ATARNotes

Specialist VCE Units 1/2

ATARNotes September Lecture Series

Presented by:
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TODAY'S PLAN

Topics to be covered

- Statistics – Hypothesis testing + probability
- Complex Numbers
- Circular Functions
- Vectors
- Kinematics and Vector Calculus

Announcements

- Please ask any questions!
- We will be covering stats in detail then revise over other topics

Overview of statistics

- Linear combinations of random variables
    - Similar to methods probability/sampling
        - Discrete, continuous, normal, normal approx of binomial, central limit theorem, confidence intervals, sampling
    - Difference between aX vs $\underbrace{X + X + \cdots + X}_{a\ times}$
- Hypothesis testing
    - Hard to understand, but honestly EASIEST marks on the exam!
    - Drawing is important: normal distribution
    - “If company claims mean is x, how true is this if we take a sample?”
        - Null/alternative hypothesis, p-value, level of significance
        - Type I and type II errors

Statistics

SUMMARY PROBABILITY FORMULAS

$$E(aX \pm b) = aE(x) \pm b$$

$$E(aX \pm bY) = aE(X) \pm bE(Y)$$

$$Var(aX \pm b) = a^2 Var(X)$$

$$Var(aX \pm bY) = a^2 Var(X) + b^2 Var(Y)$$

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

Recommend you have a page in ur book listing all the formulas for each distribution type

ALWAYS ADD VARIANCE!!! SD ONLY THRU $\sqrt{Var(X)}$

Discrete Mean	$E(X) = \sum_x x \cdot \mathrm{p(x)}$
Continuous Mean	$E(X) = \int_{-\infty}^{\infty} xf(x)\,dx$
Normal	$E(X) = \mu$
Inverse Norm	$Z = \frac{X-\mu}{\sigma}$ (ALWAYS TO THE LEFT)
Central Limit Theorem	

All of this is methods ¾ knowledge

LINEAR COMBINATIONS

- aX vs $\underbrace{X + X + \cdots + X}_{a \text{ times}}$

- Eg. The Devil of Gambling offers you two options to win money:

- 1. You roll a die <u>once</u>. You'll earn 2 times the number you roll
- 2. You roll a die <u>twice</u>. You'll earn the sum of the numbers you roll

- Which game yields more consistent income (i.e. less variability)?

LINEAR COMBINATIONS

- If X: the number that turns up when rolling a die once. Then:
  - $2X$: Roll the die **once**, but double
  - $X + X$: Roll the die **twice**, and add up the numbers rolled
- While $E(2X) = E(X + X) = 2E(X)$, the story is different for variability:
  - $Var(2X) = 2^2Var(X) = 4Var(X)$
  - $Var(X + X) = 1^2Var(X) + 1^2Var(X) = 2Var(X)$
- So the 2nd option will yield a more <u>consistent</u> income, since it has lower variability.

LINEAR COMBINATIONS

- If you do X **once**, then scale it by factor a afterwards, then $Y = aX$, but if you do X a times, without scaling by any factor, then $Y = \underbrace{X + X + \cdots + X}_{a\ times}$

Generally,

- If $Y = aX$
- $E(Y) = aE(X)$
- $Var(Y) \boxed{= a^2 Var(X)}$
- If $Y = \underbrace{X + X + \cdots + X}_{a\ times}$
- $E(Y) \boxed{= aE(X)}$
- $Var(Y) = aVar(X)$

Notice the difference in the variance and no difference in the expected value!

LINEAR COMBINATIONS

- From a given scenario, you need to be able to identify which case you are dealing with: aX or $\underbrace{X + X + \cdots + X}_{a \text{ times}}$?
  - VCAA probably won't tell you which case you're dealing with. 100% guaranteed on exam!
- Decide if the question has independent components.
  - If yes: ADD variances

    $\underbrace{X + X + \cdots + X}_{a \text{ times}}$? *"Find probability of total X and Y is…."*
  - If no: MULTIPLY

    aX *"Find probability that 3 components…."*

Statistics

VCAA 2016

Oranges grown on a citrus farm have a mean mass of 204 grams with a standard deviation of 9 grams. Lemons grown on the same farm have a mean mass of 76 grams with a standard deviation of 3 grams.

The masses of the lemons are independent of the masses of the oranges.

The mean mass and standard deviation, in grams, respectively of a set of three of these oranges and two of these lemons are

[✓ **A.**] 764, $3\sqrt{29}$
B. 636, 12
C. 764, $\sqrt{33}$
D. 636, $3\sqrt{10}$
E. 636, 33

$O \sim N(204, 9^2)$ and $L \sim N(76, 3^2)$

$$X = O + O + O + L + L$$

$$E(X) = 3E(O) + 2E(L) = 3 \cdot 204 + 2 \cdot 76 = 764$$

$$Var(X) = 3Var(O) + 2Var(L) = (3 \times 9^2) + (2 \times 3^2)$$
$$Var(X) = 261$$

$$\therefore sd(X) = \sqrt{261} = 3\sqrt{29}$$

Statistics

VCAA 2018 NHT

A farm grows oranges and lemons. The oranges have a mean mass of 200 grams with a standard deviation of 5 grams and the lemons have a mean mass of 70 grams with a standard deviation of 3 grams.

Assuming masses for each type of fruit are normally distributed, what is the probability, correct to four decimal places, that a randomly selected orange will have at least three times the mass of a randomly selected lemon?

A. 0.0062
B. 0.0828
[✓ **C.**] 0.1657
D. 0.8343
E. 0.9172

$$O \sim N(200, 5^2) \quad \& \quad L \sim N(70, 3^2)$$

$$\Pr(O \geq 3L) = \Pr(O - 3L \geq 0)$$

Set $X = O - 3L$

$$E(X) = E(O) - 3E(L) = -10$$
$$Var(X) = Var(O) + 9Var(L) = 106$$
$$\therefore X \sim N\left(-10, \sqrt{106}^2\right)$$

Now we can use this to calculate $\Pr(O - 3L \geq 0) = \Pr(Z \geq 0)$

$$= 0.1657$$

SAMPLE DISTRIBUTION

- Generally, looking at ONE sample:
- $X \sim N(\mu, \sigma^2)$

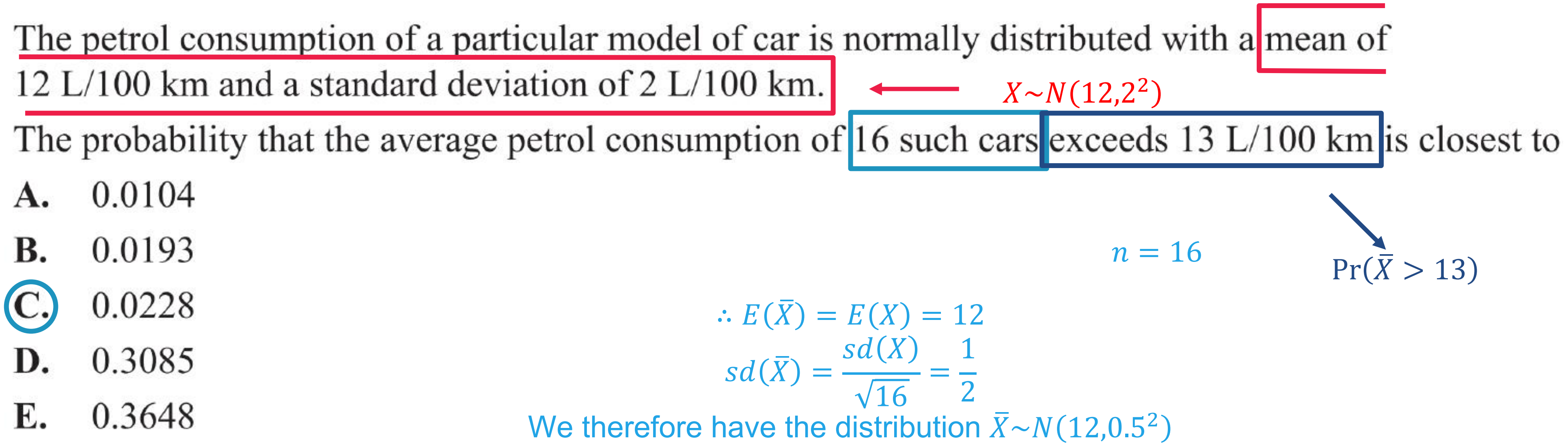
As sample size gets bigger = the more accurate the sample mean is to the population mean

- Sample distribution:
- If we take a sample of n and calculate the mean
- Then repeat for more samples of n
- This gives us: $\bar{X}$ (Sample Mean Distribution)

$$\boxed{\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)}$$

Statistics

VCAA 2017



The petrol consumption of a particular model of car is normally distributed with a mean of 12 L/100 km and a standard deviation of 2 L/100 km.

The probability that the average petrol consumption of 16 such cars exceeds 13 L/100 km is closest to

- **A.** 0.0104
- **B.** 0.0193
- [✓ **C.**] 0.0228
- **D.** 0.3085
- **E.** 0.3648

$X\sim N(12,2^2)$

$n = 16$

$\Pr(\bar{X} > 13)$

$$\therefore E(\bar{X}) = E(X) = 12$$

$$sd(\bar{X}) = \frac{sd(X)}{\sqrt{16}} = \frac{1}{2}$$

We therefore have the distribution $\bar{X}\sim N(12,0.5^2)$

$$\Pr(\bar{X} > 13) \approx 0.0228$$

CENTRAL LIMIT THEOREM

- No matter if binomial, uniform, poisson, continuous – If sample size if BIG, sampling distribution will approximate normal distribution.

$$\boxed{\bar{X} \approx N\left(\mu, \frac{\delta^2}{n}\right)}$$

- For Spesh and Methods, we can use normal approximation / central limit theorem when $\boldsymbol{n \geq 30}$

- You will need to calculate mean or standard deviation first and then put it into the sampling formulas!

CENTRAL LIMIT THEOREM

- When we have a large sample from a **large population,** the sampling distribution can be **approximately normal**
- Use the population size and “approximate” to recognise Central Limit Theorem
- Which means we have methods of determining the standard deviation and mean
- $\mu = p$

The mean is the population proportion

- $\delta = \sqrt{\frac{p(1-p)}{n}}$

Also called Standard Error

There are proofs for this in your textbooks

NORMAL APPROXIMATION OF BINOMIAL

- Binomial needs n trials and p probability

- If there is a large enough n and p not close to 0 or 1
- we will eventually get to a normal looking distribution

- Therefore:
- $\mu = np \quad and \quad \sigma = \sqrt{np(1-p)}$
- <u>AS LONG AS</u> np and n(1-p) is greater than 5

Statistics

Question 18

Consider a random variable X with probability density function

$$f(x)=\begin{cases}2x, & 0\le x\le 1\\ 0, & x<0 \text{ or } x>1\end{cases}$$

$n = 100$

If a large number of samples, each of size 100 is taken from this distribution, then the distribution of the sample means, $\bar{X}$, will be approximately normal with mean $\mathrm{E}\left(\bar{X}\right)=\frac{2}{3}$ and standard deviation $\mathrm{sd}\left(\bar{X}\right)$ equal to

[✓ **A.**] $\frac{\sqrt{2}}{60}$

B. $\frac{\sqrt{2}}{6}$

C. $\frac{1}{180}$

D. $\frac{1}{18}$

E. $\frac{\sqrt{2}}{30}$

$$sd(\bar{X})=\frac{sd(X)}{\sqrt{n}}=\frac{sd(X)}{\sqrt{100}}$$

$$Var(X)=\int_0^1 2x^3\,dx-\left(\frac{2}{3}\right)^2=\frac{1}{2}-\frac{4}{9}$$

$$Var(X)=\frac{1}{18}$$

$$\therefore sd(X)=\frac{\sqrt{2}}{6}$$

Hence, $sd(\bar{X})=\frac{\sqrt{2}}{60}$

CONFIDENCE INTERVAL

$$\left(\hat{p} - k\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + k\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

- What does a confidence interval mean?
- We can say with ___% certainty that the population proportion falls within ___ and ___

- If you have 100 samples and do a 95% confidence interval for each, 95 of the intervals that you find will **actually** contain the population proportion
- The more ‘certain’ you are that it will fall in your interval, the less useful that information becomes, as there are more possible values for population proportion

CONFIDENCE INTERVALS

- Each sample will have its associated confidence interval
- $CI = \left(\bar{x} - k\frac{s}{\sqrt{n}}, \bar{x} + k\frac{s}{\sqrt{n}}\right) = (\bar{x} - M, \bar{x} + M)$

- $\bar{x}$ Sample Mean
- n Sample Size
- s Sample SD (Note that $s \approx \sigma_X$)
- M Margin of Error

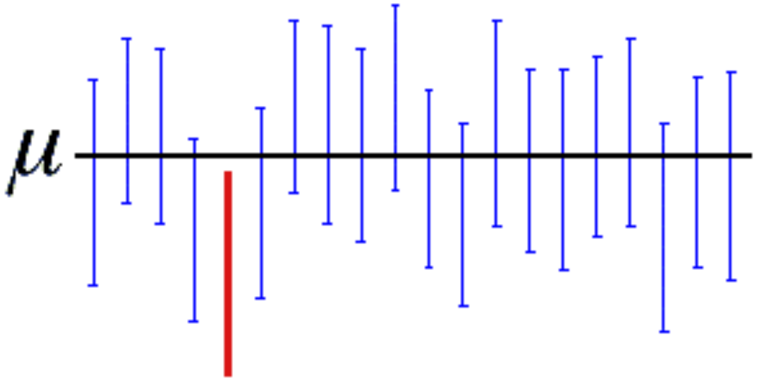
CONFIDENCE INTERVAL

Degree of Confidence	k
90%	1.645
95%	1.960
99%	2.576

It is a good idea to memorise these numbers as they can ask you to calculate the CI in Exam 1!

