{max}:

$$\lambda_{max} = \frac{\text{constant}}{T}$$

$$\Rightarrow T = \frac{\text{constant}}{\lambda_{max}}$$
$$= \frac{2.90 \times 10^{-3}\,mK}{450 \times 10^{-9}\,m}$$
$$= 6444\,K$$

The surface temperature of a star that emits light at a maximum intensity of 450 nm is 6444 K.

1.4 Exercise

1 Absorption, emission and reflection

2 Shiny surfaces are reflective and minimise the energy transfer through radiant energy. The shiny surfaces on the thermos would minimise the emission of infrared radiation (heat) from the thermos, thus minimising the heat loss from any liquid it contains.

3 **a** Reading from the graph, a star with a surface temperature of 6000 K will have a peak radiation emission that is blue with an approximate wavelength of 5000×10^{-10} m, or 500 nm.

b Spica is a blue star. As the temperature of a theoretical blackbody increases, the emission spectrum becomes bluer. The dotted line in figure 1.10 shows the trend of this change. Although the peak emission at 6000 K is also blue, the trend of the dotted line is almost vertical. Therefore, at the hotter temperature of 25 000 K, the peak emission is only slightly more blue than at 6000 K, although the overall intensity of emission is much greater.

4 As the temperature of an object increases:

- the *wavelength* of emitted radiation will decrease
- the *frequency* of emitted radiation will increase.

5 As the temperature of the metal filament increases, the peak wavelength of emitted radiation will decrease. Thus, when the filament is relatively cool, it will glow a dull red colour. As its temperature increases, the intensity of emission of shorter wavelengths of visible radiation (yellow, green, blue) will increase. This will progressively change the colour of the filament to yellow (when the peak emission is yellow) to white (when significant amounts of all visible wavelengths are being emitted) as the filament becomes hotter.

6 $\lambda_{max} = \frac{\text{constant}}{T}$

$$= \frac{2.9 \times 10^{-3}\,mK}{288\,K}$$
$$= 1.00 \times 10^{-5}\,m$$

7 $\lambda_{max} = \frac{\text{constant}}{T}$

$$\Rightarrow T = \frac{\text{constant}}{\lambda_{max}}$$
$$= \frac{2.90 \times 10^{-3}\,mK}{510 \times 10^{-9}}$$
$$= 5690\,K$$

1.4 Exam questions

1 D

Because the Sun is so hot, in addition to infrared and visible light, it also emits in the ultraviolet wavelengths. This high-energy radiation is not visible to our eyes but can cause sunburn.

2 The shiny white surfaces (that is, with an albedo close to 1.0) reflect high proportions of incoming sunlight back into space. [1 mark]

Examples include clouds and snow/ice (glaciers and icecaps) (or any other suitable example). [1 mark]

3 The colour of each star is related to its surface temperature. [1 mark]

The blue-white colour of Rigel suggests that it is much hotter (~10 000 K) compared to Betelgeuse (~4000 K). [1 mark]

4 A

At the polar regions in the middle of winter there is no sunlight, but at the equator when the Sun is directly overhead the intensity can reach 1368 W m^{-2}.

5 **a** Ratio of power $= \left(\frac{T_1}{T_2}\right)^4$

$= \left(\frac{1150 + 273}{480 + 273}\right)^4$

$= \left(\frac{1423}{753}\right)^4$ [1 mark]

$= 12.8$ times [1 mark]

b $T_1 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{4}} \times T_2$

$= (10)^{\frac{1}{4}} \times (480 + 273) = 1339\text{ K}$

$= 1339\text{ K} - 273$ [1 mark]

$= 1066\ ^\circ\text{C}$ [1 mark]

1.5 The electromagnetic spectrum

Sample problem 5

a 1.8×10^{-5} s

b 5.4×10^{2} nm

Practice problem 5

$f = \frac{c}{\lambda}$

$= \frac{3 \times 10^8}{4.5 \times 10^{-7}}$

$= 6.7 \times 10^{14}$ Hz

$T = \frac{1}{f}$

$= \frac{1}{6.7 \times 10^{14}}$

$= 1.5 \times 10^{-15}$ s

1.5 Exercise

1 $T = \frac{1}{f}$

$= \frac{1}{4.8 \times 10^{14}}$

$= 2.08 \times 10^{-15}$

$= 2.1 \times 10^{-15}$ s

2 $\lambda = \frac{c}{f}$

$= \frac{3.0 \times 10^8}{1.0 \times 10^{10}}$

$= 3.0 \times 10^{-2}$ m

$= 3.0$ cm

$\lambda = \frac{c}{f}$

$= \frac{3.0 \times 10^8}{1.0 \times 10^{12}}$

$= 3.0 \times 10^{-4}$

$= 0.30$ mm

Microwaves have wavelengths that range from fractions of a millimetre to a few centimetres.

3 $f = \frac{c}{\lambda}$

$= \frac{3.0 \times 10^8}{2.7 \times 10^{-11}}$

$= 1.1 \times 10^{19}$ Hz

$T = \frac{1}{f}$

$= \frac{1}{1.1 \times 10^{19}}$

$= 9.0 \times 10^{-20}$ s

4 **a** $f = \frac{1}{T}$

$= \frac{1}{20 \times 10^{-3}}$

$= 50$ Hz

b $\lambda = \frac{c}{f}$

$= \frac{3.0 \times 10^8}{50}$

$= 6.0 \times 10^{6}$ m

5 **a** $\lambda = \frac{c}{f}$

$= \frac{3.0 \times 10^8}{6.5 \times 10^{14}}$

$= 4.6 \times 10^{-7}$ m

b $\lambda = \frac{c}{f}$

$= \frac{2.0 \times 10^8}{6.5 \times 10^{14}}$

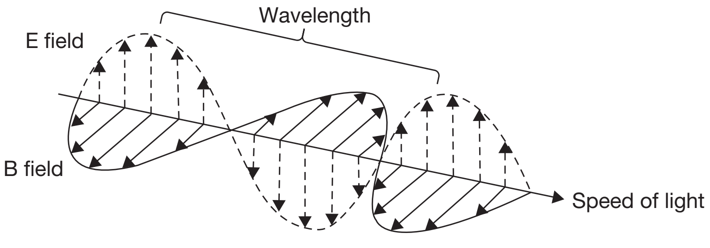
$= 3.1 \times 10^{-7}$ m

1.5 Exam questions

1 See the figure at the foot of the page*

(1 mark for correctly labelling the electric and magnetic fields, 1 mark for correctly labelling the wavelength and

1*



1 mark for correctly labelling the speed of light)
VCAA examination report note: The most common error was to indicate the wavelength incorrectly. Typically, students indicated the distance from one of the E field peaks to the next B field peak or vice versa.

2 C
Sound waves are not part of the electromagnetic spectrum (they are longitudinal pressure waves).

3 B
Radio waves have wavelengths in the range of approximately 1 m to 10^6 m. The other waves provided as options are from the ultraviolet range and beyond, and have wavelengths in the range 10^{-7} m to 10^{-14} m.

4 At the speed of light, $c = 3 \times 10^8 \text{ m s}^{-1}$ [1 mark]

5 Approximately 750 nm [1 mark]

1.6 Review

1.6 Review questions

1 A mechanical wave is a disturbance from the equilibrium of a relaxed medium. When a medium is distorted by a force at a point, it radiates energy away from the point of distortion in the form of a propagating disturbance referred to as a wave. Without a medium, there is nothing for the wave to distort, so it can't travel anywhere.

2 **a** The period, $T = \frac{1}{f}$

$$= \frac{1}{2.0}$$
$$= 0.50 \text{ s}$$

b Use $v = f\lambda$ rearranged:

$$\lambda = \frac{v}{f} = \frac{2.5}{2.0} = 1.3 \text{ m}$$

c The frequency increases and hence the wavelength decreases as $v = f\lambda$, where v is a constant; adjacent pulses are closer together. The speed of the wave is a property of the medium, which has not altered, and hence speed is unchanged.

3 **a** The period, $T = \frac{1}{f}$

$$= \frac{1}{926}$$
$$= 1.08 \times 10^{-3} \text{ s}$$

b Use $\lambda = \frac{v}{f}$

$$= \frac{340}{926}$$
$$= 0.370 \text{ m}$$

4 For the sound with a frequency of 20 Hz:

$$\lambda = \frac{v}{f} = \frac{340}{20} = 17 \text{ m}$$

For the sound with a frequency of 20 kHz:

$$\lambda = \frac{v}{f} = \frac{340}{20\,000} = 0.017 \text{ m or } 1.7 \text{ cm.}$$

Audible sound has a range of wavelengths from 1.7 cm to 17 m for a normal human.

5 Blue light: $\lambda = \frac{c}{f}$

$$= \frac{3.0 \times 10^8}{6.5 \times 10^{14}}$$
$$= 4.6 \times 10^{-7} \text{ m}$$

Yellow light: $\lambda = \frac{c}{f}$

$$= \frac{3.0 \times 10^8}{5.2 \times 10^{14}}$$
$$= 5.8 \times 10^{-7} \text{ m}$$

6 First, calculate the time required to produce 1.0×10^6 cycles.
The time required to produce one cycle is:

$$T = \frac{1}{f} = \frac{1}{6.5 \times 10^{14}} = 1.54 \times 10^{-15} \text{ s}$$

Thus, the time required to produce 1.0×10^6 cycles is:
$1.0 \times 10^6 \times 1.54 \times 10^{-15} = 1.54 \times 10^{-9}$ s
The distance between the start and finish of the pulse is:

$$d = ct = 3.0 \times 10^8 \times 1.54 \times 10^{-9} = 0.46 \text{ m}$$

The pulse of light is therefore approximately 46 cm long.

1.6 Exam questions

Section A — Multiple choice questions

1 C or D
VCAA examination report note: Both options C and D were accepted. The Doppler effect means that the frequency that Alex hears will have increased (option D). As the fire engine approaches, the amplitude of the sound Alex hears will also increase (option C).

2 B

$$\lambda = \frac{v}{f} = \frac{350}{500} = 0.70 \text{ m}$$

3 B

$$\lambda = \frac{v}{f} = \frac{330}{30} = 11 \text{ m}$$

4 B
Using: $v = f\lambda$

$$\lambda = \frac{330}{220} = 1.5 \text{ m}$$

5 B
Sound waves are longitudinal waves, so the dust particle will vibrate horizontally backwards and forwards (in the direction the sound wave is travelling).
The dust particle will vibrate with the frequency of the sound, which is 10 Hz (10 waves per second).
The dust particle will remain 10 cm from the speaker on average, because waves transfer energy through vibration without transferring matter — in this case, the dust particle.

6 B
The time taken for a complete cycle of a wave is known as the period.

7 D
They are measuring the amplitude of the wave.

8 C
If Earth did not emit infrared radiation, Earth would be too hot to support life.

9 A
Due to the surface temperature of Earth, its electromagnetic radiation is centred around the infra-red range on the electromagnetic spectrum.

10 C
The speed of sound is dependent on the medium through which it travels — it is not dependent on the source.
The frequency of sound depends on the rate of vibration of the object creating the sound. The amplitude of a sound wave depends on the size of the vibrations of the object creating the sound.

Section B — Exam questions

11 a A longitudinal wave consists of the vibration of a medium parallel to the direction of propagation, whereas for a transverse wave, the vibration of the medium is perpendicular to the direction of propagation of the wave. [1 mark] An example of a longitudinal wave is a sound wave, and an example of a transverse wave is a vibrating string such as the string on a guitar. [1 mark]

b Both types of waves transfer energy from one place to another via a medium. [1 mark] Both types of waves travel at a constant speed dependent on the properties of the medium. [1 mark] And both types of waves have the same parameters, namely frequency, amplitude and wavelength. [1 mark]

12 a The wavelength of the sound wave would be two times the distance between a compression and an adjacent rarefaction. It would therefore be at 4.50 cm. [1 mark]

b $f = \frac{v}{\lambda}$

$= \frac{38}{4.5}$

$= 8.4\text{ Hz}$ [1 mark]

c In 0.10 s, the wave will have translated to the right by $38 \times 0.10 = 3.8$ cm, so the compression marked X will be 3.8 cm to the right. [1 mark]

13 a i $\lambda_{max} = \frac{\text{constant}}{T}$

$\Rightarrow T = \frac{\text{constant}}{\lambda_{max}}$

$= \frac{2.90 \times 10^{-3}\text{ mK}}{0.40 \times 10^{-6}\text{ m}}$

$= 7250\text{ K}$ [1 mark]

ii $\lambda_{max} = \frac{\text{constant}}{T}$

$\Rightarrow T = \frac{\text{constant}}{\lambda_{max}}$

$= \frac{2.90 \times 10^{-3}\text{ mK}}{0.25 \times 10^{-6}\text{ m}}$

$= 11\,600\text{ K}$ [1 mark]

b i $\lambda_{max} = \frac{\text{constant}}{T}$

$= \frac{2.90 \times 10^{-3}\text{ mK}}{15\,000\text{ K}}$

$= 1.93 \times 10^{-7}\text{ m}$

$= 193\text{ nm}$ [1 mark]

ii $\lambda_{max} = \frac{\text{constant}}{T}$

$= \frac{2.90 \times 10^{-3}\text{ mK}}{5555\text{ K}}$

$= 5.22 \times 10^{-7}\text{ m}$

$= 522\text{ nm}$ [1 mark]

14 $v = f\lambda$

$\Rightarrow \lambda = \frac{v}{f}$

$= \frac{4600}{440}$ [1 mark]

$= 10.45\text{ m}$ [1 mark]

15 a Electromagnetic waves refer to the waves that form from the interactions between an electric and a magnetic field. An electromagnetic wave is a wave that propagates at a unique speed in uniform media. [1 mark]

b

	Wavelength (m)	**Electromagnetic wave type**
i.	1×10^{3}	Radio waves
ii.	1×10^{-2}	Microwaves
iii.	1×10^{-12}	γ -rays
iv.	1×10^{-8}	UV light

(1 mark for each correct response, maximum 4 marks)

Topic 2 — Investigating light

2.2 Refraction using Snell's Law

Sample problem 1

Speed of light $= 3.0 \times 10^8$ m s^{-1}

Distance from Sun to Earth $= 1.50 \times 10^{11}$

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$\Rightarrow \text{Time taken} = \frac{\text{distance travelled}}{\text{average speed}}$$
$$= \frac{1.50 \times 10^{11}\text{ m}}{3.0 \times 10^8\text{ m s}^{-1}}$$
$$= 0.5 \times 10^3\text{ s}$$
$$= 500\text{ s}$$
$$= 8\text{ minutes }20\text{ seconds}$$

Practice problem 1

$$\text{Time taken} = \frac{\text{distance travelled}}{\text{average speed}}$$
$$= \frac{3.8 \times 10^8 \times 2}{3.0 \times 10^8}$$
$$= 2.5\text{ s}$$

Sample problem 2

Speed of light $= 3.0 \times 10^8$ m s^{-1}

Seconds in a year $= 365.25 \times 24 \times 3600$
$= 31\,557\,600$ seconds

Distance travelled = average speed × time taken
$= 3.0 \times 10^8 \times 31\,557\,600$
$= 9.5 \times 10^{15}$ m
$= 9.5 \times 10^{12}$ km

Practice problem 2

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$
$$= \frac{200}{1.0 \times 10^{-6}}$$
$$= 2.0 \times 10^8\text{ m s}^{-1}$$

This speed is less than the speed of light in a vacuum.

Sample problem 3

Use Snell's Law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, to determine the angle of refraction:

$$1.0 \times \sin 30° = 1.45 \times \sin\theta_{\text{glass}}$$
$$\Rightarrow \sin\theta_{\text{glass}} = \frac{\sin 30°}{1.45}$$
$$= 0.3448$$
$$\Rightarrow \theta_{\text{glass}} = \sin^{-1}(0.3448)$$
$$= 20.17°$$
$$= 20°$$

Practice problem 3

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$
$$\frac{\sin 40°}{\sin 30°} = \frac{n_{\text{plastic}}}{1}$$
$$\Rightarrow n_{\text{plastic}} = 1.29$$

Sample problem 4

a Calculate the speed of light in glass:

$$1.5 = \frac{3.0 \times 10^8}{\text{speed of light in glass}}$$
$$\Rightarrow \text{Speed of light in glass} = \frac{3.0 \times 10^8}{1.5}$$
$$= 2.0 \times 10^8\text{ m s}^{-1}$$

b
$$n_{\text{glass}} \times v_{\text{glass}} = n_{\text{water}} \times v_{\text{water}}$$
$$1.5 \times 2.0 \times 10^8\text{ m s}^{-1} = 1.33 \times v_{\text{water}}$$
$$\Rightarrow v_{\text{water}} = \frac{1.5 \times 2.0 \times 10^8}{1.33}$$
$$= 2.3 \times 10^8\text{ m s}^{-1}$$

Practice problem 4

a
$$v_{\text{diamond}} = \frac{c}{n_{\text{diamond}}}$$
$$= \frac{3.0 \times 10^8}{2.42}$$
$$= 1.24 \times 10^8\text{ m s}^{-1}$$

b
$$n_{\text{diamond}} \times v_{\text{diamond}} = n_{\text{carbon disulfide}} \times v_{\text{carbon disulfide}}$$
$$\Rightarrow v_{\text{carbon disulfide}} = \frac{n_d \times v_d}{n_{cd}}$$
$$= \frac{2.42 \times 1.24 \times 10^8}{1.63}$$
$$= 1.84 \times 10^8\text{ m s}^{-1}$$

2.2 Exercise

1 $v = \frac{d}{t} \Rightarrow t = \frac{d}{v}$

a The longest time:

$$t = \frac{2.28 \times 10^{11} + 1.50 \times 10^{11}}{3.0 \times 10^8}$$
$$= 1.26 \times 10^3\text{ s}$$
$$= 21\text{ min}$$

The shortest time:

$$t = \frac{2.28 \times 10^{11} - 1.50 \times 10^{11}}{3.0 \times 10^8}$$
$$= 260\text{ s}$$
$$= 4\text{ min }20\text{ s}$$

b The longest time:

$$t = \frac{4.50 \times 10^{12} + 1.50 \times 10^{11}}{3.0 \times 10^8}$$
$$= 1.55 \times 10^4 \text{ s}$$
$$= 4 \text{ h } 4 \text{ min}$$

The shortest time:

$$t = \frac{4.50 \times 10^{12} - 1.50 \times 10^{11}}{3.0 \times 10^8}$$
$$= 1.45 \times 10^4 \text{ s}$$
$$= 4 \text{ h } 1 \text{ min } 40 \text{ s}$$

2 a $90° - 25° = 65°$

b Angle of reflection = angle of incidence = 25°

3 The angle between the incident ray and the reflected ray is:
$90° = \theta_i + \theta_r$
However, $\theta_i = \theta_r$, and so $\theta_i = \frac{90}{2} = 45°$.

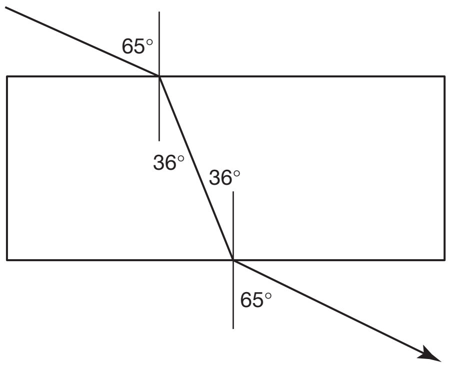
4 $\sin\theta = \frac{\sin 40°}{1.33}$
$\Rightarrow \theta = 29°$
$\sin\theta = \frac{\sin 50°}{1.33}$
$\Rightarrow \theta = 35°$
The angle of refraction increases by 6° when the angle of incidence increases by 10°.

5 $\sin 55° = n\sin 33°$

$$\Rightarrow n = \frac{\sin 55°}{\sin 33°}$$
$$= 1.5$$

6 a $\sin\theta = \frac{\sin 65°}{1.55}$
$\Rightarrow \theta = 36°$

b The angle of refraction at the bottom face is the same as the angle of incidence at the top face: 65°.



The block is rectangular, so the opposite sides are parallel. Therefore, the angle of incidence at the bottom face is the same as the angle of refraction at the top face. Because Snell's Law applies regardless of the direction of the light ray, the angle of refraction as the light emerges will be the same as the angle with which it entered the block.
This means the ray is parallel to the incoming ray, but shifted sideways.

7 $1.33 \times \sin 28° = 1.55 \times \sin\theta_r$

$$\sin\theta_r = \frac{1.33 \times \sin 28°}{1.55}$$
$$= 0.4028$$
$$\Rightarrow \theta_r = \sin^{-1}(0.4028)$$
$$= 23.8°$$

8 a $v = \frac{c}{n}$

$$= \frac{3.0 \times 10^8}{1.33}$$
$$= 2.3 \times 10^8 \text{ m s}^{-1}$$

b $v = \frac{c}{n}$

$$= \frac{3.0 \times 10^8}{1.50}$$
$$= 2.0 \times 10^8 \text{ m s}^{-1}$$

c $v = \frac{c}{n}$

$$= \frac{3.0 \times 10^8}{2.42}$$
$$= 1.2 \times 10^8 \text{ m s}^{-1}$$

9 a The different layers are parallel to each other, so the angle of refraction at the top surface of a liquid will equal the angle of incidence at the bottom surface.
This means that:

$$n_{\text{air}} \times \sin\theta_{\text{air}} = n_{\text{acetone}} \times \sin\theta_{\text{acetone}}$$
$$= n_{\text{glycerol}} \times \sin\theta_{\text{glycerol}}$$
$$= n_{\text{carbon tet}} \times \sin\theta_{\text{carbon tetrachloride}}$$

With $n_{\text{air}} = 1.0$ and $\theta_{\text{air}} = 25°$:

$$1.0 \times \sin 25° = 1.357 \times \sin\theta_{\text{acetone}}$$
$$= 1.4746 \times \sin\theta_{\text{glycereol}}$$
$$= 1.4601 \times \sin\theta_{\text{carbon tetrachloride}}$$
$$= 1.53 \times \sin\theta_{\text{glass}}$$

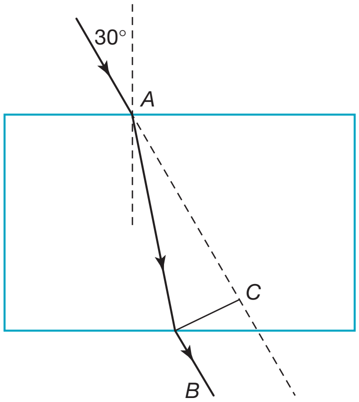
All expressions equal each other, so the angles of refraction will equal:

$$\frac{1 \times \sin 25°}{\text{refractive index of the medium}}$$

So, for acetone, the angle of refraction is 18.2°; for glycerol, it is 16.7°; for carbon tetrachloride, it is 16.8°; and for glass, it is 16.0°. Because the layers are parallel, the light ray will emerge from the glass into the air at the angle it left the air: 25°.

b When the light reflects off the mirror, the angle of reflection will equal the angle of incidence, so the light ray will leave the bottom medium at the same angle that it entered it. The light path up through the layers will be the reverse of the path coming down through the layers. All the angles will be the same.

10



The angle of refraction is given by: $\sin 30° = 1.4 \times \sin\theta$

$$\Rightarrow \theta = 20.9°$$

Length of the ray path in the glass is given by: $\dfrac{5.0}{\cos 20.9°}$

$\Rightarrow$ Sideways deflection = length of light path × sin(30.0 − 20.9)

= 0.85 cm

2.2 Exam questions

1 D

As light passes from Medium 1 to Medium 2 it refracts towards the normal. Therefore Medium 2 is more optically dense than Medium 1.

$\therefore n_1 < n_2$

As light passes from Medium 2 to Medium 3 it refracts away from the normal. Therefore Medium 2 is more optically dense than Medium 3.

The amount of refraction from Medium 2 to Medium 3 is greater than the amount of refraction from Medium 1 to Medium 2.

$\therefore \theta_3 > \theta_1$

Therefore, $n_3 < n_1 < n_2$.

The speed of light in a medium is inversely related to the refractive index $\left(n \propto \dfrac{1}{v}\right)$.

Therefore, $v_3 > v_1 > v_2$.

2 C

Refraction is the changing of speed and direction of a ray of light as it passes from one medium into another. The frequency of the light do not change.

3 $n_1\sin(\theta_1) = n_2\sin(\theta_2)$

$$\sin(\theta_2) = \frac{n_1\sin(\theta_1)}{n_2}$$

$$\Rightarrow \theta_2 = \sin^{-1}\left(\frac{n_1\sin(\theta_1)}{n_2}\right)$$

$$= \sin^{-1}\left(\frac{1.00 \times \sin(17°)}{1.47}\right) \text{ [1 mark]}$$

$$= \sin^{-1}(0.1989)$$

$$\simeq 11° \text{ [1 mark]}$$

4 $n_1\sin(\theta_1) = n_2\sin(\theta_2)$

$$\Rightarrow n_2 = \frac{n_1\sin(\theta_1)}{\sin(\theta_2)}$$

$$= \frac{1.60 \times \sin(30°)}{\sin(42°)} \text{ [1 mark]}$$

$$= 1.2 \text{ [1 mark]}$$

5 The refractive index of the two mediums must be the same. [1 mark]

It is also acceptable to identify that the speed of light in the two materials would need to be the same, or alternatively that the angle of incidence must be zero.

2.3 Total internal reflection and critical angle

Sample problem 5

$n_{air} = 1.0$; $\theta_{air} = 90°$; $n_{water} = 1.3$; $\theta_{water} = ?$

$n_{water} \times \sin\theta_c = n_{air} \times \sin 90°$

$1.3 \times \sin\theta_c = 1.0 \times \sin 90°$

$$\sin\theta_c = \frac{\sin 90°}{1.3}$$

$$= 0.7692$$

$$\Rightarrow \theta_c = \sin^{-1}(0.7692)$$

$$= 50.28°$$

$$= 50°$$

The critical angle is 50°.

Practice problem 5

a $n_1\sin\theta_1 = n_2\sin\theta_2$

$2.42\sin\theta_{cr} = 1.00\sin 90°$

$$\sin\theta_{cr} = \frac{1.00}{2.42}$$

$$\Rightarrow \theta_{cr} = \sin^{-1}\left(\frac{1.00}{2.42}\right)$$

$$= 24.4°$$

b The critical angle would now be larger. This is because light from diamond passing into water refracts less than when passing into air and hence the angle of incidence would need to be greater to achieve total internal reflection.

c $n_1\sin\theta_1 = n_2\sin\theta_2$

$2.42\sin\theta_{cr} = 1.33\sin 90°$

$$\sin\theta_{cr} = \frac{1.33}{2.42}$$

$$\Rightarrow \theta_{cr} = \sin^{-1}\left(\frac{1.33}{2.42}\right)$$

$$= 33.3°$$

Sample problem 6

a If the refractive index of the cladding was larger than the refractive index of the core, the light would be refracted *towards the normal* and internal reflection would not be possible. When the refractive index of the cladding is less than that of the core, the light is refracted *away from the normal* so that total internal reflection is possible if the angle of incidence is larger than the critical angle.

b $n_{core} \times \sin\theta_c = n_{cladding} \times \sin 90°$

$1.58 \times \sin\theta_c = 1.42 \times \sin 90°$

$$\sin\theta_c = \frac{1.42 \times \sin 90°}{1.58}$$
$$= \frac{1.42}{1.58}$$
$$\Rightarrow \theta_c = \sin^{-1}\left(\frac{1.42}{1.58}\right)$$
$$= 64.0°$$

The critical angle is 64.0°.

c Analyse the refraction of the light at the entrance of the optical fibre, the air–core boundary:

$$n_{air}v_{air} = n_{core}v_{core}$$
$$1.00 \times (3.00 \times 10^8) = 1.58 \times v_{core}$$
$$\Rightarrow v_{core} = \frac{3.00 \times 10^8}{1.58}$$
$$= 1.90 \times 10^8 \text{ m s}^{-1}$$

The speed of the laser light in the core is 1.90×10^8 m s^{-1}.

Practice problem 6

a As the refractive index for the core is smaller, the critical angle will be larger.

b $\theta_{cr} = \sin^{-1}\left(\frac{1.42}{1.56}\right)$

$= 65.5°$

c $v = \frac{c}{n}$

$$= \frac{3.0 \times 10^8}{1.56}$$
$$= 1.92 \times 10^8 \text{ m s}^{-1}$$

Therefore, the speed of light in the new optical fibre is larger.

2.3 Exercise

1 a $2.5 \times \sin\theta_c = \sin 90° = 1$

$$\sin\theta_c = \frac{1}{2.5}$$
$$= 0.4$$
$$\Rightarrow \theta_c = 24°$$

b $2.5 \times \sin\theta_c = 1.33 \times \sin 90° = 1.33$

$$\sin\theta_c = \frac{1.33}{2.50}$$
$$= 0.532$$
$$\Rightarrow \theta_c = 32.1°$$

2 a The critical angle is 45°, so the refractive index is

$\frac{1}{\sin 45°} = 1.4.$

For total internal reflection, $n \times \sin\theta_c \geq 1$.

If $n < 1.4$, then $n \times \sin\theta_c < 1$, so the ray will refract out of the glass.

$\Rightarrow n = 1.4$ is the minimum value for total internal reflection to occur for an angle of incidence of 45°.

b The rays would be inverted and swap positions (this is often used in binoculars to correct the normally inverted image). The initial ray of light would move down and refract off the surface shown, bouncing to the left at a right angle. The other ray of light would enter the block and refract at a right angle as well.

c Both rays of light would be expected to emerge at the same time, as they have identical pathways (just mirrored).

3 $1.500 \times \sin 82.0° = n \times \sin 90°$

$$\Rightarrow n = 1.500 \times \sin 82.0°$$
$$= 1.49$$

4 a Path length of reflected ray $= 2 \times \frac{r}{\cos 82°}$

$$= 2 \times \frac{0.5 \times 1 \times 10^{-6}}{\cos 82°}$$
$$= 7.18 \times 10^{-6} \text{ m}$$

Path length of straight ray $= 2 \times r\tan 82°$

$$= 2 \times 0.5 \times 1 \times 10^{-6} \times \tan 82°$$
$$= 7.11 \times 10^{-6} \text{ m}$$

Path length difference $= 7.18 \times 10^{-6} - 7.11 \times 10^{-6}$

$$= 7.00 \times 10^{-8} \text{ m}$$

b Speed of light in the glass $= \frac{c}{n}$

$$\sin\theta_c = \frac{1}{n}$$
$$\Rightarrow n = \frac{1}{\sin 82°}$$

= speed of light in the glass

$$= \frac{3.0 \times 10^8}{\sin 82°}$$
$$= 3.0 \times 10^8 \text{ m s}^{-1}$$
$$\Rightarrow \text{Time difference} = \frac{\text{distance}}{\text{speed}}$$
$$= \frac{7.0 \times 10^{-8}}{3.0 \times 10^8}$$
$$= 2.3 \times 10^{-16} \text{ s}$$

This time is small, but in an optical fibre of 1.00 km, there would be:

$$\frac{1.00 \times 10^3}{7.11 \times 10^{-6}} = 1.41 \times 10^8 \text{ reflections}$$

This corresponds to a total time difference of 3.28×10^{-8} s. This would limit the upper frequency of light pulses sent down the optical fibre. If the frequency was too high, the pulses would overlap. The problem can be overcome by using a narrower optical fibre or using an optical fibre whose refractive index gets smaller when the distance gets greater from the centre.

5 $\sin\theta_G = \frac{n_2}{n_1}$

$$n_2 = n_1 \times \sin\theta_G$$
$$= 1.33 \times \sin 70°$$
$$= 1.25$$

6 $\sin\theta_G = \frac{n_2}{n_1}$

$$\theta_G = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$
$$= \sin^{-1}\left(\frac{1.00}{2.40}\right)$$
$$= 24.6°$$

2.3 Exam questions

1 B

The point at which some light is just crossing the glass–liquid interface occurs when the angle of incidence is equal to the critical angle.

Therefore, 62.0° is the critical angle for light passing from glass into the liquid.

$$\begin{aligned} n_2 &= n_1 \sin\theta_c \\ &= 1.75 \times \sin 62.0 \\ &= 1.55 \end{aligned}$$

2 a $f = \dfrac{v}{\lambda}$

$$= \frac{3 \times 10^8}{565 \times 10^{-9}}$$

$$= 5.3 \times 10^{14}\ \text{Hz [1 mark]}$$

b $n_{co}\sin\theta_c = n_{cl}\sin 90$

$1.67\sin\theta_c = 1.45\sin 90$

$\Rightarrow \theta_c = \sin^{-1}\left(\dfrac{1.45}{1.67}\right)$ [1 mark]

$= 60.3°$ [1 mark]

VCAA examination report note: This question was generally answered well. The most common error was to invert the fraction. A significant number of substitutions showed a value of 1.65 rather than 1.67. Students must ensure that they transpose data correctly.

c $n_{air}v_{air} = n_{co}v_{co}$

$1.00 \times (3.00 \times 10^8) = 1.67 \times v_{co}$ [1 mark]

$\Rightarrow v_{co} = 1.80 \times 10^8\ \text{m s}^{-1}$ [1 mark]

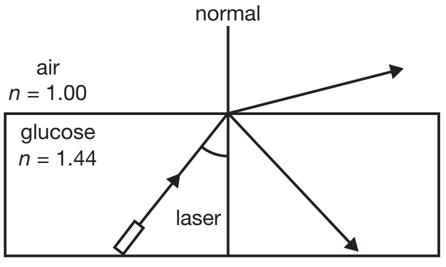
VCAA examination report note: The most common errors were to use the refractive index of the cladding rather than the core or not to give the answer to the correct number of significant figures, which was three in this case.

3 a $\sin\theta = \dfrac{n_{air}}{n_{glucose}}$

$= \dfrac{1.00}{1.44}$

$\Rightarrow \theta = 44°$ [1 mark]

b



(1 mark for drawing one refracted ray, bent away from the normal; 1 mark for drawing one reflected ray as shown)

VCAA examination report note: It was not necessary to calculate the angles of the refracted or reflected rays. While most students drew the refracted ray, very few included the reflected ray.

c All light rays from the laser are totally internally reflected at the surface. [1 mark] Hence, no light rays reach the observer above the surface, so the laser cannot be seen at X. [1 mark]

VCAA examination report note: The most common errors were not to name the phenomenon or to refer to it as refraction. This is an important concept and students should be able to name and describe it appropriately.

4 A

Total internal reflection only occurs when light is passing into a material with a lower refractive index.

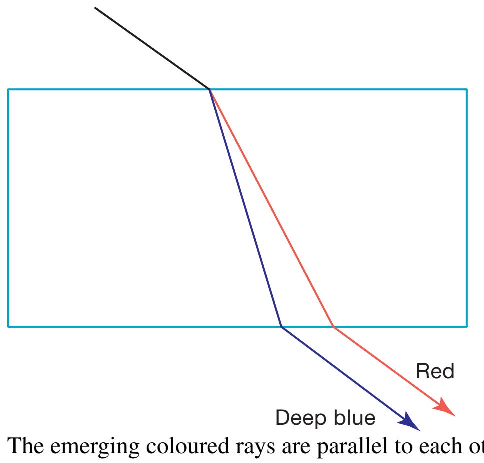
5 A

The refractive index of the cladding must be less than that of the core for total internal reflection to occur. Therefore, 1.43 is the only suitable value from the options given.

2.4 Dispersion

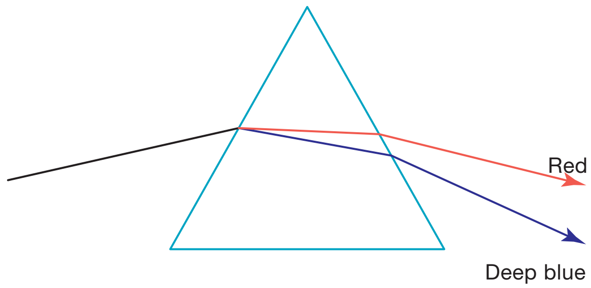
2.4 Exercise

1 a



The emerging coloured rays are parallel to each other.

b



In a triangle, the rays emerge at different angles and spread further apart the further they travel, so on a distant wall the colours are seen separately. With the rectangle, the colours stay the same distance apart regardless of how far they travel.

2 Red light travels faster through crown glass, as it has a lower refractive index:

Speed in red light $= \dfrac{c}{n_r}$

$= \dfrac{3.0 \times 10^8}{1.514}$

$= 1.982 \times 10^8 \text{ m s}^{-1}$

Speed in violet light $= \dfrac{c}{n_v}$

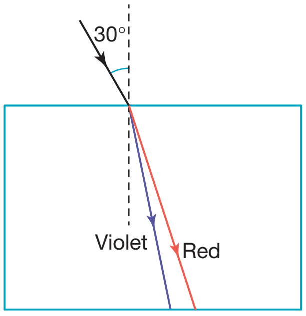
$= \dfrac{3.0 \times 10^8}{1.528}$

$= 1.963 \times 10^8 \text{ m s}^{-1}$

Difference $= 1.96 \times 10^6 \text{ m s}^{-1}$

3 Violet light is bent more as it has a greater refractive index.

4 a



b Red light:

$1 \times \sin 30° = 1.514 \times \sin\theta$

$\Rightarrow \sin\theta = \dfrac{0.50}{1.514}$

$= 0.3302$

$\Rightarrow \sin^{-1}(0.3302) = 19.28°$

Violet light:

$1 \times \sin 30° = 1.530 \times \sin\theta$

$\Rightarrow \sin\theta = \dfrac{0.50}{1.530}$

$= 0.3268$

$\Rightarrow \sin^{-1}(0.3268) = 19.07°$

c The angle between the red and violet rays is $19.28°–19.07° = 0.21°$.

5 This result indicates that red light travels faster in the prism material, as it is refracted less.

2.4 Exam questions

1 a The observed effect is known as dispersion. [1 mark] The refractive index of the glass is different for different wavelengths. [1 mark] Different wavelengths refract at different angles. Therefore, the different colours exit the prism at different angles. [1 mark]

b Colour visible at point X: Red light

Colour visible at point Y: Deep blue/violet light

(1 mark for both of the colours for points X and Y)

Red light refracts less than violet light from glass to air. That is, the relative refractive index for red light from glass to air is less than the relative refractive index for violet light from glass to air.

2 a $\dfrac{\sin i}{\sin r} = \dfrac{n_2}{n_1}$

$\dfrac{\sin 40}{\sin r} = \dfrac{1.50}{1.00}$

$\Rightarrow r = 25.4°$ [1 mark]

b $\dfrac{\sin i}{\sin r} = \dfrac{n_2}{n_1}$

$\dfrac{\sin 40}{\sin r} = \dfrac{1.52}{1.00}$

$\Rightarrow r = 25.0°$ [1 mark]

c Angle of dispersion $= 25.4 - 25.0 = 0.4°$ [1 mark]

d $\dfrac{n_2}{n_1} = \dfrac{v_1}{v_2}$

$\dfrac{1.52}{1.00} = \dfrac{3.00 \times 10^8}{v_2}$

$\Rightarrow v_2 = 1.97 \times 10^8 \text{ m s}^{-1}$ [1 mark]

3 The amount of refraction is determined with the frequency of the light, thus supporting the wave model of light. [1 mark]

4 The diamond has a higher refractive index, n [1 mark], therefore the light is dispersed more. [1 mark]

5 The majority of the light will be parallel to the incident light. [1 mark]

There will be a slight red and violet streak at the edges of the refracted light due to their differences in wavelength. [1 mark]

2.5 Optical phenomena

2.5 Exercise

1 In the morning the Sun rises in the east, and you should have your back to it; thus you should face towards the west. The water droplets should be in front of you, thus the rain should come from the west.

2 A

The order is red, orange, yellow, green, blue, indigo, violet.

3 Rainbows involve the internal reflection of light within water droplets. The spectrum from a prism is observed from the one source, whereas a rainbow is made up from light from numerous water droplets.

4 C

A mirage occurs when the ground is very hot and the air is cool.

5 This phenomenon is a mirage, occurring above a road (hotter than the air above), and is due to total internal reflection caused by atmospheric refraction. Close to ground level, the air is hot and has a refractive index close to 1. As height increases, the temperature of the air decreases and its refractive index increases. As a ray of light moves from the cooler top layer of air into hotter air near the ground, it bends away from the normal. After successive refractions, the angle of incidence exceeds the critical angle for air at that temperature and the ray is totally internally reflected. As the ray emerges, it follows a similar path, refracting towards the normal as it enters cooler air. Thus, what is observed is a refracted light from the sky, giving the impression that there is water on the road.

2.5 Exam questions

1 Emma is correct. [1 mark] The white light entering the water droplets will bend the different wavelengths different amounts. [1 mark] The red light that Emma sees in her rainbow will be from different water droplets. [1 mark]

2 White light consists of all colours (frequencies) of light across the visible spectrum, from red to violet. [1 mark] The different colours travel at slightly different speeds outside a vacuum. [1 mark] When a beam of white light strikes a material boundary, the colours are refracted by different amounts due to their differing speeds in the material. This leads to the production of a rainbow of colours, each emerging at slightly different angles. [1 mark]

3
- The Sun needs to be behind the person observing the rainbow. [1 mark]
- Some water droplets (rain, fog) need to be airborne in front of the observer. [1 mark]
- The angle between the observer, the Sun and the raindrops needs to be approximately 42°. [1 mark]

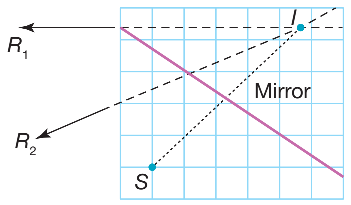
4 It is morning [1 mark], as in order for her to see a rainbow the Sun has to be behind her [1 mark]. She is looking west, so the Sun is in the east.

5 C
A mirage is an example of total internal reflection caused by atmospheric refraction.

2.6 Review

2.6 Review questions

1 Extend lines back from rays 1 and 2 to locate image of *S*. Then find the perpendicular bisector of the line *SI*. This bisector locates the mirror.



2 a Blue light: $\lambda = \frac{c}{f}$

$$= \frac{3.0 \times 10^8}{6.5 \times 10^{14}}$$
$$= 4.6 \times 10^{-7}\ \text{m}$$

Yellow light: $\lambda = \frac{c}{f}$

$$= \frac{3.0 \times 10^8}{5.2 \times 10^{14}}$$
$$= 5.8 \times 10^{-7}\ \text{m}$$

b $v = \frac{c}{n}$

$$= \frac{3.0 \times 10^8}{1.5}$$
$$= 2.0 \times 10^8 \text{ms}^{-1}$$

Hence, for blue light in the glass:

$$\lambda = \frac{v}{f}$$
$$= \frac{2.0 \times 10^8}{6.5 \times 10^{14}}$$
$$= 3.1 \times 10^{-7}\text{m}$$

Hence, for yellow light in the glass:

$$\lambda = \frac{v}{f}$$
$$= \frac{2.0 \times 10^8}{5.2 \times 10^{14}}$$
$$= 3.8 \times 10^{-7}\ \text{m}$$

3 a Use Snell's Law:

$$1 \times \sin 22° = n_{\text{perspex}} \times \sin 16°$$
$$\Rightarrow n_{\text{perspex}} = \frac{\sin 22°}{\sin 16°}$$
$$= 1.36$$

b Use Snell's Law:

$$1 \times \sin 32° = 1.36 \times \sin\theta$$
$$\sin\theta = \frac{\sin 32°}{1.36}$$
$$= 0.3896$$
$$\Rightarrow \theta = \sin^{-1}(0.3896)$$
$$= 23°$$

c Use Snell's Law:

$$1.36 \times \sin 22° = 1 \times \sin\theta$$
$$\sin\theta = 0.509$$
$$\Rightarrow \theta = \sin^{-1}(0.509)$$
$$= 31°$$

When light passes from air into perspex (refractive index is greater than 1), it bends towards the normal; the angle of refraction is thus less than the angle of incidence. However, when light passes from perspex to air, it bends away from the normal; in this case, the angle of refraction is greater than the angle of incidence.

4 a $v = \frac{c}{n}$

$$= \frac{3.0 \times 10^8}{1.63}$$
$$= 1.84 \times 10^8\ \text{m s}^{-1}$$

b Since the refractive index for salty water is less than the refractive index for carbon disulfide, and the speed of light is inversely proportional to the refractive index for that medium, it follows that the speed of light in salty water is greater than the speed of light in carbon disulfide.

c Since the light is passing from carbon disulfide into salty water — that is, from a higher refractive index into a lower refractive index — the light will bend away from the normal. The angle of refraction will be greater than the angle of incidence.

d Use Snell's Law:

$$1.63 \times \sin 45° = 1.38 \times \sin\theta_r$$
$$\sin\theta_r = \frac{1.63 \times \sin 45°}{1.38}$$
$$= 0.8352$$
$$\Rightarrow \theta_r = \sin^{-1}(0.8352)$$
$$= 57°$$

5 a The term *disperses* describes the splitting of white light into its constituent colours. This occurs when refraction into a medium has a refractive index that is dependent on colour, or frequency. Such media are known as dispersive media. In dispersive media, light of different colours will travel at different speeds and have different angles of refraction for the same angle of incidence.

b For red light:

$1 \times \sin 25° = 1.571 \times \sin\theta_r$

$$\sin\theta_r = \frac{\sin 25°}{1.571} = 0.2690$$

$$\Rightarrow \theta_r = \sin^{-1}(0.2690) = 15.61°$$

For blue light:

$1 \times \sin 25° = 1.594 \times \sin\theta_r$

$$\sin\theta_r = \frac{\sin 25°}{1.594} = 0.2651$$

$$\Rightarrow \theta_r = \sin^{-1}(0.2651) = 15.37°$$

Thus, the angle between the blue light and the red light is:

$15.61 - 15.37 = 0.24°$

c For red light:

$1 \times \sin 50° = 1.571 \times \sin\theta_r$

$$\sin\theta_r = \frac{\sin 50°}{1.571} = 0.4876$$

$$\Rightarrow \theta_r = \sin^{-1}(0.4876) = 29.18°$$

For blue light:

$1 \times \sin 50° = 1.594 \times \sin\theta_r$

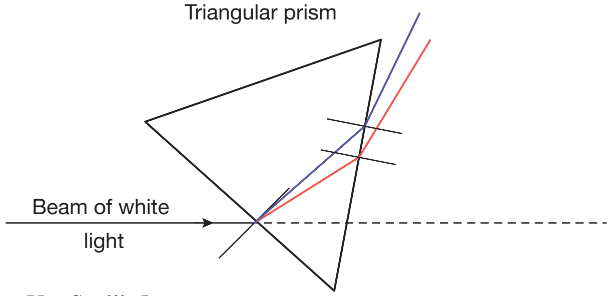
$$\sin\theta_r = \frac{\sin 50°}{1.594} = 0.4806$$

$$\Rightarrow \theta_r = \sin^{-1}(0.4806) = 28.72°$$

Thus, the angle between the blue light and the red light is:

$29.18 - 28.72 = 0.46°$

6



7 a Use Snell's Law:

$1 \times \sin 90° = n \times \sin\theta_{cr}$

$$\sin\theta_{cr} = \frac{1}{n}$$

$$\Rightarrow \theta_{cr} = \sin^{-1}\left(\frac{1}{n}\right)$$

This relationship informs us that the larger the refractive index, the smaller the critical angle. Hence, glass type Z has the smallest critical angle of the three materials.

b For material Z, $\theta_{cr} = \sin^{-1}\left(\frac{1}{1.57}\right) = 39.6°$.

c At the material Z–material X interface:

$1.53 \times \sin 90° = 1.57 \times \sin\theta_{cr}$

$$\sin\theta_{cr} = \frac{1.53}{1.57} = 0.975$$

$$\Rightarrow \theta_{cr} = \sin^{-1}(0.975) = 77.0°$$

2.6 Exam questions

Section A — Multiple choice questions

1 A

Shorter wavelengths are dispersed more than longer wavelengths. Dispersion occurs at both interfaces.

2 D

$$t = \frac{d}{v} = \frac{240}{3.0 \times 10^8} = 8.0 \times 10^{-7}\,\text{s}$$

3 A

Use Snell's Law to get: $n = \frac{\sin 35.0°}{\sin 26.0°} = 1.31$

4 B

Use Snell's Law to get:

$$\sin\theta = \frac{1.53 \times \sin 47.0°}{1.33} = 0.8413$$

$$\Rightarrow \theta = \sin^{-1}(0.8413) = 57.3°$$

5 C

The light in optic fibre Y travels slower than the light in optic fibre X. Accordingly, it will take a longer time to travel the same distance in optic fibre Y compared to optic fibre X.

6 D

Use Snell's Law:

$1.387 \times \sin\theta_c = 1 \times \sin 90°$

$$\sin\theta_c = \frac{1}{1.387} = 0.7210$$

$$\Rightarrow \theta_c = \sin^{-1}(0.7210) = 46.1°$$

7 B

Red light has the largest angle of refraction of all the visible colours. Options A, C and D are all incorrect statements.

8 C

$1.52 \times \sin\theta_c = 1.49$

$$\sin\theta_c = \frac{1.49}{1.52}$$

$$\Rightarrow \theta_c = 78.6°$$

9 A

$$n = \frac{c}{v}$$

$$\Rightarrow v = \frac{c}{n}$$

$$= \frac{3.0 \times 10^8}{1.3425}$$

$$= 2.2 \times 10^8 \text{ m s}^{-1}$$

10 B

A mirage occurs when the air is cool and the ground is very hot.

Section B — Short answer questions

11 $1.4672 \times \sin 70.0° = n \times \sin 62.5°$ [1 mark]

$\Rightarrow n = 1.56$ [1 mark]

12 **a** Use Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$1.0 \times \sin\theta_1 = 1.46 \times \sin 32 \text{ [1 mark]}$$

$$\Rightarrow \theta_1 = 51° \text{ [1 mark]}$$

b Find the critical angle for the core/cladding interface:

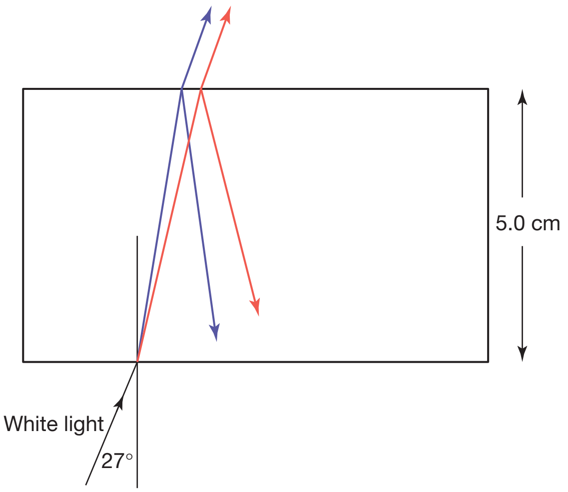
$$\theta_c = \sin^{-1}\left(\frac{1.42}{1.46}\right)$$

$$= 76.6° \text{ [1 mark]}$$

The angle of incidence is 90 – 32 = 58°. [1 mark]

Since the angle of incidence is below the critical angle, some light will be transmitted to the cladding. [1 mark]

13 **a**



(1 mark for the correct shapes; 1 mark for the correct ray colours)

b Use Snell's Law:

$1 \times \sin 27° = 1.57 \times \sin\theta_{red}$ [1 mark]

$$\sin\theta_{red} = \frac{\sin 27°}{1.53}$$

$$= 0.2967$$

$$\Rightarrow \theta_{red} = \sin^{-1}(0.2967)$$

$$= 17.3° \text{ [1 mark]}$$

c To determine the time taken, use the expression $t = \frac{d}{v}$, where d is the distance and v is the speed, both of which will be different for red and blue light.

For red light the speed is:

$$\frac{3.0 \times 10^8}{1.530} = 1.960 \times 10^8 \text{ m s}^{-1}$$

For blue light the speed is:

$$\frac{3.0 \times 10^8}{1.548} = 1.938 \times 10^8 \text{ m s}^{-1} \text{ [1 mark]}$$

Since the prism has a thickness of 5 cm (0.05 m), the distance travelled by both the red light and the blue light will be given by the expression $\frac{0.005}{\cos\theta}$, where θ is the angle of refraction. The angle can be found using Snell's Law.

For red light:

$1 \sin 27° = 1.530 \sin\theta_{red}$

$$\Rightarrow \theta_{red} = \sin^{-1}\left(\frac{\sin 27}{1.530}\right)$$

$$= 17.26°$$

For blue light:

$1 \sin 27° = 1.548 \sin\theta_{blue}$

$$\Rightarrow \theta_{bue} = \sin^{-1}\left(\frac{\sin 27}{1.548}\right)$$

$$= 17.05° \text{ [1 mark]}$$

Therefore, the time taken for red light to cross the prism is:

$$\frac{0.05}{\cos 17.26 \times 1.960 \times 10^8} = 2.671 \times 10^{-10} \text{ s}$$

The time taken for the blue light is:

$$\frac{0.05}{\cos 17.05 \times 1.960 \times 10^8} = 2.70 \times 10^{-10} \text{ s}$$

Therefore, blue light takes slightly longer. [1 mark]

14 **a** Since $n_{cladding} < n_{core}$, there exists a critical angle. [1 mark] Ideally, the refractive index of the cladding is only a little smaller than the core so that the critical angle is as large as possible. [1 mark]

b Use Snell's Law:

$n_{cladding} \times \sin 90° = 1.58 \times \sin 84°$ [1 mark]

$\Rightarrow n_{cladding} = 1.57$ [1 mark]

c $$c_{core} = \frac{3.0 \times 10^8}{1.58}$$

$$= 1.9 \times 10^8 \text{ m s}^{-1} \text{ (1 mark)}$$

d The frequency of the light is the same in both air and in the core. The change in speed is associated with a change in wavelength. [1 mark]

Hence:

$$f_{air} = f_{core}$$

$$\frac{c}{\lambda_{air}} = \frac{c}{n_{core} \times \lambda_{core}} \text{ [1 mark]}$$

$$\Rightarrow \lambda_{core} = \frac{\lambda_{air}}{n_{core}}$$

$$= \frac{400}{1.58}$$

$$= 253 \text{ nm [1 mark]}$$

15 The angle of refraction is 40.0° – 12.5° = 27.5°. [1 mark]

$$n_{kerosene} = \frac{\sin 40°}{\sin 27.5°} \text{ [1 mark]}$$

$$= 1.39 \text{ [1 mark]}$$

The absolute refractive index of the oil is 1.39.

Topic 3 — Thermal energy and its interaction with electromagnetic radiation

3.2 Explaining heat using the kinetic theory

Sample problem 1

$$\begin{aligned} T_{(\text{kelvin})} &= T_{(\text{Celsius})} + 273\,\text{K} \\ &= 0 + 273\,\text{K} \\ &= 273\,\text{K} \end{aligned}$$

Ice melts at 273 K.

Practice problem 1

$$\begin{aligned} T_{(\text{kelvin})} &= 273\,\text{K} + T_{(\text{Celsius})} \\ 3\,\text{K} &= 273\,\text{K} + T_{(\text{Celsius})} \\ T_{(\text{Celsius})} &= 3\,\text{K} - 273\,\text{K} \\ &= -270\,°\text{C} \end{aligned}$$

3.2 Exercise

1 The Kelvin scale of temperature measurement is an absolute scale. Zero degrees gives an indication that the kinetic energy of all particles is zero at this temperature. This is not reproducible in a school or ordinary laboratory. The Celsius scale is commonly used because it is based on the properties of water and is easy to reproduce anywhere in the world. The freezing of ice and the boiling of water with 100 divisions between is convenient.

2 The Kelvin scale is an absolute by which everything in the universe can be referenced. It has a zero value, which corresponds to zero average random kinetic energy of particles in a substance.

3 a The maximum temperature in Melbourne on a hot summer’s day is approximately 37 °C:

$$\begin{aligned} T_{(\text{kelvin})} &= 273\,\text{K} + T_{(\text{Celsius})} \\ &= 273\,\text{K} + 37\,°\text{C} \\ &= 310\,\text{K} \end{aligned}$$

b The minimum temperature in Melbourne on a cold, frosty winter’s morning is approximately 2 °C:

$$\begin{aligned} T_{(\text{kelvin})} &= 273\,\text{K} + T_{(\text{Celsius})} \\ &= 273\,\text{K} + 2\,°\text{C} \\ &= 275\,\text{K} \end{aligned}$$

c The current room temperature is approximately 25 °C:

$$\begin{aligned} T_{(\text{kelvin})} &= 273\,\text{K} + T_{(\text{Celsius})} \\ &= 273\,\text{K} + 25\,°\text{C} \\ &= 298\,\text{K} \end{aligned}$$

d The temperature of cold tap water is approximately 15 °C:

$$\begin{aligned} T_{(\text{kelvin})} &= 273\,\text{K} + T_{(\text{Celsius})} \\ &= 273\,\text{K} + 15\,°\text{C} \\ &= 288\,\text{K} \end{aligned}$$

e The boiling point of water is 100 °C:

$$\begin{aligned} T_{(\text{kelvin})} &= 273\,\text{K} + T_{(\text{Celsius})} \\ &= 273\,\text{K} + 100\,°\text{C} \\ &= 373\,\text{K} \end{aligned}$$

4 The large thermometer is also surrounded by the air in the school laboratory. The liquid in the test tube might be cold, but the thermometer is also measuring the surrounding air, giving a falsely high reading.

5 $T_{(\text{kelvin})} = 273\,\text{K} + T_{(\text{Celsius})}$

$$\begin{aligned} T_{(\text{kelvin})} &= 273\,\text{K} + (-78.5)\,°\text{C} \\ &= 194.5\,\text{K} \end{aligned}$$

6 $T_{(\text{Celsius})} = T_{(\text{kelvin})} - 273$

$$\begin{aligned} T_{(\text{Celsius})} &= (256 - 273)\text{ and }(166 - 273) \\ &= -17\,°\text{C and } -107\,°\text{C} \end{aligned}$$

7 The particles of the air are in constant motion; this enables the molecules that make up the aroma of the food to travel from the kitchen to the front door.

8 Translational energy describes the displacement of particles from one position to another. Other types of kinetic energy involve motion within the particles (movement within bonds of a molecule) or potential energy from atomic and subatomic interactions.

9 The particles in the hot tea are moving rapidly and experiencing many collisions with each other and the sides of the cup. The collisions with the cup transfer the kinetic energy to the cup particles and they start to move faster. The temperature of the tea drops and the temperature of the cup rises. The air particles surrounding the cup also experience similar collisions, thus kinetic energy is transferred to the air. Hence, the overall translational kinetic energy of the cup of tea is transferred to the air and the tea cools down.

10 As the translational kinetic energy of the soup particles is transferred to its surroundings, the particles slow down, thus exerting much less pressure on the cling film. The cling film is pushed into the bowl from the external air pressure.

11 Examples may include a meteorite crashing into Earth, a car hitting a wall, rubbing your hands together, a spoon stirring a cup of tea, an eraser on a piece of paper and water tumbling over a waterfall.

12 A red-hot pin has less internal energy, less kinetic energy and fewer molecules to vibrate than a nail of the same material at the same temperature. There is just less matter to vibrate, so there is less energy to transfer to the water.

13 While you are swimming in the sea, your body maintains an even core temperature of 37.6 °C. The sea around Melbourne is a lower temperature than this. The water particles do not vibrate as quickly as your molecules. Your molecules transfer their greater kinetic energy to the water. Temperature is a measure of the average random kinetic energy of the particles. Internal energy is the total energy of the particles, including the potential energy due to intermolecular forces. You have less internal energy than the sea simply because you have fewer particles than the sea.

3.2 Exam questions

1 D

The boiling point of water is 100 °C, which is equivalent to 212 °F, or 373 K.

2 **a** The Celsius scale is based on the melting point (0 °C) and boiling point (100 °C) of water. [1 mark] The Kelvin scale is based on the kinetic theory of matter, where 0 K corresponds to the lowest possible temperature such that particles completely stop moving. [1 mark]

b Both scales use the same scale increments. 1°C ≡ 1 K. [1 mark]

c An absolute scale starts at 0. There are no negative values. [1 mark]

d Kelvin does not use a degree symbol. [1 mark]

3 A
To convert kelvin to degrees Celsius, you need to subtract 273. Hence, the answer is −273 °C.

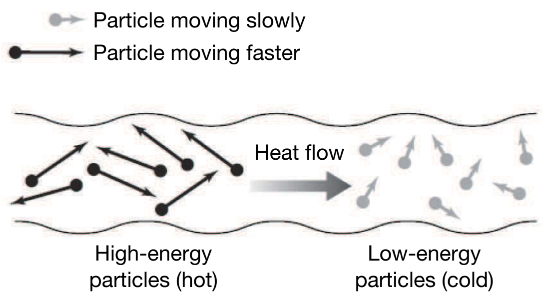
4 The atoms in a solid are tightly packed, with strong attractive forces holding them in a rigid structure. In liquids, molecules are free to move around within the volume. [1 mark] Some of the attractive forces holding the atoms together in the solid must be broken in order for the molecules to be free to move around as a liquid. [1 mark] This requires an increase in energy to break the bonds, and this energy comes from an increase in temperature. [1 mark]

5 When a tennis ball is hit by a racquet the same number of air molecules occupy a smaller volume. This increases the air pressure, causing the air molecules to collide more frequently and increase in speed. [1 mark] The increased speed of the air molecules corresponds to an increase in temperature. [1 mark] Once the ball has left the racquet and returned to its initial size and shape, the air pressure will return to normal and the temperature will return to its initial value.

3.3 Transferring heat

3.3 Exercise

1 Particles in the hot region of the substance have more random kinetic energy than those in the cooler region of the substance. Some of the kinetic energy of the particles in the hot region is transferred to the particles in the cooler region. Thus, the temperature in the cooler region increases.



2 Conduction is the transfer of heat through a substance as a result of collisions between vibrating neighbouring particles. Liquids and gases have particles that are much further apart than those in solids. The particles have to travel further to transfer kinetic energy, so the process of conduction is generally slower in gases and liquids than in solids.

3 Heat transfer through water occurs mainly by convection. That is, warmer water moves upwards as it is displaced by the denser, cooler water. Warm water does not generally move down a test tube because it is less dense than cooler water. If water was a good conductor of heat, then it wouldn't matter whether you applied the flame at the top or bottom of the test tube.

4 Convection is the transfer of heat through a substance as a result of the movement of particles between regions with different temperatures. In solids, the particles are tightly bound together and unable to move to regions of different temperatures.

5 Radiant energy moves through space at 3.0×10^8 m s^{-1}. This is significant as it is the same value as the speed of light.

6 **a** The water on the surface of a still body of water becomes warmer than the water deeper down due to it absorbing radiated energy from the Sun and heat transfer from the warm air near the surface. Although there is a temperature difference between the warm air and water surface, the water is a poor conductor, so the energy is not transferred very far into the water over a short time period. There will be no convection currents as the cooler water is under the warm water.

b If a wind is blowing, chopping up the surface, the cold water underneath mixes with the warm water on the top. The heat transfer from the surface water reduces its temperature. The wind also evaporates some of the surface water. In doing so, it uses energy from the water to change the state from liquid to vapour. This reduces the surface temperature further.

7 **a** The feeling of warmth is caused by the concrete wall radiating infrared energy due to its temperature being higher than its surroundings.

b The walls of the building would have become warm due to heat transfer by radiation from the Sun warming them and by conduction from the warm air surrounding the walls.

8 Coffee is best consumed when it is hot (around 50–60 °C). Aluminium is a good conductor. It will conduct energy from the coffee to the surrounding air. The coffee will cool quickly. The aluminium will also transfer energy to the hand holding the cup, making this an unpleasant, painful experience before it cools below 50 °C.

9 Air within a room circulates by convection, with the warm air rising up to the ceiling. Floor ducts used for heating rely on the warm air leaving the duct and rising up through the room. Ceiling ducts need more powerful blower fans to counteract the tendency of the warm air to stay at ceiling level. It needs to be forced down to mix with the cold air to warm it by conduction and convection.

3.3 Exam questions

1 A
Atoms in a solid are tightly packed in a 3D lattice and so can easily transfer kinetic energy between one another.

2 D
Metals are very good conductors.

3 The greater the temperature difference, the greater the difference in kinetic energy between colliding particles. [1 mark] More energy will be transferred in each collision, and so the rate of heat transfer is faster. [1 mark]

4 The higher-energy water at the bottom expands and becomes less dense and rises. The cooler, denser water at the top of the pot falls to the bottom. [1 mark] This motion causes a movement of material and transfers energy; that is, a convection current. [1 mark]

5 The hotter, faster vibrating particles collide with the slower vibrating particles. [1 mark] Heat is transferred from hotter to colder particles. [1 mark] This continues through the solids until both reach the same temperature; that is, thermal equilibrium is achieved. [1 mark]

3.4 Specific heat capacity

Sample problem 2

a $Q = mc\Delta T$

$m = 8.0$ kg, $c = 4200$ J kg^{-1} K^{-1} (from table 3.3)

$\Delta T = 85\,°\text{C} - 15\,°\text{C}$
$= 70\,°\text{C}$
$= 70$ K (same change)

$Q = 8.0 \times 4200 \times 70$
$= 2\,352\,000$
$= 2.4 \times 10^3$ kJ

2.4×10^3 kJ is needed to increase the temperature of 8.0 L of water from 15 °C to 85 °C.

b $Q_w = Q_s$

$m_w c_w \Delta T_w = m_s c_s \Delta T_s$

$\Delta T_w = T_f - 15\,°C$

$\Delta T_s = 120\,°C - T_f$

$0.200\,\text{kg} \times 4200\,\text{J kg}^{-1}\,°\text{C}^{-1} \times (T_f - 15\,°\text{C})$
$= 0.250\,\text{kg} \times 900\,\text{J kg}^{-1}\,°\text{C}^{-1} \times (120\,°\text{C} - T_f)$

$\Rightarrow 840T_f - 12\,600 = 27\,000 - 225T_f$

$1065T_f = 39\,600$

$T_f = \dfrac{39\,600}{1065}$
$= 37.18\,°\text{C}$

The saucepan and the water have reached a common temperature of 37 °C.

Practice problem 2

The average heat capacity of the human body is 3500 J kg^{-1} K^{-1}. Use the formula for the energy required to heat a substance.

$Q = mc\Delta T$
$= m \times 3500 \times 1$ (Answers will depend on mass.)
$= 3500\,m$

If mass is 65 kg, then:

$Q = 65 \times 3500$
$= 227\,500$ J
$= 227.5$ kJ

Sample problem 3

a $Q = mL$

$L = 2.2 \times 10^3$ J kg^{-1}, $m = 2.5$ kg

$Q = 2.2 \times 10^3$ J kg^{-1} $\times 2.5$ kg
$= 5.5 \times 10^3$ J

5.5×10^3 J is needed to completely melt 2.5 kg of aluminium.

b The temperature just after state change is still 660.3 °C.

Practice problem 3

Specific latent heat of vaporisation for water, $L = 2.3 \times 10^6$ J kg^{-1}

$m = 15$ g $= 0.015$ kg

$Q = mL$
$= 2.3 \times 10^6 \times 0.015$
$= 3.45 \times 10^4$ J

3.4 Exercise

1 **a** For a given substance and a given ΔT:

$Q \propto m$

$\Rightarrow$ If m is doubled, Q doubles.

$Q = 2 \times 200$ kJ
$= 400$ kJ

b For a given substance:

$Q \propto m\Delta T$

$\Rightarrow$ If m is tripled and ΔT is doubled, Q increases by a factor of 6 (as $3 \times 2 = 6$).

$Q = 6 \times 200$ kJ
$= 1200$ kJ

2 **a** The human body is mostly water, so you would expect the specific heat capacity to be similar to that of water. The specific heat capacity of water is quite high compared to that of other substances.

b Desert sand is dry, containing no moisture. Fertile soil is usually damp, so its specific heat capacity will be higher than that of desert sand because the specific heat capacity of water is large. It is also dependent on the percentage of water it contains — the higher the percentage of water, the larger the specific heat capacity.

c It would be expected that the steel saucepan would gain the most energy from the hotplate because its specific heat capacity is higher than that of copper. Even though aluminium has a higher specific heat capacity, its density is much less. A similar-sized aluminium saucepan has less mass.

d Metals have low heat capacities; water and things containing water have high specific heat capacities.