If exam says:
“Integer multiple of SD”

- 95% = 2



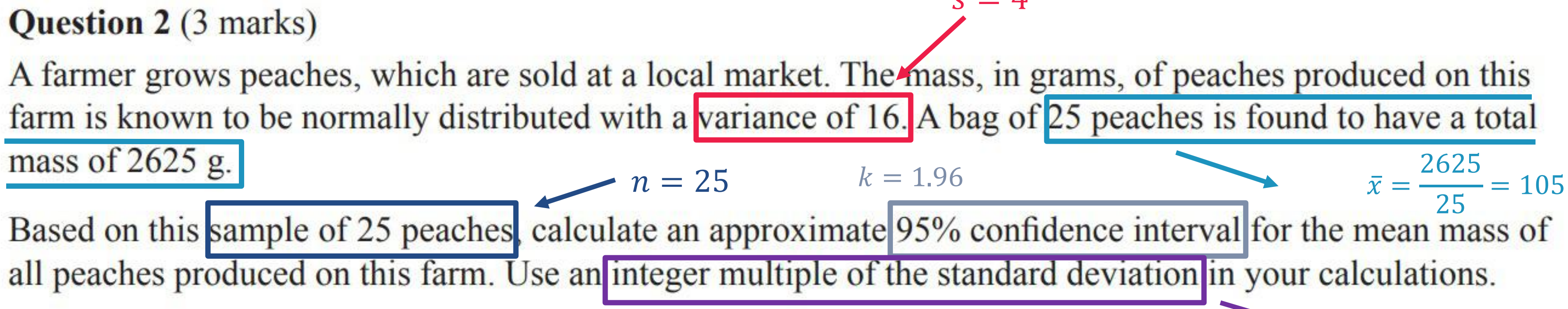
A 95% confidence interval indicates that 19 out of 20 samples (95%) from the same population will produce confidence intervals that contain the population parameter.

k VALUES

- You can determine specific k values for each C percentage of confidence using the following:

- Where $\Pr(-k < Z < k) = \frac{C}{100}$
- $2\Pr(Z < -k) = 1 - \frac{C}{100}$
- $\Pr(Z < -k) = \frac{1}{2}\left(1 - \frac{C}{100}\right)$
- $k = -\text{invNorm}\left(\frac{1}{2}\left(1 - \frac{C}{100}\right), 0{,}1\right)$

Statistics



Question 2 (3 marks)

A farmer grows peaches, which are sold at a local market. The mass, in grams, of peaches produced on this farm is known to be normally distributed with a variance of 16. A bag of 25 peaches is found to have a total mass of 2625 g.

Based on this sample of 25 peaches, calculate an approximate 95% confidence interval for the mean mass of all peaches produced on this farm. Use an integer multiple of the standard deviation in your calculations.

$s = 4$

$n = 25$

$k = 1.96$

$\bar{x} = \frac{2625}{25} = 105$

$\Rightarrow k = 2$

$$CI = \left(\bar{x} - k\frac{s}{\sqrt{n}}, \bar{x} + k\frac{s}{\sqrt{n}}\right)$$

$$CI = \left(105 - 2 \cdot \frac{4}{\sqrt{25}}, 105 + 2 \cdot \frac{4}{\sqrt{25}}\right)$$

$$CI = (103.4, 106.6)$$

HYPOTHESIS TESTING

- Some easiest questions on exam bc they can’t do anything unpredictable
- You’re seeing how likely a claim is based on a sample

- Example:
- A treatment is hypothesised to decrease the mean population size of a virus.
- We can determine whether this is true by taking a sample and comparing the probability of an anomaly mean occurring.

Statistics

IMPORTANT!

- KNOW THE DIFFERENCE BETWEEN:
  - POPULATION STANDARD DEVIATION
    - SD of the POPULATION!! NO SAMPLE SIZES HERE

$$\sigma$$

DON'T MIX THE TWO UP ITS VERY EASY

  - SAMPLE STANDARD DEVIATION
    - SD of the SAMPLE!! THERE IS A SAMPLE HERE!

$$sd(\bar{X}) = \frac{\sigma}{\sqrt{n}}$$

1. Set up statistical hypotheses

- In HT, we assume the rule of “not guilty until proven”.
- Null hypothesis is when there is NO EFFECT - the treatment is ineffective → H_0.
- $H_0\colon\ \mu = \mu_{\text{before}}$

- **Alternative hypothesis** is a DESIRED/FEARED effect – the treatment is effective.
- $H_1\colon \mu < \mu_{\text{before}}$

There are three possibilities for H_1:

$\mu > \mu_{\text{before}}$

One-tailed test: “Difference/change” $\mu < \mu_{\text{before}}$

Two-tailed test: “Above/Below” $\mu \neq \mu_{\text{before}}$

2. Set up a level of significance (p value/α significance)

- To reject the Null hypothesis, we need to prove that the probability of getting a certain sample mean is EXTREMELY SMALL, assuming the null hypothesis is true
  - Eg. If we flip a coin 20 times and we get 2 heads, can we assume the coin is fair?
- P value is the probability of getting an extreme value when assuming null is correct.
- α significance level (default is 0.05). If p-value is less than 0.05, we can state that there is a less than 5% chance of getting 2 heads if a coin is fair.

HYPOTHESIS TESTING

- **3. Calculate p-value (one tailed)**
- Predict the directionality
  - the size of virus will **decrease** after receiving treatment X
  - then $H_1: \mu < \mu_{\text{before}}$

Obvs replace the sign

- The P-value is given by:
  - $\boxed{\text{P-value} = \Pr(\bar{X} < \bar{x} | \mu = \mu_0)}$

- 3 methods of doing this

P VALUE

Method 1 (Convert into Standard Normal Distribution)

- P-value $= \Pr(\bar{X} < \bar{x} | \mu = \mu_0)$

STEPS:
1. Define Null/Alternative
2. $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ for standard normal
3. NormCdf on calc <u>Z~N(0,1)</u>

$$\Pr\left(Z \leq \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right)$$

Method 2 (NormCdf)

STEPS:
1. Define Null/Alternative
2. NormCdf on calc
3. <u>DON'T FORGET TO CONVERT TO SAMPLE SD</u>

Method 3 (Z-test)

STEPS:
1. Z-test (Menu>6>7>1)
2. Choose 'Stats' not 'Data'
3. Fill in calc, using POPULATION SD
4. Change the 'Alt Hypo' to relevant
5. Ctrl+Enter and look at the 'PVal'

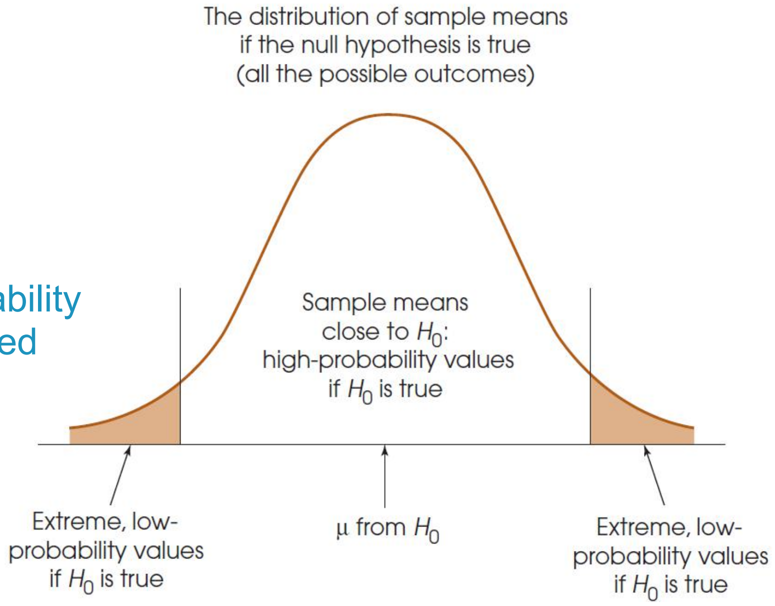
HYPOTHESIS TESTING

- **3. Calculate p-value (two tailed)**
- If we have no direction ”H_1 is different than expected”
- $H_1 : \mu \neq \mu_0$
- Basically just half of the α for one side

$$p_{2\text{ tailed}} = 2p_{1\text{ tailed}}$$
$$= 2\Pr\left(Z \geq \left|\frac{\mu_0 - \mu}{\frac{\delta}{\sqrt{n}}}\right|\right)$$

Otherwise use z-test and alt hypo as $\mathrm{u} \neq u_0$

It’s the probability of both shaded regions!



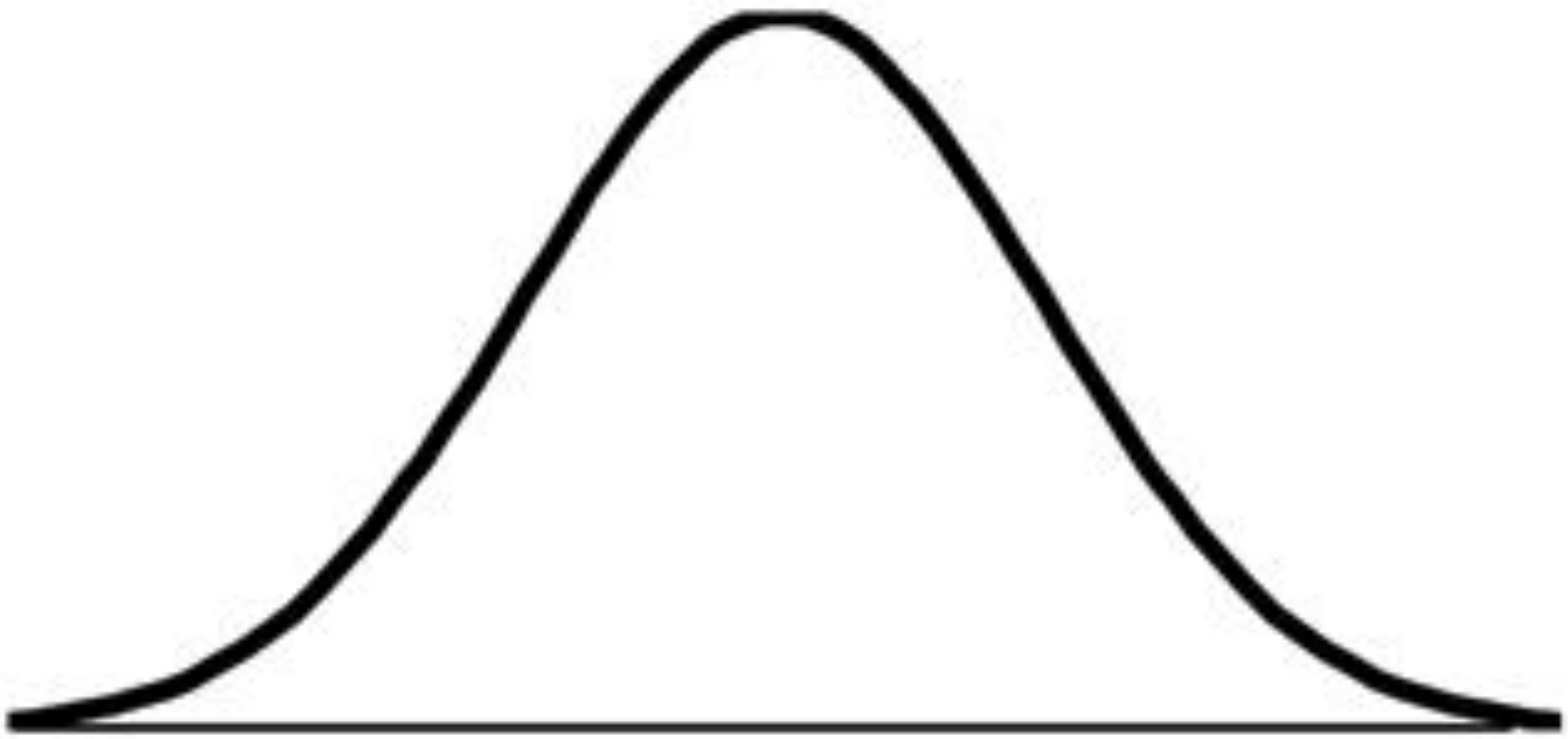
- ## 4. Making inferences

If the p value is:

- Above α = ”insufficient evidence to reject H_0”

- Below α =
    - “good evidence to reject H_0” >0.05
    - “Strong evidence to reject H_0” >0.01
    - “very strong evidence to reject H_0” >0.001

SUMMARY OF HYPO TESTING



HYPOTHESIS TESTING

- There's ALWAYS a tiny chance the p-value is erroneous/wrong due to randomness. There are 2 types of errors in HT:

	H_0 **true**	H_0 **false**
Reject H_0	Type I (False positive)	Correct conclusion
Do not reject H_0	Correct conclusion	Type II (False negative)

- Type I: Rejecting H_0 when it is true
- Type II: Not rejecting H_0 when it is false

ERRORS

You're probs not gonna need to do this for the exam but good to understand

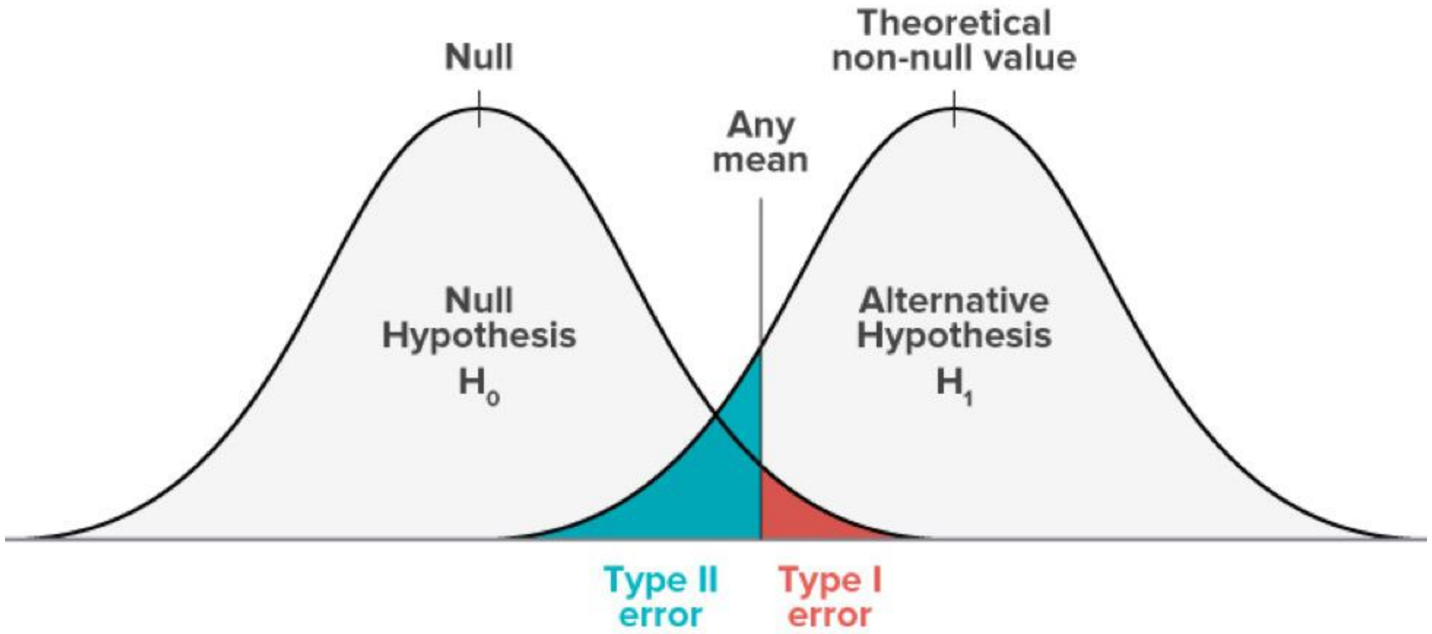
- We want to reduce the chance of both errors.
- Type I: related to significance level.
  - You reject H_0 below $\boldsymbol{\alpha}$ = 0.05, but you shouldn't have
  - Therefore, you can reduce type I error by decreasing $\boldsymbol{\alpha}$ level, but this would increase chance of Type II error