3 $Q = mL$
$= 0.500 \times 2.3 \times 10^6$
$= 1.2 \times 10^6$ J

4 **a** 230 °C. Interval BC represents the process of melting, and interval DE represents the process of evaporation. Thus, the boiling point of 230 °C can be read directly from the graph.

b The energy was used to increase the potential energy of the particles as they changed state from solid to liquid.

c Liquid

d $Q = mL$

$\Rightarrow L = \dfrac{Q}{m}$
$= \dfrac{80\text{ kJ}}{0.5\text{ kg}}$
$=$ (80 kJ read from graph: interval BC)
$= 160$ kJ kg^{-1}

e The specific heat capacity of solid candle wax is higher. More energy is required to raise the temperature of the solid wax by 20 °C than the liquid wax (100 kJ compared with approximately 40 kJ as shown on the graph).

f Over the interval DE, the temperature remains the same. The energy put into the wax does not result in an increase in the kinetic energy of the particles; it just breaks the attractive bonds between the particles as the substance changes state from liquid to gas.

5 Energy available to just melt the ice
= energy from condensing steam + energy from the cooling hot water
$= mL_{vap} + mc_{water}\,\Delta T$
$= 1 \times 2.3 \times 10^6 + 1 \times 4200 \times 100$
$= 2.3 \times 10^6 + 4.2 \times 10^5$
$= 2.72 \times 10^6$ J
$\Rightarrow m_{ice}L_{fusion} = 2.72 \times 10^6$ J
$$\Rightarrow m_{ice} = \frac{2.72 \times 10^6\ \text{J}}{3.3 \times 10^6} = 8.2\,\text{kg}$$

6 A simmering saucepan loses heat by convection and evaporation. By keeping the lid on, the water is prevented from evaporating because the air inside the pan is saturated. The evaporation of the water would tend to cool the thing you are heating. In addition, escaping steam would be replaced by cooler air, meaning that more energy would be needed to sustain the temperature of the water.

7 The evaporation of water requires energy for the water particles to change from their liquid state to the vapour state. The energy comes from the body of the liquid. Therefore, the net result is a reduction in the kinetic energy of the water particles and the temperature of the body of the liquid.

3.4 Exam questions

1 C
The specific heat capacities of ice, water and steam are all different due to all three having different amounts of kinetic and bonding energies.
Ice, water and steam do not have same bonding strengths. They all have the same chemical composition (H_2O) and their structures have little influence on internal energies.

2 B
This relationship is used to calculate how much heat energy must be transferred to change the temperature of some object (with mass m and specific heat capacity c) by ΔT degrees.

3 $Q = mc\Delta T$
$= 0.550\,\text{kg} \times 4200\,\text{J kg}^{-1}\,\text{K}^{-1} \times (89 - 34)\,°\text{C}$ [1 mark]
$= 127\,050\,\text{J}$ [1 mark]
$= 127\,\text{kJ}$ [1 mark]

4 $-Q_{water} = Q_{aluminium}$
$\Rightarrow -m_w c_w \Delta T = m_{Al} C_{Al} \Delta T$, where $(\Delta T = T_f - T_i)$ [1 mark]
$\Rightarrow -0.5 \times 4200 \times (T_f - 90) = 0.3 \times 900 \times (T_f - 20)$ [1 mark]
$-2100T_f + 189\,000 = 270T_f - 5400$
$194\,400 = 2370T_f$ [1 mark]
$T_f = 82.0\,°\text{C}$ [1 mark]

5 $Q_{total} = mc\Delta T + mL$ [1 mark]
$= 0.400\,\text{kg} \times 4200\,\text{J kg}^{-1}\,\text{K}^{-1} \times (100 - 50)°\text{C} + 0.400\,\text{kg} \times \left(2.3 \times 10^3\,\text{kJ kg}^{-1}\right)$ [1 mark]
$= 84\,000\,\text{J} + 920\,\text{kJ}$
$= 84\,\text{kJ} + 920\,\text{kJ}$ [1 mark]
$= 1004\,\text{kJ}$
$= 1.00\,\text{MJ}$ [1 mark]

3.5 Understanding climate change and global warming

3.5 Exercise

1 Some of the radiation that is emitted by the Sun is absorbed by various gases (such as ozone, nitrogen, oxygen, water vapour, carbon dioxide etc.) in the atmosphere before it reaches Earth.

2 Some gases naturally present in small proportions in Earth's atmosphere — such as water vapour, carbon dioxide, nitrous oxide, ozone and methane — absorb infrared light and re-emit it in all directions. The radiation re-emitted towards Earth's surface increases its temperature.

3 The enhanced greenhouse effect is the disruption to Earth's climate equilibrium. It is caused by the release of greenhouse gases, mainly carbon dioxide and methane, in the atmosphere due to human activities such as burning fossil fuels. This leads to an increase in global average surface temperatures.

4 Properties of water include high specific heat capacity, convective properties and the fact that it can exist in solid, liquid and gaseous states. These properties have different impacts on the climate, with ice and snow having a high albedo, water vapour being a greenhouse gas and clouds reflecting light, for example.

5 It is a threat to life on Earth as many species can survive only within specific conditions (such as temperatures and gas concentrations). The greenhouse effect may change these conditions so that they are not compatible with life on Earth.

3.5 Exam questions

1 B
Burning fossil fuels releases carbon dioxide, a greenhouse gas, into the atmosphere.

2 A
Without greenhouse gases, much of the infrared radiation given off by Earth would escape to space. With increased levels of greenhouse gases, this energy is absorbed and the gases — and so the atmosphere — heat up.

3 D
Photosynthesis is the process by which plants use sunlight to react water and carbon dioxide to grow and produce food and oxygen. Land clearing and deforestation mean fewer plants, and therefore less carbon dioxide being taken out of the atmosphere.

4 Water vapour (H_2O), carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and ozone (O_3)
(1 mark for each of the five correct responses, maximum 5 marks)

5 Sunlight heats Earth's surface, which then re-radiates infrared radiation. [1 mark] Some of this infrared radiation is absorbed by small gas molecules in the atmosphere. [1 mark] By trapping this energy, these molecules heat up and so the atmospheric temperature increases — the greenhouse effect. [1 mark]

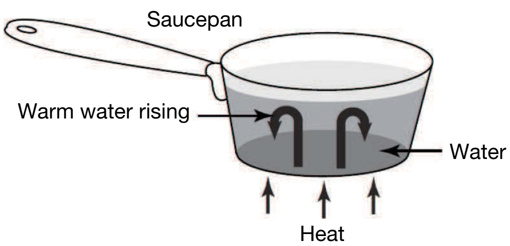
3.6 Review

3.6 Review questions

1 Testing your own temperature is a subjective exercise. If you are 'running a temperature' then your hand will be warm as well as your head, and you will not be able to detect the difference.

2 Absolute temperature is measured using the Kelvin scale. On the Kelvin scale 28 °C is not twice as hot as 14 °C (301 K and 287 K, respectively). An increase of 14 degrees from 287 K to 301 K represents a 4–5 per cent increase in temperature, not 50 per cent.

3 Particles in warm regions are further apart than those in cooler regions. The warm, less dense liquid rises, while the cooler, denser liquid sinks. This process continues throughout the container as the warm liquid rises, gradually cooling and eventually sinking again, replacing the newly heated liquid.



4 B
If two objects are of different temperatures, there will be a net transfer of heat from the hot object to the cool object. This process will continue until the objects reach the same temperature, when they are said to be in thermal equilibrium and there is no further net transfer of heat.

5 C
The joule is the SI unit of energy. Kelvin and degrees Celsius are measures of temperature, and the calorie is a non-SI unit of energy sometimes used in food science.

6 Conventional ovens have heating elements at the bottom to allow air to become warmed, and then rise. This sets up convection currents, which move around the food. Food is then warmed by conduction and convection as well as by radiation from the element. The advantage of having an oven with a fan is that the inside of the oven is kept at an even temperature, instead of having the top shelf very hot and the bottom shelf not as hot. It is a matter of choice; some cooks prefer standard ovens with the temperature difference so they can cook food that requires different temperature settings at the same time.

7 **a** Sweating reduces the temperature of the body. Sweat on the skin is basically water. The water evaporates by gaining energy from the skin to change state. Taking energy from the surface of the skin results in a reduction in the energy of the skin molecules and a reduction in the temperature of the skin.

b You feel cooler when the wind is blowing because the rate of evaporation of moisture from your skin is increased, lowering your skin temperature. Also, your body has air in contact with it; this is warmed by conduction to the same temperature. This warmed air is removed by the wind, and air at a lower temperature replaces it.

c Sweating in humid weather does not cool the body because the sweat does not evaporate. It just lies as drops of water on the skin. For evaporation to occur, the air must not be saturated with water vapour already. If there is a breeze and not total saturation, there may be some evaporation and cooling.

8 Conduction is the transfer of thermal energy through a substance as a result of collisions between neighbouring vibrating particles. Particles in solids are more tightly bound and closer together in solids than in liquids, and in liquids than in gases. Thus, solids are better conductors of heat than liquids, and liquids are better conductors of heat than gases.

9 The vaporisation of water is an example of positive feedback; the more water being vaporised, the warmer Earth gets — as water vapour is a greenhouse gas — and the warmer Earth is, the more water vaporises.
The accelerated formation of clouds following a temperature increase is an example of negative feedback, as clouds reflect sunlight and thus decrease the amount of solar energy reaching Earth's surface.

10 Ice has a high albedo; it is highly reflective. Thus, the more ice there is, the more solar energy is reflected back to space, cooling Earth, and a cooler Earth means a greater ice cover.

3.6 Exam questions

Section A — Multiple choice questions

1 D
The kinetic theory of matter states that all matter is made up of a large number of small particles; it can be used to explain the process of convection and conduction; and it is needed to explain absolute zero.

2 A
The internal energy of a substance is the sum of all energy contained within the matter of that substance, including kinetic and potential energies.

3 D
Temperature is a measure of the average of all the translational kinetic energy in a substance.

4 C
The Kelvin scale is a temperature measure based on absolute zero, when all particle movement stops.

5 B
Heat is best described as the energy transferred as a result of temperature difference.

6 C
For two substances to reach thermal equilibrium they must have the same translational kinetic energy.

7 B
The specific heat capacity is a measure of how much energy is required to raise the temperature of 1 kg of a substance by 1 °C.

8 D
This section is due to the wax changing state from liquid to solid, releasing extra energy without a temperature change.

9 A
The seasons are a cyclical process related to the orbit of Earth around the Sun. They have always existed and are not responsible for influencing climate change.

10 D
Snow is white in colour and reflects a significant amount of radiant energy.

Section B — Short answer questions

11 The energy transferred from the water will be equal to the energy transferred to the hot-water bottle. In each case the energy transferred can be expressed by the formula $Q = mc\Delta T$.
Hot-water bottle: $Q = 0.8 \times 1700 \times \Delta T_1$
Water: $-Q = 1.5 \times 4200 \times \Delta T_2$ (*Note*: The energy transferred is negative in this equation as it is transferred from the water.) [1 mark]
$\Delta T = t_{\text{final}} - t_{\text{initial}}$
Let the final temperature $= t$.
$\Delta T_1 = t - 15$
$\Delta T_2 = t - 80$
Substituting this into the initial equations:
$Q = 0.8 \times 1700(t - 15)$
$-Q = 1.5 \times 4200(t - 80)$ [1 mark]
Solving these simultaneously:
$0.8 \times 1700(t - 15) = -1.5 \times 4200(t - 80)$
$1360(t - 15) = -6300(t - 80)$
$1360t - 20\,400 = -6300t + 504\,000$
$7660t = 524\,400$
$t = 68.46\,°\text{C}$
The temperature at equilibrium is 68 °C. [1 mark]

12 To bring ice from −5 °C to 0 °C:
$Q_1 = mc_{\text{ice}}\Delta T$
$= 2.0 \times 2100 \times 5.0$
$= 21\,000$ J
To melt ice:
$Q_2 = mL_{\text{fusion}}$
$= 2.0 \times 3.3 \times 10^5$
$= 660\,000$ J
To bring water from 0 °C to 100 °C: [1 mark]
$Q_3 = mc_{\text{water}}\Delta T$
$= 2.0 \times 4200 \times 100$
$= 840\,000$ J
To evaporate water:
$Q_4 = mL_{\text{vaporisation}}$
$= 2.0 \times 2.3 \times 10^6$
$= 4.6 \times 10^6$ J [1 mark]
Total energy $= Q_1 + Q_2 + Q_3 + Q_4$
$= 6.1 \times 10^6$ J
$= 6.1$ MJ [1 mark]

13 In an oven at 300 °C, your hand is bombarded by only a limited number of fast-moving air particles, and the energy transferred is not too detrimental. [1 mark] Touching the metal tray at the same temperature brings your hand into contact with more fast-moving particles (in a solid, the molecules are closer together), which transfers more energy to your hand, thus causing injury. [1 mark]

14 **a** Sand heats in one-quarter of the time it takes to heat the same mass of water. Sand cools four times faster than the same mass of water. [1 mark]

b The temperature of dry sand increases rapidly during the day due to its low specific heat capacity. The temperature of the water increases very slowly due to its high specific heat capacity. [1 mark] In addition, there is a much greater mass of water to heat than sand. The temperature of the water on a hot day is lower than body temperature, while the temperature of the sand is greater than body temperature. [1 mark]

c The cooling effect of evaporation from bodies of water, the modifying influence of the sea breezes, and the coolness of the air above the oceans due to the high specific heat capacity of water all contribute to a lower air temperature above beach sand than desert sand. [1 mark] Thus, internal energy is transferred more rapidly from beach sand than it is from desert sand. [1 mark]

15 The temperature of Earth and the atmosphere as a whole is in balance (not allowing for global warming due to the degradation of the ozone layer and the greenhouse effect). [1 mark] The differential warming of the atmosphere and ocean causes heat transfer from the tropics to the poles by convection in the form of winds and ocean currents. [1 mark]

Topic 4 — Radiation from the nucleus and nuclear energy

4.2 Nuclear stability and nuclear radiation

Sample problem 1

Checking the periodic table, the element thorium (symbol Th, atomic number 90) has 90 protons.

90 + 144 = 234

The isotope of an atom with 90 protons and 144 neutrons is thorium-234 or ^{234}Th.

Practice problem 1

From the periodic table, the element with 26 protons is iron (Fe). The mass number is 26 + 30 = 56. The isotope is iron-56 or ^{56}Fe.

Sample problem 2

There is an attractive force called the strong nuclear force that acts between protons and neutrons.

The strong nuclear force can only hold a nucleus together when it is stronger than the electromagnetic force pushing the protons apart.

The force holding protons and neutrons together to form nuclei must be an attractive force that works between neutrons and protons and is stronger than the electromagnetic force at close range.

Practice problem 2

The strong nuclear force acts between the two protons and each proton and the neutron. It is a small nucleus, so the strong nuclear force extends to all nucleons from each of the other nucleons. The electromagnetic force also acts to repel the two protons but not as strongly as the strong nuclear force attracts them.

Sample problem 3

a $\frac{12}{6} = 2$ half-lives

$\frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

One-quarter would be left after 12 hours.

b $\frac{48}{6} = 8$ half-lives

$\left(\frac{1}{2}\right)^8 = \frac{1}{256}$

$\frac{1}{256}$ would be left after 48 hours.

Practice problem 3

$\frac{1}{16}$ will be left after four half-lives.

4 × 5.27 = 21.08 years

The cobalt-60 source will need to be replaced in just over 21 years.

4.2 Exercise

1 Sodium-23. Sodium has an atomic number of 11 and the mass number is 11 + 12 = 23.

2 **a** Hydrogen has one proton. Therefore, hydrogen-2 will have one proton and one neutron.

b Americium has atomic number 95, so it has 95 protons. Therefore, americium-241 will have 95 protons and 241 – 95 = 146 neutrons.

c Europium has atomic number 63, so it has 63 protons. Therefore, europium-164 will have 63 protons and 164 – 63 = 101 neutrons.

3 The force between a proton and neutron is the strong nuclear force, but it only acts if the proton and neutron are very close together (within about 2×10^{-15} m).

4 $\frac{10.3}{2.06} = 5$ half-lives

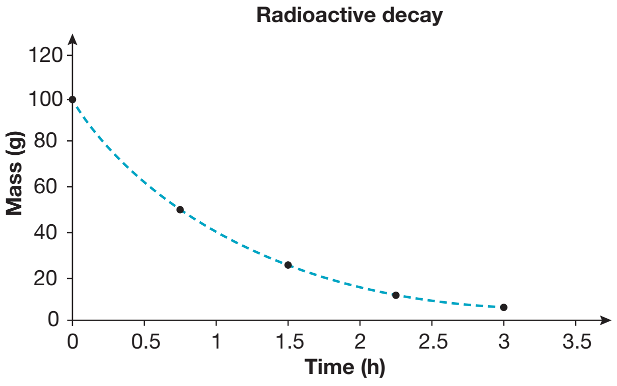
After five half-lives, $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$ of the initial sample of radioisotope remains.

5 A stable isotope is one that never decays. In practical terms, it means that no scientist has observed it to decay despite observing billions of nuclei of that isotope.

6 A nucleus can be unstable because the electromagnetic force becomes greater than the strong nuclear force at only a very small separation distance. Therefore, the sum of all of the repulsive electromagnetic forces can total more than the sum of the strong nuclear forces in large nuclei.

7 $\frac{100}{6.25} = 16 = 2^4$. The sample has halved four times so four half-lives of time have passed. One half-life is therefore $\frac{3}{4} = 0.75$ h $= 45$ minutes.

8



9 Nuclei decay at random. It is impossible to predict when any individual nucleus will decay but if a large sample of nuclei are decaying, it is possible to determine when a proportion (such as a half) of them will decay.

10 After each half-life, half of the sample remaining decays. Only half of the nuclei are left to decay so the number of decays that will happen per second will halve.

11 $\frac{24}{3} = 8$ half-lives, so $\left(\frac{1}{2}\right)^8 = \frac{1}{256}$ of the sample remains after 24 hours.

You would need to order 20 × 256 = 5120 grams or 5.12 kilograms of the isotope to have enough left when you receive it.

4.2 Exam questions

1 C

The atomic number is determined by the number of protons in the nucleus.

2 A

They have the same number of protons, so they are the same element, but they have different numbers of neutrons.

3 B

$$N = N_0\left(\frac{1}{2}\right)^{\frac{t}{6}}$$
$$= 12\,400 \times \left(\frac{1}{2}\right)^{\frac{12}{6}}$$
$$= 12\,400 \times \frac{1}{4}$$
$$= 3100$$

OR

12 days is two half-lives, hence two halvings.

4 $N_{200} = N_{\text{now}}(2)^n$ [1 mark]

$$= 10\,000 \times 2^4 \left(n = \text{number of half-lives} = \frac{200}{50}\right) \text{ [1 mark]}$$
$$= 160\,000 \text{ [1 mark]}$$

5 $\frac{N}{N_0} = \frac{1000}{8000}$

$$= \frac{1}{8}$$
$$= \left(\frac{1}{2}\right)^n \text{ [1 mark]}$$

Hence, number of half-lives $= n$

$= 3$ [1 mark]

Hence, half-life $= \frac{1}{3}$ of 1 day

$$= \frac{24}{3}$$
$$= 8 \text{ hours [1 mark]}$$

4.3 Types of nuclear radiation

Sample problem 4

a 234 – 230 = 4

92 – 90 = 2

α particle

The number of particles in the nucleus has decreased by 4, while the number of protons has decreased by 2. This implies that an α particle, or helium nucleus, has been released.

The full equation is:

$${}^{234}_{92}\text{U} \rightarrow {}^{230}_{90}\text{Th} + {}^{4}_{2}\text{He} + \text{energy}$$

b 210 – 210 = 0

82 – 83 = −1

β^- particle

This equation cannot show an α emission, as the mass number remains constant. The atomic number has increased, indicating that a proton has been formed, and therefore β^- decay has occurred. The equation becomes:

$${}^{210}_{82}\text{Pb} \rightarrow {}^{210}_{83}\text{B} + {}^{0}_{-1}\text{e} + \text{energy}$$

c 11 – 11 = 0

6 – 5 = 1

β^+ particle

The mass number stays the same, but there is one less proton, so it must be β^+ decay. The equation becomes:

$${}^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + {}^{0}_{+1}\text{e} + \text{energy}$$

Practice problem 4

a ${}^{241}_{95}\text{Am} \rightarrow {}^{237}_{93}\text{Np} + {}^{4}_{2}\text{He} + \text{energy}$

b ${}^{197}_{78}\text{Pt} \rightarrow {}^{197}_{79}\text{Au} + {}^{0}_{-1}\text{e} + \overline{v} + \text{energy}$

c ${}^{23}_{12}\text{Mg} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{+1}\text{e} + v + \text{energy}$

Sample problem 5

The longer the half-life, the more stable the nucleus.

Often, in a decay chain, the daughter nucleus has a shorter half-life than the parent. For example, uranium-238 has a much longer half-life than its daughter thorium-234.

In a decay chain, a nucleus may decay to a less stable nucleus. This, however, is a step towards increased stability as the final nucleus is always stable.

Practice problem 5

a Uranium-238 is the most stable nucleus as it has the longest half-life.

b If the decay chain started with an unstable nucleus with a shorter half-life, it would have all decayed by now. As uranium-238 has such a long half-life — one similar to the age of Earth — much of it still remains.

4.3 Exercise

1 ${}^{222}_{86}\text{Rn} \rightarrow {}^{218}_{84}\text{Po} + {}^{4}_{2}\text{He} + \text{energy}$

2 ${}^{214}_{82}\text{Pb} \rightarrow {}^{214}_{83}\text{Bi} + {}^{0}_{-1}\text{e} + \overline{v} + \text{energy}$

3 ${}^{60}_{27}\text{Co}^* \rightarrow {}^{60}_{27}\text{Co} + \gamma$

4 a 20 = 18 + 2, so the atomic number must be 2.

b 50 = 46 + 4, so the atomic mass number must be 4.

c A particle with a mass number of 4 and atomic number of 2 is an alpha particle.

5 Both the neutrino and the gamma ray have a mass number and atomic number of zero.

6 The energy released in nuclear decay is in the form of the kinetic energy of the emitted particles. The alpha, beta and neutrinos all leave the nucleus at great speed.

7 Beta particles have atomic numbers of –1 or +1; however, they are only electrons or positrons. There are no protons. In these cases, the atomic number refers to the charge of the particle. In nuclear reactions the charge must be conserved. If all of the charges on the left-hand side of the equation are added, the answer must equal the sum of the charges on the right-hand side of the equation.

4.3 Exam questions

1 B

$^{236}_{92}N = ^{4}_{2}He + ^{b}_{a}X + \text{energy}$

Hence:

$a = 92 - 2$

$= 90$

$b = 236 - 4$

$= 232$

2 A

Each alpha decay removes four nucleons from the nucleus, hence $A_x = 230 - 3 \times 4 = 218$.

3 Each α decay causes the mass number (A) to decrease by 4 (β decays cause no change in A). [1 mark]
The suggested change in A is $231 - 207 = -24$. [1 mark]
This corresponds to six α decays. [1 mark]
Hence, yes; both nuclei could be members of the same decay series. [1 mark]

4 Three alpha decays cause a change in Z of $3 \times -2 = -6$.
Two beta decays cause a change in Z of $2 \times 1 = +2$
(1 mark for the outcomes of both decays)
Total change in $Z = -4$ [1 mark]
Hence, final $Z = 92 - 4 = 88$ [1 mark]

5 $^{226}_{88}Ra = ^{4}_{2}He + ^{222}_{86}X$

Using the periodic table, X is radon, Rn.
(1 mark for the correct symbol for the alpha particle, 1 mark for $Z = 86$, 1 mark for $A = 222$, 1 mark for identifying radon)

4.4 Radiation and the human body

Sample problem 6

a Absorbed dose $= \dfrac{\text{energy absorbed}}{\text{mass}}$

$= \dfrac{0.054\ \text{J}}{60\ \text{kg}}$

$= 9 \times 10^{-4}\ \text{Gy}$

The absorbed dose is 9×10^{-4} Gy.

b Equivalent dose = absorbed dose × quality factor

$= 9 \times 10^{-4}\ \text{Gy} \times 1$

$= 9 \times 10^{-4}\ \text{Sv}$

$= 0.9\ \text{mSv}$

The equivalent dose if the energy was delivered by γ rays is 0.9 mSv.

c Equivalent dose = absorbed dose × quality factor

$= 9 \times 10^{-4}\ \text{Gy} \times 20$

$= 1.8 \times 10^{-2}\ \text{Sv}$

$= 18\ \text{mSv}$

The equivalent dose if the energy was delivered by α particles is 18 mSv.

d Equivalent dose of γ rays = 0.9 mSv
Equivalent dose of α particles = 18 mSv

$\dfrac{18}{0.9}$

The equivalent dose delivered by the α particles would cause 20 times more damage than that delivered by the γ rays.

Practice problem 6

Energy absorbed = absorbed dose × mass

$= 12 \times 80$ (assuming an astronaut weighs 80 kg)

$= 96\ \text{J}$

Each astronaut absorbed approximately 96 joules of energy.

Sample problem 7

a $0.12 \times 2.5\ \text{mSv} = 0.3\ \text{mSv}$
The effective dose is 0.3 mSv.

b $\dfrac{0.3 \times 365}{1.5} = 73$ days
It would take 73 days for Jo to be exposed to an effective dose of 0.3 mSv from the background radiation.

Practice problem 7

$0.04 \times 10 + 0.01 \times 30 = 0.3 + 0.4$

$= 0.7\ \text{mSv}$

The effective dose is 0.7 mSv.

Sample problem 8

a In one half-life, 10 mg of iodine-123 will decay. This will leave 10 mg of iodine-123. In the second half-life, 5 mg of iodine-123 will decay, leaving 5 mg of iodine-123. In the third half-life, 2.5 mg of iodine-123 will decay.
17.5 mg (10 + 5 + 2.5) of iodine will have decayed in three half-lives, or 39 hours.

b After one half-life, 10 mg of iodine-123 will decay, leaving 10 mg of iodine-123. After two half-lives, 5 mg of iodine-123 will decay, leaving 5 mg of iodine-123.
5 mg of iodine-123 will remain after 26 hours.

Practice problem 8

a $\dfrac{52}{13} = 4$; four half-lives have elapsed
Therefore, 16 mg was the starting amount.

b $\dfrac{104}{13} = 8$
Therefore, after eight half-lives, the amount remaining is 0.0625 mg.

Sample problem 9

a When half the sample has decayed, the activity will also halve. This assumes the atoms formed are not radioactive.
The time needed to reduce the activity to 4 MBq is one half-life, or 10 minutes.

b Halving the activity each half-life means that three half-lives have passed before the activity is 1 MBq.
The time taken is 30 minutes.

Practice problem 9

a Since two half-lives have elapsed, the activity would have been $8 \times 2^2 = 32$ MBq.

b A further 16 days is an extra two half-lives: $8 \times 0.5^2 = 2$ MBq.

c The initial activity when the sample was placed in storage was 32 MBq.

$\frac{32}{2} = 16$ MBq (1 half-life)

$\frac{16}{2} = 8$ MBq (2 half-lives)

$\frac{8}{2} = 4$ MBq (3 half-lives)

$\frac{4}{2} = 2$ MBq (4 half-lives)

$\frac{2}{2} = 1$ MBq (5 half-lives)

It would take five half-lives: $5 \times 8 = 40$ days.

4.4 Exercise

1 $\text{Energy absorbed} = \text{absorbed dose} \times \text{mass}$
$= 3 \times 10^{-3} \times 30$
$= 9 \times 10^{-2}$ J

2 $\text{Absorbed dose} = \frac{\text{energy absorbed}}{\text{mass}}$
$= \frac{9 \times 10^{-2}}{60}$
$= 1.5 \times 10^{-3}$ Gy
$= 1.5$ mGy

3 γ radiation quality factor = 1
Equivalent dose = absorbed dose × quality factor
$= 3 \times 10^{-3} \times 1$
$= 3 \times 10^{-3}$ Sv
$= 3$ mSv

4 Quality factor = 18
Equivalent dose = absorbed dose × quality factor
$= 3.0 \times 10^{-3} \times 18$
$= 54 \times 10^{-3}$ Sv
$= 54$ mSv

5 α particles cause a lot of localised damage. They give much of their energy quickly to nearby atoms (i.e. they are stopped over short distances).

6 Equivalent dose takes into account the different types of damage caused by different forms of ionising radiation.

7 In its early stages a foetus consists of only a few cells, which divide to form the rest of the baby. If these cells are damaged, the foetus may not be able to form properly.

8 If the mass of your body is 60 kilograms and the annual dose of radiation is 2 mSv:
Energy absorbed = absorbed dose × mass
$= 2 \times 10^{-3} \times 60$
$= 0.12$ J
Each β particle has:
$1 \text{ MeV} = 10^6 \times 1.6 \times 10^{-19}$ J
$= 1.6 \times 10^{-13}$ J of energy

$n(\beta \text{ particles}) = \frac{0.12}{1.6 \times 10^{-13}}$
$= 7.5 \times 10^{11}$ each year
$\approx 24\,000$ each second

9 Cancer cells grow more rapidly than normal tissue cells. Cells are more vulnerable to radiation damage when they are dividing. This means that more cancer cells are killed than normal cells.

4.4 Exam questions

1 C
$\text{Absorbed dose} = \frac{\text{energy absorbed}}{\text{mass}}$
$= \frac{0.009}{60}$
$= 0.000\,15$ Gy
Equivalent dose = absorbed dose × quality factor
$= 0.000\,15 \times 20$
$= 0.003$ Sv
$= 3$ mSv

2 $\text{Absorbed dose} = \frac{\text{energy absorbed}}{\text{mass}}$
$= \frac{0.016}{85}$
$= 0.000\,188$ Gy [1 mark]
$\text{Quality factor} = \frac{\text{equivalent dose}}{\text{absorbed dose}}$
$= \frac{0.002\,45}{0.000\,188}$
$= 13$ [1 mark]

3 B
The radioisotope should have a half-life that is between 5 minutes and 1 hour in order for the activity of the radioisotope to be measured. If the half-life is too short it will decay before it can be distributed to the desired organ and measured. If the half-life is too long the radioisotope will keep decaying well after the appointment, potentially exposing the patient to dangerous levels of radiation.

4 $\text{Absorbed dose} = \frac{\text{energy absorbed}}{\text{mass}}$
$1.5 \times 10^{-3} = \frac{\text{energy absorbed}}{1.4}$ [1 mark]
$\Rightarrow \text{Energy absorbed} = 1.5 \times 10^{-3} \times 1.4$
$= 2.1 \times 10^{-3}$ J [1 mark]
The energy absorbed is 2.1×10^{-3} J or 2.1 mJ.

5 **a** $\text{Absorbed dose} = \frac{\text{energy absorbed}}{\text{mass}}$
$= \frac{6.0 \times 10^{-6}}{0.50}$ [1 mark]
$= 1.2 \times 10^{-5}$ Gy [1 mark]
The absorbed dose is 1.2×10^{-5} Gy.

b Dose equivalent = absorbed dose × quality factor
$= 1.2 \times 10^{-5} \text{ Gy} \times 20$ [1 mark]
$= 2.4 \times 10^{-6}$ Sv
$= 2.4\ \mu$Sv [1 mark]
The dose equivalent for α particles is $2.4\ \mu$Sv.

4.5 Energy from mass

Sample problem 10

$m_{\text{parent}} = 3.853\,08 \times 10^{-25}$ kg

$m_{\text{products}} = 3.786\,55 \times 10^{-25}$ kg $+ 6.646\,48 \times 10^{-27}$ kg
$= 3.853\,01 \times 10^{-25}$ kg

$m_{\text{parent}} - m_{\text{products}} = 3.853\,08 \times 10^{-25}$ kg $- 3.853\,01 \times 10^{-25}$ kg
$= 7 \times 10^{-30}$ kg

$$\begin{aligned} E &= mc^2 \\ &= 7 \times 10^{-30} \times \left(3.00 \times 10^8\right)^2 \\ &= 6.3 \times 10^{-13}\ \text{J} \\ &= 3.9 \times 10^6\ \text{eV} \end{aligned}$$

3.9 MeV of energy is released when thorium-232 undergoes alpha decay.

Practice problem 10

The energy in joules is:

$$\begin{aligned} 4.2\ \text{MeV} &= 4.2 \times 10^6\ \text{eV} \\ &= 4.2 \times 10^6 \times 1.602\,176 \times 10^{-19} \\ &= 6.729\,139 \times 10^{-13}\ \text{J} \end{aligned}$$

The mass equivalent of this energy is:

$$\begin{aligned} m &= \frac{E}{c^2} \\ &= \frac{6.729\,139 \times 10^{-13}\ \text{J}}{\left(2.997\,924\,58 \times 10^8\right)^2} \\ &= 7.487\,177 \times 10^{-30}\ \text{kg} \end{aligned}$$

The mass defect of this reaction is $7.487\,177 \times 10^{-30}$ kg.

4.5 Exercise

1 The equation $E = mc^2$ expresses the fact that energy and mass are equivalent. Mass and energy cannot be considered independently.

2 $$\begin{aligned} E &= mc^2 \\ &= 0.1 \times \left(3.00 \times 10^8\right)^2 \\ &= 9 \times 10^{15}\ \text{J} \end{aligned}$$

3 $$\begin{aligned} E &= mc^2 \\ &= 2 \times 9.1 \times 10^{-31} \times \left(3.00 \times 10^8\right)^2 \\ &= 1.638 \times 10^{-13}\ \text{J} \end{aligned}$$

4 $$\begin{aligned} 4500\ \text{kJ} &= 4.5 \times 10^6\ \text{J} \\ &= \frac{\left(4.5 \times 10^6\ \text{J}\right)}{\left(1.6 \times 10^{-19}\ \text{J}\right)} \\ &\approx 2.8 \times 10^{25}\ \text{eV} \end{aligned}$$

5 $$\begin{aligned} E &= mc^2 \\ \Rightarrow m &= \frac{E}{c^2} \\ &= \frac{\left(4.5 \times 10^6\right)}{\left(3.0 \times 10^8\right)^2} \\ &= 5.0 \times 10^{-11}\ \text{kg} \end{aligned}$$

4.5 Exam questions

1 C

$$\begin{aligned} E &= mc^2 \\ &= 5.040 \times 10^{-29} \times \left(3.00 \times 10^8\right)^2 \\ &= 4.536 \times 10^{-12}\ \text{J} \end{aligned}$$

2 C

Using Einstein's mass equivalence formula, $E = mc^2$, we obtain:

$$\begin{aligned} E &= 1.09 \times 10^{-28} \times \left(3 \times 10^8\right)^2 \\ &= 9.81 \times 10^{-12}\ \text{J} \end{aligned}$$

3 B

Using Einstein's mass equivalence formula, $E = mc^2$, we obtain:

$$\begin{aligned} m &= \frac{E}{c^2} = \frac{2.83 \times 10^{-11}}{\left(3 \times 10^8\right)^2} \\ &= 3.14 \times 10^{-28}\ \text{kg} \end{aligned}$$

4 Using Einstein's mass equivalence formula, $E = mc^2$, we obtain:

$$\begin{aligned} m &= \frac{E}{c^2} \\ &= \frac{2.81 \times 10^{-12}}{\left(3 \times 10^8\right)^2}\ \text{[1 mark]} \\ &= 3.12 \times 10^{-29}\ \text{kg} \\ &= 3.1\ \times 10^{-29}\ \text{kg (to 2 s.f.) [1 mark]} \end{aligned}$$

5 Using Einstein's mass equivalence formula, $E = mc^2$, we obtain:

$$\begin{aligned} E &= 2.29 \times 10^{-29} \times \left(3 \times 10^8\right)^2\ \text{[1 mark]} \\ &= 2.06 \times 10^{-12}\ \text{J} \\ &= 2.1\ \times 10^{-12}\ \text{J (to 2 s.f.) [1 mark]} \end{aligned}$$

4.6 Energy from the nucleus

Sample problem 11

a ${}^{236}_{92}\text{U} \rightarrow {}^{148}_{57}\text{La} + {}^{85}_{35}\text{Br} + 3{}^{1}_{0}\text{n} + \text{energy}$

Binding energy of uranium-236 = 1790.415 039 MeV

Binding energy of lanthanum-148 = 1213.125 122 MeV

Binding energy of bromine-85 = 737.290 649 MeV

1213.125 122 + 737.290 649 = 1950.415 771 MeV

Energy difference = 1950.415 771 − 1790.415 039 MeV
= 160.000 732 MeV

The difference between the binding energy of the uranium-236 nucleus and the sum of the binding energies of the two fission fragments is 160.001 MeV.

b Mass of uranium-236 = $3.919\,629 \times 10^{-25}$ kg
Mass of lanthanum-148 = $2.456\,472 \times 10^{-25}$
Mass of bromine-85 = $1.410\,057 \times 10^{-25}$
Mass of a neutron = $1.674\,746 \times 10^{-27}$

$2.456\,472\ \times 10^{-25} + 1.410\,057 \times 10^{-25} + 3 \times 1.674\,746 \times 10^{-27}$
$= 3.916\,771 \times 10^{-25}$ kg

Mass difference $= 3.919\,629 \times 10^{-25} - 3.916\,771 \times 10^{-25}$
$= 2.857\,62 \times 10^{-28}$ kg

The difference between the mass of the uranium-236 nucleus and the sum of the masses of all the fission fragments is $2.857\,62 \times 10^{-28}$ kg.

c $E = mc^2$

$= 2.857\,62 \times 10^{-28} \times \left(2.997\,924\,58 \times 10^{8}\right)^2$

$= 2.568\,301 \times 10^{-11}$ J

$\dfrac{2.568\,301 \times 10^{-11}}{1.602\,176 \times 10^{-19}} = 160.300\,789$ MeV

The energy equivalent of this mass difference is $2.568\,301 \times 10^{-11}$ J or 160.300 789 MeV.

Practice problem 11

a Write out the fission equation:

$^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,^{1}_{0}n + \text{energy}$

Use table 4.8 to find the binding energy of uranium-236:
Binding energy of uranium-236 = 1790.415 039 MeV
Use table 4.8 to calculate the sum of the binding energies of the fragments:
Binding energy of barium-141 = 1173.974 609 MeV
Binding energy of krypton-92 = 783.185 242 MeV
1173.974 609 + 783.185 242 = 1957.159 851 MeV
Calculate the difference between the binding energy of the uranium nucleus and its fission fragments:
Energy difference = 1957.159 851 − 1790.415 039
= 166.744 812 MeV

b Use table 4.8 to find the mass of uranium-236:
Mass of uranium-236 = $3.919\,629 \times 10^{-25}$ kg
Use table 4.8 to find the sum of the masses of the fragments:
Mass of barium-141 = $2.339\,939 \times 10^{-25}$
Mass of krypton-92 = $1.526\,470 \times 10^{-25}$
Mass of a neutron = $1.674\,746 \times 10^{-27}$
$2.339\,939 \times 10^{-25} + 1.526\,470 \times 10^{-25} + 3 \times 1.674\,746 \times 10^{-27} = 3.916\,651 \times 10^{-25}$
Calculate the difference between the mass of the uranium nucleus and its fission fragments:
Mass difference = $3.919\,629 \times 10^{-25} - 3.916\,651 \times 10^{-25}$
$= 2.977\,62 \times 10^{-28}$ kg

c Use $E = mc^2$ to calculate the energy difference in joules:
$E = mc^2$
$= 2.977\,62 \times 10^{-28} \times (2.997\,924\,58 \times 10^{8})^2$
$= 2.676\,151 \times 10^{-11}$ J
Convert the energy to MeV:

$\dfrac{2.676\,151 \times 10^{-11}}{1.602\,176 \times 10^{-19}} = 167.032$ MeV

Sample problem 12

a Binding energy of helium-4 nucleus = 28.295 673 MeV
Binding energy of two helium-3 nuclei = $2 \times 7.864\,501$
= 15.729 002 MeV
Difference = 28.295 673 – 15.729 002
= 12.566 671 MeV
The difference between the binding energy of the helium-4 nucleus and sum of the binding energies of the two helium-3 nuclei is 12.566 671 MeV.

b Mass before fusion = $2 \times 5.006\,942 \times 10^{-27}$
$= 10.013\,88 \times 10^{-27}$ kg
Mass after fusion = $6.645\,758 \times 10^{-27} + \left(2 \times 1.674\,746 \times 10^{-27}\right)$
$= 9.995\,25 \times 10^{-27}$ kg
Mass difference = 1.863×10^{-29} kg
The difference between the sum of masses of the helium-4 nucleus and the two protons, and mass of two helium-3 nuclei is 1.863×10^{-29} kg.

c $E = mc^2$
$= 1.863 \times 10^{-29} \times (2.99\,792\,458 \times 10^{8})^2$
$= 1.674\,381 \times 10^{-12}$ J

$\dfrac{1.674\,381 \times 10^{-12}}{1.602\,176 \times 10^{-19}} = 10.45$ MeV

The energy equivalent of this mass difference is 1.674×10^{-12} J or 10.45 MeV.

Practice problem 12

a Use the table to find the binding energy of the helium-3 nucleus:
Binding energy of helium-3 nucleus = 7.864 501 MeV
Use the table to find the binding energy of the hydrogen-1 and hydrogen-2 nuclei:
Binding energy of hydrogen-1 nucleus (proton) = 0
Binding energy of hydrogen-2 nucleus = 2.224 573 MeV
Calculate the difference in binding energies:
Difference = 7.864 501 − 2.224 573
= 5.639 928 MeV

b Use the table to find the mass of the reactants:
Mass before fusion = $1.673\,351 \times 10^{-27} + 3.344\,132 \times 10^{-27}$
$= 5.017\,483 \times 10^{-27}$ kg
Use the table to find the mass of the products:
Mass after fusion = $5.006\,942 \times 10^{-27}$ kg
Calculate the difference in mass:
Mass difference = $5.017\,483 \times 10^{-27} - 5.006\,942 \times 10^{-27}$
$= 1.0541 \times 10^{-29}$ kg

c Use $E = mc^2$ to calculate the energy difference in joules:
$E = mc^2$
$= 1.0541 \times 10^{-29} \times (2.997\,924\,58 \times 10^{8})^2$
$= 9.473\,778 \times 10^{-13}$ J
Convert the energy to MeV:

$\dfrac{9.473\,778 \times 10^{-13}}{1.602\,176 \times 10^{-19}} = 5.913\,070$ MeV

4.6 Exercise

1 Fission is the name given to the process in which a very large nucleus splits into two smaller nuclei. Fusion is the process of two small nuclei combining to form a single nucleus.

2 The Sun is mostly made of small nuclei such as hydrogen (single protons) that, under the immense temperatures and pressures in the interior of the Sun, are brought close together enough to form a new larger nucleus, mostly helium-4. This is an example of fusion.

3 Energy is released in a fission or fusion reaction if the binding energy per nucleon (the energy required to remove a nucleon from the nucleus) increases in the process. When uranium-235 nuclei are split into two roughly equal-sized nuclei, the binding energy per nucleon for each of these new nuclei is greater than that in uranium-235. When fusion of two hydrogen atoms occurs, the binding energy per nucleon also increases. The nucleus with the highest binding energy per nucleon is nickel-62. As long as the products of fission or fusion have a mass number between those of the original nuclei and nickel-62, energy will be released in the process.

4 According to the graph, each nucleon in uranium-235 has 7.6 MeV. This is a total of 235 × 7.6 = 1786 MeV. Each nucleon in barium-141 has 8.4 MeV according to the graph. This is a total of 141 × 8.4 = 1184.4 MeV. Each nucleon in krypton-92 has 8.7 MeV, according to the graph. This is a total of 92 × 8.7 = 800.4 MeV. The total energy of the fission products is 800.4 + 1184.4 = 1984.8 MeV, which is 199 MeV greater than the uranium-235 nucleus. This agrees well with the measured value of 200 MeV.

5 When two small nuclei are fused together to form a larger one, energy is released because the new nucleus exists in a lower energy state than the two nuclei that formed it. Energy is conserved, so the difference in energy needs to be released in some form.

6 For light elements (mass number < 56) the graph of binding energy per nucleon is much steeper than for heavy elements. Therefore, fusion of light nucleon will usually result in more energy being released because the binding energy per nucleon changes by more with fusion than for fission.

7 Fusion occurs when the combining of two light nuclei results in an increase in binding energy per nucleon. This means that energy has been released to bind the nuclei more tightly together. The release of energy is equivalent to a release of mass, so the nucleus after fusion is lighter than the two nuclei that it was fused from.

8 The fission of uranium-236 results in two nuclei with higher binding energy per nucleon. This involves energy release, so the sum of the masses of all of the particles after the fission is less than the mass of the nucleus that underwent fission.

4.6 Exam questions

1 A
The larger the binding energy of a nucleus, the more energy is required to separate the nucleons.

2 C
A higher binding energy means that the system has moved to a lower-energy, more tightly bound, more stable state. The excess energy is released.

3 B
A fusion reaction involves the combining of two nuclei to form a larger, more stable nucleus.

4 Total b.e. (initial) $= {}^{2}_{1}\text{H} + {}^{6}_{3}\text{Li}$

$= 2.2246 + 31.9946$

$= 34.2192$ MeV [1 mark]

Total b.e. (final) $= {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He}$

$= 28.2957 + 28.2957$

$= 56.5914$ MeV [1 mark]

Energy released = gain in b.e.

$= 56.5914 - 34.2192$

$= 22.3722$ MeV [1 mark]

5 Total mass final $= 1.443\,318\,4 + 2.456\,414\,0$

$= 3.899\,732\,4 \times 10^{-25}$ kg [1 mark]

Total mass initial $= 3.902\,904\,4 \times 10^{-25}$ kg

Mass decrease $= 3.902\,904\,4 - 3.899\,732\,4$

$= 0.003\,172 \times 10^{-25}$ kg [1 mark]

Energy released $= \Delta mc^2$

$= 0.003\,172 \times 10^{-25} \times (2.997\,92 \times 10^{8})^2$

$= 2.850\,84 \times 10^{-11}$ J [1 mark]

4.7 Fission chain reactions

Sample problem 13

The element with the mass number of 137 is barium, which has a relative atomic mass of 137.3.

Practice problem 13

a From the graph, the mass number of the most common fragment of uranium-236 fission is 134.

b The element that has a relative atomic mass closest to 134 is caesium. Therefore, it is most likely caesium.

c Uranium has 92 protons. The known fragment caesium has 55 protons. So, the unknown fragment must have 92 – 55 = 37 protons. Therefore, it is rubidium.

4.7 Exercise

1 A flat mass has a much lower volume-to-surface-area ratio than a sphere. That is, for the same volume and mass as a sphere, more of the flat sheet is exposed to the air. This allows many more free neutrons to escape rather than sustain a chain reaction.

2 Kinetic energy of free neutrons and product nuclei, and some radiation

3 Neutrons are good at initiating nuclear reactions because they have no charge and can easily enter a nucleus.

4 A chain reaction occurs when more than one of the free neutrons produced in a fission reaction triggers another fission reaction.

5 Fusion has fewer waste problems and the potential to use widely available sources of fuel, such as seawater. The disadvantage is that, after about 60 years of research, a sustainable fusion reaction has still not been achieved.

4.7 Exam questions

1 B
The total number of nucleons must be in balance (this is the mass number, written to the top left of the isotope name).
On the left-hand side there are 235 + 1 = 236 nucleons.
On the right-hand side there are 140 + 94 + X = 234 + X nucleons.
Therefore, $236 = 234 + X$
$\Rightarrow X = 2$

2 B
The moderator is made from a material that slows down neutrons. As the neutrons released from the fission of uranium-235 interact with the moderator they are slowed down, and are then able to initiate fission in other uranium-235 nuclei.

3 C
On the right-hand side of the equation, the total number of protons is 42 + 52 = 94; therefore the element must be plutonium.

The balance of nucleons is $Z + 1 = 100 + 134 + 6$.
Therefore the number of nucleons must be 239.
So the isotope used was plutonium-239.

4 In a fission chain reaction the neutrons emitted from one fission reaction go on to initiate fission in surrounding atoms. [1 mark]

5 Control rods are made of materials, such as cadmium or boron, that absorb neutrons. [1 mark] Movement of the control rods into the reactor interrupts the chain of fission reactions, allowing the rate of reaction to be controlled. [1 mark]

4.8 Review

4.8 Review questions

1 The atomic number is equal to the number of protons in the nucleus and is in front of the element symbol at the bottom. The mass number is equal to the total number of protons and neutrons in the nucleus and is in front of the element symbol at the top.

a Number of protons = 30
Number of neutrons = 66 – 30 = 36

b Number of protons = 90
Number of neutrons = 230 – 90 = 140

c Number of protons = 20
Number of neutrons = 45 – 20 = 25

d Number of protons = 14
Number of neutrons = 31 – 14 = 17

2 The symbol for isotopes includes the symbol of the element with the mass number in front at the top and the atomic number in front at the bottom.

a Two protons means an atomic number of 2. From the periodic table, the element with atomic number 2 is helium, which has the symbol He. 2 neutrons and 2 protons add to a mass number of 4. The symbol is ${}^{4}_{2}\text{He}$.

b Seven protons means an atomic number of 7, which is the element nitrogen in the periodic table, with the symbol N. 13 nucleons means a mass number of 13. The symbol is ${}^{13}_{7}\text{N}$.

c 91 protons means an atomic number of 91. The element with atomic number 91 in the periodic table is protactinium, which has the symbol Pa. 143 neutrons and 91 protons add to a mass number of 234. The symbol is ${}^{234}_{91}\text{Pa}$.

3 a The mass number does not change but the atomic number increases by 1. This means that X must be a β^- particle (mass number of 0 and atomic number of –1).

b The mass number does not change but the atomic number increases by 1. This means that X must be a β^- particle (mass number of 0 and atomic number of –1).

c The mass number decreases by 4 and the atomic number decreases by 2. This means that X must be an α particle (mass number of 4 and atomic number of 2).

4 a i ${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\alpha + \text{energy}$

ii ${}^{214}_{84}\text{Po} \rightarrow {}^{210}_{82}\text{Pb} + {}^{4}_{2}\alpha + \text{energy}$

iii ${}^{241}_{95}\text{Am} \rightarrow {}^{237}_{93}\text{Np} + {}^{4}_{2}\alpha + \text{energy}$

b i ${}^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni} + {}^{0}_{-1}\beta + \overline{\nu} + \text{energy}$

ii ${}^{90}_{38}\text{Sr} \rightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}\beta + \overline{\nu} + \text{energy}$

iii ${}^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + {}^{0}_{-1}\beta + \overline{\nu} + \text{energy}$

5 The half-life is the time taken for half of the sample of radioisotope to decay. Reading from the graph, one half of the sample has decayed after approximately two hours.

6 One-eighth would remain after three half-lives. This length of time is $3 \times 5730 = 17\,190$ years.

7 The absorbed dose is the energy absorbed by each kilogram of the tissue being irradiated.

8 Extremely high temperatures and pressures — such as exist in the centre of the Sun. These conditions are required to bring the protons close enough together for the strong nuclear force to exceed the electromagnetic repulsion.

9 The critical mass of a fissionable substance is the smallest mass that will sustain an uncontrolled chain reaction.

10 a Binding energy of reactants = 1467.164 791 MeV

$$\text{Binding energy of products} = 890.065\,063 + 644.312\,162 = 1534.377\,225\ \text{MeV}$$

$$\text{Difference} = 1534.377\,225 - 1467.164\,791 = 67.212\,434\ \text{MeV}$$

b
$$\text{Mass of reactants} = 3.610\,422 \times 10^{-25} + 1.956\,117 \times 10^{-27} = 3.629\,983 \times 10^{-25}\ \text{kg}$$

$$\text{Mass of products} = 2.307\,189 \times 10^{-25} + 1.321\,596 \times 10^{-25} = 3.628\,785 \times 10^{-25}\ \text{kg}$$

$$\text{Mass difference} = 3.629\,983 \times 10^{-25} - 3.628\,785 \times 10^{-25} = 1.198 \times 10^{-28}\ \text{kg}$$

c
$$E = mc^2 = 1.198 \times 10^{-28} \times (2.997\,924\,58 \times 10^{8})^2 = 1.076\,709 \times 10^{-11}\ \text{J}$$

$$\frac{1.076\,709 \times 10^{-11}}{1.602\,176 \times 10^{-19}} = 67.202\,898\ \text{MeV}$$

4.8 Exam questions

Section A — Multiple choice questions

1 C
The correct order is strong nuclear force, electromagnetic force, weak nuclear force, gravity.

2 B
Radioisotopes decay randomly. When a large sample of nuclei of a radioisotope decay, the time for half of them to decay is called the half-life.

3 A
Two α decays would drop the mass number by eight and the atomic number by four. Two β^- decays would not change the mass number but would increase the atomic number by two, resulting in a combined effect of a drop of eight in mass number and two in atomic number.

4 D
Gamma rays have the strongest penetrating ability; they can penetrate aluminium and a few centimetres of lead.

5 B
24 days is three half-lives, therefore:

$$\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times 100 = 12.5\%$$

6 D
The Sun is mostly protons and these are most likely to fuse. They are the main source of the Sun's energy.

7 C
For fusion to occur, positively charged nuclei need to get extremely close to each other. This is only possible at extremely high temperatures where the kinetic energy of the nuclei is large enough to overcome the electric repulsion of the positive charges.

8 C
In all chemical reactions the number of protons and neutrons must be conserved. In option A the 92 protons in the reactants turns into 82 protons in the products. In option B there are 102 protons in the products and in option D there are a total of 239 nucleons in the products compared to the 236 in the reactants. Option C conserves the number of protons and neutrons, so is a possible fission reaction of uranium 235 and a neutron.

9 D
The fission equation for uranium-236 that produces a fragment with an atomic mass number around 96 is:
$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U \rightarrow ^{96}_{37}Rb + ^{137}_{55}Cs + 3^{1}_{0}n$
Therefore, the likely species is rubidium, Rb-96.

10 B
A moderator is used to slow neutrons down to speeds at which they will initiate fission.

Section B — Short answer questions

11 a The binding energy of plutonium-240 is 1813.454 956 MeV.
The binding energy of strontium-90 is 782.631 470 MeV and of barium-147 is 1204.158 203 MeV.
The sum of the binding energies of the fission fragments is:
782.631 470 + 1204.158 203 = 1986.789 673 MeV [1 mark]
The difference between the binding energies of plutonium-240 and the two fission fragments is:
1986.789 673 – 1813.454 956 = 173.334 717 MeV [1 mark]

b The mass of plutonium-240 is $3.985\ 755 \times 10^{-25}$ kg.
The masses of the fission products are strontium-90: $1.492\ 791 \times 10^{-25}$ kg; barium-147: $2.439\ 632 \times 10^{-25}$ kg; 3 neutrons: $3 \times 1.674\ 746 \times 10^{-27}$ kg.
The sum of the masses of the fission products is:
$1.492\ 791 \times 10^{-25} + 2.439\ 632 \times 10^{-25} + 3 \times 1.674\ 746 \times 10^{-27} = 3.982\ 665 \times 10^{-26}$ kg [1 mark]
The difference between the masses of plutonium-240 and all of the fission fragments is:
$3.985\ 755 \times 10^{-25} - 3.982\ 665 \times 10^{-25}$
$= 3.089\ 62 \times 10^{-28}$ kg [1 mark]

c $E = mc^2$
$= 3.089\ 62 \times 10^{-28} \times (2.997\ 924\ 58 \times 10^{8})^2$
$= 2.776\ 812 \times 10^{-11}$ J [1 mark]
$\dfrac{2.776\ 812 \times 10^{-11}}{1.602\ 176 \times 10^{-19}} = 173\ 315\ 040$ eV
$= 173.315\ 040$ MeV [1 mark]

12 a Absorbed dose $= \dfrac{\text{energy absorbed}}{\text{mass}}$
$= \dfrac{0.068}{75}$
$= 9.07 \times 10^{-4}$ Gy [1 mark]

b Equivalent dose = absorbed dose × quality factor
$= 9.07 \times 10^{-4} \times 1$
$= 9.07 \times 10^{-4}$ Sv
$= 0.907$ mSv [1 mark]

c Equivalent dose = absorbed dose × quality factor
$= 9.07 \times 10^{-4} \times 20$
$= 1.8 \times 10^{-2}$ Sv
$= 18$ mSv [1 mark]

d A dose of 18 mSv is approximately 20 times greater than that of an equivalent dose of 0.907 mSv, therefore it would cause approximately 20 times the damage. [1 mark]

13 a One half-life $= \dfrac{35}{2} = 17.5$ mg has decayed [1 mark]
Second half-life $= \dfrac{17.5}{2} = 8.75$ mg has decayed [1 mark]
Total decayed = 17.5 + 8.75
= 26.25 mg [1 mark]
It will take 2 half-lives or 12 hours.

b After 6 hours, one half-life $= \dfrac{35}{2} = 17.5$ mg [1 mark]
After 12 hours, two half-lives $= \dfrac{17.5}{2} = 8.75$ mg [1 mark]
After 18 hours, three half-lives $= \dfrac{8.75}{2}$
$= 4.375$ mg [1 mark]

14 a Adding a neutron to a stable nucleus make it more likely to undergo β^- decay. [1 mark]

b Nuclei stability follows a curve of neutron number versus proton number. Adding a neutron takes the nucleus away from that stability curve. [1 mark] Of the decay options α, β^+ and β^-, and γ, only β^- decay moves the nucleus back towards the line of stability. [1 mark] α decay drops the proton and neutron number by the same amount, so would keep the nucleus on the same side of the curve, and β^+ would move the nucleus further away from the curve. γ decay would have no impact on where the nucleus is in relation to the curve.

c Alpha emission relies on the bonding between a group of nucleons being stronger than the bonding of that group to the entire nucleus. [1 mark] As the strong nuclear force is stronger than the electromagnetic force for adjacent nucleons, this will only happen when the combined electromagnetic repulsion from a large nucleus affects protons that are less attached by the strong nuclear force. [1 mark] The strong nuclear force has a very short range so does not hold on to nucleons that are not adjacent. [1 mark] However, the strong nuclear force is very strong for an α particle, so the four particles tend to leave as one.

15 a Binding energy of uranium-236 = 1790.415 042 MeV [1 mark]
Binding energy of fragments = 1139.760 337 + 837.065 893
= 1976.826 230 MeV
Energy difference = 1976.826 230 − 1790.415 042
= 186.411 188 MeV [1 mark]

b Mass of uranium-236 = $3.919\,629 \times 10^{-25}$ kg

Mass of fragments $= 2.273\,507 \times 10^{-25} + 1.609\,304 \times 10^{-25} + 2 \times 1.674\,746 \times 10^{-27}$

$= 3.916\,306 \times 10^{-25}$ kg [1 mark]

Mass difference $= 3.919\,629 \times 10^{-25} - 3.916\,306 \times 10^{-25}$

$= 3.323 \times 10^{-28}$ kg [1 mark]

c $E = mc^2$

$= 3.323 \times 10^{-28} \times (2.997\,924\,58 \times 10^{8})^2$

$= 2.986\,563 \times 10^{-11}$ J [1 mark]

$\dfrac{2.986\,563 \times 10^{-11}}{1.602\,176 \times 10^{-19}} = 186.406\,674$ MeV [1 mark]

d Energy released per nucleon $= \dfrac{186.406\,674}{236}$

$= 0.789\,859$ MeV [1 mark]

e $m = \dfrac{E}{c^2}$

$= \dfrac{2.986\,563 \times 10^{-12}}{(2.997\,924\,58 \times 10^{8})^2}$

$= 3.322\,999 \times 10^{-29}$ kg [1 mark]

Percentage of mass transformed into energy $= \dfrac{3.322\,999 \times 10^{-29}}{3.919\,629 \times 10^{-25}} \times 100$

$= 0.008\,484\%$ [1 mark]

Topic 5 — Concepts used to model electricity

5.2 Static and current electricity

5.2 Exercise

1 Rubbing the plastic pen with wool left the pen with a net negative charge and the wool a net positive charge. When the charged plastic pen is brought close to the neutral paper the charges in the paper rearrange themselves and the positively charged paper is attracted to the negatively charged pen.

2 Rubbing the balloon on your hair has transferred electrons to the balloon, giving it a net negative charge. Your hair now has a net positive charge, which is why it is attracted to the negatively charged balloon.

3 After rubbing the balloons on your hair each balloon will have a net negative charge, which means they will repel each other when brought close together.

4 When walking across the carpet your socks rub electrons off the carpet, leaving you with a net negative charge. You get an electric shock when electrons jump from you to the metal doorknob.

5 An insulator is a substance that contains no charge carriers, making it resistant to current flow.

5.2 Exam questions

1 D
Electrons best describes the charge carriers when metals conduct electricity.

2 D
A positively charged object and negatively charged object are attracted to each other. Therefore, one object must be positively charged (containing more protons than electrons) and one object most be negatively charged (containing more electrons than protons). Option D is the most correct response. Note that while option A also suggests an unequal distribution of charges, it is incorrect as objects contain both protons and electrons, not one or the other.

3 The wires are designed to provide a pathway for the current to flow from component to component with as little loss as possible. [1 mark] Electrons in metals are delocalised, allowing them to flow freely and carry electrical current with little resistance. [1 mark]

4 Rubbing the balloon on her hair caused it to become negatively charged. [1 mark] When she brought it close to the can, the charges in the can rearranged themselves. The electrons in the can moved away from the balloon, making the side closest to the balloon positively charged and causing the can to roll towards the balloon. [1 mark]

5 **a** A conductor is a material that contains charge carriers; that is, charged particles can move and travel freely through the material. [1 mark] Examples include metals and salt solutions. [1 mark]

b Conductors always allow for charged particles to pass through, whereas semiconductors allow these particles to move under certain conditions. [1 mark]

5.3 Electric charge and current

Sample problem 1

Charge on one electron $= -1.6 \times 10^{-19}$ C

$$\frac{-6.4}{-1.6 \times 10^{-19}} = 4 \times 10^{19}$$

4×10^{19} electrons

Practice problem 1

Charge on one electron $= -1.6 \times 10^{-19}$ C

$$\frac{1.2 \times 10^{-18}}{1.6 \times 10^{-19}} = 7.5$$

7.5 is not an integer. The answer is not a whole number of electrons so the charge cannot exist.

Sample problem 2

$Q = 10$ C, $t = 5.0$ s

$$\begin{aligned} I &= \frac{Q}{t} \\ &= \frac{10\text{ C}}{5.0\text{ s}} \\ &= 2.0\text{ C s}^{-1} \\ &= 2.0\text{ A} \end{aligned}$$

The current in the conductor is 2.0 A.

Practice problem 2

$Q = 15$ C, $t = 3.0$ s

$$\begin{aligned} I &= \frac{Q}{t} \\ &= \frac{15\text{ C}}{3.0\text{ s}} \\ &= 5.0\text{ C s}^{-1} \\ &= 5.0\text{ A} \end{aligned}$$

The current in the conductor is 5.0 A.

Sample problem 3

$$I = 3.0\text{ A},\ t = 5 \times 60 + 20 = 320\text{ s}$$

$$I = \frac{Q}{t}$$

$$\begin{aligned} \Rightarrow Q &= It \\ &= 3\text{ A} \times 320\text{ s} \\ &= 960\text{ C} \\ &= 9.6 \times 10^2\text{ C} \end{aligned}$$

9.6×10^2 C charge passes through the load.

(*Note:* As the solution is quite small in magnitude, leaving the solution as 960 C instead of in scientific notation is acceptable.)

Practice problem 3

$I = 2.5\text{ A},\ Q = 7.5\text{ C}$

$$I = \frac{Q}{t}$$

$$\Rightarrow t = \frac{Q}{I} = \frac{7.5\text{ C}}{2.5\text{ A}} = 3.0\text{ s}$$

The current must flow for 3.0 seconds.

Sample problem 4

Right is the positive direction.

+ 5 C and –4 C

+ 5 – 4 = + 1 C

The overall flow of charge is 1 C to the right.

Practice problem 4

Upwards is the positive direction.

The charges are: –7 C and –(–3) = + 3 C.

–7 + 3 = –4 C

There is 4 C of charge moving downwards.

Sample problem 5

To convert mA to A, divide by 1000.

$$\frac{450\text{ mA}}{1000} = 0.450\text{ A}$$

450 mA is equal to 0.450 A.

Practice problem 5

To convert mA to A, divide by 1000.

$$\frac{280\text{ mA}}{1000} = 0.280\text{ A}$$

280 mA is equal to 0.280 A.

Sample problem 6

$$v = \frac{d}{t}$$

$$\Rightarrow t = \frac{d}{v}$$

$v = 1.0 \times 10^{-4}\text{ m s}^{-1},\ d = 2.5\text{ m}$

$$t = \frac{2.5\text{ m}}{1.0 \times 10^{-4}\text{ m s}^{-1}} = 2.5 \times 10^{4}\text{ s} = 25\,000\text{ s}$$

It would take 25 000 seconds, which is more than 7 hours.

Practice problem 6

$$v = \frac{d}{t}$$

$$\Rightarrow t = \frac{d}{v}$$

$v = 7.4 \times 10^{-5}\text{ m s}^{-1},\ d = 1.2\text{ m}$

$$t = \frac{1.2\text{ m}}{7.4 \times 10^{-5}\text{ m s}^{-1}} = 1.6 \times 10^{4}\text{ s} = 16\,000\text{ s}$$

It will take 16 000 seconds for the electron to travel to a headset from a console.

5.3 Exercise

1 Conventional current is the direction that positive charges would flow if they were free to do so. Electron current is the direction that electrons flow, from the negative terminal of a cell.

2 Direct current: The charge carriers move in only one direction.
Alternating current: The charge carriers periodically change direction (backwards and forwards).

3 $I = 2.5\text{ A},\ t = 15\text{ s}$

$$Q = It = 2.5\text{ A} \times 15\text{ s} = 37.5\text{ C}$$

4 To convert mA to A, divide by 1000.

$$\frac{45\text{ mA}}{1000} = 0.045\text{ A}$$

45 mA is equal to 0.045 A.

5 To convert A to mA, multiply by 1000 (or 10^3).

$$2.3 \times 10^{-4}\text{ A} \times 10^{3} = 2.3 \times 10^{-1}\text{ mA} = 0.23\text{ mA}$$

2.3×10^{-4} A is equal to 0.23 mA.

6 No. Electrons cannot be destroyed. They transform energy as they pass through the filament of the globe. Current electricity is the movement of charged particles along a path. Current is *not* used up in a light globe. The current flowing into a globe is equal to the current flowing out of it.

7 $I = 3.5\text{ A},\ t = 20\text{ min} = 1200\text{ s}$

$$I = \frac{Q}{t}$$

$$\Rightarrow Q = It = 3.5 \times 1200 = 4200 = 4.2 \times 10^{3}\text{ C}$$

8 The question states that the drift velocity is proportional to the current. So halving the current will also halve the drift velocity. Therefore, $v = 8.0 \times 10^{-5}\text{ m s}^{-1}$.

5.3 Exam questions

1 D

Switches allow the flow of current through the circuit to be controlled manually. They allow two possibilities: no current flow or current flow.

2 A

$t = \frac{Q}{I}$

$= \frac{2}{0.8}$

$= 2.5$ seconds

(*Note*: Time should be measured in seconds when performing calculations.)

3 Given that $1\ \mu\text{A} = 1 \times 10^{-6}$ A

$450\ \mu\text{A} = 450 \times (1 \times 10^{-6}\ \text{A})$

$= 450 \times 10^{-6}$ A

$= 4.5 \times 10^{-4}$ A [1 mark]

4 $Q = 15\,\text{C},\ t = 50\,\text{s}$

$I = \frac{Q}{t}$

$= \frac{15\,\text{C}}{50\,\text{s}}$

$= 0.30$ A [1 mark]

5 The flow of negative charge from right to left is equivalent to a flow of + 2.0 C from left to right. [1 mark]

Conventional current = 4 + 2 = 6 C per s

= 6.0 A [1 mark]

5.4 Electric potential difference

Sample problem 7

a $V = 1.5\,\text{V},\ Q = 0.50\,\text{C}$

$E = VQ$

$= 1.5 \times 0.50$

$= 0.75$ J

The battery would give 0.75 joules of energy to 0.50 coulombs of charge.

b $V = 1.5\,\text{V},\ Q = 1.6 \times 10^{-19}\,\text{C}$

$E = VQ$

$= 1.5 \times 1.6 \times 10^{-19}$

$= 2.4 \times 10^{-19}$ J

Each electron would have 2.4×10^{-19} joules of energy.

Practice problem 7

$V = 3.7\,\text{V},\ Q = 4000\,\text{C}$

$E = VQ$

$= 4.7 \times 4000$

$= 18\,800$ J

The battery originally held 18 800 joules or 18.8 kJ.

5.4 Exercise

1 $E = 1.05\,\text{J},\ Q = 0.70\,\text{C}$

$E = VQ$

$\Rightarrow V = \frac{E}{Q}$

$= \frac{1.05\,\text{J}}{0.70\,\text{C}}$

$= 1.5$ V

2 a $V = \frac{E}{Q}$

$= \frac{32\,\text{J}}{9.6\,\text{C}}$

$= 3.3$ V

b $V = \frac{E}{Q}$

$= \frac{4\,\text{J}}{670\,\text{mC}}$

$= \frac{4\,\text{J}}{0.67\,\text{C}}$

$= 6.0$ V

c $E = VQ$

$= 9.0 \times 3.5$

$= 31.5$ J

d $E = VQ$

$= 12\,\text{V} \times 85\,\text{mC}$

$= 12\,\text{V} \times 0.085\,\text{C}$

$= 1.02$ J

e $Q = \frac{E}{V}$

$= \frac{12\,\text{J}}{4.5\,\text{V}}$

$= 2.7$ C

f $Q = \frac{E}{V}$

$= \frac{7.5\,\text{kJ}}{240\,\text{V}}$

$= \frac{7500\,\text{J}}{240\,\text{V}}$

$= 31$ C

3 $E = QV$

$\Rightarrow Q = \frac{E}{V}$

$= \frac{3.6 \times 10^{-4}\,\text{J}}{6.0\,\text{V}}$

$= 6.0 \times 10^{-5}$ C

$= 60\ \mu$C

4

a 7.3 mV	0.0073 V
b 2300 V	2.3 kV
c 0.000 42 V	0.42 mV
d 932 kV	932 000 V
e 2.7 μV	0.000 002 7 V

5 A torch does not need as much energy to produce light as is needed to start a car or to run the multiple lights, sound systems and digital displays in a car. As a torch needs less energy, it can work when connected to a battery that supplies less energy to each charge. A car needs a battery that supplies more energy to each charge.

6 $E = 9.0\,\text{J},\ Q = 6.0\,\text{C}$

$$V = \frac{E}{Q} = \frac{9.0\,\text{J}}{6.0\,\text{C}} = 1.5\,\text{V}$$

5.4 Exam questions

1 The total energy is the sum of the electrical energy and light energy. [1 mark]

Total electrical energy = 40 + 8 = 48 J

$$V = \frac{E}{Q} = \frac{48}{4} = 12\,\text{V}$$ [1 mark]

2 $E = QV$

$$= 5.7\,\mu\text{C} \times 6.0\,\text{V} = 5.7 \times 10^{-6}\,\text{C} \times 6.0\,\text{V} = 34 \times 10^{-6}\,\text{J} = 3.4 \times 10^{-5}\,\text{J}$$ [1 mark]

3 D

$E = VQ$

Option A:
$1.5 \times 2.7 = 4.05\,\text{J}$

Option B:
$1.7 \times 2.5 = 4.25\,\text{J}$

Option C:
$2.0 \times 2.2 = 4.4\,\text{J}$

Option D:
$2.4 \times 1.9 = 4.56\,\text{J}$

Therefore, the battery in option D supplies more energy to an appliance.

4 A 1.5-V battery provides each charge with less energy than a 9-V battery. The 1.5-V battery would need to run for six times as long as the 9-V battery to provide the same energy to an appliance. [1 mark]

5 emf = the energy supplied to each coulomb of charge passing through the battery [1 mark]

Hence, emf = pd across the device

$$= \frac{E}{Q} = \frac{540}{60} = 9.0\,\text{V}$$ [1 mark]

5.5 Electrical energy and power

Sample problem 8

$E = 3.6 \times 10^4\,\text{J},\ I = 5.0\,\text{A},\ t = 30\,\text{s}$

$E = VIt$

$$\Rightarrow V = \frac{E}{It} = \frac{3.6 \times 10^4\,\text{J}}{5.0\,\text{A} \times 30\,\text{s}} = \frac{36\,000}{150} = 240\,\text{V}$$

The potential difference is 240 V.

Practice problem 8

$E = 1.44 \times 10^3\,\text{J},\ I = 2.0\,\text{A},\ t = 1\ \text{min} = 60\,\text{s}$

$E = VIt$

$$\Rightarrow V = \frac{E}{It} = \frac{1.44 \times 10^3\,\text{J}}{2.0\,\text{A} \times 60\,\text{s}} = \frac{1440}{120} = 12\,\text{V}$$

Sample problem 9

$V = 240\,\text{V},\ I = 5\,\text{A}$

$$P = VI = 240\,\text{V} \times 5\,\text{A} = 1200\,\text{W} = 1.2\,\text{kW}$$

The power rating is 1.2 kW.

Practice problem 9

Convert known values to correct units:

$V = 4 \times 1.5\,\text{V} = 6.0\,\text{V},\ I = 100\,\text{mA} = 100 \times 10^{-3}\,\text{A}$

$$P = VI = 6.0\,\text{V} \times 100 \times 10^{-3}\,\text{A} = 0.60\,\text{W}$$

Sample problem 10

$E = VIt$

$V = 3.7\,\text{V},\ I = 1200\,\text{mA},\ t = 1\ \text{hour}$

$$I = 1200\,\text{mA} = \frac{1200}{1000}\,\text{A} = 1.2\,\text{A}$$

$t = 60 \times 60 = 3600\,\text{s}$

$$E = 3.7\,\text{V} \times 1.2\,\text{A} \times 3600\,\text{s} = 16\,000\,\text{J} = 16\,\text{kJ}$$

There are 16 kJ of energy supplied.

Practice problem 10

$$I = \frac{Q}{t}$$

$V = 3.7\,\text{V},\ E = 14\,000\,\text{J},\ t = 6 \times 60 \times 60 = 21\,600\,\text{s}$

$E = VQ$

$$\Rightarrow Q = \frac{E}{V} = \frac{14\,000\,\text{J}}{3.7\,\text{V}}$$

$$I = \frac{\frac{14\,000}{3.7}}{21\,600} = 175\,175\,175\,2\,\text{A} = 0.175\,\text{A} = 175\,\text{mA}$$

Sample problem 11

$P = VI$

$V = 12\text{ V}, I = 2.5\text{ A}$

$$\begin{aligned} P &= VI \\ &= 12\text{ V} \times 2.5\text{ A} \\ &= 30\text{ W} \end{aligned}$$

The battery is supplying 30 W.

Practice problem 11

$P = VI$

$E = 3\text{ V}, I = 4.0\text{ A}$

$$\begin{aligned} P &= VI \\ &= 3\text{ V} \times 4.0\text{ A} \\ &= 12\text{ W} \end{aligned}$$

12 W of energy is being supplied to a 3-V light.

5.5 Exercise

1 **a** $P = 60\text{ W}, V = 240\text{ V}$

$$\begin{aligned} I &= \frac{P}{V} \\ &= \frac{60}{240} \\ &= 0.25\text{ A} \end{aligned}$$

b $P = 40\text{ W}, V = 12\text{ V}$

$$\begin{aligned} I &= \frac{P}{V} \\ &= \frac{40}{12} \\ &= 3.3\text{ A} \end{aligned}$$

c $P = 6.3\text{ W}, V = 6.0\text{ V}$

$$\begin{aligned} I &= \frac{P}{V} \\ &= \frac{6.3}{6.0} \\ &= 1.1\text{ A} \end{aligned}$$

d $P = 1200\text{ W}, V = 240\text{ V}$

$$\begin{aligned} I &= \frac{P}{V} \\ &= \frac{1200}{240} \\ &= 5.00\text{ A} \end{aligned}$$

2 $P = 600\text{ W}, E = 5.4 \times 10^4\text{ J}$

$E = Pt$

$$\begin{aligned} \Rightarrow t &= \frac{E}{P} \\ &= \frac{5.4 \times 10^4}{600} \\ &= 90\text{ s} \end{aligned}$$

3 $V = 240\text{ V}, I = 10.0\text{ A}$

$$\begin{aligned} P &= VI \\ &= 240\text{ V} \times 10.0\text{ A} \\ &= 2400\text{ W} \\ &= 2.4\text{ kW} \end{aligned}$$

4 $V = 240\text{ V}, I = 3.3\text{ A}, E = 3.2 \times 10^4\text{ W}$

$E = VIt$

$$\begin{aligned} \Rightarrow t &= \frac{E}{VI} \\ &= \frac{3.2 \times 10^4\text{ W}}{240\text{ V} \times 3.3\text{ A}} \\ &= \frac{32\,000}{792}\text{ s} \\ &= 40\text{ s} \end{aligned}$$

5 No. Mark did not use the correct units in his calculation. He needed to convert kW to W and hours to seconds in order to get the correct answer.

6 **a** A voltage drop of 240 V indicates that each coulomb of charge passing through the heater will transform 240 J of energy.

b $Q = 25\text{ C}, V = 240\text{ V}$

$$\begin{aligned} E &= QV \\ &= 25\text{ C} \times 240\text{ V} \\ &= 6000\text{ J} \\ &= 6.0 \times 10^3\text{ J} \end{aligned}$$

5.5 Exam questions

1 D

$$\begin{aligned} E &= VIt \\ &= 6 \times 10 \times 20 \\ &= 1200\text{ J} \end{aligned}$$

2 C

$$\begin{aligned} I &= \frac{E}{Vt} \\ &= \frac{150}{(3 \times 200)} \\ &= 0.25\text{ A} \end{aligned}$$

3 The sum of the power in each device is equal to the power, thus $P_R = P_{total} - P_Q - P_P$. [1 mark]

$$\begin{aligned} \text{Power of R} &= \text{total power} - \text{power in P and Q} \\ &= 30 - 15 - 7 \\ &= 8.0\text{ W [1 mark]} \end{aligned}$$

4 $t = 2.5$ minutes

$= 2.5 \times 60\text{ s}$ [1 mark]

$$I = \frac{E}{Vt}$$

$= \dfrac{3600}{(12 \times 2.5 \times 60)}$ [1 mark]

$= 2.0\text{ A}$ [1 mark]

5 **a** $V = 12\text{ V}, I = 2.0\text{ A}, t = 1800\text{ s}$

$$\begin{aligned} E &= VIt \\ &= 12\text{ V} \times 2.0\text{ A} \times 1800\text{ s} \\ &= 43\,200\text{ J} \\ &= 4.3 \times 10^4\text{ J [1 mark]} \end{aligned}$$

b $V = 12\text{ V}, I = 2.0\text{ A}$

$$\begin{aligned} P &= VI \\ &= 12\text{ V} \times 2.0\text{ A} \\ &= 24\text{ W [1 mark]} \end{aligned}$$

5.6 Electrical resistance

Sample problem 12

a Red = 2
Violet = 7
Orange = 10^3
Gold = ± 5%
Resistance = $27 \times 10^3\ \Omega \pm 5\%$
$= 27 \times 10^3\ \Omega \pm 1350\ \Omega$

b Brown = 1
Black = 0
Red = 10^2
Silver = ±10%
Resistance = $10 \times 10^2\ \Omega \pm 10\%$
$= 1.0 \times 10^3\ \Omega \pm 100\ \Omega$

Practice problem 12

a Orange = 3
White = 9
Black = 10^0
Gold = ±5%
Resistance = $39 \times 10^0\ \Omega \pm 5\%$
$= 39\ \Omega \pm 2\ \Omega$

b Green = 5
Blue = 6
Orange = 10^3
Silver = ±10%
Resistance = $56 \times 10^3\ \Omega \pm 10\%$
$= 56\,000\ \Omega \pm 10\%$
$= 56\,000\ \Omega \pm 5600\ \Omega$

c Violet = 7
Green = 5
Yellow = 10^4
Gold = ±5%
Resistance = $75 \times 10^4\ \Omega \pm 5\%$
$= 750\,000\ \Omega \pm 5\%$
$= 750\ \text{k}\Omega \pm 37.5\ \text{k}\Omega$

Sample problem 13

$$V = 6\ \text{V},\ I = \frac{300\ \text{A}}{1000} = 0.300\ \text{A}$$

$$R = \frac{V}{I} = \frac{6\ \text{V}}{0.300\ \text{A}} = 20\ \Omega$$

The resistance of the radio is 20 Ω.

Practice problem 13

$$V = 240\ \text{V},\ I = 6.0\ \text{A}$$

$$R = \frac{V}{I} = \frac{240\ \text{V}}{6.0\ \text{A}} = 40\ \Omega$$

The resistance of the radio is 40 Ω.

Sample problem 14

$$V = 6.0\ \text{V},\ R = 18\ \Omega$$

$$P = \frac{V^2}{R} = \frac{(6.0\ \text{V})^2}{18\ \Omega} = 2.0\ \text{W}$$

The radio transforms energy at 2.0 W.

Practice problem 14

$$V = 240\ \text{V},\ R = 48\ \Omega$$

$$P = \frac{V^2}{R} = \frac{(240\ \text{V})^2}{48\ \Omega} = 1200\ \text{W} = 1.2\ \text{kW}$$

The power rating of the electric jug is 1200 W or 1.2 kW.

Sample problem 15

a
$$P = VI \Rightarrow I = \frac{P}{V}$$
$$V = 240\ \text{V},\ P = 800\ \text{W}$$
$$I = \frac{800\ \text{W}}{240\ \text{V}} = 3.33\ \text{A}$$

The normal operating current is 3.33 A.

b
$$V = 240\ \text{V},\ P = 800\ \text{W}$$
$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(240\ \text{V})^2}{800\ \text{W}} = 72\ \Omega$$

The resistance is 72 Ω.

Practice problem 15

a
$$P = VI \Rightarrow I = \frac{P}{V}$$
$$V = 240\ \text{V},\ P = 600\ \text{W}$$
$$I = \frac{600\ \text{W}}{240\ \text{V}} = 2.5\ \text{A}$$

b
$$V = 240\ \text{V},\ P = 600\ \text{W}$$
$$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(240\ \text{V})^2}{600\ \text{W}} = 96\ \Omega$$

5.6 Exercise

1 $E = VIt$
$= 6\text{ V} \times 3\text{ A} \times 60\text{ s}$
$= 1080\text{ J}$
$= 1.08\text{ kJ}$

2 **a** $V = IR$
$= 8.0\text{ A} \times 4.0\,\Omega$
$= 32\text{ V}$

b $V = IR$
$= 22\text{ mA} \times 2.2\text{ k}\Omega$
$= 0.022\text{ A} \times 2200\,\Omega$
$= 48\text{ V}$

c $I = \dfrac{V}{R}$
$= \dfrac{12\text{ V}}{6.0\,\Omega}$
$= 2.0\text{ A}$

d $I = \dfrac{V}{R}$
$= \dfrac{240\text{ V}}{8.0 \times 10^4\,\Omega}$
$= 3.0 \times 10^{-3}\text{ A}$
$= 3.0\text{ mA}$

e $R = \dfrac{V}{I}$
$= \dfrac{9.0}{6.0}$
$= 1.5\,\Omega$

f $R = \dfrac{V}{I}$
$= \dfrac{1.5\text{ V}}{45\text{ mA}}$
$= \dfrac{1.5\text{ V}}{0.045\text{ A}}$
$= 33\,\Omega$

3 **a** This device is non-ohmic. The graph of current versus voltage is non-linear.

b 0 A

c $R = \dfrac{V}{I}$
$= \dfrac{0.5}{0}$
$= \infty$ (infinity)

d From the graph, when $I = 20$ mA, $V = 0.65$ V.
$V = 0.65\text{ V}$, $I = 20\text{ mA} = 0.020\text{ A}$
$R = \dfrac{V}{I}$
$= \dfrac{0.65\text{ V}}{0.020\text{ A}}$
$= 32.5\,\Omega$

4 $P = VI$
$= I^2R$
$= (0.30\text{ A})^2 \times 5.0\,\Omega$
$= 0.45\text{ W}$

5 **a** Transpose $P = \dfrac{V^2}{R}$ to make R the subject:
$R = \dfrac{V^2}{P}$
$= \dfrac{(240)^2}{60}$
$= 960\,\Omega$

b Transpose $P = \dfrac{V^2}{R}$ to make R the subject:
$R = \dfrac{V^2}{P}$
$= \dfrac{(6.0)^2}{6.3}$
$= 5.7\,\Omega$

c Transpose $P = \dfrac{V^2}{R}$ to make R the subject:
$R = \dfrac{V^2}{P}$
$= \dfrac{(12)^2}{40}$
$= 3.6\,\Omega$

6 $P = \dfrac{V^2}{R}$
$= \dfrac{(240)^2}{48}$
$= 1200\text{ W}$
$= 1.2\text{ kW}$

5.6 Exam questions

1 A
$P_2 = \dfrac{(V_2)^2}{R}$
$= \dfrac{(2V_1)^2}{R}$
$= \dfrac{4(V_1)^2}{R}$
$= 4P_1$

2 Bill is correct. [1 mark]
Ben is incorrect. [1 mark]
Ben would have to say '$V = IR$, where R is a constant'. [1 mark]

3 Yes [1 mark]
At 12.5 V:
$R = \dfrac{V}{I}$
$= \dfrac{12.5}{0.5}$
$= 25\,\Omega$ [1 mark]
At 30 V:
$R = \dfrac{30}{1.2}$
$= 25\,\Omega$, so resistance is constant [1 mark]

4 V, t and R are known, thus use $E = \dfrac{V^2 t}{R}$.
$E = \dfrac{V^2 t}{R}$
$= \dfrac{9^2 \times 100}{30}$ [1 mark]
$= 270\text{ J}$ [1 mark]

5 E, t and r are known, thus use $I = \frac{E}{Rt}$ and use the correct unit for t(s).

$$I^2 = \frac{E}{Rt}$$
$$= \frac{960}{80 \times 5 \times 60} \text{ [1 mark]}$$
$$\Rightarrow I = \sqrt{0.04}$$
$$= 0.20 \text{ A [1 mark]}$$

5.7 Review

5.7 Review questions

1 From $E = QV$:

$$V = \frac{E}{Q}$$
$$= \frac{3 \times 10^9 \text{ J}}{15 \text{ C}}$$
$$= 2 \times 10^8 \text{ V}$$

2 a From $P = VI$:

$$I = \frac{P}{V}$$
$$= \frac{2.7 \times 10^3 \text{ W}}{240 \text{ V}}$$
$$= 11 \text{ A}$$

b From $P = \frac{E}{t}$:

$$E = Pt$$
$$= 2.7 \times 10^3 \text{ W} \times 2.5 \times 60$$
$$= 405 \times 10^3 \text{ J}$$
$$= 405 \text{ kJ}$$

3 a $V = IR$

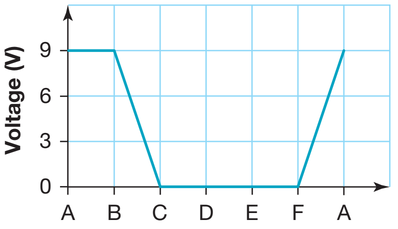
$$= 2.5 \text{ A} \times 44\,\Omega$$
$$= 110 \text{ V}$$

b The current would also increase, but at the risk of exceeding its capacity and 'burning out'.

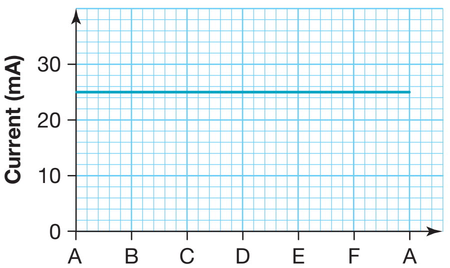
4 a $P = VI$

$$= 9 \times 25 \times 10^{-3}$$
$$= 225 \times 10^{-3} \text{ W}$$
$$= 2.25 \times 10^{-1} \text{ W}$$
$$= 0.2 \text{ W}$$

b



c



d From $V = RI$:

$$R = \frac{V}{I}$$
$$= \frac{9 \text{ V}}{25 \times 10^{-3} \text{ A}}$$
$$= 360\ \Omega$$

5 a $I = \frac{Q}{t}$

$$= \frac{15 \text{ C}}{60 \text{ s}}$$
$$= 0.25 \text{ A}$$

b $R = \frac{V}{I}$

$$= \frac{(2 \times 1.5 \text{ V})}{0.25 \text{ A}}$$
$$= 12\,\Omega$$

c $P = VI$

$$= 3.0 \text{ V} \times 0.25 \text{ A}$$
$$= 0.75 \text{ W}$$

6 a From $P = I^2R$:

$$R = \frac{P}{I^2}$$
$$= \frac{800 \text{ W}}{3.3^2 \text{ A}}$$
$$= 73\,\Omega$$

b From $P = \frac{E}{t}$:

$$E = Pt$$
$$= 800 \text{ W} \times 40$$
$$= 32\,000 \text{ J}$$
$$= 32 \text{ kJ}$$

7 a When the current is 0.2 A, the graph is linear. Using Ohm's Law, the resistance can be calculated as:

$$V = IR$$
$$\Rightarrow R = \frac{V}{I}$$
$$= \frac{3 \text{ V}}{0.2 \text{ A}}$$
$$= 15\,\Omega$$

b For voltages up to 4.5 V, there is a constant relationship between the voltage across the globe and the current flowing through it. In this range, the globe exhibits ohmic characteristics. However, for higher voltages the relationship is not constant and therefore non-ohmic.

8 a A non-ohmic device is a device that has a varying resistance.

b $I = 20 \text{ mA}$

c From the slope of the graph:

$$\frac{1}{R} = \frac{I}{V}$$
$$= \frac{(40 - 20) \times 10^{-3}}{(0.35 - 0.3)}$$
$$= \frac{20 \times 10^{-3}}{0.05}$$
$$= 0.4$$
$$\Rightarrow R = \frac{1}{0.4}$$
$$= 2.5\,\Omega$$

5.7 Exam questions

Section A — Multiple choice questions

1 A

Charge is moved in the direction of the current, both in the case of DC and AC circuits. The movement of the charge enables the energy transfer in the circuit. Even though electrons pass along the circuit, they move too slowly to suggest that the electrons pass all the way through the circuit when it is closed. The kinetic energy of the free electrons is available for conversion to other forms of energy, typically as thermal energy in a load like a light globe. The battery does not push positive charges through the circuit.

2 C

The charge carried by a single electron is equal to -1.602×10^{-19} C. 4.8×10^{-19} is a multiple of 1.602×10^{-19}.

3 D

From $I = \frac{Q}{t}$:

$$Q = It = 5.0 \times 25 = 125\,\text{C}$$

4 B

The resistance, R, is calculated from the slope of the V versus I graph:

$$R = \frac{V}{I} = \frac{4\,\text{V}}{6\,\text{A}} = 0.67\,\Omega$$

5 C

From $E = VQ$, making V the subject:

$$V = \frac{E}{Q} = \frac{1.44 \times 10^{-18}}{1.6 \times 10^{-19}} = 9\,\text{V}$$

6 D

$$E = VIt = 240 \times 1.25 \times 60 = 1.8 \times 10^{4}\,\text{J} = 18\,\text{kJ}$$

7 D

Note: This is a question about power, the rate of energy supply. The units for power are watts:

$$P = VI = 7.2 \times 1.2 = 8.64\,\text{W}$$

8 C

From $P = VI$, make I the subject.

$$I = \frac{P}{V} = \frac{1200}{240} = 5\,\text{A}$$

9 D

From $P = \frac{V^2}{R}$, make R the subject.

$$R = \frac{V^2}{P} = \frac{240^2}{1200} = 48\,\Omega$$

10 B

The circuit is closed; therefore, there is a constant current flowing.

From $P = VI$:

$$I = \frac{P}{V} = \frac{24}{12} = 2\,\text{A}$$

Section B — Short answer questions

11 **a** **i** $E = QV$

$= 1.6 \times 10^{-19}\,\text{C} \times 3.0\,\text{V}$

$= 4.8 \times 10^{-19}\,\text{J}$ [1 mark]

ii By definition 1 volt = 1 joule per coulomb. So a 3.0-V battery will supply 3.0 J to every coulomb of charge. [1 mark]

b Since $E = QV$, then:

$\frac{E}{t} = \frac{QV}{t}$ [1 mark]

$= \frac{0.04 \times 3.0}{60}$

$= \frac{0.12}{60}$ [1 mark]

$= 2.0 \times 10^{-3}\,\text{W}$ [1 mark]

12 Disagree. When a light switch is turned on, the light comes on almost instantaneously. Similarly, Luke should feel the pull on the rope almost immediately. The energy transfer is quick and he does not need to wait for the same segment of rope that was pulled on by the teacher. [1 mark] Although electrons carry the electric charge in a circuit, they move more slowly than the energy that is transferred to the loads in the circuit. [1 mark]

13 **a** From $P = VI$

$I = \frac{P}{V} = \frac{2.4\,\text{W}}{3.0\,\text{V}}$

$= 0.8\,\text{A}$ [1 mark]

b The slope of the V versus I graph is constant, so:

$R = \frac{V}{I} = \frac{2.4}{0.6}$

$= 4\,\Omega$ [1 mark]

14 a From $E = QV$:

$$Q = \frac{E}{V}$$
$$= \frac{6 \times 10^6}{18}$$
$$= 3.3 \times 10^5 \text{ C}$$
$$= 330 \text{ kC [1 mark]}$$

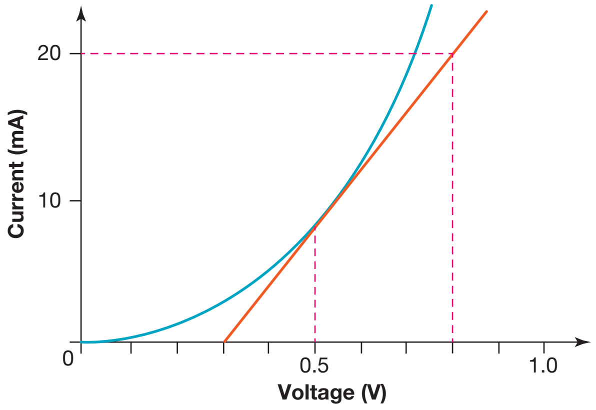
b $I = \frac{Q}{t}$

$$= \frac{3.3 \times 10^5}{8 \times 60 \times 60}$$
$$= 11.5 \text{ A [1 mark]}$$

15 a Non-ohmic; the resistance is not constant. [1 mark]

b 10 mA [1 mark]

c The gradient of the graph is: $\frac{\text{rise}}{\text{run}} = \frac{\text{current}}{\text{voltage}} = \frac{1}{\text{resistance}}$



When $V = 0.5$, the gradient is $\frac{(0.02 - 0) \text{ A}}{(0.8 - 0.3) \text{ V}} = 0.04$.

$$\frac{1}{\text{gradient}} = \frac{1}{0.04}$$
$$= 25\ \Omega \text{ [1 mark]}$$

d $P = VI$

$$= 0.5 \times (10 \times 10^{-3})$$
$$= 5 \times 10^{-3} \text{ W [1 mark]}$$

Topic 6 — Circuit electricity

6.2 BACKGROUND KNOWLEDGE Electrical circuit rules

Sample problem 1

$1.0\text{ A} + 4.0\text{ A} = 5.0\text{ A}$

$2.5\text{ A} + 1.3\text{ A} = 3.8\text{ A}$

$$I_{in} = I_{out}$$
$$5.0\text{ A} = 3.8\text{ A} + x$$
$$\Rightarrow x = 5.0\text{ A} - 3.8\text{ A}$$
$$= 1.2\text{ A}$$

The unknown current flowing out of the junction is 1.2 A.

Practice problem 1

$$I_{in} = I_{out}$$
$$3.0\text{ A} = I_a + 1.5\text{ A}$$
$$\Rightarrow I_a = 3.0 - 1.5$$
$$= 1.5\text{ A}$$

The unknown current flowing out of the junction is 1.5 A.

Sample problem 2

At a:

$$I_{in} = I_{out}$$
$$15.3\text{ mA} = 7.9\text{ mA} + a$$
$$\Rightarrow a = 15.3\text{ mA} - 7.9\text{ mA}$$
$$= 7.4\text{ mA}$$

At b:

$$I_{in} = I_{out}$$
$$7.9\text{ mA} + 7.4\text{ mA} = b$$
$$\Rightarrow b = 15.3\text{ mA}$$

At c:

$$I_{in} = I_{out}$$
$$15.3\text{ mA} = c + 2.1\text{ mA}$$
$$\Rightarrow c = 15.3\text{ mA} - 2.1\text{ mA}$$
$$= 13.2\text{ mA}$$

At d:

$$I_{in} = I_{out}$$
$$13.2\text{ mA} = d + 6.5\text{ mA}$$
$$\Rightarrow d = 13.2\text{ mA} - 6.5\text{ mA}$$
$$= 6.7\text{ mA}$$

At e:

$$I_{in} = I_{out}$$
$$2.1\text{ mA} = e$$
$$\Rightarrow e = 2.1\text{ mA}$$

At f:

$$I_{in} = I_{out}$$
$$d + 6.5\text{ mA} + e = f$$
$$\Rightarrow f = 6.7\text{ mA} + 6.5\text{ mA} + 2.1\text{ mA}$$
$$= 15.3\text{ mA}$$

Practice problem 2

$I_{in} = I_{out} \Rightarrow I_a = 11.1\text{ A}$

$$I_{in} = I_{out}$$
$$a = 11.1\text{ A}$$
$$= 3.0\text{ A} + b + 5.7\text{ A}$$
$$b = 11.1 - 3.0 - 5.7$$
$$= 2.4\text{ A}$$

Sample problem 3

V = the sum of the voltage drops
$= 9.0\text{ V}$

$$9.0\text{ V} = V_{ab} + V_{bc}$$
$$= 5.2\text{ V} + V_{bc}$$
$$\Rightarrow V_{bc} = 3.8\text{ V}$$

The unknown voltage drop is 3.8 V.

Practice problem 3

$$V_{battery} = 12\text{ V}$$
$$= V_{ab} + 4.8\text{ V}$$
$$\Rightarrow V_{ab} = 12 - 4.8$$
$$= 7.2\text{ V}$$

The unknown voltage drop is 7.2 V.

6.2 Exercise

1 **a** A connection between two or more conducting paths
b The flow of electrons through the circuit
c The amount of electrical potential energy converted in a load for every coulomb of charge passing through it
d The wire connecting the elements in an electric circuit

2 **a** $I_{in} = I_{out}$
Taking the positive direction as into the junction:
$$I_b + 2.8\text{ A} = 7.3\text{ A}$$
$$\Rightarrow I_b = 7.3 - 2.8$$
$$= 4.5\text{ A}$$
The current flows towards the junction.

b $I_{in} = 1.3\text{ A} + 4.2\text{ A} + I_c$
$= 5.5\text{ A} + I_c$

$I_{out} = 2.9\text{ A} + 3.7\text{ A}$
$= 6.6\text{ A}$

$$I_{in} = I_{out}$$
$$5.5\text{ A} + I_c = 6.6\text{ A}$$
$$\Rightarrow I_c = 6.6\text{ A} - 5.5\text{ A}$$
$$= 1.1\text{ A}$$
Ic = 0. The current flows towards the junction.

3 Components *ef* and *cd* are connected in parallel, so $V_{cd} = V_{ef}$.

$$V_{battery} = 24\text{ V}$$
$$= 8\text{ V} + V_{cd}$$
$$\Rightarrow V_{cd} = 24 - 8$$
$$= 16\text{ V}$$
$$\therefore V_{cd} = V_{ef} = 16\text{ V}$$

4 a $V_{ab} = I_1R_1$

$$= 0.20\text{ A} \times 60\,\Omega$$
$$= 12\text{ V}$$

b $V_{cd} = 12\text{ V}$

c $\varepsilon = 12\text{ V}$

d $I_2 = I_T - I_1$

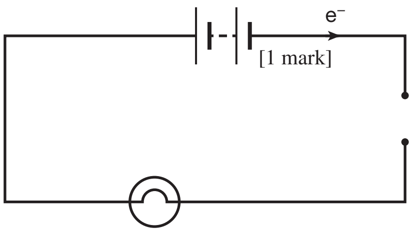
$$= 0.50\text{ A} - 0.20\text{ A}$$
$$= 0.30\text{ A}$$

e $R_2 = \dfrac{V_{cd}}{I_2}$

$$= \frac{12\text{ V}}{0.30\text{ A}}$$
$$= 40\,\Omega$$

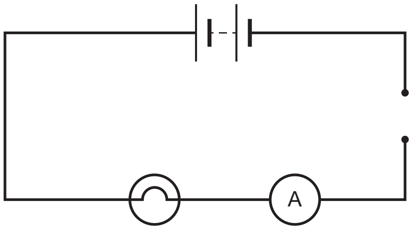
6.2 Exam questions

1 a



The direction of electron current is from the negative terminal to the positive terminal.

b



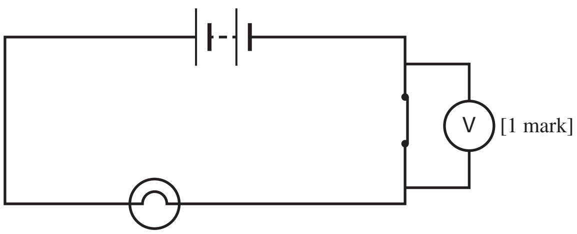
An ammeter could be placed anywhere in the circuit. [1 mark]

c From $P = VI$:

$$I = \frac{P}{V}$$
$$= \frac{72\text{ W}}{24\text{ V}}$$
$$= 3.0\text{ A} \text{ [1 mark]}$$

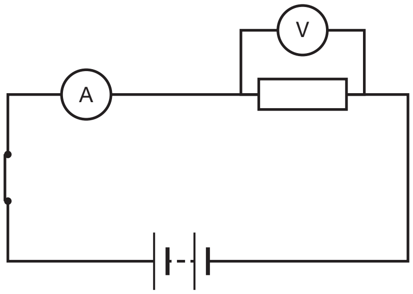
d 0 [1 mark]

e



f No current will flow in the open circuit. [1 mark]

2 a



[1 mark for correct circuit diagram]

b From $P = VI$:

$$I = \frac{P}{V}$$
$$= \frac{1.2 \times 10^3}{240}$$
$$= 5.0\text{ A} \text{ [1 mark]}$$

3 Pushing the button creates a closed circuit, which is needed for electric current to flow and operate the bell. [1 mark]

4 a Series circuit, so $R_{equivalent} = 1000 + 1500$

$$= 2500\,\Omega$$

Using $V = IR$:

$$I = \frac{V}{R}$$
$$= \frac{25\text{ V}}{2500\,\Omega}$$
$$= 0.01\text{ A}$$
$$= 10\text{ mA} \text{ [1 mark]}$$

b Substituting $V = IR$, the respective voltages are:

$1000 \times 0.01 = 10\text{ V}$

$1500 \times 0.01 = 15\text{ V}$ [1 mark]

c In parallel the voltage drop across each person will be 25 V. The current in each will be:

$$I = \frac{V}{R}$$
$$= \frac{25\text{ V}}{1000\,\Omega}$$
$$= 0.025\text{ A}$$
$$= 25\text{ mA in the first person} \text{ [1 mark]}$$
$$I = \frac{V}{R}$$
$$= \frac{25\text{ V}}{1500\,\Omega}$$
$$= 0.017\text{ A}$$
$$= 17\text{ mA in the second person} \text{ [1 mark]}$$

d In parallel the voltage drop across each person will be 25 V. [1 mark]

5 Short circuiting the resistor branch will result in the current flowing through the short circuit and not through the resistor. Under these conditions the voltage drop across the short circuit will be zero. [1 mark] Substituting in $V = IR$, where $V = 0$, R must be equal to 0. [1 mark]

6.3 Series circuits

Sample problem 4

$$R_{\text{equivalent}} = \sum_{1}^{i} R_{\text{i}}$$
$$= 15\ \Omega + 25\ \Omega + 34\ \Omega$$
$$= 74\ \Omega$$

The effective resistance of the circuit is 74 Ω.

Practice problem 4

$$R_{\text{equivalent}} = \sum_{1}^{i} R_{\text{i}}$$
$$= 1.2\ \text{k}\Omega + 5.6\ \text{k}\Omega + 7.1\ \text{k}\Omega$$
$$= 13.9\ \text{k}\Omega$$

The effective resistance of the circuit is 13.9 kΩ.

Sample problem 5

a $I_b = I_a = 1$ A
The current at point b is 1 A.

b $V_2 = IR_2$
$= 1.0\ \text{A} \times 60\ \Omega$
$= 60$ V
The voltage drop across R_2 is 60 V.

c $\varepsilon = V_1 + V_2$
$100\ \text{V} = V_1 + 60\ \text{V}$
$\Rightarrow V_1 = 40$ V
The voltage drop across R_1 is 40 V.

d $V_1 = IR_1$
$40\ \text{V} = 1.0\ \text{A} \times R_1$
$\Rightarrow R_1 = 40\ \Omega$
The value of R_1 is 40 Ω.

Practice problem 5

a The current in a series circuit is constant. So $I_b = I_a = 0.6$ A.
The current at point b is 0.6 A.

b $V_{\text{known}} = IR$
$= 0.6 \times 15$
$= 9$ V
The voltage drop across the known resistor is 9 V.

c In the series circuit the sum of the voltage drops around the circuit equals the emf of the battery:
emf $= 24$ V
$= V + 9$
$\Rightarrow V = 24 - 9$
$= 15$ V
The voltage drop across the unknown resistor, R_1, is 15 V.

d $V = IR$
$15\ \text{V} = 0.6\ \text{A} \times R_1$
$\Rightarrow R_1 = 25\ \Omega$
The value of R_1 is 25 Ω.

Sample problem 6

$$V_{\text{out}} = \frac{R_2 V_{\text{in}}}{R_1 + R_2}$$
$$\Rightarrow 4.0\ \text{V} = \frac{6.0\ \text{V} \times R_2}{2.2\ \text{k}\Omega + R_2}$$
$$8.8\ \text{k}\Omega\ \text{V} + 4.0\ \text{V}R_2 = 6.0\ \text{V}R_2$$
$$2.0\ \text{V}R_2 = 8.8\ \text{k}\Omega\ \text{V}$$
$$\Rightarrow R_2 = 4.4\ \text{k}\Omega$$

The value of the unknown resistor in the voltage divider is 4.4 kΩ.

Practice problem 6

$$V_{\text{out}} = \frac{R_2 V_{\text{in}}}{R_1 + R_2}$$
$$\Rightarrow 1.5\ \text{V} = \frac{R_2 \times 6.0\ \text{V}}{2.2\ \text{k}\Omega + R_2}$$
$$3.3\ \text{k}\Omega\ \text{V} + 1.5\ \text{V}R_2 = 6.0\ \text{V}R_2$$
$$4.5R_2 = 3.3\ \text{k}\Omega\ \text{V}$$
$$\Rightarrow R_2 = \frac{3.3}{4.5}\ \text{k}\Omega$$
$$= 0.73\ \text{k}\Omega$$
$$= 730\ \Omega$$

The value of the unknown resistor in the voltage divider in sample problem 10 if the output voltage is to be 1.5 V is 730 Ω.

6.3 Exercise

1 a $R_{\text{equivalent}} = R_1 + R_2 + R_3$
$= 12 + 20 + 30$
$= 62\ \Omega$

b $R_{\text{equivalent}} = R_1 + R_2 + R_3$
$= 1.2 + 3.2 + 11$
$= 15.4\ \text{k}\Omega$

2 One method:
$I = \frac{V_1}{R_1}$
$= \frac{3}{1}$
$= 3.0$ A
$V_2 = IR_2$
$= 3 \times 2$
$= 6.0$ V
$V_b = V_1 + V_2$
$= 3 + 6$
$= 9.0$ V
Alternative method:
$V = IR$
$\Rightarrow V_2 = 2V_1$
$= 6$ V
$\Rightarrow V_b = 3 + 6$
$= 9.0$ V

3 $V_1 = IR_1$
$= 0.4 \times 5$
$= 2.0$ V

$\Rightarrow V_2 = V_b - V_1$
$= 6 - 2$
$= 4\text{ V}$

$\Rightarrow X = R_2$
$= \frac{V_2}{I}$
$= \frac{4}{0.4}$
$= 10\,\Omega$

4 $I_a = 1\text{ A}$ (series circuit)
$V_1 = IR_1$
$= 1 \times 4$
$= 4\text{ V}$
$V_2 = 6\text{ V} - 4\text{ V}$
$= 2\text{ V}$
$R_2 = \frac{V_2}{I}$
$= \frac{2.0}{1.0}$
$= 2.0\,\Omega$

5 B
$V_{out} = V_{in} \times \frac{R_2}{R_{equivalent}}$
$= 9 \times \frac{60}{100}$
$= 5.4\text{ V}$

6 **a** $V_{out} = \frac{R_2 V_{in}}{R_1 + R_2}$
$V_{in} = 10\text{ V}$
$R_1 = 60\,\Omega$
$R_2 = 40\,\Omega$
$\Rightarrow V_{out} = \frac{40\,\Omega \times 10\text{ V}}{(60\,\Omega + 40\,\Omega)}$
$= 4.0\text{ V}$

b $V_{out} = \frac{R_2 V_{in}}{R_1 + R_2 + R_3}$
$V_{in} = 9\text{ V}$
$R_1 = 20\,\Omega$
$R_2 = 30\,\Omega$
$R_3 = 40\,\Omega$
$\Rightarrow V_{out} = \frac{30\,\Omega \times 9\text{ V}}{90\,\Omega}$
$= 3.0\text{ V}$

7 $V_{out} = V_{in} \times \frac{R_2}{(R_1 + R_2)}$
$= 12 \times \frac{3}{(3+5)}$
$= 4.5\text{ V}$

8 The equivalent resistance will increase, hence the current will decrease.
Hence, V_1 will decrease (R_1 is fixed) and hence, V_2 will increase (V_{in} is fixed).
(It is not adequate to say that $V_{out} = IR_2$, since one term increases and the other decreases.)

9 **a** $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$
$= \frac{2.2}{2.2 + 2.2} \times 6.0$
$= 3.0\text{ V}$

b $V_{out} = \frac{4.4}{2.2 + 4.4} \times 9.0$
$= 6.0\text{ V}$

6.3 Exam questions

1 C
$I = \frac{V}{R_{equivalent}}$
$= \frac{12}{(1+2+3)}$
$= \frac{12}{6}$
$= 2.0\text{ A}$

2 A
$R_{equivalent} = R_1 + R_2 + R_3$
$= 25 + 15 + 10$
$= 50\,\Omega$
$I = \frac{V}{R_{equivalent}}$
$= \frac{10}{50}$
$= 0.20\text{ A}$

3 C
$V_{out} = \frac{R_2 V_{in}}{R_1 + R_2}$
$= 4\text{ V}$
$V_{in} = 6\text{ V}$
$R_1 = 3\text{ k}\Omega$
$R_2 = R\text{ k}\Omega$
$\Rightarrow 4\text{ V} = \frac{R\text{ k}\Omega \times 6\text{ V}}{(3+R)\text{ k}\Omega}$
$\Rightarrow 12 + 4R = 6R$
$\Rightarrow 2R = 12$
$\Rightarrow R = 6$
$R = 6\text{ k}\Omega$

4 D
$V_1 = 12 - 7.5$
$= 4.5\text{ V}$
$\frac{R_1}{R_2} = \frac{V_1}{V_2}$
$\frac{X}{50} = \frac{4.5}{7.5}$
$\Rightarrow X = 30\,\Omega$
Alternatively:
$V_{out} = \frac{R_{out}}{R_{total}} V_{in}$
$7.5 = \frac{50}{50 + X} 12$
$\Rightarrow X = \frac{225}{7.5} = 30\,\Omega$

5 **a** In a series circuit:
$I_a = I_b$
$I_a = 2\text{ A}$
$\Rightarrow I_b = 2\text{ A}$ [1 mark]

b $V_{bc} = IR_2$
$= 2 \times 30$
$= 60\text{ V}$ [1 mark]

c $R_1 = \frac{V_{ab}}{I}$

$= \frac{20}{2.0}$

$= 10\ \Omega$ [1 mark]

d $R_{\text{equivalent}} = R_1 + R_2$

$= 10\ \Omega + 30\ \Omega$

$= 40\ \Omega$ [1 mark]

e $V = V_{ab} + V_{bc}$

$= 20 + 60$

$= 80\text{ V}$ [1 mark]

6.4 Parallel circuits

Sample problem 7

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{5.0\ \Omega} + \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega}$$

$$= \frac{4}{20\ \Omega} + \frac{2}{20\ \Omega} + \frac{1}{20\ \Omega}$$

$$= \frac{7}{20\ \Omega}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{20\ \Omega}{7}$$

$$= 2.9\ \Omega$$

The effective resistance of the three resistors connected in parallel is 2.9 Ω.

Practice problem 7

The four resistors are connected in parallel, so:

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$$= \frac{1}{5} + \frac{1}{5} + \frac{1}{15} + \frac{1}{20}$$

$$= \frac{12}{60} + \frac{12}{60} + \frac{4}{60} + \frac{3}{60}$$

$$= \frac{31}{60}$$

$$\Rightarrow R_{\text{equivalent}} = 2\ \Omega$$

Sample problem 8

a $V_1 = V_2 = 9\text{ V}$

The voltage drop across R_1 and R_2 is 9 V.

b $I_2 = \frac{V}{R_2}$

$= \frac{9.0\text{ V}}{10\ \Omega}$

$= 0.90\text{ A}$

I_2, the current flowing through R_2, is 0.9 A.

c $I_1 = I_T - I_2$

$= 1.35\text{ A} - 0.9\text{ A}$

$= 0.45\text{ A}$

I_1, the current flowing through R_1, is 0.45 A.

d $R_1 = \frac{V}{I_1}$

$= \frac{9.0\text{ V}}{0.45\text{ A}}$

$= 20\ \Omega$

The resistance of R_1 is 20 Ω.

e $\frac{1}{R_{\text{equivalent}}} = \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega}$

$= \frac{3}{20\ \Omega}$

$\Rightarrow R_{\text{equivalent}} = \frac{20\ \Omega}{3}$

$= 6.7\ \Omega$

The effective resistance of the circuit is 6.7 Ω.

Practice problem 8

a For the parallel circuit:

$V_1 = V_2 = \text{emf} = 24\text{ V}$

b $V_1 = R_1 I_1$

$\Rightarrow I_1 = \frac{V}{R_1}$

$= \frac{24\text{ V}}{40\ \Omega}$

$= 0.6\text{ A}$

c The current leaving the battery is given by $I_T = I_1 + I_2$.

$I_2 = I_T - I_1$

$= 0.8 - 0.6\text{ A}$

$= 0.2\text{ A}$

d $V_2 = R_2 I_2$

$\Rightarrow R_2 = \frac{V_2}{I_2}$

$= \frac{24\text{ V}}{0.2\text{ A}}$

$= 120\ \Omega$

e The resistors are in parallel, so $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$.

$\Rightarrow \frac{1}{R_{\text{equivalent}}} = \frac{1}{40} + \frac{1}{120}$

$= \frac{3}{120} + \frac{1}{120}$

$= \frac{4}{120}$

$\Rightarrow R_{\text{equivalent}} = 30\ \Omega$

Alternatively:

$R_{\text{equivalent}} = \frac{V_T}{I_T}$

$= \frac{24\text{ V}}{0.8\text{ A}}$

$= 30\ \Omega$

Sample problem 9

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{10.0\ \Omega} + \frac{1}{10\,000\ \Omega}$$

$$= \frac{1000}{10\,000\ \Omega} + \frac{1}{10\,000\ \Omega}$$

$$= \frac{1001}{10\,000\ \Omega}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{10\,000\ \Omega}{1001}$$

$$= 9.99\ \Omega$$

The effective resistance is 9.99 Ω.

Practice problem 9

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{1.2\text{ k}\Omega} + \frac{1}{4.8\text{ k}\Omega} = \frac{4}{4.8} + \frac{1}{4.8} = \frac{5}{4.8}$$

$$\Rightarrow R_{\text{equivalent}} = 0.96\text{ k}\Omega = 960\ \Omega$$

The effective resistance when a 1.2 kΩ resistor is placed in parallel with a 4.8 Ω resistor is 960 Ω.

Sample problem 10

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{0\ \Omega} + \frac{1}{10\,000\ \Omega} = \infty$$

$$\Rightarrow R_{\text{equivalent}} = 0\ \Omega$$

The effective resistance is 0 Ω.

Practice problem 10

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.2\text{ k}\Omega} + \frac{1}{4.8\text{ k}\Omega} + \frac{1}{0.5\text{ k}\Omega} = \frac{4}{4.8} + \frac{1}{4.8} + \frac{9.6}{4.8} = \frac{14.6}{4.8\text{ k}\Omega} = \frac{14.6}{4800\ \Omega}$$

$$\Rightarrow R_{\text{equivalent}} = 330\ \Omega$$

The effective resistance of the new arrangement is 330 Ω.

Sample problem 11

a Let R_{series} be the effective resistance of R_1, R_2 and R_3, which are in series.

$R_{\text{series}} = R_1 + R_2 + R_3 = 10\ \Omega$ R_{series} and R_4 in parallel, thus:

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_{\text{series}}} + \frac{1}{R_4} = \frac{1}{10} + \frac{1}{20} = \frac{3}{20}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{20}{3} = 6.67\ \Omega$$

b $I_2 = \frac{V}{R_{\text{series}}} = \frac{5}{10} = 0.5\text{ A}$

c $V_2 = I_2R_2 = 0.5 \times 7 = 3.5\text{ V}$

Practice problem 11

a Let R_{parallel} be the effective resistance of R_1, R_2 and R_3, which are in parallel.

$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1\ \Omega$ R_{parallel} and R_4 in series, thus:

$$R_{\text{equivalent}} = R_{\text{parallel}} + R_4 = 1 + 4 = 5\ \Omega$$

b $I = \frac{V}{R_{\text{equivalent}}} = \frac{12}{5} = 2.4\text{ A}$ and $I_4 = I = 2.4\text{ A}$

Thus, $V_4 = I_4R_4 = 2.4 \times 4 = 9.6\text{ V}$.

Moreover, $V_1 = V_2 = V_3 = V - V_4 = 12 - 9.6 = 2.4\text{ V}$.

Hence, $I_2 = \frac{V_2}{R_2} = \frac{2.4}{3} = 0.8\text{ A}$.

6.4 Exercise

1 a
$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{5.0} + \frac{1}{10} + \frac{1}{30} = \frac{6}{30} + \frac{3}{30} + \frac{1}{30} = \frac{10}{30}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{30}{10} = 3.0\ \Omega$$

b
$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{15} + \frac{1}{60} + \frac{1}{60} = \frac{4}{60} + \frac{1}{60} + \frac{1}{60} = \frac{6}{60}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{60}{6} = 10\ \Omega$$

2 a
$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{10}{2} = 5\ \Omega$$

b
$$I_{\text{equivalent}} = \frac{V}{R_{\text{equivalent}}} = \frac{15}{5.0} = 3.0\text{ A}$$

c
$$I_1 = \frac{V}{R_1} = \frac{15}{10} = 1.5\text{ A}$$

Similarly, $I_2 = 1.5\text{ A}$.

3 a $\frac{1}{R_{\text{equivalent}}} = \frac{1}{60} + \frac{1}{30} + \frac{1}{20}$

$= \frac{1}{60} + \frac{2}{60} + \frac{3}{60}$

$= \frac{6}{60}$

$\Rightarrow R_{\text{equivalent}} = \frac{60}{6}$

$= 10\,\Omega$

b $I = \frac{\varepsilon}{R_{\text{equivalent}}}$

$= \frac{90}{10}$

$= 9.0$ A

c $60\,\Omega \Rightarrow I = \frac{V}{R}$

$= \frac{90\text{ V}}{60\,\Omega}$

$= 1.5$ A

$30\,\Omega \Rightarrow I = \frac{V}{R}$

$= \frac{90\text{ V}}{30\,\Omega}$

$= 3.0$ A

$20\,\Omega \Rightarrow I = \frac{V}{R}$

$= \frac{90\text{ V}}{20\,\Omega}$

$= 4.5$ A

4 The current on each side of the battery is the same. So:

$I_a = 1.5$ A

$V_1 = 6$ V

$V_2 = 6$ V

$I_1 = \frac{V_1}{R_1}$

$= \frac{6.0\text{ V}}{6.0\,\Omega}$

$= 1.0$ A

$I_2 = I_a - I_1$

$= 1.5\text{ A} - 1.0\text{ A}$

$= 0.5$ A

$R_2 = \frac{V_2}{I_2}$

$= \frac{6.0}{0.5}$

$= 12\,\Omega$

5 a The resistors are connected in parallel; therefore, the voltage drop across each resistor equals 36 V.

$R_1 = 6.0\,\Omega \Rightarrow I_1 = \frac{V}{R_1}$

$= \frac{36\text{ V}}{6.0\,\Omega}$

$= 6.0$ A

$R_2 = 18\,\Omega \Rightarrow I_2 = \frac{V}{R_2}$

$= \frac{36\text{ V}}{18\,\Omega}$

$= 2.0$ A

$R_3 = 9.0\,\Omega \Rightarrow I_3 = \frac{V}{R_3}$

$= \frac{36\text{ V}}{9.0\,\Omega}$

$= 4.0$ A

b $I_T = I_1 + I_2 + I_3$

$= 6.0\text{ A} + 2.0\text{ A} + 4.0\text{ A}$

$= 12$ A

c $R_{\text{equivalent}} = \frac{V}{I}$

$= \frac{36\text{ V}}{12\text{ A}}$

$= 3.0\,\Omega$

6 a There are two parallel sections of the circuit connected together in series. Calculate the resistance of the parallel sections first:

$\frac{1}{R_{\text{section 1}}} = \frac{1}{2} + \frac{1}{3}$

$= \frac{5}{6}$

$\Rightarrow R_{\text{section 1}} = \frac{6}{5}$

$= 1.2$

$\frac{1}{R_{\text{section 2}}} = \frac{1}{1} + \frac{1}{4}$

$= \frac{5}{4}$

$\Rightarrow R_{\text{section 2}} = \frac{4}{5}$

$= 0.8$

$\Rightarrow R_{\text{equivalent}} = 1.2 + 0.8$

$= 2\,\Omega$

b $I_{\text{battery}} = \frac{V_{\text{battery}}}{R_{\text{equivalent}}}$

$= \frac{6}{2}$

$= 3$ A

c $V_4 = V_1 = V_{\text{section 2}}$

$= I_{\text{battery}} R_{\text{section 2}}$

$= 3 \times 0.8$

$= 2.4$ V

6.4 Exam questions

1 The globes will only light up if they are part of a closed circuit.

a G_1, G_2, G_3 [1 mark]

b G_1, G_3 [1 mark]

c G_1 [1 mark]

2 The 6 Ω resistor is in parallel with the 18 V power supply, so the voltage drop across the resistor is also 18 V.

$V = IR$

$\Rightarrow I = \frac{V}{R}$

$= \frac{18\text{ V}}{6\,\Omega}$

$= 3$ A [1 mark]

3 a $R_1 = \frac{V}{I_1}$

$= \frac{30}{3}$

$= 10\,\Omega$ [1 mark]

b $I_2 = \frac{V}{R_2}$

$= \frac{30}{15}$

$= 2.0\text{ A}$ [1 mark]

c $R_{\text{equivalent}} = \frac{V}{I_{\text{total}}}$

$= \frac{30}{5}$

$= 6.0\,\Omega$ [1 mark]

Alternative method: $10\,\Omega + 15\,\Omega$ resistors in parallel

4 Series branch:

$R = 2 + 4$

$= 6\,\Omega$ [1 mark]

Parallel combination of $6\,\Omega$ and $12\,\Omega$:

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{6} + \frac{1}{12}$$

$\Rightarrow R_{\text{equivalent}} = 4.0\,\Omega$ [1 mark]

5 D

$$\frac{1}{1.6} = \frac{1}{2} + \frac{1}{X}$$

$$\Rightarrow \frac{1}{X} = \frac{1}{1.6} - \frac{1}{2}$$

$= 0.625 - 0.5$

$= 0.125$

$$\Rightarrow X = \frac{1}{0.125}$$

$= 8\,\Omega$

6.5 Non-ohmic devices in series and parallel

Sample problem 12

a When the current through X is 2 A, the voltage is 10 V, so the voltage across Y is also 10 V, as X and Y are in parallel.

b When the voltage across Y is 10 V, the current through Y is seen to be 3 A.

Practice problem 12

a As A and B are in parallel, the voltage across A equals the voltage across B. From the graph, when the current through A is 80 mA, the voltage is 6 V, so the voltage across B is also 6 V.

b When the voltage across B is 6 V, the current through B is seen to be approximately 5 mA.

Sample problem 13

From the graph, at 19 °C the thermistor has a resistance of 1.5 kΩ (1500 Ω).

$$V_{\text{out}} = \left[\frac{R_2}{R_1 + R_2}\right] V_{\text{in}}$$

$$6\text{ V} = \left[\frac{1.5\text{ k}\Omega}{R + 1.5\text{ k}\Omega}\right] \times 9\text{ V}$$

Solving for R gives:

$6 \times (R + 1500) = 1500 \times 9$

$6R + 9000 = 13\,500$

$6R = 4500$

$\Rightarrow R = 750\,\Omega$

The resistance required for the fixed-value resistor to turn on the heater at 19 °C is 750 Ω.

Practice problem 13

From the graph, at 20 °C the thermistor has a resistance of 2 kΩ (2000 Ω). Substituting into the voltage divider equation gives:

$$V_{\text{out}} = \left[\frac{R_2}{R_1 + R_2}\right] V_{\text{in}}$$

$$4\text{ V} = \left[\frac{2\text{ k}\Omega}{R + 2\text{ k}\Omega}\right] \times 9\text{ V}$$

Solving for R gives:

$4 \times (R + 2000) = 2000 \times 9$

$4R + 8000 = 18\,000$

$4R = 10\,000$

$\Rightarrow R = 2500\,\Omega$

$= 2.5\text{ k}\Omega$

The resistance value required for the fixed-value resistor to turn on the warning light at 20 °C is 2.5 kΩ.

Sample problem 14

The voltage to turn on the switch will still be 6 V, so the voltage across the two resistors will be unchanged. The ratio of their resistance values will therefore also be the same. From the graph in sample problem 13, it can be seen that at 18 °C the thermistor's resistance will be greater than it was at 19 °C. So to keep the ratio the same, R must increase.

This can also be explained using current. As the resistance of the thermistor is higher at the lower temperature, there will be less current through both resistors. As the voltage drop across R is to remain the same, its resistance will need to be greater ($V = IR$).

Therefore, the resistance must be increased.

Practice problem 14

As the temperature increases, the resistance of the thermistor decreases and its share of the voltage from the power supply also decreases. Under these conditions the voltage across the fixed-value resistor will increase. Therefore, the voltage-sensitive switch should be connected to the fixed resistor so that it turns on the cooling system as the temperature increases.

From the graph, at 24 °C the thermistor has a resistance of 1.25 kΩ (1250 Ω). Substituting into the voltage divider equation gives:

$$V_{out} = \left[\frac{R_1}{R_1 + R_2}\right] V_{in}$$
$$6 = \left[\frac{R_1}{R_1 + 1250}\right] \times 9$$
$$6R_1 + 7500 = 9R_1$$
$$7500 = 3R_1$$
$$\Rightarrow R_1 = 2500\ \Omega$$
$$= 2.5\ \text{k}\Omega$$

Connect the switch across a 2.5 kΩ fixed resistor.

Sample problem 15

$$V_R = V - V_d$$
$$= 4 - 1.2$$
$$= 2.8\ \text{V}$$
$$I = I_R$$
$$= \frac{V_R}{R}$$
$$= \frac{2.8}{120}$$
$$= 0.023\ \text{A}$$

The current through the circuit is 2.3×10^{-2} A.

Practice problem 15

$$I_1 = I_2 = I_d = I = 0.6\ \text{A}$$
$$V_1 = R_1 I_1 = R_1 I = 3\ \text{V}$$
$$V_2 = R_2 I_2 = R_2 I = 1.2\ \text{V}$$
$$V = V_1 + V_2 + V_d$$
$$\Rightarrow V_d = V - V_1 - V_2$$
$$= 6 - 3 - 1.2$$
$$= 1.8\ \text{V}$$

The switch-on voltage of the diode is 1.8 V.

6.5 Exercise

1 a Reading from the graph, when $V = 100$ V, $I = 6$ mA.

b Reading from the graph, when $I = 16$ mA, $V = 140$ V.

c $R = \frac{V}{I}$

$$= \frac{140\ \text{V}}{16 \times 10^{-3}\ \text{A}}$$
$$= 8.8\ \text{k}\Omega$$

2 a $V = IR$

$$= 6 \times 10^{-3}\ \text{A} \times 5 \times 10^{3}\ \Omega$$
$$= 30\ \text{V}$$

b The current through a series circuit is constant. Therefore, $I = 6.0$ mA.

c When the current is 6 mA, the voltage drop over the non-ohmic device is 100 V (from the graph).

d $V_T = V_1 + V_2$

$$= 100\ \text{V} + 30\ \text{V}$$
$$= 130\ \text{V}$$

3 a $V = IR$

$$= 20 \times 10^{-3}\ \text{A} \times 5.0 \times 10^{3}\ \Omega$$
$$= 100\ \text{V}$$

b As the device is connected in parallel, the voltage drop across the device is equal to the voltage drop across the resistor, 100 V.

c When the voltage drop across the device is 100 V, the current through it is 6 mA.

d $I_T = 20\ \text{mA} + 6\ \text{mA}$

$$= 26\ \text{mA}$$

4 Turning the dial on a light dimmer moves the slider along a potentiometer. This has the effect of changing the resistance of the arm of the voltage divider circuit that is connected to the light.
Changing the resistance of this arm results in a changing voltage experienced by the light. If the voltage across the light decreases, the light will become dimmer.

5 a $P = VI$

$$= 0.6 \times 2.4 \times 10^{-3}$$
$$= 1.82 \times 10^{-2}\ \text{W}$$

b $P_{total} = 40 \times 1.82 \times 10^{-2}$

$$= 0.73\ \text{W}$$

c Using LEDs vastly decreases the amount of power needed to run the traffic lights. This makes traffic lights cheaper to run and decreases their environmental impact.

6.5 Exam questions

1 A
Since V_{out} is half of V_{in}, then $R_{thermistor} = R_1 = 20\ \text{k}\Omega$. From the graph, $T = 10$ °C.

2 D
If temperature increases, $R_{thermistor}$ decreases. Hence, its share of the input voltage decreases and V_{out} decreases.

3 From the graph, at 10 lx, $R_{LDR} = 6\ \text{k}\Omega$ [1 mark]
If $V_{out} = 8.0$ V :

$$8 = 12 \times \frac{6}{(6 + R)}\ \text{in k}\Omega\ \ [1\ \text{mark}]$$
$$\Rightarrow 8R + 48 = 72$$
$$8R = 24$$
$$\Rightarrow R = \frac{24}{8}$$
$$= 3.0\ \text{k}\Omega\ \ [1\ \text{mark}]$$

OR
Since the voltages divide as 4 V and 8 V, so must resistances, so:

$$R = \frac{1}{2} R_{LDR}$$
$$= 3.0\ \text{k}\Omega$$

4 As light intensity increases, R_{LDR} decreases. [1 mark] We still want the same voltage ratio, so we must keep the same resistance ratio. Hence, if R_{LDR} decreases, then so must R. [1 mark]

5 a 0 A. [1 mark] The diode is reverse biased so no current will flow. [1 mark]

b The diode needs to be connected in forward bias. [1 mark]

$V_R = 1.5 - 0.7$
$= 0.8\text{ V}$

$I_{\text{circuit}} = I_R$
$= \dfrac{0.8}{2000}$
$= 400\ \mu\text{A}$ [1 mark]

c Removing the resistor would greatly increase the current through the diode, as the resistance of the diode, and the circuit, would effectively be zero. [1 mark] This would damage the diode instead of increasing its power output. [1 mark] Hira's suggestion is not a good idea. [1 mark]

6.6 Power in circuits

Sample problem 16

$P_T = 600 + 450 + 1000$
$= 2050\text{ W}$

$I_T = \dfrac{P_T}{V}$
$= \dfrac{2050\text{ W}}{230\text{ V}}$
$= 8.91\text{ A}$

The total current flowing in the circuit is 8.91 A.

Practice problem 16

The total power being used in the circuit is:
$400 + 200 + 500 + 60 = 1160\text{ W}$

$I_T = \dfrac{P_T}{V}$
$= \dfrac{1160\text{ W}}{230\text{ V}}$
$= 5.04\text{ A}$

The total current flowing through the circuit is 5.04 A.

6.6 Exercise

1 a $R_{\text{equivalent}} = R_1 + R_2 + R_3$
$= 25 + 15 + 10$
$= 50\ \Omega$

$I = \dfrac{V}{R_{\text{equivalent}}}$
$= \dfrac{10}{50}$
$= 0.20\text{ A}$

b $25\ \Omega \Rightarrow V = IR$
$= 0.2 \times 25$
$= 5.0\text{ V}$

$15\ \Omega \Rightarrow V = IR$
$= 0.2 \times 15$
$= 3.0\text{ V}$

$10\ \Omega \Rightarrow V = IR$
$= 0.2 \times 10$
$= 2.0\text{ V}$

c $25\ \Omega \Rightarrow P = VI$
$= 5.0 \times 0.2$
$= 1.0\text{ W}$

$15\ \Omega \Rightarrow P = VI$
$= 3.0 \times 0.2$
$= 0.6\text{ W}$

$10\ \Omega \Rightarrow P = VI$
$= 2.0 \times 0.2$
$= 0.4\text{ W}$

d $P_T = P_1 + P_2 + P_3$
$= 1.0 + 0.6 + 0.4$
$= 2.0\text{ W}$

2 a $\dfrac{1}{R_{\text{equivalent}}} = \dfrac{1}{25} + \dfrac{1}{15} + \dfrac{1}{10}$
$= \dfrac{6}{150} + \dfrac{10}{150} + \dfrac{15}{150}$
$= \dfrac{31}{150}$

$\Rightarrow R_{\text{equivalent}} = \dfrac{150}{31}$
$= 4.8\ \Omega$

$I = \dfrac{V}{R_{\text{equivalent}}}$
$= \dfrac{10}{1} \times \dfrac{31}{150}$
$= 2.1\text{ A}$

b As the resistors are connected in parallel, the voltage across each resistor is equal to the voltage supplied. The voltage across each resistor = 10 V.

c $25\ \Omega \Rightarrow P = \dfrac{V^2}{R}$
$= \dfrac{(10\text{ V})^2}{25\ \Omega}$
$= 4.0\text{ W}$

$15\ \Omega \Rightarrow P = \dfrac{V^2}{R}$
$= \dfrac{(10\text{ V})^2}{15\ \Omega}$
$= 6.7\text{ W}$

$10\ \Omega \Rightarrow P = \dfrac{V^2}{R}$
$= \dfrac{(10\text{ V})^2}{10\ \Omega}$
$= 10\text{ W}$

d $P_T = P_1 + P_2 + P_3$
$= 4.0 + 6.7 + 10$
$= 21.7\text{ W}$

6.6 Exam questions

1 $P = VI$

$$0.5 = \frac{V^2}{R_{\text{equivalent}}} = \frac{V^2}{4R_1} = \frac{20^2}{4R_1} = \frac{100}{R_1}$$

$$\Rightarrow R_1 = \frac{100}{0.5} = 200\,\Omega$$

$$R_2 = 3 \times R_1 = 3 \times 200 = 600\,\Omega \text{ [1 mark]}$$

2 The total power is:
$300 + 1400 + 1000 = 2700$ W [1 mark]

$$I_T = \frac{P_T}{V} = \frac{2700}{230} = 11.7\text{ A [1 mark]}$$

3 a $P = VI$

$$= 230 \times 20 \text{ [1 mark]}$$
$$= 4600\text{ W or } 4.6\text{ kW [1 mark]}$$

b Two appliances can be connected into the circuit [1 mark]: either the fridge and dishwasher ($3 + 1.4 = 4.4$ kW), or the oven and dishwasher ($1.8 + 1.4 = 3.2$ kW) [1 mark].