- Steps:

1. Let p-value = 0.05
2. Use $\mathrm{z} = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ and solve for x

ERRORS

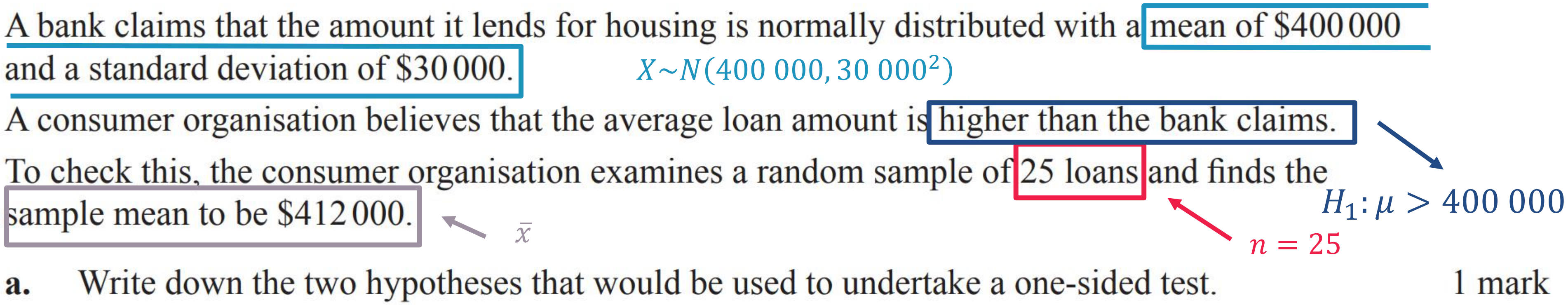
- Type II: (Decreases with sample size)



- Steps:
  - Use InvNorm to find the value where type I meets type II
  - Pr(Accept H_0|H_0 false)

Statistics

VCAA 2017 NHT



A bank claims that the amount it lends for housing is normally distributed with a mean of \$400 000 and a standard deviation of \$30 000.

$X \sim N(400\ 000, 30\ 000^2)$

A consumer organisation believes that the average loan amount is higher than the bank claims.

$H_1 : \mu > 400\ 000$

To check this, the consumer organisation examines a random sample of 25 loans and finds the sample mean to be \$412 000.

$\bar{x}$

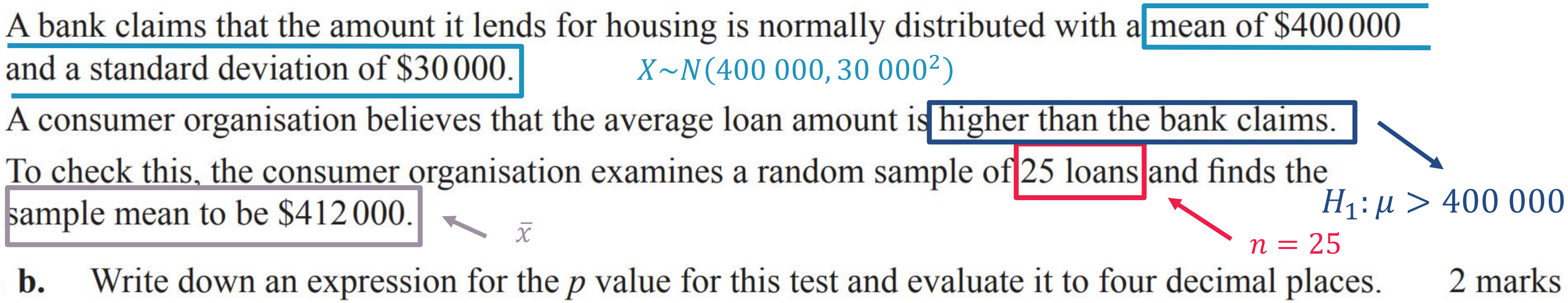
$n = 25$

a. Write down the two hypotheses that would be used to undertake a one-sided test. 1 mark

$H_0 : \mu = 400\ 000$

$H_1 : \mu > 400\ 000$

Statistics



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$H_1: \mu > 400\ 000$

To check this, the consumer organisation examines a random sample of 25 loans and finds the sample mean to be $412 000.

$\bar{x}$

$n = 25$

b. Write down an expression for the *p* value for this test and evaluate it to four decimal places. 2 marks

$p\text{-value} = \Pr(\bar{X} > 412\ 000 | \mu = 400\ 000)$

$\bar{X} \sim N\left(400\ 000, \left(\frac{30\ 000}{\sqrt{25}}\right)^2\right) \Leftrightarrow \bar{X} \sim N(400\ 000, 6000^2)$

$\therefore p\text{-value} = 0.0228$

Statistics

VCAA 2017 NHT CONT.

c. State with a reason whether the bank's claim should be rejected at the 5% level of significance. 1 mark

Since p-value $= 0.0288 < 0.05$, this means that we reject the null hypothesis which is the bank's claim.

d. What is the largest value of the sample mean that could be observed before the bank's claim was rejected at the 5% level of significance? Give your answer correct to the nearest 10 dollars. 1 mark

We want $p\text{-value} = 0.05$

$$\Pr(\bar{X} > \bar{x} | \mu = 400\,000) = 0.05$$

$$\Pr\left(Z > \frac{\bar{x} - 400\,000}{6000}\right) = 0.05 = \Pr(Z > 1.644\,\ldots)$$

$$\therefore \bar{x} \approx 409870$$

Polar Form

Complex's fancy way of using its fancy cartesian plane.

Use when multiplying/division/finding solutions with De Moivre

To convert between Cartesian and Polar form:

$$z = x + yi \quad \rightarrow \quad z = rcis(\theta) \quad or \quad z = rcos(\theta) + irsin(\theta)$$

Where:

- r is the distance from the point to the origin.

$$r = \sqrt{x^2 + y^2}$$

- θ : The angle between the positive x axis and point

$$\tan(\theta) = \frac{y}{x}$$

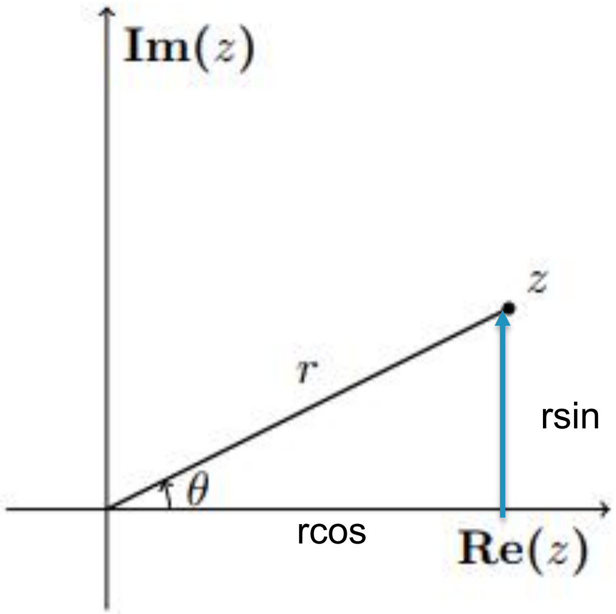
Complex Numbers

Thus, we can represent z as

$$r(\cos\theta\ +\ i\sin\theta)$$

Or, for short:

$$r\ cis\ \theta$$



SUPER important: DOMAINS

If you are given Arg (not arg) the domain is $\mathrm{Arg}(\theta)\ \epsilon\ (-\pi,\pi]$
You have to draw the CAST quadrants

Multiplication and Division

Rule: multiply/divide to r or add/subtract θ

Consider two complex numbers: (USE POLAR FOR THIS)

$$z_1 = r_1 cis(\theta_1) \quad z_2 = r_2 cis(\theta_2)$$

Multiplication:

$$\boxed{Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)}$$

$$z_1 \cdot z_2 = r_1 r_2 cis(\theta_1 + \theta_2)$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} cis(\theta_1 - \theta_2)$$

Conjugate:

$$\bar{z} = rcis(-\theta)$$

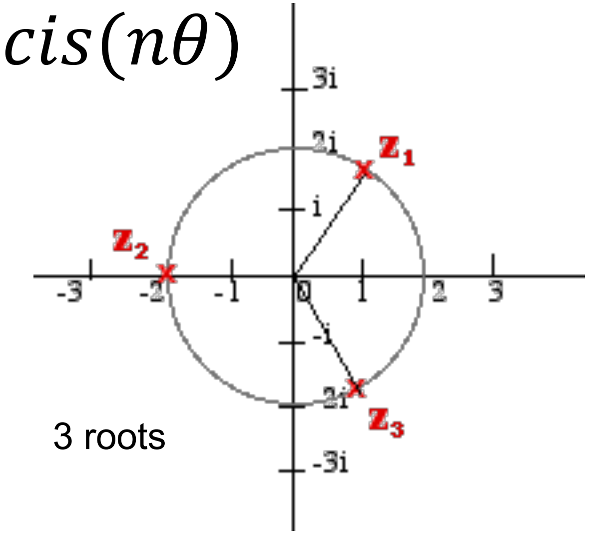
Reciprocal:

$$z^{-1} = \frac{1}{r} cis(-\theta)$$

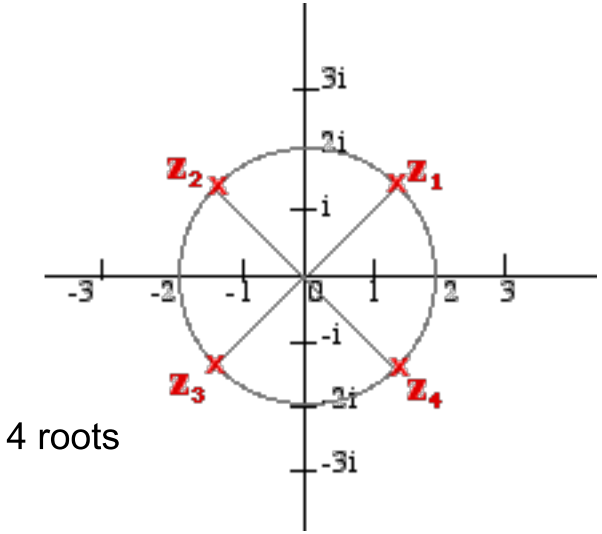
Complex Numbers

De Moivre’s Theorem

- We use this for finding nth roots of a complex number
- (eg. Numbers have can 2 square roots, 3 cube roots, etc……)
- Therefore: There are n solutions for nth roots around a circle
    - MUST BE IN POLAR FORM AND IN $Arg(\theta)\epsilon(-\pi,\pi]$
- $z^n = r^n cis(n\theta)$



3 roots



4 roots

Solving De Moivres

- Super simple! Just need to equate r variables and θ variables

- Steps:

1. Write the formula: $z^n = r^n cis(n\theta)$
2. Convert the complex number into Polar form
3. Let r^n = magnitude and $n\theta$ = arg
    - For angles if you find one angle, just $\pm$ angles equally apart
4. Make sure $Arg(\theta)\epsilon(-\pi,\pi]$
5. Convert into cartesian if needed

Polynomials

- **Conjugate root theorem:**
- If all coefficients are REAL numbers in eg
- $p = az^3 + bz^2 + cz + d$
- The solution of z=x+yi will also have $\bar{z}$=x-yi as a solution

- **Simplifying:** Let's say I have
  - $p = z^3 + 8z^2 + 25z + 26$ with a solution of (z-2)

Circular Functions

Compound Formulas

- On formula sheet, but REALLY good to memorise
- Used for trig with WEIRD angles

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$$

$$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

Double Angle Formulas:

$$\begin{aligned}\cos(2x) &= \cos^2(x) - \sin^2(x)\\ &= 1 - 2\sin^2(x)\\ &= 2\cos^2(x) - 1\end{aligned}$$

Note: sometimes you will need to use **Half Angle Formulas** to proof. Just replace with half the x

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$$

Converting Inverse Domain and Range

If we had $y = a\sin^{-1}(bx - c) + d$:

DOMAINS: UNDO

1. Take the bracket
2. Apply to the default domain of the trig
3. 'Undo' the transformations until you only have x

$$bx - c \in [-1,1]$$

$$bx \in [c - 1, c + 1]$$

$$x \in [\frac{c-1}{b}, \frac{c+1}{b}]$$

RANGES: BUILD UP

1. Rearrange equation to 'Build Up' y side
2. Let the y side equal the default range
3. 'Undo' the transformations until you only have y

$$\frac{y-d}{a} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$

$$y - d \in [\frac{-\pi a}{2}, \frac{\pi a}{2}]$$

$$y \in [d - \frac{\pi a}{2}, d + \frac{\pi a}{2}]$$

Compound Differentiation

Product Rule (use this unless stated by inversing the denominator)

$$\frac{d}{dx}(uv) = u'v + uv'$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{(u'v - uv')}{v^2}$$

Tips:

$$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$$

Add examples of differentiation that are tricky!

Calculus

Differentiation Rules

These are the most useful equations to memorize outside of formula sheet:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}(\log_e f(x)) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(a^x) = a^x \log_e a$$

$$\frac{d}{dx}(f(g(x)) = g'(x)f'(g(x)$$

This one in particular! Chain rule (work in layers)

CAREFUL OF DOMAINS

$$\frac{d}{dx}(\sin^{-1} f(x)) = \frac{f'(x)}{\sqrt{1-(f(x))^2}}$$

$$\frac{d}{dx}(\cos^{-1} f(x)) = \frac{-f'(x)}{\sqrt{1-(f(x))^2}}$$

$$\frac{d}{dx}(\tan^{-1} f(x)) = \frac{f'(x)}{1+(f(x))^2}$$

Implicit Differentiation + Concavity

Implicit Differentiation:

- If you see ‘y’ differentiate as per normal but put $\frac{dy}{dx}$ after every time

Points of Inflection/Concavity

- Used to verify TP/direction of gradient
- $f''(x) = +ve \rightarrow local\ min$
- $f''(x) = -ve \rightarrow local\ max$
- $POI: f''(x) = 0\ but\ f'(x) \neq 0$
- $SPOI: f''(x) = 0\ but\ f'(x) = 0$

Related Rates

- Look at the units in question – gives big clue to which variable is over which variable
- Is based upon Chain Rule
- Parametric is the same – derivative of both equations combined together
- Often used in applications involving time and shapes

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

INTEGRATION BY SUBSTITUTION ”LET U”

- AKA Let u when there is a STEP DOWN in power eg. ($g'(x)f\big(g(x)\big)$)
- You will make mistakes. Its ok!
- If you do the wrong sub, try the other
- Almost always more complex/denominator is the Let u

Steps:

1. Let a component = u
2. Find $\frac{du}{dx}$ and rearrange into smth that looks like other component
3. Sub u and $\frac{du}{dx}$ into back into original (if you have terminals, convert to u too)
4. Solve for u
5. Sub u original back

INTEGRATION WITH TRIG

2 cases: ANY odd powers or ALL even powers

ANY ODD POWERS ($\text{eg.}\sin^5 x \cos^2 x$)

1. Split the odd power so there is a power of 1 (a loner)

$$\sin x \sin^4 x \cos^2 x$$

2. Remember $\sin^2 x + \cos^2 x = 1$ and convert the other even powers

$$\sin x\,(1 - \cos^2 x)^2 \cos^2 x$$

3. Let u=opposite trig of odd power

$$Let\ u = \cos x, \qquad -\frac{du}{dx} = \sin x$$

4. Solve like normal

5. (If tanx is odd) Convert into $\dfrac{\sin x}{\cos x}$

INTEGRATION WITH TRIG

Super hard!

- 2 cases: ANY odd powers or ALL even powers
- **ALL EVEN POWERS** MUST MEMORISE THESE FORMULAS!
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\sin 2x = 2\sin x \cos x$$

$$\tan^2 kx = \sec^2 kx - 1$$

INTEGRATION BY PARTS/RECOGNITION

We use this technique when the integrand is in the form of

$$fg'$$

Where

- f is a function that doesn't have a standard integral, but can be differentiated
- g' is a function that can easily be integrated

Examples:

$$x^2 \log_e(x) \qquad 3x^2 \sin^{-1}(2x)$$

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx$$

INTEGRATION BY PARTS/RECOGNITION

- Used to only be three part question now can be straight :’(

The idea is:

1. You have a diff from the previous question

$$\frac{d}{dx}\left(f(x)\right) = \cdots$$

2. Then you integrate both sides

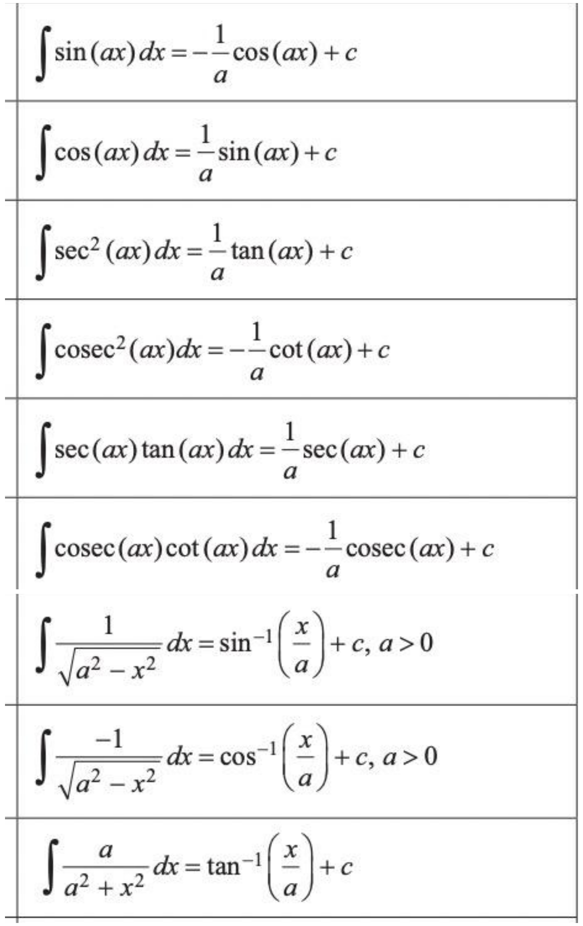
$$\int \frac{d}{dx}\left(f(x)\right)dx = \int \dots dx$$

3. Rearrange until you have what the question wants

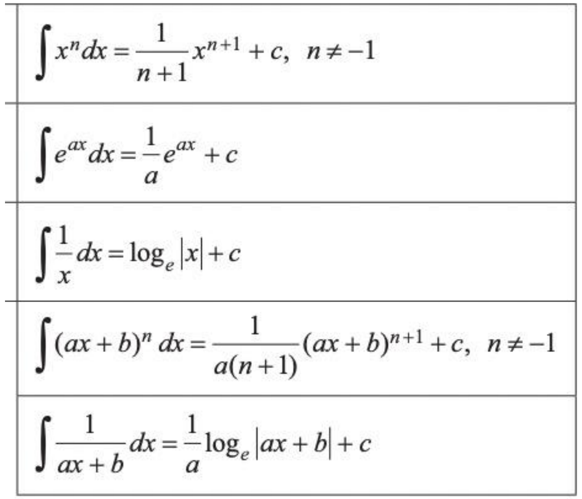
Calculus

Summary: Integration Rules

Don’t forget Modulus – must reject one side



$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\int \operatorname{cosec}^2(ax)dx = -\frac{1}{a}\cot(ax) + c$
$\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\sec(ax) + c$
$\int \operatorname{cosec}(ax)\cot(ax)\,dx = -\frac{1}{a}\operatorname{cosec}(ax) + c$
$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\int \frac{-1}{\sqrt{a^2 - x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\int \frac{a}{a^2 + x^2}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$



$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ \ n \neq -1$
$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\int \frac{1}{x}dx = \log_e\lvert x\rvert + c$
$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ \ n \neq -1$
$\int \frac{1}{ax+b}dx = \frac{1}{a}\log_e\lvert ax+b\rvert + c$

VCAA Specialist Mathematics Formula Sheet Accessed: https://vcaa.vic.edu.au/Documents/exams/mathematics/specmaths-formula-w.pdf

Integration substitutions

Integration using substitutions

- Key pattern: "step down in powers" from denominator to numerator
- Goal is to get 'u' to $\frac{du}{dx}$ to substitute (Get rid of x entirely)
- Almost always complex equations/denominator is 'u'
- Direction substitution: Convert to u terminals as well
- Linear substitution: $\frac{du}{dx} = 1$
- Special case $\int \sqrt{a^2 - x^2}\, dx$, use
  - $x = a \sin u$
  - $x = a \cos u$

Integration identities

Integration using trig identities

- Any odd powers:
  - Split the biggest power so there is a ‘loner’
  - Let u = opposite of loner
  - Use trig identities to reduce other parts:
- All even powers – use the identities from below:
  - $Sin^2(x) = \frac{1}{2}(1 - \cos(2x))$
  - $Cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
  - $2\sin(x)\cos(x) = \sin(2x)$

Integration by parts/recognition

Integration by parts

- Can now either be a 2 part (diff..hence) or 1 part question (LIATE)
  - $\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx$
  - Integrate both sides and arrange what is desired to be the subject

$\frac{ax+b}{(x-m)(x-n)}$	$\frac{A}{x-m} + \frac{B}{x-n}$
$\frac{ax+b}{(x-m)^2}$	$\frac{A}{x-m} + \frac{B}{(x-m)^2}$
$\frac{px^2+qx+r}{(x-m)(ax^2+bx+c)}$	$\frac{A}{x-m} + \frac{Bx+C}{ax^2+bx+c}$ Resolve the quadratic by completing square and integrating for tan

SPECIAL CASE: NEWTON'S LAW OF COOLING

- Rate at which a body cools(dT/dt) is PROPORTIONAL to the difference between its temp and ambient temp(T - T_s)

First Principles

Just memorise this unless

$$\frac{dT}{dt} = -k(T - T_s) \longrightarrow T = Ae^{kt} + Ts, \qquad k > 0$$

- Where:
- T_s is the temperature of the surroundings
- k is the proportional constant

SPECIAL CASE: LOGISTIC DIFFERENTIAL EQUATIONS

- Rate at which a <u>population</u> (dP/dt) is PROPORTIONAL to the <u>Population and the limiting population</u>

First Principles

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right) \longrightarrow$$

Just memorise this unless

$$P = \frac{L}{e^{c-kt} + 1}, \qquad k > 0$$

- Where:
- P = population
- L is the limiting population ($P \rightarrow L$ when $t \rightarrow \infty$)
- k = constant which changes based on the question.

Calculus

CONCENTRATION/MIXING PROBLEMS

- When you have a substance pouring into a tank, mixing with something else and then a mixture flows out.

$$\boxed{\frac{dQ}{dt} = Inflow \ - Outflow}$$

both usually in g/min

$$Inflow = \frac{Amount\ (g)}{\underline{(L)}} \times \frac{flow\ in\ \underline{(L)}}{(min)} \ = \frac{Amount(g)}{(\mathrm{min})} \times flow\ in$$

$$Outflow = \frac{Q\ (g)}{volume +\ changes\ \underline{(L)}} \times \frac{flow\ out\ \underline{(L)}}{(\mathrm{min})} \ = \frac{Q \times flow\ out}{volume +\ changes}$$

Where:

- Q is outflow substance (in grams)
- If Q is conc, remember $C = \frac{mass}{Volume}$

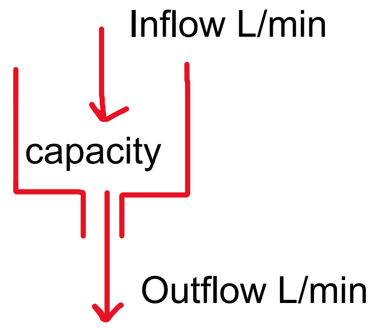
If they give you initial conditions in qn:

- *THEY DON'T COUNT FOR THIS FORMULA*
- *USE LATER FOR +c*

CONCENTRATION/MIXING PROBLEMS

- Two situations: This will change your volume in outflow.
- Inflow = outflow
- Inflow ≠ outflow

Note! If one of your substances is water, you are adding no grams/diluting another solution. Inflow will be 0.



Steps:

1. Draw the situation: inflow, outflow, capacity of tank
2. Define your outflow/inflow formulas
3. $\frac{dQ}{dt} = Inflow\ - Outflow$
4. Solve so Q is subject

SLOPE FIELDS

Steps:

1. Look at x-axis/y-axis for patterns
   a) +ve/-ve gradients, 0 gradients, undefined gradients
2. Look at the question
   a) If want $\frac{dy}{dx}$: look for TP or asymptote lines.
      a) Vertical lines: Undefined
      b) Horizontal: 0 gradient
      c) Pos/neg gradients
   b) If want y: draw curve on slope field and look at shape.
   THIS IS THE IMPLIED CURVE NOT $\frac{dy}{dx}$!

EULER'S METHOD

- $y_{n+1} = y_n + hf'(x_n, y_n)$

h is step size

- **MUST MEMORISE FORMULA**
- DON'T GET CONFUSED W/ PREVIOUS x, y TERMS
- BIGGEST ERROR IS NOT ORGANISING YOUR WORKING!!!!!

Steps:

1. Write $y_{n+1} = y_n + h\,\frac{dy}{dx}$
2. COUNT HOW MANY STEPS BEFORE SUB INTO FORMULA
3. Sub and DON'T SKIP STEPS! ORGANISE

SUMMARY BOOK - DIFFERENTIAL EQUATIONS

- Fundamental Theorem of Calculus

$$y_{final} = \int_{x_{initial}}^{x_{final}} f'(x)\,dx + y_{initial}$$

- Separation of Variables
  - Like terms: put dx with x terms, dy with y terms
  - Integrate both sides
  - +c on x side, arrange so y is subject
- Growth/Decay
  - Key words: ‘Proportional’
  - A = initial conditions, k = growth/decay, t = time

$$\frac{dy}{dt} = ky \;\rightarrow y = Ae^{kt}$$

SUMMARY BOOK - DIFFERENTIAL EQUATIONS

- Newton’s Law of Cooling
  - k = cooling/heating, T = temp, T_0 = environment temp, t = time

$$\frac{dT}{dt} = k(T - T_0) \rightarrow T = Ae^{kt} + T_0$$

- Concentration/mixing problems

$$\frac{dQ}{dt} = inflow - outflow$$

$$Inflow = \frac{amount(g)}{L} \times \frac{flow\,rate(L)}{min}$$

$$Inflow = \frac{Q(g)}{capacity \pm \Delta volume} \times \frac{flow\,rate(L)}{min}$$

$$\frac{dQ}{dt} = \frac{dQ}{dV}_{in} \times \frac{dV}{dt}_{in} - \frac{dQ}{dV}_{out} \times \frac{dV}{dt}_{out}$$

SUMMARY BOOK - DIFFERENTIAL EQUATIONS

Slope fields

1. Look at **x&y axis** for patterns (0, undefined, +/- gradients)
2. Look at **question:**
   a. If f’(x) - look for patterns(quadrants, 0, undefined, +/- grad)
   b. If y - trace the shape of the slope
3. Special:
   a. sin/cos - look at shape and period
   b. circle - look for centre

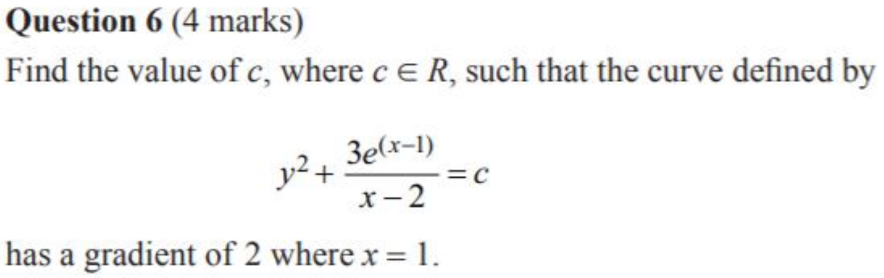
Euler’s method

- ORGANISE your working!

$$y_{n+1} = y_n + hf'(x_n, y_n)$$

- CAS: ‘euler(dy/dx, x,y,{x0,xn},y0,h)

IMPLICIT DIFFERENTIATION

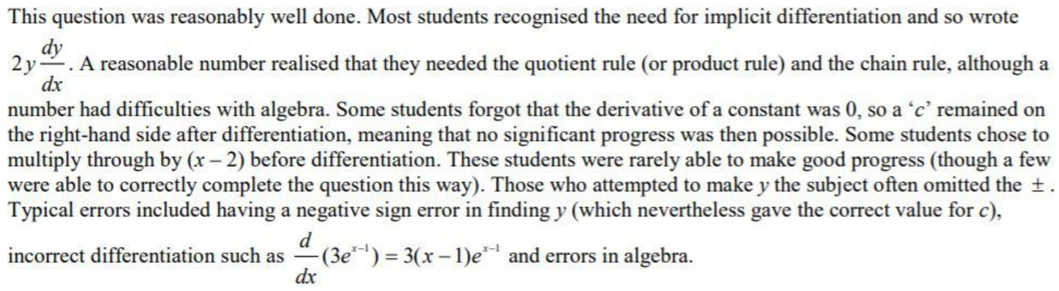


Question 6 (4 marks)

Find the value of c, where $c \in R$, such that the curve defined by

$$y^2+\frac{3e^{(x-1)}}{x-2}=c$$

has a gradient of 2 where $x = 1$.