4 To determine the power, we first need to calculate the total current flowing through the circuit.
Let's start with calculating the equivalent effective resistance of the circuit.
The equivalent resistance for the two resistors in series, R_1 and R_2, is $R_{\text{series}} = R_1 + R_2 = 4\text{ k}\Omega$. [1 mark]
The equivalent resistance, $R_{\text{equivalent}}$, of the circuit (for R_{series} and R_3 now in parallel) is such that:

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_{\text{series}}} + \frac{1}{R_3}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{4}{3}\text{ k}\Omega \text{ [1 mark]}$$

The total current, I_T, flowing through the circuit is:

$$I_T = \frac{V}{R_{\text{equivalent}}} = 9\text{ mA [1 mark]}$$

The total power output is:

$$P_T = V \times I_T = 108\text{ mW [1 mark]}$$

5 Express the effective resistance of the circuit as a function of R_x:

- For the two resistors in parallel:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_x} + \frac{1}{R_x} = \frac{2}{R_x}$$

$$\Rightarrow R_{\text{parallel}} = \frac{R_x}{2} \text{ [1 mark]}$$

- For the two resistors now in series:

$$R_{\text{equivalent}} = R_x + R_{\text{parallel}} = \frac{3R_x}{2} \text{ [1 mark]}$$

Using $P_T = V \times I_T$ and $V = R_{\text{equivalent}} I_T$ yields:

$$P_T = \frac{V^2}{R_{\text{equivalent}}}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{V^2}{P_T} = \frac{12^2}{200} = 0.72\,\Omega \text{ [1 mark]}$$

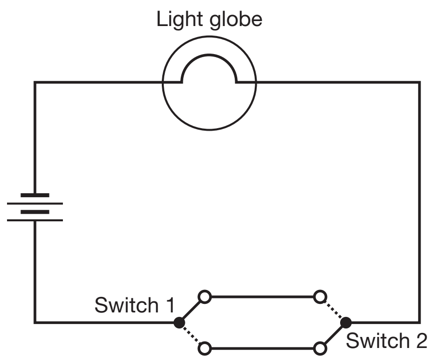
$$R_x = \frac{2}{3} R_{\text{equivalent}}$$

$$\Rightarrow R_x = 0.48\,\Omega \text{ [1 mark]}$$

6.7 Review

6.7 Review questions

1



2 a

$$V_{\text{out}} = \frac{R_2 V_{\text{in}}}{R_1 + R_2}$$

$$\Rightarrow 2.5 = \frac{10R}{10 + R}$$

$$25 + 2.5R = 10R$$
$$7.5R = 25$$
$$\Rightarrow R = 3.3\text{ k}\Omega$$

b

$$V_{\text{out}} = \frac{R_2 V_{\text{in}}}{R_1 + R_2}$$

$$\Rightarrow 6.0 = \frac{5.0 \times 9.0}{R + 5.0}$$

$$6R + 30 = 45$$
$$6R = 15$$
$$\Rightarrow R = 2.5\text{ k}\Omega$$

3 a With the switch open there is no current passing through the switch and the same current flows through the two resistors in series.
The effective resistance of the circuit is $20 + 60 = 80\,\Omega$.

$$V = IR$$

$$\Rightarrow I = \frac{V}{R} = \frac{24}{80} = 0.30\text{ A}$$

b The power dissipated can be calculated using the formula:
$P = VI = RI^2$
$\Rightarrow P(60\,\Omega) = 60 \times 0.3^2 = 5.4\,\text{W}$
$\Rightarrow P(20\,\Omega) = 60 \times 0.3^2 = 1.8\,\text{W}$

c With the switch closed the 60 Ω resistor is short circuited, so current flows through the 20 Ω resistor only.
$$V = IR$$
$$\Rightarrow I = \frac{V}{R} = \frac{24}{20} = 1.2\text{ A}$$

d The power dissipated can be calculated using the formula:
$P = VI = RI^2$
$\Rightarrow P(20\,\Omega) = 20 \times 1.2^2 = 28.8\,\text{W}$

4 a
$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{8} + \frac{1}{12} = \frac{5}{24}$$
$$\Rightarrow R_{\text{parallel}} = \frac{24}{5} = 4.8\ \Omega$$
$$R_{\text{equivalent}} = R_3 + R_{\text{parallel}} = 10 + 4.8 = 14.8\ \Omega$$

b
$$I_3 = I_{\text{battery}} = \frac{30}{14.8} = 2.03\text{ A}$$
$$V_3 = I_3R_3 = 2.03 \times 10 = 20.3\text{ V}$$

c
$$I_1 = \frac{V_1}{R_1} = \frac{V - V_3}{R_1} = \frac{30 - 20.3}{8} = 1.22\text{ A}$$

5 a $V_{\text{source}} = I(R_1 + R_2) =$ constant. So as the variable resistance is increased, the resistance of the series circuit increases. The current will therefore decrease and the lamp will dim.

b The power provided to the circuit, P, is given by $P = \frac{V^2}{R}$.
V_{source} is constant, so as the resistance of the series circuit increases, the power in the circuit will decrease.

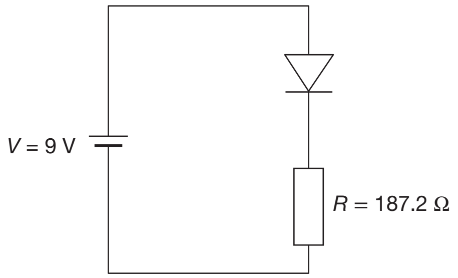
c In the parallel circuit the voltage drop across the arms of the circuit will be equal. The resistance of the lamp is constant, so the current through it will be constant: $I = \frac{V}{R}$. Increasing the resistance of the variable resistor will have no effect on the brightness of the lamp.

d Connecting the lamp and the resistor in parallel will reduce the effective resistance of the circuit. As $P = \frac{V^2}{R}$, the power consumed in the circuit will increase.

6 a From the graph, when the temperature is 150 °C, the resistance is 20 Ω.

b From the graph, when the temperature is 200 °C, the resistance is 10 Ω.
$V_{\text{in}} = 9.0\,\text{V}$, $V_{\text{out}} = 6.0\,\text{V}$
$$V_{\text{out}} = \left[\frac{R_1}{R_1 + R_2}\right] V_{\text{in}}$$
$$6 = \left[\frac{10}{R + 10}\right] \times 9$$
$$6R + 60 = 90$$
$$6R = 30$$
$$\Rightarrow R = 5\ \Omega$$

7 The current is flowing through the circuit, thus the diode is forward biased.
The same current flows through the whole circuit, thus it is in series; the circuit needs a resistor in series with the forward-biased diode.
The voltage across the resistor is: $9 - 1.7 = 7.3$ V
The resistance of the resistor needs to be:
$$R = \frac{7.3}{3.9 \times 10^{-2}} = 187.2\ \Omega$$



8 a The power used in the circuit equals the sum of the power being dissipated in each element. So:
$$P_T = 9\text{ W} + 24\text{ W} = 33\text{ W}$$
$$P = \frac{V^2}{R}$$
$$\Rightarrow R = \frac{V^2}{P} = \frac{12^2}{33} = \frac{144}{33} = 4.4\Omega$$

b No. The 24 W lamp will be brighter. Although there is the same energy dissipated per coulomb of charge within each lamp, the larger current flowing through the 24 W lamp means that more energy is being dissipated per second.

9 a R_1 and R_2 are in series with each other. R_3 and R_4 are in series with each other.
$8 + 10 = 18\ \Omega$
$12 + 15 = 27\ \Omega$
These two branches of the circuit are in parallel with each other:
$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{18} + \frac{1}{27} = \frac{5}{54}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{54}{5}$$
$$= 10.8\ \Omega$$

b The branch containing R_3 and R_4 receives the full 6 V from the battery.

$$I_3 = I_4$$
$$= \frac{V_{\text{battery}}}{R_{\text{branch}}}$$
$$= \frac{6}{27}$$
$$= 0.22\text{A}$$

c R_1 and R_2 receive the same current. This branch of the circuit receives the full 6 V from the battery.

$$I_1 = I_2$$
$$= \frac{V_{\text{battery}}}{R_{\text{branch}}}$$
$$= \frac{6}{18}$$
$$= 0.33\text{ A}$$

Using $V = IR$, the voltage across R_2 can now be found.

$$V_1 = I_1 R_2$$
$$= 0.33 \times 10$$
$$= 3.3\text{ V}$$

10 The total power being drawn from the strip is:

$$P_{\text{T}} = 100 + 60 + 28$$
$$= 188\text{ W}$$
$$I_{\text{T}} = \frac{V}{P_{\text{T}}}$$
$$= \frac{230}{188}$$
$$= 1.22\text{ A}$$

6.7 Exam questions

Section A — Multiple choice questions

1 C

$$R_{\text{equivalent}} = R_1 + R_2 + R_3$$
$$= 20 + 30 + 50$$
$$= 100\ \Omega$$

2 A

From $V = R_{\text{equivalent}} I$:

$$I = \frac{V}{R_{\text{equivalent}}}$$
$$= \frac{9\text{ V}}{100\ \Omega}$$
$$= 0.09\text{ A}$$
$$= 90\text{ mA}$$

3 C

The resistors are in series.

$$V_{\text{T}} = I(R_1 + R_2 + R_3)$$
$$= 0.09\text{ A} \times (20 + 30 + 50)\ \Omega$$
$$= 9\text{ V}$$

4 B

The resistors are in parallel. So the voltage drop across the 9 Ω resistor = 36 V.

$$V = RI$$
$$\Rightarrow I = \frac{V}{R}$$
$$= \frac{36\text{ V}}{9\ \Omega}$$
$$= 4\text{ A}$$

5 A

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{6} + \frac{1}{18} + \frac{1}{9}$$
$$= \frac{3 + 1 + 2}{18}$$
$$= \frac{6}{18}$$
$$= \frac{1}{3}$$
$$\Rightarrow R_{\text{equivalent}} = 3\ \Omega$$

6 D

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{6} + \frac{1}{18} + \frac{1}{9}$$
$$= \frac{3 + 1 + 2}{18}$$
$$= \frac{6}{18}$$
$$= \frac{1}{3}$$
$$R_{\text{equivalent}} = 3\ \Omega$$

$$V = IR$$
$$\Rightarrow I = \frac{V}{R}$$
$$= \frac{36\text{ V}}{3\ \Omega}$$
$$= 12\text{ A}$$

7 C

The equivalent effective resistance for this circuit is such that:

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{x} + \frac{1}{x} + \frac{1}{x}$$
$$= \frac{3}{x}$$
$$\Rightarrow R_{\text{equivalent}} = \frac{x}{3}\ \Omega$$

Using $V = R_{\text{equivalent}} \times I$ yields :

$$R_{\text{equivalent}} = \frac{V}{I}$$
$$= 45\ \Omega$$
$$= \frac{x}{3}$$
$$\Rightarrow x = 135\ \Omega$$

8 C

$$V_{\text{out}} = \frac{4R \times 20\text{ V}}{R + 4R}$$
$$= \left(\frac{4R}{5R}\right) 20\text{ V}$$
$$= 0.8 \times 20\text{ V}$$
$$= 16\text{ V}$$

9 C

$$V_{out} = \frac{R_1 V_{in}}{R_1 + R_2}$$
$$= 4\text{ V}$$
$$V_{in} = 6\text{ V}$$
$$R_1 = 3\text{ k}\Omega$$
$$R_2 = R$$
$$\Rightarrow 4\text{ V} = \frac{R\text{ k}\Omega \times 6\text{ V}}{(3 + R)\text{ k}\Omega}$$
$$12 + 4R = 6R$$
$$2R = 12$$
$$\Rightarrow R = 6\text{ k}\Omega$$

10 C

Thermistors are resistors that depend on temperature. As the potential difference across a thermistor increases, the temperature will not change instantaneously, so the resistance does not increase instantaneously. By Ohm's Law, the current will increase.

Section B — Short answer questions

11 **a** This is a series circuit, so:

$$R_{equivalent} = 1500 + 2500$$
$$= 4000\,\Omega$$

Using $V = IR$, $I = 3$ mA. [1 mark]

b Substituting $V = IR$, the respective voltages are:

$$V_1 = 3 \times 10^{-3} \times 1500$$
$$= 4.5\text{ V}$$
$$V_2 = 3 \times 10^{-3} \times 2500$$
$$= 7.5\text{ V}$$

(1 mark for giving both correct voltages)

c In parallel the voltage drop across each person will be 12 V. [1 mark]

d The current in each will be:

$$I_1 = \frac{V}{R_1}$$
$$= 8\text{mA in the first person}$$
$$I_2 = \frac{V}{R_2}$$
$$= 48\text{ mA in the second person}$$

(1 mark for giving both correct currents)

12 **a** The current through the unknown resistor is the same as that for the 5 kΩ resistor.

$$V = IR$$
$$\Rightarrow I = \frac{V}{R}$$
$$= \frac{8\text{ V}}{5\text{ k}\Omega}$$
$$= 1.6\text{ mA}$$

Using $V = IR$ for the unknown resistor:

$$R = \frac{V}{I}$$
$$= \frac{1}{0.0016}$$
$$= 625\,\Omega \text{ [1 mark]}$$

b
$$P = VI$$
$$= 9\text{ V} \times 0.0016\text{ A}$$
$$= 0.0144\text{ W}$$
$$= 14.4\text{ mW [1 mark]}$$

c Decreasing the resistance of the variable resistor decreases the effective resistance of the circuit, which in turn increases the current. As the current increases, V_{ab} will also increase. [1 mark]

13 **a** From the graph, when the temperature is 200 °C, the resistance is 400 Ω. [1 mark].

b From the graph, when the temperature is 100 °C, the resistance is 5000 Ω. [1 mark].

$$V_{in} = 9.0\text{ V}$$
$$V_{out} = 4.5\text{ V}$$
$$R_2 = 5000\,\Omega$$
$$R_1 = ?$$
$$V_{out} = \left[\frac{R_2}{R_1 + R_2}\right] V_{in}$$
$$4.5 = \left[\frac{5000}{R_1 + 5000}\right] \times 9$$
$$4.5R_1 + 22\,500 = 45\,000$$
$$4.5R_1 = 22\,500$$
$$\Rightarrow R_1 = 5000\,\Omega \text{ [1 mark]}$$

14 **a**
$$\frac{V_{out}}{V_{in}} = \frac{2.0}{6.0}$$
$$= \frac{1}{3}$$

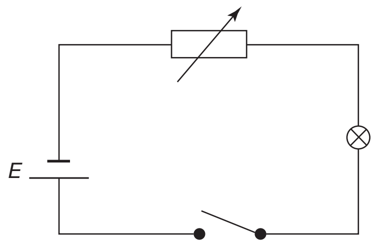
So $\frac{R_2}{R_1 + R_2} = \frac{1}{3}$ [1 mark]

$$\frac{R_2}{3.0 + R_2} = \frac{1}{3}$$
$$3R_2 = 3.0 + R_2$$
$$2R_2 = 3$$
$$\Rightarrow R_2 = \frac{3.0}{2}$$
$$= 1.5\text{ k}\Omega \text{ [1 mark]}$$

b At 100 °C the graph shows that $R_2 = 400\,\Omega$.
Substitute in the known values:

$$V_{out} = \frac{R_2 V_{in}}{R_1 + R_2}$$
$$\Rightarrow V_{out} = \frac{400}{3000 + 400} \times 6.0$$
$$= 0.7\text{ V [1 mark]}$$

15



The circuit should contain a battery, a light and a variable resistor, which will dictate the current through the light and hence, the brightness. [1 mark] The circuit consumes/dissipates more power when the light is bright since the current flowing through the globe is greater, and for the circuit $P = VI$. [1 mark]

Topic 7 — Using electricity and electrical safety

7.2 Household electricity and usage

Sample problem 1

a $P = VI$

$\Rightarrow I = \frac{P}{V}$

$= \frac{1400\text{ W}}{230\text{ V}}$

$= 6.09\text{ A}$

When operating normally the toaster draws 6.09 A.

b $V = IR$

$\Rightarrow R = \frac{V}{I}$

$V = 230\text{ V}, I = 6.09\text{ A}$

$R = \frac{V}{I}$

$= \frac{230\text{ V}}{6.09\text{ A}}$

$= 37.8\ \Omega$

The resistance of the toaster element when hot is 37.8 Ω.

Practice problem 1

a Using $P = VI$:

$I = \frac{15}{230}$

$= 65\text{ mA}$

The current through the globe when it is operating normally is 65 mA.

b Using $V = IR$:

$R = \frac{230}{0.065}$

$= 3500\ \Omega$

$= 3.5\text{ k}\Omega$

The resistance of the globe when it is operating normally is 3.5 kΩ.

Alternatively, the resistance could also have been calculated using the following relationship:

$P = \frac{V^2}{R}$

$\Rightarrow R = \frac{V^2}{P}$

$= \frac{230^2}{15}$

$= 3.5\text{ k}\Omega$

Sample problem 2

Energy (kW h) = power (kW) × time (h)

$1\text{ kW h} = 1\text{ kW} \times 1\text{ h}$

$= 1000\text{ W} \times (60 \times 60)\text{ s}$

$= 3.6 \times 10^6\text{ J}$

$= 3.6\text{ MJ}$

1 kW h represents 3.6 MJ.

Practice problem 2

$E = 8\text{ kW h}$

$= 8000\text{ W} \times 3600\text{ s}$

$= 28.8 \times 10^6\text{ J}$

$= 28.8\text{ MJ}$

28.8 MJ of energy was used.

Sample problem 3

a $P = 230\text{ V} \times 0.37\text{ A}$

$= 85\text{ W}$

The power rating of the television is 85 W.

b Power = 85 W

Time = 5 hours per day for 28 days

= 140 h

$E = 85\text{ W} \times 140\text{ h}$

$= 12\,000\text{ W h}$

$= 12\text{ kW h}$

The television consumes 12 kW h if it is operated for 5 hours a day for 4 weeks.

c Cost = 12 kW h × 16.381 cents

= 197 cents

= $1.97

The cost of running the television is $1.97.

Practice problem 3

a $P = VI$

$I = \frac{2.4}{230}$

$= 10\text{ mA}$

10 mA of current flows through the system when it is on standby.

b $E = Pt$

$= 2.4 \times 7 \times 24 \times 60 \times 60$

$= 1.45\text{ MJ}$

1.45 MJ of energy is consumed by the system when left on standby for one week.

c E (in kW h) $= 2.4 \times 7 \times 24$

$= 403\text{ kW h}$

Cost at 12 cents per kW h = $48.38

It costs $48.38 to leave the system on standby for one week.

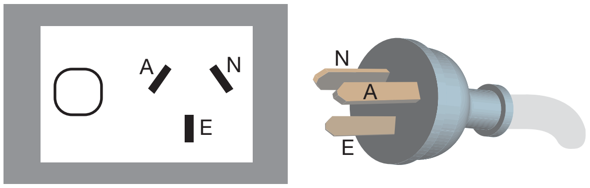
7.2 Exercise

1
- It is an alternating current.
- The voltage varies between +325 V and −325 V.
- The voltage oscillates 50 times per second.
- The voltage provides heating effects equivalent to a DC voltage of 230 V.

2 An overload occurs when too many appliances are connected to a circuit. The overall effect is to draw too much current. The excessive current can cause wires to melt or materials surrounding the wires to catch fire.

3 Active: brown
Neutral: blue
Earth: green-and-yellow striped

4



5 The neutral wire is used to provide an insulated path for the electricity to return to the generator. The earth wire connects the metal case of the appliance to earth so that, if a live active wire accidentally contacts the outer case, a low-resistance path is provided for the electricity to return to earth.

6 The earth wire is used in household lighting circuits when the light fittings have a metal case. Plastic and glass fittings are good insulators and, unlike metal cases, do not need to be earthed.

7 Cost = number of kW h × 31.28 cents/kW h

$$= \left(\frac{300 \times 365 \times 12}{1000}\right) \times 31.28$$

$= \$411.02$

8 **a** For each product:

$E = Pt$

$$= \frac{P \times 365 \times 24}{1000}$$

$= 8.76P$ kW h

Product	Energy (kW h)
Laptop computer	127.0
Modem	29.8
Cordless phone equipment	32.4
DVD player	21.0
Television	54.3

b Total energy = 264.5 kW h

264.5×1.444 kg CO_2 = 381.9 kg CO_2

7.2 Exam questions

1 B

$$R = \frac{V^2}{P}$$

$$= \frac{230^2}{3600}$$

$\simeq 15\,\Omega$

2 Period $T = \frac{1}{f}$

$$= \frac{1}{50} \text{ [1 mark]}$$

$= 0.02$ s (or 20 ms) [1 mark]

3 The neutral wire is earthed, or connected to the ground, at the switchboard. [1 mark]
Hence, there is zero volts difference between the neutral wire and a grounded person. [1 mark]

4 $R = \frac{V^2}{P}$

$$= \frac{240^2}{1200}$$

$= 48\,\Omega$ [1 mark]

$$P_{\text{USA}} = \frac{V^2}{R}$$

$$= \frac{120^2}{48}$$

$= 300$ W [1 mark]

Note: The power is not halved (to 600 W) since both the voltage and current are halved in the USA.

5 E (in kW h) = P (in kW) × t (in h)

$= 2 \times 6 \times 5$ [1 mark]

$= 60$ kW h [1 mark]

7.3 Electrical safety

Sample problem 4

a Toaster:

$$I = \frac{1000 \text{ W}}{230 \text{ V}}$$

$= 4.35$ A

Refrigerator:

$$I = \frac{312 \text{ W}}{230 \text{ V}}$$

$= 1.36$ A

Kettle:

$$I = \frac{1200 \text{ W}}{230 \text{ V}}$$

$= 5.22$ A

Microwave oven:

$$I = \frac{600 \text{ W}}{230 \text{ V}}$$

$= 2.61$ A

Juicer:

$$I = \frac{60 \text{ W}}{230 \text{ V}}$$

$= 0.26$ A

Total current in the circuit:
4.35 + 1.36 + 5.22 + 2.61 + 0.26 = 13.8 A
The fuse will not 'blow'.

b $I = \frac{2400 \text{ W}}{230 \text{ V}}$

$= 10.4$ A

The 2400-W heater will draw an additional 10.4 A, so the total current in the circuit will be 24.2 A. This is much greater than 15 A and the fuse will 'blow'.

Practice problem 4

a Using $P = VI$:

$$I = \frac{P}{V}$$

$$= \frac{(75 + 100)}{230}$$

$= 760$ mA

760 mA of current flows through the system when the fan and light globe are in use.

b Using $P = VI$:

$$I = \frac{P}{V} = \frac{(100 + 2 \times 150)}{230} = 1.7\text{ A}$$

1.7 A of current flows through the system when the fan and two heat lamps are in use.

c Using $P = VI$:

$$= \frac{P}{V} = \frac{(75 + 4 \times 150 + 100)}{230} = 3.4\text{ A}$$

3.4 A of current flows through the system when all the devices are in use.

7.3 Exercise

1 An electric shock is a violent disturbance of the nervous system caused by an electrical discharge or current through the body. Electrocution is death resulting from an electric shock.

2 Breaks or cuts in the skin, and water, oil and other fluids reduce the resistance of human skin.

3 Nerve impulses are electrical in nature. The size of the current will influence the size and type of muscle contraction.

4 Fibrillation is the disorganised rapid contraction of separate parts of the heart so that it pumps no blood.

5 Your muscles would contract and you could grip onto the victim and not be able to let go.

6 The longer the time of exposure, the more severe the shock.

7 Double insulation is a way to protect the user of handheld appliances. There are two separate layers of insulation between the functional parts of the appliance and the user.

8 Voltage doesn't flow; current does. The paper should have reported that there was a voltage drop across his body of 50 000 V. The human body is not resistant to current flow at large voltage. However, it is the size of the current that determines the amount of injury; it is not the voltage that kills.

7.3 Exam questions

1 B

A double-insulated appliance has two separate layers of insulation between the live wires and any external metal parts.

2 Wet hands and feet are more likely in the bathroom. [1 mark] This will reduce skin resistance. [1 mark] Hence, larger current will flow through the body. [1 mark]

3 Current is more important. [1 mark] Current directly affects or disrupts the actions of nerves and muscles. [1 mark] Voltage is mainly important in that higher voltage means larger current. [1 mark] (Even very high electrostatic voltages, at low current, pose relatively little danger.)

4 The earth wire acts only when the metal casing comes into contact with a live wire and so becomes live itself. [1 mark] It allows a large current to flow to earth and so 'blows' the fuse. [1 mark] This disconnects the appliance from the live wire and removes the hazard of shock to the user. [1 mark]

5 A residual current device acts when the current in the active wire is greater than that in the neutral wire. [1 mark] It disconnects the active wire:

- quickly enough to prevent dangerous shock [1 mark]
- at a value of current small enough to not be dangerous. [1 mark]

7.4 Review

7.4 Review questions

1 a $E = Pt$

$$= 3.6 \times 3 \times 365\text{ kW h} = 3942\text{ kW h}$$

b Cost $= 3942 \times \$0.28 = \$1104.00$

2 a The speakers should be connected in parallel because each speaker can be independently controlled and the failure of one does not result in a total loss of sound — the second speaker can still function.

b The power in each speaker is $P = VI$.

Total power:

$$P = 2 \times 230 \times 10 = 4600\text{ W} = 4.6\text{ kW}$$

c Cost $= 4.6 \times 6 \times \$0.32 = \$8.83$

d The fuse should fail at, or just above, the operating current of the device, so just larger than 10 A.

3 a Current $= \frac{1000}{230} = 4.35\text{ A}$

b $R = \frac{V^2}{P} = \frac{230^2}{1000} = 52.9\ \Omega$

c Cost = no. of kW h × 17 c/hour

$$= \frac{(1000 \times 5)}{1000} \times 17 = \$0.85$$

4 a The metal casing on the appliance is connected to the earth wire and not the live wiring that provides energy to the appliance. This keeps the voltage of the casing at zero.

b A shock could occur if the casing has a non-zero voltage, which could happen if the insulation between the casing and the functional parts of the appliance broke down.

5 a An electric shock occurs when electric current passes through the body. Electrocution is death caused by an electric shock.

b An electric shock is more likely to be fatal if there is a large amount of current flowing through the body, the pathway of the current is through the trunk of the body or there is an extended time of exposure to the current.

6 Both fuses and circuit breakers provide overload protection in circuits. As such, each has the potential to protect electrical equipment and reduce the likelihood of fire from overheating. A fuse comprises a metal wire that melts when its current limit is exceeded. Fuses can be used in a variety of places, including plugs, appliances and household circuits. They are also readily used in DC circuits. After the source of an electrical problem is resolved a melted fuse must be replaced, which also means that unless the electrical problem causing the problem is also resolved, replacement fuses will continue to 'blow'.

Circuit breakers comprise resettable switches activated by one of two methods — either by using the heat generated by an excessive current, or by the electromagnetic effect in a circuit to activate a switch. They are initially more expensive to install but are readily reset. They also allow the isolation of circuits within a system of circuits, as found in households.

7 **a** $P_{\text{total}} = 150 + 140 + 60 + 55 + 200$

$= 605\text{ W}$

$I = \dfrac{605}{230}$

$= 2.63\text{ A}$

b $I = \dfrac{3005}{230}$

$= 13.07\text{ A}$

No, the current is less than 15 A.

8 As the magnitude of the current passing through the body increases it takes less time to cause fibrillation. It is therefore important to safely remove a person in shock from the source.

9 Dry skin has a higher resistance than wet hands. Wet hands provide an easier pathway to earth and therefore attract a larger current that can cause damage when travelling through the body.

10 Whereas a circuit breaker provides overload protection in a circuit by detecting increasing current, a residual current device effectively detects a leakage of current, as might happen when there is a short circuit. Residual current devices rely on the magnetic properties of electric currents to detect when there is a difference in the backwards and forwards movement of AC current to then activate a switch. Residual current devices can be activated quickly and at low currents, providing good safety protection as long as the leakage of current is to earth. However, a residual current device is ineffective when a person acts as a conductor between the active and neutral wires of a circuit.

7.4 Exam questions

Section A — Multiple choice questions

1 C

The active wire carriers the largest voltage and is coloured brown.

2 B

A fuse is a short length of conducting wire or strip of metal that melts when the current through it reaches a certain value. This causes the circuit to be broken and prevents the device being damaged.

3 A

Fuses 'blow' when the current is too large, as this melts the conducting wire that forms the fuse.

4 B

Circuit breakers are switches that switch to off when the current exceeds a particular value and a short circuit occurs. They are easily reset by flicking the switch back on.

5 C

$P = VI$

$\Rightarrow I = \dfrac{P}{V}$

$= \dfrac{1840}{230}$

$= 8.0\text{ A}$

6 B

$E = Pt$

$= \dfrac{250 \times 2 \times 24}{1000}$

$= 12\text{ kW h}$

$\text{Cost} = 12\text{ kW h} \times \$0.30$

$= \$3.60$

7 D

The fuse should fail at just above the operating current of the device.

$P = VI$

$\Rightarrow I = \dfrac{P}{V}$

$= \dfrac{2000}{230}$

$= 8.69\text{ A}$ (closest to 9 A so D is correct)

8 C

The earth wire is coated in green and yellow plastic and is an important safety mechanism, connecting the circuit to the earth.

9 B

Double insulation refers to when there are two layers separating the live wire and the external metal casings. If one layer is damaged, the second layer stills allow the live wire to be separate from the metal casings, protecting against electrocution or electric shock.

10 B

It is vital to make sure an individual is not still connected to the electric source in order to prevent current continuously flowing through their body. It is vital to not touch a victim until they are no longer connected, otherwise a person treating them may also suffer an electric shock.

Section B — Short answer questions

11 There will now be two paths for current to flow from the appliance to earth: from the casing of the appliance through the earth wire in the circuit, and alternatively through the person to earth. If the resistance of the pathway through the person is high compared to the appliance to the earth wire, then the current through the person will be smaller. The smaller the current, the better, but this is an AC circuit that has the potential to cause significant injury.

A factor that would improve the safety of the person include not having wet hands or standing on a wet floor. [1 mark] It would also be better if the person was wearing shoes with an insulating sole and was not touching an easier path to earth, such as a kitchen tap. [1 mark] Letting go of the appliance is also important because a longer contact causes more harm. [1 mark] However, as the current through the person increases, it may be necessary for someone else to intervene by turning off the electricity and removing the person from the source.

12 a See the figure at the foot of the page*

(1 mark for each of the following features, maximum 3 marks: circuit breaker, switch, fuse, earth from casing)

b Any two of the following are acceptable answers:

- The earth wire in the main circuit provides a low-resistance conducting path for current should there be a short circuit.
- Insulation between the appliance and the case insulates the metal case from the live circuit.
- Separately earthing the metal case provides a low-resistance conducting path for current to flow to earth if the insulation 'barrier' breaks down.
- A fuse, circuit breaker or residual current device on the active wire between the power supply and the appliance breaks the circuit if the current to the device is excessive, or in the case of a residual current device, there is a current leakage.

(1 mark for each correct explanation; maximum 2 marks)

13 See the figure at the foot of the page*

(1 mark for each correct wire labelled; maximum 3 marks)

14 a $E = Pt$

$= \dfrac{P \times 365 \times 24}{1000}$

$= 8.76 \times P$ kW h

Product	**Power (W)**	**Power (kW h per year)**
Laptop computer	14.5	127.0
Microwave	4.2	36.8
Laser printer	8.5	74.5
Set-top box	11.2	98.1
Television	6.2	54.3

(1 mark for the correct formula; 2 marks for correctly completing the table)

b Total energy = 390.7 kW h [1 mark]

390.7×1.444 kg CO_2 = 564.2 kg CO_2 [1 mark]

c Cost = $0.50 × 390.7

= $195.35 [1 mark]

15 a If the casing of the appliance is connected to earth, a current will still flow. If the resistance of the person is larger than the path through the appliance, the current through the person will be less.

The current through the person is as follows:

$I = \dfrac{V}{R}$

$= \dfrac{230}{(600 \times 1000)\text{ A}}$

$= 0.38$ mA [1 mark]

The person will experience a shock, but provided the person does not continue to hold the appliance it should not be dangerous. [1 mark] See table 7.1 for electric shock current-versus-time parameters.

b Fibrillation could occur at 50 mA. The voltage that corresponds to a current of 50 mA can be calculated by Ohm's Law.

$V = IR$

$= 50 \times 10^{-3} \times 600 \times 10^{3}$

$= 30\,000$ V

$= 30$ kV [1 mark]

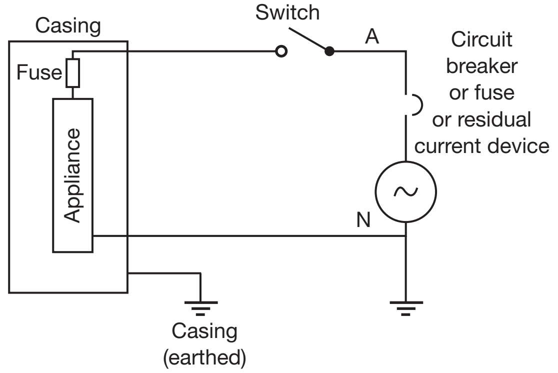
c $I = \dfrac{V}{R}$

$= \dfrac{230}{3000}$

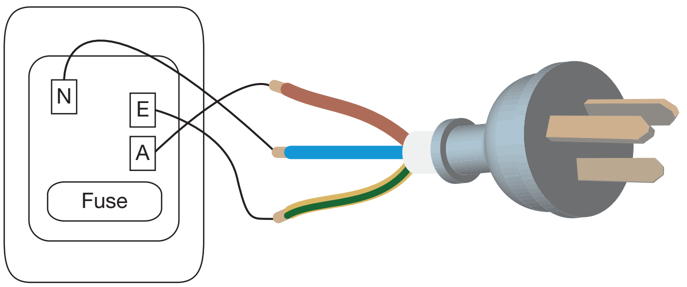
$= 76.7$ mA [1 mark]

With this current, there is a risk that the person's muscles will contract and they will be unable to let go of the appliance. Fibrillation is likely and the person must be safely removed from the source of electricity. [1 mark]

12 a*



13*



Topic 8 — Analysing motion

8.2 Describing movement

Sample problem 1

a $\Delta \boldsymbol{x} = \boldsymbol{x}_2 - \boldsymbol{x}_1$
$= 100 - 0$
$= 100$ m north
The displacement of the hare is 100 metres north.

b Distance $= 80 + 60 + 80$
$= 220$ m
The total distance that the hare travels is 220 metres.

c The tortoise travels a total distance of 100 metres.

d $\Delta \boldsymbol{x} = \boldsymbol{x}_2 - \boldsymbol{x}_1$
$= 20\text{ m} - 80\text{ m}$
$= -60$ m
The displacement is 60 metres south.

Practice problem 1

a Distance $= 4\text{ km} + 4\text{ km} + 4\text{ km} + 4\text{ km}$
$= 16$ km
The cyclist rides a distance of 16 kilometres.

b Displacement:
$c^2 = a^2 + b^2$
$\Rightarrow c = \sqrt{(a^2 + b^2)}$
$\Delta \boldsymbol{x} = \sqrt{(4.0^2 + 4.0^2)}$
$= \sqrt{32}$
≈ 5.7 km
The displacement of the cyclist before she commences her return journey is 5.7 kilometres.

c Displacement from the instant she commenced her return journey until she arrived home is 0 metres.

Sample problem 2

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

$\Rightarrow$ Distance travelled = average speed × time interval

Time interval = 3 h

Average speed $= 250\text{ m s}^{-1}$ $(\times 3.6$ to convert to km h$^{-1})$
$= 900\text{ km h}^{-1}$

Distance travelled $= 900\text{ km h}^{-1} \times 3\text{ h}$
$= 2700\text{ km h}^{-1}$

The approximate distance by air between Melbourne and Perth is 2700 kilometres.

Practice problem 2

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

$\Rightarrow$ Distance travelled = average speed × time interval

Time interval = 8 h
$= 8 \times 60 \times 60 = 28\,800$ s (convert hours to seconds)

Average speed $= 25\text{ m s}^{-1}$

Distance travelled $= 25\text{ m s}^{-1} \times 28\,800\text{ s}$
$= 720\,000$ m
$= 720$ km

The road distance from Canberra to Ballarat is 720 kilometres.

Sample problem 3

a $\text{Average speed} = \dfrac{\text{distance travelled}}{\text{time interval}}$
$= \dfrac{220\text{ m}}{20\text{ s}}$
$= 11\text{ m s}^{-1}$
The average speed of the hare is 11 m s^{-1}.

b Average velocity: $\boldsymbol{v}_{av} = \dfrac{\Delta \boldsymbol{x}}{\Delta t}$
$= \dfrac{100\text{ m north}}{20\text{ s}}$
$= 5\text{ m s}^{-1}$ north
The average velocity of the hare is 5 m s^{-1} north.

Practice problem 3

a $\text{Average speed} = \dfrac{\text{distance travelled}}{\text{time interval}}$
$= \dfrac{4.2\text{ km}}{20\text{ min}}$
$= \dfrac{4200\text{ m}}{(20 \times 60)\text{ s}}$
$= 3.5\text{ m s}^{-1}$
The triathlon participant's average speed is 3.5 m s^{-1}.

b $\Delta \boldsymbol{x} = \boldsymbol{x}_2 - \boldsymbol{x}_1$
$= 2.8\text{ km} - 1.4\text{ km}$
$= 1.4$ km
$= 1400$ m
$\boldsymbol{\Delta} t = 20$ min
$= 20 \times 60$ s
Average velocity: $\boldsymbol{v}_{av} = \dfrac{\Delta \boldsymbol{x}}{\Delta t}$
$= \dfrac{1400\text{ m east}}{1200\text{ s}}$
$= 1.2\text{ m s}^{-1}$ east
The average velocity is 1.2 m s^{-1} east.

Sample problem 4

a Average acceleration: $\boldsymbol{a}_{av} = \dfrac{\Delta \boldsymbol{v}}{\Delta t}$
$= \dfrac{10\text{ m s}^{-1}}{4\text{ s}}$
$= 2.5\text{ m s}^{-2}$

Spiro's average acceleration on his journey to cruising speed is 2.5 m s^{-2} towards the letterbox.

b Average acceleration: $\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}$

$$= \frac{-10 \text{ m s}^{-1}}{2 \text{ s}}$$
$$= -5 \text{ m s}^{-2}$$

Spiro's average acceleration when braking at the letterbox is 5 m s^{-2} away from the letterbox.

c Average acceleration: $\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}$

$$= \frac{8 \text{ m s}^{-1}}{2 \text{ s}}$$
$$= 4 \text{ m s}^{-2}$$

Spiro's average acceleration when braking at the end of his journey is 4 m s^{-2} towards the letterbox.

d The two sections of the trip where acceleration is negative are braking at the letterbox and accelerating back towards home from the letterbox.

e A decreasing speed in the negative direction is a positive change in velocity, and hence a positive acceleration.

Practice problem 4

a Average acceleration: $\boldsymbol{a}_{av} = \frac{\Delta v}{\Delta t}$

$$= \frac{30 \text{ m s}^{-1}}{2 \text{ s}}$$
$$= 15 \text{ m s}^{-2}$$

The magnitude of the cheetah's average acceleration is 15 m s^{-2}.

b **i** Average acceleration: $\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}$

$$\Delta \boldsymbol{v} = 420 \text{ km h}^{-1}$$
$$\Rightarrow \boldsymbol{a}_{av} = \frac{420 \text{ km h}^{-1}}{6.0 \text{ s}}$$
$$= 70 \text{ km h}^{-1} \text{ s}^{-1}$$

The drag-racing car's average acceleration is 70 km h^{-1} s^{-1}.

ii Average acceleration: $\boldsymbol{a}_{av} = \frac{\Delta \boldsymbol{v}}{\Delta t}$

$$\Delta \boldsymbol{v} = \frac{420\,000}{3600}$$
$$= \frac{350}{3} \text{ m s}^{-1}$$
$$\Rightarrow \boldsymbol{a}_{av} = \frac{350}{3 \times 6}$$
$$= 19.4 \text{ m s}^{-2}$$

The drag-racing car's average acceleration is approximately 19.4 m s^{-2}.

8.2 Exercise

1 **a** Scalar. Both magnitude and direction are required for a quantity to be a vector.

b Vector

c Scalar

d Vector

e Vector

2 Convert 100 km h^{-1} to m s^{-1}:

$$100 \text{ km h}^{-1} = \frac{100 \text{ km}}{1 \text{ h}}$$
$$= \frac{100\,000 \text{ m}}{3600 \text{ s}}$$
$$\approx 27.8 \text{ m s}^{-1}$$

(Alternatively, you could simply divide 100 km h^{-1} by 3.6.)

3
$$1.5 \text{ m s}^{-1} = \frac{1.5 \text{ m}}{1 \text{ s}}$$
$$= \frac{0.0015 \text{ km}}{\frac{1}{3600} \text{ h}}$$
$$= 3600 \times 0.00\,15 \text{ km h}^{-1}$$
$$= 5.4 \text{ km h}^{-1}$$

(Alternatively, you could simply multiply by 3.6.)

4 **a** Convert miles to kilometres, multiply by 1.6:

$$55 \text{ miles h}^{-1} = \frac{55 \text{ miles } \times 1.6}{1 \text{ h}}$$
$$= 88 \text{ km h}^{-1}$$

b Convert km h^{-1} to m s^{-1}:

$$88 \text{ km h}^{-1} = \frac{88\,000 \text{ m}}{3600 \text{ s}}$$
$$\approx 24 \text{ m s}^{-1}$$

5 **a** Distance = 400 m + 400 m

= 800 m

The total distance that the jogger has travelled is 800 metres.

b $\Delta \boldsymbol{x} = \boldsymbol{x}_2 - \boldsymbol{x}_1$

= 400 m − 0

= 400 m

The displacement when the jogger starts to run home is 400 metres.

c $\Delta \boldsymbol{x} = \boldsymbol{x}_2 - \boldsymbol{x}_1$

= 400 m − 400 m

= 0 m

The displacement is 0 metres, as the jogger returns to where he started.

6 **a**

Event (m)	$\frac{\text{Distance travelled}}{\text{time interval}}$	Average speed (m s^{-1})
100	$= \frac{100 \text{ m}}{10.49 \text{ s}}$	= 9.53
200	$= \frac{200 \text{ m}}{21.34 \text{ s}}$	= 9.37
400	$= \frac{400 \text{ m}}{47.60 \text{ s}}$	= 8.40
800	$= \frac{800 \text{ m}}{1 \text{ m } 53.28 \text{ s}}$ $= \frac{800 \text{ m}}{113.28 \text{ s}}$	= 7.06
1500	$= \frac{1500 \text{ m}}{3 \text{ m } 50.07 \text{ s}}$ $= \frac{1500 \text{ m}}{230.07 \text{ s}}$	= 6.52

3000	$= \frac{3000\text{ m}}{8\text{ m }6.11\text{ s}}$ $= \frac{3000\text{ m}}{486.11\text{ s}}$	= 6.17
5000	$= \frac{5000\text{ m}}{14\text{ m }6.62\text{ s}}$ $= \frac{5000\text{ m}}{846.62\text{ s}}$	= 5.91
10 000	$= \frac{10\,000\text{ m}}{29\text{ m }1.03\text{ s}}$ $= \frac{10\,000\text{ m}}{1741.03\text{ s}}$	= 5.74

b The fact that the average speed during the 100-metre event is similar to that during the 200-metre event is due to the fact that the acceleration from rest to the maximum speed takes place over a significant fraction of the time taken for the 100-metre event. Even though the maximum speed of the athlete is significantly greater during the 100-metre event, the average speed is similar to the average speed in the 200-metre event.

c $\text{Average speed} = \frac{\text{distance travelled}}{\text{time interval}}$

$\Rightarrow \text{Time interval} = \frac{\text{distance travelled}}{\text{average speed}}$

$= \frac{42\,200\text{ m}}{6.52\text{ m s}^{-1}}$

$= 6472\text{ s}$

$= 107\text{ min }52\text{ s}$

$= 1\text{ h }47\text{ min }52\text{ s}$

d Only Florence Griffith Joyner. Her event is the only one that involves straight-line motion.

7 a $\text{Average speed} = \frac{\text{distance travelled}}{\text{time interval}}$

Distance travelled = 5.0 km

Time interval = 30 min

= 0.5 hour

i $\text{Average speed} = \frac{5.0}{0.5}$

$= 10\text{ km h}^{-1}$

ii $\text{Average speed} = 10\text{ km h}^{-1}$

$= \frac{10\text{ km}}{1\text{ h}}$

$= \frac{10\,000\text{ m}}{3600\text{ s}}$

$\approx 2.8\text{ m s}^{-1}$

b $\text{Average speed} = \frac{\text{distance travelled}}{\text{time interval}}$

$\Rightarrow \text{Time interval} = \frac{\text{distance travelled}}{\text{average speed}}$

Average speed = 60 km h^{-1}

Distance travelled = 200 m

= 0.200 km

$\Rightarrow \text{Time interval} = \left(\frac{0.200\text{ km}}{60\text{ km h}^{-1}} \times 60 \times 60\right)$

$= 12\text{ s}$

8 a $\text{Average speed} = \frac{\text{distance travelled}}{\text{time interval}}$

$= \frac{3000}{196.037\text{ s}}$

$= 15.303\text{ m s}^{-1}$

$= 15.30\text{ m s}^{-1}$

b $\text{Average speed} = \frac{\text{distance travelled}}{\text{time interval}}$

$\Rightarrow \text{Time interval} = \frac{\text{distance travelled}}{\text{average speed}}$

$= \frac{151\,000\text{ km}}{15.303\text{ m s}^{-1}}$

$= 9867.35\text{ s}$

$\approx 164.46\text{ min}$

$\approx 2\text{ h }44\text{ min}$

c $\text{Time interval} = \frac{\text{distance travelled}}{\text{average speed}}$

$= \frac{151}{80}$

$= 1.89\text{ h}$

$= 1\text{ h }53\text{ min}$

d i $\text{Average speed} = \frac{\text{distance travelled}}{\text{time interval}}$

$= \frac{302\text{ km}}{4\text{ h}}$

$= 75.5\text{ km h}^{-1}$

ii $v_{av} = \frac{\Delta x}{\Delta t}$

$= 0\text{ km h}^{-1}$

Since Δx (displacement) = 0

9 $\text{Time for the tortoise} = \frac{\text{distance travelled}}{\text{average speed}}$

$= \frac{1000\text{ m}}{0.075\text{ m s}^{-1}}$

$= 13\,333\text{ s}$

$\text{Time for hare, at maximum speed} = \frac{\text{distance travelled}}{\text{average speed}}$

$= \frac{1000\text{ m}}{20\text{ m s}^{-1}}$

$= 50\text{ s}$

Duration of the hare's nap = 13 333 s − 50 s

= 13 000 s (to two significant figures)

= 221 min 23 s

= 3 h 36 min 40 s

In a tied race, the hare must have napped for approximately 3 hours 37 minutes.

10 $\text{Average speed} = \frac{\text{distance travelled}}{\text{time interval}}$

$= \frac{120 + 120 + 120}{20 + 30 + 60}$

$= \frac{360}{110}$

$\approx 3.3\text{ m s}^{-1}$

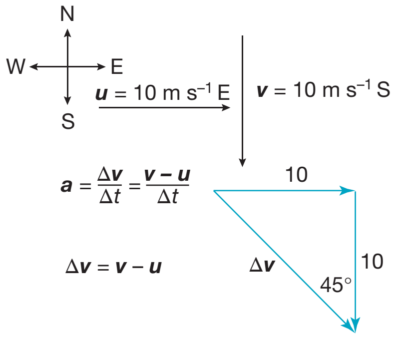
11 $\text{Average speed} = \frac{\text{distance travelled}}{\text{time interval}}$

$= \frac{300 + 300}{\frac{300}{80} + \frac{300}{100}}$

$\approx 89\text{ km h}^{-1}$

12 **a** **i** $\Delta s = s_1 - s_2$
$= 60 - 100$
$= -40\ \text{km h}^{-1}$
The driver reduces their speed by 40 km h^{-1}.
ii 40 km h^{-1} south (or –40 km h^{-1} north)
b **i** –20 m s^{-1}
ii –20 m s^{-1} in original direction (or +20 m s^{-1} opposite to original direction)
c **i** +5 m s^{-1}
ii –55 m s^{-1} in original direction (or +55 m s^{-1} opposite to original direction)

13 Yes, there is an acceleration. Even though the speed has not changed, the velocity has changed.



The magnitude of $\Delta \boldsymbol{v} = \sqrt{(10^2 + 10^2)} = 14.1\ \text{m s}^{-1}$. Its direction is south-east. The acceleration is therefore not 0.

14 This is an estimation question so responses will vary.
For a car that accelerates from rest to 60 km h^{-1} (17 m s^{-1}) in say 5 seconds:

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{17}{5} = 3.4\ \text{m s}^{-2}$$

15 This is an estimation question so responses will vary.
$\boldsymbol{v}_{av}$ for a 100-metre sprint in say 10.49 s = 9.5 m s^{-1}:
Estimate $\boldsymbol{v}_{max}$ = 12 m s^{-1} is reached after 2 seconds.

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{12\ \text{m s}^{-1}}{2\ \text{s}} = 6\ \text{m s}^{-2}$$

8.2 Exam questions

1 B
Both Q and R have the acceleration opposing the initial velocity, so decreasing the magnitude of the velocity.

2 Total displacement $= +12 - 20$
$= -8$ m [1 mark]

$$\text{Average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{-8}{4}$$

$= -2\ \text{m s}^{-1}$ (1 mark for the magnitude AND sign)

3 $\Delta \boldsymbol{x} = \boldsymbol{v}_{av}\,\Delta t$
$= 20 \times 2 \times 60$ [1 mark]
$= 2400$ m north-west [1 mark]

4 $\boldsymbol{a} = \dfrac{\Delta \boldsymbol{v}}{\Delta t}$
$= \dfrac{0 - 10}{5}$
$= \dfrac{-10}{5}$ [1 mark]
$= -2.0\ \text{m s}^{-2}$ [1 mark]

5 Yes the car is accelerating. [1 mark] A change in direction (or change in magnitude) of velocity requires an acceleration. [1 mark]

8.3 Analysing motion graphically

Sample problem 5

From the graph, the time at which Bolter Beryl's speed dropped below Steady Sam's is approximately 4.7 seconds.

Area under the graph $= 4.7 \times 6.7$
$= 31$ m

Steady Sam's displacement was 31 metres at 4.7 seconds.

Practice problem 5

a From the graph, Bolter Beryl's displacement at 2 seconds is the area under the graph at $t = 2$ seconds.
Displacement $= (12 \times 2) + \frac{1}{2}(18 - 12) \times 2$
$= 24 + 6$
$= 30$ m
Bolter Beryl's displacement at 2 seconds is 30 metres.

b Bolter Beryl's distance after 4.7 seconds is:
Distance = area under graph
$= \frac{1}{2}(18 + 7) \times 4.7$
$= 59$ m
Steady Sam's distance after 4.7 seconds is:
Distance = area under graph
$= 6.7 \times 4.7$
$= 31$ m
Beryl is ahead by $59 - 31 = 28$ m.

8.3 Exercise

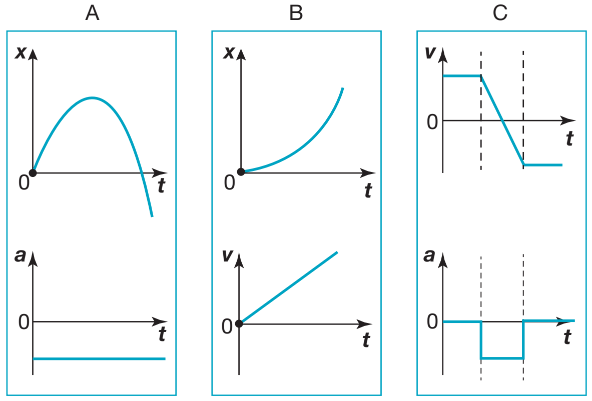
1 **a** B and C both start from the same position, but at different times.
b B and D both start from the same position at the same time.
c A and E both take the same time to travel the same distance, but in opposite directions.
d A and E are both moving towards the origin for the duration of their motion; therefore, they are moving towards each other.
e The speed is the magnitude of velocity, which is the magnitude of the gradient of the position-versus-time graph. Object D has the smallest gradient, so has the lowest speed.

2 a A: Constant negative acceleration with an initial positive velocity

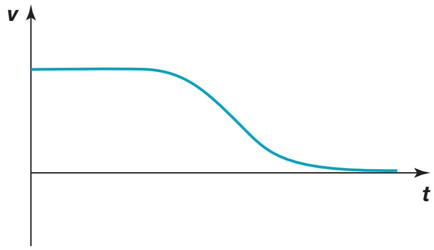
B: Constant positive acceleration from rest

C: Constant positive velocity, followed by an interval of constant negative acceleration until a negative velocity equal in magnitude to the initial velocity is reached. The velocity then remains constant.

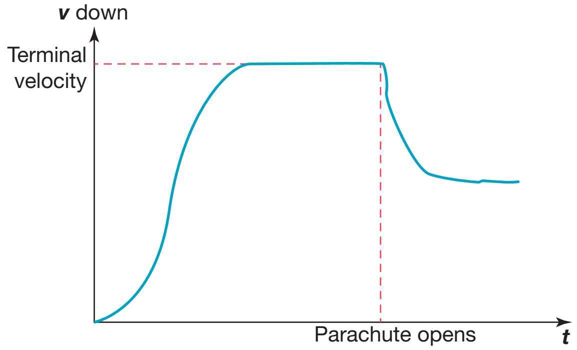
b



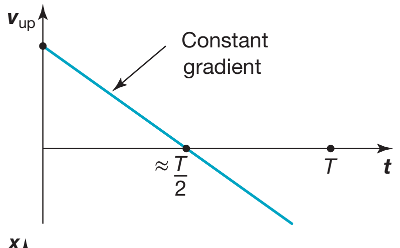
3 a



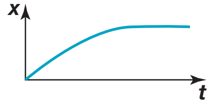
b



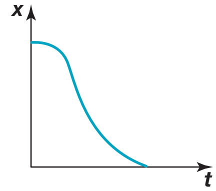
c



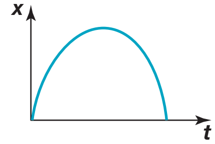
4 a



b



c



5 a The skateboarder's position did not change at interval B.

b A, D, E (the intervals in which the gradient is positive)

c 40 seconds (the first instant at which the skateboarder moves in a southerly direction)

d 20 metres north (change in position after 80 s = 20 m north of starting point)

e 260 metres (80 m in a northerly direction, followed by 120 m in a southerly direction, followed by 60 m in a northerly direction)

f D (the gradient is increasing)

g E (the gradient is decreasing)

h $\text{Average speed} = \dfrac{\text{distance travelled}}{\text{time interval}}$

$= \dfrac{260\text{ m}}{80\text{ s}}$

$= 3.3\text{ m s}^{-1}$

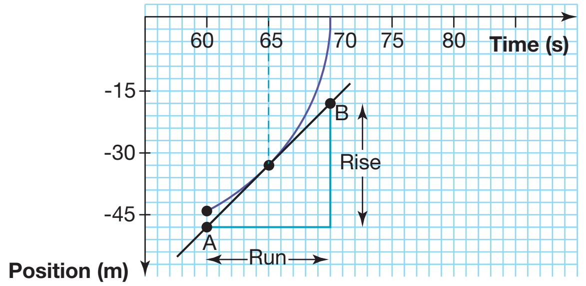
i $v = \text{gradient}$

$= \dfrac{-120\text{ m}}{20\text{ s}}$

$= -6.0\text{ m s}^{-1}$

$= 6.0\text{ m s}^{-1}$ south

j v = gradient at time t = 65 s



To find the gradient, a tangent must be drawn on the curve at t = 65 s. Two convenient points need to be drawn on the tangent (e.g. A and B).

$\text{Gradient} = \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{-18 - (-48)}{70 - 60}$ (approx.)

$= 3\text{ m s}^{-1}$

The direction is north.

6 a B, D, F (gradient = 0 during these sections; that is, there is no change in velocity)

b Displacement = total area under graph

This is most easily calculated by dividing the graphs into triangles and rectangles as shown in the figure at the foot of the page.*

$$\text{Total area} = \frac{1}{2} \times 10 \times 1 + 10 \times 1 + \frac{1}{2} \times 10 \times 1 - \frac{1}{2} \times 10 \times 1 - 10 \times 1 - \frac{1}{2} \times 5 \times 1 + \frac{1}{2} \times 5 \times 1 + 10 \times 1 + \frac{1}{2} \times 10 \times 1$$

$$= 5 + 10 + 5 - 5 - 10 - 2.5 + 2.5 + 10 + 5$$

$$= +20\,\text{m}$$

c $v_{\text{av}} = \dfrac{\Delta x}{\Delta t}$

$$= \frac{+20\,\text{m}}{80\,\text{s}}$$

$$= 0.25\,\text{m}\,\text{s}^{-1}$$

d 30 seconds (the instant that the velocity becomes negative)

e It didn't. (The negative displacement that occurs between 30 s and 55 s is not as great as the positive displacement between 0 s and 30 s.)

f C, G (when the gradient is negative)

g The first half of interval C, the first half of interval E and all of interval G (during these periods, the magnitude of the velocity is decreasing)

h A negative acceleration doesn't always decrease the speed and a positive acceleration doesn't always increase the speed. A negative acceleration increases the speed if the velocity is negative and decreases the speed if the velocity is positive. Similarly, a positive acceleration decreases the speed if the velocity is negative and increases the speed if the velocity is positive.

i a = gradient

$$= \frac{+2.0\,\text{m}\,\text{s}^{-1}}{10\,\text{s}}$$

$$= 0.20\,\text{m}\,\text{s}^{-2}$$

j $a_{\text{av}} = \dfrac{\Delta v}{\Delta t}$

$$= \frac{1.0\,\text{m}\,\text{s}^{-1}}{20\,\text{s}}$$

$$= 0.050\,\text{m}\,\text{s}^{-2}$$

k The motion of the toy robot can be described in nine different intervals.

First 10 seconds: The toy robot started from rest and increased its speed at a constant rate until reaching a speed of 1.0 m s^{-1} after 10 seconds.

10 s to 20 s: It maintained a constant speed of 1.0 m s^{-1}.

20 s to 30 s: It slowed down at a constant rate. It was at rest for an instant, 30 seconds after starting.

30 s to 40 s: It increased its speed at the same constant rate as the first interval, but in the opposite direction, to reach a maximum speed of 1.0 m s^{-1}.

40 s to 50 s: It maintained a constant speed of 1.0 m s^{-1}.

50 s to 55 s: It decelerated to rest at a constant rate.

55 s to 60 s: It increased its speed at a constant rate in the original direction. The acceleration was twice that of the first interval.

60 s to 70 s: It maintained a constant speed of 1.0 m s^{-1}.

70 s to 80 s: It decelerated to rest at a constant rate.

7 a A constant speed is reached when the acceleration becomes zero. The acceleration of the jet ski becomes zero first, after 8 seconds.

b i Δv = area under acceleration-versus-time-graph

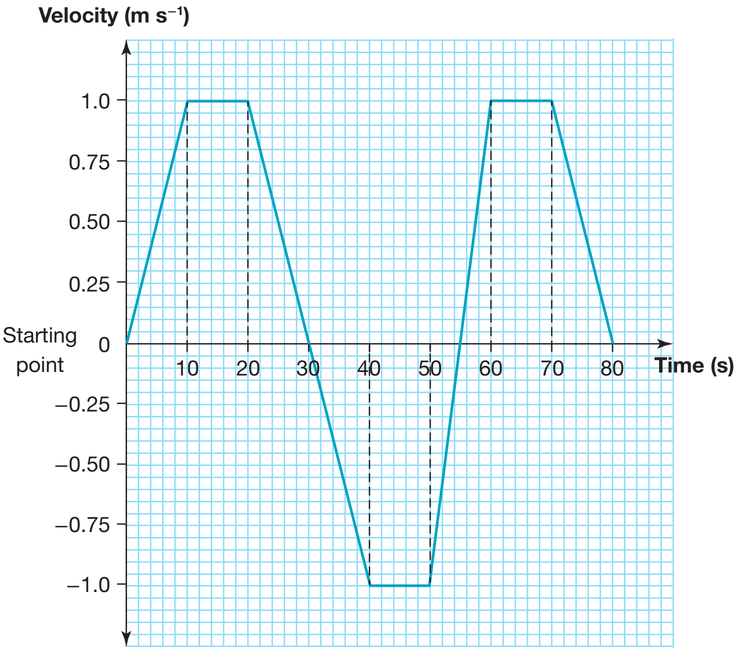
$$= \frac{1}{2} \times 8 \times 4$$

$$= 16\,\text{m}\,\text{s}^{-1}$$

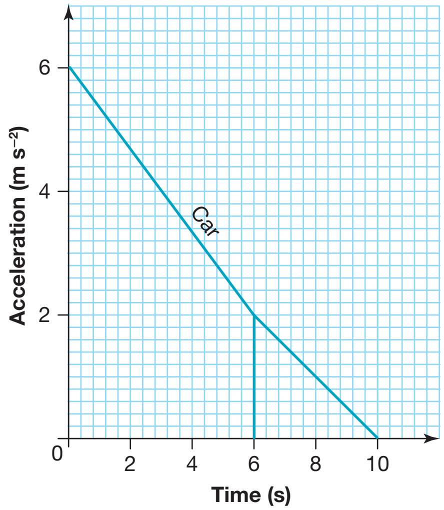
Initial $v = 5.0\,\text{m}\,\text{s}^{-1}$

$\Rightarrow v = 21\,\text{m}\,\text{s}^{-1}$ for the jet ski

6 b*



ii Divide the graph into a trapezium and triangle as shown in the following figure.



Δv = area under acceleration-versus-time-graph

$= \frac{6+2}{2} \times 6 + \frac{1}{2} \times (10-6) \times 2$

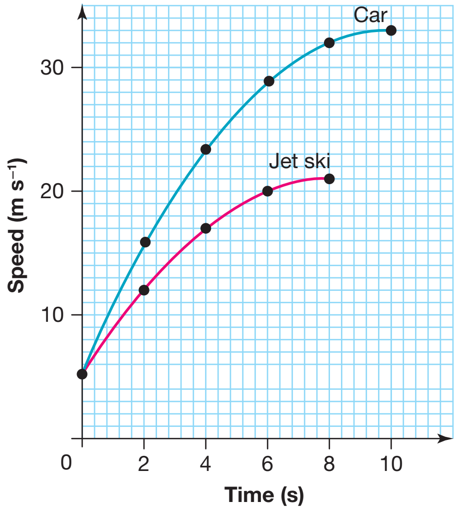
$= 24 + 4$

$= 28 \text{ m s}^{-1}$

Initial $v = 5.0 \text{ m s}^{-1}$

$\Rightarrow v = 33 \text{ m s}^{-1}$ for the car

c



8.3 Exam questions

1 D

The gradient of the v–t graph is 0.

2 A

v = gradient

$= \frac{\Delta x}{\Delta t}$

$= \frac{10-2}{4}$

$= \frac{8}{4}$

$= 2.0 \text{ m s}^{-1}$

3 s = area under the graph for 2 seconds + area under the graph for next 2 seconds

$\Rightarrow$ First 2 s: $s = \frac{1}{2} \times 2 \times 6$

$= 6$ m [1 mark]

Next 2 s: $s = 2 \times -2$

$= -4$ m [1 mark]

Total: $s = 6 - 4$

$= +2$ m [1 mark]

4 a v_{inst} = gradient

$= \frac{\Delta x}{\Delta t}$

$= \frac{-4}{1}$ [1 mark]

$= -4 \text{ m s}^{-1}$ [1 mark]

Note: The velocity is not 0; v equals $\frac{\Delta x}{\Delta t}$, not $\frac{x}{t}$.

b It occurs in the interval 4 to 5 s (just after $t = 4.5$ s). [1 mark]

Maximum instantaneous velocity occurs where the gradient of the curve has a maximum positive gradient. This occurs just after $t = 4.5$ s. [1 mark]

5 a From $t = 2$ s to $t = 3$ s, the triangular area:

$\Delta x = v\Delta t$

$= \frac{1}{2} \times 6 \times 1$

$= 3$ m [1 mark]

From $t = 3$ s to $t = 4$ s:

$\Delta x = v\Delta t$

$= \frac{1}{2} \times -6 \times 1$

$= -3$ m [1 mark]

Total displacement $= 3 - 3$

$= 0$ m [1 mark]

b Total displacement = area of rectangle + area of triangle [1 mark]

$\Rightarrow \Delta x = 6 \times 2 + \frac{1}{2} \times 6 \times 1$

$= 12 + 3$

$= 15$ m [1 mark]

$\Rightarrow v_{av} = \frac{\Delta x}{\Delta t}$

$= \frac{15}{3}$

$= 5 \text{ m s}^{-1}$ [1 mark]

8.4 Equations for constant acceleration

Sample problem 6

a $u = 0$, $a = 9.8 \text{ m s}^{-2}$, $t = 3.0$ s

$v = u + at$

$= 0 + 9.8 \times 3.0$

$= 29 \text{ m s}^{-1}$

The coin had a velocity of 29 m s^{-1} when it hit the water.

b $\boldsymbol{u} = 0$, $\boldsymbol{a} = 9.8\,\text{m s}^{-2}$, $t = 3.0\,\text{s}$

$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$
$$= 0 \times 3.0 + \frac{1}{2} \times 9.8 \times 3.0^2$$
$$= 44\,\text{m}$$

The coin fell 44 metres before hitting the water.

Practice problem 6

a $\boldsymbol{u} = 0$, $\boldsymbol{a} = 2.0\,\text{m s}^{-2}$, $\boldsymbol{v} = 12\,\text{m s}^{-1}$

Using $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$:

$$t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}}$$
$$t = \frac{12 - 0}{2.0}$$
$$= 6.0\,\text{s}$$

The car was rolling for 6 seconds.

b Using supplied data:

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$$
$$\Rightarrow \boldsymbol{s} = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{a}}$$
$$= \frac{(12^2 - 0)}{2 \times 2.0}$$
$$= 36\,\text{m}$$

Alternatively, using the answer to part **a**:

$$\boldsymbol{s} = \frac{1}{2} \times (\boldsymbol{u} + \boldsymbol{v})t$$
$$= \frac{1}{2} \times 12 \times 6.0$$
$$= 36\,\text{m}$$

The car rolled 36 metres before colliding with the wall.

Sample problem 7

a $\boldsymbol{s} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})t$

$\boldsymbol{v} = 0,\ t = 2\,\text{s},\ \boldsymbol{s} = 12\,\text{m}$

$$12 = \frac{1}{2}(\boldsymbol{u} + 0)2$$
$$\Rightarrow \boldsymbol{u} = 12\,\text{m s}^{-1}$$

The initial speed of the car was 12 m s^{-1}.

b $\boldsymbol{s} = \boldsymbol{v}t - \frac{1}{2}\boldsymbol{a}t^2$

$\boldsymbol{v} = 0,\ t = 2\,\text{s},\ \boldsymbol{s} = 12\,\text{m}$

$$12 = 0 \times 2 - \frac{1}{2}\boldsymbol{a} \times 2^2$$
$$12 = -2\boldsymbol{a}$$
$$\Rightarrow \boldsymbol{a} = -6\,\text{m s}^{-2}$$

The acceleration during the skid was –6 m s^{-2}; that is, the car was decelerating at 6 m s^{-2} during the skid.

Practice problem 7

a $\boldsymbol{u} = 24\ \text{m s}^{-1}$, $\boldsymbol{v} = 0$, $t = 1.5\ \text{s}$

$$\boldsymbol{s} = \frac{1}{2} \times (\boldsymbol{u} + \boldsymbol{v})t$$
$$= \frac{1}{2} \times 24 \times 1.5$$
$$= 18\,\text{m}$$

The car's stopping distance was 18 metres.

b $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$$\Rightarrow \boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t}$$
$$= \frac{(24 - 0)}{1.5}$$
$$= -16\,\text{m s}^{-2}$$

The car's deceleration was 16 m s^{-2}.