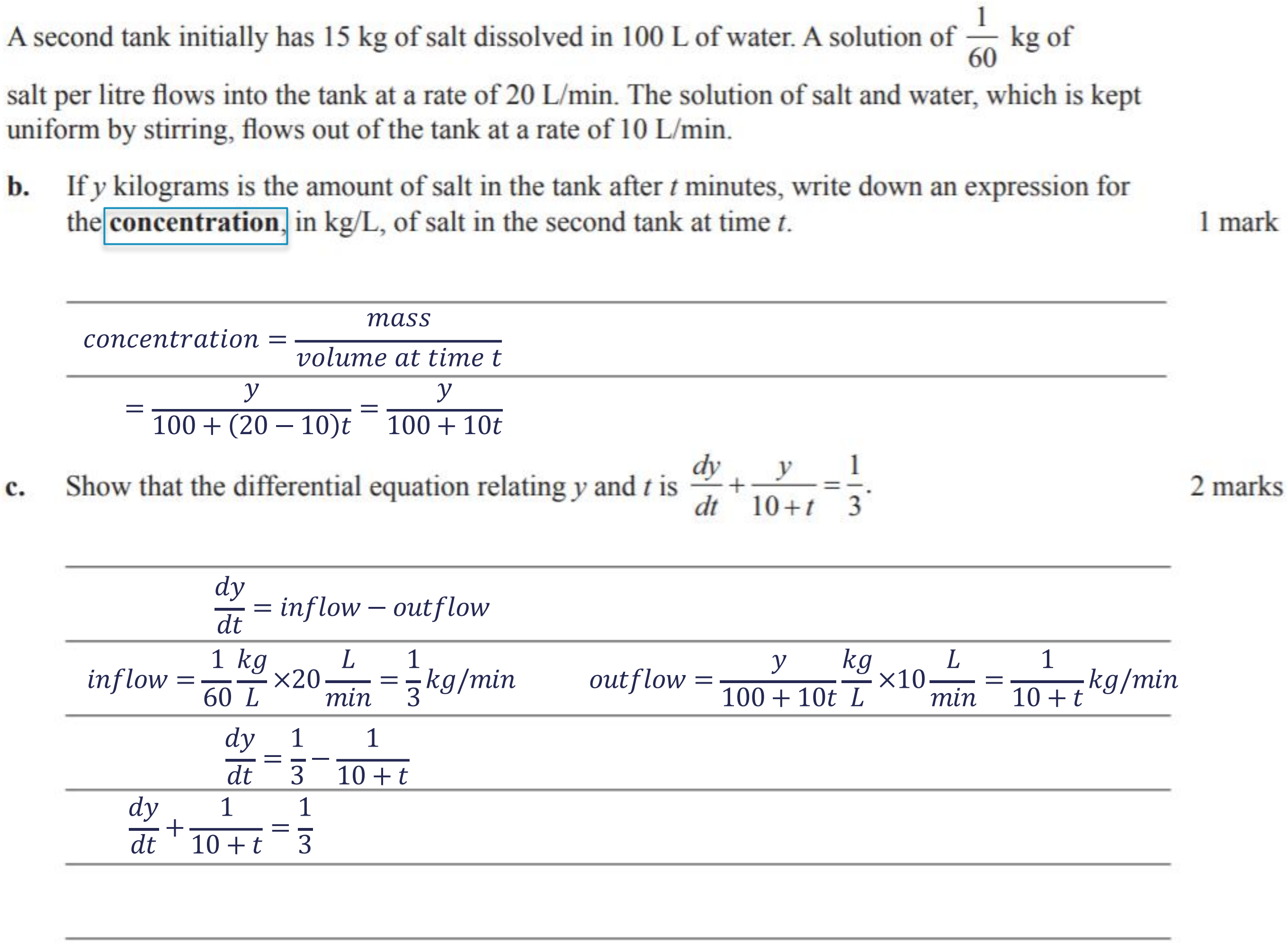


This question was reasonably well done. Most students recognised the need for implicit differentiation and so wrote $2y\frac{dy}{dx}$. A reasonable number realised that they needed the quotient rule (or product rule) and the chain rule, although a number had difficulties with algebra. Some students forgot that the derivative of a constant was 0, so a 'c' remained on the right-hand side after differentiation, meaning that no significant progress was then possible. Some students chose to multiply through by $(x-2)$ before differentiation. These students were rarely able to make good progress (though a few were able to correctly complete the question this way). Those who attempted to make y the subject often omitted the $\pm$. Typical errors included having a negative sign error in finding y (which nevertheless gave the correct value for c), incorrect differentiation such as $\frac{d}{dx}(3e^{x-1})=3(x-1)e^{x-1}$ and errors in algebra.

$(f(x))^2+\frac{3e^{(x-1)}}{x-2}=c$	$(y)^2+\frac{3e^{(x-1)}}{x-2}=c$
$2\times f'(x)\times f(x)+\frac{(x-2)\times3\times e^{(x-1)}-(1)\times3e^{(x-1)}}{(x-2)^2}=0$	$x=1$ $y=\frac{3}{2}$
$2\times\frac{dy}{dx}\times y+\frac{3(x-2)e^{(x-1)}-3e^{(x-1)}}{(x-2)^2}=0$	$\left(\frac{3}{2}\right)^2+\frac{3e^{(1-1)}}{1-2}=c$
Let $x=1,\frac{dy}{dx}=2$	$c=\frac{9}{4}+\frac{3\times1}{-1}$
$2\times2\times y+\frac{3(1-2)e^{(1-1)}-3e^{(1-1)}}{(1-2)^2}=0$	$c=\frac{9}{4}+\frac{-12}{4}=-\frac{3}{4}$
$4y+\frac{3(-1)e^0-3e^0}{(-1)^2}=0$	
$\frac{3\times-1\times1-3\times1}{1}=-4y$	
$-3-3=-4y$	
$y=\frac{3}{2}$	

Calculus

RATE IN – RATE OUT



A second tank initially has 15 kg of salt dissolved in 100 L of water. A solution of $\frac{1}{60}$ kg of salt per litre flows into the tank at a rate of 20 L/min. The solution of salt and water, which is kept uniform by stirring, flows out of the tank at a rate of 10 L/min.

b. If y kilograms is the amount of salt in the tank after t minutes, write down an expression for the **concentration**, in kg/L, of salt in the second tank at time t. 1 mark

$$concentration = \frac{mass}{volume\ at\ time\ t}$$

$$= \frac{y}{100+(20-10)t} = \frac{y}{100+10t}$$

c. Show that the differential equation relating y and t is $\frac{dy}{dt} + \frac{y}{10+t} = \frac{1}{3}$. 2 marks

$$\frac{dy}{dt} = inflow - outflow$$

$$inflow = \frac{1}{60}\frac{kg}{L} \times 20\frac{L}{min} = \frac{1}{3}kg/min \qquad outflow = \frac{y}{100+10t}\frac{kg}{L} \times 10\frac{L}{min} = \frac{1}{10+t}kg/min$$

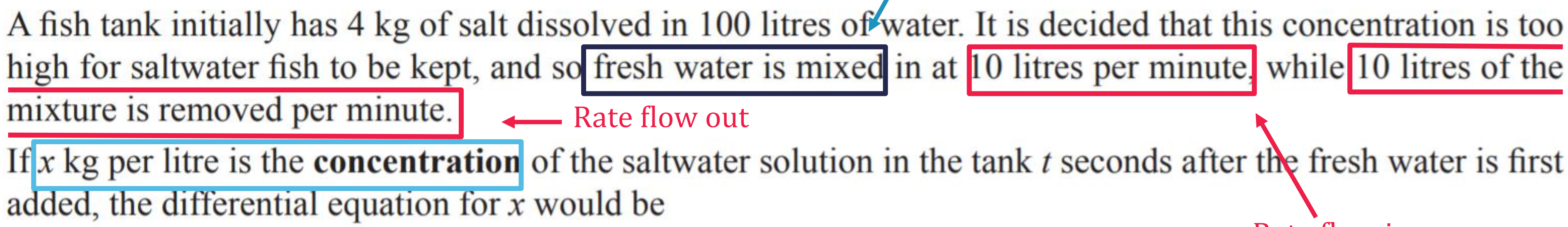
$$\frac{dy}{dt} = \frac{1}{3} - \frac{1}{10+t}$$

$$\frac{dy}{dt} + \frac{1}{10+t} = \frac{1}{3}$$

Calculus

VCAA 2009

rate in $= 0$

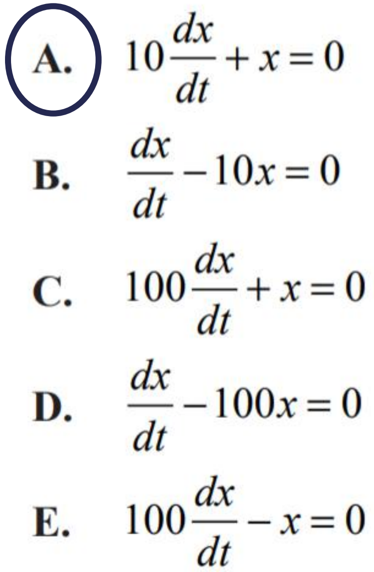


A fish tank initially has 4 kg of salt dissolved in 100 litres of water. It is decided that this concentration is too high for saltwater fish to be kept, and so fresh water is mixed in at 10 litres per minute, while 10 litres of the mixture is removed per minute.

If x kg per litre is the **concentration** of the saltwater solution in the tank t seconds after the fresh water is first added, the differential equation for x would be

Rate flow out

Rate flow in



[✓ **A.**] $10\frac{dx}{dt}+x=0$

B. $\frac{dx}{dt}-10x=0$

C. $100\frac{dx}{dt}+x=0$

D. $\frac{dx}{dt}-100x=0$

E. $100\frac{dx}{dt}-x=0$

$\frac{dx}{dt} \leftarrow$ rate of change of concentration

Volume $= 100 + 10t - 10t = 100$

$100\frac{dx}{dt} \leftarrow$ rate of change of mass $\qquad = -10x$

$100\frac{dx}{dt} =$ rate in $-$ rate out $= 0 -$ rate out

rate out $=$ concentration×rate flow out $= x \cdot 10$

$$\therefore 100\frac{dx}{dt} = -10x \Rightarrow 10\frac{dx}{dt} + x = 0$$

Calculus

VCAA 2013

The number of mobile phones, N, owned in a certain community after t years, may be modelled by $\log_e(N) = 6 - 3e^{-0.4t}$, $t \geq 0$.

$\log_e(N) = 6 - 3e^{-0.4t}$ satisfies the differential equation

$$\frac{1}{N}\frac{dN}{dt} + 0.4\log_e(N) - 2.4 = 0\,.$$

c. Using this mathematical model, find the limiting number of mobile phones that would eventually be owned in the community.

Give your answer correct to the nearest integer. 2 marks

Two ways:

1. As $t \to \infty, \log_e(N) \to 6$

OR

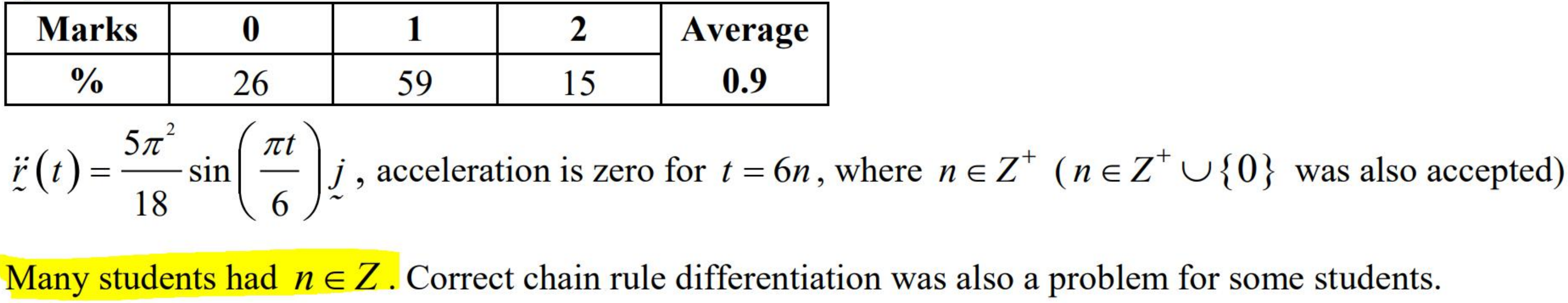
2. $\frac{dN}{dt} = 0 \Rightarrow 0.4\log_e(N) - 2.4 = 0 \Rightarrow \log_e(N) = 6 \backslash$

$$\therefore N = e^6 \approx 403$$

Calculus

TIME IS POSITIVE

- Sometimes, you may be asked to find the values of t so please ensure that the values of t you give are positive!



Marks	0	1	2	Average
%	26	59	15	0.9

$\ddot{\underset{\sim}{r}}(t)=\frac{5\pi^2}{18}\sin\left(\frac{\pi t}{6}\right)\underset{\sim}{j}$, acceleration is zero for $t=6n$, where $n\in Z^+$ ($n\in Z^+\cup\{0\}$ was also accepted)

Many students had $n\in Z$. Correct chain rule differentiation was also a problem for some students.

b. Find the times when the acceleration of the waterskier is zero. 2 marks

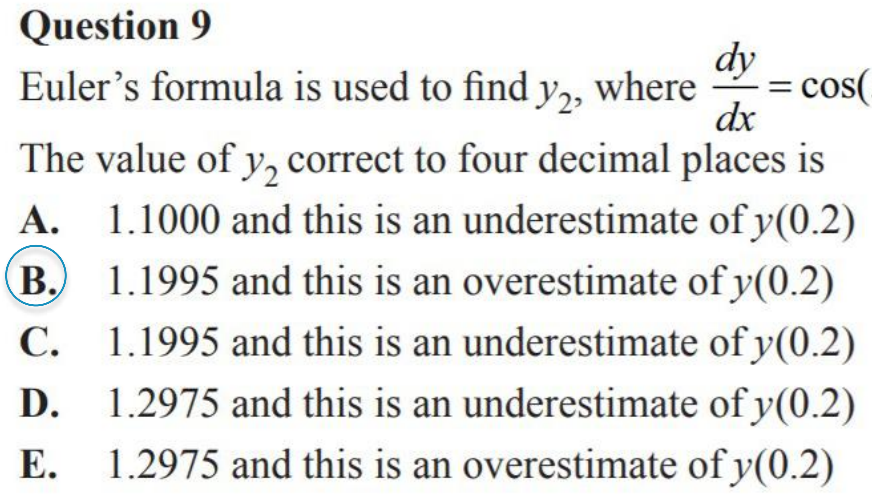
$$\dot{\underset{\sim}{r}}(t)=7.5\underset{\sim}{i}-\frac{5}{3}\cos\left(\frac{\pi t}{6}\right)\underset{\sim}{j} \qquad \Rightarrow \ddot{\underset{\sim}{r}}(t)=\frac{5}{18}\sin\left(\frac{\pi t}{6}\right)\underset{\sim}{j}$$

$$\frac{5}{18}\sin\left(\frac{\pi t}{6}\right)=0 \qquad \Rightarrow \sin\left(\frac{\pi t}{6}\right)=0 \quad \Rightarrow \frac{\pi t}{6}=n\pi \qquad ,n\in Z^+$$

$$\therefore t=6n\,,n\in Z^+$$

Calculus

EULER'S METHOD



Question 9

Euler's formula is used to find y_2, where $\frac{dy}{dx} = \cos(x)$, $x_0 = 0$, $y_0 = 1$ and $h = 0.1$

The value of y_2 correct to four decimal places is

A. 1.1000 and this is an underestimate of $y(0.2)$

[✓ **B.**] 1.1995 and this is an overestimate of $y(0.2)$

C. 1.1995 and this is an underestimate of $y(0.2)$

D. 1.2975 and this is an underestimate of $y(0.2)$

E. 1.2975 and this is an overestimate of $y(0.2)$

x	y
$x_0 = 0$	$y_0 = 1$
$x_1 = x_0 + h = 0 + 0.1 = 0.1$	$y_1 = y_0 + h \times f(x_0) = 1 + 0.1 \times (\cos(0)) = 1.1$
$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$	$y_2 = y_1 + h \times f(x_1) = 1.1 + 0.1 \times \cos(0.1) = 01.1995$

$$\frac{dy}{dx} = \cos(x) \Rightarrow y = \sin(x) + c$$

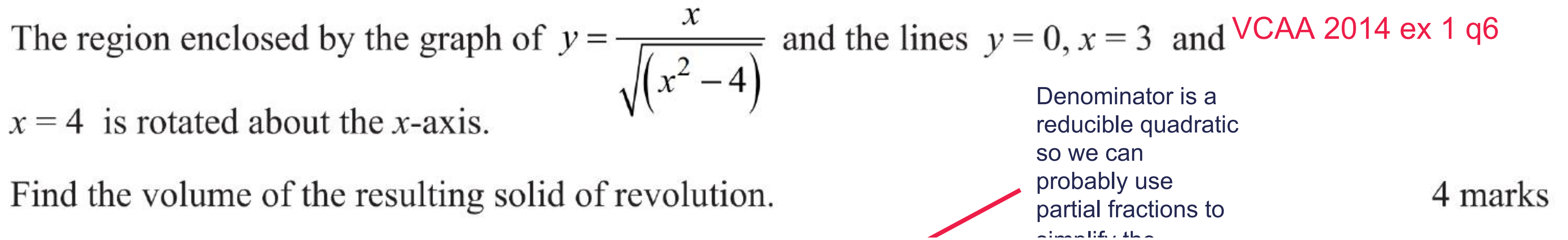
$$x = 0, y = 1$$

$$1 = \sin(0) + c \Rightarrow c = 1$$

$$y = \sin(x) + 1$$

$$y = \sin(0.2) + 1 = 1.19867$$

Calcu



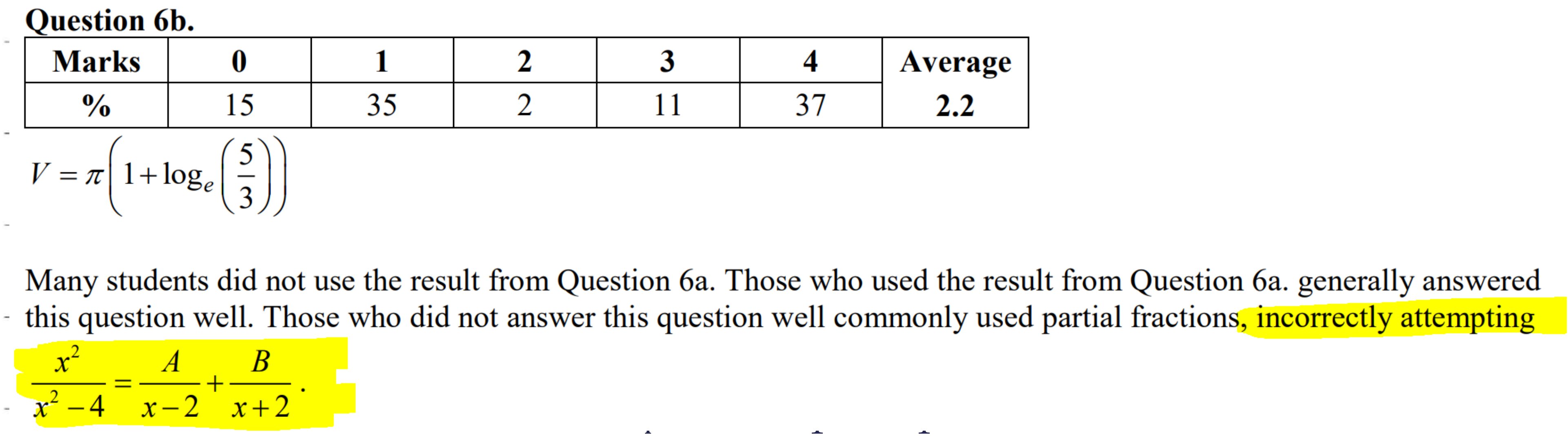
The region enclosed by the graph of $y = \frac{x}{\sqrt{(x^2-4)}}$ and the lines $y = 0, x = 3$ and $x = 4$ is rotated about the x-axis.

Find the volume of the resulting solid of revolution.

4 marks

VCAA 2014 ex 1 q6

Denominator is a reducible quadratic so we can probably use partial fractions to



Question 6b.

Marks	**0**	**1**	**2**	**3**	**4**	**Average**
%	15	35	2	11	37	**2.2**

$$V = \pi\left(1+\log_e\left(\frac{5}{3}\right)\right)$$

Many students did not use the result from Question 6a. Those who used the result from Question 6a. generally answered this question well. Those who did not answer this question well commonly used partial fractions, incorrectly attempting $\frac{x^2}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$.

$$1 + \frac{}{x^2-4} = 1 + \frac{}{x-2} - \frac{}{x+2}$$

$$\therefore V = \pi\int_3^4\left(1 + \frac{1}{x-2} - \frac{1}{x+2}\right)dx = \pi\left[x + \log_e\left|\frac{x-2}{x+2}\right|\right]_3^4 = \pi\left(1 + \log_e\left(\frac{5}{3}\right)\right)$$

INTEGRATION TECHNIQUES

- For a rational function in the form $y = \frac{f(x)}{g(x)}$ where f and g are polynomials,
- If degree of $f(x) <$ degree of $g(x)$, use partial fractions
- If degree of $f(x) =$ degree of $g(x)$, break it up and then use partial fractions
- If degree of $f(x) >$ degree of $g(x)$, break it up using long division of polynomials and then use partial fractions

INTEGRATION TECHNIQUES

- When you don’t know which integration technique to use, ask yourself:

1) Is there are derivate for a similar function given somewhere in the question? (integration by recognition)
2) Is there a trig function that I can simplify or use double angle formulae? (integration using trigonometric identities)
3) Is there a function being multiplied by a derivative of an inner function? (integration by substitution)
4) If the function is a quotient function, does the function have a reducible quadratic in the denominator? (integration by partial fractions)
5) If the function is a quotient function, does the function have a irreducible quadratic in the denominator? (integration by linear substitution or inverse trig)
6) If the function is a quotient function, does the function have a square root function in the denominator? (integration to inverse trig)

INTEGRATION TECHNIQUES

- Which integration technique should I use?

$$\int 5x^4\left(1-e^{x^5}\right)dx$$

1) Is there are derivate for a similar function given somewhere in the question? (integration by recognition)
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INTEGRATION TECHNIQUES

- Which integration technique should I use?

$$\int 5x^4(1-e^{x^5})dx$$

Is there a function being multiplied by a derivative of an inner function? (integration by substitution)

Let $u = x^5 \Rightarrow \frac{du}{dx} = 5x^4 \Rightarrow \frac{dx}{du} = \frac{1}{5x^4}$

$$\int 5x^4(1-e^{x^5})dx = \int 5x^4(1-e^u)\times\frac{1}{5x^4}du$$

$$= \int(1-e^u)du = u - e^u + c$$

$$= \int(1-e^u)du = x^5 - e^{x^5} + c$$

INTEGRATION TECHNIQUES

- Which integration technique should I use?

$$\int \frac{2+x}{\sqrt{1-x^2}}dx$$

1) Is there are derivate for a similar function given somewhere in the question? (integration by recognition)
2) Is there a trig function that I can simplify or use double angle formulae? (integration using trigonometric identities)
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INTEGRATION TECHNIQUES

- Which integration technique should I use?

$$\int \frac{2+x}{\sqrt{1-x^2}}dx = \int \frac{2}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx$$

1) Is there are derivate for a similar function given somewhere in the question? (integration by recognition)
2) Is there a trig function that I can simplify or use double angle formulae? (integration using trigonometric identities)
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INTEGRATION TECHNIQUES

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INTEGRATION TECHNIQUES

- Which integration technique should I use?

$$\int \frac{2+x}{\sqrt{1-x^2}}dx = \int \frac{2}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx$$

- Is there a function being multiplied by a derivative of an inner function? (integration by substitution)
- If the function is a quotient function, does the function have a square root function in the denominator? (integration to inverse trig)

$$\int \frac{2}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx = 2\int \frac{1}{\sqrt{1-x^2}}dx + \int \frac{x}{\sqrt{1-x^2}}dx = 2\sin^{-1}(x) + \int \frac{x}{\sqrt{1-x^2}}dx$$

Let $u = x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow \frac{dx}{du} = \frac{1}{2x}$

$$= 2\sin^{-1}(x) + \int \frac{x}{\sqrt{1-u}} \times \frac{1}{2x}du = 2\sin^{-1}(x) + \int \frac{1}{\sqrt{1-u}} \times \frac{1}{2}du$$

$$= 2\sin^{-1}(x) + \frac{1}{2}\int (1-u)^{-\frac{1}{2}}du$$

$$= 2\sin^{-1}(x) + \frac{1}{2}\int (1-u)^{-\frac{1}{2}}du = 2\sin^{-1}(x) + \frac{1}{2} \times \frac{(1-u)^{\frac{1}{2}}}{\frac{1}{2}\times -1} + c = 2\sin^{-1}(x) - \sqrt{1-x^2} + c$$

INTEGRATION TECHNIQUES

- Which integration technique should I use?

$$\int \sin^2(x)\cos^3(x)\,dx$$

1) Is there are derivate for a similar function given somewhere in the question? (integration by recognition)
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INTEGRATION TECHNIQUES

- Which integration technique should I use?

$$\int \sin^2(x)\cos^3(x)\,dx$$

Is there a trig function that I can simplify or use double angle formulae? (integration using trigonometric identities)
Is there a function being multiplied by a derivative of an inner function? (integration by substitution)

$$\int \sin^2(x)\cos^3(x)\,dx = \int \sin^2(x)\cos(x)\cos^2(x)\,dx = \int \sin^2(x)\cos(x)\left(1-\sin^2(x)\right)dx$$

$$= \int \cos(x)\left(\sin^2(x)-\sin^4(x)\right)dx$$

Let $u = \sin(x) \Rightarrow \frac{du}{dx} = \cos(x) \Rightarrow \frac{dx}{du} = \frac{1}{\cos(x)}$

$$= \int \cos(x)\left(u^2-u^4\right)\times\frac{1}{\cos(x)}\,du = \int \left(u^2-u^4\right)du = \frac{u^3}{3}-\frac{u^5}{5}+c = \frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}+c$$

INTEGRATION TECHNIQUES

- Which integration technique should I use?