8.4 Exercise

1 a $\boldsymbol{a} = 6.0\,\text{m s}^{-2}$, $\boldsymbol{u} = 17\,\text{m s}^{-1}$, $\boldsymbol{v} = 28\,\text{m s}^{-1}$

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$\Rightarrow t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}}$$
$$= \frac{28 - 17}{6.0}$$
$$= 1.8\,\text{s}$$

b $\boldsymbol{a} = 2.0\,\text{m s}^{-2}$, $\boldsymbol{u} = 0\,\text{m s}^{-1}$, $\boldsymbol{v} = 10\,\text{m s}^{-1}$

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$\Rightarrow t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}}$$
$$= \frac{10 - 0}{2.0}$$
$$= 5.0\,\text{s}$$

2 a $\boldsymbol{a} = 10\,\text{m s}^{-2}$, $\boldsymbol{s} = 36\,\text{m}$, $\boldsymbol{u} = 0$

$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$$
$$36 = 0 + \frac{1}{2} \times 10t^2$$
$$= 5t^2$$
$$\Rightarrow t = \sqrt{\frac{36}{5}}$$
$$= 2.7\,\text{s}$$

b $\boldsymbol{a} = 10\,\text{m s}^{-2}$, $\boldsymbol{s} = 36\,\text{m}$, $\boldsymbol{u} = 0$

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$$
$$= 0^2 + 2 \times 10 \times 36$$
$$= 720$$
$$\Rightarrow \boldsymbol{v} = \sqrt{720}$$
$$= 27\,\text{m s}^{-1}$$

3 a $\boldsymbol{s} = 12\,\text{m}$, $t = 2\,\text{s}$, $\boldsymbol{v} = 0$

$$\boldsymbol{s} = \frac{\boldsymbol{u} + \boldsymbol{v}}{2}t$$
$$12 = \frac{\boldsymbol{u}}{2} \times 2$$
$$\Rightarrow \boldsymbol{u} = 12\,\text{m s}^{-1}$$

b $s = 12\,\text{m},\ t = 2\,\text{s},\ v = 0$

$$s = vt - \frac{1}{2}at^2$$

$$12 = 0 - \frac{1}{2} \times a \times 4$$

$$12 = -2a$$

$$\Rightarrow a = -\frac{12}{2}$$

$$= -6\,\text{m s}^2$$

4 a $u = 100\,\text{km h}^{-1} = 27.8\,\text{m s}^{-1},\ v = 0,\ s = 48\,\text{m}$

$$v^2 = u^2 + 2as$$

$$0^2 = (27.8)^2 + 2 \times a \times 48$$

$$\Rightarrow a = \frac{(27.8)^2}{96}$$

$$= -8.0\,\text{m s}^{-2}$$

b $u = 100\,\text{km h}^{-1} = 27.8\,\text{m s}^{-1},\ v = 0,\ s = 48\,\text{m}$

$$s = \frac{u+v}{2}t$$

$$48 = \frac{27.8}{2}t$$

$$96 = 27.8t$$

$$\Rightarrow t = \frac{96}{27.8}$$

$$= 3.5\,\text{s}$$

c The reaction time of the driver needs to be known to determine the distance travelled between the instant that the branch is seen and the instant that the brakes are applied. An estimate of 0.2 seconds would be reasonable for the reaction time. At a constant speed of 100 km h^{-1} (27.8 m s^{-1}), the car would travel a distance of:

27.8 m s^{-1} × 0.2 s = 5.6 m

The total distance required to stop is therefore:

5.6 m (reacting distance) + 48 m (braking distance)
= 53.6 m

The car would not stop in time.

5 In order to make the leap, the dancer must rise and fall in 0.5 seconds.

Ignoring air resistance, the fall takes the same amount of time as the rise: 0.25 seconds.

The time taken for the dancer to rise to a height of 80 cm can be calculated:

$v = 0,\ s = 0.80\,\text{m},\ a = -10\,\text{m s}^{-2}$

$$s = vt - \frac{1}{2}at^2$$

$$0.80 = 0 - \frac{1}{2} \times -10 \times t^2$$

$$= 5 \times t^2$$

$$\Rightarrow t = \sqrt{\frac{0.80}{5}}$$

$$= 0.4\,\text{s}$$

The time taken for the dancer to fall from a height of 80 cm can be calculated:

$u = 0,\ s = -0.80\,\text{m},\ a = -10\,\text{m s}^{-2}$

$$s = ut + \frac{1}{2}at^2$$

$$-0.80 = 0 + \frac{1}{2} \times -10 \times t^2$$

$$= -5 \times t^2$$

$$\Rightarrow t = \sqrt{\frac{0.80}{5}}$$

$$= 0.4\,\text{s}$$

That is, without knowing the 'take-off' speed of the dancer, it can be shown that the leap would take 0.8 seconds. The leap is not possible.

6 It is important to remember that, at the instant that the Rolls Royce rolls off the truck, it is moving in the same direction as the truck.

After 1 minute, the truck has moved a distance of 1000 m (a constant speed of 60 km h^{-1} is equal to 1 km min^{-1}).

During the driver's reaction time, the distance moved by the truck is 8.3 metres (the distance moved in 0.5 s at a constant speed of 60 km h^{-1} = 16.67 m s^{-1} × 0.5 s).

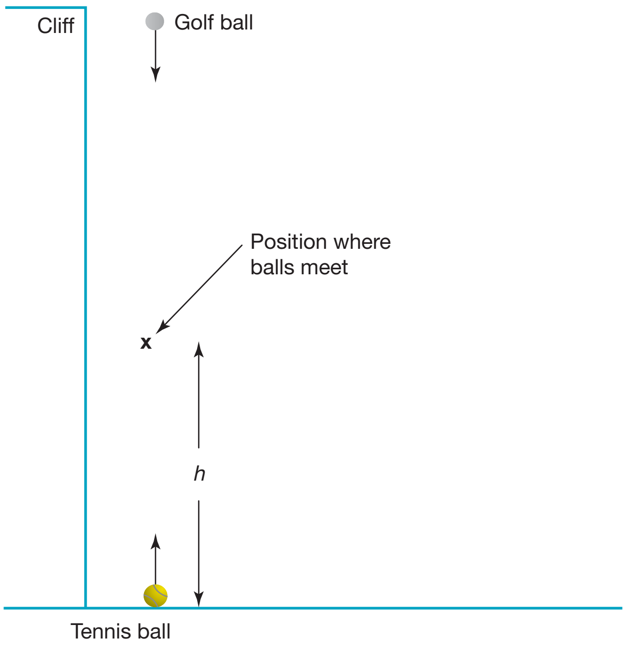
The braking distance of the truck is 25 metres.

The total distance moved by the truck is:

1000 m + 8.3 m + 25 m = 1033 m

The distance moved by the Rolls Royce is 240 metres (in the same direction as that of the truck). The Rolls Royce is therefore 793 metres behind the stopped truck.

7



a The balls collide when the tennis ball is at the top of its path at a time t_c.

For the tennis ball:

$v = 0,\ a = -10\,\text{m s}^{-2}$ (taking up as a positive), $s = h,\ t = t_c$

$$s = vt - \frac{1}{2}at^2$$

$$h = 5t_c^2 \quad [1] \quad \left(\text{since } a = -10\,\text{m s}^{-2}\right)$$

For the golf ball:

$u = 0,\ a = -10\,\text{m s}^{-2}$ (taking up as a positive),

$s = 100 - h,\ t = t_c$

$$s = ut + \frac{1}{2}at^2$$

$$100 - h = 5t_c^2 \quad [2]$$

Substitute [1] into [2]:

$$100 - \left(5t_c^2\right) = 5t_c^2$$

$$100 = 10t_c^2$$

$$\Rightarrow t_c = \sqrt{10}$$

$$= 3.16\,\text{s}$$

For the tennis ball:

$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$0 = \boldsymbol{u} - 10 \times 3.16$

$\Rightarrow \boldsymbol{u} = 32\,\text{m s}^{-1}$

b Substitute [1]:

$h = 5t^2$

$= 5 \times (3.16)^2$

$= 49.928\,\text{m}$

$\approx 50\,\text{m}$

The balls meet 50 metres from the ground.

8.4 Exam questions

1 C

$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2$

$= 4 \times 2 + \frac{3 \times 2^2}{2}$

$= 8 + 6$

$= 14\,\text{m}$

2 A

$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$

$= 3 + 4 \times 5$

$= 23\,\text{m s}^{-1}$

3 $\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$

$\Rightarrow \boldsymbol{a} = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{s}}$

$= \frac{0 - 20^2}{2 \times 40}$ (1 mark for substitution)

$= -5\,\text{m s}^{-2}$ (1 mark for sign AND magnitude)

4 In this case, $\boldsymbol{v} = 0$ and $\boldsymbol{a} = -9.8\,\text{m s}^{-2}$. [1 mark]

$\boldsymbol{s} = \boldsymbol{v}t - \frac{1}{2}\boldsymbol{a}t^2$

$= 0 - \frac{1}{2}(-9.8) \times 2^2$ [1 mark]

$= 19.6\,\text{m}$ [1 mark]

5 **a** $\boldsymbol{a} = \frac{\boldsymbol{v} - \boldsymbol{u}}{t}$

$= \frac{14 - 6}{4}$

$= 2.0\,\text{m s}^{-2}$ [1 mark]

b $\boldsymbol{s} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})t$

$= \frac{1}{2} \times 20 \times 4$ [1 mark]

$= 40\,\text{m}$ [1 mark]

c $\boldsymbol{v} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})$

$= \frac{1}{2} \times 20$

$= 10\,\text{m s}^{-1}$ [1 mark]

d $\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$ and $\boldsymbol{v} = 0$ [1 mark]

$\Rightarrow 0 = 14^2 + 2a \times 24.5$ [1 mark]

$\Rightarrow \boldsymbol{a} = -\frac{14^2}{49}$

$= -4.0\,\text{m s}^{-2}$ [1 mark]

e $t = \frac{2\boldsymbol{s}}{\boldsymbol{u} + \boldsymbol{v}}$

$= \frac{2 \times 24.5}{14 + 0}$ [1 mark]

$= \frac{49}{14}$

$= 3.5\,\text{s}$ [1 mark]

8.5 Review

8.5 Review questions

1 Distance travelled = sum of all distances

$= 10 + 5 + 7 + 9$

$= 31\,\text{m}$

Displacement = sum of changes in position

$= +10 + (-5) + 7 + (-9)$

$= +3$

$= 3$ m north

2 $t = 10$ h and 30 min

$= 10(60) + 30$ min

$= 630$ min

$= 630(60)$ s

$= 37\,800$ s

$\boldsymbol{x} = 6980\,\text{km}$

$= 6\,980\,000\,\text{m}$

$\boldsymbol{v}_{\text{av}} = \frac{\Delta \boldsymbol{x}}{\Delta t}$

$= \frac{6\,980\,000\,\text{m}}{37\,800\,\text{s}}$

$= 185\,\text{m s}^{-1}$

3 $\boldsymbol{v} = -264$, $\boldsymbol{u} = 330$

$\Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u}$

$= -264 - 330$

$= -594\,\text{km h}^{-1}$

$= \frac{-594}{3.6}\,\text{m s}^{-1}$

$= 165\,\text{m s}^{-1}$

$\boldsymbol{a}_{\text{av}} = \frac{\Delta \boldsymbol{v}}{\Delta t}$

$= \frac{165}{0.25}$

$= 660\,\text{m s}^{-2}$ away from the racquet

4 Velocity is calculated from the gradient of a position-versus-time graph:

$\boldsymbol{v} = \frac{\text{rise}}{\text{run}}$

$= \frac{80}{20}$

$= 4\,\text{m s}^{-1}$

5 Change in position is calculated from the area under a position-versus-time graph:

$\text{Area} = 7 \times 15 + \frac{1}{2} \times 3 \times 15 - \frac{1}{2} \times 2 \times 10 - 3 \times 10 - \frac{1}{2} \times 1 \times 10 + \frac{1}{2} \times 1 \times 10 + 3 \times 10$

$= 105 + 22.5 - 10 - 30 - 5 + 5 + 30$

$= 117.5\,\text{m}$

The change in position is 117.5 metres in the positive direction.
Note: The calculations could be simplified by identifying areas above and below the horizontal axis that are equivalent and cancel out, requiring fewer areas to be computed.

6 Acceleration is calculated from the gradient of a position-versus-time graph.
The largest magnitude gradient is, by inspection, in the 15- to 17-second interval.

$$\boldsymbol{a} = \frac{\text{rise}}{\text{run}} = \frac{20}{2} = 10\,\text{m s}^{-2}$$

7 See the figure at the foot of the page.*
The change in velocity is equal to the area under the acceleration-versus-time graph.
The area can be split into two triangles.

$$A_1 = \frac{1}{2}bh = \frac{1}{2} \times 0.3 \times -2.5 = -0.375\,\text{m s}^{-1}$$

$$A_2 = \frac{1}{2}bh = \frac{1}{2} \times 1.2 \times -2.5 = -1.5\,\text{m s}^{-1}$$

$$A_{\text{total}} = A_1 + A_2 = -1.875\,\text{m s}^{-1}$$

8 $\boldsymbol{s} = 50\,\text{m}$, $\boldsymbol{u} = 0\,\text{m s}^{-1}$, $t = 40\,\text{s}$,

$$\boldsymbol{v} = \frac{2\boldsymbol{s}}{t} - \boldsymbol{u} = \frac{2(50)}{40} - 0 = 2.5\,\text{m s}^{-1}$$

9 $\boldsymbol{u} = 90\,\text{m s}^{-1}$, $\boldsymbol{v} = 0\,\text{m s}^{-1}$, $\boldsymbol{a} = -9.8\,\text{m s}^{-2}$

$$\boldsymbol{s} = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{a}} = \frac{(0)^2 - (90)^2}{2(-9.8)} = \frac{-8100}{-19.6} = 413.3\,\text{m}$$

To determine the total height, add the height reached until the engine switched off (45 m) to 413.3 m.

$$\text{Total height} = 45 + 413.3 = 458.3\,\text{m above the oval}$$

10 $\boldsymbol{v} = 0\,\text{m s}^{-1}$, $\boldsymbol{s} = 27\,\text{m}$, $t = 4.5\,\text{s}$

$$\boldsymbol{u} = \frac{2\boldsymbol{s}}{t} - \boldsymbol{v} = \frac{2(27)}{4.5} - 0 = 12\,\text{m s}^{-1}$$

Convert to km h^{-1} to compare to the speed limit:

$$\boldsymbol{u} = 12\,\text{m s}^{-1} = 12 \times 3.6\,\text{km h}^{-1} = 43.2\,\text{km h}^{-1}$$

Therefore, yes, the driver was travelling faster than the speed limit when they braked.

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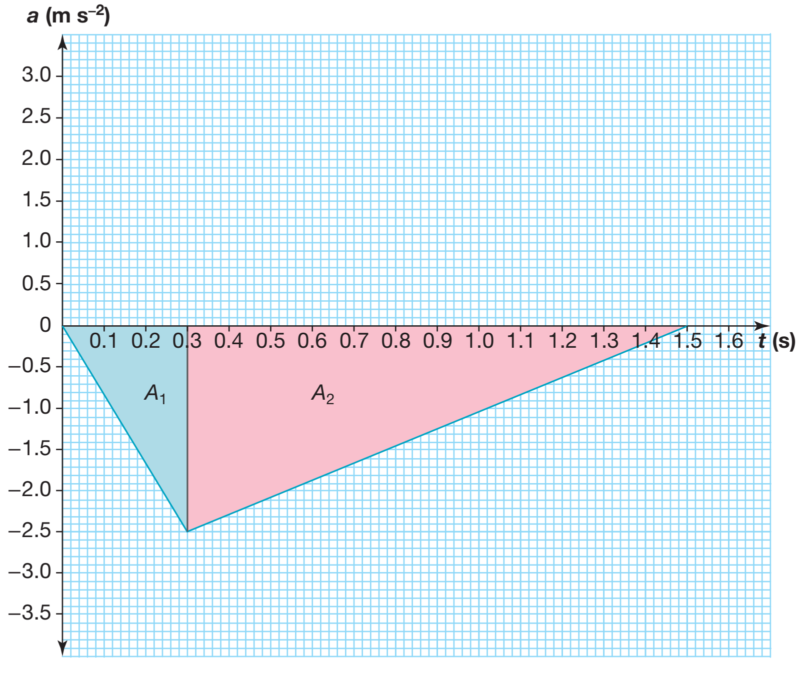
*7



8.5 Exam questions

Section A — Multiple choice questions

1 C
Vector quantities have both a size and a direction.

2 A
Displacement is the change in position. If the cyclist has returned to their starting position, that is, completed three full laps, then their displacement is zero.

3 C
We need to convert the speed from km h^{-1} to m s^{-1}.

$$\begin{aligned} \boldsymbol{v} &= 1060\text{ km h}^{-1} \\ &= \frac{1060}{3.6}\text{ m s}^{-1} \\ &= 294\text{ m s}^{-1} \end{aligned}$$

Therefore, it will travel 294 metres in a second.

4 D

$$\begin{aligned} v_{av} &= \frac{\Delta x}{\Delta t} \\ &= \frac{21}{1.9} \\ &= 11.05\text{ m s}^{-1} \end{aligned}$$

5 B
We need to convert the speed from m s^{-1} to km h^{-1}.

$$\begin{aligned} \boldsymbol{v} &= 12\text{ m s}^{-1} \\ &= 12 \times 3.6\text{ km h}^{-1} \\ &= 43.2\text{ km h}^{-1} \end{aligned}$$

6 A

$$\begin{aligned} \boldsymbol{v} &= 97\text{ km h}^{-1} \\ &= \frac{97}{3.6}\text{ m s}^{-1} \\ &= 26.94\text{ m s}^{-1} \end{aligned}$$

Then:

$$\begin{aligned} \boldsymbol{a} &= \frac{26.94}{2.28} \\ &= 11.82\text{ m s}^{-2} \end{aligned}$$

7 C
When referring to a velocity-versus-time graph, acceleration is determined from the gradient. Therefore, acceleration is 0 when the gradient of the velocity-versus-time graph is 0. The gradient is 0 for interval B, D and F.

8 D
When referring to a velocity-versus-time graph, speed can be determined from reading the velocity and taking the magnitude.

9 B
$\boldsymbol{u} = 0,\ \boldsymbol{a} = 9.8,\ t = 2$

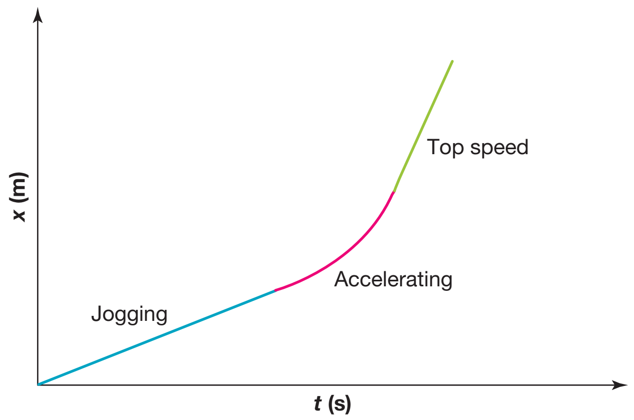
$$\begin{aligned} \boldsymbol{s} &= \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}t^2 \\ &= (0)(2^2) + \frac{1}{2}(9.8)(2^2) \\ &= 19.6\text{ m} \end{aligned}$$

10 C
$\boldsymbol{u} = 0,\ \boldsymbol{a} = 3,\ \boldsymbol{s} = 100$

$$\begin{aligned} \boldsymbol{v}^2 &= \boldsymbol{u}^2 + 2\boldsymbol{as} \\ &= (0)^2 + 2(3)(100) \\ &= 600 \\ \Rightarrow \boldsymbol{v} &= \sqrt{600} \\ &= 24.5\text{ m s}^{-1} \end{aligned}$$

Section B — Short answer questions

11



(1 mark for correct graph)

12 For an acceleration to occur there must be a change in velocity. Velocity is a vector, with a magnitude and a direction. In this instance, the magnitude of the velocity (the speed) has not changed, but the direction has. [1 mark]
Therefore, there has been a change in velocity, and thus an acceleration. [1 mark]
Alternatively, this could be explained with vector diagrams and vector subtraction to calculate the change in velocity.

13 **a** Assume that Alex and Bo are at the same position at the 3-second mark when they hand over the batons. Bo starts to run at the 1-second mark.
Determine how far Alex travels between the 1- and 3-second marks.

$$\begin{aligned} \text{Change in position} &= \text{area under velocity-versus-time graph} \\ &= 1 \times 8 + 1 \times \frac{1}{2}(8 + 7) \\ &= 8 + 7.5 \\ &= 15.5\text{ m [1 mark]} \end{aligned}$$

Determine how far Bo travels between the 1- and 3-second marks.

$$\begin{aligned} \text{Change in position} &= \text{area under velocity-versus-time graph} \\ &= \frac{1}{2} \times 2 \times 7 \\ &= 7\text{ m [1 mark]} \end{aligned}$$

The difference between Alex and Bo's change in position will be how far behind Bo Alex was when Bo started to run: $15.5 - 7 = 8.5$ m. [1 mark]

b Assume that the baton change starts at the 3-second mark and is just inside the interchange area at that point.
From the calculations for the previous question, Bo starts 7 metres before the interchange, so therefore complies with the 10-metre maximum rule. [1 mark]
The exchange happens at a constant speed of 7 m s^{-1} and takes 2 seconds, so by inspection Alex and Bo will travel 14 metres together during the interchange.
Therefore, they also comply with the 20-metre interchange area rule. [1 mark]

14 Assume that upwards motion is defined as positive.
Find the velocity at which they strike the water:
$\boldsymbol{u} = 3\text{ m s}^{-1},\ \boldsymbol{s} = -75\text{ m},\ \boldsymbol{a} = -9.8\text{ m s}^{-2}$

$$\begin{aligned} \boldsymbol{v}^2 &= \boldsymbol{u}^2 + 2\boldsymbol{a}\boldsymbol{s} \\ &= 3^2 + 2(-9.8)(-75) \\ &= 1479 \end{aligned}$$

$$\Rightarrow \boldsymbol{v} = -\sqrt{1479}$$

$$= -38.46\,\text{m s}^{-1} \text{ [1 mark]}$$

As upwards motion is positive and at the end of the dive they must be moving downwards, the value must be negative.

Now calculate the total time:

$\boldsymbol{v} = -38.46,\ \boldsymbol{u} = 3\,\text{m s}^{-1},\ \boldsymbol{a} = -9.8\,\text{m s}^{-2}$

$$t = \frac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}}$$

$$= \frac{-38.46 - 3}{-9.8}$$

$$= 4.23\text{ s [1 mark]}$$

The time taken is 4.23 seconds.

Note: Students may be inclined to break the motion into upwards and downwards segments and analyse separately; however, as the acceleration is constant and continuous throughout, this is not necessary.

15 a 3.2 seconds (can be read directly from the graph) [1 mark]

b $\boldsymbol{a}$ = gradient

$$= \frac{10\,\text{m s}^{-1}}{4\,\text{s}}$$

$$= 2.5\,\text{m s}^{-2} \text{ [1 mark]}$$

c Let T = the time at which the stuntman catches the bus.

At time T, the displacement of the stuntman is equal to the displacement of the bus.

⇒ area-under-stuntman graph = area-under-bus graph

$$\frac{1}{2} \times 4 \times 10 + 10(T - 4) = 8T \text{ [1 mark]}$$

$$20 + 10T - 40 = 8T$$

$$2T = 20$$

$$\Rightarrow T = 10\,\text{s [1 mark]}$$

d Distance = magnitude of displacement

= area under graph

= $8T$ (or $10T - 20$)

= 80 metres [1 mark]

Topic 9 — Forces in action

9.2 Forces as vectors

Sample problem 1

a $F_g = mg$

$= 50 \times 9.8$

$= 490\text{ N downwards}$

The force due to gravity acting on a 50-kilogram student on Earth is 490 N downwards.

b $F_g = mg$

$= 50 \times 1.60$

$= 80\text{ N downwards}$

The force due to gravity acting on a 50-kilogram student on the Moon is 80 N downwards.

Practice problem 1

a Force at North Pole = mg

$= 70 \times 9.832$

$= 688\text{ N}$

Force at equator = mg

$= 70 \times 9.780$

$= 685\text{ N}$

b $F_g = mg$

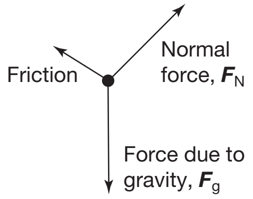
$\Rightarrow g = \dfrac{F_g}{m}$

$= \dfrac{630.08}{64.32}$

$\approx 9.796\text{ N kg}^{-1}$

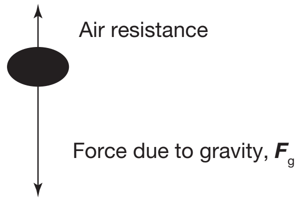
The patient is in Denver.

d

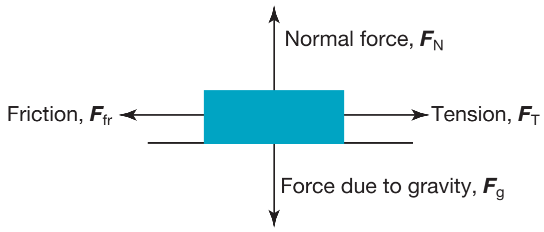


Practice problem 2

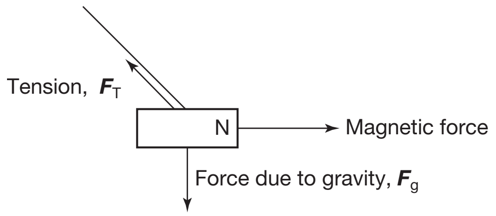
a



b

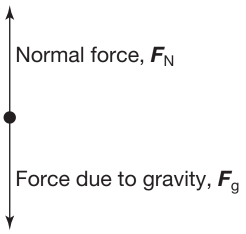


c

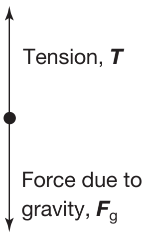


Sample problem 2

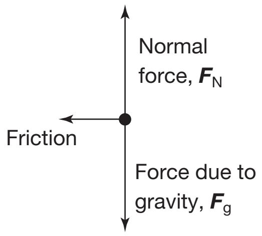
a



b



c



Sample problem 3

a $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\cos(37°) = \dfrac{F_{2x}}{25}$

$\Rightarrow F_{2x} = 25\cos(37°)$

$\approx 20\text{ N}$

The component of F_2 that acts in the x direction is 20 N.

b $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\sin(37°) = \dfrac{F_{2y}}{25}$

$\Rightarrow F_{2y} = 25\sin(37°)$

$\approx 15\text{ N}$

The component of F_2 that acts in the y direction is 15 N.

c In the x direction:

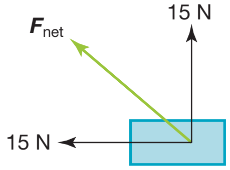
$F_{\text{net},x} = F_{2x} - F_1$

$= 20 - 35$

$= -15\text{ N}$

In the y direction:

$$\begin{aligned}\boldsymbol{F}_{\text{net}, y} &= \boldsymbol{F}_{2y}\\ &= 15\\ &= 15\text{ N}\end{aligned}$$

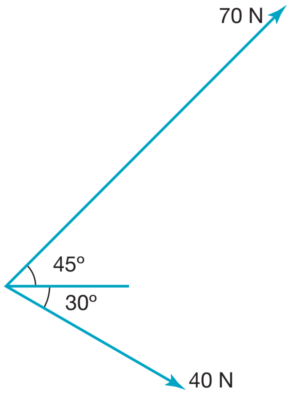


$$\begin{aligned}c^2 &= a^2 + b^2\\ \boldsymbol{F}_{\text{net}^2} &= 15^2 + 15^2\\ &= 450\\ \Rightarrow \boldsymbol{F}_{\text{net}} &= \sqrt{450}\\ &= 21.2\text{ N}\end{aligned}$$

The magnitude of the net force on the object is 21.2 N.

Note: This problem only requires the magnitude of the net force. If the direction was required it would be necessary to calculate the angle that the force is acting at.

Practice problem 3



$$\begin{aligned}\boldsymbol{F}_1(\text{vertical}) &= 70\sin(45°)\\ &\approx 49.5\text{ N}\end{aligned}$$

$$\begin{aligned}\boldsymbol{F}_1(\text{horizontal}) &= 70\cos(45°)\\ &\approx 49.5\text{ N}\end{aligned}$$

$$\begin{aligned}\boldsymbol{F}_2(\text{vertical}) &= -40\sin(30°)\\ &= -20\text{ N}\end{aligned}$$

$$\begin{aligned}\boldsymbol{F}_2(\text{horizontal}) &= 40\cos(30°)\\ &\approx 34.6\text{ N}\end{aligned}$$

$$\begin{aligned}\boldsymbol{F}_{\text{net}}(\text{vertical}) &= 49.5 - 20\\ &= 29.5\text{ N}\end{aligned}$$

$$\begin{aligned}\boldsymbol{F}_{\text{net}}(\text{horizontal}) &= 49.5 + 34.6\\ &= 84.1\text{ N}\end{aligned}$$

$$\begin{aligned}|\boldsymbol{F}_{\text{net}}| &= \sqrt{29.5^2 + 84.1^2}\\ &= \sqrt{7943.06}\\ &\approx 89\text{ N}\end{aligned}$$

The magnitude of the net force acting on the object is 89 N.

9.2 Exercise

1 Vector quantities have magnitude and direction. Scalar quantities have magnitude only.

2 Vector quantities have a magnitude and a direction. Of the quantities listed in this question, force due to gravity and gravitational field strength are the only quantities that have a direction as well as a magnitude. Therefore, the vector quantities from the list are force due to gravity (II) and gravitational field strength (III).

3 **a** $\boldsymbol{F}_g = m\boldsymbol{g}$

$$\begin{aligned}&= 1400 \times 9.8\\ &= 13\,720\text{ N}\end{aligned}$$

b $\boldsymbol{F}_g = m\boldsymbol{g}$

$$\begin{aligned}&= 1400 \times 3.6\\ &= 5040\text{ N}\end{aligned}$$

c $m = 1400$ kg anywhere. It is a measure of the amount of matter in an object or substance and does not depend on the gravitational field strength.

4 **a** $m \approx 0.100$ kg

$$\begin{aligned}\boldsymbol{F}_g &= m\boldsymbol{g}\\ &= 0.100 \times 9.8\\ &= 0.98\text{ N}\end{aligned}$$

b $m \approx 1.0$ kg

$$\begin{aligned}\boldsymbol{F}_g &= m\boldsymbol{g}\\ &= 1.0 \times 9.8\\ &= 9.8\text{ N}\end{aligned}$$

c $m \approx 80.0$ kg

$$\begin{aligned}\boldsymbol{F}_g &= m\boldsymbol{g}\\ &= 80.0 \times 9.8\\ &= 784\text{ N}\end{aligned}$$

Note: Responses will vary, depending on the estimated masses. A sample estimation was used in the solutions for this question.

5 Assume the mass of the student is m kg. If, for example, $m = 60$ kg:

a At Earth's surface:

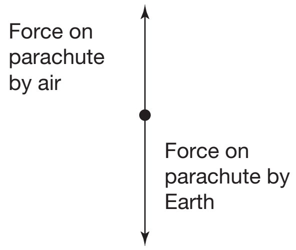
$$\begin{aligned}\boldsymbol{F}_g &= m\boldsymbol{g}\\ &= 60 \times 9.8\\ &= 588\text{ N}\end{aligned}$$

b On the surface of Mars:

$$\begin{aligned}\boldsymbol{F}_g &= m\boldsymbol{g}\\ &= 60 \times 3.6\\ &= 216\text{ N}\end{aligned}$$

c $m = 60$ kg on Mars and anywhere else. It does not depend on the gravitational field strength.

6 **a**



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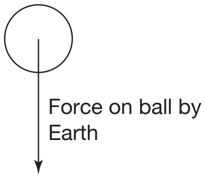

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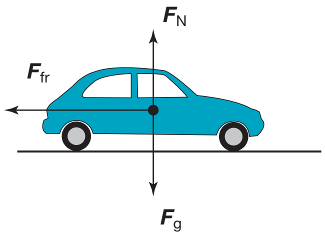

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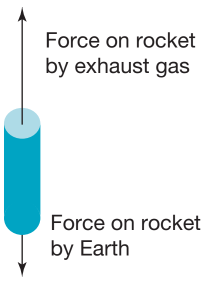
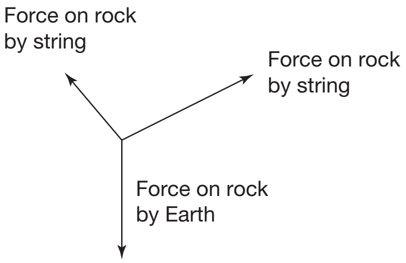


7 a



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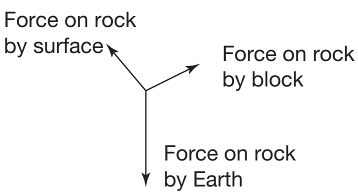

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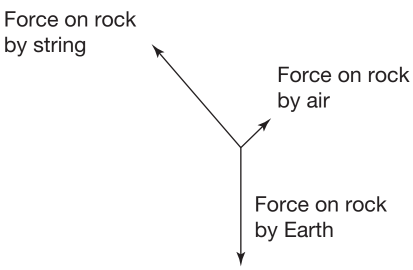

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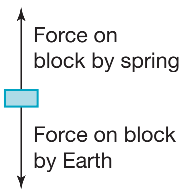
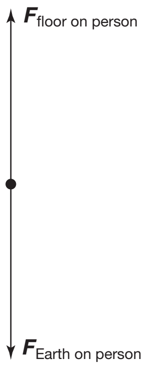
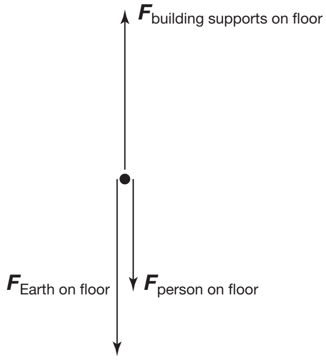


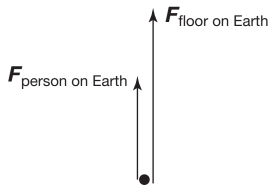
8 Forces on person:



Forces on floor:

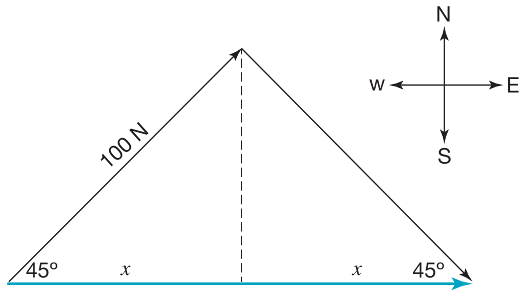


Forces on Earth:



9 a The force 3 N west is subtracted from the forces in the east direction, leaving a net force of 3 N to the east.

b



Using trigonometry we can calculate the value of x in the diagram.

$$\cos(45°) = \frac{x}{100}$$

$$\Rightarrow x = 100\cos(45°)$$

$$\approx 70.7$$

Magnitude of $\boldsymbol{F}_{\text{net}} = 2x$

$$\boldsymbol{F}_{\text{net}} = 2 \times 70.7$$

$$= 141.4\text{ N east}$$

10 a The vertical component of the 400 N force is given by:

$$\boldsymbol{F}_{\text{vertical}} = 400\cos(60°)$$

$$= 200\text{ N}$$

This is equal in magnitude and opposite in direction to the other force, so the vertical component of these forces is 0 N.

The horizontal component of the 400 N force is given by:

$F_{\text{horizontal}} = 400\sin(60°)$

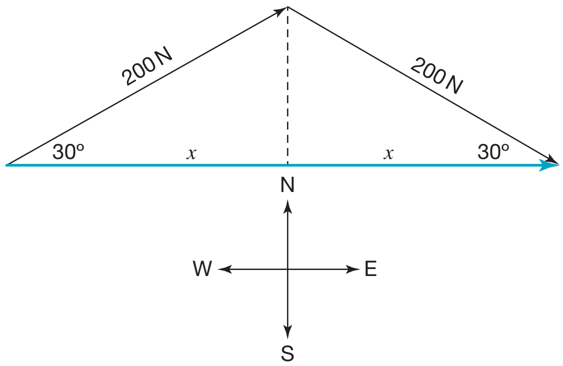
$= 346$ N west

The sum of the two forces is 346 N west.

As the net force is 0 N, the missing force must be equal in magnitude and opposite in direction to the sum of the two forces:

$\therefore \boldsymbol{F}_{\text{missing}} = 346$ N east

b The sum of the two forces acting at 30° to the horizontal can be obtained using a scale vector diagram or trigonometry.



$\cos(30°) = \dfrac{x}{200}$

$\Rightarrow x = 200\cos(30°)$

≈ 173.2 N

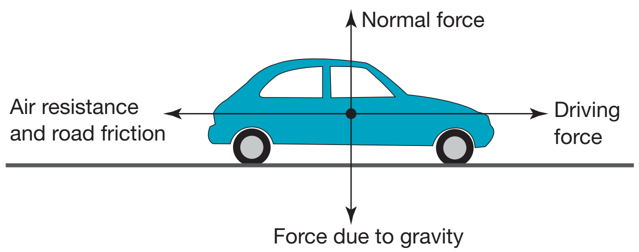
$2x = 346.4$ N east

Adding this to the 200 N force to the west gives a total force of 146.4 N east.

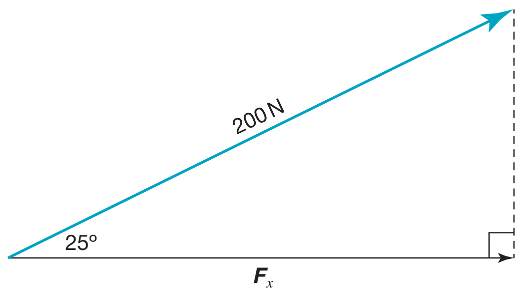
In order to obtain a net force of 200 N east, an additional force of 53.6 N east is required.

The missing force is 53.6 N east.

11



12 a

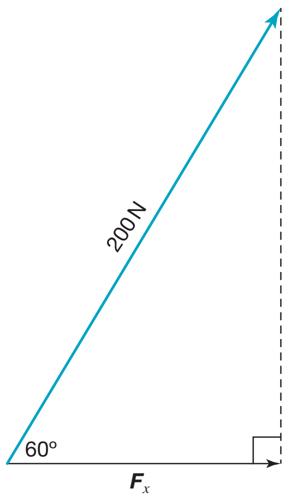


$\cos(25°) = \dfrac{\boldsymbol{F}_x}{200}$

$\Rightarrow \boldsymbol{F}_x = 200\cos(25°)$

≈ 181 N

b



$\cos(60°) = \dfrac{\boldsymbol{F}_x}{200}$

$\Rightarrow \boldsymbol{F}_x = 200\cos(60°)$

$= 100$ N

c The vector is pointing vertically upwards, so the horizontal component of the force is 0 N.

9.2 Exam questions

1 B

Net force is the hypotenuse of a right-angled triangle. Use Pythagoras to find that the hypotenuse is 5.

2 The two forces form two equal sides of a right-angled triangle. The net force is the hypotenuse.

Magnitude:

$\boldsymbol{F}_{\text{net}} = \sqrt{5.0^2 + 5.0^2}$

$= \sqrt{50}$

$= 7.1$ N [1 mark]

Direction (by symmetry) is north-east or 45° true. [1 mark]

3
- Adding $\boldsymbol{A} + \boldsymbol{B}$ gives a resultant force of 6.0 N east. [1 mark]
- Adding $(\boldsymbol{A} + \boldsymbol{B})$ to $\boldsymbol{C}$ gives a right-angled triangle with side lengths 6.0 and x, and hypotenuse 7.5. [1 mark]
- Using Pythagoras gives $x = 4.5$ N. [1 mark]

4 $m = \dfrac{\boldsymbol{F}_{\text{g Moon}}}{g_{\text{Moon}}}$

$= \dfrac{128}{1.60}$

$= 80$ kg [1 mark]

$\boldsymbol{F}_{\text{g Earth}} = mg_{\text{Earth}}$

$= 80 \times 9.8$

$= 784$ N [1 mark]

5 a 25 kg [1 mark]

b The person's mass is still 50 kg but the force due to gravity acting on them will be halved, due to the reduction in $\boldsymbol{g}$. [1 mark]

Since the calibration of the dial is unchanged, the reading for mass will be only half of the correct value. [1 mark]

9.3 Newton's First Law of Motion

9.3 Exercise

1 For a smartphone sliding across a table to maintain a constant speed there must be no unbalanced forces on the phone. Though the vertical forces cancel out, there will always be a horizontal friction force opposing the direction of motion. There will also be a small force due to air resistance that will also oppose the direction of motion.
These horizontal forces act in the same direction, so there is a non-zero net force acting on the phone. According to Newton's First Law, the net force will cause the motion of the phone to change, slowing until it comes to a stop.

2 The vehicle experiences a non-zero net force that slows it down. No such force acts on you. The net force on you is zero. Therefore, you continue in your state of constant velocity. You are not really thrown forward. You continue your motion while the vehicle slows down.

3 There is an unbalanced force on the bike and so its velocity changes. Your inertia keeps you moving forward as there is no unbalanced force to change your motion (apart from gravity).

4 The boxes are moving at a constant speed so the forces acting on them must be in balance. The 125 N force exerted by the removalist is balancing the 125 N friction force acting to oppose the motion of the boxes.

5 The horizontal forces on all the balls were balanced when inside the train so they continued in their motion along with the train and the student. Once the ball exited the window, an air resistance force of significant magnitude would have acted on it in the horizontal direction, and the net force in this direction would be non-zero. This imbalance in forces would lead to a change in the motion of the ball as observed.

9.3 Exam questions

1 Air resistance and friction act to cause the ball to slow down eventually. [1 mark]
These two forces acting to oppose the horizontal motion of the ball result in a non-zero net force upon the ball. In accordance with Newton's First Law, this will change the state of motion of the ball, which in this case means eventually slowing it to a stop. [1 mark]

2 To fall at constant velocity, in accordance with Newton's First Law, the forces acting on the skydiver must balance to produce a zero net force (and constant state of motion). The force due to gravity from Earth is acting downwards on them and is 600 N. So the air resistance must be 600 N [1 mark] in the opposite direction (upwards) [1 mark] to give zero net force.

3 Applying Newton's First Law to the motion of objects, for objects to remain in the same state of motion (constant velocity) the net force on them must be zero.
The space probe in deep space is likely experiencing very close to zero force due to gravity and no air resistance, as deep space is a vacuum. There are no forces that need to be balanced by a rocket thrust to produce a zero net force. Hence, the probe can continue at constant velocity without firing rockets. [1 mark]
An aircraft on Earth experiences significant air resistance opposing its motion, so for it to have a zero net force and continue at constant velocity the engines must produce thrust to balance out the air resistance. [1 mark]

4 $a = 0$
Hence, by Newton's First Law, the net force on the boat = 0. [1 mark]
Hence, driving force = total retarding force
$= 600 + 300$
$= 900$ N [1 mark]

5
- The passenger and car were initially moving forward at speed. [1 mark]
- The obstacle exerted a large net force on the car, which stopped very quickly. With no net force acting on the passenger, he kept moving forward at the original speed. [1 mark]
- The passenger keeps moving forward until he strikes something that exerts a net force backwards, to slow him down. [1 mark]

9.4 Newton's Second Law of Motion

Sample problem 4

$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$$
$$350 = 65\boldsymbol{a}$$
$$\Rightarrow \boldsymbol{a} = \frac{350}{65}$$
$$\approx 5.4 \text{ m s}^{-2}$$

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$$
$$= 0 + 2 \times 5.38 \times 8$$
$$= 86.08$$
$$\Rightarrow \boldsymbol{v} = \sqrt{86.08}$$
$$\approx 9.3 \text{ m s}^{-1}$$

She will be travelling at approximately 9.3 m s^{-1} at the bottom of the slide.

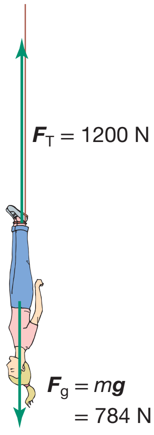
Practice problem 4

a Using Newton's Second Law:
$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$$
$$= 0.058 \times 1.2 \times 10^4$$
$$= 696 \text{ N}$$
The average force applied to the tennis ball during service is 696 N.

b Using Newton's Second Law:
$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$$
$$2 = m \times 2.5$$
$$\Rightarrow m = \frac{2}{2.5}$$
$$= 0.8 \text{ kg}$$
The mass of the toy car is 0.8 kilograms.

Sample problem 5

a
$$\boldsymbol{F}_{\text{g}} = m\boldsymbol{g}$$
$$= 80 \times 9.8$$
$$= 784 \text{ N}$$



$F_{net} = 1200 - 784$
$= 416$ N upwards

$F_{net} = ma$
$416 = 80a$
$\Rightarrow a = \frac{416}{80}$
$= 5.2$ m s^{-2} upwards

Her acceleration at that instant is 5.2 m s^{-2}.

b $u = 16$ m s^{-1}, $v = 0$, $a = -5.2$ m s^{-2}, $s = ?$

$v^2 = u^2 + 2as$
$0 = 16^2 + 2 \times -5.2 \times s$
$= 256 - 10.4s$
$10.4s = 256$
$\Rightarrow s = \frac{256}{10.4}$
≈ 24 m

The bungee jumper will not stop in time.
However, don't be upset. In practice, the acceleration of the bungee jumper would not be constant. The tension in the cord would increase as she fell. Therefore, the net force on her would increase and her upwards acceleration would be greater in magnitude than the calculated value. She will therefore almost certainly come to a stop in a distance considerably less than that calculated.

Practice problem 5

$u = 100$ km h^{-1}
$= \frac{100}{3.6}$ m s^{-1}
≈ 27.8 m s^{-1}

Using a constant acceleration formula we can determine the acceleration needed to stop in 50 metres.

$v^2 = u^2 + 2as$
$0^2 = 27.8^2 + 2 \times 50 \times a$
$\Rightarrow a = -\frac{27.8^2}{2 \times 50}$
$= -\frac{771.60}{100}$
≈ -7.72 m s^{-2}

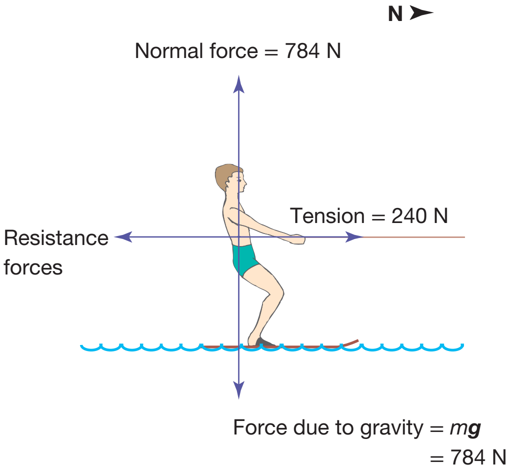
Using Newton's Second Law we can calculate the net force:
$F_{net} = ma$
$= 1200 \times -7.72$
$= -9259$ N

$F_{net} = F_{friction} + F_{brakes}$
$-9259 = -1000 + F_{brakes}$
$\Rightarrow F_{brakes} = -9259 + 1000$
$= -8259$ N

The brakes need to apply a force of at least 8259 N to stop the car in 50 metres.

Sample problem 6

a $F_g = mg$
$= 80 \times 9.8$
$= 784$ N



$v = u + at$
$12 = 0 + a \times 6$
$\Rightarrow a = \frac{12}{6}$
$= 2$ m s^{-2} north

$F_{net} = ma$
$= 80 \times 2$
$= 160$ N north

The net force on the waterskier is 160 N north.

b $F_{net} = 240$ N north
$F_{net} = ma$
$240 = 80a$
$\Rightarrow a = \frac{240}{80}$
$= 3$ m s^{-2} north

If the tension in the rope were the only horizontal force acting on the waterskier, his acceleration would be 3 m s^{-2} north.

c Sum of resistance forces $= F_{net}$ − tension
= 160 N north − 240 N north
= 80 N south

The sum of the resistance forces on the waterskier is 80 N south.

Practice problem 6

a $v^2 = u^2 + 2as$

$6^2 = 0^2 + 2 \times 9 \times a$

$36 = 18a$

$\Rightarrow a = \frac{36}{18}$

$= 2 \text{ m s}^{-2}$

The acceleration of the sled is 2 m s^{-2}.

b Using Newton's Second Law:

$F_{net} = ma$

$= 60 \times 2$

$= 120 \text{ N}$

$F_{net} = F_T - F_{fr}$

$120 = F_T - 200$

$\Rightarrow F_T = 320 \text{ N}$

The tension in the rope is 320 N.

Sample problem 7

a $a = \frac{\text{rise}}{\text{run}}$

$= \frac{-2}{6}$

$= \frac{-1}{3} \text{ m s}^{-2}$

$F_{net} = ma$

$= 45 \times \frac{-1}{3}$

$= -15 \text{ N}$

The magnitude of the net force on the girl on the concrete surface is –15 N.

b $\frac{\text{Friction force of gravel path on rollerblades}}{\text{Friction force of concrete path on rollerblades}} = \frac{F_{net} \text{ on girl while on gravel}}{F_{net} \text{ on girl while on concrete}}$

$\frac{F_{net} \text{ on girl while on gravel}}{F_{net} \text{ on girl while on concrete}} = \frac{ma \text{ on gravel}}{ma \text{ on concrete}}$

$= \frac{a \text{ (during last 4.0 s)}}{a \text{ (during first 6.0 s)}}$

$\frac{a \text{ (during last 4.0 s)}}{a \text{ (during first 6.0 s)}} = \frac{\text{gradient (for last 4.0 s)}}{\text{gradient (for first 6.0 s)}}$

$= \frac{\left(\frac{-6}{4}\right)}{\left(\frac{-1}{3}\right)}$

$= \frac{36}{8}$

$= 4.5$

The value of the ratio is 4.5.

Practice problem 7

a Acceleration = gradient of velocity-versus-time graph

$= -\frac{1}{3}$

Using Newton's Second Law:

$F_{net} = ma$

$= 1200 \times -\frac{1}{3}$

$= -400 \text{ N}$

The magnitude of the net force during the first 6 seconds is 400 N.

b Acceleration = gradient of velocity-versus-time graph

$= -1.5$

Using Newton's Second Law:

$F_{net} = ma$

$= 1200 \times -1.5$

$= -1800 \text{ N}$

The magnitude of the net force during the final 4 seconds is 1800 N.

9.4 Exercise

1 Idealisations can be made to allow the use of a simple mathematical model to solve a physical problem. For example, in order to use simple equations to analyse the motion of a falling ball, the idealisation can be made that the air resistance is insignificant and the ball does not spin.

2 **a** $F_{net} = ma$

$= 2.2 \times 10^6 \times 3.0$

$= 6.6 \times 10^6 \text{ N}$

b $F_{net} = F_{thrust} - F_g$

$\Rightarrow F_{thrust} = F_{net} + F_g$

$= 6.6 \times 10^6 + 2.2 \times 10^6 \times 9.8$

$= 6.6 \times 10^6 + 2.15 \times 10^7$

$= 2.8 \times 10^7 \text{ N}$

3 **a** Both the bowling ball and gold bar have a very small air-resistance-to-mass ratio, so they will experience a very similar acceleration. Therefore, they will fall at the same rate, hitting the ground simultaneously.

b The air resistance on the doormat is significant when compared with the force due to gravity acting on it. Therefore, the net force on the doormat is less than that of the bowling ball (which has the same mass as the doormat) and the acceleration of the doormat $\frac{F_{net}}{m}$ is smaller than that of the bowling ball (and the gold bar).

4 **a** $F_g = ma$

$= 70 \text{ kg} \times 9.8 \text{ N kg}^{-1}$

$= 686 \text{ N downwards}$

b **i** The upwards force is greater when the jumper is decelerating downwards or, in other words, accelerating upwards.

ii The force due to gravity is greater than the upwards pull when the jumper is accelerating downwards.

c The tension in the bungee cord must be equal in magnitude to the force due to gravity acting on the jumper in order for the speed to be constant; that is, 686 N. This occurs only for an instant during the fall. At this instant, the cord is extending and the tension is increasing.

5 **a** $F_{net} = 10\,000 \text{ N} - 2500 \text{ N}$

$= 7500 \text{ N}$

b $a = \frac{F_{net}}{m}$

$= \frac{7500}{1200}$

$= 6.250 \text{ m s}^{-2}$

c $v = u + at$
$= 0 + 6.25 \times 5$
$= 31.25 \text{ m s}^{-1}$

d $s = ut + \frac{1}{2}at^2$
$= 0 + \frac{1}{2} \times 6.25 \times (5.0)^2$
$\approx 78.13 \text{ m}$

6 $v^2 = u^2 + 2as$
$0^2 = 25^2 + 2 \times 360 \times a$
$0 = 625 + 720a$
$\Rightarrow a = -\frac{625}{720}$
$\approx -0.8680 \text{ m s}^{-2}$
$F_{net} = F_{fr}$
$F_{fr} = ma$
$= 8.0 \times 10^6 \times -0.868$
$= -6.944 \times 10^6 \text{ N}$
The brakes must apply a force of at least 6.944×10^6 N for the train to stop within 360 metres.

7 $v^2 = u^2 + 2as$
$0^2 = 12^2 + 2 \times 50 \times a$
$0 = 144 + 100a$
$\Rightarrow a = -\frac{144}{100}$
$= -1.44 \text{ m s}^{-2}$
$F_{net} = F_{fr}$
$F_{fr} = ma$
$= 70 \times -1.44$
$= -100.8 \text{ N}$
The skier must apply a frictional force of 100 N to stop before the cliff.

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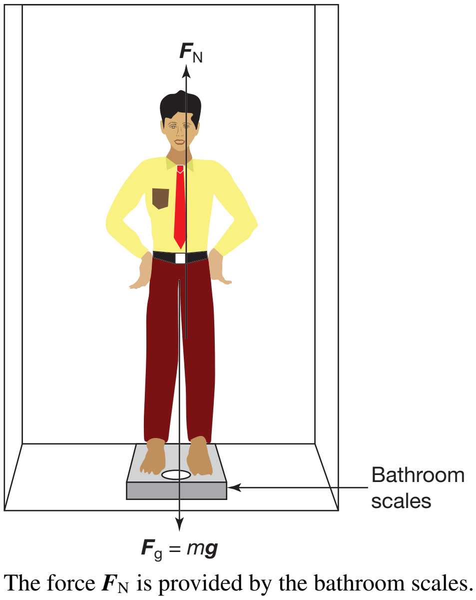


The force F_N is provided by the bathroom scales. The reading on the scales will be equal to F_N.
When the teacher is stationary, $F_N = mg = 700$ N.

a $F_{net} = 0$ since velocity is constant
$F_N - mg = 0$
$\Rightarrow F_N = mg$
$= 700 \text{ N}$

b Assign down as negative.
$a = -2.0 \text{ m s}^{-2}$
$F_N - mg = ma$
$mg = 700$
$\Rightarrow m \approx 71.4 \text{ kg (since } g = 9.8 \text{ N kg}^{-1})$
$F_N - 700 = 71.4 \times -2.0$
$\Rightarrow F_N = -142.8 + 700$
$= 557 \text{ N}$

c $a = 2.0 \text{ m s}^{-2}$
$F_N - mg = ma$
$F_N - 700 = 71.4 \times 2.0$
$\Rightarrow F_N = 142.8 + 700$
$= 840 \text{ N}$

9 **a** The ball is stationary at the top of its flight: $v = 0 \text{ m s}^{-1}$.

b The only force acting on the ball at the top of its flight is the force due to gravity.
$F_{net} = ma$
$F_g = ma$
$mg = ma$
$g = a$
$\Rightarrow a = 9.8 \text{ m s}^{-2}$
The magnitude of the acceleration of the ball at the top of its flight is 9.8 m s^{-2}.

c $F_{net} = ma$
$= 0.5 \times 9.8$
$= 4.9 \text{ N}$
The net force on the ball at the top of its flight is 4.9 N downwards.

9.4 Exam questions

1 Assigning up as positive, the net force on the lift is given by $F_{net} = F_T - F_g$, where F_T is the tension in the cable and F_g is the force due to gravity acting on the lift and its passengers.
$F_g = mg$
$= (480 + 24 \times 70) \times 9.8$
$= 2160 \times 9.8$
$= 21\,168$ N [1 mark]
$F_{net} = F_T - 21\,168$
$2160a = F_T - 21\,168$
$\Rightarrow a = \frac{F_T - 21\,168}{2160}$ [1 mark]
Max tension $= 25\,000$ N
$\Rightarrow a = \frac{25\,000 - 21\,168}{2160}$
$\approx 1.8 \text{ m s}^{-2}$ [1 mark]

2 Consider the forward motion of the block.

$F_{net} = ma$
$= 2.5 \times 2$
$= 5.0$ N forward [1 mark]

Friction on block $= 6.5 - 5.0$
$= 1.5$ N backwards [1 mark]

Hence, friction on the bench top (magnitude) is 1.5 N. [1 mark]

3 Consider only the forces acting on the trailer in the direction of motion. These are the tension force (T) forward and the retarding force backwards of 20 N.

$F_{net} = ma$
$= 50 \times 1.5$
$= 75$ N [1 mark]
$F_{net} = T - F_{retarding}$
$75 = T - 20$ [1 mark]
$\Rightarrow T = 95$ N [1 mark]

4 She has overlooked the retarding force of friction on the object by the benchtop. [1 mark]

$F_{net} = ma$
$= 2.0 \times 2.5$
$= 5.0$ N [1 mark]
$F_{net} = F_{pull} - F_{friction}$
$5 = 6 - F_{friction}$
$\Rightarrow F_{friction} = 1.0$ N [1 mark]

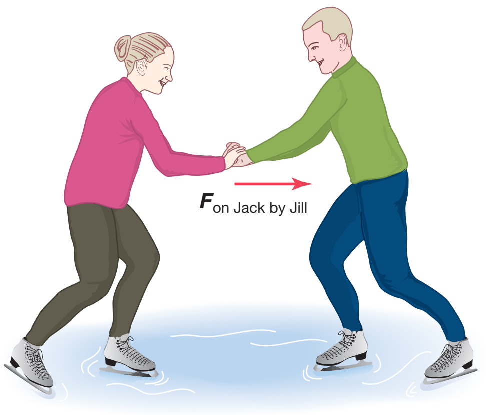
5 Consider only the forces acting on the trailer in the direction of motion. These are the tension force (T) forward and the retarding force backwards of 400 N.

$F_{net} = ma$
$= 600 \times 1.5$
$= 900$ N [1 mark]
$F_{net} = T - F_{retarding}$
$900 = T - 400$ [1 mark]
$\Rightarrow T = 1300$ N [1 mark]

9.5 Newton's Third Law of Motion

Sample problem 8

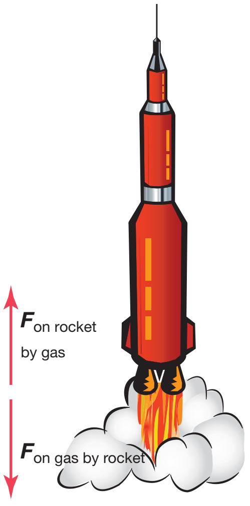
$\boldsymbol{F}_{\text{on A by B}} = -\boldsymbol{F}_{\text{on B by A}}$



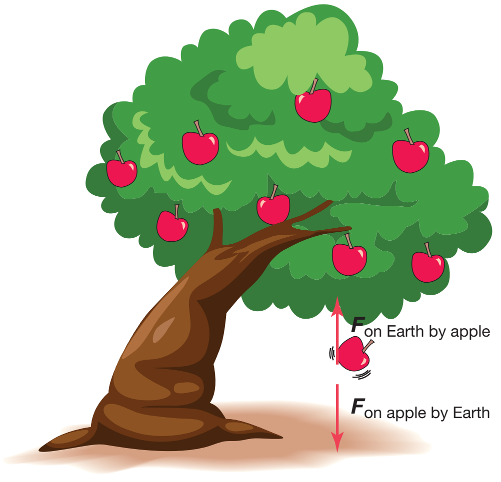
Practice problem 8

a

b



c



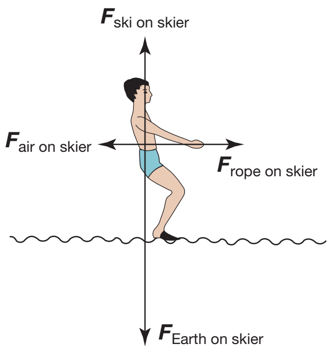
d

9.5 Exercise

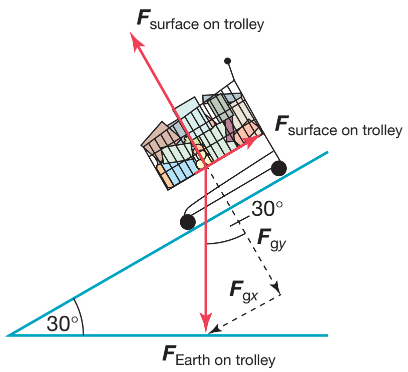
1

Force 1	Pair of force 1
You push on a wall with the palm of your hand.	The wall pushes on your palm in the opposite direction.
Your foot pushes down on a bicycle pedal.	The bicycle pedal pushes up on your foot.
The ground pushes up on your feet while you are standing.	You push down on the ground when you are standing.
Earth pulls down on your body.	Your body pulls up on Earth.
You push on a broken-down car to try to get it moving.	The broken-down car pushes on you in the opposite direction.
A hammer pushes down on a nail.	The nail pushes up on the hammer.

2 **a**



b



3 The dinghy:

Force	Action–reaction pair
Resistance forces	$\boldsymbol{F}_{\text{dinghy on air and water}}$, $\boldsymbol{F}_{\text{air and water on dinghy}}$
Gravity	$\boldsymbol{F}_{\text{Earth on dinghy}}$, $\boldsymbol{F}_{\text{dinghy on Earth}}$
Tension	$\boldsymbol{F}_{\text{rope on dinghy}}$, $\boldsymbol{F}_{\text{dinghy on rope}}$
Normal force	$\boldsymbol{F}_{\text{dinghy on water}}$, $\boldsymbol{F}_{\text{water on dinghy}}$

The boat:

Force	Action–reaction pair
Resistance forces	$\boldsymbol{F}_{\text{boat on air and water}}$, $\boldsymbol{F}_{\text{air and water on boat}}$
Gravity	$\boldsymbol{F}_{\text{Earth on boat}}$, $\boldsymbol{F}_{\text{boat on Earth}}$
Tension	$\boldsymbol{F}_{\text{boat on rope}}$, $\boldsymbol{F}_{\text{rope on boat}}$
Normal force	$\boldsymbol{F}_{\text{boat on water}}$, $\boldsymbol{F}_{\text{water on boat}}$
Driving force	$\boldsymbol{F}_{\text{boat on water}}$, $\boldsymbol{F}_{\text{water on boat}}$

4 All swimmers move forwards in the water because the water pushes them forwards. This is the unbalanced force that provides the acceleration during each stroke (Newton's First Law). The size of the forward force is equal to, and opposite in direction to, the force that the swimmer applies to the water (Newton's Third Law). A freestyle stroke pushes water back with a greater force than a breaststroke stroke. Therefore, the forward force is greater for a freestyle swimmer.

5 The student is correct but needs to make it clear that the two friction forces are not an action–reaction pair. The best way to do this is to identify the two action–reaction pairs involved. If the car is a front-wheel drive, the friction on each of the rear tyres is a reaction to the backwards push of the front tyres on the road. The rear tyres are being pulled forward. So the friction on the rear tyres is a reaction to the forward push of the rear tyres on the road.

9.5 Exam questions

1 Any two of the following or equivalent:

- $\boldsymbol{F}_{\text{on car by air (drag)}} = -\boldsymbol{F}_{\text{on air by car}}$
- $\boldsymbol{F}_{\text{on car by road (friction)}} = -\boldsymbol{F}_{\text{on road by car}}$
- $\boldsymbol{F}_{\text{on car by road (normal)}} = -\boldsymbol{F}_{\text{on road by car}}$
- $\boldsymbol{F}_{\text{on car by Earth}} = -\boldsymbol{F}_{\text{on Earth by car}}$

(1 mark for any two of the above)

2 B

If object B applies a force to object A, then object A applies an equal and opposite force to object B.

Thus, option B is false.

3 C

The forces are equal in magnitude and opposite in direction; this is a Newton's Third Law force pair.

$\boldsymbol{F}_{\text{large on small}} = -\boldsymbol{F}_{\text{small on large}}$

4 By Newton's Third Law, P and Q exert the same magnitude of force on one another. [1 mark]

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{\text{net}}}{m}$$

$$\Rightarrow \boldsymbol{a}_{\text{Q}} = \frac{\boldsymbol{F}}{m_{\text{Q}}}$$

$$a_P = \frac{F}{m_P}$$
$$= \frac{F}{2m_Q} \text{ [1 mark]}$$
$$= \frac{1}{2}a_Q$$
$$= \frac{1}{2} \times 3$$
$$= 1.5 \text{ m s}^{-2} \text{ [1 mark]}$$

5 • To cause motion, the tyres push backwards on the road and, by Newton's Third Law, the road friction pushes forward on the tyres. [1 mark]
- However, the muddy road surface can provide very little friction force on the tyres. [1 mark]
- Hence, with little force forward, the acceleration of the wheels (and car) is negligible. [1 mark]

9.6 Forces in two dimensions

Sample problem 9

$F_{net} = 0$

Friction $= F_{gx}$

$$\sin(15°) = \frac{F_{gx}}{F_g}$$
$$\Rightarrow F_{gx} = F_g \sin(15°)$$

$$F_g = mg$$
$$= 1600 \times 9.8$$
$$= 15\,680 \text{ N}$$

$$F_{gx} = F_g \sin(15°)$$
$$= 15\,680 \sin(15°)$$
$$\approx 4058 \text{ N}$$

The road friction on the car is 4058 N.

Note: It is useful to consider the effect on the net force of the angle of the incline to the horizontal. If the angle is greater than 15°, the component of the force due to gravity parallel to the slope increases and the net force will no longer be zero. The speed of the car will therefore increase. The component of the force due to gravity perpendicular to the slope decreases and the normal force decreases by the same amount.

Practice problem 9

a **i** Component of gravity down the slope:

$$mg\sin\theta = 5000 \times 9.8 \times \sin(20°)$$
$$\approx 16\,759 \text{ N}$$

ii Component of gravity perpendicular to the slope:

$$mg\cos\theta = 5000 \times 9.8 \times \cos(20°)$$
$$\approx 46\,045 \text{N}$$

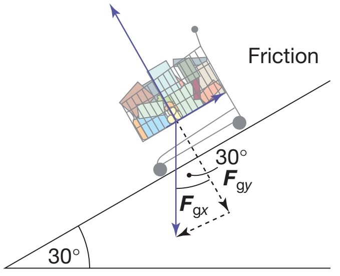
b **i** Zero, as reaction force is perpendicular to the surface

ii Reaction force $= F_g \cos(15°)$

$$= 1600 \times 9.8 \times \cos(15°)$$
$$\approx 15\,146 \text{ N}$$

Sample problem 10

a



$$F_{net} = F_{gx} - \text{friction}$$
$$= mg\sin(30°) - 270 \text{ N}$$
$$= 588 \text{ N} \times \sin(30°) - 270 \text{ N}$$
$$= 294 \text{ N} - 270 \text{ N}$$
$$= 24 \text{ N}$$

$$F_{net} = ma$$
$$24 = 60a$$
$$\Rightarrow a = \frac{24}{60}$$
$$= 0.40 \text{ m s}^{-2} \text{ down the slope}$$

$$v = u + at$$
$$= 0 + 0.4 \times 9$$
$$= 3.6 \text{ m s}^{-1}$$

The speed of the trolley at the end of its roll is 3.6 m s^{-1}.

b
$$s = ut + \frac{1}{2}at^2$$
$$= 0 + \frac{1}{2} \times 0.4 \times 9^2$$
$$= 16.2 \text{ m}$$

The trolley travels 16.2 metres before it is stopped.

Practice problem 10

a Component of the force due to gravity down the slope:

$$mg\sin(20°) = 100 \times 9.8 \times \sin(20°)$$
$$\approx 335.2 \text{ N}$$

$$F_{net} = 335.2 - 300$$
$$= 35.2 \text{ N}$$

The net force on the bicycle (including the cyclist) is 35 N down the slope.

b Using Newton's Second Law:

$$F_{net} = ma$$
$$35.2 = 100a$$
$$\Rightarrow a = \frac{35.2}{100}$$
$$= 0.35 \text{ m s}^{-2}$$

The acceleration of the bicycle is 0.35 m s^{-2} down the slope.