Given $\frac{d}{dx}[3x\tan^{-1}(2x)] = 3\tan^{-1}(2x) + \frac{6x}{1+4x^2}$, find $\int 3\tan^{-1}(2x)\,dx$

1) Is there are derivate for a similar function given somewhere in the question? (integration by recognition)
2) Is there a trig function that I can simplify or use double angle formulae? (integration using trigonometric identities)
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INTEGRATION TECHNIQUES

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Given $\frac{d}{dx}[3x\tan^{-1}(2x)] = 3\tan^{-1}(2x) + \frac{6x}{1+4x^2}$, find $\int 3\tan^{-1}(2x)\,dx$

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INTEGRATION TECHNIQUES

- Which integration technique should I use?

Given $\frac{d}{dx}[3x\tan^{-1}(2x)] = 3\tan^{-1}(2x) + \frac{6x}{1+4x^2}$, find $\int 3\tan^{-1}(2x)\,dx$

Is there are derivate for a similar function given somewhere in the question? (integration by recognition)

$$3x\tan^{-1}(2x) = \int 3\tan^{-1}(2x) + \frac{6x}{1+4x^2}dx = 3\int \tan^{-1}(2x)\,dx + \int \frac{6x}{1+4x^2}dx$$

$$3x\tan^{-1}(2x) - \int \frac{6x}{1+4x^2}dx = 3\int \tan^{-1}(2x)\,dx$$

Let $u = x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow \frac{dx}{du} = \frac{1}{2x}$

$$3\int \tan^{-1}(2x)\,dx = 3x\tan^{-1}(2x) - \int \frac{6x}{1+4u} \times \frac{1}{2x}du = 3x\tan^{-1}(2x) - \int \frac{3}{1+4u}du$$

$$3\int \tan^{-1}(2x)\,dx = 3x\tan^{-1}(2x) - \frac{3}{4}log_e|1+4u| + c$$

$$\int \tan^{-1}(2x)\,dx = x\tan^{-1}(2x) - \frac{1}{4}log_e|1+4u| + c$$

INTEGRATION TECHNIQUES

- Which integration technique should I use?

$$\int 2x\sqrt{\frac{1}{2}x-3dx}$$

1) Is there are derivate for a similar function given somewhere in the question? (integration by recognition)
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INTEGRATION TECHNIQUES

- Which integration technique should I use?

$$\int 2x\sqrt{\frac{1}{2}x - 3dx}$$

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6) If the function is a quotient function, does the function have a square root function in the denominator? (integration to inverse trig)

- Which integration technique should I use?

$$\int 2x\sqrt{\frac{1}{2}x-3}\,dx$$

Is there a function being multiplied by a derivative of an inner function? (integration by substitution)

Let $u=\frac{1}{2}x-3\Rightarrow x=2u+6\Rightarrow\frac{dx}{du}=2$

$$\int 2x\sqrt{\frac{1}{2}x-3}\,dx=\int 2(2u+6)\sqrt{u}\times 2\;du=4\int(2u+6)\sqrt{u}\;du$$

$$4\int(2u+6)\sqrt{u}\;du=4\int 2u^{\frac{3}{2}}+6u^{\frac{1}{2}}\,du=4\left(2\times\frac{u^{\frac{5}{2}}}{\frac{5}{2}}+6\times\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)+c$$

$$=\frac{16}{5}u^{\frac{5}{2}}+16u^{\frac{3}{2}}+c=\frac{16}{5}\left(\frac{1}{2}x-3\right)^{\frac{5}{2}}+16\left(\frac{1}{2}x-3\right)^{\frac{3}{2}}+c$$

Properties of Vectors

- <u>Length/Magnitude of Vector:</u>
- If a vector is r = xi + yj + zk:
- $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$
- <u>Parallel Vectors:</u>
- Two vectors, $\vec{u}$ and $\vec{v}$, are **parallel** if $\vec{u} = k\vec{v}$ where k is a scalar (this stretches/squishes the magnitude).
- <u>Unit Vectors:</u>
- Special Vectors that have a magnitude of 1 to SPECIFY direction. We just divide the vector by its magnitude.
- $\hat{u} = \frac{\vec{u}}{|\vec{u}|}$

Scalar (Dot) Product

For multiplying vectors together = gives you a real number!

Super simple: Remember to multiply like and like together and add. Eg:

$$\vec{a} = a_1\vec{i} + b_1\vec{j} \qquad \vec{b} = a_2\vec{i} + b_2\vec{j}$$

Their dot product is:

$$\vec{a} \cdot \vec{b} = a_1a_2 + b_1b_2$$

Vector Angles

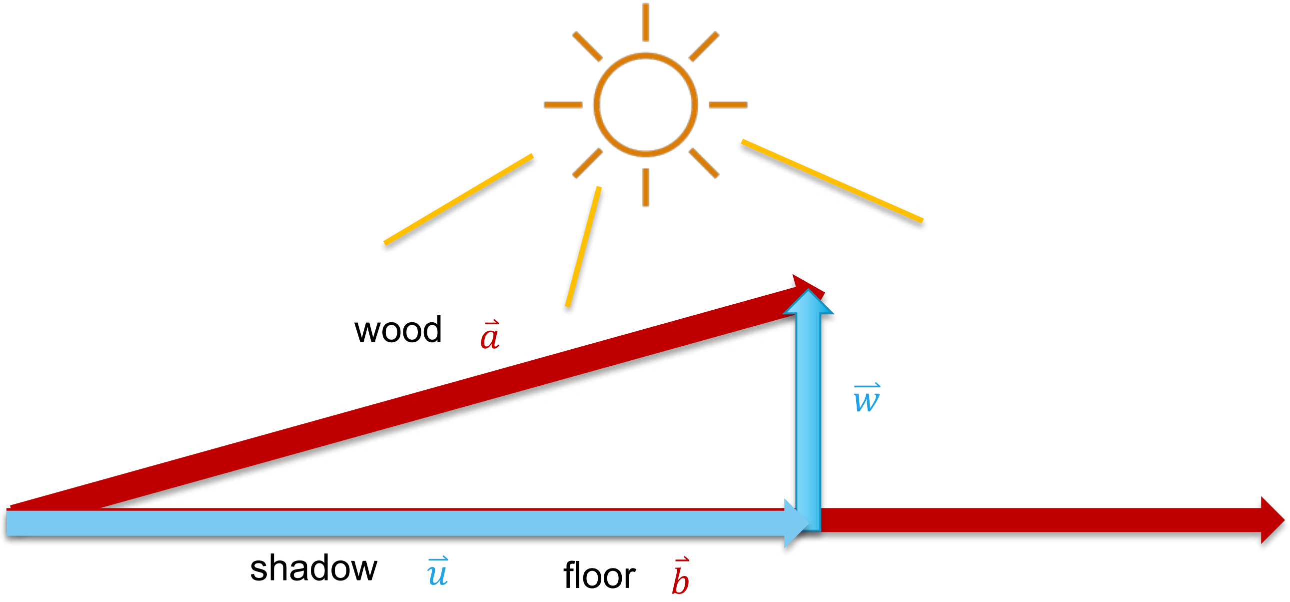
To find angles between 2 vectors, THEY MUST BE TAIL TO TAIL

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

SOME IMPORTANT PROPERTIES

- $\vec{a}\cdot\vec{a} = |a|^2$
- $\vec{a}\cdot\vec{b} = 0$ if $\vec{a}$ and $\vec{b}$ are perpendicular

Vector Projections/Resolutes



Vector Resolutes ask “What is the shadow $(\vec{u})$ cast by the FIRST vector $(\vec{a})$ on the SECOND vector $(\vec{b})$”

Basically, we are trying to find the components of vector a.

Main gimmick: Shortest distance aka perpendicular distance

Kinematics and Vector Calculus

Signed area is displacement

- **Motion Variables**

Vector	Scalar	Representation on velocity – time graph
Displacement	Distance	Area under graph
Velocity	Speed	Coordinate of point
Acceleration	Acceleration	Gradient at a point


$\frac{d}{dt}$

$\frac{d}{dt}$

$\int v\,dt$

$\int a\,dt$

- $x \rightarrow$ position
- $v \rightarrow$ velocity

- **Velocity/Speed:**

Average rate of change/ Average velocity	$\frac{displacement}{time} = \frac{x_2 - x_1}{time}$
Instantaneous velocity	$\pm\frac{dx}{dt}$
Average speed	$\frac{distance\ travelled}{time}$

- **Units:** Always ms^{-1} unless specified (divide km/hr by 3.6)

- **Acceleration**

Average acceleration	$\frac{Velocity}{time} = \frac{v_2 - v_1}{time}$
Instantaneous acc	$\pm\frac{dv}{dt}$

- **IMPORTANT:** (physics time)
- Motion direction ≠ Acceleration: eg. Moving to the right but slowing down

CONSTANT ACCELERATION

Not on your formula sheet

- THIS IS MANDATORY YOU MUST MEMORISE THESE!
- When you have **unchanging acceleration**, you can use 5 equations called SUVAT to work out displacement, initial/final velocity, acceleration and time.
- *tip: Each equation is missing a variable

Most important ones in yellow

S= displacement
U= initial velocity
V= final velocity
A= acceleration
T= time

*Don’t use previous variables you’ve found

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

SPECIAL CASE: PROJECTILE MOTION

- Almost always SUVAT related. When you drop/shoot something in the air. It will have vertical and/or horizontal movement

Horizontal:

- NO ACCELERATION UNLESS STATED (like engine propelling forward)
- Speed is ALWAYS constant unless air resistance. Newton’s First Law. No force acting on it

Vertical:

- GRAVITY! Direction is important

Just remember:

- Displacement is FINAL position – INITIAL position
- Define your positive direction of motion

OTHER ACCELERATION FORMULAS

- Pretty common question. Acceleration can be written in many ways. Just look at what info you are given in the question to decide which formula you need to use.
- $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

This one is specific to a question type

$a = f(t)$	$\frac{dv}{dt}$
$a = f(v)$	$\frac{dv}{dt}\ (inital\ has\ v, t)$
$a = f(v)\ or\ f(x)$	$v\frac{dv}{dx}\ (inital\ has\ v, x)$
$a = f(x)$	$\frac{d}{dt}\left(\frac{1}{2}v^2\right)\ (inital\ v, x)$

Kinematics and Vector Calculus

- Example) The acceleration $a\ ms^{-1}$ of a body moving in a straight line in terms of the velocity $v\ ms^{-1}$ given by $a = 4v^2$.
- Given that when $v = e$ when $x = 1$, where x is displacement of the body in metres, find the velocity of the body when $x = 2$ (VCAA – exam 1 – 2015)

$$a = v\frac{dv}{dx}$$

$$v\frac{dv}{dx} = 4v^2 \Rightarrow \frac{dv}{dx} = 4v$$

$$\frac{dx}{dv} = \frac{1}{4v}$$

$$x = \frac{1}{4}log_e|v| + c$$

Let $v = e$ when $x = 1$

$$1 = \frac{1}{4}log_e|e| + c \Rightarrow c = 1 - \frac{1}{4} = \frac{3}{4}$$

$$x = \frac{1}{4}log_e|v| + \frac{3}{4}$$

$$x - \frac{3}{4} = \frac{1}{4}log_e|v|$$

$$4x - 3 = log_e|v|$$

$$v = e^{4x-3}$$

Let $x = 2$

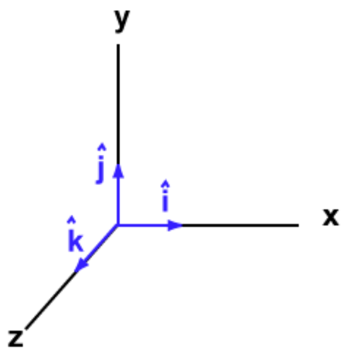
$$v = e^{4\times2-3} = e^5$$

Kinematics and Vector Calculus

Awful. Horrible. Terrible.

- Mixing vectors and calculus amazing.

- Either 2D or 3D brain is needed! (z axis only in 3D)
- Vectors with respect to time given by



$$\begin{tikzcd}
\vec{r}(t) = x(t)\vec{\imath} + y(t)\vec{\jmath} + z(t)\vec{k} \arrow[d, bend right, "\frac{d}{dt}"'] \\
\vec{v}(t) = x'(t)\vec{\imath} + y'(t)\vec{\jmath} + z'(t)\vec{k} \arrow[u, bend right, "\int v\,dt"'] \arrow[d, bend right, "\frac{d}{dt}"'] \\
\vec{a}(t) = x''(t)\vec{\imath} + y''(t)\vec{\jmath} + z''(t)\vec{k} \arrow[u, bend right, "\int a\,dt"']
\end{tikzcd}$$

Just diff/integrate normally with components separate

Kinematics and Vector Calculus

- **Total distance travel from t_0 to t_1**
- Arclength/total area under a velocity-time graph of the curve $\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$
- Distance $= \int_{t_0}^{t_1} \left|\dot{\underset{\sim}{r}}(t)\right| dt = \int_{t_0}^{t_1} \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2 + \left[\frac{dz}{dt}\right]^2}\, dt$
- **Distance between two objects**
- Distance $= |\overrightarrow{r_1}(t) - \overrightarrow{r_2}(t)|$
- **Direction of motion**
- Given by $\dot{r}(t)$ or $\underset{\sim}{v}(t)$. IT IS THE VELOCITY!!!

Kinematics and Vector Calculus

- **Two objects crossing paths:**
- Share x, y points but not necessarily at the same t

- **Two objects collide:**
- If and only if the simultaneous system have the SAME t
- $x_1(t) = x_2(t)$

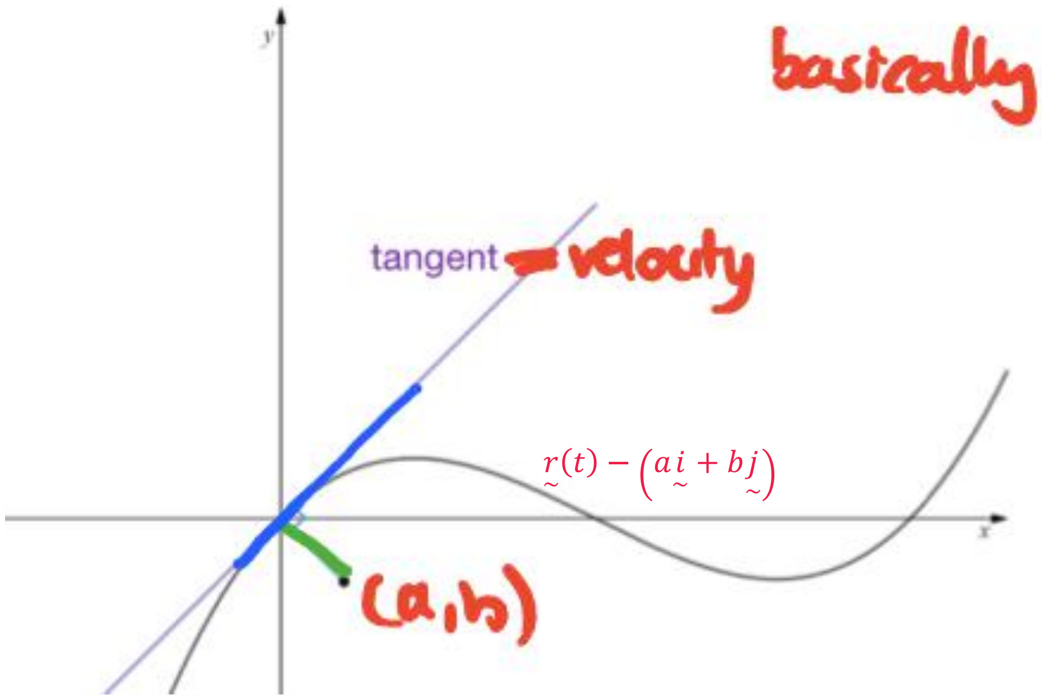
$$y_1(t) = y_2(t)$$

Kinematics and Vector Calculus

SHORTEST DISTANCE

- To find the shortest distance between a point and curve, we want to be PERPENDICULAR to the curve to go STRAIGHT THERE.
- Remember vector property:
  - dot product of 2 vectors = 0 is perpendicular

basically $\boxed{\underline{r(t)}\cdot(\underline{r(t) - ai - bj}) = 0}$



r(t)

SHORTEST DISTANCE

- Steps:

1. Convert the point into a vector
2. Subtract from curve vector
3. Multiply against velocity curve vector
4. Let dot product = 0 and solve for t
5. Distance formula $|\overrightarrow{r_1}(t) - \overrightarrow{r_2}(t)|$

Kinematics and Vector Calculus

VECTOR CALCULUS

- Example) The velocity of a particle at time t seconds is given by
- $\dot{\underset{\sim}{r}} = (4t-3)\underset{\sim}{i} + 2tj - 5\underset{\sim}{k}$ when components are measures in metres per second.
- Find the distance of the particle from the origin in metres when t=2 given that $\underset{\sim}{r}(0)=\underset{\sim}{i}-2\underset{\sim}{k}$ (VCAA - exam 1 – 2015)

$$\underset{\sim}{r}= (2t^2-3t)\underset{\sim}{i} + t^2 j - 5t\underset{\sim}{k} + c$$

$$\text{Let t} = 0,\ \underset{\sim}{r}(0) = \underset{\sim}{i} - 2\underset{\sim}{k}$$

$$\underset{\sim}{i} - 2\underset{\sim}{k} = (2(0^2) - 3\times 0)\underset{\sim}{i} + 0^2 j - 5\times 0\times\underset{\sim}{k} + c$$

$$\underset{\sim}{i} - 2\underset{\sim}{k} = (0)\underset{\sim}{i} + c \Rightarrow c = \underset{\sim}{i} - 2\underset{\sim}{k}$$

$$\underset{\sim}{r}(\mathrm{t}) = (2t^2-3t)\underset{\sim}{i} + t^2 j - 5t\underset{\sim}{k} + \underset{\sim}{i} - 2\underset{\sim}{k}$$

$$\underset{\sim}{r}(\mathrm{t}) = (2t^2-3t+1)\underset{\sim}{i} + t^2 j + (-5t-2)\underset{\sim}{k}$$

$$\underset{\sim}{r}(2) = (3)\underset{\sim}{i} + 4j + (-12)\underset{\sim}{k}$$

$$distance = \sqrt{(3)^2+(4)^2+(-12)^2} = \sqrt{169} = 13\ units$$

Kinematics and Vector Calculus

- Kinematics but with mass! Biggest tips:
- **DRAW EVERYTHING, FORCES, POSITIVE DIRECTIONS**
  - Objects as boxes, forces as arrows
- **$\boldsymbol{F = ma}$ is everything**
  - F is NET force (N), the sum of ALL forces on an object
  - m is mass (kg)
  - a is acceleration (ms^{-2})

NEWTON’S LAWS OF MOTION

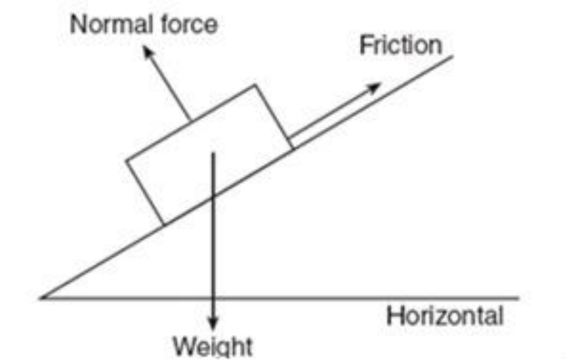
- **Newton’s First Law:**
- An object will keep moving in a straight line until an external force acts upon it.
- Inertia – depends on mass. More mass = more force needed to change motion
- **Newton’s Second Law**

THE most important equation you will ever meet

- ***F=ma***
- Can link to SUVAT if constant acceleration
- **Newton’s Third Law**
- For every action there is an EQUAL and OPPOSITE reaction
- This is called the Normal force. Its what stops my computer from falling thru the table
- Law of Conservation in momentum systems (P=m(Vf-Vi)

SPECIAL FORCES/WORDS

Force	Equation	Description
Weight	W=mg	Always down
Normal	N	PERPENDICULAR to contact
Friction	$F_R=\mu N$	Opposes motion direction. M given in qn, N is normal. Smooth is NO friction
Tension		In string, CONSTANT along whole length. Usually ignore
Gravity	$g=9.8ms^{-2}$ DOWN	Always down



Summary/Exam Tips

Exam 1:

- **Friday 3 November:** 9:00am → 10:15am (40 marks)

Exam 2:

- **Monday 6 November**: 11:45am → 2:00pm (80 marks)

Summary/Exam Tips

BOUND REFERENCE

- Aim to finish your bound reference BEFORE doing practice exams
- You bound reference needs to work for YOU not for anyone else
- Put in formula, calculator commands or mathematica code and content examples
- Do not have a huge bound reference, you will waste time trying to find things

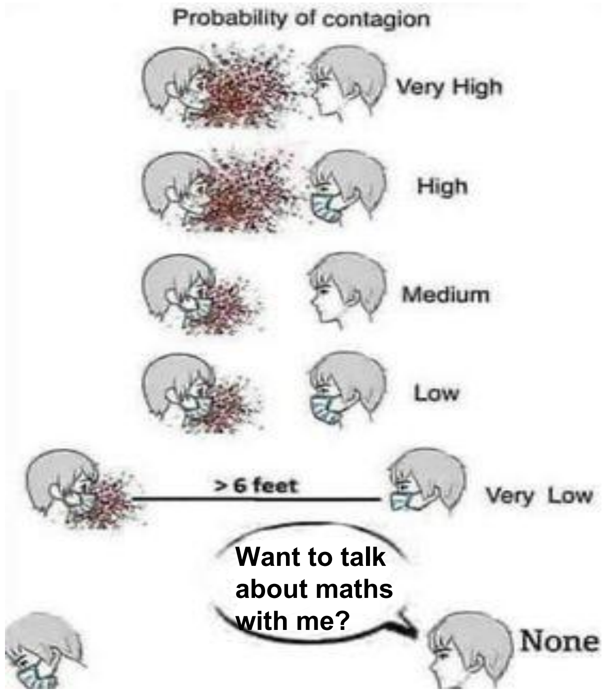
Summary/Exam Tips

- PRACTISE EXAMS!!!!
  - My recommendation is MAV, NEAP, VCAA, VCAA NHT (2016-2023)
  - Do MAV/NEAP first, VCAAs near exams so you are used to it
- Try to get ur teacher to get SOME papers bc remember VCE is school vs school not peers vs peers. Help each other out! https://vcaa.vic.edu.au/assessment/vce-assessment/past-examinations/Pages/Index.aspx
- Correct it yourself HARSHLY and analyse your performance (what were the errors you made, how can you avoid doing that next time)

Summary/Exam Tips

EXAM ADVICE - PREPARATION

- Go to sleep early every night
- Make sure you are comfortable using your calculator. Have a play around with your calculator at some point and explore the different functionality. You may discover more functions on your calculator that come in use
- Try teaching someone else. Explaining a concept to someone in your own words can help to cement your understanding
- Don't stress! A good mindset comes with good study scores!



Summary/Exam Tips

EXAM ADVICE – EXAM 1

- Exam 1: (1 hour writing time + 15 mins reading time)
- Use reading time wisely!
- During reading time, go through each question and form a plan of attack. (vector calc and closest distance → dot product being zero). Do this for as many questions as possible.
- You only have one hour to complete the exam so manage your time wisely!
- If you can't figure out what to do in 10secs, move on. Come back to it later. ~30-45min you should have attempted every page

Summary/Exam Tips

- Exam 2: (2 hours writing time + 15 mins reading time)
- Reading time
- Reading time!! USE IT TO READ EXTENDED RESPONSE
- DO extended response questions FIRST!!!! You can always guess MC if you run out of time. If worse comes to worse, damage control and get as many working/formula marks.
- USE YOUR CALC! Set up a new problem tab for EVERY ER and 1 page for MC for ease
- Each mark is ~90 sec so 30min for MC is good

Summary/Exam Tips

- Ultimately, you should do the exam in your own preferred way.
- This is why doing practice exams is valuable
- In doing many practice exams, you should develop a strategy that works for you.

Summary/Exam Tips

EXAM ADVICE – IN THE EXAM

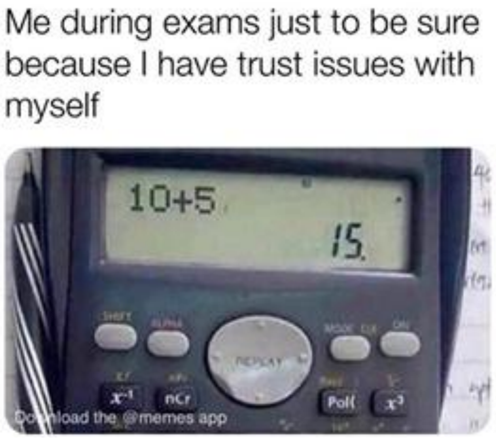
- UNDERLINE, DRAW, CHECK
- Underline IMPORTANT WORDS so you actively retain info. Look for clues (no. of marks, rounding, ‘hence’ etc.)
- Draw the qns visually if confused, could help
- Check to make sure you answered the qn, have units, correct form

- Unless stated, give answers in exact form.
- Pen for answers, pencil for graphs

Summary/Exam Tips

EXAM ADVICE – IN THE EXAM

- “If you panic, the exam is already over”.
- DO NOT EVER STRESS/PANIC. You WILL stuff up if you panic or stress. 1-2 minutes to breathe can do wonders!
- Trust yourself to do well.



EXAM ADVICE – DURING EXAM SEASON

- In between exams, don’t be afraid to pursue a lot of leisure tasks.
- Watch some stress relieving films, read a book, go out for walks often, hang out with friends
- Always make sure each night you are going over revision whether that be in the form of an exam analysis doc, ‘what to remember’ list etc.

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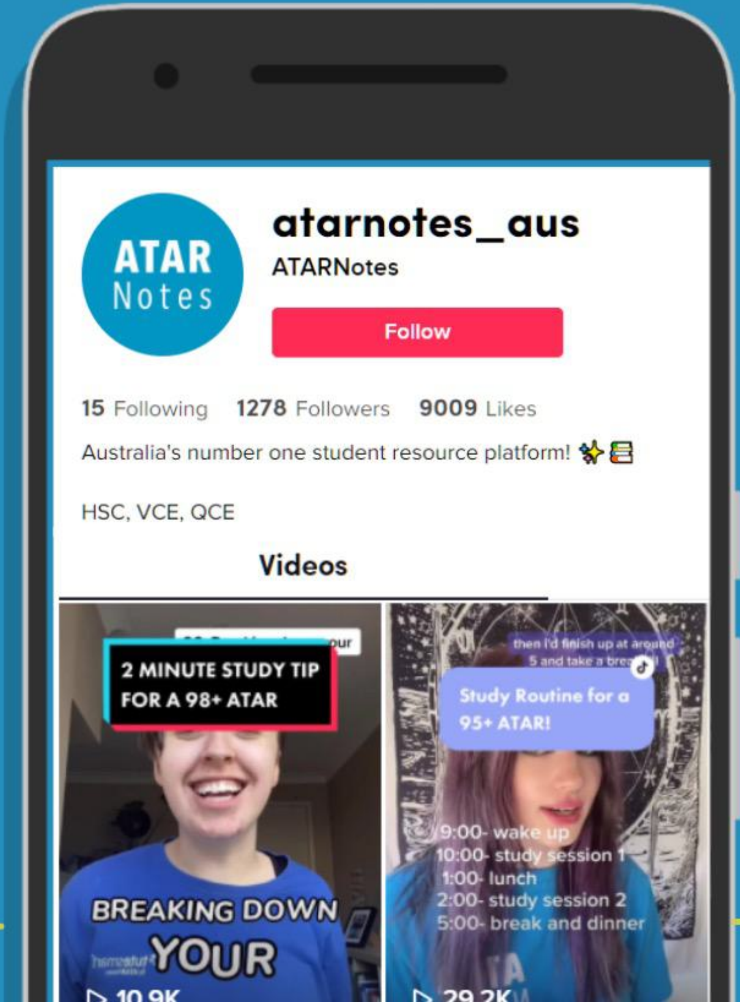
New videos every week!



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Key skills

- Statistics – Hypothesis testing + probability
- Complex Numbers
- Circular Functions
- Vectors
- Kinematics and Vector Calculus

Reminders

- Try to study smarter – study in blocks
- Good Luck!

Questions?