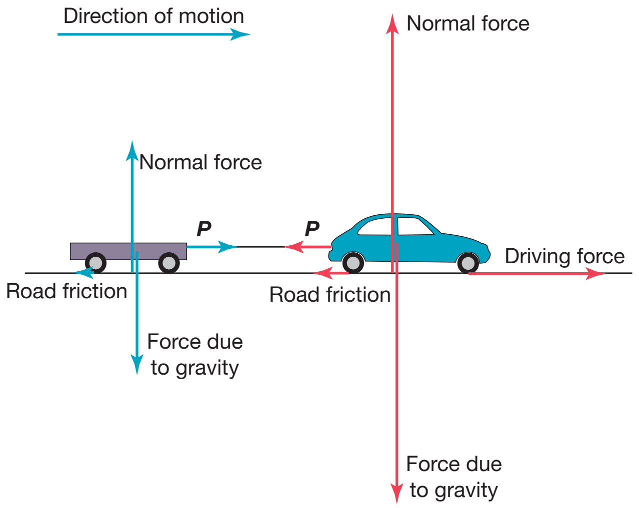
c Final velocity can be found using a constant acceleration formula.

$$v = u + at$$
$$= 0 + 0.352 \times 12$$
$$\approx 4.2 \text{ m s}^{-1}$$

The bicycle is travelling at 4.2 m s^{-1} when it reaches the horizontal surface.

Sample problem 11

a



$\boldsymbol{F}_{\text{net}}$ = driving force − road friction (car) − road friction (trailer)

= 5400 N − 800 N − 400 N

= 4200 N

$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$

$4200 = 2100\boldsymbol{a}$

$\Rightarrow \boldsymbol{a} = \dfrac{4200}{2100}$

$= 2 \text{ m s}^{-2}$ to the right

The acceleration of both the car and trailer is 2 m s^{-2} to the right.

b $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$

$\boldsymbol{P} - 400 = 700 \times 2$

$\Rightarrow \boldsymbol{P} = 700 \times 2 + 400$

$= 1800$ N

The force with which the trailer is pulled by the car is 1800 N.

Practice problem 11

a $\boldsymbol{F}_{\text{net}}$ on whole system = 4700 – (400 + 100)

= 4200 N

Using Newton's Second Law:

$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$

$4200 = 2100\boldsymbol{a}$

$\Rightarrow \boldsymbol{a} = \dfrac{4200}{2100}$

$= 2 \text{ m s}^{-2}$

The acceleration of the boat and dinghy is 2 m s^{-2}.

b Using Newton's Second Law:

$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$

$= 100 \times 2$

$= 200$ N

The net force on the dinghy is 200 N.

c $\boldsymbol{F}_{\text{net}}$ = tension – resistance

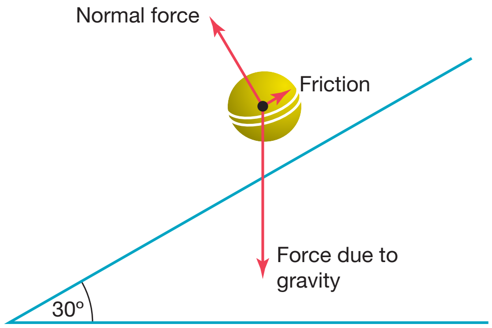
So tension = 200 + 100

= 300 N

The tension in the rope is 300 N.

9.6 Exercise

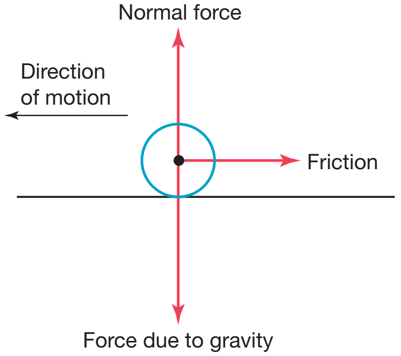
1 a



b The direction of the net force on the ball is down the hill (parallel to the hill) since the ball is speeding up as it rolls down the hill.

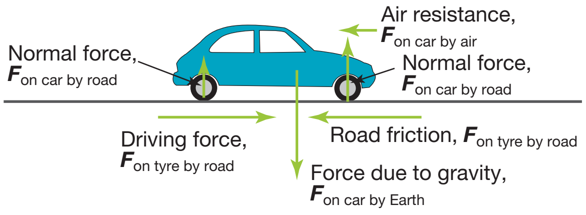
c The force due to gravity. The normal force is equal to the component of the force due to gravity that is perpendicular to the slope, so is smaller in magnitude. As the ball is accelerating down the slope the friction force is smaller than the component of the force due to gravity that is down the slope, so is smaller in magnitude.

d The ball slows to a stop because of the effect of the friction acting on the ball. On a horizontal surface, the normal reaction is equal in magnitude to the force due to gravity, so the friction force is the net force, which causes the ball to decelerate.

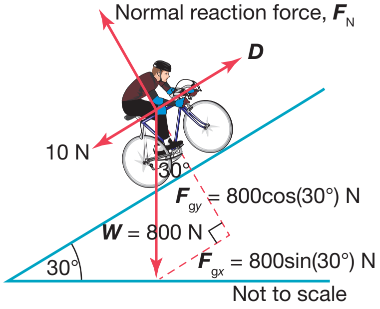


2 When you attempt to push the car you are unable to provide enough force to overcome the friction between the stationary tyres (which cannot rotate due to the handbrake being on) and the road. This friction balances out your push, so the net force remains zero.

3



4



a The net force is zero because the cyclist is moving with constant velocity.

b $\boldsymbol{F}_g = m\boldsymbol{g}$

$= 80 \text{ kg} \times 9.8 \text{ N kg}^{-1}$

$= 784 \text{ N}$

$\boldsymbol{F}_{gx} = \boldsymbol{F}_g \sin(30°)$

$= 784\sin(30°)$

$= 392 \text{ N}$

c $\boldsymbol{F}_{net} = 0$

Therefore, the sum of forces parallel to the slope is zero.

$\boldsymbol{D} - 392 - 10 = 0$

$\Rightarrow \boldsymbol{D} = 402 \text{ N}$

d The sum of the forces perpendicular to the slope is zero.

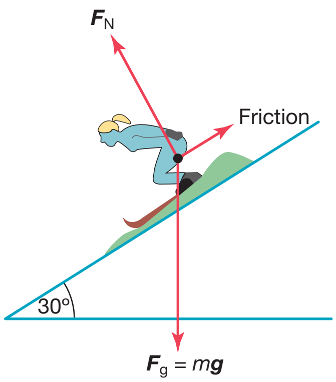
$\boldsymbol{F}_N = \boldsymbol{F}_{gy}$

$= 784\cos(30°)$

$\approx 679 \text{ N}$

5 a Since the speed is increasing down the slope, the net force must be in the same direction; that is, down the slope.

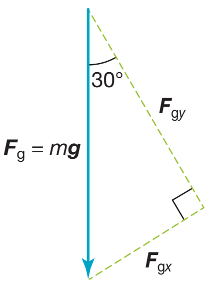
b



c $\boldsymbol{F}_{gx} = m\boldsymbol{g}\sin(30°)$

$= 60 \times 9.8 \times \sin(30°)$

$= 294 \text{ N}$

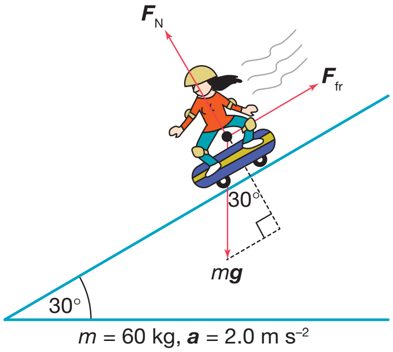


d $\boldsymbol{F}_{net} = 294 \text{ N down slope} - 8 \text{ N up slope}$

$= 286 \text{ N down slope}$

Magnitude of net force = 286 N

6



The force due to gravity $m\boldsymbol{g}$ can be resolved into components that are parallel to and perpendicular to the slope.

Considering the forces parallel to the slope:

$\boldsymbol{F}_{net} = m\boldsymbol{g}\sin(30°) - \boldsymbol{F}_{fr}$

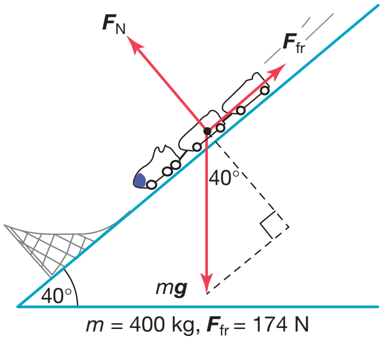
$m\boldsymbol{a} = m\boldsymbol{g}\sin(30°) - \boldsymbol{F}_{fr}$

$120 = 588\sin(30°) - \boldsymbol{F}_{fr}$

$\Rightarrow \boldsymbol{F}_{fr} = 294 - 120$

$= 174 \text{ N}$

7



The force due to gravity $m\boldsymbol{g}$ can be resolved into components that are parallel to and perpendicular to the slope.

Considering the forces parallel to the slope:

$\boldsymbol{F}_{net} = m\boldsymbol{g}\sin(40°) - 180$

$400\boldsymbol{a} = 3920\sin(40°) - 180$

$$\Rightarrow \boldsymbol{a} = \frac{3920\sin(40°) - 180}{400}$$

$\approx 5.85 \text{ m s}^{-2}$

8 a $\boldsymbol{a} = \dfrac{\Delta \boldsymbol{v}}{\Delta t}$

$= \dfrac{6}{8}$

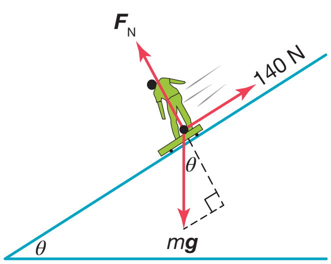
$= 0.75 \text{ m s}^{-2}$

$\boldsymbol{F}_{net} = m\boldsymbol{a}$

$= 56 \text{ kg} \times 0.75 \text{ m s}^{-2}$

$= 42 \text{ N}$

b



The force due to gravity, $m\boldsymbol{g}$, can be resolved into components that are parallel to and perpendicular to the slope.

Considering the forces parallel to the slope:

$F_{net} = mg\sin\theta - 140$

$42 = 548.8\sin\theta - 140$

$\sin\theta = \dfrac{42 + 140}{548.8}$

$\Rightarrow \theta \approx 19°$

9 **a** Traction (friction) on your blades

b The component of gravity down the slope and the force of the ground on your poles if you use them

c The tension in the rope attached to the handle that you are holding

d The traction (friction) as you push back on the ground

e The force exerted by the water on your hands, arms, legs and feet as you push back with your hands and kick

f The force exerted by the water on the oar as the oar pushes back on the water

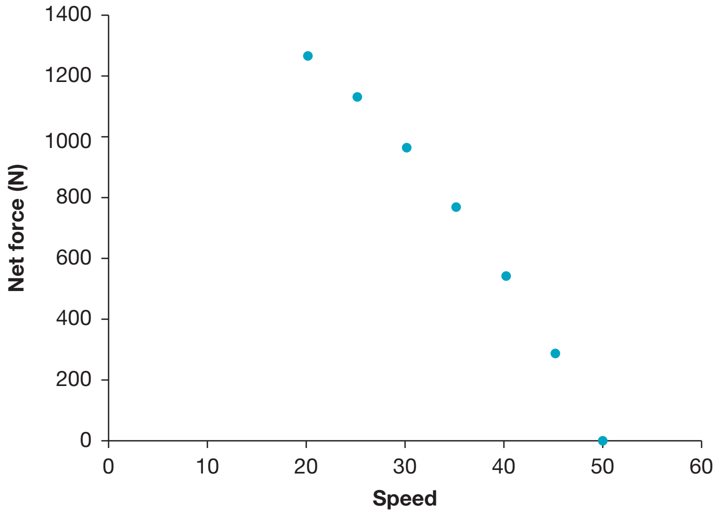
10 The driving force, the forward force applied to the tyres by the road, is a reaction to the force applied backwards to the road by the tyres. The size of the driving force is, therefore, controlled by the driver's use of the accelerator. The driving force acts on the front wheels of a front-wheel-drive vehicle, whereas it acts only on the rear wheels of a rear-wheel-drive car. The front wheels of a rear-wheel-drive car are pushed forward as a result of the driving force on the rear wheels. So the force applied to the front wheels cannot be controlled directly by the driver.

11 **a** A sample spreadsheet is shown here.

	A	B	C	D	E
1	Speed (km h^{-1})	Driving force (N)	Friction (N)	Force due to air resistance (N)	Net force (N)
2	20	1800	–300	=–0.6*A2*A2	=B2+C2+D2
3	25	1800	–300	=–0.6*A3*A3	=B3+C3+D3
4	30	1800	–300	=–0.6*A4*A4	=B4+C4+D4
5	35	1800	–300	=–0.6*A5*A5	=B5+C5+D5
6	40	1800	–300	=–0.6*A6*A6	=B6+C6+D6
7	45	1800	–300	=–0.6*A7*A7	=B7+C7+D7
8	50	1800	–300	=–0.6*A8*A8	=B8+C8+D8

	A	B	C	D	E
1	Speed (km h^{-1})	Driving force (N)	Friction (N)	Force due to air resistance (N)	Net force (N)
2	20	1800	–300	–240	1260
3	25	1800	–300	–375	1125
4	30	1800	–300	–540	960
5	35	1800	–300	–735	765
6	40	1800	–300	–960	540
7	45	1800	–300	–1215	285
8	50	1800	–300	–1500	0

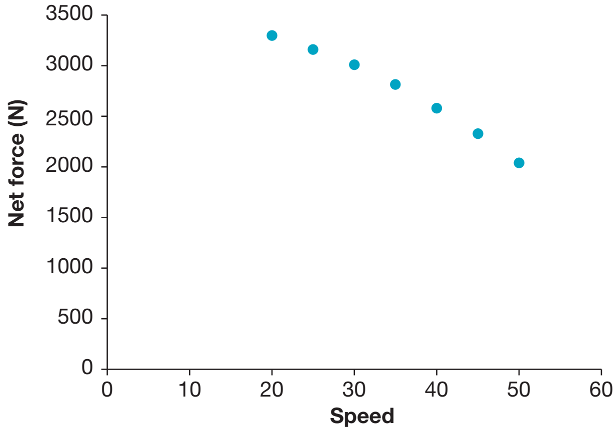
b



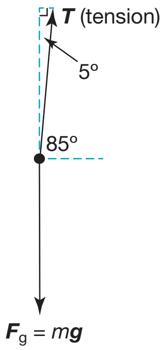
c

	A	B	C	D	E
1	Speed (km h^{-1})	Driving force (N)	Friction (N)	Force due to air resistance (N)	Net force (N)
2	20	1800	–300	=–0.6*A2*A2	=B2+C2+D2+1200*9.8*0.17365
3	25	1800	–300	=–0.6*A3*A3	=B3+C3+D3+1200*9.8*0.17365
4	30	1800	–300	=–0.6*A4*A4	=B4+C4+D4+1200*9.8*0.17365
5	35	1800	–300	=–0.6*A5*A5	=B5+C5+D5+1200*9.8*0.17365
6	40	1800	–300	=–0.6*A6*A6	=B6+C6+D6+1200*9.8*0.17365
7	45	1800	–300	=–0.6*A7*A7	=B7+C7+D7+1200*9.8*0.17365
8	50	1800	–300	=–0.6*A8*A8	=B8+C8+D8+1200*9.8*0.17365

	A	B	C	D	E
1	Speed (km h^{-1})	Driving force (N)	Friction (N)	Force due to air resistance (N)	Net force (N)
2	20	1800	–300	–240	3302.124
3	25	1800	–300	–375	3167.124
4	30	1800	–300	–540	3002.124
5	35	1800	–300	–735	2807.124
6	40	1800	–300	–960	2582.124
7	45	1800	–300	–1215	2327.124
8	50	1800	–300	–1500	2042.124



12



The acceleration of the yoyo in the horizontal direction is the same as the acceleration of the rollerblader.

$\Rightarrow ma$ (horizontal) $= T\sin(5°)$ [1]

The vertical acceleration is zero.

$mg = T\cos(5°)$ [2]

Dividing [1] by [2]:

$$\frac{ma}{mg} = \frac{T\sin(5°)}{T\cos(5°)}$$

$$\Rightarrow \frac{a}{9.8} = \tan(5°)$$

$$\Rightarrow a = 9.8\tan(5°)$$
$$= 0.9 \text{ m s}^{-2}$$

13 The friction on the rear wheels is the driving force, which pushes the bicycle forward. The driving force is a reaction to the backward push of the rear wheel on the road, which can be controlled by the cyclist. The friction on the front wheel is a retarding force, which opposes the forward rolling motion of the front wheel, which is pushed forward by the moving bicycle. When the driving force is greater than the retarding force, the bicycle accelerates. When the driving force is equal to the retarding force, the bicycle maintains a constant speed. If the driving force is less than the retarding force, the bicycle decelerates.

14

a $F_{net} = 14$ N, $m = 7$ kg

$$a = \frac{F_{net}}{m}$$
$$= \frac{14 \text{ N}}{7.0 \text{ kg}}$$
$$= 2.0 \text{ m s}^{-2}$$

b Consider the 3.0 kg trolley:

$$F_{net} = T$$
$$3.0 \text{ kg} \times 2.0 \text{ m s}^{-2} = T$$
$$\Rightarrow T = 6.0 \text{ N}$$

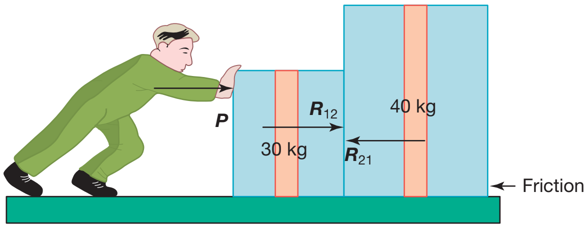
c $F_{net} = 14 \text{ N} - T$

$$= 14 \text{ N} - 6.0 \text{ N}$$
$$= 8.0 \text{ N}$$

d $F_{net} = 14$ N, $m = 4.0$ kg

$$a = \frac{F_{net}}{m}$$
$$= \frac{14 \text{ N}}{4.0 \text{ kg}}$$
$$= 3.5 \text{ m s}^{-2}$$

15



R_{12} = Force exerted by 30 kg crate on 40 kg crate
R_{21} = Force exerted by 40 kg crate on 30 kg crate

Assign direction to the right as positive.

a Applying Newton's Second Law to the system of the two crates:

$$F_{net} = ma$$
$$\Rightarrow P - \text{friction} = 70a$$
$$420 - (70 \times 2) = 70a$$
$$\Rightarrow a = \frac{420 - 140}{70}$$
$$= 4.0 \text{ m s}^{-2} \text{ to the right}$$

b Applying Newton's Second Law to the 40 kg crate:

$$F_{net} = ma$$
$$= 40 \times 4.0$$
$$= 160 \text{ N to the right}$$

c Applying Newton's Second Law to the 30 kg crate:

$$F_{net} = ma$$
$$= 30 \times 4.0$$
$$= 120 \text{ N}$$

$$F_{net} = P + R_{21} + \text{friction on 30 kg crate:}$$
$$\Rightarrow 120 = 420 + R_{21} - (30 \times 2)$$
$$120 = 360 + R_{21}$$
$$\Rightarrow R_{21} = -240 \text{ N}$$
$$= 240 \text{ N to the left}$$

d $R_{12} = -R_{21}$

$$= 240 \text{ N to the right}$$

e The net force is still P – friction.

The mass is still 70 kg.

$$\Rightarrow a = \frac{F_{net}}{m}$$
$$= \frac{420 - (70 \times 2)}{70}$$
$$= 4.0 \text{ m s}^{-2} \text{ to the right}$$

It is no easier.

9.6 Exam questions

1 The acceleration of the trailer is the same as that of the whole system of cyclist, bike and trailer. [1 mark]

$$a = \frac{F_{net}}{m_{total}} \text{ [1 mark]}$$
$$= \frac{220 - 150 - 50}{80 + 20}$$
$$= \frac{20}{100}$$
$$= 0.2 \text{ m s}^{-2} \text{ [1 mark]}$$

2 C

The driving force = the friction on the rear tyre directed forward

The net force = driving force – rolling friction – drag

The net force = ma = 0 (constant speed)
Hence, drag = driving force – rolling friction
= 40 – 10 = 30 N

3 C
$\boldsymbol{R} = mg \times \cos(\text{angle})$
$= 98 \times \cos(40°)$
$= 75$ N

4 B
The friction of the road on the rear tyre is the driving force that propels the bike forwards.

5 C
$\boldsymbol{R} = mg\cos(\theta)$
$= 4 \times 9.8\cos(30°)$
$= 33.9$ N

9.7 Momentum and impulse

Sample problem 12

$\boldsymbol{p} = m\boldsymbol{v}$
$= 8 \times 10^6 \times 15$
$= 1.2 \times 10^8$ kg m s^{-1}

The momentum of the train is 1.2×10^8 kg m s^{-1} north.

Practice problem 12

a Using $\boldsymbol{p} = m\boldsymbol{v}$:
$\boldsymbol{p} = 1200 \times 15$
$= 18\,000$ kg m s^{-1}
The momentum of the car before it accelerates is 18 000 kg m s^{-1} east.

b Using $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$:
$\boldsymbol{v} = 15 + 3.0 \times 2.0$
$= 21$ m s^{-1}
Using $\boldsymbol{p} = m\boldsymbol{v}$:
$\boldsymbol{p} = 1200 \times 21$
$= 25\,200$ kg m s^{-1}
The momentum of the car at the end of the 2-seconds acceleration is 25 200 kg m s^{-1} east.

Sample problem 13

a $\boldsymbol{p}_i = m\boldsymbol{u}$
$= 0.03 \times 15$
$= 0.45$ kg m s^{-1}
$\boldsymbol{p}_f = m\boldsymbol{v}$
$= 0.03 \times -12$
$= -0.36$ kg m s^{-1}
$\Delta\boldsymbol{p} = \boldsymbol{p}_f - \boldsymbol{p}_i$
$= -0.36 - 0.45$
$= -0.81$ kg m s^{-1}
The change in momentum of the squash ball is 0.81 kg m s^{-1} away from the wall.

b $\boldsymbol{I} = \Delta\boldsymbol{p}$
$= -0.81$ N s
The impulse on the squash ball is 0.81 N s away from the wall.

c $\boldsymbol{F}_{\text{net}} = \dfrac{\Delta\boldsymbol{p}}{\Delta t}$
$= \dfrac{-0.81}{1.5 \times 10^{-3}}$
$= 540$ N
The magnitude of the force exerted by the wall on the squash ball is 540 N.

Practice problem 13

a Change in momentum $= m \times \Delta\boldsymbol{v}$
$= 1400 \times (4.0 - (-16))$
$= 28\,000$ kg m s^{-1}
The change in momentum of the car during contact with the barrier is 28 000 kg m s^{-1}.

b Impulse = change in momentum
= 28 000 kg m s^{-1}
The impulse applied to the car by the barrier is 28 000 kg m s^{-1}.

c Using $\boldsymbol{I} = \boldsymbol{F} \times \Delta t$:
$F = \dfrac{28\,000}{1.4}$
$= 20\,000$ N
The force exerted by the barrier on the car is 20 000 N.

Sample problem 14

Magnitude of impulse = area A + area B + area C
$= \frac{1}{2} \times 1.1 \ \times 400 + 0.9 \ \times 200$
$+ \frac{1}{2} \times 0.9 \ \times 200$
$= 220 + 180 + 90$
$= 490$ N

$\boldsymbol{I} = \Delta\boldsymbol{p}$
$\boldsymbol{I} = m\Delta\boldsymbol{v}$
$490 = 40\Delta\boldsymbol{v}$
$\Rightarrow \Delta\boldsymbol{v} = \dfrac{490}{40}$
$= 12.25$ m s^{-1}

As the skater started at rest, her velocity after 2 seconds will be equal to the change in velocity.

The skater's velocity after 2 seconds is 12.25 m s^{-1}.

Practice problem 14

a Area under graph = impulse = change in momentum
Area $= \frac{1}{2} \times 400 \times 1.1$
$= 220$ kg m s^{-1}
So velocity after 1.1 s $= \dfrac{220}{40}$
$= 5.5$ m s^{-1}

b Average acceleration $= \dfrac{\Delta\boldsymbol{v}}{\Delta t}$
$= \dfrac{5.5}{1.1}$
$= 5.0$ m s^{-2}

c Total impulse = 490 N s

So average force $= \frac{490}{2.0}$

$= 245$ N

9.7 Exercise

1 $v = 60$ km h^{-1}

$= \frac{60}{3.6}$ m s^{-1}

$= \frac{50}{3}$ m s^{-1} east

$p = mv$

$= 1400 \times \frac{50}{3}$

$= 23\,000$ kg m s^{-1} east

2 **a** $40\,\text{km h}^{-1} = \frac{40}{3.6}\ \text{m s}^{-1}$

$a = \frac{\Delta v}{\Delta t}$

$= \frac{11.11 - 0}{3.2}$

$= 3.47$ m s^{-2}

$F_{net} = ma$

$= 1000\,\text{kg (estimate)} \times 3.47\,\text{m s}^{-2}$

$\approx 3 \times 10^3$ N to one significant figure

b At terminal velocity:

Air resistance = force due to gravity

$= mg$

$= 80\ \text{kg} \times 9.8\ \text{N kg}^{-1}$

$= 8.10^2$ N to one significant figure

c $p = mv$

$= 80\ \text{kg (estimate)} \times 10\ \text{m s}^{-1}$ (estimate)

$\approx 8 \times 10^2\ \text{kg m s}^{-1}$ to one significant figure

d $p = mv$

$= 1000\ \text{kg (estimate)} \times 60\ \text{km h}^{-1}$ (estimate)

$\approx 1000\ \text{kg} \times 16.7\ \text{m s}^{-1}$

$\approx 2 \times 10^4\ \text{kg m s}^{-1}$ to one significant figure

e $I = m\Delta v$

$= 70\ \text{kg} \times 8\ \text{m s}^{-1}$ (estimate)

$\approx 6.10^2\ \text{kg m s}^{-1}$ to one significant figure

f $I = m\Delta v$

$= 0.4\ \text{kg (estimate)} \times 5\ \text{m s}^{-1}$

$= 2$ N s to one significant figure

g $\Delta p = m\Delta v$

$= 0.06\ \text{kg (estimate)} \times 50\ \text{m s}^{-1}$ (estimate)

$= 3\ \text{kg m s}^{-1}$ to one significant figure

3 Taking down as positive:

a $\Delta p = m\Delta v$

$= 0.060(-6.0 - 8.0)$

$= -0.84\ \text{kg m s}^{-1}$

$= 0.84\ \text{kg m s}^{-1}$ up

b $0.84\ \text{kg m s}^{-1}$ or 0.84 N s down

The impulse applied by the tennis ball to the ground is equal and opposite to the impulse applied by the ground to the tennis ball.

c No. The ground does not move a measurable amount. The impulse applied to the ground is 0.84 kg m s^{-1} but the mass of Earth is so large that the change in velocity is negligible.

d $F_{net}\Delta t = \Delta p$

$\Rightarrow F_{net} = \frac{\Delta p}{\Delta t}$

$= \frac{0.84\ \text{N s}}{2.0 \times 10^{-3}\ \text{s}}$

$= 4.2 \times 10^2$ N up

e $F_{net} = N - mg$

$4.2 \times 10^2 = N - 0.06 \times 10$

$= N - 0.6$

$\Rightarrow N = 4.2 \times 10^2 + 0.6$

$= 4.2 \times 10^2$ N up

4 **a** $m = 75$ kg, $u = -3.2$ m s^{-1}, $v = 0$

Defining up as positive:

Impulse $= m\Delta v$

$= 75(0 - -3.2)$

$= 240$ N s

$= 240$ N s upwards

b $F_{net}\Delta t = m\Delta v$

$F_{net} \times 0.10 = 240$

$\Rightarrow F_{net} = 2400$ N upwards

$F_{net} = N - mg$

(where N = force ground applies to feet (normal force))

$2400 = N - 735$

$\Rightarrow N = 3135$ N

$= 3.1 \times 10^3$ N upwards

c $u = 0$, $v = -3.2$ m s^{-1}, $a = -9.8$ m s^{-2}

$v^2 = u^2 + 2as$

$(-3.2)^2 = 2 \times -9.8 \times s$

$\Rightarrow s = -\frac{(3.2)^2}{19.6}$

$= -0.52$ m

Height (assuming feet are motionless relative to the basketballer's centre of mass) is approximately 0.52 metres.

5 $m = 1400$ kg, $u = 60\ \text{km h}^{-1} = \frac{50}{3}\ \text{m s}^{-1}$, $v = 0$, $t = 0.080$ s

a $I = m\Delta v$

$= 1400 \times \left(0 - \frac{50}{3}\right)$

$= 2.3 \times 10^4$ N s opposite to initial direction of motion of the car

b $F_{net}\Delta t = m\Delta v$

$\Rightarrow F_{net} = \frac{m\Delta v}{\Delta t}$

$= \frac{2.33 \times 10^4}{0.080}$

$= 2.9 \times 10^5$ N opposite to initial direction of motion of the car

c The deceleration of the driver is the same as the deceleration of the car because the driver is wearing a seatbelt.

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$
$$= \frac{\frac{50}{3}}{0.080}$$
$$= -2.1 \times 10^2 \text{ m s}^{-2}$$
$$\Rightarrow \text{Deceleration} = 2.1 \times 10^2 \text{ m s}^{-2}$$

6 The airbags allow the change in momentum (impulse) of the driver's head to take place over a longer time interval than would be the case if it collided directly with the steering wheel. The average net force on (and the magnitude of the acceleration of) the driver's head is therefore less.

7 The change in momentum (impulse) on the legs takes place over a longer interval, reducing the force exerted by the ground on the knee joints and muscles, tendons and ligaments in the legs.

8 **a** The impulse is the area under the graph. The area is approximately equal to the area of a triangle with a base of 0.10 seconds and a height of 4.4×10^3 N.

$$\text{Area} = \frac{1}{2} \times 0.10 \times 4.4 \times 10^3$$
$$\Rightarrow \text{Impulse} = 2.2 \times 10^2 \text{ N s (approximately)}$$

b $\text{Impulse} = m\Delta \boldsymbol{v}$

$$2.2 \times 10^2 = 60\Delta \boldsymbol{v}$$
$$\Rightarrow \Delta \boldsymbol{v} = 3.7 \text{ m s}^{-1}$$

But $\boldsymbol{v} = 0$.

$\Rightarrow \boldsymbol{u} = 3.7 \text{ m s}^{-1}$ (approximately)

c The impulse on the unbelted occupant is greater than that on the belted occupant (the area under the force versus time graph is clearly greater).

The change in velocity, $\Delta \boldsymbol{v}$, is the same for both occupants.

Because impulse = $m\Delta \boldsymbol{v}$, the mass of the unbelted occupant must be greater.

The area under the blue curve can be estimated to be approximately 1.6 times the area under the green curve; therefore, the unbelted occupant is approximately 1.6 times heavier.

$1.6 \times 60 = 96$ kg

d The graph describing the force on the occupant with the seatbelt shows that the force is applied immediately and is applied for a relatively large amount of time compared with the force applied to the occupant without the seatbelt. The occupant without the seatbelt experiences no immediate force as she or he continues to move forward at the same speed as the car was moving before impact. The force applied to this occupant increases rapidly to a magnitude greater than the force applied to the occupant with the seatbelt. The multiple peaks in force on the second occupant can be explained by multiple impacts with the dashboard or other parts of the car.

9 **a** The impulse is the area under the graph. This can be found most easily by counting the small squares.

The area of each small square = 0.005 s × 100 N = 0.5 N s.

The number of small squares under the curve is approximately 190.

$$\text{Impulse} \approx 190 \times 0.5 \text{ N s}$$
$$\approx 95 \text{ N s upwards}$$

b Impulse applied by net force = $m\Delta \boldsymbol{v}$

$\Rightarrow$ Impulse applied by floor + impulse due to force due to gravity = $m\Delta \boldsymbol{v}$

$$\Rightarrow 95 \text{ N s} - m\boldsymbol{g} \times 0.10 = m\Delta \boldsymbol{v}$$
$$95 - 600 \times 0.10 = 60\Delta \boldsymbol{v}$$
$$\Delta \boldsymbol{v} = \frac{95 - 60}{60}$$
$$= 0.58 \text{ m s}^{-1} \text{ (taking up as positive)}$$
$$\boldsymbol{u} = 0 \Rightarrow \boldsymbol{v} = 0.58 \text{ m s}^{-1}$$

c $\boldsymbol{F}\Delta t$ = impulse applied by floor, where $\boldsymbol{F}$ = average force applied by floor

$$\Rightarrow \boldsymbol{F} = \frac{95 \text{ N s}}{0.10 \text{ s}}$$
$$= 9.5 \times 10^2 \text{ N upwards}$$

d The normal force is present as the basketballer is initially pushing down on the floor with a force equal to the basketballer's weight (force due to gravity).

10 Bouncing off during collision results in a greater change in momentum of the cars in a similar or smaller time interval. The rate of change in momentum of the cars and the resulting net force on the passengers would therefore be greater ($\boldsymbol{F}\Delta t = m\Delta \boldsymbol{v}$). In low-speed collisions with small vehicles (such as dodgem cars), this is not a problem. However, in real cars at typical road speeds, more injuries would occur.

9.7 Exam questions

1 **a**
$$\Delta \boldsymbol{p} = m\Delta \boldsymbol{v}$$
$$= 50 \times -12 \text{ [1 mark]}$$
$$= -600 \text{ kg m s}^{-1} \text{ east}$$
$$\therefore \Delta \boldsymbol{p} = 600 \text{ kg m s}^{-1} \text{ west [1 mark]}$$

b Impulse on barrier = −impulse on vehicle

= −chance in momentum of vehicle

= 600 N s east [1 mark]

2
$$\Delta p = m\Delta \boldsymbol{v}$$
$$20 = 4.0\Delta \boldsymbol{v} \text{ [1 mark]}$$
$$\Rightarrow \Delta \boldsymbol{v} = 5.0 \text{ m s}^{-1} \text{ [1 mark]}$$
$$\boldsymbol{v} = u + \Delta \boldsymbol{v}$$
$$= 6.0 + 5.0$$
$$= 11 \text{ m s}^{-1} \text{ [1 mark]}$$

Note: The change in momentum could be in the opposite direction, in which case the final velocity would be $6.0 - 5.0 = 1.0 \text{ m s}^{-1}$.

3
$$\text{Total impulse} = \text{impulse of 10 N force} + \text{impulse of } F_X \text{ [1 mark]}$$
$$130 = 4 \times 10 + F_X 6$$
$$6\boldsymbol{F}_X = 90 \text{ [1 mark]}$$
$$\Rightarrow \boldsymbol{F}_X = \frac{90}{6}$$
$$= 15 \text{ N [1 mark]}$$

4
$$\Delta \boldsymbol{p} = m\Delta \boldsymbol{v}$$
$$= 0.2(-20 - 30) \text{ [1 mark]}$$
$$= 0.2 \times -50$$
$$= -10 \text{ kg m s}^{-1} \text{ [1 mark]}$$
$$\boldsymbol{F}_{net} = \frac{\Delta \boldsymbol{p}}{\Delta t}$$
$$= \frac{-10}{0.025}$$
$$= -400 \text{ N [1 mark]}$$
$$= 400 \text{ N south [1 mark]}$$

5 Take the direction east as positive.

$$\text{Net impulse} = \boldsymbol{F}_1\Delta t_1 + \boldsymbol{F}_2\Delta t_2$$
$$= 5 \times 8 - 3 \times 20$$
$$= -20 \text{ N s [1 mark]}$$

$$m\Delta \boldsymbol{v} = \text{net impulse [1 mark]}$$
$$2.5\Delta \boldsymbol{v} = -20$$
$$\Rightarrow \Delta \boldsymbol{v} = -8 \text{ [1 mark]}$$
$$= 8 \text{ m s}^{-1} \text{ west [1 mark]}$$

9.8 Torque

Sample problem 15

$$\boldsymbol{\tau} = r_{\perp}\boldsymbol{F}$$
$$30 = 0.3 \times \boldsymbol{F}$$
$$\Rightarrow \boldsymbol{F} = \frac{30}{0.3}$$
$$= 100 \text{ N}$$

The force by the hand on the wrench is 100 N.

Practice problem 15

Torque = $r_{\perp}\boldsymbol{F}$, so $r_{\perp} = \frac{30}{30} = 1$ m

You should place your hand 1 metre along the handle to achieve a torque of 30 N m.

9.8 Exercise

1 $\boldsymbol{\tau} = r_{\perp}\boldsymbol{F}$

$= 0.25 \times 200$

$= 50$ N m

2
- They can increase the force that they are applying.
- They can increase the distance from the nut that they are applying the force at. This could be achieved by placing a hollow metal pipe over the shifter to increase its length.

3 $\boldsymbol{\tau} = r_{\perp}\boldsymbol{F}$

$$\Rightarrow \boldsymbol{F} = \frac{\boldsymbol{\tau}}{r_{\perp}}$$
$$= \frac{20}{0.25}$$
$$= 80 \text{ N}$$

4 The solution will depend on the chosen example and estimates made.
Calculate using $\boldsymbol{\tau} = r_{\perp}\boldsymbol{F}$ with estimated quantities in appropriate units (m and N). For example, turning a steering wheel in a car might require a force of 15 N acting over a distance of 0.2 metres.

5 As Sam moves beyond the left of the fulcrum, the seesaw will rotate anticlockwise. This will happen when the torque from Sam about the fulcrum is larger than the torque of the bag.

9.8 Exam questions

1 C

$$\boldsymbol{F} \times 0.3 + 20 \times 0.7 = 26$$
$$\Rightarrow \boldsymbol{F} = \frac{(26 - 14)}{0.3}$$
$$= 40 \text{ N}$$

2 B

Torque = $\boldsymbol{F}r\sin\theta$

The maximum value of $\sin\theta$ is when θ is equal to 90°.

So, an increase in θ will reduce $\sin\theta$ and hence, the torque.

3 D

$$\text{Net torque clockwise} = 6 \times 0.8 - 9 \times 0.6$$
$$= 4.8 - 5.4$$
$$= -0.6 \text{ N m}$$

∴ Magnitude is 0.60 N m.

4 $\boldsymbol{\tau} = r_{\text{perp}}\boldsymbol{F}$ [1 mark]

$= 4 \times 200$ [1 mark]

$= 800$ N m clockwise [1 mark]

5 The two torques are: clockwise = 2$\boldsymbol{F}$ N m and anticlockwise = 3 × 40 = 120 N m. [1 mark]
Net torque clockwise = 40 = 2$\boldsymbol{F}$ − 120 [1 mark]
Hence, 2$\boldsymbol{F}$ = 160 and $\boldsymbol{F}$ = 80 N. [1 mark]

9.9 Equilibrium

Sample problem 16

$$\boldsymbol{F}_{\text{net}} = 0$$
$$\boldsymbol{\tau}_{\text{net}} = 0$$

$$\boldsymbol{R} = 800 + 600$$
$$= 1400 \text{ N upwards}$$
$$\boldsymbol{\tau}_{\text{net}} = 0$$

$\Rightarrow$ Sum of clockwise torques = sum of anticlockwise torques

$$600 \times 2 = 800 \times d$$
$$\Rightarrow d = \frac{1200}{800}$$
$$= 1.5 \text{ m}$$

To balance the seesaw, person 1 must sit 1.5 metres to the left of the fulcrum.

Practice problem 16

The seesaw is balanced, so the net force and net torque are both zero.

$$\boldsymbol{\tau}_{\text{net}} = 0$$
$$3.2 \times 9.8m = 2 \times 600$$
$$31.36m = 1200$$
$$\Rightarrow m = \frac{1200}{31.36}$$
$$\approx 38 \text{ kg}$$

The mass of person 1 is 38 kilograms.

Sample problem 17

$\boldsymbol{F}_{\text{net}} = 0$

$\boldsymbol{\tau}_{\text{net}} = 0$

$\boldsymbol{R}_1 + \boldsymbol{R}_2 = 40 \times 9.8 + 60 \times 9.8$

$= 980\text{ N}$

$\boldsymbol{\tau}_{\text{net}} = 0$

$\Rightarrow$ Sum of clockwise torques = sum of anticlockwise torques

$$\Rightarrow 40 \times 9.8 \times \frac{1}{2} L + 60 \times 9.8 \times \frac{1}{4} L = \boldsymbol{R}_2 \times L$$

$$40 \times 9.8 \times \frac{1}{2} + 60 \times 9.8 \times \frac{1}{4} = \boldsymbol{R}_2$$

$$196 + 147 = \boldsymbol{R}_2$$

$$\Rightarrow \boldsymbol{R}_2 = 343\text{ N}$$

$\boldsymbol{R}_1 + \boldsymbol{R}_2 = 980$

$\boldsymbol{R}_1 + 343 = 980$

$\Rightarrow \boldsymbol{R}_1 = 980 - 343$

$= 637\text{ N}$

The magnitude of $\boldsymbol{R}_1$ is 637 N and $\boldsymbol{R}_2$ is 343 N.

Practice problem 17

a Torque by soil: $\boldsymbol{F} \times 0.80 = 6000\text{ N m}$

$$\Rightarrow \boldsymbol{F} = \frac{6000}{0.8} = 7500\text{ N}$$

b **i** $\boldsymbol{F} \times 7.5 = 6000\text{ N m}$

$$\boldsymbol{F} = \frac{6000}{7.5} = 800\text{ N}$$

ii $\boldsymbol{F}_{\text{net}} = 0$

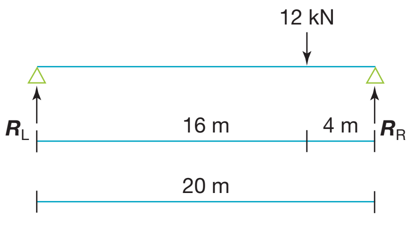
Force by ground = 7500 + 800

= 8300 N

9.9 Exercise

1 **a** The reaction from the left abutment decreases and the reaction from the right abutment increases.

b



$\sum \boldsymbol{F} = 0$:

$\boldsymbol{R}_L + \boldsymbol{R}_R = 12\text{ KN}$ [1]

$\sum \tau = 0$, taking torques about the left-hand support:

$12 \times 16 - 20\boldsymbol{R}_R = 0$ [2]

$$\Rightarrow \boldsymbol{R}_R = 12 \times \frac{16}{20} = 9.6\text{ kN}$$

Substituting [1], $\boldsymbol{R}_L = 2.4\text{ kN}$

2 **a** The wall resists the loads from the balcony and the person with an upwards force and an anticlockwise torque.

b As the person moves towards the wall, the total reaction remains constant. However, the torque on the wall decreases.

3 Taking torques about the left support:

$1.8mg - 20g(5 - 1.8) = 0$

$$\Rightarrow m = \frac{64}{1.8} = 36\text{ tonnes}$$

4 This question relies on applying the equations of equilibrium.

$\sum \boldsymbol{F} = 0$:

$2000 + 800 + 600 + 200 - \boldsymbol{R}_L - \boldsymbol{R}_R = 0$

$\Rightarrow \boldsymbol{R}_L + \boldsymbol{R}_R = 3600\text{ N}$ [1]

$\sum \boldsymbol{\tau} = 0$, taking torques about the left-hand support:

$800 \times 2 + 2000 \times 3 + 600 \times 4 + 200 \times 5 - \boldsymbol{R}_R \times 6 = 0$ [2]

$\Rightarrow 6\boldsymbol{R}_R = 11\,000$

$\Rightarrow \boldsymbol{R}_R = 1833\text{ N}$

Substituting into equation [1]:

$\boldsymbol{R}_R + \boldsymbol{R}_L = 3600\text{ N}$

$1833 + \boldsymbol{R}_L = 3600\text{ N}$

$\Rightarrow \boldsymbol{R}_L = 1767\text{ N}$

5 Pirate Bill will tip into the water when the overturning torque caused by his weight is greater than the stabilising torque of the plank's own weight. This will occur when the pirate is x metres past the edge of the boat. The centre of mass of the plank is 1 metre inside the edge of the boat.

$\sum \tau = 0$:

$-500x + 800 \times 1 = 0$

$$\Rightarrow x = \frac{800}{500} = 1.6\text{ m}$$

9.9 Exam questions

1 Taking torques about the pivot point:

$\boldsymbol{\tau}_{\perp 1} = \boldsymbol{\tau}_{\perp 2}$

$r_{\perp 1}\boldsymbol{F}_{g1} = r_{\perp 2}\boldsymbol{F}_{g2}$

$150 \times 0.75 = r_{\perp 2} \times 190$ [1 mark]

$112.5 = r_{\perp 2} \times 190$

$\Rightarrow r_{\perp 2} = 0.59\text{ m}$ [1 mark]

2 A

The torques about the top of support P balance.

$80 \times 3 = \boldsymbol{F}_Q \times 4$

$$\Rightarrow \boldsymbol{F}_Q = \frac{240}{4} = 60\text{ N}$$

3 **a** The torques about pillar A balance.

$2400 \times 5 = 2000 \times 4 + \boldsymbol{F}_g \times 8$ [1 mark]

$12\,000 = 8000 + 8\boldsymbol{F}_g$ [1 mark]

$\Rightarrow \boldsymbol{F}_g = 500\text{ N}$ [1 mark]

b Take torques about B; assume $\boldsymbol{F}_A$ is down.

$\boldsymbol{F}_A \times 5 + 2000 \times 1 = 2000 \times 3$ [1 mark]

$5\boldsymbol{F}_A = 6000 - 2000$

$= 4000$ [1 mark]

$\Rightarrow \boldsymbol{F}_A = 800\text{ N}$ [1 mark]

Thus the direction of $\boldsymbol{F}_A$ is down. [1 mark]

4 Clockwise torques must balance anticlockwise torques about the top of support B.

$$8F_A = 6 \times 400 + 4 \times 200 \text{ [1 mark]}$$

$$\Rightarrow F_A = \frac{2400 + 800}{8} \text{ [1 mark]}$$

$$= 400 \text{ N [1 mark]}$$

5 Assume that the support A applies an upward force to the beam. [1 mark]
Consider torques balancing about the top of support B.
$8F_A + 4 \times 1200 = 2 \times 1000$ [1 mark]

$$\Rightarrow F_A = \frac{2000 - 4800}{8} \text{ [1 mark]}$$

$$= -350 \text{ N [1 mark]}$$

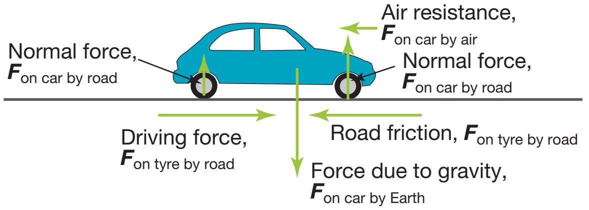
∴ The force is 350 N downward. [1 mark]

9.10 Review

9.10 Review questions

1 Forces acting on an object falling in the air (after rolling off a table):
- Gravity: the force due to gravity of Earth on the object
- Air resistance: the force of the air on the object; opposite direction to the motion

2 The car is accelerating forwards, so for the diagram to be correct the length of the driving force arrow (acting forwards) must be longer than the combined length of the air resistance and road friction arrows.



3 Horizontal component:

$$F_{\text{horiz}} = F\cos\theta$$
$$= 25\cos(30°)$$
$$= 21.7 \text{ N}$$

Vertical component:

$$F_{\text{vert}} = F\sin\theta$$
$$= 25\sin(30°)$$
$$= 12.5 \text{ N}$$

4 East–west components (taking east as positive):

$$\sum F_{\text{EW}} = 3000\cos(10°) - 2500\cos(15°) - 2400\sin(5°)$$
$$= 330 \text{ N}$$

North–south components (taking north as positive):

$$\sum F_{\text{NS}} = 3400 - 3000\sin(10°) - 2500\sin(15°) - 2400\cos(5°)$$
$$= -159 \text{ N}$$

Combine using Pythagoras to determine the magnitude of the net force:

$$c^2 = a^2 + b^2$$

$$F_{\text{net}} = \sqrt{330^2 + (-159)^2}$$
$$= \sqrt{134\,181}$$
$$= 366 \text{ N}$$

The force components are acting towards the east and south.
Use trigonometry to determine the angle from east towards south.

$$\theta = \tan^{-1}\left(\frac{159}{330}\right)$$
$$= 25.7°$$

This is a bearing of 90 + 25.7 = 115.7°.
The net force has a magnitude of 366 N and acts in the direction 115.7° true.

5 $F = ma$
$$= 9.6 \times 1645$$
$$= 15\,792 \text{ N}$$

6 $F = ma$

$$\Rightarrow m = \frac{F}{a}$$
$$= \frac{2.2 \times 10^6}{1.57}$$
$$= 1.4 \times 10^6 \text{ kg}$$

7 **a** Net force = force due to gravity – air resistance

$$F_{\text{net}} = 410 \times 9.8 - 2400$$
$$= 1618 \text{ N}$$

$$a = \frac{F}{m}$$
$$= \frac{1618}{410}$$
$$= 3.9 \text{ m s}^{-2}$$

b For constant velocity to occur, the net force must be equal to zero.
In this case that will require the air resistance to be equal in magnitude to the force due to gravity.

$$F_g = 410 \times 9.8$$
$$= 4018 \text{ N}$$

Therefore, the air resistance will need to have a magnitude of 4018 N.

8 **a** Consider the forces acting on both boxes:

$$F_{\text{net}} = 175 - 40$$
$$= 135 \text{ N (taking left to be the positive direction)}$$

$$a = \frac{135}{(25 + 50)}$$
$$= 1.8 \text{ m s}^{-1}$$

b Consider the forces acting on box B:
Removalist pushes to the left.
Box A will push to the right.
Determine the net force on box B using the acceleration calculated for the system:

$$F = 50 \times 1.8$$
$$= 90 \text{ N}$$

Take the sum of forces on box B to determine the unknown (force of box A on it):

$$\Rightarrow F_{\text{net}} = F_{\text{removalist}} - F_{\text{box A}}$$
$$90 = 175 - F_{\text{box A}}$$
$$F_{\text{box A}} = 85 \text{ N}$$

Therefore, the force acting between the boxes is 85 N.
If this calculation was repeated by analysing box A, the same result would be produced.

9 a Defining that towards the racquet as positive:

$$\Delta \boldsymbol{p} = m\Delta \boldsymbol{v}$$
$$= \frac{57}{1000} \times (-33 - 65)$$
$$= -5.6 \text{ kg m s}^{-1}$$

The negative sign indicates that the change in momentum is away from the racquet.

b $\boldsymbol{F}_{\text{av}}\Delta t = m\Delta \boldsymbol{v}$

$$\Rightarrow \boldsymbol{F}_{\text{av}} = \frac{m\Delta \boldsymbol{v}}{\Delta t}$$
$$= \frac{-5.6}{0.002}$$
$$= -2793 \text{ N}$$

The negative sign indicates that the force is away from the racquet, as expected.

10 $\boldsymbol{I}$ = area under the force − versus-time graph

$$= \frac{1}{2} \times 0.3 \times 1200$$
$$= 180 \text{ N s}$$

9.10 Exam questions

Section A — Multiple choice questions

1 B

Mass is a scalar quantity as it has magnitude only, no direction.

2 A and D

The forces acting on a basketball in motion are the force due to gravity and air resistance. The normal force acts on an object when it is in contact with another object, and the net force is the sum of all forces acting on an object; it is a theoretical force, not a physical force.

3 D

An object is not at rest or in a uniform state of motion if it is accelerating. The train accelerating uniformly as it departs the station is not at rest or in uniform motion.

4 C

Using Newton's Second Law of Motion:

$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$$
$$86 = 43\boldsymbol{a}$$
$$\Rightarrow \boldsymbol{a} = \frac{86}{43}$$
$$= 2.0 \text{ m s}^{-2}$$

5 B

Newton's Third Law states that a force pair is of the form $\boldsymbol{F}_{\text{on A by B}} = -\boldsymbol{F}_{\text{on B by A}}$. This means that the force pair involves two objects, with the forces acting from the first to the second object and from the second to the first object. The force due to gravity and the normal force both act on the person sitting on the chair, so although they are equal in magnitude and opposite in direction, they are not a Newton's Third Law force pair.

6 A

$$\boldsymbol{p} = m\boldsymbol{v}$$
$$= 2250 \times 20$$
$$= 45\,000 \text{ kg m s}^{-1}$$

7 D

$$\Delta \boldsymbol{p} = \boldsymbol{p}_{\text{f}} - \boldsymbol{p}_{\text{i}}$$
$$= m\boldsymbol{v}_{\text{f}} - m\boldsymbol{v}_{\text{i}}$$
$$= 0.058 \times \frac{155}{3.6} - 0.058 \times 0$$
$$= 2.5 \text{ kg m s}^{-1}$$

8 B

Impulse = change in momentum

$$\boldsymbol{I} = \Delta \boldsymbol{p}$$
$$= m\boldsymbol{v}_{\text{f}} - m\boldsymbol{v}_{\text{i}}$$
$$= 1850 \times 0 - 1850 \times 8$$
$$= -14\,800 \text{ kg m s}^{-1}$$
$$\boldsymbol{F}_{\text{net}} = \frac{\boldsymbol{I}}{\Delta t}$$
$$= \frac{-14\,800}{0.2}$$
$$= -74\,000 \text{ N}$$

The average net force acting on the car during the collision is 74 000 N.

9 C

Reducing the duration of the collision will cause the change in momentum to occur over a smaller time period, increasing the net force acting on the car during the collision.

10 C

$$\boldsymbol{\tau} = r_{\perp}\boldsymbol{F}$$
$$= 1.25 \times 735$$
$$\approx 919 \text{ N m}$$

Section B — Short answer questions

11 For two forces to be a Newton's Third Law pair, they must be equal and opposite in magnitude and act on different objects, such that $\boldsymbol{F}_{\text{on A by B}} = -\boldsymbol{F}_{\text{on B by A}}$.

In the given scenario, while the forces are equal and opposite, they are both acting on the book. [1 mark]

The normal force is the force of the table on the book. The pair of this is the force of the book on the table.

The force due to gravity is the force of Earth on the book. The pair of this is the force of the book on Earth.

12 Determine the acceleration of the system as a whole (bike and trailer).

$$\boldsymbol{F}_{\text{net}} = 172 - 34$$
$$= 138 \text{ N}$$
$$\boldsymbol{a} = \frac{\boldsymbol{F}}{m}$$
$$= \frac{138}{(95 + 20)}$$
$$= 1.2 \text{ m s}^{-2} \text{ [1 mark]}$$

To determine the tension force in the link, break the system at the link and analyse either the bike or trailer separately. In this solution, the bike and cyclist are analysed.

Determine the net force acting on the bike from the acceleration calculated:

$$\boldsymbol{F} = m\boldsymbol{a}$$
$$= 95 \times 1.2$$
$$= 114 \text{ N [1 mark]}$$

Take the sum of forces on the bike to determine the unknown tension force:

$$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{driving}} - \boldsymbol{F}_{\text{resistance}} - \boldsymbol{F}_{\text{tension}}$$
$$114 = 172 - 20 - \boldsymbol{F}_{\text{tension}}$$
$$\Rightarrow \boldsymbol{F}_{\text{tension}} = 172 - 20 - 114$$
$$= 38 \text{ N [1 mark]}$$

The tension in the link is 38 N.

13 In this situation, a component of the force due to gravity will act down the ramp to oppose the motion of the rocket.

$$\boldsymbol{F}_{gx} = m\boldsymbol{g}\sin\theta$$
$$= 2500 \times 9.8 \times \sin(42°)$$
$$= 16\,394 \text{ N [1 mark]}$$

To determine the acceleration of the rocket, calculate the net force along the ramp by taking the sum of forces.

$$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{thrust}} - \boldsymbol{F}_{gx}$$
$$= 18\,000 - 16\,394$$
$$= 1606 \text{ N [1 mark]}$$

Hence, calculate the acceleration of the rocket:

$$\boldsymbol{a} = \frac{\boldsymbol{F}}{m}$$
$$= \frac{1606}{2500}$$
$$= 0.64 \text{ m s}^{-2} \text{ [1 mark]}$$

14 Calculate the change in momentum.

$$\Delta\boldsymbol{p} = m\Delta\boldsymbol{v}$$
$$= 1980 \times \left(\frac{-12}{3.6} - \frac{64}{3.6}\right)$$
$$= -41\,800 \text{ kg m s}^{-1} \text{ [1 mark]}$$

Calculate the average force.

$$\boldsymbol{F}_{\text{av}}\Delta t = m\Delta\boldsymbol{v}$$
$$\Rightarrow \boldsymbol{F}_{\text{av}} = \frac{m\Delta\boldsymbol{v}}{\Delta t}$$
$$= \frac{-41\,800}{0.16}$$
$$= -261\,250 \text{ N [1 mark]}$$

15 Rotational equilibrium:

Take the sum of torques about the right-hand support, assuming that clockwise is positive.

$$\sum\tau = \boldsymbol{F}_{\text{support L}} \times 40 - \boldsymbol{F}_{\text{bridge}} \times 20 - \boldsymbol{F}_{\text{truck}} \times 9$$

Substitute for the force due to gravity on the bridge and the truck and solve.

$$\boldsymbol{F}_{\text{support L}} \times 40 = 46\,000 \times 9.8 \times 20 - 14\,500 \times 9.8 \times 9 \text{ [1 mark]}$$
$$\boldsymbol{F}_{\text{support L}} \times 40 = 10\,294\,900$$
$$\Rightarrow \boldsymbol{F}_{\text{support L}} = 257\,372.5 \text{ N [1 mark]}$$

Translational equilibrium:

Take the sum of vertical forces, assuming that upwards is positive.

$$\sum\boldsymbol{F}_{\text{vertical}} = \boldsymbol{F}_{\text{support L}} + \boldsymbol{F}_{\text{support R}} - \boldsymbol{F}_{\text{bridge}} - \boldsymbol{F}_{\text{truck}}$$
$$\Rightarrow 0 = 257\,372.5 + \boldsymbol{F}_{\text{support R}} - 46\,000 \times 9.8 - 14\,500 \times 9.8$$
$$\Rightarrow \boldsymbol{F}_{\text{support R}} = 335\,527.5 \text{ N [1 mark]}$$

Topic 10 — Energy and motion

10.2 Impulse and momentum

Sample problem 1

a $\boldsymbol{p}_A + \boldsymbol{p}_B = \boldsymbol{p}_{A+B}$
$= m_A\boldsymbol{v}_A + m_B\boldsymbol{v}_B$
$= 5 \times 4 + 3 \times -4$
$= 20 - 12$
$= 8\ \text{kg m s}^{-1}$ to the right

$\boldsymbol{p}_{A+B} = \boldsymbol{p}_A + \boldsymbol{p}_B$
$m_{A+B}\boldsymbol{v}_{A+B} = 8$
$(5 + 3)\boldsymbol{v} = 8$
$\Rightarrow \boldsymbol{v} = 1\ \text{m s}^{-1}$

The velocity of the blocks after the collision is 1 m s^{-1} to the right.

b $\Delta\boldsymbol{p}_A = m_A\Delta\boldsymbol{v}_A$
$= 5(1 - 4)$
$= 5 \times -3$
$= -15\ \text{kg m s}^{-1}$
$= 15\ \text{kg m s}^{-1}$ to the left

$\Delta\boldsymbol{p}_B = m_B\Delta\boldsymbol{v}_B$
$= 3(1 - (-4))$
$= 3 \times 5$
$= 15\ \text{kg m s}^{-1}$ to the right

The change in momentum of each of the blocks is 15 kg m s^{-1}.

c The impulse on block A is equal to the change in momentum of block A: 15 kg m s^{-1} to the left.

d $\boldsymbol{p}_{Af} + \boldsymbol{p}_{Bf} = 8$
$5 \times -0.5 + 3\boldsymbol{v}_{Bf} = 8$
$-2.5 + 3\boldsymbol{v}_{Bf} = 8$
$3\boldsymbol{v}_{Bf} = 10.5$
$\Rightarrow \boldsymbol{v}_{Bf} = \frac{10.5}{3}$
$= 3.5\ \text{m s}^{-1}$ to the right

The final velocity of block B if, instead of moving off together, block A rebounds to the left with a speed of 0.5 m s^{-1}, is 3.5 m s^{-1} to the right.

Practice problem 1

a The total momentum of the two-car system before the collision is the sum of the two cars' momentums.
Total momentum $= 5 \times 2 + 5 \times 1$
$= 15\ \text{kg m s}^{-1}$ east

The total momentum of the two-car system before the collision is 15 kg m s^{-1} east.

b By conservation of momentum, the final momentum will be equal to the initial momentum.
$(5 + 5)\,\boldsymbol{v} = 15$
$10\boldsymbol{v} = 15$
$\Rightarrow \boldsymbol{v} = \frac{15}{10}$
$= 1.5\ \text{m s}^{-1}$ east

The velocity of the model cars as they move off together after the collision is 1.5 m s^{-1} east.

c Change of momentum = final momentum − initial momentum
$\Delta\boldsymbol{p} = \boldsymbol{p}_f - \boldsymbol{p}_i$
$= (5 \times 1.5) - (5 \times 2)$
$= 7.5 - 10$
$= -2.5\ \text{kg m s}^{-1}$ east

The change in momentum of the car that was travelling faster before the collision is 2.5 kg m s^{-1} west.

d Change of momentum = final momentum − initial momentum
$\Delta\boldsymbol{p} = \boldsymbol{p}_f - \boldsymbol{p}_i$
$= (5 \times 1.5) - (5 \times 1)$
$= 7.5 - 5$
$= 2.5\ \text{kg m s}^{-1}$ east

The change in momentum of the car that was travelling slower before the collision is 2.5 kg m s^{-1} east.

e Impulse = change in momentum = 2.5 kg m s^{-1}
The magnitude of impulse on both cars during the collision is 2.5 kg m s^{-1}.

f The impulses on the two cars are equal in magnitude, but opposite in direction.

10.2 Exercise

1 By releasing a high-pressure propellant, the astronaut gains momentum in one direction while the propellant gains the same amount of momentum in the opposite direction. The total change in momentum of the astronaut and the contents of their backpack is zero.

2 a Provided that friction is negligible, there is no horizontal net force on the system of the mass and the trolley.
Total momentum before = total momentum after
$2 \times 0.6 = 4 \times \boldsymbol{v}$
$\Rightarrow \boldsymbol{v} = 0.3\ \text{m s}^{-1}$

b There is no horizontal net force on the system of the sand and the trolley.
Total momentum before = total momentum after
$4 \times 0.6 = 2 \times \boldsymbol{v}_{sand} + 2\boldsymbol{v}_{trolley}$
$\Rightarrow \boldsymbol{v}_{sand} = 0.6\ \text{m s}^{-1}$ as it falls from the moving trolley
$\Rightarrow 4 \times 0.6 = 2 \times 0.6 + 2\boldsymbol{v}_{trolley}$
$2\boldsymbol{v}_{trolley} = 1.2$
$\Rightarrow \boldsymbol{v}_{trolley} = 0.6\ \text{m s}^{-1}$

3 a Total momentum before collision = 0
Total momentum after collision = 0
$\boldsymbol{p}_L + \boldsymbol{p}_C = 0$
$m_L\boldsymbol{v}_L + m_C\boldsymbol{v}_C = 0$
$\Rightarrow m_L = \frac{-m_C\boldsymbol{v}_C}{\boldsymbol{v}_L}$
$= \frac{-50 \times 1.2}{-1.5}$
$= 40\ \text{kg}$

b Impulse on Catherine $= m_C\Delta\boldsymbol{v}_C$
$= 50 \times 1.2$
$= 60\ \text{N s}$

c Impulse on Lauren = −impulse on Catherine
$= -60$ N s
Magnitude of impulse = −60 N s

d Zero. As there is no external net force acting on the system of the two girls, momentum is conserved.

e If they pushed each other harder they would have greater speeds but still in the same ratio as before. The total momentum would remain zero as there are no external horizontal forces acting on the girls.

4 a Total momentum before collision is given by:
$m_N\boldsymbol{v}_N + m_L\boldsymbol{v}_L = 60 \times 2.0 + 70 \times 0$
$= 120 \text{ kg m s}^{-1}$
Total momentum after collision is given by:
$m_N\boldsymbol{v}_N + m_L\boldsymbol{v}_L = 60 \times 0 + 70 \times \boldsymbol{v}_L$
$= 70\,\boldsymbol{v}_L$
Total momentum is conserved.
$\Rightarrow 70\boldsymbol{v}_L = 120$
$\Rightarrow \boldsymbol{v}_L = \frac{120}{70}$
$\approx 1.7 \text{ m s}^{-1}$

b Impulse on Nick $= m_N\Delta\boldsymbol{v}_N$
$= 60 \times -2$
$= -120$ N s
Magnitude of impulse = 120 N s

c Change in momentum = impulse
$= 120 \text{ kg m s}^{-1}$

d $\Delta\boldsymbol{p}_L = -\Delta\boldsymbol{p}_N$ since total change in momentum is zero
Magnitude of Luke's change in momentum = 120 kg m s^{-1}

e They would have different speeds but the total momentum would still be conserved as there are no external horizontal forces acting on the boys.

f $\boldsymbol{p}_i = 120 \text{ kg m s}^{-1}$
$\boldsymbol{p}_f = (m_N + m_L)\boldsymbol{v}$
$= 120$
$\Rightarrow 130\boldsymbol{v} = 120$
$\Rightarrow \boldsymbol{v} = \frac{120}{130}$
$\approx 0.92 \text{ m s}^{-1}$

5 a Total momentum before collision is given by
$\boldsymbol{p}_C + \boldsymbol{p}_P$, where $\boldsymbol{p}_C$ = momentum of car and driver and $\boldsymbol{p}_P$ = momentum of police car and occupants.
$\boldsymbol{p}_C + \boldsymbol{p}_P = 1250\boldsymbol{v}_C + 1500 \times 0$
Total momentum after collision:
$(m_C + m_P)\boldsymbol{v} = 2750 \times 7.0$
$= 19\,250 \text{ kg m s}^{-1}$
Total momentum is conserved.
$\Rightarrow 1250\boldsymbol{v}_C = 19\,250$
$\Rightarrow \boldsymbol{v}_C = 15.4 \text{ m s}^{-1}$

b Impulse on police car $= m_P\Delta\boldsymbol{v}_P$
$= 1500 \times 7.0$
= 10 500 N s in the initial direction of motion of the car

c Impulse on driver $= m_d\Delta\boldsymbol{v}_d$
$= 50 \times (\boldsymbol{v} - \boldsymbol{u})_d$
$= 50 \times (7.0 - 15.4)$
$= -420$ N s
= 420 N s opposite to the initial direction of motion of the car

d $\boldsymbol{F}_{net}\Delta t$ = impulse on police car
$\Rightarrow \boldsymbol{F}_{net} = \frac{10\,500 \text{ N s}}{0.10 \text{ s}}$
$= 105\,000$ N
Average net force = 105 000 N in the initial direction of motion of the car

6 The two forces act on different objects. Forces are what objects do, rather than something that objects have, so force cannot be conserved.

10.2 Exam questions

1 Final momentum = initial momentum + change in momentum
$70\boldsymbol{v} = 3 \times 70 + 80$ [1 mark]
$= 290$
$\Rightarrow \boldsymbol{v} = 4.1 \text{ m s}^{-1}$ [1 mark]

2 Total momentum before = total momentum after
$32\,000 \times 12 + 40\,000 \times 4 = (32\,000 + 40\,000)\boldsymbol{v}$ [1 mark]
$384\,000 + 160\,000 = 72\,000\boldsymbol{v}$
$544\,000 = 72\,000\boldsymbol{v}$
$\Rightarrow \boldsymbol{v} = 7.6 \text{ m s}^{-1}$ [1 mark]

3 Conservation of momentum says that the final momentum of the combined trolleys = initial momentum of first trolley
$(6 + m) \times 1.8 = 6 \times 2.4$ [1 mark]
$6 + m = \frac{6 \times 2.4}{1.8}$ [1 mark]
$\Rightarrow m = 8 - 6$
$= 2.0$ kg [1 mark]

4 $\boldsymbol{p}_{final} = \boldsymbol{p}_{initial}$
$\Rightarrow 40 \times 2 + 20 \times v = 40 \times 6$ [1 mark]
$\Rightarrow v = 8.0 \text{ m s}^{-1}$ [1 mark]

5 Sum of momentums before collision = sum of momentums after collision [1 mark]
$\Rightarrow 2 \times 5 = 2 \times \boldsymbol{v}_R + 3 \times 4$ [1 mark]
$\Rightarrow \boldsymbol{v}_R = -1 \text{ m s}^{-1}$ moving east, or 1 m s^{-1} moving west [1 mark]

10.3 Work and energy

Sample problem 2

a $W = \boldsymbol{F}\boldsymbol{s}$
$= 150 \times 5$
$= 750$ J
750 J of work is done on the trolley by the force applied by the shopper.

b $\boldsymbol{F}_{net} = 150 - 120$
$= 30$ N
$W = \boldsymbol{F}_{net}\boldsymbol{s}$
$= 30 \times 5$
$= 150$ J
150 J of work is done on the trolley by the new force.

c $W = \boldsymbol{F}\boldsymbol{s}$
$= 120 \times 5$
$= 600$ J
600 J of work is done on the trolley by the shopper to oppose the friction force.

Practice problem 2

a Work done by worker = force × displacement

$= 300 \times 2$

$= 600$ J

600 J of work is done on the crate by the warehouse worker.

b Work done by net force = force × displacement

$= (300 - 240) \times 2$

$= 120$ J

120 J of work is done on the crate by the net force.

Sample problem 3

W = area under graph

$= \frac{1}{2}(3 + 1)\,150$

$= 300$ J

300 J of work is done.

Practice problem 3

The work done can be calculated as the area under the graph.

W = area under force-versus-distance graph

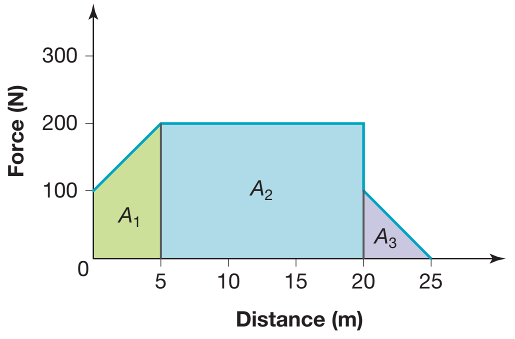
$= \frac{1}{2} \times 0.15 \times 50$

$= 3.75$ J

3.75 J of work is done.

10.3 Exercise

1 The work done can be calculated as the area under the graph.



W = area under force-versus-displacement graph

$= A_1 + A_2 + A_3$

$= \frac{1}{2}(100 + 200) \times 5 + 200 \times 15 + \frac{1}{2} \times 100 \times 5$

$= 750 + 3000 + 250$

$= 4000$ J

2 Work = force × displacement

$W = \boldsymbol{Fs}$

$= 40 \times 1.5$

$= 60$ J

3 None. As there is no displacement in the direction of the force, the work done is zero.

4 **a** Zero. The displacement after one complete revolution is zero. The force applied to keep the toy dog swinging is directed towards the centre of the circle. It is therefore perpendicular to the direction of motion at all times. In other words, there is no displacement in the direction of the applied force. Therefore, no work is done on the toy dog by the girl.

b Although the displacement is not zero after half of a full revolution, there is still no work done on the toy dog by the girl because the applied force is perpendicular to the direction of motion at all times.

5 $W = Fs\cos\theta$

$1400 = 275 \times \boldsymbol{s} \times \cos(35°)$

$1400 = 225.3 \times \boldsymbol{s}$

$\Rightarrow \boldsymbol{s} = 6.2$ m

10.3 Exam questions

1 B

$W = \boldsymbol{Fs} \times \cos(60°)$

$= 16 \times 12 \times \cos(60°)$

$= 96$ J

2 A

Pulling force: $W = 4 \times 1.5$

$= 6$ J

Normal reaction force = 0

Force and displacement at right-angles do no work.

3 W = area under graph

$= \frac{1}{2}$base × height [1 mark]

$= \frac{1}{2} \times 4.0 \times 80$ [1 mark]

$= 160$ J [1 mark]

4 Net work = area under $\boldsymbol{F}$–$\boldsymbol{x}$ graph [1 mark]

$= 40 \times 2 - 20 \times 3$ [1 mark]

$= 20$ J [1 mark]

5 $W = \boldsymbol{Fs}$

$128\,000 = 8000 \times \boldsymbol{s}$ [1 mark]

$\Rightarrow \boldsymbol{s} = 16$ m [1 mark]

10.4 Energy transfers

Sample problem 4

Mass and speed of the athlete:

Mass ≈ 70 kg

Speed ≈ 10 m s^{-1}

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 70 \times 10^2$

$= 3500$ J

Mass and speed of the family car:

Mass ≈ 1500 kg

Speed ≈ 60 km h^{-1}

≈ 17 m s^{-1}

$$E_k = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 1500 \times 17^2$$
$$\approx 217\,000 \text{ J}$$

$$\frac{217\,000}{3500} = 62$$

The family car has approximately 62 times more kinetic energy than the athlete.

Practice problem 4

a $E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 2000 \times 8.0^2$

$= 64\,000$ J

The kinetic energy of the charging elephant is 64 000 J.

b **i** A cyclist may have an approximate mass of 80 kilograms and a speed of approximately 5.0 m s^{-1}.

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 80 \times 5.0^2$

$= 1000$ J

A cyclist riding to work may have approximately 1000 J of kinetic energy.

ii A snail may have an approximate mass of 0.01 kg and a speed of approximately 0.002 m s^{-1}.

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.01 \times 0.002^2$

$= 2 \times 10^{-8}$ J

A snail crawling across a footpath may have approximately 2×10^{-8} J of kinetic energy.

Sample problem 5

$$\Delta E_k = \boldsymbol{F}_{\text{net}}\boldsymbol{s}$$
$$E_k = \boldsymbol{F}_{\text{net}}\boldsymbol{s}$$
$$= 30 \times 5.0$$
$$= 150 \text{ J}$$

$$E_k = \frac{1}{2}mv^2$$
$$150 = \frac{1}{2} \times 30 \times v^2$$
$$v^2 = \frac{150}{15}$$
$$= 10$$
$$\Rightarrow v = \sqrt{10}$$
$$\approx 3.2 \text{ m s}^{-1}$$

The trolley's final speed is 3.2 m s^{-1}.

Practice problem 5

$$\boldsymbol{F}_{\text{net}} = 150 - 120$$
$$= 30 \text{ N}$$

Work done = force × displacement

$$W = \boldsymbol{Fs}$$
$$= 30 \times 4$$
$$= 120 \text{ J}$$

The work done is equal to the change in kinetic energy.

$$W = \Delta E_k$$
$$120 = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 60v^2$$
$$v^2 = 4$$
$$\Rightarrow v = 2 \text{ m s}^{-1}$$

The final speed of the wheelbarrow is 2 m s^{-1}.

Sample problem 6

$$E_s = W$$
$$W = \text{area under graph}$$
$$= \frac{1}{2} \times 20 \times 0.25$$
$$= 2.5 \text{ J}$$

2.5 J of strain potential energy is stored in the spring when it is compressed by 25 centimetres.

Practice problem 6

a Strain potential energy is equal to the area under the force-versus-displacement graph.

$E_s = \frac{1}{2} \times 0.10 \times 8$

$= 0.40$ J

0.40 J of strain potential energy is stored in the spring when it is compressed by 10 centimetres.

b Strain potential energy is equal to the area under the force-versus-displacement graph.

$E_s = \frac{1}{2} \times 0.20 \times 16$

$= 1.6$ J

1.6 J of strain potential energy is stored in the spring when it is compressed by 20 centimetres.

Sample problem 7

a $\boldsymbol{F} = k\boldsymbol{x}$

$\boldsymbol{x} = 0.30 - 0.20 = 0.10$ m

$\boldsymbol{F} = 50 \times 0.10$

$= 5.0$ N

5 N is applied on the wooden block by the compressed spring.

b $E_s = \frac{1}{2}k\boldsymbol{x}^2$

$= \frac{1}{2} \times 50 \times 0.10^2$

$= 0.25$ J

0.25 J of strain potential energy is stored in the compressed spring.

c $W = E_s$

$= 0.25$ J

0.25 J of work was done on the spring by the wooden block.

Practice problem 7

a Recall Hooke's Law: $\boldsymbol{F} = k\boldsymbol{x}$

i $\boldsymbol{F} = k\boldsymbol{x}$

$= 60 \times 0.20$

$= 12$ N

ii Spring potential energy $= \frac{1}{2}k\boldsymbol{x}^2$

$= 0.5 \times 60 \times 0.50^2$

$= 7.5$ J

iii $\boldsymbol{F} = k\boldsymbol{x}$

$= 60 \times 0.20$

$= 12$ N

Force due to gravity = spring force

$m\boldsymbol{g} = k\boldsymbol{x}$

$m \times 9.8 = 12$

$\Rightarrow m = \frac{12}{9.8}$

≈ 1.2 kg

b Recall Hooke's Law.

$\boldsymbol{F} = k\boldsymbol{x}$

$20 = k \times 0.25$

$\Rightarrow k = \frac{20}{0.25}$

$= 80$ N m^{-1}

Sample problem 8

$\Delta E_k = \Delta E_g$

$\frac{1}{2}mv^2 = mg\Delta h$

$\frac{1}{2} \times 50v^2 = 50 \times 9.8 \times 1.5$

$25v^2 = 735$

$v^2 = \frac{735}{25}$

$\Rightarrow v = \sqrt{\frac{735}{25}}$

≈ 5.4 m s^{-1}

The speed of the skater at the bottom of the ramp is 5.4 m s^{-1}.

Practice problem 8

a Recall the formula for strain potential energy.

$E_s = \frac{1}{2}k\boldsymbol{x}^2$

$= \frac{1}{2} \times 80 \times 0.1^2$

$= 0.4$ J

b Assuming all of the strain potential energy is transformed into kinetic energy:

$E_k = \frac{1}{2}mv^2$

$0.4 = \frac{1}{2} \times 0.5v^2$

$v^2 = \frac{0.8}{0.5}$

$= 1.6$

$\Rightarrow v = \sqrt{1.6}$

≈ 1.3 m s^{-1}

10.4 Exercise

1 Work = $\boldsymbol{Fs}$

Unit = N m

But 1 N = 1 kg m s^{-2}

$\Rightarrow$ Unit = kg m s^{-2} × m

= kg m^2 s^{-2}

Kinetic energy $= \frac{1}{2}mv^2$

Unit = kg (m s^{-1})^2

= kg m^2 s^{-2}

Therefore, both kinetic energy and work are the same unit.

2 a $m \approx 750$ kg

$\boldsymbol{v} = 60$ km h^{-1}

≈ 16.7m s^{-1}

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 750 \times (16.7)^2$

$\approx 100\ 000$ J

b $\boldsymbol{v} = 100$ km h^{-1}

≈ 28 m s^{-1}

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.058 \times (28)^2$

≈ 23 J

3 $W = \Delta E_k$

$= \frac{1}{2}mv^2 - \frac{1}{2}m\boldsymbol{u}^2$

$= \frac{1}{2} \times 0.058 \times \left(\frac{200}{3.6}\right)^2 - 0$

≈ 90 J

4 Student responses will vary. A sample response is given below.

a $m \approx 70$ kg

$\Delta h \approx 2.5$ m

$$\Delta E_g = mg\Delta h = 70 \times 9.8 \times 2.5 = 1715 \text{ J}$$

b $m \approx 30$ kg

$\Delta h \approx 2$ m

$$\Delta E_g = mg\Delta h = 30 \times 9.8 \times 2 = 588 \text{ J}$$

c $m \approx 70$ kg

$\Delta h \approx 0.75$ m

$$\Delta E_g = mg\Delta h = 70 \times 9.8 \times 0.75 \approx 510 \text{ J}$$

5 a $m \approx 20$ kg

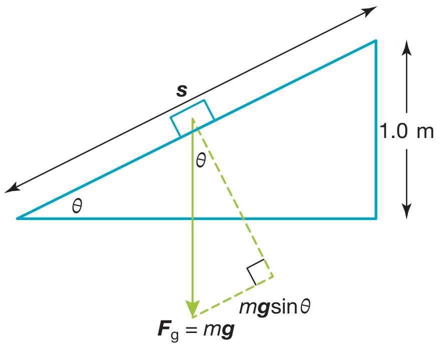
$\Delta h \approx 1$ m

$$\Delta E_g = mg\Delta h = 20 \times 9.8 \times 1 = 196 \text{ J}$$

b The same amount of work against gravity must be done in each case.

$$W = \boldsymbol{Fs} = m\boldsymbol{gs} = 20 \times 9.8 \times 1 = 196 \text{ J}$$

c



To push the crate up the ramp with a constant speed, the applied force must be $mg\sin\theta$.

$W = \boldsymbol{Fs}$

But $\sin\theta = \dfrac{1}{\boldsymbol{s}}$

$$\Rightarrow \boldsymbol{s} = \frac{1}{\sin\theta}$$

$$W = m\boldsymbol{g}\sin\theta \times \frac{1}{\sin\theta} = 20 \times 9.8 \times 1 = 196 \text{ J}$$

d It is better to use the ramp. Although the amount of work needed to move the crate is the same, the force that needs to be applied to the crate is less if the ramp is used.

6 So that as little of their kinetic energy as possible is transferred to gravitational potential energy. Subsequently, a greater proportion of their kinetic energy is available to cover the horizontal distance as fast as possible.

7 a Spring X:

$$\boldsymbol{F} = k\boldsymbol{x} = 200 \times 0.20 = 40 \text{ N}$$

Spring Y:

$$\boldsymbol{F} = k\boldsymbol{x} = 100 \times 0.20 = 20 \text{ N}$$

b Spring X:

$$E_s = \frac{1}{2}k\boldsymbol{x}^2 = \frac{1}{2} \times 200 \times 0.20^2 = 4.0 \text{ J}$$

Spring Y:

$$E_s = \frac{1}{2}k\boldsymbol{x}^2 = \frac{1}{2} \times 100 \times 0.20^2 = 2.0 \text{ J}$$

8 a The gravitational potential energy is converted into kinetic energy.

$$E_k = E_g = m\boldsymbol{g}\Delta h = 1.2 \times 9.8 \times 20 \approx 235 \text{ J}$$

b $E_k = \dfrac{1}{2}mv^2$

$$235 = \frac{1}{2} \times 1.2v^2$$

$$v^2 = \frac{235}{0.6}$$

$$\Rightarrow v = \sqrt{\frac{235}{0.6}} \approx 20 \text{ m s}^{-1}$$

9 a $E_k = \dfrac{1}{2}mv^2$

$$= \frac{1}{2} \times 450 \times (12)^2 = 32\,400 \text{ J}$$

b The kinetic energy at point B will be the kinetic energy at A, plus the gravitational potential energy.

$$E_k = 32\,400 + \Delta E_g = 32\,400 + m\boldsymbol{g}\Delta h = 32\,400 + 450 \times 9.8 \times 20 = 120\,600 \text{ J}$$

$$\frac{1}{2}mv^2 = 120\,600$$

$$v^2 = \frac{120\,600 \times 2}{450} = 536$$

$$\Rightarrow v = \sqrt{536} \approx 23 \text{ m s}^{-1}$$

The speed of the car at the point B is 23 m s^{-1}.

The kinetic energy at point C will be the kinetic energy at A, plus the gravitational potential energy.

$E_k = 32\,400 + \Delta E_g$
$= 32\,400 + mg\Delta h$
$= 32\,400 + 450 \times 9.8 \times 12$
$= 85\,320$ J

$\frac{1}{2}mv^2 = 85\,320$

$v^2 = \frac{85\,320 \times 2}{450}$
$= 379.2$
$\Rightarrow v = \sqrt{379.2}$
≈ 19.5 m s^{-1}

The speed of the car at the point C is 19.5 m s^{-1}.

c At point D the kinetic energy of the car will be the same as the kinetic energy at point B, 120 600 J.
The car will reach its maximum height when all of the kinetic energy is transformed into gravitational potential energy.

$E_k = E_g$
$120\,600 = mg\Delta h$
$120\,600 = 450 \times 9.8\Delta h$
$\Rightarrow \Delta h = \frac{120\,600}{450 \times 9.8}$
≈ 27.3 m

The car will reach a height of 27 metres.

10 a The work done by the driving force is equal to the area under the graph. This can be approximated by a triangle and a rectangle as follows.
See the figure at the foot of the page.*

$A_1 = \frac{1}{2} \times 625 \times 1600$
$= 500\,000$ J

$A_2 = 400 \times 1000$
$= 400\,000$ J

Work done by the driving force $= A_1 + A_2$
$= 900\,000$ J

b Work = force × displacement

$W = \boldsymbol{F}s$
$= 360 \times 1000$
$= 360\,000$ J

c The work done by the net force is the work done by the driving force minus the work done by the forces opposing the motion of the car.

$W_{\text{net force}} = 900\,000 - 360\,000$
$= 540\,000$ J

The kinetic energy of the car after it has travelled 1 kilometre is equal to the work done by the net force.

$E_k = 540\,000$
$\frac{1}{2}mv^2 = 540\,000$
$v^2 = \frac{540\,000 \times 2}{1200}$
$= 900$
$\Rightarrow v = \sqrt{900}$
$= 30$ m s^{-1}

11 $E_s = \frac{1}{2}k\boldsymbol{x}^2$
$= \frac{1}{2} \times 50 \times (0.1)^2$
$= 25 \times 0.01$
$= 0.25$ J

At natural length the strain potential energy is converted into kinetic energy.

$E_k = E_s$
$\frac{1}{2}mv^2 = 0.25$
$v^2 = \frac{0.25 \times 2}{0.5}$
$= 1.0$
$\Rightarrow v = \sqrt{1.0}$
$= 1.0$ m s^{-1}

12 a The force applied by the spring is proportional to its compression.

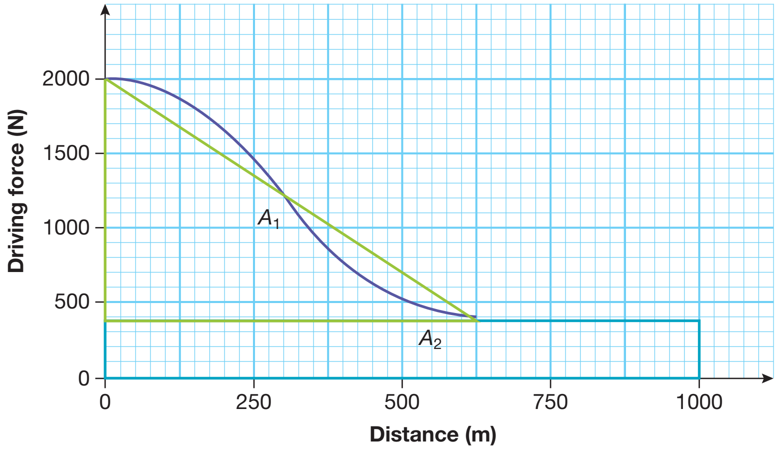
b The spring constant is the gradient of the force-versus-compression graph:

$k = \frac{\text{rise}}{\text{run}}$
$= \frac{1500}{0.08}$
$= 18\,750$ N m^{-1}

c Work done as the spring expands from maximum compression is equal to the area under the graph:

$W = \frac{1}{2} \times 0.08 \times 1500$
$= 60$ J

***10 a**



d E_k gained = work done by spring

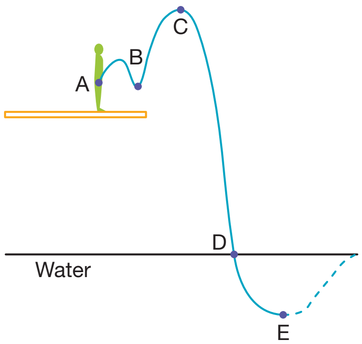
$= 60 \text{ J}$

e The maximum height gained by the child will be given by the height at which all of the kinetic energy is transformed into gravitational potential energy.

$$E_k = E_g$$
$$60 = m\boldsymbol{g}\Delta h$$
$$\Rightarrow \Delta h = \frac{60}{m\boldsymbol{g}}$$
$$= \frac{60}{30 \times 9.8}$$
$$\approx 0.2 \text{ m}$$

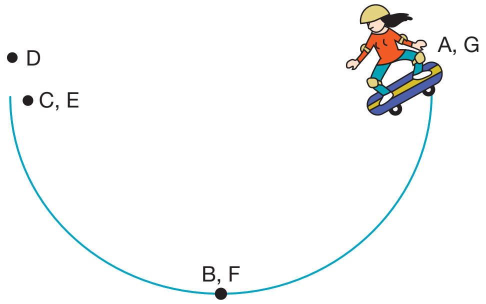
13 As the child falls through the air from maximum height, gravitational potential energy is transformed into kinetic energy.
After the child touches the trampoline after falling through the air, kinetic energy and gravitational potential energy are transformed into strain potential energy until the trampoline is at maximum extension.
After maximum extension, strain potential energy is transformed into gravitational potential energy and kinetic energy until contact is lost with the trampoline.
After contact is lost, kinetic energy is transformed into gravitational potential energy until maximum height is achieved.

14

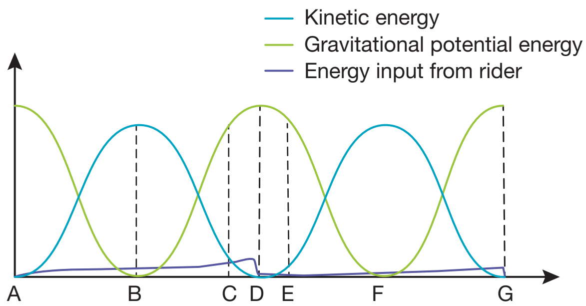


The energy transformations include the following:
A–B: Chemical energy is transformed into strain potential energy of muscles, tendons and ligaments. Strain potential energy of muscles is transformed into kinetic energy, which is then transformed into gravitational potential energy as the diver rises for the first time. Gravitational potential energy is transformed into kinetic energy as the diver descends to the end of the springboard.
B–C: Kinetic energy and some gravitational potential energy are transformed into strain potential energy of the springboard until the springboard reaches its maximum deflection. The strain potential energy is transformed into kinetic energy and gravitational potential energy until the diver loses contact with the springboard. Kinetic energy is transformed into gravitational potential energy until the diver reaches maximum height.
C–D: Gravitational potential energy of the diver is transformed into kinetic energy until the diver strikes the water.
D–E: Kinetic energy of the diver is transferred to the water (as kinetic energy) and eventually transformed into thermal energy of the water particles. Some of the diver's kinetic energy is transformed into sound energy.

15 a



The energy transformations can be displayed with a graph of energy versus time or energy versus position, as follows.



As the rider moves down and up the slope, gravitational potential energy is transformed to kinetic energy and back again to gravitational kinetic energy. However, the total mechanical energy is not quite conserved and the rider needs to provide some additional energy 'input' to reach the top of the slope and point C. Further energy input is needed from the rider in order to gain the gravitational potential energy required at point D. Gravitational potential energy is then transformed into kinetic energy as the rider returns to point F and transformed into gravitational potential energy at point G. At points A, D and G, the rider's kinetic energy is zero.

b Between points C and D, the skateboard and the rider are in free fall. They are both travelling at the same speed at point C. Even though they have different amounts of kinetic energy due to their different masses, and therefore gain different amounts of gravitational potential energy, they reach the same height. That is:

$$\frac{1}{2}m\boldsymbol{v}^2 = m\boldsymbol{g}\Delta h$$
$$\Rightarrow \boldsymbol{v}^2 = 2\boldsymbol{g}\Delta h$$
$$\Rightarrow \Delta h = \frac{\boldsymbol{v}^2}{2\boldsymbol{g}}$$

The horizontal components of the speed of both the rider and the skateboard are also the same, as long as air resistance is negligible. The rider therefore needs to make little effort to remain in contact with the skateboard. There is some skill involved in ensuring that the frictional forces made possible by the contact are used to turn the skateboard so that it lands on the ramp before the feet or any other part of the rider's body.

10.4 Exam questions

1 The work done by the retarding force = the loss of kinetic energy of the car [1 mark]

Now $\Delta E_k = \frac{1}{2}mv^2$

$= \frac{1}{2}1200 \times 14^2$

$= 1.176 \times 10^5$ J [1 mark]

$\Rightarrow \boldsymbol{F}\Delta \boldsymbol{x} = \Delta E_k$

$\Rightarrow \boldsymbol{F} = \frac{1.176 \times 10^5}{12}$

$= 9800$ N [1 mark]

2 **a** $W = \boldsymbol{Fx}$

$= 6.0 \times 2.0$

$= 12$ J [1 mark]

b The rough surface exerted an opposing force that did negative work = −12 J. [1 mark]

Hence, the net work on the block = 12 − 12 = 0.

So, there was no change in kinetic energy. [1 mark]

3 Gain of E_g = loss of E_k [1 mark]

$1.5 \times 9.8 \times h = 147$ [1 mark]

$\Rightarrow h = 10$ m [1 mark]

4 **a** Jill is correct. [1 mark]

b The weight force is vertically down. [1 mark]

To calculate the work done by the gravitational force, Jack must use the displacement parallel to this force; that is, 3.0 m, not the sloping distance of 5.0 m. [1 mark]

5 W = area under graph [1 mark]

$= \frac{1}{2} \times 0.40 \times 100$ [1 mark]

$= 20$ J [1 mark]

10.5 Efficiency and power

Sample problem 9

$\eta = \frac{\text{useful energy out}}{\text{total energy in}}$

The 'total energy in' is the initial gravitational potential energy of the ball.

$E_g = mgh$

$= mg \times 1.5$

The 'useful energy out' is the gravitational potential energy of the ball at its rebound height of 1.2 metres.

$E_g = mgh$

$= mg \times 1.2$

$\eta = \frac{\text{useful energy out}}{\text{total energy in}}$

$= \frac{1.2mg}{1.5mg}$

$= 0.8$

$= 80\%$

The efficiency is 80%.

Practice problem 9

Efficiency after fourth bounce $= (0.80)^4$

≈ 0.41

Final height $= 0.41 \times 2.0$

$= 0.82$ m

Sample problem 10

a $W = mg\Delta h$

$= 40 \times 9.8 \times 12$

$= 4704$ J

$P = \frac{W}{\Delta t}$

$= \frac{4704}{40}$

≈ 118 W

The student is doing work against the force of gravity at a rate of 118 W.

b $P = \frac{\text{energy transferred}}{\text{time taken}}$

$= 30 \text{ kJ min}^{-1}$

$= \frac{30\,000 \text{ J}}{60 \text{ s}}$

$= 500$ W

If energy is transformed by the leg muscles of the student at the rate of 30 kJ every minute, the student's power output is 500 W.

Practice problem 10

a $E_g = mg\Delta h$

$720 = 50 \times 9.8\Delta h$

$\Rightarrow \Delta h = \frac{720}{50 \times 9.8}$

≈ 1.5 m

b $P = \frac{E}{t}$

$= \frac{720}{1.2}$

$= 600$ W

10.5 Exercise

1 Answers will depend on individual measurements.

A sample solution is as follows:

2 kg mass, 60 cm arm length

As the arm length is 60 cm, the difference in height of the weight from its lowest to highest points is 2 × 60 cm = 1.2 m.

The time taken was recorded to be 30 seconds.

The energy required to lift the weight 10 times = 10 × change in gravitational potential energy.

$\Delta E_g = mg\Delta h$

$= 2 \times 9.8 \times 1.2$

$= 23.52$ J

$10 \times E_g = 253.2$ J

As human muscle has an efficiency of 20% the total work done is:

$0.2W = 253.2$

$\Rightarrow W = \frac{253.2}{0.2}$

$= 1176\,\text{J}$

$P = \frac{W}{t}$

$= \frac{1176}{30}$

$= 39.2\text{ W}$

2 The energy provided by the hammer will be equal to the gravitational potential energy before it drops.

Energy of hammer $= m\boldsymbol{g}\Delta h$

$= 500 \times 9.8 \times 5$

$= 24\,500\text{ J}$

Energy transferred = 80% of 24 500

$= 19\,600\text{ J}$

$E_\text{k} = \frac{1}{2}mv^2$

$19\,600 = \frac{1}{2} \times 700v^2$

$\Rightarrow v^2 = \frac{19\,600}{350}$

$= 56$

Using a constant acceleration formula we can find the average acceleration.

$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$

$0^2 = 7.5^2 + 2 \times 0.050 \times \boldsymbol{a}$

$\Rightarrow \boldsymbol{a} = -560\text{ m s}^{-2}$

Using Newton's Second Law of Motion we can calculate the net force.

$\boldsymbol{F}_\text{net} = m\boldsymbol{a}$

$= 700 \times -560$

$= -392\,000\,\text{N}$

Resistance $= -\boldsymbol{F}_\text{net} + \boldsymbol{F}_\text{g}$

$= 392\,000 + 700 \times 9.8$

$= 398\,860\,\text{N}$

$\approx 4.0\,\text{kN}$

Note that an alternative method to determine $\boldsymbol{F}_\textbf{net}$ is:

Energy transferred = 19 600 J

The amount of energy transferred is equal to the work done.

$W = \boldsymbol{F}_\text{net} \times \boldsymbol{s}$

$19\,600 = \boldsymbol{F}_\text{net} \times 0.05$

$\Rightarrow \boldsymbol{F}_\text{net} = 392\,000\text{ N}$

3 Work done = change in kinetic energy

$W = \Delta E_\text{k}$

$= \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.058 \times \left(\frac{200}{3.6}\right)^2$

$= 89.5$

$\approx 90\text{ J}$

$P = \frac{W}{t}$

$= \frac{89.5}{0.004}$

$= 22\,375\text{ W}$

$\approx 2.0 \times 10^4\text{ W}$

4 As the car is travelling at a constant speed the net force on the car is zero. Therefore, the force by the engine must be equal in magnitude to the sum of the air resistance and friction.

$\text{Force}_\text{by engine on car} = 570 + 150$

$= 720\text{ N}$

$P = \boldsymbol{Fv}$

$= 720 \times 20$

$= 1.4 \times 10^4\text{ W}$

5 The work done against gravity each stride is equal to the change in gravitational potential energy each stride.

$W = \Delta E_\text{g}$

$= m\boldsymbol{g}\Delta h$

$= 60 \times 9.8 \times 0.03$

$\approx 18\text{ J}$

The man's stride length is 1 metre and he walks at a speed of 2 m s^{-1}, so the time taken each stride is 0.5 seconds.

$P = \frac{W}{t}$

$= \frac{18}{0.5}$

$= 36\text{ W}$

10.5 Exam questions

1 D

12 kW input at 80% efficiency: $12 \times 0.8 = 9.6$ kW

200 kW input at 6% efficiency: $200 \times 0.06 = 12.0$ kW

9 kW input at 95% efficiency: $9 \times 0.95 = 8.6$ kW

25 kW input at 49% efficiency: $25 \times 0.49 = 12.3$ kW

25 kW input at 49% efficiency has the highest useful energy output

2 For the tractor to get up the hill it needs to gain gravitational potential energy.

$E_\text{g} = m\boldsymbol{g}\Delta h$

$= 2200 \times 9.8 \times 500$

$= 10\,780\,000\,\text{J}$

$= 10\,780\,\text{kJ}$ [1 mark]

The amount of power used to get the tractor up the hill is:

$P = \frac{E}{t}$

$= \frac{10\,780\,000}{270}$

$= 39\,926\,\text{W}$

$= 39.9\,\text{kW}$ [1 mark]

As the tractor uses 39.9 kW out of its 80 kW to get up the hill, its efficiency is $\frac{39.9}{80} \approx 50\%$. [1 mark]

3 $W = \Delta E_\text{g}$

$= m\boldsymbol{g}\Delta h$

$= 4 \times 9.8 \times 1.5$

$= 58.8\,\text{J}$ [1 mark]

$P = \frac{W}{t}$

$= \frac{58.8}{1.2}$

$= 49$ W [1 mark]

4 a $W = \Delta E_g$

$= mg\Delta h$

$= 180 \times 9.8 \times 1.8$

$= 3175.2$ J

$\approx 3.2 \times 10^3$ J [1 mark]

b $P = \frac{W}{t}$

$= \frac{3175.2}{3}$

$= 1058.4$ W

$\approx 1.1 \times 10^3$ W [1 mark]

c None. As the barbell undergoes no displacement, the work done is zero. [1 mark]

5 a As the cyclist is riding at a constant speed the net force is zero. [1 mark]

$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{driving}} - \boldsymbol{F}_{\text{friction}} - \boldsymbol{F}_{\text{air resistance}}$

$0 = \boldsymbol{F}_{\text{driving}} - 5.7 - 6.5$

$\Rightarrow \boldsymbol{F}_{\text{driving}} = 12.2$ N [1 mark]

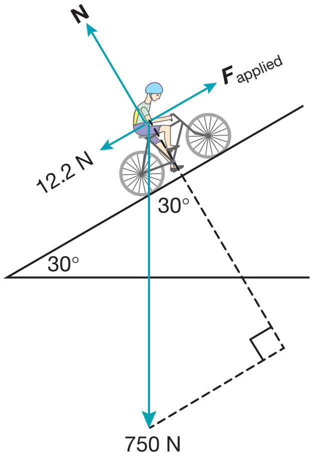
$P = \boldsymbol{Fv}$

$56 = 12.2\boldsymbol{v}$

$\Rightarrow \boldsymbol{v} = \frac{56}{12.2}$

$= 4.6$ m s^{-1} [1 mark]

b On a slope, the force applied to the bicycle to achieve a constant speed is greater (see the following diagram).



$\boldsymbol{F}_{\text{driving}} = 12.2 + \boldsymbol{mg}\sin(30°)$ [1 mark]

$= 12.2 + 75 \times 9.8 \times 0.5$

$= 379.7$ N [1 mark]

$P = \boldsymbol{Fv}$

$= 379.7 \times 4.6$ [1 mark]

≈ 1747 W

Additional power required = 1747 – 56

$= 1.7 \times 10^3$ W [1 mark]

10.6 Review

10.6 Review questions

1 a $\Delta\boldsymbol{p} = m\Delta\boldsymbol{v}$

$= \frac{58.5}{1000}(-19 - 27)$

$= 0.0585 \times -46$

$= -2.7$ kg m s^{-1}

Note: The negative sign indicates the direction, in this instance away from the wall.

b $\boldsymbol{F}_{\text{av}}\Delta t = \Delta\boldsymbol{p}$

$\boldsymbol{F}_{\text{av}}(0.05) = -2.7$

$\Rightarrow \boldsymbol{F}_{\text{av}} = -54$ N

The average force is 54 N. The force on the ball acts away from the wall.

2 $\sum \boldsymbol{p}_{\text{before}} = \sum \boldsymbol{p}_{\text{after}}$

$\boldsymbol{p}_{\text{large car}} + \boldsymbol{p}_{\text{small car}} = \boldsymbol{p}_{\text{after}}$

$(1980 \times 11) + (970 \times \boldsymbol{v}_{\text{small car}}) = (1980 + 970) \times 0$

$\Rightarrow \boldsymbol{v}_{\text{small car}} = \frac{-21\,780}{970}$

$= -22$ m s^{-1}

The negative sign indicates that, as would be expected, the smaller car was travelling in the opposite direction to the large car before the collision.

3 $W = \boldsymbol{Fs}$

$= 40 \times 0.8$

$= 32$ J

4 $E_k = \frac{1}{2}mv^2$

$= \frac{1}{2}(560\,000)\left(\frac{250}{3.6}\right)^2$

$= 1.35 \times 10^9$ J

5 a $\boldsymbol{F} = -k\boldsymbol{x}$

$= -90 \times \frac{-12}{100}$

$= 10.8$ N

b $\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{spring}} - \boldsymbol{F}_g$

As the acceleration is 0, the net force must also equal zero.

$0 = 10.8 - m \times 9.8$

$\Rightarrow m = \frac{10.8}{9.8}$

$= 1.1$ kg

c $E_s = \frac{1}{2}k\boldsymbol{x}^2$

$= \frac{1}{2}(90)\left(\frac{12}{100}\right)^2$

$= 0.65$ J

6 $E_g = mg\Delta h$

$= 72 \times 9.8 \times 185$

$= 130\,000$ J

7 a Calculate the initial gravitational potential energy.

$$E_g = mg\Delta h$$
$$= \left(\frac{420}{1000}\right) \times 9.8 \times 175$$
$$= 720.3 \text{ J}$$

Apply conservation of energy.

As it is at rest initially its initial kinetic energy is zero. The initial gravitational potential energy from its height above the ground below is transformed into kinetic energy as it falls.

$$\sum E_{\text{before}} = \sum E_{\text{after}}$$
$$E_k = 7.2 \times 10^2 \text{ J}$$

b

$$E_k = \frac{1}{2}mv^2$$
$$7.2 \times 10^2 = \frac{1}{2}\left(\frac{420}{1000}\right)v^2$$
$$\Rightarrow v = \sqrt{\frac{7.2 \times 10}{0.21}}$$
$$= 59 \text{ m s}^{-1}$$

8 Calculate the initial kinetic energy.

$$E_k = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(1500)\left(\frac{62}{3.6}\right)^2$$
$$= 222\,454 \text{ J}$$

Initial kinetic energy + work done = final kinetic energy

$$222\,454 + 550\,000 = E_{k\ \text{final}}$$
$$E_{k\ \text{final}} = 772\,454$$
$$\frac{1}{2}mv^2 = 772\,454$$
$$\frac{1}{2}(1500)v^2 = 772\,454$$
$$\Rightarrow v = \sqrt{\frac{772\,454}{750}}$$
$$= 32 \text{ m s}^{-1}$$

9 a

$$E_{k\ \text{before}} = \frac{1}{2}\left(\frac{180}{1000}\right)23^2$$
$$= 48 \text{ J}$$
$$E_{k\ \text{after}} = \frac{1}{2}\left(\frac{180}{1000}\right)10^2$$
$$= 9.0 \text{ J}$$

b

$$\eta = \frac{\text{energy out}}{\text{energy in}}$$
$$= \frac{9}{48}$$
$$= 0.19$$

10

$$P = \boldsymbol{F}\boldsymbol{v}$$
$$= 1150 \times 30$$
$$= 34\,500 \text{ W}$$
$$= 35 \times 10^3 \text{ W}$$

11 a The gravitational potential energy is converted into kinetic energy.

$$E_k = E_g = mg\Delta h$$
$$= 0.160 \times 9.8 \times 2.0$$
$$\approx 3.1 \text{ J}$$

b 32% of the kinetic energy can be stored in the cricket ball as elastic potential energy.

$$32\% \text{ of } 3.14 \text{ J} = 0.32 \times 3.14$$
$$\approx 1.0 \text{ J}$$

c Assuming 100% recovery of stored energy:

$$E_g = E_s$$
$$mg\Delta h = 1.0$$
$$0.16 \times 9.8\Delta h = 1.0$$
$$\Rightarrow \Delta h = \frac{1.0}{0.16 \times 9.8}$$
$$= 0.64 \text{ m}$$

12 a Convert 50 km h^{-1} to m s^{-1}:

$$50 \text{ km h}^{-1} = \frac{50}{3.6} \text{ m s}^{-1}$$
$$\approx 14 \text{ m s}^{-1}$$

The work done by the net force is equal to the initial kinetic energy of the car.

$$\boldsymbol{F}_{\text{net}}\boldsymbol{s} = E_k$$
$$\Rightarrow \boldsymbol{F}_{\text{net}} = \frac{E_k}{\boldsymbol{s}}$$
$$= \frac{\left(\frac{1}{2} \times 1500 \times 13.9^2\right)}{0.6}$$
$$\approx 2.4 \times 10^5 \text{ N}$$

b Average acceleration can be found using Newton's Second Law.

$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$$
$$2.4 \times 10^5 = 1500\boldsymbol{a}$$
$$\Rightarrow \boldsymbol{a} = \frac{2.4 \times 10^5}{1500}$$
$$= 1.6 \times 10^2 \text{ m s}^{-2}$$

c The work done by the net force is equal to the initial kinetic energy of the car.

$$\boldsymbol{F}_{\text{net}}\boldsymbol{s} = E_k$$
$$\Rightarrow \boldsymbol{F}_{\text{net}} = \frac{E_k}{\boldsymbol{s}}$$
$$= \frac{\left(\frac{1}{2} \times 1500 \times 13.9^2\right)}{0.1}$$
$$\approx 1.5 \times 10^5 \text{ N}$$
$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$$
$$1.5 \times 10^5 = 1500\boldsymbol{a}$$
$$\Rightarrow \boldsymbol{a} = \frac{1.5 \times 10^5}{1500}$$
$$\approx 10^3 \text{ m s}^{-2}$$

d The kinetic energy of the car is transformed into potential energy of the materials in the crumple zone, which undergo a permanent change in shape. This leaves a smaller amount of kinetic energy to be transferred to the passengers.

e One could argue that a large car is safer. For a given force applied by an obstacle or another vehicle, the deceleration of a large car is less than that of a small car. Therefore, the deceleration of the occupants inside is less. For example, consider a car of mass 1500 kg coming to rest from 20 m s^{-1} when a concrete wall applies a force of 48 000 N to the car.

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{net}}{m}$$
$$= -\frac{48\,000\text{ N}}{1500\text{ kg}}$$
$$= -32\text{ m s}^{-2}$$

The deceleration of an occupant with a correctly fitted seatbelt would be 32 m s^{-2}. Consider a car of mass 1200 kilograms coming to rest from the same speed when the same force is applied by the wall.

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{net}}{m}$$
$$= -\frac{48\,000\text{ N}}{1200\text{ kg}}$$
$$= -40\text{ m s}^{-2}$$

The deceleration of an occupant with a correctly fitted seatbelt would be 40 m s^{-2}. Without seatbelts, an occupant would strike the interior of a larger car with a smaller relative speed. Of course, these arguments are not very strong because there are so many other variables related to car design and the nature of the rigid barrier that affect the deceleration of a car.

13 **a** The kinetic energy is equal to the work done by the net force. This can be calculated by the area under the force-versus-distance graph.

$$E_k = W$$
$$= 8 \times 240$$
$$= 1920\text{ J}$$

b
$$\boldsymbol{F}_{net} = m\boldsymbol{g}\sin\theta - (\text{friction} + \text{air resistance})$$
$$\text{friction} + \text{air resistance} = m\boldsymbol{g}\sin\theta - \boldsymbol{F}_{net}$$
$$= 50 \times 9.8\sin(30°) - 240$$
$$= 245 - 240$$
$$= 5\text{ N}$$

c The kinetic energy is equal to the work done by the net force. This can be calculated by the area under the force-versus-distance graph.

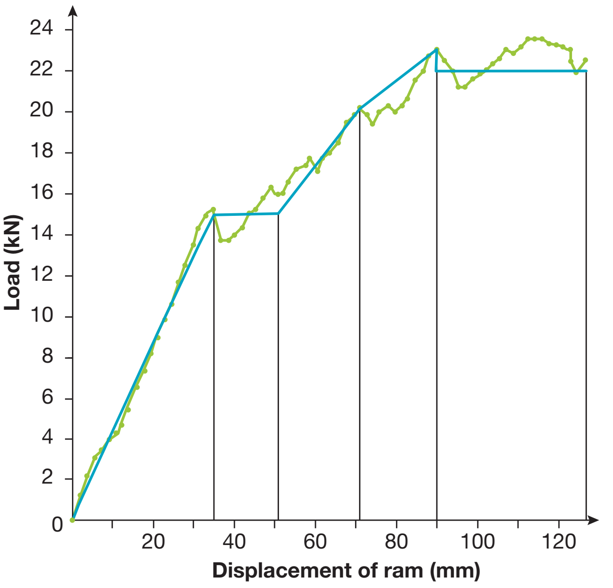
$$E_k = W$$
$$= 1920 + \frac{1}{2} \times 8 \times 120 + 8 \times 120 + \frac{1}{2} \times 4 \times 120$$
$$= 1920 + 480 + 960 + 240$$
$$= 3600\text{ J}$$

d
$$E_g = m\boldsymbol{g}\Delta h$$
$$= 50 \times 9.8 \times 20\sin(30°)$$
$$= 4900\text{ J}$$

e Some of the gravitational potential energy is transformed into thermal energy and sound, due to the frictional force and air resistance.

14 **a** W = area under load-versus-displacement graph

This area can be estimated by dividing the area into a number of triangles, trapezia and rectangles. The following graph shows one way in which this can be done.



The unit of area is J since kN × mm = N m = J.

$$W = \frac{1}{2} \times 15 \times 35 + 15 \times 16 + 19\left(\frac{15+20}{2}\right)$$
$$+ 20\left(\frac{20+23}{2}\right) + 22 \times 37$$
$$= 2079\text{ J}$$

b The car needs to have 2079 J of kinetic energy upon impact. Therefore, it needs to have 2079 J of gravitational potential energy before it is dropped.

$$E_g = m\boldsymbol{g}\Delta h$$
$$2079 = 1400 \times 9.8\Delta h$$
$$\Rightarrow \Delta h = \frac{2079}{1400 \times 9.8}$$
$$\approx 0.15\text{ m}$$

The car needs to be dropped from a height of 0.15 metres to crush its roof by 127 millimetres.

15 Speed and momentum before impact, speed and momentum after impact, change in momentum on impact, average force during impact, loss of gravitational potential energy on falling, gain in kinetic energy on falling, gain of gravitational potential energy on rebounding, loss of kinetic energy on rebounding, percentage of energy lost

10.6 Exam questions

Section A — Multiple choice questions

1 A

According to the Law of Conservation of Momentum, the sum of the momentum before the collision is equal to the momentum after the collision. As both cars have the same mass and opposite velocities before the collision, their momentums are equal and opposite. Therefore, the sum of their momentum is zero. After the collision they have zero momentum; hence, their speed after colliding is 0 m s^{-1}.

2 C

$$\boldsymbol{I}_{\text{on Terri by Steve}} = \boldsymbol{I}_{\text{on Steve by Terri}}$$
$$= m_{\text{Steve}}\Delta\boldsymbol{v}_{\text{Steve}}$$
$$= 75 \times 5$$
$$= 375\text{ N s}$$

3 B

$W = \boldsymbol{F}\boldsymbol{s}$

$= 125 \times 5$

$= 625\ \text{J}$

4 B

$E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.5 \times 25^2$

$\approx 156\ \text{J}$

5 D

$E_s = \frac{1}{2}k\boldsymbol{x}^2$

$= \frac{1}{2} \times 5 \times 0.1^2$

$= 0.025\ \text{J}$

6 B

$E_g = m\boldsymbol{g}\Delta h$

$= 80 \times 9.8 \times 25$

$= 19\,600\ \text{J}$

7 A

$\boldsymbol{F} = k\boldsymbol{x}$

$= 125 \times 0.17$

$= 21.25\ \text{N}$

8 D

$E_k = \Delta E_g$

$= m\boldsymbol{g}\Delta h$

$= 369 \times 9.8 \times 40$

$= 144\,648\ \text{J}$

9 B

$0.9 \times 42\ \text{J} = 37.8\ \text{J}$

10 C

$W = \Delta E_g$

$= m\boldsymbol{g}\Delta h$

$= 3.7 \times 9.8 \times 0.47$

$= 17.0422\ \text{J}$

$P = \frac{W}{t}$

$= \frac{17.0422}{0.8}$

$\approx 21.3\ \text{W}$

Section B — Short answer questions

11 Total energy is conserved, however, not all of the gravitational potential energy is converted into kinetic energy. The most likely cause for the discrepancy is the work done on the object by friction acting to oppose its motion down the ramp. [1 mark]
This will decrease the amount of gravitational potential energy that is transformed into kinetic energy. [1 mark]

12 a

$$\sum \boldsymbol{p}_{\text{before}} = \sum \boldsymbol{p}_{\text{after}}$$

$$\boldsymbol{p}_{\text{tram A}} + \boldsymbol{p}_{\text{tram B}} = \boldsymbol{p}_{\text{tram A\&B}}\ \text{[1 mark]}$$

$$26\,000 \times \frac{54}{3.6} + 35\,000 \times 0 = (26\,000 + 35\,000) \times \boldsymbol{v}_{\text{final}}$$

$$390\,000 = 61\,000 \times \boldsymbol{v}_{\text{final}}$$

$$\Rightarrow \boldsymbol{v}_{\text{final}} = 6.4\ \text{m s}^{-1}$$

As would be expected, the sign on the final velocity indicates that the trams continue to move in the same direction as the runaway tram after the collision, only at a lower speed (its initial speed was 15 m s^{-1}).

b $\boldsymbol{F}_{\text{av}}\Delta t = \Delta\boldsymbol{p}$

$\boldsymbol{F}_{\text{av}} \times 0.40 = 26\,000 \times 6.4 - 390\,000$ [1 mark]

$\Rightarrow \boldsymbol{F}_{\text{av}} = -559\ \text{kN}$ [1 mark]

Note: The negative sign indicates that the force on the runaway tram during the collision was in the opposite direction to its initial motion.

13 The student is not doing any work on the tray during the motion described. [1 mark].
For work to be done there must be a force applied and a displacement in the direction of that force. In this instance, the student would be exerting an upwards force on the tray to balance the downwards force due to gravity on the tray. This is perpendicular to the horizontal motion of the tray. [1 mark]
Note: If the tray was accelerating in the horizontal direction, then the forces involved would be different and work would be done. Also, if air resistance was not negligible then the student would need to apply a horizontal force to balance it, resulting in work being done.

14 $P = \frac{E}{t}$

Assume that friction loses and air resistance are negligible. Assume that the motor/generator is 100% efficient when generating electricity. The energy transformed is equal to the change in gravitational potential energy. [1 mark]

$E_g = m\boldsymbol{g}\Delta h$

$= 2500 \times 9.8 \times 450$

$= 11.025 \times 10^6\ \text{J}$ [1 mark]

Calculate the power:

$P = \frac{11.025 \times 10^6}{520}$

$= 21.2\ \text{kW}$ [1 mark]

15 Apply conservation of energy.
At point A: Initially the ball is not moving so its kinetic energy is zero. Assume that when the spring is released it transfers all of its potential energy to the ball. [1 mark]
At point B: The ball has gained kinetic and gravitational potential energy. Assuming that there are no other energy transformations taking place, this will be equal to the energy it gained from the spring. [1 mark]
Energy from spring at A = kinetic energy at B + gravitational potential energy at B [1 mark]

$$\frac{1}{2}k\boldsymbol{x}^2 = \frac{1}{2}m(v_B)^2 + m\boldsymbol{g}\Delta h \qquad \text{[1 mark]}$$

$$\frac{1}{2}(300)\left(\frac{10}{100}\right)^2 = \frac{1}{2}\left(\frac{80}{1000}\right)(v_B)^2 + \left(\frac{80}{1000}\right) \times 9.8 \times \left(\frac{6}{100}\right) \qquad \text{[1 mark]}$$

$$1.5 = 0.04(v_B)^2 + 0.047$$

$$\Rightarrow vB = \sqrt{\frac{1.453}{0.04}}$$

$$= 6\ \text{m s}^{-1} \qquad \text{[1 mark]}$$

Topic 12 — Scientific investigations

12.2 Key science skills and concepts in physics

Sample problem 1

The independent variable is the salt type.

The dependent variable is the time it takes for the water to boil.

Research question: Does the type of salt added to water affect the time it takes for water to boil?

Aim: To determine if different types of salt affect the time it takes for water to boil

Hypothesis: If table salt, sea salt, Himalayan pink salt or chicken salt is added to water, then the time taken for the water to boil will decrease. (Pure table salt will cause the largest decrease in time.)

Practice problem 1

To write a research question, the independent variable and dependent variable should first be determined. The independent variable is the surface material of the incline and the dependent variable is the speed in which the ball rolls down the incline.

Responses will vary but the following are examples.

Research question: Does the type of surface material of an incline affect the speed in which a ball will roll down the incline?

Aim: To determine if different surface materials have an impact on the speed of a ball rolling down an incline

Hypothesis: If the surface material of an incline is smooth (such as steel or aluminium) a ball will have a faster speed when rolling down the incline compared to a rougher surface (such as carpet or sandpaper).

Sample problem 2

a The decimal place would need to go between 1 and 4 to form 1.496. The decimal was moved eight spots to the left so the exponent is 8: 10^8.
1.496×10^8 km

b The decimal place would need to go between 1 and 6 to form 1.67. The decimal was moved 24 spots to the right so the exponent is –24: 10^{-24}.
1.67×10^{-24} g

Practice problem 2

a The diameter of Saturn's rings is 282 000 km. The decimal needs to be moved between the 2 and the 8 to make 2.82 (remove the extra zeros). In order to do this, the decimal (at the end of the number) needs to be moved five spots to the left; therefore, the exponent is 5. This would be written as 10^5. Therefore, in scientific notation 282 000 km is 2.82×10^5 km.

b The number of metres that sound travels in one hour is 1 235 000 m. The decimal needs to be moved between the 1 and the 2 to make 1.235 (remove the extra zeros). In order to do this, the decimal (at the end of the number) needs to be moved six spots to the left; therefore, the exponent is 6. This would be written as 10^6. Therefore, in scientific notation 1 235 000 m is 1.234×10^6 m.

c The uncertainty of a highly precise clock is 0.000 000 000 000 000 003 seconds. The decimal needs to be moved after the 3. In order to do this, the decimal needs to be moved 18 spots to the right; therefore, the exponent is –18. This would be written as 10^{-18}. Therefore, in scientific notation 0.000 000 000 000 000 003 seconds is 3×10^{-18} s.

12.2 Exercise

1 In order to be a testable question, it must link the independent and dependent variable, and be able to be investigated through scientific method and be practicable.
Responses may vary; however, example responses are given.

- **a** How much of a difference does placing water in an insulated Thermos have on the drop in temperature?
- **b** What is the difference in brightness between globes in series and parallel circuits?
- **c** Does being placed in a vacuum affect the speed in which a ping-pong ball falls to the ground?

2 A logbook is important as it shows evidence of each step of an investigation, allowing work to be validated as your own. It shows the different components of scientific approach and contains all results obtained in your practical. These may not all be shown on a scientific poster or in a final report, so the logbook is an important supplementary tool to gain an understanding to all aspects of an investigation.

3 B
This shows an acceptable hypothesis as it clearly predicts an outcome, links the independent and dependent variables, has a tentative explanation (that it will change from a liquid to a solid at a faster rate) and is testable (recording the freezing temperature).

4
- Decide on a topic.
- Formulate a question.
- Create an aim and hypothesis.
- Select equipment.
- Create a clear and reproducible method that will produce precise, accurate, valid and reliable results.
- Complete a risk assessment.
- Submit a practical proposal to your teacher.

5 **a** To investigate how the shape of a parachute affects the time it remains floating in the air

b If circular, square and triangular parachutes are dropped from a similar height, the circular parachute will take the longest to hit the ground due to its larger surface area and increased air resistance.

12.2 Exam questions

1 D
A hypothesis is not a question (ruling out option A). Option D identifies a tentative hypothesis (explanation) and a predicted outcome by which the hypothesis can be tested. Options B and C are testable predictions, but they do not include a tentative explanation.

2 B
Science is based on making observations and then constructing a hypothesis to explain an observation. Designing and conducting experiments follows.

3 D
When data does not support your hypothesis, you must conclude that the hypothesis is not supported. You will need to rethink your hypothesis.

4 **a** If the freefall speed of an object in a fluid is determined by the viscosity of the fluid, then the speed will be greater when the fluid viscosity is small. (1 mark)

b To determine whether fluid viscosity affects the freefall speed of an object (1 mark)

c Independent variable: the viscosity of the fluid. Note that the size, shape, mass and initial height of the marble dropped in the fluid could be controlled variables. (1 mark)
Dependent variable: the rate at which the marble falls down the fluid (1 mark)

5 **a** Independent variables: initial speed and angle of projection (1 mark)
Dependent variable: range of the projectile (1 mark)

b Mass, shape and initial height of the projectile; air temperature (1 mark)

c The main issue is that there are two independent variables being explored in this investigation. (1 mark) One of these (either initial speed or angle of projection) needs to be controlled. Testing both of these variables at once means it is not a fair test and thus the results cannot be seen as conclusive or valid. For example, if the projectile range in test 3 is the greatest, there is no data to suggest whether it is the initial speed or the angle of projection that is leading to those results. (1 mark) An example of a change would be to test the projectile range at the same initial speed (e.g. $2\ \text{m s}^{-2}$) but with different angles of projection to make appropriate comparisons related to the effect of angle of projection on projectile range. (1 mark) (Alternatively, the projectile range could be tested with the same angle of projection and different initial speeds, to make appropriate comparisons related to the effect of the initial speed on projectile range.)

12.3 Characteristics of scientific methodology and primary data generation

12.3 Exercise

1 **a** Scientific methodology is a technique used to make predictions and produce answers, while scientific method is a particular scientific methodology and shows the steps and the process involved for answering questions.

b Primary sources of data are from the initial source of data collection, often through direct investigation. Secondary data is a summary of analysed primary data.

c Continuous data can take any numerical value, while discrete data can only take on set values that can be counted.

2 Yes. An experiment needs to be replicated in order to be seen as reliable. Replicating a method and getting similar results (usually by multiple investigations) supports it as reliable.

3 Results would not be considered reliable if repetitions of an investigation lead to vastly different answers. An experiment would also not be considered reliable if the method is not clear enough that an individual cannot replicate the process and achieve similar results.

4 Examples of procedures that could affect the internal validity of an experiment include the following:
- Not correctly testing the variable outlined in the hypothesis
- Misinterpreting measurements
- Not controlling variables correctly.

5 A strength of quantitative data is it provides a clearer understanding of trends and patterns in results. A weakness is in that it can be harder and more time consuming to collect. A strength of qualitative data is that it is often easy and quick to collect. A weakness is that it can be subjective (i.e. one person might class a colour as blue while another might state it is purple).

12.3 Exam questions

1 The experimental groups are exposed to the changing conditions determined by the independent variable, but the control group is independent and not affected by those conditions, and thus is used as a baseline for comparison with the experimental groups. (1 mark)

2 **a** The control group is group C and the test groups are groups A, B and D. (1 mark)

b Having a control group is better. In any experiment there is a need to have a control group not affected by the independent variable, to compare with the experimental group affected by the independent variable. (1 mark)

3 Any three considerations such as:
- that the equipment is safe to be used in the investigation
- that the equipment selected will give accurate results
- what personal safety equipment needs to be used.

(1 mark for each of three suitable considerations)

4 **a** Answers may vary, but could include the mass, volume and density of the apple.
(1 mark for three pieces of quantitative data)

b Answers may vary, but could include a digital balance, water displacement can and volumetric cylinder. (1 mark)

c Answers may vary, but could include the colour, firmness and shape of the apple.
(1 mark for three pieces of qualitative data)

d None — only sense of sight and touch are used (1 mark)

5 **a** The independent variable is the temperature of the ping-pong ball and the dependent variable is the height of the first bounce. (1 mark)

b Both the temperature of the ball and the height of the first bounce are quantitative. (1 mark)

c Responses may vary. A sample response is as follows:
Aim: To investigate how the temperature of a ping-pong ball affects its coefficient of restitution (1 mark)
Hypothesis: If the temperature of a ping-pong ball affects its coefficient of restitution, then the height of its first bounce will increase when the temperature of the ball is increased. (1 mark)

d Any two variables, such as:
- the initial height of the ping-pong ball
- the ping-pong ball's initial speed
- the nature of the surface on which the ping-pong ball bounces.

(1 mark for each of two suitable variables)

e D
An experiment is the most appropriate methodology in this case, to investigate the relationship between an independent variable and a dependent variable, controlling all other variables.

12.4 Health, safety and ethical guidelines

12.4 Exercise

1 **a** The contents of batteries are corrosive so it's important to be careful in case of leakage. A battery can heat up easily, which can lead to rupture of its case. Appropriate precautions include keeping batteries in a dry location, checking for leaks and not cutting them open.

b Light globes can reach high temperatures, which may lead to burns. Light globes are made of glass, so may break or chip, leading to lacerations. Appropriate precautions include immediately reporting and sweeping up broken glass, and not touching the light globe while in use.

c Bunsen burners are sources of ignition. The blue roaring flame is extremely hot and difficult to see, and can lead to burns. Damage to the base, jet and tubes can cause flames and increased temperatures, which may also lead to burns. Safety precautions include avoiding touching the Bunsen burner while hot, especially while on the roaring flame, and tying back any loose clothing or long hair.

d Beakers, being made of glass, can break or chip, leading to possible cuts or lacerations. Appropriate precautions include immediately reporting and sweeping up broken glass (not picking out) and discarding any beakers with damage, as these are more prone to breaking.

e Thermometers are made of glass and can break, leading to cuts. Some thermometers contain mercury, which is toxic. Appropriate precautions are discarding if damaged, using a dustpan and brush if glass breaks and ensuring careful use, as they are fragile. If a mercury thermometer breaks, immediately cover with sulfur. Try to use alcohol thermometers instead of mercury ones.

2 Ethics may be important when confidentiality is involved. If an individual is conducting an opinion survey, it is important that they have all relevant information and their personal information is kept private. Ethics can also relate to personal beliefs, particularly if concepts go against the participants' own ideas and ethics.

3 Purposes of a risk assessment include:
- identifying hazards with equipment or chemicals
- suggesting standard handling procedures
- outlining the correct disposal of chemicals
- outlining any first aid information that may be required
- providing information for the practical, including the location, time and date.

4 Other things that could be added to a risk assessment include:
- information about devices used, such as specific voltages
- further information about allergies
- contacts to emergency hotlines
- clear information about how many students will be in each group and at each bench.

5 In any plan drawn, you should show:
- the emergency gas and electricity shut-offs
- the location of fire blankets and extinguishers
- the exits and windows
- the location of safety glasses
- lab coats and gloves
- the location of the fume cupboard, taps, eyewash and emergency shower.

Other safety features may include the location of locked cupboards, the location of cleaning equipment (including spill kits) and the location where broken glass should be disposed of.

12.4 Exam questions

1 C

The ethical approach is to report the mistake and get it corrected.

2 Sample response:

Respect (1 mark), control (1 mark) and attribution (1 mark)

3 Sample response:

Each capacitor is rated for a maximum voltage and exceeding this voltage can destroy the capacitor. Thus, you should make sure that the capacitor you have selected can withstand a 12 V voltage drop across it and if not, you could use a resistor to decrease the voltage drop. (1 mark) Electrolytic capacitors are polarised and should be connected appropriately, otherwise they will heat up and explode or catch fire. (1 mark) Additionally, a charged capacitor can be dangerous, and you should also avoid touching it with your bare hands anywhere except on the sides of its body. (1 mark)

4 Sample response:

Hazards:
- The bowling ball will increase its speed (and energy) down the slope and might hurt anyone standing at the bottom of the slope, or damage any equipment at the bottom of the slope. (1 mark)
- The bowling bowl will have to be picked up from the ground 10 times, and given its mass, the person picking it up could hurt their back. (1 mark)
- The bowling ball will have to be carried around (from the bottom of the slope to the top) and if dropped, could hurt someone. (1 mark)

Safety precautions:
- Place some padding material at the bottom of the slope to absorb the energy of the ball and stop it safely. (1 mark)
- Restrict access to the surroundings of the incline and clear anything that could cause the person carrying the ball to trip and fall or drop the ball. (1 mark)
- The person picking up the ball from the ground should not bend their back forward but should bend their knees, keeping their back straight, and the ball close to their centre of mass while lifting it. (1 mark)

5 Sample response:

Hazards:
- The different parts of the slingshot (like the elastic band and its attachment points) could be worn or damaged and break while in use, causing injuries. (1 mark)
- The projectiles are small but with the speed they will acquire, they can hurt people or damage equipment. (1 mark)
- The projectiles can bounce off or ricochet, which might cause them to hit something that wasn't planned. (1 mark)
- The bands are made of latex and some people are allergic to latex. (1 mark)
- The small plastic pellets, if left outside after use, could be ingested by animals and are not environmentally friendly. (1 mark)

Safety precautions:
- Regularly inspect the slingshot for damage. (1 mark)
- Wear suitable eye protection, such as polycarbonate glasses. (1 mark)

- Restrict access to the area where the slingshot will be used, and make sure no bystanders might get hurt by the projectiles. (1 mark)
- Prevent anyone with a latex allergy from manipulating the latex band. (1 mark)
- Pick up the plastic projectiles after use, or use clay pellets instead, which are biodegradable and non-toxic, and which will break on impact instead of bouncing. (1 mark)

12.5 Accuracy, precision, reproducibility, repeatability and validity of measurements

Sample problem 3

a Accuracy refers to how close a measurement is to a known value.
Student 1 had data 1.48 °C lower and 3.52 °C higher than the actual data.
Student 2 had data that was up to 5.52 °C higher.
Student 3 had data that was 0.48 °C lower and 2.02 °C higher.
Student 2 had the least accurate data, as their values were the furthest from the actual value.

b Precision refers to how close multiple measurements of the same investigation are to each other.
Student 1 had a data range of 5.0 °C.
Student 2 had a data range of 1.5 °C.
Student 3 had a data range of 2.5 °C.
Student 1 had the least precise data.

c Using the results from parts **a** and **b** it can be seen that student 3 had the most accurate data and student 2 had the most precise data.
The student who had the most precise data was not the same student who had the most accurate data. Students may have measurements very close together (precise), but they may not be accurate. This may be due to errors in the measuring device or interpretation of the melting point (when the solid turned to liquid). Data may also be accurate without being precise; you can be close to the target but with inconsistent readings. For reliable and valid results, data should be both accurate and precise.

Practice problem 3

a Accuracy refers to how close a measurement is to a known value. Student 1 had data 9.9 °C lower and 6.6 °C higher than the actual data. Student 2 had data 1.9 °C lower and 1.1 °C higher than the actual data. Student 3 had data that was 4.4 °C lower and 4.6 °C higher. Therefore, student 1 had the least accurate data (as they had the largest deviation from the actual value).

b Precision refers to how close multiple measurements of the same investigation are to each other. Student 1 had a range of data of 16.5 °C. Student 2 had a range of data of 3.0 °C. Student 3 had a range of data of 9.0 °C. Therefore, student 1 has the least precise data.

c The student with the most precise data was student 2 and the student with the most accurate data was also student 2. In this case, the student with the most precise data was also the one who had the most accurate data.

12.5 Exercise

1 Accuracy refers to how close a measurement is to a known value, while precision refers to how close multiple measurements of the same investigation are to each other.

2 Repeatability. This is how close results of successive measurements are to each other in the exact same conditions. Reproducibility is how close results are when the same variable is being measured, but under different conditions.

3 Invalid results would be those obtained if your experimental method does not clearly relate to the purpose of the investigation and care has not been taken to get precise measurements. Invalid results can also be obtained when the experiment has been poorly designed or not performed to the method.

4 An experiment can be repeated exactly the same way by two people, but if the method is unsound the results — even though they may be the same — will be invalid. If results are valid it means that the experiment is repeatable, as the method is sound and should give the same results.

5 **a** Student 2. The mean of student 2’s data is the closest to the known value.

b Student 1’s data is the most precise as their range of values is the smallest.

c The answer to part **a** does not change as student 3 is not the closest to 25 N m^{-1}. The answer to **b** does change because the range of student 3’s data is now 0.3, which is the smallest range. Student 3 would now have the most precise data.

12.5 Exam questions

1 D
Accurate results would be results close to the bullseye, and precise results would be close to each other. Only player D has precise results that are also inaccurate.

2 C
Experimental bias can occur in many ways. Ignoring data from one experimental group is an example of one of the ways biases can occur.

3 D
Calibration of a machine involves taking a measurement and comparing the measurement to a known expected value. This ensures accuracy of measurement.

4 **a** This is an example of reproducibility. The same variable is being measured under different conditions (different observer, different time, different location) and the results are close. (1 mark)

b The students’ results are accurate. The experimental values found by the students are close to the known value for the variable they investigated. (1 mark)

5 **a** Calculate the mean for each student.

	Student 1 measurements	Student 2 measurements	Student 3 measurements
Mean	0.8004	0.8164	0.8304

Student 2 has the most accurate data. (1 mark) The mean of student 2’s data is the closest to the known value. (1 mark)

b Calculate the range for each student.

	Student 1 measurements	Student 2 measurements	Student 3 measurements
Range	0.02	0.03	0.054

Student 1's data is the most precise (1 mark) as their range of values is the smallest (1 mark).

12.6 Ways of organising, analysing and evaluating primary data

Sample problem 4

a 128.93
The measurement can be between 128.925 and 128.935.
0.01 g is the smallest measurement possible.

$$\frac{0.01}{2} = 0.005\text{ g}$$

The mass is 128.93 ± 0.005 g.

b 47 °C
The measurement can be between 46.75 and 47.25.
0.5 is the smallest measurement possible.

$$\frac{0.5\text{ °C}}{2} = 0.25\text{ °C}$$

The temperature is 47 ± 0.25 °C.

Practice problem 4

a If a reading is found to be 0.12 grams, this can fall between 0.115 and 0.125. In regard to the tolerance, the smallest measurement possible is 0.01 g, so the tolerance is half of this: 0.005 g.
Therefore, the reading, including the tolerance, is 0.12 ± 0.005 g.

b If a reading is found to be 0.195 grams, this can fall between 0.1945 and 0.1955. In regard to the tolerance, the smallest measurement possible is 0.001 g, so the tolerance is half of this: 0.0005 g.
Therefore, the reading, including the tolerance, is 0.195 ± 0.0005 g.

12.6 Exercise

1 C
Of the actions identified, C is the appropriate action. An experimental result that does not appear to fit is called an outlier. An outlier is most commonly the result of a measurement error, an incorrect recording or a sampling error. However, in some cases, an outlier may be a valid data point that provides further information about the research topic being investigated.

2 A random error is chance variation in measurements and usually affects the precision of data, and can be improved by repeating an experiment. A systematic error usually affects the accuracy of an experiment and is often due to equipment errors, and therefore cannot be improved by repeating the experiment using the same equipment.
Examples of random errors are incorrect judgement when recording a volume in a measuring cylinder, or not lining up a ruler correctly with the edge of an object being measured.
Examples of systematic errors are a faulty scale that has been incorrectly calibrated, or a 1-metre ruler that was incorrectly made, and each 1 centimetre marked actually is 1.8 centimetres long.

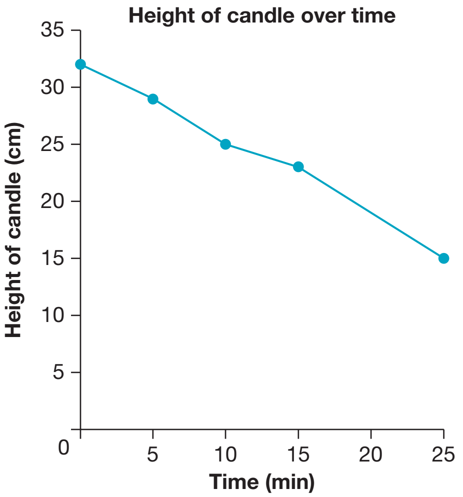
3 a A line graph would be most appropriate, as a variable (height of rocket) is being observed over time.

b A bar graph would be most appropriate, as one variable is qualitative (brand of car) and the other is quantitative (volume of airbag).

c A line graph would be most appropriate, as both variables (pH and temperature) are quantitative and a line graph would allow for trends to be seen. However, a scatterplot with a line of best fit may also be appropriate.

d A histogram would be most appropriate, as frequency and intervals are being used in the data.

4 The following line graph would be an appropriate choice.



It can be seen that as time passes, the height of the candle decreases, showing a negative correlation. On average, it decreases by around 4 centimetres per minute, except between 0 and 5 minutes and between 10 and 15 minutes, where the rate of size reduction is slower. This may be due to minor changes in room temperature or different thicknesses in the candle.

5 An outlier should be mentioned and discussed, with an outline of possible reasons. However, it usually is excluded when calculating averages or trying to add a line of best fit.

12.6 Exam questions

1 The voltage drop across an ohmic resistor is directly proportional to the current through it (Ohm's Law), thus we already know the trend the data should display. (1 mark)
A scatterplot with a line of best fit would be the most appropriate graph. Alternatively, as both variables are quantitative, a line graph would also be appropriate. (1 mark)

2 a The graph has a grid and a title, the axes are labelled with units, the scales cover the entire range of the measurement and equal divisions are marked. The only thing missing is clearly visible data points. (1 mark)

b The half-life can be read when 50 per cent of the carbon-14 atoms are remaining; thus it is approximately 6000 years. (1 mark)

3 a A bar graph would be most appropriate (1 mark) as one of the variables (material) is qualitative while the other (electrical conductivity) is quantitative.

b Significant figures need to be consistent across data sets. Here, both aluminium and platinum have their electrical conductivity given with two significant figures, instead of three like silver, gold and copper. (1 mark)

4 Sample response:
Student A's illustration could be seen as misleading; for instance, the big droplet for the USA seems to contain 7 or 8 times as much water as the droplet for China, giving the impression that the daily water consumption for the USA is 7 or 8 times as much as the daily water consumption for China when, from the data given, it is only three times as much. (1 mark)
Student B's illustration, using several droplets instead of bigger droplets, is more suitable in that regard, as the yearly water consumption ratio between countries is approximately respected. (1 mark)
However, neither illustration mentions the fact that the data illustrated is for 2009, which is old data; thus they could both mislead viewers in making the assumption that these illustrations are for more recent data. (1 mark)
Neither illustration includes actual figures or units, and thus are not giving all the information from the data in the table. (1 mark)
A more appropriate illustration of the data would be a bar graph representing the two sets of data with two different *y*-axes (one for the daily water consumption, one for the yearly) (1 mark) and a key (1 mark).

5 Sample response:
See the figure at the foot of the page*
As one of the variables (country) is qualitative while the other two are qualitative, the most appropriate type of graph would be a bar graph. (1 mark)
As one of the data sets is in tonnes and the other one in billions of tonnes, using two different *y*-axes would be the most appropriate. (1 mark)
The graph should have a title (ideally mentioning how old the data is) (1 mark), clearly labelled axes with units (1 mark) and lines (a grid is not suitable for a bar graph) (1 mark). There should also be suitable scales showing the whole range of data. (1 mark)
Actual values can be added but are not essential as lines and suitable scales are present to facilitate reading the graph.

12.7 Challenging scientific models and theories

12.7 Exercise

1 Hypotheses are supported based on the evidence we have at the time and how this is interpreted. However, the nature of evidence is that new knowledge may become available, which may affect the outcome we obtain. Therefore, we cannot definitively prove a hypothesis, but rather support it with what we have at that point in time.

2 Three examples of strong evidence are quantitative data that was obtained when investigating a single variable, data that was not influenced by bias, and data that has been obtained through a reliable method and repeated and obtained by other investigators.

3 Key findings communicated in your discussion should include information about trends and patterns in your data, the relationship to chemical concepts and theory, and a clear answer to the question of your investigation.

4 It is important to link results to concepts to show how theory can be used to support phenomena observed in a scientific investigation. This gives an investigation more validity and helps to better back up conclusions and findings drawn.

5 New knowledge is being gained, which requires models and theories to be constantly reassessed in light of this new knowledge.

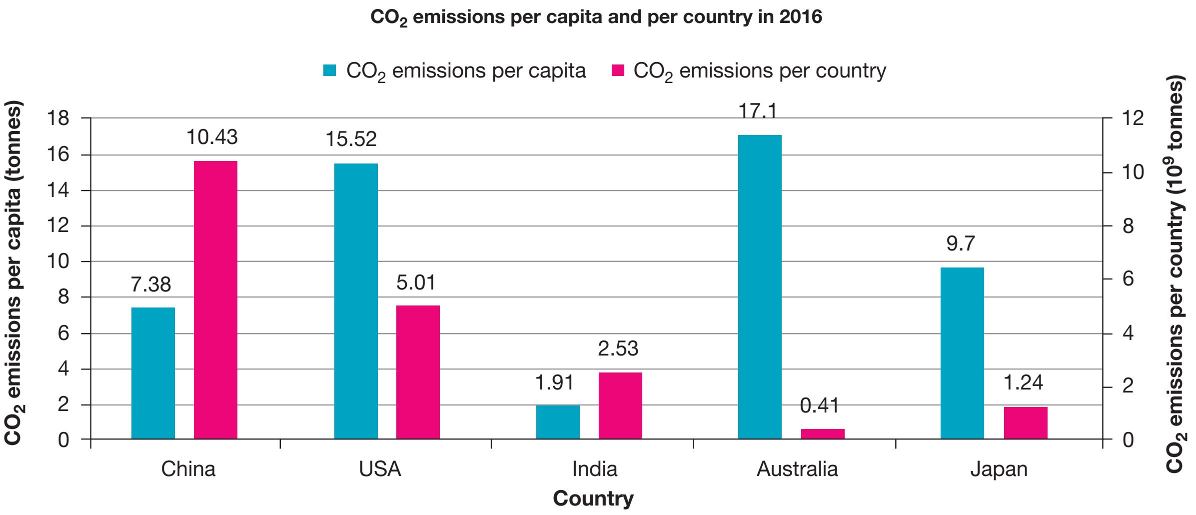
12.7 Exam questions

1 B
Strong evidence should show minimal bias and high validity, and be reproducible and reliable with a basis of scientific evidence. The research that you use in your investigation should be from peer-reviewed journals to reduce bias and improve reliability.

2 A model is a representation of an idea, process or phenomenon (1 mark), whereas a theory is a well-supported explanation (1 mark).

3 a Firsthand evidence needs to be obtained through scientific investigation using accurate, precise and valid methods. (1 mark) Results that show that the magnitude of the force acting on the same body submerged in fluids of different densities is increasing when the density of the fluid is

***5**



increasing would be consistent with the theory. (1 mark)

Results that show that for the same fluid, if the volume of the body submerged increases, then the magnitude of the force also increases, would also be consistent with the theory.

b If the evidence showed that the magnitude of the force was dependent on the mass of the body submerged instead of its volume, this would challenge the theory. (1 mark)

If the evidence showed that the magnitude of the force was independent of the gravitational field strength, this would also challenge the theory. (1 mark)

c No. The evidence would need to be based on sound scientific principles (i.e. scientific method) to be considered valid. In light of the results, the theory might be modified. (1 mark)

4 Hooke's Law states that the extension of a spring is directly proportional to the load applied to it, which in this case is directly proportional to the mass (as the magnitude of the force of gravity, the load, is directly proportional to the mass). (1 mark)

Thus, if we calculate the ratio of the spring extension to the mass, we should get approximately the same value. (1 mark)

Mass hooked to spring (g)	**Mean spring extension (cm)**	**Ratio of mean spring extension to mass (cm g^{-1})**
2	1.7	0.85
5	4.2	0.84
10	8.9	0.89
15	13.2	0.88
20	17.4	0.87
25	22.0	0.88

This is the case, therefore this experiment confirms Hooke's Law. (1 mark)

5 **a** This evidence suggests that the electric force between the two particles decreases when the distance between them increases. (1 mark)

In addition, when the distance between the particles doubles (from 2 to 4 cm or from 5 to 10 cm), the force between them is not halved, so the relationship is not linear (1 mark) but it is divided by 4, which indicates that the force is inversely proportional to the square of the distance between the charges. (1 mark)

b A scatterplot with a line of best fit should be used, with the magnitude of the force on the y-axis and $\frac{1}{r^2}$ on the x-axis. (1 mark) This would show a direct linear relation, confirming that the force is inversely proportional to the square of the distance between the charges. (1 mark)

12.8 The limitations of investigation methodology and conclusions

12.8 Exercise

1 • Models are approximations of the real world and may not represent what really happens.
- Models cannot include all the details of the processes or the things that they represent.
- Models do have some limits in their accuracy and are often simplified and stylised, using representations.

2 There may be insufficient data or evidence to enable valid conclusions to be made. Producing more data (such as repeating the experiment) and thus providing further evidence may help overcome this limitation.

3 Given the equipment available, and the location in which the experiment is performed, it may be difficult to control all variables.

Even in a school laboratory with air conditioning, the temperature in all parts of the room is not likely to be constant.

4 Some limitations may include the following:
- Not all pieces of equipment may be readily available. For example, a calorimeter may not be obtainable.
- A lack of precision equipment will cause limitations. For example, a micrometer might be needed for accurate measurement of lengths, but less accurate methods of measurement might need to be adopted.
- Although every effort may be made to identify controlled variables and keep them constant throughout the course of an experiment, it is not always possible to identify and control every one of this type of variable. As a result, the data collected may be invalid.

12.8 Exam questions

1 A conclusion can be modified if new evidence is found. (1 mark)

With new technology, new evidence can be found that either supports previous conclusions that scientists have made, or means that scientists may have to modify or change their previous conclusion. (1 mark)

2 D

Models are representations of phenomena and scientific ideas, and are usually linked to theories.

3 Examples could include:
- using slotted masses with smaller masses (such as 10 g or 5 g)
- replacing the weight hanger with a bucket and slowly pouring water in from a measuring cylinder, until the yarn breaks, to determine the mass at which the yarn breaks to the nearest mL (or g)
- repeating the same measurement multiple times (more than two trials).

(1 mark for each of two suitable examples)

4 **a** The student has assumed that the refractive index of air is exactly 1. (1 mark)

The refractive index of air can vary with the temperature, air humidity, pressure and composition, but also with the light wavelength; however, this variation is extremely small and would not affect the student's results, thus this is a fair assumption to make. (1 mark)

b The average value of the refractive index of glass is:

$$\frac{1.661 + 1.520 + 1.536 + 1.510}{4} = 1.557 \text{ (1 mark)}$$

The student's results are coherent with a glass index of 1.52 (less than 3% of difference). (1 mark)

c Sample response:

The student could improve their method by:

- increasing the range and number of incidence angles (e.g. from 5° to 60°, with a measure every 5°) (1 mark)
- repeating their measures in three trials, for instance. (1 mark)

d To easily check the reproducibility of their method, the student could repeat the experiment with a green laser or at a different location (to confirm whether the wavelength of the light, or the composition and temperature of the air, have a noticeable impact on the results). (1 mark)
Alternatively, they could have another student use their method and check whether their results are similar.

5 a Assumptions may include:

- All variables have been controlled.
- The equipment used is precise and accurate.
- Further trials would result in similar results.

(1 mark for each of two assumptions)

b Hooke's Law states that the extension of a spring is directly proportional to the load applied to it (and thus to the mass). However, from the data and graph, it is apparent that this is the case only for masses less than 100 g for the spring used. (1 mark)
This seems to indicate that there is a limit beyond which Hooke's Law is no longer applicable for a spring. (1 mark)

12.9 Conventions of science communication

Sample problem 5

a The conversion factor is $\frac{\text{millilitres}}{\text{microlitres}}$.

$\frac{10^{-3}}{10^{-6}} = 10^3$

Multiply by the value to be converted:
$10^3 \times 12.412\,\text{mL} = 12\,412$
$= 12\,412\,\mu\text{L}$
12.412 millilitres equals 12 412 microlitres.

b The conversion factor is: $\frac{\text{milligram}}{\text{decigram}}$

$\frac{10^{-3}}{10^{-1}} = 10^{-2}$

Multiply by the value to be converted:
$10^{-2} \times 26.153\,\text{mg} = 261\,53$
$= 261.53\,\text{dg}$
26 153 milligrams equals 261.53 decigrams.

c The conversion factor is: $\frac{\text{metre}}{\text{nanometre}}$

$\frac{10^{0}}{10^{-9}} = 10^9$

Multiply by the value to be converted:
$10^9 \times 8.7\,\text{m} = 87\,000\,000\,000$
$= 87\,000\,000\,000\,\text{nm}$
8.7 metres equals 87 000 000 000 nanometres.

Practice problem 5

a Determine the conversion between the units:
$\frac{\text{decigrams}}{\text{kilograms}}$
$= \frac{10^{-1}}{10^3}$
$= 10^{-4}$
Multiply the conversion by the value:
$7823 \times 10^{-4} = 0.7823$
$= 0.7823\,\text{kg}$
7823 decigrams equals 0.7823 kilograms.

b Determine the conversion between the units:
$\frac{\text{microlitres}}{\text{picolitres}}$
$= \frac{10^{-6}}{10^{-12}}$
$= 10^6$
Multiply the conversion by the value:
$213 \times 10^6 = 213\,000\,000$
$= 213\,000\,000\,\text{pL}$
213 microlitres equals 213 000 000 picolitres.

Sample problem 6

7.2136 = five significant figures

8.3 = two significant figures

The least number of significant figures = two significant figures

$\text{Density} = \frac{\text{mass}}{\text{volume}}$
$= \frac{7.2136}{8.3}$
$= 0.8691$

Round 0.8691 to two significant figures = 0.87 g mL^{-1}

The density is 0.87 g mL^{-1}.

Practice problem 6

If we look at the two values provided, 21.1 has three significant figures and 9.762 has four significant figures. We need to work with the least number of significant figures, so our answer will have three significant figures.

The density is calculated by dividing mass by volume: $\rho = \frac{m}{v}$.

$\text{Density} = \frac{9.762}{21.1}$
$= 0.462\,654\,\text{g L}^{-1}$
$= 0.463\,\text{g L}^{-1}$

The density is 0.463 g L^{-1}.

12.9 Exercise

1 A key statement allows for a quick, succinct summary that answers the investigation question, allowing for your findings to be quickly and clearly communicated.

2 a $103\,580\,000\,000\ \mu\text{g} = 1.0358 \times 10^{11}\ \mu\text{g} = 1.0358 \times 10^2$ kg ($1\ \mu\text{g} = 10^{-6}\ \text{g} = 10^{-9}$ kg)

b $12\,540\ \text{km s}^{-1} = 1.2540 \times 10^4\ \text{km s}^{-1} = 1.2540 \times 10^7\ \text{m s}^{-1}$

c $0.0307\ \text{TJ} = 3.07 \times 10^{-2}\ \text{TJ} = 3.07 \times 10^{10}\ \text{J}$ ($1\ \text{TJ} = 10^{12}\ \text{J}$)

3 a 1289.2 has five significant figures.

b 0.08234 has four significant figures. The first two zeros are not at the end of the non-zero digits, nor are they between non-zero digits, so are not included.

c 0.8003 has four significant figures. The zeros between non-zero digits are included. The zero at the start is not included.

d 121.400 has six significant figures. The zeros are at the end are after non-zero digits, so are included.

4 a $3.5 \times 3.1536 \times 10^7 = 1.10376 \times 10^8$ s

b $725 \times 4.4704 \times 10^{-1} = 3.24104 \times 10^3$ m s^{-1}

c $\frac{5}{9}(57 + 459.67) = 287.038\,888\,9$ K

d $2.701 \times 10^{-2} \times 1.6022 \times 10^{-19} = 4.327\,542\,2 \times 10^{-21}$ J

e $7.5 \times 10^{-2} \times 3.6 \times 10^6 = 2.7 \times 10^5$ J

f $3478 \times 9.4607 \times 10^{15} = 3.2904 \times 10^{19}$ m

5 a The introduction is used to: summarise key background concepts; explain any other research that has been done in this field; outline key terms; explain the purpose of your investigation (and why you chose the question); and outline the hypothesis being explored.

b The discussion is used to: outline any trends and patterns in your data; describe any outliers and how they were treated; evaluate your results and link them to theory; discuss any errors, uncertainties and limitations; describe the precision, accuracy, reliability and validity of your investigation; and suggest improvements and future investigations.

c The conclusion acts to sum up your investigation, provide a clear answer to your question, and link back to your aim and hypothesis.

12.9 Exam questions

1 B
The method must be clearly stated so that other scientists can repeat experiments and validate the results.

2 D
The reference section must contain a list of all references cited in the report. This allows the reader to find original data that was not collected in the experiment, or background information to the experiment.

3 C
The results from the experiment are used to make a conclusion. These results may support the hypothesis of the experiment or disprove the hypothesis.

4 a $4.6275 \times 5.39 \times 10^{-44} = 2.49 \times 10^{-43}$ s (1 mark)

b $2486 \times 10^3 \times 4.184 = 1.04 \times 10^7$ J (1 mark)

c $0.345 \times 10^{-3} \times 123.322 = 4.25 \times 10^{-2}$ Pa (1 mark)

d $0.879 \times 149\,597\,871 \times 10^3 = 1.31 \times 10^{11}$ m (1 mark)

5 The abstract is a brief summary of what is in the scientific paper. (1 mark) The abstract contains the purpose of the work, a brief outline of the method used and the major findings of the investigation. (1 mark)

12.10 Review

12.10 Review questions

1 a You could record the voltage, the current or the resistance.

b You would use a voltmeter, ammeter or multimeter.

c You could observe the brightness of globes in a circuit, or connect to a motor to see if it can be rotated by the voltage supplied.

d Observations would be visual; you would use instruments such as globes or motors to observe.

2 a Independent variables are manipulated by the investigator. Dependent variables are measured and are the result of changes to the independent variable.

b Qualitative data is categorical data (e.g. eye colour, type of metal), whereas quantitative data is numerical (e.g. distance, voltage).

c A control group does not have the independent variable applied to it and is a point of comparison, whereas the experimental groups have been affected by the independent variable.

d Primary sources of data are from the initial source of data collection, often through direct investigation. Secondary data is a summary of analysed primary data.

e Uncertainty involves a limit to the precision of a piece of equipment (e.g. is a measurement on a ruler 0.011, 0.012 cm). It cannot be improved by repeating an investigation. An error relates to factors affecting the accuracy of the experiment and how close results are to their true value. These can be improved by repeating an investigation.

3 a We do not count the zeros at the start as they do not follow non-zero digits. The voltage of 0.01430 has four significant figures.

b We need to move the decimal point two places to the right. In scientific notation, 0.01430 is 1.430×10^{-2}.

4 a To determine how the temperature of the hot plate and size of a water droplet affects how long the water stays on the hot plate

b The independent variables are size of droplet (numerical — discrete, as it was based on the number of droplets) and temperature setting (numerical — discrete, as it was set to a specific setting). The dependent variable is the time the droplet lasted (numerical — continuous, as it can be any value above 0 seconds).

c It can be seen that the larger the size of the water droplets, the longer the droplet will last. In regards to hot plate temperature, the higher the temperature setting, the shorter the time the droplet will last.

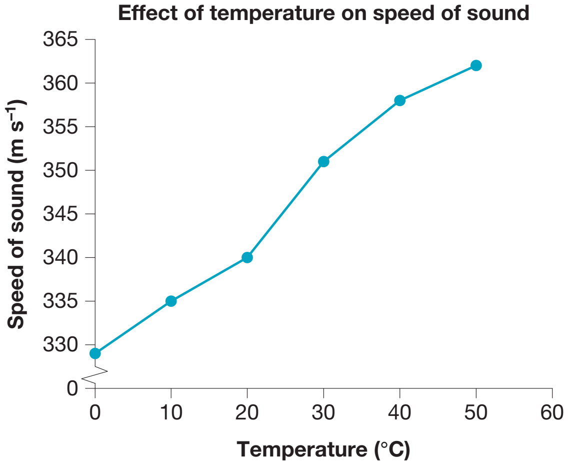
d Chris tested too many variables and did not show how he controlled variables. He also chose a method of selecting water droplet size as number of droplets, which means things such as the shape and diameter of the droplet were not well controlled. He did not record the exact temperature on the hot plate and the different settings may not be equal distances apart; for example, settings 1 and 2 may be 10 °C different, but settings 5 and 6 may be 20 °C different.

e Another independent variable might be different types of water (tap water versus bottled), water with different initial temperatures or water with different surface areas.

f Using a ruler to estimate the size would be appropriate in this case.

5 The graph should have a clear heading, scale and labelled axis.

a



b As temperature increases, the speed of sound also increases. It appears that the rate of increase is most rapid between 20 °C and 30 °C.

c Increasing temperature causes an increase in the speed of sound.

12.10 Exam questions

Section A — Multiple choice questions

1 A
The speed of the serve is being measured, so is the dependent variable.

2 B
Alex's results were the most precise, with all voltages within 0.25 V of each other. Caitlin's results were the most accurate as they were all within 0.25 V of the digital reading.

3 C
Δ represents change — this could be in a quantity such as time, temperature or force.

4 D
Even though the markings are 0.1 cm, you can tell if a measurement is halfway between two markings, so can measure to 0.05 cm. Uncertainty is half of the value, which is 0.025 cm.

5 A
Systematic errors often result in a reading being too high or too low, affecting the accuracy of the reading and how close it to the actual result.

6 C
This data is best shown in a bar graph, as one set of data is qualitative and the other is quantitative.

7 D
An aim outlines the purpose of the investigation, linking the independent and dependent variables, and the hypothesis is a statement that is a testable prediction.

8 A
874 milliseconds is equal to 874 000 microseconds (8.74×10^2). It contains three significant figures.

9 D
Theories are well-supported explanations of phenomena.

10 C
A risk assessment outlines hazards and safety equipment.

Section B — Short answer questions

11 **a** The dependent variable in this investigation is the time it took the movement of the balls in the Newton's cradle to stop. (1 mark)

b Any two of the following:
- The height the balls are pulled back and released from
- The length of the string
- The material the balls are made from
- The force in which the ball is released
- The environmental conditions (e.g. don't conduct one test outside in a windy location and the other inside).

(1 mark for any two suitable variables)

c Measuring equipment that should be used in this experiment are a ruler (to measure the balls) and a stopwatch. (1 mark)
Some factors that may affect the accuracy of the results, using this equipment, include:
- not using the ruler correctly on the curved surface
- inaccuracy with recorded reaction time
- different interpretation of when the balls have stopped
- different precision on rulers (may be to every 1 mm, while another may only be to every 5 mm).

(1 mark for any suitable factor affecting the accuracy)

d An example of a method is as follows:
1. Collect five small metal balls with a diameter of 2 cm.
2. Set up a frame to hang the balls from using two large, sturdy metal loops attached to a base.
3. Cut five pieces of string of length 30 cm.
4. Fold each string in half and attach a metal ball at the fold between the string. Attach each end of the five pieces of string to each side of the frame, ensuring that there is equal spacing between each ball.
5. Pull the ball from the furthest end back 5 cm and release, beginning the timer.
6. Record how long it takes for the movement to stop.
7. Repeat this two times and take an average.
8. Repeat steps 1 to 7 using balls of diameter 4, 6 and 8 cm.

To obtain full marks, you must:
- have a reproducible method, with clear steps and specific quantities (1 mark)
- explain how data is being collected — the data must be appropriate to the dependent variable (1 mark)
- show some form of repetition in the experiment (1 mark)
- make sure variables are controlled throughout (1 mark).

12 **a** To investigate how the current in a circuit is affected by the length of a resistor (1 mark)

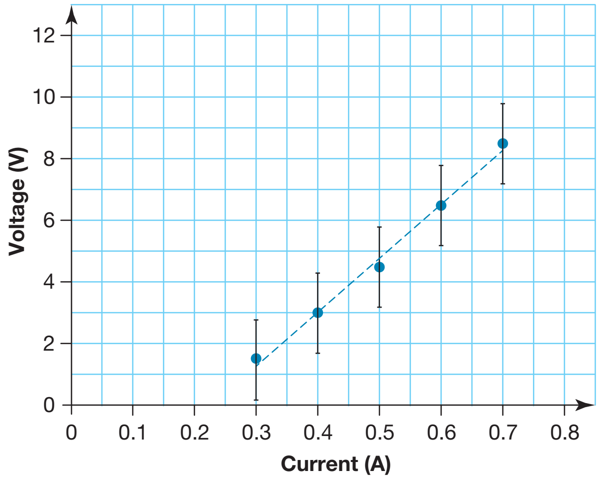
b Responses will vary; one example of a hypothesis is as follows:
If the length of a resistor in a circuit is increased, then the current will decrease, due to the increased resistance.
(1 mark; must clearly link the independent variable and dependent variable, and must be tentative and testable)

c Qualitative data could be whether the current increases or decreases. It might also be that another device (e.g. a light globe) placed in the circuit provides qualitative data through the brightness of the globe. (1 mark) Quantitative data will be the current that is recorded on the ammeter. (1 mark)

d Factors that may lead to different results by students include:
- differences in the use of the ammeter (one student may use the smaller scale and one may use the larger scale)
- differences in the material of the resistor
- differences in how the circuit is set up
- differences in the power supply provided.

(1 mark for each of two suitable factors)

13 a **The relationship between voltage and current in a light globe**



For full marks you need to have:
- clearly plotted the points (1 mark)
- shown correct scale and labelling on both axes (1 mark)
- clearly shown the relationship between points using a line of best fit (1 mark)
- shown error bars on the graph (1 mark).

b Some trends shown in the data include the following:
- There was a positive correlation (upwards trend), in which the voltage increases as current increases.
- Data was all very close to the line of best fit, but the voltage at 0.5 A was slightly lower than expected, and at 0.7 A and 0.3 A was slightly higher than expected; however, they were well within the margins of the error bars.
- The voltage increases approximately 1.75 V for every 0.1 A.
- The line of best fit suggests the data won't pass through 0, which seems unexpected.

(1 mark for each of two of the above or similar)

c The gradient of the line of best fit is:

$$\begin{aligned}\text{Gradient} &= \frac{\text{rise}}{\text{run}}\\ &= \frac{y_2 - y_1}{x_2 - x_1}\\ &= \frac{6.5 - 4.8}{0.6 - 0.5}\\ &= \frac{1.7}{0.1}\\ &= 17\end{aligned}$$

(1 mark for working, 1 mark for the result. Different values may be used, and accuracy and results may vary depending on the location of the line of best fit.)

14 a Errors that may have occurred include:
- systematic errors such as an incorrectly calibrated scale, stopwatch or a ruler that was inaccurate (markings not correct)
- random errors such as the mass being incorrectly recorded (or the scales not being set to 0 before use), the speed relying on reaction times, and different interpretations of the point just before and just after the collision.

(1 mark for an example of an error; category of error not required)

A way to avoid the error may include one of the following:
- *Faulty scales*. Make sure all scales have been calibrated, or record the mass using three different scales and get an average.
- *Faulty stopwatches*. Make sure all stopwatches have been calibrated and repeat the experiment multiple times.
- *Imprecise ruler*. Record using three different rulers and get an average.
- *Reaction times*. Repeat the experiment numerous times.
- *Recording the mass*. Ensure the scales have been set to zero.

(1 mark for listing an appropriate way to avoid the error)

b The total momentum is:

$$\begin{aligned}p_{\text{total}} &= p_{\text{car 1}} + p_{\text{car 2}}\\ &= 35.1 + 35.23\\ &= 70.33\ \text{g cm s}^{-1}\ \text{(1 mark)}\\ &= 70.3\ \text{g cm s}^{-1}\end{aligned}$$

(1 mark; must be to 1 decimal place/3 significant figures)

c According to the Law of Conservation of Momentum, the momentum before the collision should equal the momentum after the collision, so to be accurate, the data should show this. (1 mark) Joe had a 'before' momentum of 150.6 g cm s^{-1} and an 'after' momentum of 72.24 g cm s^{-1}, which has a significant difference, whereas Paul's 'before' momentum was 70.2 g cm s^{-1} and his 'after' momentum was 70.3 g cm s^{-1}, showing much more accurate results. (1 mark)

d It is important to identify the units to be converted. Seconds remains as the unit for both; however, grams is changing to kilograms and centimetres is changing to metres.

This can be done in two steps. (You may combine the steps together or do them in the opposite order.) Determine the conversion between the units:

$$\frac{\text{centimetres}}{\text{metres}} = \frac{10^{-2}}{10^{0}} = 10^{-2}$$

Multiply the conversion by the value:

$35.14 \times 10^{-2} = 0.3514$

$35.14\ \text{cm s}^{-1} = 0.3514\ \text{g m}^{-1}$ (1 mark)

We now need to convert grams into kilograms:

$$\frac{\text{grams}}{\text{kilograms}} = \frac{10^{0}}{10^{3}} = 10^{-3}$$

Multiply the conversion by the value:

$0.3514 \times 10^{-3} = 0.000\,351\,4$

$0.3514\ \text{g m s}^{-1} = 3.514 \times 10^{-4}\ \text{kg m s}^{-1}$ (1 mark)

15 a The student hasn't shown the dependent variable in their hypothesis, so it is not testable. (1 mark) An improved hypothesis might be as follows:

If aluminium foil, paper and wool are used as an insulating material on water, then aluminium foil will cause the temperature of the water to show the smallest decrease as it is the best insulating material. (1 mark)

b The experiment being conducted over three days leads to a loss of some controlled variables. (1 mark) This means that it is not a fair test, and other factors could have resulted in the change of temperature of the water (e.g. the room temperature could be very different over the three days, which would lead to errors in results). (1 mark)

c The tolerance of a device is half the smallest measurement (0.5 °C), therefore it is 0.25 °C. (1 mark) This means that there is a level of uncertainty with the data and the results obtained by each individual are not as accurate as they could be; for example, at 25 minutes, the temperatures for paper and wool may be 40.24 °C and 39.25 °C, or they may be 39.75 °C and 39.74 °C, which is a significant difference. (1 mark)

d Some limitations include:

- the inability to exactly interpret boiling point (as it is subjective)
- the inability to completely control room temperature
- the lack of accuracy in the recorded results
- the source of the boiling water not being consistent as it was done over multiple days
- the lack of repetition in the method
- the starting temperature of the water was not consistent

(1 mark for each of two of the above)

e The most appropriate graph to use would be a line graph with the three types of variables superimposed on the same graph. (1 mark) This is the best choice as both the temperature and time are quantitative, and/or line graphs are usually used when exploring a change over time. (1 mark)

f Sample response:

It was found that aluminium provides the greatest insulation for boiling water, leading to the slowest decrease in temperature (25 °C drop over 25 minutes). This is followed by wool (55.5 °C drop over 25 minutes) and finally by paper (58.5 °C drop over 25 minutes). (1 mark)
The hypothesis in this investigation was supported, as aluminium worked best in insulating water. (1 mark)

Unit 1 — Area of Study 1 Review

Practice examination

Section A — Multiple choice questions

1 A
Temperature is a measure of the average translational kinetic energy of particles.

2 D
Water freezes at 0 °C, or 273.15 K.

3 C
Reading from the graphs, the period is $T = 4$ s, and thus the frequency is $f = \frac{1}{T} = 0.25$ Hz and the wavelength is $\lambda = 2.5$ m.
Thus the speed is
$$v = f\lambda = \frac{\lambda}{T} = 0.625 \text{ m s}^{-1}$$

4 C
Using Snell's Law:
$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{1.33 \times \sin(40)^\circ}{\sin(30)^\circ} = 1.71 \text{ (to 2 decimal places)}$$

5 C
Particles are closer together in a liquid than in a gas, and they are less energetic in a liquid than in a gas.

6 B
Since the specific heat capacity of water is higher, the same amount of heat input would result in a smaller temperature rise for water than that for aluminium.

7 D
J kg^{-1}could be used as a unit for *L*. All the other options are incorrect.

8 C
The energy required is related to the change in temperature by:
$$Q = mc_{\text{Al}}\Delta T = 2.2 \times 0.92 \times 10^3 \times (44 - 22) = 4.45 \times 10^4 \text{J} = 44.5 \text{ kJ}$$

9 D
The energy supplied is related to the change in temperature by $Q = mc_{\text{w}}\Delta T$.
Thus:
$$\Delta T = \frac{Q}{mc_w} = \frac{67.0 \times 10^3}{750 \times 10^{-3} \times 4200} = 21.3\ ^\circ\text{C}$$
The final temperature:
$$T_{\text{f}} = T_{\text{i}} + \Delta T_{\text{f}} = 18.0 + 21.3 = 39.3\ ^\circ\text{C}$$

10 C
As this process involves condensation, we need to use the specific latent heat of vaporisation for water in our calculations.
$$Q = mL = 1.3 \times 2.3 \times 10^6 = 3.0 \times 10^6 = 3.0 \text{ MJ}$$

11 B
As this process involves melting, we need to use the specific latent heat of fusion for water in our calculations.
$$Q = mL$$
$$\Rightarrow m = \frac{Q}{L} = \frac{37 \times 10^9}{3.3 \times 10^5} = 1.12 \times 10^5 \text{ kg}$$

12 B
According to the Stefan–Boltzmann Law, doubling the temperature of the hotplate would increase the power by $(2)^4 = 16$. The new power output is therefore $16 \times 1200 = 19\,200$ W.

13 D
According to Wien's Law, the wavelength of the peak radiation is given by:
$$\lambda_{\text{max}} = \frac{k}{T} \text{(where } T \text{ must be in Kelvin)} = \frac{2.90 \times 10^{-8}}{2200 + 273} = 1.17 \times 10^{-6} \text{ m}$$

14 A
Using Wien's Law, the wavelength of the peak radiation is emitted from a surface with a temperature of:
$$T = \frac{k}{\lambda_{\text{max}}} = \frac{2.90 \times 10^{-8}}{6.50 \times 10^{-6}} = 446 \text{ K}$$
In degrees Celsius: $446 - 273 = 173$ °C.

15 B
Using Snell's Law:
$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{1.00 \times \sin(50)^\circ}{\sin(31.5)^\circ} = 1.479$$
Thus the speed of light in this material is:
$$v = \frac{3.00 \times 10^8}{1.479} = 2.03 \times 10^8 \text{ m s}^{-1}$$

16 C
Nitrous oxide, ozone and chlorofluorocarbon are all greenhouse gases. All other options contain at least one non-greenhouse gas.

17 D
A mirage happens when the ground is very hot and the air is cool.

18 B
Rising ocean temperatures increase evaporation, which increases the amount of water vapour. A greenhouse gas is an example of a positive feedback mechanism that enhances the effect of global warming on climate change. Option A describes a negative feedback mechanism. Options C and D are currently considered implausible.

19 D
Fluids (such as liquids and gases) that are warmer are also more buoyant, and therefore rise to displace colder and denser fluid above.

20 B
This absorption of infrared radiation is due to the fact that molecules with three or more atoms bend and stretch and can better absorb infrared radiation. The explanation in the other responses do not make sense.

Section B — Short answer questions

21 **a** According to the kinetic theory of matter, temperature is a measure of the average translational kinetic energy of particles. (1 mark) Since the beef and chicken are at the same temperature, their particles have the same average translational kinetic energy; hence, they are at thermal equilibrium. (1 mark)

b Energy is transferred from a region of high temperature (the air in the oven) to a region of lower temperature (the pieces of meat to cook) until both regions reach the same temperature. (1 mark) Since the thermometers steadily indicated 100 °C for both pieces of meat after some time, they may be considered to be in thermal equilibrium with the air in the oven near them. (1 mark) Hence, it is likely that the temperature of the air near them will be 100 °C. (1 mark)

22 **a** Using Wien's Law:

$$T = \frac{k}{\lambda}$$
$$= \frac{2.90 \times 10^{-3}}{501.7 \times 10^{-9}}$$
$= 5780$ K to the nearest kelvin (1 mark)

b Heat from the Sun is transferred to Earth through the vacuum of space by the process of radiation. (1 mark)

c This is an illustration of convection. (1 mark)

23 **a** The heat lost by the water is:

$$Q_{\text{lost}} = m_{\text{w}} c_w \Delta T$$
$$= m_{\text{w}} c_w (T_{\text{f}} - T_{\text{i}})$$
$$= 0.25 \times 4.2 \times 10^3 (100 - 90)$$
$$= 1.05 \times 10^4 \text{ J}$$

(1 mark for correctly substituting in the equation; 1 mark for the correct answer)

b The heat gained by the aluminium is the same as the heat lost by the water:

$$Q_{\text{gain}} = m_{\text{Al}} c_{\text{A1}} \Delta T$$
$$= m_{\text{Al}} c_{\text{A1}} (T_{\text{f}} - T_{\text{i}})$$
$= 1.05 \times 10^4$ J (1 mark)

Rearranging the equation, we get:

$$m_{\text{A1}} = \frac{Q_{\text{gain}}}{c_{\text{A1}} \Delta T}$$
$$= \frac{1.05 \times 10^4 \text{ J}}{0.92 \times 10^3 (90 - 23)}$$
$= 0.170$ kg (1 mark)

24 Wien's Law refers to electromagnetic radiation emitted from hot objects. (1 mark) As LEDs do not produce light due to high temperatures, Wien's Law does not apply to them. (1 mark)

25 The temperature of the heating element has increased by a factor of $\frac{900}{850} = 1.059$. Using Stefan–Boltzmann Law, the higher temperature would increase the power by $1.059^4 = 1.257$. (1 mark) The new power output is therefore $2250 \times 1.257 = 2828$ W (1 mark).

26

Wavelength (m)	Electromagnetic wave type
a. 1.0×10^{-14}	Gamma rays (1 mark)
b. 1.0×10^{-2}	Microwaves (1 mark)
c. 1.0×10^{4}	Radio waves (1 mark)
d. 1.0×10^{-8}	Ultra-violet rays (1 mark)

Unit 1 — Area of Study 2 Review

Practice examination

Section A — Multiple choice questions

1 C
Isotopes have the same number of protons but have different numbers of neutrons.

2 D
The strong nuclear force is the attractive force holding the nucleons together.

3 B
After 48 hours, which is 8 half-lives, the number of remaining nuclei is $64\,000 \times \left(\frac{1}{2}\right)^8 = 250$.

4 C
In β^- decay, a neutron transforms to a proton, and an electron is released.

5 C
Each alpha decay removes four nucleons from the nucleus. Therefore the mass number of element X is
$226 - 2 \times 4 = 218$.

6 B
The absorbed dose is the ratio of the energy absorbed to the mass and is expressed in Gy (or J kg^{-1}).

7 D
The equivalent dose is equal to the product of the absorbed dose by the quality factor and is expressed in sieverts.

8 B
$\frac{32}{8} = 4$
Therefore it takes 4 half-lives, or 2 hours, for the activity to drop from 32 MBq to 8 MBq.

9 C
The half-life of the radioisotope should be comparable to the time taken to carry out the medical examination, to minimise the dose to the patient.

10 D
$\frac{48}{6} = 8$
There is 0.05 mg of radioisotope after 8 half-lives,
thus the initial amount was $0.05 \times 2^8 = 12.8$ mg.

11 A
The energy absorbed is the product of the equivalent dose by the mass.

12 B
The energy absorbed by the technician over a year is
$75 \times 2 \times 10^{-3} = 0.375$ J and 1 MeV $= 1.6 \times 10^{-13}$ J.
Thus, the number of β particles is $\frac{0.375}{1.6 \times 10^{-13}} = 2.34 \times 10^{12}$ each year.

13 A
Using the mass-energy equivalence equation:

$$\begin{aligned} E &= mc^2 \\ &= 2 \times 9.1 \times 10^{-31} \times \left(3.0 \times 10^8\right)^2 \\ &= 1.6 \times 10^{-13}\ \text{J} \end{aligned}$$

14 D
1 MeV is equivalent to $10^6 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-13}$ J.
The energy released is therefore $\frac{3.0 \times 10^{-10}}{1.6 \times 10^{-13}} = 1875$ MeV.

15 C
The atomic number is 2, therefore there are 2 protons. The mass number is the sum of protons and neutrons. As the mass number is 4 and there are 2 protons, there must be 2 neutrons.

16 A
In beta decay, a neutron forms an electron and a proton, with the electron being released in the process. The mass number of the atom would remain unchanged and there would be one extra proton after decay. The full beta decay equation is therefore ${}^{90}_{38}\text{Sr} \rightarrow {}^{0}_{-1}\beta + {}^{90}_{39}\text{X}$.

17 C
12.5% is one-eighth of the initial amount. Therefore:

$$\begin{aligned} A_n &= A_0 \times \frac{1}{2^n} \\ \frac{A_n}{A_0} &= \frac{1}{2^n} \\ 12.5\% &= \frac{1}{8} \\ \frac{1}{8} &= \frac{1}{2^n} \\ \Rightarrow n &= 3 \end{aligned}$$

Therefore the total elapsed time would be $3 \times 29 = 87$ years.

18 C
Since there are two nucleons in each deuteron nucleus, from the mass number of 2, the total binding energy is
$2 \times 1.11 = 2.22$ MeV.

19 B
Using the mass–energy equivalence equation:

$$\begin{aligned} E &= mc^2 \\ \Rightarrow m &= \frac{E}{c^2} \\ &= \frac{1.16 \times 10^{-12}}{\left(3 \times 10^8\right)^2} \\ &= 1.3 \times 10^{-29}\ \text{kg} \end{aligned}$$

20 D
Using the half-life equation:

$$A_n = A_0 \times \frac{1}{2^n}$$

$$\frac{A_n}{A_0} = \frac{1}{2^n}$$

Substituting in the values:

$$\begin{aligned} \frac{A_n}{A_0} &= \frac{1}{2^n} \\ &= \frac{15\,000}{240\,000} \\ &= \frac{1}{16} \\ &= \frac{1}{2^4} \\ \Rightarrow n &= 4 \end{aligned}$$

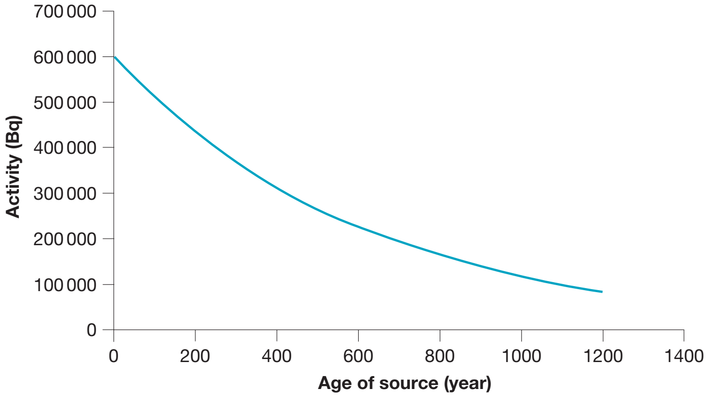
There have been 4 half-lives over the 32-hour period.
Therefore, the half-life is $\frac{32}{4} = 8$ hours.

Section B — Short answer questions

21 a The transport takes $\frac{1}{6}$ of the half-life, thus the initial amount should be $60 \times 2^{\frac{1}{6}} = 67.35$ mg. [1 mark] The hospital should order at least 68 mg. [1 mark]

b After 6 half-lives, $\left(\frac{1}{2}\right)^6 \simeq 1.56\%$ of the initial amount remains and after 7 half-lives, $\left(\frac{1}{2}\right)^7 \simeq 0.78\%$ of the initial amount remains. [1 mark] Therefore it would take between 16.5 days and 19.25 days for 99% of the initial amount of molybdenum-99 to decay. [1 mark]

22 a Reading from the graph, the time it takes for the sample to go from 6.0×10^5 Bq to 3.0×10^5 Bq is approximately (420 – 0) = 420 years. Looking at 3.0×10^5 Bq to 1.5×10^5 Bq, the time is approximately (860 – 420) = 440 years. [1 mark] Therefore, a range of between 400 and 450 years is acceptable [1 mark].



Note: Students may choose to show the points directly on the graph instead of describing them. The points on the graph must be consistent with the answer.

b Alpha decay refers to when an unstable nucleus (in this case, americium-241) ejects a relatively large particle known as an alpha particle (which consists of two protons and two neutrons). [1 mark]

c ${}^{241}_{95}\text{Am} \rightarrow {}^{4}_{2}\alpha + {}^{237}_{93}\text{Np}$

(1 mark for a correct representation of the alpha decay and alpha particle; 1 mark for the correct notation of neptunium-237. Deduct 1 mark for any errors; for example, incorrect notation of americium or alpha particle.)

23 a x is the atomic number of zirconium, and is computed from the difference between the atomic number of plutonium and xenon. [1 mark]

$x = 94 - 54 = 40$ [1 mark]

b y can be computed by subtracting the known total mass number of the reaction products from the total mass number of plutonium and the free neutron. [1 mark]

$y = (239 + 1) - (134 + 103) = 3$ [1 mark]

c Using the mass–energy equivalence equation:

$E = mc^2$

$= 3.1 \times 10^{-28} \times (3.0 \times 10^8)^2$

$= 2.79 \times 10^{-11}$ J [1 mark]

Converting this energy to electron volts:

$\frac{2.79 \times 10^{-11}}{1.6 \times 10^{-19}} = 174 \times 10^6$

$= 174$ MeV [1 mark]

24 a Using $E = mc^2$:

$m = \frac{E}{c^2}$

$\Rightarrow m = \frac{17.56 \times 10^6 \times 1.6 \times 10^{-19}}{(3 \times 10^8)^2}$

$= 3.122 \times 10^{-29}$ kg [1 mark]

b The number of fusion events per second is:

$n = \frac{4.3 \times 10^9}{3.122 \times 10^{-29}}$

$= 1.4 \times 10^{41}$ [1 mark]

c Each second, there are 1.4×10^{41} fusion events, each releasing 17.56 MeV. [1 mark] Therefore, each second the Sun releases $1.4 \times 10^{41} \times 17.56 \times 10^6 \times 1.6 \times 10^{-19} = 3.9 \times 10^{29}$ J of energy. [1 mark]

This corresponds to the energy released by burning 1.3×10^{19} tonnes of coal. [1 mark]

UNIT 1 — Area of Study 3 Review

Practice examination

Section A — Multiple choice questions

1 C

The relationship between the emf, energy supplied and number of charge is:

$$V = \frac{E}{Q}$$

$$\Rightarrow E = VQ$$
$$= 15 \times 375$$
$$= 5625 \text{ J}$$

2 B

$$R = \frac{V}{I}$$
$$= \frac{12}{2.5}$$
$$= 4.8\ \Omega$$

3 D

The energy consumed is determined from the power of the motor and the time it runs, $E = Pt$. Since the power of the motor is also the power supplied by the battery; that is, $P = IV$:

$$E = IVt$$
$$= 2.5 \times 12 \times 30$$
$$= 900 \text{ J}$$

4 C

Combining power, $P = IV$, and Ohm's law, $V = IR$, we obtain $P = I^2R$. Substituting in the values:

$$P = I^2R$$
$$= 9^2 \times 8$$
$$= 648 \text{ W}$$

5 B

Reading off the graph, 8 kΩ corresponds to 25 °C.

6 C

Since the voltage across the diode is 3.2 V at switch-on, the voltage across the resistor is:

$$V_R = 9 - 3.2$$
$$= 5.8 \text{ V}$$

The current through the resistor is:

$$I = \frac{V_R}{R}$$
$$= \frac{5.8}{290}$$
$$= 0.02 \text{ A}$$
$$= 20 \text{ mA}$$

7 C

Since this is an ohmic device, the resistance is constant across all voltages. Therefore:

$$R = \frac{V}{I}$$
$$= \frac{28.8}{3.8}$$
$$\approx 7.5\ \Omega$$

Therefore, C is correct, as the resistance is closest to 7.5 Ω, compared to the other options.

8 B

According to the circuit rule for conservation of charge, the sum of currents entering any point in the circuit must equal the sum of currents leaving that point. Since the outgoing currents total 0.8 + 4.8 = 5.6 A, the current entering is 1.2 + I = 5.6, leading to I = 4.4 A.

9 B

According to the circuit rule for conservation of energy, the sum of voltage drops in the circuit must equal the sum of voltage gains (emfs) in the circuit. The voltage gain is 9 V, thus the remaining voltage drop is found from 9 = 2.5 + 3.5 + V, leading to V = 3 V.

10 A

The equivalent resistance of parallel resistances is calculated by:

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$
$$= \frac{1}{7} + \frac{1}{12}$$
$$= 0.2262$$

$$\Rightarrow R_{\text{equivalent}} = \frac{1}{0.2262}$$
$$= 4.42\ \Omega$$

11 D

Parallel connections of two like resistors is equivalent to half the resistance:

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{R}{2}$$

Thus, two 12-Ω resistors is equivalent to 6 Ω, which when added to two 7-Ω resistors gives an equivalent resistance of:

$$R_{\text{equivalent}} = 6 + 7 + 7$$
$$= 20\ \Omega$$

12 B

The equivalent resistance of the parallel resistances is given by:

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{16} + \frac{1}{16}$$
$$= \frac{1}{8}$$

$$\Rightarrow R_{\text{equivalent}} = 8\ \Omega$$

$$I = \frac{V}{R}$$
$$= \frac{12}{8}$$
$$= 1.5 \text{ A}$$

13 B

The equivalent resistance of the series resistances is:

$$R_{\text{equivalent}} = \frac{V}{I}$$
$$= \frac{12}{1.5}$$
$$= 8\ \Omega$$

As the equivalent resistance is the sum of all series resistances:

$R_{\text{equivalent}} = 3 + R = 8\ \Omega$

$\Rightarrow R = 5\ \Omega$

14 C

The equivalent resistance of the parallel resistances is:

$$R_{\text{equivalent}} = \frac{V}{I} = \frac{12}{2.4} = 5\ \Omega$$

The equivalent resistance of the parallel resistances is calculated as:

$$\frac{1}{R_{\text{equivalent}}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

$$\Rightarrow R_{\text{equivalent}} = \frac{R}{2} = 5$$

$$\Rightarrow R = 10\ \Omega$$

15 B

The output voltage $V_{\text{out}} = \left(\frac{R_2}{R_1 + R_2}\right) V_{\text{in}}$.

Substituting the known values in, we obtain:

$$6 = \left(\frac{4}{R+4}\right) \times 9$$

$$6(R+4) = 4 \times 9$$

$$R + 4 = \frac{36}{6}$$

$$\Rightarrow R = 6 - 4 = 2\ \text{k}\Omega$$

16 D

Root mean square is the mathematical formulation that relates the heating effect of an alternating (and therefore unsteady) current to that of a steady, direct current. An alternating current supply with a peak voltage of 325 V will provide the same effect as a 230-V direct current supply. This means that responses A, B and C are incorrect.

17 B

The active wire is brown. Red is the old colour for the active wire. Green and yellow is the colour for the earth wire. Blue is the neutral wire.

18 B

The power rating of the appliance in watts must be converted into kilowatts first. Thus, the energy consumed by the dishwasher in kW h is $1.8 \times 2.5 = 4.5$ kW h. The cost is then $4.5 \times 30 = 135$ cents.

19 A

Option B is the operating principle of the fuse; option C is the operating principle of the earth wire; option D is the operating principle of a circuit breaker.

20 D

Option A is the resistance of a person immersed in a bath; option B is the resistance of a dripping wet hand; option C is the resistance of a moist hand.

Section B — Short answer questions

21 **a** An open switch connected to a battery circuit will have the full potential of the battery across it: $V_{\text{switch}} = 4.2$ V. [1 mark]

b The current is the amount of charge flowing in the circuit each second: $I = 1.4$ A. [1 mark]

c The emf of the battery is the amount of energy that the battery supplies to each coulomb of charge; hence, this battery in the circuit is supplying 4.2 J C^{-1}. [1 mark]

d A closed switch has no potential difference across it as its electrical resistance is very low. Therefore, $V_{\text{switch}} = 0$ V. [1 mark]

e The number of electrons flowing through the battery each second is equal to $6.24 \times 10^{18} \times I$.

$$n_e = 6.24 \times 10^{18} \times I = 6.24 \times 10^{18} \times 1.4 = 8.7 \times 10^{18} \text{ electrons}$$ [1 mark]

f The power of the battery is:

$$P = IV = 1.4 \times 4.2 = 5.9\ \text{W}$$ [1 mark]

g The energy transformed in 3 minutes is:

$$E = Pt = VIt = 4.2 \times 1.4 \times 3 \times 60 = 1058 \simeq 1.1 \times 10^3\ \text{J}$$ [1 mark]

h The resistance of the globe is:

$$R = \frac{V}{I} = \frac{4.2}{1.4} = 3.0\ \Omega$$ [1 mark]

22 **a** The equivalent resistance of two light globes in parallel is calculated first:

$$\frac{1}{R_{\text{equivalent}\,||}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

$$\Rightarrow R_{\text{equivalent}\,||} = \frac{6}{5} = 1.2\ \Omega$$ [1 mark]

This is then added to the 6-Ω light globe in series, to give an equivalent resistance of $R_{\text{equivalent}} = 7.2\ \Omega$. [1 mark]

b The current equals voltage divided by resistance:

$$I = \frac{V}{R} = \frac{15}{7.2} = 2.1\ \text{A}$$ [1 mark]

This is the current flowing through the battery and through the 6-Ω light globe, as it is in series with the battery. [1 mark]

c Since the 3-Ω light globe is one of the two parallel resistances, we calculate the potential difference across it using the equivalent resistance found in part a; that is, $R_{\text{equivalent}\,||} = 1.2\,\Omega$. Thus, the potential difference across the 3-Ω light globe is:

$$V = \frac{R_{\text{equivalent}\,||}}{R_{\text{equivalent}}} \times V_{\text{supply}} \text{ [1 mark]}$$

$$= \frac{1.2}{1.2 + 6} \times 15$$

$$= 2.5 \text{ V [1 mark]}$$

d The current flowing through the 3-Ω light globe is found from:

$$I_3 = \frac{V_3}{R_3}$$

$$= \frac{2.5}{3}$$

$$= 0.83 \text{ A [1 mark]}$$

e $P = VI$

$$= 2.5 \times 0.83$$

$$= 2.08 \text{ W [1 mark]}$$

23 **a** The earth wire is green and yellow. [1 mark] It acts as a safety feature by connecting the metal chassis of an appliance to the earth, which is at 0 V. This provides a lower-resistance conducting path to the earth rather than through a person, causing a fuse to blow if the active wire ever touches the metal casing of the appliance and thus preventing electric shock. [1 mark]

b

$$I = \frac{P}{V}$$

$$I_{\text{fridge}} = \frac{500}{230}$$

$$= 2.17 \text{ A}$$

$$I_{\text{coffee machine}} = \frac{800}{230}$$

$$= 3.48 \text{ A}$$

$$I_{\text{fan}} = \frac{1000}{230}$$

$$= 4.35 \text{ A}$$

Total before toaster = 10 A

$$I_{\text{toaster}} = \frac{700}{230}$$

$$= 3.04 \text{ A}$$

Total after toaster = 13.04 A

Therefore, the fuse will blow [1 mark], as the total current after the toaster is added (13.04 A) exceeds the 10.5-A fuse [1 mark].

UNIT 2 — Area of Study 1 Review

Practice examination

Section A — Multiple choice questions

1 B

The average velocity is calculated from the total displacement of the body, which is 25 – 7 = 18 m, and the total travel time of 8 + 2 + 2 = 12 s.

$$\begin{aligned} \boldsymbol{v} &= \frac{s}{t} \\ &= \frac{18}{12} \\ &= 1.5\,\text{m s}^{-1} \end{aligned}$$

2 A

The average speed is calculated from the total distance travelled by the body, which is 25 + 7 = 32 m, and the total travel time of 8 + 2 + 2 = 12 s.

$$\begin{aligned} \text{speed} &= \frac{d}{t} \\ &= \frac{32}{12} \\ &\approx 2.7\,\text{m s}^{-1} \end{aligned}$$

3 C

When considering average acceleration, it is important to note that acceleration is a vector and therefore the direction of the vector is significant.

$$\begin{aligned} \boldsymbol{a} &= \frac{\Delta \boldsymbol{v}}{\Delta t} \\ &= \frac{\boldsymbol{v}_\text{f} - \boldsymbol{v}_\text{i}}{\Delta t} \\ &= \frac{3 - (-5)}{0.8} \\ &= 10\,\text{m s}^{-2}\ \text{north} \end{aligned}$$

4 B

The gradient of a velocity-versus-time graph is the acceleration. The gradient of the graph in the first two seconds is linear, indicating a constant acceleration. Thus, taking the velocity at t = 2 s, $\boldsymbol{v}$ = 8 m s^{-1}:

$$\begin{aligned} \boldsymbol{a} &= \frac{\boldsymbol{v}}{t} \\ &= \frac{8}{2} \\ &= 4\,\text{m s}^{-2} \end{aligned}$$

5 A

The area under a velocity-versus-time graph is the displacement of the body; in this case the distance is equivalent to the displacement. The area under this graph can be calculated as the sum of the triangle in the first two seconds:

$$A = \frac{1}{2} \times 2 \times 8 = 8\,\text{m}$$

and the rectangle in the next two seconds:

$A = 2 \times 8 = 16\,\text{m}$

This gives a total of 24 m.

6 B

A body slows down when the sign of its velocity is opposite to the sign of its acceleration, which is indicated by II and III.

7 C

The formula that relates the initial velocity, the final velocity, distance and acceleration is:

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$$

$$\begin{aligned} \Rightarrow \boldsymbol{a} &= \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{s}} \\ &= \frac{9^2 - 3^2}{2 \times 24} \\ &= 1.5\,\text{m s}^{-2} \end{aligned}$$

8 B

Assuming that the acceleration is constant, the formula that relates to the initial velocity, the final velocity, distance and time is:

$$\boldsymbol{s} = \frac{1}{2}(\boldsymbol{u} + \boldsymbol{v})t$$

$$\begin{aligned} \Rightarrow t &= \frac{2\boldsymbol{s}}{\boldsymbol{u} + \boldsymbol{v}} \\ &= \frac{2 \times 24}{3 + 9} \\ &= 4\,\text{s} \end{aligned}$$

9 B

According to Newton's First Law of Motion, unless acted on by a net force, a body continues in its uniform straight-line motion. Here, the car experiences a force by the barrier, which slows it down; however, the unrestrained passenger experiences no force until they hit the windscreen.

10 C

As the lift is at constant speed, the net force on the man is zero. The force of gravity, acting downwards, on him is 750 N, and this is balanced by the force exerted on him by the floor of the lift, which is acting upwards.

11 D

The net force acting on the car can be calculated using Newton's Second Law:

$$\begin{aligned} \boldsymbol{F} &= m\boldsymbol{a} \\ &= 800 \times 2.5 \\ &= 2000\,\text{N} \end{aligned}$$

12 D

The net force acting on the truck and its load is $\boldsymbol{F} = (m_\text{truck} + M)\boldsymbol{a}$. Substituting the known values:

$$9600 = (3600 + M) \times 2$$

$$9600 = 7200 + 2M$$

$$\begin{aligned} \Rightarrow M &= \frac{9600 - 7200}{2} \\ &= 1200\,\text{kg} \end{aligned}$$

13 C

According to Newton's Third Law of Motion:

$\boldsymbol{F}_{\text{on } J \text{ by } K} + \boldsymbol{F}_{\text{on } K \text{ by } J} = 0$, or $-\boldsymbol{F}_{\text{on } J \text{ by } K} = \boldsymbol{F}_{\text{on } K \text{ by } J}$

14 A
The forces on the window-washer are the force of gravity, acting downwards, and the force of the platform, acting upwards, with the net force accelerating the window-washer upwards:

$\boldsymbol{F}_\text{g} + \boldsymbol{F}_\text{platform} = \boldsymbol{F}_\text{net}$
$m\boldsymbol{g} + \boldsymbol{F}_\text{platform} = m\boldsymbol{a}$
Substituting values in:
$60 \times (-9.8) + \boldsymbol{F}_\text{platform} = 60 \times 1.2$
$\Rightarrow \boldsymbol{F}_\text{platform} = 660\,\text{N upwards}$
According to Newton's Third Law of Motion, the force exerted on the platform by the window-washer is equal in magnitude but opposite in direction to the force exerted on the window-washer by the platform; hence, the answer is 660 N down.

15 B
The normal force acts on the box in the direction perpendicular to the surface of the ramp; thus, it is equal in magnitude to the component of the force due to gravity that is perpendicular to the slope.

$\boldsymbol{F}_\text{N} = \boldsymbol{F}_\text{g}\cos(40°)$
$= 98\cos(40°)$
$\approx 75\,\text{N}$

16 A
The friction force acts on the box in the direction parallel to the surface of the ramp; thus, it is equal in magnitude to the component of the force due to gravity that is parallel to the slope.
$\boldsymbol{F}_\text{f} = \boldsymbol{F}_\text{g}\sin(40°)$
$\boldsymbol{F}_\text{N} = 98\sin(40°)$
$\approx 63\,\text{N}$

17 D
It is important that momentum is a vector, hence, direction is important. Taking the direction east as positive, the change in momentum is given by:
$\Delta\boldsymbol{p} = m(\boldsymbol{v} - \boldsymbol{u})$
$= 80(2 - 7)$
$= -400\,\text{kg m s}^{-1}\text{ east}$
$= 400\,\text{kg m s}^{-1}\text{ west}$

18 B
The force on the jogger to slow him down can be calculated using Newton's Second Law:

$\boldsymbol{F} = m\boldsymbol{a}$
$= m\dfrac{\Delta\boldsymbol{v}}{\Delta t}$
$= m\dfrac{\boldsymbol{v} - \boldsymbol{u}}{t}$
$= 80 \times \dfrac{2 - 7}{2}$
$= -200\,\text{N}$
The magnitude of the force is 200 N.

19 C
The work done on the ball is equal to the kinetic energy of the ball.

$E_\text{k} = \dfrac{1}{2}mv^2$
$= \dfrac{1}{2} \times 0.42 \times 13.5^2$
$\approx 38\,\text{J}$

20 C
The spring constant can be found using Hooke's Law, $\boldsymbol{F} = k\boldsymbol{x}$. The force due to gravity is equal to the spring force at equilibrium.
$\boldsymbol{F}_\text{g} = \boldsymbol{F}$
$m\boldsymbol{g} = k\boldsymbol{x}$
$2.5 \times 9.8 = k(1.2 - 0.7)$
$\Rightarrow k = \dfrac{24.5}{0.5}$
$= 49\,\text{N m}^{-1}$

Section B — Short answer questions

21 **a** $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$
$\Rightarrow \boldsymbol{a} = \dfrac{\boldsymbol{v} - \boldsymbol{u}}{t}$
$= \dfrac{14 - 6.0}{4.0}$
$= 2.0\,\text{m s}^{-2}$ [1 mark]

b $\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$
$\Rightarrow \boldsymbol{s} = \dfrac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{a}}$
$= \dfrac{14^2 - 6^2}{2 \times 2}$
$= \dfrac{160}{4}$
$= 40\,\text{m}$ [1 mark]

c $\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{as}$
$\Rightarrow \boldsymbol{a} = \dfrac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{s}}$
$= \dfrac{0^2 - 14^2}{2 \times 24.5}$
$= \dfrac{-196}{49}$
$= -4.0\,\text{m s}^{-2}$
Thus the magnitude of the acceleration is $4.0\,\text{m s}^{-2}$. [1 mark]

d $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$
$\Rightarrow t = \dfrac{\boldsymbol{v} - \boldsymbol{u}}{\boldsymbol{a}}$
$= \dfrac{0 - 14}{-4}$
$= 3.5\,\text{s}$ [1 mark]

22 **a** $\boldsymbol{F}_\text{net} = m\boldsymbol{a}$
$= (150 + 150) \times 1.9$
$= 380\,\text{N}$ [1 mark]

b The net force on the van and trailer, calculated in part a, is the resultant of the driving force less the two resistive forces:
$\boldsymbol{F}_\text{net} = \boldsymbol{F}_\text{driving} - \boldsymbol{F}_\text{resistive on car} - \boldsymbol{F}_\text{resistive on trailer}$ [1 mark]
$380 = \boldsymbol{F}_\text{driving} - 30 - 20$
$\Rightarrow \boldsymbol{F}_\text{driving} = 430\,\text{N}$ [1 mark]

c Considering only the trailer, the net force on the trailer is:

$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$

$= 50 \times 1.9$

$= 95\,\text{N}$ [1 mark]

The forces acting on the trailer are the force of the rod and the resistive force:

$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{rod}} - \boldsymbol{F}_{\text{resistive}}$ [1 mark]

$95 = \boldsymbol{F}_{\text{rod}} - 20$

$\Rightarrow \boldsymbol{F}_{\text{rod}} = 115\,\text{N}$ [1 mark]

23 a Suzy's initial momentum is:

$\boldsymbol{p} = m\boldsymbol{u}$

$= 55 \times 4.8$

$= 264\,\text{kg m s}^{-1}$ east [1 mark]

b For an isolated collision, the total momentum before collision is the same as the total momentum after; that is,

$\boldsymbol{p}_{\text{final}} = 264\,\text{kg m s}^{-1}$ east [1 mark]

$\boldsymbol{p}_{\text{final}} = (m_{\text{Suzy}} + m_{\text{Kai}})\boldsymbol{v}$ [1 mark]

$264 = (55 + m_{\text{Kai}}) \times 3$

$\Rightarrow m_{\text{Kai}} = \dfrac{264}{3} - 55$

$= 33\,\text{kg}$ [1 mark]

24 a $E_{\text{k}} = \dfrac{1}{2}mv^2$ [1 mark]

$= \dfrac{1}{2} \times 600 \times 14.4^2$

$= 6.22 \times 10^4\,\text{J}$ [1 mark]

b If there is no energy lost, then the gravitational potential energy at point A is equal to the kinetic energy at point B.

$E_{\text{g}} = E_{\text{k}}$ [1 mark]

$mgh = 6.22 \times 10^4$

$600 \times 9.8 \times h = 6.22 \times 10^4$

$\Rightarrow h = \dfrac{6.22 \times 10^4}{600 \times 9.8}$

$= 10.6\,\text{m}$ [1 mark]

c The kinetic energy is converted into spring potential energy.

$E_{\text{s}} = \dfrac{1}{2}kx^2 = 6.22 \times 10^4$ [1 mark]

$\Rightarrow k = \dfrac{2 \times 6.22 \times 10^4}{x^2}$

$= \dfrac{2 \times 6.22 \times 10^4}{7.4^2}$

$= 2.27 \times 10^3\,\text{N m}^{-1}$ [1 mark]